76-15,820

OTUOMAGIE, Robert, 1943-
ANALYSIS OF PRESSURE BUILD-UP IN AN INFINITE TWO-LAYERED OIL RESERVOIR.

The University of Oklahoma, Ph.D., 1975
Engineering, petroleum

Xerox University Microfilms, Ann Arbor, Michigan 48106

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

ANALYSIS OF PRESSURE BUILD-UP IN AN INFINITE TWO-LAYERED OIL RESERVOIR

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

ROBERT OTUOMAGIE

Norman, Oklahoma

1975

ANALYSIS OF PRESSURE BUILD-UP IN AN
INFINITE TWO-LAYERED OIL RESERVOIR

APPROVED BY

D. E. Menzie

Henry Crichlow

Martin E. [illegible]

Roger Frech

L. A. E. Northen

DISSERTATION COMMITTEE

ACKNOWLEDGEMENTS

The writer expresses his sincere appreciation to Dr. D. E. Menzie and Dr. H. B. Crichlow for their encouragement and advice during the various stages of this research.

Acknowledgement is made to Dr. W. Cobb of Mississippi State University and Dr. R. Raghaven of Amoco Production Company for their interest in personal communication.

ABSTRACT

Two methods of analysis have been used in this study, namely Hartsock and Lefkovits _et al_. Equations were derived using the assumptions and boundary conditions stated by these methods. The difference between the two methods is that Hartsock has taken into consideration the force of gravity which has been neglected by Lefkovits _et al_. The equations derived taking the force of gravity into consideration suggest some analysis techniques that can be used to determine the effective permeabilities of the individual layers if the initial pressures of the two zones or their difference Δp is known. It is further suggested that it is possible to determine the initial pressure of any of the zones if either the other initial pressure or the initial pressure different Δp is known. This study suggests that it is possible to use the pressure drop equation of the upper or lower zone for such estimates.

The equations derived by neglecting the force of gravity can be used to study the effects of permeability contrast, thickness ratio, skin factor and well-bore storage in pressure build-up curves for an infinite two-layered reservoir.

Both methods can be used to determine the kh product and to study the effects of permeability and thickness ratios.

Using the pressure drop equations derived for the case where the force of gravity is not neglected, equations have been derived to determine the skin factors for the upper and lower zones.

The derivations of the equations used in this study are shown in the Appendices, and similarities between two- and single-layered reservoirs have been indicated.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ANALYSIS OF PRESSURE BUILD-UP IN AN INFINITE TWO-LAYERED OIL RESERVOIR

CHAPTER I

INTRODUCTION

The work of other investigators[4,5,14,22] has assumed that the initial pressures in dual zone reservoirs are equal. As a result, they have not been able to determine properties of the individual layers from a combined build-up curve.

The work of Lefkovits et al.[14] has shown that it is possible to determine the permeability-thickness product, well-bore damage and static reservoir pressure.

Cobb[4] investigated the behavior of a two-layer bounded reservoir with varying permeability contrast. He used the methods of Muskat, Miller-Dyes-Hutchinson, and Horner for his analysis. His results confirmed those of Lefkovits et al.[14]

Raghaven et al.[22] extended the work of Cobb by studying the effect of thickness ratio of dual zone reservoirs for various permeability contrast. Their work showed that it is possible to calculate permeability ratio for a given thickness ratio.

Earlougher et al.[5] presented results to show that there is no particular shape for the pressure build-up curve for a multiple-layered system without cross flow.

Hartsock[7] showed that it is possible to determine the effective permeability of the individual layers if the initial pressures are known or the difference in the initial pressures. In his analysis, he also showed that the initial pressure of any zone can be estimated if the initial pressure of the other zone is known or the difference in the initial pressures. In his procedure, he used the pressure drop of one of the zones.

The present study was conducted to utilize the pressure drop of either zone for the estimation of the initial pressures and effective permeabilities of the individual layers as outlined above. This study also suggested equations, derived from the pressure drop equations for the estimation of skin factors for the individual zones. The effect of permeability contrast, thickness ratio, skin factors, and well bore storage were studied. The outer boundary considered in this study is that of an infinite two-layered reservoir.

CHAPTER II

SCOPE

The purpose of this study is to obtain a set of expressions and procedures that can be used to determine the reservoir parameters from pressure build-up curves for an infinite two-layered oil reservoir. Equations were derived which define the pressure behavior at the well bore. Pressure drop equations were derived for the upper and lower zones using Hartsock's method. From these pressure drop equations other equations were derived which describe theoretical build-up for this system when the well is shut in.

From these equations it is possible to determine the initial pressure of one zone if the other initial pressure or the difference between the two initial pressures of the respective zones are known. The effective permeabilities can also be computed under the above conditions; that is, if the initial pressure in either zone is known or if the difference in pressure between the two zones is known.

Hartsock[7] used only the pressure drop of the upper zone for these derivations and determinations, but this study shows that either the pressure drop of the upper or lower zone can be used. Further, this study shows that from the pressure drop and shut-in pressure equations, other equations can be derived to determine the skin factors for the upper and lower zones.

This study also shows under what conditions an infinite two-layered oil reservoir is similar to a single-layered reservoir.

Hartsock[7] and Lefkovits et al.[14] methods have been used to show the effects of permeability and thickness ratios on pressure build-up curves. Also studied were the effects of skin and well-bore storage using the equations derived for an infinite two-layered reservoir with the boundary conditions at the well-bore outlined by Lefkovits et al.[14]

CHAPTER III

LITERATURE REVIEW

Pressure build-up analysis is one of the well test procedures usually carried out in reservoir engineering studies. Pressure data are very useful in analyzing the permeability, capacity, transmissibility, calculation of skin effect, productivity index, flow efficiency of a well, and the estimation of static and average reservoir pressure. Pressure data and hydrocarbon properties can be used together for volumetric and material balance calculation for the determination of oil or gas in place.

Several methods of pressure build-up analysis have so far been studied. These methods can be classified as:

i. Pressure build-up in single-layered reservoirs.

ii. Pressure build-up in layered reservoir with cross flow.

iii. Pressure build-up in layered reservoir without cross flow (commingled fluid production).

Methods of analysis of build-up curves are outlined as follows:

The initial work on pressure build-up analysis was started by Muskat[19] when he related the slope of the straight portion of a semilog plot of pressure versus time with the permeability and thickness of the formation.

Miller, Dyes and Hutchinson[17,18] also established a method for the estimation of average permeability, effective permeability and static reservoir pressure for a single finite layer reservoir. Their work included the estimation of the above parameters for oil and gas flow in the reservoir if the gas is distributed in the oil phase.

Hurst[9] and Van Everdingen[29] have plotted shut-in pressure versus $(t + \Delta t)/\Delta t$ for an infinite reservoir and have determined the effective permeability from the slope of that graph, and the static pressure at the point where $(t + \Delta t)/\Delta t = 1$. Both have also considered the problem of reduced permeability near the well-bore.

Arps[1] introduced a graphical method of computing the completion factor which is related to the damage around the well-bore. In the determination of the completion factor, Arps has utilized the equations suggested by Van Everdingen.

Horner[8] proposed analysis of pressure build-up for a well in an infinite reservoir, a well close to a fault but far from any other boundary and a well in a finite reservoir. His method of analysis is suitable under the above condition for a single layer reservoir to estimate effective permeability, static reservoir pressure, and the distance of a fault from a well.

Thomas[28] further utilized the method of Horner and introduced the skin effect. The skin factor is responsible for increased resistance to flow if positive, and there is increased flow rate when the skin factor is negative. In addition, the Thomas method could be used to calculate the effective permeability and static reservoir pressure.

Gladfelter et al.[6] have suggested pressure correction for after production and the "condition ratio" which accounts for the formation permeability by comparing the pressure build-up and productivity-index estimates.

Perrine[21] in his work has summarized the above methods of pressure build-up analysis and noted that the important difference is in the boundary condition assumed. He further suggested a procedure that should be followed in pressure build-up analysis.

Mathews et al.[15] developed a method whereby pressure build-up can be used to find the average reservoir pressure by volumetric averaging of the individual drainage zone pressures of each well. Their concept is based on the fact that at steady state each individual drainage volume is proportional to a well's production rate. The method suggests that the extrapolated pressure to an infinite time for a well in a bounded reservoir should be corrected to obtain the average pressure within that boundary.

Extended methods[13,24] of pressure build-up analysis can be used for the analysis of after production and the late transient portions of the build-up curve. Using these techniques, it is possible to estimate kh, transmissibility, skin factor, average reservoir pressure and contributory pore volume.

Ramey's[23] study on short-time well test data can be used to supplement conventional interpretation of the straight line portion of pressure build-up analysis. Interpretation of short time well tests before the straight line portion is reached is useful to detect the presence of skin and well-bore storage. This is done by using type curve matching. The type curve matching approach uses a set of curves plotted on a log-log paper which are compared to a published set of curves until a match is found.

In practice, production of oil from heterogeneous reservoirs is of interest to petroleum engineers. One of these heterogeneous reservoirs is the layered reservoir with cross flow.

Russell and Prats[25] investigated mathematically the performance of a bounded two layer reservoir in which flow is possible from a layer of low permeability to that of higher permeability. Their studies show that except for the early time when the reservoir behaves as that of a stratified system, the performance of the reservoir is identical to that of a single layer with the same pore volume, drainage and well-bore radii. The total "kh" and "ϕh" products are the sum of the individual layers.

Later Russell and Prats[26] showed that from the production performance and pressure response, it is possible to detect if there is interlayer cross flow between layers. They further established that if there is communication between layers, the above conclusions apply. They also concluded that production from a reservoir with interlayer cross flow has a shorter period of production life and high primary recovery.

The studies of Katz and Tek[11] confirmed those of Russell and Prats[25,26] and included the method of extending the solution to three layers. They have shown that inter-layer cross flow is affected by the vertical permeability and the ratio of system thickness to the drainage radius.

Pendergrass and Berry[20] used both analytic and numerical methods to study the interlayer crossflow in reservoirs. Their studies confirmed those of other investigators.[11,25,26] They added that it is not possible to detect the effect of stratification on reservoir transient data at long time transient performance except at the early transient period, which is so short as to be of little practical concern.

Another type of heterogeneous reservoir that is of practical interest to petroleum engineers is the stratified reservoir without crossflow, where the layers are only in communication with each other at the well-bore. Lefkovits _et al._[14] studied the behavior of bounded reservoirs composed of stratified layers. They found that when the shut in pressure P_{ws} versus $\Delta t/(t + \Delta t)$ is plotted on a semilog graph paper, a curve with a straight-line section and subsequent leveling, rising and flattening sections is obtained. Their work showed that the time necessary to reach pseudo-steady state is much longer for a two-layered reservoir than for a single layered reservoir. This is due to changing rates of production or differential depletion between layers at the transient stage of the reservoir. As a result of this longer time, it will be necessary to shut-in a well penetrating a multilayered reservoir much longer in order to obtain useful results of any analysis.

Recent investigators[3,4,22] have utilized the conventional methods of Muskat, Miller-Dyes-Hutchinson and Horner for the interpretation of pressure build-up behavior in bounded layered reservoirs. These investigators made a significant contribution by establishing that static pressure can be obtained in the same way as in single layered reservoirs. Their studies also show that the early straight line portion of Horner and Miller-Dyes-Hutchinson plots can be used for the determination of $\bar{k}\bar{h}$ product, and the extended Muskat method can be used to compute the flow capacity and the pore volume of the reservoir.

In addition, Raghaven et al.[22] showed that it is possible to determine a permeability ratio for a given thickness ratio, using the dimensionless pressure rise.

Earlougher et al.[5] recently studied the behavior of pressure build-up in a closed square layered system, with variation in porosity, permeability and thickness ratios. In their studies, they varied the number of wells and the shape of the system. Their study also included a developed system, where there is a shut in well among producing wells. They used the principle of superposition and the exponential integral for their calculation. The results of their study show that curves of build-up obtained for layered reservoirs vary and may not necessarily identify layered reservoirs.

Hartsock[7] derived expressions that will enable the reservoir engineer to compute the permeabilities and initial pressures of the drainage areas with a dual zone producing well. He utilized the pressure drop of one zone for his derivation. All that is needed to compute the above parameters is a reliable build-up curve and either the initial pressure of one of the zones or the difference in the initial pressures of the two zones.

Apart from pressure build-up analysis, other kinds of well test procedures have also been applied to study the behavior of multiple-layered reservoirs, namely, pressure limit test and pulse test.

Kazemi[12] studied pressure build-up analysis in reservoir limit test of stratified systems. He concluded that the conventional equations used in the analysis of homogeneous single layer reservoirs and heterogeneous reservoirs with crossflow are not applicable to heterogeneous reservoirs without crossflow. He established that those equations used in pressure build-up analysis for stratified systems can also be used for reservoir limit test of stratified reservoirs.

Woods[30] made a study of pulse test in a two-zone reservoir using a single layer model. The purpose of his study was to determine what errors could be caused by using a single layer model for two layers. The conclusion of his study shows that the properties of the individual layers can be obtained using a combination of single well tests provided there is no communication between layers except at the well-bore. It was further established that apparent transmissibility is either equal or greater than true transmissibility and apparent storage is either equal or less than the total storage.

CHAPTER IV

MATHEMATICAL EQUATIONS AND THEORETICAL BUILD-UP RELATIONS

A. Basic Assumptions

Figure 1 shows schematically the model of the reservoir studied. The reservoir is divided horizontally into two layers, and there is no communication between the two zones except at the well-bore. The reservoir is overlaid and underlaid by an impermeable layer. The two layers of the reservoir are infinite, homogeneous and of uniform thickness throughout. The layers are completely saturated with a single fluid. Over the range of pressures and temperatures encountered, the fluid is of constant compressibility and viscosity. The two layers are penetrated by a single well and production is commingled. The flow of fluid is axially symmetrical and obeys Darcy's law.

For a radial system the partial differential equation defining the flow of fluid is given as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_j}{\partial r}\right) = \frac{1}{\eta_j}\frac{\partial P_j}{\partial t}\,, \qquad j = 1,2 \tag{1}$$

where $P_j(r,t)$ is the pressure drop defined as:

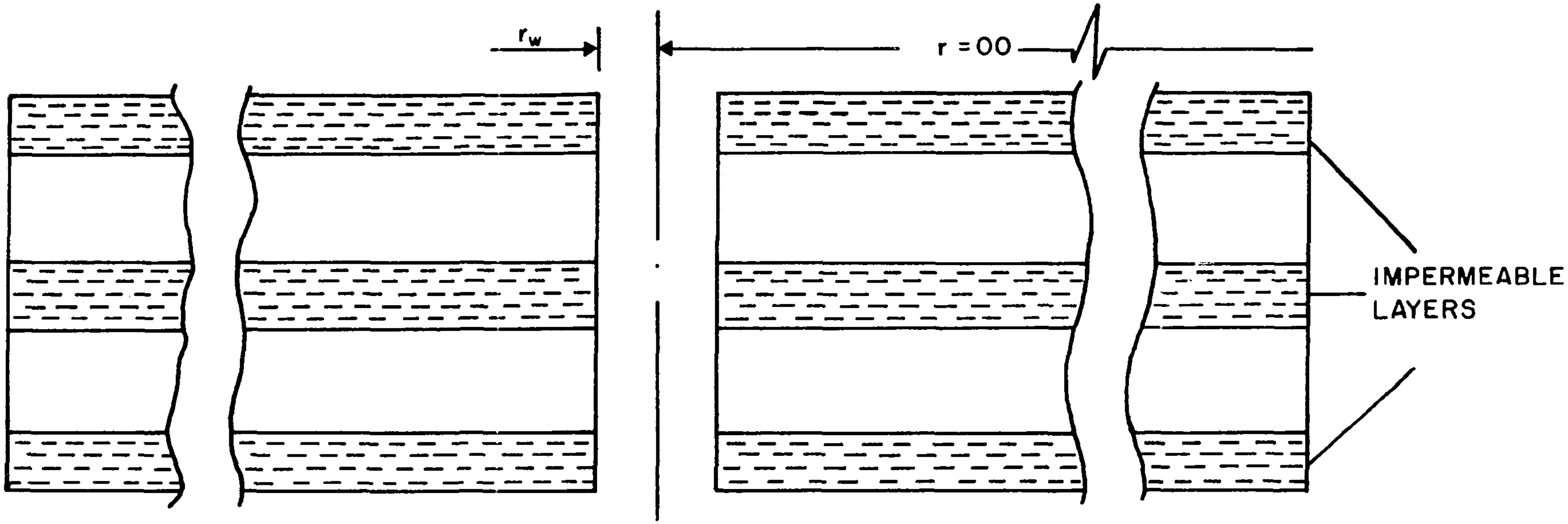

Fig. I MATHEMATICAL MODEL OF AN INFINITE TWO LAYERED RESERVOIR

$$P_j(r,t) = p_{i,j} - p_j(r,t), \qquad j = 1,2 \tag{2}$$

for the case of Hartsock's pressure drop equation, and

$$P_j(r,t) = p_i - p_j(r,t), \qquad j = 1,2 \tag{3}$$

for the case outlined by Lefkovits _et al._[14] Hartsock[7] assumed that due to the force of gravity the pressure drops for the two layers are different while Lefkovits _et al._ have neglected the effect of gravity. As a result, for the case of Lefkovits _et al._ the initial pressures in both layers are the same and the reverse for that considered by Hartsock.

For the conditions considered, equation 1 has been solved varying the boundary conditions at the well for the two layers. From the basic assumptions, it can be defined that

$$r_{w1} = r_{w2} \tag{4}$$

$$c_1 = c_2 \tag{5}$$

$$\mu_1 = \mu_2 \tag{6}$$

For simplicity of notation, Lefkovits _et al._ made the following substitutions:

$$\eta_j = \frac{k_j}{\phi_j \mu_j c_j}, \qquad j = 1,2 \tag{7}$$

$$a_j = \frac{r_{w,j}}{\sqrt{\eta_j}}, \qquad j = 1,2 \tag{8}$$

$$\beta_j = \frac{h_j k_j}{\mu_j}, \qquad j = 1,2 \tag{9}$$

and the mean permeability and porosity are defined as:

$$\bar{k} = \frac{\sum_{j=1}^{2} k_j h_j}{\bar{h}} , \quad j = 1,2 \tag{10}$$

$$\bar{\phi} = \frac{\sum_{j=1}^{2} \phi_j h_j}{\bar{h}} , \quad j = 1,2 \tag{11}$$

where

$$\bar{h} = \sum_{j=1}^{2} h_j , \quad j = 1,2 \tag{12}$$

B. Pressure Build-Up Analysis Using Hartsock's Pressure Drop Equation

In addition to the basic assumptions as given above, Hartsock took into consideration the force of gravity to obtain the pressure drop equation for the upper zone. With such an assumption, the initial pressures for the two zones are not the same. Likewise, the flowing bottom hole pressures for the two layers are not the same. He also assumed that the well has produced long enough before shut in for a build-up analysis.

i. Pressure Drop Equation

The initial and boundary conditions used in solving equation 1 are:

$$\text{At } t = 0, \quad P_j = 0 , \quad j = 1,2 \tag{13}$$

$$\text{As } r \to \infty, \quad P_j = 0 , \quad j = 1,2 \tag{14}$$

$$\text{At } r = r_{w,j} \,, \qquad p_j = P_{w,j}(t) \,, \qquad j = 1,2 \tag{15}$$

and

$$\sum_{j=1}^{2} q_j(t) = -2\pi \sum_{j=1}^{2} \beta_j \left(r \frac{\partial P_j}{\partial r} \right)_{r=r_{w,j}} = q \tag{16}$$

The solutions obtained are expressions for pressure drop equations for the upper and lower zones and are given as:

For the upper zone:

$$p_{i_1} - p_{wf_1}(t) = \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_2} (\ln t + 0.809 + \ln \bar{a}_1) + (\gamma - 1)\Delta p \right\} \tag{17}$$

and for the lower zone:

$$p_{i_2} - p_{wf_2}(t) = \frac{\frac{1}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_1} (\ln t + 0.809 + \ln \bar{a}_2) + (1 - \gamma)\Delta p \right\} \tag{18}$$

where

$$\bar{a}_j = \frac{k_j}{\phi_j \mu c r_w^2} \,, \qquad j = 1,2 \tag{19}$$

$$\gamma = \frac{\rho_o g (d_2 - d_1)}{(p_{i_2} - p_{i_1})} \tag{20}$$

and

$$\Delta p = p_{i_2} - p_{i_1} \,. \tag{21}$$

The difference between equations 17 and 18 is in the terms $(\gamma - 1)$ and $(1 - \gamma)$ respectively and also $\bar{a}_1$ and $\bar{a}_2$. Details of the derivation of equations 17 and 18 are given in Appendix A.

To obtain expressions for analyzing pressure build-up curves, the principle of superposition[16] is applied to equations 17 and 18. At the time the well is shut-in, the build-up pressure is equal to the pressure at $(t + \Delta t)$ due to flow-rate $+q$ plus pressure at Δt due to flow rate $-q$. Then equations 17 and 18 become

$$p_{i_1} - p_{ws} = \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_2} \left(\ln \frac{t + \Delta t}{\Delta t} \right) + 2(\gamma - 1)\Delta p \right\} \tag{22}$$

and

$$p_{i_2} - p_{ws} = \frac{\frac{1}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_1} \left(\ln \frac{t + \Delta t}{\Delta t} \right) + 2(1 - \gamma)\Delta p \right\} \tag{23}$$

From equations 20 and 21, it can be seen that a semilog plot of shut in pressure p_{ws} versus $(t + \Delta t)/\Delta t$ will yield a slope of

$$m = \frac{1.151\, q \, \frac{1}{\beta_1} \frac{1}{\beta_2}}{2\pi \left(\frac{1}{\beta_1} + \frac{1}{\beta_2} \right)} \tag{24}$$

Substituting equations 6, 9, 10 and 12 into equation 24 and simplifying, the following equation is obtained:

$$m = \frac{1.151\ q\mu}{2\pi \bar{k}\bar{h}} . \tag{25}$$

The intercept at the point where $(t + \Delta t)/\Delta t$ is equal to unity is given by the following equations:

$$p_1^* = p_{i_1} - 2b_1(\gamma - 1)\Delta p \tag{26}$$

and

$$p_2^* = p_{i_2} - 2b_2(1 - \gamma)\Delta p \tag{27}$$

where

$$b_1 = \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \tag{28}$$

and

$$b_2 = \frac{\frac{1}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} . \tag{29}$$

Equations 26 and 27 differ not only in terms of b_1 and b_2 but also in terms of $(\gamma - 1)$ and $(1 - \gamma)$. From equations 26 and 27 it can be seen that at the point where $(t + \Delta t)/\Delta t$ is equal to unity is not an indication of the initial pressure for an infinite two layered reservoir.

At Δt equal to unity, equations 22 and 23 can be written as:

$$p_{i_1} - p_{1\ hr} = \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_2}(\ln\ t+1) + 2(\gamma - 1)\Delta p \right\} \tag{30}$$

and

$$p_{i_2} - p_{1\,hr} = \frac{\frac{1}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_1}(\ln\, t+1) + 2(1 - \gamma)\Delta p \right\} \tag{31}$$

Subtracting equation 17 from equation 30 and equation 18 from 31, the following equations are obtained:

$$p_{1\,hr} - p_{wf_1}(t) = \frac{b_1 q}{4\pi\beta_2} \ln \frac{t+1}{t} + b_1(\gamma - 1)\Delta p - \frac{b_1 q}{4\pi\beta_2}(0.809 + \ln \bar{a}_1) \tag{32}$$

and

$$p_{1\,hr} - p_{wf_2}(t) = \frac{b_2 q}{4\pi\beta_1} \ln \frac{t+1}{t} + b_2(1 - \gamma)\Delta p - \frac{b_2 q}{4\pi\beta_1}(0.809 + \ln \bar{a}_2) \tag{33}$$

For $t >> 1$, $\frac{t+1}{t} \approx 1$. Then equations 32 and 33 simplify to:

$$p_{1\,hr} - p_{wf_1} = b_1(\gamma - 1)\Delta p - \frac{b_1 q}{4\pi\beta_2}(0.809 + \ln \bar{a}_1) \tag{34}$$

$$p_{1\,hr} - p_{wf_2} = b_2(1 - \gamma)\Delta p - \frac{b_2 q}{4\pi\beta_1}(0.809 + \ln \bar{a}_2) \tag{35}$$

ii. Application

Either equations 26 and 34 or equations 27 and 35 can be used for the analysis of pressure build-up curves. For the present illustration of the application, equations

26 and 34 are used. The following properties of a two-layered reservoir can be determined:

(1) The initial pressure of one zone can be determined if the initial pressure of the other zone is known or if Δp is known.

(2) The kh product of each zone.

Equations 26 and 34 are solved simultaneously in order to establish the above conditions. For example, if p_{i_1} is known, k_1/μ can be determined by eliminating

$$b_1(\gamma - 1)\Delta p \tag{36}$$

from both equations 26 and 34 and solving for k_1/μ. With the use of equation 24, $1/\beta_2$ can be determined. If p_{i_2} or Δp is known, replace b_1 by

$$\frac{2\pi m}{1.151\ q\ \frac{1}{\beta_2}} \tag{37}$$

and also replace k_1/μ by

$$\frac{1.151\ q}{2\pi m h_1} - \frac{\beta_2}{h_1} \tag{38}$$

in equations 26 and 34. Equations 26 and 34 are then solved to determine $1/\beta_2$ and subsequently $1/\beta_1$ and p_{i_1}. The substitution for b_1 and k_1/μ have been done using equation 24. If $p_{1\ hr}$ does not fall on the straight line portion of the semilog plot of shut-in pressure versus $(t + \Delta t)/\Delta t$, an extrapolation of the straight line portion is necessary.

iii. Determination of Skin Factor

Skin factor is a dimensionless number which expresses to what extent the permeability around the well-bore has been reduced or increased. If the permeability around the well-bore has been reduced due to infiltration of drilling mud into the formation during drilling and well completion, the skin factor is a positive dimensionless number. On the other hand, if the permeability around the well-bore has increased due to acidization or fracturing, the skin factor is a negative dimensionless number.

The skin factors for the individual layers can be determined with the use of equations 17 and 18. For the sake of completeness equations 17 and 18 are repeated.

$$p_{i_1} - p_{wf_1} = \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_2}(\ln t + 0.809 + \ln \bar{a}_1) + (\gamma - 1)\Delta p \right\} \tag{17}$$

and

$$p_{i_2} - p_{wf_2} = \frac{\frac{1}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_1}(\ln t + 0.809 + \ln \bar{a}_2) + (1 - \gamma)\Delta p \right\} \tag{18}$$

From this point only equation 17 is used for the determination of S_1 which is the skin factor for the upper zone. The same procedure can be used for the determination of S_2, which is the skin factor for the lower zone, using equation 18.

As stated by Van Everdingen, the skin factor S_1 of the upper zone relates the pressure drop in the skin to the dimensionless flow rate; that is,

$$\Delta p_{skin} = S_1\left(\frac{q_1\mu}{2\pi k_1 h_1}\right) \tag{39}$$

Equation 17 can be rewritten as:

$$p_{wf_1} = p_{i_1} - \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1}+\frac{1}{\beta_2}}\left\{\frac{q}{4\pi\beta_2}(\ln t + 0.809 + \ln \bar{a}_1) + (\gamma - 1)\Delta p\right\} \tag{40}$$

The introduction of equation 39 into equation 40 leads to:

$$p_{wf_1}(t) = p_{i_1} - \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1}+\frac{1}{\beta_2}}\left\{\frac{q}{4\pi\beta_2}\left[\ln t + 0.809 + \ln \bar{a}_1 + \frac{2S_1 q_1}{q}\,\frac{\frac{1}{\beta_1}+\frac{1}{\beta_2}}{\frac{1}{\beta_2}}\right] + (\gamma - 1)\Delta p\right\} . \tag{41}$$

Equation 40 has been reduced by the amount

$$S_1\left(\frac{q_1\mu}{2\pi k_1 h_1}\right) . \tag{39}$$

From equation 22, the build-up pressure can be expressed as:

$$p_{ws} = p_{i_1} - \frac{\frac{1}{\beta_1}}{\frac{1}{\beta_1}+\frac{1}{\beta_2}}\left\{\frac{q}{4\pi\beta_2}\left(\ln \frac{t+\Delta t}{\Delta t}\right) + 2(\gamma - 1)\Delta p\right\} \tag{42}$$

Eliminating p_{i_1} between equations 41 and 42, and substituting equations 24, 28 and 29, the following equation is obtained:

$$p_{ws} - p_{wf_1}(t) = m\left[\ln t + 0.809 + \ln \bar{a}_1 + \frac{2S_1q_1}{qb_2}\right]$$

$$+ b_1(\gamma - 1)\Delta p - m \ln\left(\frac{t + \Delta t}{\Delta t}\right) - 2b_1(\gamma - 1)\Delta p \quad (43)$$

Substituting equation 19 into equation 43 and simplifying leads to:

$$p_{ws} - p_{wf_1}(t) = m\left\{\ln\left[\frac{t\ \Delta t\ k_1}{(t + \Delta t)\phi_1\mu c r_w^2}\right] + 0.809 + \frac{2S_1q_1}{qb_2}\right\}$$

$$- b_1(\gamma - 1)\Delta p \quad (44)$$

Choosing $\Delta t = 1 << t$, then $(t + \Delta t)/t = 1$ and equation 44 becomes

$$p_{1\ hr} - p_{wf_1}(t) = m\left\{\ln \frac{k_1}{\phi\mu c r_w^2} + 0.809 + \frac{2S_1q_1}{qb_2}\right\} - b_1(\gamma - 1)\Delta p \quad (45)$$

Rearranging equation 45, an expression for the skin factor for the upper zone is:

$$S_1 = \frac{qb_2}{2q_1}\left[\frac{p_{1\ hr} - p_{wf_1}(t) + b_1(\gamma - 1)\Delta p}{m} - \ln \frac{k_1}{\phi_1\mu c r_w^2} - 0.809\right] \quad (46)$$

Similarly, an expression for the skin factor for the lower zone is obtained as:

$$S_2 = \frac{qb_1}{2q_2}\left[\frac{p_{1\,hr} - p_{wf_2}(t) + b_2(1-\gamma)\Delta p}{m} - \ln\frac{k_2}{\phi_2 \mu c r_w^2} - 0.809\right] \tag{47}$$

q_1 and q_2 can be determined by equations suggested by Lefkovits *et al.*[14] They suggested that for an infinite two-layered reservoir, the fractional production rate from each zone is equal to the kh product of each zone divided by the total kh product. For a two layered reservoir, we have

$$\frac{q_1}{q} = \frac{k_1h_1}{k_1h_1 + k_2h_2} \tag{48}$$

and

$$\frac{q_2}{q} = 1 - \frac{q_1}{q} = \frac{k_2h_2}{k_1h_1 + k_2h_2} \tag{49}$$

Equations 48 and 49 can be used to evaluate q_1 and q_2. The values of q_1 and q_2 can then be substituted into equations 46 and 47 for the determination of the skin factors.

C. Pressure Build-Up Analysis Using the Outlines of Lefkovits *et al.*[19]

In addition to the basic assumptions given above, the effect of gravity is neglected. As a result, the initial pressures and flowing bottom hole pressures are the same for both layers. With these assumptions, three cases of pressure drop equations have been considered, namely, pressure drop equation without skin and well-bore storage;

$$S_2 = \frac{qb_1}{2q_2}\left[\frac{p_{1\,hr} - p_{wf_2}(t) + b_2(1-\gamma)\Delta p}{m} - \ln\frac{k_2}{\phi_2\mu c r_w^2} - 0.809\right] \tag{47}$$

q_1 and q_2 can be determined by equations suggested by Lefkovits et al.[14] They suggested that for an infinite two-layered reservoir, the fractional production rate from each zone is equal to the kh product of each zone divided by the total kh product. For a two layered reservoir, we have

$$\frac{q_1}{q} = \frac{k_1h_1}{k_1h_1 + k_2h_2} \tag{48}$$

and

$$\frac{q_2}{q} = 1 - \frac{q_1}{q} = \frac{k_2h_2}{k_1h_1 + k_2h_2} \tag{49}$$

Equations 48 and 49 can be used to evaluate q_1 and q_2. The values of q_1 and q_2 can then be substituted into equations 46 and 47 for the determination of the skin factors.

C. Pressure Build-Up Analysis Using the Outlines of Lefkovits et al.[19]

In addition to the basic assumptions given above, the effect of gravity is neglected. As a result, the initial pressures and flowing bottom hole pressures are the same for both layers. With these assumptions, three cases of pressure drop equations have been considered, namely, pressure drop equation without skin and well-bore storage;

pressure drop equation with skin effect; and pressure drop equation with well-bore storage effect.

i. Pressure Drop Equation Without Skin and Well-bore Storage Effects

A detailed derivation of the pressure drop equation for an infinite two-layered reservoir without skin and well-bore storage effects is shown in Appendix B. Equation 1 has been solved with the following initial and boundary conditions:

At the initial time, the pressure drop is zero in the two layers; that is,

$$P_j = 0 \quad \text{at } t = 0, \qquad j = 1,2 \tag{13}$$

As the well bore radius approaches infinity, the pressure drop is equal to zero in both layers; that is,

$$P_j = 0 \quad \text{as } r \to \infty, \qquad j = 1,2 \tag{14}$$

At the well the pressure drops in all the layers are equal since the pressure in all layers at the well are also equal; that is,

$$P_j = P_{wf}(t) \quad \text{at } r = r_w, \quad j = 1,2 \tag{15}$$

The production rate which is a combined total from the two layers is constant; that is,

$$\sum_{j=1}^{2} q_j(t) = -2\pi \sum_{j=1}^{2} \beta_j \left(r \frac{\partial P_j}{\partial r} \right)_{r=r_{w,j}} = q \tag{16}$$

With these boundary conditions, equation 1 has been solved and the pressure drop obtained is:

$$p_i - p_{wf} = \frac{q\mu}{4\pi\bar{k}\bar{h}} \{\ln \gamma t - Q\} \qquad (50)$$

where

$$Q = \frac{k_1 h_1 \ln\left(\frac{\gamma^2 \phi_1 \mu c r_w^2}{4k_1}\right) + k_2 h_2 \ln\left(\frac{\gamma^2 \phi_2 \mu c r_w^2}{4k_2}\right)}{k_1 h_1 + k_2 h_2} \qquad (51)$$

γ is Eulers constant, $\gamma = 1.78$;

$$\bar{\gamma} = \ln \gamma = 0.5772 \qquad (52)$$

The dimensionless pressure drop is defined as

$$\frac{2\pi\bar{k}\bar{h}}{q\mu} (P_i - p_{wf}) = P_D(t_D) \qquad (53)$$

where t_D is the dimensionless time based on r_w and is given as

$$t_D = \frac{\bar{k}t}{\bar{\phi}\mu c r_w^2} \qquad (54)$$

Multiplying equation 50 by $2\pi\bar{k}\bar{h}/q\mu$, a dimensionless pressure drop for an infinite two layered reservoir is obtained; that is,

$$\frac{2\pi\bar{k}\bar{h}}{q\mu} (P_i - p_{wf}) = \frac{1}{2} \{\ln(\gamma\bar{\sigma}t_D) - Q\} \qquad (55)$$

where

$$\bar{\sigma} = \frac{\bar{\phi}\mu c r_w^2}{\bar{k}} . \qquad (56)$$

Equation 55 is used for the analysis of pressure build-up for the case of an infinite two-layered reservoir, where the force of gravity is neglected.

ii. Pressure Drop Equation With Skin Effect

A detailed derivation of the equation for pressure drop with the effect of skin is shown in Appendix C. The solution for equation 1 is sought with the boundary conditions similar to those for a pressure drop equation without skin and well-bore storage effects with the exception of the boundary condition at the well.

According to Lefkovits *et al.*[14] the skin factor for any layer can be stated as

$$S_j = \frac{P_{wf} - P_{fj}}{q_j(t)\mu_j/2\pi k_j h_j} \tag{57}$$

Rearranging equation 57 the boundary condition at the well is obtained as:

$$P_{wf}(t) = P_{fj} + \frac{S_j q_j(t)\mu_j}{2\pi k_j h_j}, \qquad j = 1,2 \tag{58}$$

For this case, the well-bore pressure drop is given as:

$$p_i - p_{wf} = \frac{q\mu}{4\pi\bar{k}\bar{h}}\{\ln \gamma t + Q_S\} \tag{59}$$

where

$$Q_S = \frac{k_1h_1\left[2S_1 - \ln\left(\frac{\gamma^2\phi_1\mu c r_w^2}{4k_1}\right)\right] + k_2h_2\left[2S_2 - \ln\left(\frac{\gamma^2\phi_2\mu c r_w^2}{4k_2}\right)\right]}{k_1h_1 + k_2h_2} \tag{60}$$

With the exception of the skin factors S_1 and S_2 for the upper and lower zones respectively, equation 51 is similar to 60.

Multiplying equation 59 by $2\pi\bar{k}\bar{h}/q\mu$, a dimensionless pressure drop for an infinite two layered reservoir with skin effect is obtained; that is,

$$\frac{2\pi\bar{k}\bar{h}}{q\mu}(p_i - p_{wf}) = \frac{1}{2}[\ln(\gamma\bar{\sigma}t_D) + Q_S] \qquad (61)$$

Equation 61 is used for the analysis of pressure build-up for the case of an infinite two-layered reservoir with skin effect.

iii. Pressure Drop Equation With the Effect of Well-bore Storage

In this study, the method used for investigating the effect of well-bore storage is that of flow into casing and tubing with a loading constant, $\bar{\beta}$. Van Everdingen[29] and Hurst[9] observed that in many cases the formation (or sand face) flow rate can be approximated by a formula of the type:

$$q_{sf} = q(1 - e^{-\alpha t}) \qquad (62)$$

where q is the constant flow rate from the two layers and α is a constant whose dimension is given by $1/t_D$.

Equation 62 shows that the formation flow rate starts from zero and increases exponentially until it gets to the constant rate q. A detailed derivation of the pressure drop equation for the case of a variable flow rate expressed by equation 62 is shown in Appendix D. After the pressure drop equation has been derived for a constant flow rate q, the principle of superposition[16] is applied to obtain the pressure drop equation for a variable flow rate. For the variable flow rate expressed by equation 62, the pressure drop equation obtained for an infinite two layered reservoir is given as:

$$p_i - \bar{\bar{p}}_{wf} = P_{wf}(t) - \frac{q\mu e^{-\alpha t}}{4\pi\bar{k}\bar{h}}[E_i(\alpha t) - \ln \alpha - Q] \tag{63}$$

where $P_{wf}(t)$ is the pressure drop for the case without skin and well-bore storage effects, described by equation 50; that is,

$$P_{wf}(t) = p_i - p_{wf} = \frac{q\mu}{4\pi\bar{k}\bar{h}} [\ln \gamma t - Q] \tag{50}$$

and

$$E_i(\alpha t) = \int_{-\infty}^{\alpha t} (e^u/u)\, du \tag{64}$$

is the exponential integral whose numerical values are given in Tables of Sine, Cosine and Exponential Integrals.[27]

Van Everdingen[29] showed that

$$\alpha\bar{\phi}\mu c r_w^2/\bar{k} = \bar{\beta} \tag{65}$$

and

$$\bar{\beta} t_D = \alpha t \tag{66}$$

From equation 65 and expression for α is obtained as:

$$\alpha = \frac{\bar{\beta}\bar{k}}{\bar{\phi}\mu c r_w^2} \tag{67}$$

Substituting equation 56 into 67 leads to:

$$\alpha = \bar{\beta}/\bar{\sigma} \tag{68}$$

Multiplying equation 63 by $2\pi\bar{k}\bar{h}/q\mu$ and substituting equations 66 and 69, equation 63 reduces to:

$$\bar{\bar{P}}_D(t_D) = P_D(t) - \frac{1}{2} e^{-\bar{\beta} t_D}[E_i(\bar{\beta} t_D) - \ln \bar{\beta}/\bar{\sigma} - Q] \tag{69}$$

where $P_D(t_D)$ is described by equation 53, and

$$\bar{\bar{P}}_D(t_D) = \frac{2\pi\bar{k}\bar{h}}{q\mu}(p_i - \bar{\bar{p}}_{wf}) \tag{70}$$

For a single layered reservoir, Q can be expressed as

$$Q_{SL} = \ln\left(\frac{\gamma^2 \phi\mu c r_w^2}{4k}\right) \tag{71}$$

Equation 71 can further be expressed as

$$Q_{SL} = 2\bar{\gamma} - \ln 4 + \ln \bar{\sigma} \tag{72}$$

where

$$\bar{\gamma} = \ln \gamma = 0.57722 \tag{52}$$

and

$$\bar{\sigma}_{SL} = \frac{\phi\mu c r_w^2}{k} \tag{73}$$

for a single layered reservoir.

Substituting equation 72 into the second term on the right of the equality sign of equation 69, the following equation is obtained:

$$\frac{1}{2}\, e^{-\bar{\beta} t_D}[E_i(\bar{\beta} t_D) - \ln \bar{\beta} - 2\gamma + \ln 4] \tag{74}$$

Equation 74 is the same as the second term on the right of the equality sign of equation 6 of Van Everdingen.[29]

CHAPTER V

RESULTS AND DISCUSSION

In the present study the effects of the parameters of an infinite two layered reservoir were investigated. The effects of the variation of permeability and thickness ratios were investigated using the Hartsock[7] and Lefkovits *et al.*[14] methods. In addition, the effects of skin and well-bore storage were investigated using the Lefkovits *et al.*[14] method. All the equations used for these investigations were derived in this study and they are shown in Appendices A, B, C and D.

A. The Effects of Permeability and Thickness Ratios

i. Hartsock's Method

This is the method where the force of gravity between the two layers has been taken into consideration in deriving equations to be used for the determination of the parameters for an infinite two layered reservoir. These equations have also been used to study the effects of permeability and thickness ratios.

In practical units, equation 22 can be written as:

$$p_{ws} = p_i - m \log_{10} \frac{t + \Delta t}{\Delta t} - 2\, b_1(\gamma - 1)\Delta p \qquad (22a)$$

where

$$m = \frac{162.6\ q_o\beta\ \frac{\mu}{k_1h_1}\ \frac{\mu}{k_2h_2}}{\frac{\mu}{k_1h_1} + \frac{\mu}{k_2h_2}} \tag{24a}$$

or equivalently

$$m = \frac{162.6\ q_o\mu\beta}{\bar{k}\bar{h}} \tag{25a}$$

which is the slope per cycle of the straight-line portion of the build-up curve plotted on a base ten semilog graph paper;

$$b_1 = \frac{\frac{\mu}{k_1h_1}}{\frac{\mu}{k_1h_1} + \frac{\mu}{k_2h_2}} \tag{28a}$$

is a dimensionless number, and

$$\gamma = \frac{0.433\ \rho_o(d_2 - d_1)}{(p_{i_2} - p_{i_1})} \tag{20a}$$

is also a dimensionless number.

Equation 22a is used in generating pressure build-up curves. The build-up curves obtained can be described as ideal curves. Such curves are shown in Figures 2, 3, 4 and 5. An ideal build-up curve is a straight line curve without the effects of skin and well-bore storage. Such ideal curves are similar to those of single layered reservoirs, except for the slope and intercept.

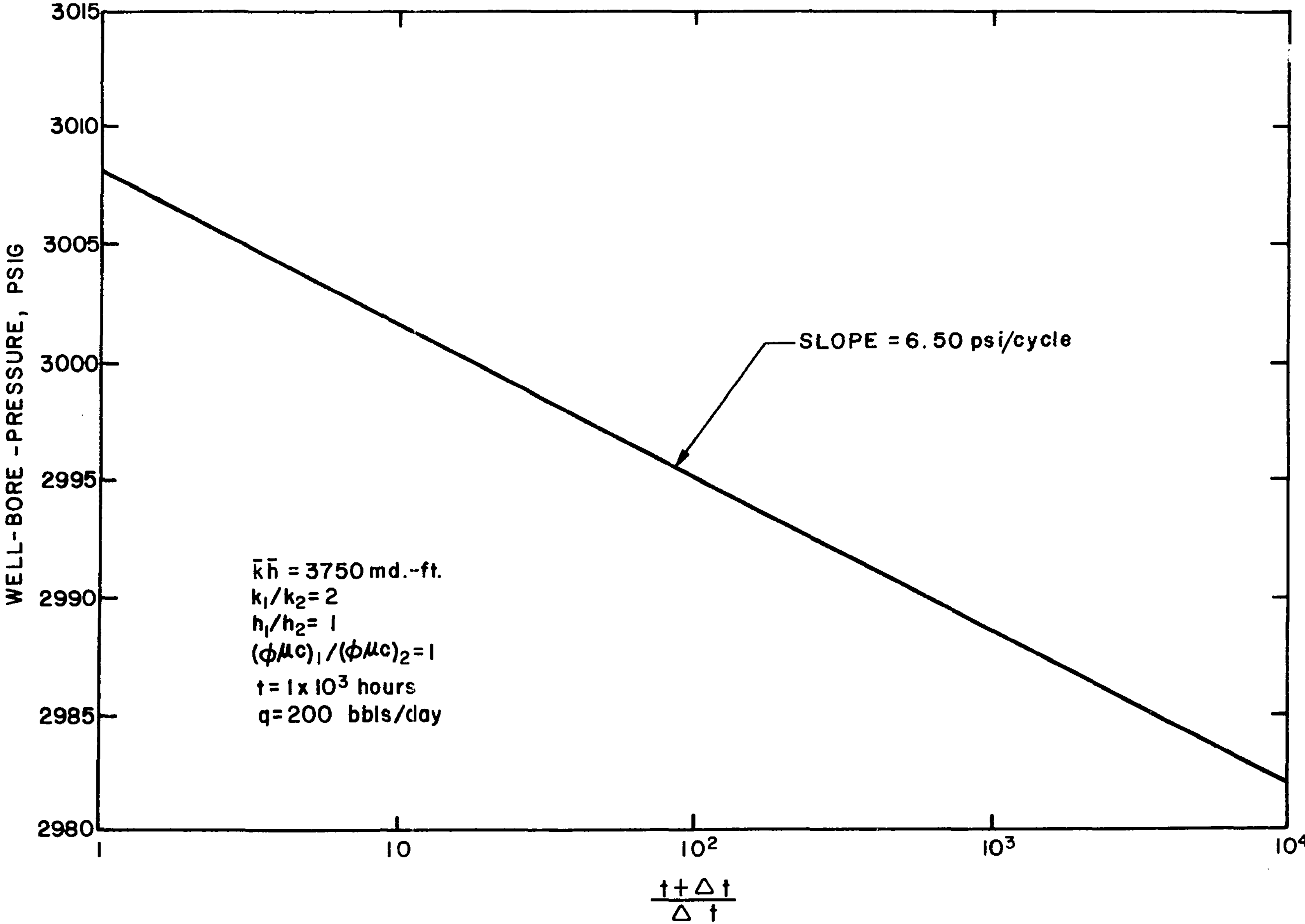

Fig. 2 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 22a.

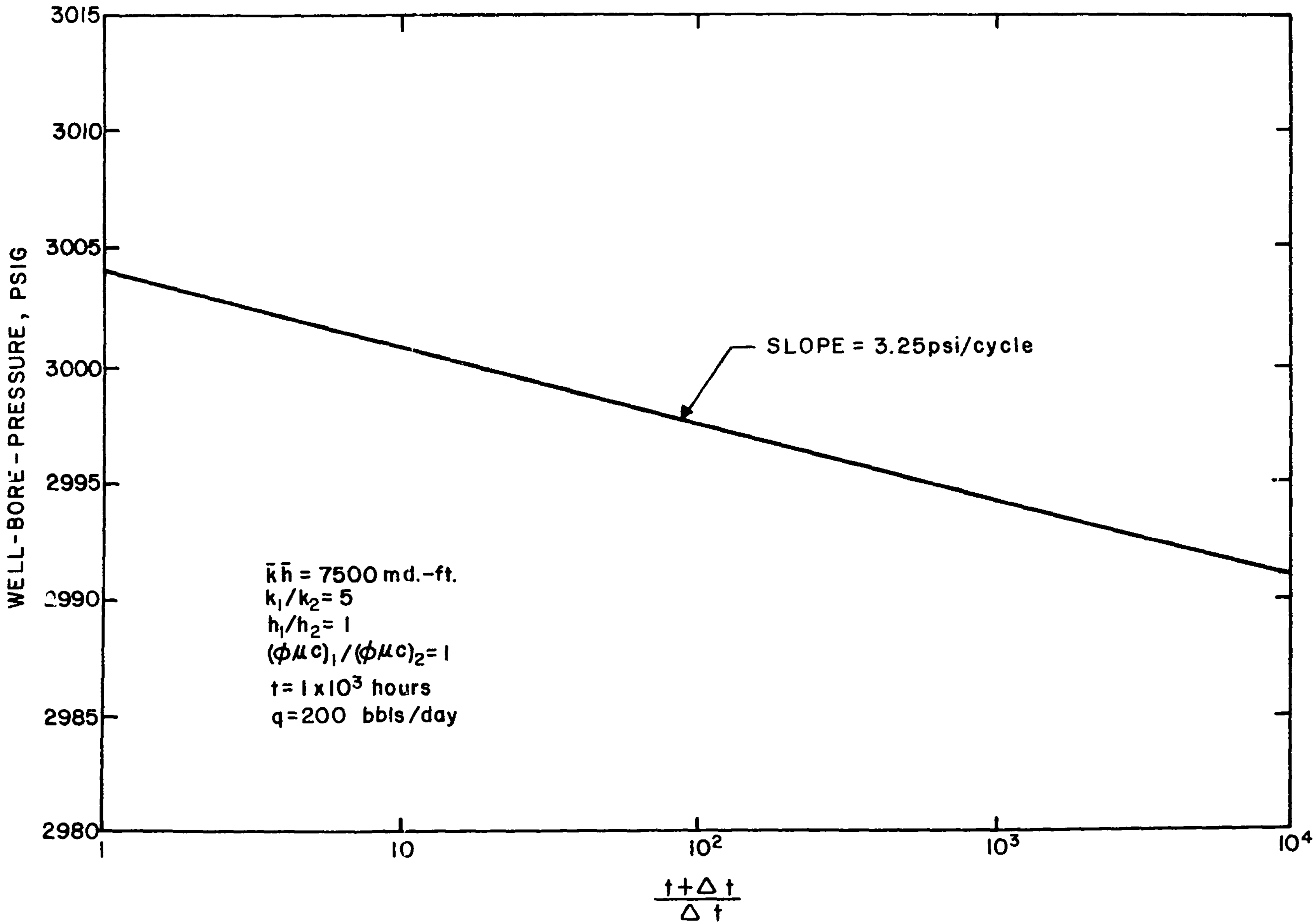

Fig. 3 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 22a.

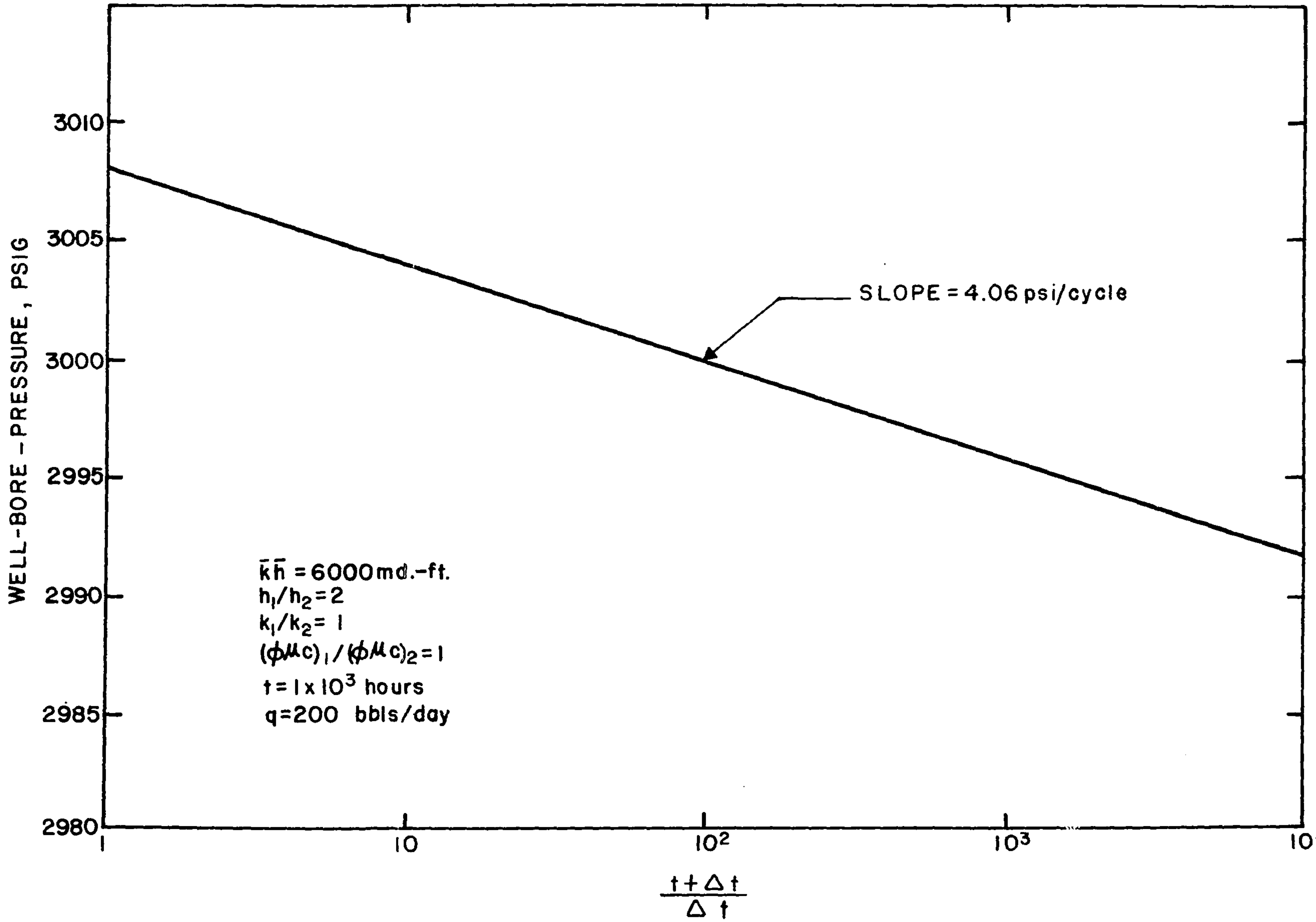

Fig. 4 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 22a.

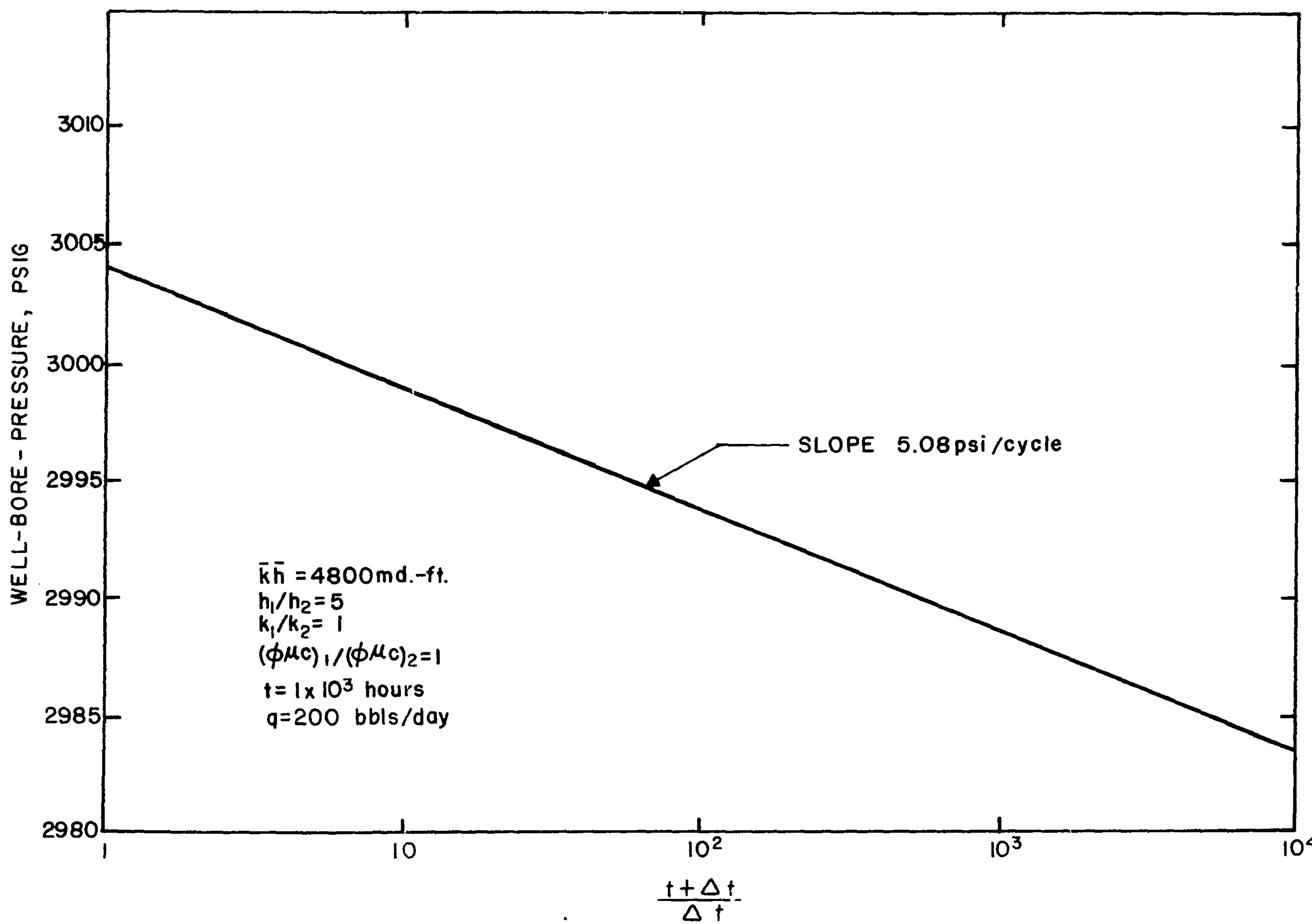

Fig. 5 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 22a.

In this study, it is observed that the slope m of a build-up curve for an infinite two layered reservoir depends more on the $\bar{k}\bar{h}$ product and not on the permeability ratio nor thickness ratio. From tables 2 and 3 it can be seen that it is the $\bar{k}\bar{h}$ product that affects the value of the slope and not the permeability nor the thickness ratio. The same observation can be made with Figure 6. From these observations it can be concluded that with an increase in the $\bar{k}\bar{h}$ product the smaller the slope and the reverse is observed with a decrease in $\bar{k}\bar{h}$ product.

The intercept of a semilog plot of shut-in pressure p_{ws} versus $(t + \Delta t)/\Delta t$ is given as:

$$p_1^* = p_{i_1} - 2b_1(\gamma - 1)\Delta p \tag{26}$$

where b_1 is described by equation 28a, γ by equation 20a and Δp by equation 21. From equation 28a, it can be seen that b_1 will depend on the sum of μ/k_1h_1 and μ/k_2h_2. The greater the sum, the smaller b_1, the greater the intercept. For the sum of μ/k_1h_1 and μ/k_2h_2 to increase, the kh product of the individual layers have to be small. Therefore, the smaller the capacity of the individual layers the greater the intercept.

In addition to the kh product of the individual layers, the term $\gamma - 1$ in equation 26 also affects the intercept. If $\gamma - 1$ is greater than zero, that is $(\gamma - 1) > 0$, the smaller the intercept. On the other hand, if $(\gamma - 1) < 0$, the greater the intercept. In the present study, the values of $\gamma - 1$ obtained are less than zero and as a result the values of the intercept are greater than 3000 psi which is the assumed initial pressure of the upper zone.

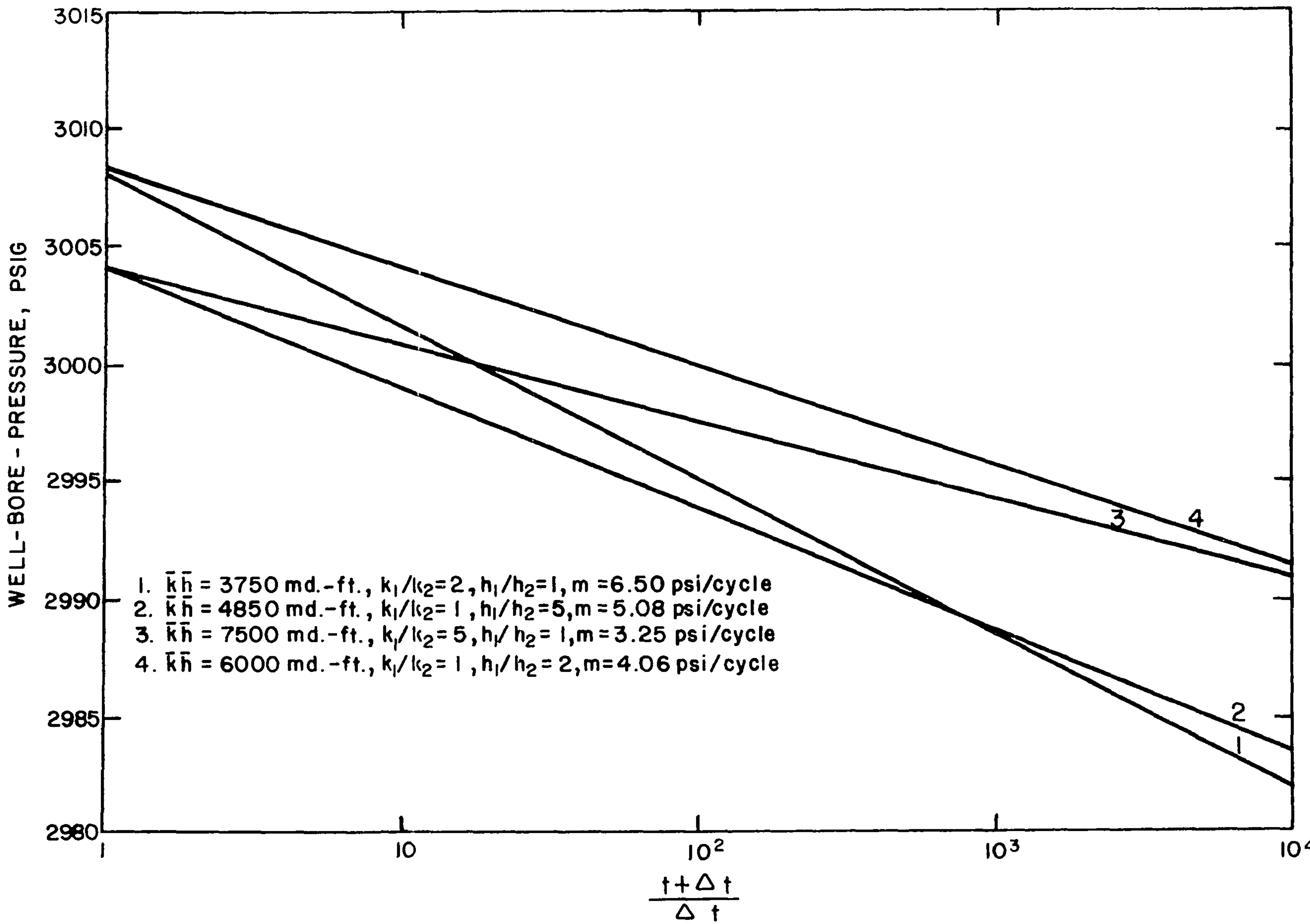

Fig. 6 PLOTS FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR SHOWING THE EFFECT OF $\bar{k}\bar{h}$ PRODUCT, USING EQUATION 22a.

In practical units equation 23 can be written as:

$$p_{ws} = p_{i_2} - m \log_{10} \frac{t + \Delta t}{\Delta t} - 2b_2(1 - \gamma)\Delta p \qquad (23a)$$

The slope m of equation 23a is the same as the slope of equation 22a described by equation 24a and 25a. So all conditions as discussed above for the slope of equation 22a will hold for equation 23a.

If equation 23a is used for the analysis of pressure build-up, then the values of the intercept,

$$p_2^* = p_{i_2} - 2b_2(1 - \gamma)\Delta p \qquad (27)$$

will depend on

$$b_2 = \frac{\frac{\mu}{k_2h_2}}{\frac{\mu}{k_1h_1} + \frac{\mu}{k_2h_2}} \qquad (29a)$$

and the term $1 - \gamma$.

Similarly, as discussed above with regards to b_1, the value of b_2 will depend on the sum of $\mu/k_1h_1 + \mu/k_2h_2$ except for the fact that the numerator of b_2 is μ/k_2h_2 unlike the numerator for b_1 which is μ/k_1h_1.

Unlike the term $\gamma - 1$ as discussed above with regards to equation 22a, the intercept (equation 27) for equation 23a will depend on the term $1 - \gamma$. If $(1 - \gamma) > 0$, the intercept value will be less. On the other hand, if $(1 - \gamma) < 0$, the intercept value will be high. In the present study, with the use of equation 23a, the intercept at

$(t + \Delta t)/\Delta t = 1$ is always less than 3030 psi which is the assumed initial pressure of the lower zone. This fact is illustrated graphically in Figures 7, 8 and 9. As a result, the intercept values obtained in this study either by using equation 22a or 23a lie between the initial pressures of the upper and lower zones.

ii. Lefkovits <u>et al.</u>[14] Method

This is the method where the force of gravity between the two layers has been neglected in the derivation of the equations used in studying the effects of permeability and thickness ratios; skin and well-bore storage for an infinite two layered reservoir.

The analysis used in this study for this method has been done with the use of dimensionless pressure and time. The dimensionless shut-in pressure used for this analysis is given as:

$$\frac{2\pi \bar{k}\bar{h}}{q\mu} (p_i - p_{ws}) = P_D(t + \Delta t)_D - P_D(\Delta t_D) \tag{75}$$

All that is needed for a semilog plot of dimensionless shut-in pressure P_{ws} versus $(t + \Delta t)/\Delta t$ is a table of dimensionless well-bore pressure and time. For an infinite two layered reservoir with the variation of permeability and thickness ratios, dimensionless well bore pressure and time are given in tables 4 and 5 for the case without skin and without well-bore storage effects. From these tables and with equation 75, semilog plots of dimensionless shut-in

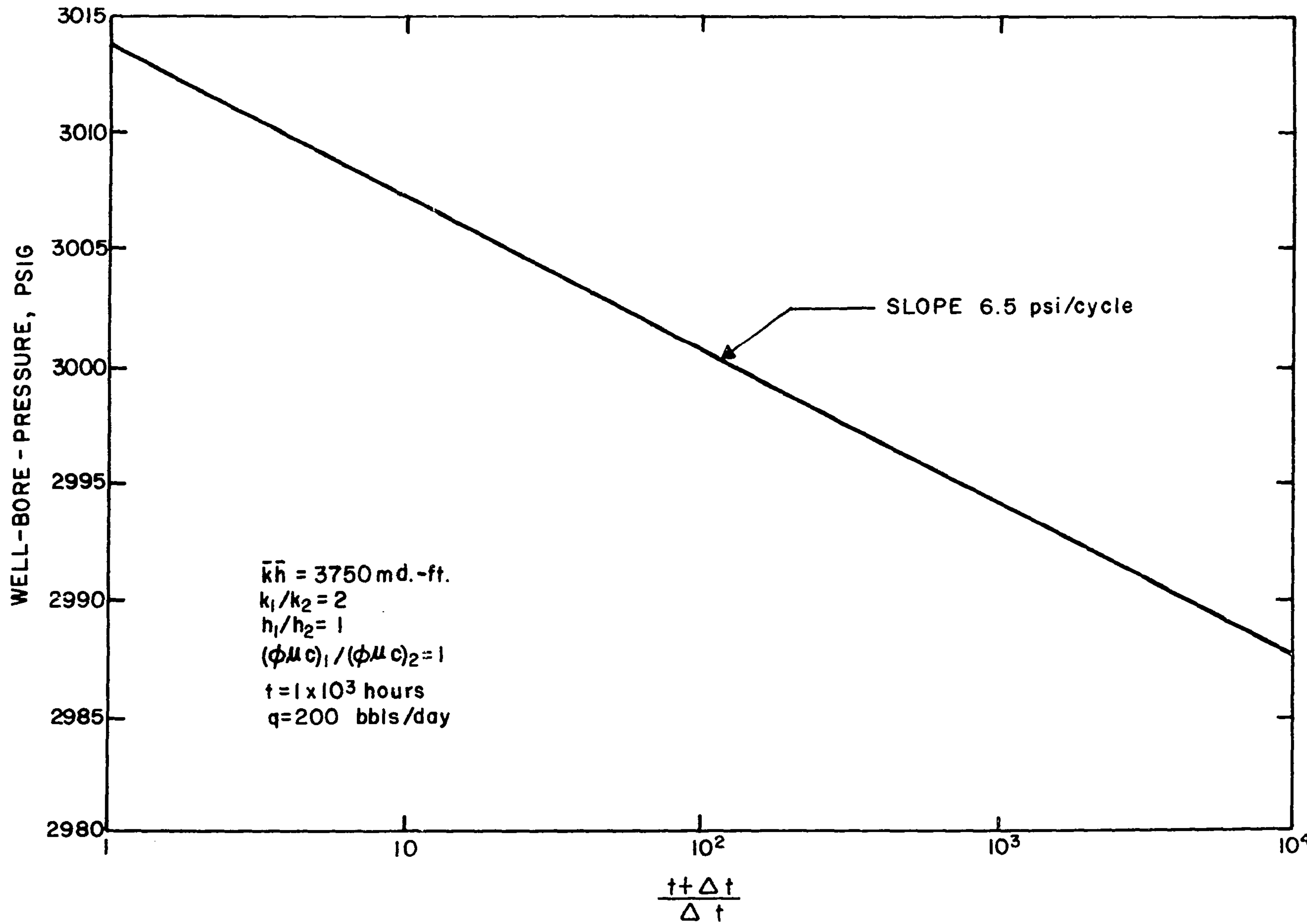

Fig. 7 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 23a.

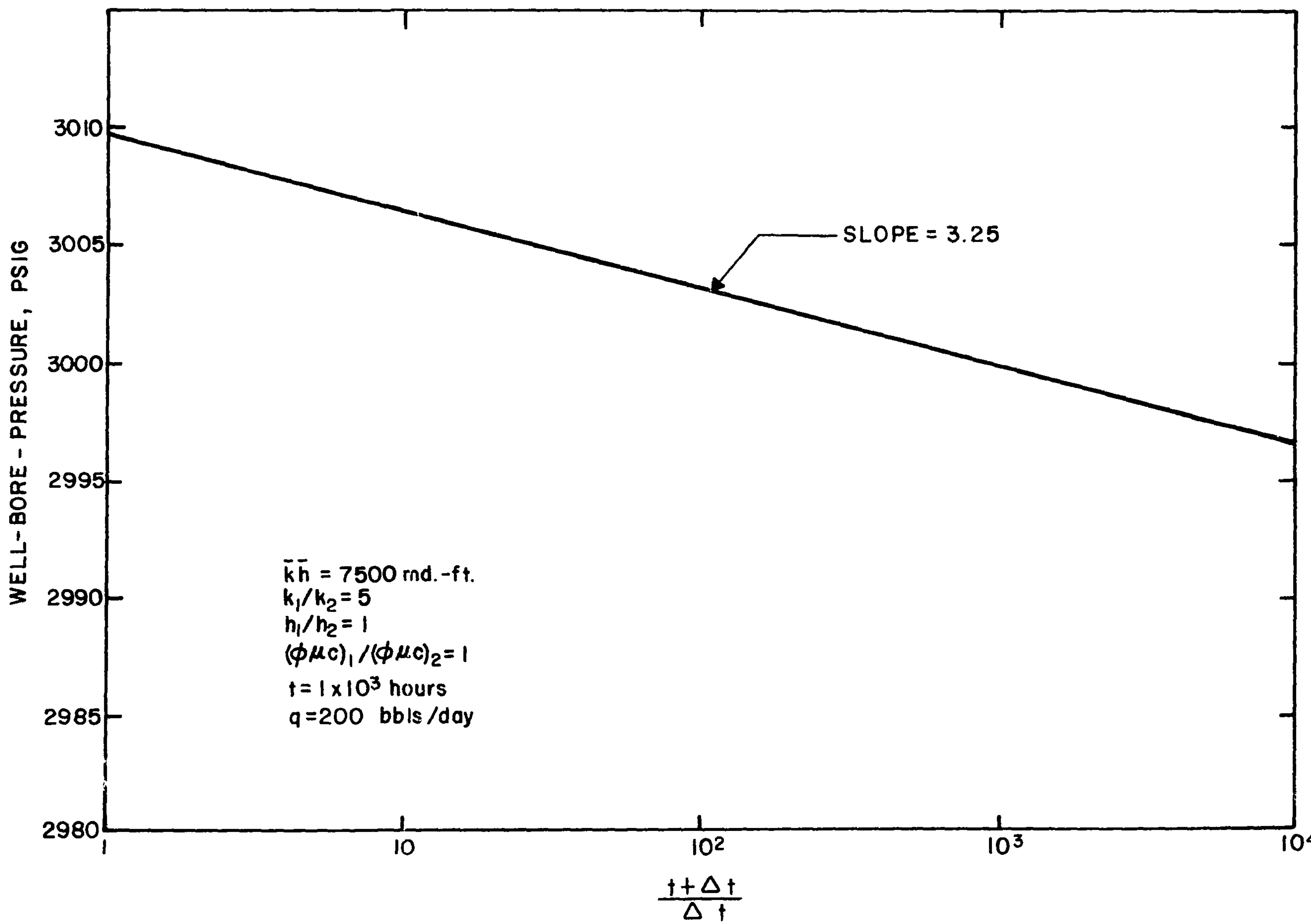

Fig. 8 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 23a.

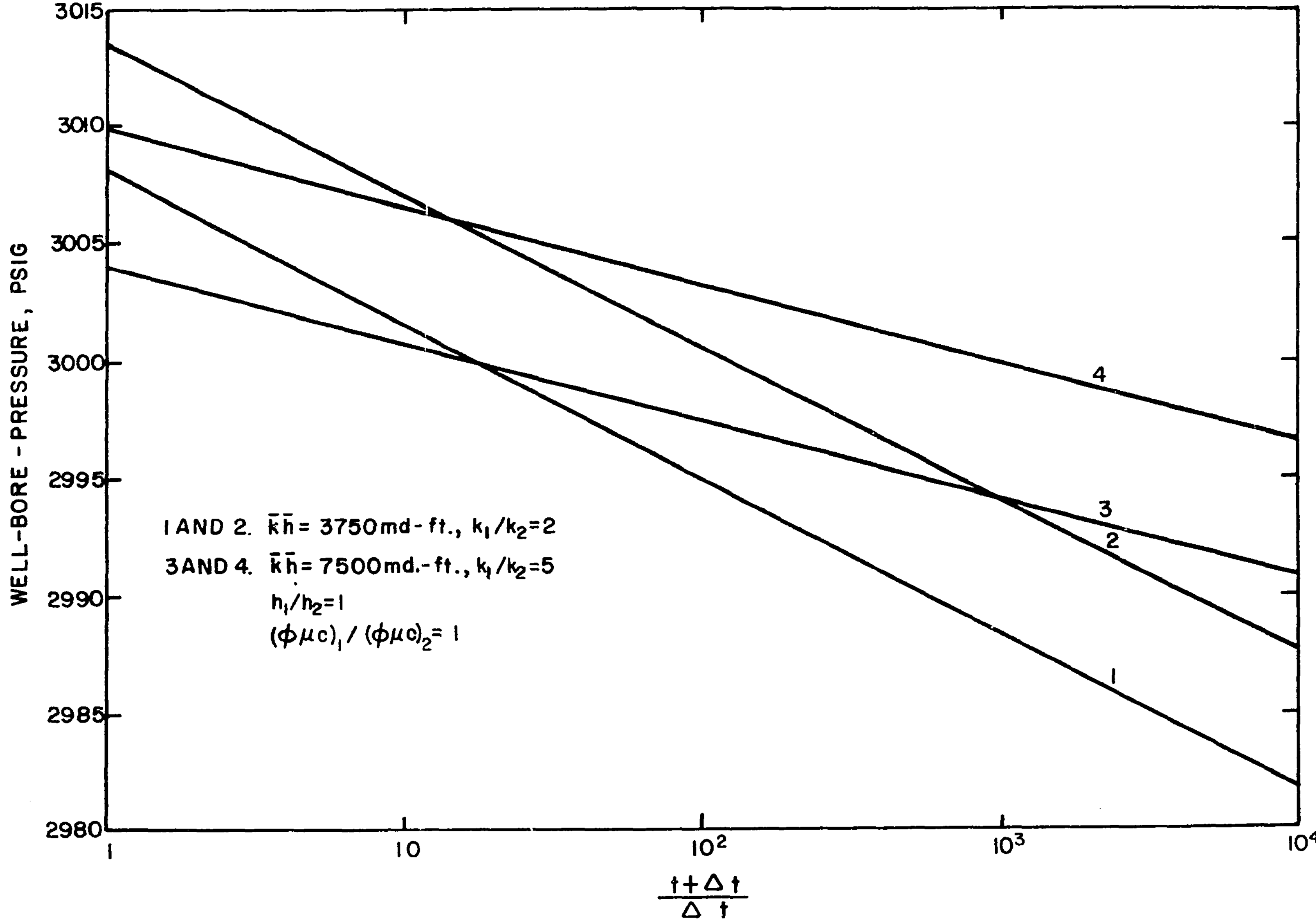

Fig. 9 1 AND 3 ARE PLOTS USING EQUATION 22a.
2 AND 4 ARE PLOTS USING EQUATION 23a.

pressure versus $(t + \Delta t)/\Delta t$ have been produced as shown in Figures 10, 11, 12 and 13. These plots are straight lines of ideal pressure build-up curves without the effects of skin and well-bore storage. The intercepts of these plots at $(t + \Delta t)/\Delta t = 1$ are all zero. This shows that at an infinite shut-in time, the shut-in pressure is equal to the initial pressure of an infinite two layered reservoir with both layers having the same initial pressures.

The slope of all the plots is 1.151. Thus a plot of the shut in pressure versus $(t + \Delta t)/\Delta t$ on a semilog paper of base 10 will yield a slope of

$$m = \frac{162.6\ q\mu\beta}{\bar{k}\bar{h}} \tag{25a}$$

Similar to Hartsock's[7] method described above, the slope for the case where the initial pressures in both layers are the same for an infinite two layered reservoir, will depend on the $\bar{k}\bar{h}$ product and not the permeability nor thickness ratios.

In this study, it has been shown that under certain conditions, an infinite two layered reservoir can behave like a single layered reservoir. Repeating equation 51 we have,

$$Q = \frac{k_1 h_1 \ln\left(\dfrac{\gamma^2 \phi_1 \mu c r_w^2}{4k_1}\right) + k_2 h_2 \ln\left(\dfrac{\gamma^2 \phi_2 \mu c r_w^2}{4k_2}\right)}{k_1 h_1 + k_2 h_2} \tag{51}$$

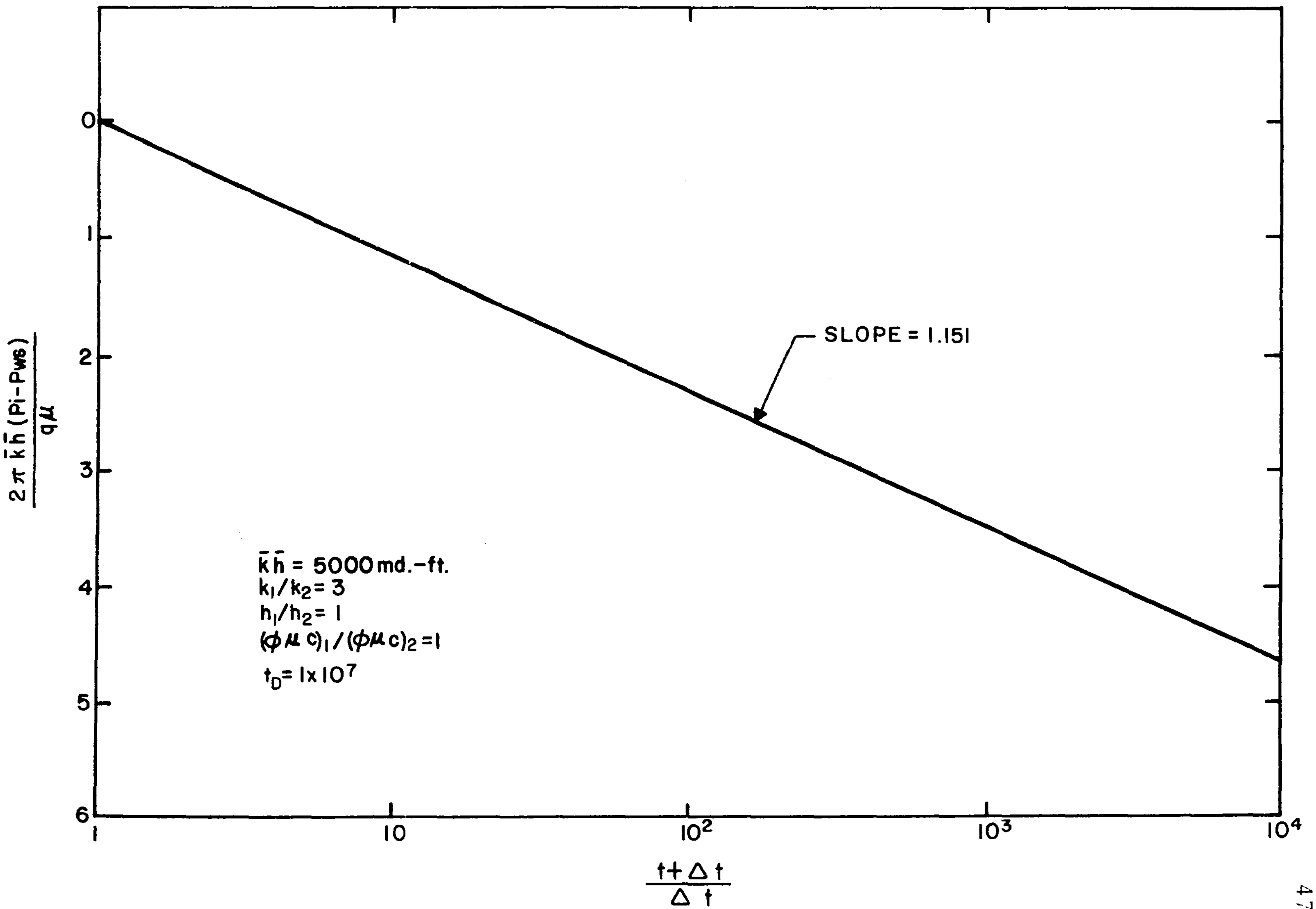

Fig. 10 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 55

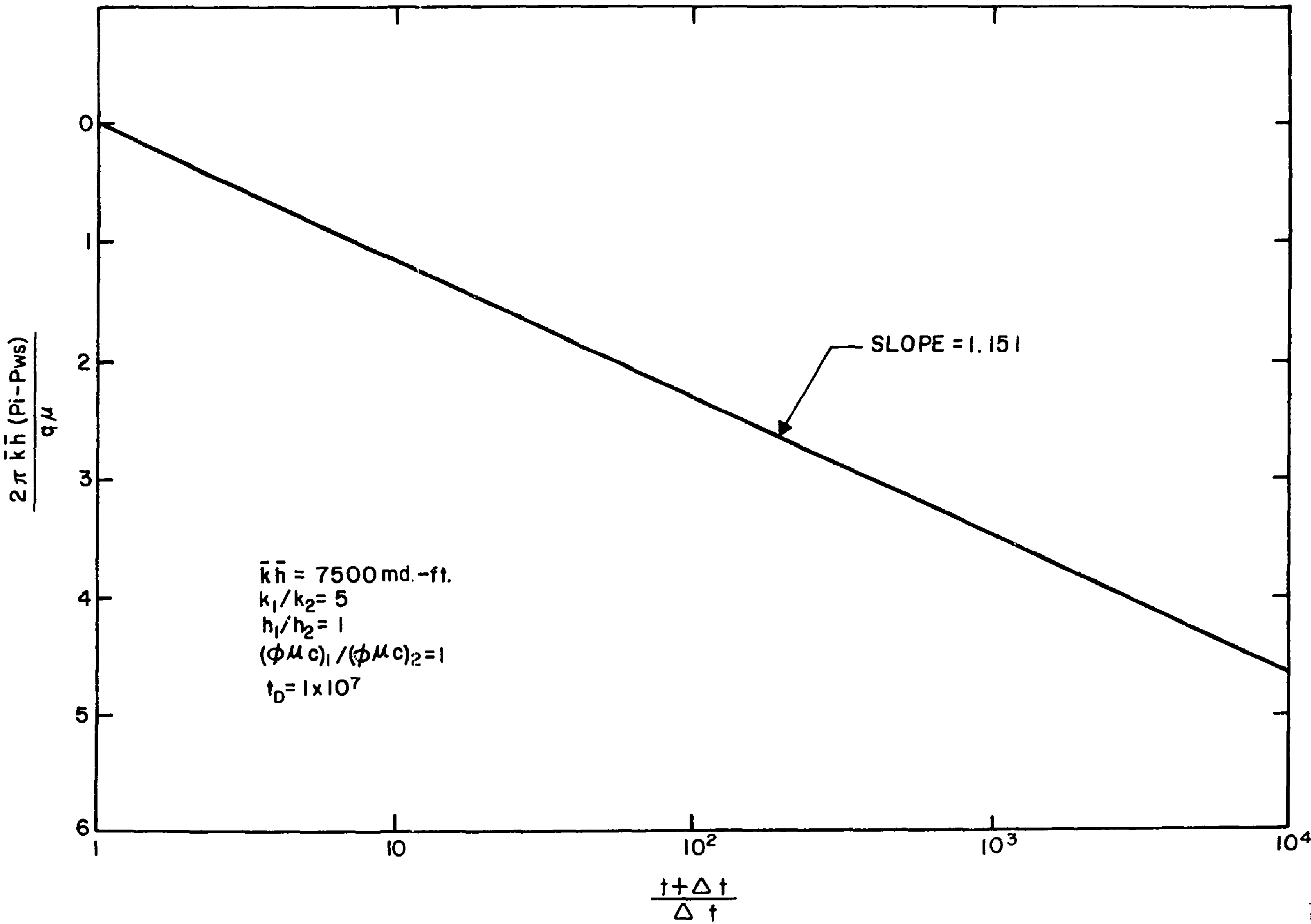

Fig. 11 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 55

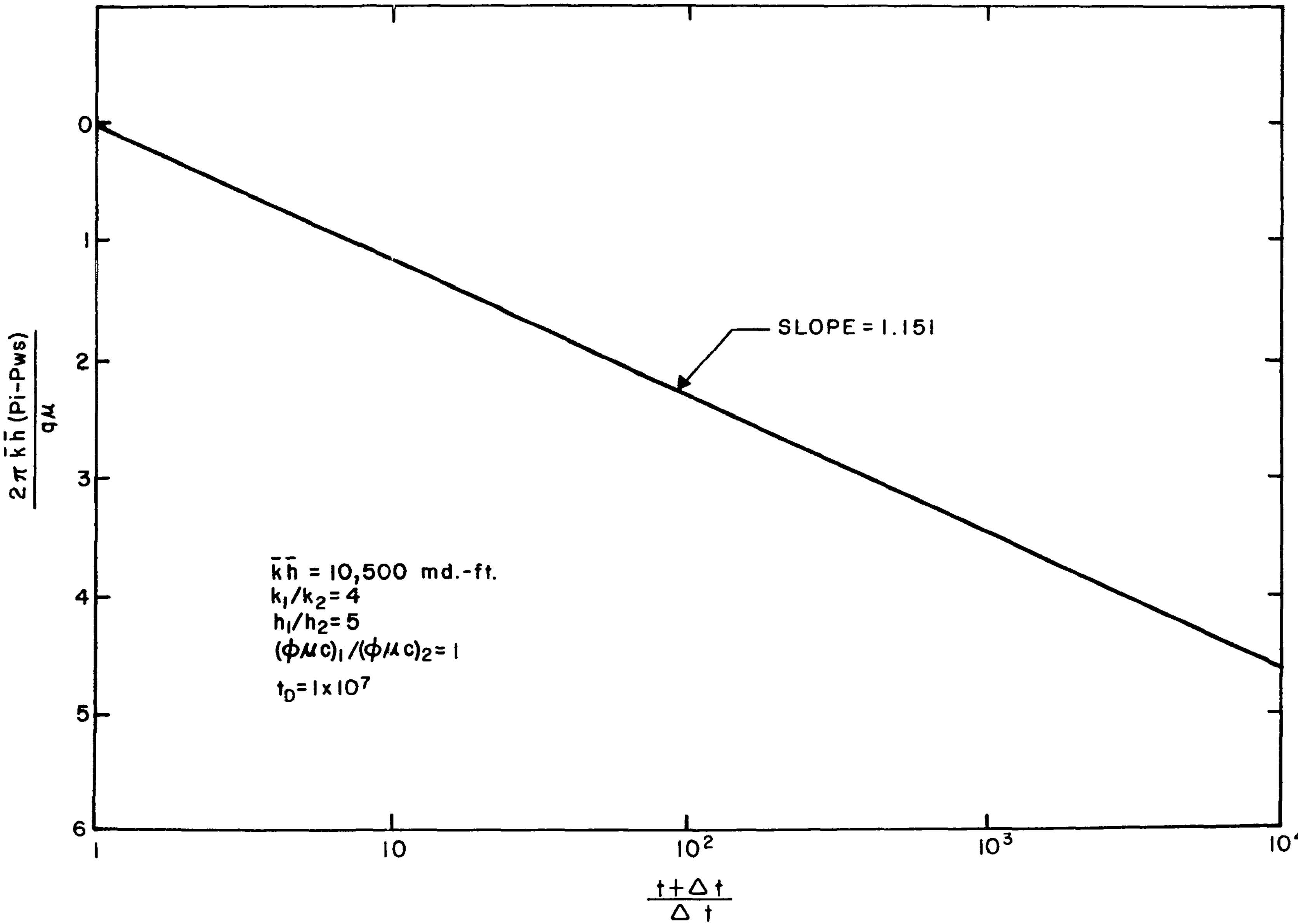

Fig. 12 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 55

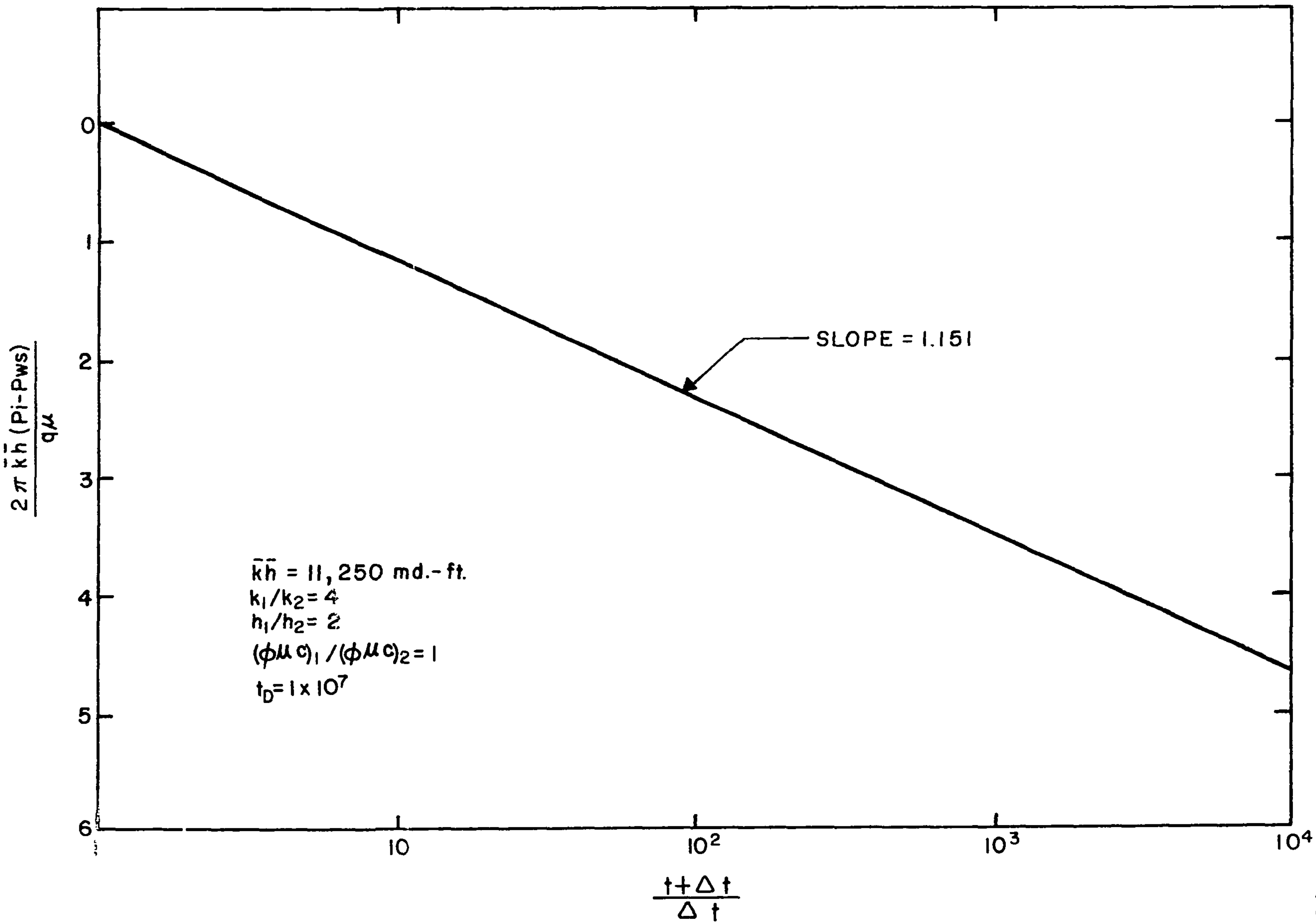

Fig. 13 PLOT FOR A CONSTANT RATE WELL IN AN INFINITE TWO LAYERED RESERVOIR USING EQUATION 55

If the following conditions hold, that is

$$r_{w1} = r_{w2} \tag{4}$$

$$C_1 = C_2 \tag{5}$$

$$\mu_1 = \mu_2 \tag{6}$$

$$k_1 = k_2 \tag{76}$$

$$\phi_1 = \phi_2 \tag{77}$$

Q reduces to:

$$Q = \ln\left(\frac{\gamma^2 \phi\mu c r_w^{\ 2}}{4k}\right) \tag{78}$$

From equation 78 it is obvious that if the above conditions hold, pressure drop from a two layered reservoir will be similar to that for a single layered reservoir irrespective of the thickness ratio, provided the initial pressures of both layers are the same. As a result, pressure drop can only depend on the thickness ratio if there is a permeability or porosity contrast of the two layers.

B. The Effects of Producing Time and Constant Production Rate

In this study it is observed that the effect of producing time is not noticeable in the semilog plots of shut-in pressure versus $(t + \Delta t)/\Delta t$ for an infinite two layered reservoir. The only effect that producing time has is that it is necessary to shut-in the well for a long time to have a good build-up.

Usually the production rate is kept constant before a well is shut-in for pressure build-up. In any case, the constant rate of production before shut-in for pressure build-up will decrease with increased drainage radius. In this study, it is assumed that four shut-in pressure build-ups were made at different times during the life of a well. At each time there is a decline in the constant production rate before shut-in. For the case shown in Figure 14, it is assumed that the decline is exponential. The curves show that with the increase in life of the reservoir, the shut-in time required for a build-up also increases. All the curves have the same intercept at an infinite shut-in time, that is at $(t + \Delta t)/\Delta t = 1$. Thus the semilog plots for all build-ups for a well in an infinite two layered reservoir will have the same intercept.

C. The Effect of Skin

A positive skin is caused by the reduction of the permeability around the well-bore due to infiltration of drilling mud and completion fluids; and the presence of mud cake, cement and high gas saturation in the producing formations. A negative skin is an increase in permeability of the producing formation due to fracturing and acidization.

In this study it has been shown that for the case of an infinite two layered reservoir where the initial pressures are not the same in both layers, the skin factors can be determined with equations 46 and 47. In practical units

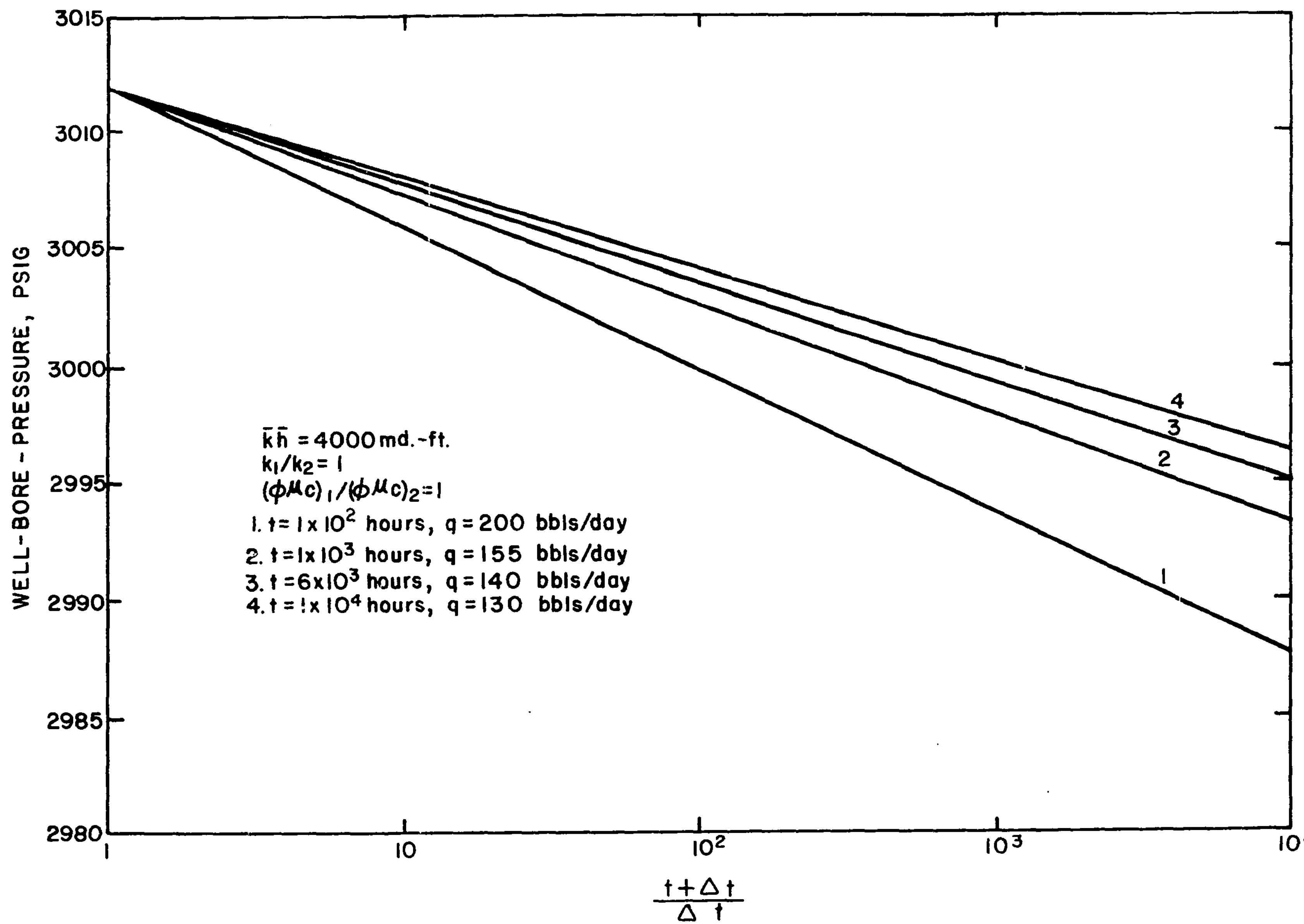

Fig. 14 SELECTED PLOTS FOR DIFFERENT CONSTANT RATES AT DIFFERENT PRODUCING TIMES USING EQUATION 22a.

these equations can be expressed as:

$$S_1 = \frac{1.151\ qb_2}{q_1}\left[\frac{p_{1\ hr} - p_{wf_1} + b_1(\gamma - 1)\Delta p}{m} - \log_{10}\frac{k_1}{\phi\mu c r_w^2} + 3.32\right] \tag{46a}$$

and

$$S_2 = \frac{1.151\ qb_1}{q_2}\left[\frac{p_{1\ hr} - p_{wf_2} + b_2(1 - \gamma)\Delta p}{m} - \log_{10}\frac{k_2}{\phi\mu c r_w^2} + 3.32\right] \tag{47a}$$

Thus using pressure drop equations as derived by Hartsock's[7] method, it is possible to obtain equations to determine skin factors for upper and lower zones of an infinite two layered reservoir.

With the outlines of Lefkovits et al.,[14] it is not possible to determine the skin factors, but it is possible to investigate their effects. Three plots of dimensionless well-bore pressure versus dimensionless time are shown in Figure 15. Curve 1 was obtained using equation 55 which does not include skin factors. Curve 2 was plotted with equation 61 with $S_1 = 1$ and $S_2 = 10$. Curve 3 has been plotted with the same equation as for curve 2 but with $S_1 = 10$ and $S_2 = 1$. The subscripts 1 and 2 refer to the upper and lower zones respectively. The permeability ratio of the upper zone to the lower zone is 10. Therefore, curve 2 is a plot of dimensionless pressure versus dimensionless time with

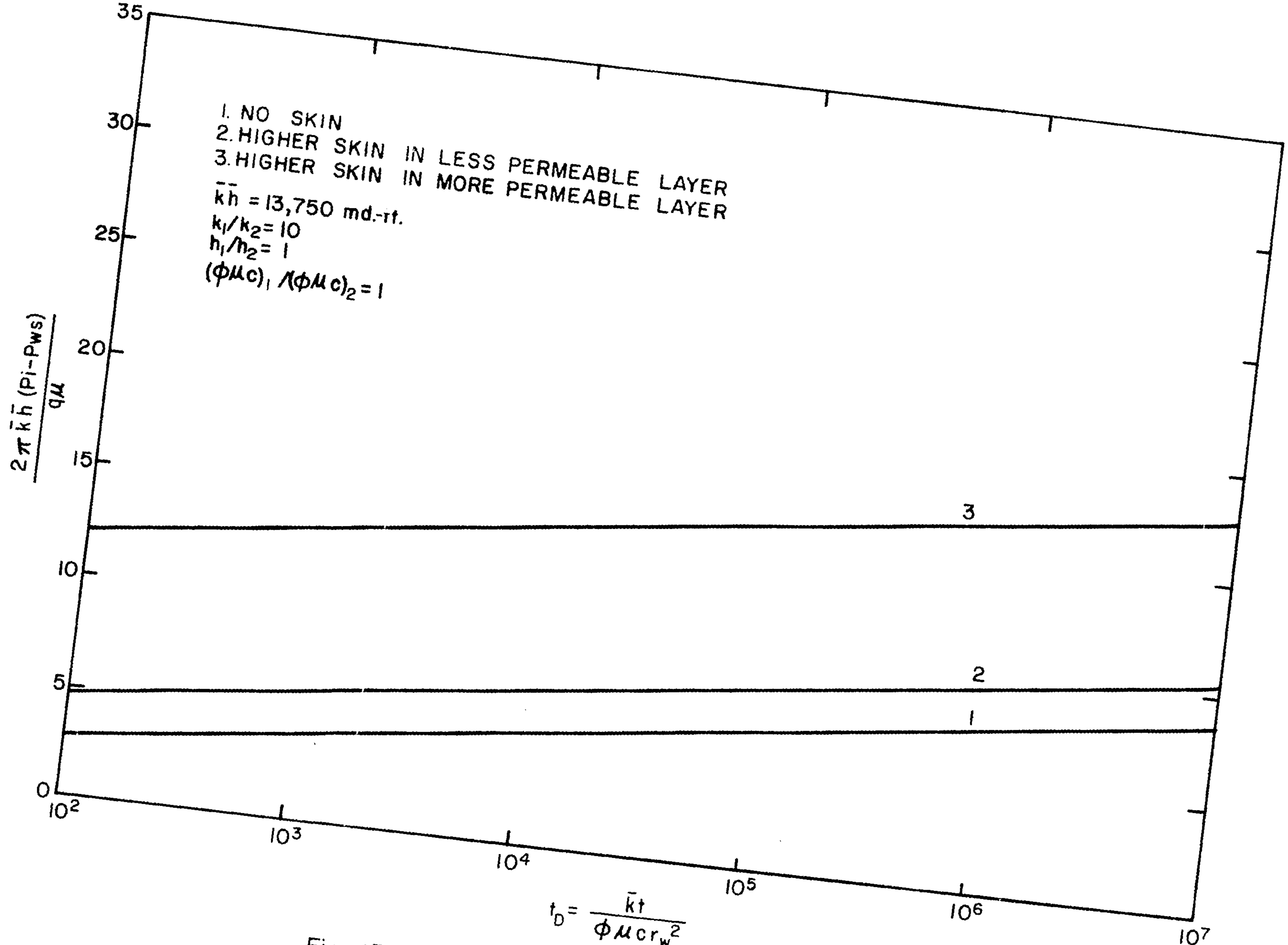

Fig. 15 EFFECT OF POSITIVE SKIN ON PRESSURE DECLINE CURVES

higher skin in the less permeable layer while curve 3 is also a semilog plot like curve 2 but with higher skin in the more permeable layer. Curve 2 shows an increase in pressure drop while curve 3 shows a further increase in pressure drop. The skin factors considered in Figure 15 are all positive.

In Figure 16 is shown the effects of negative skin factors. The permeability ratio is the same as for those used in the investigation of positive skin factors above. Curve 1 in Figure 15 is the same as curve 1 in Figure 16 which is a plot of dimensionless well-bore pressure versus dimensionless time without the effect of skin. Curve 2 was plotted with equation 61 with $S_1 = -1$ and $S_2 = -10$. Similarly curve 3 was plotted with equation 61 but with $S_1 = -10$ and $S_2 = -1$. Curve 2 is a plot of dimensionless pressure versus dimensionless time with a lower skin in the less permeable layer while curve 3 is also a semilog plot like curve 2 but with lower skin in the more permeable layer. Curve 2 shows a decrease in pressure drop and curve 3 shows a further decrease in pressure drop.

D. The Effect of Variable Flow Rate Due to Well-Bore Storage

The method used for investigating well-bore storage in this study is that of flow into casing and tubing with a loading constant, $\bar{\beta}$. Equation 69 was used to generate dimensionless pressures and dimensionless time tabulated in tables 7 and 8. Values from these tables with equation

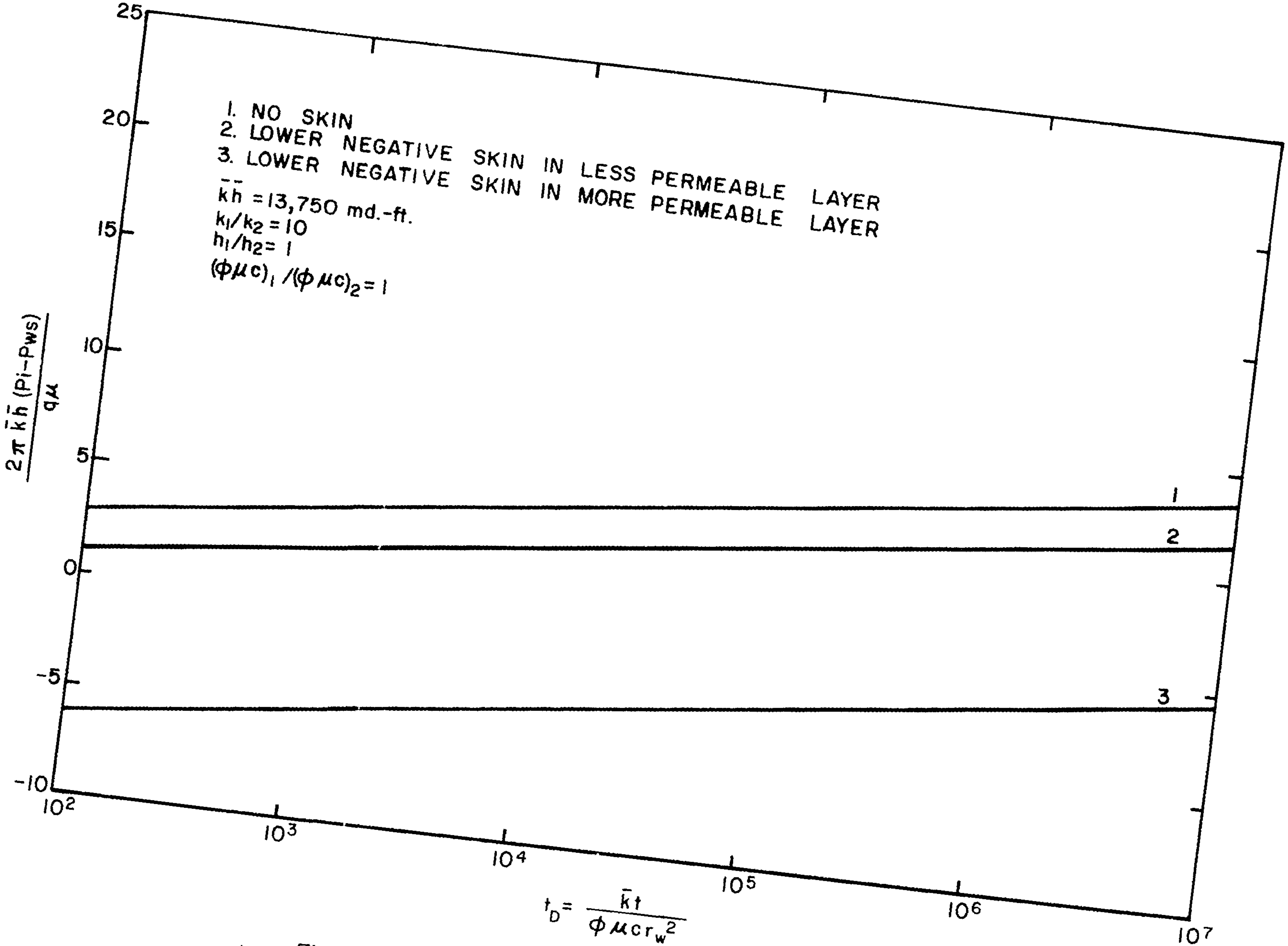

Fig. 16 EFFECT OF POSITIVE SKIN ON PRESSURE DECLINE CURVES

75 were used to plot graphs of dimensionless shut in pressure P_{ws} versus $(t + \Delta t)/\Delta t$. Figures 17, 18, 19, 20 and 21 are examples of such graphical plots. These graphs show that there is early distortion of the build-up curves due to flow into casing and tubing when the well is shut-in. Such distortions are similar to those of single layered reservoirs obtained by Hurst[9] and Van Everdingen.[29] On Figure 22 is shown that with an increase in the loading constant, pressure build-up curves approach ideal curves.

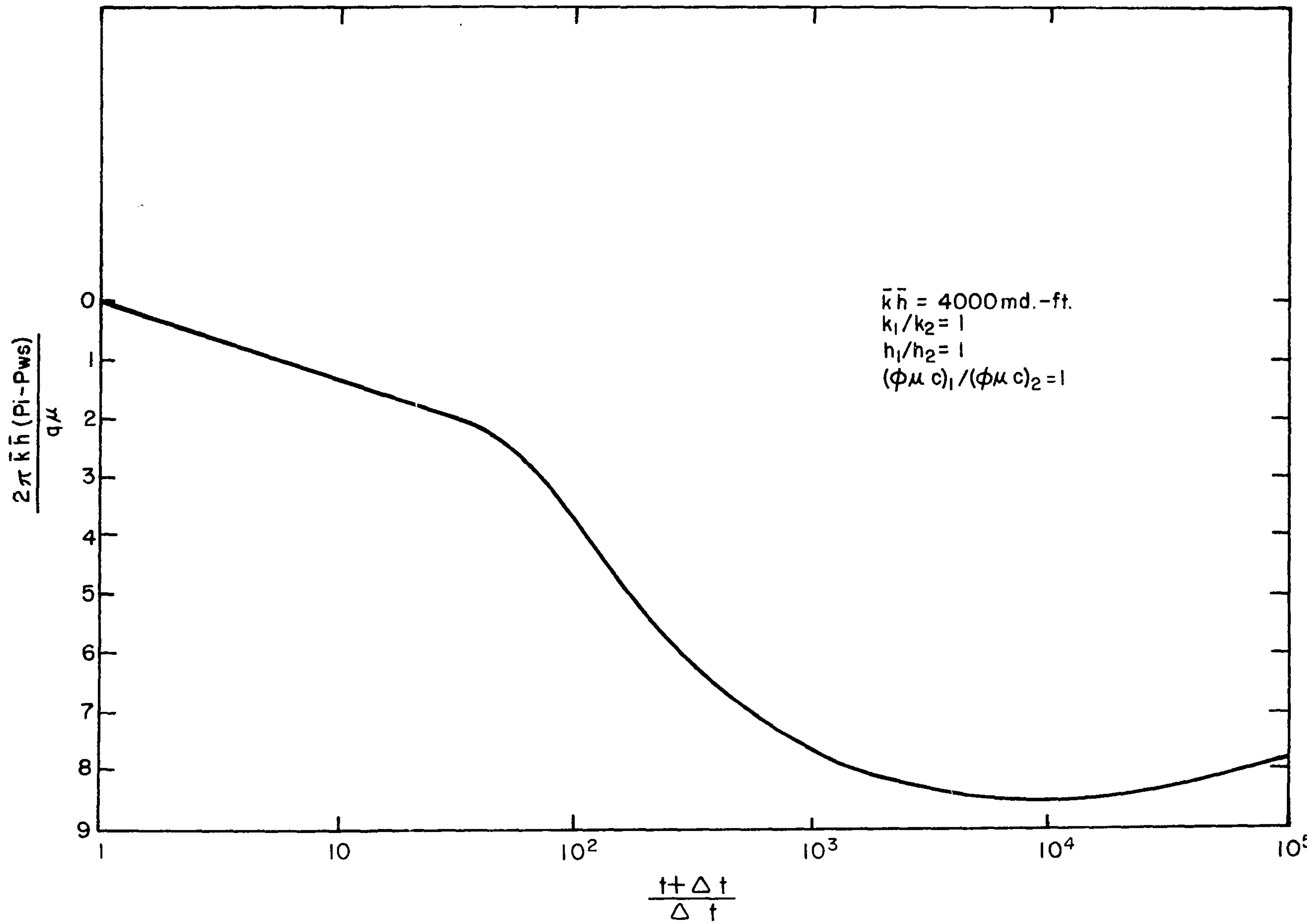

Fig. 17 PLOT SHOWING THE EFFECT OF VARIABLE FLOW RATE DURING BUILD-UP WITH A DIMENSIONLESS LOADING CONSTANT OF β = 0.00002

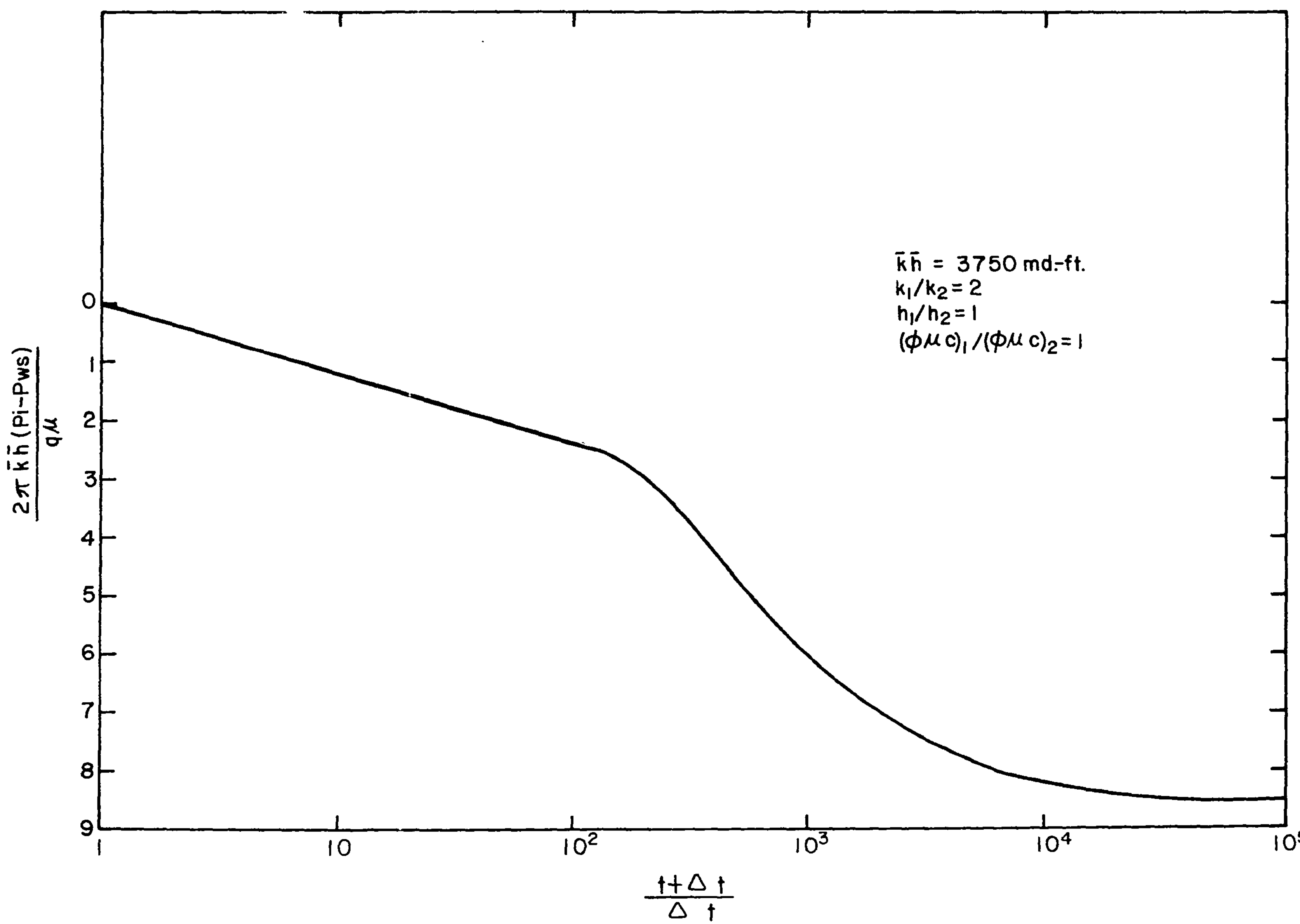

Fig. 18 PLOT SHOWING THE EFFECT OF VARIABLE FLOW RATE DURING BUILD-UP WITH A DIMENSIONLESS LOADING CONSTANT OF β = 0.00008

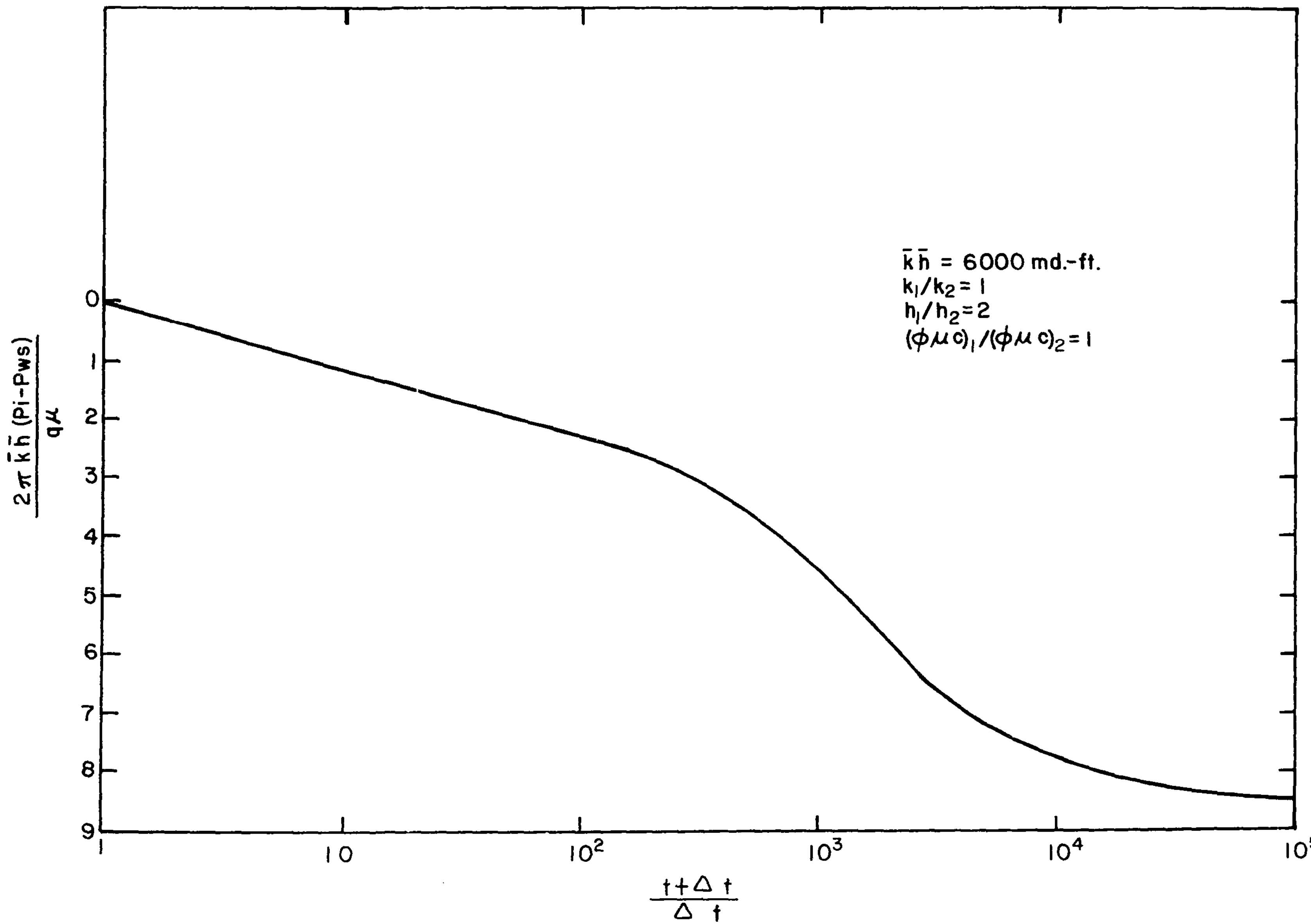

Fig. 19 PLOT SHOWING THE EFFECT OF VARIABLE FLOW RATE DURING BUILD-UP WITH A DIMENSIONLESS LOADING CONSTANT OF β = 0.0002

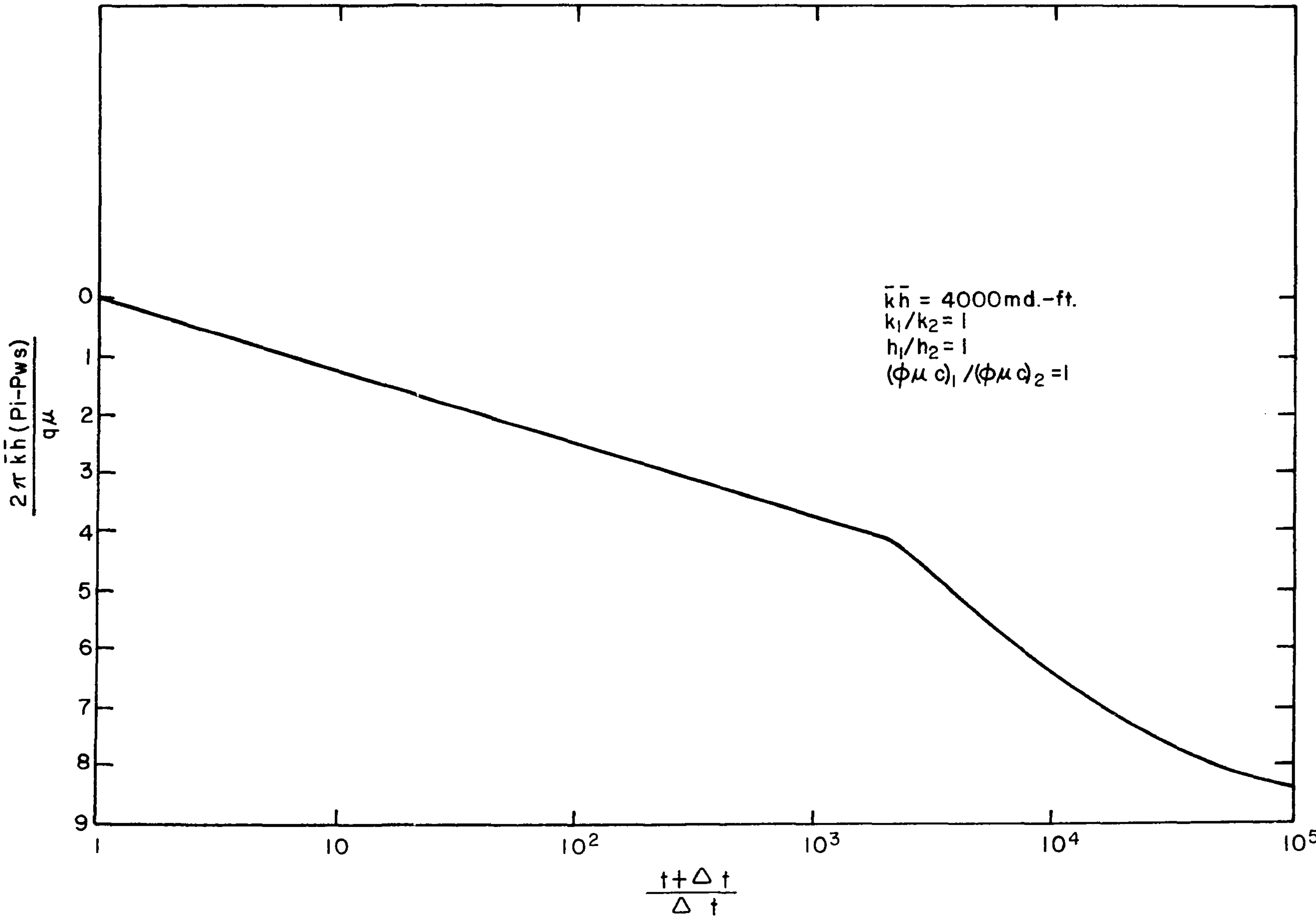

Fig. 20 PLOT SHOWING THE EFFECT OF VARIABLE FLOW RATE DURING BUILD-UP WITH A DIMENSIONLESS LOADING CONSTANT OF β = 0.0008

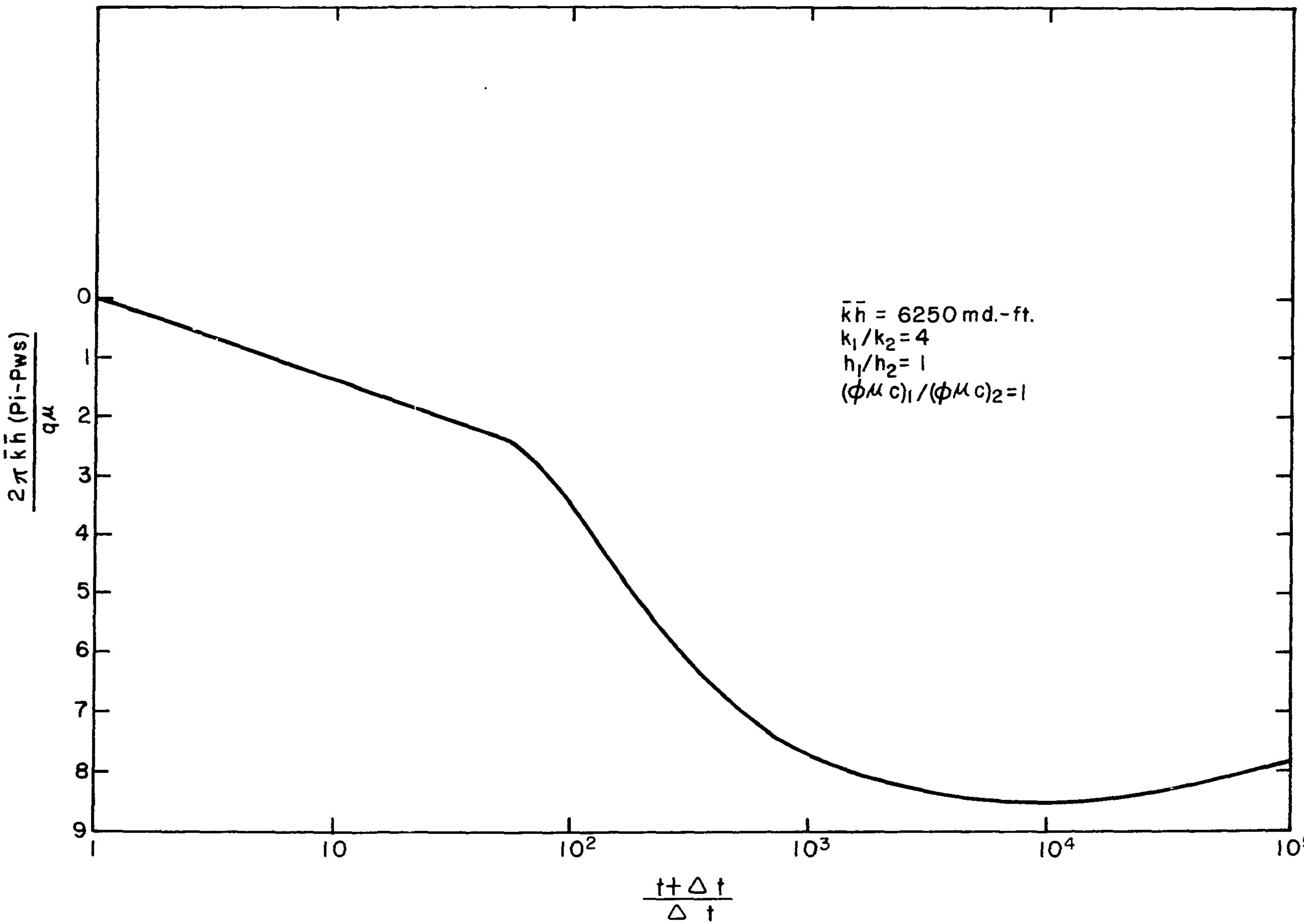

Fig. 21 PLOT SHOWING THE EFFECT OF VARIABLE FLOW RATE DURING BUILD-UP WITH A DIMENSIONLESS LOADING CONSTANT OF β = 0.00002

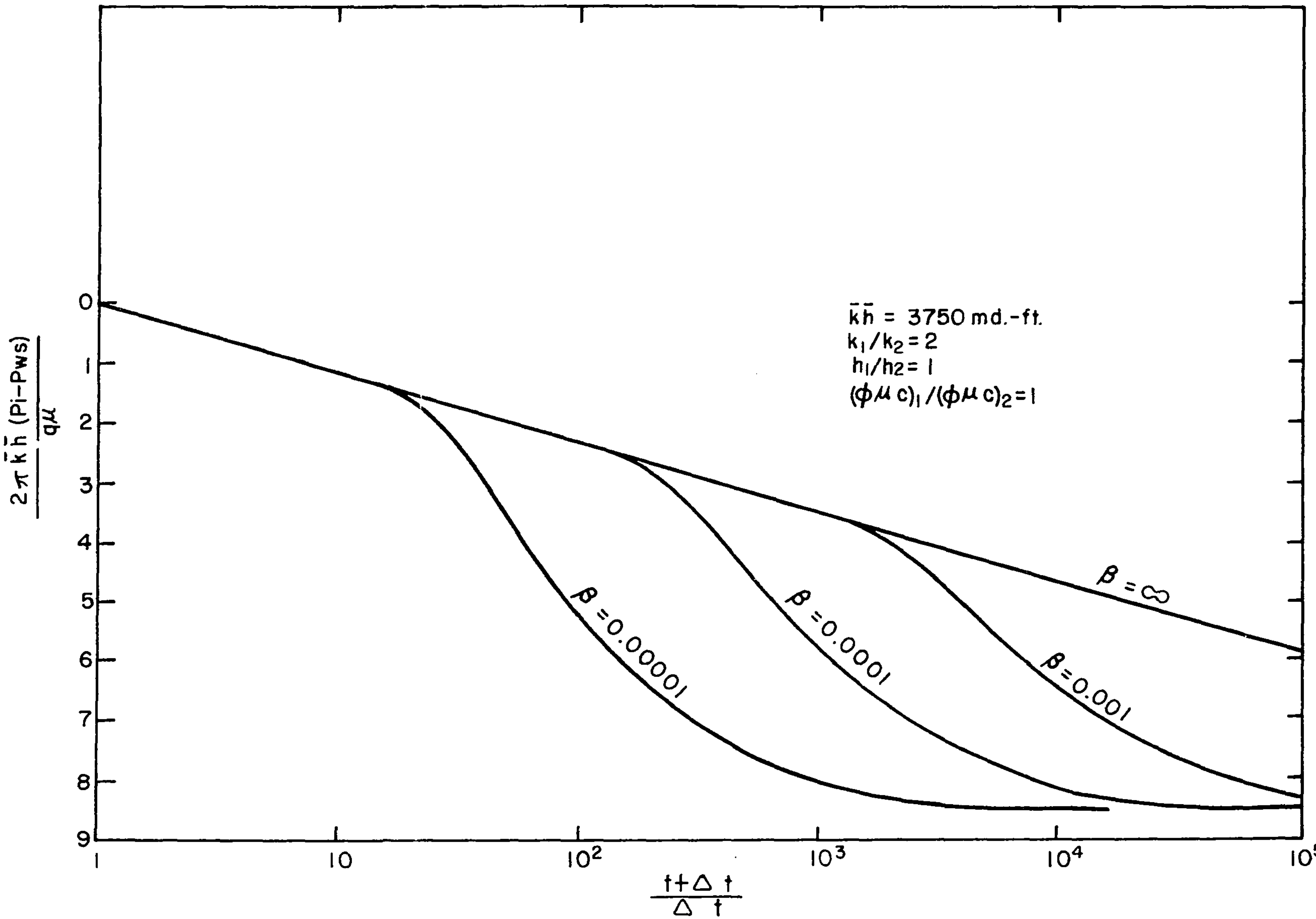

Fig. 22 PLOT SHOWING THE EFFECT OF VARIABLE FLOW RATE DURING BUILD-UP WITH A DIMENSIONLESS LOADING CONSTANT OF β = 0.00001, β = 0.0001, β =0.001, β =OO

CHAPTER VI

CONCLUSIONS

Mathematical equations have been developed that can be used to determine the parameters of the individual layers of an infinite two layered oil reservoir. In addition, the effects of the reservoir parameters, skin and well-bore storage have also been studied. The conclusions obtained in this study are given as follows:

1. In order to obtain equations to compute the reservoir parameters of an infinite two layered reservoir, the initial pressures of the two layers must be different; that is, the force of gravity between the two layers is not neglected. If the initial pressures are the same, the equations obtained can only be used to study the effects of the reservoir parameters, skin and well-bore storage.
2. Using the Hartsock's method, it is possible to determine the initial pressure of one zone and the permeabilities of the individual zones if the initial pressure of one zone or the difference in the initial pressures is known.

3. Pressure build-up curves obtained for an infinite two layered reservoir without the effects of well-bore storage are similar to those for an ideal single layered reservoir.
4. Permeability and thickness ratios for an infinite two layered reservoir do not affect the slope of the build-up curve but only the $\bar{k}\bar{h}$ product affects the slope.
5. Equations have been developed that can be used to determine the skin factors of the individual zones. Further, the study of the skin effect show that there is a greater pressure drop with a higher positive skin in the more permeable layer than a higher positive skin in a less permeable layer. Also, there is a lesser pressure drop with a lower negative skin in the more permeable zone than a lower negative in the less permeable zone.
6. Equations were derived to show the effects of well-bore storage. It was found that the effect of well-bore storage in an infinite two layered reservoir is the same for that of a single layered reservoir.
7. The results of the analysis demonstrate the validity of the equations derived for the analysis of pressure build-up in an infinite two-layered reservoir.

NOMENCLATURE

c	total compressibility, psi^{-1}
d	depth of formation, feet
e	exponential function
$E_i(x)$	exponential integral function
g	acceleration due to gravity, ft/sec^2
h	formation thickness, feet
$\bar{h}$	total formation thickness, feet. Eq. 12
I_o	modified Bessel function of the second kind of zero order
k	permeability, millidarcy
$\bar{k}$	harmonic mean of the permeabilities of the upper and lower zones, milli-darcy. Eq. 10
K_o	modified Bessel function of the first kind of zero order
ln	natural logarithm
$\log_{10}$	base ten logarithm
m	slope of plot of P_{ws} versus $(t + \Delta t)/\Delta t$, psi/cycle
p	formation pressure, psi
P	pressure drop
P_f	pressure drop at the well
p_{wf}	flowing well-bore pressure, psi

p_{wf}	pressure drop at the well-bore
$\bar{\bar{p}}_{wf}$	flowing bottom hole pressure due to well-bore storage, psi
$\bar{\bar{\bar{p}}}_{wf}$	pressure drop due to well-bore storage
p_{ws}	shut in bottom-hole pressure, psi
p^*	intercept pressure, psi. Eq. 26 and 27
Δp	pressure difference between the upper and lower zones, psi
Δp_{skin}	pressure drop in "skin" region next to well-bore, psi
Q	constant for two layers, Eq. 51
Q_S	constant for two layers with skin, Eq. 60
Q_{SL}	constant for single layer, Eq. 71
q	total flow rate from the two layers, bbls/day
r	radial distance, feet
S	skin factor, dimensionless
t	time, hours
t_D	dimensionless time for an infinite two layered reservoir based on well radius
Δt	shut in time, hours
Δt_D	dimensionless shut-in time
Z	independent variable for time in Laplace equation
α	loading constant, $1/t_D$
β_o	formation volume factor
$\bar{\beta}$	dimensionless loading constant
γ	Euler's constant, $\gamma = 1.781$, $\ln \gamma = 0.5772$
ϕ	porosity
$\bar{\phi}$	harmonic mean porosity, Eq. 11

ρ_o	oil specific gravity
μ	viscosity, cp

<u>Subscripts</u>

D	dimensionless
f	formation
1h	one hour
i	initial
j	index number for the layers
sf	sand face
1,2	numbers indicating upper and lower zones
SL	single layer
w	well

REFERENCES

1. Arps, J. J.: "How Well Completion Damage can be Determined Graphically," World Oil, 140 [5] 222 (1955).
2. Carslaw, H. S. and Jaeger, J. C.: Operational Methods in Applied Mathematics, Dover Publications, Inc., New York (1963).
3. Cobb, W. M., Ramey, H. J., Miller, F. G.: "Well-Test Analysis for Wells Producing Commingled Zones," Journal of Petroleum Technology (Jan. 1972), 27.
4. Cobb, W. M.: "Transient Flow in Two-Layer Reservoirs with Commingled Fluid Production," Ph.D. Dissertation, Stanford University, Calif., (1971).
5. Earlougher, R. C., Jr., Kersch, K. M. and Kunzman, W. J.: "Some Characteristics of Pressure Build-Up Behavior in Bounded Multiple-Layered Reservoirs Without Crossflow," Journal of Petroleum Technology (Oct. 1974), 1178.
6. Gladfelter, R. E., Tracy, G. W., and Wilsey, L. E.: "Selecting Wells Which Will Respond to Production-Stimulation Treatment," Drilling and Production Practice (1955), 117.
7. Hartsock, J. H.: "Dual-Zone Pressure Build-up," Reservoir Mechanics Division File Note, Gulf Research and Development Company, Pittsburgh, PA (March, 1959).

8. Horner, D. R.: "Pressure Build-up in Wells," Proceedings of the Third World Petroleum Congress, (11), (1951), The Hague, 503.

9. Hurst, William: "Establishment of the Skin Effect and Its Impediment to Fluid Flow into a Well Bore," Petroleum Engineer, 25 [11] B6 (1953).

10. Hurst, William: William Hurst and Associates, Petroleum Reservoir Engineer, Houston, Texas: Private Communications.

11. Katz, M. L. and Tek, M. L.: "A Theoretical Study of Pressure Distribution and Fluid Flux in Bounded Stratified Porous Systems with Crossflow," Society of Petroleum Engineers Journal, (March, 1962), 68.

12. Kazemi, H.: "Pressure Build-up in Reservoir Limit Testing of Stratified Systems," Journal of Petroleum Technology, (April, 1970), 503.

13. Larson, V. C.: "Understanding the Muskat Method of Analysing Pressure Build-up Curves," Journal of Canadian Petroleum Technology, Vol. 2, No. 3 (Fall, 1963).

14. Lefkovits, M. C., Hazebroek, P., Allen, E. E. and Mathews, C. S.: "A Study of the Behavior of Bounded Reservoirs Composed of Stratified Layers," Trans., AIME (1961), Vol. 222, 43.

15. Matthews, C. S., Brons, F., and Hazebroek, P.: "A Method for Determination of Average Pressure in a Bounded Reservoir," Trans., AIME (1954), Vol. 204, 35.

16. Mathews, C. S., and Russell, D. G.: Pressure Build-up and Flow Tests in Wells, Monograph Series, Society of Petroleum Engineers of AIME, Dallas (1967), 1.

17. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr.: "Application of Build-up Curves to Determine Reservoir Pressures and Permeabilities," Petroleum Engineer, 22 [6] B7 (1950).

18. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr.: "The Estimation of Permeability and Reservoir Pressure from Bottom-hole Pressure Build-Up Characteristics," Trans., AIME (1950) 189, 91.

19. Muskat, Morris: "Use of Data on the Build-Up of Bottom-hole Pressures," Trans., AIME (1937) 123, 44.

20. Pendergrass, J. D. and Berry, V. J., Jr.: "Pressure Transient Performance of a Multi-Layered Reservoir with Crossflow," The Society of Petroleum Engineers of AIME (Mid-Continent Section). Production Research Symposium, Tulsa, Oklahoma (April, 1962) 92.

21. Perrine, R. L.: "Analysis of Pressure Build-Up Curve," Drilling and Production Practice, API (1956), 482.

22. Raghaven, R., Topaloglu, H. N., Cobb, W. M. and Ramey, M. J.: "Well Test Analysis for Wells Producing from Two Commingled Zones of Unequal Thickness," Journal of Petroleum Technology, (Sept., 1974), 1035.

23. Ramey, M. J.: "Short-Time Well Test Data Interpretation in the Presence of Skin Effect and Well-bore Storage," Journal of Petroleum Technology, (Jan., 1970), 97.

24. Russell, D. G.: "Extensions of Pressure Build-Up Analysis Methods," Journal of Petroleum Technology, (Dec., 1966), 1624.

25. Russell, D. G. and Prats, M.: "Performance of Layered Reservoirs with Crossflow--Single-Compressible-Fluid Case," Society of Petroleum Engineers Journal, (March, 1962), 53.

26. Russell, D. G. and Pratts, M.: "The Practical Aspects of Interlayer Crossflow," Journal of Petroleum Technology, (June, 1962), 589.

27. Tables of Sine, Cosine and Exponential Integrals, Federal Works Agency, Works Projects Administration for the City of New York, (1940).

28. Thomas, G. B.: "Analysis of Pressure Build-up Data," Trans., AIME (1953) 198, 125.

29. Van Everdingen, A. F.: "The Skin Effect and Its Influence on the Productive Capacity of a Well," Trans., AIME (1953) 198, 171.

30. Woods, E. G.: "Pulse-Test Response of a Two-Zone Reservoir," Society of Petroleum Engineers Journal, (Sept., 1970), 245.

TABLE 1

RESERVOIR PARAMETERS USED IN THIS STUDY

$c_1 = c_2 = 1.5 \times 10^{-5}$ psi^{-1}

$d_1 = 5000$ ft

$d_2 = 5050$ ft

$\mu_1 = \mu_2 = 0.75$ cp

$p_{i_1} = 3000$ psi

$p_{i_2} = 3030$ psi

$q = 200$ bbl/day

$r_w = 0.25$ ft

$\phi_1 = \phi_2 = 0.20$

$\rho_o = 0.825$

$\beta_o = 1.0$

Thickness and Thickness Ratio			Permeability and Permeability Ratio		
h_1 ft	h_2 ft	h_1/h_2	k_1 md	k_2 md	k_1/k_2
50	10	5			
40	10	4	80	80	1
60	20	3	100	50	2
50	25	2	150	50	3
25	25	1	200	50	4
25	50	0.5	250	50	5
10	50	0.2	500	50	10

TABLE 2

THE EFFECT OF $\bar{k}\bar{h}$ PRODUCT ON SLOPE m, WITH VARIATION OF PERMEABILITY RATIO

k_1 md	k_2 md	k_1/k_2	$\bar{k}\bar{h}$ md-ft	$m = \frac{162.6\ q\mu\beta_o}{\bar{k}\bar{h}}$ psi/cycle
80	80	1	4000	6.1
100	50	2	3750	6.5
150	50	3	5000	4.88
200	50	4	6250	3.90
250	50	5	7500	3.25
180	30	6	5250	4.65
140	20	7	4000	6.10
320	40	8	9000	2.61
405	45	9	11250	2.17
500	50	10	13750	1.77

where $\bar{k}\bar{h} = \frac{k_1h_1 + k_2h_2}{\bar{h}}$

and $\bar{h} = h_1 + h_2$

$h_1 = h_2 = 25$ ft

$q_T = 200$ bbls/day

$\mu = 0.75$ cp

TABLE 3

THE EFFECT OF $\bar{k}\bar{h}$ PRODUCT ON SLOPE m, WITH VARIATION OF THICKNESS RATIO

h_1 ft	h_2 ft	h_1/h_2	$\bar{k}\bar{h}$ md-ft	$m = \dfrac{162.6\ q_T \mu \beta_o}{\bar{k}\bar{h}}$ psi/cycle
10	50	0.2	4800	5.08
25	50	0.5	6000	4.07
25	25	1	4000	6.10
50	25	2	6000	4.06
60	20	3	6400	3.81
40	10	4	4000	6.10
50	10	5	4800	5.08

where $\bar{k}\bar{h} = \dfrac{k_1 h_1 + k_2 h_2}{\bar{h}}$

and $\bar{h} = h_1 + h_2$

$k_1 = k_2 = 80$ md

$q_T = 200$ bbls/day

$\mu = 0.75$ cp

TABLE 4

DIMENSIONLESS PRESSURE VERSUS DIMENSIONLESS TIME WITH VARIATION OF PERMEABILITY RATIO FOR AN INFINITE TWO-LAYERED RESERVOIR WITHOUT THE EFFECTS OF SKIN AND WELL-BORE STORAGE AND WITH A THICKNESS RATIO OF ONE.

t_D	P_D					
	k_1/k_2					
	1	2	3	4	5	10
1.00E 02	2.7071	2.7260	2.7516	2.7739	2.7926	2.8521
2.00E 02	3.0537	3.0735	3.1004	3.1237	3.1432	3.2045
4.00E 02	3.4003	3.4209	3.4488	3.4729	3.4930	3.5560
6.00E 02	3.6030	3.6240	3.6524	3.6770	3.6974	3.7611
8.00E 02	3.7469	3.7681	3.7968	3.8216	3.8423	3.9065
1.00E 03	3.8584	3.8799	3.9089	3.9338	3.9546	4.0191
2.00E 03	4.2050	4.2270	4.2567	4.2822	4.3033	4.3687
4.00E 03	4.5516	4.5741	4.6043	4.6302	4.6517	4.7180
6.00E 03	4.7543	4.7770	4.8075	4.8338	4.8554	4.9220
8.00E 03	4.8982	4.9210	4.9517	4.9781	4.9998	5.0667
1.00E 04	5.0097	5.0327	5.0636	5.0900	5.1118	5.1789
2.00E 04	5.3563	5.3796	5.4109	5.4377	5.4597	5.5272
4.00E 04	5.7029	5.7265	5.7581	5.7852	5.8074	5.8754
6.00E 04	5.9056	5.9294	5.9612	5.9884	6.0107	6.0790
8.00E 04	6.0495	6.0733	6.1053	6.1326	6.1550	6.2234
1.00E 05	6.1610	6.1850	6.2170	6.2444	6.2668	6.3353
2.00E 05	6.5076	6.5318	6.5641	6.5917	6.6143	6.6830
4.00E 05	6.8542	6.8786	6.9111	6.9389	6.9616	7.0307
6.00E 05	7.0569	7.0814	7.1141	7.1420	7.1648	7.2340
8.00E 05	7.2007	7.2253	7.2581	7.2860	7.3089	7.3782
1.00E 06	7.3123	7.3370	7.3698	7.3978	7.4207	7.4900
2.00E 06	7.6589	7.6837	7.7167	7.7448	7.7678	7.8374
4.00E 06	8.0055	8.0304	8.0636	8.0919	8.1150	8.1847
6.00E 06	8.2082	8.2332	8.2666	8.2949	8.3180	8.3879
8.00E 06	8.3520	8.3771	8.4105	8.4389	8.4620	8.5320
1.00E 07	8.4636	8.4886	8.5222	8.5506	8.5738	8.6437
2.00E 07	8.8102	8.8353	8.8688	8.8972	8.9204	8.9909
4.00E 07	9.1568	9.1820	9.2156	9.2441	9.2674	9.3375
6.00E 07	9.3594	9.3848	9.4185	9.4471	9.4703	9.5406
8.00E 07	9.5033	9.5287	9.5625	9.5911	9.6144	9.6846
1.00E 08	9.6149	9.6403	9.6741	9.7027	9.7261	9.7963
2.00E 08	9.9615	9.9870	10.0209	10.0496	10.0731	10.1435
4.00E 08	10.3081	10.3337	10.3677	10.3965	10.4200	10.4905
6.00E 08	10.5108	10.5365	10.5705	10.5994	10.6229	10.6935
8.00E 08	10.6546	10.6803	10.7145	10.7433	10.7669	10.8375
1.00E 09	10.7662	10.7919	10.8261	10.8550	10.8786	10.9492
2.00E 09	11.1128	11.1386	11.1728	11.2018	11.2254	11.2962
4.00E 09	11.4593	11.4853	11.5196	11.5486	11.5723	11.6431
6.00E 09	11.6621	11.6880	11.7224	11.7515	11.7752	11.8460
8.00E 09	11.8059	11.8319	11.8663	11.8954	11.9191	11.9900

TABLE 5

DIMENSIONLESS PRESSURE VERSUS DIMENSIONLESS TIME WITH VARIATION OF THICKNESS RATIO FOR AN INFINITE TWO-LAYERED RESERVOIR WITHOUT THE EFFECTS OF SKIN AND WELL-BORE STORAGE AND WITH A PERMEABILITY RATIO OF 4.

t_D	P_D					
	h_1/h_2					
	0.2	0.5	1	2	4	5
1.00E 02	2.7737	2.7844	2.7739	2.7547	2.7365	2.7317
2.00E 02	3.1240	3.1349	3.1237	3.1035	3.0843	3.0793
4.00E 02	3.4736	3.4847	3.4729	3.4518	3.4319	3.4267
6.00E 02	3.6780	3.6890	3.6770	3.6554	3.6352	3.6299
8.00E 02	3.8229	3.8340	3.8216	3.7998	3.7790	3.7740
1.00E 03	3.9352	3.9464	3.9338	3.9118	3.8911	3.8858
2.00E 03	4.2839	4.2951	4.2822	4.2596	4.2384	4.2329
4.00E 03	4.6323	4.6436	4.6302	4.6071	4.5856	4.5800
6.00E 03	4.8360	4.8473	4.8338	4.8103	4.7886	4.7830
8.00E 03	4.9805	4.9918	4.9781	4.9545	4.9326	4.9270
1.00E 04	5.0926	5.1038	5.0900	5.0663	5.0444	5.0387
2.00E 04	5.4405	5.4518	5.4377	5.4136	5.3913	5.3856
4.00E 04	5.7883	5.7995	5.7852	5.7608	5.7383	5.7324
6.00E 04	5.9917	6.0029	5.9884	5.9638	5.9412	5.9353
8.00E 04	6.1359	6.1472	6.1326	6.1079	6.0851	6.0793
1.00E 05	6.2478	6.2591	6.2444	6.2196	6.1968	6.1909
2.00E 05	6.5953	6.6066	6.5917	6.5666	6.5436	6.5377
4.00E 05	6.9428	6.9540	6.9389	6.9136	6.8905	6.8845
6.00E 05	7.1460	7.1571	7.1420	7.1166	7.0933	7.0873
8.00E 05	7.2901	7.3013	7.2860	7.2606	7.2372	7.2312
1.00E 06	7.4019	7.4131	7.3978	7.3723	7.3489	7.3429
2.00E 06	7.7492	7.7603	7.7448	7.7192	7.6956	7.6896
4.00E 06	8.0963	8.1075	8.0919	8.0660	8.0423	8.0363
6.00E 06	8.2989	8.3106	8.2949	8.2689	8.2452	8.2391
8.00E 06	8.4430	8.4546	8.4978	8.4129	8.3891	8.3830
1.00E 07	8.5547	8.5659	8.5506	8.5245	8.5007	8.4946
2.00E 07	8.9019	8.9130	8.8972	8.8713	8.8474	8.8413
4.00E 07	9.2490	9.2601	9.2441	9.2179	9.1940	9.1879
6.00E 07	9.4520	9.4631	9.4471	9.4208	9.3968	9.3907
8.00E 07	9.5960	9.6071	9.5911	9.5647	9.5407	9.5346
1.00E 08	9.7076	9.7188	9.7027	9.6764	9.6523	9.6462
2.00E 08	10.0548	10.0658	10.0496	10.0231	9.999	9.9928
4.00E 08	10.4018	10.4128	10.3965	10.3699	10.3457	10.3395
6.00E 08	10.6047	10.6157	10.5994	10.5727	10.5485	10.5423
8.00E 08	10.7487	10.7597	10.7433	10.7167	10.6924	10.6862
1.00E 09	10.8604	10.8714	10.8550	10.8283	10.8040	10.7978
2.00E 09	11.2073	11.2183	11.2018	11.1750	11.1507	11.1444
4.00E 09	11.5542	11.5652	11.5486	11.5217	11.4973	11.4910
6.00E 09	11.7571	11.7681	11.7515	11.7245	11.7001	11.6938
8.00E 09	11.9011	11.9121	11.8954	11.8684	11.8440	11.8377

TABLE 6

DIMENSIONLESS PRESSURE VERSUS DIMENSIONLESS TIME SHOWING THE EFFECT OF SKIN FOR AN INFINITE TWO-LAYERED RESERVOIR WITH A PERMEABILITY RATIO OF 10 AND THICKNESS RATIO OF 1.

t_D	P_D		t_D	P_D	
	S_1 = 10	1		S_1 = -10	-1
	S_2 = 1	10		S_2 = -1	-10
1.00E 02	12.0339	4.6703	1.00E 02	-6.3297	1.0339
2.00E 02	12.3863	5.0227	2.00E 02	-5.9773	1.3863
4.00E 02	12.7378	5.3742	4.00E 02	-5.6258	1.7378
6.00E 02	12.9429	5.5793	6.00E 02	-5.4207	1.9429
8.00E 02	13.0883	5.7247	8.00E 02	-5.2753	2.0883
1.00E 03	13.2009	5.8373	1.00E 03	-5.1627	2.2009
2.00E 03	13.5506	6.1869	2.00E 03	-4.8131	2.5506
4.00E 03	13.8998	6.5361	4.00E 03	-4.4639	2.8998
6.00E 03	14.1038	6.7402	6.00E 03	-4.2598	3.1038
8.00E 03	14.2485	6.8849	8.00E 03	-4.1151	3.2485
1.00E 04	14.3607	6.9971	1.00E 04	-4.0029	3.3607
2.00E 04	14.7091	7.3454	2.00E 04	-3.6546	3.7091
4.00E 04	15.0572	7.6936	4.00E 04	-3.3064	4.0572
6.00E 04	15.2608	7.8971	6.00E 04	-3.1029	4.2608
8.00E 04	15.4052	8.0415	8.00E 04	-2.9585	4.4052
1.00E 05	15.5171	8.1535	1.00E 05	-2.8465	4.5171
2.00E 05	15.8649	8.5012	2.00E 05	-2.4988	4.8649
4.00E 05	16.2125	8.8489	4.00E 05	-2.1511	5.2125
6.00E 05	16.4158	9.0521	6.00E 05	-1.9479	5.4158
8.00E 05	16.5600	9.1964	8.00E 05	-1.8036	5.5600
1.00E 06	16.6718	9.3082	1.00E 06	-1.6918	5.6718
2.00E 06	17.0192	9.6556	2.00E 06	-1.3444	6.0192
4.00E 06	17.3665	10.0029	4.00E 06	-0.9971	6.3665
6.00E 06	17.5697	10.2060	6.00E 06	-0.7940	6.5697
8.00E 06	17.7138	10.3501	8.00E 06	-0.6499	6.7138
1.00E 07	17.8256	10.4619	1.00E 07	-0.5381	6.8256
2.00E 07	18.1727	10.8091	2.00E 07	-0.1909	7.1727
4.00E 07	18.5193	11.1557	4.00E 07	0.1557	7.5193
6.00E 07	18.7224	11.3588	6.00E 07	0.3588	7.7224
8.00E 07	18.8665	11.5028	8.00E 07	0.5028	7.8665
1.00E 08	18.9782	11.6146	1.00E 08	0.6146	7.9782
2.00E 08	19.3253	11.9616	2.00E 08	0.9616	8.3253
4.00E 08	19.6723	12.3087	4.00E 08	1.3087	8.6723
6.00E 08	19.8753	12.5117	6.00E 08	1.5117	8.8753
8.00E 08	20.0193	12.6557	8.00E 08	1.6557	9.0193
1.00E 09	20.1310	12.7674	1.00E 09	1.7674	9.1310
2.00E 09	20.4780	13.1143	2.00E 09	2.1143	9.4780
4.00E 09	20.8249	13.4613	4.00E 09	2.4613	9.8249
6.00E 09	21.0278	13.6642	6.00E 09	2.6642	10.0278
8.00E 09	21.1718	13.8082	8.00E 09	2.8082	10.1718

TABLE 7

DIMENSIONLESS PRESSURE VERSUS DIMENSIONLESS TIME FOR VARIABLE FLOW RATE WITH LOADING CONSTANT $\bar{\beta}$ RANGING FROM 0.00001 TO 0.00008, WITH A PERMEABILITY RATIO OF 1 AND A THICKNESS RATIO OF 1.

t_D	$\bar{\bar{P}}_D$				
	$\bar{\beta}$ (x 10^{-5})				
	1	2	4	6	8
1.00E 02	0.7934	0.7423	0.2953	0.03490	0.0444
2.00E 02	0.7430	0.2966	0.0472	0.0404	0.0440
4.00E 02	0.2980	0.0499	0.0495	0.0707	0.0928
6.00E 02	0.0426	0.04694	0.0755	0.1106	0.1463
8.00E 02	0.0527	0.0550	0.1038	0.1531	0.2020
1.00E 03	0.0334	0.0665	0.1319	0.1960	0.2590
2.00E 03	0.0734	0.1455	0.2856	0.4648	0.6455
4.00E 03	0.1591	0.3123	0.6968	0.9892	1.2168
6.00E 03	0.2482	0.5269	1.0325	1.3774	1.6555
8.00E 03	0.3389	0.7480	1.3117	1.7104	2.1238
1.00E 04	0.4303	0.9373	1.5598	2.0748	2.5420
2.00E 04	1.0001	1.6740	2.7328	3.4873	4.0140
4.00E 04	1.7883	2.9238	4.2906	4.9498	5.2709
6.00E 04	2.4790	3.8711	5.1341	5.5638	5.7206
8.00E 04	3.1145	4.5672	5.6033	5.8633	5.9422
1.00E 05	3.6521	5.0779	5.8863	6.0421	6.0855
2.00E 05	5.3776	6.2266	6.4320	6.4984	6.5074
4.00E 05	6.5668	6.7785	6.8539	6.8542	6.8542
6.00E 05	6.9358	7.0477	7.0569	7.0569	7.0569
8.00E 05	7.1249	7.2005	7.2007	7.2007	7.2007
1.00E 06	7.2555	7.3123	7.3123	7.3123	7.3123
2.00E 06	7.6589	7.6589	7.6589	7.6589	7.6589
4.00E 06	8.0055	8.0055	8.0055	8.0055	8.0055
6.00E 06	8.2082	8.2082	8.2082	8.2082	8.0282
8.00E 06	8.3520	8.3520	8.3520	8.3520	8.3520
1.00E 07	8.4636	8.4636	8.4636	8.4636	8.4636
2.00E 07	8.8102	8.8102	8.8102	8.8102	8.8102
4.00E 07	9.1568	9.1568	9.1568	9.1568	9.1568
6.00E 07	9.3595	9.3595	9.3595	9.3595	9.3595
8.00E 07	9.5033	9.5033	9.5033	9.5033	9.5033
1.00E 08	9.6149	9.6149	9.6149	9.6149	9.6149
2.00E 08	9.9615	9.0615	9.9615	9.9615	9.9615
4.00E 08	10.3081	10.3081	10.3081	10.3081	10.3081
6.00E 08	10.5101	10.5101	10.5101	10.5101	10.5108
8.00E 08	10.6546	10.6546	10.6546	10.6546	10.6546
1.00E 09	10.7662	10.7662	10.7662	10.7662	10.7662
2.00E 09	11.1128	11.1128	11.1128	11.1128	11.1128
4.00E 09	11.4593	11.4593	11.4593	11.4593	11.4593
6.00E 09	11.6621	11.6621	11.6621	11.6621	11.6621
8.00E 09	11.8059	11.8059	11.8059	11.8059	11.8059

TABLE 8

DIMENSIONLESS PRESSURE VERSUS DIMENSIONLESS TIME FOR VARIABLE FLOW RATE WITH LOADING CONSTANT $\bar{\beta}$ RANGING FROM 0.0001 TO 0.0008, WITH A PERMEABILITY RATIO OF 2 AND A THICKNESS RATIO OF 1.

t_D	$\bar{\bar{P}}_D$				
	$\bar{\beta}$ (x 10^{-4})				
	1	2	4	6	8
1.00E 02	0.02194	0.04373	0.08674	0.1290	0.17045
2.00E 02	0.05059	0.1003	0.1971	0.3346	0.4753
4.00E 02	0.1139	0.2237	0.5265	0.7436	0.9014
6.00E 02	0.1811	0.3967	0.7868	1.0294	1.2166
8.00E 02	0.2504	0.5778	0.9964	1.2715	1.5796
1.00E 03	0.3208	0.7286	1.1802	1.5554	1.9080
2.00E 03	0.7914	1.2945	2.0988	2.6828	3.0951
4.00E 03	1.4087	2.2897	3.3717	3.9029	4.1665
6.00E 03	1.9596	3.0667	4.0873	4.4439	4.5788
8.00E 03	2.4805	3.6483	4.4990	4.7215	4.7928
1.00E 04	2.9243	4.0825	4.7562	4.8937	4.9346
2.00E 04	4.3821	5.0964	5.2811	5.3471	5.3561
4.00E 04	5.4366	5.6275	5.7027	5.7029	5.7029
6.00E 04	5.7873	5.8964	5.9056	5.9056	5.9056
8.00E 04	5.9740	6.0492	6.0495	6.0495	6.0495
1.00E 05	6.1042	6.1610	6.1610	6.1610	6.1610
2.00E 05	6.5076	6.5076	6.5076	6.5076	6.5076
4.00E 05	6.8542	6.8542	6.8542	6.8542	6.8542
6.00E 05	7.0569	7.0569	7.0569	7.0569	7.0569
8.00E 05	7.2007	7.2007	7.2007	7.2007	7.2007
1.00E 06	7.3123	7.3123	7.3123	7.3123	7.3123
2.00E 06	7.6589	7.6589	7.6589	7.6589	7.6589
4.00E 06	8.0055	8.0055	8.0055	8.0055	8.0055
6.00E 06	8.2082	8.2082	8.2082	8.0282	8.2082
8.00E 06	8.3520	8.3520	8.3520	8.3520	8.3520
1.00E 07	8.4636	8.4636	8.4636	8.4636	8.4636
2.00E 07	8.8102	8.8102	8.8102	8.8102	8.8102
4.00E 07	9.1568	9.1568	9.1568	9.1568	9.1568
6.00E 07	9.3595	9.3595	9.3595	9.3595	9.3595
8.00E 07	9.5033	9.5033	9.5033	9.5033	9.5033
1.00E 08	9.6149	9.6149	9.6149	9.6149	9.6149
2.00E 08	9.9615	9.9615	9.9615	9.9615	9.9615
4.00E 08	10.3081	10.3081	10.3081	10.3081	10.3081
6.00E 08	10.5108	10.5108	10.5101	10.5101	10.5101
8.00E 08	10.6546	10.6546	10.6546	10.6546	10.6546
1.00E 09	10.7662	10.7662	10.7662	10.7662	10.7662
2.00E 09	11.1128	11.1128	11.1128	11.1128	11.1128
4.00E 09	11.4593	11.4593	11.4593	11.4593	11.4593
6.00E 09	11.6621	11.6621	11.6621	11.6621	11.6621
8.00E 09	11.8059	11.8059	11.8059	11.8059	11.8059

APPENDIX A

HARTSOCK'S METHOD OF DERIVATION OF PRESSURE DROP EQUATION FOR AN INFINITE TWO LAYERED RESERVOIR

The mathematical solution to obtain the pressure drop equation for an infinite two layered reservoir, taking into consideration the force of gravity is presented. The diffusivity equation and the boundary conditions for this derivation are given:

The diffusivity equation is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial P_j}{\partial r}\right) = \frac{1}{\eta_j}\frac{\partial P_j}{\partial t}\,, \qquad j = 1,2 \tag{1}$$

where

$$P_j(r,t) = p_{i,j} - p_j(r,t)\,, \qquad j = 1,2 \tag{2}$$

The initial condition is:

$$\text{At } t = 0, \qquad P_j = 0\,, \qquad j = 1,2 \tag{13}$$

The boundary conditions are:

$$\text{As } t \to \infty, \qquad P_j = 0, \qquad j = 1,2 \tag{14}$$

$$\text{At } r = r_{w,j}, \qquad P_j = P_{w,j}(t), \qquad j = 1,2 \tag{15}$$

Since the flowing bottom hole pressures and initial pressures are not the same for the two layers due to the force of gravity, equation 15 can be expressed for the two zones as:

$$P_1 = p_{i_1} - p_{wf_1}(t) \tag{A1}$$

and

$$P_2 = p_{i_2} - p_{wf_2}(t) \tag{A2}$$

Also at $r = r_{w,j}$:

$$\sum_{j=1}^{2} q_j(t) = -2\pi \sum_{j=1}^{2} \beta_j \left(r \frac{\partial P_j}{\partial r} \right)_{r=r_{w,j}} = q \tag{16}$$

Taking the Laplace transform of equation 1 and applying equation 13 yields

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \overline{P}_j}{\partial r} \right) - \frac{Z}{\eta_j} \overline{P}_j = 0 \ , \qquad j = 1,2 \tag{A3}$$

Taking the Laplace transform of the boundary conditions, the following equations are obtained:

$$\text{When } r \to \infty, \qquad \overline{P}_j = 0 \ , \qquad j = 1,2 \tag{A4}$$

$$\text{At } r = r_{w,j} \ , \qquad \overline{P}_j = \overline{P}_{w,j}(Z) \tag{A5}$$

and for the two zones we have

$$\overline{P}_1 = \frac{p_{i_1}}{Z} - p_{wf_1}(Z) \tag{A6}$$

and

$$\overline{P}_2 = \frac{p_{i_2}}{Z} - p_{wf_2}(Z) \tag{A7}$$

Also, at $r = r_{w,j}$,

$$\sum_{j=1}^{2} \beta_j \left(r \frac{\partial \bar{P}_j}{\partial r} \right)_{r=r_{w,j}} = - \frac{q(Z)}{2\pi} \tag{A8}$$

The general solution of equation A3 is:

$$\bar{P}(r,Z) = A_j K_o\left(r \sqrt{\frac{Z}{\eta_j}}\right) + B_j I_o\left(r \sqrt{\frac{Z}{\eta_j}}\right) \tag{A9}$$

where A_j and B_j are constants of integration and $K_o(r\sqrt{Z/\eta_j})$ and $I_o(r\sqrt{Z/\eta_j})$ are modified Bessel functions of the first and second kind respectively.

When $r \to \infty$, $I_o(r\sqrt{Z/\eta_j}) \to \infty$; see figure 23. So B_j must be zero.

Therefore,

$$\bar{P}_j(r,Z) = A_j K_o\left(r \sqrt{\frac{Z}{\eta_j}}\right) \tag{A10}$$

At $r = r_{w,j}$

$$P_{wf,j}(Z) = A_j K_o\left(r_w \sqrt{\frac{Z}{\eta_j}}\right) \tag{A11}$$

Differentiation of equation A10 with respect to r yields

$$\frac{\partial \bar{P}}{\partial r} = -A_j K_1\left(r \sqrt{\frac{Z}{\eta_j}}\right) \sqrt{\frac{Z}{\eta_j}} \tag{A12}$$

At $r = r_{w,j}$,

$$\left(r \frac{\partial \bar{P}}{\partial r}\right)_{r=r_{w,j}} = -A_j a_j K_1(a_j \sqrt{Z}) \sqrt{Z} \tag{A13}$$

where

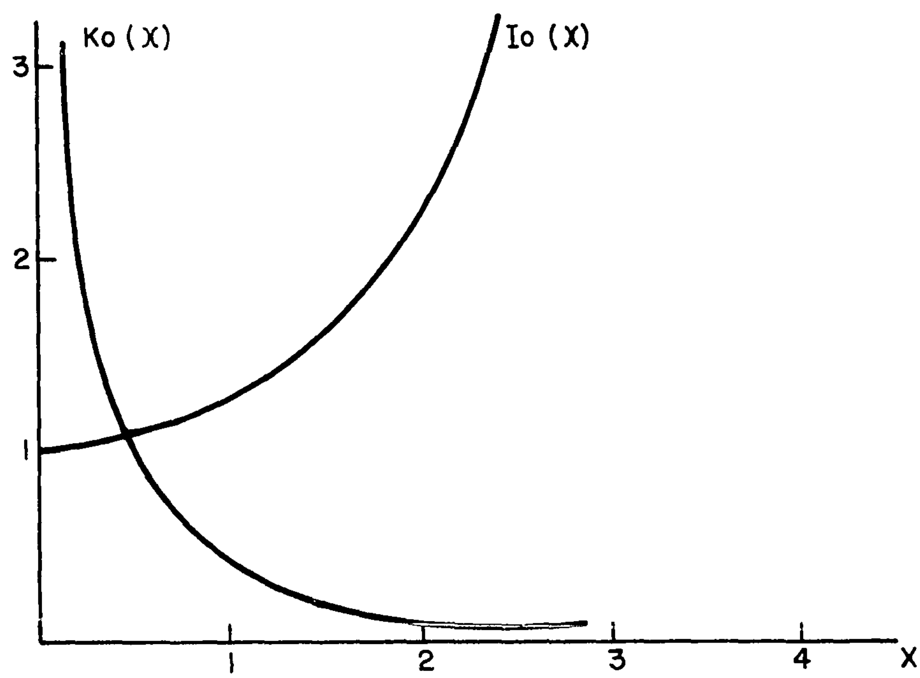

Fig. 23 GENERAL CHARACTERISTICS OF MODIFIED BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND OF ZERO ORDER

$$a_j = \frac{r_{w,j}}{\sqrt{\eta_j}} \qquad (8)$$

Multiplication of equation A13 by β_j and substituting equation A8 yields,

$$A_j \beta_j a_j \sqrt{Z} K_1(a_j\sqrt{Z}) = \frac{q(Z)}{2\pi} \qquad \text{(A14)}$$

From equation A14, A_j can be expressed as:

$$A_j = \frac{q(Z)}{2\pi\sqrt{Z}} \frac{1}{\Sigma \ \beta_j a_j K_1(a_j\sqrt{Z})} \qquad \text{(A15)}$$

Substitution of equation A15 into A11 leads to:

$$\bar{P}_{wf,j}(Z) = \frac{q(Z)}{2\pi\sqrt{Z}} \frac{K_o(a_j\sqrt{Z})}{\Sigma \ \beta_j a_j K_1(a_j\sqrt{Z})} \qquad \text{(A16)}$$

With the substitution of equations A6 and A7 into A16, expressions for the two zones are given as:

$$\frac{p_{i_1}}{Z} - p_{wf_1}(Z) = \frac{q_1(Z) K_o(a_1\sqrt{Z})}{2\pi\sqrt{Z} \ \beta_1 a_1 K_1(a_1\sqrt{Z})} \qquad \text{(A17)}$$

$$\frac{p_{i_2}}{Z} - p_{wf_2}(Z) = \frac{q_2(Z) K_o(a_2\sqrt{Z})}{2\pi\sqrt{Z} \ \beta_2 a_2 K_1(a_2\sqrt{Z})} \qquad \text{(A18)}$$

The difference between the flowing bottom hole pressures can be expressed as:

$$p_{wf_2}(t) - p_{wf_1}(t) = \text{constant} = \gamma(p_{i_2} - p_{i_1}) \qquad \text{(A19)}$$

where the subscripts 1 and 2 represent the upper and lower zones respectively, and

$$\gamma \approx \frac{\rho_o g(d_2 - d_1)}{(p_{i_2} - p_{i_1})} \tag{A20}$$

Equation 49 can be repeated as:

$$q_1 + q_2 = q = \text{constant} \tag{16}$$

The Laplace transform equations A19 and 16 give

$$p_{wf_2}(Z) - p_{wf_1}(Z) = \frac{\gamma(p_{i_2} - p_{i_1})}{Z} \tag{A21}$$

$$q_1(Z) + q_2(Z) = \frac{q}{Z} \tag{A22}$$

From here only the pressure drop for the upper zone will be considered. Similar steps can be used to determine the pressure drop in the lower zone.

Subtracting equation A17 from A18 yields

$$\frac{p_{i_2} - p_{i_1}}{Z} = p_{wf_2}(Z) - p_{wf_1}(Z) + \frac{q_2(Z)K_o(a_2\sqrt{Z})}{2\pi\sqrt{Z}\ \Sigma\beta_2 a_2 K_1(a_2\sqrt{Z})}$$

$$- \frac{q_1(Z)K_o(a_1\sqrt{Z})}{2\pi\sqrt{Z}\ \Sigma\beta_1 a_1 K_1(a_1\sqrt{Z})} \tag{A23}$$

Substitution of equation A21 into A23 leads to:

$$\frac{p_{i_2} - p_{i_1}}{Z} = \frac{\gamma(p_{i_2} - p_{i_1})}{Z} + \frac{q_2(Z)K_o(a_2\sqrt{Z})}{2\pi\sqrt{Z}\ \Sigma\beta_2 a_2 K_1(a_2\sqrt{Z})}$$

$$- \frac{q_1(Z)K_o(a_1\sqrt{Z})}{2\pi\sqrt{Z}\ \Sigma\beta_1 a_1 K_1(a_1\sqrt{Z})} \tag{A24}$$

By rearranging equation A24, the following equation is obtained:

$$\frac{\Delta p}{Z}(1 - \gamma) = \frac{q_2(Z)K_o(a_2\sqrt{Z})}{2\pi\sqrt{Z}\ \beta_2 a_2 K_1(a_2\sqrt{Z})} - \frac{q_1(Z)K_o(a_1\sqrt{Z})}{2\pi\sqrt{Z}\ \beta_1 a_1 K_1(a_1\sqrt{Z})} \tag{A25}$$

where $\Delta p = p_{i_2} - p_{i_1}$. (A26)

From A25 an expression for $q_2(Z)$ can be obtained as:

$$q_2(Z) = \beta_2 a_2 K_1(a_2\sqrt{Z})\{[2\pi\sqrt{Z}\ \beta_1 a_1 K_1(a_1\sqrt{Z})(1 - \gamma)\Delta p$$

$$+ q_1(Z)K_o(a_1\sqrt{Z})Z]/[\beta_1 a_1 K_1(a_1\sqrt{Z})K_o(a_2\sqrt{Z})Z]\} \tag{A27}$$

Eliminating $q_2(Z)$ between equations A22 and A27 and solving for $q_1(Z)$ gives

$$q_1(Z) = \frac{1}{Z}\left(\frac{M1 + M2}{M3 + M4}\right) \tag{A28}$$

where

$$M1 = q\beta_1 a_1 K_1(a_1\sqrt{Z})K_o(a_2\sqrt{Z}) \tag{A29}$$

$$M2 = 2\pi\sqrt{Z}\ \beta_1 a_1 \beta_2 a_2 K_1(a_1\sqrt{Z})K_1(a_2\sqrt{Z})(\gamma - 1)\Delta p \tag{A30}$$

$$M3 = \beta_1 a_1 K_o(a_2\sqrt{Z})K_1(a_1\sqrt{Z}) \tag{A31}$$

and

$$M4 = \beta_2 a_2 K_o(a_1\sqrt{Z})K_1(a_2\sqrt{Z}) \qquad \text{A32)}$$

Substitution of equation A28 into A17 leads to:

$$\frac{p_{i_1}}{Z} - p_{wf_1}(Z) = \frac{1}{2\pi Z}\left(\frac{MM1 + MM2}{MM3 + MM4}\right) \tag{A33}$$

where

$$MM1 = qK_o(a_1\sqrt{Z})K_o(a_2\sqrt{Z}) \tag{A34}$$

$$MM2 = 2\pi\sqrt{Z}\ \beta_2 a_2 K_o(a_1\sqrt{Z})K_1(a_2\sqrt{Z})(\gamma - 1)\Delta p \tag{A35}$$

$$MM3 = \sqrt{Z}\ \beta_1 a_1 K_o(a_2\sqrt{Z})K_1(a_1\sqrt{Z}) \tag{A36}$$

and

$$MM4 = \sqrt{Z}\ \beta_2 a_2 K_o(a_1\sqrt{Z})K_1(a_2\sqrt{Z}) \tag{A37}$$

$$\lim_{a_j\sqrt{Z}\to 0} K_o(a_j\sqrt{Z}) = -\left[\ln \frac{a_j\sqrt{Z}}{2} + \ln \gamma\right] \tag{A38}$$

where

$$\bar{\gamma} = \ln \gamma = 0.5772 \tag{52}$$

$$\lim_{a_j\sqrt{Z}\to 0} K_1(a_j\sqrt{Z}) = \frac{1}{a_j\sqrt{Z}} \tag{A39}$$

With the substitution of equations A38 and A39 into A29 and simplifying, the following equation is obtained:

$$\frac{p_{i_1}}{Z} - p_{wf_1}(Z) = -\frac{1}{4\pi\beta_1}\left(\frac{MS1 - MS2}{Z\,[MS3 + MS4]}\right) \tag{A40}$$

where

$$MS1 = q\ \ln\left(\frac{0.793\ Z}{\bar{a}_1}\right)\ \ln\left(\frac{0.793\ Z}{\bar{a}_2}\right) \tag{A41}$$

$$MS2 = 4\pi\beta_2(\gamma - 1)\Delta p\ \ln\left(\frac{0.793\ Z}{\bar{a}_1}\right) \tag{A42}$$

$$MS3 = \ln\left(\frac{0.793\ Z}{\bar{a}_2}\right) \tag{A43}$$

and

$$MS4 = \frac{\beta_2}{\beta_1} \ln\left(\frac{0.793\ Z}{\bar{a}_1}\right) \tag{A44}$$

Subsequently

$$\bar{a}_j = \frac{k_j}{\phi_j \mu c r_w^2} \tag{19}$$

Dividing the numerator and denominator of the term on the right hand side of the equality sign of equation A40 and observing that

$$\frac{\ln \dfrac{0.793\ Z}{\bar{a}_1}}{\ln \dfrac{0.793\ Z}{\bar{a}_2}} \approx 1$$

the following equation is obtained

$$\frac{p_{i_1}}{Z} - p_{wf_1}(Z) = -\frac{1}{4\pi\beta_1}\left\{\frac{q \ln\left(\dfrac{0.793\ Z}{\bar{a}_1}\right) - 4\pi\beta_2(\gamma - 1)\Delta p}{Z\left(1 + \dfrac{\beta_2}{\beta_1}\right)}\right\} \tag{A45}$$

The inverse Laplace transform of equation A45 gives the pressure drop equation for the upper zone as:

$$p_{i_1} - p_{wf_1}(t) = \frac{\dfrac{1}{\beta_1}}{\dfrac{1}{\beta_1} + \dfrac{1}{\beta_2}} \left\{\frac{q}{4\pi\beta_2}(\ln t + 0.809 + \ln \bar{a}_1) + (\gamma - 1)\Delta p\right\} \tag{17}$$

Similarly, the pressure drop equation for the lower zone can be obtained and is expressed as:

$$p_{i_2} - p_{wf_2}(t) = \frac{\frac{1}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_2}} \left\{ \frac{q}{4\pi\beta_1}(\ln t + 0.809 + \ln \bar{a}_2) + (1 - \gamma)\Delta p \right\} \quad (18)$$

APPENDIX B

DERIVATION OF PRESSURE DROP EQUATION WITHOUT SKIN AND WELL-BORE STORAGE EFFECT

The mathematical solution to obtain the pressure drop equation for an infinite two layered reservoir is presented. The method of this derivation assumes the Lefkovits et al.[14] approach whereby the initial pressures in both layers are the same and the gravitational force between the two layers is neglected. For sake of completeness the diffusivity equation and the boundary conditions for this case are repeated.

The diffusivity equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial P_j}{\partial r}\right) = \frac{1}{\eta_j}\frac{\partial P_j}{\partial t}, \qquad j = 1,2 \qquad (1)$$

where

$$P_j(r,t) = p_i - p_j(r,t), \quad j = 1,2 \qquad (3)$$

The initial condition is:

$$\text{at } t = 0, \qquad P_j = 0, \qquad j = 1,2 \qquad (13)$$

The boundary conditions are:

$$\text{as } r \to \infty\,, \qquad P_j = 0, \qquad j = 1,2 \qquad (14)$$

$$\text{at } r = r_w \qquad P_j = P_{wf}(t) \ , \qquad j = 1,2 \tag{15}$$

and

$$\sum_{j=1}^{2} q_j(t) = -2\pi \sum_{j=1}^{2} \beta_j \left(r \frac{\partial P_j}{\partial r} \right)_{r=r_{w,j}} = q \tag{16}$$

Taking the Laplace transform of equation 1 and applying equation 13 leads to:

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \bar{P}}{\partial r} \right) - \frac{Z}{\eta_j} \bar{P}_j = 0 \ , \qquad j = 1,2 \tag{A3}$$

The Laplace transform of the boundary conditions leads to:

$$\text{when } r \to \infty, \qquad \bar{P}_j = 0 \ , \qquad j = 1,2 \tag{A4}$$

$$\text{at } r = r_w, \qquad \bar{P}_j = \bar{P}_{wf,j} \ , \qquad j = 1,2 \tag{A5}$$

and

$$\sum_{j=1}^{2} \beta_j \left(r \frac{\partial \bar{P}}{\partial r} \right)_{r=r_{w,j}} = - \frac{q}{2\pi Z} \tag{A8}$$

The general solution of equation A3 is:

$$\bar{P}_j(r,Z) = A_j K_o(r\sqrt{Z/\eta_j}) + B_j I_o(r\sqrt{Z/\eta_j}) \tag{A9}$$

where A_j and B_j are constants to be determined with the boundary conditions.

As $r \to \infty$, $I_o(r\sqrt{Z/\eta_j}) \to \infty$ (see figure 23). So B_j must be zero. Therefore,

$$\bar{P}_j(r,Z) = A_j K_o(r\sqrt{Z/\eta_j}) \tag{A10}$$

At $r = r_{w,j}$

$$\bar{P}_{wf,j}(Z) = A_j K_o(r_w\sqrt{Z/\eta_j}) \tag{A11}$$

$$a_j = \frac{r_{w,j}}{\sqrt{\eta_j}} \tag{8}$$

Therefore

$$\bar{P}_{wf,j}(Z) = A_j K_o(a_j \ \overline{Z}) \tag{A10}$$

Differentiation of equation A10 with respect to r yields

$$\frac{\partial \bar{P}}{\partial r} = - A_j K_1(r\sqrt{Z/\eta_j})(\sqrt{Z}/\eta_j) \tag{A12}$$

At $r = r_{w,j}$

$$\left(r \frac{\partial \bar{P}}{\partial r}\right)_{r=r_{w,j}} = - A_j K_1(r_w\sqrt{Z/\eta_j})(\sqrt{Z}/\eta_j) r_w \tag{B1}$$

Substituting equation 8 into B1 gives

$$\left(r \frac{\partial \bar{P}}{\partial r}\right)_{r=r_{w,j}} = - A_j a_j K_1(a_j\sqrt{Z})\sqrt{Z} \tag{A13}$$

Multiplying equation A13 by β_j and substituting equation A8 leads to:

$$A_j \beta_j a_j \sqrt{Z} \ K_1(a_j\sqrt{Z}) = \frac{q}{2\pi Z} \tag{A14}$$

Therefore,

$$A_j = \frac{q}{2\pi Z^{3/2}} \frac{1}{\Sigma \beta_j a_j K_1(a_j\sqrt{Z})} \tag{A15}$$

By substituting equation A15 into equation A10, the following equation is obtained:

$$\bar{P}_{wf}(Z) = \frac{q}{2\pi Z^{3/2}} \frac{K_o(a_j\sqrt{Z})}{\Sigma \beta_j a_j K_1(a_j\sqrt{Z})} \tag{A16}$$

$$\lim_{a_j\sqrt{Z}\to 0} K_o(a_j\sqrt{Z}) = -[\ln(a_j\sqrt{Z}/2) + \ln\gamma) \tag{A38}$$

$$\lim_{a_j\sqrt{Z}\to 0} K_1(a_j\sqrt{Z}) = \frac{1}{a_j\sqrt{Z}} \tag{A39}$$

Substituting equations A38 and A39 into A16 and simplifying leads to:

$$\bar{P}_{wf}(Z) = -\frac{q}{2\pi Z}\frac{1}{\beta_j}[\ln(a_j\sqrt{Z}/2) + \ln\gamma] \tag{B2}$$

Multiplying the numerator and denominator of equation B2 by $2\Sigma\beta_j$ and simplifying leads to:

$$\bar{P}_w(Z) = -\frac{q}{4\pi Z(\Sigma\beta_j)^2}[\Sigma\beta_j(\ln\frac{a_j^2}{4} + \ln Z + \ln\gamma^2)] \tag{B3}$$

The inverse Laplace transform of equation B3 is:

$$P_{wf}(t) = \frac{q}{4\pi(\Sigma\beta_j)^2}[\Sigma\beta_j(\ln\gamma t - \ln\frac{\gamma^2 a_j^2}{4})] \tag{B4}$$

Substituting equations 7, 8, 9 and 10 into B4, the following equation is obtained:

$$p_i - p_{wf} = \frac{q\mu}{4\pi\bar{k}\bar{h}}[\ln\gamma t - Q] \tag{50}$$

where

$$P_{wf}(t) = p_i - p_{wf} \tag{B5}$$

and

$$Q = \frac{k_1 h_1 \ln\left(\frac{\gamma^2\phi_1^2 r_w^2}{4k_1}\right) + k_2 h_2 \ln\left(\frac{\gamma^2\phi_2 cr_w^2}{4k_2}\right)}{k_1 h_1 + k_2 h_2}$$

APPENDIX C

DERIVATION OF PRESSURE DROP EQUATION WITH SKIN EFFECT

The mathematical derivation of pressure drop for an infinite two layered reservoir with skin effect is shown in this Appendix. Apart from the boundary conditions, the assumptions used in this derivation are basically the same used in Appendix B. For the sake of completeness, the diffusivity equation and boundary conditions are repeated.

The diffusivity equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial P_j}{\partial r}\right) = \frac{1}{\eta_j}\,\frac{\partial P}{\partial t}\,, \qquad j = 1,2 \tag{1}$$

where

$$P_j(r,t) = p_j - p_j(r,t)\,, \quad j = 1,2 \tag{3}$$

The initial condition is:

$$\text{at } t = 0\,, \qquad P_j = 0\,, \qquad j = 1,2 \tag{13}$$

The boundary conditions are:

$$\text{as } r \to \infty, \qquad P_j = 0\,, \qquad j = 1,2 \tag{14}$$

$$\text{at } r = r_{w,j} \qquad P_{wf}(t) = P_{fj} + \frac{S_j q_j(t)\mu_j}{2\pi k_j h_j}$$

$$j = 1,2 \tag{58}$$

and

$$\sum_{j=1}^{2} q_j(t) = -2\pi \sum_{j=1}^{2} \beta_j \left(r \frac{\partial P_j}{\partial r} \right)_{r=r_{w,j}} = q \qquad (16)$$

The Laplace transform of the diffusivity equation with the application of equation 13 gives

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \bar{P}}{\partial r} \right) - \frac{Z}{\eta_j} \bar{P}_j = 0 , \qquad j = 1,2 \qquad (A3)$$

The Laplace transform of the boundary conditions are given by the following equations:

$$\text{when } r \to \infty, \qquad \bar{P}_j = 0 , \qquad j = 1,2 \qquad (A4)$$

at $r = r_{w,j}$

$$\bar{P}_w(Z) = \bar{P}_{f,j}(r,Z) + \frac{S_j \bar{q}_j(Z) \mu_j}{2\pi k_j h_j} \qquad (C1)$$

and

$$\sum_{j=1}^{2} \beta_j \left(r \frac{\partial \bar{P}}{\partial r} \right)_{r=r_{w,j}} = - \frac{q}{2\pi Z} \qquad (A8)$$

The general solution of equation A3 is:

$$\bar{P}_j(r,Z) = A_j K_o(r\sqrt{Z/\eta_j}) + B_j I_o(r\sqrt{Z/\eta_j}) \qquad (A9)$$

By substituting the equations for the boundary conditions into equation A9, the constants A_j and B_j are determined and subsequently an equation for pressure drop for an infinite two layered reservoir with skin effect is obtained.

As $r \to \infty$, $I_o(r\sqrt{Z/\eta_j}) \to \infty$ (see Figure 23). So B_j

must be zero. Therefore,

$$\bar{P}_j(r,Z) = A_j K_o(r\sqrt{Z/\eta_j}) \tag{A10}$$

At $r = r_{w,j}$

$$\bar{P}_{wf}(Z) = A_j K_o(r_w\sqrt{Z/\eta_j}) + \frac{S_j \bar{q}_j(Z)\mu_j}{2\pi k_j h_j} \tag{C2}$$

Substituting equations 8 and 9 into C2 gives

$$\bar{P}_{wf}(Z) = A_j K_o(a_j\sqrt{Z/\eta_j}) + \frac{S_j \bar{q}_j(Z)}{2\pi\beta_j} \tag{C3}$$

Differentiating equation A10 and substituting equation A8 leads to:

$$A_j \beta_j a_j \sqrt{Z}\; K_1(a_j\sqrt{Z}) = \frac{q}{2\pi Z} \tag{A14}$$

From equation A14 an expression for A_j is

$$A_j = \frac{q}{2\pi Z^{3/2}} \frac{1}{\Sigma\beta_j a_j K_1(a_j\sqrt{Z})} \tag{A15}$$

With the substitution of equation A15 into equation C3 leads to:

$$\bar{P}_{wf}(Z) = \frac{q}{2\pi Z^{3/2}} \frac{K_o(a_j\sqrt{Z})}{\Sigma\beta_j a_j K_1(a_j\sqrt{Z})} + \frac{S_j \bar{q}_j(Z)}{2\pi\beta_j} \tag{C4}$$

$$\textstyle\sum_{j=1}^{2} \bar{q}_j(Z) = \frac{q}{Z} \tag{C5}$$

Substituting equation C5 into C4 results in

$$\bar{P}_{wf}(Z) = \frac{q}{2\pi Z^{3/2}} \frac{K_o(a_j\sqrt{Z})}{\Sigma\beta_j a_j K_1(a_j\sqrt{Z})} + \frac{S_j q}{2\pi Z \beta_j} \tag{C6}$$

$$\lim_{a_j\sqrt{Z} \to 0} K_1(a_j\sqrt{Z}) = -[\ln(a_j\sqrt{Z}/2) + \ln\gamma] \tag{A38}$$

$$\lim_{a_j\sqrt{Z} \to 0} K_1(a_j\sqrt{Z}) = \frac{1}{a_j\sqrt{Z}} \tag{A39}$$

Substitution of equation A38 and A39 into equation C6 leads to:

$$\bar{P}_{wf}(Z) = -\frac{q}{2\pi Z} \frac{[\ln(a_j\sqrt{Z}/2) + \ln\gamma]}{\Sigma\beta j} + \frac{S_j q}{2\pi Z\ \Sigma\beta_j} \tag{C7}$$

Multiplication of the numerator and denominator of the terms on the right hand side of the equality sign by $2\ \Sigma\beta_j$ gives

$$\bar{P}_{wf}(Z) = \frac{-q}{4\pi Z} \frac{\Sigma\beta_j\ [\ln(a_j^2/4) + \ln Z + 2\ln\gamma]}{(\Sigma\beta_j)^2} + \frac{(2\Sigma\beta_j)S_j q}{4\pi Z(\Sigma\beta_j)^2} \tag{C8}$$

Taking the inverse Laplace transform and rearranging equation C8, the following equation is obtained:

$$P_{wf}(t) = \frac{q}{4\pi}\left\{\left[\frac{(2\Sigma\beta_j)S_j - \Sigma\beta_j\ \ln(a_j^2/4)}{(\Sigma\beta_j)^2}\right] + \frac{\ln t - \ln\gamma}{\Sigma\beta_j}\right\} \tag{C9}$$

Adding and substracting $\ln\gamma$ to the last term on the right side of the equality sign and with further simplification, equation C9 takes the form:

$$P_{wf}(t) = \frac{q}{4\pi}\left\{\left[\frac{(2\Sigma\beta_j)S_j - \ln\left(\frac{\gamma^2 a_j^2}{4}\right)}{(\Sigma\beta_j)^2}\right] + \frac{\ln \gamma t}{\Sigma\beta j}\right\} \tag{C10}$$

Expanding equation C10 leads to

$$P_{wf}(t) = \frac{q\mu}{4\pi\bar{k}\bar{h}}\left[\ln \gamma t + Q_S\right] \tag{59}$$

where

$$Q_S = \frac{k_1h_1\left[2S_1 - \ln\left(\frac{\gamma^2\phi_1\mu c r_w^2}{4k_1}\right)\right] + k_2h_2\left[2S_2 - \ln\left(\frac{\gamma^2\phi_2\mu c r_w^2}{4k_2}\right)\right]}{k_1h_1 + k_2h_2} \tag{60}$$

APPENDIX D

DERIVATION OF PRESSURE DROP EQUATION
WITH THE EFFECT OF WELL-BORE STORAGE

The solution for the diffusivity equation is sought for the case of variable flow rate due to the effect of well-bore storage. The solution for a variable flow rate is obtained by using the Duhamel theorem[16] (or principle of superposition). In this derivation, the initial pressures of both zones are assumed to be the same; that is, the gravitational force between the two layers is neglected.

Van Everdingen[29] and Hurst[9,10] have shown that for the case of flow into the casing and tubing, the variable flow rate can be approximated by

$$q_{sf} = q(1 - e^{-\alpha t}) \tag{D1}$$

where q is the constant flow rate from the two layers, and α is a constant whose dimension is given by $1/t_D$.

By the principle of superposition[16] the pressure drop equation for an infinite two layered reservoir with well-bore storage effect can be expressed in an integral form as:

$$\bar{\bar{P}}_{wf}(t) = \int_0^t e^{-\alpha t'} P_{wf}(t - t')\, dt' \tag{D2}$$

where

$$P_{wf}(t) = \frac{q\mu}{4\pi \bar{k}\bar{h}} [\ln \gamma t - Q] \tag{50}$$

Carslaw and Jaeger[2] have shown that

$$\int_0^t F_1(t - t')F_2(t')dt' = f_1(z)f_2(z) \tag{D3}$$

Therefore, equation D2 becomes

$$\bar{P}_{wf}(Z) = \frac{\alpha}{Z + \alpha} \frac{qK_o(a_j\sqrt{Z})}{2\pi Z^{3/2} \Sigma\beta_j a_j K_1(a_j\sqrt{Z})} \tag{D4}$$

Constants A and B can be found such that

$$\frac{\alpha}{Z(Z + \alpha)} = \frac{A}{Z} + \frac{B}{Z + \alpha} \tag{D5}$$

Clearing fractions and identifying coefficients of like powers, we have

$$\alpha = (A + B)Z + A\alpha$$

This is an identity if $A + B = 0$ and $A = 1$; thus $B = -1$. Therefore equation D5 can be written as

$$\frac{\alpha}{Z(Z + \alpha)} = \frac{1}{Z} - \frac{1}{Z + \alpha} \tag{D6}$$

and subsequently equation D4 takes the form

$$\bar{P}_{wf}(Z) = \frac{qK_o(a_j\sqrt{Z})}{2\pi Z^{3/2} \Sigma\beta_j a_j K_1(a_j\sqrt{Z})} - \frac{qK_o(a_j\sqrt{Z})}{2\pi\sqrt{Z}\, \Sigma\beta_j a_j K_1(a_j\sqrt{Z})} \frac{1}{Z + \alpha} \tag{D7}$$

The inversion of the first term on the right hand side of the sign of equality is the solution for constant rate which is given as equation 50.

The solution of the second term on the right hand side of the equality sign is given as follows:

$$\lim_{a_j\sqrt{Z}\to 0} K_1(a_j\sqrt{Z}) = \frac{1}{a_j\sqrt{Z}} \tag{A39}$$

Then the second term on the right hand side of the equality sign is:

$$\frac{qK_o(a_j\sqrt{Z})}{2\pi\ \Sigma\beta_j}\ \frac{1}{Z + \alpha} \tag{D8}$$

Equation D8 can be solved by using the Mellins inversion formula, namely

$$\frac{c}{2\pi i} \int_{\alpha-i\gamma}^{\alpha+i\gamma} \frac{e^{\lambda t}\ K_o(a_j\lambda^{1/2})}{(\lambda + \alpha)}\, d\lambda \tag{D9}$$

where

$$c = \frac{q}{2\pi\ \Sigma\beta_j} \tag{D10}$$

There is a singularity at

$$\lambda = \alpha e^{i\pi} \tag{D11}$$

The solution obtained at this singularity is:

$$ce^{-\alpha t}\ K_o(a_j\alpha^{1/2}e^{i\pi/2}) = -ce^{-\alpha t}\ \frac{\pi}{2}\ Y_o(a_j\alpha^{1/2}) \tag{D12}$$

However,

$$Y_o(a_j\alpha^{1/2}) = \frac{2}{\pi}\left\{J_1(a_j\alpha^{1/2})[\ln(a_j\alpha^{1/2}/2) + \ln\gamma]\right.$$

$$\left. + \left(\frac{a_j\alpha^{1/2}}{2}\right)^2 - \frac{3}{8}\left(\frac{a_j\alpha^{1/2}}{2}\right)^4 + \frac{11}{216}\left(\frac{a_j\alpha^{1/2}}{2}\right)^6 \right. \qquad \text{(D13)}$$

As $a_j\alpha^{1/2} \to 0$,

$$Y_o(a_j\alpha^{1/2}) = \frac{2}{\pi}[\ln(a_j\alpha^{1/2}/2) + \ln\gamma] \qquad \text{(D14)}$$

Therefore

$$-ce^{-\alpha t}\,\frac{\pi}{2}\,Y_o(a_j\alpha^{1/2}) = ce^{-\alpha}[-\ln(a_j\alpha^{1/2}/2) - \ln\gamma] \qquad \text{(D15)}$$

The contour of integration of equation D9 is given as shown in Figure 24. The integration on the upper half along the negative real axis of Figure 24 is given as follows:

Set $$\lambda = u^2e^{i\pi} \qquad \text{(D16)}$$

Then

$$d\lambda = 2u\,e^{i\pi}\,du \qquad \text{(D17)}$$

With the substitution of equations D16 and D17 into D9 the following equation is obtained:

$$\frac{c}{2\pi i}\int_0^\infty \frac{e^{u^2e^{i\pi}t}\,K_o(a_jue^{i\pi/2})}{(u^2e^{i\pi} + \alpha)}\,2ue^{i\pi}\,du \qquad \text{(D18)}$$

$$e^{i\pi} = -1 \qquad \text{(D19)}$$

$$K_o(a_jue^{i\pi/2}) = -\frac{\pi i}{2}\,[J_o(a_ju) - iY_o(a_ju)] \qquad \text{(D20)}$$

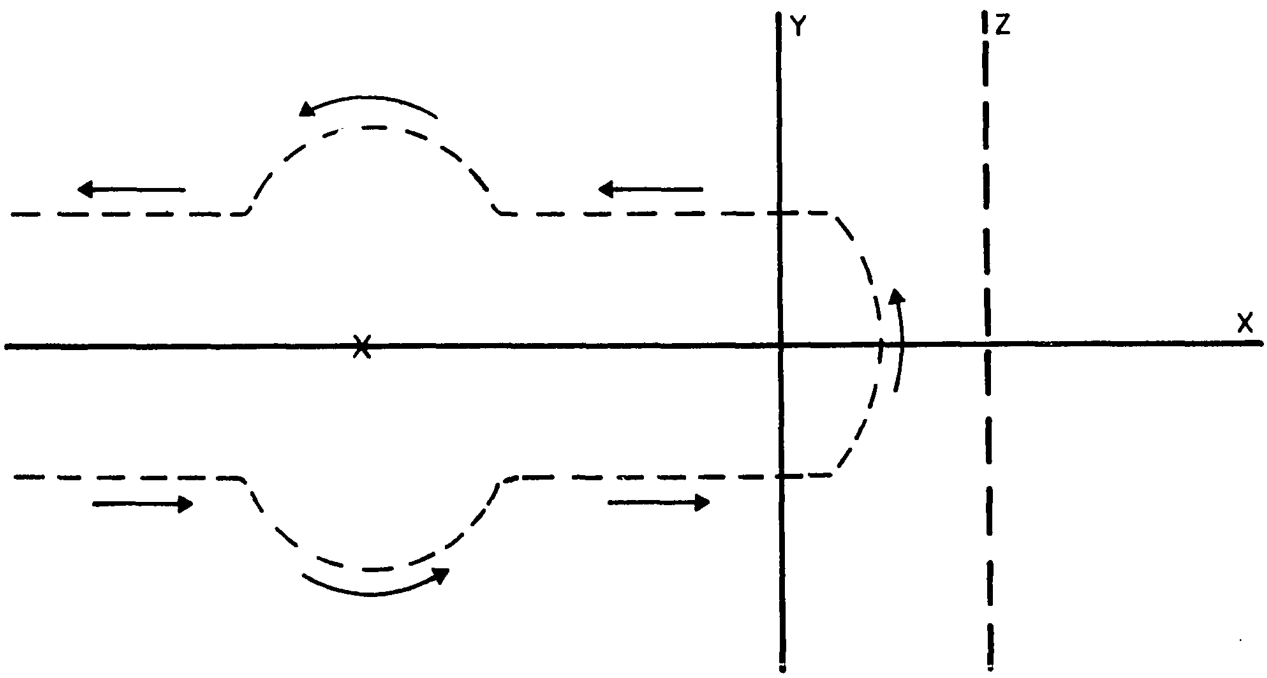

Fig. 24 CONTOUR OF INTEGRATION IN THE COMPLEX PLANE

Substituting equations D19 and D20 into D18 leads to:

$$\frac{c}{2\pi i}\int_0^\infty e^{-u^2 t}\,\frac{\{\frac{\pi i}{2}[J_o(a_j u) - iY_o(a_j u)]\}}{[\alpha - u^2]}\,2u\,du \tag{D21}$$

Considering only the real and neglecting the complex part, the integration on the upper half along the negative real axis of Figure 24 is

$$\frac{c}{2}\int_0^\infty \frac{e^{-u^2 t}\;J_o(a_j u)}{(\alpha - u^2)}\,u\,du \tag{D22}$$

The integration on the lower half along the negative real axis of Figure 24 is given as follows:

Set $$\lambda = u^2 e^{-i\pi} \tag{D23}$$

Therefore,

$$d\lambda = 2u\,e^{-i\pi}\,du \tag{D24}$$

With the substitution of equations D23 and D24 into D9, the following equation is obtained:

$$\frac{c}{2\pi i}\int_\infty^0 \frac{e^{u^2 e^{-i\pi} t}\;K_o(a_j u e^{-i\pi/2})}{(u^2 e^{-i\pi} + \alpha)}\,2u\,e^{-i\pi}\,du \tag{D25}$$

$$e^{-i\pi} = -1 \tag{D26}$$

$$K_o(a_j u e^{-i\pi/2}) = \frac{\pi i}{2}[J_o(a_j u) + iY_o(a_j u)] \tag{D27}$$

Substitution of equation D26 and D27 into D25 yields,

$$= \frac{c}{2\pi i} \int_{\infty}^{0} \frac{e^{-u^2 t} \, \{\frac{\pi i}{2}[J_o(a_j u) + iY_o(a_j u)]\}}{(\alpha - u^2)} (-2u) \, du \tag{D28}$$

Changing the limits of integration and considering only the real and neglecting the complex part, the integration on the lower half along the negative real axis of Figure 24 is:

$$\frac{c}{2} \int_{0}^{\infty} \frac{e^{-u^2 t} \, J_o(a_j u)}{(\alpha - u^2)} u \, du \tag{D29}$$

Summation of the integration on the upper and lower half along the negative real axis leads to:

$$c \int_{0}^{\infty} \frac{e^{-u^2 t} \, J_o(a_j u)}{(\alpha - u^2)} u \, du \tag{D30}$$

Let $$v = \alpha - u^2 \tag{D31}$$

Therefore $$u = \sqrt{\alpha - v} \tag{D32}$$

$$u^2 = \alpha - v \tag{D33}$$

and $$dv = -2u \, du \tag{D34}$$

Substitution of equations D31, D32, D33 and D34 into equation D30 yields

$$\frac{e^{-\alpha t}}{2} c \int_{-\infty}^{\alpha} \frac{e^{vt} \, J_o(a_j \sqrt{\alpha - v})}{v} dv \tag{D35}$$

For $a_j\sqrt{\alpha - v} \to 0$, $J_o(a_j\sqrt{\alpha - v}) \approx 1$. Therefore, equation D35 can be expressed as:

$$\frac{e^{-\alpha t}}{2} c \int_{-\infty}^{\alpha} \frac{e^{vt}}{v} dv \tag{D36}$$

If s = vt, D36 can be expressed as

$$\frac{e^{-\alpha t}}{2} c \int_{-\infty}^{\alpha t} \frac{e^{s}}{s} ds = c \frac{e^{-\alpha t}}{2} E_i(\alpha t) \tag{D37}$$

With the substitution of equation D10, the solution of equation D8 is

$$\frac{q e^{-\alpha t}}{2\pi \Sigma\beta_j} [- \ln(a_j\alpha^{1/2}/2) - \ln \gamma + E_i(\alpha t)/2] \tag{D38}$$

Multiply the denominator and numerator of D38 by $2\Sigma\beta_j$; the following equation is obtained:

$$\frac{q e^{-\alpha t}}{4\pi(\Sigma\beta_j)^2}[-2\Sigma\beta_j \ln(a_j\alpha^{1/2}/2) - 2\Sigma\beta_j \ln \gamma + 2\Sigma\beta_j E_i(\alpha t)] \tag{D39}$$

Simplification of equation D39 leads to:

$$\frac{q e^{-\alpha t}}{4\pi\bar{k}\bar{h}} [E_i(\alpha t) - \ln \alpha - Q] \tag{D40}$$

where

$$Q = \frac{k_1h_1 \ln\left(\frac{\gamma^2\phi_1\mu c r_w^2}{4k_1}\right) + k_2h_2 \ln\left(\frac{\gamma^2\phi_2\mu c r_w^2}{4k_2}\right)}{k_1h_1 + k_2h_2} \tag{59}$$

Therefore, the pressure drop equation for a variable flow rate due to well-bore storage for an infinite two layered reservoir is equation 50 minus equation D40; that is,

$$\bar{\bar{P}}_{wf}(t) = P_{wf}(t) - \frac{q\ e^{-\alpha t}}{4\pi\bar{k}\bar{h}} [E_i(\alpha t) - \ln \alpha - Q] \qquad (63)$$

APPENDIX E

EXAMPLE CALCULATIONS FOR AN INFINITE TWO LAYERED RESERVOIR

This example shows how equations 26 and 34 can be used in the analysis of pressure build-up for an infinite two layered reservoir. In practical units these equations are written as:

$$p_1^* = p_{i_1} - 2b_1\left(\frac{0.433\ \rho_o(d_2 - d_1)}{(p_{i_2} - p_{i_1})} - 1\right)(p_{i_2} - p_{i_1}) \tag{26a}$$

$$p_{1\ hr} - p_{wf_1} = b_1\left(\frac{0.433\ \rho_o(d_2 - d_1)}{(p_{i_2} - p_{i_1})} - 1\right)(p_{i_2} - p_{i_1})$$

$$- m\left[\log_{10}\left(\frac{0.0002637\ k_1}{\phi_1 c\mu r_w^2}\right) + 0.3513\right] \tag{34a}$$

Data:

A plot of shut in pressure versus $(t + \Delta t)/\Delta t$ for an infinite two layered reservoir is given in Figure 25. Stabilized production rate before shut-in is 326 bbls/day.

μ = 0.88 cp

β_o = 1.15

t_s = 9398 hours

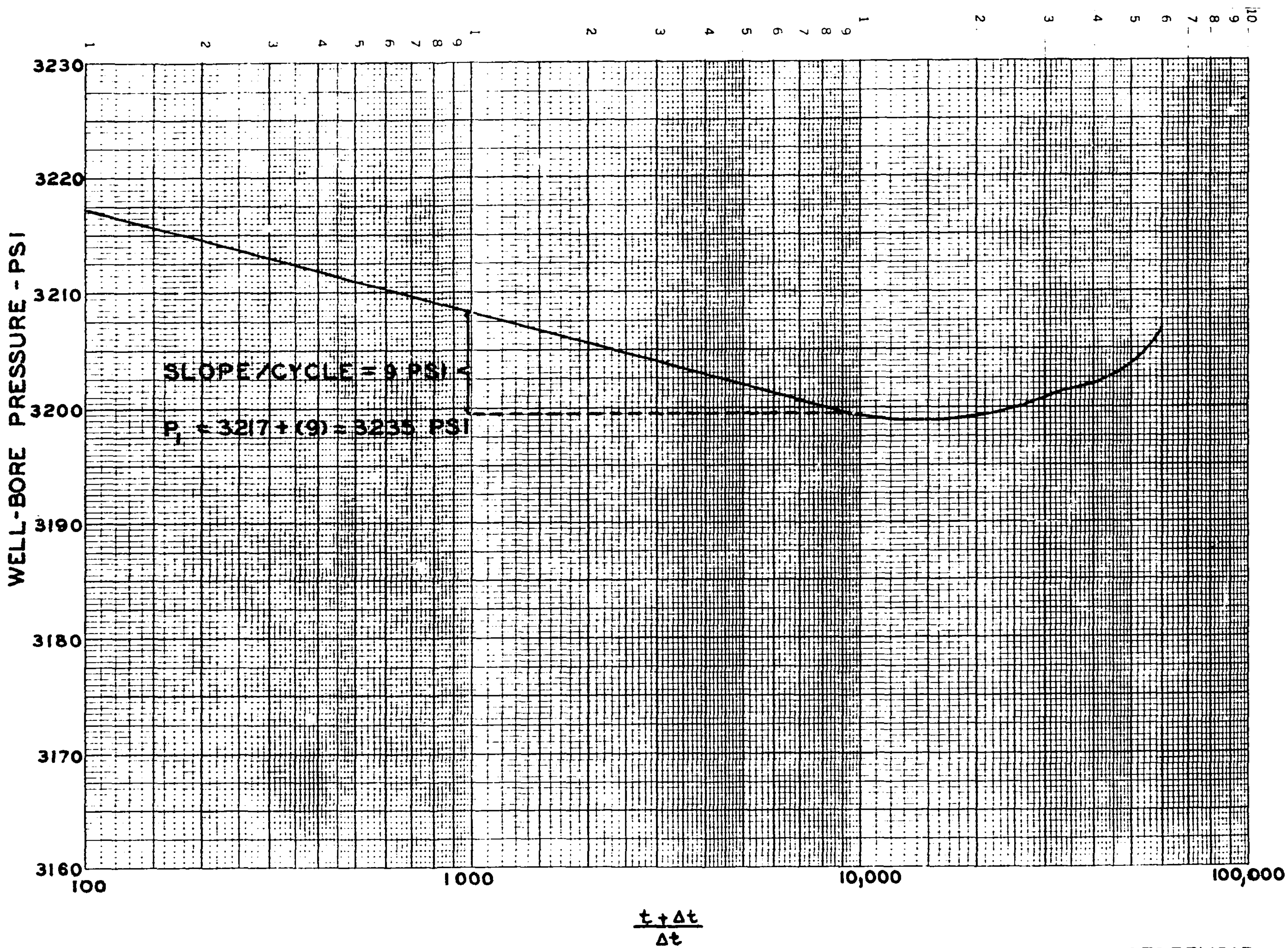

FIG. 25 EXAMPLE OF PRESSURE BUILD-UP CURVE IN AN INFINITE TWO LAYERED RESERVOIR

$h_1 = 23$ ft

$h_2 = 37$ ft

$d_1 = 5023$ ft

$d_2 = 5075$ ft

$c = 1.7 \times 10^{-5}$ vol/vol/psi

$p_{wf_1} = 3253$ psi

$p_{wf_2} = 3271$ psi

$p_{i_1} = 3226$ psi

$r_w = 0.25$ ft

$\rho_o = 0.831$

$\phi_1 = 0.27$

$\phi_2 = 0.27$

Equation 26a can be rearranged to give

$$b_1\left[\frac{0.433\ \rho_o(d_2 - d_1)}{(p_{i_2} - p_{i_1})} - 1\right]\Delta p = \frac{1}{2}(p_{i_1} - p_1^*) \tag{E1}$$

Substitution of E1 into 34a yields

$$p_{1\ hr} - p_{wf_1} = \frac{1}{2}(p_{i_1} - p_1^*) - m\left[\log_{10}\left(\frac{0.0002637\ k_1}{\phi_1 c\mu r_w^2}\right) + 0.3513\right] \tag{E2}$$

From Figure 25 we have,

$p_{1\ hr} = 3199$ psi

$p^* = 3235$ psi

$m = 9$ psi/cycle

Substituting the values of $p_{1\ hr}$, p_{wf_1}, p_{i_1}, p_1^*, m, ϕ_1, c,

μ, and r_w into E2, the value of k_1 is calculated to give,

$$k_1 = 134.90 \text{ md}$$

The equation of the slope is given as:

$$m = 162.6\ q_o\beta_o \frac{\frac{\mu}{k_1h_1}\ \frac{\mu}{k_2h_2}}{\frac{\mu}{k_1h_1} + \frac{\mu}{k_2h_2}} \tag{25a}$$

Substituting the values of m, q, β_o, μ, k_1h_i, and h_2 into 25a, the value of k_2 is calculated to give,

$$k_2 = 76 \text{ md.}$$

p_2 is determined by using equation 26a; that is,

$$p_1^* = p_{i_1} - 2b_1\left[\frac{0.433\ \rho_o(d_2 - d_1)}{(p_{i_2} - p_{i_1})} - 1\right](p_{i_2} - p_{i_1}) \tag{26a}$$

$$b_1 = \frac{\frac{\mu}{k_1h_1}}{\frac{\mu}{k_1h_1} + \frac{\mu}{k_2h_2}}$$

$$= \frac{2.836 \times 10^{-4}}{2.836 \times 10^{-4} + 3.129 \times 10^{-4}}$$

$$= 0.4754$$

$$0.433\ \rho_o(d_2 - d_1) = 0.433 \times 0.831(5075 - 5023)$$

$$= 18.71 \text{ psi}$$

Therefore, for this example equation 26a is

$$p_1^* = p_{i_1} - 0.951\left[\frac{18.71}{(p_{i_2} - p_{i_1})} - 1\right](p_{i_2} - p_{i_1}) \tag{E3}$$

Substituting the values of p_1^* and p_{i_1} into E3, the value of p_{i_2} is calculated to give

$$p_{i_2} = 3254 \text{ psi}$$

S_1 is determined by using equation 46a; that is,

$$S_1 = \frac{1.151\ qb_2}{q_1}\left[\frac{p_{1\ hr} - p_{wf_1} + b_1(\gamma - 1)\Delta p}{m} - \log_{10}\frac{k_1}{\phi\mu c r_w^{\ 2}} + 3.32\right] \qquad (46a)$$

$$b_2 = \frac{\dfrac{\mu}{k_2h_2}}{\dfrac{\mu}{k_1h_1} + \dfrac{\mu}{k_2h_2}}$$

$$= \frac{3.219 \times 10^{-4}}{2.836 \times 10^{-4} + 3.129 \times 10^{-4}}$$

$$= 0.5246$$

$$q_1 = \left[\frac{k_1h_1}{k_1h_1 + k_2h_2}\right] q$$

$$= \left[\frac{134.9 \times 23}{134.9 \times 23 + 76 \times 37}\right] 326$$

$$= 171 \text{ bbls/day}$$

$$\gamma = \frac{0.433\ \rho_o(d_2 - d_1)}{p_{i_2} - p_{i_1}}$$

$$= \frac{0.433 \times 0.831(5075 - 5023)}{3254 - 3226}$$

$$= 0.668$$

$$\Delta p = p_{i_2} - p_{i_1}$$

$$= 3254 - 3226$$

$$= 28 \text{ psi}$$

Substituting the values of q_1, q, b_1, b_2, $p_{1\ hr}$, p_{wf_1}, Δp, γ, k_1, ϕ, μ, c, and r_w into equation 46a, the value of S_1 is calculated to give,

$$S_1 = -12.53$$

S_2 is determined using equation 47a; that is,

$$S_2 = \frac{1.151\ qb_1}{q_2}\left[\frac{p_{1\ hr} - p_{wf_2} + b_2(1 - \gamma)\Delta p}{m} - \log_{10}\frac{k_2}{\phi\mu c r_w^2} + 3.32\right] \tag{47a}$$

$$q_2 = q - q_1$$
$$= 326 - 171 = 155 \text{ bbls/day}$$

Substituting the values of q_2, q, b_1, b_2, $p_{1\ hr}$, p_{wf_1}, Δp, γ, k_2, ϕ, μ, c and r_w into equation 47a, the value of S_2 is calculated to give

$$S_2 = -13.2$$