ANALYSIS OF SYMMETRICAL AND UNSYMMETRICAL CURVED MEMBER FRAMES BY MOMENT DISTRIBUTION

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DISTRIBUTION

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PREFACE

The purpose of this report is to present a new method of analyzing frames with unsymmetrical curved members by moment distribution. The slope deflection equations were derived using virtual work and relating the reactive elements to a sloped axis passing through the elastic center of the member.

In completing this final requirement for the degree of Master of Science in Civil Engineering, I wish to gratefully express my indebtedness:

To Professor Jan Tuma, for the outline of derivation of the slope deflection equations in this report and for his helpful advice throughout the graduate year.

To my wife, Roberta, whose encouragement, consideration, and understanding were the greatest forces leading me to the completion of this work.

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Stillwater, Oklahoma

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NOMENCLATURE

M_{AB}, M_{BA}	Moments at left and right ends of curved bar AB
GM_{AB}, GM_{BA}	Fictitious moments at ends of axis A'B'
$FM_{AB}^{(Load)}$, $FM_{BA}^{(Load)}$	Fixed end moments at ends of curved bar AB due to applied loads
$FM_{AB}^{(\Delta_x)}$, $FM_{BA}^{(\Delta_x)}$	Fixed end moments due to Δ_x
$FM_{AB}^{(\Delta_y)}$, $FM_{BA}^{(\Delta_y)}$	Fixed end moments due to Δ_y
$EM_{BA}^{(Load)}$, $EM_{BA}^{(\Delta_x)}$, $EM_{BA}^{(\Delta_y)}$	Modified fixed end moments of end B when end A is hinged
H	Thrust of curved bar AB
$FH^{(Load)}$, $FH^{(\Delta_x)}$, $FH^{(\Delta_y)}$	Fixed end thrusts due to loads, Δ_x and Δ_y
$EH^{(Load)}$, $EH^{(\Delta_x)}$, $EH^{(\Delta_y)}$	Modified fixed end thrusts when end A is hinged
R_{Ax}, R_{Bx}	Horizontal reactions at ends of curved bar AB
R_{Ay}, R_{By}	Vertical reactions at ends of curved bar AB
N_x, T_x, M_x	Cross-sectional elements of curved bar AB (Normal force, tangential force, and bending moment)
θ_A, θ_B	Angular rotations of ends A and B
Δ_{Ax}, Δ_{Bx}	Horizontal displacements of ends A and B
Δ_{Ay}, Δ_{By}	Vertical displacements of ends A and B

Δ_x, Δ_y	Relative horizontal and vertical displacements, respectively, of ends A and B
Basic structure	Simple curved bar AB with roller at A
$BR_{Ay}, BR_{By}, BR_{Bx}$	Reactions of the basic structure
BH_x, BV_x, BM_x	Cross-sectional elements of the basic structure (horizontal force, vertical force, and bending moment)
$B\theta_A, B\theta_B, B\Delta_{Ax}$	Displacements of the basic structure
c, d, f, L	Dimensions of curved bar AB
e_A, e_B	Lengths of fictitious rigid arms at ends A and B
a, e	Horizontal and vertical distances, respectively, of the elastic center of the curved bar from end A
α	Slope of the base line ($\overline{AB}$) of the curved bar
β	Slope of the axis A'B'
φ	Slope of the tangent to the bar
ψ_{AB}	Angular rotation of the base line $\overline{AB}$
I	Moment of inertia of the bar at any section
E	Modulus of elasticity
$K_{AB}^{(\theta)}, K_{BA}^{(\theta)}$	Angular stiffness factors of ends A and B, respectively
C_{AB}, C_{BA}	Carry-over factors of ends A and B
$K_{AB}^{(H)}, K_{BA}^{(H)}$	Sidesway stiffness factors of ends A and B

$K_{AB}^{(\psi)}, K_{BA}^{(\psi)}$ Downsway stiffness factors of ends A and B

$K_{BA}^{(\theta)'}, K_{BA}^{(H)'}, K_{BA}^{(\psi)'}$ Modified stiffness factors of end B when end A is hinged

$K_{AB}^{(\theta)''}$ Modified angular stiffness factor of a symmetrical curved bar with a symmetrical system of loads

$K_{AB}^{(\theta)'''}$ Modified angular stiffness factor of a symmetrical curved bar with an antisymmetrical system of loads

SIGN CONVENTION

Reactive elements: The sign convention of statics is used for all end moments and all horizontal and vertical reactions with the exception of the thrust in a curved bar, which is considered positive if it is a compressive force.

Cross-sectional elements: The sign convention of deformation is used for all cross-sectional elements.

Displacements: The following displacements are considered positive: (a) clockwise angular rotations, (b) a relative horizontal displacement causing a positive fixed end moment in the left end of a curved bar, (c) a relative vertical displacement causing positive fixed end moments in a curved bar, and (d) a relative tangential displacement causing positive fixed end moments in a straight bar.

PART I

DERIVATION OF SLOPE DEFLECTION EQUATIONS

1. Statics. A fixed end, unsymmetrical curved bar of variable cross section acted upon by a general system of loads will be considered (Fig. 1). From the equations of static equilibrium, R_{Ay}, R_{By}, and R_{Bx} may be expressed in terms of R_{Ax}, M_{AB}, and M_{BA}:

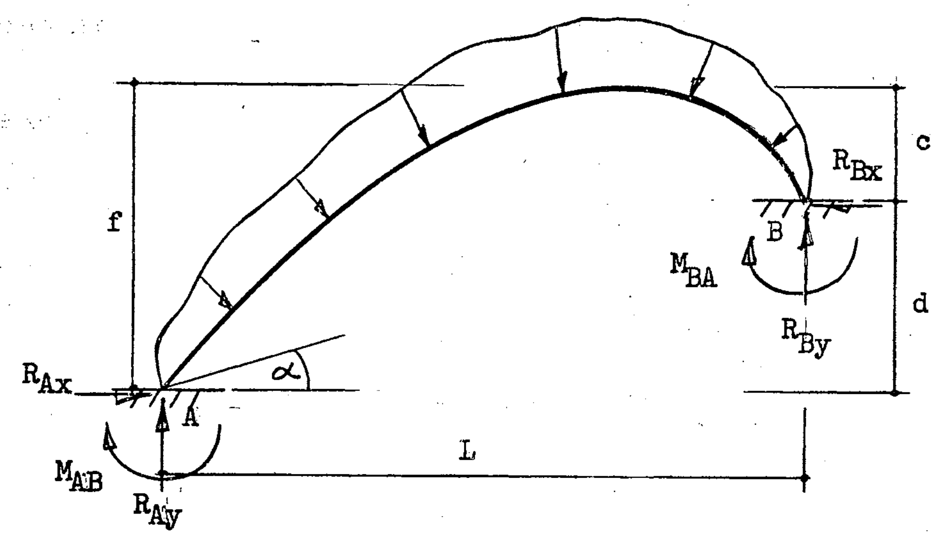

Fig. 1

$$(1)\quad R_{Ay} = BR_{Ay} - \frac{M_{AB} + M_{BA}}{L} + R_{Ax} \tan \alpha$$

$$(2)\quad R_{By} = BR_{By} + \frac{M_{AB} + M_{BA}}{L} - R_{Ax} \tan \alpha$$

(3) $R_{Bx} = R_{Ax} + BR_{Bx}$

Where BR_{Ay}, BR_{By}, and BR_{Bx} are the reactions of the basic structure (a simple curved bar with a roller at A).

The reactive elements may be related to any sloped axis passing through the elastic center of the bar (Fig. 2). If this axis is properly chosen, the derivation and final form of the slope deflection equations can be greatly simplified.

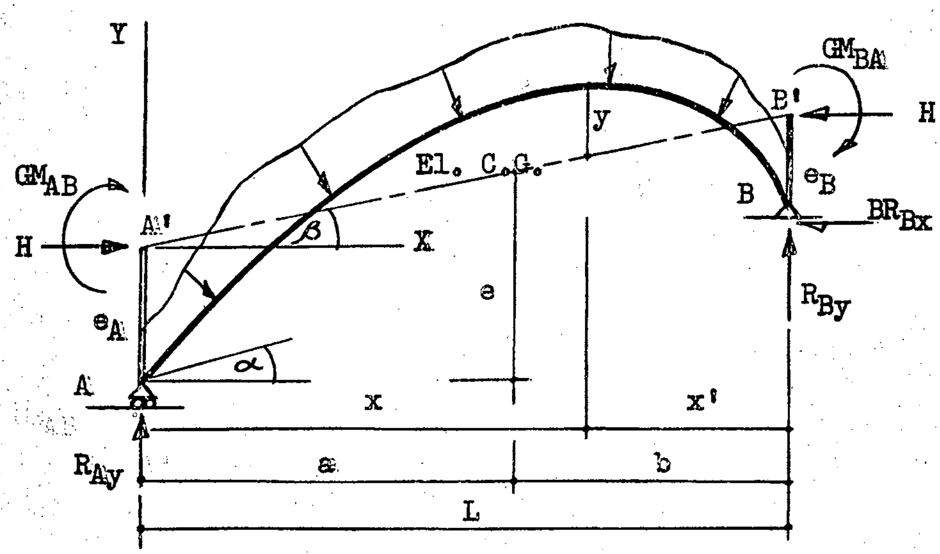

Fig. 2

Selecting the fictitious reactions GM_{AB}, GM_{BA}, and H as the redundants, R_{Ay} and R_{By} become:

$$(4)\quad R_{Ay} = BR_{Ay} - \frac{GM_{AB} + GM_{BA}}{L} + H \tan\beta$$

$$(5)\quad R_{By} = BR_{By} + \frac{GM_{AB} + GM_{BA}}{L} - H \tan\beta$$

The relationships between the real reactions and the fictitious reactions are:

(6) $R_{Ax} = H$

(7) $M_{AB} = GM_{AB} + He_A$

(8) $M_{BA} = GM_{BA} - He_B$

The cross-sectional elements in terms of the fictitious reactions are:

(9) $N_{x = 0 \to L}^{(A)} = - H \dfrac{\cos(\varphi - \beta)}{\cos\beta} - BH_x \cos\varphi - \left(BV_x + \dfrac{GM_{AB} + GM_{BA}}{L}\right) \sin\varphi$

(10) $T_{x = 0 \to L}^{(A)} = - H \dfrac{\sin(\varphi - \beta)}{\cos\beta} - BH_x \sin\varphi + \left(BV_x + \dfrac{GM_{AB} + GM_{BA}}{L}\right) \cos\varphi$

(11) $M_{x = 0 \to L}^{(A)} = GM_{AB} \dfrac{x'}{L} - GM_{BA} \dfrac{x}{L} - Hy + BM_x$

Where φ is the slope of the tangent to the bar at any point, and BH_x, BV_x, and BM_x are the cross sectional elements of the basic structure.

2. Virtual work. The strain energy equation of the bar is:

(12) $U_{Loads} + \sum Q_i q_i = \int_A^B \dfrac{N^2 ds}{2AE} + \int_A^B \dfrac{V^2 ds}{2AG} + \int_A^B \dfrac{M^2 ds}{2EI}$

Where q_i is the displacement corresponding to the reactive element Q_i. Allowing a general displacement of supports, the external work of reactive elements becomes:

(13) $\sum Q_i q_i = GM_{AB}\, \theta_A + H(e_A\theta_A + \Delta_{Ax} - e_B\theta_B - \Delta_{Bx})$

$+ GM_{BA}\, \theta_B - R_{Ay}\Delta_{Ay} - R_{By}\, \Delta_{By}$

The rates of change of reactive elements with respect to the redundants are tabulated in Table I.

Q_i	$\frac{\partial Q_i}{\partial GM_{AB}}$	$\frac{\partial Q_i}{\partial GM_{BA}}$	$\frac{\partial Q_i}{\partial H}$
GM_{AB}	1	0	0
GM_{BA}	0	1	0
H	0	0	1
R_{Ay}	$-\frac{1}{L}$	$-\frac{1}{L}$	$+\tan\beta$
R_{By}	$+\frac{1}{L}$	$+\frac{1}{L}$	$-\tan\beta$

TABLE I

Considering the deformations due to normal and shearing forces to be negligible, the equations of virtual work are:

$$(14)\quad \frac{\partial U}{\partial GM_{AB}} = 0 = \int_A^B M \frac{\partial M}{\partial GM_{AB}} \frac{ds}{EI} - \sum \frac{\partial Q_i}{\partial GM_{AB}} q_i$$

$$(15)\quad \frac{\partial U}{\partial GM_{BA}} = 0 = \int_A^B M \frac{\partial M}{\partial GM_{BA}} \frac{ds}{EI} - \sum \frac{\partial Q_i}{\partial GM_{BA}} q_i$$

$$(16)\quad \frac{\partial U}{\partial H} = 0 = \int_A^B M \frac{\partial M}{\partial H} \frac{ds}{EI} - \sum \frac{\partial Q_i}{\partial H} q_i$$

From equation (11) the partial derivatives of the bending moment with respect to GM_{AB}, GM_{BA}, and H are $\frac{x'}{L}$, $-\frac{x}{L}$, and $-y$, respectively. Using these relationships and those expressed in TABLE I, equations (14), (15), and (16) become:

$$(17)\quad GM_{AB}\int_A^B \frac{x'^2}{L^2}\frac{ds}{EI} - GM_{BA}\int_A^B \frac{xx'}{L^2}\frac{ds}{EI} - H\int_A^B \frac{x'y}{L}\frac{ds}{EI} + \int_A^B \frac{(BM_x)x'}{L}\frac{ds}{EI} = \theta_A + \frac{\Delta_{Ay} - \Delta_{By}}{L}$$

$$(18)\quad -GM_{AB}\int_A^B \frac{xx'}{L^2}\frac{ds}{EI} + GM_{BA}\int_A^B \frac{x^2}{L^2}\frac{ds}{EI} + H\int_A^B \frac{xy}{L}\frac{ds}{EI} - \int_A^B \frac{(BM_x)x}{L}\frac{ds}{EI} = \theta_B + \frac{\Delta_{Ay} - \Delta_{By}}{L}$$

$$(19)\quad -GM_{AB}\int_A^B \frac{x'y}{L}\frac{ds}{EI} + GM_{BA}\int_A^B \frac{xy}{L}\frac{ds}{EI} + H\int_A^B \frac{y^2\,ds}{EI} - \int_A^B (BM_x)y\frac{ds}{EI} = e_A\theta_A - e_B\theta_B + \Delta_{Ax} - \Delta_{Bx} - (\Delta_{Ay} - \Delta_{By})\tan\beta$$

Denoting:

$$C_1 = \int_A^B \frac{x^2}{L^2}\frac{ds}{EI} \qquad \Delta_x = \Delta_{Ax} - \Delta_{Bx}$$

$$C_2 = \int_A^B \frac{xx'}{L^2}\frac{ds}{EI} \qquad \psi_{AB} = \frac{\Delta y}{L} = \frac{\Delta_{Ay} - \Delta_{By}}{L}$$

$$C_3 = \int_A^B \frac{x'^2}{L^2} \frac{ds}{EI}$$

$$C_4 = \int_A^B y^2 \frac{ds}{EI}$$

$$C_6 = \int_A^B \frac{xy}{L} \frac{ds}{EI}$$

$$C_7 = \int_A^B \frac{x'y}{L} \frac{ds}{EI} = -C_6$$

$$B\Delta_{Ax} = \int_A^B (BM_x)\, y \frac{ds}{EI}$$

$$B\theta_A = \int_A^B \frac{(BM_x)x'}{L} \frac{ds}{EI}$$

$$B\theta_B = \int_A^B \frac{(BM_x)x}{L} \frac{ds}{EI}$$

The virtual work equations may be written:

(20) $GM_{AB}\, C_3 - GM_{BA}\, C_2 + HC_6 + B\theta_A = \theta_A + \psi_{AB}$

(21) $-GM_{AB}\, C_2 + GM_{BA}\, C_1 + HC_6 - B\theta_B = \theta_B + \psi_{AB}$

(22) $GM_{AB}\, C_6 + GM_{BA}\, C_6 + HC_4 - B\Delta_A = e_A\, \theta_A - e_B\, \theta_B$

$$+\Delta_x - \Delta_y \tan\beta$$

3. Selection of Axis. If $C_6 = 0$, the solution of equations (20), (21), and (22) will be greatly simplified. The value of $\tan\beta$ must be found which will satisfy this desired condition.

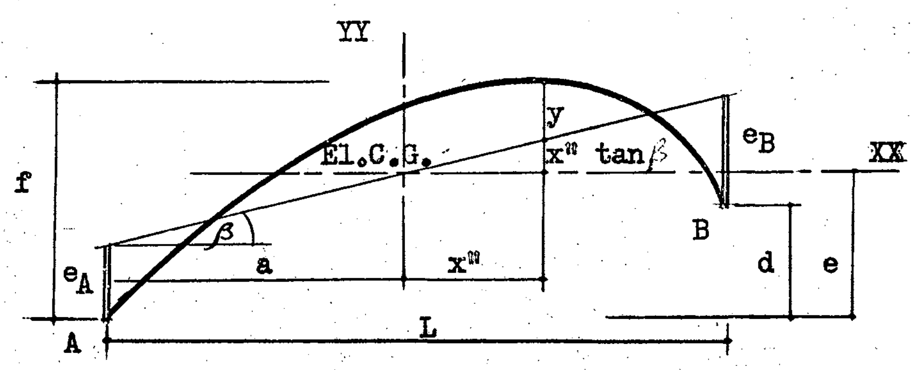

Fig. 3

$$\int_A^B xy \ \frac{ds}{EI} = 0 = \int_A^B (x'' + a)(y'' - x'' \tan\beta) \ \frac{ds}{EI}$$

$$0 = \int_A^B x''y'' \ \frac{ds}{EI} - \tan\beta \int_A^B x''^2 \ \frac{ds}{EI}$$

Denoting:

$$I_{xy} = \int_A^B x''y'' \ \frac{ds}{EI} \qquad\qquad I_{yy} = \int_A^B x''^2 \ \frac{ds}{EI}$$

(23) $\tan\beta = \frac{I_{xy}}{I_{yy}}$

4. Equations of reactive elements. Selecting $\tan\beta$ to satisfy equation (23), the virtual work equations become:

(24) $GM_{AB}\, C_3 - GM_{BA}\, C_2 = \theta_A + \psi_{AB} - B\theta_A$

(25) $-GM_{AB}\, C_2 + GM_{BA}\, C_1 = \theta_B + \psi_{AB} + B\theta_B$

(26) $HC_4 = e_A\theta_A - e_B\theta_B + \Delta_x - \Delta_y \tan\beta + B\Delta_{Ax}$

Solving these equations simultaneously for the redundants and denoting $N = C_1C_3 - C_2C_2$,

(27) $GM_{AB} = \frac{C_1}{N}\theta_A + \frac{C_2}{N}\theta_B + \frac{C_1 + C_2}{N}\psi_{AB} - \frac{C_1\, B\theta_A - C_2\, B\theta_B}{N}$

(28) $GM_{BA} = \frac{C_3}{N}\theta_B + \frac{C_2}{N}\theta_A + \frac{C_3 + C_2}{N}\psi_{AB} + \frac{C_3\, B\theta_B - C_2\, B\theta_A}{N}$

(29) $H = \frac{e_A}{C_4}\theta_A - \frac{e_B}{C_4}\theta_B + \frac{\Delta x}{C_4} - \frac{\Delta y}{C_4}\tan\beta + \frac{B\Delta Ax}{C_4}$

The equations of the reactive moments are:

$$(30)\quad M_{AB} = \left(\frac{C_1}{N} + \frac{e_A^2}{C_4}\right)\theta_A + \left(\frac{C_2}{N} - \frac{e_A e_B}{C_4}\right)\theta_B + \frac{e_A}{C_4}\Delta_x + \left(\frac{C_1 + C_2}{N} - \frac{e_A L}{C_4}\tan\beta\right)\psi_{AB} + FM_{AB}^{(Load)}$$

$$(31)\quad M_{BA} = \left(\frac{C_3}{N} + \frac{e_B^2}{C_4}\right)\theta_B + \left(\frac{C_2}{N} - \frac{e_A e_B}{C_4}\right)\theta_A - \frac{e_B}{C_4}\Delta_x + \left(\frac{C_3 + C_2}{N} + \frac{e_B L}{C_4}\tan\beta\right)\psi_{AB} + FM_{BA}^{(Load)}$$

Where:

$$(32)\quad FM_{AB}^{(Load)} = -\frac{C_1\,B\theta_A - C_2\,B\theta_B}{N} + \frac{B\Delta_{Ax}}{C_4}\,e_A$$

$$(33)\quad FM_{AB}^{(Load)} = +\frac{C_3\,B\theta_B - C_2\,B\theta_A}{N} - \frac{B\Delta_{Ax}}{C_4}\,e_B$$

It is readily observable that equations (30) and (31) are similar to the general slope deflection equations for symmetrical curved bars derived by Ungson[1], and that they reduce to these equations when applied to a symmetrical bar, $\tan\beta$ being zero.

5. Conclusions. By relating the reactive elements of any curved bar to an axis passing through the elastic center of the bar (the slope being defined by equation (23)) the general slope deflection equations

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1 Rafael G. Ungson, "Simple Slope Deflection Equations for Symmetrical Arch Structures" (unpub. M.S. report, Oklahoma A. and M. College, 1955), p. 11.

can easily be derived.

These equations are similar to those for symmetrical curved bars, having one additional term due to the unsymmetry of the bar.

By the use of these equations the solution of complex frame structures can be greatly simplified.

PART II

THE MOMENT DISTRIBUTION METHOD

6. Introduction. The application of the moment distribution method to the analysis of frames with straight members is well known. The modification of this method to the analysis of frames with curved members consists of the following steps:

a. Determine stiffness, carry-over, and distribution factors

b. Determine fixed end moments and thrusts due to loads, horizontal displacements, and vertical displacements

c. Distribute moments due to loads and to each horizontal and each vertical displacement

d. Write as many independent shear equations as there are unknown displacements, and solve these equations simultaneously

e. Compute the final thrusts and moments.

The basic difference is in the determination of shear equations. The horizontal shearing force (thrust) in a curved bar is redundant and cannot be computed directly from the end moments as can the shear in a straight bar. A new shear factor λ must be derived and three basic thrusts must be determined:

a. Fixed end thrust due to the applied loads

b. Fixed end thrust due to the relative horizontal displacement Δ_x.

c. Fixed end thrust due to the relative vertical displacement Δ_y.

7. Moment Distribution Constants. Each of the equations of reactive moments (slope deflection equations) derived in PART I consists of five basic moments:

a. Fixed end moment due to the applied loads

b. Moment due to the rotation θ_A of the left end

c. Moment due to the rotation θ_B of the right end

d. Fixed end moment due to the relative horizontal displacement Δ_x

e. Fixed end moment due to the relative vertical displacement Δ_y.

The fixed end moments due to the loads are defined by equations (32) and (33). Considering the unloaded fixed end bar and allowing only a unit rotation of the left end A, the angular stiffness and carry-over factors for that end can be determined from equations (30) and (31). The angular stiffness and carry-over factors for end B, the fixed end moments due to Δ_x, and the fixed end moments due to Δ_y can be determined from equations (30) and (31) by a similar method.

The fixed end moments due to $\Delta_x = 1$ are defined as the sidesway stiffness factors. The fixed end moments due to $\psi_{AB} = 1$ are defined as the downsway stiffness factors.

The angular stiffness factors are:

$$(34)\quad K_{AB}^{(\theta)} = \frac{C_1}{N} + \frac{e_A^{\,2}}{C_4} \qquad K_{BA}^{(\theta)} = \frac{C_3}{N} + \frac{e_B^{\,2}}{C_4}$$

The carry-over moment is:

$$(35)\quad M^{(c)} = \frac{C_2}{N} - \frac{e_A e_B}{C_4}$$

The carry-over factors are:

$$(36)\quad C_{AB} = \frac{M^{(c)}}{K^{(\theta)}_{AB}} \qquad\qquad C_{BA} = \frac{M^{(c)}}{K^{(\theta)}_{BA}}$$

The sidesway stiffness factors are:

$$(37)\quad K^{(H)}_{AB} = \frac{e_A}{C_4} \qquad\qquad K^{(H)}_{BA} = -\frac{e_B}{C_4}$$

The downsway stiffness factors are:

$$(38)\quad K^{(\psi)}_{AB} = \frac{C_1 + C_2}{N} - \frac{e_A L}{C_4}\tan\beta \qquad\qquad K^{(\psi)}_{BA} = \frac{C_3 + C_2}{N} + \frac{e_B L}{C_4}\tan\beta$$

Using these notations, equations (30) and (31) may be written:

$$(39)\quad M_{AB} = K^{(\theta)}_{AB}\,\theta_A + C_{BA}K^{(\theta)}_{BA}\,\theta_B + FM^{(\Delta x)}_{AB} + FM^{(\Delta y)}_{AB} + FM^{(Load)}_{AB}$$

$$(40)\quad M_{BA} = K^{(\theta)}_{BA}\,\theta_B + C_{AB}K^{(\theta)}_{AB}\,\theta_A + FM^{(\Delta x)}_{BA} + FM^{(\Delta y)}_{BA} + FM^{(Load)}_{BA}$$

Where:

$$(41)\quad FM^{(\Delta x)}_{AB} = K^{(H)}_{AB}\,\Delta_x \qquad\qquad FM^{(\Delta x)}_{BA} = K^{(H)}_{BA}\,\Delta_x$$

$$(42)\quad FM^{(\Delta y)}_{AB} = K^{(\psi)}_{AB}\,\psi_{AB} \qquad\qquad FM^{(\Delta y)}_{BA} = K^{(\psi)}_{BA}\,\psi_{AB}$$

8. <u>Modified Constants</u> - <u>Hinged End</u>. If end A of an unsymmetrical curved bar AB is hinged, no moment can be developed at that end. Thus $M_{AB} = 0$ in equation (39) and

$$(43)\quad \theta_A = -\frac{C_{BA}K^{(\theta)}_{BA}}{K^{(\theta)}_{AB}}\,\theta_B - \frac{K^{(H)}_{AB}}{K^{(\theta)}_{AB}}\Delta_x - \frac{K^{(\psi)}_{AB}}{K^{(\theta)}_{AB}}\,{}_{AB} - \frac{FM_{AB}}{K^{(\theta)}_{AB}}$$

Using this relationship, equation (40) becomes:

$$(44)\quad M_{BA} = K_{BA}^{(\theta)}(1 - C_{AB}C_{BA})\,\theta_B + (K_{BA}^{(H)} - C_{AB}K_{AB}^{(H)})\Delta_x + (K_{BA}^{(\psi)} - C_{AB}K_{AB}^{(\psi)})\psi_{AB} + FM_{BA} - C_{AB}FM_{AB}$$

$K_{BA}^{(\theta)}(1 - C_{AB}C_{BA})$ is defined as the modified angular stiffness factor of end B of the curved bar and is denoted by $K_{BA}^{(\theta)'}$.

Similarly, the modified sidesway and downsway stiffness factors of end B are, respectively, $K_{BA}^{(H)} - C_{AB}K_{AB}^{(H)}$ and $K_{BA}^{(\psi)} - C_{AB}K_{AB}^{(\psi)}$, denoted by $K_{BA}^{(H)'}$ and $K_{BA}^{(\psi)'}$.

$FM_{BA} - C_{AB}FM_{AB}$ is defined as the modified fixed end moment of end B and is denoted by EM_{BA}. $K_{BA}^{(H)'}\Delta_x$ and $K_{BA}^{(\psi)'}\psi_{AB}$ are defined as the modified sidesway and downsway fixed end moments and are denoted by $EM_{BA}^{(\Delta x)}$ and $EM_{BA}^{(\Delta y)}$.

9. <u>Thrust</u> <u>equation</u>. In applying the method of moment distribution to curved bars, it is necessary to obtain an expression for the thrust H in terms of the moments due to rotation of joints. From equations (29), (39), and (40):

$$(45)\quad H^{(\theta)} = \frac{e_A}{C_4}\theta_A - \frac{e_B}{C_4}\theta_B$$

$$(46)\quad M_{AB}^{(\theta)} = K_{AB}^{(\theta)}\theta_A + C_{BA}K_{BA}^{(\theta)}\theta_B$$

$$(47)\quad M_{BA}^{(\theta)} = K_{BA}^{(\theta)}\theta_B + C_{AB}K_{AB}^{(\theta)}\theta_A$$

Solving equations (46) and (47) simultaneously for θ_A and θ_B and substituting into equation (45):

$$(48)\quad H^{(\theta)} = \frac{\lambda_A}{C_4}M_{AB}^{(\theta)} - \frac{\lambda_B}{C_4}M_{BA}^{(\theta)}$$

Where:

$$(49)\quad \lambda_A = \frac{e_A}{K_{AB}^{(\theta)}} + \frac{e_B C_{AB}}{K_{BA}^{(\theta)'}}$$

$$(50)\quad \lambda_B = \frac{e_B}{K_{BA}^{(\theta)'}} + \frac{e_A C_{BA}}{K_{AB}^{(\theta)'}}$$

Using this relationship for $H^{(\theta)}$, equation (29) becomes:

$$(51)\quad H = \frac{\lambda_A}{C_4} M_{AB}^{(\theta)} - \frac{\lambda_B}{C_4} M_{BA}^{(\theta)} + FH^{(\Delta x)} + FH^{(\Delta y)} + FH^{(Load)}$$

Where:

$$(52)\quad FH^{(\Delta x)} = \frac{\Delta x}{C_4} \qquad FH^{(\Delta y)} = -\frac{\Delta y}{C_4}\tan\beta \qquad FH^{(Load)} = \frac{B\Delta Ax}{C_4}$$

10. Modified Thrust Equation - Hinged End. From equations (39), (46), and (47) and the conditions of Article 8.

$$(53)\quad M_{AB}^{(\theta)} = -FM_{AB}^{(\Delta x)} - FM_{AB}^{(\Delta y)} - FM_{AB}^{(Load)}$$

$$(54)\quad M_{BA}^{(\theta)} = M_{BA}^{(\theta_B)} + C_{AB} M_{AB}^{(\theta)}$$

Substituting these expressions for $M_{AB}^{(\theta)}$ and $M_{BA}^{(\theta)}$ into equation (51):

$$(55)\quad H = -\frac{\lambda_B}{C_4} M_{BA}^{(\theta_B)} + EH^{(\Delta x)} + EH^{(\Delta y)} + EH^{(Load)}$$

Where:

$$(56)\quad EH^{(\Delta x)} = \left(1 - e_A \frac{K_{AB}^{(H)}}{K_{AB}^{(\theta)}}\right)\frac{\Delta x}{C_4}$$

$$(57)\quad EH^{(\Delta y)} = -\left(\tan\beta + \frac{e_A}{L}\;\frac{K_{AB}(\psi)}{K_{AB}(\theta)}\right)\frac{\Delta y}{C_4}$$

$$(58)\quad EH^{(Load)} = FH^{(Load)} - \frac{K_{AB}^{(H)}}{K_{AB}^{(\theta)}}\; FM_{AB}^{(Load)}$$

Are the modified fixed end thrusts.

Equation (55) is the thrust equation for an unsymmetrical curved bar AB when end A is hinged.

11. Symmetrical Curved Bars. In the case of a symmetrical curved bar of variable moment of inertia: $C_1 = C_3$, $e_A = e_B = e$, and $\tan\beta = 0$. Therefore, from equations (34) through (38), the stiffness and carry-over factors become:

$$(59) \quad K_{AB}^{(\theta)} = K_{BA}^{(\theta)} = K^{(\theta)} = \frac{C_1}{N} + \frac{e^2}{C_4}$$

$$(60) \quad M^{(c)} = \frac{C_2}{N} - \frac{e^2}{C_4}$$

$$(61) \quad C_{AB} = C_{BA} = C = \frac{M^{(c)}}{K^{(\theta)}}$$

$$(62) \quad K_{AB}^{(H)} = - K_{BA}^{(H)} = K^{(H)} = \frac{e}{C_4}$$

$$(63) \quad K_{AB}^{(\psi)} = K_{BA}^{(\psi)} = K^{(\psi)} = \frac{C_1 + C_2}{N}$$

If end A of a symmetrical curved bar AB is hinged, the modified stiffness factors of end B are, from equations (44) and (59) through (63):

$$(64) \quad K_{BA}^{(\theta)'} = K^{(\theta)'} = K^{(\theta)} \; (1 - C^2)$$

$$(65) \quad K_{BA}^{(H)'} = - K^{(H)'} = - K^{(H)} \; (1 + C)$$

$$(66) \quad K_{BA}^{(\psi)'} = K_{BA}^{(\psi)} = K^{(\psi)} \; (1 - C)$$

From equations (49) through (52), the equation for thrust in a symmetrical curved bar is:

$$(67) \quad H = \frac{K^{(H)'}}{K^{(\theta)'}} \left(M_{AB}^{(\theta)} - M_{BA}^{(\theta)} \right) + FH^{(\Delta x)} + FH^{(Load)}$$

From equation (55), the equation for thrust in a symmetrical curved bar AB when end A is hinged is:

$$(68) \quad H = -\frac{K^{(H)'}}{K^{(\theta)'}} M_{BA}^{(\theta_B)} + EH^{(\Delta x)} + EH^{(\Delta y)} + EH^{(Load)}$$

Where the modified fixed end thrusts are, from equations (56), (57), and (58):

$$(69) \quad EH^{(\Delta x)} = \left(1 - e\,\frac{K^{(H)}}{K^{(\theta)}}\right)\frac{\Delta x}{C_4}$$

$$(70) \quad EH^{(\Delta y)} = -\frac{K^{(H)}K^{(\psi)}}{K^{(\theta)}}\,\psi_{AB}$$

$$(71) \quad EH^{(Load)} = FH^{(Load)} - \frac{K^{(H)}}{K^{(\theta)}} FM_{AB}^{(Load)}$$

PART III

PARABOLIC ARCHES

12. <u>Constants</u>. From an evaluation of the equations of PART II, the following tables are obtained for unsymmetrical parabolic arches (Fig. 4) having the variation:

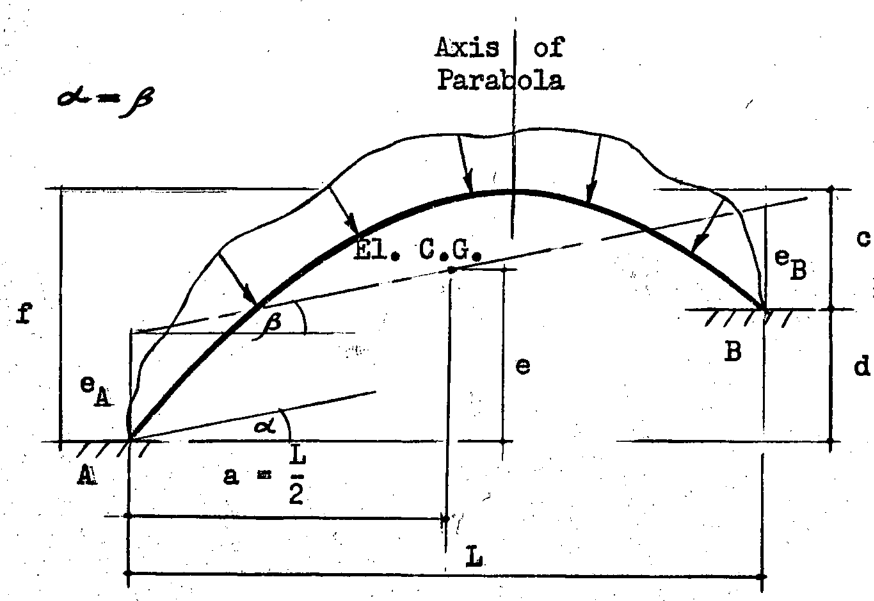
$$I_x = I_o \sec \varphi$$

Where: I_x = The moment of inertia at any section

I_o = The moment of inertia at the crown

φ = The slope of the tangent to the bar at x

In the tables $\sqrt{\frac{c}{f}}$ is denoted by γ.



<u>Fig. 4</u>

$C_1 = \int_0^L \frac{x^2}{L^2} \frac{ds}{EI}$	$\frac{L}{3EI_o}$
$C_2 = \int_0^L \frac{xx'}{L^2} \frac{ds}{EI}$	$\frac{L}{6EI_o}$
$C_3 = \int_0^L \frac{x'^2}{L^2} \frac{ds}{EI}$	$\frac{L}{3EI_o}$
$N = C_1C_3 - C_2C_2$	$\frac{L^2}{12(EI_o)^2}$
$\bar{x} = a = \frac{\int_0^L x \frac{ds}{EI}}{\int_0^L \frac{ds}{EI}}$	$\frac{L}{2}$
$\bar{y} = e = \frac{\int_0^L y \frac{ds}{EI}}{\int_0^L \frac{ds}{EI}}$	$\frac{f}{3}(2 + \gamma - \gamma^2)$
$I_{xx} = \int_0^L (y'')^2 \frac{ds}{EI}$	$\frac{f^2L}{45EI_o}(1 + \gamma)^2(4 - 7\gamma + 4\gamma^2)$
$I_{xy} = \int_0^L x''y'' \frac{ds}{EI}$	$\frac{fdL}{12EI_o}$
$I_{yy} = \int_0^L (x'')^2 \frac{ds}{EI}$	$\frac{L^3}{12EI_o}$
$\tan\beta = \frac{I_{xy}}{I_{yy}}$	$\frac{d}{L}$
$C_4 = \int_0^L y^2 \frac{ds}{EI} = I_{xx} - I_{xy} \tan\beta$	$\frac{{e_A}^2 L}{5EI_o}$
$e_A = e - a \tan\beta$	$\frac{f}{6}(1 + \gamma)^2$
$e_B = e - d + b \tan\beta$	$\frac{f}{6}(1 + \gamma)^2$

TABLE II

13. Stiffness and Carry-over Factors.

$K_{AB}^{(\theta)} = \frac{C_1}{N} + \frac{e_A^2}{C_4}$	$\frac{9EI_o}{L}$
$K_{BA}^{(\theta)} = \frac{C_3}{N} + \frac{e_B^2}{C_4}$	$\frac{9EI_o}{L}$
$M^{(c)} = \frac{C_2}{N} - \frac{e_A e_B}{C_4}$	$-\frac{3EI_o}{L}$
$C_{AB} = \frac{M^{(c)}}{K_{AB}^{(\theta)}}$	$-\frac{1}{3}$
$C_{BA} = \frac{M^{(c)}}{K_{BA}^{(\theta)}}$	$-\frac{1}{3}$
$K_{AB}^{(H)} = \frac{e_A}{C_4}$	$\frac{5EI_o}{e_A L}$
$K_{BA}^{(H)} = -\frac{e_B}{C_4}$	$-\frac{5EI_o}{e_A L}$
$K_{AB}^{(\psi)} = \frac{C_1 + C_2}{N} - \frac{e_A L}{C_4} \tan\beta$	$\left(6 - \frac{5d}{e_A}\right)\frac{EI_o}{L}$
$K_{BA}^{(\psi)} = \frac{C_3 + C_2}{N} + \frac{e_B L}{C_4} \tan\beta$	$\left(6 + \frac{5d}{e_A}\right)\frac{EI_o}{L}$

TABLE III

14. Modified Stiffness Factors - Hinged End.

$K_{BA}^{(\theta)'} = K_{BA}^{(\theta)} (1 - C_{AB}C_{BA})$	$\frac{8EI_c}{L}$
$K_{BA}^{(H)'} = K_{BA}^{(H)} - C_{AB}K_{AB}^{(H)}$	$-\frac{10EI_c}{3e_A L}$
$K_{BA}^{(\psi)'} = K_{BA}^{(\psi)} - C_{AB}K_{AB}^{(\psi)}$	$\left(8 + \frac{10d}{3e_A}\right) \frac{EI_c}{L}$

TABLE IV

15. Thrust Equations. From Equation (51) and TABLES II, III, and IV, the equation for thrust in an unsymmetrical parabolic arch is:

$$(72) \quad H = \frac{5}{12e_A} (M_{AB}^{(\theta)} - M_{BA}^{(\theta)}) + FH^{(\Delta x)} + FH^{(\Delta y)} + FH^{(Load)}$$

From equations (52) and (53) and the tables:

$$(73) \quad FH^{(\Delta x)} = \frac{5EI}{e_A^2 L} \Delta x \qquad FH^{(\Delta y)} = -\frac{5d\ EI}{e_A^2 L^2} \Delta y$$

From equation (55) and TABLES II, III, and IV, the equation for thrust in an unsymmetrical parabolic arch AB when end A is hinged is:

$$(74)\quad H = -\frac{5}{12e_A} M_{BA}^{(\theta_B)} + EH^{\Delta_x} + EH^{\Delta_y} + EH^{(Load)}$$

From equations (56), (57), and (58) and the tables:

$$(75)\quad EH^{(\Delta_x)} = \frac{20EI}{9e_A{}^2L}\Delta_x \qquad EH^{(\Delta_y)} = \frac{(30e_A - 70d)EI}{9e_A{}^2L^2}\Delta_y$$

$$(76)\quad EH^{(Load)} = FH^{(Load)} - \frac{5}{9e_A} FM_{AB}^{(Load)}$$

16. Symmetrical Parabolic Arches. For symmetrical parabolic arches, $\gamma = 1$, $d = 0$, and the constants, stiffness and carry-over factors from TABLES II, III, and IV become:

$e = \frac{2}{3} f$	$\tan\beta = 0$
$I_{xy} = 0$	$C_4 = \frac{4f^2L}{45EI_o}$

TABLE V

$K_{AB}^{(\theta)} = K_{BA}^{(\theta)} = K^{(\theta)}$	$\frac{9EI_o}{L}$
$C_{AB} = C_{BA} = C$	$-\frac{1}{3}$
$K_{AB}^{(H)} = -K_{BA}^{(H)} = K^{(H)}$	$\frac{15EI_o}{2fL}$
$K_{AB}^{(\psi)} = K_{BA}^{(\psi)} = K^{(\psi)}$	$\frac{6EI_o}{L}$

TABLE VI

STIFFNESS AND CARRY-OVER FACTORS

$K_{AB}^{(\theta)'} = K^{(\theta)'}$	$\frac{8EI_o}{L}$
$K_{BA}^{(H)'} = -K^{(H)'}$	$-\frac{5EI_o}{L}$
$K_{BA}^{(\psi)'} = K^{(\psi)'}$	$\frac{8EI_o}{L}$

TABLE VII

MODIFIED STIFFNESS FACTORS

For a symmetrical parabolic arch equation (72) becomes:

$$(77) \quad H = \frac{5}{8f}\left(M_{AB}^{(\theta)} - M_{BA}^{(\theta)}\right) + FH^{(\Delta x)} + FH^{(Load)}$$

And equation (74) becomes:

$$(78) \quad H = -\frac{5}{8f} M_{BA}^{(\theta_B)} + EH^{(\Delta x)} + EH^{(\Delta y)} + EH^{(Load)}$$

Where:

$$(79) \quad EH^{(\Delta x)} = \frac{5EI}{f^2L}\Delta x \qquad EH^{(\Delta y)} = -\frac{5EI}{fL^2}\Delta y$$

$$(80) \quad EH^{(Load)} = FH^{(Load)} - \frac{5}{6f} FM_{AB}^{(Load)}$$

PART IV

ILLUSTRATIVE EXAMPLE

17. Method of Analysis. A symmetrical rigid frame loaded by a symmetrical system of concentrated loads (Fig. 5) will be analyzed by moment distribution. The arches are symmetrical and of the variation $I_x = I_o \sec \varphi$. The moment of inertia of all columns is I_o and the modulus of elasticity is constant throughout the structure. All loads equal 6,500 lbs. and are equally spaced along the arches.

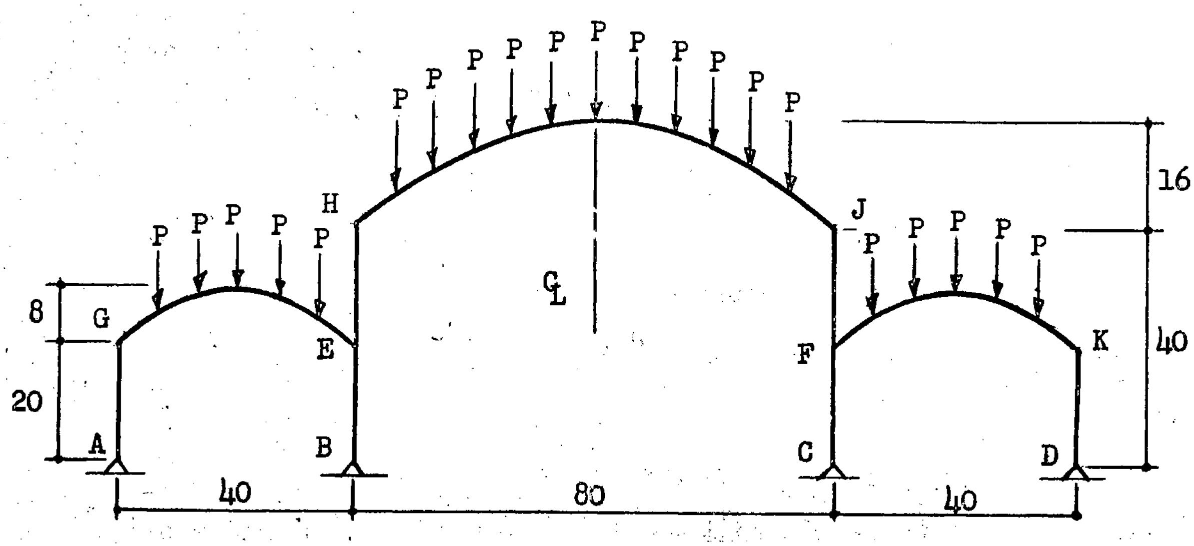


Fig. 5

By selecting bars AE, EF, and FD as members and considering symmetry, the structure can be reduced from six joints to one: joint E. By this method only two finite distributions of moments and one joint equation will be required.

18. Evaluation of Constants. (a) Bar AE: By using transfer formulas and the elastic properties of the arch and column separately:

$$C_1 = \frac{40}{3EI} \qquad \bar{x} = \frac{40}{3} \qquad I_{yy} = \frac{32000}{3EI}$$

$$C_2 = \frac{20}{3EI} \qquad \bar{y} = \frac{182}{9} \qquad \tan\beta = .383$$

$$C_3 = \frac{100}{3EI} \qquad I_{xx} = \frac{12080}{3EI} \qquad e_A = 15.1$$

$$N = \frac{400}{(EI)^2} \qquad I_{xy} = \frac{12270}{3EI} \qquad e_E = 10.44$$

$$C_4 = \frac{7380}{3EI}$$

The stiffness and carry-over factors are:

$$K_{AE}^{(\theta)} = \frac{C_1}{N} + \frac{e_A^2}{C_4} = .126\ EI \qquad K_{EA}^{(\theta)} = \frac{C_3}{N} + \frac{e_E^2}{C_4} = .1276\ EI$$

$$M^{(c)} = \frac{C_2}{N} - \frac{e_A e_E}{C_4} = -.0475\ EI$$

$$C_{AE} = \frac{M^{(c)}}{K_{AE}^{(\theta)}} = -.377 \qquad C_{EA} = \frac{M^{(c)}}{K_{EA}^{(\theta)}} = -.372$$

$$K_{AE}^{(H)} = \frac{e_A}{C_4} = .00614\ EI \qquad K_{EA}^{(H)} = -\frac{e_E}{C_4} = -.00425\ EI$$

The modified stiffness factors (hinge at A) are:

$$K_{EA}^{(\theta)'} = K_{EA}^{(\theta)}\,(1 - C_{AE}C_{EA}) = .1098\ EI \qquad K_{EA}^{(H)'} = K_{EA}^{(H)} - C_{AE}K_{AE}^{(H)} = -.00193\ EI$$

The shear factor is:

$$\lambda_{EA} = \frac{e_E}{K_{EA}^{(\theta)'}} + \frac{e_A C_{EA}}{K_{AE}^{(\theta)'}} = \frac{43.2}{EI}$$

(b) Bar EF:

$$C_1 = C_3 = \frac{140}{3EI} \qquad C_2 = \frac{40}{3EI} \qquad N = \frac{2000}{(EI)^2}$$

$$C_4 = \frac{14550}{EI} \qquad e = 23.8$$

The stiffness and carry-over factors are:

$$K_{EF}^{(\theta)} = \frac{C_1}{N} + \frac{e^2}{C_4} = .0622\ EI \qquad M^{(c)} = \frac{C_2}{N} - \frac{e^2}{C_4} = -.0322\ EI$$

$$C_{EF} = \frac{M^{(c)}}{K_{EF}^{(\theta)}} = -.518 \qquad K_{EF}^{(H)} = \frac{e}{C_4} = .001636\ EI$$

Since the loading is symmetrical, $\theta_E = -\theta_F$ and the modified angular stiffness factor is:

$$K_{EF}^{(\theta)''} = K_{EF}^{(\theta)} - M^{(c)} = .0944\ EI$$

19. Evaluation of Fixed End Moments and Thrusts. (a) Bar AE: From equations (44) and (55), the end moment and end thrust due to loads are:

$$EM_{EA} = FM_{EA}^{(Load)} - C_{AE} FM_{AE}^{(Load)} \quad \text{and} \quad EH_{EA} = \frac{C_5}{C_4} - \frac{K_{AE}^{(H)}}{K_{AE}^{(\theta)}} FM_{AE}^{(Load)}$$

These equations can be evaluated by integration, but it is simpler to compute the fixed end moments and thrust in the arch and then distribute.

Fixed end moments and thrust - arch GE:[1]

$$GM_{GE} = \sum - \frac{Pab^2}{L^2} = -121900^{\#}ft. \qquad FH^{(Load)} = \sum \frac{5Pa^2b^2}{2eL^3} = 23000^{\#}$$

$$FM_{GE}^{(Load)} = -FM_{EG} = GM_{GE} + eFH^{(Load)} = +800^{\#}ft.$$

Stiffness and distribution factors:

$$K_{GA}^{(\theta)'} = \frac{3EI}{L} : 6 \qquad D_{GA} = .4$$

$$K_{GE}^{(\theta)} = \frac{9EI}{L} : 9 \qquad D_{GE} = .6$$

Moments and thrust due to translation of G:

$$EM_{GA}^{(\Delta x)} = -\frac{3EI}{L}\frac{\Delta G}{L} : -8X$$

$$FM_{GE}^{(\Delta x)} = -FM_{EG}^{(\Delta x)} = \frac{15EI}{2fL}\Delta G : 25X$$

$$FH_{GE}^{(\Delta x)} = \frac{45EI}{4f^2L}\Delta G : 4.69X$$

Distribution of moments:

Loads

	G		E
D's	.4	.6	0
FM's	0	+800	-800
Dist	-320	-480	
C			+160
	-320	+320	-640

Δ_G

G		E
.4	.6	0
-8	+25	-25
-6.8	-10.2	
		+3.4
-14.8	+14.8	-21.6

[1]Ibid., pp. 14-15.

Joint equation:

$$\Sigma F_G = 0 \quad : \quad H_{GE} - H_{GA} = 0$$

$$H_{GE} = \frac{5}{8f}\left(M_{GE}^{(\theta)} - M_{EG}^{(\theta)}\right) + FH^{(\triangle x)} + FH^{(Load)}$$

$$= .0781\,(-\,640 - 13.6X) + 4.69X + 23000 = 22950 + 3.63X$$

$$H_{GA} = \frac{M_{GA}}{L} = -16 - .74X$$

Substituting these values into the joint equation, X = -5250 and the end moment and thrust are:

$EM_{EA} = +\ 112900^{\#}ft.$ $\qquad$ $EH_{EA} = +\ 3880^{\#}$

(b) Bar EF: By a similar procedure, the fixed end moment and thrust due to load for bar EF are:

$FM_{EF} = +\ 253000^{\#}ft.$ $\qquad$ $FH_{EF} = +\ 24500^{\#}$

20. <u>Solution of the Rigid Frame.</u> (a) Stiffness and distribution factors:

$K_{EA}^{(\theta)'} = .1098\ EI$ $\qquad$ $D_{EA} = .310$

$K_{EF}^{(\theta)''} = .0944\ EI$ $\qquad$ $D_{EF} = .266$

$K_{EB}^{(\theta)'} = \frac{3EI}{L} = .15\ EI$ $\qquad$ $D_{EB} = .424$

(b) Fixed end moments and thrusts. Due to loads:

$EM_{EA} = +\ 112900^{\#}ft.$ $\qquad$ $FM_{EF} = +\ 253000^{\#}ft.$

$EH_{EA} = +\ 3880^{\#}$ $\qquad$ $FH_{EF} = +\ 24500^{\#}$

Due to translation of E:

$$EM_{EA}^{(\Delta_x)} = K_{EA}^{(H)'}(-\Delta_E) = +.00193\ EI\ \Delta_E \quad : \quad +193X$$

$$FM_{EF}^{(\Delta_x)} = K_{EF}^{(H)}(2\Delta_E) = +.00327\ EI\ \Delta_E \quad : \quad +327X$$

$$EM_{EB} = -\frac{3EI}{L}\ \frac{\Delta_E}{L} = -.00750\ EI\ \Delta_E \quad : \quad -750X$$

$$EH_{EA}^{(\Delta_x)} = \left(1 - e_A \frac{K_{AE}^{(H)}}{K_{AE}^{(\theta)}}\right)\frac{-\Delta_E}{C_4} = -.0001076\ EI\ \Delta_E \quad : \quad -10.76X$$

$$FH_{EF}^{(\Delta_x)} = \frac{2\Delta_E}{C_4} = .0001374\ EI\ \Delta_E \quad : \quad +13.74X$$

(c) Distribution of moments.

	Loads EA	Loads EF	Loads EB	Δ_E EA	Δ_E EF	Δ_E EB
D's	.310	.266	.424	.310	.266	.424
FM's	+112900	+253000	0	+193	+327	-750
$M^{(\theta_E)}$	-113500	-97400	-155000	+71.3	+61.2	+97.5
	-600	+155600	-155000	+264.3	+388.2	-652.5

(d) Joint equation.

$$\sum F_E = 0 \quad : \quad H_{EA} + H_{EB} - H_{EF} = 0$$

$$H_{EA} = -\frac{\lambda_{EA}}{C_4} M_{EA}^{(\theta_E)} + EH^{(\Delta_x)} + EH^{(Load)}$$

$$= -.01745\ (-113500 + 71.3X) - 10.76X + 3880 = 5860 - 12X$$

$$H_{EB} = \frac{M_{EB}}{L} = -7750 - 32.62X$$

$$H_{EF} = \frac{K^{(H)'}}{K^{(\theta)'}} \left(M_{EF}^{(\theta)} - M_{FE}^{(\theta)} \right) + FH^{(\Delta x)} + FH^{(Load)}$$

$$= (.01732)\ (2)\ (-\ 97400 + 61.2X) + 13.74X + 24500$$

$$= 21140 + 15.86X$$

Substituting these values into the joint equation, X = -381 and the final moments and thrusts are:

$M_{EA} = -\ 101300^{\#}$ ft. $\qquad H_{EA} = +\ 10430^{\#}$

$M_{EF} = +\ 7800^{\#}$ ft. $\qquad H_{EF} = +\ 15100^{\#}$

$M_{EB} = +\ 93500^{\#}$ ft. $\qquad H_{EB} = +\ 4670^{\#}$

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