ANALYSIS OF CONTINUOUS BEAMS BY INFINITE SERIES

By

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PREFACE

The author of this thesis was ordered by the United States Air Force to work towards his Master of Science Degree in Mechanical Engineering with a special emphasis on structural problems.

A special agreement between the School of Mechanical Engineering and the School of Civil Engineering has been reached to make the realization of the United States Air Force's order possible.

The School of Civil Engineering, having a large structural research program sponsored by the Robberson Steel Company, Oklahoma City, Oklahoma, permitted the author to take part in this research activity and to select a topic for his thesis connected with this program.

A special three man advisory committee, appointed to supervise the author's thesis writing, was composed of William H. Easton, Professor of Mechanical Engineering and chairman of the author's thesis committee; Jan J. Tuma, Associate Professor of Civil Engineering in charge of the Robberson Steel Research Project, and Raymond E. Chapel, Assistant Professor of Mechanical Engineering.

The writer wishes to acknowledge his indebtness to Professor Easton for his early reading of this thesis and for his helpful encouragement during the entire academic year, and to Professor Tuma, who originated this type of analysis in preliminary form, for his valuable guidance and careful checking of all the writer's derivations. He extends his thanks to Roger L. Flanders, Professor and acting head of the School of Civil Engineering, and to Professor Chapel, for their reading of the manuscript and much helpful advice.

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NOMENCLATURE

a,b,c,d,e,f,g,h,j,k,m,n,p,r	Functions of New Distribution Factor
s,t,u,v,w,S,T,V	Equivalents
x,y,z	Denominators of convergency
A,B,C,D,E,F,G,H,J	Points or ends of beams
FM	Fixed end moment
EM*	Propped beam end moment
I	Moment of inertia
L	Length
K	Stiffness factor
M	End moment

SIGN CONVENTION

The slope deflection sign convention is used throughout this paper. According to this convention, all moments are positive if they act in a clockwise sense on the member, and are negative if they act in a counter-clockwise sense on the member.

Section 1

(Courtesy of Texas Highway Department)

Colorado River Bridge on State Highway 71 near Smithville, Texas.

SECTION I GENERAL THEORY

INTRODUCTION

Although the moment distribution method[1] became a very important and widely used tool of structural analysis during the last two decades, no systematic effort has been made:

1. To investigate the internal functional mechanics of this procedure;
2. To define it in general algebraic terms;
3. To replace the n-cycle procedure by one direct function, clearly expressed in terms of structural identities and loads applied.

This paper is intended to be a contribution to the above mentioned effort presenting:

1. A complete mathematical analysis of the moment distribution method applied to the computation of the end moments in continuous beams.
2. A derivation of the general end moment equations for a limited

[1]This method was developed in connection with the calculation of secondary stresses in trusses and is described in the book by O. Mohr, "Abhandlungen aus dem Gebiete der Technischen Mechanik", p. 429, 1906 and was extended to the analysis of statically indeterminate frames by K. A. Čalisev (Techničné Listy 1923 No. 17-21, Zagreb). The final form of the numerical method of successive approximations was obtained in the paper by H. Cross (Transactions, Am. Soc. C.E., Vol. 96, 1932) and is without a doubt the most important contribution to structural analysis in recent years.

number of cases (three, four, five, six, seven and eight span beams).

3. A direct, simple and workable procedure for the computation of end moments in continuous beams due to static or moving loads, due to settlement or rotation of supports.

Although only beams with a constant moment of inertia are analyzed, it is evident that all the principles apply to beams or frames with a variable moment of inertia.

Tables for twenty-four specific cases were prepared and their application demonstrated by three typical problems.

1. BASIC SERIES

If the method of moment distribution is used to compute the end moments for three or four span beams and the balancing procedure is expanded algebraically, each moment consists of a series that is:

1. Infinite (number of terms $= \infty$),
2. Convergent (ratio of two successive terms < 1),
3. Geometric (ratio of two successive terms is constant).

The sum of all the terms in each series forms the final moment, which contains two separate and independent functions[2][3]:

1. New Distribution Factor,
2. Fixed End Moment.

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[2] Jan J. Tuma, Wind Stress Analysis of One Story Bents by New Distribution Factor (Oklahoma Engineering Experiment Station Publication No. 80, May, 1951).

[3] Jan J. Tuma, Influence Lines For Frames With Constant Moment of Inertia and Sidesway Prevented (Oklahoma Engineering Experiment Station Publication No. 85, November, 1952).

2. BASIC SERIES FOR A THREE SPAN BEAM

In the following illustration of the Basic Series, the end moment equations will be derived for a three span cyclosymmetrical beam, loaded as shown in Fig. 1.

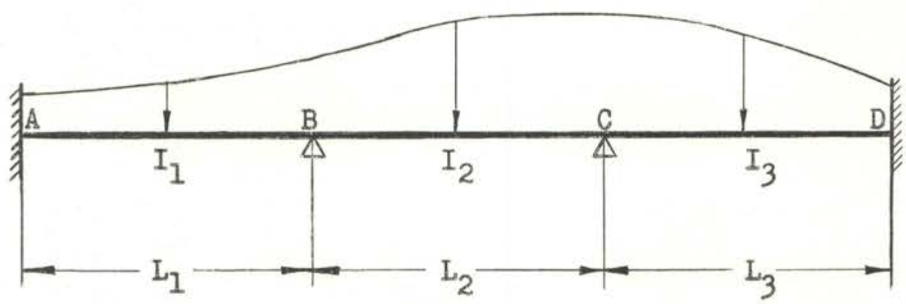

Fig. 1
THREE SPAN BEAM

Denoting the fixed end moments due to the applied loads as $-FM_{AB}$, FM_{BA}, $-FM_{BC}$, FM_{CB}, $-FM_{CD}$, and FM_{DC}, the sums of the fixed end moments are at Point B

$$B = FM_{BA} - FM_{BC}, \tag{1a}$$

and at Point C

$$C = FM_{CB} - FM_{CD}. \tag{1b}$$

The distribution factors are denoted

at Point B as

$$D_{BA} = 2a, \ D_{BC} = 2b, \tag{2a}$$

and at Point C as

$$D_{CB} = 2c, \ D_{CD} = 2d. \tag{2b}$$

The distribution of the fixed end moments at Points B and C is presented in Table 1.

TABLE 1

ALGEBRAIC DISTRIBUTION OF FIXED END MOMENTS FOR A THREE SPAN BEAM

Points:	A		B		C	D
Distribution Factors:	0	2a	2b	2c	2d	0
Fixed End Moments:	$-FM_{AB}$	FM_{BA}	$-FM_{BC}$	FM_{CB}	$-FM_{CD}$	FM_{DC}
1st Cycle		$-2aB$	$-2bB$	$-2cC$	$-2dC$	
Carry-over	$-aB$		$-cC$	$-bB$		$-dC$
2nd Cycle		$2acC$	$2bcC$	$2bcB$	$2bdB$	
Carry-over	acC		bcB	bcC		bdB
3rd Cycle		$-2abcB$	$-2b^2cB$	$-2bc^2C$	$-2bcdC$	
Carry-over	$-abcB$		$-bc^2C$	$-b^2cB$		$-bcdC$
4th Cycle		$2abc^2C$	$2b^2c^2C$	$2b^2c^2B$	$2b^2cdB$	
Carry-over	abc^2C		b^2c^2B	b^2c^2C		b^2cdB
5th Cycle		$-2ab^2c^2B$	$-2b^3c^2B$	$-2b^2c^3C$	$-2b^2c^2dC$	
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = <u>Final Moment</u>	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}

Summing all the terms in column M_{AB} of Table 1,

$$M_{AB} = - FM_{AB} - \frac{a}{x} B + \frac{ac}{x} C, \quad (3)$$

in which

$$\frac{1}{x} = 1 + bc + (bc)^2 + (bc)^3 + \ldots\ldots\ldots\ldots\ldots\ldots + 0 \quad (3a)$$

and

$$x = 1 - bc \text{ (denominator of Convergency for } \overline{AD}\text{)}. \quad (3b)$$

All moment equations similarly derived are presented in Section 2, Table C3.

3. BASIC SERIES FOR A FOUR SPAN BEAM

As a second illustration of the Basic Series, the end moment equations will be derived for a four span cyclosymmetrical beam, loaded as shown in Fig. 2.

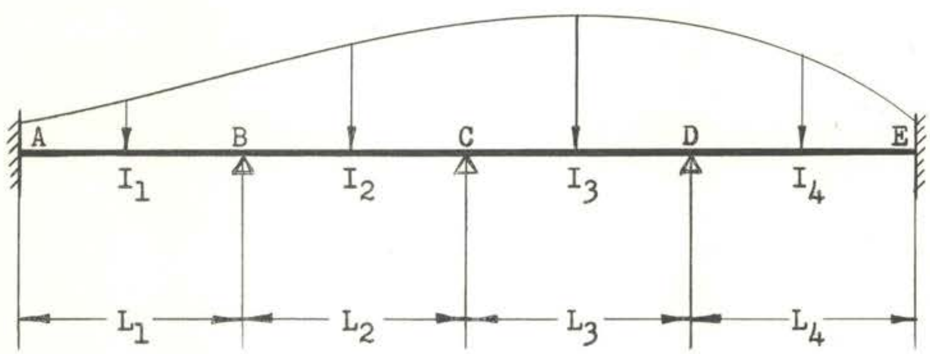

Fig. 2
FOUR SPAN BEAM

Using nomenclature similar to that in the previous chapter and summing all the terms in column M_{AB} of Table 2, the moment equation becomes

$$M_{AB} = - FM_{AB} - \frac{ay_2}{x} B + \frac{ac}{x} C - \frac{ace}{x} D \qquad (4)$$

in which

$$\frac{1}{x} = 1 + (bc + de) + (bc + de)^2 + (bc + de)^3 + \ldots\ldots\ldots + 0, \qquad (4a)$$

$$x = 1 - bc - de \text{ (denominator of convergency for } \overline{AE}), \qquad (4b)$$

$$y_1 = 1 - bc \text{ (denominator of convergency for } \overline{AD}), \qquad (4c)$$

and

$$y_2 = 1 - de \text{ (denominator of convergency for } \overline{BE}). \qquad (4d)$$

All moment equations similarly derived are presented in Section 2, Table C4.

TABLE 2

ALGEBRAIC DISTRIBUTION OF FIXED END MOMENTS FOR A FOUR SPAN BEAM

Points:	A		B		C		D	E
Distribution Factors:	0	2a	2b	2c	2d	2e	2f	0
Fixed End Moments	$-FM_{AB}$	FM_{BA}	$-FM_{BC}$	FM_{CB}	$-FM_{CD}$	FM_{DC}	$-FM_{DE}$	FM_{ED}
1st Cycle		-2aB	-2bB	-2cC	-2dC	-2eD	-2fD	
Carry-over	-aB		-cC	-bB	-eD	-dC		-fD
2nd Cycle		2acC	2bcC	2cS[4]	2dS	2deC	2dfC	
Carry-over	acC		cS	bcC	deC	dS		dfC
3rd Cycle		-2acS	-2bcS	-2ct[4]C	-2dtC	-2deS	-2dfS	
Carry-over	-acS		-ctC	-bcS	-deS	-dtC		-dfS
4th Cycle		2actC	2bctC	2ctS	2dtS	2detC	2dftC	
Carry-over	actC		ctS	bctC	detC	dtS		dftC
5th Cycle		-2actS	-2bctS	$-2ct^2C$	$-2dt^2C$	-2detS	-2dftS	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}	M_{DE}	M_{ED}

[4]Substituting bB + eD = S and bc + de = t simplifies the algebraic procedure. When summing the terms in each column, the values S and t are replaced by the original values.

4. INTRODUCTION TO SERIES OF SERIES

The Basic Series has proven itself adequate in the two preceding derivations. For beams with more than four spans, the series become very complex and it would be unadvisable to derive the moment equations by the previous procedure.

To illustrate the complexity of such a problem, consider an elementary case: a five span beam of equal spans, constant moment of inertia, and loaded as shown in Fig. 3.

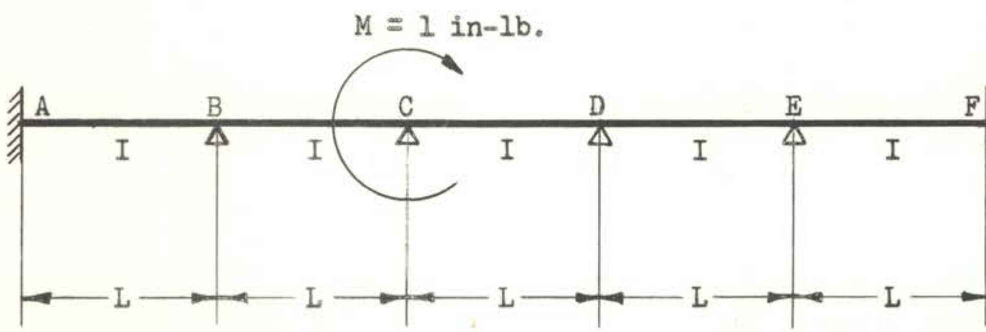

Fig. 3
FIVE SPAN BEAM - EQUAL SPANS

Denoting all the distribution factors as

$$D = 2a, \tag{5}$$

from Table 3, the moment M_{AB} is

$$M_{AB} = a^2 + 2a^4 + 5a^6 + 13a^8 + 24a^{10} + \ldots\ldots\ldots + 0. \tag{6}$$

The series forming the moment M_{AB} is evidently infinite in length and does converge, but it is not a simple geometric series. Although the algebraic terms form a perfect geometric series, their numerical factors follow a different generating law. By resolving each numerical

TABLE 3

ALGEBRAIC DISTRIBUTION OF APPLIED MOMENT (M = 1) FOR A FIVE SPAN BEAM

Points:	A	B		C		D		E		F
D's	0	2a	2a	2a	2a	2a	2a	2a	2a	0
1st Cycle				$-2a$	$-2a$					
			$-a$			$-a$				
2nd Cycle		$2a^2$	$2a^2$			$2a^2$	$2a^2$			
	a^2			a^2	a^2			a^2		
3rd Cycle				$-4a^3$	$-4a^3$			$-2a^3$	$-2a^3$	
			$-2a^3$			$-2a^3$	$-a^3$			$-a^3$
4th Cycle		$4a^4$	$4a^4$			$6a^4$	$6a^4$			
	$2a^4$			$2a^4$	$3a^4$			$3a^4$		
5th Cycle				$-10a^5$	$-10a^5$			$-6a^5$	$-6a^5$	
			$-5a^5$			$-5a^5$	$-3a^5$			$-3a^5$
6th Cycle		$10a^6$	$10a^6$			$16a^6$	$16a^6$			
	$5a^6$			$5a^6$	$8a^6$			$8a^6$		
7th Cycle				$-26a^7$	$-26a^7$			$-16a^7$	$-16a^7$	
Infinite Cycle	0	0	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}	M_{DE}	M_{ED}	M_{EF}	M_{FE}

factor into basic components:

$$\left.\begin{aligned}
T_1 &= (1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad)a^2 = 1a^2,\\
T_2 &= (1 + 1 \qquad\qquad\qquad\qquad\qquad\qquad\quad)a^4 = 2a^4,\\
T_3 &= (1 + 2 + 2 \qquad\qquad\qquad\qquad\qquad\quad)a^6 = 5a^6,\\
T_4 &= (1 + 2 + 5 + 5 \qquad\qquad\qquad\qquad\quad)a^8 = 13a^8,\\
T_5 &= (1 + 2 + 5 + 13 + 13 \qquad\qquad\qquad\quad)a^{10} = 34a^{10},\\
T_6 &= (1 + 2 + 5 + 13 + 34 + 34 \qquad\qquad\quad)a^{12} = 85a^{12},\\
T_{n-1} &= (T_1 + T_2 + T_3 + \ldots\ldots + T_{n-3} + 2T_{n-2})a^{2(n-1)},\\
T_n &= (T_1 + T_2 + T_3 + \ldots\ldots + T_{n-2} + 2T_{n-1})a^{2n}.
\end{aligned}\right\} \quad (7)$$

Equation 7 demonstrates that each final moment is a combination of an infinite number of infinite series, generating parallel to zero. In order to simplify the analysis of the series in the following chapters, the investigated beams will be divided into isolated parts.

5. CARRY-OVER SERIES α AND β FOR A FIVE SPAN BEAM

In the following illustration of the Carry-over Series, the end moment equations will be derived for a five span cyclosymmetrical beam, loaded as shown in Fig. 4.

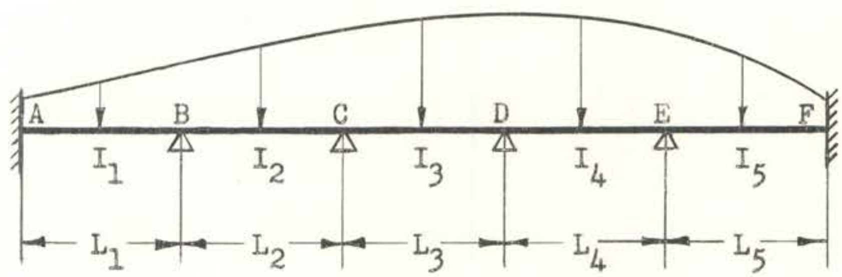

Fig. 4
FIVE SPAN BEAM

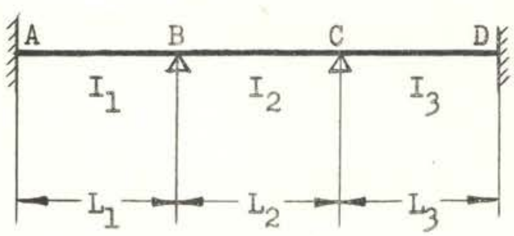

Fig. 5
ISOLATED BEAM 1

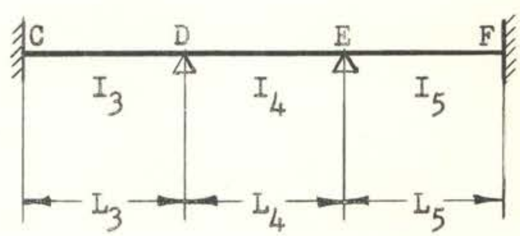

Fig. 6
ISOLATED BEAM 2

In order to simplify the algebraic procedure, only the fixed end moments at Points A and B will be considered. The same procedure must be repeated for the fixed end moments at Points C, D, E, and F, and the final moments will be the sums of the partial results.

The distribution factors are denoted by the algebraic terms shown in

TABLE 4

ALGEBRAIC DISTRIBUTION OF FIXED END MOMENTS FOR A FIVE SPAN BEAM
(ISOLATED BEAM 1)

Points:	A		B		C	D
Distribution Factors:	0	2a	2b	2c	2d	0
Fixed End Moments:	$-FM_{AB}$	FM_{BA}	$-FM_{BC}$			
1st Cycle		$-2aB$	$-2bB$			
Carry-over	$-aB$			$-bB$		
2nd Cycle				$2bcB$	$2bdB$	
Carry-over			bcB			bdB
3rd Cycle		$-2abcB$	$-2b^2cB$			
Carry-over	$-abcB$			$-b^2cB$		
4th Cycle				$2b^2c^2B$	$2b^2cdB$	
Carry-over			b^2c^2B			b^2cdB
5th Cycle		$-2ab^2c^2B$	$-2b^3c^2B$			
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}

Table 4 and the sum of the fixed end moments at the points of support are designated by their respective symbols, as was done in Chapters Two and Three. Point D is temporarily locked against rotation; thus, isolating from a five span beam $\overline{AF}$, a three span beam $\overline{AD}$ (Beam 1). From Table 4 the moments on Beam 1 are

$$M_{AB_0} = - FM_{AB} - \frac{a}{x} B, \tag{8}$$

$$M_{BA_0} = + FM_{BA} - 2\frac{a}{x} B, \tag{9}$$

$$M_{BC_0} = - FM_{BA} + 2\frac{a}{x} B, \tag{10}$$

$$M_{CB_0} = - 2\frac{bd}{x} B, \tag{11}$$

$$M_{CD_0} = + 2\frac{bd}{x} B, \tag{12}$$

$$M_{DC_0} = + \frac{bd}{x} B. \tag{13}$$

Hereafter, all series performing moments in Beam 1 will be called "Basic Series" because of their similarity with series derived in Chapters Two and Three.

By unlocking Point D and locking Point C, a new beam is isolated (Beam 2). Denoting

$$M_{DC_0} = + \frac{bd}{x} B = \alpha_0, \tag{14}$$

and summing the values in the various columns of Table 5,

$$M_{CD_1} = - \frac{e}{y} \alpha_0, \tag{15}$$

$$M_{DC_1} = + \alpha_0 - 2\frac{e}{y} \alpha_0, \tag{16}$$

$$M_{DE_1} = - \alpha_0 + 2\frac{e}{y} \alpha_0, \tag{17}$$

$$M_{ED_1} = - 2\frac{fh}{y} \alpha_0, \tag{18}$$

$$M_{EF_1} = + 2\frac{fh}{y} \alpha_0, \tag{19}$$

$$M_{FE_1} = + \frac{fh}{y} \alpha_0. \tag{20}$$

Unlocking Point C, locking Point D and denoting

$$M_{CD_1} = - \frac{e}{y} \alpha_0 = \beta_0, \tag{21}$$

the carry-over value from Table 6 is

TABLE 5

ALGEBRAIC DISTRIBUTION OF FUNCTION α_0 IN A FIVE SPAN BEAM
(ISOLATED BEAM 2)

Points: C, D, E, F

Points:	C		D		E	F
Distribution Factors:	0	2e	2f	2g	2h	0
Starting Moments:		α_0				
1st Cycle		$-2e\alpha_0$	$-2f\alpha_0$			
Carry-over	$-e\alpha_0$			$-f\alpha_0$		
2nd Cycle				$2fg\alpha_0$	$2fh\alpha_0$	
Carry-over			$fg\alpha_0$			$fh\alpha_0$
3rd Cycle		$-2efg\alpha_0$	$-2f^2g\alpha_0$			
Carry-over	$-efg\alpha_0$			$-f^2g\alpha_0$		
4th Cycle				$2f^2g^2\alpha_0$	$2f^2gh\alpha_0$	
Carry-over			$f^2g^2\alpha_0$			$f^2gh\alpha_0$
5th Cycle		$-2ef^2g^2\alpha_0$	$-2f^3g^2\alpha_0$			
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{CD}	M_{DC}	M_{DE}	M_{ED}	M_{EF}	M_{FE}

TABLE 6

ALGEBRAIC DISTRIBUTION OF FUNCTION β_0 IN A FIVE SPAN BEAM (ISOLATED BEAM 1)

Points:	A	B		C		D
Distribution Factors:	0	2a	2b	2c	2d	0
Starting Moments:					β_0	
1st Cycle				$-2c\beta_0$	$-2d\beta_0$	
Carry-over			$-c\beta_0$			$-d\beta_0$
2nd Cycle		$2ac\beta_0$	$2bc\beta_0$			
Carry-over	$ac\beta_0$			$bc\beta_0$		
3rd Cycle				$-2bc^2\beta_0$	$-2bcd\beta_0$	
Carry-over			$-bc^2\beta_0$			$-bcd\beta_0$
4th Cycle		$2abc^2\beta_0$	$2b^2c^2\beta_0$			
Carry-over	$abc^2\beta_0$			$b^2c^2\beta_0$		
5th Cycle				$-2b^2c^3\beta_0$	$-2b^2c^2d\beta_0$	
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = <u>Final Moment</u>	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}

TABLE 7

SERIES α AND β AS A FUNCTION OF α FOR A FIVE SPAN BEAM

Series α - Function α	Series β - Function α
$\alpha_0 = \left(\frac{de}{xy}\right)^0 \alpha$	$\beta_0 = -\left(\frac{de}{xy}\right)^0 \frac{e}{y} \alpha$
$\alpha_1 = \left(\frac{de}{xy}\right)^1 \alpha$	$\beta_1 = -\left(\frac{de}{xy}\right)^1 \frac{e}{y} \alpha$
$\alpha_2 = \left(\frac{de}{xy}\right)^2 \alpha$	$\beta_2 = -\left(\frac{de}{xy}\right)^2 \frac{e}{y} \alpha$
$\alpha_n = \left(\frac{de}{xy}\right)^n \alpha$	$\beta_n = -\left(\frac{de}{xy}\right)^n \frac{e}{y} \alpha$
$\sum_0^\infty \alpha = \frac{\alpha}{1 - \frac{de}{xy}}$	$\sum_0^\infty \beta = \frac{-\frac{e}{y}\alpha}{1 - \frac{de}{xy}}$

TABLE 8

SERIES α AND β AS A FUNCTION OF β FOR A FIVE SPAN BEAM

Series α - Function β	Series β - Function β
$\alpha_0 = -\left(\frac{de}{xy}\right)^0 \frac{d}{x} \beta$	$\beta_0 = \left(\frac{de}{xy}\right)^0 \beta$
$\alpha_1 = -\left(\frac{de}{xy}\right)^1 \frac{d}{x} \beta$	$\beta_1 = \left(\frac{de}{xy}\right)^1 \beta$
$\alpha_2 = -\left(\frac{de}{xy}\right)^2 \frac{d}{x} \beta$	$\beta_2 = \left(\frac{de}{xy}\right)^2 \beta$
$\alpha_n = -\left(\frac{de}{xy}\right)^n \frac{d}{x} \beta$	$\beta_n = \left(\frac{de}{xy}\right)^n \beta$
$\sum_0^\infty \alpha = \frac{-\frac{d}{x}\beta}{1 - \frac{de}{xy}}$	$\sum_0^\infty \beta = \frac{\beta}{1 - \frac{de}{xy}}$

$$M_{DC_2} = -\frac{d}{x}\beta_0 = +\frac{de}{xy}\alpha_0 = \alpha_1. \tag{22}$$

Systematically locking and unlocking Points C and D performs two functions:

1. Balancing the moments at each support,

2. Establishing equilibrium between the isolated beams.

The carry-over values from "Beam 1" to "Beam 2" and from "Beam 2" to "Beam 1" are denoted the functions "α" and "β" respectfully, and they are tabulated. Tables 7 and 8 demonstrate that these functions form perfect infinite, convergent, geometric series. Superimposing the Basic Series (Eq. 8) and the Carry-over Series (β) recently derived, the moment M_{AB} due to the unbalance at Point B is

$$M_{AB} = -FM_{AB} - \frac{a}{x}B + \frac{ac}{x}\beta_0 + \frac{ac}{x}\beta_1 + \frac{ac}{x}\beta_2 + \ldots\ldots + 0, \tag{23a}$$

or

$$M_{AB} = -FM_{AB} - \frac{a}{x}B + \frac{ac}{x}\sum_{0}^{\infty}\beta \quad , \tag{23b}$$

in which (Table 7)

$$\sum_{0}^{\infty}\beta = \frac{-\frac{e}{y}\alpha}{1 - \frac{de}{xy}} . \tag{23c}$$

With the notation 14, equation 23b becomes

$$M_{AB} = -FM_{AB} - \frac{a}{x}(1 + \frac{bc}{z})B, \tag{23}$$

in which

$$x = 1 - bc \text{ (Denominator of convergency for Beam 1)}, \tag{23d}$$

$$y = 1 - fg \text{ (Denominator of convergency for Beam 2)}, \tag{23e}$$

and

$$z = \frac{xy}{de} - 1 \text{ (Modified denominator of convergency[5] of Carry-over Series } \alpha \text{ and } \beta\text{)}. \quad (23f)$$

The above procedure is repeated for the fixed end moments at Points C, D, E and F. All moment equations similarly derived are presented in Section 2, Table C5.

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[5]The actual denominator of convergency of the Carry-over Series α and β is

$$1 - \frac{de}{xy}.$$

In order to simplify the algebraic form of the final equations, a modified term is used.

$$\frac{1 - \frac{de}{xy}}{\frac{de}{xy}} = \frac{xy}{de} - 1 = z$$

6. CARRY-OVER SERIES α, β, δ AND γ FOR A SIX SPAN BEAM

As a second illustration of the Carry-over Series, the end moment equations will be derived for a six span cyclosymmetrical beam, loaded as shown in Fig. 7. This beam could be divided into sections of unequal lengths, but the derivations of the moment equations are less complicated if the beams are isolated as shown in Figs. 8, 9, and 10.

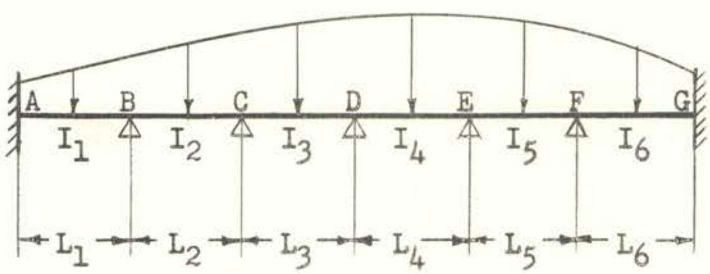

Fig. 7
SIX SPAN BEAM

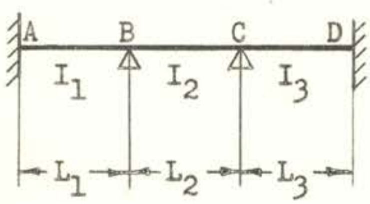

Fig. 8
ISOLATED BEAM 1

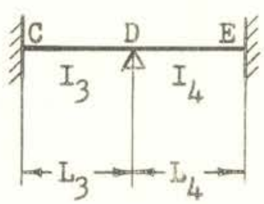

Fig. 9
ISOLATED BEAM 2

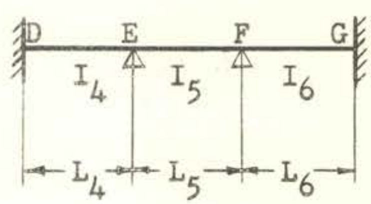

Fig. 10
ISOLATED BEAM 3

Using nomenclature similar to that of the previous chapters and temporarily locking Point D, the moments on Beam 1 due to the fixed end moments

TABLE 9

ALGEBRAIC DISTRIBUTION OF FIXED END MOMENTS FOR A SIX SPAN BEAM
(ISOLATED BEAM 1)

Points:	A		B		C	D
Distribution Factors:	0	2a	2b	2c	2d	0
Fixed End Moments:	$-FM_{AB}$	FM_{BA}	$-FM_{BC}$			
1st Cycle		$-2aB$	$-2bB$			
Carry-over	$-aB$			$-bB$		
2nd Cycle				$2bcB$	$2bdB$	
Carry-over			bcB			bdB
3rd Cycle		$-2abcB$	$-2b^2cB$			
Carry-over	$-abcB$			$-b^2cB$		
4th Cycle				$2b^2c^2B$	$2b^2cdB$	
Carry-over			b^2c^2B			b^2cdB
5th Cycle		$-2ab^2c^2B$	$-2b^3c^2B$			
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}

at Points A and B are (Table 9)

$$M_{AB_0} = - FM_{AB} - \frac{a}{x_1}B, \quad (24)$$

$$M_{BA_0} = FM_{BA} - 2\frac{a}{x_1}B, \quad (25)$$

$$M_{BC_0} = - FM_{BA} + 2\frac{a}{x_1}B, \quad (26)$$

$$M_{CB_0} = - 2\frac{bd}{x_1}B, \quad (27)$$

$$M_{CD_0} = 2\frac{bd}{x_1}B, \quad (28)$$

$$M_{DC_0} = \frac{bd}{x_1}B. \quad (29)$$

Unlocking Point D and locking Points C and E, a second beam is isolated (Beam 2). Denoting

$$M_{DC_0} = \frac{bd}{x_1}B = \alpha_0, \quad (30)$$

and summing the values in the various columns of Table 10,

$$M_{CD_1} = - e\,\alpha_0, \quad (31)$$

$$M_{DC_1} = \alpha_0 - 2e\,\alpha_0, \quad (32)$$

$$M_{DE_1} = - 2f\,\alpha_0, \quad (33)$$

$$M_{ED_1} = - f\,\alpha_0. \quad (34)$$

TABLE 10

ALGEBRAIC DISTRIBUTION OF FUNCTION α_0 IN A SIX SPAN BEAM
(ISOLATED BEAM 2)

Points:	C	D		E
Distribution Factors:	0	2e	2f	0
Starting Moments:		α_0		
1st Cycle		$-2e\,\alpha_0$	$-2f\,\alpha_0$	
Carry-over	$-e\,\alpha_0$			$-f\,\alpha_0$

Unlocking Points C and E and locking Point D, Beam 1 is again

TABLE 11

ALGEBRAIC DISTRIBUTION OF FUNCTION β_o IN A SIX SPAN BEAM
(ISOLATED BEAM 1)

Points:	A	B		C		D
Distribution Factors:	0	2a	2b	2c	2d	0
Starting Moments:					β_o	
1st Cycle				$-2c\beta_o$	$-2d\beta_o$	
Carry-over			$-c\beta_o$			$-d\beta_o$
2nd Cycle		$2ac\beta_o$	$2bc\beta_o$			
Carry-over	$ac\beta_o$			$bc\beta_o$		
3rd Cycle				$-2bc^2\beta_o$	$-2bcd\beta_o$	
Carry-over			$-bc^2\beta_o$			$-bcd\beta_o$
4th Cycle		$2abc^2\beta_o$	$2b^2c^2\beta_o$			
Carry-over	$abc^2\beta_o$			$b^2c^2\beta_o$		
5th Cycle				$-2b^2c^3\beta_o$	$-2b^2c^2d\beta_o$	
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = <u>Final</u> <u>Moment</u>	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}

TABLE 12

ALGEBRAIC DISTRIBUTION OF FUNCTION γ_0 IN A SIX SPAN BEAM
(ISOLATED BEAM 3)

Points:	D	E		F		G
Distribution Factors:	0	2g	2h	2j	2k	
Starting Moments:		γ_0				
1st Cycle		$-2g\,\gamma_0$	$-2h\,\gamma_0$			
Carry-over	$-g\,\gamma_0$			$-h\,\gamma_0$		
2nd Cycle				$2hj\,\gamma_0$	$2hk\,\gamma_0$	
Carry-over			$hj\,\gamma_0$			$hk\,\gamma_0$
3rd Cycle		$-2ghj\,\gamma_0$	$-2h^2j\,\gamma_0$			
Carry-over	$-ghj\,\gamma_0$			$-h^2j\,\gamma_0$		
4th Cycle				$2h^2j^2\gamma_0$	$2h^2jk\,\gamma_0$	
Carry-over			$h^2j^2\gamma_0$			$h^2jk\,\gamma_0$
5th Cycle		$-2gh^2j^2\gamma_0$	$-2h^3j^2\gamma_0$			
Infinite Cycle	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{DE}	M_{ED}	M_{EF}	M_{FE}	M_{FG}	M_{GF}

isolated, but also isolated is Beam 3. The following moments are denoted

$$M_{CD_1} = - e\alpha_0 = \beta_0, \tag{35}$$

$$M_{ED_1} = - f\alpha_0 = \gamma_0, \tag{36}$$

and the carry-over values from Tables 11 and 12 are

$$M_{DC_2} = - \frac{d}{x_1}\beta_0 = \alpha_1, \tag{37}$$

$$M_{DE_2} = - \frac{g}{x_2}\gamma_0 = \delta_0. \tag{38}$$

TABLE 13

ALGEBRAIC DISTRIBUTION OF FUNCTION δ_0 IN A SIX SPAN BEAM
(ISOLATED BEAM 2)

Points:	C		D	E
Distribution Factors:	0	2e	2f	0
Starting Moments:			δ_0	
1st Cycle		$-2e\ \delta_0$	$-2f\ \delta_0$	
Carry-over	$-e\ \delta_0$			$-f\ \delta_0$

The Points C, D, and E are locked and unlocked systematically and the values of α, β, δ and γ are tabulated (Table 14). Superimposing the Basic Series (Eq. 24) and the Carry-over Series (β) recently derived, the moment M_{AB} due to the fixed end moments at Points A and B is

$$M_{AB} = - FM_{AB} - \frac{a}{x_1}B + \frac{ac}{x_1}\beta_0 + \frac{ac}{x_1}\beta_1 + \frac{ac}{x_1}\beta_2 + \ldots\ldots + 0, \tag{39a}$$

or

$$M_{AB} = - FM_{AB} - \frac{a}{x_1}B + \frac{ac}{x_1}\sum_{0}^{\infty}\beta, \tag{39b}$$

in which (Table 14)

$$\sum_{0}^{\infty}\beta = - \frac{e}{(1 - T)}\alpha = - \frac{x_1}{dz_1}\alpha. \tag{39c}$$

TABLE 14

SERIES α, β, δ AND γ AS A FUNCTION OF α FOR A SIX SPAN BEAM

Series α - Function α	Series β - Function α
$\alpha_0 = \alpha$	$\beta_0 = -e\,\alpha$
$\alpha_1 = \frac{de}{x_1}\alpha$	$\beta_1 = -e\,T\alpha$
$\alpha_2 = \frac{de}{x_1}T\alpha$ (6)	$\beta_2 = -e\,T^2\alpha$
$\alpha_n = \frac{de}{x_1}T^{n-1}\alpha$	$\beta_n = -e\,T^n\alpha$
$\sum_0^\infty \alpha = \frac{\alpha}{(1-T)}(1-\frac{fg}{x_2})$	$\sum_0^\infty \beta = -\frac{e\alpha}{(1-T)}$
Series δ - Function α	**Series γ - Function α**
$\delta_0 = \frac{gf}{x_2}\alpha$	$\gamma_0 = -f\alpha$
$\delta_1 = \frac{gf}{x_2}T\alpha$	$\gamma_1 = -f\,T\alpha$
$\delta_2 = \frac{gf}{x_2}T^2\alpha$	$\gamma_2 = -f\,T^2\alpha$
$\delta_n = \frac{gf}{x_2}T^n\alpha$	$\gamma_n = -f\,T^n\alpha$
$\sum_0^\infty \delta = \frac{gf\alpha}{(1-T)x_2}$	$\sum_0^\infty \gamma = -\frac{f\alpha}{(1-T)}$

[6]Substituting $\frac{de}{x_1} + \frac{fg}{x_2} = T$.

TABLE 15

SERIES α, β, δ AND γ AS A FUNCTION OF β FOR A SIX SPAN BEAM

Series α - Function β	Series β - Function β
$\alpha_0 = -\frac{d}{x_1}\beta$	$\beta_0 = \beta$
$\alpha_1 = -\frac{d^2e}{x_1^2}\beta$	$\beta_1 = \frac{de}{x_1}\beta$
$\alpha_2 = -\frac{d^2e}{x_1^2}T\beta$	$\beta_2 = \frac{de}{x_1}T\beta$
$\alpha_n = -\frac{d^2e}{x_1^2}T^{n-1}\beta$	$\beta_n = \frac{de}{x_1}T^{n-1}\beta$
$\sum_0^\infty \alpha = \frac{d\beta}{(1-T)x_1}\left(1 - \frac{fg}{x_2}\right)$	$\sum_0^\infty \beta = \frac{\beta}{(1-T)}\left(1 - \frac{fg}{x_2}\right)$

Series δ - Function β	Series γ - Function β
$\delta_0 = -\frac{dfg}{x_1 x_2}\beta$	$\gamma_0 = \frac{df}{x_1}\beta$
$\delta_1 = -\frac{dfg}{x_1 x_2}T\beta$	$\gamma_1 = \frac{df}{x_1}T\beta$
$\delta_2 = -\frac{dfg}{x_1 x_2}T^2\beta$	$\gamma_2 = \frac{df}{x_1}T^2\beta$
$\delta_n = -\frac{dfg}{x_1 x_2}T^n\beta$	$\gamma_n = \frac{df}{x_1}T^n\beta$
$\sum_0^\infty \delta = -\frac{dfg\beta}{x_1 x_2(1-T)}$	$\sum_0^\infty \gamma = \frac{df\beta}{x_1(1-T)}$

TABLE 16

SERIES α, β, δ AND γ AS A FUNCTION OF δ FOR A SIX SPAN BEAM

Series α - Function δ	Series β - Function δ
$\alpha_0 = \frac{de}{x_1}\,\delta$	$\beta_0 = -\,e\,\delta$
$\alpha_1 = \frac{de}{x_1}\,T\,\delta$	$\beta_1 = -\,e\,T\,\delta$
$\alpha_2 = \frac{de}{x_1}\,T^2\,\delta$	$\beta_2 = -\,e\,T^2\,\delta$
$\alpha_n = \frac{de}{x_1}\,T^n\,\delta$	$\beta_n = -\,e\,T^n\,\delta$
$\sum_0^{\infty}\alpha = \frac{de\,\delta}{(1 - T)x_1}$	$\sum_0^{\infty}\beta = \frac{-\,e\,\delta}{(1 - T)}$

Series δ - Function δ	Series γ - Function δ
$\delta_0 = \delta$	$\gamma_0 = -\,f\,\delta$
$\delta_1 = \frac{fg}{x_2}\,\delta$	$\gamma_1 = -\,f\,T\,\delta$
$\delta_2 = \frac{fg}{x_2}\,T\,\delta$	$\gamma_2 = -\,f\,T^2\,\delta$
$\delta_n = \frac{fg}{x_2}\,T^{n-1}\,\delta$	$\gamma_n = -\,f\,T^n\,\delta$
$\sum_0^{\infty}\delta = \frac{\delta}{(1 - T)}(1 - \frac{de}{x_1})$	$\sum_0^{\infty}\gamma = \frac{-\,f\,\delta}{(1 - T)}$

TABLE 17

SERIES α, β, δ AND γ AS A FUNCTION OF γ FOR A SIX SPAN BEAM

Series α - Function γ	Series β - Function γ
$\alpha_0 = -\frac{deg}{x_1 x_2}\gamma$	$\beta_0 = \frac{eg}{x_2}\gamma$
$\alpha_1 = -\frac{deg}{x_1 x_2}T\gamma$	$\beta_1 = \frac{eg}{x_2}T\gamma$
$\alpha_2 = -\frac{deg}{x_1 x_2}T^2\gamma$	$\beta_2 = \frac{eg}{x_2}T^2\gamma$
$\alpha_n = -\frac{deg}{x_1 x_2}T^n\gamma$	$\beta_n = \frac{eg}{x_2}T^n\gamma$
$\sum_0^\infty \alpha = -\frac{deg\gamma}{(1-T)x_1 x_2}$	$\sum_0^\infty \beta = \frac{ge\gamma}{(1-T)x_2}$

Series δ - Function γ	Series γ - Function γ
$\delta_0 = -\frac{g}{x_2}\gamma$	$\gamma_0 = \gamma$
$\delta_1 = -\frac{fg^2}{x_2^2}\gamma$	$\gamma_1 = \frac{fg}{x_2}\gamma$
$\delta_2 = -\frac{fg^2}{x_2^2}T\gamma$	$\gamma_2 = \frac{fg}{x_2}T\gamma$
$\delta_n = -\frac{fg^2}{x_2^2}T^{n-1}\gamma$	$\gamma_n = \frac{fg}{x_2}T^{n-1}\gamma$
$\sum_0^\infty \delta = \frac{-g\gamma}{(1-T)x_2}(1 - \frac{de}{x_1})$	$\sum_0^\infty \gamma = \frac{\gamma}{(1-T)}(1 - \frac{de}{x_1})$

With notation 30, equation 39b becomes

$$M_{AB} = - FM_{AB} - \frac{a}{x_1}(1 + \frac{bc}{z_1})\ B, \qquad (39)$$

in which

$$x_1 = 1 - bc \text{ (Denominator of convergency of Beam 1)}, \qquad (39d)$$

$$x_2 = 1 - hj \text{ (Denominator of convergency of Beam 3)}, \qquad (39e)$$

$$z_1 = \frac{x_1}{de} - \frac{fgx_1}{dex_2} - 1 \text{ (Modified denominator of convergency[7] of Carry-over Series } \alpha, \beta, \delta \text{ and } \gamma. \qquad (39f)$$

The above procedure is repeated for the fixed end moments at Points C, D, E, F and G. All moment equations similarly derived are presented in Section 2, Table C6.

---

[7]The actual denominator of convergency of the Carry-over Series α, β, δ and γ is

$$1 - \frac{de}{x_1} - \frac{fg}{x_2}.$$

In order to simplify the algebraic form of the final equations, modified terms are used.

$$\frac{1 - \frac{de}{x_1} - \frac{fg}{x_2}}{\frac{de}{x_1}} = \frac{x_1}{de} - 1 - \frac{fgx_1}{dex_2} = z_1,$$

$$\frac{1 - \frac{de}{x_1} - \frac{fg}{x_2}}{\frac{fg}{x_2}} = \frac{x_2}{fg} - \frac{dex_2}{fgx_1} - 1 = z_2.$$

7. CARRY-OVER SERIES α AND β FOR A SEVEN SPAN BEAM

The derivation of the end moment equations for the seven span cyclo-symmetrical beam are considerably like those which are presented in Chapter Five. The beam will be loaded and divided as shown in Figs. 11, 12, and 13.

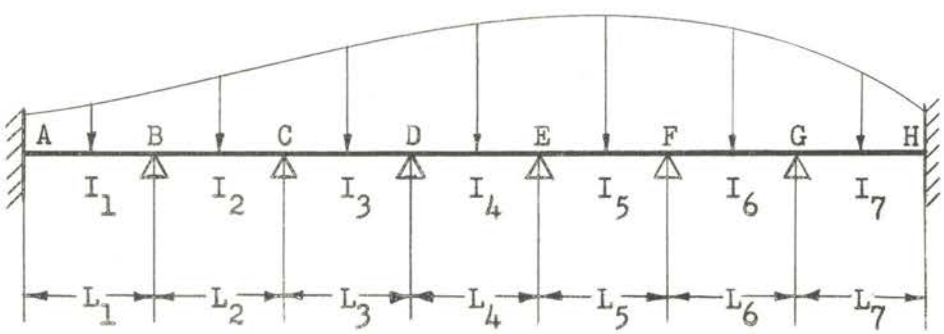

Fig. 11
SEVEN SPAN BEAM

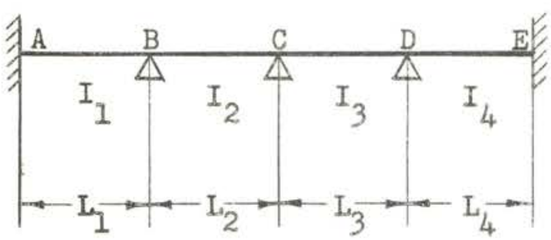

Fig. 12
ISOLATED BEAM 1

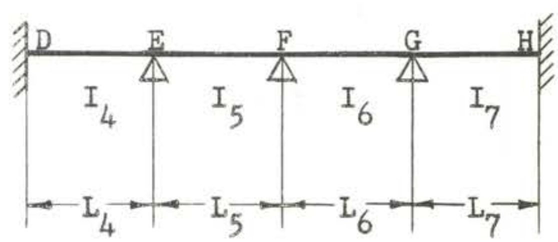

Fig. 13
ISOLATED BEAM 2

Tables 18, 19, 20, 21, and 22 are formed by the same procedure that was used in Chapter Five.

TABLE 18

ALGEBRAIC DISTRIBUTION OF FIXED END MOMENTS FOR A SEVEN SPAN BEAM
(ISOLATED BEAM 1)

Points:	A	B		C		D		E
Distribution Factors:	0	2a	2b	2c	2d	2e	2f	0
Fixed End Moments:	$-FM_{AB}$	FM_{BA}	$-FM_{BC}$					
1st Cycle		$-2aB$	$-2bB$					
Carry-over	$-aB$			$-bB$				
2nd Cycle				$2bcB$	$2bdB$			
Carry-over			bcB			bdB		
3rd Cycle		$-2abcB$	$-2b^2cB$			$-2bdeB$	$-2bdfB$	
Carry-over	$-abcB$			$-b^2cB$	$-bdeB$			$-bdfB$
4th Cycle[8]				$2bctB$	$2bdtB$			
Carry-over			$bctB$			$bdtB$		
5th Cycle		$-2abctB$	$-2b^2ctB$			$-2bdetB$	$-2bdftB$	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}	M_{DE}	M_{ED}

[8] Substituting $bc + de = t$.

TABLE 19

ALGEBRAIC DISTRIBUTION OF FUNCTION α_o IN A SEVEN SPAN BEAM (ISOLATED BEAM 2)

Points:	D	E		F		G		H
Distribution Factors:	0	2g	2h	2j	2k	2m	2n	0
Starting Moments:		α_o						
1st Cycle		$-2g\alpha_o$	$-2h\alpha_o$					
Carry-over	$-g\alpha_o$			$-h\alpha_o$				
2nd Cycle				$2hj\alpha_o$	$2hk\alpha_o$			
Carry-over			$hj\alpha_o$			$hk\alpha_o$		
3rd Cycle		$-2ghj\alpha_o$	$-2h^2j\alpha_o$			$-2hkm\alpha_o$	$-2hkn\alpha_o$	
Carry-over	$-ghj\alpha_o$			$-h^2j\alpha_o$	$-hkm\alpha_o$			$-hkn\alpha_o$
4th Cycle [9]				$2hjv\alpha_o$	$2hkv\alpha_o$			
Carry-over			$hjv\alpha_o$			$hkv\alpha_o$		
5th Cycle		$-2gh jv\alpha_o$	$-2h^2 jv\alpha_o$			$-2hk lv\alpha_o$	$-2hk mv\alpha_o$	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{DE}	M_{ED}	M_{EF}	M_{FE}	M_{FG}	M_{GF}	M_{GH}	M_{HG}

[9] Substituting $hj + km = v$.

TABLE 20

ALGEBRAIC DISTRIBUTION OF FUNCTION β_0 IN A SEVEN SPAN BEAM
(ISOLATED BEAM 1)

Points:	A		B		C		D	E
Distribution Factors:	0	2a	2b	2c	2d	2e	2f	0
Starting Moments:							β_0	
1st Cycle						$-2e\beta_0$	$-2f\beta_0$	
Carry-over					$-e\beta_0$			$-f\beta_0$
2nd Cycle				$2ec\beta_0$	$2de\beta_0$			
Carry-over			$ce\beta_0$			$de\beta_0$		
3rd Cycle		$-2ace\beta_0$	$-2bce\beta_0$			$-2de^2\beta_0$	$-2def\beta_0$	
Carry-over	$-ace\beta_0$			$-bce\beta_0$	$-de^2\beta_0$			$-def\beta_0$
4th Cycle				$2cet\beta_0$	$2det\beta_0$			
Carry-over			$cet\beta_0$			$det\beta_0$		
5th Cycle		$-2acet\beta_0$	$-2bcet\beta_0$			$-2de^2t\beta_0$	$-2deft\beta_0$	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}	M_{DE}	M_{ED}

TABLE 21

SERIES α AND β AS A FUNCTION OF α FOR A SEVEN SPAN BEAM

Series α - Function α	Series β - Function α
$\alpha_0 = \left(\frac{fgy_1y_4}{x_1x_2}\right)^0 \alpha$	$\beta_0 = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^0 \frac{gy_4}{x_2} \alpha$
$\alpha_1 = \left(\frac{fgy_1y_4}{x_1x_2}\right)^1 \alpha$	$\beta_1 = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^1 \frac{gy_4}{x_2} \alpha$
$\alpha_2 = \left(\frac{fgy_1y_4}{x_1x_2}\right)^2 \alpha$	$\beta_2 = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^2 \frac{gy_4}{x_2} \alpha$
$\alpha_n = \left(\frac{fgy_1y_4}{x_1x_2}\right)^n \alpha$	$\beta_n = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^n \frac{gy_4}{x_2} \alpha$
$\sum_0^\infty \alpha = \frac{\alpha}{1 - \frac{fgy_1y_4}{x_1x_2}}$	$\sum_0^\infty \beta = -\frac{\frac{gy_4}{x_2} \alpha}{1 - \frac{fgy_1y_4}{x_1x_2}}$

TABLE 22

SERIES α AND β AS A FUNCTION OF β FOR A SEVEN SPAN BEAM

Series α - Function β	Series β - Function β
$\alpha_0 = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^0 \frac{fy_1}{x_1} \beta$	$\beta_0 = \left(\frac{fgy_1y_4}{x_1x_2}\right)^0 \beta$
$\alpha_1 = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^1 \frac{fy_1}{x_1} \beta$	$\beta_1 = \left(\frac{fgy_1y_4}{x_1x_2}\right)^1 \beta$
$\alpha_2 = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^2 \frac{fy_1}{x_1} \beta$	$\beta_2 = \left(\frac{fgy_1y_4}{x_1x_2}\right)^2 \beta$
$\alpha_n = -\left(\frac{fgy_1y_4}{x_1x_2}\right)^n \frac{fy_1}{x_1} \beta$	$\beta_n = \left(\frac{fgy_1y_4}{x_1x_2}\right)^n \beta$
$\sum_0^\infty \alpha = -\frac{\frac{fy_1}{x_1} \beta}{1 - \frac{fgy_1y_4}{x_1x_2}}$	$\sum_0^\infty \beta = \frac{\beta}{1 - \frac{fgy_1y_4}{x_1x_2}}$

Combining the terms of Tables 18, 20 and 21, the moment M_{AB} is

$$M_{AB} = - FM_{AB} - \frac{a}{x_1}\left(y_2 + \frac{bcde}{y_1 z}\right), \tag{40}$$

in which

$$x_1 = 1 - bc - de, \tag{40a}$$

$$x_2 = 1 - hj - km, \tag{40b}$$

$$y_1 = 1 - bc, \tag{40c}$$

$$y_2 = 1 - de, \tag{40d}$$

$$y_3 = 1 - hj, \tag{40e}$$

$$y_4 = 1 - km, \tag{40f}$$

and

$$z = \frac{x_1 x_2}{fgy_1 y_4} - 1 \text{ (Modified denominator of convergency of Carry-over Series } \alpha \text{ and } \beta\text{).}^{(10)} \tag{40g}$$

All moment equations similarly derived are presented in Section 2, Table C7.

---

[10] The actual denominator of convergency of the Carry-over Series α and β is

$$1 - \frac{fgy_1 y_4}{x_1 x_2}.$$

In order to simplify the final equation, a modified term is used.

$$\frac{1 - \frac{fgy_1 y_4}{x_1 x_2}}{\frac{fgy_1 y_4}{x_1 x_2}} = \frac{x_1 x_2}{fgy_1 y_4} - 1 = z.$$

8. CARRY-OVER SERIES α, β, δ AND γ FOR AN EIGHT SPAN BEAM

As a final illustration of the Carry-over Series, the end moment equations will be derived for an eight span cyclosymmetrical beam, loaded as shown in Fig. 14.

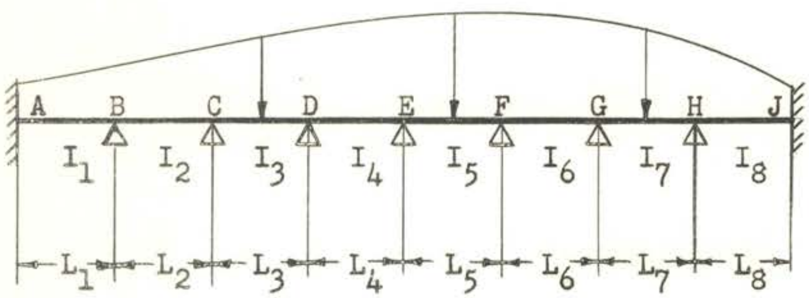

Fig. 14
EIGHT SPAN BEAM

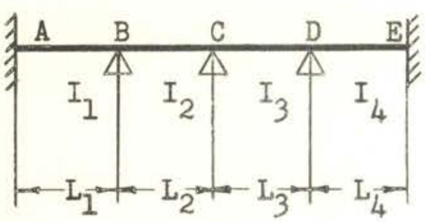

Fig. 15
ISOLATED BEAM 1

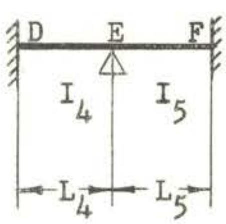

Fig. 16
ISOLATED BEAM 2

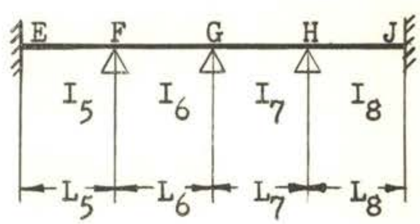

Fig. 17
ISOLATED BEAM 3

Since the series would become complicated if beams of more than four spans were isolated, it is necessary to divide the eight span beam into two four span sections and a two span section, as shown in Figs. 15, 16 and 17.

TABLE 23

ALGEBRAIC DISTRIBUTION OF FIXED END MOMENTS FOR AN EIGHT SPAN BEAM
(ISOLATED BEAM 1)

Points:	A		B		C		D	E
Distribution Factors:	0	2a	2b	2c	2d	2e	2f	0
Fixed End Moments:	$-FM_{AB}$	FM_{BA}	$-FM_{BC}$					
1st Cycle		-2aB	-2bB					
Carry-over	-aB			-bB				
2nd Cycle				2bcB	2bdB			
Carry-over			bcB			bdB		
3rd Cycle		-2abcB	$-2b^2cB$			-2bdeB	-2bdfB	
Carry-over	-abcB			$-b^2cB$	-bdeB			-bdfB
4th Cycle[11]				2bctB	2bdtB			
Carry-over			bctB			bdtB		
5th Cycle		-2ab ctB	$-2b^2$ ctB			-2bd etB	-2bd ftB	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}	M_{DE}	M_{ED}

[11] Substituting bc + de = t.

Using nomenclature similar to that of the previous chapters and temporarily locking Point E, the moments in Beam 1 due to the fixed end moments at Points A and B are (Table 23)

$$M_{AB0} = -FM_{AB} - \frac{ay}{x_1}2B, \tag{41}$$

$$M_{BA0} = FM_{BA} - 2\frac{ay}{x_1}2B, \tag{42}$$

$$M_{BC0} = -FM_{BA} + 2\frac{ay}{x_1}2B, \tag{43}$$

$$M_{CB0} = -\frac{bdu}{x_1}2B, \tag{44}$$

$$M_{CD0} = \frac{bdu}{x_1}2B, \tag{45}$$

$$M_{DC0} = 2\frac{bdf}{x_1}B, \tag{46}$$

$$M_{DE0} = -2\frac{bdf}{x_1}B, \tag{47}$$

$$M_{ED0} = -\frac{bdf}{x_1}B. \tag{48}$$

Unlocking Point E and locking Points D and F, a second beam is isolated (Beam 2). Denoting

$$M_{ED0} = -\frac{bdf}{x_1}B = \alpha_0, \tag{49}$$

and summing the values in the various columns of Table 24

$$M_{DE1} = -g\alpha_0, \tag{50}$$

$$M_{ED1} = 2h\alpha_0, \tag{51}$$

$$M_{EF1} = -2h\alpha_0, \tag{52}$$

$$M_{FE1} = -h\alpha_0. \tag{53}$$

TABLE 24

ALGEBRAIC DISTRIBUTION OF FUNCTION α_0 IN AN EIGHT SPAN BEAM (ISOLATED BEAM 2)

Points:	D	E		F
Distribution Factors:	0	2g	2h	0
Starting Moments:		α_0		
1st Cycle		$-2g\alpha_0$	$-2h\alpha_0$	
Carry-over	$-g\alpha_0$			$-h\alpha_0$

TABLE 25

ALGEBRAIC DISTRIBUTION OF FUNCTION β_0 IN AN EIGHT SPAN BEAM
(ISOLATED BEAM 1)

Points:	A		B		C		D	E
Distribution Factors:	0	2a	2b	2c	2d	2e	2f	0
Starting Moments:							β_0	
1st Cycle						$-2e\beta_0$	$-2f\beta_0$	
Carry-over					$-e\beta_0$			$-f\beta_0$
2nd Cycle				$2ce\beta_0$	$2de\beta_0$			
Carry-over			$ce\beta_0$			$de\beta_0$		
3rd Cycle		$-2ace\beta_0$	$-2bce\beta_0$			$-2de^2\beta_0$	$-2def\beta_0$	
Carry-over	$-ace\beta_0$			$-bce\beta_0$	$-de^2\beta_0$			$-def\beta_0$
4th Cycle				$2cet\beta_0$	$2det\beta_0$			
Carry-over			$cet\beta_0$			$det\beta_0$		
5th Cycle		$-2acet\beta_0$	$-2bcet\beta_0$			$-2de^2t\beta_0$	$-2deft\beta_0$	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}	M_{DE}	M_{ED}

TABLE 26

ALGEBRAIC DISTRIBUTION OF FUNCTION γ_0 IN AN EIGHT SPAN BEAM (ISOLATED BEAM 3)

Points:	E	F		G		H		J
Distribution Factors:	0	2j	2k	2m	2n	2p	2r	0
Starting Moments:		γ_0						
1st Cycle		$-2j\gamma_0$	$-2k\gamma_0$					
Carry-over	$-j\gamma_0$			$-k\gamma_0$				
2nd Cycle				$2km\gamma_0$	$2kn\gamma_0$			
Carry-over			$km\gamma_0$			$kn\gamma_0$		
3rd Cycle		$-2jkm\gamma_0$	$-2k^2m\gamma_0$			$-2knp\gamma_0$	$-2knr\gamma_0$	
Carry-over	$-jkm\gamma_0$			$-k^2m\gamma_0$	$-knp\gamma_0$			$-knr\gamma_0$
4th Cycle[12]				$2kmw\gamma_0$	$2knw\gamma_0$			
Carry-over			$kmw\gamma_0$			$knw\gamma_0$		
5th Cycle		$-2jkmw\gamma_0$	$-2k^2mw\gamma_0$			$-2knpw\gamma_0$	$-2knrw\gamma_0$	
Infinite Cycle	0	0	0	0	0	0	0	0
Sum of all the terms in each column = Final Moment	M_{EF}	M_{FE}	M_{FG}	M_{GF}	M_{GH}	M_{HG}	M_{HJ}	M_{JH}

[12] Substituting $km + np = w$.

Unlocking Points D and F and locking Point E, Beam 1 is again isolated, but also isolated is Beam 3. The following moments are denoted

$$M_{DE_1} = - g\alpha_0 = \beta_0, \tag{54}$$

and

$$M_{FE_1} = - h\alpha_0 = \gamma_0. \tag{55}$$

The carry-over values from Tables 25 and 26 are

$$M_{ED_2} = - \frac{fy_1}{x_1}\beta_0 = \alpha_1, \tag{56}$$

$$M_{EF_2} = - \frac{jy_4}{x_2}\gamma_0 = \delta_0. \tag{57}$$

TABLE 27

ALGEBRAIC DISTRIBUTION OF FUNCTION δ_0 IN AN EIGHT SPAN BEAM (ISOLATED BEAM 2)

Points:	D	E	E	F
Distribution Factors:	0	2g	2h	0
Starting Moments:			δ_0	
1st Cycle		$-2g\ \delta_0$	$-2h\ \delta_0$	
Carry-over	$-g\ \delta_0$			$-h\ \delta_0$

The Points D, E and F are locked and unlocked systematically and the values of α, β, δ and γ are tabulated (Tables 28, 29, 30 and 31). Superimposing the Basic Series (Eq. 41) and the Carry-over Series recently derived, the moment M_{AB} due to the fixed end moments at Points A and B is

$$M_{AB} = - FM_{AB} - \frac{ay_2}{x_1}B - \frac{ace}{x_1}\beta_0 - \frac{ace}{x_1}\beta_1 - \frac{ace}{x_1}\beta_2 - \ldots\ldots - 0 \tag{58a}$$

or

$$M_{AB} = - FM_{AB} - \frac{ay_2}{x_1}B - \frac{ace}{x_1}\sum_0^\infty \beta , \tag{58b}$$

in which (Table 28)

$$\sum_0^\infty \beta = \frac{- g\alpha}{1 - \frac{fgy_1}{x_1} \frac{hjy_4}{x_2}}. \tag{58c}$$

TABLE 28

SERIES α, β, δ AND γ AS A FUNCTION OF α FOR AN EIGHT SPAN BEAM

Series α - Function α	Series β - Function α
$\alpha_0 = \alpha$	$\beta_0 = -g\alpha$
$\alpha_1 = \frac{fgy_1}{x_1}\alpha$	$\beta_1 = -gV\alpha$
$\alpha_2 = \frac{fgy_1}{x_1}V\alpha$ (13)	$\beta_2 = -gV^2\alpha$
$\alpha_n = \frac{fgy_1}{x_1}V^{n-1}\alpha$	$\beta_n = -gV^n\alpha$
$\sum_0^\infty \alpha = \frac{\alpha}{1-V}(1 - \frac{hjy_4}{x_2})$	$\sum_0^\infty \beta = \frac{-g\alpha}{1-V}$
Series δ - Function α	Series γ - Function α
$\delta_0 = \frac{hjy_4}{x_2}\alpha$	$\gamma_0 = -h\alpha$
$\delta_1 = \frac{hjy_4}{x_2}V\alpha$	$\gamma_1 = -hV\alpha$
$\delta_2 = \frac{hjy_4}{x_2}V^2\alpha$	$\gamma_2 = -hV^2\alpha$
$\delta_n = \frac{hjy_4}{x_2}V^n\alpha$	$\gamma_n = -hV^n\alpha$
$\sum_0^\infty \delta = \frac{\alpha}{1-V}\frac{hjy_4}{x_2}$	$\sum_0^\infty \gamma = \frac{-h\alpha}{1-V}$

[13] Substituting $\frac{fgy_1}{x_1} + \frac{hjy_4}{x_2} = V$.

TABLE 29

SERIES α, β, δ AND γ AS A FUNCTION OF β FOR AN EIGHT SPAN BEAM

Series α - Function β	Series β - Function β
$\alpha_0 = -\frac{fy_1}{x_1}\beta$	$\beta_0 = \beta$
$\alpha_1 = -\frac{f^2gy_1^2}{x_1^2}\beta$	$\beta_1 = \frac{fgy_1}{x_1}\beta$
$\alpha_2 = -\frac{f^2gy_1^2}{x_1^2}V\beta$	$\beta_2 = \frac{fgy_1}{x_1}V\beta$
$\alpha_n = -\frac{f^2gy_1^2}{x_1^2}V^{n-1}\beta$	$\beta_n = \frac{fgy_1}{x_1}V^{n-1}\beta$
$\sum_0^\infty \alpha = \frac{-fy_1\beta}{x_1(1-V)}(1 - \frac{hjy_4}{x_2})$	$\sum_0^\infty \beta = \frac{\beta}{1-V}(1 - \frac{hjy_4}{x_2})$

Series δ - Function β	Series γ - Function β
$\delta_0 = -\frac{fhjy_1y_4}{x_1x_2}\beta$	$\gamma_0 = \frac{fhy_1}{x_1}\beta$
$\delta_1 = -\frac{fhjy_1y_4}{x_1x_2}V\beta$	$\gamma_1 = \frac{fhy_1}{x_1}V\beta$
$\delta_2 = -\frac{fhjy_1y_4}{x_1x_2}V^2\beta$	$\gamma_2 = \frac{fhy_1}{x_1}V^2\beta$
$\delta_n = -\frac{fhjy_1y_4}{x_1x_2}V^n\beta$	$\gamma_n = \frac{fhy_1}{x_1}V^n\beta$
$\sum_0^\infty \delta = -\frac{fhjy_1y_4\beta}{x_1x_2(1-V)}$	$\sum_0^\infty \gamma = \frac{fhy_1\beta}{x_1(1-V)}$

TABLE 30

SERIES α, β, δ AND γ AS A FUNCTION OF δ FOR AN EIGHT SPAN BEAM

Series α - Function δ	Series β - Function δ
$\alpha_0 = \frac{fgy_1}{x_1} \delta$	$\beta_0 = - g \delta$
$\alpha_1 = \frac{fgy_1}{x_1} V \delta$	$\beta_1 = - g V \delta$
$\alpha_2 = \frac{fgy_1}{x_1} V^2 \delta$	$\beta_2 = - g V^2 \delta$
$\alpha_n = \frac{fgy_1}{x_1} V^n \delta$	$\beta_n = - g V^n \delta$
$\sum_0^\infty \alpha = \frac{fgy_1 \delta}{x_1(1 - V)}$	$\sum_0^\infty \beta = \frac{- g \delta}{1 - V}$

Series δ - Function δ	Series γ - Function δ
$\delta_0 = \delta$	$\gamma_0 = - h \delta$
$\delta_1 = \frac{hjy_4}{x_2} \delta$	$\gamma_1 = - h V \delta$
$\delta_2 = \frac{hjy_4}{x_2} V \delta$	$\gamma_2 = - h V^2 \delta$
$\delta_n = \frac{hjy_4}{x_2} V^{n-1} \delta$	$\gamma_n = - h V^n \delta$
$\sum_0^\infty \delta = \frac{\delta}{1 - V}(1 - \frac{fgy_1}{x_1})$	$\sum_0^\infty \gamma = \frac{- h \delta}{1 - V}$

TABLE 31

SERIES α, β, δ AND γ AS A FUNCTION OF Y FOR AN EIGHT SPAN BEAM

Series α - Function Y	Series β - Function Y
$\alpha_0 = -\frac{fgjy_1y_4}{x_1x_2} Y$	$\beta_0 = \frac{gjy_4}{x_2} Y$
$\alpha_1 = -\frac{fgjy_1y_4}{x_1x_2} V Y$	$\beta_1 = \frac{gjy_4}{x_2} V Y$
$\alpha_2 = -\frac{fgjy_1y_4}{x_1x_2} V^2 Y$	$\beta_2 = \frac{gjy_4}{x_2} V^2 Y$
$\alpha_n = -\frac{fgjy_1y_4}{x_1x_2} V^n Y$	$\beta_n = \frac{gjy_4}{x_2} V^n Y$
$\sum_0^{\infty} \alpha = -\frac{fgjy_1y_4 Y}{x_1x_2(1 - V)}$	$\sum_0^{\infty} \beta = \frac{gjy_4 Y}{x_2(1 - V)}$

Series δ - Function Y	Series γ - Function Y
$\delta_0 = -\frac{jy_4}{x} Y$	$\gamma_0 = Y$
$\delta_1 = -\frac{hj^2y_4^2}{x_2^2} Y$	$\gamma_1 = \frac{hjy_4}{x_2} Y$
$\delta_2 = -\frac{hj^2y_4^2}{x_2^2} V Y$	$\gamma_2 = \frac{hjy_4}{x_2} V Y$
$\delta_n = -\frac{hj^2y_4^2}{x_2^2} V^{n-1} Y$	$\gamma_n = \frac{hjy_4}{x_2} V^{n-1} Y$
$\sum_0^{\infty} \delta = \frac{-jy_4 Y}{x_2(1 - V)}(1 - \frac{fgy_1}{x_1})$	$\sum_0^{\infty} \gamma = \frac{Y}{1 - V}(1 - \frac{fgy_1}{x_1})$

With notation 49, equation 58b becomes

$$M_{AB} = - FM_{AB} - \frac{a}{x_1}(y_2 + \frac{bcde}{y_1 z_1})\ B, \tag{58}$$

in which

$$x_1 = 1 - bc - de, \tag{58d}$$

$$x_2 = 1 - km - np, \tag{58e}$$

$$y_1 = 1 - bc, \tag{58f}$$

$$y_2 = 1 - de, \tag{58g}$$

$$y_3 = 1 - km, \tag{58h}$$

$$y_4 = 1 - np, \tag{58j}$$

and

$$z_1 = \frac{x_1}{fgy_1} - \frac{hjy_4 x_1}{fgx_2 y_1} - 1$$ (Modified denominator of convergency[14] of Carry-over Series α, β, δ and γ). (58k)

The above procedure is repeated for the fixed end moments at Points C, D, E, F, G, H and J. All moment equations similarly derived are presented in Section 2, Table C8.

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[14]The actual denominator of convergency of the Carry-over Series α, β, δ and γ is

$$1 - \frac{fgy_1}{x_1} - \frac{hjy_4}{x_2}.$$

In order to simplify the algebraic form of the final equations, modified terms are used.

$$\frac{1 - \frac{fgy_1}{x_1} - \frac{hjy_4}{x_2}}{\frac{fgy_1}{x_1}} = \frac{x_1}{fgy_1} - 1 - \frac{hjy_4 x_1}{fgy_1 x_2} = z_1,$$

$$\frac{1 - \frac{fgy_1}{x_1} - \frac{hjy_4}{x_2}}{\frac{hjy_4}{x_2}} = \frac{x_2}{hjy_4} - 1 - \frac{fgy_1 x_2}{hjy_4 x_1} = z_2.$$

9. FINAL CONCLUSIONS

A general procedure for the derivation of the end moments by moment distribution has been demonstrated by using the summations of infinite, convergent, geometric series. The results are exact, direct and general. The final moments are functions of two separate and independent variables:

1. The New Distribution Factor which is a function of the stiffness factors and the carry-over factors,
2. The Fixed End Moment which is a function of the applied loads or the performed deformation.

In general, the New Distribution Factor consists of two classes of infinite series:

1. Basic Series,
2. Carry-over Series.

The Basic Series are of two types:

1. Three Span Basic Series,
2. Four Span Basic Series.

The Carry-over Series are of two types:

1. Carry-over Series from left to right,
2. Carry-over Series from right to left.

The Basic Series consists of an infinite number of terms converging to zero and each term is a finite number.

$$\text{Basic Series} = Ar^0 + Ar^1 + Ar^2 + Ar^3 + \ldots\ldots\ldots\ldots\ldots + 0. \quad (59a)$$

The Carry-over Series also consists of an infinite number of terms converging to zero. Each term in this series is not finite; it is another infinite series.

$$\text{Carry-over Series} \begin{cases} Cr^0s^0 + Cr^1s^0 + Cr^2s^0 + Cr^3s^0 + \ldots\ldots\ldots + 0, \\ Cr^0s^1 + Cr^1s^1 + Cr^2s^1 + Cr^3s^1 + \ldots\ldots\ldots + 0, \\ Cr^0s^2 + Cr^1s^2 + Cr^2s^2 + Cr^3s^2 + \ldots\ldots\ldots + 0, \\ Cr^0s^3 + Cr^1s^3 + Cr^2s^3 + Cr^3s^3 + \ldots\ldots\ldots + 0, \\ \circ \quad \circ \quad \circ \quad \circ \quad 0, \\ \circ \quad \circ \quad \circ \quad \circ \quad 0, \\ \circ \quad \circ \quad \circ \quad \circ \quad 0, \\ 0 \quad 0 \quad 0 \quad 0 \quad 0. \end{cases} \tag{60a}$$

The sum of the Basic Series is the simple sum of an infinite, convergent, geometric series.

$$\sum_{0}^{\infty} A = \frac{A}{1 - r}. \tag{59}$$

The sum of the Carry-over Series is the sum of an infinite series of infinite series.

$$\sum_{0}^{\infty} C = \frac{C}{1 - s} r^0 + \frac{C}{1 - s} r^1 + \frac{C}{1 - s} r^2 + \ldots\ldots\ldots + 0, \tag{60b}$$

$$= \frac{C}{(1 - s)(1 - r)}. \tag{60}$$

According to the performed investigations, the continuous beams may be divided into two groups:

1. Three and four span beams containing New Distribution Factors formed by the Basic Series;
2. Five and more span beams containing New Distribution Factors formed by the Basic Series and the Carry-over Series.

Although the investigation was limited to continuous beams of three, four, five, six, seven and eight spans having a constant moment of inertia, it is evident that the principles are valid for all cases in which the Fixed End Moments may be distributed or the end angle changes may be balanced. Any beam may be divided into an integral number of two, three and four span sections and the Carry-over Series tabulated. In many cases it becomes necessary to decide which division is the most appropriate. If the isolated sections are symmetrical with respect to the center of the beam, the derivations are simpler and the resulting equations are more concise than if the isolated sections are unbalanced. It must be noted that no matter how the beam is divided, the results will always be the same, although the algebraic form may differ.

Section 2

(Courtesy of Texas Highway Department)

California Street Overpass on U.S. Highway 77 in Gainesville, Texas.

SECTION 2

1. GENERAL NOTES

The moment tables presented in Section 2 are divided into the following groups:

Group 3 - Three Span Beams,

Group 4 - Four Span Beams,

Group 5 - Five Span Beams,

Group 6 - Six Span Beams,

Group 7 - Seven Span Beams,

Group 8 - Eight Span Beams.

Each group is subdivided into three subgroups:

1. Equal Span Beams,
2. Symmetrical Beams,
3. Cyclosymmetrical Beams.

Each table is composed of the following parts:

1. Description of the beam and structural identities,
2. Illustration of beam,
3. Constant functions of New Distribution Factors,
4. Table of final moments.

In the application of the following tables, follow this procedure:

1. Select the table for the case to be investigated,
2. Determine all the stiffness factors,
3. Determine all the required functions of the New Distribution Factors (a,b,c,d .... x,y and z),

4. Determine all the required load functions (fixed end moments, sums of the fixed end moments at the points of support or the applied couples),

5. Substitute all the computed values into the Table of Final Moments,

6. Determine the final moments from the Table of Final Moments. For example, from Table C3, the final moment

$$M_{AB} = - FM_{AB} - \frac{a}{x}(FM_{BA} - FM_{BC}) - \frac{ac}{x}(FM_{CB} - FM_{CD}).$$

The proper sign is incorporated in each formula in every table, based on the principles defined in "Nomenclature".

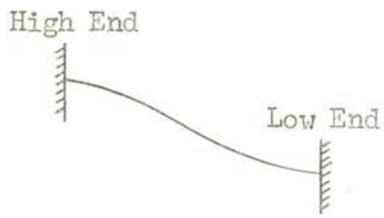
The tables may be used to compute the end moments due to the following:

1. Static or moving loads,
2. Rotations at the points of support,
3. Displacements of the supports.

All the tables were prepared to be used mainly for the computation of the end moments due to static or moving loads; therefore, the correct signs for each fixed end moment are included in each formula. Every fixed end moment is to be taken as positive when substituting into the Moment Table and the table will algebraically work out the proper sign.

If the end moments due to applied couples at the points of support are desired, the fixed end moments in the second vertical column of the selected table are placed equal to zero. The sum of the fixed end moments at the point of support where the couple is applied, is replaced by the numerical value of that couple. All couples are positive if they act in a clockwise direction, and negative if they act in a counterclockwise direction.

Sometimes the end moments due to the displacements of the supports are desired. In cases of this type a more complicated rule of signs must be followed.



1. The fixed end moment of the high end is to be taken as positive.
2. The fixed end moment of the low end is to be taken as negative

The above rules can be verified by basic relationships. If all the tables were derived for transverse loads and all the signs of the fixed end moments have been included in each formula, the multiplication factor (-1) has to be used to correct the sign for the low end. This modification must be used throughout the given table.

The tables were prepared for continuous beams with fixed ends, but may be modified for any end conditions. The modifications for two very common end conditions are explained in the following paragraphs.

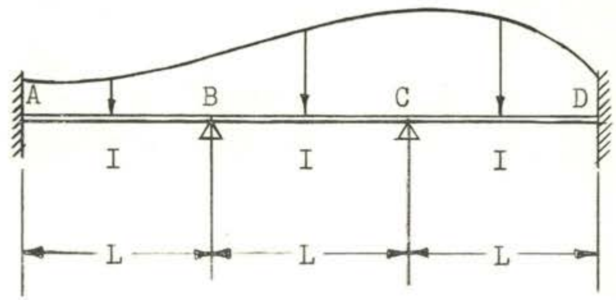
If an exterior end is freely supported, the exterior span becomes a propped beam instead of a fixed end beam. The fixed end moment at the freely supported end becomes equal to zero, and the fixed end moment at the other end of the propped span is replaced by a propped end moment. The stiffness factor of the exterior span must be replaced by a modified stiffness factor ($K' = \frac{3}{4} K$).

If the exterior span has an overhanging end, the fixed end moment at the freely supported end is equal to the cantilever moment and the fixed end moment of the other end is replaced by a propped end moment. This adjustment may be done numerically and the tabular procedure applied. The stiffness factor of the exterior span must be replaced by a modified stiffness factor ($K' = \frac{3}{4} K$).

CASE A30	THREE SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam with ends built in and freely supported at two points. Supports equally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K = \frac{I}{L}$ $\qquad a = \frac{1}{4}$

$x = \frac{15}{16}$

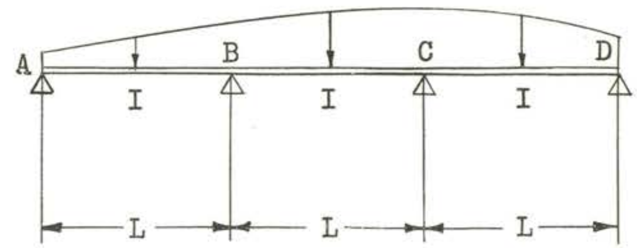
TABLE OF FINAL MOMENTS			
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$
M_{AB}	$- FM_{AB}$	- .2667	.0667
M_{BA}	FM_{BA}	- .5333	.1333
M_{BC}	$- FM_{BA}$	.5333	- .1333
M_{CB}	FM_{CD}	- .1333	.5333
M_{CD}	$- FM_{CD}$	.1333	- .5333
M_{DC}	FM_{DC}	.0667	- .2667

CASE A31	THREE SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam freely supported at four points.

Supports equally spaced.



PROPPED BEAM END MOMENTS

$$EM^{*}_{BA} = \frac{FM_{AB}}{2} + FM_{BA}$$

$$EM^{*}_{CD} = FM_{CD} + \frac{FM_{DC}}{2}$$

CONSTANT FUNCTIONS OF NDF

$$K = \frac{I}{L} \qquad K' = \frac{3I}{4L}$$

$$a = \frac{3}{14} \qquad b = \frac{2}{7}$$

$$x = \frac{45}{49}$$

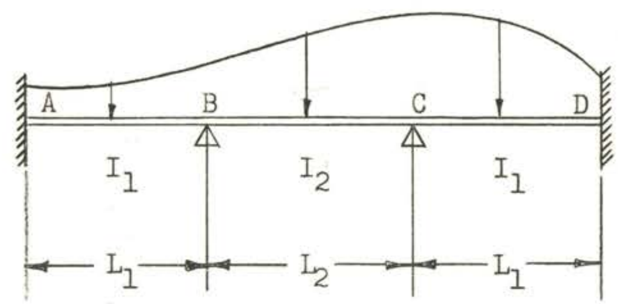
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$EM^*_{BA} - FM_{BC}$	$FM_{CB} - EM^*_{CD}$
M_{BA}	EM^*_{BA}	$-.4667$	$.1333$
M_{BC}	$-EM^*_{BA}$	$.4667$	$-.1333$
M_{CB}	EM^*_{CD}	$-.1333$	$.4667$
M_{CD}	$-EM^*_{CD}$	$.1333$	$-.4667$

CASE B3 | THREE SPAN BEAM | SYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at two points. Supports symmetrically spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$$K_1 = \frac{I_1}{L_1} \qquad K_2 = \frac{I_2}{L_2}$$

$$a = \frac{K_1}{2(K_1 + K_2)} \qquad b = \frac{K_2}{2(K_1 + K_2)}$$

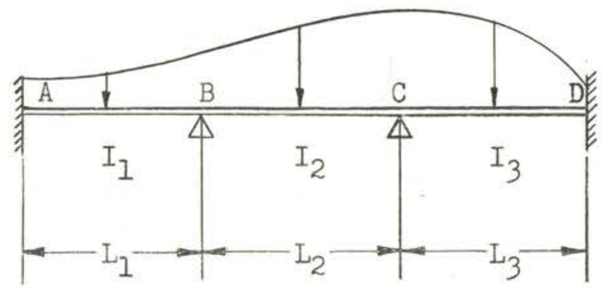
$$x = 1 - b^2$$

TABLE OF FINAL MOMENTS			
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x}$	$\frac{ab}{x}$
M_{BA}	FM_{BA}	$-2\frac{a}{x}$	$2\frac{ab}{x}$
M_{BC}	$-FM_{BA}$	$2\frac{a}{x}$	$-2\frac{ab}{x}$
M_{CB}	FM_{CD}	$-2\frac{ab}{x}$	$2\frac{a}{x}$
M_{CD}	$-FM_{CD}$	$2\frac{ab}{x}$	$-2\frac{a}{x}$
M_{DC}	FM_{DC}	$\frac{ab}{x}$	$-\frac{a}{x}$

CASE C3	THREE SPAN BEAM	CYCLOSYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at two points. Supports unequally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1}$ $\quad K_2 = \frac{I_2}{L_2}$ $\quad K_3 = \frac{I_3}{L_3}$

$a = \frac{K_1}{2(K_1 + K_2)}$ $\quad b = \frac{K_2}{2(K_1 + K_2)}$

$c = \frac{K_2}{2(K_2 + K_3)}$ $\quad d = \frac{K_3}{2(K_2 + K_3)}$

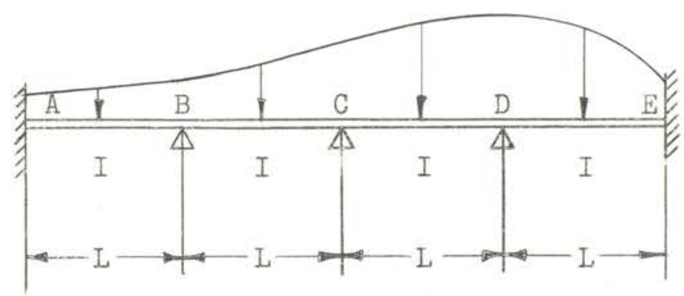
$x = 1 - bc$

TABLE OF FINAL MOMENTS			
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$
M_{AB}	$- FM_{AB}$	$- \frac{a}{x}$	$\frac{ac}{x}$
M_{BA}	FM_{BA}	$- 2 \frac{a}{x}$	$2 \frac{ac}{x}$
M_{BC}	$- FM_{BA}$	$2 \frac{a}{x}$	$- 2 \frac{ac}{x}$
M_{CB}	FM_{CD}	$- 2 \frac{bd}{x}$	$2 \frac{d}{x}$
M_{CD}	$- FM_{CD}$	$2 \frac{bd}{x}$	$- 2 \frac{d}{x}$
M_{DC}	FM_{DC}	$\frac{bd}{x}$	$- \frac{d}{x}$

CASE A40	FOUR SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam with ends built in and freely supported at three points. Supports equally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTORS

$K = \frac{I}{L}$ $\qquad a = \frac{1}{4}$

$x = \frac{7}{8}$

$y = \frac{15}{16}$

$u = \frac{7}{4}$

TABLE OF FINAL MOMENTS

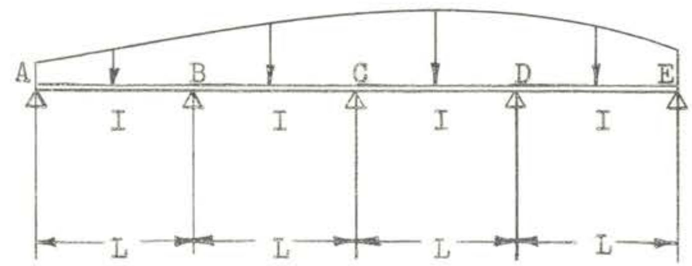
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$
M_{AB}	$-FM_{AB}$	− .2679	.0715	− .0179
M_{BA}	FM_{BA}	− .5358	.1429	− .0357
M_{BC}	$-FM_{BA}$	.5358	− .1429	.0357
M_{CB}	FM_{CB}	− .1250	− .5000	.1250
M_{CD}	$-FM_{CD}$	.1250	− .5000	− .1250
M_{DC}	FM_{DE}	.0357	− .1429	.5358
M_{DE}	$-FM_{DE}$	− .0357	.1429	− .5358
M_{ED}	FM_{ED}	− .0179	.0715	− .2679

CASE A41 | FOUR SPAN BEAM | EQUAL SPANS

DESCRIPTION:

Beam freely supported at five points.

Supports equally spaced.



PROPPED BEAM END MOMENTS

$$EM^{*}_{BA} = \frac{FM_{AB}}{2} + FM_{BA}$$

$$EM^{*}_{DE} = FM_{DE} + \frac{FM_{ED}}{2}$$

CONSTANT FUNCTIONS OF NDF

$K = \frac{I}{L}$ $K' = \frac{3I}{4L}$

$a = \frac{3}{14}$ $b = \frac{2}{7}$ $c = \frac{1}{4}$

$x = \frac{6}{7}$ $y = \frac{13}{14}$

$u = \frac{12}{7}$

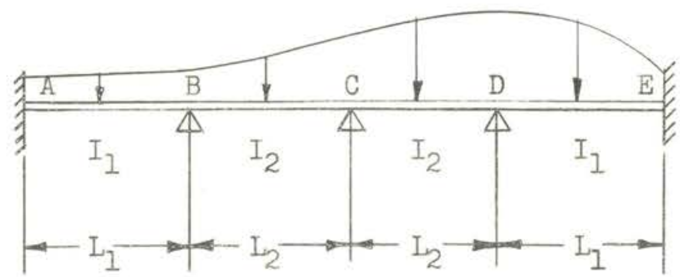
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$EM^*_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - EM^*_{DE}$
M_{BA}	EM^*_{BA}	- .4643	.1250	- .0357
M_{BC}	$- EM^*_{BA}$	.4643	- .1250	.0357
M_{CB}	FM_{CB}	- .1429	- .5000	.1429
M_{CD}	$- FM_{CD}$	.1429	- .5000	- .1429
M_{DC}	EM^*_{DE}	.0357	- .1250	.4643
M_{DE}	$- EM^*_{DE}$	- .0357	.1250	- .4643

CASE B4	FOUR SPAN BEAM	SYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at three points. Supports symmetrically spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$$K_1 = \frac{I_1}{L_1} \qquad K_2 = \frac{I_2}{L_2}$$

$$a = \frac{K_1}{2(K_1 + K_2)}$$

$$b = \frac{K_2}{2(K_1 + K_2)} \qquad c = \frac{K_2}{2(K_2 + K_2)} = \frac{1}{4}$$

$$x = 1 - \frac{b}{2}$$

$$y = 1 - \frac{b}{4}$$

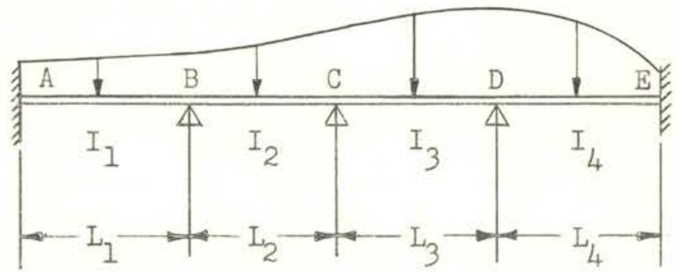
$$u = 2 - b$$

TABLE OF FINAL MOMENTS				
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$
M_{AB}	$-FM_{AB}$	$-\frac{ay}{x}$	$\frac{a}{4x}$	$-\frac{ab}{4x}$
M_{BA}	FM_{BA}	$-2\frac{ay}{x}$	$\frac{a}{2x}$	$-\frac{ab}{2x}$
M_{BC}	$-FM_{BA}$	$2\frac{ay}{x}$	$-\frac{a}{2x}$	$\frac{ab}{2x}$
M_{CB}	FM_{CB}	$-\frac{b}{2}$	$-\frac{1}{2}$	$\frac{b}{2}$
M_{CD}	$-FM_{CD}$	$\frac{b}{2}$	$-\frac{1}{2}$	$-\frac{b}{2}$
M_{DC}	FM_{DE}	$\frac{ab}{2x}$	$-\frac{a}{2x}$	$2\frac{ay}{x}$
M_{DE}	$-FM_{DE}$	$-\frac{ab}{2x}$	$\frac{a}{2x}$	$-2\frac{ay}{x}$
M_{ED}	FM_{ED}	$-\frac{ab}{4x}$	$\frac{a}{4x}$	$-\frac{ay}{x}$

CASE C4 | FOUR SPAN BEAM | CYCLOSYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at three points. Supports unequally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1}$ $K_2 = \frac{I_2}{L_2}$ $K_3 = \frac{I_3}{L_3}$ $K_4 = \frac{I_4}{L_4}$

$a = \frac{K_1}{2(K_1 + K_2)}$ $b = \frac{K_2}{2(K_1 + K_2)}$ $c = \frac{K_2}{2(K_2 + K_3)}$

$d = \frac{K_3}{2(K_2 + K_3)}$ $e = \frac{K_3}{2(K_3 + K_4)}$ $f = \frac{K_4}{2(K_3 + K_4)}$

$x = 1 - bc - de$

$y_1 = 1 - bc$

$y_2 = 1 - de$

$u_1 = 2 - b$

$u_2 = 2 - e$

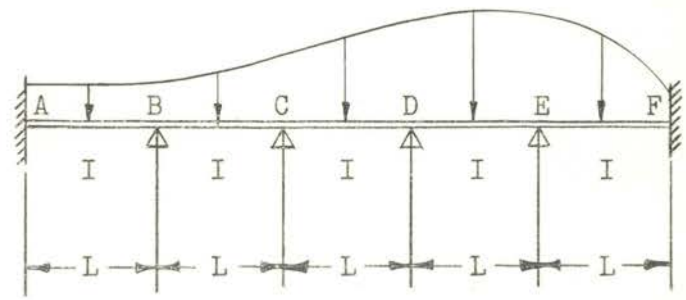
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$
M_{AB}	$-FM_{AB}$	$-\frac{ay_2}{x}$	$\frac{ac}{x}$	$-\frac{ace}{x}$
M_{BA}	FM_{BA}	$-2\frac{ay_2}{x}$	$2\frac{ac}{x}$	$-2\frac{ace}{x}$
M_{BC}	$-FM_{BA}$	$2\frac{ay_2}{x}$	$-2\frac{ac}{x}$	$2\frac{ace}{x}$
M_{CB}	FM_{CB}	$-\frac{bdu_2}{x}$	$-\frac{cu_1}{x}$	$\frac{ceu_1}{x}$
M_{CD}	$-FM_{CD}$	$\frac{bdu_2}{x}$	$-\frac{du_2}{x}$	$-\frac{ceu_1}{x}$
M_{DC}	FM_{DE}	$2\frac{bdf}{x}$	$-2\frac{df}{x}$	$2\frac{fy_1}{x}$
M_{DE}	$-FM_{DE}$	$-2\frac{bdf}{x}$	$2\frac{df}{x}$	$-2\frac{fy_1}{x}$
M_{ED}	FM_{ED}	$-\frac{bdf}{x}$	$\frac{df}{x}$	$-\frac{fy_1}{x}$

CASE A50	FIVE SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam with ends built in and freely supported at four points. Supports equally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K = \frac{I}{L}$ $a = \frac{1}{4}$

$x = \frac{15}{16}$

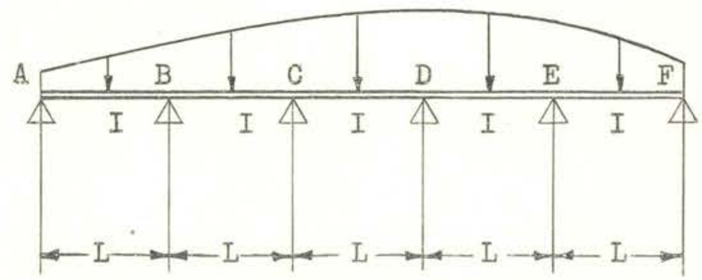
$z = \frac{209}{16}$

TABLE OF FINAL MOMENTS					
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$
M_{AB}	$- FM_{AB}$	- .2679	.0718	- .0191	.0048
M_{BA}	FM_{BA}	- .5359	.1436	- .0382	.0096
M_{BC}	$- FM_{BA}$	.5359	- .1436	.0382	- .0096
M_{CB}	FM_{CD}	- .1242	.4976	.1340	- .0335
M_{CD}	$- FM_{CD}$	.1242	- .4976	- .1340	.0335
M_{DC}	FM_{DC}	.0335	- .1340	- .4976	.1242
M_{DE}	$- FM_{DC}$	- .0335	.1340	.4976	- .1242
M_{ED}	FM_{EF}	- .0096	.0382	- .1436	.5359
M_{EF}	$- FM_{EF}$	.0096	- .0382	.1436	- .5359
M_{FE}	FM_{FE}	.0048	- .0191	.0718	- .2679

CASE A51 | FIVE SPAN BEAM | EQUAL SPANS

DESCRIPTION:

Beam freely supported at six points. Supports equally spaced.



PROPPED BEAM END MOMENTS

$$EM^{*}_{BA} = \frac{FM_{AB}}{2} + FM_{BA}$$

$$EM^{*}_{EF} = FM_{EF} + \frac{FM_{FE}}{2}$$

CONSTANT FUNCTIONS OF NDF

$K = \frac{I}{L}$ $\quad K' = \frac{3I}{4L}$

$a = \frac{3}{14}$ $\quad b = \frac{2}{7}$

$c = d = \frac{1}{4}$

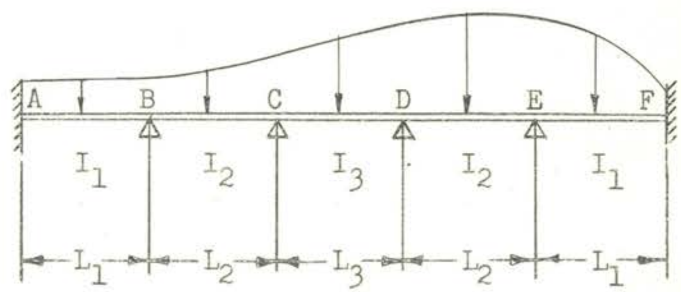
$x = \frac{13}{14}$ $\quad z = \frac{627}{49}$

TABLE OF FINAL MOMENTS					
Moment	Fixed End Moment	$EM^*_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - EM^*_{EF}$
M_{BA}	EM^*_{BA}	− 0.4641	0.1244	− 0.0334	0.0096
M_{BC}	$- EM^*_{BA}$	0.4641	− 0.1244	0.0334	− 0.0096
M_{CB}	FM_{CD}	− 0.1435	0.5024	0.1334	− 0.0383
M_{CD}	$- FM_{CD}$	0.1435	− 0.5024	− 0.1334	0.0383
M_{DC}	FM_{DC}	0.0383	− 0.1334	− 0.5024	0.1435
M_{DE}	$- FM_{DC}$	− 0.0383	0.1334	0.5024	− 0.1435
M_{ED}	EM^*_{EF}	− 0.0096	0.0334	− 0.1244	0.4641
M_{EF}	$- EM^*_{EF}$	0.0096	− 0.0334	0.1244	− 0.4641

CASE B5 | FIVE SPAN BEAM | SYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at four points. Supports symmetrically spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$$K_1 = \frac{I_1}{L_1} \qquad K_2 = \frac{I_2}{L_2} \qquad K_3 = \frac{I_3}{L_3}$$

$$a = \frac{K_1}{2(K_1 + K_2)} \qquad b = \frac{K_2}{2(K_1 + K_2)}$$

$$c = \frac{K_2}{2(K_2 + K_3)} \qquad d = \frac{K_3}{2(K_2 + K_3)}$$

$$x = 1 - bc$$

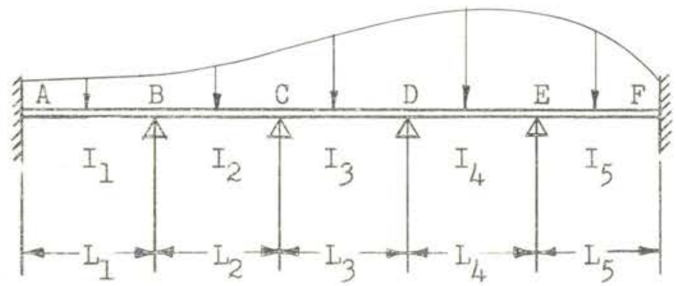
$$z = \left(\frac{x}{d}\right)^2 - 1$$

TABLE OF FINAL MOMENTS					
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x}(1 + \frac{bc}{z})$	$\frac{ac}{x}(1 + \frac{1}{z})$	$-\frac{ac}{dz}$	$\frac{abc}{dz}$
M_{BA}	FM_{BA}	$-\frac{2a}{x}(1 + \frac{bc}{z})$	$\frac{2ac}{x}(1 + \frac{1}{z})$	$-\frac{2ac}{dz}$	$\frac{2abc}{dz}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x}(1 + \frac{bc}{z})$	$-\frac{2ac}{x}(1 + \frac{1}{z})$	$\frac{2ac}{dz}$	$-\frac{2abc}{dz}$
M_{CB}	FM_{CD}	$-\frac{b}{z}(\frac{2x}{d} - 1)$	$\frac{1}{z}(\frac{2x}{d} - 1)$	$\frac{1}{z}(\frac{x}{d} - 2)$	$-\frac{b}{z}(\frac{x}{d} - 2)$
M_{CD}	$-FM_{CD}$	$\frac{b}{z}(\frac{2x}{d} - 1)$	$-\frac{1}{z}(\frac{2x}{d} - 1)$	$-\frac{1}{z}(\frac{x}{d} - 2)$	$\frac{b}{z}(\frac{x}{d} - 2)$
M_{DC}	FM_{DC}	$\frac{b}{z}(\frac{x}{d} - 2)$	$-\frac{1}{z}(\frac{x}{d} - 2)$	$-\frac{1}{z}(\frac{2x}{d} - 1)$	$\frac{b}{z}(\frac{2x}{d} - 1)$
M_{DE}	$-FM_{DC}$	$-\frac{b}{z}(\frac{x}{d} - 2)$	$\frac{1}{z}(\frac{x}{d} - 2)$	$\frac{1}{z}(\frac{2x}{d} - 1)$	$-\frac{b}{z}(\frac{2x}{d} - 1)$
M_{ED}	FM_{EF}	$-\frac{2abc}{dz}$	$\frac{2ac}{dz}$	$-\frac{2ac}{x}(1 + \frac{1}{z})$	$\frac{2a}{x}(1 + \frac{bc}{z})$
M_{EF}	$-FM_{EF}$	$\frac{2abc}{dz}$	$-\frac{2ac}{dz}$	$\frac{2ac}{x}(1 + \frac{1}{z})$	$-\frac{2a}{x}(1 + \frac{bc}{z})$
M_{FE}	FM_{FE}	$\frac{abc}{dz}$	$-\frac{ac}{dz}$	$\frac{ac}{x}(1 + \frac{1}{z})$	$-\frac{a}{x}(1 + \frac{bc}{z})$

CASE C5	FIVE SPAN BEAM	CYCLOSYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at four points. Supports unequally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1}$ $K_2 = \frac{I_2}{L_2}$ $K_3 = \frac{I_3}{L_3}$ $K_4 = \frac{I_4}{L_4}$ $K_5 = \frac{I_5}{L_5}$

$a = \frac{K_1}{2(K_1 + K_2)}$ $b = \frac{K_2}{2(K_1 + K_2)}$ $c = \frac{K_2}{2(K_2 + K_3)}$

$d = \frac{K_3}{2(K_2 + K_3)}$ $e = \frac{K_3}{2(K_3 + K_4)}$ $f = \frac{K_4}{2(K_3 + K_4)}$

$g = \frac{K_4}{2(K_4 + K_5)}$ $h = \frac{K_5}{2(K_4 + K_5)}$

$x = 1 - bc$

$y = 1 - fg$

$z = \frac{xy}{de} - 1$

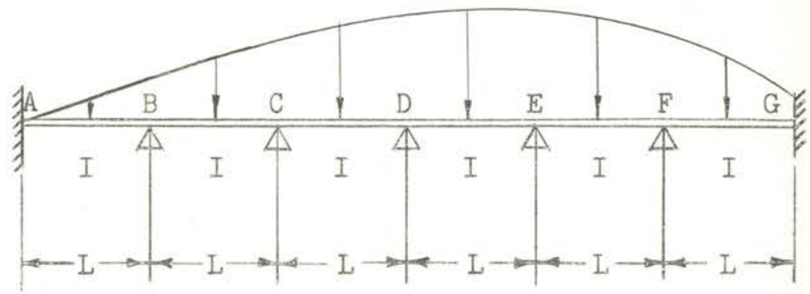
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x}(1+\frac{bc}{z})$	$\frac{ac}{x}(1+\frac{1}{z})$	$-\frac{ac}{dz}$	$\frac{acg}{dz}$
M_{BA}	FM_{BA}	$-\frac{2a}{x}(1+\frac{bc}{z})$	$\frac{2ac}{x}(1+\frac{1}{z})$	$-\frac{2ac}{dz}$	$\frac{2acg}{dz}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x}(1+\frac{bc}{z})$	$-\frac{2ac}{x}(1+\frac{1}{z})$	$\frac{2ac}{dz}$	$-\frac{2acg}{dz}$
M_{CB}	FM_{CD}	$-\frac{b}{z}(\frac{2y}{e}-1)$	$\frac{1}{z}(\frac{2y}{e}-1)$	$\frac{1}{z}(\frac{x}{d}-2)$	$-\frac{g}{z}(\frac{x}{d}-2)$
M_{CD}	$-FM_{CD}$	$\frac{b}{z}(\frac{2y}{e}-1)$	$-\frac{1}{z}(\frac{2y}{e}-1)$	$-\frac{1}{z}(\frac{x}{d}-2)$	$\frac{g}{z}(\frac{x}{d}-2)$
M_{DC}	FM_{DC}	$\frac{b}{z}(\frac{y}{e}-2)$	$-\frac{1}{z}(\frac{y}{e}-2)$	$-\frac{1}{z}(\frac{2x}{d}-1)$	$\frac{g}{z}(\frac{2x}{d}-1)$
M_{DE}	$-FM_{DC}$	$-\frac{b}{z}(\frac{y}{e}-2)$	$\frac{1}{z}(\frac{y}{e}-2)$	$\frac{1}{z}(\frac{2x}{d}-1)$	$-\frac{g}{z}(\frac{2x}{d}-1)$
M_{ED}	FM_{EF}	$-\frac{2bfh}{ez}$	$\frac{2fh}{ez}$	$-\frac{2fh}{y}(1+\frac{1}{z})$	$\frac{2h}{y}(1+\frac{fg}{z})$
M_{EF}	$-FM_{EF}$	$\frac{2bfh}{ez}$	$-\frac{2fh}{ez}$	$\frac{2fh}{y}(1+\frac{1}{z})$	$-\frac{2h}{y}(1+\frac{fg}{z})$
M_{FE}	FM_{FE}	$\frac{bfh}{ez}$	$-\frac{fh}{ez}$	$\frac{hf}{y}(1+\frac{1}{z})$	$-\frac{h}{y}(1+\frac{fg}{z})$

CASE A60 | SIX SPAN BEAM | EQUAL SPANS

DESCRIPTION:

Beam with ends built in and freely supported at five points. Supports equally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K = \frac{I}{L}$ $a = \frac{1}{4}$

$x = \frac{15}{16}$

$u = \frac{7}{4}$

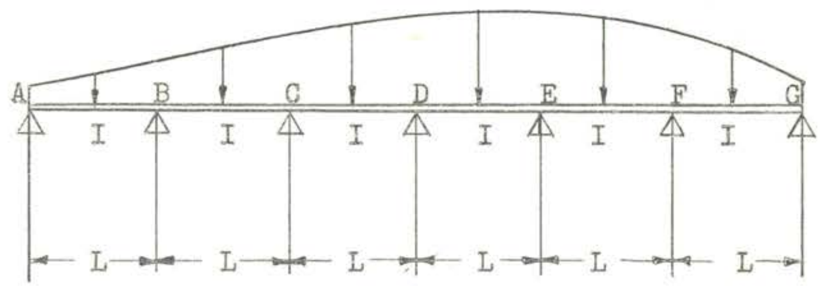
$z = 13$

TABLE OF FINAL MOMENTS						
Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$
M_{AB}	$- FM_{AB}$	- .2680	.0718	- .0192	.0051	- .0013
M_{BA}	FM_{BA}	- .5360	.1436	- .0384	.0102	- .0026
M_{BC}	$- FM_{BA}$	.5360	- .1436	.0384	- .0102	.0026
M_{CB}	FM_{CD}	- .1242	- .4973	.1345	- .0359	.0090
M_{CD}	$- FM_{CD}$	.1242	.4973	- .1345	.0359	- .0090
M_{DC}	FM_{DC}	.0333	- .1333	- .5000	.1333	- .0333
M_{DE}	$- FM_{DE}$	- .0333	.1333	- .5000	- .1333	.0333
M_{ED}	FM_{ED}	- .0090	.0359	- .1345	.4973	.1242
M_{EF}	$- FM_{ED}$	.0090	- .0359	.1345	- .4973	- .1242
M_{FE}	FM_{FG}	.0026	- .0102	.0384	- .1436	.5360
M_{FG}	$- FM_{FG}$	- .0026	.0102	- .0384	.1436	- .5360
M_{GF}	FM_{GF}	- .0013	.0051	- .0192	.0718	- .2680

CASE A61 — SIX SPAN BEAM — EQUAL SPANS

DESCRIPTION:

Beam freely supported at seven points. Supports equally spaced.



PROPPED BEAM END MOMENTS

$$EM^{*}_{BA} = \frac{FM_{AB}}{2} + FM_{BA}$$

$$EM^{*}_{FG} = FM_{FG} + \frac{FM_{GF}}{2}$$

CONSTANT FUNCTIONS OF NDF

$K = \frac{I}{L}$ $\qquad$ $K' = \frac{3K}{4L}$

$a = \frac{3}{14}$ $\qquad$ $b = \frac{2}{7}$ $\qquad$ $c = d = e = \frac{1}{4}$

$x = \frac{13}{14}$ $\qquad$ $z = \frac{90}{7}$

$u = \frac{7}{4}$

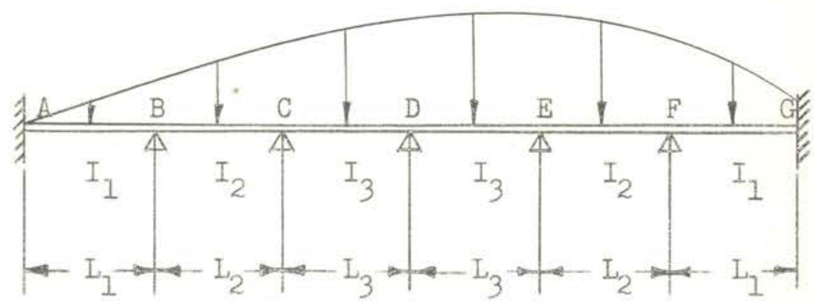
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$EM^*_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - EM^*_{FG}$
M_{BA}	EM^*_{BA}	- .4641	.1244	- .0333	.0090	- .0026
M_{BC}	$- EM^*_{BA}$	.4641	- .1244	.0333	- .0090	.0026
M_{CB}	FM_{CB}	- .1436	- .5026	.1333	- .0359	.0103
M_{CD}	$- FM_{CB}$	.1436	.5026	- .1333	.0359	- .0103
M_{DC}	FM_{DC}	.0385	- .1342	- .5000	.1342	- .0385
M_{DE}	$- FM_{DE}$	- .0385	.1342	- .5000	- .1342	.0385
M_{ED}	FM_{ED}	- .0103	.0359	- .1333	.5026	.1436
M_{EF}	$- FM_{ED}$	.0103	- .0359	.1333	- .5026	- .1436
M_{FE}	EM^*_{FG}	.0026	- .0090	.0333	- .1244	.4641
M_{FG}	$- EM^*_{FG}$	- .0026	.0090	- .0333	.1244	- .4641

CASE B6	SIX SPAN BEAM	SYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at five points. Supports symmetrically spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$$K_1 = \frac{I_1}{L_1} \qquad K_2 = \frac{I_2}{L_2} \qquad K_3 = \frac{I_3}{L_3}$$

$$a = \frac{K_1}{2(K_1 + K_2)} \qquad b = \frac{K_2}{2(K_1 + K_2)} \qquad c = \frac{K_2}{2(K_2 + K_3)}$$

$$d = \frac{K_3}{2(K_2 + K_3)} \qquad e = \frac{K_3}{2(K_3 + K_3)} = \frac{1}{4}$$

$$x = 1 - bc$$

$$u = \frac{7}{4}$$

$$z = 4\frac{x}{d} - 2$$

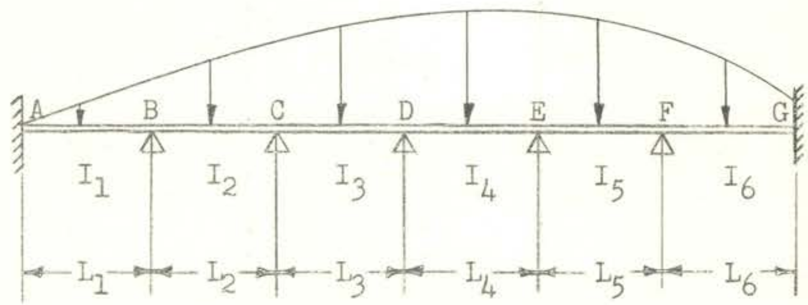
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x}(1 + \frac{bc}{z})$	$\frac{ac}{x}(1 + \frac{1}{z})$	$-\frac{ac}{dz}$	$\frac{ac}{xz}$	$-\frac{abc}{xz}$
M_{BA}	FM_{BA}	$-\frac{2a}{x}(1 + \frac{bc}{z})$	$\frac{2ac}{x}(1 + \frac{1}{z})$	$-\frac{2ac}{dz}$	$\frac{2ac}{xz}$	$-\frac{2abc}{xz}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x}(1 + \frac{bc}{z})$	$-\frac{2ac}{x}(1 + \frac{1}{z})$	$\frac{2ac}{dz}$	$-\frac{2ac}{xz}$	$\frac{2abc}{xz}$
M_{CB}	FM_{CD}	$-\frac{b}{z}(7 - \frac{2d}{x})$	$-\frac{1}{z}(7 - \frac{2d}{x})$	$\frac{1}{z}(\frac{x}{d} - 2)$	$-\frac{1}{z}(1 - \frac{2d}{x})$	$\frac{b}{z}(1 - \frac{2d}{x})$
M_{CD}	$-FM_{CD}$	$\frac{b}{z}(7 - \frac{2d}{x})$	$\frac{1}{z}(7 - \frac{2d}{x})$	$-\frac{1}{z}(\frac{x}{d} - 2)$	$\frac{1}{z}(1 - \frac{2d}{x})$	$-\frac{b}{z}(1 - \frac{2d}{x})$
M_{DC}	FM_{DC}	$\frac{bd}{2x}$	$-\frac{d}{2x}$	$-\frac{1}{2}$	$\frac{d}{2x}$	$-\frac{bd}{2x}$
M_{DE}	$-FM_{DE}$	$-\frac{bd}{2x}$	$\frac{d}{2x}$	$-\frac{1}{2}$	$-\frac{d}{2x}$	$\frac{bd}{2x}$
M_{ED}	FM_{ED}	$-\frac{b}{z}(1 - \frac{2d}{x})$	$\frac{1}{z}(1 - \frac{2d}{x})$	$-\frac{1}{z}(\frac{x}{d} - 2)$	$\frac{1}{z}(7 - \frac{2d}{x})$	$\frac{b}{z}(7 - \frac{2d}{x})$
M_{EF}	$-FM_{ED}$	$\frac{b}{z}(1 - \frac{2d}{x})$	$-\frac{1}{z}(1 - \frac{2d}{x})$	$\frac{1}{z}(\frac{x}{d} - 2)$	$-\frac{1}{z}(7 - \frac{2d}{x})$	$-\frac{b}{z}(7 - \frac{2d}{x})$
M_{FE}	FM_{FG}	$\frac{2abc}{xz}$	$-\frac{2ac}{xz}$	$\frac{2ac}{dz}$	$-\frac{2ac}{x}(1 + \frac{1}{z})$	$\frac{2a}{x}(1 + \frac{bc}{z})$
M_{FG}	$-FM_{FG}$	$-\frac{2abc}{xz}$	$\frac{2ac}{xz}$	$-\frac{2ac}{dz}$	$\frac{2ac}{x}(1 + \frac{1}{z})$	$-\frac{2a}{x}(1 + \frac{bc}{z})$
M_{GF}	FM_{GF}	$-\frac{abc}{xz}$	$\frac{ac}{xz}$	$-\frac{ac}{dz}$	$\frac{ac}{x}(1 + \frac{1}{z})$	$-\frac{a}{x}(1 + \frac{bc}{z})$

CASE C6 — SIX SPAN BEAM — CYCLOSYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at five points. Supports unequally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$$K_1 = \frac{I_1}{L_1} \quad K_2 = \frac{I_2}{L_2} \quad K_3 = \frac{I_3}{L_3} \quad K_4 = \frac{I_4}{L_4} \quad K_5 = \frac{I_5}{L_5} \quad K_6 = \frac{I_6}{L_6}$$

$$a = \frac{K_1}{2(K_1 + K_2)} \quad b = \frac{K_2}{2(K_1 + K_2)} \quad c = \frac{K_2}{2(K_2 + K_3)} \quad d = \frac{K_3}{2(K_2 + K_3)}$$

$$e = \frac{K_3}{2(K_3 + K_4)} \quad f = \frac{K_4}{2(K_3 + K_4)} \quad g = \frac{K_4}{2(K_4 + K_5)} \quad h = \frac{K_5}{2(K_4 + K_5)}$$

$$j = \frac{K_5}{2(K_5 + K_6)} \quad k = \frac{K_6}{2(K_5 + K_6)}$$

$$x_1 = 1 - bc$$

$$x_2 = 1 - hj$$

$$u_1 = 2 - e$$

$$u_2 = 2 - f$$

$$z_1 = \frac{x_1}{de} - 1 - \frac{fgx_1}{dex_2}$$

$$z_2 = \frac{x_2}{fg} - 1 - \frac{dex_2}{fgx_1}$$

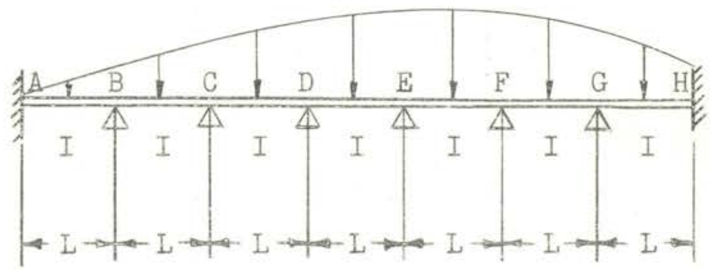
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$
M_{AB}	$- FM_{AB}$	$- \frac{a}{x_1}(1 + \frac{bc}{z_1})$	$\frac{ac}{x_1}(1 + \frac{1}{z_1})$	$- \frac{ac}{dz_1}$	$\frac{acg}{dx_2z_1}$	$- \frac{acgj}{dx_2z_1}$
M_{BA}	FM_{BA}	$- \frac{2a}{x_1}(1 + \frac{bc}{z_1})$	$\frac{2ac}{x_1}(1 + \frac{1}{z_1})$	$- \frac{2ac}{dz_1}$	$\frac{2acg}{dx_2z_1}$	$- \frac{2acgj}{dx_2z_1}$
M_{BC}	$- FM_{BA}$	$\frac{2a}{x_1}(1 + \frac{bc}{z_1})$	$- \frac{2ac}{x_1}(1 + \frac{1}{z_1})$	$\frac{2ac}{dz_1}$	$- \frac{2acg}{dx_2z_1}$	$\frac{2acgj}{dx_2z_1}$
M_{CB}	FM_{CD}	$- \frac{b}{ez_1}(u_1 - \frac{2fg}{x_2})$	$\frac{1}{ez_1}(u_1 - \frac{2fg}{x_2})$	$\frac{1}{z_1}(\frac{x_1}{d} - 2)$	$- \frac{g}{x_2z_1}(\frac{x_1}{d} - 2)$	$\frac{gj}{x_2z_1}(\frac{x_1}{d} - 2)$
M_{CD}	$- FM_{CD}$	$\frac{b}{ez_1}(u_1 - \frac{2fg}{x_2})$	$- \frac{1}{ez_1}(u_1 - \frac{2fg}{x_2})$	$- \frac{1}{z_1}(\frac{x_1}{d} - 2)$	$\frac{g}{x_2z_1}(\frac{x_1}{d} - 2)$	$- \frac{gj}{x_2z_1}(\frac{x_1}{d} - 2)$
M_{DC}	FM_{DC}	$\frac{bd}{x_1z_2}(\frac{2x_2}{g} - 1)$	$- \frac{d}{x_1z_2}(\frac{2x_2}{g} - 1)$	$- \frac{1}{z_1}(\frac{2x_1}{d} - 1)$	$\frac{g}{x_2z_1}(\frac{2x_1}{d} - 1)$	$- \frac{gj}{x_2z_1}(\frac{2x_1}{d} - 1)$
M_{DE}	$- FM_{DE}$	$- \frac{bd}{x_1z_2}(\frac{2x_2}{g} - 1)$	$\frac{d}{x_1z_2}(\frac{2x_2}{g} - 1)$	$- \frac{1}{z_2}(\frac{2x_2}{g} - 1)$	$- \frac{g}{x_2z_1}(\frac{2x_1}{d} - 1)$	$\frac{gj}{x_2z_1}(\frac{2x_1}{d} - 1)$
M_{ED}	FM_{ED}	$- \frac{bd}{x_1z_2}(\frac{x_2}{g} - 2)$	$\frac{d}{x_1z_2}(\frac{x_2}{g} - 2)$	$- \frac{1}{z_2}(\frac{x_2}{g} - 2)$	$- \frac{1}{fz_2}(u_2 - \frac{2de}{x_1})$	$\frac{j}{fz_2}(u_2 - \frac{2de}{x_1})$
M_{EF}	$- FM_{ED}$	$\frac{bd}{x_1z_2}(\frac{x_2}{g} - 2)$	$- \frac{d}{x_1z_2}(\frac{x_2}{g} - 2)$	$\frac{1}{z_2}(\frac{x_2}{g} - 2)$	$\frac{1}{fz_2}(u_2 - \frac{2de}{x_1})$	$- \frac{j}{fz_2}(u_2 - \frac{2de}{x_1})$
M_{FE}	FM_{FG}	$\frac{2bdhk}{gx_1z_2}$	$- \frac{2dhk}{gx_1z_2}$	$\frac{2hk}{gz_2}$	$- \frac{2hk}{x_2}(1 + \frac{1}{z_2})$	$\frac{2k}{x_2}(1 + \frac{hj}{z_2})$
M_{FG}	$- FM_{FG}$	$- \frac{2bdhk}{gx_1z_2}$	$\frac{2dhk}{gx_1z_2}$	$- \frac{2hk}{gz_2}$	$\frac{2hk}{x_2}(1 + \frac{1}{z_2})$	$- \frac{2k}{x_2}(1 + \frac{hj}{z_2})$
M_{GF}	FM_{GF}	$- \frac{bdhk}{gx_1z_2}$	$\frac{dhk}{gx_1z_2}$	$- \frac{hk}{gz_2}$	$\frac{hk}{x_2}(1 + \frac{1}{z_2})$	$- \frac{k}{x_2}(1 + \frac{hj}{z_2})$

CASE A70	SEVEN SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam with ends built in and freely supported at six points. Supports equally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K = \frac{I}{L}$

$a = \frac{1}{4}$

$x = \frac{7}{8}$

$y = \frac{15}{16}$

$u = \frac{7}{4}$

$z = \frac{2911}{225}$

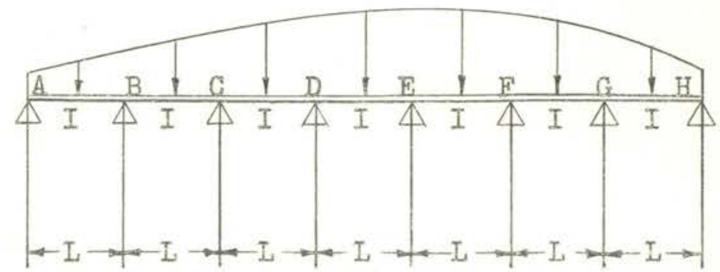
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - FM_{GH}$
M_{AB}	$-FM_{AB}$	- .2681	.0718	- .0192	.0052	- .0014	.0003
M_{BA}	FM_{BA}	- .5362	.1436	- .0385	.0103	- .0028	.0007
M_{BC}	$-FM_{BA}$	.5362	- .1436	.0385	- .0103	.0028	- .0007
M_{CB}	FM_{CB}	- .1244	- .5030	.1346	- .0361	.0096	- .0024
M_{CD}	$-FM_{CB}$	.1244	.5030	- .1346	.0361	- .0096	.0024
M_{DC}	FM_{DE}	.0333	- .1333	.5000	.1340	- .0357	.0089
M_{DE}	$-FM_{DE}$	- .0333	.1333	- .5000	- .1340	.0357	- .0089
M_{ED}	FM_{ED}	- .0089	.0357	- .1340	- .5000	.1333	- .0333
M_{EF}	$-FM_{ED}$	.0089	- .0357	.1340	.5000	- .1333	.0333
M_{FE}	FM_{FG}	.0024	- .0096	.0361	- .1346	.5030	.1244
M_{FG}	$-FM_{FG}$	- .0024	.0096	- .0361	.1346	- .5030	- .1244
M_{GF}	FM_{GH}	- .0007	.0028	- .0103	.0385	- .1436	.5362
M_{GH}	$-FM_{GH}$	.0007	- .0028	.0103	- .0385	.1436	- .5362
M_{HG}	FM_{HG}	.0003	- .0014	.0052	- .0192	.0718	- .2681

CASE A71	SEVEN SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam freely supported at eight points. Supports equally spaced.



PROPPED BEAM END MOMENTS

$$EM^*_{BA} = \frac{FM_{AB}}{2} + FM_{BA}$$

$$EM^*_{GH} = FM_{GH} + \frac{FM_{HG}}{2}$$

CONSTANT FUNCTIONS OF NDF

$K = \frac{I}{L}$ $\quad K' = \frac{3I}{4L}$

$a = \frac{13}{14}$ $\quad b = \frac{2}{7}$ $\quad c = d = e = f = \frac{1}{4}$

$x = \frac{97}{112}$ $\quad y_1 = \frac{13}{14}$ $\quad y_2 = \frac{15}{16}$

$z = \frac{8733}{676}$ $\quad u_1 = \frac{12}{7}$ $\quad u_2 = \frac{7}{4}$

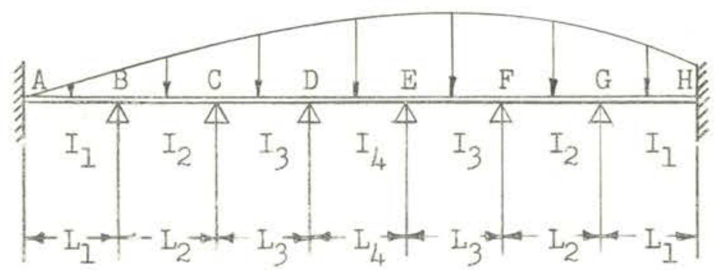
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$EM^*_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - EM^*_{GH}$
M_{BA}	EM^*_{BA}	- .4641	.1244	- .0333	.0089	- .0024	.0007
M_{BC}	$- EM^*_{BA}$	.4641	- .1244	.0333	- .0089	.0024	- .0007
M_{CB}	FM_{CB}	- .1436	- .4974	.1333	- .0357	.0096	- .0028
M_{CD}	$- FM_{CB}$	.1436	.4974	- .1333	.0357	- .0096	.0028
M_{DC}	FM_{DE}	.0385	- .1347	.5002	.1339	- .0361	.0103
M_{DE}	$- FM_{DE}$	- .0385	.1347	- .5002	- .1339	.0361	- .0103
M_{ED}	FM_{ED}	- .0103	.0361	- .1339	- .5002	.1347	- .0385
M_{EF}	$- FM_{ED}$	.0103	- .0361	.1339	.5002	- .1347	.0385
M_{FE}	FM_{FG}	.0028	- .0096	.0357	- .1333	.4974	.1436
M_{FG}	$- FM_{FG}$	- .0028	.0096	- .0357	.1333	- .4974	- .1436
M_{GF}	EM^*_{GH}	- .0007	.0024	- .0089	.0333	- .1244	.4641
M_{GH}	$- EM^*_{GH}$	.0007	- .0024	.0089	- .0333	.1244	- .4641

CASE B7	SEVEN SPAN BEAM	SYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at six points. Supports symmetrically spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1}$ $K_2 = \frac{I_2}{L_2}$ $K_3 = \frac{I_3}{L_3}$ $K_4 = \frac{I_4}{L_4}$

$a = \frac{K_1}{2(K_1 + K_2)}$ $b = \frac{K_2}{2(K_1 + K_2)}$ $c = \frac{K_2}{2(K_2 + K_3)}$

$d = \frac{K_3}{2(K_2 + K_3)}$ $e = \frac{K_3}{2(K_3 + K_4)}$ $f = \frac{K_4}{2(K_3 + K_4)}$

$x = 1 - bc - de$

$y_1 = 1 - bc$

$y_2 = 1 - de$

$u_1 = 2 - b$

$u_2 = 2 - e$

$z = \left(\frac{x}{fy}\right)^2 - 1$

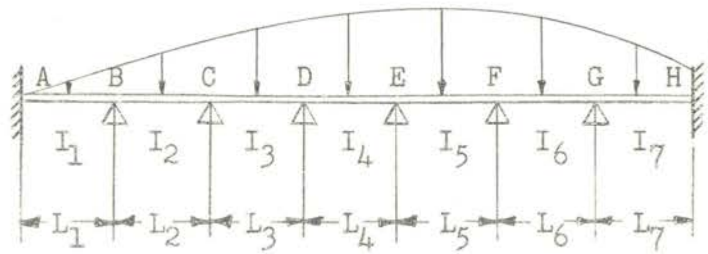
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - FM_{GH}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x}(y_2 + \frac{bcde}{y_1z})$	$\frac{ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{ace}{x}(1 + \frac{1}{z})$	$\frac{ace}{fy_1z}$	$-\frac{acde}{fy_1^2z}$	$\frac{abcde}{fy_1^2z}$
M_{BA}	FM_{BA}	$-\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$	$\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{2ace}{x}(1 + \frac{1}{z})$	$\frac{2ace}{fy_1z}$	$-\frac{2acde}{fy_1^2z}$	$\frac{2abcde}{fy_1^2z}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$	$-\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$\frac{2ace}{x}(1 + \frac{1}{z})$	$-\frac{2ace}{fy_1z}$	$\frac{2acde}{fy_1^2z}$	$-\frac{2abcde}{fy_1^2z}$
M_{CB}	FM_{CB}	$-\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$	$-\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$\frac{ceu_1}{x}(1 + \frac{1}{z})$	$-\frac{ceu_1}{fy_1z}$	$\frac{cdeu_1}{fy_1^2z}$	$-\frac{bcdeu_1}{fy_1^2z}$
M_{CD}	$-FM_{CB}$	$\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$	$\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$-\frac{ceu_1}{x}(1 + \frac{1}{z})$	$\frac{ceu_1}{fy_1z}$	$-\frac{cdeu_1}{fy_1^2z}$	$\frac{bcdeu_1}{fy_1^2z}$
M_{DC}	FM_{DE}	$\frac{bd}{y_1z}(\frac{2x}{fy_1} - 1)$	$-\frac{d}{y_1z}(\frac{2x}{fy_1} - 1)$	$\frac{1}{z}(\frac{2x}{fy_1} - 1)$	$\frac{1}{z}(\frac{x}{fy_1} - 2)$	$-\frac{d}{y_1z}(\frac{x}{fy_1} - 2)$	$\frac{bd}{y_1z}(\frac{x}{fy_1} - 2)$
M_{DE}	$-FM_{DE}$	$-\frac{bd}{y_1z}(\frac{2x}{fy_1} - 1)$	$\frac{d}{y_1z}(\frac{2x}{fy_1} - 1)$	$-\frac{1}{z}(\frac{2x}{fy_1} - 1)$	$-\frac{1}{z}(\frac{x}{fy_1} - 2)$	$\frac{d}{y_1z}(\frac{x}{fy_1} - 2)$	$-\frac{bd}{y_1z}(\frac{x}{fy_1} - 2)$
M_{ED}	FM_{ED}	$-\frac{bd}{y_1z}(\frac{x}{fy_1} - 2)$	$\frac{d}{y_1z}(\frac{x}{fy_1} - 2)$	$-\frac{1}{z}(\frac{x}{fy_1} - 2)$	$-\frac{1}{z}(\frac{2x}{fy_1} - 1)$	$\frac{d}{y_1z}(\frac{2x}{fy_1} - 1)$	$-\frac{bd}{y_1z}(\frac{2x}{fy_1} - 1)$
M_{EF}	$-FM_{ED}$	$\frac{bd}{y_1z}(\frac{x}{fy_1} - 2)$	$-\frac{d}{y_1z}(\frac{x}{fy_1} - 2)$	$\frac{1}{z}(\frac{x}{fy_1} - 2)$	$\frac{1}{z}(\frac{2x}{fy_1} - 1)$	$-\frac{d}{y_1z}(\frac{2x}{fy_1} - 1)$	$\frac{bd}{y_1z}(\frac{2x}{fy_1} - 1)$
M_{FE}	FM_{FG}	$\frac{bcdeu_1}{fy_1^2z}$	$-\frac{cdeu_1}{fy_1^2z}$	$\frac{ceu_1}{fy_1z}$	$-\frac{ceu_1}{x}(1 + \frac{1}{z})$	$\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$
M_{FG}	$-FM_{FG}$	$-\frac{bcdeu_1}{fy_1^2z}$	$\frac{cdeu_1}{fy_1^2z}$	$-\frac{ceu_1}{fy_1z}$	$\frac{ceu_1}{x}(1 + \frac{1}{z})$	$-\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$-\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$
M_{GF}	FM_{GH}	$-\frac{2abcde}{fy_1^2z}$	$\frac{2aced}{fy_1^2z}$	$-\frac{2ace}{fy_1z}$	$\frac{2ace}{x}(1 + \frac{1}{z})$	$-\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$
M_{GH}	$-FM_{GH}$	$\frac{2abcde}{fy_1^2z}$	$-\frac{2aced}{fy_1^2z}$	$\frac{2ace}{fy_1z}$	$-\frac{2ace}{x}(1 + \frac{1}{z})$	$\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$
M_{HG}	FM_{HG}	$\frac{abcde}{fy_1^2z}$	$-\frac{aced}{fy_1^2z}$	$\frac{ace}{fy_1z}$	$-\frac{ace}{x}(1 + \frac{1}{z})$	$\frac{ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{a}{x}(y_2 + \frac{bcde}{y_1z})$

CASE C7 | SEVEN SPAN BEAM | CYCLOSYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at six points. Supports unequally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1}$ $K_2 = \frac{I_2}{L_2}$ $K_3 = \frac{I_3}{L_3}$ $K_4 = \frac{I_4}{L_4}$ $K_5 = \frac{I_5}{L_5}$ $K_6 = \frac{I_6}{L_6}$ $K_7 = \frac{I_7}{L_7}$

$a = \frac{K_1}{2(K_1+K_2)}$ $b = \frac{K_2}{2(K_1+K_2)}$ $c = \frac{K_2}{2(K_2+K_3)}$ $d = \frac{K_3}{2(K_2+K_3)}$

$e = \frac{K_3}{2(K_3+K_4)}$ $f = \frac{K_4}{2(K_3+K_4)}$ $g = \frac{K_4}{2(K_4+K_5)}$ $h = \frac{K_5}{2(K_4+K_5)}$

$j = \frac{K_5}{2(K_5+K_6)}$ $k = \frac{K_6}{2(K_5+K_6)}$ $m = \frac{K_6}{2(K_6+K_7)}$ $n = \frac{K_7}{2(K_6+K_7)}$

$x_1 = 1 - bc - de$ $x_2 = 1 - hj - km$

$y_1 = 1 - bc$ $y_2 = 1 - de$

$y_3 = 1 - hj$ $y_4 = 1 - km$

$u_1 = 2 - b$ $u_2 = 2 - e$

$u_3 = 2 - h$ $u_4 = 2 - m$

$$z = \frac{x_1 x_2}{f g y_1 y_4} - 1$$

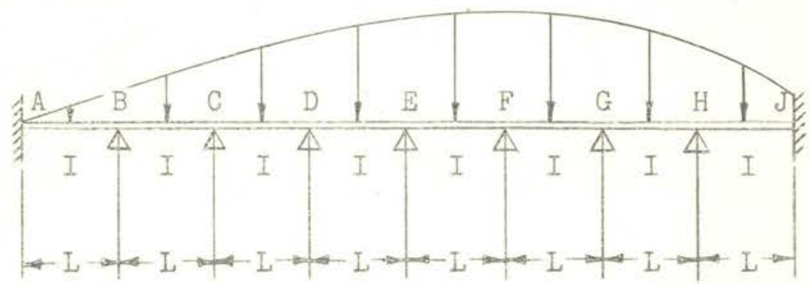
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - FM_{GH}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x_1}(y_2 + \frac{bcde}{y_1 z})$	$\frac{ac}{x_1}(1 + \frac{de}{y_1 z})$	$-\frac{ace}{x_1}(1 + \frac{1}{z})$	$\frac{ace}{fy_1 z}$	$-\frac{acej}{fy_1 y_4 z}$	$\frac{acejm}{fy_1 y_4 z}$
M_{BA}	FM_{BA}	$-\frac{2a}{x_1}(y_2 + \frac{bcde}{y_1 z})$	$\frac{2ac}{x_1}(1 + \frac{de}{y_1 z})$	$-\frac{2ace}{x_1}(1 + \frac{1}{z})$	$\frac{2ace}{fy_1 z}$	$-\frac{2acej}{fy_1 y_4 z}$	$\frac{2acejm}{fy_1 y_4 z}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x_1}(y_2 + \frac{bcde}{y_1 z})$	$-\frac{2ac}{x_1}(1 + \frac{de}{y_1 z})$	$\frac{2ace}{x_1}(1 + \frac{1}{z})$	$-\frac{2ace}{fy_1 z}$	$\frac{2acej}{fy_1 y_4 z}$	$-\frac{2acejm}{fy_1 y_4 z}$
M_{CB}	FM_{CB}	$-\frac{bd}{x_1}(u_2 - \frac{ceu_1}{y_1 z})$	$-\frac{cu_1}{x_1}(1 + \frac{de}{y_1 z})$	$\frac{ceu_1}{x_1}(1 + \frac{1}{z})$	$-\frac{ceu_1}{fy_1 z}$	$\frac{ceju_1}{fy_1 y_4 z}$	$-\frac{cejmu_1}{fy_1 y_4 z}$
M_{CD}	$-FM_{CB}$	$\frac{bd}{x_1}(u_2 - \frac{ceu_1}{y_1 z})$	$\frac{cu_1}{x_1}(1 + \frac{de}{y_1 z})$	$-\frac{ceu_1}{x_1}(1 + \frac{1}{z})$	$\frac{ceu_1}{fy_1 z}$	$-\frac{ceju_1}{fy_1 y_4 z}$	$\frac{cejmu_1}{fy_1 y_4 z}$
M_{DC}	FM_{DE}	$\frac{bd}{y_1 z}(\frac{2x_2}{gy_4} - 1)$	$-\frac{d}{y_1 z}(\frac{2x_2}{gy_4} - 1)$	$\frac{1}{z}(\frac{2x_2}{gy_4} - 1)$	$\frac{1}{z}(\frac{x_1}{fy_1} - 2)$	$-\frac{j}{y_4 z}(\frac{x_1}{fy_1} - 2)$	$\frac{jm}{y_4 z}(\frac{x_1}{fy_1} - 2)$
M_{DE}	$-FM_{DE}$	$-\frac{bd}{y_1 z}(\frac{2x_2}{gy_4} - 1)$	$\frac{d}{y_1 z}(\frac{2x_2}{gy_4} - 1)$	$-\frac{1}{z}(\frac{2x_2}{gy_4} - 1)$	$-\frac{1}{z}(\frac{x_1}{fy_1} - 2)$	$\frac{j}{y_4 z}(\frac{x_1}{fy_1} - 2)$	$-\frac{jm}{y_4 z}(\frac{x_1}{fy_1} - 2)$
M_{ED}	FM_{ED}	$-\frac{bd}{y_1 z}(\frac{x_2}{gy_4} - 2)$	$\frac{d}{y_1 z}(\frac{x_2}{gy_4} - 2)$	$-\frac{1}{z}(\frac{x_2}{gy_4} - 2)$	$-\frac{1}{z}(\frac{2x_1}{fy_1} - 1)$	$\frac{j}{y_4 z}(\frac{2x_1}{fy_1} - 1)$	$-\frac{jm}{y_4 z}(\frac{2x_1}{fy_1} - 1)$
M_{EF}	$-FM_{ED}$	$\frac{bd}{y_1 z}(\frac{x_2}{gy_4} - 2)$	$-\frac{d}{y_1 z}(\frac{x_2}{gy_4} - 2)$	$\frac{1}{z}(\frac{x_2}{gy_4} - 2)$	$\frac{1}{z}(\frac{2x_1}{fy_1} - 1)$	$-\frac{j}{y_4 z}(\frac{2x_1}{fy_1} - 1)$	$\frac{jm}{y_4 z}(\frac{2x_1}{fy_1} - 1)$
M_{FE}	FM_{FG}	$\frac{bdhku_4}{gy_1 y_4 z}$	$-\frac{dhku_4}{gy_1 y_4 z}$	$\frac{hku_4}{gy_4 z}$	$-\frac{hku_4}{x_2}(1 + \frac{1}{z})$	$\frac{ku_4}{x_2}(1 + \frac{hj}{y_4 z})$	$\frac{jm}{x_2}(u_3 - \frac{hku_4}{y_4 z})$
M_{FG}	$-FM_{FG}$	$-\frac{bdhku_4}{gy_1 y_4 z}$	$\frac{dhku_4}{gy_1 y_4 z}$	$-\frac{hku_4}{gy_4 z}$	$\frac{hku_4}{x_2}(1 + \frac{1}{z})$	$-\frac{ku_4}{x_2}(1 + \frac{hj}{y_4 z})$	$-\frac{jm}{x_2}(u_3 - \frac{hku_4}{y_4 z})$
M_{GF}	FM_{GH}	$-\frac{2bdhkn}{gy_1 y_4 z}$	$\frac{2dhkn}{gy_1 y_4 z}$	$-\frac{2hkn}{gy_4 z}$	$\frac{2hkn}{x_2}(1 + \frac{1}{z})$	$-\frac{2kn}{x_2}(1 + \frac{hj}{y_4 z})$	$\frac{2n}{x_2}(y_3 + \frac{hjkm}{y_4 z})$
M_{GH}	$-FM_{GH}$	$\frac{2bdhkn}{gy_1 y_4 z}$	$-\frac{2dhkn}{gy_1 y_4 z}$	$\frac{2hkn}{gy_4 z}$	$-\frac{2hkn}{x_2}(1 + \frac{1}{z})$	$\frac{2kn}{x_2}(1 + \frac{hj}{y_4 z})$	$-\frac{2n}{x_2}(y_3 + \frac{hjkm}{y_4 z})$
M_{HG}	FM_{HG}	$\frac{bdhkn}{gy_1 y_4 z}$	$-\frac{dhkn}{gy_1 y_4 z}$	$\frac{hkn}{gy_4 z}$	$-\frac{hkn}{x_2}(1 + \frac{1}{z})$	$\frac{kn}{x_2}(1 + \frac{hj}{y_4 z})$	$-\frac{n}{x_2}(y_3 + \frac{hjkm}{y_4 z})$

CASE A80	EIGHT SPAN BEAM	EQUAL SPANS

DESCRIPTION:

Beam with ends built in and freely supported at seven points. Supports equally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K = \frac{I}{L}$ $a = \frac{1}{4}$

$x = \frac{7}{8}$

$y = \frac{15}{16}$

$u = \frac{7}{4}$

$z = \frac{194}{15}$

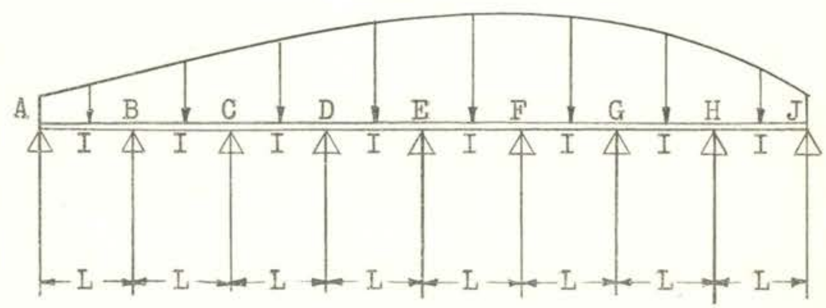
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - FM_{GH}$	$FM_{HG} - FM_{HJ}$
M_{AB}	$- FM_{AB}$	- .2681	.0718	- .0192	.0052	- .0014	.0004	- .0001
M_{BA}	FM_{BA}	- .5362	.1436	- .0385	.0103	- .0028	.0007	- .0002
M_{BC}	$- FM_{BA}$	.5362	- .1436	.0385	- .0103	.0028	- .0007	.0002
M_{CB}	FM_{CB}	- .1243	- .5022	.1346	- .0351	.0097	- .0026	.0007
M_{CD}	$- FM_{CB}$	.1243	.5022	- .1346	.0351	- .0097	.0026	- .0007
M_{DC}	FM_{DE}	.0333	- .1334	.5000	.1340	- .0359	.0096	- .0024
M_{DE}	$- FM_{DE}$	- .0333	.1334	- .5000	- .1340	.0359	- .0096	.0024
M_{ED}	FM_{ED}	- .0089	.0357	- .1340	- .5000	.1340	- .0357	.0089
M_{EF}	$- FM_{EF}$	.0089	- .0357	.1340	- .5000	- .1340	.0357	- .0089
M_{FE}	FM_{FE}	.0024	- .0096	.0359	- .1340	- .5000	.1334	- .0333
M_{FG}	$- FM_{FE}$	- .0024	.0096	- .0359	.1340	.5000	- .1334	.0333
M_{GF}	FM_{GH}	- .0007	.0026	- .0097	.0351	- .1346	.5022	.1243
M_{GH}	$- FM_{GH}$	.0007	- .0026	.0097	- .0351	.1346	- .5022	- .1243
M_{HG}	FM_{HJ}	.0002	- .0007	.0028	- .0103	.0385	- .1436	.5362
M_{HJ}	$- FM_{HJ}$	- .0002	.0007	- .0028	.0103	- .0385	.1436	- .5362
M_{JH}	FM_{JH}	- .0001	.0004	- .0014	.0052	- .0192	.0718	- .2681

CASE A81 | EIGHT SPAN BEAM | EQUAL SPANS

DESCRIPTION:

Beam freely supported at nine points. Supports equally spaced.



PROPPED BEAM END MOMENTS

$$EM^*_{BA} = \frac{FM_{AB}}{2} + FM_{BA}$$

$$EM^*_{HJ} = FM_{HJ} + \frac{FM_{JH}}{2}$$

CONSTANT FUNCTIONS OF NDF

$K = \frac{I}{L}$ $\quad K' = \frac{3K}{4L}$

$a = \frac{13}{14}$ $\quad b = \frac{2}{7}$ $\quad c = d = e = f = g = \frac{1}{4}$

$x = \frac{97}{112}$ $\quad y_1 = \frac{13}{14}$ $\quad y_2 = \frac{15}{16}$

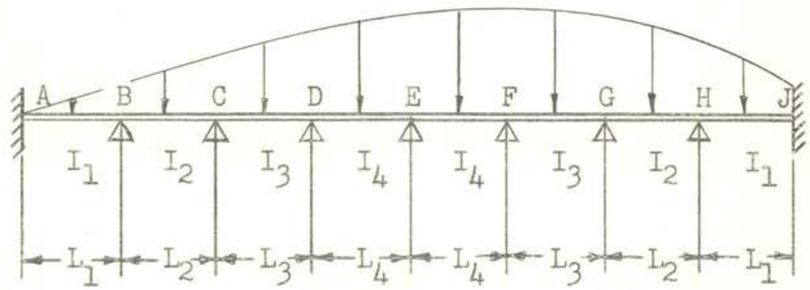
$z = \frac{168}{13}$ $\quad u_1 = \frac{12}{7}$ $\quad u_2 = \frac{7}{4}$

TABLE OF FINAL MOMENTS								
Moment	Fixed End Moment	$EM^*_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - FM_{GH}$	$FM_{HG} - EM^*_{HJ}$
M_{BA}	EM^*_{BA}	- .4641	.1244	- .0333	.0089	- .0023	.0006	- .0002
M_{BC}	$- EM^*_{BA}$	.4641	- .1244	.0333	- .0089	.0023	- .0006	.0002
M_{CB}	FM_{CB}	- .1436	- .4973	.1333	- .0357	.0096	- .0026	.0007
M_{CD}	$- FM_{CB}$	.1436	.4973	- .1333	.0357	- .0096	.0026	- .0007
M_{DC}	FM_{DE}	.0385	- .1348	.5002	.1339	- .0356	.0097	- .0028
M_{DE}	$- FM_{DE}$	- .0385	.1348	- .5002	- .1339	.0356	- .0097	.0028
M_{ED}	FM_{ED}	- .0103	.0361	- .1337	- .5000	.1337	- .0361	.0103
M_{EF}	$- FM_{EF}$	.0103	- .0361	.1337	- .5000	- .1337	.0361	- .0103
M_{FE}	FM_{FE}	.0028	- .0097	.0356	- .1339	- .5002	.1348	- .0385
M_{FG}	$- FM_{FE}$	- .0028	.0097	- .0356	.1339	.5002	- .1348	.0385
M_{GF}	FM_{GH}	- .0007	.0026	- .0096	.0357	- .1333	.4973	.1436
M_{GH}	$- FM_{GH}$	.0007	- .0026	.0096	- .0357	.1333	- .4973	- .1436
M_{HG}	EM^*_{HJ}	.0002	- .0006	.0023	- .0089	.0333	- .1244	.4641
M_{HJ}	$- EM^*_{HJ}$	- .0002	.0006	- .0023	.0089	- .0333	.1244	- .4641

CASE B8 | EIGHT SPAN BEAM | SYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at seven points. Supports symmetrically spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1} \quad K_2 = \frac{I_2}{L_2} \quad K_3 = \frac{I_3}{L_3} \quad K_4 = \frac{I_4}{L_4}$

$a = \frac{K_1}{2(K_1 + K_2)} \quad b = \frac{K_2}{2(K_1 + K_2)} \quad c = \frac{K_2}{2(K_2 + K_3)}$

$d = \frac{K_3}{2(K_2 + K_3)} \quad e = \frac{K_3}{2(K_3 + K_4)} \quad f = \frac{K_4}{2(K_3 + K_4)}$

$g = \frac{K_4}{2(K_4 + K_4)} = \frac{1}{4}$

$x = 1 - bc - de \qquad y_1 = 1 - bc$

$u_1 = 2 - b \qquad y_2 = 1 - de$

$u_2 = 2 - e$

$z = 4\frac{x}{fy_1} - 2$

$u_3 = \frac{7}{4}$

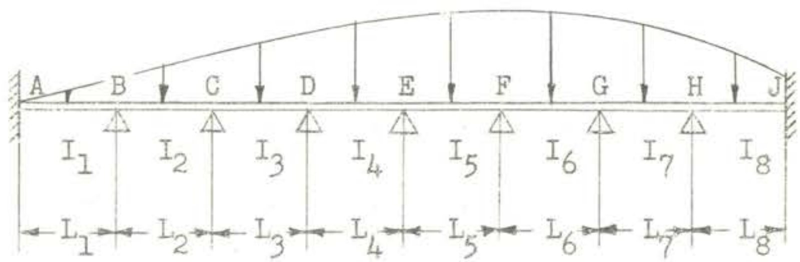
TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM_{BA} - FM_{BC}$	$FM_{CB} - FM_{CD}$	$FM_{DC} - FM_{DE}$	$FM_{ED} - FM_{EF}$	$FM_{FE} - FM_{FG}$	$FM_{GF} - FM_{GH}$	$FM_{HG} - FM_{HJ}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x}(y_2 + \frac{bcde}{y_1z})$	$\frac{ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{ace}{x}(1 + \frac{1}{z})$	$\frac{ace}{fy_1z}$	$-\frac{ace}{xz}$	$\frac{acde}{xy_1z}$	$-\frac{abcde}{xy_1z}$
M_{BA}	FM_{BA}	$-\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$	$\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{2ace}{x}(1 + \frac{1}{z})$	$\frac{2ace}{fy_1z}$	$-\frac{2ace}{xz}$	$\frac{2acde}{xy_1z}$	$-\frac{2abcde}{xy_1z}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$	$-\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$\frac{2ace}{x}(1 + \frac{1}{z})$	$-\frac{2ace}{fy_1z}$	$\frac{2ace}{xz}$	$-\frac{2acde}{xy_1z}$	$+\frac{2abcde}{xy_1z}$
M_{CB}	FM_{CB}	$-\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$	$-\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$\frac{ceu_1}{x}(1 + \frac{1}{z})$	$-\frac{ceu_1}{fy_1z}$	$\frac{ceu_1}{xz}$	$-\frac{cdeu_1}{xy_1z}$	$\frac{bcdeu_1}{xy_1z}$
M_{CD}	$-FM_{CB}$	$\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$	$\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$-\frac{ceu_1}{x}(1 + \frac{1}{z})$	$\frac{ceu_1}{fy_1z}$	$-\frac{ceu_1}{xz}$	$\frac{cdeu_1}{xy_1z}$	$-\frac{bcdeu_1}{xy_1z}$
M_{DC}	FM_{DE}	$\frac{bd}{y_1z}(7 - \frac{2fy_1}{x})$	$-\frac{d}{y_1z}(7 - \frac{2fy_1}{x})$	$\frac{1}{z}(7 - \frac{2fy_1}{x})$	$\frac{1}{z}(\frac{x}{fy_1} - 2)$	$-\frac{1}{z}(1 - \frac{2fy_1}{x})$	$\frac{d}{y_1z}(1 - \frac{2fy_1}{x})$	$-\frac{bd}{y_1z}(1 - \frac{2fy_1}{x})$
M_{DE}	$-FM_{DE}$	$-\frac{bd}{y_1z}(7 - \frac{2fy_1}{x})$	$\frac{d}{y_1z}(7 - \frac{2fy_1}{x})$	$-\frac{1}{z}(7 - \frac{2fy_1}{x})$	$-\frac{1}{z}(\frac{x}{fy_1} - 2)$	$\frac{1}{z}(1 - \frac{2fy_1}{x})$	$-\frac{d}{y_1z}(1 - \frac{2fy_1}{x})$	$\frac{bd}{y_1z}(1 - \frac{2fy_1}{x})$
M_{ED}	FM_{ED}	$-\frac{bdf}{2x}$	$\frac{df}{2x}$	$-\frac{fy_1}{2x}$	- .500	$\frac{fy_1}{2x}$	$-\frac{df}{2x}$	$\frac{bdf}{2x}$
M_{EF}	$-FM_{EF}$	$\frac{bdf}{2x}$	$-\frac{df}{2x}$	$\frac{fy_1}{2x}$	- .500	$-\frac{fy_1}{2x}$	$\frac{df}{2x}$	$-\frac{bdf}{2x}$
M_{FE}	FM_{FE}	$\frac{bd}{y_1z}(1 - \frac{2fy_1}{x})$	$-\frac{d}{y_1z}(1 - \frac{2fy_1}{x})$	$\frac{1}{z}(1 - \frac{2fy_1}{x})$	$-\frac{1}{z}(\frac{x}{fy_1} - 2)$	$-\frac{1}{z}(7 - \frac{2fy_1}{x})$	$\frac{d}{y_1z}(7 - \frac{2fy_1}{x})$	$-\frac{bd}{y_1z}(7 - \frac{2fy_1}{x})$
M_{FG}	$-FM_{FE}$	$-\frac{bd}{y_1z}(1 - \frac{2fy_1}{x})$	$\frac{d}{y_1z}(1 - \frac{2fy_1}{x})$	$-\frac{1}{z}(1 - \frac{2fy_1}{x})$	$\frac{1}{z}(\frac{x}{fy_1} - 2)$	$\frac{1}{z}(7 - \frac{2fy_1}{x})$	$-\frac{d}{y_1z}(7 - \frac{2fy_1}{x})$	$\frac{bd}{y_1z}(7 - \frac{2fy_1}{x})$
M_{GF}	FM_{GH}	$-\frac{bcdeu_1}{xy_1z}$	$\frac{cdeu_1}{xy_1z}$	$-\frac{ceu_1}{xz}$	$\frac{ceu_1}{fy_1z}$	$-\frac{ceu_1}{x}(1 + \frac{1}{z})$	$\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$
M_{GH}	$-FM_{GH}$	$\frac{bcdeu_1}{xy_1z}$	$-\frac{cdeu_1}{xy_1z}$	$\frac{ceu_1}{xz}$	$-\frac{ceu_1}{fy_1z}$	$\frac{ceu_1}{x}(1 + \frac{1}{z})$	$-\frac{cu_1}{x}(1 + \frac{de}{y_1z})$	$-\frac{bd}{x}(u_2 - \frac{ceu_1}{y_1z})$
M_{HG}	FM_{HJ}	$\frac{2abcde}{xy_1z}$	$-\frac{2acde}{xy_1z}$	$\frac{2ace}{xz}$	$-\frac{2ace}{fy_1z}$	$\frac{2ace}{x}(1 + \frac{1}{z})$	$-\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$
M_{HJ}	$-FM_{HJ}$	$-\frac{2abcde}{xy_1z}$	$\frac{2acde}{xy_1z}$	$-\frac{2ace}{xz}$	$\frac{2ace}{fy_1z}$	$-\frac{2ace}{x}(1 + \frac{1}{z})$	$\frac{2ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{2a}{x}(y_2 + \frac{bcde}{y_1z})$
M_{JH}	FM_{JH}	$-\frac{abcde}{xy_1z}$	$\frac{acde}{xy_1z}$	$-\frac{ace}{xz}$	$\frac{ace}{fy_1z}$	$-\frac{ace}{x}(1 + \frac{1}{z})$	$\frac{ac}{x}(1 + \frac{de}{y_1z})$	$-\frac{a}{x}(y_2 + \frac{bcde}{y_1z})$

CASE C8 EIGHT SPAN BEAM CYCLOSYMMETRICAL

DESCRIPTION:

Beam with ends built in and freely supported at seven points. Supports unequally spaced.



CONSTANT FUNCTIONS OF NEW DISTRIBUTION FACTOR

$K_1 = \frac{I_1}{L_1}$ $K_2 = \frac{I_2}{L_2}$ $K_3 = \frac{I_3}{L_3}$ $K_4 = \frac{I_4}{L_4}$ $K_5 = \frac{I_5}{L_5}$ $K_6 = \frac{I_6}{L_6}$ $K_7 = \frac{I_7}{L_7}$ $K_8 = \frac{I_8}{L_8}$

$a = \frac{K_1}{2(K_1+K_2)}$ $b = \frac{K_2}{2(K_1+K_2)}$ $c = \frac{K_2}{2(K_2+K_3)}$ $d = \frac{K_3}{2(K_2+K_3)}$

$e = \frac{K_3}{2(K_3+K_4)}$ $f = \frac{K_4}{2(K_3+K_4)}$ $g = \frac{K_4}{2(K_4+K_5)}$ $h = \frac{K_5}{2(K_4+K_5)}$

$j = \frac{K_5}{2(K_5+K_6)}$ $k = \frac{K_6}{2(K_5+K_6)}$ $m = \frac{K_6}{2(K_6+K_7)}$ $n = \frac{K_7}{2(K_6+K_7)}$

$p = \frac{K_7}{2(K_7+K_8)}$ $r = \frac{K_8}{2(K_7+K_8)}$

$x_1 = 1 - bc - de$ $u_1 = 2 - b$

$x_2 = 1 - km - np$ $u_2 = 2 - e$

$y_1 = 1 - bc$ $u_3 = 2 - g$

$y_2 = 1 - de$ $u_4 = 2 - h$

$y_3 = 1 - km$ $u_5 = 2 - k$

$y_4 = 1 - np$ $u_6 = 2 - p$

$z_1 = \frac{x_1}{fgy_1} - \frac{hjx_1y_4}{fgx_2y_1} - 1$ $z_2 = \frac{x_2}{hjy_4} - \frac{fgx_2y_1}{hjx_1y_4} - 1$

TABLE OF FINAL MOMENTS

Moment	Fixed End Moment	$FM'_{BA} - FM'_{BC}$	$FM'_{CB} - FM'_{CD}$	$FM'_{DC} - FM'_{DE}$	$FM'_{ED} - FM'_{EF}$	$FM'_{FE} - FM'_{FG}$	$FM'_{GF} - FM'_{GH}$	$FM'_{HG} - FM'_{HJ}$
M_{AB}	$-FM_{AB}$	$-\frac{a}{x_1}(y_2 + \frac{bcde}{y_1z_1})$	$\frac{ac}{x_1}(1 + \frac{de}{y_1z_1})$	$-\frac{ace}{x_1}(1 + \frac{1}{z_1})$	$\frac{ace}{fy_1z_1}$	$-\frac{acejy_4}{fx_2y_1z_1}$	$\frac{acejm}{fx_2y_1z_1}$	$-\frac{acejmp}{fx_2y_1z_1}$
M_{BA}	FM_{BA}	$-\frac{2a}{x_1}(y_2 + \frac{bcde}{y_1z_1})$	$\frac{2ac}{x_1}(1 + \frac{de}{y_1z_1})$	$-\frac{2ace}{x_1}(1 + \frac{1}{z_1})$	$\frac{2ace}{fy_1z_1}$	$-\frac{2acejy_4}{fx_2y_1z_1}$	$\frac{2acejm}{fx_2y_1z_1}$	$-\frac{2acejmp}{fx_2y_1z_1}$
M_{BC}	$-FM_{BA}$	$\frac{2a}{x_1}(y_2 + \frac{bcde}{y_1z_1})$	$-\frac{2ac}{x_1}(1 + \frac{de}{y_1z_1})$	$\frac{2ace}{x_1}(1 + \frac{1}{z_1})$	$-\frac{2ace}{fy_1z_1}$	$\frac{2acejy_4}{fx_2y_1z_1}$	$-\frac{2acejm}{fx_2y_1z_1}$	$\frac{2acejmp}{fx_2y_1z_1}$
M_{CB}	FM_{CB}	$-\frac{bd}{x_1}(u_2 - \frac{ceu_1}{y_1z_1})$	$-\frac{cu_1}{x_1}(1 + \frac{de}{y_1z_1})$	$\frac{ceu_1}{x_1}(1 + \frac{1}{z_1})$	$-\frac{ceu_1}{fy_1z_1}$	$\frac{ceju_1y_4}{fx_2y_1z_1}$	$-\frac{cejmu_1}{fx_2y_1z_1}$	$\frac{cejmpu_1}{fx_2y_1z_1}$
M_{CD}	$-FM_{CB}$	$\frac{bd}{x_1}(u_2 - \frac{ceu_1}{y_1z_1})$	$\frac{cu_1}{x_1}(1 + \frac{de}{y_1z_1})$	$-\frac{ceu_1}{x_1}(1 + \frac{1}{z_1})$	$\frac{ceu_1}{fy_1z_1}$	$-\frac{ceju_1y_4}{fx_2y_1z_1}$	$\frac{cejmu_1}{fx_2y_1z_1}$	$-\frac{cejmpu_1}{fx_2y_1z_1}$
M_{DC}	FM_{DE}	$\frac{bd}{gy_1z_1}(u_3 - \frac{2hjy_4}{x_2})$	$-\frac{d}{gy_1z_1}(u_3 - \frac{2hjy_4}{x_2})$	$\frac{1}{gz_1}(u_3 - \frac{2hjy_4}{x_2})$	$\frac{1}{z_1}(\frac{x_1}{fy_1} - 2)$	$-\frac{jy_4}{x_2z_1}(\frac{x_1}{fy_1} - 2)$	$\frac{jm}{x_2z_1}(\frac{x_1}{fy_1} - 2)$	$-\frac{jmp}{x_2z_1}(\frac{x_1}{fy_1} - 2)$
M_{DE}	$-FM_{DE}$	$-\frac{bd}{gy_1z_1}(u_3 - \frac{2hjy_4}{x_2})$	$\frac{d}{gy_1z_1}(u_3 - \frac{2hjy_4}{x_2})$	$-\frac{1}{gz_1}(u_3 - \frac{2hjy_4}{x_2})$	$-\frac{1}{z_1}(\frac{x_1}{fy_1} - 2)$	$\frac{jy_4}{x_2z_1}(\frac{x_1}{fy_1} - 2)$	$-\frac{jm}{x_2z_1}(\frac{x_1}{fy_1} - 2)$	$\frac{jmp}{x_2z_1}(\frac{x_1}{fy_1} - 2)$
M_{ED}	FM_{ED}	$-\frac{bdf}{x_1z_2}(\frac{2x_2}{jy_4} - 1)$	$\frac{df}{x_1z_2}(\frac{2x_2}{jy_4} - 1)$	$-\frac{fy_1}{x_1z_2}(\frac{2x_2}{jy_4} - 1)$	$-\frac{1}{z_1}(\frac{2x_1}{fy_1} - 1)$	$\frac{jy_4}{x_2z_1}(\frac{2x_1}{fy_1} - 1)$	$-\frac{jm}{x_2z_1}(\frac{2x_1}{fy_1} - 1)$	$\frac{jmp}{x_2z_1}(\frac{2x_1}{fy_1} - 1)$
M_{EF}	$-FM_{EF}$	$\frac{bdf}{x_1z_2}(\frac{2x_2}{jy_4} - 1)$	$-\frac{df}{x_1z_2}(\frac{2x_2}{jy_4} - 1)$	$\frac{fy_1}{x_1z_2}(\frac{2x_2}{jy_4} - 1)$	$-\frac{1}{z_2}(\frac{2x_2}{jy_4} - 1)$	$-\frac{jy_4}{x_2z_1}(\frac{2x_1}{fy_1} - 1)$	$\frac{jm}{x_2z_1}(\frac{2x_1}{fy_1} - 1)$	$-\frac{jmp}{x_2z_1}(\frac{2x_1}{fy_1} - 1)$
M_{FE}	FM_{FE}	$\frac{bdf}{x_1z_2}(\frac{x_2}{jy_4} - 2)$	$-\frac{df}{x_1z_2}(\frac{x_2}{jy_4} - 2)$	$\frac{fy_1}{x_1z_2}(\frac{x_2}{jy_4} - 2)$	$-\frac{1}{z_2}(\frac{x_2}{jy_4} - 2)$	$-\frac{1}{hz_2}(u_4 - \frac{2fgy_1}{x_1})$	$\frac{n}{hy_4z_2}(u_4 - \frac{2fgy_1}{x_1})$	$-\frac{np}{hy_4z_2}(u_4 - \frac{2fgy_1}{x_1})$
M_{FG}	$-FM_{FE}$	$-\frac{bdf}{x_1z_2}(\frac{x_2}{jy_4} - 2)$	$\frac{df}{x_1z_2}(\frac{x_2}{jy_4} - 2)$	$-\frac{fy_1}{x_1z_2}(\frac{x_2}{jy_4} - 2)$	$\frac{1}{z_2}(\frac{x_2}{jy_4} - 2)$	$\frac{1}{hz_2}(u_4 - \frac{2fgy_1}{x_1})$	$-\frac{n}{hy_4z_2}(u_4 - \frac{2fgy_1}{x_1})$	$\frac{np}{hy_4z_2}(u_4 - \frac{2fgy_1}{x_1})$
M_{GF}	FM_{GH}	$-\frac{bdfknu_6}{jx_1y_4z_2}$	$\frac{dfknu_6}{jx_1y_4z_2}$	$-\frac{fknu_6y_1}{jx_1y_4z_2}$	$\frac{knu_6}{jy_4z_2}$	$-\frac{knu_6}{x_2}(1 + \frac{1}{z_2})$	$\frac{nu_6}{x_2}(1 + \frac{km}{y_4z_2})$	$\frac{mp}{x_2}(u_5 - \frac{knu_6}{y_4z_2})$
M_{GH}	$-FM_{GH}$	$\frac{bdfknu_6}{jx_1y_4z_2}$	$-\frac{dfknu_6}{jx_1y_4z_2}$	$\frac{fknu_6y_1}{jx_1y_4z_2}$	$-\frac{knu_6}{jy_4z_2}$	$\frac{knu_6}{x_2}(1 + \frac{1}{z_2})$	$-\frac{nu_6}{x_2}(1 + \frac{km}{y_4z_2})$	$-\frac{mp}{x_2}(u_5 - \frac{knu_6}{y_4z_2})$
M_{HG}	FM_{HJ}	$\frac{2bdfknr}{jx_1y_4z_2}$	$-\frac{2dfknr}{jx_1y_4z_2}$	$\frac{2fknry_1}{jx_1y_4z_2}$	$-\frac{2knr}{jy_4z_2}$	$\frac{2knr}{x_2}(1 + \frac{1}{z_2})$	$-\frac{2nr}{x_2}(1 + \frac{km}{y_4z_2})$	$\frac{2r}{x_2}(y_3 + \frac{kmnp}{y_4z_2})$
M_{HJ}	$-FM_{HJ}$	$-\frac{2bdfknr}{jx_1y_4z_2}$	$\frac{2dfknr}{jx_1y_4z_2}$	$-\frac{2fknry_1}{jx_1y_4z_2}$	$\frac{2knr}{jy_4z_2}$	$-\frac{2knr}{x_2}(1 + \frac{1}{z_2})$	$\frac{2nr}{x_2}(1 + \frac{km}{y_4z_2})$	$-\frac{2r}{x_2}(y_3 + \frac{kmnp}{y_4z_2})$
M_{JH}	FM_{JH}	$-\frac{bdfknr}{jx_1y_4z_2}$	$\frac{dfknr}{jx_1y_4z_2}$	$-\frac{fknry_1}{jx_1y_4z_2}$	$\frac{knr}{jy_4z_2}$	$-\frac{knr}{x_2}(1 + \frac{1}{z_2})$	$\frac{nr}{x_2}(1 + \frac{km}{y_4z_2})$	$-\frac{r}{x_2}(y_3 + \frac{kmnp}{y_4z_2})$

Section 3

(Courtesy of Texas Highway Department)

Nueces Bay Causeway on U.S. Highway 181 near Corpus Christi, Texas.

SECTION 3

ILLUSTRATIVE EXAMPLES

1. END MOMENTS FOR A FOUR SPAN FREELY SUPPORTED BEAM

A four span beam is loaded as shown in Fig. 18. The beam has spans of 12 feet, 16 feet, 24 feet and 30 feet and has a constant moment of inertia of 4 ft^4. Determine the end moments due to the transverse loading.

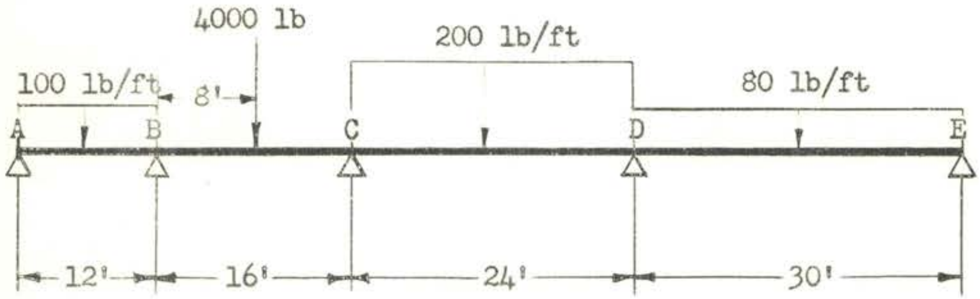


Fig. 18

SOLUTION

1. Select table: Case C4 Four Span Beam - Cyclosymmetrical

2. Determine the stiffness factors:

$$K_1 = \frac{3}{4}\frac{I_1}{L_1} = \frac{1}{4},$$

$$K_2 = \frac{I_2}{L_2} = \frac{1}{4},$$

$$K_3 = \frac{I_3}{L_3} = \frac{1}{6},$$

$$K_4 = \frac{3}{4}\frac{I_4}{L_4} = \frac{1}{10}.$$

$$\sum K_B = K_1 + K_2 = \frac{1}{2},$$

$$\sum K_C = K_2 + K_3 = \frac{5}{12},$$

$$\sum K_D = K_3 + K_4 = \frac{4}{15},$$

3. Determine the Constant Functions of the New Distribution Factor:

$$a = \frac{K_1}{2\sum K_B} = \frac{1}{4},$$

$$b = \frac{K_2}{2\sum K_B} = \frac{1}{4},$$

$$c = \frac{K_2}{2\sum K_C} = \frac{3}{10},$$

$$d = \frac{K_3}{2\sum K_C} = \frac{1}{5},$$

$$e = \frac{K_4}{2\sum K_D} = \frac{5}{16},$$

$$f = \frac{K_5}{2\sum K_D} = \frac{3}{16},$$

$$x = 1 - bc - de = \frac{69}{80},$$

$$y_1 = 1 - bc = \frac{37}{40},$$

$$y_2 = 1 - de = \frac{15}{16},$$

$$u_1 = 2 - b = \frac{7}{4},$$

$$u_2 = 2 - e = \frac{27}{16}.$$

4. Fixed End Moments:

The first and the last spans must be considered as prop spans in place of the usual fixed end spans when computing the end moments.

$$FM_{AB} = 0,$$

$$EM_{BA} = \frac{3}{2}\frac{wl^2}{12} = 1800 \text{ ft-lb},$$

$$FM_{BC} = + FM_{CB} = + \frac{Pl}{8} = + 8000 \text{ ft-lb},$$

$$FM_{CD} = + FM_{DC} = + \frac{wl^2}{12} = + 9600 \text{ ft-lb},$$

$$EM_{DE} = + \frac{3}{2}\frac{wl^2}{12} = + 9000 \text{ ft-lb},$$

$$FM_{ED} = 0.$$

5. Final Moments:

M	FM	$\sum FM_B = -6200$	$\sum FM_C = -1600$	$\sum FM_D = +600$
M_{AB}	0	0	0	0
M_{BA}	1800	-.5430	.1740	-.0543
M_{BC}	-1800	.5430	-.1740	.0543
M_{CB}	8000	-.0978	-.6080	.1904
M_{CD}	-9600	.0978	-.3914	-.1904
M_{DC}	9000	.0218	-.0870	.4022
M_{DE}	-9000	-.0218	.0870	-.4022
M_{ED}	0	0	0	0

$M_{AB} = 0,$

$M_{BA} = 1800 - (.5430)(-6200) + (.1740)(-1600) - (.0543)(600)$

$= 4857$ ft-lb.

In a similar manner

$M_{BC} = -4857$ ft-lb,

$M_{CB} = -M_{CD} = 9694$ ft-lb,

$M_{DC} = -M_{DE} = 9245$ ft-lb,

$M_{ED} = 0.$

2. INFLUENCE LINES

A single concentrated load of 10,000 pounds is to move across a five span continuous beam that is fixed at both ends and freely supported at four points. This beam has equal side spans of 25 feet each ($I_1 = 4$ ft.^{4}), equal intermediate spans of 50 feet each ($I_2 = 4$ ft.^{4}), and a center span of 50 feet ($I_3 = 4$ ft.^{4}).

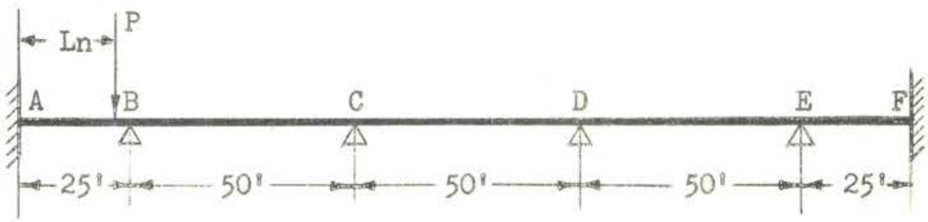


Fig. 19

Determine the influence line for moment M_{BA}.

SOLUTION

1. Select table: Case C5 Five Span Beam - Symmetrical.

2. Determine the stiffness factors:

$$K_1 = \frac{I_1}{L_1} = \frac{4}{25},$$

$$K_2 = \frac{I_2}{L_2} = \frac{4}{50},$$

$$K_3 = \frac{I_3}{L_3} = \frac{4}{50}.$$

$$\sum K_B = K_1 + K_2 = \frac{6}{25},$$

$$\sum K_C = K_2 + K_3 = \frac{4}{25},$$

3. Constant Functions of the New Distribution Factor:

$$a = \frac{K_1}{2\sum K_B} = \frac{1}{3},$$

$$b = \frac{K_2}{2\sum K_B} = \frac{1}{6},$$

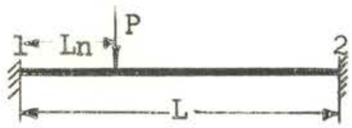
$$c = \frac{K_2}{2\sum K_C} = \frac{1}{4},$$

$$d = \frac{K_3}{2\sum K_C} = \frac{1}{4}$$

$$x = 1 - bc = \frac{23}{24}$$

$$z = \left(\frac{x}{d}\right)^2 - 1 = \frac{493}{36}$$

4. Fixed end moments due to "P":



$$FM_{12} = PLn(1 - n)^2,$$

$$FM_{21} = PLn^2(1 - n).$$

5. Influence Line equation for moment M_{BA}, load in 1st span:

$$M_{BA} = FM_{BA} - 2\frac{a}{x}(1 + \frac{bc}{z})\ FM_{BA},$$

$$= .3048\ PLn^2(1 - n) = 76,250\ n^2(1 - n).$$

n	$n^2(1 - n)$	M_{BA}
0	.000	0
.2	.032	2,440
.4	.096	7,310
.6	.144	10,970
.8	.128	9,760
1.0	.000	0

If the influence line equation is differentiated with respect to n and the result set equal to zero, the point of maximum moment is obtained.

$$\frac{\partial M_{BA}}{\partial n} = 2n - 3n^2 = 0, \text{ and } n = 0, \tfrac{2}{3}. \quad M_{BAmax} = 11,120 \text{ ft-lb.}$$

6. **Influence Line equation for moment M_{BA}, load in 2nd span:**

$$M_{BA} = 2\,\frac{a}{x}\left(1 + \frac{bc}{z}\right) FM_{BC} + 2\,\frac{ac}{x}\left(1 + \frac{1}{z}\right) FM_{CB},$$

$$= .6952\ FM_{BC} + .1865\ FM_{CB} = 348{,}000\ n(1 - n)^2$$

$$+ 93{,}200\ n^2(1 - n).$$

n	$n(1 - n)^2$	$n^2(1 - n)$	M_{BA}
0	.000	.000	0
.1	.081	.009	29,000
.2	.128	.032	47,600
.3	.147	.063	57,100
.4	.144	.096	59,100
.5	.125	.125	55,200
.6	.096	.144	46,900
.7	.063	.147	35,600
.8	.032	.128	23,300
.9	.009	.081	10,700
1.0	.000	.000	0

$$\frac{\partial M_{BA}}{\partial n} = 348{,}000\ (1 - 4n + 3n^2) + 93{,}200\ (2n - 3n^2) = 0,$$

$n = 1.201$, $.379$, and $M_{BAmax} = 59{,}120$ ft-lb.

7. **Influence Line equation for moment M_{BA}, load in 3rd span:**

$$M_{BA} = 2\,\frac{ac}{x}\left(1 + \frac{1}{z}\right)(-FM_{CD}) - 2\,\frac{ac}{dz}\,FM_{DC},$$

$$= -\ .1865\ FM_{CD} - .0487\ FM_{DC},$$

$$= -\ 93{,}300\ n(1 - n)^2 - 24{,}350\ n^2(1 - n).$$

n	$n(1-n)^2$	$n^2(1-n)$	M_{BA}
0	.000	.000	0
.1	.081	.009	-7,869
.2	.128	.032	-12,719
.3	.147	.063	-15,260
.4	.144	.096	-15,770
.5	.125	.125	-14,700
.6	.096	.144	-12,470
.7	.063	.147	-9,460
.8	.032	.128	-6,110
.9	.009	.081	-1,040
1.0	.000	.000	0

$$\frac{\partial M_{BA}}{\partial n} = -93{,}300\,(1-4n+3n^2) - 24{,}350\,(2n-3n^2) = 0,$$

$n = 1.188$, $.386$, and $M_{BAmax} = -15{,}820$ ft-lb.

8. **Influence Line equation for moment M_{BA}, load in 4th span:**

$$M_{BA} = -2\,\frac{ac}{dz}\,(-FM_{DE}) + 2\,\frac{abc}{dz}\,FM_{ED},$$

$$= 24{,}350\,n(1-n)^2 + 4{,}055\,n^2(1-n).$$

n	M_{BA}	n	M_{BA}
0	0	.6	3,220
.1	2,000	.7	2,130
.2	3,250	.8	1,300
.3	3,850	.9	550
.4	3,900	1.0	0
.5	3,550		

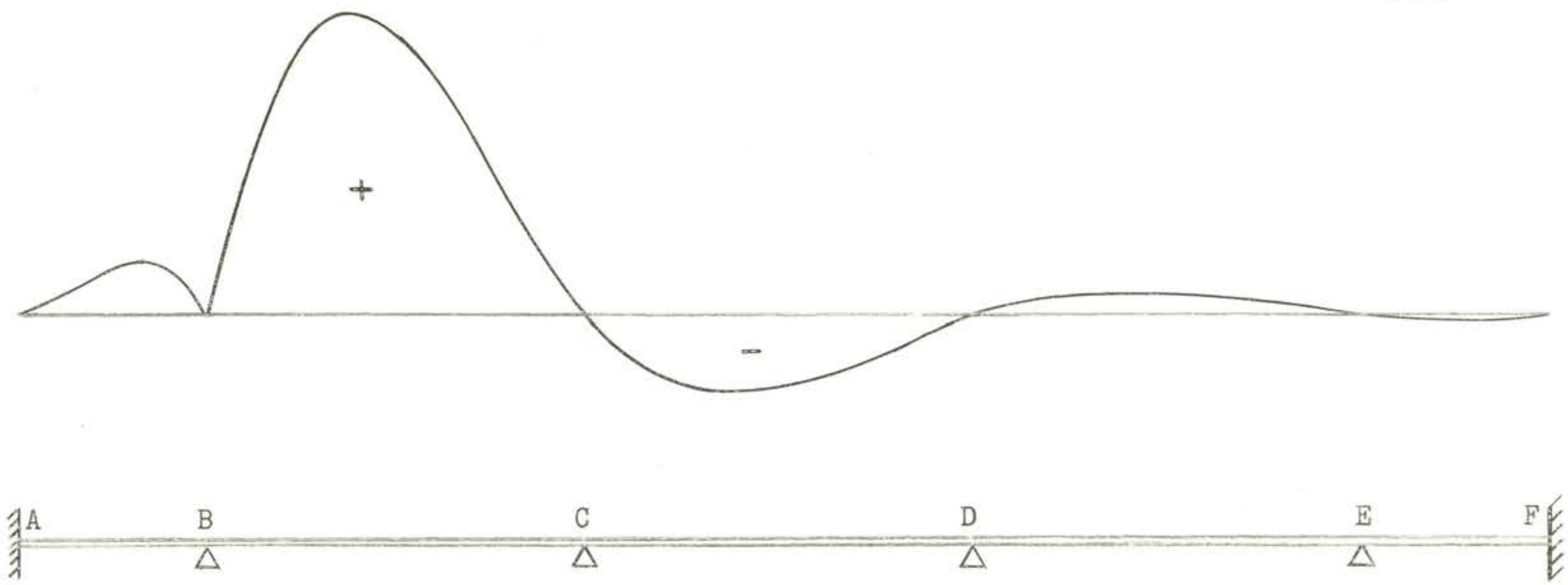
9. Influence Line equation for moment M_{BA}, load in 5th span:

$$M_{BA} = 2\,\frac{abc}{dz}\,FM_{EF} = 2028\,n(1-n)^2.$$

n	M_{BA}
0	0
.2	-254
.4	-290
.6	-195
.8	-65
1.0	0

Fig. 20
INFLUENCE LINE FOR MOMENT AT POINT B

Scale
1 in = 37,500 ft-lb



3. SETTLEMENT OF SUPPORTS

Determine the end moments of the six span beam shown in Fig. 21, due to the vertical displacements of the supports.

$\triangle_B = .5$ in,

$\triangle_C = .8$ in, $\qquad E = 30\ 10^6$ lb/in,

$\triangle_D = 1.0$ in, $\qquad I = 720\ in^4$,

$\triangle_E = .8$ in, $\qquad L = 360$ in,

$\triangle_F = .5$ in.

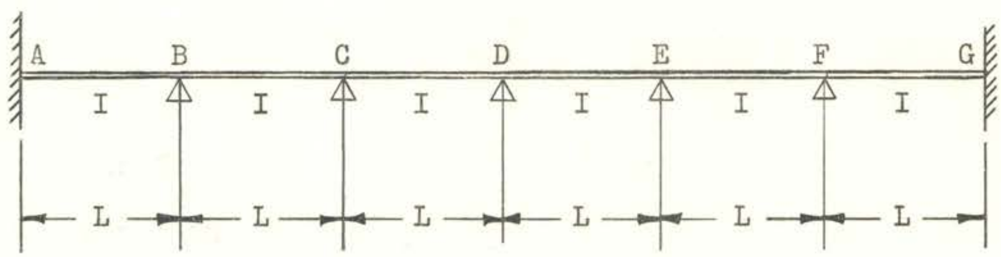


Fig. 21

SOLUTION

1. Select table: Case A60 Six Span Beam - Equal Spans.
2. Determine the fixed end moments due to the displacements:

$$FM = \frac{6EI}{L^2}\triangle = \frac{6\times30\times10^6\times720}{360\times360}\triangle = \triangle\ 10^6 \text{ lb.}$$

The fixed end moments at the low end of each span must be multiplied by (-1), then the tables may be used.

$FM_{AB} = -FM_{BA} = 5\times10^5$ in-lb, $\qquad FM_{DE} = -FM_{ED} = -2\times10^5$ in-lb,

$FM_{BC} = -FM_{CB} = 3\times10^5$ in-lb, $\qquad FM_{EF} = -FM_{FE} = -3\times10^5$ in-lb,

$FM_{CD} = -FM_{DC} = 2\times10^5$ in-lb, $\qquad FM_{FG} = -FM_{GF} = -5\times10^5$ in-lb.

M	FM	$\Sigma FM_B = -8\times10^5$	$\Sigma FM_C = -5\times10^5$	$\Sigma FM_D = 0$	$\Sigma FM_E = +5\times10^5$	$\Sigma FM_F = +8\times10^5$
M_{AB}	-5×10^5	-.2680	.0718	-.0192	.0051	-.0013
M_{BA}	-5×10^5	-.5360	.1436	-.0384	.0102	-.0026
M_{BC}	5×10^5	.5360	-.1436	.0384	-.0102	.0026
M_{CB}	-3×10^5	-.1242	-.4973	.1345	-.0359	.0090
M_{CD}	3×10^5	.1242	.4973	-.1345	.0359	-.0090
M_{DC}	-2×10^5	.0333	-.1333	-.5000	.1333	-.0333
M_{DE}	2×10^5	-.0333	.1333	-.5000	-.1333	.0333
M_{ED}	-3×10^5	-.0090	.0359	-.1345	.4973	.1242
M_{EF}	3×10^5	.0090	-.0359	.1345	-.4973	-.1242
M_{FE}	-5×10^5	.0026	-.0102	.0384	-.1436	.5360
M_{FG}	5×10^5	-.0026	.0102	-.0384	.1436	-.5360
M_{GF}	5×10^5	-.0013	.0051	-.0192	.0718	-.2680

$$M_{AB} = -5\times10^5 - .2680(-8\times10^5) + .0718(-5\times10^5)$$
$$+ .0051(5\times10^5) - .0013(8\times10^5) = -3.205\times10^5 \text{ in-lb.}$$

In a similiar manner

$M_{BA} = -M_{BC} = -1.40\times10^5$ in-lb.

$M_{CB} = -M_{CD} = .373\times10^5$ in-lb.

$M_{DC} = -M_{DE} = -1.20\times10^5$ in-lb.

$M_{ED} = -M_{EF} = .373\times10^5$ in-lb.

$M_{FE} = -M_{FG} = -1.40\times10^5$ in-lb.

$M_{GF} = 3.205\times10^5$ in-lb.

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