INVESTIGATION OF THE PARALLEL INVERTER USING

SILICON CONTROLLED RECTIFIERS

By

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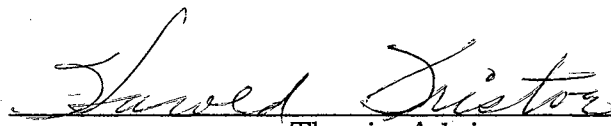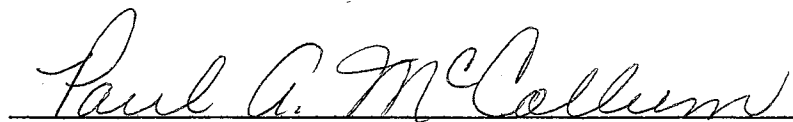the degree of
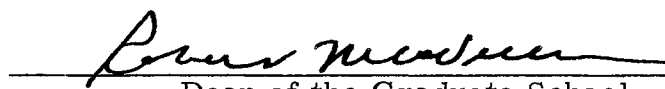MASTER OF SCIENCE
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PREFACE

(One of the major applications of the silicon controlled rectifier

is in the field of power inverters. Demands for high power-handling

capabilities with high efficiency in extreme environments have made the

SCR quite popular. In many instances the parallel inverter has proven

to be the most practical method of converting d.c. to a.c.

The first part of this investigation consists of an analysis of the

parallel inverter circuit. The results of the analysis are studied with

the aid of a digital computer. The latter portion contains a development

of design procedures and experimental verification of them.

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CHAPTER I

INTRODUCTION

Basic sources of electrical energy being developed today produce low-voltage direct current. For example, thermoelectric, thermionic, and magnetohydrodynamic devices as well as fuel cells, solar cells, and nuclear batteries are all d.c. generators. While direct current is useful in some applications, the majority of uses require alternating current. Where direct current is required, the voltage output of the d.c. generators may need to be transformed to a higher voltage.

This situation requires an efficient and economical method for converting d.c. to a.c. or higher voltage d.c., or, in other words, an inverter. Several different types of inverters are available: (1) the conventional motor-generator set, (2) the mercury-arc rectifier or ignitron inverter, (3) the thyratron inverter, (4) the transistor inverter, and (5) the silicon controlled rectifier (hereafter referred to as SCR) inverter. The first four are limited to either high, medium, or low power ranges, whereas the SCR inverter can be designed efficiently from 50 watts to 150 kilowatts with higher power units under development.

Comparison of Inverters

A brief comparison of these different inverters will stress the importance of the SCR inverter. First, the motor-generator set is a rotating device and suffers from short service life and environmental problems; also its weight and size are disadvantages in many cases, whereas the SCR inverter has roughly one half the weight and one half the volume of a motor-generator set. The SCR inverter has an efficiency of 87 - 95% compared with 75 - 88% efficiency for the motor-generator set.

The mercury-arc rectifier or ignitron inverter requires less maintenance than most motor-generator sets, but it must have vacuum pumps and other associated equipment. The efficiency of the SCR inverter in high power applications compares favorably with the mercury-arc or ignitron inverter. The SCR inverter is much more rugged than the mercury-arc or ignitron inverter.

In the medium power range the thyratron inverter is being replaced by the SCR inverter. The thyratron has the usual tube difficulties such as heater warm-up time, short life, and fragility, as well as limited frequency capabilities and lower efficiency than the SCR.

In the low power applications (up to 10 kilowatts) the transistor inverter has shown considerable promise in certain applications, but it cannot compete with the SCR inverter on the basis of efficient operation in severe environments such as high temperatures.

A final comparison that should be made is the cost. At the

present time for lower power applications the SCR inverter is more

economical in many cases. In the high power range (up to 50,000 kilo-

watts) Westinghouse predicts that although at the present time the cost

of the SCR inverter is higher, somewhere in the 1960's the SCR in-

verter should be at the motor-generator set cost level and by the 1970's

it should be comparable in cost to the ignitron inverter.[1]

SCR Inverter Applications

Many applications of the SCR inverter are immediately apparent

in view of such desirable characteristics as high efficiency, compact

size, ruggedness, long life, wide power range, and potentially lower

cost. At the present time SCR inverters are replacing transistor in-

verters in such applications as aircraft, missile and control instru-

mentation -- the transistor inverters having only recently replaced

rotary inverters, vibrator-transformer inverters, and thyratron in-

verters in these applications. The high efficiency, higher power level,

and wider temperature range of the SCR inverter are important in

these applications.

Other uses include emergency power systems where immediate

operation of the inverter is necessary. Since the SCR inverter has a

wide frequency range it can be used in high frequency lighting, new

motor designs incorporating high frequency operation, ultrasonic

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[1]E. J. Duckett, "DC to AC Power Conversion by Semiconductor Inverters," Westinghouse Engineer, XX (1960), 170-4.

generators and induction heating. SCR inverters will be ready when
new electrical energy sources are developed to convert the low voltage
d.c. to a higher voltage d.c. or to a.c.

Purpose of the Thesis

The favorable characteristics of the SCR inverter indicate an in-
creasing number of applications for the inverter. Increased use of
the SCR inverter requires an understanding of the inverter operation
and an analysis of the inverter circuit. Because of the close similarity
between the thyratron parallel inverter circuit and that of the SCR
inverter, the few articles that have been written concerning the circuit
analysis of the inverter circuit using SCR's have referred to the basic
circuit analysis for thyratron inverters done by C..F. Wagner in 1935.
To a certain extent this procedure is justifiable assuming this analysis
can be extended to include the SCR inverter. The point that has not
been discussed, however, is that there are serious disagreements be-
tween the theoretical and the actual current and voltage response of the
inverter under certain conditions using Wagner's analysis. In this
thesis the approach used by Wagner has been followed using Laplace
transformations in place of the classical method of solution. Through
the aid of an IBM 650 digital computer the derived equations have been
compared with the actual circuit and the various sources of error have
been investigated. Also, the limits of operation of an inverter using
SCR's have been studied, and conditions for various output voltage
waveshapes such as square wave, sine wave, and sawtooth wave have

been described. Specific circuit designs for each of these waveshapes have been given.

Therefore, it is the purpose of this thesis to study the basic parallel inverter, to investigate with the aid of a computer, where and why the previous analysis done by Wagner fails, and to extend the analysis to include the SCR inverter. Special attention will be given to the various output voltage waveshapes possible.

CHAPTER II

DESCRIPTION OF THE SCR

An understanding of the characteristics of the SCR is necessary

before studying the inverter. This chapter will give a brief descrip-

tion of the theory of operation of the SCR, its voltage and current

characteristics, and its switching times.

Theory of Operation

The SCR is a three-terminal semiconductor p-n-p-n device with

characteristics somewhat analogous to a thyratron. A complete dis-

cussion of the theory of operation of the p-n-p-n device has been given

in various technical papers.[2,3] Only a summary of the operation will

be given here.

First, consider the two terminal device. Figure 1 shows the

p-n-p-n structure which may be considered as a PNP and an NPN

transistor with a common collector junction. The total current follow-

ing in the p-n-p-n structure is found by summing the current at

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[2]J. T. Moll et al., "P-N-P-N Transistor Switches," Pro-
ceedings of the IRE, XLIV (1956), 1174-1182.

[3]I. M. Mackintosh, "The Electrical Characteristics of Silicon
P-N-P-N Triodes," Proceedings of the IRE, XLVI (1958), 1229-1235.
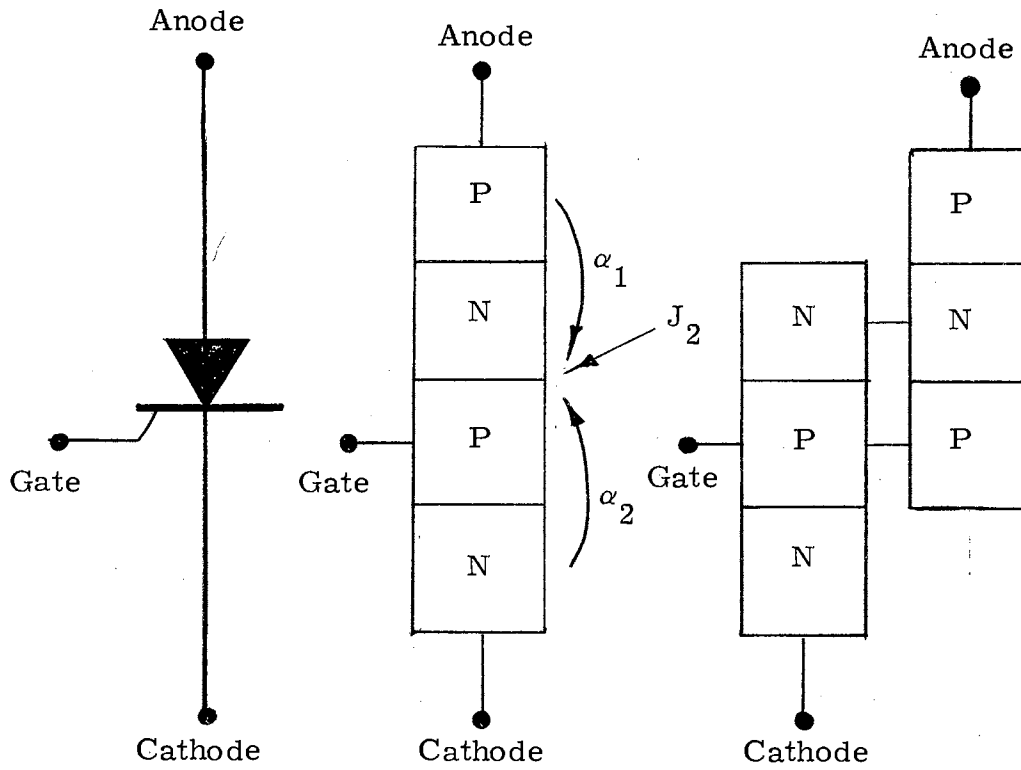
Fig. 1. The SCR Symbol and PNPN Structure

junction J_2, which must be equal to the current in the external circuit. It is given by

$$I = \frac{I_{co}}{1 - (M_n \alpha_1 + M_p \alpha_2)} = \frac{I_{co}}{1 - \alpha_T} \tag{1}$$

where M_n and M_p represent the multiplication factors for electrons and holes arriving at the collector junction. I_{co} is the collector current with the emitter open, and α_1, α_2 are the respective h_{FB}'s

of the PNP and NPN transistors. α_T is the total α. With little or no applied voltage across the p-n-p-n, α_T is less than one, approximately 0.9. Under this condition the external current which flows will be of the order of magnitude of I_{co}, and the p-n-p-n will be in the "off" state or high impedance state. By increasing α_T to unity, it can be seen from (1) that the current will be limited only by the external circuit, i.e., the device is in the "on" or low impedance state. α_T may be increased to unity by two methods. The applied voltage may be increased until the forward breakover voltage has been reached, at which time α_T will be unity. The other method is by injecting a control current into the base region with the gate, thereby increasing α_T to unity. This is the method used to turn the SCR on.

To return the p-n-p-n to the "off" or high impedance state, α_T must be decreased. This can be done by removing the external current or by applying a reverse current, as is done in the inverter.

V - I Characteristics

The typical V - I characteristics of an SCR and the various regions of operation are given in Fig. 2. The reverse region is typical of any silicon power rectifier cell, but the forward region differs considerably from the conventional PN rectifier cell. With zero gate current no appreciable forward current flow exists until the forward breakover voltage, V_{BO}, is reached. Exceeding the V_{BO} causes the device to switch from the blocking to the conducting state. Once the device is in the conducting state the forward characteristic is typical of any
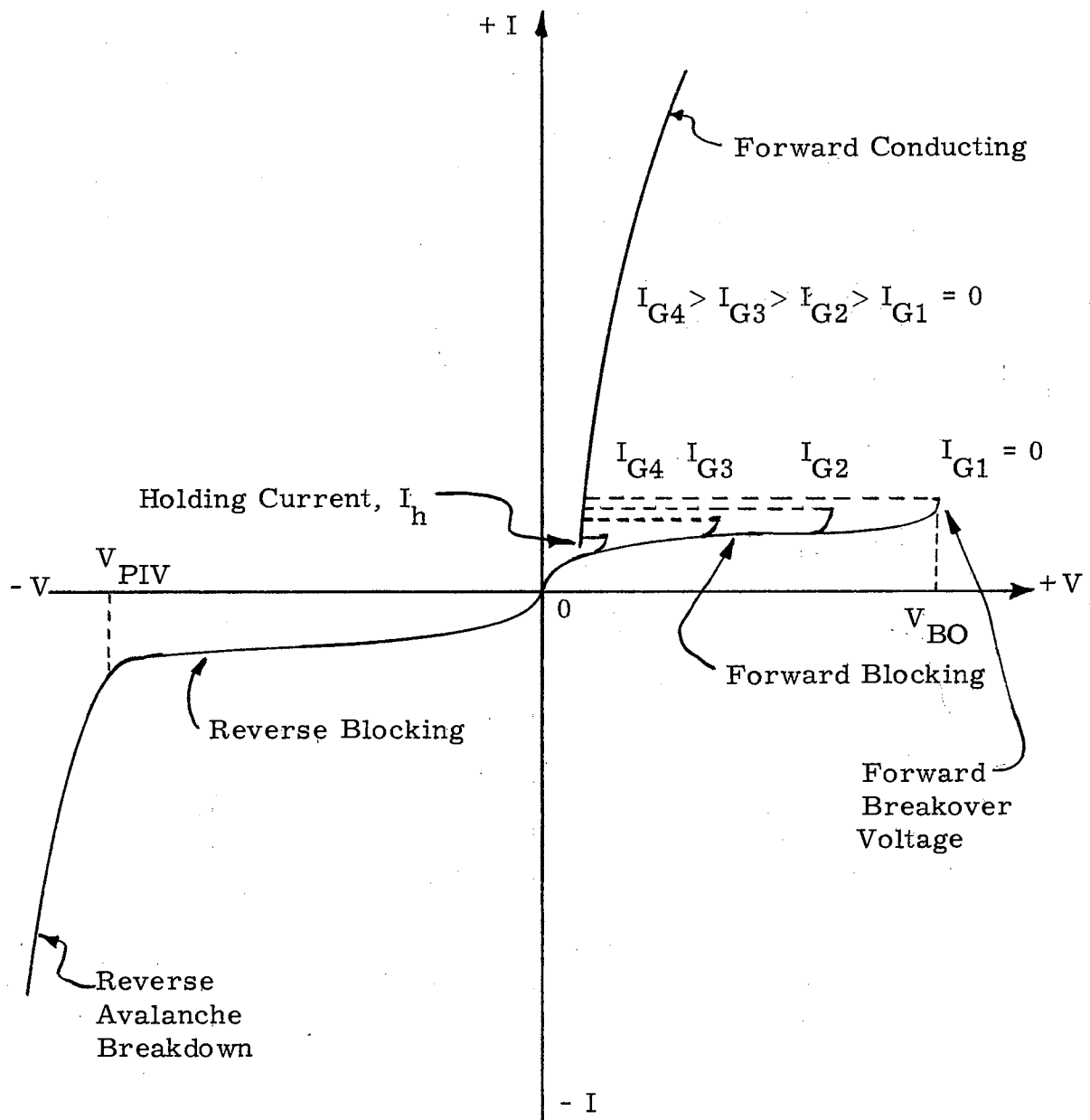
Fig. 2. SCR V - I Characteristics

medium power silicon rectifier. By increasing the gate current the forward breakover voltage is reduced. With sufficient gate current the device will have a forward characteristic similar to any silicon power rectifier. The gate no longer has any control once conduction has started.[4] This can be seen from Fig. 2. To turn the device off, the anode-cathode current must be reduced to a value below that of the holding current, I_h.

If the peak inverse voltage rating of the SCR is exceeded, it will eventually break down and will be destroyed by the resulting avalanche-type currents. Exceeding the forward breakover voltage is nondestructive and will only result in the SCR being turned on.

The forward voltage drop of the SCR in the conducting state is similar to any medium power silicon rectifier and can be approximated by the following empirical equation:[5]

$$V = 0.78 + 0.0957 \ln i + iR \qquad (2)$$

where V = instantaneous forward voltage drop, volts

i = instantaneous forward current, amperes

R = device constant of 0.011 ohm, 125° C.

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[4]At the time of writing this thesis, Texas Instruments had just introduced a new p-n-p-n device which can be turned off by applying a negative pulse to the gate. Called the silicon gate controlled switch (SGCS), it can handle 5 ampere d.c. with a turn-off gain of ten.

[5]D. K. Bisson and R. F. Dyer, "A Silicon Controlled Rectifier - Characteristics and Ratings -- I," AIEE Transactions, Communications and Electronics, LXXVIII (1959), 102 - 106.

In inverter applications the forward voltage drop is 1 to 2 volts. (This compares with 10 to 15 volts in the thyratron.)

Manufacturer's specifications present in detail the operating characteristics for any given SCR. Similar electrical characteristics exist for all SCR types with the exception of the forward current ratings and the peak inverse voltage, V_{PIV}. At the present time SCR's are available at current ratings up to 235 amperes and peak inverse voltage ratings of 600 volts. In this analysis a medium power SCR was used: the TI 2N1602 with an average forward current rating of 3 amperes and a peak inverse voltage rating of 200 volts.[6]

The turn-on characteristics of the SCR are given in terms of minimum gate voltage and current at which all units will turn on and maximum allowable voltage which will not fire the device. Peak gate voltage (forward) is 10 volts, reverse is 5 volts. Peak gate power allowable is 5 watts, average gate power is 1/2 watt. Peak gate current is 2 amperes. These values are applicable to the majority of SCR's.

The analysis in this thesis assumes that the SCR is kept within a normal operating temperature range (-40° to + 100° C). The reader is referred to the Silicon Controlled Rectifier Manual for further information regarding temperature effects and heatsinks.[7]

[6] Texas Instruments, "Diffused Silicon PNPN Controlled Rectifier, Types 2N1600 - 2N1604," Specification Bulletin No. DL-S 60414, May, 1960.

[7] General Electric Company, Silicon Controlled Rectifier Manual (2nd ed., Syracuse, 1961), pp. 222 - 234.

Switching Times

The turn-on and turn-off times of the SCR are important in inverter applications. In the inverter analysis the SCR's are assumed to be perfect switches with instantaneous switching action. In the actual circuit, power is lost and error is introduced in the predicted response of the inverter since the charged commutating capacitor is short-circuited for about 1/2 to 5 microseconds depending on the particular unit, while one SCR is turning on and the other SCR is turning off.

In general, the turn-on time is 1 to 5 microseconds and the turn-off time is 5 to 25 microseconds. Both are dependent upon several factors.[8] The turn-on time depends upon the voltage and current magnitude of the gate pulse, the pulse width and rise time, the anode voltage and current, and the temperature.

The test setup shown in Fig. 3 was used to control these parameters and to study the turn-on characteristics. Using a Heathkit Model W-6M power amplifier to amplify the output of a Tektronix Model 202 square wave generator, the anode voltage could be varied from 20 to 120 volts. By varying the load resistance, the load current could be controlled. The gate pulse was obtained from a Rutherford Model 7-B7 pulse generator, synchronized with the square wave generator to give a delayed trigger pulse after the anode voltage had been

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[8] Walt Matzen, Switching Time of NPNP Triodes (Texas Instruments, Inc. Memorandum, 1958).
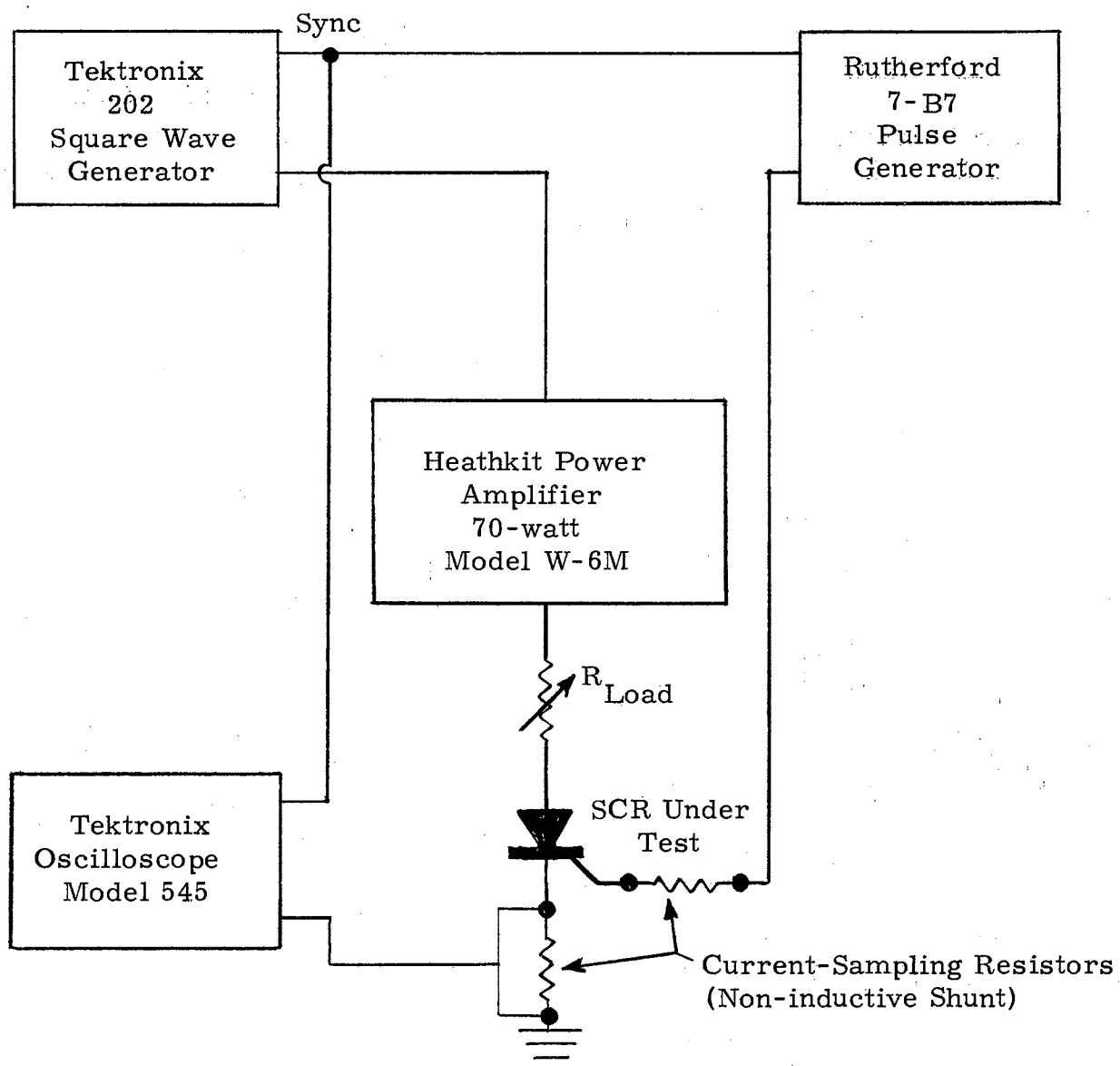
Fig. 3. Test Equipment Diagram for Study of Turn-on Time.

applied approximately 50 milliseconds. The pulse width, rise time, and magnitude of the output of the Rutherford pulse generator could be varied. A Tektronix Model 545 oscilloscope was used to observe the turn-on time. Using a 2N1602 SCR, it was observed that with a gate pulse of 200 ma, 1 microsecond width, and a risetime of 0.2 microseconds, anode voltage of 50 to 100 volts, the turn-on time was 0.5 microseconds for the better units. The other units tested ranged up to 1.5 microseconds. Increasing the magnitude of the gate pulse above 200 milliamperes had little effect. For a gate pulse of 50 ma, 5 microseconds width, risetime of 0.5 microseconds, temperature of 25° C, the turn-on time was 0.8 to 3 microseconds for the anode voltage greater than 20 volts. This is a satisfactory response for inverter applications. It must be stressed that this test was only to determine the effect of the various parameters upon the turn-on time and not a test for measuring the turn-on time, since this will vary from unit to unit and for various applications.

The turn-off time depends upon the type circuit in which the SCR is used. In the inverter circuit the SCR has a large reverse voltage applied across it which results in a very fast turn-off time. General Electric has inverter types of SCR's which are specially selected to meet inverter circuit requirements. They have a maximum limit on turn-off time of 12 microseconds on medium power units and 20 microseconds on high power units where the rate of rise of reverse current is 5 amperes per microsecond and the rate of rise

of reverse voltage is 20 volts per microsecond.

The turn-on and turn-off times of the SCR are 50 times faster than the thyratron, thereby allowing a much higher frequency of operation since the controlled rectifier does not have to be reverse biased nearly so long.

CHAPTER III

BASIC CIRCUIT ANALYSIS

Types of Inverter Circuits

There are three basic types of power inverter circuits: (1)
parallel, (2) series, and (3) bridge. These circuits are character-
ized by one or more capacitors which are alternately charged and dis-
charged to commutate the inverter SCR's. Each of the inverter cir-
cuits has its advantages and disadvantages. The series inverter is
not suitable for high power conversion or conversion from low d.c.
supply voltages. The bridge inverter is most effective for high power,
low distortion polyphase power inversion where reasonably high d.c.
supply voltage is available. The parallel inverter is best suited for
high power conversion with low d.c. source voltage. The more com-
mon circuit diagrams for these inverters are shown in Fig. 4.

This analysis will consider only the parallel inverter since it has
the widest application. For an analysis of the other types of inverters
mentioned, refer to a report by G. P. Underbrink, Temco Electronics.[9]

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[9]G. P. Underbrink, Investigation of Silicon Controlled Rectifiers
for Static Power Conversion, Armed Services Technical Information
Agency (Arlington, Virginia, 1960).
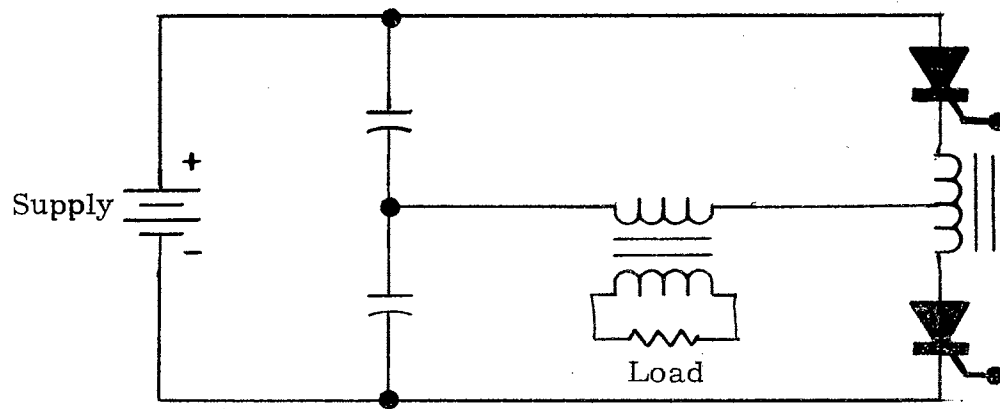
Series Inverter
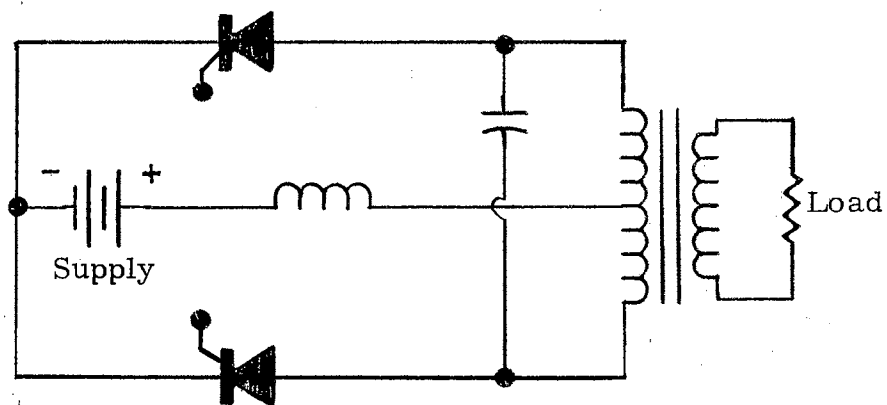
Parallel Inverter
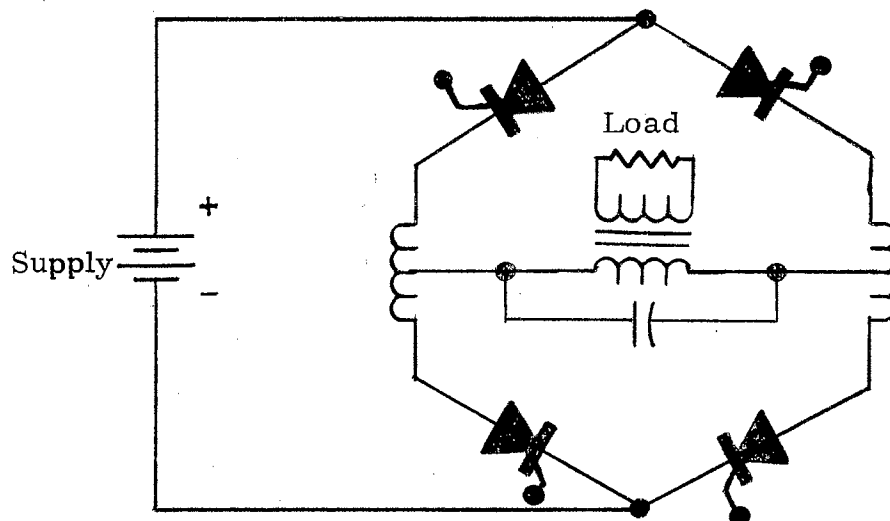
Bridge Inverter

Fig. 4. Basic Power Inverter Circuits

This chapter will include a description of the parallel inverter circuit operation and a discussion of an analysis of the circuit. A mathematical analysis of the simplified circuit with a resistive load will be given, followed by a more exact analysis for reactive loads.

Parallel Inverter Operation

An ideal inverter circuit is shown in Fig. 5, consisting of an ideal transformer with a center-tapped primary, a d.c. voltage source, and two SCR's, which may be thought of as perfect switches.
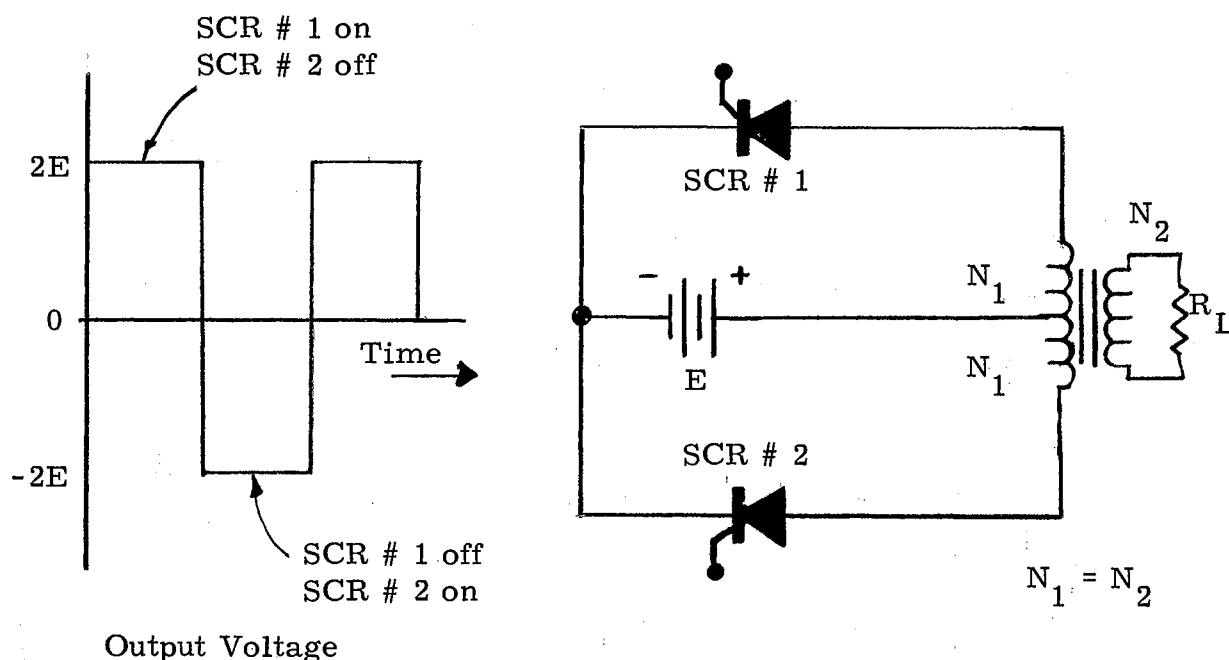
Fig. 5. An Ideal Inverter Circuit

This inverter would produce only a square wave output voltage. It would operate by alternately switching SCR # 1 and SCR # 2 by means of an external oscillator. As SCR # 1 was turned on, SCR # 2 would be simultaneously turned off and vice versa. Thus a d.c. voltage would be applied first across one half of the transformer primary and then across the other half as SCR # 1 was turned on and as SCR # 2 was turned on. (See Fig. 5.) The primary voltage is stepped up by the auto transformer action of the center-tapped primary to twice the supply voltage, E. When the second SCR is turned on, the current in the transformer reverses direction and the voltage across the load reverses from + 2E to -2E, assuming an ideal transformer with unity turns ratio. The output voltage waveform is illustrated in Fig. 5.

The only problem with the ideal inverter is that the SCR cannot be turned off by the gate as it was turned on. Even the recently announced silicon gate controlled switch, which can be turned off by the gate, has a low turn-off gain and hence would be less efficient. Also, the possibility exists that a waveshape other than a square wave might be desired. A sinewave or a sawtooth waveshape could only be obtained from the ideal inverter in Fig. 5 by some means of filtering and with loss in efficiency.

To provide the necessary turn-off action, a commutating capacitor and a ballast inductor are added as shown in Fig. 6. The resulting circuit will provide a reverse voltage across the SCR long enough to turn it off. As SCR # 1 is turned on, the capacitor charges through
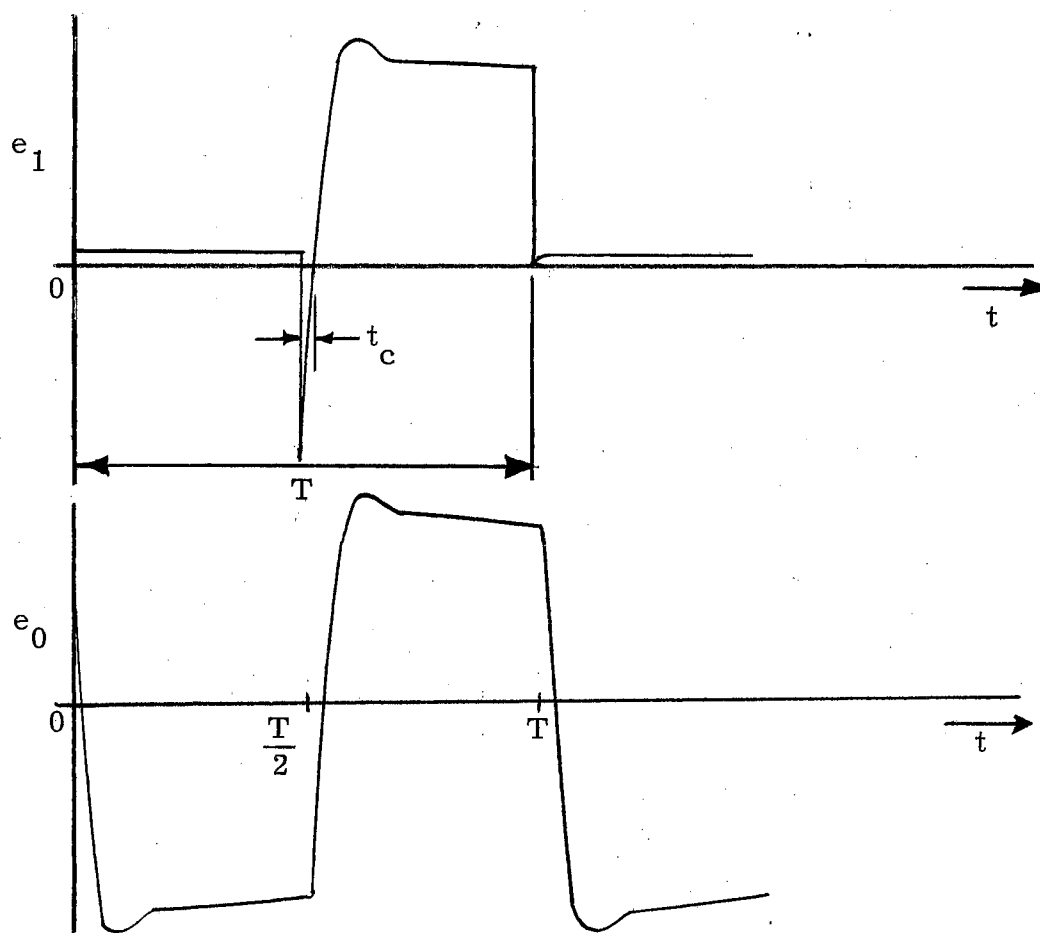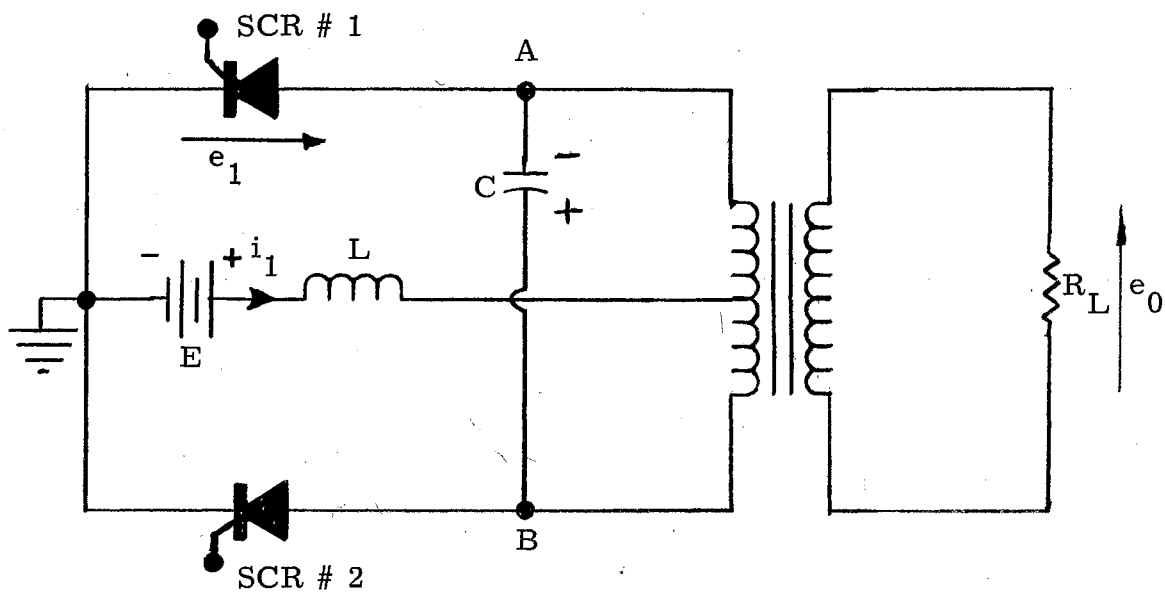
Fig. 6. SCR Parallel Inverter Circuit
and Voltage Waveforms

the inductor to a steady state value of 2E, which is the voltage across the transformer primary. (See Fig. 6.) When SCR # 2 is turned on, point B goes to ground, and the charged commutating capacitor reverse biases SCR # 1 thus turning it off. This leaves only SCR # 2 conducting, forcing the capacitor to charge to 2E with the opposite polarity. When SCR # 1 is again turned on, the commutating action will repeat and SCR # 2 will be turned off, thus completing the cycle.

If the ballast inductor were omitted and if the circuit through which the capacitor charges and discharges had no resistance, the capacitor would discharge and charge so quickly when the second SCR was turned on that the first SCR would not be turned off. This would result in both SCR's being on with no means of turning them off. In the actual circuit, a small amount of resistance exists and the transformer has some leakage inductance which allows the SCR's sufficient time to turn off. The inverter will then operate without the ballast inductor. (This would not be true in the thyratron inverter due to the much longer deionization time or turn-off time of the thyratron.)

By adding the inductor, however, the capacitor will charge at a slower rate, and therefore insure proper commutation of the SCR's. The inductor limits the capacitor charging current, thereby maintaining a reverse bias across the SCR for a period of time sufficient to turn the device off.

The inductor serves one other important purpose. Without the inductor the charging current of the capacitor is limited only by the

small resistance of the circuit. Hence, a heavy surge of current will be demanded from the d.c. supply the first part of each half cycle and will decay to a value required by the load, so that the current from the supply flows in large pulses as the capacitor requires charging current. Figure 7 illustrates the effect of placing the inductor in series with the supply. The current will be limited to a more continuous flow.

Without Inductor   With Inductor
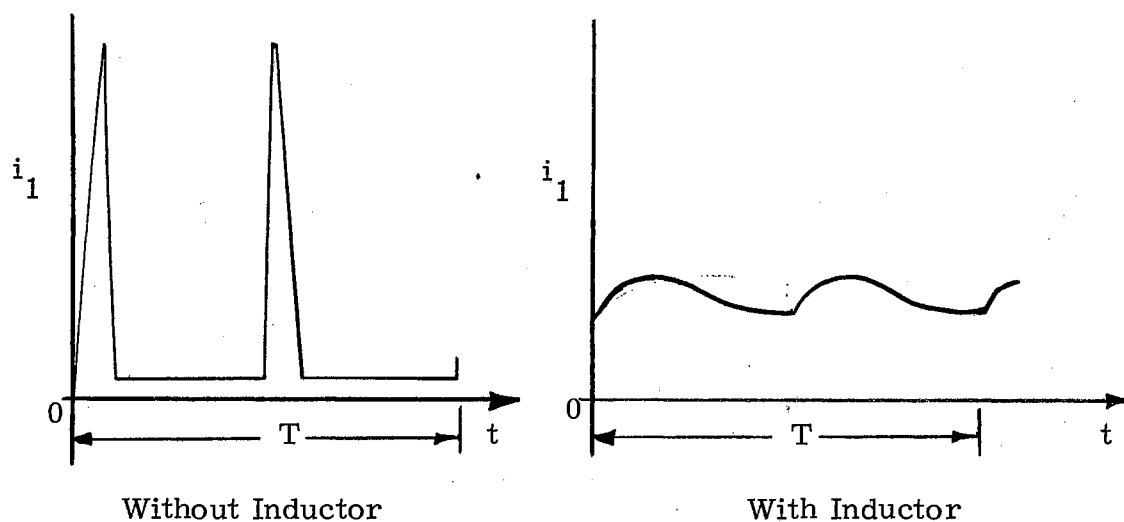
Fig. 7. Effect of the Ballast Inductor
on the Input Current

From the energy standpoint, the inductor stores energy ($\frac{1}{2} L i^2$) in its magnetic field which is absorbed by the capacitor when it charges, $\frac{1}{2} Ce^2$ being the energy stored in its electric field. With the

proper size inductor, the energy stored in the magnetic field of the inductor is transferred to the electric field of the capacitor, thereby relieving the d.c. supply of sudden current surges.

The effect of varying the load, commutating capacitor, ballast inductor, and the switching frequency on the input current and the output voltage will be discussed after the mathematical analysis has been given.

Approach to the Circuit Analysis

The most straightforward approach to the problem is to assume an ideal transformer with unity turns ratio and a resistive load.[10] A more complete analysis considers the resistance of the transformer windings, the leakage inductance of the transformer, and a capacitive or inductive load.[11] This method involves cubic equations and is difficult to solve. The complete solution of the simplified case will be given, but only the derivation of the equations and an outline of the solution will be given for the more complete analysis.

In each case the differential equations will be set up for one SCR conducting and the other SCR off. Because of symmetry in the circuit, definite initial conditions are known. Also, since the circuit operation is "symmetric", only one-half cycle will be analyzed. It will be

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[10]C. F. Wagner, "Parallel Inverter with Resistive Load," Electrical Engineering, LIV (1935), 1227 - 1235.

[11]C. F. Wagner, "Parallel Inverter with Inductive Load," Electrical Engineering, LV (1936), 970 - 980.

assumed that the ballast inductance has been made large enough so that the current flows throughout each half cycle instead of in pulses. Laplace transforms will be used to simplify the analysis.

Simplified Case - Resistive Load

The parallel inverter with a resistive load and using an ideal transformer will now be analyzed. The circuit to be used, shown in Fig. 8, can be reduced to that in Fig. 9 by making the following as-sumptions:

1.   The transformer leakage inductance, primary inductance, winding resistance, core loss, and magnetizing current can be neglected.

2.   Perfect coupling exists between each half of the primary and between the primary and secondary of the transformer.

3.   The resistance of the choke is negligibly small.

4.   The SCR's are perfect switches.

5.   The load resistance can be reflected to one half of the primary by $R = \dfrac{R_L}{n^2}$, where R_L is the load resistance and n is the turns ratio of the secondary to half the primary.

6.   The commutating capacitor can be reflected to half the primary of the transformer as 4C.

The equivalent circuit in Fig. 9 is obtained by making these assumptions. This permits a straightforward mathematical analysis.
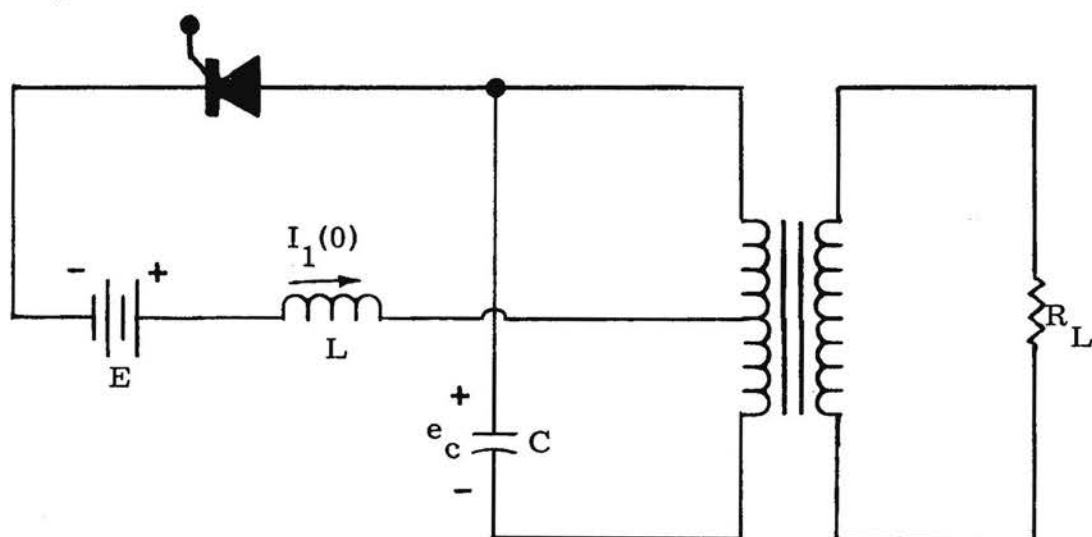
Fig. 8. Equivalent Circuit for Parallel
Inverter with Resistive Load
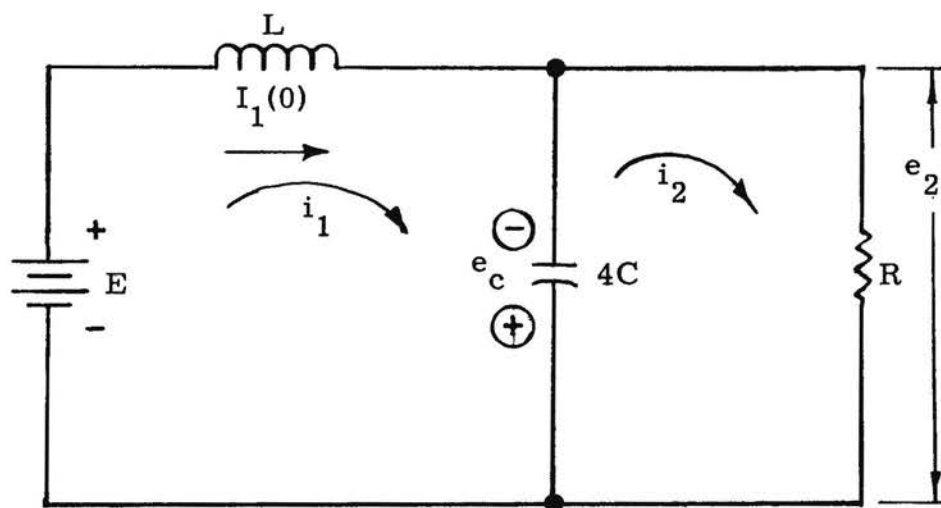
Fig. 9. Equivalent Circuit Assuming Ideal Transformer

Only the first and third assumptions are critical. The transformer winding resistance, its leakage inductance, and the resistance of the choke cannot always be neglected. This will be discussed more fully by comparison with an actual experimental inverter. The more exact analysis made later in this chapter will demonstrate the effect on the voltage and current equations when these resistances and inductances are included in the equivalent circuit.

From the explanation of the operation of the inverter circuit it can be seen that the input current, i_1, at the beginning of a period must equal the current at the end of the half period. Also, the charge on the commutating capacitor at the beginning of a period must be equal in magnitude and opposite in sign to the charge at the end of the half period. Letting $T = \frac{1}{f}$, the full period of the control frequency, it may then be stated that

$$i_1(0) = i_1\left(\frac{T}{2}\right) \tag{3}$$

and
$$e_2(0) = -e_2\left(\frac{T}{2}\right) . \tag{4}$$

These will be the boundary conditions in the following solutions.

For the equivalent circuit in Fig. 9, the following differential equations may be written by summation of voltage drops around loops 1 and 2:

$$E + e_c = L\frac{di_1}{dt} + \frac{1}{4C}\int i_1 dt - \frac{1}{4C}\int i_2 dt \tag{5}$$

$$- e_c = - \frac{1}{4C} \int i_1 dt + i_2 R + \frac{1}{4C} \int i_2 dt \tag{6}$$

where e_c is the charge on the capacitor at the time of switching,
$t = 0$.

The derivation of the following equations is given in Appendix A.
Using Laplace transformations and solving (5) and (6) for the input
current, I_1 (s), and the output voltage, E_2 (s), gives

$$I_1 (s) = \frac{I_1(0) \ s^2 + \left[\frac{E + e_c}{L} + 2\alpha \ I_1(0) \right] \ s + \frac{\beta^2 E}{R}}{s \ (s^2 + 2\alpha s + \beta^2)} \tag{7}$$

and

$$E_2(s) = \frac{- e_c s^2 + 2\alpha \ RI_1(0) \ s + \beta^2 E}{s \ (s^2 + 2\alpha s + \beta^2)} \ , \tag{8}$$

where $\alpha = \frac{1}{8RC}$, $\beta = \sqrt{\frac{1}{4LC}}$, and I_1 (0) is the initial current
in the choke.

The form of the solution of (7) and (8) depends upon the roots
of the impedance function, i.e., $s^2 + 2\alpha s + \beta^2$. If the impedance
function has complex roots the solution of (7) and (8) will contaim
damped sinusoids. For real roots of the impedance function the solu-
tion will contain exponentials. When the impedance function has equal
roots the solution will be the boundary shown in Fig. 10.

Complex Roots

The impedance function has complex roots when $\beta > \alpha$ or

$$\beta = \sqrt{\dfrac{1}{4LC}}$$

Complex
Roots

$\beta > \alpha$

$\alpha = \beta$, Equal Roots

Real Roots

$\alpha > \beta$

$$\alpha = \dfrac{1}{8RC}$$
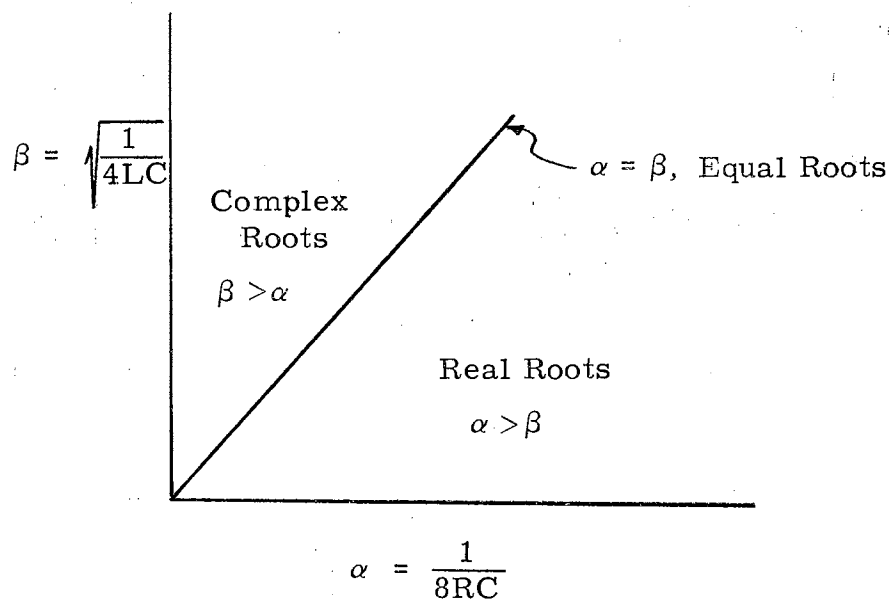
Fig. 10. Division of Solutions

$4R > \sqrt{\dfrac{L}{C}}$. Using inverse Laplace transformations, (7) and (8)

transform into

$$i_1(t) = \frac{E}{R} + \left[I_1(0) - \frac{E}{R} \right] \frac{\beta}{\omega} \epsilon^{-\alpha t} \sin(\omega t + \theta) +$$

$$+ \left[\frac{E + e_c}{L} + 2\alpha I_1(0) \right] \frac{\epsilon^{-\alpha t}}{\omega} \sin \omega t \qquad (9)$$

and

$$e_2(t) = E - \left(e_c + E \right) \frac{\beta}{\omega} \epsilon^{-\alpha t} \sin(\omega t + \theta) + 2\alpha R I_1(0) \frac{\epsilon^{-\alpha t}}{\omega} \sin \omega t \quad (10)$$

where $\qquad \omega = \sqrt{\beta^2 - \alpha^2} \quad$ and $\theta = \tan^{-1} \dfrac{\omega}{\alpha} \quad .$

To evaluate the initial conditions $I_1(0)$ and e_c, relations (3) and (4) are used. These yield

$$\frac{E}{R}\frac{\beta}{\omega}\epsilon^{-\alpha\frac{T}{2}}\sin(\omega\frac{T}{2}+\theta) - \frac{E}{L}\frac{\epsilon^{-\alpha\frac{T}{2}}}{\omega}\sin\omega\frac{T}{2} - \frac{E}{R}\frac{\beta}{\omega}\sin\theta =$$

$$I_1(0)\left[\frac{\beta}{\omega}\epsilon^{-\alpha\frac{T}{2}}\sin(\omega\frac{T}{2}+\theta) + 2\alpha\frac{\epsilon^{-\alpha\frac{T}{2}}}{\omega}\sin\omega\frac{T}{2} - \frac{\beta}{\omega}\sin\theta\right] +$$

$$e_c\left[\frac{\epsilon^{-\alpha\frac{T}{2}}}{L\omega}\sin\omega\frac{T}{2}\right] \tag{11}$$

and

$$2E - E\frac{\beta}{\omega}\sin\theta - E\frac{\beta}{\omega}\epsilon^{-\alpha\frac{T}{2}}\sin(\omega\frac{T}{2}+\theta) =$$

$$I_1(0)\left[-2\alpha R\frac{\epsilon^{-\alpha\frac{T}{2}}}{\omega}\sin\omega\frac{T}{2}\right] + e_c\left[\frac{\beta}{\omega}\sin\theta + \frac{\beta}{\omega}\epsilon^{-\alpha\frac{T}{2}}\sin(\omega\frac{T}{2}+\theta)\right],$$
$$\tag{12}$$

which are in the form

$$\left.\begin{array}{l} k_1 = a_{11}I_1(0) + a_{12}e_c \\[2ex] k_2 = a_{21}I_1(0) + a_{22}e_c \end{array}\right\} \tag{13}$$

Therefore (11) and (12) give the value of $I_1(0)$ and e_c. The other unknowns in (9) and (10) can be found from the circuit parameters. Thus, (9) and (10) represent the input current and output voltage as a function of time for the simplified case, resistive load.

Equal Roots

If the impedance function has equal roots, $\alpha = \beta$, the solution of (7) and (8) will be

$$i_1(t) = \frac{E}{R} + \left[\left(\frac{e_c - E}{L} + \alpha I_1(0)\right) t + I_1(0) - \frac{E}{R}\right] \epsilon^{-\alpha t} \qquad (14)$$

and

$$e_2(t) = E + \left[\left(\frac{I_1(0)}{4C} - \alpha(E - e_c)\right) t - e_c - E\right] \epsilon^{-\alpha t} \qquad (15)$$

As before, the values of $I_1(0)$ and e_c must be known. These can be found from the following relations:

$$\frac{E}{L} \frac{T}{2} \epsilon^{-\alpha \frac{T}{2}} - \frac{E}{R}(1 + \epsilon^{-\alpha \frac{T}{2}}) = I_1(0)\left[-1 + \epsilon^{-\alpha \frac{T}{2}}(1 + \alpha \frac{T}{2})\right] +$$

$$e_c \left[\frac{1}{L} \frac{T}{2}\right] \epsilon^{-\alpha \frac{T}{2}} \quad , \qquad (16)$$

$$E\left[(\alpha \frac{T}{2} + 1) \epsilon^{-\alpha \frac{T}{2}} - 1\right] = I_1(0)\left[\frac{1}{4C} \frac{T}{2}\right] \epsilon^{-\alpha \frac{T}{2}} +$$

$$e_c \left[-1 + (\alpha \frac{T}{2} - 1) \epsilon^{-\alpha \frac{T}{2}}\right] \quad . \qquad (17)$$

These equations are of the form (13). Therefore, if $\beta = \alpha$ the input current and output voltage can be formed from (14) and (15) using (16) and (17).

Real Roots

When $\alpha > \beta$ the impedance function has real roots. The solution of (7) and (8) will be

$$i_1(t) = \left[\frac{a^2 I_1(0) + \frac{a}{L}(E + e_c) + 2\alpha a I_1(0) + \frac{E\beta^2}{R}}{a(a-b)}\right] \epsilon^{at} +$$

$$\left[\frac{b^2 I_1(0) + \frac{b}{L}(E + e_c) + 2\alpha b I_1(0) + \frac{E\beta^2}{R}}{b(b-a)}\right] \epsilon^{bt} + \frac{E\beta^2}{abR} \qquad (18)$$

and

$$e_2(t) = \left[\frac{-e_c a^2 + \frac{a}{4C} I_1(0) + E\beta^2}{a(a-b)}\right] \epsilon^{at} +$$

$$\left[\frac{-e_c b^2 + \frac{b}{4C} I_1(0) + E\beta^2}{b(b-a)}\right] \epsilon^{bt} + \frac{E\beta^2}{ab} \qquad (19)$$

where
$$a = -\alpha + \sqrt{\alpha^2 - \beta^2}$$

$$b = -\alpha - \sqrt{\alpha^2 - \beta^2} \quad .$$

Finding the values for $I_1(0)$ and e_c yields

$$\frac{E}{a-b}\left\{\left[\frac{1}{L} + \frac{\beta^2}{Ra}\right](1 - \epsilon^{a\frac{T}{2}}) - \left[\frac{1}{L} + \frac{\beta^2}{Rb}\right](1 - \epsilon^{b\frac{T}{2}})\right\} =$$

$$\left[(\epsilon^{a\frac{T}{2}} - 1)\left(\frac{a+2\alpha}{a-b}\right) + (\epsilon^{b\frac{T}{2}} - 1)\left(\frac{b+2\alpha}{b-a}\right)\right] I_1(0) +$$

$$\left[\left(\epsilon^{a\frac{T}{2}} - \epsilon^{b\frac{T}{2}}\right)\left(\frac{1}{L(a-b)}\right)\right] e_c \qquad (20)$$

and

$$E\beta^2 \left\{ \frac{1}{a - b} \left[\frac{\epsilon^{a\frac{T}{2}}}{a} - \frac{\epsilon^{b\frac{T}{2}}}{b} \right] + \frac{1}{ab} \right\} =$$

$$\frac{1}{4C(b - a)} \left[\epsilon^{a\frac{T}{2}} - \epsilon^{b\frac{T}{2}} \right] I_1(0) + \left[1 + \frac{1}{a - b} (a\epsilon^{a\frac{T}{2}} - b\epsilon^{b\frac{T}{2}}) \right] e_c .$$

$$(21)$$

As before, (20) and (21) are in the form (13). Therefore, if $\alpha > \beta$ the input current and output voltage can be found from (18) and (19) using (20) and (21).

A More Accurate Analysis

The assumption that the transformer is an ideal transformer leads to considerable error in certain instances. Including some of the transformer characteristics in the equivalent circuit as shown in Fig. 12 complicates the solution of the differential equation by introducing cubic equations. A method of solution will be explained but the actual solution will not be carried out.

The transformer leakage inductance, its winding resistance, and the resistance of the choke have been added to the inverter circuit, as is shown in Fig. 11. The load can be either resistive or inductive. Assuming the transformer has unity turns ratio and that the SCR's are perfect switches, the circuit in Fig. 11 can be reduced to the equivalent circuit in Fig. 12, where
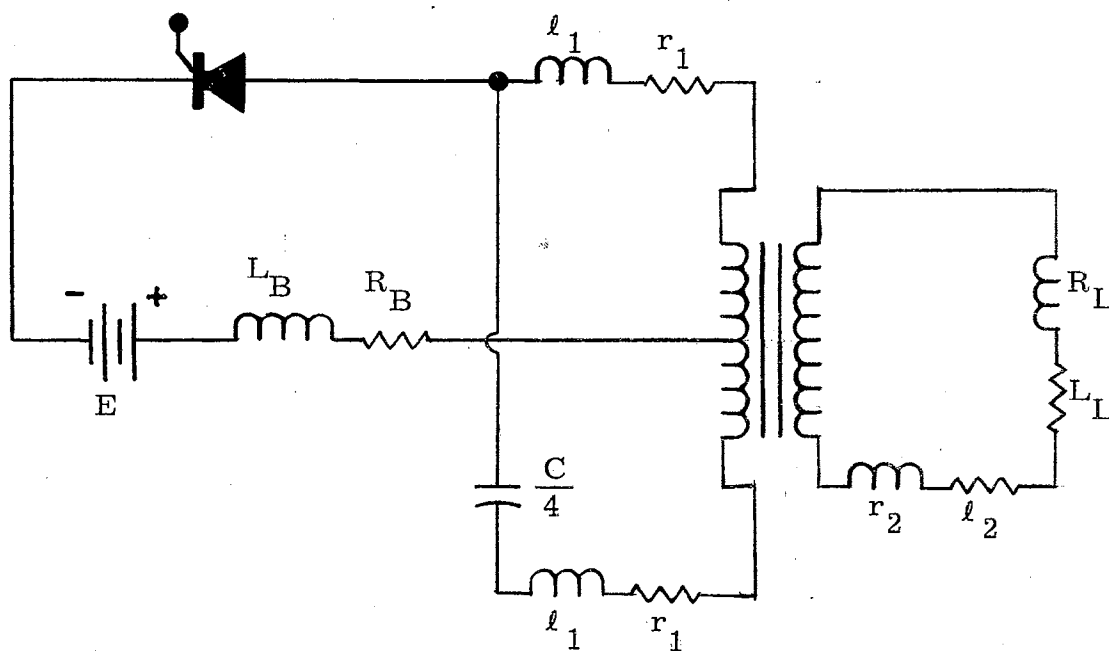
Fig. 11. Schematic Diagram of Parallel Inverter with Reactive Load
Including Transformer Characteristics
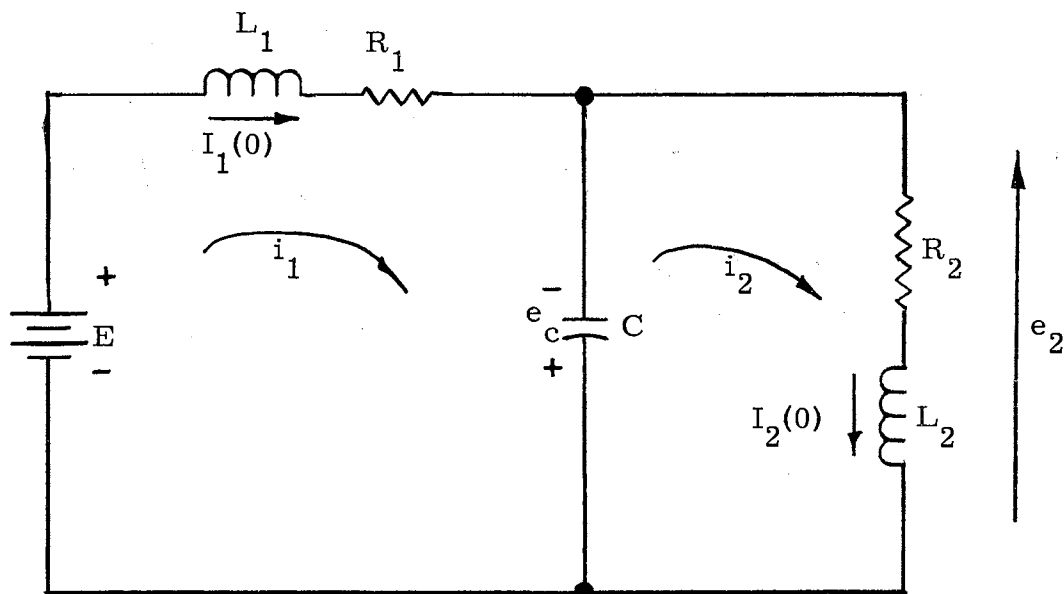
Fig. 12. Equivalent Circuit of Parallel Inverter with Reactive Load
Including Transformer Characteristics

$$R_1 = R_B + \frac{r_1}{2} \tag{22}$$

$$R_2 = R_L + r_2 + \frac{r_1}{2} \tag{23}$$

$$L_1 = L_B + \frac{\ell_1}{2} \tag{24}$$

$$L_2 = L_L + \ell_2 + \frac{\ell_1}{2} \quad , \tag{25}$$

R_B being the resistance of the choke, r_1 the resistance of one primary winding, r_2 the resistance of the secondary winding, R_L the load resistance, L_B the choke inductance, ℓ_1 the equivalent leakage inductance of one primary, ℓ_2 the equivalent leakage inductance of the secondary, and L_L the inductance of the load.

The boundary conditions will again be

$$i_1(0) = i_1\left(\frac{T}{2}\right) \tag{26}$$

$$i_2(0) = i_2\left(\frac{T}{2}\right) \tag{27}$$

$$e_2(0) = -e_2\left(\frac{T}{2}\right) \quad . \tag{28}$$

Writing the differential equations for the inverter circuit in Fig. 12 yields

$$E + e_c = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C}\int i_1 dt - \frac{1}{C}\int i_2 dt \tag{29}$$

and

$$-e_c = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C}\int i_2 dt - \frac{1}{C}\int i_1 dt \qquad . \qquad (30)$$

(The derivation of the following equations is given in Appendix A.)

Using Laplace transformations and solving these equations for $I_1(s)$

and $I_2(s)$ gives

$$I_1(s) = \frac{L_1 L_2 I_1(0) s^3 + \left[L_1 R_2 I_1(0) + (E + e_c) L_2\right] s^2}{s\left[L_1 L_2 s^3 + (R_1 L_2 + R_2 L_1)s^2 + (R_1 R_2 + \dfrac{L_1+L_2}{C})s + \dfrac{R_1+R_2}{C}\right]} +$$

$$\frac{\left[(E + e_c) R_2 + \dfrac{1}{C}\left\langle L_1 I_1(0) + L_2 I_2(0)\right\rangle\right]s + \dfrac{E}{C}}{s\left[L_1 L_2 s^3 + (R_1 L_2 + R_2 L_1)s^2 + (R_1 R_2 + \dfrac{L_1+L_2}{C})s + \dfrac{R_1+R_2}{C}\right]}$$

$$(31)$$

and

$$I_2(s) = \frac{L_1 L_2 I_2(0) s^3 + \left[L_2 R_1 I_2(0) - e_c L_1\right] s^2}{s\left[L_1 L_2 s^3 + (R_1 L_2 + R_2 L_1)s^2 + (R_1 R_2 + \dfrac{L_1+L_2}{C})s + \dfrac{R_1+R_2}{C}\right]} +$$

$$\frac{\left[\left\langle L_1 I_1(0) + L_2 I_2(0)\right\rangle \dfrac{1}{C} - R_1 e_c\right]s + \dfrac{E}{C}}{s\left[L_1 L_2 s^3 + (R_1 L_2 + R_2 L_1)s^2 + (R_1 R_2 + \dfrac{L_1+L_2}{C})s + \dfrac{R_1+R_2}{C}\right]} \qquad .$$

$$(32)$$

There will be two types of solutions to (31) and (32): the case

where the cubic impedance function has three real roots and the case

where it has one real and two complex roots.

All Real Roots

Assuming the impedance function has three real roots, (31) and (32) can be written in the form

$$F(s) = \frac{s^3 + gs^2 + hs + f}{s\,(s - b)\,(s - c)\,(s - d)} \; . \tag{33}$$

The inverse Laplace transformation will be [12]

$$f(t) = A + B\epsilon^{bt} + C\epsilon^{ct} + D\epsilon^{dt} \; , \tag{34}$$

where

$$A = \frac{f}{bcd} \; ,$$

$$B = \frac{b^3 + b^2 g + bh + f}{b\,(b-c)\,(b-d)} \; ,$$

$$C = \frac{c^3 + c^2 g + ch + f}{c\,(c-b)\,(c-d)} \; ,$$

$$D = \frac{d^3 - d^2 g + dh + f}{d\,(d-b)\,(d-c)} \; .$$

The values of $I_1(0)$, $I_2(0)$, and e_c can be formed by solving the equations (26), (27), and (28) simultaneously.

Complex Roots

If the impedance function has one real root and a pair of complex roots, the solution of (31) and (32) can be found by writing them in the form

[12] G. A. Korn and T. M. Korn, Mathematical Handbook for Scientists and Engineers (New York, 1961), pp 770 - 778.

$$F(s) = \frac{K_1 (s^2 + gs + d)}{(s - b)\left[(s - a)^2 + \omega^2\right]} + \frac{K_2}{s(s - b)\left[(s - a)^2 + \omega^2\right]} . \quad (35)$$

The Laplace transform will be [13]

$$f(t) = K_1 \left[A_1\, \epsilon^{at} \sin(\omega t + \alpha_1) + B_1\, \epsilon^{bt} \right] +$$

$$K_2 \left[A_2\, \epsilon^{at} \sin(\omega t + \alpha_2) + B_2\, \epsilon^{bt} + K_3 \right] , \quad (36)$$

where

$$A_1 = \frac{1}{\omega} \left[\frac{(a^2 - \omega^2 + ag + d)^2 + \omega^2 (2a + g)^2}{(a - b)^2 + \omega^2} \right]^{1/2}$$

$$B_1 = \frac{b^2 + bg + d}{(a - b)^2 + \omega^2}$$

$$\alpha_1 = \tan^{-1} \left[\frac{\omega (2a + g)}{a^2 - \omega^2 + ag + d} \right] - \tan^{-1} \left[\frac{\omega}{a - b} \right]$$

$$A_2 = \frac{1}{\omega} \frac{1}{(a^2 + \omega^2)^{1/2} \left[(a - b)^2 + \omega^2 \right]^{1/2}}$$

$$B_2 = \frac{1}{b\left[(b - a)^2 + \omega^2 \right]}$$

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[13]Ibid.

$$K_3 = \frac{-1}{b(a^2 + \omega^2)}$$

$$\alpha_2 = -\tan^{-1}\left(\frac{\omega}{a-b}\right) - \tan^{-1}\frac{\omega}{a} \quad .$$

As in the case for real roots, $I_1(0)$, $I_2(0)$, and e_c can be found from (26), (27), and (28).

If the load were capacitive rather than inductive, a capacitor would be added in the second loop of the equipment circuit in Fig. 12. This would not change the type of solution that was obtained for the inductive load, therefore the analysis for a capacitive load will not be carried out.

The current and voltage equations have now been derived for the simplified resistive load case and the more complete analysis concerning the reactive load with transformer losses.

CHAPTER IV

APPLICATION OF THE MATHEMATICAL ANALYSIS

The equations representing the input current and output voltage

for the simplified case, resistive load, do not readily illustrate the

effect of the circuit parameters on the voltage and current, nor are the

solutions easily obtained through numerical calculations. For these

reasons it was decided to make use of a digital computer in analyzing

the equations of Chapter III. A description of the computer program

which was written and its use will now be given.

The Use of the Digital Computer in Solving the Voltage and Current Equations of the Inverter

The digital computer will be used to find the values of the input

current and the output voltage as functions of time for the parallel in-

verter with resistive load, assuming an ideal transformer as shown in

Fig. 9. From the output of the computer the current and voltage wave-

forms can be plotted. In Chapter III the equations representing the cur-

rent and voltage were developed; also, equations for finding the initial

values for these relations were given. The computer will determine

which case described in Chapter III it has to evaluate, and then use the

following equations from Chapter III: complex roots -- initial values

from (11) and (12), voltage and current from (9) and (10); equal

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roots -- initial values from (16) and (17), voltage and current from (14) and (15); real roots -- initial values from (20) and (21), voltage and current from (18) and (19).

A computer program to solve these equations for various values of time was written in Fortran language for the IBM 650 digital computer. A listing of the program in Fortran language is given in Appendix B. It can be easily modified for use on the IBM 1620 computer. The time required to perform the necessary calculations for one set of voltage and current waveforms was less than one minute.

Flow Diagram of Computer Program

A flow diagram of the program is given in Fig. 13. As was mentioned earlier, there are three cases: complex roots, equal roots, and real roots. Each case requires a different set of equations for the solution. For each of these cases the particular value of I_1 (0) and e_c must be evaluated. The computer takes the input data containing the circuit parameters and determines which case exists. At this point it will evaluate the two appropriate simultaneous equations to obtain the initial conditions, I_1 (0) and e_c. It is then ready to calculate the input current and the output voltage as t varies from zero to T/2 for eleven values of time. The computer will then go to the next set of data.

Computer Input-Output Data

The format for the input and output cards for the computer is shown in Fig. 14. The input data consists of the following, the first symbol being the Fortran symbol and the second being that used in
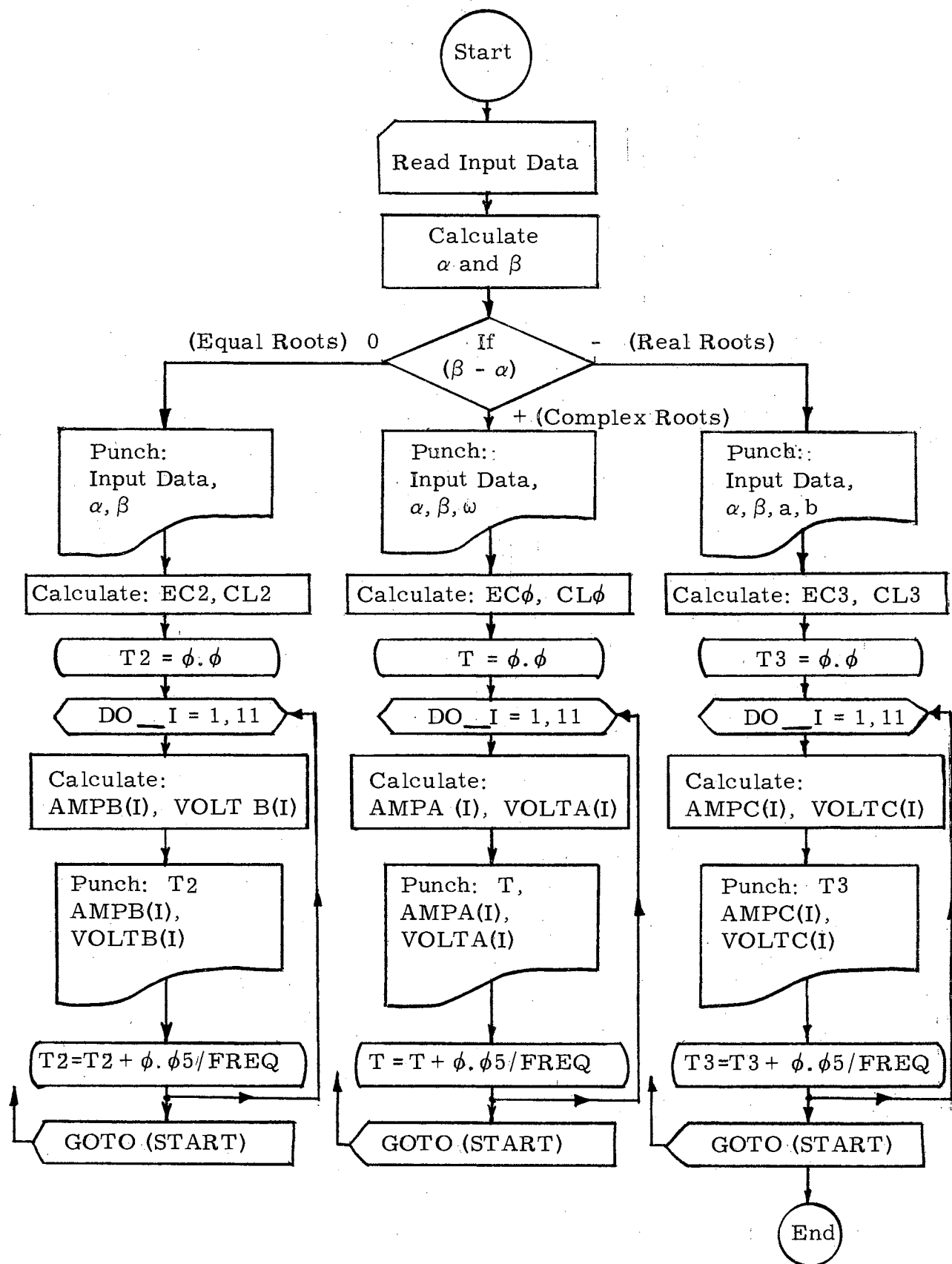
Fig. 13. Flow Diagram of Computer Program
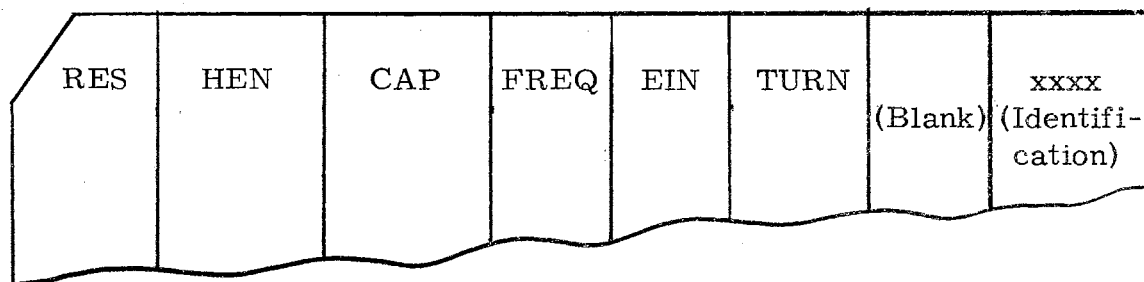
Input Data Card
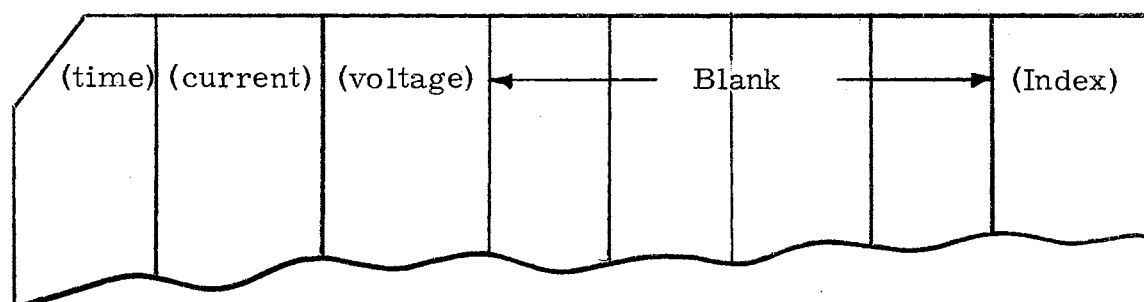
Output Data Card

Fig. 14. Data Card Formats

Chapter III:

RES = R_L, the load resistance in ohms

HEN = L, the ballast inductance in henrys

CAP = C, the commutating capacitance in farads

FREQ = f, the switching frequency, 1/T, in cycles per second

EIN   =   E, the d.c. supply voltage in volts

TURN   =   n, the turns ratio of the secondary to half the primary .

The output of the computer will be as shown in Fig. 14 where

T, T2, T3 = t, time in seconds

AMPA(I), AMPB(I), AMPC(I) = $i_1(t)$, the input current in amperes

VOLTA(I), VOLTB(I), VOLTC(I) = $e_2(t)$, the output voltage in volts.

The meaning of other symbols used in the program are given in Appendix B with the listing. The input data is also punched out with each set of calculations in the same form as it was read into the computer for reference purposes after the calculations have been performed.

The results using the computer will be compared with the experimental results in Chapter VI.

CHAPTER V

PARALLEL INVERTER DESIGN

The basic design of a parallel inverter involves selecting suitable values of commutating capacitance, C, and ballast inductance, L, for a given load and switching frequency. Given the input voltage and the required output voltage or current, the transformer can be specified. The range of L and C is limited by the condition for SCR turn-off and the condition for continuous input current. The basis for selecting L and C depends, in part, on the desired output voltage waveform, e.g., square wave, sinewave, or sawtooth. This chapter will consider these factors in establishing certain design criteria for the parallel inverter using SCR's. The design procedure developed in this chapter will be applied to an experimental inverter in the following chapter.

Condition for SCR Turn-Off

The SCR must be reverse-biased long enough for it to regain its forward blocking characteristics. Although the SCR may stop conducting in less than a microsecond, it cannot block a forward current until the semiconductor material has had sufficient time to recover from the conducting state. The turn-off time is defined as the time interval required for the gate to regain control of the forward blocking characteristic

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after forward conduction. As discussed in Chapter II, the turn-off time

may be as short as 7 microseconds, but can range up to 30 microseconds.

If t' is the time interval the SCR is reverse-biased, then t' must

be greater than the maximum required turn-off time, t_o, for turn-off

to occur. This allows sufficient time for the semiconductor material to

recover from saturation.

The equation for the voltage across the SCR was derived in

Chapter III : Equation (10) for complex roots and Equation (19) for real

roots.

An expression for t' will now be determined. The waveform be-

tween t = 0 and e = 0 in Fig. 15 will be assumed to be linear.
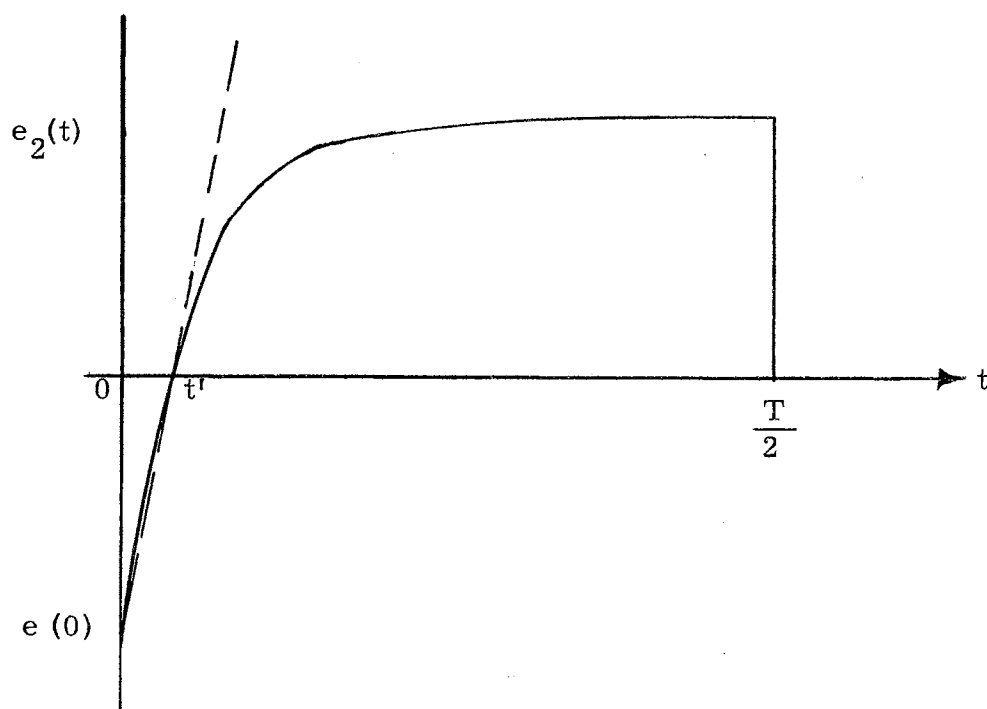
Fig. 15. Voltage Across the SCR During 1/2 Cycle of Operation

Thus, equating the slope of the waveform at $t = 0$ and the slope of the line through $e(0)$ and t' gives

$$\frac{de(t)}{dt}\bigg|_{t=0} = \frac{e(0)}{t'} \quad . \tag{37}$$

The limiting condition for SCR turn-off will only be approached for the case of real roots; t' will always be greater for complex roots. Differentiating (19) at $t = 0$ yields

$$\frac{de(t)}{dt}\bigg|_{t=0} = \frac{-e_c a^2 + \frac{a}{4C} I_1(0) + E\beta^2}{a - b} + \frac{-e_c b^2 + \frac{b}{4C} I_1(0) + E\beta^2}{b - a} =$$

$$= -e_c (a + b) + \frac{I_1(0)}{4C} \quad . \tag{38}$$

Also,

$$e(0) = \frac{-e_c a^2 + \frac{a}{4C} I_1(0) + E\beta^2}{a (a - b)} + \frac{-e_c b^2 + \frac{b}{4C} I_1(0) + E\beta^2}{b (b - a)} + \frac{E\beta^2}{ab} =$$

$$= -e_c \quad . \tag{39}$$

Substituting (38) and (39) into (37) and solving for t' gives

$$t' = \frac{-e_c}{e_c (a + b) - \frac{1}{4C} I_1(0)} \quad . \tag{40}$$

When the limiting condition for SCR turn-off is approached the output voltage will approximate a square wave, and e_c and $I_1(0)$ may be given by

$$e_c = E$$
$$\text{and} \qquad I_1(0) = \frac{E}{R} \qquad \Bigg\} . \tag{41}$$

Equation (40) now becomes

$$t' = \frac{-1}{a + b - 2\alpha} \tag{42}$$

where $a, b = -\alpha \pm \sqrt{\alpha^2 - \beta^2}$, as in Chapter III. As t' becomes small, then $\alpha^2 \gg \beta^2$, and

$$a = 0$$
$$b = -2\alpha \qquad \Bigg\} . \tag{43}$$

Therefore, the limiting condition for SCR turn-off is

$$t' = \frac{1}{4\alpha} = 2\,RC > t_o . \tag{44}$$

This is the most important relationship in basic inverter design since inability of the SCR to turn off would lead to a short-circuit condition.

If a varying load is anticipated, the full load resistance should be used in (44) since the t' for fractional loads will be greater than for a full load.

Starting Conditions

When the inverter is first turned on, the initial conditions $I_1(0)$ and e_c used previously do not apply; in fact, the initial conditions are quiescent. After the first several cycles of operation the boundary conditions given in (3) and (4) are satisfied, and $I_1(0)$ and e_c will apply.

The question arises as to whether or not the first SCR to be turned off will be reverse-biased long enough for turn-off under the starting initial conditions. Previous analysis[14] and experience with actual inverter circuits have shown that this condition is satisfied if the SCR turn-off condition described in the previous section is satisfied. The only requirement other than this is that the inductance not be too large for a given input voltage. If the inductance is too large, sufficient charge will not build up on the capacitor to permit the SCR to turn off. The greater the input voltage, the greater the charge build-up. An empirical relationship gives

$$L < 5E \tag{45}$$

where L is the ballast inductance in <u>millihenrys</u> and E the input voltage in volts. Further discussion concerning this can be found in a paper by Dr. H. T. Fristoe.[15]

It also follows that the inverter would be harder to start at higher frequencies, but this is not a significant factor unless $t_o \cong t'$. Therefore caution should be exercised when designing with t' close to t_o.

Condition for Continuous Input Current

It was explained in Chapter III that as L decreases the

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[14]R. H. Murphy and K. P. P. Nambiar, "A Design Basis for Silicon Controlled Rectifier Parallel Inverters," Institution of Electrical Engineers Proceedings, CVIII (1961), 556-562.

[15]H. T. Fristoe, "Fluorescent Lighting Utilizing the SCR Inverter," Texas Instruments Memorandum, September, 1961.

variations in the input current become greater. The variations can increase until the current drops below the holding current of the SCR before the end of the half cycle, thus turning the SCR off. (See Fig. 16.)
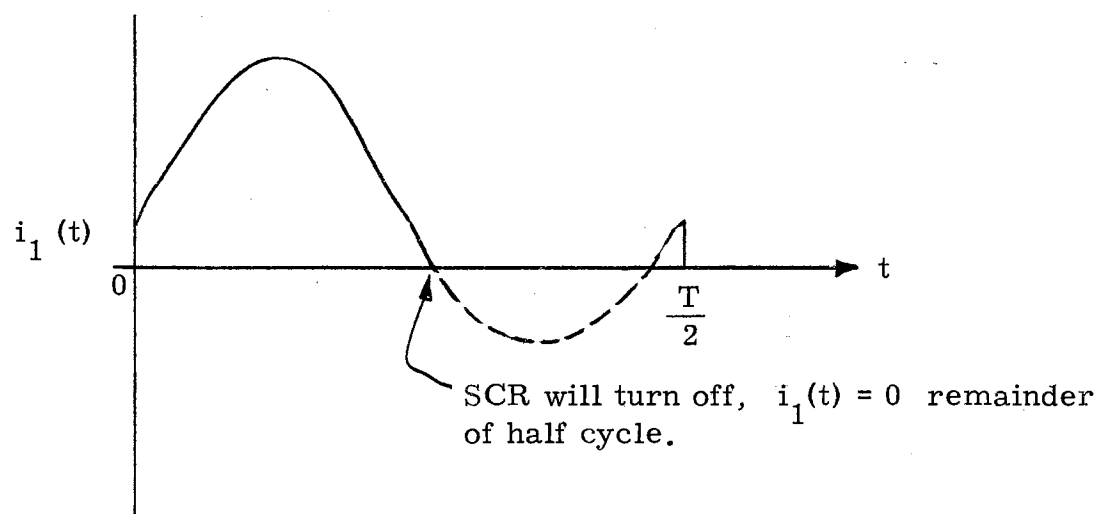
Fig. 16. Input Current Flowing in Pulses

This results in the inefficient and generally undesirable condition where the input current flows in pulses. An analytical expression derived by Wagner[16] gives the point at which the pulse current begins to occur. His approach was to find the time at which the first minimum value of input current occurred. This value of time was substituted into the expression for current and equated to zero. The result led to the relationship

[16]C. F. Wagner, "Parallel Inverter with Resistive Load," Electrical Engineering, LIV (1935), 1227 - 1235.

$$\exp\left[-\frac{\alpha}{\omega} \tan^{-1} \sqrt{\frac{\beta^2}{\alpha^2} - 1 + \pi} \right] = \frac{\alpha}{\beta} \quad , \qquad (46)$$

which was solved by trial and error to give

$$\beta^2 = 12.96\, \alpha^2 \quad . \qquad (47)$$

Therefore, the current will flow in pulses when

$$\beta > 3.6\alpha \quad . \qquad (48)$$

The possibility of the SCR being switched before the first current minimum occurred was not considered by Wagner. The natural resonant frequency of the equivalent circuit of the inverter is β in radians or

$$f_n = \frac{\beta}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{4LC}} \quad . \qquad (49)$$

This will be the frequency of the transient ringing shown in Fig. 16. It follows that the input current will flow continuously when the natural resonant frequency is less than the switching frequency, f_s, i.e., $f_n < f_s$. Also, the current will flow continuously when $f_n > f_s$ if $\beta < 3.6\alpha$.

Output Voltage Waveforms

Square Wave: Square wave operation is usually the most efficient and useful mode of operation. A basic design procedure can be formed from the following relationships. It can be seen from Fig. 17 that for a square wave output voltage t' should be made as small as possible. Therefore, one condition for square wave operation will be

$$t_o < 2RC \quad . \qquad (50)$$
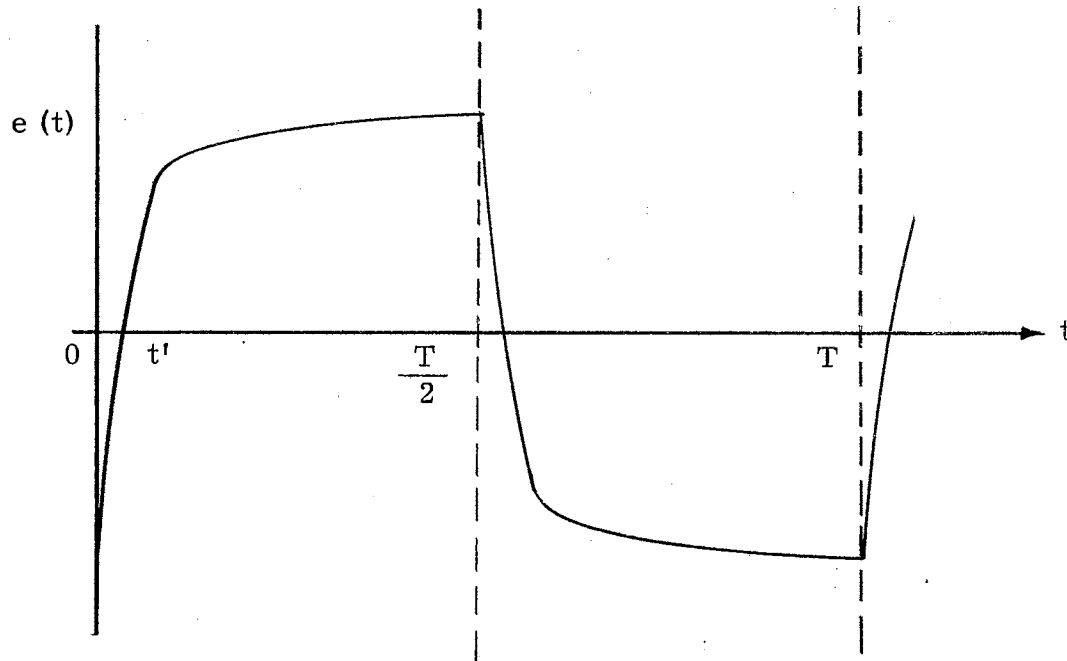
Fig. 17. Output Voltage of a Square Wave Inverter

The smaller 2RC is made, the better the square wave. Allowances

for variation in turn-off time or load, the use of an oversized inductor,

or a high switching frequency should be made in (50). Since R, the

equivalent load, is determined by the specific design and t_o is given

in the SCR specifications, the value of C can be found.

A second requirement for an acceptable square wave output,

which will determine a value for L, concerns the rate of rise of the

voltage with respect to the switching frequency. The waveshape is

primarily determined by the decaying exponential term, $\epsilon^{-\alpha t}$. The

time required for the exponential to decay to less than 5% of its initial value can be found from the following:

$$\epsilon^{-\alpha t} = 0.05$$

or
$$\alpha t = 3 \quad . \tag{51}$$

For a given α the minimum period (maximum frequency) which will allow the exponential to decay to 5% of its initial value will be

$$\alpha \left(\frac{T}{2} \right) = 3$$

$$T = \frac{6}{\alpha}$$

$$f = \frac{\alpha}{6} = \frac{1}{48RC} \quad , \tag{52}$$

where $\alpha = \dfrac{1}{8RC}$.

A minimum value of L for square wave operation can be specified as that which will resonate with C at the frequency given in (52). Therefore,

$$f = \frac{1}{48RC} = \frac{1}{2\pi} \sqrt{\frac{1}{4LC}} \quad . \tag{53}$$

Solving for L ,

$$L = \frac{144R^2 C}{\pi^2} \simeq 15R^2 C \quad . \tag{54}$$

An L smaller than this will result in overshoot. The maximum allowable value of L is determined from the starting conditions (45). The value of L is not critical and may be based on the rate of rise versus overshoot of the output voltage.

A maximum frequency for square wave operation can be found from (52). A switching frequency greater than $\frac{1}{48RC}$ does not allow the exponent to decay to 5% of its initial value, the result being a poor square wave. Therefore, the maximum frequency of operation is given by

$$f_{max} = \frac{1}{48RC} = \frac{1}{24t_o} . \qquad (55)$$

Sinewave Output: Sinewave operation of the inverter makes use of the natural resonate frequency of the L-C circuit. The inverter is switched at a frequency which resonates with the commutating capacitor, C, and the ballast inductor, L. Thus, for a given output load and frequency, the condition that

$$f_s = \frac{1}{2\pi} \sqrt{\frac{1}{4LC}} \qquad (56)$$

must be satisfied. 4C is used in (56) since the commutating capacitor appears four times as large due to the autotransformer action of the center-tapped transformer primary.

A second relationship may be obtained by considering the expression for t', the time at which the output voltage goes through zero. It was found previously that $t' = 2RC$. From Fig. 18 it can be seen that for sinewave operation

$$t' = \frac{1}{2} \left(\frac{T}{2} \right) = \frac{1}{4f_s} \qquad (57)$$

where f_s is the switching frequency. Therefore,

$$f_s = \frac{1}{8RC} . \qquad (58)$$
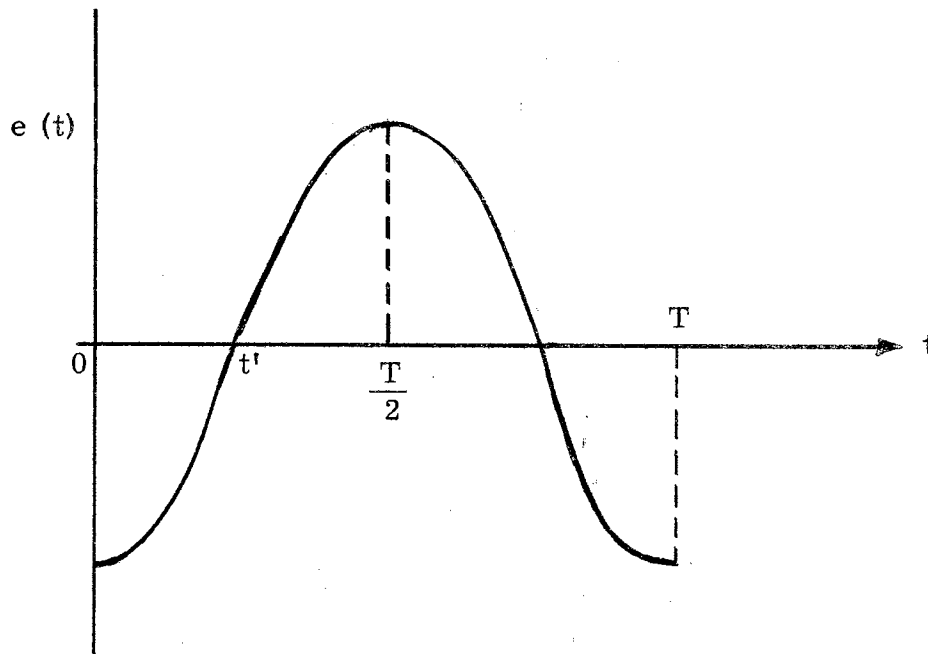
Fig. 18. Output Voltage of a Sinewave Inverter

C can be found from (58) since the frequency and load depend upon the specific design. L can be found from (56).

Sawtooth Output: For generation of a sawtooth waveform, the switching frequency, f_s, should be much greater than the natural resonate frequency, f_r. A maximum value of f_r for a sawtooth waveform may be given as

$$2 f_r \leq f_s \ . \tag{59}$$

A similar maximum condition can be put on the exponential decay time constant, $\epsilon^{-\alpha t}$:

$$2\alpha \lesssim f_s$$

or

$$f_s \geq \frac{1}{4RC} \quad . \tag{60}$$

Therefore, with R and f_s given for a specific design, a minimum C can be found from (60). Knowing C and f_s, a minimum L can then be found from (59) as

$$f_s = \frac{1}{\pi} \sqrt{\frac{1}{4LC}} \quad . \tag{61}$$

A more linear sawtooth is obtained as R, L, and C are increased. As R, L, and C are increased, the efficiency decreases. Thus the inverter design is largely dependent upon the linearity desired and the efficiency required.

Summary of Design Criteria

A summary of the equations developed in this chapter will help to simplify inverter design. Regardless of the output voltage wave-shape desired, the following conditions must be satisfied:

SCR turn-off $\qquad\qquad t_o < 2RC$

Starting Condition $\qquad\quad L < 5E \qquad$ (L in mh.)

Continuous Current $\qquad\; f_s > f_r$, $\qquad$ or if $f_s < f_r$

$$\beta > 3.6\alpha \quad .$$

For a square wave output the following relations hold:

Minimum Allowable C $\qquad C > \dfrac{t_o}{2R}$

Minimum Allowable L $\qquad L > 15R^2 C$

Maximum Allowable f_s $\qquad f_{max} = \dfrac{1}{24\, t_o} \quad .$

For a sinewave output the conditions are:

C is given by

$$f_s = \frac{1}{8RC} \quad \text{or} \quad C = \frac{1}{8Rf_s}$$

L is given by

$$f_s = \frac{1}{2\pi} \sqrt{\frac{1}{4LC}} \quad \text{or} \quad L = \frac{1}{16\pi^2 C f_s^2} \quad .$$

For a sawtooth output voltage, the limitations are:

Minimum Acceptable C

$$f_s \geq \frac{1}{4RC} \quad \text{or} \quad C \geq \frac{1}{4Rf_s}$$

Minimum Acceptable L

$$f_s \geq \frac{1}{\pi} \sqrt{\frac{1}{4LC}} \quad \text{or} \quad L \geq \frac{1}{4\pi^2 C f_s} \quad .$$

(R is the load referred to one-half of the transformer primary. f_s is the switching frequency.)

CHAPTER VI

EXPERIMENTAL SCR INVERTER

The design criteria developed in Chapter V and the voltage and current equations derived in Chapter III will be used in this chapter in designing specific parallel inverters, thus enabling a comparison to be made between the computed and experimental results. An evaluation of the preceding analysis can then be made and predictions as to when it will be in error can be stated. This chapter will include a description of the experimental inverter used, followed by a comparison of experimental and computed results and a discussion of the disagreements.

Description of Experimental Inverter

An experimental setup which would allow variation of all the parameters in the inverter circuit was desired. This led to the setup shown in Fig. 19. The d.c. voltage source could be varied from 0 to 35 volts at 5 amperes. The output of the d.c. source was shunted by a large capacitance (500 μf) to insure a low transient impedance. The load was a power resistor decade box. The SCR's used were TI 2N1603 (3 ampere capacity) mounted on 5" square, 1/16" thick aluminum plates as heatsinks.
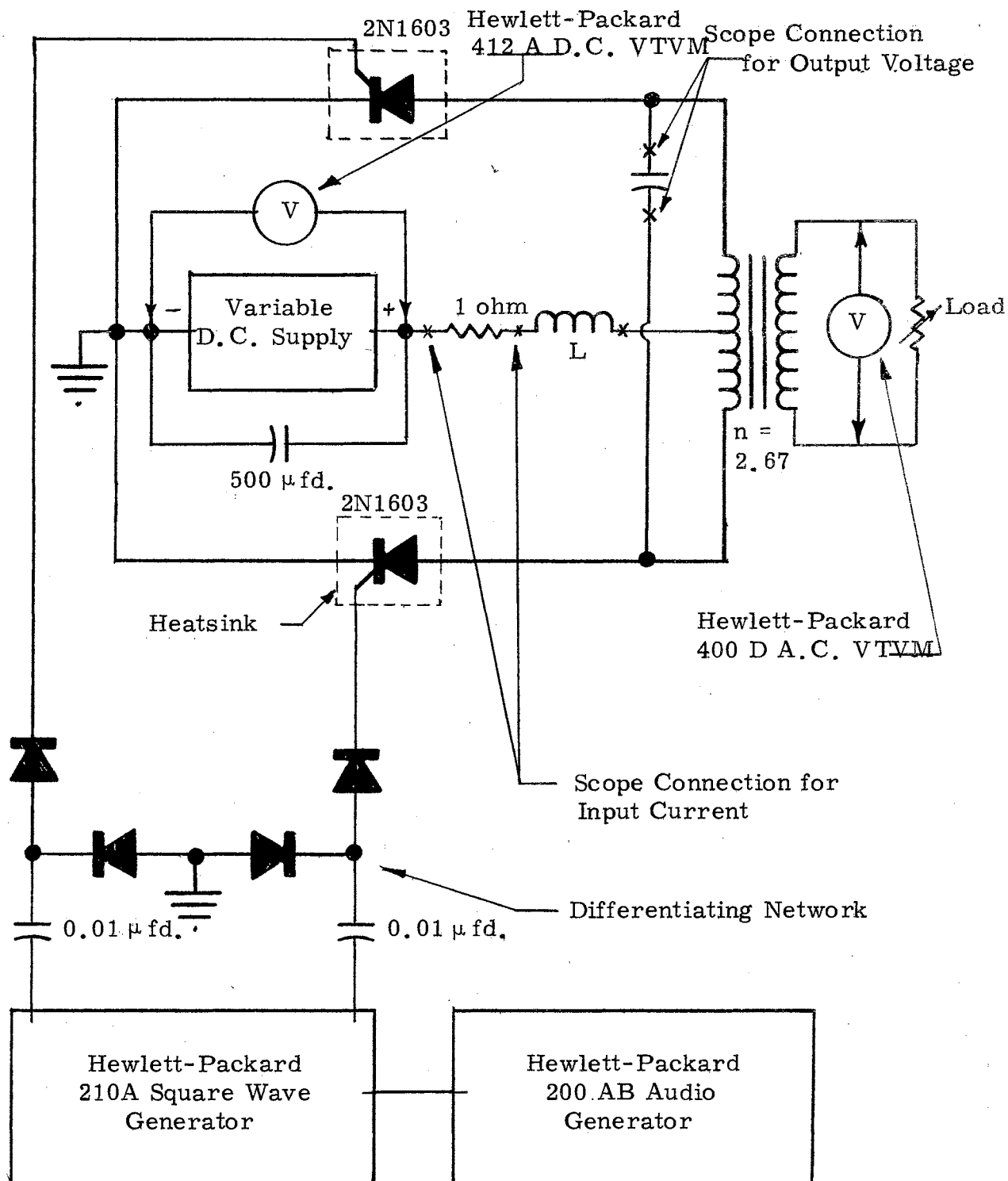
Fig. 19. Experimental Inverter Setup

The principal power transformer that was used is described in Fig. 20. The turns ratio (secondary to one-half total primary) was 2.67. Since a unity turns ratio was assumed throughout the analysis, the equivalent load resistance was found using the square of the turns ratio. Other transformers used had much higher winding resistances and leakage inductances which made them impractical for use in inverters.

Core: 7/8 Orthosil, 29 ga.

T_1 = 78 turns
#20 AWG
(Bifilar Wound)

53V RMS T_1

R_p = 0.7 ohms

T_1

T_2 = 208 turns
#22 AWG

141 Volts
T_2 RMS

R_s = 1.4 ohms

n = 2.67
160 watts
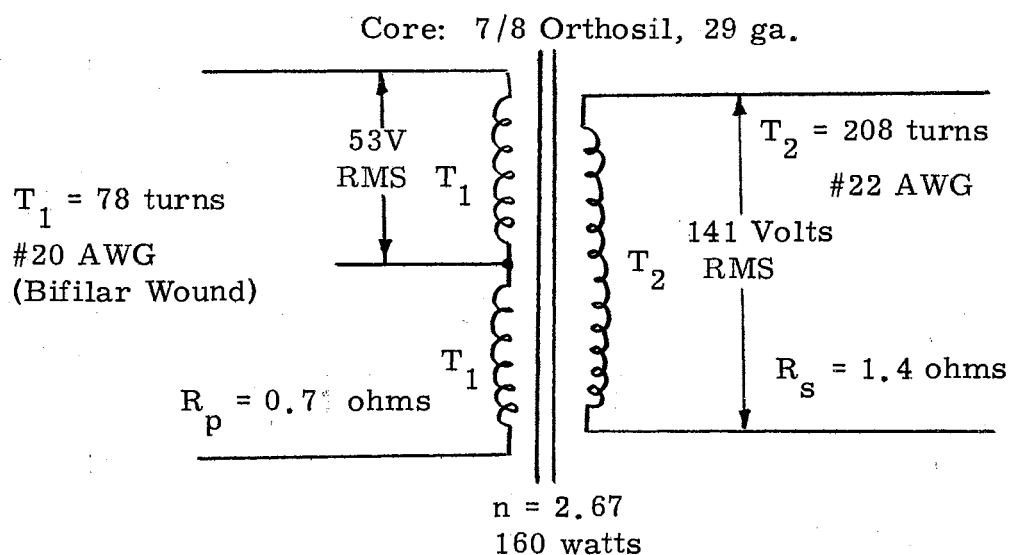
Fig. 20. Transformer Used in Experimental Inverter

The trigger source was the differentiated output of a Hewlett-Packard square wave generator driven by a Hewlett-Packard audio oscillator. This provided a wide frequency range with sufficient pulse magnitude for switching the SCR's.

The output voltage was measured across the commutating capacitor. This would be the same as the voltage across the transformer secondary if the transformer had a unity turns ratio. The calculated output voltage, $e_2(t)$, in Chapter III was the voltage across the load in the equivalent circuit; therefore, it must be multiplied by two to obtain the output voltage in the actual circuit. (This has already been done in the computer program.)

Comparison of Experimental and Computed Results

In Chapter III equations were derived for the input current and output voltage for an inverter with an ideal transformer and a resistive load. These equations were programmed on a digital computer so that the current and voltage waveforms could be plotted from the computer output. By computing waveform data for a wide variation in parameters the behavior of the equivalent inverter circuit could be analyzed and compared with the experimental inverter. An example of the computed results compared with experimental results is shown in Fig. 21 for one-half cycle of operation. The disagreements arise from the simplified inverter circuit used.

One method used to compare the voltage waveforms was to plot the peak voltage during a half cycle versus the switching frequency, the inverter components remaining the same. A typical comparison of the variation of experimental and computed peak voltages as the frequency is varied is shown in Fig. 22. At low frequencies the experimental and computed results agree. As the frequency is increased
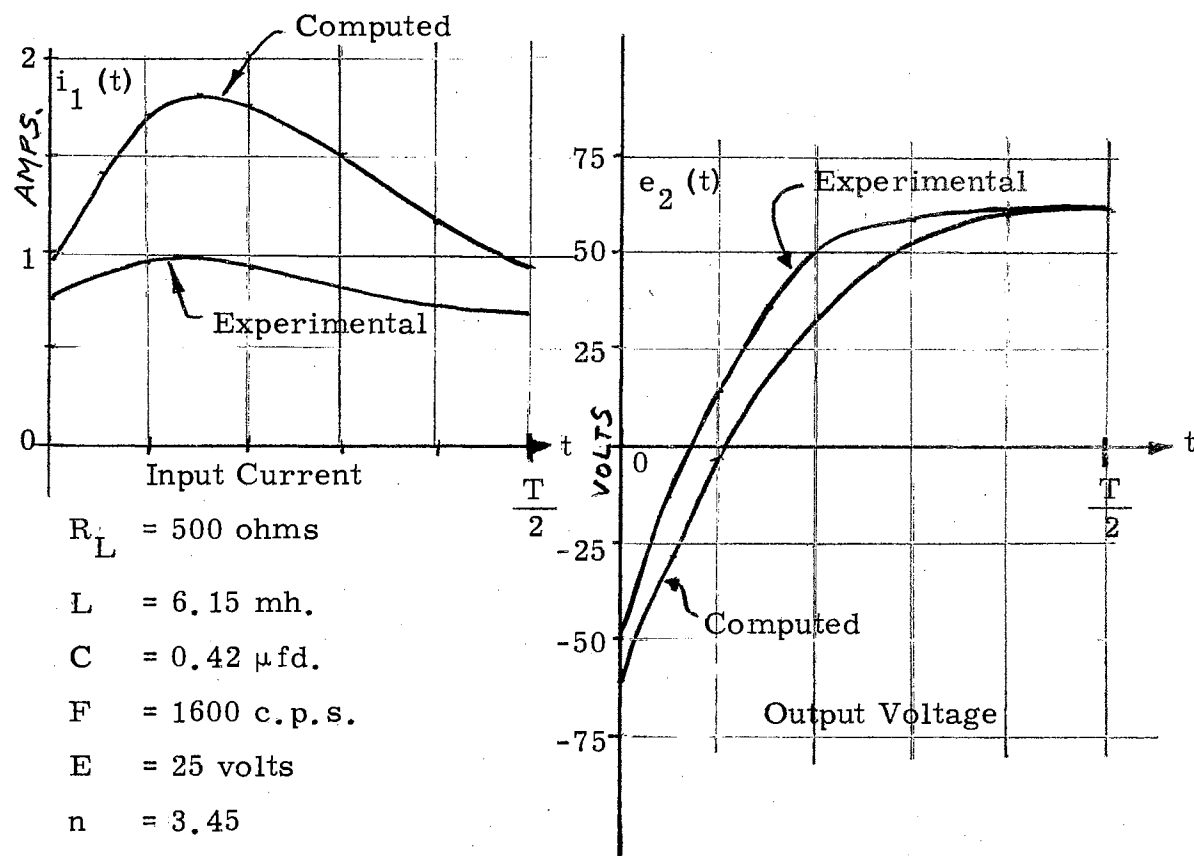
Fig. 21. Computed and Experimental Waveforms

the computed peak voltage increases to a sharp peak. At higher fre-
quencies it is attenuated to a low value. The experimental results
give a much wider peak of much smaller magnitude and occurring at a
slightly higher frequency.

Before discussing why the calculated and experimental results
differ, an explanation will be given as to the behavior of the computed
results. Figure 23 shows the equivalent circuit used in Chapter III,
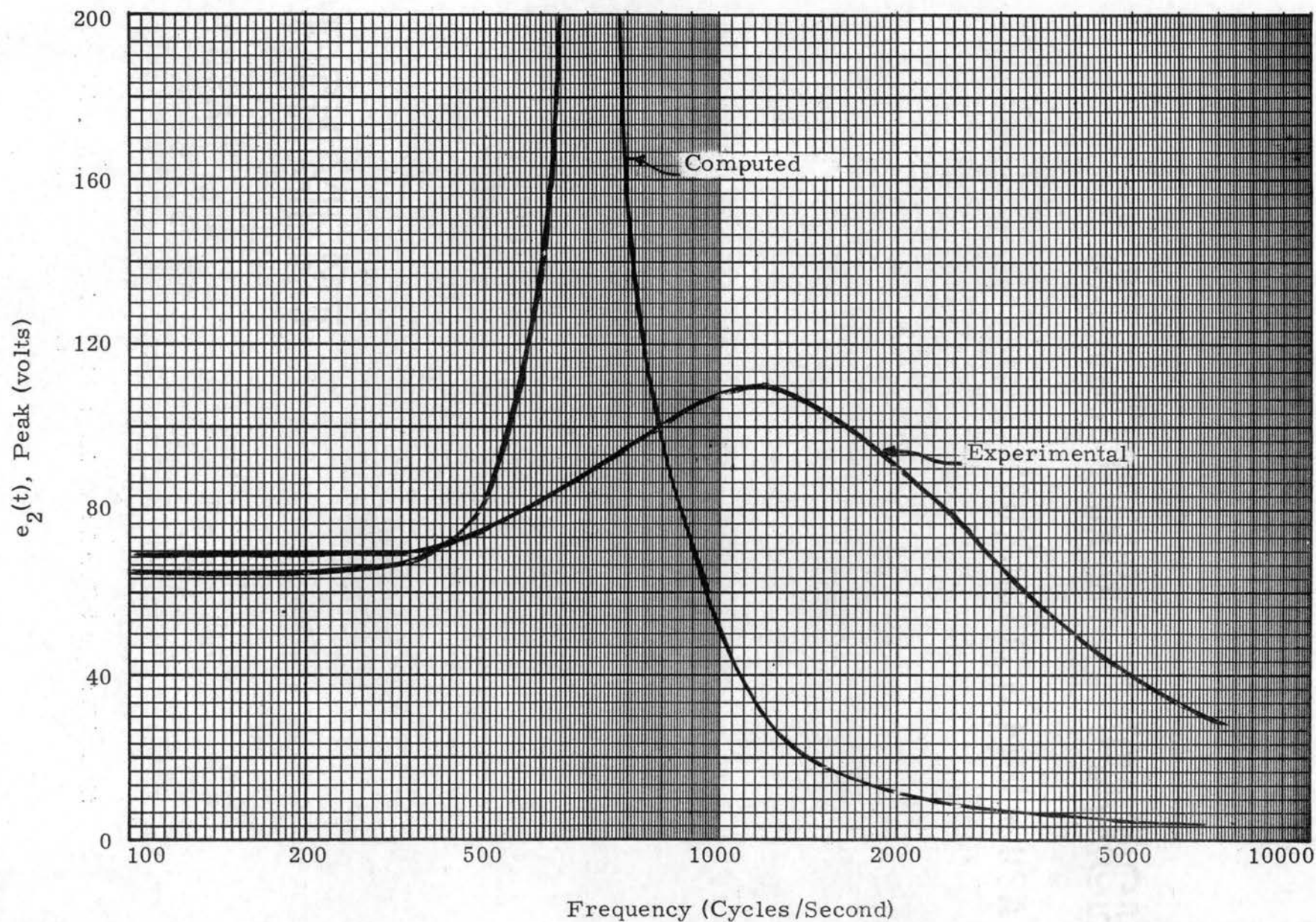
Fig. 22. Comparison of Experimental and Computed Results
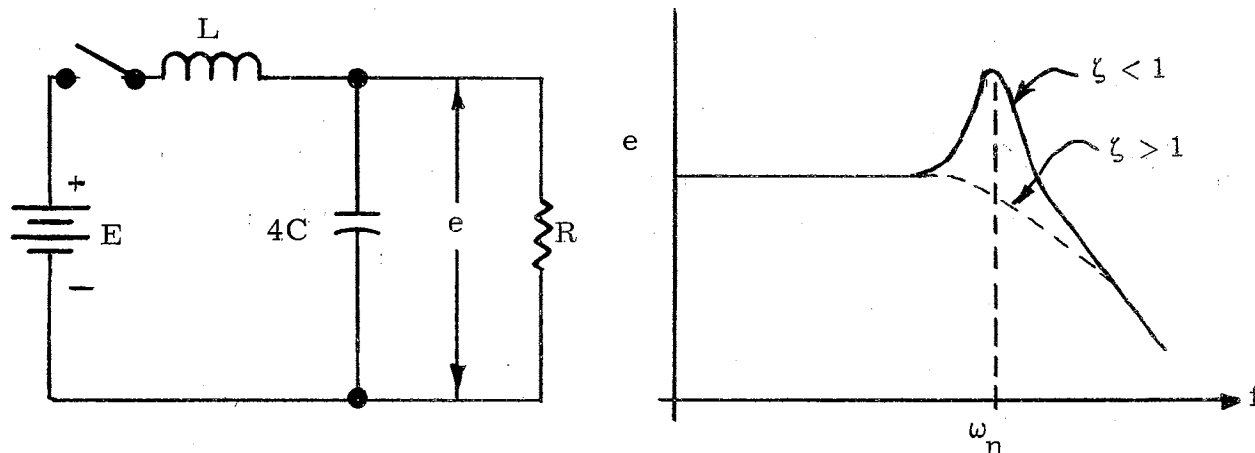
Fig. 23. Voltage Response of Equivalent Circuit

ignoring the initial conditions. The impedance function of this circuit

is $s (s^2 + \dfrac{1}{4RC} s + \dfrac{1}{4LC})$ or $s (s^2 + 2\alpha s + \beta^2)$, which is of the form

$s (s^2 + 2\zeta \omega_n s + \omega_n^2)$ where $\omega_n = \beta$ and $\zeta = \dfrac{\alpha}{\beta}$.

The Bode diagram in Fig. 23 shows a resemblance to the peak

voltage vs. frequency response of the inverter in Fig. 22 for $\zeta < 1$.

Leaving the steady-state response and looking at the transient

response of such a circuit (this time assuming an initial charge on the

capacitor), it can be seen from Fig. 24 that two possible conditions

exist: the underdamped and the overdamped case. If $\beta > \alpha$ complex

roots will exist in the impedance function and the transient will ring.

If $\beta < \alpha$ the impedance function will be a decaying exponential. For
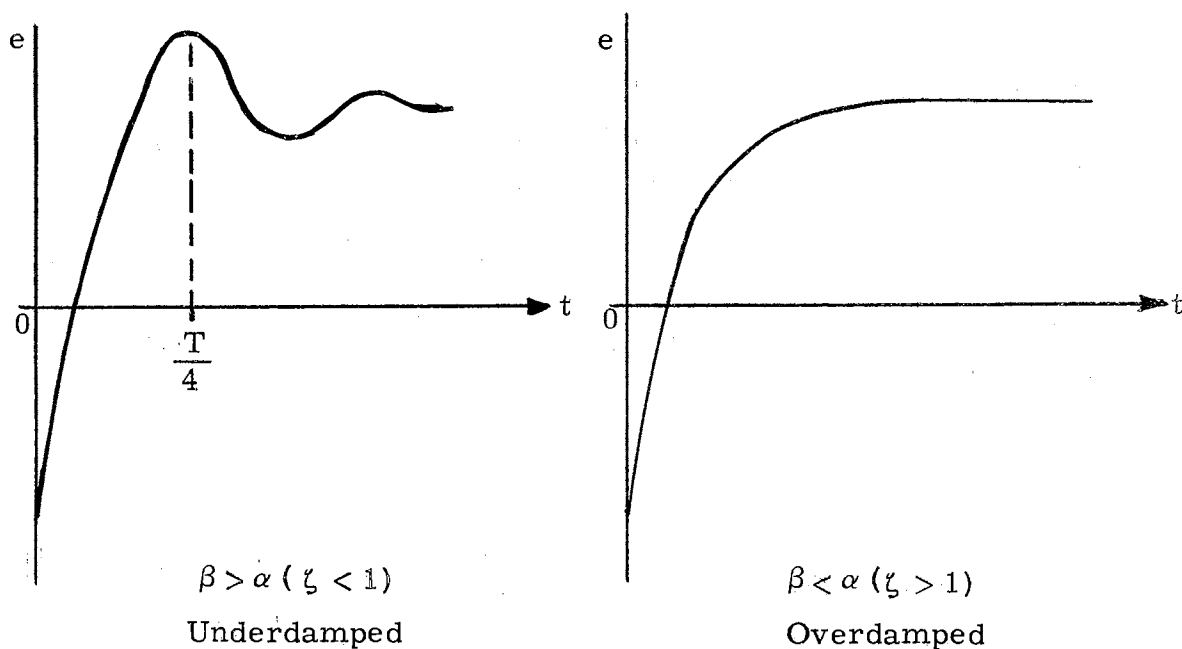
Fig. 24. Transient Response of Equivalent Circuit

$\beta > \alpha$ a maximum peak-to-peak voltage would be obtained by switching at $\dfrac{T_r}{4}$ where $T_r = \dfrac{1}{f_r}$, the natural resonant frequency of the L-C circuit. This would be one-half the period of the switching frequency, f_s. Hence, the maximum voltage would occur at

$$\frac{T_r}{4} = \frac{T_s}{2}$$

or

$$f_s = 2f_r \, , \qquad\qquad (62)$$

where $f_r = \dfrac{1}{2\pi} \sqrt{\dfrac{1}{4LC}}$. This agrees with the computed results shown

in Fig. 25. The curves in Fig. 25 represent the peak output voltage

from the inverter as the frequency is varied. The four examples shown

have resonant frequencies of 250, 500, 1,000, and 1,500 c.p.s. They

all have the same ζ of 1/2. The peaks fall slightly below the resonant

frequency for ζ greater than 0.25 and less than 1.0. For a small ζ

the peak tends to be higher and have a wider bandwidth than when ζ

approaches unity.

If the initial conditions are considered the explanation above re-

mains true, although the peak can reach an extremely high voltage.

From computed data it was found that the bandwidth of the peak varied

from less than 100 c.p.s. for heavy loads (ζ near 1.0) to almost 1,000

c.p.s. for light loads (ζ less than 0.1). For $\beta < \alpha$ or $\zeta > 1$ the output

voltage increases with frequency but no peak is reached.

The analytical results will now be compared with the experimen-

tal results. As shown in Fig. 22, the largest area of disagreement

occurs near the resonant peak. To study the cause of this disagree-

ment, different transformers were used in the inverter. The effect of

the transformer characteristics on the output voltage is shown in

Fig. 26. Three different transformers were used, the equivalent load

remaining the same. Transformers B and C had much higher wind-

ing resistances and core losses than transformer A. In the equivalent

circuit, these losses were neglected since an ideal transformer was
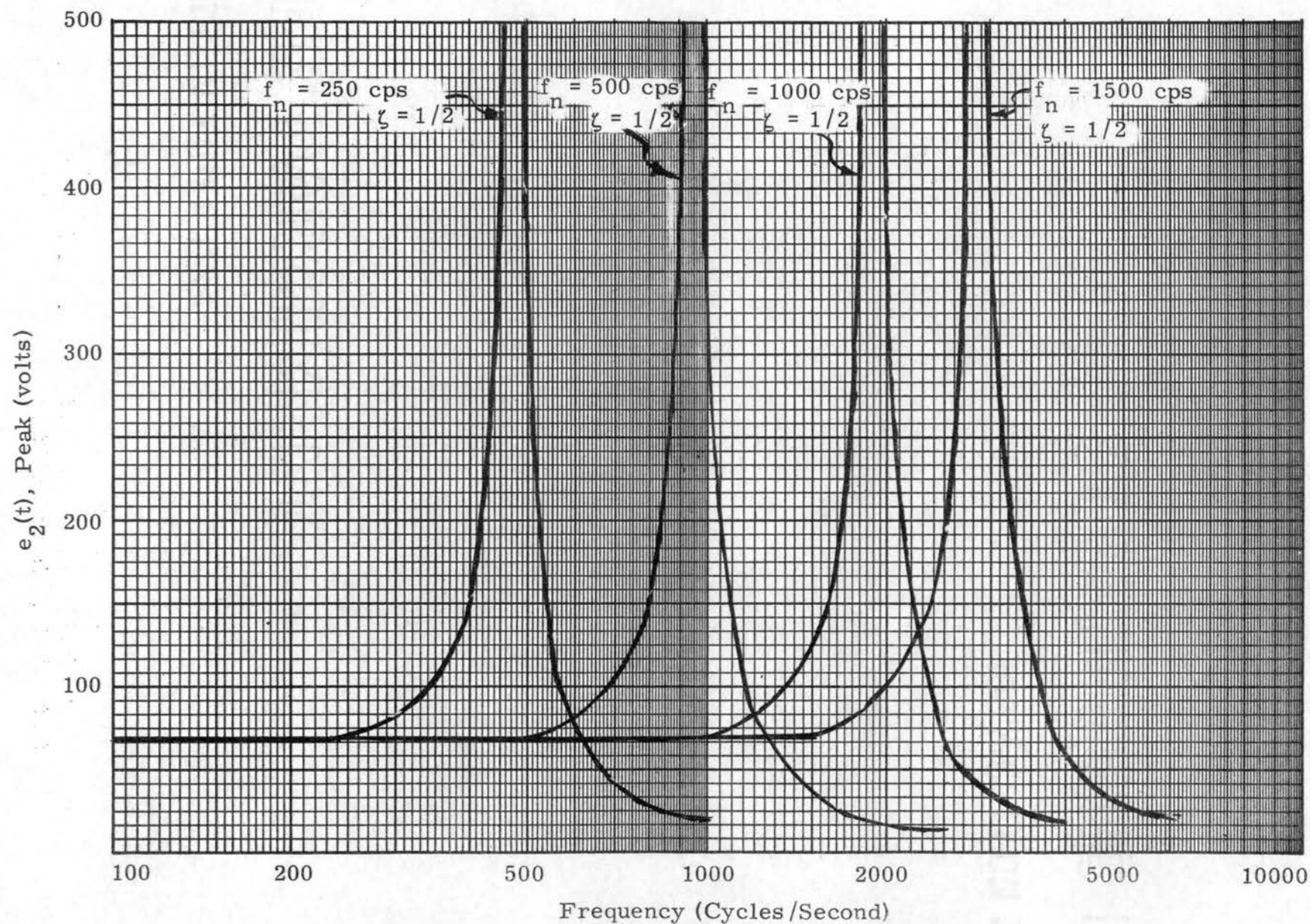
Fig. 25. Inverter Response for Various Resonant Frequencies, ζ Constant
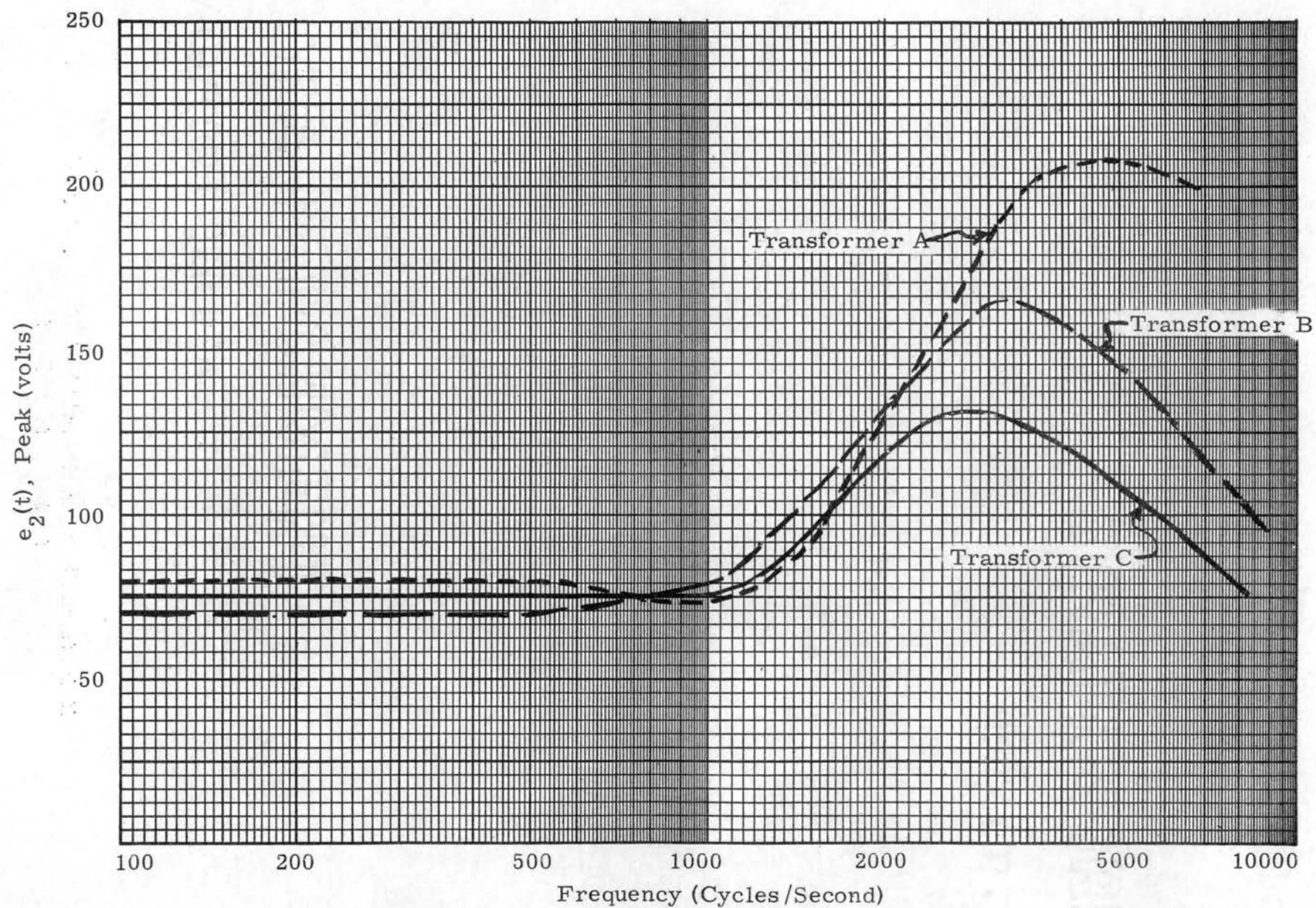
Fig. 26. Effect of Transformer Characteristics on the Inverter Output

assumed; also, circuit losses were neglected. As these various resistances are introduced into the inverter circuit, the peak shown in Fig. 22 is reduced and the bandwidth is increased. Thus, agreement in experimental and computed results could be obtained by reducing transformer and circuit losses to a minimum, or by including these losses in the equivalent circuit as was done in the more complete analysis in Chapter III.

If $\alpha > \beta$ the output voltage of the experimental inverter will increase as frequency increases until the transformer and circuit losses increase faster than the voltage. This results in a decrease in the output voltage at high frequencies.

In general, the calculated output voltage and input current were larger than the experimental values over the operating range of the inverter, particularly near the resonant peak where $f_s = 2f_r$. The differences resulted from the transformer losses, circuit losses, and the resistance of the choke.

Application of Design Criteria

Figure 27 shows the sample calculations and voltage waveforms for a square wave inverter using the design equations in Chapter V. In general, the output voltage will be 4E peak-to-peak for unity turns ratio. The reverse voltage across the SCR's would be 2E, thus determining the PIV rating required. The SCR current rating will be one-half the input current, which can be found from the output power and by assuming transformer and circuit efficiencies.
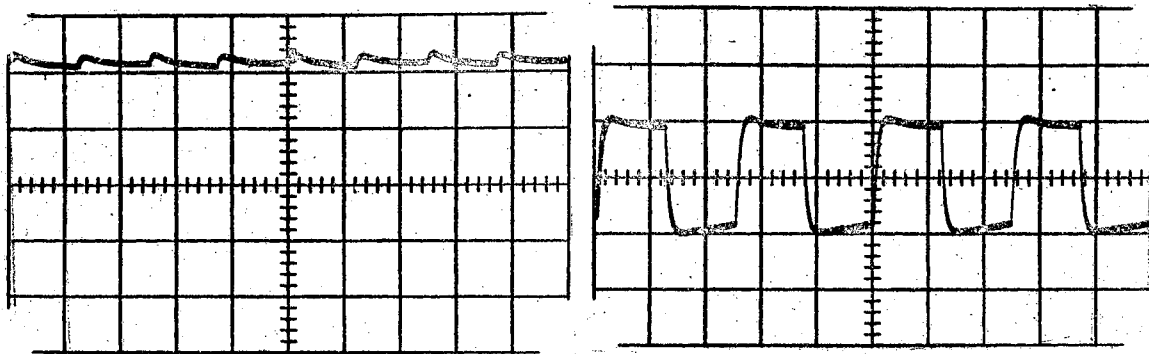
INPUT CURRENT   OUTPUT VOLTAGE

Scale: 2 amps./cm.   50 volts/cm.
          1 ms./cm.   1 ms./cm.

Sample Calculations

Given: R_L = 30 ohms   E = 25 v

          f_s = 400 c.p.s.   n = 2.67

          t_o = 30 μs.

$$R = \frac{R_L}{n^2} = \frac{30}{(2.67)^2} = 4.2 \text{ ohms}.$$

$$C > \frac{t_o}{2R} = \frac{30\mu s}{2(4.2)} = 3.6 \ \mu fd.$$

$$L > 15 R^2 C = 15(4.2)^2 (3.6\mu) = 0.95 \text{ mh}.$$

Therefore, let C = 3.6 μfd.

          L = 7.5 mh.

Output Voltage: E_o = 4E = 4(25) = 100 volts peak-to-peak
                        (Output voltage across primary as in
                         oscillogram)

Output Current: $I_o = \frac{E_o \ rms}{R_L} = \frac{50}{30} (\frac{2.67}{2})^2 = 2.25$
                        (Output current referred to primary)

Input Current: $I_{in} = \frac{E_o}{E_{in}} I_o = \frac{50}{25} (2.25) = 4.5$ amperes
                        (Compare with Oscillogram)

Fig. 27. Calculations and Waveforms of Square Wave Inverter

The output of a sinewave inverter is shown in Fig. 28 with the sample calculations. The inductor should be made slightly smaller than the calculated value of L because of the leakage inductance of the transformer which has been ignored. This gives an actual resonant frequency nearer the switching frequency. The output voltage depends upon the load. It can be as much as eight times the input voltage, E. The SCR should be chosen accordingly.

The output voltage of a sawtooth inverter is shown in Fig. 29. The effective values of R, L, and C in a sawtooth inverter result in operation near the resonant peak shown in Fig. 22. For a light load the output voltage can be many times the input voltage. The particular transformer used can influence the magnitude of the output voltage considerably due to transformer losses, leakage inductance, and distributed capacitance.

These have been examples of parallel inverters with various output voltage waveshapes. They certainly are not the only possible conditions for obtaining these waveshapes from inverters. The specific application may warrant modifications of the design criteria or changes in the inverter circuit.

It has been shown in this chapter that the transformer characteristics are important in inverter design. For example, a transformer with low winding resistance and core loss would be desired. The windings should be bifilar wound to minimize leakage inductance. The primary should be wound with fairly high inductance if the transformer
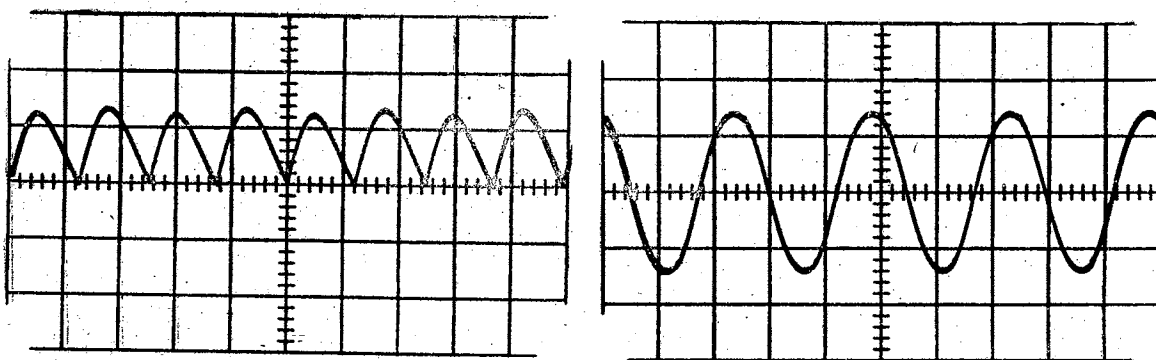
INPUT CURRENT	OUTPUT VOLTAGE

Scale: 2 amps./cm.   100 volts/cm.
         1 ms./cm.   1 ms./cm.

Sample Calculations

Given: R_L = 500 ohms   E = 25 volts

         f_s = 400 c.p.s.   n = 2.67

$$R = \frac{R_L}{n^2} = \frac{500}{(2.67)^2} = 70 \text{ ohms}$$

$$C = \frac{1}{8 f_s R} = \frac{1}{8(400)(70)} = 4.5 \ \mu\text{fd.}$$

$$L = \frac{1}{16\pi^2 f_s^2 C} = \frac{1}{16\pi^2 (400)^2 (4.5)\mu} = 8.9 \text{ mh.}$$

Therefore, let   C = 4.5 μfd.
                    L = 7.5 mh.   (allowing for transformer leakage
                                   inductance)

Fig. 28. Calculations and Waveforms of Sinewave Inverter
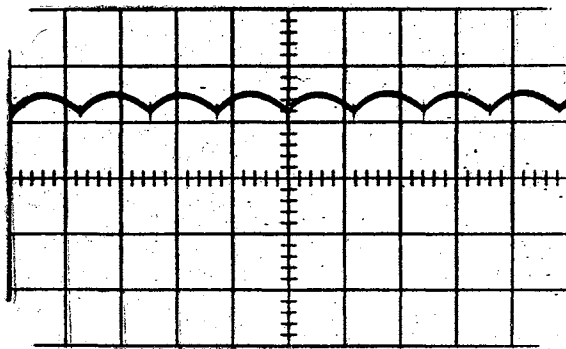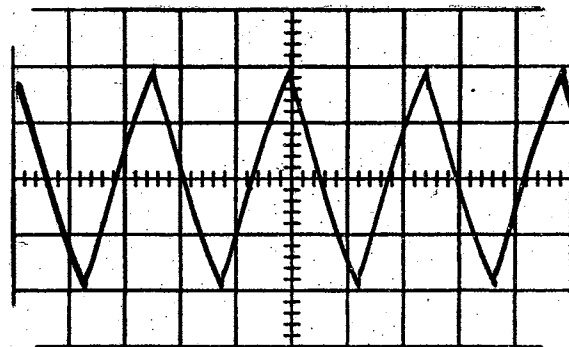
INPUT CURRENT	OUTPUT VOLTAGE

Scale: 1 amp./cm.   100 volts/cm.
         0.2 ms./cm.   0.2 ms./cm.

Sample Calculations

Given: R_L = 2K ohms   E = 25 volts

         f_s = 2Kc.   n = 2.67

$$R = \frac{R_L}{n^2} = \frac{2K}{(2.67)^2} = 280 \text{ ohms}$$

$$C \quad \frac{1}{4f_s R} = \frac{1}{4(2K)(280)} = 0.45 \ \mu\text{fd.}$$

$$L \quad \frac{1}{4\pi^2 f_s^2 C} = \frac{1}{4\pi^2 (2K)^2 (.45\mu)} = 14 \text{ mh.}$$

Therefore, let   C = 0.5 μfd.
                  L = 20 mh.

Fig. 29. Calculations and Waveforms of Sawtooth Inverter

is to carry square current pulses. If the operating frequency is high, hysteresis and eddy current losses should be minimized by the use of thin core laminations. The transformer should be designed so that the maximum flux density will be less than one-third the saturation flux density to avoid commutating difficulties.

CHAPTER VII

ADDITIONAL CONSIDERATIONS

Only the basic inverter circuit has been discussed in the pre-
ceding chapters. There are several additional topics that could be
covered in a discussion of SCR inverters, such as trigger circuits,
regulation, modifications of the parallel inverter, RFI problems, and
protection against short circuit. These are all important in the applica-
tion of the inverter, but time has not permitted a thorough study of these
problems. This chapter will summarize some of these problems and
refer to information available in the literature.

Inverter Trigger Circuits

The triggering source for an inverter must apply alternating
pulses to each SCR. Transistor multivibrators, saturable reactors,
unijunction and other transistor oscillator circuits meet the require-
ments for the triggering source. As discussed in Chapter II, there are
both minimum (or threshold) and maximum (or damage) values for gate
excitation voltage and current, depending upon the temperature, anode
voltage, and anode current which must be kept in mind when designing
the trigger source.[17] A pulse with a steep wavefront assures that

[17]E. E. Moyer and A. Schmidt, Jr., "Excitation Requirements
for Silicon Controlled Rectifiers," Electrical Manufacturing, September,
1960, p. 127.

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SCR's with different triggering sensitivities will be turned on at the same time.

A trigger source used in this investigation was an astable multi-vibrator circuit shown in Fig. 30.[18] The two outputs from the multi-vibrator give alternating pulses to each SCR. Each 0.1μfd. capacitor, in conjunction with the gate impedance of the SCR, differentiates the square wave from the emitter to give a trigger pulse with a steep wave-front. The diode at the output prevents negative pulses from reaching the gate. It also provides a low impedance path from the gate to cathode which speeds up turn-off. For experimental purposes the repetition rate was made variable from 1,500 to 5,000 pulses per second with the addition of a potentiometer and condenser (50 K ohms - 0.05 μfd.) between the cross-coupling networks.

Basic oscillator circuits that can be used for trigger sources are the UJT relaxation oscillator, the saturable core oscillator, the regulated saturable core transistor oscillator, and the L-C tuned oscillator.[19] The UJT oscillator is simple, economical, and reliable. Its pulse width (maximum width of about 50 μs.) is not long enough for use with SCR inverters which must supply low p.f. loads. The saturable core oscillator

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[18]H. T. Fristoe, "Fluorescent Lighting Utilizing the SCR Inverter," Texas Instruments Memorandum, September, 1961.

[19]G. P. Underbrink, Investigation of Silicon Controlled Rectifiers for Static Power Conversion, Report 2, Armed Services Technical Information Agency (Arlington, Virginia, 1960), pp. 42-48.

+25 volts

Pulse Equalizer

330 ohms

330 ohms

50 K ohms

0.33 μ fd.

.05 μ fd.

0.33 μ fd.

1.8 K ohms

5.6 K ohms

5.6 K ohms

Diodes
1N659

2N497

0.1μ fd.

0.1 μ fd.

220
ohms

2.2 K ohms

2.2 K ohms

220
ohms
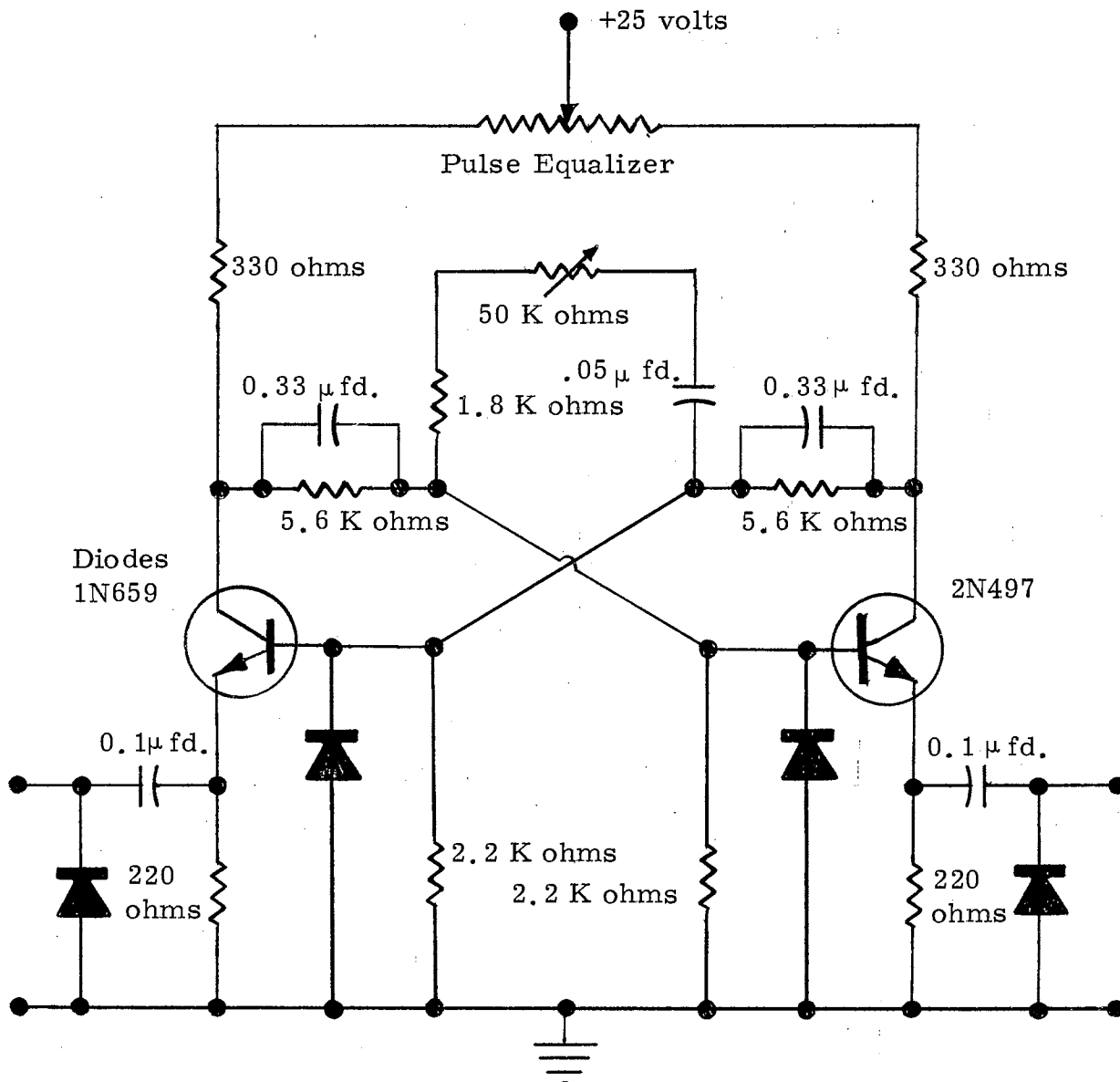
Fig. 30. Transistor Trigger Circuit

is reliable and simple, but its frequency stability depends on the power

supply regulation. This problem is overcome by the regulated satura-

ble core transistor oscillator, which is more complex. The

transistorized push-pull, L-C oscillator is most frequently used in power inverter applications requiring 1% frequency stability. It can be seen that the trigger source poses no major problem in the design of an SCR inverter.

Modified Parallel Inverter

A recent modification of the SCR parallel inverter is the square wave inverter developed by W. McMurray and B. D. Bedford of the General Electric General Engineering Laboratory shown in Fig. 31.[20] Although the circuit resembles the parallel inverter used in the preceding analysis, the method of commutation in the square wave inverter is different. The use of feedback diodes D1 and D2 makes it particularly well suited for driving inductive loads. It produces essentially a square wave output under all load conditions and does not create high voltages across the SCR's under lightly loaded or no-load conditions.

The feedback diodes perform three important functions in the circuit:

1.   Damp out circuit oscillations due to L-C resonance.

2.   Prevent the voltage across either half of the primary winding from exceeding the supply voltage. This tends to make the output of the inverter a square wave under any load condition. Also, this limits the maximum voltages impressed

[20] General Electric Company, Silicon Controlled Rectifier Manual (2nd ed., Syracuse, 1961), pp. 152-155.

across the SCR's making possible the use of lower voltage and less expensive SCR's.

3. Provide a path for reactive currents to flow back into the power supply during portions of the cycle when neither SCR is conducting.
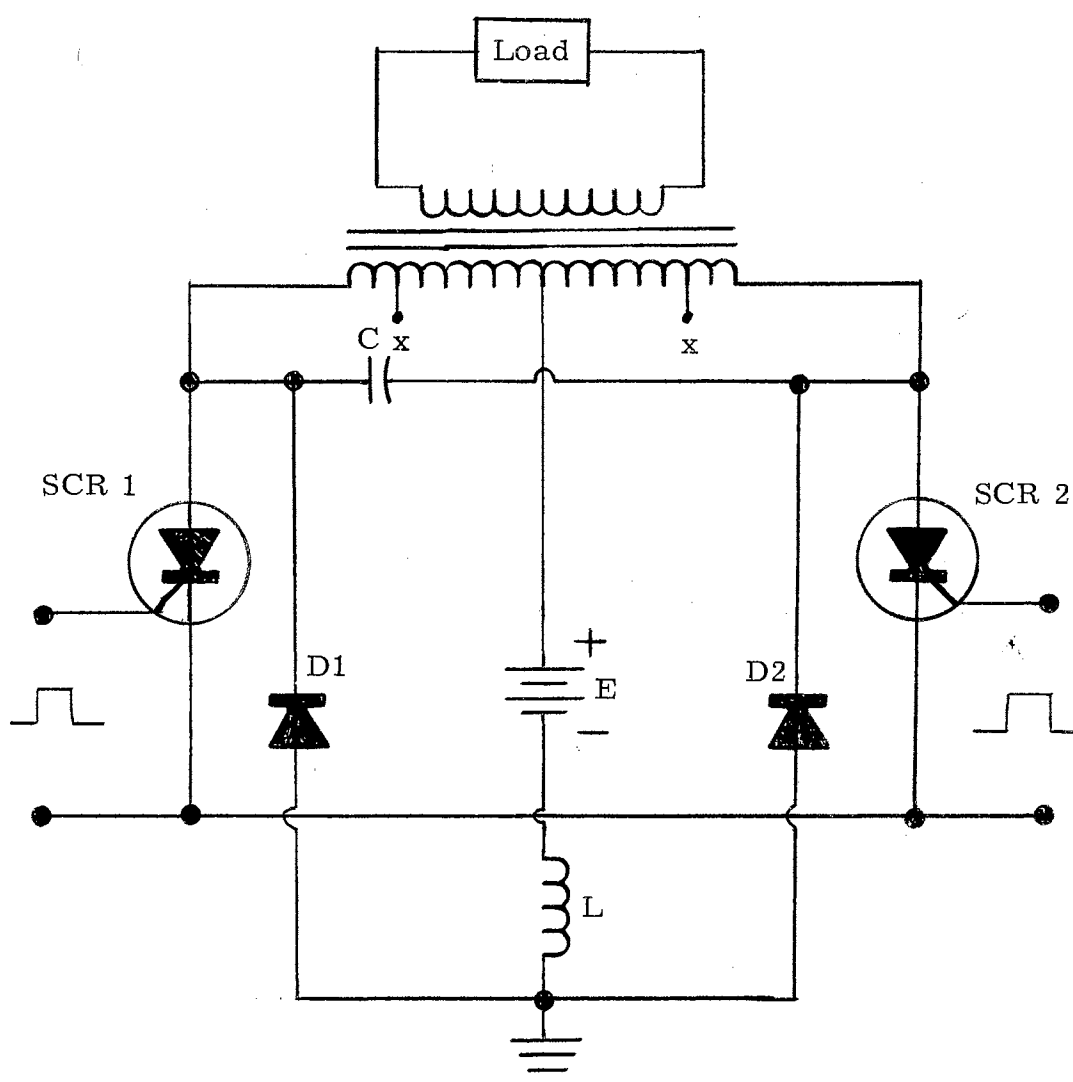
Fig. 31. McMurray-Bedford Parallel Inverter

Another advantage of the circuit is that the commutating capacitor and the choke can be made smaller in size since the capacitor is not required to compensate for the reactive current.

The d. c. supply should have a low transient impedance since it is usually required to both accept and supply power. It must accept the reactive power fed back by the diodes. A capacitor across the d. c. input of the inverter is usually required.

In applications where the feedback current results in considerable power loss, the efficiency may be improved by connecting the cathode of each feedback diode to a tap "x" on the primary of the transformer. (See Fig. 31.) This will increase the load voltage during part of the cycle and will result in a poorer square wave.

The modified inverter described above is therefore useful in inverter applications where a varying or reactive load is expected and the output waveform desired is a square wave.

Regulation Techniques

The conventional parallel inverter presents a problem with varying load conditions since the correct size of commutating capacitance depends upon the load. Under light load conditions the output voltage can rise two or three times above that at full load conditions. Also, the output voltage will vary as the power factor of the load varies.

The simplest method of obtaining regulation, but less effective than more elaborate methods, is to use a harmonic-suppressing

regulating transformer.[21] When used in conjunction with the square wave inverter described in the previous section, the output voltage will be a regulated sinewave with short-circuit protection. The harmonic-suppressing regulating transformer is difficult to use with the basic parallel inverter since the effective load at the input to the regulating transformer will vary widely in magnitude and power factor. Also, this method is applicable only to low-power inverters. (See Fig. 32.)
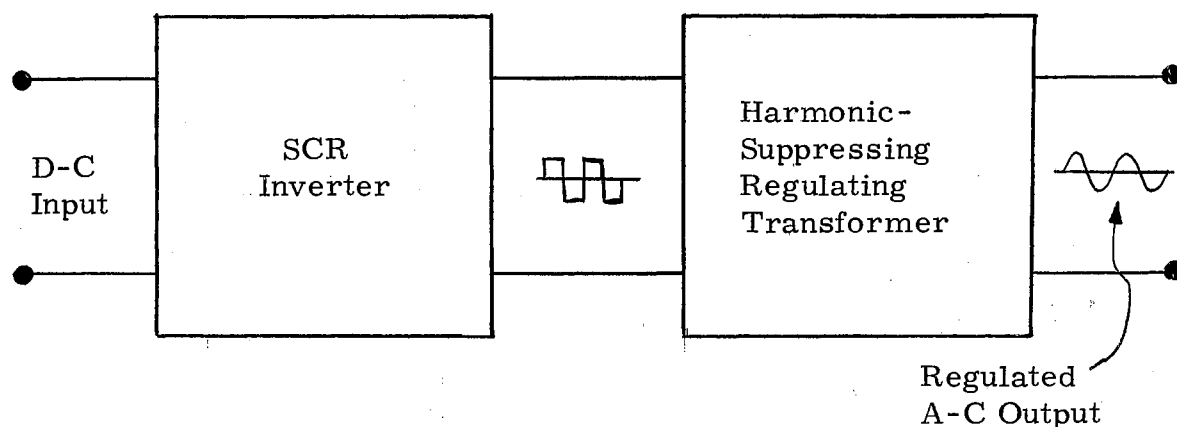
Fig. 32. Block Diagram of a Regulated DC to AC Inverter

[21]H. R. Lowry, "Voltage Regulated SCR Inverter with Sinewave Output and Current Limiting," General Electric Application Note 200.7, (1961).

Another simple method of regulation involves using a saturable transformer in the commutating circuit of the parallel inverter.[22] The SCR's are turned on by a constant frequency gate pulse and turned off by the saturation of the output transformer. Thus phase control on variable duty-cycle regulation is achieved. An experimental model described in the reference gave 1% regulation of RMS output over a 20% change in input, and an efficiency of 90% under normal conditions.

Better regulation can be obtained by one of the following methods described in the report by G. P. Underbrink:[23] (1) line boost, (2) turns-ratio boost, (3) variable duty-cycle, (4) stepped turns-ratio, and (5) pulse frequency modulation. The line boost method of regulation uses power conversion techniques to boost the supply voltage and maintain a constant output voltage over a range of input and load variations. It is a simple, reliable regulation method using passive commutation but requires a larger power transformer, more SCR's, more output filtering, and is less efficient than other methods.

The turns-ratio boost method of regulation varies the effective secondary to primary turns ratio of the inverter transformer. It is similar to the line boost method.

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[22]L. H. Walker, "Controlled Rectifier Inverter with Saturable Transformer Regulation," Conference Paper presented at the AIEE Summer and General Meeting, Denver, Colorado, June 17, 1962.

[23]G. P. Underbrink, Investigation of Silicon Controlled Rectifiers for Static Power Conversion, Report 2, Armed Services Technical Information Agency (Arlington, Virginia, 1960), pp. 6-16.

The stepped turns-ratio regulation method varies the turns ratio of the power transformer in calculated steps, thereby maintaining the output voltage within specified limits. It has high efficiency but requires an excessive number of SCR's for acceptable regulation.

The variable duty-cycle method of regulation varies the conduction time per cycle to change the RMS output voltage of the inverter. This is accomplished in the SCR inverter by independent commutation of the controlled rectifiers, as shown in Fig. 33. The output is shown in Fig. 34.
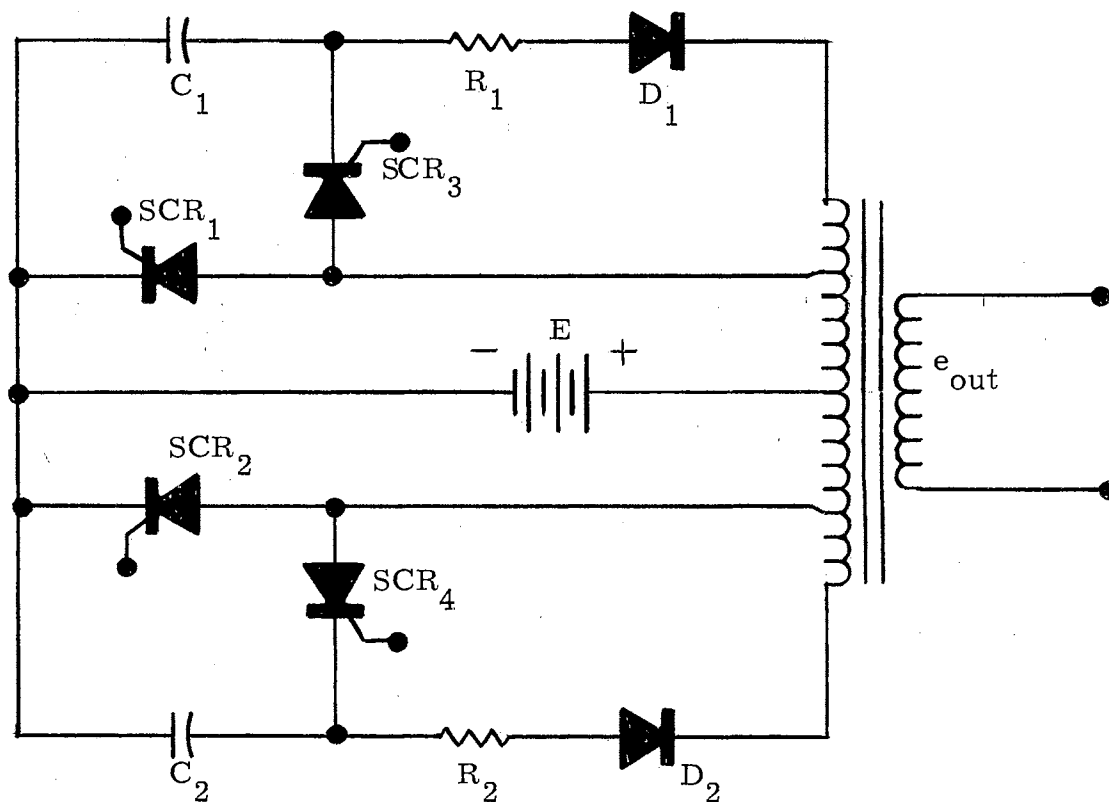
Fig. 33. SCR Inverter with Variable Duty-Cycle Regulation
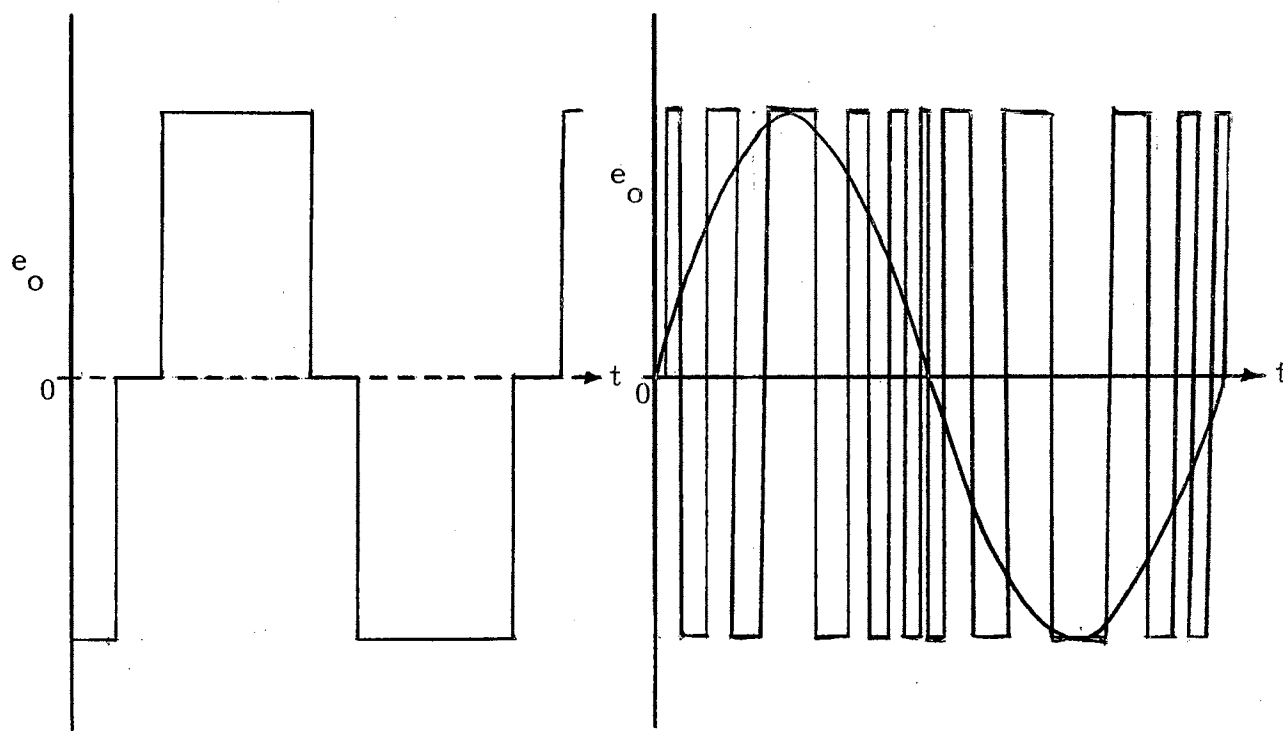
Fig. 34. Output Waveform of Variable Duty-Cycle
          Regulated Inverter and PFM Regulated
          Inverter

Pulse frequency modulation (PFM) regulation can be obtained by

simple on-off switching, the primary objective being to synthesize a

regulated sinusoidal output voltage with a minimum of filtering. (See

Fig. 34.) The principal disadvantage of this method applied to SCR

inverters is the excessive commutation losses that result from the high

chopping rate.

Thus, it can be seen that there is a wide variety of possible regulation methods for SCR inverters. The best method will depend upon the particular application.

RFI Problems

Very high switching transients are generated in the SCR inverter due to the microsecond switching of high currents. The transients result in broadband electromagnetic radiation and conducted noise on the d.c. input line. With a 100 watt inverter, current transients of over 20 amperes with 1 to 2 microsecond duration were observed. In a typical 500 watt inverter, peak switching transients in excess of 10 kilowatts are possible.

Transients on the d.c. input line can be controlled by filtering. Careful design and layout of the commutating circuit can reduce radio interference by as much as 20 db. (This can also improve the inverter efficiency in some instances.) Ordinary RFI suppression techniques can be employed to control the interference in the final construction.

CHAPTER VIII

CONCLUSIONS

An analysis of the basic parallel inverter using SCR's has been

given in the preceding chapters. The purpose was to study the in-

verter circuit and to explain why the experimental results differed

from the computed response. Due to the complexity of the equations,

it was necessary to use a computer to determine where the disagree-

ments occurred. Design criteria for various output voltage waveforms

have been given, based on the circuit analysis.

It may be concluded that the simplified analysis, assuming an

ideal transformer (as was carried out by Wagner), results in the greatest

error when the switching frequency is near twice the natural resonant

frequency, $\dfrac{1}{\pi} \sqrt{\dfrac{1}{4LC}}$. Below this frequency the computed voltages

and currents will be higher than those in the actual inverter. Above

the resonant peak the computed values fall below the experimental

values. The simplified analysis can be applied to a square wave in-

verter but not to a sinewave or sawtooth inverter for determining the

output voltage or input current.

From a comparison of experimental results using various trans-

formers and with computed data it may be concluded that the transformer

characteristics are important factors in determining the magnitude of the output voltage, especially at switching frequencies above the natural resonant frequency. The winding resistances and core losses tend to reduce the actual output voltage. The leakage inductance reduces the output voltage at higher frequencies. A desirable transformer for an inverter would have low leakage inductance, low winding resistance, and low core loss. It would not saturate (for improved commutation) and would be bifilar wound for close coupling between windings.

It can be seen from Chapter VI that the design procedure developed in Chapter V can be useful as a starting point for basic inverter design. The most important criteria derived relates the turn-off time of the SCR to the commutating capacitance and the load: $t_o < 2RC$.

The experimental inverter used to check the analytical results was a low-power inverter limited to 150 watts input due to the equipment available. The analysis given in this thesis should be compared with a higher power inverter. The computed and experimental results might compare more closely at a higher power level. Also, the most important application of the SCR parallel inverter at the present is in the medium to high power range.

The computer was a valuable tool in checking the analytic results. Without it the computation necessary for a complete evaluation of the equations would have been impractical to carry out. If the more complete analysis were to be followed, it would be necessary to program the equations on a computer before they could be applied to a specific design.

Several areas for further investigation can be stated as a result of research done for this thesis. One area concerns finding a way to vary the commutating capacitance as the load is varied. The analysis in this thesis has assumed a constant load. Also, possibly the inductance should be changed with the load.

An extension of the analysis in this thesis could be another area of investigation. The more complete analysis given in the latter part of Chapter III, which includes some of the transformer characteristics, could be studied for possible ways of simplifying the results but still maintaining useable accuracy. Also, other inverter circuits could be studied including modifications of the parallel inverter to determine a more efficient and useful circuit.

The recently developed silicon gate controlled switch, capable of being turned both on and off by the gate, has great possibilities in the inverter field. It is certainly an area for further research. It could eliminate the commutating capacitor and ballast inductor from the inverter discussed in this thesis, leaving a very simple circuit for converting d.c. power to a.c. power.

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APPENDIX A

DERIVATION OF THE VOLTAGE AND CURRENT

EQUATIONS FOR THE SIMPLIFIED INVERTER

CIRCUIT - RESISTIVE LOAD

In Chapter III the expressions for the input current and the output voltage were given. These equations are derived here in detail. The same conditions hold and the same assumptions are made as in Chapter III.

The differential equations for the circuit shown in Fig. 9 were given in (5) and (6), Chapter III, as

$$E + e_c = L \frac{di_1}{dt} + \frac{1}{4C} \int i_1 dt - \frac{1}{4C} \int i_2 dt \qquad \text{(A-1)}$$

and

$$-e_c = -\frac{1}{4C} \int i_1 dt + i_2 R + \frac{1}{4C} \int i_2 dt \quad , \qquad \text{(A-2)}$$

where e_c is the initial charge on the capacitor at $t = 0$. Taking the Laplace transform of (A-1) and (A-2) yields

$$\frac{E + e_c}{s} = L \left[I_1 s - I_1(0) \right] + \frac{I_1}{4Cs} - \frac{I_2}{4Cs} \qquad \text{(A-3)}$$

and

90

$$\frac{-e_c}{s} = -\frac{I_1}{4Cs} + I_2 R + \frac{I_2}{4Cs} \ , \tag{A-4}$$

where $I_1(0)$ is the current in the inductor at $t = 0$. I_1 is actually $I_1(s)$, the Laplace transform of $i_1(t)$, and likewise for I_2.

Equations (A-3) and (A-4) may be rewritten as

$$E + e_c + s L I_1(0) = (Ls^2 + \frac{1}{4C}) I_1 - \frac{1}{4C} I_2 \tag{A-5}$$

and

$$-e_c = -\frac{1}{4C} I_1 + (Rs + \frac{1}{4C}) I_2 \ . \tag{A-6}$$

Solving (A-5) and (A-6) for I_1 and I_2 by determinants yields

$$I_1(s) = \frac{\begin{vmatrix} e_c + E + LI_1(0) s & -\dfrac{1}{4C} \\ -e_c & Rs + \dfrac{1}{4C} \end{vmatrix}}{\begin{vmatrix} Ls^2 + \dfrac{1}{4C} & -\dfrac{1}{4C} \\ -\dfrac{1}{4C} & Rs + \dfrac{1}{4C} \end{vmatrix}}$$

$$= \frac{(E + e_c) Rs + LRI_1(0) s^2 + \dfrac{E}{4C} + \dfrac{LI_1(0)s}{4C}}{RLs^3 + \dfrac{L}{4C} s^2 + \dfrac{R}{4C} s} \ ,$$

or

$$I_1(s) = \frac{I_1(0)\, s^2 + \left[\dfrac{E + e_c}{L} + \dfrac{I_1(0)}{4RC}\right] s + \dfrac{E}{4RLC}}{s\left(s^2 + \dfrac{1}{4RC}\, s + \dfrac{1}{4LC}\right)} \quad . \tag{A-7}$$

Likewise, $I_2(s)$ is

$$I_2(s) = \frac{\begin{vmatrix} s^2 L + \dfrac{1}{4C} & (E + e_c) + L I_1(0)\, s \\[2mm] -\dfrac{1}{4C} & -e_c \end{vmatrix}}{s\left(s^2 + \dfrac{1}{4RC}\, s + \dfrac{1}{4LC}\right)(RL)}$$

$$= \frac{-e_c L s^2 + \dfrac{E}{4C} + \dfrac{L I_1(0)\, s}{4C}}{s\left(s^2 + \dfrac{1}{4RC}\, s + \dfrac{1}{4LC}\right)(RL)} \quad ,$$

or

$$I_2(s) = \frac{\dfrac{-e_c}{R}\, s^2 + \dfrac{I_1(0)s}{4RC} + \dfrac{E}{4RLC}}{s\left(s^2 + \dfrac{1}{4RC}\, s + \dfrac{1}{4LC}\right)} \quad . \tag{A-8}$$

Since the output voltage or the voltage across R is desired, then

$$E_2(s) = I_2(s)\, R = \frac{-e_c s^2 + \dfrac{I_1(0)\, s}{4C} + \dfrac{E}{4LC}}{s\left(s^2 + \dfrac{1}{4RC}\, s + \dfrac{1}{4LC}\right)} \quad . \tag{A-9}$$

Three different solutions exist for (A-7) and (A-9): complex roots, equal roots, and real roots.

Complex Roots

Let $\alpha = \dfrac{1}{8RC}$ and $\beta = \sqrt{\dfrac{1}{4LC}}$. Equation (A-7) may be re-written as

$$I_1(s) = \frac{I_1(0)s^2 + \left[\dfrac{E + e_c}{L} + 2\alpha I_1(0)\right]s + \dfrac{\beta^2 E}{R}}{s\,(s^2 + 2\alpha s + \beta^2)} . \tag{A-10}$$

Using the inverse Laplace transforms

$$\frac{1}{s\,(s^2 + 2\alpha s + \beta^2)} \longrightarrow \frac{1}{\beta^2}\left[1 - \frac{\beta}{\omega}\,\epsilon^{-\alpha t}\sin(\omega t + \theta)\right] \left.\begin{array}{c}\\[3em]\end{array}\right\}$$

$$\frac{s}{s^2 + 2\alpha s + \beta^2} \longrightarrow \frac{\beta}{\omega}\,\epsilon^{-\alpha t}\sin(\omega t + \theta) \quad\Bigg\} \tag{A-11}$$

$$\frac{1}{s^2 + 2\alpha s + \beta^2} \longrightarrow \frac{1}{\omega}\,\epsilon^{-\alpha t}\sin\omega t$$

where $\omega = \sqrt{\beta^2 - \alpha^2}$ and $\theta = \tan^{-1}\dfrac{\omega}{\alpha}$, (A-10) transforms into

$$i_1(t) = I_1(0)\frac{\beta}{\omega}\,\epsilon^{-\alpha t}\sin(\omega t + \theta) + \left[\frac{(E + e_c)}{L} + \frac{I_1(0)}{4RC}\right]\frac{\epsilon^{-\alpha t}}{\omega}\sin\omega t +$$

$$\frac{E}{4RLC\,\beta^2} - \frac{E}{4RLC\,\beta}\,\frac{\epsilon^{-\alpha t}}{\omega}\sin(\omega t + \theta) . \tag{A-12}$$

This can be reduced to

$$i_1(t) = \frac{E}{R} + \left[I_1(0) - \frac{E}{R}\right] \frac{\beta}{\omega} \epsilon^{-\alpha t} \sin(\omega t + \theta) + \left[\frac{E + e_c}{L} + 2\alpha I_1(0)\right] \frac{\epsilon^{-\alpha t}}{\omega} \sin \omega t .$$

(A-13)

Similarly, (A-9) can be written as

$$E_2(s) = \frac{-e_c s^2 + 2\alpha R I_1(0) s + \beta^2 E}{s(s^2 + 2\alpha s + \beta^2)} .$$

(A-14)

Using the inverse Laplace transforms given in (A-11), (A-14) transforms into

$$e_2(t) = -e_c \frac{\beta}{\omega} \epsilon^{-\alpha t} \sin(\omega t + \theta) + 2\alpha R I_1(0) \frac{\epsilon^{-\alpha t}}{\omega} \sin \omega t +$$

$$\frac{E\beta^2}{\beta^2} - E\beta \frac{\epsilon^{-\alpha t}}{\omega} \sin(\omega t + \theta) ,$$

(A-15)

which can be reduced to

$$e_2(t) = E - \left(e_c + E\right) \frac{\beta}{\omega} \epsilon^{-\alpha t} \sin(\omega t + \theta) + 2\alpha I_1(0) R \frac{\epsilon^{-\alpha t}}{\omega} \sin \omega t .$$

(A-16)

Boundary Conditions

Using the relationship

$$i_1(0) = i_1\left(\frac{T}{2}\right) ,$$

(A-17)

from (A-13) the following is obtained:

$$\frac{E}{R} + \left[I_1(0) - \frac{E}{R} \right] \frac{\beta}{\omega} \sin \theta = \frac{E}{R} + \left[I_1(0) - \frac{E}{R} \right] \frac{\beta}{\omega} \epsilon^{-\alpha \frac{T}{2}} \sin \left(\omega \frac{T}{2} + \theta \right) +$$

$$\left[\frac{E + e_c}{L} + 2\alpha I_1(0) \right] \frac{\epsilon^{-\alpha \frac{T}{2}}}{\omega} \sin \omega \frac{T}{2} . \tag{A-18}$$

Putting (A-18) in a more useable form gives

$$\frac{E}{R} \frac{\beta}{\omega} \epsilon^{-\alpha \frac{T}{2}} \sin \left(\omega \frac{T}{2} + \theta \right) - \frac{E}{L} \frac{\epsilon^{-\alpha \frac{T}{2}}}{\omega} \sin \omega \frac{T}{2} - \frac{E}{R} \frac{\beta}{\omega} \sin \theta =$$

$$I_1(0) \left[\frac{\beta}{\omega} \epsilon^{-\alpha \frac{T}{2}} \sin \left(\omega \frac{T}{2} + \theta \right) + 2\alpha \frac{\epsilon^{-\alpha \frac{T}{2}}}{\omega} \sin \omega \frac{T}{2} - \frac{\beta}{\omega} \sin \theta \right] +$$

$$e_c \left[\frac{\epsilon^{-\alpha \frac{T}{2}}}{L\omega} \sin \omega \frac{T}{2} \right] . \tag{A-19}$$

Substituting (A-15) into the expression

$$e_2(0) = -e_2 \left(\frac{T}{2} \right) \tag{A-20}$$

yields

$$-E + (e_c + E) \frac{\beta}{\omega} \sin \theta = E - (e_c + E) \frac{\beta}{\omega} \epsilon^{-\alpha \frac{T}{2}} \sin \left(\omega \frac{T}{2} + \theta \right) + 2\alpha R I_1(0) \frac{\epsilon^{-\alpha \frac{T}{2}}}{\omega} \sin \omega \frac{T}{2} \tag{A-21}$$

which can be rewritten in the form

$$2E - E \frac{\beta}{\omega} \sin \theta - E \frac{\beta}{\omega} \epsilon^{-\alpha \frac{T}{2}} \sin \left(\omega \frac{T}{2} + \theta \right) =$$

$$I_1(0)\left[-2\alpha R\,\frac{\epsilon^{-\alpha\frac{T}{2}}}{\omega}\,\sin\omega\frac{T}{2}\right]+e_c\left[\frac{\beta}{\omega}\sin\theta+\frac{\beta}{\omega}\,\epsilon^{-\alpha\frac{T}{2}}\sin\left(\omega\frac{T}{2}+\theta\right)\right].\quad\text{(A-22)}$$

Equations (A-13) and (A-16), along with (A-19) and (A-22), are the equations which were used in the analysis in Chapter III.

Equal Roots

For equal roots the following inverse Laplace transform is required:

$$\frac{s^2+as+b}{s(s+\alpha)^2}\;\longrightarrow\;\frac{b}{\alpha^2}+\left[\frac{a\alpha-b-\alpha^2}{\alpha}\,t+\frac{\alpha^2-b}{\alpha^2}\right]\epsilon^{-\alpha t}\;.\quad\text{(A-23)}$$

Transforming (A-7) yields

$$i_1(t)=I_1(0)\left\{\frac{\dfrac{E}{4RLCI_1(0)}}{\alpha^2}+\left[\frac{\alpha\left(\dfrac{e_c+E}{I_1(0)\,L}+\dfrac{1}{4RC}\right)-\dfrac{E}{4RLCI_1(0)}-\alpha^2}{\alpha}\,t+\right.\right.$$

$$\left.\left.\frac{\alpha^2-\dfrac{E}{4RLCI_1(0)}}{\alpha^2}\right]\epsilon^{-\alpha t}\right\}\;.\quad\text{(A-24)}$$

This reduces to

$$i_1(t)=\frac{E}{R}+\left[\left(\frac{e_c-E}{L}+\alpha I_1(0)\right)t+I_1(0)-\frac{E}{R}\right]\epsilon^{-\alpha t}\;.\quad\text{(A-25)}$$

Transforming (A-9) gives

$$e_2(t) = -e_c \left\{ \frac{\dfrac{-E}{4LCe_c}}{\alpha^2} + \left[\frac{\alpha\left(\dfrac{-I_1(0)}{4Ce_c}\right) + \dfrac{E}{4LCe_c} - \alpha^2}{\alpha} \right] t + \right.$$

$$\left. \frac{\alpha^2 + \dfrac{E}{4LCe_c}}{\alpha^2} \right] \epsilon^{-\alpha t} \right\} . \qquad \text{(A-26)}$$

Simplifying, (A-26) becomes

$$e_2(t) = E + \left[\left(\frac{I_1(0)}{4C} - \alpha(E - e_c) \right) t - e_c - E \right] \epsilon^{-\alpha t} . \qquad \text{(A-27)}$$

Boundary Conditions

Using the boundary condition in (A-17) and $i_1(t)$ in (A-25) yields

$$I_1(0) - \frac{E}{R} = \left[\left(\frac{e_c - E}{L} + \alpha I_1(0) \right) \frac{T}{2} + I_1(0) - \frac{E}{R} \right] \epsilon^{-\alpha \frac{T}{2}} \qquad \text{(A-28)}$$

which reduces to

$$\frac{E}{L} \frac{T}{2} \epsilon^{-\alpha \frac{T}{2}} - \frac{E}{R} \left[1 + \epsilon^{-\alpha \frac{T}{2}} \right] = I_1(0) \left[-1 + \epsilon^{-\alpha \frac{T}{2}} (1 + \alpha \frac{T}{2}) \right] +$$

$$e_c \left[\frac{1}{L} \frac{T}{2} \right] \epsilon^{-\alpha \frac{T}{2}} . \qquad \text{(A-29)}$$

Using the second boundary condition in (A-20) and $e(t)$ in (A-27) yields

$$\left[-E + e_c + E \right] = E + \left[\left(\frac{I_1(0)}{4C} - \alpha(E - e_c) \right) \frac{T}{2} - e_c - \right.$$

$$\left. E \right] \epsilon^{-\alpha \frac{T}{2}} , \qquad \text{(A-30)}$$

which reduces to

$$\frac{E}{R}\left[\left(\alpha\frac{T}{2}+1\right)\epsilon^{-\alpha\frac{T}{2}}-1\right]=I_1(0)\left[\frac{1}{4C}\ \frac{T}{2}\right]\epsilon^{-\alpha\frac{T}{2}}+e_c\left[-1+(\alpha\frac{T}{2}-1)\ \epsilon^{-\alpha\frac{T}{2}}\right]\ .$$

$$\text{(A-31)}$$

Equations (A-25), (A-27), (A-29), and (A-31) were used in the analysis

in Chapter III.

Real Roots

For real roots the following inverse Laplace transform is re-

quired:

$$\frac{s^2+gs+d}{s(s-a)(s-b)}\ \longrightarrow\ A\epsilon^{at}+B\epsilon^{bt}+K \qquad\qquad \text{(A-32)}$$

where

$$A=\frac{a^2+ga+d}{a(a-b)}\ ,\quad B=\frac{b^2+gb+d}{b(b-a)}\ ,\ \text{and}\ K=\frac{d}{ab}\ .$$

Transforming (A-7) yields

$$i_1(t)=I_1(0)\left[\frac{a^2+a\left(\dfrac{e_c+E}{LI_1(0)}+\dfrac{1}{4RC}\right)+\dfrac{E}{4RLCI_1(0)}}{a(a-b)}\right]\epsilon^{at}+$$

$$I_1(0)\left[\frac{b^2+b\left(\dfrac{e_c+E}{LI_1(0)}+\dfrac{1}{4RC}\right)+\dfrac{E}{4RLCI_1(0)}}{b(b-a)}\right]\epsilon^{bt}+\frac{E}{4RLCab}\ ,\quad \text{(A-33)}$$

which reduces to

$$i_1(t) = \left[\frac{a^2 I_1(0) + \frac{a}{L}(E + e_c) + 2\alpha a I_1(0) + \frac{E\beta^2}{R}}{a(a-b)}\right]\epsilon^{at} +$$

$$\left[\frac{b^2 I_1(0) + \frac{b}{L}(E + e_c) + 2\alpha b I_1(0) + \frac{E\beta^2}{R}}{b(b-a)}\right]\epsilon^{bt} + \frac{E\beta^2}{abR} \quad . \tag{A-34}$$

(A-9) can be transformed into

$$e_2(t) = -e_c\left[\frac{a^2 + a\left(\frac{-I_1(0)}{4Ce_c}\right) + \frac{-E}{4LCe_c}}{a(a-b)}\right]\epsilon^{at} -$$

$$e_c\left[\frac{b^2 + b\left(\frac{-I_1(0)}{4Ce_c}\right) + \frac{-E}{4LCe_c}}{b(b-a)}\right]\epsilon^{bt} + \frac{E}{4RLCab} \quad , \tag{A-35}$$

which reduces to

$$e_2(t) = \left[\frac{-e_c a^2 + \frac{a}{4C}I_1(0) + E\beta^2}{a(a-b)}\right]\epsilon^{at} + \left[\frac{-e_c b^2 + \frac{b}{4C}I_1(0) + E\beta^2}{b(b-a)}\right]\epsilon^{bt} + \frac{E\beta^2}{ab} \quad . \tag{A-36}$$

Boundary Conditions

From the boundary condition given in (A-17) and Equation (A-34),

it follows that

$$\frac{a^2 I_1(0) + \frac{a}{L}(E+e_c) + 2\alpha a I_1(0) + \frac{E\beta^2}{R}}{a(a-b)} + \frac{b^2 I_1(0) + \frac{b}{L}(E+e_c) + 2\alpha b I_1(0) + \frac{E\beta^2}{R}}{b(b-a)} =$$

$$\left[\frac{a^2 I_1(0) + \frac{a}{L}(E+e_c) + 2\alpha a I_1(0) + \frac{E\beta^2}{R}}{a(a-b)}\right] \epsilon^{a\frac{T}{2}} +$$

$$\left[\frac{b^2 I_1(0) + \frac{b}{L}(E+e_c) + 2\alpha b I_1(0) + \frac{E\beta^2}{R}}{b(b-a)}\right] \epsilon^{b\frac{T}{2}} . \tag{A-37}$$

Collecting like terms and simplifying, (A-37) becomes

$$E\left[\frac{\frac{a}{L} + \frac{\beta^2}{R}}{a(a-b)}\right]\left(1 - \epsilon^{a\frac{T}{2}}\right) + E\left[\frac{\frac{b}{L} + \frac{\beta^2}{R}}{b(b-a)}\right]\left(1 - \epsilon^{b\frac{T}{2}}\right) = \left[\left(\epsilon^{a\frac{T}{2}} - 1\right)\left(\frac{a+2\alpha}{a-b}\right) + \right.$$

$$\left.\left(\epsilon^{b\frac{T}{2}} - 1\right)\left(\frac{b+2\alpha}{b-a}\right)\right] I_1(0) + \left[\left(\epsilon^{a\frac{T}{2}} - \epsilon^{b\frac{T}{2}}\right)\left(\frac{1}{L(a-b)}\right)\right] e_c .$$

$$\tag{A-38}$$

The second boundary condition, (A-20), and $e_2(t)$ in (A-36) yields

$$\frac{-e_c a^2 + \frac{a}{4C} I_1(0) + E\beta^2}{a(a-b)} + \frac{-e_c b^2 + \frac{b}{4C} I_1(0) + E\beta^2}{b(b-a)} + \frac{E\beta^2}{ab} =$$

$$\left[\frac{e_c a^2 - \frac{a}{4C} I_1(0) - E\beta^2}{a(a-b)}\right] \epsilon^{a\frac{T}{2}} + \left[\frac{e_c b^2 - \frac{b}{4C} I_1(0) - E\beta^2}{b(b-a)}\right] \epsilon^{b\frac{T}{2}} - \frac{E\beta^2}{ab} .$$

$$\tag{A-39}$$

Collecting terms and simplifying, (A-39) becomes

$$E\beta^2 \left\{ \frac{1}{a-b} \left[\frac{\epsilon^{a\frac{T}{2}}}{a} - \frac{\epsilon^{b\frac{T}{2}}}{b} \right] + \frac{1}{ab} \right\} = \frac{1}{4C(b-a)} \left[\epsilon^{a\frac{T}{2}} - \epsilon^{b\frac{T}{2}} \right] I_1(0) +$$

$$\left[1 + \frac{1}{a-b} \left(a\epsilon^{a\frac{T}{2}} - b\epsilon^{b\frac{T}{2}} \right) \right] e_c \quad . \tag{A-40}$$

This concludes the derivations for the simplified inverter with an ideal

transformer and a resistive load described in Chapter III.

Derivation of Equations for an Inverter with Reactive Load

The following equations were developed in Chapter III from the

equivalent circuit in Fig. 12:

$$E + e_c = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C} \int i_1 dt - \frac{1}{C} \int i_2 dt \tag{A-41}$$

$$-e_c = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C} \int i_2 dt - \frac{1}{C} \int i_1 dt \quad , \tag{A-42}$$

where the leakage inductance of the transformer and the resistance of

its windings were included in the analysis. By using Laplace trans-

forms, (A-41) and (A-42) become

$$\frac{E + e_c}{s} = L_1 \left[I_1 s - I_1(0) \right] + \frac{I_1}{Cs} + I_1 R_1 - \frac{I_2}{Cs} \tag{A-43}$$

and

$$\frac{-e_c}{s} = -\frac{I_1}{Cs} + L_2 \left[I_2 s - I_2(0) \right] + I_2 R_2 + \frac{I_2}{Cs} \quad , \tag{A-44}$$

where $I_1 = I_1(s)$, $I_2 = I_2(s)$, and $I_1(0)$ and $I_2(0)$ are the initial currents in L_1 and L_2, respectively. Putting (A-43) and (A-44) in a more useable form yields

$$E + e_c + sL_1I_1(0) = \left[L_1s^2 + R_1s + \frac{1}{C}\right] I_1 - \frac{1}{C} I_2 \qquad (A-45)$$

and

$$-e_c + sL_2I_2(0) = -\frac{1}{C} I_1 + \left[L_2s^2 + R_2s + \frac{1}{C}\right]I_2 \; . \qquad (A-46)$$

Solving (A-45) and (A-46) for I_1 and I_2 by determinants yields the following:

$$\Delta = \begin{vmatrix} L_1s^2 + R_1s + \frac{1}{C} & -\frac{1}{C} \\ \\ -\frac{1}{C} & L_2s^2 + R_2s + \frac{1}{C} \end{vmatrix}$$

$$\Delta = s\left[L_1L_2s^3 + (R_1L_2 + R_2L_1)s^2 + (R_1R_2 + \frac{L_1+L_2}{C})s + \frac{R_1+R_2}{C}\right]$$

$$\qquad (A-47)$$

$$I_1(s) = \frac{\begin{vmatrix} (E+e_c) + sL_1I_1(0) & -\frac{1}{C} \\ \\ -e_c + sL_2I_2(0) & L_2s^2 + R_2s + \frac{1}{C} \end{vmatrix}}{\Delta}$$

$$I_1(s) = \frac{L_1L_2I_1(0)s^3 + \left[L_1R_2I_1(0) + (E+e_c)L_2\right]s^2}{\Delta} +$$

$$\frac{\left[(E + e_c) R_2 + \frac{1}{C} (L_1 I_1(0) + L_2 I_2(0)) \right] s + \frac{E}{C}}{\Delta} \tag{A-48}$$

$$I_2(s) = \frac{\begin{vmatrix} L_1 s^2 + R_1 s + \frac{1}{C} & (E + e_c) + s L_1 I_1(0) \\[2em] -\frac{1}{C} & -e_c + s L_2 I_2(0) \end{vmatrix}}{\Delta}$$

$$I_2(s) = \frac{L_1 L_2 I_2(0) s^3 + \left[R_1 L_2 I_2(0) - e_c L_1 \right] s^2}{\Delta} +$$

$$\frac{\left[(L_1 I_1(0) + L_2 I_2(0)) \frac{1}{C} - R_1 e_c \right] s + \frac{E}{C}}{\Delta} \tag{A-49}$$

(A-48) and (A-49) are the equations used in Chapter III for a more accurate analysis of the inverter circuit.

COMPUTER PROGRAM LISTING IN FORTRAN

A list of the symbols used in the Fortran statements is given on the left below and their meaning on the right.

AMPA(I), AMPB(I), AMPC(I) = i_1(t), the input current in amperes

VOLTA(I), VOLTB(I), VOLTC(I) = e_2(t), the output voltage in volts

RES = R_L, the load resistance in ohms

HEN = L, the ballast inductance in henrys

CAP = C, the commutating capacitance in farads

FREQ = f, the switching frequency in cycles per second

EIN = E, the d.c. supply voltage in volts

TURN = n, the turns ratio of the secondary to half the primary

R = R, the load referred to one half the primary

AA = α = $\dfrac{1}{8RC}$

BB = β = $\sqrt{\dfrac{1}{4LC}}$

WW = ω = $\sqrt{\beta^2 - \alpha^2}$

THETA = θ = $\tan^{-1} \dfrac{\alpha}{\omega}$

A11, A12, A21, A22; B11, B12, B21, B22; G11, G12, G21, G22 are the coefficients a_{11}, a_{12}, a_{21}, a_{22} of the initial condition equations.

C1, C2, D1, D2, H1, H2 correspond to k_1 and k_2 in the initial condition equations

DET, DET2, DET3 = determinant of coefficients above

CL$\emptyset$, CL2, CL3 = $I_1(0)$

EC$\emptyset$, EC2, EC3 = e_c

T, T2, T3 = t, time in seconds

P,Q = real roots of quadratic factor

```
C 0001 0 SCR POWER INVERTER
C 0002 0       RESISTIVE LOAD CASE
C 0003 0 CURRENT AND VOLTAGE WAVEFORMS
      5 0 DIMENSION AMPA(11),VOLTA(11)
      6 0 DIMENSION AMPB(11),VOLTB(11)
      7 0 DIMENSION AMPC(11),VOLTC(11)
     10 0 READ,RES,HEN,CAP,FREQ,EIN,TURN
     15 0 R=RES/(TURN**2)
     20 0 AA=1.0/(8.0*R*CAP)
     25 0 BB=SQRTF(1.0/(4.0*HEN*CAP))
     30 0 IF(BB-AA)400,200,35
C 0031 0       COMPLEX ROOTS
     35 0 WW=SQRTF(BB**2-AA**2)
     37 0 THETA=ATNRF(WW/AA)
     40 0 PUNCH,RES,HEN,CAP,FREQ,EIN,TUR
     40 1 N
     41 0 PUNCH,R,AA,BB,WW
     45 0 A=BB*EXPEF(-AA/(2.0*FREQ))*SIN
     45 1 F(THETA+WW/(2.0*FREQ))/WW
     47 0 B=BB*SINF(THETA)/WW
     49 0 C=EXPEF(-AA/(2.0*FREQ))*SINF(W
     49 1 W/(2.0*FREQ))/WW
     50 0 C1=EIN*(A-B)/R-EIN*C/HEN
     52 0 C2=2.0*EIN-EIN*(A+B)
     54 0 A11=A-B+2.0*AA*C
     56 0 A12=C/HEN
     58 0 A21=-2.0*AA*C*R
     60 0 A22=A+B
     62 0 DET=A11*A22-A21*A12
     64 0 CLO=(C1*A22-C2*A12)/DET
     66 0 ECO=(A11*C2-A21*C1)/DET
     70 0 T=0.0
     71 0 PAUSE
     75 0 DO 130 I=1,11
    100 0 AMPA(I)=EIN/R+((CLO-EIN/R)*BB*
    100 1 EXPEF(-AA*T)*SINF(WW*T+THETA)/
    100 2 WW)+(((EIN+ECO)/HEN+2.0*AA*CLO
    100 3 )*EXPEF(-AA*T)*SINF(WW*T))/WW
    110 0 VOLTA(I)=2.0*(EIN-((ECO+EIN)*B
    110 1 B*EXPEF(-AA*T)*SINF(WW*T+THETA
    110 2 )/WW)+2.0*AA*R*CLO*EXPEF(-AA*T
    110 3 )*SINF(WW*T)/WW)
    120 0 PUNCH,T,AMPA(I),VOLTA(I)
    130 0 T=T+0.05/FREQ
    140 0 PAUSE
    150 0 GO TO 10
C 0199 0       EQUAL ROOTS
    200 0 EX=EXPEF(-AA/(2.0*FREQ))
    202 0 PUNCH,RES,HEN,CAP,FREQ,EIN,TUR
    202 1 N
    203 0 PUNCH,R,AA,BB
    205 0 D1=EIN*EX/(HEN*2.0*FREQ)-EIN*B
    205 1 B**2*(1.0+EX)/(R*AA**2)
    210 0 D2=EIN*BB**2*((AA/(2.0*FREQ)+1
```

```
  210 1 .0)*EX-1.0)/AA**2
  215 0 B11=EX*(1.0+AA/(2.0*FREQ))-1.0
  217 0 B12=EX/(HEN*2.0*FREQ)
  219 0 B21=EX/(CAP*8.0*FREQ)
  221 0 B22=EX*(AA/(2.0*FREQ)-1.0)-1.0
  225 0 DET2=B11*B22-B21*B12
  230 0 CL2=(D1*B22-D2*B12)/DET2
  240 0 EC2=(B11*D2-B21*D1)/DET2
  250 0 T2=0.0
  251 0 PAUSE
  275 0 DO 330 I=1,11
  300 0 AMPB(I)=EIN*BB**2/(AA**2*R)+((
  300 1 (EC2-EIN)/HEN+AA*CL2)*T2+CL2-E
  300 2 IN*BB**2/(AA**2*R))*EXPEF(-AA*
  300 3 T2)
  310 0 VOLTB(I)=2.0*(EIN*BB**2/AA**2+
  310 1 ((CL2/(4.0*CAP)-EIN*BB**2/AA+A
  310 2 A*EC2)*T2-EC2-EIN*BB**2/AA**2)
  310 3 *EXPEF(-AA*T2))
  320 0 PUNCH,T2,AMPB(I),VOLTB(I)
  330 0 T2=T2+0.05/FREQ
  350 0 PAUSE
  375 0 GO TO 10
C 0399 0    REAL ROOTS
  400 0 P=-AA-SQRTF(AA**2-BB**2)
  405 0 Q=-AA+SQRTF(AA**2-BB**2)
  410 0 PUNCH,RES,HEN,CAP,FREQ,EIN,TUR
  410 1 N
  411 0 PUNCH,R,AA,BB,P,Q
  420 0 EA=EXPEF(P/(2.0*FREQ))
  422 0 EB=EXPEF(Q/(2.0*FREQ))
  430 0 H1=EIN*((1.0/HEN+BB**2/(P*R))*
  430 1 (1.0-EA)-(1.0/HEN+BB**2/(R*Q))
  430 2 *(1.0-EB))/(P-Q)
  440 0 H2=EIN*BB**2*((EA/P-EB/Q)/(P-Q
  440 1 )+1.0/(P*Q))
  450 0 G11=((EA-1.0)*(P+2.0*AA)-(EB-1
  450 1 .0)*(Q+2.0*AA))/(P-Q)
  460 0 G12=(EA-EB)/(HEN*(P-Q))
  470 0 G21=(EA-EB)/(4.0*CAP*(Q-P))
  480 0 G22=1.0+(P*EA-Q*EB)/(P-Q)
  490 0 DET3=G11*G22-G21*G12
  500 0 CL3=(H1*G22-H2*G12)/DET3
  505 0 EC3=(G11*H2-G21*H1)/DET3
  530 0 U=((P**2*CL3+P*(EIN+EC3)/HEN+2
  530 1 .0*P*AA*CL3+EIN*BB**2/R)/(P*(P
  530 2 -Q)))
  535 0 V=(Q**2*CL3+Q*(EIN+EC3)/HEN+2.
  535 1 0*Q*AA*CL3+EIN*BB**2/R)/(Q*(Q-
  535 2 P))
  540 0 X=(-EC3*P**2+P*CL3/(4.0*CAP)+E
  540 1 IN*BB**2)/(P*(P-Q))
  550 0 Y=(-EC3*Q**2+Q*CL3/(4.0*CAP)+E
  550 1 IN*BB**2)/(Q*(Q-P))
```

```
555 0 Z=EIN*BB**2/(P*Q)
600 0 T3=0.0
601 0 PAUSE
610 0 DO 660 I=1,11
630 0 AMPC(I)=U*EXPEF(P*T3)+V*EXPEF(
630 1 Q*T3)+Z/R
640 0 VOLTC(I)=2.0*(X*EXPEF(P*T3)+Y*
640 1 EXPEF(Q*T3)+Z)
650 0 PUNCH,T3,AMPC(I),VOLTC(I)
660 0 T3=T3+0.05/FREQ
670 0 PAUSE
680 0 GO TO 10
700 0 END
```

VITA

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