HYPERFINE SPLITTING OF THE $3^2S_{\frac{1}{2}}$ ENERGY LEVEL OF SODIUM FROM INTERFERENCE FRINGE RADII MEASUREMENTS

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Preface

The yellow doublet, D_1-D_2, in the sodium spectrum can be resolved to show a hyperfine structure of ten lines. The hyperfine components of both D_1 and D_2 are separated into two groups by the double nature of the $3^2S_{\frac{1}{2}}$ energy level. The splitting of this level is caused by the interaction of nuclear magnetic moment with the total magnetic moment of the valence electron. The interaction energy is proportional to the wave number difference of the two major groups of hyperfine lines of either D line. The purpose of this thesis is to demonstrate a method by which the above mentioned wave number difference can be obtained from measurements of the radii of interference fringes.

ACKNOWLEDGEMENT

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INTRODUCTION

A small prism spectrometer reveals a single yellow line in the sodium spectrum. An instrument of higher resolving power, however, shows this line to be a doublet. This complexity is generally referred to as fine structure. This doublet can be explained in terms of the three quantum numbers, s, l, and j. The quantum number s is obtained by giving the valence electron a spin of one-half a quantum unit of angular momentum $s^* h/2\pi$.[1] The quantum number l is associated with the orbital angular momentum given by $l^* h/2\pi$. The resultant $j^* h/2\pi$ is obtained by adding vectorially the spin angular momentum to the orbital angular momentum.[2] The quantum number j is associated with this resultant. The sodium doublet is governed by the selection rule that $\Delta J = \pm 1$ or 0, and is shown by Figure 1.

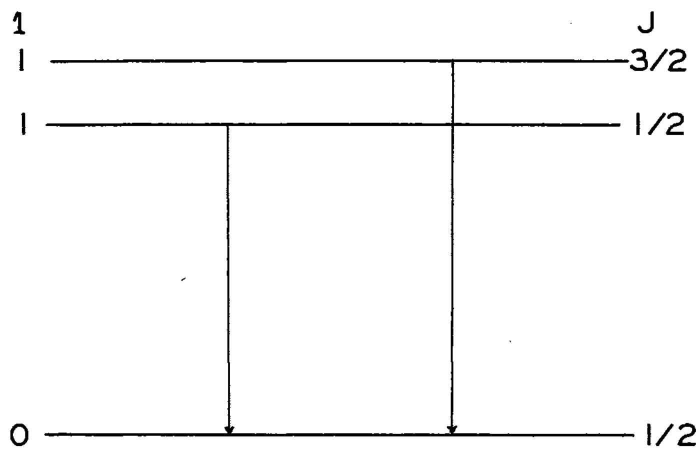

Figure 1

[1] Harvey E. White, Introduction to Atomic Spectra, pp. 121.

[2] Ibid., pp. 123.

However, when the D lines of sodium are examined with a Fabry-Perot interfrometer, all of the obtainable spectral lines can not be explained by these three quantum numbers. Pauli[3] has shown that these lines could be explained by the introduction of a small magnetic moment associated with the nucleus. This magnetic moment could be associated with a mechanical moment whose magnitude will be given by $I^{*}h/2\pi$. I is a new quantum number called the nuclear spin quantum number. The vector model for hyperfine structure is shown in Figure 2.

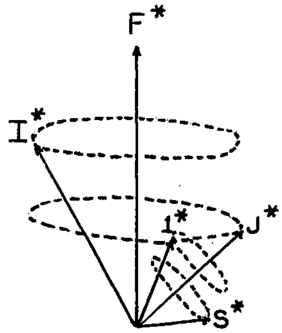

Figure 2

The vector F^{*} is the vector sum of I^{*} and j^{*}. F is the resultant quantum number and is in reality the total angular momentum quantum number for the whole atom. The j^{*} vector and the I^{*} vector will precess around the resultant F^{*} vector just as l^{*} and s^{*} precess around j^{*}.[4] The hyperfine structure pattern of a line is governed by the selection principle that $\Delta F = \pm 1$ or 0, excluding $(0 \rightarrow 0)$.[5] Figure 3 shows this rule for the hyperfine

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[3] W. Pauli, Naturwissenschaften, XII (1924), 741.

[4] R. C. Johnson, Atomic Spectra, pp. 74.

[5] White, op. cit., pp. 354.

structure of the sodium D lines where I is equal to three-halves.

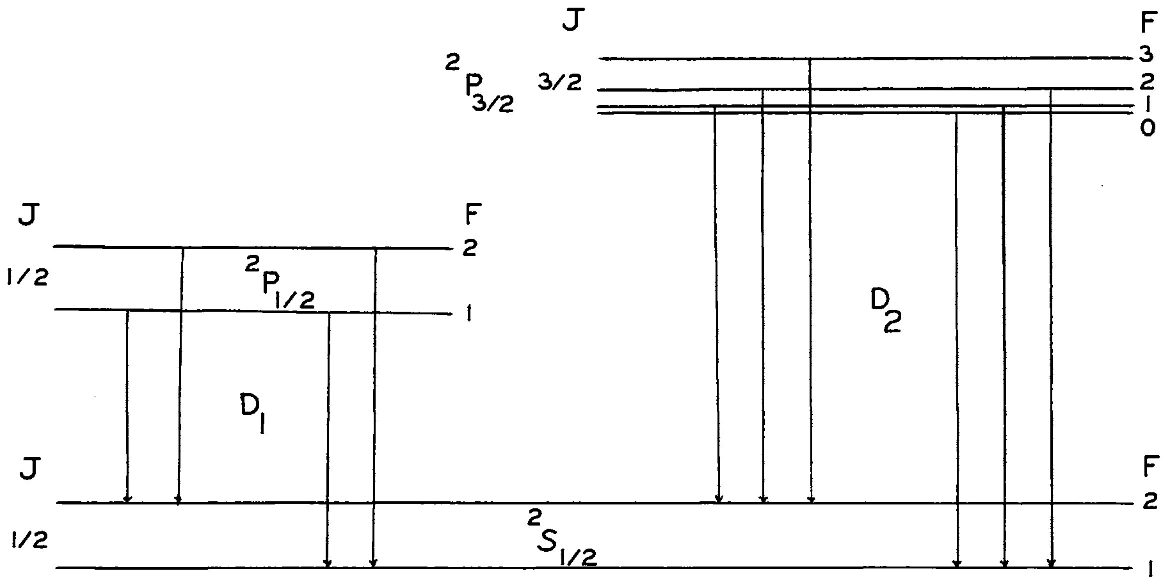

Figure 3

LANDE INTERVAL RULE [6]

Goudsmit and Back have shown that the interaction energy between the nuclear moment and the electron moment is given by

$$T_F = A' I^* J^* \cos(I^* J^*).$$

From Figure 2 it is seen that

$$I^* J^* \cos(I^* J^*) = \frac{F^{*2} - I^{*2} - J^{*2}}{2} .$$

Therefore, we obtain

$$T_F = \tfrac{1}{2} A' (F^{*2} - I^{*2} - J^{*2}) .$$

A vector diagram which illustrates graphically the interval rule for the $^2S_{\frac{1}{2}}$ levels in sodium for an s electron with l equal to zero, is shown in Figure 4.

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[6] White, op. cit., pp. 354.

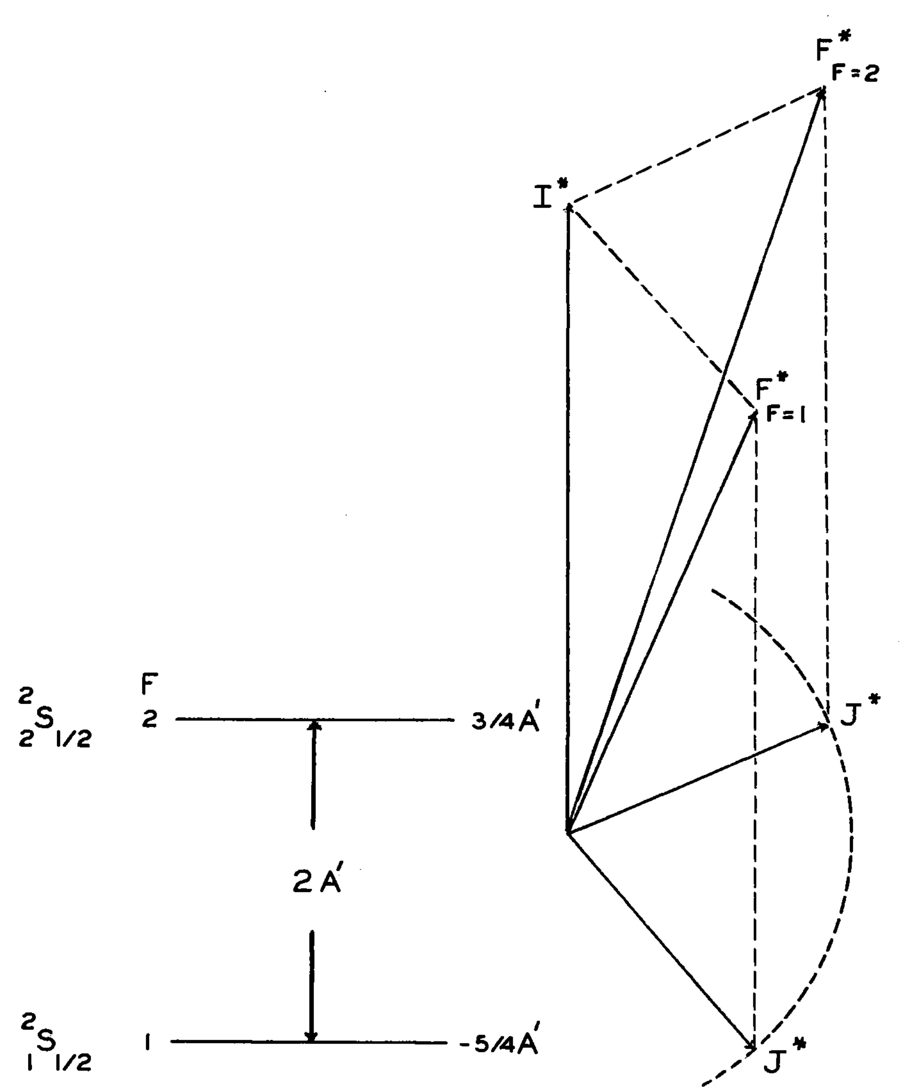

Figure 4

When I is equal to 3/2 and j is equal to ½, we find that I^* is equal to $(15/4)^{1/2}$ and that j^* is equal to $(3/4)^{1/2}$. We also find that when F is equal to one that F^* is equal to the square root of two, and when F is equal to two, then F^* is equal to the square root of six. T_{F_1} is then equal to $-$ 5/4 A', and T_{F_2} is equal to 3/4 A'. The difference in these term values is 2A'. The angle between I^* and J^* is equal to 138° when F is equal to one, and when F is equal to two, the angle is equal to 65°.

CHAPTER I

NUCLEAR INTERACTION WITH ONE VALENCE ELECTRON [1]

The interaction of an electron with the nucleus may be divided into two parts: (1) interaction of the orbital motion l^* with the nuclear spin I^*, and (2) the interaction of the electron spin s^* with I^*.

According to the classical electromagnetic theory the electric field at the nucleus due to the electron at a distance r is given by

$$\mathbf{E} = (e/r^3)\mathbf{r} .$$

The magnetic field at the nucleus due to the orbital motion of the electron is

$$\mathbf{H} = \frac{\mathbf{E} \times \mathbf{v}}{c} .$$

Bohr's relation for the angular momentum is

$$m\mathbf{r} \times \mathbf{v} = \frac{l^* h}{2\pi} .$$

Therefore,

$$H = \frac{e(\mathbf{r} \times \mathbf{v})}{r^3 c} = \frac{e}{r^3} \frac{l^* h}{2\pi mc} .$$

(m) and (e) are the mass and charge on the electron and r is the distance between the nucleus and the electron. Because r is not constant, the quantity $(1/r^3)$ must be averaged. I. Waller[2] gives

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[1] This theory was taken from White, op. cit., pp. 358.

[2] Pauling and Wilson, Introduction to Quantum Mechanics, pp. 143.

$$\overline{(1/r^3)} = \frac{Z^3}{a_0^3 \, n^3 l(l - \frac{1}{2})(l - 1)} .$$

The nucleus with a mechanical moment $I^* h/2\pi$ and magnetic moment M_I tends to carry out a Lamor precession around this field with an angular velocity w, which is the product of the field strength and the ratio between the magnetic moment and the mechanical moment of the nucleus. For this ratio we obtain

$$\frac{M_I}{I^* h/2\pi} = g_I (e/2mc) .$$

The quantity e/2mc is the classical ratio and g_I is called the nuclear g factor. Then

$$w = g_I \frac{e^2}{2m^2c^2} \frac{l^* h}{2\pi} \overline{(1/r^3)} .$$

The interaction energy is given by the product of w and the projection of $I^* h/2\pi$ on l^*,

$$W_{I,l} = g_I \, (e^2/2m^2c^2)(l^* h/2\pi)\overline{(1/r^3)}(I^* h/2\pi)\overline{\cos(I^* l^*)} .$$

Since I^* and j^* precess around F^*, and l^* precesses around j^*, $\cos(I^* l^*)$ must be averaged. l^* precesses around j^* much faster than I^* and j^* precess around F^*. We may then project l^* on j^*, then j^* on I^*. We then obtain[3]

$$\overline{\cos(I^* l^*)} = \cos(I^* j^*) \cos(l^* j^*) .$$

Therefore

$$W_{I,l} = g_I \, (e^2/2m^2c^2)(l^* h/2\pi)\overline{(1/r^3)}(I^* h/2\pi)\cos(I^* j^*)\cos(l^* j^*) .$$

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[3] White, op. cit., pp. 359.

The mutual energy[4] of two magnetic dipoles with moments M_I and M_s and distance r apart is equal to

$$W_{I,s} = (M_I M_s/r^3)\left\{\cos(M_I M_s) - 3\cos(M_I \mathbf{r})\cos(M_s \mathbf{r})\right\} .$$

The magnetic moment of the spinning electron is

$$M_s = -2(e/2mc)(s^* h/2\pi) .$$

The nuclear magnetic moment is given by

$$M_I = g_I(e/2mc)(I^* h/2\pi) .$$

By the use of the direction cosines Pauling and Goudsmit[5] have shown that the value of the term in the braces is equal to

$$-\tfrac{1}{2}\cos(I^* j^*)\left\{\cos(j^* s^*) - 3\cos(j^* l^*)\cos(s^* l^*)\right\} .$$

Therefore

$$W_{I,s} = g_I\,(e/2mc)(I^* h/2\pi)2(e/2mc)(s^* h/2\pi)(\overline{1/r^3})\tfrac{1}{2}\cos(I^* j^*) \left\{\cos(j^* s^*) - 3\cos(j^* l^*)\cos(s^* l^*)\right\} .$$

The total interaction for the spin and orbit becomes

$$\Gamma_F = W_{I,l} + W_{I,s} = a' I^* j^* \cos(I^* j^*) = \tfrac{1}{2}a'(F^{*2} - I^{*2} - J^{*2}) ,$$

where

$$a' = g_I(e^2h^2/8\pi^2m^2c^2)(\overline{1/r^3})\left\{(l^*/j^*)\cos(l^* j^*) + (s^*/2j^*)\cos(s^* j^*) - (3s^*/2j^*)\cos(j^* l^*)\cos(s^* l^*)\right\} .$$

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[5] Pauling and Goudsmit, The Structure of Line Spectra, pp. 206.

Substituting the value

$$\overline{(1/r^3)} = \frac{Z^3}{a_o^3\, n^3 l(l - \frac{1}{2})(l - 1)}$$

into the preceding formula, we obtain

$$a' = g_I \frac{Z^3}{n^3 l(l - \frac{1}{2})(l - 1)} \frac{e^2h^2}{8a_o^3\pi^2m^2c^2} \left\{ \quad \right\}.$$

Letting

$$R = \frac{2\pi^2me^4}{ch^3}, \quad \alpha^2 = \frac{4\pi^2e^4}{h^2c^2}, \quad a_o = \frac{h^2}{4\pi^2me^2},$$

then

$$a' = g_I \frac{Rhc\,\alpha^2Z^3}{n^3 l(l - \frac{1}{2})(l - 1)}.$$

Goudsmit has shown[6] that the value of the terms in the bracket could be replaced by l^{*2}/j^{*2}.

Then

$$a' = g_I \frac{Rhc\,\alpha^2Z^3}{n^3(l - \frac{1}{2})j(j - 1)}$$

$$a' = g_I \frac{R\,\alpha^2Z^3}{n^3(l - \frac{1}{2})j(j - 1)}\ cm^{-}.$$

The nuclear factor g is given by the ratio between the nuclear magnetic moment in Bohr magnetons (eh/4πmc) and the mechanical moment. Therefore if g_I is to be expressed in nuclear magnetons (eh/4πMc) then g_I must be divided by 1838, then[7]

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6 Ibid., pp. 225.

7 White, op. cit., pp. 361.

$$a' = \frac{g_I}{1838} \frac{R\alpha^2 Z^3}{n^3(l - \frac{1}{2})j(j - 1)} \text{ cm}^{-1} .$$

Z^3 is broken up into two parts Z_i and Z_o^2 where Z_i is the effective nuclear charge inside the closed electron shell and Z_o the effective nuclear charge outside.[8] This gives

$$a' = \frac{g_I}{1838} \frac{R\alpha^2 Z_i Z_o^2}{n_o^3 (l - \frac{1}{2})j(j - 1)} .$$

Breit[9] has shown that relativity corrections are different for two levels of a doublet and must be taken into account. This correction is obtained by multiplying a' by k, where

$$k = \frac{4j(j - \frac{1}{2})(j - 1)}{(4p^2 - 1)\, p} , \qquad p^2 = (j - \tfrac{1}{2})^2 - (\alpha Z_i)^2 .$$

THEORETICAL COMPUTATIONS

For s electrons l is equal to zero and j is equal to one-half, therefore, the new value of a' is given by

$$a' = (g_I/1838)(8R\alpha^2 Z_i Z_o^2/3n_o^3)k .$$

Z_i is equal to Z for s electrons and Z_o is equal to one for an alkali atom. R is equal to 109734 cm^{-1} and n_o equals 1.627.[10] α^2 is given by 5.305×10^{-5} and g_I equals 1.4. Substituting

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8 Ibid., pp. 361.

9 G. Breit, see Goudsmit, Physical Review, XLIII (1933), 636.

10 White, op. cit., pp. 90.

these values in the above formula we find that a' is equal to 0.030200 x k. We find that p^2 is equal to 0.99358. Using this value and the above values we find that k is equal to 1.0118865. Multiplying this value of k by 0.030200 we find that a' is equal to 0.030559.

Referring to Figure 3, we see that the $^2S_{\frac{1}{2}}$ separation is given by $T_{F_2} - T_{F_1}$ where T_F is given by $\frac{1}{2}a'(F^{*2} - I^{*2} - J^{*2})$. Substituting in this formula first for F equal to two and then for F equal to one, we find that $T_{F_2} - T_{F_1}$ is equal to 2a' which is the value found graphically in Figure 4. Therefore the $^2S_{\frac{1}{2}}$ separation is 0.061118 cm^{-1}.

CHAPTER II

INSTRUMENTATION FOR THE STUDY OF HYPERFINE STRUCTURE

Observation of the hyperfine structure of spectra requires instruments which are capable of high resolution. The term resolution refers generally to the ability of an optical instrument to separate two closely situated images. Interferometers are the chief instruments for resolving hyperfine structure. The one most commonly used is some modification of the parallel plate or Fabry-Perot type. Powerful as this instrument is, the over-all resolution obtainable with any experimental arrangement is limited by other factors. The origin of line spectra is in general a complex process and several factors influence the form of the "lines" in emission. The phenomena of hyperfine structure are manifested by extremely minute effects, and considerable attention must be directed to instrumentation in order that such effects may be detected.

The factors which, at the radiation source, affect the width and form of spectral lines must especially be taken into account in the study of hyperfine structure. A group of such factors is the following:[1,2]

1. Natural width.
2. Doppler effect.
3. Pressure broadening.
4. Absorption.
5. Stark effect.

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[1] S. Tolansky, High Resolution Spectroscopy, pp. 3-18.

[2] White, op. cit., pp. 419-436.

The first of these is an intrinsic property of the atoms. This natural width arises from the fact that the energy levels within the "average" atom are not too sharply defined, but are of finite width. It is to be expected, then, that a spectral line will have a finite width corresponding to the sum of the finite widths of the energy levels involved in its formation. Since the spectroscopist deals with collections of atoms and not with individual atoms, this line width is inescapable; hence, the terms "average" atom and "natural" width.

The Doppler effect exercises perhaps the largest influence on the breadth of spectral lines. The Doppler spread is a consequence of the random thermal motions of the radiating atoms. Atoms moving with velocity component (u) toward or away from the observer emit radiation which, because of the Doppler effect, would appear to be of frequency $v + dv$ and $v - dv$ respectively, where, $dv = uv_o/c$; v_o being the radiation frequency of an atom for which (u) is zero, and (c) the velocity of light.[3] The net result is a statistical broadening of the spectral lines and is a function of temperature and the mass of the atom.

Pressure broadening of lines results from the perturbing effects of neighboring atoms or molecules on the energy states of the radiating atom. The degree of perturbation depends on the nature of the perturbing particle, i.e., the particle may be charged or uncharged, like or unlike the radiating atom. The perturbation effects depend also on the closeness of

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[3] Tolansky, op. cit., pp. 6.

approach of the particles, and, hence on the temperature and density of the gas in which the radiating atom is located. This type of broadening may be reduced by use of sources in which the gas pressure and current density are maintained at low values.

The emission of a radiating atom may be absorbed by non-radiating atoms which surround it. This absorption may cause serious widening of spectral lines; especially in the case of resonance lines. The tendency for absorption is a maximum for the frequency of greatest intensity, i.e., at the center of the line. If the source is extensive or of high density, the center-line range of frequencies may be entirely absorbed and not observable. This phenomenon, known as self-reversal, results in the formation of a false "structure". For example, if the absorption at the center of a line is nearly complete, the line may appear to be a doublet, which, in reality, it is not.

The Stark effect, i.e., the interaction of a strong external electrical field with the radiating atoms, broadens the lines in that it initiates additional quantized splitting of the energy levels. The effect is more deleterious if the external field is non-uniform. In this case, radiating atoms in different parts of the field are variously affected, and the result is a "smeared" Stark broadening.

The above discussion on the factors which influence the form of spectral lines may be applied directly to the images obtained by interference methods. The spectral "lines" of the interferometer are then called fringes.

The form or intensity distribution of interference fringes is perhaps best illustrated by the following series of cross sectional diagrams.

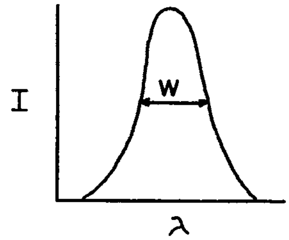

Fig. 5.--General Intensity Distribution

The breadth of a line is designated by the half-width (w) which is the distance between two points where the intensity is one-half of the maximum intensity.

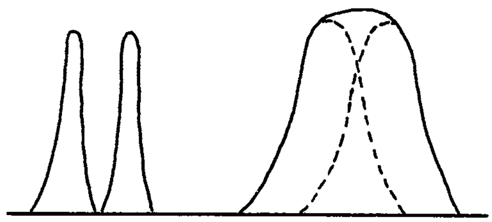

Fig. 6.--True Doublet and Masked Doublet

If the half-widths of the doublet components approaches or exceeds their separation, the structure is masked and cannot be detected.

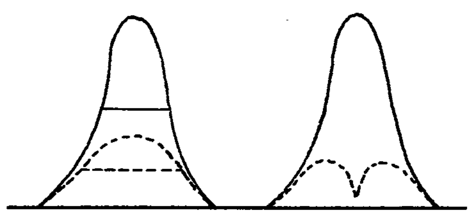

Fig. 7.--Effect of Absorption on Line Form

The solid curve represents the line as emitted. The dash curves show the effects of absorption. Note the increase in effective half-width caused by absorption. The formation of a spurious structure, when absorption at the center of the line is excessive, is also indicated.

The width of spectral lines or fringes can be controlled to a remarkable extent by the proper design and operation of the light source. The authors of this paper were, however, limited to the use of an arc-type source. With the arc source both the Doppler effect and pressure broadening are relatively large. For example, a quantitative indication of the magnitude of the Doppler effect alone may be calculated from the following expression.[4]

$$\text{Half-width} = .71 \times 10^{-6}(T/M)^{\frac{1}{2}}\ v\ \text{cm.}^{-1}$$

where, T = Absolute temperature

M = Atomic weight

v = Frequency in wave numbers

For Sodium at 500°Abs., a relatively low value, the above formula yields a half-width of .056 cm^{-1} or .02 Angstrom units. This value is of the same order of magnitude as the separation of the hyperfine structure components of one of the Sodium D lines. The properties of the arc, thus, may leave much to be desired in connection with the sharpness of its lines.

The Fabry-Perot Interferometer

This instrument consists of two plane-parallel glass plates with adjacent surfaces coated with a partially transparent film of metal which has a high coefficient of reflection.

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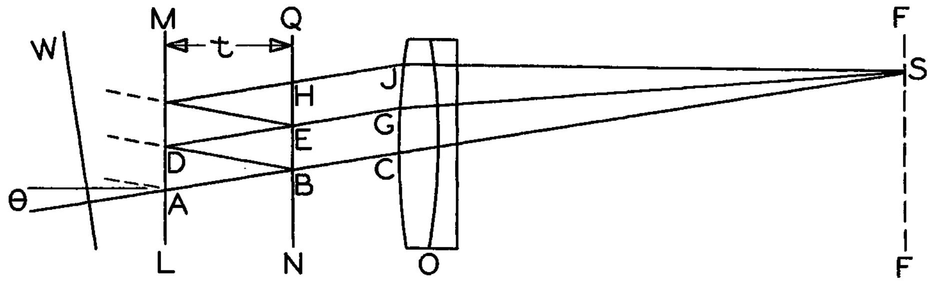
[4] Tolansky, loc. cit.

One of the plates is fixed while the other is movable normal to its reflecting surface. The following diagram will illustrate the principles of the Fabry-Perot interferometer.[5]



Let LM and NQ in the above figure represent the parallel half-silvered plates. If light of wave length λ falls on the plates at all angles, interference fringes are formed in the focal plane (FF) of the objective lens (O). All light incident along a cone of semi-angle θ will contribute to the formation of a single fringe. The pattern in the focal plane of the objective consists, then, of a set of concentric circular fringes. If R and P are the fractions of the light intensity reflected and transmitted at each plate, the amplitudes are proportional to the square roots of R and P. Then, if W is one wave front of unit amplitude, represented by its normal ray which is incident at an angle θ, the portion (AB) transmitted by the first plate has an amplitude $P^{\frac{1}{2}}$; by the second plate (BC) an amplitude of P. The amplitude associated with ray (BD) is $P^{\frac{1}{2}}R^{\frac{1}{2}}$; with (DE) it is $P^{\frac{1}{2}}R$, and with (EG) it is PR. Similarly the amplitude corresponding to the multiple-reflected portion of W which is represented by its normal (HJ) has an amplitude of PR^2. These wave

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[5] W. Ewart Williams, Applications of Interferometry, pp. 76-82.

fronts, resulting from multiple reflections, are all parallel and will be collected by the lens at the point S. When θ is small and the reflection efficiency is high, there may be over one-hundred of such wave fronts. The phase difference between successive wave fronts is given by the expression;

$$\emptyset = (2\pi/\lambda)2t\cos\theta ,$$

where (t) is the separation between plates. When this phase difference is an integral multiple of (2π), constructive interference occurs. The intensity distribution within any fringe, in terms of P, R, and $\emptyset$, is given by:[6]

$$I = \frac{P^2}{(1-R)^2} \frac{1}{1 - \frac{4R}{(1-R)^2}\sin^2\emptyset/2}$$

$$\text{or: } I = \frac{I_{max}}{1 - F\sin^2\emptyset/2}, \text{ where, } I_{max} = \frac{P^2}{(1-R)^2},$$

$$\text{and, } F = \frac{4R}{(1-R)^2}.$$

The order (n) of a fringe is $(2t/\lambda)\cos\theta$ and is a maximum at the center of the pattern. From the intensity equation and the Rayleigh criterion for resolution, an expression can be obtained for the resolving power of the Fabry-Perot interferometer.

The Rayleigh criterion is that two lines are said to be resolved when the maximum intensity of one line falls on the first minimum of the other. This situation for two lines of equal in-

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[6] Ibid., pp. 78.

tensity is shown in the figure.

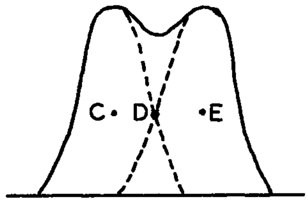

Fig. 8.--Limit of Resolution

In the theory of the diffraction grating it is shown that the intensity distribution of a spectral line may be put in the form; $I = I_{max}(\sin^2 \phi/2)/\phi^2$.[7] Since ϕ, the phase change from point C to point D, is $\pi/2$, it follows that the intensity at D is $I_D = (4/\pi^2)I_{max} = 0.405\ I_C$, and from the intensity equation:

$$\sin^2 \phi/2 = \frac{I_C - I_D}{FI_D} = \frac{1 - .405}{.405\ F} = \frac{1.469}{F}$$

$$\text{and, } \phi = 2 \sin^{-1}(1.21/F^{\frac{1}{2}}) \ .$$

The latter is the phase change between points C and D. The phase change from point C to point E is then twice this value. A change in phase of 2π is equivalent to an order change of one, i.e., from n to n - 1. The total phase change from C to E then corresponds to an order change of $dn = (2/\pi)\sin^{-1}(1.21/F^{\frac{1}{2}})$. At the center of the fringe system we have $n_0 = 2t/\lambda$, and by differentiation, $\lambda/d\lambda = -n_0/dn_0$. The resolving power is given by the ratio $\lambda/d\lambda$. The value of dn_0 is given by the above expression for the minimum resolvable change of order. Then the

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7 R. A. Houston, A Treatise on Light, pp. 171-173.

resolving power is:

$$\lambda/d\lambda = -n_o/dn_o = -n_o\pi/\ [2\ \sin^{-1}(1.21/F^{\frac{1}{2}})] \quad .$$

If the reflecting power R is high, F is large enough that the angle may be replaced by the sine, and the resolving power expression then becomes:

$$\lambda/d\lambda = -n_o\pi F^{\frac{1}{2}}/(2 \times 1.21)$$

$$= -n_o\pi \ \{R^{\frac{1}{2}}/1.21(1-R)\} \quad .$$

The quantity in braces may be called the effective number of reflections and is analogous to the number of apertures in the expression for the resolving power of diffraction gratings. It must be remarked, however, that the fringes of two radiations to be resolved are in general not of equal intensity. Also, in the case of the Fabry-Perot interferometer, the fringes are not symmetrical, but are broadened unsymmetrically toward the center of the pattern. Hence, the Rayleigh criterion cannot be applied with rigor. The above equation, however, is a workable approximation, and shows the dependance of resolving power on the central order number and the reflecting power of the half-silvered surfaces.

The variation of order (n) with wave number (v) may be obtained from the basic equation: $n = 2tv\cos\theta$, where $1/\lambda$ is replaced by (v). Near the center of the pattern; $dv = dn/2t$. Moving from one fringe to the next, dn is equal to one, and the corresponding wave number change dn is 1/2t. This last quantity is called the spectral range. It represents the range of wave number that may be examined without the overlapping of the fringes of successive orders. The range between orders may be varied at will by adjusting the plate separation.

CHAPTER III

EXPERIMENTAL PROCEDURES

The components of the apparatus used by the authors are shown in the photograph on page 23. A schematic diagram showing the placement of the components is given on page 24. A list of the specific parts of apparatus is as follows:

1. Light source-----Sodium Lab-Arc--Gaertner
2. Monochromator----Type L227 Spectrometer--Gaertner
3. Interferometer---Fabry-Perot--Gaertner
4. Short telescope--Type 508--Gaertner
5. Cathetometer-----Type M930--342--Gaertner
6. Miscellaneous----Auxiliary lenses and photographic equipment.

Light from the Sodium Lab-Arc is passed through the spectrometer in order to isolate the D lines from other lines present in the Sodium spectrum. The spectrometer also offers a means of controlling the intensity of the light falling on the interferometer. The optical system of the spectrometer (monochromator) consists of an entrance slit, collimating lens, constant-deviation prism, objective lens and exit slit. Adjustment of the monochromator slits is somewhat critical in that the optimum combination of resolution and intensity is thereby controlled. This is especially pertinant in the case of direct visual observation. Following the monochromator are the collimating and converging lenses. The convergent beam method of mounting is advantageous in that it effectively introduces a stop, i.e., the interferometer aperture is reduced.[8] With an intense source, the loss of intensity due to decrease in aperture is no problem, and errors in plate figure, mirror deposition, and plate parallelism are materially reduced. In using the interferometer, the plate separation was adjusted so that the fringes of the D_1 and D_2 lines fell alternately and evenly spaced. The plate separation for such a pattern may be calculated by setting the spectral range (1/2t) equal to $1/(n + \frac{1}{2})$ times the wave number difference of the D lines (17.3 cm^{-1}).[9] The plate separation used was 1.497 centimeters. A short telescope fitted with a micrometer eyepiece was used for visual observation of the fringe pattern. The fringes were photographed in two ways: (1) a lens of 50 cm. focal length was used, the film being placed in its focal plane, (2) the fringe pattern was first formed by the short telescope objective lens and then projected by the eyepiece on to the photographic film. By the last method, the image size obtained was equivalent to that of a single lens of 1.5 meter focal length.

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[8] Tolansky, op. cit., pp. 162.

[9] Granath and Van Atta, "The Nuclear Spin and Magnetic Moment of Sodium from Hyperfine Structure", Physical Review, XLIV (1933), 935.

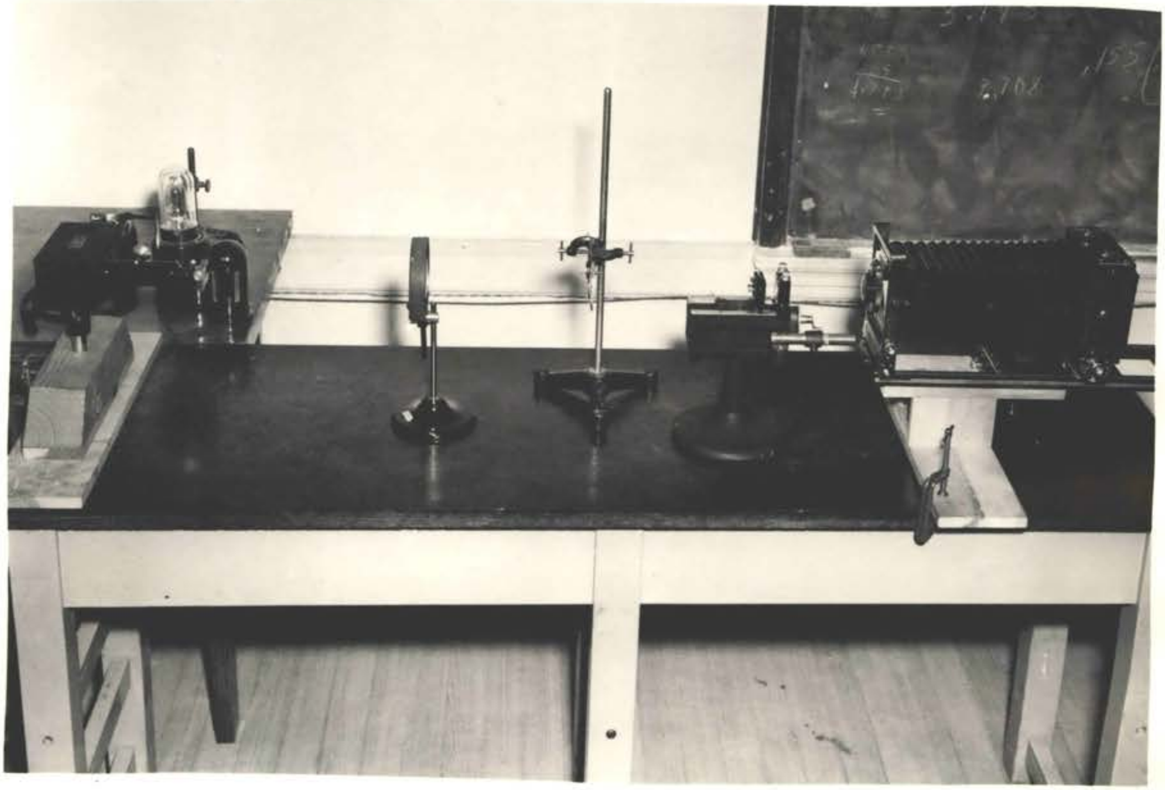
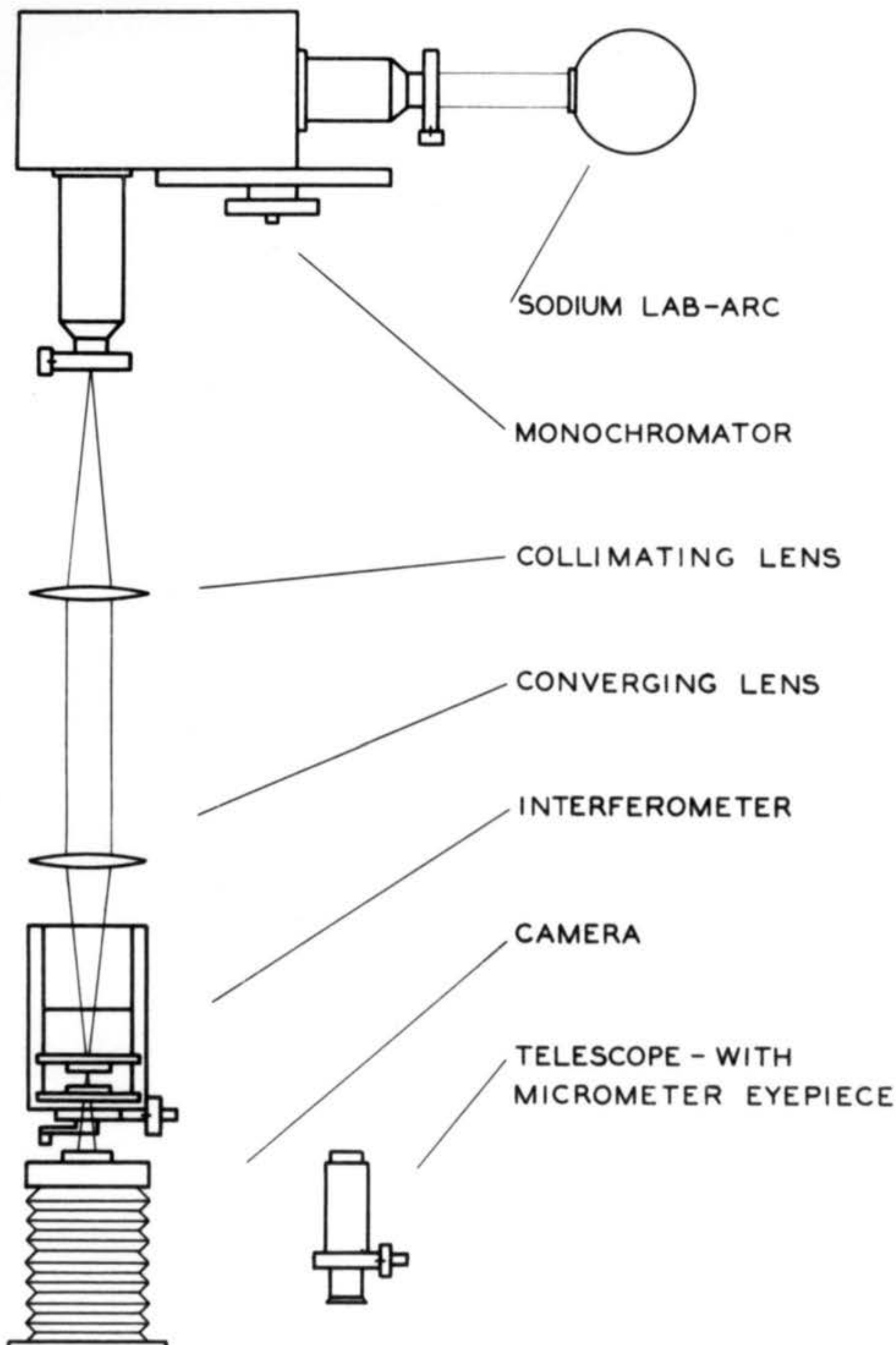


Figure 9.--Photograph of Apparatus



PLAN OF APPARATUS

Measurements

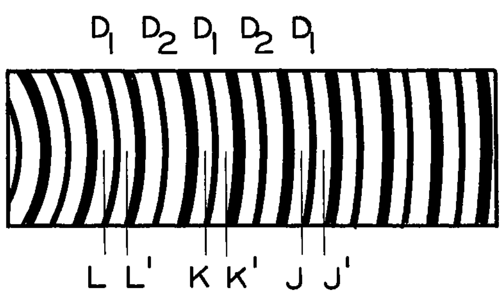
The data taken was based on reduction formulas communicated to the authors' adviser, Dr. A. V. Pershing, by Dr. Russell A. Fisher of Northwestern University. The set of three-order formulas is as follows:

$$1.\ v - v' = \frac{1}{2t} \cdot \frac{(S_J' - S_J)}{(S_K - S_L)} \cdot \frac{(S_J' - S_L)}{(S_J - S_K)} - \frac{2(S_J' - S_K)}{(S_J - S_L)}$$

$$2.\ v - v' = \frac{1}{2t} \cdot \frac{(S_K' - S_K)}{(S_J - S_L)} \cdot \frac{(S_J - S_K')}{(S_K - S_L)} - \frac{2(S_K' - S_L)}{(S_J - S_K)}$$

$$3.\ v - v' = \frac{1}{2t} \cdot \frac{(S_L' - S_L)}{(S_J - S_K)} \cdot \frac{(S_J - S_L')}{(S_K - S_L)} - \frac{2(S_K - S_L')}{(S_K - S_L)}$$

A drawing of a portion of the fringe pattern will serve to illustrate the manner in which the data for the above formulas was taken.



The S's in the formulas are simply the scale readings of the micrometer eyepiece or cathetometer as the cross-hair of the instrument was set on the hyperfine components of one of the sodium D lines. The primed and unprimed subscripts refer to particular components as labeled on the above drawing. The quantity (t) in the formulas is the plate separation in centimeters. The average of the results of these formulas then gives the desired separation of the hyperfine components of one of the sodium D lines; in wave numbers (cm^{-1}).

Data

The data in Table I was taken from a photograph; using a cathetometer. One-hundred readings were taken on each component of the D_2 line, in three successive orders. The first block of ten readings is given complete. The first and the nine remaining blocks are indicated by their respective averages.

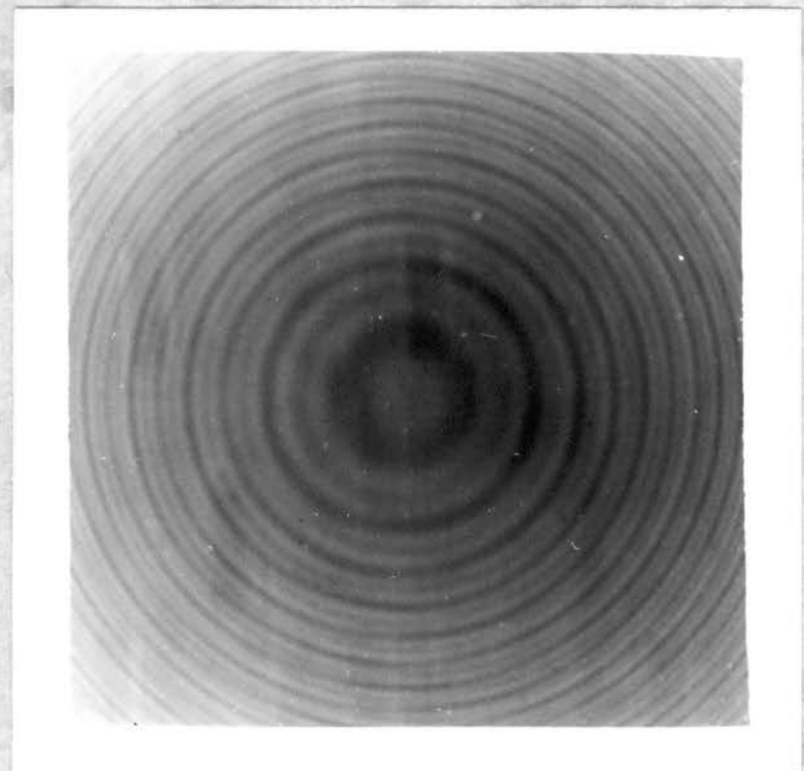


Figure 10.--Fabry-Perot Interferometer Fringes of the D_1 and D_2 Lines of Sodium

Table I

L	L'	K	K'	J	J'
4.7702 4.7718 4.7535 4.7668 4.7768 4.7627 4.7680 4.7627 4.7668 4.7648	5.0849 5.0730 5.0899 5.0835 5.0830 5.0823 5.0879 5.0792 5.0848 5.0734	5.9745 5.9873 5.9677 5.9764 5.9719 5.9702 5.9777 5.9672 5.9703 5.9643	6.1583 6.1639 6.1598 6.1735 6.1648 6.1656 6.1605 6.1748 6.1762 6.1669	6.9015 6.8919 6.8987 6.8914 6.8885 6.8921 6.8918 6.8823 6.8936 6.8863	7.0518 7.0421 7.0473 7.0554 7.0527 7.0514 7.0492 7.0442 7.0531 7.0414
4.76540	5.08209	5.97275	6.16643	6.89180	7.04886
4.75962	5.07909	5.97133	6.17357	6.88855	7.05142
4.77008	5.08460	5.97583	6.17490	6.89238	7.05510
4.75846	5.08504	5.97499	6.17376	6.88891	7.05457
4.75842	5.09215	5.98158	6.17246	6.88925	7.05584
4.78357	5.07801	5.98675	6.17639	6.89664	7.05187
4.77446	5.08670	5.97677	6.17767	6.88714	7.05562
4.78297	5.08139	5.98211	6.18047	6.89518	7.05604
4.78000	5.09729	5.98861	6.18679	6.90373	7.06666
4.77276	5.10639	5.99009	6.18877	6.90355	7.06824

Average of 100 readings.

L	L'	K	K'	J	J'
4.77059	5.08737	5.97886	6.17713	6.89371	7.05642

Experimental Results

When the values of the quantities S_L, S_L', S_K, S_K', S_J, S_J', as given in the final row of averages in Table I, are substituted into the three-order formulas and the indicated computations performed, the following results are obtained:

from formula, 1. $v - v' = 1/2t\ (.199778)$

2. $v - v' = 1/2t\ (.342531)$

3. $v - v' = 1/2t\ (.006686)$

$$v - v' = 1/2t\ (.182996)$$

The value of (t) was 1.497 centimeters. The final value for the frequency difference of the two components of the sodium D_2 line is:

$$v - v' = .182998/2.994 = .06112\ cm^{-1}\ .$$

Again, this result is based on data taken by using a cathetometer on a photograph of the fringes. Consistent results could not be obtained from data taken directly by means of the micrometer eyepiece. The unreliability of the latter method rests largely on two factors: (1) eye fatigue in the observer, and (2) the impossibility of retaining the same fringe pattern over a period of time sufficient for the collection of a reasonable amount of data. The ridgidity of the adjustable type interferometer used by the authors was inadequate with respect to the second of the above factors. Visual measurements with the micrometer eyepiece were, however, useful in the preliminary identification of the features of the hyperfine structure fringe system.

Concluding Remarks

We have seen how the theory of hyperfine structure yields a value of .061118 cm^{-1} for the spectral wave number separation corresponding to the splitting of the $3^2S_{\frac{1}{2}}$ energy level of sodium. Separations of .0612 cm^{-1} from the D_1 line and .0555 cm^{-1} from the D_2 line have been reported by Granath and Van Atta.[10] The agreement between theory and experiment found by the authors of this paper, hence, may not be qualified completely without a similar study of the D_1 line. However, the application of Dr. R. A. Fisher's reduction forumlas toward the analysis of hyperfine structure has been illustrated; and the value of .06112 cm^{-1} determined by the authors is in good agreement with the theoretically computed value. It may be expected, that, when the study of hyperfine structure is implemented by light sources having sharp lines and with a recording microphotometer which indicates the intensity contour of photographically recorded fringe patterns, accurate and rapid analysis of hyperfine structure could be accomplished by using the Fisher reduction formulas.

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10 Granath and Van Atta, loc. cit.

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