AN INVESTIGATION OF THE HEATING OF A THREE-PHASE INDUCTION MOTOR OPERATING ON UNBALANCED VOLTAGES

By

JACK EDWARD TOMPKINS

Bachelor of Science

Oklahoma Agricultural and Mechanical College

Stillwater, Oklahoma

1949

Submitted to the faculty of the Graduate School of
the Oklahoma Agricultural and Mechanical College
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1953

AN INVESTIGATION OF THE HEATING OF A THREE-PHASE

INDUCTION MOTOR OPERATING ON

UNBALANCED VOLTAGES

Thesis Approved:

Chas. F. Cameron
Thesis Adviser

A. Naeter

D. C. McIntosh
Dean of the Graduate School

PREFACE

A complete balance of load (and thus the voltage) between the three phases of a distribution system is virtually impossible to obtain, or if ever realized it would be exceedingly difficult to maintain. This is especially true of the larger distribution systems of metropolitan areas which are constantly being subjected to alteration and expansion.

One of the basic assumptions made by manufacturers in the design of three-phase induction motors is that the voltage supply to the motor will be balanced. Very often the difference in motor capabilities occasioned by failure of distribution systems to meet this ideal condition is a significant amount. The most noticeable effect will be overheating of the motor resulting in a reduction in the life of the electrical insulation. Some instances are known where practically new motors have been "burned out" for this very reason.

Since a completely balanced voltage supply is not usually available the question often arises as to what degree of unbalance in the voltage supply of an induction motor can be tolerated insofar as overheating is concerned. To the author's knowledge no means by which this question may be answered has yet been developed. It is to this end that the work presented in this thesis was undertaken.

An expression of appreciation is extended to the Staff of the School of Electrical Engineering of the Oklahoma Institute of Technology for their interest and cooperation and especially to Professor Charles F. Cameron who suggested the problem and served as advisor in preparation of the thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER II

CHAPTER III

CHAPTER IV

CHAPTER I

INTRODUCTION

The practice of generating and selling electric power has, in comparatively recent years, become big business throughout nearly the entire world. In its infancy the electric power industry had a product which was expensive, the utility of which was fairly limited, and the supply doubtful. The entire electrical industry concentrated on these shortcomings and, as a result, in a few short years electric power has gained wide usage by people of all economic levels and its supply is seemingly continuous and abundant.

The electric utilities have strived to maintain a high degree of service. In the beginning their primary problem was often one of simply maintaining service; however, with advance in technology and improvement in generation, transmission, and distribution equipment this problem has been practically eliminated. An "outage" on a modern well-kept distribution system is a very rare occurrence. With this problem nearing solution distribution engineers have directed their respective abilities and efforts to what might be called "improving the quality of service". The problems on which these engineers dwell today would have been cast aside by many engineers of yesterday as being of relatively minor importance; yet, if perfection is the goal, these problems must be solved. Oddly enough, as the problems have become more detailed they seem to have become more difficult to solve.

Malfunctioning of electrical apparatus continues today for many reasons. Low power-factor, poor voltage regulation, low voltage, unbalanced voltages, and other improper conditions which are a source of trouble, still occur. It is a study of the effect of having an unbalanced voltage supply for a three-phase induction motor that is attempted here.

EFFECT OF UNBALANCED VOLTAGES

Unbalanced voltages will result in unbalanced torque within the motor, and this unbalanced torque may, in turn, cause several noticeable defects. It could lead to vibration, increased wear on mechanical parts, "singing" or excessive noise in the motor, and reduced torque output. As will be pointed out in a later chapter, this reduction of torque can be accounted for (when analyzed by symmetrical components) as being the result of having two oppositely rotating magnetic fields in the motor. One field, due to the positive-sequence currents, rotates in one direction, the other, the field due to negative-sequence currents, rotates in a counter-direction. These oppositely rotating fields give rise to opposing torques which are directly proportional to their respective magnetic fields. The motor torque output, then, is the net of these two torques. With balanced voltages the negative-sequence current is zero, therefore the counter-torque is zero. As the degree of unbalance increases the negative-sequence current becomes greater, thus, the counter-torque becomes greater and the torque output is reduced.

A three-phase induction motor operating on unbalanced voltages will tend to restore the voltage balance. It may be shown that any three-phase device which offers less impedance to negative-sequence current than to positive-sequence current will exert a balancing effect on the unbalanced three-phase supply. This, however, is a creditable attribute and it is the undersirable effects which should be studied and remedied.

For nearly all motor applications the aforementioned effects of an unbalanced voltage supply will not be nearly so critical as will be the reduction in operating efficiency and the increased heating which will result therefrom. The following explanation accounts for this fact and testifies to the significance of the problem.

The increased heating of the motor on load is a more important consideration than the torque reduction. The value of the negative-sequence current flowing can be obtained by multiplying the negative-sequence voltage by the standstill admittance (considered constant over the motor speed range to the negative-sequence currents). Full voltage impressed on motors of normal design when locked will cause 6 to 8 times full load current to flow; that is, they have an admittance of 6 to 8 at 100 per cent slip. If these motors were to have unbalanced voltages impressed on them such that the negative-sequence component of voltage amounted to about 15 per cent of the positive-sequence component (that is, an unbalance factor of 0.15) there would result a negative-sequence current equal to full load current. Thus, on load, the torque of ordinary induction motors is not sensitive to a moderate degree of voltage unbalance, but their heating characteristics are very sensitive to such unbalance.[1]

METHOD OF ANALYSIS

In the initial consideration of this problem several possible methods of attack were considered. The first of these was an attempt to use conventional motor calculation procedures with per-unit values of impedance, volts, etc. Use of per-unit quantities in solving electrical circuits has several advantages, and it seemed very likely that their use might afford the means to a solution of the problem at hand. However, an investigation of this possibility did not make its use immediately obvious. Any future work conducted on this subject might well start by re-investigating this possibility. It was felt that time limitation did not permit as exhaustive an investigation as was warranted.

Another method considered was one based entirely on obtaining emperical data. Some experimentation was conducted along this line and it is

---

[1]Edward O. Lunn, "Induction Motors Under Unbalanced Conditions," Electrical Engineering, (April 1936), 391.

believed that it might have been followed to completion. There was one serious objection and that was its absolute empericism. Its use left a feeling of lack of scientific or proper engineering quality.

This study and work eventually led to the solution presented in this thesis. While doing this research several ideas gradually developed. It seemed that the ideal process of solving a problem related to motors would be based on principles and concepts already developed and accepted for making motor analysis. This can be accomplished by recombining methods of solutions of various motor problems, or extending old ideas and methods to cover the new situation. The work presented in this thesis is essentially the result of combining theoretical studies of two phases of electric motor design. Like all other problems, once a solution was obtained it seemed very simple. The final logic applied can be summarized as follows.

All the energy represented as an increase in motor temperature or heat given off by the motor must appear as motor losses. That is, this energy must be reflected in reduced motor efficiency. There are available methods to calculate motor performance when operating on unbalanced voltages; motor efficiency can be calculated by their use and thus motor losses can be determined. There remains, then, only to relate motor losses or efficiency to temperature rise. A study of theory of heating in electric devices pointed the way to the solution of this problem which is simply a new and somewhat modified application of some known design principles.

The work presented in this thesis may be considered in separate phases. The first phase consists of conducting tests on a motor to determine experimentally its performance and to obtain data from which the motor performance, on unbalanced voltages, can be calculated.

The second phase is to use the data obtained and calculate the motor performance characteristics and compare them with actual test results. This step is vital, for the limitations on accuracy of such motor calculations are well known. Since these calculations are an intermediate step toward the final solution it will be essential when making an overall evaluation of the method of analysis presented, to know the degree of accuracy of the intermediate steps. From these calculations the motor efficiency will be determined.

Once the efficiency is known the final phase of the problem is reached. That is to relate the motor losses to temperature rise.

CHAPTER II

MOTOR TESTS, PERFORMANCE CALCULATIONS AND COMPARISON OF RESULTS

In order to determine motor performance by calculation, certain data are needed. These data may be determined by motor tests and these tests are the blocked-rotor and no-load tests. In addition a dynamometer-load test may be performed on the motor to determine, experimentally, its characteristics.

Once these test data are known the motor performance may be calculated by one of several means. The methods for doing this will be discussed in this chapter.

EXPERIMENTAL MOTOR AND TEST EQUIPMENT

The motor used in performing the experimental phases of this work was obtained from the Westinghouse Electric Corp. It is understood that this motor was the last pilot model built by Westinghouse prior to actual production of motors of its general type. There are six copper-constantan thermocouples embedded in its field winding which makes it ideal for heat studies. The following motor data are given:

Westinghouse Electric Corp., 5 H. P., 4 poles, 220/440 volts, three-phase, 60 c.p.s., locked rotor KVA code J: 6 Imbedded copper-constantan thermocouples.

Winding Data:

36 coils (slots), 18 turns per coil with one of NO. 19-.036 and one of NO. 20-.032 enameled copper wire (one of each size in parallel). Coils are wound with a throw of 1 and 8, 3 coils per group, 12 groups, connected

parallel wye for 220 volt operation and series wye for 440 volt operation.

The following test equipment was used:

Dynamometer - Diehl Mfg. Co., 20 H. P. absorbed, 700/2100 R. P. M., 230/460 volts, d.c.

Weston Industrial Analyzer - Model 630, Type 2, No. 357.

Thermocouple Potentiometer - Type PJ - B4, No. 3246091.

EQUIVALENT CIRCUIT METHOD OF CALCULATING MOTOR PERFORMANCE

There are available two commonly used methods for calculating motor performance. One is by use of the equivalent circuit and the other is by use of a circle diagram. Since the former of these two may be more readily expanded for use in calculating motor performance with unbalanced applied voltages it will be used in this thesis.

Several textbooks on alternating-current machinery[1] give the derivation of the equivalent circuit for a three-phase induction motor. This equivalent circuit will be given with a brief explanation of its component branches, but no attempt will be made to show its derivation.

In Figure 1, the equivalent circuit of a three-phase induction motor is shown. It is customary to solve the circuit on a single-phase basis; thus, E is the applied phase voltage, R_1 and jX_1 are the stator resistance and leakage reactance in single-phase values. Similarly R_2 and jX_2 are the rotor resistance and leakage reactance, s is the slip, jXm and g_h are the magnetizing reactance and core loss conductance, respectively. I_1 and I_2 are the currents of the stator and rotor; I_m and I_h are the magnetizing

---

[1]Phillip L. Alger, Nature of Polyphase Induction Machines, John Wiley & Sons, Inc., 1951, Chapter 5.

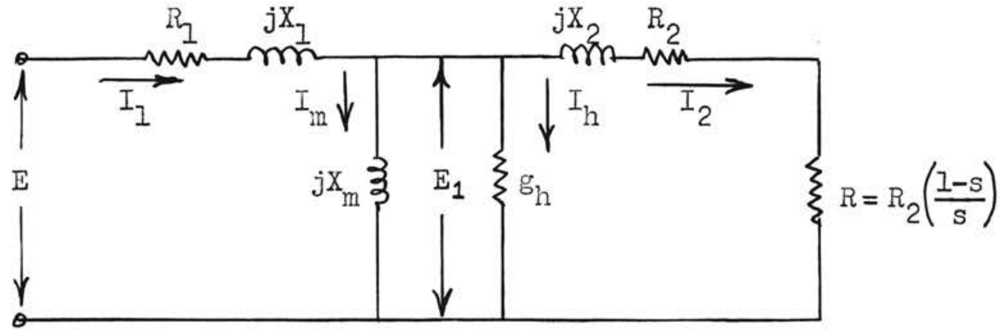


FIGURE 1
Equivalent Circuit for a Three-Phase Induction Motor

current and core loss components of the no-load current.

It should be noted that the total resistance of the rotor circuit is $R_2 + R$. Since the value of R depends upon R_2 and s, for a given equivalent circuit, it can change only with s. This points out the method by which the equivalent circuit may be used to calculate the motor performance. This method consists of assuming a value of slip and solving for I_1 and I_2. The motor output in watts is the power dissipated in R, and the power imput is $EI_1 \cos\theta\Big]_{I_1}^{E}$.

DETERMINING CONSTANTS FOR EQUIVALENT CIRCUIT

Before the equivalent circuit can be established and used it is necessary to determine the values of the circuit parameters. To do this it will be necessary to obtain the blocked-rotor and the no-load test data. These data are given in Table 2-1 and Table 2-2 respectively, and are plotted in Figure 2 and Figure 3.

The stator and rotor leakage reactances may be taken to be equal in value, with each being equal to one-half the value of X_e or

(1) $$X_1 = X_2 = 0.5X_e$$

Where X_e may be determined in the following manner.

(2) $$Z_e = \frac{E}{I_{sc}}$$

E = rated phase voltage

I_{sc} = short-circuit or blocked-rotor current with E applied

The resistive component, R_e, of Z_e, may be determined by use of one of the following three equations. In each of these equations the values of current, watts, and volt-amperes are blocked-rotor values taken with rated phase voltage, at rated frequency, applied.

(3) $$R_e = (Z_e)\,\frac{\text{in-phase component of } I_{sc}}{I_{sc}}$$

(3a) $$R_e = (Z_e)\,\frac{\text{watts}}{\text{volt-amperes}}$$

(3b) $$R_e = (Z_e)\,\frac{\text{watts}}{3I^2_{sc}}$$

Now that R_e and Z_e are known, X_e may be calculated.

(4) $$X_e = \sqrt{Z^2_e - R^2_e}$$

The resistance, R_1, of the stator may be determined directly by measurement. With a d.c. voltage applied which causes about one-quarter full-load current to flow, the voltage and current are noted and the terminal-to-terminal resistance calculated. For a wye connection the phase resistance is one-half this value. R_2 is then calculated by

(5) $$R_2 = R_e - R_1$$

There remains to be determined jX_m and g_h.

(6) $$X_m + X_1 = \frac{E}{I_{nl}}$$

where:

E = applied phase voltage

I_{nl} = the no-load current

Since X_1 is known, X_m may be calculated by equation (6).

The no-load power represents the power necessary to overcome the machine losses, namely, friction, windage, core, and stator I^2R loss. If the curve of watts vs. volts for no-load were to be extended to the ordinate, $E = 0$, the value of the intercept would represent the friction and windage loss. Since I and R_1 are known, the stator copper loss can be calculated. The sum of stator copper loss plus the friction and windage loss subtracted from the watts input gives the core loss. The voltage across the parallel shunt branches of Figure 1 may be calculated and by knowing the voltage drop and power loss, g_h may be determined in the following manner

(7) $$g_h = \frac{W_{cl}}{3E_1{}^2}$$

where

W_{cl} = total core loss watts

E_1 = applied voltage minus the stator voltage drop. (see Figure 1)

If the no-load and blocked-rotor test data are available the equivalent circuit for a three-phase induction motor may be established by use of the preceeding equations.

EQUIVALENT CIRCUIT FOR THE TEST MOTOR

The blocked-rotor and no-load test data are given in Tables 2-1 and 2-2 respectively.

TABLE 2-1

BLOCKED-ROTOR TEST DATA

VOLTS	AMPS A	AMPS B	AMPS C	INPUT KILOWATTS	SCALE LBS.	POWER-FACTOR	TORQUE LB-FT.
50	15.1	15.0	15.0	0.60	1.3	.46	1.37
100	31.0	31.0	31.0	2.75	4.0	.50	4.2
150	50.0	50.0	50.0	7.50	10.0	.59	10.5

TABLE 2-2

NO-LOAD TEST DATA

VOLTS	AMPS A	AMPS B	AMPS C	INPUT WATTS
243	7.80	7.46	7.50	350
200	5.20	4.80	4.50	240
150	3.40	3.28	3.38	160
100	2.20	2.31	2.30	110
50	1.50	1.30	1.40	70

In addition to these data which are necessary to calcualte the values of the components of the equivalent circuit, a conventional dynamometer-load test was performed on the motor. These test data are given in Table 2-3 and the performance curves are given in Figure 4. This information is included for the purpose of comparing the calculated performance with actual experimental test results.

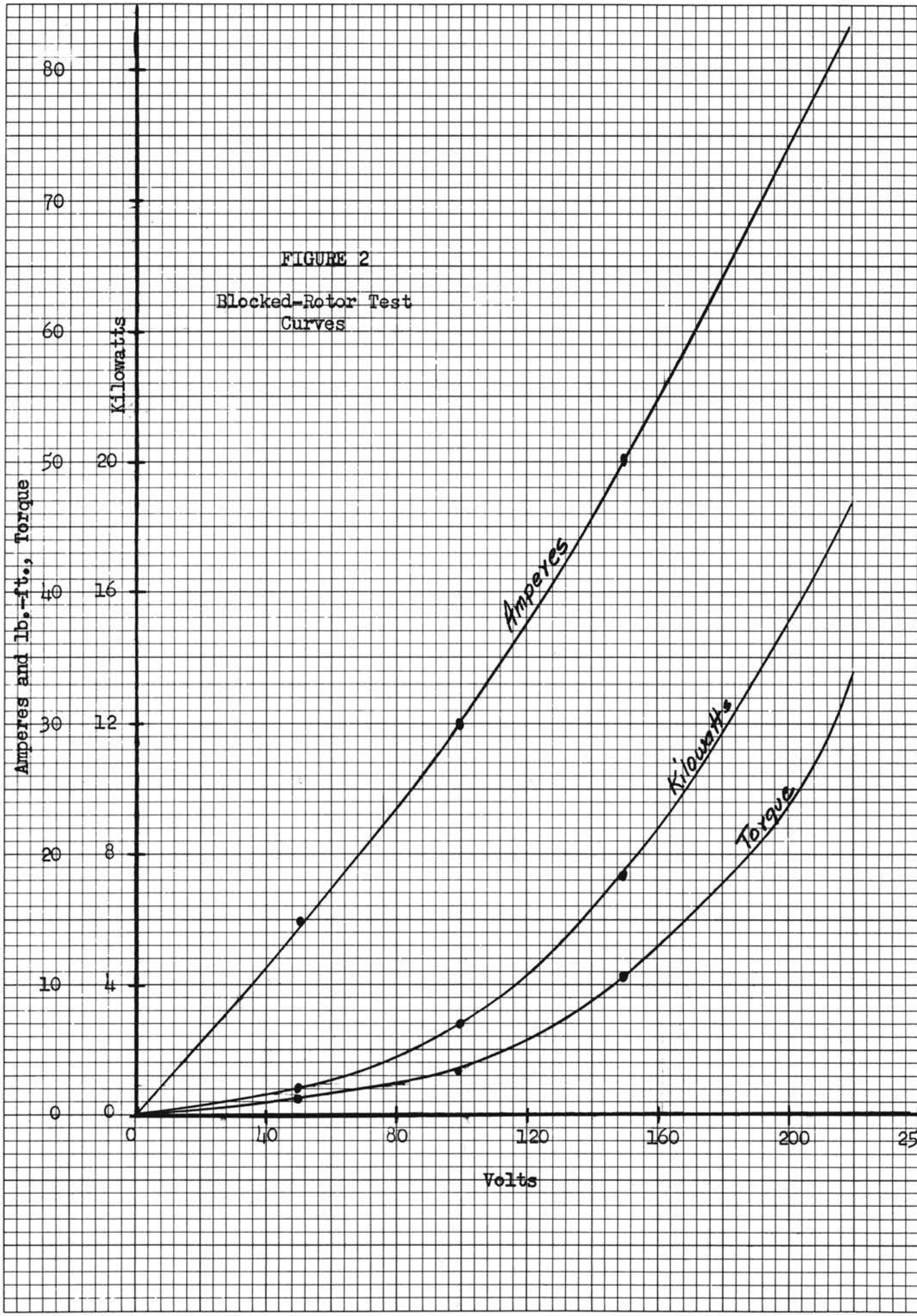


FIGURE 2
Blocked-Rotor Test
Curves
Amperes and lb.-ft., Torque
Kilowatts
80
70
60
50
40
30
20
10
0
20
16
12
8
4
0
0
40
80
120
160
200
250
Volts
Amperes
Kilowatts
Torque

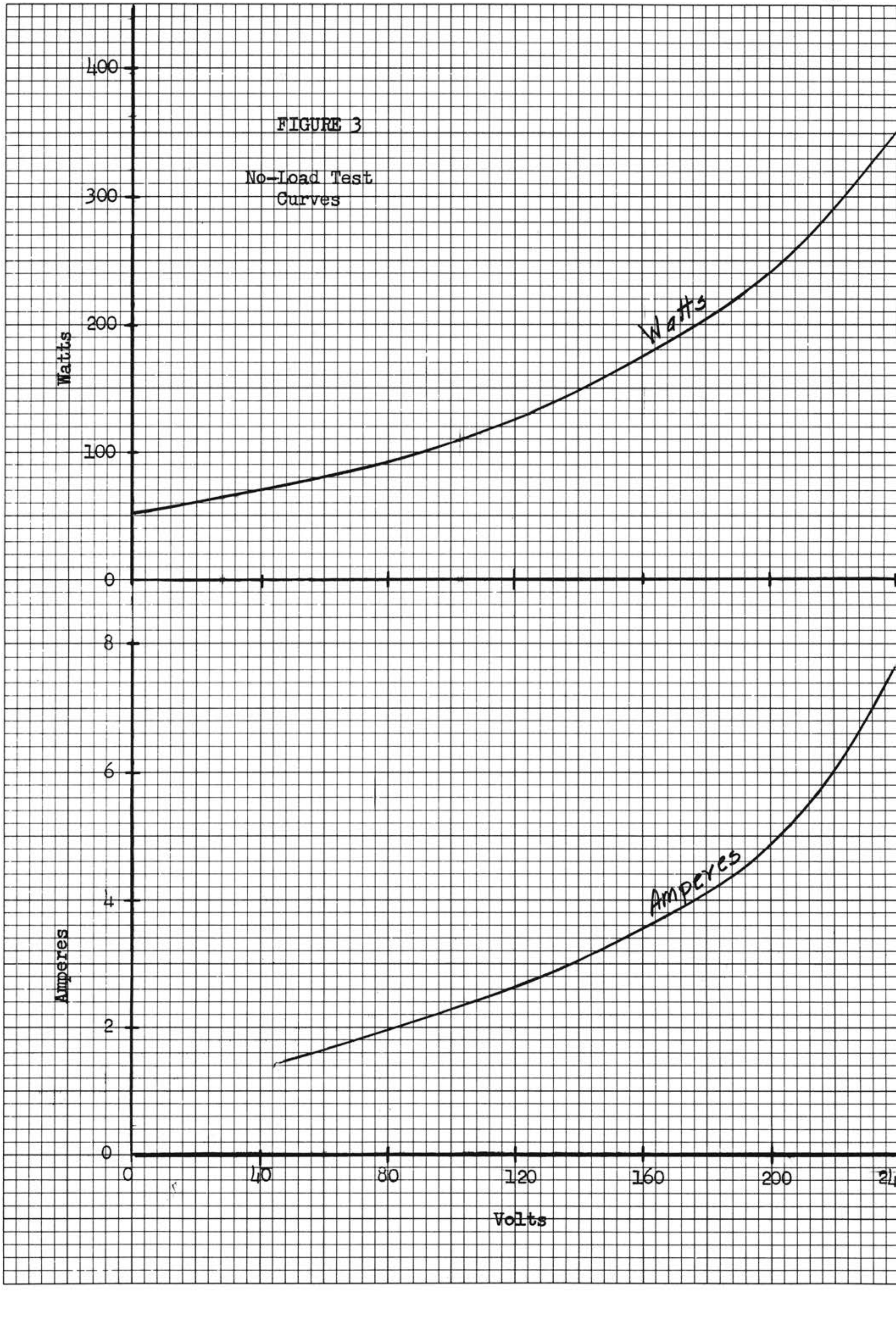


FIGURE 3
No-Load Test
Curves
Watts
Amperes
Volts
400
300
200
100
0
8
6
4
2
0
0
40
80
120
160
200
240

TABLE 2-3

DYNAMOMETER-LOAD TEST DATA

Torque lb.-ft.	Volts	Amperes	Speed R.P.M.	Scale lbs.	Power-factor	Input Watts	Output Watts	Losses Watts	Efficiency, %	Output H.P.
0.0	220*	5.8*	1798*			300*				
1.05	220	6.0	1792	1.0		600	300	32	50	.403
6.2	220	7.8	1767	5.9	.62	2000	1585	32	79.5	2.12
10.5	220	10.4	1743	10.0	.76	3150	2632	32	83.4	3.52
15.75	220	14.5	1710	15.0	.83	4750	3862	32	81.3	5.17
21.0	220	19.5	1682	20.0	.85	6450	5062	32	78.5	6.80

*The data given on the first line are for the uncoupled motor input.

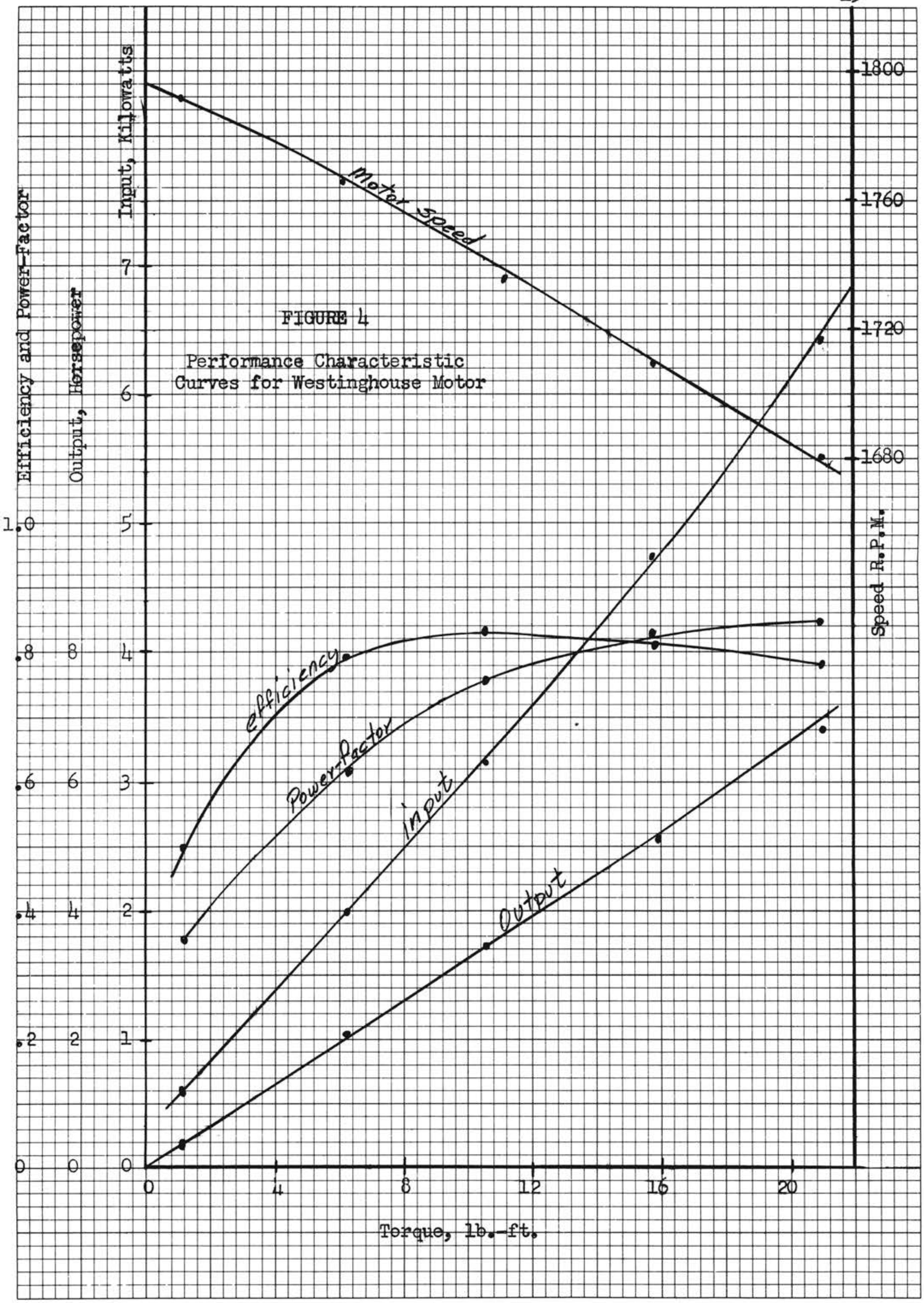


FIGURE 4
Performance Characteristic
Curves for Westinghouse Motor
Efficiency and Power-Factor
Output, Horsepower
Input, Kilowatts
Speed R.P.M.
Torque, lb.-ft.
Motor speed
efficiency
Power-factor
input
Output
1800
1760
1720
1680
0
4
8
12
16
20
1
2
3
4
5
6
7
1.0
.8
.6
.4
.2

From these data and equations the equivalent circuit for the test motor can be established.

(2) $$Z_e = {}^{E}/I_{sc}, \quad E = 127 \text{ and from Figure 2, } I_{sc} = 83$$

$$Z_e = \frac{127}{83}$$

$$Z_e = 1.53 \text{ ohms}$$

(3a) $$R_e = Z_e \frac{\text{watts}}{\text{Volt-amperes}}$$

From Figure 2, watts $= 18.8 \cdot 10^3$ and

$$\text{Volt-amperes} = \sqrt{3}\ (220)(83) = 31.7 \cdot 10^3$$

$$R_e = 1.53 \ \frac{18.8 \cdot 10^3}{31.7 \cdot 10^3}$$

$$R_e = .910 \text{ ohms}$$

(4) $$X_e = \sqrt{Z_e^{\,2} - R_e^{\,2}}$$

$$= \sqrt{1.53^2 - .910^2}$$

$$X_e = \quad 1.23 \text{ ohms}$$

R_1 was measured as .441 ohms per phase

(5) $$R_2 = R_e - R_1$$

$$R_2 = .910 - .441$$

$$R_2 = .469 \text{ ohms}$$

(1) $$X_1 = X_2 = .5X_e$$

$$X_1 = X_2 = (.5)(1.23)$$

$$X_1 = X_2 = .615 \text{ ohms}$$

(6) $$X_m + X_1 = {}^{E}/I_{nl}, \ I_{nl} = 6 \text{ amps. in Figure 3}$$

$$X_m + .615 = \frac{127}{6}$$

$$X_m = 21.1 - .615$$

$$X_m = 20.5 \text{ ohms}$$

The voltage drop in the stator is

$$V = I_{nl} \quad (R_1 + jX_1)$$

$$= 6\ (.441 + j\ .615)$$

$$= 4.54 \underline{/\ 54.4^\circ}$$

$$V = 2.64 + j3.69 \text{ volts}$$

$$E_1 = \overline{E} - \overline{V}$$

$$= 127 + j0 - 2.64 - j\ 3.69$$

$$= 124.3 - j3.69$$

$$E_1 \approx 124 \text{ volts}$$

From Figure 3, the no-load watts at rated voltage is 290 watts. The friction and windage loss is 50 watts. The stator copper loss is

$$I_{nl}^2 R_1 = 6^2\ (.441)$$

$$= 15.9 \text{ watts}$$

The sum of the friction, windage, and stator copper loss is

$$50 + 15.9 = 65.9 \text{ watts}$$

which, if subtracted from the no-load watts input will give the core loss.

$$\text{core loss} = 290 - 65.9$$

$$= 224 \text{ watts}$$

(7) $$g_h = \frac{W_{cl}}{3E_1^2}$$

$$= \frac{224}{3(124)^2}$$

$$g_h = .00485 \text{ mhos}$$

The equivalent circuit shown in Figure 5 may be drawn using the values calculated.

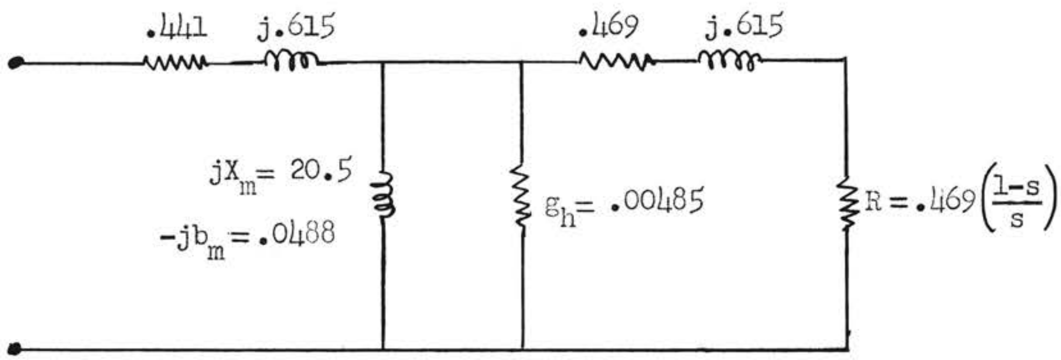


FIGURE 5
Equivalent Circuit for Westinghouse Three-Phase Induction Motor

SOLUTION OF EQUIVALENT CIRCUIT

Probably the most convenient method of solving the equivalent circuit is that given by the Alger[2]. The following steps for calculations are given

[2]Ibid, Page 120-124

TABLE 2-4

EQUIVALENT CIRCUIT CALCULATIONS

s	Slip
(8) $Z_2 = \frac{R_2}{s} + jX_2$	Secondary impedance
(9) $Y_2 = G_2 - jB_2$	Reciprocal of (8)
(10) $Y_m = g_h - jB_m$	Magnetizing admittance
(11) $Y = G - JB$	Sum of (9) and (10)
(12) $Z = R + jX$	Reciprocal of (11)
(13) $Z_1 = R_1 + jX_1$	Primary Impedance
(14) $Z_o = R_o + jX_o$	Sum of (12) and (13)
(15) $\lvert Z_o \rvert = \sqrt{R_o + jX_o}$	Phase Impedance
(16) $I_1 = {}^{E}/Z_o$	Phase Current
(17) R_o/Z_o	Power factor
(18) $I_1^{\,2}R_o$	Input per phase
(19) $\frac{I_1^{\,2}\ RG_2\ (1-s)}{G}$	Input to shaft or gross output
(20) $(W_f)(1-s)$	Friction and windage
(21) Net output	(19) minus (20)
(22) Efficiency	(21) divided by (18)
(23) Torque	(21) divided by (1-s) Sync. Speed

TABLE 2-5

EQUIVALENT CIRCUIT CALCULATIONS MADE IN ACCORDANCE WITH TABLE 2-4

		s = .00777	s = .0166	s = .033	s = .05
(8)	Z_2	60.4-j.615	28.3-j.615	14.08-j.615	9.390j.615
(9)	Y_2	.0165-jo	.0353-jo	.0710-jo	.108-jo
(10)	Y_m	.00485-j.0488	.00485-j.0488	.00485-j.0488	.00485-j.0488
(11)	Y	.0213-j.0488	.042-j.0488	.758-j.0488	.113-j.0488
(12)	Z	7.55-j17.2	10-j12.15	9.34-j6.21	7.45-j3.21
(13)	Z_1	.441-j.615	.441-j.615	.441-j.615	.441-j.615
(14)	Z_c	7.99-j17.8	10.4-j12.7	9.78-j6.82	7.89-j3.82
(15)	$\lvert Z_o \rvert$	21.9	16.4	11.9	8.78
(16)	I_1	6.07	7.75	10.65	14.5
(17)	Power Factor	.410	.624	.820	.90
(18)	Input per phase	293	623	1110	1660
(19)	Input to shaft	213	518	958	1420
(20)	$W_f(1 - s)$	49.6	49	48.5	47.5
(21)	Net Output	163.4	469	909.5	1372.5
(22)	Efficiency	55.7	75.5	81.9	82.2

The calculations outlined in Table 2-4, can be made with greatest convenience when performed in the order shown. These calculations were made for several values of slip and are shown in Table 2-5. From this table, the items necessary to plot conventional performance curves may be taken. These data are tabulated in Table 2-6, and are plotted in comparison with actual test results in Figure 6. It may be easily seen that the calculated performance curves are in very close agreement with the curves plotted from test data. This justifies the assumption that the equivalent circuit shown in Figure 5, is sufficiently accurate to be used as a basis for further calculations.

TABLE 2-6

TABULATED DATA FOR CALCULATED PERFORMANCE

Slip, %	Input Watts	Output H.P.	Power-factor	Efficiency %
.077	879	.606	.410	55.7
1.6	1869	1.89	.624	75.5
3.3	3874	3.66	.820	81.9
5.0	4260	5.51	.900	82.8

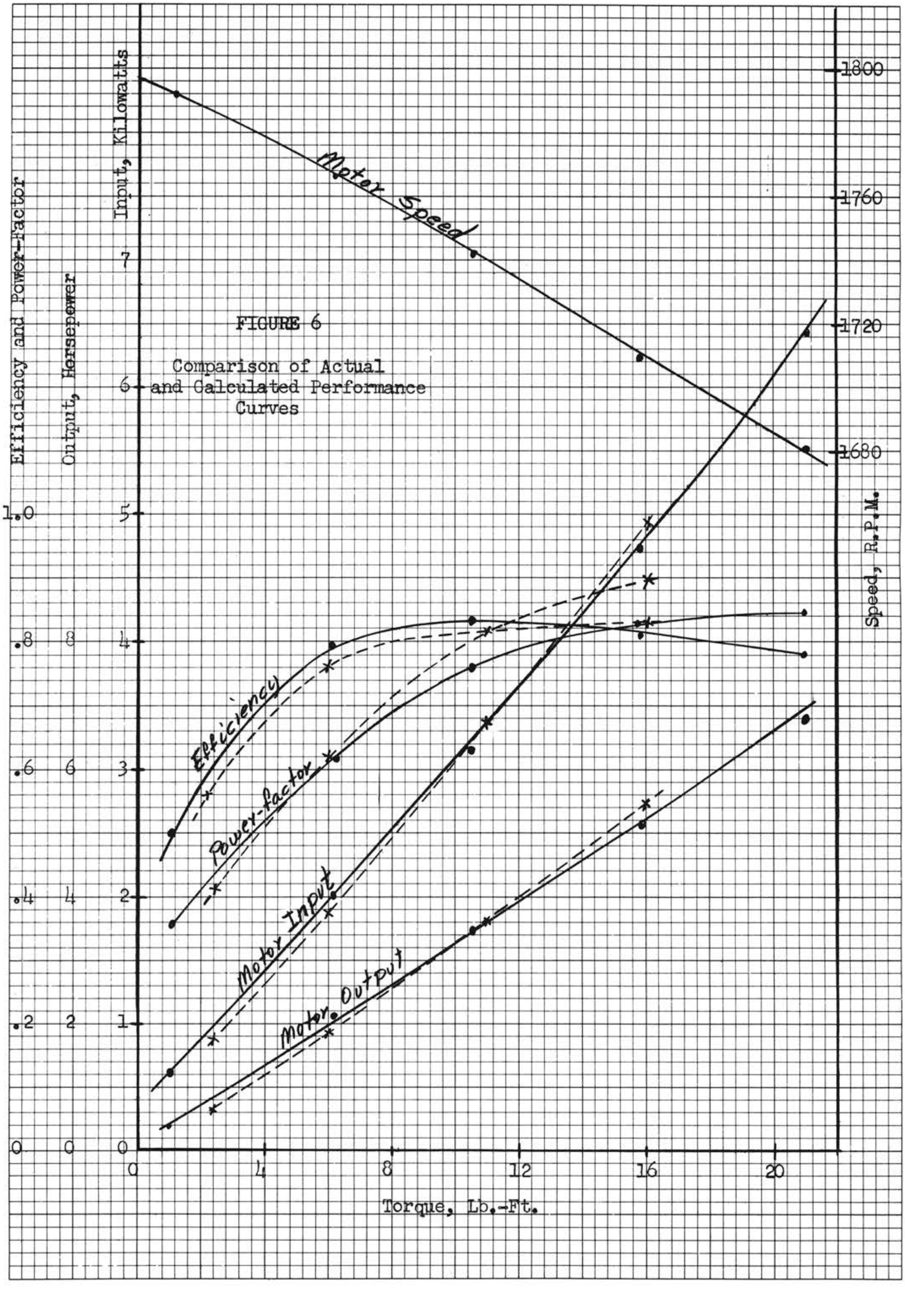


FIGURE 6
Comparison of Actual and Calculated Performance Curves
Motor Speed
Efficiency
Power-factor
Motor Input
Motor Output
Efficiency and Power-Factor
Output, Horsepower
Input, Kilowatts
Speed, R.P.M.
Torque, Lb.-Ft.
1800
1760
1720
1680
1.0
.8
.6
.4
.2
0
8
6
4
2
0
7
6
5
4
3
2
1
0
4
8
12
16
20

CHAPTER III

CALCUALTION OF MOTOR PERFORMANCE ON UNBALANCED APPLIED VOLTAGES

Unbalanced three-phase circuits are generally solved by the method of symmetrical components. This method affords a convenient means of extending the equivalent circuit solution for induction motors to enable the motor performance to be determined analytically when it is operating on unbalanced voltages.

SYMMETRICAL COMPONENTS

It can be shown that any unbalanced set of three-phase vectors, such as a set that might represent an unbalanced voltage supply to an induction motor, may be represented or resolved into three separate sets of balanced vectors.[1] One set, called the positive-sequence, is a set of three-phase vectors having the same phase rotation as the original unbalanced vectors. The second, called the negative-sequence, is a set of three-phase vectors having opposite phase rotation to that of the original set; and the third, called the zero-sequence, is composed of three equal vectors in time-phase with each other. Thus, some unbalanced three-phase vector system can be represented by the three sets of vectors shown in Figure 1.

The original unbalanced vector system represented by the symmetrical components in Figure 1, may be obtained by combining these components in

---

[1]Wagoner & Evans, Symmetrical Components, Chapter II, McGraw-Hill Book Co.

the following manner.

(1) $$E_{ab} = E_{ab1} + E_{ab2} + E_{ab0}$$

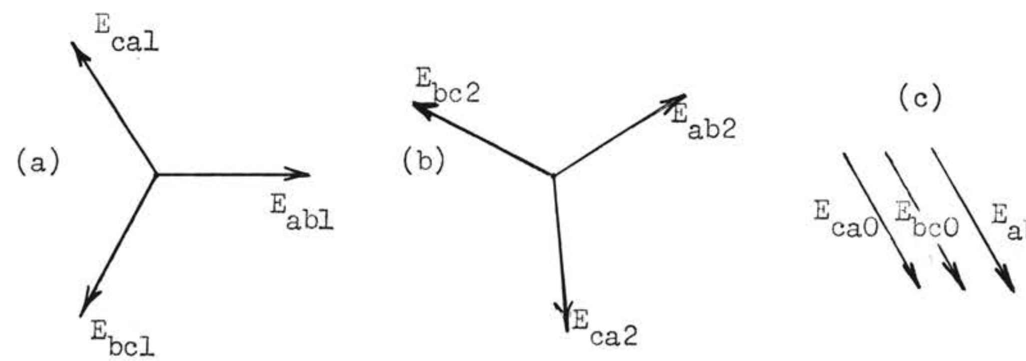


FIGURE 1
Symmetrical Components Vector System

Positive-sequence is represented in (a), negative-sequence in (b) and zero-sequence in (c).

(2) $$E_{bc} = E_{ab1} + E_{ab2} + E_{ab0}$$

(3) $$E_{ca} = E_{ca1} + E_{ca2} + E_{ca0}$$

Symmetrical components method of analysis makes use of an operator, "a". This operator represents a unit vector at an angle of 120 degrees.

(4a) $$a = 1 \underline{/120^{\circ}} = -.5 + j.866$$

(4b) $$a^2 = 1 \underline{/240^{\circ}} = -.5 - j.866$$

(4c) $$a^3 = 1 \underline{/360^{\circ}} = 1 + j0$$

From Figure 1, it may be seen that

(5a) $$E_{ca1} = aE_{ab1}$$

(5b) $$E_{bc1} = a^2E_{ab1}$$

(5c) $$E_{bc2} = aE_{ab2}$$

(5d) $$E_{ca2} = a^2E_{ab2}$$

(5e) $$E_{ab0} = E_{bc0} = E_{ca0}$$

and by proper substitution of the equalities of (5) into (1), (2), and (3), the following equations may be written.

(6) $$E_{ab} = E_{ab1} + E_{ab2} + E_{ab0}$$

(7) $$E_{bc} = a^2E_{ab2} + aE_{ab2} + E_{ab0}$$

(8) $$E_{ca} = aE_{ab2} + a^2E_{ab2} + E_{ab0}$$

These equations relate the unbalanced vector system to the three balanced systems.

DETERMINATION OF THE SYMMETRICAL COMPONENTS FOR AN UNBALANCED THREE-PHASE SYSTEM

By miltiplying equation (7) by a and (8) by a^2, and adding the results to equation (6), the following is obtained.

$$\begin{aligned} E_{ab} + aE_{bc} + a^2E_{ca} &= (E_{ab1} + E_{ab1} + E_{ab1}) + (E_{ab2} + a^2E_{ab2} + aE_{ab2}) + \\ &\quad (E_{ab0} + aE_{ab0} + a^2E_{ab0}) \\ &= 3E_{ab1} + (1 + a^2 + a)E_{ab2} + (1 + a + a^2)E_{ab0} \end{aligned}$$

since

$$1 + a + a^2 = 1 - .5 + j.866 - .5 - j.866 = 0$$

$$E_{ab} + aE_{bc} + a^2E_{ca} = 3E_{ab1}$$

(9) $$E_{ab1} = 1/3\ (E_{ab} + aE_{bc} + a^2E_{ca}).$$

By a similar method it can be shown that

(10) $$E_{ab2} = 1/3\ (E_{ab} + a^2E_{bc} + aE_{ca})$$

and by simply adding equations (6), (7), and (8) the following equation results.

(11) $$E_{ab0} = 1/3\ (E_{ab} + E_{bc} + E_{ca})$$

Three equations are now available to use in determining the symmetrical components for and unbalanced set of three-phase vectors.

OBTAINING PHASE QUANTITIES FROM LINE VALUES

When applied to the analysis of a three-phase induction motor the zero-sequence component may usually be neglected. In a three-wire three-phase connection the vector sum of the line voltages must be zero, and equation (11) shows the zero-sequence must then be zero. Most three-phase induction motors are wye connected with a buried neautral so the zero-sequence component may be neglected.

While the omission of the neutral wire simplifies unbalanced calculations to the extent of making it possible to neglect the zero-sequence, it is also a complicating factor. Motor calculations are usually made on a single-phase basis. With no neutral connection the measurement of phase voltages is difficult. This situation gives rise to the need for equations which will enable phase quantities to be calculated from line-to-line voltages, and this may be accomplished in the following manner.

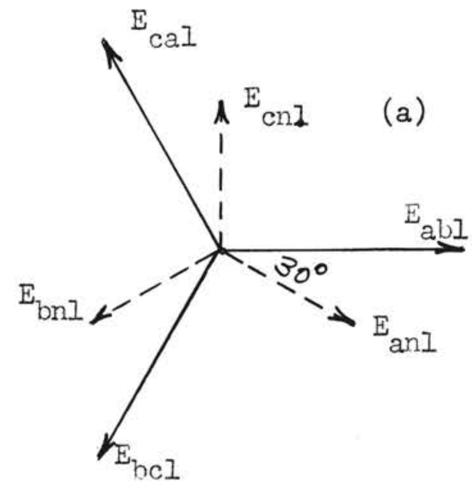
By knowing the line-to-line voltages the positive- and negative-sequence components may be calculated. Consider first, the positive-sequence set. From the balanced set of three-phase vectors the vector diagram for phase voltages may be drawn as shown in Figure 2.

From the vector diagram of Figure 2, the following relations may be stated

(12a) $$E_{ab1} = E_{an1} - E_{bn1}$$

(12b) $$E_{bc1} = E_{bn1} - E_{cn1}$$

(12c) $$E_{ca1} = E_{cn1} - E_{an1}$$



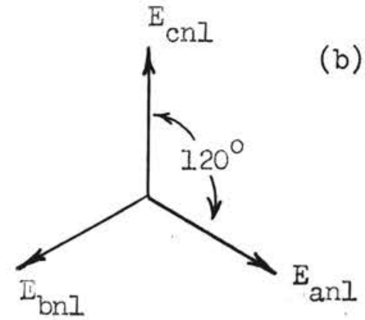


FIGURE 2
Relation of Symmetrical Components Line and Phase Values

In (a) the solid-line vectors represent the positive-sequence component and the dashed-line vectors are phase voltages of the line-to-line voltages; in (b) the phase vectors of (a) are shown removed.

and by use of the operator "a" the following equalities may be written

(13a) $$E_{bn1} = a^2 E_{an1}$$

(13b) $$E_{cn1} = aE_{an1}$$

and substituted into (12a) to give

(14) $$E_{ab1} = E_{an1} - a^2 E_{an1}$$

$$= (1-a^2)E_{an1}$$

but, $$(1-a^2) = 1 - j.866$$

$$= \sqrt{3}\ \underline{/30^\circ}$$

so,

(14a) $$E_{ab1} = \sqrt{3}\, E_{an1}\underline{/30^\circ}\ .$$

From which the expression for determining the positive-sequence phase value from the positive-sequence line value may be written.

$$E_{an1} = \frac{E_{ab1}}{\sqrt{3}} \angle -30^{\circ} \tag{15}$$

By similar method it may be shown that the negative-sequence phase value may be determined by the following equation.

$$E_{an2} = \frac{E_{ab2}}{\sqrt{3}} \angle 30^{\circ} \tag{16}$$

There now remains one last obstacle to the use of symmetrical component method for the solution of a practical motor problem. In all the equations developed the voltages are used as vectors. Generally, only a voltmeter is available to measure the line-to-line voltages of a motor, thus only the magnitude and not the angular relations of these voltages will be known. The solution of this problem is relatively simple and may be achieved in one of two different ways. One method of solútion would be by graphical means. Once the magnitudes of the voltages are known a vector can be drawn to scale to represent one voltage. With the head of this vector as a center strike an arc whose radius is a scaled representation of the magnitude of the second line-to-line voltage. Next strike an arc about the tail of this vector with a radius to scale for the third line-to-line voltage. The remaining two vectors may be drawn from the head and tail of the first vector to the point of intersection of the two arcs. This graphical solution is made possible by the fact that the vector sum of the voltage of a three-phase system must be zero. This solution is shown in Figure 3.

An analytical solution for determining the angular relationship of three line-to-line voltages is also available. The angles of the voltages

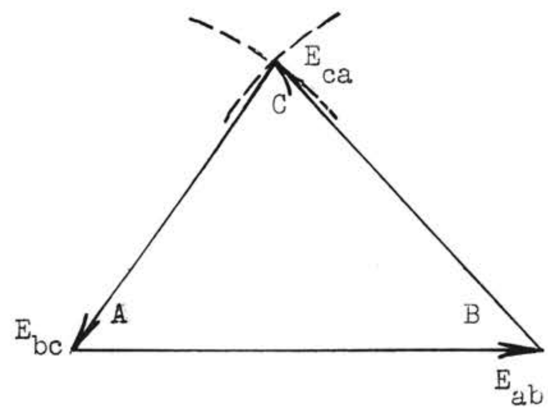


FIGURE 3
Closed-Vector Diagram for a Set of Three-Phase Vectors

in Figure 3 could have been determined by the law of cosines in the following manner.

(17) $$E_{bc}^2 = E_{ab}^2 + E_{ca}^2 - 2E_{ab}E_{ca} \cos B$$

From which it is possible to solve for the angle B.

Equation (17) may be expressed in a more convenient form since it will be used only to solve for the angle B.

$$\frac{E^2_{bc}}{2E_{ab}E_{ca}} = \frac{E_{ab}}{2E_{ca}} - \frac{E_{ca}}{2E_{ab}} - \cos B$$

$$\cos B = \frac{E_{ab}}{2E_{ca}} + \frac{E_{ca}}{2E_{ab}} - \frac{E^2_{bc}}{2E_{ab}E_{ca}}$$

(17a) $$B = \cos^{-1}\left[\frac{E_{ab}}{2E_{ca}} + \frac{E_{ca}}{2E_{ab}} - \frac{E^2_{bc}}{2E_{ab}E_{ca}}\right]$$

By use of the law of sines the other angles may be readily calculated.

(18) $$\frac{E_{ca}}{\sin A} = \frac{E_{bc}}{\sin B} = \frac{E_{ab}}{\sin C}$$

SOME ADDITIONAL EQUATIONS FOR SYMMETRICAL COMPONENTS

From the basic equations already established it is possible to derive some other equations which will be in a more convenient form insofar as their usefulness here is concerned. It will be seen later that a primary problem will be to determine the positive- and negative-sequence component of three-phase line-to-line voltages when only the magnitudes of these voltages are known. These components will have to be expressed in phase values in order to solve the symmetrical components equivalent circuit.

Given three line-to-line voltages of a three-phase system it is known their vector sum must be zero.

$$E_{ab} + E_{bc} + E_{ca} = 0$$

$$E_{ca} = -E_{ab} - E_{bc}$$

Upon substituting this value for E_{ca} into equation (9), the following is obtained.

(9a)
$$E_{ab1} = 1/3 \left[E_{ab} + aE_{bc} + a^2 (-E_{ab} - E_{bc})\right]$$
$$= 1/3 \left[(1-a^2) E_{ab} + (a-a^2) E_{bc}\right]$$

since
$$1-a^2 = \sqrt{3} \underline{/30^\circ}$$
$$a-a^2 = \sqrt{3} \underline{/90^\circ},$$
$$E_{ab1} = 1/3 (E_{ab} \sqrt{3} \underline{/30^\circ} + E_{bc} \sqrt{3} \underline{/90^\circ})$$

which may be reduced to

(19)
$$E_{ab1} = \frac{E_{ab} + E_{bc} \underline{/60^\circ}}{\sqrt{3}} \underline{/30^\circ}$$

By a similar method it may be shown that

(20)
$$E_{ab2} = \frac{E_{ab} - E_{bc} \underline{/-60^\circ}}{\sqrt{3}} \underline{/-30^\circ}$$

The greatest advantage in the use of equations (19) and (20) is realized when calculating phase values of positive- and negative-sequence components.

By substituting the right-hand part of equation (19) into (15) for E_{ab1}, the following equation may be written

$$E_{an1} = \left[\frac{\left(\frac{E_{ab} + E_{bc} \angle 60^\circ}{\sqrt{3}} \right) \angle 30^\circ}{\sqrt{3}} \right] \angle -30^\circ$$

which may be reduced to

(21) $$E_{an1} = 1/3 \; (E_{ab} + E_{bc} \angle 60^\circ)$$

and by similar substitution of (20) into (16) the equation for the phase value of the negative-sequence component may be written.

(22) $$E_{an2} = 1/3 \; (E_{ab} + E_{bc} \angle -60^\circ)$$

Since the solution of the positive- and negative-sequence equivalent circuit is based on having E_{an1} and E_{an2} as applied voltages, equations (21) and (22) are very useful and convenient.

SYMMETRICAL COMPONENTS EQUIVALENT CIRCUIT FOR AN INDUCTION MOTOR

Several alternating current machinery text books give the positive- and negative-sequence equivalent circuits for an induction motor.[2] These circuits will be given and used here without derivation or substantiation.

By use of the equivalent circuit shown in Figure 4, the total positive-sequence shaft power output may be calculated by

(23) $$P_1 = 3\left(\frac{1-s}{s}\right) R_r I^2_{r1}$$

The negative-sequence equivalent circuit is shown in Figure 5. The total negative-sequence shaft-power output may be calculated by the following equation.

---

[2]Ibid., Chapter XVII

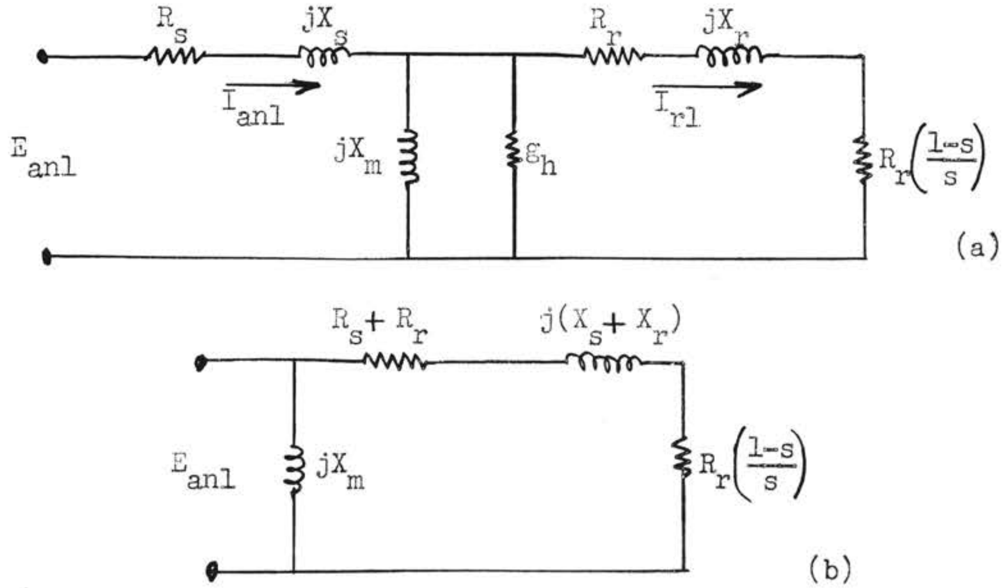


FIGURE 4
Positive-Sequence Equivalent Circuit for an Induction Motor

(a) the exact equivalent circuit, (b) the approximate or simplified equivalent circuit.

E_{an1}= pos.-seq. phase voltage jX_s = stator leakage reactance
R_s = resistance of stator jX_r = rotor leakage reactance
R_r = resistance of rotor jX_m = magnetizing reactance

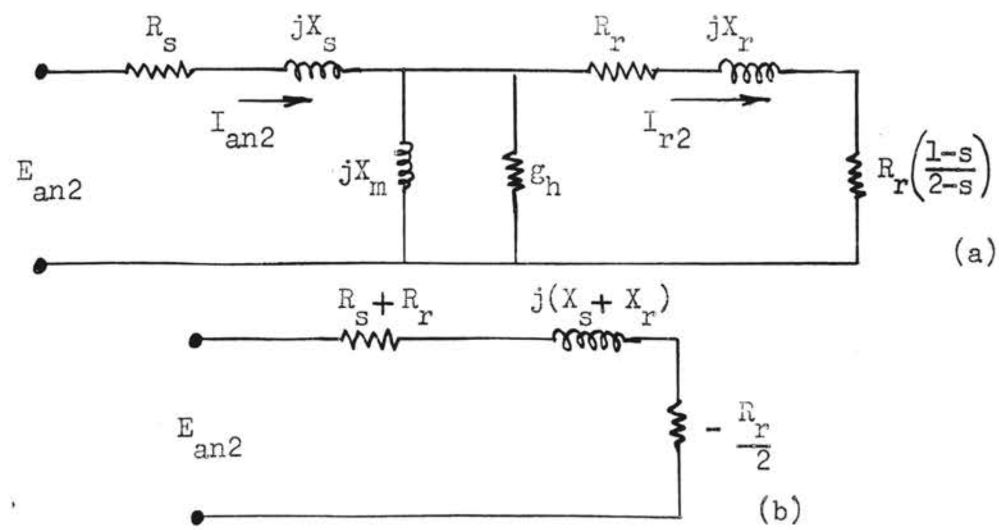


FIGURE 5
Negative-Sequence Equivalent Circuit for an Induction Motor

(a) the exact equivalent circuit, (b) the approximate or simplified equivalent circuit.

E_{an2} = neg.-seq. phase voltage jX_s = stator leakage reactance
R_s = resistance of stator jX_r = rotor leakage reactance
R_r = resistance of rotor jX_m = magnetizing reactance

(24) $$P_2 = -3\left(\frac{1-s}{2-s}\right)R_r I^2_{r2}$$

Equation (24) is for the calculation of power output when the exact negative-sequence equivalent circuit is used. If the approximate circuit is used this equation is modified only to the extent that, $\left(\frac{1-s}{2-s}\right)R_r$, is replaced by $R_r/2$.

When both positive- and negative-sequence voltages are applied to a motor (as would be the case with unbalanced applied voltages) the total shaft power output may be calculated by

(25) $$P_t = 3\left(\frac{1-s}{s}\right)R_r I^2_{r1} - 3\left(\frac{1-s}{2-s}\right)R_r I^2_{r2}.$$

SYMMETRICAL COMPONENTS EQUIVALENT CIRCUITS, THEIR SOLUTION AND COMPARISON WITH TEST RESULTS

From the equivalent circuit of Chapter II, actual values may be subsittuted in the circuits shown in Figures 4 and 5, to establish the symmetrical components equivalent circuits for the test motor. These equivalent circuits are shown in Figure 6.

The next step is to solve these circuits to obtain calculated motor performance data. Usually, the procedure used in solving these circuits to obtain performance data is to assume a value of slip, reduce the circuit, solve for branch currents, and from these currents calculate performance items. In this work these circuits are used only as a means of calculating motor efficiency at, or near, full load. The efficiency in in the region near rated load is essentially a flat curve, so any value of slip may be used for calculations which will result in approximately full-load current flow or rated horsepower output. For most induction motors the full-load slip is approximately 5 per cent. As a first try this value will be used in solving the equivalent circuits. These circuits will be solved for a particular unbalanced voltage condition and the results of the calculations compared with test data.

In Table 3-1, the data from a dynamometer-load test made with unbalanced voltages applied to the motor is given. These data are shown plotted in Figure 7. Constant values of voltages and voltage unbalance were maintained for all loads during the test.

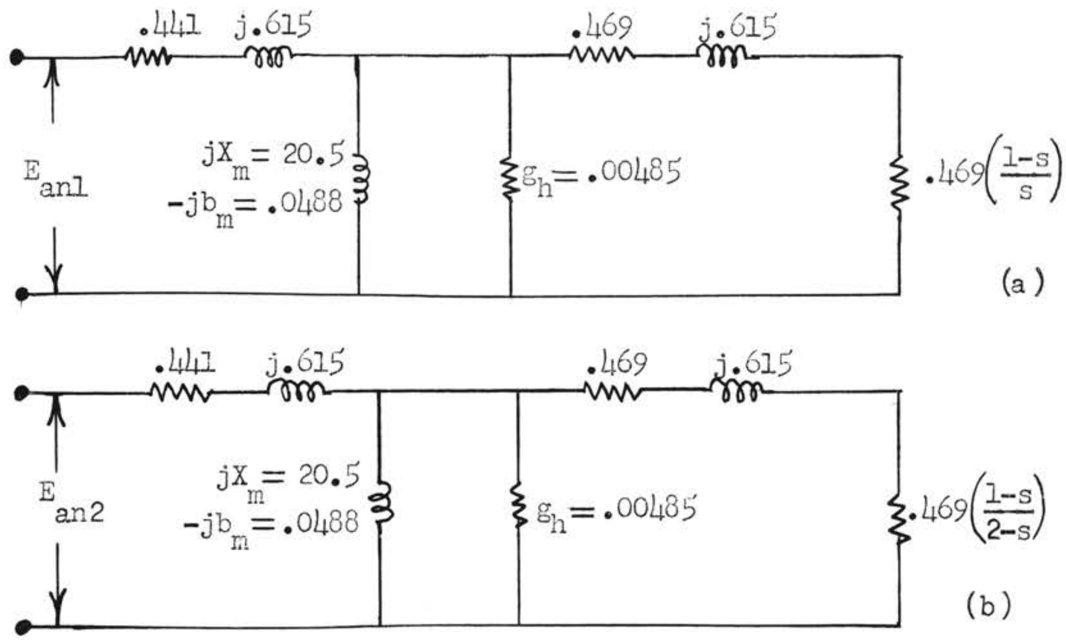


FIGURE 6
Symmetrical Components Equivalent Circuits

The positive-sequence circuit is shown in (a), and (b) shows the negative-sequence circuit.

In order to compare the calculated results with test data the equivalent circuits solutions will be made using the same line-to-line voltages as used for the test, namely

$$E_{ab} = 200 \text{ volts}$$

$$E_{bc} = 210 \text{ volts}$$

$$E_{ca} = 240 \text{ volts}$$

TABLE 3-1

DYNAMOMETER-LOAD TEST WITH UNBALANCED VOLTAGES

VOLTS			Amperes			Input	Speed	Scale	Output	Losses	Output	Effici-	Slip
AB	BC	CA	A	B	C	Watts	RPM	Lbs	Watts	Watts	H.P.	ency %	%
200*	210*	240*	9.8*	4.9*	14.6*	600*							
200	211	240	10.8	4.0	13.5	950	1780	0.40	106	32	.158	14.5	1.11
200	210	240	15.3	2.0	14.9	2700	1743	6.90	1800	32	2.48	68.0	3.27
200	210	240	18.6	4.2	16.4	3850	1727	10.0	2580	32	3.5	67.5	4.15
200	210	240	22.0	8.2	19.4	5100	1714	14.0	3580	32	4.84	70.8	4.77
200	210	240	25.0	10.6	22.0	6200	1700	17.8	4510	32	6.08	73.0	5.55

---

* Data given on the first line are for the uncoupled motor input.

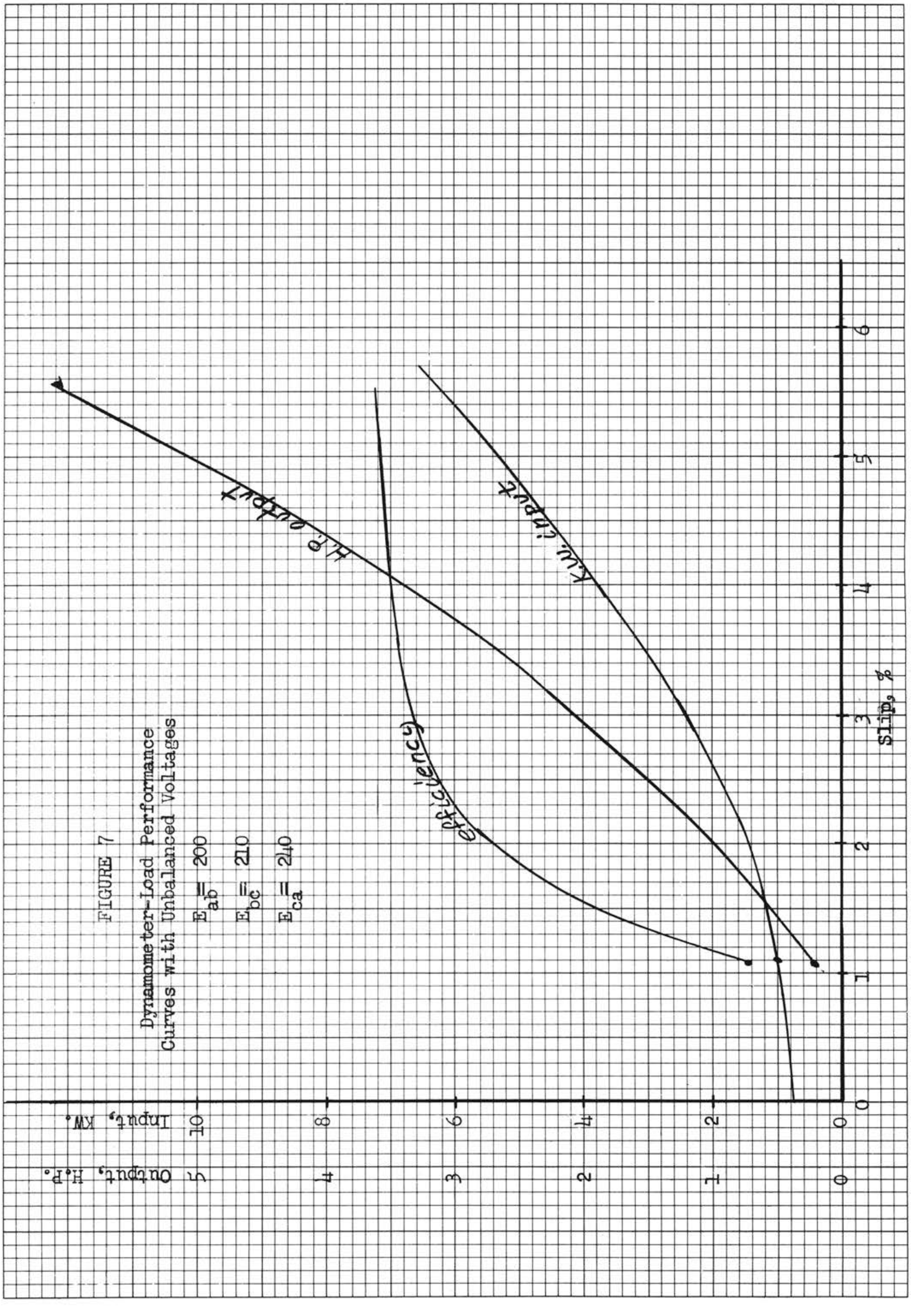


FIGURE 7

Dynamometer-Load Performance
Curves with Unbalanced Voltages

$E_{ab} = 200$

$E_{bc} = 210$

$E_{ca} = 240$

The vector relationship of these voltages must be established.

(17a)
$$B = \cos^{-1}\left[\frac{200}{480} + \frac{240}{400} - \frac{210}{2(200)(240)}\right]$$
$$= \cos^{-1}(.558)$$
$$B = 56.1^{\circ}$$

and by use of the law of sines

$$A = 71.5^{\circ}$$
$$C = 52.4^{\circ}$$

The vector diagram of Figure 8, may now be drawn for this system.

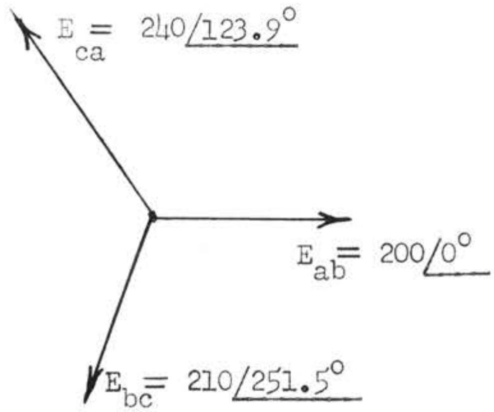


FIGURE 8
Three-Phase Vector Diagram for Unbalanced Voltages

Before solving the equivalent circuits it will be necessary to determine the positive- and negative-sequence phase voltages. By equation (21)

$$E_{an1} = \frac{1}{3}\ (200 + 210\ \underline{/251.5^{\circ} + 60^{\circ})}$$
$$= 125\ \underline{/-24.8^{\circ}}$$
$$= 113.1 - j5.24 \text{ volts}$$

and by equation (22)

$$E_{an2} = \frac{1}{3}\ (200 + 210\ \underline{/251.5^{\circ} - 60^{\circ})}$$
$$= 14.2\underline{/101^{\circ}}$$

$$E_{an2} = -2.7 - j13.9 \text{ volts}$$

TABLE 3-2

Calculations for Positive- and Negative-Sequence Equivalent Circuits

	Positive-Sequence	Negative-Sequence
(1) Rotor Impedance, Z_r	$9.4 - j.615$	$.696 + j.615$
	$9.4\angle 3.76^\circ$	$.930\angle 41.4^\circ$
(2) Rotor Admittance, Y_r	$.1063 - j0$	$.806 - j.711$
	$.1063\angle 0^\circ$	$1.075\angle -41.4^\circ$
(3) Mag. Admittance, Y_m	$.00485 - j.0488$	$.00485 - j.0488$
(4) Admittance, Y	$.111 - j.0488$	$.812 - j.760$
	$.1215\angle -23.6^\circ$	$1.11\angle -43.2^\circ$
(5) Reciprocal of (4), Z	$8.24\angle 23.6^\circ$	$.900\angle 43.2^\circ$
	$7.55 + j3.30$	$.656 + j.616$
(6) Stator Impedance, Z_s	$.441 + j.615$	$.441 + j.615$
(7) Total Impedance, Z_o	$7.99 + j3.91$	$1.097 + j1.231$
	$8.86\angle 26.2^\circ$	$1.645\angle 48.3^\circ$
(8) $I_{an} = E_{an}/Z_o$	$14.1\angle 1.4^\circ$	$8.64\angle 52.7^\circ$
(9) $I_{an}Z_s$	$10.7\angle 55.8^\circ$	$6.56\angle 107.3$
	$6.0 + j8.85$	$-1.92 + j6.29$
(10) $E_s = E_{an} - (9)$	$107 - j14$	$-78 + j7.61$
	$108\angle -7.45^\circ$	$7.61\angle 95.9^\circ$
(11) $I_{r2} = E_s/Z_r$	11.5	8.24
(12) Power Input, P_i	4750	141
(13) Power Output, P_o	3550	46.1

Total power input = 4750 - 141 = 4890 watts

Total power output = 3550 - 46.1 = 3504 watts, Horsepower Output = 4.8

Efficiency = 71%

From Figure 7, it may be seen that actual test data shows a horsepower output of 4.95 at 5 per cent slip and the efficiency for this point is 70.8 per cent. These values are in very close agreement with the calculated values shown in Table 3-2. This correlation of results justifies use of symmetrical components equivalent circuits to determine the motor efficiency when operating on unbalanced voltages.

CHAPTER IV

ANALYTICAL CALCUALTION OF MOTOR TEMPERATURE RISE

It has been shown in the previous chapter how motor performance with unbalanced applied voltages may be determined analytically. From the data obtained by these calculations it is possible to find the motor operating efficiency at any load. The losses represented by motor inefficiency is the power supply which increases the motor temperature. It remains, therefore to determine some method to relate these losses (which are usually expressed in watts) to temperature rise.

HEAT EQUATION FOR A MOTOR

Several alternating-current machinery textbooks show the temperature rise vs. time curve for an induction motor to be an exponential curve.[1] A typical curve is shown in Figure 1.

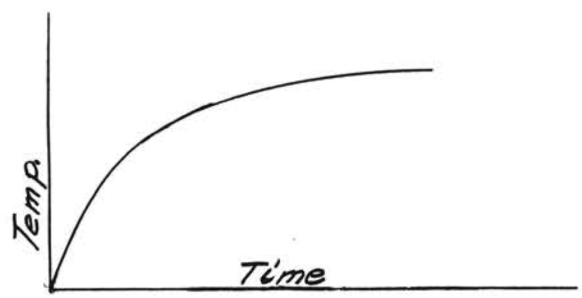


FIGURE 1

The general equation for such an exponential curve is

(1) $$y = k_1 (1 - e^{-k_2 x})$$

where x and y are the horizontal and vertical coordinate axis, respectively.

[1] B. F. Bailey & J. S. Gault, Alternating Current Machinery, McGraw-Hill Book Co., First Edition, p.p. 198.

It may be recalled that the foregoing relationship represents the transient current build-up in a R-L circuit when a direct-current voltage is applied. For this case, if current and time are represented on the y and x axis, respectively and the constants in the exponential equation are expressed in terms of the circuit parameters the resulting equation will be

$$I = \frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right). \tag{2}$$

It will be noted that k_1 of equation 1, is directly proportional to the voltage E, and inversely proportional to the resistance R. Similarly k_2 is directly proportional to R, and inversely proportional to the inductance L. By analogy with equation (2) the constants in equation (1) may be expressed in terms of other parameters when the general equation is applied in different cases. For the present purpose it is desired to express the constants of the general equation in terms of the motor thermal properties which are involved in motor heating.

In a general case the temperature of any body immersed in a homogenius medium is directly related to the energy supplied to raise its temperature and to its capacity for storing heat-energy. The body temperature is inversely related to its ability to dissipate heat to the surrounding medium; this infers an inverse relationship to size, velocity at which the surrounding medium is passing the body, etc.

Upon applying this generalized information to the problem of motor heating, the body is the motor and the surrounding medium is air. Assume for this study that there is no forced-air ventilation of the motor, i.e., the motor is surrounded by still air. Assume also that the heat conducted away from the motor through its mounting feet and shaft coupling are negligible. All the heat dissipated, then, must be given off into the surrounding still air. Represent the ability of the motor to dissipate heat through its surface by a constant, C_1, and let C_2 represent the motor ability to store heat energy.

To write the general exponential equation in terms of these motor thermal parameters, analogies may be made between them and the circuit parameters of equation (2). In equation (2) the quantity that increases exponentially with time is the current, for a motor it will be the temperature. In equation (2) the voltage, E, is the applied force which causes the current I, to increase. For the motor, the power applied to increase the temperature is the motor losses. (These are the losses reflected in motor efficiency.) The resistance R, is the parameter which represents the circuit ability to dissipate energy and the inductance L, is the parameter which determines the ability of the circuit to store energy. These may be represented in the new application as C_1 and C_2, respectively, since they are the constants which represent the motor ability to dissipate and store heat-energy. These analogies are summarized in Table 4-1.

TABLE 4-1

TABLE OF ANALOGIES

Parameter in Elec. circuit equation.		Parameter in motor thermal equation.
Current, I,	analogous to	Motor temperature, T.
Applied voltage, E,	"	Motor losses, P_L.
Resistance, R,	"	Constant, C_1.
Inductance, L,	"	Constant C_2.
Time, t,	"	Time, t.

The temperature-rise equation for the motor may now be written by substitution of these analogous items into equation (2).

$$(3) \qquad T = \frac{P_L}{C_1}\left(1 - e^{-tC_1/C_2}\right)$$

If P_L is constant, then for a given motor, the motor temperature is a function of only one variable, time (C_1 and C_2 will not vary for one motor provided the original assumptions are held). If C_1, C_2 and P_L are known then the temperature-rise vs. time curve for the motor may be plotted.

METHOD FOR DETERMINING MOTOR THERMAL PROPERTIES

When a motor is operating on unbalanced voltages its performance may be calculated by the method given in Chapter III. From these calculations motor efficiency may be determined for any load desired. Once the efficiency is known the motor losses may be calculated, so the power being supplied to raise the motor temperature is known. This is P_L of equation (3) and may be calculated from:

$$(4) \qquad P_L = (1\text{-efficiency})P_{input}$$

The constants C_1 and C_2 must now be determined. For values of time approximately equal to the value required for the motor to reach its final temperature, equation (3) may be simplified to

$$(5) \qquad T \approx \frac{P_L}{C_1}$$

$$(5a) \qquad C_1 \approx \frac{P_L}{T}$$

From this relationship C_1 may be determined with sufficient accuracy when T and P_L are known.

If the first derivitative of equation (3) were to be taken it would yield the time-rate-of-change of the motor temperature.

(3a) $$\frac{dT}{dt} = \frac{P_L}{C_2} e^{-tC_1/C_2}$$

upon subsitituing t = 0, as a boundry condition, the expression becomes

(3b) $$\frac{dT}{dt} = \frac{P_L}{C_2} \text{, when } t = 0$$

thus, the initial slope of the temperature-rise vs. time curve is P_L/C_2. If the initial slope of the curve and P_L are known, C_2 may be calculated.

It should be recognized that the motor thermal properties expressed by C_1 and C_2 are constants that depend only on the motor material and design. Regardless of the method, once they have been determined they are constants and may be used as such.

Experimental data obtained to plot the temperature-rise vs. time curve for the test motor operating on balanced voltages are given in Table 4-2, and this curve is shown in Figure 2. From these data and curves the motor thermal constants amy be determined by use of equations (4), (5a) and (3b).

In obtaining the experimental data an effort was made to keep a constant 5 h.p. load on the motor, however, this load varied slightly during the initial readings. A period of 30 seconds was used for the first four time increments in order to establish as accurately as possible the initial slope of the temperature-rise curve. The necessity for haste demanded by taking data for such small increments of time increase did not permit fine adjustment of the load. It is believed sufficient accuracy will be obtained by using the value of efficiency taken at points where the load was exactly 5 h.p., for use in calculating P_L. From data given in Table 4-2 and use of equation (4), the power loss for the motor operating at full load with balanced voltages applied may be calculated.

$$P_L = (1 - .800)\ 4650$$
$$= 931 \text{ watts}$$

TABLE 4-2

HEAT TEST WITH BALANCED VOLTAGES

Time Min.	Volts	Amperes	Watts Input	T_1	T_2	T_3	T_4	T_5	T_6
0-	220			20.0	20.0	20.0	20.0	20.0	20.0
0+	220								
.50	220		4600	31.0	31.0	33.0	33.0	33.0	33.0
1.0	220		4600	31.5	31.5	34.0	34.0	34.0	34.0
2.0	220	14.4	4700	32.0	32.0	36.0	36.0	35.0	36.0
3.0	220	14.3	4700	33.0	33.0	36.0	37.0	37.0	36.0
5.0	220	14.4	4650	35.0	35.0	37.0	38.0	38.0	36.0
10.	220	14.3	4650	39.0	38.5	39.5	41.0	41.0	37.0
15.	220	14.4	4650	42.0	41.0	42.0	43.0	43.0	40.0
20.	220	14.4	4650	44.0	43.5	43.5	45.5	45.5	42.0
30.	220	14.4	4650	48.0	47.0	47.0	49.0	49.0	45.0
40.	220	14.4	4650	50.0	49.0	48.0	50.0	50.0	47.0
60.	220	14.6	4650	52.5	50.5	50.0	51.5	52.0	49.0
100.	220	14.6	4650	54.0	52.0	51.0	53.0	53.0	50.0
130.	220	14.6	4659	54.0	52.0	51.0	53.0	53.0	50.0

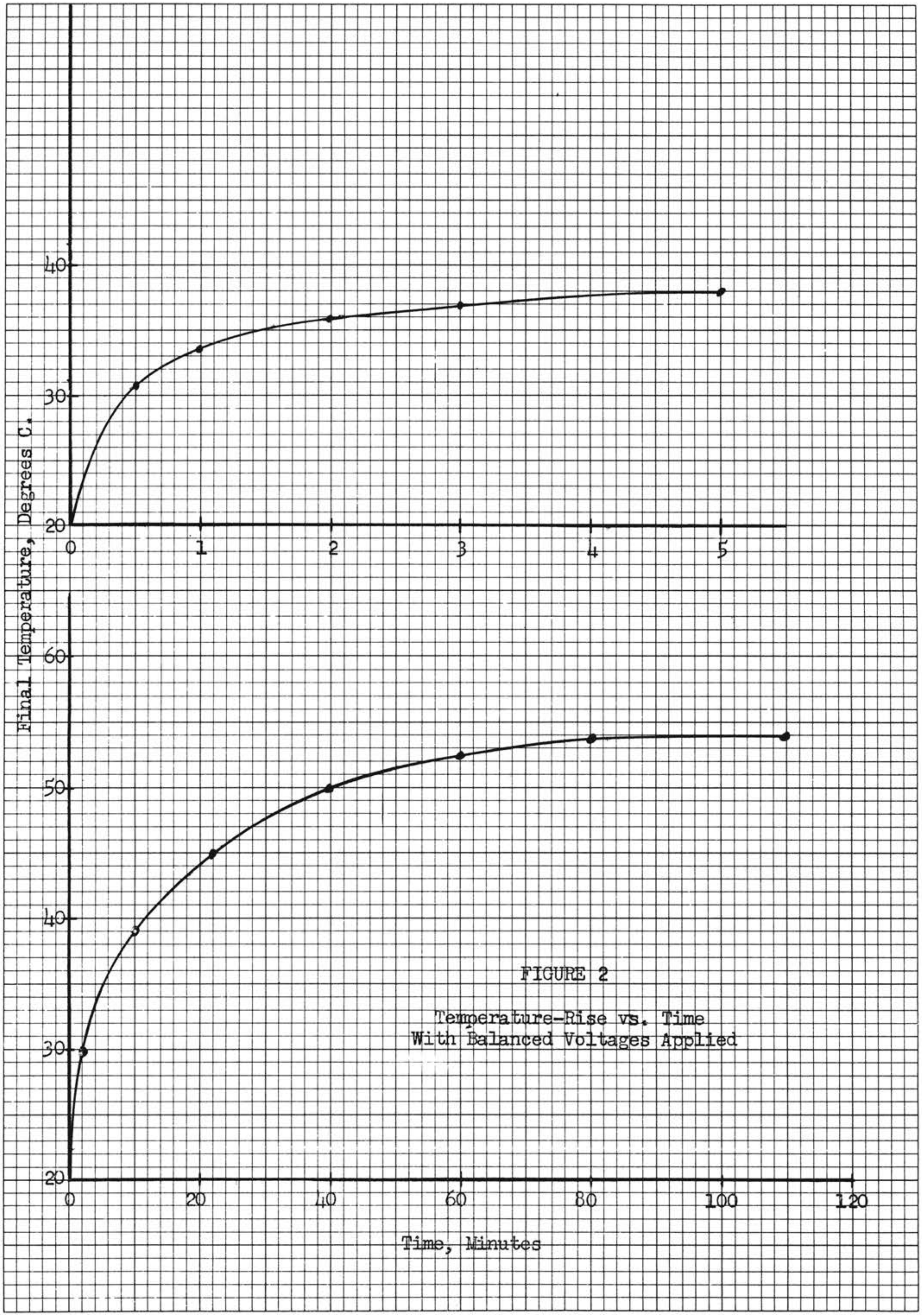


FIGURE 2

Temperature-Rise vs. Time
With Balanced Voltages Applied

The temperature rise T, may be calculated as the difference between the initial and final maximum temperature.

$$T = 54^{o} - 20^{o}$$
$$= 34^{o}C$$

and from equation (5a)

$$C_1 = \frac{931}{34}$$
$$= 27.4 \text{ watts/degree C.}$$

The initial slope of the temperature-rise curve may be determined graphically from Figure 2, and C_2 calculated by use of equation (3b). The initial slope is

$$\frac{25-20}{.30} = 16.65 \text{ degrees/min.}$$

and from equation (3b)

$$16.65 = \frac{dT}{dt} = \frac{P_L}{C_2}$$
$$C_2 = \frac{931}{16.65}$$
$$C_2 = 56 \text{ watt-min/degree C}$$

Now that the values of C_1 and C_2 are known they may be used in equation (3) to synthesize other temperature-rise vs. time curves for any value of P_L.

Ordinarily, in a practical case the only temperature of interest is the final maximum temperature. Equation (5) show that the final temperature rise depends only upon P_L and C_1, thus there is no need to calculate the values of temperature rise for small values of time. For motors on intermittent duty such as refrigerators and air conditioners the temperature-rise for short periods of operation will be of interest.

TYPICAL EXAMPLE

To illustrate the foregoing method a typical example illustrating its use is given, and a comparison made between calculated and experimental results. For the example and comparison the following unbalanced voltages will be used.

$$E_{ab} = 200 \text{ volts}$$

$$E_{bc} = 210 \text{ volts}$$

$$E_{ca} = 240 \text{ volts}$$

The experimental test data obtained using these unbalanced applied voltages are given in Table 4-3. The symmetrical components equivalent circuits were solved in Chapter III, using these unbalanced voltages and the results of these solutions will be used in this example. From these solutions it was determined that the motor efficiency, when operating near full load, was 71 per cent. When operating at this efficiency with a 5 H. P. load the motor input, in watts, will be

$$\frac{(5)(746)}{.71} = 5250 \text{ watts}$$

that the power loss P_L will be

$$P_L = (1-.71)\ 5250$$

$$= 1520 \text{ watts.}$$

By use of equation (3) with this value of P_L and the previously determined values of C_1 and C_2, the data for a temperature-rise vs. time curve may be calculated. These calculations are most easily made in columnar form.

$P_L = 1520$, $C_1 = 27.4$ watts/degree C,

$C_2 = 56$ watt-min./degree C

(3) $$T = \frac{P_L}{C_1}\left(1-e^{-tC_1/C_2}\right)$$

t	$\frac{tC_1}{C_2}$	e^{-tC_1/C_2}	$\frac{P_L}{C_1}$	$\frac{P_L}{C_1}e^{-tC_1/C_2}$	T
1	.489	.614	55.5	34	21.5°
2	.978	.376	55.5	20.8	34.7°
3	1.47	.230	55.5	12.75	42.7°
4	1.95	.143	55.5	7.94	47.6°
5	2.45	.0861	55.5	4.79	50.7°
10	4.89	.00756	55.5	4.20	51.3°
30	14.7	—	55.5	—	55.3°
60	29.4	—	55.5	—	55.3°

This calculated temperature rise curve is shown in comparison with the experimentally determined curve. The calculated curve has the same initial and final values of temperature rise as the experimentally determined curve. This indicates the proposed method for calculating motor temperature rise is satisfactory insofar as determining the final temperature but may have considerable error when it is used to calculate the temperature rise for short intervals of operation.

It should be noted, however, that the calculated curve yields pessimestic values, i.e., it indicates a higher temperature than actually exists. Therefore any decisions based on calculated temperature rise will probably be on the "safe side".

TABLE 4-3

HEAT TEST WITH UNBALANCED VOLTAGES

Time min.	Volts AB	BC	CA	Amperes A	B	C	Input Watts	T_1	T_2	T_3	T_4	T_5	T_6
0-	200	210	235					24.0	24.0	24.0			
0+	200	210	235										
.50	200	209	235				4450	43.0	44.0	42.0			
1.0	200	209	235				4440	44.0	47.0	43.0			
2.0	200	209	235				4430	49.0	49.0	41.0	54.0	53.0	43.0
3.0	200	209	235	21.0	7.7	18.0	4470	50.0	51.0	41.0	56.0	53.0	44.0
5.0	200	209	236	21.0	7.7	18.0	4440	53.0	51.0	43.0	57.0	53.0	45.0
10.	200	209	236	21.0	7.7	18.0	4460	57.0	57.0	46.0	61.0	58.0	50.0
15.	202	210	237	21.0	7.8	18.2	4500	61.0	62.0	50.0	66.0	64.0	55.0
20.	202	210	237	20.6	7.8	17.6	4480	63.0	65.0	53.0	68.0	67.0	57.0
30.	202	210	237	21.0	8.2	18.2	4950	68.0	70.0	55.5	73.0	71.0	61.5
40.	202	210	238	20.5	8.0	18.0	4900	72.0	73.0	57.0	75.0	73.0	64.0
65.	202	211	239	21.0	8.1	18.1	4950	76.0	77.0	60.0	78.0	78.0	68.0
100	201	210	237	20.6	8.1	18.0	4950	77.0	78.0	61.0	78.5	78.5	68.5
170	203	211	239	20.6	7.9	18.0	4950	77.0	78.0	61.0	78.5	78.5	69.0

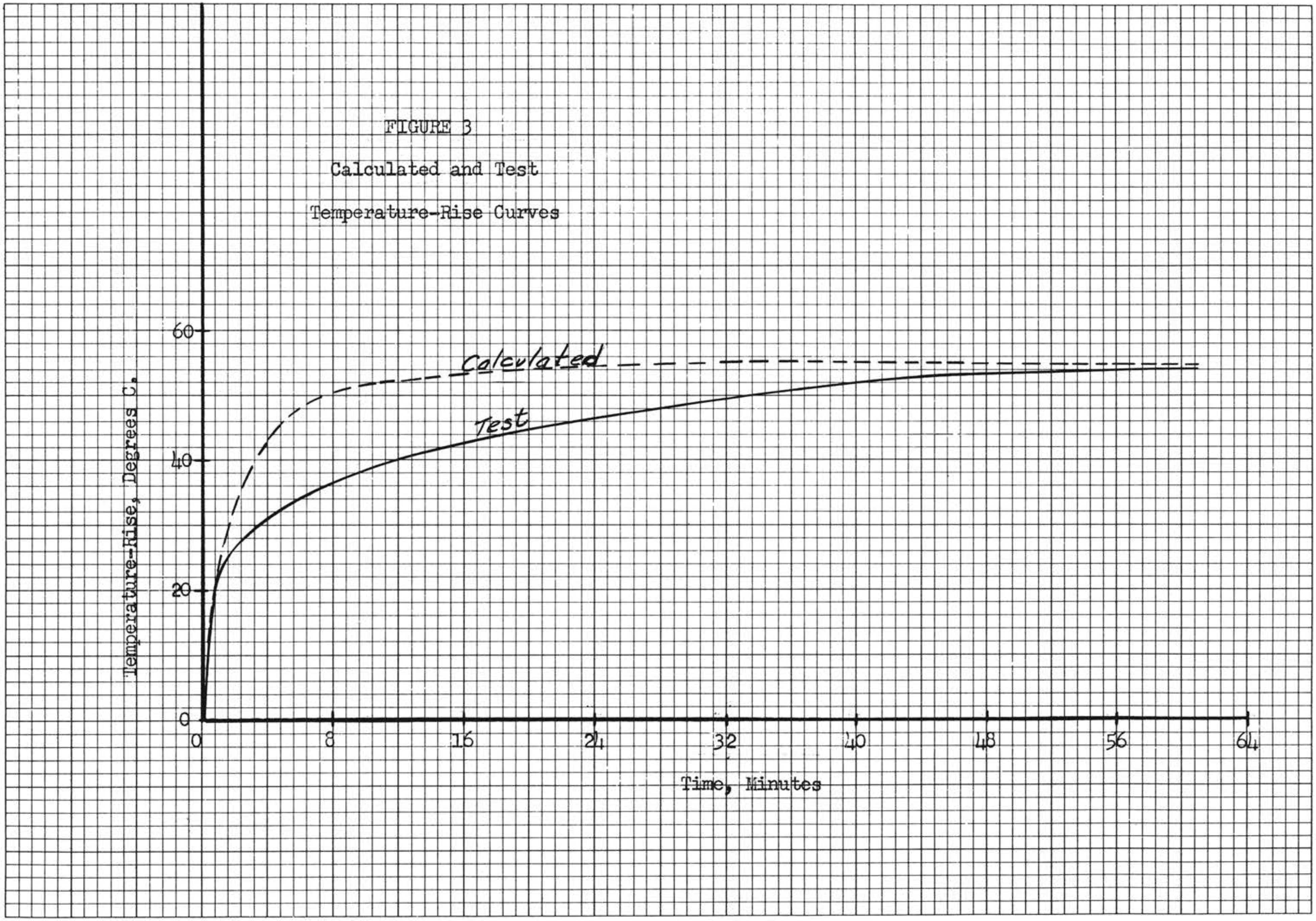


FIGURE 3

Calculated and Test

Temperature-Rise Curves

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The process for calculating motor temperature rise which has been developed may be summarized in the following steps:

1. Establish the symmetrical components equivalent circuits for the motor.
2. Solve these circuits to determine the motor efficiency for the particular unbalanced voltage condition being investigated.
3. Determine the motor thermal constants.
4. Use these constants to calculate the motor temperature rise for the efficiency calculated in step 2.

Practical limitations of using this method are immediately recognized. The equivalent circuit for a motor is not usually available nor will it be possible in many instances to determine the motor thermal constants. However, these limitations are not so serious as they may seem to be on first thought. Rarely is it necessary to know exactly what temperature a motor will attain; the primary consideration is whether the insulation will withstand the operating temperature or not. The permissable temperature ranges for various insulations is fairly broad, thus, in a practical case all that is required is to calculate whether the motor temperature will fall safely within the temperature limits of its insulation.

Motor manufacturers are forced to determine, by test, their motor performance characteristics in order to establish compliance with the standards of their industry. Therefore, all the data required to determine the motor thermal constants are available from the manufacturer. It will be remembered, the only motor thermal constant which must be known to calculate the final temperature rise is the constant C_1. If the full-load temperature-rise and efficiency are known this constant can be calculated.

This method of calculating motor temperature rise will have its usefulness in practical motor applications. The exact solution using the equivalent circuit and symmeterical components will have its usefulness from a design standpoint.

For a particular motor it will be possible to relate efficiency to temperature-rise and relate efficiency to the absolute value of the ratio of positive- to negative sequence voltages. Table 5-1 lists, for different values of unbalanced voltages, the positive- and negative-sequence voltages along with the efficiency and temperature-rise for near full-load conditions. These experimentally determined data are plotted in Figure 1.

TABLE 5-1

TABULATED DATA OF EFFICIENCY AND TEMPERATURE-RISE

FOR UNBALANCED VOLTAGES

VOLTS AB	BC	CA	E_1	E_2	EFFICI-ENCY,%	TEMP
210	220	230	210	9.73	82	63
230	220	210	230	15.6	78.8	69.5
200	210	240	206	24.3	72	78.5

The relations shown graphically in Figure 1 hold true only for the test motor, however, it illustrates what might be done to effect a solution of the problem of determining motor heating due to unbalanced volt-

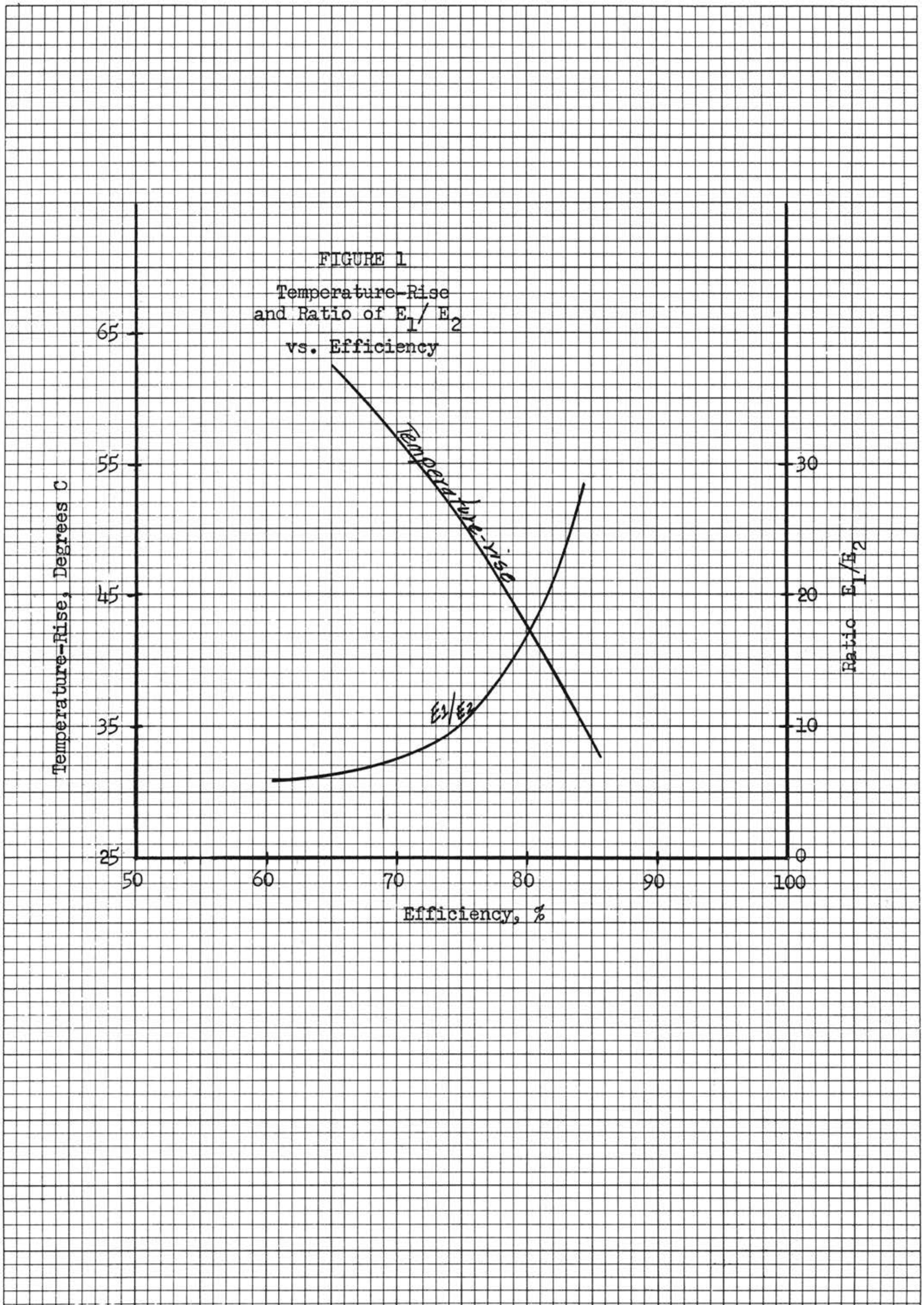


FIGURE 1
Temperature-Rise
and Ratio of E_1/E_2
vs. Efficiency
Temperature-rise
E_1/E_2
Temperature-Rise, Degrees C
Ratio E_1/E_2
Efficiency, %
25
35
45
55
65
0
10
20
30
50
60
70
80
90
100

ages.

The solution of the heating problem could well be solved by extending motor performance tests to include performance data for operation on unbalanced voltages. Information of this nature could be made available for motors and should become a point of primary consideration for application engineers. A study should be made of the voltage unbalance before a selection of equipment is made. In this connection it would be well to call attention to the following point.

During the experimental work on this thesis it became necessary to devise some method of calculating what combination of the possible voltage magnitudes would cause the greatest value of negative-sequence voltage to exist. A graphical method for solving this problem was devised and is given in Appendix A.

This graphical solution has its practical applications. Suppose investigation revealed the three line-to-line voltages for a motor supply to be of values falling in the following ranges:

line 1-2	210-220 volts
line 1-3	200-230 volts
line 2-3	210-230 volts

The problem then arises as to what combination of these voltages will cause the greatest negative-sequence voltages, and allowances made for the expected increased heating of this voltage.

RECOMMENDATIONS

With the advent of greatly increased number of three-phase motors being placed in use it seems logical the problem of motor heating due to unbalanced voltages must become a recognized problem in motor design. It further seems illogical to expect or demand an absolute balanced three-phase supply to be available for all three-phase motor applications. The solution of the problem seems to be in compromise between these two items, i.e., motor manufacturers should expect and design their motors to withstand the increased heating effect of unbalanced voltages and power companies should make every effort to provide a reasonably balanced supply.

It seems desirable to include in motor standards a minimum requirement which three-phase motors should meet in regards to operation on unbalanced voltages. This would certify to users of these motors the degree of unbalance which could be tolerated in the voltage supply. It would then be a problem of those involved in a particular application to determine the significance of any voltage unbalanced which may exist in light of the motor capabilities.

Further, application engineers should be provided with performance charts such as that given in Figure 1, which would enable them to intelligently evaluate the significance of the voltage unbalance which may exist in a specific situation. These data are of as much inportance in the field of application engineering as motor performance characteristic curves.

BIBLIOGRAPHY

Alger, Phillip L. The Nature of Polyphase Induction Machines. New York: John Wiley and Sons, Inc., 1951.

Bailey, Benjamin F. and James S. Gault. Alternating Current Machinery. New York: McGraw-Hill Book Company, Inc., 1951.

Clarke, Edith. Circuit Analysis of A-C Power Systems. Volume II. New York: John Wiley and Sons, Inc., 1950.

Cotton, H. Design of Electrical Machinery. London: Oxford University Press, 1934.

Karapetoff, V. and Boyd C. Dennison. Experimental Electrical Engineering. New York: John Wiley and Sons, Inc., 1941.

Kuhlman, John H. Design of Electrical Apparatus. New York: John Wiley and Sons, Inc., 1940.

Lunn, Edward O. "Induction Motors Under Unbalanced Conditions." Electrical Engineering, 55 (April 1936), 391.

Moore, A. D. Fundamentals of Electrical Design. New York: McGraw-Hill Book Company, Inc., 1927.

Puchstein, A. F. and T. C. Lloyd. Alternating Current Machines. Second Edition. New York: John Wiley and Sons, Inc., 1942.

Reed, H. R. and R. J. W. Koopman. "Induction Motors on Unbalanced Voltages." Electrical Engineering, 55 (November 1936), 1206.

Ricker, C. W. and Carlton E. Tucker. Electrical Engineering Laboratory Experiments. New York: John Wiley and Sons, Inc., 1940.

Slichter, W. I. Principles Underlying the Design of Electrical Machinery. New York: John Wiley and Sons, Inc., 1926.

Wagoner, C. F. and R. D. Evans. Symmetrical Components. New York: McGraw-Hill Book Company, Inc., 1933.

APPENDIX A

When considering the heating effect coused by an unbalanced voltage supply to a three-phase induction motor it is necessary to determine the values of the positive- and negative-sequence voltages. When only the mangitudes of the line-to-line voltages are known it will be necessary to determine what phase rotation and vector relationship between these voltages will cause the greatest negative-sequence voltage to exist, and make allowances for the heating caused by this maximum value. A method for solving this problem was developed and is presented below.

From the equation for the negative-sequence voltage

$$E_{ab2} = \frac{1}{\sqrt{3}} \left(E_{ab} + E_{bc} \angle{-60^\circ} \right) \angle{-30^\circ} \qquad (1)$$

it may be seen that E_{ab2} is maximum when the vector sum of $E_{ab} + E_{bc} \angle{-60^\circ}$ is maximum. This sum would have its maximum value when E_{ab} and $E_{bc} \angle{-60^\circ}$ were co-linear vectors, however it is known that with ordinary conditions of voltage unbalance on a three-phase distribution system the angular displacement between E_{ab} and E_{bc} must be approximately 120°. Equation (1) was derived for a vector system in which E_{bc} lagged E_{ab} by 120°; if it is desired to use the opposite phase rotation this equation cannot be used and another equation must be derived for this condition. However the value of E_{ab2} for both phase rotations will be equal.

Figure 1, shows an unbalanced set of three-phase vectors and the determination of E_{ab2} for the set made by graphical use of equation (1).

Suppose measurements showed the line-to-line voltages may vary between the limits of 200 and 240 volts. What particular combination of these magnitudes would give the greatest negative-sequence voltage?

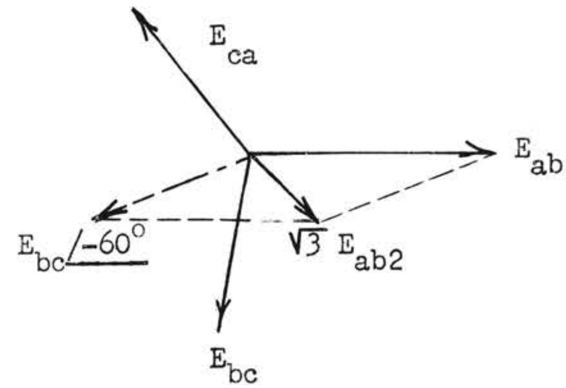


FIGURE 1
Graphical Determination of E_{ab2} for Unbalance Set of Three-phase Vectors

Co-inspection of Figure 1 and equation (1) shows the indicated vector addition turns out to be an arithmetic subtraction of the real part of E_{bc} from E_{ab}. Further, if E_{ab} has a large magnitude E_{ab2} will be large. The problem is to determine what magnitudes of E_{bc} and E_{ca} will cause the proper vector relationship to exist to provide the greatest negative-sequence. A graphical means for solving this problem is shown in Figure 2.

It is known that

$$200 \text{ volts} \leqslant |E_{ab}| \leqslant 240 \text{ volts}$$

$$200 \text{ volts} \leqslant |E_{bc}| \leqslant 240 \text{ volts}$$

$$200 \text{ volts} \leqslant |E_{ca}| \leqslant 240 \text{ volts}$$

From this information lay off, to scale, a vector $E_{ab} = 240\underline{/0^{\circ}}$. With the origin as center strike two arcs, one with a scale radius of 200 volts and the other with 240 volts. It is known that $-E_{ca}$ must terminate on or between these two arcs. Since the vector sum of the voltages must be zero the vector E_{bc} must extend from $-E_{ca}$ to the head of E_{ab}. The magnitude of E_{bc} may also vary between 200 and 240 volts so two arcs equal to the preceeding two are struck about the head of vector

E_{ab}. $-E_{ca}$ and E_{bc} must terminate on the boundry or within the area contained by the boundry 1-2-3-4.

Let

$$E_{bc} \underline{/-60^{\circ}} = E'_{bc}$$

$$E'_{bc} = -M \mp jn$$

Then E_{ab2} is maximum when $-M$ is a minimum and jn is a maximum. Two possibilities exist, one would be if the angle of E'_{bc} were -90° and the other if the angle were $+90^{\circ}$. This requires the angle of E_{bc} to be either -30° or $+ 150^{\circ}$. For ordinary conditions of unbalance these vector relations would never occur. The condition which comes closest to fulfilling these requirement will give the maximum negative-sequence voltage possible.

Obviously, from Figure 2, the conditions that must be investigated are where $-E_{ca}$ and E_{bc} terminate at points 1 and 3. These vectors are shown in Figure 2 with the added subscript 1 or 3 to denote the termination point. E_{bc1} and E_{bc2} are rotated through -60° and are shown as E'_{bc1} and E'_{bc2}. A vector from the origin to E'_{bc1} or E'_{bc2} is equal to $\sqrt{3}E_{ab2}$, from which the maximum value of E_{ab2} can be determined.

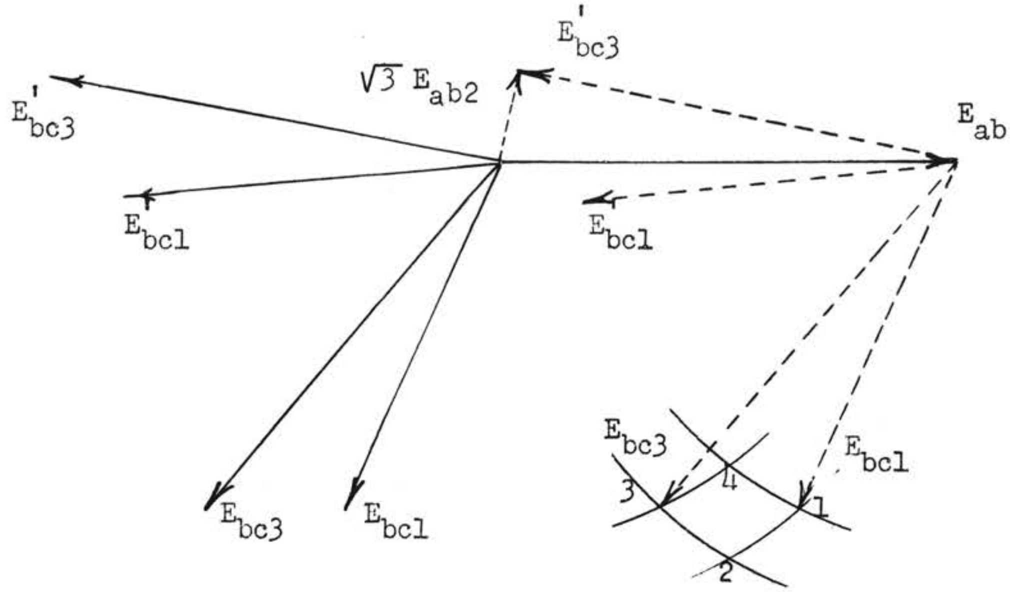


FIGURE 2
Graphical Determination of Maximum Negative-Sequence Voltage

VITA

Jack Edward Tompkins
candidate for the degree of
Master of Science

Thesis: AN INVESTIGATION OF THE HEATING OF A THREE-PHASE INDUCTION MOTOR OPERATING ON UNBALANCED VOLTAGES

Major: Electrical Engineering

Biographical and Other Items:

Born: July 19, 1924 at El Reno, Oklahoma

Undergraduate Study: Oklahoma A & M College, 1946-1949

Graduate Study: Oklahoma A & M College, 1951-1953

Experiences: U.S. Navy 1941-1945; Halliburton Oil Well Cementing Co. 1949-1950; Southwestern Power Administration 1950-1951; Staff Member School of Electrical Engineering, Oklahoma A & M College, 1951-1953.

Member of: Phi Delta Theta, Sigma Tau, Eta Kappa Nu, Associate member of American Institute of Electrical Engineers.

Date of Final Examination: May 1953

THESIS TITLE: AN INVESTIGATION OF THE HEATING OF A THREE-PHASE INDUCTION MOTOR OPERATING ON UNBALANCED VOLTAGES

AUTHOR: Jack Edward Tompkins

THESIS ADVISER: Prof. Charles F. Cameron

The content and form have been checked and approved by the author and thesis adviser. The Graduate School Office assumes no repsonsibility for errors either in form or content. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

TYPIST: Claribel L. Wadsworth