ANALYSIS OF AMPLIFIER RESPONSE THROUGH THE USE OF ASYMPTOTIC PLOTS

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PREFACE

There are many good treatise on the various aspects of design and analysis of audio amplifiers. Some of these deal strictly with the problem of frequency response and others go into the theory of feedback and other modifications. These articles deal primarily with the mathematical development of the alternating current circuit theory involved, and in order to apply them to an actual analysis over a wide band of frequencies, one must go through the laborious process of computing the output at a number of frequencies. Furthermore, they neglect to outline a complete usable system of analysis that can be applied over a wide band of frequencies. Finally, in order to gain the necessary knowledge to design amplifiers for practical use, one must read far and wide to fetter the information needed.

It is the object of this thesis to gather together as much of the available information as is necessary and outline a procedure for the analysis of audio amplifiers which will yield an acceptable response with simple calculations at only a few strategic frequencies throughout an entire wide band. At all times, the phase shift of the circuits, an often ignored factor, will be considered along with the amplitude response. This is considered important since the action of feedback applied to amplifiers is affected equally as much by phase shift as by amplitude response.

The material in this thesis is not intended for the beginner who is searching for the basic fundamentals of amplifier circuit action, but rather it is intended for the practicing engineer who is already familiar with amplifier and alternating current circuit theory and is searching for a systematic approach to the problem of design and analysis. The engineer in the field has a need for an approach to the problem that is not too involved with mathematics and which would allow a simple graphical interpretation to be applied to either new or existing equipment for the purpose of readily determining corrections to a design in order to meet the specific requirements at hand.

The procedure outlined is an amalgamation of various developments of circuit theory. Each bit within itself is not original with the writer, but taken together it gives a unique and satisfactory method for analyzing the performance of amplifiers, especially when feedback is involved.

ACKNOWLEDGMENT

The author wishes to express his most sincere appreciation to Dr. Harold T. Fristoe for his guidance and personal interest in the preparation of this thesis. He also wishes to extend thanks to Dr. Attie L. Betts for many valuable suggestions and criticism.

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CHAPTER I

INTRODUCTION

The problem of analyzing the performance of an electronic amplifier resolves itself to just one question; what happens to the amplitude and phase of the output signal voltage when the frequency of the input signal voltage is varied? This appears to be a quite simple question. However, when applied to the modern multistage amplifier it becomes a very tedious problem if attacked by conventional alternating current circuit methods. Therefore, any less laborious method, that can be satisfactorily applied, would be most welcome.

All electronic amplifier circuits contain a combination of resistance and reactance. Because of this complex impedance, the output voltage and its phase with respect to the input voltage will not be constant over a wide range of frequencies. In order to relieve the amount of computation required to determine the output of a given circuit over a wide band of frequencies, a complete network can be analyzed as a specific combination of several elementary networks, each containing only one resistance and reactance, respectively. Since the response of an elementary resistance - reactance combination always behaves in exactly the same manner, the overall response of the complete network can be determined by graphically adding together the logarithmic frequency response of the several elementary circuits involved in the complete network. The main problem, preliminary to the analysis of a network by this method, involves the arrangement of the equation for output voltage as a function of frequency in the desired form.

The results of laboratory tests on an amplifier do not always coincide too closely with the calculated performance due to unknown variations in circuit values. Resistors and condensers used in this type of work have a usual tolerance of ten to twenty per cent. Also, tube constants such as plate resistance, transconductance, and interelectrode capacities, will differ from their typical published values due to manufacturing differences and the choice of operating potentials. Due to these unpredictable variations in circuit and vacuum tube parameters, the response performance of an amplifier must be carefully checked experimently to see if it meets the requirements of the original design. Also, quite often the engineer wishes to modify an existing amplifier for a specific purpose or for an improvement in tonal and fidelity performance which also requires actual laboratory tests. The procedure for measuring the phase response of an amplifier is even more of a problem to the average engineer. There are several excellent direct reading phase meters on the market, but they are quite expensive. Furthermore, they do not respond to the wide band of frequencies required in high fidelity audio work. Included in this thesis is the description of a relatively inexpensive direct reading phase meter which will fulfill all but the more exacting requirements on audio amplifier phase measurements.

CHAPTER II

BACKGROUND THEORY

Voltage Gain and Decibel Gain

A complete amplifier usually consists of several stages in cascade, and if these individual stages are operated class A_1, then each stage may be considered independent of the following stage except for the effects of the input impedance of the tube. Therefore, the overall voltage gain of the complete amplifier is determined by the product of the gains of the individual stages.[1]

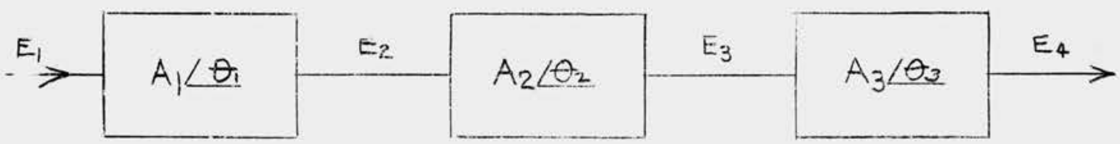

Figure 1. Block Diagram of a Three-Stage Amplifier

If the gain of each stage in Figure 1 is $A_1\underline{/\theta_1}$, $A_2\underline{/\theta_2}$, and $A_3\underline{/\theta_3}$, respectively, then the overall voltage gain would be

$$\frac{E_4}{E_1} = A_1\underline{/\theta_1} \cdot A_2\underline{/\theta_2} \cdot A_3\underline{/\theta_3} \qquad (1)$$

$$= A_1 \cdot A_2 \cdot A_3 \underline{/\theta_1 + \theta_2 + \theta_3} \qquad (2)$$

It will prove convenient to express the gain in decibels for use in the graphical system to follow in a later chapter.

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[1]Samuel Seely, Electron-Tube Circuits, p. 70.

By definition,[2] the decibel gain of an amplifier stage is proportional to the logarithm of the voltage gain. In Figure 1, the decibel gain of the first stage would be[3]

$$db_1 = 20\text{Log}\frac{E_2}{E_1} \quad (3)$$

The overall decibel gain of the amplifier shown in Figure 1 would be[4]

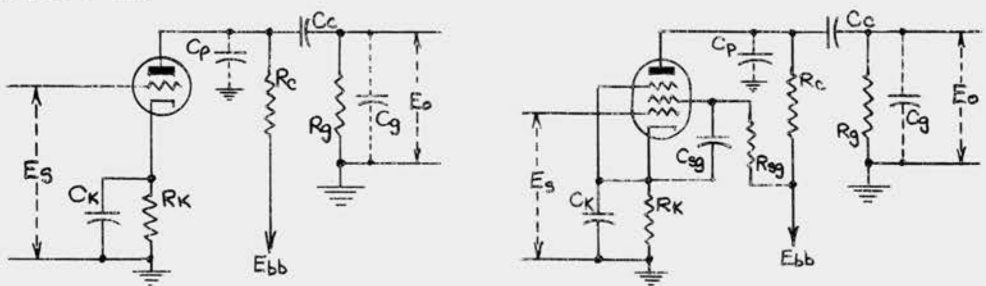
$$\text{Total db gain} = db_1 + db_2 + db^3 \quad (4)$$

$$= 20\text{Log}A_1 + 20\text{Log}A_2 + 20\text{Log}A_3 \quad (5)$$

The total phase shift of the complete amplifier would be determined in the same manner as in equation (2).

The Resistance Coupled Amplifier

The most common type of amplifier stage used in modern audio equipment is the resistance coupled amplifier. Typical circuits[5] of a triode and pentode amplifier are shown in Figure 2.



(a) Triode Amplifier (b) Pentode Amplifier

Figure 2

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[2] *Ibid.*, pp. 55-56.

[3] The use of log implies $\log_{10}$.

[4] Seely, *op. cit.*, p. 70.

[5] Fredrick E. Terman, *Radio Engineering*, 3rd Edition, p. 230.

In order to analyze the gain and frequency response of the resistance coupled amplifier, it is convenient to use the constant-current[6] equivalent circuit as shown in Figure 3. This circuit assumes that the cathode and screen condensers are fully effective in their by-passing action.

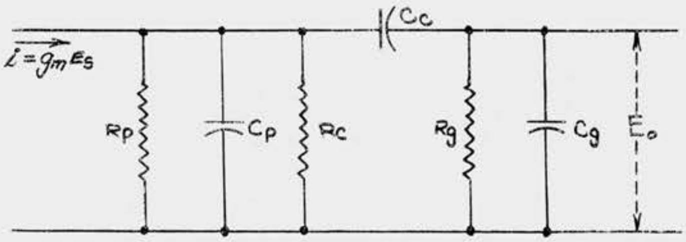

Figure 3. Equivalent Circuit of a Resistance Coupled Amplifier.

It can be shown[7] that throughout some middle range of frequencies, the coupling condenser C_c will have such a low reactance that it can be considered a short circuit, whereas the reactance of the shunting capacities will be so high as to be the practical equivalent of an open circuit. Applying these conditions, the equivalent circuit would take the form shown in Figure 4, and the voltage gain would be[8]

$$A_M = \text{Voltage gain in the middle range of frequencies} = \frac{E_o}{E_s} = g_m \cdot R_{eq} \qquad (6)$$

where R_{eq} is the parallel combination of R_p, R_c, and R_g.

[6] Loc. cit.

[7] Ibid., p. 232.

[8] Loc. cit.

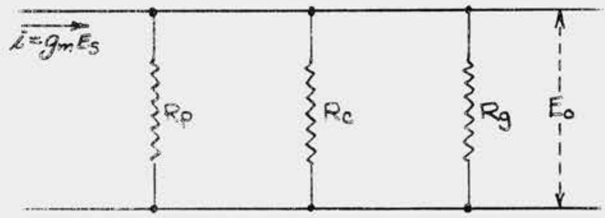

Figure 4. Practical Equivalent Circuit of a Resistance-coupled Amplifier for the Mid-range of Frequencies

At high frequencies, the reactance of the combined shunting capacities is no longer infinitely large in comparison with other shunt impedances in the circuit, and therefore must not be neglected. The practical equivalent circuit[9] effective at high frequencies is shown in Figure 5.

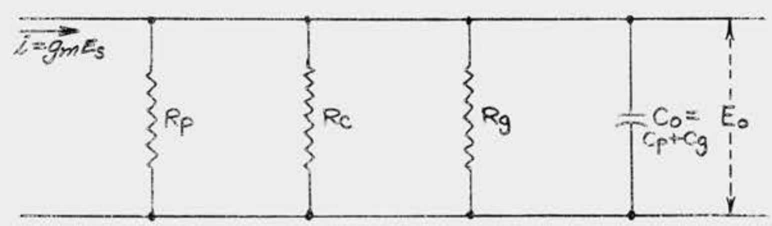

Figure 5. Practical Equivalent Circuit of a Resistance-coupled Amplifier for the High Frequencies

An analysis of this circuit shows that

$$\text{Voltage gain at high frequencies} = A_H = \frac{1}{\sqrt{1 + \left(\frac{R_{eq}}{X_o}\right)^2}} \cdot g_m R_{eq} \qquad (7)$$

where

$X_o = \frac{1}{2\pi f\, C_o}$ = reactance of total shunting capacity C_o.

R_{eq} = equivalent resistance of R_p, R_c, and R_g in parallel.

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[9]Terman, op. cit., p. 232.

It is significant to note that when the ratio R_{eq}/X_o becomes unity, the voltage gain will have decreased to 0.707 of the mid-range gain.[10] It is also important to realize that this drop in voltage amplification represents a loss in gain of 3 db, and the frequency at which X_o becomes equal to R_{eq} would be

$$f_2 = \frac{1}{2\pi f\ R_{eq}\ C_o} \qquad (8)$$

Figure 6 shows a more complete picture of how the gain and phase varies in the high-frequency region.[4]

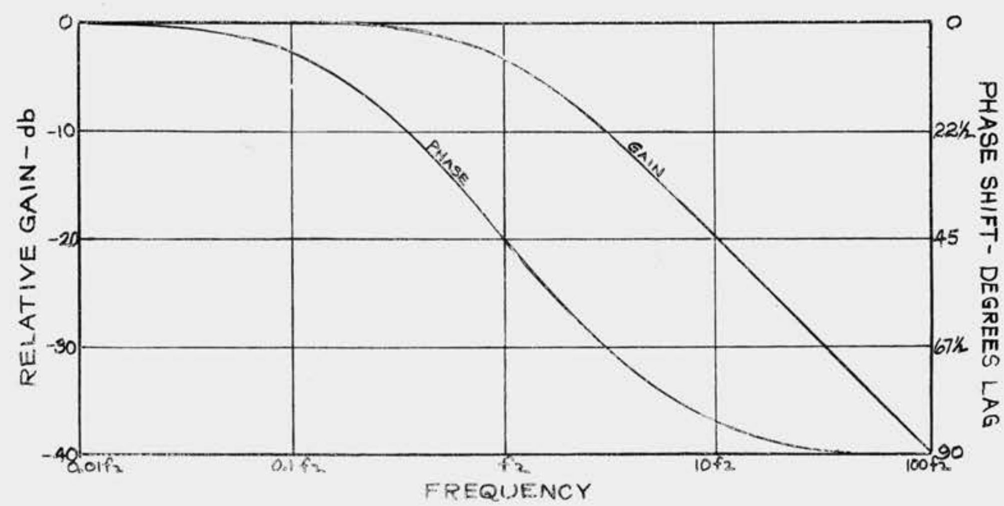

Figure 6. Relative Gain and Phase Shift of a Resistance-coupled Amplifier for High Frequencies

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[10] If $X_o/R_{eq} = 1$, then $A_H = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$ and $20\text{Log}(0.707) = -3$ db. Furthermore, when $X_o = R_{eq} = \frac{1}{2\pi f C_o}$, then solving for f yields $f = \frac{1}{2\pi R_{eq} C_o}$.

[11] Terman, op. cit., p. 235.

At frequencies below the mid-range, the shunting capacitance C_o has such a high reactance that it may be considered an open circuit without introducing appreciable error in the circuit analysis. The reactance of the coupling capacitance C_c also increases and becomes sufficient to reduce the output voltage developed across the grid resistor R_g. For this condition, the equivalent circuit[12] shown in Figure 7 (a) is used.

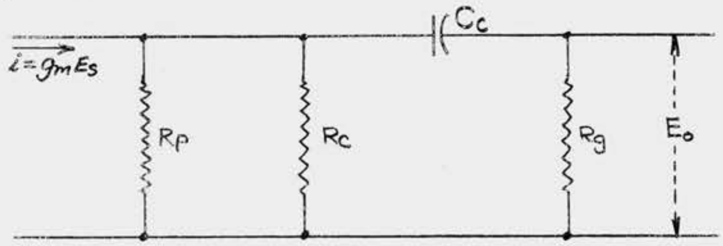

Figure 7 (a). Practical Equivalent Circuit of a Resistance-coupled Amplifier for Low Frequencies.

The gain of the circuit is now

$$\text{Voltage gain at low frequencies} = A_L = \frac{1}{\sqrt{1 + \left(\frac{X_c}{R}\right)^2}} \cdot g_m R_{eq} \qquad (9)$$

where

$$X_c = \frac{1}{2\pi f C_c}$$

$$R = R_g + \frac{R_c R_p}{R_c + R_p}$$

It will be noted that in equation (9) the voltage gain will again be reduced to 0.707 of the mid-range value when $X_c = R$, and the frequency at which this occurs would be

$$f_1 = \frac{1}{2\pi R C_c} \qquad (9A)$$

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[12]Terman, op. cit., p. 234.

Figure 7 (b) shows how the gain and phase vary in the low frequency region.

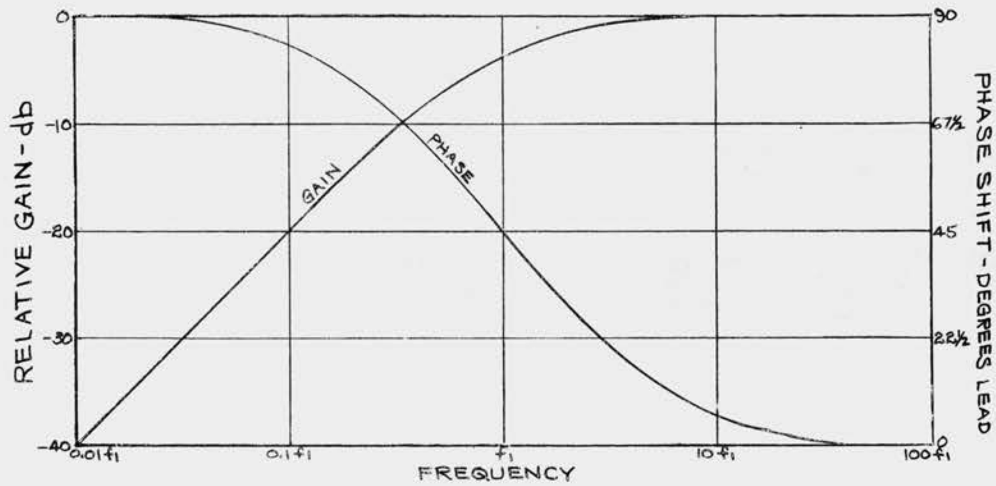

Figure 7 (b). Relative Gain and Phase Shift of a Resistance-coupled Amplifier for Low Frequencies

Other factors[13,14] which may affect the gain at low frequencies are the screen, cathode, and plate supply by-pass condensers. In order to provide nearly perfect by-passing action, the reactances of these condensers should be as low as possible, and under any circumstance, each of these reactances should be much smaller than any parallel impedance. In actual practice it is not too difficult to achieve and maintain these requirements in the screen and plate supply circuits. A screen by-pass condenser of one-tenth to one-half microfarad capacity will satisfy all but the most unusual case, while a twenty to forty microfarad electrolytic condenser will usually be adequate for the plate decoupling circuit. It might be necessary to shunt the

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[13]Terman, op. cit., p. 236.

[14]George E. Valley, Jr. and Henry Wallman, Vacuum Tube Amplifiers, pp. 87-88.

electrolytic with a small paper or mica type of condenser to insure good by-passing at high-frequencies,[15] since most of the electrolytic types show an increase in power factor and also displays an inductive characteristic at higher frequencies.

The by-passing of the cathode bias resistor presents a more involved problem. Since the cathode resistor is usually not more than a few thousand ohms, the by-pass condenser would have to be very large in order to provide a sufficiently low reactance. Large condensers of several hundred microfarads are easily obtainable, but they would be of the low voltage electrolytic types which tend to develop an increase in leakage and power factor with age. This increase in internal resistance of the condenser would also increase the cathode bias impedance and degeneration in the amplifier would result. By leaving out the cathode by-pass condenser entirely, the degeneration produced would be uniform at all frequencies, but in many instances the accompanying reduction in gain might not be tolerable. Therefore, it seems desirable to be able to analyze the effect of the cathode by-pass condenser in some detail. This action will be discussed more completely in Chapter IV.

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[15] Terman, op. cit., pp. 265-266.

Transformer-Coupled Output Amplifier

The amplifier stage providing power to a low resistance load, such as a loudspeaker, must have a low power loss coupling circuit between the tube and the load. In most all cases, a transformer with a step-down turns ratio is used for this purpose. The analysis and performance prediction of an amplifier with an output transformer is somewhat more indeterminate than the case of the resistance-coupled amplifier. This is due to several notable effects. First, the impedances of the transformer are not lumped and must be measured by more involved procedures. Second, some of the impedances of the transformer are non-linear to an extent which makes it impossible to assign a definite value for all operating conditions encountered. Third, if all of the impedances were to be included in the analysis, a very unwieldy problem would be encountered. However, by making certain simplifications in the equivalent circuit, an approximate analysis can be made which will be sufficiently accurate for practical work. The usual circuit[16] and its exact equivalent circuit is shown in Figure 8.

Since the impedances in the secondary circuit of the transformer appear to the tube to have been modified by the square of the turns ratio, the equivalent circuit should be redrawn as though the transformer was reduced to unity turns ratio. At the same time, several of the impedances can be neglected since they are either too small or too large to

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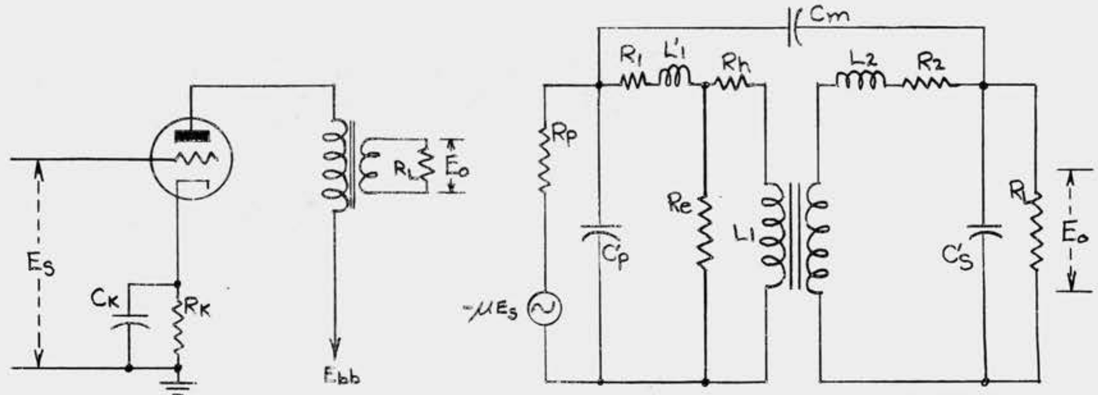
[16]Terman, op. cit., p. 291.

produce noticeable effects on the performance. After making these changes, the total equivalent circuit will be as shown in Figure 9.



(a) Actual Circuit of Transformer-coupled Output Amplifier

u - amplification factor

R_p - plate resistance

R_1 - d-c resistance of primary winding

L_1' - leakage inductance of primary

L_2 - leakage inductance of secondary

R_2 - d-c resistance of secondary winding

R_L - load resistance

(b) Exact Equivalent Circuit of Transformer-coupled Output Amplifier

C_p - shunt capacitance of primary

C_s' - shunt capacitance of secondary

R_h - hysteresis losses

R_e - eddy current losses

C_m - capacitance between primary and secondary

L_1 - primary inductance

Figure 8

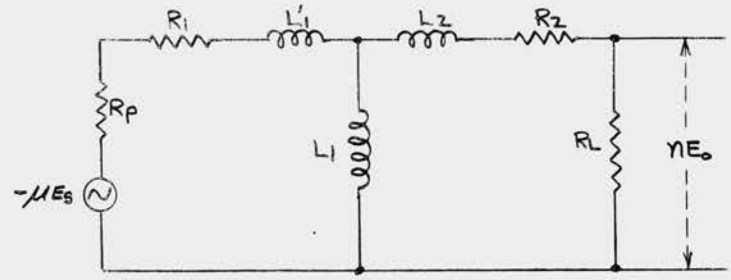


Figure 9. Practical Equivalent Circuit Reduced to Unity Turns Ratio

Throughout some middle range of frequencies,[17] the reactance of the primary inductance will be so large as to be the practical equivalent of an open circuit, while the reactance of the leakage inductance will be so small as to be considered a short-circuit. By applying these simplifications, the equivalent circuit can be drawn as shown in Figure 10.

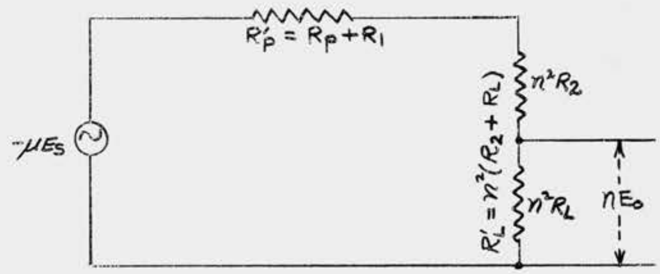


Figure 10. Equivalent Circuit of a Transformer-coupled Output Amplifier for the Mid-range of Frequencies

The voltage gain for this relatively simple circuit can be seen to be

$$\text{Voltage gain at the mid-range of frequencies} = A_M = \frac{unR_L}{R'_p + R'_L} \qquad (10)$$

where

$$R'_p = R_p + R_1$$

$$R'_L = n^2(R_2 + R_L)$$

[17]Terman, op. cit., pp. 291-292.

When the frequency of the applied signal is low, the reactance of the primary inductance will become small and must not be neglected. The circuit shown in Figure 11 (a) will be accurate in the low-frequency region.

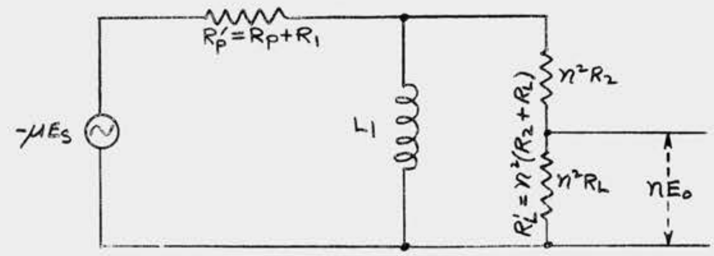


Figure 11 (a). Equivalent Circuit of a Transformer-coupled Output Amplifier for the Low-frequency Range.

An analysis of this circuit gives

$$\text{Voltage gain at low frequencies} = A_L = \left[\frac{\mu n R_L}{R_p' + R_L'}\right]\left[\frac{1}{\sqrt{1 + \left(\frac{R_1'}{X_1}\right)^2}}\right] \qquad (11)$$

where

$$R_1' = \frac{R_p' R_L'}{R_p' + R_L'}$$

$$X_1 = 2\pi f L_1$$

The frequency at which the low-frequency gain is down 3 db from its value at mid-range is

$$f_1 = \frac{R_1'}{2\pi L_1} \qquad (12)$$

In the high-frequency region, the reactance of the leakage inductance becomes appreciable and must be considered, while the primary reactance will be so large as to be an effective open circuit. These stipulations produce an equivalent circuit as shown in Figure 11 (b).

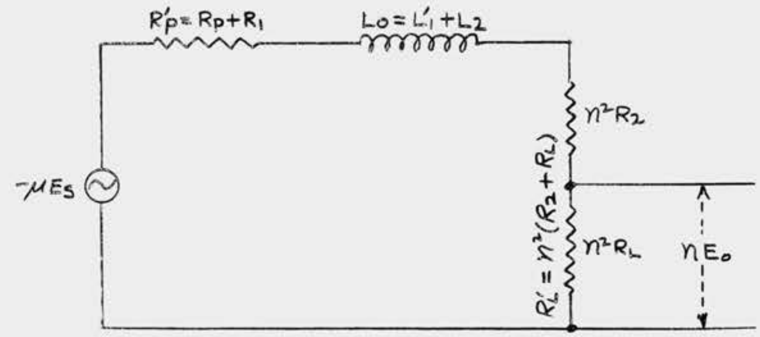


Figure 11 (b). Equivalent Circuit of a Transformer-coupled Output Amplifier for the High-frequency Range

The voltage gain for this circuit would be

$$\text{Voltage gain at high frequencies} = A_H = \left[\frac{\mu n R_L}{R'_p + R'_L}\right]\left[\frac{1}{\sqrt{1 + \left(\frac{X_o}{R'}\right)^2}}\right] \qquad (13)$$

where

$$X_o = 2\pi f L_o$$

$$R' = R'_p + R'_L$$

The frequency at which the high-frequency gain is down 3 db from its value at mid-range is

$$f_2 = \frac{R'}{2\pi X_o} \qquad (14)$$

It will be observed that in Figure 9 and Figure 11 (b) the primary shunt capacity C'_p has been disregarded. When these circuits are associated with a pentode or beam power tube, some error will be introduced when neglecting this capacity. However, for the sake of simplification at the moment, this concession will be endured.

Figure 12 shows the more complete typical gain and phase response of a transformer-coupled output amplifier stage.

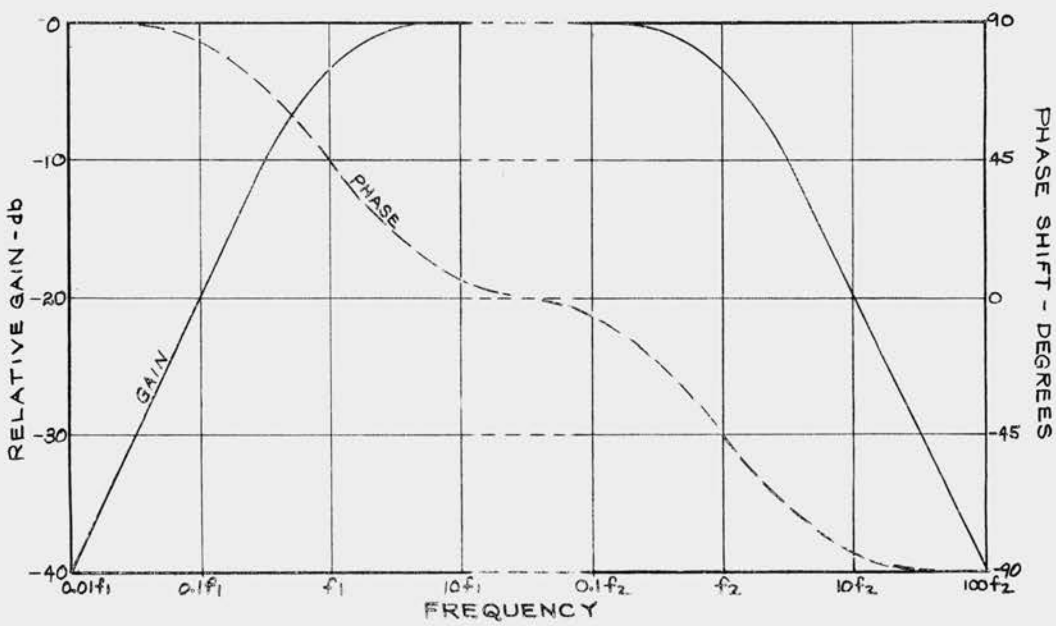


Figure 12. Typical Gain and Phase Response of a Transformer-coupled Output Amplifier Stage

CHAPTER III

THE TRANSFER FUNCTION

General Form and Operator Notation

The ratio of output voltage to input voltage of a network may properly be called a transfer function[1,2,3] and will be designated by the symbol A. The function for the gain of an amplifier stage is also defined as the ratio of output to input voltage and thus would be the transfer function of the amplifier.

When writing the transfer function of a network it is expedient to incorporate the notation used by Oliver Heavyside.[4] The Heavyside system makes use of an operator which designates the mathematical operation of differentiation as $\frac{d}{dt} = p$. Conversely, the integral operator $\int dt$ is denoted by $\frac{1}{p}$.

When analyzing a network in terms of its steady-state

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[1]Hendrick W. Bode, Network Analysis and Feedback Amplifier Design, p. 230.

[2]Harold Chestnut and Robert W. Mayer, Servomechanisms and Regulating System Design, Volume I, pp. 161-162.

[3]E. W. Tschudi, "Transfer Functions for R-C and R-L Equalizing Networks," Electronics, Volume XXII (May, 1949), pp. 116-120.

[4]Eugene Stephens, The Elementary Theory of Operational Mathematics, pp. 1 and 9-10.

response to a sine-wave signal, the operator takes on the meaning of[2] $p = +j\omega$, where j is the conventional operator so familiar in alternating current circuit theory, and ω is the angular velocity of the voltage or current which is numerically equal to $2\pi f$. The use of the operator p provides a short-hand method of writing the impedance equations of a network, and reduces the amount of work required to manipulate an algebraic expression through its various forms.

It might be of academic interest to note, that if it were desired to analyze a network for its transient response, the network equations written as a function of p would be in the proper form for the application of Laplace transformations to the solution of linear differential equations.[3]

The impedance equations and transfer functions shown in Figure 13 serve to illustrate the use of the operator p for several simple circuits.

In order to study the characteristics of a transfer function in more detail it is desirable to plot the amplitude and phase shift vs. frequency, and in order to do this, it greatly simplifies the problem to write the equation of the function in the form

$$A = K \cdot \frac{T_1P(1+T_3P)(1+T_5P)\cdot\cdot\cdot\cdot\cdot(1+T_nP)}{T_2P(1+T_4P)(1+T_6P)\cdot\cdot\cdot\cdot\cdot(1+T_{n-1}P)} \qquad (15)$$

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[2]Chestnut and Mayer, op. cit., pp. 100-102.

[3]Stanford Goldman, Transformation Calculus and Electrical Transients, pp. 41-43.

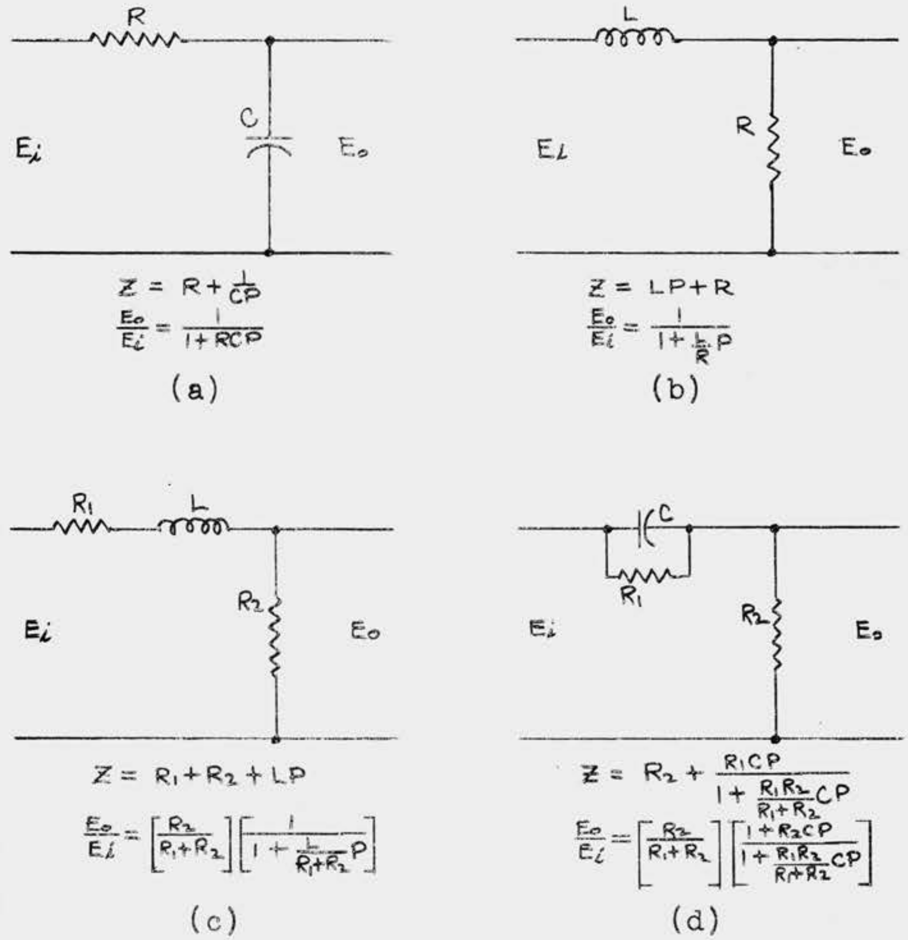


Figure 13. Common Electrical Circuits With Their Associated Impedance Equations and Transfer Functions

The transfer function for the circuit illustrated in Figure 13(d) was shown to be

$$\frac{E_o}{E_i} = \left[\frac{R_2}{R_1+R_2}\right]\left[\frac{1+R_1CP}{1+\left(\frac{R_1R_2}{R_1+R_2}\right)CP}\right] \qquad (16)$$

which is of the standard form

$$A = K \cdot \frac{1+T_1P}{1+T_2P} \qquad (17)$$

where

$$K = \frac{R_2}{R_1+R_2}$$

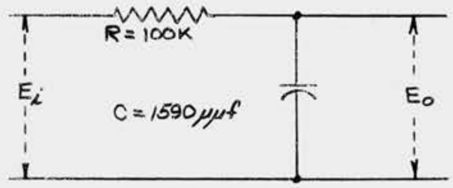
$$T_1 = R_1C$$

$$T_2 = \left(\frac{R_1R_2}{R_1+R_2}\right)CP$$

The transfer function of almost any four terminal linear network can be put in the form of equation (15) providing it is not resonant. T_n is defined as the time constant of the factor $(1 + T_np)$ in which the subscript n denotes the order of magnitude of the time constant appearing in the expression, but the exact order of the time constants depend upon each individual solution.

<u>Plotting the Amplitude Response of the Transfer Function</u>

An accepted procedure for graphical plotting of the transfer function would be to make the substitution of $j\omega$ for the operator p, and then solve for the db response at a sufficient number of frequencies to allow accurate plotting of the function. Although this method would give a true graphical representation, it would be quite tedious and would defeat one of the main objectives of this study.



$$\frac{E_o}{E_i} = \frac{1}{1+RCp} = \frac{1}{1+j\omega RC}$$

Figure 14. A Simple Circuit and Its Transfer Function

If the decibel response of the circuit shown in Figure 14 was plotted by the above method, it would appear as shown in Figure 15. An examination of this plot reveals that when

$$f = f_o = \frac{1}{2\pi RC} = \frac{1}{(6.28)(10^5)(1590 \cdot 10^{-12})} = 1000 \text{ cps} \tag{18}$$

the response is down 3 db from its maximum possible value, and as the applied frequency is increased above f_o, the response curve approaches a line which has a slope of 6 db per octave (20 db per decade) as an asymoptote. On the other hand, at frequencies progressively lower than f_o, the response curve approaches the 0 db ordinate as an asymoptote. The intersection of the two asymoptotes will be at the frequency f_o which is characterized as the corner frequency. Since the graphical plot of the factor $\frac{1}{1 + j\omega T}$ will always be of the form depited in Figure 15, and the plot of $(1 + j\omega T)$ will be precisely the inverse, the plotting of the function would be much faster if the corner frequency was first determined as in equation (18) and then the asymoptotes drawn from that point with the proper slope.[4] The actual response may be readily drawn by placing a prepared template[5] tangent to the asymoptotes. The scale factors of the template must be the same as the graph paper being used.

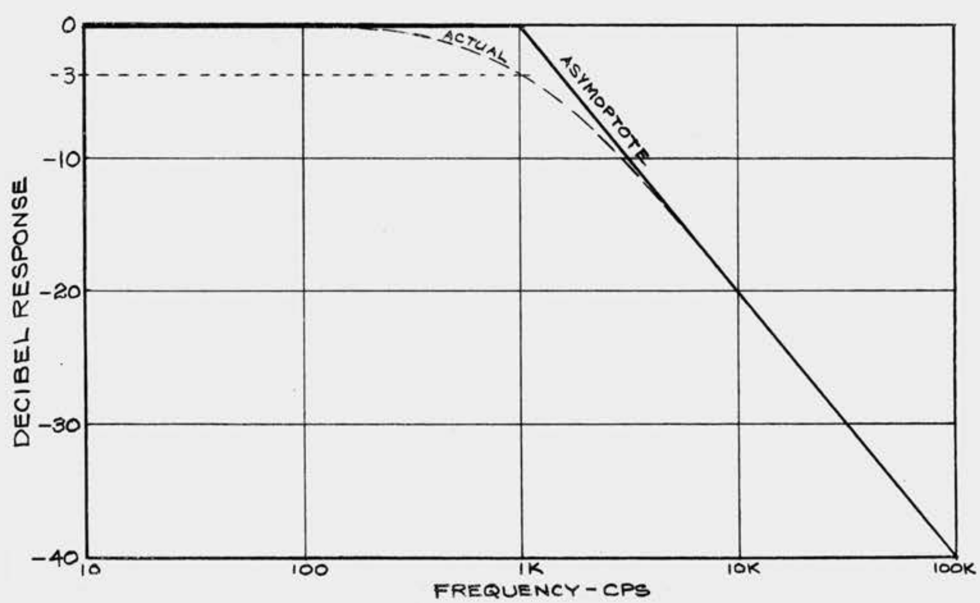


Figure 15. Amplitude Response of the R-C Circuit Shown in Figure 14.

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[4]Chestnut and Mayer, op. cit., pp. 302, et. sqq.

[5]Cf. post., p. 29.

When a transfer function is composed of more than one factor, the asymoptotes are drawn for each individual factor and are then added[6] together graphically to obtain the complete response of the function.

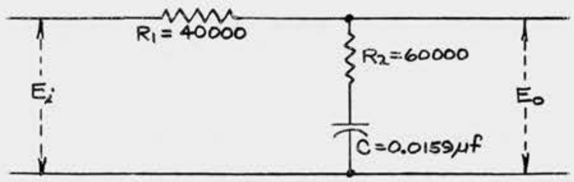

Figure 16. Typical Equalizing Circuit

The circuit shown in Figure 16 has a transfer function of

$$A = \frac{1 + R_2 CP}{1 + (R_1 + R_2)CP} \tag{19}$$

where the time constant in the denominator is obviously the larger. The corner frequency for the term in the denominator would be

$$f_1 = \frac{1}{2\pi(R_1+R_2)C} = \frac{1}{(6.28)(40000+60000)(0.0159\cdot 10^{-6})} = 100 \text{ cps} \tag{20}$$

The straight line plot corresponding to the denominator is shown in Figure 16 as curve A. Similarly, the corner frequency for the term in the numerator is

$$f_2 = \frac{1}{2\pi R_2 C} = \frac{1}{(6.28)(60000)(0.0159\cdot 10^{-6})} = 167 \text{ cps} \tag{21}$$

Curve B on the graph of Figure 16 is the asymoptote representing the numerator. The overall response, obtained by adding the curves A and B together, is shown as curve C.

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[6]Addition is used because the amplitude of the asymoptotes is plotted in decibels which is a logarithm. Logarithms representing multiplied factors are combined by addition.

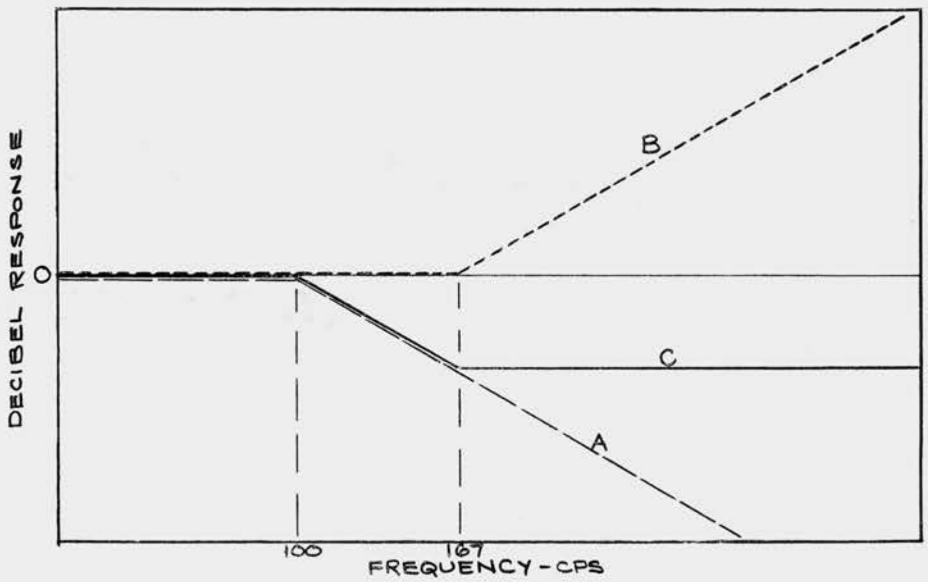

Figure 17. Amplitude Response of the R-C Circuit Shown in Figure 16

If a transfer function should have a term pT in the numerator, the true amplitude plot of this term will actually be a straight line with a positive slope of 6 db per octave. In this case, when $f = 1/2\pi T$, the value of pT, where $pT = j\omega T$ is unity and the equivalent decibel value is zero. This serves to establish the location of the straight line on the graph since it will pass through the 0 db ordinate at a frequency of $f = 1/2\pi T$. When the term pT appears in the demoninator, the plot will differ only by the algebraic sign of its slope.

Any constant term K appearing in the transfer function will have no variation with frequency, and will plot simply as a straight horizontal line with an ordinate value of $20 \cdot \text{Log} K$.

To illustrate one more example in the plotting of transfer functions, consider the circuit whose function is

$$A = K \cdot \frac{PT_1(1+PT_2)}{(1+PT_3)} \tag{22}$$

Figure 18 shows the overall response of the function as obtained from the sum of the individual asymoptote plots.

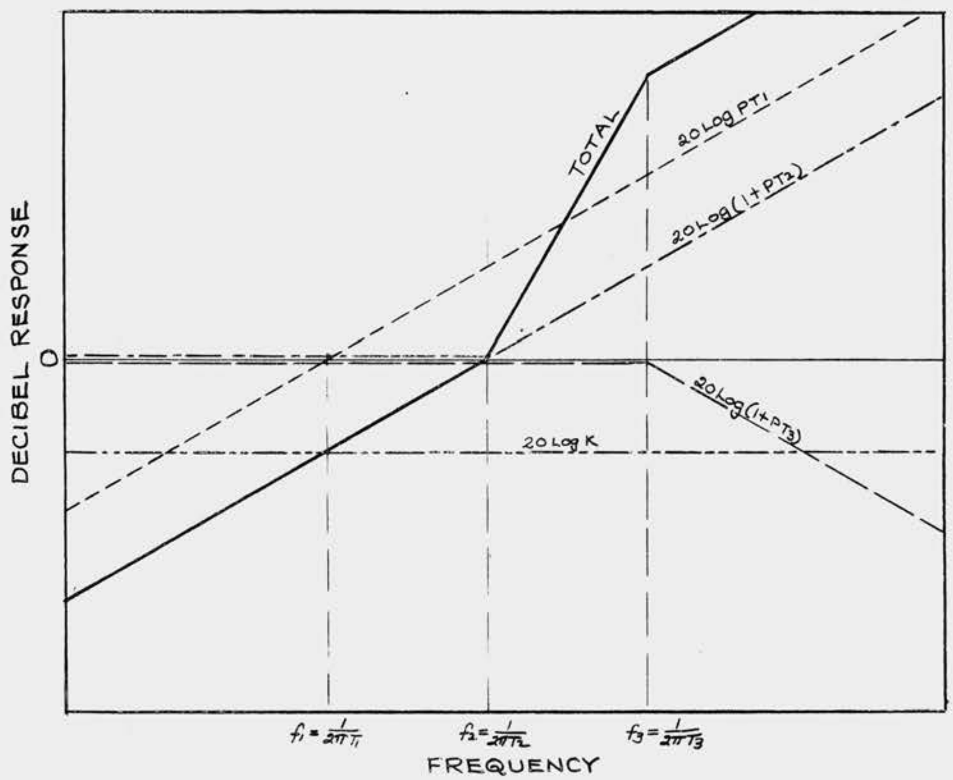

Figure 18. Amplitude Response of the Transfer Function of Equation (22)

Plotting the Phase Response of the Transfer Function

The overall phase response of the transfer function is also obtained by graphically adding together the phase plots of the individual terms in the function. The phase shift curves of the individual terms cannot be drawn as straight line asymoptotes, but must be plotted in accordance with actual computed points, however, since each of the terms in equation (15) will have a definite form of curvature as depited in Figure 19, they may be quickly drawn on the graph with the aid of a prepared phase template[7] whose scale factors agree with those of the graph paper being used.

The terms K, pT, and 1/pT have a phase shift that is constant with frequency. Factor K has a phase shift of zero degrees, while pT has a shift of +90 degrees and 1/pT has -90 degrees.

The identifying feature in locating the positions of the curves for the terms 1 + pT and $\frac{1}{1 + pT}$ is that the phase shift of each term will be 45 degrees respectively at its corner frequency $f = \frac{1}{2\pi T_n}$. The only difference in the phase shift of the two terms is the algebraic sign of the angle.

Figure 19 serves to illustrate the phase response development of the function

$$A = K \cdot \frac{pT_1(1+pT_2)}{(1+pT_3)} \tag{23}$$

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[7]*Cf.* *post.*, p. 29.

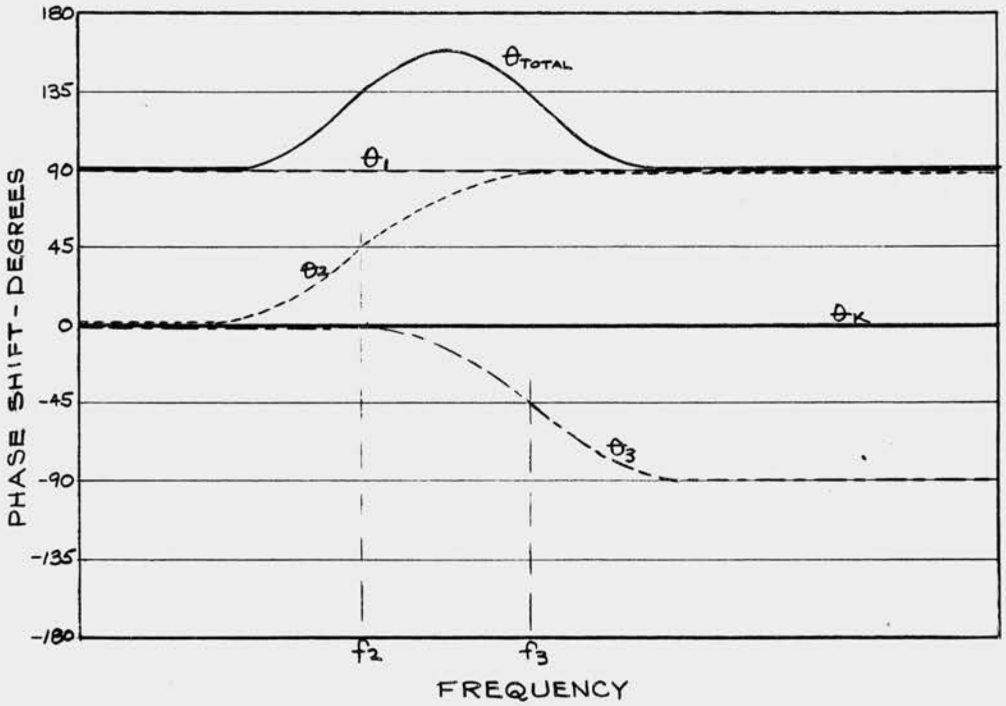

Figure 19. Phase Response of the Function $K \cdot \frac{PT_1(1+PT_2)}{(1+PT_3)}$.

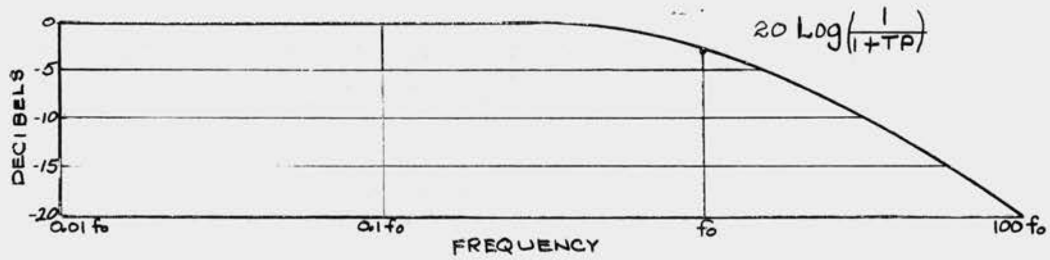

Figure 20(a). Template for Drawing Amplitude Response*

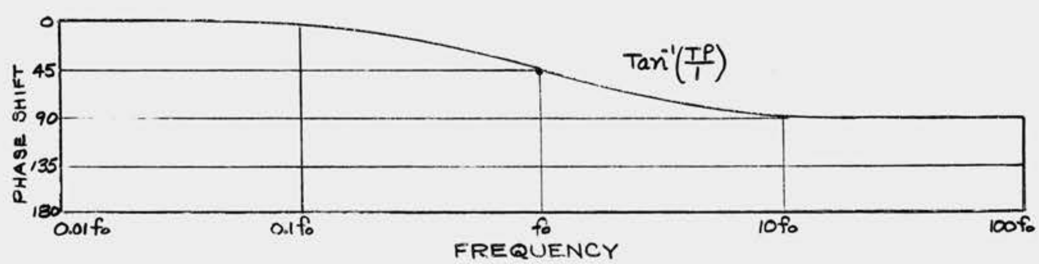

Figure 20(b). Template for Drawing Phase Shift*

*Note: Due to space limitations, the reproduction of these templates is approximately one-half scale.

Networks in Cascade

When two or more networks are in cascade, the amplitude response of the overall system is determined by simply adding the asymoptote plots of the individual networks.[8] Similarly, the total phase response is obtained by adding the phase plots of the various networks.[9] When resolving the response of networks in cascade by the prescribed procedure, it is presumed that each following network presents no loading effect on the preceding one. If loading is present, then the combination must be treated as one circuit and a transfer function developed in terms of the complete system.

When analyzing audio amplifiers, each amplifier stage is considered to be a network which has no load placed upon it by the following stage. The problem of loading is avoided by treating the shunt input capacity of the tube circuit as part of the preceding stage.

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[8] *Cf. ante.*, p. 24.

[9] *Cf. ante.*, pp. 4-5.

CHAPTER IV

TRANSFER FUNCTIONS FOR THE RESISTANCE-COUPLED AMPLIFIER

General

When analyzing the performance of an audio amplifier, it is customary to investigate the effects on gain at the low, middle, and high ranges of frequency individually. To develop a transfer function for the complete equivalent circuit would embody a great deal of needless effort. To go a step farther, it is not necessary to derive a single transfer function encompassing all of the constituents affecting low frequency gain, since the interaction of one upon the other is not serious. While there are several factors[1] influencing low frequency gain, only the major two will be considered, namely, the coupling condenser and the cathode by-pass condenser. At the high-range of frequencies, only the shunting capacity need be considered for good accuracy. Changes in high-frequency response due to inductive and resistive qualities of condensers are unpredictable and too remote to warrant a quanitative study.

Transfer Function for Mid-Range Frequencies

The gain of a resistance-coupled amplifier in the mid-range is independent of frequency and thus is a constant for a given amplifier stage.

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[1]*Cf. ante.*, p. 10

The transfer function for this condition is very simple and will not differ in form from that given in equation (6).[2] For the sake of completeness, however, it is repeated here as

$$A_M = g_m \cdot R_{eq} \qquad (24)$$

where R_{eq} is the parallel combination of R_p, R_c, and R_g. This function would be plotted on a log frequency graph as a horizontal line with an ordinate value of $db_M = 20 \mathrm{Log} A_M$.

Transfer Functions for the Low-Range of Frequencies

The gain at an R-C amplifier at any frequency can, in general, be said to equal

$$A = \alpha \beta \gamma \delta \lambda A_M \qquad (25)$$

where

α = transfer function for the coupling condenser effect on low frequency gain,

β = transfer function for the screen by-pass effect on low frequency gain,

γ = transfer function for cathode by-pass effect on low frequency gain,

λ = transfer function for plate decoupling condenser effect on the low frequency gain,

δ = transfer function for the shunt capacity effect on high frequency gain.

---

[2] Cf. ante., p. 6.

The factors β and λ are to be neglected for reasons stated previously[3] and since the δ factor is to be considered separately, equation (25) can be simplified to

$$A_L = \alpha \gamma A_M \tag{26}$$

It remains then, to develop transfer functions relating the effects of the coupling and cathode condensers to the low-frequency gain.

The transfer function exhibiting the effect of the coupling condenser C_c on the low-frequency gain is actually the same as equation (9) in Chapter II, but for the benefit of simplicity in plotting it should be written in the form of equation (15) in Chapter III. By applying basic algebraic manipulations, equation (9) can be written as

$$\alpha = \frac{PT}{1+PT} = \frac{RC_cP}{1+RC_cP} \tag{27}$$

where

$$R = R_g + \frac{R_c R_p}{R_c + R_p}$$

The corner frequency necessary for plotting the asymoptotic response of this function would be calculated from

$$f_{C_c} = \frac{1}{2\pi R C_c}$$

Figure 21 shows the typical amplitude and phase plots for the function expressed in equation (27).

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[3]*Cf.* *ante.*, p. 10.

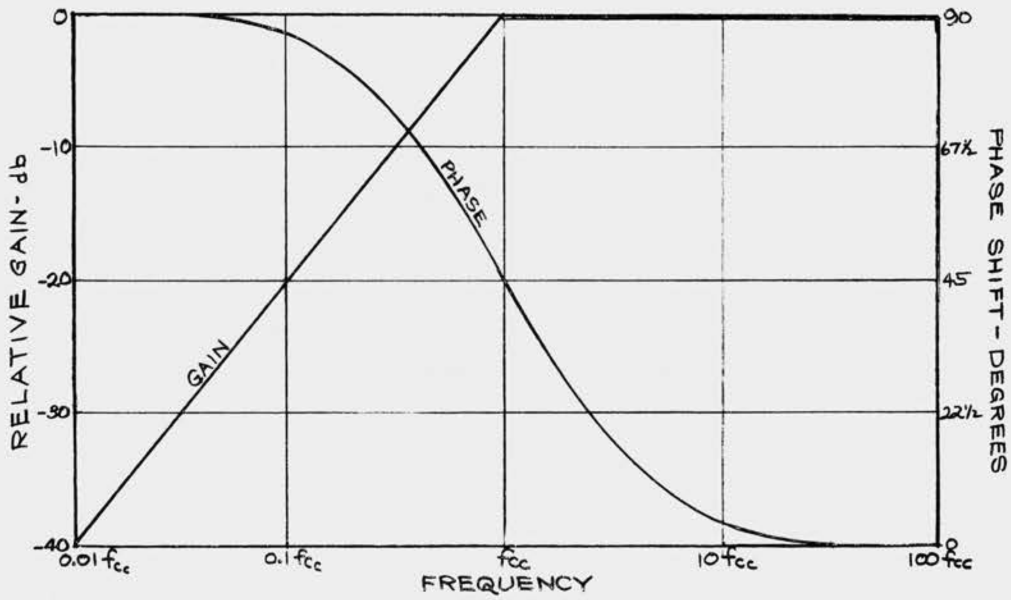
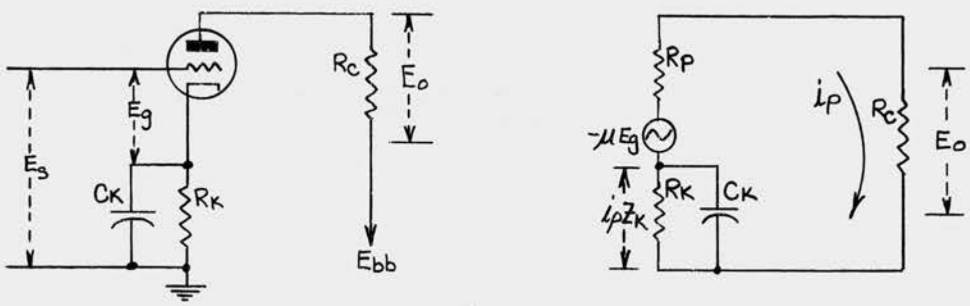


Figure 21. Typical Low-Frequency Response of an R-C Amplifier Due to Coupling Condenser Effects

It is rather obvious from an inspection of Figure 22, that when the impedance Z_k in the cathode circuit is not equal to zero that a voltage $i_p \cdot Z_k$ will appear, which will modify the effective grid voltage E_g and produce degeneration.



(a) R-C Amplifier (b) Equivalent Circuit

Figure 22

This condition will occur only at low frequencies since the reactance of C_k at high frequencies will be so small as to be considered effectively a short-circuit.

A rather complete analytical development of the transfer function relating the cathode impedance to the gain at low frequencies is given in Appendix A, therefore at this point it suffices to say that

$$\gamma = \frac{R_o}{R_o + \mu R_K} \cdot \left[\frac{1 + R_K C_K P}{1 + \left(\frac{R_o R_K}{R_o + \mu R_K} \right) C_K P} \right] \qquad (29)$$

where

$R_o = R_p + R_c$

$\mu = g_m R_p$

It follows that the corner frequencies for this function would be

$$f_{1K} = \frac{1}{2\pi R_K C_K} \tag{30}$$

$$f_{2K} = \frac{1}{2\pi \left(\frac{R_o R_K}{R_o + \mu R_K}\right) C_K} \tag{31}$$

The curves shown in Figure 23 are typical of the amplitude and phase response for the cathode function.

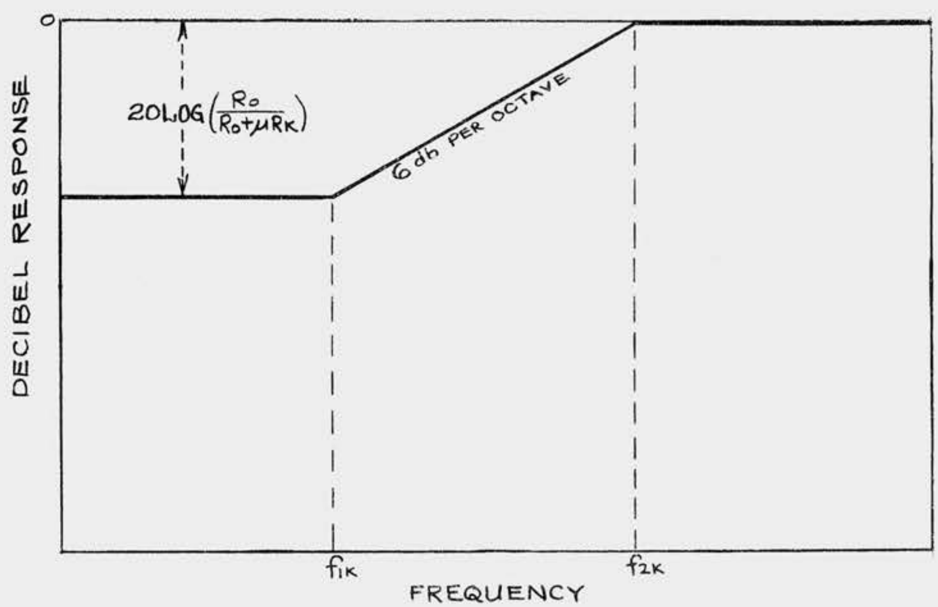


Figure 23. Response of Cathode Circuit Transfer Function

Equations (29), (30), and (31) along with Figure 23 lend themselves very readily to qualitative studies of the influence of various circuit elements on the response.

The constant term $\frac{R_o}{R_o + \mu R_k}$ in equation (29) gives the gain due to complete degeneration. If the cathode condenser was completely eliminated from the circuit, the gain of the amplifier at all frequencies would be reduced by the factor $\frac{R_o}{R_o + \mu R_k}$.

In many pentode amplifiers, especially in video circuits, $R_c \ll R_p$. In this case, equation (29) could, for all practical purposes, be simplifed to

$$\gamma = \frac{1}{1+g_m R_K} \cdot \left[\frac{1 + R_K C_K P}{1 + \left(\frac{R_K}{1+g_m R_K}\right) C_K P} \right] \tag{32}$$

and thus

$$f_{1K} = \frac{1}{2\pi R_K C_K} \tag{33}$$

$$f_{2K} = \frac{1}{2\pi \left(\frac{R_K}{1+g_m R_K}\right) C_K} \tag{34}$$

It might be of academic interest to note that, for the case of a cathode-follower where $R_c = 0$ and $C_k = 0$, equation (29) can be written as

$$\text{Voltage gain} = \frac{g_m R_k}{1 + g_m R_k} . \tag{35}$$

<u>Transfer Functions for the High-Range of Frequencies</u>

To develop a transfer function of the desired form which relates the shunt capacity to the high-frequency gain, it is only necessary to rearrange equation (7) in Chapter II to obtain

$$\delta = \frac{1}{1 + R_{eq} C_o P} = \frac{1}{1 + TP} \tag{36}$$

Where

$$R_{eq} = \frac{R_g}{1 + \frac{R_g}{R_p} + \frac{R_g}{R_c}}$$

$$C_o =$$

The corner frequency for this function is

$$f_H = \frac{1}{2\pi R_{eq} C_o} \tag{37}$$

The function of equation (36) is quite simple to plot and appears as shown in Figure 24.

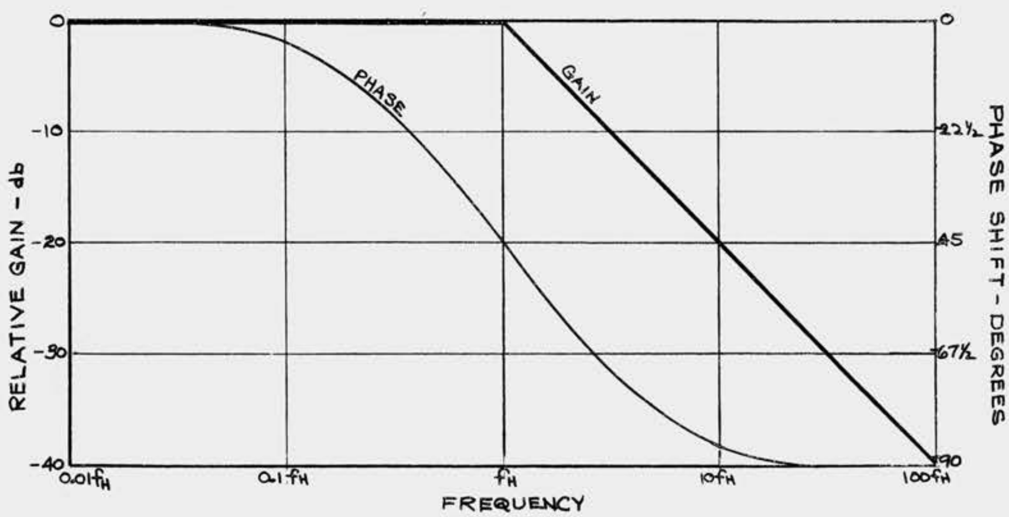


Figure 24. High Frequency Response of a Resistance-coupled Amplifier Due to the Shunting Capacity

CHAPTER V

TRANSFER FUNCTIONS FOR THE TRANSFORMER-COUPLED OUTPUT AMPLIFIER

Transfer Functions for the Mid-Range Frequencies

The development of the transfer functions for a transformer-coupled output amplifier consists chiefly of rearranging the equations for gain from Chapter II into the standard form of equation (15) in Chapter III.

The equation for the gain at mid-range frequencies is independent of frequency and is, therefore, a constant. The transfer function for the mid-range gain will be the same as equation (10), namely,

$$A_M = \mu n \cdot \frac{R_L}{R'_p + R'_L} \tag{38}$$

where

R_L = load resistance across the secondary.

$R'_L = n^2(R_L + R_2)$ = load resistance + dc resistance of secondary winding.

$R'_p = R_p + R_1$ = plate resistance of tube + dc resistance of primary winding.

$n = n_p/n_s$ = turns ratio of transformer.

μ = amplification factor of tube.

The function of equation (38) would be plotted on a log frequency graph as a horizontal line with an ordinate value of $db_M = 20 \text{Log} A_M$.

Transfer Function for the Low-Range of Frequencies

The transfer function relating the primary inductance to the low-frequency gain is a rearrangement of equation (11) in Chapter II and becomes

$$\alpha = \frac{\frac{L_1}{R'_1} P}{1 + \frac{L_1}{R'_1} P} = \frac{TP}{1+TP} \qquad (39)$$

where

L_1 = primary inductance of transformer.

R'_1 = parallel combination of R'_p and R'_L.

The corner frequency determined by this function is

$$f_L = \frac{R'_1}{2\pi L_1} \qquad (40)$$

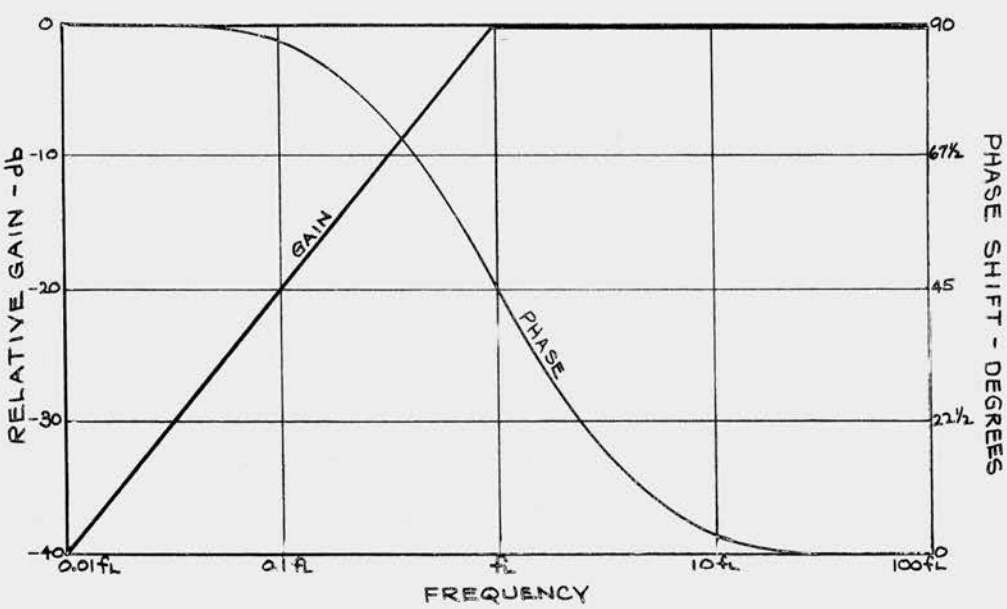


Figure 25. Typical Amplitude and Phase Response of a Transformer-Coupled Output Amplifier at Low-Frequencies

Transfer Function for the High-Range of Frequencies

By rearranging equation (13) in Chapter II, the transfer function associating the leakage inductance with the high-frequency gain results in

$$A_H = \frac{1}{1 + \frac{L_o}{R'}p} = \frac{1}{1 + Tp} \qquad (41)$$

where

L_o = Total leakage inductance

$R' = R'_p + R'_L$

The corner frequency associated with the high frequency function is

$$f_H = \frac{R'}{2\pi L_o} \qquad (42)$$

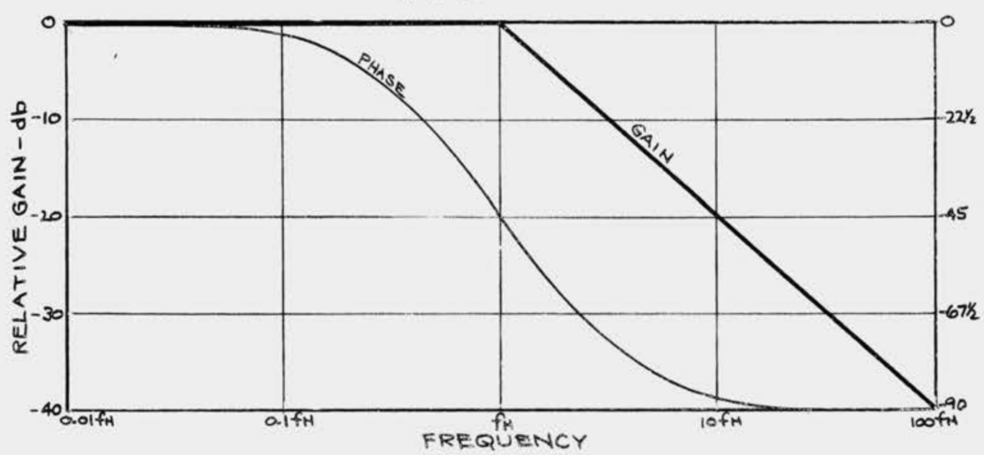


Figure 26. Typical Amplitude and Phase Response of a Transformer-Coupled Output Amplifier at High-Frequencies

The transfer functions and methods presented in this chapter will allow a satisfactory study to be made of the amplitude and phase shift of the usual type of audio amplifier. In Chapter VII, a demonstration will be made of the use of these transfer functions in analyzing the performance of an experimental amplifier.

CHAPTER VI

AMPLIFIER RESPONSE WITH FEEDBACK

General

Predicting the effect of negative feedback on the frequency response of an amplifier becomes such a complex problem that it is very discouraging to attempt a solution based on conventional network analysis. If the simplified direct form of graphical analysis discussed previously is extended properly a simple yet suitable analysis can be made of negative feedback effects, and the problem would not only become less laborious but also more enlightening.

It can be shown that the gain of an amplifier with negative feedback is equal to[1]

$$A' = \frac{A}{1 + A\beta} \qquad (43)$$

where

A = transfer function of amplifier without feedback.

β = transfer function of feedback network.

An inspection of equation (43) reveals that when $A\beta \rightarrow 1$, then all that remains to make the amplifier oscillate $[A' \rightarrow \infty]$ is for the phase angle of $A\beta$ to reverse in polarity, which may occur at the extreme high and low frequency ranges of the amplifier response.

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[1]Terman, op. cit., p. 311, et. sqq.

Quoting from Terman[2] on this matter:

> In order to realize the advantages of feedback, the amplifier and its feedback must be so arranged that oscillations do not occur. In the normal range of frequencies no problem is presented, because here the circuit arrangements are such that the feedback is negative. However, at both very low and very high frequencies, the amplifier stages produce phase shifts that cause the phase of the feedback factor $A\beta$ to differ from the phase corresponding to negative feedback. This introduces the possibility of $A\beta$ reversing its polarity, thus introducing positive feedback and directly assisting the production of oscillations. To be unconditionally stable, i.e., free of oscillations under all conditions, it is necessary that the circuit arrangements be such that, under conditions where the phase shift of the feedback factor $A\beta$ equals 180°, the feedback factor $A\beta$ will have a magnitude less than unity.

If a resistive network in the feedback loop will not allow the above conditions to be met, then the amplitude and phase characteristics of the network can be made a function of frequency. For that matter, phase correction to $A\beta$ could be accomplished by inserting an additional phase shift circuit of the appropriate type in the amplifier circuit itself. When properly proportioned, this will allow more negative feedback to be utilized than would otherwise be possible, and also satisfy the above phase requirement.

Graphical Solution

Further examination of equation (43) disclosed that when $A\beta \gg 1$ the gain with feedback is approximately equal to $1/\beta$, and when $A\beta \ll 1$ the gain is very nearly equal to A. This suggests a graphical procedure for determining the response of an amplifier when negative feedback is applied.[3]

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[2]Terman, op. cit., p. 314.

[3]Chestnut and Mayer, op. cit., p. 336, et. sqq.

Curve A in Figure 27 shows the typical middle and high-frequency response of an amplifier whose transfer function is $A = A_M \cdot \left[\frac{1}{1 + T_H p}\right]$.

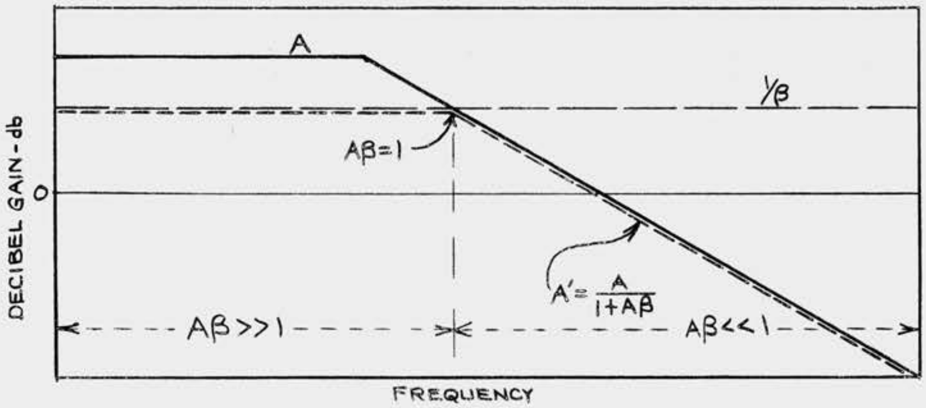

Figure 27. The Effect on Amplifier Response When $\beta = K$ (a constant)

Now if a negative feedback loop is incorporated with a transfer function β, the amplifier response with feedback can be determined by drawing the asymoptotic plot of $1/\beta$ on the same graph, then the total response will follow the locus of the lower curve at all frequencies.

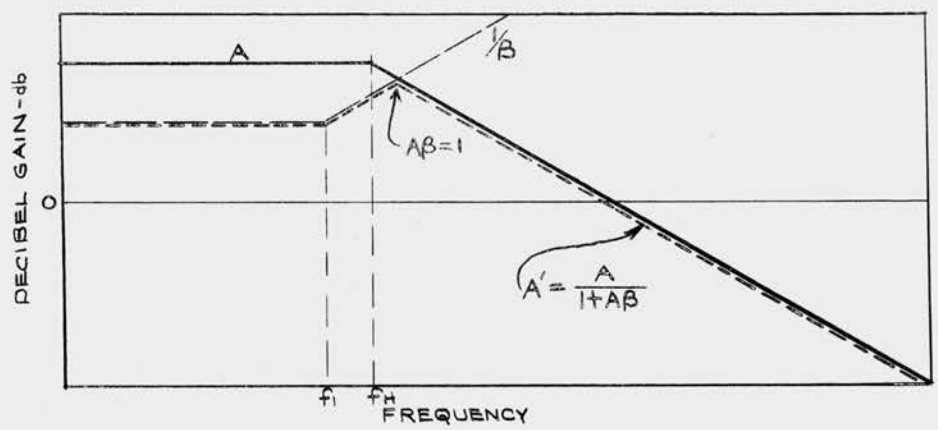

Figure 28. The Effect on Amplifier Response When $\beta = K_1\left(\frac{1}{1+T_1 p}\right)$ and $T_1 > T_H$.

The manner in which the frequency response is affected, as shown in Figure 28, will suggest to the engineer many possibilities for study of equalization and compensation of amplifiers through the use of negative feedback.

Avoidance of Oscillation

It was pointed out[4] that when the phase shift of the feedback factor $A\beta$ was equal to 180°, the magnitude of $A\beta$ must be less than unity otherwise oscillation would result. Conversely, it can be said that when the magnitude of $A\beta$ is unity, the phase shift of the feedback factor $A\beta$ must not equal 180° if oscillation is to be avoided. As a matter of fact, in any region where $A\beta = 1$ [where the graphical plot of $1/\beta$ intersects the plot of A], the phase shift of $A\beta$ should have a phase margin of safety of at least 30°.[5] Interpreting this in a practical circuit means that the magnitude of $A\beta$ should not vary too rapidly with frequency until A is considerably less than unity.

Figure 29 shows the phase shift curves corresponding to the asymoptotic plots of Figure 28. It will be noted that at the frequency f_x where $A\beta = 1$, the phase shift of $A\beta$ is approximately 150° thus providing a phase margin of 30°, which should be ample to prevent oscillation. If the high-frequency response of the amplifier had fallen at the rate of 12 db per octave instead of 6 db per octave, which could easily occur if two stages in cascade had identical

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[4]Cf. ante., p. 40.

[5]Terman, op. cit., pp. 317-319.

corner frequencies in the high-range, then the phase shift of the amplifier would pass through 90° at the corner frequency and would approach 180°. This added to the phase shift of the β function could easily result in a condition which would produce oscillation.

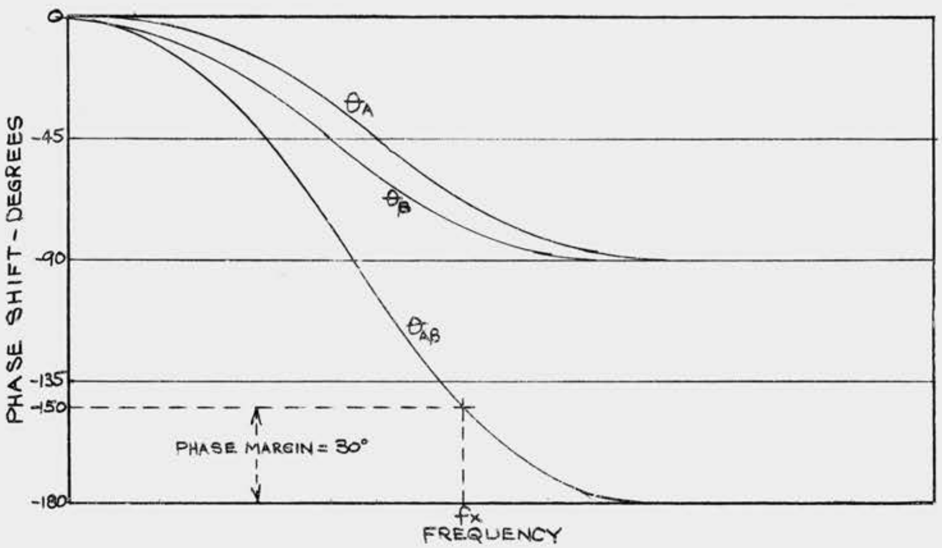

Figure 29. Phase Curves Corresponding to the Amplitude Plots of Figure 28.

Limitations

The amplitude response due to negative feedback and obtained through the use of the graphical method discussed in the foregoing paragraphs, is subject to the conditions as previously imposed.[6] The validity of the graphical solution depends on the provision that $A\beta \gg 1$ or $A\beta \ll 1$. When $A\beta$ is near unity, the error in the solution will vary similar to the manner shown in Figure 30. Actual calculation of the error when $0.1 \gg A\beta \gg 10$ would reveal that

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[6] Cf. ante., p. 43.

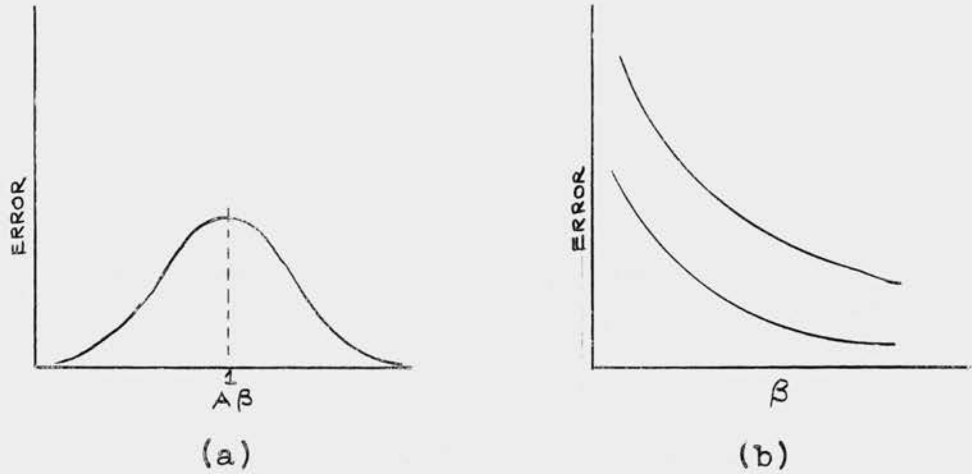

Figure 30. Relative Error in Graphical Solution for Variations in β and $A\beta$

the error is too small to be of any concern in practical design work. The region where the error would be the largest is where the asymoptotic plot of $1/\beta$ intersects the plot of A, thus accounting for one reason why a liberal phase margin should be provided in this region.

CHAPTER VII

ANALYZING THE PERFORMANCE OF AN AMPLIFIER

General

In order to more clearly demonstrate the use of asymoptotic plots in the analysis of audio amplifier performance, the amplitude and phase response of a small amplifier will be analyzed, and in a later chapter the data from laboratory tests will be presented for correlation with the calculated data.

The amplifier circuit as shown in Figure 31 was selected because it has a minimum number of required stages to allow a complete demonstration of the graphical analysis, and also because it is typical of many small commercial amplifier units.

The first step in preparing for the study of an amplifier is to make a systematic listing of all the pertinent information regarding the constants of the tubes and circuit elements. In the case of the output transformer, it might be necessary to make measurements[1] if the transformer properties are not available elsewhere.

Tables I and II contain lists of all the information necessary for analyzing the gain and phase performance of the amplifier shown in Figure 31.

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[1]Appendix B.

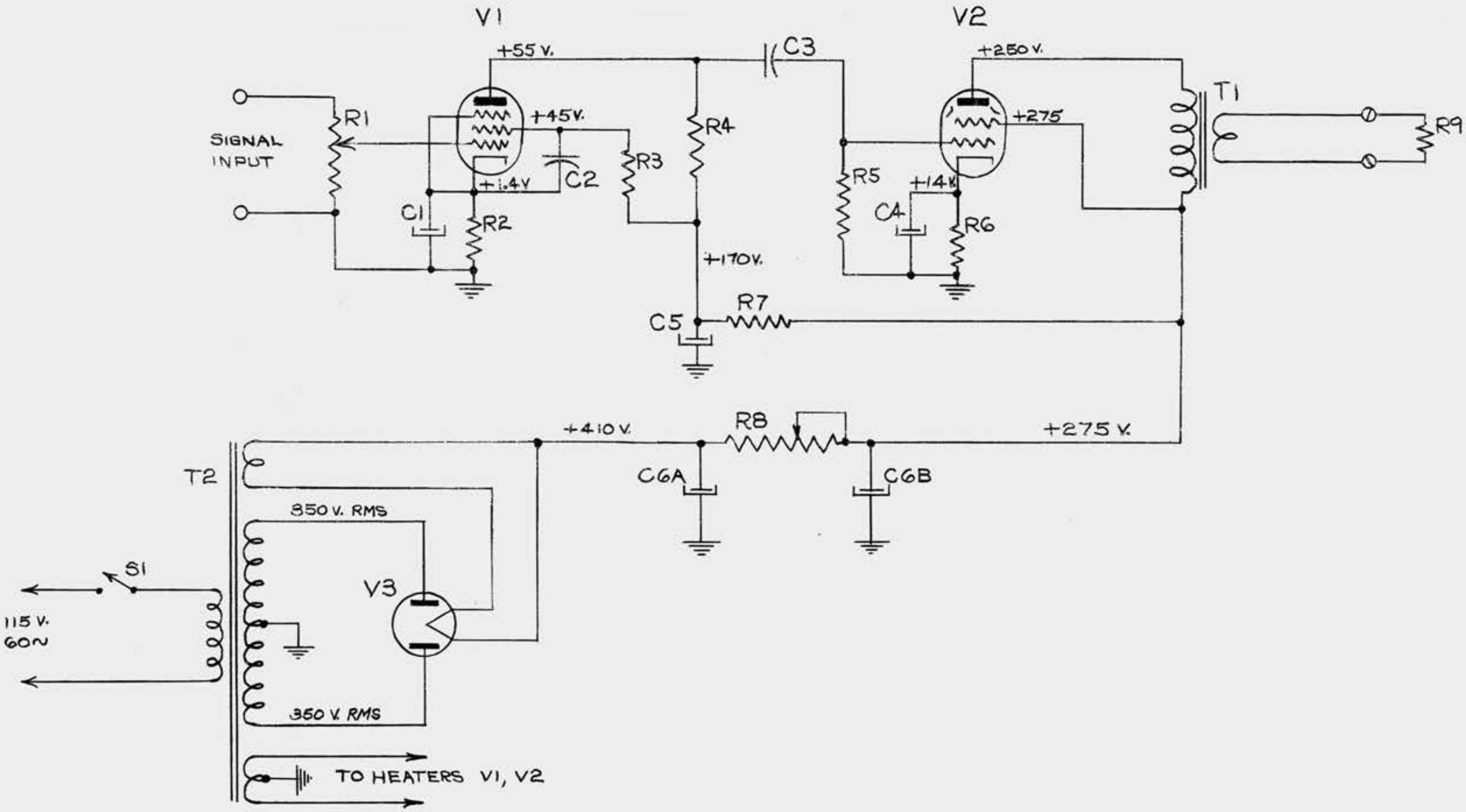

FIGURE 31. SCHEMATIC DIAGRAM OF AN EXPERIMENTAL AMPLIFIER.

TABLE I

AMPLIFIER PARTS LIST

Capacitors (All capacitances are in microfarad. All capacitors are paper unless otherwise specified.)

C1	10, 25 v. electrolytic	C4	16, 50 v. elect.
C2	0.5, 600 v.	C5	40, 450 v. elect.
C3	0.05, 600 v.		

Resistors (All resistances are ohms. K represents 1000; M represents 1,000,000. All resistors are 1/2 watt unless otherwise specified.)

R1	1M, potentiometer	R6	400, 5 watt, WW.
R2	3600	R7	185K, 1 watt.
R3	1M	R8	5K, 25 watt, WW adjustable
R4	500K		
R5	500K	R9	6.5, non-inductive

Transformers

T1	Output transformer UTC, R-27, 5000 ohm pri. 6.5 ohm sec.	T2	Power transformer 700 v. c.t., 70 ma.; 5 v., 3 a., 6.3 v., 3 a.

Tubes

V1	6AU6	V3	80
V2	6AQ5		

TABLE II

TUBE AND TRANSFORMER CONSTANTS

6AU6

g_m = 800 μmho
R_p = 1 megohm
u = 800
Input capacity = 5.5 μμf
Output capacity = 5 μμf
Stray output capacity[2] = 15 μμf

6AQ5

g_m = 4100 μmho
R_p = 52000 ohms
u = 210
Input capacity = 8 μμf
Stray input capacity[2] = 15 μμf

6AU6 circuit

$$R = R_g + \frac{R_c R_p}{R_c + R_p} = 0.83 \text{ megohm}$$

$$R_{eq} = \frac{R_g}{1 + \frac{R_g}{R_p} + \frac{R_g}{R_c}} = 0.333 \text{ megohm}$$

$$R_o = R_p + R_c = 1.5 \text{ megohm}$$

$$C_o = 5 + 15 + 8 + 15 = 43 \text{ μμf}$$

6AQ5 circuit

L_1 = 10 henry

L_o = 0.243 henry

R_1 = 580

$R'_p = R_p + R_1$ = 52580

n = 27.8/1

$R'_L = n^2(R_2 + R_L)$ = 6160

R_2 = 1.5

$$R'_1 = \frac{R'_p R'_L}{R'_p + R'_L} = 5520$$

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[1]All tube constants were determined from the RCA HB3 Tube Handbook.

[2]It has been the experience of the author that a rather generous allowance of 15 to 20 μμf per stage should be made for stray shunt capacity.

[3]Transformer constants were measured. See Appendix B.

Mid-Frequency Gain

The mid-frequency gain of the 6AU6 stage would be

$$A_M(6AU6) = g_m R_{eq} = (800 \cdot 10^{-6})(0.333 \cdot 10^{6}) = 266 \tag{44}$$

which corresponds to a decibel gain of

$$db_M(6AU6) = 20 \operatorname{Log} A_M = 20 \log 266 = 48.5 \text{ db}. \tag{45}$$

Following a similar procedure, the mid-frequency gain of the 6AQ5 stage would be

$$A_M(6AQ5) = \frac{\mu n R_L}{R_p' + R_L'} = \frac{(210)(27.8)(6.5)}{52580 + 6160} = 0.65. \tag{46}$$

When converted to decibels, the 6AQ5 gain is

$$db_M(6AQ5) = 20 \operatorname{Log} A_M = 20 \log 0.65 = -3.8 \text{ db}. \tag{47}$$

As a matter of academic interest, the total voltage gain would result as

$$A_M(\text{TOTAL}) = A_M(6AU6) \cdot A_M(6AQ5) = (266)(0.65) = 172. \tag{48}$$

The total decibel gain of the amplifier is

$$db_M(\text{TOTAL}) = db_M(6AU6) + db_M(6AQ5) = 48.5 - 3.8 \doteq 45 \text{ db} \tag{49}$$

Low-Frequency Response[2]

The determination of the low-frequency response of the 6AU6 stage includes the effects of two networks, namely, that of the coupling condenser C3 and the cathode condenser C1.

The corner frequency associated with the coupling network would be

$$f_{C_c}(6AU6) = \frac{1}{2\pi RC_3} = \frac{1}{(6.28)(0.83\cdot 10^{6})(0.05\cdot 10^{-6})} = 3.8 \text{ cps} \qquad (50)$$

and the two corner frequencies related to the cathode condenser effect are

$$f_{1K}(6AU6) = \frac{1}{2\pi R_2 C_1} = \frac{1}{(6.28)(3600)(10\cdot 10^{-6})} = 4.5 \text{ cps} \qquad (51)$$

$$f_{2K}(6AU6) = \frac{1}{\dfrac{2\pi R_o R_2 C_1}{R_o + \mu R_2}} = \frac{1}{\dfrac{(6.28)(1.5\cdot 10^{6})(3600)(10\cdot 10^{-6})}{(1.5\cdot 10^{6}) + (800)(3600)}} = 13 \text{ cps} \qquad (52)$$

The corner frequency for the primary inductance effect in the 6AQ5 stage is

$$f_{L_1}(6AQ5) = \frac{R_1'}{2\pi L_1} = \frac{5520}{(6.28)(10)} = 88 \text{ cps} \qquad (52)$$

The asymoptotic curves for all of the above effects have been drawn on the graph of Figure 32 in accordance with the procedures outlined in Chapters IV and V. The total low-frequency response of the amplifier as shown is the sum of all the individual curves. It is to be noted that the curves of Figure 32 have been drawn with reference to the mid-range gain of 45 db rather than zero db, as was

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[2] Cf. ante., Chapter II.

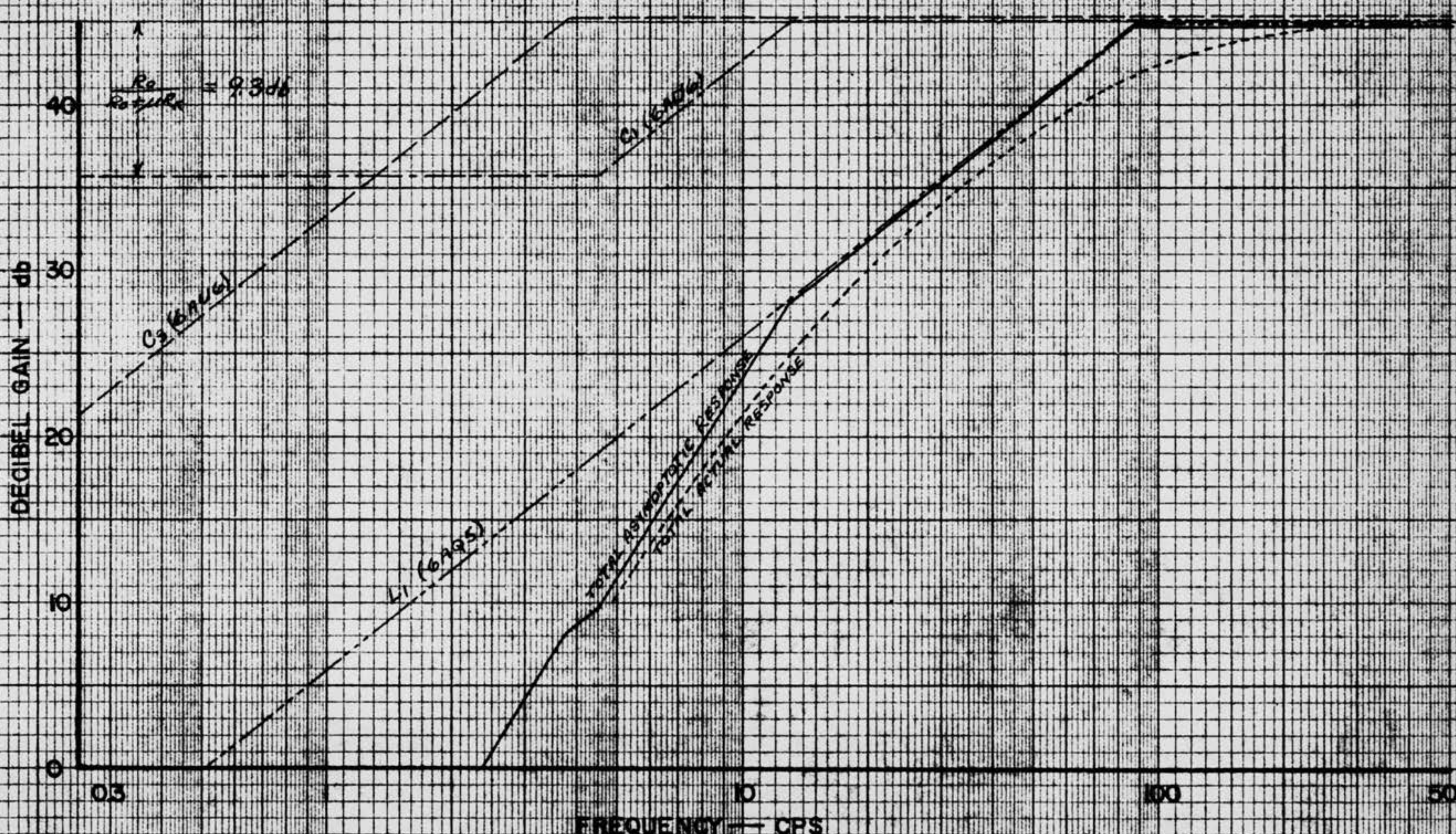

FIGURE 32. LOW-FREQUENCY RESPONSE OF AN EXPERIMENTAL AMPLIFIER.

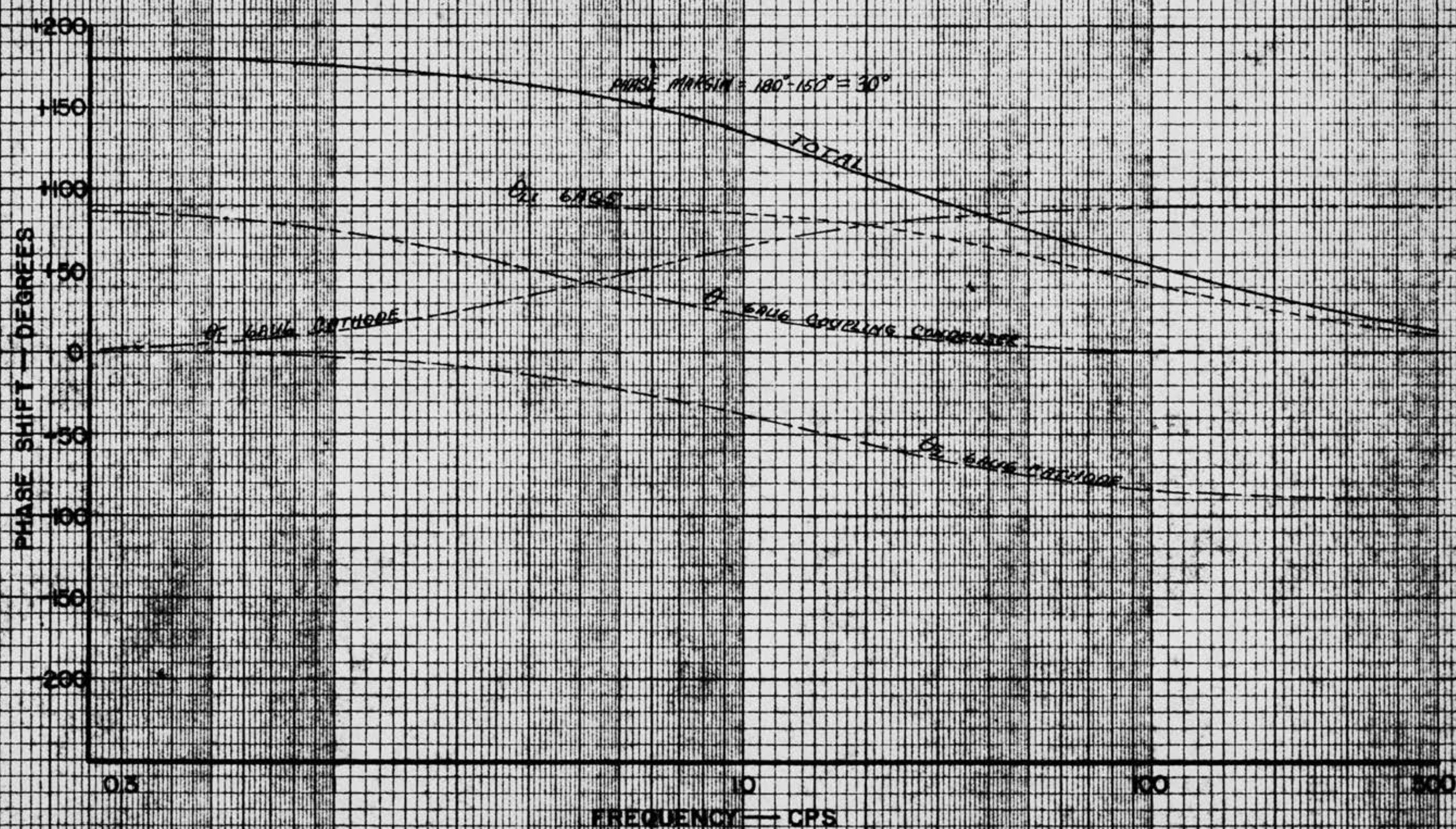

FIGURE 33. PHASE SHIFT AT LOW-FREQUENCIES OF AN EXPERIMENTAL AMPLIFIER.

illustrated in Chapters IV and V. This simply means that each point on the overall low-frequency response curve represents an absolute level of gain instead of being merely relative. In other respects, the graph shown in Figure 32 is largely self-explanatory.

In Figure 33, the phase shift curves for all the low-frequency effects have been drawn and the summation made to give the total phase shift curve of the amplifier in the low-frequency region.

<u>High Frequency Response</u>

Analysis of the high-frequency response of the amplifier is somewhat easier in that a smaller number of networks have to be considered. In the 6AU6 stage the shunt capacity regulates the high-frequency response and in the 6AQ5 stage it is the leakage inductance that dominates.

For the 6AU6 stage, the corner frequency determined by the shunt capacity effect is

$$f_H(6AU6) = \frac{1}{2\pi R_{eq} C_o} = \frac{1}{(6.28)(0.333 \cdot 10^6)(43 \cdot 10^{-12})} = 11{,}100 \text{ cps.} \qquad (49)$$

The corner frequency corresponding to the influence of the leakage inductance in the 6AQ5 circuit would be

$$f_H(6AQ5) = \frac{R'_p + R'_L}{2\pi L_o} = \frac{52580 + 6160}{(6.28)(0.243)} = 38{,}500 \text{ cps.} \qquad (50)$$

The asymoptotic curves associated with the above high-frequency effects have been plotted on the graph of Figure 34 along with the overall high-frequency response curve.

The curves shown in Figure 35 present the phase shift of the amplifier at the high-range of frequencies.

<u>Summary</u>

If negative feedback is applied around an amplifier, it was stated in Chapter VI that when the feedback function $A\beta$ is equal to unity, the phase margin at that point should be at least 30° to avoid oscillation. An inspection of the low-frequency phase shift response of Figure 33 will reveal that from 5 cycles and down, the phase margin is decreasing below 30°. At the same time, reference to the amplitude plot of Figure 32 will show that the gain at 5 cycles is still 12 db. This is interpreted to mean that if negative feed-back was applied around the amplifier with a $1/\beta$ function of 12 db or less, the amplifier would oscillate in the frequency range of about 2.5 to 5 cycles.

A similar condition also exists at the high-range of frequencies. The phase plot of Figure 35 shows the phase margin becoming less than 30° at about 90KC, at which point the gain from Figure 34 is 18 db. Again, this means that if $1/\beta$ was 18 db or less, the amplifier would oscillate in the range of 90 KC to 260 KC where the gain finally drops below zero db. Of course if $1/\beta$ was made larger than 18 db and the feedback network contributed no phase shift, then the am-plifier would not oscillate. On the other hand, if it was

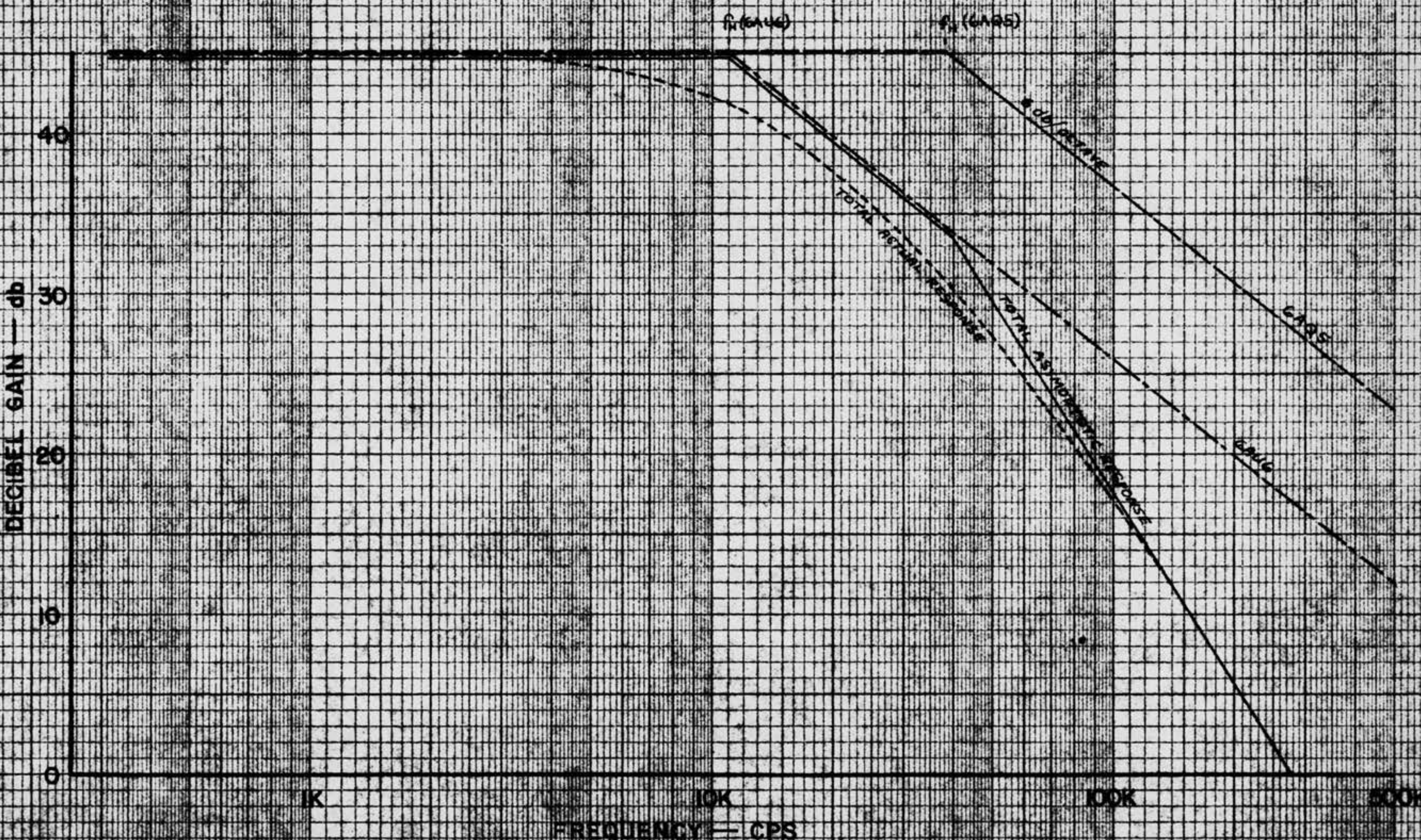

FIGURE 34. HIGH-FREQUENCY RESPONSE OF AN EXPERIMENTAL AMPLIFIER.

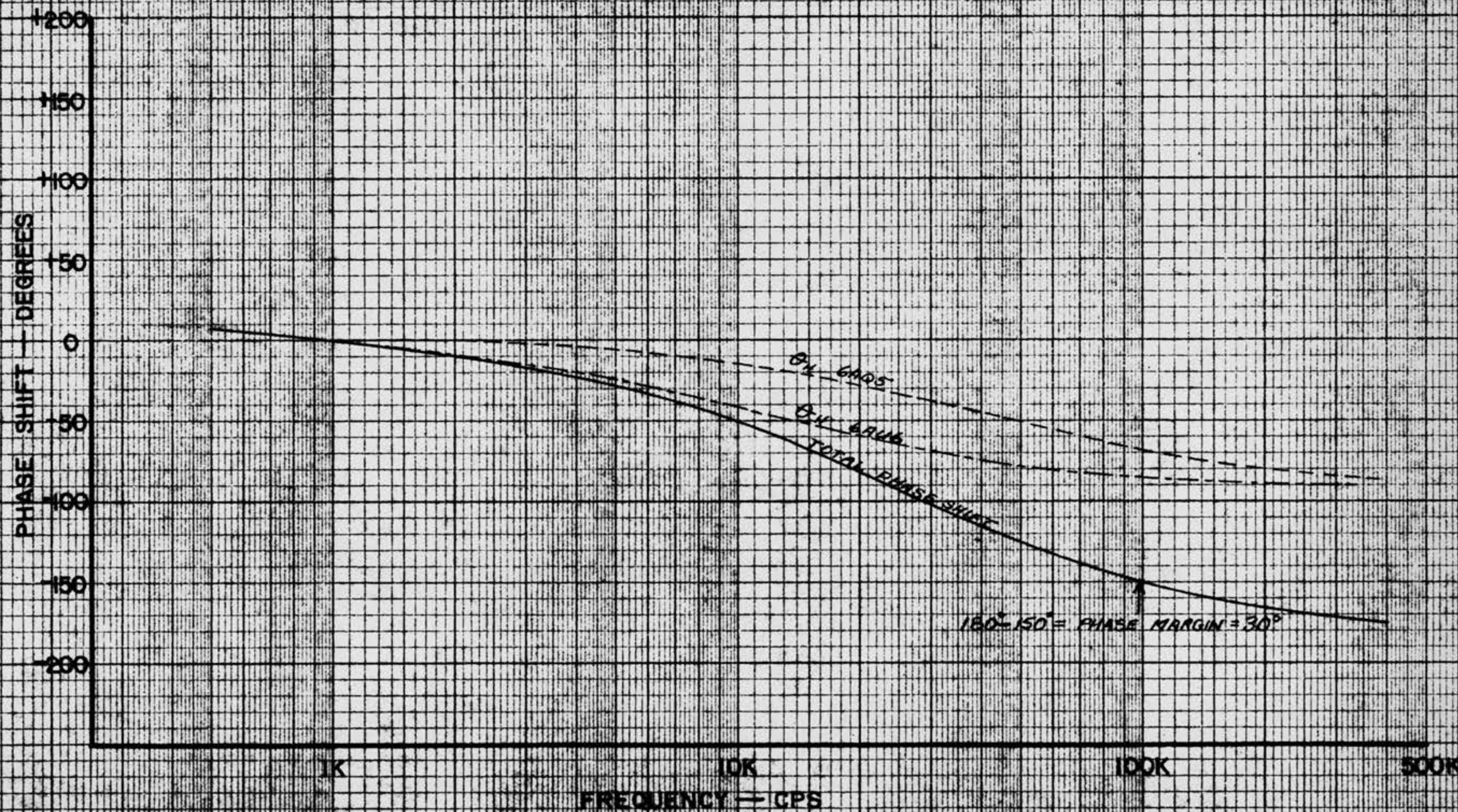

FIGURE 35. PHASE SHIFT AT HIGH-FREQUENCIES OF AN EXPERIMENTAL AMPLIFIER.

desired to make $1/\beta$ less than 18 db, then either the phase shift of the amplifier or the feedback network would have to be compensated so that when $A\beta \rightarrow 1$ the phase margin of $A\beta$ would be at least 30°.

Another interesting observation from Figure 32, is that when complete cathode degeneration is present in the 6AU6 stage, it affects the gain of the amplifier by reducing it 9.3 db. If the 6AU6 cathode condenser was completely eliminated from the circuit, the gain of the amplifier would be reduced 9.3 db throughout the mid-range. While this reduction in gain might be a disadvantage, on the other hand not only would the frequency response be extended on both ends of the operating range, but the phase shift at the low-frequencies would not approach 180° so rapidly, thus allowing more negative feedback to be applied without low-frequency oscillation.

In order for an audio amplifier to be suitable for the high-fidelity reproduction of music, its frequency response should be uniform from about 20 to 20,000 cycles. An inspection of the response curves of Figure 32 and Figure 34 show that this amplifier falls far short of meeting the specified requirement. At the low end, the major factor contributing to the early drop in response is the primary inductance of the output transformer. To correct the transformer itself would, in all probability, require a replacement. An alternative would be to use shunt feed to the 6AQ5 tube which would eliminate the dc saturation in the transformer and thus increase the primary inductance, but at the same time this would require a high inductance choke and a quite large coupling condenser which might offset the advantage of shunt feed and make replacement of the transformer seem more feasible.

At the high end, the major factor contributing to the early drop in the response in the shunt capacity of the 6AU6 stage. While the capacity itself cannot be lessened appreciably, its effect can be reduced by using a smaller d-c plate load resistor R_4 in order to cut down on the equivalent resistance R_{eq}. This modification would also lower the mid-frequency gain but this effect would have to be tolerated.

As mentioned earlier, removal of cathode by-pass condensers and application of negative feedback could also be employed to extend the frequency response at both ends of the mid-range.

Partial by-passing of a cathode resistor is many times utilized to extend the high-range response. For example, if the capacity of the cathode condenser C_1 on the 6AU6 stage was reduced until the corner frequencies f_{1k} and f_{2k} were located in the vicinity of 10 and 30 KC respectively, then the high-frequency response would be extended measurably. This too, of course, would reduce the mid-frequency gain which again must be tolerated.

In order to verify the calculated performance data that has been derived in this chapter, and to demonstrate more specifically the use of asymoptotic plots and phase shift curves for the practical design of a feedback network, the experimental amplifier circuit of Figure 31 was constructed so that measured data might be obtained. The procedure and results of these measurements is presented in Chapter IX. If complete verification of the calculated data was to be possible, some suitable means of measuring the phase shift of the amplifier had to be conceived. Consequently, as a part of this study of audio amplifier performance and design, the phase meter about to be described in Chapter VIII was designed and constructed.[3]

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[3]*Cf.* *ante.*, p. 2.

CHAPTER VIII

DIRECT READING PHASE METER

General

After an engineer in the field has designed and constructed a given amplifier circuit, laboratory tests must be conducted to see if the equipment fulfills the expectations. As was pointed out earlier, measurements of the phase response of an amplifier is somewhat of a problem since commercial phase meters are quite expensive, and many of them do not perform well above 20 kilocycles.

The main objective of the preceding material has been to present a simplified graphical solution to the problem of audio amplifier analysis. Now, in order to simplify the problem of measurement, it would be very desirable if some type of direct reading phase meter was developed which would give reasonable accuracy of measurement up to around 100 kilocycles, was simple to construct, and was low in cost. The phase meter about to be presented meets the above requirements quite well. The circuits are conventional and anyone with some experience in amplifier construction and testing should have little trouble in the assembly and calibration of this instrument. The simplicity of the phase meter about to be described is based on the fact that its use is limited to the phase measurement of sine wave signal voltages. At first, this might seem to be a great disadvantage, however, since the principle application of this meter is on amplifiers having low distortion, this limitation is not a serious drawback. When one of the signal voltages contains 10 per cent harmonic distortion the error in phase measurement is approximately 10°, and any amplifier producing more than 10 per cent distortion should be corrected in any event before more detailed measurements are attempted.

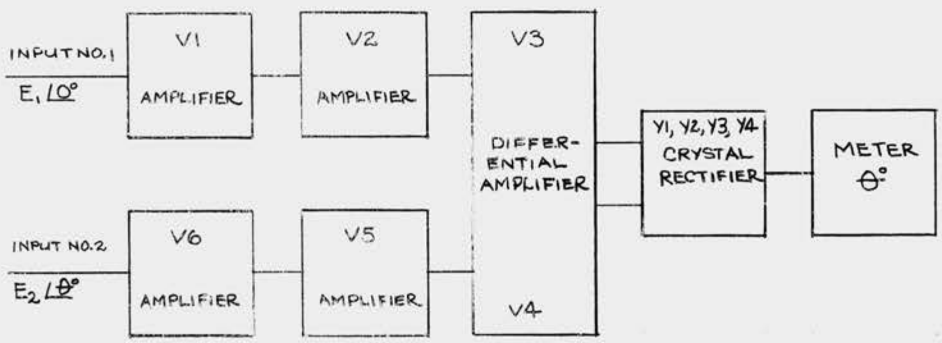

Figure 36. Block Diagram of Simple Phase Meter

TABLE III

TECHNICAL SPECIFICATIONS OF PHASE METER

Phase measurement range, 0 - $\pm 180^{\circ}$

Full-scale range, 0 - 180°

Frequency range, 5 cycles to 100 kc.
(error increases rapidly above 150 kc.)

Channel no. 1

Minimum input voltage, 5 mv.

Frequency response flat within 3 db from 2 cycles to 600 kc.

Channel no. 2

Minimum input voltage, 14 mv.

Frequency response flat within 3 db from 2 cycles to 1 mc.

Error due to waveform distortion is approximately 5° when one of the signal voltages contains 5 per cent harmonic distortion. An additional error of approximately 5° is present over the upper one-fourth part of the meter scale due to the crowded calibration.

Operating power, 115 v., 60 cycles.

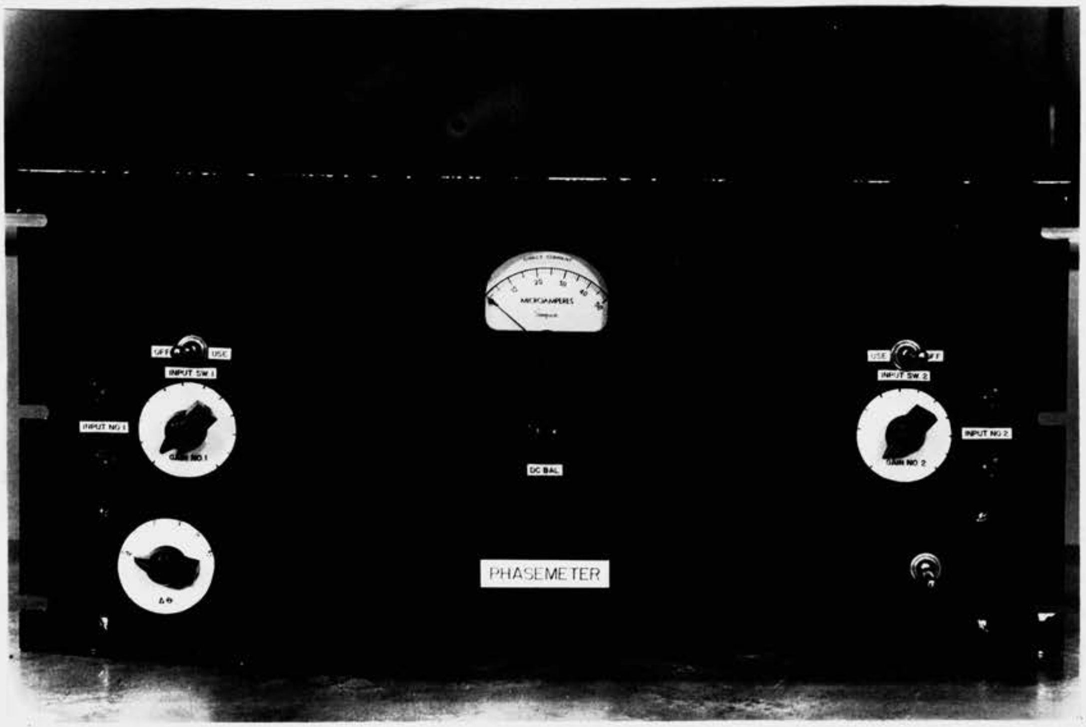

Figure 37. Front Panel View of Phase Meter.

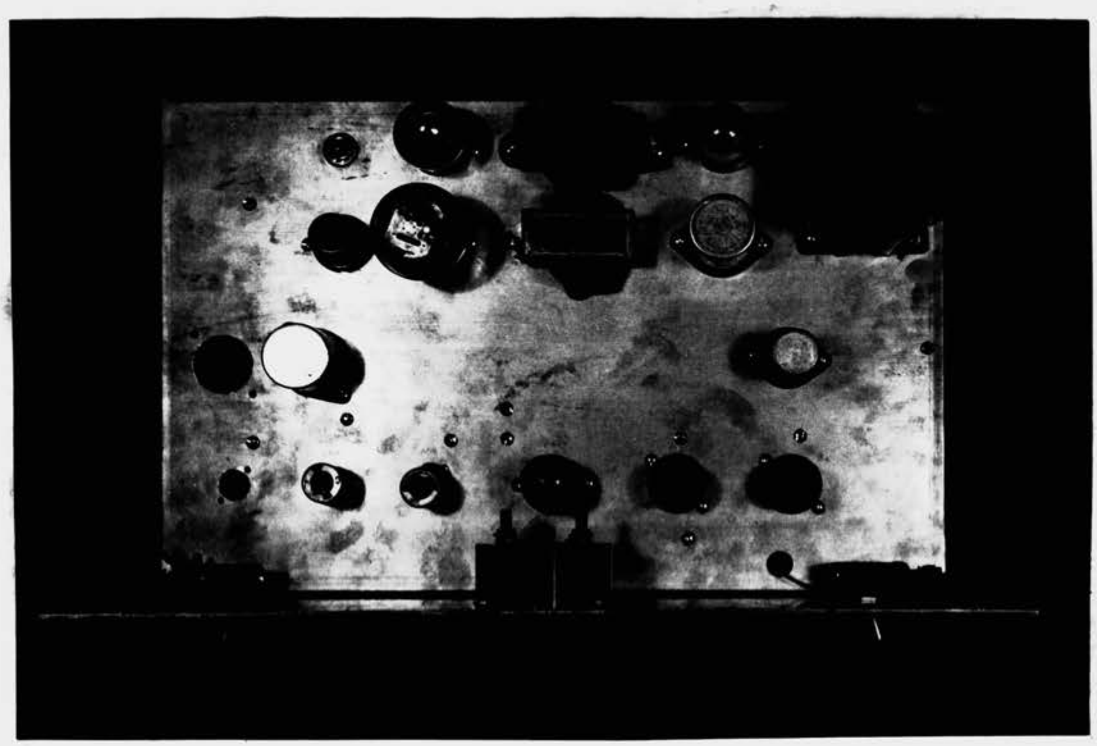

Figure 38. Top View of Phase Meter.

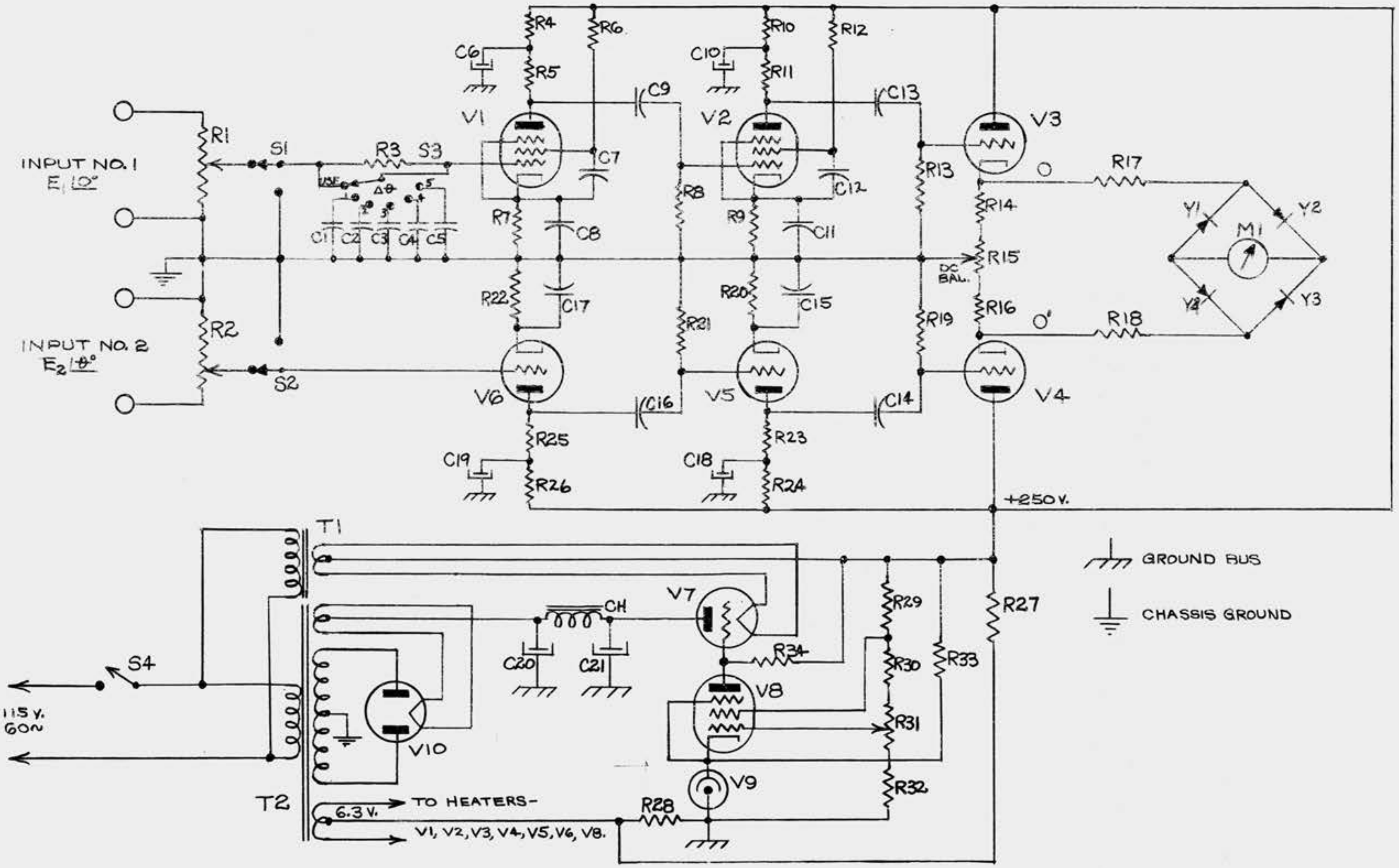

FIGURE 39. SCHEMATIC DIAGRAM OF A DIRECT READING PHASEMETER.

TABLE IV

PHASE METER PARTS LIST

Capacitors (All capacitances are microfarads. All capacitors are paper unless otherwise specified.)

C1	0.1, 600 v.	C12	0.1, 600 v.
C2	0.02, 600 v.	C13	0.1, 600 v.
C3	0.005, 600 v.	C14	0.1, 600 v.
C4	0.001, 600 v.	C15	250 μμf, 400 v. mica
C5	0.0002, 400 v.		
C6	40, 450 v. elect.	C16	0.1, 600 v.
C7	0.1, 600 v.	C17	900 μμf, 400 v. mica
C8	0.0022, 400 v. mica		
C9	0.1, 600 v.	C18	40, 450 v. elect.
C10	40, 450 v. elect.	C19	40, 450 v. elect.
C11	0.0022, 400 v. mica	C20	40, 475 v. elect.
		C21	40, 475 v. elect.

Resistors (All resistances are in ohms. K represents 1000; M represents 1,000,000. All resistors are 1/2 watt unless otherwise specified.)

R1	200, potentiometer	R18	62K
R2	200, potentiometer	R19	2M
R3	15K	R20	3000
R4	15K, 1 watt	R21	2M
R5	100K	R22	3000, 1 watt
R6	470K	R23	30K, 1 watt
R7	1000	R24	25K, 1 watt
R8	2M	R25	39K, 1 watt
R9	1000	R26	25K, 1 watt
R10	15K, 1 watt	R27	75K, 1 watt
R11	100K	R28	18K, 1 watt
R12	470K	R29	10K, 1 watt
R13	2M	R30	22K, 1 watt
R14	2K, 1 watt	R31	10K, potentiometer
R15	500, potentiometer	R32	5K, 1 watt
R16	2K, 1 watt	R33	75K, 1 watt
R17	62K	R34	470K

Transformers

T1	Filament, 6.3 v., 3 a.	T2	Power transformer 750 v., c.t., 70 ma.; 5 v., 3 a., 6.3 v., 4 a.

TABLE IV

(Continued)

Crystal Rectifiers

Y1, Y2, Y3, Y4	1N34A

Meters

M1	0-50 v. d-c microamperes full-scale

Tubes

V1	6AU6	V6	6J5
V2	6AU6	V7	6B4
V3	1/2 6SN7	V8	6SJ7
V4	1/2 6SN7	V9	0A3/VR75
V5	6J5	V10	5Y3GT

Switches

S1	Single pole double throw toggle type	S3	Single pole six position rotary
S2	Same as S1	S4	Single pole single throw toggle type

Chodes

CH	10 henry, 80 ma.

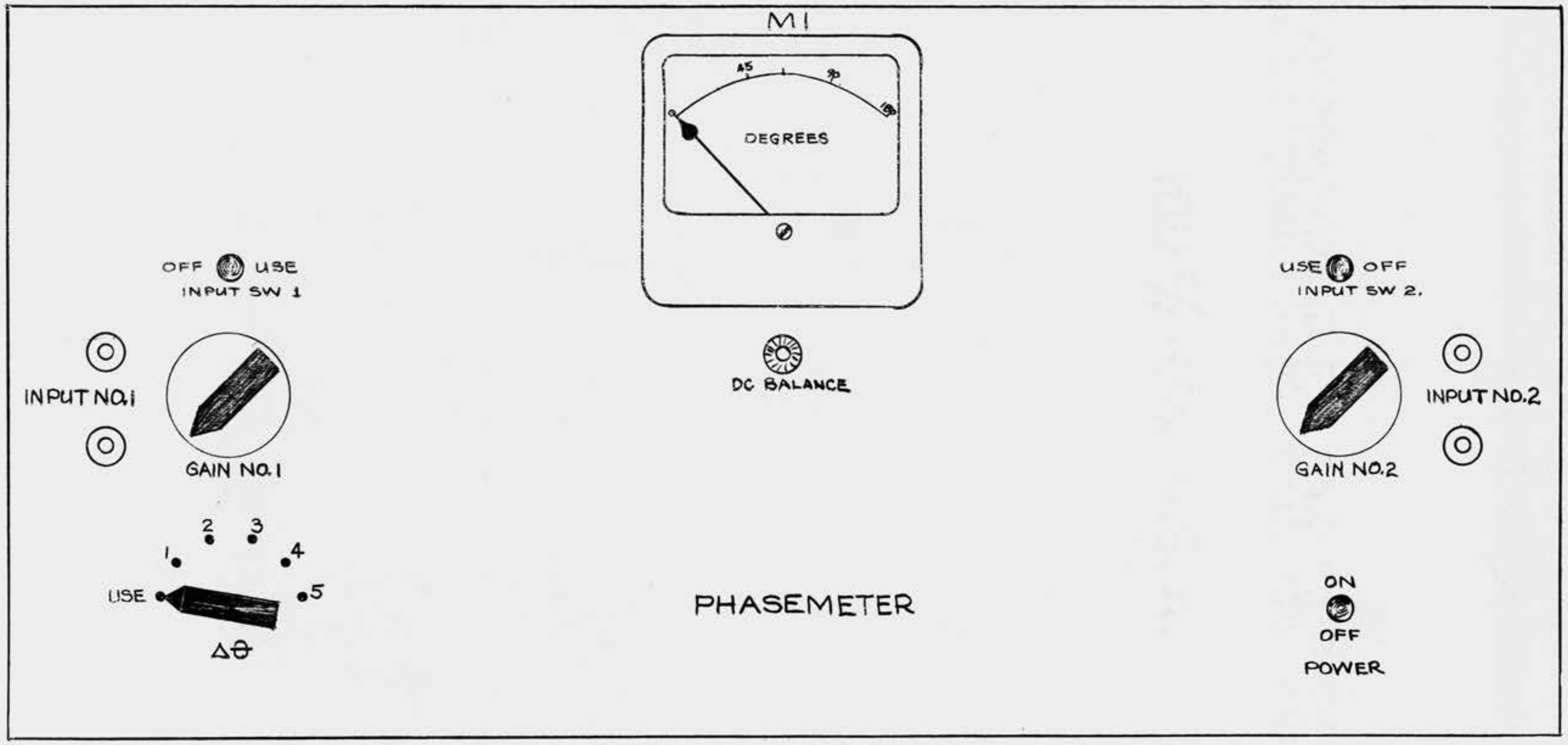

FIGURE 40. FRONT PANEL LAYOUT SHOWING ALL OPERATING CONTROLS.

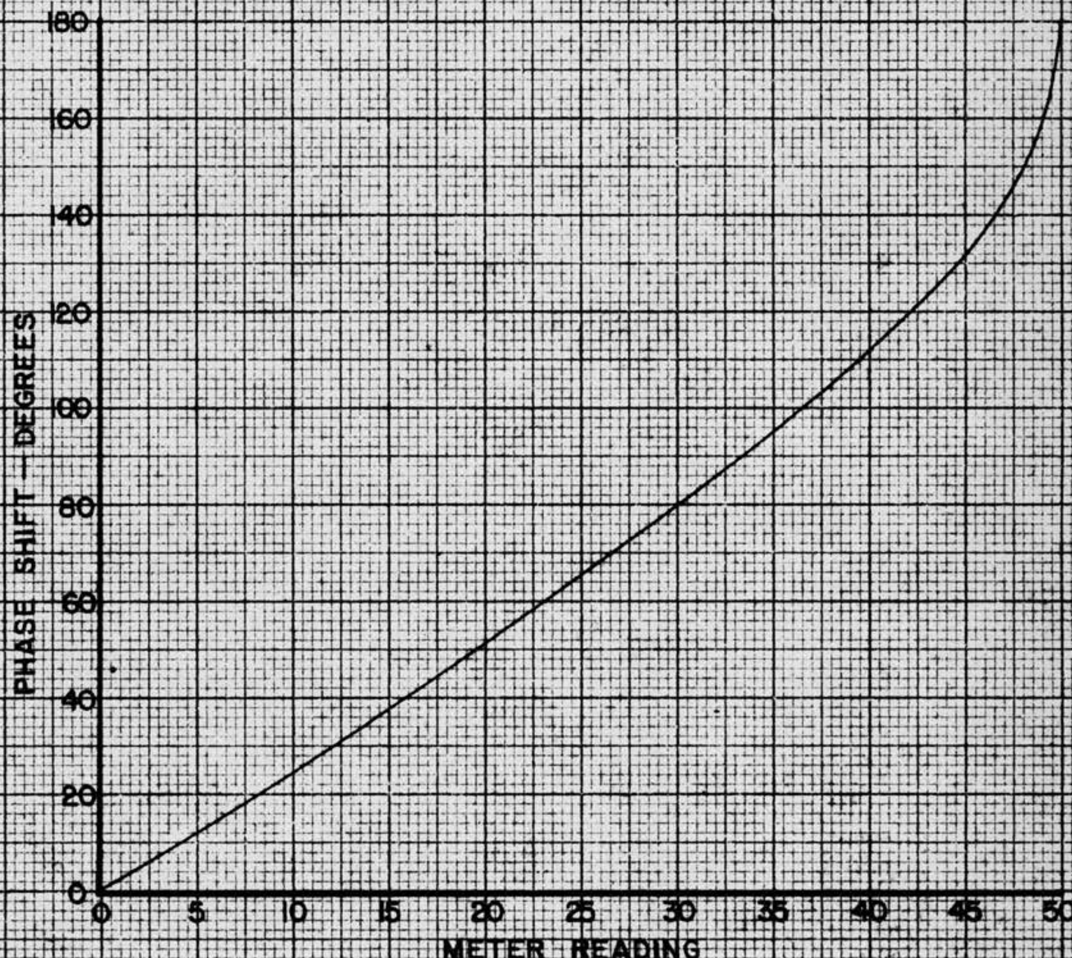

FIGURE 41. CALIBRATION CURVE FOR THE PHASE METER.

Principle of Operation

The fundamental principle of operation of the phase meter is based on the properties of a difference amplifier,[1,2] which in this case is arranged in the form of a cathode follower circuit for the purpose of providing a low output impedance. The basic circuit as shown in Figure 41 has two input voltages $e_1 = E\sin \omega t$ and $e_2 = E\sin(\omega t \pm \theta)$ whose amplitude factors E are to be made equal.

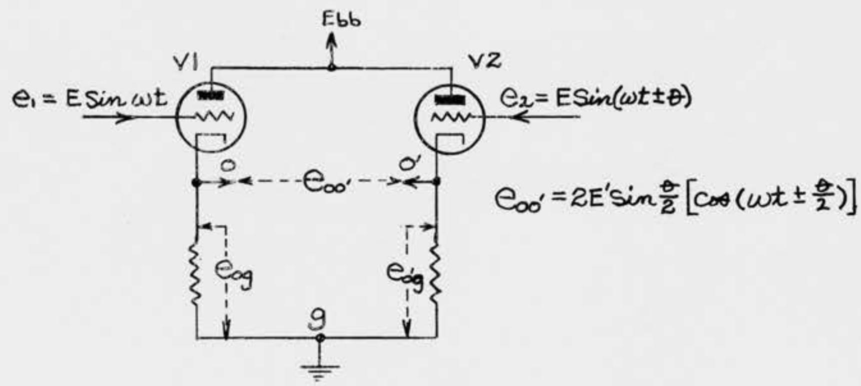

Figure 42. Basic Cathode Follower Difference Amplifier

Since the cathode follower circuits will not contribute unequal phase shift due to symmetry, the output voltages of each circuit would be

$$e_{og} = E' \sin \omega t \qquad (51)$$

$$e_{o'g} = E' \sin(\omega t \pm \theta) \qquad (52)$$

The net output voltage measured between the cathodes would be the difference between e_{og} and $e_{o'g}$.

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[1]George E. Valley, Jr. and Henry Wallman, Vacuum Tube Amplifiers, pp. 441-454.

[2]Samuel Seely, Electron-Tube Circuits, pp. 113-117.

$$e_{oo'} = e_{og} - e_{og'} = E'\sin\omega t - E'\sin(\omega t \pm \theta) \qquad (53)$$

$$= E'[\sin\omega t - \sin(\omega t \pm \theta)] \qquad (54)$$

Applying the trigonometric identity, sin A - sin B = 2·cos 1/2(A+B)·sin 1/2(A-B), equation (53) becomes

$$e_{oo'} = 2E'\sin\tfrac{1}{2}(\omega t - \omega t \pm \theta)\cos\tfrac{1}{2}(\omega t + \omega t \pm \theta) \qquad (55)$$
$$= 2E'\sin\left(\pm\tfrac{\theta}{2}\right)\cdot\cos\left(\omega t \pm \tfrac{\theta}{2}\right)$$

But $\sin \pm\frac{\theta}{2} = \sin\frac{\theta}{2}$, so

$$e_{oo'} = 2E'\sin\tfrac{\theta}{2}\left[\cos\left(\omega t \pm \tfrac{\theta}{2}\right)\right] \qquad (56)$$

Equation (56) can be interpreted to mean that the output voltage $e_{oo'}$ will be of sine wave form but having a peak or rms amplitude that is proportional to $\sin\frac{\theta}{2}$. In other words, when the two equal input voltages e_1 and e_2 differ in phase by an angle of $\pm\theta$, an a-c meter connected across the points oo' would deflect in proportion to $\sin\frac{\theta}{2}$. A maximum deflection of the meter would result when e_1 and e_2 differ in phase by 180° at which time $\sin\frac{\theta}{2} = 1$, and furthermore, this deflection would be twice as much as that produced by either input voltage alone.

These observations reveal a method by which this circuit can be used to measure phase shift between two sine wave voltages. If e_1 alone was applied and its amplitude adjusted until half-scale deflection was produced on a meter connected between the cathodes, and similarly e_2 was applied alone and varied until it also produced half-scale deflection of the meter, then when both voltages were applied simultaneously, the meter would deflect in proportion to $\sin\frac{\theta}{2}$ where full-scale deflection corresponded to 180° and zero deflection to 0°. By suitable calibration of the meter scale or by drawing a calibration curve, the circuit of Figure 41 then becomes a direct reading phase measuring instrument.

Circuit Description

The complete circuit diagram of the instrument appears in Figure 39 and incorporates several additional features to make it especially applicable to measurements on audio amplifiers and also to make it more self-contained and convenient to use. The input voltage to a typical audio amplifier is usually quite small, and the output voltage from a low impedance secondary winding on an output transformer is usually only a few volts at the mid-range frequencies and much lower at the high and low frequencies where the response falls off. These low voltages require additional amplification to be built into the phase meter so that sufficient voltage will be available to operate the difference amplifier properly. In Figure 39, a cathode to cathode voltage on tubes V3 and V4 of 8 volts was arbitrarily chosen to correspond to full-scale reading of the meter M1. This means that each half of the difference amplifier circuit must contribute one-half of this voltage or 4 volts. Due to the low value of the cathode resistances, waveform distortion starts to result when the grid signal voltage on V3 and

V4 exceeds 8 volts. This distortion might be reduced by increasing the cathode resistances R14 and R16 but this would increase the output impedance of the circuit with a resultant loss of high-frequency response. If the voltage from the difference amplifier was chosen at too low a value, then trouble would be experienced from noise interference.

The amplifiers in the phase meter must pass a considerably wider band in comparison to the operating range of the instrument if error due to internal phase shift is to be avoided. The high-frequency response of the amplifiers extends at least 5 decades above the rated operating range and the low frequency response extends 2 decades below the rated range.

The potentiometer R15 is for the purpose of balancing the d-c cathode voltages of tubes V3 and V4. By placing the switches S1 and S2 in the OFF position, no signal voltages will be applied and R15 can then be adjusted for a zero reading on the meter M1.

The circuit composed of R3, C1-5, and S5 determines the sign of the measured phase shift angle, i.e., whether e_2 leads or lags e_1. When S5 is turned to one of the $\Delta\theta$ positions, additional phase lag is introduced into channel one. If the meter reading increases, then e_2 leads e_1, and if it decreases e_2 lags e_1.

A crystal diode meter rectifier circuit was used in preference to other types because of its simplicity, ease of construction, and excellent response to the higher frequencies without special compensation. A meter movement of 50 μa full-scale sensitivity was used for the meter M1 so that the metering circuit would have a relatively high impedance and thus not appreciably load the difference amplifier circuit.

It will be noticed that an electronically regulated power supply has been incorporated to stabilize the B supply voltage for all the tubes. This helps to improve accuracy of calibration and to minimize drift. By returning the center tap of the 6.3 volt filament winding to a divider on the B supply, a positive voltage of approximately 50 volts is applied to the filament circuits which results in considerably less residual noise.

Calibration

If the deflection of the meter M1 was perfectly linear with respect to the output voltage from the difference amplifier, then the meter scale could be directly calibrated in proportion to the function sin $\frac{\theta}{2}$. Unfortunately, due to the slight nonlinearity of the crystal diodes, this direct approach to the problem of calibration could not be employed until the meter scale was recalibrated as a function of the input voltage to the a-c metering circuit.

The voltage calibration of the meter scale was accomplished by temporarily disconnecting the metering circuit from the difference amplifier and connecting it to a source of known voltage as shown in Figure 43.

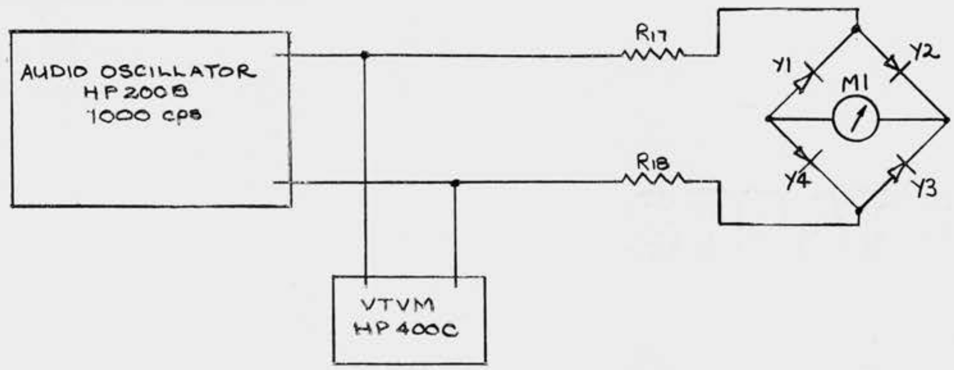

Figure 43. Voltage Calibration Circuit

Since 8 volts from the oscillator was required to produce full-scale deflection of the meter M1, other values of known voltage were applied and the reading of M1 recorded for each voltage. From this data the calibration curve of Figure 44 was drawn.

The next step in the procedure of calibration was to determine how much the meter movement should read for various values of phase shift θ. Since the actual voltage delivered to the meter circuit would be $E_{oo'} = 8 \sin\frac{\theta}{2}$, the corresponding reading of the meter for each value of θ selected was determined by referring to the voltage calibration curve of Figure 44. Values of θ were selected for every 10° between 0° and 180° and the corresponding meter readings were plotted to give the phase shift calibration curve of Figure 41.

The accuracy of calibration was checked by setting up the test circuit shown in Figure 45 and adjusting the variable resistance to give a known phase shift between e_1 and e_2.

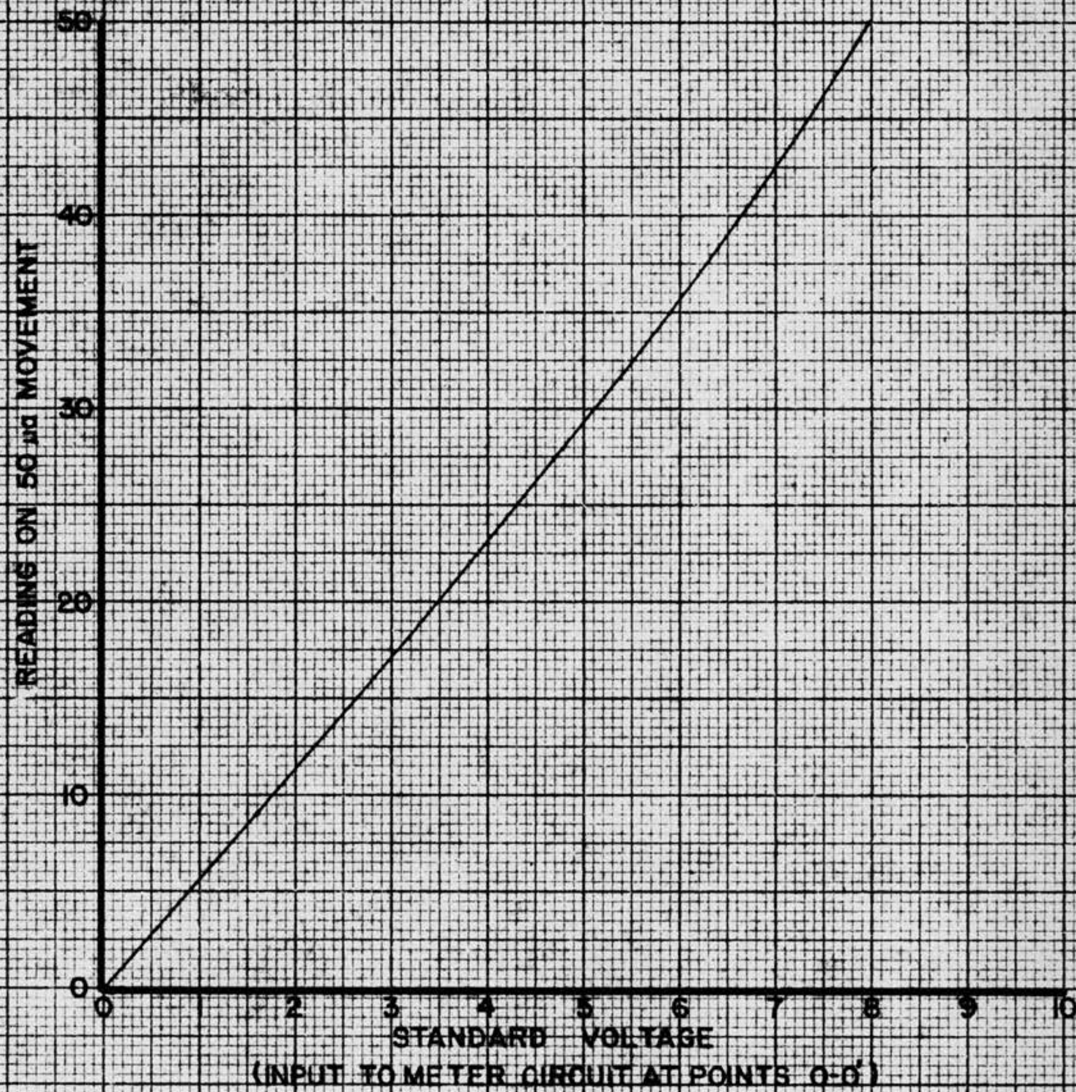

FIGURE 44. VOLTAGE CALIBRATION CURVE FOR THE A-C METER CIRCUIT IN THE PHASE METER.

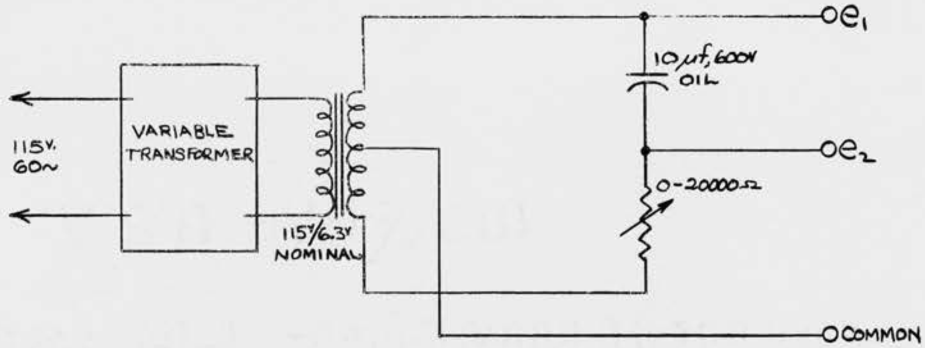

Figure 45. Test Circuit for Checking Calibration of the Phase Meter

To prevent excessive loading of the test circuit, the input potentiometers in the phase meter were temporarily disconnected and 0.5 megohm units connected in their place. The calibration checks made with the aid of the test circuit proved the calibration procedure to be satisfactory.

Connections to an Amplifier

It will be noted that the input potentiometers of the phase meter are 200 ohms each. They were made low deliberately in order to minimize noise pickup in the input circuits. However, a little judgment much be exercised when connecting the inputs to an amplifier. When the input terminals of channel no. 2 are connected across the usual speaker output leads of an amplifier no difficulty will be experienced from excessive loading. However, if the amplifier under test has an output load impedance rating of 50 to 600 ohms, then a resistance of 100 - 10,000 ohms should be placed in series with the input lead to the phase meter to prevent loading.

Input no. 1 of the phase meter may be connected to the input of the amplifier test in one of the three ways shown in Figure 46 depending upon the equipment and its requirements.

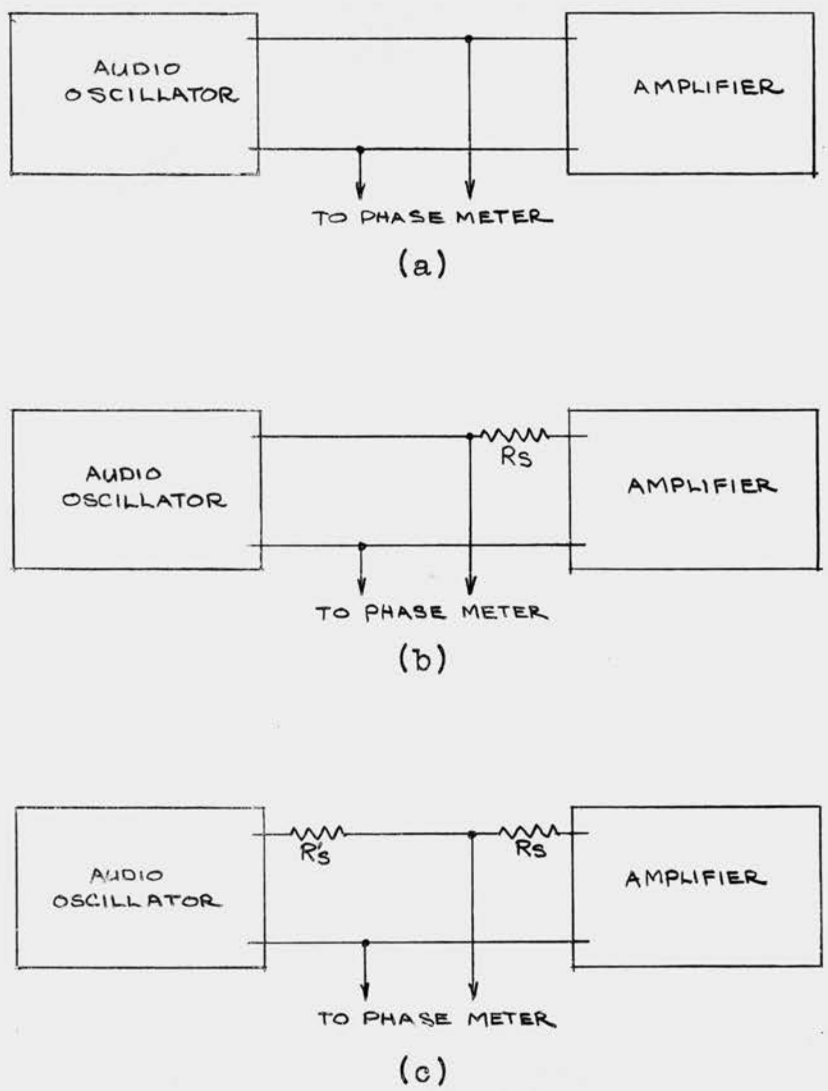

Figure 46. Connecting the Phase Meter to an Amplifier

If connected as shown in Figure 46(a) and the amplifier has a high impedance input, the 200 ohm input resistance of the phase meter will not only be the load on the oscillator but also will shunt the input capacity of the first amplifier tube. Any effect the input capacity would normally have on the high-frequency phase response of the amplifier would not be noticed when shunted by the 200 ohm potentiometer. Therefore, when the amplifier has a high input impedance, the circuit of Figure 46(b) or 46(c) is to be preferred. R_s would be made equal to the internal resistance of the driving source normally used with the amplifier. If, however, the amplifier has a low input impedance of 50 - 600 ohms, the connection of Figure 46(a) would be suitable. The connection shown in Figure 46(c) should be used when the amplifier and oscillator both have high impedance circuits or when the oscillator delivers too much signal voltage.

Operating Instructions

The instructions for placing the phase meter in operation can best be presented in the form of definite steps as follows:

1. Connect the phase meter to the amplifier under test in a suitable manner.

2. Before turning on the power and amplifier, the following controls should be in the position indicated.

Input switch no. 1	OFF
Input switch no. 2	OFF
Gain no. 1	OFF
Gain no. 2	OFF
$\Delta\theta$	0

3. Turn on the power and place the amplifier in operation.

4. After the phase meter has warmed up for a few minutes, adjust DC BALANCE for minimum deflection of the meter which should be zero.

5. Operate INPUT SW 1 to USE and advance GAIN 1 until meter reads one-half scale. One-half scale on the meter is 23 due to the nonlinearity present. Turn INPUT SW 1 to OFF.

6. Operate INPUT SW 2 to USE and advance GAIN 2 until meter again reads one-half scale or 23.

7. Operate both input switches to USE and read the meter.

8. Refer the meter reading to the calibration curve of Figure 41 to obtain the phase shift angle.

9. If the polarity of the phase shift angle is not known, advance the $\Delta\theta$ control to 1 and repeat steps 5, 7, and 8. If the new reading is larger than the original reading then e_2 leads e_1, and if the new reading is smaller then e_2 lags e_1. If the new reading is no different from the original reading then $\Delta\theta$ should be advanced to a higher number.

While the use of this phase meter is strictly limited to phase measurements on sine wave voltages, experience has shown that it is a very satisfactory low cost instrument for the purpose of testing audio amplifier performance.

CHAPTER IX

AMPLIFIER TESTING AND IMPROVEMENT OF FREQUENCY RESPONSE

General

In order to further demonstrate the use of asymoptotic plots to the problem of analysis and to extend this method to the correction of response of an audio amplifier, the amplifier whose circuit diagram is shown in Figure 31[1] was constructed and experimental data obtained to compare with the results of the theoretical analysis in Chapter VII. The laboratory testing of the experimental amplifier also offered an opportunity to demonstrate the application of the phase meter which was described in Chapter VIII.

The first step in the procedure of the laboratory tests was to determine the amplitude response and phase shift of the original circuit of Figure 31, and to inspect those results for the purpose of incorporating certain improvements in design at the least expense; second, to make several simple modifications to the original circuit so that negative feedback could be applied; third and last, to design and install a phase compensated negative feedback loop as a final approach to improving the frequency response of the amplifier. Actually, very few physical changes were made in the amplifier circuit, the object being to accomplish the improvements in performance with-

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[1]*Cf. ante.*, p. 49.

out resorting to a major redesign and expensive replacements.

The equipment necessary for thorough laboratory testing of audio amplifiers should include at least the following instruments:

1. Audio oscillator with a range of 10 cps - 100 kc and a waveform distortion of less than 0.5 per cent.

2. Vacuum tube voltmeter with a usable frequency range comparable to the oscillator.

3. Oscilloscope.

4. Phase meter.

Other instruments which are desirable but not absolutely necessary for the usual performance tests on audio amplifiers are:

1. Noise and distortion meter.

2. Square wave generator.

3. Intermodulation analyzer.

4. Wave analyzer.

The equipment which was available for the testing of the experimental amplifier included:

1. Oscillator, Hewlitt-Packard Model 650A.

2. Vacuum tube voltmeter, Hewlitt-Packard Model 400C.

3. Oscilloscope, DuMont Model 304H.

4. Phase meter as described in Chapter VIII.

A photograph of the equipment setup and the experimental amplifier are shown in Figure 47.

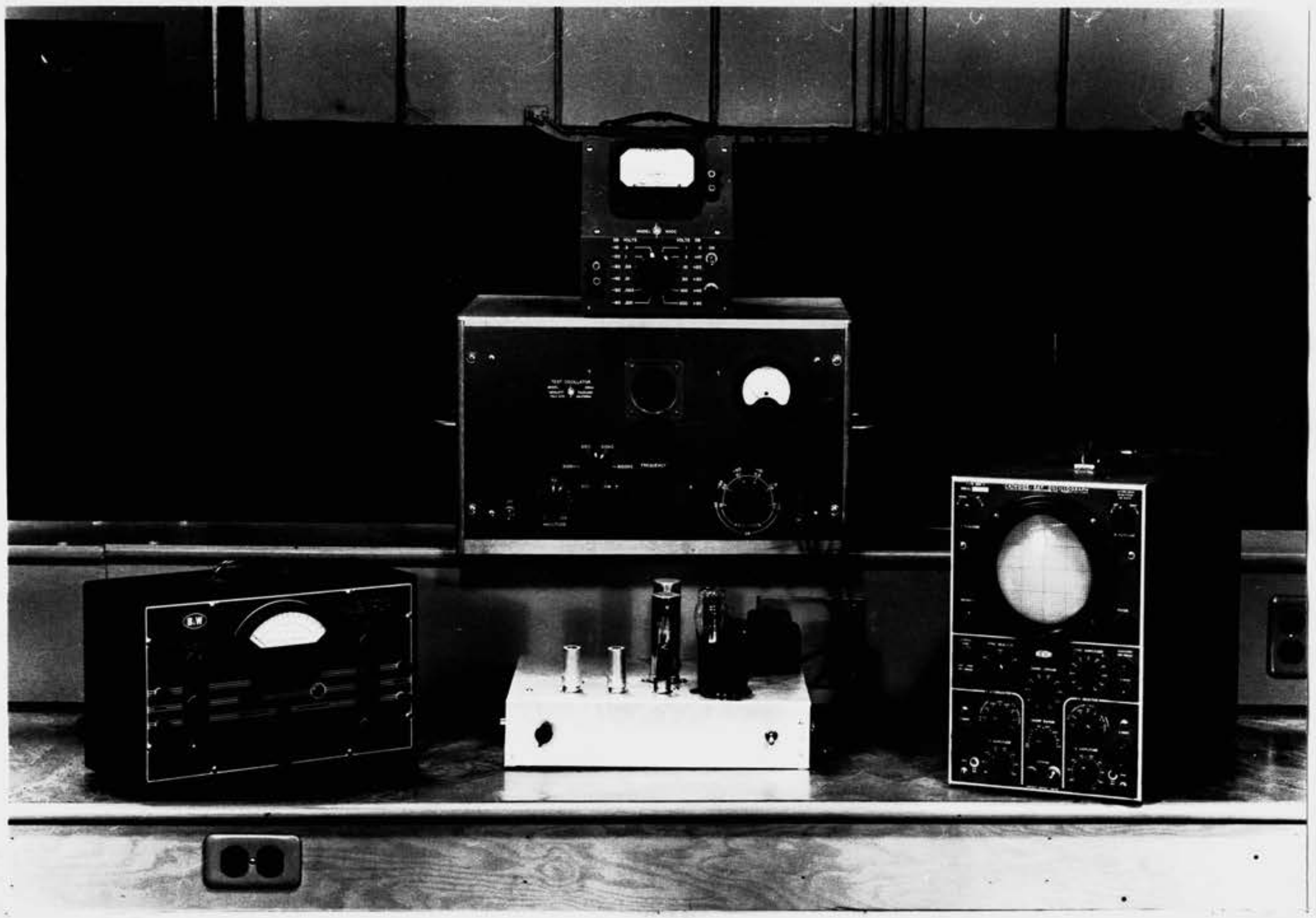

Figure 47. The Equipment Setup and Experimental Amplifier.

Amplitude Response and Phase Shift Measurements

The block diagram of Figure 48 shows the connections of the test equipment to the experimental amplifier.

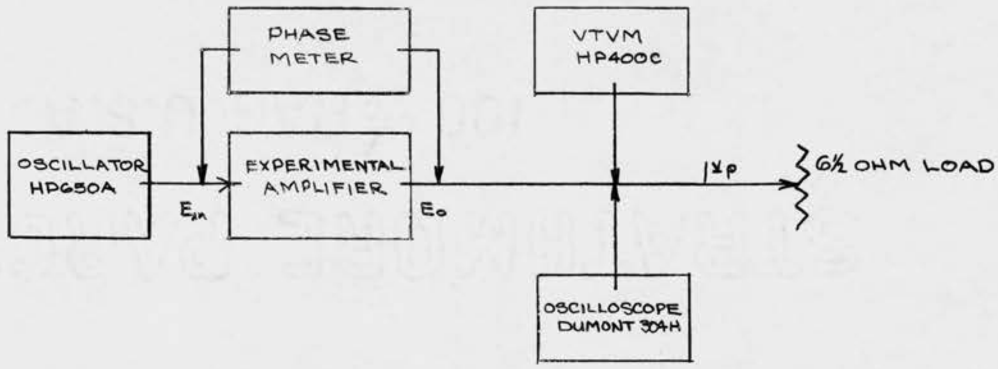

Figure 48. Connections of the Test Equipment to the Experimental Amplifier

On each response test, the built-in attenuator in the oscillator was adjusted for one volt output from the amplifier at the mid-frequency range and the corresponding input voltage to the amplifier was held constant as the frequency was then varied throughout the desired range. At each frequency selected, the output voltage from the amplifier was read on the vacuum tube voltmeter and converted to decibel gain by the equation $db = 20 \text{Log} E_o/E_{in}$.[2] Also, at each frequency the phase shift of the output voltage with respect to the input voltage was determined with the phase meter.

The first test was conducted with the amplifier circuit in its original form as shown in Figure 31 and the results of this test are shown on the graphs of Figures 49, 50, 51, and 52 along with plots of the calculated response which was obtained from the analysis of Chapter VII.

[2]Seely, op. cit., p. 56.

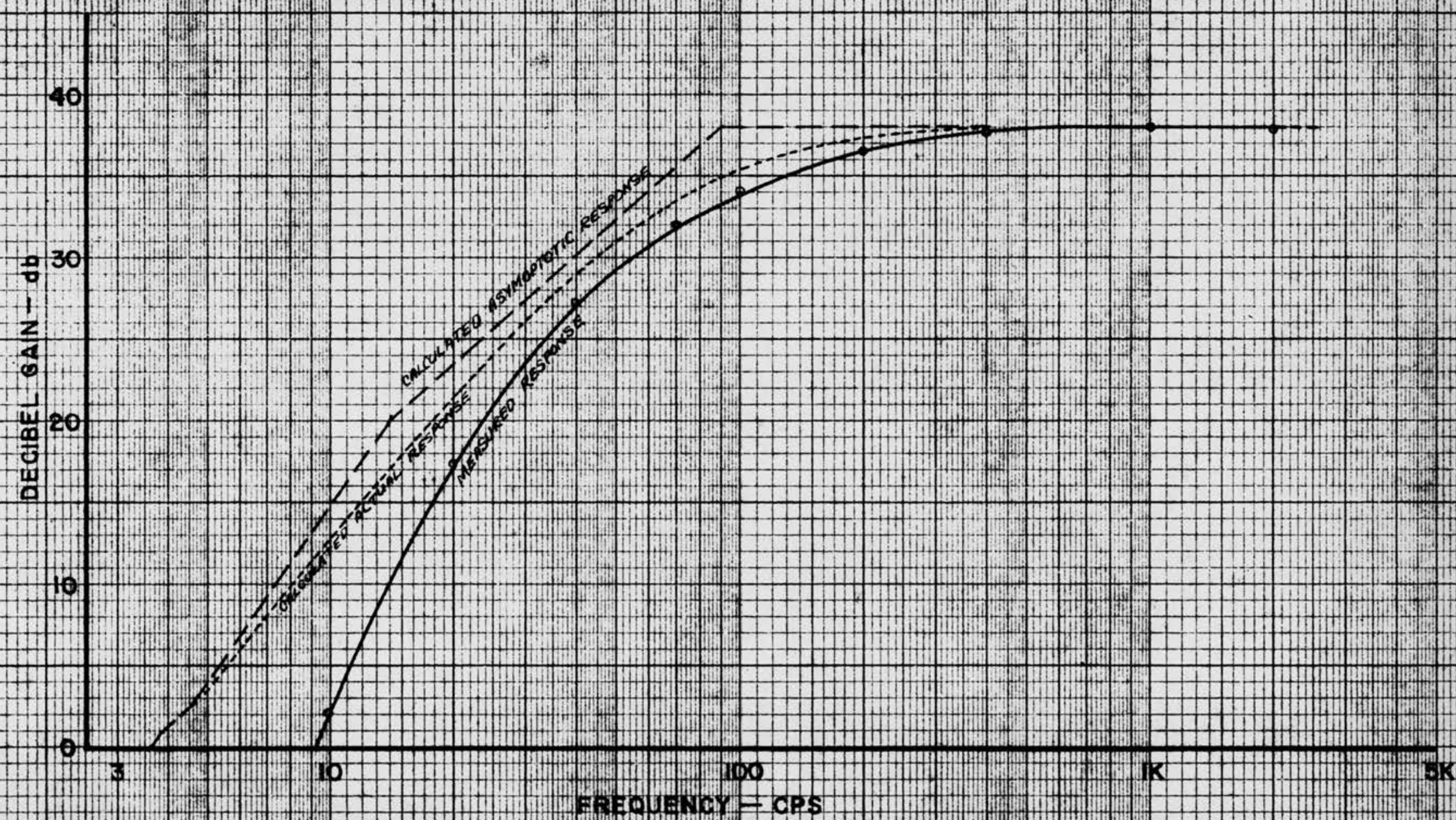

FIGURE 49. LOW-FREQUENCY RESPONSE OF THE ORIGINAL AMPLIFIER CIRCUIT

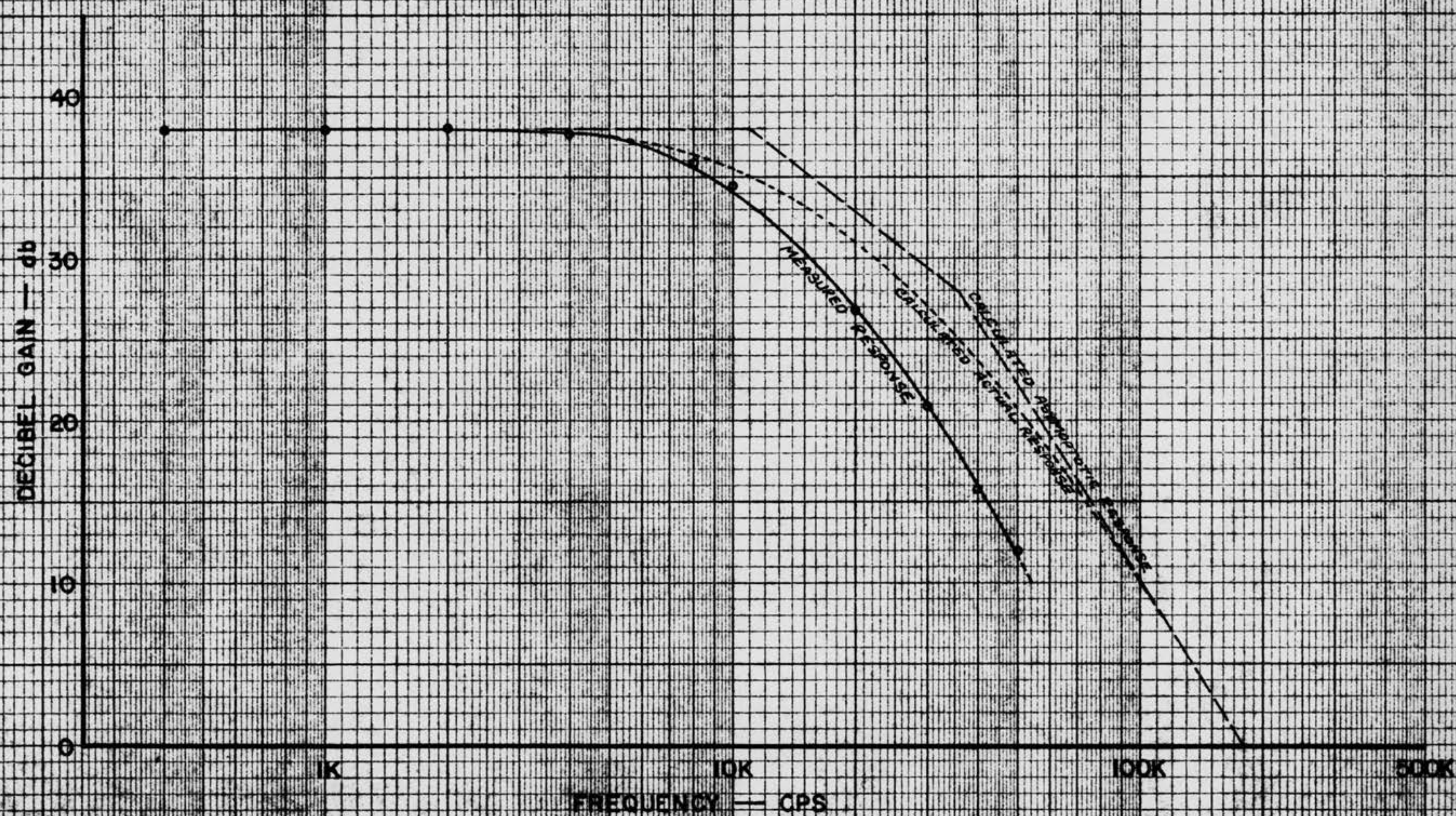

FIGURE 50. HIGH-FREQUENCY RESPONSE OF THE ORIGINAL AMPLIFIER CIRCUIT.

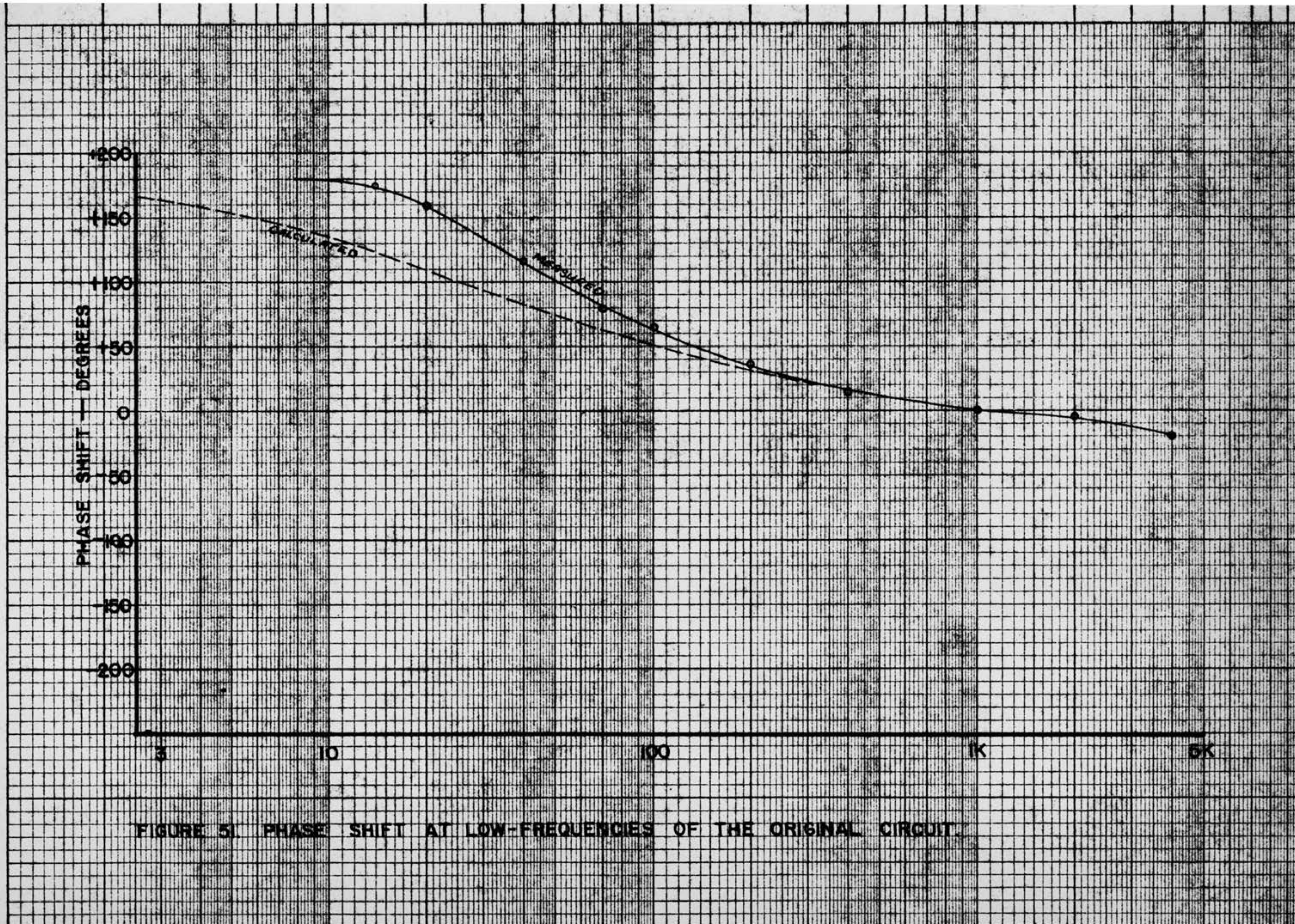

FIGURE 51. PHASE SHIFT AT LOW-FREQUENCIES OF THE ORIGINAL CIRCUIT.

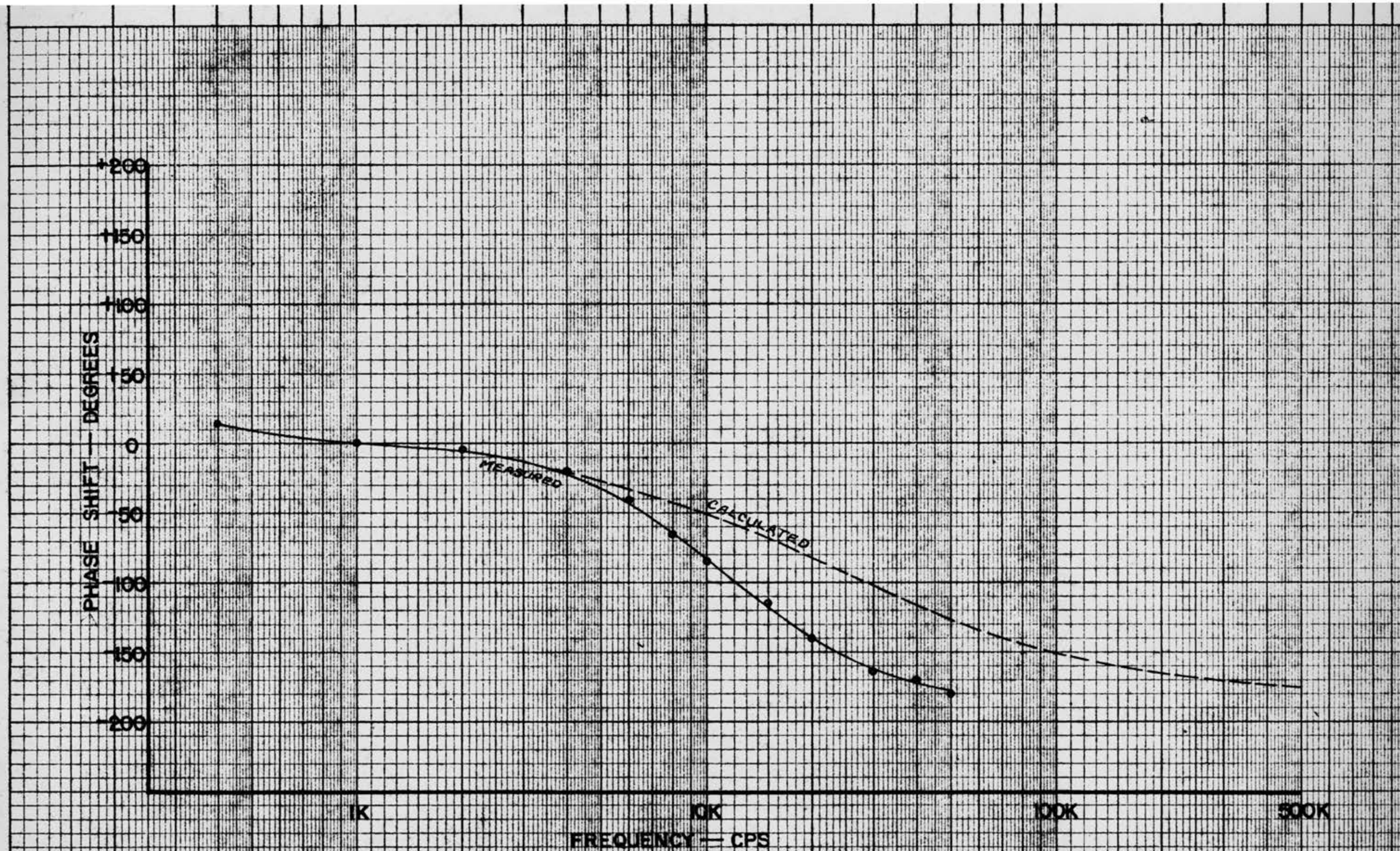

FIGURE 52. PHASE SHIFT AT HIGH-FREQUENCIES OF THE ORIGINAL AMPLIFIER CIRCUIT.

An inspection of the graphs reveal that the calculated response and measured response are in reasonable agreement throughout the range of 50 - 15000 cps. Outside this range the deviation between the two increases considerably due, no doubt, to the neglect of certain circuit effects which in reality influence the response to some extent depending upon circuit values. This, in particular, refers to the effect of the screen circuit impedance on low-frequency response and the effect of the primary capacity of the output transformer on the high-frequency response. These erros in no way defeat the objective of the calculated analysis since it does give an excellent prediction of the amplifier response and was accomplished without long and laborious calculations.

The calculated mid-range gain was 45 db,[3] however, the calculated response curves, shown in Figures 49 and 50, were drawn to the same mid-range level as the measured response so the shape of the curves could be compared more readily.

The greatest error between the calculated and measured data appears in the phase shift of the amplifier as presented by the curves of Figures 51 and 52. In the vicinity of the corner frequency of an asymoptotic curve, the phase shift of the output is changing quite rapidly. This means that if a small error existed in the calculation of the asymoptotic plot, a correspondingly larger error would exist in the phase shift plot. But again, the calculated

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[3] _Cf. ante._, p. 52.

data gives the engineer an approximate picture of the phase shift with a minimum of effort and if he will but realize that the measured phase shift will be a little more than is indicated,[4] the calculated data will be very useful in designing feedback networks for amplifiers. The error in the calculated phase shift data also emphasizes the need for a simple low cost phase meter such as the one described in Chapter VIII.

Before going into the methods of improving the performance of the amplifier, it is interesting to point out how much uncompensated negative feedback can be applied to the original circuit before oscillation occurs. A closer inspection of Figures 49, 50, 51, and 52 will show that if negative feedback was applied in the manner shown in Figure 53 such that $20 \text{Log} 1/\beta = 20\angle 0^\circ$ db, then the plot of $1/\beta$ would intersect the amplifier amplitude plot at 15 and 35,000 cps where the phase margin at each point is less than 10°.

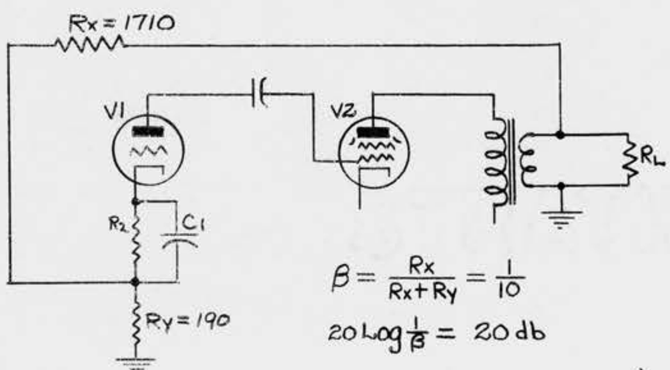

Figure 53. Uncompensated Feedback Network

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[4]Due to neglecting the effect of the screen by-pass condenser at low frequencies and the transformer shunt capacity at high frequencies.

At each point of intersection [$A\beta = 1$] a very unstable condition would exist and very likely oscillation would be the result. Actually, when the circuit of Figure 53 was applied to the amplifier with the values indicated, oscillation did occur at low and high frequencies simultaneously. Of course, if phase compensation was incorporated in either the amplifier or the feedback loop in such a manner that a reasonable phase margin of at least 30° existed at the points where $A\beta = 1$, then oscillation could be avoided.

Improvement of Frequency Response

The bandwidth between the -3 db points of the measured amplifier response is from 130 - 9200 cps which is much too limited for good reproduction of music. By referring to the graphs of Figures 32 and 34, it can be seen that the major factor contributing to poor low-frequency response is the low primary inductance of the output transformer, and at the high-frequencies it is the excessive shunt capacity in the 6AU6 circuit which produces an early drop in high-frequency response.

Improving the low-frequency response by increasing the transformer inductance would require replacement which is to be avoided, if possible, since a suitable transformer would cost several dollars. The high-frequency response could be improved by choosing a lower resistance for the d-c plate load resistor on the 6AU6. This modification would be entirely practical, however, it would also required accompanying changes in values of the cathode bias resistor and the series resistor in the screen circuit.

Negative Feedback as a Means of Improving the Frequency Response

Since it was desired to apply negative feedback to derive the benefits of distortion and noise reduction, it was decided that it should be investigated as a means of correcting the frequency response at the same time. Referring again to Figures 49 and 50, it can be seen that if a feedback factor of $20 \text{Log} 1/\beta = 20$ db was applied that the frequency response should be practically uniform from about 20 - 25,000 cps.[5] However, the discussion in the previous section pointed out that oscillation resulted when $20 \text{Log} 1/\beta$ was made equal to $20\angle 0^{\circ}$ db so that phase compensation would have to be applied to either the amplifier, the feedback loop, or both. By modifying the amplifier to the extent of replacing the 6AU6 cathode condenser C1 with a 250 μf unit, replacing the coupling condenser C3 with one of 0.1 μf, and replacing the grid resistor R5 with a resistance of 1 megohm, the low-frequency amplitude response was only slightly improved, but the phase margin in the vicinity of 10-20 cps was increased considerably. The low-frequency response and phase shift curves of the modified circuit are plotted in Figures 54 and 55 and show that at the point of intersection between $1/\beta$ and A [$A\beta = 1$] the phase margin is 50°, which is sufficient to prevent oscillation at the low-frequencies.

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[5] *Cf.* *ante.*, p. 44.

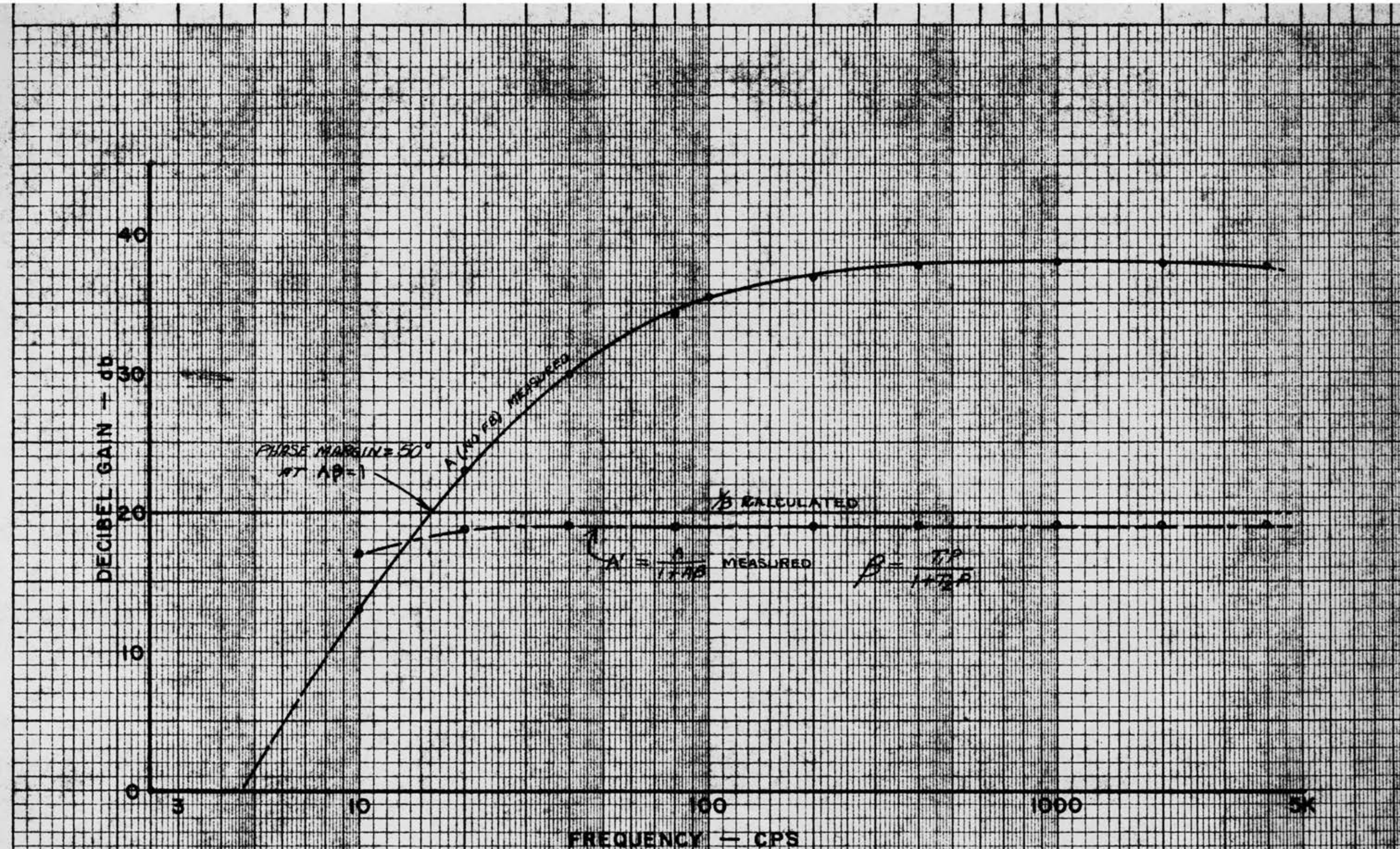

FIGURE 54. LOW-FREQUENCY RESPONSE OF MODIFIED AMPLIFIER SHOWING THE EFFECT OF NEGATIVE FEEDBACK.

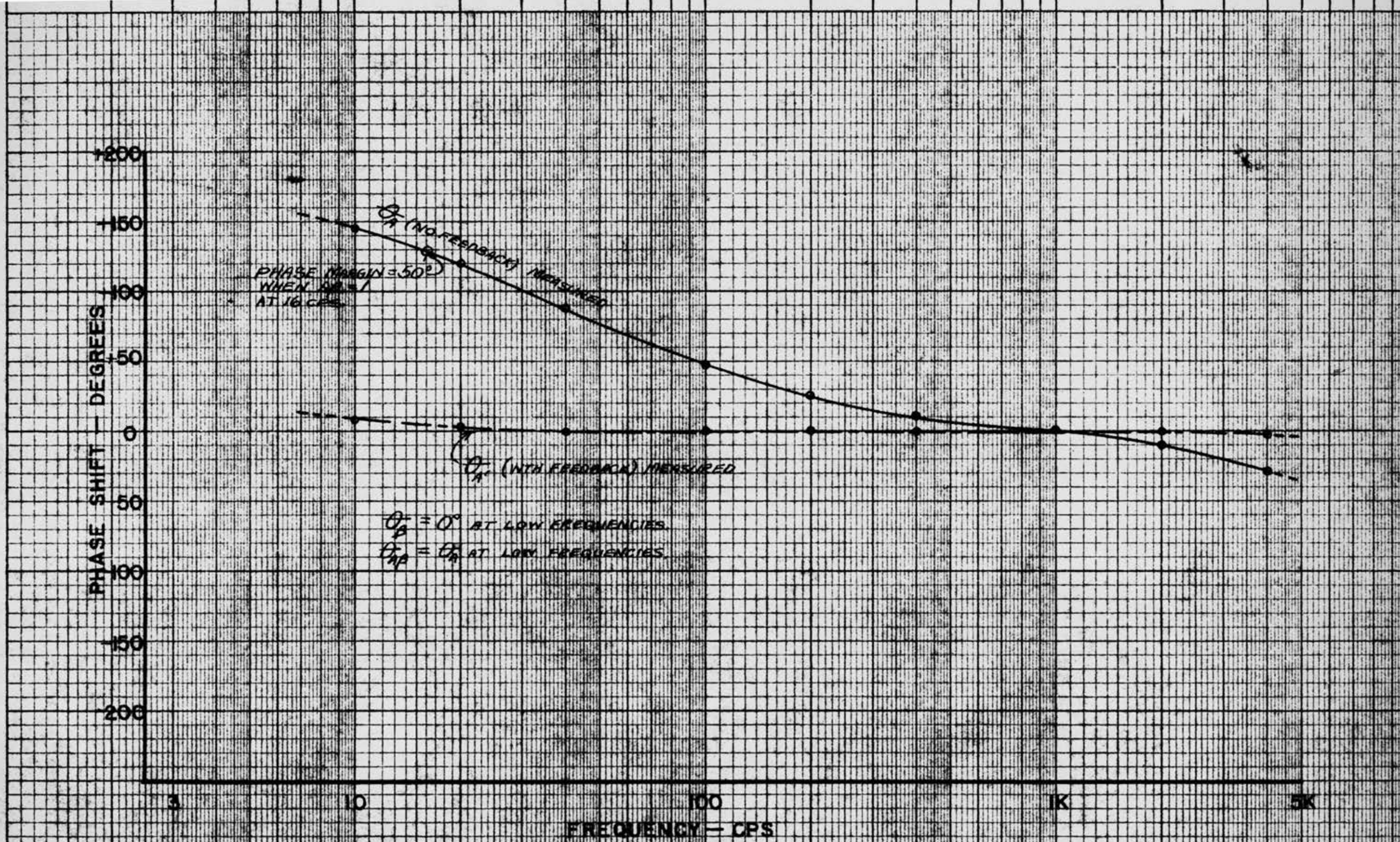

FIGURE 55. PHASE SHIFT OF MODIFIED AMPLIFIER AT LOW FREQUENCIES SHOWING THE EFFECT OF NEGATIVE FEEDBACK.

FIGURE 56. HIGH-FREQUENCY RESPONSE OF MODIFIED AMPLIFIER SHOWING THE EFFECT OF NEGATIVE FEEDBACK.

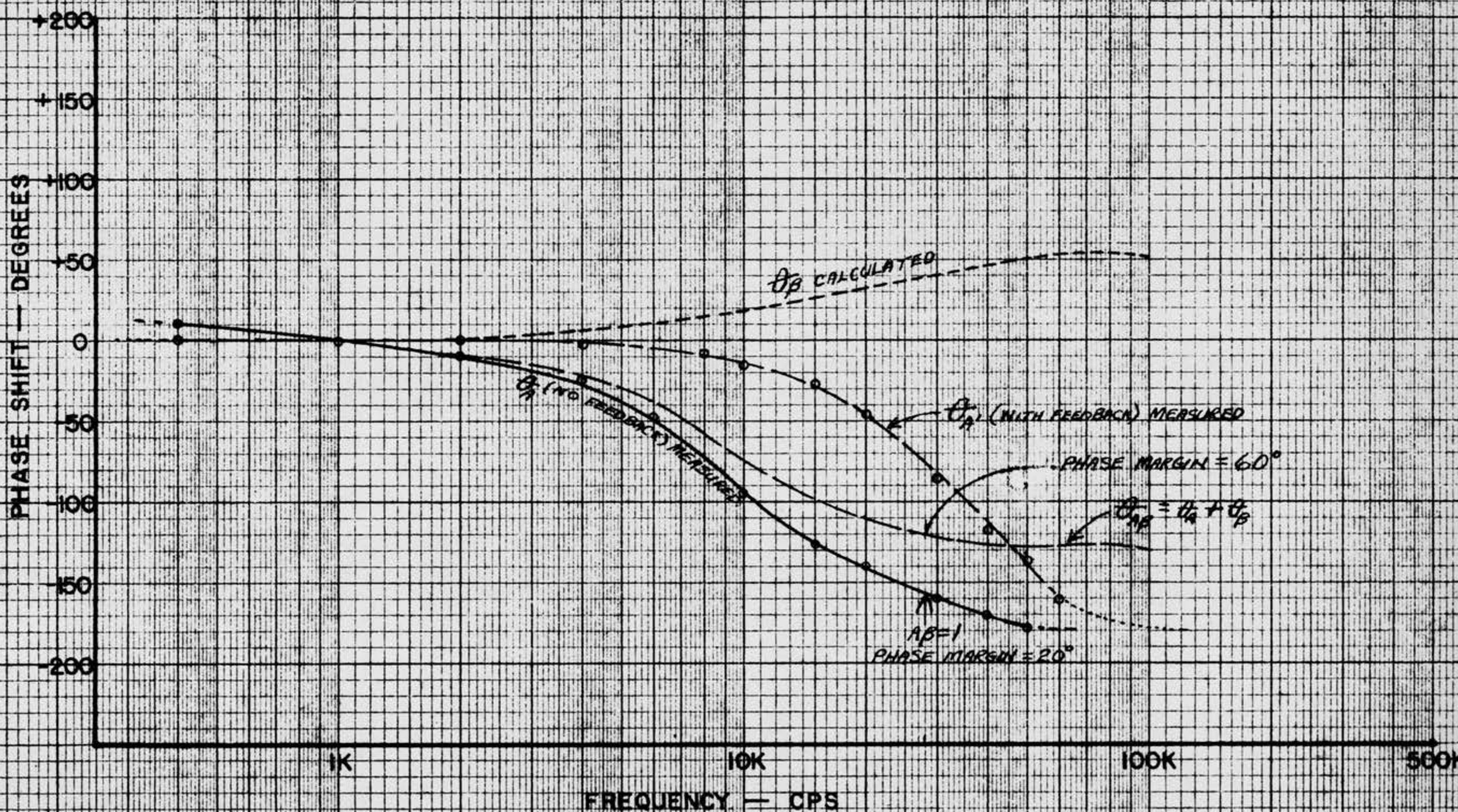

FIGURE 57. PHASE SHIFT OF MODIFIED AMPLIFIER AT HIGH FREQUENCIES SHOWING THE EFFECT OF NEGATIVE FEEDBACK.

At the high-frequency end of the response as shown in Figures 56 and 57, the phase margin at the point $A\beta = 1$ changed very little, meaning that the amplifier would still tend to be unstable if $20 \text{Log} 1/\beta = 20\underline{/0^{\circ}}$ db. By shunting R_x in the feedback loop with a small condenser as shown in Figure 58, the function β is given a leading phase angle at high-frequencies to offset part of the lagging phase shift of the amplifier, thereby decreasing the phase angle of A and increasing its phase margin to a point of stability.

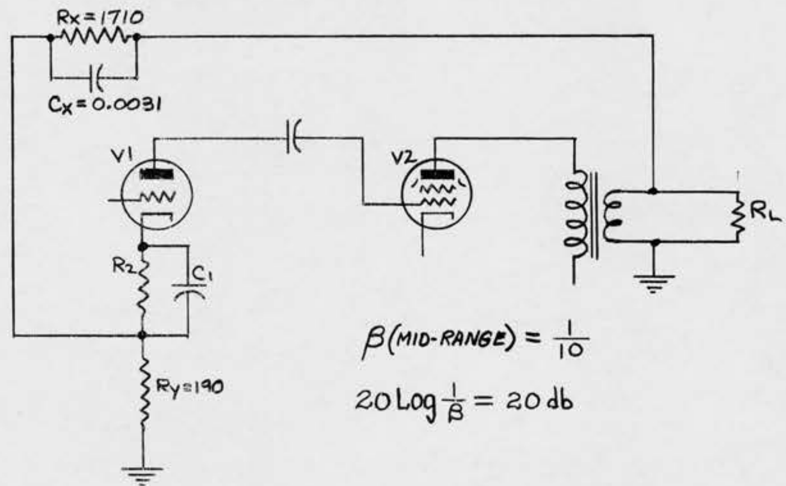

Figure 58. Phase Compensated Feedback Loop to Prevent Oscillation at High-Frequency

An R-C circuit of the type used in the feedback loop has for a transfer function

$$\beta = \frac{R_y}{R_x + R_y} \cdot \left[\frac{1 + R_x C_x P}{1 + \left(\frac{R_x R_y}{R_x + R_y} \right) C_x P} \right] = K \cdot \frac{1 + T_1 P}{1 + T_2 P} \qquad (57)$$

where

$$f_1 = \frac{1}{2\pi R_x C_x} \tag{58}$$

$$f_2 = \frac{1}{2\pi\left(\frac{R_x R_y}{R_x + R_y}\right)C_x} \tag{59}$$

The asymoptotic response and phase shift typical of this function is shown in Figure 59.

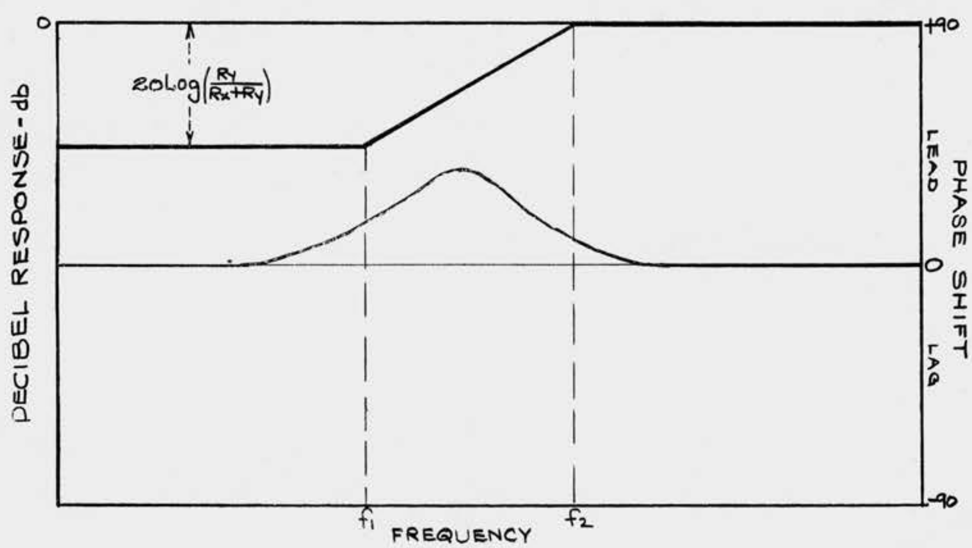

Figure 59. Typical Response of the Function

By choosing a frequency for f_1 that is slightly higher than the intersecting point of $1/\beta$ and A [see Figure 56], the increase of negative feedback at high frequencies will have little effect on the amplitude response of the amplifier, but the leading phase shift due to the addition of C_x will accomplish the goal of increasing the phase margin of $A\beta$ in the region of $A\beta = 1$. In this case a frequency of 30000 cps was chosen for f_1 and thus the necessary value for C_x can be determined from equation (58) as

$$C_x = \frac{1}{2\pi f_1 R_x} = \frac{1}{(6.28)(30000)(1710)} = 0.0031\ \mu f \tag{60}$$

The other corner frequency f_2 was calculated to be

$$f_2 = \frac{1}{2\pi\left(\frac{R_x R_y}{R_x + R_y}\right)C_x} = \frac{1}{(6.28)\left(\frac{1710 \cdot 190}{1710 + 190}\right)(0.0031 \cdot 10^{-6})} = 300\ \text{Kc} \tag{61}$$

From these calculations, the curves for the feedback loop and the resultant phase shift curve for $A\beta$ were plotted on the graphs of Figures 56 and 57. The phase shift curve of $A\beta$ now shows a phase margin of 60° to exist at the point where $A\beta = 1$, which is more than enough to prevent oscillation at high-frequency.

To complete the laboratory test on the experimental amplifier, the compensated feedback loop as shown in Figure 56 was installed and a final check made of the frequency response. The results of this test is plotted as the A' curve in Figures 54 and 56 and is shown to be very close to the expected response which should follow the lower curve on the graph. The bandwidth of the A' amplifier response curve is now 10 - 30000 cps which would provide excellent reproduction of live music or tape recordings. The reduction in the gain of the amplifier from 38 to 19 db means that more signal voltage must be applied to the amplifier input in order to develop the same output. The laboratory measurements showed that with feedback the amplifier input must be 0.13 volts in order to develop the output of 1 volt. The value of 0.13 volt is still low enough to allow the amplifier to be driven directly from a crystal phono pickup or from the preamplifier of the popular variable reluctance pickup.

While a reduction of distortion was not a primary objective of this study, it should always be considered when attempting to improve the performance of an audio amplifier. The effect of negative feedback in reducing distortion and noise of amplifiers is well known and in this case there was no exception. As measured by a Barker and Williams distortion meter, the total noise and distortion was reduced from 5 down to 1 per cent at mid-range frequencies due to the application of the negative feedback.

Many other tests and demonstrations involving compensation and equalization of amplifiers could be made, however, the simple procedures presented in this chapter indicate the possibilities of problems which can be easily and satisfactorily solved with the aid of asymoptotic plots of network transfer functions.

CHAPTER X

SUMMARY AND CONCLUSIONS

Summary

The primary objective of this study was to present procedures and methods which would aid the practicing engineer in the problems of practical design, analysis, and testing of audio amplifiers. This objective was accomplished in a two-fold manner by first showing the application of asymoptotic plots to the problem of design and analysis, and second, by describing and demonstrating the use of a low cost direct reading phase meter, which will allow a more complete laboratory analysis of amplifiers to be made quickly and with a minimum of expensive equipment.

It is desirable that the engineer should study the complete and more complex equations related to amplifier networks in order that he understand more clearly how the various circuit parameters affect the amplifier performance. However, to apply these complex equations to a practical design problem requires a great deal of time and labor which does not seem warranted when the laboratory tests of the finished model do not exactly agree with the expectations. A varying disagreement between calculated and measured data for amplifiers is to be expected, not always because of limitations in the mathematical treatment of amplifier action, but mainly because of a rather wide variation in values of circuit components and tube constants due to manufacturing tolerances and differences in operating potentials. Since these tolerances do exist, a calculated design must be followed by laboratory tests in which the final modifications to the original design are made in order that the amplifier shall meet the required specifications. For this reason, a great deal of time can be saved by applying the approximate graphical solution that has been prescribed in the preceding chapters. Also, the use of the phase meter in the laboratory tests will save the engineer time when feedback is to be applied to the amplifier under test.

Difficulties Encountered in Amplifier Design

The major difficulty facing the engineer when he prepares a design of an amplifier is that complete and exact information on component parts and tubes is not usually available. Of course, in the case of resistors and condensers, precision units can either be purchased or selected from a large stock. However, when the question arises concerning the values for tube and transformer constants, the problem is more perplexing. The information presented in tube data manuals on plate resistance, mutual conductance, and amplification factor, rarely fits the individual problem of design, especially when voltage amplifier tubes are involved where the plate and screen are operated at reduced potentials. In this case, the engineer has the alternative of estimating values for the tube constants or subjecting a number of tubes to exhaustive tests in order to arrive at an average figure for the constants.

Information concerning the internal impedances of transformers is not ordinarily available so that measurements must be made before the amplifier design progresses. Some of the questions that arise from measurements made on audio transformers are more completely discussed in Appendix B, however, it should be said at this point that due to the nonlinearity of transformers, exact values of impedances cannot always be assigned, but instead average figures must be used.

This discussion may make it appear that attempting a calculated design or analysis is futile, however, such is not the case. An intelligent use of the available data will produce a set of data that is approximately correct so that minor modifications of the test model will yield the desired results.

Recommendations for Further Study

The procedures and methods outlined in this thesis have been proven beyond any doubt to be entirely satisfactory in solving the problems associated with the analysis of audio amplifiers. However, the transfer functions which were developed do not embrace all of the circuits and variations encountered in modern amplifiers. In order to make the use of asymoptotic plots completely versatile in this respect, additional transfer functions should be derived for all of the conceivable circuit variations[1] and conditions which are found in audio amplifiers. A few of the more important circuits are:

1. Screen circuit impedance.

2. Cathode circuit impedance of a transformer-coupled amplifier.

3. Plate power supply decoupling impedance.

4. Interstage transformer-coupled amplifier.

5. Several types of phase-inverters.

6. Equalization and compensation circuits.

7. Resonant filters.

8. Video amplifiers.

With a complete set of transfer functions at hand, the graphical methods outlined in this thesis will allow the practicing engineer to determine an approximate solution to the problem of design and analysis of audio amplifiers in a fraction of the time otherwise required by classical methods.

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[1]Tschudi, loc. cit.

BIBLIOGRAPHY

Bode, Hendrik W. Network Analysis and Feedback Amplifier Design. New York: D. Van Nostrand Company, Inc., 1945.

Brown, Gordon S. and Donald P. Campbell. Principles of Servomechanisms. New York: John Wiley and Sons, Inc., 1948.

Chestnut, Harold and Robert W. Mayer. Servomechanisms and Regulating System Design. Volume I; New York: John Wiley and Sons, Inc., 1951.

Cooper, George F. "Audio Feedback Design." Radio-Electronics, XXII, no. 1, through XXII, no. 12(October, 1950 - September, 1951).

Eastman, Austin V. Fundamentals of Vacuum Tubes. 3rd Edition; New York: McGraw-Hill Book Company, Inc., 1949.

E.E. Staff, Massachusetts Institute of Technology. Applied Electronics. New York: John Wiley and Sons, Inc., 1943.

Glasgow, R. S. Principles of Radio Engineering. New York: McGraw-Hill Book Company, Inc., 1936.

Goldman, Stanford. Transformation Calculus and Electrical Transients. New York: Prentice-Hall, Inc., 1949.

Greenwood, Ivan A., J. Vance Holdam, Jr. and Duncan Macrae, Jr. Electronic Instruments. 1st Edition; New York: McGraw-Hill Book Company, Inc., 1948.

RCA Tube Handbook. HB-3, Volume 1-2; Harrison, New Jersey: Commercial Engineering Dept., Radio Corporation of America.

Read, Oliver. The Recording and Reproduction of Sound. 2nd Edition; Indianapolis: Howard W. Sams and Co., Inc., 1952.

Reed, Henry R., T. C. Gordon and George F. Corcoran. Electrical Communications Experiments. New York: John Wiley and Sons, Inc., 1952.

Reich, Herbert J. Theory and Applications of Electron Tubes, 2nd Edition; New York: McGraw-Hill Book Company, Inc., 1944.

Seely, Samuel. Electron-Tube Circuits. 1st Edition; New York: McGraw-Hill Book Company, Inc., 1950.

Stephens, Eugene. The Elementary Theory of Operational Mathematics. New York: McGraw-Hill Book Company, Inc., 1937.

Terman, Fredrick E. Radio Engineering. 3rd Edition; New York: McGraw-Hill Book Company, Inc., 1947.

Terman, Fredrick E. Radio Engineers' Handbook. New York: McGraw-Hill Book Company, Inc., 1943.

Terman, Fredrick E. and Joseph M. Pettit. Electronic Measurements. 2nd Edition; New York: McGraw-Hill Book Company, Inc., 1948.

Thaler, George J. and Robert G. Brown. Servomechanism Analysis. 1st Edition; New York: McGraw-Hill Book Company, Inc., 1953.

Tschudi, E. W. "Transfer Functions for R-C and R-L Equalizing Networks," Electronics, XXII (May, 1949), 116-120.

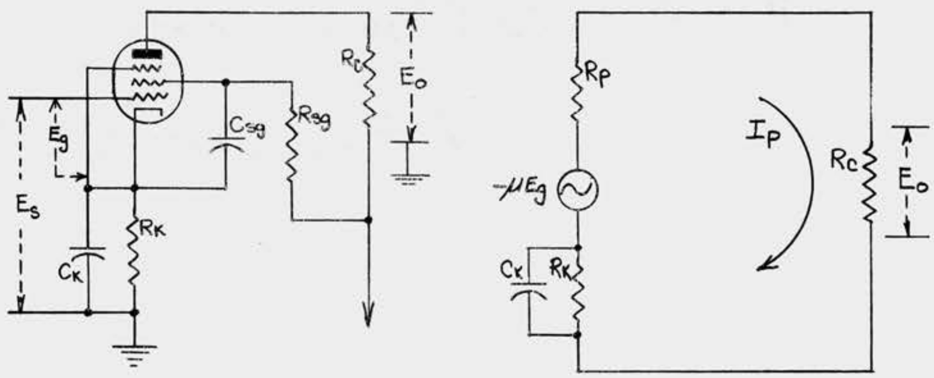
Valley, George E. Jr. and Henry Wallman, editors. Vacuum Tube Amplifiers. 1st Edition; New York: McGraw-Hill Book Company, Inc., 1948.

Zeluff, Vin, and John Markus. Electronics Manual for Radio Engineering. 1st Edition; New York: McGraw-Hill Book Company, Inc., 1949.

APPENDIXES

APPENDIX A

DEVELOPMENT OF THE TRANSFER FUNCTION RELATING THE CATHODE BIAS IMPEDANCE TO THE GAIN OF AND R-C AMPLIFIER AT LOW FREQUENCIES



(a) Actual circuit (b) Equivalent circuit

Figure A1. Resistance-Coupled Amplifier

In Figure A1 it can be seen that

$$E_g = E_s - I_p Z_K \tag{1a}$$

and $$I_p = \frac{\mu E_g}{R_p + R_c + Z_K} = \frac{\mu(E_s - I_p Z_K)}{R_p + R_c + Z_K} \tag{2a}$$

Solving for I_p

$$I_p = \frac{\mu E_s}{R_p + R_c + Z_K(1+\mu)} \tag{3a}$$

Since $E_o = I_p \cdot R_c$ and $\mu = g_m \cdot R_p$, then

$$\frac{E_o}{E_s} = \frac{I_p R_c}{E_s} = \frac{g_m R_p R_c}{R_p + R_c + Z_K(1+\mu)} \tag{4a}$$

In the case of modern vacuum tubes, $\mu \gg 1$, therefore,

$$\frac{E_o}{E_s} = A_L = \frac{g_m R_p R_c}{R_p + R_c + \mu Z_K} \tag{5a}$$

But $Z = \frac{g_m R_p R_c}{1 + R_K C_K P}$ so that

$$A_L = \frac{g_m R_p R_c}{R_p + R_c + \mu R_K} \cdot \left[\frac{1 + R_K C_K P}{1 + \left(\frac{R_p + R_c}{R_p + R_c + \mu R_K} \right) R_K C_K P} \right] \tag{6a}$$

To simplify, let $R_o = R_p + R_c$. Then equation (6a) becomes

$$A_L = \frac{g_m R_p R_c}{R_o + \mu R_K} \cdot \left[\frac{1 + R_K C_K P}{1 + \left(\frac{R_o R_K}{R_o + \mu R_K} \right) C_K P} \right] \tag{7a}$$

In order to express the function as a ratio of gain at low-frequencies to the gain at mid-range frequencies it is necessary to divide equation (9a) by $g_m \frac{R_p R_c}{R_p + R_c}$ which is the expression for the mid-range gain. Then equation (9a) becomes[1]

$$\frac{A_L}{A_M} = \gamma = \frac{R_o}{R_o + \mu R_K} \cdot \left[\frac{1 + R_K C_K P}{1 + \left(\frac{R_o R_K}{R_o + \mu R_K} \right) C_K P} \right] \tag{8a}$$

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[1] Cf. ante., p. 35.

APPENDIX B

MEASUREMENTS OF AN AUDIO OUTPUT TRANSFORMER[1]

Turns Ratio

The turns ratio of an audio transformer can be easily measured by applying a known a-c voltage to the primary of the transformer and measuring the open circuit secondary voltage with a high resistance voltmeter such as a vacuum tube voltmeter. The turns ratio is then determined by

$$N_p/N_s = E_p/E_s \quad (1b)$$

The frequency of the voltage used for this measurement should not be too high or too low in order to avoid losses due to the internal impedances of the transformer. A frequency of 400 to 1000 cps is usually satisfactory.

Total Leakage Inductance

The total leakage inductance reduced to unity turns ratio can be readily obtained by measuring the inductance between the primary terminals when the secondary is short-circuited. An audio impedance bridge such as the General Radio 650A is an excellent instrument for making this measurement. In lieu of a bridge, determining the leakage inductance by resonance with a known external capacitance is also a satisfactory method. Conclusions drawn from the results of measurements on several output transformers was that this is the simplest and least critical

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[1]Fredrick E. Terman, Radio Engineers Handbook, pp. 972.

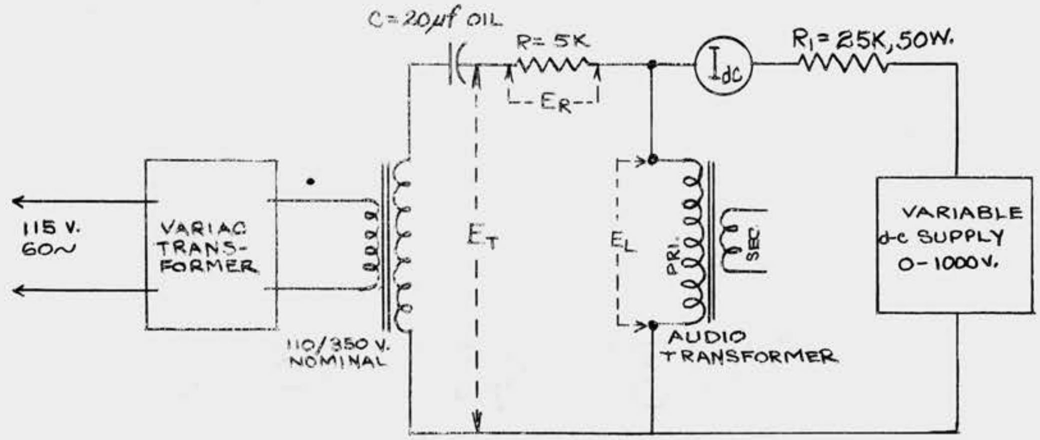
characteristic to measure. The frequency and amplitude of the applied voltage had little effect on the measured value.

Primary Incremental Inductance

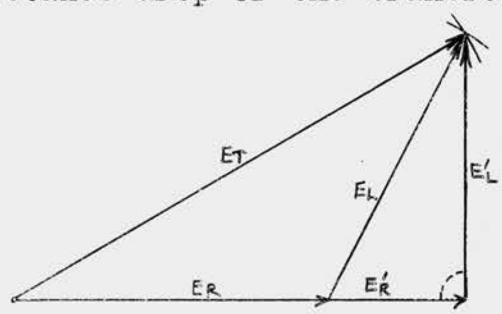
There are several methods by which the incremental inductance of a transformer can be measured.[2] The method adopted by the author is shown in Figure B1. Actually, the method used was checked by both resonance and bridge methods on occasion to be assured of correct results. The graph of Figure B2 shows the results of measurements of the UTC R-27 output transformer which was used in the experimental amplifier described in Chapters VII and IX.

The variation of primary inductance due to changes in magnitude of the applied signal presents a rather perplexing problem. Several transformers were measured, and in some cases the primary inductance increased as much as 500 per cent as the applied signal was increased from 1 volt to 100 volts. This increase is due, no doubt, to the initial increasing slope in the B-H curve of the core material. As the d-c saturation of the core was increased, the change of inductance with signal change was not so pronounced. At the rated d-c operating points of two low priced transformers, the change in inductance was small enough to be ignored by averaging the extreme values which were measured with small and large applied signals. However, when dealing with a class A push-pull transformer where no d-c saturation is present, it leaves one in doubt

[2]Ibid., pp. 909-910.



The desired d-c saturation is set by adjusting the voltage of the d-c supply and the desired signal voltage to the audio transformer is set by adjusting the variac transformer. Readings are then taken of E_T, E_r, and E_L with a vacuum tube voltmeter. The vector diagram shown below is then drawn to scale. The point of intersection of E_L and E_T is found by striking arcs with a compass. The voltages E_L' and E_R' can be determined either by graphical measurement or by trigonometry. E_L' is the internal reactance drop of the transformer and E_R' is the internal resistance drop of the transformer.



The primary inductance and resistance is then determined by

$$L_{(\text{TRANSFORMER})} = \frac{E_L' \cdot R}{2\pi f\, E_R}$$

$$R_{(\text{TRANSFORMER})} = \frac{E_R' \cdot R}{E_R}$$

Figure B1. Method of Measuring the Incremental Inductance of an Audio Transformer

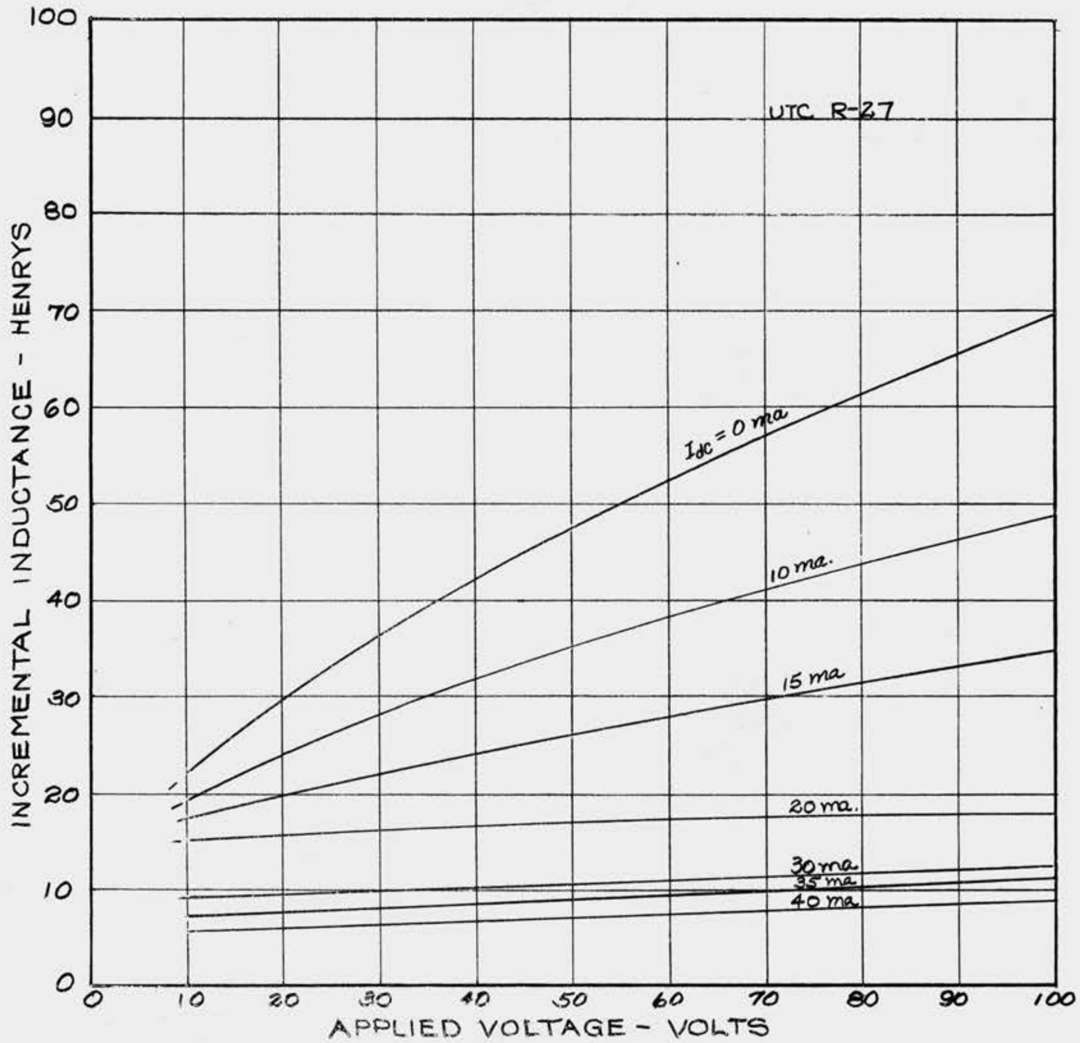


Figure B2. Effect of d-c Saturation and Applied Signal on Incremental Inductance as to what value of inductance to choose for a basis of design.

Primary Resistance

When measuring the primary impedance of output transformers, it was found that the equivalent series a-c resistance would easily vary as much as 10 to 1 since it was affected by both the magnitude and frequency of the applied signal. This variation would appear to be valid since core losses would represent a considerable portion of resistive losses in the transformers. Here again, the question of selection of a value presents itself. Fortunately, when tubes with a rather high plate resistance are used, as in the case of pentode and beam power tubes, the variation of primary resistance is not a major consideration. However, if a triode tube of low plate resistance were to be used, this variation could quite easily be much greater than the plate resistance of the tube and thus become an important factor in design.

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