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IN EIGHTH GRADE MATHEMATICS

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CHAPTER I

INTRODUCTION AND PROBLEM STATEMENT

The recognition of individual differences among students, and the provision of appropriate educational experiences which take account of such differences, comprise a broad topic which has concerned educators in all subject areas. The National Society for the Study of Education (1950, 1958, 1962, 1968) has devoted one part of each of four of its yearbooks to this topic during the post-World War II era. The most immediate, although certainly not the only, aspect of individual differences is the relation between learning and age. According to McGeoch and Irion (1952), the relation between learning and age is a function of several conditions, including (1) material learned and method of presentation and measurement, (2) amount of transfer from previous learning, (3) transfer from practice at formal learning, (4) mental age, and (5) motivation and personality characteristics.

Johnson and Rising (1967, p. 180) recognize the following dimensions along which students in a given mathematics classroom may vary:

1. Mental ability, ability to reason or think reflectively, ability to solve problems.

2. Mathematical ability, ability to use symbols, ability to do logical reasoning, ability to compute.

3. Knowledge of mathematical concepts, structures, and processes.

4. Motivations, interests, attitudes, appreciations.

5. Physical, emotional, and social maturity of the learner.

6. Special talents or deficiencies such as creativity, lack of reading skill, or retention span.

7. Learning habits, self-discipline, attention span, and organization of written work.

The focus of the research reported herein involves learner attributes which appear to fall under items (6) and (7) above. The research emphasizes material learned and method of presentation as a condition of learning, as cited by McGeoch and Irion.

Articles in The Mathematics Teacher and various yearbooks of the National Council of Teachers of Mathematics have implicitly or explicitly maintained that learning of mathematics takes place through many, if not all, of the senses. Two of the yearbooks, the eighteenth and thirty-fourth, are devoted entirely to instructional aids.

The differences in learners' preferences for the visual or verbal mode of instruction, as a concern for psychologists and educators, is not a new phenomenon. Discussion of this preference, and the possibility of relating it to instructional procedures, dates back to Francis Galton, (1822-1911) according to Waugh (1967). In her research review, Balmuth (1968) states that research on sensory modality began at least eighty years ago. During the past ten or twelve years, considerable effort has been expended toward the goal of identifying and capitalizing upon individual modality preference. The main group of learners with whom study has been concerned are those who are normal in mental ability, sensory acuity, and emotional stability, but who receive and process data in some idiosyncratic manner. (Waugh, 1971, p. 3).

Two additional expository articles may be cited as evidence that the modality concept is part of an active research area. Yamamoto (1969) notes that the matter of stimulus mode and sense modality has implications for several areas of education. Included among these areas would be the preparation of textual materials, utilization of audio-visual media, foreign language instruction, teaching of the culturally deprived and mentally retarded, and remedial reading. The author provides examples under each area.

Wepman (1967) discusses the differences among children in their use of specific modalities for learning and the necessary establishment of perceptual bases for conceptual learning. He cites clinical evidence of preference for certain input pathways on the part of children with no demonstrable neurological impairment, and stresses that the importance of the understanding of modality learning lies in the area of assisting the under-achiever.

Tests which have been devised for the purpose of detecting learners' modality preference appear to place emphasis upon perceptual aspects. In his review of research on visual and auditory modalities, Jones (1972) finds fault with the existing methods of identifying individual modality preference:

> . . . High on the list of priorities, then, should be the development of a modal preference test. Quite likely, such a test would need to consider the conceptual as well as the perceptual aspects of learning. A valid test of modal preference would also do much to strengthen the research in this area. (Jones, 1972, p. 33).

Statement of the Problem

As will be noted in the literature review, the efforts expended toward assessing the effects of verbal versus visual presentations, and toward identifying individual modality preference, have been greatly concentrated in the area of reading. The research which is reported herein is concerned with the problem of modality preference and visual versus verbal presentations in mathematics, and is directed specifically to the following questions:

1. Do students learn mathematics better when visual methods of presenting concepts are used extensively by the instructor than when such devices are not used?

2. Can interaction between selected learner aptitudes and instructional mode be observed?

3. To what extent can students with considerable modality preference be identified by differences in their performance following visually-rich instruction and following verbally-rich instruction?

CHAPTER II

REVIEW OF RELATED LITERATURE, THEORETICAL FRAMEWORK, AND HYPOTHESES

The research which has been done in areas close to that of the present study lends itself to categorization as follows: (1) basic experimental research on modality which pertains directly to the study of reading; (2) research which is somewhat classroom-oriented, and relates to a subject matter other than mathematics; and (3) a type similar to category (2), with mathematics as the subject matter.

Basic Experimental Research on Modality as Related to Reading

Problems relating to reading cannot be completely separated from research concerning learning in any academic area. Writers such as Earp (1971) point out the connection between success in learning mathematics and facility in the verbal skills of reading. Earp's article dealt specifically with pupils' vocabulary attainments in relation to the readability level of the textbooks from which they are expected to learn. Too often, the readability level is too high for a large fraction of a given class.

Jones (1972) has presented a very useful review of research dealing with modal relationships, largely auditory versus visual. Modality research has directed attention to three factors: intersensory transfer, intersensory perceptual shifting, and modal preference. Under the latter, research may be subcategorized into two types: (1) studies which compare listening and reading as input channels for the comprehension of analogous verbal and printed materials, and (2) studies which investigate individual modal preference as a factor in learning to read. The latter involve efforts to ascertain the modal preference of each subject.

Much of the basic experimental research in this area involves paired associate tasks. Williams (1970) worked with subjects at grade levels two, four, six, eight, and ten, using eight-pair lists of familiar nouns. Each subject learned two lists, one presented visually and the other aurally. Performance on tests over the visually learned material was superior to that over the aurally learned, and the anticipated improvement with increasing grade level was found.

Gaeth (1967 and 1966) conducted studies of paired associate learning in children with normal hearing and in those with severe hearing losses. The verbalness of the material and its meaningfulness in terms of the learners' experiences, were variables which were manipulated in material presented auditorily, visually, and audiovisually. The first of the studies indicated that meaningfulness was the more important independent variable. In the later study, the dimensions of meaningfulness were explored, as was the relationship to the students' choice of the audio or visual modes of presentation. It was concluded that meaningfulness is not to be viewed in an absolute fashion, but is always relative to the context in which the material is to be placed, irrespective of mode.

Ringler and associates (1971) used the New York University Modality Test to identify the modality preferences of a sample of first graders. These were then randomly assigned to an additional period of instruction via a modality of one of four types, or to a control group which received no extra instruction. Results were that the four experimental groups differed significantly from the control, but not from one another. There was no significant difference between the group whose treatment matched their preference and the group whose treatment did not.

Wheeler (1972) investigated the effect of a distinct visual stimulus on the learning of a four-component chain of nonsense syllables by young children. The speed of acquisition was unaffected, but errors were reduced. Thalberg's study (1964) dealt with the relative effectiveness of reading and listening in the learning of verbal material. Neither the ability level of the subjects nor the difficulty of the task was differentially related to either channel of presentation. Nazzaro and Nazzaro (1970) compared auditory and visual learning of Morse code triads with respect to short term retention. The task was presented in a manner which provided for close similarity between the visual and the oral stimuli. It was found that auditory patterns were learned with significantly greater rapidity. In an experiment by Duncan and Hartley (1969), college undergraduate subjects were presented a maze either visually or verbally, then asked to reproduce it in each of the visual and verbal modes. Reproduction of the maze in the same mode as it was presented was done significantly better.

Modality Research Involving Specific Subject Matter Other Than Mathematics

A study by Allen and associates (1970) investigated visual-verbal presentation modes when used for instruction in different types of learning tasks with learners of differing mental abilities. Within each of seven social studies and science content areas, five parallel experiments were conducted, each for a different learning objective or task. For each content area, stimulus sequences representing seven levels on a visual-verbal continuum were designed. It was found that motion picture sound and still picture treatments were superior to others for four of the five learning objectives. The five parallel experiments failed to verify a hierarchical pattern relating the learning tasks.

Gagné and associates (1965) did two studies using visual representations for science instruction. The first study tested the hypothesis that the use of pictorial instruction would produce higher correlation between results of visual aptitude tests and results of learning tests, and that the use of verbal instruction would produce higher correlation between results of verbal aptitude tests and results of learning tests. Aptitude measures used were (1) spatial relations,

(2) verbal reasoning, (3) abstract reasoning, and (4) intelligence. The hypothesis was not supported. The second study used pictorial representations in review sessions, dealing with the topic of mechanical advantage. Students who reviewed by pictorially presented materials retained and transferred information significantly better than those who did not review at all. Adding additional pictorial examples did not improve retention but led to better transfer than when only the original materials were reviewed.

Dwyer (1967 and 1968) carried out two experiments using visual illustrations to complement different types of instruction. In the first study, the visual illustrations complemented programed instruction with ninth graders as subjects. Various groups were given instruction which was either verbal only, programed only, or programed with either (1) line drawings as visual illustration, (2) drawings, or (3) photos. Verbal presentations were found to be superior for learning effectiveness, economy, and simplicity of production. Retention tests showed superiority for the line illustrations, however. The second study concerned the use of visual illustrations to complement oral instruction on television. The subjects were college students and the subject matter dealt with the human heart. The visual illustrations, presented via slides, were effective when the objectives were measured by a drawing test, but were unnecessary and even distracting with respect to other objectives.

In his study, Filep (1967) related learner characteristics to media stimuli and programing sequences. Three visual stimulus modes and three audio stimulus modes were used with either linear or branching programs. The subjects were eighth grade students in general science and the subject matter was classified as either non-concrete, concrete, or action process. The chief outcome of interest here was that children of low I.Q. who were non-white and from the three lowest occupational groups learned best with the branching, nonverbal, sound treatments.

Gropper (1965) conducted two multivariate experiments which used programed science demonstrations presented over television and programed verbal materials on the same subject matter given in self-paced booklets. The subjects were at the eighth grade level. Conclusions were that in the visual presentation, students performed better by actively practicing responses to features of the presentation than by remaining passive and responding afterward. When both visual and verbal modes were used in instruction, employing the visual mode before the verbal gave the best results. Short instructional sequences lend themselves to an integrated fixed-paced visual presentation with a self-paced verbal program.

Research Concerning the Interaction of Aptitude, Treatment, and Achievement in Mathematics Instruction

The interaction of individual modality preferences with classroom instruction, which is at the heart of this research, lies within the purview of aptitude-treatment interaction (ATI) research. Studies involving mathematics instruction which investigate only treatment-achievement interaction, provided they involve some sensory concern, are also deemed related.

Becker (1970) has suggested a paradigm for the presentation and interpretation of ATI research; he attributes the notion to L. J. Cronbach. The paradigm involves the simultaneous presentation of the regression lines of two or more treatment groups, showing the regression of outcome on aptitude. Values of the aptitude measure corresponding to points where the regression lines cross may then serve as bounds for assigning learners to the most desirable treatments.

The above paradigm was an important feature of at least two of the studies which were examined. One of these was the dissertation of Marvin Trask (1972). The study was concerned with the interaction of concrete manipulative versus non-manipulative experiences with aptitude, at the third grade level. A comprehensive background of learner characteristics was obtained through administration of the Otis Lennon Mental Ability Test, the Stanford Achievement Test, and the Torrance Test of Creative Thinking. Only for the S.A.T. subscale Arithmetic Computation was significant interaction found, and this only for the computational form of the learning test. Higher ranking pupils on the above subscale benefited more from the manipulative experience.

A study by Hancock (1973) investigated the interaction between the personological variables of sex difference, certain mental factors and two methods of presenting concepts and principles associated with a made-up mathematical relation having the properties of a linear order relation. This relation was studied by subjects from a college population by either a figural or a verbal program. No significant interactions of the instructional modes with any of the mental factors were found, although analysis of the main effect of mode on achievement clearly favored the verbal mode.

The mental factors referred to above were determined by a battery of nine tests developed by Guilford, and are related to Guilford's Structure of Intellect model. (Guilford and Hoepfner, 1971). This model provides an identification of abilities by an ordered triple (x,y,z), where x denotes a categorization as to content, y as to operation, and z as to products. The number of levels of the three dimensions are, respectively, four, five, and six. Some of the studies referred to below, in addition to that of Hancock, employ this model as a generator of independent variables.

An investigation by Carry (1968), and a related one by his student, Eastman (1972), examined the interaction of spatial visualization and general reasoning abilities with treatments. The subject matter was the topic of quadratic inequalities. Both studies used the Necessary Arithmetic Operations test for reasoning ability. Carry used the Paper Folding Test and Eastman the Abstract Reasoning Test to measure spatial visualization. Subjects of both studies were geometry students. Both studies employed either a graphical or analytic mode. The later research had some refinements under each type. Carry found no significant interaction, using learning scores as criterion, but did find significance when transfer was used as criterion. Eastman did find significant interaction with respect to learning scores, at the .05 level.

Davis (1967) studied aptitude-treatment interaction in connection with the derivative of an algebraic expression and the multiplication of vectors. He drew upon Guilford's model in measuring certain mental qualities. A group of college undergraduates and one of tenth graders learned from either a symbolic or a semantic program treatment. The study indicated that there does exist interaction between ability and the content form, at least for the topics studied. Both symbolic and semantic factor tests were significant predictors of achievement on semantic learning materials. Only the symbolic factor tests significantly predicted learning of the symbolic learning materials.

Dorminey (1972) investigated the interaction of the aptitudes of inductive reasoning ability and deductive reasoning ability with an Inductive-Deductive instructional sequence and a Deductive-Expository approach. The subjects were eighth graders and the topic dealt with introductory counting principles. The tests used to measure the selected aptitudes were the "Letter Sets" and "Inference" from the Kit of Reference Tests for Cognitive Factors. The first of these measured inductive and the second deductive reasoning ability. Tests for significant interaction between each aptitude and the treatments were also made, using linear regression analysis. The only significant interaction was between inductive reasoning ability and treatments on the retention test.

Of considerable interest in relation to the present research, in spite of the subject matter's not being specified in the abstract, is a dissertation by Huebner (1969). He examined the interaction between patterns of individual differences in sensory modalities of pupils and methods of classroom instruction. Subjects were second graders. The auditory and visual reception subtests of the Illinois Test of Psycholinguistic Abilities were used to measure modality preference. Interactions between sensory modality and mode of instruction and also between sensory modality and mode of test were sought. No significance was found.

Atkinson (1972) explored the effect of different geometrical surfaces and different modalities of perception upon the ability of young children (pre-kindergarten) to match geometrical models. The basic task required of a subject was to perceive two selected models and to decide whether or not they were the same shape. The modalities used included vision, touch, and combinations of these. Significant effects were attributable to both surface and modality as factors, but interaction between these was not significant.

Behr (1970), in a summary paper based on his dissertation, reported an investigation of the interaction between structure of intellect factors and two methods of presenting modular arithmetic. The subjects were students enrolled in a college course for prospective elementary teachers. The programed material was presented through either a figural-symbolic or a verbal-symbolic mode. Modest support for the hypotheses regarding interaction was found in the sense that significant interaction occurred between five of the selected fourteen mental factors and the two methods of instruction. In four of the five cases, the interaction was consistent with the expectation that figural factors would be better predictors for learning and retention by figural presentation, and semantic factors for semantic presentation.

Basis for the Present Research

The questions posed in Chapter I point to the investigation of the main effects of mode of presentation on achievement, the interaction of learner aptitudes and instructional mode, and the possibility of identifying learners having presentation modality preference. Assessment of various effects is based upon measurement of learning under the two modes of presentation.

Some of the examined research of a basic experimental nature was concerned with main effects only, and some examined relationships with mental factors of the subjects. Virtually all of it was concerned with possible implications for reading instruction. Jones (1972) refers to a review by Day and Beach (1950) of thirty-four studies dealing with listening and reading as input channels, which did not take into account learner modality preferences. A list of findings gleaned from these studies includes some contradictory results. Jones attributes the contradictions to individual differences. He cites McGeoch and Irion's identification of the following major variables as the basis for such differences: (1) practice, (2) chronological age, (3) type of material to be learned, and (4) mode of apprehension. (1952). The latter authors observe that

> . . . The subtlety and richness of the representative and reactive devices in the human subject permit so ready a translation of the material into other terms than those in which it is presented that mode of stimulation may be unimportant, except in cases of very strong habitation to a particular mode. (1952, p. 482). (underlining mine.)

It is the closing phrase in the above quote, and the existence of individuals possessing such a strong habituation, which underlie the present research, and many studies done before. Indeed, Wepman (1971) states that modality dominance tends to be overcome in most children by the age of nine. But again, the word is "most".

In the introduction to this study, a quote from Jones (1972) recommends that modality preference identification tests incorporate the conceptual with the perceptual. Hashem (1971) investigated the relationship between a nonverbal and a verbal learning profile. The results suggested that perceptual aspects may be overemphasized in the diagnosis and remediation of reading difficulties. Neither the verbal battery nor the nonverbal battery presented a differentiated modality learning profile. This information, together with McGeoch and Irion's inclusion of type of material to be learned as a major variable underlying individual differences, suggest that modality preference ought to be investigated within the framework of several different subject matters.

Of the cited studies dealing with subject matter other than mathematics, those by Dwyer and by Gropper were concerned with main effects of visual and verbal materials. The remaining ones incorporated learner characteristics in the study. It is interesting to note that Allen and associates (1970) classified the items of their criterion tests according to a hierarchical scheme. Trask (1972), dealing with mathematics instruction, classified his criterion tests as either computational or problem solving, but did not imply a hierarchy. Trask's research, and all the other studies on mathematics education, were of the ATI type.

With regard to the tests aimed at the identification of modality preference, it has been noted that questions have been raised as to the perceptual emphasis found in some such tests. The conceptual aspects need to be considered. There are research precedents for the practice of the determination of preference by researcher-designed procedures. In a dissertation by Montgomery (1972), aptitude was defined in terms of the learners' ability to master specific concepts associated with a certain unit of material. The determination of this ability was done through a procedure consisting of a pretest, a brief instructional treatment, and a posttest on the unit. Learners were categorized as to their ability, and this categorization served as an independent variable in the remainder of the study. Levin (1971) did much the same thing. Montgomery's study involved teaching a mathematical topic to second and third graders and Levin's dealt with the learning of paired associates by pupils from kindergarten and first and third grades.

Hypotheses

The present research is directed toward testing the following hypotheses in connection with the three questions posed in the introduction:

I. A group of students which has been taught a unit of material using visually-rich methods will score higher on a test over the unit than will another group which has been taught the unit using verbally-rich methods.

II. There will be interaction between mode of instruction and each of several aptitude measures from the Differential Aptitude Tests, in the sense that individuals ranking lower on a given subscale will benefit relatively more from the visually-rich instruction.

III. Using a discriminant analysis procedure, it will be possible to classify students as visually-oriented, verbally-oriented, or neutral, so as to agree with their classification by a process based simply on differences between their scores on the visual and the verbal unit.

The statistical null hypotheses are as follows:

Ho I: A group of students which has been taught a unit of material using visually-rich methods will score no higher on a test over the unit than will another group which has been taught the unit using verbally-rich methods.

Ho II: There will not be interaction between mode of instruction and each of the aptitude measures from the Differential Aptitude Tests; that is, individuals ranking lower on a given subscale will not benefit relatively more from the visually-rich instruction.

Ho III: The classification of subjects by a discriminant analysis procedure as visually-oriented, verbally oriented or neutral, will agree with their classification by a process based simply upon differences between their scores on the visual and the verbal unit, to no greater an extent than that to be expected by chance.

CHAPTER III

DESIGN

Sampling and Treatment Procedures

The unit of analysis in the present research is the eighth grade student in mathematics. The research was conducted at Patrick Henry Junior High School, Sioux Falls, South Dakota. Ten sections averaging twenty-nine students each were selected for the research, with a view toward minimizing the number of teachers involved. The students had been randomly arranged in fourteen sections following the removal of the scholastically highest ranking sixty individuals. The sample excluded approximately the highest ranking one-eighth of the class because this group was being taught different subject matter. Classes were assigned to one or the other of the two groups corresponding to each of the treatment sequences described below, subject to the condition that five classes comprised each group.

The subject matter consisted of two topics, based on the text Modern School Mathematics, Structure and Use, Book 8, by Ernest R. Duncan and others. (1972). Topic I was elementary discrete probability. Topic II followed from one to three days after Topic I; the intervening material was not involved in the research. Topic II concerned the integers, their properties, and the operations of addition and subtraction. From four to five class periods, plus testing time, were allowed for each topic.

One of the groups was taught Topic I through a visually-enriched, but verbally limited, approach, and then Topic II through a verbally enriched but visually limited approach. The other group was taught the topics in the same order, but with reversed sensory emphasis. By visually-enriched, it is meant that the presentation of each day's lesson was centered around from one to four models or projected graphical configurations which did not appear in the students' text. By verbally-enriched, it is meant that the concepts or rules being taught were further illustrated through verbal examples provided to the teachers. In the verbal emphasis mode, the teachers were instructed to deal with the visual illustrations in the text only as referred to by the written material.

Illustrations

In that which follows, examples of enrichment materials are shown, one each of a visual and a verbal type, within each of the two topics. Descriptions of all of the materials appear in Appendix A.

Topic I - Visual

The example pertains to the lesson the concept of odds. The transparency used, which is duplicated in figure 1, deals with the concept without referring to a specific numerical problem. In all such problems, however, there is a comparison of a certain subset, here symbolized by shading, with the rest of the sample space, i.e., the complement. The numerical valuation of the subset and of its complement, indicated by n(   ), is of course simply a count of elements in

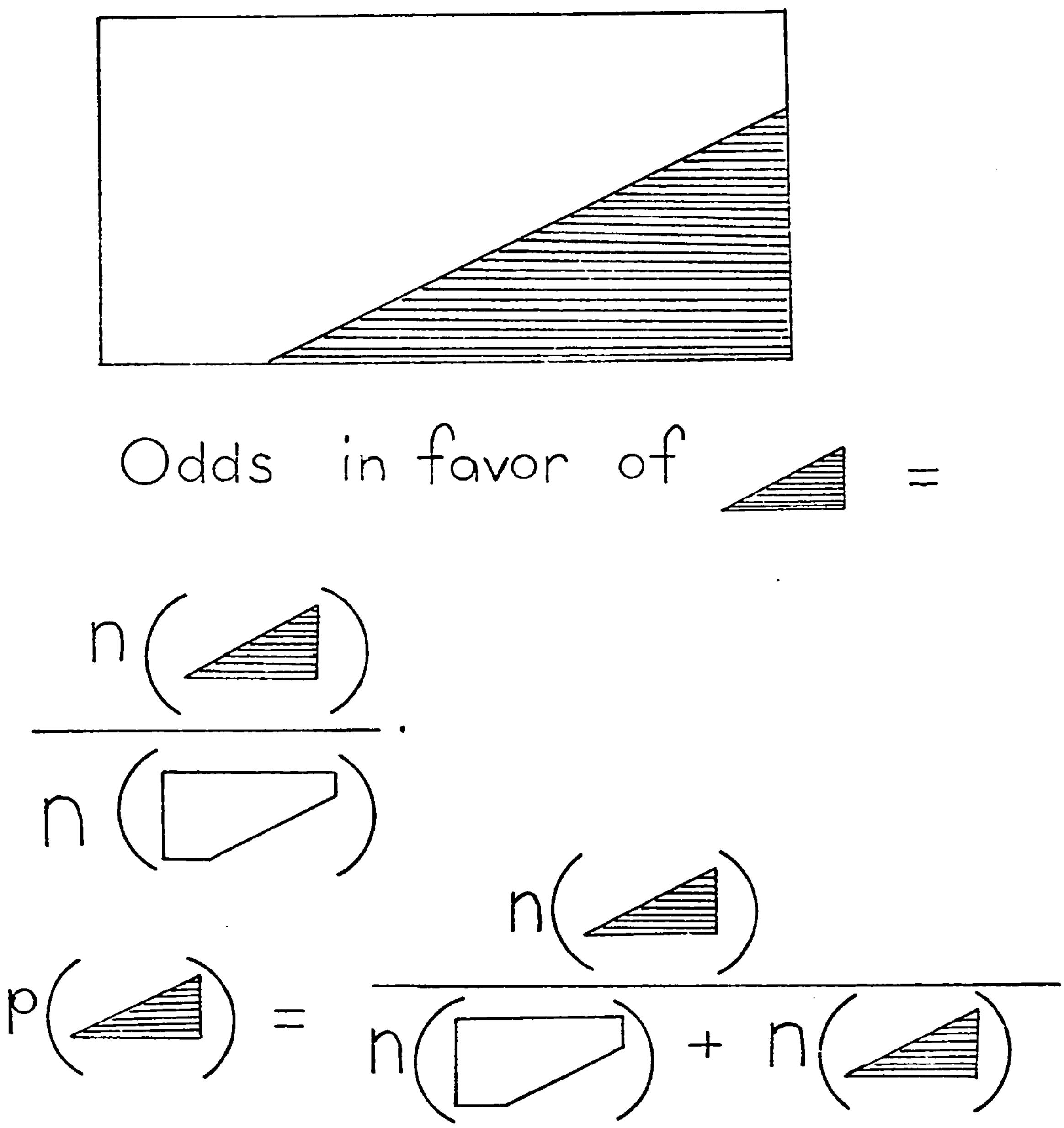


Figure 1

any specific problem. However, the use of shaded regions enables the presentation of the concept without reliance on a specific numerical example.

Topic I - Verbal

The concept dealt with is again odds. Three statements were made, in the form below:

In one throw of a die, the odds in favor of an outcome of 6 are 1/5. The odds against an outcome of 6 are 5/1.

"Odds against" is always the reciprocal of "odds in favor", so long as "odds in favor" is greater than zero.

When an event is as likely as not to happen, the odds are in favor of it are 1/1.

Topic II - Visual

This visual deals with the subtraction of a negative integer. Figure 2 is divided into three panels. The top panel, containing the same information as did one of the transparencies making up this visual, introduces positive and negative "counters" which respectively resemble the right and left halves of the numeral O. Thus, when placed together, they serve to "cancel" each other. The bottom panel shows five positive counters, which also appeared on one transparency. The middle panel shows the appearance of the figure when the latter transparency is overlaid with another containing two negative counters. At this point, the integer $^{+}3$ is displayed. Lifting the overlay "takes away" $^{-}2$ from $^{+}3$, leaving $^{+}5$.

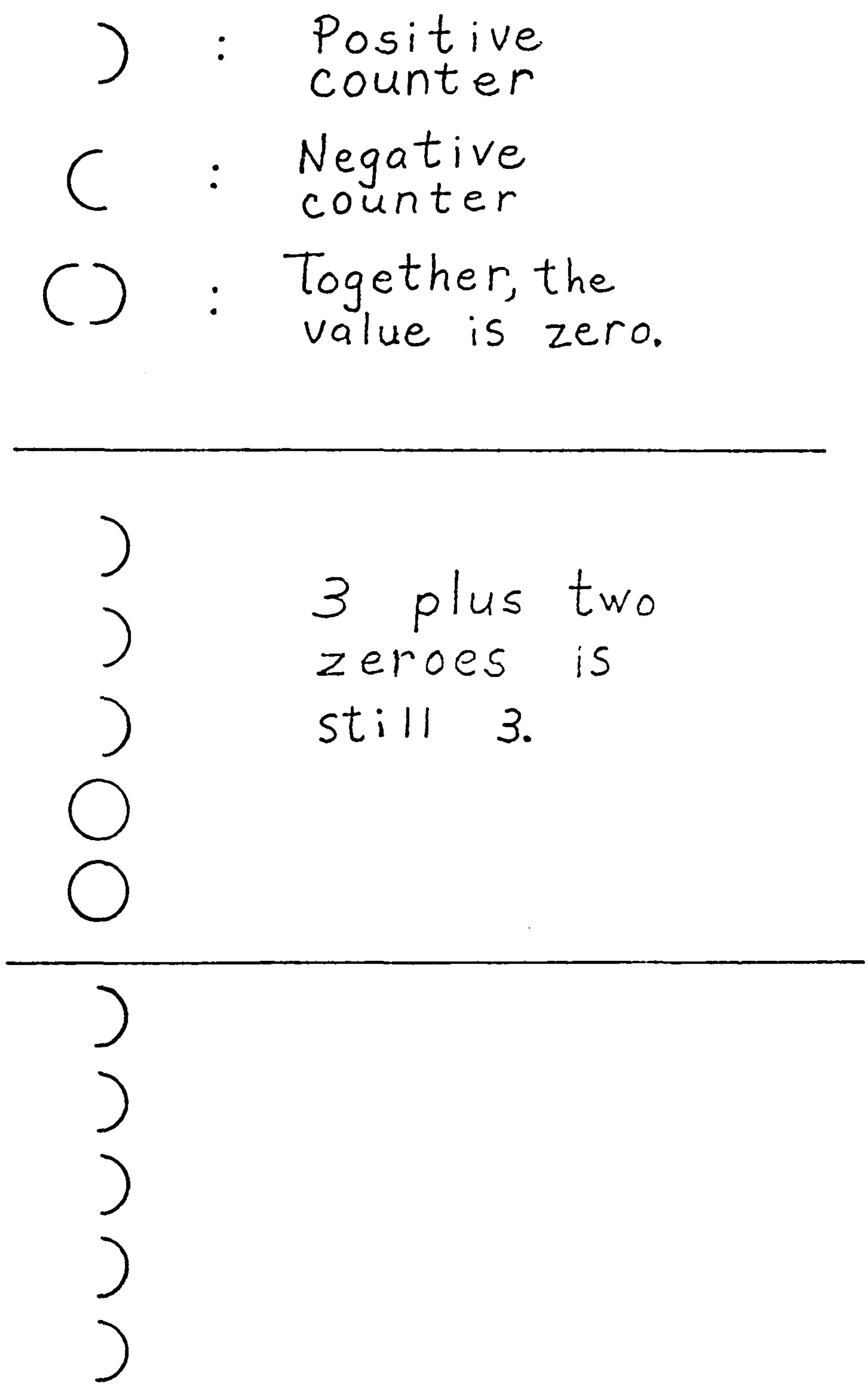

Positive counter
Negative counter
Together, the value is zero.
3 plus two zeroes is still 3.

Figure 2

Topic II - Verbal

The teaching of the subtraction of a negative integer was enriched verbally with two examples as follows:

Suppose that a company owns five grocery stores, and that one of them has been losing money for several months. If this store is sold, the effect on the total profits of the company would be just like the subtraction of a negative integer from another integer.

A large balloon filled with hot air or gases, before making a flight, may be held down with sand bags. The balloon rises when enough of the bags have been thrown out. Throwing out each bag is like the subtraction of a negative integer from another integer.

Testing and Statistical Processing

Posttests for each of the two topics were prepared by the author. The scores from these tests produced the dependent variables of the study.

Scores on the following two standardized test batteries were available: (1) Iowa Test of Basic Skills (ITBS), of which the total score on the subtests Arithmetic Concepts and Arithmetic Problem Solving was used, and (2) the Differential Aptitude Tests (DAT). The nine subscales of the latter were all used in the study. They are: (1) Verbal Reasoning, (2) Numerical Ability, (3) Verbal Reasoning + Numerical Ability, (4) Abstract Reasoning, (5) Space Relations, (6) Mechanical Reasoning, (7) Clerical, (8) Language Usage, Spelling, and (9) Language Usage, Sentences.

The null hypothesis Ho I, with reference to the statement of hypotheses at the end of Chapter II, was tested separately for each of the two topics. The analysis of covariance was the statistical procedure used to obtain a comparison of visual and verbal treatment effects. The method provides adjusted group means; the adjustment is based upon some pertinent covariable, one which is likely to be related to the dependent variable of the study. The ITBS Arithmetic subtest score was used as the covariable in this analysis.

The ideal situation for use of the analysis of covariance is one in which subjects have been randomly assigned to treatments, so that one may assume that disturbing variables other than the covariable are uniformly distributed over the groups (Cochran, 1957, p. 262). The design of this study did not permit such assignment, but the fact that students had been randomly assigned to classes could be expected to provide at least a portion of the benefits of pure random assignment.

The power of the F test in the analysis of covariance was used to establish a sample size which was used for all of the main hypothesis tests. Power may be defined as the probability of declaring a statistic to be significant when, in fact, the population parameter which it estimates is significant (Hays, 1963, p. 270). The .05 level was selected as the minimal level at which significance would be reported.

Since analysis of covariance is an analysis of variance on adjusted group means, it was deemed reasonably appropriate to use methods of power estimation related to the analysis of variance to determine the size of a subsample which would afford a specified level of power of the test for differences in adjusted group means.

The theory of power of the F test in analysis of variance is based upon the noncentral F distribution. The distribution has three parameters: the upper and lower degrees of freedom and a non-centrality parameter ϕ, which for the two-group situation of the present research may be expressed as (Hays, 1963, p. 384)

$$\phi = \sqrt{\frac{n(\alpha_1^2 + \alpha_2^2)}{2\sigma_e^2}} \quad ,$$

where n is the (equal) group size, α_1, and α_2 the treatment effects, and σ_e the common variance of the group scores. There exist tables, as in the Chemical Rubber Company's Handbook of Probability and Statistics (Beyer, 1966, p. 248) which, for various α-levels and several sample sizes, exhibit power as a function of ϕ.

In an actual implementation of these ideas, it is necessary to assign values to α_1 and α_2, in terms of σ_e. It was decided to find a sample size which would achieve detection of a difference of means of one σ_e with a probability of .90, that is, with power = .90. This suggests setting $\alpha_1 = .5$ and $\alpha_2 = -.5$. Substitution into the equation defining ϕ leads to

$$\phi = \sqrt{.25\, n} \quad .$$

Using the power table for the two-group situation, and a significance level of .05, the value n = 23 was found to yield a power as near as possible to .90.

The null hypothesis Ho II, concerning interaction of the treatments, was tested separately for each of the two posttests, with respect to each of the nine subscales of the DAT. Becker (1970) sugfested the notion of aptitude-treatment interaction. It is based upon locating the point of intersection of two regression lines. Each of these lines corresponds to the regression of some dependent variable on a certain aptitude measure, and represents one treatment group. (See figure 3).

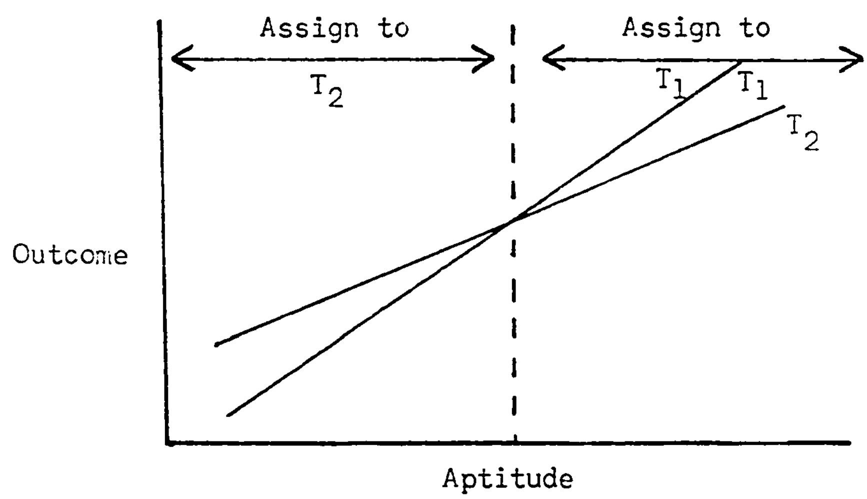


Figure 3

Assuming that the point of intersection lies within the range of the aptitude measure, a subject whose aptitude score is less than the crossover aptitude value would benefit most from treatment T_2, while one whose aptitude score is greater than the crossover value would benefit most from T_1.

As refined by Behr (1970) and employed by Trask (1972), the method calls for the rejection of the hypothesis of no interaction between treatments and a specific aptitude measure if and only if the following occur:

1. The two regression lines intersect within the range of the aptitude measure,

2. At least one of the regression coefficients is significantly different from 0, at the .05 level, and

3. The hypothesis of equal regression coefficients is rejected, also at the .05 level.

The third hypothesis, dealing with classification of learners as visually-oriented, verbally-oriented, or neutral, was tested by the extent of agreement between the classification by the upper twenty-seven per cent - lower twenty-seven per cent method, and by a discriminant analysis method. The former method classified the students on a quantity D. For each subject in the group receiving Topic I visually and Topic II verbally, D was set equal to (Topic I posttest standard score minus Topic II posttest standard score). In the other group, D equalled (Topic II posttest standard score minus Topic I posttest standard score).

This method, which was derived by Truman L. Kelly (1939), provides a split such that the difference between the means of the upper and lower groups, divided by the standard deviation of the difference, is a maximum. While the procedure was derived for use in item validation, it has been used to obtain a high-low division of a set of scores for other purposes. An example is the use of the method by Flamini in part of his dissertation study (1969).

The twenty-seven per cent having the largest D score comprised a group considered to have preference for visual presentation, the lowest twenty-seven per cent a group having preference for verbal presentation, and the middle group to have no preference. A multiple discriminant analysis procedure (Tatsuoka, 1971) was applied to the three groups, using the DAT subscale scores as classification variables.

Processing of the data was carried out by the Stepwise Discriminant Analysis program designated BMD07M, from the Biomedical Computer Program library (Dixon, 1973, p. 233). The program computes a discriminant function for each group; each such discriminant function is a linear combination of classification variables. The variables are selected one at a time in the order of the magnitude of their contribution to discriminating ability in the presence of those already in use. (Dixon, 1973, p. 242). The process continues until the potential contribution of all remaining variables to discriminating ability falls below a certain level. The terminal stage of a given program run provides a classification matrix, telling the number of subjects in each original group which the program would classify into each of the three possible groups.

The null hypothesis Ho III was tested by comparing the number of correct classifications of subjects with the number to be expected by chance. The probability model for this is the binominal, with the probability of correct classification by chance equal to 1/3.

CHAPTER IV

RESULTS

General Conditions of the Study

The study was carried out beginning January 21, 1974, at Patrick Henry Junior High School, Sioux Falls, South Dakota. The classes met four times weekly. The posttest on Topic I: Probability was given either January 29 or 30. Instruction on Topic II: The Integers began on February 5 or 6. The posttest on this topic was administered either February 13 or 14, except in the case of three of the ten sections. Because of an oversight on the part of one teacher, these three sections took the test one period later than was planned. The two posttests appear in Appendix A, each following the description of the materials used in the study.

Four teachers participated in the study. The regular teaching assignments of these teachers provided for two of them to teach three sections each and two to teach two sections each. One treatment group comprised the three sections of one teacher and the two of another. The other group then consisted of the remaining five sections taught by the other two teachers. It can be seen that only four possibilities existed as to the way the groups could be composed and be assigned to treatment sequences. This composition and assignment was done by two coin tosses.

The teachers presenting Topic I with visual emphasis and Topic II with verbal emphasis will be labelled Mrs. A and Mr. B. The former, having more than twenty years of junior high school teaching experience, taught two sections; the latter, having about ten years of experience, taught three sections. The teachers presenting the material with reversed sensory emphasis will be labelled Mr. C and Mrs. D. The former, who had five years of junior high school teaching experience taught two sections; the latter was in her second year of teaching at this level and taught three sections.

The author visited one of the classrooms of each teacher on one of the days that instruction was being done via the visual mode. Each was treating the visuals for the lesson as prescribed. Other than the late administration of one of the tests to three of the sections, the study ran smoothly. Excellent liaison, over and above personal contact with the teachers, was provided by the mathematics department chairman, who was not otherwise involved.

The total number of students on the class rolls in the ten participating sections, as of the first week of January, was 297. The first trimming of the gross sample was done on the basis of missing ITBS arithmetic scores. After the instruction had been completed and posttests given, the data from all subjects who were in any of the following categories was deleted from the study:

1. subjects who failed to take one of the posttests;

2. subjects who were absent for more than one class during either one of the topic instructional periods;

3. subjects who were present for all of the classes during the instructional period of one topic but absent for exactly one of the periods of the other topic.

At this point the sample had been reduced in size to 174; 90 were male and 84 female. The preliminary analysis described below was based on this sample. Seven more subjects were deleted because their Differential Aptitude scores were unavailable. From the resulting 167, a final sample of male and female subjects was obtained as described below in the Analysis of Covariance section.

Preliminary Analysis of Posttest Results

In order to be able to obtain a measure of the reliability of the posttests, using the split-half technique (Thorndike and Hagen, 1955, p. 128), it was necessary to determine each subject's score on the even-numbered and the odd-numbered items of each test. Table 1 presents the numerical information pertaining to these tests; for each test, a summary of item difficulty appears in Appendix A, following the test to which it pertains. The reliabilities of the probability and integers tests were .78 and .76, respectively.

The preliminary analysis was done via an author-written computer program, run on the IBM 360-40 at the University of South Dakota, Vermillion. Input-output was through the IBM 2700 series high-speed terminal at Dakota State College, Madison. All of the computer processing reported in this chapter, using both author-written and library programs, was done on the same system.

TABLE 1

POSTTEST RESULTS

No. of Items	Means			Standard Deviations			Reliability
	Odd Items	Even Items	Pooled	Odd Items	Even Items	Pooled	
Probability Test							
28	8.54	7.08	15.62	2.28	2.78	3.60	.78
Integer Test							
26	7.88	7.61	15.49	2.71	2.50	3.69	.76

Analysis of the scores of the male and female subjects separately was also provided. For the probability test, pooling the results for the odd and even items resulted in a mean and standard deviation of 16.17 and 3.69, respectively, for male subjects. For the female subjects the corresponding figures were 15.04 and 3.45. For the integers test, the male mean and standard deviation were 16.00 and 3.59, respectively. For the females, the values were 14.94 and 3.74. For each posttest, a t-test of the null hypothesis that there was no difference between the sexes in their mean scores resulted in rejection at the .01 level (Hays, 1963, p. 320). In each case the performance of male subjects was superior. Based upon these findings, all succeeding analyses were separate for the sexes.

Results of the Analysis of Covariance for Main Effects

As was indicated in Chapter III, null hypothesis Ho I, that of no superiority of the visual mode over the verbal mode of presentation, was tested by an analysis of covariance on the posttest means for the two treatment groups. This was done for each topic and separately for male and female subjects, a total of four tests.

Based upon the findings concerning power, as described in Chapter III, the analysis was carried out, using the data from 23 randomly chosen subjects within each of the two treatment groups, for each sex. The computer program used was "Analysis of Covariance", designated BMDO9V, in the Biomedical Computer Program library (Dixon, 1973, pp. 705-718). A single covariate, the ITBS arithmetic score, was employed.

A requisite to the use of the results of an analysis of covariance is the non-rejection of the hypothesis that the regression slopes of each treatment group's test scores on the covariable be equal. An acceptable significance level for this requirement is .10 (Kirk, 1968, p. 469). In this situation, the degrees of freedom were 1 and 42, leading to an F value, at the .10 level, of 2.82. The latter value was obtained from the table of the t-distribution, for a two-tailed test, at the .10 level, using the relation $t^2 = F$. With df = 1, this is appropriate, since the alternative to the null hypothesis concerning regression slopes was non-directional (Hays, 1963, p. 375).

The test of significance of differences between adjusted group means was obtained by comparing the computed F-value to the square of tabulated t-distribution, in the directional sense, at the .05 level for forty-four degrees of freedom. The critical F was 2.82.

In the investigation of the first hypothesis, as well in the following two, the posttest scores were expressed in per cent correct. The ITBS score was the percentile.

As may be seen by examining Tables 2 and 3, significant support for the hypothesis that instruction which is visually-rich yields better results than that which is verbally-rich was found only for female subjects when probability was the topic. That is, in only one of four cases was the hypothesis sustained. It should be noted also that the results of the test for males, probability, are not useable, since the regression slopes differed significantly. (This difference is indicated by the F value, for equality of slopes, at 1.847).

TABLE 2

ANALYSIS OF COVARIANCE, PROBABILITY TEST

Source of Variance	D.F.	Mean Square	F	Adjusted Means	
				Visual	Verbal
Males					
				57.766	57.760
Equality of Adjusted Cell Means	1	.031	0.000		
Zero Slope	1	3853.590	24.779		
Error	43	155.526			
Equality of Slopes	1	281.715	1.847		
Error	42	152.522			
Females					
				57.583	50.181
Equality of Adjusted Cell Means	1	623.082	4.434*		
Zero Slope	1	4978.691	35.427		
Error	43	140.534			
Equality of Slopes	1	.695	.005		
Error	42	143.864			

*Significant result, .05 level.

TABLE 3

ANALYSIS OF COVARIANCE, INTEGERS TEST

Source of Variance	D.F.	Mean Square	F	Adjusted Means	
				Visual	Verbal
Males					
				60.465	57.428
Equality of Adjusted Cell Means	1	105.324	0.712		
Zero Slope	1	3781.832	25.645		
Error	43	147.465			
Equality of Slopes	1	70.969	0.475		
Error	42	149.287			
Females					
				54.563	53.630
Equality of Adjusted Cell Means	1	9.918	0.062		
Zero Slope	1	5975.383	37.024		
Error	43	161.393			
Equality of Slopes	1	117.875	0.726		
Error	42	162.429			

Results Concerning Aptitude-Treatment Interaction

As was described in Chapter III, in addition to the requirement that the regression lines of the two treatment groups intersect within the range of any given aptitude measure, the conclusion of significant interaction required that at least one regression line have a slope significantly different from zero, and that the slopes differ significantly. Underlying the latter test is the requirement of homogeneous variance of the posttest scores of the two groups (Dixon and Massey, 1957), p. 102). Thus three statistical tests were involved in each investigation of ATI.

Pertinent quantities which underlie the three tests mentioned above are as follows:

r -- The correlation between the independent (aptitude measure) and dependent (posttest score) variables for the two groups.

a -- The "Y-intercept" for the regression equation of the two groups.

b -- The regression coefficient or slope of each of the equations.

n -- The number of subjects of a given sex in a given treatment group.

S_b -- The standard error of the regression coefficient.

Computations leading to the statistical tests in this phase of the research were done by an author-written computer program. The formulas used for r, b, and S_b were the usual ones employed when working with raw scores. (Hays, 1963, pp. 505-509).

The test, for each group with respect to each aptitude measure, of the null hypothesis: $b = 0$ was

$$t = \frac{b_{yx}\ S_x\sqrt{n-2}}{S_{y \cdot x}} . \qquad \text{(Hays, 1963, p. 521)}$$

The alternative was directional: $b > 0$.

Using the notation $S_1{}^2$ and $S_2{}^2$ for the variances of any two groups of subjects being compared, the test of the hypothesis $S_1{}^2 = S_2{}^2$ was

$$F = \frac{S_1{}^2}{S_2{}^2} .$$

S_1 denotes a visually-instructed group and S_2 a verbally-instructed one. Here, the alternative is two sided. Thus the hypothesis of homogeneity was rejected only if the observed F exceeded $F(df_1, df_2, .975)$ or the reciprocal of the observed F was less than $F(df_2, df_1, .025)$. (Hays, 1963, p. 350).

The null hypothesis $b_1 = b_2$, where b_1 is the regression coefficient of a visually-instructed group and b_2 that of a verbally-instructed one, was based on an F test in the analysis of covariance (Snedecor and Cochran, 1967, p. 432). Again, since there were only two groups involved in each test, a t-test was used with pooled error term S_p defined by

$$S_p = \sqrt{\frac{(n_1 - 1)\, S_1^2 \div (n_2 - 1)\, S_2^2}{n_1 + n_2 - 2}} \ .$$

Then the test is

$$t = \frac{b_1 - b_2}{\sqrt{2\, S_p^2}} \ .$$

The alternative was again directional: $b_1 < b_2$.

Table 4 shows the pertinent critical values of t and F, and Table 5 the observed values of F in the test of homogeneity of variances. From these it is readily noted that the latter hypothesis was sustained in each case. In the test of the null hypothesis $b_1 = b_2$, a significant result was obtained when the observed value of t was <u>less</u> than the tabulated value.

TABLE 4

CRITICAL VALUES OF t AND F FOR ATI, .05 LEVEL

Null Hypothesis	Degrees of Freedom	Critical Value
$b = 0$	21	1.72
$S_1 = S_2$	22,22	Upper F:2.36 Lower $\frac{1}{F}$: .42
$b_1 = b_2$	44	-1.68

TABLE 5

OBSERVED VALUES OF F FOR NULL HYPOTHESIS: $S_1 = S_2$

Treatment Group/Test	F
Males, Probability	1.438
Females, Probability	1.158
Males, Integers	0.604
Females, Integers	0.440

Tables 6 and 7 present the values of t for the test of regression coefficient different from zero, the value of the aptitude measure at which the regression lines cross, and the test for equality of regression coefficients. The DAT subscales are numbered the same as in the listing in the statistical portion of Chapter III.

TABLE 6

OBSERVED t VALUES AND CROSSOVER VALUE FOR ATI, PROBABILITY

DAT Subscale	t for hypothesis: b = 0		Crossover Value	t for hypothesis: $b_1 = b_2$
	Visual Group	Verbal Group		
		Males		
1	4.328	1.838	11.4	1.052
2	5.636	1.597	21.8	2.352
3	5.865	1.891	37.4	1.626
4	1.427	2.262	41.1	-0.829
5	1.703	1.655	33.8	0.485
6	3.402	0.933	44.2	1.981
7	2.532	0.855	30.3	1.240
8	1.839	1.553	***	0.024
9	3.128	1.451	14.8	0.862
		Females		
1	4.058	0.066	15.1	3.166
2	3.461	1.800	10.0	1.315
3	4.689	2.285	***	0.834
4	3.691	1.324	16.6	1.222
5	0.906	0.851	***	0.047
6	5.328	0.730	32.4	2.578
7	3.377	1.508	***	0.618
8	2.835	1.144	***	1.059
9	3.943	2.936	***	0.157

***Crossover value was not in the range of the aptitude measure.

From Table 7, support can be observed for the hypothesis that subjects ranking lower on certain aptitude measures benefit most from visually-rich instruction, and those ranking higher most from verbally-rich instruction. The contents of Table 6 are negative in regard to this hypothesis. In Appendix B, graphs of the regression lines of those cases exhibiting significant interaction in the direction hypothesized are presented.

TABLE 7

OBSERVED t VALUES AND CROSSOVER VALUE FOR ATI, INTEGERS

DAT Subscale	t for hypothesis: b = 0		Crossover value	t for hypothesis: $b_1 = b_2$
	Visual Group	Verbal Group		
Males				
1	3.593	4.132	***	-0.202
2	3.300	5.400	27.1	-1.651
3	4.065	5.547	46.1	-0.676
4	3.413	1.970	34.4	1.103
5	4.251	1.955	***	0.154
6	1.990	2.511	47.6	-0.952
7	2.320	1.812	***	-0.037
8	3.350	1.934	69.7	0.862
9	2.022	2.136	***	-0.090
Females				
1	-0.332	3.970	21.7	-3.640**
2	2.721	4.080	19.0	-1.930*
3	3.173	5.006	45.0	-1.407
4	0.920	4.217	31.6	-2.349**
5	0.889	0.358	***	0.212
6	-0.008	4.750	26.7	-3.503**
7	1.086	6.190	23.5	-2.560**
8	2.385	2.561	49.7	-0.731
9	2.384	3.979	24.2	-1.246

*Significant result, .05 level.
**Significant result, .01 level.
***Crossover value was not in the range of the aptitude measure.

Results Concerning Instructional Modality Preference

Treating the sexes separately, the pair of standard scores of each subject on the two posttests was the basis for dividing the sample by the upper twenty-seven - lower twenty-seven percent method. The quantity D was defined as the standard score on the posttest following visually-rich instruction minus that following verbally-rich instruction. This resulted in twelve subjects of each sex being placed in each of the visual and verbal preference groups and twenty-two in the no preference group.

The library computer program used in the analysis, BMD07M (Dixon, 1973, p. 233), requires a number of parameters to be specified. The parameter of primary interest in this report was that which regulates the manner in which the computation proceeds from one iteration to the next. The option elected in the computer runs was to use the value specified by the program, that is, at each step to bring into the classifying set of variables that variable which had the greatest "F to enter" at the preceding step. An additional parameter was the minimal significance level that the "F to enter" of a given variable must attain, to be eligible for inclusion in the classifying set. The value of this parameter was set at .5. Any variable not meeting this criterion, at the step at which it had the greatest "F to enter", was considered to be only a trivial contributor to the classifying ability, and was accordingly excluded. Table 8 exhibits the order in which the variables entered the classifying set. Those not listed failed to meet the criterion for "F to enter".

TABLE 8

VARIABLE ENTERED BY STEP, DISCRIMINANT ANALYSIS

Step Number	Variable Entered	F to Enter
	Males	
1	Mechanical Reasoning	2.165
2	Language Usage, Sentences	1.297
3	Abstract Reasoning	1.101
4	Language Usage, Spelling	1.323
5	Space Relations	1.210
6	Verbal Reasoning	0.720
7	Numerical Ability	0.542
	Females	
1	Clerical	9.844
2	Language Usage, Spelling	2.356
3	Language Usage, Sentences	2.402
4	Mechanical Reasoning	1.393
5	Verbal Reasoning + Numerical Ability	0.704

Note:

It is not necessarily true that the "F to enter" must show a steadily decreasing pattern from step to step. The measure of a given variable's potential contribution to classifying ability changes in relation to those variables already included.

The final classification matrices are presented in Tables 9 and 10.

From Table 9, it can be observed that twenty-seven of the forty-six male subjects were correctly classified. From Table 10, Thirty-two of the forty-six female cases were correctly classified.

TABLE 9

CLASSIFICATION MATRIX, MALE SUBJECTS

Group	Number of Cases Classified Into Group		
	Visual	No Preference	Verbal
Visual	8	1	3
No Preference	4	12	6
Verbal	3	2	7

TABLE 10

CLASSIFICATION MATRIX, FEMALE SUBJECTS

Group	Number of Cases Classified Into Group		
	Visual	No Preference	Verbal
Visual	8	4	0
No Preference	6	14	2
Verbal	0	2	10

The probability of a given case being correctly classified by chance alone is 1/3, since there are three groups. The binominal distribution is an appropriate probability model for a situation involving n independent repetitions of a two-outcome random process. (Hays, 1963, p. 143). The random classification of cases would fit this description. With n cases, the probability of r correct classifications would then be

$$p(r) = \binom{n}{r} (1/3)^r \ (2/3)^{n-r} ,$$

where $\binom{n}{r}$ is the binomial coefficient.

Computationally, the normal approximation to the binomial (Hays, 1963, pp. 227-230) offers definite advantages. Both (n x 1/3) and (n x 2/3) exceed 10 for the value n = 46, which is the number of subjects of each sex in this study; thus, the use of the approximation is appropriate. Letting r denote a given number of correct classifications, and with mean = (n x 1/3), variance = (n x 1/3 x 2/3), the z value corresponding to r is $z = \frac{r - n \times 1/3 + 1/2}{\sqrt{n \times 1/3 \times 2/3}}$.

The resulting z value for male subjects was 3.71, a result significant beyond the .001 level. For females the value was 5.37, which was also significant beyond the .001 level.

Supplementary results from the output of program BMD07M are available in Appendix C.

CHAPTER V

CONCLUSIONS

Summary of the Findings

Main Effects of Instructional Mode

Ho I: A group of students which has been taught a unit of material using visually-rich methods will score no higher on a test over the unit than will another group which has been taught the unit using verbally-rich methods.

The null hypothesis was rejected only for the group of female subjects, when the topic was probability. The significance level was .05. For the other three tests, the group which was taught via the visual mode also exceeded, although not significantly, that taught via the verbal mode, in terms of the adjusted group means.

Aptitude-Treatment Interaction

Ho II: There will not be interaction between mode of instruction and each of the aptitude measures from the Differential Aptitude Tests; that is, individuals ranking lower on a given subscale will not benefit relatively more from the visually-rich instruction.

ATI was investigated for male and female subjects, for each of the nine DAT measures. Of the thirty-six tests, only five resulted in rejection of the null hypothesis. All of these pertained to the female subjects and the integers topic (Table 7). The DAT subscales with which there was significant interaction were: Numerical Ability, at the .05 level, and Verbal Reasoning, Abstract Reasoning, Mechanical Reasoning, and Clerical, each at the .01 level.

The Investigation of Learner Preference For Visual or Verbal Instruction

Ho III: The classification of subjects by a discriminant analysis procedure as visually-oriented, verbally oriented or neutral, will agree with their classification by a process based simply upon differences between their scores on the visual and the verbal unit, to no greater an extent than that to be expected by chance.

The null hypothesis was rejected at the .001 level for both male and female subjects.

General Observations

The research reported herein has, in a broad sense, been concerned with learner differences as related to mode of instruction. A portion of it dealt only with the effects of a visual versus a verbal mode.

The two remaining questions came within the general framework of aptitude-treatment interaction. Of these, one was investigated by closely following a well-accepted paradigm, and the other by a method not widely found in reports of research.

The instruction in the verbal mode may be described as having followed a symbolic model, in the language of some writers and researchers exemplified by Fennema (1972, p. 233). The visually-rich instruction followed a concrete model, further describable as passive, since the handling of the materials was all done by the teachers rather than by the subjects.

The author was aware, both in the design of the study and in the interpretation of results, of each of the four difficulties in ATI research identified by Becker in his article which placed this subject before the mathematics education community (1970, pp. 26-27).

The Problem of Selection of Aptitude Measures

A vital factor is the selection of aptitude measures which will interact with methods of instruction. Aptitude measures are needed that reveal specific mental abilities. In order to sense a variety of specific abilities, a broad battery of tests may be needed; possibly some of these may have low correlations with others in the battery. It would appear that the Differential Aptitude Test battery qualified fairly well relative to the above criteria. Nevertheless, one cannot be sure that a battery dealing specifically with visual-verbal modality preference would not have afforded better classifying ability.

The Problem of Determining the Length of the Treatment Period

This is a most perplexing issue. The setting for ATI research is primarily the classroom. This is not to say that the findings of laboratory-type investigations cannot provide input and guidelines for classroom-based research. In the research reported herein, it was considered important that the posttest results could be substantially attributed to immediate learning, rather than to transfer or retention. Nevertheless, the instructional sequences needed to be long enough to permit treatment effects to emerge. Additionally, there is the consideration of test reliability. It would be difficult to prepare a test not entailing excessive repetition which covered a very short instructional sequence, and which would at the same time avoid the problems of chance elements in the subjects' behavior or in the particular sampling of items.

> . . . "In general, the larger a sample of a persons' behavior we have, the more reliable the measure will be." (Thorndike and Hagen, 1955, p. 129).

Becker cites evidence that a subject's response to instruction is a function of the kind of instruction he received in the past. Disturbing effects may occur in the case of a class whose teacher used visual materials extensively but then discontinues their use as called for by the research guidelines. On the other hand, the students of a teacher with the opposite propensity may be affected either positively or negatively when first presented with numerous devices for nonverbal learning.

The Problem of Determination of Achievement Measures Which Will Uncover Interactions

The design of the present research substantially dictated emphasis on immediate recall measures, or at least items which required the application of material to be recalled. The construction of the post-tests also called for fairness to each treatment group.

The Results As Related To Past Findings

The slight indication of superiority for the visual over the verbal mode of presentation appears to be in harmony with earlier findings. Trask's report (1972) on research with early elementary pupils, involving a comparison between the results of manipulative versus symbolic learning experiences, indicated no clear pattern of superiority for either. Working with a college population, Hancock (1973) reported clear superiority for a verbal over a figurative presentation mode.

Past findings in aptitude-treatment interaction research have ranged from no significant interaction with any of the aptitude measures used, such as reported by Huebner (1969), to quite positive findings exemplified by Davis' report (1967). The latter does indicate the possibility that success in uncovering interactions may be somewhat dependent upon the nature of the content being taught. The author is inclined to agree with this suggestion, in light of the modest support for the interaction hypothesis indicated by the results pertaining to the integers topic, and the rather negative findings (i.e., somewhat opposite in directionality) relative to the probability topic. These negative findings may possibly indicate a novelty effect of the visuals which seems to impair slow learners' performance more than that of faster ones. This is in line with the report of Kulm and others (1974), pertaining to the effect of pictorial materials on problem solving, and with Trask's report that new manipulative materials benefited higher-ranking pupils more than lower.

The importance of the sensory modality concept as a general factor in classroom instructional planning may be related to the age of the pupils. As noted in Chapter II, Wepman has taken the stand that modality preference tends to lose importance for most pupils by age nine. Nevertheless, that segment of the school population for whom this is not the case deserves consideration, in terms of the availability of learning opportunities.

The findings reported in Chapter IV pertaining to visual versus verbal instructional modal preference showed a statistically significant ability of the discriminant analysis procedure to classify learners in agreement with another grouping based on posttest scores. This does not automatically mean that twenty-seven per cent of a given class necessarily needs extensive opportunity for nonverbal learning. Indeed, there were some cases in which a subject with a substantially higher relative score on the visual compared to the verbal unit was misclassified, as were some showing the reverse pattern.

It is the author's opinion that the methodology applied to the investigation of modality preference as reported herein could be of value in planning mathematics instruction. Suppose that after replication on different topics and the exploration of different aptitude measures and methods of initially dividing learner groups, a combination were obtained which yielded very high success in classification over a wide range of topics at a given grade level. The data from the research subjects, or simply the discriminant classifying equations, could be retained, and pupils in subsequent classes could be screened for modality preference by simply feeding their aptitude scores into the program.

Limitations; Recommendations for Further Research

One limitation of the research reported herein is the fact that between-teacher variability was not measured. Another basic question is whether or not the nature of the posttests permitted subjects to exhibit learning which they had obtained nonverbally. In the absence of special nonverbal test items--which would be very difficult to present in a classroom situation--learners needed to rely upon some internal mediator between the nonverbal learning and the essentially verbal test items.

The discriminant analysis procedure relies upon the assumption of the multivariate normality of the classifying variables. (Tatsuoka, 1971, p. 164-65). Robustness of the various tests internal to the program to violation of the assumptions is not established at this time. This could affect the interpretation of results.

The sample used in the research came from a junior high school located in the highest-ranking socioeconomic area of the city. To the extent that such factors may affect the results of research of the type reported herein, the generalizability of the conclusions is correspondingly restricted.

The author recommends that further investigation take place along the following lines:

1. Aptitude-treatment interaction research on mathematical concepts should be pursued in laboratory-like settings with emphasis on the use of test items calling for a minimum of verbalness in the responses.

2. Verbal-visual modality preference research involving several pairs of mathematics topics, various sets of aptitude measures, and various methods of initially dividing a group of learners should be pursued by means of the methodology used in the present research.

APPENDIX A

INSTRUCTIONAL MATERIALS USED IN THE STUDY

TABLE 11

NUMBER OF VISUALS BY LESSON

Lesson		Number
	Probability	
I:	Probability - Introduction	4
II:	Independent Events	2
III:	Dependent Events	2
IV:	Naming Subsets; Ordered Subsets	2
V:	Odds	2
	The Integers	
I:	Number Sentences; Number Properties	3
II:	Integers	2
III:	Addition of Integers	3
IV:	Inverse Operations on the Integers	3
V:	Subtraction of Integers	1

Note:
The lessons are based on pages 202-211 and 221-230, respectively, of Modern Mathematics, Structure and Use, Book 8, by Duncan and others (1972).

Topic I: Probability

Descriptions or Sketches of Visual Materials

I - 1. The students were shown a die and a coin. The teachers tossed the die and the coin and tried to elicit the understanding of the equiprobability of the head and the tail, and of each of the faces of the die.

I - 2. Four transparencies were used in this pictorial representation of probability as a fraction. The bottom transparency, number 1, showed the container only; the next one, number 2, showed four red balls, and the next, number 3, two green. Attention was called to the six balls as the sample space of the experiment. Number 3 was moved upward, making the green balls "rise above" the others. Then, number 2 was also moved upward, so that the red balls lay just below the green ones. Finally, number 4, identical to number 3, was overlaid so as to bring the sample space back to full strength, thus illustrating the fraction 2/6 = 1/3 = the probability of drawing a green ball.

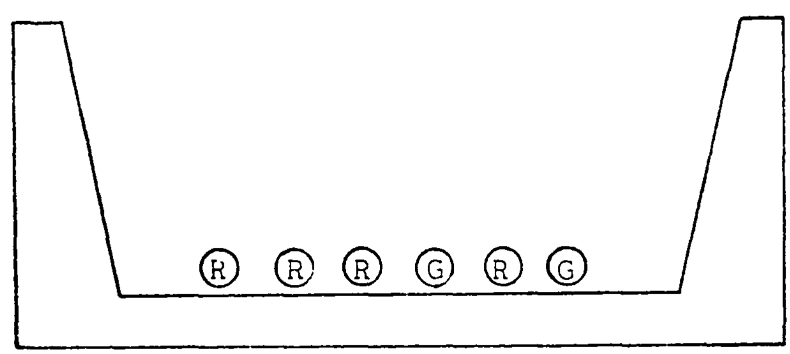


Figure 4

I - 3. The three probability expressions shown below appeared on a single transparency. They were intended to picture probabilities as ratios of measures of regions within a sample space. The intent was to suggest that a probability may be arbitrarily close to zero, about one-half, or up through one.

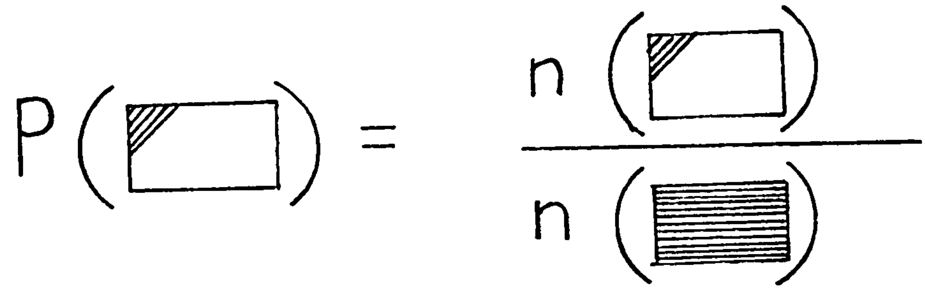


Figure 5

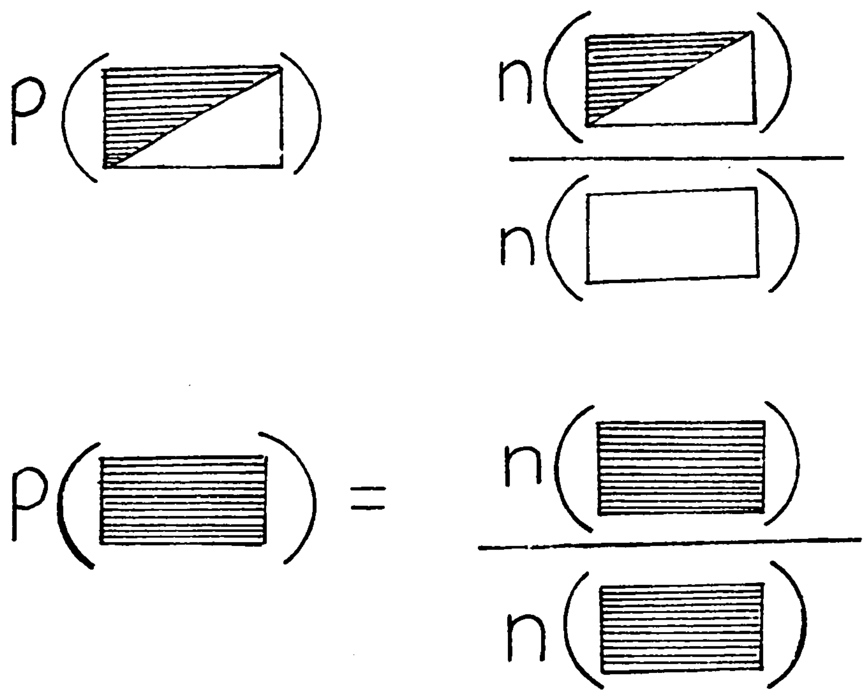


Figure 6

<u>I - 4</u>. In the sketch below, a sample space is presented, and certain subsets are specified according to visual properties of the members. The teachers demonstrated the presence or absence of straight or curved segments by motions of a straightedge. The fact that the intersection of any two of the sets is empty was noted.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Let U = {Letters in alphabet}

n (U) = 26

Let C = {Letters having no straight segments}

A = {Letters having no curved parts}

B = {Letters having some curved parts and some straight segments}

Figure 7

II - 1. This visual involved a demonstration with physical objects to show the concept of independent events. A piece of tagboard containing a single hole, three styrofoam balls of the same size but different color, and a sack for each ball comprised the materials. The probability of being able to pass any randomly chosen ball through the hole in the tagboard is independent of the ball chosen. A variation of this consisted of the introduction of a second piece of tagboard, containing 3 holes of different sizes. In this case, the probability of randomly choosing a hole through which an earlier-chosen ball will fit is again independent of the ball chosen; that is, any ball chosen would pass through the same number of holes as would any other.

II - 2.

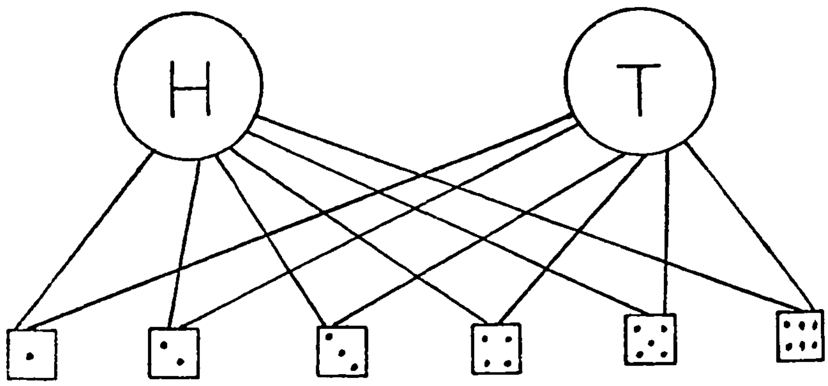


Figure 8

The sketch shown above appeared on two transparencies. The lower transparency included the head side of the coin with lines connecting it to each of the six die faces. The upper transparency showed the tail side with lines of a different color connecting it to each die face. The intent was to show that the combined sample space of two experiements if formed by pairing outcomes of the two separate experiments.

III - 1. The visual shown below employed four transparencies to convey the concept of dependent events, through emphasis on the changed nature of the sample space after one drawing without replacement. The bottom transparency, number 1, showed the container, two red, and two green balls. The next one, number 2, contained two red balls to overlay the middle two of number 1. Number 3 contained a single red ball to overlay the rightmost red one in number 1. Finally, number 4 contained all three red balls. Moving number 4 up slightly showed the fraction 3/5; number 4 was then laid aside. Number 3 was also moved up and laid aside, signifying the drawing of one red ball without replacement. Moving number 2 up, finally, showed a new fraction, 2/4, the probability of drawing a red on the second draw.

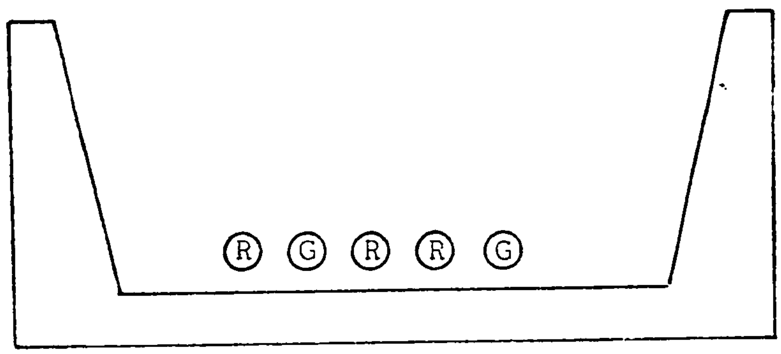


Figure 9

III - 2. This visual closely paralleled II - 1 described above. Here, however, the concept of dependent events was shown. Instead of three equal-sized balls, three of differing sizes were used. Thus, it did make a difference which ball was drawn, as to the success in passing a chosen ball through either the single hole or a chosen one of three holes.

IV - 1. The sketch below appeared on three transparencies, and was intended to show the doubling of the number of subsets of a set, when the set is formed by the inclusion of one additional element in a set previously defined. The bottom transparency contained the single-element set with its two subsets below. The middle transparency showed the two-element set with its four subsets, each new one deriving from a subset of the smaller set by adjoining the new element. Finally, the three-element set with its eight subsets appears on the top transparency; again each new subset derives from a subset of the next smaller set by the adjoining of the new element.

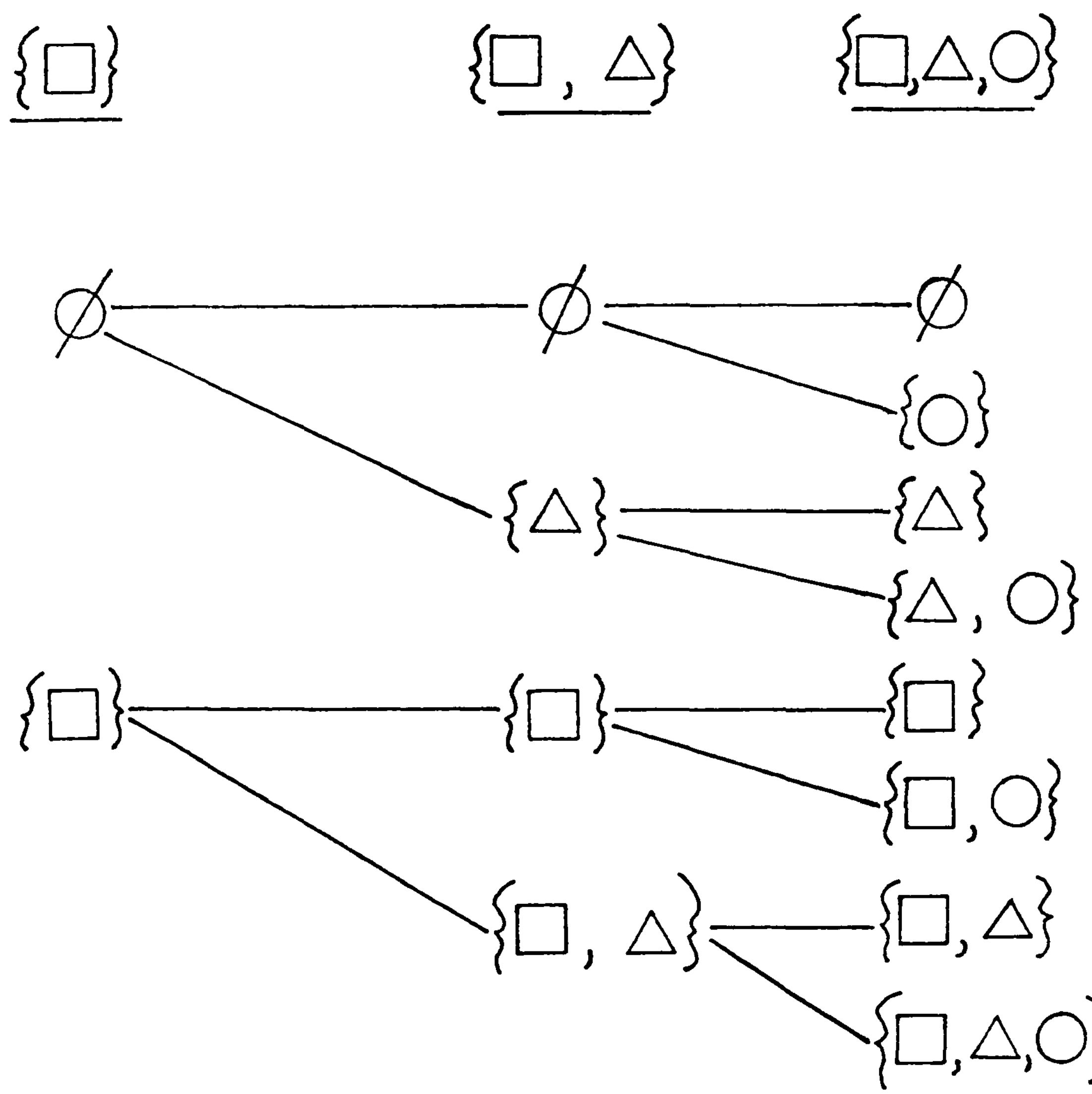


Figure 10

IV - 2. This visual is a demonstration using physical objects, with the intent of picturing the formation of two-element ordered subsets from a set of three objects. The objects were three different-shaped wood pieces. There are three ways of selecting any two of these. Given a chosen pair, there are two possible "towers" that can be made from them, i.e., two ordered subsets. A total of six, or three times two, ordered subsets results.

V - 1. This visual shows the complement of a set on a single transparency. The color red was used for the set and green for the complement.

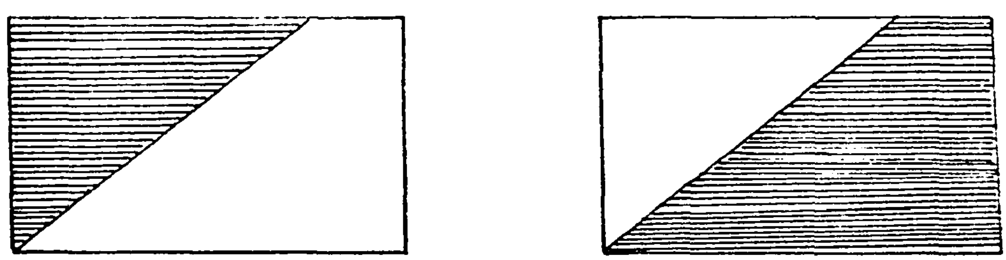


Figure 11

V - 2. (See the detailed discussion in Chapter III).

VERBAL ENRICHMENT: PROBABILITY

Lesson I

In drawing equal-sized balls from a jar, each ball has as much chance to be drawn as any other. The same can be said for the cards in the example on page 203. The drawing of a ball or a card would be done without looking.

Because a probability compares some of the outcomes with the entire set of outcomes, it must be a fraction not greater than one, nor less than zero.

At the beginning of the professional football season, the sample space of all teams that might become the Super Bowl champions contains 26 teams (elements). When regular season play has been completed, the sample size is reduced in size to 8. On the morning of the Super Bowl game, the size is 2.

A girl wishes to wear something green on St. Patrick's day. She owns one green blouse, three green dresses, and two green pairs of jeans. Assuming no preference on her part, the probability that she selects a green <u>dress</u> is 3/6 or 1/2.

Lesson II

Suppose we call two events A and B. If the happening of event A does not affect the probability of B happening, and vice-versa, the events are independent.

If two students go to different rooms and are each told to guess a number between 1 and 25, the probability that one student guesses, say, 15, is unaffected by whether or not the other guessed 15. These would be independent events.

When the weatherman says that there is a 30 per cent chance of rain in the next 24 hours, he means that the probability of rain is 3/10.

In listing the sample space for a two-part experiment, like throwing a die and a coin, we list all the possible pairs of outcomes. One member of each pair is an outcome of the die throw and the other an outcome of the coin throw.

Lesson III

Suppose that a die is thrown twice, and we want to find the probability that the total of the two outcomes is 9 or more. If the first throw is a 6,

$$P(\text{total} \geq 9) = P(\text{second} = 3) + P(\text{second} = 5) + P(\text{second} = 6) =$$
$$1/6 + 1/6 + 1/6 + 1/6 = 2/3.$$

However, if the first throw is a 4,

$$P(\text{total} \geq 9) = P(\text{second} = 5) + P(\text{second} = 6) = 1/6 + 1/6 =$$
$$2/6 = 1/3.$$

When the first outcome is a 6, the sample space for the total throw is $\{7,8,9,10,11,12\}$. But, when the first outcome is a 4, the sample space for the total is $\{5,6,7,8,9,10\}$. This shows that the outcome for the total depends on the outcome for the first die.

Suppose that you are about to meet someone for the first time, and you are wondering whether or not he has blue eyes. If you knew that both of his parents have blue eyes, you would be more likely to guess that your soon-to-be acquaintance will have blue eyes. That is, the event of a person's eyes being blue is <u>dependent</u> on the event that his parents' eyes are blue.

Lesson IV

Let us examine a set $\{A,B,C,D\}$. To form a subset having one element, there are four different elements, so there are four different one-element subsets. To get a two-element subset, we need to pick any two to include in the subset (the other two are automatically excluded). We could have $\{A,B\}$, $\{A,C\}$, $\{A,D\}$, $\{B,C\}$, $\{B,D\}$, $\{C,D\}$: a total of six two element subsets. To get a three-element subset, it's simply a matter of deciding which single element to exclude, so again there are four possibilities: $\{A,B,C\}$ (exclude D); $\{A,B,D\}$ (exclude C); $\{A,C,D\}$ (exclude B); and $\{B,C,D\}$ (exclude A). Finally, there is a single four-element subset and a single empty subset. Adding up all of these subsets, we get $16 = 2^4$.

A delegate to a two-day state meeting of student council officers has six outfits from which she could pick two to take along. She does not want to wear the same one on both days, and considers the wearing of outfit A on Friday and outfit B on Saturday to be different from the reverse order. How many choices of apparel does she have? $(6 \times 5 = 30)$.

New uniforms are being ordered for a school football team. There are two quarterbacks, Joe and Tom, who are to receive numbers between 1 and 19. How many ways can numbers be assigned to these two players, if, for example, it is different to give Joe number 11 and Tom number 14 than the reverse? $(19 \times 18 = 342)$.

Lesson V

In one throw of a die, the odds in favor of an outcome of 6 are 1/5. The odds against an outcome of 6 are 5/1.

"Odds against" is always the reciprocal of "odds in favor", so long as "odds in favor" is greater than zero.

When an event is as likely as not to happen, the odds in favor of it are 1/1.

PROBABILITY TEST

CIRCLE THE LETTER OF THE CORRECT CHOICE

1. In a box are 10 marbles; 4 are red, 3 black, and 3 yellow. What is P(black), the probability of picking a black ball?
   (a) 7/10 (b) 1 (c) .03 (d) 3/10

2. In the box containing the 10 marbles; 4 red, 3 black, 3 yellow, what is P(black or yellow)?
   (a) 1 (b) 3/5 (c) 3/10 (d) 1/5

3. In which case would the outcomes have unequal probabilities?
   (a) Getting a head or tail when tossing a coin.
   (b) Getting an odd number of an even number in tossing a die.
   (c) Picking someone in this room who is left-handed or someone who is right-handed.
   (d) Picking a black or a yellow marble from a box containing 3 black and 3 yellow marbles.

4. Which one of the numbers below could <u>not</u> be the probability of any outcome?
   (a) 5/6 (b) 6/5 (c) .74 (d) 0

5. In picking a whole number from 1 through 30, what is the probability of getting a number which ends in a 5 or a 0?
   (a) 1/5 (b) 1/6 (c) 1/30 (d) 1/11

6. A regular card deck has 13 hearts(red), 13 diamonds(red), 13 spades(black), and 13 clubs(black). What is P(black card or a heart?
   (a) 13/52 (b) 26/52 (c) 0 (d) 39/52

7. In the box containing the 4 red, 3 black, and 3 yellow marbles, how many <u>different</u> elements belong to the sample space?
   (a) 10 (b) 3 (c) 5 (d) 2

8. In the set of <u>even</u> numbers from 2 through 20, what is P(number ending in 5)?
   (a) 5/20 (b) 5/18 (c) 0 (d) 2/5

9. In the set of <u>odd</u> numbers from 3 through 24, what is P(number ending in 7)?
   (a) 25% (b) 2/11 (c) 7/21 (d) 30%

10. In the set of whole numbers from 0 through 99, what is P(number ending in 9)?
(a) 9/99 (b) 20% (c) .09 (d) 10%

11. If you toss one coin, see that it is a head, then toss a second coin, what is P(tail on 2nd coin)?
(a) 1 (b) 3/4 (c) 5/8 (d) 1/2

12. In tossing a nickel and a dime, how many outcomes are there?
(a) 2 (b) 4 (c) 15 (d) 3

13. Remember that in the box in problem 1 there were 10 marbles, of which 4 were red, 3 black, and 3 yellow. We draw a red one and do not replace it. Then, for a second drawing, what is P(red)?
(a) .4 (b) 1/3 (c) 7/10 (d) .12

14. From the box with 4 red, 3 black, and 3 yellow marbles, we draw a red one and do not replace it. Then, for a second drawing, what is P(yellow)?
(a) 3/10 (b) 6/19 (c) 1/10 (d) 1/3

15. From the box with 4 red, 3 black, and 3 yellow marbles, we draw a red one and do not replace it. Then, for a second drawing, what is P(red or yellow)?
(a) 7/10 (b) 6/19 (c) 2/3 (d) 7/15

16. Five boys of equal ability are competing for two available spots on a basketball team, one forward and one guard position. Calling the boys A, B, C, D, and E, what is P(A is chosen guard, B is chosen forward)?
(a) 1/5 (b) 1/25 (c) 1/20 (d) 1/9

17. Five boys of equal ability, named A, B, C, D, and E, are competing for one guard and one forward spot on a basketball team. What is P(A not chosen guard, D is chosen forward)?
(a) 1/4 (b) 4/9 (c) 1/5 (d) 3/20

18. Five boys of equal ability, named A, B, C, D, and E, are competing for one guard and one forward spot on a basketball team. What is P(C doesn't make the team)?
(a) 3/5 (b) 16/25 (c) 7/20 (d) 3/4

19. Let A = $\{1,2,3\}$; B = $\{9,10\}$; C = $\{99,100\}$. Which statement is correct?
    (a) C has more subsets than A.
    (b) C has more subsets than B.
    (c) A has the fewest subsets of all.
    (d) A has the most subsets of all.

20. Which of the numbers is the only one which could possibly be the number of subsets of some set?
    (a) 16 (b) 24 (c) 80 (d) 72

21. A set with 4 elements has more 2-element subsets than any other size. If another set has 10 elements, which size will be the most numerous? Those with:
    (a) 2 elements (b) 3 elements (c) 5 elements (d) 9 elements

22. From the digits $\{1,2,3,4,5\}$, how many 3-digit numerals can be made if no digit is used twice in any numeral?
    (a) 120 (b) 60 (c) 45 (d) 543

23. Making 3-digit numerals as in problem 22, what is the probability of getting a numeral which is larger than 500?
    (a) 1/3 (b) 12/500 (c) 9/500 (d) 1/5

24. Considering only the whole numbers from 1 through 20, and letting A equal $\{$all odd numbers between 1 and 20$\}$, which number is in the complement of A?
    (a) 5 (b) 15 (c) 16 (d) 17

25. Saying that the odds in favor of the Lakers' winning the championship are 2/3 means that P(Lakers win) = ?
    (a) 1/6 (b) 2/3 (c) 2/5 (d) 1/3

26. In the problem about the box containing the 4 red, 3 black, and 3 yellow marbles what are the odds against picking a red ball?
    (a) 3/2 (b) 2/3 (c) 2/5 (d) 5/2

27. In the above problem, what are the odds in favor of a ball which is either red or black?
    (a) 7/10 (b) 1/3 (c) 11/4 (d) 7/3

28. Which statement is correct?
    (a) 0 could never be the odds in favor of some outcome.
    (b) 1/1 could never be the odds in favor of some outcome.
    (c) Every number we have studied so far in this course could be the odds in favor of some outcome.
    (d) No fractional number larger than 1 could be the odds in favor of some outcome.

TABLE 12

ITEM DIFFICULTY: PROBABILITY TEST

Item Number	Number of Incorrect Responses
1	1
2	42
3	46
4	122
5	64
6	64
7	102
8	47
9	66
10	120
11	23
12	64
13	27
14	42
15	37
16	101
17	152
18	96
19	15
20	108
21	127
22	86
23	82
24	108
25	130
26	105
27	78
28	99

Note:

The number of subjects writing the test was 174.

Topic II: The Integers

Description or Sketches of Visual Materials

I - 1. In the visual shown below, the intent is to contrast the closure of a set under an operation (which is not specified) with lack of closure.

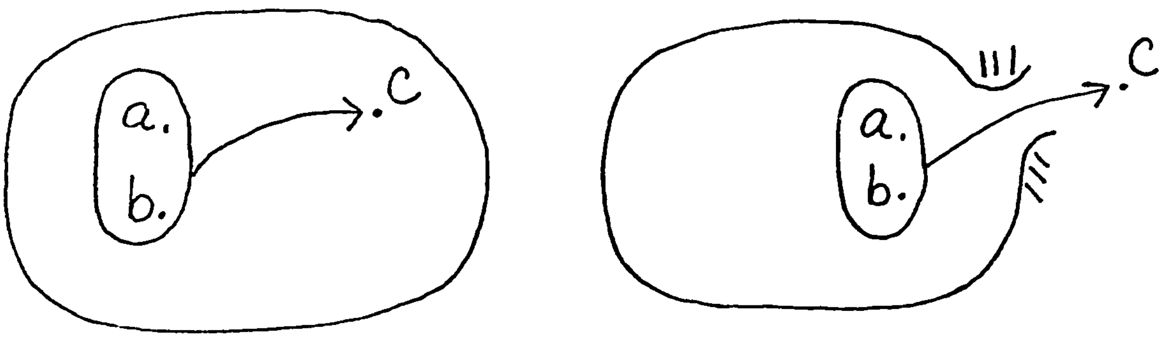


Figure 12

II - 2. A transparency was used to show an example of a non-associative operation from outside of mathematics. The operation was that of combining ingredients in the making of iced tea. The drawing first showed (tea + boiling water) + ice, which yields the iced tea. The tea was symbolized by a box labelled "tea", the boiling water by a kettle and the ice by a tray of ice cubes. Then, the drawing showed tea + (boiling water + ice), and the teachers were to point out, as needed, that this way of combining ingredients would not yield the desired result.

I - 3. The visual sketched below illustrated the distributive law by combining two stacks of blocks. The lower transparency contained the two-by-two stack, and the upper, the two-by-three, thus showing the instance 2 x 2 + 2 x 3 = (2 + 3) = 2 (5). The upper transparency was moved so as to bring the two stacks into an appearance of one longer stack. A second instance shown was 3 x 2 + 3 x 4 = 3 (2 + 4) = 3 (6).

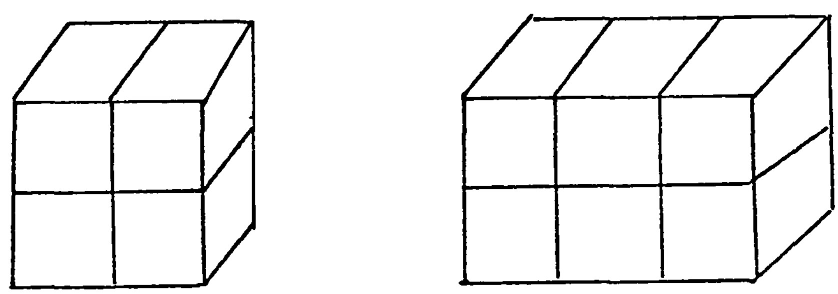


Figure 13

II - 1. This visual, in the form of a Venn diagram on a transparency, was intended to suggest that the whole numbers, labelled "W", are only part of a more inclusive number system, the integers. On the transparency, the entire diagram was outlined in black, the shading within the whole number region was green, and the boundaries of the region complementary to the whole numbers, i.e., the negative integers were outlined in red.

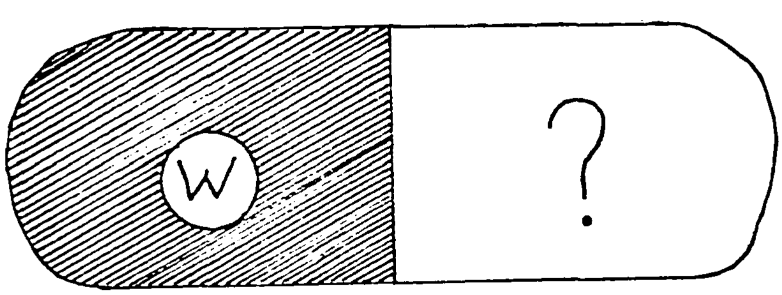


Figure 14

II - 2. The number line was drawn on a transparency, but with the additional feature of vertical bars constructed at each integer (from - 5 through + 5) extending upward or downward according as the integer is positive or negative. The positive portion of the line, and the vertical bars there, were green; the negative portion and its bars were red.

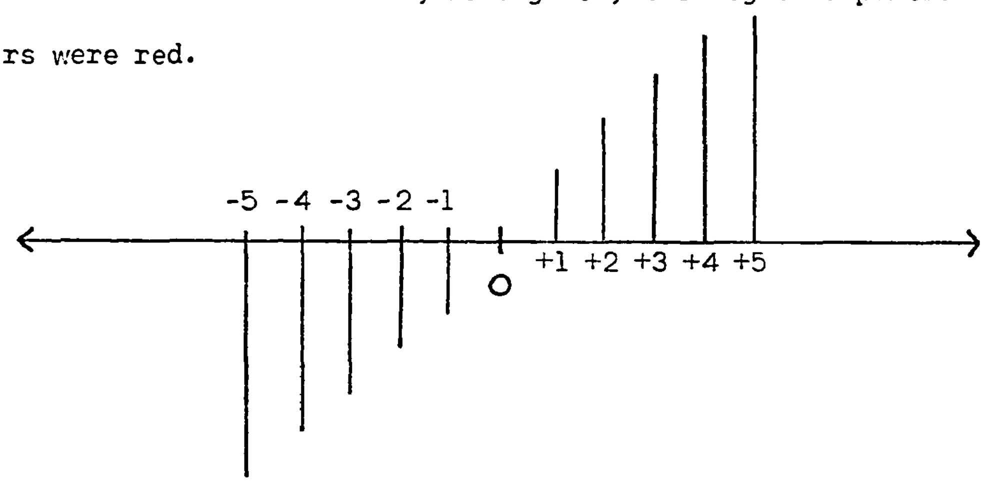


Figure 15

III - 1. An adaptation of Papy's (1968, p. 316) treatment of the addition of integers was presented in this lesson, by means of a black flannel board and green (positive) units and red (negative) units. The positive army is arrayed on one side of the board and the negatives on the other. When the armies engage, one negative and one positive annihilate one another until one of the armies is depleted. The teachers presented a number of examples of addition facts within the integers by this means.

III - 2. Two transparencies with number lines drawn on each were used to show the addition of integers. The number line on the lower transparency was positioned so that it was lower than the other in the projected view. In order to perform the addition of + 1 to + 4, the upper transparency is first moved to make the upper 0 align with the lower + 1. Then, a move of 4 units from 0 to the right reaches + 4 on the upper line, which is aligned with + 5 on the lower. This is the answer.

<u>III - 3</u>. The sketch below shows the absolute value of a number as the length of the line segment joining the number to zero on the number line. Rather than clutter up the number line, congruent segments have been shown, vertically removed from the number line. The teachers were instructed to make a point of counting out the number of unit moves from the origin to + 4 and to - 4, and that the number of units resulting from this count is the absolute value of each integer.

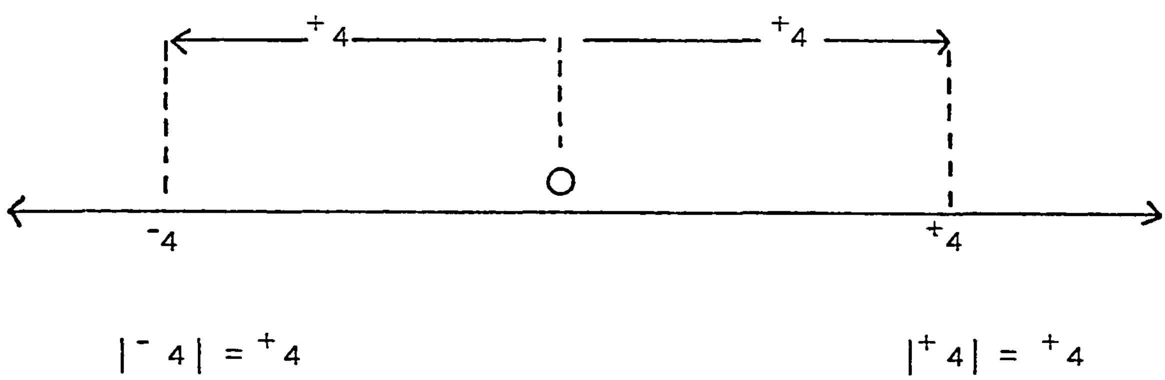


Figure 16

<u>IV - 2</u>. The net result of adding to some number a second number, and then subtracting the latter, is equal to the original number. The same can be said if subtraction takes place first, followed by addition.

The visual showed + 5 as the number being subtracted and added, with + 3 and - 2 the other numbers involved.

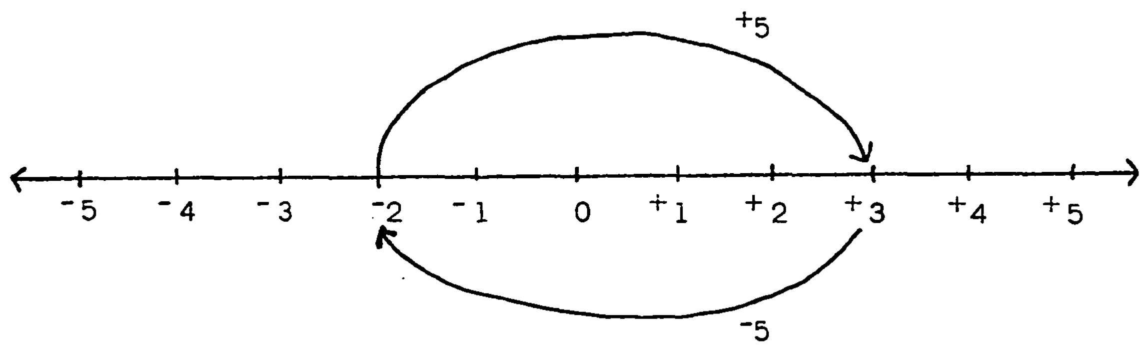


Figure 17

IV - 2. The flannel board was used as in III - 1; this time it is intended to show the identity property of addition. A certain number of green units, say three, is used along with no red ones. Since no green units are annihilated, three remain. This illustrates 3 + 0 = 3. The procedure was repeated for 3 red and no green.

IV - 3. Again the flannel board was used as in III - 1, for the purpose of showing the additive inverse property. Equal numbers of positive and negative units do battle, leaving none of either as survivors. Thus, for instance, 3 + (- 3) = 0.

V - 1. (See the detailed discussion in Chapter III).

VERBAL ENRICHMENT: THE INTEGERS

Lesson I

On page 221, problems 1-3, the reason for no solution in the whole numbers is that $5 < 6$. In problems 4-6, the reason is that no whole number is less than 0.

Pages 222-223

The commutative property for addition means that the sum of two numbers is the same regardless of which number is placed to the left of the addition symbol, and which to the right. (A similar statement may be made with regard to multiplication.)

A particular set of numbers is closed under addition provided that the sum of any two numbers of the set is a number in that same set. (A similar statement may be made for multiplication.)

Addition is a process which combines two numbers to form a third. If we wish to add three numbers, however, the associative property for addition assures us that it makes no difference whether we group the first two or the last two together to be added first. (A similar statement may be made for multiplication.)

The distributive property means that, when multiplying a number by the sum of a second and a third number, an equal result is obtained if one multiplies the first by the second and the first by the third, and then adds these products.

Lesson II

The whole numbers and the negative integers together form the integers. Each negative integer is less than each whole number.

Some additional instances of the concept of negative numbers:

A. Some land in the deserts of Southern California is below sea level.

B. When the United States buys more goods from foreign countries than it sells to them, there is a trade deficit.

C. Space explorers in a space ship re-entering the earth's atmosphere experience negative G-forces.

Some instances of situations where only whole numbers are needed:

A. The number of runs scored or points made in sports contests.

B. The number of eggs to add to a cake mix.

C. How far a student lives from school.

Lesson III

Suppose that we wish to add a first number to a second number, using the number line. If the second number is positive, start at the first and move to the right that number of units which equals the second number. The stopping point is the sum. If the second number is negative, move as many units to the left as you would have moved to the right if the second number had had a different sign, but was otherwise the same. Again the stopping point is the sum.

The absolute value of any integer is simply how far it is from zero, without paying attention to direction.

In comparing the absolute values of two numbers, the one which is farther from zero has the greater absolute value.

The first of two numbers, although less than the second, <u>may</u> have a greater absolute value.

Lesson IV

Further illustrations of operations, their inverses, and identity elements:

Operation	Inverse	Identity element
An airplane climbing 1,000 feet.	An airplane coming down 1,000 feet.	An airplane maintaining a fixed altitude.
Showing a profit of $100.	Showing a loss of $100.	Showing no profit or loss.
A club's losing five members.	A club's gaining five new members.	Size of a club remaining the same.

The sum of any pair of opposite numbers is much like a tug-of-war between two teams of exactly equal strength. The rope fails to move in one direction or the other, just as the sum of the numbers is 0.

Lesson V

Suppose that a company owns five grocery stores, and that one of them has been losing money for several months. If this store is sold, the effect on the total profits of the company would be just like the subtraction of a negative integer from another integer.

A large balloon filled with hot air or gases, before making a flight, may be held down by sand bags. The balloon rises when enough of the bags have been thrown out. Throwing out each bag is like the subtraction of a negative integer from another integer.

TEST ON THE INTEGERS

CIRCLE THE LETTER OF THE CORRECT CHOICE

1. Which statement is correct?
   (a) Subtracting a whole number from another whole number always results in a whole number.
   (b) An equation in whole numbers always has a whole number solution.
   (c) An inequality in whole numbers always has a whole number solution.
   (d) Adding whole numbers always results in a whole number.

2. Which of the following is an example of the commutative property?
   (a) $(x + 2) + 5 = (2 + 5)$ (b) $(3 + 2) + 5 = 5 + (3 + 2)$
   (c) $3 \times (2 + 5) = 3 \times 2 + 3 \times 5$ (d) $(2 + 5) \times 3 = 2 \times 3 + 5 \times 3$.

3. Which of the following is an example of the distributive property?
   (a) $5 + (2 + 4) = (5 + 2) + 4$ (b) $(5 + 2) + 4 = 4 + (5 + 2)$
   (c) $5 \times (2 + 4) = (5 \times 2) + (5 \times 4)$ (d) $5 \times (2 \times 4) = (5 \times 2) \times 4$

4. Which of the following is an example of the identity property?
   (a) $10 \times (3 - 2) = 10$ (b) $10 \times (3 \times 2) = (10 \times 3) \times 2$
   (c) $10 \times (3 + 2) = 10 \times 3 + 10 \times 2$ (d) $10 + (3 + 2) = (10 + 3) + 2$

5. Let n be a whole number. Which statement illustrates the closure property?
   (a) $5 + n$ is a whole number (b) $(5 + n) + 2 = 5 + (n + 2)$
   (c) $5 \times (n + 3) = 5 \times n + 5 \times 3$ (d) $5 + n = n + 5$

6. If we said that, for all sets A and B, $A \cup B = B \cup A$, what would you call this property of the union of sets?
   (a) commutative (b) associative (c) closure (d) identity

7. Which statement is correct?
   (a) The set of integers is the same as the set of whole numbers.
   (b) There are some integers which are not whole numbers.
   (c) There are some whole numbers which are not integers.
   (d) The largest negative integer is 0.

8. Which statement is correct?
   (a) There is some whole number which is larger than all other whole numbers.
   (b) There is some negative integer which is less than all other negative integers.

(c) Between <u>any</u> two integers there is another integer.
(d) Each negative integer is less than every whole number.

9. How many negative integers are less than $^{-}5$?
(a) 4 (b) 5 (c) 6 (d) more than 1 million

10. How many negative integers are greater than $^{-}5$?
(a) 4 (b) 5 (c) 6 (d) more than 1 million

11. If n is an integer and $n < {}^{-}3$, what can be said for sure?
(a) n is a whole number (b) $n < {}^{-}2$
(c) $n < {}^{-}4$ (d) n is positive

12. $^{+}5 + {}^{-}2 = ?$
(a) $^{-}3$ (b) $^{+}7$ (c) $^{+}3$ (d) $^{-}7$

13. $^{-}8 + {}^{-}1 = ?$
(a) $^{-}7$ (b) $^{+}7$ (c) $^{-}9$ (d) $^{+}9$

14. $|{}^{-}5 + {}^{+}3| = ?$
(a) $^{+}8$ (b) $^{+}2$ (c) $^{+}5$ (d) $^{+}3$

15. If a and b are integers, then (a + b) will be both greater than a and greater than b if
(a) a is negative and b positive (b) a is positive and b negative
(c) a is positive and b positive (d) a is negative and b negative

16. Which statement is correct, where n might be any integer?
(a) $|{}^{+}5 + n|$ has to be at least as great as 5.
(b) $|{}^{+}5 + n|$ has to be at least as great as n.
(c) $|{}^{+}5 + n| = {}^{+}5 + n$, no matter what n equals.
(d) $|{}^{+}5 + n|$ could be negative.

17. $|{}^{-}7| + |{}^{+}6| = ?$
(a) $^{-}1$ (b) $^{+}13$ (c) $^{+}1$ (d) $^{-}13$

18. $({}^{-}6 + {}^{+}6) + ({}^{-}2) = ?$
(a) $^{+}2$ (b) $^{+}14$ (c) $^{+}4$ (d) $^{-}2$

19. In the equation $(n - 6) + 6 = 11$, what is n?
(a) 23 (b) 11 (c) $^{-}1$ (d) 0

20. $10 + (5 - 5) = ?$
(a) 5 (b) 0 (c) 20 (d) 10

21. In the equation $17 + (9 - n) = 17$, what is n?
(a) 9 (b) 0 (c) 10 (d) 8

22. Which of the following true statements about the union of sets looks like an identity property to you?
(a) $A \cup B = B \cup A$
(b) $(A \cup B) \cup C = A \cup (B \cup C)$
(c) $A \cup \{ \} = A$
(d) $A \cup B$ is always a set

23. $^{+}5 - (^{-}2) = ?$
(a) $^{+}3$ (b) $^{+}1$ (c) $^{-}7$ (d) $^{+}7$

24. $^{-}7 - {}^{+}4 = ?$
(a) $^{-}3$ (b) $^{-}11$ (c) $^{+}11$ (d) $^{+}3$

25. No matter which negative integer we subtract from $^{+}2$, the answer will be:
(a) less than the negative interer.
(b) between the negative integer and $^{+}2$.
(c) greater than $^{+}2$.
(d) definitely zero.

26. $^{+}5 - {}^{+}8 = ?$
(a) $^{+}13$ (b) $^{+}3$ (c) $^{-}13$ (d) $^{-}3$

TABLE 13

ITEM DIFFICULTY: INTEGERS TEST

Item Number	Number of Incorrect Responses
1	45
2	79
3	52
4	76
5	85
6	87
7	101
8	50
9	52
10	88
11	95
12	22
13	21
14	88
15	84
16	147
17	89
18	33
19	33
20	32
21	25
22	119
23	82
24	69
25	127
26	48

Note:

The number of subjects writing the test was 174.

APPENDIX B

TABLE 14

MEANS AND STANDARD DEVIATIONS
OF THE TREATMENT GROUPS

Treatment Group	I: Probability		II: Integers	
	Mean	S.D.	Mean	S.D.
Males				
I - Visual, II - Verbal	58.539	16.442	58.194	16.564
I - Verbal, II - Visual	56.987	13.701	59.699	12.087
Females				
I - Visual, II - Verbal	58.696	16.035	53.345	13.094
I - Verbal, II - Visual	49.068	14.902	54.849	19.750

In the tables which follow in this appendix, beginning with Table 16, the simultaneous plots of the two regression lines and scattergrams for the two treatment groups are presented, for those aptitude measures with which there was significant interaction.

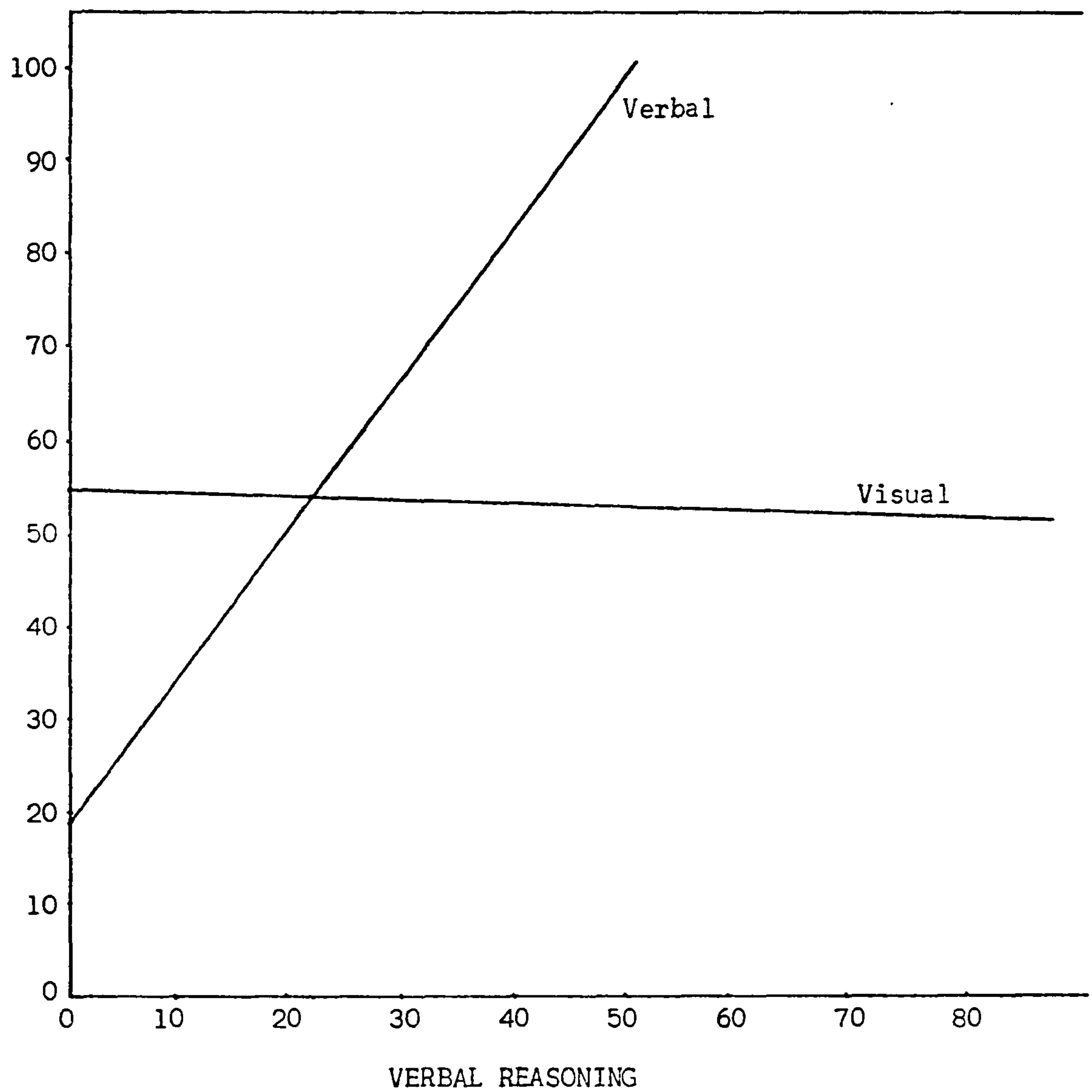
TABLE 15

CORRELATIONS OF POSTTEST SCORES
WITH DAT MEASURES

DAT Subscale	Probability		Integers	
	Visual	Verbal	Visual	Verbal
Males				
1	.687	.372	.617	.670
2	.776	.330	.584	.762
3	.788	.381	.664	.771
4	.298	.443	.597	.395
5	.348	.340	.680	.392
6	.596	.200	.398	.481
7	.484	.184	.452	.368
8	.372	.321	.590	.389
9	.564	.302	.404	.423
Females				
1	.663	.014	-.072	.655
2	.603	.366	.511	.665
3	.715	.446	.569	.738
4	.627	.278	.197	.677
5	.194	.183	.191	.078
6	.758	.157	-.002	.720
7	.593	.313	.231	.804
8	.526	.242	.462	.489
9	.652	.539	.462	.656

TABLE 16

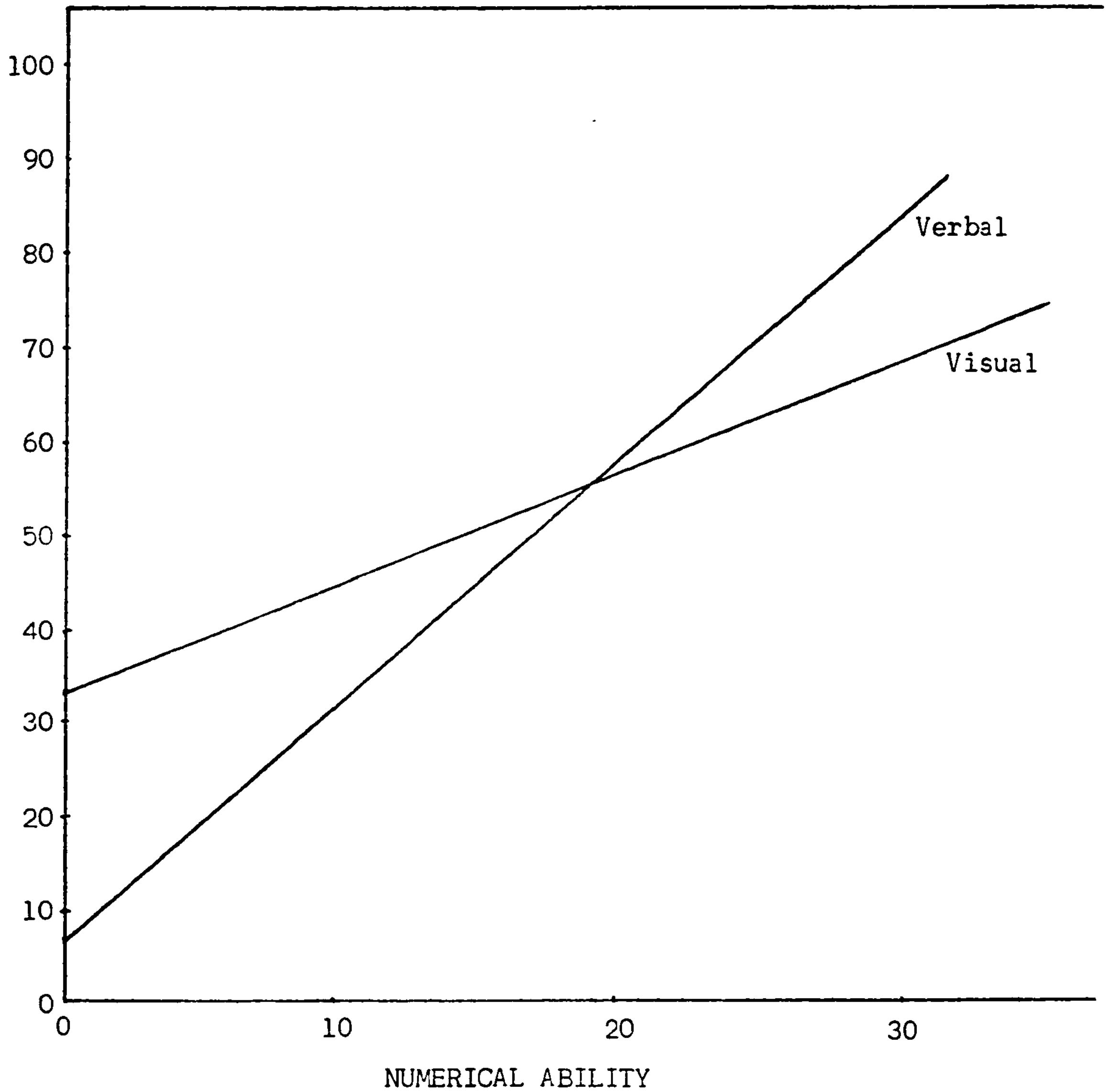
ATI FOR INTEGERS SCORES VERSUS DAT SCALE
VERBAL REASONING, FEMALES



	Intercept	Slope	Observed Range, DAT's
Visual	54.985	-0.072	10 - 78
Verbal	19.154	1.582	

TABLE 17

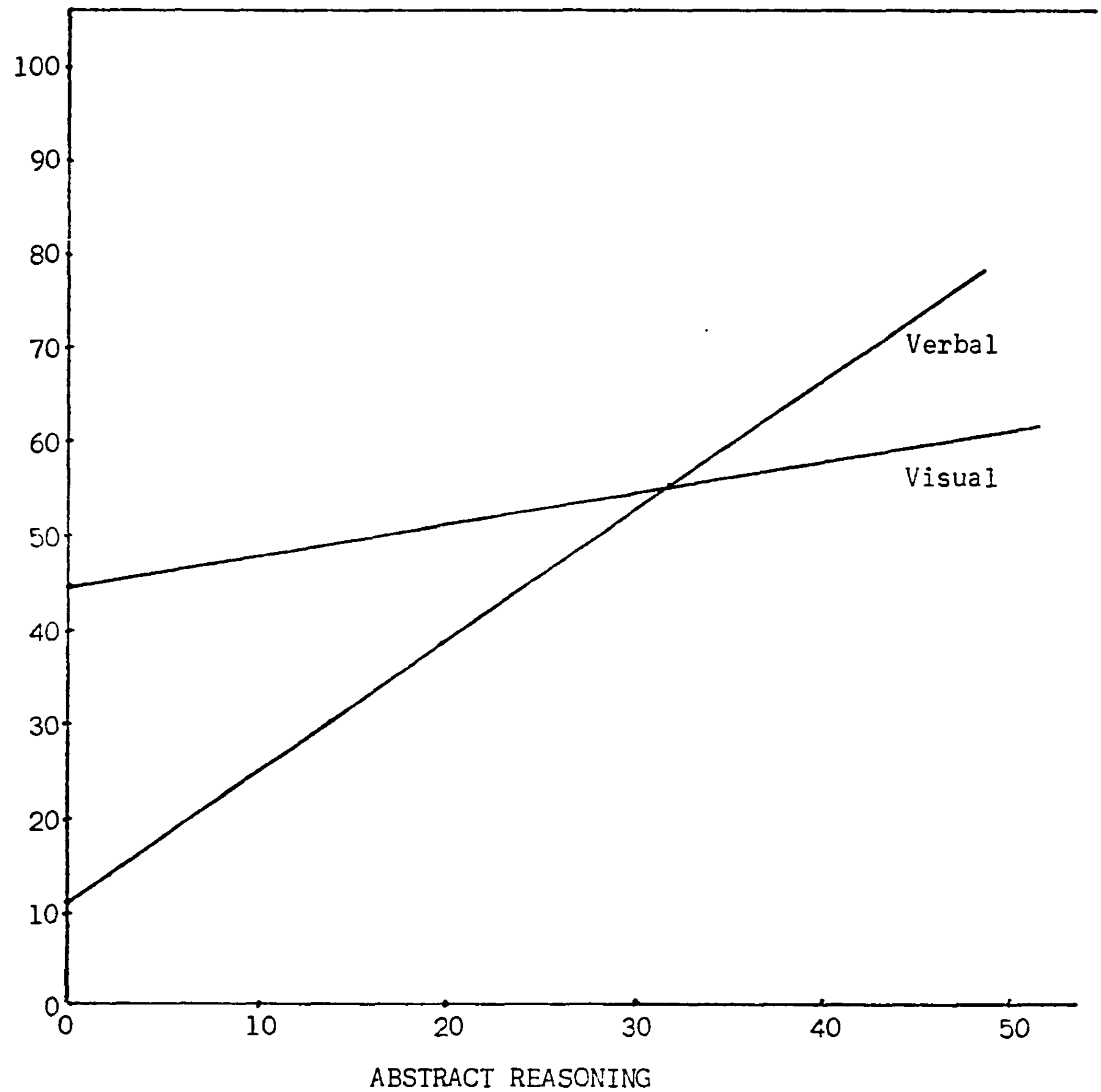
ATI FOR INTEGERS SCORES VERSUS DAT SCALE
NUMERICAL ABILITY, FEMALES



	Intercept	Slope	Observed Range, DAT's
Visual	33.566	1.083	8 - 30
Verbal	6.909	2.489	

TABLE 18

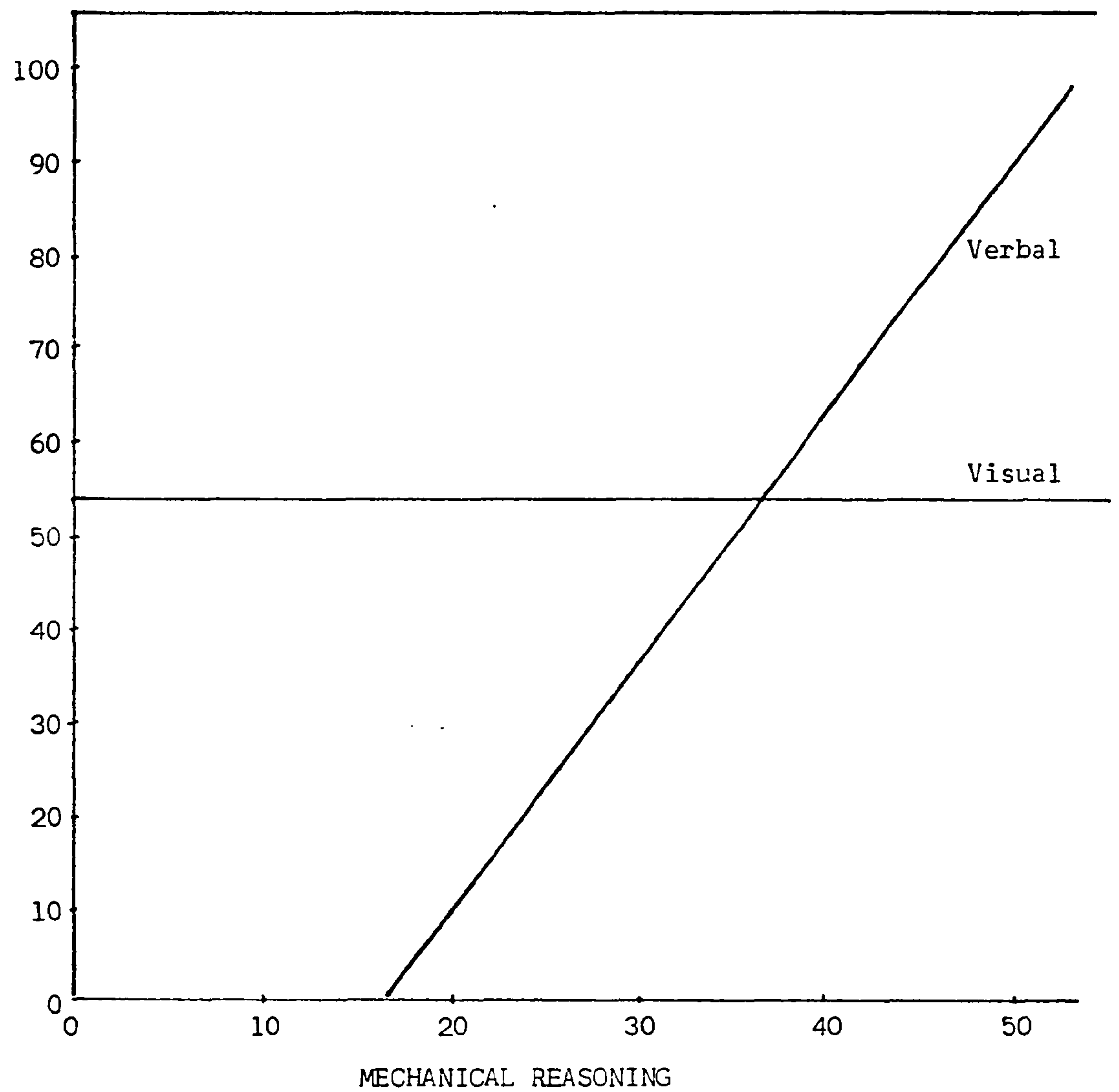
ATI FOR INTEGERS SCORES VERSUS DAT SCALE
ABSTRACT REASONING, FEMALES



	Intercept	Slope	Observed Range, DAT's
Visual	44.389	0.293	6 - 46
Verbal	10.815	1.354	

TABLE 19

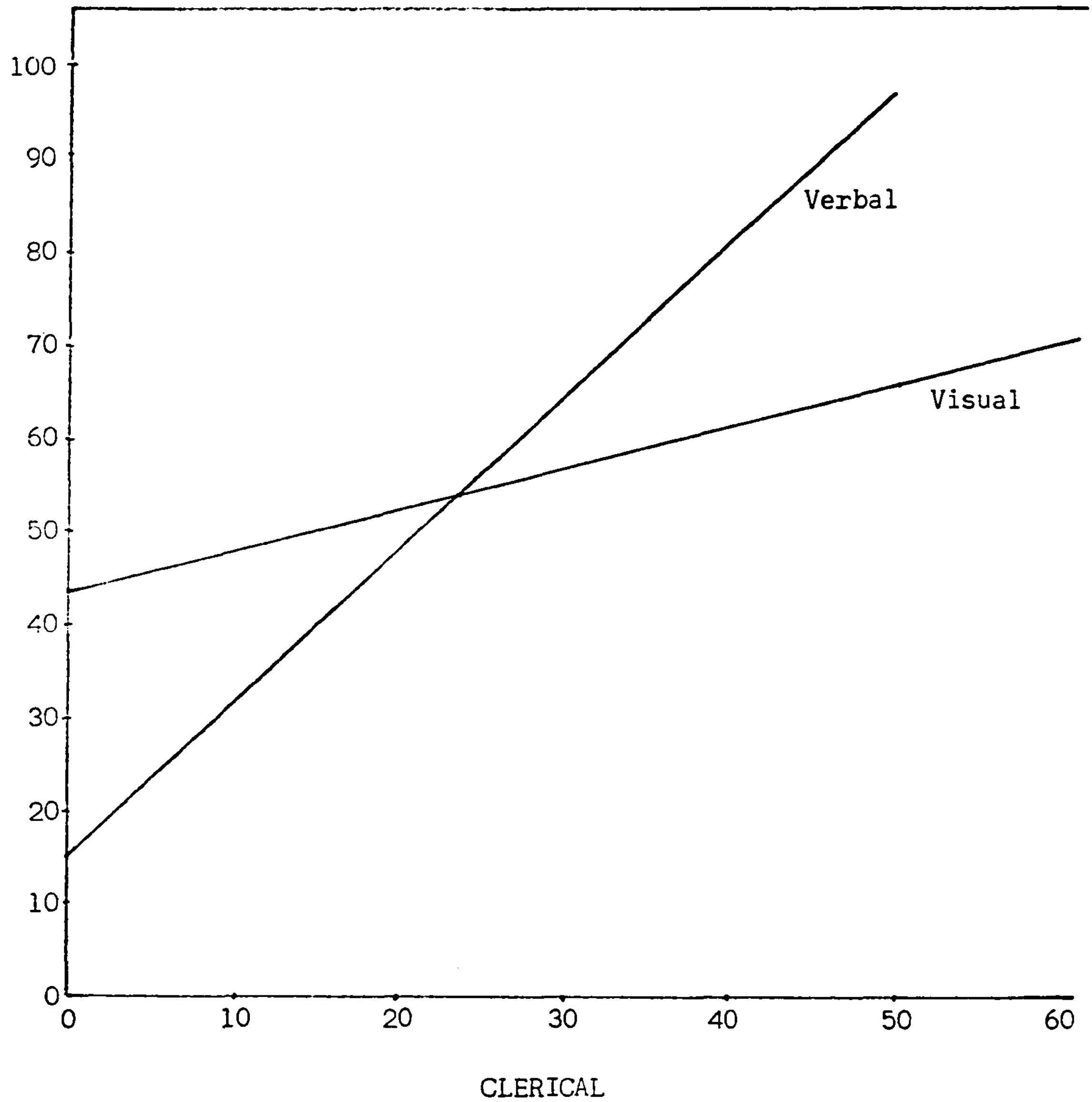
ATI FOR INTEGERS SCORES VERSUS DAT SCALE
MECHANICAL REASONING, FEMALES



	Intercept	Slope	Observed Range, DAT's
Visual	53.494	-0.004	26 - 47
Verbal	-41.869	2.581	

TABLE 20

ATI FOR INTEGERS SCORES VERSUS DAT SCALE
CLERICAL, FEMALES



	Intercept	Slope	Observed Range, DAT's
Visual	43.105	0.411	9 - 52
Verbal	15.638	1.579	

APPENDIX C

TABLE 21

MEANS AND STANDARD DEVIATIONS OF THE MODALITY GROUPS ON THE DAT MEASURES, MALE SUBJECTS

DAT Subscale	Visual Group	No Preference Group	Verbal Group
		Means	
1	23.000	22.318	20.833
2	19.750	20.909	18.083
3	42.750	43.045	38.917
4	31.333	35.636	32.833
5	40.917	43.682	38.083
6	48.583	47.500	42.583
7	33.833	33.273	25.750
8	63.417	58.727	60.333
9	24.083	22.500	19.833
		Standard Deviations	
1	9.145	7.403	9.428
2	5.786	6.480	6.156
3	11.655	12.572	13.688
4	11.911	9.189	7.184
5	6.543	10.144	10.229
6	6.374	7.866	8.618
7	10.214	13.090	9.176
8	11.381	15.923	15.168
9	5.143	6.247	7.222

TABLE 22

MEANS AND STANDARD DEVIATIONS OF THE MODALITY GROUPS
ON THE DAT MEASURES, FEMALE SUBJECTS

DAT Subscale	Visual Group	No Preference Group	Verbal Group
	Means		
1	19.917	22.773	25.000
2	18.917	17.727	20.500
3	38.833	37.364	45.500
4	29.917	29.182	37.583
5	46.833	44.909	42.250
6	36.083	36.545	38.500
7	20.417	22.863	33.000
8	68.083	69.500	66.750
9	22.750	23.682	28.167
	Standard Deviations		
1	5.696	14.419	7.337
2	6.960	4.842	6.346
3	9.806	11.155	11.674
4	9.774	9.762	6.230
5	9.806	8.535	10.814
6	5.977	5.152	6.557
7	4.033	8.498	8.257
8	12.034	12.902	18.709
9	5.707	7.530	8.277

TABLE 23

DAT WITHIN GROUPS CORRELATION MATRIX, MALES

	Variables								
	1	2	3	4	5	6	7	8	9
1	1.000								
2	.512	1.000							
3	.908	.821	1.000						
4	.338	.238	.328	1.000					
5	.286	.355	.341	.399	1.000				
6	.443	.128	.342	.276	.183	1.000			
7	.548	.463	.583	.271	.283	.558	1.000		
8	.453	.560	.566	.258	.366	- .034	.093	1.000	
9	.525	.541	.616	.104	.175	.008	.213	.706	1.000

TABLE 24

DAT WITHIN GROUPS CORRELATION MATRIX, FEMALES

	Variables								
	1	2	3	4	5	6	7	8	9
1	1.000								
2	.117	1.000							
3	.337	.797	1.000						
4	.028	.556	.624	1.000					
5	.125	.379	.311	.273	1.000				
6	.087	.359	.523	.532	.204	1.000			
7	.052	.430	.531	.560	.123	.583	1.000		
8	.107	.419	.519	.209	.218	.234	.438	1.000	
9	.298	.442	.629	.218	.154	.238	.273	.618	1.000

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