## ANNUAL REPORT.

or

## THE BOARD OF REGENTS

of the

## SMITHSONIAN INSTITUTION,

showing

THE OPERATIONS, EXPENDITURES, AND CONDITION OF THE INSTITUTION FOR THE YEAR 1871.

WASHINGTON:

In the Senate of the United States,
Ordered to lie on the table and be printed.
LETTER
FROM THE
SECRETARY OF THE SMITHSONIAN INSTITUTION,
TRANSMITTING
The annual report of the Smithsonian Institution for the year 1871.

## Smithsonian Institution,

 Washington, April 15, 1872.SIR: In behalf of the Board of Regents, I have the honor to submit to the Congress of the United States the annual report of the operations, expenditures, and condition of the Smithsonian Institution for the year 1871.
I have the honor to be, very respectfully, your obedient servant, JOSEPE HENRY, Seeretary Smithsonian Institution.
Hon. S. Colfax, President of the Senate.
Hon. J. G. Blaine,
Speaker of the House of Representatives.

## ANNUAL REPORT OF THE SMITHSONIAN INSTTTUTION FOR 1871.

This document contains: 1 . The programme of organization of the Smithsonian Institution. 2. The annual report of the Secretary, giving an account of the operations and condition of the establishment for the jear 1871, with the statistics of collections, exchanges, meteorology, \&c. 3. The report of the executive committee, exhibiting the financial affairs of the Institution, including a statement of the Smithson fund, the receipts and expenditures for the year 1871, and the estimates for 1872. 4. The proceedings of the Board of Regents. 5. A general appendix, consisting principally of reports of lectures, translations from foreign journals of articles not generally accessible, but of interest to meteorologists, correspondents of the Institution, teachers, and others interested in the promotion of knowledge.

## THE SMITHSONIAN INSTITUTION.

ULYSSES S. GRANT..... President of the United States, ex-officio Presiding Officer of
the Institution.
SALMON P. CHASE ..... Chief Justice of the United States, Chancellor of the Insti-
tution, President of the Board of Regents.
JOSEPH HENRY......... Secretary (or Director) of the Institution.

REGENTS OF THE INSTITUTION.


# EXECUTIVE COMMITTEE OF THE BOARD OF REGENTS. 

PETER PARKER.
JOHN MACLEAN.
WILLIAM T. SHERMAN.

## MEMBERS EX OFFICIO OF THE INSTITUTION.

| T | sident of the United States. |
| :---: | :---: |
| S. COLFAX | Vice-President of the United States. |
| S. P. CHASE | Chief Justice of the United States. |
| H. FISH | Secretary of State. |
| G. S. BOUTWELL | Secretary of the Treasury. |
| W. W BELKNAP. | .Secretary of War. |
| G. M. ROBESON | Secretary of the Navy. |
| J. A. J. CRESWELI | Postmaster General. |
| C. DELANO. | Secretary of the Interior. |
| GEO. H. WILLIAMS | Attorney General. |
| M. D. LEGGETT | Commissioner of Patents. |
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## OFFICERS OF THE INSTITUTION.

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JOSEPH HENRY, Secretary,
    Director of the Institution.
    SPENCER F. BAIRD,
    Assistant Secretary.
    WILLIAM J. RHEES,
        Chief Clerk.
    DANIEL LEECH,
    Corvesponding Clerk.
    CLARENCE B. YOUNG,
        Book-keeper.
HERMANN DIEBITSCH,
    Meteorological Clerk.
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HENRY M. BANNISTER, Museum Clerk.
EDWARD PALMER, Curator of the Museum.

> JANE A. TURNER, Exchange Clerk.
SOLOMON G. BROWN, Transportation Clerk.
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JOSEPH HERRON, Janitor of the Museum.
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## PROGRAMME OF ORGANIZATION

## OF THE

## SMITHSONIAN INSTITUTION.

[PRESENTED IN THE FIRST ANNUAL REPORT OF THE ADOPTED BY THE BOARD OF REGENTS, DECEMBER 13, 1847.]

## INTRODUCTION.

General considerations which should serve as a guide in adopting a Plan of Organization.

1. Will of Siutinson. The property is bequeathed to the United States of America, "to found at Washington, under the name of the Smithsonian Institution, an establishment for the increase and diffusion of knowledge among men."
2. The bequest is for the benefit of mankind. The Government of the United States is merely a trustee to carry out the design of the testator.
3. The Institution is not a national establishment, as is frequently supposed, but the establishment of an individual, and is to bear and perpetuate his name.
4. The objects of the Institution are, 1st, to increase, and 2 d , to diffuse knowledge among men.
5. These two objects should not be confounded with one another. The first is to enlarge the existing stock of knowledge. by the addition of new truths; and the second, to disseminate knowledge, thus increased, among men.
6. The will makes no restriction in favor of any particular kind of knowledge; hence all branches are entitled to a share of attention.
7. Knowledge can be increased by different methods of facilitating and promoting the discovery of new truths; and can be most extensively diffused among men by means of the press.
8. To effect the greatest amount of good, the organization should be such as to enable the Institution to produce results, in the way of increasing and diffusing knowledge, which cannot be produced either at all or so efficiently by the existing institutions in our country.
9. The organization should also be such as can be adopted provisionally; can be easily reduced to practice; receive modifications, or be abaudoned, in whole or in part, without a sacrifice of the funds.
10. In order to compensate in some measure for the loss of time occasioned by the delay of eight years in establishing the Institution, a considerable portion of the interest which has accrued should be added to the principal.
11. In proportion to the wide field of knowledge to be cultivated, the funds are small. Economy should, therefore, be consulted in the construction of the building; and not ouly the first cost of the edifice should be considered, but also the continual expense of keeping it in repair, and of the support of the establishment necessarily connected with it. There should also be but few individuals permanently supported by the Institution.
12. The plan and dimensions of the building should be determined by the plan of organization, and not the converse.
13. It should be recollected that mankind in general are to be benefited by the bequest, and that, therefore, all unnecessary expenditure on local objects would be a perversion of the trust.
14. Besides the foregoing considerations, deduced immediately from the will of Smithson, regard must be had to certain requirements of the act of Congress establishing the Institution. These are, a library, a museum, and a gallery of art, with a building on a liberal scale to contain them.

## SECTION I.

Plan of organization of the Institution in accordance with the foregoing deductions from the will of Smithson.

To increase knowledge. It is proposed-

1. To stimulate men of talent to make original researches, by offering suitable rewards $f \mathrm{r}$ memoirs containing new truths; and,
2. To appropriate annually a portion of the income for particular researches, under the direction of suitable persons.
To diffuse knowledge. It is proposed-
3. To publish a series of periodical reports on the progress of the different brauches of knowledge; and,
4. To publish occasionally separate treatises on subjects of general interest.
details of the plan to increase knowledge.

## I. By stimulating researches.

1. Facilities afforded for the production of original memoirs on all branches of knowledge.
2. The memoirs thus obtained to be pablished in a series of volumes, in a quarto form, and entitled Smithsonian Contributions to Knowledge.
3. No memoir on subjects of physical science to be accepted for publication which does not furnish a positive addition to human knowledge, resting on original research; and all unverified speculations to be rejected.
4. Each memoir presented to the Institution to be submitted for examination to a commission of persons of reputation for learning in the branch to which the memoir pertains; and to be accepted for publication only in case the report of this commission is favorable.
5. The commission to be chosen by the officers of the Institution, and the name of the author, as far as practicable, concealed, unless a favorable decision is made.
6. The volumes of the memoirs to be exchanged for thet transactions of literary and scientific societies, and copies to be given to all the colleges and principal libraries in this country. One part of the remaining copies may be offered for sale, and the other carefnlly preserved, to form complete sets of the work, to supply the demand from new institutions.
7. An abstract, or popular account, of the contents of these memoirs to be given to the public through the annual report of the Regents to Congress.

## II. By appropriating a part of the income, annually, to special objects of research, under the direction of suitable persons.

1. The objects, and the amount appropriated, to be recommended by counselors of the Institution.
2. Appropriations in different years to different objects; so that in course of time each branch of knowledge may receive a share.
3. The results obtained from these appropriations to be published, with the memoirs before mentioned, in the volumes of the Smithsonian Contributions to Knowledge.
4. Examples of objects for which appropriations may be made:
(1.) System of extended meteorological observations for solving the problem of American storms.
(2.) Explorations in descriptive natural history, and geological, magnetical, and topographical surveys, to collect materials for the formation of a physical atlas of the Uvited States.
(3.) Solution of experimental problems, such as a new determination of the weight of the earth, of the velocity of electricity, and of light; chemical analyses of soils and plants ; collection and publication of scientific facts accumulated in the offices of Government.
(4.) Institation of statistical inquiries with reference to physical, moral, and political subjects.
(5.) Historical researches, and accurate surveys of places celebrated in American history.
(6.) Ethoological researches, particularly with reference to the different races of men in North America; also, explorations and accurate surveys of the mounds and other remains of the ancient people of our country.

## details of the plan for diffusing knowledge.

I. By the publication of a series of reports, giving an account of the new discoveries in science, and of the changes made from year to year in all branches of knowledye not strictly professional.

1. These reports will diffuse a kind of knowledge generally interesting, but which, at present, is inaccessible to the public. Some of the reports may be published annually, others at longer intervals, as the income of the Institution or the changes in the branches of knowledge may indicate.
2. The reports are to be prepared by collaborators eminent in the different branches of knowledge.
3. Each collaborator to be furnished with the journals and publioan tions, domestic and foreign, necessary to the compilation of his report; to be paid a certain sum for his labors, and to be named on the titlepage of the report.
4. The reports to be published in separate parts, so that persons interested in a particular branch can procure the parts relating to it with: out purchasing the whole.
5. These reports may be presented to Gangress, for partial distribution, the remaining copies to be given to literary and scientific institutions, and sold to individuals for a moderate price.

## II. By the publication of separate treatises on subjects of general interest.

1. These treatises may occasionally consist of valuable memoirs, translated from foreign languages, or of articles prepared under the direction of the Iustitution, or procured by offering premiums for the best exposition of a given subject.
2. The treatises should, in all cases, be submitted to a commission of competent judges, previous to their publication.
3. As examples of these treatises, expositions may be obtained of the present state of the several branches of knowledge mentioned in the table of reports.

## SEOTION II.

Plan of organization, in accordance with the terms of the resolutions of the Board of Regents providing for the two modes of increasing and diffusing knouledge.

1. The act of Congress establishing the Institution contemplated the formation of a library and a museum ; and the Board of Regents, in-
cluding these objects in the plan of organization, resolved to divide the income into two equal parts.
2. One part to be appropriated to increase and diffuse knowledge by means of publications and researches, agreeably to the scheme before given. The other part to be appropriated to the formation of a library and a collection of objects of nature and art.
3. These two plans are not incompatible with one another.
4. To carry out the plan before described, a library will be required, consisting, 1st, of a complete collection of the transactions and proceedings of all the learned societies in the world; $2 d$, of the more important current periodical publications, and other works necessary in preparing the periodical reports.
5. The Institution should make special collections, particularly of objects to illustrate and verify its own publications.
6. Also, a collection of instruments of research in all branches of experimental science.
7. With reference to the collection of books, other than those mentioned above, catalogues of all the different libraries in the United States should be procured, in order that the valuable books first purchased may be such as are not to be found in the United States.
8. Also, catalogue of memoirs, and of books and other materials, should be collected for rendering the Institution a ceuter of bibliographical knowledge, whence the student may be directed to any work which he may require.
9. It is believed that the collections in natural history will increase - by donation as rapidly as the income of the Institution can make provision for their reception, and, therefore, it will seldom be necessary to purchase articles of this kind.
10. Attempts should be made to secure for the gallery of art casts of the most celebrated articles of ancient and modern sculpture.
11. The arts may be encouraged by providing aroom, free of expense, for the exhibition of the objects of the Art-Union and other similar societies.
12. A small appropriation should annually be made for models of antiquities, such as those of the remains of ancient temples, \&c.
13. For the present, or until the building is fully completed, besides the Secretary no permanent assistant will be required, exceptone, to act as librarian.
14. The Secretary, by the law of Congress, is alone responsible to the Regents. He shall take charge of the building and property, keep a record of proceedings, discharge the duties of librarian and keeper of the museum, and may, with the consent of the Regents, employ assistants.
15. The Secretary and his assistants, during the session of Congress, will be required to illustrate new discoveries in science, and to exhibit
new objects of art. Distinguished individuals should also be invited to give lectures on subjects of general interest.

The foregoing programme was that of the general policy of the Institution until 1866, when Congress took charge of the library, and since an appropriation has been made by Government for the maintenance of the museum the provisons of Section II are no longer fully observed.

## REPORT

## OF <br> PROFESSOR J0SEPH HENRY,

SECRETARY OF THE SMITHSONIAN INSTITUTION,
HOR 1871.

To the Board of Regents of the Smithsonian Institution:
Gentlemen: I have the honor to present herewith another annual report, in which I am happy to inform you that the financial affairs of the Institution intrusted to your care by the Government of the United States are still in a favorable condition, and that its operations during the year 1871 have continued to enlarge the bounds of haman knowledge and to facilitate the international exchange of scientific truths.

Finances.-The following is a general statement of the condition of the Smithson fund at the begiuning of the year 1872, as will be seen by a reference to the report of the Executive Committee:
Total permanent Smithson fund in United States Treasury. \$650, 00000
In addition to the above there remains of the extra fund, derived from savings, \&c., in Virginia bonds, at par value, $\$ 88,125.20$; now worth about

35,500 00
Cash balance in First National Bank 16,31502
Amount of congressional appropriation for the fiscal year, June $30,1872, \$ 10,000$, one-hakf of which is available January, 1872.

5, 00000
Total Smithson funds, January, 1872 ............... 706, 81502
The Virginia stock, which in 1870 was $\$ 72,760$, has been nominally increased to $\$ 88,125.18$ by the funding of the interest due, while the marketable value of the whole has declined from $\$ 48,000$ to $\$ 35,500$. This fall in the value of the Virginia stock has been due to the unsettled policy of the State in regard to its public debt. It will be recollected that all the other State stocks held by the Institution, in which the savings from the income had been invested, were sold in 1867, and the proceeds;' added, by an act of Congress, to the permanent fund, forever deposited in the Treasury of the United States. The Virginia stock was retained, with the confident expectation on the part of the majority of the Board of Regents that its value would increase. Not the slightest idea was entertained that Virginia, with
all her resources and a large amount of money in her treasurs, would hesitate to make provision for the payment of the interest on her bouds. It is still confidently expected, from recent indications, that the value of this stock will increase. I would, however, recommend that it be disposed of as sön as may be, and the proceeds added to the permanent fund.
In an institution of this kind no dependence ought to be placed upon the contingency of the fluctuation of stocks. I may, perhaps, in this connection, be allowed to mention the fact that, to meet the payments on the building during its construction, it became my duty from time to time to sell portions of the stock ii which the building fund had been invested. In doing this, by waiting a few days, in some cases a considerable profit might have been made, and in other cases a loss would have ensued. These fluctuations gave rise to considerable anxiety and an unpleasant sense of responsibility, from which I was relieved by adopting the rule always to sell on the day in which the money was actually required. A similar policy has been adopted in regard to the sale of the gold received as the semi-annual interest on the permanent fund, which is always disposed of on the day in which it is paid by the Treasury, and the proceeds placed to the credit of the Smithson account in the First National Bank.

The income from the fund during the year, including the premium on gold, was $\$ 43,192.50$. The expenditures were as follows : viz, $\$ 9,032.41$ for repairs, and reconstruction of the building, and furniture; \$11,302.64 for salaries and general expenses; $\$ 15,431,93$ for publications and researches; $\$ 8,132.95$ for museum ; $\$ 4,455.36$ for exchanges, etc. ; making an aggregate of $\$ 48,355.29$, indicating an apparent excess of, expenditures over receipts of $\$ 5,162.79$. But to balance this excess there remained in the United States Treasury, as previously stated $\$ 5,000$ of the appropriation for the museum which had not been drawn.
Besides the foregoing, $\$ 20,000$ were expended on the building, and $\$ 4,976$ for the care of the museum from appropriations by Congress, a more detailed account of which will be found in a subsequent part of this report.

As stated in the last report Congress has indicated its intention to make appropriations for the independent support of the national museum, under the care of the Institution, and hence, in giving an account of the operations of the whole establishment, it is proper to divide them into two classes, those which relate to the legitimate objects of the Smithsonian Institution and those which pertain to the care and exhibition of the specimens of the national museum. In the following account we shall adopt this division.

## OPERATIONS OF THE INSTITUTION.

Publications.-The publications of the Institution are of three classes -the Contributions to Knowledge, the Miscellaneous Collections, and
the Annual Reports. The first consist of memoirs containing positive additions to science resting on original research, and which are generally the result of investigations to which the Institution has in some way rendered assistance. The Miscellaneous Collections are composed of works intended to facilitate the study of branches of natural history, meteorologỳ, etc., and are designed especially to induce individuals to engage in studies as specialties. The Annual Reports, beside an account of the operations, expenditures, and condition of the Institution, contain translations from works not generally accessible to American students, reports of lectures, extracts from correspondence, \&c.
During the past year the seventeenth volume of the Contributions has been distributed. It consists of a single memoir, by Lewis H. Morgan, esq., of 602 quartō pages, illustrated by thirteen plates, in three parts: First, a descriptive system of relationship of the Aryan, Semitic, and Uralian families; second, the classificatory system of the Ganowanian family; and third, a classificatory system of the Turanian and Malayan families. This volume has been distributed to institutions in this country and abroad, and has met with approval as an important contribution to the science of anthropology.
The paper on "The rain-fall in the United States," referred to in the last report, has been printed, but it was found necessary to make additions and corrections, especially in the charts, which have prevented its distribution to the present time.

A short paper by Professor William Ferrel, on "Converging series, expressing the ratio between the diameter and the circumference of a circle," which was read before the National Academy of Sciences, has been printed during the past year, and will form part of the eighteenth volume of the Contributions.
The papers of General J. G. Barnard, in "Problems of rotary motion presented by the gyroscope, the precession of the equinoxes, and the pendulum ;" of Mr. J. N. Stockwell, on "Secular variations in the orbits of the eight principal planets;" and of Dr. H. C. Wood, on "Freshwater algæ," have been placed in the hands of the printers during the past year, and will also form parts of the eighteenth volume of Contributions, to be issued in 1872.
Another paper in course of publication is by Professor William Harkness, of the United States Naval Observatory. It contains the records and discussions of a series of magnetic observations by the professor during thecruise of the Monitor Monadnock, from Philadelphia to San Francisco, in 1865-'66. The investigation was undertaken because the vessel was heavily armored and the voyage extended far into both hemispheres, thas affording a favorable opportunity of submitting Poisson's theory of the deviations of compasses on iron ships to the test of rigorous observations, which had never been done before. The disturbing force acting on a compass-needle is expressed as a function of the force of terrestrial magnetism, and of certain constants peculiar to the ship upon which the
compass is situated. Hence, in addition to swinging the Monadnock, or, in other words, turning its bow in succession to every point of the horizon to determine the deviations of her compasses from the true north, it was necessary to make observations on terrestrial magnetism on shore, and these, in their turn, required the determination of time, latitude, and azimuth. The memoir is divided into five sections: 1st, introduction; 2d, description of stations; 3d, astronomical observations; 4th, observations on terrestrial magnetism; 5th, observations on the magnetism of the ship. The results obtained may be summed up as follows: The latitude of seven points was determined. The magnetic declination, inclination, and horizontal force were obtained at seventeen stations, eleven of which were in South America. The ship was swung, and the deviations of all her compasses, seven in number, were observed and compared with those deduced from theory at ten places so situated as to afford very great changes in the terrestrial magnetic elements. For all these compasses the co-efficients or quantities necessary to reduce Poisson's general equations were defermined separately, with considerable accuracy. The agreement betreen theory and observation was found to be sufficiently exact for the purposes of navigation, but not entirely satisfactory in a scientific point of view. It appears from the results that certain parts of the theory require further investigation; and from the observations it is shown that when a vessel is swung for the first time near where she was built it is impossible to make any reliable estimate of the changes which the deviations of her compasses will undergo upon a change of magnetic latitude.

The memoir of Dr. E. W. Hilgard, on "The geology of Lower Louisiana, including the Petite Anse region," mentioned in the last report, has been received from the author, and the illustrations put in the hands of the engraver.

The work of Professor S. Newcomb, on "A new orbit of Uranus as influenced by the perturbations of Neptune and other bodies," is still in progress. In the calculation of the tables for indicating the places of Uranus, the assistance of Dr. Kampf, late of Germany, has been secured at the expense of the Institution. The labors of Professor Newcomb are gratuitously given for the advance of science.
The articles for the Miscellaneous Collections mentioned in the last report, viz: DeSaussure's "Monograph of hymenoptera," Uhler's "Monograph of hemiptera," and Watson's "Botany of the region west of the Mississippi," are still in the course of preparation, and some of them will be pablished during the next year.

The "Arrangement of the families of Mollusks," by Professor Theodore Gill, described in the last report, has been published. It forms an octaro pamphlet of 65 pages, and will be of importance in arranging the specimens of the national museum, as well as those of other collectious in this country.

A fourth edition of the "List of foreign institutions in correspondence
with the Smithsonian" is now in press, as well as a similar list, embracing all the scientific, educational, and literary establishments in the United States, prepared by Mr. Rhees, chief clerk of this Institution.

New editions of the following works were printed during the year: Physical and meteorological tables, Catalogue of Smithsonian publications, Review of American Birds, Classification of coleoptera, Bibliography of North American conchology, Researches on Hydrobiinæ, Check lists of fossils, Instructions relative to shells, insects, tornadoes, Museum miscellanea, Catalogue of birds, \&c.

In addition to the above, the following new circulars of instructions have been prepared and distributed:

Circular relative to observations on thunder-storms.
Circular relative to the construction of lightning-rods.
Circular relative to collection of altitudes from railway and canal explorations.
The Institution many years ago prepared and published lists of words and phrases for collecting vocabularies of the several Indian languages of North America, which were distributed to officers of the Army, missionaries, Government exploring parties, and private individuals, and from these have been received over two hundred separate vocabularies. These include the tribes of Oregon, Washington, California, northwest coast, New Mexico, Arizona, and the prairies. They have all been placed in the hands of George Gibbs, esq., for critical study and revision, and after consultation with some of the principal philologists of the country, it has been concluded to publish them, as it were provisionally, for distribution, as materials for ethnological and linguistic investigations. Mr. Gibbs has kindly undertaken to superintend the printing, and it is proposed to put them to press immediately. They will not only be of great use to the student of ethnology, but also be of practical value to missionaries, teachers, 2nd all who are brought into intercourse with the aborigines of the cpuntry. No publication of the Institution has been called for more frequently than that of the Grammar and Dictionary of the Dakota language. Unfortunately, it was published at an early period of the Institution, and was not stereotyped; otherwise we would long since have struck off a new edition.
The Report of the Institution for the year 1870 was printed, as heretofore, at the Government expense, and we are gratified to state that a larger number of extra copies was ordered than of the previous year. The demand for these reports is, however, constantly increasing; and we would renew the recommendation made before, that Congress not only order a larger edition of the report for the coming year, but that a new edition be printed from the stereotype plates of previous volumes. In addition to the report of the Secretary, giving an account of the operations, expenditures, \&c., of the Institution, and the proceedings of the Board of Regents, the report for 1870 contains
the following articles: A eulogy on Professor Alexander Dallas Bache, late Superintendent of the Coast Survey, and president of the National Academy of Sciences, prepared by Professor Henry at the request of the Board of Regents of the Smithsonian Institution; a lecture on Switzerland, by Professor Bache, to illustrate his style, with notes, bringing the subject down to the present time, by Jno. Hitz, esq., consul general of that country; on a physical observatory, by Professor Henry ; memoirs of Arago, Sir John Herschel, Henry Gustavus Magnus, Professor Chester Dewey ; an original article on the nature and origin of force, by W. B. Taylor, of the United States PatentOffice; a discourse on induction and deduction, by Liebig; an address on the relations of food to work and its bearing on medical practice by Rev. Samuel Haughton, of Dublin; a lecture on hydrogen, by Dr. J. E. Reynolds; a lecture on the identification of the artisan and artist, by Cardinal Wiseman; the diamond and other precious stones, translated from the French of M. Belbinet; a large number of original communications on ethnology, physils, and meteorology.

The following are the rules which have been adopted for the distribution of the publications of the Smithsonian Institution:
1st. To learned societies of the first class which present complete series of their publications to the Institution.
2d. To libraries of the first class which give in exchange their catalogues and other publications; or an equivalent, from their duplicate volumes.
3d. To colleges of the first class which furnish meteorological observations, catalogues of their libraries and of their students, and all other publications relative to their organization and history.

4th. To States and Territories, provided they give in return copies of all documents published under their authority.

5th. To public libraries in this country, not included in any of the foregoing classes, containing 10,000 volumes, and to smaller libraries where a large district would be otherwise unsupplied.
6th. To institutions devoted exclusively to the promotion of particular branches of knowledge are given such Smithsonian publications as relate to their respective objects.
7th. The Reports are presented to the meteorological observers, to contributors of valuable material to the library or collections, and to persons engaged in special scientific research.

Exchanges.-The system of international exchanges has been largely increased in extent and efficiency during the past year. The number of foreign establishments to which the Smithsonian and other publications are distributed, and from which returns are received, now amounts to nearly two thousand. The system includes not only all the first-class libraries, and societies of established reputation, but also a considerable number of the minor institutions of the Old World. The following
table exhibits the number of foreign institutions in each country with which the Smithsonian is at present in correspondence:
Sweden 19 Turkey ..... 11
Norway ..... 22
Africa ..... 18
Iceland Asia ..... 36
Denmark ..... 26
Australia ..... 26
Russia ..... 154
Holland ..... 65
Germany ..... 573
Switzerland ..... 63
Belgium ..... 126
France ..... 190
Italy ..... 149
Portugal ..... 20
Spain ..... 11
Great Britain and Ireland ..... 323
Greece
New Zealand ..... 11
Polynesia ..... 1
South America ..... 33
West Indies ..... 11
Mexico ..... 8
Central America ..... 1
British America ..... 27
General ..... 5
Total ..... 1,937-

During the year, 1,778 packages, containing many thousand different articles, were transmitted to foreign countries. These packages filled 108 large boxes, having a cubical content of 772 feet and weighing 29,950 pounds. The parcels received at the Institution for parties in this country, in addition to those for the Smithsonian library, numbered 3,952.

As in previous years, the Institution has received important aid from various steamer and railroad lines in the way of free freights, without which the expense of carrying on the system would be far beyond the means at our command. Acknowledgment is again due for the liberality of the following companies : Pacific Mail Steamship, Panama Railroad, Pacific Steam Navigation, New Yetzand Mexican Steamship, New York and Brazilian Line, North German Lloyds, Hamburg American Packet, French Transatlantic, Inman Line, Cunard Line; Anchor Line, Union Pacific Railroad. The Adams Express Company also continues its liberal policy in regard to freight for the Institution.
The advantages which result from the international scientific exchanges have become so apparent that establishments similar in this respect to the Smithsonian are beginning to be formed in different parts of Europe. A central scientific bureau for the Netherlands has been established in Amsterdam, the object of which is to receive and transmit packages for different parts of the world, and in this country to 00 -operate with the Smithsonian Institution.

The international exchange is not confined alone to the transactions and proceedings of societies, but also includes scientific works of individuals. We frequently receive from persons abroad who can afford the cost, copies of works to be gratuitously distributed among institutions and libraries in this country, and also- scientific: works from
persons in this country to be distributed abroad. In most cases the list of distribution is made out by the party sending the copies, but sometimes the selection of recipients is left to the Institution. Among the articles distributed in this way which we should have mentioned in the last report, is the narrative of an exploration to Musardo, the capital of the Western Mandigoes, through the country east of Liberia, by Benjamin Anderson, a young man of pure negro blood. The narrative was printed without correction from the original manuscript at the expense of Mr. H. M. Schieffelin, of New York, and nearly the whole of the edition was presented to the Institution for distribution.
The labors of the Institution in the way of exchanges can scarcely be too highly estimated. Whatever tends to enlarge the sympathies of individuals and of nations, to render the progress of thought in each country common to all, must serve an important end in advancing the world in intelligence and morality. The works which are received through this system, by the several institutions of the United States, contain the records or the advance of science in all foreign countries at the present day. They do not consist of ordinary books, but special accounts of the actual increase of knowledge by the human family, or an account of that which constitutes the advance of man in a higher and wider intellectual development.

To afford information as to the regulations adopted for transmitting packages intended for exchange, a circular, of which the following is a copy, has been widely distributed:

1. Every package, without exception, must be enveloped in strong paper, and so secured as to bear separate transportation by express or otherwise.
2. The address of the institution for which, or the individual for whom, the parcel is intended must be written legibly on the package, and the name of the sender in one corner.
3. No single package must exceed the half of a cubic foot in bulk.
4. A detailed list of addresses of all the parcels sent, with their contents, must accompany them.
5. No letter or other communication can be allowed in the parcel, excepting such as relates exclusively to the contents of the package.
6. All packages must be delivered in Washington free of freight and other expenses.
Unless all these conditions are complied with the parcels are not forwarded from the Institution; and on the failure to comply with the first and second conditions, they are returned to the sender for correction.

The Institution recommends that every parcel should contain a blank acknowledgment, to be signed by the recipient and returned through the agent of the Institution, or, what is still better, directly by mail to the sender. Should exchanges be desired for what is sent, the fact should be explicitly stated on the accompanying circular. Much disappointment is frequently expressed at the absence of any return in kind for transmis-
sions; but unless these are specifically asked for they will fail in many instances to be made. It will facilitate the labor of the Institution very greatly if the number corresponding to the several addresses in the Smithsonian printed catalogue be marked on the face of each parcel ; and for this purpose a copy of the work will be forwarded to all who apply for it.
Specimens of natural history will not be received for transmission, unless with a previous understanding as to their character and bulk.

Library.-The accessions to the library during the last year from the foreign exchanges have not been as large as they were the year before, on account of the war between France and Germany.

The following is a statement of the books, maps, and charts received by exchange in 1871, and which have been deposited in the National Library in accordance with arrangements made several years ago, and fully explained in previous reports:
Volumes:
Quarto, or larger ..... 277
Octavo, or less. ..... 659
Parts of volumes:
Quarto, or larger ..... 625
Octavo, or less ..... 1,156
1, 781
Pamphlets:
Quarto, or larger ..... 316
Octavo, or less ..... 1, 482
1, 798
Maps and charts ..... 82
Total receipts ..... 4, 597

The following are some of the larger foreign donations received by the Smithsonian Institution in 1871:
From the Royal University of Norway, Christiania: 14 volumes, 37 pamphlets, and 3 charts.
Bergen Maseum, Bergen, Norway : 11 volumes and 31 pamphlets.
Russian government, St. Petersburg : Engineering Journal, Artillery Journal and Ordnance Magazine for 1870; Caucasian statistics, 1869; Appendix to the Oode of Laws, 1869.
Statistical bureau, Stockholm : Contributions to Swedish statistics, 26 parts, quarto.
Emperor of Germany: "Preussen's Schlösser und Residenzen," vol. xi, folio; and "Scriptores rerum Prussicarum," vol. iv.
F. Vieweg \& Son, Braunschweig : 42 volumes and 12 pamphlets.

Hungarian Academy of Sciences, Pesth : 16 volumes and 63 parts, reports transactions, \&c.

University of Pesth : 44 pamphlets, inaugural dissertations.
University of Leipsic : 104 pamphlets, inaugural dissertations.
University of Göttingen : 70 pamphlets, inaugural dissertations.
University of Bonn : 44 pamphlets, inaugural dissertations.
University of Königsberg : 144 pamphlets, inaugural dissertations.
University of Würzburg: 80 pamphlets, inaugural dissertations.
Board of Admiralty, London : 7 volumes, 36 charts, and 10 pamphlets.
British Museum : Catalogue of Syriac manuscripts, part ii; catalogue of prints ; catalogue of satires, vol. i; hand list of birds, parts ii and iii.

Royal Society, London: Philosophical transactions, vol. 160, part i; proceedings, 119-123; catalogue of scientific papers, vol. iv; Greenwich magnetic and meteorological observations, 1868.
R. L. Simmonds, London : 18 volumes and 52 pamphlets.

Thomason College Rourkee : 10 works on Civil Engineering.
Government Observatory, Sydney, Australia: Observations, 3 volumes and 55 parts.

Grand Ducal Court Library, Karlsruhe: 5 volumes and 3 parts.
University of Pisa: 22 volumes and 40 pamphlets.
The Minister of Agriculture, Industry, and Commerce, Florence: 27 volumes and 41 pamphlets.
Royal Institution for the Encouragement of Natural Sciences, Technology, \&c., Naples : Atti, second series, volumes i-viii ; quarto.
University of Chili, Santiago : 14 volumes and 5 pamphlets.
The value of the National Library still continues to be increased in the number and character of the books which are annually added to it, first by books purchased, second by the Smithsonian exchanges, and third by the deposit of books in accordance with the copyright law. As we have said in previous reports, the space for the accommo. dation of this valuable library-now the largest in the United Statesis far too circumscribed even for the wants of the present time, without regard to those of the future. It is, therefore, proper to keep the proposition of a new and separate building constantly in mind. The necessity for such a building is not alone confined to the better accommodation of the books, but also includes greater facilities for consulting them by students, as well as by general readers, in the way of greater seclusion in separate spaces, and the number of hours during which the library is open. With a separate building, certain portions of it at least might be accessible during the evening, which, perhaps, would be of greater importance to Washington than a similar arrangement in any other city, on account of the large number of educated men in the various offices of the Government, who cannot avail themselves at other hours of the great advantage which the library affords for the prosecution of study.
It may be proper to add, in this connection, that the library now de-
posited in the Army Medical Museum, numbering 20,000 volumes of works relating to medical subjects, may be considered as part of the great National Library, and is rapidly increasing in the number and value of its contents by an annual appropriation from Congress.
In accordance with the original agreement the use of these books, as well as those now in the Capitol, is free to the Smithsonian Institution, and we may perhaps indulge the hope that the new building for the library, which is now contemplated, will be erected on the Smithsonian Grounds, perhaps as an extension of the present building.
As we have said, one source of the increase of the library is the copyright system. The number of these books would be increased, we think, and their character greatly improved, if an international copyright law were established, granting to the foreign author the same protection that is afforded to our own citizens. For example, we would ask, what would be the condition of the wool-grower if the manufacturer of cloth in this country had the power to obtain surreptitiously all the wool that he uses, paying nothing but for manufacturing the article? What encouragement is there to an author to produce an original work on any branch of science when the publisher can obtain one which will equally well answer his purpose from a foreigner without paying anything? But the question ought not to be decided on considerations even of this character ; it belongs to the province of justice and morality. The results of the labors of the mind, which form the basis of all human improvement, ought not to be appropriated without remuneration, any more than the labors of the hand or of the machine.

Meteorology.-The impression has prevailed since the establishment of the meteorological system by the Government, under the direction of the Signal-Corps, that the observations which have been so long made under the direction of the Smithsonian Institution may now be discontinued. This idea is, however, erroneous. The hisect of the operations of the Signal-Service is principally one of immediate practical utility, viz , that of predicting the condition of the weather for a day or more in advance of the actual occurrence. This it is enabled to do by the fact previously established, that, as a general rule, disturbances of the atmosphere are propagated over a wide extent of the surface of the earth in an easterly direction. Besides the number of stations necessary for the practical predictions of the weather, a much more numerous series of stations and long-continued observations are required for determining the peculiarities of the climate, or for obtaining such information as may satisfy the requirements of the scientist, the physician, and the agriculturist. It is on this account that the more extended observations established by the Institution, and which have now been prosecuted for more than twenty years, are continued. It is true we would be gratified if the charge of this system were transferred to the Government, with more ample funds for its maintenance than can be afforded from the income of the Institution. But so long as an arrange-
ment of this kind is not effected, it becomes the duty of the Institution to continue the system with such improvements as the appropriation which can be made on account of it will allow. During the past year the number of stations has remained about the same, viz, 514, to which a large number of additional rain-gauges have been distributed. Besides these, meteorological observations are received from British America, Central America, Mexico, Bermuda, and some of the West Indies.
The tables and deductions of rain-fall have been printed, and are nearly ready for distribution.
The discussion of all the observations relative to the winds made under the direction of the Institution is still going on under the supervision of Professor Coffin. Like his former work on the winds of the northern hemisphere, it will consist mainly of abstracts of observations on the relative frequency of the different winds, both at the surface of the earth and in the higher regions, as indicated by the motion of the clouds, with their resultant directions and the monsoon influences by which they are affected in the different seasons, or months of the year. Where data could be obtained the actual transfer of the air in miles is also given.

Where the places of observation are sufficiently remote from each other to admit of distinct delineation of the results, on maps of the scale it is proposed to use, separate computations are made for each; in other cases they are grouped by districts. The work will embrace the following material:
I. All the observations reported to the Smithsonian Institution from the year 1854 to 1869, inclusive, with some others in the earlier years.
II. All those made at the United States military posts, and reported to the Surgeon General, from the year 1822 to 1859 inclusive; and all those from posts west of the Mississippi for the succeeding ten years, up to the end of 1869 .
III. All those at sea, collected at the United States Naval Observatory, so far as they have been published; i. e., over all the oceans between the parallels of latitude $60^{\circ}$ north and south, except a comparatively small portion of the North Pacific lying between the meridians $150^{\circ}$ east and $165^{\circ}$ west from Greenwich; and a few additional observations south of Cape Horn.
IV. Those taken at sea, beyond these limits, by Arctic and Antaretic explorers.
V. Those at several hundred stations in other parts of the globe.

This material, though very much more condensed than in his former work, will still make a considerably larger volume.
In the discussion the whole surface of the earth is divided into zones by parallels of latitude drawn $5^{\circ}$ asunder, and observations in these zones investigated in regular order from the North to the South Pole; commencing with the observations in each at the 180th meridian from Greenwich, and proceeding easterly to the same meridian again. Professor Coffin hopes to complete the tabular work in the course of two or
three months, when nothing will remain to be done but the maps and some general deductions.
To defray the cost of the labor in the preparation of this work other than that of Professor Coffin himself, an appropriation has been made from the income of the Institution. The world will not only therefore be indebted to the Institution for the publication of the work, but also for the collection of the material and a part of the expense of the reductions.

I may mention that the previous publication by the Institution of the Winds of North America has been largely made use of by the English Board of Trade in constructing their wind-charts of the northern oceans, and that the work now in process of preparation will be. of especial value for a similar purpose.
The temperature observations are still in progress of reduction, two computors being engaged upon the work. The progress of their labors has, however, frequently been interrupted by calls from different portions of the country for reports on the climate of different districts.
The following is an account of the present condition of this part of the general reductions:
The collection and tabulation, in the form of monthly and annual means, of all accessible observations of the atmospheric temperature of the American continent and adjacent islands, have been completed to the close of the year 1870, and extensive tables representing the daily extremes, or the maximum and the minimum at the regular observing hours, have been prepared.
An exhaustive discussion of all the observations available for the investigation of the daily fluctuations of the temperature has been made, and this part of the work is now ready for the printer.
The discussion of the annual fluctuations of the temperature has been commenced and carried as far as the present state of other parts of the discussion would permit.
The construction of a consolidated table giving the mean results, from a series of years, for each month, season, and the year, at all of the stations, which will probably exceed 2,500 in number, has been begun and completed for that part of the continent lying north of the United States, and also for several of the States. This is perhaps the most laborious, as it is one of the most important parts of the discussion. In many of the large cities there are numerous series, made by various observers, at different hours, all of which have to be brought together, corrected for daily variation, and combined to obtain the final mean. To give some adequate idea of the time and labor involved in the preparation of these tables, it may be mentioned that, in the State of New York alone, there are about three hundred series, which are derived from nearly two million individual observations.

The principal sources from which the general collection of results has been derived, may be enumerated as follows:

1. The registers of the Smithsonian Institution, embracing upward of three hundred large folio volumes.
2. The publications of the Institution, Patent-Office, Department of Agriculture, and public documents.
3. All the published and unpublished records of the United States Army, United States Lake Survey, and United States Coast Survey.
4. The large volume compiled by Dr. Hough, from the observations made in connection with the New York University system, the records made in connection with the Franklin Institute, and those obtained from numerous observatories and other scientific institutions.
5. The immense collection of printed slips, pamphlets, manuscripts, \&c., in the possession of the Smithsonian Institution.
The work has been somewhat retarded by the collection and tabulation of the rain-fall, to the end of 1870 for the Smithsonian stations, and to the end of 1871 for the United States military posts.
Beside the discussion of the observations on temperature, rain, and wind, there remain those relative to the pressure of the atmosphere, and its humidity; also those which are classed under the head of casual phenomena, such as thunder-storms, tornadoes, auroras, meteors, early and late frosts, progress of vegetation, opening and closing of rivers, \&c. These will be put in hand as soon as the funds of the Institution which can be devoted to meteorology will permit the requisite expenditure.

Explorations and collections.-As in previous reports, it is proper to make a distinction between the collections of the Institution and the specimens exhibited in the public museum. The former are collected as a part of the operations of the Institution, to advance science and promote general education ; they are usually in great numbers, including many duplicates of the same species. A type specimen of each species and variety is deposited in the National Museum. The remainder are reserved for distribution to foreign establishments, and to societies, colleges, and academies in this country, after they have been submitted to scientific investigation and duly assorted and labeled.

At the last session of Congress an appropriation was made of $\$ 12,000$ for the continuance of an exploration of the region of the Colorado of the West and its tributaries, by Professor J. W. Powell, to be expended under the direction of the Smithsonian Institution. The region here mentioned is one of the most interesting in a geological point of view of almost any in this or any other country. The Colorado of the West and its tributaries traverse chasms in some places over a mile below the general surface of the country, and present in different places at one view sections of the principal members of the known geological formations of the continent of North America. The region surveyed lies between the 35th and 39th parallels of latitude, and the 109th and 115th meridians of west longitude. It includes the headwaters of the Uintah, the Price, the San Rafael, the Paira, the Kanab, and the Virgin Rivers, the lower portion of the Grand, and a part of the

Colorado. In the year 1870 a general reconnaissance of the country had been made, and several routes through it explored from Salt Lake City to the Green and Colorado Rivers, and depositaries of supplies established. The operations of Professor Powell and party under his command in 1871, consisted, first, in an exploration of the Green River from the point where it is crossed by the track of the Union Pacific Railway to its junction with the Grand, or where the anion of these rivers forms the Colorado of the West, and the exploration of this to the mouth of the Paira; second, the establishment of a base-line in the valley of the Kanab, from which a system of triangles was extended westward to the valley of the Virgin River, southward and eastward to the Colorado, and northward to the Paira; third, a geological survey of the region, and the collection of a series of specimens of geology and mineralogy; fourth, an ethnological study of the Indians of the region, including their mythology, manners and customs, means of subsistence, language, \&c., together with a full collection of all their implements and articles of manufacture. The explorations and surveys of Professor Powell have furnished additions to our knowledge of a portion of our public domain previously but very imperfectly known, which, together with the extensive series of specimens which he has added to the collections of the Institution and the National Museum, fully repay the appropriation which was made from the national Treasury on this account. I have certitied to this effect to Congress, and respectfully commend the application of Professor Powell for an additional appropriation to complete the survey.
The alleged decrease of the food-fishes of the coast and lakes of the United States led to the passage of a law at the last session of Congress, directing the President to appoint a commissioner of fish and fisheries, for the purpose of making inquiries upon the subject. Professor Baird, assistant secretary of this Institution, whose attention has been directed for some time both to the scientid: and economical relationships of the fishes, received the appointment, and proceeded in June last to Wood's Hole, a convenient point on the Massachusetts coast, from which to prosecute his inquiries. With the aid of an appropriation from Congress, and facilities afforded by various departments of the Government, he was enabled to carry on an extended research during a period of several months. In this work he had the special co-operation of Professors Verrill and Smith, of Yale College, in the investigation of the invertebrate fauna of the coast in its relation to the food-fishes; of Professor Gill, of Washington, in the study of fishes themselves; and of Professor Hyatt, of the Boston Society of Natural History, Professor Jenks; of Middleborough, Dr. A. S. Packard, jr., of Salem, and W. G. Farlow, of Cambridge, in other branches of the investigation. Among other gentlemen interested in the researches, who visited Wood's Hole during the season, were Professor L. Agassiz, Professor J. Gwyn Jeffreys, of England, Colonel Lyman, Professor D. C. Eaton, Professor W. H. Brewer, Professor J. H. Trumbull, and Professor
W. D. Whitney. With this corps of helpers it was quite possible to make a rery thorough exploration of everything connected with the general economical and natural history of the fauna of the waters on the southern coast of New England; and while Professor Baird and some of his party were engaged in visiting different parts of the coast and taking testimony as to the actual condition of the fisheries, others of the party were occupied in trawling, dredging, and in otherwise collecting the various inhabitants of the sea.
A large amount of information was gathered which will have an important bearing upon the objects of the commission, and of which Professor Baird will present a reportin full to Congress atan early date. The inquiries include numerous observations in regard to currents, temperatures, distribution of life at different depths, \&c. The collections made during the exploration were very extensive, embracing a full series of all the fishes of the coast, as well as of the invertebrates, from which sets will be made up for distribution by the Institation. Among other results of the expedition should be inentioned a series of nearly three hundred photographs of a large size, cepresenting all the fishes found, in their various stages of growth, the whole constituting an almost unique collection of portraits, and especially important as relating to the larger fishes, like the sharks, rays, sturgeons, tunnys, sword-fish, \&cc.
Dr. Hayden, in the prosecution of his researches as United States geologist for the Territories, gathered very large collections of minerals, skins of mammals and birds, eggs, \&c., filling forty-five boses, illustrative of the natural history of Montana, and of the region about the head-waters of the Yellowstone, a report of which he has presented to the Secretary of the Interior. This exploration has excited a great degree of interest on account of the wonderful series of geysers and remarkable scenery, of which it has furnished an authentic description. Indeed such has been the interest manifested in the Yellowstone district that a proposition, originally made by Mr. Catlin as early as 1832, has been revived and presented to Congress, to reserve the country around these geysers as a public park. It is thought this proposition will be adopted by the Government; and if so, we doubt not that in time the Yellowstoue region will become a favorite resort for travelers from every part of the world.

After reserving a full set of the specimens for the National Museum the duplicates of Hayden's collections will be made up into sets for distribution.

Among the persons to whom the obligations of the Institation are particularly due for the magnitude and variety of contribution of specimens we should mention Mrs. James A. McMinn. She gave the valuable herbarium described in the last report, and has since presented the entire collection of objects of natural history belonging to her late husband, who was for many years a correspondent of the Institution. This gentleman had accumulated large numbers of minerals,
fossils, plants, \&cc., which filled twenty-six boxes, and were presented to the Institution to be used as it might deem best for the interest of science. Many of the specimens are duplicates, but are valuable as material for distribution.
To Mr. George A. Boardman the Institution is indebted for extensive collections of birds and skeletons from Florida, and also three complete skeletons of the moose from Nova Scotia.

To his son, Mr. Charles A. Boardman, and to Mr. S. W. Smith we owe acknowledgments for fine specimens of the moose and caribou.
Dr. Yarrow, assistant surgeon United States Army, Fort Macon, North Carolina, has sent a large collection of skulls and skeletons of the porpoises of the southern coast, as well as many Indian relics, fishes, shells, \&c.
From Professor Sumichrast we have received additional collections of birds, reptiles, \&cc., illustrative of the natural history of Tehuantepec. The name of this gentleman has frequently been mentioned in previous reports as a large contributor to the Smithsonian Collections.
Captain Charles Bryant, in charge of the fur-seal islands of Alaska, has contributed full series of skins, skulls, and skeletons of seals, walras, \&c., abounding in that region.
To the Army Medical Museum the Institution is indebted, as heretofore, for numerous specimens in ethnology and natural history, in accordance with an arrangement made several years ago, by which, in consideration of the transfer to it from the Institution of human crania, all other objects of an anthropological character received by that museum were to be placed in the Smithsonian Collection.
Some interesting specimens have also been received from the Department of Agriculture ander a similar arrangement of exchange.

Dr. Destruges has contributed the skeleton of a sloth, and Mr. Henry Hague that of a Guatemalan tapir; Profe sor Poey a skeleton, and Dr. Gundlach a specimen in alcohol of sotendodon, a rare insectivorous mammal of Cuba; Mr. Hernberg and Colonel Gibson, skeletons of buffalo; Mr. Isaac H. Taylor, of Boston, crania of South African mammals; Captain Scammon, of the United States revenue-service, skulls of whales and other cetaceans.

Although but few birds have been received, some valuable specimens from Veragua were contributed by Mr. Salvin; from Brazil, by Mr. Albuquerque; from Buenos Ayres, from the national museum under the charge of Professor Baumeister; from Labrador, from C. G. Brewster.
Mr. Strachan Jones has furnished a number of eggs from the Lower Slave Lake, and Mr. Charles R. Bree specimens of eggs of the Larus gelastes from Turkey.
The reptiles received have been principally specimens gathered by the naturalists of the Tehuantepec and Darien expedition.
Fine specimens of the celebrated Eozoon canadense have been re-
ceived from Mr. E. Billings, of Canada, and Dr. Josiah Curtis, of Chelmsford, Massachusetts.
Mr. Brittan has contributed Permian fossils from Kansas ; Mr. U. P. James, a series of Ohio Lower Silurian fossils ; Mr. S. A. Miller, fossils from Ohio, and a fossil tree-trunk of the genus Psaronius; Mr. D. M. Shafer, Lower Silurian fossils.

Specimens of woods have been presented by Mr. George Davidson, of the Coast Surves; of birds, reptiles, and fishes, from Illinois, by Mr. R. Ridgeway; fishes, reptiles, and vertebrates, by W. H. Clarke, of the Tehuantepec experition.

As usual, the amount of material received from the Old World is much less than that from our own continent, the most noteworthy being a collection of specimens in alcohol, presented by the museum of Bergen, in Norway.
Mr. Knudsen has sent a collection of human crania from the Sandwich Islands. The museum of Wellington, New Zealand, under the charge of Dr. Hector, has prevented casts of the eggs of the Dinornis and Apteryx, with casts of bones of the former animal, and various ethnological objects.
To Mr. Genio Scott, and to Messrs. Middleton \& Carman, of New York, the Institution is indebted for specimens of Cybium caballa, or Cero, a food-fish but lately indicated as occurring on our coast. The museum at Bergen has also supplied a number of fishes peculiar to the coast of Norway.
All the specimens of ethnology and natural history, not at present on exhibition in the public museum, are now stored in the west basement, and the various operations connected with unpacking, labeling, cleaning, assorting, poisoning, etc., have been transferred to that part of the building. The necessity of making this transfer in a limited space of time involved considerable derangement of the specimens, and much time has been occupied during the fall and winter in re-arranging them. This work, however, is in great measure accomplished; and Professor Baird, with assistants, is now occupied in assorting and classifying the material for the purpose of selecting duplicates to be distributed for the advance of science. A very extensive distribution of specimens has been made during the year, partly in the way of giving general series for educational purposes to colleges, academies, and scientific institutions, and partly in the way of exchanges with the principal museums at home and abroad. The amount of work done in the distribution of specimens will be shown in the following table:

Distribution of duplicate specimens to the end of 1871.

| Class. | Distribution in 1871. |  | Total to the end of 1871 . |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Species. | Specim's. | Species. | Specim's. |
| Skeletons and skulls.. | 111 | 156 | 325 | 827 |
| Mammals | 25 | 40 | 941 | 1,822 |
| Birds | 410 | 477 | 22,940 | 35,428 |
| Reptiles | 100 | 100 | 1,841 | 2,970 |
| Fishes | 42 | 100 | 2,477 | 5,311 |
| Eggs of birds. | 151 | 304 | 6,606 | 16,698 |
| Shells | 2,534 | 3,000 | 83,712 | 186,157 |
| Radiates |  |  | 583 | 778 |
| Crustaceans |  |  | 1,078 | 2,650 |
| Marine invertebrates |  |  | 1,838 | 5,152 |
| Plauts and packages of see | 3,000 | 4,000 | 18,503 | 25, 063 |
| Fossils . | 151 | 151 | 4,109 | 10,135 |
| Minerals and rocks. | 1,000 | 1,400 | 4,630 | 9,974 |
| Ethnological specimens | 152 | 152 | 1,295 | 1,342 |
| Insects | 204 | 204 | 1,836 | 3,150 |
| Diatomaceous earths | 1 | 55 | 29 | 623 |
| Total | 7,881 | 10,139 | 152, 743 | 308, 080 |

As heretofore, a great amount of labor has been expended in cataloguing the specimens received, their enumeration having been carried forward from 164,700 to 169,750 , the increase representing about the average of the last ten years.
As in previons years, the collections of the Institution have been placed freely at the service of naturalists in this ecantry and Europe, and large numbers of specimens are now in the hands of collaborators. Among these may be mentioned Dr. Elliott Coues, assistant surgeon, United States Army, who has undertaken a critical revision of a special family of Rodents of North America. This group is very extensive, embracing numerous genera and species differing entirely from the corresponding families in the Old World. The large amount of material we have placed in the hands of Dr. Coues will enable him to solve many interesting questions as to the geographical distribution and zoölogical affinities of the family in question. Dr. Coues' memoir on this group will be published by the Institution, and series of type specimens will be distributed to other museums. To Professor Cope have been intrusted, as before, the collections of reptiles, and other material has been furnished to Professor Leidy, Professor Marsh, Professor Agassiz, Dr. Stimpson, and others. Type specimens of American birds have been sent to Messrs. Sclater, Salvin, and Dresser, of London, for use by them in the preparation of descriptive works.

In accordance with the same policy a few years ago the alcoholic invertebrates were intrusted to Dr. Stimpson of the Chicago Academy of Sciences for study and distribution into sets of duplicates. Unfortunately, however, this collection, although deposited in a building supposed to be fire-proof, was destroyed in the disastrous fire of 1871. The misfortune was not alone confined to the loss of the specimens, but included also the results of years of labor of Dr. Stimpson, the great object of his scientific life, the publication of which was looked forward to with interest by all engaged in the study of natural history.

The ethnological specimens collected by the Institution to illustrate the arts, manners, and customs of the present Indians and the more ancient inhabitants of the American continent, are unsurpassed in number and variety, and are constantly increased by special efforts in the way of correspondence and small appropriations for explorations. The greatest additions to the collections received during the past year have been in this department, an account of some of the more important of which will be of interest.

From Captain C. F. Hall, the intrepid explorer, now, we trust, successfully prosecuting his researches in northern Greenland, we have received the entire series of relics of Sir John Franklin, obtained by Captain Hall during his last visit to the north, as also the relics of the Frobisher expedition, which wintered on Frobisher Bay several hundred years ago. To these were added a number of specimens illustrative of the habits and manners of the Esquimaux, and showing their relationship to, as well as their differences from, a corresponding series belong. ing to the Esquimaux of the Mackenzie's River region, furnished to the Institution by Mr. R. McFarlane and some of his colleagues of the Hudson's Bay service.

From the northwest coast of North America specimens have been furnished by Mr. George Gibbs, illustrating many points in the ethnology of the savage tribes; and specimens of dresses from Mr. Jos. T. Dyer.

Lieutenant Ring has sent specimens obtained from graves in Alaska and in British Columbia. Dr. Yates, of California, has added to his previous donations large Indian mortars and the crania obtained from sundry mounds.

Dr. Palmer collected for the Institution a very interesting series of stone implements from ancient ruins in Arizona, and Major Powell has furnished a full series of the implements, utensils, dresses, \&c., of the Indians of the valley of the Colorado. Dr. Irwin, of the Army, has also added to this series.

From Colorado Territory we have specimens from Dr. Berthoud, indicating, in his opinion, an antiquity of the human race in that region far beyond that usually ascribed to it.

Additions from New Mexico are represented by specimens of blankets and other manufactures of the Navajo Indians; as also by a loom contain-
jng a part of an unfinished blanket, showing the mode of weaving, presented by Governor Arny.

A series of bone implements of remarkable character, and different from any we had previously possessed, together with other interesting objects from ancient graves in Michigan, have been presented by Dr. Irwin.

Mr. Andrews has contributed stone implements and other objects from Tennessee; Mr. J. Fisher, very interesting copper implements, and Mr. Peter, stone objects from Kentucky. Rer. D. Thompson and Mr. Clark have furnished stone implements from Ohio. Mr. Hotchkiss, of Louisiana, has furnished a remarkable series of stone lances and knives, some of them being of very great length and of beautiful finish. Mr. Keenan, of Mississippi, has supplied a variety of Indian implements.
From Georgia we have an extensive collection made by the late Colonel Floyd, and kindly presented by his heirs through the mediation of Colonel McAdoo; and from Messrs. W. and A. F. McKinley, a general ethnological collection of great value. The accessions from Florida are quite numerous, but the most important consist of a series of implements and crania from the mounds near Sarasota, presented by Mr. J. G. Webb. Among these are broken fragments of skulls, completely silicified, and quite unique in this respect. Rev. J. Fowler, of New Brunswick, has supplied a valuable collection gathered in his vicinity. From Mexico we have received a collection of ancient vases of remarkable beauty, deposited by Mrs. General Alfred Gibbs; and another collection of a similar character, presented by the Natural History Museum of Mexico; as also some by Dr. Penatiel, one of its officers.
Mr. Riotte has furnished an interesting series of diminutive figures, dressed to represent the costumes of the aborigines of Guatemala.
Dr. Flint, of Nicaragua, has sent various specimens of aucient pottery: obtained near Omatope, and similar articles have been received from Dr. Vau Patten, obtained in Costa Rica.
From Peru the most interesting accessions are two mummies from a burial-place at Arica, accompanied by various articles, presented by Mr. Henry Meiggs, the well-known railway engineer of South America. From Brazil we have received a series of the bows and arrows used by the natives of that country, and presented by Mr. Albuquerque.
Among the most important additions to the collections should be mentioned a large number of Lacustrian implements from Switzerland, from Professor Pagenstecker, of Heidelberg, Mr. Messikomer, of Zurich, and Professor Rutimeyer, of Basle. The latter gentleman has also added an extensive series, properly identified and labeled, of the various kinds of domestic animals used by the builders of the lake dwellings.
An interesting collection was presented by Mr. di Cesnola, United States consul to Oyprus, embracing numerous specimens of pottery obtained by him in his excavations in the site of the ancient Idalium. Some of these are believed to be purely Phœenician in their character,
and others of a later date, all of them characterized by great beauty and size.
One of the most interesting additions to the department of ethnology is the cast of the Tanis stone, on which is a trilingual inscription recently obtained from some excavations made at Tanis, on the eastern or Pelusiac branch of the Nile, and belonging to the museum of Egyptian antiquities at Cairo. The original is a block six feet high, two and a half feet broad, and a foot thick, with the top arched. One side is occupied partly by hieroglyphic inscriptions, together with a Greek translation of the same, while a portion of the left side is occupied with an equivalent inscription in the Demotic character. This stone occupies a position in Egyptology similar to that of the "Rosetta stone," except that it is much more perfect, and will probably aid much in deciphering the hieroglyphics. The cast was taken by the instrumentality of Dr. Lansing for presentation to Monmouth College, Illipois, but at his request and that of Mr. S. H. Scudder, and by permission of the authorities of that college, it was sent to the Institution to be copied. Unfortunately, it was very much broken in the transit, and required patient labor on the part of a skillfal modeler to restore it to anything like its original condition. When this is accomplished a mold and casts will be taken, and the original sent to the college. In this connection we may mention that the inscriptions on the stone have been carefully studied by Dr. G. Seyffarth, an eminent Egyptologist, who visited Washington for the purpose, and will present a paper on the subject to the Institution, for publication.

Correspondence.-As we have said in previous reports, a very large amount of labor is devoted to correspondence. Beside those relating to the ordinary business of the establishment, hundreds of letters are received during the year containing inquiries on various subjects on which the writer desires information, and also many memoirs which are presented for publication. Among the former a large number are received from the five hundred meteorological observers who furnish, voluntarily, records of the weather, and who require frequent explanation of special phenomena. Among the papers submitted for publication are a large number containing speculations in reference to science which in many instances exhibit great industry and profound thought on the part of their authors, but which, nevertheless, cannot be considered as positive additions to knowledge founded on original research, and which, therefore, in accordance with the rules adopted by the Institution, cannot be accepted for publication. On account of the wide diffusion of elementary education in the United States, and the general taste for reading amongall classes, there is no other part of the world, perhaps, in which there exists a greater diffusion of elementary scientific knowledge, and, perhaps, more activity of mind directed in the line of scientific thought. Much, however, of this, from a want of proper training, and the means of experiment and observation to verify deductions from a priori con-
ceptions, is unproductive of positive results. The Institution does not discard antecedent speculations provided deductions from them are made in the form of new results which are verified by actual phenomena. It is not enough that a new hypothesis may give a general explanation of a class of phenomena in order that it may be adopted; it must do more than this. It must point out new facts and phenomena which can be readily exhibited by experiment or verified by observation. Such advances have been made in physical science within the last two hundred years that most of the phenomena which lie, as it were, on the face of nature, have been studied and referred to general principles. In order, therefore, to make advances, in general physics, at least, apparatus, as well as training in the use of it, is essential to scientific research ; and as but few, comparatively, possess the advantages of these, it rarely happens that investigations of much importance result from the speculations of the kind we have mentioned. In the line of mathematics, however, which requires no extraneous aid, and of natural history, in the study of which objects are everywhere presented, results of importance may be derived from the labors of isolated individuals who have no other assistance than books.

As a means of adult education, it may be remarked that from the first the Institution has encouraged the establishment of lyceums and scientific associations in all parts of the country, and as the number of these has coustantly increased, they have added to our correspondence, and much more largely during the past year than during any one in the history of the Institution.

Miscellaneous items.-In 1863 Congress incorporated an association, under the name of the National Academy of Sciences, which should investigate, examine, experiment, and report upon any subject of science or art on which information might be required by any department of Government. Though this society was in no way connected with the Smithsonian Institution in its inception and organizatipn, yet it is accommodated with rooms for its meetings in the Smithsonian building, and communications which are adopted by it are accepted for publication by the Institution.

A series of scientific inquiries has been referred to this society by different departments of Government, and the investigations in regard to them have principally been made under direction of members of the academy in this Institution. The organization of the scientific department of the North Polar Expedition under Captain Hall was intrusted by Congress to the National Academy, and the procuring of the instruments and the organization of the scientific corps were principally effected in connection with the Smithsonian Institution. A copy of the scientific instructions will be found in the appendix to this report.

In the law organizing the Light-House Board it is declared that it shall consist of two officers of the Army of high grade, two officers of the Navy, and two civilians of scientific reputation, whose services
might be at the disposal of the President of the United States, to gether with an officer of the Navy to act as naval secretary, and an officer of the Corps of Engineers of the Army, as engineer secretary. From the commencement of the board to the present time, the members from civil life have been the Superintendent of the Coast Survey and the Secretary of the Smithsonian Institution. During the whole period I have occupied the position of chairman of the committee on experiments, and have, with the exception of the summer I was in Europe, devoted my vacations to investigations relative to lighting-materials, fog-signals, and other duties connected with the light-house service. In October, 1871, on the retirement of Admiral Shubrick and the ordering of Admiral Jenkins to the charge of the East India squadron, I, being the oldest member, was elected chairman of the board. For the discharge of the duties of this position, in addition to the time of my summer vacation, I have mad/ arrangements for devoting one day in each week. It is proper to observe that my office as a member of the Light-House Board, although one of much responsibility, and to which I have, during the last eighteen years, devoted a large amount of labor, is accompanied with no salary, the expense of traveling and subsistence being.defrayed by an allowance of ten cents per mile.
The services which have been rendered to the Government by the Institution from its commencement to the present time are deserving of recognition. They include not only those connected with the National Academy, the Light-House Board, investigations now being carried on relative to fishes, the care of the Government collections, the organization of the natural history portions of the various exploring expeditions, the series of investigations made during the war, but also answers to the constant applications from members of Congress for information on special subjects. In no case has the Secretary or his assistants received any remuneration for labors thus performed.
In this connection I may mention that on the occasion of my visit to Europe in the summer of 1870 I was honored by the President of the United States with an appointment to represent this country at a meeting of an international commission, invited by the late Emperor of France, to consider the best means of multiplying copies for distribution of the original meter preserved in the archives of the government at Paris. Unfortunately, before the time of meeting arrived, in August, the Franco-German war commenced, preventing the attendance of a number of commissioners who would otherwise have been present. On this account it was resolved to permanently adopt no definite proposition in regard to the meter, but merely to discuss the various questions which might be connected with the general subject. The commission remained in session from the 8th to the 14th of August, and adjourned to meet again at a more favorable season.
The Institution has taken much interest in the historical phenomenon of the movementin Japan in regard to the adoption of western civilization.

A full set of the publications of the Institution has been presented to the University of Yedo, and arrangements made with itfor obtaining meteorological observations and specimens of archæology and natural history. A special request was made by the Institution in behalf of the Japanese Minister, Mr. Mori, of the principal publishers of school-books in the United States for such of their publications on education as they might see fit to present for examination to the Japanese commission. In response to this application acknowledgments are due, for liberal donations, to the following publishers: D. Appleton \& Co.; A. S. Barnes \& Co. ; Brewer \& Tileston; E. H. Butler \& Co.; Claxton, Remsen \& Haffelfinger; R. S. Davis \& Co. ; Eldredge \& Bro.; W. S. Fortescue ; Harper Bros. ; Holt \& Williams; Houghton \& Co.; Ivison, Blakeman, Taylor \& Co.; J. B. Lippincott \& Co.; Henry C. Lea; G. \& C. Merriam ; Murphy \& Co. ; Oakley, Mason \& Co. ; J. W. Schermerhorn \& Co. ; C. Scribner \& Co.; Sheldon \& Co. ; Sower, Barnes \& Potts; Thompson, Bigelow \& Brown; University Publishing Company; Wilson, Hinkle \& Co. ; Woolworth, Ainsworth \& Co.
While the Smithsonian Institution occupies ground otherwise uncultivated, it has been its policy from the begining to co-operate with all other institutions in advancing science and promoting education. There must always exist objects of importance for the promotion of which appropriations cannot be immediately obtained from Congress, and which, without aid, cannot be properly prosecuted, In England such objects to a limited extent are assisted by funds derived from the subscription list of members of the British Association, and by an annual grant from the government to the Rogal Society. These appropriations, though producing important results, are far from being adequate to the solution of problems, the number and variety of which are constantly increasing. When we consider the intimate connection of a knowledge of abstract science with modern civilization, the effect which it has had in substituting the powers of nature for slave labor, in the discovery of laws a knowledge of which enables man to predict, and in many cases to control, the future, it must le evide it that nothing can better mark the high intelligence of a people than the facilities which they afford and the means they provide for promoting investigatiors in this line. It is a matter of surprise, however, that so imperfectly is the importance of abstract science appreciated by the public generally, that unless it be immediately applied to some practical purpose in the arts it is almost entirely disregarded.

## NATIONAL MUSEUM.

An appropriation during the last two years has been made by Congress of $\$ 20,000$ for the reconstruction of parts of the building destroyed by the fire, and the fitting up of rooms for the better accommodation of the National Museum. This sum, together with about $\$ 9,000$ from the
income of the Smithsonian fund, has been devoted during the past year to this purpose.

With a view to the ultimate separation of the operations of the Smithsonian Institution from the National Museum, arrangements have been made for appropriating the east wing and range to the business - which may be considered as belonging exclusively to the essential objects of the Institution, and devoting the main building, west wing, and towers to the museum. For this purpose the large room on the first floor of the east ewing, which was formerly used as a museumlaboratory and store-room, has been fitted up with bins and conveniences for assorting and packing the literary and scientific exchanges to be sent to foreign countries. Preparation has also been made for removing the chemical laboratory from the first floor of the east range to the space immediately below it in the basement, and for applying the whole of the first floor of this part of the building to the business offices of the Secretary and his assistants in the line of what are called the active operations.

For the special accommodation of the museum the large room in the west wing, formerly occupied by the library, has been prepared for the reception of cases for mineralogical and geological specimens; while the great hall, 200 feet by 50, in the second story of the main building, has been completed and is now ready to receive the cases for the anthropological and other specimens.

Estimates are now before Congress for fitting up these rooms with cases for the reception and display of the Government collections; and it is hoped that, in the next report, we shall be able to chronicle the commencement, if not the completion, of the work.

The changes consequent upon the extension of the museum mentioned made a re-arrangement necessary of the greater part of the basement so as to obtain additional security against fire, and greater convenience for the storage of fuel, packing-boxes, and specimens. A floor was laid through the basement, and new passage-ways opened, furnishing better access from one extreme of the building to the other. In introducing the fireproof floor into the west wing, advantage was taken of the opportunity to increase the height of the room below it, and to convert it and the adjoining rooms in the west range into laboratories and store-rooms for natural history.

Furthermore, for better security, the fire-proofing of the floors of the four towers on the corners of the main building has been commenced. The rooms in the towers furnish studies and dormitories for the investigators in the line of natural history who resort to the Institution, especially during the winter, to enjoy the use of the library and the collections for special researches.

The Norman style of architecture adopted for the Smithsonian building produces a picturesque effect, and, on this account, the edifice has been much admired. It is, howerer, as I hsve frequently before
remarked, one of the most expensive buildings in proportion to its interior capacity which could have well been derised ; expensive not only in its first construction, but also in the repairs which are continually required to protect it from the influence of the weather, which is obvious when the number of projections, towers, and exposed angles is considered.
The building, which from the first has been a drain on the Smithson funds, still requires an appropriation for leating-apparatus, and for annual repairs, which, in justice to the bequest, we trust will be provided by Congress.
For defraying the expenses of the care and exhibition of the National Museum, Congress has annually, for the last two years, appropriated $\$ 10,000$. Although this appropriation was more than double that of previous years, still it fell short of the actual expenditure. The amount of items chargeable to the museum during the past year, independent of the rent which might have been charged for the rooms occupied, or for repairs of the building, was a little more than $\$ 13,000$. Deducting from this sum the $\$ 10,000$ appropriated by Congress, and there reremains $\$ 3,000$, which was paid from the income of the Smithson fund.
A statement of this deficiency has been presented to Congress, and we trust that the sum of $\$ 15,000$ will be appropriated for the same purpose for the ensuing fiscal year.
By the completion of the large room in the second story and the appropriation of the west wing and connecting range to the same purpose, the space allotted to the museum in the Smithson building has been increased to about threefold. It is proposed, as was stated in the last report, to devote the room in the west wing to specimens of geology and mineralogy, and the large room in the second story to specimens of archæology and palæontology. As preparatory to the fitting up of these rooms, a series of designs has been prepared at the expense of the Institution by B. Waterhouse Hawkins, the well-known restorer of the ancient animals which illustrate the palæontology of the Sydenham Palace, near London.
A commencement has also been made in the furnishing of the large room with casts of some of the larger extinct animals.
The cast of a skeleton of the Megatherium cuvieri, generously presented by Professor H. A. Ward, of Rochester, has been set up in the middle of the room. This gigantic fossil was first made known to the scientific world in 1789. It was discovered on the banks of the river Luxan, near the city of Buenos Ayres, and was subsequently transmitted to Madrid. The original bones, of which this specimen is a copy, were found in the same Pampean deposit, between the years 1831 and 1838, and belong partly to the Hunterian Museum of the Royal College of Surgeons, and partly to the British Museum. Cuvier, who gave it its generic title, thonght it combined the character of the sloth,
ant-eater, and armadillo. Professor Owen has, however, shown that the Megatherium was a "ground-sloth," feeding on the foliage of trees, which it uprooted by its great strength. The extreme length of the mounted skeleton is 17 feet; its height from the pedestal to the top of the spinous process of the first dorsal vertebra is 10 feet 6 inches. The length of the skull is 30 inches; the circumference of the skeleton at the eighth rib is 11 feet.
Also in association with the Mcgatheriun a cast has been placed in the same room of the Colossochelys atlas, a gigantic tortoise, a restoration from fragments discovered in the Miocene strata of the Sewalik Hills, India, and now in the museum of the Asiatic Society of Bengal. It is 8 feet 2 inches in length by 5 feet 10 in width.
In additiou to this, there has been set up a cast of the Glyptodon, a representative in Pleistocene times of the armadillos of South America, the original of which was found in 1846, near Montevideo, on the banks of the Luxan. It was presenfed by order of the Dictator Rosas to Vice-Admiral Dupolet, who gav it to the museum of his native city, Dijon, France, where it is still preserved.
The two last-mentioned specimens were purchased from Professor Ward.

The basis of the national museum is the collection of specimens of the United States exploring expedition under Captain, now Admiral, Wilkes, originally deposited in the Patent Office. It was transferred to the Institution in 1858, and since then has been very much increased by the type specimens from upward of fifty subsequent expeditions of the General Government, and contributions resulting from the operations of the institution. The character of the museum will be properly exhibited for the first time after the various articles are displayed in the new roous now in preparation for their reception. The museum is especially rich in specinens to illustrate the subject of anthropology; and it is proposed to bring these as far as possible together in the new room in the second story, and to arrange them so as to exhibit their connection and to illustrate the gradual progress of the development of the arts of civilized life.
At present a portion of the large room in the second story is used for the exhibition of the cartoons or original sketches made by the celebrated Indian traveler and explorer Mr. George Oatlin. The object of this exhibition is to induce the Government to purchase the whole collection of Indian paintings, including sketches and portraits, the result of the labors of upward of forty years of this enthusiastic and indefatigable student of Indian life. The entire collection, which comprises about twelve hundred paintings and sketches, was offered by Mr. Catlin to the Government in 1846, and its purchase was advocated by Mr. Webster, Mr. Poinsett, General Cass, and other statesmen, as well as by the principal artists and scholars of the country. A report recommending its purchase was made by the Joint Committee on the

Library of Congress, but, owing to the absorption of public attention by the Mexican war, no appropriation was made for the purpose. Mr. Catlin made no further efforts at the time, but exhibited his pictures in Europe, where, on account of an unfortunate speculation into which he was led in London, claims were brought against them which he had not the means to satisfy. At this crisis, fortunately, Mr. Joseph Harrison, of Philadelphia, a gentleman of wealth and patriotism, desiring to save the collection for our country, advanced the means for paying off the claims against the pictures and shipped them to Philadelphia, where they have since remained unredeemed. Mr. Catlin, however, retained possession of the cartoons, and has since enriched them with a large number of illustrations of the ethnology of Sonth America. Whatever may be thought of these paintings from an artistic point of view, they are certainly of great value as faithful representations of the person, features, manners, customs, implements, superstitions, festivals, and everything which relates to the ethnological characteristics of the primitive inhabitants of our country. We think that there is a general public sentiment in favor of granting the moderate appropriation asked for by Mr. Catlin, and we trust that Congress will not fail at the next session to act in accordance with this feeling. It is the only general collection of the kind in existence, and any one who has given thought to the subject cannot but sympathize with Mr. Oatlin, who, in his old age and after a life of hard labor and the devotion of all that ho possessed in the world to its formation, is now anxious to obtain the means to redeem the portion of his collection retained as security for the payment of claims against him, for the means to enable him to finish the sketches that are still incomplete, and to secure the whole from dispersion through their purchase by the Government.
Respectfully submitted.
Washington, January, 1872.

## APPENDIX T0 THE REPORT OF THE SECRETARY.

Table showing the entries in the record-books of the Smithsonian Museum at the end of the years 1870 and 1871.

| Class. | Up to the end of 1870 . | Up to the end of 1871 . |
| :---: | :---: | :---: |
| Skeletons and skulls. | 11,512 | 12, 059 |
| Mammals. | 9,773 | 9,849 |
| Birds . | 61,150 | 61,250 |
| Reptiles.. | 7,535 | 7,536 |
| Fishes.. | 7,897 | 7,983 |
| Eggs .-.... | 15,671 | 15,986 |
| Crustaceans | 1,287 | 1,287 |
| Mollusks | 22,345 | 24,792 |
| Radiates | 2,730 | 2,730 |
| Annelids. | 100 | 100 |
| Fossils | 7,380 | 7,697 |
| Minerals. | 7,154 | 7,160 |
| Ethnological specimens | 10,000 | 10,931 |
| Plants. | 175 | 390 |
| Total | 164,709 | 169, 750 |

Total entries during the year.
5, 041
Approximate table of distribution of duplicate specimens to the end of 1871.

| Class. | Distribution to the end of 1870 . |  | Distribution in 1871. |  | Total. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Species. | Specimens. | Species. | Specimens. | Species. | Specimens. |
| Skeletons and skulls. | 214 | 671 | 111 | 156 | 325 | 827 |
| Mammals ........... | 916 | 1,782 | 25 | 40 |  | 1,822 |
| Birds. | 22, 530 | 34, 951 | 410 | 477 | 22,940 | 35, 428 |
| Reptiles ............. | 1,741 | 2,870 | 100 | 100 | 1,841 | 2,970 |
| Fishes. | 2,435 | 5,211 | 42 | 100 | 2,477 | 5,311 |
| Eggs of birds.... | 6,455 | 16,394 | 151 | 304 | 6, 606 | 16,698 |
| Shells . | 81, 178 | 183, 157 | 2,534 | 3, 000 | 83, 712 | 186,157 |
| Radiates ............. | 583 | 778 |  | ........... | 583 | 778 |
| Crustaceans. | 1,078 | 2,650 |  |  | 1,078 | 2,650 |
| Marine invertebrates. | 1,838 | 5,152 |  |  | 1,838 | 5,158 |
| Plants and packages of seerls. | 15, 03 | 21, 063 | 3, 000 | 4,000 | 18, 503 | 25, 063 |
| Fossils............... | 3, 958 | 9,984 | 151 | 151 | 4,109 | 10, 135 |
| Minerals and rocks.. | 3,630 | 8,574 | 1,000 | 1,400 | 4,630 | 9,974 |
| Ethnological speci- mens............... | 1,143 | 1,190 | 152 | 152 | 1,295 | 1,342 |
| Insects. ............. | $\begin{array}{r}1,632 \\ \hline 28\end{array}$ | 2, 9646 | 204 1 | - 204 | 1,836 89 | 3, 623 |
| Total .......... | 144, 862 | 297, 941 | 7,881 | 10,139 | 152, 743 | 308, 480 |

## ADDITIONS TO THE COLLECTIONS OF THE SMITHSONIAN INSTITUTION IN 1871.

## Agricultural Department.-(See Mechling.)

Albuquerque, F., Rio Grande do Sul, Brazil.-Bow and arrows of South American Indians.
Allard, C. T., Parkinson's Landing, Ilinois.-Micaceous slate and copper pyrites, Illinois.

Alvarado, Sr. Don. J. J.--Specimen of stalactite, from Costa Rica.
Andrew, G., Knoxville, Tennessee.-Indian relics and shells, from Tennessee.
Army Medical Museum, Washington, D. C.-Ethnological specimens from Arizona and Colorado. (See also Irwin, Dr. B. J.D.; Weeds, Dr. J. F.; Otis, Dr. G. A.; and White, Dr. C. B.)

Arny, Hon. W. M .F.-Ethnological specimens, from New Mexico.
Baird, Professor S. s.-Forty-seven boxes general collections, Wood's Hole, Massachusetts.

Baird, Mrs. S. F., Washington, D. C.-Tire-bag of Indians of Hudson Bay Territory; skeleton of domestic tarkey, Washington, D. C.

Beardslee, Com. I. A.-Young flying-fishes in alcohol, Átlantic Ocean.
Bergen Museum, Bergen, Norway.-Box of natural-history collections.
Berthoud, E. L., Golden City, Colorado.-Indian relics \&c., from Crow Creek, Colorado.
Billings, E., Montreal, Canada.-Specimens of Eozoon canadense and cast of trilobite, from Canada.
Bland, Thomas, New York.-Box of shells.
Bliss, B. K. \& Co., New York.-Palmetto fiber, from South Carolina.
Boardman, G: A., Calais, Maine. $/-$ Specimens of birds, fishes, and skeletons, from Florida; skeletons $0 /$ moose, from Maine.

Boardman Charles A., and S. W. Smith.-Skin of moose, from Nova Scotia.
Bree, Dr. C. R.-Eggs of Larus gelastes, from Kustridge Turkey.
Brewster, C. G., Boston, Massachusetts.-Specimens of birds.
Brittan, H., Thayer, Kansas.-Box of Permian fossils.
Bryant, Captain Charles.-Skulls, skeletons, and skins of fur-seal, and walrus, and one box dried plants, from Saint Paul Island, Behring Sea.

Burr, C. S., Alliance, Ohio.-Box of fossil plants.
Burroughs, John, Washington, D. O.-Nest and egg of Dendroica coorules. cens, from Delaware County, New York.
Burrows, Mrs.-German horn, and small shoes made at Saint Helena.
Butcher, M., Prince Edward Island.-Stone axe. (Sent through Rev.
J. Fowler.)

Carpenter, Dr. P. P.-Box of sheils from west coast of North America.
Carpenter, W L., Mill Creek, Wyoming Territory.-Larva of insect (borer) in wood.

Cesnola, General L. P. di, United States Consul.-Ancient Phoenician pottery, from the site of the ancient Idalium, Island of Oyprus.

Chalmers, R., Konchibougonack, New Brunswich.-Arrrow-heads. (Sent by Rev. J. Fowler.)

Choate, Isaac B., Gorkam, Maine.-Specimens of minerals, ancient pottery and arrow-heads, \&c.

Christ, R. Nazareth; Pennsylvania.-Birds' eggs, from various localities.
Clarke, John, Bowling Green, Ohio.-Indian stone relics from Ohio.
Clarke, W. H., Washington, D. C.—Alcoholic collections of fishes, reptiles, and invertebrates from the Isthmus of Darien.

Clough, A., Fort Reynolds, Colorado.-Box of specimens of uatural history from Colorado.

Colonial Museum, Wellington, New Zealand, (Dr. J. Hector.)-Caists of eggs of Dinornis and Apteryx; and ethuological specimens.

Constable, Major A. G.-Skeleton of mouse.
Cortelyou, J. Gardner, Somerset County, New Jersey.-Indiąn stone implements.

Coues, Dr. Elliott, United States A/my.-Four specimens of albino birds.

Crane, E. H:, Burr Oak, Michigan.-Insects and small batrachian.
Curtis, Dr. Joseph.-Oolite from Florida, and Eozoon canadense in chelmsfordite, Chelmsford, Massachusetts.

Curtis, Rev. M. A., Hillsborough, North Carolina.-Specimen of Menopoma alleghaniense.

Darling, Major, United States Army.-Specimen of pedunculated cirrhiped in alcohol.

Davidson, Professor George.-Specimens of woods from Alaska.
De Castro, Diego.-Specimen of six-legged cat.
Destruge, A., Guayaquil, Ecuador.-Skeleton of Bradypus tridactylus.
Diokinson, E., Springfield, Massachusetts.-Birds' eggs from Springfield, Massachusetts.

Doane, Lieutenant G. C., United States Army.-Box of minerals, \&c., from Yellowstone Lake, Montana Territory.

Dodge, General.-Specimen of oolitic limestone, Oxford, Tama County, Iowa.

Dodge, S. C., Chattanooga, Tennessee.-Stone axe from Lookout Mountain, Tenuessee.

Dodt, Colonel Helenus, (through Dr. E. Palmer.)-"Helma," or workbag of Mohave Indians, Arizona.

Driver, G. W., Washington, D. C.-Specimen of Echeneis from Lower Potomac.

Dunn, A., Salmon River, New Brunswick.-Stone axe and chisel. (Sent through Rev. J. Fowler.)

Dyer, Joseph T., Washington, D. C.-Ethnological specimens, dresses, \&c., Alaska.

Eby, J. W., Indian Bureau.-Minerals and photographs;, Utah.

Edmunds, Mrs. Geo. F., Washington, D. C.-Thirty-one specimens tropical birds.
Edwards, W. H., Coalburgh, West Virginia.-Box of bird-skins.
Emmet, Dr. T. A., New York.-Box of bird-skins from Central America.
Filer, O. L., New Harmony, Utah.-Indian stone arrow head.
Fithiam, Thomas, United States consul.-Book perforated by ants, Saint Helena.
Fish, William C., East Harwich, Massachusetts.-Flint chips and arrow heads.
Fisher, Professor D., United States Naval Academy.-Shells in alcohol from Milwaukee, Wisconsin.
Fisher, J., Lexington, Kentucky.-Ethnological specimens, copper and stone, from mounds near Lexington, Kentucky.
Flint, Earl, Granada, Nicaragua.-Box of seeds and ethnological specimens, Ometepec Island, Nicarauga.
Floyd, General T. C., Georgia, Heirs of.-Indian stone implements, \&c. Ford, T. S., Columbia, Mississippi.-Stone hatchet from Mississippi.
Fowler, Rev. J., Bass River, New Brunswick.-Indian relies and shells from Nova Scotia and New Brunswick.
Fuller, J. F., Salado, Texas.-Specimen of arrow-head from Texas.
Furnas, R. W., Brownville, Nebraska.-Specimen of radiating fibrous gypsum.

Gentry, J. P., Paducah, Kentucky.-Specimen of clay.
Gibbons, J. S., Lewes, Delaware.-Section of pine trunk bored by teredo.

Gibbs, Mrs. Alfrea, New York.-Ethnological specimens. (Deposited.)
Gibbs, George, New York.-Box of Indian relics, California. Ethnological specimens from northwest coast.

Gibson, Colonel G., United States Army.-Skeleton of buffalo, Fort Hayes, Kansas.
Glasco, J. M., Gilmer, Texas.-Specimens of Indian pottery.
Goeller, C. L., Millealgeville, Georgia.-Specimen of supposed tin ore, Jefferson County, Tennessee.
Green, H. A., Ateo, New Jersey.-Specimens of fossils and minerals from New Jersey.
Green, H. N., Boston Station, Kentucky.-Weathered fossils from Kentucky.
Greer, Colonel James, Dayton, Ohio.-Artesian borings, Indian stone implements, and specimen of meteorite, from Ohio.
Gundlach, Dr. J., Havama.-Specimen of Solenodon enbanus in alcohol.
Gurley, William, Danville, Illinois.-Box of fresh-water shells from Central Illinois.
Hague, Henry.-Skeleton of tapir and box of natural history collections from Guatemala.
Hall, Captain C. F.-Collection of relics of Franklin and Frobisher expeditions, and ethuological specimens from Arctic America.

Hancock, E. M., Waukon, Iowa.-Box of minerals, fossils, aud natural history collections.

Hayden, Dr. F. V. United States Geologist.-Extensive general collections in geology, ethnology, and natural history, from the western Territories, ( 45 boxes.)

Hayes, Dr. I. I.: Philadelphia, Pennsylvania.-Bird-skins from Greenland.

Heiligbrodt, L., Austin, Texas.-Bird's eggs, and Indian arrow-heads.
Hemphill, H., Oakland, California.-Box of shells from California.
Henry, Professor Joseph.-Diatoms, \&c., from hot springs of California.

Hershey, David, Spring Garden, Pennsylvania.-Prismatic quartz crys. tal.

Hilgert, Henry, Santa Fé, New Mexico.-Nest of swallows from Albaquerque, New Mexico.

Hough, F. B., Louville, New York.-Box of birds' nests and eggs from Northern New York.

Hotchkiss, Mr., Shreveport, Louisiana.-Flint implements, pottery, \&c., from near Shreveport.

Huggins, Lieutenant.-Skeleton of Callorhinus ursinus, Alaska.
Hurlburt, General S. A., United States minister to Neu Granada.-Skins and skeletons of mountain tapir, Tolima, New Granada.

Irwin, Dr. B. J. D., United States Army, Fort Wayne, Michigair-Box of alcoholic vertebrates, Indian relics, \&c., from Arizona. (Through Army Medical Museum.)

James, U. P., Cincinnati, Ohio.-Lower Silurian fossils, (46 species,) from Ohio.

Jeffreys, J. Gwyn, London, England.-Brachiopods from the North Atlantic.

Jones, Dr. Joseph, New Orleans, Louisiana.-Specimen of prepared wood.

Jones, Strachan, Goderich, Canada.-Box of birds' nests and eggs from Lesser Slave Lake, Hudson Bay Territory.

June, L. W., Wellington, Ohio.-Indian stone relics from Ohio.
Keenan, T. J. R., Broolhaven, Mississippi.-Two boxes ethnological and natural history specimens.

Kidder, Dr. F., Leesburgh, F'lorida.-Specimens of pearl-bearing unios.
Knudsen, Valdimar, Kanui, Hawaiian 1slands.-Skulls of ancient Sandwich Islanders.

Lesher, W. T., Youngromanstown, Pennsylvania.-Indian arrow-heads, \&c.

Lewis, George H., Atlantic City, Montana Territory.-Fragment of fossil turtle.

Limpert, W. J., Groveport, Ohio.-Specimen of Sphyropicus varius.
Luce, Jason, West Tisbury, Massachusetts.-Specimens of rare fishes from Martha's Vinejard.

Macintosh, I., Welford, New Brunswick.-Arrow-heads. (Sent by Rev. J. Fowler.)

Mactier, W. L., Philadelphia, Pennsylvania.-Eggs of Bulimus hoemastoma.
Maguire, J. C., Washington, D. C.-Indian slate hatchet. (Deposited.)
Manzano, Dr. D. J., (through Dr. A. Schott.)-Human skull carved in fossil wood from Yucatan.

Mathews, Dr. Washington, United States Army.-Eggs of Archibuteo ferrugineus, with head, wings, and feet of parent, from Dakota Territory.

MoAdoo, W. G.-Stone disc from East Tennessee.
McCoy, John, Black River, New Brunswick.-Arrow-heads. (Sent by Rev. J. Fowler.)
McKinley, W. and A. T., Milledgeville, Georgia.-Box of flint implements and ancient pottery, Oconee River, Georgia.
McMinn, Mrs. J.-Twenty-six boxes geological, mineralogical, and botanical specimens, the collections of the late James McMinn.
McNaughton, R., Mumford, New York.-Calcareous tufa from Monroe County, New York.
Mechling, Mrs. F. E. D., (through Agricultural Department.)-Specimens of reptiles, fishes, birds, \&cc., from Belize, British Honduras.

Meiggs, Henry, Lima, Peru.-Two boxes Peruvian mummies.
Meigs, General M. C., Quartermaster General United States Army.Skin of Phoca pealii, from Alaska, and Indian relies from Montana; minerals Galena, flluor spari, \&c.) from Rosiclare, Illinois.
Merriam, C. Haxt, White Plains, New York.-Birds' eggs and nests from New York.

Merritt,J. C., Farmingdale, Nero York.-Arrow-heads from Long Island, New York.
Miller, F., West Farmington, Ohio.-Box of fossils.
Miller, J. Imbrie.-Splinter of calcined wood, Oogun Camp, Central India.
Miller, S. A., Cincinnati, Ohio.-Fossil wood, Lower Silurian fossils, and Indian relics from Ohio.
Morrison, E. H., Newark, New Jersey.-South African birds' eggs.
Munn, Dr. C. E., United States Army.-Package of diatoms from Fort Wadsworth, Dakota Territory.
Museo Publico, Buenos Ayres.-Box of birds, mammals, \&c.; from the Argentine Republic.
National Museum of Mexico.-Ancient pottery from Mexico.
Orton, Professor Edward, Yellowo Springs, Ohio.-Box of fossils. from Ohio.

Otis, Dr. G. A., Army Medical Museum.—Painted scapula of Buffalo.
Packard, Dr. A. S., Salem, Massachusetts.-Eggs of fish from Salem Harbor.

Pagenstecker, Professor, Heidelberg.-Box of Swiss pre-historic relics from Lake D wellings.

Palmer, Dr. E., Washington, D. C.-Seven boxes and one bale generai collections from Arizona; two boxes skulls of cetaceans from Wellfleet, Massachusetts.

Penafiel, Dr. Antonio, City of Mexico.-Ancient pottery from Mexico.
Pence, J. B., Frankfort, Indiana.-Meteoric dust from surface of snow.
Peter, Dr. R., Lexington, Kentucky.-Indian stone relics from Kentucky.
Peter's, Henry, New Smyrna, Florida.-Eggs of Ortyx virginianus.
Putton, W. T., New York.-Creosotized wood from New York Creosotiz. ing Works, 157 Broadway.

Poey, Professor Felipe, Havana.-Skeleton of Solénodon cubanus.
Pourtales, Count L. F. De.-Series of brachiopods from deep-sea dredgings in Gulf Stream.

Powell, Mr. Joseph; United States consul, Port Stanley.-Horn of wild ox from Falkland Islands.

Powell, Major J. W., Normal, Illinois.-Two boxes and one bale of Ute clothing and implements, Colorado.

Ridgway R.-Birds and reptiles from Mount Carmel, Illinois.
Ring, Lieutenant F. M., United States Army.-Two boxes Indian relics from Alaska.

Riotte, Sr. Pedro.-Twenty-seven dressed figures made by Indians of Guatemala, and representing native costumes of that country.

Rutimeyer, Professor.-Lacustrine antiquities, bones, \&c., Switzerland.
Salt Lake Museum.-Two boxes minerals, fossils, and ethnological specimens, Utah.

Salvin, O., and Sclater, P. L., London.-Specimens wf birds from Veragua, Columbia.

Sartorius, Dr. C., Huatasco, Mexico.-Box of specimens of natural history: box of living plants from Mexico.

Scammon, Captain C. M., United States Revenue Marine.-Nondescript baleen and parasites from cetaceans, North Pacific; baleen of humpback; skull and baleen of small whale from Puget Sound; general collections from Northwest coast.

Schenck, Dr: J., Mount Carmel, Illinois.-Specimen of salamander from Southern Illinois.

Schott, Dr. A., Georgetown, D. C.-Two arrows of Papago Indians of Sonora.

Schlucker, P. F., Baltimore.-Specimen of asbestos from Maryland.
Schuber, $N$., Panama.-Head of Peruvian mummy and specimens of ancient pottery from Peru.

Scott, Genio C., New York.-Fishes preserved in ice. (Cybium caballa.)
Scroggins, S. R., Baltimore, Maryland.-Specimens of fish. (Megalops thrissoides.)

Sears, Joseph C., Elast Dennis, Massachusetts.-Indian grooved stone pestle.

Schaffer, D. M., Cincinnati, Ohio.-Lower Silurian fossils from Ohio.
Shirley, James, Welford, Kent County, New Brunswick.-Stone chisel. (sent by Rev. James Fowler.)

Smith, H. H., San Irancisco, California.-Seed vessels of lily.
Spear, Dr., United States Canal survey of the Isthmus of Tehuantepeo.Three boxes of general collections, Tehuantepec.

Squier, E. G., Nevo Yorf.--Specimens of pottery from near Lima, Peru.
Stearns, R. E. O., Petaluma, Ealifornia.-Box of birds' nests and eggs, \&c.

Stephens, T. H., Jacksonville, Florida.-Skull of alligator and skins of gars, Florida.

Sterling, Dr. E., Cleveland, Ohio.-Cast of roe of muskelonge from Saginaw River, Michigan; casts of fresh-water ish.

Sternberg, C. M., Fort Harker, Kansas.-Skeleton of buffalo.
Sumichrast, Dr. F.-Two boxes natural history specimens from Mexico.
Taylor, George, Washington, D. C.-Head of Rhynehops nigra, Саре May, New Jersey.

Taylor, Isaac H., Boston, Massachusetts.-One box skulls, South African mammals. (Through G. S. Boardman.)

Thompson, Rev. D., Milnersville, Ohio.-Box of ethnological specimens, fossils, \&c.

Thompson, J. H., New Bedford, Massachusetts.-Box containing three fish.

Tilton, B. M., Chilmark, Massachusetts.-Specimen of Blepharis, in alcohol.

Treat, Mrs. M., Vineland, New Jersey.-Specimen of living serpent.
Turner, Incian, Mount Carmel, Illinois.-Fishes from Southern Illinois.
Turner, Samuel, Mount Carmel, Illinois.-Birds from Wabash County, Illinois.

University of Christiania.-Sparagmite from Norway.
University of Louisiana, Baton Rouge.-Two boxes of Indian stone relics. (Deposited.)

Van Patten, Dr.-Ancient pottery from Costa Rica.
Vaux, William S., Philadelphia.-Ethnological specimens, casts, \&c.
Verstenikoff, A., Saint Paul Island, Alaska Territory.—Skull of fox.
Vortisch, Rev. L.-Ethnological specimens, Satow, Germany.
Wallace, President D. A., Monmouth College, Illinois.-Cast of inscription faces of the Tanis stone, received from Dr. Lansing, Alexandria, Egypt.

Wallace, John.-Specimen of musk-deer in the flesh; skull of giraffe.
Ward, Professor H. A., Rochester, New York.-Casts of megatherium, glyptodon, and colossochelys.

Webb, J. G., Sarasota Bay, Florida.-Box of ethnological and natural history collections.

Webster, Professor H. E., Schenectady, New York.-Box of marine invertebrates, \&c.

Weeds, Dr.J.F.-Ethnological specimens from New Mexico. (Through Army Medical Museum.)

White, Dr. O. B.-Specimen of Podiceps cornutus from Fort Schuyler, New York. (Through Army Medical Museum.)

Wilson, L., Astoria, Oregon.-Specimen of beetles in alcohol.
Wright, J. W. A., Turlock, California.—Arrow-heads from San Joaquin Valley, California.
Yager, W. E., Oneonta, Nero York.-Reptiles in carbolic-acid solution.
Yarrow, Dr. H. C., Fort Macon, North Carolina.-Specimens of fish, cetaceans, and Indian relics from North Carolina.

Yates, Dr. L. F.-Human cranium and box of pine cones from California.

Zeledon, José C., Washington, D. O.-Twelve card photographs of Indians of Guatemala; miniature carvings by the same.

Unknowon.-Box of corals, \&c.; specimen of symplocarpus, Whatcom, Washington Territory; specimen of dark marble, Jefferson County, West Virginia; specimens of fish.

## LITERARY AND SCIENTIFIC EXCHANGES.

Table showing the statistics of the Smithsonian exchanges in 1871.

| Agent and country. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Royal Swedish Academy of Sciences, Stockholm: <br> Sweden ....... ........................... | 18 | 41 | 3 | 24 | 900 |
| Royal University of Norway, Christiania: Norway. | 22 | 39 | 2 | 16 | 600 |
| Royal Danish Society of Sciences, Copenhagen: <br> Denmark <br> Iceland | 25 1 | 44 2 | 2 $-\cdots$ | 16 | 600 |
| Total | 26 | 46 | ...... |  |  |
| L. Watkins \& Co., Saint Petersburg: Russia $\qquad$ | 93 | 160 | 4 | 32 | 1,200 |
| Frederick MUller, Amsterdam : <br> Holland <br> Belgium | 52 95 | 93 105 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 8 16 | 300 600 |
| Total | 147 | 198 | .-.. |  |  |
| Dr. Felix Fltgarl, Leipsio: Germany Iceland Switzerland | 145 1 46 | 477 1 64 | 28 <br>  <br> 2 | 224 16 | $\begin{array}{r}8,400 \\ \hline 600\end{array}$ |
| Total | 492 | 542 |  |  |  |
| Gustave Bossange, Paris: France | 132 | 147 | 6 | 48 | 1,800 |
| Reale Istituto Lombardi di Scienze e Lettere, Milan: <br> Italy ....................................... | 109 | 120 | 8 | 64 | 2,400 |
| Royal Academy of Sotences, Iisbon : Portagal. | 19 | 20 | 1 | 8 | 300 |
| Royal Academy of Sciences of Madrid: Spain. | 7 | 9 | 1 | 8 | 300 |
| Wrleam Wesley, London: Great Britain and Ireland... | 252 | 332 | 23 | 184 | 6,900 |
| Untrersity of Melbourne : Australia | 18 | 20 | 1 | 8 | 300 |
| Pardiamentary Library, Wellington : New Zealand. | 7 | 8 | 1 | 8 | 300 |
| Rest of the world | 90 | 96 | 23 | 92 | 3,450 |
| Grand total . | 1,432 | 1,778 | 108 | 772 | 28,950 |

Packages received by the Smithsonian Institution from parties in America, for foreign distribution, in 1871.

| Address. |  | Address. |  |
| :---: | :---: | :---: | :---: |
| ALBANY, NEW YORK. |  | IOWA CITY, IOWA. |  |
| Albany Institute | 1 | Professor G. Hinrichs | 57 |
| New York State Library | 124 | Dr. C. A. White.... | 34 |
| Professor James Hall BOGOTA, COLOMBIA. | 41 | JANESVILIER, WISCONSIN. |  |
| Society of Naturalists...............- | 36 | Wisconsin Institution for Educating the Blind | 16 |
| BOSTON, MASSACHUSETTS. |  | KEYTESVILLE, MISSOURI. |  |
| American Academy of Arts and Sciences.. | 163 | John C. Veatch. | 1 |
| Board of State Charities. | 228 | LIBERTY, VIrginia. |  |
| Boston Society of Natural History-- | 296 |  |  |
| Massachusetts Historical Society .. | 1 | A. H. Curtis.. | 3 |
| Perkins Institution for Blind...... | 95 |  |  |
| Mrs. Julia Ward Howe. | 140 | MONTREAL, CANADA. |  |
| BROOKLYN, NEW YORK. |  | Natural History Society ............ | 22 |
| S. S. Cutting | 1 | E. Billings .............................. | 1 |
| burlington, new jersey. |  | NEW BEDFORD, MASSACHUSETTS. |  |
| W. G. Binney | 3 | J. H. Thomson | 2 |
| CAMBRIDGE, MASSACHUSETTS. |  | NEW HAVEN, CONNECTICUT. |  |
| Museum of Comparative Zoolog | 1,345 | American Journal of Science and |  |
| Professor Asa Gray | 3 | Arts ..-............................ | 24 |
| Count L. F. Pourtales. | 1 | Connecticut Academy of Arts and |  |
| Professor J. D. Whitney ............ | 15 | Sciences. | 175 |
|  |  | Professor J. D. Dana |  |
| COLUMBUS, OHIO. |  | S. J. Smith. | 16 |
|  |  | Professor A. E. Verrill | 37 |
| Ohio State Board of Agriculture...n- | 227 | NEWPORT, VERMONT. |  |
| DORCHESTER, MASSACHUSETTS. |  | Orleans County Society of Natural |  |
| Dr. E. Jarvis | 43 | Sciences...................... ....... | 137 |
| FORT M'HENRY, MARYLAND. |  | NEW YORK, NEW YORK. |  |
| Dr. Elliott Coues | 1 | American Institute .................. | 8 |
| FOUNTAINDALE, ILLINOIS. |  | Anthropological Institute of New York | 300 |
|  |  | Argentine consul ................... | 40 |
| M. S. Bebb | 2 | Lyceum of Natural History....... | 174 |
|  |  | J. Maunsell Schieffelin.............. | 500 |
| GEORGETOWN, DIST. OF COLUMBIA. |  | OXFORD, MISSISSIPPI. |  |
| Georgetown College ... . . . . . . . . . . - | 1 | E. W. Hilgard.................... .-. | 3 |
| INDLANAPOLIE, INDIANA. |  | PAXTON, ILlinois. |  |
| Indiana Institute for Educating the <br> Deaf and Dumb. | 39 | T. N. Hasselquint. | 1 |

Paokages received from parties in America, \&ec.-Continued.

| Address. |  | Address. |  |
| :---: | :---: | :---: | :---: |
| PEORIA, ILLINOIS. |  | san francisco, californta. |  |
| Dr. F. Brendel. | 2 | R. E. C. Stearns | 2 |
| PhELadELPHLA, PENNSXLVANTA. |  | sprivgrield, ulinois. |  |
| Academy of Natural Sciences | 178 | A. H. Worthen. | 25 |
| Ammerican Philosophical Society ..... | 291 | SPRLNGFIELD, MASSACHUSETTS. |  |
| Director of the Mint. <br> House of Refuge | 6 1 |  |  |
| Wagner Free Institute of Sciences.. | 264 | S. C. S. Southw | 5 |
| Rev. E. R. Beadle.................... | 4 | toronto, canada. |  |
| Henry C. Carey J. S. Hart | 1 30 |  |  |
| Dr. Isaac Lea. | 54 | Canadian Institute |  |
| W. S. Vaux... |  | trenton, new jersey. |  |
| PORTLAND, MaINE. |  | John S. Hart | 16 |
| Portland Society of Nataral History. | 63 | UTICA, NEW YORK. |  |
| POTTSVILLE, PENNSYLVANTA. |  | E. Jewett . | 2 |
| P. W. Sheafer. | 86 | Waghington, d. c. |  |
|  |  | Board of Indian Commissio | 302 |
| quebec. |  | Bureau of Statistics. | 200 |
|  |  | Census Bureau.. |  |
| Literary and Historical Society .... | 26 | Clinio-pathological Society........ | 1 |
|  |  | Department of Agriculture General Land-Office. | 400 |
| sacramento, california. |  | Nautical Almanac Office. | $\stackrel{2}{4}$ |
|  |  | Navy Department ................. |  |
| and Dumb $\qquad$ | 25 | Office of Chief of Engineers ...... Quartermaster General's Office.... | 99 50 |
| California State Board of Health... | 1 | Quartermaster General's Office.... | 183 |
|  |  | United States Congress. | 500 |
| SAINT LOUIS, MISSOURI. |  | United States Naval Observatory.. | 59 |
|  |  | United States Patent Office ........ | 238 |
| Dr. G. Engelmann | 1 | Treasury Department.............. |  |
|  |  | Dr. Cleveland Abbe................. | 127 |
| saint paul, minnesota. |  | W. H. Dall........................ | 36 |
|  |  | Dr. F. V. Hay |  |
| Minnesota Historical Society | 10 | T. Poesche |  |
|  |  | C. R. Rangab |  |
| balem, massachusetts. |  | R. Ridgeway |  |
|  |  | Unknown... |  |
| Peabody Academy of Science ....... | 101 | Total | 7,730 |

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| Regents of New York State University | 6 | Commissioner of Fisheries.. ....... Maine Lunatic Hospital. | 2 |
| Albany Institute.... ................ | 7 |  |  |
| Board of State Charities | 1 | AUSTIN, NEVADA. |  |
| Dudley Observatory ................. | 16 |  |  |
| New York State Agricultural Society $\qquad$ | 35 | Dr. T. Storch. | 1 |
| New York State Cabinet of Natural History | 5 | austin, texas. |  |
| New York State Homoopathic Society. | 1 | Judge Julius Schultze. | 2 |
| New York State Library... | 33 | baldwin city, kansas. |  |
| Inspectors of the Penitentiary. | 1 |  |  |
| Inspectors of the State Prisons of New York. | 1 | Baker University ....... | 1 |
| Hon. Francis Barlow | 1 | BALTLMORE, MARYLAND. |  |
| Professor James Hall. | 30 |  |  |
| Professor J. W. Hough................ | 1 | American Journal of Dental Science $\qquad$ | 3 |
| ALLEGHENY CITY, PENNSYLVANIA. |  | Maryland Historical Society Mercantile Library | 6 |
| Observatory | 1 | Municipality <br> Peabody Institute | 2 |
| AMHERST, MABSACHUSETTS. |  | Peabody Institute. <br> University of Maryland | 7 |
|  |  | A. M. Carter...... | 1 |
| Amherst College. | 5 | P. R. Uhler | 2 |
| Geological Surver of Massachusetts. Professor E. S. Snell |  |  |  |
| Professor E. S. Snell..... | 1 2 | BLOOMLNGTON, INDIANA. |  |
|  |  | Indiana State University | 1 |
| anNafolis, maryland. |  | Professor D. Kirkwood.............. | 1 |
| State Library | 3 | BOSTON, MASSACHUSETTS. |  |
| United States Naval Academy...... ANN ARBOR, MICHIGAN. | 2 | American Academy of Arts and Sciences. | 114 3 |
| Observatory | 7 | American Social Science Association | 3 |
| University of Michigan | 1 | American Statistical Association.. | 8 |
| Dr. Freese. | 1 | American Unitarian Association... | 5 |
| Dr. J. C. Watson | 2 | Atheneum ...... . .................. | 3 |
| Professor A. Winchell | 7 | Board of State Charities........... | 4 |
|  |  | Boston Christian Register....... .. | 3 |
| APPLETON, WISCONSIN. |  | Boston Medical and Surgical Journal. | 1 |
| Lawrence University | 1 | Boston Society of Natural History. | 171 |
|  |  | Directors of Public Institutions... | , |
| ATHENS, GEORGIA. |  | Gynæcological Society ............. | 1 |
| University of Georgia. | 2 | Inspecturs of Massachusetts State <br> Prisons. | 1 |
|  |  | Massachusetts Historical Society.. | 9 |
| ATHENS, ILLINOIS. |  | Massachusetts Society for Prevention of Cruelty to Animals. | 1 |
| Professor Elihu Hall | 1 | Municipality ...... ................ | 1 |
| ATHENS, OHIO. |  | New England Historic, Genealogical Society. | 8 |
|  | 1 | North American Review .... ....... | 7 |

Packages received from Europe, \&cc.-Continued.


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| American Medical College. | 1 | Lutheran College. | 2 |
| Astronomical Society. .............. | 1 |  |  |
| Astronomical Observatory.......... | 24 6 | delaware, ohio. |  |
| Historical and Philosophical Society of Ohio | 1 | Wesleyan University | 1 |
| Mercantile Library | 3 | des moines, Iowa. |  |
| Municipality-...... | 1 |  |  |
| Ohio Mechanics' In | 1 | Governor of the State of Iowa State Library |  |
| Cleveland, ohio. |  | DETROTT, MICHIGAN. |  |
| Cleveland University ....... CLINTON, NEW YORK. | 1 | Inspector of Detroit House of Correction. |  |
| Litchfield Observatory | 2 | Michigan State Agricultural Society | 13 |
| Professur C.H.F. Pete | 4 | W. U. Reichert ...................... |  |
| coalburgh, west virginia. |  | dorchester, massachusetts. |  |
| W. H. Edwards. | 2 | Dr. Edward Jarvis. | 16 |
| COLUMBIA, MISSOURI. |  | east greenwich, new york. |  |
| Geological Survey of Missouri |  | Asa Fitch. | 1 |
| Missouri University . | 6 |  |  |
| Dr.E.C:Swallow |  | easton, pennsylvania. |  |
| columbia, south carolina. |  | Lafayette College <br> Professor J. H. Coffin | $\frac{1}{6}$ |
| South Carolina College | 1 | HLMMRA, NEW YORK. |  |
| columbus, ofio. |  | Elmira Academy of Sciences. | 2 |
| Ohio State Board of Agriculture | 68 3 | ERIE, PENNSYLVANTA. |  |
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| Prison.. | 1 | Edward Norton | 1 |
| credit, canada. |  | FORT M'HENRY, MARYLAND. |  |
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| crow wing, minnesota. |  | fountaindale, illinois. |  |
| Francis Pierz | 1 | M. S. Bebb | 2 |
| davenfort, IOWA. |  | Frankfort, Kentucky. |  |
| Public Library | 1 | Geological Survey of Kentacky.. | 5 |

## Packages received from Europe, dec.-Continued.



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ciety

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| INMCANSVILLE, WISCONSIN. |  |
| Wisconsin Scandinavian Society.... |  |
| IOWA CTTY, IOWA. |  |
| Geological Survey of Iowa......... |  |


| Iowa State University................... | 32 |
| :--- | :--- | :--- |
| Professor G. Hinrichs.................. | 12 |

Dr. C. A. White............................. 3


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Wisconsin Institution for Educating
the Blind..................................... 5
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Cumberland University
LEXINGTON, KENTUCKY.
Transylvania University
Professor J. H. Clarke
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LOUISVILLLE, KENTUCKY.
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Municipality................................
Richmond and Louisville Medical Journal.

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Franklin B. Hough
1
MADISON, WISCONSLN.
Skandinaviske, Presse-Forening... 1

Wisconsin State Agriculturad Soci-
ety ...............................................................
MANCHESTER, NEW HAMPSHIRE.
City Library

## Packages received from Europe dec.-Continued.

| Address. |  | Address. |  |
| :---: | :---: | :---: | :---: |

MARQUETTE, MICHIGAN.

## Bishop Ignatius Maak

MARYSVILLE, CALIFORNIA.
Dr. E. T. Wilkins
meadvilie, pennsylvania.
Observatory of Allegheny College.
Professor Langley
MILWAUKEE, WISCONSIN.
German Academy of Natural Sciences
I. A. Lapham

MONTPELIER, VERMONT.
Historical and Antiquarian Society.
State Library

## montreal, canada.

Entomological Society of Montreal.
Geological Survey of Canada.......
Historical Society
King's College
McGill College
Natural History Society
E. Billings $\qquad$
Dr. P. P. Carpenter.
Professor J. W. Dawson
Dr. T. Sterry Hunt.
Sir W. E. Logan
Dr. C. Smallwood

## NASHUA, NEW HAMPSHIRE.

Dr. B. K. Emerson
nashindue, TENNESSEE.
Geological Survey of Tennessee
University
NEENAH, WISCONSIN.
Scandinavian Library Association Scandinavian Literary Society
negaunee, michigan.
Major F. B. Brooks.
new bedford, massaceusetts.
John H. Thomson
NEW BRUNSWICK, NEW JERSEY.
Geological Survey of New Jersey

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Rutgers College ....... ............ 1

| Rutgers College |
| :--- |
| Protessor George |
| H. Cook................... |

Professor John C. Smock.
new corlin, wisconsin.
T. A. Bruhin
new haven, CONNECTICUT.

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Professor C. S. Lyman............... 2
Professor O. C. Marsh................ $\quad 4$
Professor H. A. Newton............... 5
Professor B. Silliman............... . 12
Sidney J. Smith......................... 1
Professor A. E. Verrill................. 7
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Young Men's Christian Association.
new orleans, louistana.
Manicipality
New Orleans Academy of Natural Sciences.
newport, RHODE ISLAND.
Mechanics' Library
NEWPORT, VERMONT.
Orleans County Society of Natural Sciences.

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tory
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Cooper Union ..... 1
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Journal of Psychological Medicine.

## Packages received from Europe, do-Continued.

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| :---: | :---: | :---: | :---: |
| New York, N. Y.-Continued. |  | OTTAWA, ILLINOTS. |  |
| Lyceum of Natural History | 72 | Ottawa Academy of Natural Sci- |  |
| Medical Gazette. | 2 | ences |  |
| Medical Journal | 1 |  |  |
| Medical Record | 1 | OXFORD, MISSISSIPPI. |  |
| Mercantile Library Association | 4 |  |  |
| Metropolitan Board of He Microscopical Society | 2 | E |  |
| Municipality ...... | 1 | Paxton, illinois. |  |
| New York Academy of Medicine... | 1 | T. N. Hasselquint ..................... |  |
| New York Christian Inquirer..... | 1 | T. N. Hasselquint. |  |
| New York Historical Society ....... | 5 | PENN YAN, NEW YORK. |  |
| Numismatic and Archæological Society | , | Samuel H. Wright........... |  |
| School of Mines...-.............. | 9 | Samuel H. |  |
| Secretary of American Prison Association. | 1 | PEORIA, ILLINOIS. |  |
| Society Library | 2 | Dr. F. Brendel. |  |
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| University | 3 | PHILADELPHIA, PENNSYLVANIA. |  |
| Dr. F. A. P. Barn | 1 |  |  |
| Professor Bauer | 1 | Academy of Natural Scienc | 144 |
| Thomas Bland. | 1 | American Entomological Society.. | 11 |
| Dr. Carrington Bolton | 2 | American Journal of Conchology.. | 4 |
| Professor C. F. Chandler | 3 | American Pharmaceutical Associa- |  |
| Captain J. M. Dow | 2 | tion | 35 |
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| Professor T. Egle |  | Board of Inspectors of County Pris- |  |
| A. Eilers.. | 5 |  |  |
| Samuel Elirt | 1 | Central High School |  |
| Captain John Ericss | 2 | Curator of Birds, Philadelphia Mu- |  |
| Professor Hermann Flüge | 1 | seum .-............................ |  |
| Dr. Gescheidt | 1 | Dental Cosmos | 2 |
| Henry Grinnell | 6 | Dental Enquirer |  |
| Dr. Charles Joy | 6 | Dental Times | 2 |
| Dr. James P. Kimball | 4 | Franklin Institute | 29 |
| Dr. H. J. Knapp | 1 | Historical Society of Pennsylvania | 13 |
| Dr. James Knight | 1 | Jefferson Medical College ........... |  |
| George N. Lawrence | 5 | Library Company ....... |  |
| Professor S. F. B. Mor | 1 | Medical and Surgical Reporter...- |  |
| Dr. J. S. Newberry | 8 | Medical Times |  |
| Dr. J. C. Nott | 2 | Mercautile Librar |  |
| Baron R. Ostensack | 2 | Municipality ... | 2 |
| Dr. Martyn Paine | 2 | North American Medico-Chirurgical |  |
| Messrs, Parker \& Douglas | 3 | Review ...-.............-. .-. .-. | 1 |
| Alfred Pell. | 1 | Numismatic and Antiquarian So- |  |
| Professor A. Poey | 2 | ciety...... .-.....--.............. |  |
| Professor R. Pumpelly | 5 | Observatory of Girard College..... |  |
| Dr. R. W. Raymond. | 3 | Pennsylvania Institution for Blind. | 1 |
| Professor O. M. Rood | 1 | Pennsylvania Society for Preven- |  |
| Lewis M. Rutherford | 1 | tion of Cruelty to Animals....-. |  |
| H. M. Schiefflin | 1 | Public Schools ...... | 2 |
| E. G. Squier | 4 | Society for Alleviating Miseries of |  |
| Dr. L. Tellkampf | 1 | Public Prisons..................... | 2 |
| T. C. Theaker. | 1 | Superintendent of State Peniten- |  |
| Dr. John Torrey | 1 | tiary---................ ............ |  |
| Dr. Luther Voss | 1 | University |  |
|  | 1 | Wagner Free Institute of Science. Rev. E. R. Beadle. |  |
| NORTHAMPTON, MASSACHUSETTS. |  | Lorin Blodget . . . . . . . . . . . . . . . . . . . . . - |  |
| State Lunatic Asylum. .... . | 1 | H. C. Carey....... .................... |  |

## Packages received from IIurope, dec.-Continued.

| Address. |  | Address. |  |
| :---: | :---: | :---: | :---: |
| Philadilleria, Pa.-Continued. |  | Providence, R. I.-Continued. |  |
| J. Cassin | 2 | Professor A. Caswell | 2 |
| Pliny Earl Chase | 3 | Dr. Edwin M. Snow | 4 |
| Dr. F. A. Conrad. | 1 |  |  |
| Professor E. D.. Cope | 6 |  |  |
| Dr. Bennett Dowler | 1 | QUEBEC, CANADA. |  |
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| Judge Hare. | 1 | Legisiative Library .-............. | 11 |
| Dr. M. Hare. | 1 | Literary and | 11 |
| Dr. G. H. Horn | 2 | Observatory | 2 |
| Dr. Isaac Lea | 11 | M. Joly de Lotbinierre. ................ | 1 |
| Professor J. Leidy ............................. | 15 | Abbé Provanche....... | 1 |
| J. P. Lesley ..... | 1 |  |  |
| Johnson D. Lund | 1 | Raleigh, north carolina. |  |
| B. S. Lyman. | 2 |  |  |
| Thomas Meeha | 1 | Professor W. C. Kerr. | 5 |
| J. A. Meigs. | 2 |  |  |
| Dr. C. F. Parker....................... | 2 | RICHMOND, VIRGINIA. |  |
| Thomas Stewardson, jr | 1 | RICHMOND, VIRGINLA. |  |
| George W. Tryon...................... | 1 |  |  |
| Professor Wagner. | 1 | Historical Society of Virginia..... | 1 |
| Dr. Horatio Wood, j | 7 | State Library <br> T. H. Wynne | 1 |
| PHGNIXVILLE, PENNSYLVANIA. |  | SACRAMENTO, CALIFORNLA. |  |
| Charles M. Wheatley ................. | 1 | Dr. Thomas M. Logan | 1 |
| PITTSFIELD, MASSACHUSETTS. |  | SAINT ANTHONY, MINNESOTA. |  |
| Líbrary Association | 1 | University of Minnesota........... | 1 |
| PITTSBURGH, PENNSYLVANLA. |  | SAINT JOHN, NEW BRUNSWICK. |  |
| Professor S. P. Langley.............. | 1 | Mechanics' Institute $\qquad$ <br> Natural History Society. $\qquad$ | 1 |
| PORTLAND, MAINE. |  |  |  |
| Legislature of Maine | 1 | SA |  |
| Portland Society of Natural His- |  | Humboldt Medical College | 1 |
| tory | 27 |  | 1 |
|  |  | Medical and Surgical Journal Missouri Dental Journal | 4 |
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| Miss Maria Mitchell. | 2 | Public School Library. | 1 |
|  |  | Saint Louis Academy of Sciences.. | 84 |
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| College of New Jersey .............. | 7 | Dr. Louis Engelman | 1 |
| Horticultural Saciety. | 1 | Louisa Lange | 1 |
| Pharmaceutical Society | 1. | Charles V. Riley. | 2 |
| Professor S. Alexander. | 1 | Maurice Shuster. | 1 |
| Professor A. Gayot . . . . . . . . . . . . . Providennae, rhode island. | 9 | SAINT PAUL, MINNESOTA. |  |
|  |  | Minnesota Historical Soctety ...... | 6 |
| Athenæum | 1 | Northwestern Medical and Surgi- |  |
| Brown University | 5 | cal Journal. | 1 |
| Rhode Island Historical Society | 4 | J. H. Kloos. | 7 |

Paokages received from Europe, de-Continued.

| Address. |  | Address. |  |
| :---: | :---: | :---: | :---: |
| SALEM, MASSACHUSETTS. |  | Washington, D. C.-Continued. |  |
| Essex Institute | 57. | Argentine Legation. ................. | 1 |
| Peabody Academy of Science | 40 | Board of Indian Commissioners... | 1 |
| A. S. Packard, jr. | 21 | Bureau of Navigation............... | 7 |
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|  |  | Commissioner of Agriculture...... | 1 |
| University | 1 | Department of Agriculture......... | 148 |
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| Professor H. N. Bolande | 1 | Library of Congress. | 26 |
| S. A. L. Brannan, jr | 2 | Medical Society of the District of |  |
| Dr. J. G. Cooper. | 2 | Columbia. | 1 |
| R. E. C. Stearns | 4 | Municipality. | 1 |
|  |  | Navy Department | 2 |
| SCHENECTADY, NEW YORK. |  | National Academy of Scien | 39 |
|  |  | Ordnance Bureau. | 7 |
| Professor H. E. Webster | 1 | Secretary of the Nav | 1 |
|  |  | - Secretary of War | 4 |
| SING SING, NEW YORK. |  | Signal Office... | 1 |
|  |  | State Department....... | 4 |
| Dr. G. J. Fisher | 3 | Surgeon General's Office.... .-..... | 93 |
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| Professor A. M. Mayer | 1 | United States Naval Observatory. | 75 |
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| SPRINGFIELD, ILLINOIS. |  | United States Revenue Department | 1 |
| Geological Survey of Illinois. |  | War Department................... | 5 |
| Illinois State Agricultural Society.. | 2 | Washington Public Schools .-..... | 3 |
| Illinois State University ............ | 1 | Professor Cleveland Abbe | 1 |
| Professor A. H. Worthen ............- | 7 | General H. L. Abbot...... | 1 |
| TORONTO, CANADA. |  | Mrs. C. Alexander... | 15 |
|  |  | Professor S. F. Baird. | 45 |
| Botanical Society of Toronto. | 1 | G. M. Bache . | 1 |
| Canadian Institute | 15 | Dr. H. M. Bannister. | 2 |
| Literary and Historical Society | 1 | General J. G. Barnard | 1 |
| bservatory | 5 | Professor W. P. Blake. | 3 |
| Minity Colleg | 1 | Professor J. H. C. Coffin | 2 |
| University | 3 | T. A. Craven..... | 1 |
| D. K. Winder | 1 | W. H. Dall. | 10 |
|  |  | C. H. Davis. | 1 |
| tuscaloosa, atabama. |  | Miss Dorothea Dix | 1 |
|  |  | General W. H. Emory .... .......... | 2 |
| University of Alabama............... | 1 | Dr. E. Foreman. | 2 |
|  |  | W. Q. Force. | 1 |
| UTICA, NEW YORK. |  | General J. C. Frémont | 3 |
|  |  | Edw. M. Gallaudet. | 1 |
| American Journal of Insanity....... | 5 | Baron Gerolt.... | 1 |
|  |  | Professor T. Gill. <br> Dr. F. V. Hayden | 8 |
| WASHNGMON, D. C. |  | Professor J. Henry | 20 |
| American Nautical Almanac Office.. | 8 | J. E. Hilgard... | 2. |

## Packages received from Europe, \&ec.-Continued.

| Address. |  | Address. |  |
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| Washington, D. C.-Continued. |  | WEST POINT, NEW YORK. |  |
| G. W. Hill | 3 |  |  |
| John Hitz. | 1 | Professor W. H. C. Bartlett........ |  |
| General A. A. Humphreys | 4 |  |  |
| E. B. Hunt.. | 1 | WILLIAMSBURGH, VLRGINIA. |  |
| J. C. G. Kennedy | 3 |  |  |
| Admiral Lee . | 1 | Virginia Eastern Lunatic Asylum. |  |
| Professor J. H. MoChesney | 1 |  |  |
| J. N. Maffit.... | 11 | WELLIAMSTOWN, MASSACHUSETTS. |  |
| Brigadier General A.J. M | 1 |  |  |
| Protessor S. Newcomb.. | 3 | Williams College |  |
| Dr. C. C. Parry | 1 |  |  |
| T. Poesche.. | 2 | WILMINGTON, DELAWARE. |  |
| W. J. Rhees. | 2 |  |  |
| Admiral Sands | 2 | Agricultural Society of Wilmington | 1 |
| Professor G. C. Schaeff | 1 | Wilmington Institute....... ....... | 1 |
| C. A. Schott. | 3 |  |  |
| Henry Ulke. | 2 |  |  |
| Lieutenant Colonel Wood ward | 1 | WINDSOR, NOVA SCOTLA. |  |
| C. B. Young........... | 3 | Kings College.. | 1 |
| WATERBURY, CONNECTICUT. |  |  |  |
| Brownson Library. | 1 | WORCESTER, MASSACHUSETTS. |  |
| Waterville, maine. |  | American Antiquarian Society...- | 14 |
| Waterville College....... ............ | 1 | Free Public Library ....... .......... | 3 |

Total addresses of institutions ..... 318
Total addresses of individuals ..... 255
Total number of parcels to institutions ..... 3, 049
Total number of parcels to individuals ..... 903

## LIST OF Meteorological stations and observers of the smithioNiAN institution for the year 1871.



Inst of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| CONNECTICUT. |  |
| Alcott, William P. | North Greenwich, Fairfield County. |
| Andrews, L. | Southington, Hartford County. |
| Rockwell, Charlotte | Colebrook, Litchfield County, |
| Ward, H. D. A., and John John | Middletown, (Wesleyan University,) Middlesex County. |
| Yeomans, W. G | Columbia, Tolland County. |
| DAKOTA. |  |
| Dorsey, Rev. J. Owen | Ponka Agency, Todd County. |
| DELAWARE. |  |
| Bateman, J. H | Dover, Kent County. |
| Gilman, R. H | Milford, Kent County. |
| FLORIDA. |  |
| Atwood, G. W | Saint Augustine, Saint John's County. |
| Baldwin, Dr. A. S | Jacksonville, Duval County. |
| Barker, E | Ocala, Marion County. |
| Beecher, Rev. C | Newport, Waculla County. |
| Chamberlain, S. | Mosquito Inlet, Volusia County. |
| Lowd, E. K. | New Smyrna, Volusia County. |
| Powell, Charles F | Bicalata, Saint John's County. |
| Robinson, General G. D | Pilatka, Putnam County. |
| Thralls, George R. | Welborn, Suwanee County. |
| White, W. F..... | Tampa, Hillsborough County. |
| GEORGIA. |  |
| Barker, E | Saint Mary's, Camiden County. |
| Cutter, John L | Quitman, Brooks County. |
| Deckner, F., \& Son | Atlanta, Fulton County. |
| Hillyer, H. L . | Berne, Camden County. |
| Hollifield, Horatio N | Sandersville, Washington County. |
| McClutchen, A. R | Lafayette, Walker County. |
| Sanford, S. P .... | Penfield, Greene County. |
| illinois. |  |
| Adams, W. H | Elmore, Peoria County. |
| Aldrich, Verry | Tiskilwa, Bureau County. |
| Bowman, M. B | Andalusia, Rock Island County. |
| Brendel, F. | Peoria, Peoria County. |
| Brookes, S | Chicago, Cook County. |
| Carey, Daniel | Rochelle, Ogle County. |
| Chase, Dr. D. H | Louisville, Clay County. |
| Cochrane, J | Havana, Mason County. |
| Dudley, T | Decatur, Macon County. |
| Duncan, Rev. A | Mount Sterling, Brown County. |
| Finley, Dr, T | Pana, Christian County. |
| Gramesly, C | Charleston, Coles County. |
| Grant, J. and Miss M | Manchester, Scott County. |
| Hearne, F.J. | Quincy, Adams County. |
| Henry, W. E | Mattoon, Coles County. |
| James, J. W | Marengo, McHenry County. |
| Jozefe, Dr. C | Waterloo, Monroe County. |
| Langguth, J. G | Chicago, Cook County. |
| Livingston, Professor W | Galesburgh, (Lombard University,) Knox County. |
| Marcy, Professor 0 | Evanston, (Northwestern University,) Cook County. |

List of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| Ilunois-Continued. |  |
| Mead, S. B | Augusta, Hancock County. |
| Moss, G. B | Belvidere, Boone County. |
| Murray, Peter | New Manchester, Scott County. |
| Osborn, Ethan | Hennepin, Putnam County. |
| Patterson, H. N. | Oquawka, Henderson County. |
| Puffer, A. W........... | Mattoon, Coles County. |
| Spaulding, A., and Mrs. E. D | Aurora, Kane County. |
| Spencer, W.C | Dubois, Washington County. |
| Whitaker, B. | Warsaw, Hancock County. |
| indiana. |  |
| Alden, Thomas E | Rising Sun, Ohio County. |
| Andrew, F. G | Laporte, Laporte County. |
| Applegate, J. A., and daughter | Mount Carmel, Franklin County. |
| Boerner, C. G | Vevay, Switzerland County. |
| Chappelsmith Clark, W. S.. | New Harmony, Posey Country. Beech Grove, Rush County. |
| Crosier, A. | Laconia, Harrison County. |
| Curtis, J. W | Warsaw, Kosciusko County. |
| Dawson, W | Spiceland, Henry County. |
| Deem, D. | Knighttstown, Rush County. |
| Hadley, Dr., and other | Indianapolis, Marion County. |
| Howard, J. K | Livonia, Washington County. |
| Mallow, T. H | Rensselaer, Jasper County. |
| McCoy, Dr. S. and Mise | Columbia City, Whitley County. |
| Mchenry, B. F | Merom, Sullivan County. |
| Robertson, R. | Fort Wayne, Allen County. |
| Spitler, D <br> Sutton, $\mathbf{G}$ | Kentland, Newton County. |
| Thralls, Geo. R | Warsaw, Kosciusko County. |
| Williams, Mrs. B. C | Annapolis, Parke Conuty. |
| Iowa. |  |
| Adams, Ernest | Ames, Story County. |
| Ashby, M. V | Afton, Union County. |
| Babcock, E | Boonesborough, Boone County. |
| Bryant, Mrs. J. A | Fontanelle, Adair County. |
| Collin, Professor A | Mount Vernon, Linn County. |
| Dickinson, J. P | Guttenberg Clayton County. |
| Farnsworth, P.J | Clinton, Clinton County. |
| Gilbert, A. P. | Lemars, Plymouth County. |
| Horr, Dr. Asa | Dubuque, Dubuque County. |
| Mansfield, A. A | Mount Pleasant, (University,) Henry County. |
| Marshall, Grego | Cresco, Howard County. |
| McClintock, F | West Union, Fayette County. |
| McCready, D | Fort Madison, Lee County. |
| Miller, E. and R | Grant City, Sac County. |
| Nelson, D. B.......... | Sac City, Sac County. |
| Parvin, Professor T. S | Iowa City, (University,) Johnson County. |
| Ross, F. A. | Durant, Cedar County. |
| Russell, A. M | West Branch, Cedar County. |
| Sheldon, D. S | Davenport, Scott County. |
| Smith, Rufus P | Monticello, Jones County. |
| Stern, Jacob F | Logan, Harrison County. |
| Tulbot, Benjam | Council BIuff, Pottawattomie County. |
| Townsend, N | Iowa Falls, Hardin County. |

List of meteorological stations and observers for the year 1871-Oontinued.

| Name of observer. | Address. |
| :---: | :---: |
| Iowa-Continued. |  |
| Wadey, H | Rockford, Floyd County. <br> Independence, Buchanan County. <br> Algona, Kossuth County. <br> Independence, Buchanan County. <br> Woodbine, Harrison County. <br> Bowen's Prairie, Jones County. |
| Warne, Dr. G |  |
| Wheaton, Mrs. D. B |  |
| Witter, D. K. |  |
| Woodworth; S |  |
| kansas. |  |
| Adams, Ernest. | Williamstown, Jefferson County. |
| Beckwith, W | Olathe, Johnson County. Williamstown, Jefferson County. Crawfordsville, Crawford County. Williamsburgh, Franklin County. Atchison, Atchison County. Burlingame, Osage County. <br> Baxter Springs, Cherokee County. Douglas, Butler County. Manhattan, (Agricultural College,) Riley Connty. |
| Cotton, J. M |  |
| Daniels, P |  |
| Fogle, D.. |  |
| Horn, Dr. H. B., |  |
| Hoskinson, R. M |  |
| Ingraham \& Hyland |  |
| Lamb, Dr. W. M. |  |
| Mudge, Professor B. F |  |
| Parker, J. D. | Burlington, Coffey County. <br> Plam Grove, Butler County. |
| Richardson, A. G |  |
| Shoemaker, J. G | Leroy, Coffee County. |
| Snow, Professor F. H | Lawrence, (Univ'y, ) Douglas County. |
| Stayman, Dr. J |  |
| Walrad, L. D | Paola, Miami County. |
| Walters, Dr. J | Holton, Jackson County. Council Grove, Morris County. |
| Woodworth, A |  |
| kentucky. |  |
| Beatty, 0 | Danville, Boyle County. Blandville, Ballard County. Pine Grove, Clarke County. Arcadia, Lincoln County. Springdale, Jefferson County. |
| Horr, Edw. W |  |
| Martin, Dr. S. D |  |
| Shriver, Howard |  |
| Young, Mrs. Law |  |
| lodislana. |  |
| Cleland, Rev. T. H | Delhi, Richland Parish. |
| Collins, H. C | Ponchatoula, Tangipahoa Parish. |
| Foster, Captain R | New Orleane, Orleans Parish. |
| Moore, Dr. Jos. L | Shreveport, Caddo Parish. |
| malne. |  |
| Clifford, A. J | Montville, Waldo County. |
| Clifford, J. R | Montville, Waldo County. Houlton, Aroostook County. |
| Fernald, C. H |  |
| Fernald, M. C | Órono, Penobscot County. |
| Gardiner, R. H | Gardiner, Kennebec County. |
| Guptill, G. W | Cornish, York County. <br> Bucksport, Hancock Countr. |
| Haskell, Willabe |  |
| Mayo, E. D. |  |
| Moore, Asa P. | Lishon, Androscoggin County. |
| Moulton, J. P | Standish, Camberland County. |
| Parker, J. D. | Mount Desert, Hancock County. |
| Pitman, Edwin | Barnard, Piscataquis County. |
| Reynolds, Henry | East Wilton, Franklin County.Norway, Oxford County, |
| Smith, H. D. |  |
| Tripp, Oscar H | Surry, Hancock County. |
| Wentworth, B. | Montville, Waldo County. |

List of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| Manne-Continued. |  |
| West, Silas | Cornish, York Coanty. |
| Wilbur, Ben. J | West Waterville, Kennebec County. |
| Curtise, G. | Fallston, Harford County. |
| Devilbiss, $\mathbf{F}$ | San's Creek, Carroll County. |
| Elliott, J. F | Saint Inigoes, Saint Mary's County. |
| Goodman, W. R | Annapolis, Anne Arundel County. |
| Hansliew, C. F. | Linwod, Carroll County. |
| Hanshew, J. K | Frederick, Frederick County. |
| Jourdan, Professor C. | Emmittsburgh, (Mount Saint Mary's College,) Frederick County. |
| McCormick, | Woodlawn, Cecil County. |
| Naval Hospital | Annapolis, Anne Arundel County. |
| Shepherd, H. M | Ellicott City, Howard County. |
| Shriver, E. T | Cumberland, Alleghany County. |
| Stephenson, Rev. James, and Valente, A. X. | Saint Inigoes, Saint Mary's County. Woodstock, (College,) Baltimore County. |
| MASSACHUSETTS. |  |
| Bacon, W | Richmond, Berkshire County. |
| Bixby, J. H | West Newton, Middlesex County. |
| Caldwell, J. H | Newburyport, Essex County. |
| Cunningham, George | Lunenburg, Worcester County. |
| Dewhurst, Rev. E. | Hinsdale, Berkshire County. |
| Fallon, J. | Lawrence, Essex County. |
| Frost, B. D | Hoosac Tunnel, Berkshire County. |
| Hart, George S | New Bedford, Bristol County. |
| Hopkins, Professor A., and Frede | Williamstown, (Williams College,) Berkshire County. |
| Merriam, S. A | Topsfield, Essex County. |
| Metcalf, Dr. J. G | Mendon, Worcester County. |
| Morrill, D. T., M. Bemis, and D. | Worcester, (Lunatic Hospital,) Worcester County. |
| Nason, Rev. | North Billerica, Middlesex County. |
| Nelson, H. M | Georgetown, Essex County. |
| Newcomb, G. S | Kingston, Plymouth County. |
| Perry, Mrs. S. H | Cambridge, Middlesex County. |
| Rodman, S. | New Bedford, Bristol County. |
| Snell, Professor | Amherst, (College,) Hampshire County. |
| Teele, Rev. A. K | Milton, Norfolk County. |
| Tacker, E. T | New Bedford, Bristol County. |
| MICHIGAN. |  |
| Bullard, R. | Litchfield, Litchfield County. |
| Ellis, Dr. Edwi | Ontonagon, Ontonagon County. |
| Higgins, F'. W | Detroit, Wayne County. |
| Holmes, Dr. E. | Grand Rapids, Kent Connty. |
| Howell, D. | Macon, Lenawee County. |
| Kedzie, Professor R. C | Lansing, (Agricultural College,) Ingham County |
| Kemp, Professor A. F | Olivet, (College, Eaton County. |
| Mapes, H. H. | Kalamazoo, Kalamazoo Connty. |
| Pattison, H. A | Muskegon, Maskegon County. |
| Pexton, J. W | Alpena, Alpena County. |
| Smith, Rev. G. N | Northport, Leelenau County. |
| Southworth, N. I | Coldwater, Branch County. |
| Streng, L. H. | Grand Rapids, Kent Connty. |
| Wells, L. E. | Battle Creek, Calhoun County. |

List of metearological stations and observers for the year 1871-Continued.


## MONTANA.

Goddard, E. N
Minesinger, J. M
Stuart, Granville

## NEBRASKA.

Caldwell, Mrs. E. E
Dunn, William.
Hamiĺton, Rev. W
Seltz, Charles
Smith, L. H.
Truman, George S
Zahner, $\mathbf{P}$
Virginia City, Madison Connty.
Missoula, Missoula County.
Deer Lodge City, Deer Lodge County.

Bellevue, Sarpy County.
Emerson, Otoe County.
Omaha Agency, Burt County.
De Soto, Washington County.
New Castle, Dixon County.
Santee Agency, L'Eau qui Court County.
Nebraska City, Otoe County.

List of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| NEW HAMPSHIRE. |  |
| Brewster, Alfred. | Tamworth, Carroll County. |
| Brown, Branch. | Stratford, Coos County. |
| Colby, Alfred. | Goffstown Center, Hillsborough County. |
| Couch, E. D | Contoocookville, Merrimack County. |
| Huntington, J. H | Hanover, Coos County. |
| Hurlin, Rev. W. | South Antrim, Hillsborough County. |
| Kidder, L. D | Whitefield, Coos County. |
| Odell, F. | Shelburne, Cocs County. |
| NEW JERSEY. |  |
| Beans, Thomas J. | Moorestown, Burlington County. |
| Brooks, William. | Paterson, Passaic County. |
| Chandler, Dr. W | South Orange, Essex County. |
| Cook, E. R | Trenton, Mercer County. |
| Fleming, J | Readington, Hunterdon County. |
| Green, H. A | Atco, Camden County. |
| Howard, Thomas T, jr | Jersey City, Hudson County. |
| Ingram, Dr.-J | Vineland, Cumberland County. |
| Noll, A. B. | New Germantown, Hunterdon County. |
| Palmer, Mrs. J. R | Rio Grande, Cape May County. |
| Perry, H. C. | Allowaystown, Salern County. |
| Sheppard, Miss R. C | Greenwich, Cumberland County. |
| Whitehead, W. A.. | Newark, Essex County. |
| NEW XORK. |  |
| Adriance, Chas. E | Hector, Schuyler County. |
| Albro, S. H., and Love, S. G | Jamestown, Chautanqua County. |
| Arden, T. B. | Garrison's, Putnam County. |
| Arnold, C. P | Augelica, Allegany County. |
| Baker, Gilbert | Himrods, Yates Cointy. |
| Barrett, A. J . | Lowville, Lewis County. |
| Barrows, Captain S | South Trenton, Oneida County. |
| Bartlett, E. B. | Vermillion, Oswego County. |
| Brownell, W. A | Fairfield, Herkimer County. |
| Bussing, J. W., and D. S. | Minaville, Montgomery County. |
| Clarke, B. W ...... .-. | Lockport, Niagara County. |
| Cooley, Professor J. S. | Fort Edward, Washington County. |
| Edwards, D. | Little Genesee, Allegany County. |
| Gardiner, J. H | Newburgh, Orange County. |
| Godfrey, M. P | Carlton, Orleans County. |
| Haas, H. . | Depanville, Jefferson County. |
| Hachenberg, Dr. G. P | Rochester, Monroe County. |
| Heimatreet, Jno. W.. | Troy, Rensselaer County. |
| Hendricks, D. B. | Kingston, Ulster County. |
| Howell, R. | Nichols, Tioga County. |
| Hint, G. M | North Argyle, Washington County. |
| Ingalsbe, G. M | South Hartford, Washington County. |
| Irish, Rev. Wm | Lowville, Lewis County. |
| Ives, W | Buffalo, Erie County. |
| Jones, W. Martiu | Suspension Bridge: |
| Johnson, Rev. S. W | Newark Valley, Tioga County. |
| Keese, G. P. | Cooperstown, Otsego County. |
| Lee, Leslie A | Canton, Saint Lawrence County. |
| Mack, E. T. | Flatbush, Kings County. |
| Mailler, I. P. | Brooklyn, Kings County. |
| Malcolm, W. S | Oswego, Oswego County. |

List of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| New York-Continued. |  |
| McNutt, Randolph | Warrensburgh, Warren County. Farmingdale, Queens County. <br> Furt Edward, Washington County. <br> Throg's Neck, Westchester County. <br> New York, New York County. <br> New York, New York County. <br> North Volney, Oswego County. Middleburgh, Schoharie County, Gouverneur, Saint Lawrence County. <br> Fairfield, Herkimer County. <br> Moriches, Suffolk County. <br> Cazenovia, (Seminary,) Madison County. <br> Oneida, Madison County. <br> Waterburgh, Tompkins County. <br> White Plains, Westchester County. <br> North Hammond, Saint Lawrence County. <br> Houseville, Lewis County. <br> West Day, Saratoga County. |
| Miller, J De |  |
| Morris, Miss E |  |
| Morris, Professor |  |
| Naval Hospital |  |
| Partrick, J. M |  |
| Roe, Sanford W |  |
| Russell, C. H |  |
| Sawyer, G. F |  |
| Smith, E. A., an |  |
| Spooner, Dr. S |  |
| Trowbridge, D |  |
| Willis, O. R., and daug |  |
| Wooster, C. A. |  |
| Yale, Walter D |  |
| Young, J. M |  |
| north carolina. |  |
| Adams, Professor | Goldsborough, Wayne County. |
| Allison, T. P | Statesville, Iredell County. |
| Aston, Edw. | Asheville, Buncombe County. |
| Austin, Robert | Tarborough, Edgecombe County. |
| Beall, R. L | Lenoir, Caldwell County. |
| Benton, A. A | Edenton, Chowan County |
| Greene, J. H | Bakersville, Mitchell County. |
| Hanna, George ${ }^{\text {Hardy }}$ | Charlotte, (Mint, ) Mecklenburgh County. |
| Hardy, Dr. J. | Asheville, Buncombe County. |
| Harrell, John | Weldon, Halifax County |
| Hicks, Dr. W | Oxford, Granville County. |
| Howard, | Greensborough, Guilford |
| Kron, F.J.. | Albemarle, Stanley County. |
| Lawrence, G | Fayetteville, Cumberlan |
| Murdoch, W. | Raleigh, Wake County. |
| Sherwood, Jno. | Fayetteville, Cumberland County. |
| оніо. |  |
| Ballantine, W............................  <br> Barringer, W...................... Sago, Muskingum County. <br> Bellefontaine, Logan County  |  |
|  |  |  |
| Bingman, T. J., and King, J. P | Pennsville, Morgan County. |
| Burras, O | North Fairfield, Huron County. |
| Clarke, $J$ | Bowling Green, Wood County. |
| Crane, G. W | Bethel, Clermont Connty. |
| Doyle, Joseph | Steubenville, Jeffersen County. |
| Dunn, F. K | Gambier, Knox County. <br> Painsville, Lake County. |
| Ferriss, E. J |  |
| Hammitt, Jn | Painsville, Lake County. |
| Harper, G. W | Cincinnati, Hanilton County. |
| Haywood, Pro | Westerville, (Univ'y,) Franklin County. |
| Herrick, 1. | Oberlin, Lorain County. |
| Huntingdon, G. C., and D. K |  |
| Hyde, G. A. | Cleveland, Cuyahoga County. Farmer, Defiance County. |
| Irwin, Dr: A. |  |
| Marsh, Mrs. M. | Ripley, Huron County. |
| Mathews, J. McD | Hillsborough, Highland County. Oxford, (Miami Univ'y,) Butler County |
| McFarland, Professor R. |  |
| McCune, Dr. James | Mount Gilead, Morrow County. |
| Morton, Dr. George |  |

List of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| OHIO-Continued, |  |
| Müller, Dr. R. | Carthagena, Mercer County. |
| Neill, Thomas. | Sandusky, Erie County. |
| Owsley, Dr. J. B | Jacksonburgh, Butler County. |
| Pettenger, I. M | Berea, Cuyahoga County. |
| Phillips, R. C. | Cincinnati, Hamilton County. |
| Pollock, Rev. J. E | Salem, Columbiana County. |
| Rodgers, Alexander P | Gallipolis, Gallia County. |
| Shaw, Dr. W.S. | Savannah, Ashland County. |
| Shields, J. H | Cincinnati, Hamilton County. |
| . Shreve, C. R., and Martha | Martin's Ferry, Belmunt County. |
| Smith, C. J | Hudson, Summit County. |
| Suith, Dr. C. H | Kenton, Hardin County. |
| Stillwell, C. A | Adams' Mills, Muskingum County. |
| Thompson, Rev | New Birmingham, Guernsey County. |
| True, Dr. H. A. | Marion, Marion County. |
| White, J. H. | Cincinnati, (Mount Auburn Ladies' Institute, Hamilton County. |
| Wilkinson, J. R. | Williamsport, Pickaway County. |
| Williams, Professor M. G | Urbana, (University,) Champaign County. |
| Winger, M.. | Wooster, Wayne County. |
| OREGON. |  |
| Oxer, H. A., and J. S. Reed. | Portland, Multnomah County. |
| Pearce, Thomas. | Eola, Polk County. |
| Wilson, Louis. | Astoria, Clatsop County. |
| pennsylvania. |  |
| Albree, C. and George | Pittsburgh, Allegheny County. |
| Bentley, E. T. | Tioga, Tioga County. |
| Black, Samuel A | Harrisburgh, Dauphin County. |
| Cook, Dr. W. | Carlisle, Cumberland County. |
| Corson, M. H | Plymouth Meeting, Montgomery County. |
| Cammings, J | Tarentum, Alleghany County. |
| Curtis, A. W | Catawissa, Schuylkill County. |
| Darlington, F | Parkersville, Chester County. |
| Day, Theodor | Dyberry, Wayue County. |
| Feicht, B. | Alleghany City, Alleghany County. |
| Fenton, E. | Grampian Hills, Clearfield County. |
| Grathwohl, J | Nyce's, Pike County. |
| Hance, E. | Fallsington, Buck's County. |
| Haworth, J | Hazleton, Luzerne County. |
| Hoffer, Dr. J. R | Mount Joy, Lancaster County. |
| Hubbs, Dr. J. Allen | Brownsville, Fayette County. |
| James, Professor C. S | Lewisbargh, Union Couṇty. |
| Jefferis, W. A | Westchester, Chester County. |
| Kirkpatrick, J | Philadelphia, Philadelphia County. |
| Kohler, E | Egypt, Lehigh County. |
| Lefever, Jacob | Mount Rock, Cumberland County. |
| Madlem, William F | Ephrata, Lancaster County. |
| Marsden, Dr. J. H | York Sulphur Springs, Adams County. |
| Martin, Dr, George | West Chester, Chester County. |
| McConnell, E. M | New Castle, Lawrence County. |
| Meehan, T | Germantown, Philadelphia County. |
| Naval Hospital | Philadelphia, Philadelphia County. |
| Packard, D. P | Greenville, Mercer County. |
| Peelor, D | Johnstown, Cambria County. |
| Raser, J. H | Reading, Berks Countr. |
| Sisson, R | Factoryville, Lazerne County |
| Smith, Dr. | Cannonsburg, (Col., ) Washington County. |
| Spencer, Miss Ann | Horsham, Montgomery County. |

List of meteorological stations and observers for the year 1871-Continued.


List of meteorological stations and observers for the year. 1871-Continued.


List of meteorological stations and observers for the year 1871-Continued.

| Name of observer. | Address. |
| :---: | :---: |
| Spaulding, J | Wautoma, Waushara County. |
| Tate, Andrew | Bayfield, Bayfield County. |
| Waite, M. C | Baraboo, Sauk County. |
| Whiting, W. H | Geneva, Wabash County. |
| Wright, R. M | SLurgeon Bay, Door County. |
| WYOMLNG TERRITORY. |  |
| Pierce, D. J. | Laramie City, Albany County. |

## additional meteorological material received in 1871 AND KEPT IN THE SMITHSONIAN INSTITUTION.

Albree, G., Pittsburgh, Pennsylvania.-Record of weather and indications.
Andrews, Luman, Southington, Connecticut.-Chart of auroras seen October 14, 1870.
Ballou, Nahum E., Sandwich, Illinois.-Monthly abstracts of temperature and rain-fall observations.

Annual abstract for 1871.
Barnard, A. D., San Buenaventura, California.-Account of northern light seen June 17, 1871.
Barnes, G. W., San Diego, California.-Notes of observations made on a trip to the mountains.
Barraud, A. L., Pacquette's F'erry, Iowa.-Observations of temperature and state of weather at $7 \mathrm{a} . \mathrm{m} ., 12 \mathrm{~m}$. , and $8 \mathrm{p} . \mathrm{m}$.
Bissey, Charles E., Iowa State Agricultural College, Ames, Iowa.-Account of aurora seen June 17.

Bland, T., New York.-Meteorological observations in Barbadoes October, 1871.
Boerner, Charles G., Vevay, Indiana.-Observations of August shower of meteors.

Branly, E. H., Amesville, Ohio.-Account of weather and crops.
Bryant, A. F., Fontanelle, Towa.-Account of wind-storm.
Buchner, H. F., Muco, Oreel Nation.-Thermometric observations for 1861 and 1871 at 7 a. m., 2 and 7 p. m.
Burras, O., North Fairfield, Ohio.-Account of the great tornado of July 16.
Busby, D. Beinjamin, Pomaria, South Carolina.-Report of observations of wind and rain-fall, for November, 1871.
Carlton, A. Y., Stoutville, Camden County, Missouri.-Register of temperature and direction of wind from November 13 to November 30, 1871.

Central Park, New York.-Weekly abstract of barometric and thermometric observations at 7 a. m., 2 p. m., and 9 p . m., and of the direction, force, and velocity of wind, and amount of cloud and rain.

Chase, Pliny E.-Monthly and annual rain-curves at Lisbon.
Chazaro, M. M., San Juan.-Observaciones meteorologicas en Octobre, 1871.

Clarke, John.-Weather predictions for August.
Clemson, Thomas G.-Climate of South Carolina.
Cochrane, J.-Account of tornado near Mason City.
Cockrell, Thomas J., Natchez, Mississippi.-Daily record of height of barometer and thermometer, $6 \mathrm{a} . \mathrm{m} ., 12 \mathrm{~m}$., and 6 p . m., and direction of wind. (Newspaper slips.)

Cunningham, G. A., Lunenburgh, Massachusetts.-Monthly rain-table from 1841 to 1868 and monthly means of temperature from 1838 to 1868.

Curle, T. J.-Observations in support of theory that anvil-shaped clouds always indicate rain.
Davison, C. E., Wayland, Michigan.-Weather report.
Doton, Hosea, Woodstock, Vermont.-Sketch of mountains around Killington, Vermont.
Edwoards, Daniel, Little Genesee, New York.-Account of the weather during August.

Engineers, Battalion of, Willet's Point, New York Harbor.-Horary curves-barometer, thermometer, psychrometer.

Ewing, Charles G., San Francisco, California.-Monthly report of barometric, thermometric, psychrometric, and rain-fall observations at 8.30 a. m. (Newspaper slips.)

Table showing quantities of rain falling in each month from 1865 to 1870.

Fogle, D., Williamsburgh, Kansas.-Observations of temperature and rain-fall, 7 a. m., 2 and 9 p . m.
Foster, R. W., New Orleans, Louisiana.-Weather notes for Greenville, during October, 1869.

Gatchell, H. T. F.-Climate of Colorado Springs.
Gleason, William, Arion, Maine.-Meteorological record for July, (temperature and wind observations at noon.)
Grady, B. F., Jr.-Condensed meteorological observations for August.
Grant, W. T.-Diagram of thermometrical observations.
Green, H. A., Atco, Neio Jersey.-Register of observations of temperature at Atco, New Jersey, May, 1871. Thermometric observations from. November, 1870, to March, 1871.

Greethurst, Joseph, Enterprise.-Monthly report of weather and crops.
Hannah, S. W., Washburn, Missouri.-Report of rain•fall in March, April, and May.
Higgins, F. W., Detroit, Michigan.-Table showing highest and lowest range of the thermometer, mean monthly temperature, highest and lowest daily mean in each month, amount of rain and melted snow, monthly mean of cloudiness, prevailing winds, \&c., at Woodmere Cemetery, near Detroit, Michigan, during the year 1871.
Howard, Thomas T., Jr., Jersey City, New Jersey.-General remarks to accompany meteorological reports.

Quarterly report of meteorogical observations at Jersey City, New Jersey.

Synopsis of meteorological register.
Hyland, W., Cherokee County, Kansas.-Monthly weather report.
Hough, G. W.-Description of automatic registering and printing barometer.

Jackson, George L., Vandalia, Illinois.-Account of cold.
James, J. W., Riley, Illinois.-Summary of meteorological observations for the year 1871.

Jewell, J. G., M. D., Consul at Singapore.-Copy of Sarawak Gazette containing report of temperature and rain-fall on the Quop estate in Sarawak for 1870.

Keutgen, O., Jr., Staten Island, New York.-Meteorological observaions, for the year 1871.
Synopses of meteorological observations.
King, Thomas D., Montreal, Canada.-Monthly register of thermometric and barometric observations.

King, William, Neuton Falls, Ohio.-Temperature observations at 7 a. m., 2 and $9 \mathrm{p} . \mathrm{m}$. , and diagram of same.

Langguth, J. G., Jr., Chicago, Illinois.-Account of hail-storm.
Lapham, I. A., Milwaukee, Wisconsin.-Dates of closing and opening of Milwaukee River from 1836 to 1871, and account of Wisconsin meteoric iron. (Newspaper slip.)
Leoni, George N., Clear Lake, Galveston County, Texas.-Result of meteorological observations for July, 1871.
Lewis, George H., Atlantic, Wyoming Territory.-Monthly report of temperature and rain-fall.
Logan, Thomas M., Santa Barbara.-Temperature, vital statistics, \&c., of Santa Barbara, ("Santa Barbara as a sanitarium," in Scientific Press.)

Mailler, I. P., Brooklyn, New York.-Account of the earthquake on June 19, 1871, (newspaper extract.)
Mapes, Henry H., Oshtomo, Michigan.-Monthly weather notes.
Martin, Allen.-Meteorological observations for July, 1871.
McCall, C. Olympia.-Account of rain-storm.
McOord, RR. H., Springfield, Missouri.-Acount of raill-storm June 5, 1871.

Mills, George.-"How it feels to be struck by lightning."
Moss, G. B., Belvidere, Illinois.-Monthly abstracts of register.
Abstract of register for the years 1868, 1869, 1870, 1871.
Mueller, Dr. R., Carthagena, Ohio.-Monthly record of casual phenomena, \&c. Appendix to register.
Noll, Arthur B., New Germantown, New Jersey.-Monthly reports of range of barometer.
Odell, Fletcher, Gorham, New Hampshire.-Weather notes from April to October, 1871.

Owen, Benjamin, Weston, Lewis County, West Virginia.-Monthly observations of temperature at $7 \mathrm{a} . \mathrm{m} ., 2 \mathrm{p} . \mathrm{m}$., and $9 \mathrm{p} . \mathrm{m}$.
Palmer, Dr. E.-Scrap-book containing meteorological observations for points in Nebraska.
Pastorelli \& Co.-Description of storm rain-gauge, designed by G. J. Symons, for observation of rate of fall.
Patterson, A. B., Saint Paul, Minnesota.-Monthly meteorological notes (newspaper slip.)
Payne, John K., Knowville, Tennessee.-Account of weather and crops, July and August, 1871.

Peelor, D., Johnstown, Penasylvania.-Table giving temperature of the earth at depth of 1 foot, from April, 1869, to April, 1871, inclusive.

Pettersen, Fred., San Autonio, Texas.-Diagram representing the relative frequency of the different winds, according to observations made during three years.

Mean temperature and rain-fall at San Antonio de Bexar, 1868, 1869, 1870, 1871.

Platt, Luciano, San Salvador.-Observaciones meteorologicas hechas en el laboratorio de la faculdad de medecina de San Salvador durante la semana nona de 1871. Semana segunda.

Poole, Henry, Glace Bay, Cape Breton, Nova Scotia.-Meteorological register kept during the years 1867, 1868, 1869, 1870, 1871.

Redding, Thomas B., Newcastle, Indiana--Account of aurora seen June 18.

Reed, Lyman, New York.-Lunar monthly weather predictions.
Sartwell, .H. P., Penn Yan, New York.-Register of meteorological observations. (Prepared for Yates County Whig.)

Seltz, Charles, De Soto, Nebras7ca.-Account of storm of July 28, 1871. (Newspaper slip.)

Account of rain-storm in Omaha.
Shepherd, Smiley, Hennepin, Illinois.-Monthly abstract of thermometric observations.

Signal-Office, Washington, D. C.-Daily weather maps, 7.35 a. m.; daily weather bulletins, $4.35 \mathrm{p} . \mathrm{m}$.

Barograms for November, 1871.
Aisson, Rodman, Abington, Pennsylvania.-Account of a violent hail and wind storm, and account of a terrible tornado at Hopbottom, July 9, 1872.

Account of auroral display of October 16, 1871.
Table showing mean temperature of each year from 1864 to 1870 , inclusive.

Slade, Elisha, Somerset, Massachusetts.-Monthly report of observations of maximum, minimum, and mean temperature, direction of wind, and state of weather.

Mean temperature third and fourth quarters 1871.
Smithsonian Institution, Washington, D. C.-Diagrams from recording barometer and thermometer.

Snow, F. H., Lavorence, Kansas.-Summary of meteorological observations for the year 1871.

Barometric, thermometric, and psychrometric observations, 7 a. m., 2 p. m., $9 \mathrm{p} . \mathrm{m}$.; direction of wind, amount of rain, force of vapor, \&c.

Stephenson, James, Saint Inigoes, Maryland.-Weather notes for April, 1871.

Steineman, U., Eckhart Mine, Alleghany County, Maryland.-Rain-fall observations, 1864 to 1868, inclusive.

Sutton, E. H.-Diagram of meteorological observations.

Sternberg, George M.-Description of improved anemometer.
Tayloe, Edward.-Weather report.
Tennent, Thomas, San Francisco, California.-Rain-fall for parts of 1849-1850.

Truman, George S., Santee Agency, Nebraska.-Monthly report of temperature, direction of wind, and state of weather.
Diagrams of parhelia, November 22, 1871.
Turner, Ernest, Germantown, Pennsylvania.-Effect of lightning in West Philadelphia during storm of July 11, 1871.
Wadsworth, H. L., Litchfield, Minnesota.-Diagram of halo seen from Litchfield, December 27, at 9. p. m.

Walton, F. P., Muscatine, Iowa.-Condensed report for 1871.
Webb, John G., Little Sarasta, Southern Florida.-Notes ou two cyclones in August, 1871.

Whitcomb, Thomas M., Union Ridge, Clarke County, Washington Ter-ritory.-Aurora observed July 21, 1871.
White, J. H., Cincinnati, Ohio.-Summary of meteorological observations for the year 1871, made at the Mount Auburn Young Ladies' Institute.

Whitehead, W. A., Newark, New Jersey.-Yearly meteorological report for 1871.
Wilbur, Benjamin FF., West Waterville, Maine.-Rain-fall in August, 1871.

Williams, H. C., Vienna, Virginia.-Synopsis of observations made by Franklin Williams, of Piedmont, Fauquier County, Virginia.

Williams, R. G., Castleton, Vermont.-Diagrams exhibiting comparison of Mason's and Boehlen's and Staehlen's hygrometers.

Wing, Minerva E., West Charlotte, Vermont.-Account of sunrise-phenomena.
Record of periodic phenomena at West Charlotte, Vermont.
Weekly meteorological records, (newspaper-slips.)
Witter, D. K., Woodbine, Iowa.-Account of the weather.
Wright, T. W. A.-Rain-table and remarks on climate for a portion of San Joaquin Valley, California.

## METEOROLOGICAL ARTICLES RECEIVED BY THE INSTITU. TION AND DEPOSITED IN THE LIBRARY OF CONGRESS, 1871. <br> AURORAS.

Aurore boréale et autres phénomènes météorologiques observés en Piémont le 3 janvier 1870.
Aurora polare osservata in Piemonte nel 5 Aprile 1870. P. Francesco Denza.
Le aurore boreali e la loro origine cosmica. Professor G. B. Donati.
Auroral belt of October 24, 25, 1870. American Journal of Science and Art, vol. i, pp. 73, 126.
Aurore boréale du 9 novembre. Observations faites à Brest, par M. Tarry. Comptes-rendus hebdomadaires des séances de l'académie des sciences, tome lxviii, No. 21.

Aurores boréales observées à Vendôme en 1870. E. Renon. Comptesrendus hebdomadaires des séances de l'académie des sciences, tome lxxii, No. 10.

Observations on the variation of the magnetic declination in connection with the aurora of October 14, 1870, by Alfred M. Mayer. American Journal of Science and Art, vol. i, p. 77.
Observations sur les relations qui existent entre les apparitions des aurores boréales et les variations de température. Ch. Sainte Claire Deville. Comptes-rendus hebdomadaires des séances de l'académie des, sciences, tome lxxii, No. 13.
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Sur les caractères de l'hiver 1870-71, et sur la comparaison de la température moyenne à l'observatoire de Paris et à l'observatoire météorologique central de Montsouris. Ch. Sainte Claire Deville. Comptes-rendus hebdomadaires des séances de l'académie des sciences, tome lxxii, No. 13.

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7 s 71

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## REPORT OF THE EXECUTIVE COMMITTEE.

The Executive Committee of the Board of Regents respectfully submit the following report in relation to the funds of the Institution, the receipts and expeuditures for the year 1871, and the estimates for the year 1872:

## Statement of the fund at the beginning of the year 1872.

The amount originally received as the bequest of James Smithson, of England, deposited in the Treasury of the United States, in accordance with the act of Congress of. August 10, 1846
$\$ 515,16900$
The residuary legacy of Smithson, received in 1865, deposited in the Treasury of the United states, in accordance with the act of Congress of February 8, 1867

26,210 63
Total bequest of Smithson . .......................... ....... 541,37963
Amount deposited in the Treasury of the United States, as authorized by act of Congress of February 8,1867 , derived from savings of income and inerease in value of investments

108, 62037
Total permanent Smithson fund in the Treasury of the United States, bearing interest at 6 per cent., payable semi-annually in gold
$\$ 650,00000$
In addition to the above, there remains of the extra fund derived from savings, \&c., in Virginia bonds, at par value $\$ 88,125.20$, now valued at.

35,50000
The cash balance iu First National Bank, January, 1872
$\$ 16,31502$
Amount of cóngressional appropriation for the fiscal year ending June $30,1872, \$ 10,000$, one-half of which available January, 1872.. 5, 00000

$$
21,31502
$$

Total of Smithson funds January, $1872 \ldots \ldots . . .$.
The interest due on the Virginia bonds, instead of being paid, has been funded by the State, and has thus increased the amount of the bonds from $\$ 72,760$, as stated in the last report, to $\$ 88,125.18$, as given in the foregoing statement. The market value of this stock, which was
given last year at $\$ 48,000$, has fallen, during 1871 , to $\$ 35,500$, on account of the uncertain policy of the State.

The balance at the beginning of the year 1872 , viz, $\$ 21,315.02$, is very nearly the same as that at the beginning of the year 1871, which was $\$ 21,477.81$. This balance is not invested as a part of the perma, nent fund, because it is required in order to pay cash for bills as they become due, and previous to receiving the semi-annual income.

## Statement of receipts from the Smithson fund for 1871.

$$
\text { Interest on } \$ 650,000 \text {, at } 6 \text { per cent. in gold ............... } \$ 39,00000
$$

Premium on gold, June and December, $12 \frac{5}{8}$ and $8 \frac{7}{8}$
4, 19250
Total receipts. 43,19250

Statement of expenditures from the Smithson fund for 1871.

## BUILDING.


Furniture and fixtures. . . . . . . . . . . . . . . . . . . . . $205 \cdot 29$
$\$ 9,03241$
GENERAL EXPENSES.
Meetings of the board. ............................... . $\$ 12712$
Lighting the bailding. .............................. . . . 26715
Heating the building .............................. . 7969
Postage . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 44876
Stationery . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 45255
Incidentals . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 35475
Salaries and clerk hire. . . . . . . . . . . . . . . . . . . . . . . 9,572 62

## PUBLICATIONS AND RESEARCHES.

Smithsonian Contributions, quarto ............. \$9, 753 68
Miscellaneous collections, octavo ............... 60812
Reports, octavo ..................................... . . . 73948
Meteorology, computations, rain-gauges, \&c... 2,000 55
Apparatus for researches ......................... 74403
Explorations, natural history, and archæology - 1,301 07
Lectures 28500

15,43193

## MUSEUM, LIBRARY, AND EXCHANGES.

Musenm, in addition to the sum drawn from the
appropriation by Congress, $(\$ 4,976) . . . . . .$. . $\$ 8,13295$
Literary and scientific exchanges through agencies in London, Paris, Leipsic, Amsterdam, Milan, \&c. $\$ 4,20150$
Purchase of books and periodicals.
25386
\$12, 58831

## Total expenditures, (repayments having been de- <br> ducted) <br> \$48, 35529

From the above statement, it appears that the expenditures were $\$ 5,162.79$ in excess of the receipts; but to meet this deficiency, $\$ 5,000$ of the congressional appropriation for the museum, as was stated before, is still in the Treasury of the United States. Had this sum been drawn during the year, it would have been deducted from the $\$ 8,132.95$ charged to the museum.

During the past jear the Institution has advanced money for the payment of freight on specimens and articles directed to its care, and for fitting out the expedition toward the north pole. It has also sold publications, old and useless material, and meteorological instruments, the payments for which hare been deducted from the several items of the previous accounts of expenditures, as follows:
From the museum, for repayments for freight. .............. $\$ 592 \quad 92$
From exchanges, for repayments on expense of literary and
scientific exchanges ......................................................... $945 \quad 17$
From explorations, for repayments on account of Hall's expedi-
tion toward the north pole, \&c............................... 52227
From Smithsonian contributions and miscellaneous collections, for sales of publications 52570
Building and incidentals general, repayments for old material, postage refunded, \&c. 62259
Apparatus-sale of meteorological apparatus ..... $40 \quad 00$
Total repayments and miscellaneous credits ..... 3,248 65
Appropriations and expenditures from Congress on account of the museum and care of the Government collections.

In addition to the receipts from the Smithson fund, the following amounts have been received :
From appropriation by Congress for fitting up halls for collections.
$\$ 20,00000$
From appropriation by Congress for annual care of collections, being part of the $\$ 10,000$ appropriated for the fiscal year ending June 30, 1871, ( $\$ 5,024$ having been drawn in the year 1870)

The appropriation of $\$ 20,000$ was expended, under the direction of the Secretary of the Interior, and accounted for to that Department, in ceiling, flooring, plastering, and painting the large hall in the upper story of the main building, repairing the roof, fire-proofing the west wing, and fitting up the basement of the same for the preparation of specimens and storage.

The appropriation of $\$ 4,976$ was expended for salaries, taxidermy labor, \&c., in preserving the Government collections, and was accounted for to the Interior Department.

The estimates for the year 1872 are as follows:

## RECEIPTS.

From interest on the permanent fund . . . . . . . . . . . . . . . . . . $\$ 39,00000$
Probable premium on gold, 10 per cent . . . . . . . . . . . . . . . . . $3,900 \quad 00$
42,900 00

## APPROPRIATIONS.

For building . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . $\$ 5,00000$
For general expenses . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10,00000
For publications and researches . . . . . . . . . . . . . . . . . . . . . . 20,00000
For exchanges . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5, 000 . 00
For books and apparatus . . . . . . . . . . . . . . . . . . . . . . . . . . . . 90000
For museum, additional to Congress appropriation ..... . 2,00000
42,900 00
The Executive Committee have examined seven hundred and fiftyseven receipted vouchers for payments made during the four quarters of the year 1871, both from the Smithson fund and the appropriations from Congress. In every voucher the approval of the Secretary of the Institution is given, and the certificate of an authorized agent of the Institution is appended, setting forth that the materials and property and services rendered were for the Institution, and to be applied to the purposes stated.
The quarterly accounts-current, bank-book, check-book, and ledger have also been examined and found correct, showing a balauce in bank December 31, 1871 , of $\$ 16,315.02$.
Respectfully submitted.

> PETER PARKER, JOHN MACLEAN, Executive Committee.*

March 13, 1872.

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## JOURNAL OF PROCEEDINGS

OF

## THE BOARD OF REGENTS <br> OF THE

SMITHSONIAN INSTITUTION.

Washington, D. C., January 25, 1872.

A meeting of the Board of Regents of the Smithsonian Institution was held this day in the Regents' room, at 7 o'clock p. m. Present: Hon. H. Hamlin, Hon. L. Trumbull, Hon. G. Davis, Hon. L. P. Poland, Hon. S. S. Cox, Hon. P. Parker, Hon. H. D. Cooke, and Professor Henry, the Secretary.
Mr. Hamlin was called to the chair.
The Secretary stated that an act of Congress had substituted the governor of the District of Columbia as an ex-officio Regent, in place of the mayor of Washington, the latter office having ceased to exist. Governor Cooke was then introduced as a member of the Board.
Dr. Parker, from the Executive Committee, presented a preliminary statement of accounts.
On motion of Mr. Trumbull, the report was accepted.
The Secretary made a statement relative to the Virginia stocks held by the Institution. It had been deemed advisable that the registered stock should be converted into coupon bonds, because the coupons were receivable for taxes, and the State had not paid interest on its stock for several years. The transfer had therefore been made for the Institution by Riggs \& Co.
On motion of Judge Poland, the Secretary was directed to deposit the Virginia coupon bonds, now in Riggs' Bank, in the Treasury of the United States for safe-keeping.
The Secretary gave an account of the improvements made in the building during the past year.
A communication from Dr. C. H. F. Peters, of the observatory at Clinton, New York, was read, asking the Institution to defray the expense and act as the medium of communicating discoveries of planets, comets, etc., by ocean telegraph.
The Secretary stated that he had applied to the ocean telegraph company for the free transmission of astronomical discoveries, but had not received a reply.
Several of the Regents expressed the opinion that the Institution
should have the franking privilege, to enable it to distribute scientifie reports, \&c., to libraries and other institutions of the country.

The Secretary stated that a stable had recently been erected on the grounds, with the approval of General Babcock, Commissioner of Public Buildings. This was necessary for the use of the Institution, though the horse and carriage used by the Secretary had been purchased by himself.

On motion of Mr. Trumbull, the action of the Secretary was approved.
A claim,.presented by T. R. Peale, es $\dot{q}$., of Washington, for a portrait of Washington, painted by his father, Charles Wilson Peale, now in the Smithsonian museum, was referred to the Executive Committee.

A communication was presented from Henry O'Rielly, relative to the discovery of the electro-magnetic telegraph, which, on motion of Mr. Davis, was read, and ordered to be placed in the archives of the Institution.

Adjourned to meet at the call of the Secretary.

March $28,1872$.
A meeting of the Board was called for this evening at 7 o'clock. Present: Hon. S. P. Chase, Chancellor of the Institution; Hon. L. P. Poland, Hon. J. A. Garfield, Hon. P. Parker, and Prof. Henry, the Secretary.

On account of a night session of the Senate, the Vice-President, Hon. Mr. Colfax, and Senators Trumbull and Hamlin were prevented from attending the meeting.

No quorum being present, adjourned to meet at the call of the Secretary.

APRIL 3, 1872.
A meeting of the Board of Regents was held at $70^{\circ}$ clock at the Institution. Present: Vice-President Colfax, Hon. H. Hamlin, Hon. L. Trumbull, Hon. L. P. Poland, Hon. P. Parker, Hon. H. D. Cooke, and Prof. Henry, Secretary.

Mr. Colfax was called to the chair.
The minutes of the previous meeting were read and approved.
Dr. Parker, in behalf of the Executive Committee, presented the report of the committee, which was read, and, on motion of Mr. Hamlin, accepted.

Dr. Parker stated that the Virginia coupon bonds which had been received from the State had no seal affixed to them. In regard to this, the Secretary presented the following communication from Jos. Majo, jr., treasurer of Virginia :

Commonwealth of Virginia, Treasurer's Office, Richmond, March 30, 1872.
The following coupon bonds Nos. 11521 to 11578, both inclusive, for $\$ 1,000$ each ; No. 1380 for $\$ 500$, and Nos. 4191 and 4192 for $\$ 100$ each, of

Virginia consolidated debt, exchanged December 9, 1871, for the Smithsonian Institution, and standing in its name on the books of this office, were regularly issued and are good and valid. The omission of the State seal upon them was an inadvertance, which will be corrected whenever the bonds are returned for the purpose. In fact the seal is not necessary to give validity to the bonds, though it is customary to place it upon them.

Very respectfully, yours,

> JOS. MAYO, Treasurer of Virginia.

On motion of Mr. Hamlin, it was
Resolved, That the Secretary return the bonds to Richmond for the purpose of having the State seal affixed to them.
The Secretary gave an account of Major Powell's expedition, which was authorized by Congress at its last session and had by law been placed under the direction of the Smithsonian Institution. He stated that he had addressed a communication to Congress recommending an additional appropriation for continuing the survey.
The Secretary stated that, for many years, harmonious relations had existed between the Institution and the Department of Agriculture for co-operation in advancing the science of meteorology. The blanks had been furnished and distributed by that Department, and the observers sent their returns to the Commissioner, saving a large item of expense in the way of postage. The monthly summaries of observations of rain, temperature, etc., had been published in the monthly reports of the Department, and had done much to encourage and stimulate the observers and to furnish valuable data for agricultural and scientific purposes. Judge Watts, the present Commissioner, had recently decided, however, to discontinue this publication, and this was an additional reason why the Institution should have the franking privilege. The Institution had a large number of computers at work in reducing and discussing all the meteorological observations it had collected during the last twenty years, and would soon publish the results.
The Seeretary presented his annual report for the year 1871, which was read, and, on motion of Mr. Trumbull, accepted.

A communication from F. O. J. Smith, esq., of Portland, relative to the electro-magnetic telegraph, was presented to the Board, and urdered to be placed in the archives.
The board then adjourned sine die.

## GENERAL APPENDIX

TO THE
SMITHSONIAN REPORT FOR 1871.

The object of this appendix is to illustrate the operations of the Institution by reports of lectures and extracts from correspondence, as well as to furnish information of a character suited especially to the meteorological observers and other persons interested in the promotion of knowledge.

## MEMOIR OF SIR JOHN FREDERICK WILLIAM HERSCHEL.

By N. S. Dodge.


#### Abstract

About the year 1760, as Dr. Miller, the organist, better known, perhaps, as the historian of Doncaster, England, was dining at Pontefract with the officers of the Durham militia, one of them told him that they had a young German in their band who was an excellent performer on the violin, and if he would step into another room he might judge for himself. The invitation was gladly accepted, and Miller heard a solo of Giardini's executed in a manner that surprised him. Learning afterward that the engagement of the young musician was only from month to month, he invited him to leave the band and come and live with him. "I am a single man," he said, "and we doubtless shall be happy together; beside, your merit will soon entitle you to a more eligible situation." The offer was accepted as frankly as itwas made; and the satisfaction with which the old organist always plumed himself upon this act of generous feeling is not surprising, since the German hautboy-


 player turned out at last to be Herschel the astronomer.The Jow Snetzler, a famous organ-builder a hundred years and more ago, was at this time setting up a new organ for the parish church of Halifax. Herschel, at Dr. Miller's advice, became one of the seven candidates for the place of organist. They drew lots how they were to perform in succession. Herschel drew the third. The second fell to Dr. Wainwright, of Manchester, whose rapid execution astonished the judges. "I was standing in the middle aisle with Herschel," wrote Dr. Miller, "and I said to him, 'What chance have you to follow this man?' He replied, 'I don't know; I am sure fingers will not do.' He ascended the organ-loft, however, and produced from the instrument so uncommon a fullness, such a volume of slow, solemn harmony, that I could not account for the effect. After a short extempore effusion, he finished with the old Hundredth Psalm tune, which he played better than his opponent. 'Ay, ay,' cried old Snetzler, 'tish is very goot; 'I vill luf tish man, for he gives my piphes room for to spheak.'" Having afterward asked Mr. Herschel by what means he produced so uncommon an effect, he replied, "I told you fingers would not do;" and taking two pieces of lead from his pocket, "One of these," he said, "I placed on the lowest key of the organ and the other on the octave above; thas, by accommodating the harmony, I produced the effect of four hands instead of two."

In 1780, twenty years after this, when Miller talked of his friend Herschel's great fame, and of his sister, Caroline Herschel, who, when her brother was asleep, amused herself in sweeping the sky with his twenty-
feet reflector and searching for comets, the kind-hearted old man used to wish that the science of acoustics had been advanced in the same degree as the science of opties, "For," he said, "had William constructed auditory tubes of proportionate power to his great telescope, who knows but we might have been enabled to hear the music of the spheres!" From this date, fourscore and twelve years ago, until the present time, no name among modern scientific men has attained a higher rank thau that of Herschel. Ninety volumes of the Philosophical Transactions of the Royal Society have been enriched with papers bearing the wellknown signature. Genius, though often hereditary, is quite as often wayward. It not unfrequently skips a generation. It descends sometimes to daughters. It reappears in other cases, after being dormant in children and grandchildren, in a fourth or fifth step of descent. But with the two Herschels the transmission was immediate. The original circumstances of the two great philosophers were indeed widely different. Sir William, the father, by genius and application succeeded in rising from obscurity to the proud position of the first astronomer of the age. His son, Sir John Herschel, had the advantage of the highest university training. But both were gifted with extraordinary talents, keen scientific tastes, and those great mathematical powers which so materially assist in abstruse inquiries. In the case of the subject of this memoir, the combination of high education with an extraordinary natural talent for communicating his thoughts in an attractive manner, has been one of the means of making him the most distinguished philosopher of the nineteenth century.

John Frederick William Herschel was born at Slough, March 7, 1792. His father was already famous. People came from distant lands to see the great telescope. There are traditions about the wonder with which mail-travelers used to stare, in passing, at the mechanism by which the monster tube was used. A thousand stories of its revelations passed current among the vulgar. The astronomer let nobody use his forty-foot telescope, but the fame of it could not be hidden. It went through all the civilized world. And it was under the shadow of that mysterious erection that this only child of the house-born when his father, then of twoscore and twelve years, was absorbed alike in the fame he had achieved and the wonders he was every night discovering; reared in infancy with an uncle who spent his days in adjusting instruments, and an aunt whose nights were devoted to discovering new comets in the heavens ; without a boy's associations and playmates, in a house kept quiet all the day that the star-watchers might sleep; and wandering through rooms whose silence no sports were permitted to disturb and no youthful buoyancy to interrupt-it was here that he passed his boyhood. Twelve years before the boy's birth the "Observations of the periodical star Mira Ceti," read before the Royal Society, had establishéd his father's position among scientific men, and one year later his discovery of Uranus brought him into the foremost rank of astronomical observers.

Amid such a childhood, separated from boys of his own age, suppressed in every demonstration which youthful spirits naturally give to feeling, without the school antagonisms that teach a lad his real worth, or the school rivalries that lead him to rate his fellow according to the plucky boyhood he exhibits, at the form or on the play-ground, in the dormitory or at the sparring-match, it is strange that the boy did not grow up full of eccentricities. His detractors-and even he, the gentlest of men, was not without them-say that he did. But there was in him, from first to last, no lack of manliness, no insincerity, no jealousy, no indifference even to rival merit. And then the man's life-long and conspicuous veneration for his father is perhaps the best proof of a happy childhood and youth. No want pinched the household; warm affection existed between the parents; the boy was the idol of a fond aunt and a fonder uncle; and it must have been from a happy home that he went to Eton.
At the usual period of life young Herschel entered St. John's College, Cambridge, from which he graduated B. A. in 1813, as senior wrangler, having for his competitors the late Dr. Peacock, Dean of Elr, who was second wrangler, and the late Rev. Fearon Fallows, formerly astronomer at the Cape of Good Hope, as third wrangler. The names of several other men of mark appear in the honor-list as contemporary students, such as Professor Mill, Dr. Robinson, Master of the Temple, and Bishop Carr, of Bombay. Mr. Herschel had no sooner attained his degree than he forwarded a mathematical paper to the Royal Society, "On a remarkable application of Cotes's Theorem." This was published in the Philosophical Transactions. In the same year he was elected a Fellow of the Royal Society, and though barely past his majority became at once an active member.

The early researches of Herschel were confined to pure mathematics. For papers on this subject, published in the Philosophical Transactions, the Copley medal was awarded him in 1821. In 1822 he turned bis attention to "observing" astronomy, that practical branch which descended to him as a hereditary duty. This occupation led him to associate with others in forming a special society for the general advancement of astronomical science. A few years previous to the death of his father, in consequence of the improvement in astronomical telescopes, amateur observers sprang up, who took great interest in the delineation of the heavens. It was considered an epoch favorable to the formation of a body that should be exclusively devoted to the encouragement of astronomy; and Mr. Herschel drew up an address which forms the first publication of the present Royal Astronomical Society.
all the while, however, the imagination of the young philosopher was dwelling on the last discovery of his father-the binary stars. It was a secret, won from the unknown, that opened a new view into the universe. The boy was scarcely in adolescence, the father passing into old age, when the constitution of the nebulæ was an-
nounced. It was the great achierment of the one; it was the first dictate to the young manhood of the other. Three years of conversation and thought passed away, when the son, then twenty-four, took from his father, then seventy-eight, the work of examining the double stars. The old man's end of life was gained. What of nobility was in him had descended right royally. In the space of five years the young astronomer had mapped 380 double and triple stars, obtained by abore 10,000 separate measurements. The record of these observations was acknowledged by the French Academy of Sciences in bestowing their astronomical medal, and followed by a similar reward in England. This occurred in 1824. The old astronomer had foreseen the honors which his son would win, but did not live to rejoice in them. Sir William had died two years before. With his death came great changes to the pleasant family at Slough. The good mother survived, indeed, but the strange ${ }_{4}$ ancient household was broken up. The aunt, who had watched the clock and catalogued the stars up to the last, returned to her old home in Germany. The cheerful old uncle had desisted from mechanical adjustments only when apoplexy felled him at his work, and the young inheritor of all the honors was left to perform his task alone.
To those who have had no experience in continuous astronomical observations there can be no conception of its anxious toil. Money cannot repay it, nor honors, nor fame. In the pursuit day must be turned into night, society abandoned, the round of home comforts broken in upon, intercourse with friends and neighbors discontinued; and the astronomical observer, quitting all the amenities of life, finds his compensation in the brotherhood of the stars. This self-sacrifice young Herschel made. The objects to observe required a calm atmosphere. The best time for this is between midnight and sun-rise. This continuous night-work requires health. Herschel felt the severity of it. "Should I be fortunate enough," he writes, when he was but thirty years old, " to bring this work to a conclusion, I shall then joyfully yield up a subject on which I have bestowed a large portion of my time, and expended much of my health and strength, to others who will hereafter, by the aid of those masterpieces of workmanship which modern art places at their disposal, pursue with comparative ease and convenience an inquiry which has presented to myself difficulties such as at oue period had almost compelled me to abandon it in despair."
In 1831 Mr . Herschel received the honor of knighthood from the hands of King William, in acknowledgment of his eminent scientific services.

In 1833 he was awarded the royal medal of the Royal Society for his paper "On the investigation of the orbits of revolving domble stars." The Duke of Sussex then said of him, " Sir John Herschel has devoted himself for many years, as much from filial piety, perhaps, as from inclination, to the examination of those remote regions of the universe into which his illustrious father first penetrated, and which he trans-
mitted to his son as a hereditary possession, with which the name of Herschel must be associated for all ages. He has subjected the whole sphere of the heavens within his observation to a repeated and systematic scrutiny. He has determined the position and described the character of the mostremarkable of the nebulæ. Hehas observed and registered many thousand distances and angles of position of double stars, and has shown, from comparison of his own with other observations, that many of them form systems whose variations of position are subject to invariable laws. He has succeeded, by a happy combination of graphical construction with numerical calculations, in determining the relative elements of the orbits which some of them describe round each other, and in forming tables of their motions; and he has thus demonstrated that the laws of gravitation, which are exhibited, as it were, in miniature in our own planetary system, prevail also in the most distant regions of space-a memorable conclusion, justly entitled, by the generality of its character, to be considered as forming an epoch in the history of astronomy, and presenting one of the most magnificent examples of the simplicity and universality of those fundamental laws of nature by which their great Author has shown that he is the same to day and forever, here and everywhere."
It is impossible to give any analysis of the results of the numerous researches which occupied the time of Sir John Herschel at the various periods of his life. From a rough and evidently incomplete list of his papers it would appear that out of seventy, twenty-eight are on astronomical subjects, thirteen on optics, ten on pure mathematics, eight on geology, and eleven on miscellaneous science.

There are, however, two of his astronomical works to which we may fittingly refer here, since they furnish a key which unlocks much of Sir John's personal history. These are, first, his "Catalogue of nebulæ and clusters," published in the Philosophical Transactions for the year 1833, for which the gold medals of the Royal Society and the Astronomical Society were awarded; and, second, "Results deduced from observations made at the Cape of Good Hope." For this latter work he received the Copley medal for the second time from the Royal Society, and an honorary testimonial from the Astronomical Society.
The interest which Sir John Herschel always exhibited in the minute details of nebulæ and double stars must be considered as the result of his association with his illustrious father. M. Arago, in his admirable and exhaustive biographical notice of Sir Willian Herschel, translated from the French, and published recently in the report of the Smithsonian Institution, refers gracefully to this fact. Sir John's early familiarity with his father's instruments, in familiarity with which he may be said to have grown up, and with their necessary use in making observations, had its influence doubtless in the same direction. Hence, probably, the reason why so long a period of his observing time was devoted to this section of astronomical research. One of his first communications to the
memoirs of the Astronomical Society is an account of the great nebulæ of Andromeda and Orion, accompanied by an admirable engraving of the latter. From 1825 to 1833 nearly all his astronomical energies were given to this kind of observation. The catalogue of nebulæ and clusters, previously mentioned, contains a list of more than twenty-five hundred of both; their right ascensions and declinations determined; the character of their general appearance recorded; and those which present an unusual constitution, or an extraordinary shape, (of which there are nearly one hundred,) are drawn with a precision, delicacy, and taste worthy of the most accomplished artist. The astronomer royal, on presenting the gold medal of the Astronomical Society to Captain Smyth, on behalf of Sir John Herschel, who was then residing at the Cape of Good Hope, remarks: "That one of the most important parts of this work is the division containing the engraved representations of the most remarkable nebulæ. The peculiarities they represent cannot be described by words nor by numerical expressions. These drawings contain that which is conspicuous and distinctive to the eye, and that which will enable the eyes of future observers to examine whether secular variation is perceptible. They are, in fact, the most distinct and most certain records of the state of a nebulæ at a given time."
The second series of investigations to which it is desired here to draw especial attention, is that described in the unique volume entitled "Results of Astronomical Observations made during the years 1834-1838, at the Cape of Good Hope; being the completion of a telescopic survey of the whole surface of the visible heaveus." After the publication of the catalogue of nebulæ in 1833, Sir John Herschel determined to undertake a voyage to South Africa, for the purpose of continuing his researches in another hemisphere under a new heaven. He had the same plan in view and the same instruments. It had been irksome to his honored father, and was alike fretful to his own spirit, that the clouded sky of England allowed free sweep of the great telescope along the path of the stars at a rate so niggardly. Hardly more than thirty hours in thrice that number of nights were the mysteries of the great vault exposed to his search. He resolved, therefore, to seek a clearer atmosphere and a wider field of inquiry. The southern extremity of Africa, where was an English colony, in which seclusion could be found without loss of means of communication with the philosophic world, and an unclouded sky bending above a healthy climate, seemed to offer the greatest advantages. He consequently fixed upon the Cape of Good Hope as the most fitting place for a protracted residence away from England, and the broadest field for thorough researches.

Sir John Herschel embarked at Portsmouth, in company with his family, on the 15th of November, 1833, and arrived safely at Table Bay on the 18th of January, 1834, after a pleasant royage, diversified by few nautical incidents.

No one knew so well as the great astronomer of whom we write, eten
before, while recumbent on the deck of the vessel that was bearing him through the tropic zone, he watched for hours together the shifting panorama of the star fretted vault, how the moon appeared brighter, fairer, and better defined through a more transparent atmosphere; how the planets seemed to be other orbs; how the stars, long watched in a northern sky, drooped toward the horizon, and were at length looked for in vain; how orbs, which, to his former vision, had modestly moved along the southern outskirts of visible creation, now marched majestically overhead, each

> "Walking the heavens like a thing of life,"
while new and strange bodies ascended high and higher, until the old earth had passed away and a new heaven was aloft; nor how the Via Lactea, in the neighborhood of the Centaur and the Cross, coupled with profuse collections of nebulæ and asteroids, stars and constellations, makes the southern sky the most magnificent star-view from any part of earth. Like the sources of the Nile to the untraveled geographer, or the ice-cliffs of Greenlaud to the student of arctic royages, he knew well what a personal inspection would place before him, and though the civilized world rang with applause at his sacrifice of home and its comforts, and country aud its honors, for the sake of science, yet true philosophers knew that the compensation, present and future, far outweighed the loss.

After a temporary residence at Wilterfreiden, he engaged a suitable mansion, bearing the name of Feldhausen, about four miles from Cape Town-a spot full of rural beauty, within sight of lofty hills, and situated on the last of the terraced slopes by which Table Mountain lets itself down to the lowlands and meadows near the sea. In this place, removed from all the noise of traffic and exposure to intrusion, surrounded on all sides by a grove of planted trees, he caused a suitable building to be erected for the equatorial, while the 20 -foot reflector was mounted in the open air.

The observatory at Feldhausen was situated in south latitude $33^{\circ} 58^{\prime}$ $55^{\prime \prime} 56$, and longitude $22^{\circ} 46^{\prime} 9^{\prime \prime} 11$ east from Greenwich. Its altitude was 142 feet above the level of the sea in Table Bay. During the erection of his instruments, Sir John resided at Welterfreiden, and so quickly were his plans completed, that on the 22d of February, 1834, he was enabled to gratify his curiosity by viewing, with his 20 -foot reflector. 9 Crucis, the interesting nebula about $\eta$ Argus, and ou the evening of the 5th of March to begin a regular series of observations.

After erecting his observatory and determining its geographical position, the attention of Sir John was directed to the fitting up of the telescope with which his observations were to be made. He had carried out with him three specula, one of which was made by his father, and used by him in his 20 -foot sweeps; another was made by Sir John himself, under his father's inspection and instructions, and the other, of the very same metal as the last, was ground and figured by himself alone.

They had each a clear diameter of $18 \frac{1}{4}$ inches of polished surface, and were all equally reflective when freshly polished, and perfectly similar in their performance. The operation of re-polishing, which was more frequently required than in England, was performed by himself with the requisite apparatus, which he also brought from England.

Although Sir John Herschel never exhibited-as indeed he had no occasion to do-the wonderful mechanical genius of his father, he nevertheless fully understood all the former's methods of preparing and treating specula. When it was stated at a meeting of the British Association in 1842, that Lord Ross had attained such skill in the treatment of metallic specula that he could dismount the mirror of his large telescope, repolish it, and replace it the same day, Sir John four years previously had written to Arago these words: "By following my father's rules minutely and using his apparatus, I have succeeded in a single day, without the least assistance, in polishing completely three Newtonian mirrors of nineteen-iuch aperture."

In the use of reflecting specula of considerable weight, it is of the utmost importance that the metal should be supported in its case so as not to suffer any change of figure from its own weight. Sir John found that a speculum was totally useless by allowing it to rest horizontally on three metallic points at its circumference. The image of every considerable star became triangular, throwing out long flaming caustics at the angles. Having on one occasion supported the speculum simply againsta flat board, inclined atan angle of about $45^{\circ}$, he found that its performance was tolerably good; but on stretching a thin pack-thread rertically down the middle of the board, so as to bring the weight of the metal to rest upon the thread, the images of the stars were lengthened horizontally " to a preposterous extent, and all distinct- vision utterly destroyed by the division of the mirror into two lobes, each retaining something of its parabolic figure, separated by a vertical band in a state of distortion, and of no figure at all!" The method which Sir John found the best was the following: Between the mirror and the back of the case he interposed six or seven folds of thick woolen baize, of uniform thickness and texture, stitched together. at their edges. The metal, when laid flat on this bed, was shaken so as to be concentric with the rim of the case, and two supports, composed of several strips of similar baize, were introduced so as to occupy about $30^{\circ}$ each, and to leare an arc of about $40^{\circ}$ unoccupied opposite the point which was to be lowermost in the tube. "When the case is raised into an inclined position, and slightly shaken, the mirror takes its own free bearing on these supports, and preserves its figure. It is essential, however, to the successful application of this method that many thicknesses of the baize should be employed, by which only the effect of flexure in the wooden back of the case can be eliminated."

This simple plan, adopted by Sir John Herschel, is mentioned to show how mechanical genius aided him, as it did his father before
him, in overcoming what had seemed to be insurmountable difficulties. The ingenious method by which Lord Ross afforded an equable support to a large speculum, and which is now generally adopted, was then unknown to him.
The labors of Sir John Herschel in South Africa were chiefly confined to different subjects of observation. Stellar astronomy, however, occupied his principal attention. Two of the most celebrated ne-bulæ-that in the sword-handle of Orion and that surrounding the variable star Eta Argus, as well as portions of the Milky Way, he delineated with particular care. The published drawings of these objects are acknowledged by all astronomers to be the most perfect representations of these beautiful ornaments of the southern sky. The nebula of Orion, magnificent as it is north of the equator, comes out in much grander detail in the southern hemisphere, where its great elevation in the heavens renders it comparatively free from the ill effects of an impure atmosphere. During the cooler months at the Cape of Good Hope, from May to October inclusive, and more especially in June and July, the finest opportunities for delicate astronomical observation occurred, and were quite equal to the observer's most sanguine expectations. Sir John remarks that the state of the atmosphere in these months was habitually good, and imperfect vision rather the exception than the rule. The best nights, when the stars were most steady, always occurred after the heavy rains had ceased for a day or two, when "the tranquillity of the images and sharpness of vision was such that hardly any limit was set to magnifying power, but what the aberrations of the specula necessitated."
Upon occasions like these Sir John found that optical phenomena of extraordinary splendor were produced by viewing a bright star through diaphragms of card-board or zinc, pierced in regular patterns of circular holes by machinery. These phenomena, arising from the interferences of the intromitted rays, and produced less perfectly in a moderate state of the air, surprised and delighted every one. A result of a more interesting kind was obtained when the aperture of the telescope had the form of an equilateral triangle, the center of which coincided with the center of the speculum. When close double stars were viewed with the telescope, having a diaphragm of this form, the dises of the two stars, which are exact circles, are reduced to about a third of their size, and possess a clearness and perfection almost incredible. These discs, however, are accompanied with six luminons radiations running from them at angles of $60^{\circ}$, forming straight, delicate, and brilliant lines, like illuminated threads, reaching far beyond the sea of view, and capable of being followed like real appendages to the star, long after the orb itself had left the field.

Another optical phenomenon, arising from a peculiar condition of the atmosphere, is described as "nebulous haze." The effect of it was to encircle every star of the ninth magnitude and upward with a faint
sphere of light of an extent proportioned to the brightness of the star. This phenomenon presented itself very suddenly in a perfectly clear sky, free from suspicion of mist or cloud, and disappeared as suddenly after the lapse of about a hundred seconds. Sir John Herschel stated that similar nebnlous affections occurred in England, but with less frequency of coming and going. He at first suspected that the phenomena arose from dew upon the eye-piece; but repeated observations satisfied him that they were atmospheric.

Under the favorable circumstances in which he was now placed, the opportunity of studying the grand nebula in the sword-handle of Orion was eagerly embraced. He had himself delineated this remarkable object in 1824. Four representations of it, differing essentially from his, had been subsequently published, and it therefore became of the deepest interest to discover the causes of these discrepancies, and to ascertain whether in form or light a change had taken place. The splendid drawing of this nebula, twelve inches square, is viewed with mute admiration. The mysterious assemblage of suns and systems which it sets before the observer is at first almost overlooked in his wonder at the patience and skill of the artist astronomer. No fewer than one hundred and fifty stars are accurately depicted, and the faint luminosity shades away on the picture, as in the heavens, into the dark sky. That this marvelous thing of beauty, having no relation to the stars which bespangle it and no union with the stars themselves, las recently undergone or is undergoing great and rapid changes, Sir John did not believe. He writes: "Comparing my only drawings made at epochs (1824 and 1837) differing by thirteen years, the disagreements, though confessedly great, are not more so than I am disposed to attribute to inexperience in such delineations, (which are really difficult) at an early period; to the far greater care, pains and time, bestowed upon the later drawings; and, above all, to the advantage of local situation, and the very great superiority in respect both of light and defining power in the telescope at the latter, over what it possessed at the former epoch, the reasons of which I have already mentioned. These circumstances render it impossible to bring the figures into comparison, except in points which cannot be influenced by such causes. Now there is only one such particular on which I am at all inclined to insist as evidence of change, viz: in respect of the situation and form of the 'nebula oblongata,' which my figure of 1824 represents as a tolerably regular oval. Comparing this with its present appeanance, it seems hardly possible to avoid the conclusion of some sensible alteration having taken place. No observer now, I think, looking ever so cursorily at this point of detail, would represent the broken, curved, and unsymmetrical nebula in question as it is represented in the earlier of the two figures, and to suppose it seen as in 1837, and yet drawn in 1824, would argue more negligence than I can believe myself fairly chargeable with."

The magnificent Catalogue of Nebulæ and Clusters of Stars in the

Southern Hemisphere, comprehending 4.015, was reduced, arranged, and executed by Sir John's own hands, and appears like the work of a lifetime.
In treating of the Magellanic clouds, two fine eye-sketches are given, "drawn without telescopic aid, when seated at a table in the open air, in the absence of the moon, and with no more light than was absolutely necessary for executing a drawing at all." He was compelled to this method in consequence of his attempts to represent other than very small portions of the Nubecula Major in the telescope, having been completely baffled by the perplexity of its details.
On the 25th of October, 1837, Sir John was fortunate enough to obtain a view of the anxiously expected comet of Dr. Halley. In the fifth chapter of the "Astronomical Observations" he has given the results of his notice of this singular member of our solar system. Thirteen drawings illustrate the comet. We have it as it appeared night after night. On the 1st of November he describes its nucleus as small, bright, and lighly condensed, shielded on the side next the sun by a narrow crescent of vivid, nebulous light, the front presenting an outline nearly circular, and having an amplitude of $90^{\circ}$ from horn to horv. Four days afterward it had the common appearance of a comet, with its nucleus and slightly diverging tail ; but on its return from the sun, on the 26 th of January, it assumed a new and surprising appearance. Its head was sharply terminated "like a ground-glass lamp-shade, and within thishead was seen a vividly luminous nucleus, as if a miniature comet, perfect.in itself, possessing head and tail, and considerably exceeding the surrounding head in intensity of light;" in fact, a comet within a comet. As the nights followed each other, and the stranger advanced across the heavens, its increase in dimensions was so rapid "that it might be said it was almost seen to grow." On the 26th the nucleus appeared as a star of the tenth magnitude, furred and nebulous, and more than double in size within twenty-four hours. On the 28th, upon looking through the 20 -foot reflector, Sir John exclaimed, "Most astonishing! The coma is all but gone, and there are long irregular tails everywhere." The nucleus was then a sharp point, like one of Jupiter's satellites in a thick fog of hazy light-no well defined disk could be raised upon it-and its body was clearly discernable from its coma. "I can hardly doubt," he writes, "that this comet was fairly evaporated in perihelio by the sun's heat, resolved into transparent vapor, and is now in process of rapid condensation and reprecipitation on the nucleus."
Sir John concludes his "astronomical observations" by notices of the salar spots, and conjectures of their causes. Thirteen figures, delineated from magnified images formed on a screen by means of a 7 -foot achromatic refractor, are given in a single plate. One of these spots occupied an area equal to $3,786,000,000$ square miles. Of one huge spot he makes no measurement. Of another, not one tenth in size, he says, "Its black center would have allowed the globe of our earth to drop
through it, leaving a thousand miles clear of contact on all sides of that tremendons gulf." Of his theories of the causes of these vast spots ou the surface of the sun no mention need here be made. GGalileo, Kepler, Huygens, Kant, Lambert, and others, each gave their views upon these recondite phenomena. Sir John Herschel gave his as his father had done before him. Others are giving, and others still, perhaps as accurate observers and logical reasoners as eitherof the two, will give theirs. The world can afford to wait. Astronomy advances. It may be, in the distant future, that the mysterious center around which our sun and his worlds revolve may be detected and afford a solution for other mysteries as well as these. The greatest astronomer is equipped for no more than a Sabbath-day's journey. Mountain-tops rise to his' view as he moves along, and peaks of precipices disappear beyond the horizon which he leaves behind, but the Canaan he seeks to explore is still a terra incognita.

The work from which we have taken the foregoing, entitled "Results of Astronomical Observations made during the years 1834,-'35-'36-'37, and -38, at the Cape of Good Hope, being the completion of a telescopic surv̀ey of the whole surface of the visible heavens, commenced in 1825," which occupies seven chapters, extending over four hundred and fifty pages, and illustrated by seventeen beautifully executed plates, would doubtless have appeared in a series of unconnected memoirs among the transactions of the Royal or Astronomical Societies, had it not been for the manificence of the late Duke of Northumberland, who gave a large sum for its publication as a single and separate work. The following are the subjects which are treated in the volume:

Chapter I. On the nebulæ and clusters of stars in the southern hemisphere.

Chapter II. On the double stars in the southèrn hemisphere.
Chapter III. On astronomy, or the numerical expression of the apparent magnitude of stars.

Chapter IV. Of the distribution of stars, and of the constitution of . the galaxy or milky way in the southern hemisphere.
Chapter V. Observations on Halley's comet, with remarks on its physical condition and that of comets in general.
Chapter VI. Observations on the satellites of saturn.
Chapter VII. Observations on the solar spots.
Here let us turn back for a moment to fix our attention upon the author of these marvelous works. The father, Sir William Herschel, had been not only a great astronomer, but a fortunate man. He was fortunate in having George theThird for a patron. Again he was fortunaterin laving Arago for a biographer, who, while complete master of his subject, was superior to envy and a lover of true greatness. But thrice fortunate was he in transmitting his name and fame to one, who, with the amplest intellectual resources of an accomplished scholar and philosopher, cherished the characteristic boldness of his predecessor's spirit, and
uplield that Tibserty of conjecture which is the mainspring of sagacity. It is rare that the parent's purple of intellect falls upon the child. By no culture however skillful, and no anxieties however earnest, can we transmit to our successors the qualities or the capacities of the mind. In lofty destinies father and son are rarely associated; and in the few cases where a joiut commission has issued to them, it has generally been to work in different spheres, or at different levels. In the universe of mind a double star is more rare than its prototype in the firmament, and when it does appear we watch its phases and mutations with corresponding interest. The case of the two Herschels is a remarkable one, and appears an exception to the general law. The father, however, was not çalled to the survey of the heavens, till he had passed the middle period of life, and it was but a just arrangement that the son, in his youth and manhood, should continue the labors of his sire. As has been eloquently said, "The records of astronomy do not emblazon a more glorions day than that in which the semi-diurnal are of the father was succeeded by the semi-diurnal are of the son. No sooner had the eveningluminary disappeared, amid the gorgeous magnificence of the west, than the morning star arose bright and cloudless in its appointed course." When it is considered that these two men, father and son, have carefully examined the whole starry firmament with 20 -foot telescopes-instruments of which, in their present state of perfection, the elder Herschel may be said to have been the inventor-and that they have made known to us thousands of the most interesting phenomena, it is hardly an exaggeration to say that the science of moderate siderial astronomy rests chiefly on their labors.
It is worthy of remark, in connection with Sir John Herschel's labors at the Cape of Good Hope, that his residence was productive of benefits to meteorology as well as to astronomy. While occupied there, he suggested a plan of having meteorological observations made simultaneously at different places-a plan subsequently developed at greater length in his Instructions for making and registering meteorological observations at various stations in Southern Africa, published under official authority in 1844. The result has been the almost universal adoption of a similar plan in Europe and the United States.
The record of the site of the 20 -foot reflector at Feldhausen, South Africa, has been preserved. No sooner had Sir John embarked for England, than his numerous friends at the Cape raised by subscription a sufficient sum to erect a granite obelisk on the spot. There, in the quiet dell, surrounded by trees, at the foot of Table Mountain, stands an enduring memorial, not only of "the pleasing and grateful recollections of years spent in agreeable society, cheerful occupations, and unalloyed happiness," as he gracefully expressed it, but of the discovery of thousands of nebula and double stars in the remote regions of the sidereal firmament.
Sir John Herschel returned to England in May, 1838. London re-
ceived him with enthusiasm. The whole scientific world joined in the acclamation. He was entertained at a great public dinner. At the meeting of the British Association, at Newcastle, he was honored as the principal guest. The Crown made him a baronet. Oxford conferred upon him the highest university honor; and Scotland, not to be behiud, elected him lord rector of Marischal College at Aberdeen. Withont doubt, the Duke of Sussex having vacated the office, he might have been elected president of the Royal Society, and the British Gorernment proposed to reimburse all his four years' pecuniary outlays; but he declined them both. His motives for his long expatriation had not been money, nor pleasure, nor health, nor fame, but increase and diffusion of knowledge among men. That object he had gained the means of reaching, and his largest ambition was satisfied.
Sir John was the author of the articles on "Isoperimetrical Problems," and of "Meteorology," and "Physical Geography," in tho Encyclopcedia Britannica, (the last two of which have been republished separately,) and also of several articles on scientific subjects in the Edinburg snd Quarterly Reviews, which were collected and published in a separate form in 1857, together with some of his lectures. He contributed besides to "Good Words" some popular papers on the wonders of the universe; and, two or three years before he died, he gave to the world, in the pages of "Cornhill Magazine," a poetical version of part of the Inferno of Dante. He was also one of the many sexegenarian translators of Homer's Iliad.

Sir John Herschel was either an honorary or corresponding member of the academies of Vienna, St. Petersburg, Gottingen, Turin, Bologna, Bruxelles, Nuremberg, Copenhagen, Stockholm, Prague, Warsaw, and Naples, as well as of almost all other scientific associations existing in Europe and America, Asia, and the southern hemisphere. To his other honors was added that of "Chevalier of Merit," founded by Frederick the Great, and given at the recommendation of the Academy of Sciences at Berlin.
We have hitherto confined our remarks to the principal original researches of Sir John Herschel, which are doubtless the most striking to the man of science; but still there can be no question that his popular reputation has arisen chiefly from his two well-known works, "A preliminary discourse on the study of natural philosophy" and "Outlines of astronomy," both of which contain internal evidence of his great attainments in almost every department of human knowledge, and of his high powers as a philosophical writer. We give a short extract from each of these works as examples of his style. Upon their contents it is not possible to enter here.
In the "Preliminary discourse," writing upon a subject with which he was more intimately acquainted than any man had ever been in the past, or was in the present, he says:
"Among the most remarkable of the celestial objects are the revolving double stars, or stars which, to the naked eye, or to inferior telesoopes, ap-
pear single, but if examined with high magnifying powers are found to consist of two individuals placed almost close together, and which, when carefully watched, are (many of them) found to revolve in regular elliptic orbits about each other, and, so far as we have yet been able to ascertain, to obey the same laws which regulate the planetary movements. There is nothing calculated to give a grander idea of the scale on which the sidereal heavens are constructed than these beautiful systems. When we see such magnificent bodies united in pairs, undoubtedly by the same bond of mutual gravitation which holds together our . own system, and sweeping over their enormous orbits in periods comprehending many centuries, we admit at once that they must be accomplishing ends in creation which will remain forever unknown to man; and that we have here attairied a point in science where the human intellect is compelled to acknowledge its weakness, and to feel that no onception the wildest imagination can form will bear the least comparison with the intrinsic greatness of the subject."

Eloquently and nobly said; and yet not more eloquent and noble - are the thoughts themselves, or the language that clothes the thoughts, in the passages we have quoted, than are others to be found on almost every page of the volume.

In the other volume alluded to, "The outlines of astronomy," a work clustered with brilliant thoughts thick as the stars which stud the midnight heavens, he writes:
"There is no science which, more than astronomy, draws more largely on that intellectual liberality which is ready to adopt whatever is demonstrated, or concede whatever is rendered highly probable, however new and uncommon the points of view may be in which objects the most familiar may thereby become placed. Almost all its conclusions stand in open and striking contradiction with those of superficial and vulgar observations, and with what appears to every one, until he has understood and weighed the proofs to the contrary, the most positive evidence of his senses. Thus the earth on which he stands, and which has served for ages as the unshaken foundation of the firmest structures, either of art or of nature, is divested by the astronomer of its attribute of fixity, and conceived by him as turning swiftly on its center, and at the same time moving onwards through space with great rapidity. The sun and the moon, which appear to untaught eyes round bodies of no very considerable size, become enlarged in his imagination into vast globes; the one approaching in magnitude to earth itself, the other immensely surpassing it. The planets, which appear only as stars somewhat brighter than the rest, are to him spacious, elaborate, and habitable worlds, sev. eral of them much greater, and far more curiously furnished, than the earth he inhabits, as there are also others lessso; and the stars themselves, properly so-called, which, to ordinary apprehension, present only lucid sparks or brilliant atoms, are to him suns of various and trauscendent glory, effulgent centers of life and light to myriads of unseen worlds.

So that when, after dilating his thoughts to comprehend the grandeur of those ideas his calculations have called up, and exhausting his imag. ination and the powers of his language to devise similes and metaphors illustrative of the immensity of the scale on which his universe is constructed, he shrinks back to his native sphere, he fiuds it in comparison a mere point; so lost, even in the minute system to which it belongs, as to be invisible and unsuspected from some of its principal and remoter members."
In the year 1851 Sir John Herschel accepted the appointment of master of the mint. This office, ouce held by Sir Isaac Newton, had degenerated into a place for politicians. Irrespective of qualification, the existing ministry had been accustomed for more thān a hundred years to give it to the member of the House of Commons who had served them best. From the date of Herschel's acceptance of the office its political character ceased. He brought to the duties of the position the same thorough search, conscientious dealing, and indefatigable industry that characterized his life. He abolished old charters, did away with antiquated indentures, and refused to renew contracts for meltings and coinages. His work was so thorough that it is still styled by the employés at the mint the "revolution of '51." Like all innovations, it caused alarm. A faction grew up in opposition. Members of Parliament and of the ministry took sides against his plans; but that firmness for the right which never yielded, and that gentleness toward opponents which never lost its equipoise, ultimately achieved success. The "trial plates"-he called them "fiducial pieces"-which had been used for centuries, were abandoned; standard tables for the qualities of the precious metals were prepared; the conventional purity of British coingold as 916.6 and silver as 925 -was settled; and the mathematical coincidence of the result of the pyx with the legal standard, established the correct result of the assays.

The subject of our memoir, however, was not made for office-wiork. Though present at his labors throughout every day, and with papers spread before him, revising and calculating his work far into the hours of every night, the toil was not congenial. Bodily infirmity followed. He was unable to work. His friends became alarmed. For himself he had not sought the place. Nature still needed his interpretations, and he desired to be at liberty to pass his last days in her domain. He therefore resigned his office as master of the mint in 1855, and betook himself to the well-earned repose of a veteran of science.
His mind, upou the recovery of his health, resumed its wonted activity, and though passing his life in comparative retirement at Collingwood, he prepared and published his catalogue of nebulæ and star-clusters. This splendid work was presented to the Royal Society on November 19, 1863, and contains all the nebulæ and clusters which had been anywhere described, and identified in position sufficiently to warrant their inclusion. The number of objects comprised in it is 5,078 , including all
observed by Sir William Herschel, Sir John Herschel, the Earl of Rosse, and others. This truly noble undertaking will ever remain a monument of the energy and perseverànce of Sir John Herschel, who at an age past three score and ten years found time and inclination to arrange and republish the great astronomical work of the century.

From the rank which Sir John Herschel held among scientific men, his services were in almost constant demand on committees, boards, and royal commissions, whose object was the attainment of information for the advancement of science. For many years he was one of the "visitors" to inspect aqnually the Royal Observatory. To him was made the annual report of the Astronomer-Royal on the efficiency of that establishment, and he was an important member of the royal commission appointed to prepare new standards of length and weight in lieu of those destroyed by fire in 1835. As member of the council, and one of the secretaries of the Royal Society, he was one of its leading members for years. In 1830, on the resignation of the presidency by the late Mr. Davis Gilbert, a strong effort was made to elect Sir John Herschel to the vacant chair, in opposition to the D'uke of Sussex, on the ground that his appointment would be peculiarly acceptable to men of science in Europe. But a commoner, however great, has in England little chance of success when a royal duke is his rival. There were special reasons which-influenced a large number of the fellows to support a member of the royal family, and the duke was elected. In the Royal Astronomical Society Sir John filled the office of president for six years, and in 1845 he presided over the meeting of the British Association.

It was the peculiar privilege-let us say in the conclusion of this part of our memoir-of Sir John Herschel, or peculiar gift, if the phrase be preferred, to combine with his special studies a breadth of view and power of expression that made him the Homer of science. Take, for example, what he has said of the vast practical importance of scientific knowledge, "As showing us how to avoid impossibilities, in securing us from important mistakes when attempting what is in itself possible by means either inadequate or actually opposed to the end in view; in enabling us to accomplish our ends in the easiest; shortest, most economical and most effectual manner; and in inducing us to attempt and enabling us to accomplish objects which, but for such knowledge, we would never have thought of undertaking."

Or again, "The character of the true philosopher is to hope all things not impossible, and to believe all things not unreasonable. When once embarked on any physical research, it is impossible for any one to predict where it will ultimately lead him. The true answer of science is that which again is at once the parallel and the illustration of the language of the apostle, "The mysteries of knowledge, which in other ages were not made known unto the sons of men, are now revealed, and will be still more revealed to those whom God has chosen."

Or still again, "The students of science are as messengers from Heaven to earth to make such stupendons announcements, that they may claim to be listened to when they repeat in every variety of urgent instance, that these are not the last announcements they have to communicate; that there are yet behind, to search out and to declare, not only secrets of nature which shall increase the wealth and power of men, but truths which shall ennoble the age and country in which they are divulged, and, by dilating the intellect, react upon the moral character of mankind."

We have called Sir John Herschel the Homer of science because he was its highest poet. It is the poet's function to move the soul-rousing the emotions, animating the affections, and inspiring the imagination; and all this Herschel did on almost every page of his writings. It is true that he avoids all fanciful representations of the facts of nature just as he eschews the meagerness of literal narration, but he has drawn beautiful pictures of nature's doings-so beautiful that they have disposed two generations to find their recreation and joy in science.

There is, besides, poetry of no meau order in such a life as that of Sir John Herschel-a life wholly given to lofty, unselfish aims-a life of labor, working, as he expresses it, "like a working-bee" to the very end, reserving his almost only indignation for that spirit of idleness and luxury which spends life but doẹs not use it.
There is a passage in one of Sir John's popular addresses that furnishes so admirable an insight to his own character, that it is worth transcribing. Speaking of the advantages of a taste for reading, he says: "Give a man this, and you place him in contact with the best society in every period of history-with the wisest, wittiest, tenderest, bravest, and purest of characters who have adorned humanity; you mike him a denizen with all nations, a contemporary of all ages. It is hardly possible but the character should take a higher and better tone from the constant habit of associating with thinkers above the average of humanity. It is morally impossible but that the manners should take a tinge of good breeding from having before one's eyes the ways in which the best bred and the best informed men have talked and acted."

No word he ever spoke, no sentence he ever wrote, so exactly depicts himself: He was in the utmost degree a well-bred man, not from gentle birth and careful training, not from scholarly pursuits and polite society, not from association with persons of rank and intimacy with men of taste and thought, not even from his loving nature and noble aspira-tions-not from all these together, so much as from the lofty ideal he cherished from boyhood to old age of perfect manhood. The upright form grew bent with passing years, the firm footstep staggered, the hand that poised instruments so accurately that rell-nigh impossible angles of space could be measured to a hair's breadth became tremulous, the lines of thought on his face deepened into wrinkles, the straggling, grizzled hair turned to snow-like whiteness, and the absent
expression of the eyes grew more thoughtful, but the air and manner, and bearing and address of the well-bred man never left him. He received criticisms upon his own specalations with the same equanimity that he pointed out the errors of his opponents. His action in discussion was never violent, nor his voice loud. He readily acknowledged a fault, and still more readily apologized for a wrong. To the capacity of the young, whether in May-day sports or Christmas gambols, even when past his fourthscore year, he was as yielding as he was stern against any inroad upon morals or violation of truth. He never lost his equipoise, was never betrayed into anger, shrank from injustice to others as if the pain to be endured were his own, looked beneath the rough exterior of many who approached him for honest motives, and, more than most of the best and wisest of our race, might have said truly :
"Write me as one who loves his fellow-men."
Sir John Herschel's life-long contemplation of the infinite in number and magnitude, exalting and hallowing his mind, was exhibited in its effects upon his character, The truths he had learned from the stars were converted into principles of action. Lofty thoughts promoted noble deeds. "Surely," he himself had said in a yet higher mood of the same vein of thought as that of the last passage quoted, "Surely, if the worst of men were transported to Paradise for only half an hour amongst the company of the great and good, he would come back converted."
There is one feature in Sir John Herschel's character of which some delineation cannot be omitted in any approximately correct picture of his long life. It is his filial piety. In a soul full of the gentlest feelings, his love for his father while the veteran lingered on the stage of life, and his reverence for the great and good man's memory after his departare, constituted the strongest sentiment. Perhaps there is no other instance in all history where filial affection became for so long a time the ruling motive of a life. The son was born for a successor in the line of chemistry to Sir Humphrey Davy and a rival to Michael Faraday; for his father's sake he became an astronomer. His tastes led him into discoveries of the properties of hyposulphate salts and the actinic relations of light; his reverence for his illustrious sire determined him to complete, to the abandonment of every favorite pursuit, what the latter had so nobly begun. The pursuit of astronomy was neither the roluntary choice nor the principal bias of his intellectual life. His inborn aptitude lay in another direction. Uncontrollable circumstances determined his career, and these were framed oat of impressions of the happy home of his childhood. He became a great astronomer, not through the promptings of natural taste bat by the dictates of filial piety. And no man was ever more emphatically, in thought and work, in hostility to error and search after truth, the son of his father. Over the two the eulogy of David over Saul and Jonathan might be fitly pronounced.

[^1]This deep reverence for his father's memory, and this high appreciation of the value of his discoveries-neither undeserved nor overratedpossessed Sir John Herschel to the last. His "idolatry" of the great telescope by which the sidereal heavens had been first unveiled to human sight has been called "weak in sentiment and dubious in taste." Arago did not so regard the means by which its remains were preserved, nor do other philosophers who hold the heart to be ever superior to the intellect. On the 1st of January, 1840, Sir John Herschel and his family, the old servants among the number, assembled at Slough. The metal tube had been placed horizontally in the meridian. At noon they walked in procession around the instrument, entered the capacious cylinder, seated themselves on benches previously prepared, sung a requiem, and then, ranging themselves around that-call it a piece of metal if you will-which had been the means of opening the star-world to human sight, witnessèd its hermetical sealing. "I know not," says Arago, "whether those persons who can only appreciate things from the peculiar point of view from which they have been accustomed to look, may think there was something strange in several of the details of this ceremony; I affirm, however, that the whole world will applaud the pious feeling which actuated Sir John Herschel, and that all the friends of science will thank him for having consecrated the humble garden where his father achieved̆ such immortal labors by a monument more expressive in its simplicity than pyramids or statues."

The true place of Sir John Herschel among the great lights of his age cannot be accurately fixed until this generation shall have passed away. The feelings, prejudices, and partialities of contemporaneous life warp correct judgment. Proximity is unfavorable to true appreciation. No one know this better than Biot, when he replied, in answer to the question, "Whom of all the philosophers of Europe do you regard as the most worthy successor of Laplace?" "If I did not love him so much, I should unhesitatingly say Sir John Herschel." Indeed, through his long confinement and protracted old age, the seekers after scientific truth not only in the English universities, but over all Europe, in their difficulties, anticipations, and successes, betook themselves to the aged phîlosopher of Collingwood.

Of the work done by the Herschels, father and son, during a period of almost one hundred years, it is fitting that something be said in the conclusion of this memoir. That work is not in general correctly understood. The labors of the elder Herschel are indeed associated in the public mind with those of his son, but the real end and aim of those labors, the qualities which characterized the labors of each, and the steps by which the two men moved on, each like a star in its orbit,

> "Making no haste and taking no rest,"
towards the grand consummation, it is only necessary to peruse the obituary notices which appeared upon his death to see are wholly misunderstood, even by men of intelligence.

The real work of the Herschels, then, that to which all their labors were directed, was the survey of those regions of space which lie beyond the range of the unaided vision. Other work they did which well deserves attention. Sir William Herschel, in particular, left papers describing observations of the planets, careful studies of the sun's surface, and researches into a variety of other subjects of interest. But all the work thus recorded was regarded by him rather as affording practice whereby he might acquire a mastery over his instruments than as a work to which he cared to devote his powers. Even the discovery of a planet traveling outside the path of Saturn-although, in popular estimation, this discovery is regarded as the most note-worthy achievement of Herschel's life-was in reality but an almost accicosntal result of his real work among the star-depths. It was, in truth, such an accident as he may be said to have rendered a certainty. No man can apply the powers of telescopes, larger than any before constructed, to scrutinize as he did every portion of the celestial depths, without being rewarded by some such discovery. He never swept the star-depths for an hour without meeting multitudes of hitherto unknown orbs, far mightier than the massive bulk of Uranus. These discoveries pass unrecorded save uumerically, but they tended to the solution of the noblest problem which men have yet attempted to master. It was the same with the son. All discoveries, all studies, were subordin ated to this one purpose, a knowledge of the construction of the heavens.

In the pursuit of this single end it is not strange that the great pioneer of star-observers should have formed opinions from time to timewhich he afterwards abandoned as unsupported by facts. In his paper, printed in the Philosophical Transactions of 1785, Sir William Herschel had said, "I have now viewed and gauged the milky way in almost every direction, and find it composed of stars whose number constantly increases and decreases in proportion to its apparent brightness to the naked eye. That this shining zone is a most extensive stratum of stars of various sizes admits no longer of the least doabt, and that our sun is actually one of the heavenly bodies belonging to it is evident." In the plate accompanying this paper, our sun makes one of innumerable stars, all comparable with each other in magnitude, and distributed with approach to uniformity.

In 1802, after his telescope had been asking seven years longer the secret of the skies, writing of our sun, magnificent as its system is, as only a single individual of the insulated stars, he says: "To this may be added that the stars we consider as insulated are also surrounded by a magnificent collection of innumerable stars called the milky way. For, though our sun and all the stars we see may truly be said to be in the plane of the milky way, yet I am now convinced by a long inspection that the milky way itself consists of stars differently scattered from those which are immediately about us."

Similar changes of opinion in regard to the nature of double stars, 9 s 71
to the constitution of the vast star system, and to the nature of the nebulæ, occurred, as he modified the principle of interpreting his observations. In 1811 he writes: "I find that by arranging the nebulæ in certain successire, regular order, they may be viewed in a new light, which cannot be indifferent to an iuquiring mind." He now expressed the opinion that these nebulæ did not consist of multitudes of stars, but of some self-luminous substance of exceeding tenuity. He recognized the existence of this luminous vapor amidst large tracts of the hearens, and regarded it as lying within the limits of the galaxy. Nay more, he believed this vaporous matter to be the material from which the stars were made. According to this view, vaist as has been the age of our galaxy, it has not completely formed itself into compact bodies. For many years he had held that all the nebulæ are.composed of stars. He now believed that some nebulæ were not of a starry nature; that a luminous matter existed in the universe in an elemental state; that the globular nebulæ were the earliest formed and most advanced in growth ; and that this vaporous or luminous matter lay within the line of the milky way, and formed part and parcel of its constitation.

This new view taken by Sir William Herschel of the construction of the heavens, whether as respects exteusion in space or duration in time, is singularly impressive. It implies indeed an enermous diminution of dimensions. It reduces the supposed countless millions of stars around Orion to chaotic vapor. It contracts distances, so far beyond our starsystem as not to be separately discerned by the most powerful glass, into spaces midway only between us and our galaxy. In reducing these distances many hundred times, this theory reduced the vastness of the objects many million times. But, on the other hand, it showed the milky way to be a more wonderful scheme than had ever been supposed. Vast as has been the period of its existence it had not yet entirels shaped itself into stars; over the regions where it extends, enormous masses of nebulous matter are still condensing into suns, and it becomes to the imagination a stupendous laboratory where systems of worlds have been produced and countless suns have had their genesis.

Despite the ingenuity of illustration and incontestable force of reasoning by which Sir William Herschel sought to establish this bold hysothesis, it has not won general favor since his day. Observation seems conclusively to show that the greater the optical power of the telescope the more certain is the evidence that the nebulæ are aggregations of stars. Sir John Herschel, too, with his usual reverential caution about controverting his father's dicta, seems to entertain this last opinion. "It may very reasonably be doubted," he wrote, "whether there is any essential physical distinction between clusters of stars and those nebulæ which my father regarded as composed of a shining nebulous fluid, and whether such distinction as there is be anything else than one of degree, arising merely from the excessive minuteness and multitude of the stars of which the latter compared with the former consist."

In the course of that stupendous work which has already been pointed out-the work of surveying those regions of space too distant to be seen by the naked eye-it would be a greater marvel than all their united discoveries had the Herschels never found occasion to change their views and remodel their theories. They did this, both father and son, once and again. "If it should be remarked," wrote Sir. William Herschel in 1811, "that in this new arrangement I am not entirely consistent with what I have already in former papers said on the nature of some objects that have come under my observation, I must freely confess that, by continuing my sweeps of the heavens, my opinion of the arrangement of stars and their magnitudes, and of some other particulars, has undergone a gradual change ; and, indeed, when the novelty of the subject is considered, we cannot be surprised that many things, formerly taken for granted, should on examination prove to be different from what they were generally but incautiously supposed to be. For instance, an equal scattering of the stars may be admitted in certain calculations; but when tre examine the milky way, or the closely compressed clusters of stars, this supposed equality of scattering must be given up. We may also have surmised nebulæ to be no other than clusters of stars disguised by their very great distance, but a longer experience and better acquaintance with the nature of nebulæ will not allow a general admission of such a principle, although undoubtedly a cluster of stars may assume a nebulous appearance when it is too remote for us to discern the stars of which it is composed." In fact, M. Arago's memoir of Sir William Herschel, as well as the numerous papers of himself and Sir John Herschel, which appeared from time to time, during more than three-quarters of a century, in the Transactions of the Royal Society and the Astronomical Society, show not only that the former modified his theories, gradually, indeed, but not infrequently, in accordance with newly-discovered facts, but also that Sir John Herschel's discoveries, though considerably in advance of the points reached by his father, but lying, nevertheless, strictly in the direction along which the elder had been progressing, led to the same result. Sir William modified his views about unequal double stars, concluding that the fainter orb is physically associated with the brighter one, instead of being far beyond it. He modified his views as to star-groups, regarding at last the masses of the milky way as aggregations of stars instead of depths extending into space. He had come to regard many star-clusters as part and parcel of the milky way; large numbers of nebulæ as vaporous luminous masses; and galaxies external to our system, as he once believed, a portion of the heavens with which he was familiar. Neither father nor son ever regretted to see hypotheses, though never so dearly cherished, pass beyond the field of controversy into the domain of the known.

Let us now turn to another consideration of Sir John Herschel-still necessarily but less closely, perhaps, connecting him with his fatherthe consideration of his character as a theorist in astronomy. As an
astronomical observer he was undeniably facile princeps, not merel, among the astronomers of his own country, but among all his astronomical contemporaries. His mastery extended over the widest range. In his general knowledge of the science of astronmy he was unapropached; in the mathmetical department of the science he was proficient above most; in his- knowledge of the details of observatory-work he was surpassed by none; and as a gauger of the heavens by the largest ${ }^{\dagger}$ telescopes he dwarfs into insignificance all the observational work accomplished by astronomers living or dead. He went over the whole range of his father's work through the northern skies, and then com. pleted the survey of the heavens that bend over the southern hemisphere. He alone could boast that no part of the celestial depths had escaped his scrutiny. As an interpreter of nature, he was unrivaled; as an expounder of astronomical truths he had no living peer, and as a theorist he commanded universal attention and compelled large assent..
In order to be clearly understood as to the meaning attached to the words "astronomical theorist," let us quote a passage from one of the papers of Sir William Herschel. It is taken from that noble essay contributed to the Transactions of the Royal Society, in which he first presented his ideas respecting the constitution of the celestial depths.
"First let me mention," he says, "that if "we should hope to make any progress in investigations of a delicate nature, we ought to avoid two opposite extremes, of which I can hardly say which is the most dangerous. If we indulge a fanciful imagination snd build worlds of our own, we must not wonder at our going wide from the path of truth and uature ; but these will vanish like the Cartsian vortices that soon gave way when better theories were offered. On the other hand, if we add observation to observation, without attempting to draw, not only certain conclusions, but also conjectural views from them, we offend against the very end for which only observations ought to be made."
Sir John Herschel has also described the quality primarily requisite in a theorist. "As a first preparation," the paper goes on to say, "he must loosen his hold on all crude and hastily-adopted notions, and must strengthen himself by something like an effort and a resolve for the unprejudiced admission of any conclusion which shall appear to be supported by careful observation and logical argument, even should it prove of a nature adverse to notions he may have previously formed for himself, or taken up, without examination, on the credit of others. Such an effort is, in fact, a commencement of that intellectual discipline which forms one of the most important ends of all science. It is the first movement of approach towards that state of mental purity which alone can fit us for a full and steady perception of moral beauty as well as physical adaptation. It is the 'euphrasy and rue' with which we must 'purge our sight' before we can receive and contemplate as they are the lineaments of truth and nature."

These principles Sir John Herschel strictly observed. He approached every subject on which he proposed to theorize with "enforced mental purity." He divested himself of prejudice. Previous views, preconceived notions, pride of opinion were cast aside. Like a child, he went to Nature's school to learn what she had to teach. When be entered on his astronomical labors, double stars were supposed to be two stars seen accidentally in the same direction, and his father had propounded the grandest views respecting galaxies beyond our own. Sir Jokn Herschel must have regarded these two theories with great favor, for they were associated with the name of his father. Notwithstanding this, Sir John devoted twenty-one years-eight in resurveying the fields of space which had been swept by his father's telescope, four in observation of the southern heavens, and nine in reducing his work to form -in order to confirm or overturn, as facts might warrant, these lypotheses of his father. From him we now know that double stars are not stars seen accidentally in the same direction, but are star-couples, associated by the mighty bond of common gravity. We also know that the second hypothesis did not bear the crucial test to which it was subjected. Other theories, indeed, of the elder Herschel, in their important features, were confirmed. It is not of that, however, that we speak, but of the conscientious honesty and philosophic spirit with which the son reviewed and continued his father's work, forever setting scientific truth higher than filial reverence.

Sir John Herschel was most sagacious in the interpretation of facts. Take, for example, his examination of the Magellanic clouds, those two curious patches on the southern celestial vault. He mapped their outlines, pictured their minute stars, and colored and shaded their starcloudlets. At this point others might have stopped. There was an array of interesting objects in certain regions of the heavens. What more could he say? But Sir John Herschel was not thus satisfied. He reasoned from the globular shape of the Magellanic clouds to the distance of the star-cloudlets within them, thence to the scale on which they were formed, and thus deduced the most important conclusion, perhaps, ever arrived at in astronomy by abstract reasoning, to wit, that all the orders of star-cloudlets belong to our own system.

Again, Sir John Herschel was deeply impressed with the existence of analogies throughout the whole range of creation. In a private letter written to Richard A. Procter, as late as 1869, we find him saying: "An opinion which the structure of the Magellanic clouds has often suggested to me, has been strongly recalled by what you say of the inclusion of every variety of nebulous form within our galaxy, viz, that if such be the case, that is, if these forms belong to the galactic system, then that system includes within itself miniatures of itself on an almost infinitely reduced scale, and what evidence then have we that there exists a universe beyond, unless a sort of argument from analogy, that the galaxy, with all its contents, may be but one of these miniatures of
that vast universe, and so on an infinitum, and that in that universe there may exist multitudes of other systems on a scale as vast as our galaxy, the analogues of those other nebulous and clustering forms which are not miniatures of our galaxy?"

As an illustration of his power of tracing the chain that binds cause and effect, we may refer to a passage in his Treaties on Astronomy. Tracing the connection between the central luminary of our system and terrestrial phenomena, Sir John remarks that "the sun's rays are the ultimate source of almost every motion that takes place on the surface of the earth. By its heat are produced the winds and those disturbances on the electric equilibrium of the atmosphere which give rise to the phenomena of lightning, and probably also to those of terrestrial magnetism and the aurora. By their vivifying action vegetables are enabled to draw support from inorganic matter, and become in their turn the support of animals and man, and the sources of those great deposits of dynamical efficiency which are laid up for human use in our coal strata. By them the waters of the sea are made to circulate in vapors through the air and irrigate the land, producing springs and rivers. By them are produced all disturbances of the chemical equilibrium of the elements of nature, which by a series of compositions and decompositions give rise to new products and originate transfers of material. Even the slow degradation of the solid constituents of the surface, in which its chief geological changes consist, is almost entirely due, on the one hand to the abrasion of wind and rain, and the alternation of heat and frost, and on the other hand to the continual beating of sea-waves, the result of solar radiation."

He was an admirable expounder of scientific principles. His style of writing is perhaps cumbrous, and his sentences are often long and involved. But the thought he would express, like a thread of silver running through a web of purple, is always clear. The popular taste for astronomical studies is due to his writings more than to those of all other men.

He, of all others, held mastery over pride of self-opinion. His own errors he admitted instantly they were discovered. Upon theories of others he worked as fairly and patiently as upon his own. He never struggled for a known error nor declined to accept a proven truth. With untiring patience, observing skill, and ingenious device, he sought earnestly to detect falsehood in his own opinions, and to discover truth in the opinions of others. It is said that he had a feeble grasp upon facts; that while his father clung with vise-like grip to the sure and the known, he at times allowed them to slip from his grasp. "If so, it were a grievous fault." But so few are the instances-not above two or three-cited by those who allege this, so unimportant are the facts named, so apparent is the motive, unconscious it may be to themselves, of the theorizers who urge the objection, that it would seem probable that his opinions upon the facts had been misinterpreted or his statements of them misunderstood.

Even if this blemish exists, it is but as a spot upon the sun. It argues no more than that in one particular the son was second to the father. But without more satisfactory evidence we prefer to range ourselves among the doubters, and to be among the number of those who believe that Sir John Herschel's reasoning was never in a single instance marred by a forgotten fact.

In the contemplation of the work of the two Herschels, let us remark in conclusion, and what that work has revealed to us, the mind stands appalled. Reason shrinks before the specter of boundless creation. If our sun and all his planets, primary and secondary, are in rapid motion round an invisible focus-if from that mysterious center no ray of light has ever reached our globe, then the buried relics of primeval life have taught us less of mau's brief tenure on this terrestrial paradise than we learn from the lesson of the stars. The one may date back unnumbered centuries, the other declares that from the origin of the human race to its far distant future the system to which it belongs will have described but an infinitesimal are of an immeasurable circle in which it is destined to revolve.
He married Margaret Brodie, daughter of Dr. Stewart, in 1829; she and a numerous family survive him. Two of his sons are already very favorably known in the realm of science, and their father lived to see one of them selected by the council for election to the Royal Society. Another son has an important professorship in the north of England. The eldest son, the present Sir William Herschel, occupies, with distinguished merit, a very important post in the civil service of Bengal.

Herschel's whole life, like the lives of Newton and Faraday, confutes the assertion, and ought to remove the suspicion, that a profound study of nature is unfavorable to a sincere acceptance of the Christian faith. Surrounded by an affectionate family, of which he was long spared to be the pride, the guide, and the life, John Herschel died, as he had lived, in the unostentatious exercise of a devout, yet simple, faith.
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## JOSEPH FOURIER.

BIOGRAPHY READ BEFORE THE FRENCH ACADEMY OF SCIENCES, BY M. ARAGO.

Gentlemen: In former times one Academician differed from anothe e only in the number, the nature, and the brilliancy of his discoveries. Their lives, thrown in some respects into the same mold, consisted of events little worthy of remark. A boyhood more or less studious; progress sometimes slow, sometimes rapid; inclinations thwarted by capricious or shortsighted parents; inadequacy of means, the privations which it introduces in its train; thirty years of a laborious professorship and difficult studies-such were the elements from which the admirable talents of the early secretaries of the Academy were enabled to execute those portraits so piquant, so lively, and so varied, which form one of the principal ornaments of your learned collections.
In the present day, biographies are less confined in their object. The convulsions which France has experienced in emancipating herself from the swaddling-clothes of routine, of superstition, and of privilege, have cast into the storms of political life citizens of all ages, of all conditions, and of all characters. Thus has the Academy of Sciences figured during forty years in the devouring arena, wherein might and right have alternately seized the supreme power by a glorious sacrifice of combatants and victims!
Recall to mind, for example, the immortal National Assembly. You will find at its head a modest Academician, a pattern of all the private virtues, the unfortunate Bailly, who, in the different phases of his political life, knew how to reconcile a passionate affection for his country with a moderation which his most cruel enemies themselves have been compelled to admire.

When, at a later period, coalesced Europe launched against France a million of soldiers; ; wheu it became necessary to organize for the crisis fourteen armies, it was the ingenious author of the Essai sur les Machines and of the Géométrie des Positions who directed this gigantic operation. It was again Carnot, our honorable colleague, who presided over the incomparable campaign of seventeen months, during which French troops, novices in the profession of arms, gained eight pitched battles, were victerious in one hundred and forty combats, occupied one hundred and sixteen fortified places, and two hundred and thirty forts or redoubts, enriched our arsenals with four thousand cannon and seventy thousand muskets, took a lundred thousand prisoners, and adorned the dome of the Invalids with ninety flags. During the same time
the Chaptals, the Fourcroys, the Monges, the Berthollets, rushed also to the defense of French independence, some of them extracting from our soil, by prodigies of industry, the very last atoms of saltpeter which it contained; others transforming, by the aid of new and rapid methods, the bells of the towns, villages, and smallest hamlets into a formidable artillery, which our enemies supposed, as indeed they had a right to suppose, we were deprived of. At the voice of his country in danger, another Academician, the young and learned Meunier, readily renounced the seductive pursuits of the laboratory; he went to distinguish himself upon the ramparts of Königstein, to contribute as a hero to the long defense of Mayence, and met his death, at the age of forty years only, after having attained the highest position in a garrison wherein shone the Aubert-Dubayets, the Beaupuys, the Haxos, the Klebers.

How could I forget here the last secretary of the original Academy ? Follow him into a celebrated assembly, into that convention, the sanguinary delirium of which we might almost be inclined to pardon, when we call to mind how gloriously terrible it was to the enemies of our independence, and you will always see the illustrious Condorcet occupied exclusively-with the great interests of reason and humanity. You will hear him denounce the shameful brigandage which for two centuries laid waste the African continent by a system of corruption ; demand in a tone of profound conviction that the code be purified of the frightful stain of capital punishment, which renders the error of the judge forever irreparable. He is the official organ of the Assembly on every occasion when it is necessary to address soldiers, citizens, political parties, or foreign nations in language worthy of France; he is not the tactician of any party; he incessantly entreats all of them to occupy their attention less with their own interests and a little more with public matters ; he replies, finally, to unjust reproaches of weakness by acts which leave him the onily alternative of the poison cup or the scaffold.

The French Revolution thus threw the learned geometer, whose discoveries I am about to celebrate, far away from the route which destiny appeared to have traced out for him. In ordinary times it would be about Dom* Joseph Fourier that the secretary of the Academy would have deemed it his duty to have occupied your attention. It would be the tranquil, the retired life of a Benedictine which he would have unfolded to you. The life of our colleague, on the contrary, will be agitated and full of perils; it will pass into the fierce contentions of the forum and amid the hazards of war; it will be a prey to all the anxieties which accompany a difficult administration. We shall find this life intimately associated with the great events of our age. Let us hasten to add, that it will be always worthy and honorable, and that the personal qualities of the man of science will euhance the brilliancy of his discoveries.

[^2]Fourier was born at Auxerre on the 21st of March, 1768. His father, like that of the illustrious geometer Lambert, was a tailor. This circumstance would formerly have occupied a large place in the éloge of our learned colleague; thanks to the progress of enlightened ideas, I may mention the circumstance as a fact of no importance: nobody, in effect, thinks in the present day, nobody even pretends to think, that genius is the privilege of rank or fortune.
Fourier became an orphan at the age of eight years. A lady who had remarked the amiability of his manners and his precocious natural abilities, recommended him to the bishop of Auxerre. Through the influence of this prelate, Fourier was admitted into the military school which was conducted at that time by the Benedictines of the Convent of St. Mark. There he prosecuted his literary studies with surprising rapidity and success. Many sermons very much applauded at Paris in the mouth of high dignitaries of the church were emanations from the pen of the schoolboy of twelve years of age. It would be impossible in the present day to trace those first compositions of the youth Fourier, since, while divulging the plagiarism, he had the discretion never to name those who profited by it.
At thirteen years Fourier had the petulence, the nqisy vivacity of most young people of the same age; but his character changed all at once, and as if by enchantment, as soon as he was initiated in the first principles of mathematics, that is to say, as soon as he became sensible of his real vocation. The hours prescribed for study no longer sufficed to gratify his insatiable curiosity. Ends of candles carefully collected in the kitchen, the corridors and the refectory of the college, and placed on a hearth concealed by a screen, served during the night to illuminate the solitary studies by which Fourier prepared himself for those labors which were destined, a few years afterward, to adorn his name and his country.
In a military school directed by monks, the minds of the pupils necessarily waver only between two careers in life-the church and the sword. Like Descartes, Fourier wished to be a soldier; like that philosopher, he would doubtless have found the life of a garrison very wearisome. But he was not permitted to make the experiment. His demand to undergo the examination for the artillery, although strongly supported by our illustrious colleague Legendre, was rejected with a severity of expression of which you may judge yourselves: "Fourier," replied the minister, "not being noble, could not enter the artillery, although he were a second Newton."
Gentlemen, there is in the strict enforcement of regulations, even when they are most absurd, something respectable, which I have a pleasure in recognizing; in the present instance nothing could soften the odious character of the ministerss words. It is not true in reality that no one could formerly enter into the artillery who did not possess a title of nobility: a certain fortune frequently supplied the want of
parchments. Thus it was not a something undefinable, which, by the way, our ancestors, the Franks, had not yet invented, that was wanting to young Fourier, but rather an income of a few hundred livres, which the men who were then placed at the head of the country would have refused to acknowledge the genius of Newton as a just equivalent for! Treasure up these facts, gentlemen; they form an admirable illustration of the immense advances which France has made during the last forty years. Posterity, moreover, will see in this, not the excuse, but the explanation of some of those sanguinary dissensions which stained our first revolution.

Fourier, not having been enabled to gird on the sword, assumed the habit of a Benedictine, and repaired to the Abbey of St. Benoît-sur-Loire, where he intended to pass the period of his novitiate. He had not yet taken any vows when, in 1789, every mind was captivated with beautifully seductive ideas relative to the social regeneration of France. Fourier now renounced the profession of the church; but this circumstance did not prevent his former masters from appointing him to the principal chair of mathematics in the military school of Aaxerre, and bestowing upon him numerous tokens of a lively and sincere affection. I venture to assert that no event in the life of our colleague affords a more striking proof of the goodness of his natural disposition and the amiability of his manners. It would be necessary not to know the human heart to suppose that the monks of St. Benoît did not feel some chagrin upon finding themselves so abruptly abandoned, to imagine especially that they should give up without lively regret the glory which the order might have expected from the ingenious colleague who had just escaped from them.

Fourier responded worthily to the confidence of which he had just become the object. When his colleagues were indisposed, the titular professor of mathematics occupied in turns the chairs of rhetoric, of history, and of philosophy; and whatever might be the subject of his lectures, he diffused among an audience which listened to him with delight the treasures of a varied and profound erudition, adorned with all the brilliancy which the most elegant diction could impart to them.

About the close of the year 1789 , Fourier repaired to Paris and read before the Academy of Sciences a memoir on the resolution of numerical equations of all degrees. This work of his early youth our colleague, so to speak, never lost sight of. He explained it at Paris to the pupils of the Polytechnic School; he developed it upon the banks of the Nile in presence of the Institute of Egypt ; at Grenoble, from the year 1802, it was his favorite subject of conversation with the professors of the Central School and of the faculty of sciences. This finally contained the elements of the work which Fourier was engaged in seeing through the press when death put an end to his career.

A scientific subject does not occupy so much space in the life of a man of science of the first rank without being important and difficult.

The subject of algebraic analysis above mentioned, which Fourier had studied with a perseverance so remarkable, is not an exception to this rule. It offers itself in a great number of applications of calculation to the movements of the heavenly bodies, or to the physics of terrestrial bodies, and in general in the problems which lead to equations of a high degree. As soon as he wishes to quit the domain of abstract relations, the calculator has occasion to employ the roots of these equations; thus the art of discovering them by the aid of a uniform method, either exactly or by approximation, did not fail at an early period to excite the attention of geometers.

An observant eye perceives already some traces of their efforts in the writings of the mathematicians of the Alexandrian school. These traces, it must be acknoovedged, are so slight and so imperfect that we should truly be justified in referring the origin of this branch of analysis only to the excellent labors of our countryman Vieta. Descartes, to whom we render very imperfect justice when we content ourselves with saying that he taught us much when he taught us to doubt, occupied his attention also for a short time with this problem, and left upon it the indelible impress of his powerful mind. Hudde gave for a particular but very important case rules to which nothing has since been added. Rolle, of the Academy of Sciences, devoted to this one subject his entire life. Among our neighbors on the other side of the channel, Harriot, Newton, Maclaurin, Stirling, Waring-I may say all the illustrious geometers which England produced in the last century-made it also the subject of their researches. Some years afterward the names of Daniel Bernoulli, of Euler, and of Fontaine came to be added to so many great names. Finally, Lagrange in his turn embarked in the same career, and at the very commencement of his researches he succeeded in substituting for the imperfect, although very ingenious, essays of lis predecessors, a complete method which was free from every objection. From that instant the dignity of science was satisfied; but in such a case it would not be permitted to say with the poet-
"Le temps ne fait rien à l'affaire."
Now, although the processes invented by Lagrange, simple in principle and applicable to every case, have theoretically the merit of leading to the result with certainty, still, on the other hand, they demand calculations of a most repulsive length. It remained then to perfect the practical part of the question : it was necessary to devise the means of shortening the route without depriving it in any degree of its certainty. Such was the principal object of the researches of Fourier, and this he has attained to a great extent.
Descartes had already found, in the order according to which the signs of the different terms of any numerical equation whatever succeed each other, the means of deciding, for example, how many real positive roots this equation may have. Fourier advanced a step further : he
discovered a method for determining what number of the equally positive roots of every equation may be found included between two given quantities. Here certain calculations become necessary, but they are very simple, and whatever be the precision desired, they lead without any trouble to the solutions sought for.

I doubt whether it were possible to cite a single scientific discovery of any importance which has not excited discussions of priority. The new method of Fourier for solving numerical equations is in this respect amply comprised within the common law. We ought, however, to acknowledge that the theorem which serves as the basis of this method was first published by M. Budan; that according to a rule which the principal academies of Europe have solemnly sanctioned, and from which the historian of the sciences dares not deviate without falling into arbitrary assumptions and confusion, M. Budan ought to be considered as the inventor. I will assert with equal assurance that it would be impossible to refuse to Fourier the merit of having attained the same object by his own efforts. I even regret that, in order to establish rights which nobody has contested, he deemed it necessary to have recourse to the certificates of early pupils of the Polytechnic School or professors of the University. Since our colleague had the modesty to suppose that his simple declaration would not be sufficient, why (and the argument would have had much weight) did he not remark in what respect his demonstration differed from that of his competitor?-an admirable demonstration, in effect, and one so impregnated with the elements of the question, that a young geometer, M. Sturm, has just employed it to. establish the truth of the beautiful theorem by the aid of which he determines not the simple limits, but the exact number of roots of any equation whatever which are comprised between two given quantities.

We had just left Fourier at Paris, submitting to the Academy of Sciences the analytical memoir of which I have just given a general view. Upon his return to Auxerre, the young geometer found the town, the surrounding comntry, and even the school to which he belonged, occupied intensely with the great questions relative to the dignity of human nature, philosophy, and politics, which were then discussed by the orators of the different parties of the National Assembly. Fourier abandoned himself also to this movement of the human mind. He embraced with enthusiasm the principles of the Revolution, and he ardently associated himself with everything grand, just, and generous which the popular impulse offered. His patriotism made him accept the most difficult missions. We may assert, that never, even when his life was at stake, did he truckle to the base, covetous, and sanguinary passions which displayed themselves on all sides.

A member of the popular society of Auxerre, Fourier exercised there an almost irresistible ascendency. One day-all Burgundy has preserved the remembrance of it-on the occasion of a levy of three hundred thousand men, he made the words honor, country, glory, ring so
cloquently, he induced so many voluntary enrollments, that the ballot was not deemed necessary. At the command of the orator the contingent assigned to the chief town of the Yonne formed in order, assembled together within the very enclosure of the Assembly, and marched forthwith to the frontier. Unfortunately these struggles of the forum, in which so many noble lives then exercised themselves, were far from having always a real importance. Ridiculous, absurd, and burlesque notions injured incessantly the inspirations of a pure, sincere, and enlightened patriotism. The popular society of Auxerre would furnish us, in case of necessity, with more than one example of those lamentable contrasts. Thus I might say that in the very same apartment wherein Fourier knew how to excite the honorable sentiments which I have with pleasure recalled to mind, he had on another occasion to contend with a certain orator, perhaps of good intentions, but assuredly a bad astron? omer, who wishing to escape, said he, from the good pleasure of municipal rulers, proposed that the names of the north, east, south, and west quarters should be assigned by lot to the different parts of the town of Auxerre.

Literature, the fine arts, and the sciences appeared for a moment to flourish under the auspicious influence of the French Revolution. Observe, for example, with what grandeur of conception the reformation of weights and measures was planned; what geometers, what astronomers, what eminent philosophers presided over every department of this noble undertaking! Alas! frightful revolutions in the interior of the country soon saddened this magnificent spectacle. The sciences could not prosper in the midst of the desperate contest of factions. They would have blushed to owe any obligations to the men of blood, whose blind passions immolated a Saron, a Bailly, and a Lavoisière.

A few months after the 9th Thermidor, the convention being desirous of diffusing throughout the country ideas of order, civilization, and internal prosperity, resolved upon organizing a system of public instruction, but a difficulty arose in finding professors. The members of the corps of instruction had become officers of artillery, of engineering, or of the staff, and were combating the enemies of France at the frontiers. Fortunately at this epoch of intellectual exaltation, nothing seemed impossible. Professors were wanting: it was resolved without delay to create some, and the normal school sprung into existence: Fifteen hundred citizens of all ages, dispatched from the principal district towns, assembled together, not to study in all their ramifications the different branches of human knowledge, but in order to learn the art of teaching under the greatest masters.

Fourier was one of these fifteen hundred pupils. It will, no doubt, excite some surprise that he was elected at St. Florentine, and that Auxerre appeared insensible to the honor of being represented at Paris by the most illustrious of her children. But this indifference will be readily understood. The elaborate scaffolding of calumny which it has
served to support will fall to the ground as soon as I recall to mind, that after the 9th Thermidor the capital, and especially the provinces, became a prey to a blind and disorderly reaction, as all political reactions invariably are; that crime (the crime of having changed opinionsit was nothing less hideous) usurped the place of justice; that excellent citize:as; that pure, moderate, and conscientious patriots were daily massacred by hired bands of assassins in presence of whom the inhabitants remained mute with fear. Such are, gentlemen, the formidable inflacnces which for a moment deprived Fourier of the suffrages of his countrymen ; and caricatured, as a partisan of Robespierre, the individual whom St. Just, making allusion to his sweet and persuasive eloquence, styled a patriot in music ; who was so often thrown into prison by the Decemvirs; who, at the very height of the reign of terror, offered before the revolutionary tribinal the assistance of his admirable talents to the mother of Marshal Davoust, accused of the crime of having at that unrelenting epoch sent some money to the emigrants; who had the incredible boldness to shut up at the inn of Tonnerre an agent of the committee of public safety, into the secret of whose mission he penetrated, and thus obtained time to warn an honorable citizen that he was about to be arrested; who, finally, attaching himself personally to the sanguinary proconsul before whom every oue trembled in Yonne, made him pass for a madman, and obtained his recall! You see, gentlemen, some of the acts of patriotism, of devotion, and of humanity which signalized the early years of Fourier. They were, you have seen, repaid with ingratitude. But ought we, in reality, to be astonished at it \% To expect gratitude from the man who cannot make an avowal of his feelings without danger would be to shut one's eyes to the frailty of human nature and to expose one's self to frequent disappointments.
In the normal school of the convention, discussion from time to time succeeded ordinary lectures. On those days an interchange of characters was effected: the pupils interrogated the professors. Some words pronounced by Fourier at one of those curious and useful meetings sufficed to attract attention toward him. Accordingly, as soon as a necessity was felt to create masters of conference, all eyes were turned toward the pupil of St. Florentine. The precision, the clearness, and the elegance of his lectures soon procured for him the unanimous applause of the fastidious and numerous audience which was confided to him.
When he attained the height of his scientific and literary glory, Fourier used to look back with pleasure upon the year 1794, and upon the sublime efforts which the French nation then made for the purpose of organizing a corps of public instruction. If he had ventured, the title of pupil of the original normal school would have been beyond doubt that which he would have assumed by way of preference. Gentlemen, that school perished of cold, of wretchedness, and of hunger, and not, whatever people may say, from certain defects of organization, which time and reflection would have easily rectified. Notwithstanding its short exist-
ence, it imparted to scientific studies quite a new direction, which has been productive of the most important results. In snpporting this opinion at some length, I shall acquit myself of a task which Fourier would certainly have imposed upon me, if he could have suspected that with just and eloquent eulogiums of his character and his labors there should mingle within the walls of this apartment, and even emanate from the mouth of one of his successors, sharp critiques of his beloved normal school.

It is to the normal school that we must inevitably ascend if we would desire to ascertain the earliest public teaching of descriptive geometry, that fine creation of the genius of Monge. It is from this source that it has passed almost without modification to the Polytechnic School, to founderies, to manufactories, and the most humble workshops.
The establishment of the Normal School accordingly indicates the commencement of a veritable revolution in the study of pure mathematics; with it demonstrations, methods, and important theories, búried in academical collections, appeared for the first time before the pupils, and encouraged them to recast upon new bases the works destined for instruction.
With some rare exceptions, the philosophers engaged in the cultivation of science constituted formerly in France a class totally distinct from that of the professors. By appointing the first geometers, the first philosophers, and the first naturalists of the world to be professors, the couvention threw new luster upon the profession of teaching, the advantageous influence of which is felt in the present day. 'In the opinion of the public at large, a title which a Lagrange, a Laplace, a Monge, a Berthollet, had borne, became a proper match to the finest titles. If under the empire, the Polytechnic School counted among its active professors councilors of state, ministers, and the president of the senate, you must look for the explanation of this fact in the impulse given by the Normal School.
Yon see in the ancient great colleges professors concealed in some degree behind their portfolios, reading as from a pulpit, amid the indifference and inattention of their pupils, discourses prepared beforehand with great labor, and which reappear every year in the same form. Nothing of this kind existed at the Normal School; oral lessons alone were there permitted. The authorities even went so far às to require of the illustrious savants appointed to the task of instruction the formal promise never to recite any lectures which they might hare learned by heart. From that time the chair has become a tribune where the professor, identified, so to speak, with his audience, sees in their looks, in their gestures, in their countenance, sometimes the necessity for proceeding at greater speed, sometimes, on the contrary; the necessity of retracing his steps, of awakening the attention by some incidental observations, of clothing in a new form the thought which, when first expressed, had left some doubts in the minds of his audience. And do 10 s 71
not suppose that the beautiful imprompta lectures with which the amphitheater of the Normal School resounded remained unknown to the public. Short-hand writers paid by the State reported them. The sheets, after being revised by the professors, were sent to the fifteen hundred pupils, to the members of convention, to the consuls and agents of the republic in foreign countries, to all governors of districts. There was in this something certainly of profusion compared with the parsimonious and mean habits of our time. Nobody, however, would concur in this reproach, however slight it may appear, if I were permitted to point out in this very apartment an illustrious Academician, whose mathematical genius was awakened by the lectures of the Normal School in an obscure district town.

The necessity of demonstrating the important services, ignored in the present day, for which the dissemination of the sciences is indebted to the first Normal School, has inducod me to dwell at greater length on the subject" than I intended. I hope to be pardoned; the example in any case will not be contagious. Eulogiums of the past, you know, gentlemen, are no longer fashionable. Everything which is said, everything which is printed, induces us to suppose that the world is the creation of yesterday. This opinion, which allows to each a part more or less brilliant in the cosmogonic drama, is under the safeguard of too many vanities to have anything to fear from the efforts of logic.

I have already stated that the brilliant success of Fourier at the Normal School assigned to him a distinguished place among the persons whom nature has endowed in the highest degree with the talent of public tuition. Accordingly, he was not forgotten by the founders of the Polytechnic School. Attached to that celebrated establishment, first with the title of superintendent of lectures on fortification, afterward appointed to deliver a course of lectures on analysis, Fourier has left there a venerated name, and the reputation of a professor distinguished by clearness, method, and erudition; I shall add even the reputation of a professor full of grace, for our colleague has proved that this kind of merit may not be foreign to the teaching of mathematics.

The lectures of Fourier have not been collected together. The Jourmal of the Polytechnic School contains only one paper by him, a memoir apon the "Principle of virtual velocities." This memoir, which prob:ably had served for the text of a lecture, shows that the secret of our celebrated professor's great success consisted in the combination of abstract truths, of interesting applications, and of historical details little known, and derived, a thing so rare in our days, from original sources.

We have now arrived at the epoch when the peace of Leoben brought back to the metropolis the principal ornaments of our armies. Then the professors and the pupils of the Polytechnic School had sometines the distinguished honor of sitting in their amphitheaters beside Generals Desaix and Bonaparte. Everything indicated to them then an
active participation in the events which each foresaw, and which in fact were not long in occurring.

Notwithstanding the precarious condition of Europe, the Directory decided upon denuding the country of its best troops, and launching them upon an adventurous expedition. The five chiefs of the republic were then desirous of removing from Paris the conqueror of Italy, of thereby putting an end to the popular demonstrations of which he everywhere formed the object, and which sooner or later would become a real danger.

On the other hand, the illustrious general did not dream merely of the momentary conquest of Egypt; he wished to restore to that country its ancient splendor; he wished to extend its cultivation, to improve its system of irrigation, to create new branches of industry, to open to commerce numerous outlets, to stretch out a helping hand to the unfortunate inhabitants, to rescue them from the galling yoke under which they had groaned for ages-in a word, to bestow upon them without delay all the benefits of European civilization. Designs of such inagnitude could not have been accomplished with the mere personnel of an ordinary army. It was necessary to appeal to science, to literature, and to the fine arts; it was necessary to ask the coöperation of several men of judgment and of experience. Monge and Berthollet, both members of the Institute aud professors in the Polytechnic School, became, with a view to this object, the principal recruiting aids to the chief of the expedition. Were our colleagues really acquainted with the object of this expedition? I dare not reply in the affirmative; but I know at all events that they were not permitted to divulge it. We are going to a distant country ; we shall embark at Toulon ; we shall be constantly with you; General Bonaparte will command the army, such was in form and substance the limited amount of confidential information which had been imperiously traced out to them. Upon the faith of words so vague, with the chances of a naval battle, with the English hulks in perspective, go in the present day and endeavor to enroll a father of a family, a savant already known by useful labors and placed in some honorable position, an artist in possession of the esteem and confidence of the public, and I am much mistaken if you obtain anything else than refusals; but in 1798, France had hardly emerged from a terrible crisis, during which her very existence was frequently at stake. Who, besides, had not encountered imminent personal danger? Who had not seen with his own eyes enterprises of a truly desperate nature brought to a fortunate issue? Is anything more wanted to explain that adventurous character, that absence of all care for the morrow, which appears to have been one of the most distinguishing features of the epoch of the Directory. Fourier accepted then, without hesitation, the proposals which his colleagues brought to him in the name of the commander-in-chief; he quitted the agreeable duties of a professor of the Polytechnic School to go-he knew not where; to do-he knew not what.

Chance placed Fourier during the voyage in the vessel in which Kleber sailed. The friendship which the philosopher and the warrior vowed to each other from that moment was not without some influence upon the events of which Egypt was the theater after the departure of Napoleon.

He who signed his orders of the day, the Member of the Institute, Com-mander-in-chief of the Army in the East, could not fail to place an academy among the means of regenerating the ancient kingdom of the Pharaohs. The valiant army which he commanded had barely conquered at Cairo, on the occasion of the memorable battle of the Pyramids, when the Institute of Egypt sprung into existence. It consisted of forty-eight members, divided into four sections. Monge had the honor of being the first president. As at Paris, Bonaparte belonged to the section of mathematics. The situation of perpetual secretary, the filling up of which was left to the free choice of the society, was unanimously assigned to Fourier.

You have seen the celebrated geometer discharge the same duty at ${ }^{\circ}$ the Academy of Sciences; you have appreciated his liberality of mind, his enlightened benevolence, his unvarying affability, his straightforward and conciliatory disposition; add in imagination to so many rare qualities the activity which youth, which health, can alone give, and you will have again conjured into existence the secretary of the Institute of Egypt; and yet the portrait which I have attempted to draw of him would grow pale beside the original.

Upon the banks of the Nile, Fourier devoted himself to assiduous researches on almost every branch of knowledge which the vast plan of the Institute embraced. The Decade and the Courier of Egypt will acquaint the reader with the titles of his different labors. I find in these journals a memoir upon the general solution of algebraic equations; researches on the methods of elimination; the demonstration of a new theorem of algebra; a memoir upon the indeterminate analysis; studies on general mechanics; a technical and historical work upon the aqueduct which conveys the waters of the Nile to the Castle of Cairo; reflections upon the oases; the plan of statistical researches to be undertaken with respect to the state of Egypt; programme of an intended exploration of the site of ancient Memphis, and of the whole extent of burying-places; a descriptive account of the revolutions and manners of Egypt, from the time of its conquest by Selim.

I find also in the Egyptian Decade, that, on the first complementary day of the year VI, Fourier communicated to the Institute the description of a machine designed to promote irrigation, and which was to be driven by the power of wind.

This work, so far removed from the ordinary current of the ideas of our colleague, has not been printed. It would very naturally find a place in a work of which the expedition to Egypt might again furnish
the subject, notwithstanding the many beautiful publications which it has already called into existence. It would be a description of the manufactories of steel, of arms, of powder, of cloth, of machines, and of instruments of every kind which our army had to prepare for the occasion. If, during our infancy, the expedients which Robinson Crusoe practiced in order to escape from the romantic dangers which he had incessantly to encounter, excite our interest in a lively degree, how, in mature age, could we regard with indifference a handful of Frenchmen thrown upon the inhospitable shores of Africa, without any possible communication with the mother-country, obliged to contend at once with the elements and with formidable armies, destitute of food, of clothing, of arms, and of ammunition, and yet supplying every want by the force of genius !

The long route which I have yet to traverse will hardly allow me to add a few words relatire to the administrative services of the illustrious geometer. Appointed French commissioner at the Divan of Cairo, he became the official medium between the general-in-chief and every Egyptian who might have to complain of an attack against his person, his property, his morals, his habits, or his creed. An invariable suavity of manner, a scrupulous regard for prejudices to oppose which directly would have been vain, an inflexible sentiment of justice, had given him an ascendency over the Mussulman population, which the precepts of the Koran could not lead any one to hope for, and which powerfully contributed to the maintenance of friendly relations between the inhabitants of Cairo and the French soldiers. Fourier was especially held in reneration by the Cheiks and the Ulémas. A single anecdote will serve to show that this sentiment was the offspring of genuine gratitude.

The Emir Hadgey, or Prince of the Caravan, who had been nominated by General Bonaparte upon his arrival in Cairo, escaped during the campaign of Syria. There existed strong grounds at the time for supposing that four Cheiks Ulémas had rendered themselves accomplices of the treason. Upon his return to Egypt, Bonaparte confided the investigation of this grave affair to Fourier. "Do not," said he, "submit halfmeasures to me. You have to pronounce judgment apon high personages: we must either cut off their heads or invite them to dinner." On the day following that on which this conversation took place, the four Cheiks dined with the general-in-chief. By obeying the inspirations of his heart, Fourier did not perform merely an act of humanity ; it was, moreover, one of excellent policy. Our learned colleague, M. Geoffroy Saint-Hilaire, to whom I am indebted for this aneedote, has stated in fact that Soleyman and Fayoumi, the principal of the Egyptian chiefs, whose punishment, thanks to our colleague, was so happily transformed into a banquet, seized every occasion of extolling among their countrymen the generosity of the French.

Fourier did not display less ability when our generals confided diplomatic missions to him. It is to his tact and urbanity that our army is
indebted for an offensive and defensive treaty of alliance with Mourad Bey. Justly proud of this result, Fourier omitted to make known the details of the negotiation. This is deeply to be regretted, for the plenipotentiary of Mourad was a woman, the same Sitty Néfiçah whom Kleber bas immortalized by proclaiming her beneficence, her noble character, in the bulletin of Heliopolis, and who, moreover, was already celebrated from one extremity of Asia to the other, in consequence of the bloody revolutions which her unparalleled beauty had excited among. the Mamelukes.
The incomparable victory which Kleber gained over the army of the Grand Vizier did not damp the energy of the janissaries, who had seized upon Cairo while the war was raging at Heliopolis. They defended themselves from house to house with heroic courage. The besieged had to choose between the entire destruction of the city and an honorable capitulation. The latter alternative was adopted. Fourier, charged, as usual, with the negotiations, conducted them to a favorable issue; but on this occasion the treaty was not discussed, agreed to, and signed within the mysterious precincts of a harem, upon downy couches, under the shade of balmy groves. The preliminary discussions were held in a house half ruined by bullets and grape-shot, in the center of the quarter of which the insurgents valiantly disputed the possession with our soldiers, before even it would have been possible to agree to the basis of a treaty of a few hours. Accordingly, when Fourier was preparing to celebrate the welcome of the Turkish commissioner conformably to oriental usages, a great number of musket-shots were fired from the house in front, and a ball passed through the coffee-pot which he was holding in his hand. Without calling in question the bravery of any person, do you not think, gentlemen, that if diplomatists were usually placed in equally perilous positions, the public would haveless reason to complain of their proverbial slowness?
In order to exhibit, under one point of view, the various administrative duties of our indefatigable colleague, I should have to show him to you on board the English fleet, at the instant of the capitulation of Menou, stipulating for certain guarantees in favor of the members of the Institute of Egypt; but services of no less importance and of a different nature demand also our attention. They will even compel us to retrace our steps, to ascend even to the epoch of glorious memory when Desaix achieved the conquest of Upper Egypt, as much by the sagacity, the moderation, and the inflexible justice of all his acts, as by the rapidity and boldness of his military operations. Bonaparte then appointed two numerous commissions to proceed to explore in those remote regions a multitude of monuments of which the moderus hardly suspected the existence. Fourier and Costas were the commandants of these commissions. I say the commandants, for a sufficiently imposing military force had been assigned to them; since it was frequently after a combat with the wandering tribes of Arabs that the astronomer found in the
movements of the heavenly bodies the elements of a future geographical map; that the naturalist collected unknown plants, determined the geological constitution of the soil, occupied himself with troublesome dissections; that the antiquary measured the dimensions of edifices; that he attempted to take a faithful sketch of the fantastic images with which everything was covered in that singular country, from the smallest pieces of furniture, from the simple toys of children, to those prodigious palaces, to those immense facades, beside which the vastest of modern constructions would hardly attract a look.
The two learned commissions studied with scrupulous care the magnificent temple of the ancient Tentyris, and especially the series of astronomical signs which have excited in our days such lively discus. sions; the remarkable monuments of the mysterious and sacred Isle of Elephantine; the ruins of Thebes, with her hundred gates, before which (and yet they are nothing but ruins) our whole army halted, in a state of astonishment, to applaud.
Fourier also presided in Upper Egypt over these memorable works, when the commander-in-chief suddenly quitted Alexandria, and retnrned to France with his principal friends. Those persons then were very much mistaken who, upon not finding our colleague on board the frigate Muiron, beside Monge and Berthollet, imagined that Bonaparte did not appreciate his eminent qualities. If Fourier was not a passenger, this arose from the circumstance of his having been a hundred leagues from the Mediterranean when the Muiron set sail. The explanation contains nothing striking, but it is true. In any case, the friendly feeling of Kleber toward the secretary of the Institute of Egypt, the influence which he justly granted to him on a multitude of delicate occasions, amply compensated him for an unjust omission.
I arrive, gentlemen, at the epoch so suggestive of painful recollections, when the Agas of the janissaries, who had fled into Syria, having despaired of vanquishing our troops so admirably commanded by the honorable arms of the soldier, had recourse to the dagger of the assassin. You are aware that a young fanatic, whose imagination had been wrought up to a high state of excitement in the mosques by a month of prayers and abstinence, aimed a mortal blow at the hero of Heliopolis at the instant when he was listening, without suspicion, and with his usual kindness, to a recital of pretended grievances, and was promising redress.
This sad misfortune plunged our colony into profound grief. The Egyptians themselves mingled their tears with those of the French soldiers. By a delicacy of feeling which we shonld be wrong in supposing the Mahometans not to be capable of, they did not then omit, they have not since omitted, to remark, that the assassin and his three accomplices were not born on the banks of the Nile.

The army, to mitigate its grief, desired that the funeral of Kleber should be celebrated with great pomp. It wished, also, that on that solemn day some person shpuld recount the long series of brilliant
actions which will transmit the name of the illustrious general to the remotest posterity. By unanimous consent this honorable and perilous mission was confided to Fourier.
There are very few individuals, gentlemen, who have not seen the brilliant dreams of their youth wrecked one after the other against the sad realities of mature age. Fourier was one of those few exceptions.
In effect, transport yourselves mentally back to the year 1789, and consider what would be the future prospects of the humble convert of St. Benoit-sur-Loire. No doubt, a small share of literary glory ; the favor of being heard occasionally in the churches of the metropolis; the satisfaction of being appointed to eulogize such or such a public personage. Well, nine years have hardly passed and you find him at the head of the Institute of Egypt, and he is the oracle, the idol, of a society which counted among its members Bonaparte, Berthollet, Monge, Malus, Geoffroy St. Hilaire, Conté, \&c.; and the generals rely upon him for overcoming apparently insurmountable difficulties, and the army of the East, itself so rich in adornments of all kinds, would desire no other interpreter when it is necessary to recount the lofty deeds of the hero which it had just lost.

It was upon the breach of a bastion which our troops had recently taken by assault, in sight of the most majestic of rivers, of the mag. nificent valley which it fertilizes, of the frightful desert of Lybia, of the colossal pyramids of Gizeh; it was in presence of twenty populations of different origins which Cairo unites together in its vast basin ; in presence of the most valiant soldiers that had ever set foot on a land, wherein, however, the names of Alexander and of Cæsar still resound; it was in the midst of everything which could move the heart, excite the ideas, or exalt the imagination, that Fourier unfolded the noble life of Kleber. The orator was listened to with religious silence ; but soon, addressing himself with a gesture of his hand to the soldiers ranged in battle-array before him, he exclaims: "Ah, how many of you would have aspired to the honor of throwing yourselves between Kleber and his assassin! I call you to witness, intrepid cavalry, who rushed to save him upon the heights of Koraïm, and dispelled in an instant the multitude of enemies who had surrounded him!" At these words an electric tremor thrills throughout the whole army, the colors droop, the ranks close, the arms come into collision, a deep sigh escapes from some teu thousand breasts torn by the saber and the bullet, and the voice of the orator is drowned amid sobs.
A few months after, upon the same bastion, before the same soldiers, Fourier celebrated with no less eloquence the exploits, the virtues, of the general whom the people conquered in Africa saluted with the name so flattering of Just Sultan, and who sacrificed his life at Marengo to secure the triumph of the French arms.
Fourier quitted Egypt only with the last wreck of the army, in virtue of the capitulation signed by Menou. On his return to France the
object of his most constant solicitude was to illustrate the memorable expedition of which he had been one of the most active and most useful members. The idea of collecting together the varied labors of all his colleagues incontestably belongs to him. I find the proof of this in a letter still unpublished, which he wrote to Kleber from Thebes on the 20th Vendémiaire, in the year VII. No publicact in which mention is made of this great literary monument is of an earlier date. The Institute of Cairo having adopted the project of a Work upon Egypt as early as the month of Frimaire, in the year VIII, confided to Fourier the task of uniting together the scattered elements of it, of making them consistent with each other, and drawing up the general introduction.

This introduction was published under the title of Historical Preface. Foutanes saw in it the graces of Athens and the wisdom of Egypt united together. What could I add to such an eulogium? I shall say only that there are to be found there, in a few pages, the principal features of the goverument of the Pharaohs, and the results of the subjection of ancient Egypt by the kings of Persia, the Ptolemies, the successors of Augustus, the emperors of Byzantium, the first Caliphs, the celebrated Saladin, the Mamelukes, and the Ottoman princes. The different phases of our adventurous expedition are there characterized with the greatest care. Fourier carries his scruples to so great a length as to attempt to prove that it was just. I have said only so far as to attempt, for in that case there might have been something to deduct from the second part of the eulogium of Fontanes. If, in 1797, our countryman experienced at Cairo or at Alexandria outrages and extortions which the Grand Seignior either would not or could not repress, one may in all rigor admit that France ought to have exacted justice to herself; that she liad the right to send a powerful army to bring the Turkish customhouse officers to reason. But this is far from maintaining that the Divan of Constantinople ought to have favored the French expedition; that our conquest was about to restore to him, in some sort, Egypt and Syria; that the capture of Alexandria and the battle of the Pyramids would enhance the luster of the Ottoman name! However, the public hastened to acquit Fourier of what appears hazarded in this small part of his beautiful work. The origin of it has been sought for in political exigencies. Let us be brief; behind certain sophisms the hand of the original commander-in-chief of the army of the East was suspected to be seen!

Napoleon then would appear to have participated, by his instructions, by his counsèls, or, if we choose, by his imperative orders, in the composition of the essay of Fourier. What was not long ago nothing more than a plausible conjecture has now become an incontestable fact. Thanks to the courtesy of M. Champollion-Figeac, I held in my hands, within the last few days, some parts of the first proof-sheets of the historical preface. These proofs were sent to the Emperor, who wished to make himself acquainted with them at leisure before reading them with

Fourier. They are covered with marginal notes, and the additions which they have occasioned amount to almost a third of the original discourse. Upon these pages, as in the definitive work given to the public, one remarks a complete absence of proper names; the only exception is in the case of the three generals-in-chief. Thus Fourier had imposed upon himself the reserve which certain vanities had blamed so severely. I shall add that nowhere throughout the precious proof-sheets of M . Champollion do we perceive traces of the miserable feelings of jealousy which have been attributed to Napoleon. It is true that upon pointing out with his finger the word illustrious applied to Kleber, the Emperor said to our colleague, "Some one has directed my attention to this epithet;" but, after a short pause, he added, "It is desirable that you should leave it, for it is just and well deserved." These words, gentlemen, honored the monarch still less than they branded with disgrace the some one whom I regret not being able to designate in more definite terms; one of those vile courtiers whose whole life is occupied in spying out the frailties, the evil passions of their masters, in order to make them subservient in conducting themselves to honors and fortune!
Fourier had no sooner returned to Europe than he was named (January $2 \mathrm{~d}, 1802$ ) prefect of the department of 1 'Isère. The ancient Dauphiny was then a prey to ardent political dissensions. The republicans, the partisans of the emigrants, those who had ranged themselves under the banners of the consular government, formed so many distinct castes, between whom all reconciliation appeared impossible. Well, gentlemen, this impossibility Fourier achieved. His first care was to cause the Hôtel of the Prefecture to be considered as neutral ground, where each might show himself without even the appearance of a concession. Curiosity alone at first brought the people there, but the penple returned; for in France they seldom desert the saloons wherein are to be found a polished and benevolent host, witty without being ridiculons, and learned without being pedantic. What had been divulged of the opinions of our colleague, respecting the anti-biblican antiquity of the Egyptian monuments, inspired the religious classes especially with lively apprehensions; they were very adroitly informed that the new prefect counted a saint in his family; that the blessed Pierre Fourier, who established the religious sisters of the Congregation of Notre-Dame, was his grand-uncle, and this circumstance effected a reconciliation which the unalterable respect of the first magistrate of Grenoble for all conscientious opinions cemented every day more and more.

As som as he was assured of a truce with the political and religious parties, Fourier was enabled to devote himself exclusively to the duties of his office. These duties did not consist with him in heaping up old papers to no advantage. He took personal cognizance of the projects which were submitted to him ; he was the indefatigable promoter of all those which narrow-minded persons sought to stifle in their birth; we may include in this last class the superb road from Grenoble to Turin
by Mount Genèvre, which the events of 1814 have so unfortunately interrupted, and especially the drainage of the marshes of Boargoin.

These marshes, which Louis XIV had given to Marshal Turenne, were a focus of infection to the thirty-seven communes, the lands of which were partially covered by them. Fourier directed personally the topographic operations which established the possibility of drainage. With these documents in his hand he went from village to village-I might almost say from house to house-to fix the sacrifice which each family ought to impose upon itself for the general interest. By tact and perseverance, taking "the ear of corn always in the right direction," thirty-seven municipal councils were induced to contribute to a common fund, without which the projected operation would not even have been commenced. Success crowned this rare perseverauce. Rich harvests, fat pastures, numerous flocks, a robust and happy population now covered an immense territory, where formerly the traveler dared not remain more than a few hours.
One of the predecessors of Fourier, in the situation of perpetual secretary of the Academy of Sciences, deemed it his duty, on one occasion, to beg an excuse for having given a detailed account of certain researches of Leibnitz, which had not required great efforts of the intellect: "We ought," says he, "to be very much obliged to a man such as he is, when he condescends, for the public good, to do something which does not partake of genius!" I cannot conceive the ground of such scruples; in the present day the sciences are regarded from too high a point of view, that we should hesitate in placing in the first rank of the labors with which they are adorned those which diffuse comfort, health, and happiness amidst the working population.
In presence of a part of the Academy of Inscriptions, in an apartment wherein the name of hieroglyph has so often resounded, I cannot refrain from alludiug to the service which Fourier rendered to science by retaining Champollion. The young professor of history of the faculty of letters of Grenoble had just attained the twentieth year of his age. Fate calls him to shoulder the musket. Fourier exempts him by investing him with the title of pupil of the School of Oriental Languages which he had borne at Paris. The minister of war learns that the pupil formerly gave in his resignation; he denounces the fraud, and dispatches a peremptory order for his departure, which seems even to exclude all idea of remonstrance. Fourier, however, is not discouraged; his intercessions are skillful and of a pressing nature; finally, he draws so animated a portrait of the precocious talent of his young friend, that he succeeds in wringing from the government an order of special exemption. It was not easy, gentlemen, to obtain such success. At the same time, a conscript, a member of our Academy, succeeded in obtaining a revocation of his order for departure only by declaring that he would follow on foot; in the costume of the Institute the contingent of the arrondissernent of Paris in which he was classed.

The administrative duties of the prefect of l'Isère hardly interrupted the labors of the geometer and the man of letters. It is from Grenoble that the principal writings of Fourier are dated; it was at Grenoble that he composed the Théorie Mathématique de la Chaleur, which forms his principal title to the gratitude of the scientific world.

I am far from being unconscious of the difficulty of analyzing that admirable work, and yet I shall attempt to point out the successive steps which he has achieved in the advancement of science. You will listen to me, gentlemen, with indulgence, notwithstanding several minute details which I shall have to recount, since I thereby fulfill the mission with which you have honored me.

The ancients had a taste, let us say rather a passion, for the marvelous, which caused them to forget even the sacred duties of gratitude. Observe them, for example, grouping together the lofty deeds of a great number of heroes, whose names they have not even deigned to preserve, and investing the single personage of Hercules with them. The lapse of ages has not rendered us wiser in this respect. In our own time the public delight in blending fable with history. In every career of life, in the pursuit of science especially, they enjoy a pleasure in creating Herculeses. According to vulgar opinion, there is no astronomical discovery which is not due to Herschel. The theory of the planetary movements is identified with the name of Laplace ; hardly is a passing allusion made to the eminent labors of D'Alembert, of Clairaut, of Euler, of Lagrange. Watt is the sole inventor of the steam-engine. Chaptal has enriched the arts of chemistry with the totality of the fertile and ingenious processes which constitute their prosperity. Even within this apartment has not an eloquent voice lately asserted that, before Fourier, the phenomenon of heat was hardly studied, that the celebrated geometer had alone made more observations than all his predecessors put together; that he had with almost a single effort invented a new science?

Although he runs the risk of being less lively, the organ of the Academy of Sciences cannot permit himself such bursts of enthusiasm. He ought to bear in mind that the object of these solemnities is not merely to celebrate the discoveries of Academicians ; that they are also designed to encourage modest merit; that an observer, forgotten by his contemporaries, is frequently supported in his laborious researches by the thought that he will obtain a benevolent look from posterity. Let us act, so far as it depends upon us, in such a manner that a hope so just, so natural, may not be frustrated. Let us award a just, a brilliant homage to those rare men whom nature has endowed with the precious privilege of arranging a thousand isolated facts, of making seductive theories spring from them; but let us not forget to state, that the scythe of the reaper had cut the stalks before one had thought of uniting them into sheaves!

Heat presents itself in natural phenomena, and in those which are the products of art, under two entirely distinct forms, which Fourier has
separately considered. I shall adopt the same division, commencing, however, with radiant heat the historical analysis which I am about to subinit to you.

Nobody doubts that there is a physical distinction which is eminently worthy of being studied between the ball of iron at the ordinary temperature which may be handled at pleasure, and the ball of iron of the same dimensions which the flame of a furnace has very much heated, and which we cannot touch without burning ourselves. This distinction, according to the majority of physical -inquirers, arises from a certain quantity of an elastic imponderable fluid, or at least a fluid which has not been weighed, with which the second ball has combined during the process of heating. The fluid which upon combining with cold bodies renders them hot, has been designated by the name of heat or caloric.

Bodies unequally heated act upon each other even at great distances, even through empty space, for the colder becomes more hot, and the hotter becomes more cold; for after a certain time they indicate the same degree of the thermometer, whatever may have been the difference of theiroriginal temperatures. According to the hypotheses above explained, there is but one way of conceiving this action at a distance : this is to suppose that it operates by the aid of certain effluvia which traverse space by passing from the hot body to the cold body; that is, to admit that a hot body emits in every direction rays of heat, as luminous bodies emit rays of light.

The effluvia, the radiating emanations by the aid of which two distant bodies form a calorific communication with each other, have been very appropriately desiguated by the name of radiating caloric.

Whatever may be said to the contrary, radiating heat had already been the object of important experiments before Fourier undertook his labors. The celebrated Academicians of the Cimento found, nearly two centuries ago, that this heat is reflected like light; that, as in the case of light, a concave mirror concentrates it at the focus. Upon substituting balls of snow for heated bodies, they even went so far as to prove that frigorific foci may be formed by way of reflection. Some years afterward Mariotte, a member of this Academy, discovered that there exist different kinds of radiating heat; that the heat with which rays of light are accompanied traverses all trausparent media as easily as light does; while, again, the caloric which emanates from a strongly heated, but opaque substance, as well as the rays of heat which are found mingled with the luminous rays of a body moderately incandescent, are almost entirely arrested in their passage through the most transparent plate of glass!

This striking discovery, let us remark in passing, will show, notwithstanding the ridicule of pretended savants, how happily inspired were the workmen in founderies, who looked at the incandescent matter of their furnaces only through a plate of ordinary glass, thinking by the
aid of this artifice to arrest the heat which would have burned their eyes.
In the experimental sciences, the epochs of the most brilliant progress are almost always separated by long intervals of almost absolute repose. Thus, after Mariotte, there elapsed more than a century without history having to record any new property of radiating heat. Then, in close succession, we find in the solar light obscure caloritic rays, the existence of which could admit of being established only with the thermometer, and which may be completely separated from luminous rays by the aid of the prism ; we discover, by the aid of terrestrial bodies, that the emission of caloric rays, and consequently the cooling of those bodies, is considerably retarded by the polish of the surfaces; that the color, the nature, and the thickness of the outer coating of these same surfaces exercise also a manifest influence upon their emissive power. Experience, finally, rectifying the vague predictions to which the most enlightened minds abandon themselves with so little reserve, shows that the calorific rays which emanate from the plane surface of a heated body, have not the same force, the same intensity in all directions; that the maximum corresponds to the perpendicular emission, and the minimum to the emissions parallel to the surface.
Between these two extreme positions, how does the diminution of the emissive power operate? Leslie first sought the solution of this important question. His observations seem to show that the intensities of the radiating rays are proportional (it is necessary, gentlemen, that I employ the scientific expression) to the sines of the angles which these rays form with the heated surface. But the quantities upon which the experimenter had to operate were too feeble; the uncertainties of the thermometric estimations compared with the total effect were, on the. contrary; too great not to inspire a strong degree of distrust; well, gentlemen, a problem before which all the processes, all the instruments of modern physics, have remained powerless, Fourier has completely solved without the necessity of having recourse to any new experiment. He has traced the law of the emission of caloric sought for, with a perspicuity which one cannot sufficiently admire, in the most ordinary phenomena of temperature, in the phenomena which at first sight appeared to be entirely independent of it.

Such is the privilege of genius ; it perceives, it seizes relations where vulgar eyes see only isolated facts.
Nobody doubts, and besides experiment has confirmed the fact, that in all the points of a space terminated by any envelope maintained at a constaut temperature, we ought also to experience a constant temperature, and precisely that of the envelope. Now, Fourier has estallished that if the calorific rays emitted were equally intense in all directions, if the intensity did not vary proportionally to the sine of the angle of emission, the temperature of a body. situated in the inclosure would depend on the place which it would occupy there; that the temperature
of boiling water or of melting iron, for example, would exist in certain points of a hollow envelope of.glass ! In all the vast domain of the physical sciences we should be unable to find a more striking application of the celebrated method of the reductio ad absurdum of which the ancient mathematicians made use in order to demonstrate the abstract truths of geometry.
I shall not quit this first part of the labors of Fourier without adding, that he has not contented himself with demonstrating with so mach felicity the remarkable law which connects the comparative intensities of the calorific rays, emanating under all angles from heated bodies; he has sought, moreover, the physical cause of this law, and he has found it in a circumstance which his predecessors had entirely neglected. Let us suppose, says he, that bodies emit heat not only from the molecules of their surfaces, but also from the particles in the interior. Let us suppose, moreover, that the heat of these latter particles cannot arrive at the surface by traversing a certain thickness of matter without undergoing some degree of absorption. Fourier has reduced these two hypotheses to calculation, and he has hence deduced mathematically the experimental law of the sines. After having resisted so radical a test, the two bypotheses were found to be completely verified; they have become laws of nature; they point out latent properties of caloric which could ouly be discerned by the eye of the intellect.
In the second question treated by Fourier, heat presents itself under a new form. There is more difficalty in following its movements; but the couclusions deduced from the theory are also more general and more important.
Heat excited, concentrated into a certain point of a solid body, communicates itself by way of conduction, first to the particles nearest the heated point, then gradually to all the regions of the body. Whence the problem of which the following is the enunciation.
By what routes, and with what velocities, is the propagation of leat effected in bodies of different forms and different natures subjected to certain initial conditions?
Fundamentalls, the Academy of Sciences had already proposed this problem as the subject of a prize as early as the year 1736. Then the terms heat and caloric were not in use ; it demanded the study of nature, and the propagation of fire! The word fire, thrown thus into the programme without any other explanation, gave rise to a mistake of the most singular kind. The majority of philosophers imagined that the question was to explain in what way burning communicates itself, and increases in a mass of combustible matter. Fifteen competitors presented themselves; three were crowned.
This competition was productive of very meager results. However, a singular combination of circumstances and of proper names will render the recollection of it lasting.
Has not the public a right to be surprised upon reading this academic
declaration: "The question affords no handle to geometry!" In matter of inventions, to attempt to dive into the future is to prepare for one's self striking mistakes. One of the competitors, the great Euler, took these words in their literal sense: the reveries with which his memoir abounds are uot compensated in this instance by any of those brilliant discoveries in analysis-I had almost said of those sublime inspirations-which were so familiar to him. Fortunately Euler appended to his memoir a supplement truly worthy of his genius. Father Lozeran de Fiesc and the Count of Créqui were rewarded with the high honor of seeing their names inscribed beside that of the illustrious geometer, although it would be impossible in the present day to discern in their memoirs any kind of merit, not even that of politeness, for the courtier said rudely to the Academy: "The question which you have raised interests only the curiosity of mankind."

Among the competitors less favorably treated, we preceive one of the greatest writers whom France has produced-the author of the Henriade. The memoir of Voltaire was, no doubt, far from solving the problem proposed; but ít was at least distinguished by elegance, clearness, and precision of language; I shall add, by a severe style of argument; for if the author occasionally arrives at questionable results, it is only when he borrows false data from the chemistry and physics of the epoch,sciences which had just sprung into existence. Moreover, the antiCartesian color of some of the parts of the memoir of Voltaire was calculated to find little favor in a society where Cartesianism, with its incomprehensible vortices, was everywhere held in high estimation.

We should have more difficulty in discovering the causes of the failure of a fourth competitor, Madame the Marchioness du Châtelet, for she also entered into the contest instituted by the Academy. The work of Emilia was not.only an elegant portrait of all the properties of heat known then to physical inquirers; there were remarked, moreover, in it different projects of experiments, among the rest, one which Herschel has since developed, and from which he has derived one of the principal flowers of his brilliant scientific crown.

While such great names were occupied in discussing this question, physical inquirers of a less ambitious stamp laid experimentally the solid basis of a future mathematical theory of heat. Some established that the same quantity of caloric does not elevate by the same number of degrees equal weights of different substances, and thereby introduced into the science the important notion of capacity. Others, by the aid of observations no less certain, proved that heat, applied at the extremity of a bar, is transmitted to the extreme parts with greater or less velocity or intensity, according to the nature of the substance of which the bar is composed: thus they suggested the original idea of conductibility. The same epoch, if I were not precluded from entering into too minute details, would present to us interesting experiments. We should find that it is not true that, at all derrees of the thermometer, the loss of
heat of a body is proportional to the excess of its temperature above that of the medium in which it is plunged; but I have been desirous of showing you geometry penetrating, timidly at first, into questions of the propagation of heat, and depositing there the first germs of its fertile methods.

It is to Lambert, of Mulhouse, that we owe this first step. This ingenious geometer had proposed a very simple problem, which any person may comprehend. A slender metallic bar is exposed at one of its extremities to the constant action of a certain focus of heat. The parts nearest the focus are heated first. Gradually the heat communicates itself to the more distant parts, and, after a short time, each point acquires the maximum temperature which it can ever attain. Although the experiment were to last a hundred years, the thermometric state of the bar would not undergo any modification.
As might be reasonably expected, this maximum of heat is so much less considerable as we recede from the focus. Is there any relation between the final temperatures and the distances of the different particles of the bar from the extremity directly heated? Such a relation exists. It is very simple. Lambert investigated it by calculation, and experience confirmed the results of theory.
In addition to the somewhat elementary question of the longitudinal propagation of heat, there offered itself the more general but much more difficult problem of the propagation of heat in a body of three dimensions terninated by any surface whatever. This problem demanded the aid of the higher analysis. It was Fourier who first assigned the equations. It is to Fourier, also, that we owe certain theorems, by means of which we may ascend from the differential equations to the integrals, and push the solutions, in the majority of cases, to the final numerical applications.
The first memoir of Fourier on the theory of heat dates from the year 1807. The Academy, to which it was communicated, being desirous of inducing the author to extend and improve his researches, made the question of the propagation of heat the subject of the great mathematical prize which was to be awarded in the beginning of the year 1812. Fourier did, in effect, compete, and his memoir was crowned. But, alas! as Fontenelle said, "in the country even of demonstrations, there are to be found causes of dissension." Some restrictions mingled with the favorable judgment. The illustrious commissioners of the prize, Laplace, Lagrange, and Legendre, while acknowledging the novelty and importance of the subject, while declaring that the real differential equations of the propagation of heat were finally found, asserted that they perceived difficulties in the way in which the author arrived at them. They added that his processes of integration left something to be desired, even on the score of rigor. .They did not, however, support their opinion by any arguments.
Fourier never admitted the validity of this decision. Even at the 11 s 71
close of his life he gave unmistakable evidence that he thought it unjust, by causing his memoir to be printed in our volnmes without chang. ing a single word. Still, the doults expressed by the commissioners of the Academy reverted incessantly to his recollection. From the very beginning they had poisoned the pleasure of his triumph. These first impressions, added to a high susceptibility, explain how Fourier ended by regarding with a certain degree of displeasure the efforts of those geometers who endeavored to improve lis theory. Tlisis, gentlemen, was a very strange aberration of a mind of so elevated an order. Our colleague had almost forgotten that it is not allotted to any person to conduct a scientific question to a definitive termination, and that the important labors of D'Alembert, Clairaut, Euler, Lagrange, and Laplace, while immortalizing their authors, have continually added new luster to the imperishable glory of Newton. Let us act so that this example mas not be lost. While the civil law imposes upon the tribunes the obligation to assign the motives of their judgments, the academies, which are the tribunes of science, cannot have even a pretext to escape from this obligation. Corporate bodies, as well as individuals, act wisely when they reckon in every instance only upon the authority of reason.

At any time the "Théorie Mathématique de la Chaleur" would have excited a lively interest among men of reflection, since, upon the supposition of its being complete, it threw light upon the most minute processes of the arts. In our own time the numerous points of affinity existing between it and the carious discoveries of the geologists have made it, if I may use the expression, a work for the occasion. To point out the intimate relation which exists between these two kinds of researches would be to present the most important part of the discoveries of Fourier, and to show how happily our colleague, by one of those inspirations reserved for genius, had chosen the subject of his researches.
The parts of the earth's crust which the geologists call the sedimentary formations were not formed all at once. The waters of the ocean, on several former occasions, covered regions which are situated in the present day in the center of the continent. There they deposited, in thin horizontal strata, a series of rocks of different kinds. These rocks, although superposed like the layers of stones of a wall, must not be confounded together. Their dissimilarities are palpable to the least prac:ticed eye. It is necessary, also, to note this capital fact, that each stratum has a well-defined limit; that no process of transition connects it with the stratum which it supports. The ocean, the original source of all these deposits, underwent then formerly enormous changes in its chemical composition, to which it is no longer subject.

With some rare exceptions, resulting from local convulsions, the effects of which are otherwise manifest, the order of antiquity of the successive strata of rocks which form the exterior crust of the globe ought to be
that of their superposition. The deepest have been formed at the most remote epochs. The attentive study of these different envelopes may aid us in ascending the stream of time, even beyond the most remote epochs, and enlightening us with respect to those stupendous revolutions which periodically overwhelmed continents beneath the waters of the ocean, or again restored them to their former condition. Crystalline rocks of granite upon which the sea has effected its original deposits have never exhibited any remains of life. Traœes of such are to be found only in the sedimentary strata.
Life appears to have first exhibited itself on the earth in the form of vegetables. The remains of vegetables are all that we meet with in the most ancient strata deposited by the waters; still they belong to plants of the simplest structure-to ferns, to species of rushes, to lycopodes.
As we ascend into the apper strata, vegetation becomes more and more complex. Finally, near the surface, it resembles the vegetation actually existing on the earth, with this characteristic circumstance, however, which is well deserving attention, that certain vegetables which grow only in southern climates-that the large palm-trees, for example-are found in their fossil state in all latitudes, and even in the center of the frozen regions of Siberia.
In the primitive world, these northern regions enjoyed then, in winter, a temperature at least equal to that which is experienced in the present day under the parallels where the great palms commence to appear ; at Tobolsk, the inhabitants enjoyed the climate of Alicante or Algiers:

We shall deduce new proofs of this mysterious result from an attentive examination of the size of plants.
There exist, in the present day, willow-grass or marshy rushes, ferns, and lycopodes; in Europe as well as in the tropical regions; but they are not met with in large dimensions, except in warm countries. Thus, to compare together the dimensions of the same plants is, in reality, to compare, in respect to temperature, the regions where they are produced. Well, place beside the fossil plants of our coal mines, I will not say the analogous plants of Europe, but those which grow in the countries of South America, and which are most celebrated for the richness of their vegetation, and you will find the former to be of incomparably greater aimensions than the latter.
The fossil flora of France, England, Germany, and Scandinavia offer, for example, ferns ninety feet high, the stalks being six feet in diameter or eighteen feet in circumference.
The licopodes which, in the present day, whether in cold or temperate climates, are creeping-plants, rising hardly to the height of a decimeter above the soil; which, even at the equator, under the most favorable circumstances, do not attain a height of more than one meter, had.in Europe, in the primitive world, an altitude of twenty-five meters.

One must be blind to all reason not to find in these enormous dimen-
sions a new proof of the high temperature enjoyed by our country before the last irruptions of the ocean.
The study of fossil animals is no less fertile in results. I should digress from my subject if I were to examine here how the organization of animals is developed upon the earth; what modifications, or more strictly speaking, what complications it has undergone after each cataclysm, or if I even stopped to describe one of those ancient epochs during which the earth, the sea, and the atmosphere had for inhabitants cold-blooded reptiles of enormous dimensions; tortoises, with shells three feet in diameter; lizards seventeen meters long; pterodactyles, veritable flying dragons of such strange forms that they might be classed on good grounds either among reptiles, among mammiferous animals, or among birds. The object which I have proposed does not require that I should enter into such details; a single remark will suffice.

Among the bones contained in the strata nearest the present surface of the earth are those of the hippopotamus, the rhinoceros, and the elephant. These remains of animals of warm countries are to be found in all latitudes. Travelers have discovered specimens of them even at Melville Island, where the temperature descends, in the present day, $50^{\circ}$ beneath zero. In Siberia they are found in such abundance as to have become an article of commerce. Finally, upon the rocky shores of the Arctic Ocean, there are to be found not merely fragments of skeletons, but whole elephants still covered with their flesh and skin.
I should deceive myself very much, gentlemen, if I were to suppose that each of you had not deduced from these remarkable facts a conclusion no less remarkable, to which, indeed, the fossil flora had already habituated us; namely, that as they have grown older the polar regions of the earth have cooled down to a prodigious extent.

In the explanation of so curious a phenomenon, cosmologists have not taken into account the existence of possible variations of the intensity of the solar heat; and yet the stars, those distant suns, have not the constant brightness which the common people attribute to them. Nay, some of them have been observed to diminish in a sufficiently short space of time to the hundredth part of their original brightness; and several have even totally disappeared. They have preferred to attribute everything to an internal or primitive heat with which the earth was at some former epoch impregnated, and which is gradually being dissipated in space.

Upon this hypothesis the inhabitants of the polar regions, although deprived of the sight of the sun for whole months together, must have evidently enjoyed, at very ancient epochs, a temperature equal to that of the tropical regions, wherein exist elephants in the present day.
It is not, however, as an explanation of the existence of elephants in Siberia that the idea of the intrinsic heat of the globe has entered for the first time into science. Some savants had adopted it before the discovery of those fossil animals. Thus, Descartes was of opinion that
originally (I cite his own words) the earth did not differ from the sun in any other respect than in being smaller. Upon this hypothesis, then, it ought to be considered as an extinct. sun.
Leibnitz conferred upon this hypothesis the honor of appropriating it to himself. He attempted to deduce from it the mode of formation of the different solid envelopes of which the earth consists. Buffon, also, imparted to it the weight of his eloquent authority. According to that great naturalist, the planets of our system are merely portions of the sun, which the shock of a comet had detached from it some tens of thousands of years ago.

In support of this igneous origin of the earth, Mairan and Buffon cited already the high temperature of deep mines, and, among others, those of the mines of Giromagny. It appears evident that if the earth was formerly incandescent, we should not fail to meet in the interior strata-that is to say, in those which ought to have cooled last-traces of their primitive temperature. The observer who, upon penetrating into the interior of the earth, did not find an increasing heat, might then consider himself amply authorized to reject the hypothetical conceptions of Descartes, of Mairan, of Leibnitz, and of Buffon. But has the converse proposition the same certainty? Would not the torrents of heat, which the sun has continued incessantly to launch for so many ages, have diffused themselves into the mass of the earth, so as to produce there a temperature increasing with the depth? This is a question of high importance. Certain easily satisfied minds conscientiously supposed that they had solved it, when they stated that the idea of a constant temperature was by far the most natural; but woe to the sciences if they thus included vague considerations, which escape all criticism, among the motives for admitting and rejecting facts and theories! Fontenelle, gentlemen, would have traced their horoscope in these words, so well adapted for humbling our pride, and the truth of which the history of discoveries reveals in a thousand places: "When a thing may be in two different ways, it is almost always that which appears at first the least natural."
Whatever importance these reflections may possess, I hasten to add that, instead of the arguments of his predecessors, which have no real value, Fourier has substituted proofs, demonstrations; and we know what meaning such terms convey to the Academy of Sciences.
In all places of the earth, as soon as we descend to a certain depth, the thermometer no longer experiences either diurnal or annual variation. It marks the same degree, and the same fraction of a degree, from day to day, and from year to year. Such is the fact: what says theory?

Let us suppose, for a moment, that the earth has constantly received all its heat from the sun. Descend into its mass to a sufficient depth, and you will find, with Fourier, by the aid of calculation, a constant temperature for each day of the year. . You will recognize further, that this solar temperature of the inferior strata varies from one climate to
another; that in each country, finally, it ought to be always the same, so long as we do not descend to depths which are too great relatively to the earth's radius.
Well, the phenomena of nature stand in manifest contradiction to this result. The observations made in a multitude of mines, "observations of the temperature of hot springs coming from different depths, have fall given an increase of one degree of the centigrade for every twenty or thirty meters of depth. Thus, there was some inaccuracy in the hy. pothesis which we were discussing upon the footsteps of our colleague. It is not true that the temperature of the tertestrial strata may be attributed solely to the action of the solar rays.

This being established, the increase of heat which is observed in all climates when we penetrate into the interior of the globe is the manifest indication of an intrinsic heat. The earth, as Descartes and Leibnitz maintained it to be, but without being able to support their assertions by any demonstrative reasoning,-thanks to a combination of the observations of physical inquirers with the analytical calculations of Fourier,-is an incrusted sun, the high temperature of which may be boldly invoked every time that the explanation of aucient geological phenomena will require it.

After having established that there is in our earth an inherent heata heat the source of which is not the sun, and which, if we may judge of it by the rapid increase which observation indicates, ought to be already sufficiently intense at the depth of only seven or eight leagues to hold in fusiou all known substances-there arises the question, what is its precise value at the surface of the earth ; what weight are we to attach to it in the determination of terrestrial temperatures; what part does it play in the phenomena of life?
According to Mairan, Buffon, and Bailly, this part is immense. For France, they estimate the heat which escapes from the interior of the earth at twenty-nine times in sumnier, and four hundred times in winter, the heat which comes to us from the sun. Thas, contrary to general opinion, the heat of the body which illuminates us would form only a very small part of that whose propitious influence we feel.

This idea was developed with ability and great eloquence in the Mlemoirs of the Academy, in the Epoques sur la Nature of Buffon, in the letters from Bailly to Voltaire upon the Origin of the Sciences and upon the Atlantide. But the ingenious romance to which it has served as a base has ranished like a shadow before the torch of mathematical science.
Fourier having discovered that the excess of the aggregate temperature of the earth's surface above that which would result from the sole action of the solar rays has a determinate relation to the increase of temperature at different depths, succeeded in deducing from the experimental value of this increase a uumerical determination of the excess in question. This excess is the thermometric effect which the solar heat produces at the surface. Now, instead of the large numbers adopted by

Marian, Baills, and Buffon, what has our colleague found A thirtieth of a degree ; not more.
The surface of the earth, which originally was perhaps incandescent, has cooled then in the course of ages so as hardly to preserve any seusible trace of its primitive heat. However, at great depths, the original heat is still enormous. Time will alter sensibly the internal temperature; but at the surface (and the phenomena of the surface can alone modify or compromise the existence of living beings) all the changes are almost accomplished. The frightful freezing of the earth, the epoch of which Buffon fixed at the instant when the central heat would be totally dissipated, is then a pure dream. At the surface, the earth is no longer impregnated except by the solar heat. So long as the sun shall continue to preserve the same brightness, mankind will find, from pole to pole, under each latitude, the climates which have permitted them to live and to establish their residence. These, gentlemen, are great, magnificent results. While recording them in the annals of science, historians will not neglect to draw attention to this singular peculiarity-that the geometer, to whom we owe the first certain demonstration of the existence of a heat independent of a solar influence in the interior of the earth, has annihilated the immense part which this primitive heat was made to play in the explanation of the phenomena of terrestial temperature.

Besides divesting the theory of climates of an error which occupied a prominent place in science, supported as it was by the imposing authority of Marian, of Bailly, and of Buffon, Fourier is entitled to the merit of a still more striking achievement; he has introduced into this theory a consideration which hitherto had been totally neglected; he has pointed out the influence exercised by the temperature of the celestial regions, amid which the earth describes its immense orb around the sun.
When we perceive, even under the equator, certain mountains covered with eternal snow, upon observing the rapid diminution of temperature which the strata of the atmosphere undergo during ascents in balloons, meteorologists have supposed that, in the regions wherein the extreme rarity of the air will always exclude the presence of mankind, and that especially beyond the limits of the atmosphere, there ought to prevail a prodigious intensity of cold. It was not merely by hundreds, it was by thousands of degrees, that they had arbitrarily measured it. But, as usual, the imagination (cette folie de la maison) had exceeded all reasonable limits. The hundreds, the tens of thousands of degrees, have dwindled down, after the rigorous researches of Fourier, to fifty or sixty degrees only. Fifty to sixty degrees beneath zero, such is the temperature which the radiation of heat from the stars has established in the regions furrowed indefinitely by the planets of our system.
You recollect, gentlemen, with what delight Fourier used to converse upon this subject. You know well that he thought himself sure of having assigned the temperature of space within eight or ten degrees.

By what fatality has it happened that the memoir, wherein, no doubt, our colleague had recorded all the elements of that important determination, is not to be found? May that irreparable loss prove at least to so many observers that, instead of pursuing obstinately an ideal perfection, which it is not allotted to man to attain, they will act wisely in placing the public, as soon as possible, in the confidence of their labors?

I should have yet a long course to pursue if, after having pointed out some of those problems of which the condition of science enabled our learned colleague to give numerical solutions, I were to analyze all those which, still enveloped in general formulæ, await merely the data of experience to assume a place among the most curious acquisitions of modern physics. Time, which is not at my disposal, precludes me from dwelling upon such developments. I should be guilty, however, of an unpardonable omission if I did not state that, among the formulæ of Fourier, there is one which serves to assign the value of the secular cooling of the earth, and in which there is involved the number of centuries which have elapsed since the origin of this cooling. The question of the antiquity of the earth, including even the period of incandescence, which has been so keenly discussed, is thus reduced to a thermometric determination. Unfortunately this point of theory is sulject to serious difficulties. Besides, the thermometric determination, in consequence of its excessive smallness, must be reserved for future ages.

I have just exhibited to you the scientific fruits of the leisure hours of the prefect of l'Isère. Fourier still occupied this situation when Napoleon arrived at Cannes. His conduct during this grave conjuncture has been the object of a hundred false rumors. I shall then discharge a duty by establishing the facts in all their truth, according to what I have heard from our colleague's own mouth.

Upon the news of the Emperor having disembarked, the principal authorities of Grenoble assembled at the residence of the prefect. There each individual explained ably, but especially, said Fourier, with much detail, the difficulties which he perceived. As regards the means of vanquishing them, the authorities seemed to be much less inventive. Confidence in administrative eloquence was not yet worn out at that epoch; it was resolved accordingly to have recourse to proclamations. The commanding officer and the prefect presented each a project. The assembly was discussing minutely the terms of them, when an officer of the gensdarmes, an old soldier of the imperial armies, exclaimed rudely, "Gentlemen, be quick, otherwise all deliberation will become useless. Believe me, I speak from experience; Napoleon always follows rery closely the couriers who announce his arrival." Napoleon was in fact close at hand. After a short moment of hesitation, two companies of sappers, which had been dispatched to cut down a bridge, joined their former commander. A battalion of infantry soon followed their example. Finally, upon the very glacis of the fortress, in presence of the numerous population which crowned the ramparts, the fifth regiment of the line to
a man assumed the tricolor cockade, substituted for the white flag the eagle-witness of twenty battles-which it had preserved, and departed with shouts of Vive l'Empereur! After such a commencement, to attempt to hold the country would have been an act of folly. General Marchand caused accordingly the gates of the city to be shut. He still hoped, notwithstanding the evidently hostile disposition of the inhabitants, to sustain a siege with the sole assistance of the third regiment of engineers, the fourth regiment of artillery, and some weak detachments of infantry which had not abandoned him.

From that moment, the civil authority had disappeared. Fourier thought then that he might quit Grenoble, and repair to Lyons, where the princes had assembled together. At the second restoration, this departure was imputed to him as a crime. He was very near being brought before a court of assizes, or even a provost's court. Certain personages pretended that the presence of the prefect of the chief place of l'Isère might have conjured the storm ; that the resistance might have been more animated, better arranged. People forgot that nowhere, and at Grenoble even less than anywhere else, was it possible to organize even a pretext of resistance. Let us see then, finally, how this martial city-the fall of which Fourier might have prevented by his mere pres-ence-let us see how it was taken. It is eight o'clock in the evening. The inhabitants and the soldiers garrison the ramparts. Napoleon precedes his little troop by some steps; he advances even to the gate; he knocks, (be not alarmed, gentlemen, it is not a battle which I am about to describe,) he knocks with his snuff-box! "Who is there" "cried the officer of the guard. "It is the Emperor! Open!" "Sire, my duty forbids me." "Open, I tell vou; I have no time to lose." "But, sire, even though I should open to you, I could not. The keys are in the possession of General Marchand." "Go, then, and fetch them.". "I am certain that he will refuse them to me." "If the general refuse them, tell him that I will dismiss him."

These words petrified the soldiers. During the previous two days, hundreds of proclamations designated Bonaparte as a wild beast which it was necessary to seize without scruple; they ordered everybody to run away from him, and yet this man threatened the general with deprivation of his command! The single word dismissal effaced the faint line of demarkation which separated for an instant the old soldiers from the young recruits ; one word established the whole garrison in the interest of the Emperor.

The circumstances of the capture of Grenoble were not yet known when Fourier arrived at Lyons. He brought thither the news of the rapidadvance of Napoleon; that of the revolt of two companies of sappers, of a regiment of infantry, and of the regiment commanded by Labédoyère. Moreover, he was a witness of the lively sympathy which the country people along the whole route displayed in favor of the proscribed exile of Elba.

The Count d'Artois gare a very cold reception to the prefect and his communications. He declared that the arrival of Napoleon at Grenoble was impossible; that no alarm need be apprehended respecting the disposition of the country people. "As regards the facts," said he to Fourier, "which would seem to have occurred in your presence at the very gates of the city, with respect to the tricolored cockades substituted for the cockado of Henry IV, with respect to the eagles which you say have replaced the white flag, I do not suspect your good faith, but the uneasy state of your mind must have dazzled your eyes. Prefect, return then without delay to Grenoble; you will answer for the city with your head."

You see, gentlemen, after having so long proclaimed the necessity of telling the truth to princes, moralists will act wisely by inviting princes to be good enough to listen to its language.

Fourier obeyed the order which had just been given him. The wheels of his carriage had made only a few revolutions in the direction of Grenoble, when he was arrested by hussars and conducted to the headquarters at Bourgoin. The Emperor, who was engaged in examining a large chart with a pair of compasses, said upon seeing him enter, "Well, prefect, you also have declared war against me?" "Sire, my oath of allegiance made it my duty to do so !" "A duty you say? and do you not see that in Dauphiny nobody is of the same mind? Do not imagine, however, that your plan of the campaign will frighten me much. It only grieved me to see among my enemies an Egyptian, a man who had eaten along with me the bread of the bivouac, an old friend!"

It is painful to add that to those kind words succeeded these also: "How, moreover, could you have forgotten, Monsieur Fourier, that I have made you what you are?"

You will regret with me, gentlemen, that a timidity, which circumstances would otherwise easily explain, should have prevented our colleague from at once emphatically protesting against this confusion, which the powerful of the earth are constantly endeavoring to establish between the perishable bounties of which they are the dispensers and the noble fruits of thought. Fourier was prefect and baron by the favor of the Emperor; he was one of the glories of France by his own genius.

On the 9th of March, Napoleon, in a moment of anger, ordered Fourier, by a mandate, dated from Grenoble, to quit the territory of the seventh military division within five days, under pain of being arrested and treated as an enemy of the country! On the following day our colleague departed from the conference of Bourgoin, with the appointment of prefect of the Rhone and the title of count, for the Emperor after his return from Elba was again at his old practices.

These unexpected proofs of favor and confidence afforded little pleasure to our colleague, but he dared not refuse them, although he per-
ceived very distinctly the immense gravity of the events in which he was led by the vicissitude of fortune to play a part.
"What do you think of my enterprise?" said the Emperor to him on the day of his departure from Lyons. "Sire," replied Fourier, "I am of opinion that you will fail. Let buta fanatic meet you on your way, and all is at an end." "Bah!" exclaimed Napoleon, "the Bourbons have nobody on their side, not eveu a fanatic. In connection with this circumstance, you have read in the jouruals that they have excluded me from the protection of the law. I shall be more indulgent on my part; I shall content myself with excluding them from the Taileries."

Fourier held the appointment of prefect of the Rhone only till the 1st of May. It has been alleged that he was recalled, because he refused to be accessory to the deeds of terrorism which the minister of the huudred days enjoined him to execute. The Academy will always be pleased when I collect together and place on record actions which, while homoring its members, throw new luster around the entire body. I even feel that in such a case I may be disposed to be somewhat credulous. On the present occasion, it was imperatively necessary to institute a most rigorous examinațion. If Fourier honored himself by refusing to obey certain orders, what are we to think of the minister of the interior from whom those orders emanated? Now, this minister, it must not be forgotten, was also an Academician, illustrious by his military services, distinguished by his mathematical works, esteemed and cherished by all his colleagues. Well, I declare, gentlemen, with a satisfaction which you will all sbare, that a most scrupulous investigation of all the acts of the hundred days has not disclosed a trace of anything which might detract from the feelings of admiration with which the memory of Carnot is associated in your minds.
Upon quittiug the prefecture of the Rhone, Fourier repaired to Paris. The Emperor, who was then upon the eve of setting out to join the army, perceiving him amid the crowd at the Taileries, accosted him in a friendly manner, informed him•that Carnot would explain to him why his displacement at Lyons had become indispensable, and promised to attend to his interest as soon as military affairs would allow him some leisure time. The second restoration found Fourier in the capital without employment, and justly anxious with respect to the future. He, who, during a period of fifteen years, administered the affairs of a great department; who directed works of such an expensive nature; who, in the affair of the marshes of Bourgoin, had to contract engagements for so many millions, with private individuals, with the communes, and with public companies, had not twenty thousand francs in his possession. This honorable poverty, as well as the recollection of glorious and important services, was little calculated to make an impression upou ministers influenced by political passion, and subject to the capricious interference of foreigners. A demand for a pension was accordingly repelled with rudeness. Be reassured, however, Frauce will not have to blush for
having left in poverty one of her principal ornaments. The prefect of Paris-I have committed a mistake, gentlemen; a proper name will not be out of place here-M. Chabrol, learns that his old professor at the Polytechnic School, that the perpetual secretary of the Institute of Egypt, that the author of the Théorie Analytique de la Chaleur, was reduced, in order to obtain the means of living, to give private lessons at the residences of his pupils. The idea of this revolts him. He accordingly shows himself deaf to the clanors of party, and Fourier receives from him the superior direction of the Bureau de la Statistique of the Seine, with a salary of 6,000 francs. It has appeared to me, gentlemen, that I ought not to suppress these details. Science may show herself grateful toward all those who give her support and protection, when there is some danger in doing so, without fearing that the burden should ever become too heavy.
Fourier responded worthily to the confidence reposed in him by M. de Chabrol. The memoirs with which he enriched the interesting volumes published by the prefecture of the Seine, will serve henceforth as a guide to all those who have the good sense to see in statistics something else than an indigestible mass of figures and tables.
The Academy of Sciences seized the first occasion which offered itself to attach Fourier to its interests. On the 27th of May, 1816, he was nominated a free Academician. This election was not confirmed. The solicitations and influence of the Dauphin, whom circumstances detained at Paris, had almost disarmed the authorities, when a courtier exclained that an amnesty was to be granted to the civil Labédoyère!" This word-for during many ages past the poor human race has been gorerned by words-decided the fate of our colleague. Thanks to political intrigue, the ministers of Louis XVIII decided that one of the most learned men of France should not belong to the Academy; that a citizen who enjoyed the friendship of all the most distinguished persons in the metropolis should be publicly stricken with disapprobation!

In our country the reign of absurdity does not last long. Accordingly in 1817, when the Academy, without being discouraged by the ill success of its first attempt, unanimously nominated Fourier to the place which had just been vacant in the section of physics, the royal confirmation was accorded without difficulty. I ought to add that soon afterward the ruling authorities, whose repugnances were entirely dissipated, frankly and unreservedly.applauded the happy choice which you made of the learned geometer to replace Delambre as perpetual secretary. They even went so far as to offer him the directorship of the fine arts; but our colleague had the good sense to refuse the appointment.

Upon the death of Lémontey, the French Academy, where Laplace and Cuvier already represented the sciences, called also Fourier into its bosom. The literary titles of the most eloquent of the writers connected

[^3]with the work on Egypt were incontestable; they even were not contested, and still this nomination excited violent discussions in the journals, which profoundly grieved our colleague. And yet, after all, was it not a fit subject for discussion, whether these double nominations are of any real utility? Might it not be maintained, without incurring the reproach of paradox, that it extinguishes in youth an emulation which we are bound by every consideration to encourage? Besides, with double, triple, and quadruple Academicians, what would eventually become of the justly boasted unity of the Institute? Witbout insisting further on these remarks, the justness of which you will admit if I mistake not, I hasten to repeat that the academic titles of Fourier did not form even the subject of a doubt. The applause which was lavished upon the eloquent éloges of Delambre, of Bréguet, of Charles, and of Herschel, would sufficiently evince that, if their author had not been already one of the most distinguished members of the Academy of Sciences, the public would have invited him to assume a place among the judges of French literature.

Restored at length, after so many vicissitudes, to his favorite pursuits, Fourier passed the last years of his life in retirement and in the discharge of academic duties. To converse had become the half of his existence. Those who have been disposed to consider this the subject of just reproach have, no doubt, forgotten that constant reflection is no less imperiously forbidden to man than the abuse of physical powers. Repose, in everything, recruits our frail machine; but, gentlemen, he who desires repose may not obtain it. Interrogate your own recollections and say if, when you are pursuing a new truth, a walk, the intercourse of society, or even sleep, have the privilege of distracting you from the objects of your thoughts? The extremely shattered state of Fourier'shealth enjoined the most careful attention. Aftermany attempts, he found only one means of escaping from the contentions of mind which exhausted him: this consisted in speaking aloud upon the events of his life; upon his scientific labors, which were either in course of being planned, or which were already terminated; upon the acts of injustice of which he had reason to complain. Every person must have remarked how insignificant was the state which our gifted colleague assigned to those who were in the habit of conversing with him ; we are now acquainted with the cause of this.
Fourier had preserved, in old age, the grace, the urbanity, the varied knowledge which, a quarter of a century previously, had imparted so great a charm to his lectures at the Polytechnic School. There was a pleasure in hearing him relate the anecdote which the listener already knew by heart, even the events in which the individual had taken a direct part. I happened to be a witness of the kind of fascination which he exercised upon his audience, in connection with an incident which deserves to be known, for it will prove that the word which I have just employed is not in any wise exaggerated.

We found ourselves seated at the same table. The guest from whom I separated him was an old officer. Our colleague was informed of this, and the question "Have you been in Egypt?" served as a commencement of a conversation between them. The reply was in the aftirmative. Fourier hastened to add: "As regards myself, I remained in that magnificent country until the period of its complete evacuation. Although foreign to the profession of arms; I have, in the midst of our soldiers, fired against the insurgents of Cairo; I have had the honor of hearing the cannon of Heliopolis." Hence to give an account of the battle was but a step. This step was soon made, and we were presented with four battalions drawn up in squares in the plain of Quoubbèh, and maneuvering, with admirable precision, conformably to the orders of the illustrious geometer. My neighbor, with attentive ear, with immovable eyes, and with outstretched neck, listened to this recital with the liveliest interest. He did not lose a single syllable of it ; one would have sworn that he had for the first time heard of those memorable events. Gentlemen, it is so delightful a task to please! After having remarked the effect which he produced, Fourier reverted, with still greater detail, to the principal fight of those great days: to the capture of the fortified village of Mattaryeh, to the passage of two feeble columns of French grenadiers across ditches heaped up with the dead and wounded of the Ottoman army. "Generals, ancient and modern, have sometimes spoken of similar deeds of prowess," exclaimed our colleague, "but it was in the hyperbolic style of the bulletin; here the fact is materially trueit is true like geometry. I feel conscious, however," added he, "that in order to induce you to believe it, all my assurances will not be more than sufficient."
"Do not be anxious upon this point," replied the officer, who at that moment seemed to awaken from a long dream. "In case of necessity, I might guarantee the accuracy of your statement. It was I who, at the head of the grenadiers of the 13th and 85th semi-brigades, forced the entrenchments of Mattaryeh, by passing over the dead bodies of the janissaries."

My neighbor was General Tarayre. You may imagine much better than I can express, the effect of the few words which had just escaped from him. Fourier made a thousand excuses, while I reflected upon the seductive influence, upon the power of language, which for more than half an hour had robbed the celebrated general even of the recollection of the part which he had played in the battle of giants he was listening to.

The more our secretary had occasion to converse the greater repugnance he experienced to verbal discussions. Fourier cut short every debate as soon as there presented itself a somervat marked difference of opinion, only to resume afterward the same subject upon the modest pretext of making a small step in advance each time. Some one asked Fontaine, a celebrated geometer of this Academs, how he occupied his
thoughts in society, wherein he maintained an almost absolute silence. "I observe," he replied, "the vanity of mankind, to wound it as occasion offers." If, like his predecessor, Fourier also studied the baser passions which contend for honors, riches, and power, it was not in order to engage in hostilities with them; resolved never to compromise matters with them, he yet so calculated his movements beforehand as not to find himself in their way. We perceive a wide difference between this disposition and the ardent, impetuous character of the young orator of the popular society of Auxerre. But what purpose would philosophy serve, if it did not teach us to conquer our passions? It is not that occasionally the natural disposition of Fourier did not display itself in fall relief. "It is strange," said one day a certain very influential personage of the court of Charles X, whom Fourier's servant would not allow to pass beyond the antechamber of our colleague, "it is truly strange that your master should be more, difficult of access than a minister!" Fourier heard the conversation, leapel out of his bed to which he was confined by indisposition, opened the door of the chamber, and exclaimed, face to face with the courtier, "Joseph, tell Monsieur, that if I was minister, I should receive everybody, because it would be my duty to do so; but being a private individual, I receive whomsoever I please, and at what hour soever I please!" Disconcerted by the liveliness of the retort, the great seignior did not utter one word in reply. We must even believe that from that moment he resolved not to visit any but ministers, for the plain man of science heard nothing more of him.
Fourier was endowed with a constitution which held forth a promise of long life; but what can natural advantages avail against the antihygienic habits which men arbitrarily acquire? In, order to guard against slight attacks of rheumatism, our colleague was in the habit of clothing himself, even in the hottest season of the year, after a fashion which is not practiced even by travelers condemned to spend the winter amid the snows of the polar regions. "One would suppose me to be corpulent," he used to say occasionally with a smile; "be assured, however, that there is much to deduct from this opinion. If, after the example of the Egyptian mummies, I was subjected to the operation of disembowelment,-from which heaven preserve me,-the residue would be found to be a very slender body." I might add, selecting also my comparison from the banks of the Nile, that in the apartments of Fourier, which were always of small extent and intensely heated, even in summer, the currents of air to which one was exposed resenbled sometimes the terrible simoon, that burning wind of the desert, which the caravans dread as much as the plague.
The prescriptions of medicine which, in the mouth of M. Larrey, were blended with the anxieties of a long and constant friendship, failed to induce a modification of this mortal régine. Fourier had already experienced, in Egypt and Grenoble, some attacks of aneurism of the heart. At Paris it was impossible to be mistaken with respect to the primary
cause of the frequent suffocations which he experienced. A fall, however, which he sustained on the 4th of May, 1830, while descending a flight of stairs, aggravated the malady to an extent beyond what could have been ever feared. Our colleague, nothwithstanding pressing solicitations, persisted in refusing to combat the most threatening symp. toms, except by the aid of patience and a high temperature. On the 16th of May, 1830, about four o'clock in the evening, Fourier experienced in his study a violent crisis, the serious nature of which he was far from being sensible of; for, having thrown himself completely dressed upon his bed, he requested M. Petit, a young doctor of his acquaintance, who carefully attended him, not to go far away, in order, said he, that we may presently converse together. But to these words succeeded soon the cries, "Quick, quick, some vinegar; I am fainting!" and one of the men of science, who has shed the brightest luster upon the A cademy, had ceased to live.

Gentlemen, this cruel event is too recent that I should recall here the grief which the Institute experienced upon losing one of its most important members; and those obsequies, on the occasion of which so many persons, usually divided by interests and opinions, anited together in one common feeling of admiration and regret, around the mortal remains of Fourier; and the Polytechnic School swelling in a mass the cortege, in order to render homage to one of its earliest, of its most celebrated professors; and the words which on the brink of the tomb depicted so eloquently the profound mathematician, the elegant writer, the upright administrator, the good citizen, the devoted friend. We shall merely state that Fourier belonged to all the great learned societies of the world, that they united with the most touching unanimity in the mourning of the Academy, in the mourning of all France: a striking testimony that the republic of letters is no longer, in the present day, merely a vain name. What, then, was wanting to the memory of our colleague? A more able successor than I have been, to exhibit in full relief the different phases of a life so varied, so laborious, so gloriously interlaced with the greatest events of the most memorable epochs of our history. Fortunately, the scientific discoveries of the illustrious secretary had nothing to dread from the incompetency of the panegyrist. My object will have been completely attained if, notwithstanding the imperfection of my sketches, each of you will have learned that the progress of general physics, of terrestrial physics, and of geology will daily multiply the fertile applications of the Théorie Analytique de la Chaleur, and that this work will transmit the name of Fourier down to the remotest posterity.

# ON PROFESSOR THOMAS GRAHAM'S SCIENTIFIC WORK.* 

By William Odling, M.B., F.R.S., Fullerian Professor of Chemistry, R.I.

The simple story of Mr. Graham's life, though not without its measure of interest, and certainly not without its lessons, is referred to in the following pages only in illustration of. the grander story of his work. Thomas Graham was borv in Glasgow, on the 21st December, 1805. He entered as a student at the University of Glasgow, in 1819, with a view to becoming ultimately a minister of the Established Church of Scotland. At that time the university chair of chemistry was filled by Dr. Thomas Thomson, a man of very considerable mark, and one of the most erudite and thoughtful chemists of his day. The chair of natural philosophy was also filled by a man of much learning, Dr. Meikleham, who appears to have taken a warm personal interest in the progress of his since distinguished pupil. Under these masters, Mr. Graham acquired a strong liking for experimental science, and a dislike to the profession chosen for him by his father; who, for a time at least, seems to hare exerted the authority of a parent somewhat harshly, but quite unavailingly, to effect the fulfillment of his own earnest wishes in the matter.

After taking his degree of master of arts at Glasgow, in 1826, Mr. Grahain worked for nearly two years in the laboratory of the University of Edinburgh, under Dr. Hope. He then returned to Glasgow ; and, while supporting himself by teaching, at first mathematics and afterward chemistry, yet found time to follow up the path of experimental inquiry, on which he had already entered.

His first original paper appeared in the Annals of Philosophy for 1826, its author being at that time in his twenty-first year. It is interesting to note that the sulbject of this communication, "On the absorption of gases by liquids," forms part and parcel of that large subject of spontaneous gas-movement with which Mr. Graham's nàme is now so inseparably associated; and that, in a paper communicated to the Royal Society just forty years later, he speaks of the liquefiability of gases by chemical means, in language almost identical with that used in this earliest of his published memoirs.

Having, in the interval, contributed several other papers to the scientific journals, in the year 1829 he published in the Quarterly Journal of Science-the journal, that is to say, of the Royal Institution-the first of his papers relating specifically to the subject of gas-diffusion. It

[^4]was entitled "A short account of experimental researches on the diffusion of gases through each other, and their separation by mechanical means." In the same year, he became lecturer on chemistry at the Mechanics' Institute, Glasgow ; and in the next year, 1830, achieved the yet more decisive step of being appointed professor of chemistry at the Andersonian University. By this appointment he was relieved from anxiety on the score of living, and afforded, in a modest way, the means of carrying out his experimental work.

In 1831 he read, before the Royal Society of Edinburgh, a paper "On the law of the diffiusion of gases," for which the Keith prize of the society was shortly afterward awarded him. Although several of bis earlier papers, and especially that "On the diffiusion of gases," published in the Quarterly Journal of Science, had given evidence of considerable power, it was this paper-in which he established the now well-recognized law that the velocities of diffusion of different gases are inversely as the square roots of their specific gravities-that constituted the first of what may properly be considered his great contributions to the progress of chemical science.

In 1833 he communicated a paper of scarcely less importance, to the Royal Society of London, entitled "Researches on the arseniates, phosphates, and modifications of phosphoric acid." It afforded further evidence of Mr. Graham's quiet, steady power of investigating phenomena, and of his skill in interpreting results; or rather of his skill in setting forth the results in all their simplicity, undistorted by the gloss of preconceived notions, so as to make them render up their own interpretation. It is difficult nowadays to realize the independence of mind involved in Mr. Graham's simple interpretation of the facts presented to him in this research, by the light of the facts themselves, irrespective of all traditional modes of viewing them. Their investigation let in a flood of light upon the chemistry of that day, and formed a starting-point from which many of our most recent advances may be directly traced. In this paper, Mr. Graham established the existence of two new, and, at that time, wholly unanticipated classes of bodies, namely, the class of polybasic acids and salts, and the class of so-called anhydro acids and salts. The views of Graham on the polybasicity of phosphoric acid were soon afterward applied by Liebig to tartaric acid, and by Gerhardt to polybasic acids in general, as we now recog. nize them. After a long interval, the idea of polybasicity was next extended to radicals and to metals by Williamsou and myself successively; afterward to alcohols by Wurtz, and to ammonias by Hofmann. The notion of anhydro-salts was extended by myself to the different classes of silicates; by Wurtz to the compounds intermediate between oxide of ethylene and glycol ; and by other chemists to many different series of organic bodies.

The next most important of the researches completed by Mr. Graham while at Glasgow was the subject of a paper communicated to the

Royal Society of Edinburgb, in 1835 , "On water as a constituent of salts," and of a second paper communicated to the Royal Society of London, in 1836, entitled "Inquiries respecting the constitution of salts, \&c.," for which latter a royal medal of the society was afterward awarded. The subject of hydration had yielded him such a harvest of results in the case of phosphoric acid, that it was only natural he should wish to pursue the inquiry further. Indeed, it is a curious illustration of the persistency of the man that he never seems to have left out of sight the subjects of his early labors. Almost all his subsequent original work is but a development, in different directions, of his youthful researches on gas-diffusion and water of hydration; and so completely did he bridge over the space intervening between these widely remote subjects, that, with regard to several of his later investigations, it is difficult to say whether they are most directly traceable to his primitive work on the one subject or on the other.

In 1837, on the death of Dr. Edward Turner, Mr. Grabam was appointed professor of chemistry at University College, London, then called the University of London. On his acceptance of this appointment he began the publication of his well-known Elements of Chemistry, which appeared in parts, at irregular intervals, between 1837 and 1841. Elementary works, written for the use of students, have necessarily much in common; but the treatise of Mr. Graham, while giving an admirably digested account of the most important individual suk stances, was specially distinguished by the character of the introductory chapters, devoted to chemical physics, wherein was set forth one of the most original and masterly statements of the first principles of chemistry that bas ever been placed before the English student. "The theory of the voltaic circle" had formed the subject of a paper communicated by Mr. Graham to the British Association in 1839 ; and the account of the working of the battery, given in his Elements of Chemistry, and based oil the above paper, will long be regarded as a model of lucid scientific exposition.

In 1841 the now flourishing Chemical Society of London was founded; and though Mr. Graham had been, at that time, but four years in London, such was the estimation in which he was held by his brother chemists, that he was unanimously chosen as the first president of the society. The year 1844 is noticeable in another way. Wollaston and Davy had been dead for some years. Faraday's attention had been diverted from chemistry to those other branches of experimental inquiry in which his highest distinctions were achieved; and, by the death of Dalton in this year, Mr. Graham was left as the acknowledged first of English chemists, as the not unworthy successor to the position of Black, Priestley, Cavendish, Wollaston, Davy, and Dalton.

From the period of his appointment at University College, in 1837, Mr. Graham's time was fully occupied in teaching, in writing, in advising on chemical manufactures, in investigating fiscal and other questions for
the Government, and in the publication of various scientific memoirs, several of them possessing a high degree of interest; but it was not till 1846 that he produced a research of any considerable magnitude. In that year he presented to the Royal Society the first part of a paper "On the motion of gases," the second part of which he supplied in 1849. For this research Mr. Graham was awarded a second royal medal of the society in 1850. The preliminary portion of the first part of the paper related to an experimental demonstration of the law of the effision of gases, deduced from Torricelli's theorem on the efflux of liquids -a demonstration that was achieved by Mr. Graham with much ingenuity, and without his encountering any formidable difficulty. But the greater portion of the first part, and whole of the second part, of this most laborious paper were devoted to an investigation of the velocities of transpiration of different gases through capillary tubes, with a view to discover some general law by which their observed transpiration rates might be associated with one another. Again and again, with characteristic pertinacity, Mr. Graham returned to the investigation; but, although much valuable information of an entirely novel character was acquired-information having an important bearing on his subsequent work-the problem itself remained, and yet remains, unsolved. Why, for example, under an equal pressure, oxygen gas should pass through a capillary tube at a slower rate than any other gas is a matter that still awaits interpretation.

Near the end of the same year, 1849, Mr. Graham communicated, also to the Royal Society, a second less laborious, but in the novelty and interest of its results more successful, paper "On the diffusion of liquids." It was made the Bakerian lecture for 1850, and was supplemented by further observations communicated to the society in 1850 and 1851. In his iuvestigation of this subject, Mr. Graham applied to liquids the exact method of inquiry which he had applied to gases just twenty years before, in that earliest of his papers on the subjectof gas-diffusion published in the Quarterly Journal of Science; and he succeeded in placing the subject of liquid-diffiusion on about the same footing as that to which he had raised the subject of gas-diffusion prior to the discovery of his numerical law.
In 1854 Mr. Graham communicated another paper to the Royal Society, "On osmotic force," a subject intimately connected with that of his last previous communication. This paper was also made the Bakerian lecture for the year; but, altogether, the conclusions arrived at were hardly in proportion to the very great labor expended on the inquiry. In the next year, 1855 , just five-and-twenty years after his appointment at the Andersonian University, Mr. Graham was made master of the mint; and, as a consequence, resigned his professorship at University College. During the next five years he published no original work.
Thus, at the beginning of the year 1861, Mr. Graham, then fifty-six
years of age, had produced, in addition to many less important communications, five principal memoirs; three of them in the highest degree successful ; the other two less successful in proportion to the expenditure of time and labor on them, but, nevertheless, of great originality and value. The most brilliant period, however, of his scientific career was to come. In the year 1861, and between then and his death in 1869, Mr. Graham communicated four elaborate papers to the Royal Society, three of them far exceeding in novelty, interest, and philosophic power anything that he had before produced; and the other of them, relating to a certain physical effect of that hydration of compounds, from the consideration of which his attention could never wholly be withdrawn. This least important paper, "On liquid transpiration in relation to chemical composition," was communicated to the Royal Society in 1861. Of the three greater papers, that "On liquid diffusion applied to analysis" was communicated also in 1861. For this paper more especially, as well as for his Bakerian lectures "On the diffusion of liquids" and "On osmotic force," Mr. Grabam received, in 1862, the Copley medal of the Royal Society; and, in the same year, was also awarded the Jecker prize of the Institute of France. Following in quick succession, his paper "On the molecular mobility of gases" was .presented to the Royal Society in 1863 ; and that "On the absorption and dialytic separation of gases by colloid septa," in 1866. With regard to these three great papers, two of them were each supplemented by a communication to the Chemical Society; while the third was supplemented by four successive notes to the Royal Society, containing an account of further discoveries on the same subject, hardly less remarkable than those recorded in the original paper. The last of these supplementary notes was communicated on June 10, 1869, but a few months before the death, on September 13 , of the indefatigable but physically brokendown man.
In considering Mr. Graham as a chemical philosopher and lawgiver, we find him characterized by a pertinacity of purpose peculiarly his own. Wanting the more striking qualities by which his immediate predecessors, Davy, Dalton, and Faraday, were severally distiuguished, he displayed a positive zeal for tedious quantitative work, and a wonderful keen-sightedness in seizing the points which his innumerable determinations of various kinds, condacted almost incessantly for a period of forty years, successively unfolded. His work itself was essentially that of detail, original in conception, simple in execution, laborious by its quantity, and brilliant in the marvelous results to which it led. As regards its simplicity of execution, scarcely any investigator of recent times has been less a friend to the instrument-maker than Mr. Graham. While availing himself, with much advantage, of appliances devised by Bunsen, Poiseuille, Sprengel, and others, all the apparatus introduced by himself was of the simplest character, and for the most part of laboratory construction.

Essentially inductive in his mode of thought, Mr. Graham developed his leading ideas, one after another, directly from experiment, scarcely, if at all, from the prevailing ideas of the time. As well observed by Dr. Angus Smith, "he seemed to feel his way by his work." His records of work are usually, in a manner almost characteristic, preceded each by a statement of the interpretation or conclusion which he formed; but the records themselves are expressed in the most unbiased matter-offact language. Singularly cautious in drawing his conclusions, he announces them from the first with boldness, making no attempt to convince, but leaving the reader to adopt them or not as he pleases. Accordingly, in giving an account of his various researches, Mr. Graham rarely, if ever, deals with argument; but he states succinctly the experiments he has made, the conclusions he has himself drawn, and not unfrequently the almost daring speculations and generalizations on which he has ventured. Some of these speculations, on the constitution of matter, are reproduced in his own words further ou.

Mr. Graham was elected a fellow of the Royal Society in 1837; corresponding member of the Institute of France in 1847 ; and doctor of civil law of Oxford in 1855.

The remaining pages of this abstract are devoted to an account of his principal discoveries-the generalizations they suggested to him, and the relations in which they stood to precedent knowledge.

## I.

Modifications of phosphoric acid.-At the date of'Mr. Graham's investigation of this subject, when oxy-salts were usually represented as compounds of anhydrous base with anhydrous acid, the point of greatest importance, with regard to each class of salts, was held to be the ratio borne by the oxygen of the base to the oxygen of the acid. Thus, in the carbonates, this ratio was as 1 to 2 ; in the sulphates, as 1 to 3 ; and in the nitrates, as 1 to 5 . But with regard to the pbosphates, taking common phosphate of soda as a type of phosphates in general, there was a difficulty. Dr. Thomson maintained that, in this salt, the ratio of the oxygen of the base to the oxygen of the acid was as 1 to 2 ; and his view was substantially supported by Sir Humphrey Davy. Berzelins contended, however, that the ratio was as 1 to $2 \frac{1}{2}$, or, to avoid the use of fractions, as 2 to 5 ; but, notwithstanding the excellence of the Swedish chemist's proof, and its corroboration by the researches of others, the simpler and, as it seemed, more harmonious view of Dr. Thomson prevailed very generally in this country. Anyhow, those phosphates in which the oxygen ratio Jas the same as that in phosphate of soda were takeu as the neutral salts. But phosphate of soda was found to have the peculiar and quite inexplicable property of reacting with nitrate of silver to throw down, as a jellow precipitate, a phosphate of silver, in which the proportion of metallic base exceeded that in the original phosphate of soda-the precipitation of the basic salt being
accompanied correlatively by the formation of a strongly acid liquid According to Berzelius, the ratio of the oxygen of the base to that of the acid, in this yellow precipitate, was as 3 to 5.

In 1821 Mitscherlich, then working in Berzelius's laboratory, obtained, by treating ordinary phosphate of soda with aqueous phosphoric acid, a new crystallizable phosphate of soda, in which the ratio of acid to base was twice as great as that in the ordinary phosphate. This new salt, which had a strongly acid reaction to test paper, he called the biphosphate of soda. He observed that it was a hydrated salt, and that while the ratio in it of the oxygen of the base to the oxygen of the acid, was as 1 to 5 , the ratio of the oxygen of the base to the oxygen of the water was 1 to 2 .
In 1827 Mr . Graham's fellow-townsman, and predecessor at the Mechanics' Institute, Dr. Clark, discovered another new phosphate of soda, in which the ratio of the oxygen of the base to the oxygen of the acid was identical with that in the ordinary phosphate, namely, as 2 to 5 . But whereas the ordinary phosphate crystallized with 25 proportions of water, the new phosphate crystallized with only 10 ; and whereas the ordinary phosphate gave a yellow precipitate with nitrate of silver and a strongly acid supernatant liquid, the new phosphate gave a chalkwhite precipitate with nitrate of silver and a perfectly neutral supernatant liquid. This new phosphate, being formed by heating the common phosphate to redness, was accordingly designated the pyrophosphate. By dissolution in water and evaporation of the liquid, it could be obtained in the 10 -hydrated crystalline state; and by desiccation at a sand-bath heat, the crystalline salt could be again rendered anhydrous. With regard to the 25 proportions of water belonging to the ordinary salt, Dr. Clark noticed that 24 proportions could be driven off by a sandbath heat, and that this moderate heat did not alter the nature of the salt. He found that the 25 th proportion of water, however, could only be driven off by a full red heat; and that, simultaneously with its expulsion, the change in the nature of the salt was effected. But he carefully guarded himself against being supposed to think that the change in properties of the salt was consequent upon an elimination of its water. The driving off of water from salts being, as he justly remarked, a common effect of heat, he regarded this effect as a concomitant only of the peculiar effect of heat in altering the nature of the phosphate.
Other anomalies with regard to phosphoric acid and the phosphates were also known to chemists; and, on referring now to standard chemical works written before the year 1833, the whole subject of the phosphates will be seen to be in the greatest confusion. It was in this year that Mr. Graham communicated his paper, entitled "Researches on the arseniates, phosphates, and modifications of phosphoric acid," to the Royal Society,*

In the course of these researches he established the existence of a

[^5]class of soluble sub-phosphates analogous to the yellow insoluble phosphate of silver; and he showed, with great clearness, that in the three classes of phosphates, namely, the sub-phosphates, the common phosphates, and the bi-phosphates, the ratio borne to the oxygen of the acid by the other oxygen of the salt is the same, namely, as 3 to 5 ; only that, in the three classes of salts, the non-acid oxygen is divided between different proportions of metallic base and water, thus:

| Sub-phosphate of sod | $3 \mathrm{NaO}, \mathrm{PO}_{5}$. |
| :---: | :---: |
| Common phosphate of soda | $\mathrm{HO} .2 \mathrm{NaO} . \mathrm{PO}_{5}$. |
| Bi-phosphate of soda | $2 \mathrm{HO} . \mathrm{NaO} . \mathrm{PO}_{5}$. |

He further pointed out that, to these three series of salts, there corresponded a definite phosphate of water, or,

$$
\text { Hydrated phosphoric acid............................................ } 3 \text { HO.P } \mathrm{O}_{6}
$$

Compounds of one and the same anhydrous acid with one and the same anhydrous base, in different proportions, had long been known; but it was thas that Mr. Graham first established the notion of polybasic compounds--the notion of a class of hydrated acids having more than one proportion of water replaceable by metallic oxide, and that successively, so as to furnish more and more basic salts, all preserving, as we should now say, the same type.
Mr. Graham further showed that Dr. Clark's pyrophosphate of soda, like the common phosphate, yielded an acid-salt or bi-phosphate; and that these two compounds were related to a hydrated phosphoric acid differing in composition and properties from the above-mentioned hydrate, and yielding, after neutralization with alkali, a white instead of a yellow precipitate with nitrate of silver. This series of compounds he expressed by the following formulæ:

| Clark's pyrophosphate of soda | $2 \mathrm{NaO} \mathrm{PO}_{6}$ |
| :---: | :---: |
| Acid or bi-pyrophosphate of soda. | H0.NaO. $\mathrm{PO}_{5}$. |
| Hydrated pyrophosphoric acid | $2 \mathrm{HO} \mathrm{PO}_{5}$. |

Lastly, Mr. Grabam showed that when the bi-phosphate or bi-pyrophosphate of soda was ignited, there was left a new variety of phosphate, which he called the metaphosplate, haviug the same proportions of soda and anhydrous phosphoric acid as the original compound, but differing from it in several properties, more particularly in its inability to furuish any acid salt. From this new phosphate he obtained the corresponding hydrated acid, and found it to be identical with that variety of phosphoric acid then, and still, known as glacial phosphoric acid, which had previously been noticed to possess the distinctive property of causing a precipitate in solutions of albumen. This salt and acid he represented as follows:

$$
\begin{aligned}
& \text { Metaphosphate of soda } \\
& \mathrm{Na} 0 . \mathrm{PO}_{5} \\
& \text { Metaphosphorio acid } \\
& \text { H O. P } 05 .
\end{aligned}
$$

Speaking of the acid obtainable from, and by its neutralization reconverted into, the phosphate, pyrophosphate, and metaphosphate of soda respectively, Mr. Graham remarked: "The acid, when separated from the
base, will possess and retain for some time the characters of its peculiar modification. * * * But I suspect that the modifications of phosphoric acid, when in what we would call a free state, are still in combination with their usual proportion of base, and that that base is water. Thus the three modifications of phosphoric evidence may be composed as follows :

Phosphoric acid
3 НО. $\mathrm{PO}_{5}$.
Pyrophosphoric acid.
2HO. PO
Metaphosphoric acid.
HO. $\mathrm{PO}_{5}$;
or they are respectively a tri-phosphate, a bi-phosphate, and phosphate of water." These remarks be followed up by analytical eridence, showing the existence of the three hydrates, each in its isolated state.
Just as in his demonstration of the relationship to one another of sub-phosphate of soda, phosphate of soda, bi-phosphate of soda, and common phosphoric acid, Mr. Graham originated the notion of polybasic compounds, so, in his demonstration of the nature of the pyrophosphates and metaphosphates, as bodies differing from the normal compounds by an abstraction of water or metallic base, did he originate the notion of anhydrocompounds-so did he discover, for the first time, an instance of that relationship between bodies which is now known to prevail most extensively among products of organic as well as of miueral origin.
The different properties manifested by phosphoric acid, in its different reputedly isomeric states, having been shown by Mr. Graham to be dependent on a difference of hydration; that is to say, on a difference of chemical composition, he was inclined to view the difference of properties observed in the case of other reputedly isomeric bodies as being also dependent on a difference of composition, the difference occasionally consisting in the presence of some minute disregarded impurity. Accordingly he communicated to the Royal Society of Edinburgh in 1834* a paper "On phosphureted hydrogen," in which he showed that the spontaneously inflammable and non-spontaneonsly inflammable varieties of the gas "are not isomeric bodies, but that the peculiarities of the spontaneously inflammable species depend upon the presence of adventitious matter," removable in various ways, and existing but in very minate proportion.t He further showed that the vapor of some acid of nitrogen, apparently " nitrous acid, is capable of rendering phosphureted hydrogen spontaneously inflammable when present to the extent of one ten-thousandth part of the volume of the gas." In connection with this research may be mentioned Mr. Graham's earlier experiments on the influence of minate impurities in modifying the chemical behavior of different substances. In some "Observations on the oxidation of phosphorus," published in the Quarterly Journal of Science, $\ddagger$ for 1829, he showed that the presence of $\frac{1}{650}$ of olefiant gas, and even $\frac{1}{100}$, by vol-

[^6]ume, of turpentine vapor, in air under ordinary pressure, rendered it incapable of effecting the slow oxidation of phosphorus. He also observed and recorded the influence upon the oxidation of phosphorus of various additions of gas and vapor to air, under different circumstances of pressure and temperature.

## II.

Hydration of compounds.-In the earliest of Mr. Graham's published memoirs, that "On the absorption of gases by liquids,"* he contended that the dissolution of gases in water, at any rate of the more soluble gases, is a chemical phenomenon, depending on their essential property of liquefiability being brought into play by their reaction with the solvent, that is to say by their hydration. The results of some further work on the same subject he published under the title of "Experiments on the absorption of vapors by liquids." $\dagger$

In 1827 he gave to the Royal Society of Edinburgh "An account of the formation of alcoholates, definite compounds of salts and alcohol analogous to the hydrates." $\ddagger$ In this paper, atter a description of some experiments on the desiccation of alcohol, he showed that anhydrous chloride of calcium, nitrate of lime, nitrate of magnesia, chloride of zinc, and chloride of manganese have the property of uniting with alcohol, as with water, to form definite compounds. The crystalline compound with choride of zinc, for instance, containing 15 per ceut. of alcohol, he represented by the formula $\mathrm{ZnCl} .2 \mathrm{C}_{2} \mathrm{H}_{3} \mathrm{O}$; corresponding to the modern formula $\mathrm{Zn} \mathrm{Cl}_{2} .2 \mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}$.
In 1835 Mr . Graham communicated a paper, also to the Royal Society of Edinburgh, "On water as a constituent of salts."§ In this paper he showed more particularly that the so-called magnesian sulphates, crystallizing usually with 7,6 , or 5 proportions of water, gave up all but the last proportion of water at a moderate heat, but retained this last proportiou with great tenacity. The comparatively stable mono-hydrated salts, mono-hydrated sulphate of zinc, for instance, $\mathrm{ZnO} . \mathrm{SO}_{3}$. Н O , he regarded as the analogues of crystallizable sulphuric acid $\mathrm{HO} \mathrm{SO}_{3}$. HO . He showed further that the firmly retained water of sulphate of zinc, for instance, differed from the firmly retained water of phosphate of soda, in not being basic, or replaceable, that is to say, by metallic oxide. He conceived, however, that in the double sulphates, potassio-sulphate of zinc, for instance, $\mathrm{ZnO} . \mathrm{SO}_{3}, \mathrm{~K} \mathrm{O}_{3} \mathrm{SO}_{3}$, the water of the compound, $\mathrm{ZnO} . \mathrm{SO}_{3}$. H O , was replaced by alkali-sulphate, and he accordingly designated the water of this last, and of similar compounds, by the name of saline or constitutional water.

In the following year, $1836, \mathrm{Mr}$. Graham communicated to the Royal

[^7]Society of London an elaborate paper, entitled "Inquiries respecting the constitution of salts, of oxalates, nitrates, phosphates, sulphates, and chlorides." " In it are recorded careful analyses of very many salts, more particularly in respect to their water of hydration; with remarks upon the greater or less tenacity with which the water is retained in different instances. In this paper he pat forward the notion that truly basic salts are nevertheless neutral in constitution; and that the excess of metallic base does not stand in the relation of a base to the anhydrous acid, but as a representative of the water of hydration of the neutral salt. He illustrated this position by a comparison of the defnite hydrate of nitric acid with other hydrated nitrates, thus:

| Hydrated nitric acid, sp. | HO. $\mathrm{NO}_{5} .3 \mathrm{HO}$. |
| :---: | :---: |
| Hydrated nitrate of zinc | $\mathrm{ZnO} \mathrm{NO}_{6} .3 \mathrm{HO}$. |
| Hydrated nitrate of coppe | $\mathrm{CuO}, \mathrm{NO}_{6} .3 \mathrm{HO}$. |
| Basic nitrate of copper | HO. $\mathrm{NO}_{5} .3 \mathrm{CuO}$. |

He contended that, in the last cupric salt, it is the water and not the oxide of copper which acts as a base; and, in support of this view, he remarked that if the water of the salt were water of hydration simply, it ought, in presence of so large an excess of metallic base, to be very readily expelled by heat; whereas it is actually inexpulsable by any heat whatever, short of that effecting an entire decomposition of the salt. Again, he pointed out that when the strongest nitric acid HO.NO ${ }_{5}$ is added, in no matter what excess, to oxide of copper, the basic salt is alone produced, apparently by a direct addition of the oxide of copper to the nitrate of water.

In 1841 Mr . Graham gave to the Chemical Society "An account of experiments on the heat disengaged in combination." $\dagger$ These experiments included numerous determinations of the heat evolved in the hydration of salts, and more particularly of the sulphates, including sulphate of water, or hydrated sulphuric acid. Starting from oil of vitriol $\mathrm{H} \mathrm{O} . \mathrm{SO}_{3}$, he found that each successive addition of a proportion of water HO, evolved an additional, but successively smaller and smaller increment of heat; and that, even after the addition of fifty proportions of water to the acid, the further addition of water was yet followed by a perceptible development of heat.
The relation of ether to alcohol being regarded as that of an oxide to its hydrate, and expressed by the formulæ $\mathrm{C}_{4} \mathrm{H}_{5} \mathrm{O}$, and $\mathrm{C}_{4} \mathrm{H}_{5} \mathrm{O} . \mathrm{HO}$, the conversion of alcohol into ether became a matter of dehydration; and, accordingly, could not escape the examination of Mr. Gralam, who, in 1850, presented to the Chemical Society some "Observations on etherification." $\ddagger$ The process of manufacture consisting in the distillation of a misture of alcohol with sulphuric acid, and being attended by an intermediate production of sulphate of ether or sulphethylic acid, the substitution of ether for the basic water of sulpharic acid at one

[^8]temperature, and the reverse substitution of water for the basic ether of sulphethylic acid at a higher temperature, had been represented as depending on the augmented elasticity of the ether vapor at the higher temperature. Mr. Graham showed, however, that ether could be very readily formed by heating the mixture of sulphuric acid and alcohol in sealed tubes-that is, under conditions in which the angmentation of volatility due to heat was pari passu countromalanced by the diminution of volatility due to pressure. Altogether, Mr. Grabain supported the contact theory of ether formation, as opposed to the then received reaction theory; but several of his experiments afforded clear, though indeed supererogatory, support to the reaction theory soon afterward introduced by Williamson.

In addition to the memoirs cited above, the question of hydration formed an express or incidental subject of many other of Mr. Graham's investigations. It is noteworthy that, for him, osmosis became a mechanical effect of the hydration of the septum; that the interest attaching to liquid-trauspiration was the alteration in rate of passage consequent on an altered bydration of the liquid; that the dialytic difference between crystalloids and colloids depended on the dehydration of the dialytic membrane by the former class of bodies only; and similarly in many other instauces.

## III.

Movements of liquids under pressure. Transpiration.-That the velocities with which different liquids, under the same pressure, issue from a hole in the side or bottom of a vessel should be inversely as the square roots of their respective specitic gravities is a propositiou deducible from well-known mechanical principles. As demonstrated, however, by Dr. Poiseuille, this law is not applicable to the case of liquids issuing under pressure through capillary tubes. In addition to determining experimentally the laws of the passage of the same liquid-that the velocity is directly as the pressure, inversely as the length of the capillary, and directly as the fourth power of the diameter, and that it is accelerated by elevation of temperatureDr. Poiseuille further showed that the rate of passage of different liquids through capillary tubes is for the most part a special property of the particular liquids; and that while the rate of passage of water, for instance, is scarcely affected by the presence of certain salts in solution, it is materially accelerated by the presence of chlorides and nitrates of potassium and ammonium, and materially retarded by the presence of alkalies. He also showed that while the rate of passage of absolute alcohol is much below that of water, the rate of passage of alcohol diluted with water in such proportion as to form the bydrate, $\mathrm{H}_{6} \mathrm{C}_{2} \mathrm{O}$. 3 Aq , is not only much below that of alcohol, but also below that of any other mixture of alcohol and water.

Some time after Dr. Poisenille's death Mr. Graham, starting from this last observation, took up the inquiry. Giving to the phenomenon itself
the name of "transpiration," which he had previously applied to the similar passage of gases through capillary tubes, he communicated his results to the Royal Society in a paper "On liquid transpiration in relation to chemical composition."* The method he followed in his experiments was precisely that of Dr. Poiseuille, and the principal results at which he arrived are the following:

1. That dilution with water does not effect a pari passu alteration in the transpiration velocity of certain liquids; but that dilution up to a certain point, corresponding to the formation of a definite hydrate, not unfrequently retards the transpiration velocity for increases the transpiration time) to a maximum, from which the retardation gradually diminishes with further dilution. This is well seen in the following table, giving the transpiration times of certain liquids in their undiluted state, and also the maximum transpiration times observed with the same liquids when diluted with a regularly increasing quantity of water, the particular dilution causing the maximum retardation corresponding in every case to the production of a definite hydrate:

| Water | Transpiration times. |  | $x$ Aq. |
| :---: | :---: | :---: | :---: |
|  | 1.000 | 1. 000 |  |
| Sulphuric acid.. $\mathrm{H}_{2} \mathrm{SO}_{4}$ | 21.651 | 23. 771 | $\mathrm{H}_{2} \mathrm{SO}_{4}$. Aq. |
| Nitric acid ...... $\mathrm{HNO}_{3}$. | . 990 | 2.103 | $2 \mathrm{HNO}_{3} .3 \mathrm{Aq}$ - |
| Acetic acid. . . . . $\mathrm{H}_{4} \mathrm{C}_{2} \mathrm{O}_{2}$. | 1.280 | 2.704 | $\mathrm{H}_{4} \mathrm{C}_{2} \mathrm{O}_{2} .2 \mathrm{Aq}$. |
| Alcohol ......... $\mathrm{H}_{6} \mathrm{C}_{2} \mathrm{O}$ | 1.195 | 2.787 | $\mathrm{H}_{6} \mathrm{C}_{2} \mathrm{O} .3 \mathrm{Aq}$. |
| Wood-spirit .... $\mathrm{H}_{4} \mathrm{CO}$ | . 630 | 1.802 | $\mathrm{H}_{4} \mathrm{C} 0.3 \mathrm{Aq}$. |
| Acetone $\ldots \ldots . . . . \mathrm{H}_{6} \mathrm{C}_{3} \mathrm{O}$ | . 401 | 1.604 | $\mathrm{H}_{6} \mathrm{C}_{3} \mathrm{O} .6$ Aq. |

2. That the transpiration times of homologous liquids increase reg. ularly with the complexity of the several molecules constituting terms of the same series-certain first terms of the different series, however, preseuting some anomalies, as was, indeed, to be expected. The transpiration times of the fatty ethers are given below in illustration. Similar results were obtained with the series of fatty acids and their corresponding alcohols:

Transpiration times.

In this paper Mr. Graham also recorded the results of two very full series of determinations of the transpiration rates of water at different temperatures between $0^{\circ}$ and $70^{\circ}$, and of two similar series of experiments made with alcohol. The transpiration velocity of water was found to increase uniformly from 0.559 at $0^{\circ}$ to 1.000 at $20^{\circ}$, and thence

[^9]to 2.350 at $70^{\circ}$; and correlatively the transpiration times were found to decrease in the same proportion. The results obtained with alcohol were precisely similar.
IV.

Diffusion of liquids.-Mr. Graham's early study of the spontaneous movements of gases, so as to mix with one another, naturally led him to investigate the similarly occurring movements of liquids. His results formed the subject of two papers communicated to the Royal Society, one in 1849, "On the diffasion of liquids,"* and the other in 1861, "On liquid diffusion applied to analysis." $\dagger$ In the series of experiments described in the first of these papers and in two supplementary communications an open. wide-mouthed vial, filled with a solution of some salt or other substance, was placed in a jar of water; when, in course of time, a portion of the dissolved salt, described as the diffusate, passed gradually from the vial into the external water. By experimeuting in this manner, the amounts of diffusate yielded by different substances were found to vary greatly. Thus, under precisely the same conditions, common salt yielded twice as large a diffusate as Epsom salt, and this latter twice as large a diffusate as gun-arabic. Erery substance examined was in this way found to have its own rate of diffusibility in the same liquid medium-the rate varying with the nature of the mediumwhether water or alcohol, for instance. It is noticeable that the method of vial diffusion resorted to in these experiments is exactly similar to that employed by Mr. Graham in his earliest experiments on the diffusion of gases, published in the Quarterly Jourual of Science for 1829.

In the series of experiments recorded in the paper "On liquid diffusion applied to analysis," the solution of the salt to be diffused, instead of being placed in a vial, was conveyed by means of a pipette to the bottom of a jar of water; when, in course of time, the dissolved salt gradually rose from the bottom, through the superincumbent water, to a height or extent proportional to its diffusibility. The results of this method of jar-diffusion were found to bear out generally those attained by the method of vial-diffusion; while they further showed the absolute rate or velocity of the diffusive movement. Thus, during a fourteen days. aqueous diffusion from 10 per cent. solutions of gum-arabic, Epsom salt, and common salt respectively, the gum-arabic rose only through $\frac{7}{14}$ of the superincumbent water, or to a beight of 55.5 millimeters; the Epsom salt rose through the whole $\frac{14}{14}$ of superincumbent water, or to a height of 111 millimeters ; and the common salt not only rose to the top, but would have risen much higher, seeing that the uppermost or fourteenth statum of water, into which it had diffused, contained about fifteen times as much salt as was contained in the uppermost or fourteenth stratum of water into which the Epsom salt had diffused.

[^10]But of all the results obtained, the most interesting, from their bearing on various natural phenomena, were those on the partial separation of different compounds from one another, brought about by their unequal diffusibility. Thus, with a solution of equal weights of common salt and gum-arabic placed in the diffusion-vial, for every 100 milligrams of salt, not more than 22.5 milligrams of gum were found to pass into the external water; or a separation of the salt from the gum, to this large extent, took place spontaneously by the excess of its own proper diffusive movement. Again, when a solution, containing 5 per cent. of common salt and 5 per cent. of Glauber's salt, was submitted for seven days to the process of jar-diffusion, the upper half, or $\frac{7}{14}$, of superincumbent water was found to contain 380 milligrams of common salt and only 53 milligrams of Glauber's salt; or the ratio of common salt to Glauber's salt in the upper half of the liquid was as 100 to 14 , the ratio in the original stratum of solution being as 100 to 100 . And not only a partial separation of mixed saits, but even a partial decomposition of chemical compounds, was fonnd to result from the process of liquid diffusion. Thus the double sulphate of potassium and hydrogen, when submitted to diffiusion, underwent partial decomposition into the more diffusible sulphate of hydrogen and the less diffusible sulphate of potassium; and, similarls, ordinary alum, a double sulphate of aluminum and potassium, underwent partial decomposition into the more diffusible sulphate of potassium, and the less diffusible sulphate of aluminum. Strictly speaking, perhaps, the decomposition of the original salts was not caused by, but only made evident by, the difference in diffusibility of the products.
As a general result of his experiments, Mr. Graham inferred that liquid diffusibility is not associated in any definite way with chemical composition or molecular weight. Thus he found the complex organic bodies picric acid and•sugar to have much the same diffusive rates as common salt and Epsom salt respectively. Isomorphous compounds, however, proved for the most part to be equi-diffusive; although the groups of equi-diffusive substances habitually comprehended other than those which were isomorphous.
Observing further that, in many cases, the diffusion-rates of different equi-diffasive groups stood to one another in some simple numerical relation, Mr. Graham remarked that, "In liquid diffusion we no longer deal with chemical equivalents or the Daltonian atoms; but with masses even more simply related to each other by weight." We may suppose that the cbemical atoms "group together in such numbers as to form new and larger molecules of equal weights for different substances, or * * * of weights which appear to have a simple relation to each other;" and he inferred that the relative weights of these new molecules would be inversely as the square roots of the observed diffusion rates of the substances-that is inversely as the squares of their diffüsion times. Thus the squares of the times of equal diffiusion of hydrate, nitrate, and
sulphate of potassium being 3,6 , and 12 , the densities of their diffusion molecules would be as the reciprocals of these numbers, or as 4,2 , and 1 .

Lastly, in comparing highly diffusive substances on the one hand, with feebly diffusive substances on the other, one broad dissimilarity became apparent, namely, that highly diffusible substances affected the crystalline state, while feebly diffusive substances were amorphous, and characterized, in particular, by a capability of forming. gelatinous hydrates. Hence the distinction established by Mr. Graham between highly diffusive bodies, or crystalloids, and feebly diffusive bodies, or colloids. Compounds capable of existing both in the crystalline and gelatinous states he found to be possessed of two distinct diffusive rates corresponding respectively each to each.

## V.

Dialysis and osmose.-The subject of clialysis was included in the paper "On liquid diffusion applied to analysis," referred to in the preceding section; aud some further results were communicated in 1864 to the chemical society, in a paper "On the properties of silicic acid aud other analogous colloidal substances." *

In the course of his experiments on diffusion, Mr. Graham made the curious discovery that highly diffusible crystalloid bodies were able to diffuse readily, not only into free water, but also into water that was already in a low form of combination, as in the substance of a soft solid, such as jelly or membrane. Common salt, for instance, was found to diffuse into a semi-solid mass of jelly almost as easily and as extensively as into a similar bulk of free water; but the introduction of a gelatinous substance, though not interfering in any appreciable degree with the diffiusion of a crystalloid, was found to arrest almost entirely the diffusion of a colloid. The colloid, of but little tendency to diffuse into free water, proved quite incapable of diffusing into water that was already in a state of combination, however feeble. Hence, although the partial separation of a highly diffusible from a feebly diffusible substance might be effected by the process of free diffusion into water, a much better result was obtained by allowing the diffasion to take place into, or through, the combined water of a soft solid such as a piece of membrane or parchment-paper. In the process of dialysis, then, crystalloid and colloid bodies, existing in solution together, are separated from one another by pouring the mixed solution into a shallow tray of membrane or parchment-paper, and letting the tray rest on the surface of a con'siderable excess of water, once or twice renewed. By this means the crystalloid, in process of time, diffuses completely away through the membranous septum into the free water; but the colloid, being quite incapable of permeating the membrane, however thin, is retained completely on the tray, unable to reach the free water on the other side.

[^11]By means of the process of dialysis, Mr. Graham succeeded in obtaining various colloid organic substances, such as tannin, albumen, gum, caramel, \&c., in a very pure state; some of them, indeed, in a state of purity exceeding any in which they had before been met with. But the most curious results were obtained with different mineral substances, usually thrown down from their dissolved salts in the state of gelatinous or colloid precipitates. Most of these precipitates being soluble in some or other crystaltoid liquid, on submitting the so-produced solutions to dialysis, the crystalloid constituents diffuse away, leaving the colloid substances in pure aqueous solution. By proceeding in this manner, Mr . Graham was able to obtain certain hydrated forms of silica, ferric oxide, alumina, chrome, prussian-blue, stannic acid, titanic acid, tungstic acid, molybdic acid, \&c., \&c., in the state of aqueous solution-these bodies having never before been obtained in solution, save in presence of strongly acid or alkaline compounds serving to dissolve them. Altogether, the production of these colloid solutions of substances, such as silica and alumina-in their crystalline state, as quartz and corundum, completely insoluble-threw an entirely new light upon the conditions of aqueous solution.

The colloidal solutions, obtained as above, of substances usually crystalline, wefe found to be exceedingly unstable. Either spontaneously, or on the addition of some or other crystalloid reagent, even in very minute quantity, they pectized or became converted into solid jellies. Hence Mr. Graham was led to speak of two colloidal states; the peptous or dissolved, and the pectous or gelatinized. In addition to their power of gelatinizing, their mutability, their non-crystalline habit, and their low diffusibility, substances in the colloid state were found to be further characterized by their chemical inertness and by their high combining weights. Thus the saturating power of colloid silica was only about $\frac{1}{36}$ of that of the ordinary acid.
In his supplementary paper communicated to the Chemical Society, Mr. Graham showed how the pectous forms of different mineral colloids could, in many cases, be reconverted into their peptous forms. He further showed how the water of different peptous and pectous colloids could be mechanically displaced by other liquids, as alcohol, glycerine, sulphuric acid, \&c. To the different classes of compounds so formed, he gave distinctive names. Thus, the alcoholic solution and jelly, of silicic acid for instance, he designated as the alcosol and alcogel respectively.

Closely associated with the passage of different liquids through membranes is the action known as endosmose, discovered by Dutrochet. Mr. Graham's principal results on this subject are recorded in a very elaborate paper "On osmotic force," communicated to the Royal Society, in 1854; * but a few furthēr results and a statement of his final views. are contained in the paper, referred to immediately above, "On liquid

[^12]diffusion applied to analysis." When the solution of a saline or other compound is separated from an adjacent mass of water by a membranous septum, a greater or less quantity of the water very commonly passes through the septum into the solution; and if the solution be contained in a vessel of suitable construction, having a broad membranous base and a narrow upright stem, the water, in some cases, flows into the vessel through the membrane, with a force sufficient to raise and sustain a column of 20 inches or more of liquid in the stem. The problem is to account for this flow; which, with acid fluids more particularly, takes place in the reverse direction-i. e. from the solution into the water.

In the course of his experiments Mr. Graham examined the osmotic movement produced with liquids of most diverse character, employing osmometers of animal membrane, albuminated calico, and baked earthenware. His results were, moreover, observed and recorded in very great detail. As an illustration of these results, it may be mentioned that with 1 per cent. solutions in the membranous osmometer, the liquid rose in the stem 2 millimeters in the case of common salt, 20 millimeters with chloride of calcium, 88 millimeters with chloride of nickel, 121 millimeters with chloride of mercury, 289 millimeters with prqto-chloride of tin, 351 millimeters with chloride of copper, and 540 millimeters with chloride of aluminum. Mr. Graham showed, further, in opposition to the views of Dutrochet, that the velocity of the osmotic flow was not proportional to the quantity of salt or other substance originally con: tained in the solution; and that the flow did not depend on capillarity, as Dutrochet had inferred; or yet on diffusion, as some of his own experiments might be thought to indicate. Eventually he was led to the conclusion that osmose was essentially dependent on a chemical action taking place between one or other of the separated liquids and the material of the septum. He appears to have held somewhat different views of the nature of this chemical action at different times, and not to have considered it as being in all cases of the same character.

The following extracts, expressing his latest views on the subject, are taken from the conclusion of his paper "On liquid diffission applied to analysis."
"It now appears to me that the water movement in osmose is an affair of hydration and of de-hydration in the substance of the membrane, or other colloid septum, and that the diffusion of the saline solution placed within the osmometer has little or nothing to do with the osmotic result otherwise than as it affects the state of hydration of the septum. * ** Placed in pure water, such colloids (as animal membrane) are hydrated to a higher degree than they are in neutral saline solutions. Hence the equilibrium of hydration is different on the two sides of the membrane of an osmometer. The outer surface of the membrane being in contact with pure water, tends to hydrate itself in a higher degree than the inner surface does, the latter surface being supposed to be in contact
with a saline solution. When the full hydration of the outer surface extends through the thickness of the membrano, and reaches the inner surface, it there receives a check. The degree of hydration is lowered, and the water must be given up by the inner layer of the membrane, and it forms the osmose. * * * Far from promoting this separation of water, the diffusion of the salt throughout the substance of the membrane appears to impede osmose by equalizing the condition as to saline matter of the membrane through its whole thickness. The advantage which colloidal solutions have in inducing osmose, appears to depend in part upon the low diffusibility of such solutions, and their want of power to penetrate the colloidal septum."

## VI.

Movements of Gases under pressure. Effusion and transpiration.The mechanical law of the passage of different gases under the same pressure through a mere perforation, as of the passage of differentliquids, being that the velocities are inversely as the square roots of the specific gravities, Mr. Graham subjected this law to an experimental verification, and made known his results in a paper communicated to the Royal Society in 1846. The mode of experimenting was as follows: A jar standing on the plate of an air-pump was kept vacuous by continued exhaustion, and a measured quantity of gas allowed to find its way into the jar through a minute aperture in a thin metallic plate. The admission of 60 cubic inches of dry air into the vacuous, or nearly vacuous jar, being arranged to take place in about 1,000 seconds, the times of passage of the same volume of air were found not to vary from each other by more than two or three seconds in successive experiments. Operating with different gases, the relative times of passage, or of "effusion," as it was denominated by Mr. Graham, proved to be approximatively identical with the square roots of the specific gravities of the several gases; or, in other words, their velocities of effusion were shown experimentally to be inversely as the square roots of their specific gravities. The rate of effusion of a mixed gas corresponded in most cases with the calculated mean rate of its constituents; but the rates of effusion of the light gases, marsh gas and hydrogen, were very disproportionately retarded by the admixture with them, even to a small extent, of the heavier gases, oxygen and nitrogen.

Passing from the study of the effusion of gases through a perforated plate, Mr. Graham next submitted their "transpiration" through a capillary tube to a similarly conducted experimental inquiry. His results were communicated to the Royal Society in two very elaborate papers, "On the motion of gases," Parts I and II,* the first part containing also his above-described results on the effusion of gases. With a very short capillary, the relative rates of passage of different gases were found to approximate to their relative rates of effusion; but with

[^13]every elongation of the capillary, a constantly increasing deviation from these rates was observed-the increase of the deviation, however, becoming less and less considerable with each successive increment of elongation, until, when the tube had acquired a certain length in proportion to its diameter, a maximum deviation of the relative rates of passage of the different gases from their relative rates of effusion was arrived at. These ultimate rates of passage, unaffected in relation to each other by further elongation of the capillary, constitute the true transpiration velocities of the different gases, as distinguished from their velocities of effusion. Of all the gases experimented on, oxygen was found to have the longest transpiration time, or slowest transpiration velocity. In the following table its time of transpiration is taken as unity, and the times of a few other gases compared therewith. In other columns are given the specific gravities of the same gases, referred to the specific gravity of air as unity; and the square roots of their specific gravities, which also express their relative times of effiusion.

|  | Specific gravity. | $\sqrt{\frac{\text { Specific }}{\text { gravity }}}$ | Transpiration time. |
| :---: | :---: | :---: | :---: |
| Hydrogen | . 069 | . 263 | . 437 |
| Marsh gas | . 559 | . 747 | . 551 |
| Nitrogen | . 971 | . 985 | . 877 |
| Oxygen | 1.105 | 1.051 | 1.000 |
| Carbonic gas | 1. 529 | 1.236 | . 730 |

That gas transpiration has no direct relation to gas specific gravity is shown by the transpiration times of oxygen and nitrogen exceeding the transpiration times both of the much lighter hydrogen and marsh gas, and of the much heavier carbonic gas. Again, ammonia, olefiant gas, and cyanogen, with the different specific gravities $.590, .978$, and 1.806 respectively, have the almost identical transpiration times $.511, .505$, and .506 ; or, approximatively, half the transpiration time of oxygen, 1.000. Nerertheless the trauspiration times of oxygen and nitrogen are directly as their specific gravities; and further, the specific gravities of nitrogen, carbonic oxide, and nitric oxide being .971, .968, and 1.039 , their transpiration times are $.877, .874$, and .876 respectively. But then olefiant gas, with the same specific gravity .978 , has the much shorter transpiration time .505 ; and similarly in other cases. Altogether the discordance between transpiration and specific gravity is of greater frequency than the accordance; but still the circumstance of gases haring the same, or about the same, specific gravity, having also the same, or about the same, rate of transpiration, is of too frequent occurrence to be merely accidental.

As a rule, the observed transpiration rate of a mixture of gases corresponded with the calculated mean rate of its constituents; bat the transpiration rates of the light gases, hydrogen and marsh gas, were
found to be disproportionately retarded to a greater extent even than their effusion rates by the admixture with them of heavier gases. Further, by employing mixtures of gas and vapor, Mr. Graham extended his inquiry so as to include a determination of the transpiration times of several vapors; the results being calculated on the assumption that the observed transpiration time of the mixture was the mean of the transpiration times of the permanent gas and of the coercible vapor experimented on. In this way the transpiration time of ether vapor, sp. gr. 2.586, was shown to be identical with that of hydrogen gas, sp . gr. 0.069 ; and the transpiration time of carbonic sulphide vapor, sp. gr. 2.645 , identical with that of sulphureted hydrogen gas, sp. gr. 1.191.

With respect to gas transpiration in general, the rates of transpiration of different gases were found to be independent of the nature of the material of the capillary; apparently from the capillary, of what material soever, becoming lined with a film of gas, with which alone the current of gas could come in contact; so that the friction was purely intestine, and suggestive of a sort of viscosity in the gas itself. The rate of passage was further shown to be inversely as the length of the capillary ; and directly, in some high but undetermined ratio, as its diameter. Lastly, the rate of "effusion" of a given volume of any particular gas being independent of pressure and temperature, the rate of transpiration of a given volume of any particular gas was observed to vary directly with its variation of density, whether the result of alteration of pressure or of temperature; 100 cubic inches of dense air, for example, transpiring more rapidly than 100 cubic inches of tenuous air, in proportion to the excess of density.
Speaking of the importance and fundamental nature of the physical properties manifested by bodies in the gaseous state, and of the extent of his own inquiries on gas-transpiration, Mr. Graham observed: "It was under this impression that I devoted an amount of time and attention to that class of constants (transpiration-velocities) which might otherwise appear disproportionate to their value and the importance of the subject. As the results, too, were entirely novel, and wholly unprovided for in the received view of the gaseous constitution, of which indeed they prove the incompleteness, it was the more necessary to verify each fact with the greatest care."

## VII.

Diffusion of gases.-In 1801, Dalton, in an essay "On the constitution of mixed gases, and particularly of the atmosphere," propounded the now celebrated view that "where two elastic fluids denoted by $\mathbf{A}$ and B are mixed together, there is no mutual repulsion among their particles; that is, the particles of $A$ do not repel those of $B$, as they do one another; consequently the pressure or whole weight upon any one particle arises solely from those of its own kind." During the act of admixture, "the particles of A meeting with no repulsion from those of

B . . . . would instantaneously recede from each other as far as possible under the circumstances, and consequently arrange themselves just as in a void space." At the beginning of 1803, in a supplementary paper "On the tendency of elastic fluids to diffusion through each other," he made known the remarkable action of intermixture which takes place, even in opposition to the influence of gravity, when any two gases are allowed to communicate with each other. Thus, in a particular experiment, he showed that when a vial of hydrogen is connected with a vial of carbonic gas by means of a narrow piece of tubing, so that the vial of light hydrogen may be inverted over the other vial of heavy carbonic gas, the heavy carbonic gas actually ascends through the light hydrogen, and the light hydrogen descends through the heavy carbonic gas until the uniform admixture of the two gases with each other is effected. The subject was afterward investigated by Berthelot, who, in a series of experiments performed with great care, while opposing Dalton's theoretical conclusions, corroborated his results, and indicated further the high diffusiveness of hydrogen. Here it was that Mr. Graham took up the inquiry. The first of his papers relating directly to the subject of gas-diffusion appeared in the "Quarterly Journal of Science" for 1829, under the title, " A short account of experimental researches on the diffusion of gases through each other, and their separation by mechanical means."* The mode of proceeding adopted in these researches was as follows: Each gas experimented on was allowed to diffuse from a horizontally placed bottle through a narrow tube, directed either upward or downward according as the gas was heavier or lighter than air, so that the diffusion always had to take place in opposition to the influence of gravity. The result was that equal volumes of different gases escaped in very unequal times, the rapidity of the escape having an inverse relation to the specific gravity of the gas. Thus hydrogen was found to escape four or five times more quickly than the twenty-two times heavier carbonic gas. Again, with a mixture of two gases, the lightest or most diffusible of the two was found to leave the bottle in largest proportion, so that a sort of mechanical separation of gases could be effected by means of their unequal diffusibility. Most of these last results were obtained by allowing the gaseous mịxture to diffuse into a limited atmosphere of some other gas or vapor, capable of subsequent removal by absorption or condensation.

But these methods of operating, by free or adiaphragmatic diffusion, were soon abandoned by Mr. Graham for the more practicable method of diffusion through porous septa. Once again, however, many years afterward, in a paper "On the molecular mobility of gases," to be more fully considered presently, Mr. Graham made some additional and very curious observations on the free diffusion of hydrogen and carbonic gas into surrounding air, showing the absolute velocities of the molecu-

[^14]lar movements in each of the two cases. A glass cylinder, .57 meter high, had the lowest tenth of its height filled with carbonic gas. Then, after different intervals of time, the uppermost tenth of air in the cylinder was drawn off and examined. In five minates the carbonic gas in this upper tenth of air amounted to .04 , and in seven minutes to 1.02 per cent.; or 1 per cent. of carbonic gas had diffused to the distance of half a meter in seven minutes, being at the rate of 73 mil limeters per minute. Now, the conditions of this movement always prevail in the air of the atmosphere, and, using the words of Mr. Graham, "it is certainly most remarkable that in perfectly still air its molecules should spontaneously alter their position, and move to a distance of half a meter in any direction in the course of five or six minutes." By similar experiments made with an inverted cylinder, 1 per cent. of, hydrogen was found to diffuse downward at the rate of 350 millimeters per minute, or about five times as rapidly as the carbonic gas diffused upward.

With regard to Mr. Graham's experiments on the diffusion of gases through porous septa, his earliest results on this subject were communicated to the Royal Society of Edinburgh, in a paper "On the law of the diffiusion of gases," already referred to as the first-born of what may be considered his great papers.* Prior even to Dalton's above-mentioned expermments on free diffusion, Dr. Priestly, when transmitting different gases through stoneware tubes surrounded by burning fuel, perceived that the tubes were porous; and that not only was there an escape of the gas, under pressure, from within the tube outward to the fire, but that there was also a penetration of the exterior gases of the fire into the tube, notwithstanding the superior pressure of the current of gas passing through the tube.

Mr. Graham, however, appears to have had his attention originally directed to the study of the transmission of gases through porous diaphragms by the curious observations and experiments of Döbereiner, who, having occasion to collect and store some quantities of hydrogen over water, accidentally made use of a fissured jar, and was surprised to find that the water of the pneumatic trough rose in this jar to the height of an inch and a half in twelve hours, and to not far short of three inches in twenty-four hours. Having assured himself of the constancy of the phenomenon, Döbereiner attributed it to capillary action, conceiving hydrogen to be alone attractable by, and, on account of the assumed minuteness of its atoms, admissible through the fissure. In repeating Döbereiner's experiments, however, Mr. Graham soon observed that the escape of hydrogen outward was always accompanied by a penetration of air inward, the volume of air finding an entrance through the fissure amounting to about one-fourth of the volume of hydrogen making its escape; or the fissure proved permeable to the grosser air as well as to the finer hydrogen. Having arrived at this

[^15]point, he replaced the fissured jar by an instrument admitting of much greater experimental precision. For the jar itself he substituted a piece of glass tube about half an inch in diameter, and from eight to fourteen inches long, and for the fissure in the jar he substituted a plate of stucco serving to close one end of the tube. Operating with a diffusion-tube of this kind standing in a jar of water, it was found, as in Dalton's experiments, that the two gases, say external air and internal hydrogen, exhibited a powerful tendency to intermix or change places with each other; but more than this, it was found that the air did not exchange with its own volume of hydrogen, but instead with 3.8 times its volume. Using the word diffusion-volume to express the bulks of different gases exchanging thus with one another by the process of diffusion, the diffusion-volume of hydrogen would be 3.8, that of air being taken as 1. Similarly, it was ascertained that every gas has a diffusionvolume which is peculiar to itself, and is indeed inversely as the square root of its specific gravity; and since the unequal diffusion volumes of different gases are consequences of their unequal diffusion velocities, it follows that the relative velocities at which different gases diffuse into one another, by virtue of their own inherent mobility, are identical with those at which they effuse under pressure into a vacuum-a result quite in accordance with, and indeed deducible from, Dalton's aphorism. But although the relative rates of effusion and diffusion are alike, it is important, wrote Mr. Graham, in the later paper already quoted from, "to observe that the phenomena of effusion and diffusion are distinct and essentially different in their nature. The effusion movement affects masses of gas, the diffusion movement affects molecules; and a gas is usually carried by the former kind of impulse with a velocity many thousand times as great as is demonstrated by the latter."*
Thus the result arrived at by Mr. Graham, in his original paper, was the enunciation of the now well-recognized law of the diffusion of gases; but some thirty years afterward, he again subjected the phenomena of gas-diffusion to an elaborate experimental investigation-going over the old and penetrating into new ground with an activity by no means impaired, and with intellectual powers largely expanded by increase of years. His results were communicated to the Royal Society of London, in a paper "On the molecular mobility of gases," + and it is impossible to read this and his original paper "On the law of the diffusion of gases" together, without being struck by the great advance in philosophic grasp and breadth of view which had become developed in the long interval between the publication of the two memoirs. These later experiments on gas-diffusion were made principally with septa of compressed graphite ; and it will, be well to preface their consideration by Mr. Graham's own introductory remarks. He observes:

[^16]"The pores of artificial graphite appear to be really so minute that a 'gas in mass cannot penetrate the plate at all. It seems that molecules only can pass; and they may be supposed to pass wholly unimpeded by friction, for the smallest pores that can be imagined to exist in graphite mast be tunnels in magnitude to the ultimate atoms of a gaseous body. The sole motive agency appears to be that intestine movement of molecules which is now generally recognized as an essential property of the gaseous condition of matter.
"According to the physical hypothesis now generally received, a gas is represented as consisting of solid and perfectly elastic spherical particles or atoms, which move in all directions, and are animated with difs ferent degrees of velocity in different gases. Confined in a vessel, the moving particles are constantly impinging against its sides and occasionally against each other, and this contact takes place without any loss of motion, owing to the perfect elasticity of the particles. If the containing vessel be porous, like a diffusiometer, then gas is projected through the open channels, by the atomic motion described, and escapes. Simultaneously the external air is carried inward in the same manner, and takes the place of the gas which leaves the vessel. To this atomic or molecular movement is due the elastic force, with the power to resist compression, possessed by gases. The molecular movement is accelerated by heat and retarded by cold, the tension of the gas being increased in the first instance and diminished in the second. Even when the same gas is present both within and without the vessel, or is in contact with both sides of our porous plate, the movement is sustained without abatement-molecules continuing to enter and leave the vessel in equal number, although nothing of the kind is indicated by change of volume or otherwise. If the gases in communication be different, but possess sensibly the same specific gravity and molecular velocity, as nitrogen and carbonic oxide do, an interchange of molecules also takes place without any change in volume. With gases opposed of unequal density and molecular velocity, the permeation ceases of course to be equal in both directions."
One set of novel experiments recorded in the later paper, from which the above remarks are extracted, had reference to the diffusion of single gases through porous septa, into a vacuous or partially vacuous space. The diffusion-tube was substantially the same as that formerly employed, except in the circumstance of its being closed by a plate of compressed graphite instead of by stucco, and in the further circumstance of the tabe itself being in some cases so far lengthened and otherwise modified as to admit of the production within it of a barometric vacumm of comparatively large dimensions. The mode of experimenting was as follows: The short tubes, when employed, were filled with mercury, and inverted in a mercurial trough. Then, by means of a very simple arrangement, the gas under examination was allowed to sweep over the surface of, and diffuse through, the graphite plate, so as to depress the
mercury within the tube until it stood at a height of 100 millimeters only-that is, until the external pressure exceeded the internal pressure by 100 millimeters only. Matters being in this state, the experiment consisted in observing the number of seconds required for the admission through the graphite septum, into the graduated tube, of a given volume of gas-the mercury in the tube being kept throughout at the constant height of 100 millimeters, by a gradual lifting up of the tube, effected by a mechanical arrangement originally devised and employed by Professor Bunsen. The long tubes were filled with mercury in a different manver; but the conduct of the experiments made with them differed only from that of the experiments made with the shorttubes, in that the level of mercury in the long tubes was maintained throughout at or near to the barometric height, so that the external gas diffused into the tube under full atmospheric pressure. Experimenting in this way, the relative times of permeation of equal volumes of different gases were found to be almost identical with the square roots of the specific gravities of the respective gases, as shown in.the following table:

|  | Times of equal diffusion. | Square roots of specific gravities |
| :---: | :---: | :---: |
| Oxygen .. | 1.0 | 1.0 |
| Air. | . 9501 | . 9507 |
| Carbonic gas. | 1.1860 | 1.1760 |
| Hydrogen.. | . 2505 | . 2502 |

These results are of great value from the simplicity and constancy of the conditions under which they were obtained, and from their close accordance with the induced law. By allowing the diffusion to take place into a complete or partial vacuum, instead of into an atmosphere of other gas, the results were not complicated with those of interdiffusion; and by employing a thin plate of highly compressed graphite, instead of a comparatively thick plug of more porous stucco, the results were not complicated with those of transpiration, as happened in some otherwise admirable experiments of Professor Bunsen, which led that distinguished investigator to question at one time the accuracy of Mr. Graham's law.
The absence of any transpiration of gas through the graphite wafer was made evident by the want of any approximation, in the rates of passage, to the characteristic rates of transpiration; and was consequent on the impermeability of the exceedingly minute pores of the graphite to any enforced bodily transmission of gas through them. It may be as well to state this conclusion in Mr. Graham's own words:
"The movement of gases through the graphite plate appears to be solely due to their own proper molecular motion, quite unaided by transpiration. It seems to be the simplest possible exhibition of the mole-
cular or diffusive movement of gases. This pure result is to be ascribed to the wonderfully fine (minute) porosity of the graphite. The interstitial spaces appear to be sufficiently small to extinguish capillary transpiration entirely. The graphite plate is a pneumatic sieve which stops all gaseous matter in mass, and permits molecules only to pass."
By similarly conducted experiments, a determination was also made of the difference of rate, if any, at which hydrogen diffuses through a graphite plate into a vacuum and into atmospheric air. Thus, in one minute of time, the following quantities of hydrogen passed through the graphite plate, in the two cases respectively:

> 1.289 cubic centimeters into a vacuum. 1.243 cubic centimeters into air.

These numbers indicate a close approach to equality in the velocities of passage into a vacuum and into a space of other gas-a yet closer equality being probably attainable by a modified form of experimenting.
The diffusion of hydrogen into air, as in the above-referred-to experiment, is of course accompanied by a diffasion of air into hydrogen, which had to be allowed for in calculating out the above result. Moreover, Mr. Graham made a special repetition of his early experiments on interdiffusion, operating with dry instead of moist gas, substituting mercury for water in the diffusion-tube, maintaining a constant pressure by Bunsen's mechanism instead of by a pitcher of water, and using a wafer of graphite instead of a plug of stucco as the porous diaphragm. The theoretical exchange of hydrogen for air being 3.8 volumes for 1 , and that of hydrogen for oxygen being 4.0 volumes for one, the exchanging volumes actually found were 3.876 and 4.124 respectively.
Referring to the approximatively equally rapid passage of hydrogen into a vacuous and aerial space, Mr. Graham remarks as follows on the subject of interdiffusion:
"In fine, there can be little doubt left on the mind that the permeation through the graphite plate into a vacuum, and the diffusion into a gaseous atmosphere, through the same plate, are due to the same inherent mobility of the gaseous molecule. They are the exhibition of this movement in different circumstances. In interdiffusion we have two gases moved simultaneously through the passages in opposite directions, each gas under the influence of its own inherent force; while with gas on one side of the plate, and a vacuum on the other side, we have a single gas moving in one direction only. The latter case may be assimilated to the former if the vacuum be supposed to represent an infinitely light gas. It will not involve any error, therefore, to speak of both movements as gaseous diffusion-the diffusion of gas into gas (double diffusion) in the one case, and the diffusion of gas into a vacuum (single diffusion) in the other. The inherent molecular mobility may also be justly spoken of as the diffusibility or diffusive force of gases.
"The diffusive mobility of the gaseous molecule is a property of matter, fundamental in its nature, and the source of many others. The rate
of diffusibility of any gas has been said to be regulated by its specific gravity, the velocity of diffusion having been observed to vary inversely as the square root of the density of the gas. This is true, but not in the sense of the diffusibility being determined or caused by specific gravity. The physical basis is the molecular mobility. The degree of motion which the molecule possesses regulates the volume which the gas assumes, and is obviously one, if not the only, determining cause of the peculiar specific gravity which the gas enjoys. If it were possible to increase in a permanent manner the molecular motion of gas, its specific gravity would be altered, and it would become a lighter gas. With the density is also associated the equivalent weight of a gaseous element, according to the doctrine of equal combining volumes."
In addition to the above two sets of experiments, on the diffusion of a single gas into a vacuum and on the diffusion of one gas into another, a third set of experiments was made on the diffusion of one gas away from another; or on the partial separation of mixed gases by the process of atmolysis. The experiments on this subject were conducted in several different ways, but the most striking results were obtained with what Mr. Graliam named his tube atmolyser. This instrument consists of one or more lengths of ordinary tobacco-pipe, (conveying the current of mixed gas,) surrounded by a glass tube maintained in a more or less vacuous state by exhaustion with an air-pump. The most diffusible constituent of the mixed gas passing away in largest proportion through the porous material of the tobacco-pipe, the least diffusible constituent becomes concentrated in the residue of gas passing along, and finally delivered by the pipe. By this simple contrivance the proportion of oxygen in ordinary air, transmitted by the tobacco-pipe, was increased from below 21 up to 24.5 per cent., as a result of the small superior diffusive velocity of nitrogen 1.01 , over that of oxygen 0.95 .

In experiments made with the far more unequally diffusive gases oxygen and hydrogen, mixed in equal volumes, the proportion of oxygen transmitted by the tobacco-pipe was increased from the original 50 per cent. to 90 , and even in some cases, to 95 per cent. Electrolytic gas, consisting of 33.3 per cent. oxygen and 66.6 per cent. hydrogen, was slowly transmitted through a single tobacco-pipe, in some experiments inclosed in a vacuum, in others exposed to the air. In the vacuum experiments the transmitted gas was found to consist of 90.7 per cent. oxygen and 9.3 per cent. hydrogen. In the air experiments, the transmitted gas was found to consist of 40.4 per cent. oxygen, 5.5 per cento hydrogen, and 54.1 per cent. air. In both cases it had lost its explosive character, and acquired the property of re-inflaming a glowing splinter.

This paper of Mr. Graham's "On the molecular mobility of gases" was supplemented by a communication made to the Chemical Society in 1864, entitled "Speculative ideas respecting the constitntion of matter,"* from which the following extracts are taken:

[^17]"It is conceivable that the various kinds of matter, now recognized as different elementary substances, may possess one and the same ultimate or atomic molecule existing in different conditions of movement. The esseutial unity of matter is a hypothesis in harmony with the equal action of gravity upon all bodies. We know the anxiety with which this point was investigated by Newton, and the care he took to ascertain that every kind of substance, 'metals, stones, woods, grain, salts, animal substances,' \&c., are similarly accelerated in falling, and are therefore equally heary.
"In the condition of gas, matter is deprived of numerous and varying properties, with which it appears invested when in the form of a liquid or solid. The gas exhibits only a few grand and simple features. These again may all be dependent upon atomic or molecular mobility. Let us imagine one kind of substance only to exist-ponderable matter; and further, that matter is divisible into ultimate atoms, uniform in size and weight. We shall then have one substance and a common atom. With the atom at rest the uniformity of matter would be perfect. But the atom possesses always more or less motion, due, it must be assumed, to a primordial impulse. This motion gives rise to volume. The more rapid the movement the greater the space occupied by the atom, somewhat as the orbit of a planet widens with the degree of projectile velocity. Matter is thus made to differ only in being lighter or denser matter. The specific motion of an atom being inalienable, light matter is no longer convertible into heavy matter. In short, matter of different density forms different substances-different inconvertible elements, as they have been considered.
"But farther, these more and less mobile, or light and heavy forms of matter, have a singular relation connected with equality of volume. Equal volumes of two of them can coalesce together, unite their movement, and form a new atomic group, retaining the whole, the half, or some simple proportion of the original movement and consequent volume. This is chemical combination. It is directly an affair of volume, and only indirectly connected with weight. Combining weights are different, because the densities, atomic and molecular, are different. The volume of combination is uniform, but the fluids measured vary in density. This fixed combining measure-the metron of simple sub-stances-weighs 1 for hydrogen, 16 for oxygen, and so on with the other 'elements.'
"To the preceding statements respecting atomic and molecular mobility, it remains to be added that the hypothesis admits of another expression. As in the theory of light we have the alternative hypotheses of emission and undulation, so in molecular mobility the motion may be assumed to reside either in separate atoms and molecules, or in a fluid medium caused to undulate. A special rate of vibration or pulsation originally imparted to a portion of the fluid medium enlivens that portion of matter with an individual existence, and constitutes it a distinct substance or element.
"Lastly, molecular or diffusive mobility has an obvious bearing upon the communication of heat to gases by contact with liquid or solid surfaces. The impact of the gaseous molecule upon a surface possessing a different temperature appears to be the condition for the transference of heat, or the heat movement, from one to the other. The more rapid the molecular movement of the gas, the more frequent the contact with consequent communication of heat. Hence, probably, the great cooling power of hydrogen gas as compared with air or oxygen. The gases named have the same specific heat for equal volumes, but a hot object placed in hydrogen is really touched 3.8 times more frequently than it would be if placed in air, and 4 times more frequently than it would be if placed in an atmosphere of oxygen gas. Dalton had already ascribed this peculiarity of hydrogen to the high 'mobility' of that gas. The same molecular property of hydrogen recommends the application of that gas in the air-engine, where the object is to alternately heat and cool a confined volume of gas with rapidity."

## VIII.

Passage of gases through colloid septa.-In 1830, Dr. Mitchell, of Philar delphia, discovered a power in gases to penetrate thin sheet India rubber; and, noticing the comparatively rapid transmission of carbonic gas through the rubber, associated this observation with the further one that a solid piece of India rubber is capable of absorbing its own volume of carbonic gas, when left in contact with excess of the gas for a sufficient length of time. By means of a suitable arrangement, Dr. Mitchell found that various gases passed spontaneously through a caoutchouc membrane into an atmosphere of ordinary air with different degrees of velocity-that as much of ammonia gas was transmitted in 1 minute as of carbonic gas in $5 \frac{1}{2}$ minutes, as of hydrogen in $37 \frac{1}{2}$ minutes, and as of oxygen in 113 minutes. Soon after their publication, these results were ably commented on and extended by Dr. Draper, of New York; and, altogether, they attracted considerable attention in scientific circles. One of Mr. Graham's earliest observations-having reference to the spontaneous passage of carbonic gas into a moist bladder of air, so as ultimately to burst the bladder-had obviously a very close connection with Dr. Mitchell's results, and received from Mr. Graham in 1829 the same explanation that in 1866 he gave to his own India rubber experiments, the account of which he communicated to the Royal Society in a paper "On the absorption and dialytic separation of gases by colloid septa." * In his experiments on the penetration of different gases, through septa of India rubber, into a vacuum, Mr. Graham employed a tube considerably exceeding in length the barometric column, open at one end and closed at the other by a thin film of caoutchouc stretched over a plate of highly porous stucco. On filling this tube with mercury,

[^18]and inverting it into a cup of mercury, a Torricellian vacuum was left at the top, into which the external air, or any external gas experimented on, gradually found its way by passage through the caoutchouc film, so as to cause a depression of the mercurial column. By experiments made in this manner, it was found that different gases penetrated the rubber, and entered the vacuous space with the following relative velocities, differing widely from the velocities of diffusion and transpiration of the same gases given in the other two columns of the table:

|  | Rates of passage through caoutchouc. | Transpiration velocities. | Diffusion velocities, |
| :---: | :---: | :---: | :---: |
| Nitrogen. | 1.00 | 1.14 | 1.01 |
| Marsh gas. . | 2.15 | 1.81 | 1.34 |
| Oxygen. | 2.55 | 1.00 | . 95 |
| Hydrogen.... | 5.50 | 2.29 | 3.80 |
| Carbonic gas................... | 13.58 | 1.37 | . 81 |

Bearing in mind the partial separation of gases from one another attainable by reason of their unequal diffusive velocities, the possibility of effecting a similar separation of gases by reason of their unequal velocities of transmission through India rubber was easily to be foreseen. For example, atmospheric air consisting of 20.8 volumes of oxygen and 79.2 volumes of nitrogen, and the transmission velocities of these two gases being respectively 2.55 and 1.0 , it follows that the air transmitted through India rubber into a vacuum should consist of 40 per cent. oxygen and 60 per cent. nitrogen, thus:


In subjecting this conclusion to the test of experiment, Mr. Graham availed himself of Dr. Sprengel's then newly invented mercurial pump or exhauster, an instrument which also stood him in good stead in his subsequent work, and to which he freely acknowledged his obligations. By a slight alteration in the pump, as originally constructed, Mr. Graham made it serve not only for its original purpose of creating and maintaining an almost perfect vacuum, but also for delivering pari passu any gas penetrating into the vacuum through its caoutchouc or other walls.
The caoutchouc films employed in these experiments were of various kinds; but the most readily practicable and, on the whole, successful results, were obtained with India-rubber varnished silk made up into a flat bag, exposing on each side about 0.25 meter of square surface. The interior of such a bag being in communication with the Sprengel pump, the constituents of the external air were gradually sucked through the walls of the bag and delivered by the turned-up fall-tabe of
the pump. On examining the delivered gas, it was found to contain on the average 41.6 per cent. of oxygen; and accordingly, to have the property of re-inflaming a glowing splinter. Thus, by the simple suction of atmospheric air through a caoutchouc film, the remarkable result was arrived at of nearly doubling the proportion of oxygen in the volume of air sucked through. Unfortunately for the practical application of the process, the entire volume of air sucked through proved to be very small, about 2.25 cubic centimeters per minute, per square meter of surface, at $20^{\circ} \mathrm{C}$. At $60^{\circ} \mathrm{C}$., however, the passage of air through the rubber was almost exactly three times as rapid as at $20^{\circ}$.
Instead of allowing the gases experimented on to pass through the India rubber into a vacuous space, they were in some cases allowed to pass into space already occupied with t different gas, somewhat as in Dr. Mitchell's original experiments; fut the conditions of the action were then more complex. The constituent gases of atmospheric air, for instance, pass through an Indià-rubber septum into a space containing carbonic gas at the relative velocities with which they enter a vacuous space; but throughout the experiment, not only are oxygen and nitrogen continually entering the space, but carbonic gas is continually, and very rapidly, escaping from it. Eventually, by the rapid escape of carbonic gas, the proportion or pressure of oxygen in the internal space comes to exceed that in the external air; whereupon a reverse transmission, through the India rubber, of the excess of oxygen into the external air, at once begins. But by stopping the operation at an early stage, and then absorbing the carbonic gas with caustic alkali, a residue of hyperoxygenized air was left, capable, in some cases, of re-inflaming a glowing splinter, and containing as much as 37.1 volumes of oxygen to 62.9 volumes of nitrogen.

The interpretation given by their discoverer to the above results was in accordance with his slowly-developed views on the relations of gases and liquids to each other and to soft solids. Having satisfied himself that the merest film of India rubber is quite devoid of porosity, and that oxygen is at least twice as absorbable by India rubber as by water at ordinary temperature, (the absorbability of its own volume of carbonic gas by India rubber, as by water, having been noticed by Dr. Mitchell,) Mr. Graham came to view the entire phenomenon as having a very complex character, as consisting in a dissolution of the gas in the soft India rubber; in a diffusion of the liquefied gas, as a liquid, through the thickness of the India rubber; in an evaporation of the liquefied gas from the internal surface of the India rubber; and lastly in a diffusion of the evaporated gas into the internal space. Thus, in reference to the remarks of Drs. Mitchell and Draper, he writes: "These early speculations lose much of their fitness from not taking into account the two considerations already alluded to, which appear to be essential to the full comprehension of the phenomenon, namely, that gases undergo liquefaction when absorbed by liquids and such
colloid substances as India rubber, and that their transmission through liquid and colloid septa is then effected by the agency of liquid and not gaseous diffusion. Indeed, the complete suspension of the gaseous function during the transit through colloid membrane cannot be kept too much in view." Mr. Graham seems thus to have recognized at least three distinct modes of gas transmission through a solid or semisolid septum :

1st. By a sufficient degree of pressure gases might be forced bodily, i. e. in masses, through the minute channels of a porous septum; or, in other words, might pass through such a septum by transpiration, of course in the direction only of the preponderating total pressare.
2 d . As the channels of a porous septum became more and more minute, their resistance to the bodily transmission of gas would become greater aud greater, and the quantity of gas forced through them less and less, until at length the septum would be absolutely impermeable to transpiration under the particular pressure. But such a septum, of which the individual capillary chaunels were so small as to offer a frictional resistance to the passage of gas greater thani the available pressure could overcome, might nevertheless present a considerable aggregate of interspace through which the diffusion proper of gases, consequent on their innate molecular mobility, could take place freely in both directions.
3d. A septum might be quite free from pores, of any kind or degree of minuteness, and so far be absolutely impermeable to the transmission of gas in the form of gas; but it might nevertheless permit a considerable transmission of certain gases by reason of their prior solution or liquefaction in the substance of the septum. And whereas. the mere passage of gas, by transpiration or diffusion through a porous. septum, would take place in thorough independence of the nature of the material of the septum, in this last-considered action, the transmission would take place by virtue of a sort of chemical affinity between the gas and the material of the septum-the selective absorption of the gas by the septum being a necessary antecedent of its transmission; whence it might be said the gas was trausmitted because it was first absorbed. Of course in certain transmissions two, or all three, modes of action might come into play simultaneously.

## IX.

Occlusion of gases by metals.-The experiments of Deville and Troost having made known the curious fact of the permeability of ignited homogeneous platinum and ignited homogeneous iron to hydrogen gas, and given some indication also of the permeability of ignited iron to carbonic oxide gas, Mr. Graham, in 1866, corroborated the results of the French chemists in reference to platinum; but, modifying their method by letting the hydrogen pass into a space kept vacuous by the Sprengel pump, instead of into an atmosphere of other gas, assimilated 14 \& 71
the process to that which he had employed in his India-rubber experiments. The results he obtained were communicated to the Royal Society, partly in the paper already referred to "On the absorption and separation of gases by colloid septa," and partly in four supplementary notices published in the proceedings of the society.* In carrying out the investigation forming the subject of these several communications, Mr. Graham had the advantage of being admirably seconded by his assistant, Mr. W. Chandler Roberts, whose able and zealous co-operation he repeatedly acknowledged in the warmest terms.
In the course of experiments made on the transmission of gases through ignited metallic septa, a particular platinum tube, being rendered vacuous, was found at all temperatures below redness to be quite impermeable to hydrogen; whereas, at a red heat, it transmitted 100 cubic centimeters of hydrogen in half an hour, the quantities of oxygen, nitrogen, marsh gas, and carbonic gas, transmitted under the same conditions, not amounting to .01 cubic centimeter each in half an hour. It was ascertained further that, with an ignited vacuous tube of platinum surrounded by a current of ordinary coal-gas, (a variable mixture of gases containing on the average about 45 per cent. of marsh gas, 40 per cent. of hydrogen, and 15 per cent. of other gases and vapors, ) a transmission of pure hydrogeh alone took place through the heated metal. This property of selective transmission, manifested by platinum, was so far analogous to the property of selective transmission manifested by. India-rubber, that whereas a septum of India rubber transmitted the nitrogen of the air in a much smaller ratio than the oxygen, the septum of ignited platinum transmitted the other constituents of coal-gas in an infinitely smaller ratio than the hydrogen. Hence the knowledge of the absorption by India rubber of the gases which it most freely transmitted, suggested to Mr. Graham an inquiry as to the possible absorption of hydrogen gas by platinum. Accordingly platinum, in different forms, was heated to redness, and then allowed to cool slowly in a continuous current of hydrogen. The metal so treated, and after its free exposure to the air, was placed in a porcelain tube, which was next made vacuous by the Sprengel pump. During the production and maintenance of the vacuum, no hydrogen was extracted from the metal at ordinary temperatures; or even during an hour's exposure to the temperature of $220^{\circ}$; or yet at a heat falling just, short of redness. But at a dull red-heat and upward, a quantity of hydrogen gas was given off amounting in volume, measured cold, to as much, in some cases, as 5.5 times the volume of the platinum. Thus was opened out to Mr. Graham the subject of his last, and probably greatest discovery, the occlusion of ! gases by metals. Very many metals were examined in their relations to different gases, but the most interesting results were those obtained with platinum as above described; and those obtained with silver, with iron, and, above all, with palladium.

[^19]The characteristic property of silver, heated and cooled in different atmospheres, proved to be its capability of absorbing and retaining, in some cases, as much as seven times its volume of oxygen-its absorption of hydrogen falling short of a single volume. Some silver-leaf, heated and cooled in ordinary air, and subsequently heated in a vacuum, gave off a mixture of oxygen and nitrogen gases centaining 85 per cent. of oxygen, or more than four times the proportion contained in theoriginal air. This remarkable property of solid silver to effect the permanent occlusion of oxygen gas, must be distinguished from the not less remarkable and doubtless associated property of melted silver to effect the temporary absorption of a yet larger volume of the same gas; which, on the solidification of the metal, is discharged with the well-known phenomenon of spitting.

Iron, though tolerably absorptive of hydrogen, was found to be specially characterized by its absorption of carbonic oxide. What may be called the natural gas of wrought iron, or the gas derived from the forge in which it was heated, proved to consist chiefly of carbonic oxide, and, in different experiments, was found to range from 7 to 12.5 times the volume of the metal ; so that, in the course of its preparation, iron would appear to occlude upward of seven times its volume of carbonic oxide, and to carry this gas about with it ever after. The absorbability of carbonic oxide by iron has an obviously important bearing on the theory of steel production by cementation. This process would appear to consist in an absorption of carbonic oxide gas into the substance of the iron, and in a subsequent decomposition of the absorbed gas into carbon entering into combination with the metal, so as to effect its acieration, and carbonic gas discharged from the surface of the metal, so as to produce the well-known appearance of blistering. Nor is this the only, or even the chief point of interest that was made out with regard to iron; for the study of the behavior of telluric manufactured iron naturally led Mr. Graham to the examination of sidereal native iron, that is to say, the iron of meteorites, and with the following result. A portion of meteoric iron, from the Lenarto fall, when heated invacuo, gave off 2.85 times its volume of natural gas, of which the preponderating constituent, to the extent of 85.7 per cent. of the total quantity, consisted not of carbonic oxide, but of hydrogen, the carbonic oxide amounting to only 4.5 per cent., and the remaining 9.8 per cent. consisting of nitrogen. The inference that the metcorite had been, at some.time or other, ignited in an atmosphere having hydrogen as its prevailing constituent, seems irresistible; and judging from the volume of gas yielded by the iron, the hydrogen atmosphere in which it was ignited must, in all probability, have been a highly condensed one; the charge of hydrogen extracted being fully five times as great as it was found possible to impart to ordinary iron artificially.
But it was with palladium that Mr. Graham obtained his most extraordinary results. This metal he found to have the property of transmitting hydrogen with extreme facility, even at temperatures very far
short of redness. Coincidently, at temperatures even below those requisite for transmission, palladium was found capable of absorbing many hundredtimes its volume of lyydrogen. Thus a piece of palladiumfoil maintained at a temperature of $90^{\circ}-97{ }^{\circ}$ for three hours, and then allowed to cool down during an hour and a half, while surrounded by a continuous current of hydrogen gas, gave off, on being afterward heated in vacuo, 643 times its volume of the gas, measured cold; and even at ordinary temperatures, it absorbed 376 times its volume of the gas, provided it had first been recently ignited in vacuo. In another experiment, palladium sponge, heated to $200^{\circ}$ in a current of hydrogen and allowed to cool slowly therein, afterward yielded 686 times its volume of the gas; while a piece of electrolytically deposited palladium heated only to $100^{\circ}$ in hydrogen, afterward yielded, upon ignition in vacuo, no less than 982 times its volume of the gas. The lowness of the temperature at which, under favorable circumstances, the absorption of hydrogen by palladium could thus be effected, soon suggested other means of bringing about the result. For example, a piece of palladium-foil was placed in contact with a quantity of zinc undergoing solution in dilute sulphuric acid; and, on subsequent examination, was found to have absorbed 173 times its volume of hydrogen. Again, palladium, in the forms of wire and foil, was made to act as the negative pole of a Bunsen's battery effecting the electrolysis of acidulated water; and in this manner was found to absorb from 800 to 950 times its volume of hydrogen in different experiments.

Palladium being thus chargeable with hydrogen in three different ways-namely, by being heated and cooled in an atmosphere of the gas; by being placed in contact with zinc dissolving in acid, i. e., with hydrogen in the act of evolution; and, lastly, by being made the negative electrode of a battery-correlatively, the charged metal could be freed from its occluded bydrogen by exposing it to an increase of temperature in air or vacuo; by acting on it with different feebly oxidizing mixtures; and by making it the positive electrode of a battery.

The palladium, when charged to its maximum, was frequently found to give off a small proportion of its hydrogen, though with extreme slowness, at ordinary temperatures, bath into the atmosphere and into a vacuum. But not until the temperature approached $100^{\circ}$ was there any appreciable gas-evolution; which, above that point, took place with a facility increasing with the temperature, so as to be both rapid and complete at about $300^{\circ}$. Since, however, the transmission of hydrogen through heated palladium is a phenomenon of simultaneous absorption and evolution, it follows that the property of palladium to absorb hydrogen does not cease at $300^{\circ}$, or indeed at close upon the melting-point of gold-the highest temperature at which Mr. Graham's experiments on transmission were conducted; but whereas the maximam absorption of hydrogen by palladium takes place at comparatively low temperatures, the velocity of transmission was observed to increase, in a rapid ratio, with the increase of temperature, indefinitely.

As regards the removal of hydrogen from palladium by oxygenants, the gas of the charged metal was found to manifest all the chemical activity of hydrogen in the nascent state. Thus it reduced corrosive . sublimate to calomel, combined directly with free iodine, converted ferrid into ferro cyanides, destroyed the color of. permanganates, \&uc. Moreover, the spongy metal, charged with hydrogen and exposed to the air, was apt to become suddenly hot, and so completely discharged, by a spontaneous aerial oxidation of its absorbed gas into water; while the hydrogen of a piece of charged palladium wire was often capable of being set fire to, and of burning continuously along the wire.

Lastly, the reversal of the position of the palladium plate in the decomposing cell of the battery afforded a most ready means of completely extracting its hydrogen. Iudeed, for some time after thereversal, while hydrogen was being freely evolved from the negative pole, no oxygen was observable on the surface of the palladium plate, now made the positive pole, through its rapid oxygenation of the absorbed hydrogen.

As regards the extent of the absorption of hydrogen by palladium, it was found, as already indicated, to vary considerably with the physical state of the metal, whether fused, hammered, spongy, or electrolytically deposited, for example. In one case, previously referred to, a specimen of electrolytically deposited palladium, heated to $100^{\circ}$, and then slowly cooled in a continuous current of hydrogen, was found to occlude 982.14 times its volume of the gas, measured cold. In this case the actual weight of palladium experimented with was 1.0020 gram, and the weight of hydrogen absorbed. 0073 gram, being in the ratio of 99.277 per cent. of palladium and 0.723 per cent. of hydrogen. The atomic weight of hydrogen being 1 , and that of palladium 106.5 , it is observable that the ratio of the weights of the constituents of the charged metal, hydrogen and palladium, approximates to the ratios of their atomic weights.

In another experiment some palladium wire, drawn from a piece of the fused metal, was charged electrolytically with 935.67 times its volume of hydrogen. Some idea of these enormous absorptions of hydrogen may be formed by remembering that water at mean temperature absorbs only 782.7 times its volume of that most absorbable of the common gases, ammonia.

A point of interest with regard to the different quantities of hydrogen absorbable by palladium in its different states is the gradual diminution in the absorptive power of any particular specimen of the metal with each successive charge and discharge of gas in whatever way effectedthe absorptive power, however, being partially restorable by subjecting the metal to a welding heat.

The density of palladium charged with eight or nine hundred times its volume of lyydrogen is perceptibly lowered. Owing, however, to a continuous formation of bubbles of hydrogen on the surface of the
charged metal when immersed in water, there is a difficulty in taking its exact density by comparing its respective weights in air and water with one another. There is also a difficulty in determining the density by direct measurement of the charged palladium when in the form of wire; owing to the curious property of the wire, on being discharged, of not merely returning to its original volume, but of undergoing a considerable and permanent additional retraction. But in the case of certain alloys of platinum, silver, and gold with excess of palladium, while the absorptive power of the constituent palladium is still manifested, the excess of retraction on discharge of the wires does not occur ; and the specific gravities deducible from the mere increase in length of wires of these alloys are found to accord approximatively with those deducible from the increase in length of the pure palladium wire, not above its original length, but above the length to which it retracts on discharge of its absorbed gas. It would thus appear that, simultaneously with its absorption of hydrogen, the pure palladium wire, unstably stretched by the process of drawing, suffers two opposite actions; that is to say, it undergoes a process of shortening by assuming a more stable condition of cohesion, and a process of lengthening by the addition to it of other matter-the lengthening due to the additional matter being the excess of the length of the charged above that of the discharged wire. In a particular experiment illustrative of this peculiarity, a new platinum wire took up a full charge of hydrogen electrolytically, namely, 956.3 volumes, and increased in length from 609.585 to 619.354 millimeters. With the expulsion of the hydrogen afterward, the wire was permanently shortened to 600.115 millimeters. The sum of the two changes taken together amounts to 19.239 millimeters, and represents the true increase in the length of the wire due to the addition of bydrogen. It corresponds to a linear expansion of 3.205 in 100, or to a cubical expansion of 9.827 in 100 . The original volume of the wire being .126 cubic centimeter, the volume of the condensed hydrogen would accordingly be .01238 cubic centimeter. Then, as the charged wire, on being heated in vacuo, evolved 120.5 cubic centimeters of hydrogen gas, weighing . 0108 gram, the density of the absorbed hydrogen would be-

$$
\frac{.01080}{.01238}=.872 .
$$

Calculated from the mere increase in length of the charged wire above that of the wire originally, the density of the absorbed hydrogen would be 1.708. The following table gives the densities of condensed hydrogen in different experiments made with palladium wire, in which the excess of retraction on discharge was allowed for as above; and also the densities observed in experiments made with palladium alloys in which the contraction on discharge took place to the original lengths of the wires only:

| When united with- | Density of condensed lydrogen. |
| :---: | :---: |
| Palladium. | 0.854 to 0.872 |
| Palladium and platinum. | 0.7401 to 0.7545 |
| Palladium and gold. | 0.711 to 0.715 |
| Palladium and silver | 0.727 to 0.742 |

Whether the absorption of hydrogen by palladium, alloyed or not with another metal, was large or small, the density of the occluded hydrogen was found to be substantially the same. That the excessive retraction of the palladium wire on the discharge of its absorbed hydrogen is not a mere effect of heat was shown by the charged wire undergoing a similar retraction when discharged electrolytically instead of by ignition in vacwo and also by the original wire not undergoing any sensible retraction as a result of annealing. That the retraction is merely in length was shown by the absence of any difference in specific gravity between the original and the discharged wire. Very curiously, the shortening of the wire, by successive chargings and dischargings of hydrogen, would seem to be interminable. Thus the following expansions of a particular wire, caused by variable charges of hydrogen, were followed, on expelling the hydrogen, by the contractions recorded in the other column :

|  | Elongation in millimeters. | Retraction in millimeters. |
| :---: | :---: | :---: |
| First experiment.. | 9.77 | 9.70 |
| Second experiment. | 5.765 | 6.20 |
| Third experiment | 2.36 | 3.14 |
| Fourth experiment. | 3.482 | 4.95 |
|  |  | . 23.99 |

The palladium wire, which originally measured 609.144 millimeters, thus suffered, by four successive chargings and dischargings of hydrogen, an ultimate contraction of 23.99 millimeters, or a reduction of its original length to the extent of nearly 4 per cent., each increment of contraction below the original length usually exceeding the previous increment of elongation above the original length of the wire. The alternate expansion and contraction of palladium by its occlusion and evolution of hydrogen is ingeniously shown by a contrivance of Mr. Roberts, in which a slip of palladium-foil, varnished on one side, is made to curl and uncurl itself, as it becomes alternately the negative and positive electrode of a battery, or is alternately charged and discharged of hydrogen on its free surface.

That hydrogen is the vapor of a highly volatile metal has frequently been maintained on chemical grounds; and from a consideration of the physical properties of his hydrogenized palladium, Mr. Grahain was led
to regard it as a true alloy of palladium with hydrogen, or rather hydrogenium, in which the volatility of the latter metal was restrained by the fixity of the former, and of which the metallic aspect was equally due to both of its constituents. Although, indeed, the occlusion of upward of 900 times its volume of hydrogen was found to lower the tenacity and electric conductivity of palladium appreciably, still the hydrogenized palladium remained possessed of a most characteristically metallic tenacity and conductivity. Thus, the tenacity of the original wire being taken as 100 , the tenacity of the fully charged wire was found to be 81.29; and the electric conductivity of the original wire being 8.10, that of the hydrogenized wire was found to be 5.99. In further support of the conclusion arrived at by Mr. Graham, as to the metallic condition of the hydrogen occluded in palladium, he adduced his singular discovery of its being possessed of magnetic properties, more decided than those of palladium itself, a metal which Mr. Faraday had shown to be "feebly but truly magnetic." Operating with an electromagnet of very moderate strength, Mr. Graham found that while an oblong fragment of electrolytically deposited palladium was deflected from the equatorial by $10^{\circ}$ only, the same fragment of metal, charged with 604.6 times its volume of hydrogen, was deflected through $48^{\circ}$. Thus did Mr. Graham supplement the idea of hydrogen as an invisible incondensable gas, by the idea of hydrogen as an opaque, lustrous, white metal, having a specific gravity between 0.7 and 0.8 , a well-marked tenacity and conductivity, and a very decided magnetism.

# on the relation of the pifsical soiences to science in general. 

Delivered before the University of Heidelberg, by Dr. Herman Helmholtz.

[Translated for the Smithsonian Institution, by Prof. C. F. Kroef.]

Our university renews, on the annual return of this day, her grateful remembranees of Charles Frederic, the enlightened prince who, at a time when the whole established order of Europe was revolutionized, labored most zealously and efficiently to improve the well-being and facilitate the mental development of his people, and who clearly perceived that the revival of this university would be one of the principal means for the attainment of his benevolent object. Since it is my duty on this occasion to speak of our whole university as its representative, it is proper to review the connection between the sciences and their study in general, as far as may be possible, from the circumscribed point of view of an individual observer.

It would seem indeed, to-day, as if the mutual relations of all sciences, in virtue of which we unite them under the name of a universitas litterarum, had become looser than ever before. We see the scholars of our times absorbed in a study of details of such immense magnitude that even the most industrious cannot hope to master more than a small portion of modern science. The linguist of the last three centuries found sufficient occupation in the study of Greek and Latin, and it was only for immediate practical purposes that a few modern languages were learned. Now, comparative philology has set for itself no less a task than to study all the languages of the human race, in order to deduce from them the laws of the formation of language itself, and its votaries have brought immense industry to bear upon this gigantic work. Even within classical philology they no longer confine themselves to the study of those writings which, by their artistic finish, the clearness of their thoughts, or the importance of their contents, have become the models of the poetry and prose of all times; the philologists are aware that every lost fragment of an ancient writer, every remark of a pedantic grammarian or of a Byzantine court-poet, every broken tomb-stone of a Roman official that is found in some remote corner of Hungary, Spain, or Africa, may contain some information or proof of importance in its proper place, and hence a large number of scholars are occupied in the gigantic task of collecting and cataloguing all remnants of classic antiquity so that they may be ready for use. Add to this the study of historical sources, the examination of parchments and papers accumulated in states and towns, the collection of notes scattered through me-
moirs, correspondences, and biographies, and the deciphering of the hieroglyphics and cuneiform inscriptions; add again to these the continually and rapidly augmenting catalogues of minerals, plants, and animals, living and antediluvian, and there will be unfolded before our ejes a mass of scientific material sufficient to make us giddy. In all these sciences, researches are pushed forward constantly in proportion to the improvement of our means of observation, and there is no perceptible limit. The zoölogist of the last century was generally satisfied with describing the teeth, fur, formation of the feet and other external characteristics of an animal. The anatomist, on the other hand, described the anatomy of man alone, as far as he could gain a knowledge of it by means of the knife, the saw, the chisel, or, perhaps, of injections into the tissues. The study of human anatomy was even then considered an extremely extensive and difficult branch of science. To-day we are no longer content with what is so-called descriptive human anatomy, which, although incorrectly, is considered as exhausted, but comparative anatomy, $i$. e., the anatomy of all animals, and miscroscopic anatomy, both sciences of unlimited scope, have been added and absorb the interest of the observer.
The four elements of antiquity and of mediæval alchemy have swelled to sixty-four* in our modern chemistry ; the last three have been discovered according to a method originating in our university, which leads us to expect other similar discoveries. Not only, however, has the number of the elements increased extraordinarily, but the methods for producing complex compounds have been so greatly improved, that what is so-called organic chemistry, which comprises only the combinations of carbon with hydrogen, oxygen, nitrogen, and a few other elements, has already become a separate science.
"As many as the stars in heaven," used to be the natural expression for a number which exceeds all limits of our comprehension. Pliny considered it an undertaking bordering on arrogance when Hipparchus commenced to number the stars and determine their positions. Nevertheless, the catalogues of stars up to the seventeenth century, which were made out without the use of telescopes, contained only from 1,000 to 1,500 stars of the first to the third magnitude. At present they are engaged at the different observatories in extending these catalogues down to the tenth magnitude, which will make a sum total of more than 200,000 fixed stars in the whole hearens; and these are all to be noted down, measured, and their places determined. The immediate consequence of these observations has been the discovery of many new planets. Of these only six were known in 1781, while at present we know seventyfive. $\dagger$ When we pass in review this gigantic activity in all branches of

[^20]science, the rash projects of man are, indeed, apt to astonish and frighten us, like the chorus in Antigone, when it exclaims,
"Much is surprising, but nought more surprising than man."
Who can overlook the whole, keep the connecting threads in his hand and find his way through the labyrinth. The immediate and natural consequence is that every investigator is forced to choose a field of labor constantly becoming more circumscribed, and to confine himself to a but imperfect acquaintance with the rest. We are now inclined to laugh when we hear that in the seventeenth century Kepler was called to Grätz to discharge the duties of the chair of mathematics and moral science, or that at the beginning of the eighteenth century Boerhave held at the same time the professorships of botany, chemistry, and clinical medicine, which, of course, included also pharmacy. Now, we need at least four and in many universities even seven or eight teachers for all these branches. The same is the case in other departments of science.

I have the more reason to consider the connection between the different sciences here, because I confine myself to the circle of natural sciences, which have latterly been accused of pursuing a course isolated from other sciences, correlated through mutual philological and historical studies, and of having become estranged from them. This indeed has long been perceptible, and seems to have been developed, or rather brought to notice, under the influence of the philosophy of Hegel. At the end of the last century, under the philosophy of Kant, such a separation had not been pronounced. His philosophy was on equal footing with the natural sciences, as Kant's own labors in natural science demonstrate, especially his cosmogonic hypotheses, based on Newton's law of gravitation, which was later generally received under the name of the theory of Laplace. Kant's critical philosophy was calculated only to investigate the sources and basis of our knowledge, and to create a standard for the intellectual labors of the different sciences. A law found a priori by pure thought, could, according to his doctrine, become only a rule for a method of thinking, and could not have any positive or real significance. The philosophy of identity was bolder. It proceeded from the hypothesis that the real world, that nature, and the life of man, were the result of the thoughts of a creative mind, which mind was similar to that of man. Accordingly, the human mind might undertake, even without the guidance of external experience, to think over again the thoughts of the Creator, and to find them again, through its own inner activity. In this sense the philosophy of identity endeavored to construct $\dot{a}$ priori the material results of the other sciences. This might succeed more or less easily with respect to religion, law, political economy, language, art, history, and, in short, in all sciences which are developed chiefly from is psychological basis, and which are therefore classified under the name of mental sciences. The state, the church, art and language, have for their object the satisfaction of certain spiritual and mental wants of
man. Although external obstacles, the forces of nature, accident, rivalry of other men, frequently exert a disturbing influence, the endeavors of a human mind perseveringly pursuing its object must, in the end, preponderate and triumph over planless hinderances. Under these circumstances it would not be impossible to lay out a priori the course of development of mankind with regard to the above relations, especially when the philosopher has already considerable empirical material at his command with which he can combine his abstractions. Hegel was materially aided in his attempts to solve this question by the deep philosophical insight into history and science which the philosophers and poets of the immediately preceding time had gathered, and which he needed only to arrange and combine to produce a system full of astonishing discoveries. In this manner he succeeded in gaining the enthusiastic applause of the majority of the scholars of his time, and in exciting fantastical hopes for the solution of the profoundest mysteries of human life; the latter all the more because his system was reiled in curiously abstract language, and was, perhaps, really understood and appreciated only by a small number of his admirers.

The fact that the construction of the principal results of the mental sciences was more or less successful, was, however, no proof of the correctness of the hypothesis of identity from which Hegel's philosophy proceeded. On the contrary, the facts of nature would have been the means of furnishing decisive proof. It was a matter of course that traces of the activity of the human mind and of its stages of development must be found in the mental sciences. If nature reflected the result of the thoughts of a similar creative mind, the comparatively simpler forms and processes of nature could the more easily be arranged into systems. But it was just at this point that the philosophy of identity failed, we may say, completely. The natural philosophy of Hegel, to naturalists at least, appeared absolutely senseless. Among the many excellent naturalists of that time there was not a single one who could accept his ideas. But it was of the greatest importance to Hegel to obtain the same appreciation here that he had found so abundantly in the other sciences. He waged an unusually passionate and bitter controversy, directed especially against Newton, the first and greatest representative of scientific research. He taxed the naturalists with narrow. mindedness, and they in their turn accused their opponent of absurdities. The naturalists began to lay stress upon the assertion that their labors had been free from all philosophical influences, and soon many of them, including even men of great eminence, condemned all philosophy, not only as useless, but even as injurious vagaries. We cannot deny that along with the unjust claims of the philosophy of Hegel, to subordinate the other sciences, its just claims, to criticise the sources of knowledge and determine a standard for intellectual labors, were thrown overboard.

In the mental sciences the effect was different, although it finally led
to the same result. In all branches of science relating to religion, the state, law, art, and language, enthusiastic followers of Hegel arose, each of whom sought to reform their branch according to his doctrine and to gather rapidly in a speculative way the fruits, which until then could only be obtained by means of slow and tedious. labor. Thus it was for a time that a sharp and well-defined antagonism existed between the physical sciences on the one side and the mental sciences on the other, and not infrequently was it denied that the former possessed the characteristics of a science at all.

The bitterness which existed between the two did not, however, last long. The physical sciences proved to every one, by a rapid series of brilliant discoveries and applications, that they contained a healthy germ of unusual productiveness. It was impossible not to esteem and appreciate them. In the other departments of science, conscientious investigators of facts soon raised objections against the presumptous Ica-rus-flight of speculation. That these philosophical systems produced a beneficial effect is however unmistakable; we cannot deny that since the appearance of the works of Hegel and Schelling, the attention of investigators of the different branches of mental sciences has been directed more pointedly and more perseveringly to their intellectual import and scope than in preceding times, and that therefore the results of that philosophy have not been entirely in vain.
In a measure as the empirical investigation of facts became more prominent in the other sciences, the contrast between them and the plysical sciences was dinininished. Although this contrast had been exaggerated through the influence of philosophy, we cannot deny that it is founded upon the nature of things, and thatit will assert its claims. It lies partially in the kind of mental labor involved, and partially in the subjects of the sciences, as their names, physical and mental sciences, indicate. The physicist will find some difficulty in explaining a complicated process of nature to a philologist or a lawyer. It would require on their part abstractions from the appearance of the senses and dexterity in the use of geometrical and mechanical aids, in which they could not easily follow him. Artists and theologians, on the other hand, would perbaps find the naturalist too much inclined to mechanical and material explanations, which would seem trivial to them, and which might tend to suppress the warmth of their feeling and their enthusiasm. The philologist and the historian, with whom the lawyer and the theologian are intimately associated by their common philological and historical studies, will find the physicist surprisingly indifferent to literary treasures, more indifferent perhaps than is proper and good for the advance of his own science. It cannot be denied, finally, that the mental sciences have to do directly with the dearest interests of the human mind, and with its creations in the world, while the physical sciences work with external matter, to which we may be indifferent, but we cannot neglect
because of their great practical utility, although they may not seem to hare any immediate effect in developing the mind.

Since the sciences have been separated into so many divisions and subdivisions, since very appreciable contrasts have been developed among them, and since no one man can comprehend the whole, or even a considerable part of the whole, is there any use in keeping them together in the same institution? Is the union of the four faculties in one university only a remnant of the usages of the middle ages? It has been alleged that many external advantages are gained by sending students of medicine to the hospitals of large cities, students of natural sciences to polytechnic schools, and by erecting special seminaries and schools for theologians and lawyers. Let us hope that our German universities may long be preserved from the influence of such an idea! That woulp indeed tear asunder the connection between the different sciences, a connection eminently important to seientific labor, and to the improvement of the material products of that labor, as will be seen on a brief consideration of the question.

First, as regards their formal relations, I would say that the union of the different sciences is necessary to maintain a healthy equilibrium of the mental powers. Every science exercises certain faculties of the mind, and strengthens them by continual practice. But all one-sided development has its dangers; it is detrimental to those faculties which are less exercised, it limits our view of the whole, and leads us to overestimate our own labors. He who perceives that he can perform a certain kind of mental labor much better than other men, is too apt to forget how many other things there are in which they surpass him. Over-estimation of self-let no votary of science forget it-is the greatest and worst enemy of all scientific labors. How many with great talents have not forgotten that criticism of self, so difficult and yet so necessary to the scholar, or have become discouraged and lax in their labors, because they considered dry, persevering work unworthy of them, and were bent only on producing brilliant combinations and revolutionizing discoreries! How many such men hare not concluded a melancholy life in au embittered and misanthropical state of mind, because they failed to obtain that appreciation from their fellow-men which is gained only by work and success, and not by the self.complacency of genius alone. The more isolated we are, the more we are exposed to this danger; while, on the other hand, nothing is more conducive to efficient mental labor than to be obliged to exert all our powers to gain the appreciation of men whom we ourselves are constrained to appreciate.
When we compare the kinds of mental activity required in different branches of science, we shall find certain differences due to the sciences themselves, although we cannot deny that every man of great talent has a special individual tendency which fits him for his special branch. It is only necessary to compare the works of two investigators in inti-
mately related branches, to find that the greater the men, the more decided is their mental individuality, and the less one would be able to perform the works of the other. To-day we cannot, of course, go further than to characterize the most general differences of intellectual work in the different branches of science.
I have reminded jou of the gigantic extent of the materials of our sciences. It is clear that the greater their extent, the more necessary it is to obtain a better and more accurate organization and arrangement, in order not to become hopelessly lost in the labyrinth of learning. The better the order and system, the greater may the accumulation of details become without injuring the connection. Our time becomes all the more profitable in working out details, because our predecessors have taught as the organization of science.
This organization consists, in the first place, in an external mechanical arrangement, as found in our catalogues, lexicons, registers, indexes, literary reviews, yearly reports, digests of laws, systems of natural history, etc. By the aid of these we gain only because all knowledge which it is not necessary to keep constantly in mind can be found at any moment by those who need it.
By means of a good lexicon a student of a preparatory school can accomplish much in the study of the classics that must have proved difficult to Erasmus, in spite of life-long reading. Works of this kind are, as it were, the scientific capital of mankind, with the interest of which the business is carried on. We might compare them to capital invested in lands. Like the earth, of which the lands are composed, the knowledge contained in these catalogues, lexicons, and indexes looks little inviting, and the ignorant cannot appreciate the labor and expense lavished on these acres; the work of the plowman seems excessively difficult, laborious, and tedious. Although, however, the work of the lexicographer or of the compiler of systems of natural history requires as much perseverance and diligence as that of the plowman, it must not be believed that it is of a subordinate or secondary nature, or that it is as dry and mechanical as it looks afterward, when the catalogue lies printed before us. Every single fact must be discovered by attentive observation; it must afterward be verified and compared, and the important must be separated from the unimportant. None can do this but those who have a clear understanding of the object of the collection and of the intellectual import of the science and its methods; and for such men every single fact will be of peculiar interest in its relations to the whole science. Otherwise such work would be the most intolerable drudgery that could be imagined. That the progressive development of science has its influence on these works also is seen in the fact that new lexicons, new systems of natural history, new digests of laws, new catalogues of stars, are constantly found necessary. They testify to the progress of the methods and the organization of knowledge.

But our knowledge must not remain idle in the form of catalogues;
for the fact that we must have it about us in this form, black tupon white, proves that we have not mastered it intellectually. It is not sufficient to be cognizant of facts; science results only from a knowledge of their laws and causes. The logical elaboration of these facts consists in collecting together those which are similar under one common head. Thus are formed generic ideas, which take their place in our thinking. We call them generic ideas when they comprise a number of existing things, and laws when they comprise a series of phenomena or processes. When I have discovered that all the mammalia, i. e., all warm-blooded animals which bring forth living young, breathe by means of lungs, have two chambers of the heart and at least three auricular bones, I need no longer remember these peculiarities separately for the ape, the horse, the dog, or the whale. The general rule includes an immense number of individual instances and represents them in the memory. The law of the refraction of light does not only include all cases where rays fall, at different angles, upon a smooth surface of water and show the result, but all cases where rays of any color strike a surface of any kind of any transparent substance. This law, therefore, includes such.an endless number of cases that it would have been absolutely impossible to retain them all singly in the memory. Moreover, this law does not only include those cases which we or others have already observed, but we do not hesitate to apply it to new cases, which have not yet been recog. nized, to predict the effect of the refraction of light, and our expectations will not be disappointed. In the same manner; if we should find an unknown mammal, that has never been anatomically dissected, we might conclude alnost with certainty that it had lungs, two heartchambers, and three or more auricular bones.

While we thus generalize the facts of our experience into classes and laws, we not only reduce our knowledge to a form in which it is more easily used and remembered, but we actually increase it, since we can extend the rules and laws thus found to cases which may come to our notice in future.
In the above examples the generalization of facts presents no diffculty, and the whole process is obvious. But in complicated cases we do not succeed so easily in separating the similar from the dissimilar, and in forming clear, sharply defined ideas. Suppose we know a man to be ambitious; we may predict, with tolerable certainty, that, if this man be placed in certain conditions, he will follow the promptings of his ambition and choose a certain course of action. But we can neither define with certainty how an ambitious man is to be recognized, nor how his ambition can be estimated, nor can we ascertain how great it must be to lead him, under certain circumstances, to adopt a certain line of action. We compare the observed actions of one man with those of other men who have acted similarly in similar cases, and draw our conclusion as to the result of future actions, without having either our major or our minor premise clearly defined, and even without being
aware that our predictions are founded on the described comparison. Our opiniou, in such cases, proceeds from a certain psychological fact and not from a conscious argument, although, in the main, the mental process was the same as in the instance where we predicted that the newly discovered mammal would have lungs.

The latter kind of induction, which cannot be carried out to the complete form of a logical syllogism nor to the establishment of general laws, plays a very great part in the lives of men. The whole development of our sensations is based upon it, as can be proven by an investigation of illusions. When, e. g., the nerves of our eye are disturbed by a blow, we have a sensatiou of light, because, during our whole life, the optic nerve had been affected only by light, and we had been accustomed to identify the sensation of the optic nerve with the action of light, a habit which, in the present case, leads us to an incorrect conclusion. The same kind of induction plays the principal part in psychological processes, on account of the extreme complexity of the influences which determine the formation of a man's character or momentary state of mind. In fact, by asserting that we are free agents, $i$. e., that we have the power of acting accordiug to our own free will and choice, without being forced by a strict, inevitable law, we deny the possibility of referring back at least a part of the manifestations of our soul to an inflexible law.

We might call this kind of induction artistic induction, in contradistinction to logical induction, which produces sharply defined, general conclusions; because it is pre-eminently apparent in the finest works of art. It is an essential part of artistic talent to be able to reproduce the external characteristics of a character or state of mind by means of words, forms, colors, or sounds, and to comprehend instinctively the operations of the soul without being guided by any tangible rule. On the contrary, wherever we become aware that the artist has consciously worked after general rules and abstractions, we find his production commonplace and our admiration ceases immediately. The works of great artists, bowever, depict to us characters and operations of the mind with such vivacity, such profusion of individual traits, and such convincing truthfulness, that they seem superior to real life, because no disturbing influences have entered.
When we examine the sciences with regard to the manner in which conclusions must be drawn in each, we are struck by a fundamental difference between the natural and the mental sciences. In the natural sciences, induction may usually be continued until we obtain decided general rules and laws, while in the mental sciences deductions from psychological tact preponderate. So in the historical sciences, the sources of facts must first be verified, and, when the facts are established, the more difficult and more important labor begins of investigating the complicated and various motives of peoples and individuals. Both must be done chiefly through psychological consideration. The psychological sciences, in so far as they have to do with the explanation 15 s 71
and emendation of the texts transmitted to us, and with the history of literature and art, require an intuitive perception of the sense of an author and of the secondary meaning of his words; they require a correct appreciation, both of the individuality of the author and of the genius of the language in which he wrote. All these are instances of artistic and not of logical induction. We can only form our judgment, if a large number of similar facts is ready in the memory to be quickly brought into relation with the question before us. One of the first requirements for this kind of studies is, therefore, a reliable and ready memory. Indeed, many celebrated historians and philologists have excited the astonishment of their contemporaries by the power of their memories. Of course, the mere memory would not suffice without the faculty of quickly perceiving analogies, or without a finely developed appreciation of human emotions; and this latter requisite cannot, perhaps, be acquired without a certain warmth of feeling or an interest in observing the emotions of others. While our intercourse with our fellow-men in every-day life must furnish the basis for these psychological reflections, the study of history and art serves to supplement and enrich them, since both show us men acting under unusual circumstances, and teach us the whole extent of the powers that lie slumbering in our bosoms.
The above-mentioned sciences, with the exception of grammar, generally do not succeed in obtaining strict niversal laws. The laws of grammar are established by the human will, although it may have been unconsciously and without a premeditated plan, but developing as the need of them was felt. They appear, therefore, to the learner of the language as laws established by extraneous authority.
Intimately connected with philology are theology and jurisprudence, whose preparatory and auxiliary studies in fact belong to the circle of philological sciences. The general laws, which we find in both, are also such as have been established by extraneous authority for our belief and mode of action as regarded from a moral or judicial point of view, and not laws like those of nature, which state the generalization of a mass of facts. Like the application of a law of nature, however, to a particular case, the application of a grammatical, legal, moral, or dogmatical rule, is made in the form of a conscious logical syllogism. The rule forms the major premise, and the minor premise must show whether the case in point fulfills the requirements of the rule. The solution of this latter process, as well in grammatical analysis for explaining the sense of a sentence as in a legal consideration of the truth of facts, the intentions of agents or the sense of their writings must again be of a psychological nature. We cannot deny, however, that both the syntax of civilized languages and the system of our jurisprudence, perfected by a practice of more than 2,000 years, have attained so high a degree of logical finish and consistency that cases not coming clearly under their laws are exceptional. Of course, there will always be such cases, because human laws can never hope to become as
perfect and comprehensive as the laws of nature. In such cases we have no other alternative but to divine the intention of the law-giver from the analogy of the laws for similar cases.
Grammatical and legal studies have certain advantages for cultivating the mind, because they uniformly exercise its different faculties. The education of the modern Europeans is, for this reason, based especially upon the grammatical study of foreign languages. The mother tongue and foreign languages, that are learned by practice alone, do not exercise logical thought, although they may teach us artistic beauty of expression. The two classical languages, Greek and Latin, in common with most ancient and original languages, have the advantage of an extremely artistic and logical development, and of full and distinct inflections, which clearly indicate the grammatical relation of words and sentences. By long use languages become worn down, grammatical forms are sacrificed for practical brevity and rapidity of utterance, and the result is greater indistinctness. This is clearly seen by comparing the modern European languages with the Latin. The wearing down of inflections has proceeded furthest in English. This seems to me to be the reason why modern languages are less fit for educational purposes than the ancient.
As grammar is best adapted to the education of youth, so are juridical studies a means of calture for a maturer age, even where they are not immediately necessary for practical use.
The extreme opposite of the philological and historical sciences, as far as the kind of intellectual labor involved is concerned, is found in the natural sciences. I do not mean to deny that, in many branches of these sciences, an instinctive perception of analogies and a certain artistic tact play a conspicuous part. In natural history it is, on the contrary, left entirely to this tact, without a clearly definable rule, to determine what characteristics of species are important or unimportant for parposes of classification, and what divisions of the animal or vegetable kingdom are more natural than others. It is furthermore significant that Goethe, i. e., an artist, has first suggested comparative anatomical investigations of the analogies of the corresponding argan's of different animals, and also of the analogous metamorphosis of leaves in the vegetable kingdom, thus determining materially the direction which comparative anatomy has since taken. But even in these branches, where we have to do with the effects of vital processes, as yet not understood, we can generally form comprehensive ideas and discover clear laws more easily than in cases where our judgment depends upon an analysis of the actions of the soul. The peculiar scientific character of the natural sciences appears most sharply defined in the experimental and mathematical branches, especially in pure mathematics.

The essential difference betweeu these sciences, in my opinion, consists in that it is comparatively easy in the latter to unite individual
cases which have come under our observation or experience, under general laws of absolute correctness and extensive application, while in the former such generalization usually presents insurmountable difficulties. Indeed, in mathematics, the general laws called axioms are so few, so comprehensive, and so evident that they require no proof. The whole of the pure mathematics is developed out of the following three axioms:
"Two magnitudes equal to a third are equal to each other.
"Equals added to equals produce equals.
" Unequals added to equals produce unequals."
The axioms of geometry and of theoretical mechanics are not more numerous. These sciences are developed out of these few axioms by employing every obtained conclusion in working out subsequent cases. Arithmetic is not confined to the addition of a finite number of magnitudes, but teaches in higher analysis even to add an infinite number of magnitudes, which increase or decrease in value according to the most varying laws; in other words, to solve problems which could never be done by direct methods. Here we see the conscious logical operation of our mind in its purest and most perfect form; here we learn the whole labor and great care with which it must proceed, the accuracy necessary to determine the full value of discovered general laws, the difficulty of forming and understanding abstract ideas; but we also learn at the same time to gain confidence in the certainty, scope, and fruitfulness of such mental labors.

The latter becomes still more obvious in applied mathematical sciences, especially in mathematical physics, to which must also be added physical astronomy. After Newton had once recognized, from the mechanical analysis of the motions of planets, that between all ponderable matter there exists an attraction, inversely proportional to the square of the distance, this simple law was sufficient for calculating with the greatest precision all the motions of the planets to the remotest periods of past or future time, if we only have the place, velocity, and mass of the various bodies of our system given for some point of time. We evèn recognize the effects of the same force in the motions of double stars, whose distance from us is so great that their light is years in reaching us, and in those whose distances have never been successfully measured.

This discovery of the law of gravitation aud of its consequences is the most wonderful effort of logical power of which the human mind has ever been capable. I do not assert that no men possessing powers of logical abstraction as great or greater than those of Newton or of the other astronomers, who led the way to or developed his discovery, have ever lived; but that there has never been a better opportunity, than that of solving the confused motions of the planets, which had before served only to foster a belief in astrology among the uneducated, and which were now brought under a law that was able to account for the slightest details of their motions.

Other branches of physics have also been developed according to the
above gieat model, especially optics, electricity, and maguetism. The experimental sciences have the advantage over the rest, that they can at will vary the conditions under which a result takes place, and may thus confine themselves to the observation of comparatively few characteristic cases in order to determine the law. Its correctness must, of course, be verified in more complicated cases. Thus the physical sciences have advanced with comparative rapidity after the correct methods had once been determined. They have not only enabled us to look back into the distant past when the cosmical nebulæ were consolidated to stars and became incandescent by the power of their aggregation; not only to investigate the chemical constituents of the solar atmosphere-the chemistry of the most distant fixed stars will probably soon become known also*-but they have taught us to avail ourselves of the forces of nature for our own benefit.

From what has been said, it is sufficiently evident how different the mental labor is in the two classes of sciences. The mathematician needs no memory at all for individual facts, and the physicist but little. Suppositions based on the recollection of similar cases may be useful in indicating the right direction, but they become valuable only when they have led to a precise and marked law. There is no doubt that we have to do in nature with unvarying laws. We must, therefore, work on until we have discovered them. We must not rest until we have accomplished that; for it is then only that our knowledge obtains its triumphs over time and space, and over the forces of nature.
The solid work of conscious argument requires great perseverance and care; it is generally slow, and is but rarely advanced by flashes of genius. We find in it little of that readiness with which the memory of the historian or philologer recalls past experiences. It is, indeed, the essential condition of methodical progress of thought that the mind must remain concentrated upon one point, undisturbed by side issues, by wishes or hopes, and proceed only according to its own will and determination. The celebrated logician, Stuart Mill, asserts as his conviction that the inductive sciences have done more in modern times for the progress of logical methods than philosophy itself. One great cause of this may be, that in no department of knowledge is a mistake of reasoning detected so easily by the erroneousness of the result as in these sciences, where we can most readily compare the results of our reasoning directly with the actual facts.
Although I have asserted that the natural sciences, and especially their mathematical branches, have come nearer the accomplishment of their scientific mission than the other sciences, I do not wish to be charged with underrating the latter. If the natural sciences have attained

[^21]greater perfection in their scientific form, the mental sciences have the advantage that they have treated a richer subject, and one that is of more intimate interest to mau, namely, the human mind itself, with its various desires and operations. They have the higher and more difficult task; but it is clear that the example of those branches of knowledge which have advanced further by reason of their easier subject-matter, must not be lost to them. They may learn methods from them and derive encouragement from the abundant harvest of their results. I believe, indeed, that our times have already learned much from the natural sciences. The great respect for facts and accurate collections, a certain distrust of appearances, the striving after the discovery of unvarying laws which distinguish our times from former time, seem to indicate sach an influence.
How far mathematical studies, being the representatives of conscious logical thought, should obtain a greater influence in our educational systems, I will not here consider. That is mainly a question of time. As science becomes more extended, system and organization must be improved, and students will find themselves obliged to pass through a severer course of thinking than grammar is able to afford. What I have particularly noticed in my own experience with students who pass from our grammar-schools to scientific and medical studies, is a certain laxness in the application of strict universal laws. The grammatical rules to which they were accustomed are usually farnished with long lists of exceptions; the students are, therefore, not used to trusting the certainty of the legitimate consequence of a general law without reserve. Secondly, I find them too much inclined to seek authorities where they might be able to form an opinion of their own. In philological studies, the scholar who can rarely overlook the whole field, and who frequently must depend upon an æsthetic perception of elegance of expression and of the genius of the language which require long culture, will, even by the best teachers, be referred to authorities. Both errors proceed from a certain sluggishness and an uncertainty of thinking, which will disqualify the student for later scientific studies. Mathematical studies are certainly the best remedy for both; in them there is absolute certainty of inference, and there is no authority but our own reason.
So much for the mutually supplementing tendencies of the mental labors of different sciences.

But the acquisition of knowledge is not the only object of man on earth. Although the sciences awaken and develop the most subtle powers of the human mind, yet he who studies only for the purpose of knowing, does not fulfill his destiny on earth. We often see highly gifted men who are by some fortune or misfortune placed in comfortable circumstances, without ambition or energy for action, drag out a tedious and unsatisfactory life, while they believe that they are carrying out the object of their existence by increasing their knowledge and devel-
oping their minds. Action alone ennobles a man's life, and his aim must therefore be either a practical application of his knowledge or an increase of science itself. The latter is also conducive to the progress of humanity, and this leads us to the consideration of the connection between the subjects of the sciences themselves.
Knowledge is power. No time demonstrates this more clearly than our 0 wn . We learn how to make the forces of nature, as found in the inorganic world, subservient to the needs of human life and the purposes of the human mind. The application of steam has increased the bodily power of man a thousand and even million fold; weaving and spinning machines have relieved man of labor whose monotonous regularity served only to staltify the mind. The intercourse of men with its material and intellectual consequences, has increased to a point which would never have been dreamed of when our parents were born. But it is not only by machines that human force is multiplied, and it is not only on cast-steel rifled cannon, and iron-clad vessels, or on supplies of provisions and money that the power of a nation depends, although these things have so unequivocally asserted their influence, that eren the proudest and most unyielding absolute governments of our time have been obliged to remove the shackles from industry and grant a political voice to the laboring classes. It is also the political and judicial organization of states, the moral discipline of individuals, which produces the preponderance of the civilized nations over the uncivilized, so that the latter are doomed to inevitable destruction if they cannot acquire civilization. Here everything acts reciprocally. Where there are no established laws, where the interests of the majority cannot assert themselves, there can be no development of national wealth and power. He alone can become a good soldier in whom a sense of honor and independence have been developed under just lars, and not the slave, who is subject to the whims of a capricious master.
For this reason every nation, from motives of self-preservation alone and without considering more ideal requirements, has an interest in fostering not only the natural sciences and their technical applications, but also the political, legal, and moral sciences, with all their subservient listorical and philological branches. No nation, wishing to preserve her independence and influence, can afford to remain behind. The civilized peoples of Europe are conscious of this. The public aid given to universities, schools, and scientific institutions far exceeds all that was done in this respect in former times. We also can boast again this year of a liberal donation by our government.* I spoke in my introduction of the increasing division and organization of scientific labor. In fact, men of science form a kind of organized army, endeavoring, for the good, and indeed mostly by the commission and at the expense of the whole nation, to promote such knowledge as tends to the increase

[^22]of industry, wealth, the comforts of life, and to the improvement of the political organization and the moral development of her citizens. Of course, we must not ask for immediate, apparent benefit, as the uneducated are so apt to do. Everything that gives us information concerning the forces of nature or the powers of the human mind is valuable, and will ultimately prove useful, often when we least expect it. Who could have thought when Galvani touched the thighs of frogs with different metals and saw them twitch, that eighty years later, Europe would be traversed by wires, carrying news with the rapidity of lightning from Madrid to St. Petersburg by means of the same agency, whose first indications that anatomist observed? Electric currents in his and at first also in Volta's hands, were of the feeblest kind, and could only be perceived by the most delicate instruments. If their investigation had then been abandoned because it was unpromising, the most important and interesting connection between the natural forces would to-day be wanting. When young Galileo, while a student at Pisa, observed a swinging lamp in church, and found by counting his pulse that the duration of the oscillations was independent of the size of the are, who could have foreseen that by means of this discovery we would obtain clocks measuring time with an accuracy then deemed impossible, and which would enable the mariner, tossed by storms on the remotest waters of the earth, to determine his longitude?

He who expects an immediate practical benefit in his study of science, may be pretty sure that his pursuit will be in vain. Perfect knowledge and understanding of the action of the powers of nature and mind are all that science can attain. The individual investigator must find sufficient reward in the pleasure of making new discoveries, victories of thought over refractory matter ; in the æsthetical beauty afforded by well-ordered knowledge, where a perfect connection exists between all. its parts and the whole shows the controlling power of the mind; and in the consciousness of having contributed to the ever-increasing stock of knowledge on which the dominion of man over inimical force depends. He cannot, indeed, expect always to find appreciation and reward adequate to the value of his works. It is true that many a one to whose memory a monument has been erected, would have been happy had he receired the tenth part of its cost in money during his lifetime. But we mnst also remember that the value of scientific discovery is much more readily and cheerfully appreciated by public opinion than formerly, and that cases where authors of material scientific progress are allowed to suffer want have become more and more rare ; that, on the contrary, the governments and people of Europe have recognized the duty of compensating prominent men of science by corresponding positions or national rewards provided especially for the purpose.

The sciences have then a common cause: to make the mind rule the world. While the mental sciences work directly to make intellectual life richer and more interesting, to separate the pure from the impure,
the natural sciences labor indirectly toward the same goal, by endeavoring to free man more and more from external necessities. Every single investigator performs his part and chooses such tasks as are most suited to his mental endowments and culture. But every one must remember, also, that he is able to further the great work only in conjunction with the rest, and that it is therefore his duty to make the results of his labors as clear and as accessible to them as possible. Then he will find assistance in others and they in him. The annals of science are rich in proofs of such mutual relations between sciences apparently the most remote. Historical chronology is based apon astronomical calculations of eclipses of the sun and moon, recorded in ancient histories. Conversely, many important data in astronomy, such as the time of revolution of many comets, are based upon old historic records. Latterly, Bruicke and other physiologists have found it possible to build up a system of all articulate sounds of which the human organs of speech are capable, and to base upon it. suggestions for a universal alphabet adapted to all human languages. Here, then, physiology has entered the service of the science of language, and has furnished the explanation of many curious changes of sound, which are determined not by the law of euphony, as had been before supposed, but by a similarity in the positions of the organs of speech. The science of language, in return, throws light upon the ancient relationship, separation, and migrations of tribes in prehistoric times and on the degree of civilization to which they had attained before their separation; for the names of those objects which they could name then, are found to be common in later languages. Thus the study of language furnishes us with the history of times of which we have no historical documents. Let me furthermore remind you of the assistance which anatomy can afford the sculptor and the archæologist who examines ancient statues. If I may be permitted to refer to some of my own latest works, I will mention that it is possible to demonstrate the elements of our musical system by the physics of sound and the physiology of its sensation, a problem belonging entirely to æsthetics. The physiology of the organs of sense is most intimately connected with psychology, because it proves results of psychological processes in the perceptions of sense which do not come within the scope of conscious reflection, and must, therefore, remain concealed from psychological self-observation.

I could only mention here the most striking examples of the mutual relations of sciences and those which required the fewest words, and was, therefore, obliged to choose those existing between the most remote branches. But the influence which each science exercises over the one nearest akin to it is, of course, much greater. It is self-evident; it requires no illustration; you all know it from your own experience.

I therefore consider every individual as a laborer at a common great work, touching the noblest interests of the whole human race; not as one striving to satisfy his desire of knowledge, or his own advantage,
or to shine by displaying his own abilities. The true scientist will not want the reward of his own conscience nor the appreciation of his fel-low-men. To keep alive the co-operation of all investigators and the relations of all branches of science with each other and to their common object is the great mission of universities; it is, therefore, necessary that in them the four faculties should always go hand in hand. We will constantly endeavor, as far as in us lies, to labor in this great cause.

# ALTERNATE GENERATION AND PARTHENOGENESIS IN THE ANIMAL KINGDOM. 

Lecture delivered before the Vienna Society for the Diffusion of Scientific Knowledge, by Dr. G. A. Kornauber.

## Translated for the Smithsonian Institation.

Among the various questions whose scientific explanation is the province of animal physiology, none has perhaps excited the interest of the people, as well as of scholars, to a higher degree than the propagation of organisms.
While in former times naturalists entertained the most various opinions and hypotheses, or indulged in the most chimerical speculations, modern science, armed with more perfect knowledge and greatly improved instruments, and more familiar with methods of exact research, has gradually succeeded in shedding some light on these mysterious processes.
These processes in general consist in this, that certain bodily constituents are from time to time separated from individual beings, and are developed into others of the same species. If the action of a second animal substance is necessary on such separated germs, which then show the characteristic structure of eggs, and are called ova, the process is called sexual propagation or generation; but if the germ under favorable external circumstances may become a new being without such action, this more simple though less general process is called unsexual or agamic reproduction.
To the latter belongs a series of phenomena to which I have the honor of directing your attention this evening; phenomena which have been accurately studied and verified only within the last two decades. A law has been established of the highest importance, not only to zoölogy but to all natural science, which has been named that of "Alternate Generation and Parthenogenesis."
It was the brilliant Danish naturalist Steenstrup who, in the celebrated essay on "Alternate Generation," (Copenhagen, 1842,) first showed the way that would lead to a satisfactory explanation of the complicated phenomena attending the multiplication of the lower forms of animal life.

By alternate generation, Steeustrup understood the power of an animal of producing progeny differing from the mother, but itself capable of producing young, which again return to the form and character of the first parent; so that the daughter would not resemble the mother, but the grandmother. Sometimes this return to the original form occurs only
in the third, fourth, or yet further removed generations. The peculiarity of this phenomenon not only consists in the alternation of different progeny, but also in that of sexual and sexless reproduction. One generation may consist of sexually developed males and females, and bear young from eggs, and the next may be sexless, and may bring forth young by fission, by buds or germs. These animals capable of agamio propagation were called nurses by Steenstrup, because it is their function to provide for the alimentation and development of the sexual animals. The number of sexless intermediate generations, as well as their degree of development and organization, differs in different species. They either possess provisory or temporary organs, and are therefore larvæ, or they are fally developed individuals, and already show the construction and mode of life of the sexual animals. The sexless larvo of animals, such as butterflies, which undergo simple metamorphosis, are distinguished from our nurses by their inability to multiply by agamic reproduction; so that we may, according to Leuckart, consider alternate generation with nurses as a metamorphosis combined with agamic reproduction.

Alternate generation, very aptly called metagenesis by R. Owen, was first observed in the salpæ, a kind of mollusks which are as remarkable for their form as for their mode of life. They belong to the tunicata, and are found in great numbers in the ocean, the Mediterranean, and in all southern seas. They swim about a little below the surface, and present the appearance of oval or cylindrical bodies, clear as crystal, moving about either isolated or united in long chains, by taking in water and expelling it again.
Our German lyric poet, Chamisso, remarked, in his voyage around the world, that the isolated salpæ could not be members of a severed chain, because they did not resemble the individuals of the latter. He farthermore recognized that the solitary salpæ always contained a progeny resembling the chain, while the individuals of the latter contained a fœetus formed exactly like the solitary salpæ. Chamisso published his interesting observations in 1819, at Berlin, in an essay entitled De animalibus quibusdam e classe vermium linnceana, Fasc. I. de Salpa, in which he expressed the view that the solitary salpæ proceeded from the individuals of the chain and the latter from the solitary ones. Chamisso's discovery was but little appreciated at first; it was even ridiculed as the vagary of a poet, until it was brilliantly defended by Steenstrup in 1842, and confirmed and expanded later by the accurate investigations of other zoölogists. We know now that the loosely connected chain is composed of hermaphrodite sexual animals, generating an embryo usually from one egg only, which remains connected for a time with the mother by means of a kind of placenta, and is nourished by it until, having attained a considerable size, it escapes and forms the solitary or isolated salpa-the only case of viviparity among the tunicata. The solitary salpa then generates a chain of sexually developed
individuals by gemmation from buds, which take the place of male and female organs of generation, and thus represent their nurse.
Un the coasts of the North and Baltic Seas immense swarms of clear, watery, bell-shaped creatures may be perceived in summer, swimming slowly around below the calm surface of the water, with their convex surface upward and their concave downward. These are the Aurelia aurita, $L$., a species of acraspedote, or unfringed medusa, some of which are male and some female, as is the case in all medusæ. The sexual organs are ruffle-like folds on the inner skin of four bags or folds.in the gastrical cavity, which open outward at the bottom of the stalk. By simple ciliary motion the seed of the male passes into the bags of the female and fecundates the eggs. These then pass out into the folds of the tentacles, where they are developed to embryos, which are proivided with a very tender covering of cilia, and move about freely in the water like infusoria. This phase of evolution was formerly considered as a separate species, called planula. Soon, however, the cilia falls off, and the animalcule, thus deprived of its locomotive organs, sinks to the bottom, attaches itself to firm objects, and grows longer. In the free end a cavity soon appears, which gradually increases and is developed into a mouth, from which wart-like excrescences or papillæ shoot out and are afterward converted into tentacles. The animal has now the appearance of a polypus; and it was, indeed, formerly so considered, and called hydra tuba. After some time-perhaps months-a circular depression is seen just below the crown of tentacles, followed by others behind it. These depressions become deeper and deeper, and short projections appear in their edges, which afterward also develop into tentacles. The whole now bears a distant resemblance to the so-called strobila, or fir-cone, or to a set of flat cups resting on a columnar foot, the polypus. The separate divisions of the strobila are the origin of the future medusæ. They develop more and more, one after another, separate from their pedestal, and afterwards attain their permanent form, size, and maturity. They now turn the convex surface by which they were attached, upward, while the mouth, which was before turned up, now points downward. In the aurelia there is, therefore, an intermediate or nurse generation during the polypus stage, in which the animal is multiplied in an agamic way by gemmation and fission. Each of the individuals so produced is again developed into a sexual medusa.
In medusæ of lower organization belonging to the hydroids, which Gegenbauer has called craspedote, because their disk is provided with a velum, a similar kind of alternate generation takes place, with the exception, however, that the polypoid nurse reaches a much more advanced stage of independent development after leaving the ovum. It grows to a stalk of considerable size, and puts forth numeroas polypus-buds. It is only when the colony has attained a high degree of development that
the sexual animals are formed, which separate from the stalk, swim about independently, and deposit their eggs in remote spots.
-In other hydroids the nurse acquires a still greater importance. In them, as in our sweet-water polypi, the sexual progeny appears only in the shape of globular appendages, which are not capable of being evolved into independent animals, but remain attached to the polypusstalk, and resemble organs for the production of the sexual secretions.

We may with Gegenbauer call this latter form of alternate generation imperfect metagenesis. We see another remarkable instance of it in the peculiar many-shaped colonies known as Siphonophorce, which swim about freely in the sea, and of which the vraya dipheys, Blaine, occurring in the Atlantic and the Mediterranean, may serve as an example. From the transparent ovum of this animal a ciliated larva is hatched. The plastic material contained in the body of this larva or nurse is then differentiated into a locomotory piece, (the posterior of the two swimmingbells at the beginning of the stalk of a ripe colony,) and an appendage which afterward becomes the second bell and the common stalk of the whole colony. The individuals now bud forth from this stalk in a fixed order, but do not separate. They remain so connected that their abdominal cavities all open into the canal passing through the common stalk. These individuals are not by any means formed alike, nor do they serve the same physiological purpose. The principal of the division of labor, which is carried out in the solitary animals so that their organs become constantly more numerous and more perfect, is here applied in such a manner that the various functions of animal life, motion, alimentation, defeuse, and aggression, as well as sexual reproduction, which is otherwise confined to single individuals, are here distributed among all the animals of the whole colony. In every tuft along the stalk, which sometimes numbers as many as fifty of them, we distinguish nourishers in the form of trumpet-shaped appendages with orifices called suction-tubes; aggressors, in the form of long contractile filaments or tentacles furnished with microscopic weapons (nettle-cells) at their knobs; defenders, in the form of stiff scales or helmets attached to the nourishers for purposes of defense ; reproducers, developed after all the rest, in the form of racemous diæcious cajsules swinging in small (special) swimming-bells. By the alternate contraction and expansion of the bell-shaped swimmers at the upper end of the colony, (the base,) with which the smaller special swinming-bells move in time, the whole colony is propelled through the water.
In a few other species, the physalidee and vellelidee, the sexual animals separate from their nursing stalk and have a short, independent existence like the medusa.

The alternate generation of some of the intestinal worms is attended by the most wonderful and extraordinary circumstances. The most curious opinions have prevailed until very lately about their origin and reproduction.

On account of their various wanderings through different animal bodies, the trematodes, and more especially certain species of the genus distoma, so called on account of two suckers or stomata on the flat part of their bodies, are of peculiar interest. From the egg of the distoma a ciliated embryo, resembling infasoria, is produced, which swims about in the water, attaches itself to certain sweet-water snails, (Limnæus, Planorbis, \&c., ) and penetrates into their bodies. There it grows, loses its, cilia, and develops a mouth and an alimentary tube. Its contents aggregate into cellular heaps, which gradually assume a definite shape, and are converted into small animals. These essentially possess the structure of mature trematodes, but are sexless and have a tail-like appendage; they increase slowly in size and expand the worm which contains them, and which seems to have no other function than to protect them and promote their development, i.e., to act as their nurse. When completely developed they pierce the envelope of their nurse and move about freely in the body of the snail until they pass through this also, and glide through the water with a winding motion by means of their tail. In this form they had long been known to naturalists under the name of cercaria, Nitz; but their relation to the trematodes was unknown until quite recently. The cercaria afterward seeks a new host among the many inhabitants of the water, (fish, mollusks, crabs, insect-larvæ, etc., ) penetrates them by means of its proboscis, and there loses both its tail and the sting of its proboscis, as no longer necessary to its new mode of living. It is now converted into a distoma.

If the animal finds all the conditions necessary to its perfect evolution in its new host, it continues to grow until it has attained maturity. If this is not the case, it remains small and sexless, surrounds itself with a transparent shell, which it secretes from the surface of its own body, and remains in a state of rest and inactivity like a pupa until its host is eaten up by a larger and stronger animal. Hence we find it in the intestines, the gall-bladder, the biliary ducts, the kidneys, etc., of higher animals, especially of ruminants, (in the liver of sheep, cattle, goats, and deer;) also in asses, hogs, hares, etc., and in rare cases in man. (Distoma hepaticum, L. ; Distoma hcematobium, Bilharz.*)
Sometimes it happens that the progeny of the worm-like nurse does not immediately assume the form of the cercaria, but that of the mother. In that case an intermediate generation of larvæ is produced, which act as nurses of the cercaria, so that the worm resulting from the embryo might be called the grand-nurse.
Thus the numerous and fertile multiplication of germs by means of agamic reproduction counterbalances the difficulties and obstacles which these animals have to encounter in their various migrations through other organisms before they reach their perfect form.
Formerly the tape-worm was considered nothing more than a simple

[^23]animal having a head and an articulated body. Since Steenstrup's time, however, and especially through the more recent investigations of v. Siebold and van Beneden, we know it to consist of a chain or colony of differently-formed individuals. The larger posterior joints (the socalled proglottides) represent the organs of generation, and contain many thousand eggs in their ramified ovaries. In these, microscopio embryos are developed, which are discharged when the ripe joints fall off with the excrement of the host. The embryos do not then leave the eggs at once, but remain in their envelopes, which are well fitted for resisting putrefaction or chemical agents, until the eggs are accidentally swallowed by some (usually an herbivorous) animal. In the intestines of the latter the embryo forces its way through the egg-euvelope, softened by the digestive juices, pierces the intestinal walls and neighboring tissues, until it reaches a vein and is carried by the blood to more distant organs, in whose parenchyma it remains. After losing its embryonic hooks, the tape-worm larva grows to a bladder of varying size, along the walls of which numerous buds (the later." heads") arise in such a manner that the hollow body of the tape-worm head extends into the bladder. Such colonies were long known and considered as different species of animals. When one of them gets into the intestines of a larger animal, the head or bud provided with hooks and suckers is turned inside out, the bladder is digested, and the joints of the tapeworm (the real sexual, hermaphrodite individual) begin to grow behind the head. The reproduction of the tape-worm, therefore, passes through three different phases: The worm-like embryo or grand-nurse, the socalled tape-worm head or nurse, and the mature sexual animal.
With the exception of the salpæ, we have so far only considered cases of metagenesis where the nurses are in the form of larvæ. In the arthropods, among the diptera, we also find such nursing larvæ-an entirely new and remarkable phenomenon first discovered in the fall of 1861 by Nicholas Wagner, professor of zoölogy, in Kasan. It produced no small excitement among zoölogists, and was the cause of so much astonishment that v. Siebold himself designated it as hardly credible on receiving, after some delay, Wagner's essay in the "Zeitschrift tür wissenschaftliche Zoologie," 1863, vol. xiii, p. 513. Wagner could not then describe clearly the insect-larva which he had recognized as capable of reproduction, and v. Siebold took it from the illustrations to be a cecydomyde larva. Not long after, however, Dr. F. Meinert,* of Copenhagen, not only fully confirmed his beautiful discovery, but extended it by proving the different phases of development up to the imago, which Wagner $\dagger$ had meanwhile also accurately investigated. Meinert calls it the miastor metraloas, but according to the later researches of our excellent dipterologist, Dr. Schiner, reported to the imperial zoölogical-botanical

[^24]society in February, 1865, it hardly seems to differ from the genus heteropeza Winnertz. Reproduction takes place by means of germs. From seven to ten of these arise out of the accumulated plastic material in the body of the "mother-larva," and develop to "daughter-larvæ." The former is thereby gradually destroyed, and the progeny tears her skin and passes out. After three or five days the same process of germination begins in the new larva, and this continues from August to June, when all the larvæ of the last generation simultaneously pass into the pupa state. After three or four days the perfect insect, a small reddish-brown fly, emerges from the pupa. The correctness of these observations was afterward verified by v. Bær and v. Siebold, and Professor Alexander Pagenstecher, of Heidelberg, observed the same thing in another species and accurately described it.*
Metagenesis, with mature individuals as nurses, is exemplified among the arthropods by the aphides. As early as the middle of the last century, Charles Bonnet $\dagger$ had already communicated exact observations on the peculiar and remarkable mode of reproduction of the aphides, (plantlice.) These well-known enemies of our gardens and green-houses cover the leaves, shoots, and branches of certain plants in thick swarms, and defy all our exertions to get rid of them by their extreme fecundity. During the summer there is a series of nine or ten generations of fullyformed but sexless animals, or nurses, which bring forth living young without fecundation, and even without the presence of the male. Embryos are formed immediately from germs, which do not show the structure of true ova. They separate from peculiar tubes (germinal tubes) and develop in the body of the mother. In autumn the next to the last generation produces sexually-developed males and females, which really cohabit. As in most insects, the male then perishes, while the female lays eggs, which hibernate and produce a new race of nurses the following spring. The anatomical examination of these animals, which was first undertaken by $\nabla$. Siebold, and afterwards confirmed by Leidig, shows that the viviparous individuals are well developed, and resemble the oviparous females of the last fall generation, but that they are sexless nurses, becauses they lack the seed-bladder belonging to all female insects, and are, therefore, incapable of receiving the male seed.
All the phenomena of alternate generation were also called "Partheno-. genesis" by the excellent English anatomist, Richard Owen, in 1849, $\ddagger$ and this name, although somewhat inappropriate, was generally received on account of its euphony. When, however, the surprising discoveries of the last fer decades put in question the theory that "every true egg

[^25]cannot be developed into a new individual, (animal or plant,) unless it has been previously fructified by the action of the male seed," it seemed expedient to confine the term "parthenogenesis" to the new phenomena. In this sense it was first used by the ingenious founder of this important new theory, the distinguished zoölogist of the Munich University, Karl Theodor v. Siebold, in his paper on "True Parthenogenesis in Butterflies and Bees; an Essay on the Reproduction of Animals. Leipsic, 1856."

Parthenogenesis or virginal generation, according to Siebold, comprises "those phenomena which demonstrate that true ova may be developed into new individuals without the fecundating intervention of the male seed."

There is no want of observations of former times according to which the eggs of virgin insects were said to have produced new individuals, but they were considered erroneous. Zoölogists doubted that they were made with proper care, and attempted to explain them in different forced ways, finally classing them under metageuesis. Among them are the communications of De Geer on the psychides, and of Herold on the silkworms. In 1845 the celebrated apiculturist, K. Dzierzon, a Catholic priest at Karlsmarkt, east of Brieg, in Prussian Silesia, emphatically maintained in the "Bienenzeitung," p. 113, that the eggs from which the male bees or drones originate are produced and developed by the sole inherent power of the mother bee without the action of male seed. This view at first seemed simply incredible to apiarists; they supposed that he had either deceived himself or intended to mystify others. But when Dzierzon reiterated his statement he was severely attacked, and the dispute continued for a long time.

Until 1852 Dzierzon stood alone against their attacks, but undaunted, unconquered. He could fall back on the experience of many years. Every oue knows that there are queens which produce only male progeny or drones, and never lay an egg from which mature females, queens, or stinted females, workers are developed; that there are others which may lay female eggs for a time bat afterward become like the former, and that finally there are worker-bees which lay eggs, which give birth only to male individuals.
Among the first-class Dzierzon frequently found bees whose wings were lame. They were thus prevented from making their hymenial flight from which they would otherwise have returned impregnated. Other queens which laid male eggs from the beginning were hatched either very early or very late in the year, at a time when there were either no more or only very few drones left, so that their flight was in vain. Queens which at first laid normal eggs and afterward produced only drones were older individuals, whose stock of seed had become gradually exhausted. Worker-bees, which sometimes lay eggs and never have any other male progeny, have never been and are indeed incapable of being impregnated. From these facts Dzierzon concluded that
impregnation was unnecessary to the production of drpnes. That in common normal generation, where the queen returns impregnated from her flight, the drones are developed from unfecundated eggs, i. e., from those through whose micropyles the spermatozoa have not penetrated, is proved by Dzierzon from the following fact: After the introduction of the Italian bee, (apis ligurica, ) distinguished by the light celor of its posterior abdomen, all the young drones from an Italian queen and a German father were true Italians, while the female progeny were clearly mixed.

The convincing truth of these facts and the logical conclusions drawn from them at last brought such eminent bee-masters as Pastor Georg Kleine, of Liiethorst, in Hanover, and August v. Berlepsch, of Seebach, near Gotha, into Dzierzon's camp; but they found no entrance as yet into zoölogical science, because these practical men were unable to furnish the proper scientific proof to physiologists, who either did not know or purposely ignored these phenomena.
The important discovery of the micropyle of the insect-egg, made almost simultaneously in 1854 by Meissner,* of Göttingen, and Leuckart, $\dagger$ of Giessen, raised the hope of the apiculturists, and seemed to make it probable that Dzierzon's views would be proved by scientific men. At the thirty-first meeting of German naturalists and physicians, held at Göttingen in 1854, Pastor Kleine succeeded in wipning Professor Leuckart for his cause just as the latter had demonstrated his beautiful discoveries about the eggs of insects. Leuckart had never been able to obtain any bee-eggs, and was then for the first time, according to his own confession, initiated into the mysteries and problems of bee-life.

The first direct proof of the existence of real parthenogenesis was furnished by Leuckart in the "Bienenzeitung;" 1855, p. 127, where be communicated the results of the microscopic examination of a queen-bee sent him by Baron Berlepsch. This queen had been hatched in September, 1854, a time when no drones existed. The next spring she had filled fifteen hundred cells with male progeny. On dissection it became evident that the queen had not been impregnated. She was a normally formed female with seed-pouch and eggs; but instead of spermatic filaments the former contained a perfectly clear liquid, devoid of granules or cells, just as in the pupæ of queens.

In order to establish Dzierzon's view fully it still remained to be proved that in impregnated queens laying normal eggs, the males are also developed from unfecundated eggs. For this purpose Baron Berlepsch invited Professor Leuckart to Seebach, where he could institute microscopic investigations. Leuckart went there willingly, but he could not obtain a definite result, in spite of all his long continued exertions. K. Th. v. Siebold, who went to Seebach a few months later, by invitation of Baron Berlepsch, and resumed Leuckart's researches, was more successful. He worked in vain for three days and declared that nothing

* Zeitschrift für wissemschaftliche, Zoologie, vi, 272.
† Archiv. für Anatomie u. Physiologie, 1855, p. 90.
could be discovered by means of the microscope. He was to return next morning, and the carriage was already before the door, when he appeared before the baron and asked permission to remain one day longer. He stated that he had been unable to sleep on account of his want of success, and that a new method had occurred to him, which he desired to try.* This method succeeded perfectly, and v. Siebold very frequently saw seed-filaments (thirty-one times in fifty-two, and in two of these cases mobile) in the interior of the bee-eggs. But these spermar tozoa were found exclusively in female eggs, and were entirely wanting in the male. $\dagger$ We therefore owe to Siebold's wonderful observation and laborious experiments the definitive establishment of Dzierzon's theory that the drone-eggs are developed parthenogenetically without impregnation by the male seed. This fact, abundantly confirmed by many accurate and oft-repeated investigations, and also by Leuckart's valuable work, $\ddagger$ must now be received as scientifically established.
When parthenogenetical reproduction was thus undoubtedly proved in bees, the above-mentioned more ancient statements were carefully re-examined. In the Solenobia triquetrella and the Solenobia lichenella belonging to the moth family, it was found that the females (which were brought up from the caterpillar stage in a closed box) laid numerous eggs soon after leaving the pupæ, and that these eggs produced small caterpillars. V. Siebold dissected such moths before and after they laid their eggs, and found their ovaries constituted exactly like those of other female batterlies, but not a trace of male spermatozoa could be discovered.§ The eggs could not therefore be impregnated, and must undergo spontaneous development.
Of the remarkable apterous butterfly, Psyche helix, Siebold, whose caterpillar makes a spiral bag, no one has yet been able to find the male, although it has been sought for over. fifteen years. And yet these females annually lay their eggs in the pupa envelope, which remains behind in the caterpillar bag, and in the fall these produce the caterpillars. On dissection, true eggs with micropyle, a seed-vessel, but always without male spermatozoa, and a copulating pouch are found. These peculiarities preclude the opinion that the psyche female is only a nurse.
V. Siebold and Schmid furthermore succeeded repeatedly in obtaining caterpillars from the eggs of a virgin silkworm, and from those of the Smerinthus, which became pupæ and emerged as perfect male and female insects.
A. Barthelemy|| also confirms the existence of parthenogenesis in

[^26]Bombyx mori, and furnishes various proofs. He also observed the laying of unimpregnated eggs by other butterflies, which are hatched if they belong to the first generation of the year, but never survive the winter.
Jourdan* also observed true parthenogenesis in the silk-worm.
At the forty-seventh meeting of Swiss naturalists at Samaden, de Filippi reported that healthy caterpillars were hatched from the eggs of the Japanese silk-butterlly, although they had certainly not been fecundated, and mentioned a similar observation of Curtis on the Bombyx atlas.
In certain species of coccides Leuckart (p.36) also found parthenogenetical generation. In the Lecanium and Aspidiotus, for instance, the eggs are developed in tubes without being previously impregnated, and the spermatozoa are entirely wanting. In the genus Chermes (Oh. abietis, Kaltenb., Ch. laricis, Harling, Ch. piceec, Ratzb., Phylloorra coccinea, Heyden) of the plant-lice, having, according to Leuckart, $\uparrow$ both a winter and a winged summer generation, which latter was erroneously taken for males by Ratzeburg, reproduction proceeds by means of eggs without previons impregnation. Leuckart examined two hundred animals, and never found males but always females, and they virgins. Males do not seem to exist, or if they do, parthenogenetical reproduction seems to be the rule. Less accurate observations of the same kind were made by Dr. Ormerod $\ddagger$ on the Vespa britannica, and by Stone§ on the Vespa vulgaris.
Leuckart ( $\mathrm{pp} .105-107$ ) has furthermore established the fact that in all other sociable Hymenoptera, as the bumble-bee, the wasp, and the ant, as well as in the bee, parthenogenesis prevails. Egg-laying workers, which are exceptional with bees, are the rule with these animals. Future researches must decide whether their progeny is always male, as Huber's§ observations of bumble-bees seem to indicate. No doubt we will also find parthenogenesis with many other insects, such as the termites and the gall-fly. In the gall-fly, a species of cynips, no male has yet been discovered, but only females.
The experiments of Lievin aud Zeuker, which demonstrated the spontaneous development of the daphides, have been confirmed by J. Lubbock. Millions of the females of these animals, which are scarcely a line long, may be seen in summer moving about in cisterns and other standing sweet waters. They multiply in rapidly succeeding generations by means of unimpregnated or summer eggs in a carity between

[^27]the shell and the back of the animal, where they develop into individuals exactly resembling the mother, and multiplying parthenogeneticallyf on separating from her. In the fall males are born, which cohabit with the females and produce one or two darkly-colored winter-eggs, which are surrounded by a second firm envelope called the ephippium, to protect them during their hibernation.

Although there can be no longer any doubt about the correctness of these facts, which have been established by the repeated, careful, and accurate observations of our most distinguished zoölogists, and although the existence of parthenogenesis among a number of articulate animals is proved beyond dispute, attempts are not wanting to render them suspicious, and represent them as unreliable. Every truth differing from long cherished opinions is received slowly and with difficulty.
Tigri proposed, in a paper to the Paris Academy of Sciences," to explain the parthenogenesis of the Bombyx mori by the supposition that there is a double cocoon containing two individuals, a male and a female, which might have copulated before leaving their shell. This supposition would presuppose the most extraordinary carelessness on the part of the above-mentioned observers, It amounts to charging them with not being able to distinguish a double from a single cocoon, or with neglecting to examine the organis of generation and determine the sex of the individuals. Errors of so crude a nature would hardly be committed by men but little acquainted with methods of research, much less by naturalists of high standing.
Schaum* states that he cannot receive the theory of the parthenogenesis of insects, and thinks he can explain it away by an hypothesis of Pringsheim. According to this the queen-bee, and the workers which lay eggs, might be androgynous, and possess male organs of generation besides their ovaries. In all cases where the skillful anatomists, r. Siebold and Leuckart, dissected such bees, there were no traces of testicles, so that the above supposition remains without foundation.
The existence of hermaphrodite bees, which were observed by v. Siebold in the apiaries of Mr. Engster, of Constanz, Bavaria, $\ddagger$ cannot be brought forward as a proof against parthenogenesis, but rather seems to confirm it. It was observed that the pure worker-bees drove the hermaphrodites out of the hive the moment they left their eggs, and did not even suffer them to remain on the board outside. The hermaphrodites perished in a short time, and could never have reached the egglaying stage, even if eggs had afterward formed in their originally empty ovaries. According to Pringsheim, every queen would have to be an hermaphrodite; but in the lance-winged and drone-producing queens, which were repeatedly examined by the above observers, no trace of androgynism or of spermatozoa could be found.

[^28]Dybocosky also appeared against parthenogenesis in his inaugural dissertation, "de parthenogenesi;" but his objections are unfounded, and evince neither thorough investigation nor satisfactory knowledge of the subject. The same is the case with various other objections brought forward by the opponents of parthenogenesis. None of them will stand test.

The reliability of the theory is established beyond doubt by many well-proved facts, and we may rejoice that we have thas gained a new and highly important law for the explanation of the most wonderful phenomena in the animal kingdom.

# ON THE PRESENT STATE OF OUR KNOWLEDGE OF CRYPTOGAMOUS PLANTS. 

Lecture delivered before the Vienna Society for the Diffusion of Scientific Knowledge, by Heinrich Wilhelm Reichardt.

## [Translated for the Smithsonian Ynstitation, by Professor C. F. Krosh.]

In the last few decades many leading botanists have given especial attention to the study of cryptogamous plants, for they very properly recognized the importance to their science which a more perfect knowledge of the development, growth, and propagation, as well as of the structure, of these simplest of organism would be. Through the combined labors of much talent, a large number of the most interesting discoveries have been made. An entirely new basis for this department of botany has been created, the previous views about seed-bearing plants in many respects reformed, and a very general interest excited in the subject. For this reason it seems proper for me to report to this society, whose object is the diffusion of scientific knowledge, the present state of our information with respect to the cryptogams.

It is evident that it is only possible to give a condensed view of the most important facts, and to consider even these only in their general outlines, in the short time allotted to a lecture.

The cryptogams were almost wholly unknown to the ancients. Even Theophrastus and Pedanius Dioskorides enumerate only twenty species of them in their works. In the Middle Ages no progress was made in a knowledge of them. Attention was only paid to a few species of cryptogams, to which were attributed medicinal or magical virtues. When, with the revival of classical learning and the reformation, science also received a fresh impulse, when Brunfels rejected the traditions of the old school and turned to the study of domestic plants and thereby created a new basis for botanical research, botanists were too much occupied with the observation of seminiferous plants to pay much attention to the lower orders. It was not until the beginning of the eighteenth century that two men appeared who actively took up the study of cryptogams, and who must therefore be considered as the founders of this branch of the science. They are Antonio Micheli, superintendent of the botanic garden at Florence, and Johann Jacob Dillenius, a German, who later became superintendent of the botanic garden at Eltham, and professor at the University of Oxford. I cannot enter into a detailed account of the labors of these two fathers of cryptogamic botany; let it suffice, therefore, to indicate that they represent the two chief schools which still characterize the study of cryptogams to-day.

Micheli was an excellent morphologist for his time, and made some very interesting discoveries in his line; Dillenius, however, was principally a systematizer; he knew and described almost one thousand species of algæ, lichens, mosses, and ferns.

At last Carl von Linné appeared on the scene. He is known to every man of culture as one of the greatest of botanists, and as a scholar who reformed and influenced the whole study of natural history. He proposed what is called the sexual system, under which heclassified all known plants; he introduced the nomenclature now in use ; he raised botany to the dignity of a true science. Occupied as he was with the phanerogams, he found no time, and had, perhaps, no inclination to investigate the cryptogams. He contented himself with dividing this, the twentyfourth class of his system, into the four orders of ferns, mosses, algæ, and fungi, and distributing among them the materials furnished by Dillenius and Micheli. In his Species Plantarum he mentions about eight hundred kinds of cryptogams, distributed among fifty genera. Linnés indirect influence on this class of plants is much more important, since he laid down general laws which his successors were to apply in detail. The following are some of the prominent men who carried out Linne's ideas in the treatment of the cryptogams: Gmelin, Turner, Vaucher, Dillwyn, and especially Aghard the elder, deroted themselves to the study of the algæ. Erik Acharius laid the foundation for the study of lichens, and was assisted by Flörke, Wallroth, and Ernst Meyer. Fungi were studied by Christian Persoon, with the assistance of Schaeffer, Bulliard, Bolton, and Link. Johann Hedwig inaugurated the study of mosses, and was seconded by Bridel, Schwägrichen, and for exotic mosses, by the elder Hooker. Ferns were made a specialty by Olaus Swartz, Willdenow, Kaulfuss, Schkuhr, Bernhardi, and others. Hedwig must be considered by far the most ingenious and eminent investigator of this period; he might properly be called the Linné of cryptogams. His researches are read with preference. The Austrians especially are proud of him as their fellow-countryman. It would occupy too much time to describe the researches of Hedwig and the others, and I must therefore deny myself that pleasure.

If we examine what was done in the investigation of cryptogams during the period of the Linnean systems, we shall find that the efforts of botanists were chiefly directed to the discovery of new forms, to make short diagnoses, and to classify them artificially according to certain characteristics. Hedwig and the other authors of that time furnish only a few though valuable data concerning their peculiarities, formation, and anatomical structure. It was left to the most recent epoch of botanical studies to unite these isolated materials into a harmonic whole. In this epoch, comprising scarcely more than three decades, botany, and especially the knowledge of cryptogams, has made immense progress.

The representatives of Linne's views had accumulated a mass of
comprehensively arranged material. Botanists, however, gradually became conscious that their system should not be only. an arrangement of plants according to certain arbitrary characteristics, but that their essential peculiarities and natural relations among themselves must be considered in their classification; in other words, that they must establish a natural system. Jussieu made the first successful attempt to build up such a system. Among the French, de Candolle, and among the English, Robert Brown, the two Hookers, and Lindley perfected it. In Germany, and especially in Austria, it found its most perfect expression in our genial and renowned compatriot, Professor Stephan Endlicher, with whom must be mentioned his friends and colleagues, Professors Fenzl and Unger, my highly-esteemed teachers.

The change which the natural system produced in the direction of betanical research, ever made it more necessary to study out the laws of the growth, formation, reproduction, and propagation of plants; to find out with accuracy the relations existing between their different organs, and to investigate the origin and development of the whole plant and its separate parts, down to the most elementary organisms. Thus morphology became a separate branch of botany through the endeavors of Robert Brown, Röper, Alexander Braun, Schleiden, Schacht, Hofmeister, and others.

Morphological studies naturally led to a more accurate consideration of the structure and the processes of plant-life. The microscope had meanwhile been greatly improved, and many botanists took up this branch with predilection. In this way the anatomy and physiology of plants reached a point, through the excellent labors of Hugo von Mohl, Unger, Nägeli, Schacht, and others, which had not before been thought possible.

Excursions to all parts of the world were undertaken by courageous investigators, who not only enriched the science with a great many new forms, but rendered it possible to determine the laws of the distribution of plants over the whole earth; so that Alexander von Humboldt was enabled to produce a masterly sketch of botanical geography.

In a measure, as mutual intercourse was facilitated, more life was infused into scientific research; a great number of scientific societies and periodicals were established where the results of investigations were deposited. So many of these publications appear now that it is extremely difficult, if not impossible, to examine them all. During this great progress of botany in general, the cryptogams were not neglected. Indeed, many of the most thorough scholars made a specialty of these simplest of organisms. The important discoveries became so numerous in this department that it was entirely recolutionized. I will endeavor to present to you a condensed view of the most important achievements. For this purpose the material has been divided into five groups: algæ, lichens, fungi, mosses, and ferus. In each of
these I shall first consider the most important points of their morphology and anatomy, and afterward their classification.

We will begin with the algæ. The reform in their study was inaugurated by two works which appeared almost simultaneously, Kützing's Phycologia universalis and Nägeli's latest algæ systems. Kützing pre.sents a view of his organographic and anatomical studies, and bases upon them a new system of algæ, illustrating it by means of plates. The Species Algarum and the Tabuloe phycologice, containing a description and picture of all species of algæ, may be considered as supplements to his great work. Kützing, no doubt, had greater facilities for the study of algæ than almost any other investigator. He was the first to examine the separate organs and the structure of fuci, and to found this branch of phycology. He broke up the classification of the old genera, which contained a chaotic mass of the most different forms, and separated them into natural groups. Unfortunately, Kuitzing rejected the usual nomenclature, and employed one of his own, thus making his work very difficult to understand. In his classification he splits up his material into too many untenable species, making it almost impossible to examine the whole.

Nägeli exerted a no less important influence on the study of the algæ. In his algæ systems and in his work on one-celled algæ, this renowned anatomist shows his unsurpassed acuteness of observation in his description of the structure and mode of life of those small organisms which cannot be recognized with the unaided eye. He showed that the increase of the separated cells depends upon mathematically determinable laws. These he developed for many species, and we may say that he created a sure mathematical basis for the study of the algr. Siuce laws, valid in the whole vegetable kingdom, can be educed most easily from the algæ, the simplest organisms, Nägeli's researches are of great value to the whole science of botany. Starting from his discovered principles, Nägeli planned an algæ system of his own; but here he was less successful.

Beside these two principal works, a great number of large and small dissertations have been published. Among these the following are the most important: The works of Alexander Braun on the life and development of microscopic forms, are worthy of being placed side by side with those of Nägeli. In them, and especially in the classical work on rejuvenation in the vegetable kingdom, he has produced real masterpieces of short but very attractively written monographs, calculated to excite the interest of every man of culture. Professor Cohn, another eminent scholar, has given to the world a series of masterly and thorough essays on the Volvocince, which had until then been classed as animals. De Barry's dissertation on the Conjugates does not fall short of the other essays.

The brilliant discovery of the zoöspores of algæ was made by Professor Unger, who observed the formation of these movable cells in
the Vaucheria clavata DC, and proved that they possessed cilia as organs of locomotion, and that they germinated into a plant like the parent. Many investigators have furnished further data concerning the existence and the structure of these interesting bodies, but the most complete researches were published by Thuret in his essay, "Sur les zoospores des algces." He had observed zoöspores in several hundred species, and illustrated them in a masterly manner. We learn from these investigations that the above spores are the unsexual organs of reproduction in the algæ, and may be compared to the buds of higher plants.
The interesting and instructive process of fructification in algæ has been studied with equal accuracy. Although the great physicist, Reaumur, had suspected the existence of organs of fructification in fuci, Thuret was the first to prove it directly and scientifically. Не demonstrated that the small indentations on the surface of the Fucacea, the so-ealled conceptacles, contained both the male and female organs of fructiflcation, (the antheridia and oogonia;) he observed the formation of antherozoids and the penetration of the spernatic filaments into them; he explained how the spore was developed after fructification. In fresh-water algæ, Pringsheim first succeeded in directly proving the existence of fructifying organs in the Vaucheria, Oedogonium, and Coleochate. Cohn followed with his interesting observations of the Sphaeroplea annulina and the Volvocince. These observations prove the following mode of fructification in the alga: the so-called seed filaments penetrate the membraneless mass of the antherozoids, which are then surrounded by a cellular membrane and converted into stationary spores. These are the direct opposites of zoöspores, and may be compared to the seeds of higher plants.
The results of this and many other researches have enabled us to gain sufficient insight into the growth, reproduction, and propagation of these plants, and it will be the task of coming investigators to continue the work on this basis.
If we now turn to the classification of the algæ, we shall see that excursions to the different seas of every zone have enlarged our acquaintance with the forms of this class. Excursions to the Antarctic and to the northern part of the Pacific Ocean have furnished us with the grandest specimens of lichens, and have shown us that marine vegetation does not reach its highest development in the tropical oceans, but in the Arctic and Antarctic polar seas. Kützing's and Nägeli's contributions have already been mentioned. In the third supplement to his Generibus Plantarum, Endlicher published, together with Diessing, a systematis table of this class, distinguished by the happy arrangement into families and genera. A very important work is Species genera et ordines algarum, by Aghard the younger, which, although it only contains the Fucoidece and Floridece, surpasses all other pablications in the original natural grouping of his materials, and by
happily keeping within bounds in his subdivisions. Besides Aghard's work, we must mention the publications of Harvey on the Antarctic algæ, the works of Postels and Ruprecht on the algæ of the north Pacific Ocean, and a number of monographs on single families or floras. I can only name the most important; to enumerate them all would lead me too far: the works of Smith and Ralfs on the British Diatoms and Desmids, that of De Barry on Conjugatce, the heautiful essays of A. Braun, and among the Austrians the excellent publications of Grunow, especially on Diatoms. Finally, I must not forget to mention that Dr. Rabenhorst has done much to promote the diffusion of accurate knowledge concerning the species of cryptogamous plants by his work on the Cryptogamic Flora of Germany, and by his later publications, especially his dried collection of cryptogams.

The structure of the regetative organs of the small but interesting group of Characere was investigated by the interesting labors of Bischoff and A. Braun. Thuret published important information concerning the antheridia; Carl Müller investigated fructification, and Pring. sheim germination. Their classification was improved, especially by A. Braun, from whose master hand we may expect a monograph of the Characece.

If we now turn to the lichens, we will see that the views of the period of Linne's system long remained in credit, and that reform was late and gradual. Consequently the number of eminent discoveries in this department has been smaller, and its organography is still far from being satisfactory. Speerschneider, it is true, has furnished us with some valuable data concerning the structure and manner of growth of the thallus; but we are indebted for the most accurate information on this subject to Schwendener, who has published in two dissertations the result of his investigations of shrubby and foliaceous lichens. We know now that the thallus of lichens consists of three different layers, an outer or envelope forming long fibrous cells, a middle or gonidium composed of roundish cells filled with chlorophyll, and an inner or pith of the same structure as the outer. The behavior of these three layers, which was investigated particularly by Schwendener, furnishes many points for classification. Körber has published an excellent dissertation on the gonidia or generating cells of lichens. He states that these cells break through the envelope, become changed and converted into the so-called soredia. These observations establish the fact that the soredia are the organs of generation of lichens, and correspond to the buds of higher plants.

Many have studied the bowl-shaped fruit or apothecium of lichens, but the data are scattered through different works. Tulasne's work, "Sur l'Appareil Reproducteur des Lichenes," is of special importance, since it proves that lichens have another kind of fruit, forming small dents and containing.minute, straight, and narrow cells. They are called spermagonia, and are probably the male organs of fructification.

The process of fructification has hitherto been observed with certainty by Karsten in the Coenogonium only.

The excellent works of Elias Fries and Wallroth, which date back to the sway of Linne's system, are still of great importance for purposes of classification. Von Flotow has indirectly exerted great influence on the study of lichens. His most prominent scholar, Körber, has inaugurated a great reform in his two principal works, the Lichenes Germanixe and the Parergis lichenologicis. He created a new system, resting upon an anatomical and organographic basis, and made more natural and sharply defined subdivisions. He was ably assisted in his work by our compatriot Massalongo, whose tables are unfortunately incorrect. The works of Mylander are of great value; his Synopsis Lichenum comprises all known species. Its publication is still continued. Hepp did much to make the European species known by the description of his collections and the investigation of their spores. Finally, we must not pass over the works of Krempelhuber, which are at present contined to domestic species; but this excellent scholar will soon have a more extensive field of operation.

We now come to the largest and most interesting, but at the same time the most difficult class of cryptogams-the fungi. Their sudden appearance and growth, their ephemeral nature, and the multiplicity of their forms, have always been a source of trouble to investígators, and even the most indefatigable of modern mycologists have been able to lift but partially the veil which hangs over the life and development of these organisms.

Far ahead of its time in organography stands the work of Professor Unger, on the exanthema of plants; for in it we find the first attempt to describe the development of mildew-fungi. Although the leading idea of the whole work, that these fungi were the diseased products of the plants on which they are found, was not confirmed, the rich treasure of new facts laid down in this beautiful work retains its full valne. Corda, another fellow-countryman, has also written on fungi, and discovered many interesting forms in the fungi of mold. He was thus enabled to gain some insight into the life and development of these organisms. In his principal work, the lcones Fungorum, he represents all forms of fungi known to him; but some of his observations have unfortunately been hastily made and consequently inaccurate. But we should not forget that Corda lived in unfavorable external circumstances; that for along time he had not the means of procuring a microscope, and that he finally met with a tragical death. The ship in which he had gone to Texas in 1849 foundered on his return voyage to Europe, and nothing has been heard of him since. The works of the Tulasne brothers throw new light on many chapters of this branch of study. They show that there exists a great difference between the fungi of mildew and those of mold; that in the former not only spermogonia, but also spores of different forms are produced, which had formerly been dis-
tributed among different genera. They also studied the interesting process by which the germs and spores of the mildew-fungi are developed. In their classical dissertation, "Sur vErgot de Seigle," they showed that the well-known black fungus, or germ, as well as all other similar forms hitherto classed as Sclerotics, were not perfectly developed organisms, but rather a peculiar kind of mycelium, analogous to the tubers of higher plants. It is from them that the fructifying fungi are developed. In the great work, "Fungi hypogei," the above-mentioned authors give us a more thorough acquaintance with truffles than their predecessors, and, in their essays on the Asconyceter, they lay before us many interesting points about these organisms, proving that they contain several kinds of spores, as well as spermogonia and spermatia. In their principal work, the Selecta Fungorum carpologica, the Tulasne brothers present to us a rich collection of observations, the introduction to which is of especial interest because it furnishes a view of the results of morphological researches. The tables are executed in a masterly manner, and leave all similar productions far behind. In the same department the Germans are well represented by De Barry. He considerably extended our knowledge of mildew-fungi, and was the first to make experiments on the inoculation of their spores. He succeeded in discovering the remarkable history of the development of mucusfungi. He showed that in them the mycelium is wanting, and that from the germinating spore a peculiar body is formed, which is gradually converted into plasmodium, a substance without an analogue in the vegetable kingdom, and finally into the perfect fungus. De Barry studied the potato fungus, and proved the existence of zoöspores in it, and in others of the same family. Finally, he discovered the organs of fructification of fungi in a parasite (Peronospora Alsinearum Casp.) living on the Stellaria media. The results of his brilliant discovery were fully confirped by Pringsheim's masterly observation of the Saprolegmia, in which the latter also found zoöspores and similar fructification. Corresponding results were found by Hofmeister in the fecundation of truffles. According to these observations the fructification of fungi takes place as follows: The antheridium touches the vogonium, one of its processes penetrating an opening in the membrane of the latter and discharging either seed, filaments, or its contents, which are communicated to the antherozoid. The latter, which before was membraneless, is now surrounded with a cellular membrane, and becomes the stationary spore of the plant. Hoffman has made comprehensive researches on the germination of the spores of fungi, and Pasteur's excellent works give us information on the part which fungi play in fermentation, by proving that they are nothing more than common mold-fungi in a peculiar stage of development. All these achievements, great as they may seem, are nothing more than preparatory labors for the solution of the organography of fungi, a great problem of the future.

The works of Elias Fries are the staudard on the classification of
fungi. Since the publication of his Systema mycologicum, about forty years ago, no work has appeared which includes all orders, genera, and species of this class. Indeed, the works of Fries are so excellent that they may be held up as models to all botanical authors. The writer, who passed a third of his unusually long life in the roods, where he studied fungi, acquired a wider experience than any other. He has grouped the genera naturally, and described the species with true linnéan precision. His work is, therefore, the basis of all mycological studies. The other authors contented themselves either with writing local floras or studying single orders for the purpose of furnishing materials for a future Systema mycologicum. Many excellent works of this kind have been produced, especially those of Leveillé, Bonorden, Fresenius, De Barry, and the thorough treatises on exotic forms by Montaigne and Berkeley.

In the class of the mosses, the morphological studies of many thorough scholars have progressed so far that these plants are now among the most perfectly known. Mirbel has furnished interesting data on the structure of the leaves and the development of the sporangia of the Marchantia polymorpha. The works of Bischoff on liverworts, although treating chiefly of classification, present a great many new observations on the structure and development of the fruit. The excellent natural history of liverworts by Nees von Esenbeck, to which I will revert again, furnishes many important contributions to organography. A celebrated essay of Hugo von Mohl on the spores of acrogens proves that four spores are formed in every cell, analogous to the formation of pollencells. Gottschee, finally, has pulblished very thorough essays on the structure and development of single groups of liverworts. All these writings are left in the shade, however, as far as the organography of ferns and mosses is concerned, by those of Hofmeister, the most prominent investigator of the subject. This excellent scholar has set himself the task of pursuing the development of the acrogens down to the simple cell, and he has succeeded in many cases. Through him we know how the germ of mosses is formed and grows, how the stem is developed, and how the leaves appear and form. We not only understand the structure of the antheridia perfectly, and know how the seed filaments are formed, but we have also gained an insight into the structure of the archegonia. We are able to follow exactly the process of fructification, and see how the complicated moss-fruit is developed after fructification by the archegonium, from the riccia, the most simple type, upto the most highly-developed forms, according to one fundamental idea.. Hofmeister has illustrated all these discoveries with excellent drawings, so that the study of his masterpiece, "Comparative investigations in the development of the higher cryptogams," is one of the most grateful tasks, although it is a very laborious one, on account of the peculiarmanner in which it is written.

Hofmeister's work is also the most important source for the morphological study of foliaceous mosses. Nägeli has determined the laws of 17 s 71
growth of the vegetative organs with the same mastery as in his treatise on the algæ. Hugo von Mohl explains in a very simple manner the interesting phenomena attending the vegetation of peat-mosses in a short but thorough essay on their perforated cells. Carl Müller explains the remarkable existence of lamels on the leaves of the polytrichaceæ, and Lantzius Beninga shows how the ripe capsule, the spores and the peristome are developed. Schimper's "Recherches sur l'organographie des mousses" and the introduction of his "Synopsis Muscorum europaeorum" are of great value; for, in both works, we not only find the results of organographic researches gathered, but we also find them enriched by numerous observations of his own.

Passing to the most important works on classification, we must grant the first place to Nees von Esenbeck for his excellent natural history of European liverworts, since it is the foundation of our present views of this branch of botany. He divides up the genera of his predecessors in a rery natural manner, and his descriptions of species are masterls. His distinguishing characteristics are always sharply defined. The principles applied with such excellent success on European species were also brought to bear on exotics by Nees von Esenbeck, Gottschee, and Lindenberg, who published together the Synopsis Hepaticarum, which is considered the standard work. Unfortunately, there are no illustrations of all species of this class; for the best are still to be found in "British Jungermannice," published 1820, or thereabouts, by Hooker. Lindenberg endeavored to supply the deficiency by his Species Hepaticarum, but after several excellent monographs of single genera had appeared the publication ceased. Later ones were limited to the description of new material or the better description of single genera. Among them must be mentioned the excellent treatises of Gottschee, the Hepatica Javanicce of Van der Sande. Lacosta, and the works of Montaigne, Taylor, Mitten, and De Notaris.

The appearance of the Bryologia Europea exercised a reforming influence on the study of the mosses. Several excellent scholars, with W. Ph. Schimper at the head, determined to describe and depict all species of mosses known in Europe in a manner adequate to the demands of the time. They mutually controlled their results for fitteen years, when the work was completed in six stately volumes of more than six hundred tables, and it now forms our basis for the study of these plants. In it the genera were more naturally (although sometimes weakly) divided and better arranged. In the description of the species, the organographical and anatomical points, especially the reticulation of the leaves, were for the first time considered. Excellent illustrations facilitate the recognition of the species, and make it possible in some cases which had before presented difficulties. After the appearance of the Bryologs, .Schimper published a fine monograph on the European peat-moss, and a more general Synopsis Museorum Europeorum. It is hoped that this excellent scholar will soon be able to realize his long-cherished plan, the
publication of a work on all the mosses, for we may well expect something excellent from him. The next author of importance is C. Müller, who published a synopsis of all known mosses, in two volumes. He deserves our thorough appreciation for his diligence in collecting the existing material. His views on system, however, are less happy. Led by the consideration of certain characteristics, he often classifies very different species together, and separates those closely related. Among other writings on exotic mosses, we must mentioned Dozy and Molkenboer's "Musci inediti Archipelagi Indici," and their " Bryologia Javanica," which was continued after their death by Van der Bosch, and Van der Sande Lacosta. They follow the same plan as the "Bryologia Europen," and are, therefore, of great value. The works of Sullivant, on the moss flora of North America, and those of Wilson, Mitten, and Hampe, are also of considerable importance.
In the last class, that of the ferns, a series of the most important discoveries was inaugurated by Nägeli. He observed that the antheridia. or male organs of fructification, were developed upon the prothallium, which originates directly from the germinating spore. Count Lesczye Suminski followed up his discovery by proving that the prothallium contained also the archegonia or female organs. Through these two brilliant discoveries new prospects were opened for the morphology of ferns. We recognized that in this whole class of plans fructification was effected on the small prothallium, and that the foliage, which we had been accustomed to take for the whole plant, was developed only when fructification had taken place. Schacht, Mettenius, and especially Hofmeister, deserve great credit for following up these discoveries. The brilliant researches of the latter author in particular, have made known to us the exact process of fecundation, and we now understand that the so-called large and small spores of the selaginella and water-fern are nothing more than the female and male organs of these plants. Hofmeister has furthermore ascertained with unexampled acuteness the laws according to which the leafy plant is developed from theimpregnated germ-vesicle of the archegonium, and also how the stem grows, and how the fans are formed. Although Hofmeister came to the erroneous conclusion that the latter were not true leaves, but peculiarly transformed branches, the value of the grand discoveries of this most original and thorough of all organographists of the acrogens remains unimpaired. Hugo von Mohl has drawn a masterly picture of the structure of the stem of tree-ferns, in his classical desertation, which has since been developed'more in detail, partly by himself and partly by other authors. The most thorough investigation of the development of the indusium and sporangium are due to Schacht.

Besides the older works of Kaulfuss and Kunze on the classification of ferns, we must mention especially the numerous pteridographic works of Hooker, which have considerably advanced our knowledge of the subject by their excellent illustrations. The works of K. B. Presl
are of great importance, and of especial interest to us Austrians. In his "Tentamen Petridographice," this thorough scholar has studied the reticulation of ferns more accurately than any of his predecessors, introduced new names, and endeavored to divide the class into more natural genera. Although he sometimes goes too far in this direction, we cannot but appreciate his earnestness, consistency, and extensive information. Fée attempted to follow in Presl's footsteps, but he was less successful, and his works must be used with caution. Our most distinguished pteridographist, Mettenius, successfully opposed the tendency to split up the existing material into too many untenable genera and species, in his excellent work on the ferns of the Leipsic botanic garden, and in a series of critical essays, which mostly appeared in the Senkeberg Museum. May this distinguished scholar indefatigably pursue and ultimately attain his object! Moore deserves great credit for his very critcal index of all ferns, for the introduction of many tropical specimens, and for publishing (together with Newman) the first work in which nature was successfully employed to print herself. Lowe's "British and Exotic Ferns" is also a valuable illustrated work. Besides all these there are many special publications on single species. The following are among the most important: Milde's Essays on the Equisetaceæ and Domestic Ferns; Presl Van der Bosch and Mettenius on Hymenophylleæ; Spring's Monograph of the Lycopodiaceæ; and A. Braun on Isoëteæ, and Water-Ferus in General.

This then is a condensed review of the most important achievements in cryptogamy within the last few decades. Taking them altogether, we may say that this branch of botany has made more progress in this period than in all preceding times, and that it has now indeed become a science. The study of the cryptograms is no longer confined to a few isolated scholars as formerly, but it is exciting general interest, and many excellent investigators are making it their favorite subject. Morphology was not only founded, but even completed and established for certain classes. Numerous and highly important anatomical and physiological data have been furnished; the classification has in the last period been reformed in accordance with the latest views, and various authors have endearored to obtain a natural arrangement of species, and have succeeded in many cases.

Although much has been accomplished, much still remains to be done, and we need the combined efforts of many. May, therefore, the interest in cryptogamous plants ever become more general and lively, and may, especially in Austria, many scholars and amateurs turn their attention to this branch of botany! The most grateful results will surely reward their exertions.

## recent researches on the secular variations of the planetary ORBITS.*

By John N. Stockwell.

The reciprocal gravitation of matter produces disturbances in the motions of the heavenly bodies, causing them to deviate from the elliptic paths which they would follow, if they were attracted only by the sun. The determination of the amount by which the actual place of a planet deviates from its true elliptic place at any time is called the problem of planetary perturbation. The analytical solution of this problem has disclosed to mathematicians the fact that the inequalities in the motions of the heavenly bodies are produced in two distinct ways. The first is a direct disturbance in the elliptic motion of the body; and the second is produced by reason of a variation of the elements of its elliptic motion. The elements of the elliptic motion of a planet are six in number, vis: the mean motion of the planet and its mean distance from the sun, the eccentricity and inclination of its orbit, and the longitude of the node and perihelion. The first two are invariable; the other four are subject to both periodic and secular variations.

The inequalities in the planetary notions which are produced by the direct action of the planets on each other, and depend for their amount only on their distances and mutual configurations, are called periodic inequalities, because they pass through a complete cycle of values in a comparatively short period of time; while those depending on the variation of the elements of the elliptic motion are produced with extreme slowness, and require an immense number of ages for their full development, are called secular inequalities. The general theory of all the planetary inequalities was completely developed by La Grange and La Place, nearly a century ago; and the particular theory of each planet for the periodic inequalities was given by La Place in the Mécanique Céleste.
The determination of the periodic inequalities of the planets has hitherto received more attention from astronomers than has been bestowed upon the secular inequalities. This is owing in part to the immediate requirements of astronomy, and also in part to the less intricate nature of the problem. It is true that an approximate knowledge of the secular inequalities is necessary in the treatment of the periodic inequalities; but since the secular inequalities are produced with such extreme slowness, most astronomers have been content with the supposition that they are developed uniformly with the time. This supposition is suffi-

[^29]ciently near the truth to be admissible in most astronomical investigations during the comparatively short period of time over which astronomical observations or human history extends; but since the values of these variations are derived from the equations of the differential variations of the elements at a particular epoch, it follows that they afford us no knowledge respecting the ultimate condition of the planetary system, or even a near approximation to its actual condition at a time only comparatively remote from the epoch of the elements on which they are founded. But aside from any considerations connected with the immediate needs of practical astronomy, the study of the secular inequalities is one of the most interesting and important departments of physical science, because their indefinite continuance in the same direction would ultimately seriously affect the stability of the planetary system. The demonstration that the secular inequalities of the planets are not indefinitely progressive, but may be expressed analytically by a series of terms depending on the sines and cosines of angles which increase uniformly with the time; is due to La Grange and La Place. It therefore follows that the secular inequalities are periodic, and differ from the ordinary periodic inequalities only in the length of time required to complete the cycle of their values. The amount by which the elements of any planet may ultimately deviate from their mean values can only he determined by the simultaneous integration of the differential equations of these elements, which is equivalent to the summation of all the infinitesimal variations arising from the disturbing forces of all the planets of the system during the lapse of an infinite period of time.
The simultaneous integration of the equations which determine the instantaneous variations of the elements of the orbits gives rise to a complete equation in which the unknown quantity is raised to a power denoted by the number of planets, whose mutual action is considered. La Grange first showed that if any of the roots of this equation were equal or imaginary, the finite expressions for the values of the elements would contain terms involving ares of circles or exponential quantities, without the functions of sine and cosine, and as these terms would increase indefinitely with the time, they would finally render the orbits so very eccentrical that the stability of the planetary system would be destroyed. In order to determine whether the roots of the equation were all real and unequal, he substituted the approximate values of the elements and masses which were employed by astronomers at that time in the algebraic equations, and then by determining the roots he found them to be all real and unequal. It, therefore, followed, that for the particular values of the masses employed by La Grange, the equations which determine the secular variations contain neither ares of a circle nor exponential quantities, without the signs of sine and cosine; whence it follows that the elements of the orbits will perpetually oscillate about their mean values. This investigation was valuable as a first attempt to fix the limits of the variations of the planetary elements;
but, being based upon values of the masses which were, to a certain extent, gratuitously assumed, it was desirable that the important truths which it indicated should be established independently of any considerations of a hypothetic character. This magnificent generalization was effected by La Place. He proved that, whatever be the relative masses of the planets, the roots of the equations which determine the periods of the secular inequalities will all be real and unequal, provided the bodies of the system are subjected to this one condition, that they all revolve round the sun in the same direction. This condition being satisfied by all the members of the solar system, it follows that the orbits of the planets will never be very eccentrical or much inclined to each other by reason of their mutual attraction. The important truths in relation to the forms and positions of the planetary orbits are embodied in the two following theorems by the author of the Mécanique Céleste: I. If the mass of each planet be multiplied by the product of the square of the eccentricity and square root of the mean distance, the sum of all these products will aloays retain the same magnitude. II. If the mass of each planet be multiplied by the product of the square of the inclination of the orbit and the square root of the mean distance, the sum of these products will always remain invariable. Now, these quantities being computed for a given epoch, if their sum is found to be small, it follows from the preceding theorems that they will always remain so; consequeqtly the eccentricities and inclinations cannot increase indefinitely, but will always be confined within narrow limits.
In order to calculate the limits of the variations of the elements with precision, it is necessary to know the correct values of the masses of all the planets. Unfortunately, this knowledge has not yet been attained. The masses of several of the planets are found to be considerably different from the values employed by La Grange in his investigations. Besides, he only took into account the action of the six principal planets which are within the orbit of Uranus. Consequently his solution afforded only a first approximation to the limits of the secular variations of the elements.
The person who next undertook the computation of the secular inequalities was Pontécoulant, who, about the year 1834, published the third volume of his Theorie Analytique du Systéme du Monde. In this work he has given the results of his solution of this intricate problem. But the numerical values of the constants which he obtained are totally erroneous on account of his failure to employ a sufficient number of decimals in his computation. Our knowledge of the secular variations of the planetary orbits was, therefore, not increased by his researches.
In 1839 Le Verrier had completed his computation of the secular inequalities of the seven principal planets. This mathematician has given a new and accurate determination of the constants on which the amount of the secular inequalities depend; and has also given the coefficients for correcting the values of the constants for differential variations of the
masses of the different planets. This investigation of Le Verrier's has been used as the groundwork of most of the subsequent corrections of the planetary elements and masses, and deservedly holds the first rank as authority concerning the secular variations of the planetary orbits. But Le Verrier's researches were far from being exhaustive, and he failed to notice some curious and interesting relations of a permanent character in the secular variations of the orbits of Jupiter, Saturn, and Uranus. Besides, the planet Neptune had not then been discovered; and the action of this planet considerably modifies the secular inequalities which would otherwise take place. We will now briefly glance at the results of our own labors on the subject.

On the first examination of the works of those authors who had investigated this problem, we perceived that the methods of reducing to numbers those analytical integrals which determine the secular variations of the elements, were far from possessing that elegance and symmetry of form which usually characterizes the formulas of astronomy. The first step, therefore, was to devise a system of algebraic equations, by means of which we should be enabled to obtain the values of the unknown quantities with the smallest amount of labor. It was soon found to be impracticable to deduce algebraic formulas for the constants, by the elimination of eight unknown quantities from as many linear symmetrical equations, of sufficient simplicity to be used in the deduction of exact results. It therefore became necessary to abandon the idea of a direct solution of the equations, and to seek for the best approximative method of obtaining rigorous values of the unknown quautities. This we have accomplished as completely as could be desired, and by means of the formulas which we have obtained, it is now possible to determine the secular variations of the planetary elements, with less labor, perhaps, than would be necessary for the accurate determination of a comet's orbit. The method and formulas are given in detail in a Memoir on the Secular Variations of the Elements of the Orbits of the Eight Principal Planets, now being published in vol. XVIII, of the Smithsonian Contributions to Knoucledge.

After computing anew the numerical coefficients of the differential equations of the elements, we have substituted them in these equations, and have obtained, by means of successive approximations, the rigorous values of the constants corresponding to the assumed masses and elements. The details of the computation are giveu in the memoir referred to, and it is unnecessary to speak of them here. We shall, therefore, only briefly allude to some of the conclusions to which our computations legitimately lead.

The object of our investigation has been the determination of the numerical values of the secular changes of the elements of the planetary orbits. These elements are four in number, viz: the eccentricities and inclinations of the orbits, and the longitudes of the nodes and perihelia. The questions that may legitimately arise in regard to the
eccentricities and inclinations relate chiefly to their magnitude at any time; but we may also desire to know their rates of change at any time, and the limits within which they will perpetually oscillate. In regard to the nodes and peribelia, it is sometimes necessary to know their relative positions when referred to any plane and origin of coördinates; and also their mean motions, together with the amount by which their actual places can differ from their mean places. With respect to the magnitudes and positions of the elements, together with their rates of change, we may observe that our equations will give them for any required epoch, by merely substituting in the formulas the interval of time between the epoch required and that of the formulas, which is the beginning of the year 1850. An extended tabulation of the variations of the elements does not come within the scope of our work; and we leave the computation of the elements for special epochs to those investigators who may need them in their researches. We shall here give the limits between which the eccentricities and inclinations will always oscillate, together with the mean motions of the perihelia and nodes on the fixed ecliptic of 1850 ; and shall also give the inclinations and nodes referred to the invariable plane of the planetary system.
For the planet Mercury, we find that the eccentricity is al ways included within the limits 0.1214943 and 0.2317185 . The mean motion of its perihelion is $5^{\prime \prime} .463803$; and it performs a complete revolution in the heavens in 237,197 years. The maximum inclination of his orbit to the fixed ecliptic of 1850 is $10^{\circ} 36^{\prime} 20^{\prime \prime}$, and its minimum inclination is $3^{\circ} 47^{\prime} 8^{\prime \prime}$; while with respect to the invariable plane of the planetary system, the limits of the inclination are $9^{\circ} 10^{\prime} 41^{\prime \prime}$ and $4^{\circ} 44^{\prime} 27^{\prime \prime}$. The mean motion of the node of Mercury's orbit on the ecliptic of 1850, and on the invariable plane, is in both cases the same, and equal to $5^{\prime \prime} .126172$, making a complete revolution in the interval of 252,823 years. The amount by which the true place of the node can differ from its mean place on the ecliptic of 1850 is equal to $33^{\circ} 8^{\prime}$, while on the invariable plane this limit is only $18^{\circ} 31^{\prime}$.
For the planet Venus, we find that the eccentricity always oscillates petween 0 and 0.0706329 . Since the theoretical eccentricity of the orbit of Venus is a vanishing element, it follows that the perihelion of her orbit can have no mean motion, but may have any rate of motion, at different times, between nothing and infinity, both direct and retrograde. The position of her perihelion cannot therefore be determined within given limits at any very remote epoch by the assumption of any particular value for the mean motion, but it must be determined by direct computation from the finite formulas. The maximum inclination of her orbit to the ecliptic of 1850 is $4^{\circ} 51^{\prime}$, and to the invariable plane it is $3^{\circ} 16^{\prime} .3$; while the mean motion of her node on both planes is indeterminate, because the inferior limit of the inclination is in each case equal to nothing.

A knowledge of the elements of the earth's orbit is especially interesting and important on account of the recent attempts to establish a
connection between geological phenomena and terrestrial temperatures, in so far as the latter is modified by the variable eccentricity for her orbit. The amount of light and heat received from the sun in the course of a year depends to an important extent on the eccentricity of the earth's orbit; but the distribution of the same over the surface of the earth depends on the relative position of the perihelion of the orbit with respect to the equinoxes, and on the obliquity of the ecliptic to the equator. These elements are subject to great and irregular variations; but their laws can now be determined with as much precision as the exigencies of science may require. We will now more carefully examine these elements, and the consequences to which their variations give rise.

As we have already computed the eccentricity of the earth's orbit at intervals of 10,000 years, during a period of $2,000,000$ years, by employing the constants which correspond to the assumed mass of the earth increased by its twentieth part, we here give the elements corresponding to this increased mass. We therefore find that the eccentricity of the earth's orbit will alwass be included within the limits of 0 and 0.0693888 ; and it consequently follows that the mean motion of the perihelion is indeterminate, although the actual motion and position at any time during a period of $2,000,000$ years can be readily found by means of the tabular value of that element. The eccentricity of the orbit at any time can also be found by means of the same table.

The inclination of the apparent ecliptic to the fixed ecliptic of 1850 , is always less that $4^{\circ} 41^{\prime}$; while its inclination to the invariable plane of the planetary system always oscillates within the limits $0^{\circ} 0^{\prime}$ and $3^{\circ} 6^{\prime}$. It is also evident that the mean motion of the node of the apparent ecliptic on the fixed ecliptic of 1850 , and also on the invariable plane, is wholly indeterminate.
The mean value of the precession of the equinoxes on the fixed eclip. tic, and also on the apparent ecliptic, in a Julian year, is equal to $50^{\prime \prime} .438239$; whence it follows, that the equinoxes perform a complete revolution in the heavens in the average interval of $25,694.8$ years; but on account of the secular inequalities in their motion, the time of revolution is not always the same, but may differ from the mean time of revolution by 281.2 years. We also find that if the place of the equinox be computed for any time whatever, by using the mean value of precessiou, its place when thus determined can never differ from its true place to a greater extent than $3056^{\prime} 26^{\prime \prime}$. The maximum and minimum values of precession in a Julian year are $52^{\prime \prime} .664080$ and $48^{\prime \prime} .212398$, respectively, and since the length of the tropical year depends on the annual precession, it follows that the maximum variation of the tropical year is equal to the mean time required for the earth to describe an are which is equal to the maximum variation of precession. Now this latter quantity being $4^{\prime \prime} .451682$, and the sidereal motion of the earth in a second of time being $0^{\prime \prime} .041067$, it follows that the maximum variation of the tropical year is equal to $\mathbf{1 0 8 . 4 0}$ seconds of time. In like manner, if we take
the difference between the present value of precession and the maximum and minimum values of the same quantity, we shall find that the tropical year may be shorter than at present by 59.13 seconds, and longer than at present by 49.27 seconds. We also find that the tropical year is now shorter than in the time of Hipparchus, by $\mathbf{1 1 . 3 0}$ seconds.
The obliquity of the equator to the apparent ecliptic, and also to the fixed ecliptic of 1850 , has also been determined. The variations of this element follow a law similar to that which governs the variation of precession, although the maximum values of the inequalities are considerably smaller than those which affect this latter quantity. The mean value of the obliquity of both the apparent and fixed ecliptics to the equator is $23^{\circ} 17^{\prime} 17^{\prime \prime}$. The limits of the obliquity of the apparent ecliptic to the equator are $24^{\circ} 35^{\prime} 58^{\prime \prime}$ and $21^{\circ} 58^{\prime} 36^{\prime \prime}$; whence it follows that the greatest and least declinations of the sun at the solstices can never differ from each other to anry greater extent than $2^{\circ} 37^{\prime} 22^{\prime \prime}$. And here we may mention a few, among the many happy consequences, which result from the spheroidal form of the earth. Were the earth a perfect sphere there would be no precession or change of obliquity arising from the attraction of the sun and moon; the equinoctial circle would form an invariable plane in the heavens, about which the solar orbit would revolve with an inclination varying to the extent of twelve degrees, and a motion equal to the planetary precession of the equinoctial points. The sun, when at the solstices, would, at some periods of time, attain the declination of $29^{\circ} 17^{\prime}$, for many thousands of years; and again, at other periods, only to $17^{\circ} 17^{\prime}$. The seasons would be subject to vicissitudes depending on the distance of the tropics from the equator, and the distribution of solar light and heat on the surface of the earth would be so modified as essentially to change the character of its vegetation, and the distribution of its animal life. But the spheroidal form of the earth so modifies the secular changes in the relative positions of the equator and ecliptic that the inequalities of precession and obliquity are reduced to less than onequarter part of what they would otherwise be. The periods of the secular changes, which, in the case of a spherical earth, would require nearly two millions of years to pass through a complete cycle of values, are now reduced to periods which vary between 26,000 and 53,000 years. The secular motions which would take place in the case of a spherical earth are so modified by the actual condition of the terrestrial globe that changes in the position of the equinox and equator are now produced in a few centuries, which would otherwise require a period of many thousands of years. This consideration is of much importance in the investigation of the reputed antiquity and chronology of those ancient nations which attained proficiency in the science of astronomy, and the records of whose astronomical labors are the only remaining monument of a highly intellectual people, of whose existence every other trace has long since passed away. For it is evident that, if these changes were much slower than they are, a much longer time would be required in order to produce changes of sufficient magnitude to be detected by observation,
and we should be unable to estimate the interval between the epochs of elements which differed by only a few thousand years, since they would manifestly be so nearly identical with our own that the value of legitimate conclusions would be greatly impaired by the unavoidable errors of the observations on which they were based.

The duration of the different seasons is also greatly modified by the eccentricity of the earth's orbit. At present the sun is north of the equator scarcely $186 \frac{1}{2}$ days, and south of the same circle about $178 \frac{3}{4}$ days; thus making a difference of $7 \frac{3}{4}$ days between the length of the summer and winter at present. But when the eccentricity of the orbit is nearly at its maximum, and its transverse axis also passes through the solstices, both of which conditions have, in past ages, been fulfilled, the summer, in one hemisphere, will have a period of $198 \frac{3}{4}$ days, and a winter of only $166 \frac{1}{2}$ days, while, in the other hemisphere, these conditions will be reversed; the winter having a period of $198 \frac{3}{4}$ days, and a summer of only 166 $\frac{1}{2}$ days. The variations of the sun's distance from the earth in the course of a year, at such times, is also enormous, amounting to almost one-seventh part of its mean distance-a quantity scarcely less than $13,000,000$ of miles !

Passing now to the consideration of the elements of the planet Mars, we find that the eccentricity of his orbit always oscillates within the limits 0.018475 and 0.139655 ; and the mean motion of his peribelion is $17^{\prime \prime} .784456$. The maximum inclination of his orbit to the fixed ecliptic of 1850 , and to the invariable plane of the planetary system, is $7028^{\prime}$ and $5^{\circ} 56^{\prime}$ respectively. The minimum inclination to both planes being nothing, the mean motion of the node is indeterminate.

The secular variations of the orbits of Jupiter, Saturn, Uranus, and Neptune, present some curious and interesting relations. These four planets compose a system by themselves, which is practically independent of the other planets of the system.

The maximum and minimum limits of the eccentricity of the orbits of these four planets are as follows:

Maximum eccentricity. Minimum eccentricity.

| Jupiter | 0.0608274 | 0.0254928 |
| :---: | :---: | :---: |
| Saturn. | . 0.0843289 | 0.0123719 |
| Uranus | 0.0779652 | 0.0117576 |
| Neptune | 0.0145066 | 0.0055729 |

The maximum and minimum inclinations of their orbits to the invariable plane of the planetary system have the following values:

$$
\text { Maximum inclination. } \quad \text { Minimum inclination. }
$$

Jupiter . . . . . . . . . . . . . . . $0^{0} 28^{\prime} 56^{\prime \prime} \ldots . .$. . . . . . . . . . . . . . . . . . $0^{\circ} 14^{\prime} 23^{\prime \prime}$
Saturn ................... 1 . 039 ................................ 0.0 . 4716
Uranus................... 1 1 10 ................................. 0 . 5425
Neptune !............... $0 \quad 47 \quad 21$................................ $0 \quad 0 \quad 33 \quad 43$

The perihelia and nodes of their orbits have the following mean motions in a Julian year of $365 \frac{1}{4}$ days:

Mean motion of perihelion. Mean motion of node on the invariable plane.


But the most curious relation developed by this investigation pertains to the relative motions and positions of the perihelia and nodes of the three planets Jupiter, Saturn, and Uranus. These relations are expressed by the two following theorems:
I. The mean motion of Jupiter's perihelion is exactly equal to the mean motion of the perihelion of Uranus, and the mean longitudes of these perihelia differ by exactly 1800. II. The mean motion of Jupiter's node on the invariable plane is exactly equal to that of Saturn, and the mean longitudes of these nodes differ by exactly $180^{\circ}$.

We also perceive that the mean motion of Saturn's perihelion is very nearly six times that of Jupiter and Uranus, and this latter quantity is very nearly six times that of Neptune; or, more exactly, 985 times the mean motion of Jupiter's perihelion is equal to 163 times that of Saturn, and 440 times the mean motion of Neptune's perihelion is equal to 73 times that of Jupiter and Uranus. The peribelion of Saturn's orbit performs a complete revolution in the heavens in 57,700 years; the perihelia of Jupiter and Uranus in 348,700 years; while that of Neptune requires no less that $2,101,560$ years to complete the circuit of the heavens. In like manner the nodes of Jupiter and Saturn, on the invariable plane, perform a complete revolution in 49,972 years ; that of Uranus in 444,432 years; while the node of Neptune requires $1,958,692$ years to traverse the circuinference of the heavens. The motions of the nodes are retrograde and those of the peribelia are direct; thus conforming to the same law of variation as that which obtains in the corresponding elements of the moon's motion.
We may here observe that the law which controls the motions and positious of the perihelia of the orbits of Jupiter and Uranus is of the utmost importance in relation to their mutual perturbations of Saturn's orbit. For, in the existing arrangement, the orbit of Saturn is affected only by the difference of the perturbations by Jupiter and Uranus; whereas, if the mean places of the perihelia of these two planets were the same, instead of differing by $180^{\circ}$, the orbit of Saturn would be affected by the sum of their disturbing forces. But notwithstanding this favoring condition, the elements of Saturn's orbit would be subject to very great perturbations from the superior action of Jupiter, were it not for the comparatively rapid motion of its perihelion; its equilibrium being maintained by the very act of perturbation. Indeed, the stability of Saturn's orbit depends to a very great extent upon the rapidly varying positions
of its transverse axis. For, if the motions of the perihelia of Jupiter and Saturn were very nearly the same, the action of Jupiter on the eccentricity of Saturn's orbit would be at its maximum value during very long periods of time, and thereby produce great and permanent changes in the value of that element. But, in the existing conditions, the rapid motion of Saturn's orbit prevents such an accumulation of perturbation, and any increase of eccentricity is soon changed into a corresponding diminution. The same remark is also applicable to the perturbations of the forms of the orbits of Jupiter and Uranus by the disturbing action of Saturn; for the secular variations of Jupiter's orbit depend almost entirely upon the influence of Saturn, because the planet Neptune is too remote to produce much disturbance, and the mean disturbing influence of Uranus on the eccentricity of Jupiter's orbit is identically equal to nothing, by reason of the relation which always exists between the perihelia of their orbits. We may here observe that the eccentricity of the orbit of Saturn always increases, while that of Jupiter diminishes, and vice versa.
The consequences which result from the mutual relations which always exist between the nodes of Jupiter and Saturn, on the invariable plane of the planetary system, are no less interesting or remarkable with respect to the position of the orbit of Uranus than those which result from the permanent relation between the perihelia of Jupiter and Uranus are with respect to the form of the orbit of Saturn. The mean disturbing force of Saturn on the inclination of the orbit of Uranus is about four times that of Jupiter; but as these two planets always act ou the inclinations in opposite directions, it follows that the joint action of the two planets is equivalent to the action of a single planet at the distance of Saturn and having about three-fourths of his mass; so that the orbit of Uranus might attain a considerable inclination from the superior action of Saturn if allowed to accumulate during the lapse of an unlimitéd time, at its maximum rate of variation depending on the action of this planet. But such an accumulation of perturbation is rendered forever impossible by reasou of the comparatively rapid motion of the nodes of Jupiter and Saturn, with respect to that of Uranus, on the invariable plane. By reason of this rapid motion, the secular changes of the inclination of the orbit of Uranus pass through a complete eycle of values in the period of 56,300 years. The corresponding cycle of perturbation in the eccentricity of Saturn's orbit is 69,140 years. It is the rapid motion of the orbit with respect to the forces in the one case, and the rapid motion of the forces with respect to the orbit, in the other, that gives permanence of form and position to the orbits of Saturn and Uranus.
The mean angular distance between the perihelia of Jupiter and Uranus is exactly $180^{\circ}$; but the conditions of the variations of these elements are sufficiently elastic to allow of a considerable deviation on each side of their mean positions. The perihelion of Jupiter may differ
from its mean place to the extent of $24^{\circ} 10^{\prime}$, and that of Uranus to the extent of $47 \circ 33^{\prime}$; and therefore the longitudes of the perihelia of these two planets can differ from $180^{\circ}$ to the extent of $71^{\circ} 43^{\prime}$. The nearest approach of the perihelia of these two planets, is, therefore, $108^{\circ} 17^{\prime}$.

In like manner the longitudes of the nodes of Jupiter and 'Saturn, on the invariable plane, can suffer considerable deviations from their mean positions. The actual position of Jupiter's node may differ from its mean place to the extent of $19^{\circ} 38^{\prime}$; while that of Saturn may deviate from its mean place to the extent of $707^{\prime}$. It therefore follows that their longitudes on the invariable plane can differ from $180^{\circ}$ by only $26^{\circ} 45^{\prime}$. Their nearest possible approach is $153^{\circ} 15^{\prime}$, while their present distance apart is $166^{\circ} 27^{\prime}$.
The inequalities in the eccentricity of Neptune's orbit are very small and the two principal ones have periods of 613,900 years, and 418,060 years, respectively. Strictly speaking, the periods of the secular inequalities of the eccentricities and perihelia are the same for all the planets; and the same remark is equally applicable to the nodes and inclinations. But the principal inequalities of the several planetary orbits are different, unless they are connected by some perinanent relation, similar to that which exists between the perihelia of Jupiter and Uranus, or the noles of Jupiter and Saturn. Thus the principal inequalities affecting the inclination of the orbits of Jupiter and Saturn have the same periods for each planet, and these periods are, for the two principal inequalities, 51,280 years, and 56,303 years. In like manner the principal inequalities in the eccentricities of Jupiter and Saturn depend on their mutual attraction, and have a period of 69,141 years. The secular inequalities of those orbits which have no vanishing elements are composed of terms, of very different orders of magnitude; and it frequently happens that two or three of these terms are greater than the sum of all the remaining ones. In such cases the variation of the corresponding element very approximately conforms to a much simpler law, and the maxima and minima repeat themselves according to definite and well-defined cycles. But with regard to the orbits of Venus, the Earth, and Mars, the rigorous expressions of the eccentricities and inclinations are composed of twenty-eight periodic terms, having coefficients of considerable magnitude; and this circumstance renders the law of their variations extremely intricate.

The method we have adopted for finding the coefficients of the cor. rections of the constants, depending on finite variations of the different planetary masses, consists in supposing that each planetary mass receives in succession a finite increment, and then finding the values of all the constants corresponding to this increased mass in the same manner as for the assumed masses. By this means we have a set of values corresponding to the assumed masses, and another set corresponding to
a finite increment to each of the planetary masses. Then, taking the difference between the two sets of constants, and dividing by the increment which produced it, we get the coefficient of the variation of the constants for any other finite increment of mass to the same planet; but, on account of the importance of the earth's mass, and the probable inaccuracy of its assumed value, we have prepared separate solutions corresponding to the several increments of $\frac{1}{20}, \frac{3}{20}$, and $\frac{3}{20}$ of its assumed mass; and a comparison of the values which the different solutions give for the superior limit of the eccentricity of the earth's orbit has suggested the inquiry whether there may not be some unknown physical relation between the masses and mean distances of the different planets. The same peculiarity in the elements of the orbit of Venus is also found to depend upon particular values of the mass of that planet. But without entering into details in regard to the peculiarity referred to, we here give the several values of the masses of these two planets which we have employed in our computations, and the corresponding values of the superior limit of the eccentricity of their orbits:

| Mass. | For Venus, maximum $e$ ' | Mass. | For the earth, maximum $e^{\prime \prime}$. |
| :---: | :---: | :---: | :---: |
| $m^{\prime}$ |  | $m^{\prime \prime}$ |  |
| $m^{m_{0}^{\prime}}{ }^{\prime}(1+1)$ | 0.074872 | $m^{\prime \prime}{ }_{0}$ | 0.067735 0.06339 |
| ${ }^{m^{\prime} 0}{ }^{\prime}\left(1+\frac{1}{20}\right)$ | 0.076075 | $m^{\prime \prime} 0_{0}\left(1+\frac{1}{20}\right)$ | 0.069389 0.066949 |
|  | 0.075092 0.072098 |  | ${ }_{0} 0.068089$ |
| $m_{0}(1+20)$ | 0.072998 | $m^{\prime \prime}{ }_{0}\left(1+\frac{3}{20}\right)$ |  |

These numbers show that if the mass of Venus were to be increased, the superior limit of the eccentricity of her orbit would also increase until it had attained a maximum value, atter which a further increase of her mass would diminish that limit; and the same remark is also applicable to the eccentricity of the earth's orbit.

The above numbers indicate that the superior limit of the eccentricity of the orbit of Venus is a maximum if the mass of that planet is equal to $m_{0}^{\prime}\left(1+\frac{2 \cdot 04}{20}\right)$, or, if $m^{\prime}=\frac{1}{354490}$ of the sun's mass; and the superior limit of the eccentricity of the earth's orbit is a maximum if the earth's mass is equal to $m^{\prime \prime}{ }_{0}\left(1+\frac{1.643}{20}\right)$, or, if $m^{\prime \prime}={ }_{560 \frac{1}{700}}$ of the sun's mass. But this value of the earth's mass corresponds to a solar parallax of $8^{\prime \prime} .730$, a value closely approximating to the recent determinations of that element.
If, then, the mass of Venus is equal to $\frac{\mathrm{r}}{3549 \mathrm{BD}}$, and the mass of the earth is equal to $\frac{{ }_{3} \frac{1}{07000}}{}$, it follows that the orbits of these two planets will ultimately become more eccentric from the mutual attraction of the other planets than they would for any other values of their respective masses; and we may now inquire whether such coincidence between
the superior limits of the eccentricities and the masses of these two planets has any physical significance, or is merely accidental.
Since the mean motions and mean distances of the planets are invariable, and independent of the eccentricities of the orbits, it would seem that there could be no connection between these elements by means of which the stability of the system might be secured or impaired; but a more careful examination shows that, although the mean motions or times of revolution of the planets are iuvariable, their actual relocities, or the variation of their mean velocities, depends wholly on the eccentricities; and were any of the planetary orbits to become extremely elliptical, the velocity of the planet would be subject to great variations of velocity, moving with very great rapidity when in peribelion, and with extreme slowness when in the neighborhood of its aphelion; and it is evident that when the planet was in this latter position a small foreign force acting upon it might so change its velocity as to completely change the elements of its orbit, by causing it to fall upon the sun or fly off into remoter space. A system of bodies moving in very eccentrical orbits is therefore one of manifest instability; and if it can also be shown that a system of bodies moving in circular orbits is one of unstable equilibrium, it would seem that, between the two supposed conditions a system might exist which should possess'a greater degree of stability than either. The idea is thus suggested of the existence of a system of bodies in which the masses of the different bodies are so adjusted to their mean distances as to insure to the system a greater degree of permanence than would be possible by any other distribution of masses. The mathematical expression of a criterion for such distribution of masses has not yet been fully developed; and the preceding illustrations have been introduced here, more for the purpose of calling the attention of mathematicians and astronomers to this interesting problem than for any certain light we have yet been able to obtain in regard to its solution.

# ON SOME METIIODS OF INTERPOLATION APPLICABLE TO THE GRADUATION OF IRREGULAR SERIES, SUCII AS TABLES OF MORTALITY, de., de. 

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#### Abstract

[The portions of the following methods of interpolation comprising the formulas 2 , 8 , A, B, C, D, E, F, $11,12,13,17,19,20,21,24,25,26,27,28,30,43,44,45,46,48,49$, and 50 , were presented to the Smithsonian Institution for publication in the year 1868. The method of constructing tables of mortality from two successive census enumerations was first given in January, 1869, and the formulas 40, 41, 42, 53, 54, 55, 56, and 59, in January, 1870.-J. H.]


We have no analytical formula which expresses the law of mortality with precision, and at the same time with such simplicity as to be practically useful. For all the purposes of life insurance and life annuities, it is expressed by numerical series. The law is known to vary in different localities, and even in the same locality at different epochs. That which prevails in any community, at a given period, can be ascertained by enumerating the persons living at the various ages, and the deaths which annually occur among them. Reduced to one of its usual forms, it is expressed in a statistical table, showing, out of a certain number of persons born, how many survive to complete each successive year of their age. These numbers of the living form a diminishing series of about one hundred terms, whose first differences are the numbers dying during each year of age. We have reason to believe that a true law of mortality is a continuous function of the age, free from sudden irregularities, so that in a perfect table the second, third, \&c., orders of differences of the series ought to go on continually diminishing, and each order by itself ought to show a certain degree of regularity; in other words, the table should be well graduated. But, in point of fact, all purely statistical tables are irregular, especially when the population observed has been small, and every table of mortality now in use has been graduated artificially. It was not strange that the Carlisle table, derived from records of population and deaths in a single town, should show many irregularities. They have been adjusted to some extent, but very imperfectly. The Combined Experience table, also, which was compiled from the records of seventeen British life insurance offices, owes its better graduation to art rather thau to nature. Farr's English life-table, No. 3, for males, derived from the census returns of 1841 and 1851, and from the registry of deaths in England and Wales
for the seventeen years from 1838 to 1854, though perhaps the best expression we have for the law of general mortality, is by no means well graduated. In this case the population observed was so large that if the tables had been formed directly from the enumeration of persons living and persons dying in each single year of age, and if these observations could have been relied upon as accurate, any irregularities then existing in the series might possibly have been thought to result irom something peculiar in the law of life at certain ages. But it was necessary to combine the single years of age into groups, owing to the impossibility of ascertaining ages with precision. All persons were required to give their exact ages at last birthday, but the reports state that round numbers, such as 50,60, \&c., were disproportionately numerous, showing that the ages were not always correctly given. In forming the life-table No. 3 the years of age were grouped together into decennial periods chiefly, and the whole term of life was then divided into five unequal parts, so as to form a chain of sub-series, each of the fourth order, and not continuous at their points of junction. We must conclude, then, that the great irregularities now found at certain points in the series result from imperfect distribution, and not from any irregularity in the true law of mortality.

A good system of distribution or adjustment, though not positively essential in practice, is nevertheless desirable, first, because a judiciously adjusted table probably comes nearer to the truth than an unadjusted or ill-adjusted one; that is, nearer to what the statistics would show if the population observed could be made indefinitely large, and if the numbers for each year of age could be independently determined. Secondly, if the primary table is well graduated, all the various series of numbers derived from it, forming the usual "commutation tables" and tables of premiums and valuations of assurances and annuities, will be well graduated also, and this will sometimes facilitate the computation of such tables, because a part of the tabular numbers can be accurately found by ordinary interpolation, and errors of computation can be discovered by the method of differences. Many writers on the law of mortality have treated of the subject of adjustment, as may be seen in the pages of the London Journal of the Institute of Actuaries and Assurance Magazine, and elsewhere. The rule of least squares was used to adjust the American table given in the report of the United States census of 1860. (See the Appendix on Average Rate of Mortality, pages 518 and 524.) The series there given, however, is not very thoroughly graduated, as can easily be shown by taking its successive orders of differences. In England, the "law of Gompertz" has been chiefly taken as a basis. But it is not necessary to adopt any exclusive theory respecting the precise nature of that function which expresses the law of mortality. The following system of distribution and graduation is based upon principles which apply to any continuous series of numbers, and is analogous to the ordinary methods of inter-
polation. It is not without interest when regarded from a purely mathematical point of view. The general question as to how an irregular series can be made regular is answered by means of the obvious principle that, although single terms in a series may deviate considerably from the normal standard, yet the arithmetical means of successive groups of terms will be less fluctuating, because the errors of the single terms which compose each group tend to compensate each other, and also because the means of two groups which are partly composed of the same terms must necessarily approximate toward each other as the number of terms common to both is increased. In ordinary interpolation, we proceed from some known single terms in a series to find the values of other terms; in the present case, however, all single terms are unreliable, and the problem is to determine the single terms in a series when only the arithmetical means of some groups of terms are given. To find expressions for the sum, and consequently the mean, of the terms in any group, we shall make use of the known principle that, in a continuous series whose law is given or assumed, the sum of a limited number of terms can be regarded as a definite integral, which is the aggregate of a succession of similar integrals corresponding to the terms considered.*

## FIRST METHOD OF ADJUSTMENT.

We know that when equidistant ordinates are drawn to the parabola-

$$
y=\mathrm{A}+\mathrm{B} x+\mathrm{C} x^{2}
$$

they form a series of the second order; that-is, their second differences are constant. Let $c$ represent the distance from one ordinate to another ; the area of the curve included between two such ordinates will be-

$$
\int_{x^{\prime}-\frac{1}{2} c}^{x^{\prime}+\frac{1}{c} c} y d x=c\left[\mathrm{~A}+\mathrm{B} x^{\prime}+\mathrm{C}\left(x^{\prime 2}+\frac{1}{12} c^{2}\right)\right]
$$

where $x^{\prime}$ is the abscissa corresponding to the middle ordinate of the area. Since this area is a function of the second degree in $x^{\prime}$, it follows that when values in arithmetical progression, such as $1,2,3, \& c$., are assigned to $x^{\prime}$, the resulting areas will form a series of the second order.
This being premised, let us assume any three areas, $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}$, so situated that the middle ordinates of $S_{1}$ and $S_{3}$ shall fall respectively to the left and right of the middle ordinate of $\mathrm{S}_{2}$, which is taken as the axis of Y. Let $n_{1}, n_{2}, n_{3}$, be the portions of the axis of X which form the bases of these areas, and let $a_{1}$ and $a_{3}$ be the portions of the same axis intercepted between the axis of $Y$ and the middle ordinates of $S_{1}$ and $S_{3}$ respectively. Then we have-

$$
\begin{aligned}
& \mathrm{S}_{1}=\int_{-a_{1}-\frac{1}{n}}^{-a_{1}+\frac{1}{1}} y d x=n_{1}\left[\mathrm{~A}-\mathrm{B} a_{1}+\mathrm{C}\left(a_{1}^{2}+\frac{1}{12} n_{1}^{2}\right)\right] \\
& \mathbb{S}_{2}=\int_{-\frac{1}{2} n_{2}}^{+\frac{1}{1} y} y d x=n_{2}\left(\mathrm{~A}+\frac{1}{12} \mathrm{C} n_{2}^{2}\right)
\end{aligned}
$$

[^30]$$
\mathrm{S}_{3}=\int_{a_{3}-\frac{1}{2} n_{3}}^{a_{3}+\frac{1}{2} n_{3}} y d x=n_{3}\left[\mathrm{~A}+\mathrm{B} a_{3}+\mathrm{C}\left(a_{3}{ }^{2}+\frac{1}{12} n_{3}{ }^{2}\right)\right]
$$

Let S be a fourth area whose base is $n$, and let $\dot{x}^{\prime}$ be the abscissa corresponding to its middle ordinate; then-

$$
\begin{equation*}
\mathrm{S}=\int_{x^{\prime}-\frac{1}{2} n}^{x^{\prime}+\frac{1}{n} n} y d x=n\left[\mathrm{~A}+\mathrm{B} x^{\prime}+\mathrm{C}\left(x^{\prime 2}+\frac{1}{12} n^{2}\right)\right] \ldots . \tag{1}
\end{equation*}
$$

Eliminating $\mathrm{A}, \mathrm{B}, \mathrm{C}$, from the above four equations employing $\mathrm{P}, \mathrm{Q}, \mathrm{R}$, as auxiliary letters, and dropping the accent from $x^{\prime}$, we have-

$$
\left.\begin{array}{l}
\mathrm{P}=a_{3}\left[x^{2}+\frac{1}{12}\left(n^{2}-n_{2}{ }^{2}\right)\right]-x\left[a_{3}{ }^{2}+\frac{1}{12}\left(n_{3}{ }^{2}-n_{2}{ }^{2}\right)\right] \\
\mathrm{Q}=a_{1}\left[x^{2}+\frac{1}{12}\left(n^{2}-n_{2}{ }^{2}\right)\right]+x\left[a_{1}{ }^{2}+\frac{1}{12}\left(n_{1}{ }^{2}-n_{2}{ }^{2}\right)\right] \\
\mathrm{R}=a_{3}\left[a_{1}{ }^{2}+\frac{1}{12}\left(n_{1}{ }^{2}-n_{2}{ }^{2}\right)\right]+a_{1}\left[a_{3}{ }^{2}+\frac{1}{12}\left(n_{3}{ }^{2}-n_{2}{ }^{2}\right)\right]  \tag{2}\\
\mathrm{S}=n\left[\left(1-\frac{\mathrm{P}+\mathrm{Q}}{\mathrm{R}}\right)\left(\frac{\mathrm{S}_{2}}{n_{2}}\right)+\left(\frac{\mathrm{P}}{\mathrm{R}}\right)\left(\frac{\mathrm{S}_{1}}{n_{1}}\right)+\left(\frac{\mathrm{Q}}{\mathrm{R}}\right)\left(\frac{\mathrm{S}_{3}}{n_{3}}\right)\right]
\end{array}\right\}
$$

This enables us to find the magnitude $S$ of an area whose position only is given, when the three other areas $S_{1}, S_{2}, S_{3}$, are given both in magnitude and position.

Now let each of the four areas be divided by equidistant ordinates into as many subdivisions as there are units in the bases $n_{1}, n_{2}, n_{3}$ and $n$ respectively, these bases being supposel to represent whole numbers, and let $a_{1}, a_{3}$, and $x$ be each a whole number or a whole number and a half, according as $n_{1}+n_{2}, n_{2}+n_{3}$, and $n_{2}+n$ are respectively even or odd; then all the subdivisions of the areas will be so situated that the abscissas corresponding to their middle ordinates will be terms in an arithmetical progression, and, consequently, the subdivisions themselves will be terms in a series of the second order. We may regard these subdivisions as representing not areas merely, but magnitudes of any kind, and the areas $S_{1}, S_{2}, S_{3}$, and S being the sums of groups of subdivisions, we see that formula (2) enables us to find the sum $S$ of ally group of consecutive terms in a series of the second order when the sums $S_{1}, S_{2}, S_{3}$, of the terms in any other three groups in the series are given. From the sums of the terms in each group their arithmetical means are known, and vice versa, for $n_{1}, n_{2}, n_{3}$, and $n$ are given, and these are the numbers of terms which the several groups contain. The groups may be entirely distinct, or they may overlap each other so that some terms belong to two or more of them at once. The intervals between the middle point of the group $\mathrm{S}_{2}$, and the middle points of the groups $\mathrm{S}_{1}, \mathrm{~S}_{3}$, and S are $a_{1}, a_{3}$, and $x$ respectively ; the interval between the middle points of any two consecutive terms being unity. We must regard $a_{1}$ and $a_{3}$ as always positive, while $x$ may be either positive or negative. When $n$ is made equal to unity, the formula gives the value of a single term $S$ by means of the sums $S_{1}, S_{2}, S_{3}$, of the three given groups of terms. The results are exact when the series taken is of the second order, but if it follows some other law, or is irregular, approximate or adjusted values for S will be obtained, and if the same groups
are constantly used as data, the single terms interpolated from them will themselves form a series of the second order. Assuming any three groups of terms in any given series, regular or irregular, we can construct a new series of the second order, such that the arithmetical means of the terms in the three corresponding groups in it shall be severally equal to those in the given series.

In the special case in which the three groups are consecative, and contain $n_{1}$ terms each, taking formula (1), which expresses the sum $S$ of any $n$ terms in a group, the abscissa of the middle point of the group being $x^{\prime}$, we may assign to $x^{\prime}$ its three values $-n_{1}, 0$, and $+n_{1}$ in succession, obtaining the three equations-

$$
\begin{aligned}
& \mathrm{S}_{1}=n_{1}\left(\mathrm{~A}-\mathrm{B} n_{1}+\frac{13}{12} \mathrm{C} n_{1}{ }^{2}\right) \\
& \mathrm{S}_{2}=n_{1}\left(\mathrm{~A}+\frac{1}{12} \mathrm{C} n_{1}{ }^{2}\right) \\
& \mathrm{S}_{3}=n_{1}\left(\mathrm{~A}+\mathrm{B} n_{1}+\frac{13}{12} \mathrm{O} n_{1}{ }^{2}\right)
\end{aligned}
$$

These suffice to determine the three constants $\mathrm{A}, \mathrm{B}, \mathrm{C}$; and dropping the accent from $x^{\prime}$ in (1), we have-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{24 n_{1}}\left[2 \dot{6} \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)\right] \\
\mathrm{B}=\frac{1}{2 n_{1}{ }^{2}}\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) \\
\mathrm{C}=\frac{1}{2 n_{1}{ }^{[ }}\left[\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)-2 \mathrm{~S}_{2}\right] \\
\mathrm{S}=n\left(\mathrm{~A}+1^{\frac{1}{2}} \mathrm{C} n^{2}+\mathrm{B} x+\mathrm{C} x^{2}\right)
\end{array}\right\} \text { (A) }
$$

This can be used in place of the more general formula (2), in all cases where the three groups are consecutive and of equal extent.
We have here a means of approximating to the population living within each single year of age when the statistics are given by decades or other intervals of age, as is often the case in census reports. If we take $n_{1}=10$, and let $u$ represent what S becomes when $n=1$, then formula (A) will reduce to-

$$
\begin{equation*}
u=\frac{1}{8000}\left[866 \mathrm{~S}_{2}-33\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)+40\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) x+4\left(\mathrm{~S}_{1}+\mathrm{S}_{3}-2 \mathrm{~S}_{2}\right) x^{2}\right] . . \tag{3}
\end{equation*}
$$

If, for example, $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}$ are the numbers aged 30 and under 40,40 and under 50,50 and under 60 , respectively, then giving $x$ the values $-\frac{1}{2},+\frac{1}{2},+\frac{3}{2}, \& c$., in succession, the resulting values of $u$ will be the numbers aged 44 and under 45,45 and under 46,46 and under 47 , \&c. If instead of taking $n=1$ we take $n=\frac{1}{2}$ or $n=\frac{1}{4}$, then by assigning the proper values to $x$ we may find the population living within any desired half year or quarter of a year of age. (See Milne on Annuities, Vol. 1, Ch. 3.) The same formula (3) enables us to distribute among the single years of age the deaths which occur within any three consecutive decades of age during a given period of time. If the population or deaths ,were thus distributed within every decade by means of the totals for
that decade and the two others nearest to it, the result would be a chain of sub-series of the second order extending throughout the term of life, but not forming a well-graduated series, because in general it would not be continuous at the points of junction between the decades. It might, however, be made approximately continuous afterward by means of the second method of adjustment, which will soon be explained. We must observe, too, that at the ages before 20 or after 80 the population and deaths vary so rapidly, that, in order to secure a good distribution by these methods, the data for those ages ought to be given by intervals of five years, or some other number less than a decade. In the ages of infancy they should be given for each single year.

Reverting now to the general formula (2), we observe that the quantities $\cdot \frac{S}{n}, \frac{S_{1}}{n_{1}}, \frac{S_{2}}{n_{2}}, \frac{S_{3}}{n_{3}}$, are the mean values of the ordinate within the several areas, so that the formula enables us not only to interpolate the arithmetical mean of a group of $n$ terms in a series when the means of the terms in three other groups are known, but also to interpolate the mean value of a function within any interval $n$ when its mean values within three other intervals $n_{1}, n_{2}, n_{3}$, are known; so that if we know the mean annual rate of mortality for three consecutive decades of age, we can find the mean rate for each single year of age by formula (3), since $\mathbf{S}_{1}, \mathbf{S}_{2}, \mathbf{S}_{3}$, are simply ten times the given mean rate for their respective decades.

When any one of the intervals $n_{1}, n_{2} ; n_{3}$ or $n$ is diminished, the mean value of the ordinate within such interval will evidently approximate to the value of the middle ordinate of the interval, and will become equal to it at the limit, when the interval becomes zero. Hence, making $n=0$, we have $\frac{S}{n}$ for the ordinate corresponding to the abscissa $x$, and (2) becomes-


When $S_{1}, S_{2}, S_{3}$, denote the population living within given intervals of age, the area $y d x$ may be regarded as denoting the number living at the exact age indicated by $x$, and if the population is a stationary onethat is, neither increasing nor diminishing, the product $n^{\prime} y$ will represent the number of persons who attain that exact age during the interval of time $n^{\prime}$; so that when the ages are grouped by decades, and we have $n=0$, formula (A) will give for the number of persons who annually attain the age indicated by $x$, since $n^{\prime}$ is unity, .
$y=\frac{1}{6000}\left[650 \mathrm{~S}_{2}-25\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)+30\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) x+3\left(\mathrm{~S}_{1}+\mathrm{S}_{3}-2 \mathrm{~S}_{2}\right) x^{2}\right]$.
For example, when $S_{1}, S_{2}, S_{3}$, denote the population aged 30 and under 40,40 and under 50,50 and under 60 , respectively, if we assign to $x$ the values $-1,0,+1, \& c$., in succession, the resulting values of $y$ will be the numbers annually attaining the ages $44,45,46, \& c$. It has usually been the practice to consider these numbers as being represented by
the population living between the ages $43 \frac{1}{2}$ and $44 \frac{1}{2}, 44 \frac{1}{2}$ and $45 \frac{1}{2}, 45 \frac{1}{2}$ and $46 \frac{1}{2}$, \&c., respectively, and a comparison of formulas (3) and (5) shows that the two sets of numbers would be almost identical, though not precisely so. The difference between them is-

$$
y-u=\frac{1}{24000}\left(2 \mathrm{~S}_{2}-\mathrm{S}_{1}-\mathrm{S}_{3}\right)
$$

a number so small that it will not ordinarily affect the first five significant figures of a result.

A considerably larger error is involved in the assumption that the ratio of the deaths annually occurring within any decade of age to the population living within such decade represents the annual rate of mortality at the exact middle age of the decade. (Assur. Mag., Vol.IX, p.125.)

Let $\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}$, be the deaths, and $\mathbb{S}_{1}, S_{2}, S_{3}$, the population, for any three consecutive decades, then the deaths annually occurring at the exact middle age of the middle decade are, by formula (5), making $x=0$,

$$
y d x=\frac{3}{240}\left[26 s_{2}-\left(s_{1}+s_{3}\right)\right] d x
$$

and the population living at the same age is,

$$
\mathrm{Y} d x=\frac{1}{24} \sigma\left[26 \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)\right] d x
$$

so that the annual rate of mortality at that exact age is,

$$
\begin{equation*}
\frac{y}{Y}=\frac{26 s_{2}-\left(s_{1}+s_{3}\right)}{26 \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)} \tag{6}
\end{equation*}
$$

The difference between this and the assumed value $\frac{\delta_{2}}{\mathcal{S}_{2}}$ is sufficient to alter the fourth significant figure of the quotient, and even the second and third at the older ages, as can easily be verified by assigning to $\mathbb{S}_{1}$, $s_{1}$, \&c., the numerical values for the various decades given by their $\log$ arithms in Table III of the Preface to the English Life Tables.

As regards the general accuracy of interpolations made by formula (2), it must be noted that near the middle point of the middle interval $n_{2}$ the values obtained will be more accurate than they will be at its extremities, and the accuracy attainable will diminish as we proceed out of the middle interval into either of the lateral ones. This is analogous to what we know to be the case with ordinary interpolations by second differences. And just as the degree of accuracy is increased by taking third differences into account, so here we can increase it by taking four intervals instead of three. This will give a curve of the third degree, which admits a point of inflexion, and is, therefore, better adapted than the common parabola to represent the form of a series whose second difference changes its sign.

For the sake of simplicity, let us assume that the four areas $S_{1}, S_{2}, \mathbb{S}_{3}$, $\mathbf{S}_{4}$, are symmetrically arranged with respect to the axis of $\mathbf{Y}$, so that the distances from the middle ordinates of $S_{1}$ and $S_{4}$ to that axis shall be each equal to $a_{1}$, and the corresponding distances for $S_{2}$ and $S_{3}$ each equal to $a_{2}$, while the bases of the first and fourth areas are each equal to $n_{1}$, and those of the second and third are each equal to $n_{2}$. Then taking the curve-

$$
y=\mathbf{A}+\mathbf{B} x+\mathbf{C} x^{2}+\mathbf{D} x^{3}
$$

we obtain the integral -

$$
\begin{equation*}
\mathrm{S}=\int_{x-\frac{1}{2} n}^{x+\frac{1}{2} n} y d x=n\left[\mathrm{~A}+\mathrm{B} x+\mathrm{C}\left(x^{2}+\frac{1}{12} n^{2}\right)+\mathrm{D} x\left(x^{2}+\frac{1}{4} n^{2}\right)\right] \tag{7}
\end{equation*}
$$

which expresses the sum $S$ of any $n$ terms taken in a group, the abscissa of the middle point of the group being $x$. Substituting for $n$ the four values $n_{1}, n_{2}, n_{2}, n_{1}$, in succession, and for $x$ the four corresponding values $-a_{1},-a_{2},+a_{2}$, and $+a_{1}$, we obtain the four equations-

$$
\begin{aligned}
& \mathrm{S}_{1}=n_{1}\left[\mathbf{A}-\mathbf{B} a_{1}+\mathbf{C}\left(a_{1}{ }^{2}+\frac{1}{12} n_{1}{ }^{2}\right)-\mathbf{D} a_{1}\left(a_{1}{ }^{2}+\frac{1}{4} n_{1}{ }^{2}\right)\right] \\
& \mathrm{S}_{2}=n_{2}\left[\mathrm{~A}-\mathrm{B} a_{2}+\mathrm{C}\left(a_{2}{ }^{2}+\frac{1}{12} n_{2}{ }^{2}\right)-\mathrm{D} a_{2}\left(a_{2}{ }^{2}+\frac{1}{4} n_{2}{ }^{2}\right)\right] \\
& \mathrm{S}_{3}=\dot{n}_{2}\left[\mathrm{~A}+\mathrm{B} a_{2}+\mathrm{C}\left(a_{2}{ }^{2}+\frac{1}{12} n_{2}{ }^{2}\right)+\mathrm{D} a_{2}\left(a_{2}{ }^{2}+\frac{1}{4} n_{2}{ }^{2}\right)\right] \\
& \mathrm{S}_{4}=n_{1}\left[\mathrm{~A}+\mathrm{B} a_{1}+\mathrm{C}\left(a_{1}^{2}+\frac{1}{12} n_{1}^{2}\right)+\mathrm{D} a_{1}\left(a_{1}^{2}+\frac{1}{4} n_{1}^{2}\right)\right]
\end{aligned}
$$

These are sufficient to determine the four constants $A, B, C, D$, and, arranging (7) according to the powers of $x$, we have-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{2 n_{1} n_{2}}\left(\frac{n_{1}\left(12 a_{1}{ }^{2}+n_{1}{ }^{2}\right)\left(\mathrm{S}_{2}+\mathrm{S}_{3}\right)-n_{2}\left(12 a_{2}{ }^{2}+n_{2}{ }^{2}\right)\left(\mathrm{S}_{1}+\mathrm{S}_{4}\right)}{12\left(a_{1}{ }^{2}-a_{2}^{2}\right)+n_{1}^{2}-n_{2}^{2}}\right) \\
\mathrm{B}=\frac{1}{2 a_{1} a_{2} n_{1} n_{2}}\left(\frac{a_{1} n_{1}\left(4 a_{1}^{2}+n_{1}^{2}\right)\left(\mathrm{S}_{3}-\mathrm{S}_{2}\right)-a_{2} n_{2}\left(4 a_{2}{ }^{2}+n_{2}{ }^{2}\right)\left(\mathrm{S}_{4}-\mathrm{S}_{1}\right)}{4\left(a_{1}^{2}-a_{2}{ }^{2}\right)+n_{1}{ }^{2}-n_{2}^{2}{ }^{2}}\right)  \tag{8}\\
\mathrm{C}=\frac{6}{n_{1} n_{2}}\left(\frac{n_{2}\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)-n_{1}\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)}{12\left(a_{1}{ }^{2}-a_{2}{ }^{2}\right)+n_{1}{ }^{2}-n_{2}^{2}}\right) \\
\mathrm{D}=\frac{2}{a_{1} a_{2} n_{1} n_{2}}\left(\frac{a_{2} n_{2}\left(\mathrm{~S}_{4}-\mathrm{S}_{1}\right)-a_{1} n_{1}\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)}{4\left(a_{1}^{2}-a_{2}^{2}\right)+n_{1}^{2}-n_{2}^{2}}\right) \\
\mathrm{S}=n\left[\left(\mathrm{~A}+\frac{1}{12} \mathrm{C} n^{2}\right)+\left(\mathrm{B}+\frac{1}{4} \mathrm{D} n^{2}\right) x+\mathrm{C} x^{2}+\mathrm{D} x^{3}\right]
\end{array}\right)
$$

This formula enables us to interpolate the sum $S$ of any $n$ terms in a group precisely as (2) does, but more accurately. It gives exact results when the series taken is of an order not higher than the third, and approximate or adjusted ones in other cases. With any given series, taking four groups of terms symmetrically situated on each side of a middle point which becomes the origin of coorrdinates, we can construct a new series of the third order, such that the arithmetical means of the terms in the four corresponding groups in it shall be equal to those in the given series. If the four groups are consecutive and contain $n_{1}$ terms each, we have-

$$
a_{1}=\frac{3}{2} n_{1}, \quad a_{2}=\frac{1}{2} n_{1}
$$

and the constants reduce to-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{12 n_{1}}\left[7\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{4}\right)\right]  \tag{B}\\
\mathrm{B}=\frac{1}{12 n_{1}{ }^{2}}\left[15\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{4}-\mathrm{S}_{1}\right)\right] \\
\mathrm{C}=\frac{1}{4 n_{1}^{3}}\left[\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)-\left(\mathrm{S}_{2}+\mathrm{S}_{3}\right)\right] \\
\mathrm{D}=\frac{1}{6 n_{1}^{4}}\left[\left(\mathrm{~S}_{4}-\mathrm{S}_{1}\right)-3\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)\right]
\end{array}\right\}
$$

When the sums $S_{1}, S_{2}, S_{3}, S_{4}$, denote population living or deaths occurring within forr consecutive decades of age, and $u$ denotes the numbers for a single year of age, then we have-
and consequently- $\quad n_{1}=10, \quad n=1, \quad \mathrm{~S}=u$

$$
\left.\begin{array}{rl}
u= & \frac{1}{16000} \\
+\frac{x}{240000} & {\left[933\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-133\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)\right]} \\
& +\frac{x^{2}}{40000}\left[\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)-199\left(\mathrm{~S}_{4}\right)-\left(\mathrm{S}_{2}\right)\right] \\
& +\frac{x^{3}}{60000}\left[\left(\mathrm{~S}_{3}\right)\right] \\
\left.\left.\mathrm{S}_{4}-\mathrm{S}_{1}\right)-3\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)\right]
\end{array}\right\}(9)
$$

When the values of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, are substituted in the equation of the curve, the number of persons who annually attain the age indicated by $x$ is expressed thus:

$$
\left.\begin{array}{l}
y=\frac{1}{120}\left[7\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{4}\right)\right]+\frac{x}{1200}\left[15\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{4}-\mathrm{S}_{1}\right)\right] \\
+\frac{x^{2}}{4000}\left[\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)-\left(\mathrm{S}_{2}+\mathrm{S}_{3}\right)\right]+\frac{x^{3}}{60000}\left[\left(\mathrm{~S}_{4}-\mathrm{S}_{1}\right)-3\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)\right] \tag{10}
\end{array}\right\}
$$

These last two formulas may be used instead of (3) and (5) when greater accuracy is desired. It will be easy to obtain similar ones for cases in which the ages of a population are taken by intervals of five, twenty, or any other number of years.
Let us now assume five or more groups, with a curvé of the general form-

$$
y=\mathrm{A}+\mathrm{B} x+\mathrm{C} x^{2}+\mathrm{D} x^{3}+\mathrm{E} x^{4}+\mathrm{F} x^{5}+\mathrm{G} x^{6}+\mathrm{H} x^{7}+\& \varepsilon .
$$

and, to make the case as simple as possible, let the groups be consecutive and composed of $n_{1}$ terms each. The sum of any $n$ terms in a group will be-

$$
\left.\begin{array}{l}
\mathrm{S}=\int_{x-\frac{\mathrm{q}}{} n}^{x+\frac{1}{n} n d x} \\
=n\left[\mathrm{~A}+\mathrm{B} x+\mathrm{C}\left(x^{2}+\frac{1}{12} n^{2}\right)+\mathrm{D} x\left(x^{2}+\frac{1}{4} n^{2}\right)+\mathrm{E}\left(x^{4}+\frac{1}{3} n^{2} x^{2}+\frac{1}{80} n^{4}\right)\right.  \tag{11}\\
\quad+\mathrm{F} x\left(x^{4}+\frac{5}{6} n^{2} x^{2}+\frac{1}{11} n^{4}\right)+\mathrm{G}\left(x^{6}+\frac{5}{4} n^{2} x^{4}+\frac{3}{16} n^{4} x^{2}+\frac{1}{448} n^{6}\right) \\
\left.\quad+\mathrm{H} x\left(x^{6}+\frac{7}{4} n^{2} x^{4}+\frac{7}{16} n^{4} x^{2}+\frac{1}{64} n^{6}\right)+\& \mathrm{cc} .\right]
\end{array}\right\}
$$

which, arranged according to the powers of $x$, is-

$$
\begin{align*}
\mathrm{S}= & n\left[\mathrm{~A}+\frac{1}{12} \mathrm{C} n^{2}+\frac{1}{80} \mathrm{E} n^{4}+\frac{1}{44} \mathrm{G} n^{6}+\left(\mathrm{B}+\frac{1}{4} \mathrm{D} n^{2}+\frac{1}{16} \mathrm{~F} n^{4}+\frac{1}{64} \mathrm{H} n^{6}\right) x\right. \\
& +\left(\mathrm{C}+\frac{1}{2} \mathrm{E} n^{2}+\frac{3}{16} \mathrm{G} n^{4}\right) x^{2}+\left(\mathrm{D}+\frac{5}{6} \mathrm{~F} n^{2}+\frac{7}{16} \mathrm{H} n^{4}\right) x^{3}+\left(\mathrm{E}+\frac{5}{4} \mathrm{G} n^{2}\right) x^{4}  \tag{12}\\
& \left.+\left(\mathrm{F}+\frac{7}{4} \mathrm{H} n^{2}\right) x^{5}+\mathrm{G} x^{6}+\mathrm{H} x^{7}+\& \mathrm{cc} .\right]
\end{align*}
$$

If we assume only five groups, the series will be of the fourth order, the constants F, G, \&c., will be zero, and by substituting for $x$ in formula (11) the five values $-2 n_{1},-n_{1}, 0,+n_{1}$, and $+2 n_{1}$, in succession, and putting $n_{1}$ for $n$, we shall obtain five equations by which to determine the five constants as follows:

$$
\begin{align*}
& \left.\mathrm{A}=\frac{1}{19: 20 n_{1}}\left[2134 \mathrm{~S}_{3}+9\left(\mathrm{~S}_{1}+\mathrm{S}_{5}\right)-116\left(\mathrm{~S}_{2}+\mathrm{S}_{4}\right)\right]\right) \\
& \mathrm{B}=\frac{1}{48 n_{1}{ }^{2}}\left[34\left(\mathrm{~S}_{4}-\mathrm{S}_{2}\right)-5\left(\mathrm{~S}_{5}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{C}=\frac{1}{16 n_{1}^{3}}\left[12\left(\mathrm{~S}_{2}+\mathrm{S}_{4}\right)-22 \mathrm{~S}_{3}-\left(\mathrm{S}_{1}+\mathrm{S}_{5}\right)\right]  \tag{C}\\
& \mathrm{D}=\frac{1}{12 n_{1}}\left[\left(\mathrm{~S}_{5}-\mathrm{S}_{1}\right)-2\left(\mathrm{~S}_{4}-\mathrm{S}_{2}\right)\right] \\
& \mathrm{E}=\frac{1}{24 n_{1}{ }^{6}}\left[6 \mathrm{~S}_{3}+\left(\mathrm{S}_{1}+\mathrm{S}_{5}\right)-4\left(\mathrm{~S}_{2}+\mathrm{S}_{4}\right)\right]
\end{align*}
$$

This, in connection with formula (12), enables us to express the sum $S$ of any group of $n$ terms in a series of the fourth order by means of the sums $\mathbb{S}_{1}, \mathbb{S}_{2}, \mathbb{S}_{3}, S_{4}, \mathbb{S}_{5}$, of the terms in any five consecutive groups of $n_{1}$ terms each. In case the given series is of a higher order than the fourth, or irregular, we can find adjusted values for each term, and for any given set of groups assumed these values will form a series of the fourth order. If we take $n_{1}=10$, formulas similar to (3) and (5) may be obtained, by which to interpolate numbers for each single year when statistics of population and mortality are given by decades of age.

Particular relations exist between the numerical coefticients of $\mathrm{S}_{1}, \mathrm{~S}_{2}, \& c$., in the values of the constants $\mathbf{A}, \mathbf{B}, \& c$. , in this and similar formulas. In the expression for $A$, the factor +2134 belongs to a single quantity $S_{3}$, while the factors +9 and -116 belong each to two quantities. So we have-

$$
2134+2 \times 9-2 \times 116=1920
$$

and 1920 is the numerical part of the denominator of the fraction outside the bracket. In the expression for $B$ a different relation appears. From the middle of the group $S_{2}$ to that of $S_{4}$ is a distance of two intervals, while from $S_{1}$ to $S_{5}$ there are four intervals. We bave accordingly$2 \times 34-4 \times 5=48$
and 48 is the numerical part of the denominator of the fraction without the bracket. Similar relations are found in the expressions for $C, D$, and $E$, except that the totals are equal to zero instead of to the denominator of the fraction.

Again, assuming six consecutive groups of equal extent, with a curve of the fifth degree, whose origin of coördinates is at the point of division between the third and fourth groups, and pursuing the same method as before, we find that the six constants are-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{60 n_{1}}\left[37\left(\mathrm{~S}_{3}+\mathrm{S}_{4}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{6}\right)-8\left(\mathrm{~S}_{2}+\mathrm{S}_{5}\right)\right] \\
\mathrm{B}=\frac{1}{180 n_{1}^{2}}\left[245\left(\mathrm{~S}_{4}-\mathrm{S}_{3}\right)+2\left(\mathrm{~S}_{6}-\mathrm{S}_{1}\right)-25\left(\mathrm{~S}_{5}-\mathrm{S}_{2}\right)\right]  \tag{D}\\
\mathrm{C}=\frac{1}{16 n_{1}{ }^{3}}\left[7\left(\mathrm{~S}_{2}+\mathrm{S}_{5}\right)-6\left(\mathrm{~S}_{3}+\mathrm{S}_{4}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{6}\right)\right] \\
\mathrm{D}=\frac{1}{36 n_{1}{ }^{4}}\left[1\left(\mathrm{~S}_{5}-\mathrm{S}_{2}\right)-28\left(\mathrm{~S}_{4}-\mathrm{S}_{3}\right)-\left(\mathrm{S}_{6}-\mathrm{S}_{1}\right)\right] \\
\mathrm{E}=\frac{1.0}{48 n_{1}}\left[2\left(\mathrm{~S}_{3}+\mathrm{S}_{4}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{6}\right)-3\left(\mathrm{~S}_{2}+\mathrm{S}_{5}\right)\right] \\
\mathrm{F}=\frac{1}{120 n_{1}^{6}}\left[10\left(\mathrm{~S}_{4}-\mathrm{S}_{3}\right)+\left(\mathrm{S}_{6}-\mathrm{S}_{1}\right)-5\left(\mathrm{~S}_{5}-\mathrm{S}_{2}\right)\right]
\end{array}\right\}
$$

In like manner, assuming seven groups, with a curve of the sixth degree, we find the seven constants-

$$
\begin{align*}
& \left.A=\frac{1}{107520 n_{1}}\left[121004 \mathrm{~S}_{4}+954\left(\mathrm{~S}_{2}+\mathrm{S}_{6}\right)-7621\left(\mathrm{~S}_{3}+\mathrm{S}_{5}\right)-75\left(\mathrm{~S}_{1}+\mathrm{S}_{7}\right)\right]\right) \\
& \mathrm{B}=\frac{1}{11520 n_{1}^{2}}\left[94 \tilde{5} 5\left(\mathrm{~S}_{5}-\mathrm{S}_{3}\right)+259\left(\mathrm{~S}_{7}-\mathrm{S}_{1}\right)-2236\left(\mathrm{~S}_{6}-\mathrm{S}_{2}\right)\right] \\
& \mathrm{C}=\frac{1}{3840 n_{1}{ }^{[ }}\left[3435\left(\mathrm{~S}_{3}+\mathrm{S}_{5}\right)+37\left(\mathrm{~S}_{1}+\mathrm{S}_{7}\right)-6020 \mathrm{~S}_{4}-462\left(\mathrm{~S}_{2}+\mathrm{S}_{6}\right)\right] \\
& \mathrm{D}=\frac{1}{288 n_{1}^{4}}\left[52\left(\mathrm{~S}_{6}-\mathrm{S}_{2}\right)-83\left(\mathrm{~S}_{5}-\mathrm{S}_{3}\right)-7\left(\mathrm{~S}_{7}-\mathrm{S}_{1}\right)\right]^{.}  \tag{E}\\
& \mathrm{E}=\frac{1}{576 n_{1}^{5}}\left[244 \mathrm{~S}_{4}+54\left(\mathrm{~S}_{2}+\mathrm{S}_{6}\right)-171\left(\mathrm{~S}_{3}+\mathrm{S}_{5}\right)-5\left(\mathrm{~S}_{1}+\mathrm{S}_{7}\right)\right] \\
& \mathrm{F}=\frac{1}{240 n_{1}}\left[\left[\dot{5}\left(\mathrm{~S}_{5}-\mathrm{S}_{3}\right)+\left(\mathrm{S}_{7}-\mathrm{S}_{1}\right)-4\left(\mathrm{~S}_{6}-\mathrm{S}_{2}\right)\right]\right. \\
& G=\frac{1}{720 n_{1}\left[15\left(S_{3}+S_{5}\right)+\left(S_{1}+S_{7}\right)-20 S_{4}-6\left(S_{2}+S_{6}\right)\right]}
\end{align*}
$$

So also with eight groups, and a curve of the seventh degree, the eight constants are-

In the same way we might determine the nine constants for a curve of the eighth degree, and so on ; for the operations required, though somewhat tedious, are always possible.* We have found, then, a very simple and general method by which, when any $m+1$ consecutive groups of equal extent are assumed in a given series, a new series of the $m$ th

[^31]order can be constructed, such that the arithmetical means of the terms in the $m+1$ corresponding groups in it will be severally equal to those in the original series.
Let us now proceed to apply this method to the graduation of an irregular rate of mortality. Column (a) in Table I shows the probability of dying within a year, at each age, from 20 to 79 , as experienced by the life insurance companies doing business in Massachusetts for seven years ending November 1, 1865, and given in the commissioners' report. The terms of the series are 100 times the quotients arising from dividing the number of deaths in each year of age by the number of years of life exposed to mortality at that age. For example, the number 1.98 opposite the age 59 signifies that of the insured persons who attained that age about 2 per cent. died within the following year. The great irregularity of this series is apparent at a glance. The observations on which it is based were not such as to give it rery high authority as a law of mortality, and it is introduced here merely to illustrate the method of graduation. The rate which it shows is too low throughout almost all the ages, owing mainly, no doubt, to the recent selection of most of the lives observed. The life insurance companies of America are of recent and very rapid growth, and in the present case the average duration of the policies observed probably did not mach exceed, if it equaled, three years. It is well known that in a class of persons aged fifty years, for instance, who have been recently pronounced healthy by a medical examiner, the rate of mortality may be expected to be lower than among another class of similar age, whose examination was made ten, twenty, or thirty years earlier; for some of the latter will have contracted disease in the mean time, while others, probably among the healthiest lives, will have surrendered their policies or allowed them to lapse, thus deteriorating the average vitality of the insured. The present rate, therefore, cannot be regarded as a permanently reliable one.' At the ages 20,21 , and 22 , however, the rate is too high. This may be merely accidental, owing to the fact that only a small number of lives were observed at those ages.
In the first place, let us construct a representative series of the fourth order. The sixty terms of series (a) form five groups of twelve terms each ; their sums are-
$$
S_{1}=9.15, \quad S_{2}=9.06, \quad S_{3}=13.03, \quad S_{4}=28.51, \quad S_{5}=87.84
$$
and when we take-
$$
n_{1}=12, \quad n=1, \quad \mathrm{~S}=u
$$
formulas (C) and (12) give-
$$
\mathrm{A}=\frac{2432.081}{16(12)^{2}}, \quad \mathrm{~B}=\frac{267.85}{4(12)^{3}}, \quad \mathrm{C}=\frac{67.19}{16(12)^{3}}, \quad \mathrm{D}=\frac{39.79}{(12)^{5}}, \quad \mathrm{E}=\frac{12.445}{(12)^{6}}
$$
and consequently-

Table I.

| Age. | (a) | (b) | (c) | Age. | (a) | (b) | (c) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20. | . 92 | 1. 07937 | . 71465 | 50 | . 97 | 1. 07582 | 1. 09695 |
| 21. | . 90 | . 97386 | . 74606 | 51 | 1.01 | 1. 12001 | 1. 14758 |
| 22 | . 92 | . 88650 | . 76615 | 52 | 1. 06 | 1. 17063 | 1. 20302 |
| 23 | . 67 | . 81545 | . 77713 |  | 1.31 | 1. 228884 | 1.26399 |
| 24 | . 82 | . 75897 | . 78100 |  | 1. 80 | 1. 29589 | 1. 33140 |
| 25 | . 70 | . 71543 | . 77949 | 55 | 1.21 | 1. 37314 | 1. 40629 |
| 26. | . 67 | . 68326 | . 77416 | 56 | 1.33 | 1.46206 | 1. 48989 |
| 27. | . 66 | . 66105 | . 76605 | 57 | 1.65 | 1.56419 | 1. 58836 |
|  | . 67 | . 64744 | . 75723 | 58 | 1.70 | 1. 68121 | 1. 68910 |
| 29. | . 68 | . 64119 | . 74779 | 59 | 1.98 | 1. 81487 | 1. 80815 |
| 30 | . 74 | . 64117 | . 73887 | 60 | 2.09 | 1.96703 | 1. 94283 |
| 31. | . 80 | . 64632 | . 73115 | 61 | 2.08 | 2. 13966 | 2. 09546 |
| 32. | . 68 | . 65572 | . 72520 | 62 | 1.89 | 2.33480 | 2. 26857 |
| 33. | . 60 | . 66852 | . 72144 | 63 | 2.75 | 2. 55463 | 2. 46499 |
| 34. | . 81 | . 68397 | . 72020 | 64 | 2.50 | 2.80139 | 2.68782 |
| 35 | . 74 | . 70145 | . 72173 | 65 | 3.51 | 3. 07746 | 2.94044 |
| 36. | . 63 | . 72039 | . 72617 | 66 | 3. 01 | 3.38528 | 3.22655 |
|  | . 65 | . 74038 | . 73361 | 67 | 4.02 | 3.72742 | 3. 55016 |
|  | . 82 | . 76106 | . 74408 | 68 | 4. 26 | 4.10655 | 3.91561 |
| 39. | . 85 | . 78219 | . 75756 | 69 | 3.31 | 4.52541 | 4. 32758 |
|  | . 87 | . 80363 | . 77402 | 70 | 6.80 | 4.98688 | 4.79112 |
|  | . 78 | . 82535 | . 79339 | 71 | 5.00 | 5. 49390 | 5.31163 |
| 42. | . 84 | . 84740 | . 81561 | 72 | 6.84 | 6. 04955 | 5. 89490 |
| 43. | . 79 | . 86994 | . 84062 | 73 | 6.14 | 6. 65697 | 6.54713 |
| 44 | . 91 | . 89323 | . 86838 | 74 | 4.58 | 7.31943 | 7.27489 |
| 45. | . 85 | . 91762 | . 89891 | 75 | 4. 50 | 8.04030 | $8.085 \% 1$ |
|  | . 97 | . 94359 | . 93224 | 76 | 7.53 | 8. 82302 | 8.98552 |
|  | . 92 | . 97168 | . 96849 | 77 | 11.31 | 9.67116 | 9.98373 |
| 48 | 1.03 | 1.00256 | 1. 00783 | 78 | 11. 69 | 10.58839 | 11.08818 |
|  | . 96 | 1.03699 | 1.05053 |  | 15.88 | 11.57845 | 12.30769 |

$$
\begin{gathered}
u=1.055794+.03879144 x+.002432280 x^{2}+.0001599072 x^{3} \\
+.000004167808 x^{4}
\end{gathered}
$$

This is the equation of the new series. Since the origin of coördinates is at the middle point of the middle group, if we assign to $x$ the values $-\frac{1}{2},+\frac{1}{2},+\frac{3}{2}, \& c$., the resulting values of $u$ will be the terms belonging to the ages $49,50,51, \& c$. When any five consecutive terms have been computed in this way, and their four orders of differences are taken, the rest of the series is readily constructed therefrom. The complete series is given in column (b). It will be found that the sums of the terms in the twelve-year groups $20-31,32-43$, \&c., are identical with those in series ( $a$ ), and consequently the arithmetical means of the terms in these groups are the same in the two series.
Next, let the required series be one of the fifth order. Taking six groups of ten terms each, their sums are:

$$
\begin{array}{lll}
\mathrm{S}_{1}=7.61 & \mathrm{~S}_{3}=8.95 & \mathrm{~S}_{5}=29.42 \\
\mathrm{~S}_{2}=7.32 & \mathrm{~S}_{4}=14.02 & \mathrm{~S}_{6}=80.27
\end{array}
$$

and using formula (D) we obtain the equation of the series-

$$
\begin{gathered}
u=1.0732474+.04640701 x+.001969958 x^{2}+.00007920042 x^{3} \\
+000004916667 x^{4}+.0000001071667 x^{5}
\end{gathered}
$$

The origin of coördinates is the same as in the previous case, being at the point of division between the third and fourth gromps. When any six consecutive terms have been computed and their five orders of differences are taken, the rest of the series is easily constructed. It is given in column (c). The sums of the terms for the decades 20-29, 30$39, \& c .$, are the same as in the original series.

It may seem strange that the two series (b) and (c) should differ so much as they do, especially at the earlier ages. There are two reasons for it. In the first place, they are derived from two different sets of groups; and as the original series is extremely irregular, the sums $\mathbb{S}_{1}$, $\mathrm{S}_{2}$, \&c., must vary somewhat from their normal value, and vary differently in the two series, thus affecting the values of all the single terms. This source of error, however, can be very much diminished, if not entirely removed, by making a preliminary adjustment by the second method, as will be shown hereafter. In the second place, there is an essential difference in the nature of the two series (b) and (c). In (b) the general term $u$ is expressed by a polynomial of the fourth degree in $x$. When the two values $+\infty$ and $-\omega$ are assigned to $x$, the resulting value of $u$ will have the same sign in both cases, because the highest power of $x$ is an even one. But in the equation of series $(c)$ the highest power of $x$ is odd, so that the values $x=+\omega$ and $x=-\omega$ will give contrary signs to $u$. In general, when a series of an even order, such as (b), is extended indefinitely in both directions, its terms will go on increasing algebraically at both extremities, or diminishing at both ; but a series of an odd order like (c) will increase at one extremity and diminish at the other. It is evident that the original series $(a)$ tends to increase at both ends, as also does (b), while (c) diminishes at the earliest ages and increases at the latest ones. This has a considerable effect on the form of the series. In (b) there is a minimum of .64117 at the age 30 , and no maximum at all, while (c) has its minimum of . 72020 at the age 34 , and a maximum at 24 . It appears that (b) represents (a) more faithfully than (c) does, and in like manner we may presume that in this case a series of the sixth order would be better than one of the seventh order, and, in general, that if a given series tends to increase at both ends, as any rate of mortality of this nature does, or to diminish at both ends, its representative series ought to be of an even order, while if it tends to increase at one end and diminish at the other, the new series should be made of an odd order. But there will be some exceptions to this rule, and of course, other things being equal, the greater the number of groups taken, and the higher the order of the new series, the more faithfully will the original one be represented by it.

SECOND METHOD OF ADJUSTMENT.
If in formula (2) we make $n_{2}$ an odd number, and assume-

$$
n_{3}=n_{1}, \quad a_{3}=a_{1}
$$

and let $u^{\prime}$ represent what $S$ becomes when we take-

$$
n=1, \quad x=0
$$

then $u^{\prime}$ will be the middle term of the middle group $\mathrm{S}_{2}$, and the lateral groups $S_{1}$ and $S_{3}$ will be similarly situated on each side of the middle group and its middle term. We have then-

$$
\begin{equation*}
u^{\prime}=\frac{S_{2}}{n_{2}}+\frac{n_{2}{ }^{2}-1}{12 a_{1}{ }^{2}+n_{1}{ }^{2}-n_{2}{ }^{2}}\left[\frac{\mathbb{S}_{2}}{n_{2}}-\frac{1}{2}\left(\frac{S_{1}+S_{3}}{n_{1}}\right)\right] \tag{13}
\end{equation*}
$$

This formula enables us to adjnst the value of any term in an irregular series, by taking it as the middle term with an arbitrary number of adjacent terms on each side of it, all together forming the middle group in which the sum of the terms is $S_{2}$ and their number is $n_{2}$, and taking two other arbitrary groups, $S_{1}$ and $S_{3}$, containing $n_{1}$ terms each, and situated one on each side of the middle term and equidistant from it. The distance from the middle point of the middle group to that of either lateral group is $a_{1}$. The simplest case which can arise is where we take five consecutive terms, $u_{1}, u_{2}, u_{3}, u_{4}, u_{5}$, and assume the three middle ones as the middle group and the first one and last one as the two lateral groups; then-

$$
n_{2}=3, \quad n_{1}=1, \quad a_{1}=2
$$

and formula (13) gives, as the adjusted value of the middle term $u_{3}$,

$$
\left.\begin{array}{rl}
u_{3}^{\prime} & =\frac{1}{15}\left[4 \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)\right]  \tag{14}\\
& =\frac{1}{1}\left[4\left(u_{2}+u_{3}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right]
\end{array}\right\}
$$

When se ven terms are taken, five in the middle group and two in each lateral one, so that the second and sixth terms belong to two groups each, we have-

$$
n_{2}=5, \quad n_{1}=2, \quad a_{1}=\frac{5}{2}
$$

and the formula is-

$$
\left.\begin{array}{rl}
u_{4}^{\prime} & =\frac{1}{46}\left[13 \mathrm{~S}_{2}-5\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)\right]  \tag{15}\\
& =\frac{1}{45}\left[13\left(u_{3}+u_{4}+u_{5}\right)+8\left(u_{2}+u_{6}\right)-5\left(u_{1}+u_{7}\right)\right]
\end{array}\right\}
$$

The accuracy of formulas (14) and (15) can easily be tested by trial with any series of the second order, the adjusted value of the middle term being in this case the same as its original value. A simple relation exists beween the numerical coefficients of $u_{1}, u_{2}$, \&c. For example, in formula (15) the coefficient +13 belongs to three terms, +8 to two, and - 5 to two, and we have-

$$
3 \times 13+2 \times 8-2 \times 5=45
$$

and 45 is the denominator of the fraction outside the bracket. The numerical coefficients within the bracket may therefore be regarded as the weights of the terms to which they belong, so that the weight of each of the terms $u_{3}, u_{4}$, and $u_{5}$ is 13 , that of $u_{2}$ and $u_{6}$ is 8 , and that of $u_{1}$ and $u_{7}$ is -5 .

By varying the positions of the groups in formula (13), and the num19 s 71
ber of terms in each, we might find an unlimited number of adjustment formulas, but (14) and (15) will serve as specimens. Similar results can also be arrived at by another method, which is very simple. We know that in a series of the third or any lower order the fourth differences are zero; that is, any five consecutive terms are connected by the relation-

$$
u_{5}-4 u_{4}+6 u_{3}-4 u_{2}+u_{1}=0
$$

and, consequently, we have-

$$
\begin{align*}
10 u_{3} & =4\left(u_{2}+u_{3}+u_{4}\right)-\left(u_{1}+u_{5}\right), \\
u_{3} & =\frac{1}{10}\left[4\left(u_{2}+u_{3}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right] \tag{16}
\end{align*}
$$

This is identical with formula (14), which is thus shown to hold good and to give exact results when applied to a series of the third order as well as the second. It is therefore equally well adapted for graduating any series, whether it has a point of inflexion or not. The same is true of (15) and all other formulas derived from (13).

When applied to an irregular series, such formulas can be modified so as to give adjusted values which will approximate to the original ones more or less closely, as may be desired. Take, for instance, formula (16). If we add $k u_{3}$ to both members of the equation next preceding, it will stand-

$$
(10+k) u_{3}=(4+k) u_{3}+4\left(u_{2}+u_{4}\right)-\left(u_{1}+u_{5}\right)
$$

and hence we have-

$$
u_{3}=\frac{1}{10+k}\left[(4+k) u_{3}+4\left(u_{2}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right]
$$

This formula differs from (16) in no respect except that the coefficient of $u_{3}$ within the bracket, and the denominator of the fraction without the bracket, have both been increased by the same quantity $k$. Since $k$ may have any value whatever, we see that the weight of the middle term $u_{3}$ can be increased or diminished to any desired extent, the denominator of the fraction without the bracket being increased or diminished by the same amount. Thus if we desire that the weight of $u_{3}$ slall be 9 instead of 4 , the formula will stand-

$$
\begin{equation*}
u_{3}=\frac{1}{15}\left[9 u_{3}+4\left(u_{2}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right] \ldots . \tag{17}
\end{equation*}
$$

In this way the value of each term in an adjusted series can be made to depend on, and approximate to, that of the corresponding term in the original series to any extent that may be required, and, of course, the closer this approximation, the more nearly will the form of the new series resemble that of the original one.

When more than five terms are to be included by an adjustment formula, the relative weights of the terms can be varied by combining two or more formulas together. For instance, (15) gives, if we drop the accent from $u_{4}^{\prime}$,

$$
45 u_{4}=13\left(u_{3}+u_{4}+u_{5}\right)+8\left(u_{2}+u_{6}\right)-5\left(u_{1}+u_{7}\right)
$$

and (16) may be written-

$$
10 k u_{4}=k\left[4\left(u_{3}+u_{4}+u_{5}\right)-\left(u_{2}+u_{6}\right)\right]
$$

Adding these two equations, we obtain-

$$
u_{4}=\frac{1}{45+10 k}\left[(13+4 k)\left(u_{3}+u_{4}+u_{5}\right)+(8-k)\left(u_{2}+u_{6}\right)-5\left(u_{1}+u_{7}\right)\right]
$$

Since $l$ may have any value, let us determine it so that the excess of the weight of $u_{3}$ and $u_{5}$ over that of $u_{2}$ and $u_{6}$ shall be equal to the excess of the latter weight over that of $u_{1}$ and $u_{7}$. This gives-

$$
13+4 k-(8-k)=8-k+5
$$

and, consequently, $k=\frac{4}{3}$. The formula then becomes-

$$
\begin{equation*}
u_{4}=\frac{1}{35}\left[11\left(u_{3}+u_{4}+u_{5}\right)+4\left(u_{2}+u_{6}\right)-3\left(u_{1}+u_{7}\right)\right] . \tag{18}
\end{equation*}
$$

and, if the weight of the middle term is increased by 7 , we have finally-

$$
u_{4}=\frac{1}{42}\left[18 u_{4}+11\left(u_{3}+u_{5}\right)+4\left(u_{2}+u_{6}\right)-3\left(u_{1}+u_{7}\right)\right] . . .(19)
$$

Here the weights increase in arithmetical progression, from the extreme terms to the middle one.
To obtain a similar formula including nine terms, we may proceed as follows. In a series of the third or any lower order the fourth differences are zero, and any five consecutive terms are connected by the relation-

$$
u_{5}-4 u_{4}+6 u_{3}-4 u_{2}+u_{1}=0
$$

In a series of the fifth or any lower order the sixth differences are zero, and for any seven consecutive terms we have the relation-

$$
u_{7}-6 u_{6}+15 u_{5}-20 u_{4}+15 u_{3}-6 u_{2}+u_{1}=0
$$

In a series of the seventh or any lower order the eighth differences are zero, and any nine consecutive terms are connected by the relation-

$$
u_{9}-8 u_{8}+28 u_{7}-56 u_{6}+70 u_{5}-56 u_{4}+28 u_{3}-8 u_{2}+u_{1}=0
$$

Hence, considering any nine consecutive terms in a series of the third or any lower order, we have-

$$
\begin{aligned}
& 126 u_{5}=56\left(u_{4}+u_{5}+u_{6}\right)-28\left(u_{3}+u_{7}\right)+8\left(u_{2}+u_{8}\right)-\left(u_{1}+u_{9}\right) \\
& 35 k u_{5}=15 k\left(u_{4}+u_{5}+u_{6}\right)-6 k\left(u_{3}+u_{7}\right)+k\left(u_{2}+u_{8}\right) \\
& 10 k^{\prime} u_{5}=4 k^{\prime}\left(u_{4}+u_{5}+u_{6}\right)-k^{\prime}\left(u_{3}+u_{7}\right)
\end{aligned}
$$

Adding these three equations together, we obtain-

$$
\begin{aligned}
\left(126+35 k+10 k^{\prime}\right) u_{5}= & \left(56+15 k+4 k^{\prime}\right)\left(u_{4}+u_{5}+u_{6}\right)-\left(28+6 k+k^{\prime}\right)\left(u_{3}+u_{7}\right) \\
& +(8+k)\left(u_{2}+u_{8}\right)-\left(u_{1}+u_{9}\right)
\end{aligned}
$$

which expresses a general relation between any nine consecutive terms in a series of the third or any lower order. The numbers $k$ and $k^{\prime}$ being entirely arbitrary, we may make the coefficients in the second member of the equation form an arithmetical progression by taking -

$$
\begin{gathered}
(8+k)+2\left(28+6 k+k^{\prime}\right)+\left(56+15 k+4 k^{\prime}\right)=0 \\
-1-2(8+k)-\left(28+6 k+k^{\prime}\right)=0
\end{gathered}
$$

These two conditions give the two values-

$$
k=-\frac{15}{2}, \quad k^{\prime}=15
$$

so that the equation reduces to-

$$
\frac{27}{2} u_{5}=\frac{7}{2}\left(u_{4}+u_{5}+u_{6}\right)+2\left(u_{3}+u_{7}\right)+\frac{1}{2}\left(u_{2}+u_{8}\right)-\left(u_{1}+u_{9}\right)
$$

and adding $\frac{3}{2} u_{5}$ to both members, we obtain-

$$
\begin{equation*}
u_{5}=\frac{1}{30}\left[10 u_{5}+7\left(u_{4}+u_{6}\right)+4\left(u_{3}+u_{7}\right)+\left(u_{2}+u_{8}\right)-2\left(u_{1}+u_{9}\right)\right] . \tag{20}
\end{equation*}
$$

The same result can also be reached by deriving from formula (13) any three special adjustment formulas comprising five, seven, and nine consecutive terms respectively, and then combining them together in the manner above indicated. There is evidently no limit to the number of terms which might be included in formulas found by these methods. With eleven terms, we have the following:

$$
\left.\begin{array}{c}
u_{6}={ }_{1} \frac{1}{65}\left[45 u_{6}+34\left(u_{5}+u_{7}\right)+23\left(u_{4}+u_{8}\right)+12\left(u_{3}+u_{9}\right)\right.  \tag{21}\\
\left.+\left(u_{2}+u_{10}\right)-10\left(u_{1}+u_{11}\right)\right]
\end{array}\right\}
$$

in which the weights are in arithmetical progression.*
If we consider any seven consecutive terms in a series of the fifth order, placing the sixth difference alone equal to zero, the equation thus formed will give-

$$
\begin{equation*}
u_{4}=\frac{1}{35}\left[1.5\left(u_{3}+u_{4}+u_{5}\right)-6\left(u_{2}+u_{6}\right)+\left(u_{1}+u_{7}\right)\right] . \tag{22}
\end{equation*}
$$

This might be used as an adjustment formula, possibly with good effect in continuing the graduation of a series already approximately adjusted. It will give exact results when applied to a series of the fifth or any lower order, and the weight of the middle term $u_{4}$ can be increased or diminished if desired. So, too, when the eighth difference is placed equal to zero, we obtain the formula-

$$
\begin{equation*}
u_{5}=\frac{1}{126}\left[56\left(u_{4}+u_{5}+u_{6}\right)-28\left(u_{3}+u_{7}\right)+8\left(u_{2}+u_{8}\right)-\left(u_{1}+u_{9}\right)\right] . \tag{23}
\end{equation*}
$$

which will give exact results if applied to a series of the seventh or any lower order.

The second method of adjustment can be applied to the logarithms of a series of numbers instead of to the numbers directly. If, for instance, the logarithms form a series of the third or any lower order, then for any five consecutive terms formula (16) gives-

$$
\begin{aligned}
\log u_{3} & =\frac{1}{10}\left[4\left(\log u_{2}+\log u_{3}+\log u_{4}\right)-\left(\log u_{1}+\log u_{5}\right)\right] \\
& =\frac{1}{10}\left[\log \left(u_{2} u_{3} u_{4}\right)^{4}-\log \left(u_{1} u_{5}\right)\right]
\end{aligned}
$$

and consequently-

$$
u_{3}=\left(\frac{\left(u_{2} u_{3} u_{4}\right)^{4}}{u_{1} u_{5}}\right)^{\frac{1}{10}}
$$

This relation will evidently hold good for any five consecutive terms in a geometrical progression, because their logarithms are in arithmetical progression; that is, they form a series of the first order. We can easily see how any similar adjustment formula can be transformed at

* For improved formulas of this nature, see Appendices I and II.
once in this way. The weights of the several terms become their exponents, the terms with positive weights become factors in the numerator of a fraction, while those with negative weights are factors in the denominator, and the fraction without the bracket becomes the exponent of the whole. Thus (22) is transformed into-

$$
u_{4}=\left(\frac{\left(u_{3} u_{4} u_{5}\right)^{15}\left(u_{1} u_{7}\right)}{\left(u_{2} u_{6}\right)^{6}}\right)^{\frac{1}{35}}
$$

which expresses a relation existing between any seven consecutive terms in a series whose logarithms form a series of the fifth or any lower order.
In all formulas under the second method, the weights of the several terms, depending on the position of each one with reference to the middle term whose adjusted value is sought, may be called local weights, to distinguish them from the intrinsic weight which any term may have by virtue of the relative goodness of the observations taken to determine its value. We may regard the total weight of a term as compounded of these two elements, so that if, for instance, the local weights of five consecutive terms are taken as in formula (16), and if we wish also to take the intrinsic weights $c_{1}, c_{2}, c_{3}$, \&cc., of the termsinto account, the adjusted value of $u_{3}$ will then be-

$$
\begin{equation*}
u_{3}=\frac{4\left(c_{2} u_{2}+c_{3} u_{3}+c_{4} u_{4}\right)-\left(c_{1} u_{1}+c_{3} u_{5}\right)}{4\left(c_{2}+c_{3}+c_{4}\right)-\left(c_{1}+c_{5}\right)} \cdot \cdots \tag{24}
\end{equation*}
$$

We know that this formula gives exact results when the series $u_{1}, u_{2}, \& c$., is of the third or any lower order, and the intrinsic weights $c_{1}, c_{2}$, \&c., are all equal, and we may naturally expect that the results will be approximately correct when the series $u_{1}, u_{2}, \& c$., approximates to regularity, and the intrinsic weights of the terms do not differ very much from one another; so that in such cases something will be gained in accuracy by taking the intrinsic weights into account.
By the use of formulas such as (16), (17), (19), or (20), we can graduate approximately all the terms in a series except the first two and last two. These also can be reached by means of the general formula (2). Let us take six consecutive terms in three groups, so as to have-

$$
n_{1}=3, \quad n_{2}=2, \quad n_{3}=1, \quad a_{1}=\frac{5}{2}, \quad a_{3}=\frac{3}{2}, \quad n=1
$$

Then for the first term we have-

$$
x=-\frac{7}{2}, \quad S=u_{1}
$$

and the formula reduces to-

$$
\left.\begin{array}{rl}
u_{1} & =\frac{1}{9}\left(5 \mathrm{~S}_{1}-5 \mathrm{~S}_{2}+4 \mathrm{~S}_{3}\right)  \tag{25}\\
& =\frac{1}{9}\left[5\left(u_{1}+u_{2}+u_{3}\right)-5\left(u_{4}+u_{5}\right)+4 u_{6}\right]
\end{array}\right\}
$$

For the second term we have-

$$
x=-\frac{5}{2}, \quad S=u_{2}
$$

and consequently-

$$
\left.\begin{array}{rl}
u_{2} & =\frac{1}{65}\left(14 \mathbb{S}_{1}+4 S_{2}-5 \mathbb{S}_{3}\right) \\
& =\frac{1}{65}\left[14\left(u_{1}+u_{2}+u_{3}\right)+4\left(u_{4}+u_{5}\right)-5 u_{6}\right] \tag{26}
\end{array}\right\}
$$

These formulas give exact results when applied to any series of the second order.

Let us now make $n_{2}$ an even number in formula (2), and assume as before-

$$
n_{3}=n_{1}, \quad a_{3}=a_{1}
$$

and let $y^{\prime}$ represent what $\frac{\mathrm{S}}{n}$ becomes when we take-
then $y^{\prime}$ is the middle ordinate of the middle area $\mathbb{S}_{2}$, and we have this formula :

$$
\begin{equation*}
y^{\prime}=\frac{\mathbf{S}_{2}}{n_{2}}+\frac{n_{2}{ }^{2}}{12 a_{1}{ }^{2}+n_{1}{ }^{2}-n_{2}{ }^{2}}\left[-\frac{\mathbf{S}_{2}}{n_{2}}-\frac{1}{2}\left(\frac{S_{1}+S_{3}}{n_{1}}\right)\right] \tag{27}
\end{equation*}
$$

When $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}$ denote stationary population living within three intervals of age, the two lateral intervals being of $n_{1}$ years each, and their middle points being each distant $a_{1}$ years from the middle point of the middle interval, which consists of $n_{2}$ years, then $y^{\prime}$ is an adjusted value for the number of persons who annually attain the exact middle age of the middle interval. The simplest case is where we have the populations $u_{1}, u_{2}, u_{3}, u_{4}$, living within four consecutive jears of age, and take the two middle ones as the middle group, and each of the others as a lateral group; then-

$$
n_{1}=1, \quad n_{2}=2, \quad a_{1}=\frac{3}{2}
$$

and (27) reduces to-

$$
\left.\begin{array}{rl}
y^{\prime} & =\frac{1}{12}\left[7 \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)\right] \\
& =\frac{1}{12}\left[7\left(u_{2}+u_{3}\right)-\left(u_{1}+u_{4}\right)\right]
\end{array}\right\}\{28)
$$

For example, if $u_{1}, u_{2}, u_{3}, u_{4}$, denote stationary population living within the ages 38 to 39,39 to 40,40 to 41 , and 41 to 42 , then $y^{\prime}$ is the number annually attaining the age 40 . And even if the population is not stationary, but increases or diminishes from natural causes or by migration, still, if $u_{1}, u_{2}, \& c$., denote the mean population living within the ages named during a given number of years, then $y^{\prime}$ will be the mean number annually attaining the age 40 , as before.

Adjustment formulas analogous to (13) and (27) can also be derived from (8) by taking $x=0$ and $n=1$ or $n=0$. It can be shown that (13) and (27) are particular cases under these, so that all the special adjustment formulas derived from them will give exact results when applied to series of the third order as well as the second.
If in formula (8) we take $n_{1}=n_{2}=0$, then $\frac{S_{1}}{n_{1}}, \frac{S_{2}}{n_{2}}, \frac{S_{3}}{n_{2}}$, and $\frac{\mathrm{S}_{4}}{n_{1}}$ will represent ordinates to the curve, and may be denoted by $y_{1}, y_{2}, y_{3}$, and $y_{4}$. If we also take-

$$
x=0, \quad a_{1}=\frac{3}{2}, \quad a_{2}=\frac{1}{2}, \quad n=1, \quad S=u^{\prime}
$$

then (8) reduces to-

$$
\begin{equation*}
u^{\prime}=\frac{1}{24}\left[13\left(y_{2}+y_{3}\right)-\left(y_{1}+y_{4}\right)\right] \ldots \tag{29}
\end{equation*}
$$

Here $y_{1}, y_{2}, y_{3}$, and $y_{4}$ are four equidistant ordinates to a carve of the third or any lower order, and $u^{\prime}$ is the area between the two middle ordinates. Hence, when the mean numbers of persons annually attaining each of four consecutive ages are known, the mean population living between the two middle ages can be computed by this formula. For instance, if $y_{1}, y_{2}, y_{3}$, and $y_{4}$ denote the numbers annually attaining the ages $39,40,41$, and 42 , then $u^{\prime}$ is the population living between the ages 40 and 41.
Let us now make a practical application of the second method of adjustment, in graduating the irregular rate of mortality given in column (d) of Table II. This is a new experience table quite recently published in England in an unadjusted form. It is probably correct in its essential features, and suited for practical use, having been prepared by the Institute of Actuaries, from the experience of twenty British life insurance companies, all of which had been in existence more than twenty years, so that the average daration of the policies observed was about nine years. The original publication not being at hand, the data have been taken as they are given in the Massachnsetts and New York State Insurance Reports of 1869. The probabilities of dying within a year at each age, according to these data, and multiplied by 100, are as they stand in column (d), for the ages 18 to 91 inclusive. The original series extends from the age 10 to 96 , but a few of the earliest and latest terms show such irregularities as to be evidently worthless for the purpose of graduation. This is owing to the paucity of observations at those ages. There were no deaths at all at the ages 11,16 , and 94 , and no survivors at the age 97 . The eight terms from 10 to 17 are therefore rejected here, and their places supplied by others taken from the English lifetable, No. 3, for males, reduced a little to correspond with the new rate. The sum of the terms for the eight ages 18 to 25 is 5.1862 by the new table, and is 0.6775 by the table No. 3. Accordingly, each of the first eight terms in series ( $d$ ) is taken from the table No. 3, but diminished in the ratio of 66775 to 51862 . The eight last terms, from 92 to 99 , have been obtained in a similar way, using the sums of the terms for the eight ages next preceding, so as to increase the values given by the table No. 3 in the ratio of 18436 to 18456. Series (d) thus completed, has been approximately adjusted by means of formula (20), which reaches all the terms except the first four and last four. The result is given in column (e). For instance, at the age 30 the adjusted term is-

$$
\begin{aligned}
& u_{5}=\frac{1}{3}\left[\begin{array}{l}
{[8.2341}
\end{array}+7(.74000+.72927)+4(.77808+.83635)\right. \\
&\quad+(.65324+.83200)-2(.69197+.87346)] \\
&=.77770
\end{aligned}
$$

At the ages 13 and 96 the adjustment has been made by formula (18), at the ages 12 and 97 by (16), at 11 and 98 by (26), and at 10 and 99 by (25). To diminish some irregularitiesstill existing in series (e), the adjustment has been repeated, only this time formula (16) was used throughout.

The result is shown in column $(f)$.* This is a roughly adjusted series, approximating closely to the form of the original series (d); too closely, however, for it retains at least one undulation which is abnormal, and would doubtless not have appeared if the number of observations on which the earlier portion of series $(d)$ is based had been very greatly increased. It is an acknowledged principle that after the age of 12 or 13 , at which the probability of dying within a year is a minimum, the rate of mortality ought to go on increasing continuously up to the limit of old age. But in series ( $f$ ) it increases up to the age 22, then diminishes up to 25 , then increases again continuously. To remedy this fault, and also to perfect the graduation, some further process of adjustment will be required.

Table II.

| Age. | (d) | (e) | (f) | Decade. | (g) | (h) | Age. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 4-13 | 6. 0501 |  |  |
| 10. | . 43626 | . 43244 | . 43143 | 5-14 | 5. 3587 | . 42670 | 10 |
| 11. | . 39292 | . 39460 | . 39407 | 6-15 | 4. 8884 | . 40437 | 11 |
| 12.. | . 37047 | . 37034 | . 37184 | 7-16 | 4. 6001 | . 39659 | 12 |
| 13.. | . 36576 | . 36580 | . 36420 | 8-17 | 4. 4594 | . 40030 | 13 |
| 14.. | . 37692 | . 37212 | . 37120 | 9-18 | 4. 4365 | . 41283 | 14 |
| 15.. | . 40177 | . 39639 | . 40160 | 10-19 | 4. 5052 | . 43190 | 15 |
| 16.. | . 43719 | . 45493 | . 44966 | 11-20 | 4. 6434 | . 45559 | 16 |
| $17 .$. | . 48163 | . 51192 | . 51820 | 12-21 | 4.8323 | . 48231 | 17 |
| 18.. | . 60556 | . 58421 | . 57807 | 13-22 | 5. 0562 | . 51074 | 18 |
| 19.. | . 70219 | . 62583 | . 62494 | 14-23 | 5. 3021 | . 53984 | 19 |
| $20 .$. | . 58236 | . 65223 | . 66049 | 15-24 | 5. 5597 | . 56879 | 20 |
| 21. | . 70084 | . 68776 | . 67539 | 16-25 | 5. 8210 | . 59700 | 21 |
| $22 .$. | . 62151 | . 67417 | . 68445 | 17-26 | 6. 0798 | . 62404 | 22 |
| 23.. | . 77380 | . 67688 | . 67164 | 18-27 | 6. 3318 | . 64966 | 23 |
| 24. | . 68369 | . 65849 | . 65506 | 19-28 | 6. 5743 | . 67373 | 24 |
| $25 .$. | . 51630 | . 63396 | . 64249 | 20-29 | 6. 8058 | . 69625 | 25 |
| 26.. | . 69197 | . 65258 | . 64742 | 21-30 | 7.0261 | . 71731 | 26 |
| $27 .$. | . 65324 | . 67830 | . 68305 | 22-31 | 7.2357 | . 73708 | 27 |
| $28 .$. | . 77808 | . 72668 | . 72526 | 23-32 | 7.4361 | . 75580 | 23 |
| $29 .$. | . 74000 | . 76554 | . 76056 | 24-33 | 7. 6292 | . 77374 | 29 |
| $30 .$. | . 82341 | . 77770 | . 78223 | 25-34 | 7.8176 | . 79122 | 30 |
| 31.. | . 72927 | . 79659 | . 79489 | 26-35 | 8. 0042 | . 80858 | 31 |
| 32.- | . 83635 | . 81111 | . 81229 | 27-36 | 8.1921 | . 82618 | 32 |
| 33.. | . 83200 | . 82694 | . 82432 | 28-37 | 8. 3845 | . 84437 | 33 |
| $34 .$. | . 87346 | . 83797 | . 84023 | 29-38 | 8.5848 | . 86351 | 34 |
| 35.. | . 82319 | . 86430 | . 86433 | 30-39 | 8.7964 | . 88399 | 5 |
| $36 .$. | . 87678 | . 90344 | . 90477 | 31-40 | 9. 0228 | . 90613 | 36 |
| 37.. | . 95530 | . 95256 | . 95107 | 32-41 | 9,2672 | . 93023 | 37 |
| 33. | 1. 03600 | . 99555 | . 99828 | 33-42 | 9. 5330 | . 95660 | 38 |
| 39.. | 1.05880 | 1.0312 | 1. 0240 | 34-43 | 9. 8234 | . 98582 | 39 |
| 40. | . 98504 | 1. 0310 | 1. 0345 | 35-44 | 10.142 | 1. 0180 | 40 |
| $41 .$. | 1. 0440 | 1.0387 | 1.0404 | 36-45 | 10.491 | 1. $05 \% 9$ | 41 |
| $42 .$. | 1. 0798 | 1. 0626 | 1. 0587 | 37-46 | 10.874 | 1. 0917 | 42 |
| 43.. | 1. 0540 | 1. 0936 | 1. 1011 | 38-47 | 11.295 | 1.1345 | 43 |
| 44.. | 1. 1793 | 1.1615 | 1. 1557 | 39-48 | 11.757 | 1.1812 | 44 |
| $45 .$. | 1.2447 | 1. 2210 | 1. 2207 | 40-49 | 12.263 | 1.2326 | 45 |
| 46.. | 1. 2474 | 1.2848 | 1. 2887 | 41-50 | 12. 818 | 1. 2888 | 46 |
| 47.. | 1. 4079 | 1. 3689 | 1. 3650 | 42-51 | 13.425 | 1. 3505 | 4 |
| 48.. | 1. 4147 | 1. 4501 | 1.4547 | 43-52 | 14.090 | 1. 4177 | 48 |
| 49.. | 1. 5297 | 1.5444 | 1. 5439 | 44-53 | 14.816 | 1. 4907 | 49 |
| $50 .$. | 1.6497 | 1.6220 | 1.6120 | 45-54 | 15.611 | 1.5714 | 50 |

[^32]Table II-Continued.

| Age. | (d) | (e) | (f) | Decade. | (g) | (h) | Age. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 51. | 1.7333 | 1. 6581 | 1. 6665 | 46-55 | 16. 481 | 1. 6593 | 51 |
| 52 | 1.7070 | 1. 7281 | 1. 7251 | 47-56 | 17.432 | 1. 7549 | 2 |
| 53 | 1.7221 | 1. 8234 | 1.8259 | 48-57 | 18.473 | 1.8599 | 53 |
| 54 | 1. 8996 | 1.9657 | 1.9764 | 49-58 | 19.614 | 1. 9750 | 54 |
| 55 | 2. 2966 | 2.1514 | 2. 1326 | 50-59 | 20.864 | 2.1008 | 55 |
| 56 | 2. 3045 | 2.2701 | 2.2783 | 51-60 | 22.235 | 2.2392 | 56 |
| 57 | 2.3903 | 2. 3998 | 2.3976 | 52-61 | 23.740 | 2. 3959 | 57 |
| 58 | 2.5133 | 2. 5368 | 2.5303 | 53-62 | 25. 391 | 2.5571 | 58 |
| 59.. | 2.5285 | 2. 6990 | 2.7195 | 54-63 | 27.205 | 2.7402 | 59 |
| 60. | 3.1197 | 2.9688 | 2.9541 | 55-64 | 29. 198 | 2. 9417 | 60 |
| 61 | 3. 2552 | 3. 2234 | 3.2248 | 56-65 | 31.388 | 3. 1630 | 61 |
| 62 | 3. 4551 | 3.4873 | 3. 4953 | 57-66 | 33.793 | 3. 4064 | 62 |
| $63 .$. | 3.7474 | 3.7711 | 3.7525 | 58-67 | 36. 435 | 3. 6741 | 63 |
| 64 | 4. 0101 | 4.0053 | 4. 0133 | 59-68 | 39. 334 | 3. 9679 | 64 |
| $65 .$. | 4.3602 | 4. 3065 | 4. 3256 | 60-69 | 42.514 | 4.2911 | 65 |
| $66 .$. | 4.6650 | 4.7110 | 4. 6986 | 61-70 | 45. 999 | 4. 6454 | ${ }_{6} 6$ |
| $67 .$. | 4. 8932 | 5. 0639 | 5. 0409 | 62-71 | 49.812 | 5. 0334 | 67 |
| 68. | 5.5426 | 5. 3338 | 5. 3803 | 6:-72 | 53.980 | 5. 4584 | 68 |
| 69.. | 6. 0906 | 5.7196 | 5. 6290 | 64-73 | 58.528 | 5. 9224 | 69 |
| 70.. | 5. 6156 | 5. 9548 | 6. 0544 | 65-74 | 63.482 | 6. 4286 | 70 |
| 71.. | 6.2011 | 6. 6791 | 6. 6521 | 66-75 | 68.868 | 6. 9794 | 71 |
| 72.. | 7.9809 | 7.5365 | 7.5263 | 67-76 | 74.711 | 7.5778 | 72 |
| 73. | 7.8641 | 8.4413 | 8. 4927 | 68-77 | 81.036 | 8. 2260 | 73 |
| 74 | 10.5370 | 9.4102 | 9. 3078 | 69-78 | 87.865 | 8. 9269 | 74 |
| 75. | 9. 4621 | 9.9458 | 10.000 | 70-79 | 95.221 | 9. 6824 | 75 |
| 76.. | 10.624 | 10.575 | 10.568 | 71-80 | 103.12 | 10.493 | 6 |
| 77. | 10.869 | 11.278 | 11. 269 | 72-81 | 111.58 | 11. 366 | 77 |
| 78. | 12.303 | 12.101 | 12. 101 | 73-82 | 120.62 | 12.298 | 78 |
| 79.. | 13. 594 | 13.185 | 13.250 | 74-83 | 130.23 | 13.286 | 79 |
| $80 .$. | 14.080 | 14.668 | 14.599 | 75-84 | 140. 42 | 14.336 | 80 |
| $81 .$. | 15.970 | 16.039 | 16. 058 | 76-85 | 151. 19 | 15. 452 | 1 |
| 82. | 17.214 | 17.477 | 17.578 | 77-86 | 162.53 | 16. 623 | 82 |
| $83 .$. | 20.673 | 18.968 | 18.639 | 78-87 | 174.40 | 17.849 | 83 |
| 84. | 18.030 | 19.487 | 19.930 | 79-88 | 186.79 | 19.133 | 84 |
| $85 .$. | 21. 627 | 21.294 | 21.070 | 80-89 | 191.65 | 20.465 | 85 |
| 86. | 21. 698 | 22.214 | 22.020 | 81-90 | 212.93 | 21. 845 | 86 |
| $87 .$. | 21.687 | 22. 307 | 22.747 | 82-91 | 226.56 | 23. 259 | 87 |
| $88 .$. | 28.462 | 23.571 | 23.056 | 83-92 | 240.45 | 24.703 | 88 |
| 89.. | 19.355 | 23. 608 | 23.958 | $84-93$ | 254.51 | 26. 163 | - |
| 90.. | 22.667 | 25.172 | 23.247 | 85-94 | 268.61 | 27.638 | 90 91 |
| 91. | 31.034 | 27.515 | 27.206 | $86-95$ | 28.62 | 29.100 | 91 |
| 92.. | 29. 427 | 29.141 | 29.510 | 87-96 | 296. 36 | 30.539 | 92 |
| $93 .$. | 30.979 | 31. 644 | 31. 336 | 88-97 | 309. 66 | 31.935 | 93 |
| 94.. | 32.531 | 32. 927 | 32. 921 | 89-98 | 322.28 | 33.262 | 94 |
| 95. | 34.251 | 33.975 | 34.182 | 90-99 | 333.99 | 34.500 | 95 |
| 96. | 35. 805 | 35.829 | 35.687 | 91-100 | 344.50 | 35. 618 | 96 |
| 97. | 37.541 | 37. 458 | 37.521 | 92-101 | 353.50 | 36. 584 | 97 |
| 98.. | 39.133 | 39.250 | 39.155 | 93-102 | 360.63 | 37. 363 | 98 |
| 99. | 41.089 | 40.962 | 41.226 | 94-103 | 365. 50 | 37.914 | 99 |
| 100.. |  |  |  | 95-104 | 367.66 366.62 |  | 101 |
| 101.. |  |  |  | 96-105 | 366.62 | 45.739 | 102 |
| 103.. |  |  |  |  |  | 50.404 | 103 |
| 104.. |  |  |  |  |  | 56.098 | 104 |
| 105.. |  |  |  |  |  | 62.821 | 105 |
| 106.. |  |  |  |  |  | 70.573 | 106 |
| 107.. |  |  |  |  |  | 79.353 | 107 |
| 108.. |  |  |  |  |  | -89.000 | 109 |
| 109. |  |  |  |  |  | 100.000 | 109 |

The foregoing method affords a ready means of diminishing the irregularities of a series without removing them altogether. It can be proved that in a series of the $m$ th order, if any $m+1$ or more consecutive terms are adjusted by any single formula, such as (16) or (20), the adjusted values will themselves form a series of the $m$ th order. But, although the order of the series remains unchanged, the absolute values of the differences are in general diminished, and thus an approximate graduation is secured.

## THIRD METHOD OF ADJUSTMENT.

The second method can be combined with ordinary interpolation in such a way as to furnish an adjusted series of any given order, extending to any desired number of places of decimals. For example, let the terms of series $(f)$ in Table II be grouped together by decades of age, as was done in forming $(c)$ in Table I. The ninety terms form nine groups of ten terms each. Their sums are-

| $\mathrm{S}_{1}=4.50521$ | $\mathbf{S}_{4}=12.26340$ | $\mathbf{S}_{7}=95.22130$ |
| :--- | :--- | :--- |
| $\mathrm{~S}_{2}=6.80581$ | $\mathrm{~S}_{5}=20.86420$ | $\mathbf{S}_{8}=199.65500$ |
| $\mathbf{S}_{3}=8.79641$ | $\mathbf{S}_{6}=42.51440$ | $\mathbf{S}_{9}=333.99100$ |

These nine values form a series which has eight orders of differences, as follows:

$$
\begin{array}{llll}
\Delta_{1}=2.30060 & \Delta_{3}=1.78639 & \Delta_{5}=2.38714 & \Delta_{7}=-16.70885 \\
\Delta_{2}=-.31000 & \Delta_{4}=1.87103 & \Delta_{6}=3.44640 & \Delta_{8}=-7.75719
\end{array}
$$

Using the ordinary formula for interpolation by finite differences, we can obtain nine equidistant values between every two terms of this series, so as to make 81 terms in all, forming a perfectly graduated series of the eighth order. The terms of this series are approximately the sums of the terms in $(f)$ for every possible decade of age, commencing with 10 to 19,11 to 20,12 to 21 , \&c., and ending with 90 to 99 . To construct the series, nine consecutive terms were carefully computed, their eight orders of differences were taken, and the rest of the series was constructed therefrom by simple additions and subtractions. One great advantage of this mode of procedure is, that the agreement of the values thus found for the decades $10-19,20-29$, \&c., with the given values $S_{1}, S_{2}, \& c$., furnishes a convenient test of the accuracy of the whole work. It is necessary, however, to carry out the values of the function and the differences to a large number of places of dec̣imals, otherwise the error represented by the neglected figures will accumulate so as finally to vitiate some of the results. In the present case, the decimals were carried out as far as they would go; that is, to twenty places.

The series is readily extended by the same law, so as to comprise all the possible decades of age from 4-13 to $96-105$. Thus completed, it is given in column (g). Now let $\mathbb{S}_{1}, \mathbb{S}_{2}, \mathbb{S}_{3}, \mathbb{S}_{4}$, be any four consecutive terms in it, and in formula (8) take-

$$
n_{1}=n_{2}=10, \quad a_{1}=\frac{3}{2}, \quad a_{2}=\frac{1}{2}, \quad x=0, \quad n=1, \quad \mathbf{S}=u^{\prime}
$$

then we have-

$$
\begin{equation*}
u^{\prime}=\frac{1}{80}\left[21\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-17\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)\right] \ldots . \tag{30}
\end{equation*}
$$

This formula gives an adjusted value for any term in series $(f)$ by means of the sums of the terms in the four nearest decades as given in series (g). For instance, at the age 35 the value obtained is-

$$
\begin{aligned}
& u^{\prime}=\frac{1}{80}[21(8.7964+9.0228)-17(8.5848+9.2672)] \\
& =.88399
\end{aligned}
$$

Column ( $h$ ) shows the graduated series, carried to as many places of decimals as are needed in order to give five significant figures. It is of the eighth order, and the arithmetical means of the terms in the nine decades $10-19,20-29, \& c$., are approximately equal to those in series $(f)$, though not precisely so. This method of adjustment, however, has one advantage, namely, that it enables us to divide a given series into a large number of groups, and make the graduated series of as high an order as we please, without previously obtaining formulas like (E) and (F), which require some labor when the number of groups is increased. If the number of terms in a group is other than ten, it will be easy to find a corresponding formula similar to (30). When it is an odd number the formula will be derived from (13) instead of from (8). For example, with eleven terms in a group we have-

$$
\begin{align*}
& n_{1}=n_{2}=11, \quad a_{1}=1 \\
& u^{\prime}=S_{2}-\frac{5}{I T}\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right) . . \tag{31}
\end{align*}
$$

and (13) becomes-
giving the adjusted value of a term by means of the sums of the terms in the three nearest groups of eleven terms each.

Series ( $h$ ) shows a minimum at the age 12 , and increases continuously thereafter. It terminates at the age 99 , and must not be extended farther by the same law, for since $(g)$ is a series of an even order with the final difference, $\Delta_{8}$, negative, it will, if produced far enough, diminish at both ends instead of increasing as the rate of mortality does. The limit of old age is evidently not reached until one year after the point where the probability of dying within a year becomes unity, that is, certainty. The position of the limit is very doubtful. The old Combined Experience table places it at 100, the Carlisle table at 105, the English Life Table No. 3 at 108, the French table of Deparcieux at 95 , the tables of Duvillard and De Montferrand at 110, and the United States census table of 1860 at 106. Owing to the paucity of reliable observations at the greatest ages, the termination of series ( $h$ ), or that of any other graduated table, must necessarily be somewhat artificial. This is not of much consequence in practice, for the chance of attaining any age beyond 100 is so small as to make but little difference in the value of an assurance or annuity for a person in middle life. If we assume 110 as the limit in the present case, then from the three known values of the probability for the ages 98,99 , and 109 , the values for the
ages 100 to 108 can be computed by ordinary interpolation. Formula (2) may be used for this purpose. If we take-

$$
n_{1}=n_{2}=n_{3}=1, \quad n=1, \quad S=u, \quad S_{1}=u_{1}, \quad S_{3}=u_{2}, \quad S_{3}=u_{3}
$$

that formula reduces to-

$$
\left.\begin{array}{l}
\mathbf{P}=a_{3} x\left(x-a_{3}\right)  \tag{32}\\
\mathbf{Q}=a_{1} x\left(x+a_{1}\right) \\
\mathbf{R}=a_{1} a_{3}\left(a_{1}+a_{3}\right) \\
u=\frac{1}{\mathbf{R}}\left[(\mathbf{R}-\mathbf{P}-\mathbf{Q}) u_{2}+\mathbf{P} u_{1}+\mathbf{Q} u_{3}\right]
\end{array}\right\}
$$

If $u_{1}, u_{2}, u_{3}$, denote any three terms in a series, and the origin of coördiuates is at $u_{2}$, and $a_{1}$ and $a_{3}$ denote the positive distances of $u_{1}$ and $u_{3}$ from $u_{2}$, the above formula enables us to interpolate any fourth term, $u$, whose abscissa is $x$. If we now take-

$$
a_{1}=1, \quad a_{3}=10, \quad u_{1}=37.363, \quad u_{2}=37.914, \quad u_{3}=100
$$

formula (32) becomes-

$$
u=37.914+1.0653 x+.51433 x^{2}
$$

When the values $1,2,3$, \&c., areassigned to $x$ in this equation, the resulting values of $u$ will be the desired terms for the ages $100,101,102, \& c$, as they stand in column ( $h$ ). The continuity of this added portion with the rest of the series may be improved a little by adjusting, with formula (20), a few of the terms adjacent to the point of junction. The adjusted values are as follows:


Series ( $h$ ), thus amended, is ready for practical use in the construction of commutation tables.

It is not claimed that this series is the best one which can possibly be obtained by similar methods. The preliminary adjustment by the second method admits of some variation, and repeated trials would be required to determine whether the form of the final series might not be varied with advantage by making it of some other order than the eighth, or by taking the groups between some other limits than 10 and 99 , or by both these modifications together. But it is believed that the graduation here obtained is accurate enough for practical purposes, and will compare favorably with that of any table now in use.

We do not know, and perhaps never can know, anything definite respecting the precise analytical form of that function which we call the law of mortality. Various formulas, mostly transcendental, have been devised to express it, but no one of them has yet received universal recognition as correct to the exclusion of all others. While this state
of the case continues, the problem of constructing a table of mortality must be regarded as, to some extent, an indeterminate one. Not only is absolute accuracy unattainable, but we cannot even decide, by the method of least squares, that a certain result is the most probable of any; for the true form of the function being unknown, any particular residual error, or difference between the observed and computed values of a term, will in general be the aggregate of two errors, one of them due to the difference of form between the assumed function and the true one, and the other due to the error of observation or difference between the observed value and the true value. The latter portion only can be of the nature of accidental errors, so as to be subject to that law of distribution which the method of least squares assumes, and which is derived from the theory of probabilities. Hence, we cannot infer that because we have made the sum of the squares of the residuals a minimum, the resulting values of the constants which enter into the assumed equation of the series must be the most probable values. To justify such an inference, it would be necessary to make the sum of the squares of the accidental portions of the residuals alone a minimum; but we have no means of effecting this, for we cannot separate the accidental portions from the others. When the method of least squares is applied under circumstances like these, it loses its peculiar claims to theoretical accuracy, and becomes merely a method of interpolation, whose merits are to be judged, like those of other methods, by the amount of labor required in obtaining the final results, and by the degree of accuracy with which these results represent the observations. We may presume that the best method of reduction for tables of mortality is that which will give, in the simplest manner, a graduated series conforming to those conditions which are known to govern such tables, and representing the observations with the necessary degree of accuracy. In behalf of the method here proposed, it may be said that the process of computation is comparatively simple; that the observations are represented with great accuracy throughout all the middle ages of life, which is just the portion where accuracy is most important in practice; and that a transcendental formula, if it contains not more than three or four constants, will be very likely to prove inferior in this respect.
From all the foregoing considerations we conclude that a very good way to graduate an experience rate of mortality for insured lives will be, to form a series like (d), expressing the probability of dying within a year, at each age, and to adjust it approximately, in the first place, by some formula or formulas under the second method, and then, dividing the adjusted terms into the proper number of groups, to complete the graduation by either the first or the third method. Treated in this way, the arithmetical means of the terms in the several groups will be brought nearer to their normal value than they would be if the approximate or preparatory adjustment were omitted.

In constructing a rate for general population from census returns and
registration of deaths, it will probably be best to adjust the population for each year of age at each ceusus approximately by the second method; that is, by (20) or some similar formula. The returns of two or more census enumerations thus adjusted will enable us to compute approximately, by known methods, the mean population living within each year of age during the period embraced by the registry of deaths; and from this series the mean number of persons who annually attained each year of age during that period cau be found by (28) or some similar formula. The mean number of deaths annually occurring within each year of age must also be adjusted approximately by the second method, and then we shall only have to divide these annual deaths for each year of age by the mean number of persons annually attaining such age, to obtain an approximately adjusted series expressing the probability of dying within a year at each age. The graduation of this series can be completed by either the first or the third method, and from it we can construct the usual series of the numbers who live to attain each year of age out of a given number of persons who are born.

It should be remarked, however, that in infancy and early childhood the rate of mortality varies so rapidly that the years ought not to be grouped together as in the first and third methods. But these years are unimportant so far as life insurance and annuities are concerned, and for practical purposes it will suffice to have a completely graduated series from the age of ten or fifteen up to the limit of old age, and to adjust the series at the earliest ages by the second method only, or not at all. The latter alternative is perhaps the best, since the ages of young children can be ascertained with greater certainty than those of adults.

The accuracy of a series obtained by the first or the third method will be greatest at and near the middle, and least at the extremities. If it should be found that the graduated values at either end of a table of mortality thus constructed are sensibly erroneous, they can be rejected, and their places supplied by the original values, and the adjustment of these, and their continuity with the graduated portion, can be approximately secured by the use of some formula under the second method.

## METHOD OF CONSTRUCTING A TABLE OF MORTALITY WITHOUT ANY REGISTRATION OF DEATHS.

It has been proposed to determine the law of mortality for general population throughout a whole country by means of two successive census enumerations, taken, for instance, at intervals of ten years, as is now the case in the United States and in Great Britain, together with a reg. istry of the immigration and emigration which occurs during the intervening ten-year period. If at the first census a certain population, $\mathrm{P}_{\mathrm{m}}$, is returned as aged $m$ and under $m+1$ years, then at the second census the survivors among them will be returned as aged $m+10$ and under
$m+11$ years, and the difference $\mathrm{P}_{\mathrm{m}}-\mathrm{P}_{\mathrm{m}}+10$ between these two enumerations will be the number of deaths which have occurred out of the population $P_{m}$ within the ten-year period, if there has been no immigration or emigration, or if the immigration and emigration have been equal, so as to balance each other. If we regard $P_{m}$ and $P_{m+10}$ as representing the numbers annually attaining the exact ages $m+\frac{1}{2}$ and $m+10 \frac{1}{2}$, then the fraction $\frac{P_{m}+10}{P_{m}}$ will denote the probability that a person aged $m+\frac{1}{2}$ will live ten years.

In the United States, however, the number of immigrants continually entering the country is so large as to become very important in this connection. Emigration from the country is comparatively small; but assuming, for the sake of generality, that there has been a registry kept of the ages of both immigrants and emigrants, let us denote by I the number of persons who have entered the country during the tenyear period, and who are of such age as to have been $m$ and under $m+1$ years old at the time of the first census, and let $E$ denote the number of persons of similar age who have left the country during the same period. Also let $D$ be the number of deaths which have occurred in the country out of the excess I-E of immigrants over emigrants, and let $\mathrm{P}_{\mathrm{m}}+{ }_{10}$ denote the population returned as aged $m+10$ and under $m+11$ at the second census. Then the portion of I-E surviving at the second census is $I-E-D$, and the difference $P_{m+10}-(I-E-D)$ is equal to that portion of the initial population $P_{m}$ which survives at the time of the second census. The probability that a person aged $m+\frac{1}{2}$ will live ten years is therefore expressed by-

$$
\frac{\mathbf{P}_{\mathrm{m}+10}-(\mathbf{I}-\mathbf{E}-\mathbf{D})}{\mathbf{P}_{\mathrm{m}}}
$$

All the quantities involved in this fraction are known excepting the deaths D ; and as this is a small number compared with the others, the result will not be seriously affected if we compute the value of $D$, or, what amounts to the same thing, compute the survivors (I-E-D), by means of any good table of general mortality, considering separately the excess of immigrants of the supposed age who have entered the country in each one of the ten years. (See the Assurance Magazine for April, 1867, page 289.)

We can thus obtain the probability of living ten years for the middle of every year of age throughout the whole term of life. If the statistics of population and migration are given in the first place by decades or other intervals of age, the numbers can be distributed among the single years by means of (3) or some similar formula derivable from (2), (8), or (C). On the other hand, if the statistics are given for single years, the irregularities of the series can be diminished by using some formula under the second method of adjustment. We may assume, then, that the probability of living ten years has been ascertained for the middle of each single jear of age, and that these probabilities form an approxi-
mately adjusted series. The problem which remains to be solved is, to find the probability of living one year at each age when the above-mentioned probabilities of living ten years are given.
It is an interesting point in relation to the whole subject of graduation of numerical series, that, instead of graduating a given series directly, we can take a constant function of each term in it, thus forming a new series, and, having graduated this, we can inversely derive from each of its terms a graduated value for the corresponding term in the original series. One consequence of this principle is, that if we take the logarithm of each term in the given series, and divide the series of logarithms thus formed into groups and graduate it by the first method, and then take the numbers corresponding to the graduated logarithms, we shall have a graduated series representing the given one, and possessing this property, that the products of the terms in the assumed groups in it will be severally equal to the products of the terms in the corresponding groups in the given series. This is evidently the case, because the sums of the logarithms of the terms in the assumed groups are equal in the two series. Furthermore, since the equation of the graduated series of logarithms enables us to interpolate the sum of the logarithms of the terms in any group when the sums of the logarithms of the terms in the assumed groups are given, it follows that when the products of the terms, in any assumed groups in a numerical series, are known, we can find, by interpolation, the product of the terms in any other group, or any single term.
Now let $p_{\mathrm{m}+k_{2},}, p_{\mathrm{m}+1_{k}, 9} p_{\mathrm{m}+2 k,}$, \&cc., denote the probabilities of living one year at the exact ages $m+\frac{1}{2}, m+1 \frac{1}{2}, m+2 \frac{1}{2}, \& c$. The chance of living through any one year of age is contingent upon having lived through the years which precede it, so that the probability that a person aged $m+\frac{1}{2}$ will live two years is equal to the product $p_{\mathrm{m}+\frac{1}{3}} \times p_{\mathrm{m}+1,2,9}$ and the probability that he will live ten years is equal to the continued product-

$$
p_{\mathrm{m}+2_{\mathrm{r}}} \times p_{\mathrm{m}+1_{6}} \times p_{\mathrm{m}+2 k_{2}} \times \text {. . . . . } \times p_{\mathrm{m}+9 \%_{h}}
$$

It appears, then, that the probabilities of living one year at each age form a series such that the product of any $n$ terms taken in a group is equal to the probability of living $n$ years at the age corresponding to the first term in the group; and hence, according to the principles which have been stated, we can find, by interpolation, the probabilities of living one year when the probabilities of living ten years are known.

Any twelve consecutive terms in a series will form three groups of ten terms each, and formula ( 2 ) will enable us to find any single term by means of the sums of the terms in the three groups. If we take-

$$
n_{1}=n_{2}=n_{3}=10, \quad a_{1}=a_{3}=1, \quad n=1, \quad S=u
$$

then (2) reduces to-

$$
u=\frac{1}{80}\left[74 \mathrm{~S}_{2}-33\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)+4\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) x+4\left(\mathrm{~S}_{1}+\mathrm{S}_{3}-2 \mathrm{~S}_{2}\right) x^{2}\right] \ldots(33)
$$

, Let $\mathbf{S}_{1}, \mathbb{S}_{2}$, and $\mathbf{S}_{3}$ represent the logarithms of the probabilities of living ten years at the ages $m+\frac{1}{2}, m+1 \frac{1}{2}$, and $m+2 \frac{1}{2}$, respectively; then if we
assign to $x$ the values $-\frac{1}{2},+\frac{1}{2},+\frac{3}{2}$, \&c., in succession, the resulting values of $u$ will be the logarithms of the probabilities of living one year at the ages $m+5 \frac{1}{2}, m+6 \frac{1}{2}, m+7 \frac{1}{2}, \& c$. If we take $x=0$, the value of $u$ will be the logarithm of the probability of living one year at the age $m+6$, and we shall have the simple formula-.

$$
\begin{equation*}
\log p_{\mathrm{m}+6}=\frac{1}{80}\left[74 \mathrm{~S}_{2}-33\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)\right] . . \tag{34}
\end{equation*}
$$

To illustrate the use of this by an example, and to test its accuracy at the same time, let us suppose that there is no migration, and assume that, in accordance with the English Life Table, No. 3, for males, the population living at the first census, between the ages of 54 and 55,55 and 56,56 and 57 , respectively is-

$$
P_{54}=212061, \quad P_{55}=206984, \quad P_{56}=201772
$$

and that the survivors at the second census are-

$$
\mathrm{P}_{64}=154139, \quad \mathrm{P}_{65}=147319, \quad \mathrm{P}_{66}=140299
$$

The logarithms of the probabilities of living ten years at the three ages $54 \frac{1}{2}, 55 \frac{1}{2}$, and $56 \frac{1}{2}$ are therefore-

$$
\begin{aligned}
& \mathrm{S}_{1}=\log \mathrm{P}_{64}-\log \mathrm{P}_{54}=\overline{1} .8614518 \\
& \mathrm{~S}_{2}=\log \mathrm{P}_{65}-\log \mathrm{P}_{55}=\overline{\mathrm{i}} .8523219 \\
& \mathrm{~S}_{3}=\log \mathrm{P}_{66}-\log \mathrm{P}_{56}=\overline{\mathrm{i}} .8421937
\end{aligned}
$$

and since $m=54$, we find that the logarithm of the probability of living one year at the age $m+6=60$ is-

$$
\log p_{60}=\frac{1}{80}\left[74 \mathrm{~S}_{2}-33\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)\right]=1.9856440
$$

This value differs but very little from the one which is actually given by the English table, namely-

$$
\log p_{60}=\log l_{61}-\log l_{60}=\log 176421-\log 182350=\overline{1} .9856445
$$

The method followed in the above example will be found sufficient for the determination of the probability of living one year after every birthday, except the first nine or ten of childhood and the last seven of old age. With the help of formula (33) we can find the probabilities for all the ages of childhood, except the first three or four, by assigning to $x$ the negative values $-1,-2,-3$, \&c., which will give values for $\log p_{\mathrm{m}+5}$, $\log p_{\mathrm{m}+4}, \log p_{\mathrm{m}+3}$, \&c. So, too, for the last years of life, we can find $\log p_{\mathrm{m}+7}, \log p_{\mathrm{m}+8}, \log p_{\mathrm{m}+9}$, \&c., by assigning to $x$ the positive values 1,2 , 3 , \&c. This will complete the series of values of $\log p$ from early childhood to extreme old age. As it will be already approximately adjusted, nothing more will remain but to divide it into groups of an equal number of terms each, and to make the final graduation by either the first or the third method. There will be a convenience in graduating the logarithms instead of the corresponding numbers, because $\log p$, and not $p$ itself, is what we require for computing in the most expedilious manner the numbers living to attain each year of age out of a given 20 s 71
number of persons who are born. It is quite possible, too, that the form of the series may be improved by this mode of procedure.
The foregoing method of reduction will evidently apply to cases where the interval between the two census enumerations is any whole number of years other than ten, or even a fractional number. Suppose it to be ten and one-half years for instance, and take-

$$
n_{1}=n_{2}=n_{3}=\frac{21}{2}, \quad a_{1}=a_{3}=1, \quad n=1, \quad S=u
$$

then formula (2) reduces to-

$$
\begin{equation*}
\mathrm{S}=\frac{1}{1008}\left[970 \mathrm{~S}_{2}-437\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)+48\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) x+48\left(\mathrm{~S}_{1}+\mathrm{S}_{3}-2 \mathrm{~S}_{2}\right) x^{2}\right] . \tag{35}
\end{equation*}
$$

Let $S_{1}, S_{2}$, and $S_{3}$ be the logarithms of the probabilities of living ten and one-half years at the ages $m+\frac{1}{2}, m+1 \frac{1}{2}$, and $m+2 \frac{1}{2}$ respectively; then if we assign to $x$ the values $-\frac{5}{4},-\frac{1}{4},+\frac{3}{4}, \& c$., in succession, the resulting values of $u$ will be the logarithms of the probabilities of living one year at the ages $m+5, m+6, m+7, \& c$. When $x=-\frac{1}{4}$, the formula becomes-

$$
\begin{equation*}
\log p_{\mathrm{m}+6}=\frac{1}{504}\left(482 \mathrm{~S}_{2}-211 \mathrm{~S}_{1}-223 \mathrm{~S}_{3}\right) \tag{36}
\end{equation*}
$$

from which values of $\log p$ can easily be found for all but the extreme ages of life.

If the interval is either exactly or approximately an odd number of years, there will be a slight advantage in deriving the formula of reduction from (8) rather than from (2). Suppose, for instance, that the second census is taken five years after the first one. In the series of logarithms of the probabilities of living one year at each age, any eight consecutive terms will form four groups of five terms each, and formula (8) will enable us to find any single term by means of the sums of the terms in these groups. If we take-

$$
n_{1}=n_{2}=5, \quad a_{1}=\frac{3}{2}, \quad a_{2}=\frac{1}{2}, \quad n=1, \quad S=u
$$

then (8) reduces to-

$$
\left.\begin{array}{c}
u=\frac{1}{80}\left[17\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-9\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)\right]+\frac{x}{480}\left[405\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)-103\left(\mathrm{~S}_{4}-\mathrm{S}_{1}\right)\right] \\
+\frac{x^{2}}{20}\left[\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)-\left(\mathrm{S}_{2}+\mathrm{S}_{3}\right)\right]+\frac{x^{3}}{120}\left[\left(\mathrm{~S}_{4}-\mathrm{S}_{1}\right)-3\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)\right] \tag{37}
\end{array}\right\}
$$

Let $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathbb{S}_{3}, \mathrm{~S}_{4}$, denote the logarithms of the probabilities of living five years at the ages $m+\frac{1}{2}, m+1 \frac{1}{2}, m+2 \frac{1}{2}, m+3 \frac{1}{2}$, respectively; then if $x$ takes the values $-1,0,+1, \& c$. , in succession, the resulting values of $u$ will be the logarithms of the probabilities of living one year at the ages $m+3, m+4, m+5$, \&c. For $x=0$ we have the simple formula-

$$
\begin{equation*}
\log p_{\mathrm{m}+4}=\frac{1}{80}\left[17\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-9\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)\right] \tag{38}
\end{equation*}
$$

which affords a ready means of determining $\log p$ for all the birthdays except the extreme ones of childhood and old age.

The general plan for graduating irregular series of numbers, whose application to the construction of tables of mortality has now been indicated, will undoubtedly be found useful in other directions. Every physical law is a mathematical relation between one or more variables and a function. To ascertain the form of this relation, or the law of the natural phenomenon, we must obtain, by observation or experiment, a number of values of the function corresponding to known values of the variable, and then endeavor to find some analytical formula which will connect and express them all. For a statement of the nature of this general problem, and of the graphical and tentative methods which have been employed for its solution, see the discussion of experiments for ascertaining the law of variation of the density of water at different temperatures, given by M. Jamin in the Cours de Physique de l'Ecole Polytechnique, Vol. II, pages 39 to 50. The number of observed values of the function is ordinarily much greater than the number of constants in the desired formula. If there is but one independent variable, and the observed values of the function are plotted as ordinates to a curve, the corresponding values of the variable being the abscissas, this curve will be a more or less irregular or wavy line, because the ordinates which fix successive points in it are subject to the errors of observation. In an exact equation of this line, the number of constants would, in general, be as large as the number of observations taken. The problem presented is, to simplify the equation by reducing the number of constants, while preserving a form of curve which shall approximate to the original one as closely as possible. Our first method of graduation secures such approximation by taking the ordinates of the original curve in groups, and making the arithmetical means of the ordinates in the corresponding groups in the new curve severally equal to those in the original one. The equation of the new curve can only contain as many constants as there have been groups assumed. This plan has obvious advantages over the one usually followed, which is, to select or compute as many normal ordinates to the original curve as there are to be constants in the equation of the new one, and then subject the new curve to the condition of passing through the extremities of these ordinates, thus making the accuracy of the new curve depend on that of the observations, as represented by the selected ordinates, instead of depending alike on all the observations in each group.
When it is not convenient to have the observed values of the function correspond to equidistant values of the variable in the first place, they can bẹ reduced to equidistant ones either graphically, or by ordinary interpolation with Lagrange's formula, or with (32), which is merely one form of a special case under it. The irregularities of the series may then be diminished by the second method of adjustment, and, finally, the first method will give an equation which will express the law of the
phenomenon so far as that law can be expressed by an algebraic and entire function.*
In practice, when this method is to be applied to the graduation of a particular series, it will not be essential to have the assumed groups contain an equal number of terms each, nor to make the groups consecutive. Their positions, and the number of terms they contain, may be entirely arbitrary. The integral-

$$
\mathrm{S}=\int_{x \rightarrow+n}^{x+\frac{+n}{n}}\left(\mathrm{~A}+\mathrm{B} x+\mathrm{C} x^{2}+\ldots \ldots+\mathrm{T} x^{\mathrm{m}}\right) d x
$$

expresses the sum S of the terms in any group in a series of the $m$ th order by means of the $m+1$ constants $\mathbf{A}, \mathrm{B}, \mathrm{C}, \& \mathrm{c}$. ., the number $n$ of terms which the group contains, and the abscissa $x$ of the middle point of the group, each term in the series being regarded as an area occupying, on the axis of $X$, a space equal to unity. In the case of any one of the assumed groups, we know the sum $S$ of the terms in it, and their number $n$, and the abscissa $x$ of their middle point, so that we have an equation of condition which, besides the $m+1$ constants $\mathbf{A}, \mathbf{B}, \mathbf{C}, \& c$., contains only numerical quantities. Each group assumed furnishes one such equation. By assuming $m+1$ groups we shall have as many equations as there are constants A, B, C, \&c., to be determined, and hence it will always be possible to find the numerical values of the constants. Substituting their values in the general expression for S , arranging the terms according to the powers of $x$, and putting $n=1$ and $S=u$, we shall have an equation of the form-

$$
u=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} x+\mathrm{C}^{\prime} x^{2}+\ldots . . . . \cdot+\mathrm{T}^{\prime} x^{m}
$$

which will be the equation of the graduated series, and from which that series may be constructed. It will have its $m$ th differences constant and the arithmetical means of the terms in the corresponding groups in it will be severally equal to those of the terms in the $m+1$ groups assumed in the original series.

But although the positions of the groups and the numbers of terms which they may contain are thus unlimited in theory, it will probably be best in most cases to make them consecutive and consisting each of the same number of terms. When the law of a series varies very rapidly in some places, and slowly in others, it may indeed be necessary to assume, at those portions of the series where the variation is most rapid, a larger number of groups, consisting of fewer terms each, than will be required in the portions where the variation is slow. Bat with a fixed number of groups, the process of finding the values of the constants A, $\mathrm{B}, \mathrm{C}, \& \mathrm{c}$., will be simplified if the groups are assumed so as to be symmetrically situated on either side of the origin of coördinates; that is, situated in such manner that for every group of terms whose abscissa

[^33]is $+x^{\prime}$ there shall be a group of an equal number of terms whose abscissa is $-x^{\prime}$, and vice versa.

Cases will often occur where the whole number of terms in a series is not an exact multiple of the number of groups we wish to assume, and therefore will not form the desired number of consecutive groups containing each an equal and entire number of terms. But it is not necessary that the number of terms in a group should be a whole number. If we suppose it to have a fractional part, then certain terms in the given series must be divided each into two portions, and each portion must be joined to its proper group. Every such term being geometrically represented by an area whose base is unity, and the two parts into which this unit is divided being known, the problem is, to divide the area into its two corresponding parts. We can often do this accurately enough for practical purposes by assuming that the two portions of the area are proportional to the two portions of the base; but a much closer approximation will be made by taking the term in question and the two others nearest to it as data for an interpolation by formula (A). Let $\mathrm{S}_{1}$, $S_{2}, S_{3}$, be the three terms, and let $n$ denote the first one of the two parts into which the base of $\mathrm{S}_{2}$ is divided; then if we take-
formula (A) reduces to-

$$
n_{1}=1, \quad x=-\frac{1}{2}(1-n)
$$

$$
S=\frac{n}{6}\left[2 S_{1}+5 S_{2}-S_{3}+3\left(S_{2}-S_{1}\right) n+\left(S_{1}+S_{3}-2 S_{2}\right) n^{2}\right] \ldots(39)^{*}
$$

where. $S$ is that portion of the area $S_{2}$ which corresponds to the first fractional part of the base. The other portion is of course $S_{2}-S$. For example, if we wish to divide the ninety terms of series $(f)$ into seven consecutive groups of an equal number of terms each, the number of terms in a group will be $\frac{90}{7}=12 \frac{6}{7}$. The sum of the terms in the first group will be composed of the twelve terms for the ages 10 to 21 inclusive, together with so much of the term for the age 22 as corresponds to the fractional interval $n=\frac{6}{7}$. The three terms for the ages 21,22 , and 23 are-

$$
\mathrm{S}_{1}=.67539, \quad \mathrm{~S}_{2}=.68445, \quad \mathrm{~S}_{3}=.67164
$$

and formula (39) gives for that part of $S_{2}$ which belongs to the first group the value $\mathbb{S}=.58695$, and the sum of the terms in the first group is therefore 6.42804 . The portion $\mathrm{S}_{2}-\mathrm{S}=.09750$ belongs to the second group. After the sums of the terms in all the other groups have been formed in the same way, the equation of a graduated series of the sixth order can be obtained by means of formula ( E ), just as when $n_{1}$ is a whole number. The accuracy of this last part of the work can be tested by the condition that the sum of all the terms in the graduated series must be precisely equal to the sum of all the terms in the original series ( $f$ ).

[^34]where $\Delta_{1}$ and $\Delta_{2}$ are the finite differences of the series $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}$.

We have remarked that when a series is graduated by means of formulas such as (A), (B), (C), \&c., the accuracy attained is greatest at the middle of the series and least at its extremities. The question then arises, whether the errors cannot be more equally distributed throughout the whole series by making the number of terms in a group smaller at the extremities and increasing up to the middle, instead of having the number the same for all the groups. When any particular law of increase is adopted, there will be no difficulty in finding corresponding formulas similar to (A), (B), \&c., by which to compute the values of the constants. For the results of some recent investigations by Tchebitcheff with regard to the best arrangement of the data in making ordinary interpolations, not from groups, but from single terms or ordinates, see the Traité de Calcul Différentiel of J. Bertrand, pages 512 to 521. These naturally lead to the supposition that when the method of groups is used, the best representation of a given series by another of algebraic form will be obtained by regarding the whole interval which the series occupies on the axis of X as being divided, not into equal portions, but into portions which are the projections upon it of equal divisions of a semicircle drawn upon that interval as a diameter, the number of these divisions being made equal to the number of groups assumed: Of course the number of terms in each group will in general be fractional. For a series of the second order, the numbers of terms in the three assumed groups will be-

$$
\begin{aligned}
& n_{1}=n_{3}=\frac{1}{2} N\left(1-\cos \frac{\pi}{3}\right)=\frac{1}{4} N \\
& n_{2}=N \cos \frac{\pi}{3}=\frac{1}{2} N
\end{aligned}
$$

where N denotes the whole number of terms in the series, so that $\frac{1}{2} \mathrm{~N}$ is the radius of the semicircle. In equation (1),

$$
\mathrm{S}=n\left[\mathrm{~A}+\mathrm{B} x+\mathrm{C}\left(x^{2}+\frac{1}{12} n^{2}\right)\right]
$$

we substitute for $n$ its three values $n_{1}, n_{2}$, and $n_{3}$ in succession, and for $x$ the three corresponding values-

$$
x=-\frac{3}{8} \mathrm{~N}, \quad x=0, \quad x=\frac{3}{8} \mathrm{~N}
$$

thus obtaining the three equations of condition-

$$
\begin{aligned}
& \mathrm{S}_{1}=\frac{1}{4} \mathrm{~N}\left(\mathrm{~A}-\frac{3}{8} \mathrm{BN}+\frac{7}{48} \mathrm{ON}^{2}\right) \\
& \mathrm{S}_{2}=\frac{1}{2} \mathrm{~N}\left(\mathrm{~A}+\frac{1}{48} \mathrm{CN}^{2}\right) \\
& \mathrm{S}_{3}=\frac{1}{4} \mathrm{~N}\left(\mathrm{~A}+\frac{3}{8} \mathrm{BN}+\frac{7}{48} \mathrm{CN}^{2}\right)
\end{aligned}
$$

These determine $A, B$, and $C$; and arranging the original equation according to the powers of $x$, we have the formula-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{3 N}\left[7 \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)\right] \\
\mathrm{B}=\frac{16}{3 N^{2}}\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) \\
\mathrm{C}=\frac{16}{\mathrm{~N}^{2}}\left[\left(\mathrm{~S}_{1}+\mathrm{S}_{3}\right)-\mathrm{S}_{2}\right] \\
\mathrm{S}=n\left(\mathrm{~A}+\frac{1}{12} \mathrm{C} n^{2}+\mathrm{B} x+\mathrm{C} x^{2}\right)
\end{array}\right\}(40
$$

In the same way we can find the values of four, five, \&c., constants in the general formula (12). For a series of the third order, the numbers of terms in the four groups are-

$$
\begin{aligned}
& n_{1}=n_{4}=\frac{1}{2} \mathrm{~N}\left(1-\cos \frac{\pi}{4}\right)=\frac{3}{4}(2-\sqrt{2}) \mathrm{N} \\
& n_{2}=n_{3}=\frac{1}{2} \mathrm{~N} \cos \frac{\pi}{4}=\frac{1}{4} \mathrm{~N} \sqrt{2}
\end{aligned}
$$

and the distances from the origin to the middle points of the groups are

$$
a_{1}=\frac{1}{8}(2+\sqrt{2}) \mathrm{N}, \quad a_{2}=\frac{1}{8} \mathrm{~N} \sqrt{2}
$$

When these values are substituted in formula (8), the constants reduce to-

$$
\left.\begin{array}{l}
A=\frac{1}{N^{2}}\left[(2 \sqrt{2}-1)\left(\mathrm{S}_{2}+\mathrm{S}_{3}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{4}\right)\right] \\
\mathrm{B}=\frac{4}{N^{2}}\left[3\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{4}-\mathrm{S}_{1}\right)\right]  \tag{41}\\
\mathrm{C}=\frac{24}{N^{2}}\left[\left(\mathrm{~S}_{1}+\mathrm{S}_{4}\right)-(\sqrt{2}-1)\left(\mathrm{S}_{2}+\mathrm{S}_{3}\right)\right] \\
\mathrm{D}=\frac{64}{N^{4}}\left[\left(\mathrm{~S}_{4}-\mathrm{S}_{1}\right)-\left(\mathrm{S}_{3}-\mathrm{S}_{2}\right)\right]
\end{array}\right\}
$$

For a series of the fourth order the numbers of terms in the five groups are-

$$
\begin{aligned}
& n_{1}=n_{5}=\frac{1}{2} \mathrm{~N}\left(1-\cos \frac{\pi}{5}\right)=.0954915 \mathrm{~N} \\
& n_{3}=n_{4}=\frac{1}{2} \mathrm{~N}\left(\cos \frac{\pi}{5}-\cos \frac{2 \pi}{5}\right)=\frac{3}{4} \mathrm{~N} \\
& n_{3}=\mathrm{N} \cos \frac{2 \pi}{5}=.3090170 \mathrm{~N}
\end{aligned}
$$

and proceeding as in the case of formula (40), we find that the constants are-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{\mathrm{~N}^{2}}\left[3.777709 \mathrm{~S}_{3}+\frac{1}{5}\left(\mathrm{~S}_{1}+\mathrm{S}_{5}\right)-.4111456\left(\mathrm{~S}_{2}+\mathrm{S}_{4}\right)\right] \\
\mathrm{B}=\frac{1}{\mathrm{~N}^{2}}\left[13.088544\left(\mathrm{~S}_{4}-\mathrm{S}_{2}\right)-\frac{48}{5}\left(\mathrm{~S}_{5}-\mathrm{S}_{1}\right)\right] \\
\mathrm{C}=\frac{1}{\mathrm{~N}^{3}}\left[55.33375\left(\mathrm{~S}_{2}+\mathrm{S}_{4}\right)-71.73251 \mathrm{~S}_{3}-\frac{144}{5}\left(\mathrm{~S}_{1}+\mathrm{S}_{5}\right)\right]  \tag{42}\\
\mathrm{D}=\frac{1}{\mathrm{~N}^{2}}\left[\frac{512}{5}\left(\mathrm{~S}_{5}-\mathrm{S}_{1}\right)-63.28668\left(\mathrm{~S}_{4}-\mathrm{S}_{2}\right)\right] \\
\mathrm{E}=\frac{256}{\mathrm{~N}^{5}}\left[\mathrm{~S}_{3}+\left(\mathrm{S}_{1}+\mathrm{S}_{5}\right)-\left(\mathrm{S}_{2}+\mathrm{S}_{4}\right)\right]
\end{array}\right\}
$$

We might go on in the same way to find formulas for constructing series of still higher orders. It will be noticed that in all these cases, in the expression for the final constant, the sums $\mathbb{S}_{1}, \mathbb{S}_{2}$, \&cc., have the same coefficient when taken without regard to sign, so that all the terms in a given series will be of equal weight in determining the coefficient of the highest power of $x$.

Nevertheless, such trials as have been made with this system of grouping have not resulted favorably for its use in constructing mortality tables. The series seems to be rather distorted by it. This is shown when we construct by formula (42) a series of the fourth order to represent the given series $(f)$. Here we have $N=90$, and consequently-

$$
n_{1}=n_{5}=8.594235, \quad n_{2}=n_{4}=22 \frac{1}{2}, \quad n_{3}=27.81153
$$

so that the sums of the terms in the five groups, as found by the aid of formula (39), are-

$$
\begin{array}{cr}
\mathbb{S}_{1}=3.63932 & \mathbb{S}_{3}=68.3619 \\
S_{2}=17.60021 \quad S_{4}=337.0553 \\
S_{5}=297.960
\end{array}
$$

the five constants are found to be-

$$
\begin{array}{lc}
\mathrm{A}=1.919514 & \mathrm{C}=.008277894 \\
\mathrm{~B}=.1673728 & \mathrm{D}=.0001512150 \\
\mathrm{E}=.0000006635611
\end{array}
$$

and the equation of the graduated series stands-

$$
\begin{aligned}
& u=1.920204+.1674106 x+.008278226 x^{2}+.0001512150 x^{3} \\
&+.0000006635611 x^{4}
\end{aligned}
$$

If the values $-\frac{1}{2},+\frac{1}{2},+\frac{3}{2}$, \&c., are assigned to $x$, the resulting values of $u$ are the terms in the graduated series for the ages $54,55,56, \& c$. The sum of all the terms in the series is equal to the sum of all the terms in $(f)$, as it should be. But it does not afford a good representation of $(f)$; especially in the first half. It begins at the age 10 with the value . 14024 , goes on increasing up to the age 27, where it has a maximum of .81152 , then diminishes up to the age 36 , where $i t$ has a minimum of .77662 , then increases to the close, having the value 41.690 at the age 99.

On the other hand, if we construct by formula (C) the equation of a similar series from five consecutive groups of eighteen terms each, the sums of the terms in the groups are-

$$
\begin{array}{cc}
\mathrm{S}_{1}=9.82520 & \mathrm{~S}_{3}=39.94320 \\
\mathrm{~S}_{2}=16.89333 & \mathrm{~S}_{4}=154.96600 \\
\mathrm{~S}_{5}=502.98900
\end{array}
$$

the five constants are-

$$
\begin{array}{ll}
\mathrm{A}=2.023103 & \mathrm{C}=.007188222 \\
\mathrm{~B}=.1433032 & \mathrm{D}=.0001722763 \\
\mathrm{E}=.000001434104
\end{array}
$$

and the equation of the graduated series is-

$$
\begin{aligned}
& u=2.023702+.1433463 x+.007188939 x^{2}+.0001722763 x^{3} \\
&+.000001434104 x^{4}
\end{aligned}
$$

This represents $(f)$ with a considerable approach to accuracy, commencing at the age 10 with the value .32319 , increasing continuously thereafter, and terminating at the age 99 with the value 43.443. This exam-
ple seems to indicate that so far as has yet been ascertained, the most advantageous mode of grouping is to make the groaps consecutive and composed of an equal number of terms each; a system which has, besides, the merit of greater simplicity.*
The algebraic and entire function-

$$
y=\mathrm{A}+\mathrm{B} x+\mathrm{C} x^{2}+\& \mathrm{c} .
$$

is of course not the only one which it is possible to employ for the purpose of graduating a given irregular series. If we take any other continuous function-

$$
y=\varphi(\mathbf{A}, \mathbf{B}, \mathbf{C}, . . . . . . . . T, x)
$$

then, as before, the integral-

$$
\mathrm{S}=\int_{x-+n}^{x++n} \varphi(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \ldots \ldots \mathrm{~T}, x) d x
$$

will express the sum $S$ of the terms in any group in the graduated series by means of the number $n$ of terms which that group contains, the abscissa $x$ of its middle point, and the constants A, B, O, . . . T. By assuming in the given series as many groups as there are constants, and giving to $\mathbb{S}, n$, and $x$ their numerical values taken from these several groups, we shall have as many equations of condition as there are constants to be determined; and if we can perform the operations necessary for finding the numerical values of the constants from these equations, then the equation of the graduated series can be easily formed; and the series itself can be constructed therefrom. This series will not have any one of its orders of differences constant, but it will be a graduated series nevertheless, and the arithmetical means of the terms in the corresponding groups in it will be severally equal to those in the original series. It will, no doubt, sometimes be possible to find in this way a transcendental equation which will express a given series more advantageously than an algebraic equation could do.

We may here notice a peculiarity of the circular function-

$$
\begin{aligned}
& y=\mathrm{A}+\mathrm{B} \sin \left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{C} \cos \left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{D} \sin 2\left(\frac{2 \pi x}{\mathrm{~N}}\right) \\
& +\mathrm{E} \cos 2\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{F} \sin 3\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{G} \cos 3\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\& \mathrm{c} .
\end{aligned}
$$

in which $N$ denotes the number of terms in the circular period, or the length of the period measured on the axis of X , so that if the values $x^{\prime}$, $x^{\prime}+\mathrm{N}, x^{\prime}+2 \mathrm{~N}, \& \mathrm{c}$., are successively assigned to $x$, the value of $y$ will remain unchanged. The arithmetical mean of any $n$ terms taken in a group, and also the mean value of the ordinate within any interval $n$, will be-

$$
\mathrm{M}=\frac{\mathrm{S}}{n}=\frac{1}{n} \int_{x-1 n}^{x+\frac{1 n}{}} y d x
$$

*This may be a too hasty conclusion. Other trials have since shown that (40), (41), and (42) do sometimes, and perhaps generally, give the best results.
and consequently-

$$
\begin{gathered}
\mathrm{M}=\mathrm{A}+\frac{\mathrm{N}}{\pi n} \sin \frac{\pi n}{\mathrm{~N}}\left[\mathrm{~B} \sin \left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{C} \cos \left(\frac{2 \pi x}{\mathrm{~N}}\right)\right] \\
+\frac{\mathrm{N}}{2 \pi n} \sin \frac{2 \pi n}{\mathrm{~N}}\left[\mathrm{D} \sin 2\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{E} \cos 2\left(\frac{2 \pi x}{\mathrm{~N}}\right)\right] \\
+\frac{\mathrm{N}}{3 \pi n} \sin \frac{3 \pi n}{\mathrm{~N}}\left[\mathrm{~F} \sin 3\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{G} \cos 3\left(\frac{2 \pi x}{\mathrm{~N}}\right)\right]+\& \mathrm{c} .
\end{gathered}
$$

The expressions for $S$ and $M$ are thus identical in form with the expression for $y$, the constants $\mathbf{B}$ and $\mathbf{C}, \mathbf{D}$ and $\mathbf{E}, \mathbf{F}$ and $\mathbf{G}$, \&c., being merely multiplied, in the expression for $\mathbf{M}$, by the known factors-

$$
\left(\frac{\mathrm{N}}{\pi n} \sin \frac{\pi n}{\mathrm{~N}}\right), \quad\left(\frac{\mathrm{N}}{2 \pi n} \sin \frac{2 \pi n}{\mathrm{~N}}\right), \quad\left(\frac{\mathrm{N}}{3 \pi n} \sin \frac{3 \pi n}{\mathrm{~N}}\right), \& \mathrm{c}
$$

This property has already been discovered, and utilized in forming the equations of curves representing annual variations of temperature, the observed monthly means being taken as data.* (See the Edinburgh New Philosophical Journal for July, 1861, and the American Journal of Sciences and Arts for January and September, 1863.) $\dagger$ The quantity $M$ is there regarded as the mean value of the infinite number of ordinates, or "instantancous temperatures," which fall within the interval $n$, and not as the arithmetical mean of a finite number $n$ of terms taken in a group.

In general, to obtain an expression for the sum $S$ of the terms in a group, it is not necessary that any integration should be performed. Since the form of the function $\varphi$ is arbitrary, it follows that the form of $\int y d x$ is arbitrary also, and may be assumed at pleasure. Denoting by $f(x)$ any continuous function of one variable, let us substitute in the place of the variable first $x+\frac{1}{2}$ and then $x-\frac{1}{2}$, and let the difference between the two results be-

$$
\begin{equation*}
u=f\left(x+\frac{1}{2}\right)-f\left(x-\frac{1}{2}\right) \tag{43}
\end{equation*}
$$

Let values in arithmetical progression, whose constant difference is unity, be successively assigned to $x$ in the above expression. In the series formed by the resulting values of $u$ let any group of $n$ terms be

[^35]Then, after integrating, we shall have-

$$
\begin{aligned}
S=A n & +\sin \left(\frac{\pi n}{N}\right)\left\{\mathrm{B}_{1} \sin \left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{C}_{1} \cos \left(\frac{2 \pi x}{\mathrm{~N}}\right)\right\} \\
& +\sin 2\left(\frac{\pi n}{\mathrm{~N}}\right)\left\{\mathrm{B}_{2} \sin 2\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{C}_{2} \cos 2\left(\frac{2 \pi x}{\mathrm{~N}}\right)\right\} \\
& +\sin 3\left(\frac{\pi n}{\mathrm{~N}}\right)\left\{\mathrm{B}_{3} \sin 3\left(\frac{2 \pi x}{\mathrm{~N}}\right)+\mathrm{C}_{3} \cos 3\left(\frac{2 \pi x}{\mathrm{~N}}\right)\right\}+\& \tau \mathrm{c}
\end{aligned}
$$

For other formulas, see Appendix IV.
$\dagger$ These articles are by J. D. Everest
considered, and let $a$ be the value of $x$ corresponding to the first term ; then the sum of the terms in the group is-

$$
\begin{gathered}
S=f\left(a+\frac{1}{2}\right)-f\left(a-\frac{1}{2}\right)+f\left(a+\frac{3}{2}\right)-f\left(a+\frac{1}{2}\right)+f\left(a+\frac{5}{2}\right)-f\left(a+\frac{3}{2}\right) \\
+\cdots \cdots \cdot+\left(a+n-\frac{1}{2}\right)-f\left(a+n-\frac{3}{2}\right)
\end{gathered}
$$

which cancels at once to-

$$
S=f\left(a+n-\frac{1}{2}\right)-f\left(a-\frac{1}{2}\right)
$$

Now, if $x^{\prime}$ be the value of $x$ corresponding to the middle of the group, we have-
and consequently-

$$
x^{\prime}=a+\frac{1}{2}(n-1)
$$

$$
a=x^{\prime}-\frac{1}{2} n+\frac{1}{2}
$$

so that the expression for S reduces to-

$$
S=f\left(x^{\prime}+\frac{1}{2} n\right)-f\left(x^{\prime}-\frac{1}{2} n\right) \cdot .(44)
$$

We can conceive that, by varying the form of the function $f$ and the values of the constants which it contains, the series of values of $u$ can be made to approximate more or less closely to any given series of equidistant numbers which follow some general law. Hence, to graduate such a given series, we have only to assume a function $f(x)$ of suitable form, and substituting in it first $x+\frac{1}{2} n$ and then $x-\frac{1}{2} n$ in place of the variable $x$, the difference between the two results will express the sum S of the terms in any group in the graduated series by means of the number $n$ of terms which that group contains, the abscissa $x$ of the middle point of the group referred to an assumed origin of coördinates, and the constants which are involved in the function $f(x)$. In the case of any single group the values of $n$ and $x$ are known, and the value of S being taken equal to the sum of the terms in the corresponding group in the given series, we shall have an equation of condition containing only the unknown constants and numerical quantities. By assuming as many groups as there are constants, we obtain a number of equations just sufficient to determine the values of the constants. Substituting these values in formula (43), we obtain the equation which expresses the empirical law of the given series, and from which the graduated one may be constructed. The axithmetical means of the terms in the assumed groups in the graduated series will be severally equal to those of the terms in the corresponding groups in the given one.
If we assume more groups than there are constants, there will result a number of equations of condition greater than the number of constants to be determined. The values of the constants can then be found by the method of least squares. In this way we may expect, in certain cases, to increase a little the degree of general accuracy with which the graduated series represents the given one, without at the same time increasing the number of constants and raising the degree of the equation. But of course the arithmetical means of the terms in the corresponding groups in the two series will now be only approximately
equal to each other, and the operations of finding and verifying the equation of the graduated series will become much more laborious. If we do not know beforehand what form the function ought to have, the most effectual means of increasing the accuracy of representation will be to increase the number of constants equally with the number of groups assumed. For instance, it is probable that a series of the sixth order, obtained either by the first or the third method, will represent an approximately adjusted series, such as $(f)$ in Table II, more accurately than any series of the fourth order, whether obtained with or without the aid of the principle of least squares, can possibly do.

The method of least squares can of course be used independently, for the purpose of graduating an irregular series of numbers. But every term will furnish one equation of condition, so that the number of equations will be as great as the whole number of terms in the series, and if this number is large the amount of labor required to find and verify the values of the constants becomes very considerable, while the method cannot be expected to have any advantage over the method of interpolation by groups, as regards the general accuracy of the result, except in cases where the assumed function is capable of expressing the true law of the natural phenomenon, or of approximating to it so closely that the errors resulting from the difference in the form of the function will be everywhere small enough to be neglected in comparison with the errors of observation. Applied to an algebraic and entire function, the general effect of the method of least squares will be to increase a little the accuracy of representation at the extremities of the series, at the cost of increased errors in the remaining portion. To illustrate this by an example, let us compare two equations, taken of the second degree for the sake of simplicity, each of them representing the first six terms of series ( $h$ ), the first equation being obtained by the method of groups and the second by the method of least squares. In the three consecutive groups of two terms each the sums are-

$$
\mathrm{S}_{1}=.83107, \quad \mathrm{~S}_{2}=.79689, \quad \mathrm{~S}_{3}=.84473
$$

and since $n_{1}=2$, formula (A) gives for the equation of the new series -

$$
u=.39717+.0017075 x+.0051262 x^{2}
$$

If we assign to $x$ the values $-\frac{5}{2},-\frac{3}{2},-\frac{1}{2}, \& c \cdot$, in succession, the resulting values of $u$ are the terms in the new series, as follows:

$$
\begin{array}{lll}
u_{1}=.42494, & u_{3}=.39760, & u_{5}=.41126 \\
u_{2}=.40614, & u_{4}=.39930, & u_{6}=.43348
\end{array}
$$

When these are compared with the original values in series ( $h$ ), their differences or errors, taken without regard to sign, are found to be-

| .00176, | .00101, | .00157 |
| :--- | :--- | :--- |
| .00177, | $.00100 ;$ | .00158 |

The sum of the squares of these errors is .0000132 .
Next, we form six equations of condition of the second degree from
the firstisix terms in series ( $h$ ), and find that by the method of least squares the equation of the new series is-

$$
u=.39710+.0015743 x+.0051468 x^{2}
$$

This gives for the terms in the new series-

$$
\begin{array}{lll}
u_{1}=.42533, & u_{3}=.39760, & u_{5}=.41104 \\
u_{2}=.40632, & u_{4}=.39918, & u_{6}=.43321 \\
\mathrm{e}-.0137, & .00101, & .00179 \\
.00195, & .00112, & .00131
\end{array}
$$

the errors are-
and the sum of the squares of the errors is .0000129 , which is a minimum. Comparing these results with the ones obtained by the method of groups, we see that nothing has really been gained in accuracy by employing the method of least squares, since the maximum error has been increased by it from .00177 to 00195 . Besides, the method of groups has a great advantage in the simplicity and brevity of the calculations required.*
The sum S of the terms in any group can be expressed in still another form by means of a series. When $f\left(x \pm \frac{1}{2} n\right)$ is expanded according to the powers of $\frac{1}{2} n$, it becomes-

$$
\begin{gathered}
f\left(x \pm \frac{1}{2} n\right)=f(x) \pm f^{\prime}(x)\left(\frac{n}{2}\right)+\frac{1}{2} f^{\prime \prime}(x)\left(\frac{n}{2}\right)^{2} \pm \frac{1}{2.3} f^{\prime \prime \prime}(x)\left(\frac{n}{2}\right)^{3} \\
+\frac{1}{2.3 .4} f^{\text {lv }(x)}\left(\frac{n}{2}\right)^{4} \pm \& c .
\end{gathered}
$$

where $f^{\prime}(x), f^{\prime \prime}(x)$, \&c., are the successive differential coefficients of $f(x)$. Consequently we have-

$$
\left.\begin{array}{rl}
\mathrm{S}=f\left(x+\frac{1}{2} n\right)-f\left(x-\frac{1}{2} n\right) \\
=n & {\left[f^{\prime}(x)+\frac{1}{2.3} f^{\prime \prime \prime}(x)\left(\frac{n}{2}\right)^{2}+\frac{1}{2.3 .4 .5} f^{\vee}(x)\left(\frac{n}{2}\right)^{4}\right.}  \tag{45}\\
& \left.+\frac{1}{2.3 .4 .5 .6 .7} f^{\mathrm{vi}(x)}\left(\frac{n}{2}\right)^{6}+\& c .\right]
\end{array}\right\}(4 .
$$

This series will terminate if $f(x)$ is algebraic and entire. To illustrate its application, let us assume-

$$
f^{\prime}(x)=\mathrm{A}+\mathrm{B} x+\mathrm{C} x^{2}
$$

then the other derivatives are-

$$
\begin{aligned}
& f^{\prime \prime \prime}(x)=\mathrm{B}+2 \mathrm{C} x \\
& f^{\prime \prime \prime}(x)=2 \mathrm{C}
\end{aligned}
$$

while $f^{\text {tv }}(x), f^{\tau}(x)$, \&c., are zero. We have accordingly-

$$
\begin{aligned}
\mathrm{S} & =n\left[f^{\prime}(x)+\frac{1}{2.3} f^{\prime \prime \prime}(x)\left(\frac{n}{2}\right)^{2}\right] \\
& =n\left[\mathrm{~A}+\mathrm{B} x+\mathrm{C}\left(x^{2}+\frac{1}{12} n^{2}\right)\right]
\end{aligned}
$$

[^36]which is identical with formula (1). It will be found that the general formula (11) can be obtained in this way more easily than in any other.
The particular feature of the first method of adjustment, that it makes the arithmetical means of the terms in the corresponding assumed groups in the new series precisely equal to those in the original one, is also characteristic of a method which has sometimes been employed in solving equations of condition. (See the Calculs Pratiques Appliqués aux Sciences d'Observation, by MM. Babinet and Housel, page 81.) If the law of a series is to be represented by an equation of the form-
$$
y=\mathrm{A}+\mathrm{B} \varphi(x)+\mathrm{C} \psi(x)+\& \mathrm{c} .,
$$
where $\varphi(x), \psi(x), \& c$., do not contain any constants to be determined, then there will subsist between any given terms or ordinates $y_{1}, y_{2}, y_{3}$, $\& c$. , and the corresponding abscissas $x_{1}, x_{2}, x_{3}, \& c$., the following equations of condition :
\[

$$
\begin{aligned}
& y_{1}=\mathrm{A}+\mathrm{B} \varphi\left(x_{1}\right)+\mathrm{C} \psi\left(x_{1}\right)+\& \mathrm{c} . \\
& y_{2}=\mathrm{A}+\mathrm{B} \varphi\left(x_{2}\right)+\mathrm{C} \psi\left(x_{2}\right)+\& \mathrm{c} . \\
& y_{3}=\mathrm{A}+\mathrm{B} \varphi\left(x_{3}\right)+\mathrm{C} \psi\left(x_{3}\right)+\& \mathrm{c} . \\
& \& \mathrm{cc} .
\end{aligned}
$$
\]

Let as suppose for example that there are only three constants, A, B, and $\mathbf{C}$, and that the number of terms in the given series is any greater number, for instance six. Then to reduce the six equations of condition to only three, we may add them together in pairs or groups of two, and, denoting the sums of the terms in the three groups by $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}$, we shall have-

$$
\begin{aligned}
& \mathrm{S}_{1}=2 \mathrm{~A}+\mathrm{B}\left[\varphi\left(x_{1}\right)+\varphi\left(x_{2}\right)\right]+\mathrm{C}\left[\psi\left(x_{1}\right)+\psi\left(x_{2}\right)\right] \\
& \left.\mathrm{S}_{2}=2 \mathrm{~A}+\mathrm{B}\left[\varphi\left(x_{3}\right)+\varphi\left(x_{4}\right)\right]+\mathrm{C}\left[\psi\left(x_{3}\right)+\psi\left(x_{4}\right)\right]\right] \\
& \mathrm{S}_{3}=2 \mathrm{~A}+\mathrm{B}\left[\varphi\left(x_{5}\right)+\varphi\left(x_{6}\right)\right]+\mathrm{C}\left[\psi\left(x_{5}\right)+\psi\left(x_{6}\right)\right]
\end{aligned}
$$

Here there are only as many equations as there are constants to be determined, and since $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}$, and $x_{1}, x_{2}$, \&c., are known from the original series, we can obtain the numerical values of the three constants. Let these be $\mathbf{A}^{\prime}, \mathrm{B}^{\prime}$, and $\mathrm{C}^{\prime}$; then the equation of the graduated series is-

$$
y=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \varphi(x)+\mathrm{C}^{\prime} \psi(x)
$$

and when the values $x_{1}, x_{2}, x_{3}$, \&c., are successively assigned to the variable in this equation, the resulting values of $y$ will be the terms of the graduated series, and the arithmetical means of the terms in the assumed groups will be the same in it as in the original series. This will always be the case, without regard to the number of terms in the series, or to the number of constants and groups to be assumed, or to the extent or position of the groups. It is not even necessary that the terms grouped together should be consecutive, nor that the abscissas $x_{1}, x_{2}, x_{3}$, \&e., should be in arithmetical progression.

- This method, however, labors under certain disadvantages when compared with the one which we have proposed. The computations it involves are much more laborious, especially when the number of con-
stants or the number of terms in the series is large; it does not give any general expression like (12) or (44) for the sum S of any $n$ terms taken in a group, and it does not permit the use of groups composed of a fractional number of terms.


## ADJUSTMENT OF A DOUBLE SERIES.

By methods entirely analogous to those which have been applied to functions of one variable, we can proceed to graduate an irregular double series or table of values of a function of two variables. The table is supposed to be arranged in the usual rectangular form, the successive values of each variable being equidistant. The intervals between any two such values, however, are not necessarily the same for both variables. The algebraic equation-

$$
z=\mathrm{A}+\mathrm{B} x+\mathrm{C} y+\mathrm{D} x^{2}+\mathrm{E} y^{2}+\mathrm{F} x y+\& c
$$

is the equation of a curved surface. The rectangular table being supposed to be situated in the plane of $\mathbf{X} \mathbf{Y}$, with its sides parallel to the axes of $X$ and $Y$, and its middle point coinciding with the origin of coordinates, let a series of equidistant vertical planes be drawn parallel to the plane of $Z Y$, and another series of planes in like manner parallel to the plane of Z X, so that the intersections of these planes with the plane of $X \mathbf{Y}$ shall form the divisions of the given table. Each of thése divisions is the base of a solid which is limited at the sides by the vertical planes and at the top by the curved surface. Every such solid may be regarded as representing the correśponding tabulated value of the function, and the sides of the bases are taken as unity, but the units lying in the directions of $x$ and $y$ are not necessarily equal to each other. If we assume a group of adjacent divisions of the table, situated so as to form a rectangle whose sides, parallel to the axes of $\mathbf{X}$ and $\mathbf{Y}$, consist each of $m$ and $n$ units respectively, then the solid included between this rectangular base, its limiting vertical planes, and the curved surface, will be represented by the integral-

$$
\mathrm{S}=\int_{y^{\prime}-\frac{1}{2} n}^{y^{\prime}+\frac{2}{3} n} d y \int_{x^{\prime}-\frac{1}{2} m}^{x^{\prime}+\frac{1}{2} m} z d x
$$

where $x^{\prime}$ and $y^{\prime}$ are the coördinates of the middle point of the rectangular base. Performing the integrations indicated, and omitting the accents from $x^{\prime}$ and $y^{\prime}$, we have-
$\mathrm{S}=m n\left[\mathrm{~A}+\mathrm{B} x+\mathrm{C} y+\mathrm{D}\left(x^{2}+\frac{1}{12} m^{2}\right)+\mathrm{E}\left(y^{2}+\frac{1}{12} n^{2}\right)+\mathrm{F} x y+\& c.\right]$.
This solid is evideutly the sum of the solids which belong to the several divisions of the assumed group, so that the formula expresses the sum $S$ of the terms in any rectangular group in the table by means of the numbers $m$ and $n$ of terms contained in each one of the sides of the group lying parallel to the axes of $X$ and $Y$ respectively, the coördinates $x$ and $y$ of the middle point of the group, and the constants $\mathbf{A}, \mathbf{B}$, $C$, \&c. For any group assumed we know the numerical values of $S, m$,
$n, x$, and $y$, so that every such group furnishes an equation of condition which, besides the constants A, B, C, \&c., contains only numerical quantities. By assuming as many groups as there are constants, we shall always be able to find numerical values for the constants, and substituting them in formula (46), and making-

$$
m=1, \quad n=1, \quad S=u
$$

we shall have an equation of the form-

$$
u=\mathbf{A}^{\prime}+\mathrm{B}^{\prime} x+\mathrm{C}^{\prime} y+\mathrm{D}^{\prime} x^{2}+\mathrm{E}^{\prime} y^{2}+\mathrm{F}^{\prime} x y+\& \mathrm{c}
$$

which will be the equation of the graduated table, and from which that table can be constructed by assigning to $x$ and $y$ the proper series of values differing from each other by unity, so that they shall represent in succession the coördinates of the middle point of each division of the table.

We can also make an approximate adjustment of a double series by formulas analogous to those which we have already found under the second method for adjusting an ordinary series. For example, any nine adjacent terms $u_{1}, u_{2}, u_{3}, \ldots u_{9}$ being grouped in a rectangle with three

terms on each side, as in the figure, let it be required to find a formula by which to adjust the value of the middle term $u_{5}$. Let us suppose that the equation of the curved surface is-

$$
z=\mathrm{A}+\mathrm{B} x+\mathrm{C} y+\mathrm{D} x^{2}+\mathrm{E} y^{2}
$$

then F and all the succeeding constants disappear, and formula (46) becomes-

$$
\begin{equation*}
\mathrm{S}=m n\left[\mathrm{~A}+\mathrm{B} x+\mathrm{C} y+\mathrm{D}\left(x^{2}+\frac{1}{12} m^{2}\right)+\mathrm{E}\left(y^{2}+\frac{1}{12} n^{2}\right)\right] . \tag{47}
\end{equation*}
$$

Now, in the rectangle $a a^{\prime}$ we have-

$$
S=u_{1}+u_{2}+u_{3}, \quad m=1, \quad n=3, \quad x=1, \quad y=0
$$

so that (47) reduces to-

$$
u_{1}+u_{2}+u_{3}=3\left(\mathrm{~A}+\mathrm{B}+\frac{13}{12} \mathrm{D}+\frac{3}{4} \mathrm{E}\right)
$$

So, too, in the rectangle $b b^{\prime}$ we have-

$$
\mathrm{S}=u_{4}+u_{5}+u_{6}, \quad m=1, \quad n=3, \quad x=0, \quad y=0
$$

and (47) becomes-

$$
u_{4}+u_{5}+u_{6}=3\left(\mathrm{~A}+\frac{1}{12} \mathrm{D}+\frac{3}{4} \mathrm{E}\right)
$$

Likewise the rectangle $c c^{\prime}$ gives-

$$
u_{7}+u_{8}+u_{9}=3\left(\mathrm{~A}-\mathrm{B}+\frac{13}{12} \mathrm{D}+\frac{3}{4} \mathrm{E}\right)
$$

Again, for the rectangle $a d^{\prime}$ we have-

$$
\mathrm{S}=u_{1}+u_{2}+u_{4}+u_{5}+u_{7}+u_{8}, \quad m=3, \quad n=2, \quad x=0, \quad y=\frac{1}{2}
$$

and (47) reduces to-

$$
u_{1}+u_{2}+u_{4}+u_{5}+u_{7}+u_{8}=6\left(\mathbf{A}+\frac{1}{2} \mathbf{C}+\frac{3}{4} \mathrm{D}+\frac{7}{12} \mathbf{E}\right)
$$

In like manner the rectangle $d c^{\prime}$ givos-

$$
u_{2}+u_{3}+u_{5}+u_{6}+u_{8}+u_{9}=6\left(\mathrm{~A}-\frac{1}{2} \mathrm{C}+\frac{3}{4} \mathrm{D}+\frac{7}{12} \mathrm{E}\right)
$$

We have thus obtained five equations by which to determine the five constants $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, in terms of the tabulated values $u_{1}, u_{2}, u_{3}, \& \mathrm{c}$. Now, in the middle one of the nine divisions we have-

$$
S=u_{5}, \quad m=1, \quad n=1, \quad x=0, \quad y=0
$$

and formula (47) becomes-

$$
u_{5}=\mathrm{A}+\frac{1}{12} \mathrm{D}+\frac{1}{12} \mathrm{E}
$$

Substituting in this the values of the constants $A, D$, and $E$; we arrive at the result-

$$
\begin{equation*}
u_{5}=\frac{1}{9}\left[5 u_{5}+2\left(u_{2}+u_{4}+u_{6}+u_{8}\right)-\left(u_{1}+u_{3}+u_{7}+u_{9}\right)\right] \tag{48}
\end{equation*}
$$

and this is the adjustment formula required. Its accuracy can easily be tested by trial with any table constructed from an equation of the form-

$$
u=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} x+\mathrm{C}^{\prime} y+\mathrm{D}^{\prime} x^{2}+\mathrm{E}^{\prime} y^{2}
$$

the adjusted value being in this case the same as the original one. Indeed, we shall find that the result is exact, even when the table has been constructed from a complete equation of the third degree.

Again, to adjust the value of a term occupying the middle of one side of the assumed rectangle, as $u_{2}$, for instance, we have-

$$
\mathrm{S}=u_{2}, \quad m=1, \quad n=1, \quad x=1, \quad y=0
$$

and consequently-

$$
u_{2}=\mathrm{A}+\mathrm{B}+\frac{13}{12} \mathrm{D}+\frac{1}{12} \mathrm{E}
$$

Substituting the values of $\mathrm{A}, \mathrm{B}, \mathrm{D}$, and E , we obtain the adjustment formula-

$$
\begin{equation*}
u_{2}=\frac{1}{9}\left[5 u_{2}+2\left(u_{1}+u_{3}+u_{5}+u_{8}\right)-\left(u_{4}+u_{6}+u_{7}+u_{9}\right)\right] \tag{49}
\end{equation*}
$$

In a similar way the adjusted value of a term like $u_{1}$, occupying one corner of the assumed rectangle, is found to be-

$$
u_{1}=\frac{1}{9}\left[5 u_{1}+2\left(u_{2}+u_{3}+u_{4}+u_{7}\right)-\left(u_{5}+u_{6}+u_{8}+u_{9}\right)\right] \ldots
$$

By one or other of the three formulas here given, the value of any term in an irregular table can be approximately adjusted, and, as in the case of an ordinary series, the weight of the term to be adjusted may be increased or diminished at pleasure.

21 s 71

## APPENDIX I.

## IMPROVED ADJUSTMENT FORMULAS.

We have seen that in (16) and similar formulas used for making preparatory adjustments by thesecond method, thelocal weight of the middle term can be increased or diminished if desired, and that, when the formula includes more than five terms, the weights of other terms besides the middle one can also be made to vary. We have employed this property in assigning to the several terms, weights increasing in arithmetical progression, from the extreme terms to the middle one, as in formula (20). But further investigation has shown that this arrangement of the weights, although it gives formulas which are very simple and easy of application, is not the best one in theory. To determine what the best arrangement is, we must consider that when one of these formulas is applied at any part of a series, all those terms which are not included by the formula have the weight zero; that as the adjustment progresses, when a term is first included by the formula its weight is negative; it then becomes positive, attains its maximum when the term occupies the middle position, then diminishes till it becomes negative again, and finally resumes the weight zero when the term is no longer included by the formula. To make this transition as unbroken and continuous as possible, it is evident that if we regard the weights as ordinates to a curve, the form of this curve should be as shown in the annexed figure, for a formula including seven terms whose positions 1, 2, 3, . . . 7, are laid off equidistantly on the axis of $\mathbf{X}$. The curve is symmetrical with respect to the middle ordinate or axis of Y , and is tangent to the axis
 of $X$ at the points 0 and 8, which are the positions of the two nearest terms not included by the formula. Such a curve has four points of inflexion, so that if it is of algebraic form, it must be of a degree not lower than the sixth. Assaming, then, that the series of weights from 0 to 8 inclusive is of the sixth order, and that it has maxima at the points 0 and 8 , these two conditions will sufficeto determine the two arbitrary numbers $k$ and $k^{\prime}$ in the formula-

$$
\begin{gathered}
u_{4}=\frac{1}{45+10 k+k^{\prime}}\left[\left(13+4 k+k^{\prime}\right) u_{4}+(13+4 k)\left(u_{3}+u_{5}\right)\right. \\
\left.+(8-k)\left(u_{2}+u_{6}\right)-5\left(u_{1}+u_{7}\right)\right]
\end{gathered}
$$

which holds good, as has been shown, for any seven consecutive terms in a series of the third or any lower order. Since the nine weights-
$0, \quad-5, \quad(8-k), \quad(13+4 k), \quad\left(13+4 k+k^{\prime}\right), \quad(13+4 k), \quad(8-k),-5,0$
are to form a series of the sixth order, their seventh differences will be zero, giving the equation-

$$
\begin{gathered}
-5-7(8-k)+21(13+4 k)-35\left(13+4 k+k^{\prime}\right) \\
+35(13+4 k)-21(8-k)-35=0
\end{gathered}
$$

Also, since there is to be a maximum at the initial term 0 , the differences of the series of weights must satisfy the condition-

$$
\begin{equation*}
\Delta_{1}-\frac{\Delta_{2}}{2}+\frac{\Delta_{3}}{3}-\frac{\Delta_{4}}{4}+\frac{\Delta_{5}}{5}-\frac{\Delta_{6}}{6}=0 \tag{51}
\end{equation*}
$$

giving the equation-

$$
1800+460(8-k)-472(13+4 k)+225\left(13+4 k+k^{\prime}\right)=0
$$

We have then two equations, from which the numbers $k$ and $k k^{\prime}$ are ascertained to be-

$$
k=\frac{2036}{637} ; \quad k^{\prime}=\frac{6679}{637}
$$

so that the adjustment formula becomes-

$$
u_{4}=\frac{1}{55704}\left[23104 u_{4}+16425\left(u_{3}+u_{5}\right)+3060\left(u_{2}+u_{6}\right)-3185\left(u_{1}+u_{7}\right)\right] \cdot(52)
$$

Here the nine weights-

$$
0, \quad-3185, \quad 3060, \quad 16425, \quad 23104, \quad 16425,3060,-3185,0
$$

form a series of the sixth order, and if their successive orders of differences are taken they will be found to satisfy the equation (51). The following formulas, comprising five, nine, and eleven terms respectively, possess properties similar to the above:

$$
\begin{gathered}
u_{3}=\frac{1}{1383}\left[783 u_{3}+400\left(u_{2}+u_{4}\right)-100\left(u_{1}+u_{5}\right)\right] \\
u_{5}=\frac{1}{58773}\left[19375 u_{5}+15696\left(u_{4}+u_{6}\right)+7056\left(u_{3}+u_{7}\right)\right. \\
\left.-704\left(u_{2}+u_{8}\right)-2349\left(u_{1}+u_{9}\right)\right] \\
u_{6}=\frac{1}{217350208218}\left[59567194206 u_{6}+51593437700\left(u_{5}+u_{7}\right)\right. \\
+31315296640\left(u_{4}+u_{8}\right)+8277866685\left(u_{3}+u_{9}\right) \\
\left.-6224658450\left(u_{2}+u_{10}\right)-6070435569\left(u_{1}+u_{11}\right)\right]
\end{gathered}
$$

It will be more convenientin practice to have the weights expressed by decimals, as follows:

$$
\left.\begin{array}{rl}
u_{3}=.56616 u_{3}+.28923\left(u_{2}+u_{4}\right)-.07231\left(u_{1}+u_{5}\right) \\
u_{4}=. & .1476 u_{4}+. \\
u_{5}=. & .32986\left(u_{3}+u_{5}\right)+.05494\left(u_{2}+u_{6}\right)-.05718\left(u_{1}+.26706\left(u_{4}+u_{7}\right)+.12006\left(u_{3}+u_{7}\right)\right. \\
& -.01198\left(u_{2}+u_{8}\right)-.03997\left(u_{1}+u_{9}\right) \tag{56}
\end{array}\right\}(55)
$$

Without attempting solutions in whole numbers, we can proceed in a
similar way to find decimal weights for thirteen or more terms, as in the following cases:

$$
\left.\begin{array}{rl}
u_{7}=.23466 & u_{7} \\
+.21137\left(u_{6}+u_{8}\right)+.14934\left(u_{5}+u_{9}\right)+.07003\left(u_{4}+u_{10}\right) \\
& +.00195\left(u_{3}+u_{11}\right)-.03005\left(u_{2}+u_{12}\right)-.01997\left(u_{1}+u_{13}\right) \tag{58}
\end{array}\right\}(57)
$$

In each of these formulas the sum of all the weights, taken for each term separately, is unity, as it should be. Owing to the rejection of decimals after the fifth figure, this condition would not always be exactly satisfied, and consequently the fifth figure, as above given, has been made to differ in some cases from its nearest value, to the extent of a single unit of the fifth place. Actual trials have shown that a better graduation can be made by these formulas than by any of the similar ones previously given, and it is possible that, in some cases, a table of mortality may be graduated sufficiently by this means alone, without recourse to the first or third methods of adjustment.

It will often be sufficient for practical purposes to use only three places of decimals; and in making an adjustment of a given series by any single formula, we can facilitate the multiplications by preparing in advance a table showing the product of each of the decimal weights by each of the nine digits.

There is another method, allied to the preceding, by which the weights may be determined when more than five terms are to be included in a formula. Supposing the number of terms to be seven, we may assume that their seven weights, together with the two nearest zero weights, are ordinates to a curve of the eighth degree, since such a curve can be made to pass through nine given points. We have, as before, the condition that this curve shall be tangent to the axis of $\mathbf{X}$ at the points 0 and 8 ; and to make its continuity with the axis at those points as complete as possible, we may give it a contact of the secoind order, so that its first and second differential coefficients shall both become zero at the points 0 and 8. We have thus the two conditions-

$$
\begin{gathered}
\Delta_{1}-\frac{\Delta_{2}}{2}+\frac{\Delta_{3}}{3}-\frac{\Delta_{4}}{4}+\frac{\Delta_{5}}{5}-\frac{\Delta_{6}}{6}+\frac{\Delta_{7}}{7}-\frac{\Delta_{8}}{8}=0 \\
\Delta_{2}-\Delta_{3}+\frac{11 \Delta_{4}}{12}-\frac{5 \Delta_{5}}{6}+\frac{137 \Delta_{6}}{180}-\frac{7 \Delta_{7}}{10}+\frac{363 \Delta_{8}}{560}=0
\end{gathered}
$$

By means of these we obtain the two numbers-

$$
k=\frac{37976}{6517 .} \quad k^{\prime}=\frac{135087}{6517}
$$

and the adjustment formula is found to be-.

$$
u_{4}=\frac{1}{808112}\left[371712 u_{4}+236625\left(u_{3}+u_{5}\right)+14160\left(u_{2}+u_{6}\right)-32585\left(u_{1}+u_{7}\right)\right]
$$

## With decimal weights it becomes-

$$
u_{4}=.45998 u_{4}+.29281\left(u_{3}+u_{5}\right)+.01752\left(u_{2}+u_{6}\right)-.04032\left(u_{1}+u_{7}\right)
$$

In a similar manner we might proceed to find formulas including more than seven terms. With nine terms we should assume a curve of the tenth degree, with the three conditions that its first, second, and third differential coefficients should all become zero at the positions of the two nearest zero weights.

This method of determining the weights may seem to be theoretically better than the previous one, but the labor required in obtaining the formulas is very considerably increased, especially when nine or more terms are to be included by them, and the practical advantages of the method, if it has any, must be small.* According to the theory of probability of errors, if we let $\varepsilon$ denote the probable error of each single term in a given series, then the probable error of a term adjusted by the above formula will be-

$$
\varepsilon_{0}=\varepsilon \sqrt{.45998^{2}+2\left(.29281^{2}+.01752^{2}+.04032^{2}\right)}=.62204 \varepsilon
$$

But if the adjustment were made by formula (54), the probable error would be only-

$$
\varepsilon_{0}=\varepsilon \sqrt{.41476^{2}+2\left(.29486^{2}+.05494^{2}+.05718^{2}\right)}=.59874 \varepsilon
$$

which indicates that (54) is slightly superior in the accuracy of its results. This, however, is not conclusive as regards smoothness of adjustment. If we imagine two series, such that the probable error of a single term is smaller in the first one than in the second, it is still possible that the second may be the more perfectly graduated of the two, since its errors may follow a continuous sequence or curve, while the errors of the first may be arranged irregularly or fortuitously, so as to follow a broken line. The comparative regularity of the graduation of two series obtained by using different adjustment formulas will be best ascertained by comparing their corresponding orders of differences.

The fourth difference is most convenient for this purpose, and may be obtained directly for any five consecutive terms by means of the for-mula-

$$
\Delta_{4}=6 u_{3}-4\left(u_{2}+u_{4}\right)+\left(u_{1}+u_{5}\right)
$$

Having thus computed all the fourth differences for each of the two series, we can add them together in each case without regard to sign, and the series which gives the smaller sum may be regarded as the better graduated of the two. This becomes evident when we consider that a curve of the third degree, since it admits a point of inflexion, may be taken to represent approximately a limited portion of any regular curve; and as all the formulas of the second method of adjustment give accurate results for a series of the third or any lower order, their use tends to bring the adjusted series into such a form that any

[^37]small number of consecutive terms in it will be approximately of an order not higher than the third. Hence, if any series, such as a table of mortality, is thus adjusted, its fourth differences will be small, and positive and negative values will be equally probable.

In the case of formulas like (22), which hold good for a series of the fifth or any lower order, we may fix the local weights of the terms by these two conditions, that the whole series of weights, including the two nearest zero weights, should be of the eighth order, and that it should have minima at the beginning and end, so as to satisfy the equa-tion-

$$
\Delta_{1}-\frac{\Delta_{2}}{2}+\frac{\Delta_{3}}{3}-\ldots \ldots . A_{8}=0
$$

Thus we obtain the formula-

$$
u_{4}=\frac{1}{12868}\left[7968 u_{4}+3675\left(u_{3}+u_{5}\right)-1470\left(u_{2}+u_{6}\right)+245\left(u_{1}+u_{7}\right)\right]
$$

which, with decimal weights, is-

$$
u_{4}=.61922 u_{4}+.28559\left(u_{3}+u_{5}\right)-.11424\left(u_{2}+u_{6}\right)+.01904\left(u_{1}+u_{7}\right) \ldots(59)^{*}
$$

To find formulas for adjusting the first two and last two terms of a series, we may proceed as follows: Assuming that five terms, $u_{1}, u_{2}, u_{3}$, $u_{4}, u_{5}$, form a series of the second order approximately, and taking the equation-

$$
u=\mathrm{A}+\mathrm{B} x+\mathrm{C} x^{2}
$$

with the origin of coördinates at the middle term $u_{3}$, we have the five, equations of condition-

$$
\begin{aligned}
& u_{1}=\mathrm{A}-2 \mathrm{~B}+4 \mathrm{C} \\
& u_{2}=\mathrm{A}-\mathrm{B}+\mathrm{C} \\
& u_{3}=\mathrm{A} \\
& u_{4}=\mathrm{A}+\mathrm{B}+\mathrm{C} \\
& u_{5}=\mathrm{A}+2 \mathrm{~B}+4 \mathrm{C}
\end{aligned}
$$

Combining these by the rule of least squares, we find that the values of the three constants are-

$$
\begin{aligned}
& \mathrm{A}=\frac{1}{35}\left[17 u_{3}+12\left(u_{2}+u_{4}\right)-3\left(u_{1}+u_{5}\right)\right] \\
& \mathrm{B}=\frac{1}{10}\left[2\left(u_{5}-u_{1}\right)+\left(u_{4}-u_{2}\right)\right] \\
& \mathrm{C}=\frac{1}{14}\left[2\left(u_{1}+u_{5}\right)-2 u_{3}-\left(u_{2}+u_{4}\right)\right]
\end{aligned}
$$

and consequently we have-

$$
\begin{aligned}
& u_{1}=\frac{1}{35}\left(31 u_{1}+9 u_{2}-3 u_{3}-5 u_{4}+3 u_{5}\right) \quad \ldots(60) \\
& u_{2}=\frac{1}{35}\left(9 u_{1}+13 u_{2}+12 u_{3}+6 u_{4}-5 u_{5}\right) \ldots(61)
\end{aligned}
$$

which can be used with advantage in place of (25) and (26), if the series

[^38]to be adjusted is not a very irregular one. We can proceed in a similar way to obtain formulas for adjusting the middle term in any group of five, seven, nine, or more terms, as follows:*
\[

$$
\begin{aligned}
& u_{3}=\frac{1}{35}\left[17 u_{3}+12\left(u_{2}+u_{4}\right)-3\left(u_{1}+u_{5}\right)\right] \\
& u_{4}=\frac{1}{21}\left[7 u_{4}+6\left(u_{3}+u_{5}\right)+3\left(u_{2}+u_{6}\right)-2\left(u_{1}+u_{7}\right)\right] \\
& u_{5}=\frac{1}{2} \frac{1}{31}\left[59 u_{5}+54\left(u_{4}+u_{6}\right)+39\left(u_{3}+u_{7}\right)+14\left(u_{2}+u_{8}\right)-21\left(u_{1}+u_{9}\right)\right]
\end{aligned}
$$
\]

In all these cases, the weights form a series of the second order. The probable errors are less than those given by other similar formulas; for instance, the probable error of the adjusted value of $u_{4}$ is only-

$$
\varepsilon_{0}=\frac{\varepsilon}{21} \sqrt{7^{2}+2\left(6^{2}+3^{2}+2^{2}\right)}=.57735 \varepsilon
$$

- But it has been found on trial that, as regards smoothness of adjustment, these formulas are decidedly inferior to (53), (54), \&c., or even to $(17),(19), \& c$. This is owing to the great want of continuity between the weights of the formula and the zero weights. If we apply Cauchy's method to the same series of terms as above, we get-

$$
\begin{aligned}
& u_{3}=\frac{1}{10}\left[4\left(u_{2}+u_{3}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right] \\
& u_{4}=\frac{1}{35}\left[11\left(u_{3}+u_{4}+u_{5}\right)+4\left(u_{2}+u_{6}\right)-3\left(u_{1}+u_{7}\right)\right] \\
& u_{5}=\frac{1}{2 T}\left[5\left(u_{3}+u_{4}+u_{5}+u_{6}+u_{7}\right)-\left(u_{1}+u_{2}+u_{8}+u_{9}\right)\right] \\
& u_{6}=\frac{1}{23} T\left[41\left(u_{3}+u_{4}+\cdots \cdots+u_{9}\right)-14\left(u_{1}+u_{2}+u_{10}+u_{11}\right)\right]
\end{aligned}
$$

All these, except the second, are special cases under our formula (13). The first one is the same as (14).

## ADDITIONAL FORMULAS UNDER THE FIRST METHOD.

The simplest case of all has been omitted; it is that in which the graduated series is of the first order, so that the expression for the sum of any $n$ terms in a group is-

$$
\mathrm{S}=n(\mathrm{~A}+\mathrm{B} x)
$$

Assuming any two groups composed of $n_{1}$ and $n_{2}$ terms respectively, with the origin of coördinates midway between the middle points of the groups, and denoting by $a$ the distance from the origin to either of these points, we have for the values of the constants-

$$
\left.\begin{array}{l}
\mathrm{A}=\frac{1}{2}\left(\frac{\mathrm{~S}_{1}}{n_{1}}+\frac{\mathrm{S}_{2}}{n_{2}}\right)  \tag{62}\\
\mathrm{B}=\frac{1}{2 a}\left(\frac{\mathrm{~S}_{2}}{n_{2}}-\frac{\mathrm{S}_{1}}{n_{1}}\right)
\end{array}\right\}
$$

[^39]If the assumed groups are consecutive and contain $n_{1}$ ternas each, the constants will be-

$$
\left.\begin{array}{l}
A=\frac{1}{2 n_{1}}\left(S_{1}+S_{2}\right)  \tag{63}\\
B=\frac{1}{n_{1}^{2}}\left(S_{2}-S_{1}\right)^{\prime}
\end{array}\right\}
$$

This properly commences not only the series of formulas $(\mathrm{A}),(\mathrm{B}),(\mathrm{C})$, \&c., but also the series (40), (41), (42), \&ce.
Again, when formula (12) is extended so as to include nine constants, it becomes-

$$
\left.\begin{array}{rl}
\mathrm{S}=n\left[\mathrm{~A}+\frac{1}{12}\right. & \mathrm{C} n^{2}+\frac{1}{80} \mathrm{E} n^{4}+\frac{1}{148} \mathrm{G} n^{6}+\frac{1}{2304} \mathrm{I} n^{3}+\left(\mathrm{B}+\frac{1}{4} \mathrm{D} n^{2}+\frac{1}{16} \mathrm{~F} n^{4}\right. \\
& \left.+\frac{1}{64} \mathrm{H} n^{6}\right) x+\left(\mathrm{C}+\frac{1}{2} \mathrm{E} n^{2}+\frac{3}{16} \mathrm{G} n^{4}+\frac{1}{16} \mathrm{I} n^{6}\right) x^{2}+\left(\mathrm{D}+\frac{5}{6} \mathrm{~F} n^{2}\right.  \tag{64}\\
& \left.+\frac{7}{16} \mathrm{H} n^{4}\right) x^{3}+\left(\mathrm{E}+\frac{5}{4} \mathrm{G} n^{2}+\frac{7}{8} \mathrm{I} n^{4}\right) x^{4}+\left(\mathrm{F}+\frac{7}{4} \mathrm{H} n^{2}\right) x^{5}
\end{array}\right\}
$$

If we assume nine consecutive groups, containing $n_{1}$ terms each, the values of the constants are found to be-

$$
\begin{align*}
& A=\frac{1}{10321920 n_{1}}\left[11702134 S_{5}+125884\left(S_{3}+S_{7}\right)+1225\left(S_{1}+S_{9}\right)\right. \\
& \left.-800216\left(\mathrm{~S}_{4}+\mathrm{S}_{6}\right)-17000\left(\mathrm{~S}_{2}+\mathrm{S}_{8}\right)\right] \\
& \mathrm{B}=\frac{1}{645120 n_{1}^{2}}\left[574686\left(\mathrm{~S}_{6}-\mathrm{S}_{4}\right)+33878\left(\mathrm{~S}_{8}-\mathrm{S}_{2}\right)-170422\left(\mathrm{~S}_{7}-\mathrm{S}_{3}\right)\right. \\
& \left.-3229\left(\mathrm{~S}_{9}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{C}=\frac{1}{1935360 n_{1}}\left[1912064\left(\mathrm{~S}_{4}+\mathrm{S}_{6}\right)+44480\left(\mathrm{~S}_{2}+\mathrm{S}_{8}\right)-3260110 \mathrm{~S}_{5}\right. \\
& \left.-323260\left(\mathrm{~S}_{3}+\mathrm{S}_{7}\right)-3229\left(\mathrm{~S}_{1}+\mathrm{S}_{9}\right)\right] \\
& \mathrm{D}=\frac{1}{23040 n_{1}^{4}}\left[6134\left(\mathrm{~S}_{7}-\mathrm{S}_{3}\right)+141\left(\mathrm{~S}_{9}-\mathrm{S}_{1}\right)-8614\left(\mathrm{~S}_{6}-\mathrm{S}_{4}\right)\right. \\
& \left.-1406\left(\mathrm{~S}_{8}-\mathrm{S}_{2}\right)\right] \\
& E=\frac{1}{27648 n_{1}^{5}}\left[15002 S_{5}+3908\left(S_{3}+S_{i}\right)+47\left(S_{1}+S_{3}\right)-10840\left(S_{4}+S_{6}\right)\right.  \tag{G}\\
& \left.-616\left(\mathrm{~S}_{2}+\mathrm{S}_{8}\right)\right] \\
& \mathrm{F}=\frac{1}{1920 n_{1}}{ }^{6}\left[22\left(\mathrm{~S}_{6}-\mathrm{S}_{4}\right)+26\left(\mathrm{~S}_{8}-\mathrm{S}_{2}\right)-74\left(\mathrm{~S}_{7}-\mathrm{S}_{3}\right)-3\left(\mathrm{~S}_{9}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{G}=\frac{1}{\left.\left.17280 n_{1}{ }^{[ } 752\left(\mathrm{~S}_{4}+\mathrm{S}_{6}\right)+80\left(\mathrm{~S}_{2}+\mathrm{S}_{8}\right)-970 \mathrm{~S}_{5}-340\left(\mathrm{~S}_{3}+\mathrm{S}_{7}\right) .\right]\left(S_{1}\right)\right]} \\
& \left.-7\left(\mathrm{~S}_{1}+\mathrm{S}_{9}\right)\right] \\
& \mathrm{H}=\frac{1}{10080 n_{1}{ }^{8}}\left[14\left(\mathrm{~S}_{7}-\mathrm{S}_{3}\right)+\left(\mathrm{S}_{9}-\mathrm{S}_{1}\right)-14\left(\mathrm{~S}_{6}-\mathrm{S}_{4}\right)-6\left(\mathrm{~S}_{8}-\mathrm{S}_{2}\right)\right] \\
& \mathrm{I}=\frac{1}{40320 n_{1}{ }^{2}}{ }^{[70} \mathrm{S}_{5}+28\left(\mathrm{~S}_{3}+\mathrm{S}_{7}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{9}\right)-56\left(\mathrm{~S}_{4}+\mathrm{S}_{6}\right) \\
& \left.-8\left(\mathrm{~S}_{2}+\mathrm{S}_{8}\right)\right]
\end{align*}
$$

This formula may be used advantageously in constructing a graduated rate of mortality similar to series ( $h$ ) in Table II. The simplest mode of procedure will be to obtain the equation of the graduated series of the form-

$$
u=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} x+\mathrm{C}^{\prime} x^{2}+\cdots \cdots+\mathrm{I}^{\prime} x^{8}
$$

and to compute by logarithms first the values of $\mathrm{B}^{\prime} x$ for all the ages, then the values of $\mathrm{C}^{\prime} x^{2}$ in like manner, and so on, and finally to take the aggregate of the values at each age. The accuracy of the work will be tested by the condition that the sums of the terms in the corresponding groups in the graduated series must be severally equal to those in the given one. It should be also mentioned that, to insure accuracy, the multiplications within the brackets in formula (G), such, for instance, as that of $S_{5}$ by its coefficient 11702134, \&c., ought to be performed arithmetically and not by logarithms.

## interpolation by means of an exponential function.

When values in arithmetical progression are assigned to $x$ in the ex. ponential equation-

$$
y=b \beta^{x}+c \gamma^{x}+d \delta^{x}+\& c .
$$

the resulting values of $y$ will be terms in a recurring seres, whose order is denoted by the number of constants $\beta, \gamma, \delta, \& c$. The above formula has sometimes been used for the purpose of ordinary interpolation, and represents a curve which, under certain conditions, can be made to pass through any number of given points whose ordinates $y_{0}, y_{1}, y_{2}, \& c$., are equidistant. The whole number of constants $b, c, d, \beta, \gamma, \delta, \& c$. , incladed by the formula, must be equal to the number of points given. If this is an odd number, we must write-

$$
y=a+b \beta^{x}+o j^{2}+d \delta^{x}+\& c .
$$

For the most general method of determining the values of the constants in any given case, see articles by Prony, in Vols. I and II of the Journal de $\mathbb{l}$ École Polytechnique. We may here remark that if there are not more than five constants, their values can easily be obtained in the ordinary way, first eliminating $a, b$, and $c$ from the equations of condition, then finding the values of $\beta$ and $\gamma$, and afterward finding those of $a, b$, and $c$.
Now let us write the general equation under the form-

$$
\begin{equation*}
y=\mathrm{A}+\left(\mathrm{B} \log ^{\prime} \beta\right) \beta^{x}+\left(\mathrm{C} \log ^{\prime} \gamma\right) \gamma^{x}+\left(\mathrm{D} \log ^{\prime} \delta\right) \delta^{\alpha}+\& \mathrm{c} . \tag{65}
\end{equation*}
$$

where $\log ^{\prime}$ denotes the Naperian logarithm. Integrating $y d x$ between the limits $x-\frac{1}{2} n$ and $x+\frac{1}{2} n$, we get-

$$
\begin{equation*}
\mathrm{S}=\mathrm{A} n+\mathrm{B}\left(\beta^{n} n-\beta^{-1 n}\right) \beta^{x}+\mathrm{C}\left(\gamma^{\mathrm{d}} n-\gamma^{-\mathrm{i} n}\right) \gamma^{x}+\mathrm{D}\left(\delta^{2 n} n-\delta^{-1 n}\right) \delta^{x}+\& \mathrm{c} . \tag{66}
\end{equation*}
$$

which is identical in form with the expression for $y$, so far as the abscissa . $x$ is concerned. Consequently, if we assume a series of groups contain-
ing $n_{1}$ terms each, and equidistant, so that $h$ may denote the constant interval between their middle points, and if we put $\mathrm{A}^{\prime}=\mathrm{A} n_{1}$, and-

$$
\mathrm{B}^{\prime}=\mathrm{B}\left(\beta^{\mathrm{j}} n_{1}-\beta-\mathrm{i} n_{1}\right), \mathrm{C}^{\prime}=\mathrm{C}\left(\gamma^{\$ n_{1}}-\gamma^{-\dot{+1}}\right), \mathrm{D}^{\prime}=\mathrm{D}\left(\delta \delta n_{1}-\delta^{-4 n_{1}}\right), \& \mathrm{dc} .
$$

and place the origin of coördinates at the middle of the left-hand group, then the sums of the terms in the several groups will be-

$$
\begin{aligned}
& S_{0}=A^{\prime}+B^{\prime}+O^{\prime}+D^{\prime}+\& c . \\
& \mathrm{S}_{1}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \beta^{n}+\mathrm{C}^{\prime} \gamma^{n}+\mathrm{D}^{\prime} \delta^{\alpha}+\& \mathrm{c} . \\
& \mathrm{S}_{2}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \beta^{2 x}+\mathrm{C}^{\prime} \gamma^{2 n}+\mathrm{D}^{\prime} \delta^{2 n}+\& c \text {. } \\
& \mathrm{S}_{3}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \beta^{3 x}+\mathrm{C}^{\prime} \gamma^{3 x}+\mathrm{D}^{\prime} \delta^{3 x}+\& \mathrm{c} .
\end{aligned}
$$

\&c.
\&c.
and in any given case, assuming as many groups as there are constants to be determined, we can find the values of the constants from these equations of condition, just as in ordinary interpolation from ordinates. In accordance with the general method referred to, we proceed as follows: If the number of constants is an even one, for instance, six, the groups forming a recurring series of the third order, whose scale of relation is $-A_{0},-A_{1},-A_{2}$, we shall have the three equations-

$$
\begin{aligned}
& A_{0} S_{0}+A_{1} S_{1}+A_{2} S_{2}+S_{3}=0 \\
& A_{0} S_{1}+A_{1} S_{2}+A_{2} S_{3}+S_{4}=0 \\
& A_{0} S_{2}+A_{1} S_{3}+A_{2} S_{4}+S_{5}=0
\end{aligned}
$$

These enable us to find the numerical values of $\mathrm{A}_{0}, \mathrm{~A}_{1}, \mathrm{~A}_{2}$, and we substithte them in the equation of relation-

$$
z^{3}+\mathrm{A}_{2} z^{2}+\mathrm{A}_{1} z+\mathrm{A}_{0}=0
$$

This numerical equation of the third degree being solved, its three roots will be the values of the three constants $\beta^{n}, \gamma^{n}, \delta^{n}$. Substituting them in the three equations of condition-

$$
\begin{aligned}
& \mathrm{S}_{0}=\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime} \\
& \mathrm{S}_{1}=\mathrm{B}^{\prime} \mathrm{g}^{2}+\mathrm{C}^{\prime} \boldsymbol{\gamma}^{4}+\mathrm{D}^{\prime} \boldsymbol{\gamma}^{4} \\
& \mathrm{~S}_{2}=\mathrm{B}^{\prime} / \mathrm{s}^{2 n}+\mathrm{C}^{\prime 2} \gamma^{2 x}+\mathrm{D}^{\prime} 8^{2 n}
\end{aligned}
$$

we can find the values of $\mathrm{B}^{\prime}, \mathrm{C}^{\prime}$, and $\mathrm{D}^{\prime}$, and consequently those of $\mathrm{B}, \mathrm{C}$, and D. Having thus determined all the constants in the equation-
we are enabled to interpolate the sum S of any $n$ terms taken in a group, or any single term, and to form a recurring series of the third order, such that the arithmetical means of the terms in the six assumed groups will be the same in it as in the given series. The equation of the graduated series will be of the form-

$$
u=\mathrm{B}^{\prime \prime} \beta^{x}+\mathrm{C}^{\prime \prime} \gamma^{2}+\mathrm{D}^{\prime \prime} \delta^{x}
$$

When the assumed groups are consecutive, we shall have $h=n_{1}$. The
three roots of the equation of relation must in all cases be positive; ; if any of them are negative, the inference will be that the given series cannot, for purposes of interpolation, be represented by an equation of the proposed form.
If the number of constants is odd, for instance, seven, we shall find the scale of relation from the four equations-

$$
\begin{aligned}
& A_{0}\left(S_{0}-A^{\prime}\right)+A_{1}\left(S_{1}-A^{\prime}\right)+A_{2}\left(S_{2}-A^{\prime}\right)+\left(S_{3}-A^{\prime}\right)=0 \\
& A_{0}\left(S_{1}-A^{\prime}\right)+A_{1}\left(S_{2}-A^{\prime}\right)+A_{2}\left(S_{3}-A^{\prime}\right)+\left(S_{4}-A^{\prime}\right)=0 \\
& A_{0}\left(S_{2}-A^{\prime}\right)+A_{1}\left(S_{3}-A^{\prime}\right)+A_{2}\left(S_{4}-A^{\prime}\right)+\left(S_{5}-A^{\prime}\right)=0 \\
& A_{0}\left(S_{3}-A^{\prime}\right)+A_{1}\left(S_{4}-A^{\prime}\right)+A_{2}\left(S_{5}-A^{\prime}\right)+\left(S_{6}-A^{\prime}\right)=0
\end{aligned}
$$

first eliminating $\mathrm{A}^{\prime}$ by subtracting each equation from the succeeding one. The equation of relation will be of the same degree as in the previous case, and the values of $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}, \mathrm{C}^{\prime}$, and $\mathrm{D}^{\prime}$ will be found from the four equations of condition-

$$
\begin{aligned}
& \mathrm{S}_{0}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime} \\
& \mathrm{S}_{1}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \beta^{n}+\mathrm{C}^{\prime} r^{\prime}+\mathrm{D}^{\prime} \delta^{n} \\
& \mathrm{~S}_{2}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \beta^{2 x}+\mathrm{C}^{\prime} \gamma^{2 x}+\mathrm{D}^{\prime} \delta^{2 \mathrm{~A}} \\
& \mathrm{~S}_{3}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime} \boldsymbol{\beta}^{3 x}+\mathrm{C}^{\prime} \gamma^{3 x}+\mathrm{D}^{\prime} \delta^{3 x}
\end{aligned}
$$

If the number of constants and of groups assumed were eight or nine; the mode of procedure would be precisely similar to the above. The scale of relation would contain four terms, and the four roots of the equation of relation-

$$
z+\mathrm{A}_{3} z^{3}+\mathrm{A}_{2} z^{2}+\mathrm{A}_{1} z+\mathrm{A}_{0}=0
$$

would be the values of the four constants $\beta^{n}, \gamma^{n}, \delta^{n}, \varepsilon^{n}$.
In the simplest case of all, we have the curve-

$$
y=b \beta^{x}
$$

whose equidistant ordinates are in geometrical progression. If we assume-

$$
y=\mathrm{A}+\left(\mathrm{B} \log ^{\prime} \beta\right) \mathrm{\beta}^{x}
$$

it is easy to obtain the following:

$$
\left.\begin{array}{l}
\beta=\left(\frac{S_{2}^{\prime}-S_{1}}{S_{1}-S_{0}}\right)^{\frac{1}{n}} \\
\mathrm{~B}=\frac{\mathrm{S}_{1}-\mathrm{S}_{0}}{\left(\beta^{h}-1\right)\left(\beta^{+1 n_{1}}-\beta^{-\frac{1}{2} n_{1}}\right)} \\
\mathbf{A}=\frac{1}{n_{1}}\left[S_{0}-\left(\frac{S_{1}-S_{0}}{\beta^{n}-1}\right)\right] \\
\mathrm{S}=\mathbf{A} n+\mathrm{B}\left(\beta^{\frac{1 n}{n}-\beta^{-\frac{1}{n} n}}\right) \beta^{x}
\end{array}\right\}(67)
$$

This can often be used with advantage in place of (3) or any similar
formula, in making a distribution of population or deaths at the earliest and latest ages of life, where the values vary so rapidly as to give the series an exponential rather than a parabolic torm.
But when our object is merely to graduate an irregular series whose terms are all separately given, the easiest way to put it in an exponential form will be to take the common logarithms of all the terms, as has been already suggested, and adjust them by the second and first methods, and then take the numbers corresponding to the graduated logarithms. The equation of the final series will be of the form

$$
u=10\left(a+b x+c x^{2}+\infty 0 .\right)
$$

the simplest case of which-

$$
u=10^{(a+b x)}
$$

represents a geometrical progression.

## APPENDIX II.

Among the various methods which can be used for fixing the values of the local weights in adjustment formulas, the following one is perhaps deserving of especial notice:
Assuming that the true law of a given series of numbers may be regarded as algebraic and of an order not higher than the third, and that the irregularities in the series are of the nature of accidental errors or deviations from this true law, and that deviations of a given amount are as likely to occur in one term as in another, let it be required to find that system of weights which will render the probable value of the fourth differences of the adjusted series, taken without regard to sign, a minimum.

Considering, in the first place, the most general form of an adjustment formula comprising only five terms, which may be written-

$$
\begin{equation*}
u_{3}=\frac{1}{k+6}\left[k u_{3}+4\left(u_{2}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right] \tag{68}
\end{equation*}
$$

we have for the values of five consecutive terms in the adjusted series-

$$
\begin{aligned}
& u_{3}^{\prime}=\frac{1}{k+6}\left[k u_{3}+4\left(u_{2}+u_{4}\right)-\left(u_{1}+u_{5}\right)\right] \\
& u_{4}^{\prime}=\frac{1}{k+6}\left[k u_{4}+4\left(u_{3}+u_{5}\right)-\left(u_{2}+u_{6}\right)\right] \\
& u_{5}^{\prime}=\frac{1}{k+6}\left[k u_{5}+4\left(u_{4}+u_{6}\right)-\left(u_{3}+u_{7}\right)\right] \\
& u_{6}^{\prime}=\frac{1}{k+6}\left[k u_{6}+4\left(u_{5}+u_{7}\right)-\left(u_{4}+u_{8}\right)\right] \\
& u_{7}^{\prime}=\frac{1}{k+6}\left[k u_{7}+4\left(u_{6}+u_{8}\right)-\left(u_{5}+u_{9}\right)\right]
\end{aligned}
$$

The fourth difference of these terms is-

$$
\Delta_{4}=6 u_{5}^{\prime}-4\left(u_{4}^{\prime}+u_{6}^{\prime}\right)+\left(u_{3}^{\prime}+u^{\prime}{ }_{7}\right)
$$

and consequently-

$$
\begin{gathered}
\Delta_{4}=\frac{1}{k+6}\left[(6 k-34) u_{5}-(4 k-32)\left(u_{4}+u_{6}\right)+(k-22)\left(u_{3}+u_{7}\right)\right. \\
\left.+8\left(u_{2}+u_{8}\right)-\left(u_{1}+u_{9}\right)\right]
\end{gathered}
$$

If we suppose that the series $u_{1}, u_{2}, u_{3}, \& c$., is of an order not higher than the third, the adjusted series $u^{\prime}{ }_{3}, u_{4}^{\prime}, u_{5}^{\prime}$, \&cc., will be of the same order, so that its fourth differences will be zero, and both members of the abore equation will be equal to zero. But if each of the terms $u_{1}, u_{2}, \& c$. , is liable to an accidental deviation or error, whose probable amount is denoted by $\varepsilon$, then the probable value of $\Delta_{4}$, taken without regard to sign, will be-

$$
\left(\Delta_{4}\right)=\frac{\varepsilon}{k+6} \sqrt{\left.(6 k-34)^{2}+2\left[(4]_{6}-32\right)^{2}+(k-22)^{2}+8^{2}+1\right]}
$$

which reduces to-

$$
\left(\Delta_{4}\right)=\frac{\varepsilon}{k+6} \sqrt{70 k^{2}-1008 k+4302}
$$

Regarding ( $\Delta_{4}$ ) as a function of the variable $k$, we have the equation -

$$
\frac{d\left(\Delta_{4}\right)}{d k}=0
$$

from which to find that value of $k$ which makes $\left(\Delta_{4}\right)$ a minimum. This is $k=\frac{111}{14}$; and substituting it in (68), we obtain-

$$
\begin{equation*}
u_{3}=\frac{1}{195}\left[111 u_{3}+56\left(u_{2}+u_{4}\right)-14\left(u_{1}+u_{5}\right)\right] . \tag{69}
\end{equation*}
$$

which is the adjustment formula sought.
To find a similar one including seven terms, we may take the most general form as used in obtaining (52), or, what amounts to the same thing, by proceeding as in the demonstration of formula (20), we can get-

$$
\begin{gathered}
u_{4}=\frac{1}{k^{\prime}+10 k-35}\left[\left(k^{\prime}+4 k-15\right) u_{4}+(4 k-15)\left(u_{3}+u_{5}\right)\right. \\
\left.+(6-k)\left(u_{2}+u_{6}\right)-\left(u_{1}+u_{7}\right)\right]
\end{gathered}
$$

Since $k^{\prime}$ affects only the weight of the middle term, we max, for the sake of brevity, denote that weight by $k^{\prime}$ alone, and so write-

$$
\begin{equation*}
u_{4}=\frac{1}{k^{\prime}+6 k-20}\left[k^{\prime} u_{4}+(4 k-15)\left(u_{3}+u_{5}\right)+(6-k)\left(u_{2}+u_{6}\right)-\left(u_{1}+u_{7}\right)\right] \ldots \tag{70}
\end{equation*}
$$

The expression for the fourth difference of the adjusted series then is -

$$
\begin{gathered}
\Delta_{4}=\frac{1}{k^{\prime}+6 k-20}\left[\left(6 k^{\prime}-34 k+132\right) u_{6}-\left(4 k^{\prime}-32 k+130\right)\left(u_{5}+u_{7}\right)\right. \\
+\left(k^{\prime}-22 k+100\right)\left(u_{4}+u_{8}\right)-(45-8 k)\left(u_{3}+u_{9}\right) \\
\left.+(10-k)\left(u_{2}+u_{10}\right)-\left(u_{1}+u_{11}\right)\right]
\end{gathered}
$$

and when each term is supposed to be affected by a probable error or deviation $\varepsilon$, the probable value of $\Delta_{4}$ becomes-

$$
\begin{gathered}
\left(4_{4}\right)=\frac{\varepsilon}{k^{\prime}+6 k-20} \sqrt{\left(6 k^{\prime}-34 k+132\right)^{2}+2\left[\left(4 k^{\prime}-32 k+130\right)^{2}\right.} \\
\frac{\left.+\left(k^{\prime}-22 k+100\right)^{2}+(45-8 k)^{2}+(10-k)^{2}+1\right]}{}
\end{gathered}
$$

which reduces to-

$$
\left(J_{4}\right)=\frac{\varepsilon}{k_{k^{\prime}}+6 k-20} \sqrt{70 k^{\prime} 2+4302 k^{2}-1008 k k^{\prime}+4064 k k^{\prime}-35896 k+75476}
$$

Regarding ( $\Delta_{4}$ ) as a function of the two independent variables $k$ and $k^{\prime}$, we have the two equations-

$$
\frac{d\left(\Lambda_{4}\right)}{d k}=0, \quad \frac{d\left(\Lambda_{4}\right)}{d k_{i}}=0
$$

giving the values $k=\frac{51}{10}$ and $k^{\prime}=\frac{469}{60}$, which render ( $J_{4}$ ) a minimum. Substituting these in (70), we get the adjustment formula sought-

$$
\begin{equation*}
u_{4}=\frac{T_{105}^{1}}{5}\left[469 u_{4}+324\left(u_{3}+u_{5}\right)+54\left(u_{2}+u_{6}\right)-60\left(u_{1}+u_{7}\right)\right] . \tag{71}
\end{equation*}
$$

It is found that in each of the formulas (69) and (71), the whole series of weights, taken together with the eight nearest zero weights, constitutes a series of the tenth order. By means of this property, we can construct with greater facility the following similar formulas:

$$
\begin{gathered}
\left.\begin{array}{c}
u_{5}=\frac{1}{8398}\left[2884 u_{5}+2268\left(u_{4}+u_{6}\right)+918\left(u_{3}+u_{7}\right)-132\left(u_{2}+u_{8}\right)\right. \\
\left.-297\left(u_{1}+u_{9}\right)\right]
\end{array}\right\}(72) \\
\left.\begin{array}{c}
u_{6}=\frac{1}{25 \frac{1}{194}}\left[7308 u_{6}+6160\left(u_{5}+u_{7}\right)+3410\left(u_{4}+u_{8}\right)+660\left(u_{3}+u_{9}\right)\right. \\
\left.-715\left(u_{2}+u_{10}\right)-572\left(u_{1}+u_{11}\right)\right]
\end{array}\right\}(73) \\
\left.\begin{array}{c}
u_{7}=\frac{1}{193154}\left[48636 u_{7}+42768\left(u_{6}+u_{8}\right)+27918\left(u_{5}+u_{9}\right)+10868\left(u_{4}+u_{10}\right)\right. \\
\left.-1287\left(u_{3}+u_{11}\right)-5148\left(u_{2}+u_{12}\right)-2860\left(u_{1}+u_{13}\right)\right]
\end{array}\right\}(74) \\
\left.\begin{array}{c}
u_{8}=\frac{1}{37 T_{450}}\left[82764 u_{8}+74844\left(u_{7}+u_{9}\right)+54054\left(u_{6}+u_{10}\right)+28028\left(u_{5}+u_{11}\right)\right. \\
\left.+5733\left(u_{4}+u_{12}\right)-6552\left(u_{3}+u_{13}\right)-8092\left(u_{2}+u_{14}\right)-3672\left(u_{1}+u_{15}\right)\right]
\end{array}\right\}(75)
\end{gathered}
$$

If the smallness of the fourth differences of the adjusted series is to be taken as the ultimate and only test of its regularity of curvature, it will follow that these formulas ought to be used in preference to (53), (54), (55), \&cc., from which, indeed, they do not differ greatly, as can be seen on comparing their decimal weights. The probable errors of the adiusted terms, however, are increased a little, and the weights follow a curve which is not precisely tangent to the line of the zero weights.

At all events, the same principles can be usefully employed in fixing the weight of the middle term in formula (48), so as to give greater regalarity to the adjustment of a double series. By a process precisely analogous to that by which (69) was obtained, it can be proved that in order to render the probable value of the complete sécond difference
$\Delta_{2+2}$ of the adjusted double series a minimum, the weight of the middle term must be increased from 5 to $8 \frac{1}{4}$, so that-

$$
\begin{equation*}
u_{5}={ }_{4}^{1}\left[33 u_{5}+8\left(u_{2}+u_{4}+u_{6}+u_{8}\right)-4\left(u_{1}+u_{3}+u_{7}+u_{9}\right)\right] . . \tag{76}
\end{equation*}
$$

will be the formula required.

## APPENDIX III.

Since the present memoir was written, the author has met with a small work by Schiaparelli, designed with especial reference to the reduction of meteorological observations, and entitled Sul modo di ricavare la vera espressione delle leggi della natura dalle curve empiriche; Milan, 1867. That work, it is proper to acknowledge, anticipates to a certain extent the second method of adjustment here given. It contains, in section 45, a development of the general relation, or system of conditions, which exists between the numerical coefficients or weights, in formulas for adjusting the middle one of any group of an odd number of terms in a series. The mode of demonstration is quite different from the one here followed, and its author does not obtain any of the special adjustment formulas which have here been constructed and recommended, such as (17), (19), \&c., (53), (54), \&c., or (69), (71), \&c. He gives instead, on page 17 , that special case under our formula (13) which arises when we take-

$$
n_{2}=n_{1}, \quad a_{1}=\frac{1}{2}\left(n_{1}-1\right)
$$

and also gives, on page 47 , the formulas which render the probable error of the adjusted term a minimum. We have seen that these last can be derived from equations of condition by the method of least squares; that their weights form series of the second order; and that the adjustments which they make are not nearly so smooth and regular as those made by formulas whose weights follow a curve which is continuous with the line of the zero weights. The method of least squares presupposes that the assumed algebraic equation, of a degree not higher than the third, can accurately represent the true law of the natural phenomenon throughout the whole group of terms included by the formula; and, moreover, to give full scope to the method, the number of terms included ought to be large. These conditions will be but imperfectly fulfilled in practice, and since the true law of the natural series is supposed to be continuous and not irregular or broken, it appears probable, or at least quite possible, that the system of weights which makes the smoothest adjustment will also make the most accurate one.

The method which Schiaparelli gives on pages 23 to 30 of his work, for obtaiuing the values of the constants in empirical equations of algebraic or circular form when the arithmetical means of the terms in certain groups are takeu as data, is not equivalent to the first method here proposed. It requires for completeness two sets of formulas, one to be
used when the number of terms grouped together is odd, and the other when it is even ; it regards the terms as being geometrically represented by ordinates, instead of areas, and does not permit the use of groups composed of a fractional number of terms, and it is not generally applicable to functions of other forms than those specified.

## APPENDIX IV.

## ADDITIONAL FORMULAS FOR INTERPOLATION WITH A CIRCULAR FUNCTION.

Denoting by N the whole number of terms in the circular period, let us write $\frac{2 \pi}{N}=0$; then assuming the curve-

$$
\begin{align*}
& \left.y=\mathrm{A}+\frac{1}{2} \theta\left[\mathrm{~B}_{1} \sin (x \theta)+\mathrm{C}_{1} \cos (x \theta)\right]+\frac{2}{2} \theta\left[\mathrm{~B}_{2} \sin 2(x \theta)+\mathrm{C}_{2} \cos 2(x \theta)\right]\right\} \\
& +\frac{3}{2} \theta\left[\mathrm{~B}_{3} \sin 3(x \theta)+\mathrm{C}_{3} \cos 3(x \theta)\right]+\& c . \tag{77}
\end{align*}
$$

we shall have for the sum of the terms in any group-

$$
\left.\begin{array}{rl}
\mathrm{S}=\mathrm{A} n & +\sin \frac{1}{2}(n \theta)\left[\mathrm{B}_{1} \sin (x \theta)+\mathrm{C}_{1} \cos (x \theta)\right] \\
& +\sin \frac{2}{2}(n \theta)\left[\mathrm{B}_{2} \sin 2(x \theta)+\mathrm{C}_{2} \cos 2(x \theta)\right]  \tag{78}\\
& +\sin \frac{3}{2}(n \theta)\left[\mathrm{B}_{3} \sin 3(x \theta)+\mathrm{C}_{3} \cos 3(x \theta)\right]+\& c .
\end{array}\right\}
$$

From this we can derive formulas for computing the values of the constauts $A, B_{1}, \mathbf{C}_{1}, B_{21} \mathrm{C}_{2}$, \&c., just as formulas (A), (B), (C), \&c., were derived from the algebraic formula (11); or, otherwise, we can determine the constants by treating the equations of condition in the manner pecaliar to the method of least squares. The results are the same in either case. When the N terms are divided into three consecutive groups of equal extent, we shall have-

$$
\left.\begin{array}{l}
A=\frac{1}{\mathrm{~N}}\left(\mathrm{~S}_{1}+\mathrm{S}_{2}+\mathrm{S}_{3}\right) \\
\mathrm{B}_{1}=\frac{2}{3}\left(\mathrm{~S}_{3}-\mathrm{S}_{1}\right) \\
\mathrm{C}_{1}=\frac{4}{9} \sin 60^{\circ}\left[2 \mathrm{~S}_{2}-\left(\mathrm{S}_{1}+\mathrm{S}_{3}\right)\right] \tag{a}
\end{array}\right\}
$$

With four groups, we get-

$$
\left.\begin{array}{l}
\mathrm{A} \doteq \frac{1}{N}\left(\mathrm{~S}_{1}+\mathrm{S}_{2}+\mathrm{S}_{3}+\mathrm{S}_{4}\right) \\
\mathrm{B}_{1}=\frac{1}{2}\left[\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)+\left(\mathrm{S}_{4}-\mathrm{S}_{1}\right)\right] \\
\mathrm{C}_{1}=\frac{1}{2}\left[\left(\mathrm{~S}_{2}+\mathrm{S}_{3}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{4}\right)\right] \\
\mathrm{B}_{2}=\frac{1}{4}\left[\left(\mathrm{~S}_{3}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{4}-\mathrm{S}_{1}\right)\right]
\end{array}\right\} \text { (b) }
$$

We omit the formulas for five, seven, nine, \&c., groups, which are not required in practice, the common use of monthly or hourly data in
meteorology making it convenient to have the number of groups a divisor of 24 . With six groups, the constants are-

$$
\begin{aligned}
& A=\frac{1}{N}\left(S_{1}+S_{2}+S_{3}+S_{4}+S_{5}+S_{6}\right) \\
& B_{1}=\frac{1}{3}\left[2\left(S_{5}-S_{2}\right)+\left(S_{4}-S_{3}\right)+\left(S_{6}-S_{1}\right)\right] \\
& C_{1}=\frac{2}{3} \sin 60^{\circ}\left[\left(S_{3}+S_{4}\right)-\left(S_{1}+S_{6}\right)\right] \\
& \mathrm{R}_{2}=\frac{1}{3}\left[\left(S_{4}-S_{3}\right)-\left(\mathrm{S}_{6}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{C}_{2}=\frac{2}{9} \cdot \sin 60^{\circ}\left[\left(\mathrm{S}_{3}+\mathrm{S}_{4}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{6}\right)-2\left(\mathrm{~S}_{2}+\mathrm{S}_{5}\right)\right] \\
& \mathrm{B}_{3}=\frac{1}{2}\left[\mathrm{~B}_{1}-\left(\mathrm{S}_{5}-\mathrm{S}_{2}\right)\right]
\end{aligned}
$$

With eight groups-

$$
\begin{align*}
& \mathrm{A}=\frac{1}{\mathrm{~N}}\left(\mathrm{~S}_{1}+\mathrm{S}_{2}+\cdots \cdots+\mathrm{S}_{8}\right) \\
& \left.B_{1}=\frac{1}{4}\left(2 \sin 45^{\circ}+1\right)\left[\left(S_{6}-S_{3}\right)+\left(S_{7}-S_{2}\right)\right]+\frac{1}{4}\left[\left(S_{5}-S_{4}\right)+\left(S_{8}-S_{1}\right)\right]\right) \\
& \mathrm{C}_{1}=\frac{1}{4}\left(2 \sin 45^{\circ}+1\right)\left[\left(\mathrm{S}_{4}+\mathrm{S}_{5}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{8}\right)\right]+\frac{1}{4}\left[\left(\mathrm{~S}_{3}+\mathrm{S}_{6}\right)-\left(\mathrm{S}_{2}+\mathrm{S}_{7}\right)\right] \\
& \mathrm{B}_{9}=\frac{1}{4}\left[\left(\mathrm{~S}_{5}-\mathrm{S}_{4}\right)+\left(\mathrm{S}_{6}-\mathrm{S}_{3}\right)-\left(\mathrm{S}_{7}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{8}-\mathrm{S}_{1}\right)\right]  \tag{d}\\
& \mathrm{C}_{2}=\frac{1}{4}\left[\left(\mathrm{~S}_{4}+\mathrm{S}_{5}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{8}\right)-\left(\mathrm{S}_{3}+\mathrm{S}_{6}\right)-\left(\mathrm{S}_{2}+\mathrm{S}_{7}\right)\right] \\
& \mathrm{B}_{3}=\mathrm{B}_{1}-\sin 45 \circ\left[\left(\mathrm{~S}_{6}-\mathrm{S}_{3}\right)+\left(\mathrm{S}_{7}-\mathrm{S}_{2}\right)\right] \\
& \mathrm{C}_{3}=\mathrm{C}_{1}+\frac{1}{2}\left[\left(\mathrm{~S}_{2}+\mathrm{S}_{7}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{8}\right)-\left(\mathrm{S}_{3}+\mathrm{S}_{6}\right)-\left(\mathrm{S}_{4}+\mathrm{S}_{5}\right)\right] \\
& \mathrm{B}_{4}=\frac{1}{8}\left[\left(\mathrm{~S}_{5}-\mathrm{S}_{4}\right)+\left(\mathrm{S}_{7}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{6}-\mathrm{S}_{3}\right)-\left(\mathrm{S}_{8}-\mathrm{S}_{1}\right)\right]
\end{align*}
$$

And with twelve groups-

$$
\begin{aligned}
& \mathrm{A}=\frac{1}{\mathrm{~N}}\left(\mathrm{~S}_{1}+\mathrm{S}_{2}+\cdots+\cdots+\mathrm{S}_{12}\right) \\
& \mathrm{B}_{1}=\frac{1}{3}\left(\sin 60^{\circ}+1\right)\left[\left(\mathrm{S}_{9}-\mathrm{S}_{4}\right)+\left(\mathrm{S}_{10}-\mathrm{S}_{3}\right)\right]+\frac{1}{3}\left(\sin 60^{\circ}+\frac{1}{2}\right)\left[\left(\mathrm{S}_{8}-\mathrm{S}_{5}\right)\right. \\
& \left.+\left(\mathrm{S}_{11}-\mathrm{S}_{2}\right)\right]+\frac{1}{6}\left[\left(\mathrm{~S}_{7}-\mathrm{S}_{8}\right)+\left(\mathrm{S}_{18}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{C}_{1}=\frac{1}{3}\left(\sin 60^{\circ}+1\right)\left[\left(\mathrm{S}_{6}+\mathrm{S}_{7}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{12}\right)\right]+\frac{1}{3}\left(\sin 60^{\circ}+\frac{1}{2}\right)\left[\left(\mathrm{S}_{5}+\mathrm{S}_{8}\right)\right. \\
& \left.-\left(S_{2}+S_{11}\right)\right]+\frac{1}{6}\left[\left(S_{4}+S_{9}\right)-\left(S_{3}+S_{10}\right)\right] \\
& B_{2}=\frac{1}{6}\left[2\left(\mathrm{~S}_{8}-\mathrm{S}_{5}\right)+\left(\mathrm{S}_{7}-\mathrm{S}_{6}\right)+\left(\mathrm{S}_{9}-\mathrm{S}_{4}\right)-2\left(\mathrm{~S}_{11}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{10}-\mathrm{S}_{3}\right)\right. \\
& \left.-\left(\mathbb{S}_{12}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{O}_{2}=\frac{1}{3} \sin 60^{\circ}\left[\left(\mathrm{S}_{6}+\mathrm{S}_{7}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{12}\right)-\left(\mathrm{S}_{4}+\mathrm{S}_{9}\right)-\left(\mathrm{S}_{3}+\mathrm{S}_{10}\right)\right] \\
& B_{3}=\frac{1}{6}\left[\left(S_{7}-S_{6}\right)+\left(S_{8}-S_{5}\right)+\left(S_{11}-S_{2}\right)+\left(\mathrm{S}_{12}-\mathrm{S}_{1}\right)-\left(\mathrm{S}_{9}-\mathrm{S}_{4}\right)\right. \\
& \left.-\left(\mathrm{S}_{10}-\mathrm{S}_{3}\right)\right] \\
& \mathrm{C}_{8}=\frac{1}{6}\left[\left(\mathrm{~S}_{6}+\mathrm{S}_{7}\right)+\left(\mathrm{S}_{3}+\mathrm{S}_{10}\right)+\left(\mathrm{S}_{2}+\mathrm{S}_{11}\right)-\left(\mathrm{S}_{6}+\mathrm{S}_{8}\right)-\left(\mathrm{S}_{4}+\mathrm{S}_{9}\right)\right. \\
& \left.-\left(\mathrm{S}_{1}+\mathrm{S}_{12}\right)\right] \\
& \mathrm{B}_{4}=\frac{1}{6}\left[\left(\mathrm{~S}_{7}-\mathrm{S}_{6}\right)+\left(\mathrm{S}_{10}-\mathrm{S}_{3}\right)-\left(\mathrm{S}_{8}-\mathrm{S}_{4}\right)-\left(\mathrm{S}_{18}-\mathrm{S}_{1}\right)\right] \\
& \mathrm{C}_{6}=\frac{1}{8} \sin 60^{\circ}\left[\left(\mathrm{S}_{6}+\mathrm{S}_{7}\right)+\left(\mathrm{S}_{4}+\mathrm{S}_{9}\right)+\left(\mathrm{S}_{3}+\mathrm{S}_{10}\right)+\left(\mathrm{S}_{1}+\mathrm{S}_{12}\right)\right. \\
& \left.-2\left(\mathrm{~S}_{5}+\mathrm{S}_{8}\right)-2\left(\mathrm{~S}_{2}+\mathrm{S}_{11}\right)\right] \\
& \mathrm{B}_{5}=\left(\mathrm{S}_{7}-\mathrm{S}_{6}\right)+\left(\mathrm{S}_{8}-\mathrm{S}_{5}\right)+\left(\mathrm{S}_{11}-\mathrm{S}_{2}\right)+\left(\mathrm{S}_{13}-\mathrm{S}_{1}\right)-\mathrm{B}_{1}-4 \mathrm{~B}_{3} \\
& \mathrm{C}_{5}=\left(\mathrm{S}_{6}+\mathrm{S}_{7}\right)-\left(\mathrm{S}_{1}+\mathrm{S}_{12}\right)-\mathrm{C}_{1}-2 \mathrm{C}_{3} \\
& B_{6}=\frac{1}{12}\left[\left(\mathrm{~S}_{7}-\mathrm{S}_{6}\right)+\left(\mathrm{S}_{9}-\mathrm{S}_{4}\right)+\left(\mathrm{S}_{11}-\mathrm{S}_{2}\right)-\left(\mathrm{S}_{8}-\mathrm{S}_{5}\right)-\left(\mathrm{S}_{10}-\mathrm{S}_{3}\right)\right. \\
& \left.-\left(\mathrm{S}_{19}-\mathrm{S}_{1}\right)\right]
\end{aligned}
$$

To illustrate the use of these formulas by an example, let us take the series employed in illustrating Cauchy's method of interpolation in the 22 \$ 71

United States Coast Survey Report for 1860, page 392. Columu (1) of the following table shows the terms of the given series corresponding to each hour of the day:

| Hour. | (1) | (2) | Hour. | (1) | (2) | Hour. | (1) | (2) |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | . .19 | .187 | 8 | -.15 | -.104 | 16 | -10 | -.080 |
| 1 | .17 | .176 | 9 | .01 | .000 | 17 | -.18 | -.173 |
| 2 | .05 | .114 | 10 | .10 | .109 | 18 | -.26 | -.220 |
| 3 | .04 | .019 | 11 | .19 | .192 | 19 | -.19 | -.211 |
| 4 | -.10 | -.082 | 12 | .29 | .233 | 20 | -.12 | -.148 |
| 5 | -.14 | -.161 | 13 | .19 | .214 | 21 | .01 | -.050 |
| 6 | -.19 | -.196 | 14 | .13 | .142 | 22 | .04 | .057 |
| 7 | -.14 | -.175 | 15 | .06 | .035 | 23 | .12. | .144 |

It is required to represent this series by a formula containing five constants. We will not make any preliminary adjustment by the second method, as that is not indispensable to our system of interpolation by groups, although it is generally desirable, as, indeed, it would be in a less degree with Cauchy's method, which also depends on the summation of irregular series of quantities within certain intervals. Dividing our 24 given terms into six groups of equal extent, we get-

$$
\begin{array}{lll}
\mathrm{S}_{1}=.45 & \mathrm{~S}_{3}=.15 & \mathrm{~S}_{5}=-.73 \\
\mathrm{~S}_{2}=-.57 & \mathrm{~S}_{4}=.67 & \mathrm{~S}_{6}=.05
\end{array}
$$

Computing by formula (c) the values of the first five constants, and substituting them in (78), we have-

$$
\begin{aligned}
S=.0008 & +\sin \frac{1}{2}(n \theta)[-.0667 \sin (x \theta)+.1848 \cos (x \theta)] \\
& +\sin (n \theta)[.3067 \sin 2(x \theta)+.7544 \cos 2(x \theta)]
\end{aligned}
$$

which we transform into-

$$
\begin{aligned}
\mathrm{S}=.0008 & +.1965 \sin \frac{1}{2}(n \theta) \sin \left(x \theta+109051^{\prime}\right) \\
& +.8144 \sin (n \theta) \sin \left(2 x \theta+67053^{\prime}\right)
\end{aligned}
$$

This expresses the sum S of any group of $n$ terms in the graduated series, the abscissa of the middle point of the group being $x$, and each term being supposed to occupy, on the axis of $X$, a space equal to unity. The angle $\theta$ is $\frac{2 \pi}{24}=15 \circ$. If we further take $n=1$ and $\mathrm{S}=u$, we obtain the equation of the graduated series-

$$
u=.001+.026 \sin \left(x \theta+109 \circ 51^{\prime}\right)+.211 \sin \left(2 x \theta+67 \circ 53^{\prime}\right)
$$

From this the values in column (2) are computed. The sums of the terms in its six groups are, of course, not precisely equal to those in column (1). To make them so, it would be necessary to add to the equation the term containing the sixth constant $\mathrm{B}_{3}$. This term is-

$$
+.018 \sin 3(x \theta)
$$

The origin of co-ordinates is at the middle of the series. If we wish to
transfer it to the first term, we put $x-11 \frac{1}{2}$ in the place of $x$, and thus get-

$$
u=.001+.026 \sin \left(x \theta+297021^{\prime}\right)+.211 \sin \left(2 x \theta+82053^{\prime}\right)
$$

which does not differ greatly from the equations obtained by Cauche's method and the method of least squares, as given in the Coast Surrey Report.

Similar results would be obtained by dividing the given series into eight or twelve groaps, and computing the values of the first five constants from formulas (d) or (e). These results would probably be a little more accurate than the preceding, being in accordance with the principle of least squares, as already stated:
In cases where the data for interpolation are the mean ralues $\mathrm{M}_{1}, \mathrm{M}_{2}$, $\mathrm{M}_{3}$, \&c., of the ordinate, taken within intervals formed by equal divisions of the circular period N, our formulas $(a),(b),(c)$, \&c., will still be applicable. For instance, with three intervals, we shall have-

$$
\mathrm{S}_{1}=\frac{1}{3} \mathrm{M}_{1} \mathrm{~N}, \quad \mathrm{~S}_{2}=\frac{1}{3} \mathrm{M}_{2} \mathrm{~N}, \quad \mathrm{~S}_{3}=\frac{1}{3} \mathrm{M}_{3} \mathrm{~N}
$$

Formula (a) then gives the values of the three coustants, and since $\mathrm{S}=\mathrm{M} n$, formula (78) becomes-

$$
\mathrm{M}=\mathrm{A}+\frac{1}{n} \sin \frac{1}{2}(n \theta)\left[\mathrm{B}_{1} \sin (x \theta)+\mathrm{C}_{1} \cos (x \theta)\right]
$$

which expresses the mean value $M$ of the ordinate within any interval $n$.
To illustrate this, let us take the corrected mean temperatures at New Haven (Transactions of the Connecticut Academy, Vol. I, p. 233) for intervals of four months:

$$
\begin{aligned}
& \text { January to April................... } \mathbf{M}_{\mathbf{i}}=34^{\circ} .35 \text { Fahr. } \\
& \text { May to August..................... } \quad \mathrm{M}_{2}=66^{\circ}, 84 \\
& \text { September to December . . . . . . . . . . } M_{3}=460.15
\end{aligned}
$$

To obtain from these an equation for the series of daily means, we have $\mathrm{N}=365 \frac{1}{4}$, and consequently-

$$
S_{1}=4182, \quad S_{2}=8138, \quad S_{3}=5619
$$

Formula (a) then gives-

$$
\mathrm{A}=49.11, \quad \mathrm{~B}_{1}=958, \quad \mathrm{C}_{1}=2492
$$

and (78) gives-

$$
\mathrm{M}=49.11+2670\left(\frac{1}{n}\right) \sin \frac{1}{2}(n \theta) \sin \left(x \theta+68^{\circ} 58^{\prime}\right)
$$

This equation expresses the mean temperature of any interval of $n$ days. The angle $\theta$ is-

$$
\theta=\frac{2 \pi}{365 \frac{1}{4}}=0059^{\prime} .138
$$

If we also take $n=1$, the equation of daily means is found to be-

$$
\mathrm{M}=49.11+22.91 \sin \left(x \theta+68058^{\prime}\right)
$$

The origin of co-ordinates is at the middle of the year.

# REPORT ON THE TRANSACTIONS OF THE SOCIETY OF PHYSICS AND NATURAL HISTORY, OF GENEVA, FROM JUNE, 1870, TO JUNE, 1871. 

By M. Henri de Saussure, Presidentr.

[Translated for the Smithsonian Institution.]
The year which has just passed has been marked loy events which have left but little time for the peaceful occupations of science. The war burst upon us almost at the moment that our scientific year commenced, and we can hardly yet say that it has terminated. If Switzerland has not been oppressed by belligerent armies, she has, nevertheless, been obliged to play an active part in the duties which her neutrality imposes upon her, and there are few present who during this sad period have not been in one way or another diverted from their regular occupations. Several members of the society have not hesitated to make the sacrifice of their precious time to works of charity which the evils of war have rendered every day more indispensable; in fact no one has been able to escape the preoccupations occasioned by the important events which have transpired in a neighboring theater of our frontier.
On this account the convocation of the scientific congress, announced for the second half of the year 1870, has been countermanded. The Helvetic Society of Natural Sciences, convoked at Frauenfeld for the month of August, has not been able to assemble, and a geological congress, organized at Geneva under the superintendence of MM. Favre, father and son, and of M. F. J. Pictet de la Rive, has been obliged to be postponed to some other time. We can therefore scarcely be surprised that our society should itself be somewhat affected by the exterior agitations, and that the meetings should have been less frequented than in ordinary times.
It, howerer, the catastrophes to which I have alluded, have somewhat diminished the activity of our members, they have procured us, by a kind of compensation, the inappreciable advantage of having seated among us a number of foreign savants, who, exiled from their homes through the vicissitudes of war, have found in the shelter of our neutrality a refuge both peacefal and hospitable. In attending our meetings, and in favoring us with their communications, they have cast upon our reunions a luster of which our records will preserve the remembrance. These savants were M. M. Regnault, of the Institute, and M. P. Cap, of the Academy of Medicine at Paris; M. le Professor Fée, of Strasburg; ${ }^{1}$
and M. Guenée, of Chateaudun. The assiduity with which these gentlemen have associated themselves with us in our labors, the desire which they have manifested to continue with us in relations in which theinterest of the society has been so largely increased, has induced us to confer upon them the title of honorary members; and your president before resigning lis place to his successor bad the pleasure of expressing to them the faithful interpretation of our sentiments.
To the names of the savants whom I have just mentioned, I must add those of several gentlemen who have sojourned with us only a short time, particularly M. Bigot and M. Duperrey, who have only appeared at our meetings at brief intervals. Lastly, we have welcomed in our city our emeritus member, M. Dumas, perpetual secretary of the Academy of Sciences, whom we delight to claim as one of ourselves; for none of you can forget that it was at Geneva that M. Dumas published his first works, and that he stands to-day among the elders of our society of physics.
It is very seldom, gentlemen, that a year passes without our being called upon to mourn the departure of one of our colleagues. To-day we have to lament the death of a highly esteemed savant, who was admitted into our ranks only a few short months ago. Dr. Augustus Waller was born, in 1816, at Elverton, near Ferusham, in the county of Kent, England. He pursued the study of medicine in France, and received in 1840 a diploma. of doctor of medicine from the faculty of Paris. He then returned to England and established himself at Kensington; where he practiced medicine for several jears. But the ordinary occupation of the physician was not sufficient to satisfy his investigating spirit, and he always found time to devote himself to scientific researches in the domain of anatomy and physiology. His principal investigations were directed to the nervous system, which did not fail to lead to important discoveries, and some well-known experiments which he made in London upon the degeneracy which the nerves and the nervous center undergo, obtained for he the of member of the Royal Society, and the grand prize of physiology from the Academy of Scieuces at Paris. Not finding in London an the facilities necessary to his researches, he resolved to change his residence, and did not hesitate to sacrifice to his studies a practice which had become extensive. He removed with his family to Bonn, where he had full leisure to. continue his physiological and microscopical investigations upon the nervous system.
The researches which he made in physiology, either alone or in collaboration with Professor Budge, entitled him to more honorable distinction on the part of the Academy of Sciences at Paris. He obtained for the second time the great prize of physiology on account of his discoveries relative to the functions of the great sympathetic nerve, and to the influence of the spinal marrow upon the pupil. From Bonn, Waller repaired to Paris, and after having labored for several. years in the
laboratory of Flourens, he was called to Birmingham to occupy a chair of physiology and a position as physician to the hospital of that city. He even then felt the first symptoms of the diseases which subsequently carried him off, and was obliged to give up some of his labors on account of his failing health. He next removed to Switzerland, and after having lived in the Canton Vaud for several years, he came in 1868 to reside in Geneva.

Although Waller had been obliged to abandon his regular labors, his mind, unusually active and ingenious, could not remain idle, and he never entirely ceased to occupy himself with interesting questions in physiology and medicine. At Geneva, his health laving improved, he devoted himself anew to medical practice, to which he was always much attached, and his large experience in that line rendered him especially eminent. In 1869 he was received as a member of our society. The same year he had the honor of being invited to delirer the Croonian lecture to the Royal Society of London, and for that purpose repaired to England. His health, which appeared to be confirmed, was not established. He had suffered several severe attacks of quinsy, a malady which suddenly terminated his existence on the 18th of September, 1870, at the age of fifty-five years.

It would take too much time to analyze all the labors of our lamented associate; we shall limit ourselves to a short summary of those which have excited the most interest in the scientific world, particularly his work upon the degeneracy of the nerves. The nerves which are distributed through different parts of the body are, we know, composed of different fibers, intermixed with each other-those which call into action motive-power, and those which convey impressions of sensibility. At their origin, that is to say at their point of emergence, from the spinal marrow, the motor nervous fibers are separated from the sensitive uervous fibers; the former constituting the anterior roots and the latter the posterior. After having demonstrated by experiment that when a complex nerve is cut, the outer segment, suddenly arrested, withers and degenerates, while the central segment, remaining in communication with the nervous center, continues unchanged, Waller studied the degeneration of the nerves taken at their origin. Beginning at the nervous roots, he proved that the nervous center, which maintains intact the nervous fibers of the anterior roots, is seated in the spinal marrow itself, while the nervous center, which continues intact the nervous fibers of the posterior roots, is situated in the intervertebral ganglion, united to their posterior roots. It was by means of sections of these roots taken at different distances, that Waller made these important discoveries, the application of which immediately occurred to him. The changes which take place in the structure of a uerre after the cutting are so evident that the experimenter can avail himself of it as a means of tracing the distribution of their fibers in the different tissues. It is in this way that he succeeded in perceiving the
terminations or ends of the nerves in the tongue, a study which he made for the most part upon the tongue of a living frog. This new method of investigation in regard to the nervous system, which obtained for Waller the prize of physiology from the Academy of Sciences at Parris, has been of great service. In order to give a just idea of its merits we sliall quote the words of Professor Vulpian, who in his Course of Plysiology of the Nervous System, describes with care this method, to which he proposes to give the name of the Wallerian method. After having given numerous examples from the experiments we have already cited, M. Vulpiapn adds: "To this day we have not deduced from this method all the results which it is able to furnish; but sooner or later we will institute some special researches, taking it as our point of departure, and without doubt we shall discover important and valuable truths in regard to anatomical physiology." An important discovery of Waller is that of the exudation of the white globules of the blood from their vessels. The memoir which he published upon this subject in 1846 had been forgotten, when Cohnheim and other microscopists rediscovered the facts in 1867, and from them deduced a new theory in regard to inflammation. M. Stricker, of Vienna, in an interesting article which appeared in 1869, awarded to Waller all the honor of the priority of this discovery. We have confined ourselves to the analysis of the works of Waller, and for more ample information we refer the reader to the list of his publications. It will suffice to give at least an approximation of the extent of the researches of this eminent man's investigations, all of which bear the stamp of true originality.

Waller had, indeed, a mind essentially ingenious. The experiments which he devised, the subsequent operations he employed, the new methods he put in practice, all, to the minutest details, exhibit the char acteristics of an eminently inventive genius. He also possessed the very valuable trait of never allowing himself to be carried away by hypotheses. Whatever opinions he advanced, he desired to prove mathematically. As long as there remained any doubt on his mind, he would have recourse to new experiments and imagine new methods by which it might be removed. His talent for exposition was remarkable, as we all know by experience in listeniag to the communications he made to our society. In him science has lost a man of rare merit, while Geneva was only too happy to include him among her residents.

Having rendered all due respect to the memory of our lamented colleague, I will give a rapid sketch of the labors of the society, in accordance with the plan adopted for the report of each year.

## PHYSICAL SCIENCES.

It is principally in this domain of science that we have listened to the most numerous lectures ; partly because the stranger savants who have visited us were principally physicists, partly because of the
accidintal absence of our excellent colleague, M. E. Claparede, always rich in communications on other subjects of a character to interest the society. Unfortunately the condition of his health this winter causes us the greatest anxiety.
General Dufour has given a summary of the results of the experiments upon which he has been engaged for some time in regard to the relative movement of material points, a question which is of interest to general astronomy. 1. In studying the movement of two stars around a supposed fixed point, it is demonstrated by observation that this point must be in motion. 2. The curve being plane, and the stars remaining in the same plane during their translation, it may therefore be concluded that the stars have all received an impulse resulting in a parallel movement. 3. The movement of the apsides proves that the center of gravity of the system is displaced, not following a straight line, but describing a curved one.
Professor Emile Plantamour has made this year, as formerly, a sojourn among the mountains, in order to determine the astronomical co-ordinates of the different stations of Switzerland. The Simplon was the place he selected for his operations in 1870. The latitude of this station, as derived from his observations, is $46^{\circ} 14^{\prime} 59^{\prime \prime} .4$, with a possible error of a quarter of a second.
The unusually cold winter which we have experienced has naturally attracted the attention of meteorologists, and M. Plantamoar, according to his custom, has given some results deduced from the compared course of the temperature of different years. The months of December and January of this winter have shown a mean temperature of 20.45 . This period of the winter is very similar to that of the winter of $1837-38$, of which the mean temperature was - 20.3 ; but the winter of 1829 , the remembrance of which is still traditional throughout the country, was colder still, as in December and January, the mean temperature was 40.7 .
Colonel E. Gautier has presented frequent communications relative to the constitution of the sun. In a paper read at the April meeting he gave ant account of an important memoir from Professor $L$, Respighi, director of the observatory of the capitol, upon some spectroscopical observations continued for fourteen months, and which have been made principally with reference to the protuberances of the edges of the sun. The author infers from his observations that the sun must have an exterior liquid envelope, compressing the overheated gases in its interior. These gases at times force themselves through the envelope, and occasion formidable eruptions; after which they disperse and combine with the elements of the surface of the sun. In consequence of these combinations, obscure points appear which in agglomerating form the spots on the disk of the sun. These masses float at the surface of theincandescent globe as dross a result arrived at by M. Gautier several years ago in trying to re-establish the theory of Gallileo, and of Simon

Marius. The paper of M. Gautier has been inserted in the Archives of Science 1871, volume XLI, page 27. He has continued to keep us informed in regard to important discoveries made in the domain of general astronomy.
Professor Cellérier presented a paper upon the molecular constitation of gas. According to modern hypothesis, gases are composed of molecules, endowed with a movement of translation in every direction, and freed during the major part of the duration of this movement, from all mutual action, this action only revealing itself by shocks. Whatever be the nature of the latter, their consequences, according to the general laws of mechauics, can only be similar to those which are produced by the shock of two perfectly elastic bodies. The movement after the shock depends either upon the direction of the movement before the shock, or, upon fortuitous circumstances, such as the direction of the plane of the shock. If we admit that, during a certain time, the direction of this plane is always parallel to one or the other of the three rectangular planes, the result must be that the diffusion of the densities, in all the masses would occur immediately, contrary to all experience. It would be the same for an infinity of otherdirections of the plane of the shock. M. Cellérier has therefore concluded that the theory of gases which Clausius and other physicists have proposed is not absolutely admissible, at least under this simple form. This communication has given rise to some observations by A. de la Rive, upon the impossibility of doing without the intervention of ether, in explaining the phenomena which the gases present.
Our compatriot, M. Duperrey, for a number of years professor at Paris, has taken advantage of a sojourn at Geneva, to lay before the society some researches which he has undertaken, to find a simple and practical relation between the temperature and the maximum tension of steam. He has obtained the following result, remarkable for its simplicity, that this tension represented in kilogrammes by square, centimeters, is nearly exactly equal to the fourth power of the temperature.
M. Serra Carpi, a Roman engineer, in passing through Geneva, has given some details relative to the variation of the mean temperature at different heights, a subject treated in a pamphlet, of which he has given to the society a copy. Professor Marcet, in a letter addressed from London to M. de la Rive, has given an account of the last observations of Dr. Carpenter upon the waters of the Mediterranean. These observations were extended to a depth of 3,000 meters. At this depth the water is turbulent, and contains a great quantity of dissolved gas. The density changes from $10^{\circ} .27$ at the surface, to $10^{\circ} .29$ at 2,000 meters, and to $10^{0} .28$ at 3,000 meters of depth. The denser water rests therefore upon water less dense; this singular fact can be explained by currents, of which Dr. Carpenter has without doubt confirmed the existence.
In the domain of physics, Professor Regnault has presented to the society an important communication, which occupied an entire meeting.

This distinguished academician gave his views as to the manner of understanding and studying meteorology, also as to the best form to be adopted for the instruments which are employed in this branch of science. He thinks that meteorology should be considered less as a dependence of astronomy, than as auxiliary to physiology, since it assists especially in determining the isothermal lines, and its priacipal object is to give account of the physical circumstances which favor or retard the development of organized beings. As to the instruments, he is in favor of simplifying them in order to render them accessible to the greatest number of people. He proposes particularly to attach to barometers and thermometers photographical registering apparatus moring by clock-work, which will record without trouble the variations of these instruments and enable us to read them with perfect exactness. Instruments constructed upon this model would be of great assistance in the researches within the domain of plysiology, botany, agriculture, etc.
The phenomena relative to the aurora borealis have been, as in the past, the object of different communications from Professor A. de la Rive, who continues to keep the society informed upon this sulject. The same member has given an account of the important researcies which he has made in regard to the rotatory magnetic power of liquids. After having devised the apparatus he employed, and the new methods he had adopted to avoid as much as possible all sources of error, he has studied successively different liquids in order to determine their magnetic rotatory power, such in particular as sulphurous acid, which had not previously been submitted to this kind of experiment, different mitxures of solutions, and a certain number of isomeric bodies of which none presented the same magneto-rotatory power. The influence of temperature has also been analyzed with care, and it has been to prove that it tends to diminish this power, which is evidently due to the manner in which the particles are grouped. M. de la Rive has also presented in concert with M. Edward Sarasin, a work which they have made together on the action of magnetism upon rarefied gases traversed by discharges of electricity. In operating successively uponatmosphericair, upon carbonic acid gas, and upon hydrogen, these two physicists have found that the magnetism produces in the portion of gas directly traversed by the discharge an increase of density, and besides an augmentation or a dimination of resistance to the conductibility according as the electrical jet is directed equatorially or axially between the poles of the electro-magnet. These augmentations and diminutions vary with each gas. They are nothing in certain positions of the jet with reference to the magnet, and are probably due, when they manifest themselves, to the perturbation caused by the action of magnetism in the disposition which the gaseous particles affect when they propagate electricity. (These two memoirs are inserted in the archives.) M. L. Soret read a memoir upon the polarization of light by water, as studied upon that of different lakes, upon sea-water and upon suow-water. He shows that
the phenomenon is more intense when the water is clearest, and that the polarization takes place for all parts of the spectrum equally. Disturbed or muddy waters give no polarization. The same physicist has also given an account of some experiments he has made in order to verify the results obtained by M. Christiansen and by M. Kundt, upon the abnormal dispersion of the light of bodies of superficial colors. The two works which I have mentioned have been published in the Archives of Science, and I refer you to them. M. Raoul Pictet has presented a paper on the resistance a body experiences in its motion through the air, with a uniform velocity. It would be difficult to give an analysis of it in a few words. This resistance is expressed by the formula $R=K v^{2}$, which is indicated by calculation, and experimentally verified.

The same savant has repeated, at the meetings of the society, various experiments, having for their object to show the emissive and absorbent powers of ice for heat, and the influence which they exercise upon its formation and its fusion. In order to prove experimentally the radiant power of ice for black heat, M. Pictet has made a piece of ice contract rapidly by the action of this radiancy, in immersing it at the level of the surface of water at $0^{\circ}$, and in exposing it to the air under a serene sky. From another side he has shown that ice is almost entirely diathermal for luminous heat, and altogether diathermal for black heat. In projecting a ray of luminous heat through a block of ice inclosing specks of foreign bodies there is formed around each corpuscle a drop of water, resulting from the absorption of the black heat which these bodies radiate under the luminous rays; and when these foreign bodies are sufficiently numerous the ice is disintegrated through its entire depth, and is melted. If, on the contrary, a ray of black heat is projected upon the block of ice, as this does not penetrate into the substance of the ice, it produces a fusion of the superficial stratum only, and does not affect the interior parts.

Professor Marignac has communicated to us the result of his researches upon the specific heat of saline solutions. (Inserted in the Archives, vol. XXXIX, page 217.) ${ }^{\bullet}$
M. Morin read a memoir upon the azotized substances found in the embryos of herbivorous animals, and especially in their eggs.

Our emeritus member, M. Dumas, has laid before the society various important questions, which were discussed by the Academy of Sciences at Paris during the siege of that capital. The necessity of having recourse to balloons for carrying on correspondence led to various improvements in the art of æronautics. It was necessary, on account of economy, to construct the balloons of cotton material, and in order to render this impermeable, a varnish of India rubber was used. But M. Dumas showed that India rubber is permeable to gas, and proposed to superimpose on it some substances soluble in water, especially gelatine. By superposing the two substances, a varnish was obtained impermeable both to gas and the moisture of the air. It was also observed that it was best to

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launch the balloons about 3 or 4 o'clock in the morning, because at that hour they were covered with dew, of which the gradual evaporation lightened them during the morning hours, and allowed them to maintain the same height without it being necessary to throw out ballast. Numerous trials, which seem to have some success, have been made in regard to directing balloons, but have not yet been completed.

The scarcity of food has induced many persons attempt to imitate the elements of first necessity, and M. Dumas has read on this subject a memoir in which he proves the impossibility of producing milk artificially. The fabrication of this substance has been frequently attempted and has been practiced upon a great scale, but the artificial milk can never take the place of the natural milk, for the latter exhibits an incontestable organic structure which cannot be reproduced chemically; the fat corpuscles are enveloped in a pellicle, which prevents ether from dissolving them. We find these globules with their pellicle even in the milk extracted from the lacteal vessels at the moment when the secretion of the glands takes place, which proves that they have a physiological origin. M. P. Cap, who we all know has been remarkably assiduous at our meetings, has read two papers concerning the history of chemistry. The numerous historic notices which proceed from the pen of this author are so well known to those who follow the progress of science, that it is hardly necessary to mention how peculiarly well qualified he is to treat these subjects. In his memoir upon the discovery of oxygen he has proved that this body was in the first place discovered by Bayat, a French chemist, fallen unjustly into oblivion, and that the work of Priestley and of Scheele is confined to making known the properties of oxygen, as well as those of its compounds. But Lavoisier's eminently generalizing mind gave to this discovery its true importance, and deduced from it its now recognized relations to the nomenclature and the science of chemieal combinations. M. Cap has also given an account of the discovery of iodine by Bernard Courtois, in which he particularly dwells upon the first phases of this discovery, and upon the biography of its author. These notices have appeared in the Journal of Pharmacy, so it is not necessary for us to speak of the further.

## NATURAL SCIENCES.

Aeology.-Professor Alphonse de Candolle has examined the question whether in case the flora which exists should be reduced to a fossil state. we would be able to discover any characteristic which would determine in a precise manner the geological age of the strata in which it occurs. Now, he has proved that there is no such general characteristic among the phanerogamous plants which are now found at the surface of the earth, and it is not probable there exists any among the cryptogamous plants. It has probably been the same at all other eporhs, and consequently the similarity between two geological strata,
situated in different parts of the earth, does not prove them to be of the same age. The term geological epoch, which always implies some dispinction in the flora and in the fanna, in reference to other epochs, lis, therefore, not adapted to the scientific signification for which it is intended. The above-mentioned idea is being more and more introduced into science.

Professeur D. Colladon has placed before the society some beantiful photographs, which represent cuttings of the earth upon the hill of Geneva, executed upon the Tranchées, a hill which is believed to be a product of the ancient alluvion of the river Arve. He published in 1870, in the Archives, (vol. VXXIX, page 199,) an extended notice upon this subject, and also drew attention to the study of the terraces of the southern shore of Lake Léman.
M. Ernest Favre has presented an interesting communication on the geology of the mountains of the region southwest of the canton of Fribourg, composing the chain of the Nivemont, the Moléson, the Verreaux, and that of Saint Cray; he compared the structure of this solid mass with similar formations, which have been observed in the Tyrol and in the Carpathes. (This has appeared in the Archives.)

Finally Professor Thury has measured the thickness of the section of the glacier of the Oldenhorn, such as it presents from the lake of Rhéto. He estimates it at 45 meters, and has counted from 70 to 80 horizontal strata, each one having a thickness of about 60 decimeters.

Botany.-Since the works of Darwin have attracted the attention of naturalists to the question of the origin of organic species, their descent and their affiliations, the manner of distribation of these species over the surface of the globe, which has great interest on the bearing of this question, has been studied with more attention than in the past, and is becoming every day the object of new and important researches. M de Candolle has shown that botanists have found in the flora of the Fortunate Islands hardly any plant similar to the western coast of Africa, while they contain a large number in common with those of Europe. This fact would indicate that the islands in question have been formerly united to Europe, by a terrestrial communication, while it seems to have always remained separated from Africa. It is true we are by no means certain of the flora of the high mountains of Maroc, which throws some doubt upon the conclusions we would be inclined to infer from the above observations.

Dr. Müller contributed an article, accompanied with drawings, upon a new species of hair discovered upon two Asiatic plants of the combretacious family. These hairs have the general appearance of scales or the plates of a shield, but instead of exhibiting a disk formed of numerous cells entirely radial, they are formed of a regular net-work of cells, which is only one cell in thickness, like the ordinary leaf of mosses. Dr. Müller described these curious scales and proposed to give the name of Lépide réticulée.

Professor Fée, of Strasburg, read a memoir upon the determination of plants mentioned by the ancients; in which he shows especially how excessively difficult it is to arrive at a sufficiently definite determination which would enable us with any degree of accuracy to apply the old nomenclature to the new. A recent work by M. Bubani, far from settling the inherent difficulties of this question only furnished a new proof of its complexity.

## ZOOLOGY AND PHYSIOLOGY.

Among the strangers who have attended our sessions, Messrs. Guénée and Bigot have for several months given their time to the arrangement of the entomological collections of our museum; especially the first of these gentlemen, who for six months has been at work in our laboratories. Mr. Bigot has classified the Diptera and M. Guénée the Lepidoptera. As the collections are about to be removed to the new academic buildings, where they will be properly exhibited, such a classification, by competent men, is of great importance.
M. Guénée discovered in our cases several new species of Papilio and allied genera; also a Bombicide, which exhibits a very remarkable case of hermaphrodism ; in this the organs of the two sexes, instead of being localized, are mingled and distributed through nearly all parts of the body. The article on this subject by M. Guénée will be inserted in our memoirs.
M. Claparède has studied the cysts of a féra sent to him by M. Lunel. The muscles of this fish inclosed various cysts, most of which contained a liquid greatly resembling milk. In one of them was a cheesy, whitish substance, evidently produced by the metamorphosis of a lacteous liquid, similar to that in the other cysts, but the more fluid elements of which had been re-absorbed. The constituent elements of these cysts were psorospermies, resembling each other, and composed of a head of lenticular form, and a tail double from its base. With these psorospermies there was always found a granular protoplasm, at whose expense the psorospermies were developed. These facts have been observed before, but what was especially remarkable in the féra in question was the presence of other cysts in the mucus of the gills, but with psorospermies very different, and much smaller, having a diameter of only onefourth to one-tenth of a millimeter. Their abundance gives to the entire bronchial apparatus a grayish tint. These psorosphermies were not lenticular, but perfectly spherical, and without a tail, each inclosing a spherical kernel, very refracting, and some small grains. M. Claparède thinks there must be a generic connection between the small cysts of the gills and the large cysts of the muscles. However, no observations have as yet confirmed this hypothesis. Upon one of the arches of the gills was a cyst of about a millimeter in size, of which the contour was very different from the other gill-cysts, and resembled somewhat those of the muscular cysts. These psorospermies are distinguished from
those of the large cysts by their shorter tails. However, with a great many of them the tail was bifurcated at the end. Prof. Claparède also exhibited the plates of a new work upon the histology of Annélides, and has given some details as to the process he employs for the arrangement and preservation of his preparations.
M. Herman Fol read before the society a long and important memoirupon the Appêndiculaires, a family belonging to the class of Tuniciers It confirms the near relation that several authors have established between these animals and vertebrates, and proposes to place them at the base of the genealogical tree of the latter. M. Fol has been made a member of our society on account of this work, which will be printed in Volume $\mathbf{X X}$ of our memoirs.
M. Godfrey Lunel has given some interesting facts observed at Geneva relative to the metamorphoses of the Axolotes. We know that these batracians are transformed sometimes by the loss of their bronchia, and, frem being aquatic, as they generally are, they become pulmonary animals, living in free air. Several Axolotes, placed in running water, did not experience any change; while of two others, left in a wash-basin, badly cared-for and exposed to the cold, one died, and the other was transformed by the loss of its bronchia; but, after having been replaced in a normal condition, it re-assumed its first form so perfectly as not to be distinguished. This fact, which constitutes a second transformation in a retrograde direction, is entirely new.

Dr. J. L. Prévost has given an account of experiments relative to the mode of action of anæsthetics and of chloroform upon the nervous center, and be has obtained results contrary to those of M. Cl. Bernard. This physiologist states that the chloroform, in acting upon the brain, affects not only that organ, but acts also, at a distance, upon the spinal marrow, without being in contact with it. M. Prérost has repeated the principal experiments of M. Bernard, which consist in stopping the circulation in frogs, by placing a bandage below the shoulders, then injecting diluted chloroform into one set below the skin of the anterior cut, and into the other below the skin of the posterior cut. In varying the position of the frogs, M. Prévost, after trial, has found that chloroform introduced in the posterior part can, contrary to the opinion of M. Bernard, anæsthetize the anterior part when the frog is placed with the posterior members in the air, while the clloroform introduced in the anterior part does not anæsthetize the posterior part if we are careful to place the frog with the head downward. He thinks that M. Bernard has not been sufficiently careful to guard against the filtration of the chloroform through the tissues.
M. Prévost, in applying pure chloroform to the denuded brain of a frog, of which the aorta was tied, and placed in the position above indicated, has anæsthetize the head only of the animal, leaving intact the functions of the spinal marrow. Afterward, when he has untied the aorta, these frogs lave returued to their normal state, which proves
that the chloroform acts in this experiment simply as an anæsthetic, and not as a caustic, which destroyed the brain, leaving the frog in the state of a headless animal. From these experiments M. Prévost has come to the conclusion that chloroform anæsthetizes in the nervous center only the parts with which it is directly in contact; and that it does not act at a distance, as M. Bernard believed.
M. Brown-Sequard has produced some phenomena of epilepsy upon Guinea pigs by means of hemisections of the marrow or of the section of a sciatic nerve. Dr. Prévost has obtained the same phenomena by the amputation of a thigh of one of these animals. In order to provoke a nervous attack it is sufficient to excite the zone called epileptic, which comprises the half of the surface corresponding to the member amputated, and immediately the animal is throwu into convulsions. The excitability of this zone decreases, however, with the continuation of the experiment, and it is always more difficult to provoke a new crisis. The study of this artificial epilepsy will, without doubt, throw some light upon the kind and nature of natural epilepsy.

## MEDICINE.

Dr. Lombard has been investigating for several years the climate of mountains, a subject which more than any other ought to interest the physicians of Switzerland. His later researches are directed to the effect which these climates exercise upon pulmonary phthisis, a question which he had been appointed to investigate by the commission established at Samaden, for the purpose of its elucidation. He estimated that a residence in high altitudes would prevent the development of the phthisis, and even cure it, either in developing the pulmonary emphysema, or by favoring the functional periphery activity. (The work of M. Lombard has appeared in the Medical Bulletin of Switzerland.)

Finally, M. Alphonse de Candolle read a notice which likewise deserves to be registered in the medical rubric. It is, in fact, an application to this science of the Darwinian principles deduced from natural history, inasmuch as it treats of an effect of selections rendering variable the intensity of maladies when they are very deadly. According to the author, when a disease has severely attacked that portioiz of the population not advanced in years, the following generation, descending from persons not disposed to take this disease, will also be in the same condition by an ordinary effect of the hereditary law. There is, therefore, a reason for the diminution of the epidemic. We can likewise explain why its attacks are most severe the first time it appears among a population, and why it afterward becomes rare or less fatal, which has been the case with most of the diseases of this kind. At the end of several generations, however, a population moderately. attacked by a disease resembles the condition of a population who have never had it, and the result is a double intensity. Applying these principles to the small-pox, M. de Candolle estimated that at the time when Jenner 23 s 71
introduced vaccine, the variolic affection was weakened relative to the anterior epoch. Vaccine ought, therefore, to be as much more effica; cious when it is applied in a similar condition. Small-pox having nearly disappeared in Europe, during two generations a new population appears less exempted from its attacks, and this cause of receptibility ought to-day to render vaccine less efficacious. The author does not pretend to say that this is the only acting cause, but he thinks that, independently of others, it exists as a necessity, and that it ought to be taken into account.

In giving a concise account of the labors of the society I have omitted manty communications of a less important character, serving as themes for those discourses with which our meetings generally terminate.

These familiar conversations, in which each one gives an account of his stadies, and which are often succeeded by interesting discussions, continue to occupy our meetings in the most useful and agreeable manner. They not only maintain between the members an intimaterelation which we all appreciate, but likewise establish a sort of oral bulletin of the most recent discoveries, allowing each one to follow in a general manner the progress of science outside of his own specialty.

## INTERNAL ADMINISTRATION.

Having given a summary of the papers presented at our meetings, it only remains for me, gentlemen to give you a brief account of the interior transactions of the society. Col. Emile Gautier has been elected president for 1871-72, and M. E. Sarasin has been confirmed in his position as secretary.

If we have had the misfortune to lose one of our colleagues, we have also had the satisfaction of gaining two new ordinary members in MM. Raoul Pictet and Herman Fol, and we have likewise increased the list of our free associates by the addition of MM. Georges Prévost, H. P. E. Sarasin, J. L. Micheli, and H. Barbey. The number of our ordinary members, which, in 1867, was forty-one, to-day amounts to forty-nine, but the number of our free associates, which at the same date was forty members, has decreased to thirty-eight, including the admission of several associates to the title of ordinary members. You have also nominated as honorary members, in addition, MM. Régnault, Fée, and Cap, who were mentioned above, Prof, de Notaris, of Genes, well known from his works upon botany, and the director of the Smithsomian Institution, of Washington, Professor Joseph Henry. This savan has been associated with us a long time, in relations which we cesteem infinitely precious, and assisted at one of our meetings in 1870.

As to our publications, they have followed their ordinary course. The Nociety of, Physics, pablishes each year half a volume, which they reserve as much as possibley, on account of its size, for the memoirs accompanied with plates giving to the archives of science those which do not require illustrations Itwas in the zear 1821 that the first number of our memoirs ap-
peared, and we finished the twentieth volume in 1870. You have decided to make a genoral index of this series, in order to facilitate researches which will become every day more difficult to examine in proportion as the number of our volumes are increased. This index, which will appear at the same time as the present volume, has been prepared by our colleague, Alfred Le Fort, who very obligingly devoted his time and labor to our interests. I am commissioned, in the name of the society, to tender him our sincere acknowledgments.

The recapitulation of the material contained in our first twenty volumes has shown that it includes in all three hundred and fifteen notices and memoirs, some of which constitute complete works. This publication constitutes, therefore, an important collection, which can claim a most honorable place among the scientific transactions of Europe.

Lastly, I will add that, although at an expense somewhat exceeding the means of the society, the rich herbarium, for which our city is indebted to the generosity of the family of DeLessert, has been placed in the botanical conservatory prepared for that purpose, where it is now definitely arranged in such a manner that botanists may have free access to it.

Before concluding this report, I desire, gentlemen, to communicate a circumstance which appears to me to have peculiar interest for us, as it refers to the origin of our society. In a preceding report, one of jour presidents, Dr. Grosse, proposed at the fiftieth anniversary of the first scientific congress held at Geneva to give you, with a talent jou all know how to appreciate, the history of the Society of Physics, of which his father was one of the founders. In some researches to which I have devoted myself this winter, in order to find in the papers of my family some documents relative to the history of this society during the first years of its existence, I have found a piece which appears to me worthy of your regard. It is a letter of M. A. Pictet to my grandfather, in which he announces the formation of the society and incloses the names of its founders. I will give the most important part of the letter:
"I ain commissioned, my very dear colleague, to offer to you, as likewise to your son Theodore and M. Necker, membership of a society with which I have the honor of being connected. I delayed mentioning it to you until I could send at the same time the rules, a copy of which I received yesterday. In reading them you will be informed of the obligations imposed, which I hope will not frighten you. I have already attended a meeting, and I assure you that, by the interest with which it has inspired me, I judge it will prove a favorable and useful project for the progress of natural science and the personal advantage of the individuals who compose this society.
"Below are the actual members:
"M. M. Oolladon, Tollot, Gosse, Vauché, Jurine, Gaudy de Russie,

Pictet. Members elected unanimously: M. M. de Saussure, father and son, Necker de Saussure, Sensbier, Tingrey.
"Perhaps there are one or two others whom I have forgotten to mention, as I made this catalogue from memory.
The next meeting will be the first Thursday after the 15 th, at M . Tollet's, and if you accept your election, as we all hope you will, your membership dates from the present, as well as that of your son and M. Necker, to whom I beg you to have the goodness to communicate the rules.
" Accept the sincere attachment of your devoted servant and colleague, "PICTET.
"Geneva, Saturday, October 8, 1791."
This document refers, as we see, the definite constitution of the Society of Physics to the year 1791. It shows that it was composed first of twelve savants of Geneva, and that the original meetings were held on Thursday, as in our days, though lately we have changed to Wednesday. The limited number of its members continually increased, and we now have the satisfaction of seeing it sustained at a level which tends rather to rise than to fall. The construction of new academic buildings, in proportion to the new demands, is a speaking testimony of the increasing progress of the intellectual activity of our city. The extensions which could be made in the library, the laboratories, and the museums would furnish a new element to this activity, and would not fail to contribute to the extension of the taste for science in which Geneva ought to occupy a position before the world superior to that which would be assigned her, merely taking into consideration her population and the smallness of her territory.

In concluding, we will hope that the year, so fraught with agitation, through which we have just passed may be succeeded by a period of calm, of repose, and of prosperity, in which the peaceable occupations of science may take the place of the clamorous commotion with which we have been too long disturbed. Our society will then return to its labors with new ardor, and more fully maintain the honorable position so long occupied by our country, through the memory of the men who have distinguished it, and of whom the traditions are well preserved wherever profound truth is cherished.

## Appendix to the report of the president. <br> EDWARD CLAPARÈDE.

Gentlemen: A few days after you had heard the reading of the report of your president upon the operations of the year 1870-71, we received the afflicting intelligence of the death of our excellent colleague, M. Edward Claparède. In view of the deep and unanimous regret which we all experience at the loss of one who ranked among the first savants of our city, we concluded it would be too long to wait
until next year's report for the testimony of esteem and affection in which you all desire to unite, and we think it more suitable to add to this year's report a notice which shall from this day recall the memory of Claparède.

Edward Claparède, born in 1832, was from an ancient family in Geneva. He commenced his studies at the Academy of this city, where he was even then remarkable for his pre-eminent resources. Endowed with a decided taste for natural sciences, he was the pupil of Professor Pictet de la Rive, who, by his instruction, developed in him a taste for zoology. In 1853 he went to the University of Berlin, where he studied with the distinguished Jean Müller, who was not long in recog. nizing his merits, and of whom he became one of the best pupils. Even while pursuing his studies, he composed several memoirs upon the inferior animals, one of which treats of the anatomy of Cyclostoma elegans, which served him as a thesis for the doctorate. It was also at this time that he commenced, in common with his friend Lachman, a great work upon the Infusoria and the Rhyzopodia, which made a considerable advance in the science of these animals, and which obtained for him the great prize of physical science from the Institute of France. Made Doctor of Medicine in 1857, Claparède returned to Geneva, where he continued his labors with great assiduity, notwithstanding impaired health, and sufferings which would have discouraged almost anyone else. He was soon elected to a professorship, and displayed in his instrnction the brilliant qualities which contributed to increase the reputation of our Academy. He also gave several public lectures, which always attracted a large audience, thanks to his great erudition, and to the fluency of speech which gave to his instruction an irresistible attraction.

Although his tastes led him to prefer the study of inferior animals, he was occupied with various subjects, and we find in the memoirs of the Archives de la Bibliothèque Universelle numerous articles of his upon different branches of science, in which he gave a résumé of works in foreign languages, also a number of analyses, as learned as varied, upon many subjects, which added much to the value of the bulletin. Understanding nearly all the languages of Europe, he could give an account of a great many works entirely inaccessible to others, while his critical appreciation bore the mark of a true scientific genius.
The desire to pursue his researches upon marine animals induced Ciaparède to make numerous journeys to the sea-shore, and on each occasion he collected the materials for important investigations, the results of which appeared either in Geneva, in the Memoirs of the Society of Physics, or in Germany, in the Zeitschrift für wissenschaftliche Zoologie. of Siebold and Kölliker, in the Archives of Mïller, \&c. The class of Annélides more particularly arrested his attention. Almost every year he made it the subject of some new publication, and finally devoted his great work to the Annelides of Naples, which, unfortunately,
was the last labor of his life. There is, however, still another extensive work by him, not yet printed, which will appear, treating of the history of these animals.

Besides his study of marine animals, Claparède made at Geneva very varied researches on other subjects. He published memoirs upon binocular vision, and numerous works upon the embryology of the Arthropodes. In 1860 the Society of the Sciences, of Utrecht, awarded him a gold medal for his beautiful investigations relative to the evolution of the Aranéides, which were followed by his studies upon different crustaceous and acarious animals, which include many new facts, and which are all important works in the progress of science. In fact, Claparéde, always noted for the correctness of his eye, ended by becoming an authority of the first order in all questions to be decided by the microscope, and in this respect he exercised throughout the entire world a well-merited authority. His eminent genius for observation, the clearness of his judgment, which comprehended all difficulties, naturally led Claparède to the study of Darwinism, of which he became a decided defender, and in relation to which he published several remarkable articles.
In reading the numerous and important works of Claparède, no one would imagine the sad condition of his health. Afflicted with serious organic maladies, his life was one long martyrdom. A violent disease of the heart had, from his earliest youth, caused great disturbance through the whole of his organism ; all exercise of any importance was interdicted; frequent hæmoptysies brought him several times to the verge of the grave; suffering of various kinds rendered him incapable of work during long periods, and we can hardly comprehend how, even in his best moments, he could devote himself to active research. His life was sustained by a force of energy in his latter years, and by extreme measures which no physician would have dared to advise. This condition of health did not cease to be a cause of anxiety and sadness to his friends. It prevented him from undertaking works of greatlength, and we can judge by what he has accomplished, notwithstanding so many difficulties, how much he might have done if he had been blessed with good or even moderate health.
The necessity for a warm climate, as much as his passion for the seashore, induced Claparède, in 1866, to pass the winter at Naples. This sojourn agreed with him perfectly; he devoted himself to his immense researches upon the Annelides, which fills the twentieth volume of our memoirs. This induced him, two years after, to spend a second winter in Naples, but the seriousillness of his wife made work almost impossible; the assiduous care which he lavished upon the companion of his life weakened him, and he became himself extremely ill. Nevertheless, he desired, in 1870, to again attempt a sojourn at Naples, but far from experiencing any relief he was more indisposed than ever. A hydropsy, which slowly ascended toward the vital organs, left him no hope-

He fought against it, according to his custom, with an extraordinary energy, denying himself drinks, and submitting to a treatment which the physicians believed to be beyond the endurance of a patient. He died the 31st of May, at Sienne, on his return voyage, at the age of thirtynine years, just at the time when we all had reason to hope that it would not be long before we should again welcome him to our midst.

The death of Claparède has taken from Genera one of the finest flowers from her scientific crown, and from our Academy one of its most illustrious professors. The sorrow of his death will extend beyond the extreme limit of our city, and be felt wherever science is cultivated. Claparède was one of those men who make a mark in the intellectual life of a country and who seem predestined to be the founder of a school. We recognize in him a combination of faculties rarely found united in the same individual, an extraordinary facility to assimilate the labors of others, a prodigious memory, great quickness of conception, and a certainty of observation which was never at fault. To these essential faculties were joined all the accessory qualities which facilitate work in the domain of natural sciences. He excelled in the art of fine preparations; he handled the brush with as much talent as the surgeon's knife, and drew himself the plates of his work. He understood all the languages of Europe outside of the Slavonic tongues; his studies were immense and relundant, though he made but few notes; his erudition was really wonderful. The largeness of his views struck all who approached him, and his instructions had a fascinating attractiveness, though nothing was sacrificed to eloquence. His conversation was always learned upon almost any subject, for it would have been difficult to find a specialty, scientific or literary, even among those most foreign from his ordinary studies, in which he could be taken unawares.

As for us, gentlemen, it is not only a philosopher whom we mourn, but a tried and devoted friend; a man of uprightness, one who, besides the genius of science, possessedalso all the generous qualities of the heart. I can only regret, in concluding, that the remembrance of his life among us should not be recorded in our annals by a pen more worthy than mine.

## EXPEDITION TOWARD THE NORTII POLE.

## INSTRUCTIONS TO CAPTAIN HALL, BY HON. G. M. ROBESON, SECRETARY OF THE NAVY.

## Navy Department, June 9, 1871.

SIR: Having been appointed, by the President of the United States, commander of the expedition toward the North Pole, and the steamer Polaris having been fitted, equipped, provisioned, and assigned for the purpose, you are placed in command of the said vessel, her officers and crew, for the purposes of the said expedition. Having taken command, you will proceed in the vessel, at the earliest possibledate, from the navyyard in this city to New York. From New York you will proceed to the first favorable port you are able to make on the west coast of Gree:1land, stopping, if you deem it desirable, at St. Johns, Newfoundland. From the first port made by you, on the west coast of Greenland, if farther south than Holsteinberg, you will proceed to that port, and thence to Goodharen, (or Lively,) in the island of Disco. At some one of the ports above referred to you will probably meet a transport, sent by the Department, with additional coal and stores, from which you will supply yourself to the fullest carrying capacity of the Polaris. Should you fall in with the transport before making either of the ports aforesaid, or should you obtain information of her being at, or having landed her stores at any port south of the island of Disco, you will at once proceed to put yourself in communication with the commander of the transport, and supply yourself with the additional stores and coal, taking such measures as may be most expedient and convenient for that purpose. Should you not hear of the transport before reaching Holsteinberg, you will remain at that port, waiting for her and your supplies, as long as the object of your expedition will permit you to delay for that purpose. After waiting as long as is safe, under all the circumstances as they may present themselves, you will, if you do not hear of the transport, proceed to Disco, as above provided. At Disco, if you hear nothing of the transport, you will, after waiting as long as you deem it safe, supply yourself, as far as you may be able, with such supplies and articles as you may need, and proceed on your expedition without further delay. From Disco you will proceed to Upernavik. At these two last-1amed places you will procure dogs and other Arctic outfits. If you think it of advantage for the purpose of obtaining dogs, \&c., to stop at Tossak, you will do so. From Upernavik, or Tossak, as the case may be, you will proceed across Melville Bay to Cape Dudley Digges, and thence you will make all possible progress, with vessels, boats, and sledges,
toward the North Pole, using your own judgment as to the route or routes to be pursued and the locality for each winter'squarters. Having been provisioned and equipped for two and a balf years, you will pursue your explorations for that period; but, should the object of the expedition require it, you will continue your explorations to such a further length of time as your supplies may be safely extended. Should, however, the main object of the expedition, viz, attaining the position of the North Pole, be accomplished at an earlier period, you will return to the United States with all convenient dispatch.
There being attached to the expedition a scientific department, its operations are prescribed in accordance with the advice of the National Academy of Sciences, as required by the law. Agreeably to this advice, the charge and direction of the scientific operations will be intrusted, under your command, to Doctor Emil Bessels; and you will render Dr. Bessels and his assistants all such facilities and aids as may be in your power to carry iuto effect the said further advice, as given in the instructions herewith furnished in a communication from the president of the National Academy of Sciences. It is, however, important that objects of natural history, ethnology, \&c., \&c., which may be collected by any person attached to the expedition, shall be delivered to the chief of the scientific department, to be cared for by him, under your direction, and considered the property of the Government; and every person be strictly prohibited from keeping any such object. You will direct every qualified person in the expedition to keep a private journal of the progress of the expedition, and enter on it events, observations, and remarks, of any nature whatsoever. These journals shall be considered confidential and read by no person other than the writer. Of these journals no copy shall be made. Upon. the return of the expedition you will demand of each of the writers his journal, which it is hereby ordered he shall deliver to you. Each writer is to be assured that when the records of the expedition are published he shall receive a copy; the private journal to be returned to the writer, or not, at the option of the Government; but each writer, in the published records, shall receive credit for such part or parts of his journal as may be used in said records. You will use every opportunity to determine the posi tion of all capes, headlands, islands, \&c.., the lines of coasts, take soundiugs, observe tides and currents, and make all such surveys as may advance our knowledge of the geography of the Arctic regions.

You will give special written directions to the sailing and ice master of the expedition, Mr. S.O. Buddington, and to the chief of the scientific department, Dr. E. Bessels, that in case of your death or disability-a contingency we sincerely trust may not arise-they shall consult as to the propriety and manner of carrying into further effect the foregoing instructions, which I here urge must, if possible, be done. The results of their consultations, and the reasons therefor, must be put in writing, and kept as part of the records of the expedition. In any event, how-
ever, Mr. Buddington shall, in case of your death-or disability, continue as the sailing and ice master, and control and direct the movements of the vessel; and Dr. Bessels shall, in such case, continue as chief of the scientific department, directing all sledge journeys and scientific operations. In the possible contingency of their non-agreement as to the course to be pursued, then Mr. Buddington shall assume sole charge and command, and return with the expedition to the United States with all possible dispatch.
You will transmit to this Department, as often as opportunity officrs, reports of your progress and results of your search, detailing the route of your proposed advance. At the most prominent points of your progress you will erect conspicuous skeleton stone monuments, depositing near each, in accordance with the confidential marks agreed upon, a condensed record of your progress, with a description of the route upon which you propose to advance, making caches of provisions, \&cc., if you deem fit.
In the event of the necessity of finally abandoning your vessel, you will at once endeavor to reach localities frequented by whaling or other ships, making every exertion to send to the United States information of jour position and situation, and as soon as possible to return with your party, preserving, as far as may be, the records of, and all possible objects and specimens collected in, the expedition.

All persons attached to the expedition are under your command, and shall, under every circumstance and condition, be subject to the rules, regulations, and laws governing the discipline of the Navy, to be modified, but not increased, by you as the circumstances may in your judg. ment require.
To keep the Government as well informed as possible of your progress, you will, after passing Cape Dudley Digges, throw overboard daily, as open water or drifting ice may permit, a bottle or small copper cylinder, closely sealed, containing a paper, stating date, position, and such other facts as you may deem interesting. For this purpose you will have prepared papers containing a request, printed in several languages, that the finder transmit it by the most direct route to the Secretary of the Navy, Washington, United States of America.
Upon the return of the expedition to the United States, you will transmit your own and all other records to the Department. You will direct Dr. Bessels to transmit all the scientific records and collections to the Smithsonian Institution, Washington.
The history of the expedition will be prepared by yourself, from all the journals and records of the expedition, under the supervision of the Department. All the records of the scientific results of the expedition will be prepared, supervised, and edited by Dr. Bessels, under the direction and authority of the president of the National Academy of. Sciences.

Wishing for you and your brave comrades health, happiness, and
success in your daring enterprise, and commending you and them to the protecting care of the God who rules the universe,

I am, very respectfully, yours,

Chas. F. Hall,<br>Commanding Expedition toward the North Pole.

LETTER OF PROFESSOR JOSEPH HENRY, (PRESIDENT OF THE NATIONAL ACADEMY OF SCIENCES, ) WITH INSTRUCTIONS TO CAPTAIN C. F. HALL FOR THE SCIENTIFIC OPERATIONS OF THE EXPEDITION TOWARD THE NORTH POLE.

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\text { Washington, D. O., June 9, } 1871 .
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Sir : In accordance with the law of Congress authorizing the expedition for explorations within the Arctic Circle, the scientific operations are to be prescribed by the National Academy; and in behalf of this society I respectfully submit the following remarks and suggestions:

The appropriation for this expedition was granted by Congress principally on account.of the representations of Captain Hall and his friends as to the possibility of improving our knowledge of the geography of the regions beyond the eightieth degree of north latitude, and more especially of reaching the Pole. Probably on this account and that of the experience which Captain Hall had acquired by. seven years' residence in the Arctic regions, he was appointed by the President as commander of the expedition.

In order that Captain Hall might have full opportunity to arrange his plans, and that no impediments should be put in the way of their execution, it was proper that he should have the organization of the expedition and the selection of his assistants. These privileges having been granted him, Captain Hall early appointed as the sailing-master of the expedition his friend and former fellow-voyager in the Arctic Zone, Captain Buddington, who has spent twenty-five years amid polar ice; and for the subordinate positions, persons selected especially for their experience of life in the same regions.

It is evident from the foregoing statement that the expedition, except in its relations to geographical discovery, is not of a scientific character, and to connect with it a full corps of scientific observers whose duty it should be to make minute investigations relative to the physics of the globe, and to afford them such facilities with regard to time and position as would be necessary to the full success of the object of their organization, would materially interfere with the views entertained by Captain Hall, and the purpose for which the appropriation was evidently intended by Congress.

Although the special objects and peculiar organization of this expedition are not primarily of a scientific character, yet many phenomena may be observed and specimens of natural history be incidentally collected, particularly during the long winter periods in which the vessel
must necessarily remain stationary; and therefore, in order that the opportunity of obtaining such results might not be lost, a committee of the National Academy of Sciences was appointed to prepare a series of instructions on the different branches of physics and natural history, and to render assistance in procuring the scientific outfit.

Great difficulty was met with in obtaining men of the proper scientific acquirements to embark in an enterprise which must recessarily be attended with much privation, and in which, in a measure, science must be subordinate. This difficulty was, however, happily obviated by the offer of an accomplished physicist and naturalist, Dr. E. Bessels, of Heidelberg, to take charge of the scientific operations, with such assistance as could be afforded him by two or three intelligent young men that might be trained for the service. Dr. Bessels was the scientific director of the German expedition to Spitzbergeu and Nova Zembla, in 1869, during which he made, for the first time, a most interesting series of observations on the depths and currents of the adjacent seas. From lis character, acquirements, and enthusiasm in the cause of science, he is admirably well qualified for the arduous and laborious office for which lie is a volunteer. The most important of the assistants was one to be intrusted, under Dr. Bessels, with the astronomical and magnetic observations, and such a one has been found in the person of Mr. Bryan, a graduate of Lafayette College, at Easton, Pennsylvania, who, under the direction of Professor Hilgard, has received from Mr. Schott and Mr. Keith, of the Coast Survey, practical instructions in the use of the instruments.
The Academy would therefore earnestly recommend, as an essential condition of the success of the objects in which it is interested, that Dr. Bessels be appointed as sole director of the scientific operations of the expedition, and that Captain Hall be instructed to afford him such facilities and assistance as may be necessary for the special objects under his charge, and which are not incompatible with the prominent idea of the origiual enterprise.

As to the route to be pursued with the greatest probability of reaching the Pole, either to the east or west of Greeuland, the Academy forbears to make any suggestions, Captain Hall having definitely concluded that the route through Baffin's Bay, the one with which he is most familiar, is that to be adopted. Une point, however, should be specially urged upon Captain Hall, namely, the determination with the utnost scientific precision possible of all his geographical positions, and especially of the ultimate northern limit which he attains. The evidence of the genaineness of every determination of this kind should be made apparent beyond all question.
On the return of the expedition the collections which may be made in natural history, \&cc., will, in accordance with a law of Congress, be deposited in the National Museum, under the care of the Smithsonian Institution; and we would suggest that the scientific records be discussed and prepared for publication by Dr. Bessels, with such assistance as he
may require, under the direction of the National Academy. The importance of refusing to allow journals to be kept exclusively for private use, or collections to be made other than those belonging to the expedition, is too obvious to need special suggestion.
In fitting out the expedition, the Smithsonian Institution has afforded all the facilities in its power in procuring the necessary apparatus, and in furnishing the outfit for making collections in the various departments of natural bistory. The Coast Survey, under the direction of Professor Peirce, has contributed astronomical and magnetical instruments. The Hydrographic Office, under Captain Wyman, has furnished a trausit instrument, sextants, chronometers, charts, books, \&c. The Signal Corps, under General Myer, has supplied anemometers, thermometers, aneroid, and mercurial barometers, besides detailing a sergeant to assist in the meteorological observations. The members of the committee of the Academy, especially Professors Baird and Hilgard, have, in discussing with Dr. Bessels the several points of scientific investigation and in assisting to train his observers, rendered important service.

The liberal manner in which the Navy Department, under your direction, has provided a vessel, and especially fitted it out for the parpose with a bountiful supply of provisions, fuel, and all other requisites for the success of the expedition, as well as the health and comfort of its members, will, we doubt not, meet the approbation of Congress, and be highly appreciated by all persons interested in Arctic explorations.
From the foregoing statement it must be evident that the provisions for exploration and scientific research in this case are as ample as those which have ever been made for any other Arctic expedition, and should the results not be commensurate with the anticipations in regard to them, the fact cannot be attributed to a want of interest in the enterprise, or to inadequacy of the means which have been afforded.
We have, however, full confidence, not only in the ability of Captain Hall and his naval associates to make important additions to the knowledge of the geography of the polar region, but also in his interest in science and his determination to do all in his power to assist and facilitate the scientific operations.

Appended to this letter is the series of instructions prepared by the committee of the Academy, viz, the instructions on astronomy, by Professor Newcomb; on magnetism, tides, \&c., by Professor J. E. Hilgard; on meteorology, by Professor Henry; on natural history, by Professor S. F. Baird; on geology, by Professor Meek; and on glaciers, by Professor Agassiz.

I have the honor to be, very respectfully, your obedient servant, JOSEPH HENRY, President of the National Academy of Nciences.
Hon. George M. Robeson, Secretary of the Nary.

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GENERAL DIRECTIONS IN REGARD TO THE MODE OF KEEPING RECORDS.

Records of observations.- It is of the first importance that in all instrumental observations the fullest record be made, and that the original notes be preserved carefully.

In all cases the actual instrumental readings must be recorded, and if any corrections are to be applied, the reason for these corrections must also be recorded. For instance, it is not sufficient to state the index error of a sextant; the manner of ascertaining it and the readings taken for the purpose must be recorded.
The log-book should contain a continuous narrative of all that is done by the expedition and of all incidents which occur on shipboard, and a similar journal should be kept by each sledge party. The actual observations for determining time, latitude, the sun's bearing, and all notes having reference to mapping the shore, soundings, temperature, \&c., should be entered in the log-book or journal in the regular order of occurrence. When scientific observations are more fully recorded in the note-books of the scientific observer than can be conveniently transcribed into the log-book, the fact of the observation and reference to the notebook should be entered.
The evidence of the genuineness of the observations brought back should be of the most irrefragable character. No erasures, whatever, with rubber or knife, should be made. When an entry requires correction, the figures or words should be merely crossed by a line, and the correct figures written above.

J. E. Hilgard.

## ASTRONOMX.

Astronomicalobservations.-One of the chronometers, the most valuable, if there is any difference, should be selected as the standard by which all observations are to be made, as far as practicable. The other chronometers should all be compared with this every day at the time of winding, and the comparisons entered in the astronomical notebook.

When practicable, the altitude or zenith distance of the sun should be taken four times a day-morning and evening for time; noon and midnight for latitude. The chronometer or watch times of the latitude observations, as well as of the time observations, should always be recorded. Each observation should always be repeated at least three times in all, to detect any mistake.

When the moon is visible, three measures of her altitude should be taken about the time of her passage over each cardinal point of true bearing, and the chronometer time of each altitude should be recorded.

As the Greenwich time deduced from the chronometers will be quite
unreliable after the first six months, it will be necessary to have recourse to lunar distances. These should be measured from the sun, in preference to a star, whenever it is practicable to do so.

If a sextant is used in observation, a measure of the semi-diameter of the sun or moon should be taken every day or two for index error.
The observations are by no means to be pretermitted when lying in port, because they will help to correct the position of the port.
The observations should, if convenient, be taken so near the standard chronometer that the observer can signal the moment of observation to an assistant at the chronometer, who is to note the time. If this is not found convenient, and a comparing watch is used, the watch-time and the comparison of the watch with the chronometer should both be carefully recorded.

The observations made by the main party should be all written down in full in a continuous series of note-books, from which they may be copied in the log. Particular care should be exercised in always recording the place, date, and limb of sun or moon observed, and any other particulars necessary to the complete understanding of the observation.

## S. Newcomb.

Observations at winter quarters.-The astronomical transit instrument will be set up in a suitable observatory. A meridian mark should be established as soon as practicable, and the instrument kept with constant care in the vertical plane passing through the mark, in order that all observations may be brought to bear on determining the deviation of that plane from the meridian of the places. The transits of circumpolar stars, on both sides of the Pole, and those of stars near the Equator, should be frequently observed.
Moon culminations, including the transits of both first and second limbs, shonld be observed for the determination of longitude independently of the rates of the chronometers. Twelve transits of each limb is a desirable number to obtain-more, if practicable. If any occultatious of bright stars by the moon are visible, they should be likewise observed.
The observations for latitude will be made with the sextant and artificial horizon, upon stars both north and south of the zenith.
All the chronometers of the expedition should be compared daily, as nearly as practicable about the same time.
Whenever a party leaves the permanent station for an exploration, and imnediately upon its return, its chronometer should be compared with the standard chronometer of the station.

Observations during sledge or boat journeys.-The instruments to be taken are the small Casella theodolite, or a pocket sextant and artificial horizon, one or more chronometers, and a prismatic compass, for taking magnetic bearings of the sun. In very high latitudes the time of the sun's meridian altitude is not readily determined; it will be advisable,
therefore, to take altitudes when the sun is near the meridian, as indicated by the compass, with regard to the variations of the compass, as derived from an isogonic chart. The time when the observation is taken will, of course, be noted by the chronometer. Altitudes should be taken in this way, both to the south and north of the zenith; they will enable the traveler to obtain his latitude at once very nearly, without the more laborious computation of the time.

The observations for time should be taken as nearly as may be when the sun is at right angles to the meridian, to the east and west, the compass being again used to ascertain the proper direction. This method of proceeding will call for observations of altitude at or near the four cardinal points, or nearly six hours apart inf time.

When the party changes its place in the interval between their observations, it is necessary to have some estimate of the distance and direction traveled. The ultimate mapping of the route will mainly depend upon the astronomical observations, but no pains should be spared to make a record every hour of the estimated distance traveledby $\log$, if afloat-of the direction of the route, by compass, and of bearings of distant objects, such as peaks, or marked headlands, by which the route may be plotted.
In case of a few days' halt being made when a very high latitude has been reached, or at any time during the summer's explorations, a special object of care should be to ascertain the actual rate of the chronometers with the party. To this end, a well-defined, fixed object, in any direction, should be selected as a mark, the theodolite pointed on it, and the transit of the sun over its vertical observed on every day during the sojourn at the place. If the party be only provided with a sextant, then the same angular distances of the sun from a fixed object should be observed on successive days, the angles being chosen so as to be between $30^{\circ}$ and $45^{\circ}$. For instance, set the sextant successively to $40^{\circ}$, to $40^{\circ} 20^{\prime}, 40^{\circ} 40^{\prime}$, \&c., and note the time when the sun's limb comes in contact with the object. The same distances will be found after twenty-four hours, with a correction for change in the sun's declination. The sun's altitude should be observed before and after these observations, and its magnetic bearing should be noted, as well as that of the mark. The altitude of the mark should also be observed, if practicable, either with the sextant or clinometer, but this is not essential.
J. E. Hilgard.

## MAGNETISM.

On the royage and sledge-journey, at all times when traveling, the declination or variation of the compass should be obtained by observing the magnetic bearing of the sun at least once every day on which the sun is visible. On ship-board or in boats the azimuth compass is to be used; on land the small theodolite will be found preferable.
When afloat, no valuable observations of the magnetic dip and in24 s 71
tensity are practicable. On the sledge-journey the dip-circle may be carried, and when halts are made longer than necessary to determine the place by astronomical observations, the $d i p$ and relative intensity, according to Lloyd's method, should be ascertained.
At winter quarters, in addition to the above-mentioned observations, those of absolute horizontal intensity should be made with the theodolite magnetometer, including the determination of moment of inertia. Also with the same instrument the absolute declination should be determined.
The least that the observer should be satisfied with is the complete determination of the three magnetic elements, namely, declination, dip, and horizontal intensity. At one period, say within one week, three determinations of each shonld be made.

It is advisable that the same observations be repeated on three successive days of each month during the stay at one place; and that on three days of each mouth, as the 1 st, 11 th, and 21 st, or any other days, the variation of the declination-magnet be read every half hour during the twenty-four hours; also that the magnetometer, or at least a theodolite with compass, remain mounted at all times, that the variation of the needle may be observed as often as practicable, and especially when unusual displays of aurora borealis take place.
In all cases the time, which forms an essential part of the record, shonld be carefully noted.
Not long before starting on a sledge journey from a winter station, and soon after returning, the observations with the loaded dipping needles for relative intensity should be repeated, in order to have a trustworthy comparison for the observations which have been made on the journey.

## FORCE OF GRAVITY.

As the loug winter affords ample leisure, pendulum experiments may be made to determine the force of gravity, in comparison with that at Washington, where observations have been made with the Hayes pendulum lent to the expedition. The record of the Washington observations, a copy of which is furnished, will serve as a guide in making the observations. Special care should be taken while they are in progress to determine the rate of the chronometer with great precision, by observations of numerous stars with the astronomical transit instrument, the pointing of which on a fixed mark should be frequently verified.

## OCEAN PHYSICS.

Depths.-Soundings should be taken frequently, when in moderate depths, at least sufficiently often to give some indication of the general depth of the strait or sound in which the vessel is afloat at the time. If an open sea be reached, it should be considered of the greatest importauce to get some measure of its depth, and since no bulky sounding ap-
paratus can be carried across the ice barrier, the boat party should be provided with 1,000 fathoms of small twine, marked in lengths of 10 fathoms. Stones taken on board when the boat is launched, may serve as weights.
Bottom should be brought up whenever practicable, and specimens preserved. Circumstances of time and opportanity must determine whether a dredge can be used, or merely a specimen-oup.

Temperature of the sea should be observed with the "Miller protected bulb thermometer," made by Casella, near the surface, about two fathoms below the surface, and near the bottom. When time permits, observations at an intermediate depth should be taken. These observations have a particular bearing on the general circulation of the ocean, and are of great importance.
Tides.-Observations of high and low water, as to time and height, should be made continuously at winter quarters. The method adopted by Dr. Hayes is recommended. It consists of a graduated staff anchored to the bottom, directly under the "ice-hole," by a mushroomanchor, or heavy stone and a chain, which is keptstretched by a counterweight attached to a rope that passes over a pully rigged overhead. The readings are taken by the height of the water in the "ice-hole." In the course of a few days' careful observations the periods of high and low water will become sufficiently well known to predict the turns approximating from day to day, and subsequently, observations taken every five minutes for half an hour, about the anticipated tarn, will suffice, provided they be continued until the turn of the tide has become well marked.
Tidal observations taken at other points, when a halt is made for some time, even if continued not longer than a week, will be of special value, as affording an indication as to the direction in which the tidewave is progressing, and inferentially as to the proximity of an open sea. If, as the expedition proceeds, the tide is found to be later, the indication is that the open sea is far distant, if indeed the channel be not closed. But if the tide occurs earlier, as the ship advances, the probability is strongly in favor of the near approach to an open, deep sea, communicating directly with the Atlantic Ocean.
In making such a comparison, attention must be paid to the semimonthly inequality in the time of high water, which may be approximately taken from the observations at winter quarters. Observations made at the same age of the moon, in different places, may be directly compared.

When the water is open, the tide may be observed by means of a graduated pole stuck into the bottom; or, if that cannot be conveniently done, by means of a marked line, anchored to the bottom, and floated by a light buoy, the observation being taken by hauling up the line taut over the anchor.

Currents.-It is extremely desirable to obtain some idea of the cur-
rents in the open polar sea, if such is found. No special observations can be indicated, however, except those of the drift of icebergs, if any should be seen.
Density.-The density of the sea-water should be frequently observed with delicate hydrometers, giving direct indications to the fourth decimal. Whenever practicable, water should be brought up from different depths, and its ilensity tested. The specimens should be preserved in carefully-sealed bottles, with a view to the subsequent determination of their mineral contents.

J. E. Hilgard.

## METEOROLOGY.

The expedition is well supplied with meteorological instruments, all the standards, with the exception of the mercurial barometers, manufactured by Casella, and compared with the standards of the Kew Observatory under the direction of Professor Balfour Stewart. Dr. Bessels is so familiar with the use of instruments, and so well acquainted with the principles of meteorology, that minute instructions are unnecessary. We shall therefore merely call attention, by way of remembrance, to the several points worthy of special notice.
Temperature. -The registers of the temperature, as well as of the barometer, direction of the wind, and moisture of the atmosphere should, in all cases in which it is possible, be made hourly, and when that cannot be done they should be made at intervals of two, three, four, or six hours. The temperature of the water of the ocean, as well as of the air, should be taken during the sailing of the vessel.

The minimum temperature of the ice, while in winter quarters, should be noted from time to time, perhaps at different depths; also that of the water beneath.
The temperature of the black-bulb thermometer in vacuo exposed to the sun, and also that of the black-bulb free to the air, should be frequently observed while the sun is on the meridian, and at given altitudes in the foreuoon and afternoon, and these observations compared with those of the ordinary thermometer in the shade.

Experiments should also be made with a thermometer in the focus of the silvered mirror, the face of which is directed to the sky. For this purpose the ordinary black-bulb thermometer may be used as well as the naked-bulb thermometer. The thermometer thus placed will generally indicate a lower temperature than one freely exposed to radiation from the ground and terrestrial objects, and in case of isolated clouds will probably serve to indicate those which are colder and perhaps higher.

Comparison may also be made between the temperature at different distances above the earth, by suspending thermometers on a spar at different heights.

The temperature of deep soundiugs should be taken with the ther-
mometer with a guard to obviate the pressure of the water. As the tendency, on account of the revolution of the earth, is constantly to deflect all currents to the right hand of the observer looking down stream, the variations in temperature in connection with this fact may serve to assist in indicating the existence, source, and direction of currents.

The depth of frost should be ascertained, and also, if possible, the point of invariable temperature. For this purpose augers and drills with long stems for boring deeply should be provided.

Pressure of air.-A series of comparative observations should be made of the indications of the mercurial and aneroid barometers. The latter will be affected by the variation of gravity as well as of temperature, while the former will require a correction due only to heat and capillarity.
As it is known that the normal height of the barometer varies in different latitudes, accurate observations in the Arctic regions with this instrument are very desirable, especially in connection with observations on the moisture of the atmosphere, since, to the small quantity of this in northern latitudes the low barometer which is observed there has been attributed. I think, however, it will be found that the true cause is in the rotation of the earth on its axis, which, if sufficiently rapid, would project all the air from the pole.
In the latitude of about 60 there is a belt around the earth in which the barometer stands unusually high, and in which violent fluctuations occur. This will probably be exhibited in the projection of the curve representing the normal height of the barometrical column in different latitudes.

Moisture.-The two instruments for determining the moisture in the air are the wet and dry-bulb thermometer, and the dew-point instrument, as improved by Regnault. But to determine the exact quantity in the atmosphere in the Arctic regions will require the use of an aspirator, by which a given quantity of air can be passed through an absorbing substance, such as chloride of calcium, and the increase of weight accurately ascertained. It may, however, be readily shown that the amount is very small in still air.

A wind from a more southern latitude will increase the moisture, and may give rise to fogs. Sometimes, from openings in the ice, vapor may be exhaled from water of a higher temperature than the air, and be immediately precipitated into fog.

The incouvenience which is felt from the moisture which exhales with the breath in the hold of the vessel may, perhaps, be obviated by adopting the ingenious expedient of one of the Arctic voyagers, viz, by making a number of holes through the deck and inverting over them a large metallic vessel like a pot. The exterior of this vessel being exposed to the low temperature of the air without would condense the moisture from within on its interior surface, and thus serve, on the principle of the diffusion of vapor, to desiccate the air below.

The variation of moisture in the atmosphere performs a very important part in all the meteorological changes. Its effects, however, are probably less marked in the Arctic regions than in more southern latitudes. The first effect of the introduction into the atmosphere of moisture is to expand the air and to diminish its weight ; but after an equilibrium has taken place, it exists, as it were, as an independent atmosphere, and thus increases the pressure. These opposite effects render the phenomena exceedingly complex.

Winds.-As to these the following observations are to be regularly and carefully registered, namely: The average velocity as indicated by Robinson's anemometer; the hour at which any remarkable change takes place in their direction; the course of their veering; the existence at the same time of currents in different directions as indicated by the clouds; the time of beginning and ending of hot or cold winds, and the direction from which they come. Observations on the force and direction of the wind are very important. The form of the windvane should be that of which the feather part consists of two planes, forming between them an angle of about $10^{\circ}$. The sensibility of this instrument, provided its weight be not too much increased, is in proportion to the surface of the feather planes. Great care inust be taken to enter the direction of the wind from the true meridian, whenever this can be obtained, and in all cases to indicate whether the entries refer to the true or magnetic north. Much uncertainty has arisen on account of the neglect of this precaution.
In accordance with the results obtained by Professor Coffin, in his work on the resultant direction of the wind, there are in the northern hemisphere three systems roughly corresponding with the different zones, viz, the tropical, in which the resultant motion is toward the west, the temperate, toward the east, and the Aretic, in which it is again toward the west.

In the discussion of all the observations the variation of the temperature and the moisture will appear in their connection with the direction of the wind. Hence the importance of simultaneous observations on these elements, and also on the atmospheric pressure.

Precipitation.-The expedition will be furnished with a number of rain-gauges, the contents of which should be measured after each shower. By inverling and pressing them downward into the snow, and subsequently ascertaining, by melting in the same vessel the amount of water produced, they will serve to give the precipitation of water in the form of snow. The depth of snow can be measured by an ordinary measuringrod. Much difficulty, however, is sometimes experienced in obtaining the depth of snow on account of its drifting, and it is sometirnes not easy to distinguish whether snow is actually falling or merely being driven by the wind.

The character of the snow should be noted, whether it is in small
rounded masses, or in regular crystals; also the conditions under which these different forms are produced.

The form and weight of hailstones should be noted, whether consisting of alternate strata, the number of which is important, of flocculent snow, or solid ice, or agglutinations of angular crystals, whether of a spherical form, or that of an oblate spheroid.

The color of the snow should be observed in order to detect any organism which it may contain, and also any sediment which may remain after evaporation, whether of earthy or vegetable matter.

Clouds.-The character of the clouds should be described, and the direction of motion of the lower and higher ones registered at the times prescribed for the other observations. Since the expedition is well supplied with photographic apparatus, frequent views of the clouds and of the general aspect of the sky shouild be taken.

Aurora.-Every phase of the aurora borealis will of course be recorded; also the exact time of first appearance of the meteor, when it assumes the form of au arch or a corona, and when any important change in its general aspect takes place. The magnetic bearing of the crown of the arch, and its altitude at a given time, should be taken; also, if it moves to the south of the observer, the time when it passes the zenith should be noted. The time and position of a corona are very important.

Two distinct arches have sometimes been seen co-existing-one in the east and the other in the west. In such an exhibition the position and crown of each arch should be determined. Drawings of the aurora, with colored crayons, are very desirable. In lower latitudes a dark segment is usually observed beneath the arch, the occurrence of which, and the degree of darkness, should be registered. It also sometimes happens that a sudden precipitation of moisture in the form of a haziuess is observed to cover the face of the sky during the shooting of the beams of the aurora. Any appearance of this kind is worthy of attention.

Wave motions are sometimes observed, and it would be interesting to note whether these are from east to west or in the contrary direction; and whether they have any relation to the direction of the wind at the time. The colors of the beams and the order of their changes may be important in forming a theory of the cause of the phenomena. Any similarity of appearance to the phenomena exhibited in Geissler's tubes should be noted, especially whether there is anything like stratification.

The aurora sho uld be frequently examined by the spectroscope, and the bright lines which may be seen garefully compared with one of Kirchoffts maps of the solar spectrum.
To settle the question as to the fluorescence of the aurora and its consequent connection with the electric discharge, a cone of light reflected from the silver-plated mirror should be thrown on a piece of white paper, on which characters have been traced with a brush dipped in sulphate
of quinine. By thus condensing the light on the paper, any fluorescense which the ray may contain will be indicated by the appearance of the previously invisible characters in a green color.

Careful observations should be made to ascertain whether the aurora ever appears over an expanse of thick ice, or only over land or open water, ice being a non-conductor of electricity.

The question whether the aurora is ever accompanied with a noise has often been agitated, but not yet apparently definitely settled. Attention should be given to this point, and perhaps the result may be rendered more definite by the use of two ear-trumpets, one applied to each ear.

According to Hansteen, the aurora consists of luminous beams, parallel to the dipping needle, which at the time of the formation of the corona are shooting up on all sides of the observer, and also the lower portions of these beams are generally invisible. It is, therefore, interesting to observe whether the auroral beams are ever interposed between the observer and a distant mountain or cloud, especially when looking either to the east or west.

The effect of the aurora on the magnetism of the earth will be observed by abnormal motion of the magnetic instruments for observing the declination, inclination, and intensity. This effect, however, may be more strikingly exhibited by means of a galvanometer, inserted near one end of a long insulated wire extended in a straight line, the two extremities of which are connected with plates of metal plunged in the water, it may be through, holes in the ice, or immediately connected with the ground.

To ascertain whether the effect on the needle is due to an electrical current in the earth, or to an inductive action from without, perhaps the following variation of the preceding arrangement would serve to give some indication. Instead of terminating the wire in a plate of metal, plunged in the water, let each end be terminated in a large metallic insulated surface, such, for example, as a large wooden disk, rounded at the edges and covered with tin-foil. If the action be purely inductive, the needle of the galvanometer inserted, say near one end of the wire, would probably indicate a momentary current in one direction, and another in the opposite, at the moment of the cessation of the action. For the purpose of carrying out this investigation the Smithsonian Institution has furnished the expedition with two reels of covered wire, each a mile in length, one of which is to be stretched in the direction, perhaps, of the magnetic meridian, and the other at right angles to it. It would be well, however, to observe the effect with the wires in various directions, or united in one continuous length.

Electricity.-From the small amount of moisture in the atmosphere, and the consequent insulating capacity of the latter, all disturbances of the electrical equilibrium will be seen in the frequent production of light and sparks on the friction and agitation of all partially non con-
ducting substances. Any unusual occurrences of this kind, such as electrical discharges from pointed rods, from the ends of spars, or from the fingers of the observer, should be recorded.

A regular series of observations should be made on the character and intensity of the electricity of the atmosphere by means of au electrometer, furnished with a polished, insulated, metallic ball, several inches in diameter, and two piles of Deluc to indicate the character of the electricity, whether + or -; and also supplied with a scale to measure by the divergency of a needle the degree of intensity. This instrument can be used either to indicate the electricity of the air by induction or by conduction. In the first case it is only necessary to elevate it above a normal plane by means of a flight of steps, say eight or ten feet, to touch the ball at this elevation and again to restore it to its first position, when it will be found charged with electricity of the same character as that of the air. Or the ball may be brought in contact with the lower end of an insulated metallic wire, to the upper end of which is attached a lighted piece of twisted paper which has been dried after previous saturation in a solution of nitrate of lead.
Thunder-storms are rare in the Arctic regions, although they sometimes occur; and in this case it is important to observe the point in the horizon in which the storm-cloud arises; also the direction of the wind during the passage of the storm over the place of the observer; and also the character of the lightuing-whether zigzag, ramified, or direct; also its direction-whether from cloud to cloud, or from a cloud to the eartl.

Optical phenomena.-Mirage should always be noted, as it serves to indicate the position of strata of greater or less density, which may be produced by open water, as in the case of lateral mirage, or by a current of wind or warmer air along the surface.
The polarization of the light of the sky can be observed by means of a polariscope, consisting of a plate of tourmaline with a slice of Iceland spar, or a crystal of niter cut at right angles to its optical axis, on the side farthest from the eye. With this simple instrument the fact of polarization is readily detected, as well as the plane in which it is exhibited.
Halos, parhelia, coronæ, luminous arches, and glories should all be noted, both as to time of appearance and any peculiarity of condition of the atmosphere. Some of these phenomena have been seen on the surface of the ice by the reflection of the sun's beams, from a surface on which crystals had been formed by the freezing of a fog simultaneously with a similar appearance in the sky, the former being a continuation, as it were, and not a reflection of the latter.
In the latitude of Washington, immediately after the sun has sunk below the western horizon, there frequently appear faint parallel bands of colors just above the eastern horizon, which may very possibly be due to the dispersion of the light by the convex form of the atmosphere, and also, at some times, slightly colored beams crossing the heavens
like meridians, and converging to a point in the eastern horizon. Any appearance of this kind should be carefully noted and described.

Meteors.-Shooting-stars and meteors of all kinds should be observed with the spectroscope. The direction and length of their motion should be traced on star-maps, and especial attention should be given at the stated periods in August and November. A remarkable disturbauce of the aurora has been seen during the passage of a meteor through its beams. Any phenomenon of this kind should be minutely described.

Ozone.-The expedition is furnished with a quantity of ozone test paper, observations with which can only be rendered comparable by projecting against the sensitized paper a given quantity of atmospheric air. For this purpose an aspirator should be used, which may be made by fastening together two small casks, one of which is filled with water, with their axes parallel, by means of a piece of plank nailed across the lieads, through the middle of which is passed an iron axis on which the two casks may be made to revolve, and the full cask may readily be placed above the empty, so that its contents may gradually descend into the latter. During the running of the water from the upper cask, an equal quantity of air is drawn through a small adjutage into a closed vessel, and made to impinge upon the test-paper. The vessel containing the test-paper should be united with the aspirator by means of an India-rubber tube.
Miscellaneous.-The conduction of sound during still weather, through the air over the ice, through the ice itself, and through the water, may be stadied.
Evaporation of snow, ice, and water may be measured by a balance, of which the pan is of a given dimension.
Experiments on the resistance of water to freezing in a confined space. at a low temperature, may be made with small bomb-shells closed with screw-plugs of iron. The fact of the liquidity of the water at a very low temperature may be determined by the percussion of a small iron bullet, or by simply inverting the shell, when the ball, if the liquid remains unfrozen, will be found at the lowest point. It might be better, however, to employ vessels of wrought iron especially prepared for the purpose, since the porosity of cast-iron is such that the water will be forced through the pores, $e . g$., the lower end of a gun-barrel, which, from the smallness of its diameter, will sustain an immense pressure, and through which the percussion of the inclosed bullet may be more readily heard. Water, in a thin metallic vessel, exposed on all sides to the cold, sometimes gives rise to hollow crystals of a remarkable shape and size, projecting above the level surface of the water, and exhibits phenomena worthy of study.

Experiments may be made on regelation, the plasticity of ice, the consolidation of snow into ice, the expansion of ice, its conducting power for heat, and the various forms of its crystallization. The effect of in-
tense cold should be studied on potassium, sodium, and other substances, especially in relation to their oxidation.

The melting point of mercury should be observed, particularly as a means of correcting the graduation of thermometers at low temperatures. The resistance to freezing of minute drops of mercury, as has been stated, should be tested.

Facts long observed, when studied under new conditions, scarcely ever fail to yield new and interesting results.

Joseph Henry.

## NATURAL HISTORY.

Objects of natural history of all kinds should be collected, and in as large numbers as possible. For this purpose all on board the vessel, both officers and sailors, should be required to collect, upon every favorable opportunity, and to deliver the specimens obtained to those appointed to have charge of them.

Zoölogy.-The terrestrial mammals of Greenland are pretty well known, but it is still desirable that a series, as complete as possible, of the skins should be preserverl, great care being taken to always indicate upon the label to be attached the sex, and probable age, as well as the locality and date of capture. The skeleton, and, when it is not possible to get this complete, any detached bones, particularly the skull and attached cervical vertebræ, are very desirable. Interesting soft parts, especially the brain, and also embryos, are very important. If it should be considered necessary to record measurements, they should be taken from specimens recently killed.

Of walruses and seals, there should be collected as many skeletons as possible, of old and young individuals; also skins, 'especially of the seals. Notes should be made regarding the habits in general, food, period of copulation, duration of gestation and time of migration, it being desirable to find out whether their migrations are periodical.

Of the Cetacea, when these are too large to be taken on board the vessel, the skull and cervical vertebræ, the bones of the extrenaities and penis, and whatever else may be deemed worthy of preservation, should be secured. All the animals should be examined for ecto and ento parasites, and the means by which they become affixed to the animals noted.

Collect carefully the species of Myodes (lemmings,) Arctomys and Arvicola, so as to determine the variations with locality and season. The relationship of two kinds of foxes, the blue and white, should be studied to determine their specific or other relationship. Any brown bears should be carefully collected, both skin and skeleton, to determine whether identical or not with the Old World Ursus arctos.

Reference has already been made to the seals and cetaceans; of these the Phoca cristata, the white whale, (Beluga,) and the Monodon are particularly desired.

What has been said in regard to the mammals will apply equally well to the birds, skins and skeletons being equally desirable. It is especially important that the fresh colors of the bill, cere, gums, eyes, and feet or caruncles, or bare skin, if there be any, should be noted, as the colors of these parts all change after the preparation of a specimen.
Of birds, the smaller land species are of the greatest interest, and complete series of them should be gathered. The northern range of the insectivorous species should be especially inquired into. The arctic falcons should be collected in all their varieties, to ascertain whether there are two forms, a brown and white, distinct through life, or whether one changes with age into the other.

Inquiry should be directed to the occurrence of Bernicla leucopsis, Anser cinereus, or other large gray geese, and the Camptolemus Labradora, and a large number of specimens, of the latter especially, should be obtained. Indeed the geese and ducks generally should form subjects of special examination. Among the Laridice the most important species is the Larus rossii or Rhodostethia rosea, scarcely known in collections. A large number of skins and of eggs will be a valuable acquisition. Larus eburneus is also worthy of being collected. The Alcidce should be carefully examined for any new forms, and inquiries directed in regard to the Alca impennis.

Of all birds' eggs an ample store should be gathered, and the skeletons of the Arctic raptores and Natatores generally.

It will be a matter of much importance to ascertain what is the extreme northern range of the continental species of birds, and whether, in the highest latitudes, the European forms known to occur in Greeuland cross Baffin's Bay.

Eggs and nests of birds, in as large numbers as possible, should be procured, great care being taken, however, in all cases, to identify them by the parents, which may be shot, and some portion, if not all of them, preserved, if not recognized by the collector. All the eggs of one set should be marked with the same number, that they may not be separated; the parent bird, if collected, likewise receiving the same number. It should also be stated, if known, how long the eggs have been set upon, as incubation influences very much their color; the situation of the nest also is very important. Notes on the manner of nesting, localities selected, and other peculiarities of breeding, should be carefully kept; whether they are polygamous, whether there are struggles between the males, and the manner in which the old birds feed their young; and whether these remain helpless in the nest for a given time, or whether they accompany the parents from birth. A journal of the arrival and departure of the migratory species should also be kept, to find out whether those which leave latest return earliest, and vice versa.

Of fishes that are obtained, the best specimens should be photographed, the fresh colors noted, and then they should be preserved in alcohol or carbolic acid.

Among the fishes the Salmonidce, Cottidce, Gadidæe and Olupeidce, will he of the most interest, and good series should be secured.
The terrestrial inferior animals should be all collected, each class in its appropriate way.

Try to get larvæ of insects, and observe their life, whether they are well adapted to their surroundings; for in proportion to the insects are the number of insectivorous animals, and for that reason the struggle for life would be more energetic, and therefore only those insects which are best adapted to the conditions will survive.
Inferior marine animals are usually collected by two methods, viz, with a pelagic net and by a dredge. Both these methods should beemployed whenever practicable. Especial attention should be paid to the larvæ, of which sketches should be made. The results of the dredging should be noted in blanks printed for this purpose, the specimens to be preserved as their constitution requires. Muller's liquor, glycerine, solution of alcohol and sugar, \&c.
It would be of peculiar interest to study the several deep regions, admitted by Forbes and others, to ascertain if in the Arctic regions the intensity of color increases with the depth, as has been stated to be the case with red and violet, which, if true, would be just the contrary to what is observed in the temperate and tropical regions.
Of shells two sets should be preserved, one dry and the other with the animal, in alcohol; the dry shell is necessary from the fact that the alcohol, by the acetic acid produced, is apt to destroy the color.

It is particularly important to get as full a series as possible of the members of the smaller families, with a view to the preparation of monographs.
There should be paid as much attention as possible to the fauna of fresh-water lakes, to ascertain whether they contain marine forms, as has been found to be the case with some of those in North America, Scandinavia, Italy, and other countries. From this, important conclusions regarding the rising of the coast may be arrived at.

Botany.-Plants are to be collected in two ways. Of each species some specimens should be put in alcohol to serve for studying the anatomy; the others to be dried between sheets of blotting-paper. The locality of each specimen should be noted, also its situation, the character of the soil and heightabove the sea, the season, and whether there is heliotropismus, \&c., \&c. In the general notes there should be remarks on the korizontal and vertical distribution.
S. F. Baird.

## GEOLOGY.

The most important point in the collection of geological specimenswhether they consist of rocks, minerals, or fossils-is, that on breaking or digging them from the matrix or bed, each individual specimen should be carefully wrapped separately in pliable but strong paper, with a label
designating the exact locality from which it was obtained. If two or more beds of rock (sandstone, limestone, clay, marl, or other material) occur at the locality from which specimens are taken, the label should also have a number on it corresponding to the particular bed in which it was found, as designated in a section made on the spot in a note-book. This should be done in order that the specimens from each bed may be separated from those found in others, whether the beds are separable by differences of composition, or by differences in the groups of fossils found in each ; and it is, moreover, often important that this care should be observed, even when one or more of the beds are of inconsiderable thickness, if such beds are characterized by peculiar fossils. For in such cases it often happens that what may be a mere seam at one place may represent an important formation at another.
Specimens taken directly from rocks in place are, of course, usually more instructive than those found loose; but it often happens that much better specimens of fossils can be found already weathered out, and lying detached about an outcrop of hard rock, than can be broken from it. These can generally be referred to their place in the section noted at the locality, by adhering portions of the matrix, or from finding more or less perfect examples of the same species in the beds in place; but it is usually the better plan to note on the labels of such specimens that they were found loose, especially if there are any evidences that they may have been transported from some other locality by drift agencies.
All exposures of rocks, and especially those of limestone, should be carefully examined for fossils, for it often happens that hard limestones and other rocks that show no traces of organic remains on the natural surfaces, (covered, as they often are, with lichens and mosses,) will be found to contain fossils when broken into. In cases where fossils are found to exist in a hard rock, if time and other circumstances permit, it is desirable that it should be vigorously broken with a heary hammer provided for that purpose, and as many specimens of the fossils as possible (or as the means of transportation will permit) should be collected.

Fossils from rocks of all ages will, of course, be interesting and instructive, but it is particularly desirable that organic remains found in the later tertiary and quarternary formations of these high northern latitudes, if any such exist there, should be collected. These, whether of auimals or plants, would throw much light on the question respecting the climatic conditions of the polar regions at, or just preceding, the advent of man.

Specimens illustrating the lithological character of all the rocks observed in each district explored should also be collected; as well as of the organic remains found in fossiliferous beds; also all kinds of minerals. Those of rocks and amorphous minerals should be trimmed to as nearly the same size and form as can conveniently be done-say 3 by 4 inches
wide and long, and 14 inches in thickness. Crystalline minerals ought, of course, to be broken from the matrix, rather with the view of preserving the crystals as far as possible, than with regard to the size or form of the hand specimens; and the same remark applies equally to fossils.

On an overland journey the circumstances may not always be such as to allow the necessary time to wrap carefully and label specimens on the spot where they were collected; but in such cases numbers or some other marks should be scratched with the point of a knife, or other hardpointed instrument, on each, by means of which the specimens collected at different times and places during the march can be correctly separated, labeled, and wrapped when the party stops for rest.

All specimens should be packed tightly in boxes as soon as enough have been collected to fill a box, and a label should be attached to each box indicating the particular district of country in which the collections were obtained. For this purpose empty provision boxes or packages can generally be used.

In examining sections or exposures of rocks along a shore or elsewhere, it is a good plan to make a rough sketch in a note-book, thus :

Section 1.


Then on the same or following pages, more particular descriptions of the nature and composition of the several beds should be written, referring to each by its number. Sections of this kind should be numbered $1,2,3$, and so on, in the order in which they were observed, and the specimens from each bed ought also to be numbered on its label so as to correspond. That is, specimens from the lowest bed of the first section should be, for instance, marked thus: "Section No. 1, bed No. 1, " and so on. The name of the locality, however, should also, as already suggested, be written on the labels as a provision against the possible loss of note-books.

It generally happens that an ontcrop will show only a part of the beds of which it is composed, thus:

| 5 | Unexposed. | 10 feet. |  |
| :--- | :--- | :--- | :--- | :--- |
| 4 | Limestone. | 7 feet. |  |
| 3 | Unexposed space. | 11 foet. |  |
| 2 | Limestone. | 8 feet. |  |
| 1 | Sandstone. |  | 15 feet. |

In such a case the facts should be noted exactly as seen, without any attempt to guess at the nature of the material that may fill the unexposed places; but, generally, by comparing different sections of this kind taken in the same region, the entire structure of a district may be made out.

The dip and strike of strata should also be carefully observed and noted, as well as the occurrence of dikes or other outbursts of igneous rocks, and the effects of the latter on the contiguous strata.

All evidences of the elevation or sinking of coasts should likewise be carefully observed and noted.

Especial attention should be given to glacial phenomena of every kind, such as the formation, size, movements, \&c., of existing glaciers, their abrading and other effects upon the subjacent rocks, their formation of moraines, \&c.; also, the formation, extent, and movements of icebergs, and their power of transporting masses of rock, \&e.

At Cape Frazer, between latitude $50^{\circ}$ north and longitude $70^{\circ}$ west, Dr. Hayes found some upper silurian fossils in a hard gray lipestone. This rock doubtless has a rather wide extension in the country referred to, as other explorers hare brought silurian fossils from several localities farther southward and westward in this distant northern region. Should the party visit the locality from which Dr. Hayes collected his specimens, it is desirable that as complete a collection as possible should be obtained, as most of those found ly Dr. Hayes were lost.

For making geological observations, and collecting geological specimens, very few instruments are required. For determining the elevations of mountains, and the general altitude of the country, a barometer is sufficiently accurate. For local elevations of less extent a pocket-level (Locke's) should be provided. Tape-lines are also useful for measuring vertical outcrops, and other purposes; and a good pocket-compass is indispensable. The latter should have a clinometer attached.

A good supply of well-tempered cast-steel hammers should also be provided. They should be of various sizes and forms, and ought to be made with large enough eyes to receive stout handles, of which a good number, made of well-seasoned hickory, should be prepared. Chisels of different sizes should also be prepared of well-tempered steel.

A pouch of leather or stout canvas, with a strap to pass over the shoulder, will be found useful to carry specimens for short distances.
F. B. Meek.

## GLACIERS.

The progress of our knowledge of glaciers has disclosed two sides of the subject entirely disconnected with one another, and requiring different means of investigation. The study of the structure of glaciers as they exist now, and the phenomena connected with their formation, maintenance, and movement, constitute now an extensive chapter in the physics of the globe. On the other hand, it has been ascertained that glaciers had a much wider range during an earlier but nevertheless comparatively recent geological period, and have produced during that period phenomena which, for a long time, were ascribed to other agencies. In any investigation of glaciers now-a-days, the student should keep in mind distinctly these two sides of the subject. He ought also to remember at the outset what is now no longer a mooted point-that, at different times during țhe glacial period, the accumulations of ice covering larger or smaller areas of the earth's surface have had an ever-varying extension, and that whatever facts are observed, their value will be increased in proportion as the chronological element is kept in view.
From the physical point of view, the Arctic expedition, under the command of Captain Hall, may render science great service should Dr. Bessels have an opportunity of comparing the present accumulations of ice in the Arctic regions with what is known of the glaciers of the Alps and other mountainous regions. In the Alps the glaciers are fed from troughs in the higher regions, in which snow accumulates during the whole year, but more largely during winter, and by a succession of changes is gradually transformed into barder and harder ice, moving down to lower regions where glaciers never could have been formed. The snow-like accumulations of the upper regions are the materials out of which the compact transparent brittle ice of the lower glaciers is made. Whatever snow falls upon the glaciers in their lower range during winter, melts away during summer, and the glacier is chiefly fed from above and wastes away below. The water arising from the melting of the snow at the surface contributes only indirectly to the internal economy of the glacier. It would be superfluous here to rehearse what is known of the internal structure of glaciers and of their movement; it may be found in any treatise on glaciers. Nor would it be of any avail to discuss the value of conflicting views concerning their motion. Suffice it to say that an Arctic explorer may add greatly to our knowledge by stating distinctly to what extent the winter snow, falling upon the surface of the great glacial fields of the Aretic, melts away during summer and leaves bare an old icy surface, covered with fragments of rock, sand, dust, \&c. Such an inquiry will teach us in what way the great masses of ice which pour into the Arctic Ocean are formed, and how the supply that empties annually into the Atlantic is replenished. If the winter snows do not melt entirely in the lower part of the Aretic glaciers during summer, these glaciers must exhibit a much more regular stratification than the Alpine glaciers, aud the successive falls of snow must in them
be indicated more distinctly by layers of sand and dust than in those of the Alps by the dirt bands. Observations concerning the amount of waste of the glaciers by evaporation or melting, or what I have called ablation of the surface during a given time in different parts of the year, would also be of great interest as bearing upon the hygrometric condition of the atmosphere. A pole sunk sufficiently deep into the ice to withstand the effects of the wind could be used as a meter. But it ought to be sunk so deep that it will serve for a period of many months, and rise high enough not to be buried by a snow-storm. It should also be ascertained, if possible, whether water oozes from below the glacier, or, in other words, whether the glacier is frozen to the ground or separated from it by a sheet of water. If practicable, a line of poles should be set out with reference to a rocky peak or any bare surface of rock, in order to determine the motion of the ice. It is a matter of deep interest with reference to questions connected with the former greater exteusion of glaciers, to know in what manner flat sheets of ice more on even ground, exhibiting no marked slope. It may be possible to ascertain, after a certain time, by the change of position of poles sunk in the ice, whether the motion follows the inequalities of the surface, or is determined by the lay of the land and the exposure of the ice to the atmospheric agents, heat, moisture, wind, \&c. It would be of great interest to ascertain whether there is any motion during the winter season, or whether motion takes place only during the period when water may trickle through the ice. The polished surfaces in the immediate vicinity of glacier ice exhibit such legible signs of the direction in which the ice moves, that wherever ledges of rocks are exposed the scratches and furrows upon their surface may serve as a sure register of its progress; but before taking this as evidence, it should, if possible, be ascertained that such surfaces actually belong to the area over which the adjoining ice moves during its expansion, leaving them bare in its retreat.

The geological agency of glaciers will no doubt receive additional evidence from a careful examination of this point in the Aretic regions. A moving sheet of ice, stretching over a rocky surface, leaves such unmistakable marks of its passage that rocky surfaces which lave once been glaciated, if I may thus express the peculiar action of ice upon rocks, viz, the planing, polishing, scratching, grooving, and furrowing of their surfaces, can never be mistaken for anything else, and may everywhere be recognized by a practiced eye. These marks, in connection with transported loose materials, drift, and bowlders, are unmistakable evidence of the great extension which glaciers once had. But here it is important to discriminate between two sets of facts, which have generally been confounded. In the proximity of existing glaciers, these marks and these materials have a direct relation to the present sheet of ice near by. It is plain, for instance, that the polished surfaces about the Grimsel, and the loose materials lying between the glacier of the Aar and the Hospice, are the work of the glacier of the Aar when it
extended beyond its present limits, and step by step its greater extension may be traced down to Meyringen, and, in connection with other glaciers from other valleys of the Bernese Oberland, it may be tracked as far as Thun or Berne, when the relation to the Alps becomes complicated with features indicating that the whole valley of Switzerland, between the Alps and the Jura, was once occupied by ice. On the other hand, there are evident signs of the former presence of local glaciers in the Jura, as, for instance, on the Deut de Vaulion, which mark a later era in the history of glaciation in Switzerland. Now the traces of the former existence of extensive sheets of ice over the continent of North America are everywhere most plainly seen, but no one has yet undertaken to determine in what relation these glaciated surfaces of past ages stand to the ice-fields of the present day in the Arctics. The scientific men connected with Captain Hall's expedition would render science an important service if they could notice the trend and bearing of all the glacial scratches they may observe upon denadated surfaces wherever they land. It would be advisable for them, if possible, to break off fragments of such glaciated rocks and mark with an arrow their bearing. It would be equally important to notice how far the loose materials, pebbles, bowlders, \&c., differ in their mineralogical character from the surface on which they rest, and to what extent they are themselves polished, rounded, scratched, or furrowed, and also what is the nature of the clay or sand which holds them together. It would be particularly interesting to learn how far there are angular bowlders among these loose materials, and what is their position with reference to the compacted drift made up of rounded, polished, and scratched pebbles and bowlders. Should an opportunity occur of tracing the loose materials of any locality to some rock in situ, at a greater or less distance, and the nature of the materials should leave no doubt of their identity, this would afford an invaluable indication of the direction in which the loose materials have traveled. Any indication relating to the differences of level among such materials would add to the value of the observation. I have purposely avoided all theoretical considerations, and only called attention to the facts which it is most important to ascertain, in order to have a statement as unbiased as possible.

L. AgAssiz.

## ETHNOLOGY.

## INDIAN MOUNDS NEAR FORT WADSWORTII, DAKOTA TERRITORY.

By A. J. Comfort, Acting Assistant Surgeon United States Army.

Indian mounds of the larger size were probably designed as cemeteries. They are located generally on a terrace, knoll, or elevation, at a convenient distance from the water.

It has been a custom of the American Indian, from time immemorial, to deposit the remains of the dead upon burial-scaffolds or suspend them from trees. At stated periods the bones were gathered together and interred. Among the Dakotas the custom has been, when a member of the tribe dies, after the autumnal leaves have falleu, to deposit the remains upon a scaffold, not to be removed until the leaves have unfolded in spring, and if a death occur after the leaf-buds have burst, the remains of the dead are likewise deposited, not to be removed until the leaves have fallen in autumn. When a member of the lodge of the "grand medicine" dies, the removal of the remains from the burialscaffold, and their interment, is attended with a grand " medicine dance," and the initiation of a new member to fill the racancy.

A solitary mound, occupying an elevated position upon the rolling prairie, near the eastern shore of a beautiful lake, was first selected for exploration. This site was chosen by the " mound-builders " evidently for its richness in those associations in which men in their primitive simplicity of customs especially delight. In every direction, except the west, as far as the eye can reach, lay stretched out the broad prairie of Dakota, upon which it was impossible for an enemy to lurk or a buffalo to range unperceived. By a gradual and almost uniform descent of a quarter of a mile, the largest of Kettle Lakes may be reached, abounding in fish and, at the appropriate season, water-fowl of the choicest variety, as game. Within a quarter of a mile of the shore is an island, about a mile in circumference, heavily timbered, the favorite resort of wood-ducks and cormorants during the period of incubation. Trees support the nests of the former in great numbers; geese, brant, and swan are wont to feed here in autumn, on their journey southward. It seems but reasonable that an elevated site possessing such advantages for the living savage should be selected as the place of deposit for his bones, especially when we reflect that among the aborigines there was prevalent an almost universal belief in the existence of a spirit which had intrusted to its charge the guardianship of the remains of the dead; consequently, a spot the most eligible on account of the
beauty of its scenery and the accessibility to game, important desiderata to the living, was deemed the most suitable for the haunts of the spirits of the dead.
The mound selected for exploration, east of Kettle Lakes, for convenience of reference, is termed No. 1. Upon and around it for several feet were strewn human bones of every stage of development and of either sex. The external appearance of this ancient structure bore the most unmistakable evidence of the purpose for which it was intendeda receptacle for the remains of the dead; but from this purpose it had been perverted by the fox of the prairie, which had burrowed within it, removing bones and other obstacles in the way of constructing its lair.

The form of this mound, like others of the class, was that of the frustum of a cone, the diameter of whose base measured fifty feet, that of its superior plane thirty feet; the height of the latter was three feet.

Almost covered with earth, I found a hornblende bowlder, of an irregular discoidal shape, divided into two unequal sections by a vein of granite three-quarters of an inch thick. This stone marked the center of the mound. I drew two lines in the direction of the cardinal points of the compass, quite across the mound, intersecting each other at this stone, and dividing the mound into four equal sections, which are designated for convenience of reference as the northeast, southeast, northwest, and southwest, respectively.
I commenced digging into the nortbeast and southeast sections, removing the earth, stratum by stratum, observing and noting objects of interest as they appeared. In the sontheast section I found within eighteen inches of the surface two incomplete skeletons lying upon their sides, facing each other, their feet directed to the east, their heads within six inches of each other. One of these skeletons was that of a male, the other that of a female; though apparently of young persons, they were fully developed. The earth surrounding these was less compact than elsewhere found in the mound, was of a homogeneous character, of a dark color, and furnished no protection to the bones against moisture, from which I infer that the interment was intrusive, and made by a tribe occupying the country since the dispersion of the mound-bailders.
The most southern portion of the southeast section of this mound was a locality of great interest; it is designated in my invoice of contributions to the Museum as "locality A."
While sinking an excavation to the depth of about four feet, the atten. tion of one of the party was attracted by the appearance of a small quantity of black, dry, pulverulent earth, which, being examined, was found to be in close proximity to some stone, between which an aperture was found large enough to receive the hand, but as fast as the hand was withdrawn the space was again filled with the same pulveralent dust. My impression was that the aperture commanicated with the carity of a vault, to obtain a view of which the surrounding earth was
removed with great care without disturbing the stones. This structure proved to be a work of rude masonry, three feet long, eighteen inches wide, and two and a half feet high, inclosing a rectangular space. The stones used for this purpose were the undressed bowlders of the prairie, from eight to twelve inches in diameter, and were sustained in position by earth banked around the wall externally without the appearance of the use of lime or mortar.
In this were found the imperfect skeletons of a female and child, whose attitude would indicate the relation of the former as that of mother or nurse to the latter. The posture of each was that of sitting-the mother upon the floor and the child upon her lap, supported by her right arm. Indian women, at the present day, in seating themselves draw their heels close to their nates and bring the knees quite close to the floor, either upon the right or left side; such had been the disposition of the lower extremities of the mother interred by the mound-builders. The masonry had formed a support for the thorax of both mother and child, both of whom had been placed facing the east, the body of the child slightly inclined to the mother. The cranium of the latter had fallen to the pelvis; that of the former was resting procumbent upon the thorax. Amidst the black pulverulent dust with which the mound-builders were wont to surround the remains of their dead, upon the floor, within the cavity of the pelvis, and around it, I found a number of the bones of a foetus. The lower jaw was divided at its median symphysis; a parietal bone, scarcely thicker than paper, had thrown out its osseous matter from the parietal protuberance in radiate lines; a humerus of not a finger's length, and proportionately slender and delicate, was picked up among other bones of this interesting locality. My Indian party having observed every object of interest with the closest attentiou, were able to distinguish human bones from those of animals and to designate their places in the skeleton. One of these, struck by the analogy of the foetal bones to those of the adult, placed his hand upon his abdomen and exclaimed, "Papoose cik cistina," (a very small infant.)
Immediately behind the mason work above described, that is, to the west of it, was found a triangular space, in which was placed a number of bones, chiefly those of the apper and lower extremities, a few vertebræ and ribs, crania, \&c.; these had been apparently thrown in promiscuously, as if from disarticulated limbs; those of the upper and lower extremities in a state of extreme flexion, probably with a view to economize space, as though the sole object had been to preserve the bones from destruction and remove them from sight. The relative position of the parts would indicate that no trunk was entire; in one place was deposited the pelvis and lower extremities, without the small bones of the feet; ip another, the lower extremities withont the pelvis; in another, tho thorax and part of the spinal column; in another, the thorax and upper extremities; here a right arm had been deposited, having been severed from the trunk at the shoulder-joint ; there a left lower extremity with
the bones of the pelvis. I can only account for this separation of parts on the supposition that, in most instances, the remains had been gathered in for interment from burial-scaffolds after many months of exposure, especially since the small bones of the hands and feet in almost every instance were wanting. No implements of any kind were found in this locality and no bones of animals except the skull of a beaver. The earth in part of this mound had not been disturbed by the inroads of auimals; although this skull was in close proximity to a collection of human bones. At the time, and for months after, I was unable to account for the presence of the skull of this rodent in a human sepulchre. Upon careful examination of this object of interest I perceived that the foramen inagnum had been enlarged, its margin having been broken away by an agency directed by more intelligence than the lower animals possess. The object, of course, had been to extract the brain, but why extract the brain from the skull of a beaver for deposit with the remains of the dead? No satisfactory answer to these inquiries suggested itself to me until by accident I obtained a bag of an ex-member of the "grand medicine lodge;" this consisted of the skin of a beaver, the claws and skull remaining attached, the posterior walls of the latter having been removed with the soft parts. The skull in question, then, was, in my opinion, the undecomposed part of a medicine-bag.

The bones in the triangular space, like the skeletons of the mother and child, were surrounded by a layer of dark pulverulent carbonaceous earth, constituting a stratum of the mound, one foot in thickness, surmounted by a layer of undressed bowlders, placed in as close proximity as possible without cement or mortar. The superincumbent earth removed from the layer of stones, its sides measured five, seven, and nine feet respectively, and running in the direction in the order of the above numbers, starting from the work of masonry as the southeastern angle, west of north, south of east, and due south to the point of starting. The floor of the mound, which constituted the floor of the triangle, was composed of clay of a wonderful cohesive property, and so compact that it could only be broken with violent blows of the pick. It had the appearance of having been baked, and yet there were no cracks in it as one would expect to see, produced by shrinking during the process of drying; it was quite smooth and level and bore the appearance of haring been finished prior to drying with a coating of clay in a plastic state, and smoothed with the hand or some rude substitute for a trowel. While the exploration of this mound was being prosecuted I was present in person, and when an object of interest was found my attention was immediately called to it. The earth was remored from it with great care, in order that its position might not be disturbed; if it was surrounded with a pulverulent earth, it was brushed away with a wisp of prairie-grass or a very fine brush-broom; if the earth was compact, it was carefully cut away with a knife, and the object chiseled out. Singular as it may seem, only the southeast section of this mound con-
tained any bones or other objects of interest worthy of note; the earth was removed from the remaining three sections within the circumference of the superior plane of the mound perpendicularly down to the floor, and the margin beyond, though not wholly removed, was examined in several places. The work thus far completed, I directed several excavations, about three feet in circuinference and as many deep, to be made, for the purpose of ascertaining the deeper structure of the mound. In every instance the material proved to be one homogeneous mass of dry, compact clay. In one of three excavations a few feet to the south of the masonry, and about a foot and a half below the floor, was found a cranium, which, on other bones of the same skeleton being exposed to view without their relations being disturbed, proved to be of an aged man. Few of these bones, chiefly those of the hands and feet, were wanting; the posture was that of sitting, with the body inclining forward and face directed to the east. I have seen Indians in council, or absorbed in earnest thought, assume a posture not unlike the one here represented. The thorax had been slightly compressed by the superimposed mass of earth. On the cranium and tibia I observed several small bony tumors of an almost pearly whiteness and great hardness, the largest about the size and shape of a half of a pea. These tumors which are called in surgery exostoses, are most generally the result of syphilis, though they may be attributed to other causes. A fine specimen of united fracture of one of the femurs was obtained from this skeleton, showing, from the amount of shortening, the obliquity of the axis of the fragments, and the ill-adjustment of the fractured ends, how much these people stood in need of surgical skill.
The interment in the triangular space must have been a contemporary act with that of the construction of the mound, soalso, in all probability, that within the work of masonry, while the burial of the aged man beneath the mound floor, unsurrounded with pulverulent carbonaceous earth, must have a date anterior to its construction. These facts are predicated upon the observation of the undisturbed condition of the superior strata of clay and black surface-mold of which the mound was composed. The habits of nomadic and uncivilized races afford them but limited facilities for preparing their food; their viands are usually broiled upon the live coals and eaten with theadherent ashes. Trituration being performed almost entirely by the teeth, these importantorgans of digestion are worn down to a common level at au early age. The tubercles of the molars, the points of the cuspids, and the cutting edge of the incisors are worn down by attrition to the level of one common plane. The teeth of the mound-builders differ not the least in this particular from that of modern American Indians who still adhere to their nomadic life.

Hampson's group of mounds.-On a knoll or elevation from fifty to one hundred feet above the water level, and sloping gradually to it at a distance of a quarter of a mile, is situated an interesting group of ten mounds, which for several years have borne the name of Hampson's

Mounds, in honor of Major Hampson of the Tenth United States Infantry, the present commander of the post.
Of this number two are particularly conspicuous, being nearly double the lieight of the rest and situated between them and the brow of the knoll; each is in form of the frustum of a cone, the usual shape of mounds in this vicinity. The measurement of the first is as follows : diameter of base fifty-five feet, diameter of superior plane thirty-five feet, perpendicular height, measured on one side, four feet, and six feet on the other ; this difference is owing to the ground on which it is located sloping slightly to the lake. The measurement of the second mound is as follows : diameter of base fifty feet; diameter of superior plane, twenty feet; height of superior plane, five feet.

Years ago animals have made inroads into the first of these mounds, carrying out fragments of human bones; their burrows now, however, are caved in, destroying its otherwise symmetrical appearance. The secoud mound was explored by me; all that portion of the mound being removed perpendicularly beneath the superior plane. In its center were found three imperfect skeletons whose crania were lying near together. They had evidently been buried with their feet in the direction of three cardinal points of the compass, one to the east, one to the north, the third to the west, two upon their sides, the third upon its back. A flat stone had formed a pillow for the three. Surrounding the bones was a stratum of black pulverulent carbonaceous earth, whose thickness was twelve inches; this was not different from the same found in Mound No. 1, aud in every sepulchral mound subsequently explored. I found in two or three spots of this layer an impalpable buff-colored powder, evidently the remains of some decomposed wood used in interment. Some distance from these skeletons, and a foot above them, was found a single cranium lying upon its side, beneath a ferv spinous processes of the vertebra of a buffalo. Here and there, in the upper stratum of the mound, I found the skulls of the musk-rat, skuuk, prairie-wolf, and other small animals, without the other bones of the carcass; these had been most probably attached to medicine-bags. The structure of this mound was essentially the same as that of all the sepulchral mounds explored by me, except No. 1, and consisted of four strata. The first or uppermost layer was three feet thick, and was composed of a black, moist, adhesive vegetable mold, not differing much from the surface-soil of the prairie except that it was a little darker in color, contained a little more moisture, and was more adherent to the shovel. The second layer was a foot thick, and consisted of a black, dry, pulverulent carbonaceons matter, in which human bones are usually found. The third layer was also a foot thick, and consisted of a siliceous loam. The fourth layer was a concrete composed of gravel and lime, and varied in thickness, as was required to make the upper surface quite horizontal. The two last layers had probably been dried in the sun and afterward burned.

On a line running nearly east and west, and about sixty feet further
from the lake, nearly parallel with the one joining the center of the principal mounds, are situated eight mounds smaller in dimensions and less conspicuous in appearance. One of these, whose diameter was twenty-five feet at base and fifteen feet at its superior plane, and whose perpendicular height was two feet, I opened at the same time of exploring the one previously described; it proved to be, unlike No. 2, destitute of strata, but composed of one homogeneous mass of surface-soil. At the depth of two feet was a stratum of clay three inches thick, very hard and compact. On this stratum, at its center, were found charcoal and ashes, but no bones.
I could explain this structure only on the supposition that a circular hut had once been located there, with a clay floor, with a fire-place in its center. Around the sides of the hut the earth had been banked, and when abandoned by its inmates the perishable portion had been removed, or, remaining undisturbed, had decomposed; the embankments had settled both internally and externally, and the center of thehabitation had filled up to the common leyel of its sides. That a circular earth-walled hat, of suitable dimensions, will assume the form of a frustum of a cone is shown by the group of small mounds which a few years ago might be seen near Saint Paul, where the once celebrated Black Dog's village stood. The remains of such huts I propose to call domiciliary tunnuli, in contradistinction to those of alarger size, with four characteristic strata, constructed for the purpose of interment.

The third mound of this group had a diameter of forty feet at base; that of its superior plane was fifteen feet, and its perpendicular height was two and a half feet. This mound showed a want of regularity in the circumference of its superior plane and that of its base, as well as the slope of its sides, and apparently had been the remains of a hut whose form had been a rectangle, with earth banked around its sides several feet high. The perishable part had been removed; its embankments settled, both externally and internally, and its central portion, though slightly depressed or cup-shaped, had nearly filled to the common level of its sides. A few inches below the surface I found the bones of the thorax, with upper extremities, in situ, as when interred, the external surface of the sternum directed upward, and to the east the cervical vertebro, not a foot below the surface. The axis of the thorax inclined to the horizon at an angle of about forty degrees; the bones of the lower extremities were entirely wauting. This mound was also destitute of the four characteristic strata which I found in Mound No. 2 and others afterward examined, from which it may be inferred that the burial was intrusive and by a more recent tribe, and that the mound was one of the domiciliary class. A stratum of clay, four inches thick,, , constituted the floor. Beneath the floor was found the skull and thighbones of an aged man. These bones among civilized men have emblematic significance. Can it be for a like reason the savages had deposited them in a place as secure as possible? The remaining mounds in this
interesting group, from.location, size, and general appearance, may be regarded as of the domiciliary class, and they vary in dimensions, their bases being in diameter from twenty to thirty-five feet, their superior planes being from fifteen to twenty-five feet, the height of the latter from one to two feet; they are located in a line at distances of from thirty to sixty feet apart.
On the west side of Kettle Lakes, about a mile and a half distant from the post, is a group of three mounds whose dimensions are as follows: diameter of base, sixty feet; diameter of superior plane, forty feet; height of superior plane, three feet.

In one of these the recent interment of the remains of an Indian child had taken place; another of these I explored, removing all the earth found perpendicularly beneath the superior plane. The mound proved to belong to the sepulchral class, and was composed of the four characteristic strata as were found elsewhere, viz, first, a stratum of surface-soil two feeit thick; second, a stratum of dry pulverulent carbonaceous matter one foot in thickness; third, a stratum of siliceous loam, bearing evidence of exposure to high heat, very dry and compact; fourth, a stratum of concrete one foot thick composed of clay containing a slight admixture of lime; both of these latter strata appear to have been subjected to a high degree of heat, being very dry and compact in structure, and so great is the cohesiveness of the particles that it requires smart blows of the pick to remove them. The shovel or spade makes no more impression upon the strata than upon a closely cemented pavement of bricks.
It would seem that the third and fourth layers of the mound had been leveled off singly, and an enormous pile of wood had been burned upon each for the purpose of baking it, and the ashes had been gathered up and sifted to remove the charcoal. An excavation ten or twelve inches deep, three feet in circumference, had been made in the third layer of this.mound, in which had been deposited the bones above mentioned. The sides and bottom bear the impressions of a pointed instrument, not unlike those made by a pick. The implement used probably was a sharpened stake, such as I have seen the Dakotas use in spring to dig tipsinna, or Dakota turnips. The bones found here had been divested of their soft parts and were piled in very compact cross-layers; they were as follows, none of them perfect, however, viz: two inferior maxillary bones, a number of fragments of a cranium, a number of fragments of a pelvis, six femora, four tibiæ, four fibulæ, three ulnæ, two radii, and one scapula. I also obtained about a peck of fragments of decayed wood, which had scarcely enough cohesiveness existing to enable it to retain its form, and yet the bark remained adherent. Each stick must have been five feet long and three inches thick. The wood was found between the first and second layer, surmounted by a number of large undressed bowlders in immediate proximity to it. It bears no mark of implements upon it, except that it has been split, and
most of it appears by its rings of annular growth to be the part of a trunk of a large tree.

I presume it is of a species of oak still growing in this vicinity in the ravines and places protected by their water surroundings from prairie fire. As a general rule, the mound-builders were wont to cover with wood or stone that portion of the second layer immediately enveloping the bones.

On a ridge, elevated ten or fifteen feet above the surface of the lakes, and within one-half of a mile from the post, and a few hundred féet from the water, is a group of eight mounds, whose dimensions are as follows: diameter of base, sixty feet; diameter of superior plane, forty-five or fifty feet; height of superior plane above the surface of the prairie immediately surrounding it, three feet. I explored one of this group and found its structure to be identical with the last described, that is, to be composed of four characteristic strata, the latter two bearing evidence of exposure to high heat. This mound, and apparently the whole group, had evidently been constructed for sepulchral purposes; a slight excavation had been made in the fourth layer to receive the bones which were as follows: four inferior maxillary, fourteen vertebræ, nine scapulæ, nine humeri, nine ribs, nine ulnæ, ten ossæ innominatæ, fifteen femora, thirteen tibiæ, and eight fibulæ. These were arranged in crosslayers, so as to occupy the least possible amount of space, and within a compass of three feet. They had been divested of their soft parts prior to interment, as was evident from their relative position. The radius was invariably found without the ulna to match, the tibia without the fibula. The ends of bones which would have been in proximity, if not disarticulated, were never found so; neither the head of the humerus nor the head of the femur was ever found in its socket. A number of the bones found here had been gnawed by mice or prairie-gophers.

On the south side of the post, and within one or two hundred feet of the sally-port, is a sepulchral mound, the diameter of whose base is between forty-five and fifty feet, that of its superior plane thirty and forty; the height of the superior plane, above the surface of the immediately surrounding prairie, is about two and a half feet. On the road to Fort Abercrombie, about a mile and a half from the post, upon a ridge arising about forty feet above the surface of the adjacent lakes, of which there is one on either side, is situated a group of seven mounds, all of which may be regarded as of the sepulchral class, and do not differ in size and appearance from those previously described.
Three miles from Fort Wadsworth, in a direction a little east of north, upon a hill sixty feet above the surface of an adjacent lake, and sloping quite to its water's edge, is a group of seven mounds, two of which belong to the sepulchral class. The dimensions of one of these are as follows: diameter of base, sixty feet; diameter of superior plane, fifteen feet; height of superior plane, above the sloping hill-side, on which the mound is situated, from four to eight feet. These mounds sustain the
same relative position to each other as those of Hampson's group, viz: two near the brow of the hill, with the remaining six in a line nearly parallel to one joining their centers. The six are about a handred and fifty feet farther from the lake, and, judging from size and appearance, belong to the domiciliary class; they vary from thirty to forty feet in diameter at base, and are about sixty feet apart, and may be regarded in a line as nearly straight as Indians are wont to construct their huts.

- On a strip of land adjoining the fort on the west, and between two lakes, are situated ten or twelve mounds. Upon a ridge, one-quarter of a mile in length, at various distances from each other, seven of them are located; the others occupy knolls which, from their elevation and proximity to water, seemed to the builders to furnish the most eligible sites.
The flag-staff of this post was planted in an Indian mound, occupying the center of the parade, and human bones were thrown out during the process of excavation. Another mound formerly stood in front of one of the barracks. Both now are leveled off and the locality overgrown with grass.

MOUNDS, FORTIFICATIONS, ETC., FOUND IN OTHER VICINITIES.
There is an interesting group of mounds on the north shore of White Bear Lake, near Glenwood, Pope County, Minnesota.

On a terrace arising by a gradual slope from the former bed of a river, and near the residence of the present Indian agent, is situated an interesting group of Indian mounds, two of which, from size and appearance, may be regarded as of the sepulchral class.
Mounds occurring both in groups and solitary may be seen on knolls at various distances from each other, on the shores of Lake Traverse, one of which is known to contain human bones, and is surrounded on every side except one by Indian fortifications; this side is protected from attack by the lake, from whose waters the bank arises almost perpendicularly.
About eighty miles from Fort Wadsworth, on the road to Fort Stevenson, is a hill of natural formation about thirty feet in height, somewhat conical in shape, bearing in the Dakota language the name of Hu-hu Pa-ha, (Bone Hill.) The sides of this hill are paved with bones, of a certain kind, obtained from the legs of buffaloes. Walks leading in different directions to the distance of several hundred feet are pared with the same bones placed end to end and two courses in width. The hill commands an extensive range of vision, and has been used by the Cheyennes as a point of observation.
Indian fortifications resembling rifle-pits are said to be found, first, near this post; second, near Lake Traverse, a short distance from the residence of Major Brown; third, on the Yellow Medicine, near where
the "upper agency" formerly stood. Arrow-heads, muscle-shells, and occasionally implements of bone and stone were formerly found in this locality.

Indian pottery, in addition to being found at this post, is said to be found also on the Coteau du Prairie, about thirty miles from this post.

On a granite rock situated upon a hill about a mile or two distant from the residence of Major Brown are to be seen what is called Wa-kinyan Owe, (the track of thunder,) and regarded by the Indians as a supernatural phenomenon. Two tracks of a bird, as they regard them, are impressed upon the rock, each having three anterior toes and one posterior. The tracks are about six inches long, each line representing a toe, not more than one-eighth of an iuch wide; their origin is clearly. artificial and may be explained on the supposition that centuries ago, with a piece of flint, some member of the Cheyenne Nation has exercised lis talents in engraving the tracks of a bird, in which a calcareous concretion of a different color from the original rock has since been. deposited.

To an elevation or knoll, from forty to sixty feet high, one-quarter of a mile in diameter, arising almost perpendicularly from the southern shore of one of Kettle Lakes, and sloping gradually in every direction into an erosion valley, I have applied the Dakota name of Cega Iyeyapi, (Chāga Eyāyäpee,) a name by which Fort Wadsworth and the surrounding country is familiarly known to the Indians. The term signifies in their language the place where " they found the kettle." The knoll has, probably, been for a long period the favorite campingground of the aborigines. The valley has at one time been a wide and deep ditch, communicating with one of Kettle Lakes and some adjoining sloughs, converting the hill into an island, admirably fortified by nature for defense. On the summit of this knoll was an artificial mound whose base was one hundred feet in diameter, and the perpendicular height of its superior plane, above the surface of the prairie, immediately surrounding it, was from one foot and a half to two feet. The demarkation of the circumference of the base of the mound is somewhat indistinct. At various distances from the surface to the depth of four feet were found alternate strata of clay, and what appears to be a dark vegetable mold, such as is found on the prairie elsewhere. The strata of clay are each about three inches thick, very hard and dry, and contain in their composition a slight admixture of lime, forming a sort of concrete. It would appear from this arrangement of a series of concrete floors that this locality, so admirably situated for defense, has been the favorite camping-ground of one band of aborigines after another, each renovating the locality of the former occupants by covering it with a layer of soil from eight to twelve inches thick, and covering the whole with a new concrete floor. On these floors I found the bones of birds, fish, and various edible animals. The lowest floor is about four feet deep, and is upon the natural clay soil; in this I found a number of hearths, formed by
digging an excavation about a foot deep, and three and a half or four feet in diameter. Upon these were found a quantity of ashes and charred bones, the remains of the feasts of men, and a number of stones from three to six inches in diameter, bearing evidence of exposure to a high degree of heat, and having probably been used for the purpose of boiling water. The granitic sand entering into the composition of the pottery may have been obtained from this source. Intermixed with the soil at various depths I found fragments of pottery of different sizes and patterns. The under surface or most dependent portion of each is incrusted with a white calcareous matter, deposited, no doubt, from the leachings of the soil. The sherds were evidently from some vessels no larger than a small jar or goblet, and from others whose capacity must have been four or five gallons. The color is either that of a cream or Milwaukee brick color, such as clay destitute of iron assumps when burned, or a dim or slate color of various shades; indeed, in some instances it is almost black. The receutly fractured edges of some of the pieces show a uniformity in color throughout the whole thickness; others are of a cream-color one-third of the thickness upon either surface, with a slate-colored streak running through the middle. One of these colors may be seen on the inside of a sherd with its opposite on the outside, and vice versa. I can detect no pigmentary matter upon either surface, and am of opinion that whatever has been used, whether for ornament or service, though probably the latter, has been imparted to the mass of clay prior to molding or baking, and by use has disappeared from the surface, the center retaiuing it; for while I find no black sherds whose fractures show a cream-colored substance within, the converse is true. The black sherds are the least brittle. The thickness of these shersd varies from an eighth to three-eighths of an inch, according to the size of the vessel, though few exceed one-fourth. Sand has been the only substance used to give stiffness to the mass during the process of molding and prevent the ware from cracking while burning, and has probably been obtained from disintegrated stones, some of which were found on the hearths elsewhere spoken of. I have been able to find no whole vessels, but from the fragments of the rims, sides, and bottoms, it is not difficult to form a fair conception of their slape, which, for aboriginal art, was wonderfully symmetrical, gradually widening from its neck or more constricted portion of the vessel until it attains its greatest diameter, at a distance of one-third of the height from the bottom, which is analogous, in curvature, to the crsstal of a watch. To the neck is attached the rim, about one inch in width, though sometimes two; this slopes outward at angle of about twenty degrees from a perpendicular. Of some of the smaller vessels the rim stands perpendicularly upon an offset resting upon the neck. Some patterns have no rim, but a mere lip arises from the neck of the vessel, the whole distance of its circumference, serving as a hand-hold to lift it by. Some small vessels had neither rims nor lips, their shape being spherical. I found no pieces
containing ears or handles, though an Indian informant tells me that small vessels were supplied with ears.
That the aboriginal potters of the lacustrine village of Cega Iyeyapi were fond of decoration, and practiced it in the ceramic art, is shown by the tracings confined to the rims, which consist of very smooth lines about one-twentieth of an inch in width, and as deep, drawn quite around the vessels, parallel to the margin. These are sometimes crossed by zigzag lines, terminating at the neck of the vessel and the margin of the rim. Lines drawn obliquely across the rim of the vessel, and returning so as to form the letter " $V$," with others parallel to the margin of the rim, joining its sides, the same repeated as often as space admits, constitute the only tracings on some vessels. The inside of the vessels is invariably plain.
That the ancient potters failed in the delineatory art, as modern Indians do, may readily be inferred, since no object of nature, such as a tree, a plant, a flower, or bird, has been attempted in their tracings.

To the art of glazing the aborigines seem to have been entire strangers, but they have rendered their ware durable and impervioug to moisture, by thoroughly incorporating throughout its substance a black pigment, which may be driven off by heating the sherds to redness in the bright coals of a common wood-fire. Fragments thus treated assume a yellowish color, and become very porous and brittle.
The neck of the vessels, as well as the rim, shows one uniform curvature, that of a circle, as if molded within a hoop, and is free from those twists and warps sometimes seen in biscuit and common clay ware manufactured by the whites. The outside of the vessels proper, exclusive of the rim-which is traced-bears the impression of very evenly-twisted cords running in a parallel direction and closely crowded together, the alternate swelling and depression of whose strands have left equidistant indentations in every line thas impressed. These lines run, on the sides of the vessels, in a direction perpendicular to the rim, and disappear within a half of an inch or an inch of it, each indentation becoming indistinct near the end. I have counted from ten to fifteen of these casts in the space of a linear inch, and yet some of the sherds represent much finer cords. I find no casts of woven fabric, as of cloth or basket-work, and yet I have seen diamond ( $\circlearrowleft$ ) figures formed near the bottom of the vessel, by the crossing of different layers of cords. A willow or rush fabric could not form such casts; the inside bark of a tree possibly might, but the sinews of the buffalo, such as bow-strings are made of, were most probably used. It would seem, then, that a sack or basket, formed by securing twisted cords, properly adjusted to a hoop, furnished the molds in which the aboriginal potters: shaped and dried their vessels, the external surface of which is a cast of the cords composing the sack.
Eartheu vessels were in use by the Dakotas during the childhood of men still living. I have interrogated separately, and on different occa26 s 71
sions, the principal and most reliable men of the Sissiton and Wahpeton tribes, all of whom tell the samestory of having seen earthen kettles for culinary purposes in use by their parents. They state, however, that the Dakotas never made pottery; but in this, Carver, a traveler who spent a winter among them more than a hundred years ago, contradicts them. Some say it was brought from the Missouri, having.been purchased from the Omahas, others that the Pawnees made it; others that they obtained it as booty from the Mandans, with whom they were constantly at war. In corroboration of this statement, Catlin gives an admirable account of seeing Mandan women make and use pottery when in the country of that nation, in 1832. That the Mandans, a tribe now. residing with the Rees, in permanent lodges, near Fort Bufort, and subsisting partly by agriculture, once possessed the territory around Kettle Lakes, and hence made the pottery, is probable, from the fact that the deepest hearths in the site of the excavation are such as the Mandans construct at the present day. The Cheyennes, about one hundred years since, were dispossessed of the soil by the Dakotas, and the country named Cega Iyeyapi, as previously stated. The legend of the latter tribe ascribed to the former the authorship of the artificial tumuli in this vicinity.

## ANTIQUITIES ON THE CACHE LA POUDRE RIVER, WELD COUNTY, COLORADO TERRITORY.

## By Edward S. Berthoud.

During a casual walk taken by me in July, 1867, along the cretaceous bluffs which extend on Cache La Poudre River for several miles, and while searching for some strata containing fossil-shells of that epoch, my attention was drawn to the beds of gravel and small bowlders which :appear to crown the bluffis and higherslopes. This gravel contains both sedimentary and igneons rocks, is evidently of recent origin, and was probably deposited long since the cretaceous period. We find bere not only rolled pebbles of quartz, felspathic and micaceous granite hornblende rock, sandstone, and ferruginous quartz-rock, but also conglomerate of an older period, both common and moss agates, variegated sandstone, \&c., with sometimes a pebble of hard limestone.

While continuing my examination and searching for moss agates, I found several small accumulations of agate-chips half buried in the soil, or composing a pavement in spots laid bare by the industry of numerous colonies of ants, who seem to be amateurs of all small gay-colored or bright pebbles with which to construct their nests. These chippings appearing in numerous places excited my curiosity, until both myself and companions found in one place two or three arrow-heads made from the coarse agates found there, as well as the oval stone tool which I send with the arrow-head, stone teeth for war-club or saw, and some broken
points spoiled in finishing. It thus appeared evident to me that here must have been either a casual manufactory of such offensive or defensive weapons, or that an old settlement had once here existed. Continuing my search and narrowly examining the ground for a large extent, I found numerous small circles of stones which, although more than half covered with soil and sod, still showed unmistakable signs of design and use. The stones were fire-stained, and frequently fell to pieces, the top coarser when exposed, covered with a tough yellowish-green moss, but frequently so much buried and fixed in the soil and débris, that they were difficult to trace out, and all marked apparently with great antiquity. These vestiges are found over an extent of several acres, and present an appearance of continued occupations. Indeed, one of the arrow-heads has incrusted upon it a sort of calcareousor siliceous cement similar to that found on the large pebbles and bowlders of the gravel formation, and everywhere near them we find flakes and chippings of agate similar to those noticed in England, France, and our Eastern States, and with the arrow-heads of identical pattern of those found from Maine to Georgia, or in our western mounds, the traces of a bygone race who once roamed here before its present Indian population. In future, we expect to continue these examinations and see if we can find vestiges of other larger circles.

## ANTIQUITIES'IN NEW MEXICO.

By W. B. Lyon.
Fort McRae, New Mexico, March 28, 1871.
I returned a week ago from a visit to the of pueblo referred to in a previous letter, although the limited time allowed did not permit me to make any minute explorations of the antiquities. I inclose berewith a ground-plan which is in the main correct.

The pueblo is situated nearly due west and twenty-five miles distant from the town of Socorro, on the Rio Grande. In no place were the walls left over two feet in height, and judging from their character and the amount of débris, I do not think any portion of the building or buildings exceeded one story in height. The material is a soft, coarse-grained sandstone, laid up without mortar or cement, none of the stones being over three inches in thickness. No remains of beams or timber of any kind were found. The walls are eighteen inches in thickness. Numerous fragments of colored pottery-not differing, however, from that now made by the Pueblo Indians-were picked up. In the south end of the court are two circular excavations, respectively forty-seven and twenty-five yards in circumference, and each about ten feet in depth. In the centre of the larger one I found, on digging, the top of a circular stone wall, five feet in diameter. My time did not permit me to make further explorations.

The pueblo occupies a point of land projecting into the valley, and elevated twenty-five or thirty feet above the bottom. The position seems to have been chosen more for its defensive advantages than for convenience. There is a fine spring about one hundred yards to the west, the water disappearing almost immediately after its exit.
Extensive silver mines have recently been discovered in the immediate vicinity, and a town has been laid out near the spring. The miners propose to use the stone from the pueblo for building purposes, but promise to preserve any utensils, or anything of interest they may find, for the Smithsonian. Some of the ore found in these mines is very rich. I think an average ton of the rock will yield over $\$ 100$. Evidences of ancient working of these mines exist in shatts entirely filled up with earth. One of these, on a lode containing a large proportion of copper, has been dug out to the depth of eighteen feet.
Although in close proximity to several cedar-trees, no very large roots penetrate it, and from this circumstance, as well as the extremely hard quality of the wall-rock, I do not believe that the time of working the shaft antedates the occupation of the country by the Spaniards. The ore is very refractory, and can be worked bere only by amalgamation.

A gentleman who has just returned from a trading expedition to the Little Colorado informs me that he discovered, near that stream, a remarkable fortitication, or series of six forts, built of solid masonry, united with cement, each provided with bastion, ditch, etc., and containing in the center a reservoir for water. They occupy the extremity of higl necks of land jutting into the valley, and extend for a mile and a hal along its course. Iu the bottom he found the ruins of towns built of adobes, and traces of large irrigating ditches.
The gentleman brought back with him one very slightly mutilated "olla," or jar, of curious workmanship, which he promised to give me for trausmission to the Smithsonian.

## ANTIQUITIES IN LENOIR COUNTY, NORTH CAROLINA.

By J. Mason Spainhour.

In a conversation with Mr. Michaux, of Burke County, North Carolina, on Indian curiosities, he informed me that there was an Indian mound on his farm, which was formerly of considerable height, but had gradually been plowed down ; that several mounds in the neighborhood had been excavated, and nothing of interest found in them. I asked permission to examine this mound, which was granted, and upon investigation the following interesting facts were revealed. Upon reaching the place I sharpened a stick four or five feet in length, and ran it down in the earth at several places, and finally struck a stone about eighteen inches below the surface, which, upon digging down, was found to be about eighteen inches long and sixteen inches wide, and from two to
three inches in thickness, the corners rounded. It rested on solid earth and had been smoothed on top.

I then made an excavation in the south of the mound, and soon struck another stone, which upon examination proved to be in front of the remains of a human skeleton in a sitting posture; the bones of the fin. gers of the right hand had been resting on the stone. Near the hand was a small stone about five inches long, resembling a tomahawk or Indian hatchet. Upon a further examination, many of the bones were found, though in a very decomposed condition, and upon exposure to the air they soon crumbled to pieces. The heads of the bones, a considerable portion of the skull, jaw-bones, teeth, neck-bones, and the vertebra were in their proper places. Though the weight of the earth above them had driven them down, yet the frame was perfect, and the bones of the head were slightly inclined toward the east. Around the neck were found coarse beads that seemed to be of some substance resembling chalk. A small lump of red paint, about the size of an egg, was found near the right side of this skeleton. From my knowledge of anatomy, the sutures of the skull would indicate the subject to have been twenty-five or twenty-eight years of age. The top of the skull was about twelve inches below the mark of the plow.

I made a further excavation in the west part of this mound and found another skeleton similar to the first, in a sitting posture, facing the last. A stone was on the right, on which the right hand had been resting, and on this was a tomahawk which had been about seven inches in length, broken into two pieces, and much better finished than the first. Beads were also on the neck of this one, but were much smaller and of finer quality than those on the neck of the first; the material, however, seemed to be the same. A much larger amount of paint was found by the side of this than the first. The bones indicated a person of larger frame, and I think of about fifty years of age. Ererything about this one had the appearance of superiority over the first. The top of the skull was about six inches below the mark of the plow.

I continued the examination, and after diligent search found nothing at the north part of the mound but on reaching the east side found another skeleton in the same posture as the others, facing the west. On the right side of this was a stone on which the right hand had been resting, and on the stone was also a tomahawk about eight inches in length, broken into three pieces, much smoother and of finer material than the others. Beads were also found on the neck of this, but much smaller and finer than on those of the others, as well as a large amount of paint. The bones would indicate a person of forty years of age; the top of the skull had been moved by the plow.

There was no appearance of hair discovered; besides, the principal bones were almost entirely decomposed, and crumbled when handled; these two circumstances, coupled with the fact that the farm on which this mound was found was the first settled in that county, the date of the
first deed running back about one hundred and fifty years, (the land still belonging to descendants of the same family that first occupied it,) would prove beyond doubt that it is very old.
The mound was situated due east and west, in size about nine by six feet, the line being distinctly marked by difference in color of the soil. It was dug in rich black loam, and filled with white or yellow sand, but contiguous to the skeleton was a dark-colored earth, and so decidedly different was this from all surrounding in quality and smell, that the lines of the bodies could be readily traced. The decomposed earth, which had been flesh, was similar in odor to that of clotted blood, and would adhere in lumps when enmpressed in the hands.

## acount of the old indian village kushkushkee, near newoastle, pennsylvania.

## By E. M. McConnell.

This Indian village was on the Mahoning River, on the south side of the present, town of Edinburgh, about five miles west of the city of Newcastle, Pennsylvania. It was located on the second bank, on the west side of the river, with a range of high hills to the west, forming an excellent protection from storms. The distance from the base of the hills on the west to the river is about one-third of a mile, making a beantiful valley of several miles both north and south. Christian Frederic Post, a Moravian, was sent on a mission to the Indians at this place by Geueral Forbes, in 1758. He says this village at that time "contained ninety houses and two hundred able warriors." Post, whose business it was, induced the chief, Pakankee, to attend a great conference to be held opposite Fort Duquesne, now Pittsburgh. This is the earliest knowledge we have of Kushkushkee.
Twelve years later, $\mathbf{1 7 7 0}$, at the request of Pakaukee, the Moravians removed from their settlement at Lawunakhannak on the Allegheny River, and settled on the Beaver River, five miles sonth of Newcastle, where they remained for two years, instructing the Indians in the principles of the Christian religion, establishing schools, and introducing agricultural pursuits, \&c. During this time they had intercourse with Indians at Kushkushkee, many of whom became converts to Christianity, among the number Glikkikan, a distinguished orator of the Delaware tribe.
In company with D. Craig, esq., and R. W. Clendenin, I visited the site of this ancient village the past summer to examine carefully its location and surroundings, and learn what I could of the race who inhabited it more than a hundred years ago. When I visited this place, some years ago, the sepulchral mound was in an almost perfect state of preservation, but at this time we found that three-fourths of it had been leveled to the grade of the field surrounding it, which, we
were informed, had been done by the owner of the land, with the expectation of finding some hidden treasure. It is a source of regret to those of us who value these traces of former occupation of our soil that they had not been sacredly protected and preserved. The mound was originally about fifty feet in circumference, and six feet high in the center. We found one human skeleton that had been left exposed, many of the bones being in a perfect state of preservation. This grave had been made on the surface of the ground. Flag-stones broken to the required width had been set on their edges around the body, uniform in height, and covered with flat stones, and then with earth; other bodies had been placed alongside in the same manner, and also on the top of those first interred, and in this way after many years forming the mound as we find it. A few rods south of the mound are about twenty graves of bodies buried separately, the ground over each grave showing a depression of a bout six inches, with a piece of flat stone set at the head and foot of each grave. This may have been adopted under the influence of the teachings of the Moravians as a more Christian form of burial. In examining a field of ten acres or more near the mound, we found a great quantity of flint chippings that had been broken off in making implements, large numbers of which have been gathered up here since the settlement of this valley by the whites.

Mr. James Park, who has lived here for almost seventy years, gave me a stone impletnent somewhat of the shape and size of a carpenter's hatchet, made of the blue-gray stone common in this neighborhood. I have others much the shape and size of wedges used for splitting stone.

## THE PIMA INDIANS OF ARIZONA.

By Captain F. E. Grossmann, U. S. A.
Their history and traditions.-The Pimas have but vague ideas of the doings of their forefathers, and whatever accounts may have been handed down to them have been so changed in the transmission that they cannot be deemed reliable now. Their account of the creation of the world is confused, different parties giving different details thereof. The story most generally accepted among them is that the first of all created beings was a spider, which spun a large web, oat of which, in process of time, the world was formed. They believe that the Supreme Being or Creator took a nerve out of his neck and thereof made a man and a woman. According to their traditions, the first human beings lived near the Salt River, in Arizona Territory, near the McDowell Mountain. These people multiplied rapidly, and soon populated the valleys of the Salt and Gila Rivers. There appears to be a strong probability that the Pima and Papago Indians, who speak the same language, and to all intents belong to the same nation, are the descendants of the earliest occupants of this section of the country. Still the ac-
counts of the two above-named tribes differ materially in many essential points of their early history. Both seem to have heard of a great flood, and each have their own method of explaining how their forefathers were saved from this deluge.

The Pimas relate that the coming of the flood was well known to the eagles, for these birds, soaring among the clouds, saw the gathering of the storm. One of the eagles, friendly disposed toward the Pimas, appeared to the principal prophet of the tribe, and warned him of the approaching disaster, advising him to prepare for it. At the same time a cunning wolf (coyote) conveyed the same caution to another prophet. The former and his followers paid no attention to the counsels of the eagle; while the other prophet, knowing the wolf to be a sagacious animal, at once prepared a boat for himself and made provisions to take with him all kinds of auimals then known. The Papagos claim to be the descendants of the more cautious one, the Pimas of the one who refused to be guided by the eagle. This bird appeared for the second time and repeated his caution, but the Pimas scorned his advice. At last the eagle came for the third time, violently flapped his wings at the door of the hut of the principal prophet, and with a shrill cry announced to him and his people that the flood was at hand, and then flew screaming away. Suddenly the winds arose and the rains descended in torrents, thunder and lightning were terrific, and darkness covered the world. Everything on earth was destroyed by this flood, and all the Pimas perished except one chief, named Sö'-hö, a good and brave Indian, who was saved by a special interposition in his favor by the Great Spirit.
The prophet who listened to and profited by the caution of the wolf, entered his boat, which safely rode through the storm and landed, when the flood subsided, upon the mountain of Santa Rosa. The wolf also escaped by crawling into a large hollow cane, the ends of which he closed with some resinous substance. The Papagos of to day believe that the prophet who saved himself by means of the boat was their forefather, and yearly visit the mountain and village of Santa Rosa, in Arizona Territory, in commemoration of the fortunate escape of the founder of their race. It is also said that a Papago will not kill a wolf. The Pimas, however, claim to be the direct descendants of the chief $S \ddot{o g}^{\prime}-\mathrm{hö}$, above mentioned. The children of $\mathrm{Sö}^{\prime}$-hö re-inhabited the Gila River Valley, and soou the people became numerous. One of the direct descendants of Sö'-hö, King Si'-va-no, erected the Casas Grandes on the Gila River. Here he governed a large empire, before-long before-the Spaniards were known. King Si'-va-no was very rich and powerful, and had many wires, who were known for therr personal beauty and their great skill in making pottery ware and ki'-hos, (baskets which the women carry upon their heads and backs.) The subjects of king Si'-vano lived in a large city near the Casas Grandes, and cultivated the soil for many miles around. They dug immense canals, which carried the
water of the Gila River to their fields, and also produced abundant crops. Their women were virtuous and industrious; they spun the native cotton into garments, made beautiful baskets of the bark of trees, and were particularly skilled in the manufacture of earthen ware. (Remains of the old canals can be seen to this day, and pieces of neatly-painted pottery ware are scattered for miles upon the site of the old city. There are several ruins of ancient buildings here, the best preserved one of which is said to have been the residence of King Si'va-no. This house has been at least four stories high, for even now three stories remain in good preservation, and a portion of the fourth can be seen. The house

田was built square; each story contains five rooms, one in the center, and a room on each of the outer sides of the inner room. This house has been built solidly of clay and cement; not of adobes, but by successive thick layers of mortar, and it was plastered so well that most of the plastering remains to this day, although it must have been exposed to the weather for many years. The roof and the different ceilings have long since fallen, and only short pieces of timber remain in the walls to indicate the place where the rafters were inserted. These rafters are of pine wood, and since there is no kind of. pine growing now within less than fifty miles of the Casas Grandes, this house must either have been built at a time when pine timber could be procured near the building site, or else the builders must have had facilities to transport heavy logs for long distances. It is certain that the house was built before the Pimas knew the use of iron, for many stone hatchets have been found in the ruins, and the ends of the lintels over doors and windows show by their hacked appearance that only blunt tools were used. It also appears that the builders were without trowels, for the marks of the fingers of the workmen or women are plainly visible both in the plastering and in the walls where the former has fallen off. The rooms were about six feet in height, the doors are very narrow and only four feet high, round holes, about eight inches in diameter, answered for windows. Only one entrance from the outside was left by the builders, and some of the outer rooms even had no communication with the room in the center. There are no stairs, and it is believed that the Pimas entered the house from above by means of ladders, as the Zuni Indians still do. The walls are perfectly perpendicular and all angles square.)

The empire of King $\mathrm{Si}^{\prime}$-va-no became so populous after a while that some of its inhabitants found it necessary to emigrate. One of the sons of the king, with numerous followers, went, therefore, to the Salt River Valley, and there established a new empire, which, in course of time, became very prosperous. Indeed, the inhabitants became so wealthy that they wore jewelry and precious stones upon their persons, and finally erected a beautiful throne for the use of their monarch. This throne was manufactured entirely of large blue stones, (probably silver or copper ore.)

In course of time a womari ascended this throne, She was very beautiful, and many of the warriors adored her, but she refused all offers of marriage, and seemed to be fond of no one except a pet eagle which lived in her house. The rejected suitors, jealous of the eagle, determined to kill him, but he, a wise bird, discovered their intentions, said farewell to his mistress, and flew away toward the rising of the sun, threatening destruction to those who had contemplated to take his life.
At the death of the queen, who married after the departure of the eagle, the government of the nation fell to her son, who was but a child in years, and weak and incapable. During the reign of this boy the eagle returned, conducting the Spaniards to his former home. These came, well armed and some mounted on horses, which before this time had been unknown to the Pimas.
The Spaniards approached in three strong columns; one marched down the Gila River, one came from the north, and the third one from the south. These armies of strange white men terrified the Pimas, who, without competent leader and good arms, were soon defeated. The enemy devastated the whole country, killed most of the inhabitants, and leveled their fine buildings to the ground. The throne of the king was broken into small pieces, and the birds of the air came and swallowed the small blue stones, which, afterward, they spit out wherever they happened to be. This, say the Pimas, accounts for the fact that these blue stones are found but rarely and in very different localities now. (Stones of this kind are highly prized by the Pimas, and worn as charms.) But few of the Pimas escaped the general massacre, and hid themselves in the neighboring mountains, whence they returned to the valley after the departure of the Spaniards. They found all their wealth destroyed, their towns in rains, their fields derastated, their friends and relatives slain or carried off by the enemy, and the survivors were in despair. Some few, hoping to be able to liberate some of their kindred who had been captured, followed the white men toward the south and finally settled in Sonora, where their descendants live to this day. The others remained in the Salt River Valley, increased in numbers, and again tilled the soil. But the Apaches, always bitter enemies of the Pimas, took advantage of the situation, and encroached upon their fields to such an extent that the Pimas finally returned to the Gila River Valley, where they still live. They never re-erected the stately mansions of their forefathers, but, humbled by defeat, were content to live in the lowly huts which are occupied by the Pimas of the present day.- Their women were virtuous and strong, and in the lapse of time numerous children were born ; the tribe increased in numbers, and, not many years after their defeat by the Spaniards, the Pimas were strong enough to cope with the Apaches, against whom they have carried on a bitter warfare ever since. At one time they were very poor indeed. Owing to the poverty of the tribe, their leaders never returned to the luxurious style of living of the former kings. They were simply called "chiefs,"
but the supreme control of the tribe was still in the hands of the old royal family, and descended from father to son. These head-chiefs were brave warriors, and under their leadership the Pimas achieved many victories. At one time the Comanche Indians came from the east, but the Pimas repulsed them after a bloody battle, which was fought near the present mail-station Sacaton. At last the reign descended to Shón-tarl-Kör-li, (old soldier,) the last, in a direct line, of the old royal house. He was a bold warrior, and highly esteemed by the whole tribe. During his reign the Maricopa Indians, imposed upon and persecuted by the Yumas and Mohaves, came to the country of the Pimas in two different parties, one from the southwest and the other from the northwest. The new-comers asked a home and protection, promising to aid the Pimas in their scouts against the Apaches. Their request was granted, and when the Yumas, who had given pursuit to the Maricopas, appeared near the country of the Pimas, the latter turned out in force, and, united with the Maricopas, defeated the Yumas in a battle fought near the present Maricopa Wells. Since then the Yumas have not dared to molest the Maricopas. The latter remained with the Pimas, were permitted to cultivate a small portion of their land, and have been ever since on friendly terms with them. The Maricopas of to-day have two villages on the reservation, and number three hundred and eighty-two. The Pimas have intermarried with the Maricopas; still the latter preserve their own language, which is that of the Yumas, Cocopas, and Mohaves. At last Shón-tarl-Kör'li, the chief, was fatally wounded by the Apaches, receiving a musket-ball in his forehead. Upon his death-bed this old chief, who had no sons to succeed him, recommended that Stjö'.e-teck-e-mús, oue of the sub-chiefs, who was a renowned warrior, should be elected head chief. This was done, and Stjö'-e-teck-emús, who was the father of the present head-chief, reigned for years, respected and beloved by all his tribe. Young Antonio Azul, or A-vá-at-Kájo, (the man who lifts his leg,) as he is called by the Pimas, accompanied his father, the chief, on all his scouts when he became old enough to use arms, and at one time went with him to Sonora and visited some of the Mexican towns. Stjö'-e-teck-e-mús led the Pimas many times against the Apaches, was repeatedly wouuded, but finally died in consequence of sickness. Upon his death Antonio Azul assumed the position of his father, but dissension arose in the tribe. Many claimed that Antonio had no title to the supreme command; that his father had been chosen chief on account of his boldness and wisdom; that these virtues did not necessarily descend from father to son, and that the choice of a new chief ought to be left to the warriors of the tribe. Some asserted that a distant relative of the chief proper was among the tribe, who, having the rogal blood in his veins, ought to govern.
Arispa, a petty chief, well known for his bravery in the field, and withal a crafty and unscrupulous man, took adrantage of the general confusion, and, with the intention of usurping Antonio's place, accused
the latter of witchcraft. Antonio was tried and declared not guilty, and since then has been generally recognized as head-chief. Still the followers of Árispa, who are the worst Indians on the reservation, refuse to be guided by Antonio, and the latter evidently believes his position to be insecure, and therefore temporizes with the bad men of the tribe rather than run the risk of a revolution and possible loss of his rank by compelling them to behave themselves. Of course the Indiaus know him thoroughly, and take advantage of his weakness.

Since Antonio Azul has become the head-chief of the tribe the overland road from Texas to California, which passes through the Pima land, has been established, and in consequence thereof these Indians have been thrown in contact with the Americans. In 1859 a reservation, coutaining one hundred square miles, was set aside for them by act of Congress, and upon and near it they have resided ever since. Eight years ago the small-pox raged among them to an alarming extent, and many, particularly children, died of this disease.

It is a lamentable fact that the Pimas have retrograded since the advent of the white men among them, both morally and physically. Fifteen jears ago, when Butterfield's mail-coaches first passed through their land, the Pimas were a healthy race, the men brave and honest, the women chaste. To-day foul diseases prevail to an alarming extent, many of the women are public prostitutes, and all will pilfer whenever opportunity offers.

Religion.-The Pimas believe in the existence of a Supreme Being or Creator, whom they call "Prophet of the Earth," and also in an evil spirit, (che-á-vurl.) They believe that, generally, their spirits will pass to another world when they die, and that there they will meet those who have gone before them. They say that whenever any one dies an owl carries the soul of the departed away, and hence they fear owls, (which they never kill,) and they consider the hooting of this bird a sure omen that some one is about to die. They give a confused account of some priests, (par-le,) who, they state, visited their country years ago and attempted to convert them•to Christianity. These priests were French, and to this day the Pimas call the French "par-le-sick;" plural, "pá-par-le-sick." It does not appear that these missionaries met with success. The Pimas have no form of worship whatever, and have neither idols nor images. They know that the Mexicans baptize their children, and sometimes imitate this ceremony. This baptism is applied, however, only as a charm, and in cases of extreme sickness of the child. When the ceremonies and charms of the native physicians (má-ke) fail to produce a cure, then the sick infant is taken to some American or Mexican, and even Papago when be is known to have embraced the Christian faith. Generally Mexican women perform the ceremony. If the child recovers it receives a Spanish name, by which it is known ever after; but these names are so much changed in pronunciation that strangers would hardly recoguize them. Pedro, for instance, becomes Pí-va-lo; Emanuel, Má-
norl ; Oristobal, Kís-to; Ignazio, I'-nas; Maria, Már-le, etc. It is cer tain that their religion does not teach them morality, nor does it point out a certain mode of conduct. Each Pima, if he troubles himself about his religion, construes it to suit himself, and all care little or nothing for the life hereafter, for their creed neither promises rewards in the future for a life well spent, nor does it threaten punishment after death to those who in this life act badly. They have no priest to counsel them, and the influence of their chiefs is insufficient to restrain those who are evil-disposed. The whole nation lives but for to-day, never thinks of the wants of the future, and is guided solely by desires and passions. They believe in witches and ghosts, and their doctors (már-ke) claim to know how to find and destroy witches. Almost anything is believed to be a witch. Usually it is a small piece of wood, to which is tied a piece of red flannel, cloth, or calico by means of a horse-hair. Should one of these be found in or near one of the Pima huts, the inhabitants thereof would at once abandon it and move elsewhere. They believe that all sickness, death, and misfortunes are caused by witches. If, therefore, a Pima is taken sick, or loses his horse or cow, he sends for one of the medicine-men, whose duty it becomes to find and destroy the evil spirit who has caused the mischief. The medicine-man on these occasions masks his face and disguises himself as much as possible. He then swiftly runs around the spot supposed to be infested, widening his circles as he runs, until, at last, he professes to have found the outer limits of the space of ground supposed to be under the influence of the witch. Then he and his assistants (the latter also masked) drive painted stakes into the ground all about the bewitched spot. These sticks, painted with certain colors found in the mountains, are said to possess the power of preventing the escape of the witch. Now begins the search for the witch; everything is looked into, huts are examined, fences removed, bushes cut down, until, at last, the medicine-man professes to find the witch, which usually is the above-described stick, horse-hair and red cloth. Of course, this so-called witch has been hidden previous to the search, by some of the assistants of the medicine-man. It is burned at once, and the uninitiated fondly believe that, for a time at least, they will be free from the evil influences of the witch thus destroyed. Of course, this mode of treatment seldom produces a cure of sick people, but, the Pimas know nothing whatever of medicines; their medicine-men never administer anything internally, and the above ceremony is the principal attempt made to cure the sick. Sometimes, for instance, in case of pains in the chest or stomach, they scarify the patients with sharp stones or place burning coals upon the skin, and in rare instances the patient is placed upon the ground, his head to the west, and then the medicine-man gently passes a brush, made of eagle feathers, from his head to his feet; after which he runs several paces, shakes the brush violently, and then returns to the patient to repeat, again and again, the same manœuver. They believe that, by this operation, the sickness
is drawn first into the brush and thence shaken to the winds, and bystanders keep a respectful distance for fear of inhaling the disease when it is shaken from the brush. Some doctors pretend to destroy sickness by shooting painted arrows from painted bows at imaginary evil spirits supposed to be hovering in the vicinity of the patient.

The Pimas know many herbs which they use as food at times when wheat is scarce, but they have no knowledge of medical properties of herbs or minerals, with the only exception of a small weed, called colondrina by the Mexicans, which, applied as a poultice, is a certain remedy for the bite of a rattlesnake.
It is believed that all efforts to christianize the Pimas would fail, not because any of them would oppose such attempts, but because they all would be entirely indifferent to the new teachings.

Burial of the dead.-The Pimas tie the bodies of their dead with ropes, passing the latter around the neck and under the knees, and then drawing them tight until the body is doubled up and forced into a sitting position. They dig the grave from four to five feet deep, and perfectly round, (about two feet diameter,) and then hollow out to one side of the bottom of this grave a sort of vault large enough to contain the body. Here the body is deposited, the grave is filled up level with the ground, and poles, trees, or pieces of timber placed upon the grave to protect the remains from the coyotes, (a species of wolf.) Burials usually take place at night without much ceremony. The mourners chant during the burial, but signs of grief are rare. The bodies of their dead are buried, if possible, immediately after death has taken place; and the graves are generally prepared before the patients die. Sometimes sick persons (for whom the graves had already been dug) recovered; in such cases the graves are left open until the persons for whom they were intended die. Open graves of this kind can be seen in several of their burial-grounds. Places of burial are selected some distance from the village, and, if possible, in a grove of mesquite bushes. Immediately after the remains have been buried, the house and personal effects of the deceased are burned, and his horses and cattle killed, the meat being cooked as a repast for the mourners. The nearest relatives of the deceased, as a sign of their sorrow, remain within their village for weeks; and sometimes months, the men cut off about six inches of their long hair, while the women cut their hair quite short. (The Pima men wear their hair very long; many have hair thirty-six inches long, and often braid it in strands; only the front hair is cut straight across, so as to let it reach the eyes. The women, who also cut the front hair like the men, part their hair in the middle, and wear it usually long enough to let it reach a little below the shoulders. The hair is their only head covering. The men are proud of long hair, braid it and comb it with care, and to give it a glossy appearance frequently plaster it over with a mixture of black clay and mesquite gum. This preparation is left on the hair for a day or two and is then
washed out, when it leaves the hair not only black and glossy, but also free from vermin.)

The custom of destroying all the property of the husband wheu he dies impoverishes the widow and children and prevents increase of stock. The women of the tribe, well aware that they will be poor should their husbands die, and that then they will have to provide for their children by their own exertions, do not care to have many children, and infanticide, both before aud after birth, prevails to a very great extent. This is not considered a crime, and old women of the tribe practice it. A widow may marry again after a year's mourning for her first husband; but having children, no man will take her for a wife and thus burden himself with her children. Widows generally cultivate a small piece of gronnd, and friends or relatives (men) generally plow the ground for them.

Marriages.-Marriages among the Pimas are entered into without ceremony, and are never considered as binding. The lover selects a friend, who goes with him to the hat of the parents of the girl and asks the father to give his daughter to his friend. If the parents are satisfied, and the girl makes no objections, the latter at once accompanies her husband to his hut, and remains with him as long as both feel satisfied with the compact. If, however, the girl refuses, the lover retires at once and all negotiations are at an end. Presents are seldom given unless a very old man desires a young bride. Wives frequeutly leave their husbauds and husbands their wives. This act of leaving is all that is necessary to separate them forever, and either party is at liberty to marry some one else, only at the second marriage the assistance of a friend is dispensed with. Instances of fidelity and strong affections are known, but many of the wives do not hesitate to surrender their charms to men other than their husbands, which, though possibly disagreeable to the husband, is not considered a crime by the tribe. Only the worst of the women of the tribe cohabit with the whites, but it is undeniable that the number of such women is increasing from year to year. But, though this has caused a great deal of disease in the tribe, which disease is rapidly spreading, still not one of the chiefs or old men of the nation appears to have thought it necessary to raise a warning voice or propose punishment to the offenders, and prostitutes are looked upou as inevitable, aud are by no means treated with contempt or scorn by the Pimas. Modesty is unknown both to men and women. Their conversation, even in the presence of children, is extremely vulgar, and many of the names of both men and.women are offensive.

Generally several married couples with their children live in one hut, and mauy of the men who can support more than one wife practice polygamy. The wife is the slave of the hasband. She carries wood and water, spins and weaves, has the sole care of the children, and does all the work in the field except plowing and sowing. It is the Pima
woman that, with patient hard labor, winnows the chaff from the wheat and then carries the latter upon her head to the store of the trader, where the husband-who has preceded her on horseback-sells it, spending perhaps all the money received for it in the purchase of articles intended only for his own use. Pima women rarely ride on horseback. The husband always travels mounted, while the wife trudges along on foot, carrying her child or a heavily laden kí-ho (basket) on her head and back. Women, during child-birth, and during the continuance of their menses, retire to a small hut built for this purpose in the vicinity of their own dwelling-place. Men never enter these huts when occupied by women, and the latter while here have separate blankets and eat from dishes used by no one else.

Weapons and manner of fighting.-The only weapons used by the Pimas before the introduction of fire-arms were the bow and arrow and war-club. For defensive purposes they carried a round shield, about two feet in diameter, made of rawhide, which, when thoroughly dry, becomes so hard that an arrow, eren if sent by a powerful enemy at a short distance, cannot penetrate it. These weapons are still used by them to a great extent, and, like all Indians, they are good marksmen with the bow, shooting birds on the wing and fishes while swimming in the shallow waters of the Gila River. For hunting fishes and small game they use arrows without hard points, but the arrows used in battle have sharp, two-edged points made of flint, glass, or iron. When going on a scout against the Apache Indians, their bitter foes, the Pimas frequently dip the points of their arrows into putrid meat, and it is said that a wound caused by such an arrow will never heal, but fester for some days and finally produce death. The war-club is made out of mesquite wood, which is hard and heavy. It is about sixteen inches long, half being handle, and the other half the club proper. With it they strike the enemy on the head. This weapon is even now very much used, for the Pimas rarely attack their enemies in open daylight. They usually surround the Apache rancheria at night, some warriors placing themselves near the doors of all huts ; then the terrible war-cry is sounded, and when the surprised Apaches crawl through the low doors of their huts the war-clubs of the Pimas descend upon their heads with a crushing force. The Pimas never scalp their dead enemies; in fact, no Pima will ever touch an Apache further than is necessary to kill him. Even the act of killing an Apache by means of an arrow is believed to make the Pima unclean whose bow discharged the fatal arrow. They firmly believe that all Apaches are possessed of an eril spirit, and that all who kill them become unclean and remain so until again cleansed by peculiar process of purification. The Pima warrior who has killed an Apache at once separates himself from all his companions, (who are not even permitted to speak to him,) and returns to the vicinity of his home. Here he hides himself in the bushesnear the river-bank, where he remains secluded for sixteen days, conversing with
no one, and only seeing during the whole period of the cleansing process an old woman of his tribe who has been appointed to carry food to him, but who never speaks. During the twenty-four hours immediately following the killing the Pima neither eats nor drinks; after this he partakes of food and water sparingly, but for the whole sixteen days he cannot eat meat of any kind nor salt, nor must he drink anything but riverwater. For the first four days he frequently bathes himself in the river; during the second four days he plasters his hair with a mixture of mesquite gum and black clay, which composition is allowed to dry and become hard upon his head, and is washed out during the night of the eighth day. On the ninth morning he again besmears his head with black clay without the gum; on the evening of the twelfth day he washes his hair, combs it, braids it in long strands, and ties the end with red ribbon or ashawl; and then for four days more frequently washes his whole body in the Gila River. On the evening of the sixteenth day he returns to his village, is met by one of the old men of his tribe who, after the warrior has placed himself at full length upon the ground, bends down, passes some of the saliva in his mouth into that of the warrior, and blows his breath into the nostrils of the latter. The warrior then rises, and now, and not until now, is he again, considered clean ; bis friends approach him and joyfully congratulate himi on his victory.

The Apache Indians, the most savage on the contineut, during the past twenty years have murdered hundreds of whites and Mexicans, and have thas obtained a large supply of fire-arms and ammunition. In order to cope with them successfully the Pimas have purchased many guns and pistols, and are now tolerably well armed with improved weapons. No restriction has ever been placed on the sale of arms and ammunition to these people.
The Pimas never capture Apache men. These are killed on the field, but women and girls and half grown boys are brought back to the reservation at times, though frequently all the inhabitants of the Apache village are killed. ${ }^{-}$
Apache prisoners are rarely treated in a cruel manner. For the first week or two they are compelled to go from village to village and are exhibited with pride and made to join the war-dance. Often, too, the peculiar war-whoop of the Apaches is sounded by some old Piina squaw as a taunt to the prisoners, but after the lapse of a few weeks they are treated kindly, share food and clothing with their captors; and generally become domesticated, learn the Pima language, and remain upon the reservation. Instances have occurred when Apache prisoners have attempted to escape, but they have invariably been overtaken and killed as soon as recaptured. Quite a number of captured Apache children are sold by the Pimas to whites and Mexicans. These children, if properly trained, are said to become very docile and make good house-servants.
In rare instances a Pima will even marry an Apache woman after she 27 s 71
has resided for two or three years on the reservation, but generally fullgrown Apache women become public prostitutes, and their owners appropriate the money received by these women from degraded white men.

Pima industry and food.-The men do not labor except so far as is necessary to enable them to raise a crop. Each village elects two or three old men, who decide everything pertaining to the digging of acequias and making of dams, and who also regulate the time during which each land-owner may use the water of the acequia for irrigating purposes. Each village has constructed years ago an acequia, (irrigating canal.) In order to force the water of the Gila River into their acequias the Pimas dam the river at couvenient spots by means of poles tied together with bark and raw-hide and stakes driven into the bed of the river. Small crevices are filled with bundles of willow-branches, reeds, and a weed called "gatuna." These frail structures rarely stand longer than a year and are often entirely carried away when the river rises suddenly, which occurs in the spring of the jear, if, during the winter, much snow has fallen upon the mountains whence the stream issues, and also sometimes during the sammer after heavy showers. Their acequias are often ten feet deep at the dam, and average from four to six feet in width, and are continued for miles, until finally the water therein is brought on a level with the ground to be cultirated, when the water is led off by means of smaller ditches all through their fields. Having no instruments for surveying or striking of levels, they still display considerable ingenuity in the selection of proper places for the "heads of ditches."
The Pimas and Maricopas have a reservation containing one hundred square miles and extending along the Gila River for a distance of nearly twenty-five miles; only a comparatively small part of this area, however, is available for agricultural purposes, for a portion of the soil on the rescrvation is strongly impregnated with alkali ; some spots are marshy, and all the land beyond the immediate river bottom-land so high above the level of the river that irrigation becomes impracticable, considering the limited means for making acequias at the disposal of the Pimas.

The Indians do not cultivate all the land that might be tilled, for their fields do not average more than from ten to fifteen acres to the family; nevertheless they are dissatisfied with the size of their reservation, asserting that their forefathers had always been in possession of a much larger portion of the Gila Valley, and since the valley above the reservation hạs been settled up by Americans and Mexicans, the Indians have frequently encroached upon the fields of the latter, whom they consider in the light of intruders, and it is apprehended that sooner or later serious difficulties will arise.
The Pima men plow the land with oxen and a crooked stick, as is done by the Mexicans ; they sow the seed and cut the grain; (the latter is done
with short sickles.) Horses thrash the grain by stamping. The women winnow the grain, when thrashed, by pitching it into the air by basketfuls, when the wind carries off the chaff; they convert the wheat into flour, grinding it by hand ou their metates, (a large flat stone upon which the wheat is placed, after having been slightly parched over the fire previously, and whereupon it is ground into coarse flour by rubbing and crushing with another smaller stone.) The principal crop is wheat, of which they sell, when the season is favorable, $1,500,000$ pounds per annum. They also raise corn, barley, beans, pumpkins, squashes, melons, onions, and a small supply of very inferior short cotton.

The diet of the Pimas is very simple; animal food is used only on occasions of ceremony, although they possess large numbers of beefcattle and chickens. They do not use the cow's milk, manufacture neither butter nor cheese, and do not eat the eggs of their hens. Very few will eat pork. But whenever they kill a cow, steer, or calf, they eat every part of it that can possibly be masticated, intestines included. Should an animal die, no matter what the disease, they eat its meat without apparent evil effects upon their health. At times they hunt the rabbit, which is about the only game (quadruped) in their country. Fish, during the months of April and May, are also extensively eaten.

Wheat, corn, beans, and above all, pumpkins and mesquite-beans are their principal food. The latter grow wild in abundance, and millions of pounds are gathered annually by the women of the tribe. These beans are gathered when nearly ripe, then dried hard, and when required as food first pounded in a wooden mortar and then boiled until they become soft. The water is then squeezed out, and the pulpy substance remaining molded into loares, which are baked in the hot ashes. The bread thus obtained has a sweetish taste, is very nourishing, but, being very heaviy, can hardly be easily digested.

The women also collect, in proper season, the fruit of the sawarra, (Columbia cactus,) out of which they manufacture the native whiskey, (called tiswin.) This, after one fermentation, must be used at once, for otherwise it becomes sour. All Pimas are inordinately fond of this beverage, and old and young partake of it until the whole nation are wildly dancing about in a drunken freuzy, until at last they drop to the ground overcome by the stupefying effect of the liquor.

The women also spin and weave a coarse kind of blanket, gather large quanties of hay annually, which are sold to white men, gather and carry all the fuel needed by their family, make the kího, a peculiarly constructed basket carried on the back of the head and shoulders by means of a broad straw strap fitting across the forehead, manufacture, of willows and reeds, superior baskets, which are made so perfect that they will hold water, and finally excel in the manufacture of a coarse kind of pottery-ware, making jugs, dishes, plates, and all their other household utensils.

# Indian mode of making arrow-heads and obtaining fire. 

## Extract of a letter from General George Crook, United States Army.

A great portion of the country east of the Sierra Nevada and Cascade ranges of mountains has quantities of small slivers of obsidian scattered over its surface. The Indians collect these, and by laying their flat side on a blanket, or some other substance that will yield, they will, with the point of a knife, nick off the edges of this to the desired shape with remarkable facility and rapidity, making from fifty to one hundred in an hour. In their primitive state they probably used buckskin or very soft wood instead of the blanket, and a piece of pointed horn or bone for the knife.*
The fire-sticks consist of two pieces. The horizontal stick is generally from one foot to a foot and a half long, a couple or three inches wide, and about one inch thick, of some soft dry wood, frequently the sap of juniper. The upright stick is usually some two feet long, and from a quarter to half an inch in diameter, with the lower end round or elliptical, and of the hardest material they can find. In the sage-bush country it is made of "grease wood."
When they make fire, they lay the first piece in a horizontal position, with the flat side down; and place the round end of the upright near the edge of the other stick; then taking the upright between the hands they give it a swift rotary motion, and as constant use wears a hole in the lower stick, they cut a nick in its outer edge down to a level with the bottom of the hole. The motion of the upright works the ignited powder out of this nick, and it is there caught and applied to a piece of spunk, or some other highly combustible substance, and from this the fire is started.

## ANCIENT MOUND, NEAR LEXINGTON, KENTUCKY.

By Dr. Robert Peter.

The little mound from which the accompanying specimens were taken by Mr. Fisher is on the southern bank of the North Elkhorn Oreek, in a bottom field, about 15 feet above the level of the creek at low water. The field has been cleared of its timber, covered with bluegrass, used as a pasture, trampled by cattle and rooted by hogs, as long as can be recollected by the present owner and neighbors; consequently the mound now presents only a gentle swelling on the level surface of the ground. It is about 70 feet in diameter, and rises in its center only to about $3 \frac{1}{2}$ to 4 feet above the general level. It is situated about half a mile west of the small, ancient, circular ditch, on the bluffs of the C. Shelton Moore place, described in Collins's History of

[^40]Kentucky, published in 1847, (page 294,) and about a quarter of a mile north of the larger ancient work near the dividing line, between the old military surveys of Dandridge and Meredith, described in the same work, of which I shall append a further description. About a mile and a half nearly north of tinis little mound, on the Nutter farm, is a larger mound, apparently about 15 feet higl.
The manuer in which these relies were discovered by Mr. Fisher was as follows: His attention having been drawn to the appearance of fragments of flint arrow-heads and other articles, in a hog-wallow near the center of the little mound, he dug a hole there about $3 \frac{1}{2}$ feet deep and 4 or 5 across, and discovered a bed of ashes about $2 \frac{1}{2}$ feet deep and 4 or 5 feet in diameter, in which the relics I send you were found, together with pieces of charcoal, most of which seems to have been made from simall stems. The copper articles were nearly all together, and a little to the north of the center of the bed of ashes, while the other articles were seattered throughout the same bed, in which were about a peck of flint arrow-heads, all evidently broken by the action of fire. The copper articles were found, according to Mr. Fisher's description, in the following positions: The larger of the adze-shaped edged-tools, or copper axes, was lying with the concave side downward; next immediately above it was the longest of the ornamental articles, the one with one ear broken off, and with the rust scraped off from the other. It was lying crosswise, with the ear next to the broader end of the lower piece. Above these was the second ornamental article, the one having a piece of flint arrow-head attached to it; this was lying with the flint upwart and the horn downmard. It has a fracture in the surface of the rust, on the lower side, corresponding to a piece of the same attached to the top of the charcoal on the adze-shaped article which lay below it; the ear was resting on the broad end of that article. Close to these, and with one hom under the pile described, was the largest article, nearly square in shape, with one horn corled and another broken off about three-fourths of an inch from the body. The smaller broken adze-shaped article was lying on this diagonally. The broken horn was tound near by. There were three hemispherical articles of iron fouml, of which two are sent, and several pieces of sandstone, similar to the coarsest ones sent.
The singular pieces of stone with holes bored throngh them seem to have been fractured by fire. Others, somewhat like these in shape, each with two holes, made of the native sulphate of baryta, which occurs in numerous seams in our limestone rock, are frequently found in this neighborhood on the sarface of the ground. I send two in the box, and a hemispherical piece of the same material. They may be distinguished by their whiteness from those taken from the mound.
It is remarkable that all the fragments of bones found in this mound, in Mr. Fisher's digging, are of the lower animals, and seem mostly to have been worked or carved for useful or ornamental purposes. No
human osseous remains were seen. If this mound was made to cover the dead, the bones have either been entirely destroyed in the lapse of time, or the bodies were laid in the outside circumference of the mound, around the fire, perhaps so that they were beyoud the hole made by Mr. Fisher. This question may, however, be settled by digging a trench across the diameter of the mound.

The copper of which these ax-slraped and ornamental articles are made is doubtless the native metal. I can discover no sign of any inscription or carving upon them. The great length of time during which they have been buried is shown by the couversion of the whole thickness of the copper, in some places one-fourth of an inch thick, as in the little axe, into carbonate and red oxide of copper.

As you will see, the carbonate of copper from the copper pieces has been diffused over the charcoal and other surrounding objects, so as to serve as a cement attaching them firmly together.
It is difficult to imagine the use of the flat square, or oblong square copper articles with the two curved horns at one end. Perhaps they were ornaments to be suspended from the neck! Neither can we tell the object or applications of the stone shaped like the button of a door, with the two bored holes through them.

On October 20,1838 , I made a measurement of this ancient work, partly on the Meredith farm, and I give you the subjoined extract from my notes made at that time:
"This large, nearly circular work is situated on a slight hill, where the corners of the Meredith, Breckinridge, (Dale,) and Moore farms meet near the North Elkhorn Creek. It consists of a ditch, in some places, six feet deep. The earth has been thrown up generally on the outside, but sometimes on the inside, with no raised pathway at present visible.
"This work where the native forest is still left, covered with as large timber as in any part of the surrounding country, and trees, as large and old as any, are growing in the ditch and on the embankment. Measured in a direction north $53^{\circ}$ east, it is 1,138 in diameter. In the direction south $72^{\circ}$ east it is 1,221 feet in diameter. Its circumference, taken by carrying the chain around in the middle of the ditch, is 3,679 feet.
" About 2,100 feet distant from this old circular work, in a northeast direction, on a higher hill or ridge, on the farm of C. Shelton Moore, is a smaller but better preserved work, of somewhat similar construction; the ditch is still very regular, being fully eight feet deep. The circular platform defined by this ditch is on a level with the top of the outside wall, and seems to have been raised above the common surface of the ridge. It has large trees growing on it and on the sides of the ditch. It is perfectly circular, and measures 132 feet in diameter. A raised passway on a level with the platform interrupts the ditch on the northwest side.
"In the hollow between the hills on which these two ancient works are situated is another small ditch, quite shallow, iuclosing a circle of about 82 feet in diameter."
In Collins's History of Kentucky, page 295, you will see it stated that in 1845 an ash tree, supposed to be four handred years old, growing ou the ditch of the larger work, was cut down.

Of course, time and cultivation have altered greatly the appearance of these remains since these descriptions were made, but the plow has not yet entirely obliterated the ditch, even in the places which have been the lougest in cultivation, and frequently flint arrow-heads, and pieces of pottery, etc., are observed on the surface. Once a large deposit of new arrow-heads, made of horn-stone, were plowed up.

## SHELL•HEAP IN GEORGIA.

By D. Brown, of Lambertville, New Jersey.

Your mention of receipts from "shell-heaps" reminds me of perhaps the largest shell-heap in the South, on the island of Osabaw, below Savannah. It had not been disturbed when I saw it, some thirty years ago, and may not yet have been, as the island is not in a traversed route. It is one of the largest of the sea islands, and was probably long ago a royal residence. When the island was assigned by Oglethorpe to one of his companions, Morel, ancestor of my wife, it was occupied by droves of, wild horses and cattle, with various large and small game. When afterward his sons were sent to England for education, peltry and furs from the island were exported to meet their expenses.
If the mound has not yet been disturbed persons curious in such matters might be induced to cause its excavatiou.

## bemarks on an ancient relic of maya sculpture.

By Dr. Arthur Schott.
In presenting to the Smithsonian Institution the accompanying relic of Maya antiquity, the donor wishes to add some remarks, which may be interesting to the ethnological reader.
This specimen was received from Señor D. Juan Manzano, M. D., of Valladolid, a once considerable town of Eastern Yucatan, where it was given to him some years ago, as having been picked up among the famous ruins of Chichen Stzà.

The material of which this little piece of art has been cut is a semiagatized xyolite, still bearing the marks of silicified conif-
erous wood, a fossil probably foreign to the soil of the peninsula. The mask of a human head or skull, which this relic evidently represents, measures 25 millimeters from chin to the top of the forehead, and 22 millimeters across just above the eyes. The vertical facial line is divided into three equal parts, corresponding respectively to the maxillary, nasal, and frontal regions. The space between the eye-sockets measures 7 millimeters, and the facial angle is about 80 degrees of an arc. The distinct employment of geometrical forms by which some of the details of the face are limited, is a prominent feature of the design, and invites particular notice:
Two circles of equal diameters, with their inner peripheries touching each other, form the ocular region. The point where these circles converge is assigned to the root of the nose. A straight horizontal line separating the upper and lower jaw and running right to the centers of four rings of equal size divides these latter into eight half rings, which seem to represent so many teeth, the four upper ones standing directly upon the lower. On each side of the head, and in place of the ears, two holes are bored, one lateral and the other from the back, so as to meet each other almost under a right angle. Over the temples a shallow grooved line runs toward the upper part of the eye-sockets, where it is probably intended to mark more distinctly the prominent cheek-bones.

As a work of art the specimen is much inferior to many others which have been left by the Mayas, for simple linear designs are freely substituted for real plasticity. In other respects, however, it proves a considerable degree of mechanical skill as well in the polish of so hard a material as also in the obvious application of the drill. Still more remarkable and mythologically highly interesting is a certain amount of symbolism plainly expressed in the principal details of this specimen of sculpture. Here the most striking feature is shown in the twice four teeth, for with a race like the American aborigines, so well known as close and faithful observers of natural objects, this deviation from reality can only be taken as an intentional representation of certain numerals ever recurring in their works of sculpture and architecture.
To decipher the special meaning embodied in the present piece must be left to the efforts of professioual mythologists. Suffice it to hint here at the direction in which such researches should be made.
As to the purpose for which this little piece may originally have been intended, it is only conjectured that it was once worn by some person as a badge or amulet, for the double lateral holes seem to have served for passing through strings or fastenings of some kind.

There is another fact connected with the present relic-that is, the high appreciation with which the arts of sculpture and stone-cutting have been considered among the ancient Mayas. They were, indeed, so highly esteemed that their protection had been assigned to a special deity, called "Htubtun." This name is formed from the verb tub, to cut, carve, engrave; and tun, stone or rock. The H prefixed, when used as a
name or a noun, gives it a male character. In the theogony of the Mayas Htubtun seems to have occupied the same position as Plutus did in Greek and Roman mythology, for both were the dispensers of mineral riches, especially metal and precious stones. Whether Htubtun stood in similar relation to some other kindred deity as Plutus was to Pluto, the writer has not been able to learn, though the very design of the present specimen may justify such a supposition.

## ANCIENT ILISTORY OF NORTH AMERICA.

COMMUNICATION TO THE ANTHROPOLOGICAL SOCIETY OF VIENNA, BY DR. M. MUCH.
[Translated for the Smithsonian Institution by Professor C. F. Kroesh.]
The material for the ancient history of America is already so extensive, that I must content myself with a general sketch, briefly touching upon the different views on the origin of the aborigines and their place among the races.

At first it was thought they derived their origin from the Jews, and Englishmen and Americans versed in biblical lore drew largely on the Old Testament for proofs. Soon the Carthagenians and Phcenicians took the place of the Jews, to be displaced in their turn by the Egyptians or Macedonians as the progenitors of the Indians. Finally the blood of Celts and Teutons, and even of Greeks and Romans, was said to flow in their veins. The most plausible reasons were found for such views, from which scarcely a people of any note was excluded.

The report that Greek inscriptions and remains of Roman camps had been found in America, you will, of course, immediately reject as a silly hoax. More lately, extensive remains of Norman settlements were said to have been discovered in the United States, and these were immediately employed to make up a case, with the Norse myths and songs, which unfortunately existed only in the imagination of the discoverer.

Other American scientists, especially Morton, advocated an autochthonous race of America on the sounder basis of comprehensive anthropological studies. But this view is no longer satisfactory, for the impulse to the civilization of Mexico, Central America, and Peru, mysterious as it still is to-day, not only seems to have come from without, but the people themselves seem to have been foreign and not native to the soil. The opinion, advanced a long time ago, that the original inhabitauts of America are of Mongolian extraction, is gaining more and more weight.*

According to Professor Haeckel's genealogy of the twelve races, the Mongolians separated early into three branches-a southeastern or Coreo Japanese, a southwestern or Indo-Chinese, and a northern or Ural-Altaians. These again sent out branches westward, where tbey separated into Tungusians, Samoyedes, Kalmucks, Tartars, Turks,

[^41]Fins, and Magyars. Another branch probably took an easterly direc. tion long before giving rise to the "Arctics," who first peopled Northeastern Asia, and afterward crossed Behring's Straits, and passed into America.

Perhaps the depressing influences of thousands of years had formed a, deteriorated branch in Asia, the descendants of which are still represented by the Esquimaux in the extreme north of America, while a southern and more vigorous branch chose the more temperate parts of North America, and spread in the course of time over the whole continent. In the extreme south this race was again modified by depressing natural influences similar to those which operated in the north.

The aborigines of America differ, as we all know, in their languages, and are divided into tribes; but the type of these tribes and the organic structure of their languages are essentially the same. Only the Esquimaux differ from the general type, but their language is intimately related to those of their southern neighbors. According to this view, the wave of Indian population, which in the old world advanced from east to west, must have taken a direction from north to south in the new; it is confirmed, indeed, by historical and mythical traditions, as well as by the character of the remnants of civilization found as we advance from north to south.

Greater or less portions of the population, especially in Mexico and Central America, seem, however, to have been in constant motion. This mobility is the attribute of a nation of hunters, who drive the existing population before them. Again, the migratory impulse, so to speak, seems to belong to a certain period in the development of a people. It is exemplified in our own Teutonic ancestors, whose impetuous advance not only caused the downfall of the Roman Empire of a thousand yeari' standing, but also involved the entire population of Europe in its motion.

Passing to the relics of American civilization, it must be stated in advance that the determination of their age, their order, and their whole history is as yet much more difficult than that of European remains. And this for two reasons: First, because of the very gradual development of civilization. The form and material of utensils and weapons remain the same during long intervals, and sometimes up to the historical period, whence it happens that remains, differing in age, perhaps, by thousands of years, can hardly be distinguished. Secondly, certain characteristic periods in the development of civilization, such as the appearance of metallic utensils in Europe, by which a classification might otherwise be effected, are wanting. Metals, especially copper, were long used in America; they are found in the most ancient deposits, while they are absent in the more recent; but the use of copper is no proof of a more advanced civilization in America, since it was for the most part employed in as rough a state as that of stone. Pieces of copper were broken off from the native blocks by means of
stone hatchets, and fashioned with the hammer. The natives evidently had no idea of its fusibility. For this reason, the use of the metal does not indicate greater progress, and we are thus deprived of a means of classification.
In Mexico, Central America, and Peru, it was different, however. There we have evidence of a high degree of skill in the working of metals (iron being almost the only exception) in the more recent period. They had advanced beyond the mere hammeriug of pieces of metal found by accident, and understood smelting, and eren attempted to obtain metals by miniug for ores. The American remains were, therefore, arranged according to the places where they were found, or the purposes for which they were intended. But to keep in view the progress of development, I have taken the liberty of adopting the following arrangement: I would assume a period immediately preceding the advent of the Luropeans in America, and continuing for a short time after. This would correspond to our historical age, and may be designated, in a restricted seuse, as the historic period.
A secoml epoch would include a time far removed even from the recollection of the inhabitants at the time of Columbus, and characterized by a different distribation of the population and other complete revolutions. To this period belong the great mounds, particularly those of the Ohio Valley. It might be called the mound period, and corresponds to the advanced portion of the age of stone, and the begiming of the age of brouze in Europe.
The third and most ancient period would then include those discoreries which point to the co-existence of man with extiuct species of animals. It corresponds to the age of the mammoth and the reindeer in Europe, and might be called the diluvial period.

Utensils of all kiuds, and buildings or mounds, belong to the two more recent periods. The buildings of the first or historic period are fonud chiefly in the eastern parts of the United States and Canada, in Mexico, and CentralAncrica. In the United States, Canala, and farther north, they consist of mounds and bulwarks.
The mounds of the first period are places of interment, and correspond precisely to the tumuli in Europe. They were probably used for the burial of chiefs, since they contain for the most part one or only a few skeletons. Sometimes, horrever, heaps of bodies or their skeletons are piled up, and covered with a knoll of earth. Whether these are the bodies of Indians fallen in battle, or of the victims of immense sacrifices, remains undecided. They are on an average 5 feet high, with a base 25 feet in circunference; but there are some as ligh as 15 feet, and having a circumference of 60 feet. That the Indians, even during the time of their first intercourse with the Europeans, erected such hillocks as graves for distinguished chiefs, or to commemorate important events, has been proved in several cases.

The works of defense consist of walls of earth, rarely of stone,
furnished in each case with palisades. They are for the most part near rivers and brooks, always near water, and especially at places surrounder on more than one side by water, on elevated ground, defended on one or more sides by natural strength of position.

To the age which, in America, corresponds to our historical period, belong also the remnants of those grand structures, those wonderful ruins of palaces, temples, and cities, which, even at the present day, bear witness of the high degree of civilization of their builders in numerous localities of Mexico, Yucatan, and Central America. Although they are almost destroyed, and covered with luxurious vegetation, these remains afford a wealth of scientific material, but I must content myself with merely naming them.

The characteristic structures of the second period are the mounds, and the period itself is the period of the mound-builders. These mounds are of three kinds, for burial, sacrifice, and worship, and occur in the whole Mississippi Valley, but most frequently in the Ohio Valley, in the vicinity of Chillicothe. The burial-mounds correspond to those of the Atlantic States, but are generally larger. Many are as high as 60 feet. They indicate a greater antiquity, by the more advanced stage of decomposition of the contained skeletous. Sometimes the bodies were burned and their ashes deposited in urns. Weapons, ornaments, and utensils are always found in them, but remvants of food occur only in the more recent. Signs of fire and animal bones, probably remnants of sacrifices or of "wakes," are often found under the top surface of these mounds. Sometimes the chiefs of a later period were buried in the old mounds, and in such cases the well-preserved skeleton of the new-comer is found above the crumbling one of the older. An interesting case in point came to light in December, 1870, when a mound near Saint Louis, Missouri, was opened by a scientific commission. It was 40 feet high and 300 feet long. Twenty years ago a dwelling-house was built on it and a cemetery instituted beside it. On digging, the bones of three different races were successively brought to light; first, those of white men; in the center, those of Indians of the present day; and below, those of the ancient mound-builders, who lived there before the Indians that possessed the land at the time of the white man's arrival. Their bones were deposited in two large stone chambers.

The second class of the older earth-mounds consists of those used for sacrifice. They are only a foot or two high. A small depression at the top bears evidences of burnt sacrifice on the hardened clay; and the ashes often contain objects of various kinds placed there to propitiate their deity or to atone for their misdeeds. These pbjects are almost without exception broken, and have suffered from fire and the effects of time.

The third class is that of the temple or palace mounds, the most important of all. They have generally the shape of truncated four-sided pyramids, with terraces, steps, and dam-like elevations, which are often
interrupted by smaller mounds. Their dimensions are enormous. Some are as high as 90 feet, and have a length of from 500 to 700 feet at the base. The upper surface of the great pyramid in Washington County, Missouri, contains 12,000 square feet. It is the largest of a group of eleven of such mounds. These mounds are either found alone or in groups; some are surrounded by earth-walls and others are not. Besides those of the Mississippi Valley, similar large earth-pyramids are found in the Colorado Valley, where they are considered as Aztec structures. They have unmistakable signs of former buildings upon them. Probably these earthworks had no other object than to serve as elevated bases for temples and the houses of chiefs and priests. These buildings must have been formed of lighter material, for they have entirely disappeared. Nevertheless they remind us of sinilar but more perfectly executed buildings of a later time in Mexico and Central America. All investigators agree that their builders belonged to a much higher civilization than those of the smaller grave-mounds in the east, or the Indians of the present day. It is said that the utensils front these mounds are worked with much more skill, and that some among them justified the conclusion that the builders followed agricultural pursuits. Another remarkable circumstance is, that now and then copper utensils were found in the possession of the Indians on the Atlantic coast. These, however, can only have been such as they found among the remains of the more ancient race; since investigations of the Lake Superior copper region prove that the knowledge of making use of these copper-ore deposits had already been lost at the time when the Europeans took possession of America. Indeed, copper utensils are found only in the earth-works of the older, but not in the mounds of the more recent period.
The mere presence of these large earth-works, however, with their inclosures or bulwarks, is sufficient proof of a more highly developed people, who were no longer nomadic. I cannot help thinking that the Mississippi Valley may have been at one time the lome of the Aztecs and Toltecs, who there erected, so to speak, the first crude models of their later wonderful structures, and then moved southward from unknown causes, carrying with them their higher civilization, and developing it still further in their new homes; while the inferior race, which took possession of their abandoned dwellings, remained without knowledge of the rich ore deposits.
There are also earth-works of another kind, similar to those in the Atlantic States, which doubtless served as fortifications. Some probably were inclosures of small villages; for they are usually found near single or around whole groups of mounds, and have the ditch on the inner side. They frequently inclose large areas, but not a trace is left of the dwellings, which may have been within.

A very peculiar species of earth-works are in the shape of men or various animals, the outlines of which they represent. Perhaps these partook of a religious or national character, some of the tribes being named after certain animals.

In the most recent period, there is an, enormous difference in the nature of the utensils employed by the northern and southern peoples. This difference is due to the use of metals. In the north are found almost exclusively utensils of stone, while in the south very fine utensils of copper, bronze, gold, silver, \&c., occur besides. If the report is true that arrow-heads of iron were found in possession of the inhabitants of some parts of South America, these can only have been made of meteoric iron.
The utensils occur in the same mannere as in Europe. They are found in the tomb-mounds, where they were deposited with the dead; or in the altar-mounds, where they were bronght as a sacrifice, or rather a gift of propitiation to their deity. In the latter case they are usually broken to pieces, probably on purpose, injured by fire, and mixed with the ashes of the victims. They are frequently brought to light by the plow or by violent rains, which wash away the soil, and lay bare the heavier stone utensils. The distribution of settlements is also similar; often considerable regions are without any, while they are very numerous in more favorable localities. In North Americia, they are most frequent in valleýs, where they are recognized by an abundance of fragments of vessels on the surface of the soil.

Sometimes earth-lieaps similar to the Danish Kjökkenmöddings indicate the spots where those old settlements stood. They have been lately iuvestigated in several cases by Wyman, Morse, and our indefatigable countryman, Charles Rau. Their appearance is the same as in Europe, with the difference, of course, that the animal remains belong to different species. Among the masses of broken shells, they contain more or less numerous utensils of stone and bone along with potsherds. They occur along the whole Atlantic coast. Near Keyport, New Jersey, on an island north of Du Français Inlet, at Crouch's Cove, Goose Islaud, in Casco Bay, Eagle Hill, at Ipswich, Massachusetts, Long Island, and the mouth of the Altamahar River, in Georgia. Traces are also found along the coasts of Massachusetts, Newfoundland, Nova Scotia, Florida, and California. A portion of the city of New York is said to be built upon such deposits. To what period they belong, or whether they belong to different periods, has not yet been determined. Finally, we must mention the relics of human civilization found by the German North Pole Expedition in Greenland, and brought home by it from the abandoned huts of the Esquimaux. They probably belong to a comparatively recent time.

The tools, weapons, vessels, and ornaments of the inhabitants of America probably remained unchanged for very long periods of time. Only the Mexicans made considerable progress in the latest period; but we know that even they had not yet given up their knives of obsidian, although they might have made them of bronze. Montezuma himself wielded the terrible Mexican sword, the edge of which was composed of pieces of obsidian, and you can even to-day admire his stone battleax in the Ambras collection.

The objects found in the North are chiefly arrow-heads, as might be expected in the case of a people of hunters and warriors. In the collection before you, there are specimens of the various shapes, some scarcely an inch long and having a rounded point, whileothers are more than three inches in length. Precisely similar in shape and material, (the latter being pure quartz, flint, chalcedony, jasper, rock-crystal, ) only larger, are the lance-heads. The royal mineralogical cabinet is in possession of a magnificent arrow-head of pure rock-crystal, evidently of American origin. It is remerkable that many lance and arrow heads slant unequally on the two edges, so that the arrow or lance would assume a rotary motion on being discharged.
The knives were also made of flint and obsidian by breaking them off from suitalle blocks by means of a single blow. They differ in no way from the European. The Indian wedges are also like those found in Europe, a circumstance that need not surprise us in an instrument of so primitive a nature. The specimen before you, with its rounded sides, was taken directly in the hand, and used to skin larger animals.
The hatchets, of which three specimens are before you, are of a shape peculiar to America. They are provided with a deep groove under the neck running around the sides, into which was fitted a forked branch forming the handle. From their frequent occurrence we conclude'that they were the most usual weapon, which was later and only gradually supplanted by the iron tomahawk. Hammers with holes to receive the handle are rare.

Among the other stone instruments, the grindstones differ also from the European, being of the shape of stones used for rubbing up colors. Larger disks, concave on both sides, were probably used in games, and smaller ones of various shapes, and pierced with holes, may have served as ornaments. The oval stone before you, with a groove running all around it, may either have been a piece of ornament, or a sinker for a net.

Which shapes and which material belong to the earlier, and which to the later times, will probably be determined only after long researches. Dr. Dickerson, of Philadelphia, claims an age of three thousand five hundred years for these arrow-heads, which were found in one of the Mississippi States. Among them you perceive half-finished and spoiled pieces. Those made of quartz correspond in shape with the iron arrows of the present day, of which you also have a specimen before you; they are, therefore, very likely the more recent. Among the metallic atensils, we must first mention the copper hatchet, an imitation of the stone wedge attached to a club like the Celtic ax, a chisel, and lance-heads. Among the ornaments are perforated copper plates, concave disks, objects reserobling buttons, small round disks of thin copper plate, or wire for stringing on a thread, like pearls. The copper was doubtless taken to Central America from the Lake Superior copper region.

The inhabitants of Mexico and Central America had made great prog-
ress in the working of gold, silver, eopper, and tin. They made not only weapons and ornaments of metal, but also vessels showing a high degree of skill. They alloyed copper and tin, and manufactured bronzed utensils, to which they imparted considerable hardness by hammering. But the arrow-heads and knives of obsidian remained in use at the same time; the latter probably in consequence of their being used, in the terrible human sacrifices, to open the breast of the victim and cut ont the heart. Immense numbers of such obsidian knives, as well as arrowheads and chips, are still found in various localities. A mountain distinguished for the large number of these objects is still called "the mountain of knives." The inhabitants of the Mississippi Valley obtained the obsidian arrow-heads from Mexico in exchange for other articles.
The pipes are peculiar to America. They are called mound-pipes, on account of their being found almost exclusively in the altar-mounds. The Indians were in all probability the first smokers, and so great was the esteem in which they held the enjoyment derived from it that they devoted more labor and skill upon their pipes than upon their weapons. The pipes are of stone, with a base in some cases 5 inches long, one end of which forms the stem. The bowls are in the center of the base and are about 1 or $1 \frac{1}{2}$ inches high. These bowls are in most cases fashioned in imitation of human heads, with all the characteristics of the Indian race upon them, or various animals, which are so faithful that they can be recognized at once; a fact which is the more surprising, since the pipes are fashioned of a single piece of very hard stone. The pipes of baked clay found in New York and elsewhere seem to belong to a later period.
The Indians of to-day also devote considerable attention to the adornment of their pipes. Many are cut from the red pipe-clay of the West, which was discovered by the celebrated artist and ethnographer Catlin. The beds of this clay were considered as on neutral ground by the Indians.
Among the other objects, which I have only time to name, are needles and bodkins of bone and horn, pearls of bone and of various shells, genuine pearls, perforated claws of eagles and bears, teeth of wild-cats and of the shark, perforated bits of mica, and the like. The vessels of clay, however, require a more detailed consideration. They also show some resemblance to the products of the corresponding era in Europe. They were all fashioned without the potter's wheel; in many cases baskets of willow or rushes served as models. They were corered inside with clay and placed with it in the fire. Thus the wickerwork left its impression on the outside of these vessels. This method, according to Catlin, was still practiced in the present century. In some southern localities pumpkins were covered with clay on the outside, and the whole placed in the fire.

A great number of the vessels, like the older European ones, had a round bottom, and could only be used for hanging up by means of a
projecting edge. Their forms are as various as their dimensions. The material consists of a black clay, mixed, as in Europe, with quartz sand, or, as is the case more frequently, with more or less finely pulverized shells. Sometimes the clay is used without any admixture. In the West Indies the pulverized bark of two trees, Hirtella silicea and Moquilca, is used in the place of sand. This bark is rery rich in silica, and produces vessels of a very fine grain, fragments of which are found in large numbers in all settlements, and especially at the places of manufacture. One of the latter was discovered and described some years ago by Charles Rau.
That the Mexican porcelain vessels should show a higher order of skill, might be expected after what has been said. A Portuguese writer, during the first years of the Spanish rule, declared that they were in no way inferior to those of Europe.
Although it is not denied that there is no reason for distinguishing in America between a palæolithic and a neolithic period, there are, nevertheless, authenticated facts which might lead to such a distinction. One of these is the discovery of human bones together with those of extinct species of animals near Natchez, Mississippi. Another is the discovery of a human skeleton under several layers of submerged forest formatiou in the Mississippi delta, near New Orleans. Still auother is the presence of human bones in a limestone conglomerate forming a part of the coral reefs of Florida, whose age is estimated at ten thousand years by Agassiz.

Unfortunately there has been so much exaggeration in America, along with trustworthy reports, that caution is necessary in accepting as true unusual statements, even when they have a scientific coloring.

From the report of the German archæologist, Dr. Koch, on the mastodonn* found in Gasconade County, Missouri, it is beyond doubt that man existed in America as early as that animal. In another case flint arrow-heads were found along with bones of the mastodou in an undisturbed deposit; and at the Pomme de Terre River, Missouri, a mastodon skeleton, together with an arrow-point, as found covered with 15 feet of alluvium.

Finally, I must state that there is scarcely a subject which excites the interest of American scientific men so much as the ancient history of their continent. Let me call your attention to the liberal support which they enjoy, the existing collections in every large city of America, the efforts of the Smithsonian Institution, and the donation of the great philanthropist, Peabody, who appropriated $£ 100,000$ sterling to the establishment of a museum of Indian antiquities.
The greatest collection of American antiquities in Europe is that at Salisbury, England ; and in America that of the Smithsonian Institution. Dr. Dickerson has also a very large collection of which he is about to publish a catalogue.

[^42]
## ON THE LANGUAGE OF THE DAKOTA OR SIOUX INDIANS.

By F. L. O. Rehrig.

In the year 1866 the writer of this article spent the interval from the 4th of July to the 26th of November in constant intercourse with the Dakota or Sioux Indians, near Fort Wadsworth, Northern Dakota Territory.

Previously to his going to that out-of-the-way region he had happened to make himself in some measure acquainted with the languages of several of the Indian tribes, particularly with the Chippewa tongue ; and he then at once directed his attention to the language of those Indians in whose immediate neighborhood he was going to reside for a while, namely, the Sioux Nation, or Dakotas.

It would take a whole volume to record his varied experience with those interesting tribes and the result of his ethnological and linguistic researches during the time he lived among them. On this occasion, however, he will content himself with presenting to the reader only a very few faint and cursory glimpses of merely such matters as may arise in his recollection, and as pertain to the language of these people. It is hoped that his elucidation of desultory topics of this nature will not prove altogether uninteresting to the ethnologist or philological inquirer.

Whenever any new truth is presented for our comprehension, or any new subject for our study and investigation, almost invariably the first thing for the human mind to do, and that, too, from an inherent craving for logical classification, is to inquire as to what other known truth the less known can possibly be linked; to what chain or series of analogous phenomena it necessarily belongs; in what accredited system it has to take its place; with what whole or totality it is connected as a part; and we seem never to be fairly at ease before we have arrived at ${ }^{\text {. }}$ the point of grouping or classifying the matter in some way or other. This applies also and particularly to languages. As soon as a new language begins to attract our attention, we feel at once an eager desire to classify it, so much so that we often cannot patiently wait even during the time necessary to collect the indispensable material from which alone we could possibly draw any legitimate conclusions in this respect. We at once ask what other tongue such language is like; with what other it may be compared; where among the languages of the world it has to take its place, \&e., and hence the often over-hasty classifications based upon mere casual and apparent resemblances. It is first of all necessary, in such cases, to be able fairly to survey a language in all
its relations; in its manifold diversities, its dialects, and, if possible, also in its various and successive phases of development, in its primary forms or its original condition.
So far as we know, the Dakota language, with several cognate tongues, constitutes a separate class or family among American Indian languages, of which we may speak on some other occasion. But the question at present is, whence does the Dakota, with its related American tongues, come? From what trunk or parent stock is it derived? Ethnologists are wont to point us to Asia as the most probable source of the prehistorical immigration from the Old World to this continent. Hence, they say, many if not all of our Indians must have come from Eastern or Middle Asia, and in considering their respective tongues, one must still find somewhere in that region some cognate, though perhaps very remotely related set of languages, however much the affinity existing between the Indian tongues and these may have gradually become obscured, and in how many instances soever, through a succession of ages, the old family features may have been impaired. But they further allow, of course, that these changes may have taken place to such an extent that this affinity cannot be easily recognized, and may be much, even altogether, obliterated.
When we consider the languages of the great Asiatic continent, of its upper and eastern portions more particularly, with a view of discovering any remaining trace, however faint, of analogy with or similarity to the Dakota tongue, what do we find? Very little; and the only group of Asiatic languages in which we could possibly fancy we perceived any kind of dim and vague resemblance, an occasional analogy or other perhaps merely casual coincidence with the Sioux or Dakota tongue, would probably be the so-called "Ural-Altaic" family. This group embraces a very wide range, and is found scattered in manifold ramifications through parts of Eastern, Northern, and Middle Asia, extending in some of its more remote branches even to the heart of Europe, where the Hungarian and the numerous tongues of the farspread Finnish tribes offer still the same characteristics, and an unmistakable impress of the old Ural-Altaic relationship.
In the following pages we shall present some isolated glimpses of such resemblances, analogies, \&c., with the Sioux language as strike us, though we need not repeat that no conclusions whaterer can be drawn from them regarding any affinity, ever so remote, between the UralAltaic languages and the Dakota tongue. This much, however, may perhaps be admitted from what we have to say, that at least an Asiatic origin of the Sioux or Dakota Nation and their language may not be altogether an impossibility.

In the first place, we find that as in those Ural-Altaic languages, so in a like manner in the Sioux or Dakota tongue, there exists that remarkable syntactical structure of sentences which we might call a constant inversion of the mode and order in which we are accustomed to
think. Thas, more or lesis, the people who speak those languages would begin sentences or periods where we end ours, so that our thoughts would really appear in their mind as inverted.

Those Asiatic languages have, noreover, no prepositions, but only postpositions. So likewise has the Daku'ra tongue.

The polysynthetic arrangement which prevails throughout the majority of the American Indian languages is less promiment, and decidedly less intricate in the Dakota tongue than in those of the other tribes of this continent. But it may be safely asserted that the above-mentioned languages of Asia also contain at least a similar polysynthetic tendency, though merely in an incipient state, a rudimental or partially developed form. Thus, for instance, all the various modifications which the fundamental meaning of a verb has to undergo, such as passive condition, causation, reflexive action, mutuality, and the like, are embodied in the verb itself by means of interposition, or a sort of intercalation of certain characteristic syllables between the root and the grammatical endings of such verb, whereby a long-continued and united series, or catenation, is often obtained, forming apparently one huge word. However, to elucidate this any further here would evidently lead us too far away from our present subject and purpose. We only add that postpositions, pronouns, as well as the interrogative particle, \&c., are also commonly blended into one with the nouns, by being inserted one after the other, where several such expressions occur, in the manner alluded to, the whole being closed by the grammatical terminations, so as often to form words of considerable leugth. ${ }^{1}$ May we not feel authorized to infer from this some sort of approach, in however feeble a degree, of those Asiatic languages-through this principle of catena-tion-to the general polysynthetic system of the American tongues?

We now proceed to a singular phenomenon, which we should like to describe technically as a sort of "reduplicatio intensitiva." It exists in the Mongolian and Turco-Tartar branches of the Ural-Altaic group, and some vestiges of it we found, to our great surprise, also in the language of our Sioux Indians.

This reduplication is in the above-mentioned Asiatic languages applied particularly to adjectives denoting color and external qualities, and it is just the same in the Daкota language. It consists in prefixing to any given word its first syllable in the shape of a reduplication, this syllable thus occurring twice-often adding to it (as the case may be) a " $p$," \&c.

The object-at least in the Asiatic languages alluded to-is to express therebs, in many cases, a higher degree or increase of the quality. An example or two will make it clear. Thus we have, for instance, in Mongolian, khara, which means black, and KHAp-lkhara with the meaning of very black, entirely black; tsagan, white, TSAP-tsagan, entirely white, \&c., and in the Turkish and the so-called Tartar (Tatar) dialects of Asiatic

Russia, kara, black, and Kap-kara, very black; sary, yellow, and sapsary, entirely yellow, \&c.
Now, in Daкотa, we find sapa, black, and with the reduplication, sapsapa. The reduplication here is, indred, a reduplication of the syllable $s a$, and not of sap, the word being $s a-p a$, and not sap-a. The " $p$ " in sapsapa isinserted after the reduplication of the first syllable, just as we have seen in the above kara and кар-kara, \&c.
In the Ural-Altaic languages " $m$ " also is sometimes inserted after the first syllable; for instance, in the Turkish beyaz, white, and BEm-beyaz, very vohite, \&c. If we find, however, similar instances in the Daкотa language, such as ćepa, ${ }^{2}$ which mèans fleshy, (one of the external qualities to which this rule applies,) and ćem-ćepa, \&cc., we must consider that the letter " $m$ " is in such cases merely a contraction, and replaces, moreover, another labial letter (" $p$ ") followed by a vowel, particularly "a." Thus, for instance, ćom is a contraction for copa, gam for gapa, hiam for liapa, skem for skepa, om for opa, tom for topa, \&c. So is ćem, in our example, only an abridged form of cepa; hence " $m$ " stands here for " $p$ " or " $p a$," and belongs essentially to the word itself, while in those Asiatic languages the " $m$ " is added to the reduplication of the first syllable, like the " $p$ " in кap-kara, \&c. We hare, therefore, to be very careful in our conclusions.
The simple doubling of the first syllable is also of frequent occurrence in Dakota; for instance, g̀i, brown, and g̀iǧi, (same meaning;) sni, cold, and snisni; ko, quick, and koko, \&cc.

There are also some very interesting examples to be found in the Daкота language, which strikingly remind us of a remarkable peculiarity frequently met with in the Asiatic languages above adverted to. It consists in the antagonism in form, as well as in meaning, of certain words, according to the nature of their voucels; so that when such words contain what we may call the strong, full, or hard vowels, viz: $a, o, u$, (in the continental pronunciation,) they generally denote strength, the male sex, affirmation, distance, \&cc., while the same words with the weak or soft vowels $e, i$,-the consonantal sketeton, frame, or ground-work of the word remaining the same,--express weakness, the feriale sex, negation, proximity, and a whole series of corresponding ideas.
A few examples will demonstrate this. Thus, for instance, the idea of "father" is expressed in Mantchoo (one of the Ural-Altaic languages) by $a \mathrm{ma}$, while "mother" is eme. ${ }^{3}$ This gives, no doubt, but a very incomplete idea of that peculiarity, but it will, perhaps, be sufficient to explain in a measure what we found analogous in the Daкотa language. Instances of the kind are certainly of rare occurrence in the latter, and we will content ourselves with giving here only a very few examples, in which the above difference of signification is seen to exist, though the significance of the respective vowels seems to be just the reverse; which would in no wise invalidate the truth of the preceding statement, since
the same inconsistent alteration or auomaly frequently takes place also in. the family of Ural-Altaic languages. [For further developments, see the Notes at the end of this article.]
Thus we find in the Dakota or Sioux language, hepan, (second son of a family, and hapan, (second daughter of a family;) éIn, elder brother, évn, elder sister ; ${ }^{4}$ ćınksi, son, ćUnksi, daughter, \&c. Also, the demonstratives kon, that, and kin, this, the, (the definite articles,) seem to come, in some respects, under this head.
To investigate the grammatical structure of languages from a comparative point of view is, however, but oue part of the work of the philologist; the other equally essential part consists in the study of the words themselves, the very material of which languages are made. We do not, as jet, intend to touch on the question of Dakota words and their possible affinities, but reserve all that pertains to comparative etymology for some other time. The identity of words in different languages, or simply their affinity, may be either immediately recognized, or rendered evident by a regular process of philological reasoning, especially when such words appear, as it were, disguised, in consequence of certain alterations due to time and to various vicissitudes, whereby either the original vowels, or the consionants, or both, have become changed. Then, also, it frequently happens that one and the same word, when compared in cognate languages, may appear as different parts of speech, so that in one of them it may exist as a noun, and in another only as a verb, \&c. Moreover, the same word may have become gradually modified in its original meaning, so that it denotes, for instance, in one of the cognate languages, the genus, and in another, merely the species of the same thing or idea. Or it may also happen that when several synonymous expressions originally existed in what we may call a mother language, they have become so scattered in their descent that only one of these words is found in a certain one of the derived languages; while others again belong to other cognate tongues, or even their dialects, exclusively.

The foregoing is sufficient to account for the frequent failures in establishing the relationship of certain languages in regard to the affinity of all their words.
Ou this occasion it.will be enough to mention, in passing, as it were, one or two of the most frequently used words, such as the names of father, mother, \&c.

In regard to these most familiar expressions, we again find a surprising coincidence between the tongues of Upper Asia (or more extensively viewed, the Ural-Altaic or Tartar-Finnish stock of languages) and the Dakota.

Father is in Dakota ate ; in Turco-Tartar, ata; Mongolian and its branches, etsï, etsige; in the Finnish languages we meet with the forms attje, atü, \&c.; they all having at (=et) as their radical syllable. Now, as to mother, it is in the Dakota language ina; and in the Asiatic tongues just mentioned it is ana, aniya, ine, eniye, \&c.

Again, we find in the Dakota or Sioux language tanin, which means to appear, to be visible, manifest, distinct, clear. Now, we have also in all the Tartar dialects $t_{\text {an }}$, tang, which means, 1st, light; hence, dawn of the morning; 2d, understanding. From it is derived tani, which is the stem or radisal part of verbs meauing to render manifest, to make known, to know; it also appears in the old Tartar verb-stems tang-(la), meaning to understand, and in its mutilated modern (and western) form, ang. (la), without the initial " $t$," which has the same signification. We may mention still mama, which in Dakota denotes the female breast. We might compare it with the Tartar meme, which has the same meaning, if we had not also in almost all European languages the word mamma, (=mama,) with the very same fundamental signification, the children of very many different nations calling their mothers, instinctively, as it were, by that name, $(\text { mamma }=\text { mama, \&c. })^{5}$

We may also assert that even in the formation of words we find now and then some slight analogy between certain characteristic endings in the languages of Upper Asia and the Dakota tongue. Thus, for instance, the termination for the "nomen agens," which in the Dakota language is $s a$, is in Tartar tsi, si, and dchi; Mongolian tchi, \&c. We also find in Dakota the postposition $t a$, (a constituent part of ekta, in, at,) which is a locative particle, and corresponds in form to the postpositions ta and $d a$, and their several varieties and modifications, in the greater part of the UralAltaic family of languages. The same remark applies in a measure to the Dakota postposition $e$, which means to, toward, \&c. ${ }^{6}$

In pointing out these various resemblances of the Sioux language to Asiatic tongues we in no wise mean to say that we are inclined to believe in any affinity or remote relationship among them. At this early stage of our researches it would be wholly preposterous to make any assertions as to the question of affinity, \&c. All that we intended to do was simply to bring forward a few facts from which, if they should be further corroborated by a more frequent recurrence of the phenomena here touched upon, the reader might perhaps draw his own conclusions, at least so far as a very remote Asiatic origin of the Dakota language is concerned. Further investigations in the same direction might possibly lead to more satisfactory results.

After having hitherto considered the Dakota or Sioux language somewhat in connection with other tongues, we shall now say a word more about that language viewed independently, in its own natural growth and development.

Vowel changes, although far less important in themselves than consonantal permutations, occur very abundantly in the DaKota language. Changes of that kind bear to each other nearly the same relation that the English " $a$ nd" bears to the German " $u$ nd," \&c., only that those forms exist, and are contemporaneously used, in one and the same language. Thus, for instance, the .Dakota Indians call the Iowa tribe "ayúkba," as well as "iýthba," (the sleepers); the verb "to mind" is in Dakota "awaćin"
as well as "ewaćin;" "yukanpi," as well as "yakonpi," is used to express are, (of the verb "to be.") We have also double forms of words, differing only in the vowel they contain, such as kpa , kpe, (lasting, durable, \&c. ;) kta, kte, (to kill ;) spa, spe, \&c.

Sometimes, however, the difference of a vowel occasions also some slight modification in the meaning; for instance, onataka and inataka, both implying the same idea, only the former being the verb, the latter the noun; wowinihan, axe; wawinihay, awful; oskopa, arch; and aśkopa, arched, \&c.
In the Dakota language, we must add, it is of the highest importance that the philologist sbould, when comparing words with different vowels, be exceedingly careful not to see in them always merely double forms of one and the same expression. For, in this language it often happens that syllables which differ only in their vowels are nevertheless sometimes of an essentially different origin, and may denote ideas wholly heterogeneous, and thus enter as parts into compounds in all else similar to each other. Thus, for instance, wada s'a means a beggar ; woda s'a means the same. Nevertheless, they are different compounds, the former meaning simply one who asks for something, who begs, while the first syllable of the latter, namely, wo, is an entirely different word from wa, and means food; so that woda s'a alludes to begging food, beg. ging for something to eat. Equal caution is necessary when comparing words like the following, which in their constituent parts are by no means identical, viz: yawaskte and yuwaśte, both meaning to bless. They have both the word waste, good, in common; but ya-waste means literally to call good, and $y u$-waste to make good. The same is the case with yatanin and yutanin, which means to disclose; yaonihan and yuonihan, to glorify; yakepa and yuliepa, to imbibe, and a great niany others.

We close these remarks with a few words on the harmonious character of this language. Vowels undergo changes not only for the purpose of expressing various modifications of the original meaning, but also for mere euphonic reasons. There is, in fact, a greater tendency in the Dakota language to bring about a constantly harmonious, smooth, graceful, and easy flow of speech than in almost any other of the known Indian tongue. Thus, we frequently find the vowel $a$, for the sake of euphony, changed to $e$; and for the same reason, any possible hiatus carefully avoided by elisions, while semi-vowels are frequently inserted where two vowels would otherwise come into immediate contact with each other and impair the harmoniousness of the sound. Contractions are also used for the same purpose, and the accent or stress of voice moves, according to certain laws, from one syllable to the other in the inflectional changes which a word undergoes, whereby the language becomes often very pleasing and majestic. Indeed, if a comparison were allowed of the widely-different but far more flexible and varied Chippewa, and our more slowly-moring, grave, and manly Dakota lan-guage, we would veuture to compare, as far as euphony and sonorous.
ness are concerned, the former with the Greek and the latter with the Latin language. In regard to the accent, we may also mention that in some instances difference of accentuation in a word is, in Dakota, resorted to as a means of distinguishing homophonous expressions with different mewnings, such as, for instance, would be in English présent and to presént or in German "gébet," (give ye,) and "gebét," (prayer;) or in Greek ध\&ôroxos and ษeotóxus, \&c. Thus, in Dakota, húta means the root of a tree or plant, while hutá denotes the shore of a river or lake, also the edge of a prairie or wood. Consonants also often undergo changes merely for the sake of euphony; thus, gutturals become palatals, and the change of $\mathfrak{k}$ to $\dot{c}(\mathrm{tch})$ is of frequent occurrence, though in all such cases care is taken not to obscure thereby the indication of any etymological changes which words may have undergone, either by combination or inflection.
We often find double forms of a word simultaneously existing, one of them, however, being the older, the more complete; the other, the more recent but-already decaying and impaired form, which finally will supersede the former, and remain alone in use. Thus, to give a simple instance, choseu from a great number of similar examples, frequently very complex, intricate, and obscure, wipi, in Dakota, means full; but in the coexisting form, ipi, full, the " $w$ " has already begup to disappear, although both forms, wipi and $i p i$, are used, and will be until the former (wipi) becomes by degrees obsolete. ${ }^{7}$ Other instances are, Woniya and oniya, (breath;) Wipata and ipata, (ornament;) wihdi and ihdi, (grease, ointment ;) Wożuha and ożuha, (a bag,) \&c. We must, however, be very careful not to mistake the significance of " $w$ " in such forms where, in one, its presence constitutes simply an addition to the word, a sort of formative prefix, and, in the other, its absence is in nowise an elision, for it is frequently found used as an element in the iormation of certain derivatives or compounds. Thus, for instance, the preflx " wa" before a word commencing with a vowel becomes reduced to a simple " $w$, " in conséquence of the elision of " $a$," for euphonic reasons. It may also happen that the " $w$ " serves to distinguish certain modifications in the meaning of a word, so that the two forms, though closely related, can no longer be considered as altogether identical. Instances of this kind are, wopeton and opeton, two verbs which are, indeed, often confounded with each other, and used indiscriminately to. express trading; while, howerer, strictly speaking, opeton means to purchase, to buy, to hire, and Wopeton, to buy, but also to buiy and sell, to trade. Wowa, to paint, to write, forms, by the addition of "pi," the usual ending of verbal nouns, the word wowoapi, which means a writing, a book; while owapi means more particularly a picture, something that is painted or lettered, though these differences do not always seem to be kept distinct, wowapi being, in the Dakota dialects, used also for painting, picture, for a letter, a sheet of paper, \&c. The letter " $h$, " at the beginning of words, frequently disappears likewise; thus we have the double forms $\mathbf{H} i$ and
i, (to come; ; Hećon and ećon, (to do;) Hnaśka and naśkn, (a frog;) Heçen and één, (such as,) \&c. We also find, in some instances, that conso. nants are dropped at the end of words, as in the double forms hektass and hekta, (back,) \&c.; " $k$ " also disappears not unfrequently, which accounts for the double forms $\mathrm{K} u$ and $u$, (to come,) \&c. K may disappear also in the middle of words; thus we have kaкi and kai, (to carry,) \&c. It sometimes happens that when " $k$, ," in the middle of a word, is followed by " $i$ " this syllable "ki" is dropped; hence, we have double forms, such as iкrun and iun, (to anoint;) iiктуuwi and iiyuwi, (to bridle,) \&c. But the greatest care is necessary not to confound this " $k i$ " with the grammatical syllable "ki," which is inserted in verbs to impart to them a more definite meaning, and is particularly incorporated in verbs indicating a special relation to or for whom anything is done; as, for instance, oyaka, (to tell;) oкryaka, (to tell to one, to somebody;) thus, oтаккyaka, (tell me,) \&c.

We have in the Dakota language also a very interesting system of consonantal permutations. Thus, among the liquids, a frequent (and often almost optional) interchange of $l$ and $n$; for inștance, boy is in the Dakota hoksiciLa and hoksiva, ( $l$ and $n$;) or, if we wish to compare the dialects of that language with one another, we have in Yanktonais liL $a$ for " very;" in the Titon dialect the same ; in Sissiton NiNa; ( $l$ and $n$ again interchanged.) Also the liquids $n$ and $m$ are interchangeable, often ad libitum, even within the limits of one and the same Dakota dialect; thus, for instance, the English preposition "on," "upon," is in Dakota " akan" as well as "akam," \&c.

We have in the Dakota language also a frequent interchange of $k$ and $t,{ }^{8}$ as, for instance, $i \mathbb{K} p i$ and $i \tau p i$, both forms being used to denote belly, abdomen. Thus, ćexpa, which means navel, twin, may assume a double form in the compounds hokśićekpa and hoksiciéppa, where $k$ and $t$ interchange with each other without affecting the signification of the word in any way whatever. Other examples are oкраza and otpaza, meaning darkness, night; wiyaкракрa and wiyатpaтpa, signifying to glisten, to glitter, \&c. This change takes place especially where the $k$ or $t$ is immediately followed by $p$. The permutation above adverted to, between $k$ and $c^{\prime},(t c h$,$) is also of frequent occurrence. It not only takes place in con-$ sequence of certain euphonic laws, but it would seem to be also optional, as we find double forms of one and the same word, the one with $k$, the other with ©; as, for instance, iкute and ićute, meaning ammunition, \&c. $K$ interchanges also with $y$, as, for instance, in the double forms кamna and yamna, meaning to acquire, \&c. Then, again, $y$ interchanges with ć; thus hoksiviopa and hoksícóopa, ${ }^{9}$ meaning child. K interchanges, moreover, with $p$; for instance, Kasto and Pasto, (brush,) \&c. $K$ interchanges also with $b$, as Katonta and вatonta, (notch,) \&c. Then, we furthermore observe that labials interchange with each other; for instance, $b$ with $p$, as Bago and Pago, two forms of one and the same verb, meaning to carve. Also, the labials $p$ and $m$ are seen to interchange with
each other; thus, napkawin and nawkawin; (to beckon with the hand,) \&e. There are also instances of a permutation between $p$ and $t$, such as petuspe and petusTe, (a fire-brand,) \&c. Also $t$ and s' sometimes interchange with one another, as in $k T a_{1}$ and $k s a_{n}$, which mean curved, whence the compounds yukTan and yuksian, meaning literally to make curved or to bend, \&c. It now and then happeus that such consonantal iuterchanges take place, and are, moreover, accidentally complicated by a transposition of the consonants in question; for instance, ортаye and ośpaye, \&c. It is important to take all these various changes into careful consideratiou when we wish to identify words in their different appearances, their innumerable protean transformations, and often surprising modes of disguise, and to trace their origin, derivation, and various affinities.

In regard to the derivation and composition of words, the Dakota or Sioux language is particularly clear and transparent. Derivations can be traced with great facility, and in the matter of the formation of compound words, this language is remarkably apt and flexible. We will take this opportunity to present but a few instances of Dakota etymologies, which will, however, be sufficient to enable the reader to form some idea of this particular subject. Ti means to dwell, to live in, and as a noun the same word means $a$ dwelling-place, a house. With the addition of the substantive ending $p i$, (tipi, it means a tent, such as the Sioux Indians inhabit; while when combined with the verb opa, which signifies to go in, to enter, to go to, it forms tiyopa, (for tiopa,) which is a substantive and designates a door, a gate, an entrance. Da is a verb which means to form an opinion, to think; its longer form is daka, with the same meaning. This word added to the adjective waste, good, forms the compounds wasteda and wastedaka, which mean to deem good, to think well of; hence, to love. On the contrary, when combined with siće, bad, it forms the compounds sićeda and sićedaka, which meau to consider bad, and, by a natural transition, to hate.

The word hoksi gives rise to a number of derivatives, of which we will here mention but a few. The word itself does not appear to be used independently; but we mas, perhaps, infer its fundamental meaning, when we consider a compound expression like hoksit-ćekpa, which not ouly means twins, but, in its probably more original signification, applies to a flower, and denotes a blue wild flower which appears first in the spring, the earliest spring-flower, thus alluding to the first beginning of floral vegetation. In a similar acceptation, it seems to enter as the principal constituent part into all words expressive of the idea of infancy and childhood, as hokśiyopa, a child =hoksiopa, the verb opa, most probably, with its meaning of following, going along with; hoksidain, a boy, dan being a very common diminutive termination, alluding here, it seems, simply to the youth and small stature of a male during childhood, \&c.; hoksivin and hoksiwinna, a virgin. In the latter expression we distiuguish in the ending: the word win, that by itself means female, woman, and winna, which is its diminutive, and stands to it somewhat in the
same relation as the German fräulein, a young unmarried woman, to frau, a woman.

The word $\dot{g} u$ means to burn; $\dot{\mathrm{g}} u \mathrm{ya}$ is a causative form of $\dot{\mathrm{g}} u$, and means to cause to burn, to make burn. This word appears also, and, it seems, in a more definite sense, under the form $a \dot{g} u$, (with prefixed $a$, to burn, and aguya, to cause to burn. With the usual substantive-ending of verbal nouns, viz: pi, ag̀uyapi, means bread, as it were, something burned or baked. With a similar import the radical letters $b r$ in our English word bread, German brod, seem to refer to the same idea, as they appear also in вRennen, BRand, BRaten, BRühen, BRauen, BRïten, BRunst, \&ic., in all of which expressions the idea of heat, if not of fire, is evidently implied. ${ }^{10}$
Interrogatives, which also in this language coincide in their form with relative and indefinite pronouns, present here the peculiarity of commencing, in the greatest number of instances, with $t$ or $d$, while the demonstratices begin with $k$. For example : tuve, who ; raku, what ; tohan, when; тоhan, where; tona, how many, \&c. And of the demonstratives we may mention к $a$, that ; кalki, there ; каna, these. Sometimes we find also the guttural softened down to a simple $h$; as, for instance, нena, the equivalent of каna, these; нehan, which means there, and answers to the above-mentioned tohan, where; and Hehan, which means then, and responds to tohan, when. We may observe here, by the way, that in most of the other languages which come under our ordinary observation precisely the contrary takes place, viz: guttural letters (which are also sometimes found replaced by their equivalent labials) serving to express the interrogative; while $t, d$, , th, commonly occur in the demonstratives. Thus, we have in Latin talis, tantus, tot, tam, tum, tunc, \&c. ; in Greek, tó, tóoos, tóze, \&c. ; in English, thts, that, thus, there, then, \&c.; and with the gutturals, in Latin, quis, quid, qualis, quantus,
 $\tau \varepsilon \rho 05$, \& c. ${ }^{11}$ The same phenomenon is remarked also, in a measure, in a great many other languages widely different from those last mentioned. We may state here, as a curious fact, that the Dakota mode of expressing the more essential part in interrogatives by $t$ or $d$, and what corresponds thereto in demonstratives by $k$, obtains also in the language of Japan, where it constitutes indeed an eminently striking feature. It is true, $k$ and $t$ are interchangeable, and, in many instances, convertible elements in languages generally, but their functions are kept distinct and apart in the particular matter under consideration.
We pass on to the Dakota word akan, which means above. It is the same as akam, and if not identical with, is at least related to akay ; just as we see, for instance, the double forms kahan and kahay, which mean then, there, so far, and one of which has $n$ where the other has $\eta$; that is, $n$, with ouly a nasal pronunciation. Now, the akan, as an adjective, means also old, implying, no doubt, the idea of above, of superior to, (in stature or in years,) just as the Latin altus reappears in the German alt, English eld, old. This akan, or, per aphoeresin, simply kan, appears also
in the form of $w a k a n k a_{9}{ }^{12}$ an old woman. Akan reappears also under the forms (w)akan and wankan, meaning likewise above, up, high, superior, and being undoubtedly closely connected with the form (w)akan, since $n$ and $n$ are interchangeable terms, (as shown in the above kahan and $k a h a_{y}$ ); and since certain derivates, moreover, are seen to confirm their intimate relationship, such as wakaniciidapi, pride, haughtiness, where wakan evidently refers to real or fancied superiority, similarly to the Latin superbus, the French altier, \&c. Perhaps wakapa also comes under this head, its meaning being to excel, to surpass, to be superior to, or to be above; wakapa standing, according to all appearance, for wakankapa, the latter part of which would be the verb kapa, to pass by, to go beyond. Thus the primary and fundamental meaning of wakay (=akay, akam, akan) would be what is superior or above, a superior something or being; hence it means a spirit, a ghost, and, as an adjective, spiritual, supernatural, divine. It gives rise to the following expressions: mini-wokan, which signifies alcohol, brandy; as it were, spirit-water, or spirituous liquor; ${ }^{13}$ wakan tanku, the Great spirit, meaning God; wakan sića, evil spirit, meaning demon, devil ; wowapi wakav, literally spirit book, or spiritual, divine book, the Dakota name for the Bible; tipi-wakan, which means a chapel or church, literally spirit house, sacred house ; wićaśte-wakan, a clergyman, priest, literally a spiritual man; \&c. Thus, also, the lightning is called wakanhdi, from wakan (spirit) and $h d i$, (to come, meaning, as it were, the coming down or arrival of a spirit. Also, the famous dance of the Sioux Indians, which is described as the Medicine-dance, viz: wakan wacipi, simply means spirit-dance or sacred dance, and, as Rev. S. R. Riggs expressly informs us in his Dictionary, is thus called especially from the fact that the high priests of the ceremonies spend the night previous in singular magic practices, and are holding communion with the spirit world. Then, again, we have the word wakan in compound verbs, such as wakan kago, which means literally to make wakail, as it were, to attend the acts of worship or divine service; and wakanećong means to perform supernatural acts, to do tricks of jugglery, of magic. A great error has been committed by travelers generally, who, resorting, perhaps for information, to the stolid half-breed Sioux Indians, who are often still more ignorant, if possible, of English than the travelers are of the Dakota tongue, have identified the idea expressed by the word wakan and everything therewith connected with that of the healing art, or medicine. To be sure, healing a disease, restoring a sufferer from sickness to health, is in the opinion of the wild Indian always and preëminently a supernaturai, wonderful act, in which beings of a higher order directly participate, and which is generally brought about by means of magical performances, conjuring, necromancy, and sorcery, rather than by the administration of remedies or other medical appliances. There is no such thing as a "medicine man" among these Indians, and they have not even a word for it ; for wićaśte-vakail, which has been erroneously taken for such, simply means a supernatural
man, a spirit man, a magician, and the like, and has come subsequently to be applied to the priest, clergyman, or missionary. An Indian doctor is called wapiye among the Dakotas, which simply means a conjurer, and is derived from the verb coapiya, to conjure the sick, which in its turn comes from pikiya, to conjure. A physician, or one who cures diseases by means of medicine, is always called peżihuta-wićaśte, from pezi, which means grass, also dry grass, herb, and huta, which denotes the root of trees or plants, so that the compound pesithuta, which properly means medicine, ${ }^{14}$ would signify literally herbs and roots, and pezihutawićaśte a herb-and-root man; which epithet is almost exclusively applied to American doctors resident in the vicinity of those Indians and to military surgeons at the forts in their territory'. Among these people the gathering of herbs and root, and the administration of such medicines are, indeed, not in anywise uncommon; it is, however, not at all the occupation of men, but of women.

The word for mouth is $i$, whence is derived the verb $i a$, to speak, which in its turn gives rise (by the addition of the ending $p i$ so common in the formation of verbal nouns) to the substantive iapi, speech, language. (Thus Dakota iapi, the Dakota language, properly the language of the companions, friends, or allies.)

The verb $\mathrm{l} a$ means to curl. It is also used with the reduplication, viz: kaha, as an adjective especially, to denote curling, curled. The same when combined with mini, ${ }^{15}$ water, signifies curling water; and thus mini$\mathrm{k} a \mathrm{~h} a$ is the usual word for a waterfall, a cascade generally. Often haha alone is used to designate a waterfall; mini (water) being understood, just as we are accustomed in English to employ simply the word "falls" in the same sense. Thus the word hahatuŋ we is used, meaning those who dwell or live at the falls, the people around the waterfalls, an expression which has become among the Dakotas the ordinary name of the Chippewa Nation. ${ }^{16}$

To translate the word minihatia (or erroneously written ${ }^{6}$ minnehaha" $)^{17}$ by laughing waters, seems to be a gross mistake, most probably the result of imperfect information derived from some half-breed Sioux who was perhaps asked, (the inquirer wrongly analyzing the word,) "What is meant by minne ?" To which the response was doubtless, "Mini means water." "And what does ihaha siguify?" The answer to which must have been: "Ihalia means to laugh." (No doubt $i$ signifying mouth, and ha, to curl; iika and ikaha mean to curl the mouth or the lips, that is, to laugh.) When Rev. S. R. Riggs, in his otherwise very excellent Dakota Dictionary, explains ihaha by "to laugh along as rapid water, the noise of waterfalls, ${ }^{98}$ he is unconsciously led astray by that current popular error. In fact, such an interpretation is founded on nothing, and is prima facie quite contrary to all right etymology. ${ }^{19}$ And to do justice to Mr. Riggs, for whom we profess the highest esteem, and who is without any comparison the best grammarian and lexicographer who has ever yet appeared in the dornain of American Indian philology, we
will state that he likewise explains (in his dictionary) haka by "water. falls, so called from the cURLING waters."

Our views on this subject, as on various other similar matters, were, moreover, fully approred by Rev. T. S. Williamson, another distinguished missionary, and a highly respectable authority as regards the Dakota language, with whom we had many a long conversation on such topics every time we happened to meet with him in the territory.

Much might yet be dope in investigating that most interesting language, in a strictly philological manner, and also tracing particularly the many Dakota names of mountains, hills, rivers, lakes, \&c., to their true origin and meaning. They almost always contain some attractive allusion, something legendary or traditional, which might lead to most valuable results in regard to the history, religious ideas, ancient usages, \&c., of this largest and most powerful of all the Indian tribes of North America.

We now say, in conclusion, that on this continent, researches in philology, ethnology, and history should have for their main object the languages and nations of America. The field is comparatively new and exceedingly interesting ; an immense deal has to be done in this domain, the real labors of thorough and exhaustive investigation having not even yet begmo. If these unpretending pages, contributed by the author as his first mite to that kind of research which he wishes to see undertaken by the scholars of this country, serve as an incentive to others to interest themselves in these studies and devote some of their time and exertions to the same, his object will have been successfully attained.

## NOTES.

${ }^{1}$ Such intercalations are, in a measure, almost analogous to the usual insertion of the many incidental clauses in long Latin or German sentences, if we are allowed that comparison.
${ }^{2}$ c stands in the present transcription of the Dakota language for tch ; ś for sh; y for nasaln; dotted letters indicate a peculiar emphasis in their utterance, for which we have no precise equivalent in English.
${ }^{3}$ Other examples in Mantchoo are kaka, meaning male, cock, while keke means hen, \&c. These phenomena are, in their last analysis, reducible to a fixed principle, which still prevails, to some extent, in the above-mentioned group of Asiatic languages, and which we have some reason to believe once formed an essential part of many other tongues. We might perhaps not improperly recognize in that antagonism something of polar opposition, some law of polarity. There are distinct and polarly-opposite correlative vowel-classes, viz: $a, o, u$, in the continental pronunciation, which are, as it were, positive, and $e, i$, which are negative. Sometimes, however, the reverse takes place, so that $e, i$, have the power and significance of $a, o, u$, and vice versa, (a quasi " inversion of the poles.") This division is not an arbitrary one, but-we remark
this by the way-the classification results quite naturally from a certain antagonistic relation of these vowels, respectively, to the guttural letters, their very test and touchstone. According to the nature of these vowels, the word receives often its characteristic meaning in those Asiatic languages; hence, only vowels of the same class occur in one aud the same word. It would lead us too far from our present subject if we should now elucidate more fully the phenomenon under consideration. We wish to make only a few remarks more. This peculiarity extends to adjectives and to verbs-qualities, (positive or negative, as the case may be,) actions, and states of being ; eveu to postpositions, \&c., (direction, tendency, \&c.) We could, indeed, illustrate it by hundreds of examples, especially in the Central-Asiatic languages, eveu in the Celtic tongues, particularly the lrish. We might point out a very considerable num ber of such instances finally depending on a certain principle of vowelharmony. Even in our own ancient and modern languages we can now and then discover some slight and obscure vestiges of that perhaps originally quite extensive phenomenon of significant vowel antagonism.

 cal-idus and gel-idus; perhaps, also, in the fundamental form homin and femin, (implying hemin: $\mathrm{f}=\mathrm{h}$, as in Span. hembra; ) in Hebrew, Nף and N•ה; Arabic 四 and $\mathcal{E}$; h $u$ and hi, \&c., and other expressions of contrast, negation, or opposite tendencies generally. We also find in German stumm and stimm-referring to the voice or its absence; in English, the rerbs to step and to stop, \&c.
${ }^{4}$ Though it is almost evident that ćun has not a separate and independent existence in the language, but is always found combined with pronominal suffixes, such as ćuŋku, (her elder sister,) we nevertheless meet also compounds like the following: ćugya, to have for an elder sister. We may, therefore, safely conclude that iun in ćunku and the verb ćunya is the word which designates an elder sister. Moreover, the form ćunluu has a parallel expression in ćinću, which means his elder brother ; and as $k u$ is identical with ć $u$ in consequence of a very common consonantal permutation, it becomes obvious that ćun, indeed, means elder sister, as ćin is known to signify elder brother.
${ }^{5}$ In the Grusinian language, mama means father-an apparent anomaly, owing, perhaps, to a mere interchange of the labials, passing here over into their extremes. Another shifting of the labials, though less in extent, we find in the Asiatic tongues, where we also meet with baba for father, fafa for mother, \&c.
${ }^{6}$ By means of such postpositions the declension of nouns is effected in the Ural-Altaic languages. The Dakota cases of declension, if ${ }^{\text {* }}$ we can use this term, amount likewise to a very rude sort of agglutination, or rather simple adding of the postpositions to the nouns. There can be here no question of any real inflection or declension, since there is throughout only a kind of loose adhesion, and nowhere what we might call a true cohesion. The postpositions are in the written language
added to the nouns without being conjoined to them in writing, (except the plural ending $p i$, as is also the case in the Mongolian language, the Turco-Tartar dialects, and other tongues of this class.
${ }^{7}$ We see in the historical development of our own modern languages an abundance of similar phenomena; thus in respect of the mere quasimonumental, and, as it were, fossil existence of labials, such, for instance, as $b, p$; and in regard to English words like debt, which in French long ago became dette. In English the $b$ of debt (=debitum) has become only silent, while in Freach, ou the contrary, it has now no tolerance whatever, even as an historical landmark. There is, in fact, more conservatism in English. The French appears a more volatile, changeable element, even in the minor details of the language. Thus, again, we have in English the word doubt, with petrified silent b, which they seem unwilling, as yet, to let go, while in French we have doute without that $b$. Many other examples might be adduced in support of this very simple and common fact in all languages. In sept, (seren,) the French still neglect ridding their language of that now useless silent $p$. They do, it seems, not affect such antiquities, and will, most likely, do with words like sept as they have done with clef, (clavis,) where the final labial $f$ became gradually silent but was left untouched. It is even now allowed to remain, but another form has already come into use at the same time with it, and a key is now a-days clef and clé.
${ }^{8}$ This interchange is seen in almost all languages of one and the same family, when compared with each other ; thus, for instance, the use of $k$ instead of $t$ constitutes one of the characteristic differences between the Hawaiian tongue of the Sandwich Islands and the language of Tahiti, the Marquesan, Rarotangan, \&cc., both groups, however, belonging to the Malayo-Oceanic, or more particularly the Micronesian stock.
${ }^{9}$ é stands here for a letter that does not strictly belong to the word, viz. $y$, which is merely inserted euphonically between hokśsi and opa.
${ }^{10}$ We venture this derivation so much the more boldly, inasmuch as the etymology. of bread, brod, \&c., is, in a degree, still an open question, Grimm connecting it-though not particularly insisting thereon-with brocken, brecken, to break, \&c., while Anglo-Saxon scholars endeavor to trace the English word bread to breadan, (to nourish,) which, however, seems rather to be a denominative verb, such as lighten from light, Their etymological attempts being mere opinions, mere assertions without proof, we feel encouraged to maintain ours.
${ }^{11}$ The $\tau$ in the Greek $\tau i$ is only an apparent exception to it, as is well understood by those conversant with the facts of comparative grammar.
${ }^{12}$ There is some room left for an attempt to derive wakanka direct from wakan. The ideas possibly underlying such a derivation would appear to us rather far-fetched and fanciful.
${ }^{13}$ Other Indian tribes call alcoholic liquor fire-water instead of spiritwater, as, for instance, the Chippewas, in whose language it is ishkode wabu, \&c.
${ }^{14}$ The word pezihuta is also applied to various other vegetable essen29 s 71
ces, beverages, \&c. Thus, coffee is called pezihuta sapa, literally, black medicine; just as the Chippewas express it in their language by makade mashkiki woabu, (black medicine water.)
${ }^{15}$ The word mini (water) is the same which is contained also in the name of Minnesota, (properly mini-sota,) meaning whitish water, and referring to the Wakpa minisota, which is the Minnesota or St. Peter's River, and also to the Mde minisota, the so-called "Clear Lake."
${ }^{16}$ It is often the case that Indians give to other nations names simply derived from some entirely external, merely accidental, and altogether unessential circumstance or quality in these strangers, which at first principally struck their attention. Thus, for instance, the inhabitants of the United States are called by the Dakotas Isantanka, meaning Big Knives; by the Chippewas, kitchimokoman; which likewise signifies Big Knives, probably from the swords of the United States soldiers in the Territories.
${ }^{17}$ Just in the same way, the erroneous orthography of "Minnesota" was introduced for the more correct Minisota ; and this is seen againwe mention it in passing-in that monstrous Dakota-Greek compound, "Minneapolis," meaning " Watertown."
${ }^{18}$ Any such meanings of iliaka, as "to bubble" and making a noise like that of waterfalls must be considered simply as secondary, as a mere extension of the original signification of that word, viz. laughing, $i+$ halia, mouth-curling, as it were; nothing whatever being contained in the constituents of that word which could have even the remotest reference to water or a cascade. The word itself seems to follow this deviation from its proper import, being even differently accentuated in that sort of figurative acceptation, viz. ihalia instead of $i \mathrm{i}$ ílika.
${ }^{19}$ Similar blunders frequently occur. Thus, in the erroneous and unmeaning English translation of Indian names generally-for instance, of "Hole-in-the-Day"-in which word it was intended to express simply one who (as a powerful archer) perforates the sky with his arrows, which we could easily place beyond any doubt, if it would not lead us too far from our present subject. So have travelers, too, themselves put the words:" squaw," "papus," \&c., into the mouths of the Dakotas, though these words belong exclusively ta widely different tribes, and are on other occasions again repeated by the Dakota Indians to strangers, as they simply suppose such words to be English, and, therefore, more intelligible to the latter! The same applies to the Chippewa word " nibo," (he died or is dead,) which travelers, probably deeming it the general and only Indian term for that idea, taught, as it were, to the Dakotas, who constantly make use of it in their conversation with Americans, mistaking it in turn and in like manner for an English word, or something more easily accessible to the mind of the strangers.

## METEOROLOGY.

[The following notes, derived from correspondence or from observation and reflection, are especially intended for the meteorological observers of the Institution principally in the way of answering queries, which bave been frequently propounded. . They may, however, be found of interest to the general reader.-J. H.]

## METEOROLGGY OF PORTO RICO,

Mr. George Latimer, from Philadelphia, one of the correspondents of the Institation, who has resided on the island of Porto Rico (rich in gold) since 1834, informs us that the northeast trade-winds prevail on the island every day of the year from about 9 o'clock in the morning until sunset; while at night there is a strong land-breeze toward the ocean on all sides of the island. The latter is stronger, however, on the west end and on the north side, which is probably owing to the greater slope of the land toward the sea in these parts.
During the rainy season, which is from the end of May to the end of October, the rain falls every day on the western portions of the island from 2 o'clock until sunset. This, however, is not the case on other parts of the island, which is divided longitudinally by a range of mountains 3,000 or 4,000 feet in elevation. These mountains turn up the current of the trade-wind air containing vapor into the colder regions, and cause its precipitation in rain on the northern slope, while on the south the land often suffers from drought for more than a year without interruption. On this side of the island irrigation is resorted to, and for this purpose there even exists a project to tunnel the mountains to conduct the water of one of the rivers from the north to the south.
Mr. Latimer states that occasionally there is a cessation of the ordinary trade-wind when the air becomes almost entirely caln or light winds arise, which go entirely around the compass in the course of a few hours. This state of things frequently contiques several days, and from these, as signs, Mr. Latimer has alwass been able to predict that a gale is blowing at the north. After the existence of a calm of ocean and air there invariably comes a heavy rolling sea from the north, so heavy that vessels cannot leave the harbor of Saint John, or load in any of the other ports on the northern side of the island. Also after this, in the course of a few hours, or in other cases after two days, comes
a strong northerly wind, the return of the regular trade-wind, with much greater intensity than usual, and vessels arriving after short passages bring the intelligence of the predicted gale and its disastrous consequences.

Colored bands diverging from the setting sun in the west, and converging to all opposite point in the east, are frequently seen throngh the summer and autumn in great beauty.

Remarks.-The rainy season in the northern tropics takes place when the sun, having a northern declination, heats in the greatest degree the land during the day, producing ascending columns of air, which, carrying up the vapor it contains into higher and colder regions cause it to be precipitated in rain, the precipitation commencing as soon as the heat from the sun begins to diminish a little after midday. The phenomenon mentioned by Mr. Latimer in regard to the occasional cessation of the trade-winds may possibly be connected with the occurrence of storms on the cohtinent of North America, or perhaps with the remarkable wind known in Texas as the "norther." This wind prevails from the Mississippi River to the Rio Grande and commences about the 1st of September and ends about the 1st of May. The day previous is marked by an unusual warmth and closeness of the atmosphere and an almost perfect calm. The first appearance of the tempest is a cloud in the north, which approaches the observer sometimes with great and at other times with less velocity, and frequently passes over his head in a series of arches composed of dense clouds separated by lighter portions. The thermometer frequently falls 30 degrees. On one occasion recorded the temperature fell in the course of three hours from $75^{\circ} \mathrm{F}$. to a degree sufficient to produce ice an inch thick. After a day or two the norther is followed by an unusual cold wind from the south, as if the norther were returning. It is said to be most intense near Corpus Christi, Texas, and that it does not occur in Florida.
The norther is probably due to a stratum of air along the border of the Gulf, abnormally moist and consequently heated, produced by a surface current from the south, which gradually attaining a state of unstable equilibrium is suddenly forced upward into a higher region by a heavier wind from the north. The violence of the wind, and consequently the intensity of the cold, will depend upon the distance northward to which the moist stratum extends previous to its overturn by the heavy air from the north. The norther, it is said, is not felt at sea in the Gulf. This would indicate what we would readily suppose, that the greatest rarefaction of air due to heat and moisture takes place over the land along the borders of the water.-[J. H.]

## METEOROLOGY OF THE GREEN RIVER COUNTRY.

By Colonel Collins.

Colonel Collins has been for three years in the Wind and Green River country. The Green River becomes the great Colorado of the west, which empties into the Gulf of California, and the Wind River becomes the Big Horn, and runs into the Yellowstone, which in turn empties into the Missouri. It often happens that rivers in the western part of the United States have different names in different parts of their course, and this appears to be especially the case when a river passes through a cañon; the fact not being known before exploration that it is the same stream at the two ends of the chasm.

The climate in the 'region above mentioned is very dry, electrical appearances being manifest in currying horses or brushing clothes, and dew is very seldom seen. Along the Wind River range the storms come from the northwest and follow the chain to the southeast. On some of the high peaks of this region there is often seen a cloud-cap remaining stationary sometimes for a day or more, while a high wind is prevailing at the same time on the plains and valleys below, with a clear atmosphere in all other parts of the sky. The cap appears compact and distinct in outline and perfectly stationary. The peaks of the Wind River range are all covered with perpetual snow. There are no trees on the plains, or anywhere in the vicinity, except on the mountain-sides from their base up to near the snow-line.
Frost at the foot of the mountains and in the valleys occurs almost every night during the summer. On the 4th of July, 1862, at the camp at the head of Sweet-Water River, the ice was formed from half to threequarters of an inch thick. The summer frost, although it does not kill the hardy grasses, will not allow the cultivation of grains and vegetables. Heat and moisture, the two essential conditions of growth, are wanting, though, in the very deepest valleys, perhaps, grain could be raised by irrigation, since the temperature in these is considerably higher than on the mountains.
The winter was exceedingly cold; at Fort Laramie in 1864 the mercury was frozen and continued solid on the 4th of January for four hours; on the 5 th fifteen, and on the 6th for twelve hours, while in the warmest part of each day the thermometer never rose above minus $20^{\circ}$. "I had command," says Colonel Collins, "at the time, of Fort Laramie, and had great difficulty in keeping the garrison warm. Fuel had to be drawn a distance of about fifteen miles. Every winter a number of men were frozen to death, being usually overtaken by snow-storms. When the greatest cold occurs the air is perfectly still and rery trans-parent-the transparency is'so perfect that objects are seen a long way off with such distinctness as to give rise to mistakes as to their actual distance.
"It should not be forgotten that the base of the Wind River Mountains is about 8,000 feet above the level of the ocean, and hence the coldness dryness and rarity of the air. Notwithstanding the great elevation of the region there are some very hot days in summer, though the mornings and evenings are cool.
"The general course of the wind is from the west, especially when it is violent. The currents are, however, modified by the mountain ranges. In some of the higher gorges a strong wind constantly prevails from the west, which is especially the case at Fort Halleck at the foot of Medicine Bow Butte, at the main head of Medicine Bow River. This fort is at about 8,300 feet above the level of the sea, and situated in a pass, with a high mountain on the south, and elevated land on the north. The direction of the wind is continually the same in winter and summer, namely, from the west, or that of the return trade, probably somewhat modified by the configuration of the surface. In the plains between the mountains the snow is immediately blown into the ravines by the violent wind, leaving the general surface bare. So constant and annoying is the wind that I advised that Fort Halleck should be abandoned. It is impossible to secure hay for the cattle; as soon as the grass is cut it is blown away. For the same reason great care is required in drying clothes.
"The storms are terrific, and in some cases, when they occur, it is impossible to ride against the wind. The snow is extremely fine, mingled with air, moving with the currents, and presenting no appearance of falling flakes. It cuts the face like fine sand, and blinds the traveler. The horse or mule cannot be made to face the blast, particularly the latter, but will always turn from it.
"The streams, fed by the perpetual snow, are always full in summer. In the winter they are frozeu solid. Thunder-storms are not frequent, but when they occur they are often attended with hail. The quantity of water which falls is small. Evaporization is very rapid. When game is killed it can be hung up and soon becomes so dry at the surface that flies cannot lay their eggs in it; a quarter of deer will in this way remain sweet for a week in the warmest weather. The soldiers rely very much on deer, buffalo, ducks, and geese, which are readily preserved. When going on a march they prepare a supply of what is called jerked meat, which consists of flesh cut into thin strips and placed over a smouldering fire to drive away the insects and afford a small quantity of smoke. The meat dries so rapidly that it becomes as hard as a stick in the course of two or three days.
"The most violent storm I experienced occurred about the last of February, 1862, when we made an excursion to the southwest after the Indians, who had made an attack upon the mail-line and one of the military posts. The storm commenced on the third day of the journey. It was not very severe at first, but increased in intensity until the third day of its continuance, when it was truly terrific. The party consisted
of one hundred men; two were frozen to death, and upward of thirty badly frostbitten in their extremities. The snow filled the air to such an extent that the course could only be followed by keeping at a certain angle with the wind, or, in other words, by adopting the direction of the wind as a course of reference.
"The mule is a less hardy animal than the horse, and often freezes standing, so that at first sight, and at a little distance, they appear alive and ruminating, but might be pushed over in a solid condition, the legs stretched out like the legs of an overturned table. In summer the horses and mules are fed on grass, which is very sweet and nutritious. I had about eight hundred head of oxen, and one thousand sheep. The best meat was that from the old cattle which had been pastured for about a year."

Remarks.-The facts which Colonel Collins has here stated are interesting in regard to general meteorology. The existence of the constant wind from the west, in these elevated passes, is in strict accordance with the assumption of a return trade-wind, giving rise to a constant westerly current at elevated points in the temperate zone. It is this wind which carries all the meteorological phenomena eastward in the temperate zone, and thus forms the basis of the prediction of the weather.

That the snow should be very fine is also in accordance with the fact of the small quantity of moisture in the air and the intense cold. The snow, for the same reason, is small in quantity on the plains. The absence of thunder-storms is also in accordance with the fact of the small amount of moisture in the air.

The cloud-cap mentioned is probably produced in a similar manner to that at Table Mountain at the Cape of Good Hope, by a moist wind blowing over the top of the mountain, which, on ascending to a certain elevation, precipitates its moisture in the form of visible vapor, which is again dissolved on descending the other side, producing the appearance of a stationary cloud, though it is constantly in the process of forming on one side and dissolving on the other.-[J. H.]

## DISTINCTION BETWEEN TORNADOES AND TEMPESTS.

Lamark, in a paper published many years ago in the Journal de physique et chimie, points out the distinctions between a tornado and a tempest. The following, according to him, are the characteristics of the tornado:

1. The effects produced at the surface of the earth take place under an isolated cloud which moves with the storm, and is in some way connected with the disturbance of the atmosphere which constitutes the phenomenou.
2. The tornado moves over the surface of the earth in a narrow path,
the middle of which is marked by the greatest destructive effect of the motion.
3. The effects of the tornado at any one place are produced in a very short time. It passes over different points of its path with great rapidity.
4. It commences at a given place with a crash, and passes off as suddenly into a calm.
5. The tornado, even the most violent, seldom lowers the barometer but little, and sometimes produces no appreciable effect in this way.
6. The tornado is generally accompanied with discharges of electricity, with large quantity of rain falling in a few minutes, and frequently with hail, (sometimes in two tracks, one on each side of the path of the meteor.)

Character of tempests according to the same author :

1. Tempests are of great extent; they are not accompanied by an isolated cloud as is the case with the tornado, but with one of apparently unlimited extent.
2. Moderate tempests continue sometimes ten or twelve hours, while the most violent ones in some cases continue thirty-six hours, with slight intermissions in the greatest inteusity.
3. All tempests are connected with the falling of the barometer, eren to the extent in some instances of an inch and a half.
4. The tempest does not come on suddenly, but manifests its approach by a gradual fall of the barometer, and an increase of the velocity of the wind.

Remarks.-The fact stated in regard to the fall of the barometer in the case of the tempest, and not in regard to the tornado, is very important as bearing on the different characters of the two meteors. It would appear to indicate that the tornado is not only of limited extent horizontally, but also in a vertical direction; that it consists of a violent overturn of two strata of different density, the one rushing upward through a circumscribed space, and the other descending probably around the sane space, so that the sum of the two pressures remains the same, while in the case of the tempest the air rises over a large space, and flows over at the top of the atmosphere.-[J. H.]

## account of a tornado which occurred in spruce creek valley, CLENTRE COUNTY, PENNSYLVANIA.

By the Rev. J. B. Meek.

Spruce Creek Valley is situated in the Alleghany range, and extends in a southwest and northeast direction between Tussey's Mountain on the northwest and Bald Eagle Mountain on the southeast. My residence was in the bottom of this valley near the south side. The fore part of the day on which the tornado took place was very warm, moist,
and sultry, or what is called close. A friend who had been our guest, prepared to leave our house a little after 12 o'clock at noon to cross Bald Eagle Mountain into Stone Valley, which lies next to Spruce Creek Valley on the south. I had concluded to go with him, when my wife advised that, if we did go we should take with us umbrellas and overcoats, for she was sure, from the feeling of the atmosphere, that a storm was impending. Her warning was not disregarded in reference to the protections from wet and cold, and we had good cause before my return to be thankful for her forethought. We left the house about half past twelve and commenced to ascend the side of the valley by a steep, path on horseback; the air was very oppressive and our progress slow. When we got about two-thirds of the way up the side of the mountain we heard heavy thunder at a distance, and saw the reflection of vivid lightning in a northwesterly direction from over the other side of the dividing ridge which separates the ralley in which we were from the one next on the north. These indications of a storm continued with increasing intensity uutil we reached the crest of the mountain, when, turning around, we were presented with the appearance of a dark circumscribed cloud at a distance of about eight or nine miles. It occupied about 15 or 20 degrees of the horizon, and exhibited such an unusual and threatening appearance that we almost involuntarily remained stationary, as if spell-bound by the phenomenon. It was very dense, and strangely agitated by a rapid vertical commotion near the middle of the mass, while it was almost incessantly traversed with discharges of electricity in different directions, mostly vertically, accompanied with heavy peals of thunder. Its direction of motion was diagonally across the valley from the northwest to the southeast. As it came over the crest of the opposite mountain it appeared to touch the surface of the ground; no clear sky was seen between it and the earth. From the crest of the ridge it seemed to precipitate itself suddenly down the slope of the mountain, and almost instantly to hide from our view all objects on that side of the valley; as it came near our point of view the character of the internal commotion became more apparent, and when it was directly opposite us, or in that point of its path which was at right angles to our line of vision, we perceived that the wind, which before, while the cloud was approaching us, had been blowing from us toward the tornado, was now moving in the opposite direction, and that the commotion in the interior of the cloud was much more astonishing. It consisted of a violent and very rapid shonting upward in the middle, turning outward and downward on the exterior of columns of mist. The velocity of the upshooting columns was exceedingly great, even as they appeared from our point of view at a distance of four miles. The mass of the cloud had a dark leaden hue, but the tops of the upmoving columns, where they projected above the general surface, were white. The whole presented the appearance of a boiling caldron violently agitated. When the tornado was directly oppo-
site to us it did not appear as dark as when it was approaching us, which would indicate that it was not of equal dimensions, but of greater width in the line of its motion.

The movement of the tornado across the valley was exceedingly rapid; it did not occupy certainly thirty minutes in traversing a line nearly straight of about fifteen miles in length. The ridge of the mountain on the side of which we stood was not above 600 feet above the bottom of the valley, and the storm-cloud did not appear more than double that height above us. During the passage of the tornado our ears were constantly impressed with a heavy roaring sound, like that of the Falls of Niagara, in unison with which peals of thunder in rapid succession were mingling. The cloud appeared to be generated in place as the tornado advanced; indeed, it might be likened to an immense locomotive-engine passing rapidly over the valley, belching forth smoke and steam. After the tornado had disappeared over the opposite ridge, the whole valley was left covered with a cloud, from which rain continued to fall during the night.
The path of the tornado was marked on the ground of the bottom of the valley by prostrate trees and other evidences of violent action. It was variable in width, being from 100 to 150 yards across. The trees were mostly thrown down on each side of the axis of the path, a larger number on the north side than on the south, about, perhaps, in the ratio of three to one. The path was generally straight and of uniform width, with occasional short bends, as if the tornado had in some places made a sudden lateral movement. Although the principal violence of the meteor was confined to the breadth mentioned, yet on each side, for a quarter of a mile, trees were thrown down in the direction in which the storm was advancing. The effects on the northern side or slope, where the tornado entered the valley, were scarcely perceptible, while on the southern slope, or where it left the valley, they were very marked. On the northern side it appeared to leap down from above to the bottom of the valley immediately below; at this point its first prominent mark was made upon a mill-pond, which it entirely emptied of water, sweeping it completely out, and even throwing up from the bottom sticks and stones which had long been sunk in the mud. The most striking effects were, however, those produced in the lowest parts of the valley, some traces of which could be seen several years afterward. Its fury was not spent in Spruce Valley, but it left traces of its power for at least twenty miles on the other side of the ridge, in the adjacent valley.
Remarks.-The account of this tornado, which was observed from a very unusually favorable position, is very instructive in regard to the cause of the phenomenon. The two causes to which these remarkable commotions of the atmosphere have been referred, are electricity and a disturbance of the pneumatic equilibrium of the atmosphere due to an abnormal condition in regard to temperature and moisture. It is true
that intense electrical excitement generally accompanies tornadoes; but, while it is easy to see how this may be the effect of a commotion of the atmosphere, it is very difficult to understand, on the known principles of electricity, how it can be the cause of such violent phenomena. Electricity generally exists in nature in a state of equilibrium, and the discharges which we witness are due to the restoration of the equilibrium, while, on the other hand, as it appears to me, all the phenomena which are exhibited find a ready explanation on well-known thermal and pneumatic principles. Let us first consider the condition of the atmosphere previous to the coming on of the tornado. The air was close and sultry; that is, it was surcharged with moisture, which, absorbing the rays of the sun, rendered it unusually warm and abnormally light. If, in this condition, we suppose a stratum of colder wind from the northwest, the direction from which the meteor moved, to be passing above, we shall have a condition of atmosphere possessing the potential energy requisite to produce the phenomena observed. As the upper wind passed over the earth at a considerable elevation, the natural equilibrium would be distarbed, a heavier stratum being above, a lighter one below. The equilibrium would be of an unstable character, and the slightest irreg. ularity at a given spot would induce the rushing up of the air at the point of least resistance, and a descent around this point of the heavier stratum. The column of agitation would be more circumscribed if a whirling motion were given the mass, and the whole would be carried forward by the motion of the upper current. The moist air would rush in below from all sides, and, ascending in the vortex and mingling with the colder stratum above, would instantly be converted into visible vapor. If the moist stratum had been sufficiently thick and the upward motion sufficiently violent to carry the vapor above the snow-line of the latitude of the place, the drops of water would have been frozen, and probably thrown out on each side of the vortex, giring rise to two - tracks of hail. According to this hypothesis the electricity is due to the condensation of the vapor, or, more definitely, to the formation of a vertical water-conductur, of which the natural electricity is disturbed by the induction of the plus electricity of space, and the minus electricity of that of the earth below. The great mechanical effects which are exhibited in tornadoes are readily accounted for on the principle of continued pressure or a succession of impulses, as an illustration of which we may mention the effect produced by blowing on a light ball in a hollow tube. In this case the ball is followed by a continued pressure from one end of the tube to the other; at every moment it receives a new impulse, which it retains by its own inertia, and finally leaves the tube with the accumulated effect of the force which is applied to it through its whole course. In like manner, a stratum of air set in motion by the removal of pressure in front of it, while a pressare is continued in the rear, is impelled forward with an accumulating velocity, and finally acquires an energy sufficient to overcome obstacles of astonishing
resistance. The results will be the less surprising when we recollect that a cubic yard of air at the surface of the earth weighs about two pounds avoirdupois, and that, consequently, a stream of this fluid a quarter of a mile long, moving with high velocity, must possess an immense energy.

## EFFECT OF THE MOON ON THE WEATHER.

In answer to a letter on the subject.
Since the form of the orbit of the earth is affected by the attraction of Venus and the other planets, as well as by our satellite the moon, they inust in some degree also affect the form of the atmospheric covering of the globe, and tend to produce tides which are of greatest magnitude when they are in opposition or conjunction with the sun : but whether these disturbances of the atmosphere or those produced by the moon are of such a character as to give rise to the violent atmospheric commotious denominated storms, is a question which has long agitated the scientific world.

The times and peculiarities of the meteorological occurrences are more varied and less definitely remembered than almost any other natural phenomena, and hence the large number of different rules for predicting the changes of the weather. The only way of accurately ascertaining the truth of any hypothesis in regard to atmospheric changes, is that of having recourse to trustworthy records of the weather through a long series of years, and it is one of our objects in collecting meteorological statistics at the Sinithsonian Institution to obtain the means of proving or disproving propositions of the character you have advanced.

The moon, being the nearest body to the earth, produces the highest tide in the waters of the ocean, and must also produce the greater effect on the aerial covering of the earth. It has, however, not been satisfactorily proved that the occurrence of the lunar tides is connected with appreciable changes in the barometrical or thermometrical condition of the atmosphere. The less pressure of the air, at a given place, or account of the action of the moon, is just balanced by the increased height of the aerial column.

The principal causes of the violent changes of the atmosphere are, I think, due to its instability produced by the formation and condensation of vapor. It is not impossible, however, that when the air is in a very unstable condition on account of the heat and moisture of the lower strata, that the aerial tide may induce an overturning of the tottering equilibrium at some one place in the northern or southern hemisphere nore unstable than the others, and thus commence a storm which, but for this extraneous cause, would not have happened. To detect, therefore, the influence of the moon, it will be necessary to com-
pare simultaneously the records of the weather from day to day throughout all the northern and southern temperate zones, and to ascertain whether the maximum of these changes have any fixed relation in time to the changes of the moon. The fact that the prollem has not been considered from this point of view, may account for the failure, in the study of a series of records at a single place, to furnish evidence of the action of the moon.

The changes of the moon take place at a given moment on every part of the earth; the greatest effect of a lunar tide ought, therefore, to be felt in succession entirely around the earth in the course of about twenty. four and one-half hours.

The problem, however, has not been solved and cannot be determined by such casual observations as those which you narrate. I have not the least idea that the attraction of Venus produces any appreciable effect. It is too small to produce a result which would be indicated by any of our meterological instruments.

I am far from subscribing to the justice of your remarks in regard to Mr. Espy, since I have a great respect for his scientific character, notwithstanding his abberation, in a practical point of view, as to the economical production of rain. The fact has been abundantly proved by observation that a large fire sometimes produces an overturn in the unstable equilibrium of the atmosphere and gives rise to the beginning of a violent storm, but it was not wise in him to insist on the possibility of turning this principle to an economical use.-[J. H.]

## CONNECTION OF GALES OF WIND AND APPEARANCE OF THE AURORA.

By R. T. Knight, of Philadelphia.

"An officer of the British navy states that from eleven years" observation, six years in the Arctic regions and five years in the north of Scotland, he has ascertained that tremendous gales follow from twelve to twenty-four hours after the appearance of the aurora borealis." I never thought proper to call your attention to the above extract from the Philadelphia Ledger of the 4 th instant, because it agrees with what I published in 1864, and also in 1865.

Remarks.-We have had frequent communications from observers suggesting a connection in the time of the appearance of the aurora borealis and the occurrence of storms of wind and other meteorological phenomena; but on referring to our records we have never been able to verify the existence of such connection. Ou the receipt of the foregoing communication the records of the Institution were examined in relation to this subject, with the following results :

1. From the log-book of the brig Advance, Haven's Arctic expedition, forty-six appearances of the aurora were followed by four storms.
2. From the log-book of the yacht Fox, Sir Leopold McClintock's Arctic Exploration, eighty-nine appearances of the aurora were followed by eighteen storms within the time specified in the foregoing rule; or, in other words, the cases in favor of the rule were eighteen, while those agaiust it were seventy-one.
3. In an examination of the records of the observations of Professor Caswell at Providence, Rhode Island, it was found that in seventy-two cases the assumed rule failed, while ouly in seventeen cases did it ap. pear to be sustained.-[J. H.]

## ACCOUNT OF A STORM IN BUTLER COUNTY, KANSAS, JUNE 23, $\mathbf{1 8 7 1}$,

## By Wm, Harrison, of Eldorado, Kangas.

The storm came from the northwest, from the plains, striking the northwest corner of Butler County. It seemed to be about ten or twelve miles wide. Many forest-trees were blown down and twisted off; houses and crops were very much injured or entirely destroyed. The violence of the storm seemed to be greatest about the town of Eldorado, in which almost every house was more or less injured. I think at least fifty houses were entirely destroyed. The walls of the court-house, which are of stone, withstood the storm, but the roof, which was of tin, was blown off' entire, and covered up a blacksmith-shop about a huudred yards distant. Many people in Eldorado were injured, and two children were killed. The injury was not done by blowing people away, but by dashing them riolently to the earth. Its violence was so great that no one could stand on his feet. It passed Eldorado in a southeast direction, doing great injury to the crops, and blowing down almost every house which was directly in its path. The storm consisted of rain and hail as well as of wind. The rain was unprecedented in this region. No wooden-built house, however well constructed, was proof against its driving intensity. The water in the streets of Eldorado was a foot deep. I can form no estimate of the damage to buildings, fences, cattle, crops, $\& c$. , but it is very great. Almost every one in the path of the storm was more or less injured. One house was blown down in Chelsea. I had a small house in Eldorado which was demolished, a part of it carried three hundred yards to the river, and then carried down the stream.




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[^0]:    * Major General W. T. Sherman, member of committee, absent, in Europe.

[^1]:    "They were lovely and pleasant in their lives, And in their death they were not divided : They were swifter than eagles; they were stronger than lions."

[^2]:    *An abbreviation of Dominus, equivalent to the English prefix Reverend.-Translator.

[^3]:    * In allusion to the military traitor, Colonel Labedoyere, who was condemned to death for espousing the cause of Napoleon.-Translator.

[^4]:    *From the proceedings of the Royal Institution, London.

[^5]:    * Philosophical Transactions, 1833, p. 253.

[^6]:    *Edinburgh Royal Society Transactions, xiii, 1836, p. 88.
    $\dagger$ It was afterward isolated by P. Thenard.
    $\ddagger$ Quarterly Journal Science, ii, 1829, p. 83.

[^7]:    *Thomson, Annals of Philosophy, xii, 1826, p. 69.
    $\dagger$ Edinburgh Journal of Science, viii, 1828, p. 326.
    $\ddagger$ Edinburgh Royal Society Transactions, xi, 1837, p. 175.
    § Ibid., xiii, 1836, p. 297.

[^8]:    * Philosophical Transactions, 1837, p. 47.
    † Chemical Society Memoirs, i, p. 106.
    $\ddagger$ Chemical Society Journal, iii, p. 24.

[^9]:    * Philosophical Transactions, 1861, p. 373.

[^10]:    * Philosophicel Transactions, 1850, pp. 1, 805 ; 1851, p. 483.
    $\dagger$ Ibid., 1861, p. 183.

[^11]:    * Chemical Society Journal, xvii, p. 318.

[^12]:    "Pbĭlosophical Trlnsactions, 1854, p. 177.

[^13]:    *Philosophical Transactions, 1846, p. 573; 1849, p. 349.

[^14]:    * Quarterly Journal of Science, ii, 1829, p. 83.

[^15]:    * Edinburgh Royal Society Transactions, xii, 1834, p. 222.

[^16]:    *The motions of effusion under pressure, and of spontaneous diffusion, would appear to be alike traceable to the elasticity of the gas itself, exerted under the conditions to which it is exposed at the time.
    $\dagger$ Philosophical Transactions, 1863, p. 385.

[^17]:    *Chemical Society Journal, xvii, p 368.

[^18]:    * Philosophical Transactions, 1866, p. 399.

[^19]:    *Royal Society Proceedings, xv, p. 502; xvi, p. 422; xvii, p. 212, p. 500.

[^20]:    * With Indium, recently discovered, sixty-five.
    $\dagger$ In the latter part of November, 1864, the eighty-second of the asteroids, Alemene, was discovered. Add to this the eight large planets, and the whole number of planets known will amount to ninety. [1871, 120.]

[^21]:    * Most interesting discoveries have already been made. They are found in the work of W. Huggins and W. A. Miller, published April, 1864, in which the analysis of Aldebaran and a Orion is given, and proof furnished that certain nebulæ are incandescen globes of gas.

[^22]:    * Means for erecting new buildings for scientific institutes, and sualler sums for hospitals and geological collections.

[^23]:    * Zeitschrift fuir wissenschaftliche Zoölogie, 1853, vol. iv, pp. 53-76 and 454-456.

[^24]:    * Zeitschrift für wissenschaftliche Zoologie, vol. xiv, p. 394.
    $\dagger$ Vol. xv, p. 106.

[^25]:    * Zeitschrift für wissedschaftliche Zoologie, xiv, p. 400. Further investigation of this subject is communicated by Leuckart, in Troschel's Archiv., year XXXI, No. 3.
    $\dagger$ Traité d’Insectologie, tome I: Paris, 1845.
    $\ddagger$ On Parthenogenesis; a disoourse introductory to the Hunterian Lectures on generation and development for 1849. Delivered at the Royal College of Surgeons of England: London, 1849.

[^26]:    * Bienenzeitung, 1863, p. 222.
    $\dagger$ True Parthenogenesis, etc., p. 111.
    $\ddagger$ Zur Kenntniss des Generations wechselsund der Parthenogenese, etc., Frankfort, 1858, p. 51.
    § Also Luckart, idem, p. 45.
    \|Etudes et Considérations Générales sur la Parthénogénèse, (Annales des Sciences Naturales, XII, p 307.)

[^27]:    * Compt. Rend., 1861, tome 53, p. 1093.
    $\dagger$ Troschel's Archives, vol. 25, p. 208. Schizoneura seems to have only an oviparous fall generatiou.
    $\ddagger$ Zoölogist, 1859; and Entomol. Annual for 1860, p. 87.
    $\S$ Proceedings Entomological Society, 1859, p. 86; Smith in Entomol. Annual for 1861, p. 39.
    \| Transactions of Linn. Society, 1802, vol. 6, p. 288.

[^28]:    * Compt. Rend., lv, 1862, p. 106.
    $\dagger$ Berliner Entom. Zeitschrift, viii, p. 93.
    $\ddagger$ C. Th. v. Siebold on Androgynous Bees, Zeitschrift für wissenschaftliche Zoologie, vol. xiv, No. 1, and in the Eichstädter Bienenzeitung, year xix, p. 223.

[^29]:    * Introduction to a memoir to be published in the "Smithsonian Contributions to Knowledge."

[^30]:    * See a note by M. Prouhet, appended to Vol. II of Sturm's Cours d'Analyse de l'École Polytechnique.

[^31]:    * See formula (G) in Appendix I.

[^32]:    *In all the terms of series (d), (e), and ( $f$ ), the fifth figure might as well have been neglected. It has no real value, and does not assist the graduation.

[^33]:    * The constant difference of the abscissas or arguments is here assumed to be unity. But if we wish to regard it as any other quantity $h$, we shall merely have to substitute, in the final equation, $\frac{x}{h}$ in the place of $x$.

[^34]:    * This formula can also be written-

    $$
    S=\frac{n}{2}\left(S_{1}+S_{2}+\dot{n} \Delta_{1}-\frac{1-n^{2}}{3} \Delta_{2}\right)
    $$

[^35]:    * For the purposes of our present method, it will be most convenient to write-

    $$
    \begin{aligned}
    & y=A+\frac{\pi}{N}\left\{B_{1} \sin \left(\frac{2 \pi x}{N}\right)+C_{1} \cos \left(\frac{2 \pi x}{N}\right)\right\}+\frac{2 \pi}{N}\left\{B_{2} \sin 2\left(\frac{2 \pi x}{N}\right)\right. \\
    & \left.+C_{2} \cos 2\left(\frac{2 \pi x}{N}\right)\right\}+{ }_{N}^{3 \pi}\left\{B_{3} \sin 3\left(\frac{2 \pi x}{N}\right)+C_{3} \cos 3\left(\frac{2 \pi x}{N}\right)\right\}+\& c
    \end{aligned}
    $$

[^36]:    * There is still another method of interpolation, devised by Cauchy, which can be used in cases of this kind. It is, however, more laborious than the method here proposed, and trials which have been made indicate that it does not secure any greater accuracy. For some account of it, see the American Journal of Science for July, 1862, and Lionville's Journal, vol. 18, page 299.

[^37]:    * Subsequent trials have shown that it has none.

[^38]:    * Thís formula may be used when the law of a given series varies so rapidly that five consecutive terms cannot be regarded as forming a series of an order not higher than the third.

[^39]:    * In like manner, it can be shown that formulas (48), (49), and (50) are in accordance with the principle of least squares.

[^40]:    *The Klamath River Indians often made arrow-heads from broken junk-bottles.-G.

[^41]:    "I find this view still further supported in the interesting lecture of Professor Fr. Müller on the inhabitants of Alaska, in which he points to the similarity of religions views in the northeastern tribes of Asia and the Indians of Alaska.

[^42]:    * I mnst state, however, that Lyell assigns much less antiquity to the American than to the European mastodon.

