THE DERIVATION OF AN EQUATION FOR THE ELASTIC RESTRAINT OF THE TOP AND SEAT-ANGLE TYPE OF SEMI-RIGID CONNECTION

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1950

Submitted to the Faculty of the Graduate School of the Oklahoma Agricultural and Mechanical College in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

1953

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PREFACE

Since the results of experiments for the semi-rigid connection were published in Professor Cyril Batho's "First, Second and Final Reports".[1], the problem and the importance of the semi-rigid connection have been of much interest to structural engineers. In January 1935 the first paper[2] relating to semi-rigid-beam-to-column connections by Professor J. Charles Rathbun was published in the Proceedings of the American Society of Civil Engineers and in the Transactions of the American Society of Civil Engineers, Volumn 101, 1936. Professor John E. Lothers proposed the method[3] of obtaining the semi-rigid connection constant, Z, by analysis and computation instead of by the expensive and time-consuming laboratory tests. Professor Lothers derived an equation for Z for the web-angle type of connection in his paper "Elastic Restraint Equations For Semi-Rigid Connections".

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[1] Professor Cyril Batho, First, Second and Final Reports, Steel Structures Research Committee, Department of Scientific and Industrial Research. H.M. Stationery Office, London, 1931-1936.

[2] J. Charles Rathbun, "Elastic Properties of Riveted Connections." Transactions of the American Society of Civil Engineers. Vol. 101, (1936), pp. 524-596.

[3] John E. Lothers, "Elastic Restraint Equations for Semi-Rigid Connections." Transactions of the American Society of Civil Engineers. Vol. 116, (1951), pp. 480-503.

The writer's decision to derive the Z-equation for the top and seat angle type of connection came about as a result of courses taken under Professor J. E. Lothers of the School of Architectural Engineering at Oklahoma A. and M. College.

Grateful acknowledgement is due to Professor Lothers for his advice and encouragement as well as for the procedure laid down in his paper[4].

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[4] J. E. Lothers, Transactions of the American Society of Civil Engineers, Vol. 116, pp. 480-503.

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INTRODUTION

In the analysis of semi-rigid connected building frames the same method of analysis may be employed as that used in rigid connected buildings. The method is modified, however, to compensate for the strain in the semi-rigid connection by means of a connection constant, Z. The latter constant has been established through laboratory measurements for many semi-rigid connections. These have been published[5,6,7] and are available to the structural designer. Obviously it would be impracticable and very expensive to measure every semi-rigid connection. In the thesis which follows an equation has been derived by which the connection constant, Z, may be computed for one type of semi-rigid connection; namely the top and seat angle connection.

The reciprocal of Z for a given semi-rigid connection is the slope of the moment-rotation curve for the same connection. The test of the accuracy of an equation for Z,

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[5]Professor Cyril Batho, First, Second and Final Reports, 1931-1936.

[6]J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 524-596.

[7]Robert A. Hechtman and Bruce G. Johnston, Riveted Semi-Rigid Beam-to-Column Building Connections, Progress Report Number 1, Committee on Steel Structures Research, American Institute of Steel Construction, 1947.

then, may be established by using the equation in the computıon for Z for a given semi-rigid connection and then plotting its reciprocal ($^1/_Z$) on the laboratory based moment-rotation curve for the same connection. As indicated above, many such curves are available to the structural designer[8,9,10]. Professor Rathbun's moment-rotation curves[11] have been used to establesh the accuracy of the equation derived on the following pages, see Figures 12, 13 and 14.

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[8] Professor Cyril Batho, First, Second and Final Reports, 1931-1936.

[9] J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 524-596.

[10] Robert A. Hechtman and Bruce G. Johnston, Riveted Semi-Rigid Beam-to-Column Building Connections, Progress Report Number 1, 1947.

[11] J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 540 and 541.

ANALYSIS

I. Introduction.

In order to derive an equation for the elastic restraint of the top and seat angle type of semi-rigid connection, it is assumed that the analysis of the connection is based on the bending strength of the connecting angles.[12,13]

Due to the bending moment of the beam the connecting angles are subjected to a pull in the beam connected leg, (Fig. 1), and the deflection, Δ_B (Fig. 1), at the heel of the top angle away from the column is a maximum.

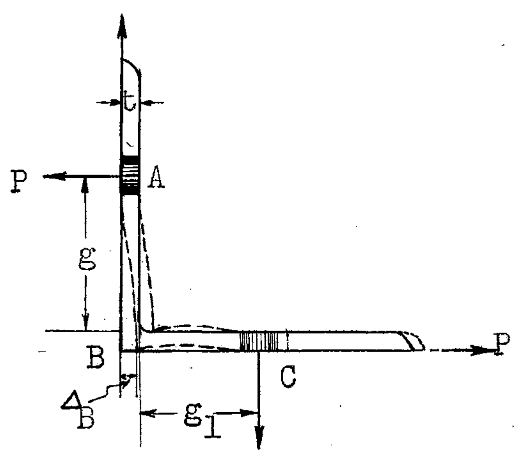

Figure 1.

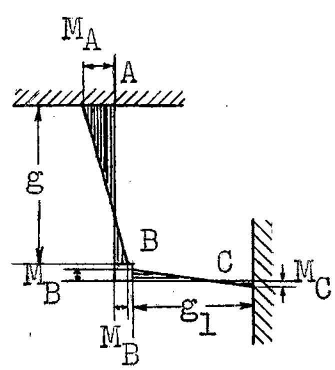

Figure 2.

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[12] J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 524-596.

[13] J. E. Lothers, Transactions of the American Society of Civil Engineers, Vol. 116, pp. 480-503.

By applying the slope deflection method, the amount of this deflection may be derived. After the neutral axis being located, the angle rotation or the angle of strain, ϕ, can be expressed.

Finally the semi-rigid connection constant, or the angle change for unit moment, Z, will be derived as ϕ being divided by the resisting moment, M, of the connection.

II. The Critical Moment in the Legs of the Connection Angle.

The notation representing the dimensions of the angle are shown in Figure 1. By the slope deflection method the bending moments at points A, B, and C are expressed by the known values, P, g, and g_1, and the ratio of the deflection of the heel of the angle with respect to point A to the length of point AB, r, is also expressed by P, g and g_1.

A. Slope Deflection Equations.

The effective length of AB and BC are g and g_1 respectively. From Figure 1 the known values are:

$$L_{AB} = g \qquad L_{BC} = g_1$$

$$\theta_A = 0 \qquad \theta_C = 0$$

and

$$r_{BC} = 0.$$

The basic slope deflection equations become:

$$M_{AB} = \frac{2EI}{L_{AB}}(2\theta_A + \theta_B - 3r_{AB}) = \frac{2EI}{g}(\theta_B - 3r_{AB}) \tag{1}$$

$$M_{BA} = \frac{2EI}{L_{BA}}(\theta_A + 2\theta_B - 3r_{BA}) = \frac{2EI}{g}(2\theta_B - 3r_{AB}) \tag{2}$$

$$M_{BC} = \frac{2EI}{L_{BC}}(2\theta_B + \theta_C - 3r_{BC}) = \frac{2EI}{g_1}(2\theta_B) \qquad (3)$$

$$M_{CB} = \frac{2EI}{L_{CB}}(\theta_B + 2\theta_C - 3r_{CB}) = \frac{2EI}{g_1}(\theta_B) \qquad (4)$$

B. Mechanics Equations.

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

and

$$M_{AB} + M_{BA} = P\,g \qquad (6)$$

C. Calculation.

Equations (1) and (3) are substituted into Equation (5) which becomes:

$$\frac{2EI}{g}(2\theta_B - 3r_{AB}) + \frac{2EI}{g_1}(2\theta_B) = 0$$

which may be simplified into:

$$3g_1 r_{AB} = 2\theta_B(g + g_1)$$

and therefore:

$$r_{AB} = \frac{2}{3}\theta_B \frac{g + g_1}{g_1} \qquad (7)$$

Equations (1) and (2) are substituted into Equation (6) which becomes:

$$\frac{2EI}{g}(\theta_B - 3r_{AB}) + \frac{2EI}{g}(2\theta_B - 3r_{AB}) = P\,g$$

which is simplified into:

$$2EI\theta_B - 6EI\,r_{AB} + 4EI\theta_B - 6EI\,r_{AB} = P\,g^2$$

and therefore:

$$6EI\theta_B - 12EI\, r_{AB} = P\, g^2 \qquad (8)$$

The value of r_{AB}, Equation (7), is substiteted in Equation (8):

$$6EI\theta_B - 12EI\left(\frac{2}{3}\,\theta_B \frac{g + g_1}{g_1}\right) = P\, g^2$$

which is simplified into:

$$6EI\, g_1\theta_B - 8EI\, g\, \theta_B - 8EI\, g_1\theta_B = P\, g^2 g_1$$

then the value of θ_B is:

$$\theta_B = -\frac{P\, g^2}{2EI}\,\frac{g_1}{4g + g_1} \qquad (9)$$

The value of θ_B is substituted into Equation (7):

$$r_{AB} = \frac{2(g + g_1)}{3\, g_1}\left(-\frac{P\, g^2}{2EI}\,\frac{g_1}{4g + g_1}\right)$$

the value of r_{AB} is:

$$r_{AB} = -\frac{P\, g^2}{3EI}\,\frac{g + g_1}{4g + g_1} \qquad (10)$$

D. The Critical Moment.

These values of θ_B and r_{AB} are substituted in Equations (1), (2), (3) and (4) to solve for the critical moment M_A, M_B and M_C.

$$M_A = M_{AB} = \frac{2EI}{g}\,(\theta_B - 3r_{AB})$$

and

$$M_A = \frac{2EI}{g}\left[-\frac{P\, g^2 g_1}{2EI(4g + g_1)} - 3\,\frac{-P\, g^2(g + g_1)}{3EI(4g + g_1)}\right]$$

which is simplified:

$$M_A = P\, g \frac{2g + g_1}{4g + g_1} \tag{11}$$

and

$$M_B = M_{BC} = M_{CB} = \frac{2EI}{g_1}(2\theta_B)$$

$$= \frac{2EI}{g_1}(2)\left[-\frac{P\, g^2 g_1}{2EI(4g + g_1)}\right]$$

Which is simplified:

$$M_B = P\, g \frac{2g}{4g + g_1} \tag{12}$$

and

$$M_C = M_{CB} = \frac{2EI}{g_1}(\theta_B)$$

$$= \frac{2EI}{g_1}\left[-\frac{P\, g^2 g_1}{2EI(4g + g_1)}\right]$$

which is simplified:

$$M_C = P\, g \frac{g}{4g + g_1} \tag{13}$$

III. The Deflection, Δ_B, of the Heel of the Top Angle.

By definition, r is the ratio of the deflection of the heel of the angle to the length of g, i. e. $r_{AB} = \frac{\Delta_B}{g}$.

The value of r_{AB} is substituted in

$$\Delta_B = r_{AB} \times g$$

and the minus sign is ignored.

$$\Delta_B = \frac{P\, g^3}{3EI} \frac{g + g_1}{4g + g_1} \tag{14}$$

IV. The Horizontal Pull, P.

The connection is based on the strength of the connecting angles and the critical bending moment is at the rivet line A which is shown in Figures 1, 4 and 5. The resistant moment, M, at the rivet line A of the top angle is:

$$M_A = \frac{I\ s}{c} = \frac{I\ s}{t/2} \tag{15}$$

substituting the value of M_A from Equation (15) into Equation (11) and solve for P,

$$\frac{I\ s}{t/2} = P\ g \frac{2g + g_1}{4g + g_1}$$

from which

$$P = \frac{2I\ s}{g\ t} \frac{4g + g_1}{2g + g_1} \tag{16}$$

and substituting $I = \frac{bt^3}{12}$ into Equation (16), then,

$$P = \frac{bt^2 s}{6g} \frac{4g + g_1}{2g + g_1} \tag{17}$$

V. The Angle of Strain, Ø.

Referring to Figure 5,

$$\emptyset = \frac{\Delta_B}{y-g-t} \tag{18}$$

The Value of Δ_B from Equation (14) is substituted into Equation (18) and Ø becomes:

$$\emptyset = \frac{P\ g^3}{3EI(y-g-t)} \frac{g + g_1}{4g + g_1} \tag{19}$$

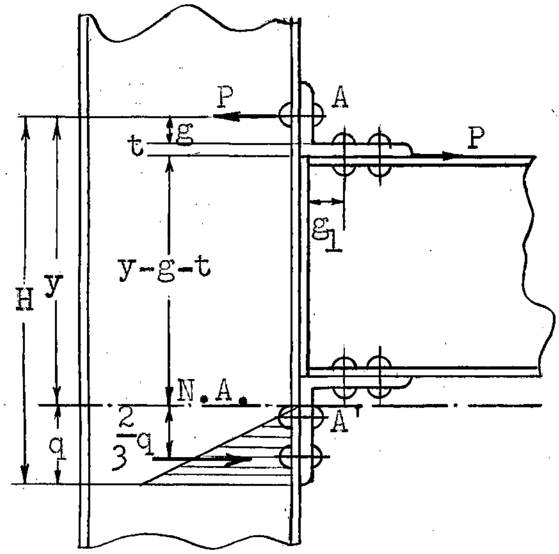

Figure 3.

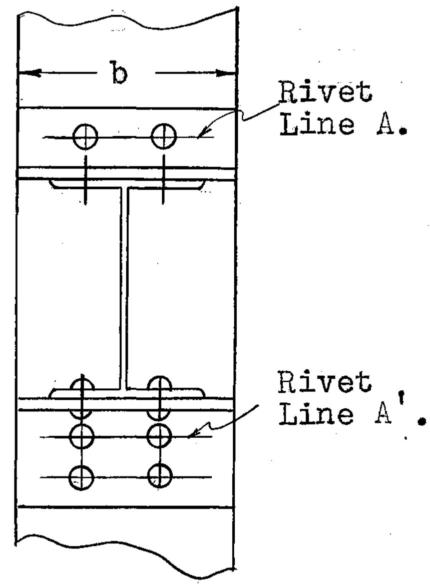

Figure 4.

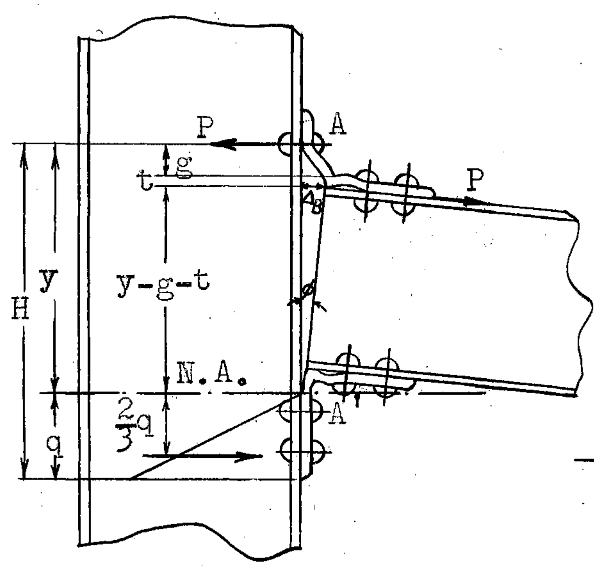

Figure 5.

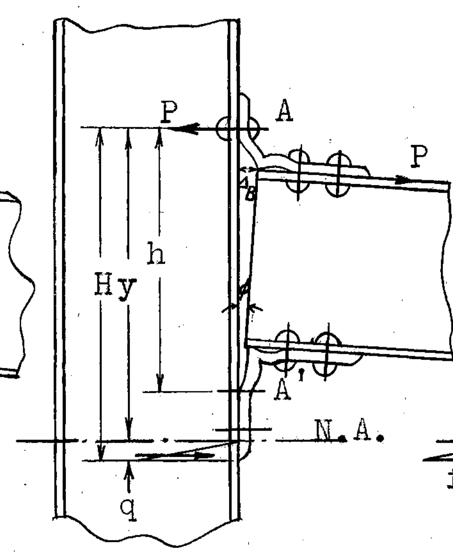

Figure 6.

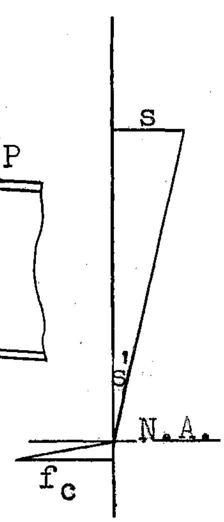

Figure 7.

The value of P from Equation (16) is substituted into Equation (19), then,

$$\phi = \frac{2I\ s}{g\ t}\ \frac{4g + g_1}{2g + g_1}\ \frac{g^3}{3EI(y-g-t)}\ \frac{g + g_1}{4g + g_1}$$

which may be simplified into:

$$\phi = \frac{2\ s\ g^2}{3E\ t(y-g-t)}\ \frac{g + g_1}{2g + g_1} \qquad (20)$$

VI. The Resisting Moment, M, of the Connection.

Since the strength of the angle governs the resisting moment of the connection. The value of P in Equation (17) is a maximum for the safe resisting moment, M, of the connection.

From Figure 3 the following relation can be obtained:

$$M = P(y + \frac{2}{3}\ q) \qquad (21)$$

The value of P from Equation (17) is substituted in Equation (21):

$$M = \frac{b\ s\ t}{6\ g}\ \frac{4g + g_1}{2g + g_1}(y + \frac{2}{3}\ q) \qquad (22)$$

VII. The Neutral Axis.

The same method as the transformed section in reinforced concrete beam analysis to locate the neutral axis[14] is used.

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14 J. E. Lothers, Transaction of the American Society of Civil Engineers, Vol. 116, pp. 480-503.

The shear area, $\underline{A_s}$, along rivet line A is:

$$A_s = b\ t$$

and the compression area, $\underline{A_c}$, of the bottom seat angle is:

$$A_c = b(H - y)$$

But the relation between the bending stress and the shearing stress is that the ratio of the bending stress, s, to the shearing stress is equal to n.

$$n = \frac{s}{v}$$

and v is equal shearing force divided by the area, $\underline{A_s}$:

$$v = \frac{P}{b\ t}$$

substituting the value of P from Equation (17), then,

$$v = \frac{1}{b\ t}\ \frac{b\ s\ t^2}{6g}\ \frac{4g + g_1}{2g + g_1} = \frac{s}{n}$$

simply and solve for n:

$$n = \frac{6g(2g + g_1)}{t(4g + g_1)} \qquad (23)$$

By the transformed area method the compression area becomes:

$$A_c' = n\ b(H - y)$$

In order to locate the neutral axis expressed by the distance, y, down from the rivet line A, the equation of the static moment of the shear area, $\underline{A_s}$, about the neutral axis and that of the transformed compression area is established as following:

$$\sum M_{N.A.} = 0$$

Case I: The neutral axis is above the rivet of the column connected leg of the seat angle (Figures 3 and 5).

$$b\ t\ y = n\ b\ (H - y)\left(\frac{H - y}{2}\right) \qquad (24)$$

Simplifying into a quadratic equation:

$$n\,y^2 - 2(nH + t)y + n\,H^2 = 0$$

which is solved for y

$$y = \frac{(n\,H + t) \pm \sqrt{(n\,H + t)^2 - (n)\,(n\,H^2)}}{n}$$

the positive sign gives an unreasonable value for y and the negative sign is used:

$$y = \frac{n\,H + t - \sqrt{(2n\,H + t)t}}{n} \tag{25}$$

Case II: The neutral axis is under the rivet of the column connected leg of the seat angle (Figure 6). Referring Figure 7 the shear stress is proportional to the distance from the neutral axis.

$$\frac{s'}{s} = \frac{y - h}{y}$$

Then the transformed shear area for rivet line A' is :

$$b\,t\,\left(\frac{y - h}{y}\right)$$

Applying $\sum M_{N.A.} = 0$

$$b\,t\,y + b\,t\,\left(\frac{y - h}{y}\right)(y - h) = n\,b\,(H - y)\left(\frac{H - y}{2}\right)$$

simplying into:

$$b\,t\,y + b\,t\,\frac{(y - h)^2}{y} = n\,b\,(H - y)\left(\frac{H - y}{2}\right) \tag{26}$$

Where $\frac{(y - h)^2}{y}$ is very small as compared to y. Its importance is less than that of other factors neglected in derivation of Equations (24) and (26). Among the latter may be considered rivet slip, bending in the column flange, etc. Therefore, for all practical considerations, the second term of Equation (26) may be assumed to vanish which makes it identical with Equation (24). The latter equation will be used in locating the neutral axis.

VIII. The Semi-Rigid Connection Factor Z.

By defination, the factor Z is the angle of strain for unit moment, therefore:

$$Z = \frac{\phi}{M} \qquad (27)$$

The values of ϕ and M from Equations (20) and (22) are substituted in Equation (27):

$$Z = \frac{\dfrac{2\,s\,g^2(g + g_1)}{3E\,t(y - g - t)(2g + g_1)}}{\dfrac{6g(2g + g_1)}{b\,s\,t^2\,(4g + g_1)(y + {}^2/_3\,q)}}$$

which is simplified into:

$$Z = \frac{4\,g^3}{Et^3 b(y - g - t)(y + \frac{2}{3}\,q)}\;\frac{g + g_1}{4g + g_1} \qquad (28)$$

IX. The Comparison of the Equation with the Published Laboratory Results.

As a check on the effectiveness of Equation (28), it will be used to compute the Z-values for specimens 8, 9 and 10, (Figure 8, 9, 10 and 11) of Professor Rathbun's paper, "Elastic Properties of Riveted Connections", published in the Transactions of the American Society of Civil Engineers, Volume 101, 1936. Also the resulting slope of

Specimen 8, 9and 10.[15]

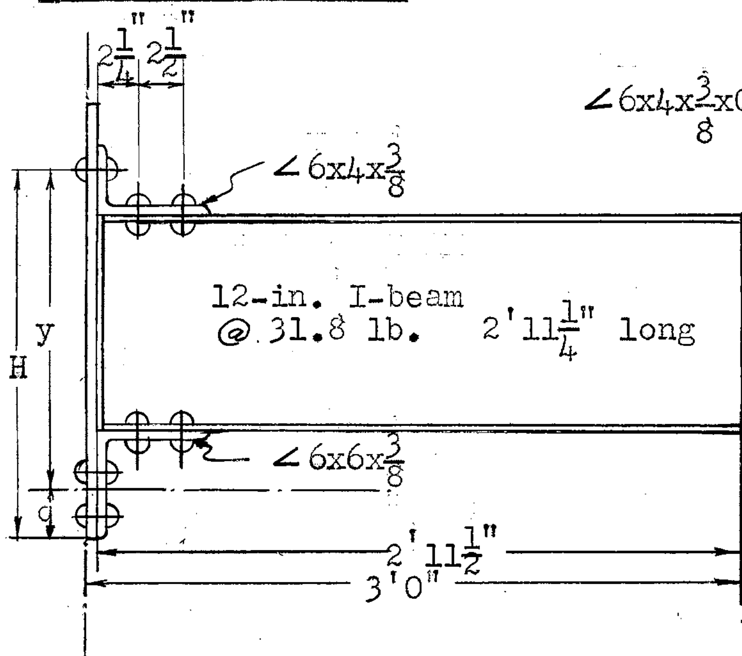

Figure 8.

Specimen 8.

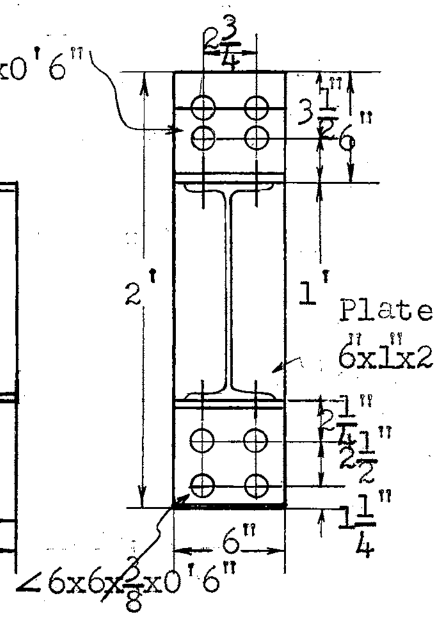

Figure 9.

Specimen 9.

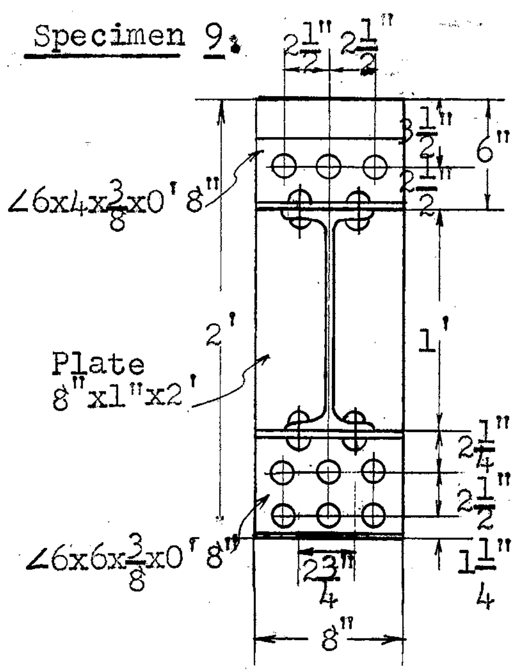

Figure 10.

Specimen 10.

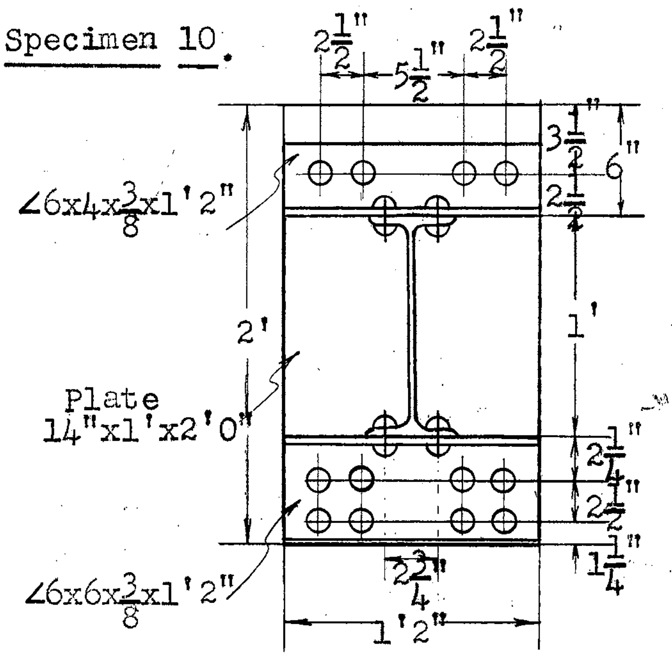

Figure 11.

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[15] J. Charles Rathbun, *Transactions of the American Society of Civil Engineers*, Vol. 101, p. 527.

the moment-rotation curve ($1/Z$) will be plotted on reproductions of Professor Rathbun's curves[16] for the above mentioned specimens in Figures 12,13 and 14.

Referring to Figures 8, 9, 10 and 11, the following data can be obtained which are common to all three specimens:

$t = \frac{3}{8}$ in. = 0.375 in.

$g = 2\frac{1}{2} - t = 2.5 - 0.375 = 2.1250$ in.

$g_1 = 2\frac{1}{4} - t = 2.25 - 0.375 = 1.8750$ in.

and

$H = g + t + 12 + 6 = 2.5 + 12 + 6 = 20.50$ in.

From Equations (23), (25) and (28), the values of $\underline{Z}$ and $\frac{1}{\underline{Z}}$ may be obtained.

From Equation (23):

$$n = \frac{6g(2g + g_1)}{t(4g + g_1)} = \frac{6 \times 2.1250(2 \times 2.1250 + 1.8750)}{0.375\ (4 \times 2.1250 + 1.8750)}$$

$$= 20.07$$

From Equation (25):

$$y = \frac{n\ H + t - \sqrt{(2\ n\ H + t)\ t}}{n}$$

$$= \frac{20.07 \times 20.50 + 0.375 - \sqrt{(2 \times 20.07 \times 20.50 + 0.375)0.375}}{20.07}$$

$$= 19.64 \text{ in.}$$

From Equation (28):

$$Z = \frac{4\ g^3}{E\ t^3 b(y - g - t)(y + {}^2/_3\ q)}\ \frac{g + g_1}{4g + g_1}$$

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16 J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 524-596.

whence:

$$\frac{1}{Z} = \frac{E\,t^3 b(y - g - t)(y + \frac{2}{3}q)}{4\,g^3} \cdot \frac{4g + g_1}{g + g_1} \qquad (29)$$

The latter equation is applied to Professor Rathbun's specimens 8, 9 and 10 below.

Specimen 8.

b = 6 in.

From Equation (29):

$$\frac{1}{Z} = \frac{29 \times 10^6 (0.375)^3 6(19.64 - 2.125 - 0.375)(19.64 + \frac{2}{3} \times 0.857)}{4(2.125)^3}$$

$$\times \frac{4 \times 2.125 + 1.875}{2.125 + 1.875}$$

$$= 214.8 \times 10^6$$

Specimen 9.

b = 8 in.

The values are the same as specimen 8 except b, and the value of $\frac{1}{Z}$ is proportional to b. Therefore:

$$\frac{1}{Z} = 214.8 \times 10^6 \times \frac{8}{6} = 286.4 \times 10^6$$

Specimen 10.

b = 1 ft. 2 in. = 14 in.

$$\frac{1}{Z} = 214.8 \times 10^6 \times \frac{14}{6} = 501.2 \times 10^6$$

The slopes of the moment-rotation curve, $1/Z$, calculated by the derived Equation (28) or Equation (29) are plotted on Professor Rathbun's curves reproduced in Figures 12, 13 and 14.

As an additional check on the effectiveness of Equation

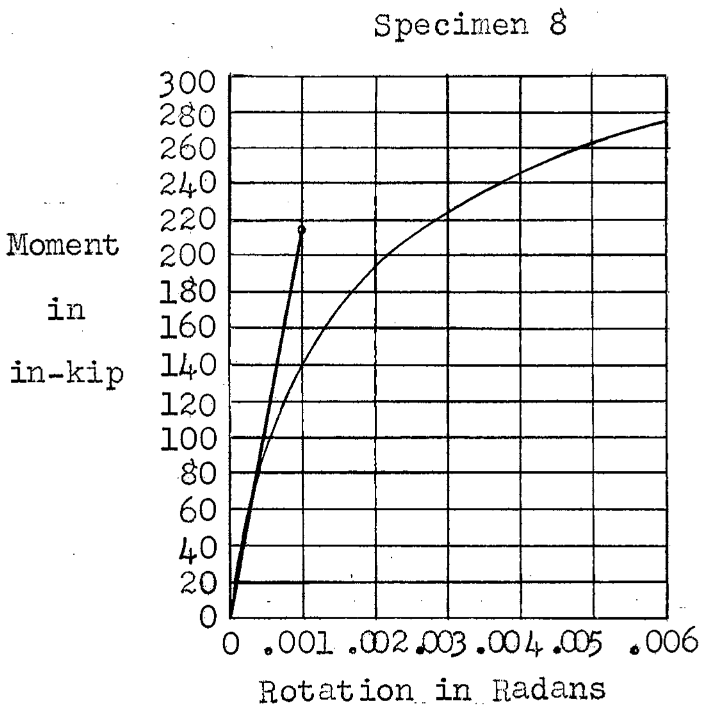

Figure 12.

<u>Specimen</u> <u>8</u>:

Ø = 0.001

M = 214800 in-lb.

<u>Specimen</u> <u>9</u>:

Ø = 0.002

M =572800 in-lb.

<u>Specimen</u> <u>10</u>:

Ø = 0.001

M = 501200 in-lb.

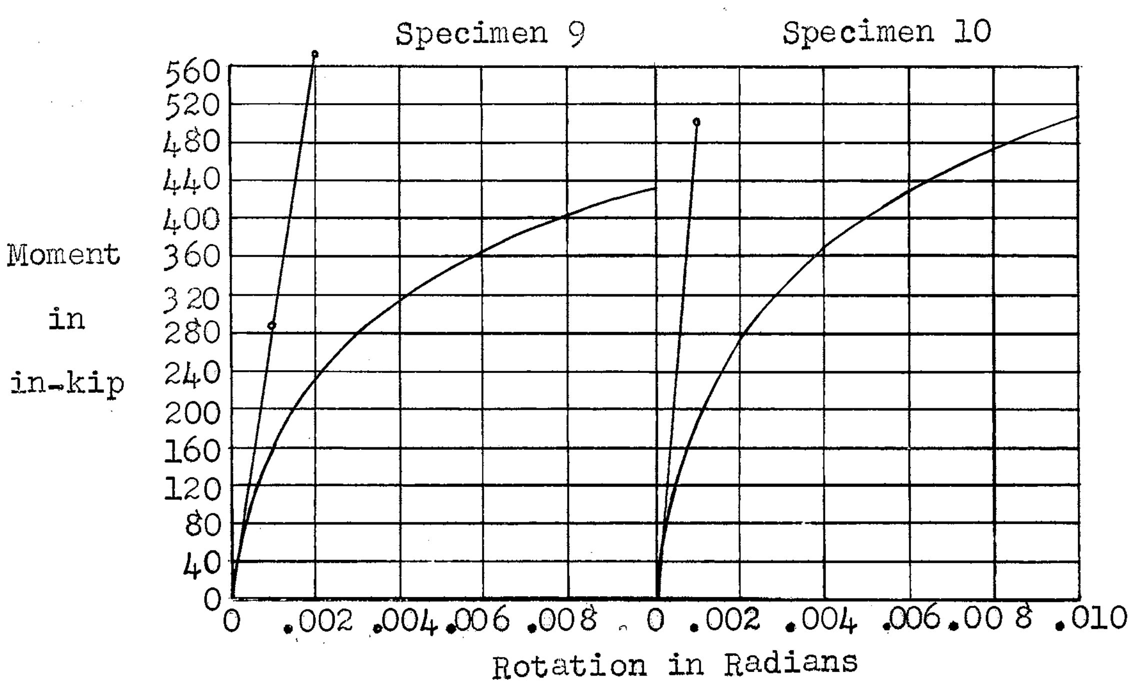

Figure 13. Figure 14.

(29) it was applied to several of the moment-rotation curves of Messrs. Hechtman and Johnston[17] with equally good results.

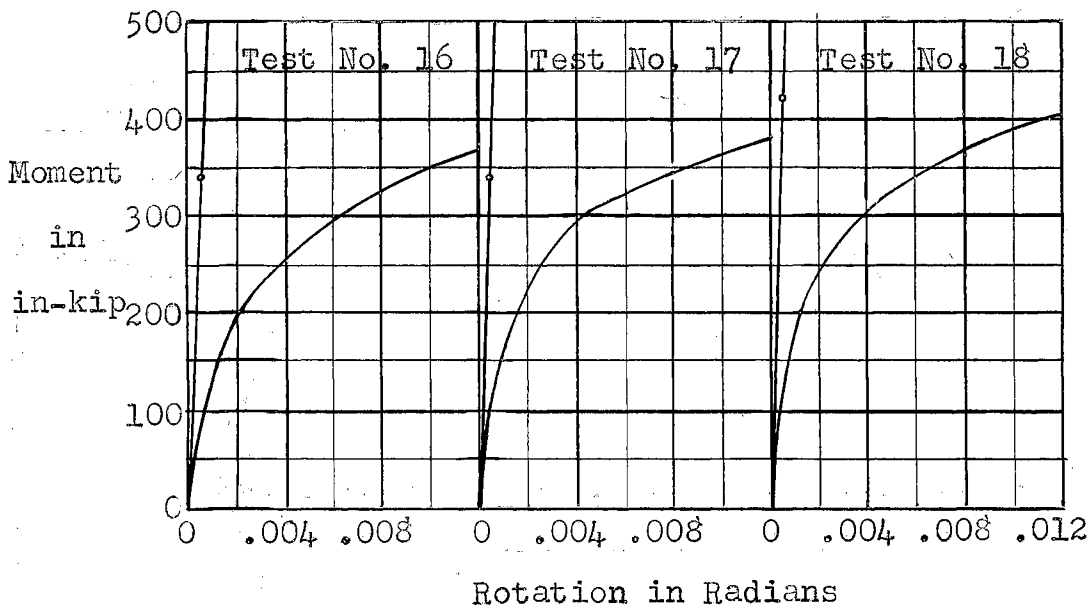

Figure 15.

The data of these specimens are in Table I:

Table I.

Specimen	t	g	g_1	H	n	y	b	$\frac{1}{Z\ 10^6}$	w*
Test No. 16	$\frac{1}{2}$	2.000	1.750	20.625	14.15	19.42	$6\frac{3}{4}$	682	
Test No. 17	$\frac{1}{2}$	2.000	1.750	20.625	14.15	19.42	$6\frac{3}{4}$	682	
Test No. 18	$\frac{1}{2}$	2.000	1.750	21.000	14.15	19.85	8	845	
Test No. 11	$\frac{1}{2}$	2.000	1.750	26.625	14.15	25.05	10	1742	$7\frac{1}{2}$
Test No. 23	$\frac{5}{8}$	1.875	1.625	24.750	10.60	22.65	10	3000	$7\frac{1}{4}$
Test No. 35	$\frac{5}{8}$	1.875	1.625	20.625	10.60	19.12	$8\frac{1}{2}$	1995	

*The length of the seat angle is different from the top

17 Robert A. Hechtman and Bruce G. Johnston, Riveted Semi-Rigid Beam-To-Column Building Connections, Progress Report Number 1, 1947. pp. 90-97,102,103,114 and 115.

angle and then the equation for locating the neutral axis, Equation (25), should be derived as follows:

Let b = the length of the top angle.

w = the length of the seat angle .

Equation (24) is change to:

$$b\ t\ y = n\ w\ (H - y)\left(\frac{H - y}{2}\right)$$

Simplifying into a quadratic equation:

$$n\ w\ y^2 - 2(w\ n\ H + b\ t)\ y + n\ w\ H^2 = 0$$

which is solved for y

$$y = \frac{n\ w\ H + b\ t - \sqrt{(2\ w\ n\ b\ H + b^2\ t)t}}{n\ w} \qquad (30)$$

Equation (30) is applied for locating the neutral axis of the specimens of Test No. 11 and Test No. 23.

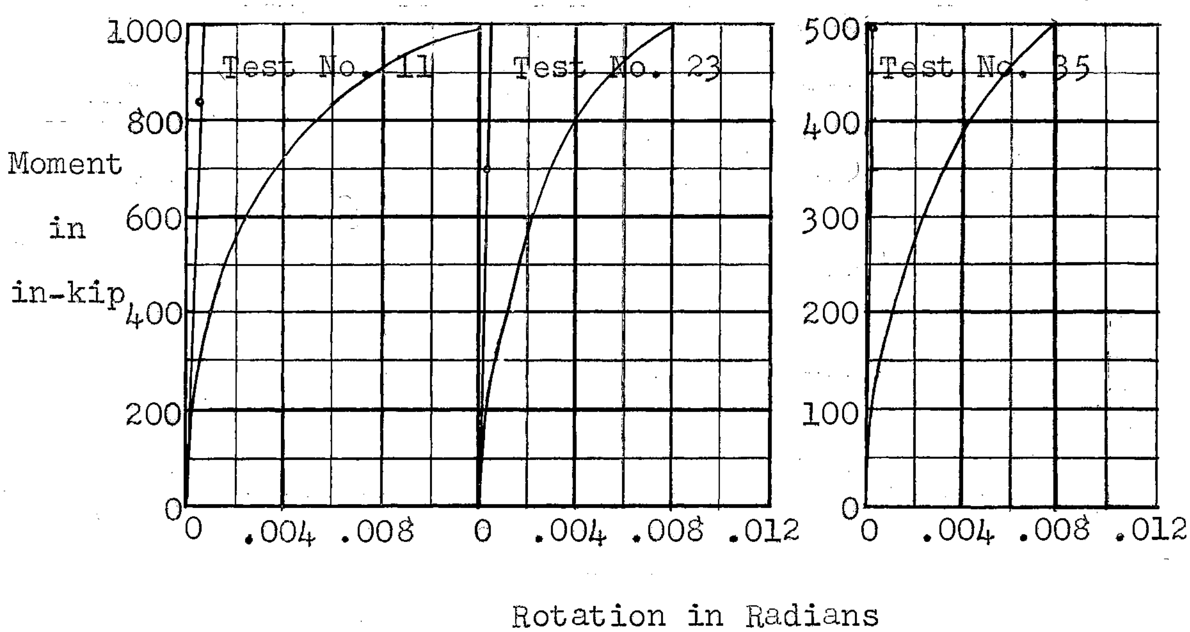

Figure 16.

APPLICATION

In order to get a comparison between rigid and semi-rigid structures, a single stroy, single bay bent with wind and unsymmetrical gravity loads, as shown in Figure 17 will be designed, First with rigid connections at B and C, and then with semi-rigid connections at the same joints.

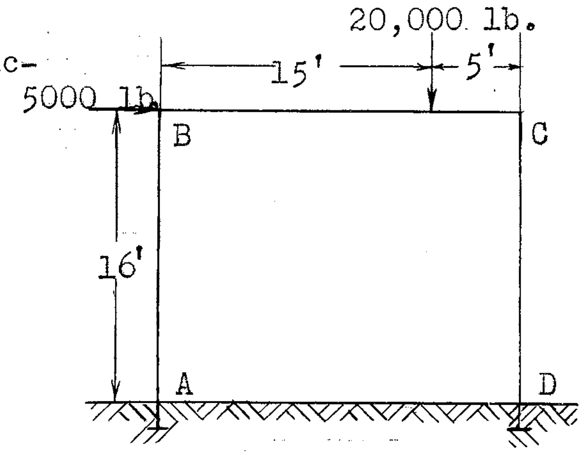

Figure 17.

I. Design the Approximate Section.

A. Beam BC.

The beam is considered as a fixed end beam.

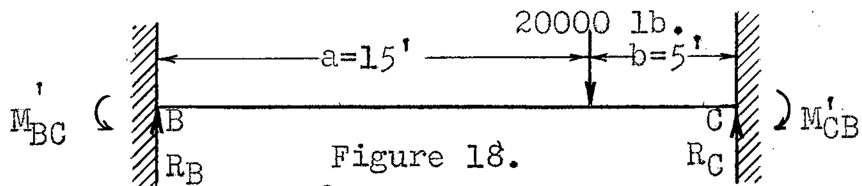

Figure 18.

$$M'_{BC} = \frac{-P\ a\ b^2}{L^2} = -\frac{20000 \times 15 \times 5^2}{20^2} = -18750 \text{ ft-lb.}$$

$$M'_{CB} = \frac{-P\ a^2 b}{L^2} = -\frac{20000 \times 15^2 \times 5}{20^2} = -56250 \text{ ft-lb.}$$

$\Sigma M_C = 0$: $(R_B \times 20) - M'_{BC} - (20000 \times 5) + M_{CB} = 0$

$20\ R_B - 18750 - 100000 + 56260 = 0$

therefore: $R_B = 3125$ lb.

And $R_C = 20000 - 3125 = 16875$ lb.

The moment under load $= (R_B \times a) - M'_{bc} = 15 \times 3125 - 18750$

$= 28125$ ft-lb.

The maximum moment $= M'_{CB} = 56250$ ft-lb $= 675000$ in-lb.

$$\frac{I}{c} = \frac{M}{s} = \frac{675000}{20000} = 33.75 \text{ in}^3$$

Then 12WF27 is required.

B. <u>Columns</u>.

It is considered as rigid frame and the moments in the columns are found by the Portal Method.

The moment in the column AB $= 20000 + 18750 + \dfrac{27 \times 20^2}{12}$

$= 39650$ ft-lb.

The moment in the column CD $= 20000 + 57150 + \dfrac{27 \times 20^2}{12}$

$= 77150$ ft-lb.

Try 8WF31 for column AB and 10WF39 for column CD for the primary sections.

C. <u>The Moment Inertia and the K-Value</u>

Column AB: 8WF31

$I_x = 109.7$ in^4.

$$K_{AB} = \frac{I_{AB}}{L_{AB}} = \frac{109.7}{16} = 6.86$$

Beam BC: 12WF27

$I_x = 204.1$ in^4

$$K_{BC} = \frac{I_{BC}}{L_{BC}} = \frac{204.1}{20} = 10.20$$

Column CD: 10WF39

$I_x = 209.7$

$$K_{CD} = \frac{I_{CD}}{L_{CD}} = \frac{209.7}{16} = 13.11$$

II. Rigid Connections at Joints B and C.

A. Slope Deflection Method.

The Known Values:

$\theta_A = 0,\qquad \theta_D = 0,\qquad R_{AB} = R_{CD} = R,$

and $R_{BC} = 0.$

Let $E = 1.$

Slope Deflection Equation:

1. $M_{AB} = 2EK_{AB}(2\theta_A + \theta_B - 3R) = 2 \times 6.86(\theta_B - 3R)$

$= 13.72\ \theta_B - 41.16\ R$

2. $M_{BA} = 2EK_{BA}(\theta_A + 2\theta_B - 3R) = 2 \times 6.86(2\theta_B - 3R)$

$= 27.44\ 0_B - 41.16\ R$

3. $M_{BC} = 2EK_{BC}(2\theta_B + \theta_C - 3R) - \dfrac{P\ a\ b^2}{L^2} - \dfrac{w\ L^2}{12}$

$= 2 \times 10.20(2\theta_B + \theta_C) - \dfrac{20000 \times 15 \times 5^2}{400} - \dfrac{27 \times 20^2}{12}$

$= 40.80\ \theta_B + 20.40\ \theta_C - 19650$

4. $M_{CB} = 2EK_{CB}(\theta_B + 2\theta_C - 3R) + \dfrac{P\ a^2 b}{L^2} - \dfrac{w\ L^2}{12}$

$= 2 \times 10.20(\theta_B + 2\theta_C) + \dfrac{20000 \times 15^2 \times 5}{400} + \dfrac{27 \times 20^2}{12}$

$= 20.40\ \theta_B + 40.80\ \theta_C + 57150$

5. $M_{CD} = 2EK_{CD}(2\theta_C + \theta_D - 3R) = 2 \times 13.11(2\theta_C - 3R)$

$= 52.44\ \theta_C - 78.66\ R$

6. $M_{DC} = 2EK_{DC}(\theta_C + 2\theta_D - 3R) = 2 \times 13.11(\theta_C - 3R)$

$= 26.22\ \theta_C - 78.66\ R$

Mechanics Equation:

a. $M_{BA} = -M_{BC}$ or $M_{BA} + M_{BC} = 0$

b. $M_{CB} = -M_{CD}$ or $M_{CB} + M_{CD} = 0$

c. $M_{AB} + M_{BA} + M_{CD} + M_{DC} + 5000 \times 16 = 0$

<u>Calculation</u>:

(1)
$$\begin{array}{rrrr} 27.44\,\theta_B & & -\,41.16\,R & \\ 40.80\,\theta_B & +\,20.40\,\theta_C & & -\,19650 \\ \hline 68.24\,\theta_B & +\,20.40\,\theta_C & -\,41.16\,R & =\,19650 \end{array}$$

(2)
$$\begin{array}{rrrr} 20.40\,\theta_B & +\,40.80\,\theta_C & & +\,57150 \\ & 52.44\,\theta_C & -\,78.66\,R & \\ \hline 20.40\,\theta_B & +\,93.24\,\theta_C & -\,78.66\,R & =-57150 \end{array}$$

(3)
$$\begin{array}{rrrr} 13.72\,\theta_B & & -\,41.16\,R & \\ 27.44\,\theta_B & & -\,41.16\,R & \\ & 52.44\,\theta_C & -\,78.66\,R & \\ & 26.22\,\theta_C & -\,78.66\,R & \\ & & & +\,5000 \times 16 \\ \hline 41.16\,\theta_B & +\,78.66\,\theta_C & -\,239.64\,R & =\,-\,80000 \end{array}$$

<u>Solution</u>:

By Gauss Method and Iteration.[18,19]

This gives three equations with three unknowns. To evalute these unknowns the Gauss method is used for obtaining approximate values. These values are then used in the Iteration method to obtain more accurate values.

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[18] John R. Parcel and george Alfred Maney, <u>Statically Indeterminate Structures</u>. (New York, 1947), pp. 224, 225 232 and 235.

[19] Hale Sutherland and Harry Lake Bowman, <u>Structural Theory</u>. (New York, 1944), p. 235.

Table II. Rigid Connections

OPERATION	EQUA. NO.	UNKNOWN TERMS θ_B	θ_C	R	CONSTANT TERM	CHECK TERM
	(1)	68.240	20.400	- 41.160	+19650.000	+19697.480
	(2)	20.400	93.240	- 78.660	-57150.000	-57115.020
	(3)	41.160	78.660	-239.640	-80000.000	-80119.820
(1)÷68.240	(1')	1.000	0.299	- 0.603	+ 287.954	+ 288.650
(2)÷20.400	(2')	1.000	4.571	- 3.856	- 2801.470	- 2799.755
(3)÷41.160	(3')	1.000	1.911	- 5.822	- 1943.635	- 1946.546
(2')-(1')	(4)		4.272	- 3.253	- 3089.424	- 3088.405
(2')-(3')	(5)		2.660	+ 1.966	- 857.835	- 853.209
(4)÷ 4.272	(4')		1.000	- 0.761	- 723.180	- 722.941
(5)÷ 2.660	(5')		1.000	+ 0.739	- 322.494	- 320.755
(5')-(4')	(6)			1.500	+ 400.686	- 402.186
(6)÷ 1.500	(6')			1.000	+ 267.124	+ 268.124
Gauss Solution		604.480	-519.899	267.124		
Iteration		604.495	-519.839	267.028	1st.	Approximation
		604.421	-519.903	266.994	2nd.	Approximation
		604.418	-519.931	266.984	3rd.	Approximation
		604.420	-519.940	266.981	4th.	Approximation
		604.421	-519.943	266.980	5th.	Approximation
		604.421	-519.943	266.980	6th.	Approximation

The actual values:

$$\theta_B = \frac{604.421}{29 \times 10^6}(12) = 0.0002501$$

$$\theta_C = \frac{-519.943}{29 \times 10^6}(12) = -0.0002152$$

$$R = \frac{266.980}{29 \times 10^6}(12) = 0.0001105$$

<u>The End Moment</u>:

1. $M_{AB} = 13.72\ \theta_B - 41.16\ R = 13.72 \times 604.421 - 41.16 \times 266.980$

$= -\ 2696.3$ ft-lb.

2. $M_{BA} = 27.44\ \theta_B - 41.16\ R = 27.44 \times 604.421 - 41.16 \times 266.980$

$= 5596.3$ ft-lb.

3. $M_{BC} = 40.80\ \theta_B + 20.40\ \theta_C - 19650$

$= 40.80 \times 604.421 + 20.40\ (-519.943) - 19650$

$= -5596.4$ ft-lb.

4. $M_{CB} = 20.40\ \theta_B + 40.80\ \theta_C + 57150$

$= 20.40 \times 604.421 + 40.80\ (-519.943) + 57150$

$= 48266.5$ ft-lb.

5. $M_{CD} = 52.44\ \theta_C - 78.66\ R = 52.44(-519.943) - 78.66 \times 266.980$

$= -48266.5$ ft-lb.

6. $M_{DC} = 26.22\ \theta_C - 78.66\ R = 26.22(-519.943) - 78.66 \times 266.980$

$= -34633.6$ ft-lb.

B. <u>Cross-Morris Method</u> (<u>Moment Distribution Method</u>).

<u>Fixed End Moment</u> : Beam BC:

$M_{BC} = -19650$ ft-lb. $M_{CB} = +57150$ ft-lb.

Column AB and CD:

$$\text{Shear in AB} = 5000 \times \frac{6.86}{6.86 + 13.11} = 1717.6 \text{ lb.}$$

Shear in CD = 5000 - 1717.6 = 3282.4 lb.

FEM in AB = 1717.6 x 8 = - 13741 ft-lb.

FEM in CD = 3282.4 x 8 = - 26240 ft-lb.

$K_{AB} = 6.86$

$K_{BC} = 10.20$

$K_{CD} = 13.11$

$K_{AB} + K_{BC} = 6.86 + 10.20 = 17.06$

$K_{BC} + K_{CD} = 10.20 + 13.11 = 23.31$

The results of the end bending moment are in Table III.

C. Comparison of the End Bending Moments as Found by the Slope Deflection and Cross Methods.

The end bending moment for the rigid connection computed by Slope Deflection method and Cross method is very close. The comparison between them is tabulated as follows:

Table III.

End of Member	End Bending Moment (ft-lb.) Slope Deflection Method	Cross Method
M_{AB}	- 2696.3	- 2696
M_{BA}	+ 5596.3	+ 5597
M_{BC}	- 5596.4	- 5597
M_{CB}	+48266.5	+48268
M_{CD}	-48266.5	-48268
M_{DC}	-34633.6	-34639

D. Design the sections of the frame.

Design of the Beam BC.

Maximum moment, M = 48266.5 ft-lb. = 579198 in-lb.

$$\frac{I}{c} = \frac{M}{s} = \frac{579198}{20000} = 28.96 \text{ in}^3.$$

Then 12 WF 27 is required and is the most economical.

Design of the Column AB and CD.

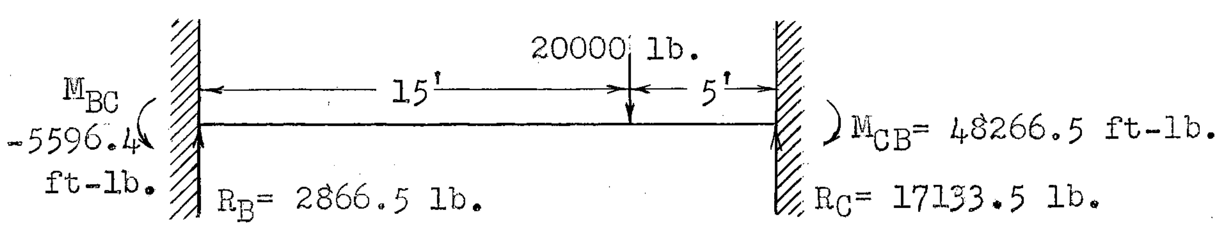

Figure 19.

$\sum M_C = 0$:

$R_B \times 20 + 48266.5 - 5596.4 - 20000 \times 5 = 0$

Therefore:

$R_B = 2866.5$ lb.

$R_C = 20000 - 2866.5 = 17133.5$ lb.

Column AB.

$M_{BA} = 5596.3$ ft-lb. = 67155.6 in-lb.

$P = R_B = 2866.5$ lb.

Try 4 WF 13: $A = 3.82 \text{ in}^2.$ $\frac{I}{c} = 5.45 \text{ in}^3.$ $r = 1.72$ in.

$$s_c = 17000 - 0.485(L/r)^2$$

$$= 17000 - 0.485\left(\frac{16 \times 12}{1.72}\right)^2 = 10950 \text{ lb./in.}^2$$

$$s_b = 20000 \text{ lb./in.}^2$$

$$1 = \frac{P}{A\ s_c} + \frac{M\ c}{I\ s_b} = \frac{2866.5}{3.82 \times 10950} + \frac{67155.6}{5.45 \times 20000} = 0.685$$

Therefore 4 WF 13 is sufficient.

Column CD.

$M_{CD} = 48266.5$ ft-lb. = 579198 in-lb.

$P = R_C = 17133.5$ lb.

Try 10 WF 33: $A = 9.71 \text{ in}^2.$ $\frac{I}{c} = 35 \text{ in}^3.$ $r = 4.2$ in.

$$s_c = 17000 - 0.485\left(\frac{16 \times 12}{4.2}\right)^2 = 15990 \text{ lb./in.}^2$$

$$1 = \frac{17133.5}{9.71 \times 15990} + \frac{579198}{35 \times 20000} = 0.9378$$

Therefore 10 WF 33 is sufficient.

III. Semi-Rigid Connection at Joint B and C.

A. Design the Connection at Joint B.

Try the top angle $6x4x\frac{3}{8}x\ 8''$ and the seat angle $6x6x\frac{3}{8}x\ 8''$ and the other dimensions are shown in Figure 20.

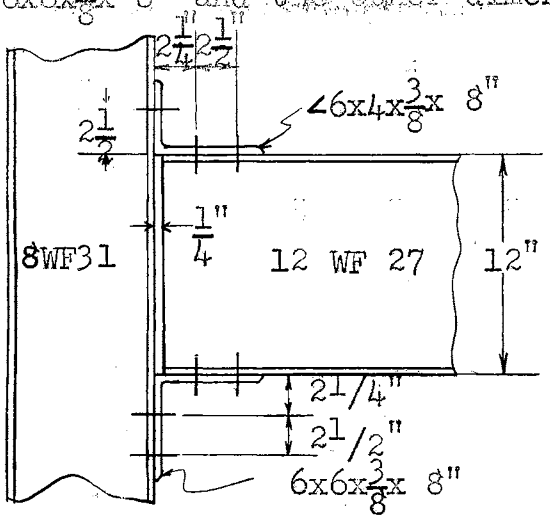

Figure 20. Joint B.

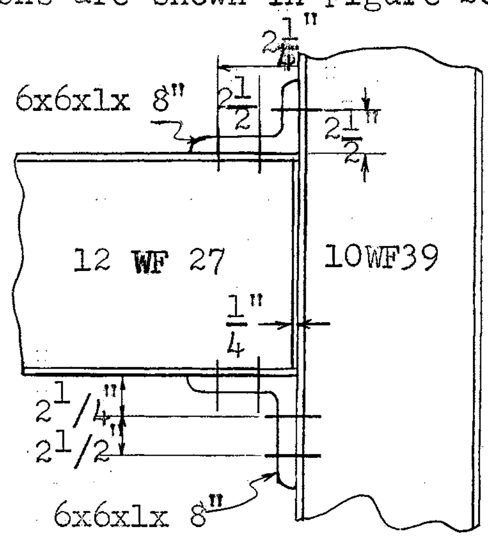

Figure 21. Joint C.

From Equation (22), the resisting moment of the connection is:

$$M = \frac{b\ s\ t^2}{6g} \frac{4g + g_1}{2g + g_1} (y + \frac{2}{3} q)$$

substituting the known values:

$$M = \frac{8 \times 24(3/8)^2}{6 \times 2.125} \frac{4 \times 2.125+1.875}{2 \times 2.125+1.875} (19.64 + \frac{2}{3} \times 0.857)$$

$= 72.6$ in-kip.

which is less than the maximum moment, $M_{BC} = 67.16$ in-kip, obtained by the slope deflection method of rigid connection. therefore the angles are sufficient.

B. Design the Connection at Joint C.

Try the top angle and the seat angle $6x6x1x\ 8''$ and the other dimensions are shown in Figure 21.

From Equation (22) the resisting moment of the connection is:

$$M = \frac{b\ s\ t^2}{6\ g} \frac{4g + g_1}{2g + g_1} \left(y + \frac{2}{3} q \right)$$

Where: $g = 2\frac{1}{2} - 1 = 1.50$ in.

$g_1 = 2\frac{1}{4} - 1 = 1.25$ in.

Substituting in Equation (23):

$$n = \frac{6\ g\ (2g + g_1)}{t\ (\ 4g + g_1)} = \frac{6 \times 1\frac{1}{2}\ (2 \times 1.5 + 1.25)}{1\ (\ 4 \times 1.5 + 1.25)}$$

$= 5.275.$

From Equation (25):

$$y = \frac{n\ H + t - \sqrt{(2\ n\ H + t)\ t}}{n}$$

$$= \frac{5.275 \times 20.5 + 1 - \sqrt{(2 \times 5.275 \times 20.5 + 1) \times 1}}{5.275}$$

$= 17.90$ in.

$q = H - y = 20.5 - 17.90 = 2.60$ in.

Substituting these value in Equation (22):

$$M = \frac{8 \times 24\ (1)^2}{6 \times 1.5}\ \frac{4 \times 1.5 + 1.25}{2 \times 1.5 + 1.25} \left(17.90 + \frac{2}{3} \times 2.60 \right)$$

$= 714$ in-kip.

which is less than the maximum moment, $M_{CB} = 579.2$ in-kip, obtained by the solpe deflection method of rigid connection therefore the angles are sufficient.

C. <u>The Value of Z.</u>

<u>Joint B.</u>

From the Equation (28):

$$Z = \frac{4\ g^3}{E\ t^3 b\ (y - g - t)(y + 2/3\ q)}\ \frac{g + g_1}{4g + g_1}$$

$$Z_{BC} = \frac{4(2.125)^3}{29 \times 10^6 (0.375)^3 8(19.64-2.125-0.375)(19.64+\frac{2}{3}\times 0.857)} \times \frac{2.125 + 1.875}{4 \times 2.125 + 1.875}$$

Therefore:

$Z_{BC} = 0.000000003485$

Joint C.

$$Z_{CB} = \frac{4 \times 1.5^3}{29 \times 10^6 (1)^3 8(17.90-1.5-1.0)(17.90 + \frac{2}{3} \times 2.60)} \times \frac{1.5 + 1.25}{4 \times 1.5 + 1.25}$$

Therefore:

$Z_{CB} = 0.000000000073$

D. Slope Deflection Method.

Slope Defledtion Equations.

$\theta_A = 0, \quad \theta_B = 0, \quad R_{AB} = R_{CD} = R, \quad R_{BC} = 0.$

$L = 20$ ft.$=240$ in. $\quad E = 29{,}000{,}000\ ^{lb}/_{in^2}.$

$L_{BC} = L + 3EI_{BC}Z_{BC} = 240+3 \times 29\times10^6 \times 204.1 \times 0.000000003485$

$= 240 + 62 = 302$ in.

$L_{CB} = L + 3EI_{BC}Z_{BC} = 240+3 \times 29\times10^6 \times 204.1 \times 0.000000000073$

$= 240 + 1.3 = 241.3$ in.

1. $M_{AB} = 2E\frac{I_{AB}}{L_{AB}}(2\ \theta_A + \theta_B - 3R) = 2 \times 29\times10^6 \times \frac{109.7}{16 \times 12}(\ \theta_B - 3R)$

$= 33100000\ \theta_B - 99300000\ R$

2. $M_{BA} = 2E\frac{I_{BA}}{L_{BA}}(\theta_A + 2\ \theta_B - 3R) = 2 \times 29\times10^6 \times \frac{109.7}{16 \times 12}(2\ \theta_B - 3R)$

$= 66200000\ \theta_B - 99300000\ R$

3. $$M_{BC} = 6EI_{BC}\,\frac{2L_{CB}(\theta_B - R) + L(\theta_C - R)}{4\,L_{BC}L_{CB} - L^2} - \frac{6\,A}{L}\,\frac{2\bar{b}\,L_{CB} - \bar{a}\,L}{4L_{BC}L_{CB} - L^2}$$

$$= 6 \times 29\text{x}10^6 \times 204.1 \times \frac{2 \times 241.3(\theta_B - 0) + 240(\theta_C - 0)}{4 \times 302 \times 241.3 - (240)^2}$$

$$- \frac{6(\frac{1}{2}\,L\,\frac{P\,b\,a}{L})}{L} \times \frac{2\,\frac{L+b}{3}\,241.3 - \frac{L+a}{3}\,240}{4 \times 302 \times 241.3 - (240)^2}$$

$$- \frac{6(\frac{2}{3}\,L\,\frac{w\,L^2}{8})}{L} \times \frac{2\,\frac{L}{2}\,241.3 - \frac{L}{2}\,240}{4 \times 302 \times 241.3 - (240)^2}$$

$$= 73300000\,\theta_B + 36450000\,\theta_C - 180200$$

4. $$M_{CB} = 6 \times 29\text{x}10^6 \times 204.1 \times \frac{2 \times 302(\theta_C - 0) + 240(\theta_B - 0)}{4 \times 302 \times 241.3 - (240)^2}$$

$$- \frac{3 \times 20000 \times 15\text{x}12 \times 5\text{x}12(\frac{2}{3} \times 300 \times 241.3 - \frac{1}{3} \times 420 \times 240)}{240\,(4 \times 302 \times 241.3 - 240^2)}$$

$$- \frac{\frac{1}{2} \times 27 \times 20^3 \times 12^2\,(240 \times 241.3 - \frac{240}{2} \times 240)}{240\,(4 \times 302 \times 241.3 - 240^2)}$$

$$= 36450000\,\theta_B + 91600000\,\theta_C + 322000$$

5. $$M_{CD} = 2E\frac{I_{CD}}{L_{CD}}(2\,\theta_C + \theta_D - 3R) = 2 \times 29\text{x}10^6 \times \frac{209.7}{16 \times 12}(2\,\theta_C - 3R)$$

$$= 126700000\,\theta_C - 190000000\,R$$

6. $$M_{DC} = 2E\frac{I_{DC}}{L_{DC}}(\theta_D + 2\,\theta_D - 3R) = 2 \times 29\text{x}10^6 \times \frac{209.7}{16 \times 12}(\theta_C - 3R)$$

$$= 63350000\,\theta_C - 190000000\,R$$

Mechanics Equation:

a. $M_{BA} = -M_{BC}$ or $M_{BA} + M_{BC} = 0$

b. $M_{CB} = -M_{CD}$ or $M_{CB} + M_{CD} = 0$

c. $M_{AB} + M_{BA} + M_{CD} + M_{DC} + 5000 \times 16 \times 12 = 0$

Calculation:

(1)

$$\begin{array}{rrrr} 66200000\ \theta_B & & -\ 99300000\ R & \\ 73300000\ \theta_B & +\ 36450000\ \theta_C & & -\ 180200 \\ \hline 139500000\ \theta_B & +\ 36450000\ \theta_C & -\ 99300000\ R & =\ 180200 \end{array}$$

(2)

$$\begin{array}{rrrr} 36450000\ \theta_B & +\ \ 91600000\ \theta_C & & +\ 322000 \\ & 126700000\ \theta_C & -\ 190000000\ R & \\ \hline 36450000\ \theta_B & +\ 218300000\ \theta_C & -\ 190000000\ R & =\ -\ 322000 \end{array}$$

(3)

$$\begin{array}{rrrr} 33100000\ \theta_B & & -\ \ 99300000\ R & \\ 66200000\ \theta_B & & -\ \ 99300000\ R & \\ & 126700000\ \theta_C & -\ 190000000\ R & \\ & 63350000\ \theta_C & -\ 190000000\ R & \\ & & & +\ 960000 \\ \hline 99300000\ \theta_B & +\ 190050000\ \theta_C & -\ 578600000\ R & =\ -\ 960000 \end{array}$$

Solution:

Table IV. Semi-Rigid Connections

OPERATION	EQUA. NO.	UNKNOWN TERMS $\theta_B \div 1000$	$\theta_C \div 1000$	$R \div 1000$	CONSTANT TERM	CHECK TERM
	(1)	139500	36450	- 99300	+18020	+ 94670
	(2)	36450	218300	- 190000	-32200	+ 32550
	(3)	99300	190050	- 578600	-96000	-385250
(1)÷139500	(1')	1.000	0.261	- 0.712	+0.129	+ 0.678
(2)÷ 36450	(2')	1.000	5.989	- 5.213	-0.883	- 0.893
(3)÷ 99300	(3')	1.000	1.914	- 5.827	-0.967	- 3.880
(2')-(1')	(4)		5.728	- 4.501	-1.012	+ 0.215
(2')-(3')	(5)		4.075	+ 0.614	+0.084	+ 4.773
(4)÷ 5.728	(4')		1.000	- 0.786	-0.177	+ 0.037
(5)÷ 4.075	(5')		1.000	+ 0.151	+0.021	+ 1.172
(5')-(4')	(6)			+ 0.937	+0.198	+ 1.135
(6)÷ 0.937	(6')			+ 1.000	+0.211	+ 1.211
Gauss Solution		0.002821	-0.000110	0.002110		
Itertion		0.002823	-0.000110	0.002108		
		0.002821	-0.000112	0.002107		
		0.002821	-0.000112	0.002107		

The End Moment.

1. $M_{AB} = 33100000\ \theta_B - 99300000\ R$

$= 33100000 \times 0.002821 - 99300000 \times 0.002107$

$= -\ 9654$ ft-lb.

2. $M_{BA} = 66200000\ \theta_B - 99300000\ R$

$= 66200000 \times 0.002821 - 99300000 \times 0.002107$

$= -\ 1873$ ft-lb.

3. $M_{BC} = 73300000\ \theta_B + 36450000\ \theta_C - 180200$

$= 73300000 \times 0.002821 + 36450000 \times (-0.000112) - 180200$

$= +\ 1875$ ft-lb.

4. $M_{CB} = 36450000\ \theta_B + 91600000\ \theta_C + 322000$

$= 36450000 \times 0.002821 + 91600000 \times (-0.000112) + 322000$

$= +\ 34547$ ft-lb.

5. $M_{CD} = 126700000\ \theta_C - 190000000\ R$

$= 126700000 \times (-0.000112) - 190000000 \times 0.002107$

$= -\ 34543$ ft-lb.

6. $M_{DC} = 633500000\ \theta_C - 190000000\ R$

$= 63350000 \times (-0.000112) - 190000000 \times 0.002107$

$= -\ 33952$ ft-lb.

E. Design of the Section of the Semi-Rigid Connection Frame.

Design of the Beam BC.

$M_{max} = M_{CB} = 34547$ ft-lb. $= 414564$ in-lb.

$$\frac{I}{c} = \frac{M}{s} = \frac{414564.0}{20000} = 20.73 \text{ in}^3$$

Then 10 WF 21 is required.

Design of the Column AB and CD.

$\sum M_C = 0$:

$$20\,R_B + 1875 + 34547 - 20000 \times 5 = 0$$

threrfore:

$$R_B = \frac{63578}{20} = 3178.9 \text{ lb.}$$

$$R_C = 20000 - 3178.9 = 16821.1 \text{ lb.}$$

<u>Column AB</u>.

$$M_{max} = M_{AB} = 9654 \text{ ft-lb.} = 115848 \text{ in-lb.}$$

$$P = R_B = 3178.9 \text{ lb.}$$

Try 5 WF 16: $A = 4.70$ in.^{2} $\frac{I}{c} = 8.53$ in.^{3} $r = 2.13$ in.

$$s_c = 17000 - 0.845\left(\frac{16 \times 12}{2.13}\right)^2 = 13060 \text{ lb./in}^2$$

$$1 = \frac{P}{A\,s_c} + \frac{M\,c}{I\,s_b} = \frac{3178.9}{4.70 \times 13060} + \frac{115848}{8.53 \times 20000} = 0.7307$$

Therefore 5 WF 16 is sufficient.

<u>Column CD</u>.

$$M_{max} = M_{CD} = 34543 \text{ ft-lb.} = 414516 \text{ in-lb.}$$

$$P = R_C = 16821.1 \text{ lb.}$$

Try 10 WF 25: $A = 7.35$ in.^{2} $\frac{I}{c} = 26.4$ in.^{3} $r = 4.26$ in.

$$s_c = 17000 - 0.845\left(\frac{16 \times 12}{4.26}\right)^2 = 16015 \text{ lb./in}^2$$

$$1 = \frac{16821.1}{7.35 \times 16015} + \frac{414516}{26.4 \times 20000} = 0.143 + 0.785 = 0.928$$

Therefore 10 WF 25 is sufficient.

IV. <u>Comparison Between Rigid and Semi-Rigid Connection</u>.

The weight of steel for the rigid frame is:

20(27) + 16(13 + 33) = 540 + 736 = 1276 lb.

The weight of steel for the semi-rigid frame is:

20(21) + 16(16 + 25) = 420 + 656 = 1076 lb.

Therefore the total average saving for this example is:

$$\frac{1276 - 1076}{1276} = 15.7\ \%.$$ but in general, may be 20 %.

CONCLUSION

By the above procedures the equation for the connection constant, Z, of the top and seat angle type of semi-rigid connection is derived. It may be mentioned that the value of Z is inversely proportional to the cubic power of the thickness and to the length of the top connecting angle, and is not much affected by the other dimensions. The slopes of the moment-rotation curve, 1/Z, calculated by the derived Equation (29) are plotted on Professor Rathbun's curves and several of Messrs. Hechtman and Johnston (six specimens are shown in the thesis) with good results. By applying this type of semi-rigid connection to the design problem shown in the thesis, it is evident that a saving of approximately 16% may be achieved.

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The content and form have been checked and approved by the author and thesis adviser. "Instructions for Typing and Arranging the Thesis" are available in the Graduate School office. Changes or corrections in the thesis are not made by the Graduate School office or by any committee. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

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