A COMPARISON OF INVENTORY COSTS
IN LOGISTICS MANAGEMENT
UNDER FLUCTUATING DEMAND CONDITIONS

By

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Scope and Method of Study: The purposes of this study are to indicate the important role of the inventory control element in Logistics Management, both in business and military, and to illustrate inventory costs (under fluctuating demand conditions) obtained from Basic E.O.Q., Wagner-Whitin, and Modified E.O.Q. Each method has been employed in deriving a purchasing plan which minimized the total inventory costs, under the fluctuating demand rate of a single item. A comparison of the results from each method is shown in order to aid the decision maker to select the best solution in this situation. A computer program for Basic E.O.Q. and Wagner Whitin was developed.

Findings and Conclusions: Under fluctuating demand conditions, Basic E.O.Q. did not provide the optimal solution, especially when there is a substantial variation of demand rate in each time period. The Wagner-Whitin method is more acceptable and more satisfactory in achieving major savings (in replenishment and carrying costs). The Modified E.O.Q. provided significant cost savings in this situation when compared to Basic E.O.Q. model.

ADVISER'S APPROVAL ____________________

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CHAPTER I

INTRODUCTION

Logistics Management and Its Meaning

The concept of management principally entails planning, directing, controlling and organizing a specific function or functions. Logistics management is primarily concerned with controlling the flow of materials and products, and the development of an effective organizational structure. It involves administering an activity that is interdisciplinary by nature because all aspects of a firm and the domestic economy are affected. Figure 1, illustrated the flow of inputs and outputs of an industrial logistical system.

Basically, there are many definitions of logistics. However, while most definitions of logistics and physical distribution are similar, variation are encountered. The National Council of Physical Distribution Management basically excludes production from its definition in the following:

"Logistic is a term employed in manufacturing and commerce to describe the broad range of activities concerned with the efficient movement of finished products from the end of the production line to the consumers, and in some cases includes the movement of raw materials from the source of supply to the beginning of the production line. These activities include freight transportation, warehousing, material handling, inventory control, plant and warehouse site selection, order processing, market forecasting, and customer service."[1]

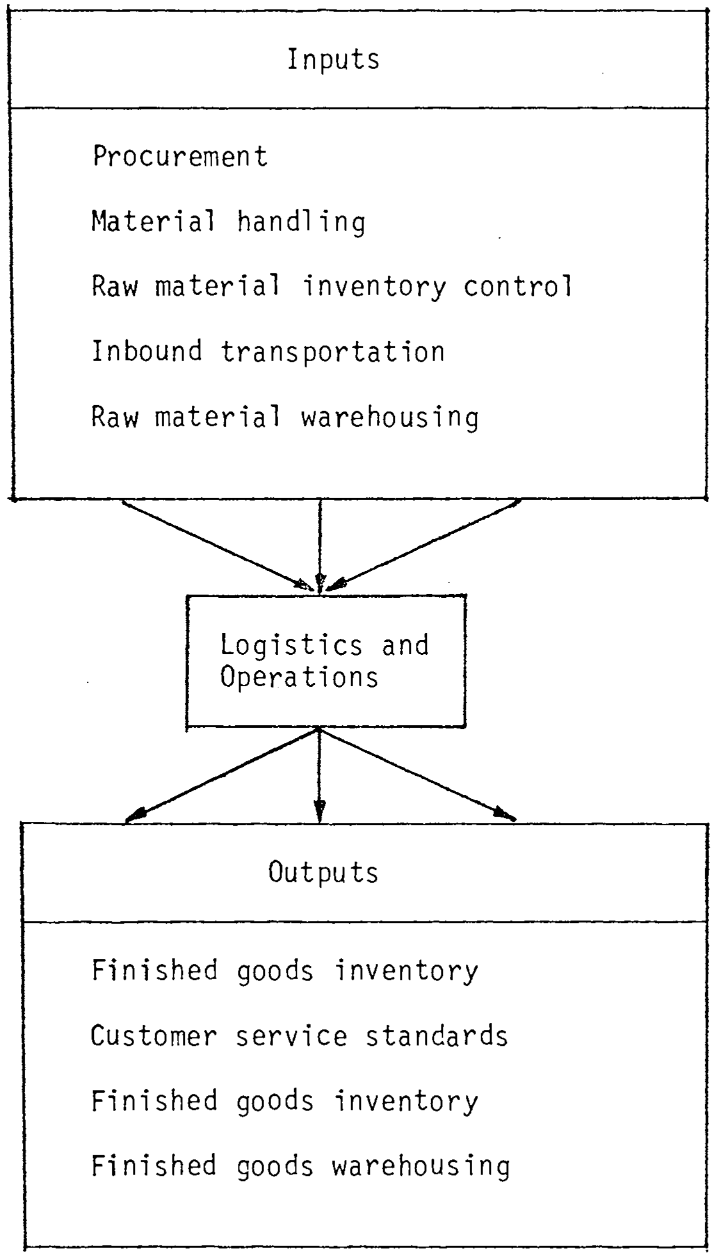

Figure 1. Flow of Inputs and Outputs of an Industrial Logistical System

Heskett, Ivie and Glaskowsky defined logistic as the management of all activities which facilitate movement and the coordination of supply and demand in the creation of time and place utility in goods.[2]

Another definition according to Ballou is "Logistic management is the planning, organizing, and controlling of all move-store activities that facilitate product flow from the point of raw material acquisition to the point of final consumption, and of the attendant information flows, for the purpose of providing a sufficient level of customer service consistent with the costs incurred for overcoming the resistance of time and space in providing the service."[3]

From all of the above definitions we can see the essential ingredients of logistic management which can be defined in its most concise form, as the physical movement of goods from supply points to final sale to customers and the associated transfer and holding of such goods at various intermediate storage points, accomplished in such manner as to contribute to the explicit goals of the organization.

Logistics activities, whether they take place in the military or in business enterprise, commonly involve movement and storage for the purpose of having the desired object of flow at the right place at the right time.

The content of logistics within a firm varies considerably with the type of business and how management perceives the scope of logistics and associated decision problems. A representative list of logistics elements for a firm with substantial logistics costs is as follows:

Key elements:

1. Transportation
   a) mode and service selection
   b) carrier routing
   c) vehicle scheduling

2. Inventories
    a) finished goods stocking policies
    b) record keeping
    c) supply scheduling
    d) short-term sales forecasting
3. Facility location and customer service
4. Order processing and information flows
    a) sales order procedures
    b) information collection, storage, and manipulation

Supporting activities:

1. Warehousing
2. Material handling
3. Protective packaging

Inventory Control in Logistics Management

1. Under Business Enterprise Aspects

Typically, the sizable financial investment in finished product and raw material inventory constitutes a significant cost of doing business. Not only is inventory considered a valuable asset, but in business lexicon it is also an investment. Therefore, inventory control is considered to be an important element of logistics management. In addition, inventory control exists in the logistics system because it would be either too expensive or impossible to provide the products at the time they are desired by consumers, besides, the business and industrial operations could not function without this element. Without the proper assortment of inventories available, serious marketing problems can develop with respect to revenue generation and customer relations. Raw material shortages can force the production line to be shut down or the production schedule to be modified, which in turn, introduces considerable added expenese and a potential shortage of the finished products. Moreover, excessive or overstocked inventories might create serious problems. For example, overstocks increase cost and reduce profitability as a result of added warehousing, capital tieup, product deterioration, excessive insurance, added taxes and product obsolescence. Inventory control is therefore properly viewed as the attainment of balance between a shortage of stock and an excess of stock within a planning environment characterized by risk and uncertainty.

The basic function of inventory is simply to increase profitability through manufacturing and marketing support. The theroetically ideal concept of inventory commitment of a zero-inventory manufacturing-distribution system is obviously not practical to consider. Inventory consists of a major area of asset deployment which should be required to provide an adequate return on investment. The lack of sophistication in the measurement of inventory investment means in part that it is difficult to identify the proper inventory level in a complex organization. Financial management has a natural tendency to want inventories to be reduced so as to improve cash flow. Marketing desires abundant finished goods inventories to protect against stockouts or back order. The manufacturing department is inclined to desire large stockpiles of raw materials and components to assure that there will be no disruption of plans designed to achieve maximum economy of production. These are examples of substantial conflicts exist within the business organization concerning the appropriate level of inventory commitment and allocation.

Four prime functions that underline inventory decisions in business enterprise:[4]

1. Geographical Specialization - which its function is also related to physical distribution.

2. Decoupling - to provide maximum efficiency of operations housed at a single geographical location.

3. Balancing Supply and Demand - which concerns about elapsed time between consumption and manufacturing.

4. Safety stock - which dealing with short-range variation in either demand or the operational capability to replenish inventories.

2. Under Military Aspects

Perhaps the term logistic is seemed to be more familiar to the military than to the business enterprise. However, for the inventory control element, the fact that the number of items stocked by the military is much larger than those held by even the largest of private enterprises (e.g., the military alone maintains inventories value at about one-third of those held by all US manufacturers). Also the inventory control problems of the military are complicated further by the size and complexity of military organization. Therefore, the military was and continues to be a source of experience from which the business sector may benefit. Though the problems of each are not identical, e.g., the military has the objective to win wars and as a result often establish a "customer service" level higher than that usually found among business, and the maintenance activity is not generally a logistics responsibility in the business sector. However, business enterprise and military economic objectives are identical in that both try to minimize costs to achieve a given objective. Common military logistics activities including determining requirements, procurement, storage, transportation, and inventory management, all of which are included in the business logistics functions. Furthermore, the military establishment has no profit mechanism to aid it in making rational decisions about its inventory level. In business enterprise some of the costs of depletion may be established, and a comparison of the costs of depletion with the cost of stocking and additional item aids the businessmen in deciding whether or not he should add to his inventory. Military decisions must be based on the same considerations, but the costs of depletion in the case of the military are frequently unknown.[5] For example, the effect of inventory depletion is vastly differ for peacetime and wartime. Therefore, these values have an extremely arbitrary nature. Yet some values must be attached and military decisions, implicitly or explicity, must be made on the basis of just such vague information.

The demand for various items in the military economy drastically changes from peacetime to wartime, and varies with changes in public opinion and politics in times of semi-mobilization such as the present. The uncertainty that surrounds questions like whether and when war will start, how long war will last, what sort of war will be fought, etc., are all reflected in an extremely complicated demand situation. In some situations, the proper levels of inventory are greatly influenced by the strategy of the enemy. Before national decisions concerning inventory levels can be made, all possible enemy strategies must be enumerated along with an evaluation of the probability of occurance of each of them. In some areas same theory can be applied.

Another extremely difficult problem that confronts military planners is the choices between quality and quantity of equipment (e.g., the innovation and invention of new and technically superior weapons).

A rational answer to this problem can be made only if the time, duration, and nature of the war is known, and if exact information is available about line production and how long it takes to make changes in it.

The most important thing is to design a logistics system that provides the important items of equipments when needed, i.e. when the fate of the nation is at stake. Even if the equipment is available at some other base, this base usually is not as accessible as the alternative sources of supply of goods and services are for the ordinary consumers. The lack of immediate or extremely rapid accessibility of some items of military equipment could in some cases, be as bad as its nonexistence, and could lead to consequences of disaster. However, an insufficient inventory policy will result in a large amount of money wasted in unnecessary inventory costs.

In the military system, the factors used to decide the inventory decision are as follows:

1) Requirements necessary to fulfill demands of a recurring nature. These requirements are usually estimated on the basis of past issues, outstanding obligations, and an evaluation of the conditions likely to be prevalent in the period during which the material is issued.

Specific consideration is to be made of these factors:

a) trend of demand rate

b) changes in operational plans

c) production status

d) seasonal demand

2) A safety allowance to provide for unknown factors that may influence the requirements in:

a) changing usage rates over time

b) different usage rates for different stations

c) changes in the rate of operations

d) change in the end use to which particular items are put.

3) Requirements necessary to carry out programs which had been planned.

4) Reserve requirements need for contingencies not included in any other requirement which are determined from the following factors:

a) modern operational plans which may effect a substantial change in logistics requirements in the future.

b) prospective production situation

c) availability of materials

Based upon the analysis thusfar, inventory control is one of the most important elements in logistics management. One of inventory cost is carrying cost which varies extensively from industry to industry, and from firm to firm. Nevertheless, any business enterprise or military system that maintains any level of inventory incurs a carrying charge. Alford and Bangs points out that the annual cost of maintaining a production inventory average 25 percent of the value of the inventory.[6] Substantiating this observation, Buffa indicated that it is not uncommon for a manufacturing concern to have 25 percent of its capital invested in inventories. For instance, Eastman Kodak and Lockheed, at one time each had inventories exceeding $185 million[7] or 30 percent of total assets.

W. Evert Welch discovered that annual carrying cost average somewhat over 20 percent of the inventory value. He also found that inventory carrying cost ranges from 10 to 34 percent of inventory value which can be defined as follows:[8]

Interest charge on investment	4 to 15%
Insurance cost	1 to 3%
Property taxes	1 to 3%
Shortage cost	0 to 3%
Obsolescence and deterioration	4 to 10%
Total carrying charges	10 to 34%

Another inventory cost is ordering cost or production set-up cost. This cost is consists of the estimated cost of all the operations involved for each additional replensihment order. The estimate would include the wages, material and equipment used for the operation. Generally, there is a fixed charge or expense per order or set-up regardless of the size of the order or set-up. The cost estimated for ordering some item since it is quoted per order will reflect how total ordering cost will charge with order frequency. Therefore, total ordering cost will depend on the purchasing pattern which achieves the objective of purchasing the optimal quantity of goods necessary for supplying continuous production, i.e., the purchase policy of minizing the sum of inventory carrying cost and ordering cost.

In the past business have been able to achieve a reasonably balanced and effective inventory policy largely through an intuitive understanding of the natures of the business. However, as a firm grows and as business executives become more and more specialized in their jobs, or further removed from direct operations, achieving an economical balance through intuition increasingly becomes difficult. Thus firms have found quantitative analysis of inventory systems an attractive approach to manage inventory control, especially under conditions of uncertainty.

Forthcoming Coverage

The second chapter will introduce three quantitative analysis methods of inventory system (i.e., Basic E.O.Q., Wagner-Whitin, and Modified E.O.Q) using fluctuating demand conditions, to identify the purchasing pattern which results in the minimum inventory cost. Also, the results obtained from each method are analyzed to compare the method to handle variation in demand rate among time periods. Chapter three will illustrate the application of these three algorithms applied to an example related to the Military Logistics Management situation. The fourth chapter will present a developed computer program used to compute the total inventory cost and purchasing pattern for a variable number, n, periods when demand of each time period is known. The output of this computer program for the examples in chapter II and III is given in the Appendix A and B respectively. Finally, chapter five will present the concluding remarks of this paper including a recommendation for further study.

ENDNOTES

[1]National Council of Physical Distribution Management, Chicago: Illinois.

[2]J. L. Heskett, Robert M. Ivie, and Nicholas A. Glaskowsky, Jr. "Management of Physical Supply and Distribution." Business Logistics, 1964, p. 21.

[3]Ronald H. Ballou. "Broadening and Unifying Marketing Logistics." The Logistics Review, Vol. 6 (Winter 1970), p. 201.

[4]Donald J. Bowersox. "Components of Logistical Systems: Elements of Inventory." Logistical Management, 1974, pp. 182-185.

[5]Thomas M. Whitin. "Inventory Control Problems of the National Military Establishment." The Theory of Inventory Management, 1957, pp. 165-175.

[6]L. P. Alford and J. R. Bangs (eds.) Production Handbook (New York: The Ronald Press, 1955), pp. 396-97.

[7]Elwood S. Buffa. Modern Production Management. 2nd ed. (New York: John Wiley & Sons, Inc., 1965), p. 469.

[8]W. Evert Welch. Tested Scientific Inventory Control. (Greenwich, Conn.: Management Publishing Corporation, 1956), p. 64.

CHAPTER II

CONCEPTS AND ALGORITHM OF THE METHODS

Economic Order Quantity (Basic EOQ)

Since the introduction of the basic economic order quantity (EOQ) or square root EOQ formula had been developed by Wilson in 1915, the concept has become a powerful theoretical tool with widespread applications. The concept of EOQ is to balance the cost of maintaining inventory against the cost of ordering. That is, the EOQ is the quantity at which total cost is minimized, where total cost equals inventory carrying cost plus ordering cost. The key to understanding the basic relationship is to remember that average inventory is equal to one-half order quantity. Therefore, the larger the order quantity the larger the average inventory. Likewise, the larger the order quantity, the fewer orders required per planning period and consequently the lower total ordering cost.

The EOQ formulation finds the exact order quantity at which the annual combined total cost of ordering and maintenance is at the lowest point for a given sale volume (shown in Figure 2-1). An aggregate view of EOQ is illustrated in Figure 2-2. Notice that the demand rate is constant and inventory is replenished instantaneously when the reorder point is reached. Average inventory, moreover, is Q/2, or a straight line.

The exact quantity that should be ordered to enjoy economical relationships can be determined by dividing the number of orders into

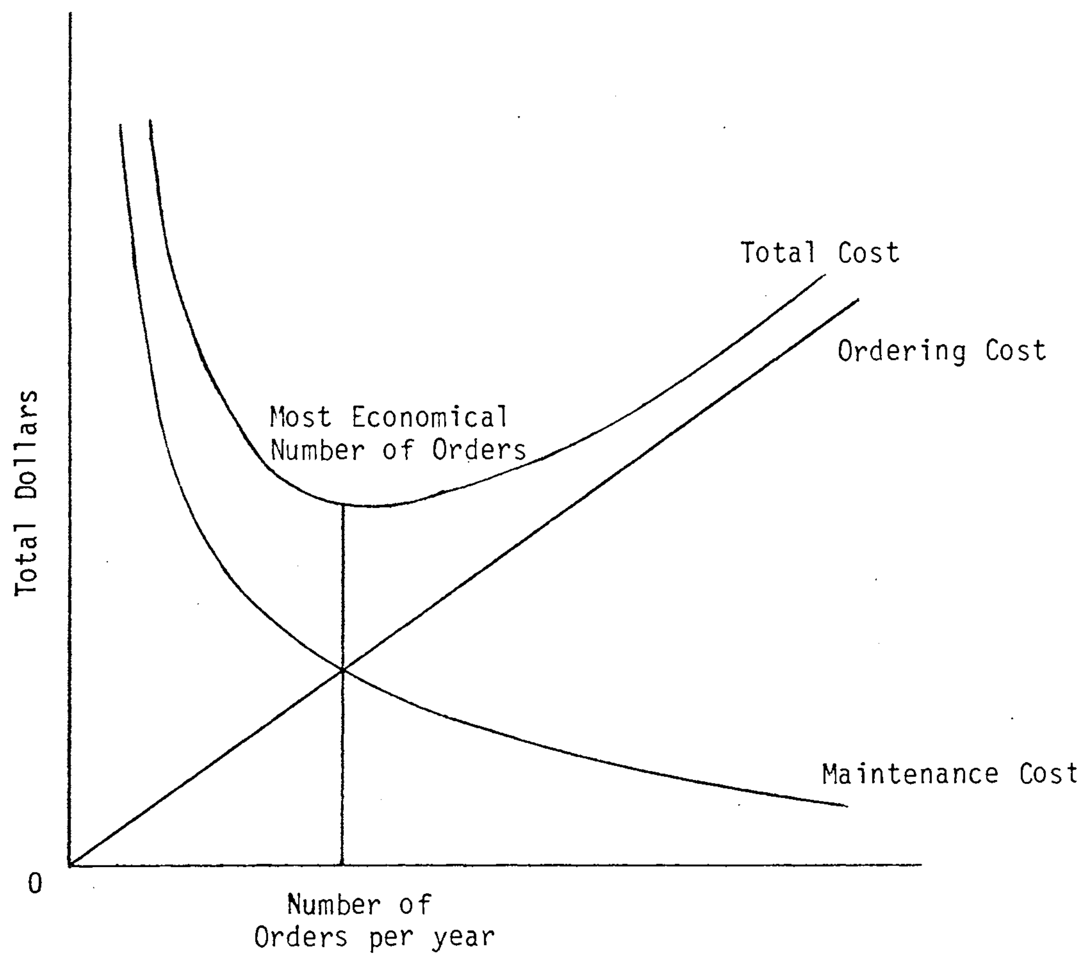

Figure 2-1. Economic Order Quantity

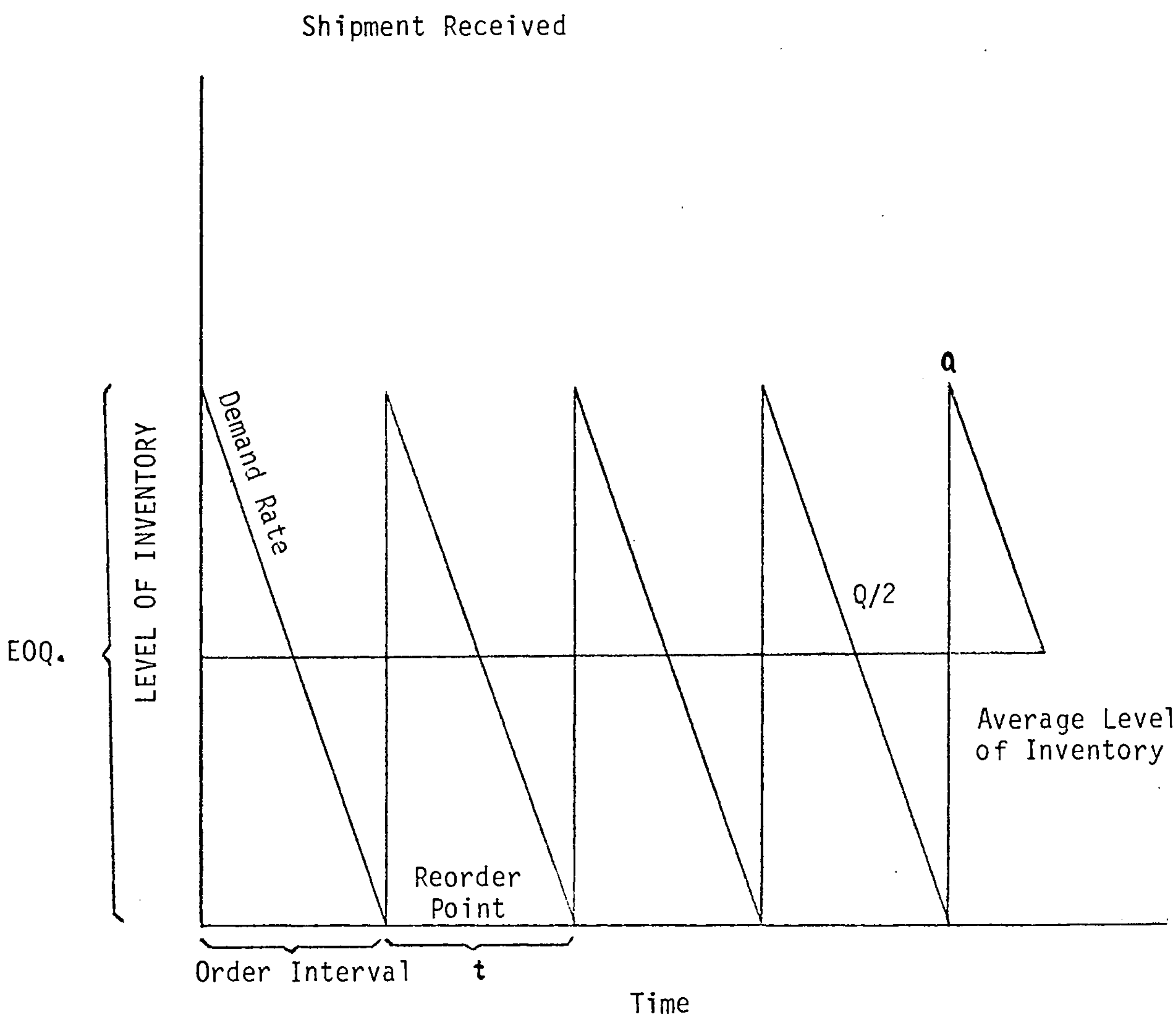

Figure 2-2. Inventory Order-Quantity Diagram.

the annual volume. EOQ is one of the most popular methods for determining the inventory reorder point, while simultaneously minimizing ordering cost.

EOQ is derived through minimizing the total cost of the purchasing decision with respect to the quantity ordered. In the most basic case, the total cost formula contains only two components:

Total cost = fixed preparation costs (order cost) + holding cost

This total cost formula could be expressed in terms of variables as follows:

$$TC\ (Q) = \frac{DS}{Q} + IC\ \left(\frac{Q}{2}\right)$$

where:

TC (Q) = annual total relevant inventory cost depending on the value of Q

Q = sizes of each order to replenish inventory

D = annual demand requirements

S = ordering cost

C = value of a unit carried in inventory

I = carrying cost as a percentage of C

The optimum Q is found by taking the first derivative of TC (Q) setting the derivative equal to zero, and solving for Q.

Taking the derivative and setting it equal to zero, will obtain

$$\frac{dTC\ (Q)}{dQ} = -\frac{DS}{Q^2} + \frac{IC}{2} = 0$$

$$** \quad Q = \sqrt{\frac{2DS}{IC}}$$

Three simplifications have been performed to reduce all of the cost of purchasing, handling, and storing inventory into this total cost formula.

1) Those costs which do not vary with the quantity ordered are irrelevant and have been eliminated.[1] Prichard and Eagle concluded that estimating all relevant costs in the EOQ model which based solely on estimates does not necessarily invalidate the accuracy of the derived EOQ. They proved, through the use of a mathematical formula, that a 20 percent error in order cost will cause only a 10 percent error in total cost, suggesting that the total incremental cost arrived at with the EOQ model is not overly sensitive to errors in input parameters.[2]

2) Some costs have been assumed away. Costs which may vary with the quantity ordered but are difficult to express in a functional relationship are assumed to be constant or part of the general overhead cost.

3) Many costs have been combined into a single component. For example, the holding cost in particular, is composed of dozens of separate costs associated with having items in inventory.

Generally, the basic EOQ formula can be used in any purchasing lot-size or production lot-size decision, but only at the cost of meeting the following assumptions which must be made in using the basic EOQ model:

1) Demand (usage) is relatively stable, i.e., disbursements are made at a constant rate (assumed linear depletion of inventory).

2) Holding cost is incurred on the basis of the average number of items in inventory.

3) An order is received in one shipment. Demand for a period must be on hand at the beginning of the period for instantaneous

shipment, i.e., orders are placed so that demand must be met.

4) The cost (price) of the item is independent of the size of the order, i.e., there are no price breaks.

The following data[3] will be applied in each method throughout this chapter due to the purpose of comparison the results:

S = ordering cost = $300 per order

D = the forecast annual demand requirement = 1,105 units

C = value of unit in inventory = $120

I = annual carrying cost rate = 20%

Using Basic EOQ Algorithm:

$$Q = \sqrt{\frac{2(1105)(300)}{(0.2)(120)}}$$

$$= \sqrt{27625}$$

$$= 166.21$$

$$\approx 167 \text{ units}$$

$$TC = \frac{(1105)(300)}{167} + \frac{(120)(0.2)(167)}{2}$$

$$= \$3,989$$

Considering the result obtained from this algorithm, we can conclude that there will be inventory replenishment 7 times with the size of each order equal to 167 units which will have ending inventory 64 units, using total inventory cost $3,989 (result also shown in computer output in Appendix A).

Wagner-Whitin Algorithm

In general, the basic EOQ formula used under the assumption of a steady-state demand rate is well known. However, in order to cope with more realistic situation when the amounts demanded in each period are known but are different, and furthermore, when inventory cost vary from period to period, the basic EOQ technique perhaps no longer assumes a minimum cost solution. Therefore, the dynamic programming algorithm, i.e., Wagner-Whitin technique is developed to be used in the cases where there is a high fluctuation in demand or usage rate in each time period. This technique stated that for each period one of two situations will occur:

1) Either purchase in that period, or

2) Purchase in a previous period and incur carrying charges for having the material on hand earlier than needed.

The choice of which alternative to select is evaluated with respect to the most recent optimal decision. Even though there might be a minimum cost which occurs in some periods but this cost is not corresponding to the most recent optimal decision, this minimum cost can't be acceptable.

Using this algorithm with the previous data including a demand requirement of each time period (which varies among time periods but the total amount is still equal to 1,105 units).

There are two approaches to calculate the total inventory costs using Wagner-Whitin algorithm:

1) inventory carrying cost for average inventory of each demand period is added when computed inventory cost for that period (result shown in Table I). This total inventory cost can be directly used in comparison with the total inventory cost obtained from the Basic EOQ.

2.) inventory carrying cost for average inventory of each demand period is ignored when computed inventory cost for that period (result shown in Table II). But before comparing with the total inventory cost from the basic EOQ, the carrying cost for the average of the total demand requirement (D/2 x carrying charge per unit per period) must be added to this total inventory cost.

The following explanation of Wagner-Whitin algorithm is provided for obtaining the result shown in Table I.

The first entry, row 1 and column 1, indicated that a purchase is made in period 1 for use in period 1 (assume immediate delivery).

Period 1, there is only one optimal decision obtained by adding ordering cost ($300) and inventory carrying charge of $10 (determined by multiplying the $2 carrying charge per unit per period times the average inventory for that period ($2 x 5 units)):

or, $\$310 = \$300 + (\$2 \times 10/2)$

Period 2, there are two alternatives for procuring material in this period as follows:

1) Buy the material for period 2 in period 1 and pay the carrying cost. The additional charges then are $2 per unit for the 10 units required for period 2 carried from period 1 (or $20), plus the average inventory in period 2 (10/2 = 5 units), times $2 per unit (or $10). These additional charges, $30 ($20 + $10), is added to the previous solution ($310), totalling $340:

or, $\$340 = \$310 + (\$20 + \$10)$

2) Entail initiating a purchase in period 2. The additional charges then are $300 for the ordering cost plus the carrying charges for the average inventory in period 2 (10/2 x $2 = $10). These new charges then are added to the previous optimal solution, summing to $620:

or, $620 = $310 + $300 + ($2 x 10/2)

It is obviously for a rational decision maker to select the optimal solution obtained from the alternative 1. ($340) which suggested purchasing in period 1 for period 1 and 2.

Period 3, there are three possibilities as follows:

1) Buy in period 1 for periods 1, 2, and 3

or, $415 = $310 + $2(25 + 20 + 15/2)

2) Buy in period 2 for periods 2 and 3

or, $665 = $310 + $300 + $2(20 + 15/2)

3) Buy in period 3 for period 3

or, $655 = $340 + $300 + $2(15/2)

The optimal solution for this period ($415) obtained by purchasing in period 1 for periods 1, 2, and 3.

Following these procedures and concepts, calculating the solution of the remaining periods which will obtain the results shown in Table I. Finally, we can obtain a purchasing pattern and total inventory cost from this method (shown in Table III). Regardless of the approach adopted for computation (results in Table I or Table II) the final results obtained will be the same.

Validation of Costs under Wagner-Whitin Method

After a pattern of purchasing has been decided, it is possible to validate total inventory cost obtained from Wagner-Whitin procedure by evaluating the inventory carrying charges in each period and then adding the carrying charges to the ordering cost (details shown in Table IV).

TABLE I

DETAILS OF WAGNER-WHITIN RESULTS
(INCLUDING AVERAGE INVENTORY CHARGED IN EACH PERIOD)

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	240	230	40	0	10
1	310	340	415	555	1,185							
2		620	665	765	1,255							
3			655	715	1,065							
4				735	945							
5					925	1,465						
6						1,405	2,155					
7							1,955	2,765				
8								2,525	3,215			
9									3,055	3,175	3,175	3,245
10										3,395	3,395	3,445
11											---	---
12												3,485
Total Cost	310	340	415	555	925	1,405	1,955	2,525	3,055	3,175	3,175	3,245

TABLE II

DETAILS OF WAGNER-WHITIN RESULTS
(EXCLUDING AVERAGE INVENTORY CHARGED IN EACH PERIOD)

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
1	300	320	380	500	1,060							
2		600	630	710	1,130							
3			620	660	940							
4				680	820							
5					800	1,160						
6						1,100	1,600					
7							1,400	1,940				
8								1,700	2,160			
9									2,000	2,080	2,080	2,140
10										2,300	2,300	2,340
11												---
12												2,380
Total Cost	300	320	380	500	800	1,100	1,400	1,700	2,000	2,080	2,080	2,140

	Total inventory cost (unadjusted)	$2,140
Add:	Carrying cost for period of using	$1,105
	Total inventory cost (actual)	$3,245

TABLE III

SUMMARY RESULTS OF WAGNER-WHITIN METHOD

Purchase in Period	For Use in Period(s)	Quantity
1	1, 2, 3, 4	55
5	5	70
6	6	180
7	7	250
8	8	270
9	9, 10, 12	280*

Total inventory cost = $3,245

*(No ending inventory)

TABLE IV

VALIDATION OF COSTS UNDER

WAGNER-WHITIN METHOD

Order Period	Order Quantity	Period	Demand	Average Inventory	Inventory Carrying Cost
1	55	1	10	50	100
		2	10	40	80
		3	15	27.5	55
		4	20	10	20
2	70	5	70	35	70
3	180	6	180	90	180
4	250	7	250	125	250
5	270	8	270	135	270
6	280	9	230	165	330
		10	40	30	60
		11	0	10	20
		12	10	5	10
Total	1,105		1,105		$1,445
		Purchase 6 times @ $300			$1,800
		Total inventory cost			$3,245

Modified EOQ

Silver and Meal developed the modified EOQ procedure, which using a simple formula, closely related to the usual form of the basic EOQ, take account of variations in the demand rate and can achieve major savings (in replenishment and carrying costs) when compared to the use of basic EOQ model. Furthermore, they pointed out that their method requires a shorter time horizon for planning purposes and less computational effort than does the Wagner-Whitin algorithm.

The details of the modified EOQ algorithm is as follows:

1) Determine the timing and quantities of all replishments during a whole period desired.

2) Assumed that there was no inventory on hand at the beginning of the first period, hence the first replenishment had to made at that time.

Let us take the zero of time to be the time at which stock replenishment is required, and let T be the length of time (measured in periods) that the current replenishment should last. We select T such that:

$$T = \sqrt{\frac{2S}{CIF(T)}} \qquad \text{-----------} \qquad (1)^4$$

where:

F(T) = is the demand rate at time T, expressed in units per period

S = is the ordering cost or set-up cost

C = is the standard unit cost

I = is the carrying charge, expressed as a decimal fraction per period

From equation (1) can be rewritten as:

$$T^2F(T) = \frac{2S}{CI}$$

Under the assumption that $F(T_i)$ is constant during the i^{th} period, by calculating the left side of the equation for steadily increasing integer values of T_i until

$$T_i^2F(T_i) \geq \frac{2S}{CI}$$

for the first time. Then we solve the equation

$$T = \sqrt{\frac{2S}{CIF(T_i)}}$$

to find the required time supply.

Numerical illustrations

Illustration 1, Consider and item with characteristics such that 2S/CI = 300 (the units are period-pieces), and suppose a replenishment is required at time 0 and the known demand pattern for this whole period (one year) is:

Time	0	1	2	3	4	5	6	7	8	9	10	11
Period	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10

For $T_1 = 1$ $\quad F(T_1) = 10 \quad T_1^2F(T_1) = 10 < 300$

$T_2 = 2 \quad F(T_2) = 10 \quad T_2^2F(T_2) = 4(10) < 300$

$T_3 = 3 \quad F(T_3) = 15 \quad T_3^2F(T_3) = 9(15) < 300$

$T_4 = 4 \quad F(T_4) = 20 \quad T_4^2F(T_4) = 16(20) > 300$

Therefore, T value is in the range 3 to 4

$$T^2 = \frac{300}{F(T_4)}$$

$$= \frac{300}{20} = 15$$

$$T = \sqrt{15} = 3.87$$

This indicated that the replenishment amount at time zero is the quantity that will last through time 3.87, namely

$$10 + 10 + 15 + 20(0.87) = 52 \text{ units}$$

Illustration 2, For the size of the second replenishment, the first replenishment last until time 3.87 which is now the T = 0 base for the second replenishment. The first period ending point to try is $T_1 = 0.13$ (which obtained from 4 - 3.87 = 0.13):

For $T_1 = 0.13$ $F(T_1) = 20$ $T_1^2 F(T_1) = (0.13)^2(20) < 300$

$T_2 = 1.13$ $F(T_2) = 70$ $T_2^2 F(T_2) = (1.13)^2(70) < 300$

$T_3 = 2.13$ $F(T_3) = 180$ $T_3^2 F(T_3) = (2.13)^2(180) > 300$

Therefore, T value will be in the range 1.13 to 2.13, we find

$$T^2 = \frac{300}{F(T_3)}$$

$$= \frac{300}{180} = 1.67$$

$$T = \sqrt{1.67} = 1.29$$

The amount of the replenishment is the total demanded from time 3.87 to time (3.87 + 1.29) = 5.16, namely

$$(0.13)(20) + 70 + (0.16)(180) = 102 \text{ units}$$

Following these procedures to calculate the size of the remaining replenishments, we will obtain the total output for this demand pattern as shown in Table V.

After obtaining total month-pieces per year (799 units) from the calculation shown in Table VI, we can calculate the inventory carrying cost for the whole period as follows:

Inventory carrying cost = (Total month-pieces per year) x (value per piece) x (carrying charge per month)

= 799 x $120 x (0.2/12)

= $1,598

Since there are six replenishments but the sixth last beyond the end of the year, there are 19 pieces left at the end of the year. With an annual demand rate of 1,105 units, these represent 0.017 year of supply. Hence, the annual replenishment costs are approximately:

Annual replenishment costs = (number of replenishments) x (cost per replenishment) / number of year covered

$$= \frac{6 \times \$300}{1.017}$$

= $1,770

The total inventory cost is the sum of the carrying cost and replenishment cost, i.e.,

$1,598 + $1,770 = $3,368

Comparison of the total inventory cost from each method

According to the same data and a demand pattern used in each algorithm, the total inventory cost obtained from each method can be compared as the following:

Basic EOQ	Modified EOQ	As % of Basic EOQ	Wagner-Whitin	As % of Basic EOQ
$3,989	$3,368	84.4	$3,245	81.3

TABLE V

SUMMARY RESULTS OF MODIFIED EOQ METHOD

Time	Quantity
0	52
3.87	102
5.16	216
6.26	269
7.31	289
8.45	196*

Total inventory cost = $3,368

*(Ending inventory = 19 units)

TABLE VI

DETAILS OF INVENTORY CARRYING COSTS

Interval			Inventory Level			
Start (1)	End (2)	Duration (3)=(2)-(1)	Start (4)	End (5)	Average $(6)=\frac{(4)+(5)}{2}$	Product (7)=(3)(6)
0	1.00	1.00	52	42	47	47
1.00	2.00	1.00	42	32	37	37
2.00	3.00	1.00	32	17	24.5	24.5
3.00	3.87	0.87	17	0	8.5	7.4
3.87	4.00	0.13	102	99	100.5	13.1
4.00	5.00	1.00	99	29	64	64
5.00	5.16	0.16	29	0	14.5	2.3
5.16	6.00	0.84	216	65	140.5	118.0
6.00	6.26	0.26	65	0	32.5	8.5
6.26	7.00	0.74	269	84	176.5	130.6
7.00	7.31	0.31	84	0	42	13.0
7.31	8.00	0.69	289	103	196	135.2
8.00	8.45	0.45	103	0	51.5	23.2
8.45	9.00	0.55	196	69	132.5	72.9
9.00	10.00	1.00	69	29	49	49
10.00	11.00	1.00	29	29	29	29
11.00	12.00	1.00	29	19	24	24
						798.7 ≈ 799 (month-pieces)

From the results, we can see that the basic EOQ did not provide the optimal solution when there is a substantial variation of demand requirement among time periods. Therefore, the users must realize the assumptions applied for each technique and assure that these assumptions are not violated before using a result obtained from each particular technique as a decision making for inventory control policy.

ENDNOTES

[1]William A. Rush. "Special Purpose EOQ Formulas." Journal of Purchasing, Vol. 9, May 1973, pp. 34-36.

[2]James W. Prichard and Robert H. Eagle. Modern Inventory Management (New York: John Wiley & Sons., Inc., 1965), p. 51.

[3]R. A. Kaimann. "Re-visiting a Fallacy of EOQ Ing." Production and Inventory Management, Vol. 9, November 4, pp. 12-13.

[4]Edward A. Silver and Harlan C. Meal. "A Simple Modification of the EOQ for the Case of a Varying Demand Rate." Production and Inventory Management, 4th Qtr., 1969, pp. 61-62.

CHAPTER III

CREATING SITUATION IN MILITARY LOGISTICS MANAGEMENT

Application of Each Method

In this chapter, I would like to present the situation in the Royal Thai Navy System from my experiences, and using the three techniques mentioned in chapter II applied to this situation. A primary reason of applying these techniques to this particular situation is to illustrate that why and how the better inventory control policy is one of the most important element in the military logistics system.

Before considering the details of this assuming situation, I would like to emphasize again that inventory control policy is different in some aspects between business enterprises and military system as mentioned in chapter I. Even though in the military itself, inventory control policy drastically changes from peacetime to wartime. Especially, for a wartime there are so many factors involved that the planners can hardly control and inventory policy will be dramatically changed.

In this situation, I will assume only for a peacetime senario as follows:

Suppose you were an officer who is in charge of controlling inventory policy of X-Supply Center which has to supply various unit activities in your area (shown in Figure 3). Every unit will submit demand requirements for various items required for each fiscal year to X-Supply Center through its supply unit, according to the operational plans each year.

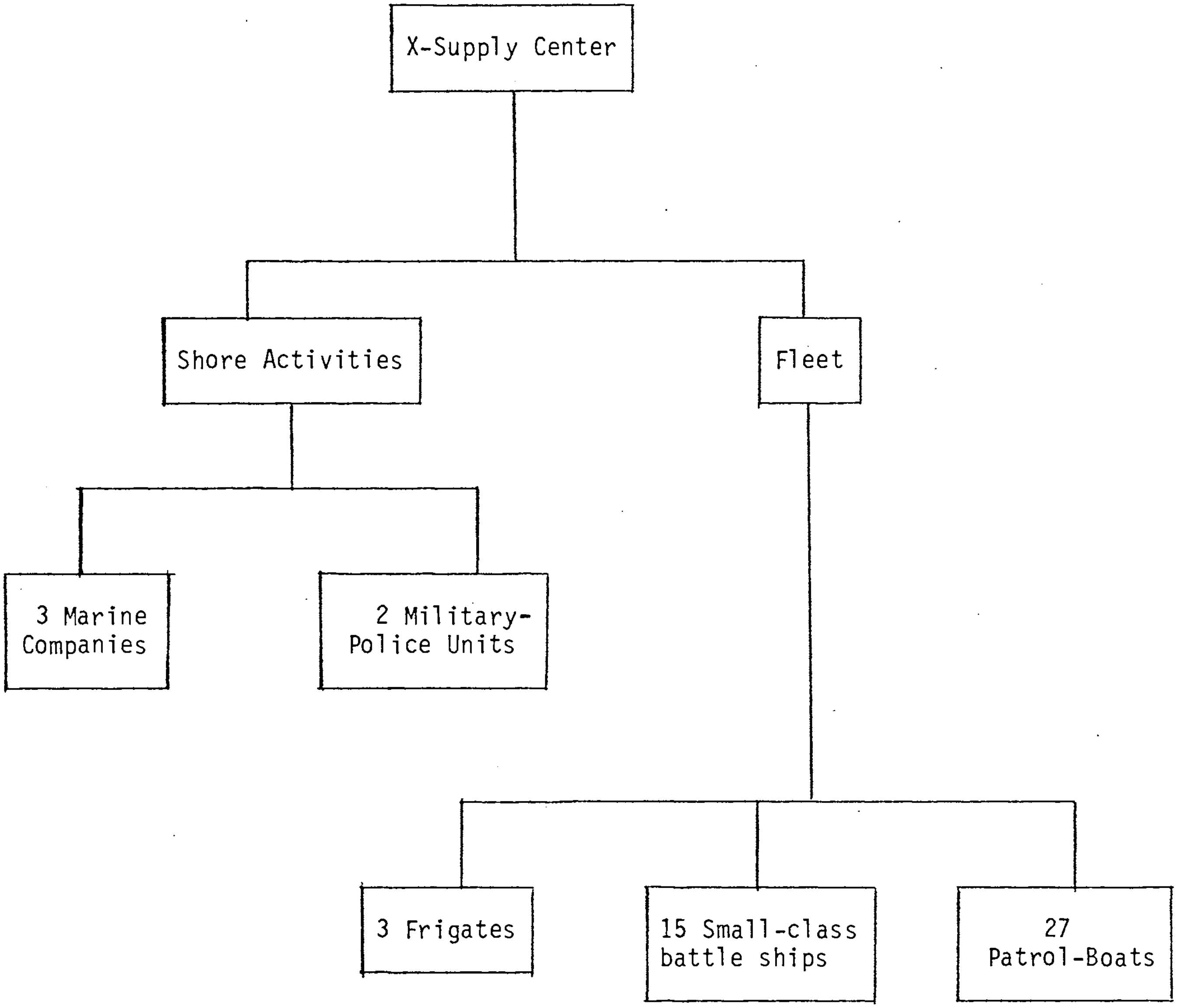

Figure 3. Supplying Areas of X-Supply Center

X-Supply Center will collect demand requirements from each unit and procure to supply these demands. However, there are some important items which have to be considered carefully in purchasing policy in order to get optimal quantity of products necessary for supplying continuous activities while simultaneously minimizing inventory carrying costs. For this example I will select a single item, such as the ammunition for 20 mm. automatic machine gun, which has a very high rate of consumption, large amount of demand rate and both ordering cost and carrying cost are substantially high. According to a particular characteristic of any item like this, inventory decision making must be prudently planned. After considering demand requirements of this item based on the operational plans, the demand pattern has been decided including the other required data in the algorithm are as follows:

Demand pattern

- 1 year plan
- demands are subdivided into a four-week period which totally are 13 periods:

Period	1	2	3	4	5	6	7	8	9	10	11	12	13
Demand	48	72	105	35	43	52	116	28	37	32	146	35	10

S = ordering cost = $5,500 per order

D = the forecasted annual demand requirement = 759 units

C = value of unit in inventory = $2,250

I = annual carrying charge rate = 22%

Applying these data into each method to calculate the purchasing pattern and total inventory cost of the ammunition for 20 mm. automatic machine gun for a one-year plan, and selected the optimal total inventory cost obtained from one of these three methods to be an inventory decision making.

Using Basic EOQ Algorithm

$$Q = \sqrt{\frac{2DS}{IC}}$$

$$Q = \sqrt{\frac{2(759)(5500)}{(0.22)(2250)}}$$

$$= \sqrt{16866.67}$$

$$= 129.87$$

$$\approx 130 \text{ units}$$

$$TC = DS/Q + IC(Q/2)$$

$$= \frac{(759)(5500)}{130} + \frac{(0.22)(2250)(130)}{2}$$

$$= \$64,286.54$$

The summary of the result of this method is shown in Table VII.

Using Wagner-Whitin Algorithm

Following the step of algorithm explained in chapter II, by using the second alternative (adding the total carrying cost for the average of total demand requirement to the unadjusted total inventory cost, previously shown in Table II), we can obtain the results from this algorithm as shown in Table VIII. And the summary of final result of Wagner-Whitin method is shown in Table IX (results also shown in Appendix B).

Using Modified EOQ Algorithm

In order to make it easier for the readers to catch up with the results of this algorithm, the numerical illustrations will be illustrated and there will be no ending inventory left at the final period to cut down the procedure of adjusting the annual replenishment cost due to the remainder of ending inventory (details in chapter II).

TABLE VII

SUMMARY RESULTS OF BASIC

EOQ METHOD

Purchase in Period	For use in Period(s)	Quantity
1	1,2	130
3	3,4	130
5	5,6	130
7	7,8	130
9	9,10	130
11	11,12,13	130*

Total inventory cost = $64,286.54

*(Ending inventory = 21 units)

TABLE VIII

DETAILS OF WAGNER WHITIN RESULTS
(EXCLUDING AVERAGE INVENTORY CHARGED IN EACH PERIOD)

Time Period	1	2	3	4	5	6	7	8	9	10	11	12	13
Demand	48	72	105	35	43	52	116	28	37	32	146	35	10
1	5500	8241.5											
2		11000	14998.1	17663.5									
3			13741.5	15074.2	18348.8								
4				19241.5	20878.8	24838.8							
5					20574.6	22554.2*							
6						23848.8	28265.8	30398.1	34624.6	39498.5			
7							29348.8	30415					
8								33765.8					
9									35898.1	37116.5*			
10										40124.6			
11											44998.5	46331.1	47092.
12												50498.5	
13													51831.
Total Cost	5500	8241.5	13741.5	15074.2	18348.8	23848.8	28265.8	30398.1	34624.6	39498.5	44998.5	46331.1	47092.

*This minimum cost can't be acceptable because it is not respected to the most recent optimal decision.

TABLE IX

SUMMARY RESULTS OF WAGNER-WHITIN METHOD

Purchase in Period	For use in Period(s)	Quantity
1	1,2	120
3	3,4,5	183
6	6,7,8,9,10	265
11	11,12,13	191*

Total inventory cost (unadjust)	$47,094.8
Add: Carrying cost for period of using	$14,451.4
Total inventory cost (actual)	$61,546.2

*(No ending inventory)

Time 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Period	1	2	3	4	5	6	7	8	9	10	11	12	13
Demand	48	72	105	35	43	52	116	28	37	32	146	35	10

Let $2S/CI = 500$

First replenishment

For $T_1 = 1$ $\quad F(T_1) = 48$ $\quad T_1^2\ F(T_1) = 48 < 500$

$T_2 = 2$ $\quad F(T_2) = 72$ $\quad T_2^2\ F(T_2) = 4(72) < 500$

$T_3 = 3$ $\quad F(T_3) = 105$ $\quad T_3^2\ F(T_3) = 9(105) > 500$

Therefore, T value is between 2 and 3

$$T^2 = 500/F(T_2)$$

$$= 500/105 = 4.76$$

$$T = \sqrt{4.76} = 2.18$$

The quantity that will last through time, 2.18, is

$$48 + 72 + 105(0.18) = 138.9$$

$$\approx 139 \text{ units}$$

Second replenishment

For $T_1 = 0.82$ $\quad F(T_1) = 105$ $\quad T_1^2F(T_1) = (0.82)^2(105) < 500$

$T_2 = 1.82$ $\quad F(T_2) = 35$ $\quad T_2^2F(T_2) = (1.82)^2(35) < 500$

$T_3 = 2.82$ $\quad F(T_3) = 43$ $\quad T_3^2F(T_3) = (2.82)^2(43) < 500$

$T_4 = 3.82$ $\quad F(T_4) = 52$ $\quad T_4^2F(T_4) = (3.82)^2(52) > 500$

T value is between 2.82 and 3.82

$$T^2 = 500/52 = 9.62$$

$$T = \sqrt{9.62} = 3.1$$

$$T(2) = 2.18 + 3.1 = 5.28$$

The quantity that will last through time 5.28, is

$$(0.82)(105) + 35 + 43 + (0.28)(52) = 178.66$$

$$\approx 179 \text{ units.}$$

Third replenishment

For $T_1 = 0.72$	$F(T_1) = 52$	$T_1^2F(T_1) = (0.72)^2(52) < 500$
$T_2 = 1.72$	$F(T_2) = 116$	$T_2^2F(T_2) = (1.72)^2(116) < 500$
$T_3 = 2.72$	$F(T_3) = 28$	$T_3^2F(T_3) = (2.72)^2(28) < 500$
$T_4 = 3.72$	$F(T_4) = 37$	$T_4^2F(T_4) = (3.72)^2(37) > 500$

T value is in the range 2.72 to 3.72

$$T^2 = 500/37 = 13.51$$

$$T = \sqrt{13.51} = 3.68$$

$$T(3) = 5.28 + 3.68 = 8.96$$

The quantity that will last through T(3) is

$$(0.72)(52) + 116 + 28 + (0.96)(37) = 216.96$$

$$\approx 217 \text{ units}$$

Fourth replenishment

For $T_1 = 0.04$	$F(T_1) = 37$	$T_1^2F(T_1) = (0.04)^2(37) < 500$
$T_2 = 1.04$	$F(T_2) = 32$	$T_2^2F(T_2) = (1.04)^2(32) < 500$
$T_3 = 2.04$	$F(T_3) = 146$	$T_3^2F(T_3) = (2.04)^2(146) > 500$

T value is in the range of 1.04 to 2.04

$$T^2 = 500/146 = 3.42$$

$$T = \sqrt{3.42} = 1.85$$

$$T(4) = 8.96 + 1.85 = 10.81$$

The quantity that will last through T(4) is

$$(0.04)(37) + 32 + (0.81)(146) = 151.74$$

$$\approx 152 \text{ units}$$

And for the fifth replenishment which is the last one of this year (with no ending inventory) is equal to 72 units. Using period-piece (958 units) obtained from Table X calculate inventory carrying cost as follows:

Inventory carrying cost = (958)($2,250)(0.22/13)

= $36,477.69

Annual replenishment cost = 5($5,500)

= $27,500

Total inventory cost = $36,477.69 + $27,500

= $63,977.69

The summary results of the modified EOQ is shown in Table XI.

The results obtained from each method can be compared as the following:

	Basic EOQ	Wagner-Whitin	Modified EOQ
Total inventory cost	$64,286.54	$61,546.2	$63,977.69

After considering the results from each algorithm, we can obviously see that when demand requirement for each period is fluctuated, Wagner-Whitin technique seems to be the better tool for handling the situation in this case.

However, computational procedures are somehwat more complicated and consumed a great deal of time when number of time periods are inerased, included more complicated in related costs figures. Therefore, using computer to calculate these desired outputs will be much more convenient.

In the following chapter a computer program for Basic EOQ and Wagner-Whitin method was developed to compute the total inventory cost and purchasing plan for a variable number, n, periods when demand of each time period is known.

TABLE X

DETAILS OF INVENTORY CARRYING COSTS

Interval			Inventory Level			
Start (1)	End (2)	Duration (3)=(2)=(1)	Start (4)	End (5)	Average $(6)=\frac{(4)+(5)}{2}$	Product (7)=(3)(6)
0	1.00	1.00	139	91	115	115
1.00	2.00	1.00	91	19	55	55
2.00	2.18	0.18	19	0	9.5	1.71
2.18	3.00	0.82	179	93	136	111.52
3.00	4.00	1.00	93	58	75.5	75.5
4.00	5.00	1.00	58	15	36.5	36.5
5.00	5.28	0.28	15	0	7.5	2.1
5.28	6.00	0.72	217	180	198.5	142.92
6.00	7.00	1.00	180	64	122	122
7.00	8.00	1.00	64	36	50	50
8.00	8.96	0.96	36	0	18	17.28
8.96	9.00	0.04	152	151	151.5	6.06
9.00	10.00	1.00	151	119	135	135
10.00	10.81	0.81	119	0	59.5	48.20
10.81	11.00	0.19	72	45	58.5	11.12
11.00	12.00	1.00	45	10	27.5	27.5
12.00	13.00	1.00	10	0	5	5
						957.41 ≈ 958 (period-pieces)

TABLE XI

SUMMARY RESULTS OF MODIFIED EOQ METHOD

Time	Quantity
0.	139
2.18	179
5.28	217
8.96	152
10.81	72*

Total inventory cost = $63,977.69

* (No ending inventory)

CHAPTER IV

A COMPUTER PROGRAM FOR BASIC EOQ AND WAGNER-WHITIN ALGORITHMS

After studying the algorithms of each method applied in case of the fluctuating demand rate among time periods as illustrated in an example in both chapter II and III, we can obviously see how effective of each method to achieve major savings (in replenishment cost and carrying cost) comparing to one another. Moreover, considering the computational effort required to compute for more and more time periods including more complicated figures of related costs, based upon the algorithms thus far, we experienced that how difficult it was to get the correct results by manual. Besides it is likely to increase the probability of making more errors in the algorithm which will lead to an unefficient inventory decision making. Consequently, this chapter will present a computer program for Basic EOQ and Wagner-Whitin method which I believe will obviously indicate a major savings of inventory costs comparing between these two methods. Nevertheless, I did not develop a computer program for Modified EOQ algorithm in this paper because I personally experienced that Wagner-Whitin method itself can be considered as one of the best techniques to cope with variations in demand rate between time periods. However, I did introduce the modified EOQ, developed by Silver and Meal earlier in this paper, to be the other alternatives in comparison because the more alternatives you have the better chances you can select. Again due to a scope of developing computer program in this paper, only a computer program for Basic EOQ and Wagner-Whitin algorithms was developed.

Details of Program

Data required in this program:

-ordering or set-up cost

-the forecast amounts of total quantity used (depended on the range of the total time periods desired (e.g. six month, one year, etc.)

-value of a unit carried in inventory

-carrying charge rate as a percentage of a unit cost carried in inventory

-number of periods desired

-maximum time periods desired for inventory to be carried.

There are two parts in the main program. The first part is for Basic EOQ algorithm and the second one is for Wagner-Whitin algorithm.

The list of variable names used in the program, details of the structure of the program, and the output required for both methods applied to an example in chapter II and chapter III are shown in Appendix A and B respectively.

CHAPTER V

CONCLUDING REMARKS

The objectives of this paper have been fulfilled by pointing out the important role of inventory control element in Logistics Management for both in business enterprise and military system, and also illustrating a more efficient method of inventory model to aid the decision maker responsible for inventory control policy to determine a purchasing plan, which achieves the objective of purchasing the optimal quantity for supplying continuous activities, and minizing the sum of inventory carrying cost and ordering cost under fluctuating demand conditions among time periods.

The computation techniques of Basic EOQ, Wagner-Whitin, and Modified EOQ have been employed in deriving an appropriate purchasing pattern with the minimum inventory costs, due to the demand pattern and other data in the examples in chapter II and III. After all, we can obviously see that under the more significant degrees of variability in the demand pattern, the less acceptable in the use of Basic EOQ, but the more accpetable in the use of dynamic programming methodology (i.e., Wagner-Whitin) or Modified EOQ. However, in other studies indicated that where variability in the demand pattern is not severe (or significantly varied) or in cases where the demand is fairly stable, the basic EOQ methodology appear to provide a solution that is either equal to or even slightly better than the dynamic programming methodology. We can also observe that Modified EOQ method could provide significant inventory cost savings in situation where the demand rate changes appreciably from period to period. In other words, Basic EOQ which has been proved useful and satisfactory in mininizing total ordering costs and inventory carrying costs under a stable demand rate, did not provide optimal solutions and even unreasonable to be used when demand rate varies significantly as we experienced from the solution of Basic EOQ in Appendix A. Therefore, the users should keep in mind the characteristics and assumptions of each method before the application.

The outputs obtained from this computer program using the assigned data provided the same solutions as that previously obtained from the calculation shown in chapter II and III, which confirmed the accuracy of this computer program, and indicated the great benefits that could be obtained from using this program.

Finally, a recommendation for further study is to consider the case of changing some related costs due to the inflation or other factors. For example, a value of unit in inventory may be changed if inventory has been carried for a long period of time, or some constraints of adjusting ordering cost etc., which require more complicated procedures. Furthermore, computer programs for other methods that can be used (e.g., Modified EOQ, etc.) should be developed in order to reassure a decision maker that a more effective inventory policy has been made.

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APPENDIX A

A COMPUTER PRINT-OUT USING DATA IN CH. II

(Assume that the number of maximum time period inventory supposed to be carried is equal to 5)

```
C
C
C    COMPUTER PROGRAM FOR BASIC E.O.Q. & WAGNER-WHITIN ALGORITHMS
C
C
C
C    LIST OF VARIABLES
C       DN - ANNUAL DEMAND   (UNITS)
C       S  - ORDERING OR SET-UP COST   ($/ORDER)
C       C  - VALUE OF A UNIT CARRIED IN INVENTORY   ($/UNIT)
C       CI - CARRYING CHARGE AS A PERCENTAGE OF C
C       QE - SIZE OF EACH ORDER TO REPLENISH INVENTORY   (UNITS)
C       TC - ANNUAL TOTAL RELEVANT INVENTORY COST OF BASIC E.O.Q.
C
C    ADDITIONAL VARIABLES FOR WAGNER-WHITIN
C       CAR - CARRYING COST OF A UNIT/PERIOD   ($)
C       N   - NUMBER OF PERIODS
C       MAX - MAXIMUM PERIOD INVENTORY ASSUMED TO BE CARRIED
C       D(J) - DEMAND REQUIRED IN PERIOD I
C       P(I,J) - INVENTORY COST OCCURED WHEN ORDER IN PERIOD I FOR PERIOD J
C       PI(J) - INVENTORY COST INCURRED UP TO PERIOD J
C       TCW - ANNUAL TOTAL INVENTORY COST OF WAGNER-WHITIN
C
C
C
C
C    BASIC E.O.Q. ALGORITHM
         DIMENSION P(20,20),D(20),PI(20),DP(20,20)
         DATA DN/1105.00/,S/300.00/,C/120.00/,CI/0.20/,N/12/,MAX/5/
         READ(5,900) (D(J),J=1,N)
         QR = SQRT((2.0*DN*S)/(CI*C))
         IC = QR+0.9
         QE = IC
         TC = (DN*S/QE)+(CI*C*(QE/2.0))
         WRITE(6,100)
         WRITE(6,200)
         DEM = 0.0
         DO 41 J = 1,N
           IF(J .EQ. 1) WRITE(6,150) J,QE
           DEM = DEM+D(J)
           IF(DEM .LT. QE) GO TO 41
           WRITE(6,150) J,QE
           DEM = DEM-QE
   41    CONTINUE
         WRITE(6,400) TC
         EI = QE-DEM
         WRITE(6,250) EI
C
C
C    WAGNER-WHITIN ALGORITHM
         CAR = CI*C/FLOAT(N)
         P(1,1) = S
         P(2,2) = P(1,1)+S
         WRITE(6,300)
         NY = N-MAX
         DO 22 I = 1,NY
           L = I+MAX
           K = I+1
           JD = I
           DP(I,JD) = D(I)
```

```
      DO 11 J = K,L
        JM = J-1
        DM = J-I
        P(I,J) = P(I,JM)+(DM*D(J)*CAR)
        DP(I,J) = DP(I,JM)+D(J)
      WRITE(6,800) I,JM,DP(I,JM),P(I,JM)
11    CONTINUE
      IM = I+1
      J = IM
      JB = IM-1
      IB = I-1
      IC = I-2
      ID = I-3
      IF(J .EQ. 2) GO TO 50
      IF(J .EQ. 3) GO TO 10
      IF(J .EQ. 4) GO TO 20
      IF(J .GE. 5) GO TO 30
10    IF(PI(JB) .EQ. P(IB,JB)) GO TO 50
      GO TO 60
20    IF(PI(JB) .EQ. P(IC,JB)) GO TO 40
      IF(PI(JB) .EQ. P(IB,JB)) GO TO 50
      IF(PI(JB) .EQ. P(I,JB))  GO TO 60
30    IF(PI(JB) .EQ. P(ID,JB)) GO TO 39
      IF(PI(JB) .EQ. P(IC,JB)) GO TO 40
      IF(PI(JB) .EQ. P(IB,JB)) GO TO 50
      IF(PI(JB) .EQ. P(I,JB))  GO TO 60
39    IF(P(ID,J) .LE. P(IM,J)) PI(J) = P(ID,J)
      IF(P(IM,J) .LE. P(ID,J)) PI(J) = P(IM,J)
      GO TO 70
40    IF(P(IC,J) .LE. P(IM,J)) PI(J) = P(IC,J)
      IF(P(IM,J) .LE. P(IC,J)) PI(J) = P(IM,J)
      GO TO 70
50    IF(P(IB,J) .LE. P(IM,J)) PI(J) = P(IB,J)
      IF(P(IM,J) .LE. P(IB,J)) PI(J) = P(IM,J)
      GO TO 70
60    IF(P(I,J) .LE. P(IM,J))  PI(J) = P(I,J)
      IF(P(IM,J) .LE. P(I,J))  PI(J) = P(IM,J)
70    IA = J+1
      JA = J+1
      P(IA,JA) = PI(J)+S
22  CONTINUE
    KA = N-(MAX-1)
    NX = N-1
    DO 44 I = KA,NX
      K = I+1
      JD = I
      DP(I,JD) = D(I)
      DO 33 J = K,N
        JM = J-1
        DM = J-I
        P(I,J) = P(I,JM)+(DM*D(J)*CAR)
        DP(I,J) = DP(I,JM)+D(J)
      WRITE(6,800) I,JM,DP(I,JM),P(I,JM)
33    CONTINUE
      WRITE(6,800) I,N,DP(I,N),P(I,N)
      IM = I+1
      J = IM
      JB = IM-1
```

```
      IF(PI(JB) .EQ. P(ID,JB)) GO TO 35
      IF(PI(JB) .EQ. P(IC,JB)) GO TO 45
      IF(PI(JB) .EQ. P(IB,JB)) GO TO 55
      IF(PI(JB) .EQ. P(I,JB))  GO TO 65
35    IF(P(ID,J) .LE. P(IM,J)) PI(J) = P(ID,J)
      IF(P(IM,J) .LE. P(ID,J)) PI(J) = P(IM,J)
      GO TO 75
45    IF(P(IC,J) .LE. P(IM,J)) PI(J) = P(IC,J)
      IF(P(IM,J) .LE. P(IC,J)) PI(J) = P(IM,J)
      GO TO 75
55    IF(P(IB,J) .LE. P(IM,J)) PI(J) = P(IB,J)
      IF(P(IM,J) .LE. P(IB,J)) PI(J) = P(IM,J)
      GO TO 75
65    IF(P(I,J) .LE. P(IM,J))  PI(J) = P(I,J)
      IF(P(IM,J) .LE. P(I,J))  PI(J) = P(IM,J)
75    IA = J+1
      JA = J+1
      P(IA,JA) = PI(J)+S
44 CONTINUE
   TCW = PI(N)+(DN/2.0)*CAR
   DP(N,N) = D(N)
   WRITE(6,300) N,N,DP(N,N),P(N,N)
   WRITE(6,700)
   PI(1) = P(1,1)
   DO 77 J = 1,N
      WRITE(6,600) J,PI(J)
77 CONTINUE
   WRITE(6,500)
   DO 88 I = 1,KA
      L = I+(MAX-1)
      DE = 0.0
      DO 66 J = I,L
         IF(P(I,J) .NE. PI(J)) GO TO 66
         DE = DE+D(J)
66    CONTINUE
      IF(DE .EQ. 0.0) GO TO 88
      WRITE(6,150) I,DE
88 CONTINUE
   KT = N-(MAX-2)
   DO 31 I = KT,N
      DE = 0.0
      DO 21 J = I,N
         IF(P(I,J) .NE. PI(J)) GO TO 21
         DE = DE+D(J)
21    CONTINUE
      IF(DE .EQ. 0.0) GO TO 31
      WRITE(6,150) I,DE
31 CONTINUE
   WRITE(6,400) TCW
   WRITE(6,350)
100 FORMAT(1H1,15X,'*** COMPARISON OF PURCHASING PATTERN & TOTAL INVEN
   $TORY COST ***'///)
150 FORMAT(1H0,24X,I2,18X,F9.2)
200 FORMAT(1H0,9X,'** BASIC E.O.Q. **'///12X,'* PURCHASING PATTERN *'/
   $//17X,'PURCHASE IN PERIOD',12X,'QUANTITY')
250 FORMAT(1H0/18X,'* ENDING INVENTORY',F9.2,'   UNITS *')
300 FORMAT(1H1,10X,'* WAGNER-WHITIN *'///9X,'INVENTORY COST FOR EACH P
   $ERIOD'///17X,'PURCHASE IN PERIOD',5X,'FOR PERIODS',5X,
```

```
      350 FORMAT(1H1,5X)
      400 FORMAT(1H0/15X,'** TOTAL INVENTORY COST          $',F9.2,'    **')
      500 FORMAT(1H1,15X,'*** SUMMARY FOR WAGNER-WHITIN RESULTS ***'///10X,
         $'* PURCHASING PATTEFN *'///17X,'PURCHASE IN PERICD',
         $12X,'QUANTITY')
      600 FORMAT(1H0,24X,I2,12X,'$',F9.2)
      700 FORMAT(1H0///9X,'SELECTED OPTIMAL INVENTORY COST FOR PURCHASING P
         $ATTERN'///23X,'PERIOD',8X,'INVENTORY COST')
      800 FORMAT(1H0,24X,I2,18X,I2,7X,F9.2,8X,'$',F9.2)
      900 FORMAT(12F6.2)
          STOP
          END

  $ENTRY
```

*** COMPARISON OF PURCHASING PATTERN & TOTAL INVENTORY COST ***

** BASIC E.O.Q. **

* PURCHASING PATTERN *

PURCHASE IN PERIOD	QUANTITY
1	167.00
6	167.00
7	167.00
8	167.00
9	167.00
10	167.00
11	167.00

** TOTAL INVENTORY COST $ 3989.03 **

* ENDING INVENTORY 64.00 UNITS *

INVENTORY COST FOR EACH PERIOD

PURCHASE IN PERIOD	FOR PERIODS	QUANTITY	INVENTORY COST
1	1	10.00	$ 300.00
1	2	20.00	$ 320.00
1	3	35.00	$ 380.00
1	4	55.00	$ 500.00
1	5	125.00	$ 1080.00
2	2	10.00	$ 600.00
2	3	25.00	$ 630.00
2	4	45.00	$ 710.00
2	5	115.00	$ 1130.00
2	6	295.00	$ 2570.00
3	3	15.00	$ 620.00
3	4	35.00	$ 660.00
3	5	105.00	$ 940.00
3	6	285.00	$ 2020.00
3	7	535.00	$ 4020.00
4	4	20.00	$ 680.00
4	5	90.00	$ 820.00
4	6	270.00	$ 1540.00
4	7	520.00	$ 3040.00
4	8	790.00	$ 5199.99
5	5	70.00	$ 800.00
5	6	250.00	$ 1160.00
5	7	500.00	$ 2160.00
5	8	770.00	$ 3780.00
5	9	1000.00	$ 5619.99
6	6	180.00	$ 1100.00

6	8	700.00	$ 2680.00
6	9	930.00	$ 4060.00
6	10	970.00	$ 4379.99
7	7	250.00	$ 1400.00
7	8	520.00	$ 1940.00
7	9	750.00	$ 2860.00
7	10	790.00	$ 3100.00
7	11	790.00	$ 3100.00
8	8	270.00	$ 1700.00
8	9	500.00	$ 2160.00
8	10	540.00	$ 2320.00
8	11	540.00	$ 2320.00
8	12	550.00	$ 2400.00
9	9	230.00	$ 2000.00
9	10	270.00	$ 2080.00
9	11	270.00	$ 2080.00
9	12	280.00	$ 2140.00
10	10	40.00	$ 2300.00
10	11	40.00	$ 2300.00
10	12	50.00	$ 2340.00
11	11	0.00	$ 2380.00
11	12	10.00	$ 2400.00
12	12	10.00	$ 2380.00

SELECTED OPTIMAL INVENTORY COST FOR PURCHASING PATTERN

PERIOD	INVENTORY COST
1	$ 300.00

4	$ 500.00
5	$ 800.00
6	$ 1100.00
7	$ 1400.00
8	$ 1700.00
9	$ 2000.00
10	$ 2080.00
11	$ 2080.00
12	$ 2140.00

*** SUMMARY FOR WAGNER-WHITIN RESULTS ***

* PURCHASING PATTERN *

PURCHASE IN PERIOD	QUANTITY
1	55.00
5	70.00
6	180.00
7	250.00
8	270.00
9	280.00

** TOTAL INVENTORY COST $ 3245.00 **

APPENDIX B

A COMPUTER PRINT-OUT USING DATA IN CH. III

(Assumed that the number of maximum time period inventory supposed to be carried is equal to 5)

```
C
C
C   COMPUTER PROGRAM FOR BASIC E.O.Q. & WAGNER-WHITIN ALGORITHMS
C
C
C
C   LIST OF VARIABLES
C     DN - ANNUAL DEMAND   (UNITS)
C     S  - ORDERING OR SET-UP COST   ($/ORDER)
C     C  - VALUE OF A UNIT CARRIED IN INVENTORY   ($/UNIT)
C     CI - CARRYING CHARGE AS A PERCENTAGE OF C
C     QE - SIZE OF EACH ORDER TO REPLENISH INVENTORY   (UNITS)
C     TC - ANNUAL TOTAL RELEVANT INVENTORY COST OF BASIC E.O.Q.
C
C   ADDITIONAL VARIABLES FOR WAGNER-WHITIN
C     CAR - CARRYING COST OF A UNIT/PERIOD   ($)
C     N   - NUMBER OF PERIODS
C     MAX - MAXIMUM PERIOD INVENTORY ASSUMED TO BE CARRIED
C     D(J) - DEMAND REQUIRED IN PERIOD I
C     P(I,J) - INVENTORY COST OCCURED WHEN ORDER IN PERIOD I FOR PERIOD J
C     PI(J) - INVENTORY COST INCURRED UP TO PERIOD J
C     TCW - ANNUAL TOTAL INVENTORY COST OF WAGNER-WHITIN
C
C
C
C   BASIC E.O.Q. ALGORITHM
      DIMENSION P(20,20),D(20),PI(20),DP(20,20)
      DATA DN/759.00/,S/5500.00/,C/2250.00/,CI/0.22/,N/13/,MAX/5/
      READ(5,900) (D(J),J=1,N)
      QR = SQRT((2.0*DN*S)/(CI*C))
      IQ = QR+0.9
      QE = IQ
      TC = (DN*S/QE)+(CI*C*(QE/2.0))
      WRITE(6,100)
      WRITE(6,200)
      DEM = 0.0
      DO 41 J = 1,N
        IF(J .EQ. 1) WRITE(6,150) J,QE
        DEM = DEM+D(J)
        IF(DEM .LT. QE) GO TO 41
        WRITE(6,150) J,QE
        DEM = DEM-QE
   41 CONTINUE
      WRITE(6,400) TC
      EI = QE-DEM
      WRITE(6,250) EI
C
C
C   WAGNER-WHITIN ALGORITHM
      CAR = CI*C/FLOAT(N)
      P(1,1) = S
      P(2,2) = P(1,1)+S
      WRITE(6,300)
      NY = N-MAX
      DO 22 I = 1,NY
        L = I+MAX
        K = I+1
        JO = I
```

```
            JM = J-1
            DM = J-I
            P(I,J)  = P(I,JM)+(DM*D(J)*CAR)
            DP(I,J)  = DP(I,JM)+D(J)
          WRITE(6,800)  I,JM,DP(I,JM),P(I,JM)
   11     CONTINUE
          IM  =  I+1
          J  =  IM
          JB  =  IM-1
          IB  =  I-1
          IC  =  I-2
          ID  =  I-3
          IF(J  .EQ.  2)  GO  TO  60
          IF(J  .EQ.  3)  GO  TO  10
          IF(J  .EQ.  4)  GO  TO  20
          IF(J  .GE.  5)  GO  TO  30
   10     IF(PI(JB)  .EQ.  P(IB,JB))  GO  TO  50
          GO  TO  60
   20     IF(PI(JB)  .EQ.  P(IC,JB))  GO  TO  40
          IF(PI(JB)  .EQ.  P(IB,JB))  GO  TO  50
          IF(PI(JB)  .EQ.  P(I,JB))   GO  TO  60
   30     IF(PI(JB)  .EQ.  P(ID,JB))  GO  TO  39
          IF(PI(JB)  .EQ.  P(IC,JB))  GO  TO  40
          IF(PI(JB)  .EQ.  P(IB,JB))  GO  TO  50
          IF(PI(JB)  .EQ.  P(I,JB))   GO  TO  60
   39     IF(P(ID,J)  .LE.  P(IM,J))  PI(J)  =  P(ID,J)
          IF(P(IM,J)  .LE.  P(ID,J))  PI(J)  =  P(IM,J)
          GO  TO  70
   40     IF(P(IC,J)  .LE.  P(IM,J))  PI(J)  =  P(IC,J)
          IF(P(IM,J)  .LE.  P(IC,J))  PI(J)  =  P(IM,J)
          GO  TO  70
   50     IF(P(IB,J)  .LE.  P(IM,J))  PI(J)  =  P(IB,J)
          IF(P(IM,J)  .LE.  P(IB,J))  PI(J)  =  P(IM,J)
          GO  TO  70
   60     IF(P(I,J)  .LE.  P(IM,J))   PI(J)  =  P(I,J)
          IF(P(IM,J)  .LE.  P(I,J))   PI(J)  =  P(IM,J)
   70     IA  =  J+1
          JA  =  J+1
          P(IA,JA)  =  PI(J)+S
   22  CONTINUE
       KA  =  N-(MAX-1)
       NX  =  N-1
       DO 44  I  =  KA,NX
          K  =  I+1
          JD  =  I
          DP(I,JD)  =  D(I)
          DO  33  J  =  K,N
            JM  =  J-1
            DM  =  J-I
            P(I,J)  =  P(I,JM)+(DM*D(J)*CAR)
            DP(I,J)  =  DP(I,JM)+D(J)
          WRITE(6,800)  I,JM,DP(I,JM),P(I,JM)
   33     CONTINUE
          WRITE(6,800)  I,N,DP(I,N),P(I,N)
          IM  =  I+1
          J  =  IM
          JB  =  IM-1
          IB  =  I-1
```

```
      IF(PI(JB) .EQ. P(ID,JB))  GO TO 35
      IF(PI(JB) .EQ. P(IC,JB))  GO TO 45
      IF(PI(JB) .EQ. P(IB,JB))  GO TO 55
      IF(PI(JB) .EQ. P(I,JB))   GO TO 65
35    IF(P(ID,J) .LE. P(IM,J)) PI(J) = P(ID,J)
      IF(P(IM,J) .LE. P(ID,J)) PI(J) = P(IM,J)
      GO TO 75
45    IF(P(IC,J) .LE. P(IM,J)) PI(J) = P(IC,J)
      IF(P(IM,J) .LE. P(IC,J)) PI(J) = P(IM,J)
      GO TO 75
55    IF(P(IB,J) .LE. P(IM,J)) PI(J) = P(IB,J)
      IF(P(IM,J) .LE. P(IB,J)) PI(J) = P(IM,J)
      GO TO 75
65    IF(P(I,J) .LE. P(IM,J))  PI(J) = P(I,J)
      IF(P(IM,J) .LE. P(I,J))  PI(J) = P(IM,J)
75    IA = J+1
      JA = J+1
      P(IA,JA) = PI(J)+S
   44 CONTINUE
      TCW = PI(N)+(DN/2.0)*CAR
      DP(N,N) = D(N)
      WRITE(6,800) N,N,DP(N,N),P(N,N)
      WRITE(6,700)
      PI(1) = P(1,1)
      DO 77 J = 1,N
        WRITE(6,600) J,PI(J)
   77 CONTINUE
      WRITE(6,500)
      DO 88 I = 1,KA
        L = I+(MAX-1)
        DE = 0.0
        DO 66 J = I,L
          IF(P(I,J) .NE. PI(J)) GO TO 66
          DE = DE+D(J)
66      CONTINUE
        IF(DE .EQ. 0.0) GO TO 88
        WRITE(6,150) I,DE
   88 CONTINUE
      KT = N-(MAX-2)
      DO 31 I = KT,N
        DE = 0.0
        DO 21 J = I,N
          IF(P(I,J) .NE. PI(J)) GO TO 21
          DE = DE+D(J)
21      CONTINUE
        IF(DE .EQ. 0.0) GO TO 31
        WRITE(6,150) I,DE
   31 CONTINUE
      WRITE(6,400) TCW
      WRITE(6,350)
  100 FORMAT(1H1,15X,'*** COMPARISON OF PURCHASING PATTERN & TOTAL INVEN
     $TORY COST ***'///)
  150 FORMAT(1H0,24X,I2,18X,F9.2)
  200 FORMAT(1H0,9X,'** BASIC E.O.Q. **'///12X,'* PURCHASING PATTERN *'/
     $//17X,'PURCHASE IN PERIOD',12X,'QUANTITY')
  250 FORMAT(1H0/18X,'* ENDING INVENTORY',F9.2,'   UNITS *')
  300 FORMAT(1H1,10X,'* WAGNER-WHITIN *'///9X,'INVENTORY COST FOR EACH P
     $ERIOD'///17X,'PURCHASE IN PERIOD',5X,'FOR PERIODS',5X,
```

```
  350 FORMAT(1H1,5X)
  400 FORMAT(1H0/15X,'** TOTAL INVENTORY COST          $',F9.2,'    **')
  500 FORMAT(1H1,15X,'*** SUMMARY FOR WAGNER-WHITIN RESULTS ***'///10X,
     $'* PURCHASING PATTERN *'///17X,'PURCHASE IN PERIOD',
     $12X,'QUANTITY')
  600 FORMAT(1H0,24X,I2,12X,'$',F9.2)
  700 FORMAT(1H0///9X,'SELECTED OPTIMAL INVENTORY COST FOR PURCHASING P
     $ATTERN'///23X,'PERIOD',8X,'INVENTORY COST')
  800 FORMAT(1H0,24X,I2,18X,I2,7X,F9.2,8X,'$',F9.2)
  900 FORMAT(13F6.2)
      STOP
      END

$ENTRY
```

** BASIC E.O.Q. **

* PURCHASING PATTERN *

PURCHASE IN PERIOD	QUANTITY
1	130.00
3	130.00
4	130.00
7	130.00
9	130.00
11	130.00

** TOTAL INVENTORY COST $ 64286.54 **

* ENDING INVENTORY 21.00 UNITS *

INVENTORY COST FOR EACH PERIOD

PURCHASE IN PERIOD	FOR PERIODS	QUANTITY	INVENTORY COST
1	1	48.00	$ 5500.00
1	2	120.00	$ 8241.54
1	3	225.00	$ 16237.69
1	4	260.00	$ 20235.76
1	5	303.00	$ 26784.99
2	2	72.00	$ 11000.00
2	3	177.00	$ 14998.07
2	4	212.00	$ 17663.46
2	5	255.00	$ 22575.38
2	6	307.00	$ 30495.38
3	3	105.00	$ 13741.54
3	4	140.00	$ 15074.23
3	5	183.00	$ 18343.84
3	6	235.00	$ 24288.84
3	7	351.00	$ 41956.52
4	4	35.00	$ 19241.54
4	5	78.00	$ 20878.84
4	6	130.00	$ 24838.84
4	7	246.00	$ 38089.60
4	8	274.00	$ 42354.21
5	5	43.00	$ 20574.23
5	6	95.00	$ 22554.22
5	7	211.00	$ 31388.07
5	8	239.00	$ 34586.53
5	9	276.00	$ 40221.91
6	6	52.00	$ 23848.84

6	7	168.00	$ 28265.76
6	8	196.00	$ 30398.07
6	9	233.00	$ 34624.60
6	10	265.00	$ 39498.45
7	7	116.00	$ 29348.84
7	8	144.00	$ 30414.99
7	9	181.00	$ 33232.68
7	10	213.00	$ 36888.07
7	11	359.00	$ 59124.98
8	8	23.00	$ 33765.76
8	9	65.00	$ 35174.61
8	10	97.00	$ 37611.53
8	11	243.00	$ 54289.21
8	12	278.00	$ 59619.98
9	9	37.00	$ 35898.07
9	10	69.00	$ 37116.53
9	11	215.00	$ 48234.98
9	12	250.00	$ 52233.06
9	13	260.00	$ 53756.13
10	10	32.00	$ 40124.60
10	11	178.00	$ 45683.83
10	12	213.00	$ 48349.21
10	13	223.00	$ 49491.52
11	11	146.00	$ 44998.45
11	12	181.00	$ 46331.14
11	13	191.00	$ 47092.67
12	12	35.00	$ 50498.45
12	13	45.00	$ 50879.21
13	13	10.00	$ 51831.14

SELECTED OPTIMAL INVENTORY COST FOR PURCHASING PATTERN

PERIOD	INVENTORY COST
1	$ 5500.00
2	$ 8241.54
3	$ 13741.54
4	$ 15074.23
5	$ 18348.84
6	$ 23848.84
7	$ 28265.76
8	$ 30398.07
9	$ 34624.60
10	$ 39498.45
11	$ 44998.45
12	$ 46331.14
13	$ 47092.67

*** SUMMARY FOR WAGNER-WHITIN RESULTS ***

* PURCHASING PATTERN *

PURCHASE IN PERIOD	QUANTITY
1	120.00
3	183.00
6	265.00
11	191.00

** TOTAL INVENTORY COST $ 61542.86 **

VITA

Wittaya Wongtirdtam

Candidate for the Degree of

Master of Business Administration

Report: A COMPARISON OF INVENTORY COSTS IN LOGISTICS MANAGEMENT UNDER FLUCTUATING DEMAND CONDITIONS

Major Field: Management

Biographical:

Personal Data: Born in Samutsongkram, Thailand, December 27, 1954, the eldest son of Tawat and Neeyom Wongtirdtam.

Education: Graduated from Armed Forces Academy Preparatory School, Bangkok, Thailand, April, 1972; received the Bachelor of Science degree from Royal Thai Naval Academy, Samutprakan, Thailand, February, 1977; completed requirements for the Master of Business Administration degree at Oklahoma State University, December, 1981.

Professional Experience: Controlling officer attached to the Dockyard Department, Royal Thai Navy; Planning officer in Data Processing Division, Comptroller Department, Royal Thai Navy.