DESIGN OF WALLS

TO

RESIST BOMB BLASTS

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CHAPTER I

THE PROBLEM AND ITS SCOPE

The Problem

The main purpose of this study is to present the methods by which a structural designer would design walls to resist bomb blasts. This involves four main steps: (1) Calculation of the frequency of the wall panel, (2) determination of the equivalent static load that would produce the same stresses in the wall as the bomb blast, (3) analysis of the wall panel to determine the critical moment, shears, etc., and (4) the design of the wall, including the new important principles introduced by the blast factor.

It would be undesirable and practically impossible to attempt, within a single report, an illustration of the design of all types of walls to resist blast. For this reason only the more common types of panels are considered. The types of wall panels illustrated in this report include steel and concrete panels with simple supported edges, steel panels with fixed edges, continuous concrete panels, and cantilever walls.

Need for the Study

Although it was said in each of the last two wars, "This is the war to end all wars," there may be another war in the future. In every war there is a need to protect vital machinery, installations, and people from being destroyed by whatever method of destruction the enemy deems to use, such as bombs, gas, bacteriological warfare, and others. In the last war more damage was created by bombs dropped from airplanes than by any other method of destruction. In bomb damage, generally speaking, the worst damage is not due to the direct impact and explosion of the bomb, but to the indirect effects of the explosion, that is, the blast produced by the explosion and the fragments of the metal bomb case that are flung out from the center of the explosion at tremendous velocities.[1] Protection of human life and vital machinery requires walls that will resist the effects of these lateral forces. Although this paper deals primarily with the design of walls to resist bomb blast, the same principles introduced here can be used in the design of walls to resist explosions that occur in industry in peace time.

Any designer can determine the stresses in common curtain walls with ease since only vertical loads are considered. However, the critical stresses in walls designed to resist blast are not only more difficult to compute, but also are entirely different from the critical stresses in curtain walls. This report presents only the ways for calculating the critical stresses in bomb blast walls and the new structural problems that are present due to lateral impact loads. Once the critical stresses have been determined, and with a knowledge of the new important principles to be considered, any designer can design a blast wall.

Delimitations

This problem is limited to the design of walls to resist the blast of high explosive bombs only. The blast resulting from other types of explosions, such as steam, gasoline, and natural gas are not considered. The effects of the atomic bomb, also, are not considered. Very little is known about the effects of the blast pressure from atomic bombs, and a large amount of data such as these are being kept a state secret. However, a few reports have been made public on the effects of the two atomic bombs dropped on Japan in the last war. It was found that the initial destructive force was the blast from the bombs, while

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[1] O. Bondy, "Air Attack Versus Steel Construction," The Iron Age, CXLVIII (August 1941), 42-45.

later damage was caused by fires that burned unchecked. Only modern well built reinforced concrete buildings, located a short distance from the points of the explosion of the bombs, survived.[2] Very little more is known about the effects of the atomic blast. Until such information is released to the public, only those connected with atomic research can design atomic bomb resistant structures.

No attempt has been made to explain the method by which the blast pressures are measured and the equivalent static loads calculated. To do this would have necessitated the presentation of extra material and calculations that are not needed to fulfill the scope of this paper.

History

The first modern structures designed and built to withstand large explosive impact loads were constructed by the French. Even before World War I they built huge reinforced concrete forts as protection against artillery shells. Many principles that they devised and used in such constructions were so fundamental that they were used in the last war in the construction of massive reinforced concrete bomb shelters. In the first world war only a small number of bombs, compared to the number dropped in the last war, were used and little effort was made to provide protection against aerial bombardment.

No important progress in air raid protection was made until after the beginning of the Spanish Civil War in 1936. With the increasing number of air raids, the Spanish people devised and built a number of different types of air raid shelters. Many were deep underground with connecting tunnels. Neutral observers noticed the damage being done by the air raids and so in the major countries of the world experiments were being conducted on the resistance of different types

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[2] E. H. Praeger, "Behavior of Concrete Structures Under Atomic Bombing," Journal of American Concrete Institute, XVII (June 1946), 709-720.

of building materials to aerial bombardment. In 1939 the British developed a method to measure the force of the blast from different bombs and determined the equivalent static loads on different structures.

Many structures were designed and built to resist aerial bombs during the last war. Some were successful and some were not. Small prefabricated family size shelters provided the necessary protection only when buried and properly covered with earth. These could not be occupied for long periods of time. The most successful surface shelters built by the British had reinforced brick masonry walls with reinforced concrete roofs. These shelters were built so they could move horizontally when subjected to blasts. The largest bomb resistive structures were massive reinforced concrete buildings built by the Germans to protect their submarines. Even with nine foot thick roofs, these structures were not completely bomb proof. A few of the heaviest bombs dropped by the Allied Air Forces penetrated the roofs and caused internal damage.

Although little information has been published since the end of the last war on bomb resistant structures, buildings are still being built and experiments are still being conducted on the resistance of different types of materials and structures to bomb attack. The Corps of Engineers of the United States Army are at present conducting experiments on the resistance of materials to high explosive bombs. Building codes in Switzerland now specify that all new buildings constructed must have air raid shelters. The Australian government is building all new administrative buildings with thick reinforced concrete walls to resist bomb blast and all floors to carry a very large debris load. The United States government and other countries may be conducting additional research along these lines, but little information is being released to the public.

CHAPTER II

BOMBS AND BOMB BLAST

The main offensive destructive weapon of the airplane is the aerial bomb. Two main types of aerial bombs are generally used, high explosive and incendiary. The incendiary bomb is used to start fires which will destroy the target, while the high explosive bomb destroys the target by explosion and blast.

These bombs are carried by planes and are released near the target area. Some types of bombs have wings and radio controls and are glided to the target, while others are power propelled through the air and strike the target with a greater impact than a bomb in free fall. This is to allow the bomb to penetrate deeper into the target before it explodes. But by far the greatest number of bombs that are dropped have a free fall to the target. These bombs range in weight from 100 to 4,000 pounds. The size of the bombs carried by the airplane depends upon the target attacked. Five hundred pound bombs usually are dropped on industrial targets, because a plane can carry a large number of these bombs and can blanket a large target area. The lighter bombs are dropped on residential areas, while the heavier bombs are used to attack such targets as underground factories and concrete bomb resistant structures (sub-pens).

FIGURE 1. BOMBER DROPPING 500-POUND HIGH EXPLOSIVE BOMBS

A bomb usually consists of four main parts: the high explosive matter, the case to hold the explosive charge, fins to stabilize the direction of the fall, and detonators to explode the bomb.

Two types of detonators are used. An instantaneous fuse causes the bomb to explode on contact with any surface, while a delayed action fuse allows an interval of time to elapse between the time of impact and the explosion of the bomb. The type of detonator used depends upon the target attacked. Delayed fuses of about .01 of a second are used to destroy structural buildings, since this allows the bomb to penetrate into the building where its explosion will produce the most damage.

The impact of the bomb causes the detonator to explode, which in turn detonates the main content of the bomb. As the bomb explodes, the solid explosive matter is changed into a gas at high pressure and high temperature, which causes the metal case of the bomb to burst into many fragments. The gas continues to expand rapidly and exerts an intense pressure on the air surrounding the point of explosion. However, the gas cools very rapidly and has a relatively small expansion. When exploded on a flat surface, a 500 pound bomb will have a radius of expansion of only about 25 feet.[1] This has been proven by the relatively small area of snow that melted when bombs burst on flat, snow covered areas.

The rapidly expanding gases exert an enormous pressure on the surrounding air, with the result that the air is suddenly compressed and forms an intense blast wave. This blast wave is similar to a sound wave, except that it has a higher initial amplitude and velocity. The amplitude and velocity gradually decrease as the wave travels away from the point of explosion and the blast wave

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[1] Harold Wessman and William Rose, Aerial Bombardment Protection, p. 106.

degenerates into a sound wave.[2]

Each blast wave consists of an initial positive pressure phase, followed by a suction phase. The positive phase of the blast, although only momentary, produces a very high compressive pressure. A 500 pound bomb, 50 feet from the target will exert a positive pressure of about 6 pounds per square inch or 864 pounds per square foot, and is about .005 of a second in duration. The suction phase has opposite characteristics. It lasts for a longer period of time (about .03 of a second) and produces a tension pressure about one-fourth that of the maximum positive pressure. The total duration of the blast wave is about 1/30 of a second.[3]

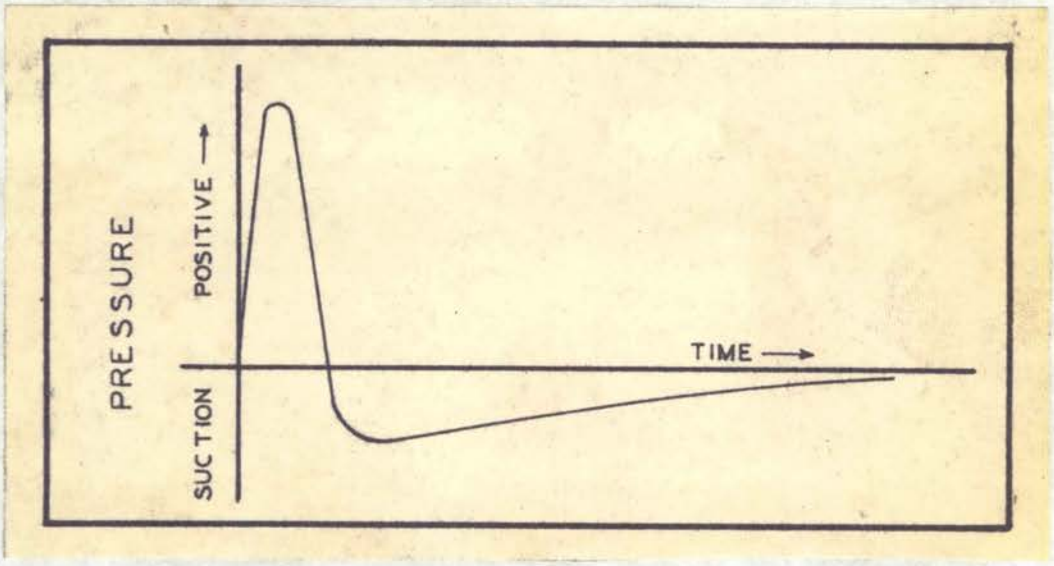

FIGURE 2. TYPICAL TIME-PRESSURE CURVE FOR 500-POUND BOMB

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[2] Anonymous, Air Raid Precautions, Handbook 5, Structural Defense, p. 19.

[3] A. M. Prentiss, "Bomb Defense," Army Ordnance, XXIII (July-August, 1942), 54-58.

Both pressures decrease as the distance from the explosion increases. These pressures have been measured at different distances from the explosions by means of an accurate, though complicated device,[4] and the variation is shown in the following graph.[5]

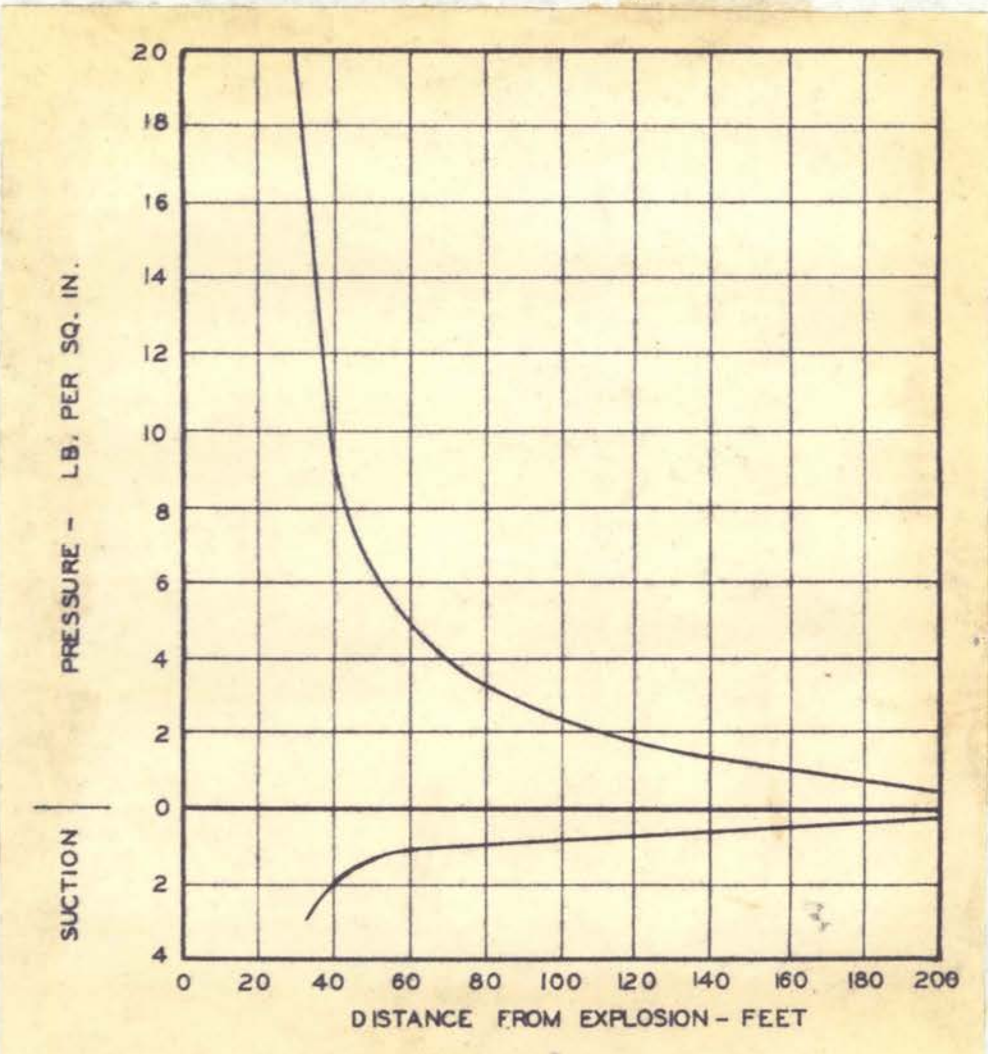

FIGURE 3. PRESSURE-DISTANCE CURVE FOR A 500-POUND BOMB[6]

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[4] Wessman and Rose, op. cit., p. 110.

[5] Anonymous, Air Raid Precautions, Handbook 5, Structural Defense, p. 10.

[6] Ibid., p. 10.

CHAPTER III

FRAGMENTATION

As a bomb explodes, the case of the bomb bursts and small fragments of the steel case are blown in all directions. The number, shape, weight, and velocities of the fragments depend not only upon the size of the bomb, but also upon the depth of penetration into the target or into the ground. The deeper the penetration of the bomb, fewer fragments are projected into the air. In fact some delayed action bombs bury themselves so deeply in the ground that when the bombs burst there are no flying fragments. Initially these fragments have a velocity of around 7,000 feet per second. Being of light weight and irregular in shape, the fragmentary velocities decrease rapidly and their effective range is around 500 feet.[1] Even though the major portion of the fragments weigh less than an ounce, at the velocity they travel, they are as deadly as any rifle bullet.

Table I, page 9, gives the recommended thickness of various materials to withstand fragmentation penetration of a 500 pound bomb. Suitable protection can be given by sections of more than one material by proportionate thickness. For instance a 6 inch reinforced concrete wall will give protection from fragmentation if covered with a three-fourths inch steel plate.

TABLE I

THICKNESS OF MATERIALS FOR PROTECTION AGAINST BOMB FRAGMENTS[2,3]

Material	Thickness
Mild steel plates	$1\frac{1}{2}$ inches
Reinforced concrete-3,000 p.s.i. or better	12 inches
Solid brick wall-cement or cement-lime mortar	13 inches
Plain concrete	15 inches
Sand or gravel between wood sheathing or corrugated iron	24 inches
Earth or sand bag wall	30 inches

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[1] Anonymous, Air Raid Precautions, Handbook 5, Structural Defense, p. 19.

[2] Ibid., p. 20.

[3] Anonymous, Protective Construction, p. 6.

CHAPTER IV

CALCULATIONS OF STATIC LOAD

The design of structures to resist blast is actually an energy problem. Simple structures, such as beams, can easily be designed to withstand energy loads. However, in the more complicated structures, such as slabs and plates, the stress is non-uniform and, when using energy loads, the design is difficult. Thus, as in the design of other complicated engineering structures that are subjected to energy or impact loads, an equivalent static load is used in blast design. Once the equivalent static load has been computed, the easier normal design methods may be used.

An equivalent static load may be defined as that steadily applied uniformly distributed load that produces the same maximum stress in a structure as the actual load.

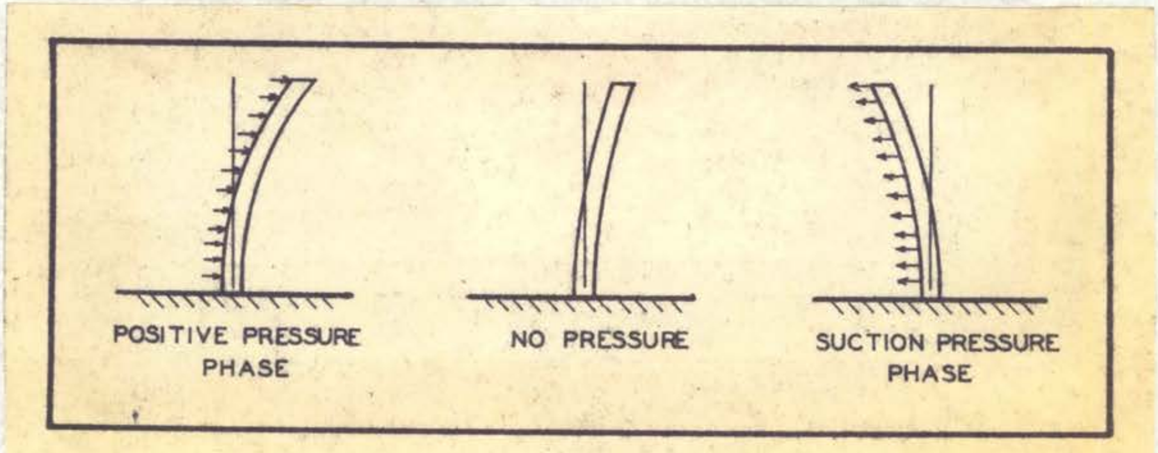

FIGURE 4

Even though an equivalent static load will give the same or even a greater stress than the actual maximum stress, there is still the probability that the material may react differently under a rapidly reversing load than the assumed static uniform load. For instance, it would seem that since the maximum positive pressure produced by a blast wave is about five times larger than the maximum suction pressure, a structure would always fail during the positive pressure phase of the blast. This is not the case. Any structure, for example a cantilever wall, will begin deflecting as soon as it is subjected to the positive blast pressure (Figure 4, page 10). If the wall is deflected to its rupture deflection before the maximum positive pressure has been applied, the wall will fail. If, however, the wall has such vibration characteristics that allow the wall to deflect an amount less than its rupture pressure, at the time of the maximum positive pressure, then the wall will begin to vibrate. The force pulling the wall back into its original position will accentuate the suction pressure and, even though the maximum negative pressure is less than the maximum positive pressure, the deflection of the wall toward the point of explosion will be greater than that away from the explosion. If this deflection is greater than the rupture deflection of the wall, the wall will fail. If not, the wall will vibrate with decreasing amplitude until it comes to rest in its original position.

The calculation of the equivalent static load must, therefore, include the vibration characteristics of the structure. Such a mathematical formula has been developed, but the derivation and the solution is long and tedious. To ease this situation the results of this analysis usually are shown in the form of a graph. Figure 5, page 12, gives the relationship between the equivalent static pressure and the lowest normal mode of vibration of the structure. Note that the dash lines represent the equivalent static pressure and the solid lines indicate the equivalent suction pressure. This graph bears out the fact that structures of higher frequency are stressed greater during the suction phase of the blast than during the positive phase. The data for this curve were calculated from the pressure produced by a 500 pound bomb at a distance

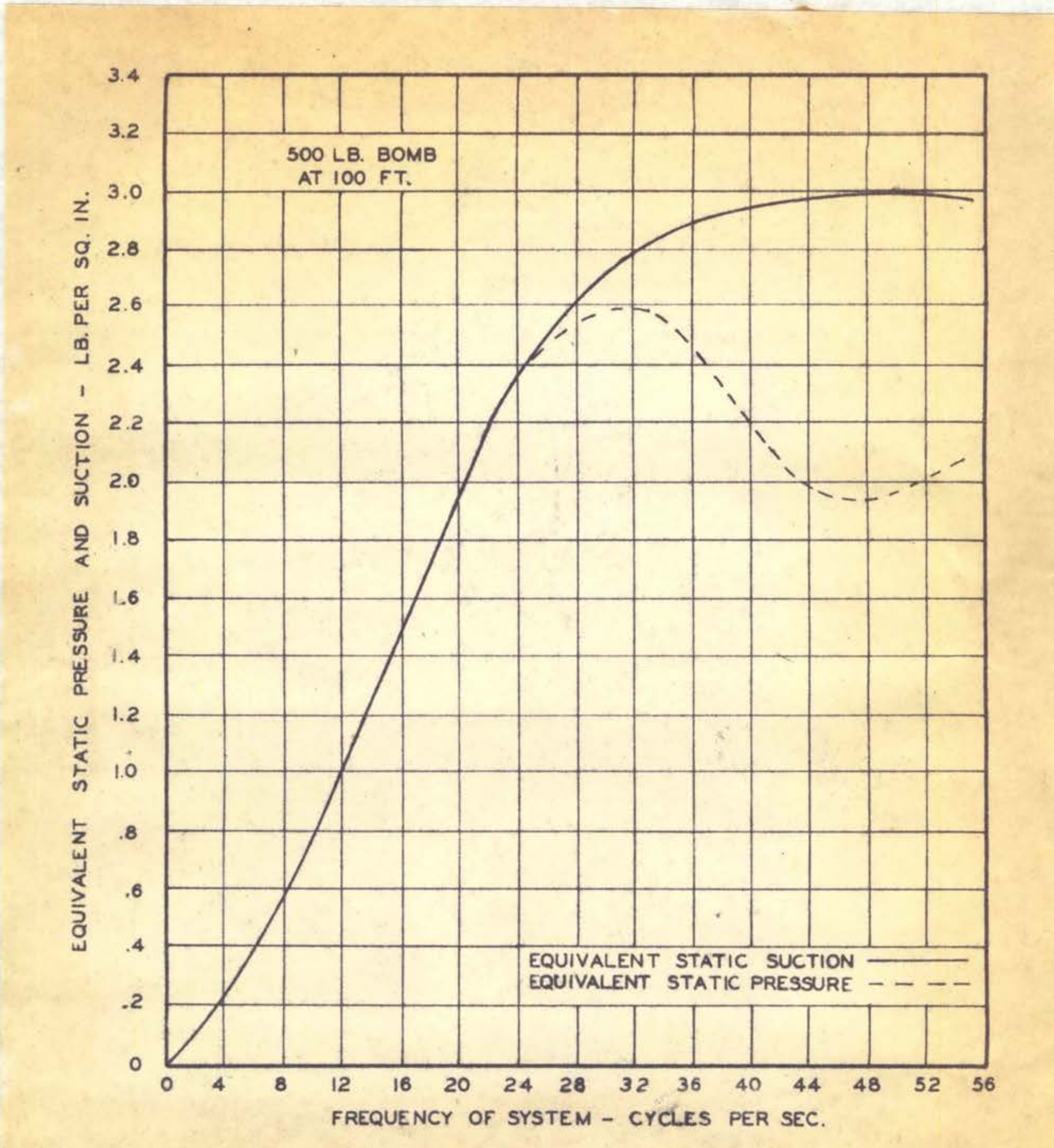

FIGURE 5. EQUIVALENT STATIC PRESSURE-FREQUENCY CURVE[1]

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[1] Anonymous, Air Raid Precautions, Handbook 5, Structural Defense, p. 11.

of 100 feet. The equivalent static pressure at any distance can be calculated by multiplying the equivalent static pressure at 100 feet by the ratio of the pressure at 100 feet and at the required distance obtained from Figure 3, page 8. Also, it is assumed that the equivalent static pressure varies directly with the weight of the bomb; that is, a 250 pound bomb will only produce one-half the equivalent static pressure of a 500 pound bomb at the same distance.

It would seem from the values in Figure 5, page 12, that all structures having a frequency greater than 30 cycles per second would fail during the suction phase of the blast. However, Figure 3 shows that both the positive pressure and the suction pressure vary with the distance from the explosion. The positive pressure increases more than the suction pressure as the distance from the blast is decreased. Thus at 100 feet from the explosion some structures will fail during the suction phase of the blast. As the distance decreases the positive pressure increases, and the suction pressure decreases. Nearer the explosion, the structure is subjected to a larger positive pressure than suction pressure. This is indicated by tests conducted on panes of glass set at different distances from the point of explosion of a bomb. The broken pieces of glass near the explosion fell away from the bomb site, those farthest away fell toward the bomb site, while in between the glass fell in both directions.[2]

Calculation of Frequency

As stated before the equivalent static pressure is dependent upon the frequency of the system. In Table II will be found the equations for calculating the frequency for a number of structures that are usually built to resist blast.

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[2] Ibid., p. 11.

TABLE II

VIBRATION FREQUENCIES FOR STRUCTURAL ELEMENTS[3,4]

The following symbols are used in this table.

f - frequency of the system-cycles per second.
L - length of beam-inches.
A - cross sectional area-square inches.
γ - weight per unit volume of the material.
g - force of gravity - 32.2 feet per second per second.
E - modulus of elasticity - pounds per square inch.
I - moment of inertia.
a - length of panel-longer dimension-inches.
b - width of panel-inches.
t - thickness of panel-inches.
m - Poisson's ratio.

Cantilever Beam: $$f = \frac{3.515}{2\pi}\sqrt{\frac{E\,I\,g}{A\,\gamma\,L^4}} = 11\sqrt{\frac{E\,I}{A\,\gamma\,L^4}}$$
Simple Supported Beam: $$f = \frac{\pi}{2}\sqrt{\frac{E\,I\,g}{A\,\gamma\,L^4}} = 30.88\sqrt{\frac{E\,I}{A\,\gamma\,L^4}}$$
Beam—One End Fixed—Simple Supported at Other: $$f = \frac{3.927^2}{2\pi}\sqrt{\frac{E\,I\,g}{A\,\gamma\,L^4}} = 48.25\sqrt{\frac{E\,I}{A\,\gamma\,L^4}}$$
Beam—Both Ends Fixed: $$f = \frac{4.73^2}{2\pi}\sqrt{\frac{E\,I\,g}{A\,\gamma\,L^4}} = 70\sqrt{\frac{E\,I}{A\,\gamma\,L^4}}$$

Continued on next page

Table II - Continued

Square Plate--All Edges Simple Supported: $$f = \frac{\pi}{a^2}\sqrt{\frac{g\,D}{\gamma\,t}} = \frac{61.76}{a^2}\sqrt{\frac{D}{\gamma\,t}}$$
Rectangular Plate--All Edges Simply Supported: $$f = \frac{\pi}{2}\sqrt{\frac{g\,D}{\gamma\,t}}\left[\frac{1}{a^2}+\frac{1}{b^2}\right] = 30.88\sqrt{\frac{D}{\gamma\,t}}\left[\frac{1}{a^2}+\frac{1}{b^2}\right]$$ Where: $$D = \frac{E\,h^3}{12\,(1-m^2)}$$
Square Plate--All Edges Fixed: $$f = \frac{14.1}{a^2}\sqrt{\frac{g\,D}{\gamma\,t}} = \frac{277.6}{a^2}\sqrt{\frac{D}{\gamma\,t}}$$
Rectangular Plate--All Edges Fixed: No formula yet has been derived for this case. A rectangular plate with fixed edges will have a higher frequency than the same plate simply supported. A square plate with fixed edges has a frequency fourteen times greater than a square plate simply supported. Generally the frequency of a rectangular plate with fixed edges is so great that it is assumed to be greater than the frequency at which the maximum equivalent load occurs.

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[3] Stephen P. Temoshenko, Vibration Problems in Engineering, pp. 297-320.

[4] D. Laughame Thornton, Mechanics Applied to Vibrations, pp. 297-365.

Illustrative Examples of Calculating Equivalent Static Pressure

Example I.

A 10 inch 35 pound American Standard steel beam 10 feet long is used in building a cantilever wall to resist blast.

1. Problem

   (a) Determine the equivalent static positive pressure and suction pressure on the steel beam produced by the blast from a 500 pound bomb 100 feet away.

   (b) Determine the equivalent static pressures produced on this beam by the explosion of a 1000 pound bomb at 100 feet distance.

   (c) What will be the equivalent static pressure and suction pressure on this beam if a 500 pound bomb explodes 40 feet away?

2. Solution

   (a) From Table II, pages 14 and 15, the formula for calculating the frequency of a cantilever beam is:

$$f = 11\sqrt{\frac{E\ I}{A\ \gamma\ L^2}}$$

From the steel handbook[5]

$I = 145.8$ inches^4 about the x-x axis.

$A = 10.22$ square inches.

γ steel $= \frac{490}{12^3} = .2835$ pounds per cubic inch.

$E = 30{,}000{,}000$ pounds per square inch.

$L = 10$ feet $= 120$ inches.

$$f = 11\sqrt{\frac{30 \times 10^6 \times 145.8}{10.22 \times .2835 \times (120)^4}} = 29.6 \text{ cycles per second.}$$

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[5] Anonymous, Steel Construction, p. 28.

From Figure 5, page 12, the equivalent static positive pressure $P\ (+)$ and the equivalent suction pressure $P\ (-)$ produced by a 500 pound bomb at 100 feet are:

$$P\ (-) = 2.7 \text{ pounds per square inch.}$$

$$P\ (+) = 2.56 \text{ pounds per square inch.}$$

(b) The equivalent static pressures produced by a 1000 pound bomb at 100 feet are just twice the equivalent static pressures produced by a 500 pound bomb at 100 feet. Thus:

$$P\ (-) = 2 \times 2.7 = 5.4 \text{ pounds per square inch.}$$

$$P\ (+) = 2 \times 2.56 = 5.1 \text{ pounds per square inch.}$$

(c) The following pressures produced by the blast of a 500 pound bomb are taken from Figure 3, page 8.

At 40 feet $\quad P\ (+) = 9$ pounds per square inch.

$\qquad\qquad\quad P\ (-) = 2$ pounds per square inch.

At 100 feet $\quad P\ (+) = 2.2$ pounds per square inch.

$\qquad\qquad\quad P\ (-) = 0.8$ pounds per square inch.

Thus the equivalent suction pressure produced on this cantilever beam by the explosion of a 500 pound bomb 40 feet away is:

$$P\ (-) = 2/0.8 \times 2.7 = 6.75 \text{ pounds per square inch.}$$

The equivalent positive pressure is:

$$P\ (+) = 9/2.2 \times 2.56 = 10.5 \text{ pounds per square inch.}$$

Note that at 100 feet the bomb will produce a slightly larger equivalent suction load. However, at 40 feet the equivalent positive pressure is much larger than the suction load.

Example II.

A one inch thick steel plate 5 feet by 7 feet is subjected to the blast of a 500 pound bomb 100 feet away.

1. Problem

   (a) Calculate the equivalent positive pressure and suction pressure if the panel has all edges simply supported.

   (b) What would be the equivalent static pressure, if the panel had all edges fixed?

2. Solution

   (a) From Table II, pages 14 and 15, the formula for the frequency of a rectangular plate simply supported is:

$$f = 30.88\sqrt{\frac{D}{\gamma t}}\left[\frac{1}{a^2} + \frac{1}{b^2}\right] \quad \text{where: } D = \frac{E\,t^3}{12(1 - m^2)}$$

Let: $m = \frac{1}{4}$ $\qquad$ $h = 1$ inch

$E = 30 \times 10^6$ pounds per square inch. $\quad a = 5$ feet $= 60$ inches

$\gamma = .2835$ pounds per cubic inch. $\quad b = 7$ feet $= 84$ inches

$$D = \frac{30 \times 10^6 \times 1^3}{12(1 - .25^2)} = 2.667 \times 10^6$$

$$f = 30.88\sqrt{\frac{2.667 \times 10^6}{.2835 \times 1}}\left[\frac{1}{60^2} + \frac{1}{84^2}\right] = 39.8 \text{ cycles per second.}$$

The equivalent positive and suction pressures on the panel when subjected to the blast of a 500 pound bomb 100 feet away are found in Figure 5, page 12.

$P\,(-) = 2.95$ pounds per square inch.

$P\,(+) = 2.2$ pounds per square inch.

(b) If this panel had all edges fixed its frequency would be so much larger than 50 cycles per second, (the frequency at which the maximum positive pressure occurs) that it would be unnecessary to attempt to calculate it. An approximate value of the frequency could be obtained by assuming the panel square, 6 feet on a side. This would give a frequency of around 230 cycles per second. In general most panels with fixed edges should be designed for the maximum positive and suction pressures which are:

$P\ (-) = 3$ pounds per square inch.

$P\ (+) = 2.6$ pounds per square inch.

CHAPTER V

DESIGN OF STEEL PANELS WITH SIMPLE SUPPORTED EDGES

Three main methods of analysis have been used in determining the stresses in steel and iron panels. (1) The panel is analyzed by considering it as divided into two strips which cross each other at right angles and act as beams. The mathematical derivation of the moment equation by this method is the simplest of all of the methods and is the easiest understood. However, as it disregards the fact that adjacent strips will carry part of the load, the moments given by these equations error considerably on the side of safety. (2) Actual experiments are conducted on the panels and the dangerous section and critical stresses are determined. Then emperical moment equations are formulated by multiplying the average bending moment at a section, found by mathematical analysis, by a factor that will give the same values of the stresses in the panel as found by the experiments. Since these equations are simple and based on the results of actual tests of the plate, this method of analysis is the most widely used. The moment equations used in this paper are derived by this method. (3) More recently another mathematical analysis based on the theory of elasticity has been used. This is the longest and most complicated analysis. A number of incorrect assumptions are made in some cases by this analysis, and as it is difficult to apply to rectangular panels, this method is rarely used.

Most moment equations for simple supported panels uniformly loaded have been derived by considering the critical section to be at the middle of the panel parallel to the long side. However, actual tests on panels have indicated that the critical section is approximately along a diagonal.

It will be assumed then that the maximum stress occurs along the diagonal M O in Figure 6, page 21, and that the critical section is M N O. This assumption is nearly correct provided that the ratio of the long side of the panel to the short does not exceed two and one half or three.

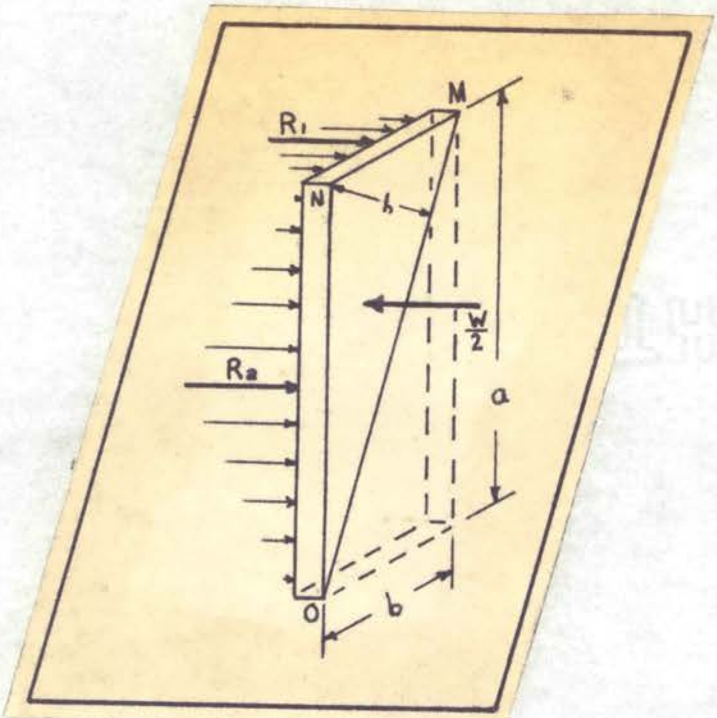

FIGURE 6

In Figure 6, let a be the length of the long side of the panel, b the length of the short side and t the thickness of the panel. If w is the load per unit area on the panel, then the total load W is w a b. Only one half of the total load W acts on the critical section under consideration and its resultant W/2 is applied at the center of gravity of the triangular area, which is h/3 from the diagonal M O. The exact distribution and the amount of the resultant of the reactions on each side of the panel are not known and need not be calculated. Both resultants R_1 and R_2 are located at h/2 from the diagonal M O and their sum is equal to one half of the total uniform load on the panel. The total bending moment on the section M O, then is:

$$M = \tfrac{1}{2}\, w\, a\, b\, (h/2) - \tfrac{1}{2}\, w\, a\, b\, (h/3) = 1/12\, w\, a\, b\, h$$

As $h = ab/\sqrt{b^2 + a^2}$ and $AC = \sqrt{b^2 + a^2}$ the average bending moment per inch of width of the diagonal is:

$$M = \frac{1}{12}\ \frac{w\,a\,b\,h}{A\,C} = \frac{1}{12}\ \frac{w\,a\,b}{\sqrt{a^2 + b^2}} \times \frac{a\,b}{\sqrt{a^2 + b^2}} =$$

$$\frac{1}{12}\ \frac{w\,a^2\,b^2}{a^2 + b^2} \quad \text{or} \quad M = k\,w\,b^2$$

where the moment coefficient $k = \dfrac{1}{12}\ \dfrac{a^2}{a^2 + b^2}$

At the corners of the panel, perpendicular to the diagonals, are points of high stresses. As the load on the panel increases, all stresses in the panel increase, with the stress at the corners being the first to reach the yield point. If the material is ductile, as the load continues to increase the stress will not increase at the points of high stress, but at these points the material will yield slightly without the rest of the panel yielding. Thus, there occurs a redistribution of stress which gives the plate an additional elastic strength. This fact was not taken into account in the above mathematical derivation. The values for the stress obtained by this equation will always be greater than the actual stress determined by experiment.

This difference can be seen in Figure 7, page 23, where the dotted line is the value of $\dfrac{1}{12}\ \dfrac{a^2}{a^2 + b^2}$ and the solid curves represent the results obtained by Westergaard[1] after modifying the values he obtained, by a thorough mathematical analysis, so they would more nearly represent the values obtained by experiment. The values of M diagonal as obtained from Westergaard's equation $M = \dfrac{1}{8}\ \dfrac{w\,b^2}{2 + \alpha^2}$ differ from the moment given by the unmodified mathematical equation $M = \dfrac{1}{12}\ \dfrac{w\,a^2\,b^2}{a^2 + b^2}$ from 7.8 per cent when $\alpha = 0$, a long narrow panel, to 10.1 per cent for a square panel ($\alpha = 1$).

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[1] H. M. Westergaard and W. A. Slater, "Moments and Stresses in Slabs," <u>Procedures of the American Concrete Institute</u>, XVII (February, 1921), 431.

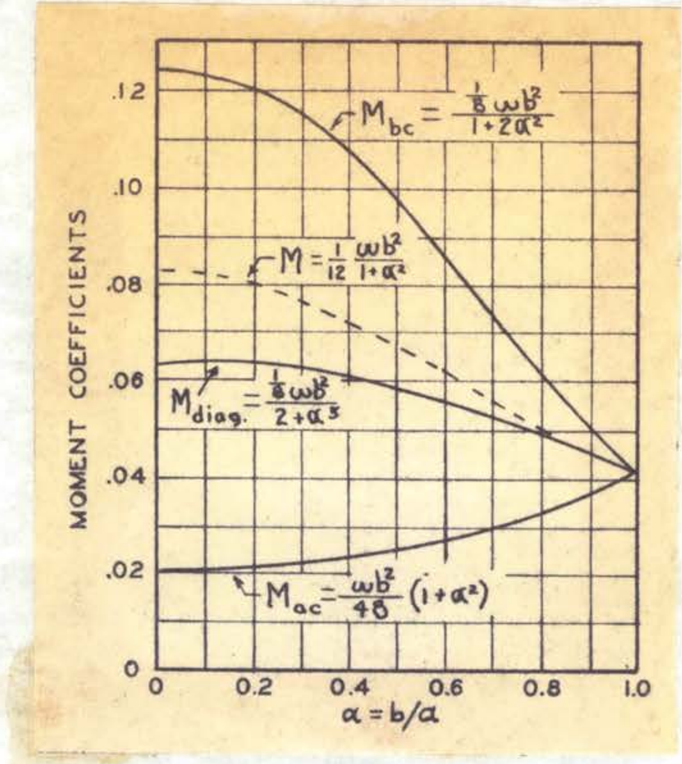

FIGURE 7

BENDING MOMENTS PER UNIT WIDTH IN RECTANGULAR PANELS WITH SIMPLE SUPPORTED EDGES[2]

Where a - length of the long side of the plate.
b - length of the short side of the plate.
w - uniformly distributed load per unit area.
α - b/a.
k - moment coefficient = M/wb^2 or $M = kwb^2$.
M_{bc} - moment per unit width at the center of the panel parallel to the shorter edge.
M_{ac} - moment per unit width at the center of the panel parallel to the longer edge.
M_{diag} - moment per unit width at a corner across a diagonal.

Actually the above solid curves and their equations are not those that are the results obtained after modification, but curves with nearly the same values that have simple analytical equations that are always on the side of safety.

Westergaard, Slater,[3] and others in their experiments found that panels designed by using the maximum moment at the center of the short span, actually

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[2] Ibid., p. 431.

[3] Ibid., p. 488.

were much stronger than their mathematical analysis indicated. Thus, the equation for the moment across the diagonal (M_{diag}) is recommended and generally used in computing the strength of panels made of ductile materials.

Once the maximum moment in inch pounds over a strip one inch wide is computed, the required thickness t of the panel to resist the bending moment can be found as follows:

$$f = \frac{M\,c}{I} = \frac{M \times t/2}{1/12 \times 1 \times t^3} = \frac{6\,M}{t^2} \quad \text{or} \quad t = \sqrt{\frac{6\,M}{f}}$$, in which f is the allowable unit stress in bending. The A.I.S.C. Code[4] recommends an allowable unit stress of 20,000 pounds per square inch for steel. This allowable stress is materially less than the elastic limit of the material. So in the interest of economy, when designing plates to resist blast, the safety factor usually used in ordinary design is reduced. An allowable unit stress of 30,000 pounds per square inch is an increase of fifty per cent over the A.I.S.C. specification and is still below the elastic limit of steel. This is the unit stress that will be used in the following designs.

Illustrative Example

Example I.

Design a 10 feet by 14 feet simple supported steel panel to resist the blast from a 500 pound bomb 50 feet away.

Since the thickness of the panel is not known, the frequency of the panel can not be calculated and the equivalent static load found. However, assume that the frequency will be around 30 cycles per second. The bomb, if exploded at 100 feet, will produce an equivalent suction pressure of about 2.7 pounds per square inch and an equivalent positive pressure of 2.58 pounds per square

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[4] Anonymous, Steel Construction, p. 286.

inch. If the bomb explodes at 50 feet, the equivalent pressures are:

$$P\ (-) = \frac{2.7 \times 1.1}{0.8} = 3.71 \text{ pounds per square inch.}$$

$$P\ (+) = \frac{2.58 \times 6}{2.2} = 7.03 \text{ pounds per square inch.}$$

Thus $w = 7.03 \times 12 = 84.4$ pounds per foot over a one inch width.

As $\alpha = b/a = 10/14 = .714$ the maximum moment on the plate is:

(From Figure 7, page 23)

$$M = \frac{1/8\ wb^2}{2 + \alpha^3} = \frac{1/8 \times 84.4 \times 10^2 \times 12}{2 + .714^3} = 5{,}350 \text{ inch pounds.}$$

As a check the moment coefficient $k = .0525$. So

$M = .0525 \times 84.4 \times 10^2 \times 12 = 5{,}325$ inch pounds, which is fairly close.

Thus:

Required $t = \sqrt{\dfrac{6\ M}{f}} = \sqrt{\dfrac{6 \times 5{,}350}{30{,}000}} = 1.035$ inches or 1 1/16 inches.

The frequency of this panel then is:

$$D = \frac{E\ t^3}{12(1 - m^2)} = \frac{30 \times 10^6 \times 1.0625^3}{12(1 - .25^2)} = 3.202 \times 10^6$$

$$f = 30.88 \sqrt{\frac{3.202 \times 10^6}{.2835 \times 1.0625}} \left[\frac{1}{120^2} + \frac{1}{168^2}\right]$$

$$= 30.88 \times 3260 \times .00010481 = 10.22 \text{ cycles per second.}$$

This frequency is less than the assumed frequency of 30 cycles per second, so the section will resist the bending moment. However, it must be remembered that the recommended minimum thickness for protection against fragmentation is $1\frac{1}{2}$ inches, so the thickness will be changed to $1\frac{1}{2}$ inches. As the thickness of the panel is increased, the frequency also will increase. The frequency of the panel, with the new thickness, needs to be checked to see that it is not greater than the assumed 30 cycles per second. Thus:

$$D = \frac{E\ t^3}{12(1 - m^2)} = \frac{30 \times 10^6 \times 1.5^3}{12(1 - .25^2)} = 9 \times 10^6$$

$$f = 30.88 \sqrt{\frac{9 \times 10^6}{.2835 \times 1.5}} \left[\frac{1}{120^2} + \frac{1}{168^2}\right]$$

$$= 30.88 \times 4598 \times .0001048 = 14.88 \text{ cycles per second.}$$

This still is less than the assumed 30 cycles per second, so the panel will withstand the bending moment.

The panel must also be checked for shear. The proportionate amount of the load that is carried in each direction is not known, as equations for computing the shear have never been developed. Thus an accurate value of the shear can not be found. However, if it be assumed that none of the 88.3 pounds per foot of load is carried by adjacent panels, a value can be obtained that is larger than the actual shear. Thus:

$$V = \tfrac{1}{2}\ 84.4 \times 10 = 422 \text{ pounds.}$$

Using this value of the shear, the shearing unit stress is:

$$v = \frac{V\ Q}{I\ t} = \frac{422 \times 1 \times 3/4 \times 3/8}{1/12 \times 1 \times 1.5^3 \times 1} = 422 \text{ pounds per square inch.}$$

This is less than the allowable 13,000 pounds per square inch. In most cases the shearing unit stress is very low and does not need to be checked. This panel then will withstand the explosion of a 500 pound bomb at 50 feet. In fact as its frequency is slightly less than 15 cycles per second, this panel will withstand the explosion of a 500 pound bomb at a closer distance than 50 feet.

<u>Example II</u>.

Design a 10 foot by 18 foot simply supported steel panel to resist the blast from a 500 pound bomb 30 feet away.

Again, assume that the frequency of the panel is about 30 cycles per second. At 100 feet the bomb will cause an equivalent positive pressure of about 2.58 pounds per square inch. If the bomb explodes at 30 feet the equivalent pressure then will be:

$$P = \frac{2.58 \times 20}{2.2} = 23.45 \text{ pounds per square inch.}$$

or $w = 23.45 \times 12 = 281.3$ pounds per foot over a one inch width.

$$\alpha = \frac{b}{a} = \frac{10}{18} = .556 \qquad \text{and so} \quad k = .0575$$

Thus:

$$M = 0.575 \times 281.3 \times 10^2 \times 12 = 19{,}420 \text{ inch pounds.}$$

Required $t = \sqrt{\dfrac{6M}{f}} = \sqrt{\dfrac{6 \times 19{,}420}{30{,}000}} = \sqrt{3.88} = 1.97$ inches.

Use 2 inches.

The frequency of this panel is then:

$$D = \frac{E\,t^3}{12(1 - m^2)} = \frac{30 \times 10^6 \times 2^3}{12(1 - .25^2)} = 21.35 \times 10^6$$

$$f = 30.88 \sqrt{\frac{21.35 \times 10^6}{.2835 \times 2}} \left[\frac{1}{120^2} + \frac{1}{216^2}\right]$$

$$= 30.88 \times 61.35 \times 9.08 \times 10^{-5} = 17.2 \text{ cycles per second.}$$

As this is slightly more than half the assumed frequency, the two inch thickness can be reduced. It should not be reduced to less than 1 1/2 inches, though, as this is the minimum thickness needed as protection against fragmentation. Assuming, then, a 1 1/2 inch thick panel, its frequency is:

$$f = 30.88 \sqrt{\frac{9 \times 10^6}{.2835 \times 1.5}} \left[\frac{1}{120^2} + \frac{1}{216^2}\right]$$

$$= 30.88 \times 4598 \times 9.08 \times 10^{-5} = 12.9 \text{ cycles per second.}$$

The corresponding equivalent static load, if the bomb explodes at 100 feet, is 1.08 pounds per square inch; and at 30 feet the load is:

$$P = \frac{1.08 \times 20}{2.2} = 9.8 \text{ pounds per square inch.}$$

and $$w = 9.8 \times 12 = 117.5 \text{ pounds per foot.}$$

The maximum moment is:

$$M = .0575 \times 117.5 \times 10^2 \times 12 = 8{,}110 \text{ inch pounds.}$$

Thus the required thickness is:

$$t = \sqrt{\frac{6 \times 8110}{30{,}000}} = \sqrt{1.62} = 1.27 \text{ inches.}$$

This is less than the assumed thickness. As there is no need to check the shearing unit stress, a panel 10 feet by 18 feet by $1\frac{1}{2}$ inches will withstand the pressure from the blast of a 500 pound bomb at a distance of 30 feet.

CHAPTER VI

DESIGN OF STEEL PANELS WITH FIXED EDGES

The majority of steel panels constructed to resist blast are riveted or welded to their supporting beams and columns. The supporting edges of these panels are held rigidly from bending, and for all practical purposes, may be considered as fixed. As in the previous chapter, the stresses in this type of panel will be calculated from formulas derived from both theoretical and experimental analysis. The results of Westergaard's[1] investigation of rectangular panels with fixed edges subjected to a uniformly distributed load are shown in Figure 8.

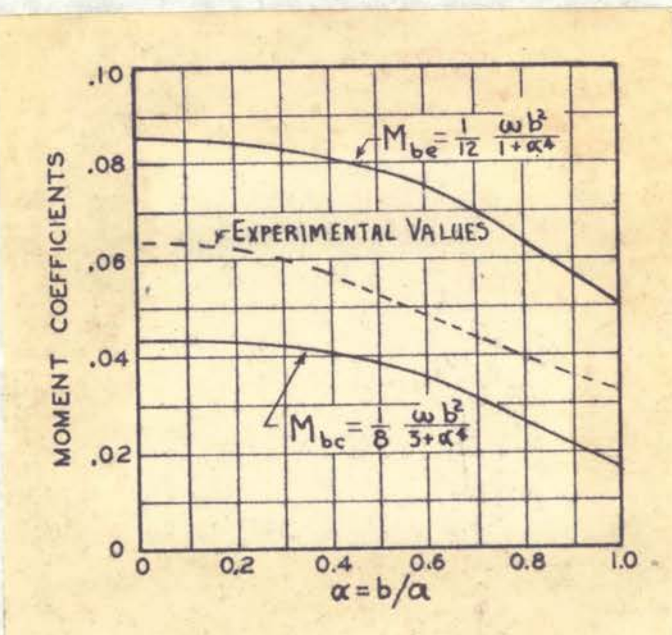

FIGURE 8. BENDING MOMENT PER UNIT WIDTH IN RECTANGULAR PANELS WITH FIXED EDGES

Where a - length of the long side of the panel.
b - length of the short side of the panel.
w - uniformly distributed load per unit area.
α - b/a.
k - M/wb^2.
M_{be} - the negative moment per unit width at the center of the long edge acting perpendicular to the long edge.
M_{bc} - the positive moment per unit width at the center of the panel parallel to the shorter edge.

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[1] H. M. Westergaard and W. A. Slater, "Moments and Stresses in Slabs," Procedures of American Concrete Institute, XXIV (February, 1928), 435-436.

These equations, like those for the rectangular panel simply supported, do not give the same values as those determined from the complicated mathematical analysis. They are simpler algebraic equations that give values that are either identical or greater than the exact values.

The largest moment produced in the panel is the negative moment at the center of the long edge acting parallel to the shorter edge (M_{be}). This negative moment is always approximately twice the value of the maximum positive moment which is located at the center of the panel acting in the same direction as the maximum negative moment. As the panel becomes long and narrow ($b/a = 0$), the negative moment in a panel with fixed edges is, as expected, less than the positive moment in the same panel simply supported. However, as the shape of the rectangle approaches a square ($b/a = 1$) this difference decreases until they are equal when $b/a = .75$. From this shape to b/a equals 1 (a square panel) the negative moment in a panel with fixed edges is greater than the positive moment in the same panel simply supported. Thus as the shape of a rectangular panel nears a square ($b/a = 1$), the simply supported panel will withstand a greater load than the same panel with fixed edges.

As ductile materials yield slightly at points of maximum stress and redistribute these high stresses, a panel made of ductile material will withstand a greater load than that given by the formulas. From the results of his experiments Bach as reported by Seely[2] suggests that the values of the moment coefficients given by the dotted line in Figure 8, page 29, be used in the design of plates made of ductile material. If one does not wish to use a table to determine the allowable moment, the average of M_{be} and M_{bc} as calculated from the formulas given in Figure 8 can be used.

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[2] Fred B. Seely, Advance Mechanics of Materials, p. 143.

The values of the moment at the center of the short edge and at the center of the panel acting parallel to the long edge are not given in Figure 8. They are not critical and so were eliminated from the diagram.

Illustrative Example

I. Design an 11 foot square steel panel with fixed edges to resist the blast of a 500 pound bomb at a distance of 50 feet.

Again the frequency of this panel can not be calculated, since the thickness of the panel is not known. However, since the plate has fixed edges, the frequency probably is greater than the 50 cycles per second at which the maximum equivalent suction load occurs. Therefore, the maximum equivalent pressure will be used in this design.

The maximum equivalent suction pressure produced by a 500 pound bomb at 100 feet is 3 pounds per square inch and the maximum equivalent positive pressure is 2.6 pounds per square inch. The pressures, then, caused by a 500 pound bomb at half that distance are:

$$P(-) = \frac{3 \times 1.1}{0.8} = 4.13 \text{ pounds per square inch.}$$

$$P(+) = \frac{2.6 \times 6}{2.2} = 7.09 \text{ pounds per square inch.}$$

Thus $w = 7.09 \times 12 = 85.1$ pounds per foot over a one inch width.

As $\alpha = \frac{11}{11} = 1$, the moment coefficient (from Figure 8, page 29) is .0315.

So the maximum moment is:

$$M = .0315 \times 85.1 \times 11^2 \times 12 = 3{,}890 \text{ pounds.}$$

Thus:

$$\text{Required } t = \sqrt{\frac{6 \times 3890}{30{,}000}} = \sqrt{.78} = .884 \text{ inch.}$$

This thickness is less than the required thickness to resist fragmentation, so it will be increased to $1\frac{1}{2}$ inches.

The frequency for this panel then is:

$$D = \frac{E\ t^3}{12(1 - m^2)} = \frac{30 \times 10^6 \times 1.5^3}{12(1 - .25^2)} = 9 \times 10^6$$

$$f = \frac{277.6}{a^2} \sqrt{\frac{D}{\gamma\ h}} = \frac{277.6}{121^2} \sqrt{\frac{9 \times 10^6}{.2835 \times 1.5}}$$

$$= \frac{277.6}{121^2} \times 4600 = 87.2 \text{ cycles per second.}$$

This is greater than the 50 cycles per second at which the maximum equivalent pressure occurs, so the panel will withstand the bending moment. Since the shear does not need to be checked, an 11 foot square panel $1\frac{1}{2}$ inches thick will withstand the blast of a 500 pound bomb at 50 feet.

No formula has been derived for calculating the frequency of a rectangular panel with fixed edges. However, this is not a problem, since the frequency of most panels will be larger than the frequency at which the maximum equivalent pressure occurs. Always use the maximum equivalent pressure in designing rectangular panels with fixed edges. If the frequency of the panel is less than the frequency at which the maximum equivalent pressure occurs, the panel is still safe, because a larger equivalent load than necessary was used in calculating the thickness.

CHAPTER VII

DESIGN OF CONCRETE PANELS WITH SIMPLY SUPPORTED EDGES

In some cases panels built to resist blast are constructed so that if a blast occurs inside the building, the panel will be blown outward from the building. This prevents the blast from being confined and allows it to escape outside, thus, decreasing the interior damage. This type of panel is known as a "blow-out" panel and is built so that all four edges are simply supported, that is, they are not constructed monolithically with their supporting beams and columns. These panels usually are reinforced in two directions (two way panels), so that the blast load will be carried by both the beams and columns. The calculation of the amount of the total load that is distributed in each direction is a statically indeterminate problem that is complicated by many factors. However, by disregarding certain factors, the problem can be simplified to a statically determinate problem.

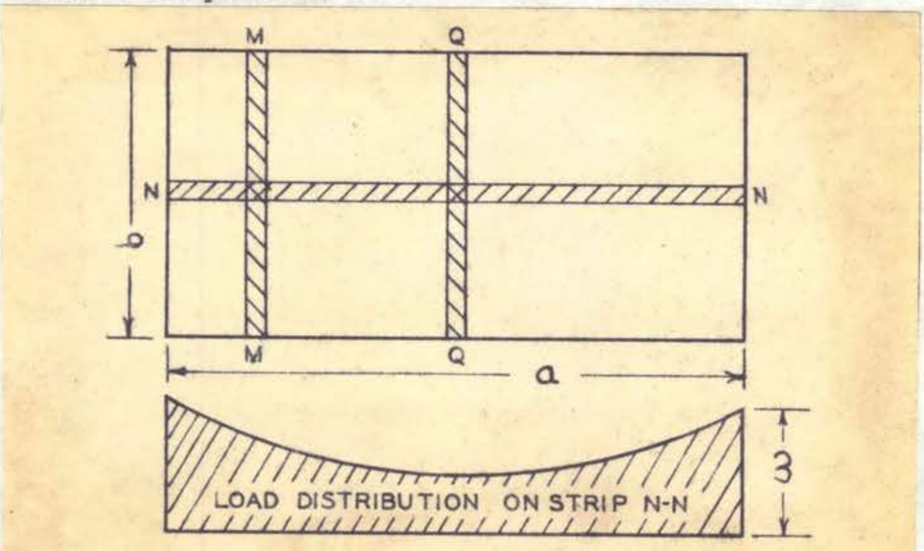

FIGURE 9

Let the rectangle in Figure 9 represent a panel of length **a** and **b** width with NN and QQ being imaginary strips one foot wide crossing at the center of the panel. The deflection of both strips at the center of their intersection is always the same. If each strip is uniformly loaded and simply supported the deflection at the center is $(5/384)\ w\,L^4/E\,I$. Letting w_A be that portion of the load carried along strip NN (the long span) and w_B the part of the load transmitted in the short direction:

$$\Delta_A = \frac{5}{384}\ \frac{w_A\ a^4}{E\ I} \qquad\qquad \Delta_B = \frac{5}{384}\ \frac{w_B\ b^4}{E\ I}$$

Since the deflections at the center are equal:

$$w_A\ a^4 = w_B\ b^4 \qquad \text{or} \qquad w_A/w_B = b^4/a^4$$

The proportion of the total load carried by each strip is thus inversely proportional to the fourth power of the length of that strip. The moments calculated by using this method of analysis will always be greater than the actual moment as this method disregards the fact that adjacent strips will carry some of the load. At points near the edges of the short side of the panel the load on the long strip will be greater than that computed by this method and near the long edges it will be less.

A similar method of analysis can be found in Kirkham's "Reinforced Concrete."[1] In addition to making the same basic assumptions as above, this method of analysis allows for the variation of load across the span. The values of the moments calculated by this method of analysis are less than that given by the above analysis, but are still greater than the actual moments.

The 1946 American Concrete Institute Building Code[2] presents a method of calculating the moment in two way panels that was developed by DiStasio and Van Buren.[3] The values of the moment calculated by this method vary, depending

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[1] John Edward Kirkham, Reinforced Concrete, pp. 81-98.

[2] Anonymous, Building Regulations for Reinforced Concrete, p. 584.

[3] J. DiStasio and M. P. VanBuren, "Slabs Supported on Four Sides," Procedures of American Concrete Institute, XXXVII (September, 1935-June, 1936), 350-364.

upon the shape of the panel, from 5 per cent to 7.5 per cent less than that given by the methods described in the above paragraphs. This seems logical because DiStasio and Van Buren[4] have taken into account the basic facts that were disregarded in the above analysis. The following formulas give results close to those obtained by their experiments conducted on two way panels:

$$r_A = \frac{1}{1 + (a/b)^3} \quad \text{and} \quad r_B = \frac{1}{1 + (b/a)^3} = 1 - r_A$$

r_A is the proportion of the total load carried in the a direction of the panel and r_B is the proportion of the total load carried at right angles in the b direction.

Once the load carried in each direction (r_A w and r_B w) is calculated, the bending moment and shear in the panel can be computed as in any one way slab. Thus the bending moment at the center of the short span is $M_{bc} = 1/8\ r_B\ w\ b^2$ and at the center of the long span $M_{AC} = 1/8\ r_A\ w\ a^2$. The maximum shear along the short side of the panel is $\frac{1}{2}\ r_A\ w\ a$ and along the side the shear is $\frac{1}{2}\ r_B\ w\ b$.

After the bending moment and shear have been calculated the panel can be designed as any one way floor slab. It must be kept in mind though, that a panel to resist blast is subjected to reversals of stress, and that two way reinforcing must be put on both sides of the panel. It makes little difference as to the spacing of the reinforcing steel. It can be placed at uniform spacing completely across the panel, or it can be placed closer at the center of the span and farther apart near the edges. Also, it has been found that reinforcing steel placed parallel to the sides is more effective than reinforcing placed

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[4] Ibid, p. 350-364.

parallel to the daigonals.[5] The following is an example of the design of a two way panel simply supported.

Illustrative Example

I. Design a 10 foot by $12\frac{1}{2}$ foot blowout panel to resist the blast of a 500 pound bomb at 50 feet.

Unit Stresses:	$f'_c = 3{,}000$ psi.	$f_s = 20{,}000$ psi.
	$f_c = 1{,}350$ psi.	$u = 150$ psi.
	$v = 90$ psi.	

The frequency of the panel is not known, so the actual equivalent static load can not be found. Assume then that the frequency is 50 cycles per second, which has an equivalent positive static load of 2.6 pounds per square inch. The equivalent static load, when the bomb is exploded at 50 feet is:

$$P = \frac{2.6 \times 6}{2.2} = 7.09 \text{ pounds per square inch.}$$

(a) Design of the panel in short direction

The proportionate amount of the total load carried by the short span is:

$$r_B = \frac{1}{1 + (b/a)^3} = \frac{1}{1 + (10/12.5)^3} = .662$$

The maximum moment is:

$$M = 1/8\ r_B\ w\ b^2 = 1/8 \times .662 \times 1022 \times 10^2 \times 12$$
$$= 101{,}500 \text{ inch pounds.}$$

Assuming the panel to have tension steel in only one side, the required depth of the wall would be: (assume $k = .4$ and $j = .87$)

$$\text{Required } d = \sqrt{\frac{M}{f_c/2\ j\ k\ b}} = \sqrt{\frac{101{,}500}{1350/2 \times .87 \times .4 \times 12}}$$
$$= \sqrt{36.0} = 6 \text{ inches}$$

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[5] H. M. Westergaard and W. A. Slater, "Moments and Stresses in Slabs," Procedures of American Concrete Institute, XXII (February, 1926), 431.

and the required area of tension steel is:

$$A_s = \frac{M}{j\ d\ f_s} = \frac{101{,}500}{.87 \times 6 \times 20{,}000} = .972 \text{ square inches.}$$

Checking the required thickness for shear:

$$V = \frac{r_B\ w\ b}{2} = \frac{.703 \times 1022 \times 10}{2} = 3{,}590 \text{ pounds.}$$

$$\text{Required } d = \frac{V}{v\ j\ b} = \frac{3{,}590}{90 \times .87 \times 12} = 3.82 \text{ inches.}$$

The bending moment controls the design. The minimum required thickness of the panel probably will be from 4 to 6 inches less than the thickness required for protection against bomb fragmentation. Either the thickness of the panel should be increased to 12 inches or a 7 inch thick panel should be faced with brick to make the necessary 12 inches. Both would afford the necessary protection against flying fragmentation. The thickness to be used, though, depends upon labor and material costs and the architecture of the building. In this example the 7 inch thick wall will be designed.

Two way reinforcing must be placed on both sides of the wall as shown in Figure 10. The effective depth of the steel is greater in one direction than it is in the other by a bar diameter. As the effective depth increases, the re-

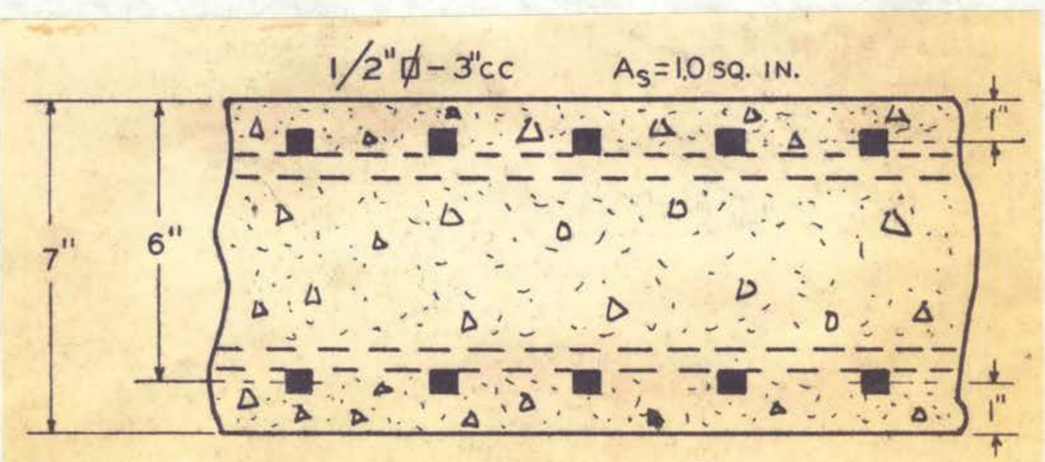

FIGURE 10

quired amount of steel decreases. So the reinforcing steel to resist the maximum moment will be placed outside the other steel at right angles.

Using the above information the section in Figure 10 is assumed and will be checked to see if it will withstand the bending moment.

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
2.5 x 12	30.00	1.25	37.50
1 x (10-1)	9.00	1	9.00
1 x 10	10.00	6	60.00
$\Sigma A = 9.00$		$\Sigma \bar{Y}.A = 106.50$	
.35 x 12	-4.20	2.325	-9.77
$\Sigma A = 44.80$		$\Sigma \bar{Y}.A = 96.73$	

Try kd = 2.5 inches

$$kd = \frac{106.5}{49.0} = 2.175 \text{ inches}$$

Try 2.15 inches

$$kd = \frac{96.73}{44.8} = 2.157 \text{ inches}$$

So use kd = 2.16 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
			Moment to be resisted	=	101,300
2.16 x 12	25.91	1350/2 = 675	17,500	5.28	92,500
Required Area of Comp. Steel = .124		14,500	1,800	5	9,000
Required Area of Tens. Steel = .965		20,000	19,300		

Since the required area of tension and compression steel is less than that furnished, the section will withstand the maximum bending moment at the center of the short span.

Checking the bond stress: $\Sigma 0 = 8$ inches

$$u = \frac{V}{(\Sigma 0)\ j\ d} = \frac{3590}{8 \times .824 \times 6} = 90.8 \text{ pounds per square inch.}$$

This is smaller than the allowable 150 pounds per square inch.

The frequency of the panel now needs to be checked:

$$D = \frac{E\ t^3}{12\ (1 - m^2)} = \frac{3 \times 10^6 \times 7^3}{12\ (1 - .25^2)} = 9.15 \times 10^6$$

$$f = 30.88 \sqrt{\frac{9.15 \times 10^6}{.0868 \times 7}} \left[\frac{1}{120^2} + \frac{1}{150^2}\right]$$

$$= 30.88 \times 3880 \times .0001139 = 13.7 \text{ cycles per second.}$$

Since the concrete is reinforced the actual frequency will be slightly greater than 13.7 cycles per second. However, it will be less than the 50 cycles per second assumed, so the panel need not be redesigned.

As the bending moment decreases from the maximum in the center of the span to zero at the outer edge, the spacing of the steel can be increased toward the outer edges of the panel. The moment at b/4 is:

$M = 3/32\ r_A\ w\ b^2 = 3/32 \times .662 \times 1022 \times 10^2 \times 12 = 76{,}100$ inch pounds.

The moment is decreased not quite one fourth, so change the steel to one half inch round bars and space them the same.

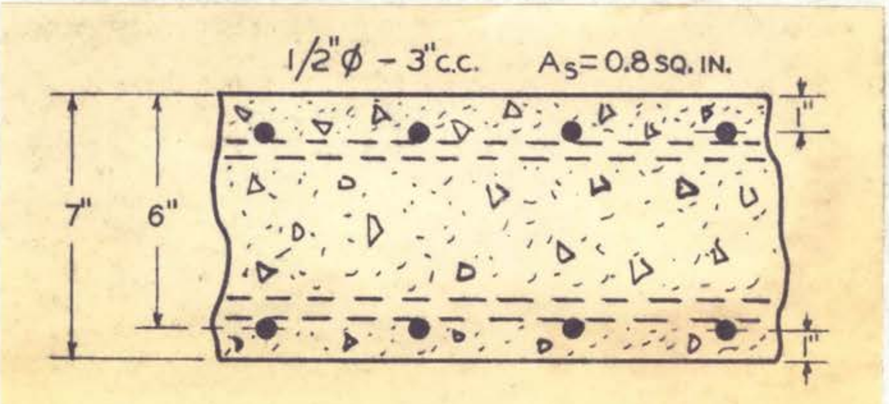

FIGURE 11

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
2 x 12	24.00	1	24.0
.8 (9)	7.20	1	7.2
.8 x 10	8.00	6	48.0
$\Sigma A = 39.20$		$\Sigma \bar{Y} A = 79.2$	
.03 x 12	+ .36	2.15	+ .77
$\Sigma A = 39.56$		$\Sigma \bar{Y} A = 79.97$	

Assume kd = 2.0 inches

$$kd = \frac{79.2}{39.2} = 2.025 \text{ inches}$$

Try kd = 2.03 inches

$$kd = \frac{79.97}{39.56} = 2.02 \text{ inches}$$

Use kd = 2.02 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
2.02 x 12	24.24	675	16,360	5.33	87,200

Disregarding the compression steel, the concrete will withstand the bending moment alone.

$$\text{Centroid of compression area} = \frac{(2.02 \times 12)\ 1.01 + (7.20)\ 1}{(2.02)\ 12 + 7.2} = 1.007 \text{ inches.}$$

$$j = \frac{6 - 1.007}{6} = .833$$

The required area of tension steel is then:

$$A_s = \frac{76,100}{.833 \times 6 \times 20,000} = .762 \text{ square inches.}$$

This is less than the area of steel furnished, so the assumed section is correct except for checking for bond stress. $\Sigma 0 = 6.28$ inches.

$$u = \frac{3,590}{6.28 \times .833 \times 6} = 114.4 \text{ pounds per square inch.}$$

As the allowable bond stress is 150 pounds per square inch, this section is satisfactory.

(b) Design of the panel in the long direction.

The proportionate amount of the total load carried in the long direction is:

$$r_A = 1 - r_B = 1 - .662 = .338$$

The maximum moment is:

$$M = 1/8\ r_A\ w\ a^2 = 1/8 \times .338 \times 1022 \times 12.5^2 \times 12 = 81{,}000 \text{ inch pounds.}$$

This moment is slightly greater than the moment at b/4 from the ends of the short span, so the area of steel will be increased. One half inch square bars spaced $3\frac{1}{2}$ inches apart will provide .86 square inches of steel per foot. The following section is assumed and will be checked:

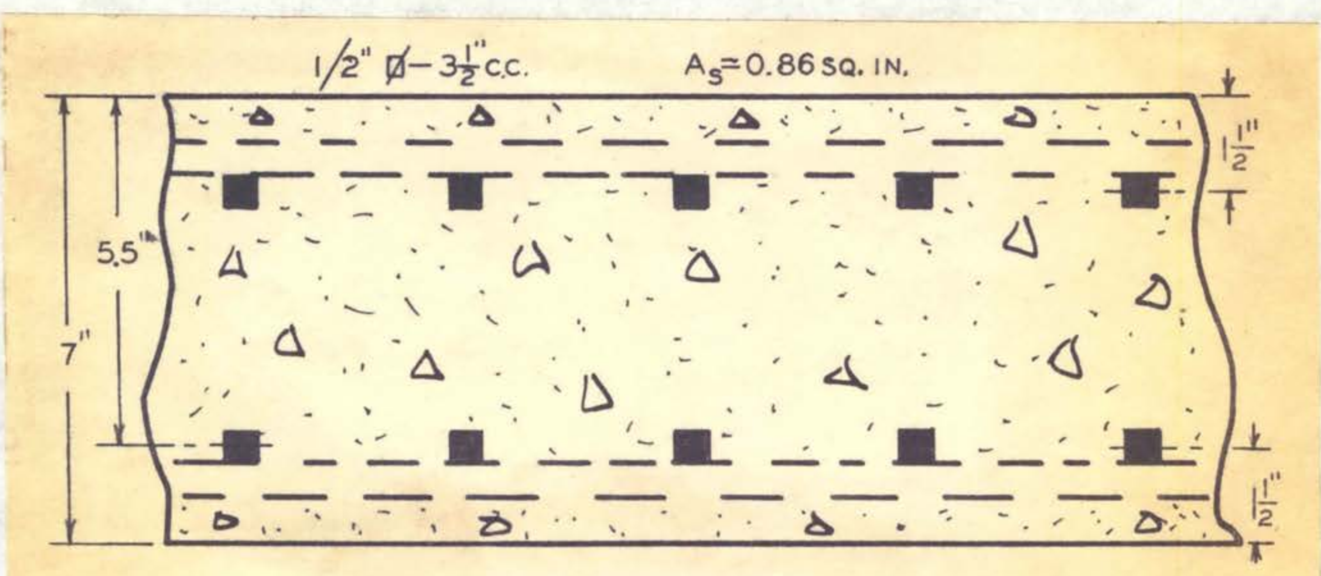

FIGURE 12

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
2.6 x 12	31.20	1.3	40.55
.86 (10-1)	7.74	1.5	11.61
.86 (10)	8.60	5.5	47.30
$\Sigma A = 47.54$		$\Sigma \bar{Y} A = 99.46$	
.55 x 12	-6.60	2.325	15.34
$\Sigma A = 40.94$		$\Sigma \bar{Y} A = 84.12$	

Assume kd = 2.6 inches

$$kd = \frac{99.46}{47.54} = 2.092 \text{ inches}$$

Assume kd = 2.05 inches

$$kd = \frac{84.12}{40.94} = 2.053 \text{ inches}$$

Use kd = 2.05 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
			Moment to be resisted		= 81,000
2.05 x 12	24.6	675	16,600	4,817	80,000
Required Area of Comp. Steel	= .034	7240	250	4	1,000
Required Area of Tens. Steel	= .8425	20,000	16,850		

Both areas of steel required to resist tension and compression are less than the area furnished, so the section needs only to be checked for bond stress: ($\Sigma 0 = 6.86$ inches.)

$$V = 1/2 \times .338 \times 1022 \times 12.5 = 2160 \text{ pounds.}$$

$$u = \frac{V}{(\Sigma 0)\, j\, d} = \frac{2160}{6.86 \times .793 \times 5.5} = 72.2 \text{ pounds per square inch.}$$

This bond stress is less than the allowable 150 pounds per square inch, so the middle section of the panel in the long direction is satisfactory.

Again the amount of steel can be decreased toward the outer edges. At a/4, one half inch round bars spaced three and one half inches apart will provide ample area. The latter required area was determined by the same procedure as above and will not be repeated here.

CHAPTER VIII

DESIGN OF CONTINUOUS CONCRETE PANELS

Most concrete panels built to resist blast, not only are constructed monolithically with their supporting beams and columns, but also are built continuous with adjacent panels. Panels of this kind do not act the same as single panels with fixed edges and should not be analyzed as such.

The first formulas for continuous panel design were developed from mathematical analysis and experiments conducted on panels of the same shape, size, and thickness.[1] A designer could determine the approximate negative moments at the supporting beams and the positive moments at the center of the panels by moment coefficients, if, and only if, all of the panels were of the same shape and arranged in a specified order. These equations, then, had limited use as few buildings were built with all panels the same. The 1946 American Concrete Institute Building Code[2] presents a simple approximate method of determining the moments in continuous irregular shaped panels. Data used as the basis of this code were determined by DiStasio and Van Buren.[3] Since no exact theoretical solution has been developed and as this method is conservative, it will be used herein.

This method of analysis differs from the other methods in that, not only can the moment be computed by moment coefficients, if need be, but also it can be determined by any method of analysing continuous structures with which the

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[1] H. M. Westergaard, "Formulas for the Design of Rectangular Floor Slabs and Supporting Girders," Procedures of American Concrete Institute, XXII (February, 1926), 26.

[2] Anonymous, Building Regulations for Reinforced Concrete, p. 584.

[3] J. DiStasio and M. P. Van Buren, "Slabs Supported on Four Sides," Procedures of American Concrete Institute, XXXII (January-February, 1936), 350-364.

designer is familiar. The bending moment in any two-way panel can be determined the same as in any one-way slab construction after loading the different panels with an equivalent uniform load. This equivalent uniform load per unit area is determined by the equation, $e_A\ r_A\ w$.

Where a - the length of the span in the direction considered.

w - the amount of uniform load per unit area.

r_A - the proportion of the total load carried by span a.

e_A - the modifying factor.

Before the proportion of the total load carried by span a is determined, the distance between inflection points must be computed. This distance is $F_A a$ in the a direction of the span and $F_B b$ in the b direction, where F_A is determined by the following formulas and F_B by substituting b for a.

End spans, continuous one end only

$$F_A = 1 - \frac{0.25}{1 + \frac{7\ K_a}{8\ K_{ar}}}$$

Interior continuous span

$$F_A = \sqrt{1 - \frac{1}{1.5 + \frac{7\ K_a}{8\ K_{ar}}}}$$

K_a is the stiffness factor I/a for the span considered and K_{ar} is the stiffness of the adjoining span. Whenever the K_{ar}'s for the left and right side of an interior span differ, the F_A in the following three formulas should be the average of the two F_A's for both sides of the panel.

The above formulas were derived on the assumption that all spans were of the same length. However, it has been found that F_A and F_B so defined, vary little with irregular span lengths and the final value is sufficiently accurate for continuous panels of any length.

Now the proportion of the total load carried in the a direction of the panel (r_A) can be determined by the following emperical formula:

$$r_A = \frac{1}{1 + \left[\frac{F_A a}{F_B b}\right]^3}$$

Similarly the proportion of the total load carried at right angles in the b direction is:

$$r_B = \frac{1}{1 + \left[\frac{F_B b}{F_A a}\right]^3} = 1 - r_A$$

The total load on a span r_A w a, is not uniformly distributed along the entire span, but varies from a minimum in the center to a maximum at the edges.

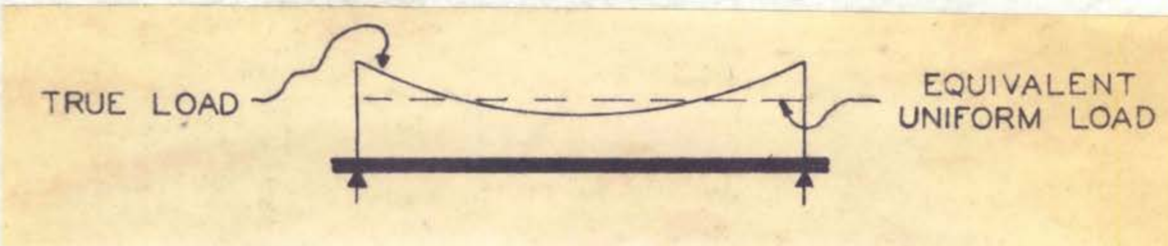

FIGURE 13. INTENSITY OF LOADING ON A PANEL

As in the analysis of other difficult problems, a modified equivalent uniform load is used that will produce the same stresses in the panels as the actual loading. This modified equivalent uniform load is given by e_A r_A w a, where the modifying factor e_A is:

$$e_A = \frac{2}{4 - \frac{F_B b}{F_A a}} \qquad e_B = \frac{2}{4 - \frac{F_A a}{F_B b}}$$

Once the equivalent uniform load is computed in one direction, the panels are considered as one way panels and the bending moment and shears are calculated by whatever method of analysis is to be used.

DiStasio and Van Buren[4] found the above formulas to give results closest to those obtained by their experiments conducted on two way slabs. These results are almost identical with the values obtained by Westergaard,[5] and, when applicable, both are about as simple to use. The main difference being in Westergaard's allowance for redistribution of stress and in a slightly different basic assumption on the distribution of the dead load. Both will give values that are slightly larger than the actual moment.

The A.C.I. Code[6] makes no limitation on the ratio of the length of the long side to the short side of a two way panel. However, investigations made by Temoshenko[7] indicate that very little load is carried in the long direction when this ratio is three or more. He suggests that when the ratio of the long span to the short span of a panel is two or greater, the panel be designed and constructed as a one way panel (main reinforcing in the short direction only).

In addition to the horizontal blast pressure the panels also will have to support some vertical load. This vertical load will not be very great, since the dead weight of the next panel above and the reactions from the adjacent floor slabs will be carried by the supporting beam at the top of the panel to its supporting columns. If the vertical load in a panel is included in a design, it will be found that the additional stress is very small compared to the stresses produced by the lateral bomb pressures and may be neglected.

The recommended minimum thickness of reinforced concrete panels for protection against bomb fragmentation is twelve inches. This thickness is usually

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[4] Ibid., p. 350-364.

[5] Westergaard, loc. cit.

[6] Anonymous, Building Regulations for Reinforced Concrete, p. 584.

[7] Stephen P. Temoshenko, Strength of Materials, Part II, Advance Theory and Problems, p. 157.

greater than the minimum thickness of the panel needed to resist the bomb blast only. Either the panels should have a minimum thickness of twelve inches and be greatly understressed or they should be made the thickness required to resist the blast only and be faced with stone or brick to the minimum twelve inches. If architecturally a series of panels are to be built to their minimum thickness and then faced to the required twelve inches, there is no need to design and construct each panel to its actual minimum thickness. Economically, all panels in a series of panels should be constructed the same thickness.

In the following two examples it will be assumed that the wall panels are solid panels with no window openings. Openings necessitate extra framing which decreases the size of the panels. This will increase the frequency of the panel and consequently the equivalent static load and the minimum thickness of the panel will be increased.

Illustrative Example

Example I.

Design the six reinforced concrete panels shown in Figure 14 to resist the

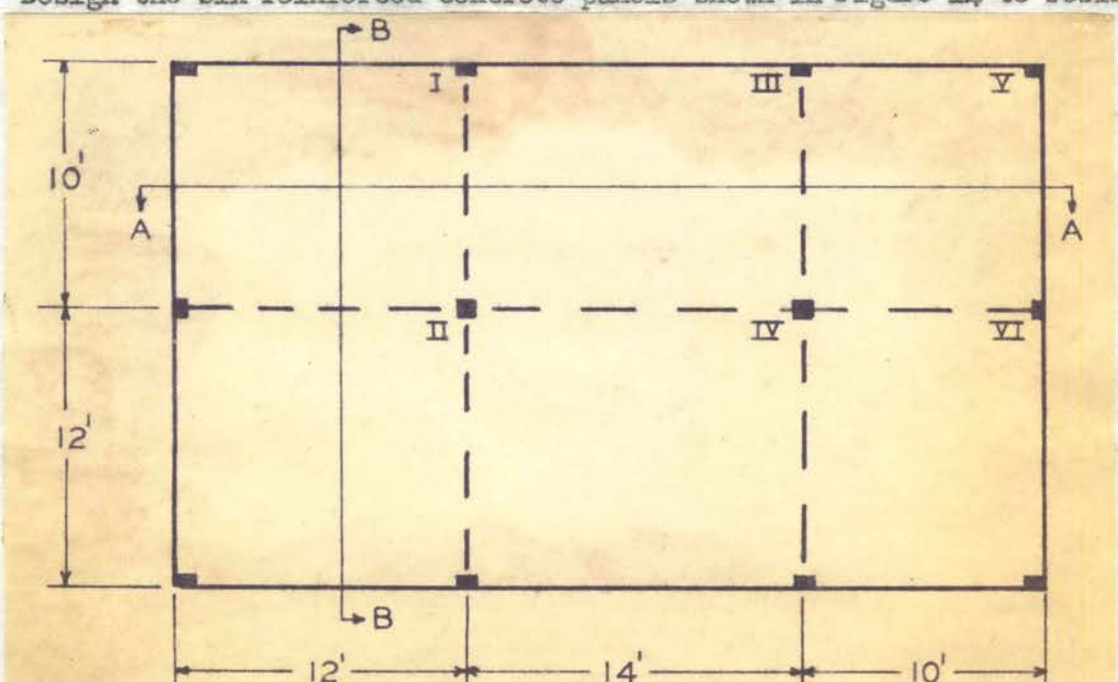


FIGURE 14

blast from a 500 pound bomb at 40 feet. All of the panels are to be two way panels and constructed monolithically with their supporting beams and columns. For architectural purposes, the walls are to be faced with brick.

Unit Stresses:	$f'_c = 3,000$ psi.	$f_s = 20,000$ psi.
	$f_c = 1,350$ psi.	$u = 150$ psi.
	$n = 10$	$v = 90$ psi.

(a) Determination of Stresses

The frequencies of the panels are not known and, since no frequency equations have been developed for panels with all edges fixed, the frequencies can not be calculated mathematically. However, the frequencies of the different panels are greater than the 50 cycles per second at which the maximum equivalent positive static load occurs, so the maximum equivalent load of 2.58 pounds per square inch will be used in this design. Thus the equivalent pressure at forty feet will be:

$$P\ (+) = \frac{2.58 \times 9}{2.2} = 10.55 \text{ pounds per square inch}$$

and $w = 10.55 \times 144 = 1,520$ pounds per square foot.

Since all of the panels will be faced with a course of brick, assume that all of the panels have a thickness of eight inches. Some of the panels will be under stressed but it is not economical to build all of the panels of different thickness and then increase them to the same twelve inch thickness.

The calculations of the equivalent static load carried in each direction by each panel may be found in TABLES III, IV, V and VI.

Now that the equivalent static loads in both directions on every panel have been determined, the moments in the panels can be calculated.

TABLE III

CALCULATION FOR F_a

PANEL	LENGTH a	THICKNESS Inches	I	K = I/a	K_{ar} LEFT	K_{ar} RIGHT	F_A
I, II	144	8	512	3.56	0	3.05	.876
III, IV	168	8	512	3.05	3.56	4.27	.737
V, VI	120	8	512	4.27	3.05	0	.888

TABLE IV

CALCULATION FOR F_b

PANEL	LENGTH b	THICKNESS Inches	I	K = I/b	K_{br} LEFT	K_{br} RIGHT	F_B
II, IV, VI	144	8	512	3.56	0	4.27	.855
I, III, V	120	8	512	4.27	3.56	0	.878

TABLE V

CALCULATIONS OF EQUIVALENT UNIFORM LOAD IN THE a-DIRECTION

PANEL	LENGTH a Inches	LENGTH b Inches	F_A	F_B	$F_A a/F_B b$	r_A	e_A	$e_A\ r_A$	$e_A\ r_A\ W$
I	144	120	.876	.878	1.197	.368	.632	.2327	354
II	144	144	.876	.855	1.025	.481	.661	.3180	483
III	168	120	.737	.878	1.175	.381	.635	.2420	368
IV	168	144	.737	.855	1.006	.496	.666	.3305	502
V	120	120	.888	.878	1.012	.491	.665	.3266	496
VI	120	144	.888	.855	.866	.606	.703	.4260	648

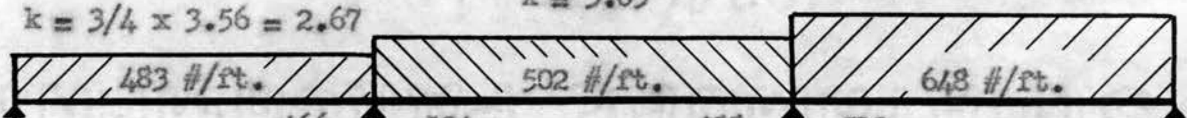
TABLE VI

CALCULATIONS OF EQUIVALENT UNIFORM LOAD IN THE b-DIRECTION

PANEL	$F_B b/F_A a$	r_B	e_B	$e_B\ r_B$	$e_B\ r_B\ w$
I	.835	.632	.713	.4509	685
II	.976	.518	.672	.3481	529
III	.851	.612	.708	.4333	658
IV	.994	.504	.668	.3368	512
V	.988	.509	.669	.3408	518
VI	1.155	.394	.638	.2513	382

TABLE VII

CALCULATIONS OF THE MOMENTS IN THE a-DIRECTION



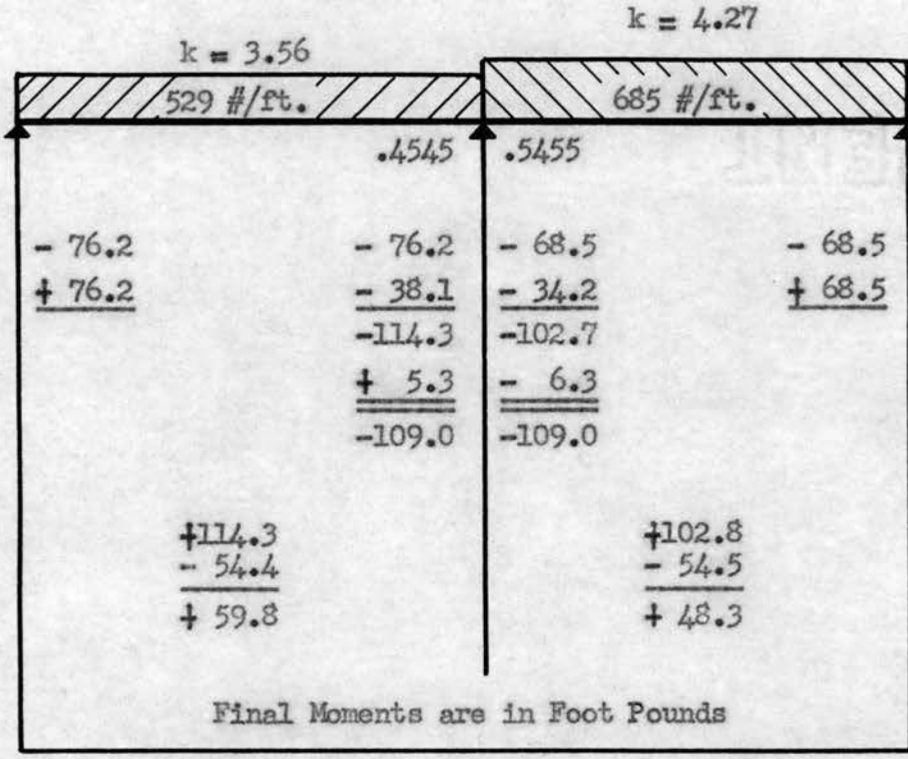
	.466	.534	.488	.512	
- 69.6	- 69.6	- 98.4	- 98.4	- 64.8	- 64.8
+ 69.6	- 34.8	0	0	- 32.4	+ 64.8
	-104.4	- 98.4	- 98.4	- 97.2	
	+ 2.8	- 3.2	+ .6	- .6	
	0	- 0.3	+ 1.6	0	
	- 0.1	+ .2	- 0.8	+ 0.8	
	-101.7	-101.7	- 97.0	- 97.0	

+104.4	+147.6	+ 97.2
- 50.8	- 99.3	- 48.5
+ 53.6	+ 48.3	+ 48.7

Final Moments are in Foot Pounds

TABLE VIII

CALCULATIONS OF THE MOMENTS IN THE b-DIRECTION



Final Moments are in Foot Pounds

The maximum moments are computed in the a and b directions only across that series of panels that will have to resist the greatest moments. These moments, determined by the Moment Distribution Method of Analysis,[8] are calculated in TABLES VII and VIII.

(b) Design of Wall

Since the vertical steel in the b direction of the panel will have to resist the greatest moment, it will be placed outside the lateral steel. The amount of vertical steel shown in Figure 15 is assumed and will be checked to see if it will withstand the 109,000 inch pounds of negative moment across the horizontal supporting beam.

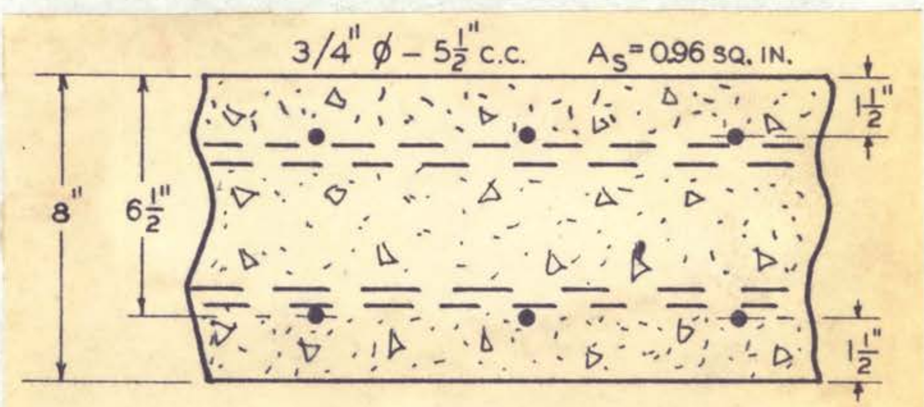


FIGURE 15

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[8] Hardy Cross and Newlin D. Morgan, Continuous Frames of Reinforced Concrete, pp. 98-103.

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
2.4 x 12	28.80	1.2	34.55
.96 x 9	8.64	1.5	12.96
.96 x 10	9.60	6.5	62.40
$\Sigma A = 47.04$		$\Sigma \bar{Y} A = 109.91$	
.07 x 12	- .84	2.365	-1.99
$\Sigma A = 46.20$		$\Sigma \bar{Y} A = 107.92$	

Try kd = 2.4 inches

$$kd = \frac{109.91}{47.04} = 2.338 \text{ inches}$$

Try kd = 2.33 inches

$$kd = \frac{107.92}{46.20} = 2.334 \text{ inches}$$

Use kd = 2.33 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
			Moment to be resisted		= 109,000
2.33 x 12	28.98	675	18,880	5.72	108,000
Required Amt. of Comp. Steel = .021		9,640	200	5	1,000
Required Amt. of Tens. Steel = .954		20,000	19,080		

Because the required area of the tension and compression steel is less than that furnished, the section will resist the maximum moment. However, the bond and shearing unit stresses must be checked.

Using $V = \frac{68.5 \times 10}{2} + \frac{109{,}000}{10 \times 12} = 4334$ pounds.

The unit shearing stress is:

$$v = \frac{V}{b\ j\ d} = \frac{4334}{12 \times .809 \times 6.5} = 68.7 \text{ pounds per square inch}$$

which is less than the allowable 90 pounds per square inch.

The bond stress is: $(\Sigma 0 = 5.66 \text{ inches})$

$$u = \frac{V}{\Sigma 0 \; j \; d} = \frac{4334}{5.66 \times .809 \times 6.5} = 145.6 \text{ pounds per square inch.}$$

This is also less than its allowable unit stress of 150 pounds per square inch, thereby, the section is satisfactory.

Even though the vertical steel could be reduced at the middle of the panels it is rarely done because of construction difficulties. The same amount of vertical steel over the middle support is extended on through the panel to the outer edges and are terminated in hooks. Although in the analysis the edges of the panels were assumed to be simply supported, they are prevented from rotating to a certain degree, but the exact amount of moment at the outer supports is not known. The American Concrete Institute Code[9] specifies that the negative moment reinforcing at an edge must be at least one-half of the amount of the positive steel at the center of the span. Therefore, if all the steel required to resist bending at the middle support is extended to the outer edges, there will be more than enough steel to resist the negative moment at the outer edges.

Now the moment across the other panels in the b direction can be found and the required amount of steel computed. This will not be done in this example, since it would be a repetition of the above analysis.

The amount of lateral steel needed over the vertical supporting columns will now be determined. The area of steel in Figure 16, page 58, is assumed and will be checked.

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[9] Anonymous, Building Regulations for Reinforced Concrete, p. 586.

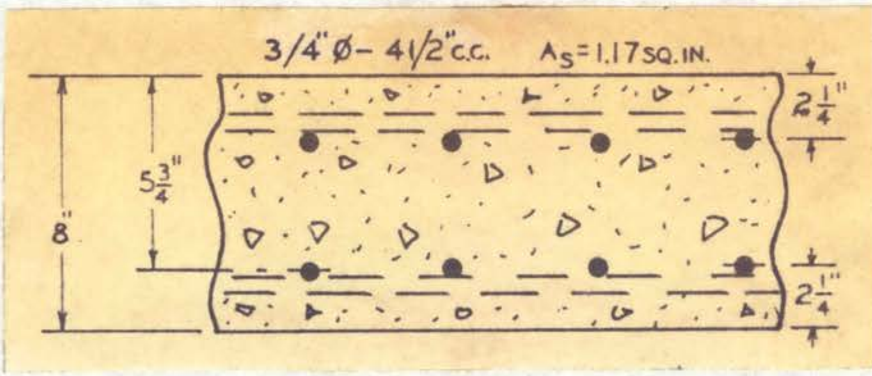

FIGURE 16

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
2.6 x 12	31.20	1.3	40.55
1.17 x 9	10.53	2.25	23.70
1.17 x 10	11.70	5.75	67.25
	ΣA = 53.43		$\Sigma\bar{Y}$ A = 131.50
.14 x 12	-1.68	2.53	-4.25
	ΣA = 51.75		$\Sigma\bar{Y}$ A = 127.25

Try kd = 2.6 inches

$$kd = \frac{131.50}{53.43} = 2.461 \text{ inches}$$

Try kd = 2.46 inches

$$kd = \frac{127.25}{51.75} = 2.46 \text{ inches}$$

Use kd = 2.46 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
			Moment to be resisted		= 101,700
2.46 x 12	29.52	675	19,920	4.93	98,200
Required Amt. of Comp. Steel	= .435	2,300	1,000	3.5	3,500
Required Amt. of Tens. Steel	=1.046	20,000	20,920		

The section will resist the bending moment, since the required amounts of tension and compression steel are both less than that furnished. The maximum shear in the a direction on the panels is:

$$V = \frac{648 \times 10}{2} + \frac{97,000}{10 \times 12} = 4048 \text{ pounds}$$

and the shearing unit stress is:

$$v = \frac{V}{b\ j\ d} = \frac{4048}{12 \times .74 \times 5.75} = 79.3 \text{ pounds per square inch}$$

which is less than the allowable 90 poundsper square inch.

The bond stress is: $(\Sigma 0 = 6.29 \text{ inches})$

$$u = \frac{V}{\Sigma 0\ j\ d} = \frac{4048}{6.29 \times .74 \times 5.75} = 151.2 \text{ pounds per square inch.}$$

This is slightly larger than the allowable 150 pounds per square inch, but the difference is small and the section does not need to be redesigned.

If desired, the lateral steel could be reduced at the center of the panels. The calculation of the required amount of steel at the center in not difficult. The moment is so small, no compression steel is needed. Using $j = .86$ the required amount of tension steel at the center of the span is:

$$\text{Required } A_s = \frac{M}{j\ d\ f_s} = \frac{53,600}{.86 \times 5.75 \times 20,000} = .542 \text{ square inches.}$$

One half inch round rods placed every four inches would provide ample area. Be sure though, that all rods are extended at least twelve diameters beyond the point at which they are not needed to resist stress and that all steel terminates in standard hooks at the edges. To be on the side of safety and to help simplify construction, the positive steel in the center of the panels could be extended to the outer edges and be the negative steel at the supports to care for the reversed blast load.

The reinforcing steel in the a direction in the other panels is less than the amount calculated above and can be computed by the same method as described above. After this is done, the design is complete.

Example II.

Design the five panels shown in Figure 17 to resist the blast from a 500 pound bomb at 40 feet. All are to be built as two way panels and constructed monolithically with their supporting beams and columns.

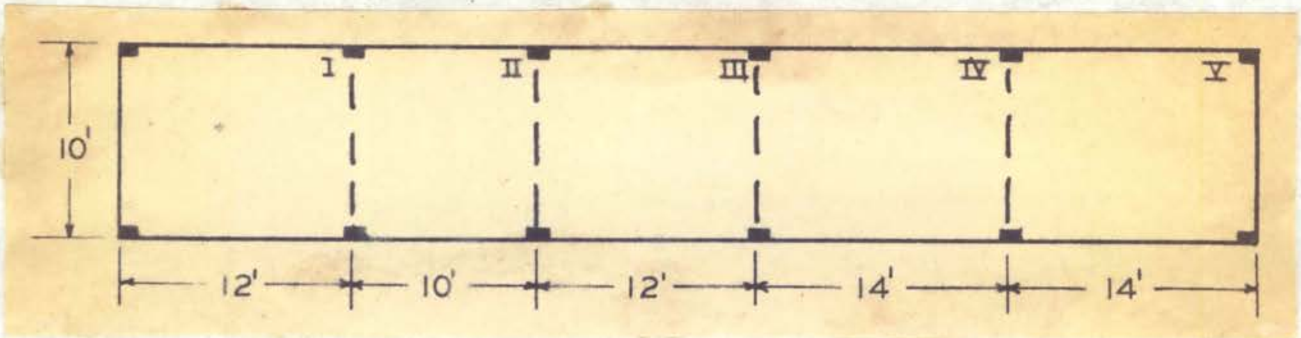

FIGURE 17

Unit Stresses:	$f'_c = 3{,}000$ psi.	$f_s = 20{,}000$ psi.
	$f_c = 1{,}350$ psi.	$u = 150$ psi.
	$n = 10$	$v = 90$ psi.

Again no equations have been developed for determining the frequency of these types of panels. However, this is not important since the frequency of these panels is greater than the 50 cycles per second at which the maximum equivalent load occurs. This maximum positive equivalent load, then, if the bomb is exploded at 40 feet is:

$$P\ (+) = \frac{2.58 \times 9}{2.2} = 10.55 \text{ pounds per square inch}$$

and $w = 10.55 \times 144 = 1520$ pounds per square foot.

Because no special facing is to be used on these walls, the thickness will be that of the minimum twelve inches needed for protection against flying bomb fragmentation.

The calculations of the equivalent static load carried in each direction by each panel may be found in TABLES IX, X, XI, and XII.

With the equivalent static loads in each direction calculated, the moment in the a direction of the span can be determined by Moment Distribution. These calculations can be found in TABLE XIII. The amount of rotation of the supporting beams in the b direction of the spans and the end moments are not known. If the panels are considered to be simply supported in the b direction, although not correct, the value of the moment will be greater than the actual positive moment. Thus the maximum positive moment in the b direction of the panel is:

$$M = 1/8\ w\,L^2 = 1/8 \times 1{,}032 \times 10^2 \times 12 = 154{,}900 \text{ inch pounds.}$$

As this is more than triple the amount of the maximum moment in the a direction, the steel in the b direction or the vertical steel will be placed outside the lateral steel.

The area of steel in Figure 13 is assumed to resist the bending moment in the b direction and will be checked.

TABLE IX

CALCULATIONS OF F_A

PANEL	LENGTH a Inches	THICKNESS Inches	I	$k_A = I/A$	K_{ar} LEFT	K_{ar} RIGHT	F_A LEFT	F_A RIGHT	AVERAGE F_A
I	144	12	1728	12	0	14.4	0	.855	.855
II	120	12	1728	14.4	12	12	.780	.780	.780
III	144	12	1728	12	14.4	10.28	.742	.776	.759
IV	168	12	1728	10.28	12	10.28	.746	.762	.754
V	168	12	1728	10.28	10.28	0	.867	0	.867

TABLE X

CALCULATIONS OF F_B

PANEL	LENGTH b Inches	THICKNESS Inches	I	$k_B = I/B$	K_{br} LEFT	K_{br} RIGHT	F_B LEFT	F_B RIGHT	AVERAGE F_B
All	120	12	1728	14.4	0	0	.75	.75	.75

TABLE XI

CALCULATIONS OF EQUIVALENT UNIFORM LOAD IN THE a-DIRECTION

PANEL	LENGTH a Inches	LENGTH b Inches	F_A	F_B	$F_A a/F_B b$	r_A	e_A	$e_A\ r_A$	$e_A\ r_A\ w$
I	144	120	.855	.75	1.368	.281	.612	.1721	262
II	120	120	.780	.75	1.040	.471	.659	.3105	472
III	144	120	.759	.75	1.215	.358	.630	.2258	343
IV	168	120	.754	.75	1.408	.264	.608	.1605	244
V	168	120	.867	.75	1.618	.191	.591	.1129	172

TABLE XII

CALCULATIONS OF EQUIVALENT UNIFORM LOAD IN THE b-DIRECTION

PANEL	$F_B b/F_A a$	r_B	e_B	$e_B\ r_B$	$e_B\ r_B\ w$
I	.731	.719	.760	.5463	831
II	.962	.529	.675	.3570	543
III	.823	.642	.718	.4610	700
IV	.710	.736	.771	.5675	863
V	.618	.809	.839	.6785	1032

TABLE XIII

CALCULATIONS OF THE MOMENTS IN THE a-DIRECTION

Span 1		Span 2		Span 3		Span 4		Span 5	
k = 3/4 x 3.56 = 2.67		k = 4.27		k = 3.56		k = 3.05		k = 3/4 x 3.05 = 2.29	
262 #/ft.		472 #/ft.		343 #/ft.		244 #/ft.		172 #/ft.	
	.385	.615	.545	.455	.539	.461	.571	.429	
- 37.7	- 37.7	- 47.2	- 47.2	- 49.4	- 49.4	- 47.8	- 47.8	- 33.7	- 33.7
+ 37.7	- 18.8	0					0	- 16.8	+ 33.7
	- 56.5	- 47.2	- 47.2	- 49.4	- 49.4	- 47.8	- 47.8	- 50.5	
	+ 3.6	- 5.7	- 1.2	+ 1.0	+ 0.9	- 0.7	- 1.5	+ 1.2	
	0	+ 0.6	+ 2.8	- 0.4	- 0.5	+ 0.7	+ 0.3	0	
	+ 0.2	- 0.4	- 1.7	+ 1.5	+ 0.6	- 0.6	- 0.2	+ 0.1	
	0	+ 0.8	+ 0.2	- 0.3	- 0.7	+ 0.1	+ 0.3	0	
	+ 0.3	- 0.5	- 0.3	+ 0.2	+ 0.4	- 0.4	- 0.2	+ 0.1	
	- 52.4	- 52.4	- 47.4	- 47.4	- 48.7	- 48.7	- 49.1	- 49.1	

Span 1	Span 2	Span 3	Span 4	Span 5
+ 56.6	+ 70.8	+ 74.1	+ 71.8	+ 50.5
- 28.3	- 49.9	- 48.0	- 48.9	- 24.5
+ 18.3	+ 20.9	+ 26.1	+ 22.9	+ 26.0

Final Moments are in Foot Pounds

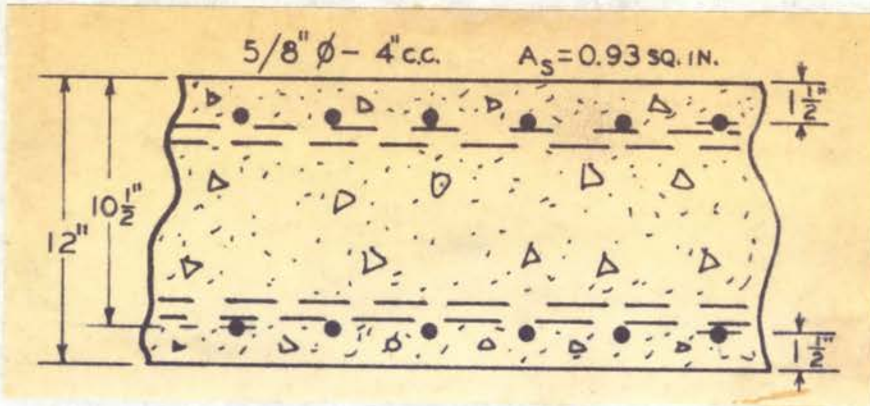

FIGURE 18

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
3 x 12	36.00	1.5	54.00
.93 x 9	8.37	1.5	12.55
.93 x 10	9.30	10.5	97.62
	ΣA = 53.67	$\Sigma \bar{Y}$ A = 164.17	
.06 x 12	.72	3.03	2.18
	ΣA = 54.39	$\Sigma \bar{Y}$ A = 166.35	

Try kd = 3 inches

$$kd = \frac{164.17}{53.67} = 3.058 \text{ inches}$$

Try kd = 3.06 inches

$$kd = \frac{166.35}{54.39} = 3.06 \text{ inches}$$

Use kd = 3.06 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
3.06 x 12	36.72	675	24,780	9.48	235,000

The concrete by itself will more than carry the compression load and no compression steel is needed. The area of steel needed, though, to resist the positive moment is:

$$\text{Required } A_s = \frac{M}{j\ d\ f_s} = \frac{154,900}{.855 \times 10.5 \times 20,000} = .863 \text{ square inches.}$$

Thus the section will resist the bending stresses, as this area is less than that furnished.

The end shear load is:

$$V = \frac{1032 \times 10}{2} = 5160 \text{ pounds}$$

and the unit shearing stress is:

$$v = \frac{V}{b\ j\ d} = \frac{5160}{12 \times .855 \times 10.5} = 48 \text{ pounds per square inch,}$$

which is about one half the allowable shearing unit stress.

Checking the bond stress: $(\Sigma 0 = 5.88 \text{ in.})$

$$u = \frac{V}{\Sigma 0\ j\ d} = \frac{5160}{5.88 \times .855 \times 10.5} = 98 \text{ pounds per square inch,}$$

and this also is less than the allowable bond stress of 150 pounds per square inch.

As it is difficult in concrete construction to change the size of the vertical steel, the vertical steel used at the center of the panel is extended to the outer edges and is terminated in hooks. Even though the negative moment at the outer edges of the panel is not known, this large amount of steel at the outer edges is much greater than is needed, and the tension steel at the outer edges need not be checked.

Since no vertical compressive steel is needed, the amount of vertical tension steel in the other panels is easily determined. For instance, in the middle panel the maximum moment is:

$$M = 1/8\ w\ L^2 = 1/8 \times 700 \times 10^2 \times 12 = 105{,}000 \text{ inch pounds.}$$

The required area of the steel is: ($\underline{j}$ will increase as the amount of steel decreases, so .86 is a good working value of $\underline{j}$.)

$$\text{Required } A_s = \frac{105{,}000}{.86 \times 10.5 \times 20{,}000} = .582 \text{ square inches.}$$

One half inch round rods spaced every four inches will provide ample area. As the bond and shearing unit stresses are smaller than their allowables, the assumed steel rods are sufficient.

In the a direction across the panels, the maximum moment is a negative 52,400 inch pounds and occurs at one of the supports. This is a relatively small moment to be resisted by so deep a panel (ten inches) and the amount of tension steel required very easily could be less than the minimum amount of steel allowed by the codes. According to the American Concrete Institute Code[10] the minimum area of steel reinforcing shall not be less than 0.0025 bd. For this section then, the area of the tension steel must be greater than 0.0025 x 12 x 10 = 0.3 square inches. Three eights inch round rods placed every four inches will provide 0.33 square inches of steel. This area then is assumed and will be checked to see if it will withstand the blast load.

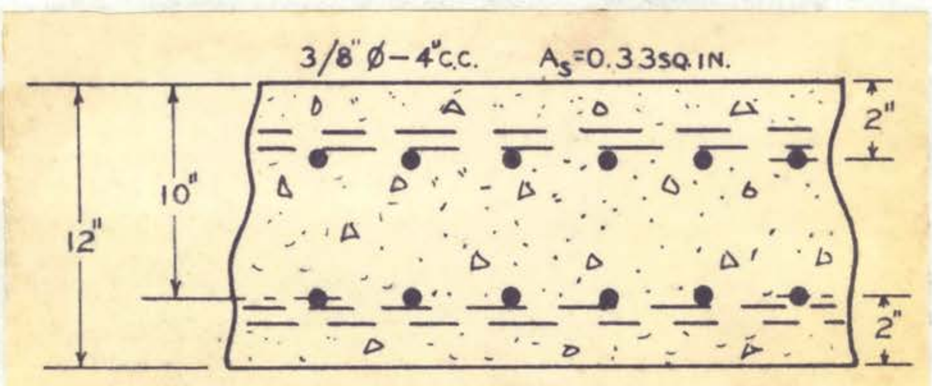

FIGURE 19

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10 Ibid., p. 581.

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
3 x 12	36.00	1.5	54.00
.33 x 9	2.97	2	5.94
.33 x 10	3.30	10	33.00
$\Sigma A = 42.27$		$\Sigma \bar{Y} A = 92.94$	
.84 x 12	-10.08	2.58	-26.02
$\Sigma A = 32.19$		$\Sigma \bar{Y} A = 66.92$	
.09 x 12	- 1.08	2.115	2.28
$\Sigma A = 31.11$		$\Sigma \bar{Y} A = 64.64$	

Try kd = 3 inches

$$kd = \frac{92.94}{42.27} = 2.2 \text{ inches}$$

Try kd = 2.16 inches

$$kd = \frac{66.92}{32.19} = 2.08 \text{ inches}$$

Try kd = 2.07 inches

$$kd = \frac{64.64}{31.11} = 2.077 \text{ inches}$$

Use kd = 2.08 inches

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
2.08 x 12	24.96	675	16,850	9.31	156,900

There is no need, then, for compression steel and the required amount of tension steel is:

$$\text{Required } A_s = \frac{52,400}{.884 \times 10 \times 20,000} = .297 \text{ square inches.}$$

This is less than the area of steel furnished and, note, that it is also less than the minimum area allowed by the Code.[11] The maximum shear in the <u>a</u> direction is:

$$V = \frac{472 \times 10}{2} + \frac{5,000}{10 \times 12} = 2402 \text{ pounds.}$$

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[11] <u>Ibid</u>., p. 581.

Thus the shearing unit stress is:

$$v = \frac{2402}{12 \times .884 \times 10} = 22.7 \text{ pounds per square inch}$$

and the bond stress is: $(\Sigma 0 = 3.54 \text{ inches.})$

$$u = \frac{2402}{3.54 \times .884 \times 10} = 76.8 \text{ pounds per square inch.}$$

Therefore, both the shearing unit stress and the bond stress are less than their allowable of 90 and 150 pounds per square inch respectively.

The moments in the center of the panels in the a direction are about one half that of the moments over the supports. However, the area of the steel can not be decreased at the middle of the spans since there would be a smaller area of steel than the allowable minimum area specified by the Code.[12] The same amount of lateral steel then should extend from one end of the panel to the other and terminate in hooks. Be sure that the end of each rod overlaps the end of the next rod enough so that both bars develop their full bond stress. It is of utmost importance that all reinforcing steel, in structures subjected to bomb blast, be securely anchored.

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[12] Ibid, p. 581.

CHAPTER IX

DESIGN OF CANTILEVER BLAST WALLS

Cantilever walls are especially well adapted to withstand bomb blasts. Being free at one end, they vibrate at a very low frequency and thus are subjected to lower equivalent static loads than other types of walls. They are also simple to analyze and design, but can not be universally used. They are used outside of buildings to protect large openings, such as doors and windows from blast. However, inside of buildings with large open floor space, they protect vital machinery and personnel and tend to localize the effects of bombs that explode inside the building.

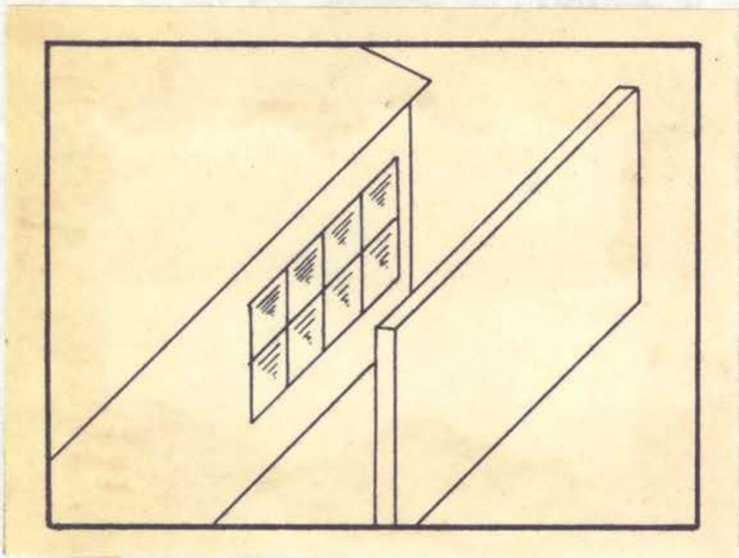

FIGURE 20. CANTILEVER BLAST WALL

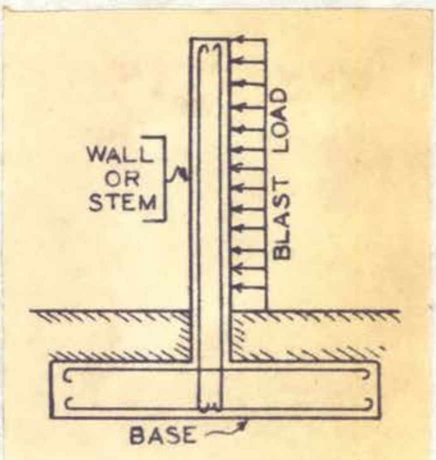

FIGURE 21

Each cantilever wall consists of two main parts (see Figure 21), the vertical cantilever wall that is subjected to the blast, and the horizontal base that prevents the wall from overturning. All tension in the wall is taken by the vertical reinforcing steel, which extends from the top of the wall down into the base. The parts of the base that extend out from the wall also act as cantilever beams and must resist the upward pressure of the earth. This type of wall can fail as a whole by sliding horizontally on its base, overturning, or by excessive settling. It can also fail by the collapse of its component parts. In a well balanced design, the wall should be equally safe in all respects.

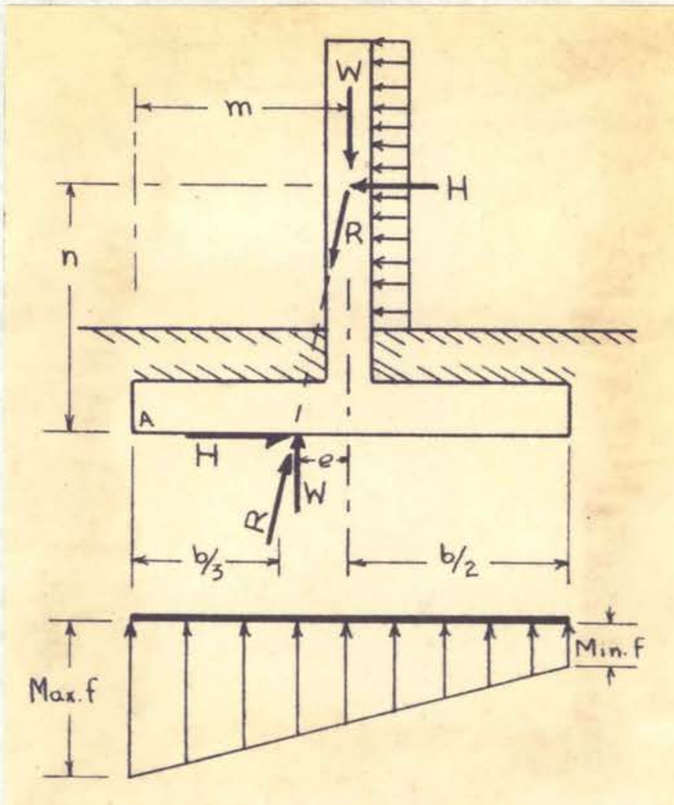

FIGURE 22

To illustrate, let W in Figure 22 represent the resultant of all the vertical forces, including the weight of the wall, base, and the earth resting on the base; and let H be the resultant of the horizontal forces on the wall due to the bomb blast. The horizontal force H tends to overturn the entire wall about the point A, but the vertical force W will prevent it from tipping. If the moment Hn is greater than the moment Wm, the resultant of the two forces R will pass outside the base and the wall will tip over. For stability the resultant of W and H must fall inside the base.

Not only the overturning moment, but also the amount of the pressure on the base will depend upon the location of the point of intersection of the resultant and the base. For yielding soils the base should bear on the earth along its entire length. This can be only if the resultant R falls within the middle third of the base. If the resultant strikes the base outside the middle third, tension will exist at one end of the base. So for satisfactory design the resultant of W and H should intersect the base within the middle third, provided the safe bearing capacity of the soil is not exceeded.[1]

However, sometimes when the walls are placed on rock or other unyielding soils and part of the base is in tension, the compressive pressure at the other end of the base will remain within the allowable limits. This case is difficult to analyze as the exact distribution of the pressure on the base is not known and will not be used herein.

The pressures on the base when the resultant falls within the middle third easily can be computed from the well known equation:

$$f = \frac{W}{A} \pm \frac{Mc}{I}$$

W/A is the uniform pressure on the base due to the vertical load W and Mc/I is the stress due to the overturning moment We. (e is the horizontal distance from the middle of the base to the point of intersection of the resultant R and the base. See Figure 22) As a strip of the wall one foot wide is considered, the total pressure on the base is:

$$f = \frac{W}{1 \times b} \pm \frac{We \times (b/2)}{1/12 \times 1 \times b^3} = \frac{W}{b} \left[1 \pm \frac{6\,e}{b}\right]$$

where b is the width of the base. The positive sign is used to determine the maximum pressure at one end of the base; while the negative sign will give the minimum pressure at the other end.

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[1] Clarence W. Dunham, The Theory and Practice of Reinforced Concrete, p. 217.

In designing a cantilever wall to resist bomb blast it is especially difficult to make the resultant intersect the base within the middle third. Lateral loads will be applied in both directions necessitating a wall that is symmetrical about the center. The resultant of all the vertical forces W is at the center of the wall and unless the base is very wide or the vertical weight very large, the resultant R will fall outside the middle third. Usually it is not economical to widen the base, so more weight is added. This can be done in many ways and perhaps the most common will be to cover the base with earth as in Figure 23. Not only will the earth produce a vertical pressure but also it will apply a lateral force to the base of the stem.

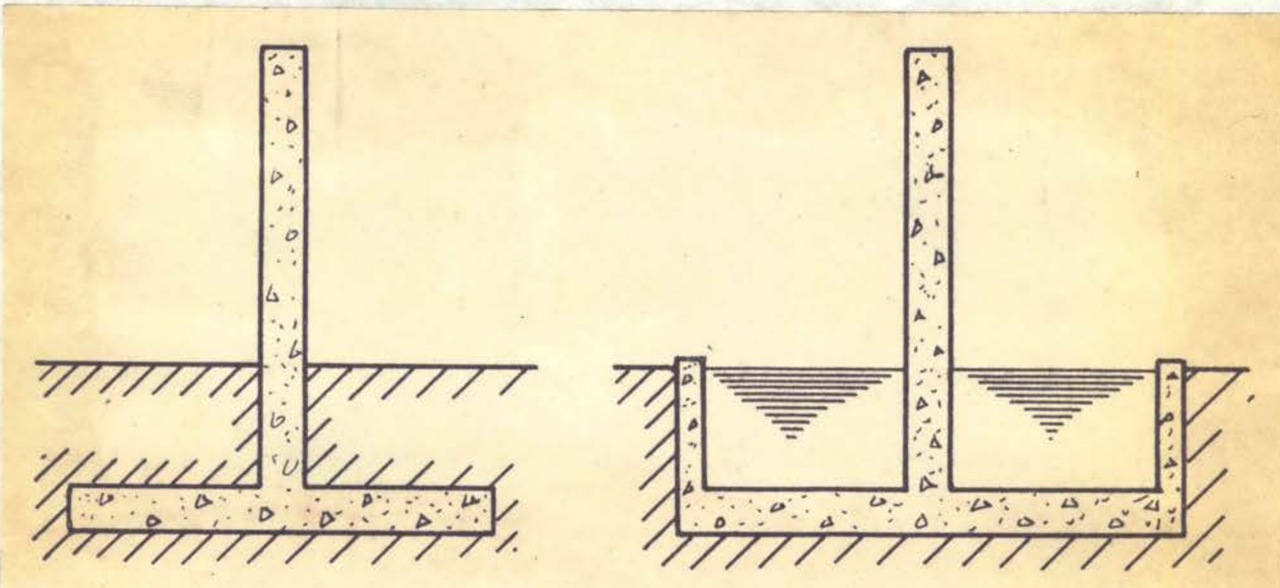

FIGURE 23. TWO TYPES OF CANTILEVER BLAST WALLS

The amount of this lateral force will depend upon the type of soil used to cover the base. Clay will tend to shrink away from the stem and will apply very little force except close to the base. Sand on the other hand will not shrink away from the stem and will apply a force that increases from zero at the surface to a large force at the base. This lateral pressure on both sides of the wall, then, will put an additional restraint on the wall and cause it to bend about a plane somewhere between the base and the top of the earth fill. The exact location of this point is not known, but is probably somewhere between one third and one half of the depth of the fill from the top of the base. In any case, to be on the side of safety, assume that there is no lateral earth pressure and design the stem to resist the bending moment at the top of the base. Also because the frequency of a cantilever wall will increase as the height of the wall is decreased, in calculating the equivalent static load, use as the height of the wall, the distance from the middle of the earth fill to the top of the wall.

Another way to provide additional weight on the base is to construct sides on the ends of the base and fill the space with water. (See Figure 23, page 74) Not only will this provide additional vertical weight, but also it will serve as an emergency water supply. Many times in bomb raids all water mains are broken and only that water stored near-by can be used to fight fires.

Illustrative Example:

Example I.

Design a ten foot concrete cantilever wall to resist the blast of a 500 pound bomb at 50 feet. The base is to be covered with earth to provide the necessary weight to resist overturning.

Unit Stresses:	$f'_c = 3{,}000$ psi.	$f_s = 20{,}000$ psi.
	$f_c = 1{,}350$ psi.	$u = 150$ psi.
	$n = 10$	$v = 90$ psi.
	Safe Bearing Value of the Earth is 2,000 pounds per square foot.	

(a) Design of Stem

The base must have an additional weight on it to prevent the wall from overturning. Assuming then that three feet of earth fill will be needed, the length of the stem will be $10 + 3 = 13$ feet.

(See Figure 24)

Since the frequency of a cantilever wall is low, the equivalent static load corresponding to a frequency of 12 cycles per second will be used. This load is one pound per square inch for a 500 pound bomb at 100 feet. At 50 feet the 500 pound bomb will produce an equivalent static load of:

P = 1 x 6/2.2 = 2.73 pounds per square inch = 393 pounds per square foot.

The moment then at the base of the stem is:

M = 393 x 10 x 8 x 12 = 377,000 inch pounds.

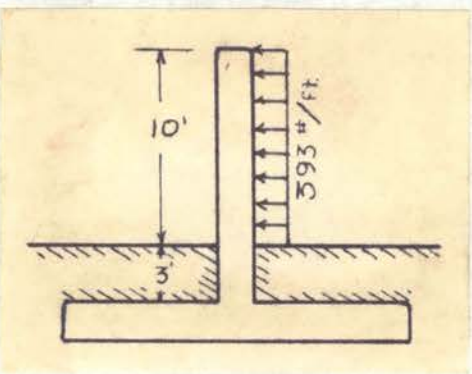

FIGURE 24

If the wall were to have reinforcing steel on the tension side only, the required depth of the wall due to the bending moment would be: (Assume k = .4, j = .87)

$$\text{Required } d = \sqrt{\frac{M}{f_c/2 \times j\ k\ b}} = \sqrt{\frac{377{,}000}{1350/2 \times .87 \times .4 \times 12}}$$

$$= \sqrt{133.7} = 11.6 \text{ inches.}$$

Assume the depth then to be 11.5 inches. With a two inch covering of the rods, the wall will be thicker than the minimum 12 inches needed as protection against fragmentation. Again, assume the area of the steel shown in Figure 25, page 77. This section now will be checked to see if it will resist the bending moment.

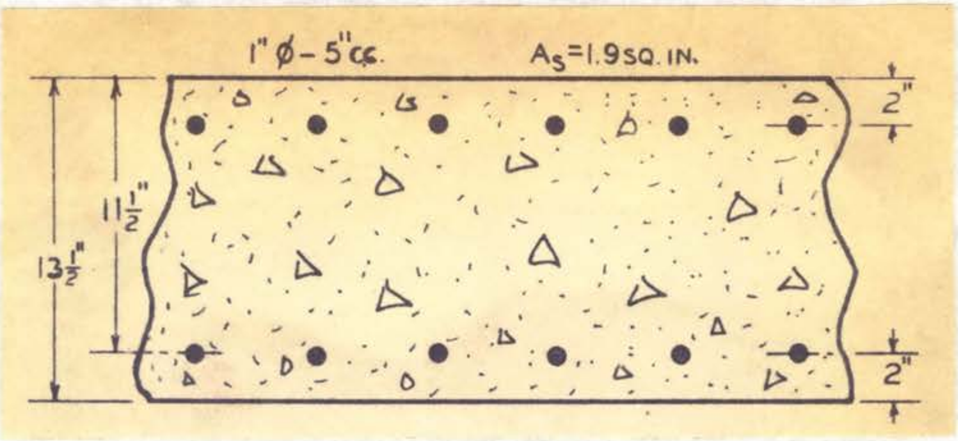

FIGURE 25

DIMENSION Inches	AREA Sq. In.	$\bar{Y}$ Inches	$\bar{Y}$ A
4.4 x 12	52.80	2.20	116.20
1.9 x(10-1)	17.10	2.00	34.20
1.9 x 10	19.00	11.50	218.50
ΣA = 88.90		$\Sigma \bar{Y} A$ = 368.90	
.26 x 12	- 3.12	4.37	- 13.63
ΣA = 85.78		$\Sigma \bar{Y} A$ = 355.27	

Assume kd = 4.4 inches

$$kd = \frac{368.9}{88.9} = 4.15 \text{ inches}$$

Try kd = 4.14 inches

$$kd = \frac{355.27}{85.78} = 4.142 \text{ inches}$$

Use kd = 4.14 inches.

DIMENSION Inches	AREA Sq. In.	STRESS Lbs. per Sq. In.	COMPRESSION FORCE - Pounds	ARM Inches	MOMENT In. Lbs.
			Moment to be resisted		= 377,000
4.14 x 12	39.69	675	33,520	10.12	339,200
Required Amt. of Comp. Steel	= .305	13,050	3,980	9.5	37,800
Required Amt. of Tens. Steel	=1.875	20,000	37,500		

The required areas of both the tension and compression steel are less than the 1.9 square inches furnished, so the section will resist the bending moment.

The bond and shearing unit stresses at the base of the stem are:

$$v = \frac{V}{b\ j\ d} = \frac{393 \times 10}{12 \times .822 \times 11.5} = 34.6 \text{ pounds per square inch,}$$

$$u = \frac{V}{\Sigma 0\ j\ d} = \frac{3930}{7.54 \times .822 \times 11.5} = 55.1 \text{ pounds per square inch.}$$

Both are less than their allowables.

Now that the thickness of the wall is known, the actual frequency of the panel can be calculated and the equivalent static load checked. The equation for the frequency of a cantilever beam is:

$$f = 11\sqrt{\frac{E\ I}{A\ \gamma\ L^4}}$$

For this wall:

$E = 3{,}000{,}000$ pounds per square inch
$I = 1/12 \times 12 \times 11.5^3 = 1522$ in.^{4}
$A = 12 \times 11.5 = 138$ square inches
$\gamma = 150/12^3 = .0868$ pounds per cubic inches
$L = 11.5$ feet $= 138$ inches

$$f = 11\sqrt{\frac{3 \times 10^6 \times 1522}{138 \times .0868 \times 138^4}} = 8.82 \text{ cycles per second.}$$

This is less than the twelve cycles per second used in calculating the equivalent static load, so the stem is satisfactory. The actual equivalent static load then on the wall is:

$$P = \frac{.7 \times 6}{2.2} = 1.91 \text{ pounds per square inch} = 275 \text{ pounds per square foot.}$$

It is unnecessary to extend all of the reinforcing steel the full height of the wall. The required area of steel at any point can be found with sufficient accuracy by assuming $j = 0.86$ and designing the wall for bending stress alone, neglecting the compressive steel. The bending moment at any distance X in feet from the top of the wall, providing X is not greater than ten feet is:

$$M = \frac{393 \times X^2 \times 12}{2} = 2358 \times X^2 \text{ inch pounds.}$$

The required area of tension steel is:

$$\text{Required } A_s = \frac{M}{j\ d\ f_s} \text{ or } M = 2{,}358 \times X^2 = A_s\ j\ d\ f_s$$

Thus the distance from the top of the wall to the section where one rod can be cut off is:

$$X^2 = \frac{A_s\ j\ d\ f_s}{2358} = \frac{1.11 \times .86 \times 11.5 \times 20{,}000}{2358} = 93.1$$

$X = 93.1 = 9.66$ feet $=$ 9 feet, 8 inches from the top of the wall.

The second rod may be stopped at:

$$X^2 = \frac{0.32 \times .86 \times 11.5 \times 20{,}000}{2358} = 26.8$$

$X = 26.8 = 5.18$ feet $=$ 5 feet, 2 inches from the top of the stem. The other rods, however, should be extended completely to the top of the wall. All rods must be terminated in standard hooks or extended far enough beyond that point at which they are no longer needed to resist stress so that full bond is developed. In addition to the vertical steel in the wall used to resist tension, there must be horizontal steel to prevent cracks due to shrinkage and temperature changes. These are usually one-half inch round rods placed two feet apart, on both sides of the wall, just inside the main vertical reinforcing.

(b) Base Pressure and Sliding

Before the base of the wall is designed in detail, its sizes are assumed and the wall is checked for sliding and base pressure. Usually the thickness of the base must be equal to or slightly greater than the thickness of the stem and the width of the base, depending upon the amount of earth fill, probably will be as long or greater than the height of the stem. So in this case assume the thickness of the base to be 1½ feet and the width of the base to be 15 feet.

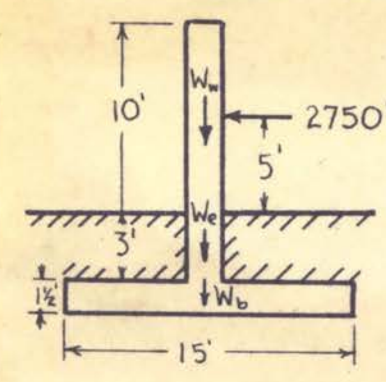

FIGURE 26

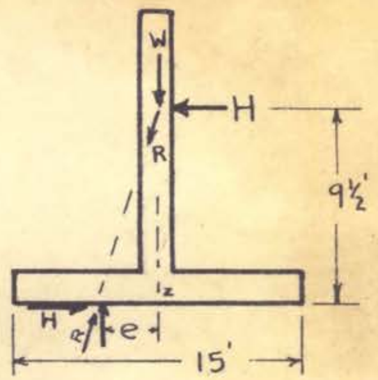

FIGURE 27

The forces acting on the wall are pictured in Figure 26.

Weight of wall (W_w) = 150 x 13 x 13.5/12 = 2192 pounds.

Weight of base (W_b) = 150 x 15 x 1.5 = 3375 pounds.

Weight of earth (W_e) = 100 x 14 x 3 = 4200 pounds.

Total Weight (W) = 9767 pounds.

The horizontal force H due to the blast is 2750 pounds and acts five feet from the top of the wall.

The resultant of the total weight W and the horizontal force H, must be checked to see if it intersects the middle third of the base. The distance e from the center of the base to the point where the resultant intersects the base is found by taking moments about the center of the base.

$$\Sigma M_z = 0 = 9767 \times e - 2750 \times 9.5$$

$$e = \frac{2750 \times 9.5}{9767} = 2.68 \text{ feet.}$$

As this is greater than $15/6 = 2.5$ feet, the resultant intersects the base outside of the middle third and there will be tension at one end of the base. The resultant must be made to fall inside the middle third. This can be done by either lengthening the base or by covering the base with more earth. Each has advantages and disadvantages. If more earth is added to that covering the base, the stem will have to be lengthened and might necessitate redesigning the stem. However, an extra foot of earth on the base will provide more additional vertical weight than that provided if the base is lengthened a foot. If the base is lengthened, the middle third is increased and the stem will not have to be redesigned.

In this example the base will be lengthened one foot. The new section is shown in Figure 28.

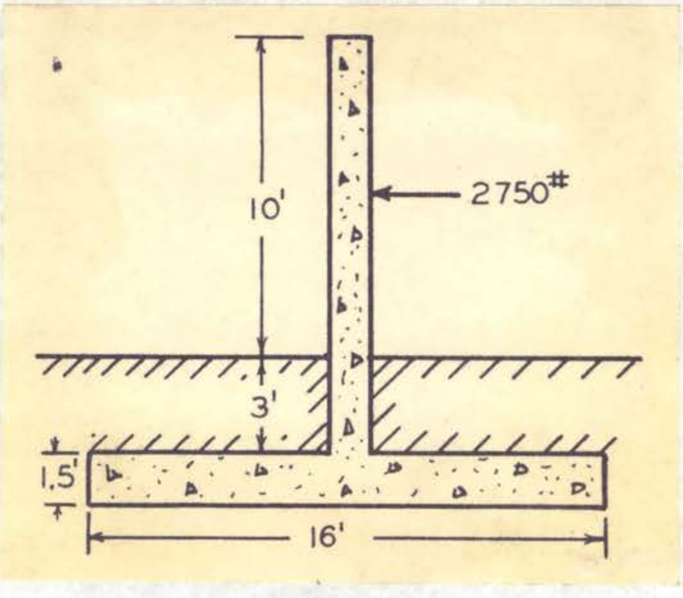

FIGURE 28

Weight of wall (W_W) = 150 x 13 x 13.5/12 = 2192 pounds.

Weight of base (W_b) = 150 x 16 x 1.5 = 3600 pounds.

Weight of earth (W_e) = 100 x 15 x 3 = 4500 pounds.

Total Weight (W) = 10,292 pounds.

The distance e from the center of the base to the point of intersection of the resultant and the base is:

$$e = \frac{2750 \times 9.5}{10,292} = 2.54 \text{ feet}$$

Since this is less than 16/6 equals 2.66 feet, the resultant intersects the base within the middle third. Therefore, the pressures on the base are:

$$\text{Maximum } f = \frac{W}{b}\left[1 + \frac{6e}{b}\right] = \frac{10,292}{16}\left[1 + \frac{6 \times 2.54}{16}\right] = 1.255 \text{ pounds per}$$

square foot.

$$\text{Minimum } p = \frac{W}{b}\left[1 - \frac{6e}{b}\right] = 31 \text{ pounds per square foot.}$$

The maximum base pressure is less than that allowed, hence, the base has a satisfactory width.

There is no need to check the tendency of the lateral blast load to slide the wall as a whole horizontally. Not only is there a large vertical load to cause a large frictional force between the base and the earth, but also there are four and one half feet of earth in front of the wall to prevent it from moving laterally.

(c) Design of the Base

Since the size of the base has already been assumed, it will now be designed for strength.

Each side of the base will be subjected to tension stress in both the top and bottom fibers and will act as cantilever beams. In Figure 29, page 83, are shown the pressures that one side of the base

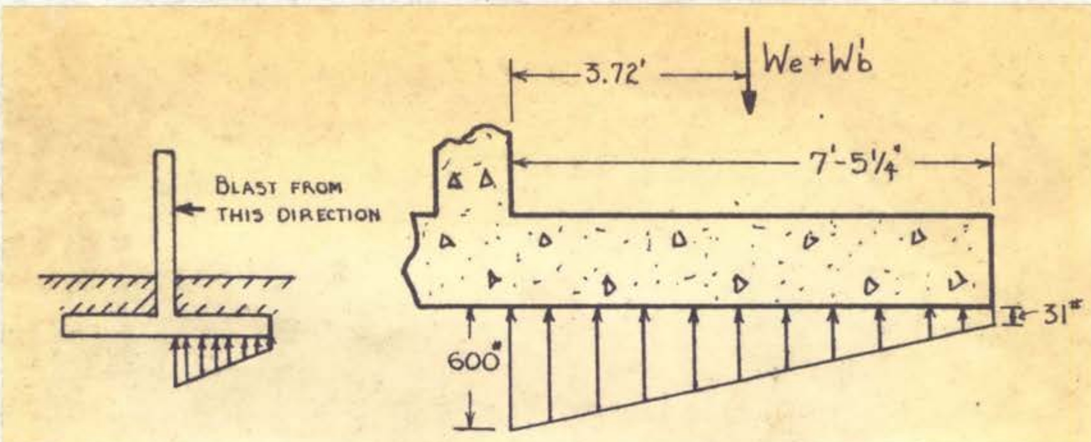

FIGURE 29. EARTH PRESSURE ON ONE-HALF OF BASE

is subjected to when the blast is so applied as to cause tension in the top fibers.

$$W_e = 100 \times 7.44 \times 3 = 2{,}232 \text{ pounds.}$$

$$W_b' = 150 \times 7.44 \times 1.5 = 1{,}675 \text{ pounds.}$$

W_b' is the weight of this part of the base and W_e is the weight of the earth above it. Taking moments about the section of the base where it joins the wall:

$$M = (W_e + W_b') \times \text{lever arm} - \text{area of pressure diagram} \times \text{lever arm.}$$

$$M = -(2232 + 1675) \times 3.72 + \frac{31 \times 7.44^2}{2} + \frac{569 \times 7.44}{2} \times \frac{7.44}{3}$$

$$= -8{,}442 \text{ foot pounds} = -101{,}400 \text{ inch pounds.}$$

As this moment is relatively small and as the base is very deep, the concrete will carry all of the compression load. Thus the steel rods in the bottom of the base will be neglected.

For a balanced design:

$$k = \frac{n\, f_c}{n\, f_c + f_s} = \frac{1350 \times 10}{13500 + 20{,}000} = .403 \quad \text{and}$$

$$j = 1 - 1/3\, k = 1 - .403/3 = .866$$

The depth of the base must be at least:

$$\text{Required } d = \sqrt{\frac{M}{f_c/2 \times j\ k\ b}} = \sqrt{\frac{101{,}400}{1{,}350/2 \times .866 \times .403 \times 12}}$$

$$= 5.99 \text{ inches.}$$

This is less than the actual depth of 15.5 inches.

$$\text{Required } A_s = \frac{M}{j\ d\ f_s} = \frac{101{,}400}{.866 \times 15.5 \times 20{,}000} = .377 \text{ square inches.}$$

One half inch round rods placed six inches center to center will not interfere with the vertical reinforcing in the wall and will provide 0.4 square inches of steel. Checking these rods then for bond and shearing unit stress:

$$V = W_e + W_b' - \text{area of pressure diagram.}$$

$$V = 2232 + 1675 - 31 \times 7.44 - 569 \times 7.44/2 = 1{,}558 \text{ pounds.}$$

$$u = \frac{1558}{3.14 \times .866 \times 15.5} = 37 \text{ pounds per square inch}$$

$$v = \frac{1558}{12 \times .866 \times 15.5} = 10 \text{ pounds per square inch}$$

Since those are less than their allowables the base will resist the negative moment.

When the wall is subjected to the other phase of the blast wave, this same part of the base will be subjected to tension in the bottom fibers. The pressures on the base, in this case, then, are shown in Figure 30, page 85.

The moment and shear on the section of the base where it joins the wall are:

$$M = -(2{,}232 + 1{,}675) \times 3.72 + \frac{686 \times 7.44^2}{2} + \frac{569 \times 7.44}{2} \times \frac{2 \times 7.44}{3}$$

$$= + 14{,}950 \text{ foot pounds} = 179{,}500 \text{ inch pounds.}$$

$$V = 2{,}232 + 1{,}675 - 686 \times 7.44 - \frac{569 \times 7.44}{2} = 3{,}333 \text{ pounds.}$$

The required depth of the base must be less than the actual 15.5 inches.

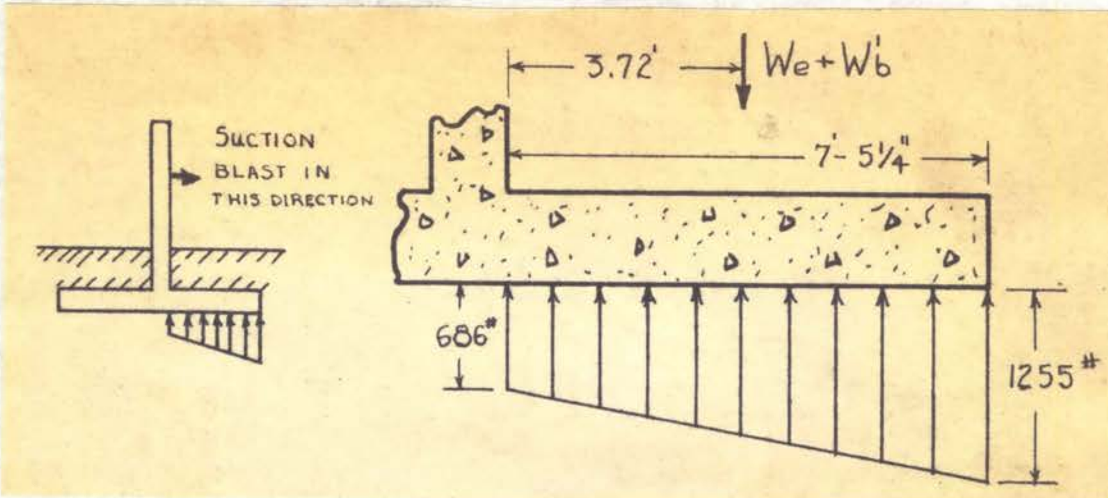

FIGURE 30. EARTH PRESSURE ON ONE-HALF OF BASE

$$\text{Required } d = \sqrt{\frac{179{,}500}{1{,}350/2 \times .866 \times .403 \times 12}} = 7.97 \text{ inches}$$

$$\text{Required } A_s = \frac{179{,}500}{.866 \times 15.5 \times 20{,}000} = .669 \text{ square inches.}$$

Five eights inch round rods placed every five inches will provide 0.74 square inches of steel. This area now will be checked for bond and shearing unit stress:

$$u = \frac{3{,}333}{4.7 \times .866 \times 15.5} = 53 \text{ pounds per square inch.}$$

$$v = \frac{3{,}333}{12 \times .866 \times 15.5} = 21 \text{ pounds per square inch.}$$

These unit stresses are satisfactory.

All of the reinforcing steel in the base must extend from one end of the base to the other and terminate in standard hooks. (See Figure 21, page 71) These hooks can be placed around the longitudinal steel for more stability. The longitudinal steel, one-half inch round rods, 18 inches center to center, are to prevent cracks in the base due to changes in temperature or shrinkage and to aid in distributing the pressure from the earthen fill on the base.

The base is much thicker than necessary and the stresses are quite low. It could be redesigned with a smaller thickness, but such would decrease the vertical weight and more earth would have to be put on the base or the base lengthened. The changes would result in no appreciable saving, so the base will be left as is.

CHAPTER X

BRICK WALLS

Contrary to most structural designers' expectations, brick walls, when properly laid with good bond, are highly resistant to blast and fragmentation. A British government publication reports:[1]

> Some preliminary experimental work has been done on the lateral resistance of brick panels from which it has been concluded tentatively that a nine inch panel, nine feet square, will resist ten pounds per square inch. This assumes brick work in cement mortar in good condition and good workmanship....Most brick walls are able to withstand blast from a five hundred pound medium case bomb at fifty feet. This is partly because high stresses cannot arise unless the load is applied to the structure long enough to allow the strains to be built up and partly because the lateral strength of brickwork is higher than is commonly supposed.

In 1940 and 1941 the United States Army conducted tests to determine the resistance of different types of wall panels to fragmentation and the blast effect of bombs.[2] One of the panels tested was a twelve inch solid brick wall supported by a concrete frame. A 600 pound bomb, exploding at 50 feet, damaged the panel very little. However, a 300 pound bomb detonated at 25 feet later blasted a large hole in the top of the panel. No fragments penetrated the entire thickness of the wall and the small part of the wall that failed was due to bond failure, rather than to brick failure.

Although there are considerable data on the strength of brick construction in compression, there is very little information on the tensile or flexural strength of brickwork. Until extensive tests have been made on panels laid with different bonds and with various kinds of mortar, brick panels can not

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1 Anonymous, Air Raid Precautions, Handbook 5, Structural Defense, p. 11.

2 Anonymous, Report of Bomb Tests on Materials and Structures, p. 3.

FIGURE 31. TWELVE INCH BRICK WALL AFTER SUBJECTED TO BOMB BLAST

be designed with any degree of certainty.

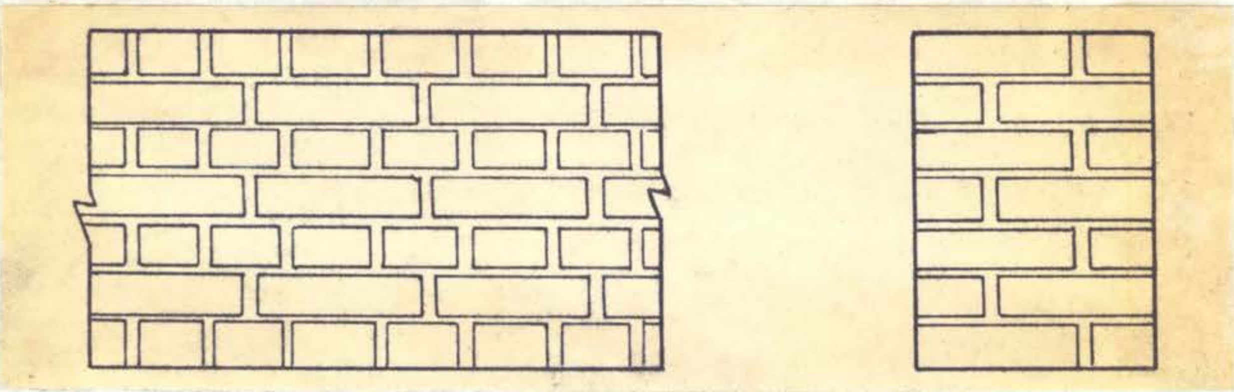

FIGURE 32. ENGLISH BOND BRICK WORK

However, if constructed correctly, a 12½ inch solid brick wall will resist all but the most severe blast. The walls should be fairly well supported on all four sides and laid with a good bond. English or Flemish type bond that has a high percentage of headers increases the resistance of the brick work to blast and should be used in preference to common bond. It is also advisable to use cement mortar of very good quality and to make sure all joints are thoroughly filled. In most of the brick walls, the line of failure followed the mortar joint, instead of extending through the brick. This indicates an inherent weakness in the mortar which could be strengthened by vertical reinforcing steel.

Brickwork has some advantages over other types of building materials. Clays, suitable for brick, are found almost universally. Brick weighs approximately 120 pounds per cubic foot, so there is some saving in weight over concrete for the same wall thickness. Its greatest advantage over steel and reinforced concrete, though, is the ease with which repairs can be made on brickwork. In some instances this is a very important point to be considered. In time of war steel becomes very scarce and brickwork, that requires a minimum of steel, becomes a very important construction material.

CHAPTER XI

SUMMARY AND CONCLUSIONS

Summary

There are three main steps that must be followed before a wall to resist bomb blast may be designed. They are: (1) calculation of the frequency of vibration of the wall panel, (2) determination of the equivalent static load that will produce the same stresses in the wall as the bomb blast, and (3) analysis of the wall panel to determine the critical moments, shears, and stresses.

Once the critical stresses have been determined the blast wall can be designed as any common wall by considering a few new principles introduced by a lateral impact load. These are: (1) Wall panels should be designed equally strong to resist the suction phase of the blast as well as the positive phase of the blast. (2) For a practical, economical design, a blast wall should never be designed to resist a heavier bomb than 500 pounds or for an explosion closer than 40 feet away. (3) To provide full protection against both blast and flying fragments, blast walls should never be less than the recommended thickness to resist bomb fragments. (4) Reinforced concrete walls must have reinforcing steel on both sides, since a blast causes both positive and negative moments on the wall. (5) A low allowable bond stress should be used and all reinforcing steel, crossing each other, should be fastened together, preferable by welding. The rapid increase in the stresses in reinforcement and the quick reversal of these stresses may cause breaking of the bond which under ordinary conditions would be quite satisfactory. (6) An additional amount of weight must be provided on both sides of the base of a cantilever wall to prevent it from overturning in either direction.

Conclusions

It is quite impractical to provide complete bomb proof protection for the entire population during an air raid. The cost of building massive reinforced concrete shelters or deep underground shelters is enormous. In addition there are always other essential jobs that workmen could be doing if they were not building bomb proof shelters. There must be then a balanced program of reasonable protection. In principle, the protection to be furnished to any town or industry should bear relation to its liability to air attack, its vulnerability to such attacks, and its value to the nation's war effort.

It is uneconomical and impracticable to make all walls bomb proof against direct hits of the larger size bombs. The major portion of the damage to buildings in the last war was due, not to the result of direct hits, but to the blast and fragmentation of near-missed bombs. In general, protection should be probided against blast from a 500 pound bomb bursting 40 feet away. This does not mean that heavier bombs will not be used in air attacks, nor does it mean that bombs will not explode at a closer distance. Walls can be designed and built using these values at a reasonable increase in cost. The cost of walls built to resist the blast of larger bombs nearer than 40 feet will increase materially. In that case the walls probably should be designed to resist direct bomb hits instead of blast. In any case, the design should be based on the importance of the object being protected.

The types of material to be used in a blast wall must be based on many considerations. The availability of materials and workmen during a war is not the same as in peacetime. Steel may become very scarce and concrete might be subjected to military priorities. Workmen become scarce because they go to work in more critical wartime industries. The nations's transportation system may be so overloaded or destroyed that materials may not be shipped long distances.

Therefore some materials may be plentiful in one part of the country while a shortage may occur in others. These and many more considerations must be taken into account by a designer when selecting the type of material to be used in building a blast wall.

The majority of buildings can be built to resist the effects of bomb blast at only a slight increase in cost. In peacetime, however, since new buildings must compete with existing buildings of the same type and purpose, building contractors are quite reluctant to build structures that will resist bomb blast at even a slight increase in cost. At present, building materials and labor costs are very high. By adding an additional amount, the cost would be far out of reach of the average business concern. The larger business companies can afford to build blast resistant buildings and in some localities are doing this. Just what should be done is not known. One solution, of course, would be for the government to subsidize this type of construction in peacetime, since it will greatly benefit from it in time of war.

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