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ECONOMIC OPTIMIZATION MODELS OF WINDPOWER SYSTEMS

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ABSTRACT

This study develops models for the economic optimum design of large scale windpower systems. Two basic models are developed: (1) systems without storage--all power generated is fed directly into the network, and (2) systems with storage--the systems can then be operated as "base-load" or "peak-load" capacity. The objective of the models is to maximize the total net value of electricity generated under assumed operating rules for the windpower systems and general conditions regarding wind speed and demand variations.

In the model for windpower systems without storage, optimization is carried out with respect to the total capacity of windturbines that vary in the values of design parameters such as rotor diameter, tower height and wind speed at which maximum power is generated. Wind data consists of discrete probability distributions of wind speed for several seasons and for several periods in a day. The power transmitted to the network is valued by the fuel costs saved on existing power plants.

The model for windpower systems with storage is developed for the case in which the average wind speeds in successive time increments show very low correlation. The model is then modified for the case of low serial correlation. An analytical storage model is used as a basis of representing storage requirements for a given system.

Separable programming is used as the solution technique in both models, and limited computational results based on available cost estimates and wind and demand data from Oklahoma are presented to illustrate the use of the model. In the model for systems without storage, separable programming will either give a global or a local optimal solution depending on the cost functions used. However, in the model with storage, the problem structure is such that a globally optimal solution cannot be guaranteed.

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ECONOMIC OPTIMIZATION MODELS OF WINDPOWER SYSTEMS

CHAPTER I

INTRODUCTION

The objective of this research is to develop economic models of windpower systems that can be used in evaluating the economic feasibility of this non-depletable energy source. The models treat a windpower system as part of an existing power network, with the objective of optimizing the economic value of the wind-generated power. The operating conditions treated by the model include wind speeds and total electricity demand that varies with both the time of day and the season of the year. The approach taken in this research is to model the entire system as an optimization problem.

In this chapter a brief description is given of the history and development of wind energy utilization, wind turbine mechanisms, energy storage techniques, wind behavior, and the economic considerations of designing and operating large windpower systems. Depending on the need for storage capability, the problem can be structured as a model for windpower systems without storage or as a model for systems with storage. A precise description of the two alternative system models is given at the end of this chapter.

Chapter II presents a literature review on (a) wind analyses and models applicable to windpower research, and (b) economic feasibility studies of wind energy use for power generation. It is worth noting that most of the feasibility studies done until now have been simple cost/benefit types of analysis based on various restrictive assumptions. There have been only a few studies that were similar in objective and scope to this study. For this reason, the review of literature on previous economic studies will be a rather limited one.

The development of a model for windpower systems without storage is presented in Chapter III and includes the general approach, problem formulation, solution technique used, and data generation.

Chapter IV presents variations of basic storage models to incorporate conversion losses and leakage factors into the modeling of storage systems for use with windpower.

An economic model for windpower systems with storage is developed in Chapter V. Two possibilities are examined in developing the model: one is serially independent wind speeds and the other is serially dependent wind speeds.

Chapter VI gives the results of test computations (with some sensitivity analyses) using various cost estimates and wind and electricity demand data for Central Oklahoma.

A summary of this research and the areas of further research are given in Chapter VII.

Finally, the Appendices list various supporting data, estimates, and data analyses that are referred to in the text.

History and Recent Development of Wind Energy Use

Until the steam engine was invented and brought to wide use, men had relied on wind for sailing on the ocean, milling grain and pumping water for irrigation and flood control. The Dutch and the English were the heaviest users of windmills. When electricity was introduced for providing light and powering motors, experiments were done to generate electricity with windmills. In the 1920's some rural Americans used windmills and batteries for lighting purposes before power lines could reach them. In those days the demand for electricity was low and fuel for power generation was abundant and cheap. Thus, wind-generated electricity could not compete with the electricity obtained from coal-fired steam turbines in scale and economy, or in meeting the need for storage to provide a stable supply of power. A study done in Germany during a period of short coal supply after World War II found it more economical to improve the efficiency of steam power plants by such measures as combining heating and power plants and installing high-pressure steam turbines than to install high cost wind turbines (20).

As the demand for electricity grew rapidly, some scientists in European countries began to look into the feasibility of using wind as an alternative source of energy. They built and tested large experimental windmills having rated power outputs ranging from 100 Kw to 1 Mw. The largest and most famous prototype unit was built in the United States in 1941. It was called the Smith-Putnam Wind Turbine and was erected on a hill ("Grandpa's Knob") in Vermont. This windmill generated a maximum power of 1.25 Mw at wind speeds of 29 mph. The power generated from the turbine was synchronized and transmitted to a grid served by a Vermont utility company. It was operated for four years until one of the blades failed due to fatigue and its manufacturer abandoned the project for financial reasons. Putnam, the designer of the unit, summarized the entire experience in his book Power from the Wind (30).

Because of recent sharp increases in fossil fuel costs and mounting pollution problems, wind energy has again stirred up considerable interest among those who seek economical and non-polluting alternative sources of energy. In the last few years, there has been significant development in windpower utilization in the United States. The federal government began to promote research and development efforts under the direction of the National Science Foundation. The proceedings of a workshop held in 1973 (26) and the report on solar energy for the Project Independence Blueprint (34) give excellent reviews of recent technology advancements and the goals and direction set for continuing research and development efforts. According to the latter report, by the year 2000, windpower systems may be developed to supply five to twenty-three percent of the projected electricity demand in the U.S. (Ibid., p. IV-1).

Technical Description of Wind Energy Conversion System

A windpower system consists of a set of windmills, inverters, connecting cables and an optional energy storage device. The system is linked to a network via transmission lines. The performance and economy of the system depends not only on its design but also on the wind characteristics at the site. In fact, wind behavior is the most important factor considered in deciding on the installation of a system. Brief descriptions of wind-driven generators (including power conversion theory), energy storage techniques, and the variations of wind speed and electricity demand follow.

Wind-Driven Generators

The components of a wind turbine are the rotor, step-up gear, generator and the tower supporting the turbine assembly (see Figure 1). Several types of rotors have been used to drive turbine shafts, including a conventional fixed- or variable-pitch propeller, a two-bladed hollow rotor which induces air flow through a turbine inside the supporting mast, a squirrel-cage rotor which directs the wind with stationary outer blades to inner rotor blades, and others. A step-up gear is used to increase the rotational speed of the rotor shaft for the generator. The generator may be a DC or an AC type. If a DC generator is used, the DC is

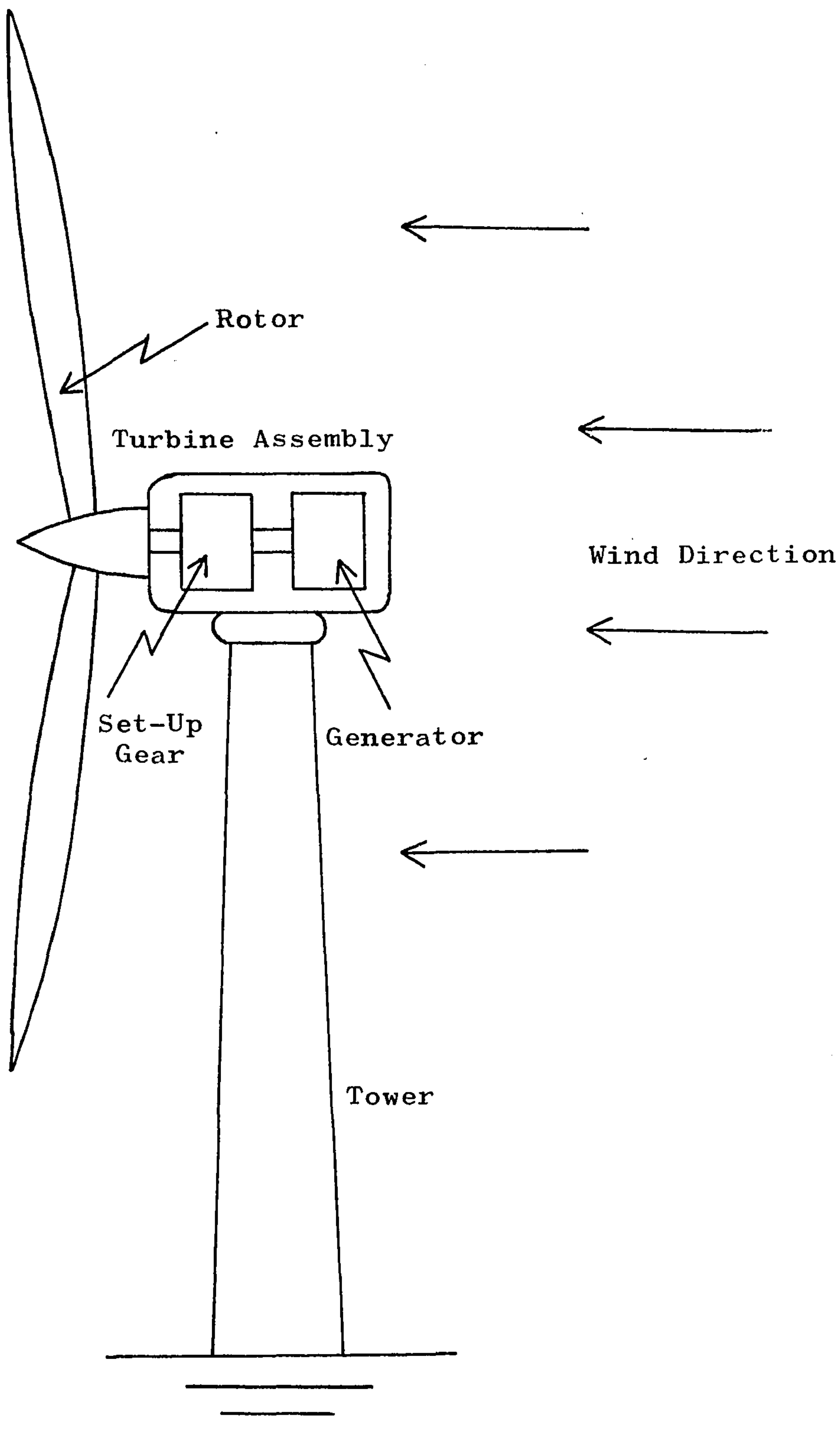


Figure 1. Diagram of a wind turbine.

changed to AC for introduction into existing AC systems. The DC power can be fed directly to storage; however, if an AC generator is used, the AC may have to be converted to DC for storage--this depends on the particular conversion technique used for storing energy. In general, a DC system is believed to be more efficient overall than an AC system. Parallel use of DC and AC for electric home appliances has been suggested by some advocates of the use of wind-generated electricity.

Technically, windmills are used to convert the kinetic energy of moving air in the atmosphere to mechanical energy in a turbine shaft and then to electricity. The kinetic energy of a mass m moving at velocity V is expressed as

$$K.E. = \tfrac{1}{2} \cdot m \cdot V^2. \tag{1}$$

Power is defined as energy per unit of time. Then the power in the air of density d passing through an area A at velocity V is given by

$$P = \frac{d}{2g} \cdot A \cdot V^3 \tag{2}$$

where g is the acceleration of gravity. Setting d equal to 1.22 Kg/m^3 and g to 9.81 m/sec^2, and converting Kg·m/sec to Kw we get

$$P = 0.000615 \cdot A \cdot V^3 \text{ Kw}, \tag{3}$$

where A is in m^2 and V in m/sec. Of this power, a maximum of 16/27 or about 60 percent may be converted, in theory, to shaft power with a propeller. Actually, the overall conversion efficiency η may be only about half of this theoretical maximum. This low efficiency is the result of additive or multiplicative effects of blade efficiency, friction losses in the step-up gear, generator efficiency, and power consumed by control mechanisms. In terms of efficiency η, rotor diameter D, and velocity V, the power output of a windmill can be expressed by

$$P = 4.00 \times 10^{-6} \cdot \eta \cdot D^2 \cdot V^3 \text{ Kw}, \qquad (4)$$

where D is in ft. and V in mph.

For technical and economic reasons windmills are not designed to generate power at all wind speeds. They start generating power at the cut-in speed (V_c) after overcoming system losses and develop full rated power (P_r) at the flat-rate speed (V_r). Above this speed, a blade pitch control keeps the turbine turning at a constant speed. For protection against strong winds, the turbines are stopped above feathering speed (V_f), thus cutting out the power. Figure 2 shows a hypothetical windmill response curve. Appendix A lists the designs and control methods of the Smith-Putnam Wind Turbine and three other smaller units built in Europe in the 1950's.

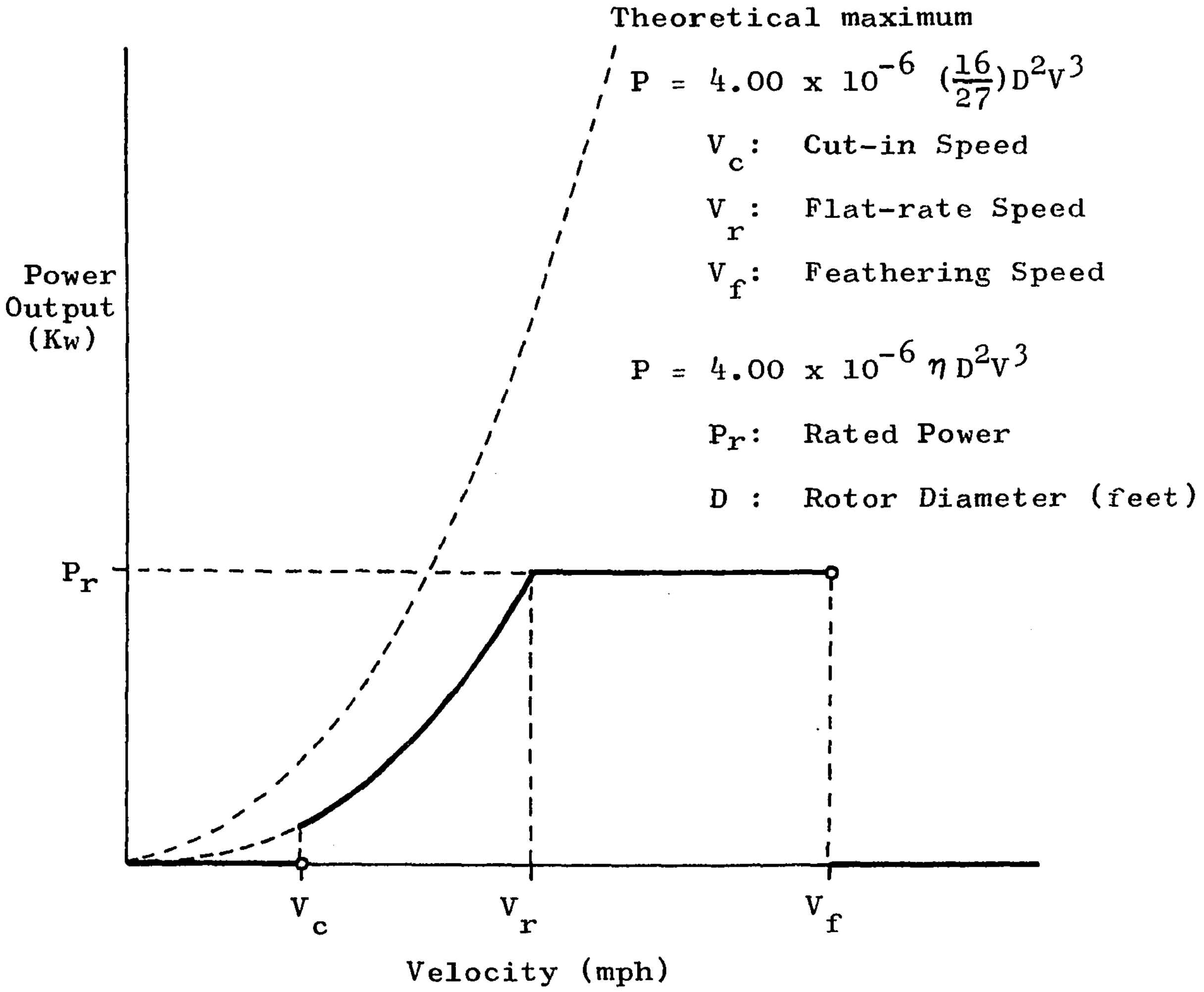


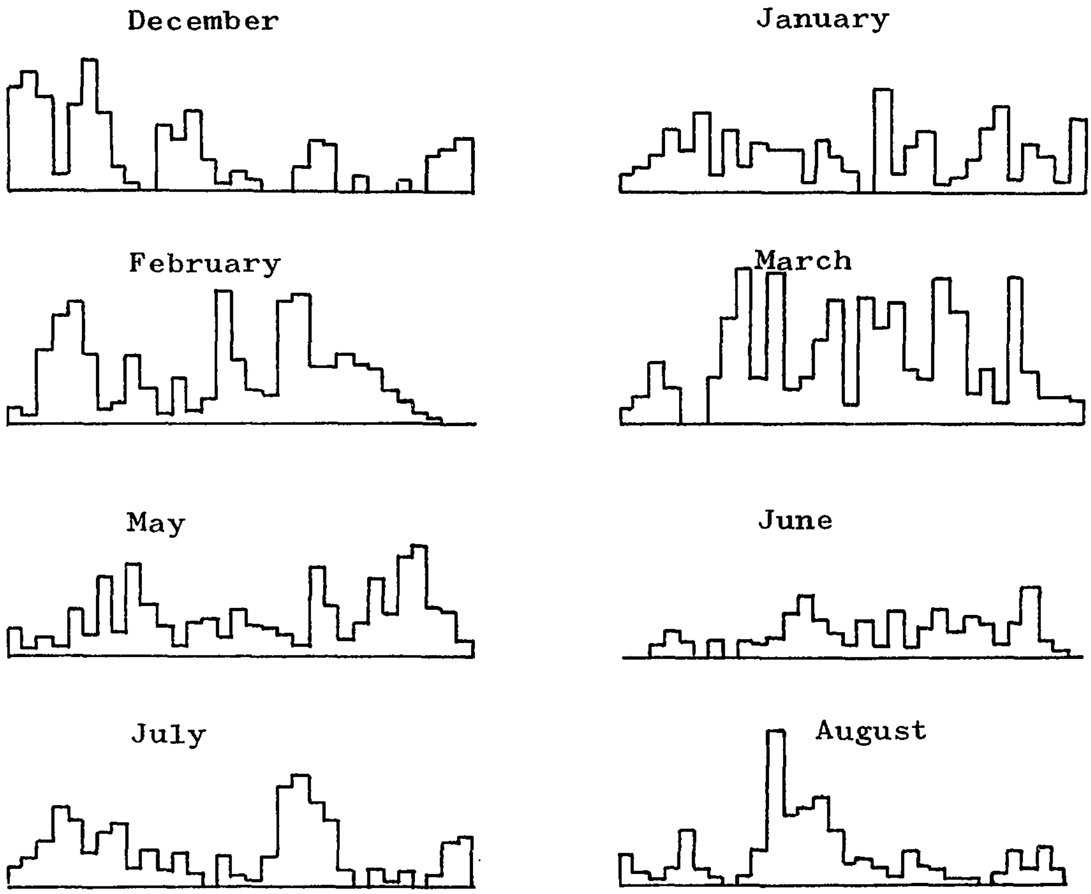
Figure 2. Windmill power output as a function of velocity.

Energy Storage Techniques

If wind is to be used as a reliable supplementary power source in meeting the demand for electricity, storage (or something equivalent such as a fast-starting backup power generator) must be provided for the instance of low wind energy. A production record of an experimental windmill shown in Figure 3 illustrates how severely the output can fluctuate on a daily basis.

Several methods have been used for converting electrical energy into a storable energy form. Some of these methods are: acid battery storage, electrolysis of water for the production of hydrogen, pumped-hydro, fly-wheels, compressed-air storage in depleted oil or gas reservoirs. Among these techniques only pumped-hydro storage has found a large scale use for energy storage by utility companies. For example, a large pumped-storage facility having a 1,872 Mw rated output and a 16,300 Mwh storage capacity has been built on the shore of Lake Michigan (1, p. X-27). The hydrogen produced by the electrolysis of water may be stored and then reconverted into electrical power with the use of fuel cells or by combustion at the power plant. With further development, this "hydrogen cycle" is expected to become an economical way of storing and retrieving electrical energy. A comprehensive study of a hydrogen energy system is found in an American Gas Association publication (1).

As a partial alternative to installing a costly storage system, windpower could be shared throughout a multiregional power grid as base-load capacity (35). This



The above figure is an approximate enlargement of the drawing in the reference, which is small and not reproducible.

Figure 3. Daily production record of an experimental windmill.
Source: Juul (18).

method requires dispersion of "windfarms" over a sufficiently broad area to ensure that the wind is blowing in at least one location most of the time.

For the purpose of modeling, a storage system will be considered to have three components: an input conversion facility, a storage tank or reservoir, and an output conversion facility. The hydrogen storage system fits this description. Other systems, such as flywheels and pumped-storage having combined motor-generators for input and output, can also be described by this generalization. In the storage models introduced in Chapters IV and V, each component will be characterized by its efficiency and capacity.

Wind Speed and Demand Variations

The average hourly wind speed varies with the season and the time of day. The pattern of these seasonal and diurnal variations differs from location to location and also with height above ground. The demand for electricity also fluctuates according to definite seasonal and daily patterns. In Oklahoma the average wind speed reaches its highest point in April and dips to its lowest point in July and August. On the other hand, the demand for electricity is highest in July and lowest in March and April. Figure 4 illustrates such seasonal variations in Central Oklahoma. The average monthly wind speeds are based on hourly observations over a 10-year period at an Oklahoma City

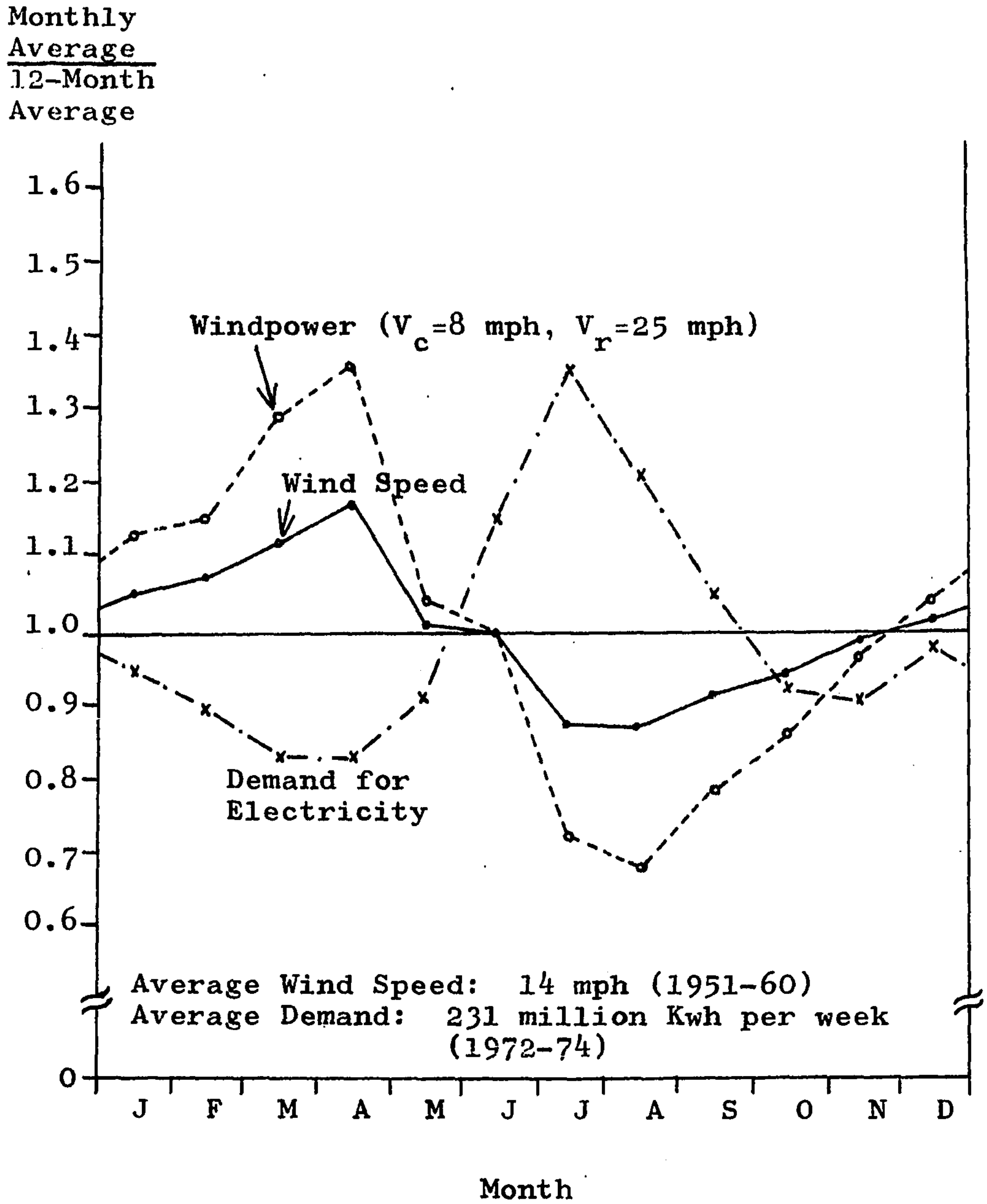


Figure 4. Seasonal variations of wind speed, windpower and electricity demand in Central Oklahoma.

airport (36). The expected power output in each month relative to the annual average was obtained from the wind speed distribution of the month assuming 8 mph cut-in and 25 mph flat-rate speed. The demand curve represents the monthly averages of net power output from the Oklahoma Gas and Electric Company (O.G.&E.) plants during the 1972-74 period. Note that all three curves are plotted on a ratio scale to show how the monthly averages vary in relation to the annual average.

The diurnal variation of wind speeds shown in Figure 5 is based on one year of data gathered at an Oklahoma City television tower by the National Severe Storms Laboratory. This figure, which is adapted from Crawford and Hudson (8), shows a significant daytime increase in wind speed at the surface level. The increase becomes less significant at the 146 ft. level. At the 296 ft. level, the pattern is inverted. Notice that wind speed increases with height. Also the pattern of diurnal variation changes with the season. Such seasonal effect is illustrated with the 146 ft. level data in Figure 6. Figure 7 shows hourly demand variations within the O.G.&E. service region in peak and off-peak seasons. The points on the curves represent the average values of a peak-day in each of peak or off-peak months in 1974.

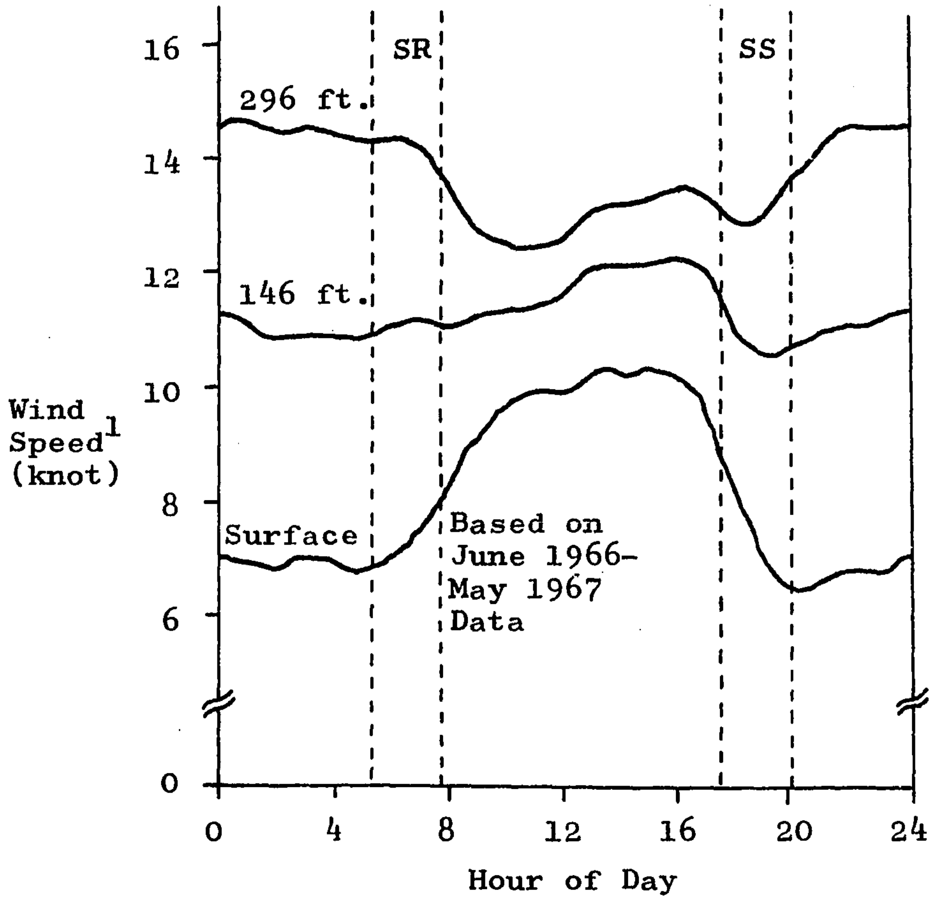


Figure 5. Diurnal variations of mean wind speed in Central Oklahoma.

[1]The wind speed represents one-year average of hourly observations for each of 24 hours.

Source: Adapted from Crawford and Hudson (8) Figure 11.

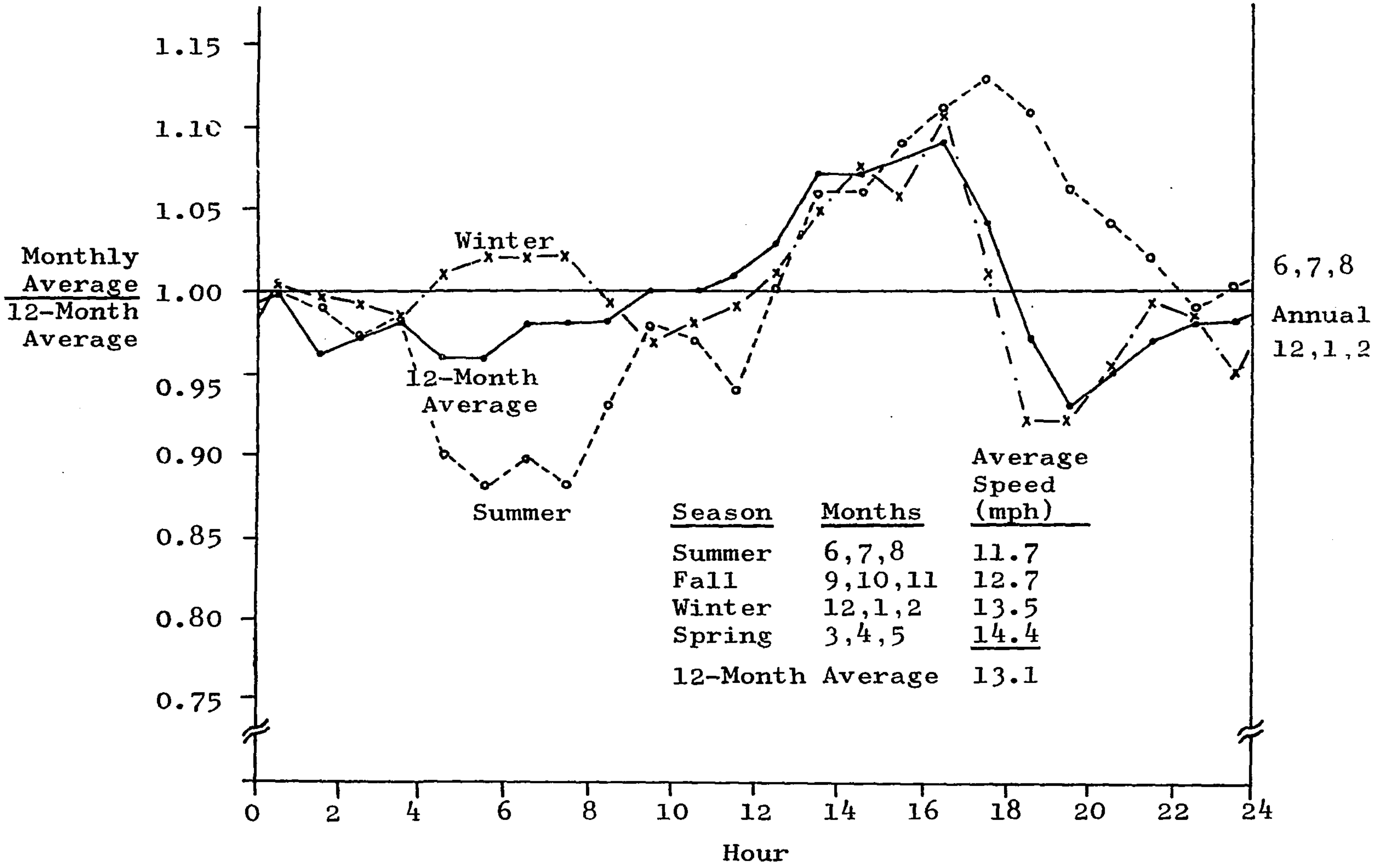


Figure 6. Seasonal effect on diurnal variation of wind speed based on 146 ft. level Oklahoma City wind data.[1]

[1]The data was provided by the National Severe Storms Laboratory in Norman, Oklahoma. The data is the same as that analyzed by Crawford and Hudson (8).

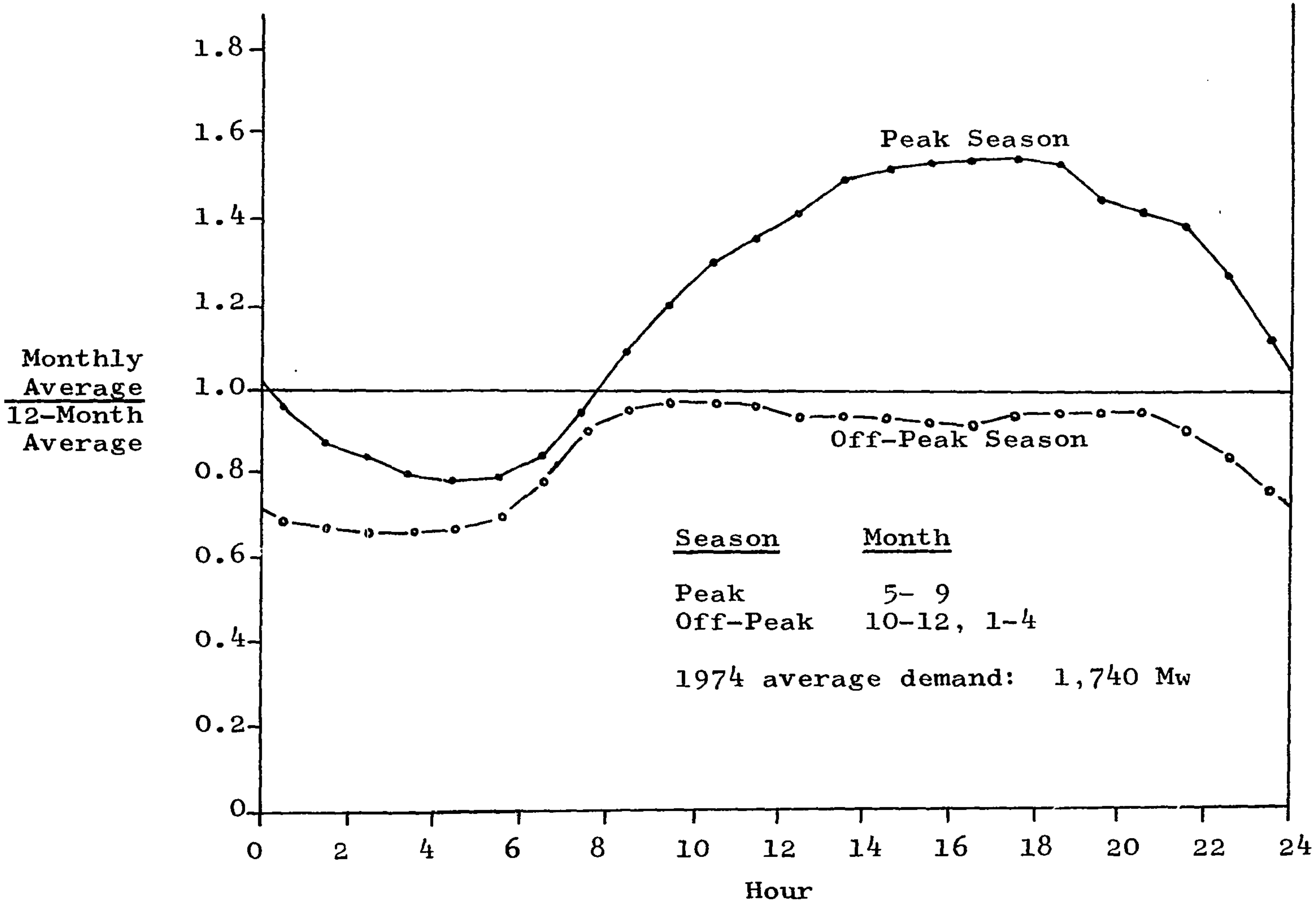


Figure 7. Hourly variation of electricity demand based on a 1974 Oklahoma Gas and Electric Company record.

Considerations in Designing Windpower Systems

The design and installation of windpower systems does not require major new technologies. Therefore, it is reasonable to assume that large windpower systems can be built in the near future as long as they are economically justifiable. The factors affecting windpower economics and the trade-offs between the cost and the magnitude of output of windmills are discussed below.

As the first step of economic analysis, a system designer must study in detail the wind characteristics of the potential sites. His selection criteria for the installation site should include the magnitude and stability of windpower at the site, the distance from the demand center, and the availability and cost of land. Once the site is chosen, the next step is selecting the windmills that will generate electricity most economically under the given wind conditions. Furthermore, if storage is to be installed for meeting demands, a decision must be made regarding the type and capacity of storage and the control of daily output to achieve maximum economy.

Of all design problems, the selection of windmills appears to be the most complex. It is a well known fact that the velocity and stability of wind increases with height above the ground. Therefore, more power can be obtained from wind by erecting taller towers; however, the tower cost also increases with its height. Thus, height selection should be based on economic grounds. The power output of a windmill is proportional to the square of the blade diameter. Therefore, doubling the blade diameter can result in four times more power obtainable. If, for example, the cost of the rotor is proportional to the diameter cubed,[1] then it would be cheaper on a per Kw basis to purchase windmills having small rotors. However, some fixed costs, including tower, installation and land costs, apply to all windmills. Thus, the rotor diameter like the tower height should be treated as an economic decision variable.

The rated power (P_r) is a function of the overall efficiency (η), rotor diameter (D), and the flat-rate speed (V_r). Although the efficiency differs from machine to machine and varies with wind speed, let us assume that it is a fixed value in all cases. Then, the rated power is a function of V_r alone for a fixed D. A windmill having a high flat-rate speed will obviously generate more electricity than one having a low flat-rate speed. When the wind blows over 25 mph, a windmill with V_r = 25 mph will generate twice the amount of power than the one with V_r = 20 mph according to the cubic law. However, the probability that the wind blows at such a high speed is usually small; and the difference in total electricity generated from the two windmills may turn

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[1]A cubic relationship was suggested in an article on home electricity generation with windmills (Bryson (6)).

out to be small. If the diameters are the same, the windmill with a higher flat-rate speed can be expected to cost more because it would have higher rated power (i.e., a larger generator).[1] In short, for a given diameter the flat-rate speed, which determines the rated power, is an economic decision variable to be considered along with all other variables.

A similar observation may be made in regard to the cut-in speed. However, the cut-in speed, as compared to the flat-rate speed, has a negligible effect on the total electricity generated by a windmill unless it is set too high (e.g., above the prevailing wind speed at the site). Again, this is because of the cubic relationship between the power and the speed. Although there appears to be an inverse relationship between the cost and the cut-in speed, little information is available on how they are actually related.

When a windpower system has no storage capability, the output from the system is highly variable (as shown, for example, in Figure 3). The utility company could not rely on the system to meet demand at any particular point in time, and thus it would still need to secure the necessary capacity with conventional power plants. Therefore, the windpower would only be used to save fuel costs on the existing power plants, and no savings on capital costs would result.

It follows then that the value of windpower at any given time can be determined, at least theoretically, from

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[1]A linear relationship between the cost and the flat-rate speed was used in a study done by investigators at the United Nations (33).

electricity demand at the time and the power plant fuel cost as a function of the load. Since utility companies use some type of economic load scheduling by which the total load is assigned to various plants in an optimal fashion, the designer of a windpower system needs to know the functional relationship between the total fuel cost and the total load. In theory, this functional relationship can be obtained from a set of incremental fuel cost functions of individual power plants [illegible]tching rules with respect to the load le[illegible] pioneering study on economic power [illegible] by Kirchmayer (19). [illegible] hypothetical aggregate fuel cost [illegible] the power plants operating in a [illegible]so illustrates how various plants mi[illegible]three different levels of the total load assigne[illegible]. The incremental fuel costs of the individual power plants are shown in Figure 9. In the case of no storage, the worth of wind-generated electricity in a given time period is simply the difference between the expected total fuel cost on the existing plants with no windpower and that with windpower taking up part of the load. In the South and Southwest the value of windpower would be greatest on hot summer days when air-conditioning systems are in full operation, because the windpower would replace peaking units that burn relatively expensive fuel. In short, the use of aggregate fuel cost functions is a convenient way

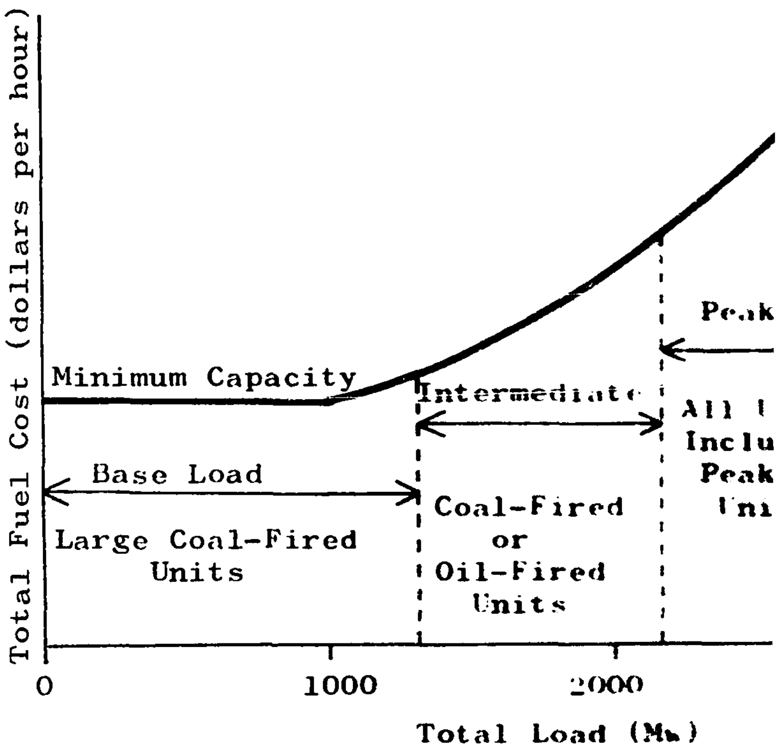

Figure 8. Total system fuel cost curve.

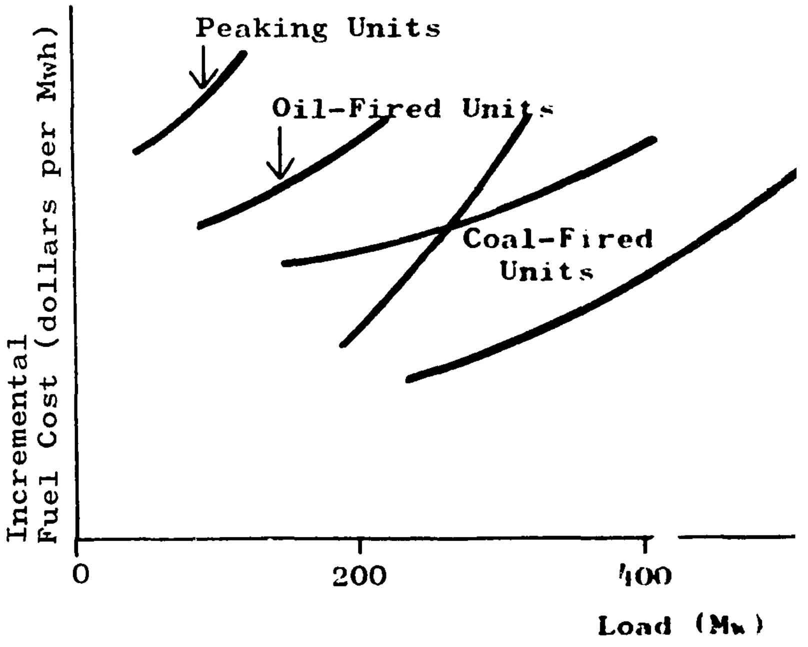

Figure 9. Incremental fuel cost curves of indi

electricity demand at the time and the power plant fuel cost as a function of the load. Since utility companies use some type of economic load scheduling by which the total load is assigned to various plants in an optimal fashion, the designer of a windpower system needs to know the functional relationship between the total fuel cost and the total load. In theory, this functional relationship can be obtained from a set of incremental fuel cost functions of individual power plants if the dispatching rules with respect to the load level are specified. A pioneering study on economic power plant operation was done by Kirchmayer (19).

Figure 8 gives a hypothetical aggregate fuel cost function representing all the power plants operating in a network. The figure also illustrates how various plants might operate under three different levels of the total load assigned to them. The incremental fuel costs of the individual power plants are shown in Figure 9. In the case of no storage, the worth of wind-generated electricity in a given time period is simply the difference between the expected total fuel cost on the existing plants with no windpower and that with windpower taking up part of the load. In the South and Southwest the value of windpower would be greatest on hot summer days when air-conditioning systems are in full operation, because the windpower would replace peaking units that burn relatively expensive fuel. In short, the use of aggregate fuel cost functions is a convenient way

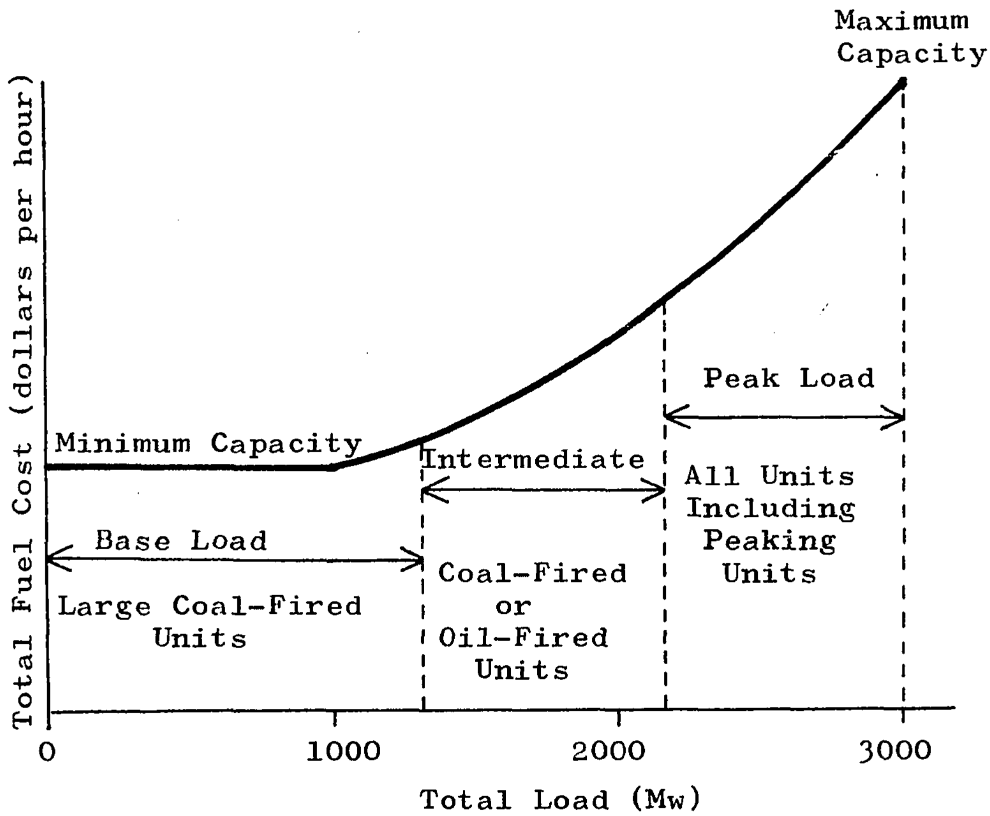

Figure 8. Total system fuel cost curve.

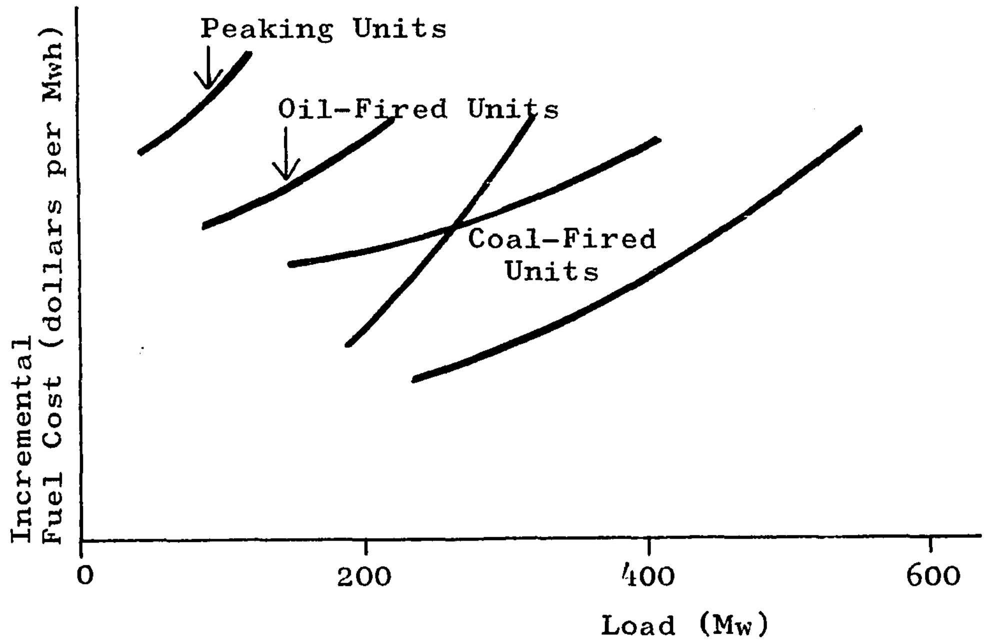

Figure 9. Incremental fuel cost curves of individual plants.

of attaching a value to windpower when no capital cost savings on conventional power plants can be realized with the installation of a windpower system.

When storage capability is added to the windpower system, two general cases need to be considered to determine its economic value. One case (Case A) assumes that even with storage, the windpower system is still not reliable enough to displace any conventional power plants. Therefore, as was the case with no storage, the economic value of windpower is determined by the fuel cost savings. The difference between the system with no storage and this system lies in the controllability of the output power. Since the power is controllable, it should be scheduled to achieve a maximum fuel cost savings.[1]

The other case (Case B) assumes that the windpower system is reliable enough so that it can be considered as part of the capacity expansion of the total power system or as a displacement of existing units. That is, the system would supply specified power levels with a sufficiently low probability of shortage. In this case, the per Kwh generation cost of the windpower system becomes the meaningful economic criteria. For example, a utility company might want to find a minimum cost design of a windpower system that

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[1]The controllability given by storage may also be necessary for engineering reasons. For example, in systems with no storage there will be a limit to the power fluctuation from the windpower system that the network can tolerate.

would be capable of delivering at least 100 Mw of power during the peak period 98 percent of the time. The other two percent of the time, when the wind is not blowing sufficiently and the storage is depleted, it is assumed that the power needed can be purchased from a nearby power grid. In this case, the power system is considered as an addition to the total power capacity. The per Kwh generation cost of windpower should be compared with that of other alternative power sources, and the economic feasibility of the windpower system determined.

Statement of the Problem

The problem this research is concerned with is to develop two economic models, one with storage and one without storage, for large scale windpower systems. The objective is to optimize the system design and operation under assumed operating rules and general conditions of wind and demand variations. Certain assumptions are made in regard to the availability of technology and the costs involved in building and installing wind turbines and storage systems.

The economic criteria used in the optimization of the windpower system are as follows:

1. The windpower system with no storage:

   The objective is to maximize the net savings in fuel costs of existing power plants that would result from the installation of the system. The net savings is defined as the difference between the annual fuel cost savings on power plants and the annual equivalent of the fixed and variable costs of the windpower system.

2. The windpower system with storage:

<u>Case A</u>

The objective is the same as for the system with no storage.

<u>Case B</u>

The objective is to minimize the per Kwh generation cost of the system installed as part of a capacity expansion or as a displacement of existing units.

The operating rules adopted for the two models are as follows:

1. The windpower system with no storage:

All the power generated by wind turbines is directly transmitted to the network. It is assumed that the network can use as much power as is transmitted by the windpower system. The problem associated with the output fluctuation is resolved by some means other than storage. Such means might be ample spinning reserve of loaded power plants or fast starting backup units.

2. The windpower system with storage:

<u>Cases A and B</u>

All the power generated from the system is first converted for storage. Then, some stored energy is retrieved from storage, reconverted into electricity and delivered to the network. Within each period of time, the power transmitted is kept constant unless the storage is depleted. The capacity of storage is such that the probability of shortage occurring in any day is less than a prespecified value.

The operating conditions considered in modeling the systems are:

1. wind characteristics at the proposed site,
2. fuel costs of existing power plants, and
3. expected demand for electricity that varies with the time of the day and with the season.

CHAPTER II

STATE-OF-THE-ART

This chapter presents a review of previous research pertinent to the economic modeling of windpower systems as described in Chapter I. Since there has been little work concerned directly with economic optimization of windpower utilization, this review only covers (a) wind and windpower analysis, and (b) economic feasibility studies including an evaluation model for a large offshore windpower system. The mathematical models of storage systems applicable to windpower systems are covered separately in Chapter IV.

Wind and Windpower Analysis

The World Meteorological Organization (WMO) has long supported the utilization of wind energy through its publications. In the early 1950's it conducted an extensive survey of favorable sites for windmill installations and prepared a comprehensive bibliography on windpower analyses and systems (37). According to this WMO report, the total power potential in the atmosphere is approximately 3×10^{14} Mw, and of this quantity approximately 2×10^{7} Mw is available for power generation. A U.S. Weather Bureau survey contained in the report defines favorable and unfavorable topographies for windmill installation. The suitable sites in the U.S. are to be found "along the coasts and in the region of the Great Plains east of the Rocky Mountains where the flat relief affords little obstruction to wind flow."

According to an estimate made by an NSF/NASA solar energy panel, the annual production of wind-generated electricity in a 300-by-1,300 mile stretch in the Great Plains area may reach 210 x 10^6 Mwh by year 2000 (25, p. 69). Although there are no established procedures for assessing windpower potential at a given site, it is thought that at least one year of continuous measurements, at the height sufficient for power generation, is necessary for a reliable assessment. These measurements should be compared to local climatic data to see whether the observed data could be considered representative of the true wind conditions at the site. These continuous observations can be used in finding the seasonal and diurnal variations of wind speed as well as the monthly distributions. The analysis may be carried out further to find how the average speed changes from hour to hour and from day to day, i.e., to find the serial correlation between the speeds in two or more successive increments of time. This serial correlation of wind speeds becomes an important factor that must be considered in determining the storage size for a system. The persistence of wind may be represented either with an autocorrelation function or with velocity transition matrices for various time lags as done by Crawford and Hudson (already referred to in Chapter I). Detailed discussion of this subject is deferred until the latter part of Chapter V.

Assessment of windpower potential based on average windspeeds (e.g., daily or monthly averages) may result in a gross underestimation of the true potential. This is because the windpower is proportional to the velocity cubed. Because of this cubic law, the mean of a windpower distribution is always greater than the power available at the mean of the corresponding velocity distribution. According to a windpower study done at a California site (3), the daily average wind energy based on "on-the-hour" velocities was approximately 2.2 times as much as the value based on the annual average velocity. An interesting observation made in the study was that although the average of hourly velocities measured in one month at one site was lower than that in the following month, the average energy content was approximately fifty percent higher in the first month. The study suggests that the true indicator of energy potential is Equivalent Energy Velocity, which can be expressed as

$$\text{E.E.V.} = \left(\sum_{i=1}^{N} V_i^{3}/N \right)^{1/3} \qquad (4)$$

where V_i = average velocity of the observation made during the i^{th} time interval (usually one hour or less), and

N = number of time intervals.

The annual E.E.V. at one site was 21.9 mph while the actual average velocity was 16.9 mph.

Prior to the California study, Bergey (4) had examined the feasibility of windpower generation in Central Oklahoma using the 146 ft. level NSSL data mentioned in Chapter I. He used the term Mean-Energy Velocity to mean the same thing as E.E.V. His calculation showed that the annual E.E.V. (or M.E.V.) at the tower was 16.2 mph whereas the mean annual velocity was 13.0 mph.

Simulation models are often used to simulate wind behavior. Within a short time period (e.g., one hour) during which the mean speed does not shift significantly, simulation may be done rather simply using a probability distribution. One example of such short-time simulation is the wind model developed by Gibson at Oklahoma State University for simulating the response of windmills (13). Gibson's model takes into account both the duration and the magnitude of gusty winds occurring in a period shorter than a few minutes. In the model, the magnitude of gusts is assumed to be normally distributed with a zero mean and a variance determined by the surface roughness and the average wind speed at the tower height. The duration of gusts is assumed to be exponentially distributed. The simulation involves (a) the generation of gusts varying in magnitude, (b) the generation of the delay times for the gusts, (c) the placement of the gusts on a discrete time scale according to the delay, and (d) the straight line interpolation for the wind speeds between any two gust arrivals.

Wind simulation for a prolonged period of time (e.g., one year) becomes more difficult if the effects of seasonal and diurnal variations must be considered in generating the wind speeds. In windpower systems long-term simulation is done primarily for determining the storage requirement under varying windpower and demand; this is much like the type of simulation done in inventory control. Simulation models may also be used in the case where analytical solutions are difficult to obtain. Such a case arises where the input, i.e., the windpower, is serially correlated. Methods of generating auto-correlated input are found in Brown (5). To the author's knowledge there have been few long-term simulation models developed for windpower analysis that are of any significance.

The relationship between wind velocity and height above the ground (vertical wind profile) can be expressed by

$$V = KH^{\alpha} \tag{5}$$

where V is the velocity, H the height, and K and α are constants unique to the site considered. Davenport (10) compiled and analyzed published data on vertical wind profile and gave the average values of α for the following three terrain categories:

Description of the Terrain	Value of α
Open country, flat coastal belts, prairie grass lands, etc.	1/7
Wooded countryside, parkland towns, outskirts of large cities	1/3.5
Centers of large cities	1/2.5

The expression for vertical wind profile may be used in finding the relationship between the tower height and the expected power output from a windmill.

Economic Feasibility Studies

Most of the studies of windpower economics done in the past were simple economic analyses in which the expected per Kwh cost of wind-generated electricity was compared to the average per Kwh fuel cost of electricity generated by conventional power plants. These analyses were simple in that the variation of per Kwh fuel cost with demand level were not taken into account. The expected per Kwh cost of wind-generated electricity is usually obtained by dividing the annual cost of windmills by the expected value of the total electricity generated in one year. The latter is based on annual wind distribution. Some of the early feasibility studies done by designers in European countries evaluated the economics, but this was not their primary concern.

Heronemous (16) of the University of Massachusetts has proposed a large off-shore windpower system for the New England area which would give an economically viable alternative to nuclear power in meeting future increases in electricity demand. His conceptual design of the system consists of a large number of wind stations that are grouped into many wind units. Within each unit the wind stations form orbital rings with an electrolyzer plant and a distillation plant at the center. These units together with a deep-water, high-pressure hydrogen storage tank and a compressor/reducer station are connected by pipelines to an onshore distribution system. The onshore system comprises a network of hydrogen distribution pipelines and fuel cell stations that use hydrogen for power generation. Of the three configurations of wind stations Heronemous envisioned, the one with 34 rotors each having 100 Kw rated power would provide the cheapest electricity. The system, when installed, would deliver power to New England for as little as 25 mills per Kwh by 1990.

Following the work of Heronemus, Dambolena (9) developed a simulation model to find the economic implications of altering the basic assumptions on the technological advancement in windmills, fuel cells, and electrolyzers. His model receives as input various design specifications and assumptions on technologies, generates wind speeds for a number of years, and computes the number of wind stations required to meet the future demand. The result for a base case simulation showed that power could be obtained from the offshore system for 20 mills per Kwh. With improvements in fuel cells and electrolyzers and higher rated power and lower cut-in speed, the system would supply power for 12 mills per Kwh. The basic drawback in this type of modeling is that the model may be used only to evaluate a given system with fixed total capacity. The models to be developed in the following chapters may be used to optimize both design and operation of a windpower system.

CHAPTER III

THE MODEL FOR WINDPOWER SYSTEMS WITHOUT STORAGE

This chapter will develop an optimization model for windpower systems with no storage capacity. The chapter contains sections on the general approach taken in developing the model, the definition of notation used, model formulation, the characteristics of the model and solution techniques, and data generation.

The General Approach

As described more fully in Chapter I, the purpose of this research is to optimize the design and operation of a windpower system which operates as part of an existing power network. The objective of the optimization model is to maximize the net savings, which is defined as the difference between the annual fuel cost savings on the existing power plants and the annual cost of the windpower system.

Some of the most important decisions in such a system's design are related to the individual windmills, and in developing the optimization model the following approach is used: The user specifies a set of alternative designs which differ from each other on the basis of rotor diameter, flat-rate speed, cut-in speed, tower height and conversion efficiency. The user can fix any of these design parameters or make it dependent upon other parameters. For example, the tower height may be expressed as a function of the rotor diameter. The decision variables of the model are "how many windmills of each design should be incorporated into the system".

In general, each windmill design will have different operating characteristics as a function of wind speed. To illustrate, Figure 10 gives a hypothetical power output curve for two different designs, i and j. From the figure, Design i will generate more electricity than j when the wind velocity is low. If the installation cost is the same for both designs and the winds are consistently low, the economic choice would be Design i. On the other hand, if the winds are high a large part of the time, the choice would be Design j because it generates more power than Design i at high wind speeds. If the wind velocities are low in one period of time and high in the next period, the optimal choice might be a mixture of the two designs.

The value of the power generated from wind during a given period is determined by the level of demand during that period and the plant fuel cost function. The expected values of windpower and demand vary with the season and the time of the day. These variations are incorporated into the model by dividing a year into I seasons and a day into J periods. The model requires the specification of the number of days in each season and the number of time increments in each period. The

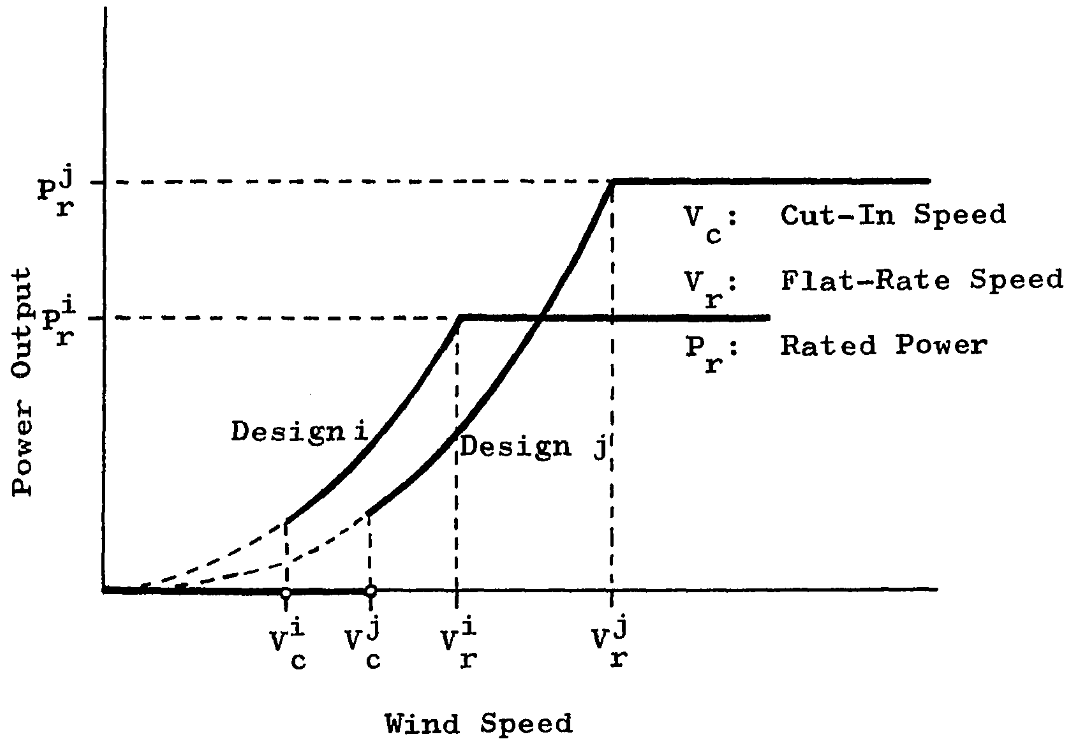

Figure 10. Hypothetical power output curves of two windmill designs.

user must determine the appropriate seasonal divisions for a year and periodic divisions for a day, taking into account wind speed and demand patterns.

Variables, Constants, and Functions

Before presenting the general structure of the model, the variables, constants and functions used in the model are first defined.

1. Variables

x^n = number of windmills of design n (n = 1, ..., N)

y_{ij} = expected power (Kw) to be transmitted from the windpower system to the network in season i, period j (i = 1, ..., I; j = 1, ..., J)

p = capacity (Kw) of the windpower system, which is also the capacity of the transmission system (dependent on x^n's)

2. Constants

a. Windpower system design

N = total number of designs

D^n = rotor diameter (ft.) of design n

V_c^n = cut-in speed (mph) of design n

V_r^n = flat-rate speed (mph) of design n

η^n = conversion efficiency of design n

P_r^n = rated power (Kw) of design n

$= 4.00 \times 10^{-6} \eta^n (D^n)^2 (V_r^n)^3$

H^n = tower height (ft.) of design n

A^n = land used (acres) by one unit of design n

b. Windpower system costs

C_w^n = installation cost (dollars) of one unit of design n

O_w = ratio of the annual operating and maintenance costs of a windmill to its installation cost (assumed fixed for all windmills)

b_w = economic life of a windmill (assumed the same for all windmills)

C_a = price of land (dollars per acre)

c. Power transmission

λ = transmission efficiency (a fixed distance assumed)

b_t = economic life of the facility

d. Wind data, expected power output, and demand

I = number of seasons in a year

J = number of periods in a day

K = maximum number of velocity increments used in approximating wind distributions

d_i = number of days in season i

h_j = number of hours in period j

v_o = velocity increment (mph) of the wind distribution at the height of the measurements

$\overline{V}_k$ = median speed (mph) of the k^{th} interval of wind distributions

R_{ijk} = $Pr[(k-1)v_o \leq V_{ij} < kv_o]$, where V_{ij} is the wind velocity in season i, period j

H_o = reference height (ft.) at which winds are measured

α = exponent of the wind profile function given by $V_h = V_o(H/H_o)^{\alpha}$, where V_h and V_o are the velocities at height H and H_o, respectively

Q^n_{ij} = expected power (Kw) generated by one unit of design n in season i, period j (The expression for computing this value from input data is given in the section on model formulation.)

E_{ij} = expected demand (Kw) in season i, period j

e. Other constants

P^m = minimum allowable capacity (Kw) of the windpower system

P^M = maximum allowable capacity (Kw) of the windpower system

A^M = acres of land available for the system installation

r = annual interest rate applicable to all capital investments

$(a/p)^b_r$ = capitalization factor for economic life b and interest rate r

$= \frac{r(1+r)^b}{(1+r)^b-1}$ assuming no salvage value at the end of economic life

3. Functions

g(p) = installation cost of the transmission system as a function of its capacity

f(E) = aggregate plant fuel cost (dollars/hour) as a function of the total load E (Kw) ($E^m \le E \le E^M$,

α = exponent of the wind profile function given by $V_h = V_o(H/H_o)^{\alpha}$, where V_h and V_o are the velocities at height H and H_o, respectively

Q^n_{ij} = expected power (Kw) generated by one unit of design n in season i, period j (The expression for computing this value from input data is given in the section on model formulation.)

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A^M = a[illegible] the system instal-la[illegible]

r = an[illegible]cable to all capital inve[illegible]

$(a/p)^b_r$ = capitaliz[illegible]tor for economic life b and interest rate r

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3. Functions

g(p) = installation cost of the transmission system as a function of its capacity

f(E) = aggregate plant fuel cost (dollars/hour) as a function of the total load E (Kw) ($E^m \leq E \leq E^M$, where E^m and E^M are the minimum and maximum capacities of the existing plants.)

Formulation of the Model

The formulation of the model involves (a) calculating the expected power output and costs of various windmills, (b) obtaining an expression for the difference between the plant fuel cost savings and the costs of windpower and transmission systems, and (c) setting up the model as a mathematical programming problem with appropriate constraints.

In calculating the expected power output from a windmill in a given period, the wind data used needs to be adjusted for the difference between the height at which the winds are measured (i.e., the reference height H_o) and the tower height. In adjusting the data for the height difference, it is assumed that within the period the coefficient of variation (i.e., the ratio of the standard deviation to the mean) of the "hourly" average wind speed is the same for all tower heights. Under this assumption, if the wind velocity at the reference height is normally distributed, then the velocity at any tower height would also be normally distributed with the mean and the standard deviation proportional to those at the reference height. The wind velocity distribution input into the model consists of a set of probabilities (R_{ijk}) assigned to K equal-length increments. The adjustment of the distribution is done in sucy a way that the number of increments stays the same but the range that each increment represents is either increased or decreased. This scaling of velocity increments utilizes a relationship between the velocities at two different heights. The relationship is implied in the expression for wind profile in Chapter II and is given by

$$\frac{V_h}{V_o} = \left(\frac{H}{H_o}\right)^{\alpha} . \qquad (6)$$

In terms of the velocity increments v_h and v_o at the heights H and H_o, the above becomes

$$\frac{v_h}{v_o} = \left(\frac{H}{H_o}\right)^{\alpha} . \qquad (7)$$

For example, if $v_o = 2$ mph, $H_o = 50$ ft., and $\alpha = 1/2$, then v_h corresponding to H = 200 ft. would be 4 mph. The interval (or increment) in which a given velocity V falls at the height H can be found from a function

$$\begin{aligned} k(V,H) &= \lfloor V/v_h \rfloor + 1 \\ &= \left\lfloor \left(\frac{V}{v_o}\right) / \left(\frac{H}{H_o}\right)^{\alpha} \right\rfloor + 1 \end{aligned} \qquad (8)$$

where the symbol $\lfloor \ \rfloor$ is used to represent a round-off to the nearest integer value. The median speeds in the velocity distribution corresponding to the tower height H are obtained by multiplying those at the reference height by a factor $(H/H_o)^{\alpha}$. Finally, the expected power output (Kw) from one unit of design n in season i, period j can be obtained from the expression below:

$$Q^n_{ij} = 4.00 \times 10^{-6} \cdot \eta^n \cdot (D^n)^2 [(H^n/H_o)^{3\alpha} \cdot \sum_{k=k^n_1}^{k^n_2-1} (\overline{V}_k)^3 \cdot R_{ijk}$$

$$+ (V^n_r)^3 \cdot \sum_{k=k^n_2}^{K} R_{ijk}], \qquad (9)$$

where $k^n_1 = k(V^n_c, H^n)$, or the interval of the velocity distribution at the height H^n, in which the cut-in speed V^n_c falls (see Equation 8), and $k^n_2 = k(V^n_r, H^n)$, or the interval in which the flat-rate speed V^n_r falls.

The costs of a windmill consist of the installation cost, operating and maintenance cost, and land use cost. On an annual basis, the windmill costs are given by

$$[(a/p)_r^{b_w} + O_w] \cdot C^n_w + C_a \cdot A^n \cdot r. \qquad (10)$$

In the above expression, the interest rate r is considered as representing the "true" cost of capital, including income tax effects. It is also assumed that the windmills will have no salvage value at the end of the b_w years of operation, and the annual operating and maintenance costs are charged as a fixed percentage of the purchase costs. The land has a perpetual life; therefore, the expression $C_a \cdot A^n \cdot r$ is appropriate for the annual land cost.

The annual equivalent cost of the transmission system is given by

$$(a/p)_r^{b_t} \cdot g(p), \tag{11}$$

where g(p) is the initial cost of the system having a capacity of p Kw. The function g is specified by the user of the model, as in the case of the windmill installation costs. The transmission capacity is set equal to the capacity of the windpower system which is given by

$$p = \sum_{n=1}^{N} P_r^n \cdot x^n. \tag{12}$$

The expected power output from all windmills in a given time period of a given season becomes a function of the number of windmills of various designs once Q_{ij}^n's are determined. This total power output must be greater than or equal to y_{ij}, the expected power transmitted in season i, period j. That is,

$$y_{ij} \leq \sum_{n=1}^{N} Q_{ij}^n \cdot x^n. \tag{13}$$

After taking into account the transmission losses, the expected value of per hour plant fuel cost savings in season i, period j, can be expressed by

$$f(E_{ij}) - f(E_{ij} - \lambda \cdot y_{ij}), \tag{14}$$

where f is a user-provided fuel cost function defined over (E^m, E^M).

Other constraints that might be imposed on the system design are concerned with total system capacity and land availability. A utility company may want to limit the maximum capacity of a windpower system to P^M(i.e., $p \leq P^M$) so that the windpower would not cause a severe stability problem to the network. A minimum capacity constraint $p \geq P^m$ can also be added to the problem formulation. A constraint on land use is represented by

$$\sum_{n=1}^{N} A^n \cdot x^n \leq A^M. \tag{15}$$

A^n might be determined by the rotor diameter via a functional relationship provided by the user.

The generalized model for windpower systems without storage now follows:

$$\begin{aligned} \text{Maximize } & \sum_{i=1}^{I} \sum_{j=1}^{J} [f(E_{ij}) - f(E_{ij} - \lambda y_{ij})] \cdot h_j \cdot d_i \\ & - \sum_{n=1}^{N} \{[(a/p)_r^{b_w} + O_w] \cdot C_w^n + C_a \cdot A^n \cdot r\} \cdot x^n \\ & - (a/p)_r^{b_t} \cdot g(p) \end{aligned} \tag{16}$$

subject to

$$y_{ij} \leq \sum_{n=1}^{N} Q_{ij}^{n} \cdot x^{n} \text{ for all } i,j \tag{17}$$

$$p = \sum_{n=1}^{N} P_{r}^{n} \cdot x^{n} \tag{18}$$

$$p \leq P^{M} \tag{19}$$

$$p \geq P^{m} \tag{20}$$

$$\sum_{n=1}^{N} A^{n} \cdot x^{n} \leq A^{M} \tag{21}$$

$$\lambda \cdot y_{ij} \leq E_{ij} - E^{m} \text{ for all } i,j \tag{22}$$

$$x^{n} \geq 0 \qquad \text{for all } n \tag{23}$$

The first term in the objective function represents the annual expected fuel cost savings. Constraint 22 is necessary because the total load taken up by the existing plants $E_{ij} - \lambda . y_{ij}$ may not be less than the minimum capacity E^{m}.

Characteristics of the Model and Solution Technique

The model, as it is formulated in the preceding section, contains IJ + N + 1 variables and 2IJ + 4 linear constraints on x^{n}'s. Half of the 2IJ + 4 constraints are simply the bounds on variables. The objective function is composed of IJ + 1 single variable functions involving y_{ij}'s and p, and N linear terms for x^{n}'s. It is noted that $f(E_{ij})$ in the objective function is only a constant term and thus may be deleted without affecting the solution. Since the objective function is composed of single variable terms, it can be characterized as non-linear and separable unless the functions f and g are linear.

Taking advantage of the separability of the objective function, the problem can be solved using a separable programming technique for an approximate solution. This technique involves the substitution of the separable, non-linear terms in the objective function or in the constraints with a set of approximating linear terms and solving the resulting linear programming problem. For example, a function $h(x)$ can be represented by a K segment piecewise linear function as below:

Let b_k = k^{th} breaking point on the x-axis of the function $h(x)$, and

w_k = non-negative weight assigned to the k^{th} breaking point such that $\sum_{k=1}^{K} w_k = 1$.

Then,

$$h(x) \approx \sum_{k=1}^{K} w_k \cdot h(b_k) \tag{24}$$

$$x = \sum_{k=1}^{K} w_k \cdot b_k. \tag{25}$$

In addition, the approximation requires that only two adjacent w_k's may be positive. The separable programming technique that uses the Simplex algorithm requires that no more than two adjacent w_k's may enter the basis.

Another way of approximating the function $h(x)$ is by introducing the variables x_k's to represent the increments of x in the range (b_{k-1}, b_k) and expressing $h(x)$ as the sum of the linear terms involving x_k's. In this,

$$h(x) \approx \sum_{k=1}^{K} s_k \cdot x_k + h(b_0), \tag{26}$$

$$x = \sum_{k=1}^{K} x_k, \tag{27}$$

where s_k is the slope of the k^{th} line segment. The conditions that are necessary in solving the problem as an LP problem are given by

$x_k = 0$ if $x_{k-1} < b_{k-1} - b_{k-2}$, $k = 2, \ldots, K$,

$x_k \geq 0$ for all k.

The two methods are essentially the same in that both require the restricted basis entry in using the Simplex method for solution. The separable programming technique only gives a local optimal solution except in special cases where a global optimum is obtained. The special case arises when all the component terms in the objective function are concave[1] in a maximization problem and the solution space is convex. The solution space will be convex if (a) all the terms associated with the "≤" constraints (or "≥" constraints) are convex (or concave) and (b) all the terms in the "=" constraints are linear. In such a special case, the slopes (s_k's) of any approximating piecewise linear function in the objective function decrease with the variable x of the

[1]By definition, linear terms are both concave and convex.

original function h(x). This makes the variables x_k's of the approximating function enter the basis in sequence, thus eliminating the necessity of the restricted basis entry. Full discussions of this separable programming technique are given in Hadley (15) and Taha (32).

Returning to the problem formulation in the preceding section, we see that the objective function will be concave if the functions f and g are both convex (or -f and -g are both concave). The fuel cost function f is most likely convex in any power system. However, the transmission cost function g is more likely concave because of the economies of scale in the installation of the facility. If the transmission cost is estimated to be a linear function of the capacity, then the separable programming technique will give a global optimum. Otherwise, a globally optimum solution cannot be guaranteed.

Data Generation

The model consists of two parts in terms of required calculation. The first part involves inputting the raw data, computing the various "constants" such as Q^n_{ij}'s and P^n_r's from the data, and setting up a problem matrix that contains the piecewise linear approximations of the non-linear terms in the objective function. The second part is simply solving the problem with an available linear programming code and, if necessary, performing a sensitivity analysis with a series of modifications on the problem matrix.

A computer program has been developed to do the first part of the calculation. The second part utilizes the IBM 360 Mathematical Programming System (17). The IBM code can handle large linear/separable programming problems.

The whole approach to modeling (aside from the computer program) is that the user can arbitrarily change the specific assumptions made in the model without affecting its general structure. In this respect, the model developed in this chapter can be regarded as a general model. The computational experience with this model based on actual and estimated data is presented in Chapter VI.

CHAPTER IV

ANALYSIS OF ENERGY STORAGE SYSTEMS

Introduction

In this chapter energy storage systems are modeled so that the results can be used in developing the optimization model for windpower systems with storage in Chapter V. The analysis consists of modifying and expanding an existing simple mathematical storage model to incorporate input/output conversion losses and reservoir leakage. The storage systems in this chapter represent the dam type of system in which the input is variable and the output is controlled. Thus, they are different from the inventory type of system in which the reverse is true. The main objective in studying a storage system is to find the probabilities of shortage or overflow. These probabilities are usually obtained from a probability distribution of the storage level. Therefore, the problem is how to derive such a distribution under the known input/output characteristics and storage capacity. The usual approach to solving the problem is to develop an analytical model representing the storage system.

Although some mathematical storage models were developed in the past, there has been little application of the models to real world problem solving, especially in the field of energy storage. The reasons for this lack of application appear to be that (a) the potential users of the models are mostly unaware of the existence of such models, and (b) even though the users know of their existence, the models are often not applicable because they are developed for specific input and output distributions. In many instances, however, if one cannot find an appropriate "dam" model for analyzing his system, he may be able to use one of the queueing models to represent his storage system. The use of a queueing model for a storage system analysis is possible because of the one-to-one correspondence between the queueing and the storage systems (see Ghosal (12), p. 3). The existing queueing models are found in Prabhu (29) and Gross and Harris (14).

Three storage models are introduced in this chapter. The first model represents a simple storage system originally developed by Moran (22,23,24) and later reviewed and expanded by Prabhu (27,28) and Ghosal (12). The model is developed on a discrete time scale, and under the assumption that the input consists of independent, identically distributed random variables. The second and third models are the results of modifying the simple model to take into account the losses during input and output conversions and the leakage from the reservoir. Both models are based on random input and fixed output. More specifically, the second model represents the case in which all the input goes through a conversion process where part of the input is lost and in which a fixed amount is retrieved from storage after reconversion in each time period. The third model is for systems in which only the difference between the variable input and the constant output is either stored or retrieved from storage. The input and output conversion losses will be less in this system than in the system in which all the input is first stored. For convenience, the two types of storage systems represented by the second and third models will be called "System 1" and "System 2", respectively. The notation used in studying these systems is that used by Ghosal.

Simple Storage Model

In a basic discrete-time storage model, the storage level at the beginning of a time interval is determined by the input, the output and the storage level in the previous time interval. Let us denote

Z_t = storage level at the beginning of the interval $(t,t+1)$,

X_t = input during the interval, and

Y_t = output at the end of the interval, and

assume that one or both of X_t and Y_t are random variables. Then, a sequence of Z_t's (i.e., $Z_1, Z_2, Z_3, \ldots$) forms a Markov chain represented by

$$Z_{t+1} = \min\{k, \max[0, Z_t+X_t-Y_t]\}, \tag{28}$$

where k is the capacity of storage. The objective of the analysis is to describe the steady state behavior of Z_t

with a probability density function (p.d.f.), or equivalently a cumulative distribution function (c.d.f.). To find the limiting c.d.f. of Z_t, let us define

$$F_t(y) = Pr(Z_t \le y) \text{ for } 0 \le y \le k$$

$$U_t = X_t - Y_t, \text{ and}$$

$$H = \text{the c.d.f. of } U_t$$

If Z_t has a limiting distribution independent of time t, we drop t from $F_t(y)$ to indicate F(y) is a stationary c.d.f. Then, the integral equation can be written as follows:

$$F(y) = -\int_0^{\infty} F(x)dH(y-x). \tag{29}$$

Using the equations F(x)=1 for $x \ge k$ and $H(-\infty)=0$, this becomes:

$$\begin{aligned} F(y) &= -\int_0^{k} F(x)dH(y-x) - \int_k^{\infty} dH(y-x) \\ &= H(y-k) - \int_0^{k} F(x)dH(y-x). \quad (0 \le y \le k) \end{aligned} \tag{30}$$

The conditions under which a unique solution exists for the above equation are given in Ghosal (12, p. 21).

It is not difficult to obtain a discrete analogue of Equation 30 for the system in which the input and the output are in discrete quantities. In such a discrete system, the problem is solving a set of simultaneous linear equations.

Now, let us consider a system in which the output is fixed at a constant quantity m. We represent the system by

$$Z_{t+1} = \min\{k-m, \max[0, Z_t+X_t-m]\}. \tag{31}$$

Let $\quad G$ = the c.d.f. of X_t.

Since $dH(u) = dG(u+m)$ and $F(x) = 1$ for $x \geq k-m$, we get

$$F(y) = -\int_0^\infty F(x)dH(y-x)$$

$$= -\int_0^\infty F(x)dG(y+m-x)$$

$$= -\int_{k-m}^\infty dG(y+m-x) - \int_0^{k-m} F(x)dG(y+m-x)$$

$$= G(y+2m-k) - \int_0^{k-m} F(x)dG(y+m-x). \quad (0 \leq y \leq k-m) \quad (32)$$

One way of solving this integral equation is to first write a discrete analogue of the system and then solve it as a system of simultaneous equations.[1] To obtain a discrete analogue, let us define

$$F_i = \Pr(Z_t \leq i) \text{ as } t \to \infty,$$
$$g_i = \Pr(X_t = i), \text{ and}$$
$$G_j = \sum_{i=0}^{i} g_i \text{ if } j \geq 0,$$
$$= 0 \quad \text{if } j < 0.$$

Then, we write
$$F_i = G_{i+2m-k} + \sum_{j=0}^{k-m-1} g_{i+m-j} \cdot F_j \quad \text{for } i = 0, 1, \ldots, k-m-1,$$
$$= 0 \quad \text{for } i < 0,$$
$$= 1 \quad \text{for } i \geq k-m. \quad (33)$$

---

[1]Another method of solving an integral equation is replacing the integral by a Gaussian quadrature formula and solving the resulting linear equations. See Stroud and Secrest (31).

For example, when k=8 and m=3, Equation 33 can be written in matrix form as:

$$\begin{bmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} g_3 & g_2 & g_1 & g_0 & 0 \\ g_4 & g_3 & g_2 & g_1 & g_0 \\ g_5 & g_4 & g_3 & g_2 & g_1 \\ g_6 & g_5 & g_4 & g_3 & g_2 \\ g_7 & g_6 & g_5 & g_4 & g_3 \end{bmatrix} \times \begin{bmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ G_0 \\ G_1 \\ G_2 \end{bmatrix}$$

The probabilities of shortage and overflow are given by:

$$\Pr(\text{shortage}) = \Pr(Z_t \le 0) = F(0) \approx F_0, \tag{34}$$

$$\begin{aligned} \Pr(\text{overflow}) &= \Pr(Z_t \ge k-m) \\ &= 1 - F(k-m-) \approx 1 - F_{k-m-1}. \end{aligned} \tag{35}$$

It should be noted that Moran and Prabhu based their storage analyses on finding the stationary distribution of Z_t+X_t instead of Z_t. The reason for this approach is that although Z_t lies in the range $[0,k]$, Z_t+X_t lies in $[0,\infty]$, which makes it less complex to solve the integral equation for some specific input distributions without resorting to a linear approximation. The shortage and overflow probabilities in the case of fixed output are obtained from the c.d.f. of Z_t+X_t.

By defining

$F'(y) = \Pr(Z_t+X_t \le y)$ as $t \to \infty$, we get

$$\Pr(\text{shortage}) = F'(m), \text{ and} \tag{36}$$

$$\Pr(\text{overflow}) = 1 - F'(k-). \tag{37}$$

Equations 30 and 32 may be modified to include storage leakage in the analysis. Let us first consider the case in which a fixed quantity q leaks from the reservoir at the end of each time interval. The integral equation corresponding to this case is

$$F(y) = -\int_0^{\infty} F(x)\,dH(y+q-x)$$

$$= -\int_0^{k-q} F(x)\,dH(y+q-x) - \int_{k-q}^{\infty} dH(y+q-x)$$

$$= H(y+2q-k) - \int_0^{k-q} F(x)\,dH(y+q-x) \qquad (38)$$

When the output is fixed, i.e., $Y_t = m$, we define U_t as $U_t = X_t - (m+q)$. Then, from Equation 32,

$$F(y) = G(y+2(m+q)+k) - \int_0^{k-m-q} F(x)\,dG(y+m+q-x). \qquad (39)$$

Equations 38 and 39 indicate that the leakage can be treated as part of the output; therefore, no separate analysis is necessary for this type of leakage.

The second case represents a variable leakage whereby a quantity proportional to the amount stored is lost at the beginning of each time interval. Let $1-e_0$ denote the leakage factor. The Markov chain corresponding to this case is

$$Z_{t+1} = \min\{k, \max[0, e_0 Z_t + X_t - Y_t]\}. \qquad (40)$$

The steady state distribution of Z_t is given by

$$F(y) = -\int_0^{\infty} F(x)\,dH(y - e_0x)$$

$$= H(y-e_0k) - \int_0^{k} F(x)\,dH(y-e_0x). \qquad (41)$$

When the output is fixed, i.e., Y_t=m, we have $F(x)=1$ if $x \geq k-m$. Then the c.d.f. of Z_t can be expressed in terms of G and F:

$$F(y) = G[y+(1+e_0)m-e_0k] - \int_0^{k-m} F(x)\,dG(y+m-e_0x). \qquad (42)$$

System 1

In this system, the input goes through an input process having an efficiency e_1 before entering the storage and the quantity released from the storage passes through an output process having an efficiency e_2 before leaving the system. The input X_t's are mutually independent random variables following a certain p.d.f., and the output Y_t's are fixed to m. The diagram of this system is shown below.

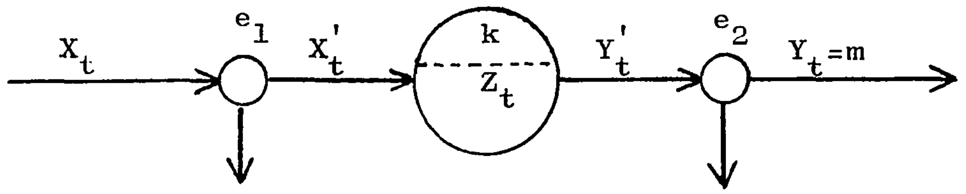

Figure 11. Diagram of System 1.

In the above diagram,

$$X_t' = e_1 X_t \text{ and } Y_t' = Y_t/e_2 = m/e_2.$$

Let G be the c.d.f. of X_t and G' the c.d.f. of X_t'. Then, G' is related to G by $G'(x) = G(x/e_1)$.

Let $\quad U_t = X_t' - Y_t' = X_t' - m/e_2$, and

$\quad H = $ the c.d.f. of U_t.

Since $H(U_t) = G'(U_t + m/e_2)$, we write the c.d.f. of Z_t as in Equation 32:

$$F(y) = G'(y+2m/e_2-k) - \int_0^{k-m/e_2} F(x)dG'(y+m/e_2-x). \quad (43)$$

Although not necessary, we can substitute G' with G to get

$$F(y) = G\left(\frac{y+2m/e_2-k}{e_1}\right) - \int_0^{k-m/e_2} F(x)dG\left(\frac{y+m/e_2-x}{e_1}\right). \quad (44)$$

This storage model will be applied in developing the model for windpower systems with storage, in which all the power generated from the windmills is stored and a fixed quantity of power is retrieved from storage every day.

If the unit time for describing storage operation is one day, the input X_t represents the total daily input from the windpower system and the output m represents the total Kwh of electricity transmitted each day. The total output m may be divided in any way and transmitted during different time periods of the day.

System 2

In this system, storage is used as a means of balancing the fluctuations of the input to keep the output constant. Some form of switching system would be necessary to operate this type of storage system. The control is done in such a way that when the input is greater than the required output, the excess quantity is stored; and when the input is below the output, the difference is extracted from storage. As in System 1, losses occur during the input and output stages of storage. The input and output losses are represented by the conversion efficiencies e_1 and e_2, respectively. The schematic diagram of this system is shown below.

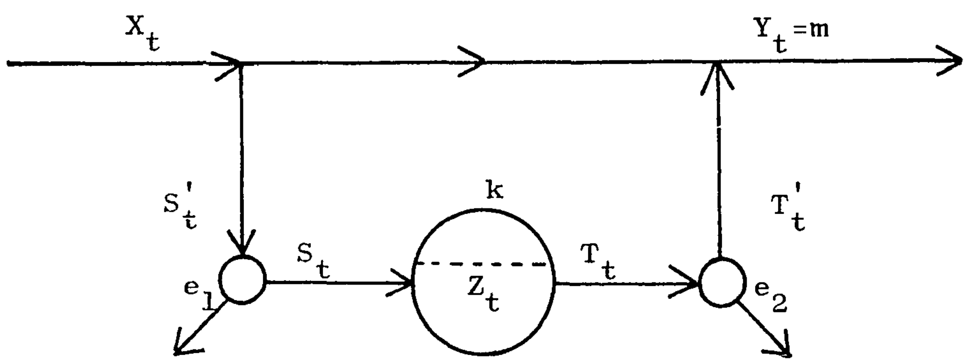

Figure 12. Diagram of System 2.

The four new variables in the diagram are related to X_t and m by

$$S_t' = \begin{cases} X_t - m \text{ if } X_t \geq m, \\ 0 \qquad \text{otherwise} \end{cases}$$

$$T_t' = \begin{cases} m-X_t & \text{if } X_t < m, \\ 0 & \text{otherwise} \end{cases}$$

$$S_t = e_1 S_t'$$

$$T_t = T_t'/e_2$$

The Markov chain corresponding to this system is

$$Z_{t+1} = \min\{k, \max[0, Z_t+S_t-T_t]\}. \tag{45}$$

Let G = the c.d.f. of X_t,

$U_t = S_t - T_t$,

H = the c.d.f. of U_t, and

$F(y) = \Pr(Z_t \le y)$ as $t \to \infty$.

Then, the c.d.f. of Z_t in terms of H and F is given by

$$F(y) = \int_0^\infty F(x)dH(y-x). \tag{46}$$

However, the above expression is not directly applicable for the analysis of System 2 type storage because H is difficult to derive. Therefore, the equation will be given in terms of G and F.

We first write

$$\begin{aligned} H(u) &= \Pr(S_t - T_t \le u) \\ &= \Pr(e_1 S_t' - T_t'/e_2 \le u) \\ &= \Pr\{(X_t - m)\cdot[e_1 \cdot \underset{[m,\infty]}{I(X_t)} + \frac{1}{e_2}\cdot \underset{[0,m]}{I(X_t)}] \le u\} \end{aligned} \tag{47}$$

where $\underset{[a,b]}{I(X_t)} = \begin{cases} 1 & \text{if } a \le X_t \le b, \\ 0 & \text{otherwise.} \end{cases}$

From the above equation, we get the relationship between H and G:

$$H(u) = \begin{cases} Pr(X_t - m \le e_2 u) = G(m+e_2 u) & \text{if } u < 0, \\ Pr(X_t - m \le u/e_1) = G(m+u/e_1) & \text{if } u \ge 0. \end{cases} \tag{48}$$

In substituting H in Equation 46 with G, we must consider two cases, $y-x < 0$ and $y-x \ge 0$, separately. We get

$$F(y) = -\int_0^\infty F(x)dH(y-x)$$

$$= -\int_0^y F(x)dH(y-x) - \int_y^\infty F(x)dH(y-x)$$

$$= -\int_0^y F(x)dG\left[m+\frac{y-x}{e_1}\right] - \int_y^\infty F(x)dG[m+e_2(y-x)] \,. \tag{49}$$

Using the fact that $F(x)=1$ for $x \ge k$,

$$F(y) = -\int_0^y F(x)dG\left[m+\frac{y-x}{e_1}\right] - \int_y^k F(x)dG[m+e_2(y-x)]$$

$$+ G[m+e_2(y-k)] \,. \tag{50}$$

In a special case where $e_1 = e_2 = 1$, the above equation becomes

$$F(y) = G(y+m-k) - \int_0^k F(x)dG(y+m-x). \tag{51}$$

Comparing this equation with Equation 44 for the case where $e_1 = e_2 = 1$, we see that System 2 requires less storage capacity than System 1 to give the same shortage probability. The difference is the fixed output m.

When the input is wind-generated power, System 2 represents a storage system that is used for keeping the output power constant without converting all the input power for storage and then retrieving it. The advantage of this system over System 1 is that less energy is lost during conversion by directly transmitting part of the power generated.

Since the analysis is done only for the case of constant power output, the model for System 2 cannot be incorporated into the optimization model developed in Chapter V where the total daily output is fixed, but the power transmitted varies with the time of the day. To be able to incorporate the storage model into the model in Chapter V, the output Y_t must be allowed to vary. A more detailed analysis of the System 2 type storage appears necessary to obtain usable results.

CHAPTER V

THE MODEL FOR WINDPOWER SYSTEMS WITH STORAGE

Introduction

Chapter III developed a model for windpower systems having no storage capacity. Chapter IV presented two mathematical storage models (System 1 and System 2) that might be used in analyzing energy storage systems with variable input and fixed output. In terms of windpower storage, the first model represents the case where all of the energy generated by the windmills is first stored and then a predetermined amount is released during certain periods of the day. The second model is for the case where storage is used primarily for keeping the output power constant. This chapter develops a model for windpower systems with storage that operates in the same way as in System 1.

Since the output from storage can be controlled, the windpower system with storage may be either base-loaded or peak-loaded, whichever is a more economical way of operating the system. In modeling the system, two general cases are considered: Case A and Case B (see the last two sections of Chapter I for the description of the cases). For Case A, as for the no storage case, the value of the output energy will be greater during peak hours than off-peak hours, because the per Kwh fuel costs during peak hours are generally higher due to the fact that peaking units often burn more expensive fuels. For Case B, no reference to the economic value of the fuel saved is made in modeling the system. Instead, the objective of the model is to find a minimum cost design that can deliver specified levels of power to the network with sufficiently low probability of shortage. When the optimal design is found, the per Kwh cost of windpower can be compared with that of other alternative power sources to determine the economic feasibility of the windpower system.

Some discussion is necessary regarding the divisions of time used in the model of this chapter. These divisions are, in decreasing order of length: year, season, day, period, and time increment. As in the model without storage, year is the time unit used in comparing the expected fuel cost savings with the windpower system costs. A year is divided into one or more seasons to account for the seasonal variations in wind velocity and demand. Day is the time unit for the storage system; therefore, the input from the windmills must be expressed in terms of the total electricity generated during a day. Similarly, the output to the network would be in terms of the total electricity transmitted during a day. A day may be divided into periods of varying lengths in two ways: one way to represent the diurnal variations of wind and the other to represent demand variations during the day. The length of each season and each period must be specified by the user. The model developed in this chapter also requires the user to choose the length of a time increment; however, in most instances, one hour will be the user's choice since the wind data available from local weather stations are usually measured on an hourly basis. In this chapter, the unit "hour" will sometimes be used interchangeably with "time increment."

The chapter is organized into two parts. The first part develops a general model that can be used for both Case A and Case B under the condition where there is very little correlation between average wind speeds in successive time increments. The second part discusses the condition where there is significant correlation between wind speeds in successive time increments.

Development of the Model for the Condition of Very Low Serial Correlation of Wind Speeds

This part of the chapter is organized in the following way. First, a proof is provided that the daily output from windpower systems is normally distributed when the wind speeds between the time increments show very low correlation. This normality is used then to derive a functional relationship between the shortage probability and the storage design variables. The system optimization model is then formulated in a manner similar to that used for the no storage model in

Chapter III. This formulation is then discussed with respect to its mathematical properties, the solution techniques, and the data generation procedure.

Normality of the Daily Output Distribution

Since the unit time interval used for the storage system is one day and the storage model requires the specification of a probability distribution for power input to storage, it is necessary to derive the probability distribution of the total daily electricity generated by the windmills. The daily output from the windmills is determined by the wind conditions, the characteristics of windpower conversion of various designs, and the number of windmills. It will be shown below that when the wind speeds in successive time increments show little correlation (i.e., the wind speeds are nearly random), both the daily output from the individual windmills and the total daily output from all windmills are approximately normally distributed regardless of the probability distribution for wind speeds.

Let us consider a windpower system consisting of only one windmill. Let

J = number of periods in a day having different wind speed distributions,

h_j = number of time increments in period j ($j = 1, \ldots, J$),

m_j = mean of the distribution of the "hourly" power output (Kwh) in period j,

v_j = variance of the distribution of the "hourly" power output in period j,

Q_{jt} = observed power output in the t^{th} time increment of period j, and

$\overline{Q}_{jt}$ = average of Q_{jt}'s ($t = 1, \ldots, h_j$).

If we treat the hourly power output as a random variable, then Q_{jt}'s can be considered as a random sample of size h_j taken from the "population" of the power output. Then, according to the Central Limit Theorem $(\overline{Q}_{jt} - m_j)/(v_j/h_j)^{1/2}$ is the value of a random variable whose distribution approaches the standard normal distribution as h_j tends to infinity. Since

$$\frac{\overline{Q}_{jt}-m_j}{\sqrt{v_j/h_j}} = (\sum_{t=1}^{h_j} Q_{jt}-h_jm_j)/\sqrt{h_jv_j} \ ,$$

if we let

$$Q_j = \sum_{t=1}^{h_j} Q_{jt} \text{ or the total output in period j,}$$

then Q_j will be approximately normally distributed with mean h_jm_j and variance h_jv_j when h_j is sufficiently large. In notation,

$$Q_j \sim N(h_jm_j, h_jv_j) \text{ for large } h_j. \tag{52}$$

For the purpose of approximating the daily output distribution, h_j may be considered large enough to apply the Central Limit Theorem if it is greater than five or six time increments. Let us define

$$Q_o = \sum_{j=1}^{J} Q_j, \text{ or}$$

$\quad$ = total output (Kwh) in one day.

Since Q_j's are independent, normally distributed random variables, Q_o is also normally distributed[1] with mean

$$m_o = \sum_{j=1}^{J} h_j m_j \text{ and} \tag{53}$$

variance

$$v_o = \sum_{j=1}^{J} h_j v_j. \tag{54}$$

In notation,

$$Q_o \sim N(\sum_{j=1}^{J} h_j m_j, \sum_{j=1}^{J} h_j v_j), \text{ or}$$

$$\sim N(m_o, v_o). \tag{55}$$

Now, let n, used as a superscript, denote the design of the windmill, and let

x^n = number of windmills of design n, and

$Q^n = x^n Q_o^n$, or

$\quad$ = total daily output from all windmills of design n.

Then

$$E(Q^n) = x^n m_o^n, \text{ and} \tag{56}$$

$$V(Q^n) = (x^n)^2 v_o^n. \tag{57}$$

Since x^n acts as a scalar for the distribution of Q_o^n, we get

$$Q^n \sim N(x^n m_o^n, (x^n)^2 v_o^n). \tag{58}$$

Denoting the mean and variance of Q^n by m^n and v^n, respectively,

$$Q^n \sim N(m^n, v^n). \tag{59}$$

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[1]This can easily be shown using the moment generating function of normal distribution. See Clark and Disney (7), p. 156 and p. 190.

As a final step for the proof of the normality of the daily output from all windmills, let us denote

$$Q = \sum_{n=1}^{N} Q^n, \text{ or}$$

= total daily output from all windmills of all designs.

Then,

$$E(Q) = \sum_{n=1}^{N} E(Q^n)$$

$$= \sum_{n=1}^{N} m^n \qquad (60)$$

$$V(Q) = \sum_{n=1}^{N} V(Q^n) + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} \mathrm{Cov}(Q^i, Q^j)$$

$$= \sum_{n=1}^{N} V(Q^n) + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} r_{ij} \sqrt{V(Q^i)} \sqrt{V(Q^j)}$$

$$= \sum_{n=1}^{N} v^n + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} r_{ij} \sqrt{v^i} \sqrt{v^j}, \qquad (61)$$

where r_{ij} = correlation between the total daily output from the windmills of design i and those of design j.

Since all the windmills are located in the same site, they are subject to the same wind conditions. Therefore, the daily output from different designs should be highly correlated. To illustrate, hypothetical probability distributions of the daily output from two groups of the windmills, design i and design j, are shown below:

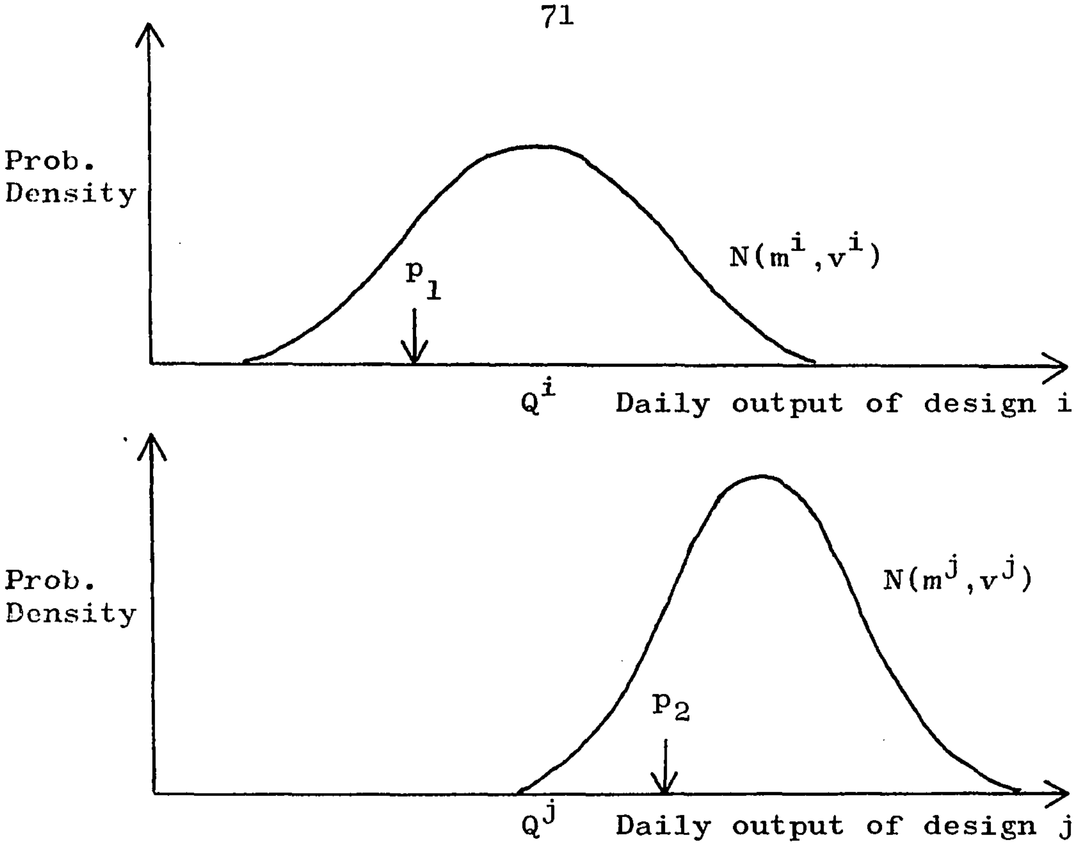

Figure 13. Hypothetical probability distributions of the daily output from two windmill designs.

In the above figure, p_1 and p_2 are the points on the Q-axes such that $Pr(Q^i \le p_1) = Pr(Q^j \le p_2)$. The reasoning is that if the output from design i is p_1, the output from design j should be near p_2 regardless of the shapes of power output curves. A simple computer simulation was used to make inferences on the degree of correlation of the output from different designs. The result shown in Appendix B suggests that the correlation would be close to 1.

When $r_{ij} \approx 1$, the variance of the output from all windmills given in Equation 61 becomes

$$V(Q) = \sum_{n=1}^{N} v^n + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} \sqrt{v^i} \sqrt{v^j}$$

$$= \left(\sum_{n=1}^{N} \sqrt{v^n} \right)^2. \qquad (62)$$

Also, when $r_{ij} \approx 1$, Q^n's are linearly dependent to one another (i.e., $Q^i = a_{ij}Q^j + b_{ij}$ for all i,j). This means that Q is also a linear function of any Q^n, and thus Q is normally distributed.[1] Then, from Equations 60 and 62,

$$Q \sim N\left(\sum_{n=1}^{N} m^n, \left(\sum_{n=1}^{N} \sqrt{v^n} \right)^2 \right), \text{ or} \qquad (63)$$

in terms of individual windmills,

$$Q \sim N\left(\sum_{n=1}^{N} x^n m_o^n, \left(\sum_{n=1}^{N} x^n \sqrt{v_o^n} \right)^2 \right). \qquad (64)$$

The result of this analysis is summarized below.

Let

Q = total daily output (Kwh) from all windmills,

Q_o^n = total daily output from one unit of design n (n = 1, ..., N),

x^n = number of windmills of design n

Q_j^n = total output from one unit of design n in period j (j = 1, ..., J)

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[1]This normality can also be explained with the application of the Central Limit Theorem to the sum of the "hourly" output from the windpower system as a whole.

h_j = number of time increments in period j

m = mean of the probability distribution of Q

m_o^n = mean of the probability distribution of Q_o^n

m_j^n = mean of the probability distribution of Q_j^n

v_n = variance of the probability distribution of Q

v_o^n = variance of the probability distribution of Q_o^n

v_j^n = variance of the probability distribution of Q_j^n.

Then Q is normally distributed with mean

$$m = \sum_{n=1}^{N} x^n m_o^n, \text{ and variance} \tag{65}$$

$$v = \left(\sum_{n=1}^{N} x^n \sqrt{v_o^n} \right)^2, \tag{66}$$

where

$$m_o^n = \sum_{j=1}^{J} h_j m_j^n, \text{ and} \tag{67}$$

$$v_o^n = \sum_{j=1}^{J} h_j v_j^n. \tag{68}$$

For the above relations to hold, there should be little correlation of the wind speeds between successive days, between successive periods in each day, and between successive time increments in each period.

Determination of Storage Requirements

In the preceding section, it was shown that the daily output from the windpower system would be normally distributed if the wind speeds show very little serial correlation. This normality of the total daily output is quite useful in computing the storage requirements since any normal distribution can be transformed into a standardized normal distribution. In this research, a storage system is completely described by defining its input distribution, fixed output, storage capacity and the probability of shortage occurrences. In addition, the conversion efficiencies in the input and output processes are also considered in characterizing the storage system (See System 1 in Chapter IV). In actual modeling of windpower systems with storage, however, it is more convenient to define the input to storage as the "net" input after conversion and the output from storage as the "gross" output before conversion. The following discusses the standardization of input which is normally distributed, the evaluation of shortage probabilities under a "standard"[1] normal input, and a method of using the result of the evaluation in economic decision making.

Any normal distribution can be transformed into a "standard" normal distribution with mean w and variance 1. Let

X_t = net input to storage in day t, following a normal distribution with mean W and standard deviation D.

M = fixed gross daily output from storage

K = storage capacity.

We also denote the input, output and storage capacity

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[1]" " is used to signify that the distribution is different from the standard normal distribution with mean 0 and variance 1.

corresponding to the standardized normal input by x_t, m and k, respectively. A transformation of a hypothetical input distribution to a "standard" normal distribution is illustrated below:

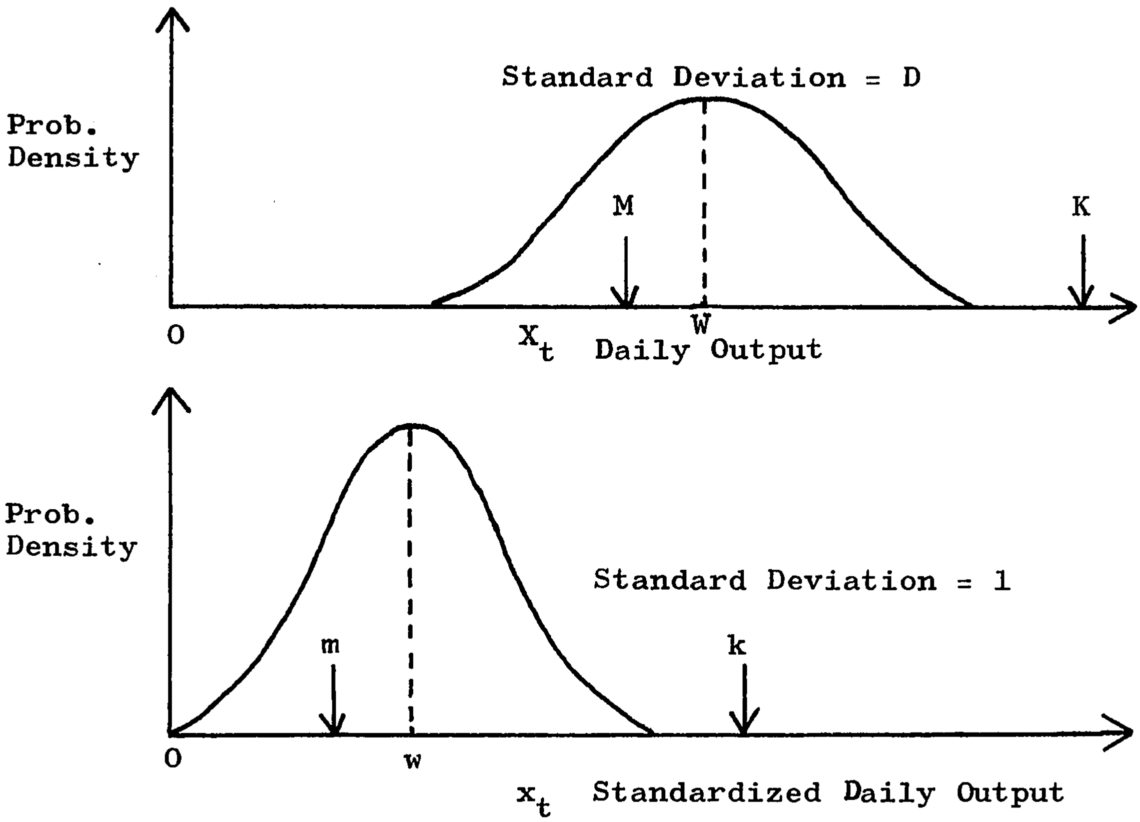

Figure 14. Standardization of a normally distributed input.

In the lower diagram, w is a value such that $Pr(x_t \leq 0) \approx 0$. Any value around 3 or greater will give a probability close to 0. In the upper diagram, the value of X_t corresponding to $x_t = 0$ is W - wD. The relationships between M and m, and between K and k are

$$M = W - (w-m)D, \text{ and} \tag{69}$$

$$K = W + (k-w)D. \tag{70}$$

The shortage probability for the case of a "standard" normal input and fixed k and m can be obtained by solving the simultaneous equations expressed by Equation 33 in Chapter IV. Since the input X_t is a continuous variable, Equation 32 ought to be used; however, a closed form solution has not been worked out for the input that is normally distributed.

The curves shown in Figure 15 are the result of solving many sets of simultaneous equations for the distribution of storage level and plotting the probability that the storage level is less than or equal to zero. Since the standardized capacity k and output m are really economic decision variables, the shortage probabilities are computed for various combinations of k and m. A discrete probability distribution that approximates a "standard" normal distribution with mean 3.75 and variance 1 is used in obtaining the plots. Actually, it is not necessary to separately compute the shortage probabilities for the cases where m is greater than the mean w. This is because in any symmetric input distribution,

$$\Pr[\text{shortage when } m = w+d]$$
$$= \Pr[\text{overflow when } m = w-d]$$

where d is a positive quantity. The proof is provided in Appendix C. As the plots show, the shortage probability is quite sensitive to the output, especially when the difference between the capacity and the output is small. The use of the plots in designing an economical storage system will be discussed in the formulation of the model.

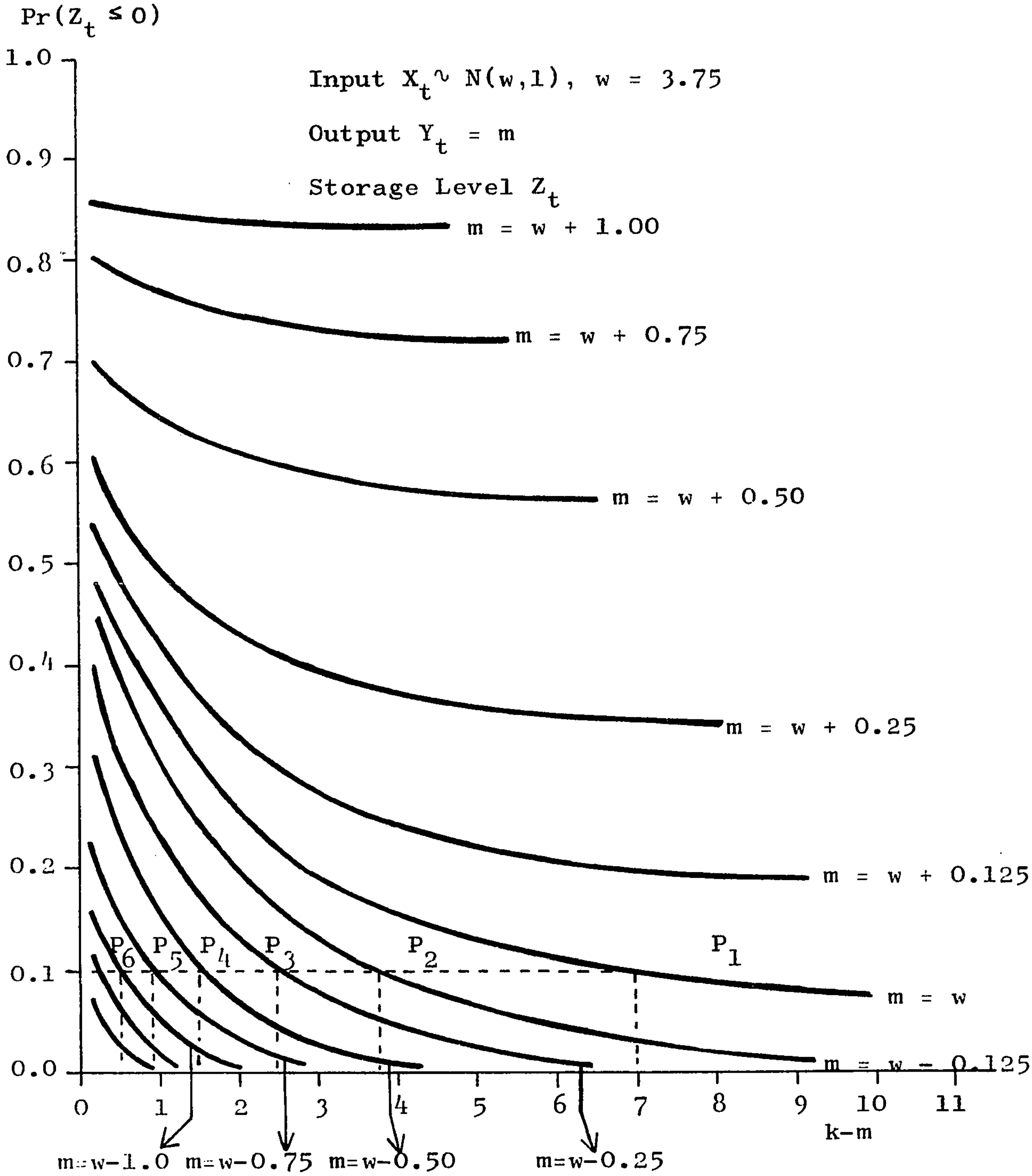

Figure 15. Probability of shortage under a standard normal input and fixed output.

Variables, Constants and Functions

Before presenting the general model formulation, the variables, constants and functions are first defined. In defining the notation used, the subscript $\bar{j}$ refers to the period of the day with respect to the diurnal variation of wind speeds, and j refers to the period of the day with respect to the demand variation.

1. Variables

x^n = number of windmills of design n (n = 1, ..., N)

y_{ij} = expected power (Kw) to be transmitted from the storage system to the network in season i, period j (i = 1, ..., I; j = 1, ..., J)

S_0 = capacity (Kwh) of the storage system

S_1 = capacity (Kw) of the input conversion facility for storage

S_2 = capacity (Kw) of the output conversion facility for storage

p = capacity (Kw) of the transmission system (dependent on S_2)

W_i^o = expected daily output (Kwh) from the windpower system in season i (dependent on x^n's)

D_i^o = standard deviation (Kwh) of the daily output from the windpower system in season i (dependent on x^n's)

U = expected total electricity transmitted (Kwh) in a year from the windpower system (dependent on y_{ij}'s)

m_i = daily gross output (Kwh) from storage in season i, corresponding to a normally distributed net input with mean w and variance 1

k_i = storage capacity (Kwh) in season i for a normally distributed net input with mean w and variance 1 (Although the actual storage capacity S_0 is fixed for all seasons, the "relative" capacity, in terms of the "standard" normal input, changes from season to season due to differences in variance of the daily output.)

2. Constants

a. Windpower system design

N = total number of designs

D^n = rotor diameter (ft.) of design n

V_c^n = cut-in speed (mph) of design n

V_r^n = flat-rate speed (mph) of design n

η^n = conversion efficiency of design n

P_r^n = rated power (Kw) of design n

$= 4.00 \times 10^{-6}\ \eta^n (D^n)^2 (V_r^n)^3$

H^n = tower height (ft.) of design n

A^n = land use (acres) by one unit of design n

b. Windpower system costs

C_w^n = installation cost (dollars) of one unit of design n

O_w = ratio of the annual operating and maintenance costs of a windmill to its installation cost (assumed fixed for all wind-mills)

b_w = economic life of a windmill (assumed the same for all windmills)

C_a = price of land (dollars per acre)

c. Storage system

e_1 = input conversion efficiency of storage

e_2 = output conversion efficiency of storage

O_1 = per Kwh operating cost of the input facility

O_2 = per Kwh operating cost of the output facility

b_O = economic life of the storage

b_1 = economic life of the input facility

b_2 = economic life of the output facility

d. Power transmission

λ = transmission efficiency (a fixed distance assumed)

b_t = economic life of the facility

e. Wind data, expected power output and demand

I = number of seasons in a year

$\bar{J}$ = number of periods in a day as broken down by the wind variation

J = number of periods in a day as broken down by the demand variation

K = maximum number of velocity increments used in approximating wind distributions

$\bar{d}_i$ = expected number of days in season i, in which no shortage occurs

$h_{\bar{j}}$ = number of hours in period $\bar{j}$ (with respect to the wind variation)

h_j = number of hours in period j (with respect to the demand variation)

v_o = velocity increment (mph) of the wind distribution at the height of the measurements

$\bar{V}_k$ = median speed (mph) of the k^{th} interval of wind distributions

$R_{i\bar{j}k}$ = $Pr[(k-1)v_o \leq V_{i\bar{j}} < kv_o]$, where $V_{i\bar{j}}$ is the wind velocity in season i, period $\bar{j}$

H_o = reference height (ft.) at which winds are measured

α = exponent of the wind profile function given by $V_h = V_o(H/H_o)^{\alpha}$, where V_h and V_o are the velocities at height H and H_o, respectively

$E(q^n_{i\bar{j}})$ = expected value of the power output (Kw) from one unit of design n in season i, period $\bar{j}$

$V(q^n_{i\bar{j}})$ = variance of the power output (Kw) from one unit of design n in season i, period $\bar{j}$

W^n_i = expected daily output (Kwh) from one unit of design n in season i

D^n_i = standard deviation (Kwh) of the daily output from one unit of design n in season i

E_{ij} = expected demand (Kw) in season i, period j

f. Other constants

P^m = minimum allowable capacity (Kw) of the windpower system

P^M = maximum allowable capacity (Kw) of the wind-power system

A^M = acres of land available for the system installation

r = annual interest rate applicable to all capital investments

$(a/p)_r^b$ = capitalization factor for economic life b and interest rate r

$= \frac{r(1+r)^b}{(1+r)^b-1}$ assuming no salvage value at the end of economic life

3. Functions

g(p) = installation cost of the transmission system as a function of its capacity

f(E) = aggregate plant fuel cost (dollars/hour) as a function of the total load E (Kw) ($E^m \leq E \leq E^M$, where E^m and E^M are the minimum and maximum capacities of the existing plants

$\phi_0(S_0)$ = installation cost of the storage system

$\phi_1(S_1)$ = installation cost of the input conversion facility for storage

$\phi_2(S_2)$ = installation cost of the output conversion facility for storage

ψ(w-m) = functional relationship between k-m and w-m giving a certain shortage probability, where k is the storage capacity and m is the fixed output in a standardized storage system (see Figure 17).

Formulation of the Model

The models for Case A and Case B are very similar, and therefore this section will develop the model for Case A followed by a description of the simple modifications required to handle Case B. The formulation of the model for Case A involves

(a) expressing the mean and the standard deviation of the daily windpower system output as functions of those of the individual windmills,

(b) deriving a functional relationship between the storage capacity and the storage output level for a specified probability of shortage under a normal input, and

(c) setting up the model as a mathematical programming problem with the objective of maximizing the net fuel cost savings.

It was previously shown that the total daily output from the windpower system is normally distributed if the wind speeds show a very low serial correlation. From Equations 65 and 66, we can express the mean (W_i^o) and the standard deviation (D_i^o) of the daily output from the whole system as linear functions of those from individual windmills as follows:

$$W_i^o = \sum_{n=1}^{N} W_i^n \cdot x^n \qquad (71)$$

$$D_i^o = \sum_{n=1}^{N} D_i^n \cdot x^n \qquad (72)$$

From Equations 67 and 68, we get

$$W_i^n = \sum_{j=1}^{\bar{J}} h_{\bar{j}} \cdot E(q_{i\bar{j}}^n) \tag{73}$$

$$D_i^n = \left(\sum_{j=1}^{\bar{J}} h_{\bar{j}} \cdot V(q_{i\bar{j}}^n) \right)^{1/2} \tag{74}$$

The calculation of $E(q_{i\bar{j}}^n)$ based on the wind data and the windmill design parameters is given by Equation 9 in Chapter III. It is repeated below:

$$E(q_{i\bar{j}}^n) = 4.00 \times 10^{-6} \cdot \eta^n \cdot (D^n)^2 \cdot [(H^n/H_o)^{3\alpha} \cdot \sum_{k=k_1^n}^{k_2^n - 1} (\bar{V}_k)^3 \cdot R_{i\bar{j}k} + (V_r^n)^3 \cdot \sum_{k=k_2^n}^{K} R_{i\bar{j}k}], \tag{75}$$

where $k_1^n = \lfloor (V_c^n/v_o)/(H^n/H_o)^{\alpha} \rfloor + 1$ and (76)

$$k_2^n = \lfloor (V_r^n/v_o)/(H^n/H_o)^{\alpha} \rfloor + 1. \tag{77}$$

The variance of the power output is given by

$$V(q_{i\bar{j}}^n) = (4.00 \times 10^{-6} \cdot \eta^n)^2 \cdot (D^n)^4 \cdot [(H^n/H_o)^{6\alpha} \sum_{k=k_1^n}^{k_2^n - 1} (\bar{V}_k)^6 \cdot R_{i\bar{j}k} + (V_r^n)^6 \cdot \sum_{k=k_2^n}^{K} R_{i\bar{j}k}] - [E(q_{i\bar{j}}^n)]^2, \tag{78}$$

where k_1^n and k_2^n are defined in Equations 76 and 77. The assumptions made in this model regarding the variability of wind speed with respect to the tower height are the same as those made in Chapter III.

The annual cost of a windmill consists of the installation cost, the operating and maintenance cost, and the land cost, and is equal to

$$[(a/p)_r^{b_w} + O_w] \cdot C_w^n + C_a \cdot A^n \cdot r. \tag{79}$$

The user of the model must either specify the cost (C_w^n) and the land usage (A^n) for each windmill design, or provide the expressions for relating C_w^n and A^n to the design parameters (e.g., D^n and V_r^n).

The capacities of the storage input and output facilities and the transmission facility are given by

$$S_1 = \sum_{n=1}^{N} P_r^n \cdot x^n, \tag{80}$$

$$S_2 = \frac{1}{e_2} \max_{i,j}[y_{ij}], \text{ and} \tag{81}$$

$$p = \max_{i,j}[y_{ij}] = e_2 S_2. \tag{82}$$

The above equations are actually part of the constraints in the final problem formulation. The equalities may be replaced with "≥" type inequalities without affecting the solution value as long as the costs of the facilities increase with their capacities. The annual equivalent installation costs of the three facilities are given by

$$(a/p)_r^{b_1} \cdot \phi_1(S_1), \tag{83}$$

$$(a/p)_r^{b_2} \cdot \phi_2(S_2), \tag{84}$$

$$(a/p)_r^{b_t} \cdot g(p), \tag{85}$$

where ϕ_1, ϕ_2 and g are the functions provided by the user. The annual operating and maintenance (O&M) cost of the input conversion facility is obtained by

$$O_1 \cdot \sum_{i=1}^{I} \overline{d}_i \cdot W_i^o, \tag{86}$$

where $\overline{d}_i$ is equal to 1 - Pr(shortage) <u>times</u> the number of days in season i (d_i), and O_1 is the per Kwh O&M cost. The annual O&M cost of the output conversion facility is, on the basis of the input to the facility,

$$(O_2/e_2) \cdot U, \tag{87}$$

where U is the expected total output from the facility in a year, excluding the days when shortage occurs. More precisely,

$$U = \sum_{i=1}^{I} \overline{d}_i \left(\sum_{j=1}^{J} h_j \cdot y_{ij} \right), \tag{88}$$

where the inner summation term represents the gross amount of the daily input to the network divided into J periods.

The expected annual fuel cost savings can be expressed by

$$\sum_{i=1}^{I} \sum_{j=1}^{J} [f(E_{ij}) - f(E_{ij} - \lambda y_{ij})] \cdot h_j \cdot \overline{d}_i, \tag{89}$$

where the terms inside the brackets represent the expected per hour fuel cost savings when the demand is E_{ij} Kw and the net received power from the storage system is λy_{ij} Kw. At any time, the power from the storage system must be less than the difference between the demand and the minimum capacity of the power plants, or

$$\lambda y_{ij} \le E_{ij} - E^m \text{ for all } i,j. \tag{90}$$

Other constraints that are of minor importance, as far as the formulation is concerned, are those on the land availability and the desired minimum and maximum capacities of the windpower system. They are

$$\sum_{n=1}^{N} A^n . x^n \le A^M, \tag{91}$$

$$S_1 \ge P^m, \text{ and} \tag{92}$$

$$S_1 \le P^M, \tag{93}$$

where A^M is the maximum available land, and P^m and P^M are the minimum and the maximum capacities of the windpower system.

In this model, the daily discharge from storage varies from season to season, while the shortage probabilities are kept the same for all seasons (the user of the model specifies the shortage probability). Therefore, for a given shortage probability and the characteristics of the daily output from the windmills, the storage capacity S_0 and the daily discharge from storage must be determined simultaneously. To illustrate how this can be done, let us take a two-season storage problem in which the input is normally distributed and the output is constant in each season. Let

W_1, W_2 = means of the input distributions in season 1 and season 2,

D_1, D_2 = standard deviations corresponding to W_1 and W_2, and

Y_1, Y_2 = daily output from storage in Season 1 and Season 2 to be determined along with S_0.

Figures 16-a and 16-b illustrate a possible way of operating the storage system. To find how Y_1, Y_2 and S_0 are interrelated, we first determine the "relative" output m_1 and m_2 as indicated in Figure 16-c from

$$W_1 - Y_1 = (w-m_1)\cdot D_1, \text{ and} \tag{94}$$

$$W_2 - Y_2 = (w-m_2)\cdot D_2. \tag{95}$$

With m_1 and m_2 determined, the "relative" capacities k_1 and k_2 are obtained from the relationship

$$k_i - m_i = \psi(w-m_i), \text{ for } i = 1,2, \tag{96}$$

where ψ sets the shortage probabilities in all seasons to a pre-fixed value. Figure 17 gives an example of such a function for a 0.1 shortage probability. The points on the graph are obtained from Figure 15 by drawing a horizontal line for a 0.1 shortage probability and reading the intersections between the line and the curves. The relationships between S_0 and Y_i's are given by

$$S_0 - Y_1 = (k_1-m_1)\cdot D_1, \text{ and} \tag{97}$$

$$S_0 - Y_2 = (k_2-m_2)\cdot D_2. \tag{98}$$

For the given input distributions and shortage probability, the interrelationships described by Equations 94-98 characterize the operation of the storage system.

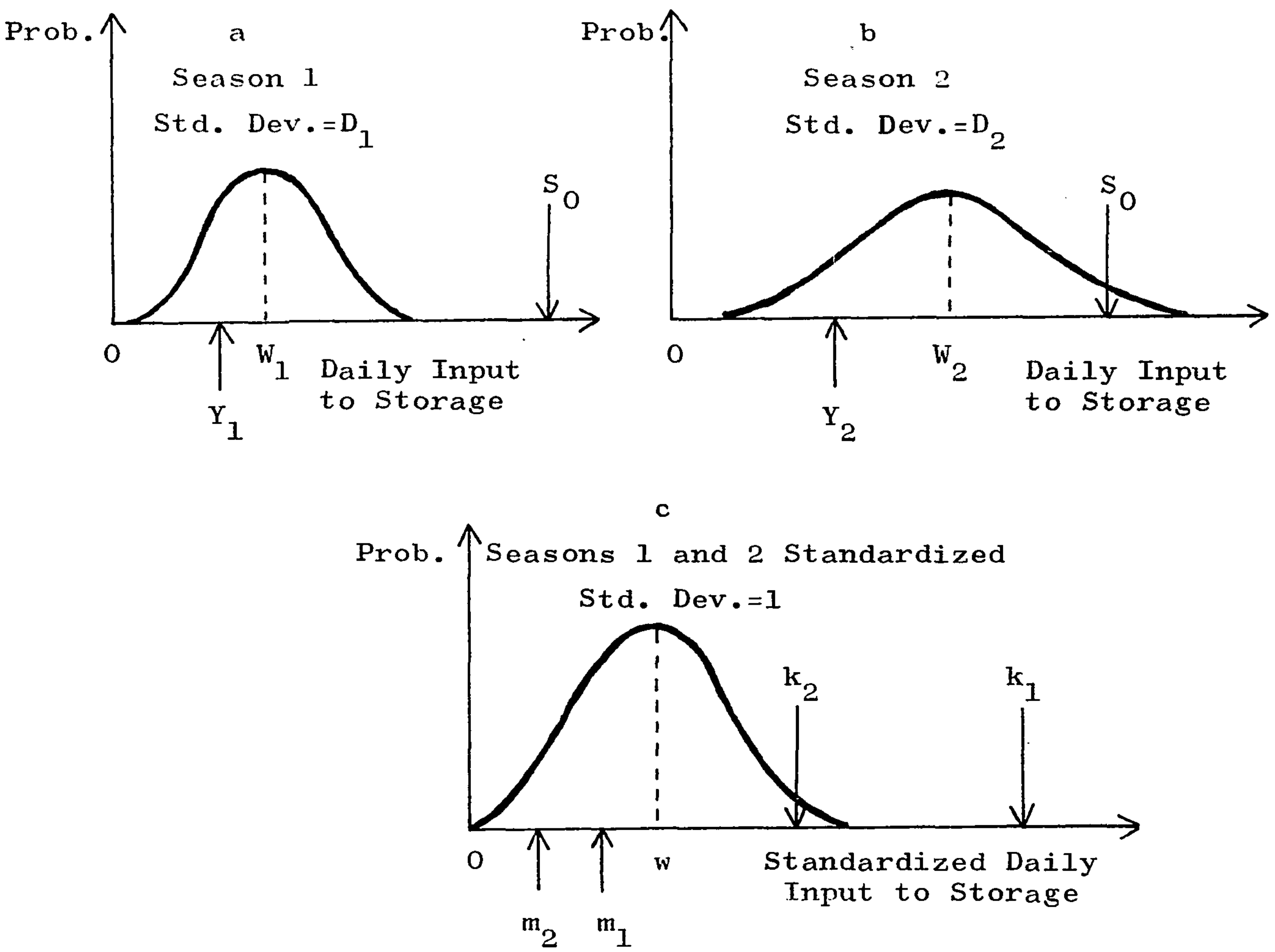

Figure 16. Standardization of storage operation with fixed capacity in two seasons.

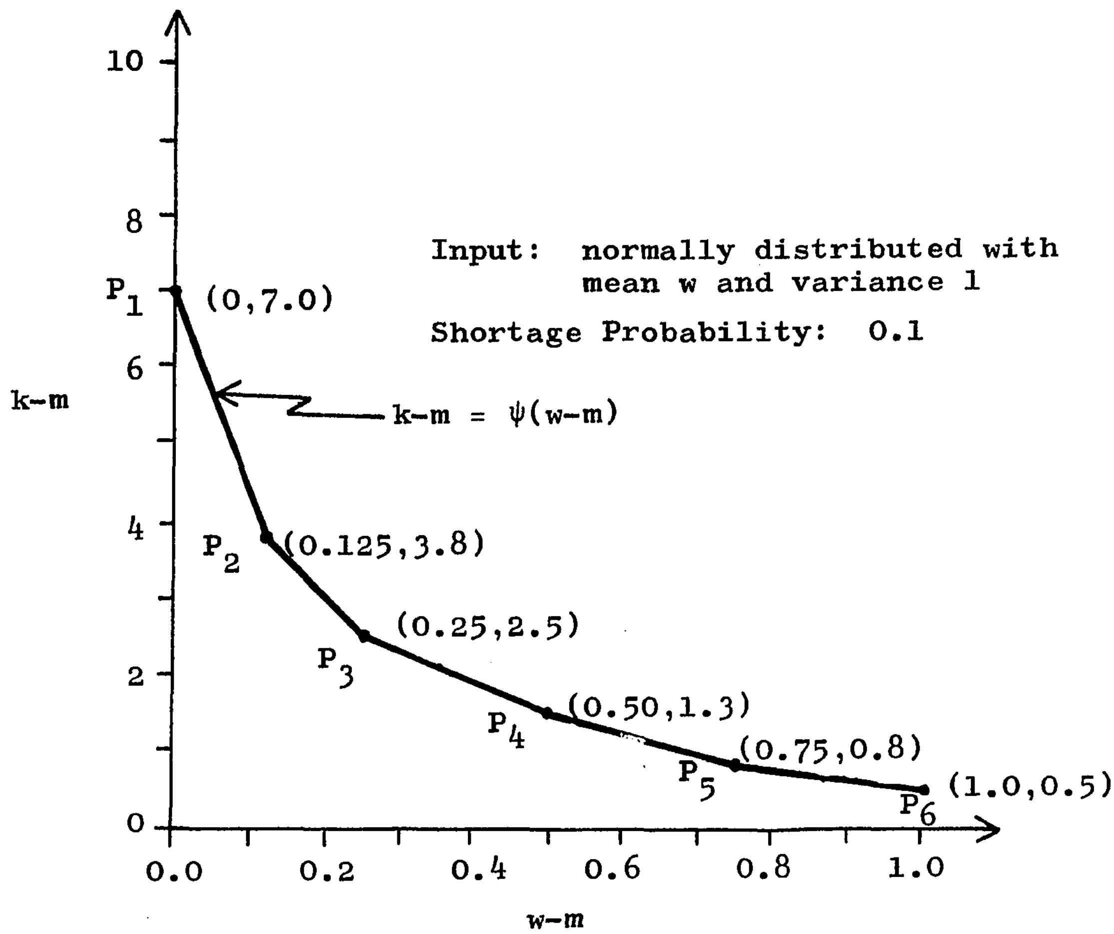

Figure 17. Relationship between the capacity and the output for 0.1 shortage probability.

In generating the equations which describe the storage system optimization model, we must incorporate the input/output conversion efficiencies into Equations 94-98 and substitute Y_i's with

$$\sum_{j=1}^{J} h_j \cdot y_{ij}.$$

The complete formulation of the model is as follows:

$$\text{Maximize} \sum_{i=1}^{I} \sum_{j=1}^{J} \{[f(E_{ij})-f(E_{ij}-\lambda y_{ij})] \cdot h_j \cdot \overline{d}_i\}$$

$$- \sum_{n=1}^{N} \{[(a/p)_r^{b_w}+O_w] \cdot C_w^n + C_a \cdot A^n \cdot r\} \cdot x^n$$

$$- [(a/p)_r^{b_O} \cdot \phi_O(S_O) + (a/p)_r^{b_1} \cdot \phi_1(S_1) + (a/p)_r^{b_2} \cdot \phi_2(S_2)]$$

$$- [\sum_{i=1}^{I} O_1 \cdot \overline{d}_i \cdot W_i^o + \frac{1}{e_2} \cdot O_2 \cdot U]$$

$$- (a/p)_r^{b_t} \cdot g(p) \qquad (99)$$

$$\text{subject to} \sum_{n=1}^{N} W_i^n \cdot x^n = W_i^o \quad \text{for all } i \qquad (100)$$

$$\sum_{n=1}^{N} D_i^n \cdot x^n = D_i^o \quad \text{for all } i \qquad (101)$$

$$\sum_{n=1}^{N} P_r^n \cdot x^n = S_1 \qquad (102)$$

$$\sum_{n=1}^{N} A^n \cdot x^n \le A^M \qquad (103)$$

$$S_1 \ge P^m \qquad (104)$$

$$S_1 \le P^M \qquad (105)$$

$$\frac{1}{e_1 e_2} \cdot \sum_{j=1}^{J} h_j \cdot y_{ij} + (w-m_i) \cdot D_i^o$$

$$= W_i^o \qquad \text{for all } i \tag{106}$$

$$\frac{1}{e_1 e_2} \cdot \sum_{j=1}^{J} h_j \cdot y_{ij} + (k_i-m_i) \cdot D_i^o$$

$$= \frac{1}{e_1} \cdot S_0 \qquad \text{for all } i \tag{107}$$

$$k_i - m_i = \psi(w-m_i) \qquad \text{for all } i \tag{108}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} (h_j \cdot y_{ij}) \cdot \bar{d}_i = U \tag{109}$$

$$\frac{1}{e_2} \cdot y_{ij} \le S_2 \qquad \text{for all } i,j \tag{110}$$

$$e_2 \cdot S_2 = p \tag{111}$$

$$\lambda \cdot y_{ij} \le E_{ij} - E^m \text{ for all } i,j \tag{112}$$

$$x^n \ge 0 \qquad \text{for all } n. \tag{113}$$

The objective function contains the terms representing the expected fuel cost savings, windmill and land costs, storage system costs and transmission facility cost, all in annual equivalent costs or savings. No assumptions are made in the formulation regarding the characteristics of the cost functions f, ϕ_i's and g.

All the constraints in the above formulation were discussed in detail during the model development. Therefore, only brief descriptions of the constraints are provided below:

Constraint	Description
100	Mean of the daily output from the windmills
101	Standard deviation of the daily output from the windmills
102	Total capacity of the windpower system (which is equal to the storage input capacity)
103	Land availability
104	Minimum capacity of the windpower system
105	Maximum capacity of the windpower system
106	Relationship between the daily input to the storage and the output from the storage
107	Relationship between the storage capacity and the daily output from the storage
108	Functional relationship between the storage capacity and the daily output used to specify the probability of shortage
109	Annual total electricity transmitted to the network
110	Storage output capacity
111	Transmission capacity
112	Maximum windpower usable in fuel cost saving in the given time period

In Case B, the objective is to find a minimum cost design for the windpower systems that will deliver specified levels of power. Minor modifications of the model developed for Case A are required to handle Case B. The changes required are:

1. eliminating the terms in the objective function that give the annual fuel cost savings and dropping Constraint 112, and
2. adding whatever constraints are necessary in specifying the levels of power to be transmitted to the network.

The power levels can be specified in many ways. For example, the user may simply want to set a lower bound on U, the total amount of electricity delivered by the windpower system in a year; or he may want to set limits on some or all of the y_{ij}'s, i.e., set limits on the Kw power generated by the system in season i, period j. In either case, only trivial changes are necessary to use the model for Case B.

Characteristics of the Model and Solution Techniques

The problem formulated contains I(J+4) + N + 5 variables and I(2J+5) + 6 constraints (not counting the non-negativity constraints). The objective function is <u>separable</u> since it is composed of single variable terms or functions. We can treat $w - m_i$ and $k_i - m_i$ in Constraints 106-108 as single variables by substituting them with new variables. The feasible solution space is <u>non-convex</u> because of the non-linear equality constraints 106 and 108. This non-convexity of the feasible region resulting from the non-linear equality constraints makes it difficult to solve for a globally optimal solution. Some of the computationally efficient non-linear programming algorithms that can be used in solving the problem are the penalty function method (or the Sequential Unconstrained Minimization Technique) discussed in Zangwill (38) and the generalized Benders partitioning procedure (11). However, an in-depth search for the most computationally efficient algorithm for the problem is outside the scope of this research. Instead, the model will be tested using separable programming after separating the product terms in Constraints 106 and 107 by logarithmic transformation. To do this, the following substitutions are made:

$$\frac{1}{e_1 e_2} \cdot \sum_{j=1}^{J} h_j \cdot y_{ij} + T_i = W_i^o \tag{114}$$

$$\frac{1}{e_1 e_2} \cdot \sum_{j=1}^{J} h_j \cdot y_{ij} + Z_i = \frac{1}{e_1} \cdot S_0 \tag{115}$$

$$\log T_i - \log (w-m_i) - \log D_i^o = 0 \tag{116}$$

$$\log Z_i - \log (k_i-m_i) - \log D_i^o = 0 \tag{117}$$

With the above constraints replacing Constraints 106 and 107 in the original formulation, the problem can now be solved as a separable programming problem. However, due to the nonlinearity of the equality constraints 116 and 117, a globally optimal solution cannot be guaranteed using this technique. (See Chapter III for a further discussion on separable programming.)

Data Generation

The general discussion given in Chapter III for the no storage model also applies to the model developed in this chapter. However, the generation of the data matrix is more complex in this model because of the linear approximations of the non-linear functions in the constraints. A computer program has been developed to generate the problem matrix which can be solved using the IBM 360 mathematical programming system (17) (see Appendix G for the documentation of the program). Computational experience with this programmed model will be presented in Chapter VI.

Consideration of the Condition of Significant Serial Correlation of Wind Speeds

The preceding part of this chapter showed (a) how a very low serial correlation of wind speeds led to normally distributed daily output for every windmill regardless of its particular design characteristics and (b) how a storage model discussed in Chapter IV could be incorporated into the wind-power systems model in Chapter III for the case of this very low serial correlation. This section deals with the wind condition in which the velocities in successive time increments indicate a significant degree of correlation.

Some elaboration is necessary at this point concerning the methods of representing the persistence of wind speeds. For the purpose of this research, the effects of seasonal and diurnal variations of wind speeds must be distinguished from their persistence. The seasonal effect can be separated out by simply measuring the correlation with each season. The difficulty lies in distinguishing the "true" persistence from the effect of diurnal variation. Two methods are commonly used in representing the correlation in any time series: transition matrices and auto-correlation coefficients for various time lags. Of the two methods of representing the serial correlation, auto-correlation as a function of time appears to be easier to work with in windpower analysis. Whichever method is used, one must consider an inherent assumption made in expressing the correlation of a time series. The assumption is that the time series is stationary, i.e., its underlying probability distribution is the same for all time increments. For the wind characteristics that show low diurnal variation, one may directly use the measured data in constructing the velocity transition matrices or computing the auto-correlation coefficients. However, if the wind speeds show a significant diurnal variation, the data need to be treated in someway so that the resulting time series may become stationary. Two conceivable ways of doing this are to (a) take the differences between the actual "hourly" measurements and their expected values and compute the correlation in the differences and (b) express the measurements on a relative scale by dividing the actual "hourly" wind speeds by their expected values and compute the correlation with the normalized data.

The remainder of this section discusses how the previous model can be modified for the case of low but significant serial correlation and what analysis could be done for the case where the correlation is high between the hours, but very low between the days. If the daily averages are correlated, the storage models in Chapter IV do not apply. Since little work has been done in storage systems analysis with correlated inputs, the last case would probably require a simulation approach. This case will not be addressed in this research.

Modification of the Model for Low Serial Correlation

The following presents a method of modifying the model using auto-correlation coefficients and the Central Limit Theorem for finitely dependent variables in a stationary time series.

Given a time series $\{X_i, i = 1, \ldots, n\}$, the auto-correlation coefficients r_t's for time lags $t = 0,1, \ldots, L-1$ are obtained by first calculating the auto-covariances v_t's of the series and then dividing v_t's by v_0. The formula for calculating v_t's is given by

$$v_t = \frac{\sum_{i=1}^{n-t} X_i \cdot X_{i+t}}{n - t} - \left(\frac{\sum_{i=1}^{n} X_i}{n}\right)^2 \quad \text{for } t = 0,1, \ldots, L-1. \tag{118}$$

The auto-correlation coefficients r_t's are given by

$$r_t = \frac{v_t}{v_0} \text{ for } t = 0,1, \ldots, L-1. \tag{119}$$

If wind data show insignificant diurnal variation and the computed auto-correlation approaches zero within several hours, the daily output from a wind power system will be approximately normal. In this case, the model developed in Chapter V may be used for system optimization. This extended application of the model is based on the Central Limit Theorem for finitely dependent variables in a stationary stochastic process. Anderson (2) states the theorem as:

Let $y_1, y_2, \ldots$ be a stationary stochastic process such that for every integer n and integers $t_1, \ldots, t_m$ $(0 < t_1 < \ldots < t_n)$ $y_{t_1}, \ldots, y_{t_n}$ is distributed independently of $y_1, \ldots, y_{t_1-m-1}$ and $y_{t_n+m+1}, \ldots$. If $E_{y_t} = 0$ and $E_{y_t^2} < \infty$, then $\sum_{t=1}^{T} y_t/\sqrt{T}$ has a limiting normal distribution with mean 0 and variance

$$E_{y_1^2} + 2E_{y_1y_2} + \ldots + 2E_{y_1y_{m+1}}. \tag{120}$$

Using the notations X_i's and v_t's, the theorem may be restated as: if the time series $\{X_i\text{'s}\}$ is stationary and v_t approaches zero for t much shorter than some time length N, then $\sum_{i=1}^{N} X_i$ is approximately normally distributed with

$$\text{mean} = N \cdot \overline{X} \quad \text{and} \tag{121}$$

$$\text{variance} = N \cdot (v_0 + 2 \sum_{t=1}^{t_0} v_t), \tag{122}$$

where $\overline{X}$ = expected value of X_i, and

t_0 = shortest time lag beyond which r_t approaches zero.

In terms of v_0 and r_t's as defined in Equation 119,

$$\text{variance} = N \cdot (1 + 2 \sum_{t=1}^{t_0} r_t) \cdot v_0. \tag{123}$$

In applying the theorem to the case of low serial correlation and insignificant diurnal variation of wind speeds, only minimal changes are necessary in the original model formulation. The changes are:

1. Subscript $\bar{j}$ used in Equations 73-78 is dropped since in the absence of diurnal variation of wind speeds, only one wind distribution is necessary for each season.
2. The equation used to compute the standard deviation of daily output from one unit of design n (Equation 74) is changed to

$$D_i^n = [h\cdot(1+2 \sum_{t=1}^{t_0} r_{t_i})\cdot V(q_i^n)]^{1/2}, \tag{124}$$

where h = number of time increments in a day,

$V(q_i^n)$ = variance of power output (Kw) from one unit of design n in season i,

r_{t_i} = auto-correlation between the power output separated by t time increments in season i, and

t_0 = shortest time lag beyond which r_t becomes zero.

The difficulty in using Equation 124 is that the auto-correlations of power output r_{t_i}'s really depend on the power response characteristics of the windmill. Therefore, to obtain the exact values of r_{t_i}'s, the wind speeds in successive time increments must be transformed into the expected power output and a separate set of r_{t_i}'s must be calculated for each windmill design. One way of avoiding this is to approximate r_{t_i}'s with the auto-correlation coefficients of the wind speeds. The accuracy of the approximation would vary with the cut-in and flat-rate speeds of windmills. The r_{t_i}'s may also be approximated by the auto-correlation coefficients of expected power output based on some "average" cut-in and flat-rate speeds (e.g., 10 mph cut-in and 20 mph flat-rate speed).

The preceding discussion on modifying the model applies to the case of an insignificant diurnal variation and a low serial correlation of wind speeds. When the wind speeds show a significant diurnal variation in average value, but <u>not</u> in variance, we can compute the auto-covariances v_t's of time series $\{x_i, i = 1, ..., n\}$, where x_i is the difference between the observed and the expected velocities in time i. If the result shows a low correlation, then we can proceed to calculate the variance of the daily output given in Equation 124. However, the expected value of the daily output should be computed using Equation 75. If both average wind speed and variance change with the hour of day, the model would not be applicable. This case may be treated in the same way as the condition of high serial correlation discussed next.

An Approach to Modeling for High Serial Correlation

If the wind data indicate a high serial correlation, the total daily output from the windmills may no longer be normally distributed. Because the model in this chapter is based on a normal daily output, other methods would have to be used to solve the problem. One method might be fixing the cut-in and flat-rate speed of windmills and optimizing over the diameter and the number of windmills, storage capacity, and the electricity transmitted each day. Also, this method can only be applied to the case where the daily average wind speeds are not correlated in any given season because the storage models in Chapter IV require independent, random input. The general steps involved in this method are as follows:

1. Fix the cut-in speed, flat-rate speed and efficiency of the windmills, and using the actual hourly observations of wind speeds, generate the distribution of the expected daily output per unit-rotor area for each season.
2. Compute the shortage probabilities based on the per-unit-area daily output distributions in much the same way as the shortage probabilities for a normal distribution were computed, and estimate the functional relationship ψ for each season (see Figure 17).
3. Construct a model similar to the model developed in this chapter.

Notice in Steps 1 and 2 that the daily output distributions are on a basis of per unit-rotor area. This should not cause any particular problem in modeling because of the direct, linear relationship between the system daily output and the per-unit-area daily output.

The method discussed above requires an extensive data analysis and tedious calculations of shortage probabilities. Thus, the optimization of windpower systems for the condition of high correlation of wind speeds appears to be very difficult to obtain. Appendix D presents an analysis of a set of Oklahoma wind data, carried out in line with the discussions given in this section.

CHAPTER VI

COMPUTATIONAL RESULTS

This chapter presents the results of solving test problems using the computer programs developed from the economic models in Chapters III and V. The purpose of presenting the computational results is to demonstrate the use of the programmed models in studying the economics of large-scale windpower systems--it is not to judge or predict the economics of wind-generated electricity at the present time in any particular location. The documentations of the programs, used as matrix generators in setting up the separable programming problems with the input data, are provided in Appendices F and G, along with example input data and the output matrix structures.

The Test Data

Appendix E gives a set of base case data required for the computations. The data were compiled solely for testing the models, and do not necessarily represent any existing or planned windpower systems. Computational results were obtained by solving a series of test problems created by changing part of the base case data. Figure 18 shows the plots of

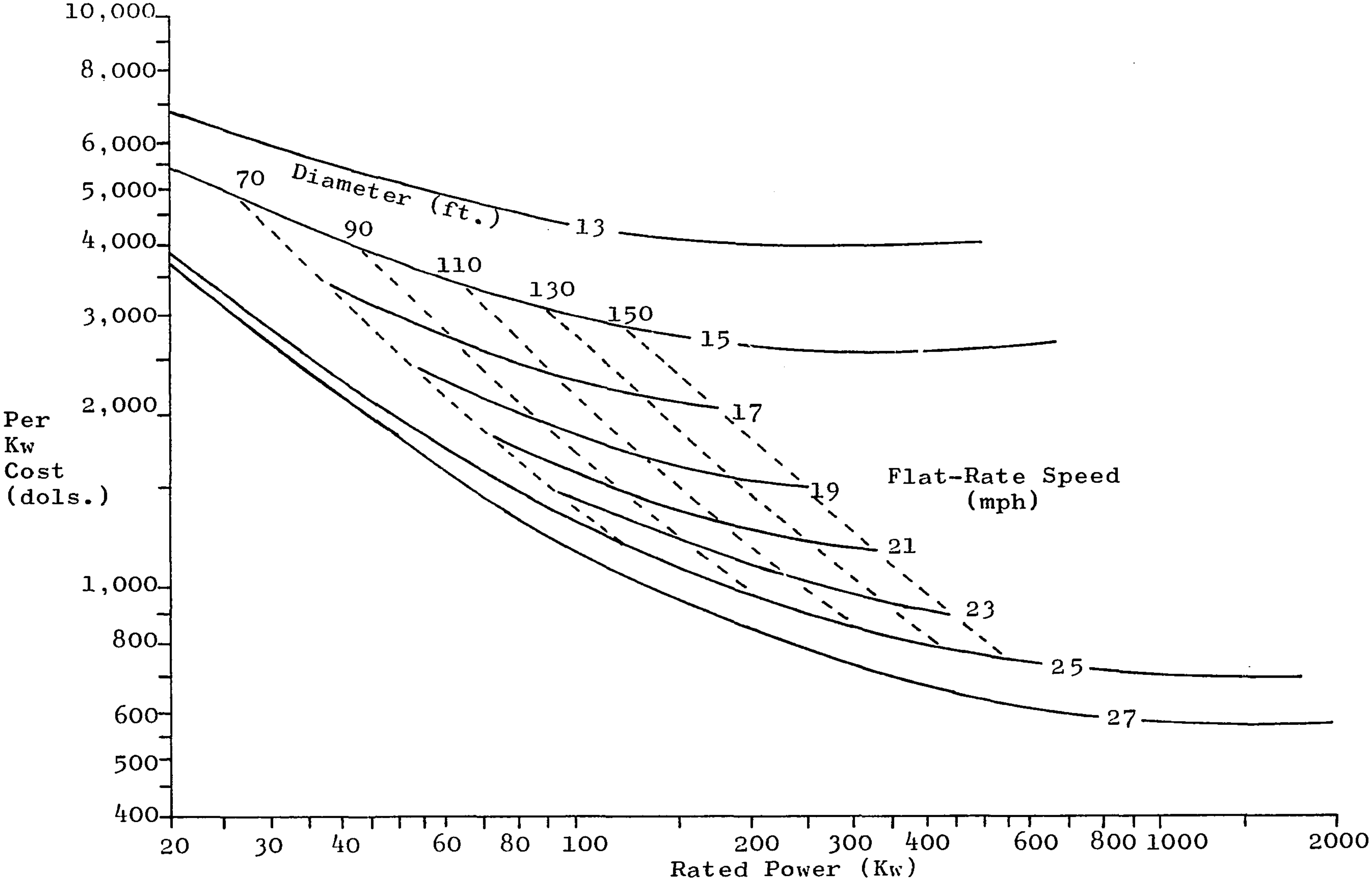

Figure 18. Windturbine installation cost as a function of rated power.

per Kw wind turbine installation cost for the base case. The cost data are the results of interpolating the data produced in a current windpower systems research project at Kaman Aerospace Corporation (Meier (21)). In the figure, the cost is represented as a function of rated power which is in turn a function of rotor diameter and flat-rate speed. The land cost is excluded from the installation cost shown in the figure. Figure 19 shows the total plant fuel cost function for the base case. The marginal fuel cost corresponding to the total fuel cost increases from four mills per Kwh at 1,000 Mw load to twelve mills per Kwh at 3,500 Mw capacity load.[1] The maximum average demand in the test data is 2,840 Mw that occurs in the afternoon of the summer season. The marginal fuel cost at this level of load is approximately ten mills per Kwh. A complete list of the test data including the demand, wind speed distribution and storage costs is provided in Appendix E. The summaries of the test problems and their solutions are tabulated in Tables 1 and 2 for the model without storage and in Tables 3 and 4 for the model with storage.

Windpower System with No Storage

Column 1 of Table 1 numbers the test problems for the no storage model starting with 1 for the base case. Columns 2

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[1] Assuming a 38 percent conversion efficiency, a fuel cost of 12 mills per Kwh corresponds to a fuel input cost of 1.34 dollars per million Btu's.

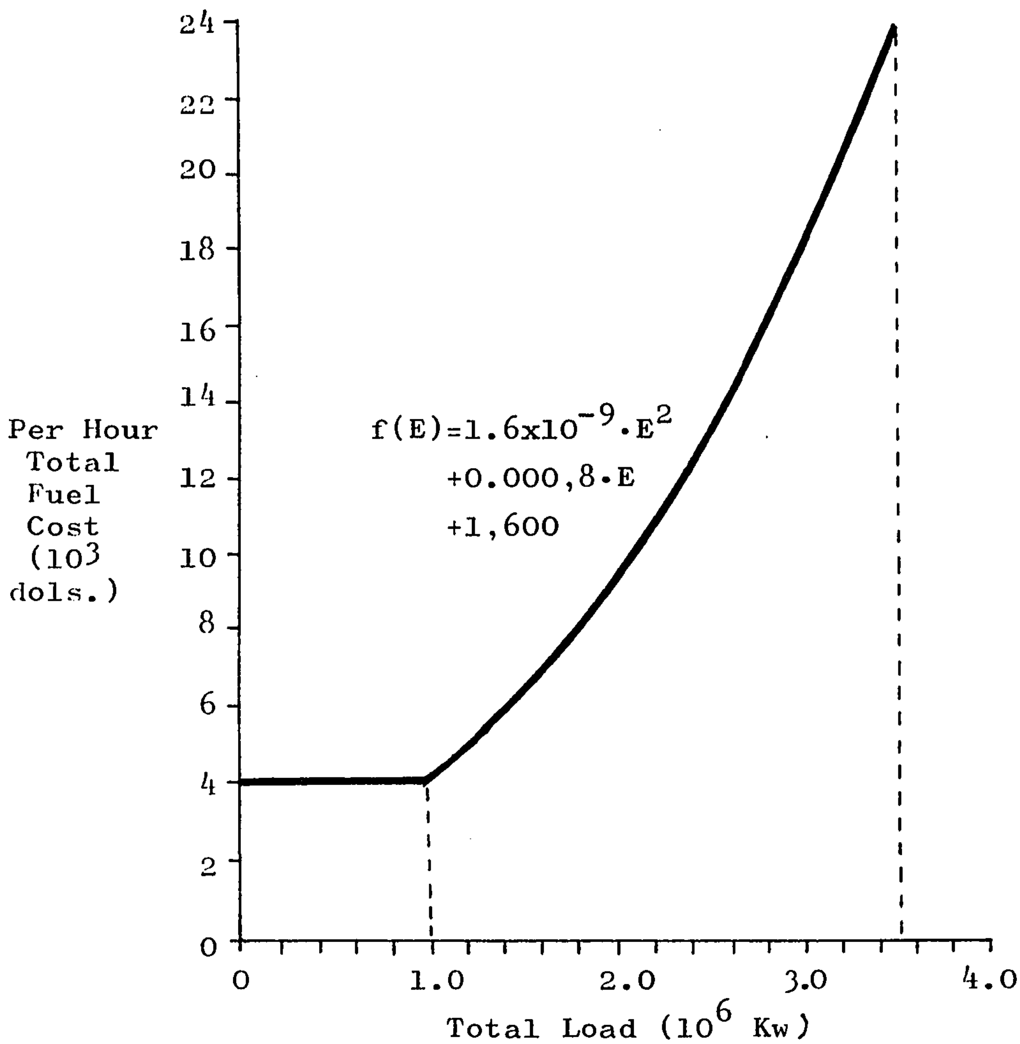

Figure 19. Total plant fuel cost as a function of the total load.

TABLE 1

TEST PROBLEMS FOR THE MODEL WITHOUT STORAGE

1 Problem Number	2 Fuel Cost Multiplier	3 Mean Vel. at 146 ft. (mph)	4 Minimum System Capacity (Mw)	5 Tower Height (ft)	6 Cut-in Speed (mph)	7 Windmill Per Kw Install. Cost
1	1	13.0	0	D	7	$f(P_r \| V_r)$
2 to 5	3,5,7,9					
6	5		100			
7	5	16.3	100			
8	5	19.5	100			
9	5	9.8	100			
10	5		100		$0.5V_r$	
11	5	16.3				$0.5f(500 \| V_r)$
12	5	16.3				$0.5f(P_r \| V_r)$
13	5	16.3		100		$0.5f(P_r \| V_r)$
14	5	16.3		100		$0.5f(500 \| V_r)$

TABLE 2

SOLUTIONS OF THE TEST PROBLEMS FOR THE MODEL WITHOUT STORAGE

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Problem Number	Annual System Cost (MM dols.)	Annual Fuel Cost Savings (MM dols.)	Annual Net Savings (MM dols.)	Windpower System Design				Annual Per Unit Windmill Cost (M dols.)	Annual Transmission Cost (M dols.)	Average Load Factor (Percent)			
				Dia. (ft.)	Flat-Rate-Speed (mph)	Rated Power (Kw)	Number Installed			Sum	Fall	Win	Spr
1	0	0	0	–	--	–	–	–	0	--	--	--	--
2 to 5	0	0	0	–	--	–	–	–	0	--	--	--	--
6	11.32	5.80	-5.52	150	25	563	178	63.5	12.1	18	25	28	31
7	11.32	9.21	-2.11	150	25	563	178	63.5	12.1	32	39	42	48
8	42.78	44.41	1.63	150	25	563	673	63.5	45.8	46	44	53	60
9	11.32	2.70	-8.62	150	25	563	178	63.5	12.1	8	12	13	15
10	11.32	5.36	-5.96	150	25	563	178	63.5	12.1	15	23	26	30
11	33.86	50.10	16.24	150	23	438	1,142	29.6	60.5	37	38	47	51
12	28.69	44.91	16.22	150	25	563	889	32.2	60.5	32	35	43	48
13	28.69	36.42	7.73	150	25	563	889	32.2	60.5	24	29	35	40
14	33.99	42.52	8.53	90	23	158	3,171	10.7	60.5	29	33	40	46

Average solution time on IBM 370 computer: 3.5 seconds.

to 7 list the differences between the base case problem and the other problems. The blanks in each column indicate no changes from the base case. The fuel costs changes from the base case are made by multiplying the base case fuel cost function by some factor, which is specified in Column 2. Column 3 shows the mean of the wind speed distribution used in each problem. The wind distributions for the problems have the same coefficient of variation (i.e., the ratio of the standard deviation to the mean). Different wind distributions are obtained by multiplying a 2 mph increment of the base case distribution by 0.75, 1.25 and 1.5 to get 9.8, 16.3 and 19.5 mph average velocities. Column 4 specifies the minimum required capacity of windpower system. The maximum capacity is set equal to 500 Mw in all problems. The tower height of wind turbines is shown in Column 5, and is set equal to the rotor diameter in Problems 1 to 12 and 100 feet in Problems 13 and 14. The cut-in speed shown in Column 6 is fixed to 7 mph for all windmills in all problems except in Problem 10 in which it is set equal to one-half of the flat-rate speed. The per Kw capacity installation cost of windmills is shown in Column 7. The notation $f(P_r|V_r)$ is used to represent that the per Kw installation cost is a function of rated power for given flat-rate speed (see Figure 18). In Problems 11 and 14, the per Kw cost depends only on the flat-rate speed, and is set equal to one-half of the base case cost of a 500 Kw unit. In Problems 12 and

13, the installation cost is simply reduced from the base case cost by half.

Table 2 gives the solutions of the problems in Table 1. Column 2 shows the annual equivalent cost of the windpower system described in Columns 5 to 8. Column 3 gives the expected annual fuel cost savings. Column 4 shows the net savings or losses, which is the difference between Column 2 and Column 3. The annual per unit windmill cost in Column 9 consists of the annual equivalent installation cost (25 year economic life, 10 percent interest rate), the annual equivalent land cost (10 percent of the purchase cost), and the annual O&M cost (4 percent of the installation cost). The average load factors in Columns 11 to 14 represent the ratio of the average power transmitted to the network in each season to the capacity of the windpower system. According to the model formulation in Chapter III, the average power transmitted may be less than the average power generated. This case arises when the expected windpower generated in some period is greater than the amount that the power system can use for fuel cost savings. Although not shown in Table 2, in Problems 6, 11 and 12, excess power is generated in the first period of Seasons 2 and 4.

The solutions to Problems 1 to 5 indicate that it is not economical to build a windpower system under the conditions stated in the problem.

In Problem 6, the minimum capacity is set to 100 Mw in addition to increasing the fuel costs by a factor of four. Under this condition, the annual losses would be 5.52 million dollars with the installation of 178 units of windmills with the largest diameter (150 ft.) and the highest flat-rate speed (25 mph) considered in the problem. From Figure 18 the per Kw installation cost of these windmills is approximately 740 dollars. Including the land and O&M costs, each unit will cost 63,500 dollars on a yearly basis. The average load factor would vary from 18 percent in the summer to 31 percent in the spring. The annual transmission cost is almost negligible as compared to the annual total system costs.

In Problems 7, 8 and 9, the average wind speed is varied to see how the wind velocity distribution affects the solution. The losses in Problem 7 is shown to be less than the losses in Problem 6. With the high average wind speed (19.5 mph) in Problem 8, a 379 Mw windpower system would yield 1.63 million dollar net savings, meaning that the return on the capital investment would be greater than 10 percent. With a one-third decrease in average wind speed in Problem 9, the losses would be 56 percent greater than the losses indicated in Problem 6. Problem 10 is different from Problem 6 in the cut-in speeds of the windmills. From Column 4, it is seen that the changes in cut-in speed from 7 mph to one-half of the flat-rate speed do not have a significant effect on the solution.

In Problem 11, the per Kw installation cost of windmills is made to depend only on the flat-rate speed. For a given V_r, the per Kw cost is set equal to half the cost at 500 Kw capacity shown in Figure 18. In addition, the fuel cost is increased four times and 16.3 mph average wind speed is used in the problem. In this case, the optimal capacity of the windpower system is 500 Mw, the maximum allowed for the system. The optimal design of windmills in this problem is shown to be 150 ft. diameter and 23 mph flat-rate speed. Problem 12 is different from Problem 11 in that the per Kw installation cost varies with the capacity as in the base case. The optimal solution indicates the installation of a 500 Mw capacity system. Problem 13 is the same as Problem 12 except that the tower height is fixed to 100 feet. The resulting solution gives a smaller value of net fuel cost savings. In Problem 14, which is the same as Problem 11 except in the way the tower height is set, the optimal solution indicates the installation of 500 Mw system consisting of 90 ft. diameter, 23 mph flat-rate speed units.

In the solutions giving the design of 150 ft. diameter and 25 mph flat-rate speed windmills, the average load factors are shown to increase with the average wind speed. In the case of the same rotor diameters, but different flat-rate speeds as in Problems 11 and 12, the load factors are shown to decrease with increasing flat-rate speed. However, the latter case may not be generalized because the effect will depend on the wind and the demand data used.

The average computer time required to solve the problems was approximately 3.5 CPU seconds on the IBM 370/158J at the University of Oklahoma. The number of Simplex pivots taken before the optimal solution was found ranged from 68 for Problem 11 to 77 for Problem 14 with an average of 71 pivots.

Although the data used in the computations are realistic to a certain degree, the results shown should not be considered as indicative of the economics of windpower systems at the present time in any specific site. The problems were solved merely for the purpose of demonstrating the use of the programmed model.

The approach for modeling the system is general enough for the user to make many desired modifications on the model without much effort. The changes might include adding further constraints to the model, using a different method of determining the value of windpower, and incorporating quantity discount factors for the windturbines. In most instances, only minor changes in the input data will be required. The model is relatively inexpensive to run, and therefore sensitivity analysis on important parameters can be performed.

Windpower System with Storage

Table 3 lists ten Case A and five Case B test problems for the model with storage. The base problem

TABLE 3

TEST PROBLEMS FOR THE MODEL WITH STORAGE

1 Problem Number	2 Case	3 Fuel Cost Multiplier	4 Windmill Install. Cost Multiplier	5 Mean Vel. at 146 ft. (mph)	6 Shortage Prob. (Percent)	7 Minimum Elect. Trans. Per Year[1] (MM Kwh)	8 Scale Factor for D_i^o
1	A	1	1	13.0	10	0	10
2,3	A	3,5					
4	A	5				394.2	10^4
5	A	5			15	372.3	10^4
6	A	5			20	350.4	10^4
7	A	5	1/2	16.3		394.2	10^4
8	A	5	1/2	19.5		394.2	10^4
9	A	3	1/3	19.5		394.2	10^4
10	A	3	1/3	19.5	15	372.3	10^4
11	B	-	1	13.0	5	416.1	10^4
12	B	-		16.3			
13	B	-	1/2	19.5			
14	B	-	1/2	16.3			
15	B	-				--	

[1] In Problem 15, a lower limit of 100 Mw is set on the average power transmitted during the third and fourth periods of the summer season.

for Case A is numbered 1 and that for Case B is numbered 11. Column 3 gives the fuel cost multiplier used in each problem. This column does not apply to the Case B problems which are concerned with finding a minimum cost design. Column 4 gives the factors by which the windmill installation cost is multiplied in the problems. The blanks in this and the following columns are used to mean that the data has the same value as the corresponding base problem. Column 5 lists the changes in average velocity from the base problem. Column 6 indicates the probability of shortage which is specified. For Case A, the shortage probability was varied between 0.1 to 0.2, while in Case B, it was set equal to 0.05. Column 7 specifies the minimum amount of wind-generated energy which must be supplied to the network in a year. The values in the column correspond to an average of 50 Mw power transmitted to the network in the days when no shortage occurs. However, Column 7 does not apply to Problem 15. In that problem, the objective is to find a minimum cost design that will transmit at least 100 Mw of power during the summer peak periods (with a probability of 0.95).

Column 8 gives the factors used to scale down the standard deviation of daily output from windpower system (i.e., D_i^o). The scaling is necessary to avoid potentially large errors resulting from making a linear approximation of logarithmic functions. The logarithmic functions are used to separate the variable products involving D_i^o in the model formulation (see Equations 101, 106 and 107). Based on the computational experience, for a good approximation the scaling factor for D_i^o should be such that the reduced value of D_i^o lies between 1 to 100 Kwh. The programmed model in Appendix G internally uses another factor to scale up $w-m_i$ and k_i-m_i. The method used to make a linear approximation of logarithmic functions, along with the uses of scaling factors, is discussed more fully in Appendix G.

Table 4 presents the solutions of the problems. The solutions listed may not be globally optimal due to the non-convexity of feasible region. However, attempts were made to obtain better solutions for some of the problems by initially setting the values of the approximating variables at their upper bounds in the first run and then at their lower bounds in the next, but no improvements on the solutions were obtained. This raises the probability that the solutions obtained were actually the global optimums.

Column 2 of Table 4 gives the annual equivalent cost of the "optimal" windpower system, which is described in Columns 7 to 15. This annual cost of the system consists of the annual equivalent windmill cost, storage cost and transmission cost. Columns 3 and 4 give the annual expected fuel cost savings and the net savings (or loss) including the windpower system cost. These two columns do not apply to Case B. Column 6 gives the Kwh cost and is obtained by dividing Column 2 by Column 5. As in Table 2, the annual

TABLE 4

SOLUTIONS OF THE TEST PROBLEMS FOR THE MODEL WITH STORAGE

1	2	3	4	5	6	7	8	9	10	11
						Windpower System Design				
Problem Number	Annual System Cost (MM dols.)	Annual Fuel Cost Savings (MM dols.)	Annual Net Savings (MM dols.)	Expected Annual Elec. Transmitted (MM Kwh)	Per Kwh Generation Cost (Cents)	Dia. (ft.)	Flat-Rate Speed (mph)	Rated Power (Kw)	Number Installed	Annual Per Unit Windmill Cost (M dols.)
1	0	0	0	0	-	-	-	-	-	-
2,3	0	0	0	0	-	-	-	-	-	-
4	44.19	11.01	-33.18	394.2	11.2	150	23	438	657	60.0
5	45.86	10.53	-35.33	372.3	12.3	150 150	21 23	333 438	639 72	58.2 60.0
6	44.28	9.89	-34.39	350.4	12.6	150	23	438	659	60.0
7	16.01	11.14	-4.87	394.2	4.06	150	25	563	371	32.2
8	12.42	11.21	-1.21	394.2	3.15	150	25	563	275	32.2
9	9.46	6.65	-2.81	394.2	2.40	150	25	563	275	21.8
10	9.40	6.26	-3.14	372.3	2.52	150	25	563	277	21.8
11	41.29	-	-	416.1	9.92	150	23	438	608	60.0
12	26.52	-	-	416.1	6.37	150	25	563	354	63.5
13	12.19	-	-	416.1	2.93	150	25	563	273	32.2
14	15.44	-	-	416.1	3.71	150	25	563	356	32.2
15	54.40	-	-	539.3	10.09	150	21	333	868	58.2

TABLE 4--Continued.

1	12	13	14	15	16	17	18	19	20	21
	Storage System Design			Annual Storage and Trans. Costs (MM dols.)	Mean of Daily Output from Windpower System (Mwh)		St. Dev. of Daily Output from Windpower System (Mwh)		Daily Output from Storage (Mwh)	
Problem Number	Storage Capacity (Mwh)	Input Conversion Capacity (Mw)	Output Conversion Capacity (Mw)		Summer	Spring	Summer	Spring	Summer	Spring
1	-	-	-	-	-	-	-	-	-	-
2,3	-	-	-	-	-	-	-	-	-	-
4	2,156	288	94	4.77	1,514	2,571	314	456	950	1,360
5	1,953	245	95	4.35	1,574	2,521	300	405	988	1,365
6	1,832	289	96	4.74	1,519	2,579	315	457	951	1,381
7	2,112	209	95	4.06	1,581	2,419	297	377	993	1,369
8	2,154	155	95	3.56	1,693	2,236	250	273	1,067	1,362
9	2,154	155	84	3.46	1,693	2,236	250	273	1,067	1,362
10	2,003	156	84	3.36	1,700	2,245	251	274	1,072	1,359
11	3,216	266	76	4.81	1,402	2,381	291	422	879	1,460
12	3,232	199	75	4.04	1,505	2,302	283	359	945	1,436
13	2,519	153	72	3.40	1,676	2,214	248	270	1,051	1,382
14	2,869	200	75	3.98	1,514	2,315	284	360	948	1,434
15	4,183	289	125	3.88	1,911	3,039	361	481	1,200	1,870

Average solution time on IBM 370 computer: 25 seconds.

equivalent per unit cost of windmills in Column 11 includes land and O&M costs. The input conversion capacity in Column 13 is the same as the total capacity of windmills. In Columns 16 to 21, the values are given only for summer and spring because of space limitations. The values for fall and winter generally lie between the values for spring and summer.

Problem 1 is the base case problem. The solution indicates that it is best to not install a windpower system. In Problems 2 and 3, the fuel cost is multiplied by a factor of 3 and 5, respectively. The optimal solution again indicates that it is best not to install a windpower system under the conditions specified. In Problem 4, a lower limit is set to the wind-generated electricity transmitted to the network. In this case, the installation of a system consisting of 657 units of 150 ft. diameter windmill and a 2,156 Mwh storage would result in a 33 million dollar loss per year. The system will input an average of 50 Mw power to the transmission line 90 percent of time. The standard deviation of the daily output from this system would be approximately 21 percent of the average in summer and 18 percent in spring. Out of the average 1,514 Mwh daily output from windmills during the summer, 20 percent is lost during input conversion and the rest is stored. Each day of the summer, 1,188 Mwh equivalent energy is retrieved from the storage and is reconverted into electricity. The output conversion efficiency used is

80 percent, thus during the summer only 950 Mwh is fed into the transmission line per day. The daily gross output from storage during the summer (1,188 Mwh) is approximately 98 percent of the average net input (1,211 Mwh). In spring, the daily gross output from storage is approximately 83 percent of the average net input. In both seasons, the probability of shortage occurring is 10 percent.

In Problem 5, the probability of shortage is set equal to 15 percent. The solution indicates that the installation of a mixture of 150 ft. D, 21 mph V_r units and 150 ft. D, 23 mph V_r units and a storage system would cause a 35 million dollar annual loss in transmitting 50 Mw power with a 15 percent shortage probability. In Problem 6, the probability of shortage is increased to 20 percent. The result is a 34 million dollar annual loss. As expected, Column 12 shows the decrease in storage capacity requirement with the increase in the probability of shortage in Problems 4, 5 and 6. In comparing Problems 4 and 6 with respect to the daily output from storage, we see that the output is approximately the same for both problems. The difference is in the total wind-generated electricity used to save the fuel cost in each year. In formulating the model in Chapter V, it is assumed that if the quantity left in storage is less than the fixed daily output, the stored energy is not transmitted. Thus, the average 50 Mw power transmission is done only during the days when no shortage occurs.

In Problems 7 and 8, the average annual wind speed is increased to 16.3 mph and 19.5 mph, respectively. In addition, the windmill cost is reduced to one-half of the base cost. The decrease in losses shown in Column 4 is largely attributable to the reduced windmill installation cost. From Columns 10 and 13, the capacity of the system necessary to meet the minimum electricity transmitted under a 19.5 mph average wind speed (Problem 8) is approximately 74 percent of the capacity necessary under a 16.3 mph average wind speed (Problem 7).

In Problems 9 and 10, the windmill installation cost is reduced to one-third of the base cost and the fuel cost function is multiplied by a factor of 3. In addition, the annual average wind speed is adjusted to 19.5 mph. The losses resulting from setting minimum values for the electricity transmitted to the network are shown as 2.81 and 3.14 million dollars for a 10 and a 15 percent shortage probability, respectively.

Although not shown in Table 4, the daily total output from storage in each season is optimally divided by the model into four six-hour periods. In the presence of an output conversion facility of which the installation cost depends on its capacity, the division is determined by the capacity of the output conversion facility as well as by the per Kwh fuel cost that varies with the time of the day. For example, in Problem 4 the daily output from storage after the output conversion is 950 Mwh in summer and 1,360 Mwh in spring. The 950 Mwh total output is divided into four periods such that the actual power transmitted is 0 Mw in Period 1, 7.0 Mw in Period 2, 75.6 Mw in Periods 3 and 4. In spring, the power transmitted is 0 Mw in Period 1 and 75.6 Mw in Periods 2, 3 and 4. The 75.6 Mw power corresponds to 80 percent of the output conversion capacity shown in Column 14.

Problems 11 to 15 are Case B problems in which minimum cost designs are sought. The 5 percent probability of shortage in Column 6 of Table 3 is assumed to be low enough to consider the windpower system as an addition to the total capacity of the power system. In this case, the generation cost shown in Column 6 of Table 4 becomes an appropriate measure for determining the system economics.

The solution for Problem 11 indicates that the minimum cost design given in Columns 7 to 10 and 12 to 14 will supply 416.1 Mwh electricity per year at 9.92 cents per Kwh.

Problem 12 is the same as Problem 11 except that the average wind velocity is increased 25 percent from the base data. From Column 6, this higher wind velocity can reduce the per Kwh generation cost by 36 percent. An average 19.5 mph wind coupled with a 50 percent decrease in windmill installation cost, as specified for Problem 13, indicates that windpower can be generated for less than three cents. In Problem 14, the average wind velocity is increased to

16.3 mph and the windmill installation cost is decreased by half from the base data. The 3.71 cents per Kwh generation cost of this problem together with the generation costs of Problems 11, 12 and 13 illustrates the sensitivity of the solution to the windmill installation cost and the average wind velocity.

In Problem 15, the power transmitted in Periods 3 and 4 during the summer season is specified instead of setting a lower limit on the total electricity transmitted per year as in the preceding problems. Except the specification of the summer loading, the problem is the same as Problem 11. The solution shows that to supply 100 Mw power to the network in the peak periods of summer with a reliability of 95 percent the system requires a 289 Mw total windmill capacity (Column 13). The minimum cost windmill design for this problem is 150 ft. in diameter and 21 mph in flat-rate speed. The relatively low flat-rate speed is apparently suitable for power generation in the summer when the average wind speed is low.

The average computer time required to solve the problems was approximately 25 CPU seconds. The number of Simplex pivots required to obtain an optimal solution varied from 208 for Problem 6 to 390 for Problem 2 with an average of 265 pivots.

As has been demonstrated, the model can be used to determine the optimal economic design of windpower systems with storage, which are either installed for fuel cost savings or for capacity expansion. More specifically, the model can be utilized in selecting the sites, finding an optimal system design for the sites, and determining the system economics by the expected "net" fuel cost saved or by comparing the expected per Kwh generation cost with that of other alternative power sources. One should recall that the model was developed for the case in which the wind conditions are such that the "hourly" average wind speeds are nearly random or show a low serial correlation. If wind speeds in successive time increments show a high correlation, the model in Chapter V cannot be used and therefore some other approach to modeling the windpower systems may be required.

CHAPTER VII

SUMMARY AND FURTHER RESEARCH

Summary

Two economic models of windpower systems, one for systems without storage and the other for systems with storage, are developed in this research. The models are designed for use in assessing the economics of large scale windpower systems that are operated in conjunction with existing power networks.

In the model without storage presented in Chapter III, all the power generated by windmills is transmitted to the network. The fluctuation of output power caused by the intermittence of wind is assumed to pose no severe problem to the whole power system. The wind-generated power delivered to the network is valued in terms of the fuel costs saved on the existing power plants. The optimization is carried out with respect to the design and the total capacity of windmills. The required input data include site wind conditions and per Kwh fuel cost that varies with demand. The model is formulated as a non-linear programming problem, and separable programming is used as the solution technique.

Before developing the model for systems with storage, Chapter IV presents two analytical storage models adapted from Moran's original model. In terms of wind energy storage the first model represents a system in which all the energy is first stored and a fixed amount of energy is then released in each interval of time. The second model represents a system in which only part of the energy is stored and the rest is output directly. In both systems, losses occur during input and output conversions. The purpose of this analysis is to determine the functional relationship between storage capacity, level of output, and the probability of shortage, so that the optimization model for windpower systems with storage can be developed.

Following the storage systems analysis, Chapter V incorporates the first storage model into the model developed in Chapter III for the site wind condition in which the daily output from the windpower system is normally distributed. This condition arises when the wind speeds in successive time increments indicate very low serial correlation. Two cases are considered in modeling the system: Case A where the objective is to maximize the difference between the fuel cost saved and the windpower system cost, and Case B where the objective is to find a minimum cost design in meeting specified levels of demand. The optimization model formulated is considerably more complex than the model in Chapter III due to the non-linear constraints associated with storage. As in the model without storage, separable programming is used as the solution technique. Chapter V also discusses how to deal with situations in which the wind speeds are significantly correlated.

Computational results are presented in Chapter VI to demonstrate the use of the models in studying the economics of large scale windpower systems. The test problems used in the computations include a base case problem for each of the two models and a range of other problems created by changing part of the base case data. The results show that the values of the solutions are most sensitive to the changes in the average wind speed and in the windturbine installation cost. Mainly due to the decrease in the per Kw installation cost of windturbines with the increase in the rated power, large windturbines having high flat-rate speeds are selected under the conditions stated in the problems. The results indicate that the programmed models based on separable programming can solve a variety of economic windpower system design problems and are computationally efficient.

Further Research

Further research can be done in the following areas:

1. Modeling of the systems with storage for the condition of highly correlated wind speeds.

Under this wind condition, the daily output from the windpower system may not be normally distributed. The major effort involved in modeling for this situation will probably be (a) deriving a power output distribution from the given wind speed distribution and the conversion characteristics of the windturbines and (b) finding the relationship among the shortage probability, the storage capacity and the output from storage.

2. Modeling of windpower systems in which only part of the power generated is stored.

A basic analytical model for this mode of storage operation is presented in Chapter IV. Therefore, for the case of normally distributed daily input to storage, the analytical storage model can be relatively easily developed, and then incorporated into the overall system model, following the approach taken in Chapter V.

3. Modeling of multi-location windpower systems.

Under this system operation, the "windfarms" are dispersed over a sufficiently broad region, thereby increasing the probability that the wind is blowing in one or more locations at any particular point in time. To analyze this system, one would first have to do a detailed study of wind data to determine the possible correlation between the windpower available at the dispersed locations. Then, based on the probability distribution for the total power output, one must decide what percentage of the expected total power output could be considered as capacity expansion, with the remainder valued by the fuel cost savings.

4. Incorporation of a variable leakage factor into the model for systems with storage.

This is an extension of the model developed in Chapter V to consider the energy lost during storage. To handle this leakage factor, an analytical storage model similar to those in Chapter IV would first have to be developed, and then incorporated into the general model in Chapter III.

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APPENDIX A

TABLE 5

EXPERIMENTAL WINDMILLS

Design	Builder, Year			
	Smith-Putnam Vermont, U.S.A. 1941	St. Albans Dorset, England 1953	Alps, FRG 1957	Gedser, Denmark 1957
Diameter (ft.)	175	79	112	79
Number of Blades	2	2	2	3
Blade Tip Speed (mph)	179	268	168	87
Rotor rpm	29	95	42	30
Rated Power (Kw)	1,250	100	100	200
Flat-Rate Speed (mph)	29	31	18	34
Efficiency (percent)[1]	44	22	63	35
Cut-in Speed (mph)	17	13	--	13
Tower Height (ft.)	117	98	72	79
Type of Generator	Synch.	Synch.	Synch.	Asynch.
Generator rpm	600	--	--	750
Step-Up Method	Gear Dr.	Pneumatic	Gear Dr.	Chain
Step-Up Ratio	21	--	--	25
Blade Pitch Regulation	Entire blade	Entire blade	Entire blade	Blade tip
Blade Pitch Servo	Hydraulic	Hydraulic	Hydraulic	Pneumatic
Rotor Position Relative to Tower	Lee	Lee	Lee	Wind
Rotor Direction Servo	Elec.	Elec.	Elec.	Elec.

Source: Södergård, B., "Analysis of the Possible Use of Windpower in Sweden. Part I--Wind-Power Resources, Theory of Wind-Power Machines," December 1973. The figures in the original table are given in MKS units.

[1]The efficiency of the Smith-Putnam turbine refers to the overall conversion efficiency, while the efficiencies of the other three windmills are most likely given in terms of the percent of the theoretical maximum (i.e., 60 percent).

APPENDIX B

A SIMULATION OF DAILY OUTPUT FROM VARIOUS WINDMILLS

The purpose of this simulation is to make inferences about the kind of correlation that the daily output from any two windmills in a windpower system would show. To accomplish this, four arbitrary probability distributions of power output are devised and the daily output from windmills is simulated based on these distributions. The hypothetical distributions are made very different from one another as shown below:

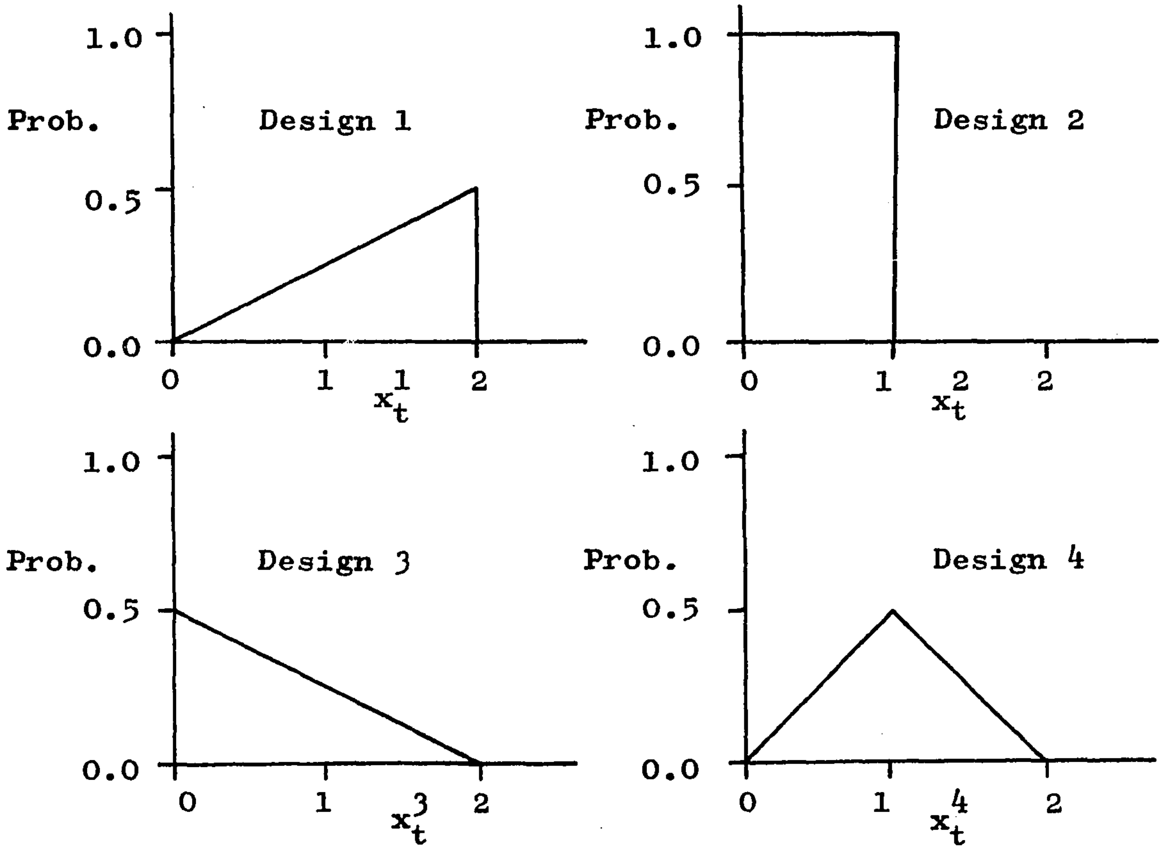

Figure 20. Hypothetical power output distributions.

In the above figure, x_t^i represents the power output from windmill i during the time increment t. For simplicity, let us assume that there are n time increments in a day and that the wind speed distributions are the same in all time increments. Let

$$X^i = \sum_{t=1}^{n} x_t^i \text{ for } i = 1,2,3,4$$

$$= \text{total daily output from windmill } i.$$

Then, we can generate the daily totals (X^i's) for many days using a random number generator and compute the correlation between X^i's. The procedure is: first, x_t^i's are determined from the probability distributions using n random numbers that apply to all i ; then, x_t^i's are summed over t for each i to get X^i for the day.

The result of a simulation for n = 10 and 500 replications is given in the correlation matrix below:

		Design 1	Design 2	Design 3	Design 4
Design	1	1.0	0.98	0.93	0.98
	2		1.0	0.98	0.99
	3			1.0	0.98
	4				1.0

The correlation matrices for n = 20 and n = 30 differed little from the above.

APPENDIX C

RELATIONSHIP BETWEEN THE SHORTAGE AND OVERFLOW PROBABILITIES UNDER SYMMETRICAL INPUT AND FIXED OUTPUT

Let X_t = input during the time t, following a symmetrical probability distribution over [0,M],

Y_t = output during the time t which is the same as the output in all other times,

Z_t = storage level at the end of time t, and

k = capacity of storage.

Then, in a steady state

$$Pr(\text{shortage} | Y_t = M - m) = Pr(\text{overflow} | Y_t = m),$$

where m is a value between 0 and M.

The proof is provided below:

$\{Z_t\}$ is a stochastic process that can be represented by

$$Z_{t+1} = \min\{k-m, \max[0, Z_t + X_t - Y_t]\}. \quad (1)$$

Let Z_t^1 = storage level corresponding to $Y_t = m$, and

Z_t^2 = storage level corresponding to $Y_t = M - m$.

$S_t = (k-m) - Z_t^2$

$T_t = M - X_t$.

Then,

$$Z_{t+1}^2 = (k-m) - S_{t+1}$$
$$= \min\{k-m, \max[0, (k-m) - S_t + (M-T_t) - (M-m)]\}. \quad (2)$$

Subtracting k − m from both sides of the equation,

$$\begin{aligned} -S_{t+1} &= \min\{0,\max[-(k-m),-S_t - T_t + m]\} \\ &= \min\{0,-\min[k-m,\ S_t + T_t - m]\} \\ &= -\max\{0,\min[k-m,\ S_t + T_t - m]\} \\ &= -\min\{k-m,\max[0,\ S_t + T_t - m]\}. \end{aligned} \qquad (3)$$

Rewriting the above,

$$S_{t+1} = \min\{k-m,\max[0,S_t + T_t - m]\}. \qquad (4)$$

From the definition of S_t,

$$\Pr(Z_t^2 \leq 0) = \Pr(S_t \geq k-m). \qquad (5)$$

But, we recognize, from Equation 4 and Equation 1 with $Y_t = m$, that $\{S_t\}$ and $\{Z_t^1\}$ are equivalent processes because the "input" T_t in Equation 4 has the same probability distribution as X_t in Equation 1. Therefore,

$$\Pr(S_t \geq k-m) = \Pr(Z_t^1 \geq k-m). \qquad (6)$$

From Equations 5 and 6,

$$\Pr(Z_t^2 \leq 0) = \Pr(Z_t^1 \geq k-m), \text{ or}$$

$$\Pr(\text{shortage} \mid Y_t = M-m) = \Pr(\text{overflow} \mid Y_t = m).$$

APPENDIX D

ANALYSIS OF A SET OF OKLAHOMA WIND DATA

The purpose of this analysis is to test the applicability of the model developed in Chapter V for systems having site wind characteristics similar to those in the data analyzed here. The data come from the one-year hourly observations made by the National Severe Storms Laboratory in Norman, Oklahoma, at a television tower outside Oklahoma City during the June 1966-May 1967 period. The data represent the five minute averages of the instantaneous wind velocities measured on the hour at the 146 ft. level of the tower. A small percentage of the possible 8,760 data points were not gathered due to instrument malfunctions, but were filled with smoothed values by the author. Figures 21-a through l show the monthly percentage frequency distributions of the measurements. The percentage frequency distributions are based on 4 mph velocity increments. For comparison, the monthly average wind speeds taken from an Oklahoma City census data[1] are given in the figures along with the monthly averages of the NSSL data.

Figures 22-a through d show the computed autocorrelation coefficients of wind velocities (V_t's), velocity cubes (V_t^3's), and a series (P_t's) defined by

$$P_t = \max\{0, \min(V_t - 10, 20)\}.$$

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[1]U.S. Weather Bureau, "Decennial Census of United States Climate--Summary of Hourly Observations, Oklahoma City, Oklahoma, 1951-1960." Washington, D.C.: Government Printing Office, 1963.

m_1 = mean of the hourly observations of the NSSL data

m_2 = mean of the hourly observations of the Census data

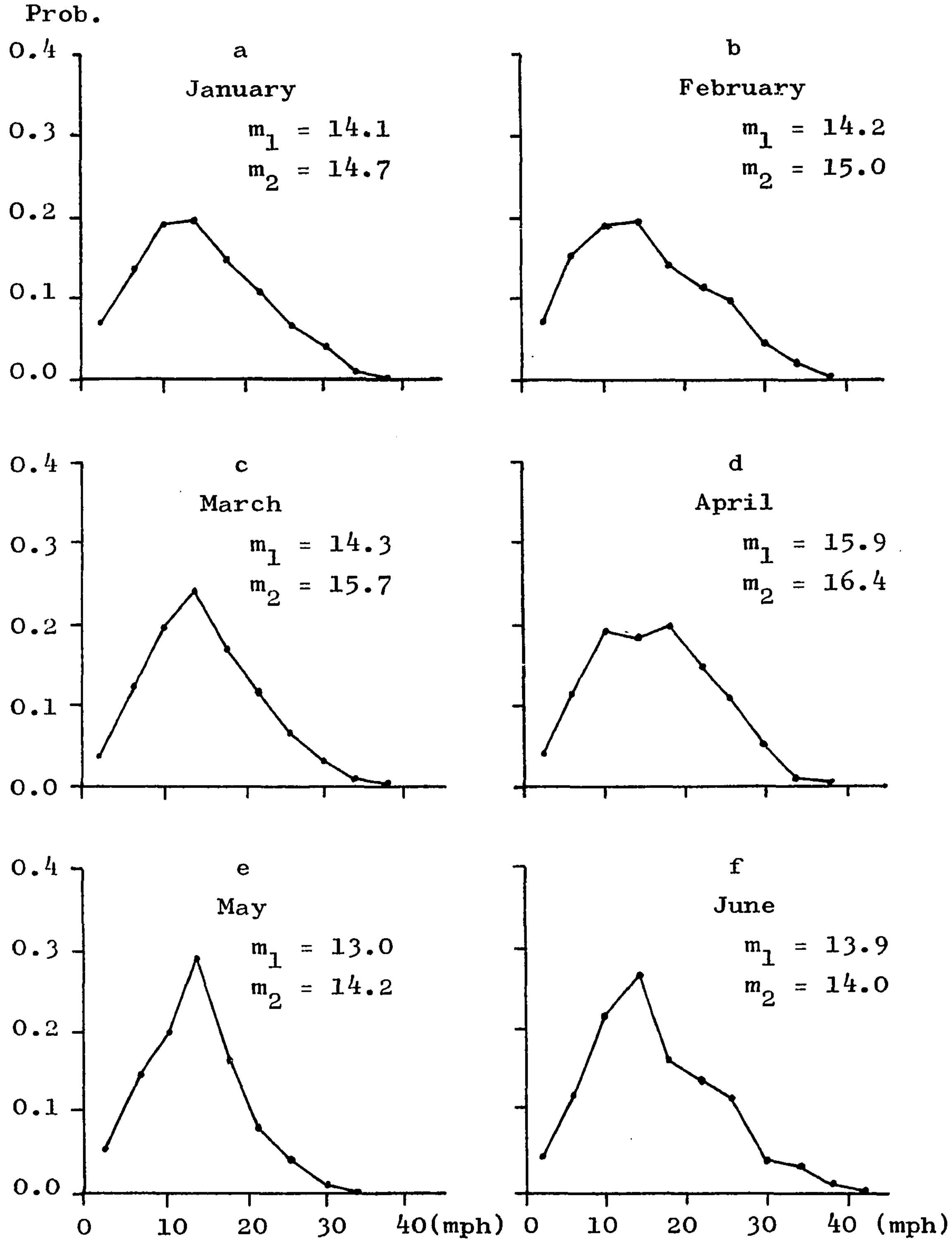

Figure 21. Monthly distributions of wind velocities in Central Oklahoma based on one-year NSSL data.

m_1 = mean of the hourly observations of the NSSL data
m_2 = mean of the hourly observations of the Census data

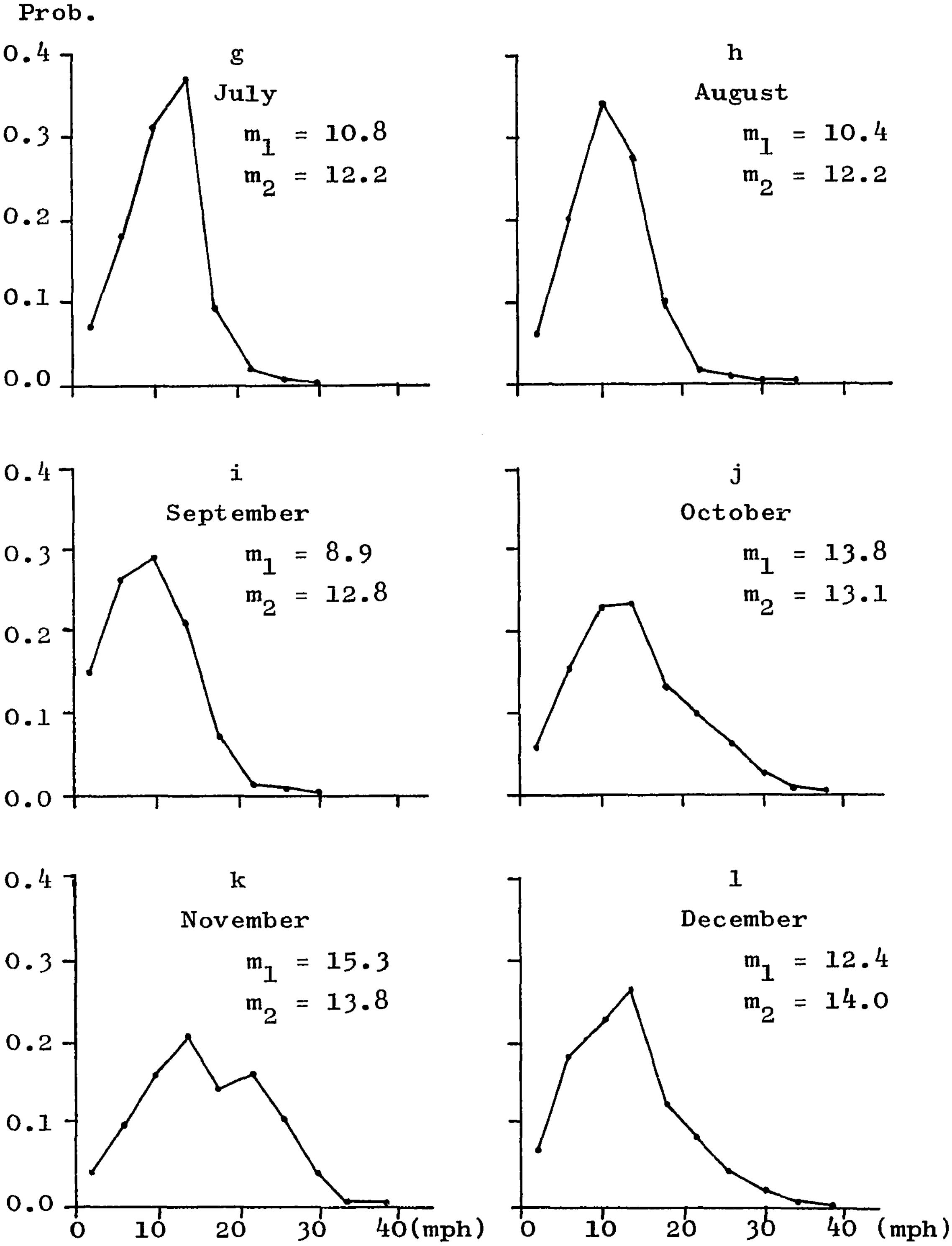

Figure 21. Continued.

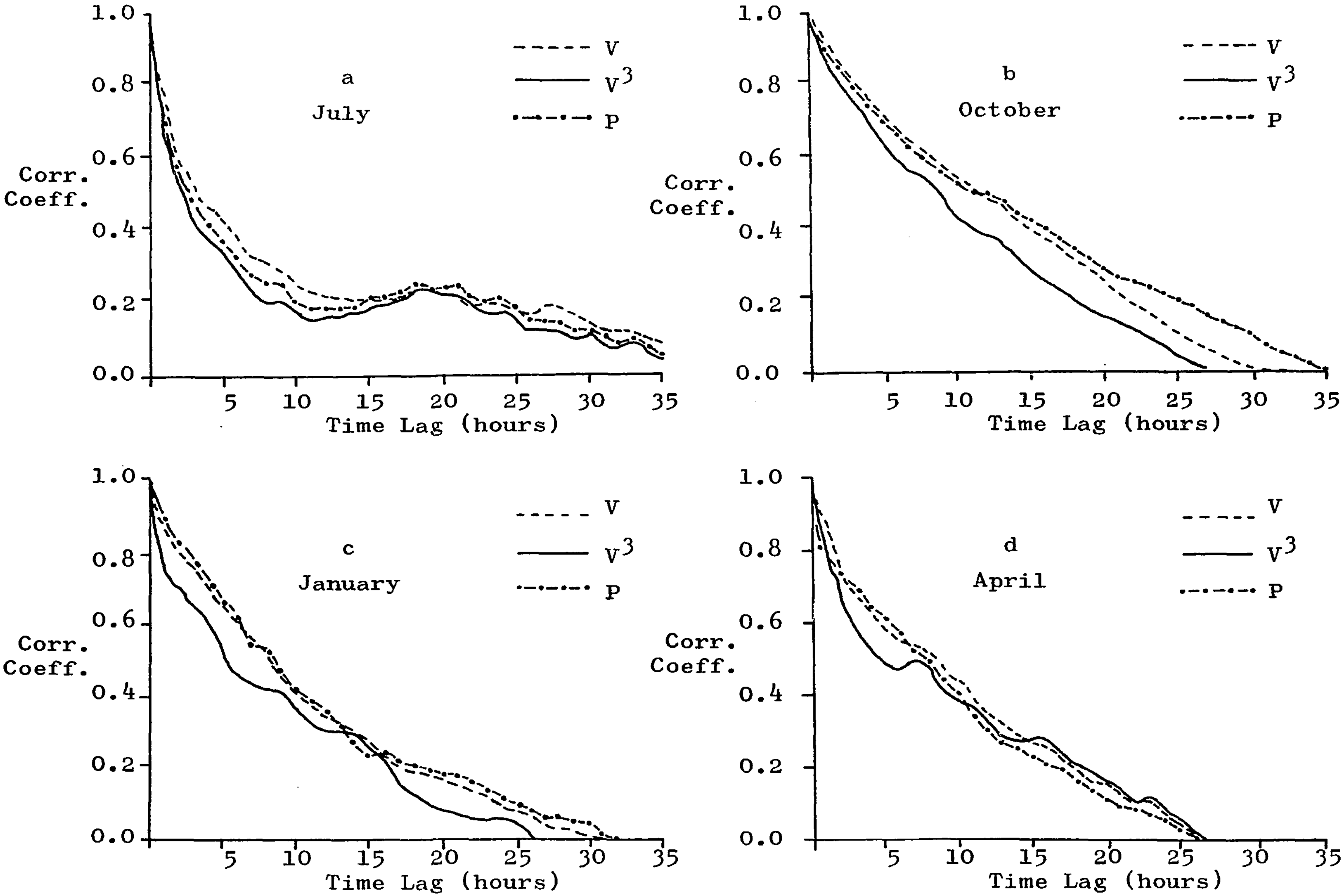

Figure 22. Auto-correlations in wind speeds, power content and power output under 10 mph cut-in and 20 mph flat-rate speed.

The velocity cubes signify the power content of the wind, and the P_t's represent the power converted by windmills having a 10 mph cut-in speed and a 20 mph flat-rate speed. Notice the straight line approximation of the power output between the cut-in and the flat-rate speed. The auto-correlation coefficients were computed without considering the effects of the diurnal variation which set a cyclic trend in the hourly averaged wind speeds (see Figure 6 in Chapter I).

The plots suggest the following:

1. The serial correlation of the power output can be closely approximated by the serial correlation of the wind speed. This closeness will probably not be affected by the variations in cut-in and flat-rate speeds.
2. The wind data show a high serial correlation. Because of this, the model developed in the first part of Chapter V cannot be applied to windpower systems under the wind conditions similar to those of Oklahoma City.

When the wind speeds are highly correlated between successive hours, but not significantly correlated between days, we may be able to model the windpower system if we can obtain the probability distribution of the daily total power output. For this particular data, a test was made to see whether the expected power output in successive days was significantly correlated. The test used is a runs test described in Miller and Freund[1] and is based on counting the

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[1] I. Miller and J. E. Freund, Probability and Statistics for Engineers, Englewood Cliffs: Prentice-Hall, 1965.

number of runs above and below the median of daily average velocities. The use of daily average velocities instead of expected power output is justified by the closeness of the auto-correlations in the two series. The summary of the runs test is presented in Table 6. The results indicate that the daily power output was not significantly correlated except in two summer months and in November; therefore, it is reasonable to treat the daily output P_t's as independent random variables.

As the next step of the analysis, the probability distributions of daily output were generated for two pairs of cut-in and flat-rate speeds. Because of the small number of data points in each month, the one-year data was divided into four seasons. Figures 23-a through h show the frequency distributions for two pairs of cut-in and flat-rate speeds: (7 mph,25 mph) and (10 mph,20 mph). (The horizontal axes of the graphs are not scaled.) From the graphs, it is obvious that no single probability distribution function, such as gamma distribution, can fit all of the distributions without additional data points which will give smoother distributions. The analysis was therefore stopped at this point without attempting to fit the distributions.

TABLE 6

RUNS TEST ON DAILY AVERAGE WIND SPEEDS

Month	Median (mph)	Number of Runs			Standard Normal Deviate
		Observed	Expected	Std. Dev.	
June[1]	13.3	8	16	2.74	-2.19
July	10.5	13	16	2.74	-1.09
Aug.[1]	10.6	9	15.5	2.64	-2.46
Sept.	8.3	13	15	2.6	-0.77
Oct.	12.3	12	16	2.74	-1.46
Nov.[1]	14.8	10	16	2.74	-2.19
Dec.	11.7	13	15.5	2.64	-0.95
Jan.	12.0	12	16	2.74	-1.46
Feb.	13.0	14	15	2.6	-0.38
Mar.	13.2	15	16	2.74	-0.36
Apr.	16.2	13	15	2.6	-0.77
May	12.9	14	15	2.6	-0.38

[1]The daily average wind speeds in these months are not random at a 95 percent confidence level. The 95 percent level corresponds to a negative 1.96 standard normal deviate.

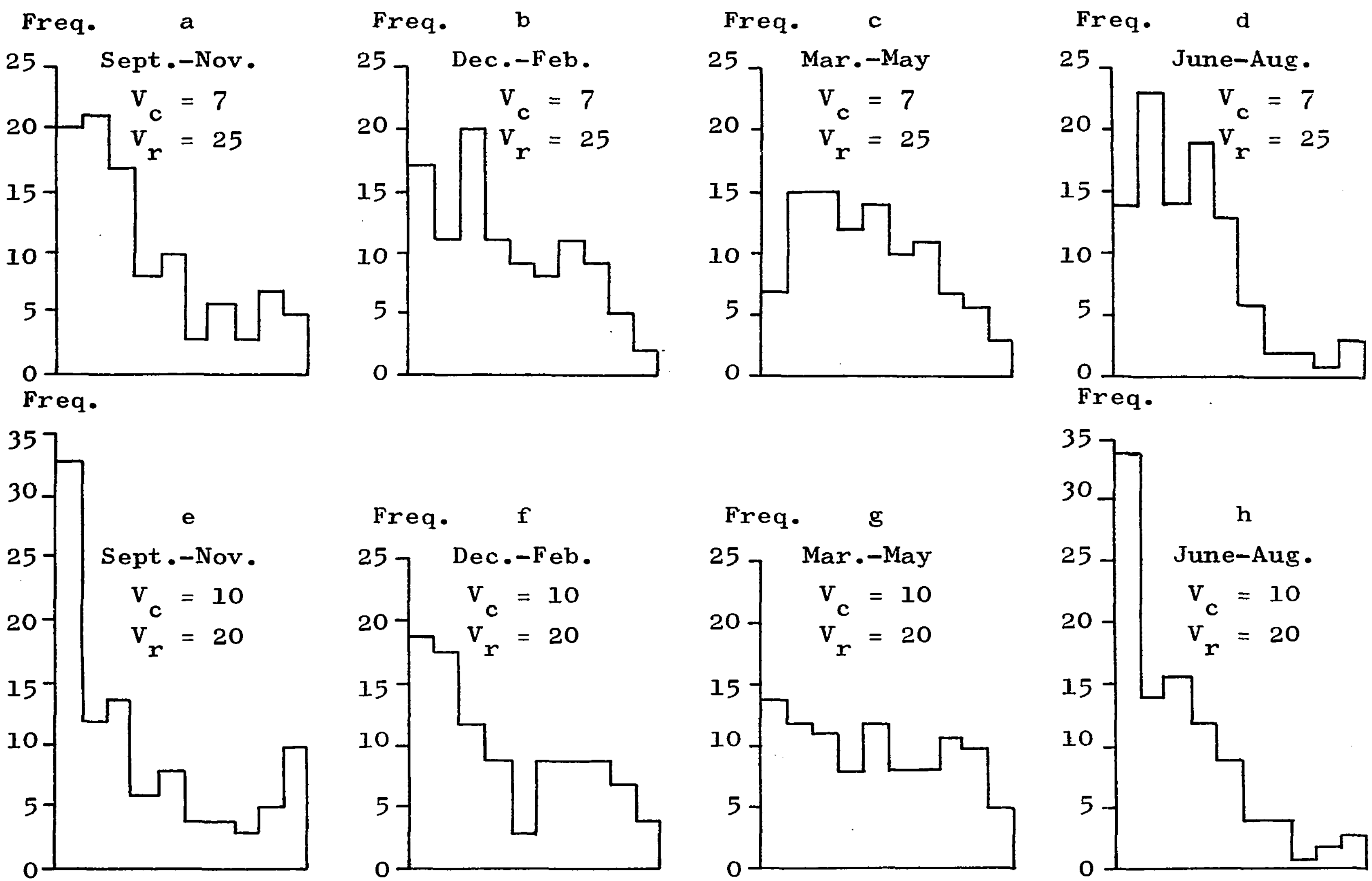

Figure 23. Variations in expected daily output distribution--by season and by design.

APPENDIX E

TEST DATA

The data listed in this appendix are just example test data prepared for testing the models in Chapter VI. Although attempts were made to obtain as much published data as possible, it was necessary to make reasonable approximations for part of the required data. The listing is divided into four parts: (1) windpower systems design and costs, (2) storage design and costs, (3) electricity demand and fuel costs, and (4) wind speed distributions.

1. Windpower systems design and costs.

   a. Windmill design.

      $\left.\begin{array}{l}\text{Efficiency } (\eta) = 0.40 \\ \text{Cut-in speed } (V_c) = 7 \text{ mph}\end{array}\right\}$ Assumed fixed for all windmills

      Range of rotor diameter (D^m, D^M) = (70,150) ft. at 10 ft. fixed increments

      Range of flat-rate speed (V_r^m, V_r^M) = (15,25) mph at 2 mph fixed increments

      Tower height (H) = rotor diameter (D)

   b. Windmill costs

      First cost (C_w): determined by interpolating the cost estimates produced in a research project by Kaman Aerospace Corporation.[1]

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[1]Richard C. Meier, "Concept Selection and Analysis of Large Wind Generator Systems," presented at the 31st Annual National Forum of the American Helicopter Society, Washington D.C., May 1975. Preprint No. S-997.

See Figure 18 and the program listing in Appendix F for the approximate per Kw installation cost.

Economic life (b_w) = 25 years

Annual O&M costs = 4 percent of the first cost

Interest rate = 10 percent (applicable to all the facilities in the system)

c. Land use.

Occupied area (A) = $(8D)^2/43560$ acres (i.e., each windmill occupies a square land area 8D units on a side)

Land cost (C_a) = 300 dollars per acre

Available land (A^M) = unlimited

d. Windpower system capacity.

Minimum (P^m) = 0 Kw

Maximum (P^M) = 500,000 Kw

e. Power transmission.

Efficiency (λ) = 0.90

Installation cost ($g(p)$) = $1.2 \cdot p$ dollars assuming a 10-mile distance to the nearest connection point

Economic life (b_t) = 50 years

2. Storage design and costs.

The storage system adapted for testing the model in Chapter V is a hydrogen storage system, in which (1) the input power is converted by Allis-Chalmers type electrolyzers delivering 1,700 psig hydrogen; (2) the hydrogen is stored in an above ground pressure vessel operating at 1,700 psi and at a negligible boil-off rate (0.05% per day); and (3) a fuel cell system burning a mixture of hydrogen and air is used to generate power. The figures tabulated below are partially based on an AGA study.[1] The conversion factors used in expressing the storage capacity in Kwh, and the input/output capacities in Kw are 325 Btus per standard cubic foot of hydrogen and 3413 Btus per Kwh, respectively.

	Input Conversion	Storage Tank	Output Conversion
Purchase cost (dollars)	$75\ S_1$	$2.7\ S_0$	$75\ S_2$
Capacity range	$(0,5x10^5)$ Kw	$(0,1x10^7)$ Kwh	$(0,5x10^5)$ Kw
Efficiency	0.80	1.00	0.80
Economic life	15	50	15
O&M Costs (mills/Kwh)	0.5	0	0.3

3. Electricity demand and fuel costs.

a. Electricity demand (E_{ij} in Mw)

	Period			
	1	2	3	4
Season	0-6 A.M.	6-12 noon	0-6 P.M.	6-12 P.M.
1 (June-Aug.)	1590	2080	2840	2560
2(Sept.-Nov.)	1095	1545	1830	1710
3 (Dec.-Feb.)	1245	1725	1715	1670
4 (Mar.-May)	1200	1635	1775	1625

Source: A partial 1974 Oklahoma Gas and Electric Company record.

[1]American Gas Association, A Hydrogen-Energy System prepared by the Institute of Gas Technology, 1973.

b. Aggregate plant fuel cost (dollars per hour)

$$f(E) = 1.6 \times 10^{-9} \cdot E^2 + 0.000{,}8E + 1{,}600$$

for E in the range of (1.0,3.5) million Kw (see Figure 19).

This function gives an average fuel cost of 4 mills per Kwh at E = 1,000 Mw and 6.86 mills per Kwh at E = 3,500 Mw. At the 1,740 Mw annual average demand as in the demand data, the function gives 4.5 mills per Kwh. The average fuel cost of the O.G.&E. system in 1974 was approximately 3.8 mills per Kwh. It is noted that in 1974 almost all O.G.&E. plants burned natural gas. These cost figures are lower than the national averages[1] as shown below:

Fuel Type	Feb. 1974	Feb. 1975
Coal	5.7 mills per Kwh	8.2
Residual Fuel Oil	18.6	20.2
Natural Gas	4.0	6.5
Weighted Average[2]	8.2	10.6

4. Wind speed distributions.

Data: The Oklahoma City television tower data.

Seasonal and periodic breakdown: the same as in the electricity demand data.

Reference height (H_o) = 146 ft.

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[1]Based on a 10,000 Btus/Kwh conversion. Source: Federal Energy Administration, "Monthly Energy Review," Washington D.C.: National Energy Information Center, July, 1975.

[2]Weighted in proportion to the total Btus of the fuel used for power generation in the United States.

Wind profile function used: $\frac{V_h}{V_o} = (\frac{H}{H_o})^{\alpha}$,

where α is estimated to be 0.26 (see the calculation below).

Level (ft.)	Annual Average Velocity (mph)	Estimate of α	
40	9.3	0.257	
146 (H_o)	13.0 (V_o)	--	} = 0.26
296	15.8	0.272	
581	18.0	0.234	
873.5	19.8	0.234	
1,166	21.4	0.239	

Maximum number of velocity increments (K) = 16

Velocity increment (v_o) = 2 mph

The probability distributions of wind speeds in all seasons and periods are shown in Table 7. They are obtained by smoothing the actual frequency distributions of the one-year measurements.

TABLE 7

TEST WIND DATA

Vel. (mph)	June-Aug.				Sept.-Nov.				Dec.-Feb.				Mar.-May			
	p.1[1]	p.2[1]	p.3[1]	p.4[1]	p.1	p.2	p.3	p.4	p.1	p.2	p.3	p.4	p.1	p.2	p.3	p.4
0- 2	.010	.020	.015	.005	.025	.025	.020	.015	.015	.015	.020	.020	.015	.010	.010	.015
2- 4	.025	.045	.030	.020	.045	.055	.055	.040	.035	.035	.045	.045	.025	.020	.025	.030
4- 6	.060	.075	.060	.035	.070	.075	.085	.065	.060	.065	.080	.075	.045	.040	.040	.060
6- 8	.120	.110	.090	.085	.090	.090	.100	.115	.075	.090	.095	.115	.070	.065	.055	.090
8-10	.165	.135	.115	.130	.110	.105	.110	.150	.100	.110	.105	.125	.095	.095	.065	.125
10-12	.180	.160	.135	.200	.120	.115	.110	.145	.120	.115	.100	.115	.125	.115	.085	.130
12-14	.185	.150	.140	.190	.130	.110	.095	.125	.125	.110	.090	.105	.130	.125	.100	.115
14-16	.125	.125	.130	.135	.115	.095	.080	.095	.115	.095	.080	.090	.115	.120	.110	.100
16-18	.060	.080	.115	.075	.086	.080	.065	.065	.090	.080	.075	.075	.100	.105	.105	.085
18-20	.025	.050	.070	.050	.065	.065	.060	.050	.070	.065	.070	.065	.080	.085	.095	.070
20-22	.015	.025	.040	.025	.050	.055	.050	.040	.060	.055	.060	.050	.060	.070	.080	.055
22-24	.010	.015	.025	.020	.040	.050	.040	.030	.045	.045	.050	.040	.045	.050	.070	.040
24-26	.005	.005	.015	.015	.025	.035	.035	.020	.035	.035	.045	.030	.035	.040	.055	.030
26-28	.005	.005	.010	.010	.015	.025	.030	.015	.025	.025	.035	.020	.025	.030	.045	.020
28-30	.000	.000	.005	.005	.010	.015	.025	.010	.020	.020	.025	.015	.015	.020	.030	.015
30-32	.000	.000	.005	.000	.005	.005	.040	.020	.010	.020	.025	.015	.020	.010	.030	.010
$\bar{v}$[2]	11.1	10.9	12.5	12.1	12.5	12.8	13.2	12.1	13.6	13.5	14.2	12.9	13.9	14.6	15.7	13.4
$\bar{s}$[3]	4.8	5.7	5.7	5.0	6.5	7.4	8.0	6.7	6.9	7.2	8.3	7.0	6.3	6.6	7.3	6.5
$\bar{s}/\bar{v}$	.43	.52	.46	.41	.52	.58	.60	.55	.51	.53	.58	.54	.45	.45	.46	.49

[1]P.1: 0-6 A.M. P.2: 6-12 noon p.3: 0-6 P.M. P.4: 6-12 P.M.

[2]Average velocity of the original data before smoothing.

[3]Standard deviation corresponding to $\bar{v}$.

APPENDIX F

DOCUMENTATION OF THE COMPUTER PROGRAM FOR THE MODEL WITHOUT STORAGE

This documentation includes:

1. List of variables, constants and functions.
2. Column (variable) names in the output matrix.
3. Setup of the output matrix.
4. List of the program including the test data in Appendix E and the MPS/360 statements used for solving the problem.

1. List of variables, constants and functions.

Program Notation	Definition or Chapter III Notation
A(N)	A^n
ALPHA	α
AMAX	A^M
AN(DIA)	Function used to compute A^n
AP(B,RI)	$(a/p)_r^b$
BT	b_t
BW	b_w
CA	C_a
CW(N)	C_w^n
CWN(PR,VR)	Function used to compute C_w^n
C1	C_1
DD(N)	D^n
DELD(I)	d_i
DELH(J)	h_j
DELY(I,J)	Increments of y_{ij}
E(I,J)	E_{ij}
EMAX	E^M
EMIN	E^m
ENUM	Number of increments in E^M-E^m
ETA	η (Fixed in the program.)
F(PE)	f(E)
G(PT)	g(p)
H	Output unit
HN(DIA)	Function used to compute H^n

HØ	H_o
II	I
INT	r
JJ	J
KK	K
K1N	k_1^n
K2N	k_2^n
LUM	λ
NN	N
NUMY(I,J)	Number of increments of y_{ij}
OBJ(·)	Coefficients of the objective function
OW	O_w
PMAX	P^M
PMIN	P^m
PNUM	Number of increments of p
PR(N)	P_r^n
Q(I,J,N)	Q_{ij}^n
R(I,J,K)	R_{ijk}
VC	V_c (Fixed in the program.)
VH	v_h
VR(N)	V_r^n
VVV(K)	$(\overline{V}_k)^3$
VØ	v_o

2. Column (variable) names in the output matrix.

Program Notation	Definition or Chapter III Notation
Yijm	Variable associated with the m^{th} increment of y_{ij}
Yij	y_{ij}
Xn	x^n
Pm	Variable associated with the m^{th} increment of p
P	p
R	Right hand side of the constraints

3. Setup of the output matrix.[1]

Columns

Rows	Y^m_{ij}	Y_{ij}	X^n	P^m	P		RHS
OB							
RA_{ij}	1	$-\frac{1}{\Delta y_{ij}}$				=	0
RB_{ij}		-1	Q^n_{ij}			≥	0
RC1			P^n_r		-1	=	0
RC2				1	$-\frac{1}{\Delta P}$	=	0
RC3			A^n			≤	A^M
Bounds min.	0	0	0	0	P^m		
Bounds max.	1	$\frac{E_{ij}-E^m}{\lambda}$		1	P^M		

[1]The elements in the matrix are actually vectors or matrices themselves. The superscripts m and n vary before the subscripts i and j, and j varies before i. Refer to the problem formulation in Chapter III for the correct dimensions of the elements. The notation used the first time in this problem setup is:

Δy_{ij} = DELY(I,J)

ΔP = PMAX/PNUM

4. List of the program.

```
C
C   **************************************************
C   * THE MODEL FOR WINDPOWER SYSTEMS WITHOUT STORAGE *
C   **************************************************
C
      INTEGER H,NUMY(6,8)
      REAL INT,LUM,VR(100),DELH(8)
      REAL DELD(6),R(6,8,40),Q(6,8,100),PR(100),DD(100)
      REAL CW(100),OBJ(400), A(100),VVV(16),DELY(6,8),E(6,8)
      DIMENSION I2(2),J2(2),Q2(2),PM(2),UL(2)
      COMMON H
      DATA C1/4.00E-6/,R/1920*0.0/
      DATA EQ,GT,LT,NOC/' E  ',' G  ',' L  ',' N  '/
      DATA RA,RB,RC,OB/'RA','RB','RC','OB'/
      DATA K1,K2,K3,K0,K9/1,2,3,0,9/
      DATA Y,X,P,R000,BLK/'Y','X','P','R',' '/
      DATA PM/'E+','E-'/,UL/'UP','LO'/
      AN(DIA) = (DIA*8.0)**2/ 43560.
      HN(DIA)=DIA
      AP(B,RI) = (RI*(1.0+RI)**B)/((1.0+RI)**B-1.0)
      F(PE)=(1.6E-9)*PE**2+0.0008*PE+1600.0
      G(PT) = 1.2*PT
C
C   IF H=6 (I.E., THE OUTPUT UNIT IS THE PRINTER), THE FIRST
C   CHARACTERS IN THE LINES PRINTED ACCORDING TO THE FORMATS
C   112, 135, 205, 220 AND 240 MUST BE REPLACED WITH BLANKS.
C
      H=10
C
C   READ THE INPUT DATA
C
      READ (5,5) II,JJ,KK,NN
    5 FORMAT (10I5)
      READ (5,15) (DELD(I),I=1,II)
      READ (5,15) (DELH(J),J=1,JJ)
      READ (5,15) ETA,LUM,VC
   15 FORMAT (10E8.0)
      READ (5,17) ((DD(N),VR(N)),N=1,NN)
   17 FORMAT (20F4.0)
      READ (5,15) OW,CA,BW,BT,INT
      READ (5,15) EMIN,EMAX,ENUM,PMIN,PMAX,PNUM,AMAX
      READ (5,15) V0,H0,ALPHA
      DO 30 I=1,II
      DO 30 J=1,JJ
   30 READ (5,35) (R(I,J,K),K=1,KK)
   35 FORMAT (16F5.0)
      DO 40 I=1,II
   40 READ (5,15) (E(I,J),J=1,JJ)
C
C   COMPUTE THE CUBES OF THE MEDIAN WIND SPEEDS.
C
```

```
      DO 62 K=1,KK
   62 VVV(K) = ((FLOAT(K) - 0.5)*V0)**3
      DO 80 N=1,NN
C
C  COMPUTE Q(I,J,N), THE EXPECTED POWER OUTPUT IN SEASON I,
C  PERIOD J FROM ONE UNIT OF DESIGN N.
C
      HF=(HN(DD(N))/H0)**ALPHA
      HF3=HF**3
      VR3=VR(N)**3
      CED=C1*ETA*DD(N)**2
      VH=V0*HF
      K1N=IFIX(VC/VH)+1
      K2N=IFIX(VR(N)/VH)+1
      K2NM=K2N-1
      DO 75 I=1,II
      DO 75 J=1,JJ
      QN=0.0
      DO 65 K=K1N,K2NM
   65 QN = QN+VVV(K)*R(I,J,K)
      QN= QN*HF3
      QNPR=0.0
      IF (K2N .GT. KK) GO TO 75
      DO 70 K=K2N,KK
   70 QNPR=QNPR + R(I,J,K)
      QNPR=QNPR*VR3
   75 Q(I,J,N) = (QN+QNPR)*CED
C
C  COMPUTE THE RATED POWER (PR), INSTALLATION COST (CW) AND
C  THE LAND REQUIREMENT (A) OF ONE UNIT OF DESIGN N.
C
      PR(N) = CED*VR3
      CW(N)= CWN(PR(N),VP(N))
      A(N)=AN(DD(N))
   80 CONTINUE
C
C  EXPRESS THE EXPECTED ANNUAL FUEL COST SAVINGS AS A
C  PIECEWISE LINEAR FUNCTION OF THE DEMAND MINUS THE
C  DELIVERED WINDPOWER.
C
      NC=0
      DELE = (EMAX-EMIN) / ENUM
      DO 100 I=1,II
      DELDI = DELD(I)
      DO 95 J=1,JJ
      YMAX = (E(I,J)-EMIN) / LUM
      NYM =IFIX(YMAX/DELE*LUM) + 1
      DLY=YMAX/FLOAT(NYM)
      NUMY(I,J) = NYM
      DELY(I,J) = DLY
      EIJ=E(I,J)
```

```
      DLE=DLY*LUM
      DO 90 M=1,NYM
      NCM=NC+M
      TEMP=F(EIJ-FLOAT(M-1)*DLE)-F(EIJ-FLOAT(M)*DLE)
      OBJ(NCM)=TEMP*DELDI*DELH(J)
   90 CONTINUE
      NC=NCM
   95 CONTINUE
  100 CONTINUE
      NYMT = NC
      IJ=II*JJ
      NC = NYMT + IJ
C
C  COMPUTE THE ANNUAL EQUIVALENT COST OF ONE UNIT OF
C  DESIGN N.
C
      APW=AP(BW,INT)
      DO 105 N=1,NN
      NCN=NC+N
  105 OBJ(NCN)=-APW*CW(N) - CA*A(N)*INT - OW*CW(N)
C
C COMPUTE THE ANNUAL EQUIVALENT TRANSMISSION COST.
C
      APT=AP(BT,INT)
      NC=NC+NN
      NPM=PNUM
      DELPO=PMAX/FLOAT(NPM)
      DO 110 M=1,NPM
      NCM=NC+M
  110 OBJ(NCM)=-APT*(G(FLOAT(M)*DELPO)-G(FLOAT(M-1)*DELPO))
  111 WRITE (H,112)
  112 FORMAT ('NAME',10X,'MODEL1',60X,/'ROWS',76X)
C
C  OUTPUT THE CONSTRAINT TYPES FOR THE ROWS SECTION.
C
      WRITE (H,113) NOC,OB,KO,KO
  113 FORMAT (A4,A2,2I1,72X)
      DO 115 I=1,II
      DO 115 J=1,JJ
  115 WRITE (H,113) EQ,RA,I,J
      DO 120 I=1,II
      DO 120 J=1,JJ
  120 WRITE (H,113) GT,RB,I,J
      WRITE (H,113) EQ,RC,K1,KO
      WRITE (H,113) EQ,RC,K2,KO
      WRITE (H,113) LT,RC,K3,KO
C
C  BEGINNING OF THE COLUMNS SECTION.
C  THE NON-ZERO COEFFICIENTS OF THE PROBLEM MATRIX ARE
C  OUTPUT IN A COLUMN ORDER.
C
```

```
  132 WRITE (H,135)
  135 FORMAT ('COLUMNS',73X)
      NC=0
      ONE=1.0
      ONEM=-1.0
C
C  OUTPUT YIJM COLUMNS.
C
      DO 160 I=1,II
      DO 160 J=1,JJ
      WRITE (H,140) Y,I,J,KO
  140 FORMAT (4X, 'MK',A1,3I1,4X,'''MARKER''',17X,
     1                                 '''SEPORG''',33X)
      MM=NUMY(I,J)
      DO 150 M=1,MM
      NC=NC+1
  150 CALL PRNT (2,Y,I,J,M,OB,0,0,OBJ(NC),1,RA,I,J,ONE,0)
  160 CONTINUE
      NMARK=1
      WRITE (H,170) NMARK
  170 FORMAT (4X,'ENDMK',I1,4X,'''MARKER''',17X,
     1                               '''SEPEND''',33X)
C
C  OUTPUT YIJ COLUMNS.
C
      DO 180 I=1,II
      DO 180 J=1,JJ
      CA=-1.0/DELY(I,J)
  180 CALL PRNT (2,Y,I,J,0,RA,I,J,CA,1,RB,I,J,ONEM,0)
      NC=NC+IJ
C
C  OUTPUT XN COLUMNS.
C
      DO 195 N=1,NN
      N1=N/10
      N2=N-10*N1
      CALL PRNT(1,X,N1,N2,0,OB,0,0,OBJ(NC+N),1,BLK,0,0,0.,0)
      K=1
      DO 190 I=1,II
      DO 190 J=1,JJ
      I2(K)=I
      J2(K)=J
      Q2(K)=Q(I,J,N)
      IF (K .LT. 2) GO TO 188
      CALL PRNT (2,X,N1,N2,0,RB,I2(1),J2(1),Q2(1),1,
     1                         RB,I2(2),J2(2),Q2(2),1)
      K=1
      GO TO 190
  188 K=2
  190 CONTINUE
      IF (K .EQ. 1) GO TO 193
```

```
      CALL PRNT (1,X,N1,N2,0,RB,I2(1),J2(1),Q2(1),1,
     1                              BLK,0,0,C,0,0)
  193 CALL PRNT (2,X,N1,N2,0,RC,1,0,PR(N),1,RC,3,0,A(N),1)
  195 CONTINUE
      NC=NC+NN
C
C  OUTPUT PM AND P COLUMNS.
C
      WRITE (H,140) P,KO,KO,KO
      DO 200 M=1,NPM
  200 CALL PRNT (2,P,M,0,0,OB,0,0,OBJ(NC+M),1,RC,2,0,ONE,0)
      NMARK=NMARK+1
      WRITE (H,170) NMARK
      CA=-1.0/DELPO
      CALL PRNT (2,P,0,0,0,RC,1,0,ONEM,0,RC,2,0,CA,1)
C
C OUTPUT THE NON-ZERO ELEMENTS IN THE RIGHT HAND SIDE.
C
  203 WRITE (H,205)
  205 FORMAT ('RHS',77X)
      CALL PRNT (1,R000,0,0,0,RC,3,0,AMAX,1,BLK,0,0,0,0,0)
C
C  OUTPUT THE BOUNDS ON THE VARIABLES.
C
      WRITE (H,220)
  220 FORMAT ('BOUNDS',74X)
      DO 225 I=1,II
      DO 225 J=1,JJ
      MM=NUMY(I,J)
      DO 225 M=1,MM
  225 CALL PRNTB (Y,I,J,M,ONE,0,1,0)
      DC 230 I=1,II
      DO 230 J=1,JJ
      YMAX=(E(I,J)-EMIN)/LUM
  230 CALL PRNTB (Y,I,J,0,YMAX,1,1,0)
      DO 235 M=1,NPM
  235 CALL PRNTB (P,M,0,0,ONE,0,1,0)
      CALL PRNTB (P,0,0,0,PMAX,1,1,0)
      IF (PMIN .LT. 0.001) PMIN=0.001
      CALL PRNTB (P,0,0,0,PMIN,1,2,0)
      WRITE (H,240)
  240 FORMAT ('ENDATA',74X)
      IF (H .LE. 7) GO TO 999
      ENDFILE H
      REWIND H
  999 STOP
      END
```

```
      FUNCTION CWN(PR,VR)
C
C  THIS IS A FUNCTION FOR CALCULATING THE WINDMILL
C  INSTALLATION COST.  THE USER MUST PROVIDE A METHOD OF
C  CALCULATING THE COST AND REPLACE THIS PORTION OF THE
C  PROGRAM WITH IT.
C
      DIMENSION CIJ(6,7),VI(6),PJ(7)
      DATA VI/13.,15.,18.,20.,23.,28./
      DATA PJ/1.301,1.699,2.0,2.301,2.699,3.0,3.301/
      DATA CIJ/3.83,3.74,3.65,3.62,3.60,3.57,
     1         3.70,3.57,3.43,3.37,3.32,3.25,
     1         3.63,3.47,3.30,3.22,3.15,3.06,
     1         3.61,3.42,3.24,3.13,3.03,2.91,
     1         3.60,3.42,3.20,3.07,2.94,2.77,
     1         3.64,3.45,3.20,3.07,2.93,2.74,
     1         3.70,3.48,3.23,3.10,2.94,2.73/
      P=ALOG10(PR)
      DO 5 I=1,6
      IF (VI(I) .GE. VR) GO TO 6
    5 CONTINUE
    6 DO 10 J=1,7
      IF (PJ(J) .GE. P) GO TO 11
   10 CONTINUE
   11 F1=(VI(I)-VR)/(VI(I)-VI(I-1))
      F2=(PJ(J)-P)/(PJ(J)-PJ(J-1))
      C1=F1*(CIJ(I-1,J-1)-CIJ(I,J-1))+CIJ(I,J-1)
      C2=F1*(CIJ(I-1,J)-CIJ(I,J))+CIJ(I,J)
      C=(C1-C2)*F2+C2
      CWN=PR*10.0**C
      RETURN
      END


      SUBROUTINE PRNT (N,COL,I,J,K,ROW1,L1,M1,W1,ICON1,
     1                            ROW2,L2,M2,W2,ICON2)
C
C  THIS ROUTINE IS USED TO OUTPUT THE COLUMNS SECTION IN
C  MPS/360 INPUT FORMAT.  IT OUTPUTS ONE OR TWO NON-ZERO
C  COEFFICIENTS PER LINE AS SPECIFIED BY THE PARAMETER N.
C
      DIMENSION PM(2)
      INTEGER H
      COMMON H
      DATA PM/'E+','E-'/
      IS1=1
      IN1=0
      V1=W1
      IF (ICON1 .NE. 0) CALL CONV (V1,IS1,IN1,W1)
      IF (N .NE. 1) GO TO 10
```

```
      WRITE (H,5) COL,I,J,K,ROW1,L1,M1,V1,PM(IS1),IN1
    5 FORMAT (4X,A1,3I1,3X,2(3X,A2,2I1,6X,F9.5,A2,I1),19X)
      RETURN
   10 IS2=1
      IN2=0
      V2=W2
      IF (ICON2 .NE. 0) CALL CONV (V2,IS2,IN2,W2)
      WRITE (H,5) COL,I,J,K,ROW1,L1,M1,V1,PM(IS1),IN1,
     1                       ROW2,L2,M2,V2,PM(IS2),IN2
      RETURN
      END


      SUBROUTINE PRNTB (COL,I,J,K,W,ICON,IUL,IONE)
C
C   THIS ROUTINE WRITES THE BOUNDS SECTION IN MPS/360
C   INPUT FORMAT.
C
      DIMENSION PM(2),UL(2)
      INTEGER H
      COMMON H
      DATA PM/'E+','E-'/,UL/'UP','LO'/
      IS=1
      IN=0
      V=W
      IF (IONE .EQ. 1) GO TO 10
      IF (ICON .NE. 0) CALL CONV (V,IS,IN,W)
      WRITE (H,5) UL(IUL),COL,I,J,K,V,PM(IS),IN
    5 FORMAT (1X,A2,1X,'BOUND',5X,A1,3I1,6X,F9.5,A2,I1,44X)
      RETURN
   10 DO 15 II=1,I
      DO 15 JJ=1,J
   15 WRITE (H,5) UL(IUL),COL,II,JJ,K,V,PM(IS),IN
      RETURN
      END


      SUBROUTINE CONV (V,IS,IN,W)
C
C   THIS ROUTINE IS USED TO CONVERT A NUMBER INTO AN 'E'
C   FORMAT.  THE FORTRAN 'E' FORMAT MAY NOT BE USED BECAUSE
C   THE MPS/360 DOES NOT ALLOW A BLANK BETWEEN 'E' AND THE
C   NON-NEGATIVE EXPONENT.
C
      DOUBLE PRECISION X,RN
      X=ABS(W)
      IS=1
      RN=DLOG10(X)
      IF (RN .GE. 0.0) GO TO 10
```

```
      RN=RN-1.0
      IS=2
   10 IN=RN
      V=W/10.0**IN
      IN=IABS(IN)
      RETURN
      END
```

```
*************************
* THE MPS/360 STATEMENTS *
*************************
```

THE DATA NAME FOR THE MODEL WITH STORAGE IS 'MODEL2'.

```
PROGRAM
TITLE ('NOSTOMOD')
INITIALZ
MOVE (XDATA,'MODEL1')
MOVE (XPBNAME,'PBFILE')
CONVERT ('SUMMARY')
MOVE (XOBJ,'OBOO')
MOVE (XRHS,'ROOO')
SETUP ('BOUND','BOUND','MAX')
PRIMAL
SOLUTION
EXIT
PEND
```

```
*******************************************
* THE DATA FOR THE MODEL WITHOUT STORAGE *
*******************************************

   4     4    16    54
  92.+0    91.+0    90.+0    92.+0
   6.+0     6.+0     6.+0     6.+0
 0.40+0   0.90+0   7.00+0
 70. 15. 70. 17. 70. 19. 70. 21. 70. 23. 70. 25. 80. 15. 80. 17. 80. 19. 80. 21.
 80. 23. 80. 25. 90. 15. 90. 17. 90. 19. 90. 21. 90. 23. 90. 25.100. 15.100. 17.
100. 19.100. 21.100. 23.100. 25.110. 15.110. 17.110. 19.110. 21.110. 23.110. 25.
120. 15.120. 17.120. 19.120. 21.120. 23.120. 25.130. 15.130. 17.130. 19.130. 21.
130. 23.130. 25.140. 15.140. 17.140. 19.140. 21.140. 23.140. 25.150. 15.150. 17.
150. 19.150. 21.150. 23.150. 25.
 0.04+0  300.+0   25.+0    50.+0  0.10+0
  1.0+6   3.5+6   5.0+0    0.0+0  500.+3    1.0+0    1.0+6
  2.0+0  146.+0  0.26+0
 .010 .025 .060 .120 .165 .180 .185 .135 .060 .025 .015 .010 .005 .005 .000 .000
 .020 .045 .075 .110 .135 .160 .150 .125 .080 .050 .025 .015 .005 .005 .000 .000
 .015 .030 .060 .090 .115 .135 .140 .130 .115 .070 .040 .025 .015 .010 .005 .005
 .005 .020 .035 .085 .130 .200 .190 .135 .075 .050 .025 .020 .015 .010 .005 .000
 .025 .045 .070 .090 .110 .120 .130 .115 .085 .065 .050 .040 .025 .015 .010 .005
 .025 .055 .075 .090 .105 .115 .110 .095 .080 .065 .055 .050 .035 .025 .015 .005
 .020 .055 .085 .100 .110 .110 .095 .080 .065 .060 .050 .040 .035 .030 .025 .040
 .015 .040 .065 .115 .150 .145 .125 .095 .065 .050 .040 .030 .020 .015 .010 .020
 .015 .035 .060 .075 .100 .120 .125 .115 .090 .070 .060 .045 .035 .025 .020 .010
 .015 .035 .065 .090 .110 .115 .110 .095 .080 .065 .055 .045 .035 .025 .020 .020
 .020 .045 .080 .095 .105 .100 .090 .080 .075 .070 .060 .050 .045 .035 .025 .025
 .020 .045 .075 .115 .125 .115 .105 .090 .075 .065 .050 .040 .030 .020 .015 .015
 .015 .025 .045 .070 .095 .125 .130 .115 .100 .080 .060 .045 .035 .025 .015 .020
 .010 .020 .040 .065 .095 .115 .125 .120 .105 .085 .070 .050 .040 .030 .020 .010
 .010 .025 .040 .055 .065 .085 .100 .110 .105 .095 .080 .070 .055 .045 .030 .030
 .015 .030 .060 .090 .125 .130 .115 .100 .085 .070 .055 .040 .030 .020 .015 .010
 1590.+3 2080.+3 2840.+3 2560.+3
 1095.+3 1545.+3 1830.+3 1710.+3
 1245.+3 1725.+3 1715.+3 1670.+3
 1200.+3 1635.+3 1775.+3 1625.+3
```

APPENDIX G

DOCUMENTATION OF THE COMPUTER PROGRAM FOR THE MODEL WITH STORAGE

This documentation includes:

1. List of variables, constants and functions.
2. Column (variable) names in the output matrix.
3. Piecewise linear approximation of logarithmic functions.
4. Setup of the output matrix.
5. List of the program including the test data in Appendix E. (The subroutines are not listed because they are the same as in the model without storage.)

1. List of variables, constants and functions.

Program Notation	Definition or Chapter V Notation
A(N)	A^n
ACOR	$1 + 2 \sum_{t=1}^{t_o} r_t$, where
	r_t = auto-correlation coefficient
ALPHA	α
AMAX	A^M
AN(DIA)	Function used to compute A^n
AP(B,RI)	$(a/p)_r^b$
BS(s+1)	b_s (s = 0,1,2)
BT	b_t
BW	b_w
CA	C_a
CW(N)	C_w^n
CWN(PR,VR)	Function used to compute C_w^n
C1	C_1
D(I,N)	D_i^n
DD(N)	D^n
DELD(I)	d_i
DELHE(J)	h_j
DELHW(J)	$h_{\bar{j}}$
DELOG(M)	m^{th} increment of log X (X ≥ 1)
DELS(s+1)	Increment of S_s (s = 0,1,2)
DELY(I,J)	Increments of y_{ij}
DELX(M)	m^{th} increment of the variable X in log X (see DELOG(M))

E(I,J)	E_{ij}
EMAX	E^M
EMIN	E^m
ES(s+1)	e_s (s = 0,1,2)
ETA	η (Fixed in the program.)
F(PE)	f(E)
G(PT)	g(p)
H	Output unit
HN(DIA)	Function used to compute H^n
HØ	H_o
II	I
INT	r
JE	J
JW	$\bar{J}$
KK	K
KMM(M)	m^{th} increment of $\psi(w-m)$
K1N	k_1^n
K2N	k_2^n
LUM	λ
NN	N
NUMY(I,J)	Number of increments of y_{ij}
OBJ(·)	Coefficients of the objective function
OS(s+1)	O_s (s = 0,1,2)
OW	O_w
PHIO(SO)	$\phi_0(S_0)$
PHI1(S1)	$\phi_1(S_1)$

PHI2(S2)	$\phi_2(S_2)$
PMAX	P^M
PMIN	P^m
PNUM	Number of increments in $P^M - P^m$
PR(N)	P_r^n
PSØ	Shortage probability
R(I,J,K)	R_{ijk}
SCALE1 } SCALE2	Scale factors used in logarithmic transformation (i.e., z_1 and z_2)
SIØ	$\psi(0)$
SMAX (s+1)	Maximum of S_s ($s = 0,1,2$)
SNUM (s+1)	Number of increments of S_s ($s = 0,1,2$)
TUNIT	Unit of time
UMIN	Minimum allowable value of U
VC	V_c^n (Fixed in the program.)
VH	v_h
VR(N)	V_r^n
VVV(K)	$(\overline{V}_k)^3$
VØ	v_o
W(I,N)	W_i^n
WMM(M)	m^{th} increment of w-m in $\psi(w-m)$ (See KMM(M))

2. Column (variable) names in the output matrix.

Program Notation	Definition or Chapter V Notation
Yijm	Variable associated with the m^{th} increment of y_{ij}
Ssm	Variable associated with the m^{th} increment of S_s $(s = 0,1,2)$
Yij	Y_{ij}
Xn	X^n
Wi	W_i^o
Di	D_i^o
Ss	S_s $(s = 0,1,2)$
U	U
Dim Tim Zim Mim Kim	Variables respectively associated with the m^{th} increments of D_i^o, T_i, Z_i, $w-m_i$ and k_i-m_i that are scaled and then logarithmically transformed.
Nim	Variable associated with the m^{th} increment of $w-m_i$ as the dependent variable of k_i-m_i in the function ψ.
R	Right hand side of the constraints

3. Piecewise linear approximation of logarithmic functions.

In using separable programming as the solution technique the product terms $D_i^o(w-m_i)$ and $D_i^o(k_i-m_i)$ in the model formulation are made separable by logarithmic transformation. In order to reduce the errors in approximating the resulting logarithmic functions, the scaling factors z_1 and z_2 are used to scale down D_i^o and scale up $w-m_i$ and k_i-m_i. Defining

$$D_i = D_i^o/z_1,$$

$$M_i = (w-m_i)z_2,$$

$$K_i = (k_i-m_i)z_2,$$

$$T_i = D_i \cdot M_i, \text{ and}$$

$$Z_i = D_i \cdot K_i,$$

we write

$$\log T_i = \log D_i + \log M_i \text{ and} \tag{1}$$

$$\log Z_i = \log D_i + \log K_i. \tag{2}$$

For the simplicity of programming, the variables defined above are restricted to at least 1. The effect of this restriction on the solution of the problem can be kept at a minimal level if appropriate z_1 and z_2 are used for scaling. Using "X" to represent any of the variables in Equations 1 and 2, we can write the variable X as

$$X = 1 + \sum_m \Delta^m x^m \qquad m = 1,2, \ldots, \tag{3}$$

where Δ^m = m^{th} increment of X for $X \geq 1$,

x^m = variable associated with the m^{th} increment of X $(0 \leq x^m \leq 1)$.

Then, the function log X can be approximated by

$$\log X \approx \sum_m d^m x^m \qquad m = 1,2, \ldots \tag{4}$$

where $d^m = \log\left(1 + \sum_{i=1}^{m} \Delta^i x^i\right) - \log\left(1 + \sum_{i=1}^{m-1} \Delta^i x^i\right)$.

The computer program uses the same set of increments (Δ^m's) for all variables. The first five increments of the variable X and the corresponding values of d^m are given below:

m:	1	2	3	4	5
Δ^m:	1.72	4.67	12.70	34.51	98.81
d^m:	1.0	1.0	1.0	1.0	1.0

To illustrate how the linear approximation of the logarithmic transformation might affect the solution, let us take an example case where $D_i^o = 200{,}000$ and $w\text{-}m_i = 0.5$ and T_i is to be approximated using Equations 1, 3 and 4. Considering the magnitudes of D_i^o and $w\text{-}m_i$, we select $z_1 = 10{,}000$ and $z_2 = 10$ to get $D_i = 20$ and $M_i = 5$. Then, using the values of Δ^m and d^m listed above and Equations 1, 3 and 4,

$$\begin{aligned}\log T_i &= \log D_i + \log M_i \\ &\approx 2.99 + 1.49 = 4.48\end{aligned}$$

Then, working backwards to find T_i corresponding to $\log T_i \approx 4.48$, we get $T_i \approx 102.03$, which is close to the exact value 100.

4. Setup of the output matrix.[1]

Rows	Y_{ij}	Y^m_{ij}	S^m_o	S^m_1	S^m_2	X^n	W^o_i	D^o_i	S_0	S_1	S_2	U	D^{om}_i	T^m_i	Z^m_i	M^m_i	N^m_i	K^m_i		RHS
OB																				
RAi	$\frac{-h_j}{ez}$						$\frac{1}{z}$							$-\Delta^m$					$=$	1
RBi						W^n_i	-1												$=$	0
RCi						D^n_i		-1											$=$	0
RD1						P^n_r				-1									$=$	0
RD2						A^n													$\leq$	A^M
RD3	$\bar{d}_i \cdot h_j$											-1							$=$	0
REi	$\frac{-h_j}{ez}$								$\frac{-1}{e_1 z}$						$-\Delta^m$				$=$	1
RFij	-1										e_2								$\geq$	0
RGi	$\frac{-1}{\Delta y_{ij}}$	1																	$=$	0
RH1			1						$\frac{-1}{\Delta S_0}$										$=$	0
RH2				1						$\frac{-1}{\Delta S_1}$									$=$	0
RH3					1						$\frac{-1}{\Delta S_2}$								$=$	0
RIi								$\frac{1}{z_1}$					$-\Delta^m$						$=$	1
RJi													$-d^m$	d^m		$-d^m$			$=$	0
RKi													$-d^m$		d^m			$-d^m$	$=$	0
RLi																$-\Delta^m$	w^m		$=$	1

RMi																		$-k^m$	Δ^m	$= k^o - 1$

Bounds:

min.	0	0	0	0	0	0	0	0	0	P^m	0	U^m	0	0	0	0	0	0
max.	$\frac{E_{ij} - E_m}{\lambda}$	1	1	1	1					P^M			1	1	1	1	1	1

[1]The elements in the matrix are actually vectors or matrices themselves. The superscripts m and n vary before the subscripts i and j, and j varies before i. Refer to the problem formulation in Chapter V for the correct dimensions of the elements. The notation used the first time in this problem setup is:

Δy_{ij}	= DELY(I,J)	Δ^m	= DELX(M)
e	= $e_1 \cdot e_2$	d^m	= DELOG(M)
z	= SCALE1/SCALE2	w^m	= WMM(M)·SCALE2
z_1	= SCALE1	k^m	= KMM(M)·SCALE2
ΔS_s	= DELS(s+1)	k^o	= SIØ·SCALE2

5. List of the program.

```
C
C    ***********************************************
C    * THE MODEL FOR WINDPOWER SYSTEMS WITH STORAGE *
C    ***********************************************
C
      INTEGER H,NUMY(6,8)
      REAL ES(3),OS(3),BS(3),SMAX(3),SNUM(3),DD(100),VR(100)
      REAL INT,LUM,WMM(10),KMM(10),DELX(10),DELOG(10)
      REAL DELD(6),R(6,8,40),W(6,100),D(6,100),PR(100)
      REAL CW(100),OBJ(400), A(100), VVV(16),DELY(6,8),E(6,8)
      DIMENSION PM(2),UL(2),DELHW(8),DELHE(8),DELS(3)
      COMMON H
      DATA C1/4.0E-6/,OBJ/400*0./,PM,UL/'E','E-','UP','LO'/
      DATA EQ,GT,LT,NOC/' E  ',' G  ',' L  ',' N  '/
      DATA RA,RB,RC,RD,RE,RF,RG,RH,RI/'RA','RB','RC','RD',
     1     'RE','RF','RG','RH','RI'/,OB/'OB'/
      DATA RJ,RK,RL,RM,T,Z,WM,KM,WMP/'RJ','RK','RL','RM',
     1     'T','Z','M','K','N'/
      DATA K0,K1,K2,K3,K9/0,1,2,3,9/,R/1920*0.0/
      DATA X,W0,D0,S,Y,U/'X','W','D','S','Y','U'/
      DATA R000/'R'/,BLK/' '/
      DATA MMDO,MMWM,MMKM,MMT,MMZ/ 7,4,5,7,7/
      DATA DELX/1.72,4.67,12.70,34.51,93.81,948.22,7006.45,
     1     3*0.0/
      DATA DELOG/5*1.0,2.0,2.0,3*0.0/
      DATA KMNUM,PS0,SCALE1,SCALE2,SI0/6,0.1,10.   .10.,7./
      DATA WMM/2*0.125,3*0.25,0.280,4*0.0/
      DATA KMM/-3.2,-1.3,-1.2,-0.5,-0.3,-0.5,4*0.0/
      HN(DIA)=DIA
      AN(DIA) = (DIA*8.0)**2/ 43560.
      AP(B,RI) = (RI*(1.0+RI)**B)/((1.0+RI)**B-1.0)
      F(PE)=((1.6E-9)*PE**2+0.0008*PE+1600.0)*1.0
      G(PT) = 1.2*PT
      PHI0(S0)=2.7*S0
      PHI1(S1)=75.0*S1
      PHI2(S2)=75.0*S2
C
C  IF H=6 (I.E., THE OUTPUT UNIT IS THE PRINTER), THE FIRST
C  CHARACTERS IN THE LINES PRINTED ACCORDING TO THE FORMATS
C  205, 252, 530, 540 AND 600 MUST BE REPLACED WITH BLANKS.
C
      H=10
C
C  READ THE INPUT DATA
C
      READ (5,5) II,JW,JE,KK,NN
    5 FORMAT (10I5)
      READ (5,15) (DELD(I),I=1,II)
      READ (5,15) ACOR
      READ (5,15) (DELHW(J),J=1,JW),TUNIT
      READ (5,15) (DELHE(J),J=1,JE)
```

```
      READ (5,15) ETA,LUM,VC
   15 FORMAT (10E8.0)
      READ (5,17) ((DD(N),VR(N)),N=1,NN)
   17 FORMAT (20F4.0)
      READ (5,15) OW,CA,BW,BT,INT
      READ (5,15) EMIN,EMAX,ENUM,PMIN,PMAX,UMIN,AMAX
      DO 21 I=1,3
   21 READ (5,15) ES(I),OS(I),BS(I),SMAX(I),SNUM(I)
      READ (5,15) V0,H0,ALPHA
      DO 23 I=1,II
      DO 23 J=1,JW
   23 READ (5,25) (R(I,J,K),K=1,KK)
   25 FORMAT (16F5.0)
      DO 30 I=1,II
   30 READ (5,15) (E(I,J),J=1,JE)
C
C  COMPUTE THE CUBES OF THE MEDIAN WIND SPEEDS.
C
      DO 50 K=1,KK
   50 VVV(K)=((FLOAT(K)-0.5)*V0)**3
C
C  COMPUTE W(I,N) AND D(I,N), THE MEAN AND THE STANDARD
C  DEVIATION OF THE DAILY OUTPUT IN SEASON I FROM ONE UNIT
C  OF DESIGN N.
C
      DO 90 N=1,NN
      HF=(HN(DD(N))/H0)**ALPHA
      HF3=HF**3
      VR3=VR(N)**3
      CED=C1*ETA*DD(N)**2
      VH=V0*HF
      K1N=IFIX(VC/VH)+1
      K2N=IFIX(VR(N)/VH)+1
      K2NM=K2N-1
      DO 85 I=1,II
      WIN=0.0
      DIN=0.0
      DO 80 J=1,JW
      WN=0.0
      DO 55 K=K1N,K2NM
   55 WN=WN+VVV(K)*R(I,J,K)
      WN=WN*HF3
      WNPR=0.0
      IF (K2N .GT. KK) GO TO 65
      DO 60 K=K2N,KK
   60 WNFR=WNPR+R(I,J,K)
      WNFR=WNPR*VR3
   65 WN=(WN+WNPR)*CED
      DN=0.0
      DO 70 K=K1N,K2NM
   70 DN=DN+(CED*HF3*VVV(K)-WN)**2*R(I,J,K)
```

```
      IF (K2N .GT. KK) GO TO 77
      DO 75 K=K2N,KK
   75 DN=DN+(CED*VR3-WN)**2*R(I,J,K)
   77 WIN=WIN+WN*DELHW(J)*TUNIT
   80 DIN=DIN+DN*DELHW(J)*TUNIT
      W(I,N)=WIN
   85 D(I,N)=SQRT(DIN*ACOR)
C
C  COMPUTE THE RATED POWER (PR), INSTALLATION COST (CW) AND
C  THE LAND REQUIREMENT (A) OF ONE UNIT OF DESIGN N
C
      PR(N)=CED*VR3
      CW(N)= CWN(PR(N),VR(N))
      A(N)=AN(DD(N))
   90 CONTINUE
C
C  EXPRESS THE EXPECTED ANNUAL FUEL COST SAVINGS AS A
C  PIECEWISE LINEAR FUNCTION OF THE DEMAND MINUS THE
C  DELIVERED WINDPOWER.
C
      IJ=II*JE
      NC=IJ+NN+2*II+4
      DELE=(EMAX-EMIN)/ ENUM
      DO 105 I=1,II
      DELDI=DELD(I)*(1.0-PSO)
      DO 100 J=1,JE
      YMAX=(E(I,J)-EMIN)/LUM
      NYM=IFIX(YMAX/DELE*LUM)+1
      DLY=YMAX/FLOAT(NYM)
      NUMY(I,J)=NYM
      DELY(I,J)=DLY
      EIJ=E(I,J)
      DLE=DLY*LUM
      DO 95 M=1,NYM
      NCM=NC+M
      TEMP=F(EIJ-FLOAT(M-1)*DLE)-F(EIJ-FLOAT(M)*DLE)
      OBJ(NCM)=TEMP*DELDI*DELHE(J)
   95 CONTINUE
      NC=NCM
  100 CONTINUE
  105 CONTINUE
      NYMT=NC
      NC=IJ
C
C  COMPUTE THE ANNUAL EQUIVALENT COST OF ONE UNIT OF
C  DESIGN N.
C
      APW=AP(BW,INT)
      DO 108 N=1,NN
      NCN=NC+N
  108 OBJ(NCN)=-APW*CW(N) - CA*A(N)*INT - OW*CW(N)
```

```
      NCS=NYMT
      APT=AP(BT,INT)
C
C  COMPUTE THE ANNUAL EQUIVALENT INSTALLATION COST OF THE
C  STORAGE SYSTEM INCLUDING THE RESERVOIR AND THE INPUT/
C  OUTPUT FACILITIES.  ADD THE TRANSMISSION COST TO THE COST
C  OF THE OUTPUT FACILITY.
C
      DO 120 I=1,3
      DELS(I)=SMAX(I)/SNUM(I)
      MM=SNUM(I)
      APS=AP(BS(I),INT)
      DO 116 M=1,MM
      FMD=FLOAT(M)*DELS(I)
      FM1D=FLOAT(M-1)*DELS(I)
      GO TO (111,113,115),I
  111 OBJ(NCS+M)=-APS*(PHI0(FMD)-PHI0(FM1D))
      GO TO 118
  113 OBJ(NCS+M)=-APS*(PHI1(FMD)-PHI1(FM1D))
      GO TO 118
  115 OBJ(NCS+M)=-APS*(PHI2(FMD)-PHI2(FM1D))
     1              -APT*(G(FMD*ES(3))-G(FM1D*ES(3)))
  118 CONTINUE
      NCS=NCS+MM
  120 CONTINUE
  200 WRITE (H,205)
  205 FORMAT ('NAME',10X,'MODEL2' ,60X,/'ROWS',76X)
C
C  OUTPUT THE CONSTRAINT TYPES FOR THE ROWS SECTION.
C
      WRITE (H,210) NOC,OB,K0,K0
  210 FORMAT (A4,A2,2I1,72X)
      DO 215 I=1,II
  215 WRITE (H,210) EQ,RA,I,K0
      DO 220 I=1,II
  220 WRITE (H,210) EQ,RB,I,K0
      DO 225 I=1,II
  225 WRITE (H,210) EQ,RC,I,K0
      WRITE (H,210) EQ,RD,K1,K0,LT,RD,K2,K0,EQ,RD,K3,K0
      DO 230 I=1,II
  230 WRITE (H,210) EQ,RE,I,K0
      DO 235 I=1,II
      DO 235 J=1,JE
  235 WRITE (H,210) GT,RF,I,J
      DO 240 I=1,II
      DO 240 J=1,JE
  240 WRITE (H,210) EQ,RG,I,J
      DO 245 I=1,3
  245 WRITE (H,210) EQ,RH,I,K0
      DO 246 I=1,II
  246 WRITE (H,210) EQ,RI,I,K0
```

```
      DO 247 I=1,II
  247 WRITE (H,210) EQ,RJ,I,KO
      DO 248 I=1,II
  248 WRITE (H,210) EQ,RK,I,KO
      DO 249 I=1,II
  249 WRITE (H,210) EQ,RL,I,KO
      DO 250 I=1,II
  250 WRITE (H,210) EQ,RM,I,KO
C
C  BEGINNING OF THE COLUMNS SECTION.
C  THE NON-ZERO COEFFICIENTS OF THE PROBLEM MATRIX ARE
C  OUTPUT IN A COLUMN ORDER.
C
      WRITE (H,252)
  252 FORMAT ('COLUMNS',73X)
      ONE=1.0
      ONEM=-1.0
      NMARK=1
      NC=IJ+NN+2*II+4
C
C  OUTPUT YIJM COLUMNS.
C
      DO 265 I=1,II
      DO 265 J=1,JE
      WRITE (H,255) Y,I,J,KO
  255 FORMAT (4X,'MK',A1,3I1,4X,'''MARKER''',17X,
     1                              '''SEPORG''',33X)
      MM=NUMY(I,J)
      DO 260 M=1,MM
      NC=NC+1
  260 CALL PRNT (2,Y,I,J,M,OB,0,0,OBJ(NC),1,RG,I,J,ONE,0)
  265 CONTINUE
C
C  OUTPUT SIM COLUMNS.
C
      DO 275 J=1,3
      I=J-1
      WRITE (H,255) S,I,KO,KO
      MM=SNUM(J)
      DO 270 M=1,MM
      NC=NC+1
  270 CALL PRNT (2,S,I,M,KO,OB,0,0,OBJ(NC),1,RH,J,KO,ONE,0)
  275 CONTINUE
      WRITE (H,280) NMARK
  280 FORMAT (4X,'ENDMK',I1,4X,'''MARKER''',17X,
     1                              '''SEPEND''',33X)
C
C  OUTPUT YIJ COLUMNS.
C
      DO 290 I=1,II
      DO 290 J=1,JE
```

```
      CA=-DELHE(J)/ES(2)/ES(3)/SCALE1*SCALE2
      CB=DELD(I)*(1.0-PSO)*DELHE(J)
      CALL PRNT (2,Y,I,J,0,RA,I,0,CA,1,RD,3,0,CB,1)
      CALL PRNT (2,Y,I,J,0,RE,I,0,CA,1,RF,I,J,ONEM,0)
      CA =-1.0/DELY(I,J)
      CALL PRNT (1,Y,I,J,0,RG,I,J,CA,1,BLK,0,0,0,0,0)
  290 CONTINUE
      II2=(II/2)*2
      II1=II-II2
C
C  OUTPUT XN COLUMNS.
C
      DO 305 N=1,NN
      N1=N/10
      N2=N-10*N1
      CALL PRNT(1,X,N1,N2,0,OB,0,0,OBJ(IJ+N),1,BLK,0,0,0.,0)
      IF (II .EQ. 1) GO TO 296
      DO 295 I=1,II2,2
      IP1=I+1
  295 CALL PRNT (2,X,N1,N2,0,RB,I,0,W(I,N),1,
     1                           RB,IP1,0,W(IP1,N),1)
      IF (II1 .EQ. 0) GO TO 298
  296 CALL PRNT (1,X,N1,N2,0,RB,II,0,W(II,N),1,BLK,0,0,0.,0)
      IF (II .EQ. 1) GO TO 301
  298 DO 300 I=1,II2,2
      IP1=I+1
  300 CALL PRNT (2,X,N1,N2,0,RC,I,0,D(I,N),1,
     1                           RC,IP1,0,D(IP1,N),1)
      IF (II1 .EQ. 0) GO TO 303
  301 CALL PRNT (1,X,N1,N2,0,RC,II,0,D(II,N),1,BLK,0,0,0.,0)
  303 CALL PRNT (2,X,N1,N2,0,RD,1,0,PR(N),1,RD,2,0, A(N),1)
  305 CONTINUE
C
C  OUTPUT WI AND DI COLUMNS.
C
      DO 310 I=1,II
      CA= -DS(2)*DELD(I)*(1.0-PSO)
      CB=ONE/SCALE1*SCALE2
      CALL PRNT (2,WO,I,0,0,OB,0,0,CA,1,RA,I,0,CB,1)
      CALL PRNT (1,WO,I,0,0,RB,I,0,ONEM,0,BLK,0,0,0,0,0)
  310 CONTINUE
      CA=1.0/SCALE1
      DO 315 I=1,II
  315 CALL PRNT (2,DO,I,0,0,RC,I,0,ONEM,0,RI,I,0,CA,1)
C
C  OUTPUT S0,S1,S2 COLUMNS.
C
      CA=1.0/ES(2)/SCALE1*SCALE2
      CB=-1.0/DELS(1)
      DO 325 I=1,II
  325 CALL PRNT (1,S,0,0,0,RE,I,0,CA,1,BLK,0,0,0,0,0)
```

```
      CALL PRNT (1,S,0,0,0,RH,1,0,CB,1,BLK,0,0,0.0,0)
      CA=-1.0/DELS(2)
      CALL PRNT(2,S,1,0,0,RD,1,0,ONEM,0,RH,2,0,CA,1)
      DO 330 I=1,II
      DO 330 J=1,JE
  330 CALL PRNT (1,S,2,0,0,RF,I,J, ES(3),0,BLK,0,0,0.0,0)
      CA=-1.0/DELS(3)
      CALL PRNT (1,S,2,0,0,RH,3,0,CA,1,BLK,0,0,0.0,0)
      CA=-OS(3)/ES(3)
      CALL PRNT (2,U,0,0,0,OB,0,0,CA,1,RD,3,0,ONEM,0)
C
C  OUTPUT DIM,TIM,ZIM,MIM,NIM,AND KIM COLUMNS.
C
      DO 404 I=1,II
      WRITE (H,255) DO,I,K0,K0
      DO 404 M=1,MMDO
      CALL PRNT (1,DO,I,M,0,RI,I,0,-DELX(M),1,BLK,0,0,0.0,0)
  404 CALL PRNT (2,DO,I,M,0,RJ,I,0,-DELOG(M),1,
     1                          RK,I,0,-DELOG(M),1)
      DO 408 I=1,II
      WRITE (H,255) T,I,K0,K0
      DO 408 M=1,MMT
  408 CALL PRNT (2,T,I,M,0,RA,I,0,-DELX(M),1,
     1                         RJ,I,0,DELOG(M),1)
      DO 412 I=1,II
      WRITE (H,255) Z,I,K0,K0
      DO 412 M=1,MMZ
  412 CALL PRNT (2,Z,I,M,0,RE,I,0,-DELX(M),1,
     1                         RK,I,0,DELOG(M),1)
      DO 416 I=1,II
      WRITE (H,255) WM,I,K0,K0
      DO 416 M=1,MMWM
  416 CALL PRNT (2,WM,I,M,0,RJ,I,0,-DELOG(M),1,
     1                          RL,I,0,-DELX(M),1)
      DO 420 I=1,II
      WRITE (H,255) WMP,I,K0,K0
      DO 420 M=1,KMNUM
  420 CALL PRNT (2,WMP,I,M,0,RL,I,0,WMM(M)*SCALE2,1,
     1                           RM,I,0,-KMM(M)*SCALE2,1)
      DO 422 I=1,II
      WRITE (H,255) KM,I,K0,K0
      DO 422 M=1,MMKM
  422 CALL PRNT (2,KM,I,M,0,RK,I,0,-DELOG(M),1,
     1                          RM,I,0,DELX(M),1)
      NMARK=NMARK+1
      WRITE (H,280) NMARK
C
C OUTPUT THE NON-ZERO ELEMENTS IN THE RIGHT HAND SIDE.
C
      WRITE (H,530)
  530 FORMAT ('RHS',77X)
```

```
      DO 532 I=1,II
  532 CALL    PRNT (1,R000,0,0,0,RA,I,0,ONE,0,BLK,0,0,0.0,0)
      CALL PRNT (1,R000,0,0,0,RD,2,0,AMAX,1,BLK,0,0,0.0,0)
      DO 534 I=1,II
  534 CALL PRNT (1,R000,0,0,0,RE,I,0,ONE,0,BLK,0,0,0.0,0)
      DO 536 I=1,II
  536 CALL PRNT (1,R000,0,0,0,RI,I,0,ONE,0,BLK,0,0,0.0,0)
      DO 537 I=1,II
  537 CALL PRNT (1,R000,0,0,0,RL,I,0,ONE,0,BLK,0,0,0.0,0)
      DO 538 I=1,II
      CA=SIO*SCALE2-1.0
  538 CALL PRNT (1,R000,0,0,0,RM,I,0,CA,1,BLK,0,0,0.0,0)
C
C  OUTPUT THE BOUNDS ON THE VARIABLES.
C
      WRITE (H,540)
  540 FORMAT ('BOUNDS',74X)
      DO 545 I=1,II
      DO 545 J=1,JE
      MM=NUMY(I,J)
      DO 545 M=1,MM
  545 CALL PRNTB (Y,I,J,M,ONE,0,1,0)
      DO 550 I=1,II
      DO 550 J=1,JE
      YMAX=(E(I,J)-EMIN)/LUM
  550 CALL PRNTB (Y,I,J,0,YMAX,1,1,0)
      DO 555 I=1,3
      MM=SNUM(I)
      J=I-1
      DO 555 M=1,MM
  555 CALL PRNTB (S,J,M,0,ONE,0,1,0)
      CALL PRNTB (S,1,0,0,PMAX,1,1,0)
      IF (PMIN .LT. 0.001) PMIN=0.001
      CALL PRNTB (S,1,0,0,PMIN,1,2,0)
      IF (UMIN .LT. 1.0) UMIN=1.0
      CALL PRNTB (U,0,0,0,UMIN,1,2,0)
      CALL PRNTB (DO,II,MMDO-1,0,ONE,0,1,1)
      CALL PRNTB (T,II,MMT-1,0,ONE,0,1,1)
      CALL PRNTB (Z,II,MMZ-1,0,ONE,0,1,1)
      CALL PRNTB (WM,II,MMWM-1,0,ONE,0,1,1)
      CALL PRNTB (WMP,II,KMNUM,0,ONE,0,1,1)
      CALL PRNTB (KM,II,MMKM-1,0,ONE,0,1,1)
      WRITE (H,600)
  600 FORMAT ('ENDATA',74X)
      IF (H .LE. 7) GO TO 999
      ENDFILE H
      REWIND H
  999 STOP
      END
```

```
****************************************
* THE DATA FOR THE MODEL WITH STORAGE *
****************************************

    4     4     4    16    54
  92.+0   91.+0   90.+0   92.+0
  1.0+0
   6.+0    6.+0    6.+0    6.+0    1.+0
   6.+0    6.+0    6.+0    6.+0
 0.40+0  0.90+0  7.00+0
 70. 15. 70. 17. 70. 19. 70. 21. 70. 23. 70. 25. 80. 15. 80. 17. 80. 19. 80. 21.
 80. 23. 80. 25. 90. 15. 90. 17. 90. 19. 90. 21. 90. 23. 90. 25.100. 15.100. 17.
100. 19.100. 21.100. 23.100. 25.110. 15.110. 17.110. 19.110. 21.110. 23.110. 25.
120. 15.120. 17.120. 19.120. 21.120. 23.120. 25.130. 15.130. 17.130. 19.130. 21.
130. 23.130. 25.140. 15.140. 17.140. 19.140. 21.140. 23.140. 25.150. 15.150. 17.
150. 19.150. 21.150. 23.150. 25.
 0.04+0  300.+0   25.+0   50.+0  0.10+0
  1.0+6   3.5+6   5.0+0   0.0+0  500.+3   0.0+0   1.0+6
 1.00+0  0.00+0  50.0+0  1.00+7  1.00+0
 0.80+0  0.50-3  15.0+0  5.00+5  1.00+0
 0.80+0  0.30-3  15.0+0  5.00+5  1.00+0
  2.0+0  146.+0  0.26+0
```

THE PROBABILITY DISTRIBUTIONS OF WIND SPEEDS AND THE DEMAND DATA ARE THE SAME AS THOSE LISTED IN THE DATA FOR THE MODEL WITHOUT STORAGE.