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KENNETH L. ELLIS

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THE EFFECTS OF INDIVIDUALIZED MATHEMATICS INSTRUCTION ON THE ACADEMIC ACHIEVEMENT OF SEVENTH-GRADE STUDENTS

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

One of the educational innovations of the New Technology has been the application of modern learning theory to teaching/learning techniques. Such programs usually start with specific objectives which can be measured in terms of the students' performance, record some measure of the students' ability to perform the objective prior to its being taught, teach the objectives in small incremental steps, usually by presenting materials in the form of programmed lessons and measure the students' ability to perform the objective after it has been taught. The primary advantage of such a system of instruction is that it is self-teaching and each student can proceed with the prescribed learning process at his own pace.

Educators have attempted to apply individualized instruction to almost every area of education. A handbook for teachers from the National Education Association (1964) reported that individualized instruction booklets, usually in the form of programmed texts, are available in more than five thousand areas. However, the success of individualized instruction as opposed to traditional teaching methods is still a moot question. An _Education U. S. A. Special Report_ was prepared in 1971 concerning the effectiveness of individualized instructional techniques. Programs being taught by individualized instruction methods were studied in forty-six (N = 46) school systems. Seventeen of the programs studied reported that those students participating in individualized instruction programs performed better than students who were taught the same materials via the traditional approach. However, further clarification of these results seems to be in order. First, student success was measured in terms of their performance on criterion-referenced tests developed specifically for testing students enrolled in the individualized instruction programs. Since these tests were prepared especially to measure the progress of the students through particualr individualized instructional programs, they may be biased in favor of these programs. Second, feedback from the questionnaires also revealed that most of the programs were not compared through the use of a scientific statistical approach (Appendix C). The lack of a pretest, inability to randomly select participants and the inability to control other extraneous variables prevented objective evaluations of most of the programs.

The Duluth Minnesota Public Schools compared students' academic achievement scores by administering the Iowa Test of Basic Skills (ITBS) and found no significant difference between the achievement scores of students participating in individualized instruction classes and achievement scores of students taught in the traditional manner. However, according to Giroux, Director of Planning and Evaluation for the Duluth School System, the evaluation was "not conducted in the context of a controlled research design with statistical analysis" (Giroux, 1974).

How effective are individualized instruction programs? While most educators seem to favor individualized instructional methods over traditional, there is not sufficient evidence to cause us to know that this type of instruction is more effective. Do the academic achievement scores of students who are participating in individualized instructional programs compare favorably with the academic achievement scores of students who are taught by traditional methods? If so, do the two groups' achievement scores compare favorably for students with different levels of ability? These are some of the questions which were investigated in the present study.

Statement of the Problem

The problem investigated in this study was to compare the academic achievement of students participating in an individualized instruction program with the achievement of students taught in a traditional manner. Stated more specifically, the purpose of the present study was to compare the mathematics achievement of seventh-grade students who were participating in an individualized mathematics program (Continuous Progress Mathematics) with the mathematics achievement of seventh-grade students who were not participating in the individualized mathematics program but who were taught by traditional methods.

Hypotheses Tested in the Study

In order to investigate the problem of the study, several hypotheses were tested. The null form of these hypotheses was as follows:

Ho_1 There is no statistically significant difference between the mathematics achievement gain shown for those seventh-grade students who participated in the individualized mathematics program and the mathematics achievement gain shown for those seventh-grade students who did not participate in the individualized mathematics program but were taught by the traditional method.

Ho_2 There is no statistically significant difference among the mathematics achievement gain scores recorded for seventh-grade students from three different ability groups.

Ho_3 There is no statistically significant interaction between the two independent variables of Type-of-Teaching (individualized instruction or traditional) and Ability Level (superior, average or below average) as reflected in the students' mathematics achievement gain scores.

Ho_4 There is no statistically significant difference among the mathematics achievement gain scores recorded for seventh-grade students from three different ability groups who had participated in an individualized mathematics program.

Definition of Terms

In order to avoid multiple interpretations of certain terms used in the present study, the following explanations and definitions are given:

1. Individualized-Instruction Mathematics Teaching: The method of teaching mathematics used by part of the

participating instructors which was proposed by the Continuous Progress Mathematics program and explained in Chapter III.

2. Traditional Mathematics Teaching: The method of teaching mathematics used by part of the participating instructors which involved the customary method of teaching-lecture, assignments, review and test.
3. Pretest Mathematics Achievement Score(s): The achievement percentiles recorded for the student participants from the first administration of the *Metropolitan '70 Achievement Test* (Metro '70) (Form F).
4. Posttest Mathematics Achievement Score(s): The achievement percentiles recorded for the student participants from the second administration of the *Metropolitan '70 Achievement Test* (Form G).
5. Mathematics/Achievement Gain Score: The arithmetic difference between the pretest mathematics achievement score and the posttest mathematics achievement score.
6. Experimental Group/CPM Group: Those seventh-grade students who participated in the Continuous Progress Mathematics program.
7. Control Group/Traditional Group: Those seventh-grade students who participated in the study but who were not participating in the Continuous Progress Mathematics program.
8. CPM Instructors: Those participating instructors who

taught mathematics with the Continuous Progress Mathematics system.

9. Traditional Instructors: Those participating instructors who taught mathematics by the Traditional Teaching method of lecture-assignment-test.
10. High Ability Students: Those student participants who scored above the 75th percentile (DIQ = 111) (national norms) on the *Otis-Lennon Mental Ability Test*.
11. Average Ability Students: Those student participants who scored between the 40th percentile (DIQ = 96) and the 75th percentile (DIQ = 111), range 40 through 75, on the *Otis-Lennon Mental Ability Test*.
12. Low Ability Students: Those student participants who scored below the 40th percentile (DIQ = 96) on the *Otis-Lennon Mental Ability Test*.

Assumptions Made in the Experiment

Certain assumptions were made about the students, the data collection instruments and the teaching methods used in the study. The most important of these assumptions were as follows:

(1) It was assumed that if one accepts the individual student as being unique and different from all other students and believes that students learn and retain different amounts of content at different rates, then it seems logical to assume that instruction for different students should be

individualized insofar as possible. Theoretically, individualized instruction which allows students to progress at their own pace should be less frustrating and more productive to students.

(2) It was assumed that the two populations of students were a true representation of the seventh-grade students enrolled at the public school system sponsoring the experiment. It was further assumed that the randomly-selected samples of high, average and low mental-ability students were a true representation of their parent populations.

(3) It was assumed that the data collection instrument, the <u>Metropolitan '70 Achievement Test</u> (Advanced Level: Forms F and G), was a valid and reliable instrument for measuring the mathematics achievement gain scores of the seventh-grade students in the experimental and control populations. It was also assumed that the items contained on the mathematics subtests of the Metro '70 measured the types of materials being taught in the experimental and control classes.

Limitations of the Study

Certain limitations were placed on the study in order to make it a reality. Without these limitations, the parameters of the data collection could not be properly set. The following limitations were established for the study:

Population Limitations

First, the student populations were limited to those seventh-grade students who were enrolled in and attending (full time) the public school system where the study was conducted. The experimental population, those students who were taught with the CPM method, contained approximately five hundred twenty-five (N = 525) students; and the control population, those students who were taught by the traditional methods, contained approximately one thousand (N = 1,000) students.

Instrument Limitations

Second, the students' achievement scores were limited to only three subtests of the Metropolitan '70 Achievement Test (Advanced Level: Forms F and G). These three subtests were (1) mathematics computation, (2) mathematics concepts and (3) mathematics problem solving. There is no doubt that the teaching methods used may have affected the students' achievement in other academic areas.

General Procedures Used in the Study

Two groups of seventh-grade students were used to compare the differences between mathematics achievement of students taught by two different methods. The first group, experimental students (N = 525), was taught mathematics by the Continuous Progress Mathematics (CPM) system. The second group, control students (N = 1,000) was taught mathematics by the traditional method. In order to determine the amount of achievement gained by each group, the Metropolitan '70 Achievement Test was given as a pretest (Form F) at the beginning of the school year and again as a posttest (Form G) at the end of the year.

A computer was used to randomly select one hundred fifty (N = 150) students from the experimental and control populations. Randomly-selected students were further divided into three mental ability groups: (1) high mental ability (above the 75th percentile on national norms), (2) average mental ability (between the 40th and 75th percentile on national norms) and (3) low mental ability (below the 40th percentile on national norms). Pretest-posttest mathematics achievement gain scores were compared for the different mental ability groups that were taught by the two different methods. A more detailed explanation of the methods and procedures used in the study is presented in Chapter III.

CHAPTER II

REVIEW OF RELATED RESEARCH

One of the greatest challenges to the individual classroom teacher has been to meet the individual differences that exist among pupils. Individual differences have long been recognized by such educators as Piaget (1963), Clymer and Kearney (1962) and Goodlad and Anderson (1963).

Piaget (1963) inspired the study of the development of children's thinking and thus a more realistic view toward readiness for learning. He was concerned primarily with the developmental stages through which children pass as they learn to accommodate the various stimuli which confront them. Piaget suggested that it is important to find each child's level of ability to function on the symbolic level as well as through concrete manipulations. Students at the junior high level are usually considered to be in the stage of development known as "Formal Operational Thought".

Johnson (1955) summarized the importance of Piaget's work in relation to individual differences in the following statements: First, the child must reach his own understandings; they cannot be handed to him ready-made. Second, mere acquisition of concrete experiences will not yield understandings; the elements of the experiences must be identified and processed (categorized). Third, in spite of the child's apparently innate capacity for acquiring some understandings, he may need considerable help in learning to process his experiences. Fourth, this processing will require that the child handle a variety of types of stimuli in an integrated rather than an isolated fashion. Fifth, because of his restricted experiences and thinking abilities, there are limits beyond which a child cannot progress through a particular stage of development (pp. 565-578).

To further complicate the matter, students develop at different rates at different times during their life. Such factors as their backgrounds, sex, race, social class, goals, needs, motivations and abilities can affect their levels of development. This would create no real problem if children were grouped on these factors, but they start to school according to only one factor--age. Consequently, most classes contain many different levels of ability and development. Educators recognize this fact and have tried to make some adjustments for individual differences. Three of the most common methods of coping with individual differences among students are discussed in this section of the literature.

Advancing Gifted Students: "Skipping" Grades

One of the most common practices in times past was the advancement of gifted students to a level commensurate with their academic/mental ability. This was commonly known as "skipping" grades. Educators soon discovered, however, that the social and interpersonal adjustment problems which the advanced students encountered were many and varied and the benefits gained in the academic areas were far outweighed by the losses encountered in social adjustment (Sigel, 1958). Sears (1959) offered an alternative reason for the advancement of gifted students in the following statement:

> For many years now educators have advanced gifted children under the guise of 'letting them proceed at their own rate'. Perhaps we should reexamine our motives when advancing gifted students. Almost without exception I have found that the students were seldom consulted and rarely in favor of being advanced. In nearly all of the cases studied (N = 727), the final decision to advance the students was made either to satisfy the child's parents, the teacher or both. (p. 132)

Most educators now believe that it is best, especially for the social development of children, to have them advance through the school grades with their agemates. Washburn (1953) states: "If the children have gotten along fairly well together and can work and play as a team, the teacher should have no hesitancy about letting them continue their group experiences together year after year."

Intraclass Grouping

An alternate approach to handling individual differences has been intraclass grouping of students. The several small groups into which primary-grade teachers subdivide their classes for reading is probably the most familiar example of intraclass grouping. The use for such grouping varies, however--in purpose, in the content children study and in the criteria applied in organizing the subgroups. Its usual purpose is to give children at low, intermediate (average) and high levels of intelligence or achievement the guidance each group needs for mastering the basic concepts and skills of the curriculum. Purposes of intraclass grouping may also include providing opportunities for each child to participate in projects suited to his talents, interests and social needs.

In addition to the advantages just mentioned, there are several disadvantages to intraclass grouping. Some of the most objectionable ones are mentioned by Frandsen (1970) as follows: (1) Intraclass grouping can result in a stigma being attached to a particular person or group of persons; (2) Intraclass grouping can result in much more work for the teacher and much less time spent with the entire class; (3) Intraclass grouping can stifle students' motivation if they feel they do not have to strive to keep up with the group norm; (4) Intraclass groups are usually formed on the students' achievement levels rather than their ability levels.

In summarizing intraclass grouping, it may be said that intraclass grouping does not enjoy the popularity it once had. However, most classes in any school system are comprised of either defined or undefined subgroups.

<u>Individualized Instruction</u>

Although intraclass grouping may simplify the problem of teaching heterogeneous groups, it does not meet all the needs of every child. For example, a particular fourth-grade child may need systematic, patient, individualized guidance in learning first-grade level phonetic and visual analysis techniques for identifying unfamiliar words. Such a situation would require individual help. Many teachers now use a variety of procedures to individualize instruction to each student's ability and needs.

This section of the literature survey contains the basic assumptions behind a program of individualized instruction and gives several examples of programs now in operation.

Duker (1972) has cogently summarized and contrasted the assumptions underlying individualized instruction. He states these five basic assumptions as follows:

> (1) Learning is promoted by giving the learner a part in determining what shall be learned, how it shall be learned and when it shall be learned.
>
> (2) The amount learned should be consonant with the ability, the motivation and the interests of the learner.
>
> (3) The pace of learning should also be suitable to the ability, motivation, pace and interests of the learner.
>
> (4) Evaluation should be based on the progress of the learner rather than on a comparison with other learners or with arbitrary standards.
>
> (5) Learning should never be an entirely solitary task so that there should be ample opportunity for sharing one's unique learning experiences with others (p. 30).

These five basic assumptions are behind all individualized programs of learning, and differences among programs of individualized education are more in terms of techniques than philosophy. It is obvious that these five basic assumptions are in contrast with the implicit assumptions underlying the conventional approach to instruction. These were stated by Duker as follows:

> (1) There are certain fundamental skills and knowledges which must be mastered by everyone.
>
> (2) There is one logical sequence into which these skills fall and the primary skills preceding any particular secondary skill must be mastered before that skill can be taught.
>
> (3) Any kind of fundamental differentiation of the above principles in the case of a particular individual is basically undemocratic.
>
> (4) Since the child when he matures will have to be a part of many groups as he takes his place and role in society, group instruction is the most satisfactory way of learning and constitutes the soundest preparation for life.
>
> (5) Since we live in a competitive society, it is beneficial to evaluate a child's work in terms of the accomplishments and achievements of others (p. 30 and 31).

These two sets of basic assumptions are established on logical and empirical bases and both have been utilized by educators with varying degrees of success. Some of the different types of individualized educational programs are described in the studies presented in this section of the literature.

The idea of individualized instruction is not really a new concept in education. The Winnetka (Illinois) Plan and the Dalton (Massachusetts) Plan were developed during the 1920's. In both the Winnetka and Dalton Plans, students were expected to undertake assignments or contracts and work at them at their own rates for as long as necessary to complete them within an allotted or contractual period. As students demonstrated their ability to complete contracts on time they were given more responsibility.

In the Dalton Plan, as operated in New York City during the 1950's, students were given monthly assignments that were uniform for a specific grade level. Since the Dalton School enrolled children from high-income families, uniform assignments based on grade-level standards were given. Where individual differences in pupil ability made it unlikely that a particular child would be successful in completing an assignment, teachers were given the authority to lower their expectations and modify the assignments for that child so that he could continue to participate with the class as a whole. However, most parents found that slower pupils needed to be tutored (out of school) to keep them from falling behind the school's achievement standards (Spaulding, 1970).

The Southside School in Durham, North Carolina, has adopted a modified version of the Dalton Plan for its students to follow. Students are grouped into units (N = 20) called "prides", which meet and plan their learning schedules.

Children are assigned to each pride on the basis of academic achievement, learning rate, sex, race and degree of socialization. Each student plans his daily schedule (with the help of his pride teacher) and attends the classes, performs the activities and meets the constraints needed to complete the daily plan. The students' daily plan schedules are approved as they move from one work station to another. Positive comments for quality work, creative ideas or products, or developing skills are also entered on the plan sheets. Individualization is achieved in this educational program by giving students several options for meeting the requirements established for different areas of learning such as reading, mathematics, spelling, science, social studies and health.

Another program of individualized education is The Borel Individualized System of Instruction (ISI) which was introduced and implemented in the Borel Middle School of San Mateo, California (Kramer, 1971). This individualized system of instruction consists of planning and conducting with each student a program of studies that is tailored to his learning needs and his characteristics as a learner.

The course of study is defined and organized through a series of sequential learning contracts. Each contract contains a specific learning objective stated in terms of what the student must do to demonstrate accomplishment of the objective (performance objective). The contract indicates the learning materials and procedures required. The teacher serves more often as an educational consultant to each child rather than as an imparter of knowledge. The student initiates and completes each contract consulting with the teacher as needed. Each student must successfully complete one learning contract before he may progress to the next.

This section of the literature review has contained examples of programs of individualized instruction which have been implemented by different school systems. However, none of the programs presented was limited to the area of mathematics, but programs of individualized instruction in mathematics are presented in the next section of literature.

Studies Concerning Individualized Instruction in Mathematics

The purpose of the present study was to examine the differences between the achievement levels of students who were taught mathematics with an individualized mathematics program and students who were taught mathematics with a traditional (conventional) mathematics program. Perhaps detailed definitions of the two types of mathematics programs are in order (Fisher, 1967).

Individualized Mathematics Program

An individualized mathematics program is one that is actually tailored to fit the needs of the individual student. It should include the following:

(1) Each student receives instruction at his own pace.

(2) A wide range of instructional materials is available in the classroom.

(3) Each child is permitted to progress at his own rate.

(4) Each child is permitted to meet with the teacher individually or in small group of students with similar problems.

(5) Each child is permitted to check his own work as he works through the assignments.

(6) The slow learner is not required to meet the standards of the group, and the bright child is permitted to explore areas of mathematics in which he is interested.

Conventional Mathematics Program

A conventional mathematics program has the following characteristics:

(1) Instruction is given to the entire class at one time and is in the form of lectures, demonstrations and/or discussions.

(2) All students receive instruction from the same textbook.

(3) No differentiation is made in assignments to meet individual needs or abilities.

(4) All students are expected to progress through the development of a concept at the same rate

and to cover a set amount of material in a given year.

(5) The student's daily work is checked by the teacher and returned one or two days after the student has completed it.

(6) All students, regardless of mathematical ability, compete against one another for grades within a pre-determined set of standards.

Programs of individualized instruction in mathematics have been based on the guidelines established by the first definition. Some of the studies regarding such programs are presented in this section of the literature.

Individually Guided Education (IGE)

Multiunit Individually Guided Education (IGE) is an individualized program of study at the elementary level (Grades 1 through 6) in the Janesville (Wisconsin) Public School System. Four elementary schools within this system were randomly selected to determine the relative effectiveness of the IGE program. Students from two of the schools that had been using the IGE program for three years (three-year schools) were compared with students from two other schools that were just beginning to implement the IGE program (transitional schools). The schools being compared had been equated on such factors as socio-economic status of students, number of students enrolled and mental ability of the students being compared. Comparisons were based on percentile scores taken from a pretest-posttest administration of the Metropolitan '70 Achievement Test. Second- and sixth-grade students from the three-year program schools showed five and seven percentile (5 - 7%ile) gains on the mathematics subtests while the transitional schools showed a comparable amount of loss on their mathematics achievement scores (-5 to -7%ile) (Loofboro, 1972).

Final results of the Janesville study showed that the children in the two multiunit schools achieved higher mean test scores than children in the transitional schools at all levels of instruction, except spelling at the sixth-grade level. Further comparisons among the other grades within the multiunit and transitional schools showed even greater areas of gain for the multiunit schools.

Individually Prescribed Instruction (IPI)

Without a doubt, the introduction of Individually Prescribed Instruction (IPI) has been the most elaborate and extensive experiment in the field of individualization of instruction since its inception. This program was originated at the University of Pittsburgh by Dr. J. R. Fisher in 1961.

The IPI program was developed to emphasize inquiry, exploration and discovery. The tests were designed to help pupils move from one idea to another; and throughout this procedure, the pupils are encouraged to explore and discover relationships among phenomena.

Some of the strategies used in Individually Prescribed Instruction Mathematics and Continuous Progress Mathematics are identical. Behavioral objectives have been developed in both programs, and educational materials (solutions to exemplary problems with explanations) are then used to facilitate learning. Other similar aspects of both programs include small group work of two or more students with interaction between these groups and the teacher and interaction between the individual students and the teacher. Consumable materials are also used for instructional purposes, and curriculum embedded tests and sub-goal tests are used to determine whether the student has mastered the subject matter at a required level. Since Individually Prescribed Instruction is similar to Continuous Progress Mathematics and because there are no previous scientific statistical evaluations of Continuous Progress Mathematics, the researcher has included a number of comparative studies made with Individually Prescribed Instruction.

Research for Better Schools Incorporated reported results of seventeen different studies comparing mathematics achievement of Individually Prescribed Instruction with the traditional mathematics teaching method (Scanlon and Becker, 1971). One of these studies was conducted by the Learning Research and Development Center of the University of Pittsburgh. Differences between students' mathematics achievement for 420 elementary students were not statistically significant as measured by standardized tests.

Six of the reported studies were conducted by individuals working with school systems, and results of these studies are summarized as follows:

(1) Gallagher and Lewy in separate studies found no significant difference;

(2) Highlands Elementary School reported an improvement for both groups with the control group showing greater improvement;

(3) Hoeltze and Gilchrist in a study comparing grades two through six reported the only significant difference at the sixth-grade level and this was only on arithmetic concepts;

(4) Hestwood reported that an experimental group made twice the achievement gain made by the control group;

(5) Sandvick reported greater gains for the control group in grades one through six but more gain for the experimental group in grade seven.

Research for Better Schools also conducted eleven other studies comparing Individually Prescribed Instruction with the traditional mathematics teaching method. Tests used in these comparisons were the Iowa Test of Basic Skills, a standardized test, and the Individually Prescribed Instruction Tests, which are criterion referenced tests. One or both of these tests were administered in the eleven studies to students ranging from grades three through six. On the _Iowa Test of Basic Skills_ only one of the studies indicated that the group using Individually Prescribed Instruction scored higher. Two of these studies reported no significant difference, and five of the studies reported the students using the traditional mathematics teaching method scored higher on arithmetic concepts and problem solving.

Comparisons of the two different methods of instruction using the _Individually Prescribed Instruction Tests_ reported the Individually Prescribed Instruction group scoring consistently higher on six of the studies. Two other comparisons using these tests were in favor of the group using the _Individually Prescribed Instruction Tests_; however, the results were mixed.

Another comparison of Individually Prescribed Instruction and the traditional mathematics teaching method was conducted with selected sixth-grade students by Verheul (1971). Primary purpose of the experiment was to determine whether the Individually Prescribed Mathematics program would cause a significant improvement in mathematics learning. Other areas studied in the experiment included the relationship between mathematics achievement, I. Q. scores and selected self-concept factors. The instruments used for pretesting and posttesting were the arithmetic portion of the _Comprehensive Tests of Basic Skills_, the _California Short-Form Test of Mental Maturity_ and the _How I See Myself Scale_.

Results of the experiment at the .05 level of confidence indicated no significant difference in mean score gains between the two groups for arithmetic concepts and arithmetic problem solving. There was a significant difference at the .05 level in favor of the students who used the traditional mathematics teaching method in the areas of arithmetic computation and total arithmetic. Students using the traditional mathematics teaching method made significantly higher mean scores on the pretest for arithmetic concepts and problem solving, and on the posttest these students scored higher in all four arithmetic areas.

In another study, Thomas (1972) compared mathematics achievement of 373 fifth- and sixth-grade students from three different schools with students who had been using the traditional mathematics teaching method. Students using Individually Prescribed Instruction Mathematics were subjects in the Individually Prescribed Mathematics Instruction Group and had used this method for a two-year period, while subjects in the traditional mathematics classes had used only the traditional mathematics teaching method.

Pretest and posttest scores on the Comprehensive Test of Basic Skills were used to measure achievement. The Prescriptive Mathematics Inventory was also used to determine mathematics deficiencies.

The Individually Prescribed Instruction Mathematics group scored significantly higher in only one instance, and this was on the Prescriptive Mathematics Inventory Test at the sixth-grade level while the fifth-grade students using the traditional mathematics teaching method scored higher than the experimental group on the same test.

Where significant differences were found with the Comprehensive Test of Basic Skills, all of the students using the traditional mathematics teaching method excelled.

Teacher Prescribed Individualized Mathematics Instruction Studies

Broussard (1971) compared the arithmetic achievement of inner-city school fourth graders, who were economically and educationally deprived, by using two equated groups who were selected from 495 subjects. The design of the study was the non-randomized, control-group pretest-posttest design. This study compared students using the traditional mathematics teaching method with students who were given individually prescribed work through independent study, small group discussions, large group activities and teacher-led discussions. Students in both groups were exposed to the same mathematics content with the only difference being the change in the method of instruction.

Results of the study indicated that sex, racial and ethnic differences did not significantly affect the academic achievement in mathematics computational skills of the subjects. The students in the individualized mathematics group achieved significantly higher achievement gains in the area of computational skills and arithmetic concepts than the group using the traditional mathematics teaching method. There was no significant difference between the two groups in the area of problem solving. The individualized mathematics group also scored significantly higher on total mathematics than the control group using the traditional mathematics teaching method.

Broussard stated that participating teachers, specialists, instructional aides, pupils and parents were very positive in their statements concerning the individualization of the mathematics instruction.

Grant (1964) conducted a longitudinal study of the effects of an individualized program of mathematics for fourth-, fifth- and sixth-grade students. Grant compared the achievement test scores of students over a three-year period. Results of the comparisons made between the pretest and posttest scores taken from the Stanford Achievement Test were not statistically significant. Differences in achievement gain, however, did suggest that there was a trend favoring the students in the individualized program of mathematics.

In a similar study, Riedesel (1962) prepared mathematics lessons with problems having two different levels of difficulty. Students in the experimental classes were allowed to choose the level of difficulty they wished to solve. The sixth-grade students in the experimental classes were matched with students in the control classes on mental ability. Results of the experiment showed that students in the experimental classes made significantly higher gains than students in the control classes.

The studies presented in this section have reported some of the research efforts which have been conducted with individualized programs of mathematics instruction involving students at the elementary and junior high school levels. Studies of individualized mathematics instruction and the students' mental ability levels are presented in a later section.

Programmed Instruction

Programmed instruction does not meet the guidelines established by the definition of individualized instruction earlier in this chapter, since there is usually less interaction between students and teachers; and in some cases there may be no student-teacher interaction in programmed instruction. Materials that are used in programmed instruction may consist of only a programmed text or some other type of programmed instruction while individualized instruction usually has a wide range of supplementary material. However, programmed instruction is self-paced. Since self-pacing is one aspect of all individualized instruction, the researcher has included two studies which were conducted at the junior high level.

Blair (1963) compared the mathematics achievement of students using the traditional mathematics teaching method in Algebra I at the ninth-grade level in five different junior high schools. The achievement scores of two classes in each of the five schools were found to be statistically equal on four different variables: eighth-grade scholastic mathematics grades; intelligence quotients, both language and non-language; mathematics achievement; and an Algebra I pretest. One experienced teacher in each of the five schools then taught the two equated classes using programmed instruction with one class and the traditional mathematics teaching method with the other.

Students and teachers kept records of gains made and the amount of time required to achieve those gains. A significant difference in mathematics achievement ($p < .01$) was found in favor of the group using the traditional mathematics teaching method. There was also a significant difference ($p < .05$) in time required to complete different areas of work which favored the traditional mathematics teaching method.

The second study of programmed instruction is included in the review of literature titled "Studies Concerning Mathematics Achievement and Mental Ability Levels".

Studies Concerning Mathematics Achievement and Mental Ability Levels

The effectiveness of programmed instruction was compared with the effectiveness of the traditional mathematics teaching method using 179 low achieving seventh-grade students from three different schools (Tanner, 1965). Subjects used in the study were classified as belonging to the lowest of three achievement groups on the basis of standardized tests.

No significant difference was found in gains on arithmetic computation for students from any of the three schools. However, there was a significant difference ($p < .05$) in arithmetic concepts in favor of the group using the traditional mathematics teaching method in two of the schools, and in the third school there was a significant difference in arithmetic concepts favoring this same group ($p < .01$).

Teacher Prescribed Instruction and Mental Ability Levels

Check (1959) used a sample of 120 fifth-grade students to compare the retention rates of mentally retarded (N = 40), average intelligence (N = 40) and high intelligence (N = 40) students. All students were given problems in mathematics computation and mathematics problem solving which were equated with their achievement level at the beginning of the experiment. After testing the hypotheses, Check concluded that retention rates for the three groups were not significantly different among the three groups after five minutes, seven weeks and fourteen weeks when the original task for each child was graded to his achievement level.

In a later study, Klausmeier and Laughlin (1961) compared the problem solving techniques employed by sixth-grade students of high, average and low mental ability.

They found a great deal of similarity between the techniques used by the students of average mental ability and the students in both the high and low mental ability groups. On the other hand, discrepancies were noted between the techniques used by the high mental ability group and the low mental ability group. The bright children corrected their own mistakes, arrived at unifying solutions and took logical approaches to the solution of problems. On the other hand, the "slow" children lacked persistence, gave incorrect solutions and used random processes in solving problems.

However, Schippert (1964) was able to show more encouraging results from a study he conducted with 688 seventh-grade students. He compared the effects of two discovery methods of teaching mathematics at the seventh-grade level. Pretest-posttest scores taken from the Iowa Test of Basic Skills (ITBS) were used to determine the students' gains after a one-year period and again two years later. Comparisons were also made between the two groups' attitudes about the teaching techniques used in the experiment. Results of the study showed that the students who were taught by the laboratory discovery method made significantly greater gains than those students who were taught by the abstract discovery method. The laboratory discovery groups after the end of the first year and again after two years had passed. The laboratory students also showed significantly better attitudes toward mathematics than those students taught by the abstract discovery methods. Further investigation of the student groups' achievement on three levels of mental ability showed that in general the brighter students were able to handle the abstract discovery method better, while the slower students responded much better to the laboratory method of teaching mathematics.

Nix (1969) conducted a study with eighth-grade students who had used only traditional mathematics teaching methods prior to the study. At the beginning of the school year three eighth-grade classes were selected to receive the individualized instruction and three to receive the traditional mathematics teaching method.

Students using the individualized instruction method were issued textbooks of the same series that were at different grade levels. Grade level of the textbook issued to the individual student was determined by his score from the mathematics achievement test. Algebra books of a different series were used by students that advanced above the eighth-grade level. Each student studied each unit in his textbook at his own rate of speed and was tested after completion of the unit. A passing score allowed the student to progress to the next unit, and a failing score required additional work on problems of the type missed.

All of these students were given the _Otis Quick-Scoring Mental Ability Test_ and the _Stanford Achievement Test_ at that time. This information as well as sex and age was used to divide the control group and the experimental group into subgroups according to sex, age, intelligence, overall achievement and mathematics achievement.

The mathematics portion of the Stanford Achievement Test was used as a pretest and posttest to determine the change in mathematics achievement for each student. Mean change scores of corresponding subgroups were used to determine if students of different age, sex and previous levels of achievement achieved more with individualized instruction or the traditional mathematics teaching method.

Different students in corresponding subgroups, in some cases, achieved more with the traditional mathematics teaching method and in others with the individualized instruction. However, students of average mathematical ability, students of below average intelligence and boys achieved significanly more under individualized instruction than the corresponding subgroups using the traditional mathematics teaching method.

Crangle (1971) conducted a study similar to Nix's study at Northwest Junior High School in Salt Lake City. The control group using the traditional mathematics teaching method was composed of 31 subjects who were randomly selected from the total enrollment of 914 students. The experimental group of 31 subjects were individually matched to the control group subjects according to age, sex, intelligence quotient and mathematics achievement.

The *Stanford Achievement Arithmetic Computation Subtest* (Form X) and the *Iowa Test of Basic Skills* (Test W, Form 1) were administered as pretests and this information was used to match the two groups. This pretest data was used with the posttest scores to determine if significant differences existed between the pretest and posttest scores for the two groups.

Findings indicated that there was a significant difference in favor of the group receiving the traditional mathematics teaching method in mathematics achievement. There was no significant difference in mathematics achievement between students of lower intelligence in the two groups, and there was also no significant difference in work-study skills achievement between all of the students in the two groups. Students using the traditional mathematics teaching method also used significantly less time to complete the study.

Nabors (1968) also compared the mathematics achievement of 316 fifth-grade students in five elementary schools that had been randomly divided into an experimental group which used individualized instruction and a control group that used the traditional mathematics teaching method. The experimental group and the control group were then separated into three subgroups based on sex, reading ability and intelligence.

Problems were prepared in sets having eleven different levels of difficulty taking into consideration the level of mathematics and reading difficulty. Students in the individualized mathematics group were asked to solve problems from these eleven different levels according to their own ability while the control group used only assignments from the regular fifth-grade mathematics textbook.

Two different forms of the Iowa Test of Basic Skills were used as a pretest and posttest, and the experiment was conducted for a period of 10 weeks. Test results indicated no significant difference between the two groups on arithmetic concepts, and there was also no significant difference on arithmetic concepts when the subgroups were compared.

However, score gains on the problem solving test indicated a significant difference ($p < .05$) favoring pupils of average intelligence who were in the individualized instruction group, and boys of average intelligence in this group also scored significantly higher.

Individually Prescribed Instruction (IPI) and Mental Ability

Fielder (1971) conducted a study which compared Individually Prescribed Instruction on student achievement in mathematics with the achievement of students that were in the regular instruction program of the Abilene Public Schools, Abilene, Texas. This investigation surveyed all students in grades 3 and 4 in 1968 and those in grades 5 and 6 in 1970. Arithmetic subsections of the Stanford

Achievement Battery were utilized as pretest and posttest data and were used to measure computation, concepts and problem solving. Students in one of the elementary schools were subjects in the experimental group, and the control group was randomly selected from the total school population.

The null hypothesis of no significant difference between the experimental group (Individually Prescribed Instruction students) and the students in the control group using the traditional mathematics teaching method was rejected in favor of the control group in arithmetic computation ($p < .002$) for the fifth grade and at ($p < .001$) in the sixth grade. Posttest scores in concepts and problem solving were also higher for the students using the traditional mathematics teaching method; however, these differences were not significant. Pretest scores for the students in both groups were similar, while all of the mean scores on the posttest were in favor of the control group.

Fielder concluded that computation skills were affected more adversely by usage of the Individually Prescribed Instruction than concepts and problem solving skills. However, students in the Individually Prescribed Instruction group scored lower on all of these subtests. Results of this study did not indicate that the Individually Prescribed Instruction was better for low achievers or high achievers.

According to Fielder the isolation of students using the Individually Prescribed Instruction and a lack of interaction seemed to adversely affect their achievement. He also stated that strategies of Individualized Prescribed Instruction aimed at individualizing instruction seemed largely confined to those of readiness and rate.

In a study somewhat comparable to the one conducted by the researcher, Deep (1966) compared the mathematics achievement of fourth-, fifth- and sixth-grade children who had been grouped into high (IQ of 111 or higher), average (90-110 IQ) and low (IQ of 89 or less) intelligence. The results of the study showed that the higher ability students mastered more skills and units than did the lower ability students. However, despite the suggestion that the bright students progressed faster and mastered more material than the slower students, the results of the standardized tests used in the study seemed to raise a contradiction. These results indicated no significant difference among the high, average and low ability students in arithmetic computation or problem solving scores whenever the pretest performance was taken into account.

Publishers of the Individually Prescribed Instruction (IPI) Program reported at least one evaluation of their system. The Urbana (Illinois) Public School System compared the reading and mathematics scores of 200 pupils from schools using the IPI Program with the reading and mathematics scores of 200 pupils from schools not using the IPI Program. Pupils were given the California Achievement Test at the beginning and end of the 1966-67 academic year to determine the amount of achievement experienced during that time. Students from both groups had been subgrouped into three mental ability categories: high intelligence (111 or higher IQ), average intelligence (90-110 IQ) and low intelligence (below 90 IQ).

Results of the comparisons showed that students in the IPI Program scored higher than their counterparts in all areas and at all three mental ability levels. In most cases, students in the IPI groups scored seven to nine percent (7-9%) higher than the traditional groups. However, in a few instances the IPI pupils scored lower than the non-IPI pupils. These results were confined to the low intelligence groups (Downey, 1974).

Summary of Individualized Mathematics Studies With Teacher Prescribed Materials and Programmed Instruction

Broussard (1971) compared the achievement of fourth-grade students using the same mathematics content, the only difference between the two groups in this case being the method of presentation. One group used the traditional mathematics teaching method and the other group used independent study, small group discussions, large group discussions, large group activities and teacher-led discussions. Students using individualized instruction scored significantly higher on computational skills, arithmetic concepts and total mathematics. There was no significant difference found in the area of arithmetic problem solving.

Another study similar to Broussard's was conducted by Nix (1969) with eighth-grade students in general mathematics. Significant differences were reported for students of average and below average ability and for boys with the difference favoring the individualized mathematics group.

A third study by Crangle (1971) with eighth-grade students reported an overall significant difference in mathematics teaching method. However, there was no significant difference in achievement between students of lower intelligence in the two groups.

Nabors (1968) found no significant difference for fifth-grade students on arithmetic concepts when individualized instruction was compared with the traditional mathematics teaching method. However, there was a significant difference for pupils of average intelligence in problem solving which favored the individualized instruction group.

Grant (1964), in a three-year longitudinal study found no significant difference for students using individualized mathematics and those using the traditional mathematics teaching method. However, the difference which was not significant, was in favor of the group using the individualized mathematics.

Riedesel (1962) found students in experimental classes using teacher prescribed materials achieved significantly more than students in control classes.

Results of studies of individualized instruction using different types of teacher prescribed materials are inconsistent. Broussard (1971), Nix (1969), Nabors (1968), Grant (1969) and Riedesel (1962) reported some differences favoring individualized instruction while Crangle (1971) found significant differences favoring the traditional mathematics teaching method.

Summary of Programmed Instruction

Comparisons of programmed instruction with the traditional mathematics teaching method were made by Blair (1963), who compared ninth-grade students in Algebra I, and Tanner (1965), who compared lower ability seventh-grade students in an arithmetic class. Blair's findings indicated a significant difference favoring the Algebra I students using the traditional mathematics teaching method. In Tanner's study no significant differences were found among the arithmetic computation scores of students from the three schools used in the experiment. However, there was a significant difference among students' arithmetic concepts scores at all of the schools. Differences favored the traditional mathematics teaching method.

Summary of Comparisons of Individually Prescribed Instruction and Individually Guided Education With Traditional Instruction

Fielder's study (1971) indicated that the traditional mathematics teaching method was more effective for total mathematics and also for the subtests: concepts, computation and problem solving. Verheul's comparisons (1971) indicated no significant difference between the two teaching methods for arithmetic concepts and problem solving. However, there was a significant difference favoring the traditional mathematics method for arithmetic computation and total arithmetic.

Thomas (1972) found no significant difference between achievement gains when comparisons were made; however, the control students excelled where differences occurred. Downey's study (1974) indicated a significant difference favoring the experimental group at the high, average and low ability level.

Deep (1966) compared the achievement gains of students using two different methods of instruction and found no significant difference. His results were similar to those reported by Thomas as control students scored greater gains where differences occurred.

Loofboro (1972) compared the mathematics achievement of students using Individually Guided Education with students in control classes and reported the experimental classes gained from five to seven percentile points (5-7%), and the control classes showed a loss of five to seven percentile points (5-7%).

Studies reported by Research for Better Schools (Scanlon and Becker, 1971) indicated that students using Individually Prescribed Instruction did as well as control students on standardized tests. However, these results differed from those reported by Deep (1966), Fielder (1971), Verheul (1971) and Thomas (1972). Students using Individually Prescribed Instruction did seem to score significantly higher on the Individually Prescribed Instruction Placement Tests than students in control classes. Comparisons of Individually Guided Education also seemed inadequate to establish the total effectiveness of their program.

The evidence presented above indicates that there were many inconsistencies among research findings. Effectiveness of different types of individualized mathematics instruction in increasing achievement gain for students when mental ability levels are disregarded remains an unanswered question.

It seems important that educators continue using innovations that will individualize instruction to accommodate the many differences in individual learners. However, all educational programs should be evaluated constantly with the intent of improving instruction and determining total effectiveness. Continuous Progress Mathematics, another attempt to individualize and improve the teaching of mathematics, has not been evaluated through the use of scientific and statistical methods. A comparative study of the effectiveness of the Continuous Progress Mathematics Program seems appropriate at this time.

CHAPTER III

METHODOLOGY

In the present study, seventh-grade students from a large public school system acted as subjects to determine possible differences between two methods of teaching mathematics at the junior high school level. One group, the experimental population, was taught mathematics with an individualized program of instruction termed "Continuous Progress Mathematics" (CPM). The other group, the control population, was taught mathematics by the traditional methods of lecture, review and test. Three groups of students were selected from both the experimental population and the control population. These groups, representing three different levels of mental ability, were compared for the amount of gain shown after a pretest-posttest administration of the Metropolitan '70 Achievement Test (Advanced Level: Forms F and G). It had been hypothesized that the experimental population, those taught with the CPM system, would achieve significantly more than those students taught by the traditional method. It was further hypothesized that the students with different levels of mental ability would experience varying amounts of pretest-posttest achievement gain.

The methods and procedures used in conducting the present study are presented in this chapter.

Pre-Experimental Procedures

All of the tasks completed before the data were gathered are pre-experimental procedures and the most important of these are explained in the following sections.

Selection of Research Design

Research design has as its purpose "obtaining answers to research questions and controlling variance." Design helps the researcher obtain answers to his questions as it includes the plan, structure and strategy used to control variance. This procedure includes the framework for testing relations between variables, describes the observations that are to be made and explains how to make them. Lehmann and Mehrens (1971) make the following statements concerning research design:

> An adequate research design tells us what variable(s) is to be manipulated and how this variable is to be manipulated; it tells us what statistical analysis to use to analyze the data (and hence, depending upon the kind of statistic to be used, it tells us what assumptions must be met and therefore how our data are to be collected); and indirectly, it tells us the kinds of conclusions and inferences that can be made from the data. (p. 343)

Major purpose of the research design is the control of as many sources of variation as possible except the variation accounted for by the treatment. Figure one is a paradigm of the research design used in this experiment. The following steps are shown in Figure I:

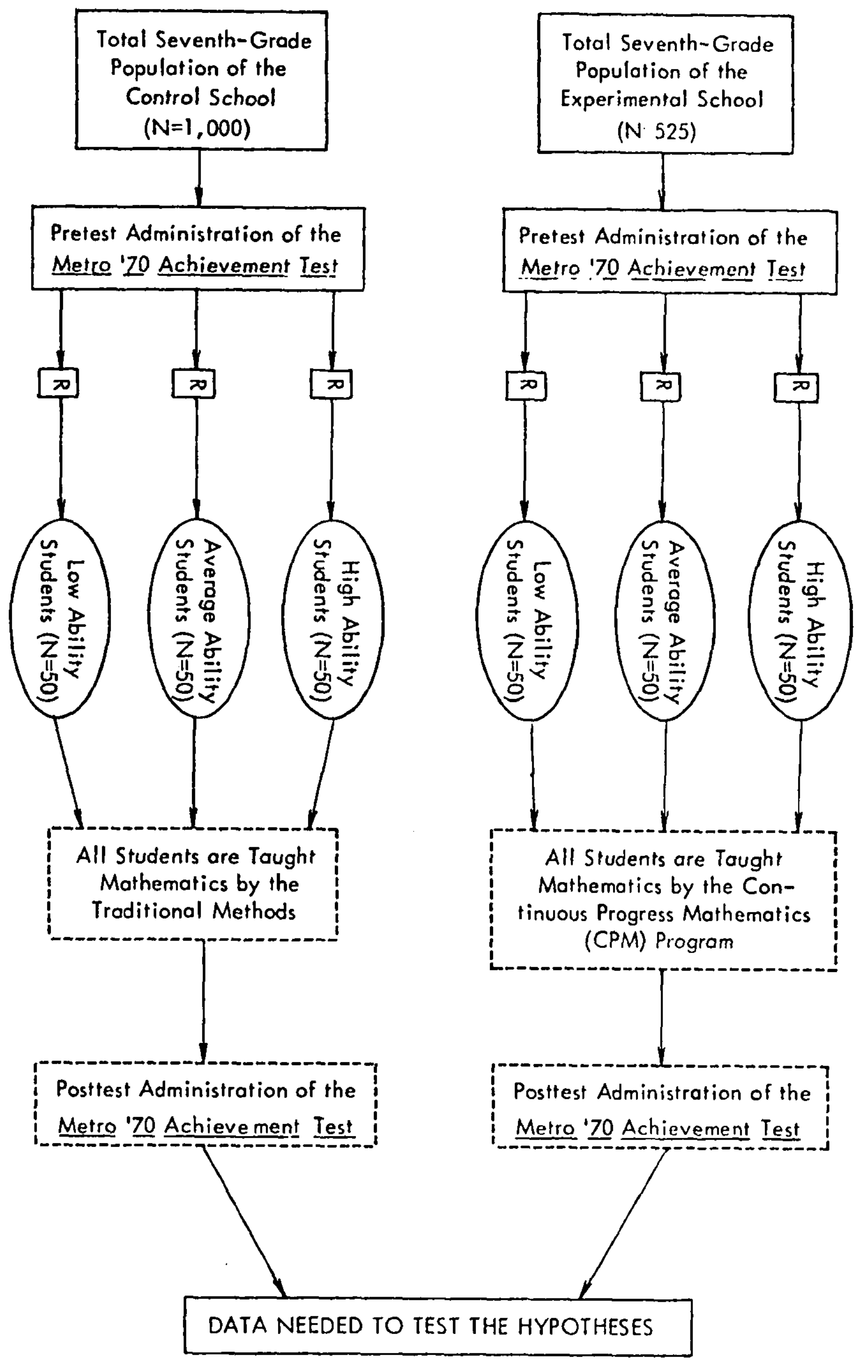

Figure 1: Research Design Used in Conducting the Study.

1. Total populations of approximately 525 seventh-grade students in the experimental group (those receiving CPM instruction) and 1,000 seventh-grade students in the control group (those being taught mathematics by the traditional method) were given the Metropolitan '70 Achievement Test (Advanced Level: Form F). Mathematics scores taken from the Metro '70 were regarded as the pretest measure.

2. Three groups of fifty (N=50) students each were randomly selected R from each of the student populations. One group had mental ability scores of 111 or higher (75th percentile) and were regarded as the High Ability Group. The second group had mental ability scores ranging from 96 (40th percentile) to 111 (75th percentile) and were regarded as the Average Ability Group. The third group had mental ability scores below 96 (40th percentile) and were regarded as the Low Ability Group.

3. All seventh-grade students in the experimental populations were taught mathematics by the CPM method, and all students in the control population were taught mathematics by the traditional method.

4. Both the experimental and control populations were administered an alternate form of the Metropolitan '70 Achievement Test (Form G). Mathematics scores taken from this administration of the Metro '70 were regarded as the posttest measure of mathematics achievement.

5. Pretest-posttest gain scores for the High, Average and Low Ability Groups were used to test the hypotheses stated in Chapter I.

Selection and Control of Independent Variables

The next step in the pre-experimental procedures was the choice and control of independent variables. In conducting any study the researcher must decide which independent variables will affect the measures to be recorded and take the necessary steps to control the effects of these variables. It should be noted that the term "control", when used in this sense, does not mean "to eliminate" but rather "to be able to account for the effects of the variable." For example, the measure taken in the present study was mathematics achievement. Therefore, the researcher had to determine all of the variables which are believed to affect the students' mathematics achievement scores and make some preparation for controlling the effects of these variables. The independent variables which are thought to affect academic achievement are presented in Figure 2. This Figure also shows the method to be used in controlling the effects of each variable.

The independent variables which are believed to affect mathematics achievement scores can be classified into three general categories: (1) student variables or characteristics, (2) teacher variables or characteristics and (3) environmental or situational variables.

The student's ability also has an effect on his mathematics achievement. Differences in ability were controlled by establishing three levels of ability in the research design.

Equating the actual samples used in the experiment involves obtaining internal validity. External validity refers to the problem of ensuring that the subjects used in the experiment are representative of a broader group

Source of Independent Variables	Method Used to Control the Effects of the Variable	Measure(s) Taken (Dependent Variable)
STUDENT VARIABLES		The Amount of Gain Made in Mathematics Achievement as Determined from a Pretest-Posttest Administration of *The Metropolitan '70 Achievement Test*
1. Student's Ability (Intelligence)	(Built into design)	
2. Student's Attitude	(Random)	
3. Race	(Random)	
TEACHER VARIABLES		
1. Teacher's Ability and Knowledge of the Subject	(Same-to-all)	
2. Teacher's Attitude	(Same-to-all)	
ENVIRONMENTAL VARIABLES		
1. Type of Mathematics Program Used	(Built into research design)	
2. Socioeconomic Status	(Random selection)	

Figure 2: The Independent variables controlled in the study, and the methods and procedures used in controlling each.

to which one may wish to generalize. Campbell and Stanley (1973) say that randomization is the best way of achieving both types of validity, internal validity by randomly assigning subjects to the several treatment combinations and external validity by randomly selecting our pool of experimental subjects from the larger population to which we wish to generalize.

Subjects could not be randomly assigned to the different treatments since the school attended by the student was the factor determining the type of treatment received. However, subjects used in the experiment were randomly selected by the computer after first being categorized into their appropriate ability level groups.

Attitude and race may affect achievement scores, but the random selection of participants would also make all groups statistically equal on these two variables.

The teacher's ability, knowledge of the subject and attitude can also affect students' mathematics scores. These factors were controlled in different ways in the two programs. Team teaching used in the Continuous Progress Mathematics program exposed all of these students to the teaching abilities of different teachers. Students in the control group were taught by a single teacher; however, random selection from a finite population (approximately 1,000 students) caused the effects of poor and good teaching to be intermingled with average teaching abilities producing an overall averaging effect for the 150 students used in this group.

Some research studies have indicated that socio-economic status can affect achievement scores. Random selection of students would also control the effects of this phenomenon since proportionate numbers should be selected from all socio-economic levels.

The type of program used to teach mathematics may also affect the students' mathematics achievement scores. In anticipation of this fact, the researcher established a research design which allowed the comparison of the scores of students who had been taught by the two different methods. Such a comparison acted as a control for the differences in mathematics scores caused by the two different teaching methods.

Instruments Used in Measuring Mathematics Achievement

Another important step in the pre-experimental procedures was the selection of an instrument suitable for determining the students' pretest and posttest mathematics achievement scores. The instrument chosen for the present study was the Metropolitan '70 Achievement Test (Form F and G).

The Metropolitan '70 Achievement Test (Metro '70) is an achievement battery which is composed of the following subtests: (1) Word Knowledge, (2) Reading, (3) Total Reading, (4) Language, (5) Spelling, (6) Mathematics

Computation, (7) Mathematics Concepts, (8) Mathematics Problem Solving, (9) Total Mathematics, (10) Science and (11) Social Studies. For the purposes of this study only the mathematics subtests scores were used.

The test-retest reliability of the (Metro '70) varies slightly from one form to the next. Harcourt, Brace, Jovanovich, Inc., publishers of the Metro '70, report the test-retest reliability as ranging from a low of 0.88 for Form H to a high of 0.92 for Form F. Comparable findings are reported by Buros in the Mental Measurements Yearbook (Seventh Edition).

The predictive validity of the Metro '70 is reported by the test publishers as ranging from a low of 0.81 for Form G to a high of 0.86 for Form F. However, Buros reports the predictive validity of the Metro '70 as ranging from a low of 0.73 for Form G to a high of 0.82 for Form F. Buros further reports that the predictive validity of the mathematics achievement section of the Metro '70 ranges from a low of 0.81 for grades one and two to a high of 0.89 for grades nine and ten. He reports the predictive validity of the Metro '70 mathematics scores for seventh-grade students as ranging from 0.83 to 0.86.

The reliability and validity of the Metropolitan '70 Achievement Test (Metro '70) (Forms F and G) were sufficient for the present study.

Selection of an Individualized-Instruction Program in Mathematics

The final step of the pre-experimental procedures was the selection of an individualized-instruction program for teaching those seventh-grade students in the experimental group. Before the final selection was made, several systems were examined. Programs were evaluated according to the following criteria: (1) simplicity of design, (2) adaptability to the classroom situation, (3) content validity and reliability, (4) ancillary services and materials available and (5) total cost of the program. The program chosen was Continuous Progress Mathematics (CPM).

Description of the Continuous Progress Mathematics (CPM) System

The CPM system has many of the characteristics of the Individualized Mathematics System (IMS) and Individually Prescribed Instruction (IPI). All of these programs have behavioral objectives stated for each mathematical concept to be taught and achievement is measured by unit tests given at prescribed intervals. Students also progress at their own rate (self-pacing) in all three programs.

The CPM system is composed of 150 behavioral objectives written for seventh- and eighth-grade mathematics classes and 131 objectives written for ninth-grade Algebra I classes.

Conducting a Survey of the Individualized Programs of Mathematics Instruction

One of the pre-experimental procedures completed by the researcher was to conduct a nation-wide survey of individualized programs of mathematics instruction. The primary purposes of the survey were as follows: (1) to gather as much information about individualized-instruction mathematics programs as possible, (2) to determine the extent of the evaluation being conducted with the various programs now in operation and (3) to determine the feasibility of conducting a pretest-posttest evaluation of students' mathematics achievement.

The National School Public Relations Association issued a publication in which they listed forty-six (N=46) school systems that were utilizing individualized learning programs. Nineteen (N=19) of the programs described seemed to be similar to the CPM program utilized in the present study, and the letter shown in Appendix B and the questionnaire shown in Appendix C were sent to these 19 schools. A list of the school systems contacted is presented in Appendix A.

Responses from the inquiries revealed that only two (N=2) of the school systems utilizing the individualized instruction mathematics programs had compared academic achievement of students involved in these programs with academic achievement of students taught by traditional methods. The results reported by both of these school systems were reviewed in the related research. The two school systems were the PLAN system utilized by the Hicksville (Ohio) School System and the Individually

Guided Education (IGE) program utilized by the Janesville (Wisconsin) School System. It should be noted, however, that nearly all school systems reported some program for evaluating their individualized instruction mathematics program, but most lacked the controls, measurements, research design and/or statistical analysis needed to be considered "defensible" evaluations.

Experimental Procedures

The experimental procedures consisted of all those tasks which were completed during the course of the experiment. These tasks consisted primarily of teaching the seventh-grade mathematics classes and measuring the results by administering the Metropolitan Achievement Test (Metro '70) on a pretest-posttest basis. The most important of the tasks involved in the Experimental Procedures are described in the following sections.

Teaching the Continuous Progress Mathematics (CPM) Classes

Students who were enrolled in seventh-grade mathematics for the first time were administered a 38-problem pretest which included one problem for each of the first 38 behavioral objectives. A test of this type is administered only when the student is coming into the program for the first time. Progress charts are kept updated throughout the year showing all of the problems missed on the test and also all work completed by students.

Problems missed on the pretest correspond with numbered practice sets which the student had to work to correct indicated deficiences. For example, if the student correctly answered problems 1, 5, 9 and 13 on the pretest, then practice sets 1, 5, 9 and 13 were not worked by the student. However, the student was required to work the remaining 34 practice sets contained in the 38-set unit. Next, the student selected the practice set which coincided with the first problem missed on the pretest and proceeded to solve each problem in the practice set. Problem solving was facilitated through the use of examples at the beginning of each set. Students either worked alone, with another student or in small groups (three or four students). The instructor was available to help the students; however, the teacher did not function in a traditional sense as a lecturer or disseminator of information. His primary function was to encourage, explain and reinforce the students as they worked with each practice set.

After completing the required practice sets within a given unit, the students proceeded to the next step of the CPM evaluation procedure. A flow chart of these procedures is shown in Figure 3. A score of 90 or higher allowed the student to progress to the next unit of the program. If the student scored below 90 on the unit evaluation, he was given additional work on the problems of the type that he missed, and the unit test was readministered whenever the student felt that he was ready. This procedure was repeated

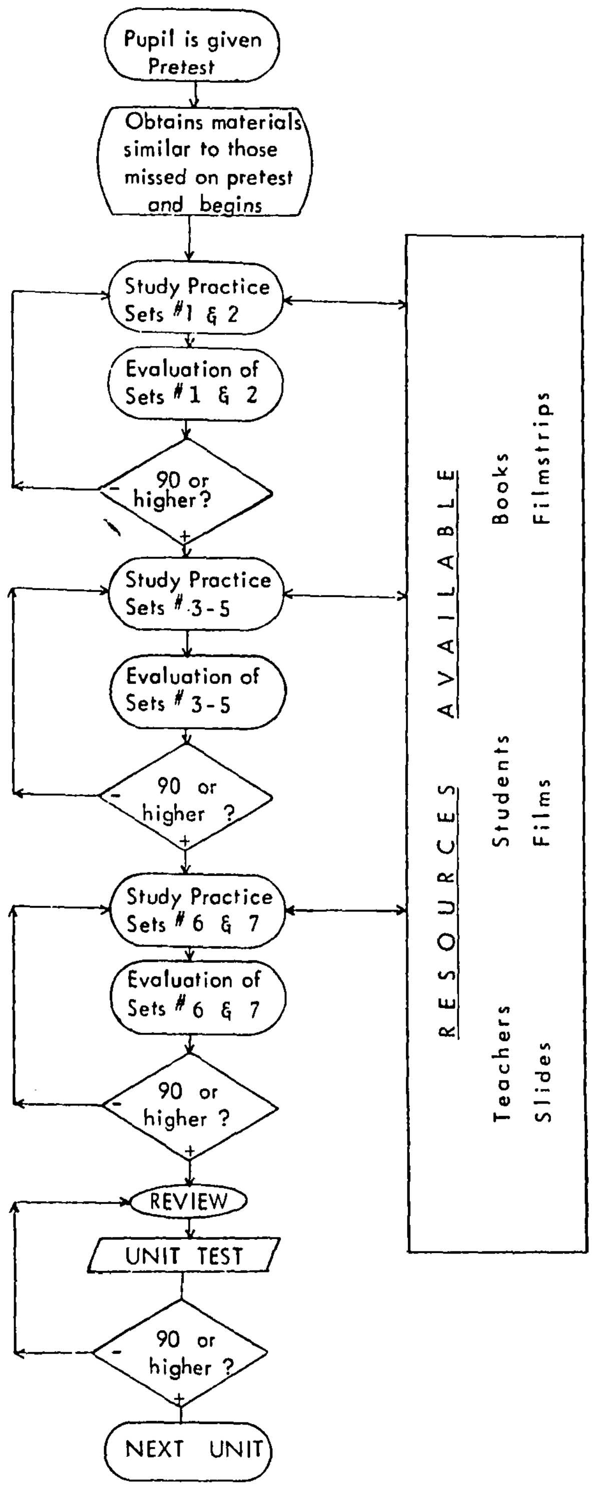

Figure 3: A flow chart of the steps taken in the Continuous Progress Mathematics (CPM) Program.

until all unit evaluations and practice sets were completed for that unit.

After completing all the evaluations and practice sets in a particular unit, the student completed a review sheet of the unit material, corrected all errors and solved any problems missed on the review material. The student then presented the progress chart, practice sets and review sheet to the instructor as evidence that he was ready to attempt the posttest composed of unit material. The student was given one of the two remaining forms of the posttest to solve. Posttests were corrected by the instructor or an aide, and the student was asked to review and solve any problems missed. The numbers of problems missed on the unit test referred the student back to the practice sets which contained problems of that particular type. For example, if a student missed problem six on the unit test this referred him back to practice set number six. All of the CPM material is color coded and studied in this order: (1) practice sets - pink, (2) evaluation test - blue, (3) review test - yellow and (4) unit test - Form A, B and C - white. Students are required to pass only one form of the unit test. These colored, numbered sheets are stored in consecutive order and can be found easily by the students. See Appendix D for an example of one complete unit.

The instructors also developed work pages of problems which were used to supplement the CPM curriculum. Sometimes more work pages were prescribed, teachers gave individual help and small group sessions were used if several students were experiencing the same type of difficulties. It was also permissable for the students to work together in small groups (two to six students) and receive help from other students in their group.

Satisfactory completion of the unit test (a score of 90 or higher) allowed the student to progress to the next major unit, and the procedures described earlier were repeated. A summary of the procedures used to teach the students in the experimental groups is as follows:

(1) Take CPM unit pretest

(2) Observe problems missed

(3) Complete practice sets corresponding to the problems missed on the pretest

(4) Take evaluation test over practice sets

  (a) Passing score on evaluation test allowed student to move to next practice set

  (b) Failing score on evaluation test required student to study additional problems like those missed on the evaluation test

(5) Satisfactory completion of all practice sets in unit allowed student to take unit review test

(6) Proficiency on the unit review test permitted the student to take the CPM unit posttest

(7) A score of 90 or higher on the CPM unit posttest allowed the student to proceed to the next unit of work in the series

Students in the experimental groups moved through a series of small steps toward the accomplishment of specific, measurable objectives (behavioral objectives). Use of the CPM unit pretest allowed the students enrolling later in the school year to move into the program at the appropriate level of learning.

Frequent testing was an integral part of the CPM teaching system. The tests served as a means of reinforcing information that the students learned and became a means for proving success rather than a psychological block to student learning. Students and their instructors kept a continuous progress chart showing completion dates for specific units.

Teaching the Traditional Mathematics Classes

Seventh-grade students in the control population were taught mathematics by the traditional methods. Basically, the content of the materials taught was identical to the materials being taught to the experimental population of students. However, there were two basic differences in the way the two groups were taught. These two differences were as follows: (1) Students in the control population were not allowed to progress at their own speed as were the students in the experimental population. (2) Students in the control population were not allowed to skip material even though they had mastered the concepts being taught.

Seven instructors were involved in team teaching with the 525 experimental students while the teachers working with the control group had approximately twenty-five students each in self-contained classrooms. As was expected, the methods used by the different instructors varied somewhat. However, those instructors who taught the mathematics classes in the traditional manner agreed that the following procedures were a good summary of the approach used:

> <u>Presentation of Materials</u>: New materials were presented primarily by lecturing, explaining the examples in the textbook, working examples of problems in class, assigning problems for homework, answering the problems posed in the textbook and answering students' questions in class.
>
> <u>Review of Material</u>: Reviewing the materials taught was accomplished by telling the students to review certain chapters or units which were to be included on each examination, scoring and returning homework, reviewing the primary concepts taught in each unit during class periods and having students work problems similar to those presented in the textbook.
>
> <u>Examination of Students</u>: Student progress was assessed and evaluation of results was accomplished by giving pop quizzes, giving extra credit for homework, giving teacher-made tests at different times during the school year and

a pretest-posttest administration of the _Metropolitan '70 Achievement Test_ (Advanced Level: Forms F and G).

These same basic methods were used by all traditional mathematics teachers. Methods and procedures used by the CPM teachers are described in another section.

Collecting the Mathematics Achievement Data

Since the data resulting from the present study were to be used in deciding future curriculum changes, every effort was made to insure a properly conducted experiment between the experimental and control populations of students. This was especially true in the collection of the mathematics achievement test scores at the beginning (pretest) and end (posttest) of the experiment.

Before the tests were administered, the researcher and the research director for the public school system conferred with the principals of the schools participating in the experiment. They were given concise and explicit directions concerning the administration of the Metro '70 on a pretest-posttest basis. Principals then met with their respective teachers prior to testing and briefed them concerning the testing procedures to be followed. Teachers were instructed to follow precisely the administration procedures outlined in the _Teacher's Directions_. Special attention was given to the time of administration and the monitoring of the students' progress during the testing sessions.

Data Analysis Procedures

The final phase of the methods and procedures was the data analysis procedures. These procedures consisted of those tasks performed after the data had been collected from the experimental and control groups. Data analysis procedures consisted of scoring the achievement tests, coding and preparation of the data, selection of statistical procedures and actual testing of the hypotheses.

Achievement tests were scored through the facilities of the school system sponsoring the study. Final results of the scoring procedures yielded a standard score and a percentile rank for each student participant, and these scores were used in testing the hypotheses.

Statistical Computations Made in Analyzing the Data and Testing the Hypotheses

The final step of the data analysis procedures was to test the hypotheses stated in Chapter I. Null hypotheses one, two and three were tested with a two-way analysis of variance (ANOVA) for two fixed variables (Glass and Stanley, 1970). This particular testing statistic yields three F values: one for the comparison of the columns (In the present study, the comparison was of the gain scores of the experimental and control groups, Ho_1.); one for the comparison of the rows (In the present study, the comparison was of the gain scores of the students with different levels of mental ability, Ho_2.); and one for the interaction of the two independent variables (In the present study this included the type of teaching method and level of mental ability, Ho_3.). Significant F values were followed with studentized range statistics as a means of locating specific mean differences among the various groups and subgroups. The Newman-Keuls Test was the range statistic used in making all post-hoc comparisons (Kirk, 1970).

The fourth null hypothesis was tested by comparing the achievement gain scores of the experimental (Continuous Progress Mathematics) students at all three ability levels with a one-way analysis of variance. Again, the calculated F value was followed by a studentized range statistic as a means of further comparing the gain scores of the three student groups.

CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

Pretest-posttest change scores taken from two administrations of the Metropolitan '70 Achievement Test were used to compare the amount of total mathematics achievement change experienced by two groups of seventh-grade students who were taught mathematics by different methods. One group, the experimental group (N = 150) was taught by an individualized program of mathematics which was primarily self-teaching and allowed the individuals to proceed at their own rate of speed. The second group, the control group (N = 150), was taught by the traditional method of lecture, review and test. Students from each group were also divided into high, average and low mental ability groups.

Total mathematics achievement change scores were computed by comparing changes in the pretest-posttest scores using all problems on both the pre- and posttest. These total mathematics achievement change scores were used to test the four null hypotheses.

The first three null hypotheses (Ho_1, Ho_2, Ho_3) were tested with a two-way analysis of variance, and the fourth null hypothesis was tested using a one-way analysis of variance. This statistic was used to determine any significant differences between teaching methods and also as a check for interaction between teaching methods and the three different levels of mental ability. Null hypothesis number four (Ho_4) was used to test for differences among students in the three different mental ability groups that were taking Continuous Progress Mathematics.

The mathematics subtests of the _Metropolitan '70 Achievement Battery_ measure the following areas: (1) mathematics computation, (2) mathematics concepts and (3) mathematics problem solving. Additional comparisons were made on each subtest to determine if students taught by the two different methods were achieving more in particular areas of mathematics. A two-way analysis of variance was also used to test the subtests.

This chapter contains the results of testing the four null hypotheses using total mathematics achievement and concomitant findings obtained by comparing achievement on the subtests. A summary of all the results and findings is located at the end of this chapter.

Information Concerning Pretests and Posttests

Pretests were administered during the first few weeks of the school year (1974-75) on the following dates:

Control school #1 September 24, 25 and 26
Control school #2 October 1, 2 and 3
Control school #3 October 8, 9 and 10
Experimental school October 8, 9 and 10

Posttests were administered at the end of the school year and in the same sequence to ensure that students would be exposed to the same learning period.

Results of Testing the Null Hypotheses

Results of Testing Null Hypothesis Number One

The null form of the first hypothesis was stated and tested as follows:

> Ho_1 There is no statistically significant difference between the total mathematics achievement change scores shown for the seventh-grade students who participated in the individualized mathematics program and the total mathematics achievement change scores shown for the seventh-grade students who did not participate in the individualized mathematics program but who were taught by the traditional method.

This hypothesis was tested by comparing the total mathematics achievement change scores computed for students in the experimental group with the total mathematics achievement change scores computed for students in the control groups. Means and standard deviations computed for the two groups' change scores are presented in Table 1. Mean values computed for the two groups were compared with a two-way analysis of variance testing statistic. The ANOVA results are presented as the first F value in Table 2.

Results of testing the first null hypothesis show that the computed F value was not significant ($F = 0.642$, $df = 1/294$; $p > .05$). These results would not allow the researcher to reject the first null hypothesis, and it was concluded that there was not a significant difference between the mathematics achievement change scores computed for the experimental and control groups.

TABLE 1

MEANS AND STANDARD DEVIATIONS COMPUTED FOR THE PRETEST, POSTTEST, AND CHANGE SCORES OF MATHEMATICS ACHIEVEMENT AS COMPUTED FOR THE EXPERIMENTAL AND CONTROL GROUPS

	Areas of Mathematics Achievement	Pretest Scores		Posttest Scores		Change Scores	
		Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Experimental Group (N=150)	Computation	42.31	20.17	59.01	23.11	16.69	9.84
	Concepts	48.15	21.09	61.27	26.16	13.12	10.32
	Problem Solving	48.99	15.85	55.81	16.62	6.82	9.04
	TOTAL	45.31	14.39	60.29	20.95	14.97	14.63
Control Group (N=150)	Computation	47.59	23.25	65.63	21.22	18.04	11.22
	Concepts	53.22	16.39	66.19	20.61	12.97	9.66
	Problem Solving	50.05	21.07	59.63	17.44	9.58	7.81
	TOTAL	49.49	19.70	65.69	18.35	16.20	12.58

TABLE 2

SUMMARY TABLE OF THE TWO-WAY ANALYSIS OF VARIANCE USED TO TEST THE FIRST THREE NULL HYPOTHESES

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Value	Significance Level
SS_A (Type of Instruction)	112.86	1	112.86	0.642	$> .05$
SS_B (Mental Ability Level)	3,334.75	2	1,667.38	9.480	$< .001$
$SS_{A \times B}$ (Instruction x Mental Ability)	386.78	2	193.39	1.099	$> .05$
SS_{Within} (Error)	51,711.66	294	175.89		
SS_{TOTAL}	55,545.00	299			

Results of Testing Null Hypothesis Number Two

The null form of the second hypothesis was stated and tested as follows:

> Ho_2 There is no statistically significant difference among the total mathematics achievement change scores recorded for seventh-grade students from three different ability groups who had participated in the study.

This hypothesis was tested by comparing the total mathematics achievement change scores computed for students who had been randomly selected for the high, average and low mental ability groups. The means and standard deviations computed for the three groups' change scores are presented in Table 3. Mean values computed for the three groups were compared with an analysis of variance testing statistic. The analysis of variance (ANOVA) results are shown as the second F value in Table 2.

Results of testing the second null hypothesis show that the computed F value was significant ($F = 9.48$, $df = 2/294$; $p < .001$). These results allowed the researcher to reject the second null hypothesis, and it was concluded that there was a significant difference among the mathematics achievement change scores computed for students from the three mental ability groups.

Since the F value computed in testing the second null hypothesis was significant, it was necessary to make further comparisons among the means in order to locate specific differences. A Newman-Keuls Test was used to make the post-hoc

TABLE 3

MEANS AND STANDARD DEVIATIONS OF PRETEST, POSTTEST, AND CHANGE SCORES OF MATHEMATICS ACHIEVEMENT AS COMPUTED FOR STUDENTS IN THE HIGH, AVERAGE, AND LOW MENTAL ABILITY GROUPS

	Areas of Mathematics Achievement	Pretest Scores Mean	Pretest Scores Standard Deviation	Posttest Scores Mean	Posttest Scores Standard Deviation	Change Scores Mean	Change Scores Standard Deviation
High Mental Ability (N=100)	Computation	70.04	22.40	85.60	29.32	15.56	14.10
	Concepts	77.60	23.70	87.42	23.96	9.82	11.84
	Problem Solving	74.93	23.01	83.79	28.22	8.86	6.67
	TOTAL	74.62	21.74	89.23	28.62	14.61	11.52
Average Mental Ability (N=100)	Computation	43.63	26.81	64.60	31.35	20.97	12.03
	Concepts	52.33	27.93	68.70	29.34	16.37	11.16
	Problem Solving	48.24	26.44	59.06	30.82	10.82	9.93
	TOTAL	46.77	28.73	66.84	30.11	20.07	16.27
Low Mental Ability (N=100)	Computation	21.18	28.32	36.75	25.51	15.57	12.64
	Concepts	22.13	29.04	35.08	29.19	12.95	13.09
	Problem Solving	25.04	32.62	30.32	26.82	4.92	6.63
	TOTAL	20.82	30.71	32.90	28.15	12.08	11.40

comparisons. Results of the Newman-Keuls Test are presented in Table 4. The results presented in this table indicate that students in the average mental ability group made significantly greater gains in mathematics achievement than students in the high and low mental ability groups. It was also noted that students in the low mental ability group made greater gains in mathematics achievement than students in the high mental ability group, but the differences were not significant. Average students in the control group made greater gains than the average experimental students; however, the difference was not significant.

TABLE 4

SUMMARY TABLE FOR THE NEWMAN-KEULS TEST AMONG THE MATHEMATICS ACHIEVEMENT CHANGE SCORES COMPUTED FOR THE THREE DIFFERENT MENTAL ABILITY GROUPS

Rank-Ordered Mean Values		$\bar{X}_3$ (Low)	$\bar{X}_1$ (High)	$\bar{X}_2$ (Average)
Low Mental Ability Group	$\bar{X} = 12.08$	----	2.53	7.99**
High Mental Ability Group	$\bar{X} = 14.61$		----	5.46*
Average Mental Ability Group	$\bar{X} = 20.07$			----

$MS_{Error} = 175.89$

*$p < .01$

**$p < .001$

Results of Testing Null Hypothesis Number Three

The null form of the third hypothesis was stated and tested as follows:

> Ho_3 There is no statistically significant interaction between the two independent variables of Type-of-Teaching (individualized instruction or traditional) and Ability Level (high, average or low) as reflected in the students' mathematics achievement gain scores.

This hypothesis was tested by comparing the total mathematics achievement change scores computed for students in both the experimental and control groups at all three levels of mental ability. The means and standard deviations of the six groups' mathematics achievement change scores are presented in Table 5. Mean values computed for the groups were compared with an analysis of variance testing statistic. The ANOVA results are presented as the third F value in Table 2.

Results of testing the third null hypothesis show that the computed F value was not significant ($F = 1.099$, $df = 2/294$; $p > .05$). These results would not allow the researcher to reject the third null hypothesis, and it was concluded that there was not a significant interaction between the type of instruction used in teaching the classes and the levels of mental ability possessed by the students.

The interaction between the two independent variables was graphed as a means of further showing the lack of significant interaction. This graph is shown in Figure 4.

TABLE 5

TOTAL MATHEMATICS ACHIEVEMENT CHANGE SCORES COMPUTED FOR STUDENTS IN BOTH THE EXPERIMENTAL CONTROL GROUPS AT THREE MENTAL ABILITY LEVELS

Student Groups	Mathematics Achievement Change Scores	
	Mean	Standard Deviation
High Mental Ability x Experimental Teaching Method	15.38	12.79
High Mental Ability x Traditional Teaching Method	13.84	12.16
Average Mental Ability x Experimental Teaching Method	18.06	16.51
Average Mental Ability x Traditional Teaching Method	22.08	15.95
Low Mental Ability x Experimental Teaching Method	11.48	13.86
Low Mental Ability x Traditional Teaching Method	12.68	8.35

Results of Testing Null Hypothesis Number Four

The null form of the fourth hypothesis was stated and tested as follows:

> Ho_4 There is no statistically significant difference among the total mathematics achievement scores recorded for seventh-grade students from three different ability groups who had participated in an individualized mathematics program.

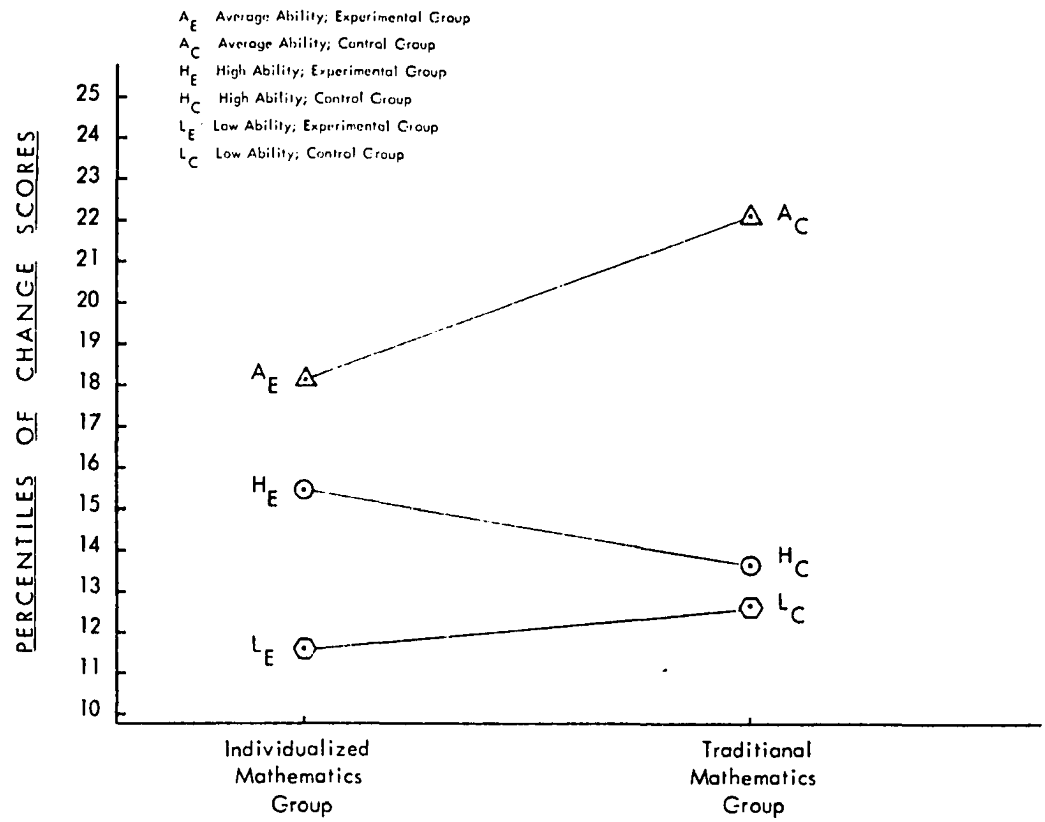

Figure 4: A graphic representation of the lack of interaction between the type of teaching technique used and the students' level of mental ability.

The fourth null hypothesis was tested by comparing the total mathematics achievement change scores computed for students from three different ability levels who had been taught by an individualized mathematics program (experimental group). Means and standard deviations computed for the three groups' pretest, posttest and change scores are presented in Table 6, while the results of the one-way analysis of variance comparing the mean values are presented in Table 7.

The results of testing the fourth null hypothesis show that the computed F value was not significant ($F = 2.658$, $df = 2/147$; $p > .05$). These results would not allow the researcher to reject the fourth null hypothesis, and it was concluded that there were no significant differences among the achievement gain scores computed for students from the three ability levels.

A further explanation of these results seems in order, since they appear to contradict the results of testing hypothesis number two. Results of testing hypothesis number two indicated that there was a significant difference among the achievement scores of students from different ability levels, while the results of testing hypothesis number four indicate that there was not a significant difference among the achievement gain scores of students from three ability levels who were taught by the experimental method. This is basically the result of separating the experimental and control groups. First, students in the control group showed more achievement gain than students in the experimental group. Thus, when the control group was eliminated, any significant differences among the group means were also eliminated. Second, the reduction of the number of persons compared in hypothesis number two ($N = 300$) to the number compared in hypothesis number four ($N = 150$) reduced the probability and magnitude of a significant difference.

TABLE 6

MEANS AND STANDARD DEVIATIONS OF PRETEST, POSTTEST, AND CHANGE SCORES OF MATHEMATICS ACHIEVEMENT AS COMPUTED FOR STUDENTS OF HIGH, AVERAGE, AND LOW MENTAL ABILITY TAUGHT BY THE EXPERIMENTAL METHOD

		Pretest Scores		Posttest Scores		Change Scores	
	Mathematics Achievement	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
High Mental Ability (N=50)	Computation	64.70	16.81	79.42	21.80	14.72	11.42
	Concepts	75.02	17.94	85.34	19.62	10.32	12.10
	Problem Solving	71.68	17.99	81.62	21.17	9.94	8.22
	TOTAL	70.54	17.73	85.92	20.03	15.38	10.67
Average Mental Ability (N=50)	Computation	42.34	21.03	62.12	19.07	19.78	17.16
	Concepts	48.96	16.64	65.08	21.96	16.12	17.23
	Problem Solving	49.00	18.89	56.84	18.02	7.84	8.15
	TOTAL	45.34	19.93	63.40	20.22	18.06	12.11
Low Mental Ability (N=50)	Computation	19.90	16.44	35.48	21.42	15.58	13.14
	Concepts	20.48	17.08	33.40	19.75	12.92	11.82
	Problem Solving	26.30	19.25	28.98	20.69	2.68	6.55
	TOTAL	20.06	18.28	31.54	21.13	11.48	10.23

TABLE 7

RESULTS OF THE ANALYSIS OF VARIANCE COMPARING THE TOTAL MATHEMATICS ACHIEVEMENT SCORES OF STUDENTS WITH HIGH, AVERAGE, AND LOW MENTAL ABILITY WHO WERE TAUGHT BY THE EXPERIMENTAL METHOD (N=150)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Value	Significance Level
$SS_{Between}$	1,094.81	2	547.41	2.685	$>$ .05
SS_{Within}	30,275.08	147	205.95		
SS_{TOTAL}	31,369.89	149			

Additional Findings

Several additional comparisons were made among the various student groups' subtest achievement scores. These comparisons were made in an attempt to determine whether particular areas of mathematics achievement were more affected by the two different teaching methods than others. The results of all additional subtest comparisons are presented in this section of the dissertation.

Comparisons of Mathematics Achievement on the Subtest of Mathematics Computation

The first additional comparison was made between the mathematics computation change scores calculated for students who were taught by the two different methods and who were grouped according to mental ability level. A two-way analysis of variance was used to make the statistical comparisons. Results of the analysis of variance calculations are presented in Table 8.

The results of comparing the various groups' mathematics computation change scores indicate that neither the type of instruction received nor mental ability level made significant differences in the students' achievement scores on the mathematics computation subtest. The results presented in Table 8 also indicate that there was not a statistically significant interaction between the two independent variables of type of instruction and mental ability level.

TABLE 8

RESULTS OF THE ANALYSIS OF VARIANCE COMPARING THE MATHEMATICS COMPUTATION ACHIEVEMENT SCORES OF STUDENTS WITH HIGH, AVERAGE, AND LOW MENTAL ABILITY WHO WERE TAUGHT MATHEMATICS BY TWO DIFFERENT METHODS

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Value	Significance Level
SS_A (Type of Instruction)	136.016	1	136.016	0.395	> .05
SS_B (Mental Ability Level)	1,947.610	2	973.805	2.826	> .05
$SS_{A \times B}$ (Instruction x Mental Ability)	116.94	2	58.247	0.169	> .05
SS_{Within} (Error)	101,310.550	294	344.594		
SS_{TOTAL}	103,510.670	299			

Comparisons of Mathematics Achievement on the Subtest of Mathematics Concepts

An additional comparison was made between the mathematics concepts change scores calculated for students who were taught by the two different methods and among the change scores of students who had been categorized into one of three mental ability levels. Again, a two-way analysis of variance was the testing statistic used to make the comparisons. Results of the analysis of variance calculations are presented in Table 9.

The results of comparing the various groups' mathematics concepts change scores indicate that the type of instruction received did not make a significant difference in the two groups' achievement test scores. However, students in the average mental ability group scored significantly higher in mathematics concepts than students in the high and low mental ability groups.

The results in Table 9 further indicate that there was very little interaction between the two independent variables of type of instruction and mental ability level.

Comparisons of Mathematics Achievement on the Subtest of Mathematics Problem Solving

A third, and final comparison was made between the mathematics problem solving change scores calculated for students who were taught by the two different methods and among the change scores of students who were categorized

TABLE 9

RESULTS OF THE ANALYSIS OF VARIANCE COMPARING THE MATHEMATICS CONCEPTS ACHIEVEMENT SCORES OF STUDENTS WITH HIGH, AVERAGE, AND LOW MENTAL ABILITY WHO WERE TAUGHT MATHEMATICS BY TWO DIFFERENT METHODS

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Value	Significance Level
SS_A (Type of Instruction)	1.616	1	1.616	0.005	$> .05$
SS_B (Mental Ability Level)	2,146.530	2	1,037.265	3.138	$< .05$
$SS_{A \times B}$ (Instruction x Mental Ability)	29.624	2	14.812	0.045	$> .05$
SS_{Within} (Error)	97,169.580	294	330.509		
SS_{TOTAL}	99,347.350	299			

into one of three mental ability levels. Again, a two-way analysis of variance was used to make the comparisons. Results of the analysis of variance calculations are presented in Table 10.

Results of comparing the various groups' mathematics problem solving change scores indicate that the type of instruction received did not make a significant difference in the two groups' achievement test scores. However, students in the average mental ability group scored significantly higher mathematics problem solving change scores than students in the high and low mental ability groups.

Results presented in Table 10 also indicate that there was very little interaction between the two independent variables of type of instruction and mental ability level.

Summary of Results

The results of testing the hypotheses and the additional comparisons are summarized in the following passages. A more concise summary is presented in the first part of Chapter V.

Results of testing the first hypothesis indicated that there was not a significant difference between the amount of total mathematics achievement gain shown by students taught with the CPM method and those taught by the traditional method. However, students taught by the traditional method did score slightly higher than those taught by the CPM method.

TABLE 10

RESULTS OF THE ANALYSIS OF VARIANCE COMPARING THE MATHEMATICS PROBLEM SOLVING ACHIEVEMENT SCORES OF STUDENTS WITH HIGH, AVERAGE, AND LOW MENTAL ABILITY WHO WERE TAUGHT MATHEMATICS BY TWO DIFFERENT METHODS

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Value	Significance Level
SS_A (Type of Instruction)	571.320	1	571.320	2.076	$> .05$
SS_B (Mental Ability Level)	1,805.840	2	902.920	3.282	$< .05$
$SS_{A \times B}$ (Instruction x Mental Ability)	935.120	2	467.560	1.699	$> .05$
SS_{Within} (Error)	80,891.720	294	275.142		
SS_{TOTAL}	84,204.000	299			

Results of testing the second hypothesis indicated that students in the average mental ability group made significantly greater gains in their mathematics achievement gain scores than students in the high and low mental ability groups. Students in the high mental ability group also showed more progress than students in the low mental ability group, but the differences between the two groups' scores were not significant.

Results of testing the third hypothesis showed that there was very little interaction between the two variables of type of instruction and level of mental ability.

Concomitant findings resulting from the additional comparisons may be summarized as follows: (1) There was no significant difference between the amount of achievement gain shown by students taught by the CPM method and the amount of achievement gain shown by students taught by the traditional method on any of the mathematics subtests of computation, concepts or problem solving. (2) Students in the average mental ability group showed significantly greater achievement gains than students in the high and low mental ability groups on the two mathematics subtests of concepts and problem solving. (3) There were no significant differences among the achievement gains shown by students in the three mental ability groups on the mathematics subtest of computation.

In conclusion, the type of instruction received did not make a difference in the amount of achievement gain shown, but students in the average mental ability group showed more achievement gain than students in the high and low mental ability groups. The results indicated that there was very little interaction between the type of instruction received and the level of mental ability.

CHAPTER V

SUMMARY, CONCLUSIONS AND IMPLICATIONS FOR FURTHER RESEARCH

This study proposed to compare the mathematics achievement gain scores of seventh-grade students who were participating in an individualized mathematics program (Continuous Progress Mathematics) with the mathematics achievement gain scores of seventh-grade students who were not participating in the program but were taught by traditional methods.

Three hundred seventh-grade students were randomly selected from a large public school system to act as subjects to determine the effects of two different methods of teaching mathematics at the junior high level. One group, the experimental group (N = 150), was taught mathematics by an individualized program called "Continuous Progress Mathematics" (CPM). CPM is basically a self-teaching method which allows students to proceed at their own pace. Another group of students, the control group (N = 150), was taught by the traditional method of lecture, review and test. Students in each group were also divided into one of three mental ability groups as follows: high, average and low. The six subgroups, representing three levels of mental ability and two different teaching techniques, were given the Metropolitan '70 Achievement Test (Advanced Level) on a pretest-posttest basis. The pretest (Form F) was administered during September and October, 1974, while the posttest was given during May, 1975.

Total mathematics achievement change scores were then used to test four null hypotheses. The researcher had hypothesized that students taught by the individualized program would progress more than students taught by the traditional method. It was further hypothesized that students in the high and average mental ability groups would progress more than those in the low mental ability group.

Additional comparisons were also made in an attempt to determine any differences among students' pretest-posttest change scores on the following subtests: (1) mathematics computation, (2) mathematics concepts and (3) mathematics problem solving.

Results of testing the first hypothesis indicated that there was not a significant difference between the amount of total mathematics achievement gain shown by students taught with the CPM method and those taught by the traditional method. However, students taught by the traditional method did score slightly higher than those taught by the Continuous Progress Mathematical method.

Results of testing the second hypothesis indicated that students in the average mental ability group made significantly greater progress than students in either the high mental ability group or the low mental ability group.

Results of testing the third hypothesis showed that there was very little interaction between the two variables of teaching method and mental ability level.

The concomitant findings resulting from additional comparisons of the Continuous Progress Mathematical Group with the Control Group may be summarized as follows: (1) There were no significant differences in achievement gains on the subtests when students at the same ability level were compared. (2) Students of average mental ability in both groups made significantly greater achievement gains in mathematics computation than high ability and low ability students. (3) Students of average mental ability in both groups made significantly greater achievement gains in mathematics concepts than high and low ability students. (4) There were no significant differences noted between the achievement gains in mathematics problem solving when students at the same ability level were compared.

Conclusions

The conclusions which were drawn from the results of the study are presented as a logical extension of the data. While there is always a temptation to over generalize results, the conclusions presented in the following sections are limited to those which can be supported by the experimental data and research design. In an attempt to avoid confusion, the hypotheses and conclusions are presented in chronological order.

Conclusion Number One

Results of testing the first null hypothesis led to the following conclusion:

> The individualized method of teaching mathematics did not cause students in the experimental group to progress more in mathematics than students taught by traditional methods.

The findings of Verheul (1971), Thomas (1972) and Deep (1966) were similar to those of the researcher as they reported no significant differences between the effectiveness of the two methods of instruction when students' total mathematics achievement scores were compared. Nix (1969), Nabors (1968) and Grant (1964) also reported no significant differences; however, those differences which were observed were in favor of the individualized mathematics group.

Broussard (1971), Check (1959) and Downey (1974) reported significant differences favoring individualized instruction while Fielder (1971), Tanner (1965) and Crangle (1971) reported significant differences favoring the traditional method of teaching mathematics.

Conclusion Number Two

Results of testing the second null hypothesis led to the following conclusion:

> The average mental ability students (N = 100) taught by both methods progressed at a rate which was significantly greater than the lower and higher mental ability groups.

Higher ability students completed more units of work and scored higher; however, the percentile change score between pretest and posttest was greater for the average group.

There were inconsistencies in the studies reviewed as Nix (1969) reported a significant difference in achievement gain for average and below average students. Nabors (1968) and Deep (1966) reported greater gains for average ability students; however, these gains were not significant.

Conclusion Number Three

Results of testing the third null hypothesis led to the following conclusion:

> There was very little interaction between the two variables of teaching method and mental ability level.

This test was made to determine if students at different mental ability levels can progress more rapidly using Continuous Progress Mathematics or through the traditional method of instruction. The average ability students in both groups made significantly greater gains; however, there was very little interaction. Neither method of teaching proved to be superior for a particular mental ability group.

Conclusion Number Four

Results of testing the fourth null hypothesis led to the following conclusion:

> The individualized method of instruction used in teaching the three different ability level groups did not cause any particular group to achieve at a

> significantly greater rate, and it was concluded that this type of instruction could be used equally well with students in any of these groups. Average students made greater percentile gains; however, this gain was not significant.

The findings in conclusions number three and four are similar to those obtained by Deep (1966) and Crangle (1971) as they reported no significant difference in achievement gains for either method of instruction or for students at different mental ability levels. However, Nix (1969) reported students of average and below average ability using individualized instruction scored significantly higher than those who were taught by traditional methods at the same mental ability levels. Nabors (1968) also found significant differences in achievement gains for average ability students and below average ability boys using individualized instruction, but this was only in mathematics problem solving.

Findings by Tanner (1965) differed from the preceding as he reported lower ability students using the traditional mathematics teaching method scored significantly higher than lower ability students who were taught by individualized instruction.

Conclusions From Additional Findings

With the exception of Deep's study (1966) other studies reviewed in the literature compared achievement gains in computation, concepts and problem solving for the total groups using two different methods of instruction. However, Deep's study (1966) and the present study compared achievement gains on these three subtests at three different mental ability levels.

The researcher has related the results and conclusions of the present study to the results derived from previous similar studies.

Conclusion Number Five

> In considering the type of instruction received and the mental ability level of the students there was little difference in the rate of progress in mathematics computation, and it was concluded that the individualized method of instruction did not cause greater gains.

Results of studies by Deep (1966) and Tanner (1965) also indicated no significant differences for mathematics computation when they were compared. However, Verheul (1971) and Fielder (1971) reported significant differences in achievement for the group using the traditional mathematics teaching method.

Broussard (1971) reported results differing from the preceding studies as the individualized mathematics group scored significantly higher.

Conclusion Number Six

> Results of comparing the two different methods of instruction with students at different levels of ability (high, average and low) on the subtest, mathematics concepts, did not indicate that

> either method of instruction was superior. However, average students using both methods scored significantly higher than low and high ability students.

The findings of Verheul (1971) and Deep (1966) also indicated no significant differences for mathematics concepts.

Broussard (1971) reported a significant difference favoring the individualized mathematics group while Tanner (1965) and Fielder (1971) reported significant differences favoring the traditional mathematics teaching method.

Conclusion Number Seven

> Results of comparing progress of students as measured by the problem solving subtest did not indicate that the individualized or the traditional method of instruction was superior. However, the average ability students in both groups scored significantly higher than the low and high ability group when the pretest-posttest change scores were considered.

Verheul (1971), Broussard (1971) and Deep (1966) reported no significant differences in arithmetic problem solving. However, Fielder's results (1971) indicated a significant difference favoring the group using the traditional mathematics teaching method.

Discussions

There was no significant difference between the over-all rate of gain for students in the Continuous Progress Mathematics classes or the traditional classes. As a whole all of the students using both methods of instruction made unusually large gains. When the pretest was administered, 53 percent of the subjects in the experimental group were underachievers and 44 percent of those in the traditional group were underachievers. Posttest results indicated that only 38 percent of the experimental and 29 percent of the control students were underachievers (below 50th percentile). Average gain for the Continuous Progress Mathematics group was 14.97 percentile points while average gain for the traditional mathematics group was 16.20 percentile points. The researcher is surmising that this large growth may have been at least partially caused by the following:

1. Subjects involved in the study were exposed to mathematics teachers that were teaching only mathematics; whereas, during their first six years a single teacher was teaching all of the different subjects.

2. All of the teachers were aware that the comparison was being made and this may have increased the probability of the Hawthorne Effect. Knowing that student progress was being checked might have caused teachers to increase their efforts. However, this possibility should not have an adverse effect concerning the validity of the experiment because the test was to determine the relative merits of the two types of instruction at different ability levels.

Implications for Further Research

Duplication of this study with different testing instruments could be an area for further investigation. (The Metropolitan '70 Achievement Test (Metro '70) is a nationally normed achievement test and is geared to analyzing the achievement in school systems consisting largely of graded structure and using the traditional method of instruction). This requires that the tests be written within a framework of what is taught in a particular grade. Since the measuring instrument was geared to measure instruction in the graded structure, there is a definite possibility that students using the Continuous Progress Mathematics (CPM) method were learning mathematics that was not measured by the instruments used. A detailed study of the Continuous Progress Mathematics program and the traditional program with the subsequent development of a new testing instrument might reveal areas of mathematics achievement that were not measured.

Other research in this area might be done with older students. Subjects in this study were seventh-grade students and practically all of them were twelve years old. Piaget (1963) describes this age (11 or 12) as the beginning period of a stage of development which he calls formal operations. During the early stages of this period, he says children can deal with many variables simultaneously and understand abstract relationships. However, this was the experimental group's first exposure to individualized instruction, and older students might master this type of instruction more easily.

A longitudinal study comparing the Continuous Progress Mathematics program during the seventh-, eighth- and ninth-grade years should also yield additional insight into its total effectiveness. This could be done by measuring achievement gains for students used in this experiment at the end of the eighth- and ninth-grade year.

The role of the teacher and interaction that occurs between the teacher and students should be examined with the intent of improving the instructional process. A large part of the responsibility for teacher-student communication seems to lie with the student. Knowledge of the percent of the communications between a single student and the teacher that are student initiated and those that are teacher initiated might give some additional insight which would allow the program to be improved.

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APPENDIX A

A LIST OF SCHOOL SYSTEMS CONTACTED CONCERNING THEIR INDIVIDUALIZED MATHEMATICS PROGRAM

Appendix A

A List of School Systems Contacted Concerning Their Individualized Instruction Mathematics Program

Principal
Granada Community School
Forte Madera, California 94925

Principal
G. S. Skiff Elementary School
Phoenix, Arizona 85000

Principal
Nova Public Schools
Fort Lauderdale, Florida 33300

Principal
Southside Elementary School
Durham, North Carolina 27701

Principal
Wilson Elementary School
Janesville, Wisconsin 53545

Superintendent of Schools
Duluth Public Schools
Duluth, Minnesota 55801

Principal
Parkside Elementary School
Murray, Utah

Elementary Principal
Urbana Public Schools
Urbana, Illinois 61801

Principal
Southwest High School
Green Bay, Wisconsin 54301

Appendix A (Cont'd.)

Principal
Roy High School
Roy, Utah 84067

Principal
Hillsdale High School
San Mateo, California 94402

Superintendent of Schools
Milton
Pennsylvania 17847

Superintendent of Schools
Evanston
Illinois 60204

Superintendent of Schools
Pendleton
Oregon 97801

Principal
Brittan Acres Elementary School
San Carlos, California 94077

Principal
Grandview High School
Grandview, Idaho 83624

Principal
John Murray Junior High School
Pendleton, Oregon 97801

Aiken Elementary School
West Hartford
Connecticut 06107

Superintendent of Schools
Temple City
California 91780

APPENDIX B

LETTER SENT TO THE SCHOOL SYSTEMS CONTACTED DURING THE PRELIMINARY SURVEY

Appendix B

Letter Sent to the School Systems Contacted During the Preliminary Survey

Eisenhower Junior High School
Fifty-Seventh and Gore
Lawton, Oklahoma 73501

Our school is using an individualized mathematics program called Continuous Progress Mathematics (grades 7-8-9). At the present time we are in the process of evaluating the effectiveness of this program.

Your school was listed in a publication of the National School Public Relations Association as using individualized learning programs and I would like to know whether or not your mathematics program has been evaluated. If the program has been evaluated, I am particularly interested in the instruments that were used for evaluation (standardized tests, teacher-made tests, opinionnaire, etc.). Do the measuring instruments used evaluate academic achievement as well as attitude changes, self motivation and independence in students?

I would also like to know whether or not the data were treated statistically and if there was firm proof of achievement.

I would like you to send me any information, brochures, etc. that you have describing your individualized instruction program, the tests and instruments that you used in your evaluation, and the results obtained. I will reciprocate by mailing results of our evaluation to you when it is completed if you want me to.

Please send this material at your earliest convenience. I have enclosed a sheet which will facilitate your answering and a self-addressed stamped envelope for your convenience.

Thank you.

Sincerely yours,

Kenneth Ellis, Coordinator
Material Center

APPENDIX C

QUESTIONNARIE SENT TO NINETEEN SCHOOL SYSTEMS CONCERNING THE EVALUATION OF THEIR INDIVIDUALIZED INSTRUCTION MATHEMATICS PROGRAM

Appendix C

Questionnaire Sent to Nineteen School Systems
Concerning the Evaluation of Their
Individualized Instruction Math Program

<u>Directions</u>: Please complete the following questionnaire concerning your individualized mathematics program and enclose the information and materials sought in questions 6 and 7.

_____1. Our mathematics program has been evaluated.

_____2. Academic achievement for our mathematics program was measured by the following method:

_____3. Academic achievement of our students was compared with the achievement of students in a traditional lecture class by using the following tests:

_____4. The data obtained in the evaluation was treated statistically in the following manner:

_____5. Other types of evaluations (attitude changes, self-motivation and independence) have been measured in the following manner:

_____6. I have enclosed tests used to measure progress.

_____7. List the names and addresses of any schools (grades 7-12) that you know who are using an individualized mathematics program.

APPENDIX D

FIRST COMPLETE UNIT OF CONTINUOUS PROGRESS MATHEMATICS

Practice Set (Color coded pink)

SETS
Practice Set 1

(Terms)

Name____________________

Section____________________

Date____________________

Given a list of terms relating to sets, the learner can write a definition and example for each term.

1. set

2. member

3. empty set

4. one-to-one correspondence

5. cardinal number

6. ordinal number

7. even numbers

8. odd numbers

9. subset

10. intersection

11. disjoint sets

12. union

Practice Set (Color coded pink)

SETS
Practice Set 2

(Comparing Numbers)

Name ____________________

Section ____________________

Date ____________________

Given pairs of numbers, the learner can compare the numbers using greater than(>), less than(<), and equal to(=).

Examples: $5 < 8$, $8 > 5$, $5 = 5$.

1. 12 18

2. 21 3

3. 714 865

4. 95 105

5. 4 +7 11

6. 15 x 0 15 + 0

7. $\frac{21}{7}$ 3

8. 45,643 38,007

9. 69 6.9

10 3 x 4 5 x 6

11. Jim is 5' 10" tall and Bill is 69 inches tall.
    Jim's height is ____________________ Bill's height.

12. A ton of bricks is ________________ a ton of feathers.

Evaluation Test (Color coded blue)

SETS

EVALUATION 1

Name________________________

Section_____________________

Date________________________

1. A. Define Set.

   B. Illustrate what is meant by the term one-to-one correspondence by using the following sets.

      A. $A = \{a,b,c\}$ B. $B = \{x,y,z\}$

   C. Rewrite the members of this set $\{1,4,6,3,5,2\}$ so that they will fit the definition of ordinal numbers.

   D. If a set has no members, it is called the____________

   E. The set of numbers $\{2,4,6,8,10...\}$ are called the ____________________.

2. A. The symbol > means ____________________

   B. The symbol < means_____________________

   C. Using the numbers 8 and 5 and the symbol >, write a true number sentence ____________________.

   D. Using the numbers 8 and 5 and the symbol <, write a true number sentence ____________________________.

   E. In the following number sentence, replace the Δ with >,< or = to make a true number sentence.

      $6 \times 9 \Delta 8 \times 7$

Practice Set (Color coded pink)

SETS

Practice Set 3

(Set Designation)

Name________________________

Section________________________

Date________________________

Given written descriptions of sets, the learner can designate the sets by listing.

Example: Description - The days of the week that begin with the Letter T.

List - {Tuesday, Thursday}

1. The months of the year that begin with the letter J.

2. The set of states of the United States that are completely surrounded by water.

3. The set of odd numbers between 4 and 11.

4. The set of letters in ''Mathematics''.

Given sets with the members listed, the learner can designate the sets by description.

Example: List {John Kennedy, Lyndon Johnson, Richard Nixon}

Description - The set of Presidents of the U.S. during the 1960's

5. {March, May}

6. {Canada, Mexico, United States}

7. {Erie, Superior, Huron, Michigan, Ontario}

8. {2,4,6,......}

Practice Set (Color coded pink)

SETS
Practice Set 4
(Subsets)

Name______________________

Section____________________

Date______________________

Given a list of sets, the learner can list the indicated subsets.

1. From {A,B,C,D} list a subset with a cardinal number of 2.

2. From {1,2,3,4,5} list the subset of even numbers

3. Frpm,{1,2,3,4,5} list the subset of odd numbers.

4. From {1,2,3,4,5} list the subset of multiples of 7.

5. From {1,2,3,4,......} list the subset of even numbers.

6. From {1,2,3,4,5,6,7,8} list the subset of numbers divisible by 4.

7. From {1,2,3,4} list the subset of numbers less than 5.

Practice Set (Color coded pink)

SETS

Practice Set 5

(Number Sentences)

Name____________________

Section____________________

Date____________________

Given a list of number sentences, the learner can write whether the following sentences are true or false.

Example: $3 + 2 < 7$
$5 < 7$ True

1. $27 + 3 > 4 \times 5$
2. $227 + 15 > 776 + 5$
3. $35 \times 52 < 667 - 75$
4. $150 \times 7 = 112 \times 9$
5. $483 - 99 < 344 - 29$
6. $2877 \times 3 = 5000 \times 2$
7. $15 \times 2 = 2 \times 15$
8. $6 \times 6 < 6 + 6$
9. $18 \div 3 = 3 + 18$
10. $15 \times 3 \times 2 \times 0 = 90$

Evaluation Test (Color coded blue)

SETS
Evaluation 2

Name____________________

Section____________________

Date____________________

5. Designate the following sets by listing.

   a. The set of the days of the week that begin with the letter S.

   b. The set of letters in "Mississippi".

Designate the following sets by description.

   c. { 1,3,5,7...}

   d. {January, June, July}

4. a. From the set { 1, 2, 3, 4, 5, 6 } list the subset of numbers divisible by 3__________.

   b. From the set { 1, 2, 3, 4, 5, 6 } list the subset of the multiples of 5 __________.

5. Write whether the following sentences are True (T) or False (F).

   a. $25 - 19 = 64 \div 8$ _______

   b. $24 \div 3 < 3 \div 24$ _______

   c. $7 \times 6 > 42 \times 0$ _______

   d. $472 - 27 < 427 + 72$ ____

Practice Set (Color coded pink)

SETS

Practice Set 6

(Solving Open Sentences)

Name________________________

Section________________________

Date________________________

Given a list of open sentences, the learner can solve the open sentences for the missing number.

1. $21 \div 3 = N + 5$

2. $6 + 4 = 8 + N$

3. $4 + 7 = N + 4$

4. $6 + 0 = 8 - N$

5. $12 - 7 = N - 4$

6. $3 \times 9 = 27 \times N$

7. $14 \times 0 = \Delta \times 7$

8. $17 - N = 10 + 3$

9. $24 + \Delta = 28 - \Delta$ (use the same number for both frames)

10. $48 + N = N \times 3$ (use the same number for both frames)

Practice Set (Color coded pink)

SETS
Practice Set 7

(Union and Intersection)

Name______________________

Section______________________

Date______________________

Given pairs of sets, the learner can designate the union (U) or intersection (∩) of each pair.

Note: Union means to unite or combine two sets to make one large set. Intersection of two sets contains only the elements that are in both sets.

Examples: $\{A,B,C,D\} \cup \{B,D,F\} = \{A,B,C,D,F\}$
$\{A,B,C,D\} \cap \{B,D,F\} = \{B,D\}$

1. $\{1,3,5,7\} \cup \{2,4,6,8\} = ?$

2. $\{1,2,3\} \cap \{2,4,6,8\} = ?$

3. $\{A,B,C\} \cup \{D,E,F\} = ?$

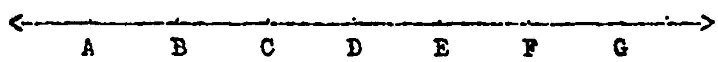
4. $\{A,B,C\} \cup \{1,2,3,4\} = ?$

5. $\{1,2,3,4\} \cap \{6,8,10\} = ?$

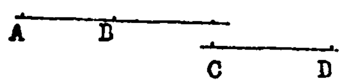
6. $\{2,4,6,\ldots\} \cap \{10,12,14\} = ?$

Use the line to answer questions (7-12).

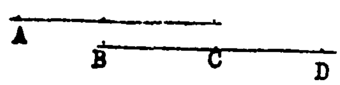
Note: Union means to unite the line segments to make one longer segment. Intersection means where the two segments cross over or overlap.



Example: $\overline{AC} \cup \overline{CD} = ?$



Example: $\overline{AC} \cap \overline{BD} = ?$



(Only the two end letters are used to name a line segment)

7. $\overline{AC} \cup \overline{BE} = ?$

8. $\overline{AD} \cap \overline{BE} = ?$

9. $\overline{AE} \cup \overline{BD} = ?$

10. $\overline{AE} \cap \overline{BD} = ?$

11. $\overline{AB} \cap \overline{DE} = ?$

12. $\overline{AC} \cap \overline{EF} = ?$

Evaluation (Color coded blue)

SETS
Evaluation 3

Name______________________

Section______________________

Date______________________

6. Solve the following sectences for the missing number.

   a. $16 + \Delta = 20 - \Delta$ $\Delta =$ __________ (use the same number for both frames)

   b. $18 \div 6 = \Delta + 3$ $\Delta =$ __________

   c. $27 \times \Delta = 9 \times 3$ $\Delta =$ __________

   d. $50 \div 5 = 2 \times \Delta$ $\Delta =$ __________

7. Designate the union ($\cup$) or intersection ($\cap$) of the following.

   a. $\{1,3,5,7\} \cup \{1,3,4,6\}$

   b. $\{A,B,C\} \cap \{C,D,E\}$

   c. $\{1,2,3,4\} \cap \{6,8,10\}$

Answer d,e, and f, using the line below.

   d. $\overline{AD} \cup \overline{DE} = ?$

   e. $\overline{AC} \cap \overline{BD} = ?$

   f. $\overline{AE} \cap \overline{BD} = ?$

Unit Review Sheet (Color coded yellow)

SETS

REVIEW SHEET

Name______________________

Section______________________

Date______________________

1. Define:

   a. cardinal number

   b. intersection

   c. empty set

2. Compare using >, <, or =.

   a. 8;5

   b. 4;10

   c. 576;593

   d. 8 + 3; 8 + 4

3. Designate by listing.

   a. The set of all odd numbers between 7 and 18.

   b. The set of days that begin with the letter O.

   Designate by description.

   c. {1,2,3,4,...}

   d. {June, July, January}

4. a. List a subset of {A,B,C,D,E} that has four members.
   b. List the subset of {0,1,2,3,4,...} that has as members multiples of 3.

5. Label each of the following sentences true or false.

   a. $4 \times 8 \times 5 < 16 \times 5 \times 0$

   b. $400 - 15 = 385$

   c. $5 \times 6 > 4 \times 7$

   d. $21 \div 3 = 3 \div 21$

6. Solve for the missing number. (Use only one number for each sentence)

   a. $N + 19 = 19 + N$

   b. $36 \div \Delta = \Delta \div 4$

   c. $\Delta \times 5 = \Delta \times 3$

   d. $31 - \Delta = 23 + \Delta$

   e. $6 \times 7 = 42 \times \Delta$

   f. $28 \div 7 = 4 \times N$

7. a. $\{1,3,5,7,...\} \cap \{1,2,3,4,5,6\} = ?$

   b. $\{A,B,C,D\} \cup \{B,C,D,E,F\} = ?$

   c. $\{A,B,C\} \cup \{A\} = ?$

   Use the line below to answer d,e,f.

   d. $\overline{AE} \cap \overline{BF}$

   e. $\overline{AD} \cap \overline{CF}$

   f. $\overline{BC} \cap \overline{DF}$

Unit Test Form A (Color coded white)

SETS
Test Form A

Do not write on test copy.
Show all work on answer sheet.

1. Define.
   a. subset
   b. odd numbers
   c. union

2. Compare using > or <.
   a. 6;11
   b. 4;7
   c. 736; 687
   d. 2+5;4+5

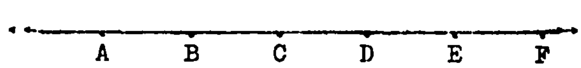
3. Designate by listing.
   a. The set of even numbers between 6 and 15.
   b. The set of days that begin with the letter C.
   Designate by description.
   c. {3,6,9,12}
   d. { March, May}

4. a. List a subset of {A,B,C,D} that has 3 members.
   b. List a subset of {1,3,5,7,9...} that has even numbers as members.

5. Label each of the following sentences true or false.
   a. $4 \times 9 \times 7 < 13 \times 5 \times 0$
   b. $100 - 86 = 14$
   c. $5 \times 7 > 6 \times 7$
   d. $63 \div 9 = 9 \div 63$

6. Solve for the missing number.
   a. $N + 11 = 11 \times 1$
   b. $32 \div \Delta = \Delta \div 2$
   c. $\Delta \times 13 = \Delta \times 5$
   d. $120 - \Delta = 110 + \Delta$
   e. $4 \times 6 = 24 \times \Delta$
   f. $64 \div 4 = 4 \times \Delta$

7. a. $\{1,2,3,4...\} \cap \{3,6,9,12...\} = ?$
   b. $\{A,B,C,D\} \cup \{A,C,E,F\} = ?$
   c. $\{A,B,C\} \cup \{1,2,3\} = ?$
   Use the line below to answer d,e,f.



   d. $\overline{BD} \cap \overline{DF}$
   e. $\overline{BF} \cap \overline{CD}$
   f. $\overline{AD} \cup \overline{CF}$

Unit Test Form B (Color coded white)

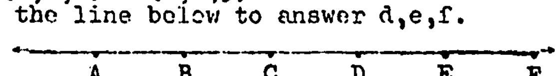
SETS
Test Form B

Do not write on test copy.
Show all work on answer sheet.

1. Define.
   a. cardinal number
   b. even number
   c. empty set
2. Compare using > or <.
   a. 5;8
   b. 3;12
   c. 876; 593
   d. 3 + 8; 7 + 9
3. Designate by listing.
   a. The set of all odd numbers between 5 and 16.
   b. The set of months that begin with the letter A.

   Designate by description.
   c. {5, 10, 15, 20}
   d. {Saturday, Sunday}
4. a. List a subset of {A,B,C,D} that has four members.
   b. List the subset of {0,2,4,6,8...} that has as members multiples of 4.
5. Label each of the following sentences true or false.
   a. $7 \times 8 \times 6 < 6 \times 12 \times 0$
   b. $200 - 16 = 184$
   c. $4 \times 5 > 3 \times 6$
   d. $28 \div 7 = 7 \div 28$
6. Solve for the missing number.
   a. $N + 18 = 18 \times 1$
   b. $48 \div \Delta = \Delta \div 12$
   c. $\Delta \times 8 = \Delta \times 12$
   d. $36 - \Delta = 28 + \Delta$
   e. $5 \times 7 = 35 \times \Delta$
   f. $48 \div 6 = 4 \times N$
7. a. $\{2,4,6,\ldots\} \cap \{1,2,3,4,5,6\} = ?$
   b. $\{A,B,C,D\} \cup \{B,C,D,E\} = ?$
   c. $\{A,B,C\} \cap \{1,2,3\} = ?$

   Use the line below to answer d,e,f.



   d. $\overline{AD} \cap \overline{DF}$
   e. $\overline{AE} \cap \overline{CF}$
   f. $\overline{BD} \cup \overline{CE}$

Unit Test Form C (Color coded white)

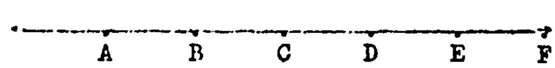
SETS
Test Form C

Do not write on test copy.
Show all work on answer sheet.

1. Define.
   a. ordinal number
   b. intersection
   c. subset
2. Compare using > or <.
   a. 7;9
   b. 5; 13
   c. 659; 703
   d. 4 + 5; 8 + 3
3. Designate by listing.
   a. The set of even numbers between 0 and 2.
   b. The set of months of the year that begin with the letter M.

   Designate by description.
   c. {2,4,6,8}
   d. {Tues.,Thurs.}
4. a. List a subset of {A,B,D,C} that has 2 members.
   b. List a subset of {3,6,9,12...} that has as members even numbers.
5. Label each of the following sentences true or false.
   a. 6 x 5 x 4 < 14 x 13 x 0
   b. 100 - 86 = 24
   c. 3 x 6 > 2 x 8
   d. 16 ÷ 8 = 8 ÷ 16
6. Solve for the missing number.
   a. N + 14 = 14 x 1
   b. 54 ÷ Δ = Δ ÷6
   c. Δ x 7 = Δ x 5
   d. 18 - Δ = 12 + Δ
   e. 3 x 8 = 24 x N
   f. 36 ÷ 6 = 3 x N
7. a. {1,2,3,4...} ∩ {2,4,6,8} = ?
   b. {A,B,C} ∪ {C,D,E,G} =?
   c. {A,B,C} ∪ {A,B,D} = ?

   Use the line below to answer d,e,f.



d. $\overline{AB} \cap \overline{CD}$
e. $\overline{AC} \cap \overline{BD}$
f. $\overline{AD} \cup \overline{CE}$

APPENDIX E

MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY RAW SCORES RECORDED FOR STUDENTS WITH HIGH, AVERAGE, AND LOW MENTAL ABILITY TAUGHT BY TWO DIFFERENT METHODS

TABLE 11
MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY SCORES* RECORDED FOR THE HIGH ABILITY GROUP WHO WERE TAUGHT BY THE CONTINUOUS PROGRESS MATHEMATICS (CPM) METHOD (EXPERIMENTAL GROUP)

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_1	91	18	26	52	30	77	78	86	84	59	52	34	54
S_2	81	90	96	90	92	94	94	98	98	4	-2	8	6
S_3	81	46	94	56	64	84	94	46	82	38	0	-10	18
S_4	81	89	72	78	80	77	86	82	86	-12	14	4	6
S_5	98	40	64	84	64	70	74	78	78	30	10	-6	14
S_6	95	84	64	90	80	94	88	94	94	10	24	4	14
S_7	84	86	98	88	92	94	86	94	94	8	-12	6	2
S_8	87	68	84	70	70	70	88	92	89	2	4	22	19
S_9	88	40	72	32	48	74	96	82	89	34	24	50	41
S_{10}	93	86	98	88	92	94	98	96	98	8	0	8	6
S_{11}	89	72	64	88	76	80	74	74	80	8	10	-14	4
S_{12}	90	22	72	74	58	80	92	86	90	58	20	12	32

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{13}	87	68	94	82	80	96	80	89	92	28	-14	7	12
S_{14}	81	40	48	38	40	74	56	62	68	34	8	24	28
S_{15}	90	64	90	78	76	77	88	86	88	13	-2	8	12
S_{16}	81	50	86	74	70	1	88	86	58	-49	2	12	-12
S_{17}	87	72	84	84	78	94	92	82	94	22	8	-2	16
S_{18}	89	46	40	60	50	94	94	92	96	48	54	32	46
S_{19}	87	68	58	42	54	77	74	58	72	9	16	16	18
S_{20}	84	36	38	52	40	46	76	66	66	10	38	14	26
S_{21}	99	72	96	88	86	88	88	98	94	16	-8	10	8
S_{22}	97	54	72	78	68	74	88	70	82	20	16	-8	14
S_{23}	90	76	90	90	86	80	88	94	92	4	-2	4	6
S_{24}	99	94	99	90	96	98	99	99	99	4	0	9	3
S_{25}	89	54	68	52	58	84	86	78	86	30	18	26	28
S_{26}	89	86	84	90	88	90	88	94	94	4	4	4	6

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{27}	79	36	40	60	48	50	76	50	62	14	36	-10	14
S_{28}	84	36	64	16	36	70	86	70	80	34	22	54	44
S_{29}	89	54	58	60	58	74	80	66	78	20	22	6	20
S_{30}	90	84	64	56	68	80	88	66	84	-4	24	10	16
S_{31}	91	50	96	60	70	66	78	92	82	16	-18	32	12
S_{32}	89	68	96	70	78	98	96	74	94	30	0	4	16
S_{33}	91	84	96	98	92	77	86	89	88	-7	-10	-9	-4
S_{34}	93	86	90	82	86	94	99	96	98	8	9	14	12
S_{35}	84	40	72	78	64	94	66	82	86	54	-6	4	22
S_{36}	79	50	68	64	62	60	86	74	78	10	18	10	16
S_{37}	87	68	72	56	64	94	88	70	89	26	16	14	25
S_{38}	93	64	84	88	78	88	92	99	96	24	8	11	18
S_{39}	94	90	86	90	90	99	80	92	94	9	-6	2	4
S_{40}	93	89	94	84	89	90	98	89	96	1	4	5	7

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{41}	81	76	68	84	78	74	88	82	86	-2	20	-2	8
S_{42}	84	54	72	38	54	66	66	82	76	12	-6	44	22
S_{43}	95	80	90	84	86	84	96	89	94	4	6	5	8
S_{44}	83	84	68	78	78	90	86	86	90	6	18	8	12
S_{45}	92	89	96	98	94	94	99	99	99	5	3	1	5
S_{46}	83	68	68	70	68	46	80	54	66	-22	12	-16	-2
S_{47}	81	50	68	82	68	80	76	96	89	30	8	14	21
S_{48}	83	68	28	52	48	66	66	70	70	-2	38	18	22
S_{49}	94	92	94	88	92	96	92	78	94	4	-2	-10	2
S_{50}	91	54	68	60	62	80	86	74	84	26	18	14	22
MEANS. . .	88.20	64.70	75.02	71.68	70.54	79.42	85.34	81.62	85.92	14.72	10.32	9.94	15.38
STANDARD DEVIATIONS. .		20.07	19.13	13.66	14.26	17.44	9.64	13.51	10.26	19.55	15.62	15.10	12.79

*All Achievement and Mental Ability Scores are given in terms of National Percentiles. Average for the National Percentile is 50.

TABLE 12

MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY SCORES* RECORDED FOR THE AVERAGE ABILITY GROUP WHO WERE TAUGHT BY THE CONTINUOUS PROGRESS MATHEMATICS (CPM) METHOD (EXPERIMENTAL GROUP)

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_1	69	22	36	56	36	70	62	46	62	48	26	-10	26
S_2	40	14	38	38	28	74	36	40	52	60	-2	2	24
S_3	75	64	72	74	68	90	74	62	80	26	2	-12	12
S_4	57	26	58	28	36	70	66	46	62	44	8	18	26
S_5	50	46	48	60	50	54	50	20	42	8	2	-40	-8
S_6	48	36	20	20	22	48	40	32	38	12	20	12	16
S_7	57	68	72	56	64	48	56	28	44	-20	-16	-28	-20
S_8	57	14	8	16	10	46	70	62	62	32	62	46	52
S_9	48	18	8	20	11	42	62	36	44	24	54	16	33
S_{10}	45	50	40	64	54	54	78	74	76	4	38	10	22
S_{11}	35	6	58	42	30	22	56	70	48	16	-2	28	18
S_{12}	57	18	22	28	22	54	70	54	62	36	48	26	40

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{13}	43	36	54	12	30	66	52	16	44	30	-2	4	14
S_{14}	50	36	64	70	58	70	76	62	72	34	12	-8	14
S_{15}	62	46	36	56	44	46	78	62	66	0	42	6	22
S_{16}	48	50	48	56	50	70	70	62	70	20	22	6	20
S_{17}	69	72	68	60	68	70	88	66	80	-2	20	6	12
S_{18}	52	36	96	42	62	66	88	36	70	30	-8	-6	8
S_{19}	67	68	64	70	64	88	76	82	86	20	12	12	22
S_{20}	67	10	84	48	44	54	74	50	62	44	-10	2	18
S_{21}	60	22	36	48	34	34	22	46	32	12	-14	-2	-2
S_{22}	69	86	72	90	86	94	86	82	90	8	14	-8	4
S_{23}	60	30	38	70	48	94	74	74	84	64	36	4	36
S_{24}	40	50	36	24	34	77	56	66	70	27	20	42	36
S_{25}	73	54	36	64	50	77	88	54	78	23	52	-10	28
S_{26}	62	50	64	64	62	48	62	58	58	-2	-2	-6	-4

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{27}	52	2	8	24	4	38	46	36	38	36	38	12	34
S_{28}	48	72	78	88	78	80	78	66	80	8	0	-22	2
S_{29}	71	46	40	38	40	66	70	62	68	20	30	24	28
S_{30}	67	26	78	56	54	70	74	82	78	44	-4	26	24
S_{31}	65	64	64	60	62	88	66	89	84	24	2	29	22
S_{32}	52	22	6	42	18	80	80	58	78	58	74	16	60
S_{33}	69	8	26	38	22	30	56	54	44	22	30	16	22
S_{34}	69	54	54	28	44	34	50	46	42	-20	-4	18	-2
S_{35}	48	36	54	48	44	50	50	40	48	14	-4	-8	4
S_{36}	43	40	38	52	44	48	30	58	44	8	-8	6	0
S_{37}	50	14	64	42	36	66	70	82	76	52	6	40	40
S_{38}	45	68	64	70	64	54	46	20	38	-14	-18	-50	-26
S_{39}	73	68	58	42	54	84	66	50	70	16	8	8	16
S_{40}	40	46	38	48	44	50	80	70	72	4	42	22	28

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{41}	67	89	72	60	76	88	74	62	78	-1	2	2	2
S_{42}	73	64	68	48	58	60	86	50	70	-4	18	2	12
S_{43}	65	22	22	38	24	46	52	40	44	24	30	2	20
S_{44}	65	64	72	48	62	90	86	86	90	26	14	38	28
S_{45}	57	8	20	28	18	30	52	46	42	22	32	18	24
S_{46}	62	64	40	52	50	34	52	66	52	-30	12	14	2
S_{47}	55	72	72	74	70	88	76	86	86	16	4	12	16
S_{48}	75	50	48	38	44	70	78	89	82	20	30	51	38
S_{49}	62	68	68	74	68	66	76	86	80	-2	8	12	12
S_{50}	55	22	20	38	24	70	50	32	52	48	30	-6	28
MEANS. . .	57.76	42.34	48.96	49.00	45.34	62.12	65.08	56.84	63.40	19.78	16.12	7.84	18.06
STANDARD DEVIATIONS. .		22.99	22.14	18.29	19.30	14.85	23.16	19.15	16.75	20.98	21.34	19.96	16.51

*All Achievement and Mental Ability Scores are given in terms of National Percentiles. Average for the National Percentile is 50.

TABLE 13
MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY SCORES* RECORDED FOR THE LOW ABILITY GROUP WHO WERE TAUGHT BY THE CONTINUOUS PROGRESS MATHEMATICS (CPM) METHOD (EXPERIMENTAL GROUP)

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_1	33	18	16	10	12	46	50	32	42	28	34	22	30
S_2	8	8	2	32	8	28	6	12	14	20	4	-20	6
S_3	16	14	1	16	6	16	6	16	10	2	5	0	4
S_4	14	30	28	24	28	38	52	40	42	8	24	16	14
S_5	21	8	6	20	6	34	40	46	38	26	34	26	32
S_6	35	26	22	70	36	46	36	16	30	20	14	-54	-6
S_7	25	26	14	38	22	38	50	46	42	12	36	8	20
S_8	25	30	48	32	36	50	56	28	44	20	8	-4	8
S_9	25	68	54	64	62	46	76	82	72	-22	22	18	10
S_{10}	29	8	28	24	18	48	40	32	38	40	12	8	20
S_{11}	9	22	10	10	10	22	12	6	11	0	2	-4	1
S_{12}	35	50	58	32	48	77	66	54	68	27	8	22	20

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{13}	13	6	10	28	11	11	12	24	14	5	2	-4	3
S_{14}	33	18	40	42	34	60	74	24	54	42	34	-18	20
S_{15}	4	4	22	32	16	11	12	16	11	7	-10	-16	-5
S_{16}	19	40	40	28	36	66	26	36	42	26	-14	8	6
S_{17}	25	18	26	24	22	42	36	36	36	24	10	12	14
S_{18}	29	8	10	12	8	54	40	10	32	46	30	-2	24
S_{19}	25	26	20	32	24	38	66	40	48	12	46	8	24
S_{20}	23	26	36	12	22	30	12	36	26	4	-24	24	4
S_{21}	12	6	8	2	2	28	2	16	14	22	-6	14	12
S_{22}	23	2	4	2	1	8	12	10	8	6	8	8	7
S_{23}	35	14	10	2	6	38	66	58	54	24	56	56	48
S_{24}	38	46	16	56	36	50	70	94	78	4	54	38	42
S_{25}	35	64	16	82	50	66	40	20	42	2	24	-62	-8
S_{26}	25	18	10	38	18	42	22	24	28	24	12	-14	10

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{27}	9	2	1	4	1	6	1	2	1	4	0	-2	0
S_{28}	16	22	28	16	22	46	40	20	32	24	12	4	10
S_{29}	10	1	4	12	2	4	10	10	4	3	6	-2	2
S_{30}	9	6	6	4	2	6	1	4	2	0	-5	0	0
S_{31}	19	50	40	48	48	42	18	40	32	-8	-22	-8	-16
S_{32}	5	26	14	28	20	22	30	32	26	-4	16	4	6
S_{33}	29	18	20	38	22	6	22	10	10	-12	2	-28	-12
S_{34}	16	26	40	60	44	24	10	24	18	-2	-30	-36	-26
S_{35}	16	10	14	24	14	42	30	40	36	32	16	16	22
S_{36}	8	2	1	1	1	11	2	28	10	9	1	27	9
S_{37}	25	18	2	12	6	46	18	28	30	28	16	16	24
S_{38}	27	40	40	28	36	74	40	28	48	34	0	0	12
S_{39}	31	40	22	42	34	54	50	32	44	14	28	-10	10
S_{40}	13	10	16	12	11	18	26	1	10	8	10	-11	-1

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{41}	31	18	40	38	30	46	76	32	52	28	36	-6	22
S_{42}	29	22	64	56	48	38	70	50	52	16	6	-6	4
S_{43}	25	1	1	4	1	18	12	4	10	17	11	0	9
S_{44}	12	2	1	2	1	16	46	10	20	14	45	8	19
S_{45}	31	2	10	20	6	8	10	32	14	6	0	12	8
S_{46}	33	1	14	12	2	42	40	10	28	41	26	-2	26
S_{47}	25	22	54	52	40	50	56	66	62	28	2	14	22
S_{48}	14	26	22	24	22	54	46	28	42	28	24	4	20
S_{49}	16	4	1	10	2	34	26	24	28	30	25	14	26
S_{50}	21	22	14	4	10	34	10	40	28	12	-4	36	18
MEANS. . .	21.68	19.90	20.48	26.30	20.06	35.48	33.40	28.98	31.54	15.58	12.92	2.68	11.48
STANDARD DEVIATIONS. .		16.33	16.95	19.70	24.13	18.80	22.77	19.38	18.28	14.70	18.89	20.85	13.86

*All Achievement and Mental Ability Scores are given in terms of National Percentiles. Average for the National Percentile is 50.

TABLE 14
MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY SCORES* RECORDED FOR
THE HIGH ABILITY GROUP WHO WERE TAUGHT BY THE
TRADITIONAL METHOD (CONTROL GROUP)

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_1	98	72	94	78	80	99	99	99	99	27	5	21	19
S_2	84	72	99	82	88	90	88	86	92	18	-11	4	4
S_3	97	50	90	88	78	88	96	94	96	38	6	6	18
S_4	89	50	64	64	62	88	88	82	89	38	24	18	27
S_5	84	50	72	48	58	94	88	74	90	44	16	26	32
S_6	99	92	98	94	96	99	99	98	99	7	1	4	3
S_7	83	80	64	70	70	96	88	74	92	16	24	4	22
S_8	77	54	58	70	62	90	86	86	90	36	28	16	28
S_9	79	54	72	48	58	94	94	78	94	40	22	30	36
S_{10}	97	84	94	64	80	94	88	92	94	10	-6	28	14
S_{11}	88	84	86	88	86	98	98	94	98	14	12	6	12
S_{12}	99	90	98	90	94	90	94	96	96	0	-4	6	2

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{13}	91	90	98	78	90	90	96	96	96	0	-2	18	6
S_{14}	84	64	72	78	70	98	80	92	94	34	8	14	24
S_{15}	96	46	72	74	64	94	80	82	90	48	8	8	26
S_{16}	90	80	64	64	70	98	80	92	94	18	16	28	24
S_{17}	84	89	72	56	74	80	99	89	94	-9	27	33	20
S_{18}	98	94	90	84	92	94	86	89	92	0	-4	5	0
S_{19}	96	72	99	84	88	94	92	99	98	22	-7	15	10
S_{20}	96	72	94	82	82	98	88	89	96	26	-6	7	14
S_{21}	81	92	90	90	92	98	99	94	99	6	9	4	7
S_{22}	93	64	90	78	76	98	80	89	94	34	-10	11	18
S_{23}	79	72	72	78	74	99	99	82	98	27	27	4	24
S_{24}	89	68	68	82	70	94	99	98	99	26	31	16	29
S_{25}	94	72	86	64	74	99	99	82	98	27	13	18	24
S_{26}	94	92	98	98	98	99	99	96	99	7	1	-2	1

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{27}	98	76	98	74	86	94	98	94	98	18	0	20	12
S_{28}	77	72	86	84	80	94	94	92	96	22	8	8	16
S_{29}	81	68	40	84	64	84	88	74	86	16	48	-10	22
S_{30}	77	26	28	48	34	54	74	62	68	28	46	14	34
S_{31}	96	84	86	96	89	96	88	96	96	12	2	0	7
S_{32}	77	68	68	42	58	77	70	62	72	9	2	20	14
S_{33}	81	86	98	98	94	98	96	78	96	12	-2	-20	2
S_{34}	84	50	58	90	68	90	88	98	94	40	30	8	26
S_{35}	90	80	64	70	70	94	80	58	84	14	16	-12	14
S_{36}	83	76	68	60	68	80	76	82	82	4	8	22	14
S_{37}	83	80	78	88	82	98	88	74	92	18	10	-14	10
S_{38}	81	99	72	98	96	99	94	94	99	0	22	-4	3
S_{39}	77	46	54	70	58	60	78	66	72	14	24	-4	14
S_{40}	79	90	84	78	86	96	78	86	90	6	-6	8	4

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{41}	79	94	84	94	92	94	96	94	98	0	12	0	6
S_{42}	94	76	86	82	80	90	92	82	92	14	6	0	12
S_{43}	77	80	84	88	82	70	86	70	80	-10	2	-18	-2
S_{44}	92	92	84	94	92	99	78	92	94	7	-6	-2	2
S_{45}	79	76	78	64	74	94	80	92	92	18	2	28	18
S_{46}	93	92	98	64	88	98	98	74	96	6	0	10	8
S_{47}	98	98	99	90	98	99	98	99	99	1	-1	9	1
S_{48}	92	96	94	99	98	96	99	89	98	0	5	-10	0
S_{49}	96	89	94	90	92	94	88	94	94	5	-6	4	2
S_{50}	95	76	72	90	80	88	88	74	89	12	16	-16	9
MEANS. . .	87.96	75.38	80.18	78.18	78.70	91.78	89.50	85.96	92.54	16.40	9.32	7.78	13.84
STANDARD DEVIATIONS. .		16.40	10.69	15.03	6.79	13.39	9.20	13.35	18.90	13.39	13.87	12.95	10.16

*All Achievement and Mental Ability Scores are given in terms of National Percentiles. Average for the National Percentile is 50.

TABLE 15
MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY SCORES RECORDED FOR THE AVERAGE ABILITY GROUP WHO WERE TAUGHT BY THE TRADITIONAL METHOD (CONTROL GROUP)

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_1	67	50	58	60	58	66	76	62	72	16	18	2	14
S_2	65	68	86	38	64	74	80	46	72	6	-6	8	8
S_3	55	46	40	20	34	66	66	40	58	20	26	20	24
S_4	48	72	64	52	62	84	76	74	82	12	12	22	20
S_5	57	72	86	70	74	80	88	82	88	8	2	12	14
S_6	69	46	84	74	68	80	88	82	89	34	4	8	21
S_7	50	54	84	52	64	74	62	78	76	20	-22	26	12
S_8	67	72	68	74	70	99	96	70	94	27	28	-4	24
S_9	62	40	48	56	48	50	52	46	52	10	4	-10	4
S_{10}	65	54	78	60	64	66	88	50	76	12	10	-10	12
S_{11}	75	92	78	74	82	88	92	86	92	-4	14	12	10
S_{12}	67	50	36	42	40	84	78	78	84	34	42	36	44

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{13}	71	64	40	60	54	77	70	40	66	13	30	-20	12
S_{14}	52	40	78	48	54	80	86	54	78	40	8	6	24
S_{15}	40	46	64	52	54	42	78	66	66	-4	14	14	12
S_{16}	45	54	26	20	30	77	74	58	72	23	48	38	42
S_{17}	48	72	98	78	82	48	76	58	66	-24	-22	-20	-16
S_{18}	62	8	38	64	34	74	76	66	76	66	38	2	42
S_{19}	52	6	22	24	14	30	18	46	30	24	-4	22	16
S_{20}	62	8	40	32	24	74	78	46	70	66	38	14	46
S_{21}	65	36	64	60	54	66	78	82	80	30	14	22	26
S_{22}	73	80	68	70	74	42	62	62	54	-38	-6	-8	-20
S_{23}	57	40	68	38	48	77	86	58	78	37	18	20	30
S_{24}	69	6	22	38	18	42	78	46	58	36	56	8	40
S_{25}	52	18	40	52	36	84	76	66	80	66	36	14	44
S_{26}	48	40	20	20	24	46	30	46	38	6	10	26	14

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{27}	55	22	1	28	10	34	88	46	58	12	87	18	48
S_{28}	71	46	40	28	36	48	76	36	54	2	36	8	18
S_{29}	50	18	22	38	24	30	50	50	42	12	28	12	18
S_{30}	67	36	68	38	48	60	70	70	70	24	2	32	22
S_{31}	71	46	84	52	62	90	80	89	90	44	-4	37	28
S_{32}	55	54	54	12	36	48	66	62	62	-6	12	50	26
S_{33}	75	50	84	88	74	94	78	92	92	44	-6	4	18
S_{34}	75	22	36	64	40	98	86	89	94	76	50	25	54
S_{35}	43	30	38	48	36	80	62	62	72	50	24	14	36
S_{36}	60	64	84	74	70	54	80	74	76	-10	-4	0	6
S_{37}	79	72	58	28	50	84	74	62	78	12	16	34	28
S_{38}	65	30	26	28	28	50	56	62	58	20	30	34	30
S_{39}	45	54	72	84	70	50	70	74	68	-4	-2	-10	-2
S_{40}	43	40	68	38	48	70	76	62	72	30	8	24	24

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{41}	67	46	48	42	44	46	66	70	62	0	18	28	18
S_{42}	75	72	54	42	54	66	88	50	72	-6	34	8	18
S_{43}	73	50	72	56	62	88	80	50	78	38	8	-6	16
S_{44}	48	46	90	52	64	84	88	78	89	38	-2	26	25
S_{45}	48	26	22	20	22	50	66	40	54	24	44	20	32
S_{46}	48	8	16	48	20	98	46	58	76	90	30	10	56
S_{47}	71	36	54	16	34	74	78	40	68	38	24	24	34
S_{48}	69	46	84	48	58	80	86	74	84	34	2	26	26
S_{49}	43	72	64	42	58	70	76	40	66	-2	12	-2	8
S_{50}	48	26	48	32	34	38	22	46	32	12	-26	14	-2
MEANS. . .	59.74	44.92	55.70	47.48	48.20	67.08	72.32	61.28	70.28	22.16	16.62	13.80	22.08
STANDARD DEVIATIONS. .		22.38	24.91	18.49	16.18	26.38	16.58	15.35	16.59	24.98	21.49	15.50	15.41

*All Achievement and Mental Ability Scores are given in terms of National Percentiles. Average for the National Percentile is 50.

TABLE 16
MATHEMATICS ACHIEVEMENT AND MENTAL ABILITY SCORES RECORDED FOR
THE LOW ABILITY GROUP WHO WERE TAUGHT BY THE
TRADITIONAL METHOD (CONTROL GROUP)

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_1	10	26	10	32	18	42	18	28	28	16	8	-4	10
S_2	27	4	28	24	16	22	36	32	28	18	8	8	12
S_3	14	18	22	16	18	46	22	4	23	28	0	-12	5
S_4	23	8	28	12	16	22	36	28	26	14	8	16	10
S_5	40	10	26	12	16	16	76	28	36	6	50	16	20
S_6	23	22	14	32	20	50	26	24	32	28	12	-8	12
S_7	38	50	68	56	58	84	78	82	86	34	10	26	28
S_8	5	30	26	4	18	48	52	28	42	18	26	24	24
S_9	31	50	38	70	54	54	46	58	54	4	8	-12	0
S_{10}	12	14	20	10	12	38	30	20	28	24	10	10	16
S_{11}	9	6	8	6	2	16	22	16	14	10	14	10	12
S_{12}	19	30	40	24	30	42	50	40	42	12	10	16	12

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{13}	38	22	28	16	22	34	36	20	28	12	8	4	6
S_{14}	33	26	36	48	34	84	46	50	62	58	10	2	28
S_{15}	38	54	20	24	30	48	46	40	44	-6	26	16	14
S_{16}	14	26	38	20	28	30	62	50	44	4	24	30	16
S_{17}	13	10	28	2	11	34	6	12	16	24	-22	10	5
S_{18}	38	68	58	48	58	77	78	46	72	9	20	-2	14
S_{19}	17	6	22	6	10	11	26	16	14	5	4	10	4
S_{20}	25	36	14	12	18	38	36	28	32	2	22	16	14
S_{21}	2	2	4	10	2	30	30	6	20	28	26	-4	18
S_{22}	38	30	22	24	24	30	46	50	38	0	24	26	14
S_{23}	16	14	8	24	11	30	18	6	16	16	10	-18	5
S_{24}	23	18	22	12	18	28	10	16	16	10	-12	4	-2
S_{25}	27	18	14	24	18	48	66	40	52	30	52	16	34
S_{26}	33	14	14	24	16	28	26	46	30	14	12	22	14

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{27}	35	1	1	16	1	24	1	16	10	23	0	0	9
S_{28}	35	8	10	48	18	18	36	36	28	10	26	-12	10
S_{29}	31	6	14	32	12	28	26	50	32	22	12	18	20
S_{30}	23	6	36	38	22	11	10	58	23	5	-26	20	1
S_{31}	25	54	38	42	44	54	66	74	68	0	28	32	24
S_{32}	17	50	16	24	28	46	56	32	44	-4	40	8	16
S_{33}	33	64	28	42	44	66	70	58	68	2	42	16	24
S_{34}	17	26	16	42	24	24	26	36	28	-2	10	-6	4
S_{35}	35	6	1	12	2	74	12	10	30	68	11	-2	28
S_{36}	27	30	40	42	36	54	50	58	54	24	10	16	18
S_{37}	33	54	58	38	50	74	74	28	62	20	16	-10	12
S_{38}	35	4	10	12	6	30	26	28	28	26	16	16	22
S_{39}	19	6	1	2	1	22	26	6	14	16	25	4	13
S_{40}	8	14	14	32	18	8	30	10	12	-6	16	-22	-6

Student Number	Mental Ability Percentile	Achievement Pretest				Achievement Posttest				Change Scores			
		Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL	Comp.	Conc.	Prob-Solv.	TOTAL
S_{41}	16	46	48	42	44	54	50	62	58	8	2	20	14
S_{42}	14	1	4	1	1	2	12	1	2	1	8	0	1
S_{43}	8	8	14	4	6	30	1	12	8	22	-13	8	2
S_{44}	21	1	28	2	4	24	12	6	12	23	-16	4	8
S_{45}	12	14	22	20	18	38	36	12	28	24	14	-8	10
S_{46}	21	6	10	20	10	24	18	28	23	18	8	8	13
S_{47}	31	26	28	28	28	28	70	32	42	2	42	4	14
S_{48}	17	40	22	32	30	70	18	36	42	30	-4	4	12
S_{49}	3	18	36	6	18	30	26	40	30	12	-10	34	12
S_{50}	35	22	38	56	36	38	62	40	44	16	24	-16	8
MEANS. . .	23.14	22.46	23.78	24.50	21.58	38.02	36.76	31.66	34.26	15.56	12.98	7.16	12.68
STANDARD DEVIATIONS. .		28.16	14.24	16.50	15.31	19.69	21.51	19.29	17.12	14.34	16.51	13.47	8.35

*All Achievement and Mental Ability Scores are given in terms of National Percentiles. Average for the National Percentile is 50.