EFFECTS OF A MENTAL ROTATIONS CURRICULUM ON MATHEMATICAL CONCEPTUALIZATION AND MATH ANXIETY IN EIGHTH GRADE MALE AND FEMALE STUDENTS

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CHAPTER 1

Introduction

Mathematics Achievement, Math Anxiety, and Gender Differences

As technology expands and our society becomes more computer oriented, mathematical reasoning skills are becoming more and more important for competitiveness in the global job marketplace. The growing importance of mathematics is quite evident in our society, especially for school advancement and career opportunities. A strong mathematical background is essential for admission to many college majors and subsequent entry into many professional occupations. Remick and Miller (1978) conducted a study at the University of Washington and found that incoming freshmen who lacked four years of high school math had as a choice of majors only five out of sixteen offered at the beginning of their college education. In recognition of this trend, greater emphasis is being placed on improving mathematics curricula and raising mathematics achievement test scores in our schools.

Recent research (Lefevre, Kulak, & Heymans, 1992: Singer & Stake, 1986) indicates most women avoid majors that require even moderate amounts of math and do not choose math-related careers, even when they are as skilled at math as males and have taken equivalent amounts of math in high school. The decision to limit one's mathematical training has serious consequences which eventually impacts career options. The Bureau of Labor Statistics indicates that men and women enter the scientific, mathematical, and technical fields as career choices in disproportionate numbers (Meece, Parsons, Kaczala, Goff, & Futterman, 1982). A two-year study by Armstrong (1980) on achievement and participation patterns of women in mathematics indicates that women in engineering represent one-tenth of one percent of the total number of engineers in the United States and women in physics represent two percent of all physicists.

Consistent gender differences have been shown to exist in mathematical background and achievement. After reviewing the literature on gender-related differences and math achievement from the years 1960 to 1974., Fennema (1974) found that when gender differences were apparent they favored males. In a more recent study, Pearson and Ferguson (1989) found significant differences between the genders on ACT Math scores. This study involved a sample of undergraduate students. Men consistently outscored women in ACT Math. In results of a meta-analyses, Friedman (1989) concluded that when gender differences existed, they favored males.

The age at which consistent gender differences first appear remains controversial, however adolescence appears to be a critical age for studying gender differences in math performance (Engelhard, 1989). Junior high boys begin to surpass girls in math achievement and to express greater interest in the subject (Aiken, 1975). In numerous studies using subjects ranging in age from grade eight through college, there is a plethora of evidence supporting male superiority in mathematical achievement (Fennema, 1974, Kimball, 1989; Maccoby & Jacklin, 1974; Pearson & Ferguson, 1989; Randhawa, 1991; Steel, 1978).

According to Fennema and Sherman (1977, 1978) the difference in math background is one of the most important contributing factors in determining gender related differences in mathematics achievement. From the middle of high school and beyond girls choose not to study mathematics. Females are inclined to select themselves out of advanced math courses both in high school and college even when these students are placed in special programs for the gifted (Fox, 1975). Math backgrounds of incoming university freshmen show that a significant imbalance favoring males exists. Citing 1972 statistics at the University of California at Berkeley, Elmore and Vasu (1980) found that 57 percent of incoming males compared to eight percent of incoming females had taken four years of high school math. This represents a difference of more than seven to one. Females tend to be less invested in mathematics because they perceive mathematics as not being useful to them and tend not to enroll in elective math courses (Singer & Stake, 1986). A growing body of research on gender differences in math has shown that fear of math combines with traditional societal influences on women to create a disproportionate number of math avoiding and/or math anxious females (Donady & Tobias, 1977; Lefevre et al., 1992; Singer & Stake, 1986).

Aside from the basic skills necessary for mathematics achievement, affective variables, most prevalent of which is math anxiety, have been shown to have a large impact on mathematics achievement. Several research studies examining the impact of affective variables on mathematics learning have found that math anxiety may contribute to mathematics avoidance and poor mathematics performance (Lefevre, et al., 1992; Wigfield & Meece, 1988). Math anxiety has been shown to be negatively correlated with both mathematics achievement and mathematics participation (Dew, Galassi & Galassi, 1983; Engelhard, 1989; Frary & Ling, 1983). Studies investigating gender differences associated with math anxiety, have found that females generally exhibit greater math anxiety than do males (Lupkowski & Schumacker, 1991; Shanklin, 1978; Skiba, 1990).

The Relationship of Spatial Ability to Math Achievement and Math Anxiety

The relationship between visual-spatial skills and achievement in mathematics has been examined by a number of researchers from both education and psychology with conflicting findings being reported concerning the relationship, particularly in the earlier research. A relationship between mathematical ability and spatial ability has been established (Maccoby & Jacklin 1974). A study by McCallum, Smith and MacFarlane in 1979 (cited in Murray, 1987) provides support for an association between spatial abilities and mathematics achievement. Factor analysis in this work indicated that a measure of spatial orientation loaded most strongly with achievement in mathematics. Other studies do not support the hypothesis that a relationship between spatial abilities and mathematics exists (Hills, 1957). No correlation between performance on spatial tests and tests of mathematics achievement for boys was found by Fennema and Tartre (1985) and Sherman

(1980), however a relationship between these two variables was found for girls. In a review of the literature regarding this relationship Denno (1982) has concluded that performance on spatial measures alone is not a predictor of success in mathematics for normal students.

Perhaps the nature of both spatial ability and mathematics achievement, being complex abilities consisting of several component skills, is responsible for the mixed findings reported in the literature. Factors which have been suggested for the inconsistency in the literature concerning the relationship between spatial ability and mathematics achievement are definitional problems of mathematics achievement, definitional problems of spatial ability, lack of validity on spatial instruments, the age of subjects, and gender differences.

One current need in research is to identify specific processes used in solving mathematical problems that might be related to spatial skills (Tartre, 1990). Many of the earlier studies (Fennema & Tartre, 1985; Guay & McDaniel, 1977; Hills, 1957; Sherman, 1980) used overall mathematics achievement as the criterion measure, whereas mathematics achievement is generally considered to consist of three, or sometimes four, specialized areas - math concepts, math problem solving, math computation, and sometimes math applications. Each of these areas of mathematics achievement represents different processes. If a consistent relationship between spatial skills and math skills is to be determined, then each of these areas of mathematics needs to be investigated separately in relation to spatial ability.

In studies that examined these separate areas of mathematics achievement (Brinkman, 1963; Bruininks & Mayer, 1979; Burnett, et al., 1979), the area of mathematical conceptualization showed the strongest relationship to spatial ability. In 1950 Vernon found a correlation between spatial ability and high level math ability (Rekdal, 1982). Harris (cited in Murray, 1987) indicates that spatial abilities may not be as strongly related to calculation as to the other skills, such as conceptualization. In a review of research by Zimmerman (1954), it was found that the more difficult spatial tests correlated quite highly with reasoning tests. These studies indicate the strongest relationship may exist between spatial ability and the more abstract mathematical conceptualization and reasoning skills. These findings suggest that visual-spatial skills may be necessary for the development of higher-level mathematical skills, particularly mathematical conceptualization.

Intuitively, achievement in higher-level mathematics would seem to require a thorough understanding of mathematical concepts, including the ability to understand the number system and the terms and operations used in higher-level mathematics. Based on the studies cited previously and the suggested need to examine specific aspects of mathematics achievement in order to better isolate the relationship of mathematics achievement to spatial abilities, mathematical conceptualization was selected as the specific aspect of mathematics achievement to be examined in this study.

The possibility that certain aspects of spatial skills are more strongly associated with some categories of mathematics achievement than others has not yet been fully explored by research. Definitive conclusions about a relationship between spatial ability and mathematics are difficult to reach when there is a lack of agreement on a single, formal definition of spatial ability (Linn & Petersen, 1985; Michael, et al., 1957; Pellegrino, et al., 1984; Thurstone, 1938). Focusing research efforts on a specific process of spatial ability, and examining its relationship to a specific aspect of mathematics achievement, has been suggested as one way to help to alleviate this definitional confusion (Gallagher & Johnson, 1992). One process of spatial ability which is present in every definition of spatial ability cited above, and is defined as a separate category of spatial ability by Linn and Petersen (1985) is that of mental rotation. Intuitively, the skills and strategies involved in mental rotation seem very similar to those required in geometry and other higher-level mathematical reasoning tasks.

The definition of mental rotation (the mental manipulation of two-dimensional and/or three-dimensional objects in space) suggests that it is related to geometry and calculus. It was suggested by Piemonte (1982) that, of the visual spatial skills he found related to mathematics achievement, spatial visualization is the skill most related. In more recent work, Pearson and Ferguson (1989) found significant correlations between two spatial ability tests, the Mental Rotations Test (MRT) and the Differential Abilities Test (DAT), and standardized ACT Math achievement test scores for both men and women. The MRT is specifically designed to measure mental rotation spatial ability and is the instrument which was selected to measure mental rotation ability in this study. Pearson's and Ferguson's study is unique in that it specifically examined mental rotation and its relationship to mathematics, and found a significant correlation. Pearson and Ferguson concluded that the specific spatial visualization skill of mental rotation was more strongly related to mathematics achievement than other types of spatial abilities.

This study examined the specific spatial process of mental rotation as it relates to mathematical conceptualization. The rationale for choosing mental rotation is based on the fact that the relationship between mental rotation and mathematics achievement appears to be strong in the Pearson and Ferguson (1989) study, yet few studies have examined this relationship; the process of mental rotation is included in most definitions of spatial ability and intuitively, the skills and strategies involved in mental rotation seem very similar to those required in geometry and other higher-level mathematical reasoning tasks; and the process of mental rotation seems to show some of the strongest gender differences and training effects of any of the spatial processes, as discussed in the following sections.

While numerous studies have shown an association between high math anxiety and low mathematics achievement (Frary & Ling, 1983; Suinn & Edwards, 1982), very little research has been done investigating the relationship between spatial abilities and math anxiety. The possibility exists that a lack of proficiency in certain skills necessary for understanding mathematical concepts could be a contributing factor in the development of math anxiety. One of these skills could be spatial ability, since it is significantly correlated with mathematics achievement yet, at the present time, this skill is rarely taught at any level of instruction in the school curriculum. The limited research in this area (Hadfield et al., 1992; Hoz, 1981; Kagan, 1987) suggests that spatial ability may be related to math anxiety.

Gender Differences in Spatial Ability and Mathematics Learning

Gender-related differences in spatial abilities have been well documented in the literature and these differences show a parallel pattern to gender-related differences in mathematical ability. In the vast majority of studies reporting significant gender differences in spatial ability, males generally outperform females on spatial tasks (McGee, 1979b; Sanders et al, 1982). It was found by Oetzel (cited in Burstein et al., 1980) that males scored better than females on spatial tests in 14 out of 19 studies, with no gender differences found in the other five studies. None of these studies reported females outperforming males.

Early studies of gender differences in spatial ability indicated that differences do not emerge until adolescence (Emmett, 1949; Fruchter, 1954). Males outperform females on spatial tasks from the elementary grades on (Kerns & Berenbaum, 1991). Although there is disagreement as to the age at which these gender differences occur, there is a belief that the magnitude of differences in spatial ability between genders widens at adolescence.

Gender differences in spatial skills have been suggested by a number of researchers as a possible reason for gender differences in both math ability and math anxiety (Bock & Kolakowski, 1973; Hadfield, 1992). Pearson and Ferguson (1989) concluded that the MRT and DAT spatial ability tests were significantly correlated with math achievement for both men and women, and that it is important to address gender differences in spatial ability as they relate to gender differences in math achievement scores. The possibility that innate gender differences in spatial ability affect mathematical achievement differently for females than for males was hypothesized by McGlone (1980). Spatial skill may function as an intervening variable in mathematics and play a differentiating role for the sexes (Friedman, 1989). In his meta-analyses, Friedman found spatial visualization to be a significant predictor of mathematical success for girls but not for boys. Interestingly, it has been found that when the effects of spatial skills have been partialed out, gender differences in mathematical achievement can be significantly reduced or eliminated (Burnett et al., 1979; Fennema & Sherman, 1977).

The current research focuses on the specific spatial ability of mental rotation, and the role of gender in its relationships with mathematics conceptualization, and math anxiety. The largest and most consistent differences found between males and females have been shown in the spatial process of mental rotation (Masters & Sanders, 1993; Stumpf, 1993; Vandenberg, 1975). In studies reporting differences in other categories of spatial ability, no conclusive results have been reached. It has been suggested by Smith (1964) that while spatial ability may not be related to mathematics ability at the beginning stages of mathematics, the ability to learn higher level mathematics increasingly depends on spatial ability. This would also account for the fact that gender differences in both mathematical achievement and spatial ability are not consistently seen until adolescence.

Some researchers have stated that gender differences in spatial skill account for gender differences in math ability and math anxiety (Bock & Kolakowski, 1973; Fennema & Sherman 1977; Hadfield et al., 1992; Kagan, 1987). Although the results are inconclusive on whether females suffer more math anxiety than males, a number of studies indicate that greater numbers of women are math anxious (Dreger & Aiken, 1957; Lupkowski & Schumacker, 1991; Skiba, 1990; Suinn & Richardson, 1972; Szetela, 1973). Adolescents who reported higher levels of math anxiety tended to have lower math performance scores (Engelhard, 1989) . Adolescence appears to be a critical age for not only studying gender differences in spatial ability and math performance, but also for studying gender differences in math anxiety. The possibility that a relationship may exist between the specific spatial ability of mental rotation, gender, math anxiety, and mathematics conceptualization forms the underlying rationale for the current research.

A Possible Role for Spatial Ability Training in Mathematics Learning

Since gender differences in spatial abilities were first noted, the question of whether training in spatial ability could mitigate these differences has been the focus of much research (Alderton, 1989; Burnett & Lane, 1980; Churchill et al., 1942). Training to improve spatial ability was thought to be a way to increase an individual's educational and occupational opportunities. In a study by Burnett and Lane (1980), the relationship between college courses/majors taken and spatial ability was examined. Students who majored in mathematics and the physical sciences performed better on spatial tests. Practice obtained through coursework in these subject areas was thought to be partially responsible for increasing spatial test scores.

While some studies (Blade & Watson, 1955; Burnett & Lane, 1980) have shown improvement in spatial scores through training in general spatial ability, more dramatic effects have been reported (Alderton, 1989; Brinkmann, 1966) in training specific spatial processes. Since the mid-1960's, the majority of research studies have reported training effects when training and measuring the specific spatial skill of mental rotation (Linn & Petersen, 1985). It appears that this aspect of spatial ability is most amenable to training and, as stated earlier, accounts for much of the variability in spatial ability differences between men and women.

The evidence that spatial visualization ability is related to mathematics achievement and may play a role in contributing to gender differences in the area of mathematics, coupled with the success of several studies in training spatial abilities, provides a compelling rationale for further research to develop training programs in spatial ability to improve scores on mathematics achievement tests and possibly reduce gender differences. A number of researchers who have recognized the moderate to high correlations between mathematics achievement and spatial ability have suggested that spatial training may be a way to improve math performance (Conner & Serbin, 1985; Linn & Hyde, 1986; Maccoby & Jacklin, 1974; McGee, 1979a; Smith, 1964). Some of the studies designed to improve mathematical problem solving through spatial training (Baldwin, 1984; Tillotson, 1984) have not been highly successful; but they have also been few in number.

The apparent lack of understanding of how specific spatial abilities are related to mathematics learning provides a strong rationale for further investigation in this area. Since the specific spatial skill of mental rotation appears to contribute meaningfully to mathematics achievement, and gender differences in spatial visualization can be reduced through training, it seems reasonable that implementation of a mental rotation training curriculum may be one way of alleviating female inequality in achievement in mathematics.

Specificity of the spatial curriculum can be considered to be an important variable. In the spatial abilities training literature, Baenninger and Newcombe (1989) concluded that spatial training needs to be test specific. Since this is considered important in the spatial training literature to improve spatial ability, it can be assumed to be an important variable for training spatial ability to improve mathematics achievement. Two of the successful studies, Moses (1980) and Rekdal (1982), had very specific curricula and instruments which measured specific spatial skills.

In summary, it is evident that the relationships between these four key variables - spatial ability, gender, mathematics achievement, and math anxiety - are complex and not well understood. The focus of this research is to investigate certain aspects of spatial ability and gender as they relate to mathematics achievement and math anxiety. Specifically, this study examines the effects of a mental rotations spatial training curriculum on mathematics conceptualization skills and math anxiety, and the influence of gender on that relationship.

Purpose Of The Study

The purpose of this research is to investigate whether incorporating a specific curriculum of mental rotations training into the conventional mathematics curriculum in a self-contained classroom setting at the Middle School (8th Grade) level can affect mathematical concepts achievement scores and/or math anxiety, and whether there is any impact of gender on this relationship. The study addresses the following questions:

1. Can implementing a specific curriculum of mental rotation affect mathematics conceptualization scores?
2. Can implementing a curriculum of mental rotation affect math anxiety?
3. Will differences in mathematical conceptualization occur between the male and female students in this study?
4. Will differences in math anxiety be observed between the male and female students in this study?
5. Will differences in math concepts scores occur by gender as a result of implementation of a Mental Rotation Curriculum?
6. Will differences in the level of math anxiety be reported by gender as a result of implementing a Mental Rotation Curriculum?

Because of the consistent gender differences in mental rotation skills evidenced in the literature review, it was necessary to control for different ability levels of mental rotation skills when addressing these research questions. This was done in this study by using mental rotation as a covariate in the statistical design, as discussed in Chapter III..

The study focuses on gender differences and treatment/control group differences in response to the mental rotations training curriculum. Subjects were divided into two groups, a treatment group and a control group. The treatment group received a specific mental rotations curriculum to supplement the normal mathematics curricula. The control group received only the normal mathematics curriculum.

Significance Of The Study

An understanding of relationships between spatial ability, gender, mathematics achievement, and math anxiety would be of great educational significance in developing curricula to improve mathematics achievement for all students, and in reducing gender differences in cognitive abilities and math anxiety. If mathematics achievement can be affected by implementing a mental rotations training curriculum, then performance differences between genders could be mitigated in all three areas: spatial ability, mathematics achievement, and math anxiety. This would have a strong impact on educational opportunities and choices in both educational and career decisions.

Operational Definitions

For the purpose of this study, spatial ability is defined as *mental rotation* - the ability to mentally rotate a two- or three-dimensional figure rapidly and accurately. This is the ability referred to as Mental Rotation in the definition of spatial ability proposed by researchers Linn and Petersen (1985).

For the purpose of this study, mathematics achievement is defined as *mathematical concepts*. Achievement in mathematics can be divided into three or four categories, as seen on standardized tests of mathematics achievement. These categories include concepts, computation, and applications/problem solving. Intuitively, achievement in higher-level mathematics would seem to require a thorough understanding of mathematical concepts, including the ability to understand the number system and the terms and operations used in higher-level mathematics.

For the purpose of this study, math anxiety is defined as a state of worry (cognitive) and emotionality (affective, anxiety, loss of ability to concentrate, and/or other emotional symptoms) brought about in an individual when faced with any mathematical task. The dimensionality of math anxiety has not yet been explored fully. In a study by Wigfield and Meece (1988), two components of math anxiety emerged from factor analyses - worry and emotionality. Worry is a cognitive component of anxiety which consists of self-deprecatory thoughts about one's performance. Emotionality is an affective component of anxiety which include feelings of nervousness, tension, and unpleasant physiological reactions to the testing situation.

Premises

The following assumptions are made concerning this study: (a) all participants answered items on the Math Anxiety Questionnaire (Wigfield & Meece, 1988) honestly, (b) no Anxiety Disorder (DSM IV 300.02) existed among the participants during the course of this study, and (c) communication among subjects during other daily activities did not appreciably affect the dependent measures.

Limitations

The following limitations apply to this research study: (a) Since this study is based on a population of average to high-average intelligence, no generalizations can be made for gifted, low, or low-average intelligence populations; (b) the only aspect of spatial ability measured is that of mental rotation, so if differences exist in other aspects of spatial ability they cannot be known; (c) this study deals with the eighth grade population, so any differences in mathematics achievement found cannot be generalized to a younger or older population; (d) the student population utilized in this study is from private Parochial schools, so generalizations of the results may not apply to all public school students of the same age; (e) use of three groups of students removed from three classrooms limits the randomization of subjects -- this restriction could not be controlled by the investigator due to scheduling and classroom availability constraints imposed by the individual schools.

Outline Of This Study

The remainder of this study is organized into four more chapters. Each chapter is described below.

Chapter II reviews the literature related to research in the areas of spatial ability and the relationship between spatial ability, mathematics achievement, math anxiety, and gender differences which exist in all these areas.

Chapter III describes the pilot study, research hypotheses, and methods used in this study. Descriptions of the research design, subjects, procedures, curriculum, and measures employed are included as part of this chapter.

Chapter IV discusses the statistical techniques used and the findings of the study. The findings involving each research question are discussed.

Chapter V provides a summary of the research problem, methodology, and findings. Conclusions and implications for further study are discussed.

CHAPTER II

Review of Relevant Literature

This literature review provides an historical perspective and background information pertinent to the areas of spatial ability and spatial ability training, and their relationships to mathematics achievement and math anxiety. Specific focus is placed on studies investigating the role of gender differences across all of these areas. The review is divided into seven sections: (a) spatial ability, (b) gender differences in spatial ability, (c) spatial ability training, (d) the relationship between spatial ability and mathematics achievement, (e) gender differences in mathematics achievement, (f) spatial training to improve mathematics achievement, and (g) the relationship between spatial ability and math anxiety.

The first section provides an historical perspective and general overview of spatial ability and discusses the confusion surrounding its definition. The review focuses on mental rotation as the key aspect (and operational definition) of spatial ability examined in this study. The various instruments which measure mental rotation skills are discussed. Separate sections are included to review studies of gender differences in spatial ability and in mathematics achievement.

The sections on spatial ability training and its relationship to mathematics achievement review studies reporting positive training results, as well as a number of studies reporting no significant benefits from training. The discussion focuses on the premises and environmental variables in the prior studies which may be responsible for the conflicting findings. These include the specific aspect (or definition) of spatial ability investigated in the prior work, the instruments used to measure the variables, the nature of the spatial training curriculum, the length of the training period involved, and the age and gender of the subjects.

In the last section, studies investigating the relationship of affective variables to mathematics learning are reviewed. Specific focus is placed on those studies examining the relationship of spatial ability to math anxiety and the role of gender in this relationship. Review of this literature found only a small number of research studies having investigated the question of whether a relationship might exist between these variables which could impact mathematics learning and achievement.

A concluding section summarizes the rationale for this study. The premises, approach, and selection of subjects for this study are based in findings from this review of the literature.

Spatial Ability

General Definition and Historical Perspective

The study of spatial ability began as a study of the factors involved in mechanical ability. The advent of factorial analysis brought about the study of individual constructs, such as spatial ability, within tests as well as among tests. It was during this time that British psychologists isolated a factor that they termed "practical" that had mechanical and spatial tests loaded on it (Fruchter, 1954). Since 1925, numerous factor analytical studies have yielded a spatial factor mathematically distinct from verbal ability (McGee, 1979a). This factor was thought to be distinct from general intelligence as well as verbal ability and was thought to be a single ability. Individuals who possessed this ability were described as being proficient at judging spatial relationships and utilizing spatial images.

While spatial abilities have been consistently noted as separate from other abilities in studies of intellectual function, they have eluded clear definition (Burstein, Bank & Jarvik, 1980). In some of the earliest work investigating spatial abilities, Thurstone (1938) developed a study of primary mental abilities in which he identified a "Space" factor which represented an ability to operate mentally on spatial or visual images. Zimmerman (1953) re-analyzed Thurstone's data and derived two spatial factors. The first was identical to Thurstone's space factor and seemed to involve mentally manipulating objects and object relationships. This factor was named the "Spatial Relations" factor. The second factor was called "Visualization" and the tests loading on this factor tended to be more difficult and less speeded than those loading on the Spatial Relations factor (Pellegrino, Alderton, & Shute, 1984).

In a summary of several research studies, Michael, Guilford, Fruchter and Zimmerman (1957) described spatial ability as consisting of three factors. These included spatial relations/orientation, spatial visualization, and kinesthetic imagery. Spatial relations is the ability to comprehend the nature of the arrangement of elements within a visual stimulus pattern primarily with respect to an individual's body as the frame of reference. Visualization is the mental manipulation of visual objects involving a specified sequence of movements. These movements require an individual to mentally rotate, turn, twist, or invert one or more objects or parts of a configuration according to explicit directions. Kinesthetic Imagery is the ability to represent a left-right discrimination with respect to the location of the human body.

A major shortcoming of the factor analytic approach to mental abilities is its inappropriateness for discovering the cognitive processes underlying an individual's performance on an intellectual task. An information processing approach to the analysis of spatial abilities has been used in an attempt to overcome some of the limitations inherent in the factor analytic approach. The goal of information processing analysis is to obtain an understanding of the basic processes and process coordination used in solving a specific problem (Pellegrino, et al., 1984). Because spatial ability is so difficult to define, an information processing perspective, which looks at the cognitive processes involved, is an attractive alternative for researchers and, hopefully, can provide answers regarding the nature of spatial ability that 70 or more years of research have failed to provide.

Based on an information processing perspective, Lohman (cited in Alderton, 1989) and Pellegrino, et al. (1984) defined spatial ability as consisting of two categories -- spatial relations and spatial visualization. Spatial relations is described as the accurate and rapid mental rotation of objects in either two or three dimensions. Spatial visualization is the execution of several cognitive processes or the repeated execution of several processes.

Linn and Petersen (1985) described spatial ability as consisting of three categories: spatial perception, mental rotation, and spatial visualization. Spatial perception involves the ability to determine spatial relationships with respect to the orientation of one's own body regardless of distracting information. Mental rotation is the ability to mentally rotate a two or three dimensional figure rapidly and accurately. Spatial visualization involves complicated multistep manipulations of spatially presented information. These operations often involve the processes required for both spatial perception and mental rotation, but can be distinguished by the multiple solution strategies needed to solve these tasks.

A summary of the categories of spatial ability based on factor analysis and information processing analysis, along with their definitions according to the following researchers include:

Thurstone(1938)

Space-an ability to operate mentally on spatial or visual images.

Michael, Guilford, Fruchter and Zimmerman(1957)

Spatial Relations/Orientation-the ability to comprehend the nature of the arrangement of elements within a visual stimulus pattern with respect to one's body as a frame of reference.

Spatial Visualization-the mental manipulation of visual objects involving a specified sequence of movements.

Kinesthetic Imagery- the ability to represent a left-right discrimination with respect to the location of the human body.

Pellegrino, Alderton, and Shute (1984)

Spatial Relations-the accurate and rapid mental rotation of objects in either two or three dimensions.

Spatial Visualization-the execution of several cognitive processes or the repeated execution of several processes.

Linn and Petersen(1985)

Spatial Perception-the ability to determine spatial relationships with respect to the orientation of one's own body.

Mental Rotation-the ability to rotate a two or three dimensional figure rapidly and accurately.

Spatial Visualization-complicated multiple step manipulations of spatial information.

Most of these researchers (Linn & Petersen, 1985; Pellegrino, et al., 1984; Michael, et al., 1957) agree on a category called *spatial visualization*, but its exact definition varies according to the researcher cited. Michael, et al. describe *spatial visualization* in the same way that Thurstone describes the category *space*, i.e. as the mental manipulation of visual objects. A category termed *spatial visualization* is also described in both the Pellegrino, et al.(1984) and the Linn and Petersen(1985) definitions; however their descriptions of the skills represented by this term are quite different from that of the other researchers.

Other categories of spatial ability such as *spatial relations, spatial orientation, spatial perception, and kinesthetic imagery* are less well defined. Linn and Petersen describe *spatial perception* as a category of spatial ability but it is defined in the same way as Michael et al. define their category of *spatial relations/orientation*. *Kinesthetic imagery* is described in Michael et al.'s categorization of spatial ability as the ability to represent a left-right discrimination with respect to the location of the human body. This definition is similar to the definition of *spatial perception* according to Linn and Petersen and *spatial relations* according to Michael, et al.

Spatial relations is a category of spatial ability which is described in both the Michael, et al. and Pellegrino, et al. categorizations of spatial ability. Pellegrino, et al. describe *spatial relations* as the mental rotation of objects in either two or three dimensions, while Linn and Petersen include ***mental rotation*** as a separate category of spatial ability.

Confusion regarding the definition of spatial ability can be readily seen in the preceding discussion. The categories of spatial ability are not the same and the definitions of individual categories are different depending on the researcher cited. Thurstone's definition is quite vague and does not describe the process skills involved. Other researchers describe various categories of spatial abilities that involve the use of multiple processes, however these processes are not included or described in the definitions. Different, even conflicting, terminology and different organizational strategies are used in the various approaches to a formal definition of spatial ability. The underlying problem seems to stem from the fact that spatial ability is not a unitary skill, but consists of multiple components (Denno, 1982).

In summarizing the literature review on spatial ability, it is apparent there is a lack of agreement on a single, formal definition of spatial ability. However, one process of spatial ability which is present in every definition of spatial ability and is defined as a separate category of spatial ability by Linn and Petersen is that of ***mental rotation***. Intuitively, the skills and strategies involved in mental rotation seem very similar to those required in geometry and other higher-level mathematical reasoning tasks.

Mental Rotation

Mental rotation is the process that allows an individual to rotate a two- or three-dimensional object when presented with visual stimuli. When rotating two-dimensional objects, an individual determines if an object in a two-dimensional picture is the same when shown in different orientations. Mental rotation of three dimensional objects enables an individual to determine if two-dimensional projected images portray objects of the same three-dimensional shape when these objects are depicted in very different orientations (Shepard & Metzler, 1971).

The most common instruments that measure mental rotation in two dimensions include the Space Relations subtest of the Primary Mental Abilities Test (PMA), (Thurstone, 1938), and both the Card Rotations test and the Form Board test from the Kit Of Factor-Referenced Cognitive Tests (Ekstrom, 1976). The most common instruments that measure mental rotation in three dimensions are the Spatial Relations subtest of the Differential Aptitude Test (DAT), (Baldwin, 1984), the Cube Comparisons test from the Kit of Factor-Referenced Cognitive Tests, and the Vandenberg and Kuse Mental Rotations Test (MRT), (Vandenburg & Kuse, 1978).

The instrument most widely used to measure mental rotation in research (Batey, 1986; Bouchard & McGee, 1977; Defries et al. 1975; Kuse, 1977; Vandenberg, 1975; Masters & Sanders, 1993) is the Vandenberg and Kuse Mental Rotations Test (MRT). This test is based on a study done by Shepard and Metzler (1971) in which eight adult subjects were presented with 1600 pairs of drawings which portrayed objects of three-dimensional shape. The subjects were to determine if the objects were the same or different. The objects consisted of ten solid cubes attached face to face to form a rigid, arm-like structure with exactly three right-angled joints connecting the arms. "Different" pairs were mirror images of each other. "Same" pairs were the objects rotated along a vertical or depth axis by 0° to 360°. These objects were chosen because they were unfamiliar and meaningless in their overall three-dimensional shape, which was thought to prevent subjects from discovering some distinctive feature possessed by only one of the two objects. This way the subject would reach a decision whether an object was the same or different by having to carry out a mental rotation.

Shepard and Metzler (1971) demonstrated that reaction times in making same/different spatial decisions regarding two pictorially presented objects were linearly related to the degree of rotation in orientation between the two objects. Reaction times increased as the angular difference increased between the objects. Another finding from this study is that no difference in reaction time existed for rotations of the two-dimensional drawings when rotated on the depth axis or rotated on the vertical axis. How the subjects went about determining the identity of the shapes was determined by introspective reports, which need to be interpreted with caution. Shepard and Metzler concluded that subjects determined the identity of the shapes by some internal strategy for an external rotation. The findings of Shepard and Metzler provide support for the existence of a mental rotation ability.

The information processing literature has developed models for studying the processes and strategies used by an individual for mental rotation. To solve a typical spatial rotation problem, an individual must encode, represent, rotate, compare, and respond (Pellegrino, et al., 1984). The strategies used in the process of rotation are of the great interest to researchers because it is thought that the choice of strategy affects performance on mental rotation tests. High spatial ability individuals exhibit a more efficient solution strategy in solving rotation problems than those individuals with low spatial ability (Pellegrino et al., 1984).

The strategy of mental rotation that is most frequently discussed in the psychological literature is that of mental rotation around standard axes (Shepard & Metzler, 1971). The standard axes are the horizontal (x) axis , the vertical (y) axis and the depth (z) axis measured relative to the observers' viewpoint. Often the object is mentally rotated in the plane of the picture so that the axis of rotation is the depth (z) axis. An object can be rotated in the horizontal (x) axis and sometimes the object is mentally rotated in depth so that the rotation axis is the vertical (y) axis (Just & Carpenter, 1985). Other strategies used to solve rotation problems have been suggested by Just and Carpenter based on a study with eight students. The strategies were reported by the students they studied. Since the number of students was small and the process of introspection so tentative, the strategy used in the majority of the psychological literature was selected as the strategy of choice for solving two- or three- dimensional mental rotation tasks.

Some researchers have suggested that mental rotation in two dimensions is easier and may reflect a different process than mental rotation in three dimensions (Linn & Petersen, 1985). Shepard and Cooper (cited in Linn & Petersen, 1985) disagree with this claim. They found no effect of dimension when rotation of two-and three-dimensional objects were compared. A two-dimensional test requires mental rotation of a stimulus object along one axis only, while a three-dimensional task requires the stimulus object to be rotated along two or more axes (Rekdal, 1982). The complexity of the object and the number of axes in which the object is to be rotated, increase the reaction time and mandate the use of an appropriate strategy to efficiently solve the problem. Appropriate strategies are required for the execution of mental rotation tasks even though different processes may or may not be involved.

Dimensionality does not affect the processes involved in solving rotation tasks. The same process is involved whether an individual is dealing with two-dimensional or three-dimensional objects. Developing an appropriate strategy and using that strategy as an analog is necessary for successful execution of mental rotation tasks.

Gender Differences In Spatial Ability

Psychologists have been interested in spatial abilities because male superiority on tasks requiring these abilities is among the most persistent of individual differences in all the abilities literature (Anastasi, 1958; Garai & Scheinfeld, 1968; Maccoby & Jacklin, 1974; and McGee, 1979b). Gender differences on Thurstone's Primary Mental Abilities Test (PMA) reported by Hobson (1947) indicated ninth grade boys exceeded girls in the Space (S) factor, as measured by the PMA spatial subtests, despite the higher IQ of the girls. In another study of gender differences using the PMA test with high school seniors, the most significant difference appeared on the space factor, favoring boys (Herzberg & Lepkin, 1954).

Of the abilities measured by the Differential Aptitude Test (DAT), boys tended to score higher than girls on space relations and mechanical reasoning, while girls score higher than boys on clerical speed, spelling, and language usage (McGee, 1979a).

In a review of literature by Oetzel (cited in Burstein et al., 1980) boys scored better than girls on spatial tests in 14 out of 19 studies. In five studies no significant gender difference was found. There were no studies where girls outperformed boys.

Shepard and Metzler (1971) reported a series of studies of mental rotation that seem to utilize spatial factors. They presented subjects with two-dimensional representations of three-dimensional figures, one of which had been rotated with respect to the other. They found that the reaction time for same/different judgments on such pairs was linearly related to the degree of rotation involved. In one sense this task measured the accuracy and speed with which a mental rotation could be performed.

Tapley and Bryden (1977) report that Shepard and Metzler had suggested that female subjects tended to show longer overall reaction times and slower rates of mental rotation than did males. Goldberg and Meredith (1975) found an immense gender difference on a paper-and-pencil modification of the Shepard and Metzler task with primary school children.

Tapley and Bryden (1977) developed a study to see if a gender difference on a mental rotation task existed. Real three-dimensional objects were used rather than pictures. Subjects were tested under three conditions (a) a match condition, in which both objects were viewed in frontal orientation, (b) a physical rotation condition in which subjects turned the rotated object back to where it started, and (c) a mental rotation condition in which subjects had to visualize the rotation. Accuracy of the same/different judgment and the response time were measured. Women performed very similar to men except in the mental rotation condition where women showed an overall slower response time. Accuracy was much poorer under the mental rotation condition than under the physical rotation condition.

Vandenberg and Kuse (1978) developed another modified paper-and-pencil test version of the Shepard and Metzler task. In large samples in which this test has been used, consistent gender differences in favor of males over every age range investigated have been reported (Bouchard & McGee, 1977; DeFries et al. 1975; Kuse, 1977; Masters & Sanders, 1993; Pearson & Ferguson, 1989; Vandenberg, 1975).

A meta-analysis of studies looking at gender differences in spatial ability examined differences in spatial perception, spatial visualization and mental rotation utilizing categories of spatial ability as defined by the researchers Linn and Petersen (1985). Spatial perception requires subjects to determine spatial relationships with respect to the orientation of their own bodies. Spatial visualization requires subjects to manipulate spatially presented information in many steps. Mental rotation requires the subjects to rotate a two- or three-dimensional figure quickly and accurately. What Linn and Petersen (1985) found was that gender differences in spatial ability are large for mental rotation, moderate for spatial perception, and small for spatial visualization.

Most of the studies of spatial ability utilized either the PMA Space subtest, the DAT Space Relations subtest, or the Vandenberg and Kuse Mental Rotations Test (MRT). The larger effects at all ages were shown on the Vandenberg and Kuse (1978) version of the Shepard-Metzler mental rotation tasks than for any other measures of mental rotation.

The magnitude of the gender difference in spatial ability varies according to the test used to measure it. The MRT has been shown in a number of studies to have a large effect size for gender and significant differences in performance with males outperforming females (Goldstein et al., 1990; Masters & Sanders, 1993; Stumpf, 1993; & Sanders et al, 1982).

Tasks on instruments thought to reflect spatial abilities are numerous and diverse. Examples of these tasks include mazes, form board, and block counting from the DAT and from the PMA. All these tasks are thought to represent distinct spatial abilities and they are tasks where males have been observed to perform better than females through the life span (Burstein, Bank, & Jarvik, 1980). The tasks are as numerous as the definitions.

Even though each study claims to be measuring spatial ability, the measures used are quite diverse and have never demonstrated either convergent or construct validity. The earlier gender difference literature on spatial ability suggests a male superiority in spatial ability but several more recent studies suggest that the magnitude of these differences is not that large, except in the area of mental rotation (Alderton, 1989; Denno, 1982; Linn & Petersen, 1985).

Earlier studies of the spatial gender difference indicated that this difference did not emerge until adolescence (Emmett, 1949; Fruchter, 1954). Maccoby and Jacklin (1974) stated that the male superiority in spatial tasks begins in early adolescence and continues into adulthood. Supporting Maccoby and Jacklin's findings was a study by Smith and Schroeder (1979) which found no gender differences among fourth grade children. In his review of the spatial ability literature, McGee (1979a) concluded that evidence suggests wide gender differences in spatial orientation skills, except in children under the age of eleven or twelve.

Contradictions to this conclusion have been found in more recent research. Strauch (cited in Baldwin, 1984) found males to be superior in spatial ability consistently across ages six to sixteen. Garai and Sheinfeld (1968) concluded that males outperform females on spatial tasks from the elementary grades through college. Gender differences favoring boys in grades two through seven were found to be significant by Guay and McDaniel (1977). Two samples of children were utilized ranging in ages from nine to thirteen years of age in a study by Kerns and Berenbaum (1991) found gender differences in mental rotation skills in both samples.

Sherman (1971) indicated that gender differences are not commonly found until the early school years, but that a wider gap develops in this difference during adolescence. This wider gap could be an explanation of why earlier studies only detected differences in adolescence and beyond. It is extremely difficult to form conclusions about results in the elementary grades.

The majority of studies involving spatial ability are done with adults and results are sometimes generalized to children. Studies with young children are few and are laden with the contradictory research results and definitional problems discussed earlier. This study chose to work with adolescent students based on the belief that, for whatever reason, gender differences are accentuated during adolescence.

Explanations for gender differences in spatial ability have ranged from differences in learning strategies (Just & Carpenter, 1985), environment (Pearson & Ferguson, 1989), neurological (McGlone, 1980), hormonal (Hier & Crawley, 1982) and genetic (Vandenberg, 1969).

Regardless of why gender differences exist, attempts have been made to try to mitigate these differences because certain occupations require a high degree of spatial ability. These occupations can be described in four categories which include; engineers, scientists, draftsmen and designers. Males make up the majority of employees in these careers. Females are at a disadvantage for opportunities in these careers because of a lack of spatial ability skills. In order to mitigate gender differences so that women can have more career opportunities, training to improve spatial abilities was thought to be a way to provide the solution.

Spatial Ability Training

Since gender differences were first noted, the question of whether training in spatial ability would mitigate these differences has been the focus of much research (Batey, 1986; Blade & Watson, 1955; Burnett & Lane, 1980; Churchill et al., 1942; Sharps & Price, 1994). Training to improve spatial ability was thought to be a way to increase an individual's ability which, in turn, would increase occupational opportunities, thus providing an additional rationale for research.

Some early training studies looked at how college courses/majors affected spatial ability. Blade and Watson (1955) administered the Space Relations Subtest of the College

Entrance Examination Board to 793 male engineering students and 201 male and female non-engineering students upon their enrollment at three universities. This spatial test measured the mental manipulation of visual images, which the authors preferred to call spatial visualization rather than spatial relations (Blade & Watson, 1955). The spatial test was administered again after one year of study. The posttest scores of the engineering students were significantly higher than the non-engineering students. These results lasted through graduation. The experiences in hobbies. high school instruction in mechanical drawing, and work experience of the high and low achieving students was examined. These studies consistently showed that achievement on the spatial test was related to the number of years of instruction in mechanical drawing in high school, work experience of a mechanical or technical nature, and hobbies which require the use of hand or machine tools and construction of objects (Blade & Watson, 1955). The conclusion was reached that persons with good spatial skills could improve those skills if provided relevant academic experience.

Churchill, et al. (1942) reported significantly greater gains on the Surface Development Test, which involved matching drawings in two and three dimensions, by an experimental group of engineering students who received training in mechanical drawing than by an untrained control group. These results contrasted with those of Faubion, et al. (1942) who found no effect in response to mixed training in mechanical drawing, blueprint reading, and metalworking. Subjects in the Faubion study received only 40 hours of training in mechanical drawing, whereas the subjects in Churchill's study received 400 hours of training exclusively in drawing. The amount and intensity of training could be a contributing factor for the lack of results in the Faubion study. Batey (1986) reported that Ranucci in 1952 found no differences in scores on four tests of spatial ability between an experimental group who received training in high school geometry and a control group.

In a later study, Burnett and Lane (1980) looked at how spatial ability could be improved through experience of relevant college courses. The instruments chosen for this study were based on the two-factor definition of spatial ability (spatial relations/orientation and spatial visualization) of Michael, et al. (1957) . Two spatial visualization tests, the Guilford-Zimmerman Spatial Visualization Test (Michael et al., 1957) and the Identical Blocks Test, were administered to a group of 142 male and female college students. After four semesters, the Spatial Visualization Test was readministered. Students who majored in the humanities and the social sciences improved less than those majoring in mathematics and the physical science. Females majoring in physical science improved more than males majoring in physical science. A significant correlation was found to exist. between improvement in tested spatial ability and the number of mathematics courses taken.

The training studies previously discussed used high school and/or college courses as the training curriculum for students. These subjects were all adults. The time period for training was quite lengthy and could have been an important variable for the significant results of these studies. The definitions of spatial ability differed in each study, but all of them involved the mental manipulation of visual images (mental rotation). These studies examined spatial ability as an important component of the mental processes required for success in engineering college course work.

In addition to the course work taken by these students their other non school experiences, such as their work and hobbies, involved the use of spatial skills and provided extra practice in these skills. The courses taken, hobbies, and work experiences suggest that experiential and school practice may be the reason for gender differences in spatial ability .

One of the most prominent cultural/environmental theorists is Sherman (1967). She hypothesized that gender differences in spatial ability are largely due to cultural factors which result in differential learning for males and females. Pointing to the fact that females are more verbally proficient at a very young age Sherman suggests that females develop a reliance on a verbal approach to problems and that spatial skills may or may not be utilized and developed. Males cannot communicate as effectively and could be using their ability to actively explore and problem solve by action rather than words. What Sherman is suggesting is that perceived gender roles influence the amount and kind of spatial practice that an individual receives.

This hypothesis of Sherman is call the Differential Experience Hypothesis and is based on the fact that boys take more math courses than girls and have more extensive experience with math outside the classroom. The greater number of courses and more experience with math outside the classroom gives boys an advantage in knowledge of math, confidence in doing mathematics and in taking standardized tests (Kimball, 1989).

This Differential Experience Hypothesis provided another issue of gender differences to be investigated. If the differences between males and females is due to cultural factors then training in spatial ability should be more beneficial to females than males. This issue of differential learning according to gender became an important issue to examine in subsequent studies done on spatial ability.

Smith and Shroeder (1979) examined two research questions. These included: does a difference in spatial ability exist among fourth grade boys and girls and will spatial visualization be differentially affected by instruction? The instrument used to measure spatial visualization, defined as the mental manipulation of visual images, was the Spatial Abilities Visualization Test. This instrument required the subjects to rotate simple two-dimensional geometric shaped pieces into a specified pattern. This task is similar to the puzzles found in tangrams. The curriculum developed for this training was similar to the task required on the test. Subjects were instructed in ten 1/2 hour tangram lessons every other school day for four weeks. Training time totaled five hours. The students were encouraged to develop their own strategies. Greater gain scores among fourth graders in the trained experimental group were reported. There was no differential learning effect between gender reported.

Based on a meta-analysis of the literature, Baenninger and Newcombe (1989) reported that spatial experience is related to good spatial test performance. The magnitude of this effect is very small possibly due to the unreliability of measures of activity participation. Another conclusion reached was that spatial training in general improves spatial test scores. Three or four sessions are needed and training needs to be test specific. Training in general spatial ability does not improve scores as much. Males and females do not differ in improvement.

In a follow up study by Smith and Litman (1979) with adolescents, greater scores in the experimental condition were reported for boys only. It was concluded that the timing of instruction of spatial visualization appeared to be crucial. One of the main differences between this follow up study and the study by Smith and Schroeder (1979) was that the training period was shorter. Another conclusion that could be reached is that the length of the training period could be a crucial variable in the attainment of this skill.

Three hypotheses were developed for a study by Blatter (1983). These were: (1) at pretest boys will outperform girls in spatial ability, (2) specific training in spatial ability will result in improved scores at posttest, and (3) girls will show more improvement than boys. The subjects involved included 48 ninth-grade students who had been identified gifted and placed in gifted programs. An experimental group consisting of 17 boys and 13 girls and a control group of 6 boys and 13 girls were utilized. The instrument used in this study to measure spatial ability was the DAT Space Relations subtest. This instrument is one of the commonly used measures of spatial ability in studies of spatial visualization.

Blatter's training program consisted of ten 1-hour sessions of instruction in two-dimensional and three-dimensional spatial reasoning, which included two sessions on two-dimensional visualization, four sessions on three-dimensional visualization, two sessions of block counting, and two sessions of graphing in three dimensions. The total training time was ten hours. Both the experimental and control groups improved significantly with the experimental group improving most. Why the control group improved could not be determined. As a support to Sherman's Differential Experience Hypothesis, girls improved more than boys although the hypothesis that training improves spatial ability was not supported.

One of the most widely cited studies in spatial training is that of Brinkman (1966). The subjects involved in this study were eighth grade students who were randomly assigned to two groups, an experimental and a control group. The spatial curriculum involved a course in basic elements of geometry. The concepts to be learned were line, point rays, angle, simple plane figures and simple solids. An object kit was included which contained geometric solids in the form of cubes, rectangular solids, pyramids, cones and other combinations. Pattern folding as well as manipulation were designed to provide tactual-kinesthetic as well as visual feedback.

This curriculum used a linear format. A linear format exists when the concepts and operations to be learned are taught in successive approximations from the easiest to the most difficult. This spatial curriculum was a programmed approach developed by Brinkman himself. Each class period for three weeks utilized the instructional model. The control group received no specific spatial instruction.

At the end of the study, the Space Relations subtest of the DAT was readministered. The experimental group showed significant improvement in visualization skills. No significant gender differences were reported but Brinkman noticed that girls in the experimental group scored an average of two raw points higher than boys.

Burnett and Lane (1980) reported a differential response favoring females. Stericker and LeVesconte (1982), Smith and Litman (1979), and Smith and Schroeder (1979) all reported no differential response favoring either gender. There is clearly no consensus within the training literature regarding differential learning by gender. Further investigation of this question is needed.

Curricula were developed in studies by Brinkman (1966), Smith and Litman (1979), Smith and Schroeder (1979),and Stericker and LeVesconte (1982) to closely match the skills needed to perform well on spatial tests. These curricula were different than those developed for other studies in that they were more specific. There is support for the contention that specificity in training is important (Baenninger & Newcombe, 1989; Brinkman, 1966; Smith & Litman, 1979; Smith & Schroeder, 1979).

Most of the studies since 1960 that have involved mental rotation of visual images have provided evidence that mental rotation scores can be greatly improved through specific training (Alderton, 1989; Batey, 1986; Linn & Petersen, 1985; Pellegrino, Alderton, & Shute, 1984). This training effect has been observed on the part of both genders.

Stericker and LeVesconte (1982) reported significant gains among college students on two different mental rotation tasks as well as two other spatial tasks for a trained experimental group when compared with an untrained group. McCloskey (cited in Batey, 1986) failed to find a response to training mental rotation but the majority of studies have reported training effects for this skill. It would stand to reason then that any training curriculum used should include mental rotation and that mental rotation would be an important aspect of spatial ability to know.

The instruments used to measure spatial ability in these studies described in this review were quite different from each other. The reason for the difference is in the many different definitions of spatial ability. Instrumentation could have affected the results of these studies. If the instruments measured the researcher's definition of spatial ability then the results could be considered valid.

In the studies conducted at a later date, the definitions of spatial ability, the curriculum developed and the instruments were well matched. These studies also reported significant differences in training effect.

Other issues which are important in training programs for spatial ability are the length of training and the age of subjects. The length of training affected the results of the Smith and Litman (1979) study and the Smith and Schroeder (1979) study. The results of the Smith and Schroeder study reported more significant findings with both genders and utilized a longer training period. Results are inconclusive about how age affects the trainability of a subject. Many of the significant results reported were from studies involving adults. There were significant results reported in three studies (Brinkman, 1966; Smith & Litman, 1979; Smith & Schroeder, 1979) which involved younger subjects. Further studies need to involve training specific skills with subjects of all ages to be able to determine what spatial ability skills are trainable and at what ages.

One issue regarding the experimental designs of the studies cited that has not been mentioned is that of practice effects. These effects are the learning that takes place when a subject takes a test twice. These practice effects can greatly inflate the results of a study. The use of a control group can help disentangle practice effects but one cannot be sure that the use of a control group has totally eliminated this problem. Krumboltz and Crystal (1960) studied practice effects with four treatment groups and found these practice effects to be significant when using the same test twice or using an alternative form of the test. Krumboltz and Crystal concluded that using alternative types of spatial tests would be the only way to eliminate this problem. This is problematic in that alternative types of spatial tests could be measuring different aspects of spatial ability.

Spatial ability is trainable and the impact of this has yet to be shown in increased opportunity in careers. Gender differences persist in occupations and intellectual abilities. Because of the consistent gender differences reported in spatial ability researchers began looking at what is the relationship between spatial ability and other intellectual abilities, one of which is mathematics. There is some speculation that spatial skill might function as an intervening variable in math achievement (Friedman, 1989).

The Relationship of Spatial Ability To Mathematics Achievement

Developing an understanding of the relationship of spatial ability and mathematics achievement has become an active area of research interest for both educators and psychologists. As early as 1935 Hamley, a mathematician and psychologist, stated that mathematical ability is a combination of general intelligence, visual imagery, and an ability to perceive number and space configurations and to retain such configurations as mental patterns (McGee, 1979a).

In 1944 Thurstone factor analyzed test data of 194 University of Chicago undergraduate volunteers (Rekdal, 1982). He found correlations of $r=.35$ between perceptual speed and mathematical reasoning, and a correlation of $r=.39$ between speed and strength of closure and mathematical reasoning.

Rekdal (1982) reports that Vernon in 1950 found a correlation between spatial ability and high level math ability. This finding suggests a relationship between spatial ability and upper levels of abstract conceptualization. Zimmerman (1954) in a review of research found that the more difficult spatial tests correlated quite highly with reasoning tests.

A study by McCallum, Smith and MacFarlane in 1979 provides support for an association between spatial abilities and mathematics achievement (Murray, 1987). A series of spatial tests and other nonverbal items were administered to British students at two different intervals. These tests were first administered at age 12 and then again at age 14. The results on these tests were compared to the students' achievement scores from national examinations administered at age 14. A factor analysis indicated that a measure of spatial orientation at both ages loaded most strongly with achievement in mathematics. It was concluded that spatial abilities are associated with the understanding of mathematics and this relationship remains stable.

In a study conducted by Connor and Serbin (1985) the relationship between visual-spatial skills and mathematics was examined with seventh and tenth grade male and female students. Correlations between the Geometry, Algebra, Arithmetic sections of a mathematics achievement test, the Math Concepts, Math Computation, Math Application subtests of the Scholastic Aptitude Test (SAT), and the DAT Space Relations subtest were reported separately for boys and girls in each grade.

For seventh grade boys significant correlations between the math tests and the Space Relations subtest of the DAT were reported for Geometry, Algebra and all of the SAT math subtests. Significant correlations for seventh grade girls were reported for Algebra only.

For tenth grade boys significant correlations were reported for the Arithmetic Concepts subtest of the Stanford Achievement Test. Significant correlations for tenth grade girls were reported in Algebra, Arithmetic Computation and Arithmetic Applications of the Stanford Achievement Test. The scores for the tenth grade students on the math achievement tests were taken from the students' sixth grade records

This study suggests that the relationship between visual-spatial skills and mathematics is significantly greater for boys than girls. Although the correlations between math achievement and visual-spatial skills were reported for the tenth grade students, in reality the actual scores were results from the students' sixth grade records. This could be responsible for misleading results. Another factor in this study which could be misleading and produce inconsistent results was the fact that the tests used to measure math concepts, math computation and math application were different for seventh grade students than for the tenth grade students.

Other studies do not support the hypothesis that a relationship between spatial abilities and mathematics exists. In an early study by Hills (1957) studied the relationships between various ability tests and performance in college mathematics. The instrument used consisted of two spatial tests, one of visualization and the other one of orientation. This Instrument came from the Guilford-Zimmerman Aptitude Survey. The subjects were 148 college students from three universities. The two spatial tests had correlations of r=.23 visualization and r=.22 orientation compared with the correlations of the verbal and reasoning tests administered which showed correlations of r=.06 for both orientation and visualization. Hills concluded that spatial ability is more important than verbal ability for achievement in college mathematics.

Fennema and Tartre (1985) and Sherman (1980) found that for boys no correlation between performance on spatial tests and tests of mathematics achievement exist. A relationship between these two variables existed for girls. Denno (1982) in a review of the literature regarding this relationship has concluded that performance on spatial measures alone is not a predictor of success in mathematics for normal students.

Most of the studies conducted have been with secondary school or college level students. Little is known on how elementary school mathematics achievement and spatial ability are related. Generalizations have been made that there is no relationship (Smith, 1964). Guay and McDaniel (1977) conducted a study to test this hypothesis. This study investigated the relationship between elementary school mathematics achievement and high and low level spatial abilities among males and females in an elementary school. The low level spatial abilities involved visualizing two-dimensional configurations. High level spatial abilities required the visualization of three-dimensional configurations and the mental manipulation of these visual images. The instruments used for this study to measure spatial ability were developed by the experimenters. Internal reliability estimates ranged from 0.56 to 0.76 and were considered adequate for use in this experiment. Subjects were classified according to two factors: mathematics achievement and gender. The children were separated into high and low math achievers on the basis of their total mathematics achievement scores on the Iowa Tests of Basic Skills, which was developed by Hieronymus et al. (1986). The findings indicated that among elementary school children high mathematics achievers have greater spatial ability than low mathematics achievers.

Although these studies refute the relationship between these variables, a moderate correlation ($r=.50$) between spatial skills and mathematics achievement has been reported in several studies (Burnett, Lane, & Dratt, 1979; Fennema & Sherman, 1977, 1978). Maccoby and Jacklin (1974) stated that a relationship between mathematical ability and spatial ability has been established.

The findings on any relationship between spatial ability and mathematics are inconsistent and the reason for this inconsistency could be that these studies have not considered that certain aspects of mathematics could be related to certain aspects of spatial abilities.

Harris (cited in Murray, 1987) indicated that spatial abilities may not be as strongly related to calculation as they are to mathematical concepts and problem solving. Piemonte (1982) had related visual spatial skills to math concepts.

Support for this relationship comes from a longitudinal study by Bruininks and Mayer (1979). This study tested 58 children on verbal and spatial measures. The instrument used was Thurstone's Primary Mental Abilities Test (PMA). Both the verbal and spatial measures, Space Relations Part I and Part II, were subtests of this instrument. These results were compared to the same children's scores in sixth grade on the Iowa Test of Basic Skills. One of the best correlations (r=.74) in this study was between Mathematics Concepts and the PMA Spatial Relations subtest.

Brinkman (1963) studied the effects of residual factors on performance on the DAT Space Relations subtest, the instructional program developed for his study, and the Iowa Tests of Basic Skills. The Math Concepts subtest of the Iowa Tests of Basic Skills was significantly correlated to both the pretest (r=.53) and the posttest (r=.58) of the DAT Space Relations subtest.

Studies have not yet determined whether certain aspects of spatial skills are more strongly associated with mathematics achievement. One could argue that the specific processes involved in spatial skills need to be identified in order to determine the relationship these processes might have to mathematical skills.

Piemonte (1982) suggests that of the visual spatial skills he found related to mathematics achievement spatial visualization is the skill most related. McGee (1979a) suggests that a relationship between spatial visualization and mathematics achievement may be stronger than the relationship between spatial orientation and mathematics achievement. Neither Piemonte nor McGee cite any research to support their statements.

Pearson and Ferguson (1989) found significant correlations between two spatial ability tests (the MRT and the DAT) and ACT scores for both men and women. Pearson and Ferguson concluded that spatial visualization was related more to mathematics achievement than other types of spatial ability. The MRT measures mental rotation ability. Pearson's and Ferguson's study is unique in that it was looking at mental rotation and its relationship to mathematics and found a significant correlation. The definition of mental rotation (the mental manipulation of two-dimensional and/or three-dimensional objects in space) suggests that it is related to geometry and calculus. This process of mental rotation is included in many definitions of spatial visualization and in some definitions of spatial relations, as well as having its own definition.

Factors which have been suggested to be causing the inconsistency in the literature concerning the relationship between spatial ability and mathematics achievement are definitional problems of spatial ability, definitional problems of mathematics achievement, lack of validity on spatial instruments, the age of subject and gender. Spatial ability has been defined in different ways by different researchers. The researchers cited in this review who have used definitions of spatial visualization are Guay and McDaniel (1977), Hills (1957), and McGee (1979a, 1979b). The researchers who used definitions of spatial relations were Brinkman (1963) and Bruininks and Mayer (1979). Spatial orientation was used as a definition for spatial ability by Hills (1957) and McGee (1979a, 1979b). Conclusions about a relationship between spatial ability and mathematics are difficult to reach when the definitions of spatial ability vary from study to study.

Examining specific processes of spatial ability and each process's relationship to mathematics achievement should help alleviate this definitional confusion. This study examined the specific spatial process of mental rotation The rationale for this is based on the fact that the relationship between mental rotation and mathematics achievement appears to be strong in Pearson's and Ferguson's study yet few studies have examined this relationship, the hypothesis that spatial ability (mental rotation) could be related to geometry, calculus, and higher level math courses, and that the process of mental rotation is included in most definitions of spatial ability.

Instruments used to measure spatial ability are diverse and varied as to the type of spatial ability they are measuring. These instruments report reliability data but little to no validity data. This is problematic in that one needs to know what an instrument is measuring before making statements about results of the test information. As the definitions of spatial abilities are narrowed, individual instruments to measure these specific definitions should prove to be more valid. This provides the rationale for this study to use the Mental Rotations Test to measure the specific spatial ability, mental rotation.

Many of the studies cited (Fennema & Tartre, 1985; Guay & McDaniel, 1977; Hills, 1957; Sherman, 1980) used overall mathematics achievement as the criterion measure. Mathematics achievement consists of three or sometimes four specialized areas. These include math concepts, math problem solving, math computation and sometimes math applications. Each of these three areas are different yet related to each other. Any one student can perform better in one area than another. If the relationship between spatial skills and math skills is to be determined, then each of these areas of mathematics needs to be investigated in relation to spatial ability.

Tartre (1990) stated that one need in research is to identify specific processes used to solve math problems that might be related to spatial skills. Each of the areas of mathematics achievement represent different processes. In studies that examined the separate areas of mathematics achievement (Brinkman, 1963; Bruininks and Mayer, 1979), math concepts showed the strongest relationship to spatial ability. Based on these studies and the need suggested to examine certain aspects of mathematics achievement in order to better isolate the relationship of mathematics achievement to spatial abilities, math concepts was selected as the specific process of mathematics achievement examined in this study.

Many early studies (Hills, 1957; Thurstone, 1944) of spatial ability and mathematics achievement were conducted with secondary school students and/or college level students. These have provided contradictory results, but overall there appears to be a relationship between these two variables.

It was assumed that a relationship did not exist with younger children (Smith, 1964). Studies by Brinkman (1963), Bruininks and Mayer (1979), and Harris (cited in Murray, 1987), have contradicted that assumption. Since a relationship between spatial ability and mathematics does exist, the way in which these variables are related, the processes involved in the relationship and the ages in which these processes are developed are crucial to understand so that curriculum in mathematics can be developed to enable every individual to reach his/her potential in mathematics.

Male superiority in understanding geometric principles and concepts has been reported by Saad and Storer in 1960 and has suggested that this gender difference may be another manifestation of the gender difference in spatial ability (McGee, 1979a). Gender differences have been widely documented in the mathematics achievement literature.

Gender Differences in Mathematics Achievement

Male superiority in mathematics has been documented almost as frequently as male superiority in spatial ability. Maccoby and Jacklin (1974) state:

> there appear to be no gender differences in performance on tasks requiring measures of number conservation followed by enumeration during the preschool years or in the mastery of numerical operations and concepts during the early school years except in disadvantaged populations. Data from large studies conducted with Head Start actually show girls to be ahead. The majority of studies show no gender differences up to adolescence but when differences are found they tend to favor boys. (p. 85)

Fennema (1974) reviewed the literature on gender-related differences and math achievement from the years 1960 to 1974. She found that when gender differences were apparent they favored males. These differences were not always apparent. Three out of four preschool studies reported no significant differences between males and females' math achievement when simple counting and number identification were involved. There was one study where females outperformed males. Nine studies of elementary age students in grades one to three failed to show any gender differences in math achievement. Twenty studies of preadolescent and early adolescent students in grades four to eleven indicated that females outperformed males consistently in computation but males outperformed females on arithmetic reasoning.

Randhawa (1991) studied gender differences in math achievement of male and female students in grades 4, 7 and 10. This study looked at math concepts, math computation, and mathematics problem solving between the years of 1978 to 1985. Results suggest that in grade 4 females increased their achievement advantage in math concepts between 1978 and 1985. In grade 7 the female achievement advantage in math concepts was reduced in 1978 but was significantly reduced in 1985 and by grade 10 males outperformed females in all three areas of math achievement and this advantage did not significantly change between 1978 and 1985.

Pearson and Ferguson (1989) found significant differences between the genders on ACT Math scores. This study involved a sample of undergraduate students. Men consistently outscored women in ACT Math.

Although gender differences are noted in many studies there has been some indication that gender differences in mathematical achievement have been decreasing over the years (Elliott, 1993). Results of a meta-analyses by Friedman (1989) indicate that the average gender difference is very small but does exist. When it does exist this difference is in favor of males. This meta-analyses indicated that sex differences in performance are decreasing. Jacklin (1989) and Caporrimo (1990) agreed that it appears that gender differences are decreasing in most areas of mathematics but stated that one exception to this trend exists. At the highest end of the mathematics achievement continuum the ratio of boys outscoring girls has remained constant.

Although there appears to be no difference in mathematical achievement in favor of boys among preschool and early school aged children, differences in favor of girls have been found in the early grades (Aiken, 1974). The age and/or grade at which gender differences first appear in favor of boys has been a debatable issue.

Fennema and Sherman (1977) state that differences in favor of boys begin to appear by grade six. Hall and Hoff (1988) found no significant differences in mathematical achievement among second, fourth and sixth grade students. They did find a trend for males in grades four and six to perform slightly higher in terms of the three areas of math performance. This provides some evidence that gender differences in math achievement begin to emerge at the junior high school level. Adolescence appears to be a critical age for studying gender differences in math performance (Engelhard, 1989).

From grade eight through college there has been a plethora of evidence supporting male superiority in mathematical achievement (Fennema, 1974, Kimball, 1989; Maccoby & Jacklin, 1974; Pearson & Ferguson, 1989; Randhawa, 1991; Steel, 1978). Aiken (1975) states that junior high boys begin to surpass girls in math achievement and to express greater interest in the subject.

These gender differences often result in women choosing not to study mathematics (Fennema, 1975; Fox, 1975). From the middle of high school and beyond girls choose not to study mathematics. Fox (1975) states that females are inclined to select themselves out of advanced math courses both in high school and college even when these students are placed in special programs for the gifted. Math backgrounds of incoming university freshmen show that an significant imbalance favoring males exists. Elmore and Vasu (1980) cited 1972 statistics at the University of California at Berkeley and found that 57 percent of incoming males compared to eight percent of incoming females had taken four years of high school math. This represents a difference of more than seven to one. Females tend to be less invested in mathematics because they perceive mathematics as not being useful to them and tend not to enroll in elective math courses (Singer & Stake, 1986). The impact for women of a limited mathematical background is great.

A strong mathematical background is essential for admission to many college majors, which are necessary for entering many professional occupations. Remick and Miller (1978) conducted a study at the University of Washington and found that incoming freshmen who lacked four years of high school math had as a choice of majors only five out of sixteen offered at the beginning of their college education. Seventy-two percent of all incoming female freshmen did not qualify for admission to almost 70 percent of the undergraduate fields of study offered at this university. In a more recent study by Lefevre, Kulak, and Heymans (1992) it was found that most women avoided majors that required even moderate amounts of mathematics even when they were as skilled at math as males and had taken equivalent amounts of math in high school.

According to Fennema and Sherman (1977, 1978) the difference in math background is one of the most important contributing factors in determining gender related differences in mathematics achievement. When the number of courses in math is controlled gender differences are erased in some cases and diminished in others.

The decision to limit one's mathematical training has serious consequences which eventually impacts career options. Bureau of Labor Statistics in 1980 indicate that men and women are still entering the scientific mathematical and technical fields as career choices in disproportionate numbers (Meece et al., 1982).

A two-year study by Armstrong (1980) on achievement and participation patterns of women in mathematics indicates that women in engineering represent one-tenth of one percent of the total number of engineers in the United States and women in physics represent two percent of all physicists. These are 1980 statistics and the numbers have changed somewhat but not that much despite efforts to improve this situation through affirmative action and scholarship programs. More recent research has indicated that most women avoided majors that required even moderate amounts of math and to not choose math-related careers (Lefevre, Kulak & Heymans, 1992; Singer & Stake, 1986).

Refuting the differential experience hypothesis is a study by Benbow and Stanley (1980) of mathematically gifted seventh, eighth, and ninth grade students. Benbow and Stanley concluded that gender differences in mathematical aptitude and achievement are a result of superior mathematical ability in males. The intention of these researchers was to examine mathematical aptitude in approximately 10,000 male and female students prior to differential course-taking. Six talent searches were conducted. To be eligible for the program students had to be in the upper three percent in mathematical ability as determined by a standardized achievement test. Both male and female participants were selected by equal criteria for high mathematical ability before participating. Female students represented 43 percent of the participants. The students took both parts of the Scholastic Aptitude Test (SAT). These included the mathematics (SAT-M) section and the verbal (SAT-V) section of this instrument. The SAT is intended to measure math reasoning and is designed specifically for juniors and seniors in high school.

The students participating in this study were taking a test designed for students who were four to five years older. Both genders from each grade level performed equally well on the SAT-V. A large gender difference in mathematical ability in favor of boys was observed and mean scores of boys were on an average 35 points higher than girls (Fox, 1975). This difference remained across all six talent searches. Benbow and Stanley felt that this data contradicted the hypothesis that differential course-taking accounts for gender differences in math ability. They concluded that males were superior in mathematical ability which could be related to male superiority in spatial ability.

The importance of math is quite evident in our society especially for school advancement, career opportunities and choices. Because of this importance researchers have been focusing on the possible causal factors of gender differences in mathematics achievement.

Gender-related differences in spatial abilities have been well documented in the literature and these differences show a parallel pattern to gender-related differences in mathematical ability. These abilities appear to show a logical relationship to each other.

Werdelin (1961) investigated the factorial structure of and gender differences in geometry and spatial ability utilizing a battery of 23 tests of geometry, reasoning and spatial ability. The sample consisted of 453 high school male and female students from schools limiting entry to students with superior academic performance.

This factor analysis resulted in the following conclusions: (1) spatial factors were highly related to geometrical construction and abstraction for both genders; (2) spatial ability is highly related to problem solving ability in males; (3) a genuine gender difference is evident in spatial ability; (4) two factors load significantly on geometrical tests --reasoning and spatial visualization; (5) females are superior to males in verbal and numerical fields and the ability to prove geometric theorems; and (6) males are superior to females in reasoning, spatial ability, abstraction, geometric construction and problem solving. Although Werdelin was not willing to definitely conclude that the data indicate that spatial visualization ability and geometry ability are related, he felt that there is a strong reason to believe a connection between the ability to visualize and geometric ability exists.

Guilford, Green and Christensen (cited in Sherman, 1967) in 1951 concluded that spatial visualization ability aided in solving mathematics problems, regardless of gender. French (cited in Sherman, 1967) in 1951 and again in 1955 showed that successful achievement in mathematics for both males and females depends to some extent on the use of spatial visualization skills.

Smith (1964) has suggested that while spatial ability may not be related to mathematics ability at the beginning stages of mathematics, the ability to learn higher level mathematics increasingly depends on spatial ability. Adolescence is typically viewed as a period in which the "higher" cognitive processes become part of an individual's cognitive repertoire (Kail, Pellegrino, & Carter, 1990) This would account for the fact that gender differences are not seen until adolescence.

Fennema (1975) disagrees with this suggestion by Smith. She feels that a major change has taken place in most K-12 mathematics curricula. Geometry has become a part of the entire mathematics curriculum and more emphasis is placed on the structure of mathematics and its underlying principles. In 1964 spatial ability was not necessary for elementary mathematics, but because of the changes in today's curriculum requirements, the need to develop spatial skills is becoming increasingly important. Fennema does not base this idea on any empirical evidence. Fennema (1975) does state:

> Little is known concerning the impact of spatial ability on the acquiring of these prerequisite mathematical ideas in which all later mathematical knowledge is based. It appears to be of the utmost importance. (p.40)

In a study by Pearson and Ferguson (1989) reliable gender differences were found between two spatial tests (the Mental Rotations Test and the DAT) and the ACT Math achievement scores. It was concluded that the spatial ability tests were significantly correlated with math achievement for both men and women and that it is important to address gender differences in spatial ability as they relate to gender differences in achievement scores.

Friedman (1989) stated that spatial skill might function as an intervening variable in mathematics and play a differentiating role for the sexes. In his meta-analyses he found spatial visualization to be a significant predictor of mathematical success for girls but not for boys.

While the majority of studies confirm a relationship between mathematics achievement and spatial ability which explain the reason for gender differences, a study by Armstrong (1980) did not find a difference in male and female spatial abilities but did find differences in mathematical reasoning.

A longitudinal study by Fennema and Tartre (1985) investigated how sixth, seventh and eighth grade boys and girls used spatial visualization skills in solving word problems and fraction problems. All differences found between genders were small. Evidence from a meta-analyses of the literature indicates that gender differences on cognitive tasks, such as math, are small and declining and gender differences in spatial ability are heterogeneous and declining (Linn & Hyde, 1986).

A difference in research results between studies that support a strong relationship between mathematics achievement and spatial ability as an explanation for gender differences in math achievement and studies that do not support this relationship can be explained in terms of the variations in the studies. Differential effects of age groups, different test instruments used, level and/or category of spatial ability measured and methods and procedures all can affect research studies.

Some evidence has indicated that spatial visualization ability contributes to mathematics achievement and plays a role in contributing to gender differences in the area of mathematics. A lack of understanding of spatial ability and how it is related to mathematics provides a strong rationale for further investigation in an attempt to reduce gender differences. Since the skill of spatial visualization appears to contribute meaningfully to mathematics achievement, and gender differences in spatial visualization can be reduced through training then it seems logical that the trainability of visual skills should be considered as one possible way of alleviating female inequality in achievement in mathematics.

Spatial Training To Improve Mathematics Achievement

A number of researchers who have recognized the moderate to high correlations between mathematics achievement and spatial ability have suggested that spatial training may be a way to improve math performance (Conner & Serbin, 1985; Linn & Hyde, 1986; Maccoby & Jacklin, 1974; McGee, 1979a; Smith, 1964).

Moses (1980) conducted a 12-week study with approximately 170 students in grades five, nine and college. Her study involved two groups, an experimental and a control group. An instructional program was developed by Moses to teach visual thinking skills. She examined the instructional effects of visual thinking on spatial visualization, mathematical problem solving and reasoning tasks.

During the instructional period Moses administered a sequence of lessons concerned with visual thinking tasks to the experimental classes; the control classes received the normal mathematics instruction by the regular teacher. The first part of the instructional phase began with the study of the cube. Students compared cubes with other three-dimensional shapes and sorted out relevant from irrelevant characteristics of the various shapes. Perspective experience and drawing tasks were included. The cube was taken apart and the different arrangements of six squares were studied in an attempt to decide which ones form cubes. Rotations of these arrangements were made.

The second part of the instructional period involved two-dimensional configurations where students used geoboards to construct specified shapes and used the method of enlarging a n-sided polygon to a (n+1)-sided polygon. Area formulas were studied and analyzed with complex polygons. This instructional unit involved hands-on manipulation and drawing.

The total amount of time spent in training was not specified. It is assumed that since the researcher developed a 12 week training program 11 of which were spent in training that the minimum number of training hours were six hours. More than six hours were probably devoted to training.

Instrumentation included seven tests. Four were spatial visualization tests, two were reasoning tests and one was a problem-solving test. The spatial tests included: Mental Rotations Test by Stafford (cited in Moses, 1980), and the Punched Holes, Form Board, and Hidden Figures tests from the Kit of Factor-Referenced Cognitive Tests. Spatial ability was defined as spatial visualization which in this study includes the mental rotation of two-and three-dimensional images as part of its definition.

The reasoning tests included the Nonsense Syllogisms test from the Kit of Reference Tests for Cognitive Factors. This test measures the ability to reason from givens, two statements with nonsensical content, to a conclusion. The Reasoning Test (Werdelin, 1961) was developed to measure inductive reasoning and was similar in nature to a logic test. The Reasoning Test correlated significantly with the problem solving test (correlation coefficient .47, $p < .0001$).

The Problem-Solving Test was constructed by Moses to measure problem-solving performance and types of solution processes. There were ten mathematics problems on this instrument. A subject was instructed to demonstrate everything he/she thought about as he/she attempted to solve the problems

This training program resulted in significant positive effects on the Mental Rotations Test and the Reasoning Test but did not result in any effects for problem-solving performance. Significant effects of instruction were noted for females on the Mental Rotations and Form Board Tests. Moses concluded that these findings provided evidence that spatial ability can be altered through instruction and that females in particular benefit from this training. It was assumed from these results that this training would affect math reasoning since the Reasoning Test was significantly correlated with the problem solving test.

Other conclusions that were reached by Moses included:

1. Age plays a significant role on problem-solving, spatial visualization and on reasoning tasks. The older the student the better they performed.
2. Gender plays a significant role on spatial visualization and reasoning tasks with males performing better than females on spatial tasks and females performing better on reasoning tasks.

3. The gap between males and females got noticeably wider from fifth-grade to ninth grade to university students on spatial visualization tasks. These results did not carry over to reasoning and problem solving in this study.

Rekdal (1982) studied the effects of a linear versus a randomly sequenced spatial curriculum on the improvement of spatial and mathematical problem-solving ability on 39 academically gifted fifth and sixth grade male and female students. The subjects were divided into two groups: one group received the linear spatial curriculum and the other group received the randomly sequenced curriculum.

Rekdal defined spatial ability as consisting of three abilities. These include: Static Analysis, mental imagery requiring the separation or location of an item or figure embedded within a more complex pattern or design in which no movement is involved; Kinetic Analysis, spatial orientation where one recognizes an object which has been rotated along one or more of its axes; and Transformational Analysis, spatial visualization where movement of part or parts of an object occurs.

The instruments used to measure spatial ability as defined by Rekdal included: Hidden Patterns test and the Hidden Figures test, the Card Rotations test, the Cube Comparisons test, Form Board test, Paper Folding test and Surface Development test. The math test used was the Arithmetic Aptitude test which consisted of word problems which required general reasoning ability. All test instruments used were from the Kit of Factor-Referenced Cognitive Tests. The students were pretested with these instruments and after a 18 week instructional program were posttested with the same instruments.

The spatial training program, which was developed by the investigator, consisted of activities which involved all three spatial abilities and ranged from easy tasks to more difficult tasks. In the linear sequence the spatial activities moved from the lowest level of spatial skill (Static Analysis) to the next highest level (Kinetic Analysis) and then to the highest level (Transformational Analysis). The training activities consisted of three times a week with the length of time varying according to the activity involved.

Significant differences were found between pretest measures and posttest measures of spatial ability, which suggests that the, linear curriculum could significantly improve spatial abilities to a greater degree than a randomly sequenced curriculum. Significant math improvement in both groups provided evidence that spatial training could affect math ability. Rekdal states that this evidence should be interpreted cautiously since a control group had not been used which makes it difficult to separate out practice effects from actual instructional effects of the curriculum.

A training study by Connor and Serbin (1985) found that after a short training session spatial visualization contributed significantly to predicting mathematics achievement. Recommendations by these researchers were that further research needs to be done relating visual-spatial skills to mathematics achievement and that educators need to concern themselves in developing spatial skills.

A study with college students by Ferrini-Mundi (1987) incorporated a training period of eight weeks with several hundred students enrolled in a calculus class. The results indicated that the training improved the ability of women to visualize when solving calculus problems. It was concluded that spatial training could benefit women more than men.

A number of studies that have attempted to show mathematical problem solving can be improved through spatial training have not been highly successful. Tillotson (1984) developed a study to investigate how instruction in spatial visualization affected a student's ability levels for spatial visualization and math problem solving. Tillotson defined spatial visualization as:

> the ability to recognize the relationship between parts of a given visual configuration and the ability to mentally manipulate one or more of those parts. (p.27)

The instruments used to measure spatial visualization were the Card Rotation, Cube Comparison and Punched Holes tests from the Kit of Factor-Referenced Cognitive Tests.

The problem solving inventory was developed by the investigator using problem sets from other instruments used in research studies. Items were either spatial, analytic, or spatial and analytic.

This eight week instructional unit was administered to 102 sixth grade students. The students were divided into an experimental group and a control group. The experimental group received instruction in spatial skills consisting of one 45 minute session for eight weeks, which involved 51 total hours of instruction. The instructional unit included activities which required students to manipulate three dimensional models, imagine movement of these models, practice rotation with two-dimensional drawings and do some problem solving activities. Tillotson found that spatial visualization is a trainable skill but visualization instruction did not significantly affect problem solving performance.

Baldwin (1984) studied whether the practice effects due to spatial skills training would transfer to improve mathematics achievement for intermediate elementary students. Baldwin defined spatial ability as spatial orientation, the ability to remain unconfused in the presence of a complex visual pattern, and spatial visualization, the ability to mentally manipulate objects.

The subjects included 88 fifth and sixth grade students from four intact classes. These students were divided into two groups, an experimental group and a control group.

The instructional unit consisted of ten lessons in each of the two defined spatial skills. The instruction consisted of twenty or thirty-minute periods for twenty days. This training period totalled between three to ten hours of instruction.

The instruments utilized in this study were the Group Embedded Figures test (to measure spatial orientation), the DAT Spatial Relations subtest (to measure spatial visualization), the Cognitive Abilities test (used to measure intelligence) and the Iowa Tests of Basic Skills (used to measure mathematics achievement). The sum of the two subtests, math concepts and problem solving was used as the indicator of mathematics achievement. These subtests were examined separately.

Several conclusions from this study were reached. These included: (a) spatial skills can be improved through instruction, (b) instruction in spatial skills significantly improves the performance of female subjects, and (c) that instruction in spatial skills showed no effect on mathematics achievement.

These studies reviewed have shown conflicting results. Moses (1980) and Rekdal (1982) found that instruction in spatial ability improved reasoning. Connor and Serbin (1985) found that after training spatial visualization was a significant predictor of mathematics achievement and Ferrini-Mundi (1987) found that training spatial ability resulted in improvement for women in calculus.

Tillotson (1984) and Baldwin (1984) found no improvement in mathematics achievement in their studies. The instruments used to measure mathematics achievement differed in each study even more than the instruments used to measure spatial ability. This makes it difficult to determine if these instruments were measuring the same math skill.

Problem solving was the skill that these research studies attempted to improve. The improvements were shown in math reasoning and overall achievement. Problem solving is only one aspect of mathematics achievement and other aspects need to be looked at. Baldwin's study came the closest to looking at more than one aspect of mathematics achievement. Both math concepts and math problem solving were measured in her study but the composite of these two subtests was used instead of examining each subtest individually. Problem solving may not be the aspect of mathematics achievement that needs to be examined.

Mathematical conceptualization was found to be related to spatial ability and it was suggested in several studies that this relationship could be stronger than the relationship of spatial ability to the other areas of mathematics (Brinkman, 1963; Bruininks & Mayer, 1979; Murray, 1987; Piemonte, 1982). In this research, math concepts was chosen as the specific aspect of mathematics achievement for study because of the significant correlations found between math concepts and spatial ability in the Bruininks and Mayer study (r = .74) and in the Brinkman study (r = .58).

The instruments used to measure spatial ability in Moses, Rekdal, Tillotson and Baldwin's studies were very similar. Moses used a mental rotations test that the others did not use. Many of the instruments were the same in several of the studies reported; such as the Hidden Figures, Card Rotation, Cube Comparisons and Punched Holes. The instruments used to measure spatial ability in these studies probably did not contribute to the differences in results observed between them.

Definitions of spatial ability differed in each of the studies reviewed. Rekdal (1982) defined spatial ability in a different way than any of the research studies reviewed. She used three abilities-static analysis, kinetic analysis, and transformational analysis. Her definitions of these three abilities are not much different than Linn and Petersen's (1985) three categories of spatial ability- spatial perception, mental rotation, and spatial visualization.

The category of spatial visualization was included as the definition or part of the definition of spatial ability in all of the studies reviewed and is one of the most common categories of spatial ability seen in research studies (Alderton, 1989; Baker, 1990; Burnett & Lane, 1980; Connor & Serbin, 1985; Moses, 1980; Rekdal, 1982; Smith & Litman, 1979; Smith & Shroeder 1979). The definitions of spatial ability were somewhat different but yet similar enough that definitional confusion was minimal and accounted for few of the differences in the results reported in these reviewed studies.

The subjects in these studies ranged from fifth grade students to college. The subjects of these studies were very similar except for the college students used by Moses and Ferrini-Mundi and probably were not a contributing factor in the different results obtained by the studies.

Moses, Rekdal and Baldwin all included fifth grade students in their studies while Rekdal, Tillotson, and Baldwin also included sixth grade students. Connor and Serbin involved eighth grade students in their study. Significant findings with these children were found in Moses Rekdal's and Connor and Serbin's studies.

Moses (1980) and Ferrini-Mundi (1987) used college students in their studies. Ferrini-Mundi found that training visual spatial skills improved female achievement in calculus. Moses found that age does play a significant role on problem-solving tasks, on spatial visualization tasks and on reasoning tasks..

These findings suggest that although gender differences in spatial and mathematical performance can be seen at younger ages a wider gap develops as children get older. This wider gap has been suggested to occur at adolescence by a number of researchers (Engelhard, 1989; Sherman, 1971; Randhawa, 1991; Sanders et al., 1982). Based on the premise that wider gaps in spatial performance occur at adolescence this study chose to use eighth grade students as subjects.

The curricula of these studies included many of the same objectives. All of the studies involved manipulation of two- and three-dimensional objects. Moses included the drawing of three-dimensional objects and related these objects to math concepts by studying the area of figures. Moses and Tillotson included hands-on manipulation with three-dimensional objects. Baldwin included "visual disembedding" tasks requiring a subject to discriminate figures in pictures where these figures are surrounded by additional stimuli. The curriculum for each study, which included many of the same objectives with a few exceptions, did not appear to result in the differences observed between the studies.

Specificity of the spatial curriculum can be considered to be an important variable. In the spatial abilities training literature, Baenninger and Newcombe (1989) concluded that spatial training needs to be test specific. Since this is considered important in the spatial training literature to improve spatial ability, it can be assumed to be an important variable for training spatial ability to improve mathematics achievement. Two of the successful studies, Moses (1980) and Rekdal (1982), had very specific curricula and instruments which measured specific spatial skills.

One of the main differences between Moses and Rekdal's study and Tillotson and Baldwin's study was in the time period for the training program. Moses had a 12 week training program but the amount of contact hours was not specified in her paper. Rekdal had a 18 week training program which involved her subjects meeting three times a week. She did not state in minutes the number of contact hours but one could assume at least 30 minutes 3 times a week, probably more. This would amount to approximately 51 total hours of instruction.

Tillotson's study lasted eight weeks with her students meeting once a week for 45 minutes. This totalled approximately five hours of instruction. Baldwin's study lasted for twenty days and consisted of twenty to thirty-minute meeting periods. This training period totalled between three to ten hours of instruction. The length of the training period was significantly shorter in these two studies which could account for no improvement in mathematics.

Several training studies (Blade & Watson, 1955; Churchill, et al., 1942) to improve spatial ability which were significant had a lengthy training program. It would seem that if the length of time a training program is administered is an important variable for the improvement of spatial ability, then it would also be an important variable for the improvement of math scores. Based on this premise, this study chose to use an eighteen week training period, similar to the one used in Rekdal's study.

Spatial ability has been suggested to be one of the cognitive skills that contributes to mathematics achievement. Training specific types of spatial ability has been successful in reducing gender differences. Gender differences in mathematical achievement parallel gender differences in spatial ability. Linn and Hyde (1986) suggest that if spatial ability were found to contribute to mathematics performance, gender differences could be reduced through training. Visual-spatial training has shown some promise in reducing gender differences in math achievement.

Another variable which has a large impact on math achievement is math anxiety. Math anxiety is known to be a contributing factor for the avoidance of mathematics and the poorer performance in mathematics achievement on the part of women (Englehard, 1990; Hyde et al., 1990; Meece et al., 1982). Gender has been shown to be associated with math anxiety. It has generally been found that females exhibit more math anxiety than do males (Betz, 1978; Donady & Tobias, 1977; Lupkowski & Schumacker, 1991; Shanklin, 1978; Skiba, 1990).

If all of the following statements are true: that spatial ability is a necessary skill for the learning of mathematics; that significant gender differences in mathematics achievement, spatial ability and math anxiety exist; that females have more difficulty with spatial skills and exhibit more math anxiety, then an intriguing question remains. Could a deficiency in spatial ability be a contributing factor in the development of math anxiety in female students?

The Relationship of Spatial Ability To Math Anxiety

Several research studies examining the effects of affective variables on mathematics learning by Aiken (1970, 1976) Lefevre et al. (1992), Englehard (1989) and Fennema and Sherman (1977) have suggested that math anxiety may be the single most important factor to mathematics avoidance and poor mathematics performance. Math anxiety involves emotional and/or physical discomfort when faced with any mathematical task. Individuals suffering math anxiety often complain of emotional symptoms such as increased heart rate, perspiration, and loss of ability to concentrate when confronted with mathematical tasks (Posamentier & Stepelman, 1986; Tobias, 1990). Often these individuals will go to any length to avoid any task dealing with numbers. Math anxiety is not only detrimental to a person's physical health but often interferes with mathematics achievement.

Numerous studies have shown an association between high math anxiety and low mathematics achievement (Frary & Ling, 1983; Suinn & Edwards, 1982). Englehard (1989) found an inverse relationship between math anxiety and performance. Students who reported higher levels of math anxiety tended to have lower scores on a math performance test.

Hadfield, Martin, and Wooden (1992) examined a number of predictors of math anxiety for Navajo middle school students. Native Americans are known to not participate in mathematical and scientific careers, and achievement in mathematics is relatively poor. Hadfield, Martin, and Wooden found that the predictors examined were inversely related to math anxiety for Navajo middle school students. These predictors included spatial skills, discriminatory skills, and persistence orientation. The results indicated that persistence orientation had the strongest negative relationship to math anxiety and spatial and discriminatory skills followed in that order (Hadfield, et al., 1992). Although these results cannot be generalized to other groups these results suggest that an inverse relationship between spatial ability and math anxiety could exist and provide a rationale for further research.

A study was reported by Kagan (1987) to test the hypothesis that math anxiety is associated with rigidity in problem solving and with deficiencies in spatial skill. The subjects involved 204 college students. What she found was that spatial skills among females were related both to anxiety and to scores on math tests. Anxiety was the strongest factor with regard to the performance of females on math and reasoning tests. These same results did not apply to males.

In a preceding study by Hoz (1981) a scale was designed to investigate the relationship between cognitive rigidity and poor math performance a spatial element was found. Rigidity has been related to math anxiety. The cognitively rigid learner can be characterized as relatively tense, compulsive, conservative, and conformist (Dean & Garabedian, 1981).

Hoz developed a unique test of rigidity as it applies to geometry. Rigidity in problem solving is defined as an unwillingness to change strategies during a task even when an alternative strategy would be more efficient. What Hoz found was that math performance was poorer for those who only used one strategy to solve all problems. It appears that Hoz's findings create parallels between strategy use for math performance and strategy use for spatial ability performance.

Some researchers have stated that gender differences in spatial skill account for gender differences in math ability and math anxiety (Bock & Kolakowski, 1973; Fennema & Sherman 1977; Hadfield et al., 1992; Kagan, 1987). A growing body of research on gender differences in math have been showing that fear of math combines with traditional societal influences on women to create disproportionate number of math avoiding and/or math anxious females (Donady & Tobias, 1977; Lefevre et al., 1992; Singer & Stake, 1986). Although the results are inconclusive on whether females suffer more math anxiety than males, a number of studies indicate that greater numbers of women are math anxious (Dreger & Aiken, 1957; Lupkowski & Schumacker, 1991; Skiba, 1990; Suinn & Richardson, 1972; Szetela, 1973).

There is some disagreement in the literature about the relationship of math anxiety to gender and mathematics achievement. Rounds and Hendel (cited in Olson, 1985) and Singer and Stake (1986) concluded that math anxiety is not limited to females but is prevalent among all students who are poorly prepared in math and that reduction in math anxiety is not related to mathematics achievement when math grades are used as the criterion for achievement.

Adolescence appears to be a critical age for not only studying gender differences in math performance but studying gender differences in math anxiety. Engelhard (1989) reported that adolescents who reported higher levels of math anxiety tend to have lower scores on math performance. Berstein (1992) reported that at age 12, males felt slightly more math anxiety than females did, but by age 14 females were more math anxious than males were.

Summary

A number of pertinent generalizations can be drawn from this review of literature. These include: (a) Spatial ability is important for math achievement: (b) Gender differences exist in the areas of spatial ability, mathematics achievement and math anxiety; (c) These gender differences become noticeable in the adolescent years; (d) The use of strategy is important in both mathematics achievement and spatial performance; (e) Since spatial ability is not included in math curricula, the development of strategy is left entirely up to each individual; (f) Men and women differ on the strategies used to solve spatial problems which has been suggested to result in poorer performance on tests of spatial ability for women; (g) Training in a specific type of spatial ability can help develop strategies for problem solving and could increase performance in mathematics achievement; (h) Training in a specific type of spatial ability may result in a reduction of anxiety as a result of development of strategies for solving spatial problems and increased mathematics achievement; and (i) There has been very little research done examining the possibility that spatial ability could be related to math anxiety.

These generalizations provide the rationale and form the major premises for this study, as detailed in the next chapter. This study investigated whether implementing a specific mental rotations spatial training program could improve mathematics achievement and reduce math anxiety, either generally or gender specifically, in adolescent male and female students.

CHAPTER III

Method And Research Design

This chapter discusses and describes the research questions addressed by this study, the research hypotheses formulated for the study, the statistical design, and the method. Included in the method section are a discussion of the pilot study, the procedures followed in carrying out the current research, the assessment instruments, the spatial training curriculum, and data analysis.

Research Questions

In reviewing the literature presented in Chapter II, it is evident that a relationship exists between spatial ability, math achievement, and math anxiety. There is still considerable confusion in understanding this relationship. Reasons for the confusion of the findings include the fact that spatial ability is not a unitary skill but consists of numerous related but disparate skills. Further, math achievement consists of at least three or four related but separate skills. This confusion has led to conflicting findings as to the role of spatial ability training in relation to mathematics learning, math anxiety, and the gender differences observed across all of these areas. This study addresses this confusion by investigating the following six specific research questions which focus on this research problem:

1. Can implementing a specific curriculum of mental rotation affect mathematics conceptualization scores?
2. Can implementing a curriculum of mental rotation affect math anxiety?
3. Will differences in mathematical conceptualization occur between the male and female students in this study?
4. Will differences in math anxiety be observed between the male and female students in this study?

5. Will differences in math concepts scores occur by gender as a result of implementation of a Mental Rotation Curriculum?
6. Will differences in the level of math anxiety be reported by gender as a result of implementing a Mental Rotation Curriculum?

Because of the consistent gender differences in mental rotation skills evidenced in the literature review, it was necessary to control for different ability levels of mental rotation skills when addressing these research questions. This was done in this study by using mental rotation as a covariate in the statistical design.

Research Hypotheses

In order to address the research questions previously discussed, the following null hypotheses are formulated.

Null Hypothesis H1: It is hypothesized that there will be no significant differences on mathematical conceptualization and math anxiety scores (compared simultaneously) between the treatment and control groups, while controlling for mental rotation.

Null Hypothesis H2: It is hypothesized that there will be no significant differences on mathematical conceptualization and math anxiety scores (compared simultaneously) between males and females, while controlling for mental rotation.

Null Hypothesis H3: It is hypothesized that there will be no significant interaction between treatment condition and gender on mathematical conceptualization and math anxiety (compared simultaneously), while controlling for mental rotation.

Statistical Design

This research examined the following variables: mental rotations training, gender, mathematical conceptualization, and math anxiety. The treatment/control condition (with the treatment group receiving training in mental rotation and the control group receiving no training) is one independent variable, and the other independent variable is gender. The dependent variables are math concepts and math anxiety. In order to control for the different ability levels of mental rotation skills in the subjects, this specific skill was used as the covariate.

The design selected for this research employed two conditions (treatment, nontreatment) x two gender groups (male, female). In order to control for differences between subjects, mental rotation was used as the covariate. Two dependent variables, math concepts and math anxiety, were measured. This design is known as a 2x2 factorial multivariate analysis of covariance (MANCOVA). Unequal numbers of subjects initially were assigned to each group.

Because of the possible interrelatedness of all the variables and the use of two dependent variables, a multivariate design was selected for this study. A multivariate design uses one or more independent variables and two or more dependent variables. Other reasons for choosing a multivariate design as suggested by Stevens (1992) include:

1. When using multiple dependent variables a more complete and detailed description of the phenomenon under investigation occurs. This study provides a more detailed description of the effects of treatment and gender on math concepts and the effects of treatment and gender on math anxiety.
2. Less error exists since the number of F tests conducted is minimized. A major problem resulting from the performance of a series of F tests on a set of data is the fact that the more comparisons one conducts, the more type I errors one will make when the null hypothesis is true. Reducing the number of F tests conducted reduces the probability of making a Type I error.

Method

Pilot Study

A pilot study was conducted for the following reasons: (1) to develop procedures which could be utilized in carrying out this pilot study, (2) to provide evidence that the Mental Rotations Test and the Math Anxiety Questionnaire (Wigfield & Meece, 1988) are appropriate for use with the middle school population, (3) to use correlational analysis to determine whether a relationship exists between mental rotation, as measured by the Mental Rotations Test, and math concepts, as measured by the Math Concepts subtest of the Iowa Tests of Basic Skills, (4) to use correlational analysis to determine whether a relationship exists between mental rotation, as measured by the Mental Rotations Test, and math anxiety, as measured by the Math Anxiety Questionnaire, (5) to use correlational analysis to determine the relationship between math anxiety, as measured by the Math Anxiety Questionnaire, and math concepts, as measured by the Math Concepts subtest of the Iowa Tests of Basic Skills, (6) to provide evidence for the use of mental rotation as a covariate for a MANCOVA design and the use of this specific design for the current research, and (7) to select a population sample from a private school to determine if this sample of students would vary enough on their test performance so as to provide a representative sample for the current research.

Bruininks and Mayer (1979) found significant correlations between the Space Relations subtest of the DAT and the Iowa Tests of Basic Skills. Pearson and Ferguson (1989) found significant correlations between both the Mental Rotations Test and the Space Relations subtest of the DAT and ACT scores for both men and women. Gallagher (1992) states that the partitioning of spatial ability into component parts related to specific areas of math achievement will be helpful in determining the relationship between visual spatial skill and mathematics. The skill of mental rotation was selected as one of the variables to be examined in this pilot study for the following reasons: (1) mental rotation is a specific skill of spatial ability which is included in all of the definitions of spatial ability, (2) Pearson and Ferguson found significant correlations between mental rotation and ACT scores utilizing the Mental Rotation Test, and (3) Gallagher suggests partitioning spatial ability into its component parts in order to study its relationship to mathematics.

Brinkman (1963) found significant correlations between the Space Relations I and II subtests of the Primary Mental Abilities Test and the Math Concepts subtest of the Iowa

Tests of Basic Skills. Tartre (1990) suggested that certain processes used to solve math problems might be related to spatial skills. Since Gallagher (1990) stated that the partitioning of spatial ability into its component parts related to specific areas of math achievement could be helpful in determining the relationship between visual spatial skill and mathematics, it can be reasoned that math achievement should be partitioned into its component parts for effective study. Earlier, Brinkman (1963) had found significant correlations between spatial ability and the Math Concepts subtest of the Iowa Tests of Basic Skills. Based on the findings of Gallagher and Brinkman discussed above, the specific skill of mathematical conceptualization was selected as one of the variables to be examined in this pilot study.

An inverse relationship between spatial ability and math anxiety was found in a study by Hadfield, Wooden, and Martin (1992). A study by Kagan (1987) found that spatial skills among females were related to math anxiety, but this relationship was not found among males. Very little research has been conducted to determine if a relationship between spatial ability and math anxiety exists. Since a relationship is thought to exist between spatial ability and math anxiety, math anxiety was selected as one of the variables to be examined in this pilot study.

Numerous studies have shown math anxiety to be inversely related to math achievement (Lefevre et al., 1992). Engelhard (1989) found that students who reported higher levels of math anxiety tended to have lower scores on a math performance test. Few studies have examined the relationship between math anxiety and the specific aspects of math achievement. This pilot study examined the relationship between math anxiety and math concepts.

Based on the review of literature, the following research questions were formulated for the pilot study:

Research Question 1: Is there a significant relationship between mental rotation and math concepts?

Research Question 2: Is there a significant relationship between mental rotation and math anxiety?

Research Question 3: Is there a significant inverse relationship between math anxiety and math concepts?

The following procedures were followed in conducting the pilot study: (1) subjects were selected, (2) instruments to test the variables of mental rotation, math concepts, and math anxiety were selected, (3) the tests were administered, and (4) correlation coefficients were computed to provide answers to the research questions.

The subjects for this pilot study included sixth, seventh, and eighth grade students from St. John's Catholic School in Bartlesville, Oklahoma. These students ranged in age from twelve years to fourteen years of age. Since this is a private school, students attending this school tend to come from middle to upper income families. Permission to conduct this study was obtained from both the Superintendent of the Catholic Diocese of Tulsa, and from the Principal of St. John's School. A parent letter (Appendix A), which included a consent form to be signed and returned by each subject within a week, was sent home with each subject prior to their participation in the study. All of the consent forms were returned signed within the specified period of time, which meant that all of the subjects selected were able to participate in the study. The total number of students participating in this study was twenty; the subjects consisted of thirteen girls and seven boys. This sample included 8 sixth grade subjects (2 males and 6 females), 5 seventh grade subjects (3 males and 2 females), and 7 eighth grade subjects (2 males and 5 females).

Three instruments were selected for use in the pilot study. These included the Math Concepts subtest of the Iowa Tests of Basic Skills, the Vandenberg and Kuse Mental Rotations Test (MRT), and the Math Anxiety Questionnaire. The Math Concepts subtest of the Iowa Tests of Basic Skills was chosen for this pilot study because of correlations ranging from .58 to .75 which were found between spatial ability and math concepts in studies conducted by Brinkman (1966) and Bruininks and Mayer (1979). These are considered moderate to good correlations. The Vandenberg and Kuse Mental Rotations Test (MRT) is reported to have a reliability coefficient of .88 and has been widely used to measure mental rotation skills. This instrument has provided consistent results in a number of studies reviewed in Chapter II, which is why this instrument was chosen for the pilot study. It is not known whether the MRT has been utilized in studies to examine its relationship to math concepts. Conner and Serbin (1985) and Pearson and Ferguson (1989) suggest a relationship between mental rotation and math achievement. This present study examined whether a relationship between these variables exist.

The Math Anxiety Questionnaire (MAQ) was chosen for this pilot study because this instrument has been reported to have good internal consistency reliability (Wigfield & Meece, 1988; Williams, 1994) and because it is short, easy to administer, and easy to score. Although a number of studies (Aiken, 1970; Berstein et al., 1992, Thorndike-Christ, 1991) have already been conducted looking at the relationship between math anxiety and mathematics achievement, there have been few studies reported which specifically focused on the relationship between math concepts and math anxiety. This pilot study was used to examine whether a correlation exists between the Math Anxiety Questionnaire and the Math Concepts subtest of the Iowa Tests of Basic Skills.

Immediately before the two instruments were administered, the purpose of the pilot study and the directions for each test were explained to all the subjects. The MRT was administered first followed immediately by the Math Anxiety Questionnaire. The subjects' scores from the Math Concepts subtest of the Iowa Tests of Basic Skills were obtained from their cumulative records.

Correlation coefficients were computed to provide answers to Research Question 1, Research Question 2, and Research Question 3. Relationships were studied between mental rotation and math anxiety, between mental rotation and math concepts, and between math anxiety and math concepts. Each of these relationships was examined with the total population of participants, but because of the small number of subjects, the analyses were not reexamined looking at differences in each correlation according to gender. Pearson product moment correlation coefficients were estimated and reported for each analysis. The correlation coefficient reported between mental rotation and math anxiety was -.49, between mental rotation and math concepts was .50 and between math anxiety and math concepts was -.53. All of these correlations were significant at the .05 level.

Based on the results of the pilot study, it was found that significant correlations existed between the Vandenberg and Kuse Mental Rotations Test, the Math Concepts subtest of the Iowa Tests of Basic Skills, and the Math Anxiety Questionnaire. The reliability estimates reported for each of the instruments used in this pilot study provided a rationale for use of these instruments in the current research. The moderate to moderately high correlations provided support for a relationship between the variables, as was suggested in the literature review. The significant correlations found in this pilot study also provided evidence supporting the use of mental rotation as a covariate and the use of MANCOVA as the statistical design in the current research.

One area of concern to the researcher was the use of a private school population of students. If students attend private school, it is sometimes thought that they are of higher intelligence and come from upper middle to upper class families. The concern in the use of this population was that they would all achieve high scores on the tests and constrict the range of variance, which would lead to spurious results. This result was not observed in this pilot study. A wide range of variability was seen in the performance of these students. This provided support for the use of this population of students in the current research.

Instrumentation

The significant correlations between the Vandenberg and Kuse Mental Rotations Test, the Math Concepts subtest from the Iowa Tests of Basic Skills, and the Math Anxiety Questionnaire, provided the rationale for use of these instruments in the current research.

Further support for the use of these instruments was provided by the reliability and validity data reported in the following discussions of each instrument.

Spatial Ability

The Vandenberg and Kuse Mental Rotations Test (MRT) was chosen to be one of the instruments to be used in this study. The MRT is a paper and pencil test which was based on Shepard and Metzler's (1971) mental rotation tasks. This instrument contains 20 items. Each item has a pictorially represented three-dimensional stimulus object and four pictorially represented three-dimensional response options - two correct alternatives and two incorrect ones. The correct alternatives are always identical to the criterion figure in structure but are shown in a rotated position. The task required of each item is to mentally rotate the stimulus object into congruence with each response option so that the subject can make comparisons. The examinee must find two correct response options and mark his/her answers directly in the test book. Administration time for students is approximately 10 minutes.

The recommended procedure for scoring is to count each line as correct if both choices are correct and to give no credit otherwise. This eliminates the need for correction of guessing (Vandenberg & Kuse, 1978). Each line is worth two points so no examinee can get a score of one. The highest possible score is 40. A raw score of 16 represents the 51st percentile for females and the 11th percentile for males. A raw score of 26 represents the 84th percentile in females age 14 to 20 and the 50th percentile for males age 14 to 20. The norms were derived differently for each gender because in the pilot study (Vandenberg & Kuse, 1975), which represented 210 subjects (99 male and 110 female), the means were significantly different between male and female students (22.30 for females and 28.49 for males). This difference was replicated in a study by Defries, et al. (1975) based on 3,268 individuals living in Hawaii. In this study, the means observed on the MRT were 13.17 for females and 19.06 for males. These gender differences have been reported in all studies utilizing the MRT.

The reliability of the test based on the Kuder-Richardson 20 formula was reported to be .88 by Defries, et al. (1975) using the sample size of 3,268 individuals ranging in age from 14 to 60 years old. Kuse (1977) reported a test-retest reliability of .83 for the MRT, after a one-year interval based on a sample of 336 subjects. Vandenberg (1975) reported a Kuder Richardson reliability of .88 for the MRT. This estimate was based upon 46 university students. Vandenberg and Kuse (1978) reported Spearman Brown split-half reliability coefficients of .79 based upon a sample of 197 female and 115 male undergraduates at the University of Colorado.

More recent reliability estimates were reported by Stumpf (1993), who administered the MRT to 233 subjects (146 males and 87 females), aged 12 to 17. This study was conducted to explore whether test-taking styles could contribute to an explanation of gender related differences on several tests of spatial ability. Stumpf reported reliability estimates of .88 for the MRT. Most of the reliability estimates reported involve subjects ranging in age from age 14 to adults. Stumpf's study reports reliability estimates for subjects as young as twelve years of age. Since there is little reliability information for the MRT with subjects under the age of fourteen, internal consistency reliability coefficients were estimated from the data analyzed for this study. A reliability estimate of .81, which is very similar to the other reliability estimates, was reported and provides evidence that this instrument has good internal consistency reliability.

Several researchers have reported the validity of the MRT by reporting coefficients of correlation between the MRT and other standardized tests of spatial ability. Baker (1990) reported a correlation coefficient of .54 between the MRT and the Surface Development Test which was modeled on Thurstone's test of the same name to measure spatial visualization. Baker's study involved 180 undergraduate students. In research conducted by Pearson and Ialongo (1984), a correlation coefficient of .59 was reported between the MRT and the DAT

Spatial Relations subtest. The subjects included 199 women and 154 men from introductory psychology classes. Stericker and LeVesconte (1982) reported that the MRT correlated .54 with the DAT-SR using a sample of 50 female and 41 male college students. This test has shown good reliability in the few studies reporting reliability. Many of the reliability studies involved the originator of this test, which may bias the reliability estimates.

Kuse (1977) reported a correlation coefficient of .62 between the MRT and the Card Rotation Test (CRT) and, in a separate study, reported a correlation of .68 between the MRT and the Identical Blocks Test. Both of the other instruments, the Identical Blocks Test and the Card Rotation Test, involved a mental rotation task. Vandenberg and Kuse (1978) reported a correlation of .50 with the Spatial Relations test of the Differential Aptitude Test (DAT-SR). The sample included 197 female and 115 male undergraduate students at the University of Colorado. The validity studies which have been conducted report moderate coefficients. Little is known about the validity of this instrument with the adolescent population. This instrument has an advantage over other spatial tests in that more reliability and validity studies could be found.

Mathematics Achievement

Math achievement was measured using the Math Concepts subtest from the Iowa Tests of Basic Skills (ITBS). The rationale for selecting the Math Concepts subtest for this research included: (1) significant correlations between spatial ability and the Math Concepts subtest of the Iowa Tests of Basic Skills were reported in Brinkman (1966) and Bruininks and Mayer (1979) studies, and (2) significant correlations were found in the pilot study conducted for the present study. The Math Concepts subtest was administered after the instructional curriculum had been completed.

The Iowa Tests of Basic Skills are a group of achievement tests which measure skills in the areas of vocabulary, reading, writing, language, and mathematics for students in Grades K to 9. The format of the ITBS consists of multiple-choice items Three parallel forms are available - Forms G, H and J. Forms G and H were published in 1985 and Form J was published in 1988. Form G of the ITBS was selected for use in this study. Form J, the newer version of this instrument, is a state-mandated form of the ITBS and is not available to be administered by any individual outside of a school system. There was no particular reason why Form G was selected over Form H, since both are parallel forms and were published in the same year. Form G consists of three batteries which are organized into 10 levels of skills development and correspond roughly to chronological age and grade level.

Level 14 of The Math Concepts subtest of the Iowa Tests of Basic Skills and Form G is the appropriate level to be used with eighth and ninth grade students according to the administration manual for the ITBS. Since eighth grade students participated in this study, this was the level of the test which was administered. Mathematics Concepts (referred to as M-1 in administration manual of the ITBS) is a test of how well one understands the number system and the terms and operations used in mathematics. This test takes approximately 30 minutes to administer and consists of 42 items (see Appendix B for sample items). This instrument is intended to be administered by teachers (sometimes with the help of proctors) to students in their classroom once a year and can be administered annually. Whether these tests are given annually is to be determined by individual school districts. Since no single test can be equally well suited to the entire range of achievement, individualized testing with one or more subtests is a viable alternative as suggested by the Teacher's Guide for the Iowa Tests of Basic Skills.

Reliability coefficients for the Math Concepts subtest of the Iowa Tests of Basic Skills were reported for both fall and spring administration. These coefficients were based on the 1985 norming sample of approximately 10,000 students for each grade who were involved in the fall norming and 1/3 of that sample size for the spring norming. The reliability coefficient reported for the Fall administration of the Math Concepts subtest is .89 and the reliability for the Spring administration is .90. Internal consistency reliability coefficients were estimated from the data analyzed for this current research. The reliability coefficient reported is .81 which is slightly smaller than the other two estimates referenced, but indicates that this instrument has good internal consistency reliability.

Math Anxiety

The instrument used to measure math anxiety in this research was the Math Anxiety Questionnaire (MAQ). The rationale for choosing this instrument for this research included: (1) the MAQ has been reported to have good internal consistency reliability (Wigfield & Meece, 1988; Williams, 1994), (2) significant correlations were reported between the MAQ and both the MRT and the Math Concepts subtest, and (3) it is short, easy to administer, and easy to score.

The Math Anxiety Questionnaire consists of 11 items with 7-point Likert-type scales (Appendix B). The student is to read each statement and then circle the number of the statement to indicate how he/she generally feels regarding math. A response of "one" represents that he/she does not feel a certain way at all, while a response of "seven" represents that he/she feels this way very much. The third question is worded in the opposite order from the other questions. This is to prevent a student from response set. A response set occurs when an examinee responds in a certain way to a particular item format regardless of content. Administration time takes approximately 15-20 minutes.

Internal consistency reliability coefficients are reported by Wigfield and Meece (1988) for the two scales, Cognitive Scale and Affective Scale. These are .82 for the seven-item emotionality scale and .76 for the four-item worry scale. An internal consistency reliability coefficient for the entire math anxiety scale was not reported. Wigfield, Meece and Eccles (1990) reported an internal consistency reliability coefficient of .77 for the Math Anxiety Questionnaire. Since this instrument is new, internal consistency reliability coefficients were estimated for the Affective Scale and the Cognitive Scale from the data analysis in the current study to determine if the coefficients reported would be similar to those reported by Wigfield and Meece. The internal consistency reliability coefficient reported in this study was .80 for the Affective Scale and .79 for the Cognitive Scale. The reliability coefficients for each scale obtained in this research were very similar to those reported by Wigfield and Meece. An internal consistency reliability coefficient for the entire Anxiety Scale was estimated from this study and it is reported to be .87, which indicates that the Math Anxiety Questionnaire has good internal consistency reliability. This instrument is relatively new and more research is needed for validity. One recent validity study by Williams (1994) failed to support the two dimensions of this instrument, but did show good internal consistency.

Subjects

Sixty-one eighth grade students (thirty-one females and thirty males) ranging from thirteen to fourteen years of age participated in this research. Initially the subjects were selected from three private parochial schools in Oklahoma. After the administration of the MRT, it became evident that nine of the subjects who had been selected for the control group, obtained the highest score possible on this instrument. It was determined by the researcher that this would constrict variability and could lead to spurious results. Within several weeks, nine different subjects were selected from a fourth Parochial School.

The researcher attempted to include a diverse population of subjects in this experiment, which would more closely resemble the general population of students. Forty-nine of the subjects were Caucasian, six of the subjects were black and one subject was Asian. Thirteen of the subjects attended Holy Family School in Tulsa, Oklahoma. Although this school is private, tuition is often waived or reduced significantly for low income students. Holy Family is an inner city school with a large black student population. Twenty-two of the subjects attended Christ the King Marquette Catholic School in Tulsa, Oklahoma. Marquette is a private school comprised of middle to upper class students. Nine of the control subjects attended Monte Casino Catholic School in Tulsa, Oklahoma. Monte Casino is a private catholic school comprised of upper middle to upper class students.

Fourteen of the subjects attended Wesleyan Christian School in Bartlesville, Oklahoma. This school is comprised of middle to upper middle class students.

Thirty students were used as a control group from four classrooms of students. Thirty-one students from three classrooms (approximately ten per classroom) volunteered to participate in the experimental group. Unequal numbers of subjects were selected based on the probability of attrition. Four of the subjects withdrew from the experimental group and one subject withdrew from the control group. At the end of the experiment there were twenty-seven students in the experimental group and twenty-nine students in the control group, a total of fifty-six subjects.

Procedures

Sixty-One students enrolled in the eighth grade of three parochial schools were asked to participate in this research. A letter to parents explaining the purpose of the research and an informed consent form were sent home with all the eighth grade students from the three schools in November of 1993 (Appendix B). All of the consent forms were returned and signed giving permission for all of the students to participate. Several parents asked to talk with the researcher so that they could understand what the experiment was about. After the signed consent forms were returned and the discussions were held with the concerned parents, the researcher met with the math teachers for the eighth grade and the administrators of each of the schools. The length of time the experiment would take place and the time when the experimental group met with the researcher were decided.

After scheduling had been determined, the principals from each of the schools assisted in the selection of students for the experimental and control groups. Each principal asked for volunteers from the eighth grade students who had turned in their signed consent forms to participate in an experimental group. The number of students needed for the experimental group, asked for by the researcher, were then selected from the students who volunteered. Thirty-one students were selected from the group of students who volunteered for the experimental group. The experimental group consisted of approximately 10 students per school. One additional subject who volunteered for the experimental group was placed in that group because of the possibility that students could become frustrated with the curriculum and quit or for other reasons attrition could occur. The thirty remaining students from the three different classrooms were used for the control group. The subjects were selected from three different classrooms to control for possible confounding affects of "class," "teacher," or from regular math instruction.

Several weeks after the experiment began, a fourth school was selected because a problem was found with the scores on the MRT of nine subjects from the Marquette control group. These nine subjects obtained the highest possible score, a score of 40, on this test. This resulted in the variability of scores within this control group being small, and when this happens constriction of range occurs. Constricting the range of test scores reduces the estimate of a test's reliability, minimizes any correlation statistic, and violates the assumptions of normality and equal variances for the population sample. Because of the statistical problems constriction of range would cause the nine subjects from the Marquette control group, who were the only subjects with the highest score on this test, were dropped from the experiment and nine new control subjects were found by selecting subjects from another parochial school. Permission letters and Informed Consent Forms were sent home with all the students, who were enrolled in the eighth grade average math class. When the Informed Consent Forms were returned, the principal selected nine subjects , six female and three male, from a group of fourteen students who returned their consent forms.

The Holy Family experimental group, which consisted of five female subjects and one male subject, met for instruction from 12:30 pm until 1:15 pm on Tuesday and Thursday. The control group, which consisted of three female subjects and four male subjects met with their homeroom teachers during that period to do activities agreed upon by both the teacher and the students. The Marquette experimental group, which consisted of eight female subjects and seven male subjects, met for instruction from 2:15 to 3:00 on

Tuesday and Thursday. The control group, which consisted of three female subjects and two male subjects, was scheduled in another class at that time. The period of time allotted to the experimental group was a regular math period, but the math teacher for these students revised her objectives so that these objectives would be met at another time. The Wesleyan experimental group, which consisted of six male subjects, met before school from 8:15 to 8:45 on Monday, Wednesday, and Friday. The control group, which consisted of two female subjects and six male subjects, were beginning to arrive at school for the day. The time of day in which the experimental group met varied because of scheduling problems for both the schools and the researcher and could not be controlled. This is a limitation of this research.

Prior to test administration all students were assigned a number. This number was their identification number. Instead of names written on the test protocols, numbers were used to insure confidentiality. All students were administered the Vandenberg and Kuse Mental Rotations Test as a pretest to determine the entry level of student spatial abilities on mental rotation. This data was used to help control for variations in abilities between groups through the use of covariate statistics.

Although the author of the mental rotation curriculum recommended that the students make their own set of nine figure cubes out of 39 small wooden cubes (Appendix D), this was not feasible due to time constraints. The researcher made all the sets of figure cubes for the subjects in the experimental group. On the first day the experimental group met, each student received their own set of figure cubes These were used for all lessons in every unit. The students were told that at the conclusion of the experiment that they would be able to take the figure cubes home with them.

The experimental group worked with the researcher on the instructional program of mental rotation for approximately 18 weeks. Five units were presented which consisted of a number of lessons and independent activities. Each lesson and/or independent activity took approximately thirty minutes to complete per session. These lessons did not interfere with regular classroom math instruction. These lessons were presented three times a week for 30 minutes a session for the Wesleyan experimental group and twice a week for 45 minutes a session for both the Marquette experimental group and the Holy Family experimental group. The total instructional time and the content covered each week was the same for all instructional groups. The instructional period ended after eighteen weeks of instruction.

The amount of instructional time per week was ninety minutes for each group, which totalled approximately 27 hours of instruction. Although the total amount of instructional time was not directly stated in Rekdal's 1982 study which reported significant results, an estimate of 27-30 hours could be made based on the number of training hours and the number of weeks of instruction Rekdal indicated in her research. The instructional time for the current research parallels the instructional time for Rekdal's study. When one of the experimental subjects was absent, every effort was made for individualized instruction so that the subject could make up the lesson that he/she missed. There were times when school holidays occurred on a training day and when that happened the training session would be rescheduled.

Use of a control group was one method employed in this research for prevention of the Hawthorne effect. The Hawthorne effect occurs when any experimental treatment, because of the attention or lack of attention it creates, causes the subjects to change (Kerlinger, 1973). Besides utilizing the control group to prevent this effect, the students assigned to the control condition remained in their normal classes and pursued special educational activities or other classes specified by their teachers. The special activities the control group students pursued were considered a placebo method and referred to as "special" so that attention to students being pulled out for the experimental treatment was lessened. The Wesleyan control group did not view the experimental group as receiving special attention since their school day began thirty minutes earlier. The activities pursued by the control groups were not considered relevant to mental rotation ability.

At the end of the 18-week instructional training period all students were administered the Math Anxiety Questionnaire and Math Concepts subtest of the Iowa Tests of Basic Skills. Many of the subjects were administered the Mental Rotations Test again for the purposes of a follow-up analysis. Both the instruction and the tests were administered by the researcher in order to control for intraclass correlation. Following data collection, students in the experimental group were given a pizza party. The findings of the research and the implications were discussed with the administrators who participated in this study.

Spatial Training Curriculum

The mental rotation (spatial ability) curriculum that was used in this study was "Power Cubes; Adventures in Spatial Perspectives". This material is published by Stamina Associates and is commercially available through three different companies The rationale for use of the Power Cubes curriculum for this research included: (1) Stanton and Miller (1987a) showed a six month gain on pre-posttest data of over five test items (on each of two 26 item Structure of Intellect tests) for a figure-cube trained group over control groups, (2) the growing use and popularity of this curriculum, and (3) the use of mental rotation as the primary skill being taught, which is then applied to mathematics.. Power Cubes was developed by Rebeca Stanton and Elizabeth A. Miller. teachers in California. Several California counties and other schools in the United States and Canada have adopted the Power Cubes as required instructional strategy (R. E. Stanton. personal communication, September 2, 1993). Considering the growing popularity of the Power Cubes it is surprising that little statistical research has been conducted looking at the effectiveness of this curriculum.

This program enables students to learn to visualize known figures, mentally rotate them in space, and then draw them in three-dimensional form from a variety of new perspectives. Students learn to analyze shapes from their component parts, create new shapes, and record them for others to analyze. Letters are used first to represent their figures. these letters are then replaced with numbers which form mathematical sentences, and then these sentences are solved. This curriculum teaches a standard strategy for rotation along the vertical and horizontal axes. The standard axes are the horizontal (x) axis , the vertical (y) axis, and the depth (z) axis measured relative to the observers' viewpoint. Often the object is mentally rotated in the plane of the picture so that the axis of rotation is the depth (z) axis. An object can be rotated in the horizontal (x) axis and sometimes the object is mentally rotated in depth so that the rotation axis is the vertical (y) axis (Just & Carpenter, 1985). This strategy was recommended as the strategy to be used to solve the Shepard and Metzler (1971) mental rotation tasks.

The instructional curriculum, as implemented for use in this research (examples can be seen in Appendix D) consisted of five units. The first unit introduced three-dimensionality. In this unit students discovered the relationships between cube and figure-cube, drew the cube in three-dimensional form, identified nine figure-cubes, used representational notations to identify the figure-cubes, drew the figure-cube in three-dimensional form, and drew the figure-cubes from four different views. The second unit dealt with using an isometric grid. In this unit students used an isometric grid to take out simple cubes, to draw simple three-dimensional figures using the isometric grid, to observe that all of the elements of a simple figure are sometimes visible and sometimes not visible from all views and to draw complex three-dimensional figures using an isometric grid.

The third unit involved independent problem solving activities presented in six activity centers. Some of these problem solving activities included matching figure-cubes with three-dimensional drawings, solving figure-cube puzzles and recording the values as addition problems, making puzzle shapes by combining figure-cubes, solving puzzle shapes created by other students and building pictured structures using figure-cubes. Each of these activity centers required the student to work independently. Unit four consisted of extended activities in problem solving. In this unit the students created their own structures from the figure cubes and drew them on paper. Unit five used parallel lesson activities. These activities included the making and solving of numerical problems using all nine figure-cubes in a variety of combinations and the solving of algebraic equations.

This Power Cubes curriculum demonstrated similar instructional objectives and lessons to the first part of the instructional phase in the study conducted by Moses (1980). In her study students compared cubes with other three-dimensional shapes and sorted out characteristics of the various shapes. Perspective experience and drawing tasks were involved, which are included in this mental rotation curriculum. The cube was taken apart and different arrangements of the six squares were viewed in an attempt to decide which ones form cubes. Rotation of these different views were made. Moses included the study of area in the second phase of her study which is included in the power cubes curriculum. Significant results in reasoning were reported in Moses' (1980) study but not in math problem solving. The current research examined how this Power Cubes curriculum, which is similar to Moses's curriculum, related to math concepts.

Data Analysis

A two groups x two gender factorial multivariate analysis of covariance was the statistical design selected for this study based on the complex relationships which exist between mental rotation, math conceptualization and math anxiety. Support for these relationships was provided by evidence presented in the literature review and in the pilot study. Based on the findings in the research that gender differences in favor of males exist in the spatial skill of mental rotation, this variable was determined to be the covariate. Mental Rotation skill among subjects was controlled by using this variable as a covariate.

The pre-treatment measure for mental rotation was the Vandenberg and Kuse Mental Rotations Test. The means and standard deviations derived from the scores on the MRT were examined and compared with the means and standard deviations of the MRT from the pilot study.

Pearson correlation coefficients were obtained between mental rotation and math anxiety, mental rotation and math concepts, and math anxiety and math concepts as a means of assessing the strength of their relationship and to see if the overall results of the pilot study would be replicated. Since little reliability data was reported for the MRT for the specific age group of the current research, internal consistency reliability coefficients were estimated for all the instruments utilized in this research.

This research utilized two dependent variables, math concepts and math anxiety. Because this is a multivariate design utilizing more than one dependent variable, a multivariate analysis of covariance (MANCOVA) was performed in order to test the hypotheses. Interaction effect, main effect of group and main effect of gender were examined at the multivariate level. Post hoc analyses of variance were then performed on each dependent variable to determine which dependent variable contributed to multivariate significance.

Although examining the effect of the mental rotation curriculum on mental rotation ability was determined not to be an integral part of this research, looking at possible differences in mental rotation ability due to treatment was determined to be a related question. This was examined in order to verify the effectiveness of the training program. A posttest of the MRT instrument was administered to twenty-nine of the fifty-six subjects in the experiment. A pre-postest design was used to analyze the data from the follow up analysis. The minimum level for statistical significance was set at $p < .05$ for all of the statistical analyses reported in this study.

Chapter IV

Results

This chapter contains an analysis of the data collected during this study. The analysis addresses the six research questions which led to the development of the multivariate null hypotheses. This chapter is divided into the following four parts: Preliminary Analysis, Hypotheses Testing and Post Hoc Research, Follow-Up Analysis, and Summary of Research Results. The first section substantiates the use of mental rotation as a covariate, validates the use of the MANCOVA design chosen for this research, and includes descriptive statistics for the covariate. The second section includes the results of the data analysis conducted to test the null hypotheses and subsequent analysis conducted to identify where significant differences occurred. The third section includes the results from a follow-up analysis to determine if the mental rotations curriculum effected any change in the mental rotation skills of the experimental subjects.

Preliminary Analysis

Mental rotation was chosen as the covariate in this study to control for differences in ability levels of mental rotation skills in the subjects and to reduce the error variance. The main criterion for a covariate is a significant correlation with the dependent variables (Keppel, 1991). The pilot study provided information that mental rotation correlated significantly with math anxiety and math concepts. Since the sample size in the pilot study was small, additional Pearson product moment correlation coefficients were obtained for the larger population in this research to substantiate the pilot correlation results and to provide a more stable correlation estimate for use in the MANCOVA design. Because the experiment could have changed the relationship of the variables to each other, only the scores from the twenty-eight control subjects were used to estimate the correlation coefficients. Pearson product moment correlation coefficients from both the pilot study and the current research are presented in Table 1.

Table 1

Pearson correlations of the Math Anxiety Questionnaire and the Math Concepts subtest of the Iowa Tests of Basic Skills with The Mental Rotations Test

Mental Rotations	Math Anxiety	Math Concepts	Number of Subjects
Pilot Study	-.49*	.50*	N=19
Current Research	-.45*	.57**	N=28

* = p < .05 ** = p < .01

Both of the correlation coefficients reported from the current research were significant. The correlation coefficient between mental rotation and math concepts was .57 which was significant at the .01 level. This relationship was stronger than what was seen in the pilot research. The correlation coefficient reported between mental rotation and math anxiety was smaller than what was reported in the pilot research but was significant at the .05 level. The relationship between mental rotation and math anxiety is an inverse relationship as can be observed in the negative correlation coefficient, which means that as mental rotation skill increases math anxiety decreases. Correlation coefficients were estimated to examine the relationship between math anxiety and math concepts in both the pilot study and the current research. An inverse relationship was observed between math anxiety and math concepts as was seen in the negative correlation coefficients reported for the pilot study, -.53, and for the current research, -.45. These correlations are significant at the .01 level for the pilot study, and .05 level for the current research. These correlations provide evidence that a significant relationship exists between these variables and validate the use of mental rotation as the covariate in this research design.

The Mental Rotations Test was used to measure mental rotation skill in this research. The Math Anxiety Questionnaire and the Math Concepts subtest of the Iowa

Tests of Basic Skills were used to measure math anxiety and mathematical conceptualization, respectively. Reliability coefficients were estimated from the data for each of the instruments used in this study. Internal consistency coefficients reported were .81 for the Mental Rotations Test, .81 for the Math Concepts subtest of the Iowa Tests of Basic Skills, and .87 for the Math Anxiety Questionnaire. All of these estimates indicate that these instruments demonstrate good internal consistency reliability. These reliability coefficients substantiate the use of these instruments in this research.

In the current research, the Mental Rotations Test was administered a week before the experiment started. The descriptive statistics for the Mental Rotations Pretest for all subjects except the nine control subjects who were excluded are presented in Table 2.

Table 2

Descriptive Statistics For the Mental Rotations Pretest

Group	Mean	Standard Deviation	Range	N
Total	14.96	8.63	34	56
Males	16.61	8.95	34	28
Females	13.32	8.11	28	28
Experimental	16.07	8.21	34	27
Control	13.00	8.67	31	29

A difference of 3.29 points was observed between the means of male and female subjects, with males scoring higher than females. These results are consistent with the research findings cited in Chapter II, where males were found to consistently score higher than females on the Mental Rotations Test. The covariate design was used to adjust treatment effects for differences in mental rotation skills between the treatment groups that existed before the experimental treatments were administered.

Hypothesis Testing and Post Hoc Research

A multivariate analysis of covariance (MANCOVA) was the statistical technique utilized to test the following null hypotheses:

Null Hypothesis H1: It is hypothesized that there will be no significant differences on mathematical conceptualization and math anxiety scores (compared simultaneously) between the treatment and control groups, while controlling for mental rotation.

Null Hypothesis H2: It is hypothesized that there will be no significant differences on mathematical conceptualization and math anxiety scores (compared simultaneously) between males and females, while controlling for mental rotation.

Null Hypothesis H3: It is hypothesized that there will be no significant interaction between treatment condition and gender on mathematical conceptualization and math anxiety (compared simultaneously), while controlling for mental rotation.

Table 3

MANCOVA Summary Table

Source	Wilk's lambda	F-Ratio	Hypoth df	Error df	p
Group (H1)	.9711	.7420	2	50	.48
Gender (H2)	.8669	3.836	2	50	.02
Group x Gender (H3)	.9609	1.016	2	50	.36

The results of the multivariate analysis of covariance are presented in Table 3. The interaction effect was examined first and found not to be significant at the multivariate level. Null Hypothesis H3 was retained, which indicates there were no differences in math concepts scores by gender to the treatment/control condition, and no differences in the level of math anxiety reported by gender to the treatment/control condition. These findings answer the following research questions: "Will differences in math concepts scores occur by gender as a result of implementation of a Mental Rotations Curriculum?" and "Will differences in the level of math anxiety be reported by gender as a result of implementing a Mental Rotations Curriculum?".

Since the interaction was not significant at the multivariate level, the main effect of group was examined to determine if differences between the experimental and control groups could be found among the two dependent variables. The main effect of group was not significant at the multivariate level. Null Hypothesis H1 was retained, which indicates there were no significant differences in math anxiety or math concepts scores between the experimental and control groups. This finding answers the following two research questions: "Can implementing a specific curriculum of mental rotations affect mathematics conceptualization scores?" and "Can implementing a curriculum of mental rotations affect math anxiety?".

Since the main effect of group was not significant at the multivariate level, the main effect of gender was examined next to determine if differences between male and female subjects could be found among the two dependent variables. The main effect of gender was significant, Wilk's lambda = .86696, $F(2,50) = 3.83627$, $p<.05$. This resulted in rejection of Null Hypothesis H2, which indicates that differences between male and female subjects in either math concepts, math anxiety, or both contributed to multivariate level significance. Consequently, post hoc univariate analyses of variance were performed on each dependent variable to determine which dependent variable(s) contributed to multivariate significance. The results of these analyses are presented in Table 4.

Table 4

Analysis of Variance on Math Concepts and Math Anxiety Scores of the Male and Female Students

Variable	df	SS	MS	F	p
Math Concepts	1,51	29.501	36.888	.79972	.375
Math Anxiety	1,51	918.786	205.556	4.46974	.039*

* = $p < .05$

The analysis of variance conducted on math concepts yielded no significant results, which indicates that no differences were found between the math concept scores of male and female students. This finding answers the research question "Will differences in mathematical conceptualization occur between the male and female students in this study?".

Table 5 presents descriptive statistics for the math concept scores of the total sample and for the male and female subjects. The mean math concepts scores of male and female subjects differed by less than one point. The analysis of variance conducted on math anxiety was significant, $F(1,51) = 4.46974$, $p<.05$. These results provide evidence that significant differences in the levels of math anxiety were reported by male and female subjects. This finding answers the research question "Will differences in math anxiety be observed between the male and female students in this study?".

Table 5

Descriptive Statistics For the Math Concepts Scores

Group	Mean	Standard Deviation	Range	N
Total	26.86	6.67	27	56
Males	26.64	6.58	26	28
Females	27.07	6.88	24	28

The mean math anxiety scores for the total sample, for males and females, and for the experimental and control groups are presented in Table 6. It is observed that the females in all groups showed higher levels of math anxiety.

Table 6

Descriptive Statistics For the Math Anxiety Scores

Group	Mean	Standard Deviation	Range	N
Total	42.27	15.39	64	56
Males	37.68	14.17	57	28
Females	46.86	15.42	53	28
Experimental	43.70	15.02	58	27
Males	36.29	11.48	39	14
Females	51.69	14.59	52	13
Control	40.93	15.86	64	29
Males	39.07	16.76	57	14
Females	42.67	15.36	53	15

Even though no interaction effects (Group x Gender) were found at the multivariate level, it is interesting to note that the largest numerical mean differences in overall math anxiety scores (Table 6) were observed by inspection between males and females within the experimental group. This was a difference of 15.40 points, and is larger than the 9.18 point difference between males and females in the total sample, and much larger than the 3.60 point difference between males and females in the control group. These mean differences were not tested statistically because of problems related to family wise error rates when a number of F or t-tests are conducted. These differences observed suggest that future research examine whether differential treatment effects occur in math anxiety as a result of training mental rotation.

Parallel gender numerical differences in mean scores were observed by inspection on the overall Math Anxiety Questionnaire and on both the Cognitive and Affective scales. No statistical analysis was conducted because of problems related to family wise error. The mean scores for the total sample, for the male and female subjects, and for the experimental and control groups are presented in Table 7. The experimental and control groups are further subdivided by gender in this table for the purpose of comparing differences in the means of these groups by gender on the Cognitive, Affective, and overall math anxiety scores.

Some of the more interesting observations include:

1. Females obtained a higher numerical mean score than males in every group (total, experimental, and control).
2. Females in the experimental group obtained the highest numerical mean score and males in the experimental group obtained the lowest numerical mean score.
3. The mean numerical difference by gender for the experimental groups was much larger than the mean numerical difference by gender for the control groups.

Table 7

Mean Scores for the Math Anxiety Questionnaire and the Cognitive and Affective Scales

Group	Math Anxiety Questionnaire	Cognitive Scale	Affective Scale
Total	42.27	24.00	18.07
Males	37.68	21.50	16.07
Females	46.86	26.57	20.07
Experimental	43.70	20.42	18.66
Males	36.29	30.00	15.85
Females	51.69	23.10	21.69
Control	40.93	23.10	17.51
Males	39.07	22.57	16.28
Females	42.67	23.60	18.66

Follow Up Analysis

Evidence was provided in the literature that mental rotation was a skill that could be improved by training. Although improvement in mental rotation skill was not an integral part of this experiment, the researcher decided to conduct a pre-posttest analysis of variance in order to determine if the curriculum had any effect on the mental rotation skills of the eighth grade subjects. Because of the small number of students who were involved in the pre-post test experiment, the results of these analyses need to be interpreted with caution.

Tests of Differences

An analysis of variance conducted on mental rotation yielded significance for the treatment group, $F(1,40) = 7.483$, $p<.01$. The results of this analysis can be seen in Table 8. These results indicate that a significant difference in mental rotation scores was found between the experimental and control groups. The Mental Rotations Curriculum appears to have had an effect on the mental rotation skills of the eighth grade subjects in the experimental group. This provides support for the evidence presented in Chapter II that mental rotation is a skill that can be improved through training.

Table 8

Analysis of Variance Summary Table

Source	SS	df	MS	F
Group	699.333	1	699.33	7.48366*
Within Group	3644.47	39		
Total	4343.804	40		

* $= p < .01$

Descriptive statistics are presented in Table 9 for the Mental Rotations posttest. There was a difference of 8.57 points between the experimental group and the control group. The experimental females obtained the highest numerical mean score by inspection. Female subjects overall obtained higher numerical mean scores than male subjects on the mental rotations posttest. Gender differences on the posttest scores were not analyzed because the degrees of freedom for the pretest (df 2,55) were much higher than the degrees of freedom for the posttest (df 2,40). Because of the small number of subjects in each group, further analysis was not conducted.

Table 9

Descriptive Statistics For the Mental Rotations Posttest

Group	Mean	Standard Deviation	Range	N
Males	20.41	9.42	32	24
Females	23.92	11.70	34	16
Experimental	24.30	8.72	34	26
Males	23.14	7.90	26	14
Females	25.66	9.75	34	12
Control	15.73	11.15	34	15
Males	15.81	10.21	32	11
Females	15.50	15.26	32	4

Observations for Pre-Posttest Data

Since the Mental Rotations Test was administered twice a comparison of pretest-posttest means of the Mental Rotations Test, is presented in Table 10. Several interesting observations can be noted by inspecting the numerical differences in these means:

1. All of the means for the posttest were higher than for the pretest, which indicates that by taking the Mental Rotations Test twice the subject would perform better.
2. The difference between the means of the experimental and control groups was higher for the posttest than that observed on the pretest (8.57 points vs 3.07 points).
3. On the posttest, females obtained a numerically higher mean score than males, which was the opposite of the results obtained on the pretest. Overall, females increased their numerical mean score from pretest to posttest by 9.80 points,

whereas males increased their numerical mean score by only 3.80 points. This suggests that female subjects may have responded more to the Mental Rotations Curriculum than did males.

4. Females in the experimental group had the highest numerical mean score on the posttest and increased their numerical mean score by 10.51 points. Males in the experimental group increased their numerical mean score from pretest to posttest by 4.29 points. It appears that females in the experimental group may have benefitted most from mental rotation training.

Table 10

Comparison of the Pre-Posttest Means of the Mental Rotations Test

	Pretest Mean	Posttest Mean
Males	16.61	20.41
Females	13.32	23.12
Experimental	16.07	24.30
Males	18.85	23.14
Females	15.15	25.66
Control	13.00	15.73
Males	14.50	15.80
Females	12.00	15.50

These observations suggest that future research focus on differential effects in mental rotation skill by gender to mental rotations training. These observations may reflect pretest sensitization. It is difficult to separate out the practice effects from taking the same test twice from the learning that takes place from training in a specific skill, such as mental rotation.

Summary

Preliminary correlational analysis was conducted to corroborate the findings of the pilot study, and to substantiate the use of the MANCOVA design for this research and the instruments selected for this study. Significant correlations were found between the Mental Rotations Test and both the Math Anxiety Questionnaire and the Math Concepts subtest of the Iowa Tests of Basic Skills. These results provided support for the previous findings from the pilot study that relationships exist between these variables and substantiated the use of a covariate and the MANCOVA design.

A multivariate analysis of covariance (MANCOVA) was used to test the null hypotheses. The first effect to be examined was the interaction effect. It was important to examine this effect first because, if there had been an interaction, then the main effects could not be interpreted. The result did not show significance at the multivariate level. Since there was not an interaction effect, the next effect to be examined was the main effect of group to determine if differences among the two dependent variables could be found between the experimental and control groups. There were no significant differences found at the multivariate level. The next main effect to be examined was gender. This effect was found to be significant at the multivariate level, Wilk's lambda = .86696, $F(1,51) = 3.84$, $p<.05$. This resulted in rejection of Null Hypothesis H2, which was the only null hypothesis rejected.

Since differences were found between gender at the multivariate level, post hoc univariate analyses of variance were performed on each dependent variable to determine if math concepts and/or math anxiety contributed to multivariate significance. The analysis of variance conducted on math concepts did not result in significant findings. The analysis of variance conducted on math anxiety was significant at the .03 level. The mean differences observed for the math anxiety scores indicated that females were more math anxious than males. When inspecting numerical differences in the mean scores for all of the groups it was observed that the females in the experimental group obtained the highest math anxiety numerical mean score while the males in the experimental group obtained the lowest. Parallel gender numerical differences in mean scores were observed by inspection on the Math Anxiety Questionnaire and its two scales, Cognitive and Affective.

A follow up analysis was conducted to determine if the Mental Rotations Curriculum effected any change in the mental rotation skills of the eighth grade subjects in the experimental group. A pre-posttest analysis of variance was performed to determine if there were any differences between the experimental and control groups. A significant difference was found at the .009 level. These results suggest that the Mental Rotations curriculum had a significant effect on the mental rotation skills of the subjects in the experimental group. Further analyses were not conducted because of the small number of subjects in several of the groups which had not been administered the posttest.

Chapter V

Discussion

This chapter is divided into five sections. The first section reviews the purpose. methodology, procedures, and results of the study. This review provides the basis for the second section, the conclusions derived from the results of this research. This section first presents a list of the main conclusions, followed by a discussion of these conclusions. This discussion is divided into five parts, which represent the analyses from which these conclusions were drawn. These five parts include: Preliminary Analysis, Interaction Effect, Main Effect-Group, and Main Effect-Gender and Follow up Analysis. The third section discusses the limitations of the study. The fourth section discusses the observations and implications from this study for current practice modifications and future research. The final section is a chapter summary.

Review of Method and Procedures

The purpose of this study was to investigate whether incorporating a specific curriculum of mental rotation into the mathematics curriculum of eighth grade male and female students could affect mathematical conceptualization and/or math anxiety. Six research questions were outlined in Chapter III. In order to answer these questions, three multivariate null hypotheses were formulated. A two conditions by two gender groups Factorial Multivariate Analysis of Covariance (MANCOVA) was utilized to test the null hypotheses.

Before the experiment began a pilot study was conducted to provide evidence for the use of mental rotation, as measured by the Mental Rotations Test, as a covariate for the MANCOVA design and to validate the use of the Math Anxiety Questionnaire and the Math Concepts subtest of the Iowa Tests of Basic Skills for this experiment. A correlational analysis was conducted between these instruments and significant correlations were found between mental rotation and both math concepts and math anxiety. This provided support for a relationship between these variables, for the use of mental rotation as a covariate and for the use of the MANCOVA design in the current research.

After the pilot study was conducted the subjects for the experiment were selected. Initially 61 subjects were asked to participate. Five subjects dropped out of the Experiment. The remaining subjects in this current study included 56 eighth grade students from four parochial schools in Oklahoma. This sample included 28 female subjects and 28 male subjects, who were divided into four different groups. Unequal number of subjects were assigned to these four groups, which included a male experimental group, a male control group, a female experimental group and a female control group.

Once group assignment was made then a pretest, the Mental Rotations Test, was administered to all the subjects participating in the experiment. After the pretest had been administered, the researcher met with the experimental groups for instruction in mental rotation two or three times a week, depending on the schedules of each school. The instructional periods ranged from thirty to forty-five minutes. The treatment program lasted for approximately eighteen weeks. At the end of the eighteen week instructional training period all students, both treatment and control groups, were administered the Math Anxiety Questionnaire and the Math Concepts subtest of the Iowa Tests of Basic Skills. Some of the subjects were administered the Mental Rotations Test, as a posttest, for a follow up analysis.

After the instruments were administered the data were analyzed by several procedures. Each procedure utilized was determined by the results of the previous procedure. Preliminary correlational analyses were conducted which replicated the findings of the pilot study. A MANCOVA was the statistical procedure used to test the null hypotheses. Interaction effect, main effect of group, and main effect of gender were examined at the multivariate level. No interaction effect was found and no significant main effect for group was found. A significant main effect of gender was found. Post Hoc univariate analyses of variance was then used to analyze whether gender differences had occurred among math concepts and/or math anxiety. These results found significant differences between males and females in math anxiety and no significant differences in math concepts. By examining the means on math anxiety it was concluded that females reported more math anxiety than males. Inspecting numerical differences of these means resulted in several interesting observations. It was observed that the females in the experimental group obtained much larger numerical mean scores in math anxiety than males in both the experimental and control groups. The females in the control group obtained numerical mean scores that were only a few points higher than the males in either the experimental or control groups. The mean performance of the subjects on the Cognitive and Affective Scale of the Math Anxiety Questionnaire was inspected to observe whether paralell patterns in performance on these scales could be observed. The results indicated that parallel patterns in performance were present on both scales and the overall Math Anxiety Questionnaire. A follow up analysis was done examining whether the Mental Rotations Curriculum effected any change in the mental rotation skills of the eighth grade subjects in the experimental group. The results of an Analysis of Variance showed a significant difference between the experimental and control groups suggesting that the curriculum did effect change in the mental rotation skills of these students.

Results and Conclusions

This section first presents a list of the conclusions which were derived from the results of this study and then is divided into five parts. Each part contains a discussion of each conclusion derived, how this conclusion relates to the literature review, and possible explanations for the findings. The following conclusions were derived as a result of this study.

1. Significant relationships exist between mental rotation and both math concepts and math anxiety.

2. There was no difference in the math concepts scores between male and female subjects in their response to the treatment or control group.
3. There was no difference in the math anxiety reported between male and female subjects regardless of whether they were in the treatment or control group.
4. There were no significant differences in math concept scores between the treatment and the control group.
5. There were no significant differences in math anxiety reported between the treatment and the control group.
6. No significant differences in math concept scores were found between the male and the female subjects in this research.
7. There were significant differences in math anxiety found between male and female subjects.
8. Mental rotation skills were changed as a result of the experimental group.

Preliminary Analyses

Preliminary correlational analyses found that significant relationships between mental rotation, one specific skill of spatial ability, and both math conceptualization and math anxiety exist. This finding supports similar relationships found in the literature for spatial ability and math achievement. As early as 1944 Thurstone found correlations between spatial ability and mathematics reasoning. Bruininks and Mayer (1979) found significant correlations between the Space Relations subtest of the DAT and the Iowa Tests of Basic Skills. Piemonte (1982) suggested that certain aspects of spatial skills are related to mathematics achievement. Pearson and Ferguson (1989) found significant correlations between both the Mental Rotations Test and the Space Relations subtest of the DAT and ACT scores for both men and women. Tartre (1990) suggested that certain processes used to solve math problems might be related to spatial skills. Brinkman (1963) found significant correlations between the Space Relations I and II subtests of the Primary Mental Abilities

Test and the Math Concepts subtest of the Iowa Tests of Basic Skills. Gallagher (1992) states that the more precise partitioning of spatial ability into components related to specific areas of math achievement will be helpful in determining the relationship between visual spatial skill and mathematics. The results of this preliminary analysis supports the findings of the research mentioned and the partitioning of these skills. Evidence from these analyses provides support that a relationship between mental rotation, a specific spatial skill and math concepts, a specific mathematics skill, exists. The nature of this relationship has yet to be determined.

One explanation for these findings is that mental rotation is a skill that is necessary for achievement in mathematics, especially achievement in higher level math (e.g., Geometry, Calculus). This could provide further explanation for gender differences observed in math achievement. Another explanation for these findings could be that mental rotation could be indirectly related to mathematics through another variable. Mental rotation skill could affect another variable, such as math anxiety, which has an even stronger relationship to mathematics achievement, which in turn would affect math achievement. It would appear that examining the nature of this relationship between mental rotation and math concepts could provide explanations for achievement in mathematics.

Several researchers have stated that gender differences in spatial skill account for gender differences in math ability and math anxiety. Hadfield et al (1992) found that spatial skill had a strong inverse relationship to math anxiety with Navajo Middle School students. A study reported by Kagan (1989) found that spatial skills among females was related to math anxiety, but this relationship was not found among males. An inverse relationship between mental rotation and math anxiety, as indicated by a negative correlation coefficient, was found in the preliminary analysis. This preliminary analysis did not examine whether a different relationship exists between mental rotation and math anxiety for males or females as did Kagan's study. Very few studies have been conducted examining the relationship between these two variables, spatial ability and math anxiety. There have been even fewer studies which have examined whether a relationship between the component parts of spatial ability (e.g., mental rotation) and math anxiety exists. This preliminary analysis was based on the following premise: just as the more precise partitioning of spatial ability into its component parts is helpful in determining its relationship with mathematics as was suggested by Gallagher (1992), so to would this partitioning of spatial ability be helpful in determining its relationship to math anxiety. The evidence from this analysis provides support for Hadfield's research and for this partitioning of spatial ability into its component part of mental rotation. It was concluded that a significant inverse relationship does exist between mental rotation, a specific spatial skill, and math anxiety.

Since mental rotation is a skill that is useful for achievement in higher level mathematics, such as geometry and calculus, it would seem possible that deficiencies in that skill could cause math anxiety. If that were true then females would be at more of a disadvantage than males and would exhibit more math anxiety. Since deficiencies in spatial ability, such as mental rotation, have been suggested to result in math anxiety and since it is known that when an individual is more math anxious that anxiety affects overall math achievement, the suggestion can be made that the inverse relationship between mental rotation and math anxiety, which was found in the Preliminary analysis, indirectly effects math achievement.

Interaction Effect

The interaction effect was examined at the multivariate level and was not statistically significant. It can be concluded that there were no differences in the math concepts scores between male and female subjects in response to their treatment condition. Moses (1980) conducted a 12-week study designed to teach spatial skills in order to improve math performance. This instructional program resulted in significant positive effects on the

Mental Rotations Test and the Reasoning Test, but did not result in any effects for problem-solving performance. Significant effects were noted particularly for females. Ferrini-Mundi (1987) incorporated a training period of eight weeks with several hundred college students enrolled in a calculus class. The results indicated that training improved the ability of women to visualize when solving calculus problems. The type of spatial training used in both studies included training the skill of mental rotation, as well as training in other spatial skills. Both of these studies concluded that spatial training could benefit women more than men. Moses's and Ferrini-Mundi's studies both incorporated a training curriculum to improve mathematics achievement. Both researchers found significant results and found that females responded more to training than males. Although their studies involved math reasoning and solving calculus problems, it was reasoned that the math conceptualization scores of females could increase in response to mental rotations training more than males math conceptualization scores based on the results from their studies. The findings from the multivariate analysis did not support Moses's or Ferrini-Mundi's findings that female subjects who were exposed to the treatment condition would respond to that treatment more than the males.

One possible explanation why the training program did not change math conceptualization scores more for the female subjects is that not enough time elapsed for change to take place. Another reason could be that the math concepts subtest did not include many items that required the use of mental rotation skills. The math concepts subtest for upper grades might include more of these items and differences would then be seen. It is possible that gender differences in math achievement have been reduced over the last several years and because of this, training in spatial ability would not affect differences between males and females on math achievement tests.

Multivariate analysis revealed that there was no difference in the math anxiety reported between male or female subjects regardless of the treatment group they were in. Parallel gender differences have been noted in mental rotation, math achievement and math anxiety. Since training studies, Moses (1980) and Ferrini-Mundi (1987), have shown favorable results in the performance of the female subjects on math achievement tests and since women have benefitted more from training in spatial ability, it was reasoned that training in a specific type of spatial ability, e.g., mental rotation, could result in a reduction of anxiety for female subjects. This premise was based not only on the research studies mentioned but on the fact that there appears to be an inverse relationship between mental rotation and math conceptualization as observed in the preliminary correlational analysis. The results of the multivariate analysis did not support this premise.

One explanation for the conclusion, that the levels of math anxiety in the female subjects from the experimental group were not significantly different than the levels of math anxiety in the male subjects from both groups or the female subjects from the control group, is that math anxiety could be resistant to change. This could be especially true if subjects had been math anxious for many years, then eighteen weeks would not be enough time to reasonably effect a change. Another reason could be that the subjects in this experiment did not accurately report the math anxiety they felt. Adolescents often do not want others to know that they are anxious or scared and this could have affected how these individuals responded on the Math Anxiety Questionnaire. Another possibility is that math anxiety is not limited to gender and that both males and females who are poorly prepared in math experience greater math anxiety. Unless math anxiety is reduced in math anxious students, avoidance of math courses and math majors often results. The decision to limit one's mathematical training has serious consequences which eventually impacts career options.

Main Effect-Group

The main effect of group did not reach significance at the multivariate level, so it can be concluded that there were no significant differences between the math concepts scores of subjects in the treatment or the control groups. A number of researchers (e.g., Conner & Serbin, 1985; Linn & Hyde, 1986; & Smith, 1964) who recognized the moderate to high correlations between mathematics achievement and spatial ability suggested that spatial training may be a way to improve math performance. Rekdal (1982) found that training in spatial abilities resulted in higher math scores. A training study by Conner and Serbin (1985) found that after a short training session, spatial visualization contributed to predicting mathematics achievement. In each of these studies described, spatial ability had a broad definition but included in that definition was the skill of mental rotation. Math achievement in Rekdal's study was measured by the Arithmetic Aptitude test, which was a reasoning test. Math achievement in the Conner and Serbin study involved the use of an instrument developed by the researchers which included standardized test items from math concepts, computation and applications subtests. Although these studies did not look at math concepts alone, this skill was assumed to be a part of the Math Reasoning test from Rekdal's study and was a component in the test designed for Conner and Serbin's study. The premise underlying the current research was that since math concepts correlated significantly with mental rotation and that since training in spatial ability, including mental rotation, resulted in significant differences in math achievement scores as described in the studies mentioned, then training in mental rotation would effect a change in the math concepts scores of the treatment group. The results of the multivariate analysis did not support the conclusions from Rekdal's and Conner and Serbin's research or the premise underlying the current research.

Possible explanations for the lack of significant differences noted in this analysis are similar to the explanations provided for the lack of an interaction effect. These explanations include: the training time allotted for this research was too short to effect any change in this skill, there were few test items on the math concepts subtest for the eighth grade which utilized the skill of mental rotation, and it is possible that there has been a reduction in gender differences over the last several years. Another possibility is that mental rotation is related more to other areas of math, such as calculation and/or math applications, and a training program in mental rotation would effect differences in these areas more than the area of math conceptualization. It is possible, as was suggested by Randhawa (1991), that gender differences in math achievement appear around the tenth grade.

Since the main effect of group was not significant, it was concluded that there were no significant differences in the math anxiety reported between the treatment and the control group. Hadfield, Martin, and Wooden (1992) found that spatial skills were inversely related to math anxiety for Navajo middle school students. A relationship between spatial ability and math anxiety has not yet been established. The current research is unique in that it examined the relationship between mental rotation, one component of spatial ability, and math anxiety; as well as the effect a Mental Rotations Curriculum would have on math anxiety. The premise was made that since a significant inverse relationship between spatial ability and math anxiety was found in Hadfield, Martin, and Wooden's study, that since a significant inverse relationship between mental rotation and math anxiety was found in the preliminary analysis, and that since training in spatial ability would effect changes in math performance, then training in mental rotation could effect changes in math anxiety. The results of this analysis did not support this premise.

The explanations for the lack of differences found in math anxiety between the treatment and control group include: the resistance of subjects with math anxiety to change, not enough time to reasonably effect a change, and inaccurate reporting by the subjects of the math anxiety they felt. The level of math anxiety was not measured before the experiment, which means that differences could have been affected, but because math anxiety was only measured once these differences were not detected. This study attempted to change math anxiety in the students from the experimental group by implementing a mental rotations curriculum. Math anxiety is not addressed by the schools. Unless schools start developing programs to reduce math anxiety, math anxious students will not reach their potential in math achievement.

Main Effect-Gender

The main effect of gender was examined at the multivariate level and was found to be significant. Gender differences have been widely documented in both the mathematics achievement literature and the math anxiety literature. In order to determine where the differences occurred Post Hoc univariate analyses of variance were conducted on each dependent variable separately.

Univariate analysis revealed that there were no significant differences in the math concept scores of the male and female subjects. This conclusion is in contrast to the findings of Macoby and Jacklin (1974), who state that male superiority in mathematics achievement has been well documented. Fennema and Sherman (1977) found that differences in favor of boys begin to appear by grade six. Pearson and Ferguson (1989) found significant differences between the genders on ACT Math scores. Randhawa (1991) concluded that in grade four females had higher achievement scores in math concepts than males, but by grade seven the female achievement advantage in math concepts was reduced significantly. By grade 10 males outperformed females in all three areas of math achievement. The results of the univariate analysis do not support Macoby and Jacklin's, Fennema and Sherman's or Pearson and Ferguson's conclusions that males outperform females in math.

The only aspect of math achievement utilized in this experiment was math concepts, so whether males outperform females in other areas of math achievement, such as math calculation and/or applications, cannot be determined by this analysis, but is a possibility. Another explanation is that gender differences would become evident in later grades as in Randhawa's study. At the junior high level math achievement between genders was the same, but by tenth grade males outperformed females in all areas of math achievement.

Univariate analysis of variance indicated that there were significant differences found in the levels of math anxiety reported by male and female subjects. A growing body of research on gender differences in math have been showing that the fear of math combines with traditional societal influences on women to create disproportionate number of math avoiding and/or math anxious females (Donady & Tobias, 1977; Lefevre et al., 1992; Singer & Stake, 1986). There is some disagreement in the literature about the relationship of math anxiety to gender and mathematics achievement. Singer and Stake (1986) concluded that math anxiety is not limited to females but is prevalent among all students who are poorly prepared in math. Engelhard (1989) reported that adolescence appears to be a critical age for studying gender differences in math anxiety. He found that adolescents who reported higher levels of math anxiety tend to have lower scores on math performance. The results of this univariate analysis found that females overall were more math anxious than males, which would support the findings of Donady and Tobias, Lefevre, and Singer and Stake. Whether this anxiety developed around adolescence could not be determined by this study, but the fact that the adolescent females reported more math anxiety suggests that there may be some credibility to Engelhard's claim, that adolescence may be a critical age for studying gender differences in math anxiety.

One explanation for female subjects reporting higher levels of math anxiety is that they were more honest in their answers than were the male subjects. Another reason could be that females become more math anxious as they enter into higher levels of mathematics, which occurs around adolescence, and this could be why math anxiety is first noted around this age. It is possible that deficiencies in specific skills related to math achievement, such as mental rotation, would result in more math anxiety.

Upon inspecting the numerical differences in the means of the math anxiety scores, it was noted that females in the experimental group had a nine point higher numerical mean score than the females in the control group, and their mean score was the highest score obtained on the Math Anxiety Questionnaire. The males in the experimental group obtained the lowest numerical mean score on this test. Although no interaction effect was found at the multivariate level, an interaction could have been present but was not detected. Based on observation the suggestions could be made that females in the experimental group were more math anxious than males in both the experimental or control group, and that females in the experimental group were slightly but not significantly more math anxious than females in the control group. Another suggestion which can be made from these observations is that females in the control group were not more math anxious than males in either the experimental or the control groups. These suggestions from inspecting the mean scores of the Math Anxiety Questionnaire lend only partial support to the findings of Donady and Tobias and Lefevre et al. Although the univariate analysis indicated significant differences with females reporting higher levels of math anxiety, inspection of numerical differences in mean scores suggested that females in the experimental group were the most math anxious. This does not provide support for the conclusion reached by Singer and Stake that math anxiety is not limited to females but is prevalent among all students who are poorly prepared in math.

The explanations reached from inspection of the numerical differences in mean scores suggest that the differences found in gender were found between the females and the males in the experimental group. Since the females in the control group were only several points higher in their numerical mean scores on the Math Anxiety Questionnaire and since the females in the experimental group were considerably higher in their mean scores on that instrument than any other group, the combination of the scores from the two female groups was large enough that it appeared females overall were more math anxious. An interaction effect is suggested by these observations. A further suggestion could be made that the Mental Rotations Curriculum affected the math anxiety of the experimental group; females became more math anxious and males became less math anxious. Whether the experiment affected math anxiety cannot be known, since math anxiety was only measured at the end of the experiment and it is not known what the math anxiety levels of the subjects were before the treatment. It is possible that because mental rotation is a skill that is more familiar for males, training in this skill actually decreased their anxiety. Mental rotation is a skill that is less familiar to females as was indicated by an number of researchers (e.g., Linn & Petersen and Vandenberg & Kuse) in the literature review. The possibility exists that when this unfamiliar skill is introduced to females, initially they become more math anxious. An eighteen week study might not have been enough time to see the full effect of a Mental Rotations Curriculum.

Follow Up Analysis

A follow up pre-posttest analysis of variance was conducted to determine if the Mental Rotations Curriculum had an effect on the mental rotation skills of the subjects in the experimental group. It was concluded that this curriculum changed the mental rotation skills of the eighth grade subjects who were in the experimental group. Smith and Schroeder (1970 found a greater gain in spatial scores for fourth graders in a trained experimental group. There was no differential learning effect between gender reported. In a study by Smith and Litman (1979) with adolescents, greater scores in an experimental condition were reported for boys only. Blatter (1983) tested the following three hypotheses: (1) at pretest boys will outperform girls in spatial ability, (2) specific training in spatial ability will result in improved scores on the posttest, and (3) girls will show more improvement than boys. Blatter found that both the experimental and control groups improved significantly with the experimental group improving the most. Girls improved more than boys but this improvement did not reach significance. Brinkman (1966) incorporated a spatial training curriculum with eighth grade students utilizing both an experimental and a control group. At the end of the study, the experimental group showed significant improvement in visualization skills, which included in its definition the specific skill of mental rotation. No significant gender differences were reported but Brinkman noticed that girls in the experimental group scored an average of two raw points higher than boys. The conclusions from the pre-posttest analysis of variance support the results from all of the studies mentioned that training in spatial ability improves spatial test scores. These results from the pre-posttest analysis are very similar to the results that Blatter's and

Brinkman's studies reported. On the pretest males scored higher than the females but this difference was not significant. Spatial training, for this study mental rotation training, resulted in improved scores on the posttest. Although the difference between male and female subjects' scores on the posttest was not statistically tested because of the low number of subjects who took the posttest, the numerical difference between the means was ten points with females scoring higher than males. This suggests that females could have responded more to the mental rotation training than did the males.

A plausible explanation for the results of the follow up analysis is that mental rotation skills improve with practice. This conclusion has been reached by a number of researchers (e.g., Alderton, 1989; Batey, 1986, Linn & Petersen, 1985; and Pellegrino, Alderton, & Shute, 1984) reviewed in Chapter II. Another possible explanation is that since the experimental group started out with higher mean scores on the Mental Rotations Test, the difference between these scores was found not to be significant, they could have had the ability to learn more through training than the control group would have learned if they had been trained in mental rotation skills. It is possible that females were able to gain more from mental rotation training, and because of this differential effect contributed more to the variance of scores of the experimental group, which resulted in the significant differences that were found between the two groups. The results of this pre-posttest analysis needs to be interpreted with caution. There are a number of problems inherent in pre-post test designs which often lead to inflated scores. The most important of these problems is practice effects resulting from taking a test twice. The use of a control group helped control inflated scores due to practice effects but it is very difficult to separate out the practice effects from taking the Mental Rotations Test twice and the practice effects from the treatment condition, which can often lead to erroneous results.

Limitations of the Study

The generalizability of the findings of this research is limited by several factors. This study was based on a population of students who were average to high-average in intelligence. No generalizations should be made for gifted, low or low-average students. Because the sample included students from private parochial schools, these results cannot be generalized to a normal public school student population. Since this research utilized eighth grade students, any differences in mental rotation or math anxiety found in this study cannot be generalized to a younger or older population. The only aspect of spatial ability measured is that of mental rotation so if differences exist in other aspects of spatial ability they cannot be determined by the results of this research. Although math achievement and math concepts are sometimes discussed interchangeably, the only aspect of math achievement that was utilized in this research was math concepts so the results of this research cannot be generalized to other areas of math achievement, such as math calculation or math applications.

The use of four intact groups of students removed from their classrooms for the purposes of this research limits any randomization of subjects. This restriction could not be controlled by the investigator due to scheduling and classroom availability constraints imposed by the individual schools. The small number in the population sample due to attrition decreases the confidence with which generalizations could be made from the results of this research.

The instruments used to measure the constructs of math conceptualization, mental rotation, and math anxiety are limited by the problems found on each instrument and may not measure the true performance of an individual in these domains. The reliability estimates reported for the Mental Rotations Test for subjects under fourteen years of age are few. This could decrease the confidence in interpreting the test results for the subjects in this experiment who were thirteen years of age.

Because of a problem found in one of the control groups of students, nine new subjects were selected from another parochial school. This selection occurred several weeks after the experiment began, which introduces an intervening variable, maturation. Maturation was a variable that could not be controlled, and could have affected the control students' performance on the three tests administered. This limits the confidence with which the results of this study can be interpreted.

Due to scheduling and classroom availability constraints imposed by the individual schools, the schedules in which the treatment was administered were different. Two of the schools had training sessions of 45 minutes, two times a week, and the third school had a training session of 30 minutes, three times a week. These training sessions amounted to 90 minutes of training a week. Whenever any of the experimental subjects was absent, every effort was made for individualized instruction so that the subject could make up the lesson that he/she missed. It cannot be determined if this resulted in reduced learning for the subjects who were absent. This difference could not be controlled and could have affected the results of this study.

The Mental Rotations Curriculum that was utilized in this research taught a standard strategy for mentally rotating objects. It was assumed that all students could benefit from learning the same strategy. This curriculum was assumed to teach the same skill, which was measured on the Mental Rotations Test. If these assumptions were false for any reason then the curriculum could have negatively impacted the results of this study.

The conclusions which were derived from the statistical analysis need to be interpreted with caution for a number of reasons. The low number of subjects limits the confidence with which the results can be interpreted. This is particularly true in the case of the small sample, (df 1, 40), utilized in the pre-posttest follow up analysis of variance. Practice effects which result from taking the same instrument twice often produce inflated results, and are hard to separate out from the practice effects which result from the treatment. Whenever numerous post hoc analyses, as was done in this research, are conducted more error can result, which could produce misleading results.

Observations and Implications

Several of the male students in the experimental group obtained a low score on the pretest of mental rotation. Since the researcher observed an inverse relationship on the preliminary analysis between mental rotation and math anxiety, it was assumed these students were quite math anxious. Their scores on the Math Anxiety Questionnaire did not reflect this assumption. During the training sessions these students would often be talking and not doing their tasks. The researcher would spend time trying to get them back on task. These males were sometimes late to the training session for no apparent reason. It appeared that these subjects were avoiding the training tasks, especially as these tasks became more difficult.

The implication of this observation is that these students did not honestly respond to the questions on the Math Anxiety Questionnaire, which could have resulted in erroneous findings. The reasons for these males not acknowledging anxiety could range from embarrassment for admitting their true feelings to peer pressure to fit in with the group. Adolescence is a time in the life of an individual when peer pressure is the greatest. If adolescent males do not respond honestly to the items on anxiety questionnaire then studies, using an adolescent population, reporting more math anxiety for females could be misleading.

Researchers using this population need to concern themselves with the possibility of subjects giving false information on math anxiety questionnaires. One way to prevent this for future research would be to have a student and an adult, preferably someone who knows the student (e.g., an interviewer), fill out the questionnaire together so the student will understand what the questionnaire is asking and respond appropriately to the questions.

An implication for the conclusion, that a significant relationship exists between mental rotation and math achievement, is that females are at a slight disadvantage because this skill of mental rotation is not as familiar to them as it is to males, and this skill is rarely taught in the public school math curriculum. The only curriculum which contains exercises in spatial concepts would be geometry units in math textbooks and geometry courses. Because of the relationship between math concepts and mental rotation and because this skill appears to be modifiable, curriculum directors, administrators, and teachers should begin to consider including it in the regular curriculum. One possibility would be to administer a mental rotation test to students in a classroom and then to provide a mental rotation training program for those students who scored low on the test.

Since mental rotation was found to have an inverse relationship with math anxiety and since math anxiety is related to how well one achieves in math, then the implication can be made that individuals who are weak in mental rotation skills would be more math anxious. Females tend to be more math anxious as was found in this study and do not perform as well on tests of mental rotation when they have had no specific training. Deficiencies in these skills could account for some of the math anxiety observed in female students. This math anxiety needs to be addressed by teachers, principals and parents so that math anxious students can reduce their anxiety in order to achieve to their potential in mathematics. Ways in which math anxiety could be addressed in a classroom setting include identifying students who are math anxious through formal or informal assessment procedures, relaxation techniques, cognitive behavioral strategies to deal with the anxiety, and/or other intervention strategies which have been developed for students who exhibit significant math anxiety.

Future research should consist of studies which examine the relationships between mental rotation and both math concepts and math anxiety, in the following ways. Correlational studies could be conducted examining the relationships of mental rotation with all areas of math achievement to determine if mental rotation is related to only math concepts or to all areas of math achievement. Other correlational studies could be conducted examining the relationships of mental rotation with math anxiety for males and females separately. Perhaps the relationship between these two variables is different for females than males. Because of the significant correlations found between these three variables, a regression analysis could be conducted to determine if math concepts scores and/or other areas of math achievement could be accurately predicted using mental rotation and math anxiety as predictors.

Since significant findings were not found in the math concepts scores of the treatment and control groups and between male and female subjects, the implication can be made that there are no differences. This is not necessarily true since math concepts is only one area of math achievement. This study could be replicated using the other areas of math achievement as dependent variables before the implication, that there are no differences in math achievement, can be made.

Significant differences in math anxiety were found with female subjects reporting greater math anxiety. This finding implies that female subjects are more math anxious. This implication could be misleading for two reasons. Math anxiety was only measured once and what the math anxiety levels of the students were before this experiment cannot be known. It is possible that the experiment changed the math anxiety levels of the subjects. The second reason this implication could be misleading is that the numerical mean scores of the females in the experimental group were much higher by inspection than males in both the experimental and control groups and that the means of the females in the control group were only a few points higher than the males in both groups.

The males in the experimental group obtained the lowest numerical mean score by inspection in math anxiety. The implication can be made that the treatment could have increased the math anxiety in the female subjects from the experimental group, and decreased the math anxiety in the male subjects in the experimental group. Because mental rotation is an unfamiliar skill for females, introducing that skill could have produced anxiety; whereas because mental rotation is a familiar skill for males, introducing that skill could have reduced anxiety. Higher levels of math anxiety could explain why more females avoid higher levels of math in high school and college as well as avoid math majors. So that math anxiety in females can be acknowledged and reduced, implications for classroom modifications should include identification of math anxious students, training in the skill of mental rotation, and interventions designed to reduce math anxiety.

If the treatment induced anxiety in the female subjects from the experimental group and reduce the math anxiety of the male subjects from the experimental group, then an implication can be made that it is possible that as female subjects become more adept at their mental rotation skill then over time their anxiety could be reduced, which was one of the goals of the current research. Future research should utilize a longitudinal study examining the effects of a Mental Rotations Curriculum on all areas of math achievement and math anxiety. A repeated measures design could be useful in determining the change both in math anxiety and the areas of math achievement, as well as controlling for the error that is found in studies utilizing pre-posttest designs. This type of study could determine if more time is needed to effect math achievement and/or math anxiety by training in the skill of mental rotation.

One interesting finding from examining the means of the follow up analysis was that the females in the experimental group obtained the highest numerical mean score by inspection on the mental rotations posttest. Since this group obtained the highest numerical mean score on math anxiety, the implication can be made that recognition of their improvement in mental rotation skill was not commensurate with their anxiety level. Their anxiety level was high, despite the fact that their performance on the mental rotation posttest was better than the performance of any of the other groups. This could be due to the fact that not enough time elapsed for the female subjects to feel comfortable with their improved performance in mental rotation.

The use of self-efficacy measures in the classroom could provide educators with information regarding how students view their abilities in comparison to how they are achieving. If a discrepancy exists between how a student views himself and what his abilities are and how he is achieving, then intervention strategies can be formulated and implemented to enable students to realistically perceive their abilities. Future research should consider the use of these self-efficacy measures when utilizing female subjects, or other subjects exhibiting high math anxiety and/or low math achievement for studies which measure the constructs of math achievement and/or math anxiety.

Summary

The first section of this chapter reviewed the purpose, methodology and results of the study. Interaction effect and main effect-group were not statistically significant at the multivariate level. Significant results were found for the main effect of gender at the multivariate level, which resulted in rejection of Null Hypothesis H2. By examining the mean scores on the Math Anxiety Questionnaire the conclusion was reached that female subjects were more math anxious than male subjects. The second section presents a list of eight conclusions which were derived from the preliminary analyses, examination of interaction effect, main effect-group, and main effect-gender and the Post hoc analyses on each dependent variable, and the follow up analyses conducted. Conclusions derived from each analysis, how these conclusions relate back to the literature review, and explanations for these conclusions are all discussed in this section.

The conclusions reached from the preliminary analyses included the following: a significant relationship exists between mental rotation and math concepts, and a significant inverse relationship exists between mental rotation and math anxiety. Support from the literature review was cited and possible explanations for these findings were provided.

Another significant result was found when examining the main effect of gender. Female subjects reported more math anxiety than male subjects. This conclusion was supported by several researchers. Possible explanations for this finding include the fact that female subjects were more honest in their responses to the Math Anxiety Questionnaire, the female subjects could become more anxious as they enter into higher levels of mathematics and female subjects have deficiencies in specific skills related to math achievement, such as mental rotation which result in greater math anxiety. It was observed from inspecting the numerical differences in mean scores that females in the experimental group appeared to be more math anxious than males in both the experimental group and the control group. The females in the control group were no more math anxious than the males in both the experimental and the control group. Two possible explanations for these observations are: first, an interaction effect was present but did not show up in the statistical analysis, and second, the Mental Rotations Curriculum effected a change in the math anxiety of the females in the experimental group.

The conclusion reached from the follow up analysis was that the Mental Rotations Curriculum changed the mental rotation skills of the experimental group. Possible explanations for this conclusion include: (a) mental rotation skills often improve with practice, (b) female subjects respond to training more than do male subjects, and their improvement in mental rotation contributed significantly to the differences found between the experimental and control groups, and (c) subjects in the experimental group started out with a higher mean score in mental rotation, and because of this were able to learn more from instruction than the control group would have if they had been trained in mental rotation skills.

Limitations of this research include (a) the population of students were average to high-average in intelligence so no generalizations can be made for gifted, low or low-average students, (b) the population sample included students from four parochial schools which limits the generalizations made for a normal public school student population, and (c) only one aspect of spatial ability, mental rotation, and one aspect of math achievement, math concepts, were measured so if differences exist in the other aspects of these two constructs they cannot be determined by this research. Other limitations involved the use of intact groups, problems inherent in the instruments utilized in this study, intervening variables which could not be controlled, scheduling constraints imposed by the individual schools, problems inherent in the Mental Rotations Curriculum utilized in this research, and weaknesses in the statistical techniques used for the pre-posttest analysis of variance.

Implications for current practice modifications include closer observations on the part of teachers for identifying math anxious students, and for developing and implementing interventions to reduce math anxiety. In order to determine if students are weak in mental rotation skills, pretesting these students and then training the students who score low on the pretest in mental rotation, could provide a successful approach to eliminating differences between students. Inclusion of a mental rotation unit in the mathematics curriculum in elementary school could help develop these skills, so that when these skills are needed for developing higher level math skills, they will be developed and will not hinder students' achievement. Use of self-efficacy measures could be utilized to determine if the way a student views his/her accomplishments are the same as the way a student views his/her ability and performance in the area of mathematics achievement.

Suggestions for future research include: further correlational analyses, which examine the nature of the relationship between mental rotation and both math achievement and math anxiety separately for males and females, and a regression analysis, to determine if math achievement scores could be predicted using mental rotation and math anxiety as predictors. Since the females in the experimental group reported greater math anxiety and since the implication was made that the treatment could have increased their math anxiety levels, a longitudinal study could be conducted to determine if, over time, the females who are trained in mental rotation would experience a reduction in math anxiety, and possibly a change in math achievement scores.

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APPENDIXES

APPENDIX A

PILOT STUDY LETTER AND CONSENT FORM

Dear Parent,

I am a certified school counselor, certified school psychometrist, and am currently a doctoral candidate in School Psychology at Oklahoma State University. I am conducting a pilot study to aid in developing a better understanding of the abilities underlying the learning of mathematics and how these abilities affect math anxiety. This research is to be conducted as part of a Ph.D. dissertation project under the direct supervision of Professor Paul Warden, Applied Behavioral Studies in Education. We hope that this study will yield information concerning the skills necessary for mathematical conceptualization as well as their relationship to math anxiety. Hopefully this information will aid in the development of future curriculum and instructional methods more closely matched to the learners' needs.

I would like to request permission for your child to participate in this pilot study investigating the relationship between visual-spatial abilities, math anxiety, and mathematical conceptualization. Each child who participates will be given a visual-spatial test, and a math anxiety questionnaire. Their math concepts scores from the Iowa Tests of Basic Skills will be used. The testing does not involve any risk to your child. No child will be forced to participate. You are free to withdraw your child from this study at any time. All information will be considered confidential; no names will be used in any written material.

If you or your child have any questions or would like further information about any aspects of this project, please feel free to contact me.

Sincerely,

Carolyn A. Harpole, M.A.

Carolyn A. Harpole, M.A.

I have been fully informed of the above-described procedure and I agree to allow my child to participate in this study. I understand that nothing done in this study would be harmful to my child and that I may withdraw my consent at any time. I also understand that all information obtained regarding my child will be held confidential, and that I may contact Carolyn Harpole at any time to obtain further information regarding this study.

Parent Signature____________________________________

APPENDIX B

CURRENT STUDY LETTER AND CONSENT FORM

Dear Parent,

I am a certified school counselor, certified school psychometrist, and am currently a doctoral candidate in School Psychology at Oklahoma State University. I am conducting research to aid in developing a better understanding of the abilities underlying the learning of mathematics and how these abilities affect math anxiety. This research is to be conducted as part of a Ph.D. dissertation project under the direct supervision of Professor Paul Warden, Applied Behavioral Studies in Education. We hope that this study will yield information concerning the skills necessary for achievement in mathematics as well as their relationship to math anxiety. Hopefully this information will aid in the development of future curriculum and instructional methods more closely matched to the learners' needs.

I would like to request permission for your child to participate in a study investigating the relationship between visual-spatial abilities, math anxiety, and achievement in mathematics. Each child who participates will be given a visual-spatial test. Half of the students will be selected to participate in an enrichment curriculum consisting of three thirty-five minute instructional periods per week. Participation in this enrichment curriculum will not interfere with your child's involvement in the regular mathematics curriculum. This enrichment curriculum is being used in a number of schools in California and Canada. Those students selected to participate will be working with manipulatives, developing spatial skills and relating these skills to mathematics. This instructional curriculum will take approximately 18 weeks. After the 18 week instructional period, all students will be administered a math achievement test and a math anxiety test.

The testing does not involve any risk to your child. No child will be forced to participate. You are free to withdraw your child from this study at any time. All information will be considered confidential; no names will be used in any written material. The results of this study will be shared with you when the data have been analyzed and the conclusions drawn.

If you or your child have any questions or would like further information about any aspects of this project, please feel free to contact me.

Sincerely,

Carolyn A. Harpole, M.A.

Carolyn A. Harpole, M.A.

INFORMED CONSENT

I have been fully informed of the above-described procedure and I agree to allow my child to participate in this study. I understand that nothing done in this study would be harmful to my child and that I may withdraw my consent at any time. I also understand that all information obtained regarding my child will be held confidential, and that I may contact Carolyn Harpole at any time to obtain further information regarding this study.

Signature of Student____

Signature of Parent/Guardian________________________________

Date_______________________________

APPENDIX C

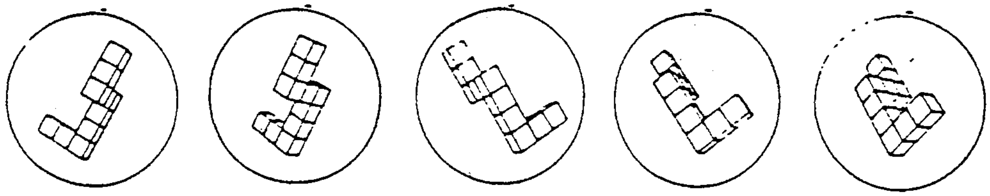
INSTRUMENTATION

M.R.T. Test Date ______________________

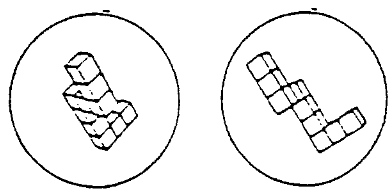
Mental Rotations Test

____

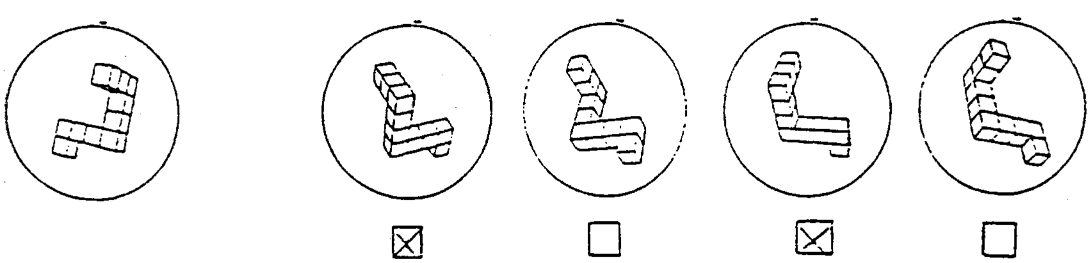
This is a test of your ability to look at a drawing of a given object and find the same object within a set of dissimilar objects. The only difference between the original object and the chosen object will be that they are presented at different angles. An illustration of this principle is given below, where the same single object is given in five different positions. Look at each of them to satisfy yourself that they are only presented at different angles from one another.



Below are two drawings of new objects. They cannot be made to match the above five drawings. Please note that you may not turn over the objects. Satisfy yourself that they are different from the above.



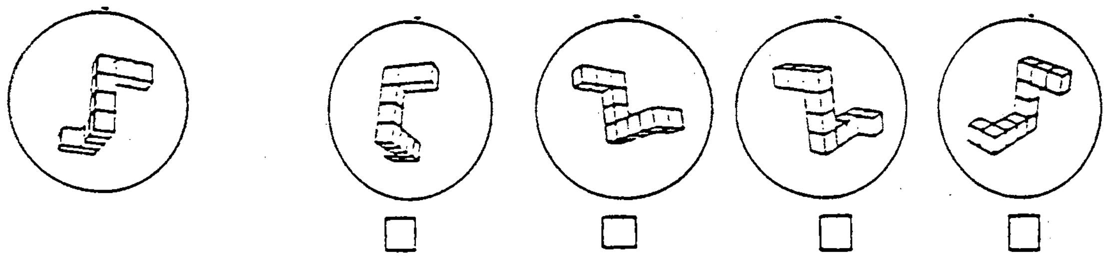
Now let's do some sample problems. For each problem there is a primary object on the far left. You are to determine which two of four objects to the right are the same object given on the far left. In each problem always two of the four drawings are the same object as the one on the left. You are to put Xs in the boxes below the correct ones, and leave the incorrect ones blank. The first sample problem is done for you.

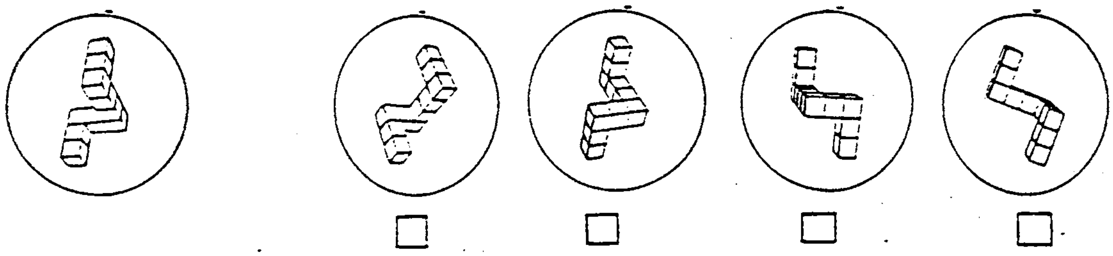


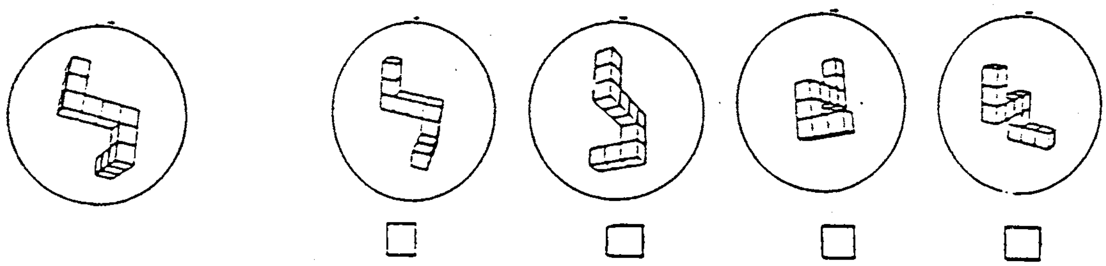
Go to the next page

Adapted by S.G. Vandenberg, University of Colorado, July 15, 1971
Revised instructions by H. Crawford, U. of Wyoming, September, 1979

Do the rest of the sample problems yourself. Which two drawings of the four on the right show the same object as the one on the left? There are always two and only two correct answers for each problem. Put an X under the two correct drawings.







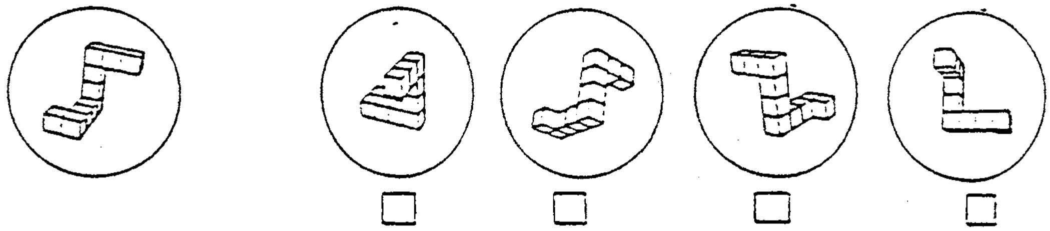
Answers: (1) first and second drawings are correct
(2) first and third drawings are correct
(3) second and third drawings are correct

This test has two parts. You will have 3 minutes for each of the two parts. Each part has two pages. When you have finished Part I, STOP. Please do not go one to Part 2 until you are asked to do so. Remember: There are always two and only two correct answers for each item.

Work as quickly as you can without sacraficing accuracy. Your score on this test will reflect both the correct and incorrect responses. Therefore, it will not be to your advantage to guess unless you have some idea which choice is correct.

DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO

1.



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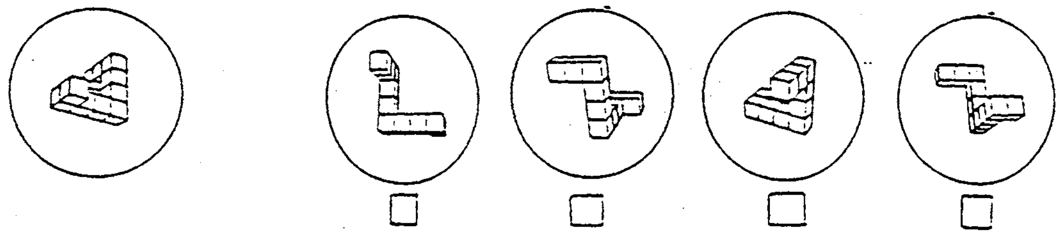

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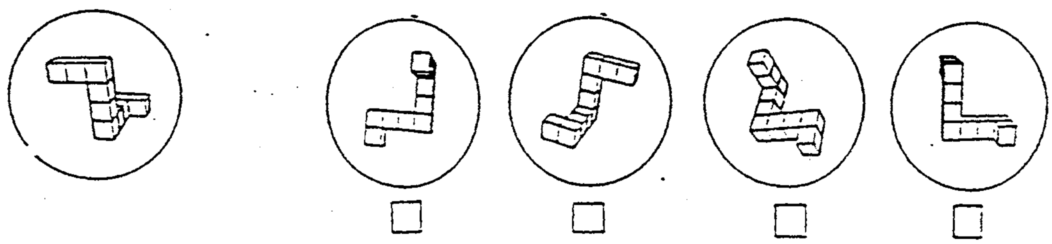

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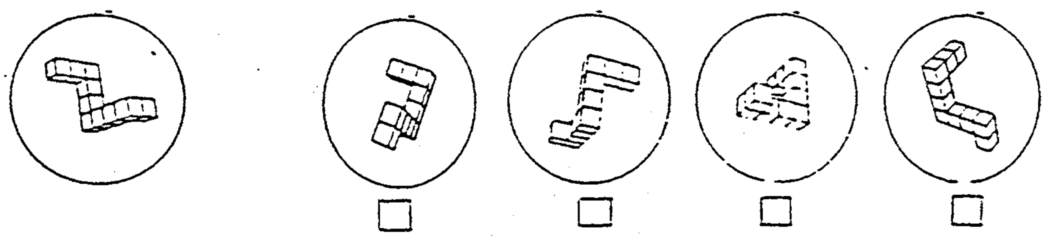

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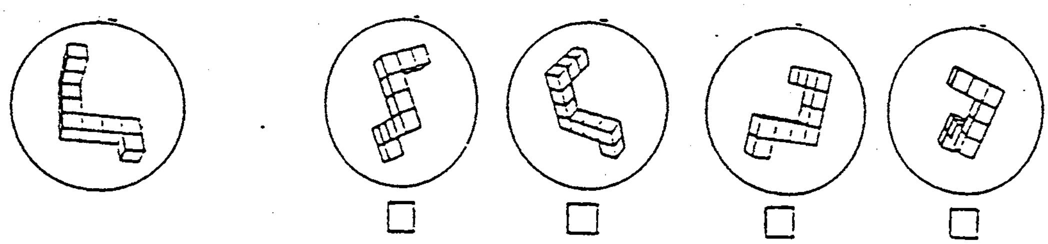

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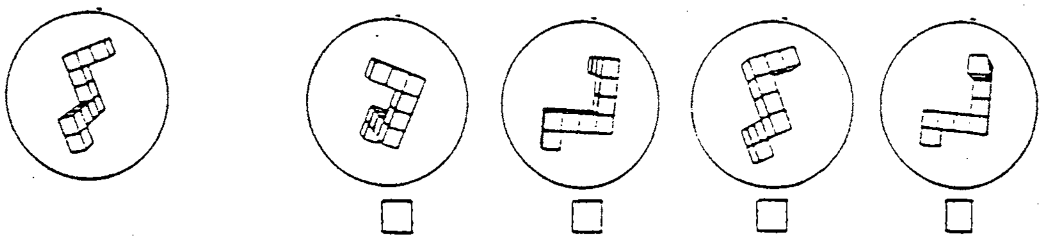

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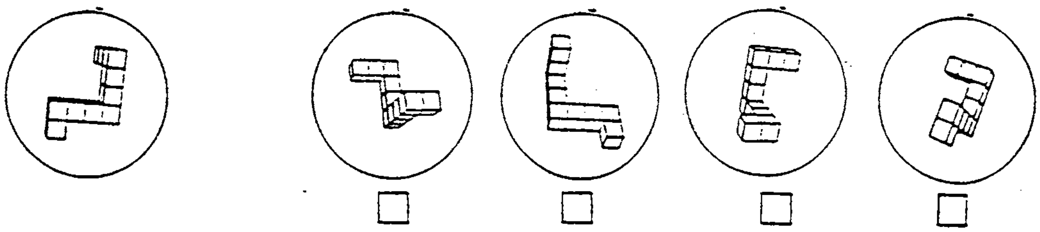

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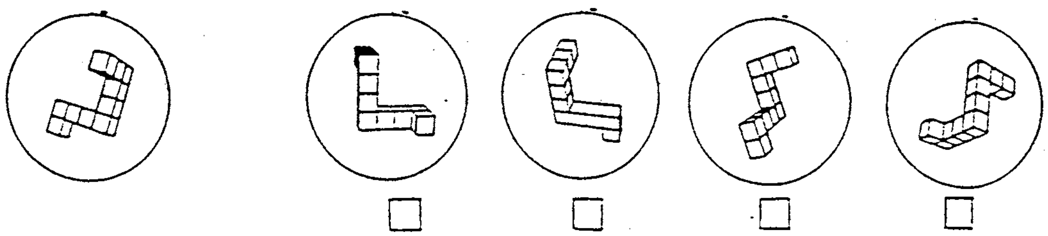

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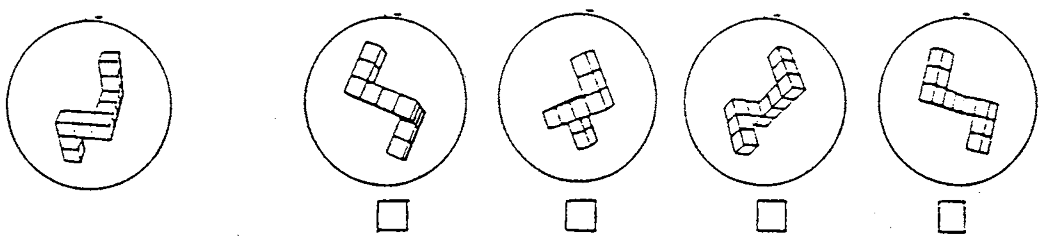

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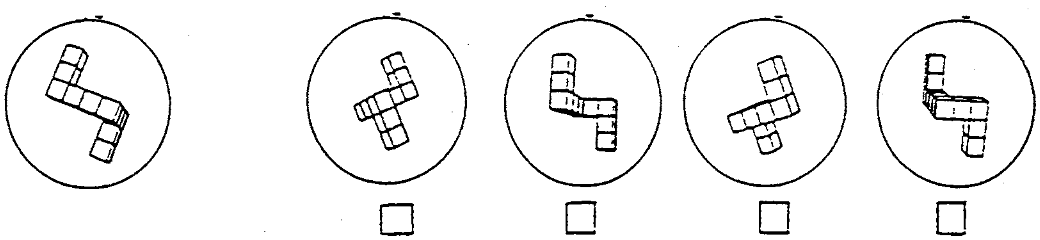

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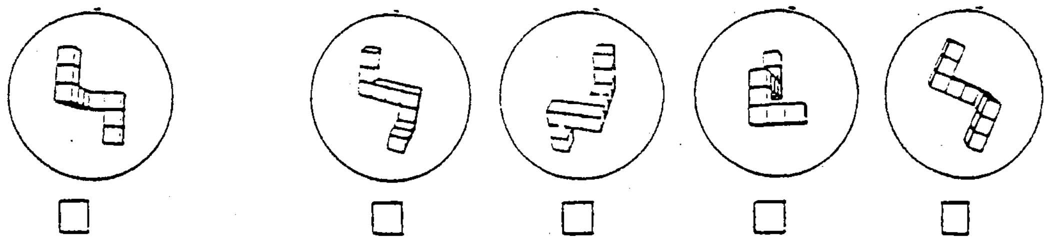
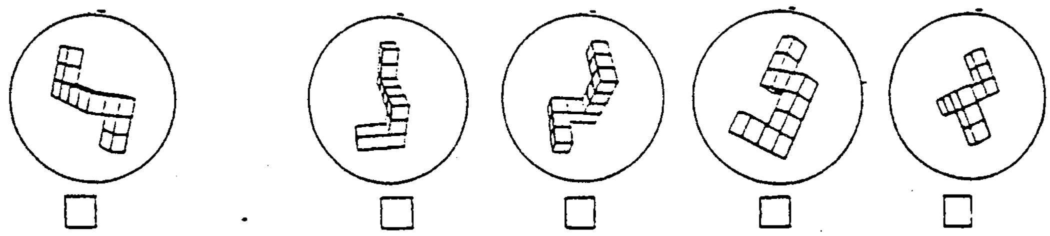
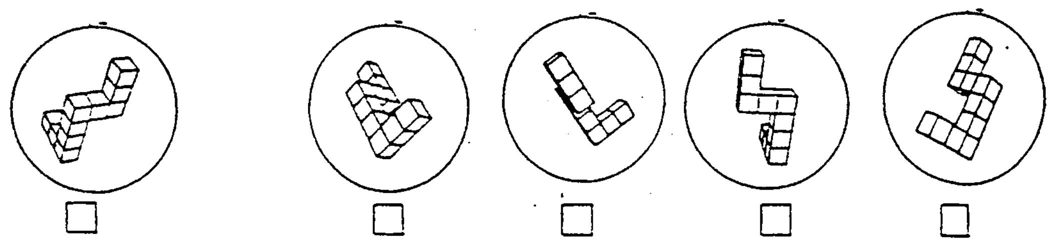
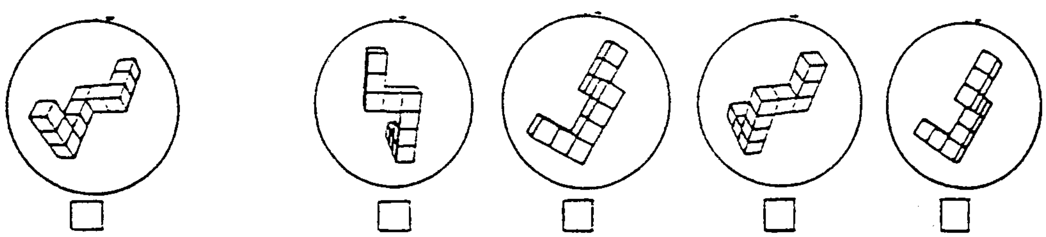
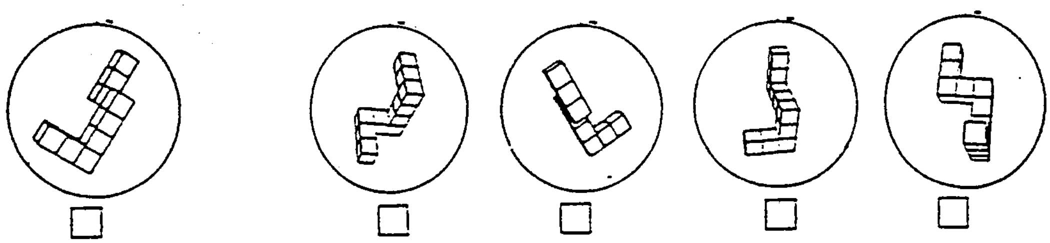
11. 

12. 

13. 

14. 

15. 

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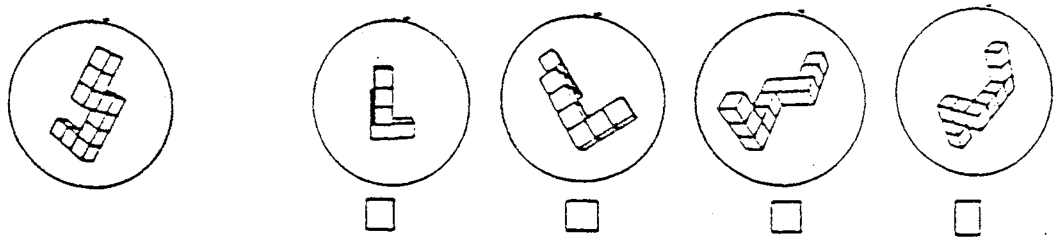
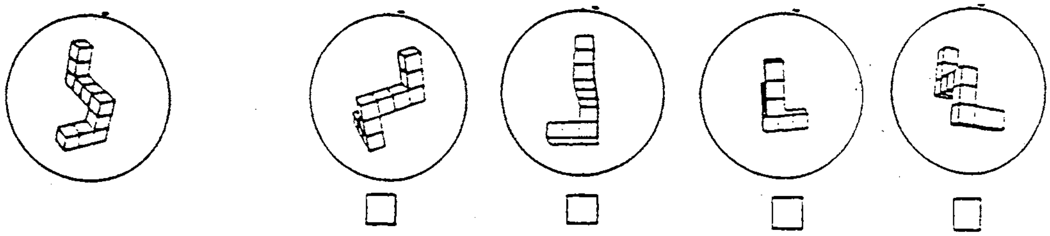
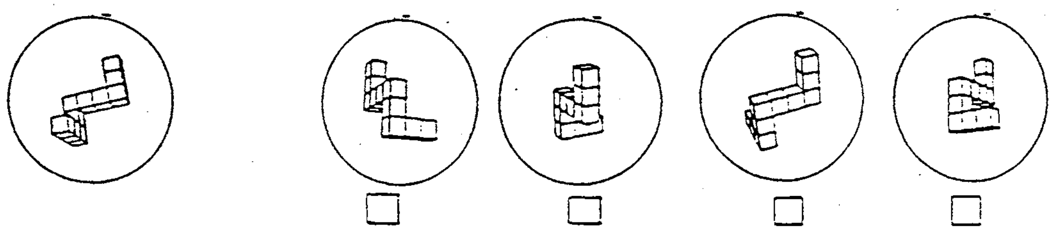
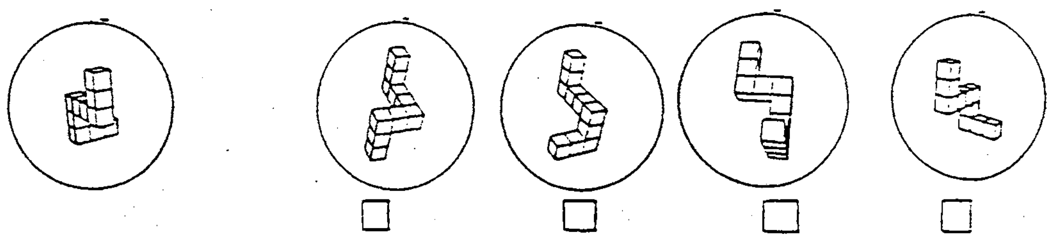
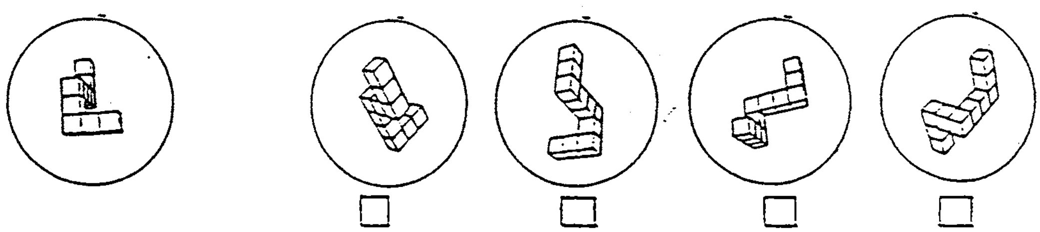
16. 

17. 

18. 

19. 

20. 

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Math Anxiety Questionnaire - Wigfield + Meece, 1988

Read each statement and then circle the number of the statement to indicate how you generally feel. The only correct responses are those that are true for you.

When the teacher says s/he is going to ask you some questions to find out how much you know about math, how much do you worry that you will do poorly?

Not at all	1	2	3	4	5	6	7	Very much

When the teacher is showing the class how to do a problem, how much do you worry that other students might understand the problem better than you?

Not at all	1	2	3	4	5	6	7	Very much

When I am in math, I usually feel at ease and relaxed.

Not at all at ease and relaxed	1	2	3	4	5	6	7	Very much at ease and relaxed

When I am taking math tests, I usually feel at ease and relaxed.

Not at all nervous and uneasy	1	2	3	4	5	6	7	Very nervous and uneasy

Taking math tests scares me.

I never feel this way	1	2	3	4	5	6	7	I very often feel this way

I dread having to do math.

I never feel this way	1	2	3	4	5	6	7	I very often feel this way

It scares me to think that I will be taking advanced high school math.

Not at all	1	2	3	4	5	6	7	Very much

In general, how much do you worry about how well you are doing in school?

Not at all	1	2	3	4	5	6	7	Very much

If you are absent from school and you miss a math assignment, how much do you worry that you will be behind the other students when you come back to school?

Not at all	1	2	3	4	5	6	7	Very much

In general, how much do you worry about how well you are doing in math?

Not at all	1	2	3	4	5	6	7	Very much

Compared to other subjects, how much do you worry about how well you are doing in math?

Much less than other subjects	1	2	3	4	5	6	7	Much more than other subjects

SAMPLE ITEMS - MATHEMATICS CONCEPTS TEST M-1
IOWA TESTS OF BASIC SKILLS
(Sample items taken from ITBS Form G, Level 14)

1. What fraction of a pound is 8 ounces?

1)	1/3	3)	2/3
2)	1/2	4)	4/5

2. What is the ratio of one hour to two days?

1)	1/2	3)	1/24
2)	1/12	4)	1/48

3. What is the greatest common factor of 50 and 325?

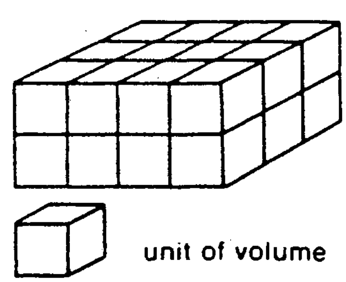
1)	5	3)	25
2)	10	4)	50

4. If $x > 230$ and $x < 250$, which of the following is a possible value of x?

1)	225	3)	250
2)	241	4)	263

5. What is the volume of the rectangular prism shown below?

1) 9 units
2) 12 units
3) 24 units
4) 26 units



APPENDIX D

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COPYRIGHT RELEASE

Carolyn Harpole is planning to publish her doctoral dissertation which is titled "Effects Of A Mental Rotations Curriculum On Mathematics Conceptualization And Math Anxiety In Eighth Grade Male And Female Students". She would greatly appreciate your permission to use the following material from "Power Cubes; Adventures in Spatial Perspectives" written by Rebeca Stanton and Elizabeth A. Miller. The materials include examples of Instructional Unit Lessons And Activities from Unit 1- Lesson 1 and 2, Unit 2-Lesson 6 and Activity Sheet 7, Unit 3-Activity Center 1, Unit 4-Activity Centers 7 and 8 and Unit 5. This material is included in Appendix C of the Dissertation Document.

Sept. 18, 1995
Date

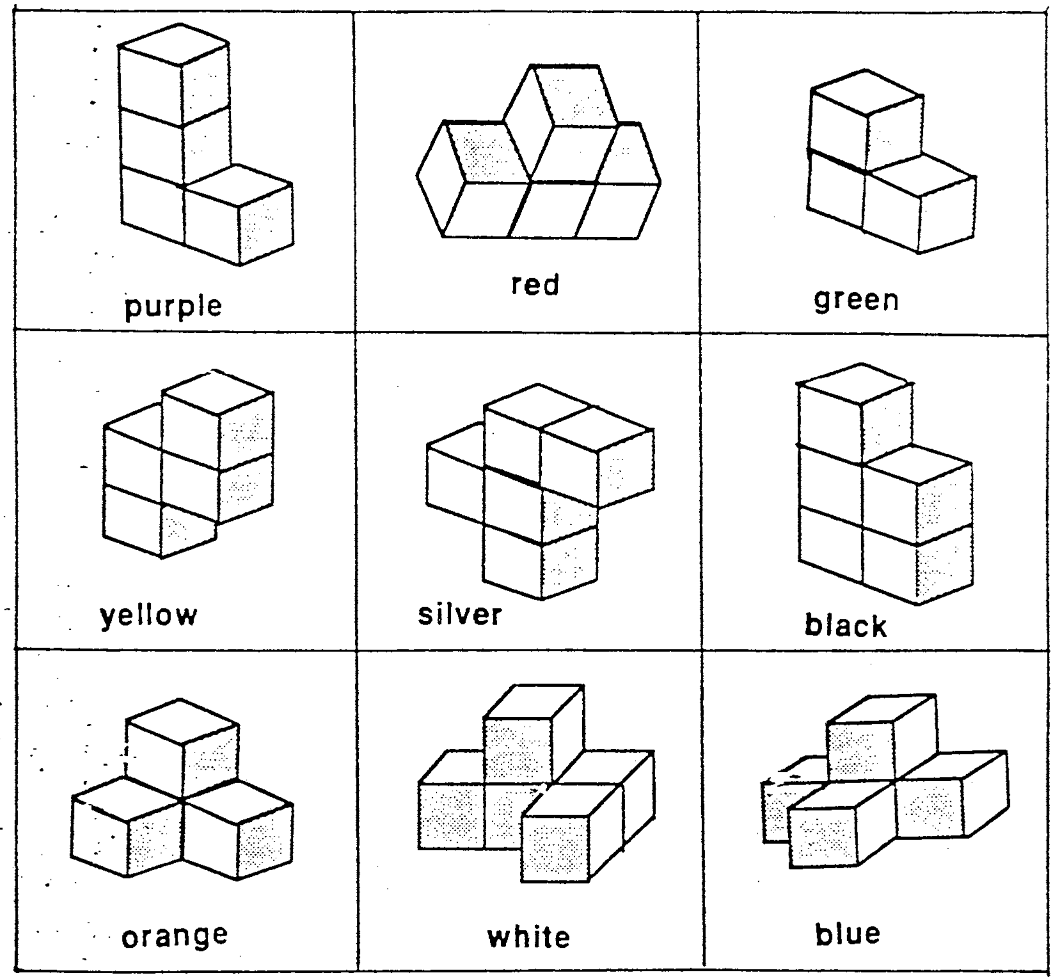
Rebeca E. Stanton
Signature
Representative for Stamina
Associates

APPENDIX E

INSTRUCTIONAL UNIT LESSONS AND ACTIVITIES

Figure Cubes

To make the Figure Cubes you will need 39 small wooden cubes, wood glue and nine different colors of paint. These colors are: purple, red, green, yellow, silver, black, orange, white, and blue. Glue the cubes together so that you have the nine different figures illustrated below. When the glue has dried, spray paint the figures according to the instructions.



You may purchase a set of 100 wooden cubes for $18.50 plus tax and freight charges. Please use the order form on the back of this page.

UNIT 1

Introduction to Three-Dimensionality

LESSON 1 1 DAY; 30 MINUTES

TEACHER NOTES

It is natural for children to play and build with blocks. When time is provided for this activity, students are then more receptive to learning during the subsequent teacher-directed activities.

Use the terms "cube" and "figure-cube" as you pass out the cubes and figure-cubes. When collecting them, show the children where the cubes and figure-cubes will be kept and what the distribution method will be in the future.

Objective: The students will explore the figure-cubes prior to a directed lesson.

Materials

Teacher and Students: One set of figure-cubes per student and teacher; two to three individual cubes per student and teacher.

Vocabulary:

cube
figure-cube (each of the shapes created by the colored cubes)

ACTIVITY DAY 1; 30 MINUTES

Allow 1/2 hour for the students to play with the cubes and figure-cubes.

LESSON 2 1 DAY; 30 MINUTES

Objective: The students will:

1. discover the relationship between cube and figure-cube
2. learn the vocabulary that defines the attributes of a cube
3. draw the cube in three-dimensional form

Materials

Teacher: overhead projector, blank transparencies and colored pens, single cube, purple figure-cube

Students: blank paper, pencil, single cube, crayons, student folders

Vocabulary:

cube (the figure)
face (each side of the cube)
angle (two lines with the same end-point)
three-dimensional (a figure with height, length, and width)
parallel lines (lines in the same plane that do not intersect; lines that never meet)
vertical lines (lines going up and down)
horizontal lines (lines going across)
perspective (the aspect from which an object is viewed)

TEACHER NOTES

The students should explore the cube by "feeling" the vocabulary. When defining a "face," have the students run their hands over each of the cube faces. When defining an "angle," have the students feel the corners.

ACTIVITY DAY 1; 30 MINUTES

Ask:

1. "What do the cube and the "figure-cube" have in common?
   (They both have sides, faces, height, length, width, and three dimensions.)
2. "How would you describe this cube?"
   (Accept any reasonable answer.)

TEACHER NOTES

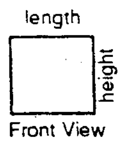
Introduce terms "horizontal" and "vertical" by equating them with the length and height of the cube. The concept of height should be taught first, length second and width last, because this matches the order of the terms in the formula for volume, $v = h \times l \times w$.

3. Say, "We are going to learn to draw a cube so that we can see its three dimensions."

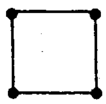
TEACHER NOTES

Use the overhead projector to demonstrate the following activities. Tell the students what you are doing in each phase, as written below. Students should perform the activities with you on their own papers.

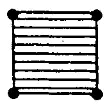
4. Place the cube on paper and trace around it with a pencil.



a. Label this "Front View." Mark the vertical plane as height and the horizontal plane as length.



b. Put a dot in each corner of the front view.



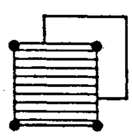
c. Shade the front view with horizontal lines.

TEACHER NOTES

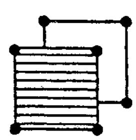
Use the vocabulary terms with each drawing. It is important to go slowly through these initial steps so that students will internalize the concept of three-dimensionality.

Students who are having difficulty with any drawing should be given additional practice before moving on to the next step.

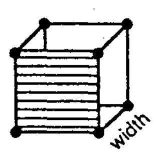
5. Now move the cube a little up and to the right of the drawing we just completed.



a. Trace around the cube again, but do not go over the shaded area.



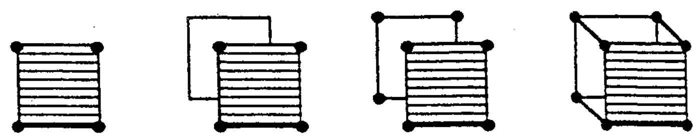
b. Put a dot in each corner.



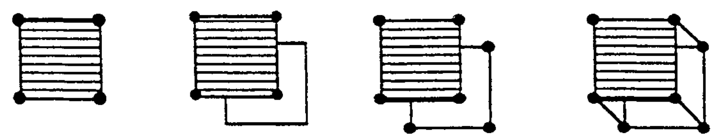
c. Connect the corner dots of the front view with the corner dots of the second view.

6. Instruct the students to repeat the drawing sequence in the previous exercises, moving the cube in the following directions:

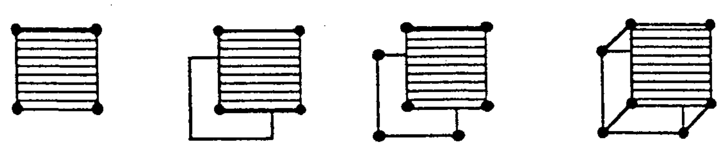
   a. up and to the left.



   b. down and to the right



   c. down and to the left



7. Say, "When you look at the same object from different points of view, you are looking at it from different perspectives." Students should begin to understand the concept of perspective.

TEACHER NOTES

At the end of this lesson, pass out the folders, explain to the students that they will keep all of their work in their folder. A place should be set aside for storing folders. The folder should be retrieved by the students at the beginning of each lesson and put away at the end of each lesson.

UNIT 2

Using an Isometric Grid

LESSON 6 2 DAYS; 30 MINUTES EACH

Objective: Students will use an isometric grid to *"take out"* simple cubes.

Materials

Teacher: Overhead projector, transparency of Activity Sheet 7 (Isometric grid with numbers), blank transparencies, colored pens, **64 single cubes**

Students: blank white paper, pencil, single cubes, copy of Activity Sheet 7, crayons, student folder (with Record-Keeping sheet)

TEACHER NOTES

Activity Sheet 7 is found at the end of this lesson. Make student photocopies of Activity Sheet 7, and an overhead projector transparency of Activity Sheet 7 for your use.

Vocabulary:

isometric grid (a network of horizontal and vertical lines forming a series of squares)
parallel lines (lines in the same plane that do not intersect; lines that never meet)

ACTIVITY DAY 1; 30 MINUTES

1. Place the transparency of Activity Sheet 7 (Isometric Grid with numbers) on the overhead with a blank transparency over it. Pass out the students' materials. Set up the 64 single cubes to replicate the isometric grid.

2. Now instruct the students as follows:

   a. "This is an isometric grid. It is a drawing representing these 64 single cubes. I will show you how to take cubes 'out' of your isometric grid by drawing around them."

   b. "Place a piece of blank paper over your copy of Activity Sheet 7 (the isometric grid). Place your cube to cover the square marked #1 on the isometric grid."

   c. "Trace around the cube with your pencil and then color the square."

TEACHER NOTES

Outline the square and color the square on the transparency as students are working on their paper. Also demonstrate the following exercises on the transparency.

   d. Remove the cube from the 64 cubes that correspond to the #1 place on the grid. Say, "Notice that if you trace only the front face of the cube it does not look like these 64 cubes with this one removed. You must show the three dimensions."

TEACHER NOTES

The isometric grid with the cubes shaded is shown below.

2. Repeat step 1 above for cubes numbered #2, #3 and #4 on Activity Sheet 7.

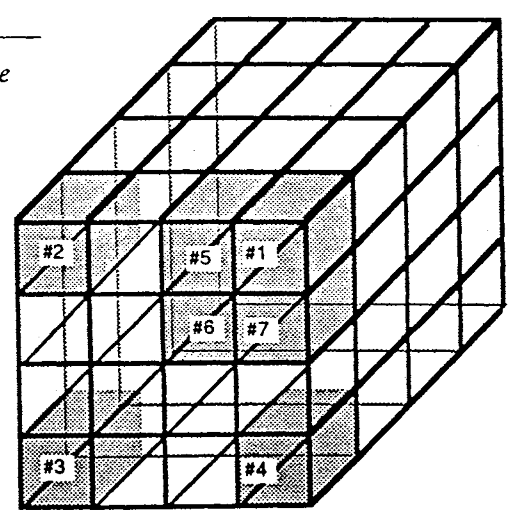
TEACHER NOTES

Cubes #3 and #4 can only be pointed to, thereby moving the students slowly from the concrete action to the abstract concept.

3. Hold up the cubes corresponding to those squares numbered #1, #5, #6 and #7 on the isometric grid. Instruct the students to trace over the area of these four cubes on the isometric grid, saying, "Imagine that you have four cubes. Trace over the lines showing where they would be, and then color them."

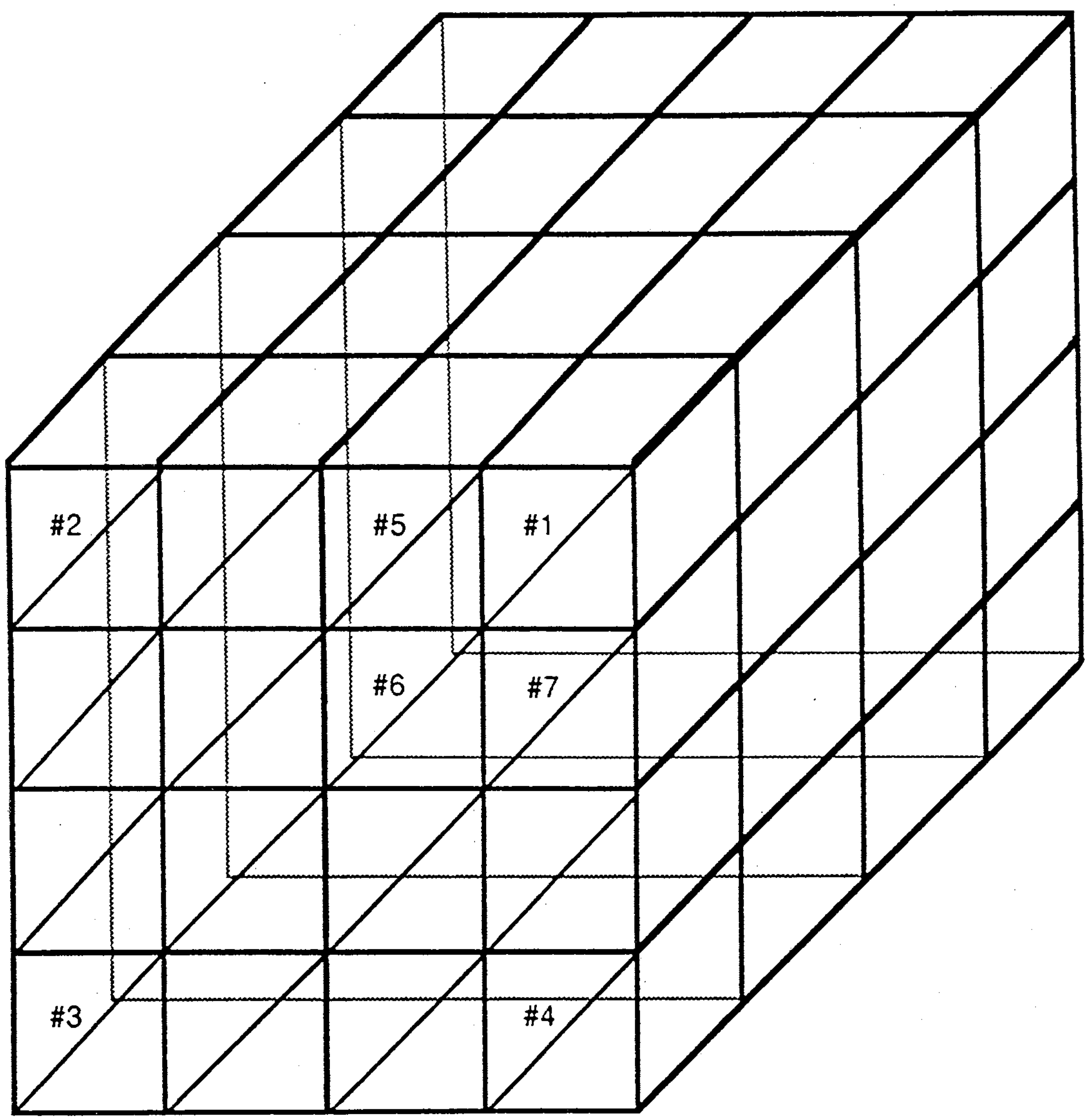
TEACHER NOTES

The tracing of cubes #1, #5, #6 and #7 returns to the concrete but at a more complex level, preparing the students for the remainder of the activities.



ISOMETRIC GRID (WITH NUMBERS)

ACTIVITY SHEET 7



UNIT 3

Centers for Problem Solving

The problem solving activities presented in this unit can be used as part of the regular math program. For example:

Group 1 meets with teacher for a directed lesson in the regular math text.

Group 2 works on independent practice assignment in the math text.

Group 3 works in a figure-cube problem solving center.

Groups can thus rotate through the three activities during math time. Only one activity requires direct teacher involvement.

Center directions are included, ready to duplicate for student use.

Problem Solving Activities

Field testing has shown that students who have mastered the previous lessons are now able to continue the figure-cube activities without direct teacher instruction. Those students who show signs of anxiety or ask for more direction may need to repeat some of the earlier lessons until they master them.

As with any independent activity, it is important to discuss organizational and behavioral expectations while introducing the activities. Students should also be made aware of the consequences of not meeting those expectations. For most students, being removed from group activity for varying lengths of time has been punishment enough. Never remove a student from a group for more then two days at a time unless the student is not able to function without direct supervision. Students may find it difficult to keep up with their group if they are removed for too long.

Rules should be as follows:

1. **Please walk quietly to the center.**
   Not: Do not run in the classroom.

2. **Please use a quiet voice in the center.**
   Not: No loud talking.

3. **Please put all the materials away before leaving the center.**
   Not: Don't leave a mess.

By stating rules in a positive way, behavioral reminders may take a gentle form, such as:

"What kind of voice do we use in the center?"
"A quiet voice?"
"Yes, thank you."

The activity center directions and activity cards may be cut from this book, mounted on sturdy board, and placed in the center (photocopy the materials first).

Students who have mastered one center may assist their peers by explaining activities and/or checking for correctness.

ACTIVITY CENTER #1

1. Match the figure-cubes with the correct drawing.

2. Draw the figure-cubes on the graph paper.

3. Label your figure-cubes with the correct letter and number.

4. Color the drawing.

5. Place your work in your folder.

6. Put your folder away.

ACTIVITY CENTER #1 5 DAYS; 20 MINUTES EACH

Objective: The students will:

1. match the figure-cubes with the three-dimensional drawings, and
2. make drawings of the figure-cubes on graph paper.

Materials: Figure-cubes for each student, graph paper (quadrille 1/2") or photocopies of Activity Sheet 25, pencils, crayons, Activity Center #1 cards 1-20, directions for Activity Center #1, rulers.

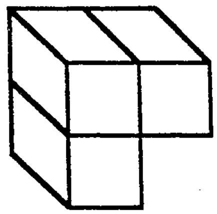
TEACHER NOTES

Center directions are on the next page. Photocopy and paste on sturdy board.

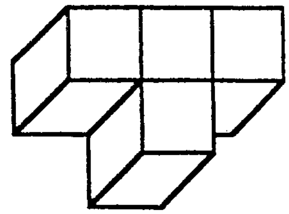
Activity card masters for Activity Center #1 begin on page 51. Photocopy the pages, then cut apart and paste on sturdy board.

Activity Sheet 25 (graph paper, quadrille 1/2") is on page 99.

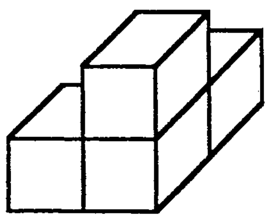
Place all materials in the activity center.



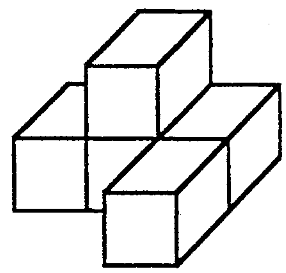
Activity Center #1 1



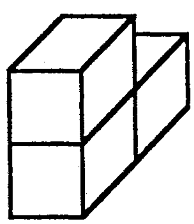
Activity Center #1 2



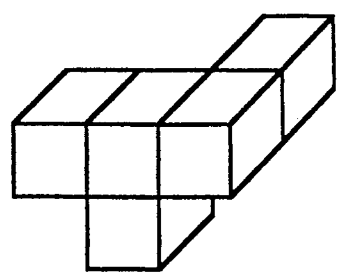
Activity Center #1 3



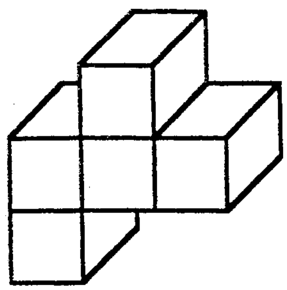
Activity Center #1 4



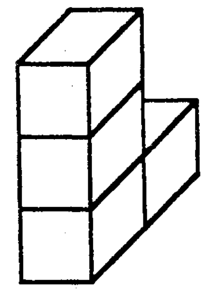
Activity Center #1 5



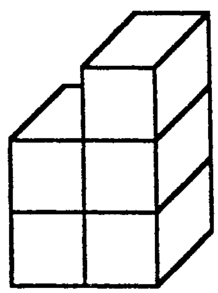
Activity Center #1 6



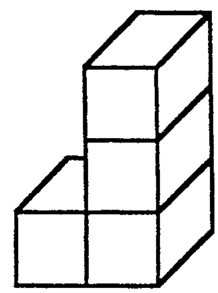
Activity Center #1 7



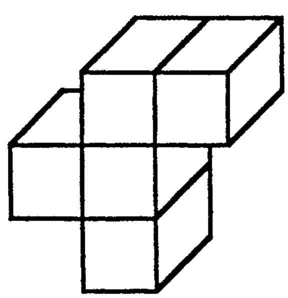
Activity Center #1 8



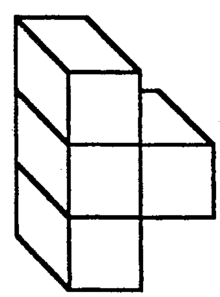
Activity Center #1 9



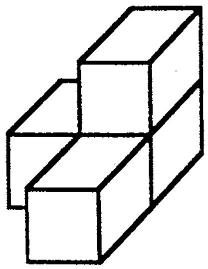
Activity Center #1 10



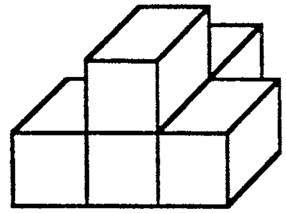
Activity Center #1 11



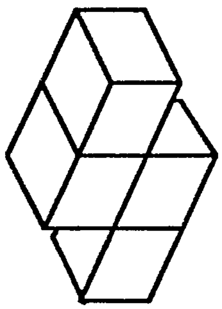
Activity Center #1 12



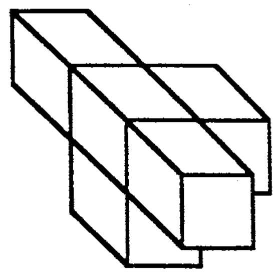
Activity Center #1 13



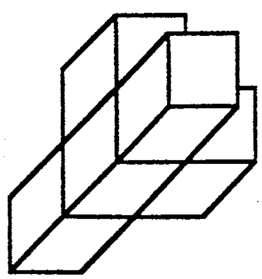
Activity Center #1 14



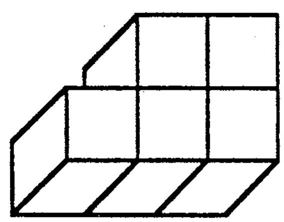
Activity Center #1 15



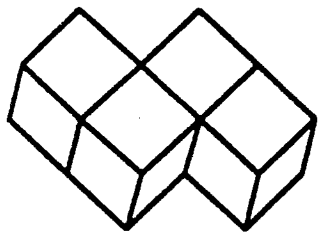
Activity Center #1 16



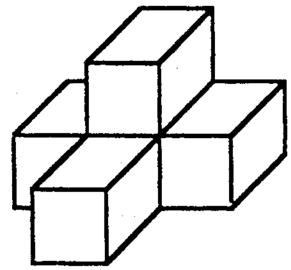
Activity Center #1 17



Activity Center #1 18



Activity Center #1 19



Activity Center #1 20

UNIT 4

Extended Activities In Problem Solving

ACTIVITY CENTER #7 1-4 WEEKS; 20 MINUTES EACH DAY

TEACHER NOTES

This Extended Activity lesson is designed for those students who have moved quickly through the previous material. It is not expected that all students will have the ability to do the more difficult tasks, especially in the time allotted.

Objective: The students will create their own structures and draw them on graph paper.

Materials: Figure-cubes for each student in the group, graph paper (1/2" quadrille) or copies of Activity Sheet 25 (found on page 99), pencils, crayons, rulers, directions for Activity Center #7.

TEACHER NOTES

Center directions are on the next page. Photocopy, then cut out and paste on sturdy board.

ACTIVITY CENTER #7

1. Build your own structures using two or more figure-cubes.

2. Draw your structures on graph paper.

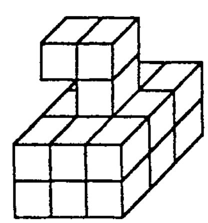
3. Color the drawing to show which figure-cube you used.

4. Beside each drawing, record the values of the figure-cube as an addition sentence and solve.

5. Put your work in your folder.

6. Put your folder away.

EXAMPLE



k + s + o + g + p = n

5 + 5 + 4 + 3 + 4 = 11

ACTIVITY CENTER #8 1-4 WEEKS; 30 MINUTES EACH DAY

TEACHER NOTES

This Extended Activity lesson is designed for those students who have moved quickly through the previous material. It is not expected that all students will have the ability to do the more difficult tasks, especially in the time allotted.

Objective: The students will draw figure-cubes combinations from more than one view.

Materials: Figure-cubes for each student in the group, graph paper (1/2" quadrille) or copies of Activity Sheet 25 (found on page 99), pencils, crayons, rulers, directions for Activity Center #8.

TEACHER NOTES

Center directions are on the next two pages (one page is an example for students to follow). Photocopy, then cut out and paste on sturdy board.

Additional examples of student work are shown on the front cover of this book.

ACTIVITY CENTER #8

1. Build your own structure using two or more figure-cubes.

2. Draw the structure on graph paper.

3. Color each figure-cube in the drawing to show how the structure was made.

4. Beside each drawing, record the values of the figure-cubes as an addition sentence and solve.

5. Rotate the structure and draw it from a different perspective.

6. Repeat steps #4 and #5 as many times as you can.

7. Put your work in your folder.

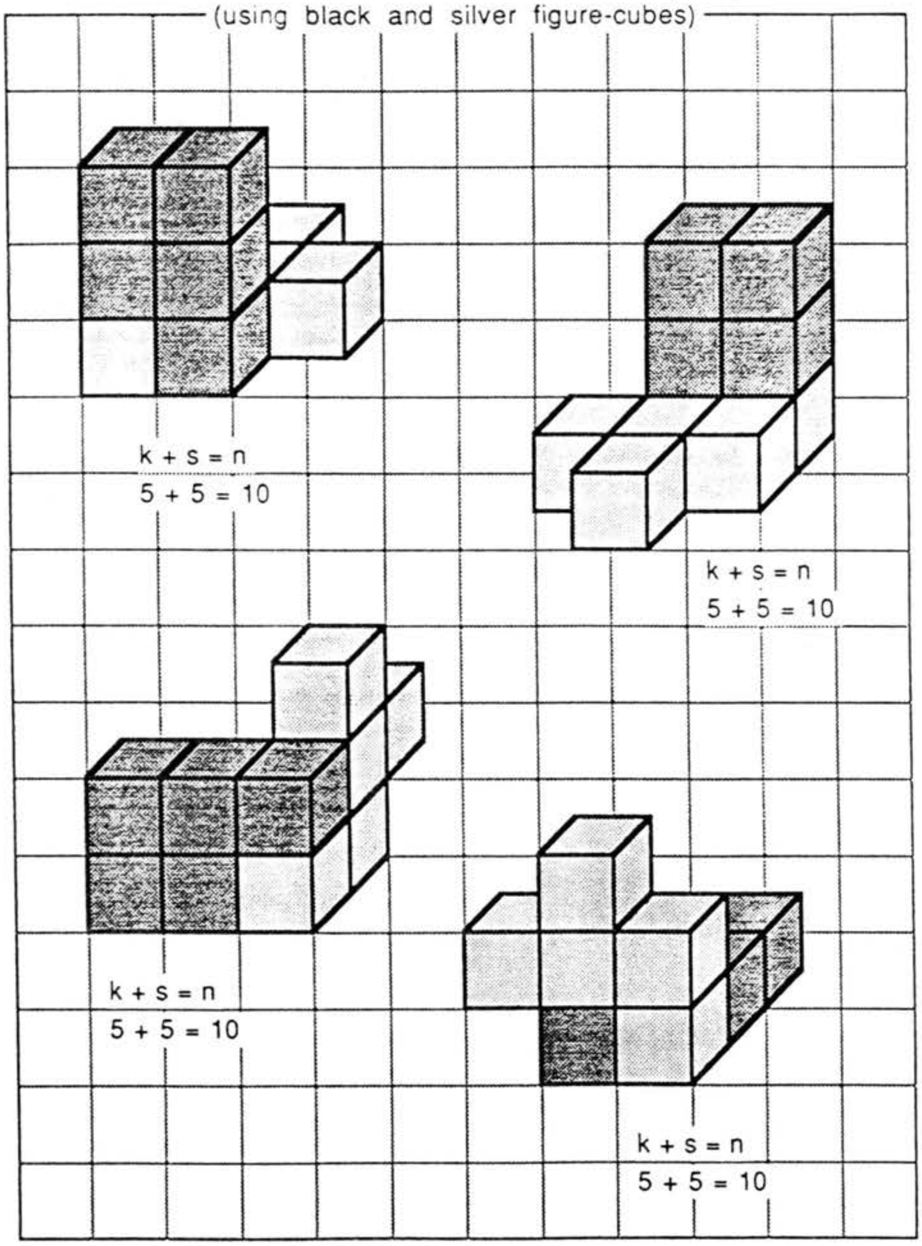
8. Put your folder away.

ACTIVITY CENTER #8

EXAMPLE

The same structure drawn from different perspectives

(using black and silver figure-cubes)



UNIT 5

Parallel Lesson

TEACHER NOTES

Students are ready for the parallel lesson activities at the end of Unit 2. These parallel activities may progress at the student's own pace and/or teacher's discretion.

The activities may be used with the whole class or with small groups.

Objective: The students will:

a. make and solve numerical problems using all nine figure-cubes in a variety of combinations, and

b. solve algebraic equations.

Materials

Teacher: Overhead projector (for use with whole class), transparencies of Activity Sheets 27-31, transparency of Activity Sheet 26, set of figure-cubes.

Students: set of figure-cubes, copies of Activity Sheets 27-31, copies of Activity Sheet 26 and 32, pencil.

ACTIVITY

Use the questions and answer format presented below to introduce the concepts of addition and multiplication. You will be asking students to combine the figure-cubes and write the number-sentences. When all the figures have been used, students can prove the accuracy of their work by adding the totals for the sum of all the figure-cubes; the proof will total 39.

Demonstrate by doing one set of problems. Be sure to remove each figure-cube as you use it, and to use them *all*.

1. Ask:
   a. "How many cubes are there in all nine figure-cubes?" (39.)
   b. "How did you find that out?" (Answers may vary, but they should be based on addition or multiplication.)

2. Say, "In lesson 9 you wrote addition sentences to describe your figure-cube combinations. Today we are going to try to make as many different number sentences as we can, using all the figure-cubes each time."

3. Ask:
   a. "If you use *all* nine figure-cubes, what number should *all* your totals add up to?" (39.)
   b. "Why?" (Because there are 39 cubes altogether in the nine figure-cubes.)
   c. "What will always be a good way to prove that you have solved all the number sentences in each set correctly?" (Add your answers together.)

TEACHER NOTES

Activity Sheet 26, found on page 111, is a blank master to use for additional student work.

Activity Sheets 27 - 31 are on pages 112 - 116. Make photocopies for each student in the group.

Activity Sheet 32 is a blank master to use for additional student work.

Samples of student work are shown on the following pages.

PROBLEM SOLVING WITH FIGURE-CUBES ACTIVITY SHEET 26

Name S A M P L E

1.	PROOF
4 + 4 = 8	8
4 + 5 = 9	9
5 + 3 = 8	8
5 + 5 + 4 = 14	14
	39

2.	PROOF
4 + 0 = 4	4
4 + 4 = 8	8
5 + 5 = 10	10
5 + 3 = 8	8
5 + 4 = 9	9
	39

3.	PROOF
4 + 0 = 4	4
3 + 4 + 5 = 12	12
5 + 4 = 9	9
5 + 5 = 10	10
4 + 0 = 4	4
	39

4.	PROOF
4 + 4 = 8	8
5 + 5 = 10	10
5 + 3 = 8	8
4 + 4 = 8	8
5 + 0 = 5	5
	39

5.	PROOF
5 + 5 + 4 = 14	14
4 + 5 + 5 = 14	14
4 + 4 + 4 = 11	11
	39

6.	PROOF
4 + 4 = 8	8
4 + 0 = 4	4
5 + 4 = 9	9
5 + 0 = 5	5
5 + 5 + 3 = 13	13
	39

7.	PROOF
4 + 4 = 8	8
4 + 4 + 5 + 5 = 18	18
5 + 3 = 8	8
5 + 0 = 5	5
	39

8.	PROOF
4 + 4 + 4 = 12	12
5 + 5 + 3 = 13	13
5 + 5 + 4 = 14	14
	39

2

VITA

Carolyn Annetta Harpole

Candidate for the Degree of

Doctor of Philosophy

Thesis: EFFECTS OF A MENTAL ROTATIONS CURRICULUM ON MATH CONCEPTUALIZATION AND MATH ANXIETY IN EIGHTH GRADE MALE AND FEMALE STUDENTS

Major Field: Applied Behavioral Studies

Biographical:

Personal Data: Born in Logansport, Indiana, April 14, 1950, the daughter of Casper and Frances Rausch. Married Kenneth J. Harpole, March 19, 1977. Children: Jared K. Harpole and Kyle W. Harpole.

Education: Graduated from Winamac High School, Winamac, Indiana in May, 1968; received Bachelor of Science degree in Elementary Education from Indiana University in August, 1972; received Master of Arts degree from the University of Tulsa in Elementary Counseling in May, 1977. Completed the requirements for the Doctor of Philosophy at Oklahoma State University, Stillwater, Oklahoma in December 1995.

Experience: Elementary Teacher, Liberty Elementary School, North Judson, Indiana, August 1972 to May 1976; Elementary Teacher, Lone Star Elementary School, Sapulpa, Oklahoma, March 1977 to May 1977; Elementary Counselor, Indian Springs Elementary School, Broken Arrow, Oklahoma, August 1977 to May 1980; Homebound Teacher, Bixby Public Schools, Bixby, Oklahoma, January 1981 to May 1982; Psychology Instructor, Tulsa Junior College, Tulsa, Oklahoma, August 1986 to January 1988; Psychological Assistant Intern, Lloyd E. Rader Center, Sand Springs, Oklahoma, September 1994 to present.

Professional Memberships: National Association of School Psychologists, Oklahoma School Psychological Association.

OKLAHOMA STATE UNIVERSITY
INSTITUTIONAL REVIEW BOARD
FOR HUMAN SUBJECTS RESEARCH

Date: 09-20-93 **IRB#:** ED-94-016

Proposal Title: EFFICACY OF IMPLEMENTING A SPECIFIC MENTAL ROTATIONS CURRICULUM TO EFFECT IMPROVEMENT OF MATHEMATICAL CONCEPTUALIZATION AND REDUCTION OF MATH ANXIETY IN EIGHTH GRADE MALE AND FEMALE STUDENTS

Principal Investigator(s): Dr. Paul Warden, Carolyn Harpole

Reviewed and Processed as: Exempt

Approval Status Recommended by Reviewer(s): Approved

APPROVAL STATUS SUBJECT TO REVIEW BY FULL INSTITUTIONAL REVIEW BOARD AT NEXT MEETING.
APPROVAL STATUS PERIOD VALID FOR ONE CALENDAR YEAR AFTER WHICH A CONTINUATION OR RENEWAL REQUEST IS REQUIRED TO BE SUBMITTED FOR BOARD APPROVAL. ANY MODIFICATIONS TO APPROVED PROJECT MUST ALSO BE SUBMITTED FOR APPROVAL.

Comments, Modifications/Conditions for Approval or Reasons for Deferral or Disapproval are as follows:

Comments:

Both student and parent must consent to participation in study, not just one or the other.

Signature: ______________________ Date: September 21, 1993
Chair of Institutional Review Board