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A COMPARATIVE STUDY OF AN ADVANCE ORGANIZER IN MATHEMATICS TO DETERMINE ITS EFFECTIVENESS ON KNOWLEDGE ACQUISITION AND RETENTION

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WALDO P. CAPONECCHI

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A COMPARATIVE STUDY OF AN ADVANCE ORGANIZER IN MATHEMATICS TO DETERMINE ITS EFFECTIVENESS ON KNOWLEDGE ACQUISITION AND RETENTION

APPROVED BY

DISSERTATION COMMITTEE

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CHAPTER I

INTRODUCTION TO THE PROBLEM

Background of the Problem

One of the principal functions of pedagogy is to facilitate the transmission of knowledge by presenting ideas and information in an effective manner so that clear, stable, and unambiguous meanings emerge and are retained over a period of time. The selection of a teaching strategy and instructional materials that can facilitate this function is a problem that has long confronted teachers. A commonly accepted teaching strategy is that of teaching the material in a manner which will make the learning meaningful for the students. David P. Ausubel asserts that meaningful learning takes place if the learning task can be related in nonarbitrary, substantive fashion to what the learner already knows, and if the learner adopts a corresponding learning set to do so (Ausubel, 1963). This learning set refers to the learner's disposition to learn or perform in a particular way; in meaningful learning, the learner has a set to relate substantive aspects of new material to relevant aspects of his cognitive structure so that significant relationships will be formed and incorporated. Cognitive structure refers to an individual's organization, stability, and clarity of knowledge in a particular subject-matter field at any given time (Ausubel, 1963). A human being rarely starts out with a "blank tablet" when approaching a new problem; at the very least he is directed by assumptions formed on the basis of his past experience. Ausubel regards an individual's existing cognitive structure in a given field as "the major factor influencing the learning and retention of meaningful new material in this same field" (1963, p. 26). He proceeds from the basis that the principal factor affecting meaningful learning and retention is the cognitive structure of the learner at the time he meets the material to be learned. Most new materials that students encounter in a school setting are related in a nonarbitrary and substantive manner to previously learned meaningful concepts, and most curricula are organized so as to introduce new facts and ideas as smoothly and efficiently as possible. The learning and retention of potentially meaningful material are influenced by the concepts in cognitive structure with which they will interact. Ausubel asserts that "it is precisely this interaction of new learning tasks with existing cognitive structure that is the distinctive feature of meaningful learning" (1963, p. 7).

Knowledge may be transmitted by showing how ideas and phenomena are logically related. These relations may be hierarchical, wherein some elements are subsumed under more generalized concepts. Ausubel views cognitive structure as being hierarchically organized with the organization ranging from regions of greater to lesser inclusiveness. The subsumption process connects these regions. As meaningful material is learned, he theorizes that it is subsumed by a concept in cognitive structure which is more inclusive. In accord with this view, he employs the principle of progressive differentiation, wherein the most general and inclusive ideas of a discipline are presented first; these are then progressively differentiated in terms of their detail. A more explicit way of stating this principle is to say that new ideas and information can be efficiently learned and retained when more inclusive and appropriately relevant concepts are available in the learner's cognitive structure to serve a subsuming role (Ausubel, 1963).

Ausubel feels that it is highly unlikely that a learner will spontaneously have available the most relevant and proximate subsuming concepts in a particular learning situation. From the foregoing discussion, it follows that the most efficient way to facilitate learning and retention is to introduce appropriate subsumers and make them part of cognitive structure prior to the actual presentation of the learning task. These introduced subsumers become "advance organizers" for the reception of new material. Advance organization is thus a strategy for deliberately manipulating cognitive structure by employing introductory materials prior to the presentation of the learning task. The organizers must be written at a higher level of abstraction, generality, and inclusiveness than the learning task itself. They differ from summaries and overviews in that the latter are ordinarily presented at the same level of abstraction, generality, and inclusiveness as the learning material itself. These simply emphasize the salient points of the material by omitting less important information, and achieve their effect primarily through repetition. The function of the organizer is to provide what Ausubel refers to as ideational scaffolding, which will allow for the incorporation and retention of the more detailed and differentiated material which will follow in the learning passage. The organizer is also designed to affect the discrimination of the learning task from the concepts which subsume it. This is of importance since the ability to maintain similarities and differences between established concepts and new materials is a major factor affecting the retention of a learning task. The advance organizer should provide the learner with a generalized overview of all the major similarities and differences between the new material in the learning passage and his previous concepts before he encounters the new concepts in a more detailed and particularized form.

Thus Ausubel's theory of advance organization accomplishes the objectives of increasing the functional retention of new subject matter knowledge by enhancing the organizational strength of a student's existing knowledge, and discriminating the new material from the conceptual systems that subsume it. In addition to being at a higher level of inclusiveness and abstraction, the advance organizers should be stated in familiar terms and employ appropriate illustrations. As Ausubel states:

> . . . if an organizer can first delineate clearly, precisely, and explicitly the principal similarities and differences between the ideas in a new learning passage, on the one hand, and existing related concepts

> in cognitive structure, on the other, it seems reasonable to postulate that the more detailed ideas and information in the learning passage would be grasped with fewer ambiguities, fewer competing meanings, and fewer misconceptions suggested by the learner's prior knowledge of the related concepts; and that as these clearer, less confused meanings interact with analogous established meanings during the retention interval, they would be more likely to retain their identity (1963, p. 83).

Statement of the Problem

This investigation treats the following problem: Does the use of an advance organizer in presenting the topic of matrices contribute to greater student achievement and retention of the material when compared with the use of an introductory overview and a control treatment?

Need for the Study

Mathematics is a discipline which has a great deal of structure, and in which many relations are hierarchical. It thus lends itself to the subsumption model of teaching and learning as proposed by Ausubel which suggests that central unifying ideas of a discipline be taught first and that less inclusive ideas and information be related clearly and logically to the unifying ideas by subsumption. These central unifying ideas thus become advance organizers. There are various concepts in mathematics which can serve a unifying purpose. Much of the curriculum revision undertaken during the past two decades in mathematics has been centered around some of these concepts. In this study, use will be made of one concept as an advance organizer for the material included in the learning passage.

If significant differences are found in favor of the advance organizer treatment, this would lend credence to Ausubel's theoretical position as regards his hierarchical view of cognitive structure and his theory of subsumption. In addition, it might lead one to attempt to determine if other major mathematical topics can be efficiently handled in a similar manner. This could result in a major change in the traditional method of introducing new subject matter knowledge in the mathematics classroom.

At this point in time, few studies involving advance organization have been conducted in mathematics. The results of some of these studies have limited application for the typical classroom situation due to the methods employed. Studies need to be conducted which approximate the normal classroom situation wherein the teaching of the material is spread over several class periods, the material is not of the self-instructional type, and there is interaction between the students and the teacher during the course of the instruction. This study will be constructed to follow as closely as possible the position set forth by Ausubel. It is designed to give sufficient control so that any differences which may be found can be attributed to their proper causes. It is also hoped that further information will be obtained on the differential effects of advance organizers across ability levels, since some conflicting results have been found in this area.

Definition of Variables and Terms

Cognitive structure refers to an individual's organization, stability, and clarity of knowledge in a particular subject-matter field at any given time.

Reception learning takes place when the entire content of what is to be learned is presented to the learner in final form. The learner is required to internalize the material presented to him so that it is available and reproducible at a future date.

Meaningful learning takes place if the learning task can be related in nonarbitrary, substantive fashion to what the learner already knows, and if the learner adopts a corresponding learning set to do so.

Advance organizers are introductory materials employed prior to the presentation of the actual learning task with the goal of deliberately manipulating cognitive structure so as to enhance proactive facilitation or minimize proactive inhibition. They consist of introductory material at a higher level of abstraction, generality, and inclusiveness than the learning task itself. Their function is to provide ideational scaffolding for the stable incorporation and retention of the more detailed and differentiated material that follows in the learning passage, as well as to increase discriminability between the learning passage and related, interfering concepts in cognitive structure. An expository organizer is used to provide proximate subsumers in the case of completely unfamiliar material; a comparative organizer is employed when the learning material is relatively familiar to the learner and is used to integrate new concepts with basically similar concepts in cognitive structure.

<u>Introductory overviews</u> are materials which are ordinarily presented at the same level of abstraction, generality, and inclusiveness as the learning material itself. They do not create an ideational scaffolding in which the future learning will be imbedded. They emphasize the salient points of the material by omitting less important information, and achieve their effect largely through repetition, condensation, selective emphasis on central concepts, and prefamiliarization of the learner with certain key words.

The <u>ability levels</u> (high, medium, low) were determined by using the sum of the English and mathematics scores each subject obtained on the ACT test. Both scores were used since the student's ability to comprehend the experimental materials would be affected by his verbal and mathematical aptitudes.

Experimental Design

The experimental design for this study is illustrated in Figure 1 which graphically depicts the nine-cell two-part analysis of variance design for analyzing the influence of treatments, ability levels, and the interaction between the two main effects on the posttest and retention test. This three by three factorial analysis had unequal observations in each cell.

The subjects were classified into three experimental groups with the following sequences of instruction and testing:

Group I	T_1	AO	L	T_2	T_3
Group II	T_1	IO	L	T_2	T_3
Group III	T_1	C	L	T_2	T_3

Figure 1: Multiple Classification Analysis of Variance for the Experimental Design.[a]

Levels \ Treatments	Advance Organizer	Introductory Overview	Control	
High	Cell #1 $n_{11} = 14$	Cell #2 $n_{12} = 9$	Cell #3 $n_{13} = 9$	$n_{1.} = 32$
Medium	Cell #4 $n_{21} = 9$	Cell #5 $n_{22} = 12$	Cell #6 $n_{23} = 9$	$n_{2.} = 30$
Low	Cell #7 $n_{31} = 6$	Cell #8 $n_{32} = 13$	Cell #9 $n_{33} = 10$	$n_{3.} = 29$
	$n_{.1} = 29$	$n_{.2} = 34$	$n_{.3} = 28$	$N = 91$

[a]In all tables in succeeding chapters, cell sizes, column sums, and row sums are the same as in Figure 1, and they will not be repeated.

where "AO" represents the advance organizer, "IO" represents the introductory overview, and "C" represents the control treatment, while "L" denotes the classroom learning situation, "T_1" represents the pretest, "T_2" represents the posttest, and "T_3" represents the retention test.

Hypotheses

A comparison of the effect of an advance organizer will be made with that of an introductory overview and a control treatment in teaching the topic of matrices in an undergraduate mathematics course at the University of Oklahoma. Since research suggests some contradictory results concerning the effects of an advance organizer on students of different ability levels, this study will probe for differential effects of the organizer on ability levels if significant interaction is present. The hypotheses to be tested at the 0.05 level of significance are:

Hypothesis 1: There are no significant differences in mean learning test scores among the three treatments.

Hypothesis 2: There are no significant differences in mean retention test scores among the three treatments.

Hypothesis 3: There are no significant differences in mean learning test scores among the three ability levels.

Hypothesis 4: There are no significant differences in mean retention test scores among the three ability levels.

Hypothesis 5: There are no significant interactions between the treatments and ability levels as measured by mean learning test scores.

Hypothesis 6: There are no significant interactions between the treatments and ability levels as measured by mean retention test scores.

If Hypothesis 5 is rejected, then Hypothesis 1 and Hypothesis 3 will be tested as simple main effects; otherwise, they will be tested as main effects. If Hypothesis 6 is rejected, then Hypothesis 2 and Hypothesis 4 will be tested as simple main effects; otherwise, they will be tested as main effects.

If Hypothesis 1 is rejected as a main effect, the following hypotheses will be tested:

Hypothesis 1a: There is no significant difference in mean learning test scores between the advance organizer treatment and the introductory overview treatment.

Hypothesis 1b: There is no significant difference in mean learning test scores between the advance organizer treatment and the control treatment.

Hypothesis 1c: There is no significant difference in mean learning test scores between the introductory overview treatment and the control treatment.

If Hypothesis 2 is rejected as a main effect, the following hypotheses will be tested.

Hypothesis 2a: There is no significant difference in mean retention test scores between the advance organizer treatment and the introductory overview treatment.

Hypothesis 2b: There is no significant difference in mean retention test scores between the advance organizer treatment and the control treatment.

Hypothesis 2c: There is no significant difference in mean retention test scores between the introductory overview treatment and the control treatment.

If Hypothesis 3 is rejected as a main effect, the following hypotheses will be tested:

Hypothesis 3a: There is no significant difference in mean learning test scores between the high ability level group and the medium ability level group.

Hypothesis 3b: There is no significant difference in mean learning test scores between the high ability level group and the low ability level group.

Hypothesis 3c: There is no significant difference in mean learning test scores between the medium ability level group and the low ability level group.

If Hypothesis 4 is rejected as a main effect, the following hypotheses will be tested:

Hypothesis 4a: There is no significant difference in mean retention test scores between the high ability level group and the medium ability level group.

Hypothesis 4b: There is no significant difference in mean retention test scores between the high ability level group and the low ability level group.

Hypothesis 4c: There is no significant difference in mean retention test scores between the medium ability level group and the low ability level group.

CHAPTER II

REVIEW OF RELATED LITERATURE

The review of related literature will be presented in two parts. The first part will review, in chronological order, those articles published in journals which were not derived from a doctoral dissertation. The second part will review, in chronological order, the research presented in Dissertation Abstracts.

Journal Articles

Experimentation with advance organizers began when Ausubel (1960) studied the effect of an advance organizer with senior undergraduate students on a learning passage dealing with the metallurgical properties of steel. He employed an experimental and a control group, the latter reading a historical passage on the steel-making process. On a posttest, the advance organizer treatment group performed significantly better than the control group, from which he concluded that the organizer had a facilitating effect.

Ausubel and Fitzgerald (1961) conducted a second study with senior undergraduate students dealing with selected Buddhist concepts. They employed a comparative organizer, an expository organizer, and a control treatment. On knowledge acquisiton, the comparative organizer treatment yielded statistically superior results; on a one week retention test, both organizer treatments were statistically superior to the control group, but there was no statistical difference between the two organizers. Most of the differences obtained were attributed to that segment of the population who scored below the median on a pretest of Christianity. Those students who performed well on this pretest obtained significantly higher scores on the Buddhism retention test; this upheld the hypothesis that the learning and retention of unfamiliar verbal material varies positively with its discriminability from related, previously learned concepts in cognitive structure.

Ausubel and Fitzgerald (1962) investigated the effects of an advance organizer, antecedent learning, and general background knowledge on the learning and retention of two unfamiliar sequential passages about endocrinology with predominantly senior undergraduate students. The organizer facilitated the learning and retention of the first pubescence passage, with practically all of the obtained difference between the experimental and control groups coming from subjects in the lower third of verbal ability scores. The organizer did not enhance the learning and retention of the second pubescence passage, but there was a suggestion of a positive interaction between the effects of the organizer and of general background knowledge which enabled the subjects to put their background knowledge to effective use in structuring the unfamiliar new material in the second passage. The antecedent learning (the first passage) had a significant facilitating effect on the subsequent learning (the second passage). Also, general background knowledge in endocrinology facilitated the learning of unfamiliar material in the subject-matter field.

Fitzgerald and Ausubel (1963) conducted an experiment in which the main purpose was to re-examine the effect of attitudes on the learning and retention of controversial material. Concurrent with this analysis, the efficacy of an organizer in facilitating the learning and retention of controversial materials was tested. The experiment was conducted with junior high school students as one unit in a sequence dealing with the causes of the Civil War. Both an organizer and a control set of materials were employed. The organizer treatment was significantly effective in facilitating learning and retention of controversial material. Most of the benefit derived from the organizer was manifested in relation to retention rather than to learning. Also, those subjects who scored in the upper third on a pretest had significantly higher scores on the retention test. The hypothesis that positive attitudinal bias facilitates and that negative attitudinal bias inhibits the learning of controversial materials was rejected; the differences were in the predicted directions but were not statistically significant. The hypothesis that attitudinal bias has no effect on the retention of controversial materials was supported.

Wittrock (1963) investigated the possibility that simple learning sets, which contain no explicit information about the content to be learned, may affect the learning of Buddhism by actively involving the subjects in comparing and contrasting their related information (Christianity to Buddhism). The study was conducted with upper division college students. Materials identical to those used by Ausubel and Fitzgerald (1961) were employed, except that in place of the advance organizer, four different learning sets were employed. Written instructions were designed to eliminate explicit information about Buddhism but to establish specific learning sets to: (1) compare Buddhism and Christianity, (2) contrast Buddhism and Christianity, (3) compare and contrast Buddhism and Christianity, (4) understand and remember the content on Buddhism. Each learning set consisted of two sentences, with appropriate word substitutions for each of the sets. The second and third groups performed significantly better than the fourth group on both the posttest and retention test of three weeks. The author concluded that some types of sets may enhance the learning and retention of connected discourse material.

Ausubel and Youssef (1963) studied the effects of previously learned background knowledge (Christianity) and two comparative organizers (one pointing out the similarities and differences between Buddhist and Christian doctrines and the second performing the same function for Buddhist and Zen Buddhist doctrines) on the learning and retention of two similarly written passages dealing with the principal concepts of Buddhism and Zen Buddhism. A control group was employed. The subjects were predominantly senior undergraduates who were all enrolled in teacher education curricula at the secondary level. The previously learned background knowledge had a facilitating effect on the learning and retention of the Buddhism material, with those subjects in the upper part of the distribution of scores on a Christianity test making higher retention scores on the Buddhism test. The first organizer significantly facilitated the learning and retention of the Buddhism passage, but there was no interaction between Christianity scores and the organizer. The interaction between the organizer and verbal ability levels was not significant, although there was a trend for the organizer to benefit the lower ability subjects. The second organizer did not facilitate the learning and retention of the second passage (Zen Buddhism), and there was no significant interaction between the organizer and level of Buddhism knowledge. However, knowledge of the newly learned Buddhism material significantly facilitated the learning and retention of the Zen Buddhism passage, with subjects who made high, average, or low scores on the Buddhism test tending to make corresponding scores on the Zen Buddhism test.

Scandura and Wells (1967) compared historical and model introductions to group theory and combinatorial topology to determine their effects on learning efficiency with undergraduate elementary education majors. The organizers were presented in the form of mathematical games in which the structure of the material to be learned was presented, but in which the proper mathematical terms were not employed. The control group studied historical material on the men who developed the topics. The experiment was conducted during one class period. Overall performance in the organizer groups as measured by a posttest was superior to that of the historical groups. The differences due to materials were significant, but the interactions between type of introduction and material were not, although the organizer effect seemed to be stronger with the topology material. Since group theory concepts may be more familiar to students, they concluded that the effectiveness of advance organizers may decrease with increasing familiarity of the models.

Grotelueschen and Sjogren (1968) manipulated the structure of the introductory material and the sequencing of the learning task (number base concepts) and studied the effects of these manipulations on learning and transfer with a small sample of paid adults of superior intelligence. They found that the advance organizers may produce facilitative effects on the learning of a complex task when used with adults of superior ability. This suggests that complexity of the learning task is a variable which must be considered when evaluating the effect of introductory materials on subsequent learning and transfer. This may account for the apparent disparity with Ausubel and Fitzgerald's results (1962) wherein only the lower verbal ability subjects were assisted by the organizers.

Proger, et al (1970), compared four different types of advance organizers on learning material dealing with nonreligious aspects of Amish life with twelfth grade social studies students. Two of these organizers constituted "overt responses" (a completion pretest and a true-false pretest), and the other two constituted "covert responses" (sentence outline and paragraph abstract). The classes were homogeneously grouped according to ability levels. The posttest included eight concepts which were stressed in the organizer materials and twelve concepts that were not covered in them. On the eight items which were stressed in the organizer materials, the covert response treatment groups performed significantly better for those subjects of lowest ability, low ability, and average ability. No significant differences were found for the above average and highest ability subjects. On these eight items, boys performed significantly better than girls. No significant differences were found for either the overt or covert response treatments on the twelve concepts which were not included in the organizer materials. The interaction between treatment and ability levels acted in the same way as that found in Ausubel and Fitzgerald (1961, 1962).

Romberg and Wilson (1973), using eleventh grade algebra classes, examined the effect of three kinds of information related to lesson content (advance organizer, cognitive set, and post organizer) on the acquisition and retention of self-instructional material pertaining to radioactive decay. The term cognitive set was used to identify information given to students prior to instruction that informs them of anticipated associations they can expect to acquire in the instruction. The advance organizer consisted of ten sentences read prior to instruction that related the new material to the learner's general background. The cognitive set was one sentence indicating that the student "should know the general law of radioactive decay and be able to solve simple problems based on the application of this law" (p. 71). The post organizer, given to the students after instruction, was eleven sentences similar in content to the advance organizer which related the content of instruction to the learner's background knowledge. No significant results were obtained on the posttest. On the one week retention test, the mean scores for the main effect of cognitive set were significant, and the interaction of the advance organizer and post organizer was significant (with significantly lower scores occurring when both treatments were applied). It should be noted that the concept of an advance organizer that was used in this study was developed to be free of a cognitive set, and as such was not the same as that of Ausubel and Fitzgerald (1961) which, as Wittrock (1963) observed, contains the cognitive set as a part of the advance organizer.

Peterson, et al (1973), tested the hypothesis that the learning and retention of self-instructional materials dealing with the mathematical concept of network tracing can be facilitated by providing the learner with an advance organizer, a post organizer, or knowledge of the behavioral objective (KBO). The KBO is very similar in nature to the cognitive set of Romberg and Wilson. The advance and post organizer treatment materials consisted of short discussions of a specific network problem, with the advance organizer containing the information that the problem should be solved diagrammatically and the post organizer containing the diagrammatic solution. The KBO stated that the learner should be able to solve simple problems requiring application of the general rule for tracing networks. The subjects, in the three independent replications made in this experiment, were either eighth grade students or college students enrolled in a mathematics course for elementary school teachers. For the posttest, none of the main effects were significant, and the only interaction which was significant was between the advance and post organizers (consistent with Romberg and Wilson). On a retention test given one week later, none of the main effects or interactions were significant. As indicated in their report, "studies should be designed to approximate normal classroom situations more closely and to include longer instructional spans with a more natural involvement of the teacher than is possible with self-instructional materials" (p. 83).

Dissertations

Blackhurst (1966) investigated the effects of an orally presented expository organizer with educable mentally retarded adolescents on the learning and retention of oral information pertaining to passing legislation in the United States Congress. He found no significant differences among his organizer, introductory, and control groups on learning or twelve day retention tests.

Schulz (1966) studied the role of organizers in an elementary school science unit. The first organizer was used to provide ideational anchorage for the subsequent work, whereas the second organizer, presented between the learning tasks, explicitly related the two learning tasks. His control group studied the same unit as the experimental group, but did not employ the two organizers. While not finding conclusive evidence regarding the general role of organizers, his study did suggest that advance organizers facilitate learning when pupils are lacking in analytic ability to reorganize information clearly into cognitive structure.

Woodward (1966) undertook a comparative study of teaching strategies involving advance and post organizers and discovery and nondiscovery techniques where the instruction is mediated by computer. Four treatment groups were thus employed, with no control group present. The subject matter was modulus 11 arithmetic with the subjects drawn from two college courses. On learning test scores, on transfer test scores, and on time to complete the learning program, the author found no significant differences between organizer groups, no significant differences between program groups, and no significant interactions between type of program and type of organizer.

Jerrolds (1967) investigated the relative effects upon delayed retention of specific facts of advance organizers as described by Ausubel and modified advance organizers formulated around main idea concepts at the ninth grade level. The groups were furthur divided as to whether or not instruction in the use of the organizers was given. No significant differences in retention were indicated between the two types of organizers. None of the groups using advance organizers differed significantly from the control group which did not employ an organizer. The only significant difference revealed relative to IQ levels was that those above average IQ subjects using the modified organizer with prior instruction did better than above average IQ subjects using the modified organizer without prior instruction.

Kuhn (1967) performed two experiments involving advance organization in an elementary college biology course, one of which involved a careful sequencing of the material to be learned. He concluded that the advance organizer technique, in comparison to a control treatment, enhanced the acquisition and retention of the material, and that the ability of the individual to acquire and retain information is highly related to his analytic ability. He found some evidence to indicate that organizers are particularly effective with individuals of low analytic ability. The use of the organizer may have a positive effect on the acquisition and retention of carefully sequenced material.

Neisworth (1967) investigated the effects of advance organizers with educable mentally retarded adolescents on the learning and retention of a learning passage dealing with accidental poisoning. He found no significant differences between the organizer and control treatments on either evaluation.

Triezenberg (1967) studied the relative effectiveness of three levels of abstraction (verbal, sketch, mechanical model) in the use of the concept of equilibrium as an advance organizer in teaching ecological systems by televised instruction in grades seven and nine. He tested at three cognitive process levels: knowledge, comprehension, and application. He found differential effects among the levels of abstraction at all levels of testing. Among his results was that at the comprehension level, the use of working models was significantly superior to verbal reference or the use of sketches, and there were no significant differences among the treatments at the knowledge and application levels. In both grades the pupils of high ability earned significantly higher test scores than pupils of average ability.

Farman (1968) investigated the relative effects of two sequences of presentation of a comparative organizer on subject retention on two parallel tasks selected from descriptive statistics. In one treatment, the organizer was interpolated between the parallel tasks; in another treatment, the organizer was presented subsequent to the parallel tasks and coupled with some further directions. He employed three other experimental conditions for further analysis. No significant inter-treatment differences were found in the subjects' performance on one of the tasks. On the other task, a significant difference between the two treatment groups employing advance organizers was found for subjects at intermediate and lower levels of quantitative aptitude, with the difference being in favor of the subsequent presentation of the organizer. The treatment involving simple overlearning yielded a mean performance score that was significantly better than the combined average score of the other four treatment groups.

Allen (1969) studied the effects on learning and retention of advance organizers and memory level or higher order questions with social studies material at the ninth grade level. Both advance organizers and type of question seemed to have an effect on delayed retention. Other tests suggested that advance organizers enhanced the effect of treatment questions for average and below average students and resulted in general facilitation of learning for above average students.

Billey (1969) undertook an analysis of the lecture method with the use of advance organizers in a college level educational psychology course. It was found that the experimental group had a significantly higher mean score on a three week posttest than the control group on one of the two topics taught. On the topic for which a significant difference was not obtained, the posttest was given eight weeks after the organizer was presented. No interaction between the type of introduction and ability levels was found.

Brovey (1969) examined the effects of advance organizers on the acquisition and retention of geological information acquired in the field. Subjects who received advance organization did not show significantly greater acquisition or retention than subjects receiving an historical introduction prior to the field experience. It was suggested that the use of concrete examples (in the field) of the expository materials may have reduced the efficacy of these materials.

Davis (1969) constructed three levels of advance organizers which he inserted either prior to or after the learning session dealing with the uses of sources of information. The study was conducted with eighth grade students. He did not employ a control treatment. There was no significant contribution to criterion scores by treatment, sequence, or the interaction of treatment and sequence. He found that most of the differences he obtained could be attributed to mental ability, and that the organizers were not sufficiently powerful to overcome individual differences in mental ability.

Hustuft (1969) examined the effect of advance organizers upon college student decision making in a simulated environment, the advance organizers being videotaped classroom incidents. The groups which used the organizers elicited posttest behavior which was significantly different from those who did not use the organizers. The use of the organizers had a significant effect upon posttest scores. The temporal position of the advance organizers within the instructional strategy (contiguous to the lecture and two days before the lecture) made a significant difference in terms of posttest scores when compared to the group which did not receive the organizers, the difference being in favor of the organizer treatment.

Townsend (1969) studied the effects an advance organizer may have on learning to graphically analyze straight line kinematics in a college physics course within the two instructional modes of an autotutorial printed program and classroom presentation by an instructor. No significant differences were found between the advance organizer and the traditional introduction treatment. No significant interaction between ability levels and the type of introduction or instruction was found. A significant interaction was found showing a positive effect of the advance organizer under programmed instruction.

Weisberg (1969) inquired whether advance organizers in the form of visual aids might serve the same function as verbal organizers when teaching an earth science concept to eighth grade students. He employed a control group. He found a significant difference between the two types of organizers, the difference being in favor of the visual materials. Students grouped into categories of high, medium, and low prior knowledge showed significant differences in improvement with all types of organizers, with the middle category showing the greatest improvement relative to the other two categories. No interactions were found between treatment and prior knowledge categories.

Gubrud (1970) investigated the effect of an advance organizer and a concrete experience on learning the concept of vectors in junior and senior high school. There was some evidence that the organizer facilitated the learning of the material on vector addition. It was conjectured that the organizer can be usefully employed only by individuals with relatively high abstract thinking ability. The most general finding was that achievement in this subject matter area was nearly linearly related to grade level.

Kirkwood (1970) investigated the use of advance organizers (defined as overviews) and "typical" introductions (defined as motivational passages) in a classroom presentation in industrial arts with undergraduate elementary education majors. He also employed a control group. The three groups did not differ significantly from each other on scores achieved on a posttest. It was found that those with a high ability level (as measured by SAT scores) attained a significantly higher mean score than those with a low ability level, but there was no interaction of treatment with ability level.

Malone (1970) studied the effectiveness of a cybernetic model as an advance organizer in teaching physiological regulation in a community college biology course. Two separate studies were conducted. In the first study, the subjects were retested after three weeks, and in the second study, after one week. There was no significant difference between the treatment groups (organizer and historical introduction), between males and females (in the second study), and no interactive effects between treatment and sex on both knowledge acquisition and retention.

Ratzlaff (1970) studied the relative effectiveness of advance organizers in the acquisition, retention, and transfer of seventh grade, base five mathematics. He employed three treatment groups: advance organization, concurrent organization, and minimal organization. The latter two treatments employed historically relevant material in the introductions, with the concurrent organization treatment having the lessons taught in the same meaningful, principle-related fashion used with the advance organizer treatment. The minimal organization group was taught the unit in a rote, mechanical manner. The data revealed no significant differences on any of the criteria variables as measured by the posttests.

Ryder (1970) undertook to determine the effects of experience background and an advance organizer on third and fifth grade pupils' understanding of two science concepts. The findings revealed that grade, sex, and treatment (an orally presented organizer and a control treatment) significantly affected pupil understanding of the two concepts, and that experience backgrounds ("good" and "poor") had no statistically significant effect. Statistically significant results were found for the interaction of grade and treatment. It was concluded that the advance organizer is most advantageous to pupils with rich experience backgrounds.

Smith (1970) examined the influence of advance organizers, overview-summary statements, and vagueness on the comprehension of oral instructional messages with elementary school children. A control group was employed. The results indicated no significant differences in comprehension of the oral messages among the three treatment groups.

Steinbrink (1970) researched the effectiveness of advance organizers for teaching geography to disadvantaged rural black elementary students. Analysis of the data indicated that the group who was taught the unit with advance organizers scored significantly higher on the posttest than did the control treatment group which did not employ the organizer.

Thelen (1970) studied the effect of advance organizers and guide material in viewing science motion pictures in a ninth grade classroom. She employed four treatment groups, including a control treatment. The use of advance organizers and guide material when used alone or in combination did not result in statistically significant differences in learning or two week retention. The interaction of advance organizers and guides was found to be nonsignificant. Students not using advance organizers demonstrated a significant difference in attitude change towards motion pictures as instructional tools, the change being negative.

Bertou (1971) evaluated the effect of advance and post organizers and interspersed questions and combinations thereof as mechanisms for facilitating attainment and retention of material from a televised lecture with ninth grade students. Eight treatment groups were employed, including a control group. As measured by test scores, the acquisition and retention of knowledge from the video-taped lessons was not significantly affected by the use of advance or post organizers, but was significantly affected by the use of interspersed questions. No interaction effects were found between the three treatment factors.

Dvergsten (1971) studied the effect of the use of advance organizers combined with guided discovery on achievement and retention in tenth grade biology. One group used advance organizers coupled with guided discovery, and the other used guided discovery alone. The two treatments were equally effective in teaching facts, concepts, and principles of biology, understanding of methods and processes of science, and developing critical thinking abilities. The treatment groups were equally effective in retaining acquired facts, concepts, and principles as measured after eight weeks. The students taught by the guided discovery method developed more positive attitudes toward science and science related concepts.

Ethirveerasingam (1971) compared the effect of organizers to that of overview and summary statements in learning and retaining complex verbal material by eleventh grade vocational agriculture students. He did not employ a control group. The data revealed no significant differences among his treatments, and there were no interactions between treatment and two and nine day retention scores. It was concluded that organizers and overview and summary, if they contribute to the learning and retention of complex verbal material, do so to the same extent.

Hershman (1971) studied the efficacy of advance organizers and behavioral objectives for improving achievement in a college physics course. He included a control treatment. No significant differences were found that could be attributed with assurance to the three treatment effects. An analysis of ability groupings within treatment groups indicated wide variability among the low ability groups from test to test, which was not consistently in keeping with previous research. The behavioral objectives were more able to help the lower ability student in most of the cases.

Kahle (1971) studied the effect of an advance organizer and the predictive ability of micro-learning tasks in conjunction with four sequenced audio-tutorial units in a college biology course for elementary education majors. One group received the advance organizer variable prior to the instructional treatment; the other group did not. The micro-learning tasks, problem solving situations were used across the two groups. No significant differences were found due to the effect of the advance organizer.

Munford (1971) investigated the effect of an advance organizer with college students when it was positioned before the learning passage, and positioned after the learning passage. He also employed a control group which read an historical passage before the learning passage was presented. He obtained no significant differences among the groups in the amount of initial learning or retention.

Lucas (1972) studied the effects that three types of advance organizers (audio, visual, and written organizers) had upon the learning of a biological concept in seventh grade science. A control group was utilized. The results indicated that the use of the three types of advance organizers did not significantly affect the learning of the concept, and that no interactive effects of IQ, abstract reasoning, and sex were found. It was found that high, medium, and low IQ groups and high, medium, and low abstract reasoning groups were not affected by the treatment.

Nixt (1972) investigated the relative effects of frequent use of advance organizers and structured reviews in a college mathematics course for students who were not science, engineering, or mathematics majors. The mathematics content of analytic geometry, vectors, and matrix algebra was presented through televised lectures supplemented with recitation sections. Four advance organizers were administered to one group during a period of 31 class days, each of which was read prior to the televised lectures on that content. Four structured reviews were given to another section during the same time period, with these being read after the televised lectures on that content had been completed. A control group was employed. Statistical analyses revealed no significant differences for treatment effects, recitation instructor effects, or interactions on learning or short-term retention.

Schnell (1972) attempted to determine if the use of an organizer would significantly affect reading comprehension of prose material in educational psychology with community college students. He examined the placement of the organizer (before, after, and both before and after the prose material), and he employed a control group. The findings indicated that the use of an organizer, regardless of placement, resulted in higher scores on a posttest than the treatment of no organizers. The post-organizer group scored higher than the pre-organizer and pre- and post-organizer groups. There was no interaction between placement of the organizer and the variables of intelligence or prior reading ability.

Price (1973) investigated the possibility of main effects and interactions among advance organizers, cognitive style (as identified by Ausubel's Cognitive Style instrument), and ability (as measured by ACT scores) on acquisition and retention of meaningful verbal information. The study included community college freshmen. The statistical analysis did not reveal any significant interactions or main effects except for that of ability on either acquisition or retention.

CHAPTER III

RESEARCH DESIGN

Selection of the Sample

The subjects for this study were selected from the population of students enrolled in Elementary Functions (Mathematics 1444) at the University of Oklahoma during the Spring semester of the 1972-1973 academic year. The textbook used for this course is *Foundations of Mathematics with Application to the Social and Management Sciences* by Grace A. Bush and John E. Young (McGraw-Hill Book Company, 1968).

No effort was made to control the enrollment for the sections of the course which were selected for use in the study. At the beginning of the semester, each of the instructors for the various sections was approached and asked if he would be willing to participate in the study. Initially, ten sections were obtained. This number was reduced to eight sections when a holiday was called by the University of Oklahoma Student Congress. The holiday coincided with the beginning of the instructional sequence for the topic selected for the investigation. It was therefore necessary to postpone the study for two class periods. During these two class periods, another topic (unrelated to the one chosen for the study) was taught.

Two of the volunteered sections had to be withdrawn. One of these sections had already covered the substituted topic; the other needed an extra day to finish the previous chapter's work.

The final sample was determined by including only those students who had been present in class for all phases of the data collection procedures, and for whom ACT scores were available. The final sample size was 91 students. This consisted of 67 males and 24 females. There were 73 freshmen, 15 sophomores, 2 juniors, and 1 senior.

Creation of the Materials

The construction of the reading materials was a crucial part of the study. As pointed out earlier, Ausubel suggests that the advance organizer (1) must be of a relatively high level of inclusiveness and abstraction (and in this way is different from an ordinary overview), (2) should be stated in familiar terms, and (3) should use appropriate illustrations.

The comparative advance organizer written for the topic of matrices was an abstract discussion of a mathematician's definition of an operation and of a specific type of mathematical system known as a ring. An operation was viewed as an assignment of a unique element of a set to an ordered pair of elements from the same set. Addition and multiplication of whole numbers served to illustrate this definition. An operation defined on the real numbers was created and five illustrations given. Also, an example of an operation that was not closed (subtraction on the set of whole numbers) was presented. It was pointed out that a mathematician looks for certain properties an operation may have. The six properties which define the mathematical concept of a ring were then listed, and each was illustrated with real number examples. In addition, the whole numbers and the integers were examined to determine if all the ring properties were satisfied in these subsets. Three additional properties were then listed and illustrated with real number examples. It was pointed out that if all nine of these properties hold in a particular set, the set of elements is referred to as a field.

Of the properties given, those which are satisfied by the set of matrices were then indicated. Those which are satisfied only part of the time were also indicated. The organizer stated that a certain class of matrices satisfied all of the ring properties. The students were told to keep in mind the similarities and differences of this system when compared to the real number system with which they were familiar.

Attached to the organizer was a series of questions. The first question had several parts all of which dealt with the operation that was introduced in the organizer. The remaining questions were structured so as to have the students verify that the rational numbers form a field.

An example of a ring is the set of all nxn matrices with the operations of addition and multiplication suitably defined. Hence, as related to the learning material, the organizer was written so as to meet the conditions of being both abstract and inclusive. Care was taken to state this organizer in terms familiar to the learner, and to employ appropriate illustrations. Thus, in the opinion of the author, the organizer written for the matrix topic met the requirements of an advance organizer as defined by Ausubel. (See Appendix E).

The introductory overview written for the matrix topic began with a discussion of a matrix as a rectangular array of numbers, with examples of a non-square and a square matrix presented. The relation of equality of matrices was then given and illustrated. The operation of addition was presented next and illustrated, and an example of two matrices that could not be added was shown. Scalar multiplication was then defined and illustrated. The operation of multiplication was illustrated with two examples, and the general procedure for multiplying two matrices then indicated. The multiplicative identity for 2x2 and 3x3 matrices was presented. The concept of a multiplicative inverse was defined, and an example of a 2x2 matrix and its inverse was given. Also included was a matrix which did not possess a multiplicative inverse.

Attached to the overview was a series of questions which dealt with many of the topics covered in the reading material.

As pointed out earlier, Ausubel suggests that an introductory overview (1) be presented at the same level of abstraction and inclusiveness as the learning material, (2)emphasize the salient points by omitting less important information, and (3) achieve its effect largely through repetition and emphasis on central concepts. Hence, as related to the learning material, the introductory overview was written to meet the conditions as set forth by Ausubel. (See Appendix E).

The control material was a discussion of the life and mathematical contributions of Arthur C. Cayley, who introduced the concept of matrices into the mathematical literature. The reading dealt with Cayley's early life and education, and the careers he carved for himself in law and mathematics. As such, the control material was historical in nature. It was methodologically important to provide an historical introduction for the control group in order that any obtained differences in learning or retention outcomes among the experimental (advance organizer and introductory overview) and control groups could be attributed to the particular nature of the materials rather than to their presence per se.

The questions attached to the control material dealt with aspects of Cayley's life as presented in the reading. (See Appendix E).

Creation of the Instruments

A pretest was administered to all subjects enrolled in the eight sections which were included in the investigation. This test (Appendix D) was given during the third week of classes. Its sole purpose was to identify those students who had previously studied matrices so that they could be eliminated from the study.

The posttest of achievement (Appendix D) consisted of 45 questions pertaining to the material taught. All of the questions were multiple choice items. On the first 29 questions, the student had to select the correct response from five given choices, one of which was "none of the above." The purpose of these questions was to determine the students' ability to perform the basic matrix operations and identify particular matrices, and to apply the concept to systems of linear equations. The last 16 questions were concerned with the properties that matrices possess with respect to the operations of addition and multiplication, and were directed toward the structure of matrices when they are viewed as a mathematical system. The choice of responses for these items was "always true," "sometimes true," and "never true."

This test of achievement was designed to determine if the concepts taught had been learned by the subjects. One method for determining if a test, in particular an achievement test, measures what it purports to measure is to ascertain if it possesses content validity. The validity of an achievement test is the extent to which the content of the test represents a balanced and adequate sampling of the outcomes (knowledge, skills, and so forth) of the instructional program it is intended to cover; it is best evidenced by a comparison of the test content with courses of study, instructional goals, and by critical analysis of the processes required in responding to the items (Lennon, p. 6).

This comparison was undertaken by distributing the test items for review by persons with competence in the areas of secondary, undergraduate, and graduate levels of teaching, and in designing courses of study for secondary and undergraduate school mathematics. They were asked to judge the test items using the criteria (1) is each item representative of the concept it seeks to measure? (2) has the concept been adequately tested? (3) is each item clear? (4) are the "choice items" well selected? The test items were revised until each judge felt that the items satisfied the four criteria.

The retention test (Appendix D) followed the same format as the posttest of achievement. It employed matrices of the same dimensions as were used on the posttest; however, the matrix elements were changed.

Collection of the Data

The pretest was given during the week of January 29. Five minutes was allotted to the administration of this instrument. These tests were returned to the investigator by each of the instructors, and the papers were reviewed to determine those students who would be eliminated from the study.

On Tuesday, February 27, the three sets of materials were distributed in each of the classes. In order to control for the effects of instructor, situational, and classroom climate variables in the eight sections, students within each section were equally divided among the three treatment groups. This was accomplished by alternating the three sets of materials so that no three students sitting in consecutive seats read the same set of materials.

The students read and studied their material and responded to the set of questions which was attached at the end of their reading copy. The purpose of these questions was to ensure that the student had indeed read his paper. The student was allowed to refer to the material he had read when responding to these questions. One complete class period was devoted to reading the materials and responding to the questions. No exact record of the amount of time students spent in reading and answering their questions was recorded, but in discussions with the instructors it was noted that a few students (primarily those who received the control set of materials) were able to finish within ten or fifteen minutes, while the majority of students finished after approximately 30 minutes. Some students remained for the entire hour. The materials and questions were returned to the instructor before the students left the classroom.

Upon the return of the materials to the investigator, all of the papers were examined to determine if each student had responded to his set of questions. Other than an occassional mathematical or copying error, it was concluded that each student had responded to his questions properly and had thus read the material he had been given.

The chapter on matrices was begun the following class period on Thursday, March 1 (the course did not meet on

Wednesdays). A total of eight class periods was allocated to the teaching of the textbook material, with one of these days given to a review of the chapter.

The posttest of achievement was given in the eight sections on Thursday, March 15. Copies of the examination were distributed to all instructors on Monday, March 12. The posttests were collected by the investigator and graded. The examinations were returned to each instructor on Tuesday, March 20.

The test of retention was given on Thursday, April 5, three weeks after the administration of the posttest. This three week period included the annual week-long spring vacation. The retention tests were returned to the investigator and graded.

The final sample was determined by selecting only those students who had been present for all phases of the data collection as outlined above, and for whom ACT scores were available.

Selection of the Statistics

Two criterion measures were selected for each of the 91 subjects. These measures were (1) posttest scores, and (2) retention test scores. Scores on the English and mathematics portion of the ACT test were obtained for all subjects.

The analysis of the data involved a reliability analysis and a factorial analysis of variance. The method used to perform the reliability analysis was that developed by Kuder and Richardson (Kuder and Richardson, 1937, pp. 151-160). The Kuder-Richardson formula is a measure of the internal consistency of test material, and it yields a unique estimate of the reliability coefficient. The

Kuder-Richardson formula (14) was employed.

The principal statistical analysis used in analyzing the data was the three by three multiple-classification analysis of variance for main effects and simple main effects. The application of this procedure was made after homogeneity of variance was verified both on an intersectional as well as an intergroup basis by applying a series of F_{max} tests on the data. The variances of the three treatment groups were calculated for the posttest and retention test and the largest variance was divided by the smallest variance for each test. The quotients yielded from the divisions were F_{max} values which were interpreted for statistical significance. A similar procedure was applied to the variances of the classes on each of the tests.

Comparison to Previous Research Designs and Results

In previous studies, conflicting results have been found on the effect of the materials across the ability levels. This study is designed to probe for differential effects of the materials with ability levels so that further information will be obtained. Also, studies have been conducted which compare an organizer to a control group, or which compare the organizer with other types of materials but in which a control treatment is not present. Thus even though significant results may be obtained, in the first case one cannot be certain that the organizer is superior to some untried approach, and in the second case one cannot conclude whether the differences are due to the content of the materials or to their presence per se. The addition of a control treatment with the comparative study undertaken here strengthens the design.

CHAPTER IV

ANALYSIS OF THE DATA

Preliminary Discussion

The analysis of the data involves (1) a reliability study of the posttest, (2) a presentation of the ACT data, (3) tests of the hypotheses which pertain to the posttest, (4) tests of the hypotheses which pertain to the retention test.

The Reliability Study

The reliability study is concerned with the reliability of the total posttest given to all students in the eight sections which participated in the investigation. Since each problem was given a score of zero if it was answered incorrectly and a score of one if it was answered correctly, it was possible for each of the 172 students who took the test to obtain a score in the range 0 to 45. The frequency distribution of total scores is given in Table 1.

The reliability coefficient for the data was computed by means of the Kuder-Richardson formula (14), wherein the data required is the number of items in the test, the difficulties of the items, and the standard deviation of the test (Kuder and Richardson, 1937, pp. 156-157). Table 2 presents the component values necessary to apply the Kuder-Richardson formula (14).

The reliability formula is given by the equation:

Table 1: Frequency Distribution of Posttest Scores.

Total Score	Frequency	Total Score	Frequency
0	0	23	4
1	0	24	3
2	0	25	4
3	0	26	6
4	0	27	4
5	0	28	8
6	0	29	10
7	0	30	11
8	0	31	7
9	0	32	9
10	0	33	13
11	0	34	8
12	0	35	8
13	1	36	10
14	0	37	10
15	0	38	8
16	0	39	9
17	0	40	12
18	1	41	9
19	1	42	5
20	3	43	1
21	1	44	1
22	5	45	0

Table 2: Components for the Kuder-Richardson Formula (14) Derived from Posttest Scores (n = 172).

Variance	Σpq	$\Sigma\sqrt{pq}$	$(\Sigma\sqrt{pq})^2$
28.6337	6.6934	16.7751	281.4073

$$r_{tt} = \frac{s^2 - \Sigma pq}{(\Sigma\sqrt{pq})^2 - \Sigma pq} \cdot \frac{(\Sigma\sqrt{pq})^2}{s^2}$$

where p denotes the item difficulty (defined as the number of correct responses to the item divided by n = 172), q = 1 - p, and s^2 represents the variance of scores on the posttest. The reliability of the posttest was computed to be .8440.

The standard error of measurement is related to the reliability coefficient by the formula:

$$s_e = s\sqrt{1 - r_{tt}}$$

where s is the standard deviation of the posttest and r_{tt} is the reliability coefficient. The standard error of measurement for the posttest was computed to be 2.11. Thus it may be said that the odds are about 2 to 1 that a student's obtained score on the posttest is no more than one standard error of measurement (2.11) from his true score and about 19 to 1 that this difference is no more than two standard errors of measurement (4.22).

Presentation of the ACT Data

Scores on the English and mathematics portions of the ACT test were obtained for each person in the sample. The two scores were summed and three ability levels were determined by these summed scores. For the 91 subjects in the sample, the mean and standard deviation of the combined scores were 41.44 and 8.18, respectively. Those students with scores in the range 46 to 58 inclusive were placed in the "high ability" level. Those students with scores in the range 38 to 45 inclusive were placed in the "medium ability" level, and those with scores in the range 21 to 37 inclusive were placed in the "low ability" level. The ACT data is summarized in Tables 3, 4, and 5.

Table 3: ACT English and Mathematics Summed Score Means and Standard Deviations for the Nine Cells.

	Advance Organizer		Introductory Overview		Control	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
High	50.43	4.05	49.00	3.24	51.22	3.53
Medium	41.67	1.87	40.50	1.98	40.78	2.17
Low	34.00	4.00	31.77	3.77	31.80	5.45

Table 4: ACT English and Mathematics Summed Score Means and Standard Deviations for the Three Treatment Groups.

	Advance Organizer	Introductory Overview	Control
$\bar{x}$	44.31	39.41	40.93
s	7.43	7.58	9.01

Table 5: ACT English and Mathematics Summed Score Means and Standard Deviations for the Three Ability Levels.

	High	Medium	Low
$\bar{x}$	50.25	40.93	32.24
s	3.68	2.00	4.40

Tests of Hypotheses: Posttest

The hypotheses for the posttest, in null form, which were tested by the investigation are:

Hypothesis 1: There are no significant differences in mean learning test scores among the three treatments.

Hypothesis 3: There are no significant differences in mean learning test scores among the three ability levels.

Hypothesis 5: There are no significant interactions between the treatments and ability levels as measured by mean learning test scores.

The scoring of the posttest was done by the author, a perfect score consisting of 45 points. The results of the performance on this test are given in Tables 6, 7, and 8. The mean and standard deviation for the posttest scores were 34.36 and 5.35, respectively.

Table 6: Posttest Means and Standard Deviations for the Nine Cells.

	Advance Organizer		Introductory Overview		Control	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
High	36.57	5.29	37.89	3.26	39.22	2.54
Medium	33.44	3.01	33.67	6.77	33.78	3.03
Low	36.17	3.71	32.46	3.82	27.30	4.81

Table 7: Posttest Means and Standard Deviations for the Three Treatment Groups.

	Advance Organizer	Introductory Overview	Control
$\bar{x}$	35.52	34.32	33.21
s	4.48	5.31	6.12

Table 8: Posttest Means and Standard Deviations for the Three Ability Levels.

	High	Medium	Low
$\bar{x}$	37.69	33.63	31.45
s	4.17	4.74	5.25

One of the assumptions underlying the analysis of variance is homogeneity of variance among the treatment groups and among the sections. This assumption was verified by applying a series of F_{max} tests on the data. Since analysis of the data showed that homogeneity of variance prevailed on an intersectional as well as an intergroup basis for both the posttest and retention test scores (See Appendix C), it was considered justifiable to treat the sets of scores on each of these instruments as comparable random samples drawn from the same population.

Since unequal cell frequencies occurred which were not due to the nature of the particular treatments used in the experiment, and the cell frequencies were not proportional, an unweighted means analysis was used in the analysis of variance (Kirk, 1969, p. 202). The formulas employed to obtain the sum of squares are presented in Appendix B. Table 9 is the analysis of variance table for the posttest scores.

Table 9: Analysis of Variance for the Unweighted Means Analysis for Posttest Scores.

Source	SS	DF	MS	F	$F_{0.05}$
Treatments	56.47	2	28.24	1.45	3.11
Levels	534.11	2	267.05	13.67	3.11
Interaction	356.46	4	89.11	4.56	2.48
Within	1602.48	82	19.54		

The F-ratio obtained for interaction is significant at the 0.05 level of significance. Hypothesis 5 was thus rejected and the conclusion made that there is significant interaction between the treatments and ability levels.

The F-ratio obtained for the three treatments is not significant at the 0.05 level, while the F-ratio obtained for the three ability levels is significant at the 0.05 level. This indicates that there is no significant difference in achievement as measured by mean scores among the three treatment groups, but that there is a significant difference in achievement as measured by mean scores among the three ability levels. However, these conclusions, while valid, must be viewed in terms of the significant interaction effect. Additional insights concerning the results of the experiment can be obtained by computing tests of simple main effects. Each sum of squares for simple main effects contains a portion of the corresponding interaction. Instead of testing Hypothesis 1 over all treatment groups or Hypothesis 3 over all ability levels, tests of the two hypotheses are performed at each treatment level and each ability level respectively. The level of significance for the tests of simple main effects is 0.01. The choice of this value comes from taking the original level of significance (0.05) and dividing by three (since there are three treatments and three ability levels). The result was rounded down to the 0.01 level for convenience in reading the tabulated values. The analysis of variance table for the unweighted means analysis of the simple main effects is presented in Table 10. In this table, the variable A represents the ability classification and variable B denotes the treatment classification. The variables a_1, a_2, and a_3 represent the high, medium, and low ability levels, while the variables b_1, b_2, and b_3 represent the advance organizer, the introductory overview, and the control treatment.

Table 10: Analysis of Variance for the Unweighted Means Analysis for Simple Main Effects for Posttest Scores.

Source	SS	DF	MS	F	$F_{0.01}$
A	534.11	2	267.05	13.67	
A at b_1	55.40	2	27.70	1.42	4.88
A at b_2	155.34	2	77.67	3.97	4.88
A at b_3	679.83	2	339.92	17.39	4.88
B	56.47	2	28.24	1.45	4.88
B at a_1	33.52	2	16.76	0.86	4.88
B at a_2	0.54	2	0.27	0.01	4.88
B at a_3	378.87	2	189.44	9.69	4.88
AB	356.46	4	89.11	4.56	
Within	1602.48	82	19.54		

Two of the F-ratios in Table 10 are significant: A at b_3 and B at a_3. It was therefore concluded that treatment b_3 (the control material) was significantly affected by the ability classification, and that the low ability groups' performance on the posttest was significantly affected by the materials read. Comparisons among the means for these significant simple main effects were made following Scheffé's procedure. The results of these comparisons are presented in Table 11.

With regard to the control group (A at b_3), we may conclude that the high ability students obtained a significantly higher mean score than either the medium or low ability students, and that the medium ability students obtained a significantly higher mean score than the low ability students. In the initial analysis of variance (Table 9), a significant F-ratio was obtained for the factor of ability levels. The analysis of the simple main effects has now located the significant differences in that over-all test.

Table 11: Comparisons Among the Means for Significant Simple Main Effects by Scheffé's Procedure.

Source	Comparison Groups	F	$F' = (k-1)F_{0.05}$
A at b_3	High, Low	34.44	6.22
	High, Medium	6.84	6.22
	Medium, Low	10.15	6.22
B at a_3	Organizer, Control	15.10	6.22
	Organizer, Overview	2.89	6.22
	Overview, Control	7.70	6.22

With regard to the effect of the treatments among the low ability students (B at a_3) the Scheffé procedure yields two significant results. Both the advance organizer treatment and the introductory overview treatment yielded a significantly higher mean score on the posttest than the control treatment for students in this ability level. There was no significant difference in mean scores between the two experimental treatments, although the direction of the difference favored the advance organizer treatment. Thus even though there were no significant differences among the treatment groups in the initial analysis of variance when computed over all ability levels, the investigation of the simple main effects as a result of the significant interaction has located a differential effect of the materials among students classified in the low ability category.

Tests of Hypotheses: Retention Test

The hypotheses for the retention test, in null form, which were tested by the investigation are:

Hypothesis 2: There are no significant differences in mean retention test scores among the three treatments.

Hypothesis 4: There are no significant differences in mean retention test scores among the three ability levels.

Hypothesis 6: There are no significant interactions between the treatments and ability levels as measured by mean retention test scores.

The scoring of the retention test was done by the author, a perfect score consisting of 45 points. The results of the performance on this test are given in Tables 12, 13, and 14. The mean and standard deviation for the retention test scores were 33.54 and 5.62, respectively.

Table 12: Retention Test Means and Standard Deviations for the Nine Cells.

	Advance Organizer		Introductory Overview		Control	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
High	35.43	4.24	38.22	2.91	39.33	3.64
Medium	33.00	5.22	32.75	4.25	33.56	3.84
Low	31.00	4.10	31.85	5.51	26.60	6.10

Table 13: Retention Test Means and Standard Deviations for the Three Treatment Groups.

	Advance Organizer	Introductory Overview	Control
$\bar{x}$	33.76	33.85	32.93
s	4.73	5.13	7.02

Table 14: Retention Test Means and Standard Deviations for the Three Ability Levels.

	High	Medium	Low
$\bar{x}$	37.31	33.07	29.86
s	4.02	4.31	5.82

The analysis of variance for the unweighted means analysis for the retention test is presented in Table 15.

Table 15: Analysis of Variance for the Unweighted Means Analysis for Retention Test Scores.

Source	SS	DF	MS	F	$F_{0.05}$
Treatments	23.96	2	11.98	0.57	3.11
Levels	888.58	2	444.29	21.14	3.11
Interaction	208.02	4	52.00	2.47	2.48
Within	1723.55	82	21.02		

The F-ratio obtained for interaction is not significant at the 0.05 level of significance. Thus Hypothesis 6 cannot be rejected.

The F-ratio obtained for the treatments is not significant at the 0.05 level. Therefore, Hypothesis 2 cannot be rejected, and the conclusion is made that there is no significant difference among the treatment groups as measured by mean scores on the retention test.

The F-ratio obtained for the three ability levels is significant at the 0.05 level. Therefore Hypothesis 4 may be rejected, and the conclusion is made that there are significant differences among the ability levels as measured by mean scores on the retention test. Scheffé's procedure for making a complete set of comparisons between the three ability levels was employed to test Hypotheses 4a, 4b, and 4c, and the results are presented in Table 16.

Table 16: Comparisons Among the Ability Level Mean Retention Test Scores by Scheffé's Procedure.

Comparison Groups	F	$F' = (k-1)F_{0.05}$
High, Low	40.19	6.22
High, Medium	13.26	6.22
Medium, Low	7.23	6.22

All of the comparisons achieve significance at the 0.05 level. Thus it may be concluded that the high ability group obtained a significantly higher mean score on the retention test than either the medium or low ability groups, and the medium ability group obtained a significantly higher mean score on the retention test than the low ability group.

CHAPTER V

SUMMARY

Overview

Purpose of the Study. The primary purpose of this study was to examine the effects of an advance organizer on the acquisition and retention of meaningful material within the limits of a subject matter discipline in a normal classroom situation. David P. Ausubel believes that efficient learning and retention of new material occurs when more inclusive relevant concepts exist and are readily available in the cognitive structure of the learner. He has hypothesized that the cognitive structure of the learner can be positively affected by the presentation of advance material at a suitably high level of abstraction and inclusiveness when it is presented in terms which are familiar to the learner. These materials are called advance organizers and are to be kept distinct from introductory overviews. The latter is material which is presented at the same level of abstraction and inclusiveness as the learning material itself. This paper is based on an application of Ausubel's theory of advance organizers in a mathematics classroom, and is further directed to a comparative evaluation of the effects of an advance organizer, an introductory overview, and a historical set of materials on the acquisition and retention of mathematical material.

Procedures. The study was conducted with students enrolled in a course in elementary functions during the spring semester of the 1972-73 academic year at the University of Oklahoma, Norman, Oklahoma. The final sample contained 91 students.

At the beginning of the semester, the students were given a pretest to determine those who had previously been introduced to the topic selected for the investigation so that these individuals could be eliminated from the analysis. Two days prior to the introduction of the matrix topic in the classroom, each student received one of three specially prepared sets of materials to read during class. The students had available the entire hour to read their material. The classroom instruction on matrices began the next class session, and lasted for eight class periods. An examination was given over the matrix topic immediately after the conclusion of the instruction (sixteen days after the reading of the materials). A retention test was given twenty one days after the initial examination.

The posttest was given to two mathematics instructors to assure that the test items reflected the content of the material taught and that an adequate sampling of the instructional program had been included. The retention test was very similar in nature to the posttest, with the difference between the two being that different matrices were employed.

The materials which the students read prior to the start of the instructional program were carefully prepared to follow the precepts set forth by Ausubel.

Experimental Design. A 3x3 treatments by levels design was used in the investigation. The three treatments corresponded to the advance organizer, the introductory overview, and the control reading materials. The three ability levels were determined by the combined English and mathematics ACT scores for each subject.

The three sets of materials were assigned within each classroom and an analysis of variance was utilized to evaluate the results. All tests of significance for main effects were made at the 0.05 level; tests of significance for simple main effects were made at the 0.01 level. Tests of homogeneity of variance were conducted prior to the use of the analysis of variance.

Findings

The hypotheses of the investigation were tested by interpreting the results obtained from the analysis of variance (main effects and simple main effects) model. As a result of these tests the investigator found:

1. Hypothesis 5 was rejected. There was significant interaction between the treatments and ability levels as measured by mean scores on the posttest. As a result of this significant interaction, Hypotheses 1 and 3 were tested as simple main effects.

2. Although in the over-all analysis of variance Hypothesis 1 was not rejected, the test of this hypothesis as a simple main effect located a differential effect of the treatments among low ability

students. Those low ability students who received either the advance organizer or introductory overview treatment obtained a significantly higher mean score on the posttest than those low ability students who received the control treatment. There was no significant difference between those students in the low ability category who received the advance organizer treatment and the introductory overview treatment.

3. In the over-all analysis of variance Hypothesis 3 was rejected. The test of this hypothesis as a simple main effect found that high ability students in the control group achieved a significantly higher mean posttest score than either the medium or low ability students in the control treatment. The medium ability students in the control treatment achieved a significantly higher mean posttest score than the low ability students in the control group.

4. Hypothesis 6 was not rejected. There was no significant interaction between the treatments and ability levels as measured by mean retention test scores.

5. Hypothesis 2 was accepted. There is no significant difference among the treatment groups as measured by mean scores on the retention test.

6. Hypothesis 4 was rejected. The high ability group obtained a significantly higher mean score on the retention test than either the medium or low ability groups, and the medium ability group obtained a significantly higher mean score on the

retention test than the low ability group.

Conclusions

Conclusions drawn from the study are applicable to the population from which the sample was selected and are based upon the evaluative instruments used in the investigation. Generalizations to other situations must be drawn with care.

1. The performance of the subjects on the retention test was quite good. If one compares the means of the nine cells on both tests (Tables 6 and 12) there are several cells in which a slight increase in knowledge occurred as measured by the mean scores. The mean scores of the three treatment groups (Tables 7 and 13) and the three ability levels (Tables 8 and 14) dropped slightly. The difference in mean scores for the two tests was 0.82, which is not significant at the 0.05 level. The tendency of the subjects to perform well on the retention test may be due to several factors:

a. It could be the result of practice, since the retention test asked the same questions as the posttest. Moreover, matrices of the same dimension were employed on both tests, although the matrices on the two tests were not identical.

b. The posttests were redistributed to the students after they were graded, and many students may have read their papers carefully to note their mistakes.

c. There may have been interaction among the students in the days following the learning test.

2. The advance organizer employed in this study does

not appear to be superior to either the introductory overview or control materials employed in the study on either learning or retention when viewed over all subjects, and thus does not lend support to Ausubel's conception of the advance organizer. However, the advance organizer and introductory overview are both better than the control materials when read by low ability students as measured by the posttest. There are several possible explanations as to why the organizer did not have a greater effect on the learning and retention of the matrix topic:

a. Scandura and Wells (1967) found that group theory concepts were served less well by an advance organizer than was topological material. They concluded that since group theory concepts may be more familiar to students as the result of their previous arithmetic and mathematics background, the effectiveness of the organizers may decrease with increasing familiarity of the models. The same phenomenon may have occurred in the study undertaken here. The concepts employed in the organizer may have been familiar to the students as a result of previous work in mathematics, and to this extent the organizer would not contribute significantly to the learning and retention of the material.

b. The nature of mathematics as a discipline may allow another interpretation for the results observed in the study. Mathematics has a great deal of structure, and is a discipline in which many relations are hierarchical. Many topics are introduced and taught within the structure of the discipline, wherein the teaching proceeds from regions of greater to lesser inclusiveness by successive differentiation of the material. The subsumption model of teaching and learning may be easily adapted to a normal mathematics classroom. To the extent to which the inherent structure of mathematics is employed in teaching a specific concept, the effect of an advance organizer may be reduced, for one would be attempting to organize material which is already fairly well organized. It seems reasonable to assume that the payoff one can expect in learning as a result of organizing material has a finite upper limit. Thus if in teaching a topic advantage is taken of any inherent organizing principles that are readily available, then it may be expected that any further efforts at organization may result in only a minimal contribution. Such may have been the case for the topic selected for this investigation.

c. One of the objectives which advance organization is designed to accomplish is increasing the functional retention of new subject matter knowledge by enhancing the organizational strength of a student's existing knowledge. If a student possesses a cognitive structure which already has strong organizational characteristics in mathematics, the effectiveness of the organizer may be correspondingly reduced. In the past decade, emphasis has been placed on the teaching of mathematics with respect to central concepts which may be met in a variety of mathematical situations. This may have resulted in the enhancement of some students' cognitive structure, and may explain why the organizer employed in this investigation did not have as great an effect as expected.

d. A second function of advance organization is to discriminate new material from the conceptual systems that subsume it. If the new material is of such a nature that this discrimination can be made by students on their own, then the advance organizer might not contribute as much to the learning or retention of the new material as might have been anticipated. It may be that matrix concepts are sufficiently different from the related concepts already present in cognitive structure so that a learner can actively make his own comparisons of the systems and effectively discriminate between them. If such was the case, the efficacy of the organizers in this study would be diminished.

e. It is also possible that the advance organizer written for the study did not achieve the goal of conforming to Ausubel's criteria. The concept of an advance organizer as defined by Ausubel seems very clear, but the problem of applying this theory in a particular situation is difficult. One of the real problems in investigations of this type is the creation of the advance organizer. The theory presented by Ausubel seems very logical and simple, but the actual creation of the instrument is quite complex. The advance organizer criteria that Ausubel sets forth are somewhat vague, and the judgments involved in their construction tend to be subjective.

In all likelihood, all of the above factors may have been involved to some extent, but it is difficult to suggest any one factor as dominant.

3. An unexpected result was the significant simple main effect of the ability levels within the control treatment on the posttest. This result may be explained by noting the mean cell scores in Table 6. Of the nine cell means, the largest cell mean is found in the high ability control group cell, and the smallest

cell mean is found in the low ability control group cell. The occurrence of the largest and smallest of the nine cell means within the same column resulted in that column (the control treatment) contributing the major portion of the significant differences among the ability levels as measured by mean scores on the posttest.

Recommendations for Further Research

Relatively few studies have been conducted which compare the effects of an advance organizer to an introductory overview as defined by Ausubel. In the few studies which have been conducted along these lines, most have found no significant difference between the two approaches. More research needs to be done in all academic areas to test the relative effects of these two methods of enhancing learning and retention. For the area of mathematics, the following specific recommendations are made:

1. A replication of the current study should be conducted with students in high school. In testing at an earlier age level, the effects of the organizer may differ from those evidenced in the current investigation.

2. It is possible that the effects of an organizer may become more pronounced when used in conjunction with more advanced topics in mathematics. Investigations should be constructed in the fields of calculus and abstract mathematics to gauge the relative effectiveness of an organizer and an introductory overview.

3. The possibility of using a sequence of advance organizers, each introduced at a key point in the learning process, should be

investigated. Only one study (Schulz, 1966) has been conducted along this line. This approach might be considered for a more advanced mathematical topic.

4. It would be instructive to construct a study comparing the effects of an advance organizer and a list of behavioral objectives in teaching a mathematics concept. Much research has been conducted in the area of behavioral objectives, and a meaningful contribution could be made in comparing this technique with the advance organizer method.

Research over a broad spectrum of mathematics must continue before any firm conjectures concerning the efficacy of advance organizers in the learning and retention of mathematical material can be made. It is important that such research be designed to approximate normal classroom situations and include a reasonable instructional span.

BIBLIOGRAPHY

Books

Ausubel, David P. The Psychology of Meaningful Verbal Learning. New York: Grune and Stratton, Inc., 1963.

Bell, E.T. Men of Mathematics. New York: Simon and Schuster, 1937.

Downie, N. and R. Heath. Basic Statistical Methods. New York: Harper and Row, 1965.

Ferguson, George A. Statistical Analysis in Psychology and Education. New York: McGraw-Hill, Inc., 1966.

Kirk, Roger E. Experimental Design: Procedures for the Behavioral Sciences. Belmont, California: Brooks Cole Publishing Co., 1969.

Scheffé, Henry. The Analysis of Variance. New York: John Wiley and Sons, Inc., 1963.

Wood, Dorothy Adkins. Test Construction. Columbus: Charles E. Merrill Books, Inc., 1960.

Booklet

Lennon, Roger T. Test Service Notebook Number 13: A Glossary of 100 Measurement Terms. Harcourt, Brace, and World, Inc.

Journals

Allen, D. Ian. "Some Effects of Advance Organizers and Level of Question on the Learning and Retention of Written Social Studies Material," Journal of Educational Psychology, Vol. 61 (1970), 333-339.

Ausubel, David P. "The Use of Advance Organizers in the Learning and Retention of Meaningful Verbal Material," Journal of Educational Psychology, Vol. 51 (1960), 267-272.

Ausubel, David P., and Donald Fitzgerald. "The Role of Discriminability in Meaningful Learning and Retention," Journal of Educational Psychology, Vol. 52 (1961), 266-274.

Ausubel, David P., and Donald Fitzgerald. "Organizer, General Background, and Antecedent Learning Variables in Sequential Verbal Learning," Journal of Educational Psychology, Vol. 53 (1962), 243-249.

Ausubel, David P., and Mohammed Youssef. "Role of Discriminiability in Meaningful Parallel Learning," Journal of Educational Psychology, Vol. 54 (1963), 331-336.

Fitzgerald, Donald, and David P. Ausubel, "Cognitive Versus Affective Factors in the Learning and Retention of Controversial Material," Journal of Educational Psychology, Vol. 54 (1963), 73-84.

Grotelueschen, Arden, and Douglas D. Sjogren. "Effects of Differentially Structure Introductory Materials and Learning Tasks on Learning and Transfer," American Educational Research Journal, Vol. 5 (1968), 191-201.

Kuder, G.F., and M.W. Richardson. "The Theory of Estimation of Test Reliability," Psychometrika, Vol. 2 (1937), 151-160.

Kuhn, David J., and Joseph D. Novak. "A Study of Cognitive Subsumption in the Life Sciences," Science Education, Vol. 55 (1971), 309-320.

Kuhn, David J., and Joseph D. Novak. "A Study of Varying Modes of Topical Presentation in Elementary College Biology to Determine the Effect of Advance Organizers in Knowledge Acquisition and Retention," Journal of Research in Science Teaching, Vol. 7 (1970), 249-252.

Pella, Milton O., and Henry J. Triezenberg. "Three Levels of Abstraction of the Concept of Equilibrium and Its Use as an Advance Organizer," Journal of Research in Science Teaching, Vol. 6 (1969), 11-21.

Peterson, John C., and H. Laverne Thomas, C.J. Lovett, George W. Bright. "The Effect of Organizers and Knowledge of Behavioral Objectives on Learning a Mathematical Concept," Journal for Research in Mathematics Education, Vol. 4 (1973), 76-84.

Proger, B.B., and R.G. Taylor, L. Mann, J.M. Coulson, R.J. Bayuk, "Conceptual Pre-Structuring for Detailed Verbal Passages," Journal of Educational Research, Vol. 64 (1970), 28-34.

Romberg, Thomas A., and James W. Wilson. "The Effect of an Advance Organizer, Cognitive Set, and Post Organizer on the Learning and Retention of Written Materials," Journal for Research in Mathematics Education, Vol. 4 (1973), 68-76.

Scandura, Joseph M., and Jay N. Wells. "Advance Organizers in Learning Abstract Mathematics," American Educational Research Journal, Vol. 4 (1967), 295-301.

Weisberg, Joseph S. "The Use of Visual Advance Organizers for Learning Earth Science Concepts," Journal of Research in Science Teaching, Vol. 7 (1970), 161-165.

Wittrock, M.C. "Effect of Certain Sets Upon Verbal Learning," Journal of Educational Psychology, Vol. 54 (1963), 85-88.

Dissertations

Allen, Donald I. "Effects on the Learning and Retention of Written Social Studies Material of the Use of Advance Organizers with Memory Level or Higher Order Questions." (University of California at Berkeley, 1969).

Bertou, Patrick D. "An Analysis of the Relative Efficacy of Advance Organizers, Post Organizers, Interspersed Questions and Combinations Thereof in Facilitating Learning from Televised Instruction." (Columbia University, 1969).

Billey, John E. "An Experimental Analysis of the Lecture Method with the Use of Advance Organizers." (West Virginia University, 1969).

Blackhurst, Albert E. "Auditory Learning of Retarded Adolescents in a Subsumption Theory Context." (University of Pittsburgh, 1966).

Brovey, Daniel J. "The Effectiveness of Advance Organizers in Acquiring Geologic Knowledge in the Field." (Columbia University, 1969).

Davis, Everett E. "The Effect of Advance Organizers on the Verbal Learning of Eighth Graders." (Arizona State University, 1969).

Dvergsten, Kenneth C. "The Effect of the Use of Advance Organizers Combined with Guided Discovery on Achievement and Retention in High School Biology." (University of Minnesota, 1971).

Ethirveerasingam, Nagalingam. "The Effect of Advance Presentation of Organizers on Complex Verbal Learning and Retention by Vocational Agriculture Students in New York State." (Cornell University, 1971).

Farmen, William J. "Comparative Organizer, Instructional Set, and Overlearning in the Acquisition and Retention of Two Parallel Tasks." (Case Western Reserve University, 1968).

Gubrud, Allan R. "The Effect of an Advance Organizer and a Concrete Experience on Learning the Concept of Vectors in Junior and Senior High School." (Purdue University, 1971).

Hershman, Kenneth E. "The Efficacy of Advance Organizers and Behavioral Objectives for Improving Achievement in Physics." (Purdue University, 1971).

Hustuft, Harry D. "The Effect of Advance Organizers on Student Decision Making in a Simulated Environment." (Indiana University, 1969).

Jerrolds, Bobby W. "The Effects of Advance Organizers in Reading for the Retention of Specific Facts." (University of Wisconsin, 1967).

Kahle, Jane B. "The Effect of an Advanced Organizer and the Predictive Ability of Micro-Learning Tasks when Utilized with Carefully Sequenced Audio-Tutorial Units." (Purdue University, 1971).

Kirkwood, James J. "A Comparative Study of Advance Organizers in a Classroom Presentation in Industrial Arts." (Purdue University, 1970).

Kuhn, John D. "A Study of Varying Modes of Topical Presentation in Elementary College Biology to Determine the Effect of Advance Organizers in Knowledge Acquisition and Retention." (Purdue University, 1967).

Lucas, Stephen B. "The Effects of Utilizing Three Types of Advance Organizers for Learning a Biological Concept in Seventh Grade Science." (Pennsylvania State University, 1972).

Malone, Kathryn M. "The Effectiveness of a Cybernetic Model as an Advance Organizer in Teaching Physiological Regulation." (Columbia University, 1970).

Munford, Paul R. "Advance Organizers and the Enhancement of Meaningful Verbal Learning and Retention." (University of Southern California, 1971).

Neisworth, John T. "Use of Advance Organizers with the Educable Mentally Retarded." (University of Pittsburgh, 1967).

Nixt, Henry Charles. "The Relative Effects of Frequent Use of Advance Organizers and Structured Reviews as Alternative Uses of Recitation Time in College Mathematics for Non-Physical Science Students." (Ohio State University, 1972).

Price, Donis Williams. "Interaction of Advance Organizers, Cognitive Style and Ability on Acquisition and Retention of Meaningful Verbal Information." (Arizona State University, 1973).

Ratzlaff, Harold C. "The Relative Effectiveness of Advance Organizers in the Acquisiton, Retention, and Lateral Transfer of Seventh Grade, Base Five Mathematics." (University of Oregon, 1970).

Ryder, Extie M.C. "The Effects of Experience Background and an Advance Organizer on Elementary Pupils' Understanding of Selected Science Concepts." (University of Michigan, 1970).

Schnell, Thomas R. "The Effect of Organizers on Reading Comprehension of Prose Materials." (Southern Illinois University, 1972).

Schulz, Richard W. "The Role of Cognitive Organizers in the Facilitation of Concept Learning in Elementary School Science." (Purdue University, 1966).

Smith, Roy L. "The Influence of Advance Organizers, Overview-Summary Statements and Vagueness on the Comprehension of Oral Instructional Messages." (Washington University, 1970).

Steinbrink, John E. "The Effectiveness of Advance Organizers for Teaching Geography to Disadvantaged Rural Black Elementary Students." (University of Georgia, 1970).

Thelen, Judith N. "Use of Advance Organizers and Guide Material in Viewing Science Motion Pictures in a Ninth Grade." (Syracuse University, 1970).

Townsend, Ronald D. "The Effects of an Advance Organizer on Learning to Graphically Analyze Straight Line Kinematics by Classroom Instruction or Programed Instruction." (Florida State University, 1969).

Triezenberg, Henry J. "The Relative Effectiveness of Three Levels of Abstraction Representing the Conceptual Scheme of Equilibrium as an Advance Organizer in Teaching Ecological Systems by Televised Instruction." (University of Wisconsin, 1967).

Weisberg, Joseph S. "The Use of Visual Advance Organizers for Learning Earth Science Concepts." (Columbia University, 1969).

Woodward, Ernest L. "A Comparative Study of Teaching Strategies Involving Advance Organizers and Post Organizers and Discovery and Nondiscovery Techniques Where the Instruction is Mediated by Computer." (Florida State University, 1966).

APPENDIX A

RAW DATA

Table 17: Raw Data for the Advance Organizer Subjects

Subject	ACT Summed Score	Posttest	Retention Test
101	58	41	40
102	55	40	37
103	55	31	30
104	53	39	37
105	53	25	32
106	52	42	38
107	52	42	41
108	49	36	37
109	47	37	40
110	47	33	36
111	47	37	28
112	46	44	39
113	46	32	29
114	46	32	29
115	45	34	33
116	43	36	38
117	43	35	32
118	42	33	27
119	42	36	41
120	41	28	32
121	40	37	36
122	40	32	34
123	39	30	24
124	37	40	36
125	37	38	34
126	37	38	33
127	34	31	25
128	32	32	28
129	27	38	30

Table 18: Raw Data for the Introductory Overview Subjects

Subject	ACT Summed Score	Posttest	Retention Test
130	56	36	39
131	52	36	35
132	50	39	41
133	48	41	42
134	48	31	35
135	48	41	38
136	47	40	36
137	46	40	42
138	46	37	36
139	45	40	35
140	43	23	29
141	42	32	32
142	41	39	35
143	40	39	38
144	40	33	28
145	40	36	33
146	40	27	27
147	39	40	39
148	39	40	37
149	39	21	27
150	38	34	33
151	37	38	31
152	37	32	26
153	35	36	34
154	35	29	20
155	34	30	29
156	33	30	35
157	31	31	29
158	31	30	27
159	30	37	40
160	30	33	37
161	28	30	35
162	26	39	37
163	26	27	34

Table 19: Raw Data for the Control Subjects

Subject	ACT Summed Score	Posttest	Retention Test
164	56	43	45
165	56	41	40
166	53	40	42
167	53	41	38
168	51	40	39
169	49	38	41
170	49	36	41
171	48	39	35
172	46	35	33
173	44	29	32
174	44	35	33
175	42	34	38
176	41	39	40
177	40	33	34
178	40	37	33
179	39	34	35
180	39	31	28
181	38	32	29
182	37	35	18
183	37	31	38
184	35	33	26
185	35	29	30
186	34	22	30
187	33	24	23
188	33	28	26
189	29	20	19
190	24	26	32
191	21	25	24

APPENDIX B

STATISTICAL FORMULAS

Table 20: Computational Procedures for Main Effects Sum of Squares in the Analysis of Variance with Unequal Cell Sizes.

Let A and B denote the two factors, with p and q denoting the number of levels of each factor, respectively. Denote the sum of all observations in row r and column c by T_{rc}, the mean score in row r and column c by $\bar{T}_{rc}$, the number of observations in row r and column c by n_{rc}. Let X_{rci} denote the ith entry in row r and column c.

Let $$\tilde{n} = \frac{pq}{\sum_{1}^{p} \sum_{1}^{q} 1/n_{rc}}$$

Then:

$$SS_A = \tilde{n} \left[\frac{\sum_{1}^{p} \left(\sum_{1}^{q} \bar{T}_{rc} \right)^2}{q} - \frac{\left(\sum_{1}^{p} \sum_{1}^{q} \bar{T}_{rc} \right)^2}{pq} \right]$$

$$SS_B = \tilde{n} \left[\frac{\sum_{1}^{q} \left(\sum_{1}^{p} \bar{T}_{rc} \right)^2}{p} - \frac{\left(\sum_{1}^{p} \sum_{1}^{q} \bar{T}_{rc} \right)^2}{pq} \right]$$

$$SS_{AB} = \tilde{n} \left[\sum_{1}^{p} \sum_{1}^{q} \bar{T}_{rc}^2 - \frac{\sum_{1}^{p} \left(\sum_{1}^{q} \bar{T}_{rc} \right)^2}{q} - \frac{\sum_{1}^{q} \left(\sum_{1}^{p} \bar{T}_{rc} \right)^2}{p} + \frac{\left(\sum_{1}^{p} \sum_{1}^{q} \bar{T}_{rc} \right)^2}{pq} \right]$$

$$SS_W = \sum_{1}^{p} \sum_{1}^{q} \sum_{1}^{n_{rc}} X_{rci}^2 - \sum_{1}^{p} \sum_{1}^{q} T_{rc}^2/n_{rc}$$

Note that the total sum of squares is not included because in an unweighted means analysis the sum of squares for A,B, AB, and within cell do not add up to the total sum of squares.

Table 21: Computational Procedures for Simple Main Effects Sum of Squares in the Analysis of Variance with Unequal Cell Sizes.

Let A and B denote the two factors, with p and q denoting the number of levels of each factor, respectively. Let a_r ($1 \leq r \leq p$) and b_c ($1 \leq c \leq q$) denote the specific levels of each factor. Denote the mean scores in row r and column c by T_{rc}, the number of observations in row r and column c by n_{ra}.

Let $$\tilde{n} = \frac{pq}{\sum_{1}^{p} \sum_{1}^{q} 1/n_{rc}}$$

Then:

$$SS_A \text{ at } b_c = \tilde{n} \left[\sum_{1}^{p} \bar{T}_{rc}^2 - \frac{\left(\sum_{1}^{p} \bar{T}_{rc} \right)^2}{p} \right]$$

$$SS_B \text{ at } a_r = \tilde{n} \left[\sum_{1}^{q} \bar{T}_{rc}^2 - \frac{\left(\sum_{1}^{q} \bar{T}_{rc} \right)^2}{q} \right]$$

As a computational check, $\sum_{1}^{q} SS_A$ for $b_c = SS_A + SS_{AB}$

and $\sum_{1}^{p} SS_B$ for $a_r = SS_B + SS_{AB}$

APPENDIX C

$\underline{F}_{max}$ TESTS

Table 22: F_{max} Ratio for the Treatment Groups on the Posttest.

Treatment Group	Variance	F_{max}
Control	37.43	
Advance Organizer	20.04	1.87
Introductory Overview	28.16	

Table 23: F_{max} Ratio for the Treatment Groups on the Retention Test.

Treatment Group	Variance	F_{max}
Control	49.32	
Advance Organizer	22.34	2.21
Introductory Overview	26.96	

Table 24: F_{max} Ratio for the Sections on the Posttest.

Section Number	Variance	F_{max}
7	48.99	4.54
2	10.80	
1	30.64	
3	24.53	
4	27.34	
5	21.07	
6	36.91	
8	22.81	

Table 25: F_{max} Ratio for the Sections on the Retention Test.

Section Number	Variance	F_{max}
4	39.19	1.95
5	20.13	
1	36.08	
2	26.20	
3	27.97	
6	38.86	
7	33.51	
8	33.46	

APPENDIX D

TESTS

PRETEST

Name________________________________

Class time:___________

1. Have you ever enrolled in Math 1513 (College Algebra)?__________
   If so, did you complete the course?___________

2. Have you previously enrolled in Math 1444?___________
   If so, did you complete the course?___________

3. Add the following matrices:

   a) $\begin{bmatrix} 2 & 3 \\ 4 & -6 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 0 & 5 \\ 6 & 2 \end{bmatrix} =$

   b) $\begin{bmatrix} 2 & -1 & 5 \\ 4 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 4 \\ 5 & 1 & 6 \\ 1 & 8 & 3 \end{bmatrix} =$

4. If $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 9 & -2 \end{bmatrix}$, then $3A =$

5. Multiply the following matrices:

   $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \text{X} \begin{bmatrix} 4 & 6 & 2 \\ 1 & 3 & 2 \end{bmatrix} =$

6. The dimension of the matrix: $\begin{bmatrix} 2 & 7 & 9 \\ 3 & 4 & 6 \end{bmatrix}$ is _______________

7. The determinant of $\begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$ is ____________________

POSTTEST

Name________________________

Instructions: This is a multiple choice test. Please be sure that you have circled one of the responses for each problem. If no response has been circled on a problem, or if more than one response has been circled, the problem will be counted as incorrect.

Given the following matrices:

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 6 & 0 \\ 2 & -1 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 4 \\ 2 & 0 \\ 1 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & -2 \\ 3 & 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 5 & -3 \end{bmatrix} \quad F = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$$

$$G = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 4 & 5 \\ -1 & 2 & -3 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & 5 \end{bmatrix}$$

1. In G, element a_{23} is:

   a) 2 b) 6 c) 5 d) 4 e) None of the above.

2. Which of the following pairs of matrices can be added?

   a) B,C b) B, H c) B, D d) E, F c) None of the above.

3. Compute: D + G

   a) $\begin{bmatrix} 5 & 1 & 2 \\ 6 & 4 & 3 \\ 3 & 3 & -3 \end{bmatrix}$ b) $\begin{bmatrix} -5 & -6 & -3 \\ -1 & -4 & -3 \\ -2 & -3 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 6 & 3 \\ 1 & 4 & 3 \\ 2 & 3 & -3 \end{bmatrix}$

   d) $\begin{bmatrix} 5 & 6 & 3 \\ 1 & 4 & 3 \\ 2 & 3 & 3 \end{bmatrix}$ e) None of the above.

4. Which of the following products is defined?

a) AE b) CD c) FE d) EF e) None of the above

5. Compute the product AF.

a) $\begin{bmatrix} 7 & 21 \\ -6 & -18 \end{bmatrix}$ b) $\begin{bmatrix} 6 & 1 \\ -24 & 6 \end{bmatrix}$ c) $\begin{bmatrix} 12 & 6 \\ 4 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 12 & 4 \\ 6 & 2 \end{bmatrix}$

e) None of the above.

6. The dimension of B + H is:

a) 2x3 b) 3x2 c) 2x6 d) 6 e) None of the above.

7. The dimension of FG is:

a) 2x3 b) 3x2 c) 2x2 d) 6 e) None of the above.

8. The dimension of EH is:

a) 4 b) 2x2 c) 3x1 d) 1x3 e) None of the above.

9. Which of the following matrices is equal to F?

a) $\begin{bmatrix} -2 & -1 \\ -6 & -3 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e) None of the above.

10. The determinant of A is:

a) 10 b) -10 c) 2 d) -2 e) None of the above.

11. The determinant of D is:

a) -22 b) -18 c) 22 d) 18 e) None of the above.

12. The determinant of C is:

a) -9 b) -8 c) 10 d) 11 e) None of the above.

13. In D, the minor of 2 is:

a) 1   b) -1   c) $\begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}$ d) $-\begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}$ e) None of the above

14. In G, the cofactor of 5 is:

a) 5   b) -11   c) 8   d) 11   e) None of the above.

15. If 2A + K = 0, then matrix K is equal to:

a) $\begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 8 \\ -2 & -4 \end{bmatrix}$ c) $\begin{bmatrix} -3/2 & -1/2 \\ 1/2 & -1 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 8 & -4 \end{bmatrix}$

e) None of the above.

16. The multiplicative inverse of A is:

a) $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -3/10 & -4/10 \\ 1/10 & -2/10 \end{bmatrix}$ d) $\begin{bmatrix} 2/10 & -1/10 \\ 4/10 & 3/10 \end{bmatrix}$

e) None of the above.

17. The multiplicative identity for D is:

a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 & -2 \\ 1 & 0 & 2 \\ -3 & -1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

e) None of the above.

18. The multiplicative inverse for C is:

a) $\begin{bmatrix} 0 & -4 \\ -2 & 3 \\ 1 & 5 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -4/8 \\ -2/8 & -5/8 \\ 1/8 & 5/8 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & -4/8 \\ -2/8 & -3/8 \end{bmatrix}$

e) None of the above.

19. Which of the following sets of matrices are all square matrices?

a) A, D, H   b) D, F, G   c) B, C, H   d) E, F, G   e) None of the above.

20. The augmented matrix for the system of equations:

$$\begin{aligned} 4x - 5y + 2z &= 8 \\ 3x + 2y - 5z &= 4 \\ x - y + 3z &= 6 \end{aligned}$$ is:

a) $\begin{bmatrix} 4 & -5 & 2 \\ 3 & 2 & -5 \\ 1 & -1 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 4 & -5 & 2 \\ 4 & 3 & 2 & -5 \\ 6 & 1 & -1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 4 & -5 & 2 & 8 \\ 3 & 2 & -5 & 4 \\ 1 & -1 & 3 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 4 & -5 & 2 & -8 \\ 3 & 2 & -5 & -4 \\ 1 & -1 & 3 & -6 \end{bmatrix}$ e) None of the above.

21. If the augmented matrix associated with a system of equations reduces to: $\begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & -1 & 5 \end{bmatrix}$, then the solution to the system is:

a) (2,3,-1) b) (1,2,-5) c) (2,6,5) d) (0,0,0) e) None of the above.

22. For the system of equations: $$\begin{aligned} 5x + 4y + 4z &= 9 \\ x + 3y - z &= -8 \\ 2x + 3z &= 11 \end{aligned}$$, the inverse of the co-efficient matrix is:

$\begin{bmatrix} 9 & -12 & -16 \\ -5 & 7 & 9 \\ -6 & 8 & 11 \end{bmatrix}$. Which of the following will yield the solution to this system?

a) $\begin{bmatrix} 9 & -12 & -16 \\ -5 & 7 & 9 \\ -6 & 8 & 11 \end{bmatrix}\begin{bmatrix} 9 \\ -8 \\ 11 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 4 & 4 \\ 1 & 3 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ -8 \\ 11 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 4 & 4 \\ 1 & 3 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 9 & -12 & -16 \\ -5 & 7 & 9 \\ -6 & 8 & 11 \end{bmatrix}$ d) $\begin{bmatrix} 9 & -12 & -16 \\ -5 & 7 & 9 \\ -6 & 8 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -8 \\ 11 \end{bmatrix}$

e) None of the above.

23. F^{-1} is:

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ e) None of the above.

24. If cA = $\begin{bmatrix} -9 & -3 \\ 12 & -6 \end{bmatrix}$, then c is equal to:

a) -3 b) 1/3 c) 3 d) -1/3 e) None of the above.

25. Solve for matrix X: $2X + H = B + 4X$.

a) $\begin{bmatrix} 4 & -4 & 4 \\ -2 & -2 & 1 \end{bmatrix}$ b) $\frac{1}{6}\begin{bmatrix} 4 & -4 & 4 \\ -2 & -2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -2 & 2 & -2 \\ 1 & 1 & -1/2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -2 & 2 \\ -1 & -1 & 1/2 \end{bmatrix}$

e) None of the above.

26. The additive inverse of B is:

a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 3 & -6 & 0 \\ -2 & 1 & -4 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -2 \\ -6 & 1 \\ 0 & -4 \end{bmatrix}$ d) $\begin{bmatrix} -2 & 1 & -4 \\ 3 & -6 & 0 \end{bmatrix}$

e) None of the above.

27. If $B + X = B$, then X equals:

a) $\begin{bmatrix} -3 & 6 & 0 \\ 2 & -1 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 3 & -6 & 0 \\ -2 & 1 & -4 \end{bmatrix}$

e) None of the above.

28. The additive identity for H is:

a) $\begin{bmatrix} -1 & -2 & -4 \\ 0 & 3 & -5 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

e) None of the above.

29. B - H is equal to:

a) $\begin{bmatrix} 4 & -4 & 4 \\ -2 & -2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 8 & 4 \\ 2 & -4 & 9 \end{bmatrix}$ c) $\begin{bmatrix} -4 & 4 & -4 \\ 2 & 2 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

e) None of the above.

Let A, B, C represent 2x2 matrices. Determine if the following statements are always true (AT), sometimes true (ST), or never true (NT).

30.	$(AB)C = A(BC)$	AT	ST	NT
31.	A^{-1} exists	AT	ST	NT
32.	$-A + A = 0$	AT	ST	NT
33.	$A0 = I$	AT	ST	NT
34.	det A exists	AT	ST	NT
35.	$AI = A$	AT	ST	NT
36.	$A(B + C) = AB + AC$	AT	ST	NT
37.	$(2A)(3A) = 6A$	AT	ST	NT
38.	$A + (B + C) = (A + B) + C$	AT	ST	NT
39.	$3A = A + A + A$	AT	ST	NT
40.	$A + B = B + A$	AT	ST	NT
41.	$AB = BA$	AT	ST	NT
42.	$(AB)^3 = A^3B^3$	AT	ST	NT
43.	$A + 0 = A$	AT	ST	NT
44.	$A + B = A + C \Longrightarrow B = C$	AT	ST	NT
45.	$AB = AC \Longrightarrow B = C$	AT	ST	NT

RETENTION TEST

Name________________________

Instructions: This is a multiple choice test. Please be sure that you have circled one of the responses for each problem. If no response has been circled on a problem, or if more than one response has been circled, the problem will be counted as incorrect.

Given the following matrices:

$$A = \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -4 & 5 & 0 \\ 3 & -2 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 5 \\ 3 & 0 \\ 2 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 4 & 3 \\ -2 & 0 & -1 \\ 4 & 2 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 6 & -4 \end{bmatrix} \quad F = \begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix} \quad G = \begin{bmatrix} 5 & 4 & 2 \\ 3 & 5 & 7 \\ -2 & 3 & -4 \end{bmatrix} \quad H = \begin{bmatrix} 2 & 3 & 4 \\ 0 & -5 & 6 \end{bmatrix}$$

1. In G, element a_{32} is:

   a) 4 b) 3 c) -2 d) 6 e) None of the above.

2. Which of the following pairs of matrices can be added?

   a) B,D b) B,C c) E,F d) B,H e) None of the above.

3. Compute: D + G

   a) $\begin{bmatrix} -7 & -8 & -5 \\ -1 & -5 & -6 \\ -2 & -5 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 7 & 8 & 5 \\ 1 & 5 & 6 \\ 2 & 5 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 7 & 8 & 5 \\ 1 & 5 & 6 \\ 2 & 5 & -4 \end{bmatrix}$ d) $\begin{bmatrix} 7 & 1 & 2 \\ 8 & 5 & 5 \\ 5 & 6 & -4 \end{bmatrix}$

   e) None of the above.

4. Which of the following products is defined?

   a) CD b) AE c) FE d) EF e) None of the above.

5. Compute the product AF.

a) $\begin{bmatrix} 16 & 1 \\ -16 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 17 & 34 \\ -5 & -10 \end{bmatrix}$ c) $\begin{bmatrix} 24 & 6 \\ 16 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 24 & 16 \\ 6 & 4 \end{bmatrix}$ e) None of the above.

6. The dimension of B + H is:

a) 2x3 b) 3x2 c) 2x6 d) 6 e) None of the above.

7. The dimension of FG is:

a) 2x3 b) 3x2 c) 2x2 d) 6 e) None of the above.

8. The dimension of EH is:

a) 1x3 b) 2x2 c) 3x1 d) 4 e) None of the above.

9. Which of the following matrices is equal to F?

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}$ d) $\begin{bmatrix} -4 & -1 \\ -8 & -2 \end{bmatrix}$ e) None of the above.

10. The determinant of A is:

a) -10 b) 10 c) 14 d) -14 e) None of the above.

11. The determinant of D is:

a) 32 b) -32 c) 24 d) -24 e) None of the above.

12. The determinant of C is:

a) -5 b) -15 c) 18 d) 14 e) None of the above.

13. In D, the minor of 3 is:

a) $\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix}$ b) $-\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix}$ c) 4 d) -4 e) None of the above.

14. In G, the cofactor of 7 is:

a) 7 b) -23 c) -7 d) 23 e) None of the above.

15. If 2A + K = 0, then matrix K is equal to:

a) $\begin{bmatrix} -4 & -1 \\ 2 & -3 \end{bmatrix}$ b) $\begin{bmatrix} -8 & -2 \\ 4 & -6 \end{bmatrix}$ c) $\begin{bmatrix} 8 & 2 \\ -4 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 1/2 \\ -1 & 3/2 \end{bmatrix}$

e) None of the above.

16. The multiplicative inverse of A is:

a) $\begin{bmatrix} 3/14 & -1/14 \\ 2/14 & 4/14 \end{bmatrix}$ b) $\begin{bmatrix} -4/14 & -2/14 \\ 1/14 & -3/14 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ e) None of the above.

17. The multiplicative identity for D is:

a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -2 & -4 & -3 \\ 2 & 0 & 1 \\ -4 & -2 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ e) None of the above.

18. The multiplicative inverse for C is:

a) $\begin{bmatrix} 0 & -5 \\ -3 & 4 \\ 2 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -5/15 \\ -3/15 & 4/15 \\ 2/15 & 6/15 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & -5/15 \\ -3/15 & 4/15 \end{bmatrix}$

e) None of the above.

19. Which of the following sets of matrices are all square matrices?

a) A,D,H b) B,C,H c) E,F,G d) D,F,G e) None of the above.

20. The augmented matrix for the system of equations:

$$\begin{aligned} 3x - 4y + 3z &= 7 \\ 2x + y - 4z &= 5 \\ x - 2y + 5z &= 8 \end{aligned}$$ is:

a) $\begin{bmatrix} 3 & -4 & 3 \\ 2 & 1 & -4 \\ 1 & -2 & 5 \end{bmatrix}$ b) $\begin{bmatrix} 7 & 3 & -4 & 3 \\ 5 & 2 & 1 & -4 \\ 8 & 1 & -2 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -4 & 3 & 7 \\ 2 & 1 & -4 & 5 \\ 1 & -2 & 5 & 8 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -4 & 3 & -7 \\ 2 & 1 & -4 & -5 \\ 1 & -2 & 5 & -8 \end{bmatrix}$ e) None of the above.

21. If the augmented matrix associated with a system of equations reduces to:

$$\begin{bmatrix} 2 & 0 & 0 & 8 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 3 & 9 \end{bmatrix},$$ then the solution to the system is:

a) (2,-1,3) b) (4,-3,3) c) (8,3,9) d) (0,0,0) e) None of the above.

22. For the system of equations: $\begin{aligned} -2x - 3y + 4z &= 11 \\ 2x + 2y - 3z &= -7 \\ x + 2y - 2z &= -6 \end{aligned}$, the inverse of the co-efficient matrix is:

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}.$$

Which of the following will yield the solution to this system?

a) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ -6 \end{bmatrix}$ b) $\begin{bmatrix} -2 & -3 & 4 \\ 2 & 2 & -3 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ -6 \end{bmatrix}$

c) $\begin{bmatrix} -2 & -3 & 4 \\ 2 & 2 & -3 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \\ -6 \end{bmatrix}$

e) None of the above.

23. F^{-1} is:

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -1 \\ -8 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ e) None of the above.

24. If $cA = \begin{bmatrix} -16 & -4 \\ 8 & -12 \end{bmatrix}$, then c is equal to:

a) 1/4 b) -4 c) -1/4 d) 4 e) None of the above.

25. Solve for matrix X: $4X + H = B + 6X$.

a) $\begin{bmatrix} 6 & -2 & 4 \\ -3 & -3 & 0 \end{bmatrix}$ b) $\frac{1}{10}\begin{bmatrix} 6 & -2 & 4 \\ -3 & -3 & 0 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 1 & -2 \\ 3/2 & 3/2 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -1 & 2 \\ -3/2 & -3/2 & 0 \end{bmatrix}$ e) None of the above.

26. The additive inverse of B is:

a) $\begin{bmatrix} -3 & 2 & -6 \\ 4 & -5 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -5 & 0 \\ -3 & 2 & -6 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 4 & -3 \\ -5 & 2 \\ 0 & -6 \end{bmatrix}$ e) None of the above.

27. If B + X = B, then X equals:

a) $\begin{bmatrix} -4 & 5 & 0 \\ 3 & -2 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -5 & 0 \\ -3 & 2 & 6 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

e) None of the above.

28. The additive identity for H is:

a) $\begin{bmatrix} -2 & -3 & -4 \\ 0 & 5 & -6 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 3 & 4 \\ 0 & -5 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

e) None of the above.

29. B - H is equal to:

a) $\begin{bmatrix} 6 & -2 & 4 \\ -3 & -3 & 0 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 8 & 4 \\ 3 & -7 & 12 \end{bmatrix}$ c) $\begin{bmatrix} -6 & 2 & -4 \\ 3 & 3 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

e) None of the above.

Let A, B, C represent arbitrary 2x2 matrices. Determine if the following statements are always true (AT), sometimes true (ST), or never true (NT).

30.	$A + 0 = A$	AT	ST	NT
31.	$(AB)C = A(BC)$	AT	ST	NT
32.	$3A = A + A + A$	AT	ST	NT
33.	A^{-1} exists	AT	ST	NT
34.	$AB = BA$	AT	ST	NT
35.	det A exists	AT	ST	NT
36.	$A0 = I$	AT	ST	NT
37.	$A + (B + C) = (A + B) + C$	AT	ST	NT
38.	$(2A)(4A) = 8A$	AT	ST	NT
39.	$AB = AC \Longrightarrow B = C$	AT	ST	NT
40.	$AI = A$	AT	ST	NT
41.	$-A + A = 0$	AT	ST	NT
42.	$A(B + C) = AB + AC$	AT	ST	NT
43.	$A + B = A + C \Longrightarrow B = C$	AT	ST	NT
44.	$A + B = B + A$	AT	ST	NT
45.	$(AB)^3 = A^3B^3$	AT	ST	NT

APPENDIX E

READING MATERIALS

ADVANCE ORGANIZER

This is an introduction to a unit that you will be studying shortly, a unit on matrices and their operations. A set of elements together with some operations defined on those elements create a mathematical system. The sets that we will be discussing here are sets of numbers. You have had lots of experience working with the real numbers and the operations of addition, subtraction, multiplication, and division.

A mathematician has a very precise concept in mind when he talks about an "operation" on a set of elements. To understand how a mathematician thinks about an operation, we must begin with the idea of an ordered pair of elements. As the name implies, an ordered pair refers to a pair of elements in which the choice of which element is written first is of prime importance. For example, if the elements are 2 and 5, then (2,5) denotes the ordered pair where 2 is first and 5 is second. A mathematician thinks of an operation as a process whereby an element of a given set is assigned to an ordered pair of elements from the same set. For example, if you were asked to tell which element is assigned to the ordered pair (2,5) when the operation is addition, you would undoubtedly give the correct response of 7. Similarly, you would say that 10 is assigned to (2,5) when the operation is multiplication. The idea of assigning elements of a set to ordered pairs of elements from the set is really not so new to you; you've actually been doing it for years but simply not thinking of it this way or writing it this way. Instead of saying: "7 is assigned to

(2,5) under addition," you wrote (much more briefly): $2 + 5 = 7$.

When working with an operation, we have to have some means of determining which element (if any) is to be assigned to a given ordered pair. We are usually given some rule for doing this. Consider the following operation defined on the set of real numbers:

> With any ordered pair (a,b), assign the real number that is half-way between the two if a is less than b; if a is greater than or equal to b, assign the number a.

Accordingly, with (2,4) we assign 3; with (5,9) we assign 7. But with (4,2) we assign 4 and with (9,5) we assign 9. With the ordered pair (4,4), we assign 4. If we label our operation with the symbol, #, we could write the above assignments as:

$$2 \# 4 = 3; \qquad 5 \# 9 = 7; \qquad 4 \# 2 = 4; \qquad 9 \# 5 = 9$$

$$4 \# 4 = 4 .$$

For a given operation, a specific ordered pair has <u>at most</u> one element from the set assigned to it. As an example, consider the set of whole numbers, $W = \{0,1,2,3,4,5, \ldots\}$. Under addition, 7 is the only element of the set assigned to (2,5); under multiplication, 10 is the only element of the set assigned to (2,5). In some cases, there is <u>no</u> element <u>in the set</u> assigned to some ordered pairs. For the ordered pair (2,5), we would ordinarily assign -3 when the operation is subtraction (i.e., $2 - 5 = -3$). But -3 is <u>not</u> a whole number. We would have to extend the set under consideration to include the negative whole numbers before the operation of subtraction could always be performed. Some care must be taken to be sure that the operation can be performed with certain ordered pairs; one cannot always proceed without some caution.

When a mathematician studies an operation, he tries to determine what kind of rules or properties the operation has. Many times a set of elements will have two operations defined on it, and in such cases the mathematician will investigate the system for properties that show relationships between the operations. Also, he examines a mathematical system looking for elements which have unique properties with respect to the operations. Specifically, in the course of your work with the real number system, you learned many important properties or rules which these numbers obeyed with respect to the operations of addition and multiplication. You have used these properties frequently, although you may not have been aware of them at times. There are, however, some important mathematical systems which do not obey all of these rules, and in which care must be taken before the rules can be applied. In particular, the following properties among others hold on the set of real numbers for the operations of addition and multiplication:

If a, b, c denote arbitrary real numbers:

1. Commutative law of addition........... $a + b = b + a$. For example, $3 + 2 = 2 + 3$. The number 5 is assigned to both ordered pairs (3,2) and (2,3).
2. Associative law of addition........... $(a+b)+c = a+(b+c)$. That is, when adding three numbers, we may proceed in either of two ways:
   $$(7 + 8) + 5 = 15 + 5 = 20$$
   OR
   $$7 + (8 + 5) = 7 + 13 = 20.$$
3. Associative law of multiplication..... $(ab)c = a(bc)$. Three factors in a product may be associated in either of two ways:
   $$(3\cdot 5)4 = (15)4 = 60 \text{ OR } 3(5\cdot 4) = 3(20) = 60.$$

4. Distributive law............. $a(b + c) = ab + ac$. Thus, $7(6 + 3) = 7\cdot(6) + 7\cdot(3)$. This property gives us a means of relating the operations of addition and multiplication.
5. Identity element of addition......... There exists a unique number, 0, with the property: $a + 0 = 0 + a = a$.
6. Inverse law of addition.......... Every a has an opposite, $-a$, such that: $a + (-a) = -a + a = 0$.

In mathematics, a special word has been coined to describe systems, all of whose elements satisfy the above six properties. The word "ring" is used to describe such a system. Since the set of real numbers satisfies all of these properties, it forms a ring. One may speak of the "ring of real numbers," and in so doing he implies that the real numbers satisfy the above laws.

We might check some of the subsets of the real numbers to see if they form a ring. Does the set of whole numbers, $W = \{0,1,2,3, \ldots\}$, form a ring? Certainly properties 1 through 4 will hold, since every whole number is a real number and these properties do hold for the real numbers. Property 5 is also satisfied since 0 is a member of W. What about property 6 (additive inverses)? Consider the whole number 4; there is no member <u>of the set W</u> which will allow this property to be satisfied for the number 4 (of course, -4 is the element we need, but it is <u>not</u> a member of W). Thus W does not form a ring.

Does the set of integers, $I = \{\ldots, -3,-2,-1,0,1,2,3,\ldots\}$, form a ring? Again, properties 1 through 4 will immediately be true, as was the case with the whole numbers. Also, property 5 is satisfied for all integers, since 0 belongs to I. Now we <u>do</u> have additive inverses for all of our numbers, so property 6 is satisfied. So the set I does form a ring.

Thus not all sets of numbers with the operations of addition and multiplication form what we call a ring. A ring is a fairly sophisticated mathematical topic, and the real numbers are just one example of this concept. There are other examples, but none are as familiar to you as are the real numbers. The real numbers also satisfy certain other rules with respect to addition and multiplication which we have often used as well as the above six. In particular, they meet the following three requirements:

7. Commutative law of multiplication ...... $ab = ba$. For example, $5(3) = 3(5)$. The number 15 is assigned to both ordered pairs (5,3) and (3,5).
8. Identity element of multiplication .... There exists a unique number, 1, with the property: $a \cdot 1 = 1 \cdot a = a$.
9. Inverse law of multiplication .... Every nonzero number a has a reciprocal, $1/a$, such that $a(1/a) = (1/a)a = 1$. So we have $1/4(4) = 1$ and $25(1/25) = 1$.

If a set of elements satisfies the six ring laws <u>and</u> the seventh property just listed, it is called a commutative ring. If we have a ring which also satisfies property 8, we call it a ring with identity. The set of integers, I, forms a commutative ring with identity, since this set satisfies the seventh and eighth requirements. If a set of elements should happen to satisfy all nine laws which we have listed, it is given another special name; it's called a "field."

It is worth noting again that not all sets of elements upon which have been defined the operations of addition and multiplication will satisfy all or even some of the above rules. You will shortly be studying such a set of elements, the set of matrices.

The system of matrices will provide an excellent comparison to the system of real numbers. We will see that there are very obvious similarities between these two sets of elements and their operations. You will also note that there are some striking differences between the two systems. We wish to take special note of these similarities and differences.

A matrix is a collection of real numbers which is arranged into rows and columns. We keep track of matrices by the number of rows and number of columns each contains. A matrix will in general contain many real numbers, with each number entered in a row and column position. In defining addition and multiplication of real numbers, these operations are so defined that <u>any</u> two real numbers can be added or multiplied. These operations will be defined on matrices in such a way that <u>not</u> all matrices can be added to each other, and <u>not</u> all matrices can be multiplied. However, for those matrices which can be added, the operation will be commutative [$A + B = B + A$], associative [$(A+B)+C = A+(B+C)$], have an identity element, and each matrix will have an additive inverse. For those matrices that can be multiplied, we will always have the associative property [$(AB)C = A(BC)$], and the distributive law will always be satisfied [$A(B + C) = AB + AC$].

Multiplication of matrices will, in general, not be commutative. There will exist matrices which can be multiplied and for which $AB \neq BA$. Similarly, a multiplicative identity will exist for some (but not all) matrices, and a multiplicative inverse will exist for some (but not all) matrices.

The set of matrices under the operations of addition and multiplication will not form a ring, since not all matrices can be added or multiplied. However, there is a certain type of matrix which will satisfy all six of the ring properties, and the set of all matrices of this type will form a ring.

When you begin your study of the matrix system, the ideas presented here should be valuable to you. Try to keep in mind the similarities and differences of this system when compared to the real number system with which you are familiar.

Name____________________

The following list of questions pertains to the material which you have just read. In responding to these questions, you may use the material you have read. These questions do not constitute a "test;" they are simply designed to help you in understanding the material which you have read. Please respond to each question before proceeding to the next question.

1. For the operation, #, which was introduced in your reading at the bottom of page 1, complete the following:

   a) 6 # 10 = ________

   b) 10 # 6 = ________

   c) 6 # 10 $\neq$ 10 # 6. The operation does not satisfy which property?__________________

   d) (6 # 10) # 4 = __________

   e) 6 # (10 # 4) = __________

   f) The answers in parts (d) and (e) are the same (namely, 8). This is an example of the operation satisfying which property?_____________________

The set of rational numbers, Q, consists of all positive and negative whole numbers and all positive and negative fractions.

2. For the rational numbers 5/2 and 7/4, is it true that: 5/2 + 7/4 = 7/4 + 5/2?

   _____Yes _____No

3. For any two rational numbers x and y, it is true that x + y = y + x. So the set of rational numbers, Q, satisfies the ____________________ property for addition.

4. For the rational numbers 2, 5, and 7, is it true that: (2 + 5) + 7 = 2 + (5 + 7)?

   ____Yes ____No

5. For any three rational numbers x, y, and z it is true that (x + y) + z = x + (y + z). So Q satisfies the _____________ property for addition.

6. For the rational numbers ½, 4, and 3, is it true that: ½(4•3) = (½•4)3?

   ____Yes ____No

7. For any three rational numbers x, y, and z, it is true that $x(yz) = (xy)z$. So the rational numbers satisfy the ________________ property for multiplication.

8. For the rational numbers, $\frac{1}{2}$, 6, and 8, is it true that: $\frac{1}{2}(6 + 8) = \frac{1}{2}(6) + \frac{1}{2}(8)$?

   ____Yes ____No

9. For any three rational numbers x, y, and z, it is true that $x(y + z) = xy + xz$. So Q is said to satisfy the ________________ property.

10. The set of rational numbers thus satisfies the commutative and associative properties for addition, the associative property for multiplication, and the distributive property (these are the correct answers for problems 3, 5, 7, and 9 respectively). Q also contains the special number 0. This number (0) is called the ________________ ________________ because $x + 0 = 0 + x = x$ for all rational numbers x.

11. The additive inverse for the rational number 2/3 is the number ______________.

12. The additive inverse for the rational number x is the rational number $-x$, because $x + (-x) = 0$, where 0 is the additive identity (see problem 10). Thus Q has an additive identity and contains inverses for all of its elements. Does Q form a ring?

    ____Yes ____ No

13. Since the six properties which define the concept of a ring are all satisfied by Q (problems 3, 5, 7, 9, 10 and 12 verify this) then Q does form a ring. Furthermore, since Q also satisfies the commutative law for multiplication ($xy = yx$), we may call it a ________________ ring.

14. Q has a multiplicative identity, namely 1 ($1 \cdot x = x$). The multiplicative inverse of 3 is 1/3 (since $3(1/3) = 1$). The multiplicative inverse of $-8/5$ is $-5/8$. Does every nonzero element in Q have a multiplicative inverse?

    ____Yes ____No

15. For any nonzero element x in Q, there is another element in Q (namely, $1/x$) such that $x(1/x) = 1$, so that all nonzero elements in Q do have a multiplicative inverse. Q thus satisfies the additional properties numbered 7, 8, and 9 given on page 4. Hence Q satisfied all nine properties given on pages 3 and 4 of your reading material, so we give Q the even more special name of ____________________ .

INTRODUCTORY OVERVIEW

This is an introduction to a unit that you will be studying shortly, a unit on matrices and their operations. This introduction should prove useful in understanding the material you will be studying.

A matrix is a collection of real numbers which are arranged into rows and columns; that is, it is a rectangular array of real numbers. We keep track of matrices by observing the number of rows and number of columns contained in the matrix. One way to describe a matrix is to write down the number of rows and number of columns which appear in it, with the number of rows always stated first. So if we have a collection of numbers which is arranged into m rows and n columns, we call the matrix an mxn (read: m by n) matrix. In general, a given matrix will be referred to by its dimension, mxn. If a matrix should happen to have the number of its rows equal to the number of its columns (that is, n rows and n columns), we have an nxn matrix, and we call such a matrix a square matrix. Square matrices are of particular importance in certain mathematical applications. A matrix is usually denoted by a capital letter. An example of a 2x3 matrix would be the matrix $A = \begin{bmatrix} 2 & 3 & -4 \\ 7 & 8 & 0 \end{bmatrix}$. An illustration of a square matrix would be $B = \begin{bmatrix} 5 & 7 \\ 2 & -6 \end{bmatrix}$, which is a 2x2 matrix.

We can define several relations and operations on the set of matrices. One important relation we will consider is that of equality of matrices. Two matrices are said to be equal if, and only if, they have the same number of rows and the same number of columns and their entries in corresponding positions are equal. Thus, matrices which differ in the number of their rows or in the number of their columns cannot be equal. If even one entry in a given position in matrix A is not equal to the entry in the same position in matrix B, then the two matrices are not equal. If A = $\begin{bmatrix} 1 & -4 \\ 3 & 7 \\ 2 & 8 \end{bmatrix}$ and we wish to write matrix B so that A = B, then B must be the following 3x2 matrix: B = $\begin{bmatrix} 1 & -4 \\ 3 & 7 \\ 2 & 8 \end{bmatrix}$.

One important operation we wish to define on the set of matrices is that of addition. If two matrices have the same number of rows and the same number of columns, then we can add them and form their sum. If matrix A and matrix B have the same number of rows and the same number of columns, then A + B is the matrix each of whose entries is the sum of the corresponding entries of A and B. For example, to get the entry in the 2nd row and 3rd column of A + B, we simply take the number in the 2nd row and 3rd column of A, the number in the 2nd row and 3rd column of B, and add them. The matrix A + B will thus have the same number of rows and the same number of columns as does A or B. As you can probably tell, the operation of addition of two matrices is defined in a fairly obvious manner. If two matrices do not have the same number of rows or if they do not have the same number of columns, then we cannot add them; their sum is not defined. To illustrate this operation, let:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 5 & -3 \\ 7 & 9 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix}$$

Then: $A + B = \begin{bmatrix} 1+2 & 3+5 & 2+(-3) \\ 2+7 & 0+9 & 4+8 \end{bmatrix} = \begin{bmatrix} 3 & 8 & -1 \\ 9 & 9 & 12 \end{bmatrix}$ which, like A and B, is a 2x3 matrix. The sum A + C is not defined since A and C do not have the same dimension.

If we add a matrix to itself, we will obtain a new matrix each of whose entries is twice the corresponding entry of the original matrix: $A + A = 2A$. We are therefore led to define the product of a matrix by a real number. To multiply a matrix by any real number, we simply multiply each entry in the matrix by that number. Thus if A is an mxn matrix and c represents an arbitrary real number, then cA is the mxn matrix each of whose entries is c times as great as the corresponding entry of A. For example, if $A = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, then $4A = \begin{bmatrix} 8 & 12 \\ -16 & 20 \end{bmatrix}$.

A second important operation we wish to define on the set of matrices is that of multiplication. We shall define the product of two matrices, but in a somewhat unique manner. We would like to have the concept of matrices assist us in as wide a variety of practical situations as possible. It turns out that in order to be most useful, the definition of the product of two matrices is somewhat different than you might first expect. When adding two matrices and forming their sum, remember that the two matrices had to have the same number of rows, and they each had to have the same number of columns. However, in order to multiply two matrices A and B, and form their product, AB, we must require that matrix A have the number of its columns equal to the number of rows in matrix B. The product matrix which results, AB, is a matrix which has the same number of rows as A and the same number of columns as B. That is, the product of the mxn matrix A and the nxr matrix B (taken in the order given, thus forming the matrix AB as versus the matrix BA) is the mxr matrix AB. To illustrate, if A = $\begin{bmatrix} 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 6 & -1 \\ 5 & 0 & 7 \end{bmatrix}$, then A is a 1x2 matrix and B is a 2x3 matrix, so the product AB <u>is</u> defined and its dimension is 1x3. The rule for multiplying two matrices is cumbersome to write out in words, but the multiplication is not difficult to perform. For the two given matrices, we proceed as follows:

$$AB = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 4 & 6 & -1 \\ 5 & 0 & 7 \end{bmatrix} = [(3x4) + (2x5),\ (3x6) + (2x0),\ (3x-1) + (2x7)]$$
$$= [12 + 10 \quad 18 + 0 \quad -3 + 14]$$
$$= [22 \quad 18 \quad 11] \ .$$

Notice that we went across the row of A and down the first column of B and summed the two products that were formed, which gave us the first element in the product matrix. Then we went across the row of A and down the second column of B and summed the two products, which gave us the second element in the product matrix. Proceeding across the row of A and down the third column of B, we obtained the products whose sum is the third element in the matrix AB. Let's take another example. If C = $\begin{bmatrix} 2 & 4 \\ 7 & 9 \end{bmatrix}$ and D = $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 6 & 4 \end{bmatrix}$, then

CD is defined (it will be a 2x3 matrix), and:

$$CD = \begin{bmatrix} 2 & 4 \\ 7 & 9 \end{bmatrix} \begin{bmatrix} 5 & 3 & 1 \\ 2 & 6 & 4 \end{bmatrix} = \begin{bmatrix} (2\times5)+(4\times2), & (2\times3)+(4\times6), & (2\times1)+(4\times4) \\ (7\times5)+(9\times2), & (7\times3)+(9\times6), & (7\times1)+(9\times4) \end{bmatrix}$$

$$= \begin{bmatrix} 10+8 & 6+24 & 2+16 \\ 35+18 & 21+54 & 7+36 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 30 & 18 \\ 53 & 75 & 43 \end{bmatrix}.$$

We went across the rows of C and down the columns of D, summing the products formed. Since C has two rows, we had to go through the process twice (once for each row). Thus to multiply two matrices A and B, the elements of the rows of A must be multiplied with the elements of the columns of B and the resulting products are summed, as indicated in the two examples given.

Not all matrices have a multiplicative identity. Only the class of square matrices (those matrices for which the number of rows is equal to the number of columns) have an identity matrix under multiplication. The identity matrix, I, for all square matrices with n rows and n columns, is that nxn matrix such that AI = IA = A. The principal diagonal of a square matrix consists of all those elements whose row and column positions are equal (those elements which are entered in the first row and first column, second row and second column, third row and third column, and so forth). The nxn identity matrix is the matrix which has ones entered on its principal diagonal, and zeros everywhere else.

If a matrix is not a square matrix, it does not have a multiplicative identity. If A = $\begin{bmatrix} 2 & 4 \\ -3 & 6 \end{bmatrix}$, then since A is a 2x2 matrix, the identity matrix is I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. You may actually verify this by performing the multiplication. For a 3x3 matrix, such as B = $\begin{bmatrix} 10 & 4 & 3 \\ -8 & 7 & 6 \\ 4 & 24 & 9 \end{bmatrix}$, the multiplicative identity is I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

For a given nxn matrix A, if there exists an nxn matrix B such that AB = I, then B is called the multiplicative inverse of A. If a matrix is not a square matrix, it does not have a multiplicative inverse. However, not all square matrices have a multiplicative inverse. The fact that a matrix is a square matrix is not sufficient to guarantee that a multiplicative inverse exists. We will want to learn how to determine whether a given square matrix has a multiplicative inverse, and then be able to compute this inverse matrix when we know that it exists. For the moment, we can illustrate this concept for the matrix A = $\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$. Letting B = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ we obtain:

$$AB = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} (5\times3) + (-2+7), & (5\times2) + (-2\times5) \\ (-7\times3) + (3\times7), & (-7\times2) + (3\times5) \end{bmatrix}$$

$$= \begin{bmatrix} 15 - 14 & 10 - 10 \\ -21 + 21 & -14 + 15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

Thus AB = I, so that B is the multiplicative inverse of A.

On the other hand, for matrix C = $\begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$ there does <u>not</u> exist a 2x2 matrix B such that CB = I, even though C is a square matrix.

When you begin your study of matrices, the ideas presented here should be valuable to you.

Name_________________________

The following list of questions pertains to the material which you have just read. In responding to these questions, you may use the material you have read. These questions do not constitute a "test;" they are simply designed to help you in understanding the material which you have read. Please respond to each question before proceeding to the next question.

1. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 6 \\ 3 & 5 \end{bmatrix}$, then its dimension is: a) 2x3 b) 6 c) 3x2 d) 3x6

2. If $B = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$, how many rows does B have?_____. How many columns?_____. Is B a square matrix? _______.

3. Which of the following matrices is square?

   a) $\begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & -1 \\ -1 & 4 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 4 \\ 1 & 6 \\ 3 & 5 \end{bmatrix}$

4. If $C = \begin{bmatrix} 4 & 1 & -3 \\ 2 & -1 & 0 \end{bmatrix}$, which of the following matrices is equal to C?

   a) $\begin{bmatrix} 4 & 2 \\ 1 & -1 \\ -3 & 0 \end{bmatrix}$ b) $\begin{bmatrix} -4 & -1 & 3 \\ -2 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 1 & 0 \\ 2 & -1 & -3 \end{bmatrix}$ d) $\begin{bmatrix} 4 & 1 & -3 \\ 2 & -1 & 0 \end{bmatrix}$

5. Add the following matrices: $\begin{bmatrix} 2 & 3 & 1 \\ 8 & 6 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -1 & 3 \\ 0 & 6 & 2 \end{bmatrix} =$

6. If $\begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix} + X = \begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$ then X is equal to a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$

7. If $F = \begin{bmatrix} 2 & -3 & 5 \\ 1 & 6 & 4 \end{bmatrix}$, compute $-3F =$

8. If $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then B is equal to: a) $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -2 & -3 \\ 1 & -4 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$, compute AB.

10. The 4x4 identity matrix for multiplication is:

a) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

11. The additive inverse for $H = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ is:

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} -2 & -3 \\ 1 & -4 \end{bmatrix}$

12. If $\begin{bmatrix} 1 & 6 & -4 \\ 2 & 0 & 3 \end{bmatrix} + B = \begin{bmatrix} 3 & 9 & -2 \\ -4 & 8 & 3 \end{bmatrix}$, then $B =$

CONTROL

This is a brief introduction to a unit that you will be studying shortly, a unit on matrices and their operations and their use in a mathematical situation. This introduction should prove useful in understanding how this particular concept was developed.

Matrices were introduced into the body of mathematical literature by Arthur Cayley (1821-1895), a very prolific and inventive English mathematician. For both quantity and quality of his contributions to mathematics, Cayley is surpassed by few other mathematicians. His range, analytical power and originality rank him high among 19th-century mathematicians. On his father's side, Cayley could trace his ancestry back to the days of the Norman conquest (1066). The family was a talented one, and Cayley seems to have inherited some of his gifts. He spent his first eight years chiefly in St. Petersburg (now Leningrad), Russia, where his father was a merchant. In 1829, he returned with his parents to live near London. After attending a private school, he enrolled in King's College, London University, at age fourteen. His mathematical genius showed itself very early; he developed an amazing skill in long numerical calculations which he did for amusement. Upon entering into the formal study of mathematics, he quickly outstripped his classmates. His teachers recognized his ability from the beginning and gave him strong encouragement; they felt he was a born mathematician who should make mathematics his career. His father, who initially opposed his son's entry into the field of mathematics, eventually gave his consent, his blessing, and his money, and Cayley went off to Cambridge.

He entered Trinity College, Cambridge, at age seventeen, where along with his study of mathematics he developed a particular passion for novel-reading (reading through the works of Scott, Jane Austen, Byron, Shakespeare). He was fluent in Greek, French, German, and Italian. By the end of his third year at Cambridge, Cayley was so far in front of the other students in mathematics that he was placed in a class by himself. He captured several top awards during his four years at Cambridge. Upon his graduation, he received a fellowship from his college at Cambridge, and was elected as an assistant tutor for a period of three years. These two honors allowed him to do pretty much as he pleased, as his duties were light almost to the point of nonexistence. He taught a select number of pupils, and continued the mathematical researches which he had begun as an undergraduate student. Much of his research grew out of his study of the masters of previous generations.

In these three years in which he had complete control over his work, Cayley published prolifically. He wrote eight papers the first year, four the second, and thirteen the third. These papers (all published before he was twenty five years old) map out much of the work that would occupy him for the next fifty years.

During this period, Cayley did not become a stuffy professor isolated in his ivory tower. He embarked upon numerous vacations to the continent where he took up mountaineering and water-color sketching. With his love of literature, travel, painting, and architecture, he had sufficient activities to keep him busy and from degenerating into a "mere mathematician."

His appointment at Cambridge ended in 1846, when he was twenty five. He could have retained his position by taking religious orders, but this he declined to do. Unable to find another position as a mathematician, he was attracted to the study of law. He entered into a three year period as an apprentice and was admitted to the bar in 1849, when he was twenty eight. For fourteen years he stuck to the practice of law, making an ample living but deliberately turning away the opportunity to smother himself in money and the renown that comes to prominent lawyers. He took this course of action so that he might earn enough to enable him to get on with his real work: the study of mathematics. While practicing law he met another important 19th-century mathematician, J.J. Sylvester, and the two spent much time discussing mathematics and the particular areas in which they were the pioneers. Cayley left the practice of law at the first opportunity which presented itself. But during this period he had published between two and three hundred mathematical papers, many of which are now considered classics.

In 1863, Cayley was able to leave his law practice when Cambridge University established a new professorship of mathematics and offered him the post, which he promptly accepted. Cayley thus became the first Sadlerian professor of pure mathematics at Cambridge. Although he made less money as a professor than he had as a lawyer, Cayley never regretted his change. This same year, at the age of forty two, he married and subsequently had two children. Cayley thoroughly enjoyed his position at Cambridge, and was always generous with his help, encouragement, and advice to those entering careers in mathematics. At Cambridge, he continued his research, often in collaboration with his friend Sylvester.

It is an interesting footnote to observe that during his professorship the higher education of women was a hotly contested issue. Cayley threw all of his influence on the side of women and largely through his efforts women were at last admitted as students (in their own buildings) at Cambridge.

His growing international reputation in mathematics brought to Cayley an invitation to lecture at Johns Hopkins University in Baltimore. He lectured there for a half year in 1881-82.

Cayley made many outstanding contributions to mathematics during his lifetime, and he developed several new mathematical inventions. Mathematics can be subdivided into many areas (such as algebra, geometry, analysis). Cayley made significant contributions to many of these fields. One of his most outstanding inventions is that of matrices and their algebra. This subject had its origin in a memoir which he wrote in 1858. The concept grew directly out of some simple observations he had made on another mathematical theory on which he had been working. Cayley meticulously undertook the creation and study of this mathematical concept and defined the necessary operations on matrices which would satisfy his observations. The creation of a mathematical concept and the rules and operations this concept satisfies is a significant contribution. As frequently happens in the history of science, the full use and significance of his discovery of matrices was not appreciated for many years. Matrix theory has found application in many branches of mathematics, and in astronomy, mechanics, electric circuit theory, quantum mechanics, relativity, nuclear physics, and aerodynamics.

Much of what Cayley did has passed into the main current of mathematics, and it is probable that much more in his massive Collected Mathematical Papers (thirteen large volumes of about 600 pages each, comprising 966 papers which treat of nearly every subject of pure mathematics as well as theoretical dymamics andastronomy) will suggest profitable lines of research for adventurous mathematicians for some time to come. Although he published only one book (in 1876), his record of over 900 published papers is almost unmatched in the history of mathematics.

As he gradually aged, Cayley's mind remained as vigorous as ever and his nature became, if anything, gentler. Cayley was an omnivorous reader of other mathematicians' work, and he seemed to know a lot about everything. His advice as a referee and arbiter was sought by authors and editors from all over Europe. Cayley continued in creative activity up to the week of his death, which occurred after a long and painful illness, on January 26, 1895. Cayley's lectures at Cambridge attracted few students; among them, however, was A.R. Forsyth, who succeeded him in the Sadlerian chair. It was Forsyth who brought English mathematics back into the main stream, from which it had been diverted after Newton's time. Thus, indirectly, Cayley played a great part in founding the modern British school of pure mathematics. To quote the closing sentences of Forsyth's biography of Cayley: "But he was more than a mathematician. With a singleness of aim, which Wordsworth would have chosen for his 'Happy Warrior,' he persevered to the last in his nobly lived ideal. His life had a significant influence on those who knew him: they admired his character as much as they respected his genius: and they felt that, at his death, a great man had passed from the world."

You will shortly be undertaking the study of matrices and their algebraic operations, and their use in a particular mathematical context. The basis for your study was laid over one hundred years ago by Arthur Cayley, and is of sufficient importance to be included in almost every course in algebra today.

Name___________________________

The following list of questions pertains to the material which you have just read. In responding to these questions, you may use the material you have read. These questions do not constitute a "test;" they are simply designed to help you in understanding the material which you have read. Please respond to each question before proceeding to the next question.

1. Cayley was a Russian mathematician. ____True ____False

2. At what age did he enter Cambridge University?_________.

3. How many languages was Cayley fluent in?_________.

4. What duties did Cayley assume immediately after graduating from Cambridge?

5. How many papers did Cayley publish (total) in the first three years after his graduation from Cambridge?_________.

6. When he left his first appointment at Cambridge, how old was he?___________.

7. In what year was he admitted to the bar?_____________.

8. For how many years did he practice law?____________.

9. A close mathematical associate of his while he was practicing law was __________________.

10. During his years of law practice, how many mathematical papers did he publish?________________.

11. In what year did Cayley return to Cambridge?____________.

12. When he returned to Cambridge, Cayley was appointed as a professor of mathematics. He was the first__________________.

13. Cayley was a male chauvinist. _____True ____False.

14. He was a guest lecturer at what famous American university?__________________.

15. The concept of matrices grew out of a memoir which he wrote in what year?____________.

16. How many papers did Cayley write in his lifetime?___________.

17. With what subjects, other than pure mathematics, were some of his published papers concerned?__________________________.

18. How many books did he publish?____________.

19. How old was Cayley when he died?______________.

20. Cayley was succeeded by what mathematician in his post at Cambridge?__________________________.