

LENGTH OF SAMPLE PERIOD AND OPTIMAL PORTFOLIOS

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1977

Submitted to the Graduate Faculty of the
College of Business Administration
Oklahoma State University
in partial fulfillment of
the requirements for the Degree of
MASTERS OF BUSINESS ADMINISTRATION
July, 1981

ABSTRACT

ame: Chiang-Chien Ho Date of Degree: July, 1981
nstitution: Oklahoma State University
ocation: Stillwater, Oklahoma
itle of Study: LENGTH OF SAMPLE PERIOD AND OPTIMAL
PORTFOLIOS.

ages in Study: 43 Candidate for Degree of Masters
of Business Administration

ajor Field: Finance

cope and Method of Study: This study is designed to examine the effect of length of sample period on the selection of optimal portfolios which are derived by using the historical records as the sources of data. The major data are the S&P(500) Index and the prices of twenty-five securities which are selected from the 500 Largest Industries in 1980 ranked by FORTUNE. The sample periods in this study range from two years (1979-1980) to twenty years (1961-1980). The major models for the selection of optimal portfolios are the single index model and the constant correlation coefficient model. Also, the regression analysis methods are used for determining the significance of time effects on the selection of optimal portfolios.

indings and Conclusions: In general, there is no relationship between the length of the sample period and estimated security characteristics under the single index model and the constant correlation coefficient model. Moreover, the length of the sample period does not affect the composition of optimal portfolio.

ADVISER'S APPROVAL

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LENGTH OF SAMPLE PERIOD AND OPTIMAL PORTFOLIOS

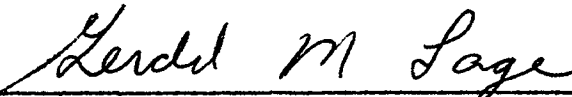
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* SIM=Single Index Model SSA=Short Selling Allowed
CCCM=Const. Corr. Coef. Model SSNA=Short Selling Not
Allowed

CHAPTER I

INTRODUCTION

A portfolio is a set of securities that belong to an investor. The investor's goal is to obtain the highest return for a given level of risk. He tries to accomplish this goal by using the tools of portfolio management. Portfolio management consists of the following steps (see, e.g., Smith 1)):

1. Portfolio planning, which includes specifications of the investor's wealth and attitudes toward risks and the establishment of investment criteria;
2. Investment analysis, which includes economic, industrial, and security analysis;
3. Portfolio selection, which includes selection models and criteria to determine the optimal portfolio;
4. Portfolio evaluation, which includes performance measurement and performance comparison;
5. Revision of portfolio.

This study will concern the problems associated with the third of these steps--portfolio selection.

Modern portfolio-selection theory dates from Marko-

Markowitz's (2)(3) pioneering articles published in 1952 and his subsequent books. Expanded from Markowitz's theory, a large amount of theories and models were developed. But almost every approach to portfolio selection utilizes, more or less, the historical records of stock prices and dividends as the bases for forecasts. These historical data thus become the important sources of information in the portfolio-selection process. But there are some problems involved when an analyst is trying to apply the modern portfolio theory for the selection of optimal portfolios. The major problems mentioned by Elton, Gruber and Manfred (4)(5) are:

1. The difficulty in accurately estimating the types of input data necessary;
2. The time and cost necessary to generate efficient portfolios (solve the quadratic-programming problems); and
3. When analysts use the historical records such as stock prices and market indexes to determine the optimal portfolios under different models, the proper length of the sample period is hard to determine because its effect on the optimal portfolio is unknown and may lead to erroneous results.

This study focusses on the third problem--that is, the effect of the sample period on the selection of optimal portfolios. The basic data required for this study are stock prices of securities and Standard and Poor's(500) Index from

NYSE Daily Stock Prices Report. First, we use random-sampling procedures to select twenty five companies from the 500 Largest Industries in 1980 ranked by FORTUNE (6) and assume that they are the securities included in the optimal portfolios. In order to get more data to test the effects of sample period on the selection of optimal portfolios, we estimate the variables such as mean return, standard deviation, etc. of the twenty five selected securities in different sample periods which range from two years (1979-1980) to twenty years (1961-1980). Then under two different models with the option of short selling, we derive a set of optimal portfolios in each sample period. At the end of this study, we use the regression analysis methods to examine the relationship between length of sample period and optimal portfolio and the significance of this relationship.

The main models applied for the selection of optimal portfolios are the single index model developed by Sharpe (7) and the constant correlation coefficient model developed by Elton and Gruber (5). In both models, the simple criterion developed by Mao (8) is also used to decide which securities should be included in the optimal portfolios with the option of short selling. And the conditions applied for determining the optimality of portfolios were developed by Lintner (9) and Kuhn-Tucker (10).

The reasons to apply the single index model and the

constant correlation coefficient model for the selection of optimal portfolios are: 1) The types of input data are easy to determine; 2) All the necessary variables can be easily computed by the SAS (Statistical Analysis System) package; 3) These two models can provide more accurate results than those from the linear programming approximations and those from the quadratic programming models.

This study will begin in Chapter II with a review of literature concerned with portfolio selection. Chapter III will discuss the methodology utilized in the examination of effect of length of sample period on the selection of optimal portfolios. The results will be covered in Chapter IV. Chapter V will show the conclusions of this study.

CHAPTER II

REVIEW OF LITERATURE

Risk is "the uncertainty of future outcomes" or "the probability of an adverse outcome." Almost any kind of investment involves different degrees of risk. One basic assumption of portfolio theory is that any investor wishes to maximize the returns from his investments. In order to adequately deal with such an assumption, certain ground rules must be laid. The first of these is that the portfolio being considered by an individual should include all of his assets and liabilities. Not just stocks or even just marketable securities, but also such items as cars, houses, coins, etc., should be included. We also normally assume that investors are risk averse, and it appears to be a reasonably accurate generalization. Any rational investor would prefer a higher return to a lesser return; unfortunately, a higher return normally involves a higher degree of risk and, as a result, an investor is continually faced with a compromise. Therefore, the continuous decision-making to derive the "optimal" trade-off between the expected return and expected risk become the "core" of the portfolio theory.

The pioneering article on portfolio selection was that of Markowitz (3), who provided the basic theoretical framework for the subsequent developments in portfolio-selection theory. Undoubtedly, his work gave us an "insight" and promoted later studies in this field. Therefore, we will use some space here to introduce Markowitz's model. Markowitz's model is based on several assumptions regarding investor behavior:

1. Investors consider each investment alternative as being represented by a probability distribution of expected returns over several holding periods.
2. Individuals estimate risk on the basis of the variability of expected returns.
3. Investors base decisions solely on expected return and risk, i.e., their utility curves are a function of expected return and variance (or standard deviation) of returns only.
4. For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk.

Under above assumptions, a single asset or portfolio of assets is considered to be "efficient" if no other assets or portfolio of assets offers higher expected return with the same (or lower) risk or lower risk with the same (or higher) expected return. In order to derive the set of ef-

efficient portfolios, he developed the following formulae:

$$E = \sum_{i=1}^n X_i R_i \qquad \sum_{i=1}^n X_i = 1$$

$$V = \sum_{i=1}^n \sum_{j=1}^n X_i X_j \sigma_{ij}$$

where

σ_{ij} = correlation coefficient of security i, j

X_i = relative amount invested in security i

E = expected return from portfolio

V = variance of portfolio

R_i = expected return on security i

The maximum return portfolio and minimum variance portfolio became the "end points" of the efficient frontier. The optimal portfolio is the efficient portfolio with the highest utility. This will be found at the point of tangency between the efficient frontier and the curve with the highest possible utility for a given investor.

Martin (11), basen on Markowitz's E-V model, developed the quadratic programming for the portfolio selection. In his study, he used a real-world investment problem to formulate his model:

$$\text{Min}_{X_i} \left[\theta = V + \lambda_1 \left(\sum_{i=1}^n X_i R_i - E \right) + \lambda_2 \left(\sum_{i=1}^n X_i - 1 \right) \right]$$

Then did the partial derivatives $\partial \theta / \partial X_i$ and solved the e-

quations with $E = \sum_{i=1}^n X_i R_i$ and $\sum_{i=1}^n X_i = 1$. At the practical level, the formulation of precise probability beliefs about securities under consideration entails problems. Almost, any real-world applications of this theory would include a large number of securities. The cost of necessary clerical, processing, and analytical activities required in such an undertaking would preclude individuals and even large institutional investors from using this model.

There is little question that the most significant and most popular developments in portfolio-selection theory since the Markowitz's mean-variance approach have been the diagonal model (also called single index model) introduced by Sharpe (7). The major characteristic of the diagonal model is the assumption that the returns of various securities are related only through common relationships with a basic underlying factor. Sharpe proposed the following model of the return from a risky security:

$$R_i = A_i + B_i + C_i \quad (i = 1, \dots, n)$$

$$I = A_{n+1} + C_{n+1}$$

where R_i is the return on a risky security i , A_{n+1} and the A_i are constants, and C_{n+1} and C_i are random variables with expected values of zero and variances Q_{n+1} and Q_i , respectively, and the covariances between C_i and C_j are zero for all values of i and j ($i \neq j$).

Helliwell and Mao (12) discussed the problems about building the simple criterion for selection of optimal portfolios. Before them, Latane and Young (13) tested four criteria: 1) the mean of all portfolios, 2) market elasticity, 3) pure-risk yield, and 4) expected value of security as the ranking rules to find out the optimal portfolios. Latane and Young's conclusion solved the problem: If one's optimal portfolio does not include all available securities, how many securities should it include? Then, Evans and Archer (14), using the method of simulation, solved the problem: Given that N securities are in the optimal portfolio, which n of the available securities are to be included?

Finally, in 1970, Mao (8) assumed that the pairwise correlation coefficient of all securities was a constant and used Lintner's (9) conclusions as the conditions of optimality of portfolios to form a formula for the calculation of the number of securities to be included in the optimal portfolio. He also suggested a simple criterion-- μ_i/b_i -- for electing the proper securities into optimal portfolios where μ_i is the expected return of security i and b_i (Beta) is an index of the nondiversifiable risk of security i .

Another important study is from Treynor and Black (15) the main viewpoint in their study is that of an individual investor who is attempting to trade profitably on the difference between his expectations and those of a monolithic

arket so large in relation to his own trading that market prices are unaffected by it. They also ignored the costs of buying and selling so that they could treat the portfolio selection problem as a single-period problem (implicitly assuming a one-period utility function as given), in the tradition of Markowitz, Sharpe, and others. The conclusions of their study are abridged as follows:

1. It is useful in balancing portfolios to distinguish between two sources of risk: market, or systematic risk on the one hand, and appraisal, or insurable risk on the other. In general, it is not correct to assume that optimal balancing leads either to negligible levels of appraisal risk or to negligible levels of market risk.
2. The overall portfolio can usually be improved by taking a long or short position in the market as a whole.
3. The rate at which the portfolio earns risk premium depends only on the total amount of market risk undertaken and is independent of the size of the investor's equity and of the composition of his active portfolio.
4. Optimal selection in the active portfolio depends only on appraisal risk and appraisal premiums.
5. The appraisal ratio depends only on 1) the quality of security analysis and 2) how efficiently the active portfolio is balanced.

Another topic discussed in Lintner's (9) and Kuhn-

ucker's (10) studies is the condition to deal with the problem about short selling in the selection of optimal portfolio. From their studies, we obtain the useful condition which can be applied in this study.

The studies which are summarized in the preceding paragraphs are the major references for this study. All the above studies are the important articles in the field of portfolio selection. In addition the others are:

1. Evans' study (16) to discuss the comparison between the Fixed-Investment-Proportion-Maintenance (FIRM) strategy and Buy-and-Hold (B&H) strategy for portfolio management.

2. Fama's (17) Mean-Semivariance (E-S) approach for the selection of portfolio.

3. Baumol (18) suggested the Expected-Gain-Confidence Limit (E-L) Criterion for the selection of portfolio.

4. Roy (19) suggested the "Safety First" theory for the portfolio management.

5. Jean (20) developed the Multidimensional-Portfolio-analysis techniques for the selection of optimal portfolio.

In the next chapter, we will introduce the methodology of this study.

CHAPTER III

METHODOLOGY

(A) Sample and Data

The sample of securities comes from the 500 Largest Industrials ranked by sales in 1980 by FORTUNE. The major sources of data are from the monthly stock prices of the twenty five selected securities and the market index which, in this study, is the Standard and Poor's(500) Index. The monthly stock prices and index are drawn from the close prices and the average S&P(500) index on the last trading day of New York Stock Exchange (NYSE) in a month.

In order to examine the effect of length of sample period on the selection of optimal portfolios, different sample periods are taken. Table-1 shows the fourteen sample periods.

Based on these stock prices and S&P(500) indexes in each period, we can derive all the estimated variables applied in the single index model and the constant correlation efficient model for the selection of optimal portfolios. The variables are mean return (expected return) on security, standard deviation of return, beta coefficient, and residual

TABLE 1

Sample Periods

No.	Period	No. of Months
1	1979 - 1980	24
2	1978 - 1980	36
3	1977 - 1980	48
4	1976 - 1980	60
5	1975 - 1980	72
6	1974 - 1980	84
7	1973 - 1980	96
8	1972 - 1980	108
9	1971 - 1980	120
10	1969 - 1980	144
11	1967 - 1980	168
12	1965 - 1980	192
13	1963 - 1980	216
14	1961 - 1980	240

error (residual risk) from the regression line which describes the relationship between security and market index. All the computations of estimated variances of each stock have been derived with the use of the computer package--SAS.

Throughout, all the figures are on the monthly basis and we will assume the existence of a riskless asset. This implies that the separation theorem holds and that the investor should maximize the ratio-excess return on a portfolio divided by the standard deviation of the portfolio. Also, throughout this paper we will make the blanket assumption that there is at least one security in the set of all investment opportunities whose expected return is strictly greater than the return on the riskless asset.

(B) The Single Index Model and the Construction of Optimal Portfolios

(i) The Standard Single Index Model

First, we shall assume that the standard single index model is an accurate description of reality. That is

1. $R_i = \alpha_i + \beta_i I + \epsilon_i$
2. $I = A_{n+1} + \epsilon_{n+1}$
3. $E(\epsilon_{n+1} \epsilon_i) = 0 \quad i = 1, \dots, n;$
4. $E(\epsilon_i \epsilon_j) = 0. \quad i = 1, \dots, n; j = 1, \dots, n; i \neq j.$

here R_i = the return on security i

I = a market index

α_i = the return on security i that is independent of changes in the market index

β_i = a measure of the responsiveness of security i to changes in the market index (beta)

ϵ_i = variable with a mean of zero and variance (residual risk)

σ_m^2 = the variance of the market index

The last two equations characterize the approximation of the standard single index model to the variance-covariance structure. The assumption implied by these equations is that the only joint movement between securities comes about because of a common response to a market index.

(ii) The Optimal Portfolio with Short Selling

The optimal portfolio in the single index model is the portfolio with the highest excess return to standard deviation (θ). That is

$$\text{Max } \theta = \frac{\bar{R}_p - R_f}{\sigma_p}$$

$$\text{d } \bar{R}_p = \sum_{i=1}^n X_i (\bar{R}_i - R_f) + R_f$$

$$\sigma_p = \left(\sum_{i=1}^n X_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^n \sum_{j=1}^n X_i X_j \beta_i \beta_j + \sum_{i=1}^n X_i^2 \sigma_{\epsilon_i}^2 \right)^{1/2}$$

here R_f = the riskless lending-borrowing rate

X_i = the relative weight we place on each security (
 $X_i > 0$ for a long position, $X_i < 0$ for a short position)

R_p = return of the portfolio (\bar{R}_p is the expected value of R_p)

σ_p = the standard deviation of the return on the portfolio.

About the way to treat short sellings, we are following Lintner's (7) suggestion. This is that the short seller pays any dividends which accrue to the person who lends the stock to him and gets a capital gain (or loss) which is the negative of any price appreciation. In addition the short seller is assumed to receive interest at the riskless rate on both the money loaned to the owner of the borrowed stock and the money placed in escrow when the short selling is made. To find the set of X_i 's which satisfy the optimality of portfolio, we define $Z_i = (\bar{R}_p - R_f / \sigma_p^2) X_i$ and solve this expression for any Z_i . Then we get:

$$Z_i = \frac{\beta_i}{\sigma_{\epsilon_i}^2} \left[\frac{\bar{R}_i - R_f}{\beta_i} - C \right]$$

$$X_i = \frac{Z_i}{\sum_{i=1}^N |Z_i|}$$

here:

$$C = \frac{\sigma_m^2 \sum_{i=1}^{25} \left[\frac{\bar{R}_i - R_f}{\sigma_{\epsilon_i}^2} \beta_i \right]}{1 + \sigma_m^2 \sum_{i=1}^{25} \frac{\beta_i^2}{\sigma_{\epsilon_i}^2}}$$

and $\sum_{i=1}^{25} |X_i| = 1$ to assure that we have invested 100% of our fund. In this model we can derive a set of X_i 's for each sample period and use the equation mentioned at the beginning of this section to decide the expected return on optimal portfolio (\bar{R}_p).

(iii) Optimal Portfolios when Short Sales are not Allowed

If short selling is not allowed then we must introduce the constraints that all $X \geq 0$. This requires employing the Kuhn-Tucker conditions. That is:

$$Z_i = \frac{\beta_i}{\sigma_{\epsilon_i}} \left[\frac{\bar{R}_i - R_f}{\beta_i} - C \right] + \mu_i$$

here:

$$Z_i \geq 0, \mu_i \geq 0, \text{ and } \mu_i Z_i = 0 \text{ for all } i$$

Since $\mu_i \geq 0$, including μ_i can only increase the value of Z_i . Thus, if Z_i is positive with $\mu_i = 0$, the including of μ_i can never make it zero. Hence, if Z_i is positive when $\mu_i = 0$, the security should be included. If $Z_i < 0$ when $\mu_i = 0$, positive values of μ_i can increase Z_i . However, since the product of μ_i and Z_i must equal zero, positive values of μ_i imply $Z_i = 0$. Hence any security with $Z_i < 0$ when $\mu_i = 0$ must be rejected. In other words, we will reject the securities which can not satisfy the constraint-- $X_i \geq 0$. In order to do these selection tests, we apply Mao's(11) simple criterion. First, let:

$$Q = \frac{\bar{R}_i - R_f}{\beta_i} \quad CK = \frac{\bar{R}_i - R_f}{\beta_i} - C$$

Then we rank all securities with β_i 's ≥ 0 by the value of Q in decreasing order and test CK to see whether it is less than zero. This tests will start from the first security which has the highest Q ($i=1$), then the first two securities ($i=2$), then the first three securities ($i=3$), etc. If $i=k+1$ which makes $CK < 0$ then the tests stop and we know that the first k securities are included in the optimal portfolio. When no more positive or zero β stocks are included, stocks with negative β 's should be tried in reverse order. Then, use the same formula in the last section to form the optimal portfolio. We repeat these procedures in each sample period.

(C) The Constant Correlation Coefficient Model
and Construction of Optimal Portfolios

(i) The Constant Correlation Coefficient Model

In this model, we assume that all pairwise correlation coefficients are equal. While this probably does not represent the true pattern one can find in the economy, it is very difficult to obtain a better estimate. Elsewhere (4), we have known that this assumption produces better estimates of future correlation coefficients than those produced from other models. As mentioned earlier, the optimal portfolio is that maximizes the ratio of excess return on the portfolio to its standard deviation of return. Letting:

1. σ_{ij} = covariance between security i and j
2. σ_i^2 = the variance of security i
3. π = the correlation coefficient between any two securities
4. all other as before.

and

$$\bar{R}_p = \sum_{i=1}^n X_i (\bar{R}_i - R_f) + R_f$$

$$\sigma_p^2 = \sum_{i=1}^n X_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n X_i X_j \sigma_{ij}$$

(ii) The Optimal Policies when Short Sales Are Allowed

In this case, the optimal portfolio can be derived without restricting the sign of X_i and by using the following formulae:

$$Z_i = \frac{1}{\sigma_i} \frac{1}{1-\pi} \left[\frac{\bar{R}_i - R_f}{\sigma_i} - \frac{\pi}{1-\pi+25\pi} \sum_{j=1}^{25} \frac{\bar{R}_j - R_f}{\sigma_j} \right]$$

$$X_i = \frac{Z_i}{\sum_{j=1}^{25} |Z_j|}$$

(iii) The Optimal Policies When
Short Sales Are Not Allowed

Again, if short selling is not allowed, then we have to rely on the Kuhn-Tucker conditions. These conditions which maximize θ are:

1. $\bar{R}_i - R_f - Z_i \sigma_i^2 - \sum_{\substack{j=1 \\ j \neq i}}^n Z_j \sigma_{ij} + \mu_i = 0$
2. $Z_i \geq 0, \mu_i \geq 0$
3. $Z_i \mu_i = 0$

olving for Z_i , we have:

$$Z_i = \frac{1}{1-\pi} \frac{1}{\sigma_i} \left[\frac{\bar{R}_i - R_f}{\sigma_i} - \frac{\pi}{1-\pi+k\pi} \sum_{j=1}^k \frac{\bar{R}_j - R_f}{\sigma_j} \right]$$

$$X_i = \frac{Z_i}{\sum_{j=1}^k Z_j}$$

The sign of Z_i depends on the terms in the brackets. Since the last term in the brackets is a constant for any k , if a security with a particular rate $(\bar{R}_i - R_f)/\sigma_i$ has a positive Z_i , then all securities with a higher ratio must also be included. Therefore, we can apply Mao's simple criterion again to decide the securities which should be included in the optimal portfolios and then derive the desired optimal portfolios.

After reviewing all pairwise correlation coefficients of twenty-five selected securities, we assume that the constant correlation coefficient is 0.4. Throughout, the month-riskless rate of return is also assumed to be 0.45 which is near to the present interest rate for saving accounts for all the calculations in each model.

(D) The Examination in the Effect of the Length of Sample Period on the Optimal Portfolios

From section (B) and section (C), we derive four sets of optimal portfolios. Each set consists of fourteen different optimal portfolios for each sample period.

First, we examine the relationship between sample period and estimated variables--mean return, beta coefficient, and residual error of each security. Second, we examine the relationship between length of sample period and optimal portfolio in each model with the option of short

elling. These examinations will be done by regression method and F-test.

CHAPTER IV

RESULTS

(A) Estimated Variables

The major estimated variables used in the single index model and the constant correlation coefficient model include mean or expected return on securities and on the S&P (500) Index, variances of expected returns, beta coefficient, residual errors for securities which are assumed to have a response to the market index, and standard deviations of securities and the S&P(500) Index. Here we define the expected return of security i as:

$$\bar{R}_i = \left(\sum_{k=1}^k \frac{P_t - P_{t-1}}{P_{t-1}} \right) / k$$

where: \bar{R}_i = expected return on security i

P_t = stock price of security i in month t

k = length of sample period

In this study, we do not consider the dividends paid any month. Also, all the estimated variables are calculated in fourteen sample periods. Table 2 shows the results the calculations for the sample period 1979-1980.

TABLE 2

Estimated Variables

Period: 1980 - 1979

Company	\bar{R}_i	β_i	σ_{ϵ}^2	σ_i
A	0.991	0.702	86.928	9.669
B	2.217	1.214	58.458	9.327
C	5.617	1.957	57.806	11.635
D	2.242	1.303	53.200	9.827
E	1.970	1.373	38.430	8.733
F	1.425	1.272	66.927	9.839
G	-0.054	0.670	35.597	6.591
H	1.000	0.348	87.078	9.319
I	0.038	0.925	16.755	5.823
J	1.433	0.536	84.728	9.418
K	0.767	0.251	49.988	7.009
L	3.371	1.118	67.662	9.588
M	2.929	2.020	74.186	12.497
N	4.287	1.373	39.813	8.805
O	0.588	0.879	46.973	7.815
P	6.454	1.919	196.693	16.279
Q	0.913	1.214	140.781	12.848
R	4.304	1.075	39.807	7.896
S	4.863	1.277	77.090	10.335
T	1.100	1.502	69.150	10.646
U	0.242	0.658	31.443	6.255
V	-1.375	1.454	125.967	12.829
W	1.953	1.480	72.285	10.726
X	0.446	0.782	28.385	6.319
Y	0.513	0.976	26.208	6.707

(B) Optimal Portfolios

We have derived four sets of optimal portfolios. Each set consists of fourteen optimal portfolios which are from the fourteen different sample periods. In the first case that short selling is allowed, all the twenty-five selected securities are assumed to be included in the optimal portfolio. All the securities with negative X's are those held short. Table 3 and Table 4 are the results of the selection of optimal portfolios for the sample period 1979-1980.

In the second case that short selling is not allowed, we rejected some securities to make sure that all X's are greater than or equal to zero. In other words, all securities included in the optimal portfolios must be held long. Table 5 and Table 6 show the results for sample period 1979-1980. Also, Table 7 shows the expected return on optimal portfolio in each sample period.

From Table 7, we have found that the expected return on optimal portfolio in the second case that short selling not allowed are higher than those in the first case. These differences result from the use of Mao's criterion to the selection tests in the second case. These tests have rejected some securities whose expected returns are low and stable (i.e. higher variance of return) and have resulted in reallocation of weights placed on remaining securities.

TABLE 3

Optimal Portfolio Under Single Index Model
(Short Selling Allowed)

Period: 1980 - 1979

Company	X_i (%)
A	-0.6
B	0.9
C	8.3
D	0.7
E	-0.5
F	-1.4
G	-6.2
H	0.3
I	-15.4
J	0.7
K	0.1
L	4.0
M	0.1
N	9.4
O	-3.3
P	3.2
Q	-1.2
R	11.0
S	6.4
T	-2.8
U	-5.4
V	-4.8
W	-0.6
X	-5.6
Y	-7.2

$$\sum |X_i| = 100.0$$

$$\bar{R}_p = 2.418\%$$

\bar{R}_p = expected return on optimal portfolio

TABLE 4

Optimal Portfolio Under Constant
Correlation Coefficients Model
(Short Selling Allowed)

Period: 1980 - 1979

Company	X_i (%)
A	-2.0
B	1.5
C	6.6
D	1.2
E	1.2
F	-0.8
G	-7.8
H	-1.9
I	-8.6
J	-0.7
K	-3.1
L	4.4
M	1.3
N	8.5
O	-3.6
P	3.6
Q	-1.8
R	11.1
S	7.0
T	-1.7
U	-6.5
V	-5.3
W	0.2
X	-5.2
Y	-4.5

$$\Sigma |X_i| = 100.0$$

$$\bar{R}_p = 2.405\%$$

TABLE 5

Optimal Portfolio Under Single Index Model
(Short Selling Not Allowed)

Period: 1980 - 1979

Company	X_i (%)
A	1.0
R	17.4
C	10.7
S	11.5
N	13.8
P	6.3
L	8.7
M	4.1
B	5.3
D	4.8
E	5.1
W	3.2
J	2.5
F	2.2
T	1.0
H	1.1
K	0.9
Q	0.3
$\Sigma X_i = 100.0$	

$$\bar{R}_p = 3.817\%$$

TABLE 6

Optimal Portfolio Under Constant
Correlation Coefficients Model
(Short Selling Not Allowed)

Period: 1980 - 1979

Company	X_i (%)
R	21.8
S	12.7
P	6.5
N	19.7
C	17.7
L	8.5
J	1.8
H	0.8
B	3.5
D	3.5
K	0.6
M	2.7

$$\Sigma X_i = 100.0$$

$$\bar{R}_p = 4.384\%$$

TABLE 7

Expected Returns on Optimal Portfolios

nit: %

Period	Simple Index Model		Const. Corr. Coef. Model	
	Case 1	Case 2	Case 1	Case 2
1980-1979	2.418	3.817	2.405	4.384
1980-1978	1.894	3.170	1.958	3.694
1980-1977	1.699	2.619	1.750	2.811
1980-1976	1.763	3.063	1.841	3.043
1980-1975	1.792	3.121	1.762	2.874
1980-1974	1.752	2.485	1.755	2.534
1980-1973	1.426	1.905	1.420	1.873
1980-1972	1.441	1.910	1.432	1.905
1980-1971	1.448	2.017	1.405	1.960
1980-1969	1.078	1.324	1.073	1.325
1980-1967	1.037	1.384	1.036	1.369
1980-1965	1.132	1.479	1.098	1.441
1980-1963	1.099	1.279	1.043	1.384
1980-1961	0.958	1.248	0.950	1.305

Case 1 = Short selling is allowed.

Case 2 = Short selling is not allowed.

(C) The Relationship between the Length of
Sample Period and Estimated Variables

By regression analysis methods, we have examined the relationship between the length of sample period and the estimated variables. In addition, we have plotted all the actual values of each variable against their predicted population values. These graphs are good references for determining whether the regression lines are fit.

Table 8 shows the regression statistics of mean return versus the sample period of each security. In the statistics, F Value is the ratio produced by dividing MS (Model) by MS (Error). It tests how well the model as a whole accounts for the dependent variable's behavior. If the significance probability, labeled PR>F, is small, it indicates significance. R-SQ measures how much variation in the dependent variable can be accounted for by the model. In general, the larger the value of R-SQ, the better the model's fit. We refer to the F Value, R-SQ, and PR>F to determine the relationship between mean return and the length of sample period. Not as might be expected, the regression results show no evidence that mean returns of security are strongly correlated with the sample period. But we find that some securities which have stable and growing returns are strongly correlated with sample period.

Also in Table 9 and Table 10, we do not find any po-

TABLE 8

Regression Analysis

Sample Period vs. Mean Return

Dependent Variable: Mean Return of Security i

Independent Variable: No. of Month

Company	SS.	R-SQ	C.V.	F-Value	PR > F
A	5.3	0.07	68	0.9	0.3622
B	4.4	0.52	57	12.9	0.0037
C	21.8	0.70	27	28.5	0.0002
D	4.7	0.66	22	23.5	0.0004
E	2.7	0.29	41	4.9	0.0464
F	13.0	0.39	30	7.7	0.0167
G	3.4	0.46	64	10.2	0.0076
H	3.0	0.36	41	6.8	0.0232
I	1.6	0.70	50	27.5	0.0002
J	1.6	0.12	37	1.6	0.2350
K	2.5	0.30	75	5.1	0.0440
L	6.2	0.28	48	4.7	0.0514
M	7.0	0.55	28	14.6	0.0024
N	12.6	0.57	31	16.2	0.0017
O	3.2	0.01	68	0.2	0.7039
P	28.2	0.65	38	22.4	0.0005
Q	6.2	0.10	708	1.3	0.2711
R	9.7	0.58	24	16.8	0.0015
S	16.3	0.63	31	20.1	0.0008
T	1.5	0.01	43	0.2	0.7056
U	1.4	0.04	43	0.6	0.4720
V	7.0	0.00	306	0.1	0.8131
W	1.7	0.44	19	9.5	0.0094
X	3.1	0.58	50	16.6	0.0016
Y	3.3	0.14	144	2.0	0.1881

TABLE 9

Regression Analysis
Sample Period vs. Beta

Dependent Variable: Beta

Independent Variable: No. of Month

Company	SS.	R-SQ	C.V.	F-Value	PR > F
A	0.32	0.14	15	1.9	0.1888
B	0.21	0.48	9	10.9	0.0063
C	0.13	0.07	5	0.9	0.3529
D	0.10	0.13	6	1.9	0.1966
E	0.12	0.12	8	1.7	0.2190
F	0.19	0.08	11	1.0	0.3420
G	0.25	0.62	10	19.5	0.0008
H	0.57	0.48	19	11.0	0.0062
I	0.20	0.31	11	5.5	0.0369
J	0.22	0.31	14	5.3	0.0394
K	0.36	0.45	21	9.8	0.0088
L	0.08	0.01	7	0.1	0.8090
M	2.17	0.42	22	8.6	0.0125
N	0.07	0.00	5	0.0	0.5714
O	0.18	0.61	8	18.9	0.0010
P	0.92	0.67	12	23.7	0.0004
Q	0.12	0.12	8	1.6	0.2242
R	0.46	0.55	9	14.7	0.0024
S	0.20	0.02	13	0.3	0.5944
T	0.53	0.23	17	3.6	0.0819
U	0.15	0.12	14	1.7	0.2214
V	0.98	0.69	13	27.1	0.0002
W	0.49	0.71	11	28.9	0.0002
X	0.46	0.45	14	9.8	0.0088
Y	0.12	0.00	9	0.0	0.9529

TABLE 10

Regression Analysis

Sample Period vs. Residual Error

Dependent Variable: Residual Error

Independent Variable: No. of Month

Company	SS.	R-SQ	C.V.	F-Value	PR > F
A	16435	0.02	30	0.2	0.6324
B	4193	0.21	32	3.3	0.0957
C	3519	0.61	13	19.0	0.0009
D	8205	0.69	21	27.1	0.0002
E	2878	0.51	23	12.6	0.0040
F	470	0.00	8	0.0	0.9919
G	6987	0.76	20	37.7	0.0001
H	2414	0.06	16	0.7	0.4155
I	3725	0.50	38	11.8	0.0049
J	2384	0.00	18	0.0	0.9615
K	3120	0.31	24	5.3	0.0403
L	1563	0.09	18	1.2	0.2904
M	5011	0.16	18	2.2	0.1603
N	2582	0.62	16	19.4	0.0009
O	1598	0.44	15	9.5	0.0095
P	12690	0.44	23	9.4	0.0098
Q	5975	0.57	14	15.7	0.0015
R	5483	0.60	20	18.1	0.0011
S	3042	0.25	19	4.0	0.0698
T	5788	0.25	25	4.1	0.0657
U	3801	0.00	27	0.1	0.7887
V	41491	0.55	24	14.7	0.0024
W	1389	0.52	13	12.8	0.0038
X	1222	0.42	20	8.7	0.0122
Y	1293	0.73	14	32.8	0.0001

3

sitive connection between the length of sample period and beta coefficient or residual error of security. In other words, all the results are opposite of what were expected and give no indication that a relationship between the length of sample period and each estimated variable exists.

(D) The Relationship between the Length of
Sample Period and Optimal Portfolio

There are three factors that should be taken into consideration for determining the effect of the length of sample period on the optimal portfolio. These factors are:

1. The changes of securities included in the optimal portfolio. (It is not valid in the case that short selling is allowed because we have assumed that all twenty-five securities are included in the optimal portfolio.);
2. The changes of weights placed on the securities in the optimal portfolio; and
3. The changes of expected returns on the optimal portfolios.

After referring to the compositions of all the optimal portfolios in each sample period, we find:

1. No matter what sample period we choose, the securities selected for the optimal portfolios are almost the same.

2. In general, the weights placed on the securities in the optimal portfolio do not have significant changes when we alter sample periods.

We also regress the sample period against the expected return on the optimal portfolio. The results of regression statistics are shown from Table 11 to Table 14. From above tables, it appears that there is a strong correlation between the length of sample period and the expected return on the optimal portfolio.

In addition to the preceding results, the other thing we want to mention before we draw any conclusions is the limitations of this study. The limitations which may create some deviations from our conclusions are as follows:

1. The sample size in this study is small. Therefore, representation of this sample to the population may be incomplete.

2. The population considered in this study is narrowed to the 500 Largest Industries in 1980. This may limit the effects of diversification on optimal portfolios.

3. Only fourteen observations are available for all the regression analyses.

4. Dividends are not included in the calculation of expected return on each security.

TABLE 11

Regression Analysis

Sample Period vs. Return on Optimal Portfolio

Single Index Model

Short Selling Permitted

General Linear Models Procedure

Dependent Variable: \bar{R}_p

Source	DF	SS.	MS.	F Value	PR > F
Model	1	1.845	1.845	54.82	0.0001
Error	12	0.404	0.034		
Corrected Total	13	2.249			

TABLE 12

Regression Analysis
 Sample Period vs. Return on Optimal Portfolio
 Single Index Model
 Short Selling Not Permitted
 General Linear Models Procedure

Dependent Variable: \bar{R}_p

Source	DF	SS.	MS.	F Value	PR > F
Model	1	7.142	7.142	23.18	0.0004
Error	12	3.697	0.308		
Corrected Total	13	10.839			

TABLE 13

Regression Analysis
 Sample Period vs. Return on Optimal Portfolio
 Constant Correlation Coefficient Model
 Short Selling Permitted
 General Linear Models Procedure

Dependent Variable: \bar{R}_p

Source	DF	SS.	MS.	F Value	PR > F
Model	1	2.071	2.071	69.00	0.0001
Error	12	0.360	0.030		
Corrected Total	13	2.431			

TABLE 14

Regression Analysis

Sample Period vs. Return on Optimal Portfolio

Constant Correlation Coefficient Model

Short Selling Not Permitted

General Linear Models Procedure

Dependent Variable: \bar{R}_p

Source	DF	SS.	MS.	F Value	PR > F
Model	1	9.476	9.476	40.04	0.000
Error	12	2.840	0.237		
Corrected Total	13	12.316			

CHAPTER V

SUMMARY AND CONCLUSIONS

This study is designed to examine the effect of the length of sample period on the optimal portfolio which is derived by utilizing the historical records as the sources of information under the single index model and the constant correlation coefficient model. Our hypothesis is that this effect will be significant. In general, the results are opposite of what we expected.

The major findings and analyses can be summarized as follows:

1. There is no significant relationship between the length of sample period and beta coefficient or residual error of security. In every sample period, the fluctuation on the beta coefficient is very little. This implies that the beta coefficient of each security tends to be constant even when the sample period of the historical records change.
2. Within the twenty-five selected securities, only a few securities which possess continuous, stable and increasing rates of growth (decline) in returns (i.e. stock prices --especially in this study) show a strong relationship be-

een their return and the length of sample period. The ones whose mean return is unstable does not show any connection between the length of sample period and their mean turn.

3. In the case that short selling is allowed and all twenty-five securities are assumed to be included in the optimal portfolio, over eighty percent of the fund is found to be vested in almost the same securities with continuous, stable, and increasing rates of growth (decline) in returns in each sample period.

4. In the case that short selling is not allowed--that is, all securities included in the optimal portfolio are held long, the optimal portfolio for each sample period always consists of the same securities with continuous, stable, and increasing rates of growth in returns.

5. The regression statistics show a strong relationship between the expected return on optimal portfolio and the length of sample period.

Based on the results shown above, we have reached the following conclusions. First, it is clear that the effect of the length of sample period on the beta coefficient or residual error of security is not significant when we use the historical records as the bases for the selection of optimal portfolio. Second, the effect of sample period on the mean return on securities vary from one security to another.

In general, sample period does not have effect on most securities except those with long-term growths or declines in prices. Third, no matter what the length of sample period we choose, it will not change much the composition of the optimal portfolio. Fourth, statistically, it seems that the length of sample period has an effect on the expected return on optimal portfolio. This implies that the shorter sample period we choose for the historical records, the higher expected return on optimal portfolio we would derive. Actually, this conclusion is in contradiction to the above conclusions. Why does it happen? As we mentioned before, the optimal portfolio in each sample period always consists of the securities with long-term increasing expected return. The expected return on security is calculated on the average basis. Therefore, the most recent expected return on a security always tends to be the highest one. Moreover, owing to the expected return on the optimal portfolio which is determined by the mean return of individual securities included in the optimal portfolio, it appears that the optimal portfolio which is from the shorter sample period would have higher expected. Yet, in essence, the higher return does not result from the choice of shorter sample period, but from the mean return of individual securities. Therefore, we can conclude that the effect of the length of sample period on the optimal portfolio is also not significant.

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A P P E N D I X A

COMPANY NAMES AND COMPUTER PROGRAMS

B = AMERICAN CYANAMIDE
 C = AVNET ELECT
 D = BECKMAN INSTRUMENT
 E = BENDIX
 F = BOEING
 G = CBS
 H = DAN RIVER MILLS
 I = EMERSON ELECTRIC
 J = GRUMMAN AIRCRAFT
 K = HERSHEY FOODS
 L = JOY MANUFACTURING
 M = NORTHROP
 N = PERKIN ELMER
 O = REICHOLD CHEMICALS
 P = SHELL OIL
 Q = SINGER
 R = SUNSTRAND CORP.
 S = SUPERIOR OIL
 T = TEXTRON
 U = UNILEVER N.V.
 V = UNITED MERCHANTS & MANUF.
 W = UNIVERSAL LEAF TOBACCO
 X = WHIRLPOOL
 Y = XEROX
 Z = MARKET INDEX-STANDARD AND POOR'S-500

COMPUTE THE PORTFOLIOS AND ITS
 EXPECTED RETURN UNDER THE SINGLE INDEX
 MODEL (SHORT SELLING PERMITTED)
 WRITTEN BY CHIANG-CHIEN HO 5/30/81

NAME= THE ALPHABETS FROM A TO Z STAND FOR THE NAME OF EACH SECURITY

R = MEAN RETURN ON SECURITY I
 B = BETA COEFFICIENT OF SECURITY I
 E = RESIDUAL ERROR OF RETURN OF SECURITY I
 CR = CONTRIBUTION TO TOTAL RETURN ON OPTIMAL PORTFOLIO FROM
 SECURITY I
 TR = TOTAL RATE OF RETURN ON OPTIMAL PORTFOLIO
 X = RELATIVE WEIGHT OF SECURITY I IN OPTIMAL PORTFOLIO
 N = THE NUMBER OF INPUT SECURITY
 Q = TEMPORARY VARIABLE
 Z = TEMPORARY VARIABLE
 Q1 = TEMPORARY VARIABLE
 Q2 = TEMPORARY VARIABLE
 TC1 = TEMPORARY VARIABLE
 TC2 = TEMPORARY VARIABLE
 TC3 = TEMPORARY VARIABLE

V = STANDARD DEVIATION OF MARKET INDEX (S&P-500)

RF = RISK-FREE RATE OF RETURN

DIMENSION R(14,25),B(14,25),E(14,25),Z(14,25),X(14,25),

EXPECTED RETURN UNDER THE CONSTANT
 CORRELATION COEFFICIENT MODEL (SHORT
 SELLING IS PERMITTED)
 WRITTEN BY CHIANG-CHIEN HO 5/30/81
 VARIABLE
 R = MEAN RETURN ON SECURITY I
 SD = STANDARD DEVIATION OF SECURITY I
 X = RELATIVE WEIGHT ON SECURITY I IN OPTIMAL PORTFOLIO
 N = THE NUMBER OF INPUT SECURITY
 Q1 = TEMPORARY VARIABLE
 TQ1 = TEMPORARY VARIABLE
 Z = TEMPORARY VARIABLE
 TR = TEMPORARY VARIABLE
 CR = STANDARD DEVIATION OF SECURITY I
 CONSTANT
 RF = RISK-FREE RATE
 CC = CORRELATION COEFFICIENT BETWEEN ANY TWO SECURITIES

```

1  DIMENSION R(100),SD(100),Z(100),X(100),NAME(100),CR(100)
2  WRITE(6,1)
3  1 FORMAT(1H1,4X,'1980 - 1978')
4  TC1 = 0.0
5  A = 0.0
6  TCR = 0.0
7  RF = 0.45
8  CC = .4
9  DO 110 I = 1,N
10     READ(5,102) NAME(I),R(I),SD(I)
11     102 FORMAT(A1,F8.3,F10.3)
12     110 CONTINUE
13     DO 120 I = 1,N
14         Q1 = (R(I) - RF)/SD(I)
15         TC1 = TC1 + Q1
16     120 CONTINUE
17     DO 130 I = 1,N
18         Z(I) = ((R(I) - RF)/SD(I) - (CC*TQ1)/(1 - CC + N*CC))/
19             (SD(I)*(1 - CC))
20         A = A + Z(I)
21     130 CONTINUE
22     WRITE(6,131) A
23     131 FORMAT(1H0,4X,'A',2),'= ',2),F10.3)
24     WRITE(6,132)
25     132 FORMAT(1H0,4X,'NAME',6X,'X',6X,'CR')
26     DO 140 I = 1,N
27         X(I) = Z(I)/A
28         CR(I) = (R(I) - RF)*X(I)
29         WRITE(6,135) NAME(I),X(I),CR(I)
30     135 FORMAT(1H0,6X,A1,4X,F6.3,2X,F6.3)
31     TCR = TCR + CR(I)
32  140 CONTINUE
    TR = TCR + RF
  
```

```

1  A(P) = C.C
2  TCF(M) = C.C
3  TQ2(M) = C.C
4  TQ1(M) = C.C
5
6  10  READ(5,10) YE(1),YE(1)
7      FORMAT(14,14)
8  15  WRITE(6,15) YE(1),YE(1)
9      FORMAT(1H1,4X,14,1X,'-',1X,14)
10  20  READ(5,20) V(1)
11      FORMAT(F7.3)
12  40  WRITE(6,40)
13      FORMAT(1HC,4X,'NAME',4X,'MEAN RETURN',5X,'BETA',8X,'RES. ERROR
14  1
15      DO 110 J=1,N
16          READ(5,104) NAME(1,J),R(1,J),E(1,J),E(1,J)
17          FORMAT(A1,F8.3,F6.3,F10.3)
18          WRITE(6,105) NAME(1,J),R(1,J),E(1,J),E(1,J)
19          FORMAT(1HC,6X,A1,5X,F8.3,7X,F6.3,6X,F10.3)
20  110  CONTINUE
21      WRITE(6,121)
22      FORMAT(1H1,6X,'NAME',6X,'BETA',7X,'C')
23  121  DO 130 J=1,N
24          C(1,J) = (R(1,J) - RF)/E(1,J)
25          WRITE(6,122) NAME(1,J),B(1,J),C(1,J)
26          FORMAT(1HC,7X,A1,6X,F6.3,4X,F6.3)
27  122  130  CONTINUE
28      DO 150 J = 1,N
29          C1(1,J) = ((1,J)*R(1,J) - RF)/E(1,J)
30          TQ1(1) = TQ1(1) + C1(1,J)
31          C2(1,J) = (B(1,J)**2)/E(1,J)
32          TQ2(1) = T(2(1) + C2(1,J)
33  150  CONTINUE
34      C(1) = ((V(1)**2)*TQ1(1))/((V(1)**2)*TQ2(1) + 1)
35      DO 180 J=1,N
36          Z(1,J) = (R(1,J) - RF - B(1,J)*C(1))/E(1,J)
37          A(1) = J(1) + AES(Z(1,J))
38  180  CONTINUE
39      WRITE(6,191) A(1)
40      FORMAT(1H0,4X,'A',2X,'=',2X,F10.2)
41  191  WRITE(6,192)
42      FORMAT(1H1,6X,'NAME',5X,'X',7X,'CR')
43  192  DO 200 J=1,N
44          X(1,J) = Z(1,J)/A(1)
45          CR(1,J) = (R(1,J) - RF)*X(1,J)
46          WRITE(6,193) NAME(1,J),X(1,J),CR(1,J)
47          FORMAT(1HC,7X,A1,4X,F6.3,3X,F6.3)
48  193  TCR(1) = TCR(1) + CR(1,J)
49  200  CONTINUE
50      TR(1) = TCR(1) + RF
51      WRITE(6,211) TR(1)
52      FORMAT(1H0,7X,'TOTAL RETURN ON OPTIMAL PORTFOLIO',2X,'=',
53      1  2X,F8.2)

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A P P E N D I X B

VARIABLES

STATISTICAL ANALYSIS SYSTEM

1

BS	MO	R1	R2	R3	R4	AM	AB	AE	BM	BO	BE	CM	CB	CE
1	24	2.418	3.817	2.405	4.384	0.991	0.702	86.928	2.217	1.214	58.458	5.617	1.957	57.806
2	36	1.894	3.170	1.958	3.694	0.325	1.000	74.510	1.428	1.306	41.138	3.981	1.965	74.278
3	48	1.699	2.610	1.750	2.811	0.481	0.847	63.504	1.135	1.244	36.699	2.817	1.886	61.836
4	60	1.763	3.063	1.841	3.043	0.713	0.948	52.995	0.872	1.029	38.698	3.478	2.133	69.373
5	72	1.792	3.121	1.762	2.874	2.626	1.410	87.098	1.013	1.014	33.895	4.335	2.102	76.342
6	84	1.752	2.485	1.755	2.534	2.048	1.010	145.658	1.019	0.908	37.317	3.125	1.726	86.495
7	96	1.426	1.905	1.420	1.873	1.119	0.976	142.905	0.316	0.971	37.814	2.376	1.862	94.301
8	108	1.441	1.919	1.432	1.905	0.851	0.958	129.421	0.215	0.986	47.182	2.190	1.883	94.307
9	120	1.448	2.017	1.405	1.960	0.833	1.018	120.379	0.245	0.996	49.377	2.278	1.926	96.947
10	144	1.078	1.324	1.073	1.325	0.588	1.129	108.799	0.285	0.953	73.452	1.232	1.936	94.766
11	168	1.037	1.384	1.036	1.369	0.929	1.118	95.900	0.388	0.946	76.821	1.762	1.951	89.077
12	192	1.132	1.479	1.098	1.441	0.643	1.087	86.789	0.276	0.942	76.107	1.736	1.915	88.066
13	216	1.099	1.279	1.043	1.384	0.669	1.083	79.282	0.403	0.958	79.091	1.323	1.856	87.001
14	240	0.958	1.248	0.950	1.305	0.412	1.033	102.023	0.438	0.949	30.419	1.378	1.901	121.968

BS	DM	CB	DE	EM	EB	EE	FM	FB	FE	GM	GB	GE	HM	HB
1	2.242	1.303	53.200	1.970	1.373	38.430	1.425	1.272	66.927	-0.054	0.670	35.597	1.000	0.348
2	2.183	1.341	44.685	1.364	1.241	31.179	3.847	1.261	90.299	0.075	0.797	28.261	0.947	0.718
3	2.010	1.188	40.191	0.625	1.191	25.910	3.542	1.107	72.510	-0.283	0.813	25.794	1.213	0.503
4	1.843	1.041	38.095	0.773	1.190	33.408	3.943	1.062	69.967	0.228	0.930	26.310	1.580	0.916
5	3.049	1.258	46.308	1.726	1.215	36.330	3.985	1.047	69.487	1.106	0.755	53.378	2.167	1.043
6	2.099	1.246	48.337	1.312	0.957	38.392	3.798	0.870	71.204	1.273	0.653	54.316	1.245	0.738
7	1.465	1.290	61.207	0.546	1.082	45.830	2.689	1.049	80.765	0.483	0.773	55.720	0.704	0.828
8	1.434	1.306	69.315	0.639	1.081	46.188	2.732	1.086	79.099	0.522	0.782	66.977	0.983	0.841
9	1.748	1.307	74.471	0.998	1.087	55.239	2.853	1.185	75.059	0.848	0.793	81.724	0.949	0.879
10	1.189	1.394	87.001	0.517	1.148	61.120	1.724	1.297	73.934	0.422	0.919	83.422	0.372	1.038
11	1.250	1.298	104.623	0.792	1.182	66.981	1.458	1.230	77.512	0.635	1.070	80.639	0.539	0.983
12	1.168	1.282	102.270	0.918	1.159	65.902	1.736	1.264	71.433	0.808	1.022	82.569	0.497	1.025
13	0.911	1.254	111.892	0.739	1.149	72.409	1.906	1.236	71.130	1.262	1.031	83.811	0.664	1.019
14	0.942	1.377	72.954	0.575	1.117	38.890	1.738	1.150	78.145	1.299	1.013	80.331	0.633	0.962

BS	HE	IM	IB	IE	JM	JB	JE	KM	KB	KE	LM	LB	LE
1	87.078	0.038	0.925	16.755	1.433	0.536	84.728	0.767	0.251	49.988	3.371	1.118	67.662
2	73.340	0.236	0.851	16.953	0.892	0.971	74.889	0.675	0.397	41.651	1.933	1.122	53.745
3	74.042	0.131	0.780	14.611	0.542	0.905	60.470	0.265	0.397	34.245	0.683	1.142	42.147
4	83.352	0.125	0.756	13.788	0.785	0.679	60.494	0.763	0.395	39.126	1.232	1.298	45.480
5	82.375	0.658	0.961	19.930	1.288	0.806	64.438	1.631	0.663	52.935	1.319	1.178	48.954
6	96.058	0.060	1.148	25.636	1.592	0.673	95.927	1.131	0.679	49.601	1.002	1.076	65.045
7	91.090	0.027	1.099	27.560	1.334	0.747	103.206	0.384	0.751	52.704	1.413	0.980	77.101
8	84.495	0.119	1.081	41.244	0.879	0.724	92.724	0.222	0.754	67.823	0.787	1.038	75.767
9	80.926	0.285	1.087	45.577	0.783	0.825	85.500	0.283	0.714	69.875	0.975	1.093	72.408
10	80.101	0.491	1.064	54.236	0.386	0.852	79.701	0.259	0.686	66.634	1.131	1.048	65.708
11	109.408	0.724	1.007	54.627	0.571	0.898	72.493	0.249	0.687	77.107	1.058	1.065	64.009

BS	MM	MB	ME	NM	NE	NE	QM	QB	QE	PM	PB	PE	QM
1	2.929	2.020	74.186	4.287	1.373	39.813	0.588	0.879	46.973	6.454	1.919	196.693	0.913
2	3.536	2.119	77.141	3.914	1.351	46.882	-0.003	0.795	40.741	4.222	1.736	138.826	-0.097
3	1.246	2.073	80.925	2.865	1.318	42.345	-0.281	0.723	32.787	2.917	1.682	108.666	-0.050
4	2.348	1.953	85.294	2.543	1.452	48.126	0.765	0.883	46.150	3.163	1.421	97.134	1.633
5	2.221	1.549	104.543	2.636	1.586	52.857	1.279	0.975	48.401	2.707	1.112	98.101	1.229
6	2.330	0.995	113.618	1.748	1.475	51.417	1.619	0.709	64.879	2.131	1.425	102.097	-0.157
7	1.864	1.113	120.084	1.424	1.494	52.815	0.994	0.855	64.008	2.109	1.327	97.269	-0.690
8	1.771	1.103	136.186	1.512	1.506	54.307	1.274	0.854	68.727	2.038	1.349	99.051	-0.662
9	1.601	1.108	127.878	1.894	1.506	58.680	0.973	0.869	63.181	1.843	1.296	96.171	-0.402
10	1.059	1.265	119.666	1.116	1.461	70.531	0.626	0.980	61.307	1.400	1.206	94.936	-0.368
11	1.330	1.284	113.810	1.404	1.438	75.145	0.870	1.036	63.361	1.279	1.080	97.735	0.014
12	1.383	1.308	112.487	1.588	1.395	76.615	0.753	1.051	61.935	1.143	1.123	95.890	-0.198
13	1.203	1.245	106.798	1.497	1.354	87.328	0.746	1.045	65.771	1.242	1.117	101.455	-0.059
14	1.195	1.197	90.862	1.435	1.411	53.928	0.438	1.060	60.693	1.085	1.087	54.118	0.246

BS	QB	QE	RM	RB	RE	SM	SR	SE	TM	TB	TE	UM	UR	UE
1	1.214	140.781	4.304	1.075	39.807	4.863	1.277	77.090	1.100	1.502	69.150	0.242	0.658	31.443
2	1.238	101.095	3.356	1.343	39.827	4.022	0.981	66.562	0.722	1.368	54.033	0.181	0.582	31.122
3	1.085	82.570	2.565	1.268	35.722	3.213	1.014	56.691	0.473	1.243	43.405	0.965	0.566	91.163
4	1.487	114.714	3.418	1.328	47.977	3.247	0.911	56.107	0.950	1.225	40.825	0.995	0.687	79.081
5	1.212	129.167	3.579	1.450	62.908	2.676	0.759	58.500	1.433	1.312	52.035	1.446	0.950	75.716
6	1.103	122.554	2.308	1.342	63.882	1.713	0.858	57.578	0.995	0.758	92.322	0.599	0.889	70.966
7	1.190	113.324	2.027	1.531	91.854	1.398	0.879	69.557	0.403	0.909	55.764	0.500	0.840	71.315
8	1.155	110.717	2.050	1.571	88.885	1.863	0.909	74.753	0.495	0.925	94.571	0.817	0.858	70.665
9	1.153	102.134	2.210	1.665	82.609	1.807	0.992	81.581	0.656	0.957	93.570	1.100	0.879	67.940
10	1.174	99.305	1.386	1.675	78.858	1.503	1.060	90.476	0.258	1.082	96.555	0.775	0.877	73.318
11	1.126	89.824	1.623	1.678	76.415	1.689	1.023	94.398	0.643	1.061	92.631	0.996	0.766	69.785
12	1.125	83.146	2.053	1.682	83.025	1.488	1.036	90.927	0.973	1.066	90.563	0.858	0.753	68.781
13	1.134	78.546	1.826	1.629	79.381	1.506	1.034	96.630	1.121	1.081	87.040	0.794	0.753	66.851
14	1.173	63.191	1.799	1.493	93.648	1.289	1.080	57.034	1.149	1.017	69.275	0.682	0.762	47.498

BS	VM	VB	VE	WM	WB	WE	XM	XB	XE	YM	YB	YE
1	-1.375	1.454	125.967	1.958	1.480	72.285	0.446	0.782	28.385	0.513	0.976	26.208
2	1.900	1.712	195.924	1.461	1.198	77.698	0.215	0.771	24.063	0.789	1.074	23.826
3	-0.263	1.508	287.037	1.298	1.080	61.308	-0.243	0.753	20.144	0.142	1.007	21.466
4	-0.460	1.581	238.986	1.957	1.026	63.264	-0.061	0.695	25.070	0.543	1.148	30.752
5	-0.164	1.448	205.926	2.146	1.034	55.302	1.181	1.028	36.160	0.550	1.305	33.559
6	-0.444	1.058	191.414	1.860	0.926	50.978	0.702	1.126	45.314	-0.331	1.269	38.224
7	-0.674	1.058	171.908	1.328	0.968	48.766	0.291	1.184	43.809	-0.471	1.203	40.700
8	-0.756	1.088	155.743	1.302	0.934	51.427	0.486	1.190	45.777	-0.249	1.183	41.386
9	-0.645	1.056	142.157	1.190	0.966	50.590	0.752	1.185	45.468	0.106	1.191	39.771
10	-0.641	1.100	132.612	1.053	0.906	59.116	0.754	1.178	47.678	0.151	1.064	35.311
11	-0.074	0.925	119.815	1.267	0.756	58.957	1.145	1.102	48.146	0.148	1.050	36.032

FOR APPENDIX B:

M0 = Number of Months

R1 = Expected Returns on Optimal Portfolios under Single
Index Model (Short Selling Allowed)

R2 = Expected Returns on Optimal Portfolios under Single
Index Model (Short Selling Not Allowed)

R3 = Expected Returns on Optimal Portfolios under Cons-
tant Correlation Coefficient Model (Short Selling
Allowed)

R4 = Expected Returns on Optimal Portfolios under Con-
stant Correlation Coefficient Model (Short Selling
Not Allowed)

AM = Mean Return of Company A

AB = Beta Coefficient of Company A

AE = Residual Error of Company A

A P P E N D I X C

OPTIMAL PORTFOLIOS

(Short Selling Allowed)

Unit: %

1980-1978		1980-1977		1980-1976		1980-1975		1980-1974		1980-1973		1980-1972	
Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i
A	-2.7	A	-1.2	A	-2.0	A	2.2	A	2.0	A	0.9	A	0.1
B	-1.1	B	0.3	B	-2.1	B	-5.0	B	-0.6	B	-4.7	B	-5.7
C	4.4	C	4.3	C	4.4	C	6.2	C	5.4	C	5.3	C	4.7
D	2.1	D	4.5	D	3.4	D	7.9	D	5.1	D	3.6	D	3.0
E	-1.5	E	-3.1	E	-3.9	E	-0.5	E	1.6	E	-2.2	E	-2.2
F	4.7	F	6.5	F	8.5	F	10.4	F	12.4	F	8.9	F	10.4
G	-7.6	G	-8.3	G	-7.9	G	-1.0	G	2.2	G	-1.6	G	-1.5
H	-0.5	H	1.3	H	1.1	H	2.1	H	0.9	H	-0.1	H	1.1
I	-11.4	I	-9.1	I	-14.6	I	-12.9	I	-15.1	I	-11.0	I	-8.0
J	-1.2	J	-1.1	J	-0.7	J	-0.2	J	2.2	J	2.3	J	0.7
K	-0.7	K	-2.1	K	0.1	K	2.6	K	1.3	K	-2.3	K	-3.4
L	1.5	L	-1.5	L	-1.1	L	-2.7	L	-1.0	L	3.1	L	-0.4
M	2.7	M	-0.6	M	1.0	M	0.2	M	3.3	M	3.3	M	2.9
N	9.0	N	7.7	N	4.5	N	2.5	N	1.6	N	3.4	N	3.9
O	-5.6	O	-6.2	O	-1.8	O	-1.5	O	3.3	O	1.5	O	3.2
P	3.0	P	2.8	P	3.7	P	3.2	P	2.1	P	4.8	P	4.7
Q	-3.2	Q	-2.4	Q	0.1	Q	-1.3	Q	-3.6	Q	-5.2	Q	-6.2
R	7.9	R	7.7	R	9.0	R	7.4	R	4.6	R	4.4	R	4.9
S	7.6	S	7.4	S	8.3	S	7.2	S	3.7	S	3.6	S	6.3
T	-3.5	T	-2.6	T	-2.4	T	-2.7	T	0.0	T	-1.4	T	-1.5
U	-5.0	U	0.5	U	0.0	U	0.2	U	-2.2	U	-1.2	U	0.2
V	-0.1	V	-1.0	V	-2.0	V	-3.3	V	-2.8	V	-3.3	V	-4.6
W	-0.3	W	0.9	W	2.5	W	3.0	W	4.8	W	4.3	W	4.2
X	-7.6	X	-10.0	X	-9.3	X	-3.4	X	-3.9	X	-4.9	X	-4.1
Y	-5.3	Y	-7.1	Y	-5.7	Y	-10.5	Y	-14.2	Y	-12.5	Y	-12.2

(Short Selling Not Allowed)

Unit: %

[illegible]

(Short Selling Allowed)

Unit: %

[illegible]

(Short Selling Allowed)

Unit: %

1980-1971		1980-1969		1980-1967		1980-1965		1980-1963		1980-1961	
Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i
A	-1.2	A	-0.8	A	-0.3	A	-2.1	A	-2.5	A	-4.3
B	-7.1	B	-5.4	B	-6.9	B	-8.2	B	-7.2	B	-6.7
C	4.5	C	2.3	C	3.2	C	3.2	C	1.0	C	1.3
D	4.6	D	3.8	D	3.9	D	2.3	D	-0.5	D	-0.0
E	1.1	E	-1.6	E	-0.2	E	1.3	E	-1.7	E	-3.8
F	10.0	F	7.3	F	4.3	F	5.8	F	7.1	F	7.2
G	-0.4	G	-2.8	G	-2.1	G	-0.8	G	2.3	G	3.5
H	-0.3	H	-2.6	H	-3.0	H	-3.5	H	-2.7	H	-2.8
I	-7.4	I	-2.2	I	-0.2	I	2.4	I	3.0	I	2.3
J	-1.4	J	-2.4	J	-2.6	J	-1.0	J	-1.2	J	-0.0
K	-6.4	K	-5.8	K	-8.6	K	-11.8	K	-13.5	K	-7.9
L	-0.3	L	3.6	L	1.6	L	-0.8	L	1.6	L	-1.3
M	2.6	M	2.1	M	2.6	M	2.7	M	1.2	M	1.8
N	6.1	N	4.2	N	5.3	N	7.0	N	5.9	N	5.4
O	0.3	O	-0.4	O	0.2	O	-1.1	O	-1.8	O	-5.0
P	4.4	P	5.5	P	3.7	P	2.7	P	3.5	P	2.7
Q	-7.6	Q	-8.1	Q	-7.3	Q	-9.5	Q	-9.6	Q	-6.8
R	4.5	R	3.5	R	3.5	R	6.0	R	4.7	R	5.4
S	6.6	S	8.4	S	9.0	S	6.5	S	6.6	S	4.9
T	-2.2	T	-3.5	T	-2.0	T	0.7	T	1.6	T	2.6
U	1.7	U	1.6	U	2.8	U	0.7	U	-0.8	U	-2.1
V	-7.0	V	-7.9	V	-6.5	V	-7.1	V	-6.7	V	-7.4
W	3.0	W	5.6	W	7.5	W	7.0	W	5.7	W	5.0
X	-1.4	X	1.5	X	4.4	X	2.6	X	5.9	X	6.1
Y	-7.7	Y	-7.0	Y	-9.2	Y	-3.4	Y	2.7	Y	3.5
ΣX_i	100.0		100.0		100.0		100.0		100.0		100.0

Unit: %

[illegible]

121010 U 000000Z 110600Z 110600Z 110600Z 110600Z 110600Z

Unit: %

1980-1971		1980-1969		1980-1967		1980-1965		1980-1963		1980-1961	
Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i	Co.	X_i
F	20.0	S	12.7	S	10.7	F	13.1	F	14.0	F	10.8
S	9.3	F	18.8	W	10.9	W	10.1	S	7.1	R	8.8
P	7.4	P	10.2	F	9.3	S	8.2	W	11.1	G	6.2
R	10.7	W	9.7	P	5.9	R	13.7	R	10.5	W	8.8
M	4.5	L	9.8	U	5.2	N	9.9	G	5.6	S	8.4
D	8.5	R	10.3	R	10.2	M	5.2	N	6.9	X	10.8
N	11.8	D	7.2	M	5.1	C	8.9	P	4.3	N	9.9
C	9.0	M	4.1	C	9.6	P	4.2	X	10.4	T	5.4
W	6.0	N	7.2	N	8.2	D	3.8	T	3.8	Y	6.7
U	3.8	C	5.6	X	9.0	U	3.1	Y	6.4	M	4.1
O	2.6	U	2.7	D	4.7	X	6.8	M	3.4	P	5.7
H	1.8	X	1.8	L	5.5	T	2.8	L	5.6	J	2.4
E	2.2			A	2.2	I	3.1	I	3.3	C	3.1
G	1.1			O	2.7	E	2.7	C	3.7	I	5.9
L	1.4			E	0.8	J	1.5	U	1.8	D	1.7
						G	1.2	J	1.8	U	0.7
						L	1.2			L	0.5
						O	0.7				
$\Sigma X_i = 100.0$		100.0		100.0		100.0		100.0		100.0	

VITA

Chiang-Chien Ho

Candidate for the Degree of
Master of Business Administration

port: LENGTH OF SAMPLE PERIOD AND OPTIMAL
PORTFOLIOS

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Biographical:

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