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GRADUATE COLLEGE

ECONOMIC OPTIMIZATION OF OFFSHORE OIL FIELD DEVELOPMENT

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

LESTER CLAUD FRAIR

Norman, Oklahoma

1973

ECONOMIC OPTIMIZATION OF OFFSHORE OIL FIELD DEVELOPMENT

APPROVED BY

DISSERTATION COMMITTEE

DISSERTATION

ECONOMIC OPTIMIZATION OF OFFSHORE OIL FIELD DEVELOPMENT

This study is concerned with the optimal development of offshore oil fields. A general offshore development model is formulated. Algorithms for the solution of this general model are developed and are shown to be computationally efficient.

The general offshore model considers the development questions of, (1) the number of platforms needed, (2) the size of each platform, (3) the location of each platform, (4) the assignment of wells to platforms and (5) the schedule of placing the platforms and drilling the wells. The objective of this model is the maximization of discounted after-tax cash flows subject to (1) platform and well constraints, (2) production limiting constraints and (3) depletion constraints.

Two different procedures are examined to obtain solutions to the general offshore development model, namely (1) the dependent subproblems approach and (2) the independent subproblems approach. Both solution procedures require that the general model be decomposed into sub-models. This is necessary because the size and nonlinear nature of the general development model make it impossible to effectively apply any known solution procedure to the model as a whole.

Computational results are presented for the independent subproblems procedure for two example offshore development problems. These results, when compared with an intuitive approach of developing the offshore property as fast as possible, appear very encouraging.

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CHAPTER I

INTRODUCTION

With most economists forecasting great increases in the worldwide demand for oil and gas over the next 20 years, it is apparent that offshore areas will play an ever-increasing role in supplying the world's fossil fuel requirements. This trend is already apparent, with "offshore investment by the petroleum industry approaching $20 billion and . . . growing at the rate of $3 billion a year" (28). Because of this it is speculated that petroleum companies will invest an ever-increasing portion of their research and development funds in offshore development. Not only will these companies seek and extensively produce oil and gas offshore, but they will probably also be among the first to carry out large-scale commercial offshore mining for minerals such as gold, platinum, sulfur, iron and tin.

History of Offshore Oil Development

Early offshore development took place over inland waters such as Caddo Lake, Louisiana, in May of 1911 and

Lake Maracaibo, Venzeuela, in the early 1920's. The presence of shipworms in Lake Maracaibo require that pilings, initially built of timber, be replaced every six to eight months. This lead to the first use of concrete pilings for platforms in 1927. Later, production and drilling operations moved from inland to costal waters. The first offshore drilling of any consequence in the Gulf of Mexico took place in the late 1930's when the first well was brought in 6000 feet off Creole, Louisiana. But according to one author (13) the offshore age didn't truly begin until the summer of 1947, when the Kerr-McGee Oil Company erected a drilling platform 10.5 miles from land. This well was significant because now, for the first time the drilling platform was essentially independent of land-based facilities.

Since that time great strides have been taken in all aspects of offshore drilling. Drilling platforms to fill just about every need have been designed and built; drilling and production have become commonplace out to 300 feet of water. Automation and computerization of equipment for both drilling and production are now widespread. In fact, the oil industry no longer questions its ability to produce oil from nearly any coastal waters, but it now questions the economics of such a production (28).

In the future, as present oil reserves are depleted and demand for petroleum products increase, the oil industry will move further and further from shore in order to tap the remaining oil reserves beneath the sea. This will necessitate operating in much greater depths of water (usually structures are now placed in less than 300 feet of water). The oil industry is aware of this and has been investigating methods and equipment which will allow them to operate in greater depths of water. Theoretically they now are capable of

1. building fixed platforms in water depths up to 1000 feet.
2. drilling in 20,000 feet of water from ships.
3. establishing complete production, separation, and storage facilities underwater. (28)

Offshore Oil--Search and Drilling

The exploration department of oil companies is usually charged with the responsibility of searching for new sources of oil. This search is most often begun by selecting those areas of the world in which, at some period in geologic time, the environment was conducive to the formation of hydrocarbons. Regional areas are then studied by means of such geophysical methods as magnetic and gravimetric surveys, as well as sonic readings. These studies then allow the geologist to construct detailed maps of the subsurface structure and thereby locate sites in which the probability of finding commercial deposits of oil is relatively high (15).

Once oil has been discovered by an exploratory well, step-out wells are then drilled to determine the physical characteristics of the field (size, pressure, temperature, porosity, etc.). At this point top management makes the decision as to whether or not to develop the field. If the decision is to continue with development, then the geologists and petroleum engineers, utilizing the information received from exploratory and step-out wells, decide on the wells to be drilled as production wells (10).

Presently most offshore fields are developed by drilling directionally from fixed platforms to various targets. Once the targets are defined and the oil company decides on the number, size, and location of the fixed platforms, as well as the time schedule for their placement, the drilling of individual wells is begun. The cost of drilling each well is dependent on the length of the drilled hole and the angle at which the hole is drilled (10).

Problem Statement and Major Contributions of the Research

As more and more funds are committed to the development of offshore petroleum resources, investment decision makers will need quantitative techniques that indicate how to optimally develop offshore oil and gas reserves. These quantitative techniques usually take the form of mathematical models that optimize the company's objectives (as realistically as possible) subject to physical, financial, and market constraints. In the development of offshore oil properties, management would like to utilize mathematical models that answer such questions as:

1. Where are fixed drilling platforms to be located?
2. How many and what size offshore drilling platforms are needed?
3. Which platform should drill each well?
4. How is the drilling of wells and the placement of platforms scheduled?

The primary concern of this research is to develop a model capable of answering the questions posed above. This entails the formulation of a general optimization model and the determination of relatively efficient solution procedures. Solution procedures must be fairly efficient so that sensitivity analyses may be obtained at realistic costs. Such analyses demonstrate how susceptible the model is to errors in pertinent information that must be estimated. For example, the investment decision makers will certainly be interested in how sensitive solutions are to inaccuracies in forecasted production rates.

This research will contribute to the field of optimum offshore oil development in the following ways:

1. Depletion is included in after-tax optimization models. It is shown that not incorporating depletion or using an approximation for the depletion

term in an after-tax optimization can result in nonoptimal investment decisions.

2. The problems of the development of offshore oil fields and the scheduling of this development are investigated. The problems are then formulated mathematically in the form of a general optimization model whose objective is to maximize the net discounted after-tax cash flow.
3. Computationally efficient algorithms, which allow sensitivity analyses to be performed, will be developed to solve the general optimization model for the development and scheduling of offshore development activities.

Review of Development Drilling Models

"When one considers that a single well drilled in the offshore areas of Louisiana and California costs approximately three million dollars, the risk to capital is staggering" (15). It is no wonder that investment managers operating under such risks seek decision-making assistance in the form of development drilling models, the solutions of which specify the most profitable way to develop offshore oil fields. The following chart presents a synopsis-type description of optimization models that have been applied to petroleum development activities. Several of these optimization models are then discussed in more detail.

One of the first approaches was that of Aronofsky

Model Identification	Development Questions Considered: Schedule of Development	Location of Development Facilities	Pipeline Development	Well Spacing	Secondary Recovery	Facility Expansion	Type of Model	Taxes Considered	Mathematical Model — Applicable To: Single Well	Multiple Well	Single Reservoir	Multiple Reservoir	Objective: Maximize Profit	Minimize Cost	Other	Constraints: Physical	Production Limitations or Requirements	Transportation	Financial and/or Regulatory	Applicable Time Span: Single Period	Multiple Period	Solution Procedures Adopted: Deterministic	Heuristic	Simulation	Utilization of Model: Planning Tool	Operational Tool	Phase of Development Process to Which the Model is Applicable: Property Acquisition and Exploration	Development	Production	Transportation
Aronofsky & Lee (Ref. 1)	X						Linear Programming	Yes		X			X			X	X	X			X	X			X			X	X	X
Aronofsky & Williams (Ref. 2)	X						Linear Integer Programming	Yes		X	X		X			X	X				X	X			X	X		X	X	X
Berilgen (Ref. 3)	X					X	Systems Model Utilizing Linear Integer Optimization	Yes		X	X		X			X	X		X		X	X		X	X		X	X	X	
Bohannon (Ref. 6)	X		X		X	X	Linear Mixed-Integer Programming	Yes				X	X			X	X	X	X		X	X			X			X	X	X
Coats (Ref. 9)	X						Systems Model Involving Partial Differential Equations	No		X				X		X	X				X	X			X	X			X	
Devine & Lesso (Ref. 11)		X					Nonlinear Mixed-Integer Programming	No		X	X	X		X		X				X		X	X		X	X		X		
Dougherty & Thurnau (Ref. 12)	X		X		X	X	System Model Utilizing Linear & Nonlinear Optimization	Yes		X	X	X	X			X	X	X	X		X	X		X	X		X	X	X	X
Frair	X	X					Nonlinear Mixed-Integer Programming	Yes		X	X	X	X			X	X		X		X	X	X		X	X		X	X	
Hartsock & Greaney (Ref. 16)	X						Stochastic Programming	No		X	X	X		X							X	X						X	X	
Rosenwald & Green (Ref. 26)	X			X			Reservoir Model in Conjunction with Mixed-Integer Programming	No		X					X	X	X				X	X		X	X	X		X	X	
Rowen & Warren (Ref. 27)	X						Systems Model Involving Differential Equations	No			X		X								X	X			X			X		
Teichroew, Lesso, Rice & Wright (Ref. 31)	X						Dynamic Programming	Yes	X	X			X				X			X		X			X				X	

and Lee (1) who developed a linear programming model that scheduled oil production from a collection of inland oil reservoirs located in the same geographical area. The model's objective is to maximize the before-tax cash flow. The variables of the linear program are the average production rates of the reservoirs during discrete time periods. The constraints are primarily material balance relationships. This early model seems unrealistic in that

1. It is only valid for a reservoir in which all the wells have been drilled.
2. It does not allow for the production scheduling of individual wells in the reservoir.

Aronofsky and Williams' (2) extended the work of Aronofsky and Lee to multi-well systems and in addition developed a new linear programming model to schedule production. This approach assumes that production from a particular well will follow a specified production-rate decline curve. The objective of this new model is to maximize the total discounted before-tax cash flow subject to constraints on rig dynamics and material balance relationships. This model may be criticized because:

1. It assumes the production rate of each new well is independent of the number of wells already present.
2. It assumes the user of the model is capable of specifying "a priori" how production will decline with time.

A paper by Coats (9) considers the problem of determining an optimum drilling schedule for a gas field that is already partially developed. The objective of the model is to meet a field productivity requirement with as few wells as possible. The solution procedure employed is to utilize a dynamic programming algorithm in order to minimize the number of wells drilled at each successive time increment (stage). The algorithm obtains the numerical solution of a particular differential equation describing semi-steady-state gas flow in a reservoir. Coates' assumption that the reservoir rock properties are known with sufficient accuracy to specify the coefficients of the governing, differential equation seems questionable.

Rowan and Warren (27) consider the problem of what drilling and production policy should be implemented subject to practical constraints for a new or partially developed reservoir so as to maximize profit. "The reservoir is considered to be a dynamic system, the behavior of which is reflected in the pressure-production history" (27). This concept leads to a system model which has production as the decision variables and reservoir pressure as a state variable. The authors state that this system approach to obtaining a solution has the advantage of dealing with coefficients of a governing differential equation rather than with variables which are subject to a considerable amount of uncertainty.

Possible disadvantages of this approach are:

1. It doesn't allow random production schedules, but rather requires that they be in the form of specified algebraic equations.
2. This model is applicable to a single reservoir only.

Hartsock and Greaney's (16) development drilling model is analogous to an inventory problem in that it consists of setup costs and shortage costs. "The setup costs reflect the costs incurred in providing additional crude oil production capacity, while the shortage costs represent a penalty for having insufficient crude oil production capacity to meet demand" (16). Production rates and demand are treated as random variables of the uncertainty associated with each quantity. The objective of this model is to minimize the total cost function, which contains expected values for the random variables. The function was found to be nonlinear and was solved by Hooke-Jeeves pattern search method. It appears that further work on the model should include:

1. redefining the objective function so that it measures after-tax cash flow.
2. placing constraints on the number of wells drilled and completed in any given time period.

The work of Devine and Lesso (11) considers the problem of finding the proper number, size and location of drilling platforms, and the allocation of targets to platforms so as to develop an oil field at minimum cost. The solution procedures adopted by the authors are heuristic in nature in that they provide "good" answers but not necessarily optimal ones. This model could be made more realistic if:

1. transportation costs were included
2. production rates and crude oil demand were forecast so that the objective of the model could be to maximize the after-tax cash flow.

Bohannon (6) presents a linear mixed-integer programming model that finds the optimum development plan for a multi-reservoir system. This model determines (1) the annual production rate for each reservoir, (2) the number of development wells to be drilled each year in each reservoir, and (3) the timing of major capital investments such as tying in unconnected fields, initiating secondary recovery projects and expanding pipeline facilities. Values for these variables are determined so as to maximize the total discounted "net cash" flows subject to reservoir production constraints, pipeline constraints and facility expansion constraints. This model is open to the following criticisms:

1. The objective function of the author's model is not an exact measure of the profit resulting from the development of multi-reservoir systems.

2. The time and expense required to solve reasonable size development problems with Bohannon's linear mixed-integer programming model may be prohibitive.

CHAPTER II

OFFSHORE DEVELOPMENT PARAMETERS AND TAX CONSIDERATIONS

Offshore Development Parameters

There are a number of parameters that influence offshore development activities. For our models, we will assume that these parameters have been fixed prior to solving the model. It may be important, however, to determine how sensitive the solutions of our model are to variations in the parameters. The parameters are:

1. An upper limit on the amount of oil produced during any future time period.
2. The location of the individual sources of oil (targets) to be tapped.
3. The production capability of wells drilled to the individual targets.

Upper Limit on the Amount of Oil Produced During any Future Time Period

There are various items that an oil company should consider when attempting to fix an upper limit on the amount of oil to be produced in any future time period. One consideration would be the projected demand for oil in these future time periods. Other considerations would be the projected capability of (1) transportation facilities, (2) storage facilities, and (3) oil refinery processing facilities.

Location of Individual Targets

The location and extent of oil sources are determined primarily through seismic readings and exploratory drillings. These location tools can be used to provide the oil field developer with a graphical representation of the sub-surface strata. This enables him to define subsurface regions (strata) that are likely to contain hydrocarbons. The hydrocarbons present in the subsurface strata usually occur in the form of large continuous reservoirs or small individual pools.

If the oil bearing stratum is in the form of a large continuous reservoir the well spacing that will most effectively drain the reservoir must first be determined. This determination depends on such things as the rock properties of the reservoir, the characteristics of the petroleum fluids, etc. Muskat (23) discusses this problem and how it may be approached. Second, since the production rate of each well, in most cases, is subject to legal constraints (allowables), the selection of additional targets so as to meet demand may be required.

Some oil fields are composed of a number of distinct and relatively small pools of trapped oil. Usually these small individual pools are considered as separate targets each of which is tapped with just one well.

Production Capability of Individual Reservoirs

The production capability of a reservoir is a function of a number of factors. Some of the most important are formation extent, formation physical properties, and recovery methods.

The extent of the producing formation is usually determined by means of electric logs, radioactivity logs, core analysis and local geological information. The physical properties of the producing formation include such things as composition, porosity, permeability, pressure, temperatures, etc. (23). The recovery methods used depend upon the reservoir energy available for moving the oil to the surface. This energy is a function of the ambient physical state of the oil reservoir, i.e., the properties of the formation. Examples of primary recovery methods (those methods that utilize only natural reservoir energy for recovery) are gas cap drive, depletion drive, and water drive (7).

Production capability may be forecast through the use of production rate-decline curves. This method of forecasting the future behavior of oil fields is used extensively throughout the oil industry (7). Decline curves are usually based on a combination of the following:

1. reservoir engineering methods that predict such

things as volumetric oil reserves and flow characteristics of the trapped fluids

2. data from comparable wells

3. general regional geological data

A typical production-rate-decline curve is presented in Figure

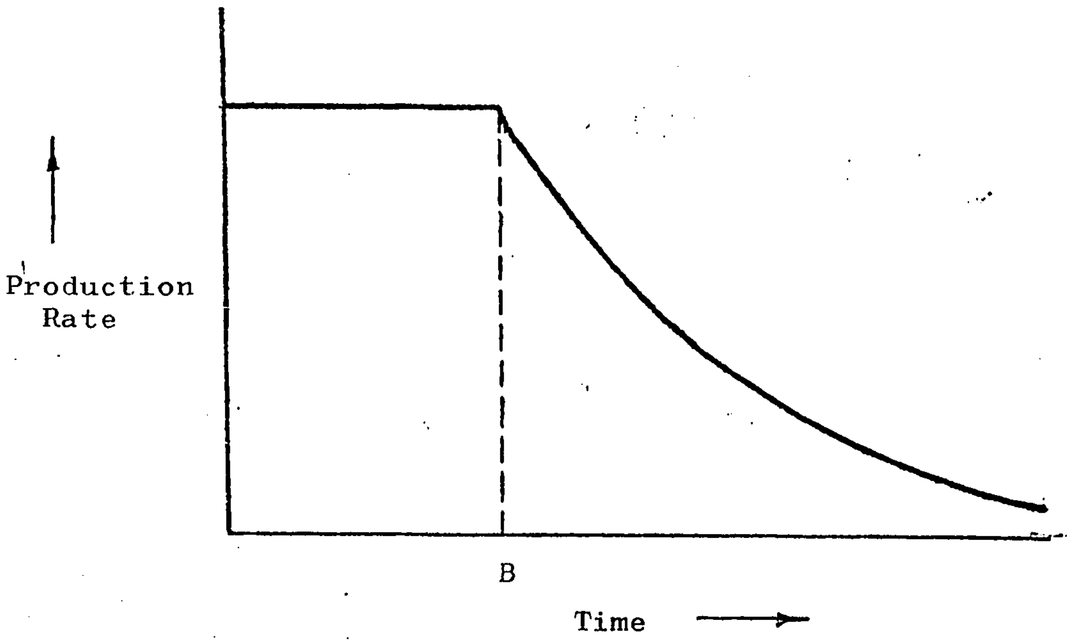

Figure 1. A Typical Production-Rate-Decline Curve.

Theoretically the production-rate capability of a reservoir should start declining immediately after it is placed on production if it is operated at its maximum production rate. However, many reservoirs are produced at a rate less than the maximum possible because of economic and/or governmental restrictions such as

1. state or federal allowable productions

2. transportation capabilities

3. storage and refinery capacities
4. marketing considerations

This results in essentially a constant production rate over a period of time (from time 0 to time B of Figure 1). Once the production rate of a reservoir starts decreasing the decline with time can most often be characterized as exponential or hyperbolic (from time B on of Figure 1) (25). The author proposes to utilize applicable production-rate-decline curves in his development model to forecast future production. The production-rate-decline method was chosen because:

1. It is a simple method which is a familiar and trusted tool to those who make investment drilling decisions.
2. This method of forecasting production capability is relatively simple to incorporate into the proposed development model.

The accuracy of the production capability forecasts depends most notably on

1. when within the development phase of a field the forecasts are prepared. The later the preparation of forecasts the more accurate the available data should be.
2. the type of forecasts. If, for instance, daily forecasts are attempted with any appreciable lead time, the forecasts will probably get more inaccurate as the lead time increases. On the other hand,

forecasts of average yearly (or even monthly) production rates should be fairly accurate.

It appears that the forecasts of primary interest are those which predict average production rates for a time period of one year or longer.

Tax Considerations

The consideration of taxes in optimization models serves in most cases, if not all, as a complicating factor. This is especially the case in oil field development models in which tangible costs and depletion must be taken into account.

For income tax purposes tangible cost items are those which must be depreciated. This is in contrast to intangible costs which may be expensed (22). These tangible costs complicate the optimization model since their calculation requires:

1. specifying which allowable method of depreciation is to be used.
2. an estimate of salvage values (as functions of time) for such things as well equipment and casings.

Extractive industries must also consider the allowable depletion they may claim to reduce their income taxes. "Allowable depletion is the amount in dollars which may be deducted from taxable income in a year. It is the greater of cost depletion or percentage depletion" (22).

1. Cost depletion (CD) is a fraction of the undepleted

investment in a property (initial investment less the previously deducted depletion). The fraction is the ratio of the amount of reserves produced during the current year to the total amount of reserves remaining

CD = cost depletion = $\frac{U}{R} \cdot P$

U = undepleted investment in property

P = amount of oil, gas, or mineral produced during the year

R = amount of oil, gas or mineral estimated to be left in the reservoir or mine plus the current year's production

2. Percentage depletion (PD)

"The discoverer of an oil or gas field did not buy a producing property. He recovers his capital through percentage depletion which is 22% of gross income--not to exceed 50% of the net income from the property" (22).

PD = percentage depletion

GI = gross income (the total revenue derived from the property in a year)

NI = net income (gross income less expenses)

In summary, the allowable depletion (DA) is

$$DA = \text{maximum} \begin{bmatrix} CD = \frac{U}{R} \cdot P \\ \\ PD = \text{minimum} \begin{bmatrix} 22\%\ GI \\ \text{maximum} \begin{bmatrix} 50\%\ NI \\ 0 \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (1)$$

In the past oil field development models have usually ignored tax considerations. An example of this is (2) in which the authors maximize before-tax cash flow. Other models bypass tax considerations by minimizing costs (10), (16). Simplifying assumptions were made in other development models that permitted taxes to be included in the model.

One model of maximizing after-tax earnings which incorporate depletion allowances is presented by Teichroew, Lesso, Rice, and Wright (31). This paper develops dynamic programming algorithms for computing the optimal production level subject to a specified demand for each of a number of properties. Although these algorithms theoretically guarantee an optimal solution, the nature of the solution technique, dynamic programming, limits their application in two ways:

1. The number of state variables required by the dynamic programming model quite often becomes large enough to make this technique impractical because of limited computer time and storage.
2. The dynamic programming procedure can only consider a discrete number of alternate production levels. This could lead to a nonoptimal solution (depending on the levels selected for consideration).

Bohannon (6) includes an "effective tax rate" constant to account for the effects of taxes and depletion in his formulation of a mixed 0-1 integer, continuous-variable, linear programming model for development of multi-reservoir pipeline systems. This effective tax rate is obtained by consulting past income tax records to find what percentage of net income from oil actually went for taxes under a given tax rate. This technique for including taxes in a development model, although very simple, can result in a nonoptimal decision if the objective is to maximize discounted after-tax cash flow, as in Bohannon's model. This is illustrated in the following example:

A company must decide between two feasible plans for developing a piece of lease property. In each plan it is assumed that each oil well requires an initial investment of $15,000,000 and can produce for 3 years.

<u>PLAN I</u>

This plan calls for drilling of three wells during the first year of development. The estimated yearly data of importance are in Table 1.

TABLE 1

PLAN I DRILLING INFORMATION

Year	Production (10^6 barrels)	Gross Income (10^6 \$)	Cost (10^6 \$)	Undepleted Investment (10^6 \$)	Reserves (10^6 barrels)
1	12	36	63	45	90
2	24	72	21	39	78
3	15	45	15	27	54

PLAN II

This alternative calls for drilling two wells during the first year and one well during the second year. Yearly data for this plan are in Table 2.

TABLE 2

PLAN II DRILLING INFORMATION

Year	Production (10^6 barrels)	Gross Income (10^6 \$)	Cost (10^6 \$)	Undepleted Investment (10^6 \$)	Reserves (10^6 barrels)
1	8	24	40	30	60
2	20	60	34	41	82
3	18	54	17	31	62
4	5	15	5	22	44

The projected after-tax earnings are then calculated for both plans using two different methods. Method I assumes an effective tax rate of 36% to account jointly for taxes and depletion whereas Method II utilizes the depletion formula (1). If a discount factor of 20% is used, the total discounted after-tax cash flows are as given in Table 3.

TABLE 3

AFTER-TAX COMPARISONS OF TWO INVESTMENT PLANS

	Method I	Method II
Plan I	$19,378,000	$26,003,000
Plan II	19,812,000	25,892,000

Thus, if the investment decision is made by assuming an effective tax rate to account for taxes and depletion (Method I), then the development should proceed according to Plan II since its discounted net cash flow is greater than in Plan I (see Table 3). On the other hand, actually calculating depletion and taxes separately (Method II) would indicate Plan I should be followed for development of the property. Grouping depletion and taxes together and applying an effective tax rate, then, can lead to non-optimal decisions, as shown in the above example.

In contrast to the above-noted models depletion will be incorporated into the model developed here via linear integer programming. This may be accomplished if it is assumed that the allowable depletion may be modeled as a piecewise linear function of a number of decision variables. Cost depletion (defined p. 18), a nonlinear function, will be omitted from our model. This is done to simplify the model and is justified by:

1. In general cost depletion usually has meaning only early in the life of a property when net income is

very small or negative and/or undepleted investment is very large (22).

2. Intuitively the exclusion of cost depletion from our model should have little effect on the final solution since its objective is to compare the relative attractiveness of alternate plans via after-tax cash flows rather than determining specific amounts of profit for each plan.

The exclusion of cost depletion from the allowable depletion formula (1) leaves

$$D = \text{minimum} \begin{bmatrix} 22\%\ GI \\ \text{maximum} \begin{bmatrix} 50\%\ NI \\ 0 \end{bmatrix} \end{bmatrix} \qquad (2)$$

where D is a piecewise continuous linear function if GI and NI are continuous linear functions. For example, if GI and NI are functions of the single variable P, then D, a function of GI and NI, may be expressed graphically as a function of P as in Figure 2.

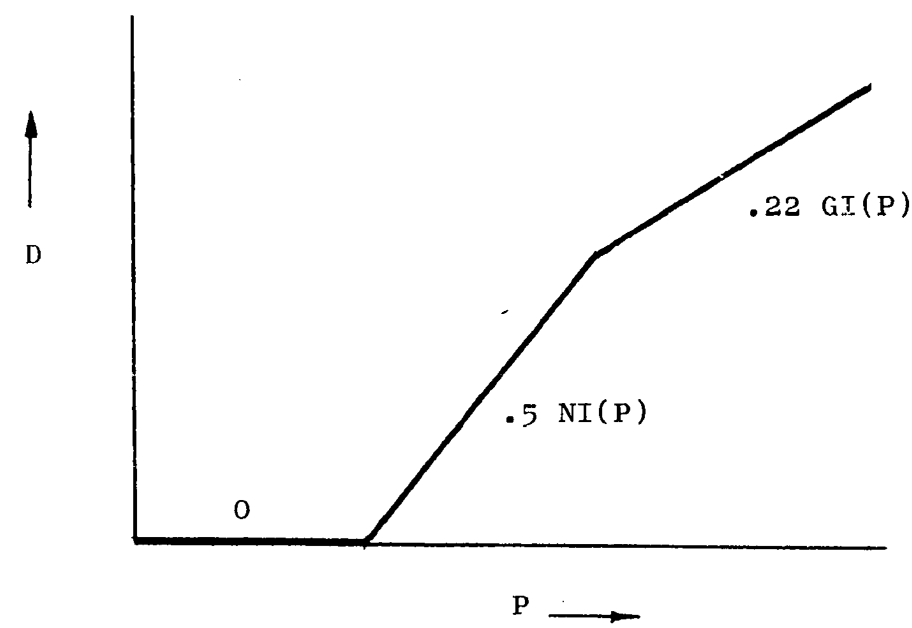

Figure 2. Depletion (D) as a Function of a Single Variable, P.

Since after-tax earnings (ATE) may be represented by the formula

$$\begin{aligned} ATE &= NI(P) - r(NI(P) - D) \\ &= (1-r)NI(P) + rD \end{aligned}$$

where r = the applicable income tax rate,

then graphically ATE as a function of P would appear in Figure 3.

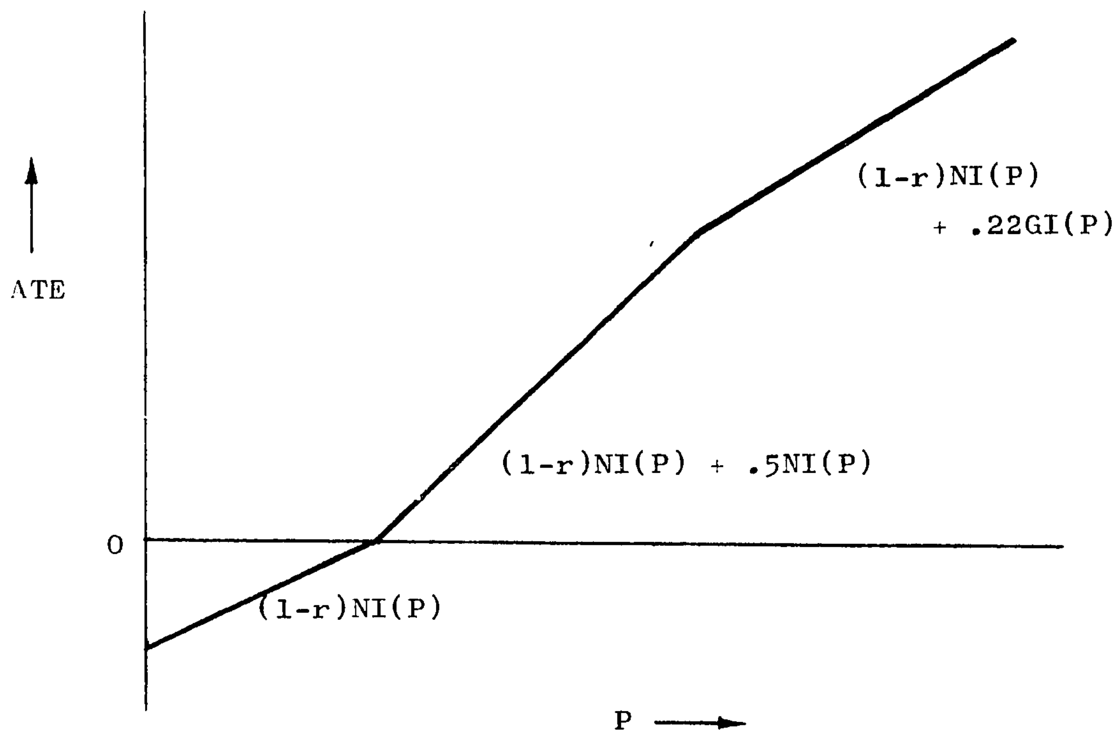

Figure 3. After-Tax Earnings (ATE) as a Function of a Single Variable, P.

Thus, if the objective of the model is to maximize after-tax earnings (ATE) we see from Figure 3 that we should be able to model this function with a linear mixed-integer (0-1) continuous programming formulation.

If $D = \text{minimum} \begin{bmatrix} .22GI \\ \text{maximum} \begin{bmatrix} .5\ NI \\ 0 \end{bmatrix} \end{bmatrix}$, the model would be:

Maximize ATE = $(1-r)NI + rD$

subject to

$$D \le .22GI$$
$$D \le .5NI + M(1-z)$$
$$NI \le zM$$
$$D \le zM$$
$$D \ge 0$$

z = 0-1 variable; M = very large number

where The value of M is selected so that it will be greater than any possible value NI could obtain.

D = allowable depletion

GI = gross income

NI = net income before taxes

r = applicable income tax rate.

It should be pointed out that cost depletion (CD) may be included in a linear, mixed-integer, continuous variable model for maximizing after-tax cash flow under special circumstances. From page 18, CD = $\frac{U}{R} \cdot P$. If we let $\delta = \frac{U}{R}$, the ratio of two linear functions, then δ may be considered a constant if the ratio of initial investment required to the amount of reserves initially discovered remains constant over all possible development plans. In this case, allowable depletion (DA) as a function of P would appear as in Figure 4.

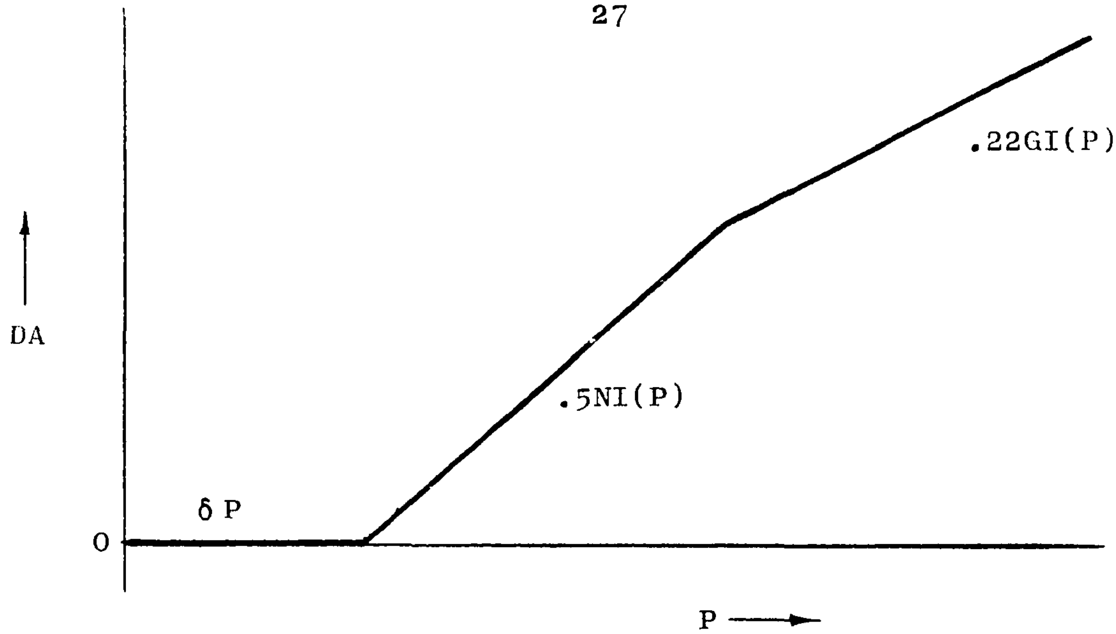

Figure 4 . Allowable Depletion (DA) as a Function of a Single Variable, P.

After-tax earnings (ATE) as a function of a single variable P where cost depletion is considered would appear as in Figure 5.

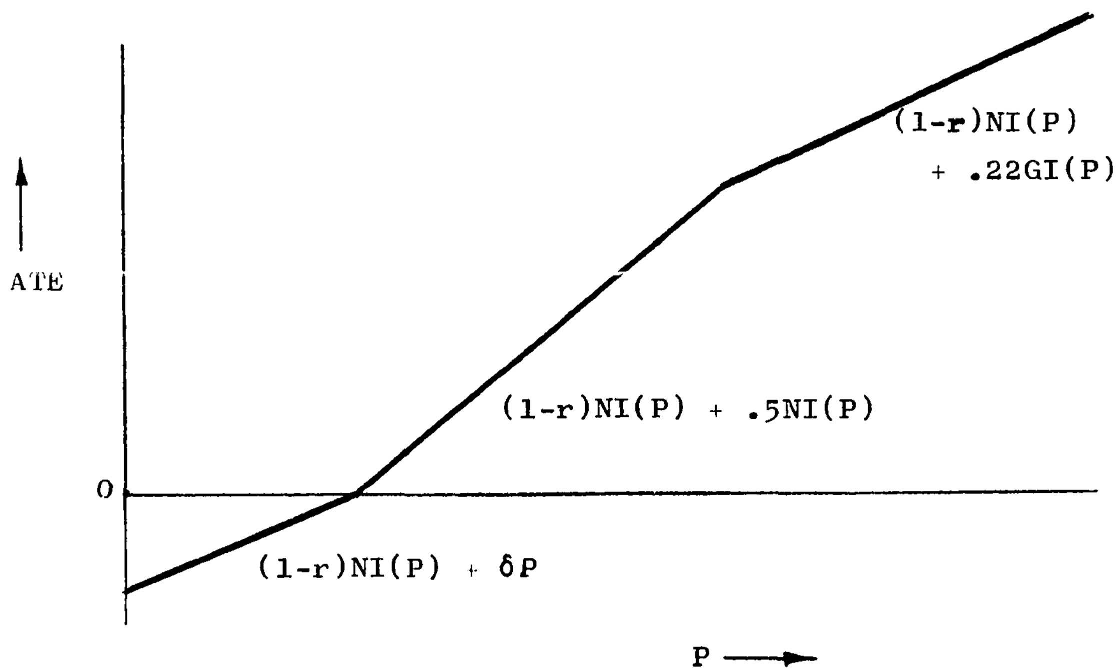

Figure 5 . After-Tax Earnings (ATE) as a Function of a Single Variable P, where Cost Depletion Is Considered.

If DA = maximum $\begin{bmatrix} CD - \delta P \\ D(\text{equation } 2) \end{bmatrix}$, where $\delta = \frac{U}{R}$, then the mixed-integer programming formulation of the function presented in Figure 5 is:

Maximize ATE = (1-r)NI + rDA

subject to

$$D \leq .22GI$$

$$D \leq .5NI + M(1-z)$$

$$NI \leq zM$$

$$D \leq zM$$

$$DA \leq M(1-y) + D$$

$$DA \leq My + \delta P$$

$$D \geq 0$$

$$DA \geq 0$$

y and z = 0-1 variables

M = very large number selected so that it will be greater than any possible value NI could attain.

The use of the suggested after-tax model, even though it gives a more realistic measure of the profit of possible investments, may be criticized because it is more complex than the before-tax model. This, for example, would be the case for the model presented by Aronofsky and Williams (2) if it were modified so as to maximize after-tax cash flow. Their original model could be solved with linear programming but the consideration of taxes and depletion in their model would require a linear mixed-integer solution algorithm.

In general, it is not evident whether the complexity of an after-tax model is justified by its capability to determine improved solutions.

CHAPTER III

THE DEVELOPMENT MODEL

The proposed development model presented in this chapter determines

1. the number of platforms needed
2. the size of each platform
3. the location of each platform
4. the assignment of wells to platforms
5. the schedule of placing the platforms and drilling wells,

so as to maximize the total discounted after-tax cash flow over a specific time period, while not exceeding a specified upper limit on production. After the presentation of the model some discussion is given concerning the information which must be supplied to the model.

General Development Model

The general offshore development model is now presented. First we define the following:

Constants

(a_i, b_i, c_i) = location coordinates for target i

NR = total number of reservoirs

NW = total number of targets

NT^m = total number of targets in reservoir m

NP = total number of platforms to be placed

N = number of time periods each of length $\Delta\tau_k$ $(k = 1, \ldots, N)$

NN = total number of time periods over which platform placement may take place

B = very large number selected so that its value is greater than any possible net income value for any period

τ_k = elapsed time from beginning of development to period k

C_k = revenue per barrel of crude received in period k

U_k = upper limit on production during time period k

b_k = well cost factor at the k^{th} period (relative to cost in the initial time period)

p_k = platform cost factor at the k^{th} time period (relative to cost in the initial period)

po_j = operating cost per period for platform j

r = the applicable income tax rate

α = the applicable discount factor

Functions

$f(d_{ij})$ = drilling cost function which can be expressed as a function of the horizontal distance between target i and platform j

where

d_{ij} = horizontal distance between target i and platform j which = $((a_i - x_j)^2 + (b_i - y_j)^2)^{1/2}$

$P(M_j, x_j, y_j)$ = cost of platform j as a function of its size (M_j) and its location (x_j, y_j)

$P(M_j)$ = cost of platform j as a function of its size (M_j) only

where

Variables

(x_j, y_j) = location coordinates for platform j

t^m_{ijk} = 1 if target i of reservoir m is drilled from the platform j in time period k

= 0 otherwise

μ_{jk} = 1 if platform j is placed in period k

= 0 otherwise

q^m_k = the average production rate from reservoir m during time period k

D_k = allowable depletion for time period k

The author plans to use applicable production decline curves to forecast future production of oil reservoirs. The basic assumption made in adapting production decline curves into the development model of this chapter is that if the reservoir is produced at its maximum capacity, the production rate of the reservoir follows an exponential decline curve with time. Usually, a reservoir is not produced at its maximum capacity initially, but rather production is constrained for some time, resulting in a constant production over this period; then it is produced at the maximum rate causing an exponential decline of production rate with time (see Figure 6).

For any given reservoir subject to exponential decline, if we assume that q_k the average production rate for any time period is equal to the mid-period instantaneous rate, then the following two sets of linear constraints essentially model the relationship depicted in Figure 7. In the following the reservoir superscript has not been included since the formulation is for any given reservoir.

The horizontal portion of the curve (from 0 to P_a) is represented by

$$q_k \quad Q_A \qquad k = 1, \ldots, N$$

The exponential portion of the reservoir curve (from P_a over) is modeled by

$$q_k \quad \frac{Q_I - \overset{k-1}{\underset{r=1}{S}} q_r \quad r}{(1 + \frac{S}{2} \quad k)} \qquad k = 1, \ldots, N$$

In addition to the exponential decline constraints, q_k, the average production rate during the k^{th} time period,

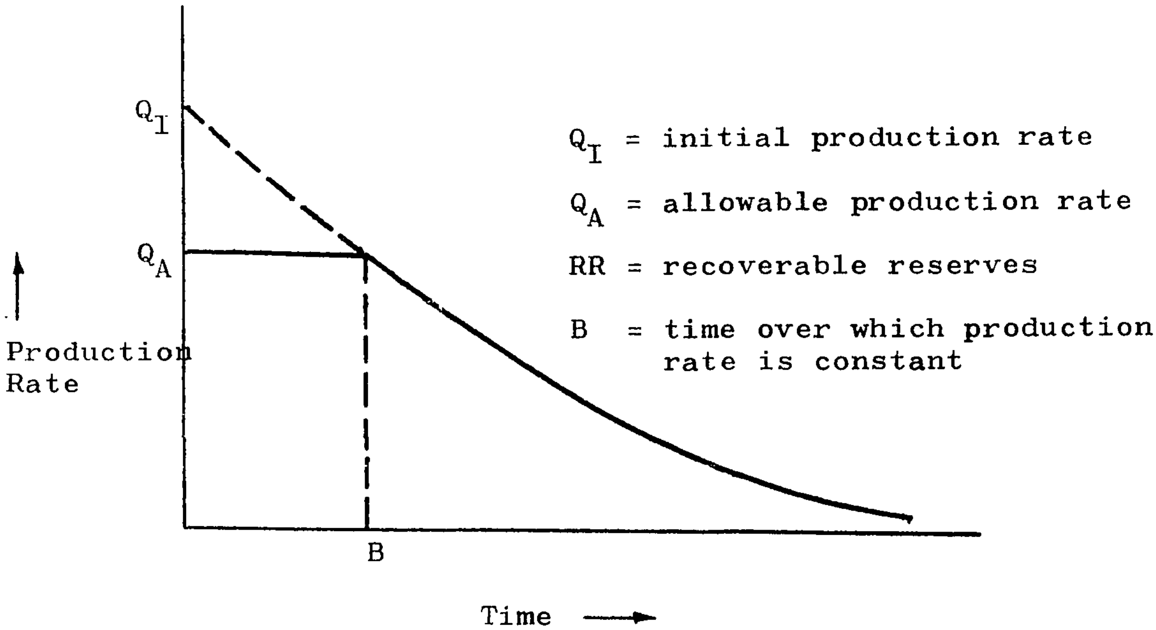

Figure 6. A Typical Reservoir Exponential Decline Curve when Production Is Restricted

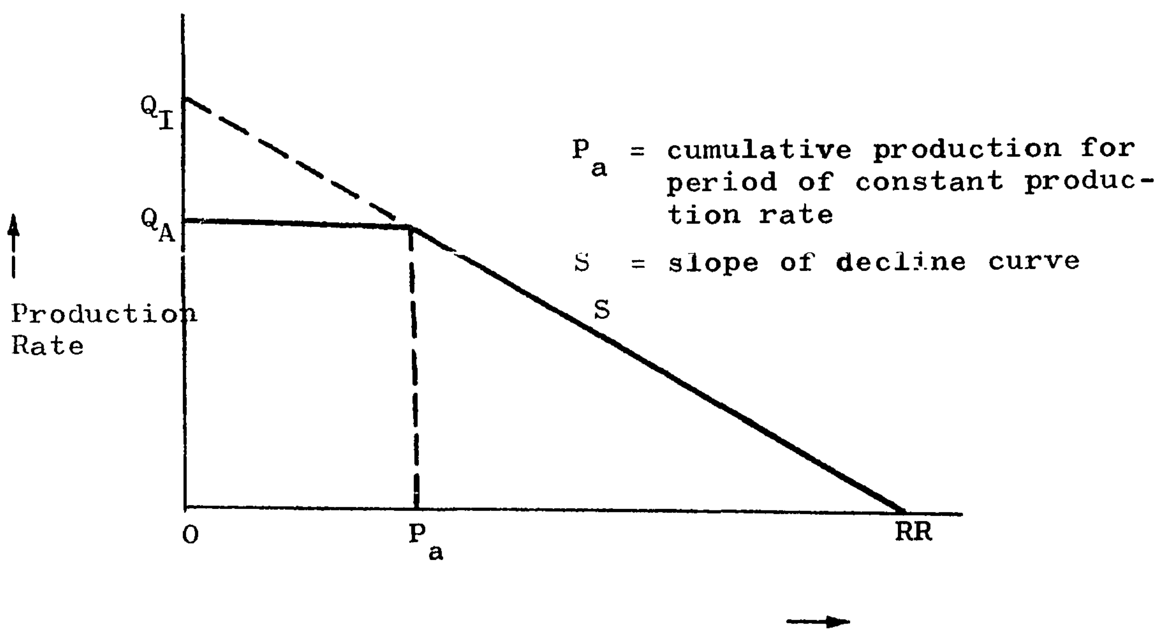

Figure 7. A Typical Production Rate--Cumulative Production Curve for an Exponential Decline Reservoir

must also be constrained by the production possible due to development drilling that has been carried out in all previous time periods as well as during the present time period k.

$$q_k \leq \sum_{r=1}^{k-1} V_{kr} \sum_{j=1}^{NP} \sum_{i=1}^{NT} t_{ijr} + \frac{V_{kk}}{2} \sum_{j=1}^{NP} \sum_{i=1}^{NT} t_{ijk} \qquad k = 1, \ldots, N$$

where

V_{kr} = the average per well production rate for wells drilled in period r and producing in period k

NT = the number of targets in the reservoir

In order to arrive at a final objective function for the proposed model that maximizes the total discounted after-tax cash flow, we proceed in the following manner:

First determine the total revenue generated during any given time period. The revenue generated by the mth reservoir during the kth time period is $C_k \Delta\tau_k q_k^m$. The total revenue generated by all reservoirs during the kth time period is

$$cf(k) = C_k \Delta\tau_k \sum_{m=1}^{NR} q_k^m$$

The costs incurred during any given time period are now formulated. The drilling cost in the kth time period is

$$dc(k) = \sum_{m=1}^{NR} \sum_{j=1}^{NP} \sum_{i=1}^{NT^m} t_{ijk}^m \, b_k \, f(d_{ijm})$$

The platform cost in the kth time period is

$$pc(k) = \sum_{j=1}^{NP} \mu_{jk} P_k P_j(M_j, x_j, y_j) + \sum_{r=1}^{k} \mu_{jr} \, po_j \, \Delta\tau_k$$

Thus, the before-tax cash flow is

$$BTCF = \sum_{k=1}^{N} cf(k) - dc(k) - pc(k)$$

To determine the after-tax cash flow, the net taxable income (NI) must first be determined. The net taxable income is a function of

$g \cdot dc(k)$ = that portion of the drilling cost which must be capitalized during the k^{th} time period,

$(1-g) \cdot dc(k)$ = that portion of drilling cost which may be expensed during the k^{th} time period,

$h \cdot pc(k)$ = that portion of platform costs which must be capitalized for the k^{th} time period,

$(1-h) \cdot pc(k)$ = that portion of platform costs which may be expensed for the k^{th} time period

Using the sum-of-the-years' digits depreciation method we find the net taxable income for the k^{th} time period to be

$$NI(k) = cf(k) - (1-g)dc(k) - (1-h)pc(k) - \sum_{r=1}^{k} \frac{(N-k+1)}{\frac{N-r+1}{2}\,(N-r+2)} [g \cdot dc(r) + h \cdot pc(r)]$$

Finally, the objective function that maximizes the total discounted after-tax cash flow, and incorporates depletion is

$$\text{Maximize} \sum_{k=1}^{N} [cf(k) - dc(k) - pc(k) - r(NI(k)-D_k)]e^{-\alpha\tau_k} \quad (3)$$

The constraints of this after-tax model fall into three general groupings.

PLATFORM AND WELL CONSTRAINTS

$$\sum_{m=1}^{NR} \sum_{k=1}^{N} \sum_{i=1}^{NT^m} t^m_{ijk} - M_j \leq 0 \qquad j = 1, \ldots, NP \qquad (4)$$

$$\sum_{k=1}^{N} t^m_{ijk} \leq 1 \qquad \begin{matrix} i = 1, \ldots, NT^m \\ j = 1, \ldots, NP \\ m = 1, \ldots, NR \end{matrix} \qquad (5)$$

$$\sum_{k=1}^{N} \mu_{jk} = 1 \qquad j = 1, \ldots, NP \qquad (6)$$

$$\sum_{r=1}^{k} \sum_{m=1}^{NR} \sum_{i=1}^{NT^m} t^m_{ijr} \leq B \sum_{r=1}^{k} \mu_{jr} \qquad \begin{matrix} j = 1, \ldots, NP \\ k = 1, \ldots, N \end{matrix} \qquad (7)$$

$$\mu_{jk};\ t^m_{ijk} = 0, 1 \qquad \begin{matrix} i = 1, \ldots, NT^m \\ j = 1, \ldots, NP \\ k = 1, \ldots, N \\ m = 1, \ldots, NR \end{matrix}$$

PRODUCTION LIMITING CONSTRAINTS

$$\Delta \tau_k \sum_{m=1}^{NR} q^m_k \leq U_k \qquad k = 1, \ldots, N \qquad (8)$$

$$q^m_k \leq Q^m_A \qquad \begin{matrix} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{matrix} \qquad (9)$$

$$q^m_k \leq \frac{Q^m_T - S^m \sum_{r=1}^{k-1} q^m_r \, \Delta \tau_r}{1 + \frac{S^m}{2} \Delta \tau_k} \qquad \begin{matrix} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{matrix} \qquad (10)$$

$$q_k^m \le \sum_{r=1}^{k-1} V_{kr}^m \sum_{j=1}^{NP} \sum_{i=1}^{NT^m} t_{ijr}^m + \frac{V_{kk}}{2} \sum_{j=1}^{NP} \sum_{i=1}^{NT^m} t_{ijk}^m \qquad \begin{array}{l} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{array} \qquad (11)$$

DEPLETION CONSTRAINTS

$D_k \le .22cf(k)$ $\qquad k = 1, \ldots, N \qquad (12)$

$D_k \le .5NI(k) + B(1-z_k)$ $\qquad k = 1, \ldots, N \qquad (13)$

$NI(k) \le z_k \cdot B$ $\qquad k = 1, \ldots, N \qquad (14)$

$D_k \le z_k \cdot B$ $\qquad k = 1, \ldots, N \qquad (15)$

$D_k \ge 0$ $\qquad k = 1, \ldots, N$

$z_k = 0\text{-}1$ $\qquad k = 1, \ldots, N$

The constraints serve the following purposes:

(4) defines the size of each platform

(5) restricts each well to one drilling

(6) restricts each platform to one placement

(7) does not allow a well to be drilled from a platform until that platform has been placed

(8) forces production during period k to be less than some upper limit

(9) requires that the production rate from reservoir m be ≤ the allowable production rate for reservoir m

(10) requires that the maximum production rate follow an exponential decline curve with time

(11) restricts production to that possible due to development drilling that has been carried out previously

(12) forces depletion for the k^{th} time period to be less than or equal to 22% of gross income at all times

(13) forces $z_k = 0$ if net income is less than or equal to zero

(14) forces $z_k = 1$ if net income for time period k is greater than zero

(15) forces $D_k = 0$ if $z_k = 0$

The model contains:

$2NP + N \cdot NR + N$	Continuous variables
$N(NW \cdot NP + NP + 1)$	0-1 variables
$NP(NW + 2) + 6N + 3N \cdot NR$	Constraints

This means that a moderate size problem of 10 periods (N), 3 platforms (NP), 5 reservoirs (NR) and 50 wells (NW) would be formulated as a nonlinear mixed-integer programming model with 66 continuous variables, 1540 zero-one variables and 366 constraints. No existing solution procedure is capable of producing an optimal answer for such a model, within reasonable cost limitations.

Two significant factors that influence development activities are the drilling cost function and the platforms cost function. A drilling cost function may be developed for a particular area by means of a multivariate curve fit for historical cost data taken from an area with similar geological features. This approach would, most likely, be used successfully only by those companies already involved in development drilling because

1. They are the only ones that possess the necessary confidential information (cost data).
2. They can readily interpret the available cost information and place it in the context of their drilling activity.

A drilling cost function may also be developed if we consider the following important factors

1. the depth of the well
2. the maximum angle of deviation at which a well is drilled

Usually, as the depth of the well increases, more than proportionate increases in costs are incurred because, (1) the deeper formations are often more difficult to drill and (2) as the well becomes longer more time is needed to change bits, add deviational equipment, make directional surveys, etc. (10). The maximum angle of deviation at which a well is drilled is an important factor in influencing drilling cost for two reasons. First, it takes a considerable amount of time just to build up to the desired deviated angle. Second, the drilling rate will decrease with increasing angles of deviation. Recognizing the importance of these factors Devine (10) suggested that drilling engineers estimate total costs for various depths and horizontal deviations. "These estimates would be made on the basis of experience, after considering the length of the hole, maximum drilling angle, etc., for each combination of depth and horizontal deviation" (10). From such cost estimates a least squares regression equation could then be derived. Devine (10) using several "rules of thumb" obtained from Mr. Ralph Brumley of Whipstock, Inc., developed a program to calculate expected total drilling costs for various depths and horizontal deviations. A least squares regression equation found for a set of generated data is

$$DC = 122.6 - 21.43C + 2.39C^2 + 12.24HD \tag{17}$$

where

DC = drilling cost in thousands of dollars

C = depth of target in thousands of feet

HD = horizontal deviation in thousands of feet

This relationship is then modified by adding another term that severely penalizes any drilling at angles above 45 degrees. This modified equation is

$$DC = 122.6 - 21.43C + 2.39C^2 + 12.24HD + 5.0(HD/(C-DK))^{10} \tag{18}$$

where

DK = depth in thousands of feet at which angled drilling is begun

Note that this penalty term, $5.0(HD/(C-DK))^{10}$, becomes very large whenever HD becomes greater than C-DK (i.e., indicates drilling angles of 45 degrees or more). This penalty term is included in the drilling cost function because:

1. The data used to arrive at equation (17) don't contain any cases in which the drilling angle is greater than 45 degrees.
2. Costs increase very rapidly when drilling at high angles.
3. It is deemed desirable to have the drilling cost function of the development model applicable to all horizontal deviations rather than constraining the size of the drilling angle, i.e., there really is no absolute limit; it is simply a matter of rapidly increasing cost for very high angles.

The author proposes to use this drilling cost function, equation (18), or one quite similar to it in working example problems.

In general, the platform cost will depend primarily on its size (the number of wells it can accommodate). The platform cost can also depend on its location due to variations in depth and bottom conditions. The discussion of a platform cost function will be postponed until later since its composition will influence the type of solution procedure needed to solve the proposed model.

Use of the Proposed Model for Investment Decisions

It should be stressed that this proposed model will not replace the investment decision maker(s) but rather should serve as a guide in the evaluation of investment alternatives. Proper use of this method to plan capital investments in offshore oil field development would dictate the solution of the proposed model a number of different times during the development period. This is necessary for a number of reasons:

1. The model essentially assumes that no dry wells are drilled. The occurrence then of one or more dry holes during a given time period could cause the existing development plan for the remaining time periods to no longer be the best possible.
2. The original development plan is obtained by utilizing estimates for production rates and demand. Since the accuracy of these estimates will improve as later and more meaningful information is obtained, the development plan for the remaining time periods should be checked periodically by resolving the development model. This is done to determine if the development plan for the remaining periods should be changed based on updated information.
3. The objectives and policies of a company may change with time so that criteria used for making decisions in the past may no longer be completely applicable.

This would be the case, for example, if a company's investment funds have been diluted and it is decided that the present discount rate applied to maximizing profit in the investment models is insufficient considering the limited amount of investment funds they now have available. Thus, since "the program which yields maximum profit discounted at 10% will usually be different from the program which yields maximum profit discounted at 20%" (12), the investment model should be resolved under such circumstances.

CHAPTER IV

SOLUTION PROCEDURES FOR THE GENERAL DEVELOPMENT MODEL

In the previous chapter it was pointed out that for realistic size problems there are no existing solution procedures capable of producing optimal solutions for the general development model, within reasonable cost limitations. Thus, the purpose of this chapter is to examine approaches for obtaining solutions to the general development problem that may prove to be computationally feasible. To accomplish this the solution procedures developed in this chapter partitions the overall problem into a set of related subproblems and then attempts to find solutions to the subproblems. Although the solutions to these subproblems are not necessarily optimal solutions to the general model, computationally they will produce "good" answers to the offshore development problem. The approach of this chapter is to break the overall development problem down into four component problem areas:

1. platform location problem
2. well assignment problem
3. platform placement schedule problem
4. well drilling schedule problem

Solutions are obtained for combinations of the component problems and then combined to specify an approximate solution to the overall offshore development problem. Two possible approaches for obtaining solutions to the development problem are:

1. dependent subproblems solution procedure
2. independent subproblems solution procedure

Dependent Subproblems Solution Procedure

One general approach for obtaining a solution to the development model of Chapter III would be to decompose it into two dependent subproblems. The solution to the first subproblem (platform location and placement schedule) specifies the location of each platform as well as the schedule for the placement of each platform given an assignment of targets to platforms and a drilling schedule for the targets. The second subproblem (target assignment and drilling schedule) is then solved to determine the best assignment of targets to platforms and the schedule for the drilling of the targets given the platform locations and placement schedules determined via subproblem one. The solution procedure would continue to alternately solve subproblem one and subproblem two until it has been determined that no better solution can be obtained by continuing.

Dependent Subproblem One--Platform Location and Platform Placement Schedule

This subproblem determines the best

1. location for each platform

2. schedule for the placement of the platforms

for a given assignment of wells to platforms and well drilling schedule. The platform location and platform placement schedule subproblem may be modeled as NP separate nonlinear mixed integer programming problems (i.e., a separate model for each platform).

The model for the j^{th} platform is

$$\text{Minimize} \quad \sum_{k=1}^{N} dc(k) + pc(k) \tag{19}$$

$$\text{subject to} \quad \sum_{k=1}^{NN} \mu_k = 1 \tag{20}$$

$$\sum_{r=1}^{k} \sum_{i \in \phi_j} t_{ir} \leq B \sum_{r=1}^{k} \mu_r \quad k = 1, \ldots, NN \tag{21}$$

FUNCTIONS

Drilling Cost Function; $dc(k) = \sum_{i \in \phi_j} t_i \cdot f(d_{ij})$

Platform Cost Function; $pc(k) = \mu_k P_j(M_j) + po_j \Delta\tau_k \cdot \sum_{r=1}^{k} \mu_r$

where

ϕ_j = set of all targets assigned to platform j

$f(d_{ij})$ = the drilling cost function (see page 32)

CONSTANTS

$P_j(M_j)$ = platform cost of platform j as a function of its size, M_j

po_j = operating cost per period for the j^{th} platform

t_{ik} = 0, 1 specified constants that indicate whether the i^{th} target is scheduled to be drilled in the k^{th} period or not

$\Delta\tau_k$ = length of time period k

NN = the number of time periods over which platform placement may take place

VARIABLES

μ_k = 1 if the j^{th} platform is placed in period k

= 0, otherwise

(x_j, y_j) = location coordinates for platform j

The constraints serve the following purposes:

(20) requires that the j^{th} platform be placed once and only once

(21) requires that the j^{th} platform be placed before any drilling is scheduled to occur for platform j

Since NN is usually small, one feasible solution procedure would be to solve the j^{th} platform model for each feasible platform placement schedule. For instance, if it is decided that the platforms must all be placed before the end of the third period, NN = 3, then the separate platform models would each be solved three times. This approach would necessitate solving an unconstrained optimization problem a number of times. Solutions to this type of problem can be obtained by a direct search procedure along the negative gradient of the drilling cost function (see Reference 5).

Dependent Subproblem Two--Target Assignments and Drilling Schedule

This subproblem determines the best

1. assignment of targets to platforms
2. drilling schedule for each platform

for a given location of each platform and a specified platform placement schedule.

The target assignment and drilling schedule model is

$$\text{Maximize} \quad \sum_{k=1}^{N} [cf(k) - dc(k) - pc(k) - r(NI(k) - D_k)]e^{-\alpha\tau_k} \tag{22}$$

$$\text{subject to} \quad q_r^m \leq Q_A^m \qquad \begin{array}{l} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{array} \tag{23}$$

$$q_k^m \leq \frac{Q_I^m - S^m \sum_{r=1}^{k-1} q_r \Delta\tau_r}{(1 + \frac{S^m}{2}\Delta\tau_k)} \qquad \begin{array}{l} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{array} \tag{24}$$

$$q_k^m \leq \sum_{r=1}^{k-1} V_{k,r}^m \sum_{j=1}^{NP} \sum_{i=1}^{NT^m} t_{ijr}^m \qquad \begin{array}{l} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{array}$$

$$+ \frac{V_{k,k}^m}{2} \sum_{j=1}^{NP} \sum_{i=1}^{NT^m} t_{ijk}^m$$

$$\sum_{k=1}^{N} \sum_{m=1}^{NR} \sum_{i=1}^{NT^m} t^m_{ijk} \leq M_j \qquad j = 1, \ldots, NP \qquad (26)$$

$$\sum_{k=1}^{N} t^m_{ijk} = 1 \qquad \begin{array}{l} j = 1, \ldots, NP \\ m = 1, \ldots, NR_m \\ i = 1, \ldots, NT^m \end{array} \qquad (27)$$

$$\Delta\tau_k \sum_{m=1}^{NR} q^m_k \leq U_k \qquad k = 1, \ldots, N \qquad (28)$$

and the depletion constraints of (12) through (15) of Chapter III.

The constraints, functions and variables of this model are defined on pages 31 and 32. This target assignment and well drilling schedule model contains

$N(NR + 1)$	Continuous variables
$N(NW \cdot NP + 1)$	0-1 variables
$NP(NW + 1) + 3N \cdot NR + 5N$	Constraints

A comparison of the size of this subproblem model with the size of the general development model for a realistic size problem of

10 periods; $N = 10$

3 platforms; $NP = 3$

5 reservoirs; $NR = 5$

50 targets; $NW = 50$

is presented in Table 4.

TABLE 4

SIZE COMPARISON OF ASSIGNMENT AND DRILLING SCHEDULE SUBPROBLEM MODEL WITH GENERAL DEVELOPMENT MODEL

	Column 1 General Development Model	Column 2 Assignment and Drilling Schedule Subproblem Model	Difference between Column 1 and Column 2
Continuous Variables	66	60	6
0-1 Variables	1540	1510	30
Constraints	366	353	13

As can be seen from Table 4, the target assignment and drilling schedule subproblem is very nearly the same size as the general development model for any specific problem. It should be pointed out that the Assignment and Drilling Schedule Subproblem is linear in nature whereas the General Development Model is nonlinear. Nevertheless the size of the Assignment and Drilling Schedule Model for realistic development problems will make the determination of an optimal solution for these problems very difficult, if not computionally prohibitive.

Independent Subproblems Solution Procedures

Another interesting approach to solving the offshore development model would be to decompose it into independent subproblems. The first subproblem (location-assignment subproblem) would determine the number, size, and location of each platform and would assign wells to various platforms so as to minimize platform and drilling cost. The second subproblem (scheduling subproblem) would then schedule the placement of platforms, the drilling of wells and the production so as to maximize the after-tax cash flow.

The Location-Assignment Subproblem

The solution to the location-assignment subproblem would include

1. the number of platforms needed
2. the location of each platform
3. the size of each platform (i.e., the number of wells assigned to each platform)
4. the assignment of wells to the various platforms

This problem has been examined by Devine and Lesso (11), who have developed relatively efficient solution procedures. Their approach is to decompose this problem into two interrelated subproblems that are solved in an iterative manner. The first subproblem (location subproblem), given an assignment of targets to a fixed number (NP) of platforms, consists of solving NP single platform location problems to define the initial locations of the platforms. The formulation of a single platform location problem is

$$\text{Minimize} \sum_{i=1}^{NT} f_i(x,y) + P(x,y). \tag{29}$$

where

(x,y) = coordinates of the platform

$f_i(x,y)$ = cost to drill target i as a function of the platform location

$P(x,y)$ = platform cost as a function of location

NT = number of targets to be drilled from this platform.

After these NP individual problems are solved, the next step is to solve what is termed the allocation subproblem. Given fixed locations for each of the NP platforms, the minimum cost allocation of wells to platforms is found by solving the following model:

$$\text{Minimize} \quad \sum_{m=1}^{NR} \sum_{i=1}^{NT^m} \sum_{j=1}^{NP} c_{ij}^m \, t_{ij}^m + \sum_{j=1}^{NP} P(M_j) \qquad (30)$$

$$\text{subject to} \quad \sum_{j=1}^{NP} t_{ij}^m = 1 \qquad \begin{array}{l} m = 1, \ldots, NR \\ i = 1, \ldots, NT^m \end{array} \qquad (31)$$

$$\sum_{m=1}^{NR} \sum_{i=1}^{NT^m} t_{ij}^m - M_j \le 0 \qquad j = 1, \ldots, NP \qquad (32)$$

$$t_{ij}^m = 0\text{-}1 \qquad \begin{array}{l} m = 1, \ldots, NR \\ i = 1, \ldots, NW \\ j = 1, \ldots, NP \end{array}$$

The constants and variables above were defined on pages 31 and 32 but note that $c_{ij} = f(d_{ij})$ = a constant since platforms are fixed. The solution procedure needed to solve the above problem depends on the form of the platform cost function ($P(M_j)$). For example, consider the case where the platform cost function increases in finite steps (see Figure 8), as would be the case when one must choose different size platforms from a list of currently available models.

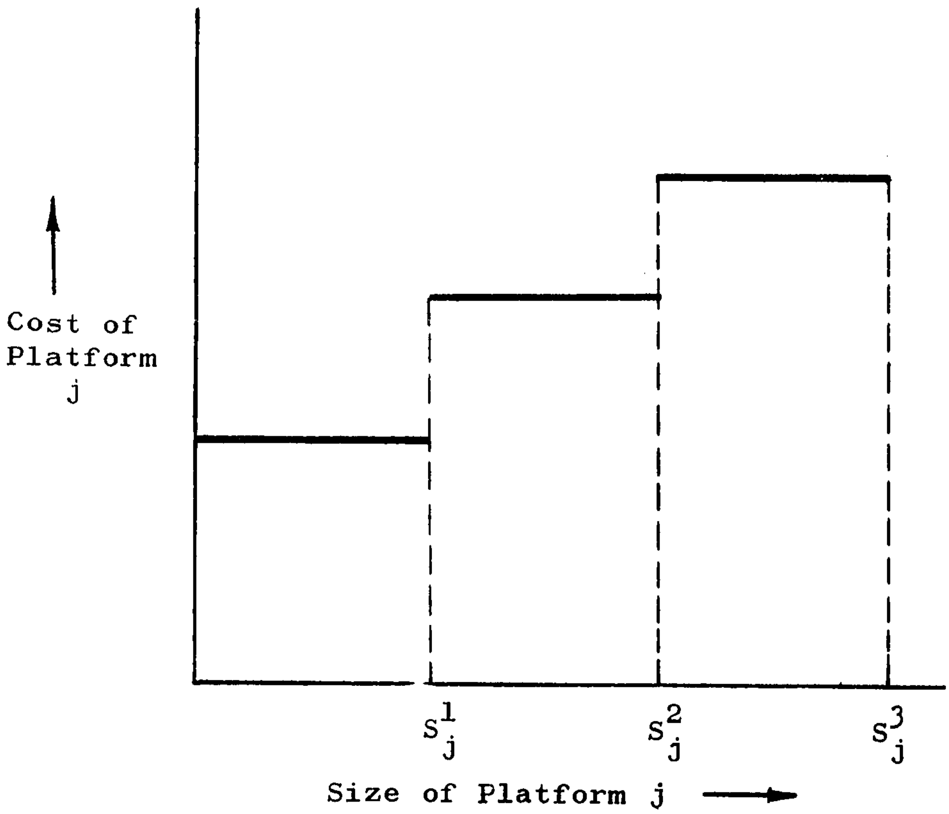

Figure 8. Example Platform Cost Function.

For such a platform cost function the allocation subproblem can be modeled as a 0-1 linear integer programming problem as follows:

$$\text{Minimize} \quad \sum_{j=1}^{NP} \sum_{m=1}^{NR} \sum_{i=1}^{NT^m} c_{ij}^m t_{ij}^m + \sum_{j=1}^{NP} \sum_{k=1}^{K_j} w_j^k v_j^k \tag{33}$$

$$\text{subject to} \quad \sum_{j=1}^{NP} t_{ij}^m = 1 \qquad \begin{array}{l} m = 1, \ldots, NR \\ i = 1, \ldots, NT^m \end{array} \tag{34}$$

$$\sum_{m=1}^{NR} \sum_{i=1}^{NT^m} t_{ij}^m \le \sum_{k=1}^{K_j} s_j^k v_j^k \qquad j = 1, \ldots, NP \tag{35}$$

$$\sum_{k=1}^{K_j} v_j^k = 1 \qquad j = 1, \ldots, NP \qquad (36)$$

$$t_{ij}^m = 0\text{-}1 \qquad \begin{array}{l} i = 1, \ldots, NW \\ j = 1, \ldots, NP \\ m = 1, \ldots, NR \end{array}$$

$$v_j^k = 0\text{-}1 \qquad \begin{array}{l} j = 1, \ldots, NP \\ k = 1, \ldots, k_j \end{array}$$

CONSTANTS

K_j = number of "steps" in the function for platform j (i.e., the number of different size platforms)

w_j^k = cost of kth size for platform j (see Figure 8)

s_j^k = capacity of the kth size for platform j (see Figure 8)

VARIABLES

v_j^k = 1 if the kth size of platform j is used,
= 0 otherwise

A special heuristic approach has been developed that jointly obtains solutions to the location and allocation subproblem for discontinuous platform cost functions such as the one illustrated in Figure 8. A flow diagram and the basic steps of this algorithm are given in Appendix A.

In many problems, however, the platform cost function is such that it is relatively easy to obtain an optimal solution to the allocation subproblem for a given set of platform locations. In this case, once the allocation subproblem is solved, then the location subproblem is resolved; upon finding this new solution, the allocation subproblem is resolved with the new platform locations in order to see if a lower cost allocation of wells to platforms is possible. The algorithm thus continues to alternately solve the location subproblem and allocation subproblem until it is determined that the best possible solution has been obtained. The block diagram of Figure 9 provides a graphical representation of the procedure followed by this algorithm to determine a solution to the location-allocation problem.

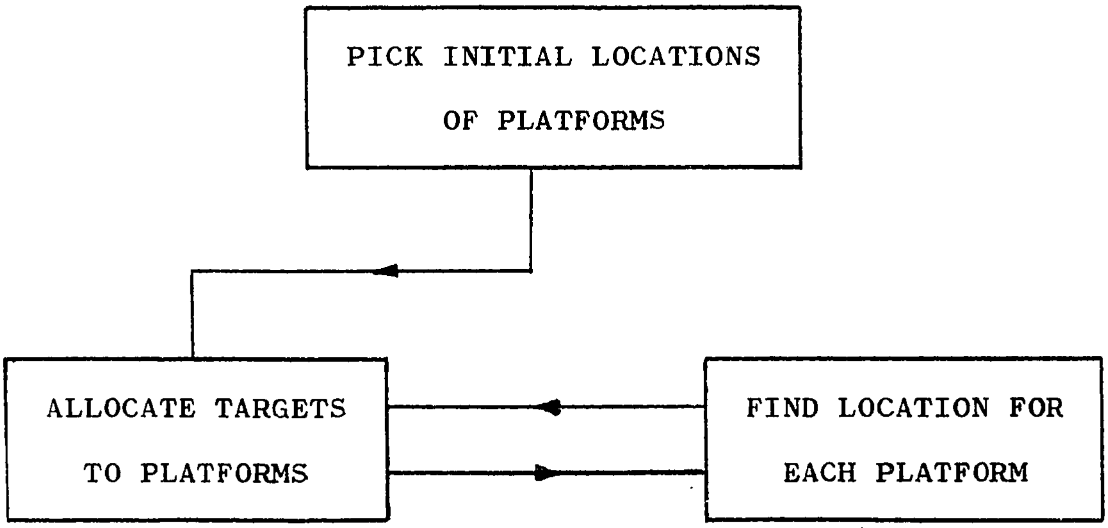

Figure 9. Block Diagram of the Location Allocation Algorithm.

The number of platforms is assumed fixed in both of these subproblems since

> the problem becomes easier to formulate and solve. For most problems, the range on the possible values of NP is rather small, usually three or four. Thus, it is reasonable to solve the problem for each possible value of NP and then to pick the best. (10)

The Scheduling Subproblem

Once the number, size, and location of platforms are determined and each well is assigned to a platform then $f(d_{ij})$, $P(M_j,x_j,y_j)$, d_{ij}, M_j and (x_j,y_j) are constants. Incorporation of these constants along with the variables,

η^m_{ik} = 1 if well i is drilled in reservoir m in time period k
= 0 otherwise

δ_{jk} = 1 if platform j is placed in time period k
= 0 otherwise

q^m_k = production rate from the mth reservoir in period k.

into our original model leaves us with a scheduling subproblem that can be formulated as a linear mixed-integer programming problem as follows:

$$\text{Maximize} \quad \sum_{k=1}^{N} (cf(k)-dc(k)-pc(k)-r(NI(k)-D_k))e^{-\alpha\tau_k} \tag{37}$$

$$\text{subject to} \quad \sum_{k=1}^{N} \eta^m_{ik} = 1 \qquad m = 1, \ldots, NR; \quad i = 1, \ldots, NT^m \tag{38}$$

$$\sum_{k=1}^{NN} \delta_{jk} = 1 \qquad j = 1, \ldots, NP \tag{39}$$

$$\sum_{r=1}^{k} \sum_{m=1}^{NR} \sum_{i\in\phi_m(j)} \eta^m_{ir} \le B \sum_{r=1}^{k} \delta_{jr} \qquad j = 1, \ldots, NP; \quad k = 1, \ldots, N \tag{40}$$

$$q^m_k \le Q^m_A \qquad m = 1, \ldots, NR; \quad k = 1, \ldots, N \tag{41}$$

$$q_k^m \le \frac{Q_I^m - S^m \sum_{r=1}^{k-1} q_r^m \Delta\tau_r}{\left(1 + \frac{S^m}{2}\Delta\tau_k\right)} \qquad \begin{array}{l} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{array} \tag{42}$$

$$q_k^m \le \sum_{r=1}^{k-1} V_{rk}^m \sum_{i=1}^{NT^m} \eta_{ir}^m + \frac{V_{kk}^m}{2} \sum_{i=1}^{NT^m} \eta_{ik}^m \qquad \begin{array}{l} m = 1, \ldots, NR \\ k = 1, \ldots, N \end{array} \tag{43}$$

$$\Delta\tau_k \sum_{m=1}^{NR} q_k^m \le U_k \qquad k = 1, \ldots, N \tag{44}$$

$$D_k \le .22cf(k) \qquad k = 1, \ldots, N \tag{45}$$

$$D_k \le .5NI(k) + B(1-z_k) \qquad k = 1, \ldots, N \tag{46}$$

$$NI(k) \le z_k B \qquad k = 1, \ldots, N \tag{47}$$

$$D_k \le z_k B \qquad k = 1, \ldots, N \tag{48}$$

$$D_k \ge 0 \qquad k = 1, \ldots, N$$

$$\eta_{ik}^m, \delta_{jk}, z_k = 0\text{-}1 \qquad \begin{array}{l} j = 1, \ldots, NP \\ k = 1, \ldots, Nj \\ m = 1, \ldots, NR \\ i = 1, \ldots, \phi_m(j) \end{array}$$

where

$\phi_m(j)$ = the set of targets in the m^{th} reservoir that are assigned to platform j

$$cf(k) = c_k \Delta\tau_k \sum_{m=1}^{NR} q_k^m \qquad k = 1, \ldots, N$$

$$dc(k) = \sum_{m=1}^{NR} \sum_{i=1}^{NT^m} \eta_{ik}^m \, b_k \, f(d_{im}) \qquad k = 1, \ldots, N$$

$$pc(k) = \sum_{j=1}^{NP} \delta_{jk} p_k P(M_j, x_j, y_j) + \sum_{r=1}^{k} \delta_{jr} po_j \ \Delta\tau_k \qquad k = 1, \ldots, NN$$

$$NI(k) = cf(k) - (1-g)dc(k) - (1-h)pc(k) - \sum_{r=1}^{k} \frac{(N-k+1)}{\frac{N-r+1}{2}(N-r+2)} [g \cdot dc(k) + h \cdot pc(k)] \qquad k = 1, \ldots, N$$

The remaining quantities are defined on pages 31 and 32. In matrix form the scheduling subproblem excluding the tax portion would be

maximize $\quad c'q - d'\eta - p'\delta$

or

minimize $\quad -c'q + fy$

subject to $\quad Aq + Dy \geq b \qquad q \geq 0, \quad y = 0\text{-}1$ variable

where

$c' = (C_k \Delta\tau_k)$ which is a 1 by N•NR matrix

$q = q_k^m$ which is an N•NR by 1 matrix

$d' = (b_k \cdot f(d_{im}))$ which is a 1 by N•NW matrix

$\eta = (\eta_{ik}^m)$ which is an N•NW by 1 matrix

$p' = (\delta_{jk} P_k P(M_j, x_j, y_j) + \sum_{r=1}^{k} \delta_{jr} po_j \ \Delta\tau_k)$ which is a 1 by N•NP matrix

$\delta = (\delta_{jk})$ which is an N•NP by 1 matrix

$f = \binom{d'}{p'}$ which is a 1 by N(NW+NP) matrix

$y = \binom{\eta}{\delta}$ which is an N(NW + NP) matrix

A is N(3NR + NP + 1) + NP + NW by N·NR

D is N(3NR + NP + 1) + NP + NW by N(NW + NP)

b is N(3NR + NP + 1) + NP + NW by 1

This problem has been segregated into a continuous portion and a binary portion. The reason for rewriting this problem in such a manner is to put it into the notational form most often used when applying Bender's Partitioning Algorithm to such a mixed integer programming problem. See (18) and (20) for a description of Bender's Partition Algorithm. Bender's algorithm might prove to be a computationally feasible solution technique for this problem if the structure of the problem is such that the algorithm will converge after a relatively few iterations and/or the problem solutions at each iteration are relatively easy to determine. However, nothing definite may be stated a priori about how many iterations of the Bender's Algorithm will be required to solve the scheduling subproblem.

Another characteristic that would encourage one to apply Bender's Algorithm to this scheduling problem is the possibility that the A matrix might be easy to solve, e.g., a transportation type matrix, thus indicating the continuous subproblem of Bender's would be relatively easy to solve at each iteration. This is not the case with the scheduling subproblem as can be seen by examining the structure of equation (42) type constraints.

The scheduling model contains N(NR + 1) linear continuous variables, N(NP + NW + 1) zero-one variables, and 5N + 3N•NR + N•NP + NW + NP) linear constraints. This means that a 10 period (N), 3 platform (NP), 5 reservoir (NR), 50 well (NW) problem would necessitate solving a linear mixed-integer programming problem with 60 continuous variables, 540 zero-one variables and 283 constraints. Although certain sophisticated mixed-integer programming (MIP) packages might be able to handle this problem (see reference 29 for a discussion of computational experience with one sophisticated MIP package), it is believed that a simplified version of the scheduling model presented (equations 37 through 48) should be developed to solve this problem. There are three reasons for this.

1. Extended access to a sophisticated MIP code is not available.
2. It is believed that even with such an MIP computer program, the amount of computer time required to solve most such problems would be much too expensive.
3. The examination of the structure of the Scheduling Model in hopes of identifying characteristics of the problem which would encourage one to apply Bender's Decomposition Algorithm proved unfruitful.

One approach could be to solve the simplified problem of determing the number of wells to be drilled from each platform to each reservoir per period rather than the more complex problem modeled by equations 37 through 48. Although this approach is not as realistic as the mixed-integer model formulated previously, this simplified model for the scheduling subproblem will be employed to obtain sufficiently efficient solution procedures.

This simplified scheduling model is:

$$\text{Maximize} \quad \sum_{k=1}^{N} [cf(k) - dc(k) - pc(k) - r_e NI(k)] e^{-\alpha\tau_k} \tag{49}$$

$$\text{subject to} \quad q_k^m \le Q_A^m \qquad m = 1, \ldots, NR; \; k = 1, \ldots, N \tag{50}$$

$$q_k^m \le \frac{Q_I^m - S^m \sum_{r=1}^{k-1} q_r^m \Delta\tau_r}{\left(1 + \frac{S^m}{2}\Delta\tau_k\right)} \qquad m = 1, \ldots, NR; \; k = 1, \ldots, N \tag{51}$$

$$q_k^m \le \sum_{r=1}^{k-1} V_{rk}^m \sum_{j=1}^{NP} \eta_{rj}^m + \frac{V_{kk}^m}{2} \sum_{j=1}^{NP} \eta_{kj}^m \qquad m = 1, \ldots, NR; \; k = 1, \ldots, N \tag{52}$$

$$\sum_{k=1}^{N} \sum_{m=1}^{NR} \eta_{kj}^m \le M_j \qquad j = 1, \ldots, NP \tag{53}$$

$$\sum_{k=1}^{NN} \delta_{jk} = 1 \qquad j = 1, \ldots, NP \tag{54}$$

$$\sum_{r=1}^{k} \sum_{m=1}^{NR} \eta_{rj}^m \le B \cdot \sum_{r=1}^{k} \delta_{jr} \qquad j = 1, \ldots, NP; \; k = 1, \ldots, N \tag{55}$$

<u>VARIABLES</u>

η_{kj}^m = the number of targets of reservoir m drilled from platform j in period k.

$_{jk}$ = 1, if platform j is placed in period k.

0, otherwise.

q_k^m = the average production rate from reservoir m in period k.

<u>CONSTANTS</u>

d_{jm} = the average drilling cost for wells drilled to reservoir m from platform j.

r_e = effective tax rate that accounts jointly for income taxes and depletion allowance.

The remaining constants and functions were previously defined on pages 31 and 32.

The simplified scheduling model contains N•NR linear continuous variables, N•NP zero-one variables, N•NR•NP integer variables, and (3N•NR + N•NP + 2NP) linear constraints. This means that a 10 period (N), 3 platform (NP), 5 reservoir (NR), 50 well (NW) problem would necessitate solving a linear mixed-integer programming problem with 50 continuous variables, 30 zero-one variables, 150 integer variables and 186 constraints. When these example size figures are compared with the corresponding figures for the original scheduling model, it is seen that the only appreciable difference is in the number of integer variables. The original scheduling model has 540 zero-one variables, whereas the simplified scheduling model has only 30 zero-one variables and 150 integer variables for this size example problem.

Of the two approaches that may be utilized in order to obtain a solution to the general development model as presented in Chapter III, the independent subproblem approach seems to be the best because:

1. The very nature of this approach, independent subproblems, will result in considerable savings in time and cost when obtaining solutions as compared to the dependent subproblem approach. This is because each major problem of the independent subproblem approach has to be solved only one time. On the other hand, the two major subproblems of the dependent subproblem approach have to be solved a number of times before a final solution is reached.
2. Any approach which subdivides the general development model into subproblems should insure that the subproblems are appreciably easier to handle computionally than the general development. This is the case with the independent subproblem approach whereas it is not the case with the dependent subproblem approach.

In summary, this chapter presents two different approaches for obtaining solution procedures for the general offshore development model presented in Chapter III.

The two solution procedures are labeled

1. Dependent Subproblems Solution Procedure
2. Independent Subproblems Solution Procedure

Mathematical models are developed for each procedure and the relative size and nature of each model are determined. It appears that the independent subproblem approach is the most promising of the two.

CHAPTER V

COMPUTATIONAL RESULTS FOR OFFSHORE OIL FIELD DEVELOPMENT

Computer programs have been implemented to find solutions to the problem of offshore oil field development via the independent subproblem approach with the simplified scheduling subproblem. The intent of this chapter is not to parade an extensive collection of computational results and corresponding computer processing times, but rather to exhibit the fact that worthwhile, computationally feasible solution procedures have been developed. Computational results are presented for two development problems. The results obtained for these example problems via the author's proposed solution procedure are then compared with solutions obtained from an intuitive approach. Finally, sensitivity analyses are presented for two different postoptimality problems.

Computational Results via the Independent Subproblem Approach

This approach, as explained in Chapter IV, breaks the general development problem down into two independent subproblems. A computer program has been implemented to obtain solutions to the first subproblem, the location-assignment subproblem. The drilling cost function used in the examples is:

$$D_i = 122.6 - 21.43\ C_i + 2.39\ C_i^2 + 12.24\ H_i$$
$$+ 5.0\ \ H_i/(C_i - 1.5)^{10}$$

where:

D_i = drilling cost for target i in thousands of dollars

C_i = depth of target i in thousands of feet

H_i = horizontal distance from the platform to target i in thousands of feet.

The platform cost function increases in finite steps (see Figure 8, Chapter IV), as would be the case when one must choose different size platforms from a list of currently available models. The computer program implemented to solve this specific location-assignment problem is a Fortran code of the heuristic algorithm described in Appendix A. This algorithm utilizes a gradient search algorithm to solve each platform location problem and an out-of-kilter algorithm to solve the transportation problem for reassignment of targets to platforms after a change in well capacity of one platform.

Once a solution is obtained to the location-assignment problem, then IBM's Mixed Integer Programming (MIP) package is utilized to determine a solution to the simplified scheduling problem. The MIP program is basically a branch and bound type algorithm (4).

It is assumed that each oil reservoir considered follows an exponential decline of production rate with time if the reservoir were produced at its maximum capacity. Typically an oil reservoir does follow an exponential or hyperbolic decline of production rate with time (23). The hyperbolic decline could be approximated with an exponential type curve if it were divided into several time segments and then each segment fitted with an exponential curve.

Twenty-Four Well Example

In this example twenty-four wells are to be drilled to a total of three oil reservoirs on an offshore tract of land approximately five square miles in area. Wells are assigned depths between 4000 feet and 7000 feet (See Appendix B). Each well drilled is a single completion type (i.e., drilled to only one oil reservoir). The oil reservoir parameters used in the optimization model are tabulated in Table 5.

TABLE 5

OIL RESERVOIR PARAMETERS (EXAMPLE ONE)

	Reservoir 1	Reservoir 2	Reservoir 3
Estimated Recoverable Reserves (Barrels)	3,000,000	5,000,000	2,400,000
Maximum Production Rate (Barrels/Year)	1,000,000	2,000,000	1,200,000
Allowable Production Rate (Barrels/Year)	700,000	700,000	700,000
Slope of Decline Curve	0.333	0.400	0.500

Three solutions are obtained for the location-allocation subproblem, the best of which indicates that two platforms should be located on the offshore tract of land. This solution is summarized in Table 6.

TABLE 6

LOCATION-ALLOCATION SOLUTION (EXAMPLE ONE)

	Platform 1	Platform 2
Capacity (Wells)	6	18
Cost	\$700,000	\$1,590,000
Drilling Cost	\$587,679	\$2,097,123
Location Coordinates* (Thousands of Feet)	x = 9.8; y = 9.48	x = 6.22; y = 5.53
Targets Assigned	11, 12, 13, 14, 15, 16	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 17, 18, 19, 20, 21, 22, 23, 24

*The origin of the offshore tract of land for reference purposes was specified as the southwest corner of the property and all coordinate locations are relative to that corner. The location coordinates for the individual targets are listed in Appendix C.

Once the solution to the location-allocation subproblem is obtained (Table 6), the simplified version of the scheduling subproblem model is utilized to determine a schedule for the placement of platforms and the drilling of wells. The total time period over which development activities were to be considered for this example problem is six years spread over four separate time periods. The first two periods each have a duration of one year while the last two time periods are each for a duration of two years. The solution to this problem indicates that both platforms should be placed at the beginning of period one. The schedule for well drilling is given in Table 7. The corresponding production rates for each reservoir are given in Table 8.

TABLE 7

WELL DRILLING SCHEDULE (EXAMPLE ONE)

	To Reservoir 1	To Reservoir 2	To Reservoir 3
Number of Wells Drilled From			
Platform 1 in Period			
1	0	4	0
2	0	0	0
3	0	1	0
4	0	0	0
Platform 2 in Period			
1	6	6	6
2	0	0	0
3	0	0	0
4	0	0	0

TABLE 8

PRODUCTION RATES (EXAMPLE ONE)

Period	Production Rate in Thsds Barrels/Year		
	Reservoir 1	Reservoir 2	Reservoir 3
1	435.000	700.000	450.000
2	700.000	700.000	700.000
3	425.000	700.000	340.000
4	253.384	628.000	190.027

The solution to example problem one, as listed in Tables 6 and 7 , corresponds to a total discounted (15% discount factor) net income (before taxes) of $7,690,480.

The computer processing time required to solve example problem one is given in Table 9.

TABLE 9

COMPUTER PROCESSING TIME* (EXAMPLE ONE)

Subproblem	Number of Runs	Average Run Time (Minutes)	Total Run Time (Minutes)
Location-Assignment	9	.2128	1.9152
Simplified Scheduling	1	3.1000	3.1000
Total Run Time (Minutes)			5.0152

*For IBM 360-50.

Fifty-Seven Well Example

The fifty-seven wells of this example problem are to be drilled to a total of three oil reservoirs located on an offshore tract six square miles in area. Wells are to be drilled to a total of 81 targets in the three oil reservoirs. Reservoir one and two each have 24 targets. These two reservoirs are located in approximately the same offshore grid location with reservoir two at a greater depth than reservoir one. Thus, the targets of reservoir one and reservoir two are dually completed, i.e., a well is drilled through both reservoirs tapping one target from each of the two reservoirs. It should be noted that the proposed development model does not consider the general dual completion question of which targets should be dually completed. But, if the targets of a few reservoirs are matched for multiple completion a priori, then the model can handle this special situation. Target coordinates and platform sizes considered for this problem are listed in Appendix C. The oil reservoir parameters used in the optimization model are given in Table 10.

TABLE 10

OIL RESERVOIR PARAMETERS (EXAMPLE TWO)

	Reservoir 1	Reservoir 2	Reservoir 3
Estimated Recoverable Reserves (Barrels)	10,000,000	11,000,000	13,000,000
Maximum Production Rate (Barrels/Year)	3,600,000	4,900,000	6,200,000
Allowable Production Rate (Barrels/Year)	2,200,000	2,900,000	3,700,000
Slope of Decline Curve	0.356	0.445	0.457

Three solutions are obtained for the location-allocation subproblem of example two, the best of which indicates that three platforms should be located on the offshore tract of land. This solution is given in Table 11.

TABLE 11

LOCATION-ALLOCATION SOLUTION (EXAMPLE TWO)

	Platform 1	Platform 2	Platform 3
Capacity (Wells)	20	20	18
Cost	$1,700,000	$1,700,000	$1,590,000
Drilling Cost for	$3,185,040	$2,336,300	$2,304,800
Location Coordinates* (Thousands of Feet)	x = 3.62 y = 6.26	x = 11.85 y = 3.97	x = 11.22 y = 10.52
Targets Assigned	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 24	20, 25, 26, 29, 30, 32, 36, 39, 40, 44, 45, 46, 49, 50, 51, 53, 54, 55, 56, 57	19, 22, 23, 27, 28, 31, 33, 34, 35, 37, 38, 41, 42, 43, 47, 48, 52

*The origin of the offshore tract of land for reference purposes was specified as the southwest corner of the property and all coordinate locations are relative to that corner. The location coordinates for the individual targets are listed in Appendix C.

The total time period over which development activities are to be considered for this example problem is ten years spread over five separate time periods. The duration of each time period is:

Time Period 1 - - - 1 year duration

Time Period 2 - - - 1 year duration

Time Period 3 - - - 2 years duration

Time Period 4 - - - 2 years duration

Time Period 5 - - - 4 years duration

The simplified scheduling model solution for example two indicates that all three platforms should be replaced at the beginning of period one. The schedule arrived at by the solution of the simplified scheduling model for example two is presented in Table 12. The corresponding production rates for each reservoir are given in Table

TABLE 12

WELL DRILLING SCHEDULE (EXAMPLE TWO)

	To Reservoir 1	To Reservoir 2	To Reservoir 3
Number of Wells Drilled From			
Platform 1 in Period			
1	15	15	0
2	5	0	0
3	0	0	0
4	0	0	0
5	0	0	0
Platform 2 in Period			
1	5	5	15
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
Platform 3 in Period			
1	1	1	14
2	0	0	3
3	0	0	0
4	0	0	0
5	0	0	0

TABLE 13

PRODUCTION RATES (EXAMPLE TWO)

Period	Production Rate in Thsds Barrels/Year		
	Reservoir 1	Reservoir 2	Reservoir 3
1	1260.000	1312.500	1957.500
2	2200.000	2307.500	3021.300
3	1746.490	1810.600	2037.400
4	829.454	1088.700	1082.500
5	312.014	374.986	446.751

The solution to example problem two, as listed in Tables 11 and 12, corresponds to a total discounted (15% discount factor) net income (before taxes) of $34,222,157.

The computer processing time required to solve example problem two is given in Table 14.

TABLE 14

COMPUTER PROCESSING TIME* (EXAMPLE TWO)

Subproblem	Number of Runs	Average Run Time (Minutes)	Total Run Time (Minutes)
Location-Assignment	8	1.3250	10.6000
Simplified Scheduling	1	7.2950	7.2950
Total Run Time (Minutes)			17.8950

*For IBM 360-50.

An Intuitive Development Procedure

In order to exemplify the usefulness of the proposed model and solution procedures, an intuitive policy for off-shore development is examined. The intuitive approach locates a platform over the center of each reservoir and develops the reservoir as fast as possible. The one restriction placed on this development policy is that the maximum number of wells that could be drilled per period from any given platform is

15 dual completion wells

20 single completion wells.

The development plans arrived at by means of the intuitive approach for the two example problems are presented in the following two tables.

TABLE 15

INTUITIVE DEVELOPMENT POLICY (EXAMPLE ONE)

Platform Placement - - - Place a platform over each reservoir at the beginning of Period One.

Well Drilling - - - drill

6 wells to reservoir one in period one.
10 wells to reservoir two in period one.
8 wells to reservoir three in period one.

Production Rates in Thsds Barrels/Year

Period	Reservoir 1	Reservoir 2	Reservoir 3
1	435.0	700.0	600.0
2	700.0	700.0	700.0
3	466.3	700.0	366.7
4	233.2	516.0	122.2

TABLE 16

INTUITIVE DEVELOPMENT POLICY (EXAMPLE TWO)

Platform Placement - - - Place a platform over the two reservoirs that are to be dually compled (reservoirs one and two) and one over the third reservoir, at the beginning of Period One

Well Drilling - - - drill

15 wells to reservoir one and two in period one.
9 wells to reservoir one and two in period two.
20 wells to reservoir three in period one.
13 wells to reservoir three in period two.

Production Rates in Thsds Barrels/Year

Period	Reservoir 1	Reservoir 2	Reservoir 3
1	900.0	937.5	1350.0
2	1918.8	1852.5	2527.2
3	1914.8	1809.0	2370.4
4	909.4	1122.9	1344.4
5	342.1	544.8	425.3

This intuitive approach results in a total discounted (15% discount factor) net income (before taxes) for

Example Problem One of $7,146,410

Example Problem Two of $31,217,897

The net income figures of the intuitive approach are compared with the corresponding net income figures arrived at by the author's proposed solution technique in Table 16.

TABLE 17

RESULTING NET INCOME FOR INTUITIVE APPROACH vs. PROPOSED SOLUTION TECHNIQUE

	Intuitive Approach 1	Proposed Solution Technique 2	Percentage Difference $\frac{2 - 1}{1}$
Example Problem 1	\$7,146,410	\$7,690,480	7.6
Example Problem 2	\$31,217,897	\$34,222,157	9.6

These computational results indicate that the proposed solution technique is better than a rather perceptive intuitive approach.

Sensitivity Analysis

It is important that any solution procedure utilized to determine offshore development policy should allow for a sensitivity analysis to be performed upon this development policy. Sensitivity analysis is desirable because:

1. It allows for the creation of contingency plans that can be adapted if some characteristic(s) of the operating environment change.
2. It can indicate how sensitive the determined solution is to inaccuracies in the various parameters used in determining the development solution.

An illustration of the case in which contingency plans could be advisable is when an oil development project is currently subject to regulated production but it is felt that limitations on production will be lifted in the future. To illustrate the capability of the proposed solution procedure to perform a sensitivity analysis, contingency plans are determined for the situation described above. This is accomplished by resolving the simplified scheduling model with the maximum allowable production rate constraint deleted for example problems one and two. The solution to example problem two is essentially the same as before (with a maximum production rate constraint) except that a slightly higher discounted net income is achieved. However, the development policy for example problem one does change. The development schedule for example one with no maximum allowable production is

drill 6 wells to reservoir 1 in period 1

drill 12 wells to reservoir 2 in period 1

drill 6 wells to reservoir 3 in period 1

which results in a total discounted (15% discount factor) net income (before taxes) of $8,770,047.

Another possibility for a sensitivity analysis is to ascertain what effect changes in the schedule for placement of platforms has on the adopted development policy. It is quite likely that unexpected circumstances could dictate that platform placement for one or more offshore platforms be rescheduled. Thus, a sensitivity analysis is performed on the development policy of example problem one by determining what effect various delays in the placement of platforms would have on the development policy. The results of this sensitivity analysis are presented in Table 18.

TABLE 18

SENSITIVITY ANALYSIS ON PLATFORM PLACEMENT SCHEDULE

Alternative Schedule for		
Platform 1 (6 wells)	Platform 2 (18 wells)	Discounted Net Income*
2	1	$7,536,204
3	1	$7,615,604
1	2	$6,371,854
1	3	$5,205,155

*Discount factor is 15% and net income is a before taxes figure.

When the discounted net income figures for the alternative platform placement schedule given in Table 18 are compared with the net income of $7,690,480 associated with the previously determined development policy, it appears that

1. Rescheduling the 6-well capacity platform (platform one) has little effect on the net income derived from the development activities).
2. Rescheduling the 18-well capacity platform (platform two) does have a significant effect on the net income associated with the development activities.

The primary reasons that rescheduling platform one placement has little effect on the development solution whereas the schedule for placement of platform two has considerable influence are

1. Platform two is assigned three times as many wells as platform one. Thus, it is expected that platform two will be utilized to produce much more oil than platform one.
2. Platform two is assigned to drill wells to all three reservoirs but platform one is assigned to drill wells to only one reservoir. Thus, the placement of platform two alone allows for the immediate production from all three reservoirs.

The sensitivity analysis performed has been dealing with the scheduling aspects of the development solution. See reference 10 for a sensitivity analysis on the location-assignment problem solution. It appears that the sensitivity of the development solutions to changes in important parameters depends greatly on the individual characteristics of each development problem.

CHAPTER VI

SUMMARY AND RECOMMENDATIONS FOR FURTHER RESEARCH

Summary

The problem of optimal development of offshore oil fields has been analyzed. Chapter I presents general background material on oil field development and describes briefly how the search and drilling for offshore oil are carried out. Attention is also given to reviewing optimization techniques that have been applied to development drilling problems in the past.

The major topics of Chapter II are offshore development parameters and tax considerations. A brief discussion of two offshore development parameters,

1. upper limit on the amount of oil produced during any future time period,
2. location of individual oil sources (targets)

is presented. A third offshore development parameter, production capability of individual wells, is examined in more detail with considerable discussion presented concerning the use of production-rate-decline curves to forecast future production capability of oil reservoirs. Tax considerations of offshore development activities are also examined with an emphasis on the handling of the allowable depletion allowance extractive industries may claim in order to reduce their income taxes. Mathematical programming models that take into consideration the depletion allowance are presented.

Chapter III presents the mathematical programming formulation of a general development model that determines

1. the number of offshore platforms needed
2. the size of each offshore platform
3. the location of each offshore platform
4. the assignment of wells to the offshore platforms
5. the schedule of placing the platforms and drilling wells

so as to maximize the total discounted after-tax cash flow. A discussion of the drilling cost function and how it may be determined for any given development is also given in Chapter III.

Two different approaches for obtaining solution procedures for the general offshore development model of Chapter III, namely

1. dependent subproblems solution procedure
2. independent subproblems solution procedure

are presented in Chapter IV. Mathematical programming models are also developed for each procedure and the relative size and nature of each model are determined. From an examination of the size and nature of the two solution procedures it appears that the independent subproblem approach offers the greatest promise for computationally efficient solution procedures.

Computational results are presented in Chapter V for the independent subproblems solution procedure for a twenty-four target problem and a fifty-seven target problem. These results, when compared with a perceptive intuitive approach of developing the offshore property as fast as possible, appear very encouraging. The feasibility of performing sensitivity analyses with the adopted solution procedures is demonstrated by conducting sensitivity analyses on the

1. effect a maximum allowable production restriction has on the solution.
2. schedule for placement of platforms.

This research contributes to the field of optimum offshore oil field development in the following ways:

1. It is shown that not incorporating depletion or using an approximation for the depletion in an after-tax optimization model can result in non-optimal investment decisions. After-tax optimization models are developed that incorporate depletion terms.
2. The problems of the development of offshore oil fields and the scheduling of this development are formulated mathematically in the form of a general optimization model whose objective is to maximize the net discounted after-tax cash flow.

3. Computationally efficient algorithms, which allow sensitivity analyses to be performed, are developed to solve the general optimization model for the development and scheduling of offshore development activities.

Further Research

One problem that should be investigated further is what path a well drilled at an angle should take. This path is determined by such things as kick-off point, rate of angle build-up, maximum drilling angle, point at which the drill should turn back vertical (if at all), etc.

Another problem for investigation would be the determination of the schedule for drilling bit replacement. This would entail ascertaining how often a drill bit should be replaced so as to balance the cost of operating inefficiently with a worn bit versus the cost of replacement bits and the time lost for replacement (21).

The consideration of secondary recovery projects in the general development model is also a possibility for further research. This extension would give management an estimate of the return on investment they could expect by engaging in secondary recovery projects.

Transportation cost considerations, either for offshore pipelines or oil tanker, in conjunction with an offshore development model would also be a topic for further research. This would involve developing a transportation model for estimating crude oil costs between a particular field and market and incorporating or linking this model with a general offshore development model. It is quite possible that increasing the complexity of the offshore development model with transportation considerations may necessitate turning partially to simulation to determine solutions to such an offshore model.

BIBLIOGRAPHY

1. Aronofsky, J. S., and Lee, A. S. "A Linear Programming Model for Scheduling Crude Oil Production." Journal of Petroleum Technology, Vol. 9, No. 7 (July, 1957).

2. Aronofsky, J. S., and Williams, A. C. "The Use of Linear Programming Models in Underground Oil Production." Management Science, Vol. 8, No. 4 (July, 1962), pp. 394-407.

3. Berligen, B. A. Economic Optimization/Simulation Model for Offshore Oil Field Development. Masters Thesis, The University of Oklahoma, September, 1973.

4. Benichou, M.; Gauthier, J. M.; Girodet, P.; Hentges, G.; Ribière, G.; and Vincent, O. "Experiments in Mixed-Integer Linear Programming." Mathematical Programming, Vol. 1 (1971), pp. 76-94.

5. Beveridge, G. S. G., and Schechter, R. S. Optimization: Theory and Practice. New York, New York: McGraw-Hill, Inc., 1970.

6. Bohannon, J. M. "A Linear Programming Model for Optimum Development of Multi-Reservoir Pipeline Systems." Journal of Petroleum Technology, Vol. 22, No. 11 (November, 1970), pp. 1429-1436.

7. Campbell, J. M. Oil Property Evaluation. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1959.

8. Charnes, A., and Cooper, W. W. Management Models and Industrial Applications of Linear Programming, Volume II. New York, New York: John Wiley and Sons, Inc., 1961.

9. Coats, K. H. "An Approach to Locating New Wells in Heterogeneous, Gas Producing Fields." Journal of Petroleum Technology, Vol. 21, No. 5 (May, 1969), pp. 549-558.

10. Devine, M. D. Minimum Cost Development of Offshore Oil Fields. Ph.D. Dissertation, The University of Texas at Austin, July, 1969.

11. Devine, M. D., and Lesso, W. G. "Models for the Minimum Cost Development of Offshore Oil Fields." Management Science, Vol. 18, No. 8 (April, 1972), pp. B378-B387.

12. Dougherty, E. L., and Thurnau, D. H. "A Computerized System for Planning Major Investment Decisions in Oil." A paper presented at the 44th Annual Fall Meeting of the Society of Petroleum Engineers of AIME, Denver, Colorado, September 28-October 1, 1969.

13. Editor. "Man, Oil and the Sea." Offshore, Vol. 32, No. 10 (October, 1972), pp. 56-79.

14. Efroymson, M. A., and Ray, T. L. "A Branch-Bound Algorithm for Plant Location." Operations Research, May-June, 1966, pp. 361-368.

15. Hartsock, J. H. A Stochastic Inventory Model for Scheduling Development Drilling. Ph.D. Dissertation, University of Pittsburgh, 1970.

16. Hartsock, J. H., and Greaney, W. A. "A Stochastic Inventory Model for Scheduling Development Drilling." Society of Petroleum Engineers Journal, September, 1971.

17. Higgins, R. V., and Lechtenberg, H. J. "How to Predict Oil-Field Performance." The Oil and Gas Journal, Vol. 68, No. 37 (September 14, 1970).

18. Hu, T. C. Integer Programming and Network Flows. Reading, Mass.: Adison-Weseley, 1970.

19. Jones, P. J. Petroleum Production, Volume II, The Optimum Rate of Production. New York, New York: Reinhold Publishing Company, 1946.

20. Lasdon, L. S. Optimization Theory for Large Systems. New York, New York: Macmillian, 1970.

21. Lilien, G. "A Note on Oil Field Development Problems and Suggested Solutions." To appear in Management Science.

22. Megill, R. E. An Introduction to Exploration Economics. Tulsa, Oklahoma: The Petroleum Publishing Co., 1971.

23. Muskat, M. Physical Principles of Oil Production. New York, New York: McGraw-Hill, Inc., 1949.

24. Nind, T. E. W. Principles of Oil Well Production. New York, New York: McGraw-Hill, Inc., 1964.

25. Ramsey, H. J., Jr., and Guerro, E. T. "The Ability of Time-Rate Decline Curves to Predict Production Rates." Journal of Petroleum Technology, Feb., 1969, pp. 139-141.

26. Rosenwald, G. W., and Green, D. W. "A Method for Determining the Optimum Location of Wells in an Underground Reservoir using Mixed Integer Programming." A paper presented at the 47th Annual Meeting of the Society of Petroleum Engineers of AIME, San Antonio, Texas, October 8-11, 1972.

27. Rowan, G., and Warren, J. E. "A Systems Approach to Reservoir Engineering: Otpimal Development Planning." The Journal of Canadian Petroleum Technology Vol. 6, No. 3 (July-September, 1967), p. 84.

28. Sheffer, L. "Offshore Enters New Era in the 70's." Offshore, Vol. 32, No. 1 (January, 1970), pp. 61-67.

29. Smith, H. V. "Computational Experience with the MIP Feature of MPSX." A talk given at the XIX International Meeting of the Institute of Management Science, Houston, Texas, April 7, 1971.

30. Taha, H. A. Operations Research: An Introduction. New York, New York: Macmillan, 1971.

31. Teichroew, D.; Lesso, W.; Rice, K.; and Wright, G. "Optimizing Models of After-Tax Earnings Incorporating Depletion Allowances." Journal of Financial and Quantitative Analysis, Vol. 2, No. 3 (September, 1967), pp. 265-298.

APPENDIX A

HEURISTIC ALGORITHM FOR THE LOCATION-ASSIGNMENT SUBPROBLEM WITH A STEPWISE PLATFORM COST FUNCTION

This heuristic algorithm was designed by Devine (10) to handle the location-assignment problem where the associated platform cost function increases in finite steps, as illustrated by Figure 8 of Chapter IV.

The following description of this algorithm is derived from reference 10.

<u>Step 1</u>: Pick initial starting points for the platforms.

<u>Step 2</u>: The iterative algorithm described on page 56 and labeled the "Alternate Location-Allocation (ALA) Algorithm" by the author of reference 10 is used with each target being allocated to the "closest" platform until the solution converges.

By simply assigning wells to the closest platform, the solution found by this procedure is locally optimum in drilling cost only, since the size of each platform is not considered.

<u>Step 3</u>: If the drilling cost found by this procedure is below some prespecified cut-off value, then from this solution the procedure tries to reduce the

total cost by reducing the costs of the platforms. (Go to Step 4.) If the drilling cost is above the cut-off, then the procedure starts over again at Step 1.

It is necessary to establish this cut-off level because the ALA Algorithm with assignments made on the "closest" basis tends to get stuck on poor local optimums. Thus, without some cut-off value on the total drilling cost, effort would be wasted trying to improve the platform cost on a solution which already has a high drilling cost. One way to determine an acceptable cut-off level would be to solve the problem several times and then pick the minimum drilling cost found as the cut-off.

Step 4: All platforms are included in the set of platforms whose sizes are subject to reduction (i.e., all platforms are placed under consideration).

There are a variety of ways to approach the problem of improving the total cost by reducing the platform cost. The basic objective is to reduce the platform cost without forcing a drastic reassignment of wells to platforms, which might increase the drilling cost significantly. Let the number of wells to be drilled from platform j (as defined by the starting point solution found by the Case 1 method) be denoted by N_j. Let S_j^r be the necessary capacity for platform j, i.e.,

$$S_j^{r-1} < N_j \leq S_j^r$$

The excess capacity of platform j (denoted by u_j) is simply

$$u_j = S_j^r - N_j.$$

Step 5: From all the platforms under consideration, pick the one with the largest excess capacity, and decrease its capacity to the next lowest level; i.e., from S_j^r to S_j^{r-1}. In case of a tie, pick the one which will give the greatest decrease in platform cost. Denote this platform as I*.

Step 6: Is the total capacity greater than or equal to the number of targets? If yes, go to Step 7. If no, go to Step 11.

Step 7: For the altered platform sizes, find the optimal reassignment of targets to platforms by solving the transportation problem.

Step 8: For the resulting assignment of targets to platforms, solve the NP location subproblems. Calculate the new total cost.

Step 9: Has the total development cost decreased? If yes, the procedure continues by returning to Step 4. If no, go to Step 10.

Step 10: Since the total cost did not decrease, the previous solution is retained and the algorithm stops.

Step 11: This point in the algorithm is reached when decreasing the capacity of platform I* has made the total capacity insufficient. For each platform except I*, determine the "incremental capacity" (IC_j) when the platform is increased to its next largest size; i.e.,

$$IC_j = S_j^{r+1} - S_j^r.$$

Step 12: Search for the single platform whose "incremental capacity" will make the total capacity greater than or equal to the number of targets. Is there such a platform? Is yes, go to Step 13. If no, go to Step 16.

Step 13: In case of a tie between two platforms in Step 12, pick the one with the smallest excess capacity. Denote this platform as J*.

Step 14: Increase platform J* to its next largest size.

Step 15: Has the total platform cost decreased? If yes, continue the procedure by going to Step 7. If no, the algorithm stops.

Step 16: This point is reached from Step 12 if there is no platform J* whose incremental capacity will make the total capacity greater than or equal to the number of targets. Raise platform I* back to its original capacity; remove platform I* from the set of platforms under consideration for size reduction; and continue by returning to Step 5.

In this procedure, generally the transportation problem must be solved several times, and thus it is advantageous to use the out-of-kilter algorithm, so that information from previous solutions can be used.

A flow diagram of the steps in this heuristic algorithm is presented in Figure B-1.

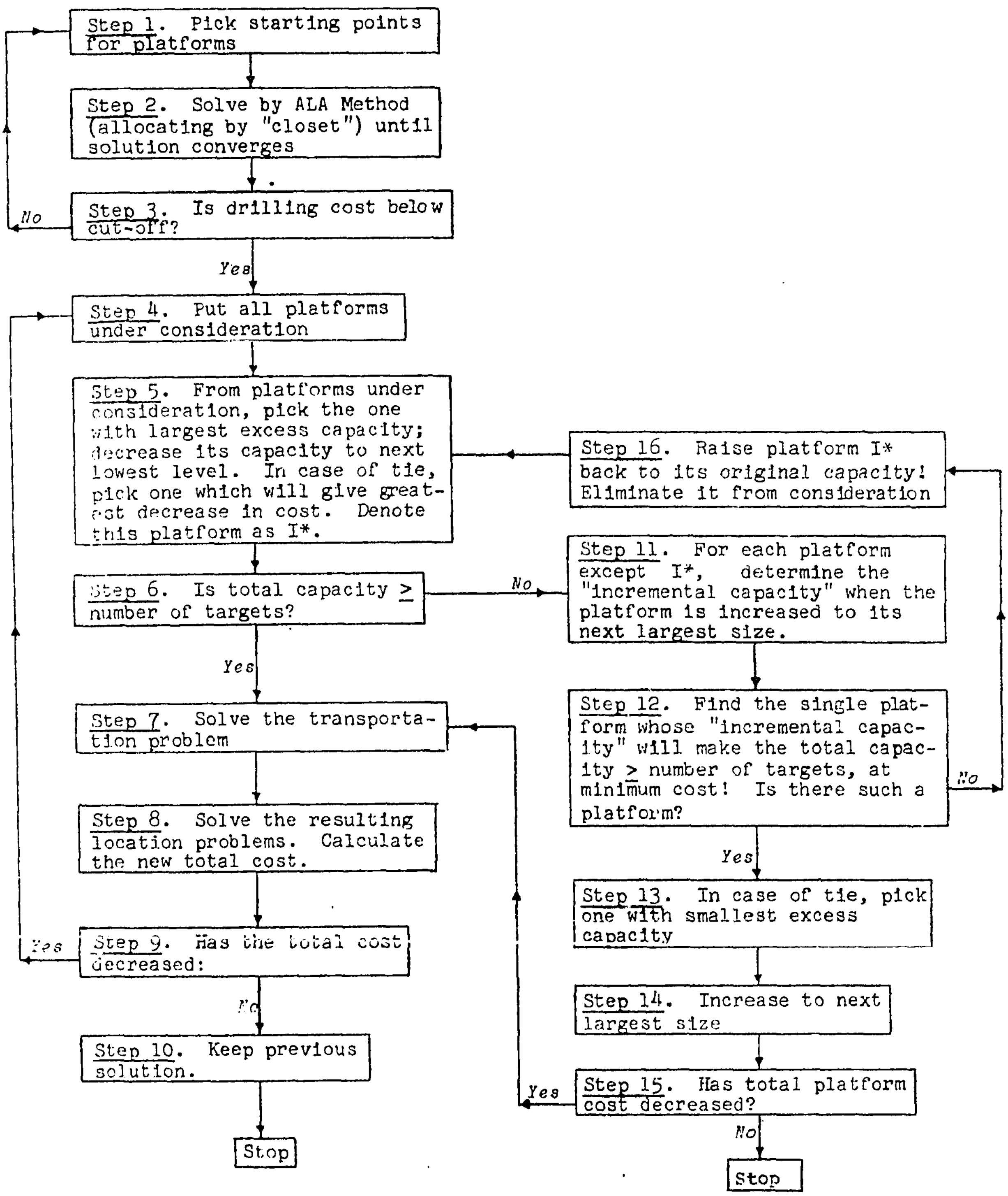

Figure A-1. Flow Diagram for Heuristic Algorithm.

APPENDIX B

This appendix contains

1. the target location coordinates
2. the platform cost functions

for the two example problems solved in Chapter V.

x	y	c
3.00	4.50	4.50
4.00	5.50	4.50
4.00	7.00	4.70
5.20	7.50	4.70
4.50	8.50	4.90
5.50	9.00	4.90
6.00	5.00	5.50
7.00	6.00	5.70
8.00	6.00	5.80
8.50	7.00	5.80
9.00	7.20	5.90
9.50	8.20	6.00
9.50	8.90	6.00
10.00	10.00	6.00
10.00	11.00	6.00
11.00	11.50	6.00
7.50	2.80	6.30
7.70	3.20	6.30
8.00	3.00	6.30
8.30	3.50	6.30
9.10	3.30	6.50
9.60	4.00	6.50
10.10	3.80	6.50
11.00	4.50	6.50

EXAMPLE ONE

Target Coordinates

x,y -- horizontal coordinates in thousands of feet

c -- depth of target in thousands of feet

EXAMPLE ONE PLATFORM

SIZE	COST (Thsds of Dollars)
0	0.0
6	700.0
9	1000.0
12	1285.0
15	1454.0
18	1590.0

x	y	c
1.00	6.00	8.20
1.00	8.00	8.30
2.00	3.00	8.20
2.00	5.00	8.40
2.00	7.00	8.70
2.00	9.00	8.80
3.00	6.00	8.80
3.00	10.00	9.20
4.00	1.00	9.00
4.00	3.00	9.10
4.00	5.00	9.30
4.00	7.00	9.50
4.00	9.00	9.60
5.00	2.00	9.00
5.00	6.00	9.20
5.00	10.00	9.70
6.00	4.00	9.40
6.00	7.00	9.70
6.00	11.00	9.90
7.00	2.00	9.10
7.00	6.00	9.80
7.00	9.00	10.00
7.00	12.00	10.20
4.00	8.00	9.50
9.00	2.00	5.50
9.00	7.00	5.80
9.00	9.00	6.00
9.00	13.00	6.30
10.00	4.00	5.60
10.00	6.00	6.00
10.00	12.00	6.40
11.00	2.00	5.70
11.00	8.00	6.30
11.00	10.00	6.40
11.00	14.00	6.70
12.00	5.00	6.30
12.00	7.00	6.40
12.00	12.00	6.80
13.00	1.00	5.80
13.00	3.00	5.90
13.00	8.00	6.60
13.00	10.00	6.90
13.00	14.00	7.20
14.00	2.00	6.10
14.00	5.00	6.70
14.00	6.00	6.80
14.00	9.00	7.10
14.00	11.00	7.20
15.00	1.00	6.30
15.00	4.00	6.80
15.00	7.00	7.00
15.00	12.00	7.30
9.00	5.00	5.70
11.00	4.00	5.90
11.00	6.00	6.10
12.00	3.00	6.20
13.00	4.00	6.10

EXAMPLE TWO

Target Coordinates

x,y --horizontal coordinates in thousands of feet

c --depth of target in thousands of feet

EXAMPLE TWO PLATFORM

SIZE	COST (Thsds of $)
0	0.0
6	700.0
9	1000.0
12	1285.0
15	1454.0
18	1590.0
20	1700.0
25	2000.0