A DYNAMIC OPTIMAL HEDGING MODEL UNDER PRICE, BASIS, PRODUCTION AND FINANCIAL RISK

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CHAPTER I

INTRODUCTION

Empirical research usually finds optimal hedge ratios close to one (Ederington; Howard and D'Antonio; Kolb and Okunev; Mathews and Holthausen; Peck). Recently, Lapan and Moschini added basis and yield risk and found lower, but still high, optimal hedge ratios. The reality is that producers hedge much less. The theoretical and empirical models used in past research have made simplifying assumptions that restrict them from explaining what farmers actually do. In a sample of 539 Kansas farmers, Schroeder and Goodwin found that, depending upon the crop, only 2% to 10% of the producers raising crops hedged. Tomek argues the hedge ratio is overestimated due to omission of important costs from the specification of farmers' objective function (i.e. yield risk and transaction costs). Lence finds that under realistic conditions the optimal hedging strategy is simply not to hedge. Shapiro and Brorsen found that next to income stability (i.e., low income variability), the most important factor explaining the use of futures markets is the individual's debt position. An appropriate model might be one in which the farmer's debt position is accounted for in the model. The model should make explicit distinction between a low-leveraged farmer who has little financial risk, and may have no need to hedge, and a high-leveraged farmer who would hedge more because of his/her higher expected bankruptcy and liquidity costs.

Firms are normally assumed risk averse. However, empirical evidence shows that risk preferences are not significantly related to hedging (Shapiro and Brorsen). Schroeder and Goodwin found that risk preferences of crop producers did not influence forward pricing. Williams, Smith and Stulz, and Brorsen show that risk aversion is not necessary for firms to hedge. Rather than assuming risk aversion, this study assumes that incentives exist to maximize firm's equity. The objective function is concave for reasons other than risk aversion. The model allows interest rates to vary according to the probability of bankruptcy, accounts for the trade off between the tax-reducing benefits of hedging and the cost of hedging, and also allows hedging to be a source of meeting cash flow requirements. It is shown that progressive tax rates are an important incentive to hedge when the firm is not close to bankruptcy. As Smith and Stulz argue, tax-reducing benefits of hedging become more significant as the function yielding the after-tax income becomes more concave. The model presented here will allow debt, and therefore leverage, to be determined endogenously.

It has been widely accepted that output and price risk should be considered together when estimating optimal hedge ratios. Yet few studies have considered both. Exceptions are Chavas and Pope, Grant, Lapan and Moschini, Losq, and Rolfo. Also, little has been done to incorporate financial risk into optimal hedge models. Schroeder and Goodwin found that leverage was positively related with forward pricing, and Harris and Baker's survey indicates that hedging increases a farmer's loan limits. Turvey and

Baker argue that hedging reduces the expected liquidity cost. Brorsen develops a theory where interest rates are a nonlinear function of initial wealth, debt, and the variability of ending wealth. The new theoretical model explicitly allows hedging to reduce bankruptcy and liquidity costs and tax liabilities. Also, the nonlinear interest rate function is estimated within the model by determining the expected bankruptcy losses to debt holders. This makes the model easier to estimate empirically.

General Objective

The general objective of this study is to derive a new theoretical model of hedging to determine the relative importance of factors that influence farmer's hedging behavior.

Specific Objective

To determine optimal hedge ratios for a wheat and stocker steer producer and their relationship with yield risk, price variability, basis risk, and financial risk, using alternative assumptions about the value of the parameters.

Procedures

An empirical example is provided to show how changes in assumptions affect optimal hedge ratios for a wheat and stocker steer producer. Because of the complexity of the analytical model, no comparative statics can be derived. Therefore, the effects of various factors are determined numerically for a specific example. The factors considered are: variance of both cash and futures prices, basis risk, yield variance, progressive tax rates, cost of hedging, liquidity cost, off farm income and dynamics. Also, simulations are run to test the sensitivity of the optimal hedge ratio to changes in the distributional assumption of yields and prices, the randomness of output, and the inclusion of tax carry back. Reaction functions are used to determine an optimum dynamic hedging strategy. These functions are called reaction functions because they enable the decision maker to react to information prevailing at the time decisions are made. Hedging for the period is a function of random variables realized in the previous period (Venkateswaran and Preckel). The objective function could not be integrated analytically and, therefore, Monte Carlo integration is used.

CHAPTER II

THEORY

The theory chapter is divided into two parts. In the first section entitled theoretical background, the existing theory of hedging is reviewed. In the second part, a new theory of hedging is developed.

Theoretical Background

Model of Johnson and Stein

The model of Johnson and Stein has been used widely. This model assumes that decision makers minimize the variance of farm's income, assuming no budget constraints, no imputed interest cost and no brokerage commission. Let Y and F be respectively, the number of units of cash commodity and of futures contract held, P_T^c the uncertain cash price at future time T, P_0^c the initial cash price, P_T^f the uncertain futures price at future time T, and P_0^f the initial futures price. The uncertain profit of the hedger who holds Y units of the commodity and hedges F futures contracts is:

(2.1) $$\pi = (P_T^c - P_0^c)Y + (P_T^f - P_0^f)F$$

where $(P_T^c - P_0^c)$ is the random price change per unit of commodity and $(P_T^f - P_0^f)$ is the random price change in the futures contract.

The minimum-variance hedge (MVH) ratio is the hedge that minimizes the variance of profit (2.1). The variance of profit is:

(2.2) $$\sigma_\pi^2 = \sigma_{p^c}^2 Y^2 + \sigma_{p^f}^2 F^2 + 2FY\sigma_{p^c p^f}$$

Where $\sigma_{p^c}^2$ is the variance of the cash price, σ_{p^f} is the variance of the futures price, and $\sigma_{p^c p^f}$ is the covariance between the cash and the futures price. Taking the derivative of (2.2) with respect to the futures contracts F and setting it equal to zero, we obtain

(2.3) $$\frac{\partial \sigma_\pi^2}{\partial F} = 2\sigma_{p^f}^2 F + 2Y\sigma_{p^c p^f} = 0$$

Then the MVH ratio (F^*/Y) is:

(2.4) $$\frac{F^*}{Y} = -\frac{\sigma_{p^c p^f}}{\sigma_{p^f}^2}$$

which depends on the covariance between the cash and the futures price relative to the variance of the futures price.

Commonly, the MVH ratio is estimated with ordinary least squares (i.e., Elam; Ederington; Heifner; Carter and Lyons; Benninga, Eldor and Zilcha; Franckle). The optimal hedge ratio ($\sigma_{p^c p^f} / \sigma_{p^f}^2$) is the slope coefficient when the cash price change ($\Delta P_t^c = P_t^c - P_{t-1}^c$) is regressed against the futures price change ($\Delta P_t^f = P_t^f - P_{t-1}^f$)

(2.5) $$\Delta P_t^c = \alpha_0 + \alpha_1 \Delta P_t^f + \epsilon_t$$

Data used have been both price levels and price changes, but in most studies price change data are used to reduce statistical problems (Blank; Brown). The intercept in (2.5) represents the basis when the futures market is unbiased $(E(\Delta P_t^f) = 0)$, the expected value of the error term is zero $(E(\epsilon_t) = 0)$, and convergence holds $(E(P_t^c) = P_{t-1}^f)$ (assuming the commodity is deliverable and is held to maturity). To see this, take the expected value of (2.5):

(2.6) $$E(\Delta P_t^c) = \alpha_0 = E(P_t^c - P_{t-1}^c) = E(P_t^c) - P_{t-1}^c = P_{t-1}^f - P_{t-1}^c$$

where $(P_{t-1}^f - P_{t-1}^c)$ represents the basis. The error term reflects the basis risk, which is the random fluctuation in the difference between the cash market price and the futures contract price. This risk is not eliminated by hedging.

The shortcomings of the MVH ratio are that the agent is assumed (implicitly or explicitly) to have a quadratic utility function (or profits are distributed normally) and that the determined hedge ratio is not an optimal hedge ratio, but rather one which minimized the variance of the producer's income. Even when the producer is assumed to have a quadratic utility function, there is no reason to believe that the utility will be maximized when the variance of the spot-futures position is minimized (Benninga, Eldor and Zilcha).

Variance minimization in the context of the expected-utility maximization is equivalent to infinite risk aversion. Infinite risk aversion is unrealistic since, for example, there is an intrinsic contradiction in having infinitely risk-averse agents holding risky cash positions.

Benninga, Eldor, and Zilcha, cited by Lence, show the conditions under which MVH would be consistent with expected utility-maximizing hedge ratios. The basic assumptions are that (i) the decision maker is not allowed to borrow, lend, or invest in alternative activities, (ii) there are neither safety margins nor futures brokerage fees, (iii) production is deterministic, (iv) random cash prices can be expressed as a linear function of futures prices plus an independent error term, and (v) futures prices are unbiased $(E(\Delta P_t^f) = 0)$. All assumptions, except for assumption (v) are relaxed in this study.

The Mean-Variance Approach

The mean variance approach has been used extensively in the literature to determine the optimal hedge ratio (Peck; Kahl; Bond and Thompson; Chavas and Pope; Berck; Karp; Levy; etc). The model presented to illustrate the mean-variance approach is similar to the model of Feder, Just and Schmitz; Danthine; and Holthausen (Robinson and Barry). These authors incorporate the possibility of buying and selling futures into the model of the competitive firm under price uncertainty developed by Sandmo, and Batra and Ullah. In this model, the production decision and the decision to trade are choice variables. Output is assumed non random, and they do not impose efficient markets $(E(P_t^c) \neq P_t^f)$; i.e., implicitly allowing for transaction costs. The objective of the producer is to maximize his/her utility of profits. Define profit as:

(2.7) $$\pi = P^c(Y - F) + P^f F - C(Y) - B$$

where π, Y, P^c, F, and P^f have been defined above. $C(Y)$ is variable cost, where $C'(Y)>0$ and $C''(Y)>0$, and fixed costs are B. Note that there is neither production nor basis risk in this model. The expected profit ($E(\pi)$) and variance of profits ($V(\pi)$) are:

$$E(\pi) = \bar{P}^c(Y-F) + P^f F - C(Y) - B \tag{2.8}$$
$$\sigma_\pi^2 = (Y-F)^2 \sigma_{P^c}^2$$

where $\bar{P}^c = E(P^c)$. The certainly equivalent model can be now formulated as:

$$\pi_{CE} = E(\pi) - \frac{\lambda}{2}\sigma_\pi^2 \tag{2.9}$$

where λ is a constant risk attitude measure. Substituting (2.8) into (2.9) yields:

$$\max\ \pi_{CE} = \bar{P}^c(Y-F) + P^f F - C(Y) - B - \frac{\lambda}{2}(Y-F)^2\sigma_{P^c}^2 \tag{2.10}$$

Differentiate (2.10) with respect to Y and F to find the optimal output and the optimal amount to hedge:

$$\frac{\partial \pi_{CE}}{\partial Y} = \bar{P}^c - C'(Y) - \lambda(Y-F)\sigma_{P^c}^2 = 0 \tag{2.11}$$
$$\frac{\partial \pi_{CE}}{\partial F} = -\bar{P}^c + P^f + \lambda(Y-F)\sigma_{P^c}^2 = 0$$

Adding the two expressions, one finds the solution for Y:

$$P^f - C'(Y) = 0 \tag{2.12}$$

This indicates that the firm should produce up to the point where marginal cost equals the futures price. This makes sense in the absence of basis risk. Cash price variability is, therefore, eliminated through hedging. The firm behaves as if there is no risk involved, and the decision to produce does not depend on the level of risk aversion (λ). The optimal condition in (2.12) no longer holds when output is uncertain and correlated with the spot price (Grant).

The optimal hedge from (2.11) is

$$F = Y - \frac{\bar{P}^c - P^f}{\lambda \sigma^2_{P^c}} \tag{2.13}$$

which shows how the firm would hedge more the more risk averse it is. Under the assumption of no basis risk and efficient markets, the decision maker will hedge all that is produced (Peck).

One drawback of the E-V framework is that it assumes the farmer's expected utility is a function only of expected income and variance of income. This implies that the utility function is quadratic (or that income is normally distributed), and that risk aversion increases with wealth. Both assumptions are unrealistic.

Yield Risk

Farmers face both price and output uncertainty, yet few studies have considered both factors. Lence and Tomek argue that one of the most important restrictions of the minimum-variance hedge ratios seems to be that production is deterministic (the other important restriction is that there are no alternative investments). Little participation of farmers in the futures markets can be in part explained by the fact that farmers cannot adequately forecast the size of the harvest even after all production decisions have been made (Rolfo). Chavas and Pope and Lapan and Moschini assumed production risk is multiplicative, and found that production and hedging decisions may be affected by the spot price. Recall from (2.11) that production was not affected by the spot price. Under production certainty and basis risk, the higher the cash price, the less the firm will hedge. Chavas and Pope and Grant could not, however, determine the effect of output price, price risk or risk aversion on hedging under production risk. When production uncertainty was assumed additive rather than multiplicative (Robinson and Barry), neither the spot price nor the risk factor affects production decisions.

Production risk provides an additional explanation of optimal hedge ratios being below unity (Rolfo; Chavas and Pope; Lapan and Moschini; Losq). Lence concludes that stochastic production reduces significantly both optimal hedges and zero-hedging opportunity costs. Also, the optimal hedge is a decreasing function of the variance of the production disturbance (Lapan and Moschini, Lence). With the exception of Lence, most studies assume yields are normally distributed even though crop yields are known to fall in the range from 0 to a maximum possible value and crop yield distributions might be significantly skewed either to the right or to the left (Nelson and Preckel). In this study yields and animal weight follow a beta distribution. The disadvantage of this is that yields are not correlated with prices . This is because of the difficulty to correlate a beta random variable with prices that follow a lognormal distribution.

Output Assumed as Given

There is some concern in the literature on whether the risk parameter affects the decision to hedge (Berck). Peck concludes that the optimal hedge ratio depends on the individual's risk attitudes when the cash position is assumed as given so that optimal cash and futures positions are not determined simultaneously. However, Kahl does not assume the cash position as given and argues that the optimal hedge ratio is independent of the individual's risk aversion. Berck finds that the simultaneous choice of crops and futures has a large effect on the amount of hedging. When including a nonlinear cost of storage in the expected cash market return, Bond and Thompson found that the optimal hedge ratio does depend on the risk parameter. In a more recent study, Lapan and Moschini found that the expected utility maximizing hedge ratio, in contrast with the mean-variance solution, does depend on risk attitudes.

This study assumes resource constraints and limited choices. By fixing output to a certain level, the only choice variables in the model are how much to hedge and the level of debt so that one can better determine the sensitivity of the optimal hedge ratio given changes in a certain parameter.

Basis Risk

Hedging eliminates risk as long as fluctuations of cash and futures prices are perfectly correlated and no financial and production risk exist. In practice, these prices are not perfectly correlated, so that a new risk associated with hedging, called basis risk, must be considered (Robinson and Barry). In Working's view, hedgers place a hedge if they believe that futures prices reflect an attractive profit opportunity when compared to the spot price. The size of the basis at the time of hedge initiation, influences the decision to hedge or not (Castelino; Peck). A hedger presumably has no intention to deliver since actual delivery from a given area to another location may be difficult or the contract is cash settled so it is not possible to deliver on it (i.e., feeder contract) , and the basis (the difference between the cash and the futures prices) may not be zero. This situation is easily handled by including in the analytical model the basis price (Peck).

Basis risk has a significant effect on the minimum-variance hedge ratio (Castelino). If basis risk is zero (assuming futures markets are unbiased and output is certain), then the minimum-variance hedge ratio is one, but if basis risk exists, the minimum-variance hedge ratio will not be one (Castelino). Robinson and Barry, based on previous developments by Johnson, Ward and Fletcher, Heifner, Peck, and Kahl, derived the optimal hedge ratio assuming output certainty and basis risk in the context of the certainty equivalent model. Results indicated that the optimal hedge ratio depends on the expected returns and variances for the cash and futures positions, as well as on the correlation of returns from these positions. However, no conclusions were drawn as to what the relationship between the optimal hedge ratio and basis risk is. Lapan and Moschini found that basis risk does not by itself affect the optimal hedge. When both basis and production risk are present, the analysis is complicated because of the interaction between the two effects; however, the authors conclude that an increase in pure basis risk will increase the optimal futures hedge assuming the regression coefficient of yield on futures price is negative (the authors assumed away the possible effect of the cash price on output). This contradicts what one would expect since the higher the basis risk the less effective the hedge becomes. As Castelino points out, the hedge ratio should be reduced in response to increasing basis risk.

Leverage and Taxes

According to Turvey, there are at least two possible explanations for why hedging increases with debt. The first is that hedging decreases financial risk (i.e. leverage) and the second is that hedging reduces liquidity risk. Another reason, that is going to be very important in explaining the findings in this paper, is the tax-reducing benefits of hedging.

It is possible to find a debt-equity mix that maximizes the market value of the firm (or that maximizes utility, for our purposes) since interest charges are tax deductible, and a tax advantage to debt exists (Hirshleifer). If there are significant "leverage-related" costs, such as bankruptcy costs and interest rates, then there exist a trade off between these costs and the tax advantage of debt (Bradley, Jarrell and Kim). Several researchers have developed models that capture the trade off between the tax advantage of debt and the debt-related costs (i.e., interests, cost of bankruptcy) (Kraus and Litzenberger; Kim; Bradley, Jarrell and Kim). In these models, it is argued that if the firm can pay its current liabilities, financial leverage decrease the firm's income tax liability and increases its after-tax operating earnings. Smith and Stulz incorporate hedging and conclude that: (i) hedging reduces the variability of pre-tax firm values, which in turn reduces the expected tax liability, increasing the expected post-value of the firm; and (ii) hedging benefits shareholders by reducing the expected transaction costs of bankruptcy and increasing the expected after-tax firm value net of bankruptcy costs. These models, though, do not incorporate progressive tax rates, like the U.S. tax code. This is important since the tax-reducing benefits of hedging may increase if the function that yields after-tax income becomes more concave (Smith and Stulz).

Several authors (Brorsen; Lence; Turvey; Turvey and Baker) have found that hedging increases with leverage. However, these models do not incorporate any tax structure. In Brorsen's model, interest rates are a nonlinear function of initial wealth, debt, and the variability of ending wealth. However, the ability of hedging to reduce bankruptcy costs, liquidity costs and tax liabilities are not incorporated in Brorsen's model. The new theoretical model developed in this study also incorporates a nonlinear interest rate function, but this function is estimated differently by determining the expected bankruptcy losses to debt holders. This makes the model easier to estimate empirically as will be shown later.

Liquidity is the motivation for farmers' use of futures in Turvey, and Turvey and Baker's model. Where liquidity refers to the farm's capacity to generate sufficient cash to meet financial commitments when they occur. With variable yields and prices, the ability of farms to generate these funds is not certain (Turvey and Baker). The cash flow derived from hedging is the difference between the net price received with the hedge in place and the cash price which would have been received without the hedge. High debt farms are more likely to have liquidity constraints and would hedge more. The probability of cash flows falling bellow a critical level decreases as hedging increases (Turvey). Turvey and Turvey and Baker include liquidity cost when firms have to sell off long-term assets to cover losses.

The optimal hedge ratio is also an increasing function of debt in Lence's model because agents use debt to enlarge their holdings of the alternative investment. This is only true when returns in futures are positively correlated with returns on the alternative investment. An alternative investment increases risk, and this risk can be partly offset by additional hedging.

The Model

Assume that because of unpredictable conditions (i.e., weather variability), the decision maker cannot predict output with certainty at the time of the production decision. The technology of the firm can then be represented by the stochastic production function:

$$y_t = f(K_{t-1}, X_{t-1}, \epsilon_{1t}) \qquad \textbf{(2.14)}$$

where $\mathbf{y}_t$ is an $\mathbf{n}$-dimensional vector of production levels at time t (letting $t = 1$ be the harvest time and $t - 1$ the time where decisions are made), f is an increasing function of K_{t-1} and X_{t-1}. Where X_{t-1} and K_{t-1} are respectively vectors of variable inputs and capital assets, and ϵ_{1t} is an $n \times 1$ vector of error terms.

Assume a competitive producer whose only available hedging instrument is a futures contract. Let F_{t-1} be an n-dimensional vector of the amounts hedged in the futures market. For each unit of F_{t-1}, the firm makes $(P^f_{t-1} - P^f_t)$, the difference between the selling and the buying prices of the futures contracts. In the cash market, the firm makes $(P^f_t + b_t)$ dollars for each unit of output sold, where the basis (b_t) is the difference between the cash price (P_t^c) and the futures price at harvest (P_t^f). The firm's variable costs consist of the cost of inputs $(P_{t-1}^{x\prime} X_{t-1})$, the cost of hedging $(h_{t-1}^{\prime} F_{t-1})$, interest on debt $(i_t D_t)$, and depreciation (αK_{t-1}), where P_{t-1}^{x} is a vector of input prices, $\mathbf{h}_t$ is the cost of hedging, i_t is the interest rate, D_t is total liabilities, and α is a (constant) rate of depreciation. The firm's profit at time t (π_t) can then be represented by

$$\begin{aligned}\pi_t = {} & y_t^{\prime}(P_t^f + b_t) + F_{t-1}^{\prime}(P_{t-1}^f - P_t^f) - P_{t-1}^{x\prime} X_{t-1} - h_{t-1}^{\prime} F_{t-1} \\ & - i_t D_t - \alpha K_{t-1} - \gamma L_t\end{aligned} \tag{2.15}$$

A liquidity fee (γL_t) has been included in (2.15) to account for situations when the firm cannot honor its current obligations; where γ is a liquidity fee rate, and L_t is the portion of current liabilities that the firm could not pay when liquid capital was not available. As assumed by Turvey and Baker, firms incur liquidity costs when they must sell off long-term assets to cover losses. Taxable income (TI_t) can be obtained by subtracting the corresponding standard deductions (STD) and exemptions (EXM) from (2.15) $(TI_t = \pi_t - STD - EXM)$. Note that because all payments to debt claims are assumed to be tax deductible, interest is subtracted in (2.15). Total liabilities (D_t) subject to interest charges can be determined by computing the liabilities from the previous period, plus the capital needed to finance inputs, hedging costs (margin calls are also included here), and investment, minus current assets from period $t-1$ (CA_{t-1}), minus any payment made to the principal in period $t-1$ (A_{t-1})

$$D_t = D_{t-1} + P_{t-1}^{x\prime} X_{t-1} + h_{t-1}^{\prime} F_{t-1} + (K_t - K_{t-1}) - CA_{t-1} - A_{t-1} \tag{2.16}$$

where $(\boldsymbol{K}_t - \boldsymbol{K}_{t-1})$ is capital investment.

Denote $\tau(TI_t)$ as the firm's income tax rate, which is an increasing function of taxable income (TI_t); therefore, net income (NI_t) or after tax income is

$$\text{(2.17)} \qquad NI_t = \begin{cases} [1-\tau(TI_t)]TI_t; & TI_t > 0 \\ \pi_t; & \pi_t < 0 \\ -W_{t-1}; & \pi_t < -W_{t-1} \end{cases}$$

indicating that taxes are zero whenever taxable income is negative, and net income can be negative whenever profits are negative, but its absolute value cannot be less than beginning equity (i.e., the firm cannot lose more capital than it has available).

The firm's current assets (CA_t) are the cash flow after the cash and the futures positions have been settled, assuming the decision maker does not keep any inventory. Current assets equal initial current assets (CA_{t-1}), plus net income (NI_t), plus depreciation $(\alpha \boldsymbol{K}_{t-1})$, plus any long term capital borrowed in period t $(LTD_t - LTD_{t-1})$, minus investment $(\boldsymbol{K}_t - \boldsymbol{K}_{t-1})$, and minus any payment made to the principal (A_{t-1}):

$$\text{(2.18)} \qquad \begin{aligned} CA_t = CA_{t-1} + NI_t + \alpha \boldsymbol{K}_{t-1} + (LTD_t - LTD_{t-1}) \\ - (\boldsymbol{K}_t - \boldsymbol{K}_{t-1}) - A_{t-1} - L_{t-1} \end{aligned}$$

In (2.18), depreciation is added back because it is not really a cash outflow for the firm, and L_{t-1} is the portion of current liabilities that could not be paid in the previous period. Assume that the farmer borrows money whenever current assets minus current expenses are not enough to pay the capital investment needed at time period t. Therefore long term debt (LTD_t) is defined as debt carried from previous period (LTD_{t-1}), plus investment $(\boldsymbol{K}_t - \boldsymbol{K}_{t-1})$, minus amortizations ($A_{t-1}$) minus any current assets available after paying variable cost $(\boldsymbol{P}_t^{c'}\boldsymbol{X}_t + \boldsymbol{h}_t'\boldsymbol{F}_t)$:

$$LTD_t = \begin{cases} LTD_{t-1} + (\boldsymbol{K}_t - \boldsymbol{K}_{t-1}) - A_{t-1} - (CA_{t-1} - \boldsymbol{P}_t^{c'}\boldsymbol{X}_t + \boldsymbol{h}_t'\boldsymbol{F}_t); \\ \qquad\qquad\qquad\qquad\qquad\qquad CA_{t-1} > \boldsymbol{P}_t^{x'}\boldsymbol{X}_t + \boldsymbol{h}_t'\boldsymbol{F}_t \\ LTD_{t-1} + (\boldsymbol{K}_t - \boldsymbol{K}_{t-1}) - A_{t-1}; \qquad CA_{T-1} < \boldsymbol{P}_t^{x'}\boldsymbol{X}_t + \boldsymbol{h}_t'\boldsymbol{F}_t \end{cases} \tag{2.19}$$

The level of current assets (CA_t) in (2.18) must be positive, otherwise the firm would not have liquid capital to pay current liabilities. Assume debt holders charge a fee (γL_t) whenever the firm does not have enough liquid capital to pay its current liabilities. Notice that if the negative of (2.18) is positive, this amount corresponds to the portion of current liabilities that could not be paid in the present period (L_t):

$$\begin{aligned} L_t = L_{t-1} + A_{t-1} - [CA_{t-1} + NI_t + \alpha \boldsymbol{K}_{t-1} \\ + (LTD_t - LTD_{t-1}) - (\boldsymbol{K}_t - \boldsymbol{K}_{t-1})] \end{aligned} \tag{2.20}$$

As stated in (2.15), a liquidity fee equal to $\gamma\, L_t$ would be subtracted from profits whenever (2.20) is positive.

Let us now further define some of the variables introduced so far in the model. The decision maker knows at time $t-1$ the vector of futures prices $(\boldsymbol{P}_{t-1}^f)$, the price vector of inputs $(\boldsymbol{P}_{t-1}^x)$ and the cost of selling in the futures market ($\boldsymbol{h}_t$). However, because of production lags the farmer does not know with certainty the vector of output prices in the cash market $(\boldsymbol{P}_t^c)$, or the futures price at harvest $(\boldsymbol{P}_t^f)$.

The random variables, (P_t^f) and (P_t^c), can be defined as following a random walk

$$P_t^c = P_{t-1}^c + \varepsilon_{2t} \tag{2.21}$$
$$P_t^f = P_{t-1}^c + \varepsilon_{3t}$$

where ε_{2t} and ε_{3t} are correlated vectors of mean zero errors.

The firm's net worth would be net income plus beginning equity:

$$W_t = NI_t + W_{t-1} \tag{2.22}$$

Since the firm cannot lose more capital than it has available, ending wealth (W_t) must be positive and net income is bounded to be greater than the negative of beginning equity ($NI_t > -W_{t-1}$). Note that the firm will be bankrupt when profits are negative and less than the negative of initial wealth ($\pi_t < -W_{t-1}$). Also note that when profits are equal to the negative of initial wealth ($\pi_t = -W_{t-1}$), net income equals initial wealth ($NI_t = -W_{t-1}$) (2.17) and the firm will be at the boundary of going bankrupt (i.e., $W_t = 0$) (2.22). When ($W_t = 0$) and the amount ($W_{t-1} - \pi_t$) is positive, the firm is bankrupt and the losses to debt holders (LS_t) are:

$$LS_t = C(D_t) + W_{t-1} - \pi_t; \;\; \pi_t < -W_{t-1} \tag{2.23}$$

where $C(D_t)$, the bankruptcy fee, is an increasing function of the debt level (2.16).

Farmers face risk (i.e., yield risk, basis risk, price risk); therefore, bankruptcy is possible since the condition ($\pi_t < -W_{t-1}$) (2.17) can occur with positive probability.

Assuming banks are risk neutral, lenders will charge the firm a premium which would be a function of the expected bankruptcy losses. Hence a firm with a higher probability of bankruptcy would have a higher expected rate of interest on its debt than a firm with lower financial risk (Barry, Baker, and Sanint). In practice, interest rates can vary by as much as five percentage points (Turvey and Baker). It is assumed here that banks have enough information about the firm to calculate the expected bankruptcy losses and charge a premium to firms likely to go bankrupt. The firm's expected interest rate on debt (i_t) will then equal the prime interest rate (r^*) plus a premium (PR_t)

(2.24) $$i_t = r^* + PR_t$$

where the prime interest rate (r^*) is assumed constant over time.

Given the bankruptcy losses (LS_t) in (2.23), the premium (PR_t) can be defined as the expected ratio of the losses to total liabilities (LS_t/D_t):

(2.25) $$PR_t = \int\int\int I\left[\pi_t < -W_{t-1}\right] \frac{LS_t}{D_t} \, f(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}) d\epsilon_{1t} d\epsilon_{2t} d\epsilon_{3t}$$

where I[.] is an indicator function that takes the value of one if the firm goes bankrupt and zero otherwise. $f(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t})$ is the probability density function of the error terms defined in equations (2.14) and (2.21). The premium (PR_t) at time t would be zero if the probability of the firm going bankrupt is zero so that the ratio of losses (LS_t) to the debt level (D_t) equals zero.

Assume that the hedger aims to maximize the expected net present worth (W_t), given the expected interest rate (2.24) and the non bankruptcy condition ($\pi_t > -W_{t-1}$), yielding the objective function:

$$\max_{F_t} E[W_t] = W_0 + \sum_{t=1}^{n} \int\int\int \beta^t NI_t I[\pi_t > -W_{t-1}] \tag{2.26}$$

$$f(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t})\; d\epsilon_{1t} d\epsilon_{2t} d\epsilon_{3t}$$

where β^t is the discount factor $(0 > \beta^t < 1)$, and W_0 is the exogenous initial wealth. Notice that the model is dynamic, and it is defined for n production periods. In (2.26), when net worth $(\pi_t < -W_{t-1})$ is negative, W_t will equal zero, indicating that equity holders get nothing if the firm enters bankruptcy. At $t-1$, the farmer has already decided the input levels ($\mathbf{X}_{t-1}$), the level of capital ($\mathbf{K}_{t-1}$) and the level of output. The only choice variable is how much to hedge (F_t). The debt level is determined by (2.16), and it will change as beginning equity changes.

CHAPTER III

DATA AND PROCEDURES

Because of the complexity of the analytical model (2.26), no comparative statics results were derived. Therefore, the effects of various factors are determined numerically for a specific example. Simulations are performed for a wheat and stocker steer producer. The model is simulated assuming constant returns to scale production and a single period. Also, government programs for wheat are not included to simplify the model and because the future of government programs is uncertain. The producer has made the decision to produce 1000 acres of wheat, and graze 296 steers on the winter wheat pasture. The wheat is planted in September and harvested in June, and the farmer buys steer calves in October and sells them in March. The only decision left to make is how much of the expected output of wheat and steers should be hedged in the futures market. The data for the base model is set forth in table 3.1.

The amount of capital borrowed in (2.16) will be dependent on the cost of hedging which depends on the amount hedged in the futures market and on beginning current assets (CA_{t-1}). There is only one investment made at period 0 (k_0), so that long term debt in (2.19) for period t-1 was defined as:

Table 3.1. Wheat for Grain, Owned Harvest Equipment, Budget Per Acre and Stocker Steers on Wheat Pasture Cost/Returns Per Head[a]

Variable	Units	Value	Standard Deviation
Wheat			
Variable Cost	$/acre	78.32	-
Capital investment	$/acre	171.47	-
Yield	Bu./acre	35.00	5.00[b]
Cash price	$/bu.	2.90	0.55
Futures price at planting	$/bu.	3.20	-
Futures price at harvest	$/bu.	3.20	0.50
Cost of hedging	$/bu.	0.02[c]	-
Steers			
Variable Cost	$/head	70.87	-
Capital investment	$/head	29.09	-
Steers calf Weight	cwt	4.36	-
Sale weight	cwt	6.65	0.50[d]
Price of steers calves	cwt	92.00	-
Cash price	$/cwt	82.00	5.50[e]
Futures price at planting	$/cwt	85.00	-
Futures price at harvest	$/cwt	85.00	5.00
Cost of hedging	$/cwt	0.25	-
Other Variables			
Bankruptcy fee	%	30.00	-
Interest rate	%	8.50	-

[a] The costs for production were taken from the OSU Enterprise Budgets developed by Oklahoma State University, Department of Agricultural Economics.

[b] Source: Schroeder and Goodwin

[c] Source: Brorsen, Coombs and Anderson

[d] Source: Koontz and Trapp

[e] Source: USDA. Calculated as the Standard Deviation of the cash price changes from October to march, 1980-1991.

(3.1) $$LTD_0 = \boldsymbol{K}_0 - (W_0 - \boldsymbol{P}_0^{x\prime}\boldsymbol{X}_0 - \boldsymbol{h}_0^{\prime}\boldsymbol{F}_0); \quad W_0 > \boldsymbol{P}_0^{x\prime}\boldsymbol{X}_0 + \boldsymbol{h}_0^{\prime}\boldsymbol{F}_0$$

Assume the farmer will pay the long-term debt in seven payments of equal amount, so A_t in (2.15) is:

(3.2) $$A_t = \frac{LTD_0}{7}$$

and outstanding debt subject to interest payments in period t is:

(3.3) $$LTD_t = LTD_0 - (t-1)A_{t-1}$$

Simulations are first performed by changing initial wealth (W_0) and solving for the optimal hedge ratio $\boldsymbol{F}_t$ in (2.26). Then initial wealth (W_0) is set at \$150,000 (ending debt to assets ratio is approximately 0.61), and numerical derivatives were used to obtain the response of the optimal hedge ratio to variations in the parameters of the model. The parameters to be changed in the model are: variance of both cash and futures price, basis risk, yield variance, tax rate, cost of hedging, liquidity cost and off farm income. Also, simulations are run to test the sensitivity of the optimal hedge ratio to changes in the distributional assumption of yields and prices, the randomness of output, and the inclusion of tax carry back.

Monte Carlo Integration

Equations (2.24) and (2.25) cannot be integrated analytically, therefore Monte Carlo Integration was used. Assume ϵ_{1t} in (2.14), ϵ_{2t} and ϵ_{3t} in (2.21) are distributed with probability density functions $beta(\epsilon_{1t})$, $lognormal(\epsilon_{2t})$, and $lognormal(\epsilon_{3t})$ respectively. A total of s random numbers for yields (2.14) $(y_{1t}, y_{2t}, \ldots, y_{st})$ were generated from a beta distribution. Cash and futures prices in (2.21) were generated from a lognormal distribution: $(P^c_{1t}, P^c_{2t}, \ldots, P^c_{st})$ and $(P^f_{1t}, P^f_{2t}, \ldots, P^f_{st})$; where s is the sample size equal to 500. Profits $(\pi_{it}(y_{it}, P^c_{it}, P^f_{it}))$ in (2.14), total debt in (2.15) $(D_{it}(y_{it}, P^c_{it}, P^f_{it}))$, net income in (2.17) $(NI_{it}(y_{it}, P^c_{it}, P^f_{it}))$ and bankruptcy losses in (2.20) were then define as functions of the random numbers for yields and prices. The risk premium (2.25) and the objective function (2.26) were approximated by:

$$PR_t = \frac{\sum_{i=1}^{s} \frac{LS_{it}(y_{it}, P^c_{it}, P^f_{it})}{D_{it}(y_{it}, P^c_{it}, P^f_{it})} [\pi_{it} < -W_{i,t-t}]}{s}$$

(3.4)

$$\max_{F_t} E[W_t] = W_0 + \frac{\sum_{t=1}^{n} \sum_{i=1}^{s} \beta^t NI_{it}(y_{it}, P^c_{it}, P^f_{it}) [\pi_{it} > W_{i,t-t}]}{s}$$

where the discounting factor is $\beta^t = \frac{1}{(1+0.085)^t}$. The model was solved to obtain the optimal hedge ratio (F_t) by means of the nonlinear algorithm in GAMS (Brooke, Kendrik, and Meeraus), which uses analytical derivatives.

Beta Random Numbers

Assume that wheat yields and the sale weight of the animals follow a beta distribution with mean and standard deviations shown in table 3.1. Crop yields may be distributed as a beta random variable since crop yields are known to fall in a range from 0 to some maximum possible value, and crop yields distributions might be significantly skewed either to the right or the left (Nelson and Preckel). The beta distributions were generated by first drawing two independent samples from a Gamma distribution using the Phillips generator (Shannon, p.365). These two samples are then used to get beta random numbers (Naylor).

The beta density function is a two-parameter density function defined over the closed interval $0 \le y \le 1$. However, the beta density function can be defined on the interval $c \le y^* \le d$ by using the following transformation (Mendenhall, Wackerly and Scheaffer):

(3.5) $$y^* = \frac{y-c}{d-c}; \quad 0 \le y \le 1; \; c \le y^* \le d$$

where, c and d are respectively the lower and upper bound of the variable y^*. If y is a beta-distributed random variable with parameters α and β, then the expected value of y ($E(y)$) and variance of y ($V(y)$) are respectively:

(3.6) $$E(y) = \frac{\alpha}{\alpha+\beta}; \quad V(y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

A beta distributed variable is a function of two gamma variables, $g_1(\alpha,\delta)$ and ($g_1(\alpha,\delta)$ + $g_2(\beta,\delta)$). Where α, β and δ are the parameters that define the gamma random variables. Variables $g_1(\alpha,\delta)$ and $g_2(\beta,\delta)$ are both independent gamma variables with identical values of δ and parameters α and β respectively (Naylor). The Phillips generator (Appendix ? lines ??) described in detail by Shannon (p. 365) was used to generate gamma random numbers . As an example, the following steps were used to generate the beta random numbers for wheat yield:

- Wheat yields were assumed to be in the closed interval $0 \leq y^* \geq 55$ bu./acre, where $E(y) = 35$ and $V(y) = 25$ (table 3.1). Therefore, using (3.5), $y^* = \frac{y}{55}$ and

$$E(y^*) = \frac{E(y)}{55} = \frac{35}{55} = 0.64$$

$$V(y^*) = \frac{V(y)}{55^2} = \frac{25}{55^2} = 0.0083$$

- By (3.6) one knows that

$$E(y) = \frac{\alpha}{\alpha + \beta} = 0.64$$

$$V(y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = 0.0083$$

The values for α and β were obtained by solving these two equations simultaneously, which yields $\alpha = 9.676,\ \beta = 17.2021$

- α, β and δ are needed to generate two independent gamma random variables. Make $\delta = E(y)/V(y) = 35/25 = 1.4$. Note that δ is obtained solving simultaneously $E(y) = \alpha/\delta;\ V(y) = \alpha/\delta^2$, which correspond, respectively, to the mean and variance of a gamma distributed variable.

Using the Phillips generator, $g_1(\alpha,\delta)$ was generated with parameters α and δ and $g_2(\beta,\delta)$ with β and δ. Finally, the beta random variable for wheat yields is given by:

$$y^* = \frac{g_1(\alpha,\delta)}{(g_1(\alpha,\delta) + g_2(\beta,\delta)}\ 55;\ 0 \leq y^* \leq 55$$

The same procedure was followed to generate the beta-distributed sale weights of the steers.

Lognormal Cash and Futures Prices

If price vector $\mathbf{p}_t$ follows a lognormal distribution with mean θ and variance-covariance matrix Ψ:

$$P_t \sim lognormal(\theta, \Psi)$$

then the logarithm of $\mathbf{p}_t$ ($\ln \mathbf{p}_t$) is distributed normally with mean $\boldsymbol{\mu}$ and variance-covariance matrix Ω (Johnson and Kotz):

(3.7)

$$\begin{aligned}
&\ln \boldsymbol{P} \sim Normal(\boldsymbol{\mu}, \Omega) \\
&\mu_i = \ln \theta_i - \frac{1}{2} \ln(1 + \frac{\Psi_{ij}}{\theta_i^2}) \ \forall i = j \\
&\Omega_{ij} = \ln(1 + \frac{\Psi_{ij}}{\theta_i^2}); \ \forall i = j \\
&\Omega_{ij} = \ln\left(\frac{\Psi_{ij}}{\exp(\mu_i + \mu_j + \frac{1}{2}(\Omega_{ii} + \Omega_{jj}))} + 1 \right) \ \forall i \neq j
\end{aligned}$$

where i and j are respectively the row and column number or a given matrix. Since prices are assumed to follow a random walk, the mean is actually the price itself lagged one period (i.e., $\theta = \boldsymbol{P}_{t-1}$). To generate the lognormal random numbers the following formula was calculated (Naylor et al.):

(3.8)

$$P_{it} = \exp(\ln P_{i,t-1} - \frac{1}{2} \ln(1 + \frac{\Psi_{ii}}{P_{i,t-1}^2}) + \bar{\Omega}_{ii} N_i(0,1) + \bar{\Omega}_{ij} N_j(0,1))$$

where $\bar{\Omega}$ is the Cholesky decomposition of Ω and $\mathbf{N(0,1)}$ is a row vector of normal random numbers with mean zero and variance one.

This is better illustrated with an example. From table 3.1, one knows that ($\theta_1 = 2.9$) is the cash price for wheat, ($\theta_2 = 3.2$) is the futures price for wheat, ($\Psi_{11} = 0.3025$) is the variance of the wheat cash price, ($\Psi_{22} = 0.25$) is the variance of the wheat futures price, and ($\Psi_{12} = \rho\sqrt{\Psi_{11}}\sqrt{\Psi_{22}} = 0.2475$) is the covariance between the wheat cash and futures price, where ($\rho = 0.90$) is the correlation between the cash and futures price. From (3.7):

$$\mu_1 = \ln\theta_1 - 0.5\ln(1 + \frac{\Psi_{11}}{\theta_1^2}) = 1.04704$$

$$\mu_2 = \ln\theta_2 - 0.5\ln(1 + \frac{\Psi_{22}}{\theta_2^2}) = 1.15109$$

$$\Omega_{11} = \ln(1 + \frac{\Psi_{11}}{\theta_1^2}) = 0.03533$$

$$\Omega_{22} = \ln(1 + \frac{\Psi_{22}}{\theta_2^2}) = 0.02412$$

$$\Omega_{12} = \Omega_{21} = \ln\left(\frac{\Psi_{12}}{\exp(\mu_1 + \mu_2 + 0.5(\Omega_{11} + \Omega_{22}))} + 1\right) = 0.02632$$

The Cholesky decomposition of Ω is then (Mapp, p. 160-163):

$$\bar{\Omega}_{22} = \sqrt{\Omega_{22}} = 0.1553;\ \bar{\Omega}_{12} = \frac{\Omega_{12}}{\bar{\Omega}_{22}} = 0.1669;\ \bar{\Omega}_{11} = \sqrt{\Omega_{11} - \bar{\Omega}_{12}{}^2} = 0.08645$$

Finally, series of random cash (P_1) and futures (P_2) prices were generated using (3.8):

$$P_1 = \exp(\ln \mu_1 + \bar{\Omega}_{11} N_1(0,1) + \bar{\Omega}_{12} N_2(0,1))$$

$$= \exp(0.0625 + 0.08703 N_1(0,1) + 0.08703\, N_2(0,1))$$

$$P_2 = \exp(\ln \mu_2 + \bar{\Omega}_{22} N_2(0,1)) = \exp(0.1407 + 0.1553 N_2(0,1))$$

Cash and futures prices were generated assuming that futures markets are efficient $(E(P_t^c) = E(P_t^f))$. The mean and variance of the generated numbers were calculated to check whether they conform with the original numbers.

Antithetic Variates

Antithetic variates, among other variance-reduction techniques, is useful to increase precision for a given sample size. The underlying idea is to have two estimators, x_1 and x_2 of an unknown parameter y, such that x_1 has a negative correlation with x_2. The estimate of y is then equal to $(x_1 + x_2)/2$ and has variance of $1/4(\sigma^2_{x_1} + \sigma^2_{x_2}) + \sigma_{x_1 x_2}$. With negative correlation, this variance is smaller than that possible when x_1 and x_2 were independent estimates (Shannon). Letting A_i be uniformly distributed (used for example to produce a gamma random variable), random numbers for the first half of the sample $(A_1, A_2, \ldots, A_{s/2})$ were drawn, where s is the sample size. Then the second half of the sample was calculated such that it was negatively correlated with the first half: $A_i = 1 - A_{i-s/2}$. For example, $A_{21} = 1 - A_1$ if sample size was 40. To generate lognormal random variates, normal random variates ($N_i(0,1)$) were first generated. Then, $N_i = -N_{i-s/2}$ yielded the negative correlated sample of normal random numbers.

Bankruptcy

Handling bankruptcy requires introducing discrete variables into the model and makes the problem difficult to solve. A dummy variable is needed to let the program know when the firm is bankrupt so that the firm will not generate more income and would have to pay a bankruptcy fee. To get around discrete variables, a differentiable 0-1 variable is necessary. Let the bankruptcy-dummy variable Z_t be equal one when the firm is bankrupt (equity $(W_t = 0)$) and zero otherwise $(W_t > 0)$. Define Z_t as:

(3.9) $$Z_t = \exp[-MW_t]; \quad W_t \geq 0$$

where M is a large number. Notice that when the firm is bankrupt $(W_t = 0)$, and $Z_t = 1$. On the other hand, whenever the firm is not bankrupt $(W_t > 0)$, the program will be calculating the exponential of a large negative number which makes $Z_t = 0$. Therefore; the only possible values for Z_t are one and zero, given that M is large enough and W_t is greater or equal to zero.

Even though equity (W_t) is restricted to be positive, GAMS would allow W_t to be negative while reaching the optimal solution. If W_t becomes negative, Z_t would approach infinity and the program will fail to converge. To get around this problem, Z_t in (1) were restricted to be less than exp(1) :

(3.10) $$Z_t = \exp[1 - 0.5(\sqrt{(1+MW_t)^2 + 1} + MW_t)]$$

This formula works the same way as (3.9), except that Z_t would not be allowed to be greater than exp(1), and the computer will continue solving even when equity goes negative. For example, if $(W_t = -1)$, then:

$$Z_t = \exp[1 - 0.5(\sqrt{(1-M)^2} + 1 - M)] = \exp[1 - 0.5(M - 1 + 1 - M)] = \exp(1)$$

Assume the firm goes bankrupt in period one and $Z_t = 1$. The firm's equity level is then:

$$W_1 = W_0 + NI_1 = 0$$

$$W_2 = W_1 + NI_2(1 - Z_1) = W_1 = 0$$

In the event of bankruptcy at period one, equity will be zero in both the first and second period. The bankruptcy fee is then calculated as:

$$C_1(D) = 0.3D_1 Z_1 = \begin{cases} 0 & if\ Z_1 = 0 \\ 0.3D_1 & if\ Z_1 = 1 \end{cases}$$

Progressive Tax Rates

Assume the farmer is married with two children. According to the 1994 instructions of tax form 1040 (Internal Revenue Service (IRS)), the standard deductions equal \$9800 (\$2400 for each) and total exemptions equal \$3175. These quantities are subtracted from profits (π_t) to get taxable income (TI_t). To calculate the tax, Schedule y-2 (IRS) for married filling separately (table 3.1) was used.

As an example, assume TI_t is less than 19000 dollars, then expression

$((0.15\sqrt{TI_t^2}+TI_t)/2)$ of (3.12) will be the tax, and the rest of the formula will be zero.

Note that if TI_t is negative, say -1000, $\tau_t(TI_t)=\frac{1}{2}0.15(\sqrt{-1000^2}-1000)=0$. If TI_t is greater than 19000 but less than 45925, say 20000, the tax would be:

$$Tax=0.15\frac{\sqrt{20000^2}+20000}{2}+0.13\frac{\sqrt{(20000-19000)^2}+20000-19000}{2}$$

$$Tax=0.15\cdot 20000+0.13\cdot 1000=3130$$

Tax Carry Back

If the farmer's standard deductions (STD) are more than the income for the year (π_t), then there is a net operating loss (NOL) (IRS, publication 536) equal to:

(3.13) $$NOL_t=-(\pi_t-STD); \qquad STD>\pi_t$$

The NOL can be used by deducting it from income in another year or years. Assume the farmer can only carry back his/her tax losses to the previous year, then the tax losses (TXLS) carry back are:

(3.14) $$TXLS_t=\tau(TI_{t-1})\,TI_{t-1}-\tau(TI_{t-1})(TI_{t-1}-NOL_t)$$

Assuming the farmer stays in the same income bracket, the tax rate ($\tau(TI_{t-1})$) is the same before and after deducting the NOL, then (3.14) becomes::

(3.15) $$TXLS_t=\tau(TI_{t-1})NOL_t$$

Table 3.2. Schedule y-2. Tax Rate Schedule for Married Filling Separately (1994)

If Taxable Income over-	But not Over	Tax	Of Amount over
0	19000	-----------15.0%	0
19000	45925	2850.00+28.0%	19000
45925	70000	10389.00+31.0%	45925
70000	125000	17852.25+36.0%	70000
125000	------	37652.25+39.6%	125000

Source: IRS. Instructions for Form 1040. 1994

Notice that the tax schedule in table 3.2 is really a step function that can be represented as:

(3.11)

$$\begin{aligned} Tax &= 0.15\, TI_t I[TI_t > 0] + 0.13\, TI_t I[TI_t > 19] + .03\, TI_t I[TI_t > 45.925] \\ &\quad + 0.05\, TI_t I[TI_t > 70] + 0.036\, TI_t I[TI_t > 12.5] \end{aligned}$$

where numbers are in thousands of dollars, and $I[.]$ is an indicator function equal to one when the condition is true, and zero otherwise. To make (3.11) differentiable, the following transformation was made:

(3.12)

$$\begin{aligned} Tax &= 0.15 \frac{\sqrt{TI_t^2} + TI_t}{2} + 0.13 \frac{\sqrt{(TI_t - 19)^2} + TI_t - 19}{2} \\ &\quad + 0.03 \frac{\sqrt{(TI_t - 45.925)^2} + TI_t - 45.925}{2} + 0.05 \frac{\sqrt{(TI_t - 70)^2} + TI_t - 70}{2} \\ &\quad + 0.036 \frac{\sqrt{(TI_t - 12.5)^2} + TI_t - 12.5}{2} \end{aligned}$$

This is the amount that should be refunded to the farmer and added to net income (NI_t) in (2.17).

Reaction Functions

In this study a two period dynamic model is estimated by computing the parameters of a reaction function. Reaction functions enable the decision maker to react to information prevailing at the decision time (Venkateswaran and Preckel). Decision makers are assumed to make their hedging decisions at the beginning of each production period. At decision period 1, the farmer chooses a fixed futures position (**F_1**). At decision period 2, the decision maker has information regarding wealth (W_1) realized in period 1 and makes a hedging decision based on this information. Therefore, the reaction functions representing the quantity of futures hedging at the beginning of the second period are:

(3.16)
$$F_2^{wheat} = \alpha_1 + \beta_1 W_1$$
$$F_2^{steers} = \alpha_2 + \beta_2 W_1$$

For the second period, instead of choosing the quantities of futures contracts (**F_2**), decision makers in this study choose the parameters of the reaction functions i.e, α's and β's to maximize the present value of terminal wealth.

CHAPTER IV

RESULTS

Simulation results show that the farm's capital structure, bankruptcy and liquidity cost, cost of hedging, off-farm income, profitability, cash and futures price variability, basis risk, production risk, and marginal tax rates are all factors that affect the decision to hedge.

Debt to Assets Ratio and the Optimal Hedge Ratio

Results clearly show that decisions at a certain level of income and debt to asset ratio depend on whether the farmer is solvent or not and depend on the potential for bankruptcy. The model yields optimal hedge ratios for wheat in the range of 0.19 to 0.44 as the debt to assets ratio increases from 0.8 to 0 respectively (fig. 4.1). However, if average profits go up, wheat hedge ratios would increase from a low of 0.13 to a high of 0.31 as the debt to assets ratio increases from 0.15 to 0.8 respectively (fig. 4.2). In figure 4.2, the probability of having positive liquidity and bankruptcy costs is zero for the entire range. For a sample of Kansas farmers with an average debt to assets ratio of 0.4, Schroeder and Goodwin find that the average percent of wheat sold using futures hedging was 22%. The results in this study predict closely their findings. Hedge ratios for wheat are higher than those for steers since wheat is relatively riskier given that the time between planting and harvest for wheat production is longer than the time the steers are owned. Results indicate that the farmer would choose to hedge steers only when the probability of bankruptcy is positive (figs. 4.1 and 4.3). Bankruptcy is an extreme case since it only happens when the debt to assets ratio is higher than 0.85 (fig. 4.3).

Contrary to what one would expect, the model shows that hedging decreases with increasing leverage (fig. 4.1) when the firm has zero probability of going bankrupt (fig.4.3). However, the shape of the curve changes with a different level of income through the effect of taxes. With an increase in prices by 15% (an average increase in profits from $5,043 to $44,283), the profitability of the production of wheat and cattle rises, and the firm hedges more as the proportion of debt increases (fig. 4.2). With low profitability, the firm pays almost no taxes after deductions and exemptions, which means that the tax reduction with increasing hedging is minimal. As profitability increases, the firm pays more taxes and has more incentives to hedge. Hedging reduces the variability of profits and the expected tax liability which in turn increases the expected ending wealth.

When the firm's probability of bankruptcy is positive (fig. 4.3), hedging increases significantly (from 0.19 to 0.8) (fig. 4.1) to reduce bankruptcy losses. The nonlinearity clearly displayed in figure 4.2 is most likely due to the tax schedule, which is a step function (table 2.2). The response of hedging to changes in the debt to assets ratio increases (the slope of the function is steeper) as the farmer moves up from one income bracket to another.

Before the firm goes bankrupt, it will first encounter liquidity problems, and hedging should serve as a source of cash flow (Turvey and Baker). This is true as long as the cost of hedging is lower than the liquidity cost. The liquidity cost in figure 4.1 is 1% of outstanding short-term liabilities, not high enough for the firm to hedge more. As soon as the firm experiences both insolvency and bankruptcy (fig. 4.3) the firm is willing to pay the cost of trading more in the futures market (fig. 4.1) and reduce the relatively higher liquidity and bankruptcy costs. Note that the results hold under the assumption of risk neutrality.

Interest Rates

As Collins and Karp argue, increases in leverage also increases the probability of a disaster which cause increased risk of loss for the lender, and the cost of borrowing also increases with leverage. This relationship is shown in figure 4.4. When the probability of bankruptcy is non-zero, the interest rate rises above the riskless interest rate. For this simulation example, the firm shows positive probability of bankruptcy at relatively high debt to assets ratios (above .85), where the cost of borrowing begins to rise.

Cost of Hedging

Traditionally optimal hedging models introduced by Johnson and Stein and variations of their approach (i.e., Myers and Thompson) assume away the cost of hedging. Here the cost of hedging influences greatly the decision of whether or not to hedge and how much to hedge (figs. 4.5 and 4.6). Similar results were obtained by Lence and

Berck. The cost of hedging has to be lower than 15 cents/cwt (the base cost was 25 cents/cwt) for a cattle producer to hedge. A cattle producer would hedge more than 50% of the steers if the cost of hedging is as low as 5 cents/cwt. The cost of hedging alone might be the reason why a cattle producer would not hedge at all. For a wheat producer, the optimal hedge ratio varies from a high of .52 to 0 when the cost of hedging increases respectively from 1 cent/bu to 28 cents/bu.

Clearly, transaction costs of hedging may well exceed the tax-reducing benefits of hedging, causing optimal hedge ratios to decrease. These results apply for a debt to asset ratio of 0.61, a point at which the farmer does not encounter any liquidity or bankruptcy cost.

Off-Farm Income and Profitability

In accordance with Lence's results, optimal hedge ratios are greatly influenced by off-farm income (fig. 4.7). As off-farm income increases, the tax-reducing benefits of hedging become more attractive and the wheat producer would hedge more. This result could not be found by Lence since tax effects were ignored in his study.

After a level of off-farm income of $20,000 (the average farm income is $6,470), the optimal hedge ratio begins to decline. This result may not be due to what Lence calls "the dilution effect." In Lence's work, as the share of the alternative investment to initial wealth increases, the cash position becomes relatively less and less important to the farmer, and the incentive to hedge decreases. In this study, though, as income increases, the farmer moves up to a higher income bracket where marginal tax rates are lower and the incentive to hedge is little.

The effect of higher profitability works much the same way as off-farm income (fig. 4.8). As the cash position becomes more profitable, benefits from futures trading become relatively negligible, making hedging unattractive. With a 25% increase in prices, profitability increases to an average of $59,335, a point at which wheat hedging is zero.

Variance of Cash and Futures Prices

For a wheat producer, the optimal hedge ratio is positively related to the variance of cash and futures prices. As the cash and futures price risk increases (one at a time), the optimal hedge ratio increases. Under the assumptions of production certainty and basis risk, Robinson and Barry, Peck, and Kahl and others found the same result. In this study, however, results are found under the assumptions of risk neutrality and production risk.

An increase of either the cash price variance or the futures price variance leads to an increase in basis risk, which would be a disincentive to hedge (fig. 4.11). However, the increase in the price variance more than offsets the increase in the basis variance, and this provides an incentive to hedge more. The zero hedging position for steers did not change with variations in the variance of prices.

Taking the derivative of the minimum-variance hedge ratio in (2.3) with respect to the variance of the futures price, one obtains that the minimum-variance hedge ratio increases as the variance of the futures price increases ($\frac{\partial F^*}{\partial \sigma^2_{pf}} = \frac{\sigma_{p^c p^f}}{\sigma^4_{pf}} > 0$). However, the minimum-variance hedge ratio does not depend on the cash price variance. The results in this study confirm partially the predictions by Robinson and Barry (2.12) and the empirical findings of Peck and Nahmias where the optimal hedge ratio increases with increases in cash price variance. It is shown here, though, that the optimal hedge ratio also increases with increases in the futures price variance (not true in Robinson and Barry's model).

The optimal hedge ratio is more sensitive to changes in cash price variance than it is to changes in the futures price variance. The curve is almost flat when the variance of futures price reaches 0.30 (a coefficient of variation of 5.28), a point at which the optimal hedge ratio starts to fall. Hedging becomes less attractive as the variability of the futures price increases.

Basis Risk

There is a significant response of hedging to changes in basis variance (figure 4.11). As basis variance increases, the correlation between futures and cash prices goes down, and producers will have less incentive to hedge. Wheat optimal hedge ratios fall from a high of 0.30 to 0.12 when the correlation of cash and futures decreases from 0.9 to 0.80. For steers, further decrease in basis risk (from the base model, table 1) would not be enough incentive to hedge. The transaction cost of hedging must be too high compared to the expected gain from hedging steers. The variance of prices and production risk are held constant.

Deterministic Production

When randomness of output was eliminated, the optimal hedge ratio for wheat increased from 0.30 to 0.40, a 32.4 % change. A recommended hedge ratio based on deterministic output can be suboptimal whenever the firm faces production uncertainty (as in most agricultural activities). As predicted by several authors (Chavas and Pope; Grant; Rolfo; Lence; Lapan and Moschini), production risk causes the optimal hedge ratio to be lower than in the case when output is nonrandom. Results explain why grain elevators find it more attractive to hedge than not to hedge. These firms hold more inventories, which have little randomness.

Progressive Tax Rates and Tax Carry Back

In this model, taxes play a very important role in the decision to hedge, especially when the firm is not close to bankruptcy. A decrease in the marginal tax rate of 30% can make the optimal hedge ratio for wheat go to zero (Fig. 4.12). On the other hand, an increase in the marginal tax rate of 30%, can bring the optimal hedge ratio to a high of 0.50. The tax reducing benefits of hedging provided the concavity of the objective function when the likelihood of positive bankruptcy and liquidity costs is zero. The agent need not be risk averse to hedge, instead, he/she hedges to reduce taxes and increase after-tax income.

With a low profitability, the changes of having net operating losses (NOP) are high. Assuming all these losses could be carried back, the optimal hedge ratio would be zero (fig. 4.13). Even if the farmer was able to carry back only 40% of his/her NOL, the optimal hedge ratio would be less than 10%. Tax losses carry back increase income in the current period and decrease the variability of income; therefore, reducing the incentive to hedge.

Two Period Dynamic Model

Dynamics matters more when income is low. When beginning equity is $150,000 (debt to assets ratio of 0.61), the farmer would hedge 41% and 70% of the wheat in the first and second periods respectively (table 4.4). However, when income increases (i.e, off-farm income is $50,000), the dynamic optimal hedge ratio is lower and almost the same for both periods (0.27 and 0.25 respectively). Steers are still not hedge in the dynamic model.

The optimal hedge ratios for wheat are higher in the dynamic model than the static optimal hedge ratios. For example, when beginning equity equals $150,000 and off-farm income is $50,000, the dynamic optimal hedge ratio for wheat in the first period is 0.27 while the static optimal hedge ratio is 0.18 (table 4.3 and 4.4). Similarly, for the case where beginning equity is $200,000, the optimal hedge ratio is 0.33 in the static case, and 0.50 in the dynamic model. As a business activity becomes riskier (i.e, more variability of income), more incentives would exist to trade in the futures markets. In the dynamic model more uncertainty is introduced. Income in the second period is uncertain not only because of price, basis, and yield risk, but because beginning equity is also uncertain (depends on first period's outcome).

In the static model, the cost of hedging affects significantly the decision to hedge and how much to hedge. This response is less dramatic in the dynamic model. A 20% increase in the cost of hedging makes the dynamic optimal hedge ratio for wheat in the first period drop 32% (from 0.41 to 0.28). The same increase in the cost of hedging wheat caused the static optimal hedge ratio to decline by 60%. The benefits of hedging in the dynamic model are more obvious and the optimal hedge ratios are relatively less responsive to changes in the cost of hedging. Hedging increases expected after tax income for the period, and this makes next period's beginning equity increase.

The slope in the reaction function for wheat is always positive (table 4.4). An increase in realized income in the first period would lead to an increase in futures trading in the second period. However, the response of dynamic optimal hedge ratios is lower as income increases. For example, when off-farm income is zero, the slope in the reaction function for wheat is 0.56, but when off-farm income increases to $50,000, the slope is 0.29. This shows that since marginal tax rates are lower at high income levels, the optimal hedge ratio should be less responsive to income changes.

Appendix 4.1

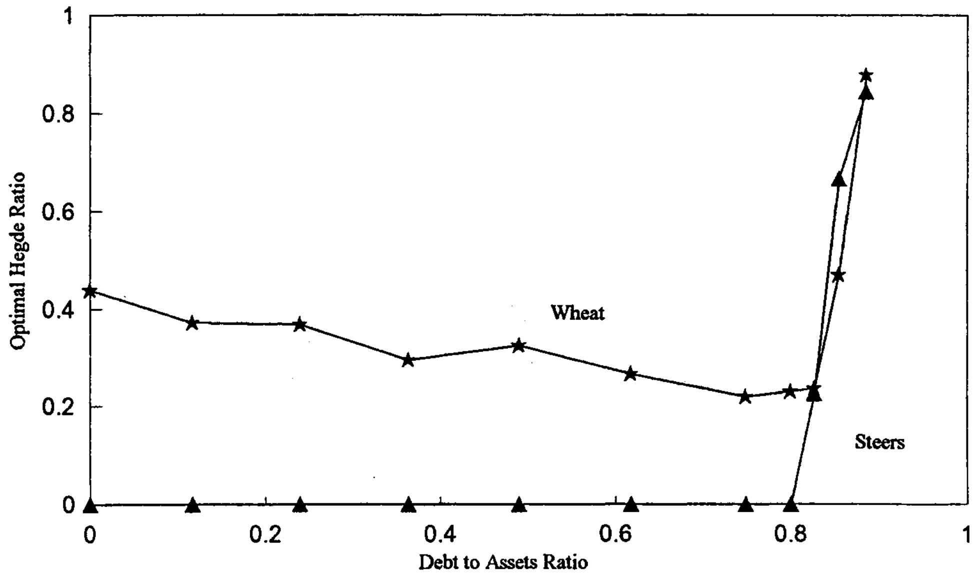

Figure 4.1. Optimal Hedge Ratio and the Debt to Assets Ratio for a Wheat and Stocker Steer Prod

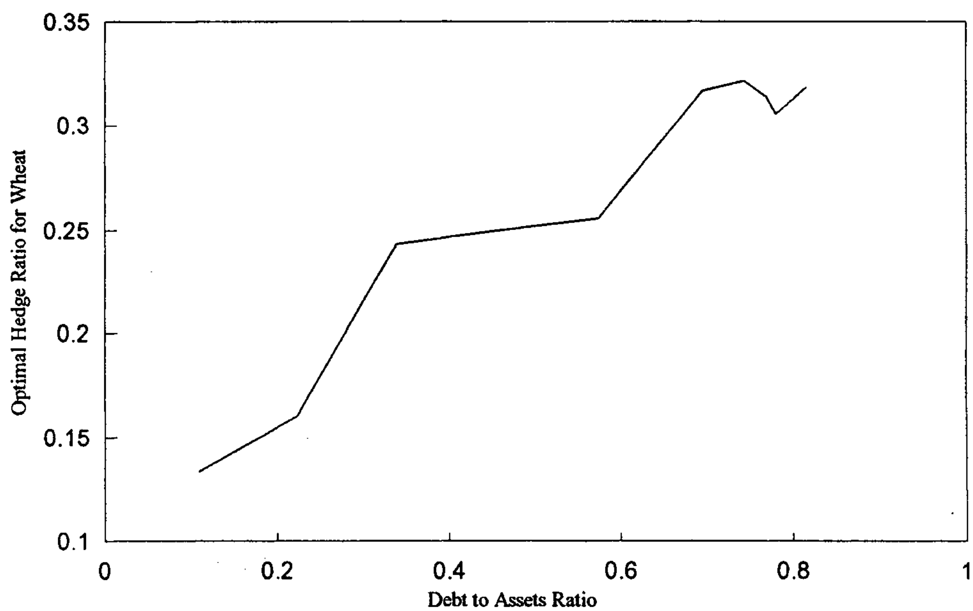

Figure 4.2. Debt to Assets Ratios and and the Optimal Hedge Ratio when Prices Go Up by 15%

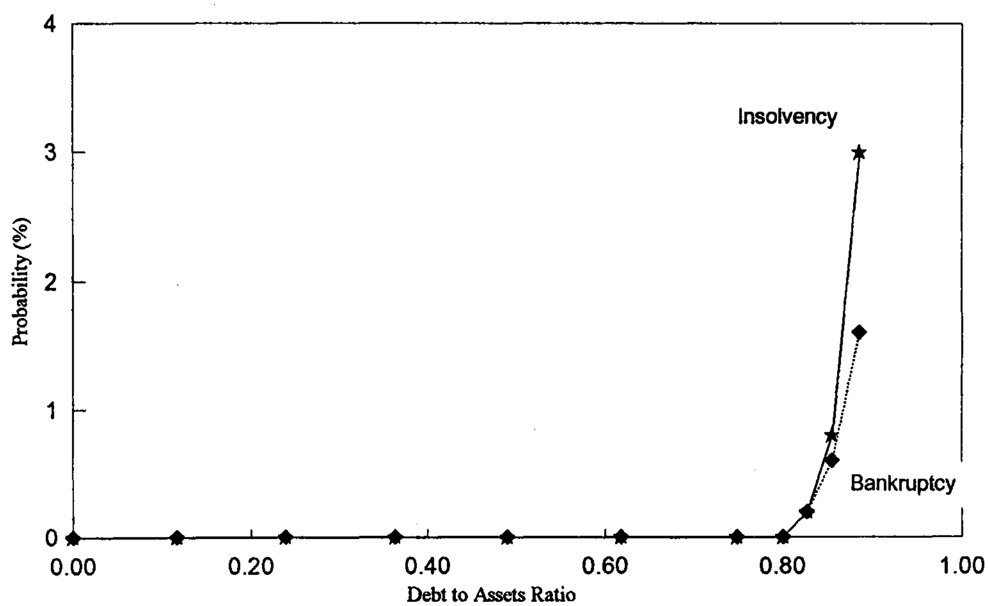

Figure 4.3. Debt to Assets Ratio vs. the Probability of Insolvency and Bankruptcy (In %).

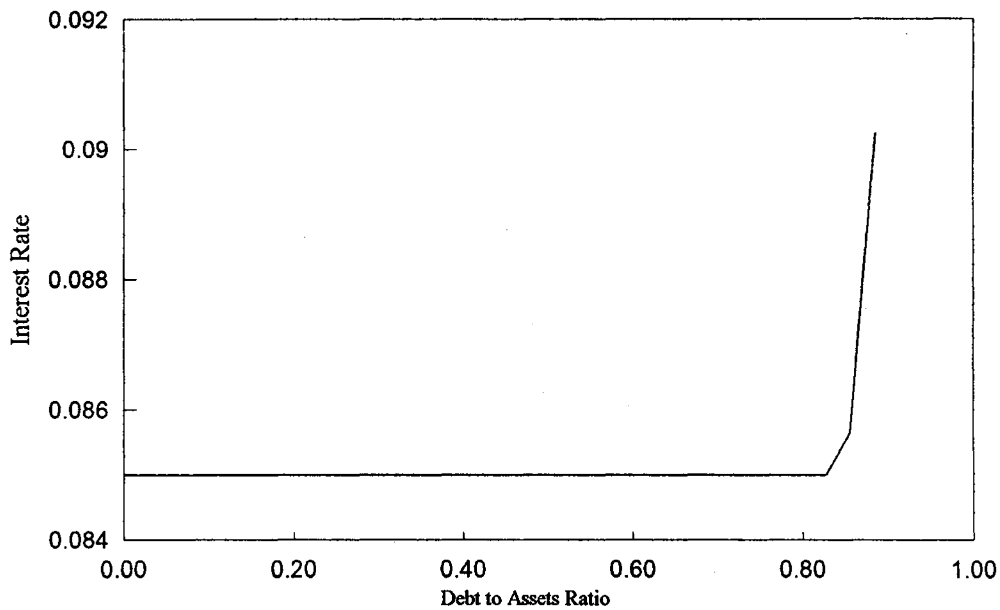

Figure 4.4. Debt to Assets Ratio vs. the Interest Rate

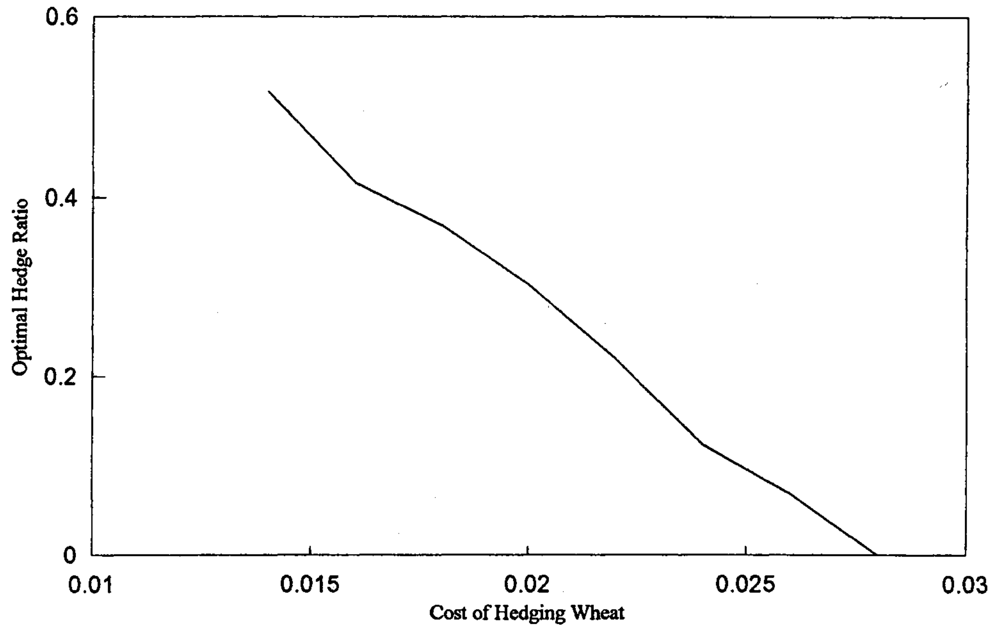

Figure 4.5. Optimal Hedge Ratios and the Cost of Hedging Wheat

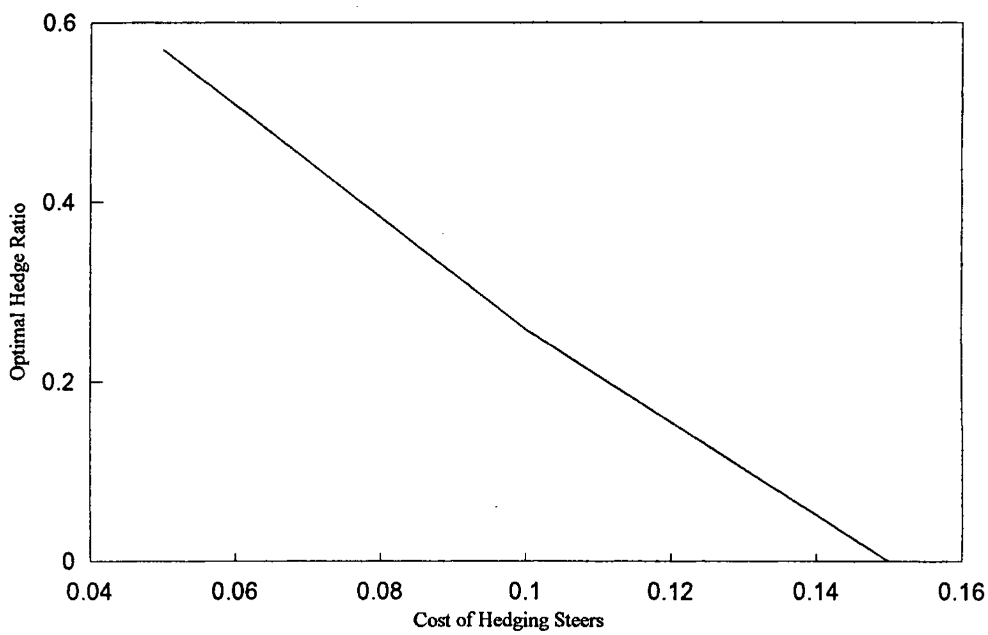

Figure 4.6. Optimal Hedge Ratios and the Cost of Hedging Steers

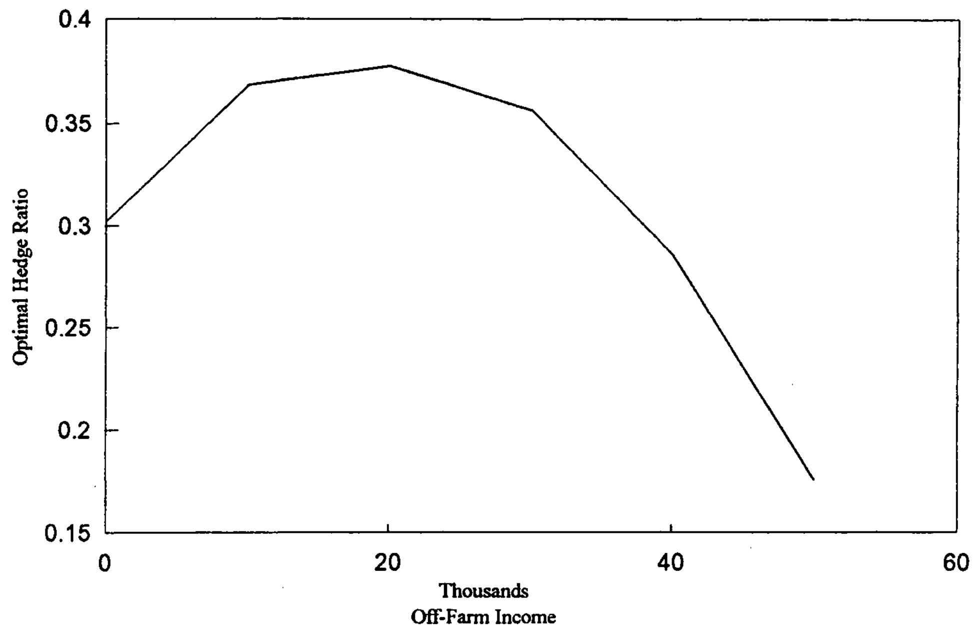

Figure 4.7. The Optimal Hedge Ratio for Wheat and Off-Farm Income

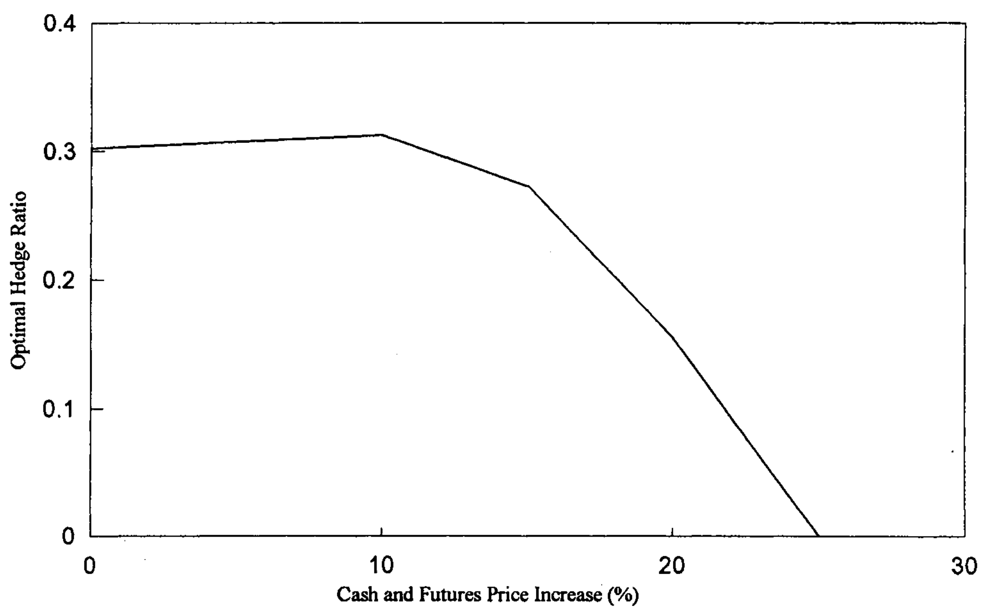

Figure 4.8. The Optimal Hedge for Wheat and the Profitability of Wheat and Cattle

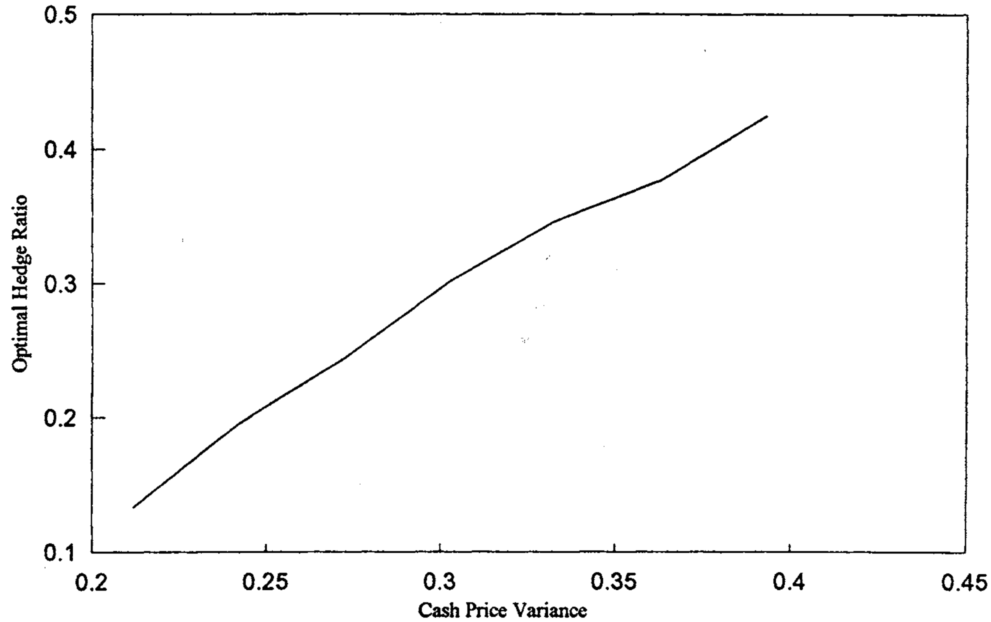

Figure 4.9. The Optimal Hege Ratio for Wheat and the Cash Price Variance

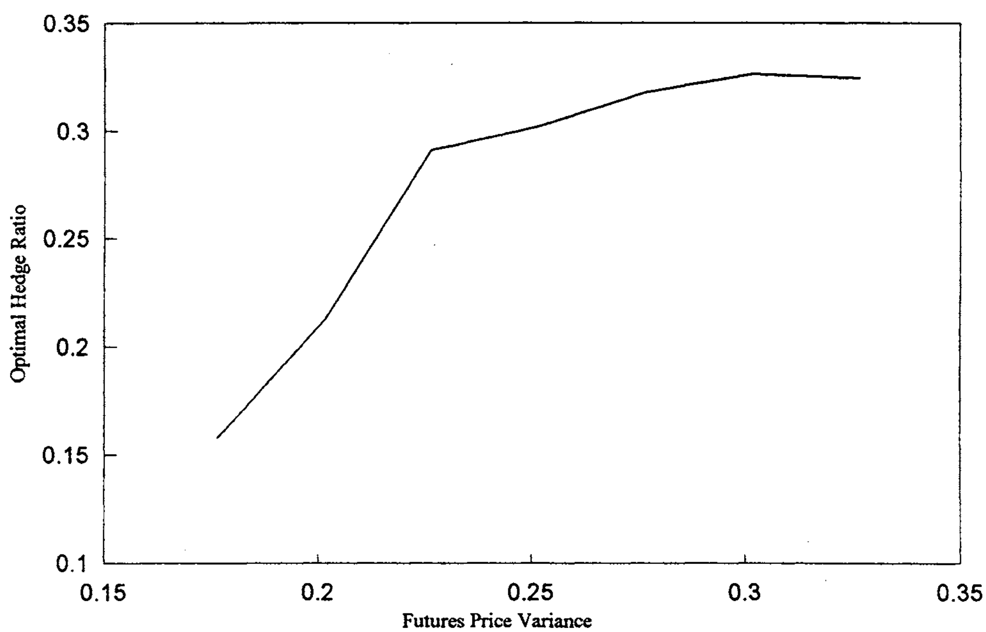

Figure 4.10. The Optimal Hege Ratio for Wheat and the Futures Price Variance

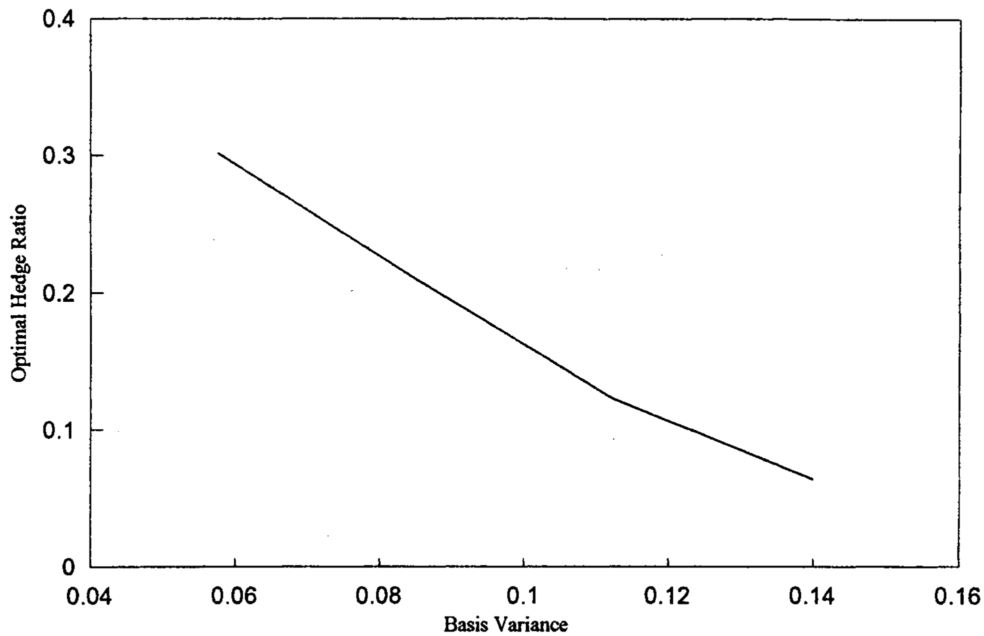

Figure 4.11. Basis Risk and Optimal Hedge Ratio for Wheat

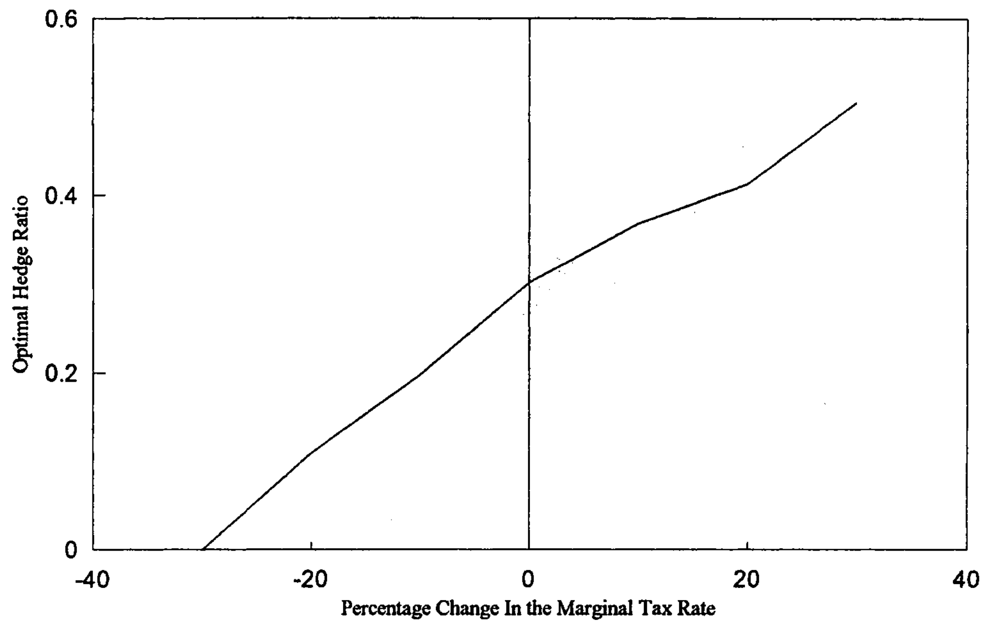

Figure 4.12. Percentage Change in the Marginal Tax Rate and the Optimal Hedge Ratio for Wheat

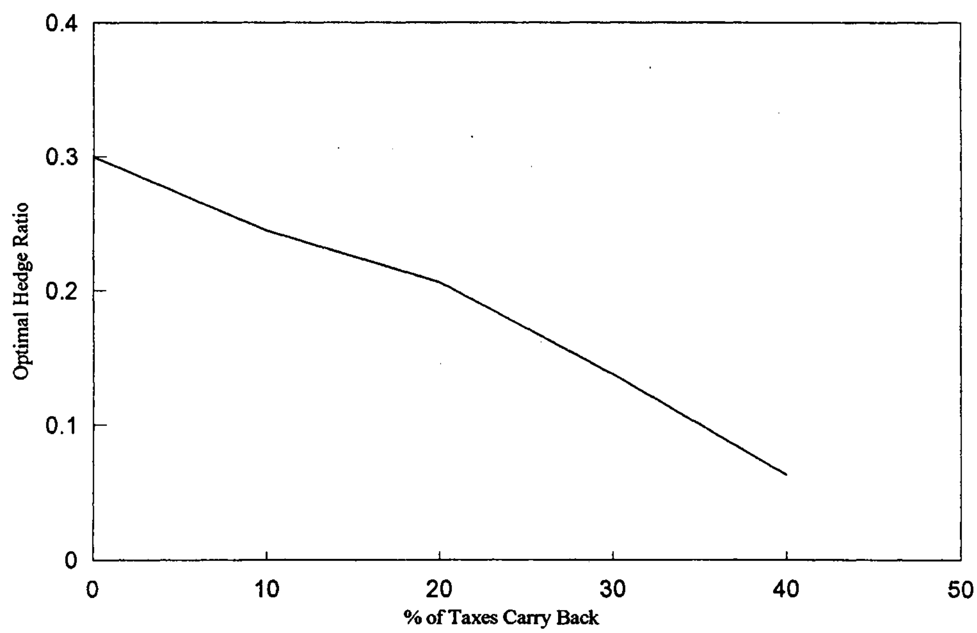

Figure 4.13. Percentage of Tax Carry Back and the Optimal Hedge Ratio

Table 4.1. Simulation Results: Debt to Assets Ratio vs. the Optimal Hedge Ratios for Wheat and Steers, Probability of Insolvency and Bankruptcy, and Interest Rates.

Debt to Assets Ratio	Optimal Hedge Ratio		Probability (%) of		Interest Rate	Objective Function Value ($1000)
	Wheat	Steers	Bankruptcy	Insolvency		
0.00	0.44	0.00	0.0	0.0	0.085	420.000
0.12	0.37	0.00	0.0	0.0	0.085	366.542
0.24	0.37	0.00	0.0	0.0	0.085	313.197
0.36	0.30	0.00	0.0	0.0	0.085	259.786
0.49	0.33	0.00	0.0	0.0	0.085	206.451
0.62	0.27	0.00	0.0	0.0	0.085	153.300
0.75	0.22	0.00	0.0	0.0	0.085	100.108
0.80	0.23	0.00	0.0	0.0	0.085	79.418
0.83	0.24	0.23	0.2	0.2	0.085	68.619
0.85	0.47	0.67	0.6	0.8	0.086	57.512
0.89	0.88	0.85	1.6	3.0	0.089	45.538

Table 4.2. Debt to Assets Ratio vs. Optimal Hedge Ratios for Wheat When Prices Go Up by 15%

Leverage	Optimal Hedge Ratio	Objective function Value ($1000)
0.11	0.13	394.786
0.22	0.16	342.003
0.34	0.24	289.174
0.46	0.25	236.406
0.57	0.26	183.786
0.69	0.32	131.124
0.74	0.32	110.049
0.77	0.31	99.504
0.79	0.30	88.960
0.82	0.32	78.412

Table 4.3. Optimal Hedge Ratios vs. Cost of Hedging, Off-Farm Income, Price Variance, Basis Risk, Progressive Tax Rates and Tax Loss Carry back.

Variable	Optimal Hedge Ratios	Objective function Value ($1000)
Cost of hedging Wheat ($/bu.)		
0.014	0.52	154.60
0.016	0.42	154.57
0.018	0.37	154.54
0.020	0.30	154.52
0.022	0.22	154.50
0.024	0.12	154.49
0.026	0.07	154.48
0.028	0.00	154.48
Cost of Hedging Steers ($/lb.)		
0.05	0.57	154.57
0.10	0.26	154.53
0.15	0.00	154.52
Off-Farm Income ($) and the Optimal Hedge Ratio for Wheat		
10000	0.37	162.86
20000	0.38	170.82
30000	0.36	178.37
40000	0.29	185.54
50000	0.18	192.39
% Increase in Prices vs. the Optimal Hedge Ratio for Wheat		
10	0.31	175.52
15	0.27	185.03
20	0.16	194.03
25	0.00	202.62
Variance of Wheat Cash Price vs. the Optimal Hedge Ratio for Wheat		
0.21	0.13	154.49
0.24	0.20	154.50
0.27	0.24	154.51
0.33	0.35	154.53
0.36	0.38	154.54
0.39	0.43	154.55

Table 4.3 (Continue)

Variable	Optimal Hedge Ratios	Objective function Value ($1000)
Variance of Wheat Futures Price vs. the Optimal Hedge Ratio for Wheat		
0.18	0.16	154.66
0.20	0.21	154.61
0.23	0.29	154.57
0.28	0.32	154.48
0.30	0.33	154.44
0.33	0.32	154.40
Basis Variance for Wheat vs. the Optimal Hedge Ratio for Wheat		
0.058	0.30	154.52
0.085	0.21	154.49
0.113	0.12	154.46
0.140	0.06	154.43
% Change in the Progressive Tax Rates		
-30	0.00	155.00
-20	0.11	154.83
-10	0.20	154.68
10	0.37	154.38
20	0.41	154.24
30	0.51	154.11
% of Tax loss Carry back		
10	0.24	154.43
20	0.21	154.56
30	0.14	154.70
40	0.06	154.83

Table 4.4. Dynamic Optimal Hedge Ratios for Wheat and Steers vs. Income and the Cost of Hedging.

Parameter	Simulation 1	2	3	4	5	6
Beginning Equity ($1000)	150	150	150	200	200	150
Off-Farm Income ($1000)	0	20	50	0	50	0
Increase in the Cost of Hedging Wheat (%)	0	0	0	0	0	20
Optimal Hedge Ratio						
Wheat						
1st period	0.41	0.46	0.27	0.50	0.22	0.28
2nd period	0.70	0.63	0.25	0.66	0.26	0.57
Steers						
1st period	0.00	0.04	0.00	0.00	0.00	0.00
2nd period	0.00	0.06	0.00	0.00	0.00	0.00
Reaction Functions						
Wheat						
Intercept (α_1)	-3.84	-6.92	-3.90	-9.23	-5.74	-4.85
Slope (β_1)	0.56	0.65	0.29	0.66	0.30	0.56
Steers						
Intercept (α_2)	0.00	0.96	0.00	0.00	0.00	0.00
Slope (β_2)	0.00	-0.04	0.00	0.00	0.00	0.00

CHAPTER V

CONCLUSIONS

Theoretical and empirical models usually find optimal hedge ratios close to one while farmers hedge much less. In this study, a new theoretical model of hedging is derived. An empirical example is provided to show how changes in assumptions affect the optimal level of hedging for a wheat and stocker steer producer.

Results explain why some firms do not hedge and some hedge more than others. Firms with higher expected income would tend to hedge less than a low-income farm. Provided expected operating losses are low, a high-levered firm would hedge more than a low-levered firm. Profitability ensures that income is positive and the firm can take advantage of the tax-reducing benefits of hedging. As profitability increases, though , farmers move up to a higher income bracket where marginal tax rates are lower and the incentive to hedge is little. The benefits of trading in the futures markets would be more significant if the marginal tax rates were higher. If the firm can carry back its net operating losses, no motivation for hedging exists. The more tax losses the farmer can carry back, the less incentive he/she would have to hedge. For high-income farms, income averaging would have a similar effect than the tax-loss carry back.

When the probability of insolvency and bankruptcy is positive, the motivation to hedge is less tax related. The farmer is willing to bear the cost of hedging to reduce the relatively higher liquidity and bankruptcy costs. Hedging decreases the variability of profits, reducing expected liquidity and bankruptcy costs, and reducing interest rates changed by banks.

Traditionally, the cost of hedging has been considered small and ignored in most studies. Even when the cost of hedging is a small proportion of total cost, farmers may have no incentives to hedge when this cost is higher than the reduction in tax payments and the firm is in good financial position (i.e. low debt to assets ratios). For high levered firms, hedging reduces expected bankruptcy losses, and this effect may be considerably greater than the cost of hedging, making hedging very attractive. The benefits of hedging in the dynamic model are more obvious and the optimal hedge ratios are relatively less responsive to changes in the cost of hedging. Hedging increases expected after tax income for the period, and this makes next period's beginning equity increase.

Price, basis and production risk also explain differences in hedging behavior among farmers. The more variable the cash and the futures prices are the more the farmer would hedge. Hedging, however, becomes less and less effective in reducing profit variability as basis risk increases. With high basis variance, the optimal decision may well be not to hedge at all. As previous research has shown, firms with little production uncertainty would hedge less than firms with high yield variability. A grain elevator would hedge more than a farmer. Grain elevators hold more inventories which have little randomness while farmers are exposed to unexpected weather conditions and therefore to higher production risk. Also, it is likely that grain elevators have lower transaction costs and would hedge more.

The optimal hedge ratios for wheat are higher in the dynamic model than the static optimal hedge ratios. As a business activity becomes riskier (i.e, more variability of income), more incentives would exist to trade in the futures markets. In the dynamic model more uncertainty is introduced. Income in the second period is uncertain not only because of price, basis, and yield risk, but because beginning equity is also uncertain (depends on first period's outcome).

A major implication of the results in this study is that risk-averse preferences are not necessary for farmers to hedge. Tax rates, liquidity costs and bankruptcy losses provide the concavity of the objective function necessary to motivate firms to hedge. Risk aversion might provide an extra incentive to hedge.

The empirical example shows that a farmer would hedge some of the wheat and would not hedge steers. The optimal hedge ratio for wheat with a debt to assets ratio of 0.45 was 0.25 (for an average profit of $48,000). These results are similar to the findings of Shroeder and Goodwin. They found that for a sample of Kansas farmers, the average percent of wheat sold using the futures markets was 22% (debt to assets ratio of 0.4). In the example provided, the optimal hedge ratio for steers was zero probably because the steers are owned for a short period while the time from planting to harvesting the wheat is longer. The potential to reduce risk using wheat futures hedging is presumably higher.

With empirical research usually finding optimal hedge ratios close to one, some believe farmers should be taught the benefits of trading in the futures markets. The new theoretical model provides optimal hedge ratios close to those used by farmers and provides an explanation of why farmer would hedge so little. Decisions at a certain income level depend on whether the farmer is solvent or not and depend on the potential for bankruptcy. Extension economists should not treat every farmer the same and give them the same hedging recommendations. Depending on the type of business, product diversification, income level, cost of hedging, off-farm income, price, basis and production risk, the optimal strategy might be not to hedge or to hedge very little. The most important motivation for hedging is the progressive tax rate and some policy implications are suggested. Progressive tax rates introduce costs inefficiencies by motivating farmers to hedge more. A government official should favor a flatter tax schedule to reduce inefficiencies in the market while futures exchange should favor progressive tax rates. Tax loss carry back, on the other hand, can eliminate the need for hedging when the firm experiences net operating losses. Similarly, for a high-income farm, income averaging would make farmers hedge less. It is possible that accounting tricks might balance income more cheaply than hedging. For example, if expected income for the year is high, farmers may chose to buy inputs for next year before taxes are filed.

Theoretical and empirical model used in past research have made simplifying assumptions that restrict them from explaining what farmers actually do. Future research on hedging should no longer ignore the following factors that affect significantly the decision of whether or not to hedge and how much to hedge: i) the farmer's income level and leverage, ii) the profitability of the cash position and off-farm income, iii) the transaction cost of hedging, iv) solvency and probability of bankruptcy, v) price, basis and production risk, and vi) marginal tax rates and tax loss carry back. Assuming away any of these factors when true would yield suboptimal recommendations.

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