MATHEMATICS USED BY STUDENTS OF ENGINEERING AND BY PROFESSIONAL ENGINEERS

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PREFACE

It has long been recognized that the engineering student, and the professional engineer, need a good mathematical background, although the precise areas and topics needed have been only fragmentarily explored. It is the purpose of this study to provide a partial answer to the question, still unanswered, as to what mathematics the engineer needs.

The problem has been attacked in a two-fold manner; representative engineering textbooks, on the undergraduate level, have been analyzed for mathematical content, and professional engineers have designated, by means of a questionnaire, their use of mathematics.

Indebtedness ia acknowledged to Drs. M. R. Chauncey, James H. Zant, J. M. Richardson, Roy Gladstone, and M. S. Wallace for their valuable guidance and assistance in helping carry this study to completion; acknowledgement is also made to Mr. Edwin F. Coppage, of Tenafly, New Jersey, a secretary for the American Institute of Electrical Engineers, and to Mr. Arthur Thigpen, Professor of Civil Engineering at Louisiana Polytechnic Institute, Ruston, Louisiana, for their aid in facilitating the sending of questionnaires to professional engineers.

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CHAPTER I

INTRODUCTION

<u>Statement of the Problem</u>. The problem taken for this study is to determine the specific mathematical principles and processes used by engineers. The first phase of the investigation seeks to discover the mathematics used in professional undergraduate courses in engineering, since engineers of today must complete such courses in recognized institutions as a prerequisite to securing a professional license. This study analyzes, therefore, the mathematics that is used in basic engineering textbooks. The second phase of the study seeks to determine the needs of engineers in mathematics by collecting and analyzing opinions of practicing professional engineers as to which phases of mathematics they find most useful in their professional work. Accordingly, a representative group of engineers was asked to give their opinions on the subject by filling out questionnaires or checklists in which the major mathematical concepts and techniques were listed.

A synthesis of the two types of analysis just described should help to reveal the phases of mathematics that are most valuable for the professional engineer. Furthermore, the study should prove of value to those who teach mathematics to students of engineering -- either through lending support for traditional subject matter and emphasis in mathematics courses or through suggesting new subject matter or new distribution of emphasis.

Need for the Study. The importance of mathematics for engineers has long been recognized. In fact, as the field of engineering grows in scope and importance there is inevitably a concomitant tendency to place greater emphasis on mathematics.[1] However, very little work has been done to discover precisely which phases of mathematics are most valuable to the engineer. Among the most logical places to look for information on the subject would be the publications of the National Council of Teachers of Mathematics and the American Society of Engineering Education. A careful study of the volumes of The Mathematics Teacher, The Yearbooks of the National Council of Teachers of Mathematics, and the Journal of the American Society of Engineering Education discloses no systematic studies of the particular needs of engineers in the field of mathematics.

It is difficult to understand why no such studies have been made in the field. Apparently it has been assumed that the mathematical needs of engineering students do not differ substantially from the mathematical needs of other students. If this study should reveal that traditional courses in mathematics are highly satisfactory in meeting the needs of engineers, the viewpoint will be justified. On the other

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1. For example, Arvid W. Jacobson states, "Many individuals in engineering, business, and education have felt that steps should be taken to enlarge the role of mathematics in order to meet the needs of more exacting technological standards." "The Emergence of Mathematical Needs in an Industrial Center," The Mathematics Teacher (Jan., 1953), p. 1.

hand, if our analysis should disclose substantial differences between the needs of engineering students and those of non-engineering students, the study should be of considerable significance.

Assumptions. The following assumptions are made in this study: (1) That the textbooks used in the engineering courses are a valid guide to the mathematics used by the students in those courses; (2) that the institution in which this study is made is a representative one; (3) that the questionnaire, as administered, has validity in determining the specific uses engineers make of mathematics.

These assumptions are, of course, open to question. To cite three of them: teachers do not always limit the mathematics used in courses to that contained in textbooks; it would be difficult to prove that the institution chosen for the study is a completely representative one, and the results obtained from questionnaires must almost always be accepted with reservations.[2] Although these objections have sufficient weight to necessitate some caution in evaluating the findings of this study, they do not, it is believed, invalidate the essential sssumptions and procedures employed.

The Sources of the Data in this Study. The data for this study are confined narrowly to two sources: (1) Material from prevailing textbooks in engineering, and (2) a questionnaire. Only rarely are sources other than the aforementioned used, and then only concomitantly.

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2. See, for example, C. V. Good, A. S. Barr, and D. E. Scates, The Methodology of Educational Research, D. Appleton-Century Company, (New York, 1936), pp. 330-337.

(1). The Survey of Textbooks. The chief sources of data in this study are the mathematical principles and processes used in the textbooks in the engineering courses at Louisiana Polytechnic Institute, Ruston, Louisiana. This institution offers the Bachelor of Science degrees in the fields of civil engineering, mechanical engineering, electrical engineering, and petroleum engineering. The textbooks used in these fields at this institution are standard texts and are widely used throughout other schools of engineering. The textbooks are analyzed in this study to determine the range of subject matter and the frequency with which certain principles and processes occur.

(2). The Questionnaire. A questionnaire was prepared and sent to a representative group of engineers. It was prepared after an exhaustive analysis of the content of the entire curriculum in undergraduate mathematics. The questionnaire lists ninety-three topics in undergraduate mathematics. Those who responded evaluated the importance of each of the topics in professional engineering.

The Procedure in the Analysis of the Textbooks. Each text has been covered in the order in which it was written, and a chapter by chapter examination made of the mathematics that is involved in the discussions and exercises. The results, however, are not analyzed by the chapter, but by the text as a whole. The study begins, in each respective branch of engineering, with texts on the elementary level, and proceeds to the more advanced. Two facets to the information are to be considered: (1) the Descriptive, and (2) the Quantitative. These are explained in turn, as follows:

(1) The Descriptive. Mathematics which occurs with considerable frequency, within a given text, is reproduced throughout this study, and is referred to as the descriptive mathematics of such a text. That is, material is reproduced which is recurrent in each text, thus typifying the mathematics within that book. It is, of course, out of the question to attempt to reproduce all, or even a major protion of the material.

It should be emphasized that this is not a random process, as each text was considered in its entirety before any descriptive selections were made. Then in deference to certain recurrent material, portions were taken from each book and reproduced in this study. In some instances, segments are reproduced verbatim, although generally only formulas, graphs, and various mathematical operations and computations are offered as typifying the mathematics in a specific text.

(2) The Quantitative. The quantitative value of the material is given through the medium of tables at the conclusion of each book considered. These tables were prepared after a complete examination of all the texts in each of the five respective branches of engineering. They contain a tabulation of occurrences of the various phases of algebra, trigonometry, calculus, and higher mathematics.

The same table occurs, with a slight modification, at the conclusion of each sub-chapter, that is, for each branch of engineering. Then, there is an over-all quantitative summary for the texts in all branches.

These modified tables contain not only a tabulation of occurrences of certain phases of mathematics, but also the number of books in which those phases occurred within the given branches of engineering.

By consulting these tables one can determine whether a given topic occurred frequently in many texts, or sparsely in a few; that is, the occurrences of the topic can be weighed against the number of texts in which the topic appeared.

It is difficult to prepare a table inclusive enough without, at the same time, becoming too detailed; and since over thirty books were to be reviewed, some consideration had to be given to brevity. The following means of classification are followed for the tables:

Algebra

1. Elementary processes and formulas -- This includes multiplication, division, addition, and subtraction of algebraic expressions; formulas which contain algebraic expressions; removal of parentheses, and addition and subtraction of fractions; the use of simple factors; ratio, proportion, and variation.

2. Solution of equations -- This includes the solution of all types of equations in any number of variables.

Trigonometry

1. Formulas containing trigonometry -- This includes any formula, whether applied or not, in which trigonometry occurs.

2. Use of logarithms -- This includes the laws of logarithms, as well as numerical computation.

3. Numerical solution of triangles -- This includes both the oblique and right triangles.

4. Use of radian measure -- This includes the relationship between arc length, radius, and central angle; between size of wheel, angular and linear velocity of a point on the rim.

5. Trigonometric identities -- This includes all use of such identities.

6. Use of vectors -- This includes figures in which vectors occur prominently; the resolution of vectors into components; addition and multiplication of vectors.

7. Inverse functions -- This includes every use of such functions.

8. Graphs of the trigonometric functions -- This includes the occurrence of such graphs, as well as references to them.

Differential Calculus

1. Differentiation of algebraic expressions -- This includes only the actual differentiation of such expressions.

2. Differentiation of transcendental expressions -- This includes only the actual differentiation of such expressions.

3. Applications of the derivative -- This includes slopes, maxima and minima, and so forth.

4. Partial derivatives -- This includes the process of differentiation, as well as their application.

Integral Calculus

1. Integration of algebraic expressions -- This includes not only the actual integration of such expressions, but also the indicated operation.

2. Integration of transcendental expressions -- This includes the indicated operation, as well as the actual integration of such expressions.

3. Application of the definite integral -- This includes computation of areas, volumes, moments, centroids, and fluid pressure.

4. Multiple integrals -- This includes the occurrence of such integrals, whether the actual integration is made or not.

Higher Mathematics

1. Differential equations, Fourier series, and so forth -- This includes any topic beyond the level of undergraduate calculus.

The Procedures in the Analysis of the Questionnaires.

The questionnaires were sent to over three hundred professional engineers. In order to make the sampling as representative as possible, each branch of the profession was included in almost equal numbers, and the geographic distribution was such that most states east of the Rocky Mountains were included.

The questionnaire contains ninety-three topics in undergraduate mathematics, to which the recipient was asked to respond, as follows: to mark the topic (1) if the subject is used frequently; to mark the topic (2) if the subject is used occasionally; and to mark it (3) if it is never used.

Answers to the questionnaires were tabulated and analyzed, and this analysis was compared ~~to~~ with the analysis of the textbooks.

<u>The Arrangement of the Material</u>. Chapter II contains the analyses of the texts, with sub-chapters in the following respective order: electrical engineering, chemical and petroleum engineering, civil engineering, and mechanical engineering. This order of the several fields is purely arbitrary and represents in no sense a ranking in value, although, as previously stated, the order of texts within each sub-chapter proceeds from the elementary to the more advanced.

At the beginning of each sub-chapter is a short introduction, including a list of the texts, in the order in which they are to be reviewed. At the conclusion of each sub-chapter is a summary of all the texts occurring within it.

Chapter III contains the questionnaire, with the tabulated results. Certain tables are given, and an analysis follows.

Chapter IV concludes the study with the findings summarized, and some recommendations for a limited modification of the mathematics curriculum.

CHAPTER II

A SURVEY OF THE MATHEMATICS IN THE ENGINEERING TEXTBOOKS

Part 1. A Survey of the Mathematics in the Electrical Engineering Textbooks.

Introduction. The texts considered here in Part 1 are given here in the order in which the student studies them. They are, as follows: Introduction to Electrical Engineering, R. P. Ward; Electric Machinery, M. Liwschitz-Garik and C. C. Whipple; Alternating Current Circuits, Second Edition, K. Y. Tang; Electrical Engineering, E. E. Kimberly; Electronic Engineering Principles, J. D. Ryder; Industrial Electricity, Second Edition, C. L. Dawes; Electric Circuit and Machine Experiments, Part II, F. W. Hehre and J. A. Balmford; Design of Electrical Apparatus, Third Edition, J. H. Kuhlmann; Electrical Illumination, Second Edition, J. O. Kraehenbuehl; and Alternating Current Machines, Second Edition, A. F. Puchstein and T. C. Lloyd.

The procedures are followed as explained in Chapter I. As the conclusion of each text will be found a short summary and a table. We begin with Introduction to Electrical Engineering by R. P. Ward.

Scientific notation is introduced; e.g., $\frac{1}{4} \times 10^7$. An equation of current is given as follows: $I = \frac{2}{U}\frac{SF}{L}$.

When values are given for the constants, the equation yields, $I = \frac{2\ (0.1)(4.44)}{4\ \times 10^{-7}(40)} = 236$ amp.

The following wave forms are copied from the text, to illustrate a need for certain types of curves:

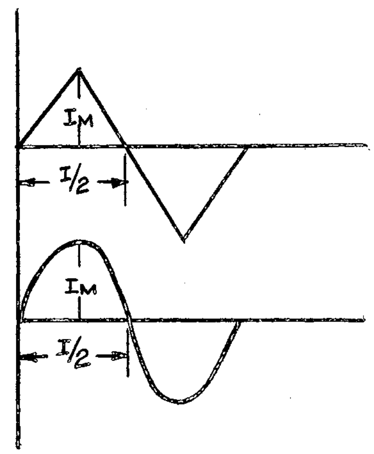


Fig. 1

Trigonometry is introduced in the early part of the text and reciprocal numbers are stressed. The trigonometry, however, is on a very elementary level. Only the definitions of the functions are used. The following illustrate:

$$
\begin{aligned}
1 &= I_m \sin 2\pi ft. \\
&= I_m \sin 2\pi \times 60 \times .001 \\
&= I_m \sin 21.6^\circ = .368 \text{ amp.}
\end{aligned}
$$

It can be seen that the student here needs to know angular measure, that is, radians to degrees, and conversely.

The mathematics continues to be very elementary, involving simple formulas, and deriving numerical results from them. The following is quite typical:

$$R_p = \frac{R_2 \; R_3}{R_2 + R_3} = \frac{20 \times 30}{20 + 30} = 12 \text{ ohms.}$$

In one instance, three linear equations involving three variables are set up and solved.

The following set of four linear equations occur:

$$23 = 26I_1 - 3I_2 + 17I_3$$
$$13 = -3I_1 + 20I_3 - 8I_4$$
$$0 = -17I_1 - 36I_2 - 13I_3$$
$$8 = -8I_1 + 13I_2 + 33I_4$$

The following formula, taken from the text, is also very typical of the formulas given in the chapter:

$$E_1 + E_3 + E_s = R_1 I_1 + R_3 I_3 + R_8 I_8 - (R_9 + R_5)\, I_5$$
$$18 + 11 + 15 = (6 \times 1.873) + (9 \times 1.358) + (12 \times 1.067) - (6 \times -1.272).$$

The mathematics in the early chapters continues to be elementary, consisting principally of formulas of the type,

$$R_x = \frac{V_1 - V_2}{V_2} R_g,$$
$$R_x = \frac{120 - 112.5}{112.5} \times 1500 = 1000 \text{ ohms.}$$

Scientific notation is used, as is the concept of proportion and variation. The following are good examples:

$$R \text{ is proportional to } \frac{1}{A},$$
$$R \text{ is proportional to } L,$$

and introducing a proportionality constant, ρ,

$$R = \frac{\rho L}{A}. \qquad (146)$$

Now given, $L = 50$ cm. $= 0.5$ m.,
$A = 2$ sq. cm. $= 2 \times 10$ sq. m. .

Substituting in (146),

$$4.31 \times 10 = \frac{(0.5)}{2 \times 10}$$
$$= 1.724 \times 10 \text{ ohm-m.}$$

The text also utilizes percentages and areas of circles. The formula, $A = \frac{\pi d^2}{4}$, for the area of a circle is given.

The student will need to know the relationship between the radius, diameter, circumference, and area of a circle. Also algebraic formulas, laws of exponents, and numerical computations are among his needs. The following is typical:

$$\frac{\mu_0 H_1 I_1 A}{L} = \frac{(4 \times 10^{-7})(398)(5)(5 \times 10^{-4})}{(25 \times 10^{-4})} = 50 \times 10^{-7}.$$

Beginning near the middle of the book, the mathematics is somewhat different. For the first time, the derivative occurs in the formulas, as does trigonometry a little more extensively than previously.

It should be pointed out, however, that neither the calculus, nor the trigonometry employed, is other than that of the most elementary type. The simple definition of the sine and cosine is given, along with the definition of a derivative and a very elementary integration. The following will illustrate:

$$\phi = 2\ xaB \cdot \sin t,$$

$$e = \lim_{\Delta t \to 0} N \frac{\Delta\phi}{t} = N \frac{d\phi}{dt} = \frac{d\left[2xaB\ \sin\omega t\right]}{dt} = 2\omega\, xaB\ \cos\omega t.$$

Consider, again, the following:

$$p = \sum_{0}^{\frac{c}{2}} \frac{2\omega^2 B^2 \cos^2\omega t \cdot abx^2 \Delta x}{} = \left[\frac{2\omega^2 B^2\ \cos^2\omega t\ abx^3}{3\rho}\right]_0^{\frac{c}{2}}.$$

Substituting the limits and putting $\omega = 2\pi f$, we get

$$p = \frac{\pi^2 f^2\ B^2\ \cos^2\omega t\ abc^3}{3\rho}.$$

The mathematics speaks for itself here, and it is evident that the student needs to know how to compute some of the more elementary definite integrals.

The following sinusoidal waves are taken from the text:

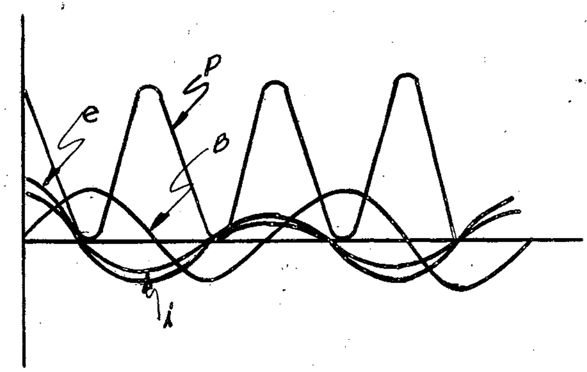


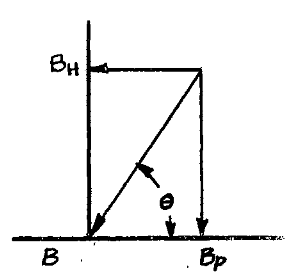
Fig. 2

These show the need for the student's mastery of terms such as period and amplitude.

The mathematics continues to stress algebraic formulas, but it also involves some trigonometry, particularly sinusoidal waves. The following formula is typical:

$$F = B^2 A = \frac{(1.395)^2(6.45 \times 10^{-4})}{4 \times 10^{-7}} = 1000 \text{ newtons.}$$

Algebraic formulas, involving exponents, are used extensively, but trigonometry and calculus also occur often. The following figure with accompanying formulas are given:



$$B_p = B \sin \theta$$

$$B_h = B \cos \theta$$

Fig. 3

The trigonometric functions of 30°, 45°, 60°, as well as the quadrantal values 0°, 90°, 180°, 270°, are used. For example:

$$e = 3.77 \times 10^{-3} \sin 0^\circ = 0.$$
$$e = 3.77 \times 10^{-3} \sin 45^\circ = 2.67 \times 10^{-3}.$$
$$e = 3.77 \times 10^{-3} \sin 90^\circ = 3.77 \times 10^{-3}.$$
$$e = 3.77 \times 10^{-3} \sin 135^\circ = 2.67 \times 10^{-3}.$$

The sine and cosine waves are constantly referred to, as is rotary motion, involving the relation between linear and angular velocity. Radian measure is prominent. The following will illustrate some of the differential calculus:

$$e = \lim_{\Delta t \to 0} - N \frac{\Delta \phi}{T} = -N \frac{d\phi}{dT} . \qquad (254)$$

Since $\phi = \phi_m \cos \theta$, (254) becomes

$$e = -N \frac{d(\phi_m \cos \theta)}{dt} ,$$

Therefore, $e = N \phi_m \sin \theta$.

The following formulas are taken from the text, as a further illustration:

$$N \frac{d\phi}{dt} = L \frac{di}{dt} ,$$

from which, $L = N \frac{d\phi}{dt}$.

The following definite integral occurs:

$$W = \int_0^I Li \, di = \frac{LI^2}{2} .$$

The calculus, especially the differential calculus, becomes slightly more complex toward the latter part of the book. The following differential equation is given and solved:

$$E - \frac{q}{c} = R\,\frac{dq}{dt}.$$

Separating the variables, we obtain

$$\frac{dq}{q - cE} = -\,\frac{dt}{Rc}.$$

Integrating, we obtain, $\log(q - cE) = -\frac{t}{Rc} + K.$

Also typical is the following:

$$i = C\,\frac{de}{dt},$$

$$e = E\sin\omega t.$$

Combining, we have,

$$i = C\,\frac{d(E\sin\omega t)}{dt} = \omega CE\cos\omega t.$$

The mathematics toward the conclusion of the text involves some very complex algebraic forms.

The following typify:

$$V_f = \sqrt{\frac{2 \times 1.59 \times 10}{8.95 \times 10}}\cdot\sqrt{E} = 5.98 \times 10\sqrt{E}\ \text{m. per sec.}$$

$$m = \frac{mr}{1 - \left(\frac{V}{C}\right)^2}.$$

$$V_f = C\sqrt{1 - \frac{1}{(1 + 1.97 \times 10^{-6}E)^2}}.$$

The text stresses algebraic formulas, complex computations, laws of exponents, sine and cosine wave curves, differential of sine and consine functions, and radian measure. The following table summarizes more completely:

Table 1. Quantitative Survey of the Mathematics in Introduction to Electrical Engineering, R. P. Ward.

	Algebra	No. of Occurrences
1.	Elementary processes and formulas.	48
2.	Solution of equations.	6
	Trigonometry	
1.	Formulas containing trigonometry.	6
2.	Use of logarithms.	1
3.	Numerical solution of triangles.	
4.	Use of radian measure.	4
5.	Trigonometric identities.	
6.	Use of vectors.	1
7.	Inverse functions.	3
8.	Graphs of trigonometric functions.	8
	Differential Calculus	
1.	Differentiation of algebraic expressions.	
2.	Differentiation of transcendental expressions.	7
3.	Applications of the derivative.	8
4.	Partial derivatives.	
	Integral Calculus	
1.	Integration of algebraic expressions.	2
2.	Integration of transcendental expressions.	4
3.	Application of the definite integral.	
4.	Multiple integrals.	

The second text in electrical engineering which is analyzed for mathematical content is Electrical Machinery, by Liwschitz-Garik and Whipple.

This textbook, which is essential to electrical engineers is replete with mathematics. All levels of mathematics are to be found including a great deal of calculus and trigonometry.

The formula, $\oint H_l \, dl = NI$, typifies a type of integral which occurs in the early part of the text. Derivatives are also used, as is shown by the following formula:

$$\oint E_l \, de = - \frac{d\phi}{dt} 10^{-8} \text{volts.}$$

The text makes extensive use of radian measure, the relation between linear and angular velocity, the sine and cosine functions, and their resulting sinusoidal waves. The following figure is taken from the text:

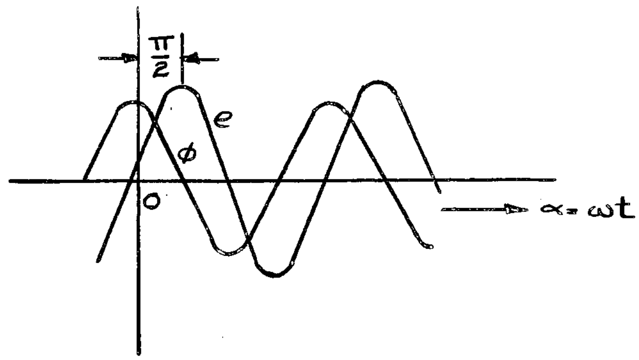


Fig. 4

The following formulas are characteristic of much of the mathematics throughout the text:

$$R\,\frac{1}{T}\int_0^T i^2 dt = R\, I_m^2 \frac{1}{T}\int_0^T \sin^2 \omega t\; dt = \frac{I_m^2 R}{2}$$

$$v = RI_m \;\; \sin \omega t + \omega L\, I_m \cos \omega t$$

$$= R \cdot I_m \; \sin \omega t + \omega L\, I_m \sin\,(\omega t + \frac{\pi}{2}).$$

$$i = C\,\frac{dv}{dt} = \omega C\; V_m \cos \omega t = \omega C\; V_m \sin\,(\omega t + \frac{\pi}{2})$$

$$V = I\sqrt{R^2 + (\;L - \frac{1}{\omega})^2} = \; IZ.$$

Inverse trigonometric functions occur, as the following formula attests:

$$= \tan^{-1} \frac{L - \frac{1}{\omega C}}{R} \; .$$

Logarithms occur in the text, and the student should know their laws. The following demonstrate:

$$\phi_d = 2M \times \frac{2.3}{0.8\pi} \log \left(1 + \frac{\pi}{2}\right).$$

$$kv = \frac{2.33\ (5 \times .218 + 0.375)}{2.33\ (5 \times 0.218 + 0.375) - (\frac{2}{3})0.375} = 1.04.$$

Some algebraic formulas are introduced here, inasmuch as they are somewhat typical of many others in the text:

$$\lambda = \left(\frac{h_1}{2b} + \frac{h_2}{3b} + \frac{2h_3}{b+b_o} + \frac{h_4}{b_o}\right)$$

$$\lambda = \left(h_1 + 3h_2 + \frac{h_1}{4S}\right).$$

Definite integrals are not infrequent. The following are characteristic:

$$e_1 = \frac{1}{r}\int_o^r e\ dx = -10^{-8}\int_o^r Bl\ dx.$$

$$s_1 = \frac{1}{r}\int_o^s e\ dx = -10^{-8}\int_o^s Bl\ dx.$$

The following typifies the numerical computation, which is not, however, very extensive in this text:

$$T = \frac{7.04}{1739}\ (221.2 \times 76) = 68\ \text{ft. - lb.},$$

$$A = \frac{\frac{1046}{6} \times 6 \times 72}{\pi \times 22.5} = 1060.$$

Although differential equations do not occupy a prominent place in this text, the following are examples:

$$e_s = L\frac{di}{dt} = L\frac{2i}{T},$$

$$e_p = R\frac{di}{dt} = R\frac{2i}{T}$$

The following differential equation is given:

$$\frac{dv}{dt} = \frac{V_m - V}{T},$$

which leads to the solution,

$$t = - T \ln (v_m - v) + C.$$

To emphasize the student's need for calculus, the following definite integrals are taken from the text:

$$\epsilon_a = \frac{1}{\pi}\int_0^{\pi} V \, d\alpha = \frac{1}{\pi}\int_0^{\pi} I_a \; x \, d\alpha = \int_0^{\pi} \frac{I_1 - I_2}{2}\left(\frac{2Yx - x^2}{2r}\right) d\alpha .$$

The text emphasizes logarithms, sine and cosine functions and their graphs, radian measure, and differential and integral calculus. The following table summarizes the frequencies more completely:

Table 2. Quantitative Survey of the Mathematics in Electrical Machinery, by Liwschitz-Garik and Whipple.

	Algebra	No. of Occurrences
1.	Elementary processes and formulas.	138
2.	Solution of equations.	
	Trigonometry	
1.	Formulas containing trigonometry.	9
2.	Use of logarithms.	9
3.	Numerical solution of triangles.	
4.	Use of radian measure.	3
5.	Trigonometric identities.	
6.	Use of vectors.	6
7.	Inverse functions.	2
8.	Graphs of trigonometric functions.	3
	Integral Calculus	
1.	Integration of algebraic expressions.	5
2.	Integration of transcendental expressions.	1
3.	Application of the definite integral.	2
4.	Multiple integrals.	
	Higher Mathematics	
1.	Differential equations, Fourier series, etc.	10

The third text in electrical engineering which is analyzed for mathematical content is Alternating Currents, by K. Y. Tang.

The mathematics in this text is quite complex. It involves differential equations, complex definite integrals, sinusoidal waves, radian measure, definitions of sine and cosine functions, and algebraic formulas of varying complexity.

The following are examples, taken from the text:

$$e_i = - N \frac{d\phi}{dt} \times 10^{-8},$$

$$P = \frac{1}{2\pi}\int_0^{2\pi} p \, d\alpha = \frac{EI}{2\pi}\int_0^{2\pi}\left[\cos\theta - \cos\,(2\alpha + \theta)\right] d\alpha$$

$$= \frac{EI}{2\pi}\int_0^{2\pi}\left[\cos\theta - \cos 2\alpha \cos\theta + \sin 2\alpha \sin\theta\right] d\alpha$$

$$= EI \cos\theta .$$

$$S = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\alpha \, d\alpha .$$

The following is one of the many graphs taken from the text:

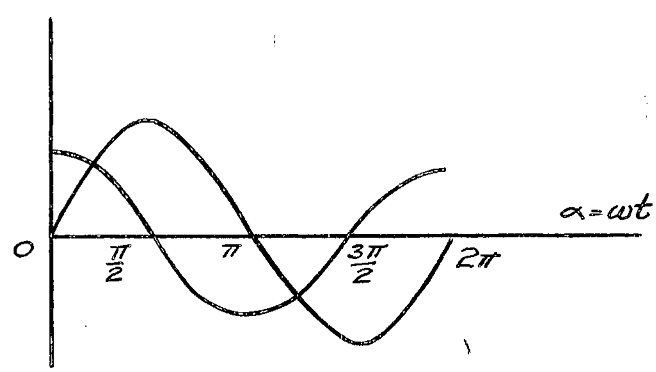


Fig. 5

This text makes such extensive use of sine and cosine waves that the student must have a mastery of them before proceeding far into the book. Areas under curves, and differential equations are also frequent. The following is typical of many formulas throughout the text:

$$i = \frac{dq}{dt} = c\,\frac{de}{dt} \text{ amperes},$$

$$e_i = -\left(N\,\frac{d\phi}{dt} \times 10^{-8}\right)\frac{di}{dt}$$

$$= -N\,\frac{d\phi}{dt} \times 10^{-8}.$$

$$W = N \times 10^{-8}\int_{\phi_1}^{\phi_2} i\,d\phi,$$

$$e_c = \frac{1}{c}\int_0^t i\,dt = \frac{q}{c}.$$

Sinusoidal waves occur quite frequently, and terms like amplitude and period should be thoroughly familiar to the student; radian measure should also be a topic familiar to him.

The integral calculus is also quite extensive, with particular emphasis on such forms as $\int \sin \omega t\,dt$, $\int \cos \omega t\,dt$, $\int \sin^2 \omega t\,dt$, $\int \cos^2 \omega t\,dt$.

The following figure is taken from the text:

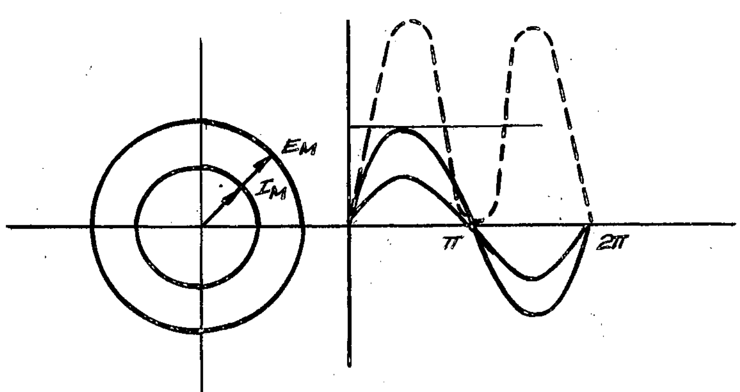


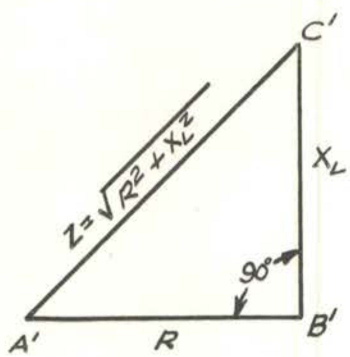
Fig. 6

The following is taken from the text:

$$W = \int_o^t p \quad dt = \int_o^t RI_m^2 \quad \sin^2\omega t \quad dt$$

$$= RI_m^2 \int_o^t \frac{1 - \text{Cox } 2\omega t}{2} \, dt = RI^2T \text{ watt-seconds.}$$

This text is also replete with vectors and their resolution into horizontal and vertical components. Hence, the definitions of the trigonometric functions, and the Theorem of Pythagorus should be well known by the student. The following figure, and formulas will illustrate:



$$\text{TAN} = \frac{X_L}{R}$$

$$\text{COS} = \frac{R}{Z}$$

$$\text{SIN} = \frac{X_L}{Z}$$

Fig. 7

The treatment of vectors is quite comprehensive. It would benefit the student, greatly, if at this point, he knew the <u>symbolic</u> <u>method</u> of representing complex numbers such as a + bi. He should also know how to add, subtract, multiply, and divide two such numbers, and understand the vectorial connotations involved. He should know the full definition of i, as given by, $i^2 = -1$, which in engineering assumes the symbol j. He should also be able to compute readily powers of the symbol, such as j^2, j^3, j^8, and so forth.

The following figure, demonstrating vectorial addition, is taken from the text:

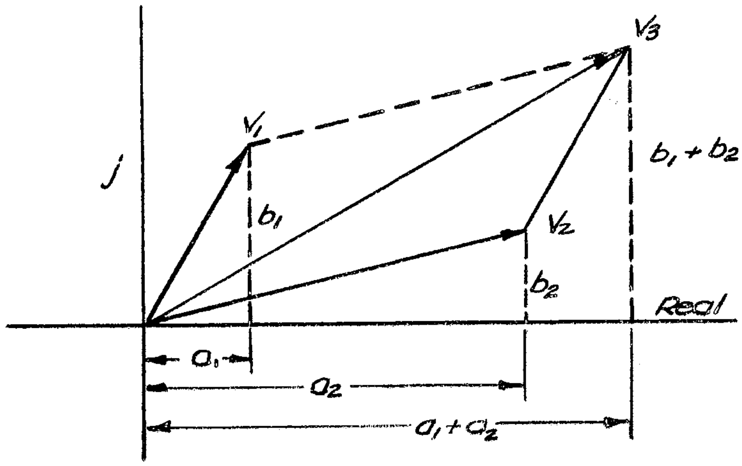


Fig. 8

The sine and cosine functions for angles of 30°, 45°, and 60°, are used in this chapter, as are the same functions for the quadrantal angles 0°, 90°, 180°, and so forth.

The Maclaurin expansion for $\sin\theta$, $\cos\theta$, and e^{θ} is given and applied. That is,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \frac{\theta^7}{7!} + \ldots,$$

and so forth.

The inverse trigonometric functions occur frequently throughout the text. The mathematics even includes such advanced topics as Fourier series. All phases of mathematics occur and the following table summarizes:

Table 3. Quantitative Survey of the Mathematics in <u>Alternating Currents</u>, by K. Y. Tang.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	256

Table 3. -- Continued

Algebra	No. of Occurrences
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	112
2. Use of logarithms.	
3. Numerical solution of triangles.	
4. Use of radian measure.	14
5. Trigonometric identities.	6
6. Use of vectors.	110
7. Inverse functions.	52
8. Graphs of trigonometric functions.	60
Differential Calculus	
1. Differentiation of algebraic expressions.	
2. Differentiation of transcendental expressions.	6
3. Application of the derivative.	
4. Partial derivatives.	
Integral Calculus	
1. Integration of algebraic expressions.	12
2. Integration of transcendental expressions.	44
3. Application of the definite integral.	4
4. Multiple integrals.	
Higher Mathematics	
1. Differential equations, Fourier series, etc.	50

The fourth text in electrical engineering which is analyzed for mathematical content is Electrical Engineering, by E. E. Kimberly.

This textbook is replete with mathematics on all levels. There is a great deal of algebra to be found, particularly algebraic formulas. As would be expected, trigonometry occurs with considerable frequency, especially radian measure, sine and cosine functions, and sinusoidal waves. Calculus occurs through all levels: ordinary differentials, definite integrals, and differential equations.

The following is characteristic of some of the purely algebraic formulas in the text:

$$P = \frac{\frac{1}{10} \times 10^8}{1}$$

$$= 10^7 \text{ dyne - cm per sec.}$$

The laws of exponents speak for themselves, in the foregoing. Again, from the text:

$$I = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right).$$

$$I = \frac{E}{R^2 + X_c^2}.$$

The following algebraic formulas are given, with the numerical computations following:

$$X_c = \frac{1}{2\pi f c}$$

$$= \frac{1}{2\pi \times 60 \times .00005}$$

$$= 53.2 \text{ ohms.}$$

$$I = \frac{E}{\sqrt{R^2 + X_c^2}}$$

$$= \frac{220}{\sqrt{30^2 + 53.2^2}}$$

$$= 3.6 \text{ amp.}$$

The text makes use of the imaginary unit, i, and employs complex numbers, both graphically and algebraically. Thus the product $(R + jx)(R - jx) = R^2 + x^2$ is used, where j, which is a standard symbol in electrical problems, has been substituted for i.

Sinusoidal waves occur frequently throughout the text, of which the following is a rather typical example:

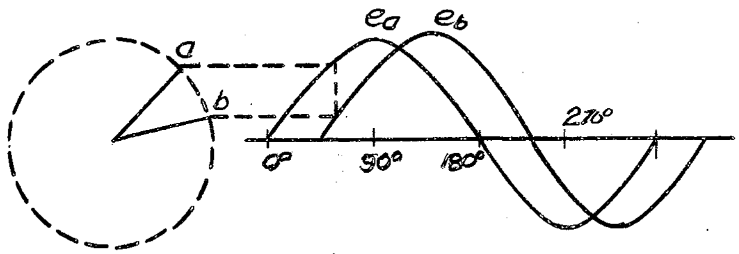


Fig. 9

Trigonometry occurs frequently, especially functions involving the sine and cosine. The following figure, followed by a trigonometric formula, and numerical computations, is a very good illustration:

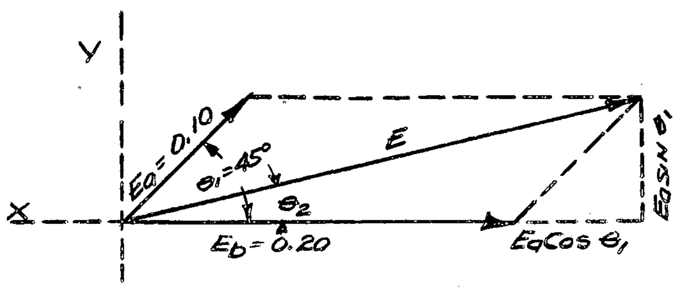


Fig. 10

$$E = \sqrt{(E_b + E_a \text{ Cos } \theta_1)^2 + (E_a \sin \theta_1)^2}$$

$$= \sqrt{(0.20 + 0.10 \cos 45°)^2 + (0.10 \sin 45°)^2}$$

$$= 0.28 \text{ volts.}$$

$$\theta_2 = \tan^{-1}\frac{E_a \sin \theta_1}{E_b + E_a \cos \theta_1} = \tan^{-1}\frac{0.10 \sin 45°}{0.20 + 0.10 \cos 45°} = 14° \ 38'$$

The student makes use of the functions of the quadrantal angles, as illustrated by the following formulas from the text:

$$E_1 = E_1 \cos 180^\circ + j \sin 180^\circ.$$

$$E_2 = E_2 \cos 60^\circ + j \sin 60^\circ.$$

Calculus occurs on many levels, the following being a typical example of differential calculus:

$$e = \frac{N}{10^8} \frac{d\phi}{dt} \quad .$$

The following is another good example, as it shows the process of differentiation, and its application:

$$- e_g = L \frac{di}{dt},$$

and if $i = I_m \sin \omega t$,

$$- e_g = L \frac{d\,(I_m \sin \omega t)}{dt}$$

$$= L \omega I_m \cos \omega t.$$

The following differential equations are taken from the text:

$$N \frac{d\phi}{dt} 10^{-8} = L \frac{di}{dt} ,$$

$$dW = i^2 R \, dt + L \frac{di}{dt} i \, dt.$$

Definite integrals occur occasionally, as do indefinite integrals. The following is a good example from the text, in that it shows the student's needs, both for trigonometry, and the calculus:

$$P = \frac{E_m \int_0^{2\pi} I_m \sin \omega t \sin (\omega t - e) \, d(\omega t)}{2}$$

$$= \frac{E_m I_m}{2} \int_0^{2\pi} \sin\omega t \ \sin\ (\omega t - e)\ d(\omega t)$$

$$= \frac{\sqrt{2}\ I_e \sqrt{2}\ E_e}{2} \int_0^{2\pi} (\sin\omega t\ \cos\theta - \cos\omega t\ \sin\theta)\ \sin\omega t\ d(\omega t)$$

$$= \frac{EI}{\pi} \int_0^{2\pi} (\sin^2\omega t\ \cos\theta - \sin\omega t\ \cos\omega t\ \sin\theta)\ d(\omega t)$$

$$= \frac{EI}{\pi} \left[\cos\theta \left(\frac{\sin\omega t\ \cos\omega t}{2} + \frac{\omega t}{2} \right) + \frac{\sin\theta\ \ \cos 2\omega t}{4} \right]_0^{2\pi}$$

$$= \frac{EI}{\pi}\ (\pi \cos\theta\ )$$

$$= EI\ \cos\theta\ .$$

The following is another good example of the use of integral calculus:

$$W_L = \int_0^I L\ \frac{di}{dt}\ i\ dt$$

$$= \frac{1}{2}\ LI^2 \quad \text{watt - seconds.}$$

The mathematics may be summarized by stating that considerable emphasis is given to trigonometry and calculus. Sinusoidal waves, sine and cosine functions and their numerical values for 30°, 60°, 45°, 90°, and 180° should be known by the student. The differential and integral forms for sine and cosine are frequent.

The following table summarizes the frequencies more completely:

Table 4. Quantitative Survey of the Mathematics in Electrical Engineering, by E. E. Kimberly.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	32
2. Solution of equations.	4

Table 4. -- Continued

Trigonometry	No. of Occurrences
1. Formulas containing trigonometry.	15
2. Use of logarithms.	
3. Numerical solution of triangles.	3
4. Use of radian measure.	2
5. Trigonometric identities.	3
6. Use of vectors.	40
7. Inverse functions.	11
8. Graphs of trigonometric functions.	15
Integral Calculus	
1. Integration of algebraic expressions.	3
2. Integration of transcendental expressions.	5
3. Applications of the definite integral.	
4. Multiple integrals.	
Higher Mathematics	
1. Differential equations, Fourier series, etc.	5

The fifth text in electrical engineering which is analyzed for mathematical content is <u>Electronic Engineering Principles</u>, by J. D. Ryder.

This textbook involves mathematics, perhaps as complex as any to appear in any undergraduate course in engineering. Complex algebraic equations occur, in which radicals and fractional exponents are common. The trigonometry stresses sinusoidal waves, inverse functions, radian measure, and logarithms, both to base e and base 10. The calculus is quite complex, even exceeding that acquired in the undergraduate courses of the subject. The text is replete with differential equations, of which many are quite complex. It is certainly obvious that only a student who is mature in mathematical techniques can read this textbook.

At the very outset, the book begins with topics ordinarily offered in the second year of calculus. The relation between force, acceleration, velocity, energy, and the like, are set forth early in the text. This, of course, is ordinarily offered in undergraduate calculus. Newton's second law of motion is stated mathematically as,

$$f = \frac{d(mV)}{dt},$$

anf if the mass of the body is a constant,

$$f = m\,\frac{dv}{dt} = ma.$$

If the acceleration is known, the instantaneous velocity may be determined by,

$$dv = a\,dt,$$

or $V = \int a\,dt + V_0$; $V = V_0$, if $t = 0$. (2 - 2).

Since velocity is the rate of change of displacement, s,

$$ds = V\,dt,$$

and $S = \iint a\,dt\,dt + \iint V_0\,dt + S_0$; $S = S_0$, if $t = 0$. (2 - 3).

Equation (2 - 2) and (2 - 3) reduce to

$$V = at + v_0 , \qquad (2 - 4)$$

and

$$S = \frac{at^2}{2} + V_0 t + S_0 . \qquad (2 - 5)$$

The foregoing are given here, as illustrative of the complexity with which this text begins. It reveals how a mastery of this topic in the calculus will aid the engineering student. By definition of a radian,

$$\theta = \frac{V dt}{r},$$

and $$\omega = \frac{d\theta}{dt} = \frac{V}{r}, \qquad (2 - 10)$$

where, ω, is the angular velocity, in radians per second.

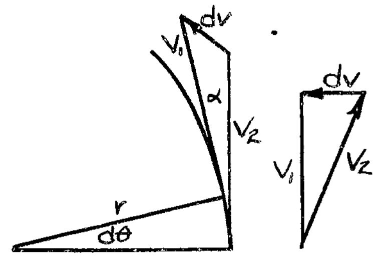


Fig. 11

Now, referring to the figure, it is seen that if $d\theta$ becomes very small, the angle α approaches $d\theta$; and dv, the change in velocity, is perpendicular to a line bisecting $d\theta$. Therefore,

$$dV = V d\theta$$

and dividing by dt,

$$\frac{dV}{dt} = a = V \frac{d\theta}{dt}, \qquad (2 - 11)$$

where a is the center-directed acceleration.

Using equation (2 - 10),

$$a = \frac{v^2}{r} = r m \omega^2/\text{sec}. \qquad (2 - 12)$$

Again, it is evident that the student's background in the calculus is essential for the reading of this book. The relations between linear and angular velocity, arc length, radian measure, and the like, are essential.

The following differential equation appears in the text, and the following solution given:

$$\frac{dx}{dt} = \sqrt{\frac{2e}{m} E_b} \left(\frac{x}{d}\right)^{2/3},$$

Separation of variables yields,

$$d^{2/3}\int_0^d \frac{dx}{x^{2/3}} = \int_0^t \frac{2eE_b}{m}\, dt ,$$

or

$$d = \frac{\sqrt{\frac{2eE_b}{m}}}{3}\, t.$$

The following equations are taken from the text, as further examples of how complex the mathematics is.

$$\frac{d^2r}{dt^2} = \left[\frac{d^2r}{dt^2} - r\left(\frac{dr}{dt}\right)^2 + j\,\frac{i}{r}\,\frac{d}{dt}\left(r^2\,\frac{d\theta}{dt}\right)\right] \cdot (\cos\theta + j\sin\theta),$$

$$f\theta = ma\theta = \frac{m}{r}\frac{d}{dt}\left(r^2\,\frac{d\theta}{dt}\right) = Be\,\frac{dr}{dt},$$

$$I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} L_b^2\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}\left(\frac{E}{r_p + R}\right)^2 \sin^2\omega t\right] d(\omega t) + \frac{1}{2\pi}\left[\int_{\pi}^{2\pi} Qd(\omega t)\right]}$$

On a more elementary level, the following equations show the need for the laws of logarithms:

$$I = k\,E^a, \tag{5 - 16}$$

Rewriting equation (5 - 16),

$$\log I = \log K + a \log E. \tag{5 - 17}$$

Also, on an elementary level, the following equations show the need for the laws of exponents:

$$\frac{V^{3/2}}{x^2} = \frac{E_b^{3/2}}{d^2}, \tag{5 - 18}$$

Solving for V,

$$V = E\left(\frac{x}{d}\right)^{4/3} \tag{5 - 19}$$

The equation, $j = -kV\sin\omega t + k\epsilon_0\frac{V}{d}\cos\omega t$, taken from the text, shows a two-fold need. The student should have a comprehension of the sine and cosine functions, in order to analyze the equation. Also, occurring here is the traditional symbol for angular velocity, as is t, for time. Thus, the student needs to know radian measure, and the relation between linear and angular velocity.

The following sinusoidal waves are taken from the text:

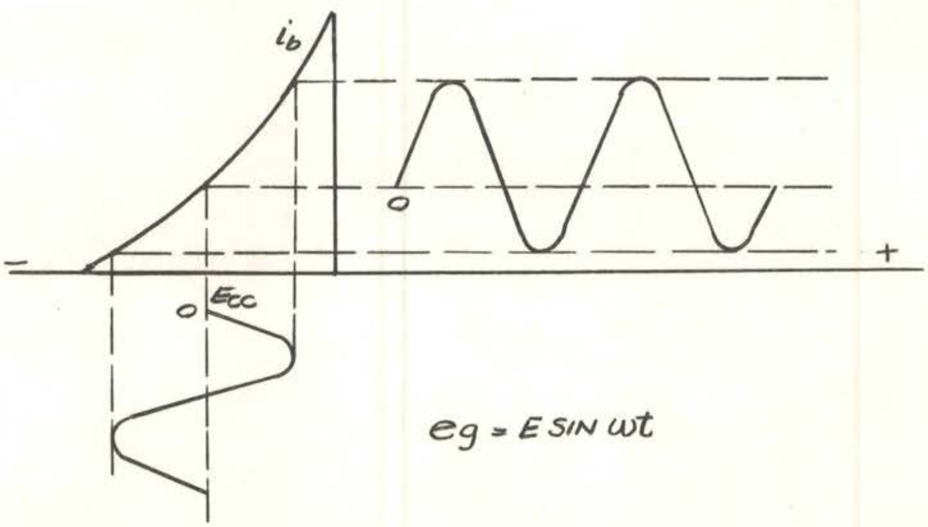


Fig. 12

Improper integrals occur in the text. The following is an example:

$$V = \int_0^r f\,dr = \int_0^r \frac{-e}{4\pi\epsilon_0 r^2}\,dr = \frac{e}{4\pi\epsilon_0 r}\ .$$

The following simple equations show the manner in which inverse trigonometric functions are used:

$$\omega t^2 = \tan^{-1}\omega RC;\ \alpha_1 = \sin^{-1}\frac{E_o}{E_m}\ .$$

In summary, it may be stated unequivocably that this text is among the more complex that the student will study. It stresses algebra of a very complicated nature.

Both the differential and integral calculus are in abundance, and are rather complicated. Radian measure and logarithms are also used extensively. The table below summarizes the frequencies more completely.

Table 5. Quantitative Survey of the Mathematics in Electronic Engineering Principles, by J. D. Ryder.

	Algebra	No. of Occurrences
1.	Elementary processes and formulas.	105
2.	Solution of equations.	
	Trigonometry	
1.	Formulas containing trigonometry.	74
2.	Use of logarithms.	20
3.	Numerical solution of triangles.	
4.	Use of radian measure.	5
5.	Trigonometric identities.	4
6.	Use of vectors.	12
7.	Inverse functions.	14
8.	Graphs of trigonometric functions.	40
	Differential Calculus	
1.	Differentiation of algebraic expressions.	16
2.	Differentiation of transcendental expressions.	4
3.	Applications of the derivative.	8
4.	Partial derivatives.	8
	Integral Calculus	
1.	Integration of algebraic expressions.	54
2.	Integration of transcendental expressions.	18
3.	Applications of the definite integral.	
4.	Multiple integrals.	
	Higher Mathematics	
1.	Differential equations, Fourier series, etc.	10

The sixth text in electrical engineering which is analyzed for mathematical content is Industrial Electricity, by C. L. Dawes.

This text does not contain any very intricate mathematics. There is no calculus at all, though trigonometry is extensive.

The author gives recognition to the preponderant quantity of trigonometry, as the titles to the appendix attests. They are as follows: Circular Measure -- The Radian; Trigonometry -- Simple Functions; Functions of Angles Greater than 90°; Graphical Representation of Trigonometric Functions; Trigometric Formulas.

The author, however, does not rely on the student's background entirely, as the following from the text will show:

Example: Find the sine of $\frac{4\pi}{3}$ radians.

$$2\pi \text{ radians} = 360^\circ$$
$$4\pi \text{ radians} = 720^\circ$$
$$\frac{4\pi}{3} \text{ radians} = 240^\circ$$
$$\sin 240^\circ = -\sin 60^\circ = -0.866. \text{ Ans.}$$

The author, throughout a large portion of the text, attempts to aid the student by analyzing the mathematics for him, which is quite unusual for textbooks in engineering. In discussing sine curves, for instance, the author gives the explanations with his figures. The following figure shows how much aid the student is given in understanding the addition of sine curves.

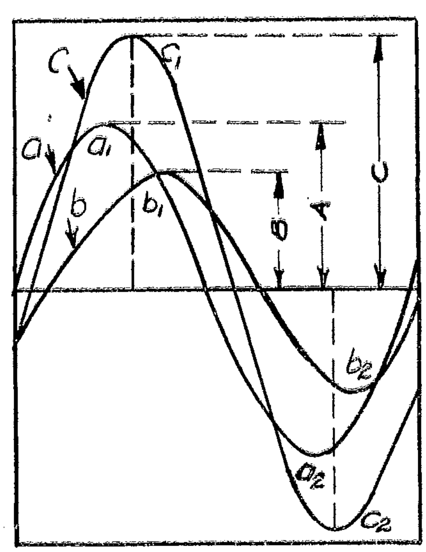
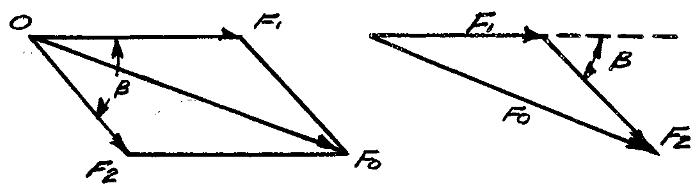


Fig. 13

Vectors are used quite often, which give rise to solutions of triangles. The following figures show vector addition by parallelogram and triangle method:



(a) (b)

Fig. 14

The equation, $i = I_m \sin\omega t = I_m \sin\pi$ ft., reveals the student's need for radian measure. With ω, in the foregoing, as the usual symbol for angular velocity, and f as frequency, it is apparent that radian measure, degrees to radians, and the relation between angular and linear velocity is one of the real needs of the student.

The following equation, from the text, is recognizable as a statement of the law of cosines:

$$E_z^2 = E^2 + E_r^2 - 2\ EE_r\ \cos\theta\ . \tag{40}$$

Solving (40) for cos θ,

$$\cos\theta = \frac{E^2 + E_r^2 - E_z^2}{2EE_r}\ . \tag{41}$$

There are, of course, some numerical computations in the text, which are generally not very complex. A few typical examples are given here.

$$X_c = \frac{1}{2\pi\, 60 \times 0.000025} = 106 \text{ ohms.}$$

$$Z = \sqrt{50^2 + (56.6 - 106)^2} = \sqrt{50^2 + (-49.4)^4}$$

$$= 2500 + 2440 = 70.2 \text{ ohms.}$$

Right triangles are solved. The Theorem of Pythagorus is employed, as are the trigonometric functions, sine, cosine, and tangent. The other functions are never employed. The following equations show the use of these trigonometric functions:

$$\cos\theta = \frac{R}{\sqrt{R^2 + (X_L - X_c)^2}} = \frac{R}{Z},$$

$$\tan\theta = \frac{IX_L - IX_c}{IR} = \frac{X_L - X_c}{Z}.$$

There is some employment of the concept of proportion, and some of the laws relative to ratio and proportion are used. For example, the following is taken from the text:

$$N_1 I_1 = N_2 I_2,$$

therefore,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1},$$

also,

$$V_1 I_1 = V_2 I_2,$$

hence,

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

In order to read this book, the student needs a basic understanding of logarithms, their definition, their laws of operation, and the meaning of the term, base. The following equation is given in the text:

$$i = AT^2 \epsilon^{-b/t},$$

where ϵ is the natural logarithmic base.

The Pythagorean relation quite frequently gives rise to some computations which the student should be able to make. The following are examples:

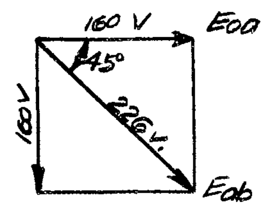


Fig. 15

$$E_{ab} = \sqrt{E_{ao}^2 + E_{ob}^2} + \sqrt{160^2 + 160^2} = 226 \text{ volts.}$$

The following graphical method of constructing the sine curve is taken directly from the text. It is given here, since it shows in such graphic form how the trigonometry teacher can directly aid the engineering student.

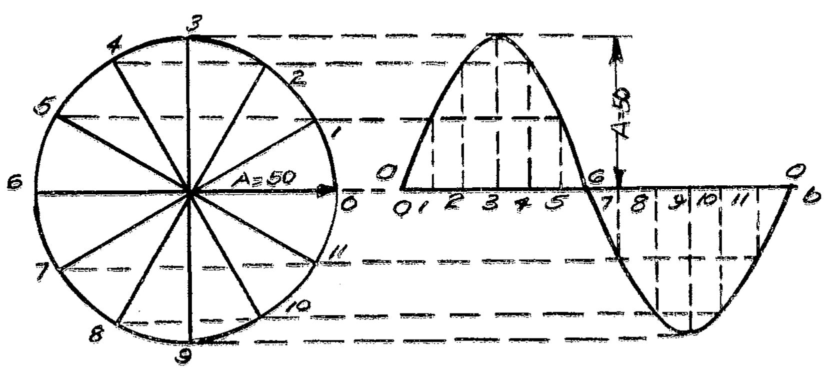


Fig. 16

In summary, it can be stated that the mathematics within this text is less complicated than in most texts. Only algebra and trigonometry occur, with some emphasis on the use of the sine and cosine curves, and on vectors. Radian measure is also a topic which occurs, though not very prominently. The following table summarizes the frequencies more completely:

Table 6. Quantitative Survey of the Mathematics in Industrial Electricity, by C. L. Dawes.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	42
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	32
2. Use of logarithms.	3
3. Numerical solution of triangles.	4
4. Use of radian measure.	4
5. Trigonometric identities.	
6. Use of vectors.	34
7. Inverse functions.	
8. Graphs of trigonometric functions.	40

The seventh text in electrical engineering which is analyzed for mathematical content is Electric Circuit and Machine Experiments, by F. W. Hehre and J. A. Balmford.

Although this is a laboratory manual, it contains a good sampling of the mathematics to be found in most of the electrical engineering texts. Mathematics on nearly every level is to be found; algebra, trigonometry, graphs, vectors, and differential equations all occur.

Quite a number of purely algebraic formulas are present, of which the following, from the text, are typical:

$$X = \frac{1}{I - \frac{E}{R_v}},$$

$$KD_1(R_g + R_s) = KD_2(R_g + R_s + X),$$

$$X = \frac{(D_1 - D_2)(R_g + R_s)}{D_2}$$

$$I_2 = \frac{E_2}{R_2^2 + X_2^2} \; ,$$

$$T = \frac{K \phi_s E_2 R_2}{R_2^2 + S^2 X_2^2} \; .$$

The student is expected to supply values for the unknowns in the above formulas, and compute desired numbers.

This text contains many trigonometric expressions, a few graphs, and some figures involving vectors and sinusoidal waves. The following figures, with the accompanying formulas are taken from the text:

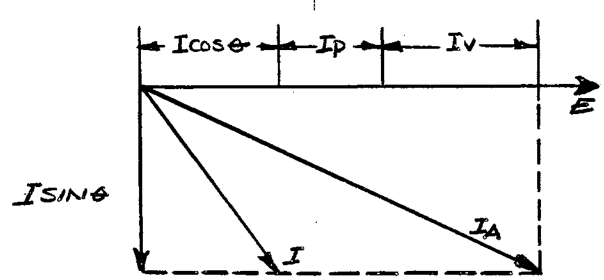


Fig. 17

$$I \cos\phi = \frac{P}{E},$$

$$I \sin\phi = \sqrt{I_A^2 - (I\cos\phi + I_v + I_p)^2} \; ,$$

$$I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2} \; .$$

In a number of places throughout the text there are formulas in which trigonometric functions occur. The following are quite characteristic of such formulas:

$$W_A = E\, I_A \cos(30^\circ + \phi) \; ,$$

$$W_c = E\, I_c \cos(30^\circ - \phi) \; .$$

It is apparent, from the foregoing, that the student will need to know the expansion formulas for cos (A + B), and cos (A - B).

The following are other examples, somewhat typical, of formulas in which trigonometry occurs:

$$T = K\omega\, I_2 \cos \theta_2,$$

$$P = E\, I \cos\omega,$$

Typical formulas in which differential equations, or calculus occurs, are reproduced here, as follows:

$$L \frac{di}{dt} = N \frac{d\theta}{dt} ,$$

$$E_p = N_p \frac{d\theta}{dt} + I_o\, R_p .$$

In summary, the text stresses algebraic formulas, trigonometric functions, especially sines and cosines; differential equations occur, although not prominently. The following table gives a more careful summary of the mathematical frequencies.

Table 7. Quantitative Survey of the Mathematics in _Electric Circuits_ and Machine Experiments, by F. W. Hehre and J. A. Balmford

	Algebra	No. of Occurrences
1.	Elementary processes and formulas.	30
2.	Solution of equations.	
	Trigonometry	
1.	Formulas containing trigonometry.	14
2.	Use of logarithms.	
3.	Numerical solution of triangles.	
4.	Use of radian measure.	
5.	Trigonometric identities.	2
6.	Use of vectors.	11
7.	Inverse functions.	
8.	Graphs of trigonometric functions.	
	Higher Mathematics	
1.	Differential equations, Fourier series, etc.	2

The eighth text in electrical engineering which is analyzed for mathematical content is Design of Electrical Apparatus, by J. H. Kuhlmann.

This textbook is a little unique among all the books in electrical engineering. It does not stress sinusoidal waves, nor radian measure, as do most of the other texts in this branch of engineering. Only a small amount of the mathematics involves calculus, most of it containing only algebra.

It does not mean, however, that the mathematics in the text is not complex. The author leaves all of the mathematics to the student, and complex algebraic formulas and computations are prolific. It is obvious that the author presumes a high degree of mathematical maturity on the part of the student. For example, he does not explain to the student how he obtains a step from the previous one. As a further example of the complexity of this text, the formulas numbered in the book do not exceed formula (230). A comparison with comparable texts reveal that numbered formulas are frequently twice that great. In Alternating Current Machines, by Puchstein and Lloyd, a book comparable to this one in content and size, the formulas number up to formula (479).

It is, in part, the writing and construction of this text that make it difficult, and not entirely the material in it. For example, the following problem is taken from the text, precisely as it occurs.

[The reactance voltage per coil is calculated by formula 101. The data required for this formula are:

$ATP_a = 5440$, $p = 6$, $t_a = 1.0$, $n_s = 15$, $m = 2$, $b = 2.52$,
$A = b + 1 - \frac{a}{p} = 2.42 + 1 - \frac{6}{6} = 2.42$, $l = 1.0$,
$l = 10.5$, $d_s = 1.56$, $w_s = 0.31$, $l_i = 7.0$, $t_1 = 0.97$,
$w_i = 1.75$, $s = 0.25$, $l = L - l = 27.1 - 10.5 = 16.6$
$4.25\, l \frac{d_s}{w_s} = 4.25 \times 10.5 \frac{1.56}{0.31} = 224$
$9.35(l - li) \log \frac{2t - w_s}{w_s} = 9.35(10.5 - 7) \log_{10} \frac{2 \times 0.97 - 0.31}{0.31} = 23.6$
$2.03\, li \frac{w - w}{s} = 2.03 \times 7 \frac{1.75 - 0.31}{0.25} = 81.9$
$8.12\, l_s = 8.12 \times 16.6 = 135$.][1]

The foregoing problem is worked, as we see, without any identifying numbered formulas or without explanations on the part of the author. Nothing but numbers and algebraic symbols meets the eye of the student. The numbers obtained in the problem, 224, 23.6, 81.9, 135, 1.708, are not labeled, though they are later used in a formula.

This type of problem occurs, again and again, in the text, making it one of the most difficult encountered.

Although calculus is not prominent, the following example is taken from the text:

---

1. J. H. Kuhlmann, Design of Electrical Apparatus, Third Edition (New York, 1950), p. 117.

$$\phi = 4 \times 3.2 \times 2 \int_0^{B/2} \frac{h\,dx}{d_1 + (x\pi)}$$

$$= 25.6 \frac{h_s}{\pi} \log_e \frac{d_1 + \frac{\pi B}{2}}{d_1}$$

$$= 19\ h_s\ \log_{10} (1 + \frac{\pi B}{2d_1}) .$$

The student will find only three basic needs in these foregoing examples: First, he will have to know the basic integration for $\int \frac{dv}{v}$. Secondly, he will have to know how to change logarithmic values, from one base to another. Finally, some algebraic skill will be necessary in substituting the limits, and arriving at the final form.

In a few places, differential equations occur, although there is no solution given for them. The following are examples:

$$e_r = (L + M) \frac{di}{dt} ,$$

$$\frac{di}{dt} = \frac{2i_a}{t} ,$$

$$\frac{di}{dt} = \frac{m}{A + m - 1} K \cdot n_s (2\ I_a) .$$

The symbols, $>$, $<$, $\leqq$, occur with enough frequency, in texts of this type, to merit some attention. The following are taken from the text:

$$w_i \leqq \frac{1 - \ell_t}{2} ,$$

$$w_i \geqq 1.5\ t_1 .$$

Although trigonometry does not play a dominant part in this text, it occurs on an elementary level in quite a few places. The following, from the text, is typical:

$$\sin\alpha = \frac{d}{t_1} = \frac{0.297}{0.766} = 0.518,$$

$$= 31.2^{\circ}, \text{ and } \cos \; = 0.855, \; \tan \; = 0.605.$$

$$k_d \; k_p = \frac{2 \sin 45^{\circ} - 2 \sin 75^{\circ}}{4} = 0.836,$$

and $$k = \sin\left(\frac{4}{6} \times 90^{\circ}\right) = 0.966.$$

In the following, the trigonometric identity, $\frac{\sin\theta}{\cos\theta} = \tan\theta$, is employed:

$$S = C \sin\alpha + b + d_s = \frac{1}{2} \frac{\pi (D + d_s)}{\cos\alpha} P \sin\alpha + b + d_s$$

$$= \frac{\pi (D + d_s)}{2p} P \tan\alpha + b + d_s \text{ in.}$$

There are some formulas in which trigonometric functions occur, but in which no numerical examples are worked. The following, are typical.

$$K_a = \frac{\pi fd + \sin \pi fd}{4 \sin fd \frac{\pi}{2}}$$

$$A_1 = \frac{4\nu + 1}{5} - \frac{\sin \nu\pi}{\pi}$$

The following, somewhat complex figure is taken from the text, and trigonometric equations derived from them:

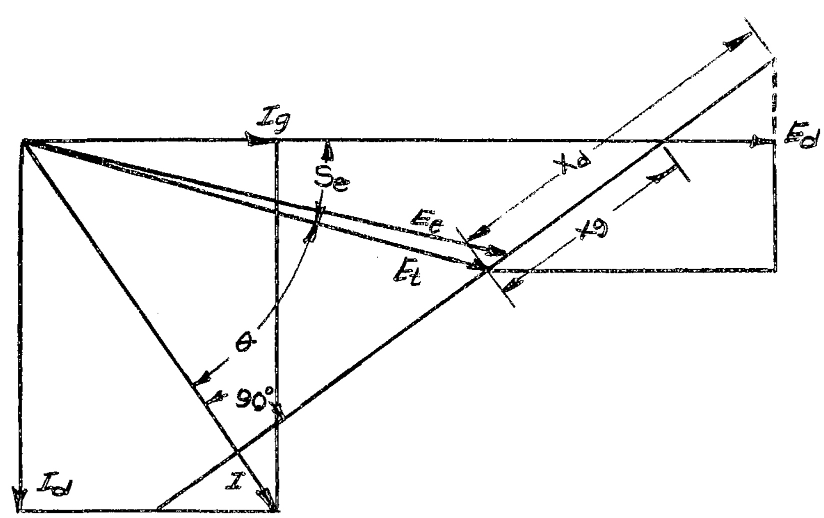


Fig. 18

From the figure, the following equation is derived:

$$\delta_e = \sin^{-1} \frac{X_q \cos}{\sqrt{1 + X_q^2 + 2 X_q \sin\theta}}$$

Another formula involving trigonometry is given here, together with a numerical application:

$$P = \cos\theta\, P_r + \sin\theta\, P_x + \frac{(\cos\theta\, P_x - \sin\theta\, P_r)^2}{200}$$

$$= 0.80 \times 0.844 + 0.60 \times 4.37$$

$$+ \frac{(0.80 \times 4.37 - 0.60 \times 0.844)^2}{200}$$

$$= 3.376$$

Although the text is written in such manner that the mathematics is difficult to follow, only once does the author bring in a topic beyond that given in undergraduate courses.

Fourier series and wave analysis occurs in one place in the text. The following figure, and equations demonstrate the nature of the topic.

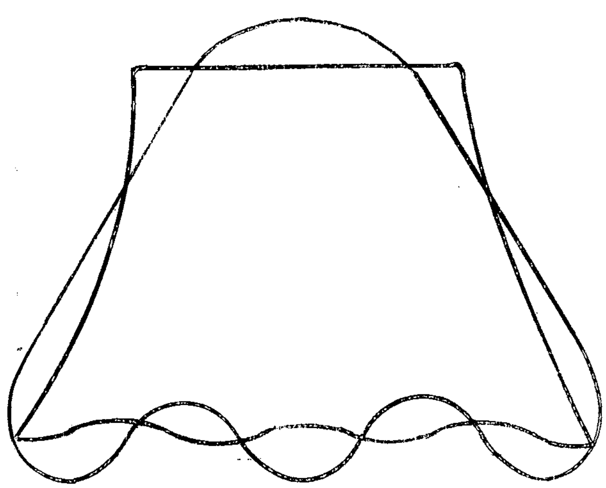


Fig. 19

$$B_x = B_1 \sin x + B_3 \sin 3x + B_5 \sin 5x + B_7 \sin 7x$$

$$B_a = \frac{1}{\pi}\int_0^{\pi} B_x \, dx = \frac{2}{\pi}\left(B_1 + \frac{1}{3} B_3 + \frac{1}{5} B_5 + \frac{1}{7} B_7\right)$$

In summary, it can be stated that in this, as in many of the other texts, algebra is predominant, although trigonometry occurs rather prominently. Of more than usual interest is the occurrence of Fourier series and wave analysis. Most of the trigonometry is only by way of using formulas, and there is not a great amount of the more intricate operations of trigonometry. The following table summarizes the mathematical frequencies more completely:

Table 8. Quantitative Survey of the Mathematics in <u>Design of Electrical Apparatus</u>, by J. H. Kuhlmann

Algebra	No. of Occurrences
1. Elementary processes and formulas.	280
2. Solution of equations.	3
Trigonometry	
1. Formulas containing trigonometry.	65
2. Use of logarithms.	9
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	3
7. Inverse functions.	2
8. Graphs of trigonometric functions.	2
Integral Calculus	
1. Integration of algebraic expressions.	2
2. Integration of transcendental expressions.	3
3. Application of the definite integral.	
4. Multiple integrals.	
Higher Mathematics	
1. Differential equations, Fourier series, etc.	7

The ninth text in electrical engineering which is analyzed for mathematical content is <u>Electrical Illumination</u>, by J. O. Kraehenbuehl.

This is one of the few textbooks in electrical engineering in which little mathematics occurs. Nothing beyond trigonometry is to be found, no calculus of any description.

The following are some examples of algebraic formulas that are taken from the text:

$$OP = D^2 + L^2 ,$$

$$I = \frac{F}{W} ,$$

therefore $$F = IW .$$

The following excerpt, taken from the text, illustrates:

(a) The area of the circumscribed cylinder,

$$A_{cc} = 2\pi h r^2$$

(b) The area of the hemisphere,

$$A_{hs} = 2\pi r^2$$

(c) The area of the zone,

$$A_2 = 2\pi rh$$

The area of the sphere is the same as the area of the surface of the circumscribed cylinder with the same altitude. In trigonometric terms, the solid angle of the zone will be,

$$= \frac{2\pi r^2(\cos\theta_2 - \cos\theta_1)}{r^2} = 2 (\cos\theta_2 - \cos\theta_1)$$

since the solid angle is the area of the zone divided by the radius square.][2]

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2. J. O. Kraehenbuehl, Electrical Illumination (New York, 1942), p. 104.

Solid geometric considerations are shown again, from the following figures and formulas:

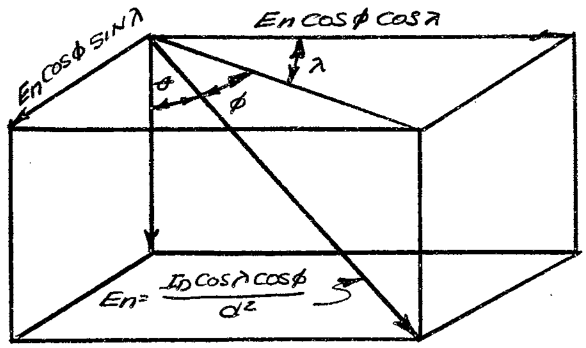

Fig. 20

The following equations are copied, and are a little more complex than others within the text:

$$I_t = \sqrt{(I_L \cos\theta_L + I_M \cos\theta_M)^2 + (I_L \sin\theta_L + I_M \sin\theta_M)^2},$$

$$I_t = \sqrt{(50 \times 1 + 86 \times 0.84)^2 + (50 \times 0 + 86 \times 0.543)^2},$$

$$I_t = \sqrt{(122.24)^2 + (46.70)^2} = 130.9 \text{ amps.}$$

In summary, the mathematics in this text is very elementary, including only simple algebra, trigonometry, and geometry. The following table summarizes the mathematical frequencies a little more completely:

Table 9. Quantitative Survey of Mathematics in <u>Electrical Illumination</u>, by J. D. Kraehenbuehl.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	34
2. Solution of equations.	2
Trigonometry	
1. Formulas containing trigonometry.	25

Table 9 -- Continued

Trigonometry	No. of Occurrences
2. Use of logarithms.	
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	1
7. Inverse functions.	3
8. Graphs of trigonometric functions.	

The tenth text in electrical engineering which is analyzed for mathematical content is Alternating Current Machines, by A. F. Puchstein and T. C. Lloyd.

This textbook contains mathematics, even more complex than that found in the ordinary courses in undergraduate mathematics. There are to be found, for example, problems leading into Fourier series, differential equations of higher order, as well as some very complex definite integrals.

The author presumes the student's background in mathematics to be such that he can follow the material. Complex steps are taken, where the student is expected to supply the details from his own knowledge of mathematics.

Formulas involving complex algebra, fractional exponents and radicals, and functions of sine, cosine, and tangent occur quite frequently. Sinusoidal curves are in the text, as is vector addition, though neither as prevalent as in some of the other texts.

As would be expected, the numerical computations are quite complex. Although algebra and trigonometry dominate the text, this does not imply that the calculus, even advanced topics of calculus, do not occur in some abundance.

The formula, $e = -N \frac{d}{dt} f(\theta,\phi)$, presumes a considerable maturity on the part of the student. In the first place, it presumes that he knows the meaning of a function. Also, in order for the symbol $\frac{d}{dt}$ to designate total derivative, he must know that θ and ϕ are both functions of time, t. The following, from the text, is an application of this formula:

$$e = -N \frac{d(\phi_m \cos \omega t)}{dt}$$

$$= \omega N \phi_m \sin \omega t.$$

This solution is left entirely to the student. The forms for $\frac{d}{dv} \sin u$, and $\frac{d}{dv} \cos u$, is used in this solution, and occurs quite frequently throughout this text. Also, ω, here, is the usual symbol for angular velocity; its value is given, $\omega = 2\pi f$, where f is the frequency. Thus, radian measure is seen to be quite important to the understanding of many electrical engineering formulas.

The imaginary unit, j, given by the relation, $j^2 = -1$, plays a prominent part in many equations throughout this text. The following formula, followed by a numerical example, is taken from the text:

$$E_o = E_t + I\,(R_e + jX_s)\,(\cos\theta - j\sin\theta),$$

$$E_o = 254 + 236\,(0.048 + j\,0.503)\,(0.8 - j\,0.6)$$

$$= 346.$$

A formula, in which only algebraic symbols occur is taken from the text, as it is typical of many others:

$$R = (1 + 0.004\, t_c^{\circ}) \times 9.7 \frac{l_c\; N_a}{A \times n}$$

where t_c = temperature in degrees Centigrade.
l_c = mean length of one turn in feet.
N_a = series turn per phase.
A = area of one conductor in circular mils.
n = number of parallel paths per phase.

The following, from the text, shows some of the needs of the student; especially does it show his needs in calculus and algebra:

$$A_1 = A \int_0^{\frac{\tau\psi}{2} + \frac{\theta}{90}\frac{\tau}{2}} \sin \frac{x}{\tau}\pi \, dx$$

$$= A \frac{\tau}{\pi}\left[1 - \cos \frac{\pi}{2} \left(\psi + \frac{\theta}{90}\right)\right].$$

$$A_2 = A \int_0^{\frac{\tau\psi}{2} - \frac{\theta}{90}\frac{\tau}{2}} \sin \frac{x}{\tau}\pi \, dx$$

$$A_1 - A_2 = A \frac{\tau}{\pi}\left[1 - \cos \frac{\pi}{2} \left(\psi - \frac{\theta}{90}\right)\right].$$

The following, from the text, shows the needs of the student. No intermediate steps are supplied, it being presumed that the student can supply them from his own background of mathematical experience:

$$e = - N \frac{d\phi}{dt} 10^{-8} \text{ volts.} \qquad (61)$$

If ϕ is assumed to follow the sine law,

$$\phi = \phi_{max} \sin 2\pi ft. ,$$

$$e = - N \frac{d(\phi_m \sin 2\pi ft.)}{dt} 10^{-8}$$

$$= - N \phi_m \cos 2\pi ft. \times 2\pi ft. \times 10^{-8} .$$

The following is taken from the text, to show a typical vector figure:

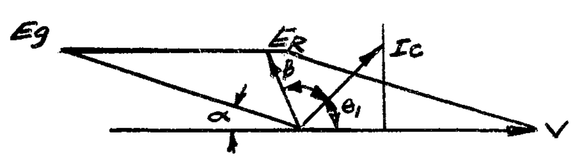

Fig. 21

The following definite integral, from the text, shows again certain needs on the part of the student:

$$\int_0^{\frac{\phi}{2}} \frac{4\pi^2\phi^2 f^2 D}{d^2 K} 10^{-16} x^2 \, dx$$

$$= \frac{\pi^2 \phi^5 f^2 D \times 10^{-16}}{6K} \text{ watts.}$$

There are some problems within this text, as in some of the others, in which ratio and proportion is used extensively. The following illustrate:

$$I_1 : N_{ac} = I_2 : N_{cb} \tag{143}$$

and $$\frac{N_{ac}}{N_{cb}} = \frac{I_2}{I_1} = a \; . \tag{144}$$

By Kirchhoff's law, $$I_2 = I_{ab} + I_{cb} \; . \tag{145}$$

Transposing, $$I_{cb} = I_2 - I_{ab}.$$

The ratio, $$\frac{I_{cb}}{I_{ab}} = \frac{I_2 - I_{ab}}{I_{ab}}. \tag{146}$$

Since, from (144), $$\frac{I_2}{I_{ab}} = \frac{I_2}{I_1} = a,$$

then, $$\frac{I_{cb}}{I_{ab}} = a - 1.$$

The following formulas, from the text, show the need for inverse functions:

$$\phi_o = \tan^{-1} \frac{I^o r \sin\theta}{E_o - I_o r \cos\theta} \; ,$$

$$\phi_s = \tan^{-1} \frac{I_s r \sin\theta}{E_s - I_s r \cos\theta} \; .$$

Most of the integrals which occur in this text are of the form, $\int \sin v \, dv$, $\int \cos v dv$, $\int dv$, or $\int v^n \, dv$, as follows:

$$\frac{2\pi W}{g}\int_{R_2}^{R_1} r^3\ dr = \frac{2\ W}{4g}\ (R_1^4 - R_2^4)\ .$$

The derivatives are also of the form, $\frac{d}{dv}$ (sin u), $\frac{d}{dv}$ (cos u), and $\frac{d}{dv}$ (u^n). The following are examples:

$$\alpha_m = A_1\ \cos\beta t + B_1 \sin\beta t + \frac{T}{k_2}\ .$$

The second derivative is,

$$\frac{d^2\alpha_m}{dt^2} = -\beta^2 A_1\ \cos\beta t - \beta^2 B_1 \sin\beta t.$$

The student, here, is expected to begin with this function and derive its second derivative.

Several examples of some of the numerical computations are given here, as each text seems to have its unique types.

$$r = 0.0000785 + \frac{2 \times 0.00000527 \times 1.43}{\left(2 \sin \frac{\pi}{\frac{48}{4}}\right)^2}$$

$$= 0.0001343 \text{ ohm.}$$

$$k = 2.3 \times 10^{-18} \frac{(8 \times 2{,}010{,}000)^2}{0.0001343} \times 48$$

$$= 212 \text{ lb - ft.}$$

This latter computation reveals the student's needs for the laws of exponents.

The following definite integrals are given here. Although some of this type have been previously given, they occur so frequently that they merit some emphasis.

$$E_d = \frac{Z}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} E_m \cos\alpha\ d\alpha$$

$$= \frac{2\ Z\ E_m}{\pi}\ .$$

$$E = E_m \frac{p}{2\pi} \int_{-\pi/p}^{\pi/p} \cos\theta \, d\theta$$

$$= E_m \frac{p}{\pi} \quad \sin \frac{\pi}{p} .$$

$$R = \frac{1}{\pi} \int_{-\pi/p}^{\pi/p} I \cos h\theta \, d\theta$$

$$= \frac{2I}{h\pi} \sin \frac{h\pi}{p} .$$

There are at least two chapters in the text which contain mathematics that is completely out of the undergraduate level.

While it is barely within the scope of this study, the differential equation for oscillation with damping action is given. This is, of course, in all elementary courses in differential equations. It is, as follows:

$$J \frac{d^2\alpha_m}{dt^2} + k \frac{d\alpha_m}{dt} + k_2\alpha = \frac{I}{R} .$$

Its solution is,

$$\alpha_m = AE^{(-\alpha+\beta)t} + B^{(-\alpha+\beta)t} + \frac{I}{K_2} ,$$

where, $\alpha = \frac{k_1}{2J}$, $\beta = \sqrt{(\frac{k_1}{2J}) - \frac{k_2}{J}}$.

On a level even higher than this is the text's use of the Fourier series. This topic does not occur ordinarily on the undergraduate level but is found usually on a lower graduate level. The expansion of functions into their Fourier series is accomplished in this text. The following expansion occurs:

$$y = \frac{4}{\pi} \left[\sin x + \frac{1}{3} 3 x + \frac{1}{5} \sin 5 x + \ldots\right] NI.$$

In summary, it can be stated that in this, as in so many of the other texts, algebra and trigonometry predominate. Of some interest, however, is the occurrence of some higher mathematics, particularly Fourier series. Vectors, logarithms, radian measure, sine and cosine waves, and exponents and radicals deserve special mention. The following table summarizes the mathematical frequencies more completely.

Table 10. Quantitative Survey of the Mathematics in Alternating Current Machines, by A. F. Puchstein, and T. C. Lloyd.

		No. of Occurrences
	Algebra	
1.	Elementary processes and formulas.	122
2.	Solution of equations.	
	Trigonometry	
1.	Formulas containing trigonometry.	47
2.	Use of logarithms.	12
3.	Numerical solution of triangles.	5
4.	Use of radian measure.	14
5.	Trigonometric identities.	8
6.	Use of vectors.	45
7.	Inverse functions.	6
8.	Graphs of trigonometric functions.	12
	Differential Calculus	
1.	Differentiation of algebraic expressions.	
2.	Differentiation of transcendental expressions.	2
3.	Application of the derivatives.	
4.	Partial derivatives.	
	Integral Calculus	
1.	Integration of algebraic expressions.	6
2.	Integration of transcendental expressions.	7
3.	Application of the definite integral.	
4.	Multiple integrals.	
	Higher Mathematics	
1.	Differential equations, Fourier series, etc.	13

This completes the texts for electrical engineering. It is of some value to consider them compositely, so a summary is here included.

SUMMARY OF THE TEXTS IN ELECTRICAL ENGINEERING. The texts in electrical engineering are replete with mathematics of all levels. To be sure, some of them contain much more than others. A comparison of K. Y. Tang's, Alternating Current Circuits, and J. O. Kraenhenbuehl's, Electric Illumination, are good contrasts, with the former containing very much more mathematics than does the latter. Trigonometry is stressed, especially the use of vectors. It is important to note the occurrence of higher mathematics, especially Fourier series. Also, the integral calculus is a topic of prime importance.

The following table gives a dual summary: (1) the number of texts in which the given items occurred, and (2) the total occurrences for all the texts.

Table 11. Quantitative Summary of the Ten Texts in Electrical Engineering

	No. of Texts Occurring.	Total of Occurrences.
Algebra		
1. Elementary Processes and formulas.	10	1087
2. Solution of Equations.	4	15
Trigonometry		
1. Formulas containing trigonometry	10	399
2. Use of logarithms.	6	54
3. Numerical solution of triangles.	3	12
4. Use of radian measure.	7	46
5. Trigonometric identities.	5	23
6. Use of vectors.	10	263
7. Inverse functions.	8	93
8. Graphs of trigonometric functions.	8	180
Differential Calculus		
1. Differentiation of algebraic expressions.	1	16
2. Differentiation of transcendental expressions.	4	19

Table 11 -- Continued

	No. of Texts Occurring.	Total of Occurrences.
Differential Calculus		
3. Application of the derivatives.	2	16
4. Partial derivatives.	1	8
Integral Calculus		
1. Integration of algebraic expressions.	7	84
2. Integration of transcendental expressions.	7	82
3. Application of the definite integral.	2	6
4. Multiple integrals.		
Higher Mathematics		
1. Differential equations, Fourier series, etc.	7	97

Part II

A Survey of the Mathematics in the Chemical and Petroleum Engineering Textbooks

Introduction. The texts considered within this chaper are given here in the order in which the student studies them. They are as follows: Inorganic Chemical Technology, Second Edition, W. L. Badger and E. M. Baker; Industrial Stoichiometry, W. K. Lewis and A. H. Radasch; Elements of Chemical Engineering, Second Edition, W. L. Badger and W. L. McCabe; Volumetric and Phase Behaviour of Hydrocarbons, B. H. Sage and W. N. Lacey.

The procedures are followed, as explained in Chapter 1. At the conclusion of each text will be found a short summary and a table. We begin with Inorganic Chemical Technology, Second Edition, by W. S. Badger and E. M. Baker.

This textbook contains a great many chemical equations, but little mathematics. There are formulas given, however, which involve logarithms, and even differential calculus. There are no integrations, nor is there any trigonometry.

The following formula is taken from the text:

$$K = \frac{x + c}{(a - x)\,\frac{(b - 0.5x\ \ )}{100 - 0.5x}}$$

Of course, this is purely algebraic, but the following formulas involve logarithms:

$$\log\ K = \frac{8775}{t} = 4.46.$$

$$\log\ K = \frac{216{,}000}{4.58T} + 1.75 \log T + 8.4.$$

The following equations involving calculus are taken from the text:

$$\frac{dw}{d\theta} = KA \Delta p,$$

$$E = \frac{Q}{JF} + \frac{de}{dt} .$$

The following simultaneous equations occur in the text:

$$117.8Y + 10 + 12.5Z = 105.3X + 12.5X$$

$$117.8Z + 12.5X = 117.8Y + 12.5X$$

$$12.5Y = 117.8Z + 12.5Z$$

Solving these equations gives:

$$X = 0.095$$

$$Y = 0.0101$$

$$Z = 0.00098$$

In summary, the text contains almost no mathematics. Almost all in the text has been reproduced. The following table summarizes the mathematical frequencies more completely:

Table 12. Quantitative Survey of the Mathematics in Inorganic Chemical Technology, Second Edition, by W. L. Badger and E. M. Baker

Algebra	No. of Occurrences
1. Elementary processes and formulas.	3
2. Solution of equations.	1
Trigonometry	
1. Formulas containing trigonometry.	
2. Use of logarithms.	3
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	
7. Inverse functions.	
8. Graphs of trigonometric functions.	
Differential Calculus	
1. Differentiation of algebraic expressions.	2

Differential Calculus	No. of Occurrences
2. Differentiation of transcendental expressions.	
3. Applications of the derivative.	
4. Partial derivatives.	

The second of the texts in chemical and petroleum engineering which is analyzed for mathematical content is Industrial Stoichiometry , by W. K. Lewis and A. H. Radasch.

This textbook is concerned, largely, only with the elementary mathematics involved in balancing chemical formulas, setting up proportions and solving for a variable, and other such simple algebraic operations. Only linear equations occur, although simultaneous linears occur with two, and even three, variables. The arithmetic computations are not difficult, and do not include the extraction of roots. There is no trigonometry or calculus.

The following typifies the type of arithmetic computation occurrring throughout the text:

$$\frac{623\ (29.92)}{29.92 - 0.87} = 642 \text{ cu. ft.,}$$

$$15.90 \times \frac{21.45}{78.55} = 4.34 \text{ tons,}$$

$$\frac{100\ (7.1)}{21.21 - 7.1} = 50.3\%$$

$$\frac{100\ (21.21 - 14.11)}{14.11} = 50.3\%.$$

From the foregoing, it is obvious that the computations in the text are not complex. Moreover, they are not more difficult than these taken directly from the text.

The following algebraic equations are taken from the text:

$$\frac{100x}{6.66 - x - (7.62 - x)\frac{79}{21}} = 4,$$

$$x = 1.74 \text{ mols.}$$

$$19.3 + 18.78 + 0.0109x = 0.0817x + 0.1848x$$

$$7595 = (97{,}900 + 934x) + 82{,}800 + 1{,}515{,}000 + 3{,}088{,}000,$$

$$x = 718 \text{ lb.}$$

Since the foregoing equations typify the linear equations in the single variable, it is apparent that the student should have no difficulty here. In fact, an average student in elementary algebra should be able to solve such equations.

The following, from the text, are examples of equations involving two and three variables:

$$0.324x + z = 27.75$$

$$(0.1955 - \frac{0.169x}{2} + 0.21y + z = 30$$

$$0.563x + 0.79y = 70$$

These equations when solved give:

$$x = 45.1$$
$$y = 56.5$$
$$z = 13.13$$

Another example, is as follows:

$$(0.10)\ (0.2004)x + 0.4197y = 63.42$$
$$(0.45)\ (0.2004x + 0.1300y = 20.80$$
$$(0.45)\ (0.2004)x + 0.0700y = 11.42$$

The text stresses linear equations in one, two, or three variables. The following table summarizes the mathematical frequencies more completely:

Table 13. Quantitative Survey of the Mathematics in Industrial Stoichiometry, by W. K. Lewis and A. H. Radasch.

Algebra	No. of Occurrences
1. Elementary process and formulas.	12
2. Solution of equations.	19

The third text in chemical and petroleum engineering which is analyzed for mathematical content is Elements of Chemical Engineering, Second Edition, by W. S. Badger and W. L. McCabe.

This textbook in chemical engineering contains only a moderate amount of mathematics. At no place is the mathematics very complex, though it should be stressed that it occurs on nearly all undergraduate levels, even including differential equations. It is difficult to state precise areas of mathematics covered, since the book is large, and a very wide area is taken.

The numerical computations are not complex; there is some attention given to logarithms, especially that of changing from one base to another; algebraic formulas are prevalent, especially those involving fractional exponents, or radicals; differentiation and integration occurs only on an elementary level.

The following numerical computations are, perhaps, a little more complex than that found throughout most of the text:

$$h = 0.725 \sqrt[4]{\frac{(0.393^3\,(59.88)^2\,(4.18 \times 10^8)(960.6)}{0.1096 \times 0.694 \times 37}},$$

$$h = 1.711.$$

$$U = \frac{1}{\frac{1}{10.58} - \frac{0.00401}{226} - \frac{1}{236.4}}$$

$$= \frac{1}{0.09450 + 0.0000177 + 0.00423}$$

$$= \frac{1}{0.09875} = 10.13.$$

The following are taken from the text to show the student's needs for logarithms:

$$y = ax^n . \qquad (9)$$

Equation (9) may be rewritten in the form,

$$\log y = \log a + n \log x.$$

Another equation is given, as follows:

$$Hp. = K \log \frac{D}{d} . \qquad (336)$$

Under certain conditions (336) becomes, $7.5 = K \log \frac{0.87}{0.31}$, from which, K = 16.75.

In order to solve the foregoing for K, the student should know the laws of logarithms.

The following algebraic formulas are taken from the text to demonstrate the degree of complexity of some of them:

$$h = 0.0255 \frac{k}{D} \left(\frac{Du}{\mu}\right) 0.8 \left(\frac{Cu}{k} \right)^{0.4}.$$

$$\frac{D}{Bg} = 0.023 \left(\frac{Du}{\mu}\right)^{0.83} \left(\frac{\mu}{Pb_g}\right)^{0.44}$$

In several places, simultaneous linear equations occur, and even in one place, with four variables. The following are typical:

$$946w + 55,000 (217 - 70) = 967x$$
$$967x + (55,000 - x) (217-186) = 986y$$
$$986y + (55,000 - x - y) (186 - 125) = 1,022z$$
$$x + y + z = 44,000$$

From these equations:

$$x = 13{,}610 \text{ lb.}$$
$$y = 14{,}660 \text{ lb.}$$
$$z = 15{,}730 \text{ lb.}$$
$$w = 22{,}460 \text{ lb.}$$

In the foregoing set of linear equations, the answer, and not the solution, is supplied by the text. It is assumed that the student's algebraic background is such that he can supply the missing steps.

This is an unusual engineering text, in that trigonometry occurs infrequently. The following equations are taken from the text:

$$\frac{V}{\theta} = 2.505 \left(\tan \frac{\alpha}{2}\right)^{0.996} H^{2.47} ,$$

$$m = r \sin\alpha ,$$

$$e = t \cos\alpha = ur \cos\alpha ,$$

$$ur \cos\alpha > r \sin\alpha ,$$

$$u > \tan\alpha .$$

$$\cos\alpha = \frac{AB}{AC} = \frac{r + d}{r + R} .$$

These formulas, and only a few others, occur throughout the entire text. The inequality signs, ∠, and >, occur here, as in several other places throughout the text.

The text gives a discussion for the computation of an area under a curve. The following is taken from the text:

> [It should be remembered from the first principles of integral calculus that the value of a definite integral, f(x)dx, is the area bounded by the curve of f (x), the ordinates $x = x_a$ and $x = x_b$, and the X - axis.][3]

3. W. L. Badger and W. L. McCabe, Elements of Chemical Engineering, Second Edition (New York, 1936), p. 11.

The following figure accompanies this discussion:

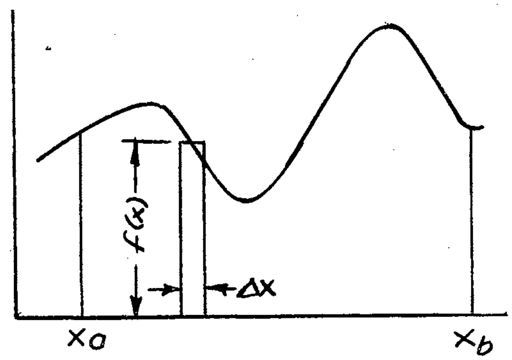

Fig. 22

The following differential equation occurs:

$$q = k \frac{dt}{dr} (2\pi rN) . \qquad (51)$$

In order to integrate equation (51), it is necessary only to separate the variables t and r, as follows:

$$\frac{dr}{r} = \frac{2\pi Nk}{q} dt. \qquad (52)$$

Equation (52) can be integrated, as follows:

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi Nk}{q} \int_{t_1}^{t_2} dt \quad ,$$

$$\ln r_2 - \ln r_1 = \frac{2\pi Nk}{q} (t_2 - t_1) \quad ,$$

$$q = \frac{k(2\pi N)\ (t_2 - t_1)}{\ln \frac{r_2}{r_1}} .$$

In several places throughout the text, the form $\int \frac{dv}{v}$ occurs, as in the foregoing example, and the following:

$$wc\int_1^2 \frac{dt}{T_1 + \frac{wct}{WC} - (\frac{wc}{WC} + 1)} = Ua\int_0^1 dL \qquad (81)$$

From (81), the following is derived:

$$-\frac{wc}{\frac{wc}{WC}+1}\ln\frac{T_1 + \frac{wct}{WC} - (\frac{wc}{WC}+1)\ t_2}{T_1 + \frac{wct}{WC} - \ \frac{wc}{WC}+1)\ t_1} = UaL_1 \ .$$

The following differential equation makes use of the same form for its solution:

$$wc\int dt = Ua\int(T - t)\ dL. \qquad (90)$$

Separating the variables in (90),

$$\int\frac{wc}{Ua(T - t)}\ dt = \int dL,$$

from which, the following is derived,

$$\int_1^2 \frac{wc}{Ua(T - t)}\ dt = \int_0^{L_1} dL = L_1\ . \qquad (91)$$

With few exceptions, the integrals are of the form $\int\frac{dv}{v}$. The following demonstrate:

$$\beta_f = \int_o^f d\theta = \frac{DF_c}{AR_c}\int_{f_2}^{f_1}\frac{dF}{F} = \frac{DF_c}{AR_c}\ln\frac{F_1}{F_2} \quad ,$$

$$\frac{G}{k}\int_{H_1}^{H_2}\frac{d(kH)}{H_s - H} = A_c = \frac{G}{k}\ln\frac{H_s - H_1}{H_s - H_2}$$

$$\int_{W_1}^{W_o}\frac{dW}{W} = \int_{X_1}^{X_o}\frac{dx}{y-x} = \ln\frac{W_o}{W_1}$$

The mathematics here is not difficult for the student who has been through the calculus. However, the algebra is of some difficulty, and the undetermined constants make these forms more difficult than would ordinarily be the case.

In summary, it can be stated that the bulk of the mathematics is concerned with algebra although there is a considerable use of logarithms. Also, there is a little mathematics to be found on all levels. The following table summarizes the mathematical frequencies more completely:

Table 14. Quantitative Survey of the Mathematics in _Elements of Chemical Engineering, Second Edition_, by W. L. Badger and W. L. McCabe.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	38
2. Solution of equations.	12
Trigonometry	
1. Formulas containing trigonometry.	8
2. Use of logarithms.	14
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	
7. Inverse functions.	7
8. Graphs of trigonometric functions.	
Differential Calculus	
1. Differentiation of algebraic expressions.	6
2. Differentiation of transcendental expressions.	
3. Applications of the derivative.	4
4. Partial derivatives.	
Integral Calculus	
1. Integration of algebraic expressions.	3
2. Integration of transcendental expressions.	8
3. Applications of the definite integral.	
4. Multiple integrals.	
Higher Mathematics	
1. Differential equations, Fourier series, etc.	6

The fourth text in chemical and petroleum engineering, which is analyzed for mathematical content is _Volumetric_ and Phase Behaviour of Hydrocarbons, by B. H. Sage and W. N. Lacey.

This text contains more intricate mathematics than any text reviewed. It deals with rates of change of temperature, with respect to time, for volumes of various gases. Since the volume occurs so often as a function of several variables, the partial derivative occurs repeatedly. In fact, so important is this topic that the author devotes an entire chapter to what he titles, Some Mathematical Concepts.

The subtitles of this chapter will give some idea to the mathematics encountered. They are, as follows: Variables; Space Co-ordinates; Slopes; Differentials; Partial Derivatives; Integration; Analytic Methods; Partial Differential Equations involving Several Independent Variables; Application of the General Equation; Graphical Methods; Co-ordinate Scales; Residual Methods of Graphical Representation; Graphical Differentiation; Simple Differentiation; Numerical Differentiation; Residual Differentiation.

One could obtain an idea of the mathematics from the entire text, by studying just this one chapter. A few direct quotes from the text prove to be of interest to the mathematician. The following is from the text:

> [The rectangular cartesian system utilizes axes at right angles to each other meeting at a point called the origin. Cartesian co-ordinates in three-dimensional space permit the simulataneous representation of three variables.] [4]

---

4. B. H. Sage and W. N. Lacey, Volumetric and Phase Behaviour of Hydrocarbons (Houston, 1949), p. 12.

Again, from the text:

> [A slope may be defined as the instantaneous rate of change in one variable with that in another variable. The slope of a curve at any given point on the curve at that point. The slope of the tangent can be readily evaluated, since it is a straight line, by ascertaining the corresponding change in value of one variable for a unit change in the second, which is ordinarily the independent variable][5]

From the topic on partial derivatives, the following is taken:

> [When only one independent variable is permitted to change, all other independent variables being kept constant, the ratio of the infinitesimal increment of the dependent variable to that of the independent variable is a partial derivative. In this case the symbol ∂ is used in place of the d used in total differentials][6]

Several graphs are given to demonstrate the meaning of the derivative, in terms of the curve. The following, is one such three-dimensional graph:

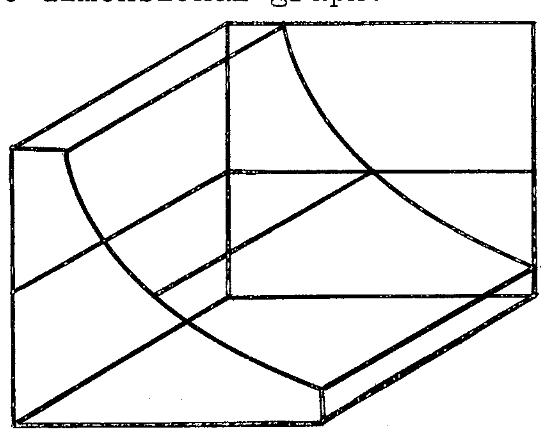

Fig. 23

5. Ibid., p. 16.
6. Ibid., p. 17.

The following are taken from the chapter titled, Some Mathematical Concepts:

$$\text{Slope} = \left(\frac{\partial V}{\partial P}\right)_T = \frac{V_b - V_a}{P_b - P_a},$$

$$\left(\frac{\partial P}{\partial T}\right) = - \frac{\left(\frac{\partial V}{\partial T}\right)_{P.t}}{\left(\frac{\partial V}{\partial T}\right)} - \frac{\left(\frac{\partial V}{\partial T}\right) + \left(\frac{\partial T}{\partial T}\right)}{\left(\frac{\partial V}{\partial P}\right)}$$

The following formula is taken from the text:

$$\text{Log } P = a + \frac{b}{T},$$

where a and b are constants. If both sides of the equation are differentiated, the resulting equation is:

$$\frac{d(\text{Log } P)}{d\left(\frac{1}{T}\right)} = b$$

> [Since the rate of change of Log P with respect to a change in 1/T is a constant, a graph in which Log P is used as one co-ordinate while 1/T is the other, both being plotted on linear scales, would show the relation in the form of a straight line.] [7]

The foregoing are all taken from the chapter on mathematical concepts. It was offered in this study, as it is, of course, typical of the mathematics in the text. It is noteworthy that the mathematics occurs almost entirely on the level of analytical geometry and above, there being no trigonometry, and the algebra occurring on a very elementary level. There are no very complicated numerical computations involved.

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7. Ibid., p. 23.

In order to give some idea of the mathematics within the text itself, the following is given:

$$\frac{dV}{dT} = \left(\frac{\partial V}{\partial T}\right)_p + \left(\frac{\partial V}{\partial P}\right)_T \cdot \frac{dP}{dT},$$

$$\frac{dV}{dP} = \left(\frac{\partial V}{\partial P}\right)_T + \left(\frac{\partial V}{\partial T}\right)_p \cdot \frac{dT}{dP}.$$

To demonstrate a few of the formulas occurring, which are purely algebraic, the following are taken from the text:

$$n = \frac{AD}{AD + BD + DE} = \frac{AD}{GJ}$$

$$V = n_1 V_1 + n_2 V_2 + n_3 V_3 + \ldots + n_n V_n$$

The following is taken from the text, with the cryptic statement after it:

$$\left[\frac{dm}{m} = \frac{dx_1}{y_1 - x_1} = \frac{dx_2}{y_2 - x_2} = \frac{dx_3}{y_3 - x_3}\right. \quad (78)$$

> This equation cannot be integrated unless the composition of the coexisting phases is known as a function of the prevailing pressure. It may be treated graphically....
> Equation (78) may be written in the following way to permit its ready solution by a progressive graphical or mechanical integration.

$$\frac{dx_1}{dx_2} = \frac{y_1 - x_1}{y_2 - x_2} \quad (79)$$

Upon integration, this expression yields:

$$\left. x_1 = \frac{y_1 - x_1}{y_2 - x_2}\, dx_2 \right]$$ [8]

The process of integration occurs in several places throughout the text.

8. Ibid., p. 156.

Logarithms occur in several places throughout the text, as the following will demonstrate:

$$\sigma = a + \frac{1.075 - a}{.10} \text{Log} (\eta - 38) ,$$

$$T = d + \frac{138 - d}{0.6812} \text{Log} (\eta - 38) ,$$

$$\text{Log } d = 1.580a + 0.598 .$$

In summary, the mathematics in the text is quite complex, with emphasis on graphing, partial and total derivatives, and integral calculus. The following table gives a more complete summary of the mathematical frequencies.

Table 15. Quantitative Survey of the Mathematics in Volumetric and Phase Behaviour of Hydrocarbons, by B. H. Sage and W. N. Lacey.

	No. of Occurrences
Algebra	
1. Elementary processes and formulas	38
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	
2. Use of logarithms.	
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	
7. Inverse functions.	
8. Graphs of trigonometric functions.	
Differential Calculus	
1. Differentiation of algebraic expressions.	8
2. Differentiation of transcendental expressions.	12
3. Application of the derivatives.	
4. Partial derivatives.	24
Integral Calculus	
1. Integration of algebraic expressions.	12
2. Integration of transcendental expressions.	16
3. Application of the definite integral.	
4. Multiple integrals.	

Table 15 -- Continued --

Higher Mathematics	No. of Occurrences
1. Differential equations, Fourier series, etc.	4

This text concludes those of chemical and petroleum engineering. The following summary for the entire chapter follows:

Summary. The fields of chemical and petroleum engineering contain relatively little mathematics. Algebra is the only topic that occurs consistently. One topic, however, which was found to recur was partial derivatives. This is the only field in which this topic occurs profusely.

The following table gives an overall total, that is, it gives the total number of texts in which the given topics occurred, as well as the total occurrences for all the texts:

Table 16. Quantitative Summary of the Four Texts in Chemical and Petroleum Engineering

Algebra	No. of Texts Occurring	Total of Occurrences
1. Elementary Processes and formulas.	4	91
2. Solution of Equations.	3	32
Trigonometry		
1. Formulas containing trigonometry.	1	8
2. Use of logarithms.	2	17
3. Numerical solution of triangles.		
4. Use of radian measure.		
5. Trigonometric identities.		
6. Use of vectors.		
7. Inverse functions.	1	7
8. Graphs of trigonometric functions.		
Differential Calculus		
1. Differentiation of algebraic expressions.	3	16
2. Differentiation of transcendental expressions.	1	12

Table 16. -- Continued --

Differential Calculus	No. of Texts Occurring	Total of Occurrences
3. Application of the derivatives.	1	4
4. Partial derivatives.	1	24
Integral Calculus		
1. Integration of algebraic expressions.	2	15
2. Integration of transcendental expressions.	2	24
3. Application of the definite integral.		
4. Multiple integrals.		
Higher Mathematics		
1. Differential equations, Fourier series, etc.	1	6

Part III

A SURVEY OF THE MATHEMATICS IN THE CIVIL ENGINEERING TEXTBOOKS

Introduction. The texts considered within this chapter are given here in the order in which the student studies them. They are, as follows: Short Course in Surveying, R. E. Davis and J. W. Kelly; Engineering Mechanics, F. L. Singer; Hydraulics, Second Edition, E. W. Schoder and F. M. Dawson; Strength of Materials, Fifth Edition, J. E. Boyd and S. B. Folk; Field Engineering, Twenty-first Edition, W. H. Searles and H. C. Ives; Substructure Analysis and Design, Paul Andersen; Structural Design, Hale Sutherland and H. L. Bowman; Structural Drafting, Second Edition, C. T. Bishop.

The procedures are followed as explained in Chapter 1. At the conclusion of each text will be found a short summary, and a table. We begin with Short Course in Surveying, by R. E. Davis and J. W. Kelly.

This book is hardly a textbook, in the usual sense, but more of a handbook for students in elementary surveying. The course is taken by all students in the school of engineering, and normally comes within their freshman year.

There is little mathematics in the book and, as would be expected, that which occurs is directed toward mensuration. Angles, triangles, sides of plane figures, and the like, occurs frequently. Except for a few trigonometric formulas, ordinary high school mathematics is all that is required to be able to read this book. There is no calculus within the text, and only a little trigonometry. The following figure, together with the accompanying formulas, is taken from the text.

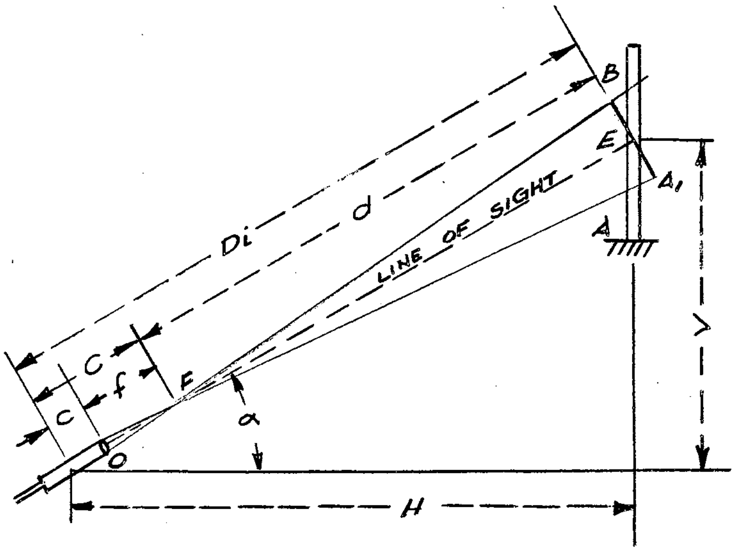


Fig. 24

$$D = Ks\cos\alpha + C.$$

$$H = Ks\cos\alpha + C\cos\alpha .$$

$$V = Ks\cos\alpha\sin\alpha + C\sin\alpha .$$

$$V = \tfrac{1}{2}\,Ks\sin 2\alpha + C\sin\alpha .$$

$$H = Ks\cos\alpha \text{ (approx.)}.$$

$$V = \tfrac{1}{2}\,Ks\sin 2\alpha \text{ (approx.)}.$$

The following purely algebraic formulas are taken from the text:

$$C_p = \frac{(P - P_0)\,L}{AE} .$$

$$C = \frac{w^2L^3}{24P^2} = \frac{W^2L}{24P^2} .$$

$$d' = c + \frac{(a - b) + (d - c)}{2}$$

In summary, this text is more of a handbook than a textbook. Its mathematics includes only elementary algebra and trigonometry. The following table gives a more complete summary of the mathematical frequencies.

Table 17. Quantitative Survey of the Mathematics in Short Course in Surveying, by R. E. Davis and J. W. Kelly.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	23
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry	14
2. Use of logarithms.	
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	8
6. Use of vectors.	
7. Inverse functions.	
8. Graphs of trigonometric functions.	

The second of the texts in civil engineering which is analyzed for mathematical content is Engineering Mechanics, by F. L. Singer.

This book is taught to all beginning students in engineering at Louisiana Polytechnic Institute, since it contains material basic to the student's future needs. It contains, perhaps, as varied a curricula of mathematics as any of the textbooks under consideration.

Scalar and vector quantities are introduced in the early part of the text. Dimensional checks, conversion of units, numerical accuracy and rounding off of numbers are then introduced. The double angle formula, $\sin 2\theta = 2 \sin\theta \cos\theta$ is used, as is the general solution for quadratic equations.

The following figure is given, together with the formulas:

$$F_x = F \cos \theta_x \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F \sin \theta_x \qquad \tan \theta_x = \frac{F_y}{F_x}$$

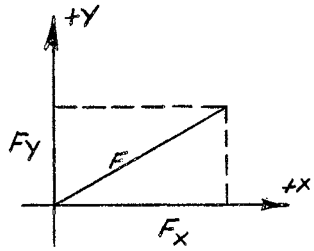


Fig. 25

It is thus evident that the Pythagorean Theorem, the rectangular co-ordinate system, and the definition of the trigonometric functions are needed early within this course. The solution of triangles by trigonometry, including the law of sines, also occurs early in the text. The simultaneous solution of linear equations, in two and three variables, is also a topic occurring.

Vectors, and their resultants, in three dimensions is prominent. The following figure, accompanied by the formulas will demonstrate:

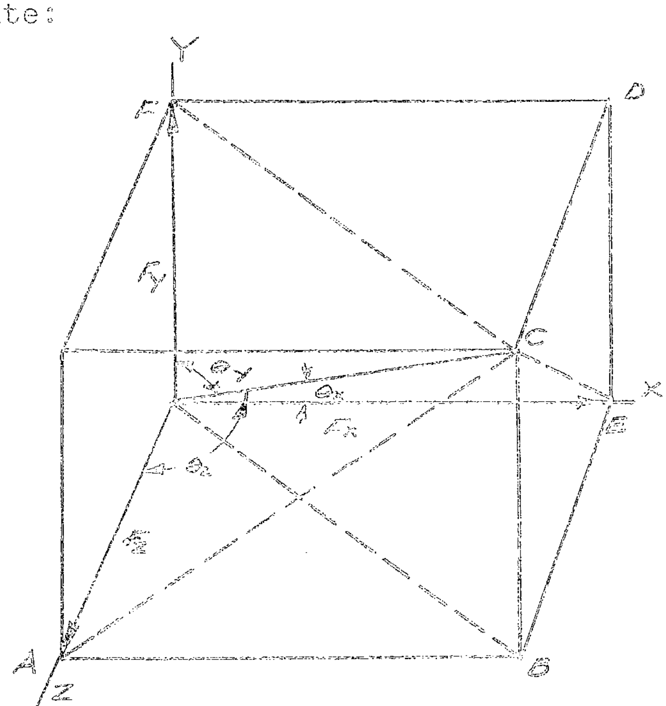


Fig. 26

$$R = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$\begin{cases} F_x = R \cos \theta_x \\ F_y = R \cos \theta_y \\ F_z = R \cos \theta_z \end{cases}$$

The foregoing, is introduced here, in complete form, as an example of the stress that should be laid on the co-ordinate system of three dimensions, especially the meaning of the direction cosines of a straight line in space.

A differential equation occurs in the early part of the text, and, without including all of the author's steps, the following will illustrate the mathematics requisite to an understanding of this topic:

$$\text{(a)}\ dN - (T + dt) \sin \frac{d\theta}{2} - T \sin \frac{d\theta}{2} = 0,$$
$$\text{(b)}\ dN = T\, d\theta\ ,$$
$$\text{(c)}\ dF = fT\, d\theta\ ,$$
$$\text{(d)}\ dT = fT\, d\theta\ ,$$
$$\text{(e)}\ \int_{T_2}^{T_1} \frac{dT}{T} = f \int_{o}^{B} d\theta\ ,$$
$$\text{(f)}\ \log \frac{T_1}{T_2} = f\beta\ .$$

The foregoing differential equation is developed in consequence of a pulley and belt. It is clear then, at this point, that the student needs to know something of circular motion, central angles, radii of circles, and their relation to tangents drawn to the point of contact. The student needs to know the meaning of differentials, how to integrate elementary forms, and the computation of definite integrals; an understanding of the nature of logarithms should be known, as well as the laws of logarithms.

Chapter VII of this text is titled, Centroids and Centers of Gravity. The treatment does not vary from that given by the calculus texts. It begins with a definition of center of gravity, then continues with a treatment of centroids determined by integration. The following formulas are given:

$$A\bar{x} = \int x \, dA$$
$$A\bar{y} = \int y \, dA \tag{1}$$

The text then continues to the computation of centroids for the following: arcs of circles, triangles, circular sectors, and parabolic segments. The following definite integrals occur:

$$\int_{-\alpha}^{\alpha} \cos\theta \, d\theta \, , \int_{0}^{h} (h - y) \, y \, dy, \int_{0}^{a} k \, x^3 \, dx.$$

The Theorem of Pappus is introduced, and illustrative problems offered, finding the surface area of a cone, and the volume of a hemisphere, by revolving a straight line and a semicircle, respectively, about a fixed line.

The chapter continues with the centers of gravity of solid bodies. Formulas for three dimensions, analogous to (1) are given, for example,

$$V\bar{x} = \int x \, dV$$
$$V\bar{y} = \int y \, dV$$
$$V\bar{z} = \int z \, dV$$

Chapter VIII is titled, Moments of Inertia. By way of definition, the following formulas are given,

$$I_x = \int y^2 \, dA.$$
$$I_y = \int x^2 \, dA.$$

A definition of polar moment of inertia is given, thus at least an elementary concept of polar co-ordinates is needed.

Formulas for radius of gyration and the transfer formulas for moments of inertia are given. Only elementary algebra is needed to follow these formulas.

The text then continues by using illustrative examples to compute moments of inertia for rectangles, triangles, and circular sectors. These are done by setting up differentials of moment, and integrating. The following definite integrals occur:

$$\int_0^d h^2 \; b \; dy, \int_{-d/2}^{d/2} y^2 \, b \; dy; \int_0^h \; y^2 \, x \; dy = \int_0^h y^2 \frac{b}{h} \, (h - y) \; dy.$$

These examples are followed by one in polar co-ordinates, that of finding the moment of a circular sector. Here, double integration is used. The following integrals are set up and solved:

$$\int_0^r \int_0^{2\pi} P^3 \; \sin^2\theta \; d\theta \quad dP = \frac{r^4}{4}\int_0^{2\pi} \sin^2\theta \, d\theta \, .$$

The text next considers moments of inertia, with respect to inclined axes. Here, the formulas for rotation of axes are used, as are some double-angle trigonometric identities. The following figure and formulas will help illustrate:

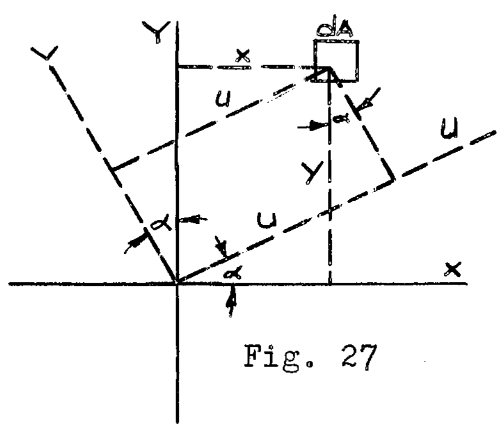


Fig. 27

$$v = y \cos\alpha - x \sin\alpha$$

$$u = y \sin\alpha + x \cos\alpha$$

$$\cos 2\alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\sin 2\alpha = \frac{1 - \cos 2\alpha}{2}$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\alpha - P_{xy} \sin 2\alpha$$

$$I_v = \frac{I_x + I_y}{2} + \frac{I_y - I_x}{2} \cos 2\alpha + P_{xy} \sin 2\alpha$$

The following, familiar, trigonometric identity occurs:

$$\cos (2\alpha + 2\theta) = \cos 2\alpha \cos 2\theta - \sin 2\alpha \sin 2\theta .$$

The text continues with a definition of radius of gyration, and the formula, $k = \sqrt{\frac{I}{M}}$, is given. This is offered in some calculus textbooks, but perhaps most engineering students are unfamiliar with this formula at this stage of their course work.

Next, the transfer formula for mass moments of inertia is developed, and given in the following form:

$$I = I_o + Md. \qquad (2)$$

In order to develop this form, the author uses the equation,

$$I = \int P^2 \, dM. \qquad (3)$$

By making use of the Pythagorean theorem, and squaring the binomial, $(a + b)^2$, the author makes a substitution for P^2, from which he derives (2).

Chapter VIII is concluded with some illustrative examples on this topic.

The next few chapters deal with dynamics, and contain many formulas, familiar to the physicist. For instance the following forms for velocity and acceleration are given:

$$\text{(a)} \quad v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\text{(b)} \quad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$(c)\ a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$

$$(d)\ v\,dv = a\,ds.$$

The text then gives Newton's second law in equation form,

$$R = \frac{W}{g}\,a \quad . \qquad (4)$$

Then treating (4) as a vector equation, and resolving into components, he obtains,

$$\begin{cases} x = \frac{W}{g}\,a_x \\ y = \frac{W}{g}\,a_y \\ z = \frac{W}{g}\,a_z \end{cases} \qquad (5)$$

A knowledge of vectors, three-dimensional rectangular co-ordinates, and direction numbers will make (5) clear. The following familiar forms appear:

$$(a)\ dv = a\,dt,$$

$$(b)\ \int_{v_o}^{v} dv = a\int_{o}^{t} dt,$$

$$(c)\ v = v_o + at.$$

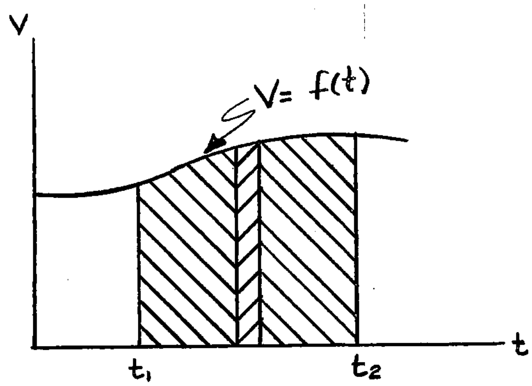
$$(d)\ ds = v\,dt,$$

$$(e)\int_{o}^{s} ds = \int_{o}^{t} v\,dt = \int_{o}^{t}(v_o + at)\,dt,$$

$$(f)\int_{v_o}^{v} v\,dv = a\int_{o}^{s} ds,$$

$$(g)\ v^2 = v_o^2 + 2\,as.$$

The text gives a treatment of motion curves, and area under a curve is considered. The following figure and formula will illustrate:



$$s = \int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v \; dt.$$

Fig. 28

The following quadratic equation occurs, and is given here, as it shows a certain mathematical need:

$$-300 = 1414 \sin 45^{\circ} \times t - 16.1 t^2.$$

The following formulas occur, showing the need of both algebra and trigonometry:

$$\tan\theta = \frac{\frac{W}{g}\frac{v^2}{r}}{W} = \frac{v^2}{gr}.$$

$$\cos^2\theta + \frac{v^2}{gL}\cos\theta - 1 = 0.$$

$$t = 2\pi\sqrt{\frac{L}{g}}$$

In summary, this text contains a great deal of mathematics on all levels, and the following table summarizes the mathematical frequencies more completely:

Table 18. Quantitative Survey of the Mathematics in _Engineering Mechanics_, by F. L. Singer.

Algebra	No. of Occurrences
1. Elementary processes and formulas	336
2. Solution of equations.	37

Table 18. -- Continued --

Trigonometry	No. of Occurrences
1. Formulas containing trigonometry.	63
2. Use of logarithms.	12
3. Numerical solution of triangles.	38
4. Use of radian measure.	44
5. Trigonometric identities.	29
6. Use of vectors.	212
7. Inverse functions.	4
8. Graphs of trigonometric functions.	3
Integral Calculus	
1. Integration of algebraic expressions.	40
2. Integration of transcendental expressions.	18
3. Applications of the definite integral.	22
4. Multiple integrals.	6
Higher Mathematics	
1. Differential Equation, Fourier series, etc.	15

The third text in civil engineering which we analyze for mathematical content is Hydraulics, Second Edition, by E. W. Schoder and F. M. Dawson.

This text contains a great deal of mathematics, and although it contains some calculus and trigonometry, the greater portion is algebra. There are a great many formulas that are purely algebraic; logarithms occur in a few places, and even differential equations. The following algebraic equations are taken at random from the text:

$$h = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + (z_1 - z_2),$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g},$$

$$W\frac{V_1^2}{2g} + W\frac{p_1}{w} + Wz_1 = W\frac{V_2^2}{2g} + W\frac{p_2}{w} + Wz_2,$$

$$\frac{V_1^2}{2g} + \frac{p_1}{w} + z = \frac{V_2^2}{2g} + \frac{p_2}{w} + z_2,$$

$$v_2 = \sqrt{\frac{1}{1 - (\frac{D_2}{D_1})^4}} \sqrt{2gh} \,.$$

The foregoing are typical of a great many formulas in the text. The following are also quite common, and are given here, as they show the occurrence of exponents:

$$b = 0.815\sqrt{h^2 + 4800} \,,$$

$$b = \sqrt{\frac{1}{N}}\, h \,,$$

$$Q = 3.33(L - 0.2h)h^{3/2},$$

$$Q = 3.33L(h + a\frac{V_2}{2g})^{3/2},$$

$$h = J(\frac{u}{p})^{0.25} + L\frac{V^{1.75}}{D^{1.25}} \,,$$

$$R = \frac{DVP}{u} \,.$$

The following graph is taken from the text:

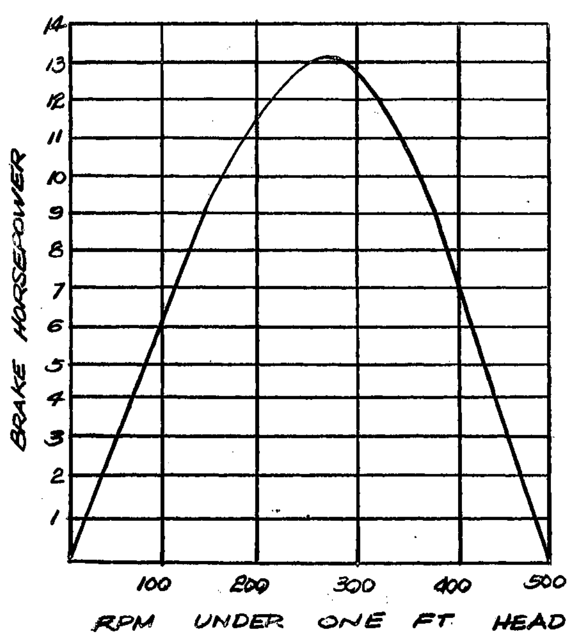


Fig. 29

Two of the laws of logarithms are given to aid the student. The following differential equation is taken directly from the text, and the solution given verbatim.

> The differential equation then becomes $dQ = dA \times v$, or $dQ = (Ldy - L\frac{v}{h}dy)\ 2gy = L\ 2g\ (y^{1/2}dy - \frac{1}{h}y^{3/2}dy)$, which upon integration gives $Q = \frac{4}{15}LH\ 2gh$. Writing this in the form $Q = AV$, we have $Q = \frac{L}{2} \times \frac{8}{15}\ 2gh.$[9]

Another very interesting differential equation is taken from the text:

> Then $P_o = x^2(p_1 - p_2) = 2\ x\ L\ \frac{dv}{dx}$.
>
> $$(p_1 - p_2)\ x\ dx = 2\ L\ dv.$$
>
> Integrating between the radius limits of o and x and the corresponding velocity limits v_c and v_x, where v_c is the center velocity and v_x the velocity at radius x, noting that in going out from the center dv is negative,
>
> $$(p_1 - p_2)\frac{x^2}{2} = 2L\ (v_c - v_x),$$
>
> whence $$v_x = v_c - \frac{x^2(p_1 - p_2)}{4L}$$ [10]

Since the text is in hydraulics, there are many topics in the book, normally taught in the student's last term of calculus. For example, there are problems involving water pressure on dams, first and second moments, pressure centers,

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9. E. W. Schoder and F. M. Dawson, Hydraulics, Second Edition (New York, 1934), p. 107.
10. Ibid., p. 243.

centroids, and so forth. The following definite integrals are taken at random from the text:

$$Q = \int_0^h L\,dy\,\sqrt{2\,gy} = L\sqrt{2g}\int_0^h y^{1/2}dy =$$

$$Lh\,\frac{2}{3}\sqrt{2\,gh}.$$

$$V = \frac{1}{\pi\,r_2}\int_0^r 2\pi\,x\,dx\,v_x\,,$$

$$P = w\,\sin\theta\int_{z_1}^{z_2} dA\;z^2.$$

Some formulas involving trigonometry are reproduced here:

$$\text{Horizontal distance} = \frac{2}{3}\,b\left[\frac{1}{\cos\theta} - \frac{1}{2}\tan\theta\sin\theta\right],$$

$$a = \frac{V - V\cos a}{t}\,,$$

$$P_x = (1 - \cos a)\,\frac{Qw}{g}\,V\,,$$

$$P_y = \sin a\,\frac{Qw}{g}\,V\,,$$

$$U = \frac{V}{2\cos z}\,.$$

In summary, this text contains some trigonometry, calculus, and even differential equations. Its chief emphasis is algebra, however, and the following table summarizes the mathematical frequencies more completely.

Table 19. Quantitative Survey of the Mathematics in Hydraulics, Second Edition, by E. W. Schoder and F. M. Dawson.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	165
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	21
2. Use of logarithms.	6
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	3
6. Use of vectors.	

Table 19 -- Continued--

Trigonometry	No. of Occurrences
7. Inverse functions.	
8. Graphs of the trigonometric functions.	
Differential Calculus	
1. Differentiation of algebraic expressions.	2
2. Differentiation of transcendental expressions.	
3. Applications of the derivatives.	
4. Partial derivatives.	
Integral Calculus	
1. Integration of algebraic expressions.	8
2. Integration of transcendental expressions.	
3. Applications of the definite integral.	
4. Multiple integrals.	

The fourth text in civil engineering, which is analyzed for mathematical content is Strength of Materials, Fifth Edition, by J. E. Boyd and S. B. Folk.

This text contains more mathematics, on all levels, than any of the books reviewed for this study. It contains mathematics through double integration and second order differential equations. One of our students has declared, "It contains more mathematics than all of the college math books." This is scarcely an exaggeration.

Numerous integrations, and differentiations are made, principally on algebraic, rather than transcendental expressions. There is no stress on radian measure and sinusoidal waves, such as occurred in the texts in electrical engineering; in fact, trigonometry is not so prevalent, as is the calculus. Elementary algebra, including the solution of equations, occurs frequently. The algebra and calculus are so woven in together within the framework of many of the problems, that the book appears almost as a mathematical text.

The following is taken verbatim from the book and is an entire unit of the text:

[Internal Work in a Beam. The unit stress at a distance v from the neutral axis of a beam is $\frac{Mv}{I}$. The internal work or resilience per unit volume is $\frac{S^2}{2E}$. If an element has a cross section dA and length dx, its volume is dA dx and the internal energy is

$$dU = \frac{S^2}{2E} dA\ dx = \frac{Mv^2}{2EI^2} dA\ dx$$

Total work in beam = $\iint \frac{M^2}{2EI^2} v^2\ dA\ dx$

The integration of this equation with respect to v as the variable gives the work done upon the volume of length dx between two vertical planes. Throughout this volume x, M and I are constant. The integral of v^2 dA across the beam from the bottom to the top is I.

$$\text{Work} = \int \frac{M^2}{2EI}\ dx$$

This equation is used to calculate the internal work in any beam. Unless M and I are constant, they must be expressed as functions of x before integrating.

For a uniform beam under constant moment M, Eq. (163.3) becomes

$$U = \frac{M^2}{2EI}\int_0^l dx = \frac{M^2}{2EI}\left[x\right]_0^l = \frac{M^2 l}{2EI}$$

For a beam supported at the ends with uniformly distributed load,

$$M = \frac{wlx}{2} - \frac{wx^2}{2}.$$

$$U = \frac{w^2}{8EI}\int_0^l (l^2x^2 - 2lx^3 + x^4)\ dx$$

$$U = \frac{w^2}{8EI}\left[\frac{l^2x^3}{3} - \frac{lx^4}{2} + \frac{x^5}{5}\right]_0^l = \frac{w^2 l^5}{240\ EI}$$

For a cantilever with a load on the free end, M = - Px. When the section is constant,

$$U = \frac{P^2}{2EI}\int_0^l x^2 dx = \frac{P^2}{6EI}\left[x^3\right]_0^l = \frac{P^2 l^3}{6EI}$$

For a beam supported at the ends with a load P at a distance, a, from one end and at a distance, b, from the other, the reaction at the end of the length a is $\frac{Pb}{I}$ and the moment in this length $\frac{Pbx}{I}$. The work in this part of the beam is

$$U = \frac{P^2 b^2}{2EII^2} \int_0^a x^2 dx = \frac{P^2 b^2 a^3}{6EII^2}$$ [11]

The foregoing was taken from the text, as it typifies so much of the mathematics. None of the above integrations were transcendental, all being algebraic. The following graph is taken from the text:

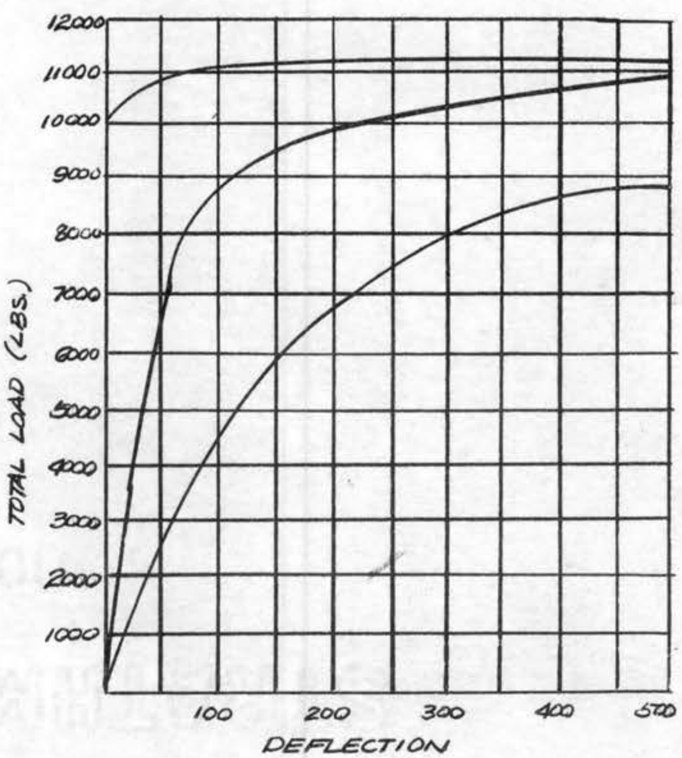

Fig. 30

The following series of differential equations are given from the book, their continuity being undisturbed from the text:

11. J. E. Boyd and S. B. Folk, Strength of Materials, Fifth Edition (New York, 1950), p. 334.

[Successive Differentiation of Equation of Elastic Line. It has been shown in preceding articles that the derivative of the deflection is the slope, the derivative of the slope is the moment, and the derivative of the moment is the sheer. To show some of these relations a series of equations may be written as follows:

$$EI\,\frac{d^4y}{dx^4} = EI\,\frac{d^3\theta}{dx^3} = \frac{d^2M}{dx^2} = \frac{dV}{dx} = w^1$$

in which w' is the load per unit length.

$$EI\,\frac{d^3y}{dx^3} = EI\,\frac{d^2\theta}{dx^2} = \frac{dM}{dx} = V = \text{sheer equation.}$$

$$EI\,\frac{d^2y}{dx^2} = EI\,\frac{d\theta}{dx} = M = \text{moment equation.}$$

$$\frac{dy}{dx} = \theta = \text{slope equation.}$$

y = equation of elastic line.][12]

The following is also verbatim from the text:

[The differential equation is

$$EI\,\frac{d^2y}{dx^2} = -Px$$

$$DI\,\frac{dy}{dx} = -\frac{Px^2}{2} + C_1$$

At the wall, where x = 1 the beam is horizontal and $\frac{dy}{dx} = 0$.

$$EI\,\frac{dy}{dx} = -\frac{Px^2}{2} + \frac{Pl^2}{2}$$

$$EIy = -\frac{Px^3}{6} + \frac{Pl^2x}{2} + C_2$$

At the wall x = 1, y = 0.

$$0 = -\frac{Pl^3}{6} + \frac{Pl^3}{2} + C_2$$

$$C_2 = -\frac{Pl^3}{3}$$

12. Ibid, p. 197.

$$EIy = -\frac{Px^3}{6} + \frac{PI^2}{2} - \frac{PI^2}{3}$$

$$y = -\frac{P}{6EI}(2\ 1^3 - 3\ 1^2 x + x^3)$$

The maximum deflection is at the free end, where x = o.

$$y_{max} = -\frac{PI^3}{3EI}$$

If x = kl, in which k is a fraction less than unity,

$$y = -\frac{PI^3}{6EI}(2 - 3k + k^3)]$$ 13

The following definite integrals are taken at random through the text:

$$T = 2\pi\ S\int_b^a r^2 dr = \frac{2\pi\ S}{3}(a^3 - b^3),$$

$$EIy = -P\int_a^1 x(x-a)dx = -P\left[\frac{x^3}{3} - \frac{ax^2}{2}\right]_a^1 =$$

$$-\frac{p}{6}(2L^3 - 3\ aL^2 + a^3).$$

$$E_y = -\frac{P}{I_m}\int_0^1 x\ dx = -\frac{PI3}{2Im},$$

$$Ey = \frac{-wl^2}{2I_m}\int_a^1 (x-a)dx = -\frac{wl^2(l-a)^2}{4lm}.$$

$$Ey = -30\int_0^{18} x^2 dx - 540\int_{18}^{60} x\ dx = -942,840.$$

$$\text{Total energy} = \frac{\pi k^2 l}{G}\int_0^a r^3\ dr = \frac{\pi l\ K^2\ a^4}{4G}.$$

$$U = \frac{1}{2EI}\int_0^1 (P^2x^2 + Pwx^3 + \frac{w^2x^4}{4})\ dx =$$

$$\frac{I}{2EI}\left[\frac{P^2x^3}{3} + \frac{Pwx^4}{4} + \frac{w^2x^5}{20}\right]^1 = \frac{P^2l^3}{6EI} + \frac{Pwl^4}{8EI} + \frac{w^2l^5}{40EI}.$$

$$U = \frac{Pw}{2EI}\int_a^b (x^3 - ax^2)dx = \frac{Pw}{2EI}\left[\frac{x^4}{4} - \frac{ax^3}{3}\right]_a^1$$

The text treats extensively topics such as moments of inertia, radius of gyration, centroids, and so forth. These

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13. Ibid., p. p. 161 - 162.

topics, of course, are normally given in the student's last term in calculus. The following is taken directly from the text.

> [Center of Gravity of Some Areas. The location of the center of gravity of an area is determined by the expressions
>
> $$\bar{x} = \frac{\int x\, dA}{A} \qquad \bar{y} = \frac{\int y\, dA}{A}$$
>
> The center of gravity of an area is frequently called the centroid. Example: Locate the centroid of the area which is bounded by the X axis, the oridinate $x = x_1$, and the curve $y = x^n$.
>
> $$\text{Area} = \int_0^{x_1} x\, dx = \frac{\left[x^{n+1}\right]_0^{x_1}}{n+1} = \frac{x_1^{n+1}}{n+1}$$
>
> $$\text{Moment of area} = \int_0^{x_1} x^{n+1} dx = \left[\frac{x^{n+2}}{n+2}\right]_0^{x_1} = \frac{x^{n+2}}{n+2}$$
>
> $$\bar{x} = \frac{n+1}{n+2} x_1$$][14]

The following familiar formulas are taken from the text:

$$\frac{1}{p} = \frac{y''}{\left[1 + (y')^2\right]^{3/2}}$$

$$I_x = \int y^2\, dA, \qquad Iy = \int x^2 dZ_x ,$$

$$J = \int (x^2 + y^2)\, dA = \int x^2 dA + \int y^2 dA ,$$

$$I = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - H \sin 2\theta .$$

Although most of the material in this text is algebraic, rather than transcendental, the following formulas, in which trigonometry occurs quite prominently, are taken from the text:

$$S^1 A \sec\theta = SA \sin\theta - SA \cos\theta + SA \tan\theta \sin\theta ,$$

$$S^1 = \tfrac{1}{2} S_x(1 + \cos 2\theta) + S_s \sin 2\theta ,$$

$$P \sin\theta = P_u \cos\theta + A \sec\theta .$$

14. Ibid, p. 383.

In summary, it can be stated that this book, together with F. L. Singer's Engineering Mechanics, contains more complicated mathematics than the other civil engineering texts combined. Elementary algebra, including the solution of linear equations, is widely used, as are formulas in which trigonometric functions appear. Radian measure and trigonometric identities occur frequently. Probably the most noteworthy thing, however, is the predominance of calculus, involving even differential equations. Differentiation and integration of algebraic expressions are particularly prolific. The following table summarizes the mathematical frequencies more completely.

Table 20. Quantitative Survey of Mathematics in Strength of Materials, Fifth Edition, by J. E. Boyd and S. B. Folk.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	150
2. Solution of equations.	22
Trigonometry	
1. Formulas containing trigonometry.	33
2. Use of logarithms.	
3. Numerical solution of triangles.	
4. Use of radian measure.	15
5. Trigonometric identities.	18
6. Use of vectors.	
7. Inverse functions.	
8. Graphs of trigonometric functions.	
Differential Calculus	
1. Differentiation of algebraic expressions.	38
2. Differentiation of transcendental expressions.	8
3. Applications of the derivative.	16
4. Partial derivatives.	

Table 20 -- Continued --

Integral Calculus	No. of Occurrences
1. Integration of algebraic expressions.	58
2. Integration of transcendental expressions.	4
3. Applications of the definite integral.	12
4. Multiple integrals.	5
Higher Mathematics	
1. Differential equations, Fourier series, etc.	12

The fifth text in civil engineering which is analyzed for mathematical content is Field Engineering, Twenty-first Edition, by W. H. Searles and H. C. Ives.

This is not a textbook, in the usual sense, but rather a handbook for the surveyor. Much the greater part of the book comprises only tables. Surveying techniques, useful figures, formulas, and the like, constitute about one-third of the book, the remainder is in useful graphs and tables.

It is difficult to describe the mathematics involved, except to say that it is all with mensuration as an end. As a surveyor's handbook, it simply describes how to measure, and compute.

The authors, in the preface, make the following statement, in regard to mathematics:

> [An elementary knowledge of algebra, geometry, and trigonometry on the part of the reader has been taken for granted, as a command of these instrumentalities is deemed essential to the education of the civil engineer.][15]

15. W. H. Searles and H. C. Ives, Field Engineering, Twenty-first Edition (New York, 1936), p. p. XI, XII.

It is especially important that the student know geometry and trigonometry, especially solutions of triangles, both right and oblique. The versed sine appears prominently throughout the book, as do the functions, secant, and cotangent.

The following figure is taken from the text:

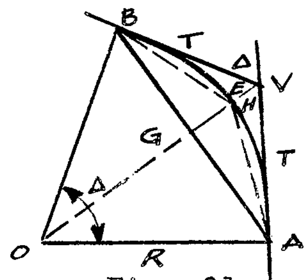


Fig. 31

In the right-angled triangle, VOA, we have,

$$VA = OA \times \tan VOA,$$

$$T = R \tan \tfrac{1}{2} \quad .$$

Referring to the same figure, and formula, the following is from the text:

[Example: What is the tangent distance of a 4° curve with a central angle of 30°?

$D = 4^\circ$ R (Table I)	log	3.156151
$\Delta = 30^\circ$, $\frac{1}{2} = 15^\circ$	logtan	9.428052
Ans. T = 383.89 feet	log	2.584203]

[16]

The mathematics in the foregoing is quite a good example of that required throughout the text. The geometry, trigonometry, logarithms, and arithmetic computation needed for the solution is quite evident.

Since most of the mathematics is repetitious, it is needless to point out many explicit formulas, or problems. The following, however, are formulas, taken at random from

16. Ibid., p. 44.

the text:

$$\sin \tfrac{1}{2} d = \frac{c}{100} \sin \tfrac{1}{2} D ,$$

$$t_2 = (100 - c) \sin \tfrac{1}{2} (D - d) ,$$

$$M^1 = R \text{ vers } \tfrac{1}{2} (\Delta - 2D) ,$$

$$R - R^1 = \frac{E - E^1}{e \sec \tfrac{1}{2}\Delta} ,$$

$$R^1 = R \tan \tfrac{1}{2}\Delta \cot \tfrac{1}{2}\Delta.$$

In summary, the text contains formulas, both geometric, and trigonometric, designed as an aid in measurements. The following table summarizes the mathematical frequencies more completely.

Table 21. Quantitative Survey of the Mathematics in Field Engineering, Twenty-first Edition, W. H. Searles and H. C. Ives.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	120
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	116
2. Use of logarithms.	65
3. Numerical solution of triangles.	12

The sixth text in civil engineering which is analyzed for mathematical content is Substructure Analysis and Design, by Paul Andersen.

The mathematics occurring in this book is rather complex, even covering such topics in calculus as hyperbolic functions. In general, however, it does not go beyond that covered in the undergraduate course in the calculus.

There is extensive use of algebra, especially algebraic formulas. The trigonometry involved is not very complex, consisting largely of definitions of sine, cosine, and tangent. The trigonometric expressions, $\tan^2 (45 - \frac{\theta}{2})$, $\tan^3 (45 - \frac{\theta}{2})$, $\tan (45 - \frac{\theta}{2})$, occur, again and again, although there is not an instance in which such expressions are expanded. The calculus is, on the whole, rather elementary, with the exception just referred to, namely, the occurrence of hyperbolic functions.

The following figure is taken from the text.

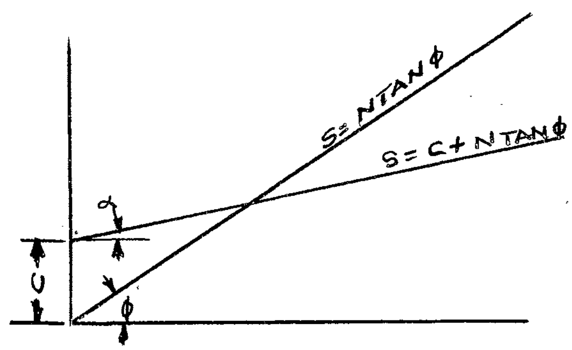


Fig. 32

The symbols, $>$ and $<$, occur often, not only in this text, but in numerous others. For example, the expression, $P_t > P_n \tan\phi + C$ occurs in the text, which also shows the need for the student's understanding of undetermined constants.

The following formula, involving trigonometry is taken from the text:

$$p = q \tan^2 (45 - \frac{\phi}{2}) - 2c \tan (45 - \frac{\phi}{2}), \quad (1 - 12).$$

The same equation, solved for q, gives,

$$q = p \tan^2 (45 + \frac{\phi}{2}) + 2c (\tan 45 + \frac{\phi}{2}). \quad (1 - 13)$$

Substituting in (1 - 12), $p = o$, $q = 100x$, $\phi = 20^\circ$, $C = 400$, the following equation occurs:

$$100x \tan^2 (45 - \tfrac{1}{2}\, 20) - 2 \cdot 400 \tan (45 - \tfrac{1}{2}\, 20) = 0,$$

$$x = 11.4 \text{ ft.}$$

The skills involved in this example are self-evident, and reveal certain algebraic and trigonometric needs of the student. Since this is offered in the text, without any intervening steps, it is evident that the student is expected to supply those steps from his own background of algebra and trigonometry.

The following, are additional formulas, in which trigonometry occurs quite prominently:

$$x = \frac{1}{\tan v} = \tan\phi + \sqrt{1 + \tan^2\phi - \frac{\tan \delta}{\sin\phi \cos\phi}} \ ,$$

$$p_a = \frac{Wh^2}{2}\left[\frac{\cos\phi}{1 + \sqrt{\sin\phi\,(\sin\phi - \cos\phi \tan\delta)}}\right]^2 ,$$

$$EC = h\left[\frac{1 + \sqrt{\sin\phi(\sin\phi - \cos\phi \tan\delta}}{\sin\phi - \cos\phi\tan\delta}\right] ,$$

$$FE = \frac{h \cos\phi}{1 - \sqrt{\sin\phi(\sin\phi - \cos\phi\tan\delta}} .$$

As in most of the engineering texts, there are some rather complex numerical computations. The following, from the text, are typical:

$$\frac{6000}{\frac{\pi}{4}\, 22^2} + \frac{150 \text{ X } 100}{1000} = 30.8 \ ,$$

$$h = \sqrt[3]{\frac{24,000 \text{ X } 46.8}{2 \text{ X } 62.5}} = 20.8 \text{ ft.} \ ,$$

$$B = \frac{3 \text{ X } 2,870,000}{100 \text{ X } 64^2 \text{ X } \tan^2(45^\circ - \frac{1}{2}\, 30^\circ)\ (\tan 30^\circ + 0.3)} = 72 \text{ ft},$$

$$P_3 = \frac{0.2441}{0.3470} \text{ X } 10^4 \ .$$

The following trigonometric equation is given, the solution coming from the student's background:

$$\tan^2 (45 - \frac{\phi}{2}) = \frac{1}{2},$$

$$= 19^\circ\ 30'.$$

Again the student must rely on his background, as the text offers the following quadratic equation:

$$2800\ a^2 + 1200 \times 4a = 94,000,$$

$$a = 5 \text{ ft.}$$

The calculus employed involves both differentials and integrals, of trigonometric, and purely algebraic functions. A few differential equations are solved, and maximum and minimum values are computed. As an example of the latter, the following is taken from the text:

$$P_A = \frac{Wh^2}{2} \times \frac{\tan v - \tan^2 v \tan \phi}{\tan v + \tan \phi}.$$

Another equation of interest is taken from the text:

$$P_A = \frac{Wh^2}{2} \times \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{wh^2}{2} \times \tan^2 (45 - \frac{\phi}{2}).$$

Since the student is expected to supply all the omitted material, from the foregoing maximum computation, it is certainly evident that it is expected of him, that he be proficient in the calculus, trigonometry, and algebra.

The following is taken from the text to illustrate the student's basic needs for differentials:

$$v\ b\ dx = dT; \quad v = \frac{1}{b}\frac{dT}{dx}, \qquad (4 - 6)$$

but

$$T = \frac{M}{jd}; \text{ so } \frac{dT}{dx} = \frac{1}{jd}\frac{dM}{dx},$$

therefore

$$\frac{dM}{dx} = jd\frac{dT}{dx}, \qquad (4 - 7)$$

then Eq. (4 - 6) becomes $v = \frac{1}{bjd} \frac{dM}{dx}$ (4 - 8)

The integration involved throughout the text is not very complex, but, as previously, the student is expected to supply details. The following is typical:

$$\Delta = \frac{1}{EA}\int_0^L (p - \tfrac{1}{2} f_x x)\, dx \,. \qquad (6 - 27)$$

Since $f_x = \frac{x}{L} f_t$, and $\frac{1}{2} f_t L = P$, (6 - 28)

then (6 - 27) yields, $\Delta = \frac{1}{EA}\int_0^L (P - \frac{x}{L^2} P)\, dx$

$$= \frac{1}{EA} \times \frac{2}{3} L \times P.$$

In summary, the text stresses algebra and trigonometry, and to a lesser extent, differential and integral calculus. Solution of algebraic and trigonometric equations, both occur. The computation of numerical values from a given formula also occurs frequently. The student is expected to have a good basic understanding of algebra and trigonometry in order to be able to read this text. The following table summarizes the mathematical frequencies more completely.

Table 22. Quantitative Survey of the Mathematics in *Substructure Analysis and Design*, by Paul Andersen.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	118
2. Solution of equations.	40
Trigonometry	
1. Formulas containing trigonometry.	56
2. Use of logarithms.	
3. Numerical solution of triangles.	1
4. Use of radian measure.	
5. Trigonometric identities.	

Table 22 -- Continued --

Trigonometry	No. of Occurrences
6. Use of vectors.	
7. Inverse functions.	3
8. Graphs of trigonometric functions.	
Differential Calculus	
1. Differentiation of algebraic expressions.	1
2. Differentiation of transcendental expressions.	
3. Applications of the derivative.	1
4. Partial derivatives.	
Integral Calculus	
1. Integration of algebraic expressions.	2
2. Integration of transcendental expressions.	
3. Applications of the definite integral.	
4. Multiple integrals.	
Higher Mathematics	
1. Differential equations, Fourier series, etc.	7

The seventh text in civil engineering which is analyzed for mathematical content is Structural Design, by Hale Sutherland and H. L. Bowman.

This book contains a moderate amount of mathematics. It emphasizes trigonometry, and algebraic formulas, in which radicals and fractional exponents occur.

Although the mathematics is, generally, not very complex, it has a few rather unique characteristics; for example, it makes use of hyperbolic functions, a topic which only occurs rarely in textbooks of engineering. Also, the secant function is used, along with sines, cosines, and tangents, which is again a little unusual for engineering books. Cartesian coordinates are used, with rotation of axes.

It should not be inferred, however, because of these unique characteristics, that the text contains intricate mathematics. With a few exceptions, it is quite moderate. Further, the mathematics is not extensive, as the early part of the text, which is the theoretical part, contains virtually all the mathematics employed.

In some places, the arithmetic computation is difficult, though, perhaps, no more so than most of the books. The following are taken from the text:

$$h = 60.5 - 2\left[2.23 - \frac{20.25\ (2.23 + 0.56)}{20.25 + 19.22}\right]$$

$$= 58.90 \text{ in.}$$

$$s = P\ \left(\frac{0.108}{2}\right) + \sqrt{\left(\frac{0.108}{2}\right)^2 + (0.167)^2}$$

$$= P\ (0.054 + 0.175) = 0.229P\ .$$

Some of the algebraic formulas are a little unusual. The following are taken from the text:

$$V = P\ \frac{2r^2\left(\frac{ec}{r_2}\right)}{LC}\ ,$$

$$\frac{M_1 - M_2}{h_1} = \frac{Vp}{h_2}\ ,$$

$$S_b = \frac{P(D + e)c}{I}\ .$$

A great many of the formulas contain radicals, or fractional exponents. The following are examples from the text:

$$d = \sqrt{\frac{3M}{asx}} = \sqrt{\frac{3M\left(1 + \frac{a}{b}\right)}{as}}\ ,$$

$$r = \sqrt{\left(\frac{Vp}{h}\right)^2 + (mp)^2} = p\ \sqrt{\left(\frac{V}{h}\right)^2 + m^2},$$

$$a = 0.806g\sqrt{\frac{I_y}{k}}\ .$$

The following, from the text, is an excellent example of some of the algebraic skills needed by the student:

$$\frac{a(xd)^2}{2} = \frac{bd^2(1 - x)^2}{2} .$$

Taking the square root, $x\sqrt{\frac{a}{b}} = 1 - x$,

and solving for x, $x = \frac{1}{1 + \sqrt{\frac{a}{b}}}$

The following equations of rotation are given, and several examples worked by them:

$$x_1 = \cos\phi + y \sin\phi$$
$$y_1 = \cos\phi - x \sin\phi$$

The following equations show the use of hyperbolic functions:

$$Q = \tanh \frac{L}{2a} - \frac{2a}{L} \left(1 - \frac{1}{\cosh \frac{L}{2a}}\right) ;$$
$$R = \frac{L}{2a} - \tanh \frac{L}{2a} ,$$
$$\psi = \frac{Ta}{KE} \left[- \tanh \frac{L}{a} + \frac{a}{L} \operatorname{sech}\left(\frac{L}{a} - 1\right) + \frac{L}{2a}\right] .$$

Trigonometry occurs so frequently, that it is probably safe to say that it is the dominant topic of the text. The following equations are given:

$$M \sin\theta = s_1 I_x \cos\alpha \qquad (A)$$

$$M \cos\theta = s_1 I_y \sin\alpha \qquad (B)$$

Dividing (B) by (A), we obtain,

$$\tan\alpha = - \frac{I_x}{I_y} \cot\alpha . \qquad (1 - 3)$$

The algebraic, and trigonometric, skills are self-evident here. The student must know, at least, the fundamental identities of trigonometry. The following formulas, in which trigonometric functions occur, are taken from the text:

$$A = \frac{P}{A} + \frac{P}{A}\frac{ec}{r2} \sec \frac{L}{2r}\sqrt{\frac{P}{EA}} ,$$

$$S = \frac{(M \sin\theta)y}{I_x} + \frac{(M \cos\theta)x}{I_y} .$$

$$S = \frac{N}{A} + \frac{(Ne \sin\theta)y}{I_x} + \frac{(Ne \cos\ )x}{I_y}$$

Although calculus does not play a prominent part in the text, it does occur in a few places. The following are taken from the text:

$$M \sin\theta = \int S_1\ v\ dA\ (y) ,$$

Since $v = y \cos\alpha - x \sin\alpha$, this expression becomes,

$$M \sin\ = \int S_1 (y^2 \cos\alpha - xy \sin\alpha)\ dA.$$

Only one differential equation occurs in the entire text. The student is expected to solve the equation from his own background of experience, as the text does not do so for him. The following equation is given:

$$\frac{d^2y}{dx^2} = - \frac{P(y + e)}{EI} .$$

The general integral of this differential equation is

$$y = C_1 \sin\alpha x + C_2 \cos\alpha x - e .$$

Since $y = 0$ when $x = 0$,

$$C_2 = e.$$

Also, since $y = o$ when $x = L$,

$$0 = C_1 \sin\alpha L + e \cos\alpha L - e,$$

$$C_1 = \frac{e(1 - \cos\alpha L)}{\sin\alpha L}$$

$$= e \tan\frac{\alpha L}{2} .$$

The general integral then takes the form,

$$y = e\ (\tan \frac{\alpha L}{2} \sin \alpha x + \cos \alpha x - 1)\ .$$

In summary, the student's needs for this text are principally algebra and trigonometry, although some calculus occurs on all levels. The following table summarizes the mathematical frequencies more completely:

Table 23. Quantitative Survey of the Mathematics in Structural Design, by Hale Sutherland and H. L. Bowman.

	Algebra	No. of Occurrences
1.	Elementary processes and formulas.	85
2.	Solution of equations.	
	Trigonometry	
1.	Formulas containing trigonometry.	20
2.	Use of logarithms.	
3.	Numerical solution of triangles.	
4.	Use of radian measure.	
5.	Trigonometric identities.	5
6.	Use of vectors.	
7.	Inverse functions.	
8.	Graphs of trigonometric functions.	
	Differential Calculus	
1.	Differentiation of algebraic expressions.	4
2.	Differentiation of transcendental expressions.	
3.	Applications of the derivative.	
4.	Partial derivatives.	
	Higher Mathematics	
1.	Differential equations, Fourier series, etc.	5

The eighth text in civil engineering which is analyzed for mathematical content is Structural Drafting, Second Edition, by C. T. Bishop.

This book does not contain much mathematics. It is in the nature of a handbook, though not entirely that, either. Nothing beyond algebraic formulas and arithmetic computations occur. There is no trigonometry, or calculus, whatever.

The following algebraic formulas are typical of those of the entire book:

$$\frac{VB}{Dr} + 12B = \frac{V}{12Dr} = \frac{V}{dr} ,$$

$$mr = \frac{fI}{c} = \frac{2fI}{d} .$$

The following arithmetic computations are also typical:

$$3{,}270 = \frac{4{,}290}{.25(7 - 2 \times 7/8)} ,$$

$$0.6 = \frac{36 \times 43 \times 304}{2 \times 29{,}000{,}000 \times 442} .$$

The following table summarizes the mathematical frequencies more completely.

Table 24. Quantitative Survey of the Mathematics in Structural Drafting, Second Edition, by C. T. Bishop.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	60
2. Solution of equations.	9
Trigonometry	
1. Formulas containing trigonometry.	3
2. Use of logarithms.	8

Since this completes the texts in civil engineering, they are now summarized, both quantitatively and qualitatively.

Summary. Civil engineering is replete with mathematics, as these texts attest. It is not unlike that of electrical engineering, with the stress on algebra, trigonometry, and integral calculus. Of the eight texts considered, five of them contained calculus, and all of them trigonometry.

The following table gives an overall total, that is, it gives the total number of texts in which the given topic occurred, as well as the total occurrences for all the texts.

Table 25. Quantitative Summary of the Eight Texts in Civil Engineering

	Algebra	No. of Texts Occurring	No. of Occurrences
1.	Elementary Processes and formulas.	8	1057
2.	Solution of Equations.	4	108
	Trigonometry		
1.	Formulas containing trigonometry.	8	326
2.	Use of logarithms.	4	91
3.	Numerical solution of triangles.	3	51
4.	Use of radian measure.	2	59
5.	Trigonometric identities.	5	63
6.	Use of vectors.	1	212
7.	Inverse functions.	2	7
8.	Graphs of trigonometric functions.	1	3
	Differential Calculus		
1.	Differentiation of algebraic expressions.	4	45
2.	Differentiation of transcendental expressions.	1	8
3.	Application of the derivatives.	2	17
4.	Partial derivatives.		

Table 25 -- Continued

Integral Calculus	No. of Texts Occurring	No. of Occurrences
1. Integration of algebraic expressions.	4	108
2. Integration of transcendental expressions.	2	22
3. Application of the definite integral.	2	34
4. Multiple integrals.	2	11
Higher Mathematics		
1. Differential equations, Fourier series, etc.	4	39

PART IV

A SURVEY OF THE MATHEMATICS IN THE MECHANICAL ENGINEERING TEXTBOOKS

Introduction. The texts considered within this chapter are given here in the order in which the student studies them. They are, as follows: Elementary Heat Power, H. L. Solberg, O. C. Cromer, and A. R. Spalding; Thermodynamic Properties of Steam, J. H. Keenan and F. G. Keyes; Internal Combustion Engines, Second Edition, E. F. Obert; Power Plant Testing, Fourth Edition, J. A. Moyer; Applied Kinematics, Third Edition, J. H. Billings; Design of Machine Elements, Second Edition, M. F. Spotts; Steam Power Stations, Third Edition, G. A. Gaffert; Applied Thermodynamics, V. M. Faires; Refrigeration, Second Edition, J. A. Moyer and R. V. Fittz; Steam Turbines, Second Edition, E. F. Church, Jr.; Heating, Ventilating, and Air Conditioning Fundamentals, Second Edition, W. H. Severns and J. R. Fellows.

The procedures are followed as explained in Chapter 1. At the conclusion of each text will be found a short summary, and a table.

The first of the texts analyzed for mathematical content is Elementary Heat Power, by H. L. Solberg, O. C. Cromer, and A. R. Spalding.

This text does not contain much mathematics. Algebra occurs only within algebraic formulas; a little use is made of vectors, a few formulas occur using trigonometry, and only in one place does calculus appear at all.

The following formula for work, due to a change in volume of gas is given:

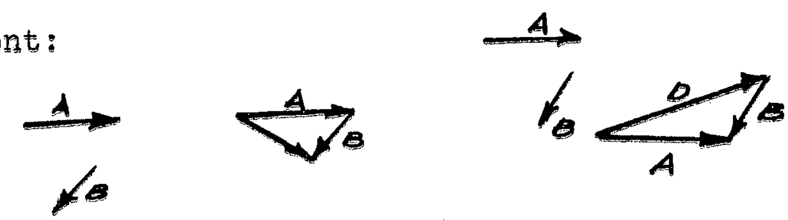
$$W = 144\int_{v_1}^{v_2} p\ dV\ .$$

Applying this formula,

$$W = 144\int_{v_1}^{v_2} p\ dV = 144p\int_{v_1}^{v_2} dV = 144\ p\ (V_2 - V_1)\ ,$$

and, $W = 144 \times 100(2 - 1) = 14{,}400$ ft-lb.

The foregoing consists of all the calculus in the text. The following figure, from the text, introduces vectors to the student:



Vector A + Vector B = Vector C + Vector A - Vector B = Vector D

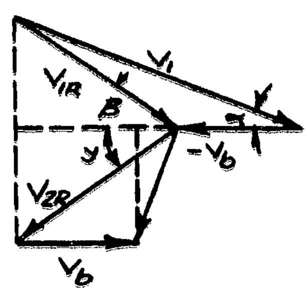


Fig. 33

From this figure, the following formula, involving trigonometry is given:

$$V_b = \frac{V_1 \cos\alpha}{2},\ \text{or}\ \frac{V_b}{V_1 \cos\alpha} = \tfrac{1}{2}.$$

Applying this formula when $\alpha = 20^{\circ}$, and $V_1 = 4240$ fps.,

$$V_b = \frac{4240 \cos 20}{2}$$

$$= 1990 \text{ fps.}$$

This is typical of the trigonometry in this text. It occurs only in a few formulas, and then, very simply.

The following graph is taken from the text, as several of this nature occur:

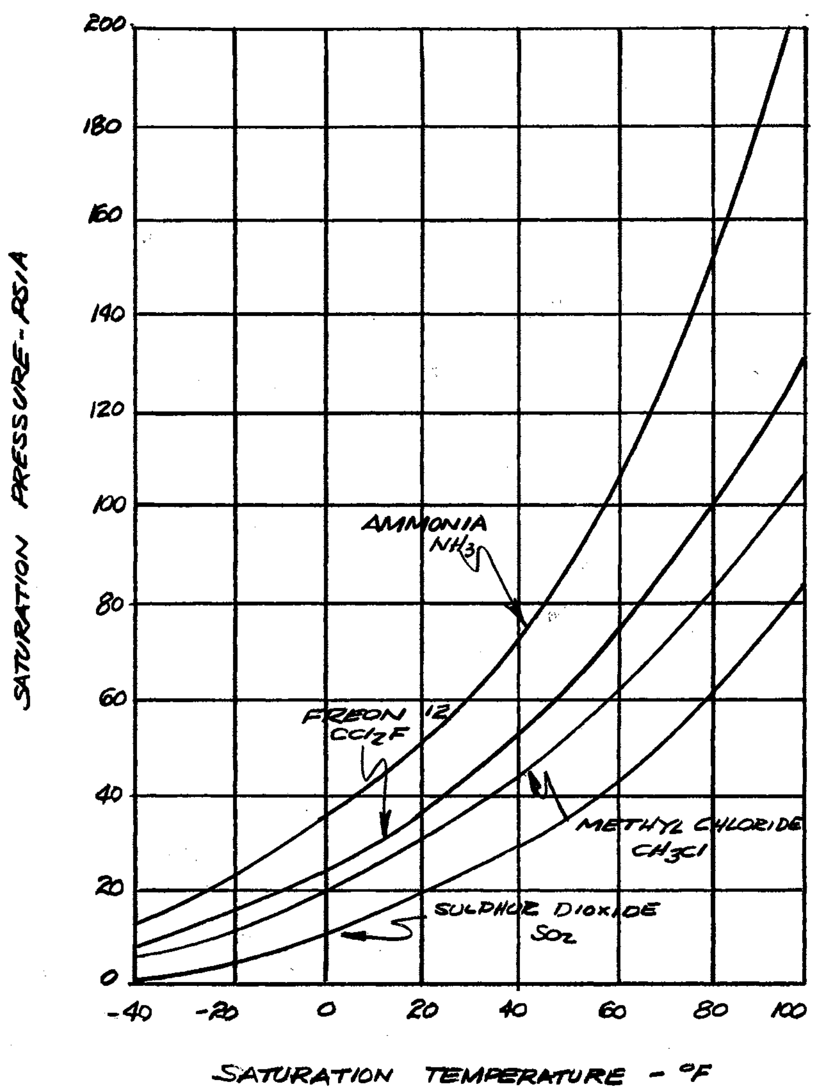


Fig. 34

The algebra in this text is not very complex. It occurs in formulas, and numerical substitutions are frequently made. The following is characteristic:

$$A_x = \frac{V_f - V_i}{T} ,$$

$$F_x = \frac{w}{g} (V_f - V_i) \frac{1}{T} ,$$

$$\text{Work} = \frac{w}{g} (V_f - V_i) \frac{S}{T} .$$

In summary, this text contains very little mathematics, almost no trigonometry or calculus, and only very elementary algebra. The following table summarizes the mathematical frequencies more completely:

Table 26. Quantitative Survey of the Mathematics in Elementary Heat Power, by H. L. Solberg, O. C. Cromer and A. C. Spalding.

	No. of Occurrences
Algebra	
1. Elementary processes and formulas.	80
2. Solution of equations.	3
Trigonometry	
1. Formulas containing trigonometry.	6
2. Use of logarithms.	
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	13
Integral Calculus	
1. Integration of algebraic expressions.	2

The second of the texts which is analyzed for mathematical content is Thermodynamic Properties of Steam, by J. H. Keenan and F. G. Keyes.

This is not a textbook in the usual sense. It is, rather, a short book of steam tables, which serve as materials from which this course is taught. However, in the introductory chapter, the authors have written many formulas containing rather complex mathematics. Most of these are here reproduced, and numbered precisely as they are in the text:

$$h = u + pv, \text{ or } h_1 - h_2 = u_1 - u_2 + (pv)_1 - (pv)_2 \quad (1)$$

$$\left.\begin{aligned} dh &= T\, ds + v\, dp \\ dh &= cp\, dT - \left[T\left(\frac{dv}{dT}\right)_p - v\right] dp \\ &= cp\, dT + \left(\frac{\partial (v\tau)}{\partial \tau}\right)_p dp \\ dh &= \left[c_v + v\left(\frac{\partial p}{\partial T}\right)_v\right] dT + \left(\frac{\partial (v\tau)}{\partial \tau}\right)_p \left(\frac{\partial p}{\partial v}\right) dv \end{aligned}\right\} . \quad (2)$$

$$\left.\begin{aligned} C_p &= \left(\frac{\partial u}{\partial T}\right)_p + p\left(\frac{\partial v}{\partial T}\right)_p ; \quad c = \left(\frac{\partial u}{\partial T}\right)_u \\ C_{sat} &= c_p - T\left(\frac{\partial v}{\partial T}\right)_p \frac{dp}{dt} = c + T\left(\frac{\partial p}{\partial T}\right)_v \frac{dv}{dt} \end{aligned}\right\} . \quad (3)$$

$$(\text{h constant}); \left(\frac{\partial T}{\partial p}\right) = \mu = - \frac{\left(\frac{\partial (v\tau)}{\partial \tau}\right)}{c_p} , \quad (4)$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p ; \quad -\mu c_p = \left(\frac{\partial h}{\partial p}\right) = \left(\frac{\partial (v\tau)}{\partial \tau}\right)_p . \quad (5)$$

$$c_p = c_1 - \int \tau \left(\frac{\partial^2 v}{\partial T^2}\right)_p dp = c_1 - \frac{\partial}{\partial T} \int \left(\frac{\partial (v\tau)}{\partial \tau}\right)_p dp . \quad (6)$$

$$\left(\frac{\partial s}{\partial p}\right)_t = - \left(\frac{\partial v}{\partial T}\right)_p ; \quad \left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T} . \quad (7)$$

$$v = \left[v_r \frac{T}{T_r} - T \int \mu c_p \, d\tau\right]_p . \quad (8)$$

$$L = T \frac{dp}{dT} (V_g - V_f) . \quad (9)$$

$$C_g - C_f = \frac{dL}{dT} + (\mu_p)_g - (\mu_p)_f \frac{dp}{dT} . \quad (10)$$

$$\log_{10} \frac{p_c}{p} = \frac{x}{T} \left[\frac{a + bx + cx^3 + ex^4}{1 + dx}\right] . \quad (11)$$

$$\log_{10} \frac{p_c}{p} = \frac{x}{T} \left[\frac{a^1 + b^1 x + c^1 x^3}{1 + d^1 x}\right] . \quad (12)$$

The following graph is taken from the text:

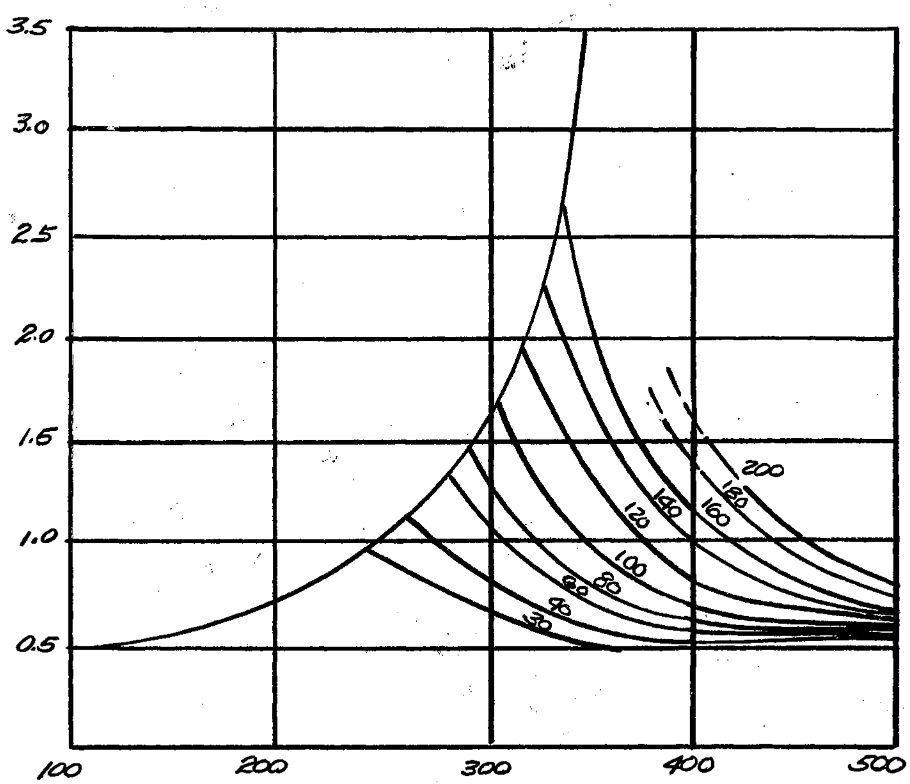


Fig. 35

The foregoing formulas, and graph, is almost in itself a summary of the mathematics in the text, but the following table gives a more complete summary of the mathematical frequencies.

Table 27. Quantitative Survey of the Mathematics in <u>Thermodynamic Properties of Steam</u>, by J. H. Keenan and F. G. Keyes.

	No. of Occurrences
Algebra	
1. Elementary processes and formulas.	28
2. Solution of equations.	3
Trigonometry	
1. Formulas containing trigonometry.	3
2. Use of logarithms.	3

Table 27 -- Continued --

Differential Calculus	No. of Occurrences
1. Differentiation of algebraic expressions.	23
Integral Calculus	
1. Integration of algebraic expressions.	8
2. Integration of transcendental expressions.	6

The third text in mechanical engineering which is analyzed for mathematical content is Internal Combustion Engines, Second Edition, by E. F. Obert.

There is a great deal of mathematics in this text, but for the most part, on an elementary level. There is no trigonometry although there are some definite integrals occurring. The following numerical computations occur in the early part of the text:

$$\text{(a) hp.} = \frac{2 \times 440 \times 2 \times 1{,}140}{33{,}000} = 191.$$

$$\text{(b) fhp.} = \frac{PN}{3000} = \frac{30.3 \times 3{,}300}{3{,}000} = 33.3$$

$$\text{(c) } \frac{\text{bhp.}}{\text{ihp.}} = \frac{\text{bhp.}}{\text{bhp.} + \text{fhp.}} = \frac{90}{90 + 33.3} = 0.731 = 73.1\%.$$

As an example of the kind of graphs the student is called on to read, the following is taken from the text:

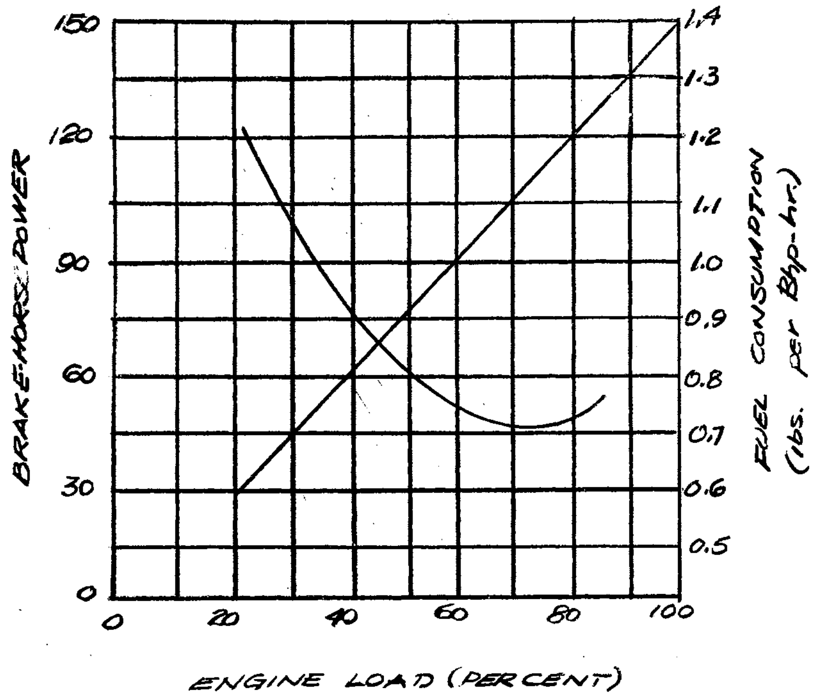


Fig. 36

Work and energy formulas are developed, making use only of algebra. The following formulas are developed by physical laws:

$$Q - W = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2g_c J} ,$$

$$m_2 = \frac{A_2 V_2}{V_2} .$$

This is typical of much of the algebra appearing in the text. Calculus is used in the early part of the book. Definite integrals occur within formulas, the following of which typifies:

$$C = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} c_x \, dT .$$

$$\Delta u = \int_{T_1}^{T_2} c \, dT .$$

There are also many differential equations. The following are typical examples:

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v} .$$

$$T\ ds + v\ dp = c_p\ dT.$$

Many formulas occur where an understanding of proportion, or variation, is necessary. For example, the following gas equations are given:

$$\frac{pv_1}{T_1} = \frac{p_2 v_2}{T_2}, \text{ or } \frac{T_2}{T_1} = \frac{p_2 v_2}{p_1 v_1} .$$

Similar equations are, as follows:

$$\frac{p_1}{p_2} = (\frac{v_2}{v_1})^n, \text{ or } \frac{v_1}{v_2} = (\frac{p_2}{p_1})^{1/n}.$$

$$\frac{T_2}{T_1} = (\frac{v_1}{v_2})^{n-1}, \text{or } \frac{T_2}{T_1} = (\frac{p_2}{p_1})^{\frac{n-1}{n}}.$$

These latter equations show the need for an elementary understanding of fractional exponents. The following equation also shows the need for a knowledge of exponents and radicals:

$$\frac{V_2}{v_2} = 224 \sqrt{\frac{C_p T_1}{V_2} \quad (\frac{p_2}{p_1})^{2/R} - (\frac{p_2}{p_1})^{R+1/R}} .$$

The following algebraic equation is taken from the text:

$$\frac{(1 - x)(2 + x)^{1/2}}{x^{3/2}} = 5 .$$

After squaring each side and reducing, the following cubic equation is obtained,

$$24x^3 + 3x = 2 ,$$

the solution of which is $x = .344$.

Since no solution is offered for this equation, it presumes that the student knows already how to solve it. This is, by far, the most complex algebra to occur in this text.

Ratio and proportion continue rather extensively, as the following formulas from the text reveal:

$$\frac{Q_a + Q_r}{Q_A} = 1 - \frac{T_a - T_a}{T_c - T_b} = 1 - \frac{T_a}{T_b}\,\frac{\left(\frac{T_d}{T_a} - 1\right)}{\left(\frac{T_c}{T_b} - 1\right)} .$$

$$\frac{T_a}{T_b} = \left(\frac{V_b}{V_a}\right)^{k-1} = \left(\frac{V_c}{V_d}\right) = \frac{T_d}{T_c} .$$

There is a great deal of mathematics of the type where either logarithms or a slide-rule would be needed. The following typifies:

$$14.7(8)^{1.4},$$
$$1318\left(\frac{1}{8}\right)^{1.285}.$$

The absolute value sign is used in the text, as are the symbols, $>$, $<$. The following demonstrate:

$$1 - \frac{|Q_k|}{Q_a} = \frac{W}{Q_a} = \frac{W}{W + |Q_r|} .$$

More formulas occur in which the definite integral is used, as in the following:

$$W = \int_c^d p\ dv = \int_a^b p\ dv ,$$

$$S_2 - S_1 = n\int_{T_1}^{T_2} c_p \frac{dT}{T} - n\ R_o \ln \frac{p_2}{p_1} .$$

The student is expected to know scientific notation, as the following reveals:

$$\frac{1.218 \times 10^{-3}}{1 + 15} = 7.6 \times 10^{-5}\ \text{lb.}$$

The integrations involved in this text are both transcendental and algebraic. Some of them occur with some complexity, and the algebra involves exponents, to fractional and negative powers.

To show how complex the algebra may become, the following formulas are given:

$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{1-R},$$

$$m_a = \frac{1.62\ c_a p_1 d_2}{\sqrt{T_1}} \sqrt{\left(\frac{p_2}{p_1}\right)^{1.43} - \left(\frac{p_2}{p_1}\right)^{1.71}} \ .$$

These formulas give numerical results for V_2 v_2 and m_a, respectively, after the various constants are obtained from tables in the text. The need for logarithmic computation is evident. The following algebraic formulas are taken from the text:

$$p = \left|C\ (2N - \frac{N}{2})\right| = \frac{3}{2}\ CN \quad ,$$

$$p = \left|C\ (2\ \frac{N}{2} - \frac{N}{2})\right| = \frac{1}{2}\ CN \quad .$$

In summary, the text stresses the development, by algebraic methods, of formulas to explain physical phenomena. Numerical computation, including the use of logarithms, occurs frequently. Algebra, including exponents and radicals, also occurs frequently. Graphs of various kinds occur in every chapter. Calculus is not used much, although definite integrals of a very elementary type occur in a few places. There is no trigonometry. The following table summarizes the mathematical frequencies more completely:

Table 28. Quantitative Survey of the Mathematics in <u>Internal Combustion Engines, Second Edition</u>, by E. F. Obert.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	101
2. Solution of equations.	8

Table 28 -- Continued --

Integral Calculus	No. of Occurrences
1. Integration of algebraic expressions.	13
2. Integration of transcendental expressions.	8
3. Application of the definite integral.	4
Higher Mathematics	
1. Differential equations, Fourier series, etc.	3

The fourth text in mechanical engineering which is analyzed for mathematical frequency is <u>Power Plant Testing, Fourth Edition</u>, by J. A. Moyer.

This text contains only a limited amount of mathematics, none of which is very complex. There is virtually no calculus and not much trigonometry; the greater portion of the mathematics is comprised of algebra. Logarithms occur in several places, as do a few formulas involving trigonometry.

Chapter IV of the text is titled, <u>Measurement of Areas</u>. This chapter involves a little elementary geometry, and includes such well known formulas as $A = \pi r^2$, $C = \pi d$, and so forth. It includes a treatise on approximation of areas, for example, the trapezoidal rule, and Simpson's rule. Also, the chapter makes use of polar co-ordinates.

A few algebraic formulas are reproduced here, to demonstrate some of the types:

$$\text{bhp.} = \frac{3.18}{10} n^2 d^5 ,$$

$$\text{hp.} = \frac{144k}{33,000(k-1)} pv \left[\left(\frac{p_b}{p_a}\right)^{0.29} - 1\right] ,$$

$$\text{hp.} = \frac{(h_1 - h_2) \times W_a \times 778}{33,000 \times 60} ,$$

$$T_1 - T_2 = \frac{wR}{r},$$

$$T_1 = \frac{\frac{Wb}{a} \quad \frac{wR}{r}}{2}.$$

The following equations are taken from the text to show how trigonometry occurs in various formulas:

$$WX = PW^1 \; X_c \; X \cos \; W^1 \; WX,$$

$$PW^1 \cos PW^1X = PO^1 \cos PO^1 Y - W^1O^1.$$

$$\text{area } TPT^1 = \tfrac{1}{2}c\,(\overline{PO}^2 + \overline{OT}^2 + 2\,\overline{PO} \cdot \overline{OT} \cos a),$$

$$f = \frac{W^1R \sin a}{rP}.$$

Although logarithms are not for computations, they do occur in several formulas throughout the text, and the student will need to know their laws. The following, from the text is taken to show their occurrence:

$$\text{Logarithmic mean difference} = \frac{t_e - t_i}{\log_e \frac{t_e - t_i}{t_e - t_o}}.$$

$$K = \frac{W}{S} \log_e \frac{t_s - t_i}{t_s - t_o},$$

$$f = \frac{\log \frac{T_1}{T_2}}{0.434c}.$$

The following graph is taken from the text:

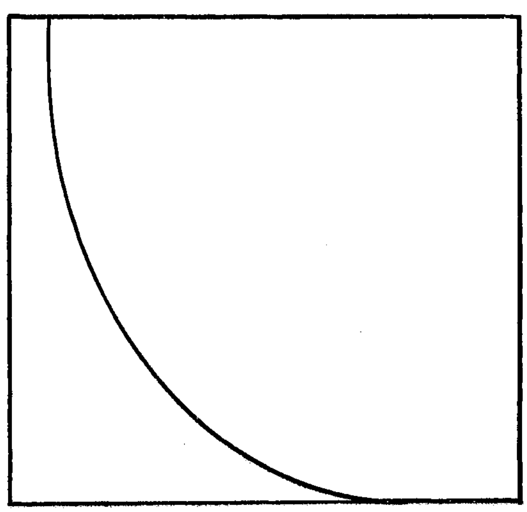


Fig. 37

The algebra employed is largely composed of formulas, and few algebraic operations are employed. For instance, there are no solutions of equations, and no use made of special factors.

In summary, this text contains, largely, only elementary algebra. The following table summarizes the mathematical frequencies a little more completely:

Table 29. Quantitative Survey of the Mathematics in *Power Plant Testing, Fourth Edition*, by J. A. Moyer.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	147
Trigonometry	
1. Formulas containing trigonometry.	10
2. Use of logarithms.	3
Integral Calculus	
1. Integration of algebraic expressions.	1

The fifth text which is analyzed for mathematical content is *Applied Kinematics, Third Edition*, by J. H. Billings.

While this text does not contain such a great quantity of mathematics, it does occur on fairly complex levels. There is some algebra, trigonometry, and differential calculus. There is no integral calculus to be found in the entire text. This book considers such engineering topics as gears, cams, wheels, pulleys, and so forth. As would be expected from such topics, trigonometry, radian measure, angular acceleration and velocities are used often. Also, vectors occur quite frequently.

The following familiar forms are taken from the text:

[Linear | Angular |
---|---|---
$V = \frac{ds}{dt}$ | $= \frac{de}{dt}$ | (1)
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ | $\alpha = \frac{dw}{dt} = \frac{d^2}{dt^2}$ | (2)

If the velocity is uniform,

$S = Vt$	$\theta = \omega t$	(3)
$V = \frac{2\pi rN}{60} = y\omega$	$\omega = \frac{2\pi N}{60} = \frac{V}{r}$	(4)

If the acceleration is uniform,

$V = V_1 + at$	$\omega = \omega_1 + \alpha t,$	(5)

subscript one denoting initial velocity.

$s = V_1 T + \frac{1}{2} at^2$	$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$	(6)][17]

The following figure and vector equation are taken from the text:

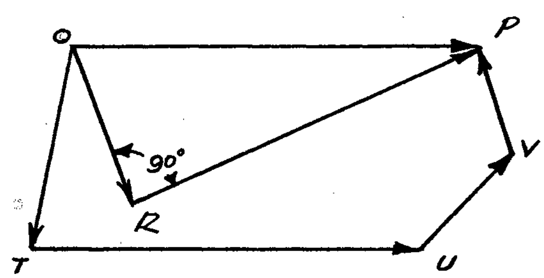


Fig. 38

[In the figure, with a right angle at R, the vectors OR and RP are rectangular components of OP. They are sometimes called normal components. The figure also shows the vector addition,

$$OT \mapsto TU \mapsto UV \mapsto VP = OP$$

If any number of vectors placed in addition yields a closed polygon, the vector sum is zero. The vectors are not in addition,

---

17. J. H. Billings, Applied Kinematics, Third Edition (New York, 1953), p. 8.

however, unless the arrows are all in the same sense around the polygon. Manifestly, the vector sum of any number of vectors, plus their resultant with reversed sense, equals zero.] 18

Another example is taken verbatim from the text.

[For a maximum value of the pressure angle ϕ, $\frac{d\phi}{dt} = 0$.

$$\tan\phi = \frac{at}{qw} \qquad (4)$$

$$\frac{d}{dt}\tan\phi = \frac{d}{dt}\left(\frac{at}{qw}\right)$$

$$\sec^2 \frac{d\phi}{dt} = \frac{a}{qw} - \frac{at}{q^2 w}\frac{dq}{dt} \qquad (5)$$

If q_o is the value of q when the acceleration begins,

$$q = q_o + \tfrac{1}{2} at^2$$

$$\frac{dq}{dt} = at \qquad (6)$$

From (5),

$$\frac{d\phi}{dt} = \cos^2\phi\left(\frac{a}{qw} - \frac{a^2 t^2}{q^2 \; w}\right) = 0 \qquad (7)$$

Dividing by $\frac{a}{qw}\cos^2\phi$ which is not zero,

$$1 - \frac{at^2}{q} = 0 \qquad (8)$$

or

$$q_m = at_m^2 \qquad (9)$$

where q_m is the value of q at the pitch point at which ϕ is maximum.] 19

The following formulas are taken at random from the text, to demonstrate the occurrence of trigonometry:

---

18. Ibid., pp. 8-9
19. Ibid., p. 134.

$$a_2 = \sqrt{(r_1 \sin e + r_2 \sin e)^2 + r_2^2 \cos^2 e} - r_2 \; ,$$

$$a_2 = \sqrt{(r_1^2 + 2r_1 r_2) \sin^2 0 \; r_2^2} - r_2 \; ,$$

$$a_2 = r_2 \left[\sqrt{\left(\frac{r_1^2}{r_2^2} + 2\,\frac{r^1}{r^2}\right) \sin 0 + 1} - 1 \right] .$$

From the text, the following parametric equations of the ellipse are given:

$$\begin{cases} x = a \cos\theta \\ y = b \sin\theta \; , \end{cases}$$

from which,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2\theta + \cos^2\theta = 1 \; .$$

The following graph, of the nature of a sinusoidal wave is copied from the text:

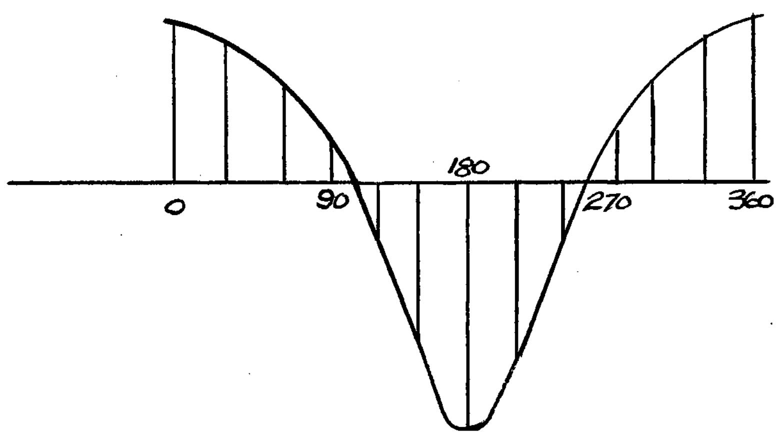


Fig. 39

In summary, the text contains algebra, trigonometry, radian measure, vectors, and differential calculus. The following table summarizes the mathematical frequencies more completely:

Table 30. Quantitative Survey of the Mathematics in Applied Kinematics, Third Edition, by J. H. Billings.

		No. of Occurrences
	Algebra	
1.	Elementary processes and formulas.	72
2.	Solution of equations.	
	Trigonometry	
1.	Formulas containing trigonometry.	27
2.	Use of logarithms.	
3.	Numerical solution of triangles.	
4.	Use of radian measure.	59
5.	Trigonometric identities.	13
6.	Use of vectors.	20
7.	Inverse functions.	2
8.	Graphs of trigonometric functions.	5
	Differential Calculus	
1.	Differentiation of algebraic expressions.	7
2.	Differentiation of transcendental expressions.	5
3.	Appplications of the derivatives.	5
	Higher Mathematics	
1.	Differential equations, Fourier series, etc.	1

The sixth text in mechanical engineering which is analyzed for mathematical content is Design of Machine Elements, Second Edition, by M. F. Spotts.

This textbook is among those replete with mathematics. It is found on all levels, with algebra, trigonometry, differential and integral calculus, and differential equations, all occurring profusely. The student will need a good mathematical background in order to read this text, as the author leaves most of the mathematics to the student, expecting that his background will be such that he can read it.

This text contains very much of the material covered in Singer's, Engineering Mechanics. This is, for instance, a great deal about bending moments, centroids, moments of inertia, and the like. As an example of some of the algebra taken from the text, the following are formulas taken at random:

$$S_{max} = \frac{1}{2}(S_1 - S_2) = \sqrt{\left[\frac{S_x - S_y}{2}\right]^2 + S_{xy}^2} ,$$

$$S_{max} \frac{0.5S_y}{FS} = \frac{16}{\pi d_3} \sqrt{(K_m M)^2 + (K_t T)^2} ,$$

$$K = \frac{d^4 G}{64 R_3 N} ,$$

$$hp. = \frac{(T_1 - T_2)V}{33,000} = \frac{(T_1 - T_2)v}{550} ,$$

$$WV = \left[W + \frac{W_b}{3}\right] V_a , \text{ or } V_a = \frac{1}{1 + \frac{W_b}{3W}} V ,$$

$$KE = \frac{WV^2}{2g}\left[\frac{1}{1 + \frac{W_b}{3W}}\right] ,$$

$$S = \frac{V}{A} \sqrt{\frac{kW_b}{g}} \quad \frac{W}{W_b} \sqrt{\left[\frac{1}{1 + \frac{W_b}{3W}}\right]} .$$

These algebraic formulas are characteristic of many throughout the text, and they create some very complex computations. The following are taken from the text:

$$S = \sqrt{22,000^2 + 2000^2 - 22,000 \times 2,000} ,$$

$$f = \frac{1}{2\pi} \sqrt{\frac{386(80 \times 0.07922 + 120 \times 0.06262)^2}{80 \times 0.07922 + 120 \times 0.06262}}$$

$$= 11.80 \text{ cycles/sec.} .$$

The following graph is taken from the text:

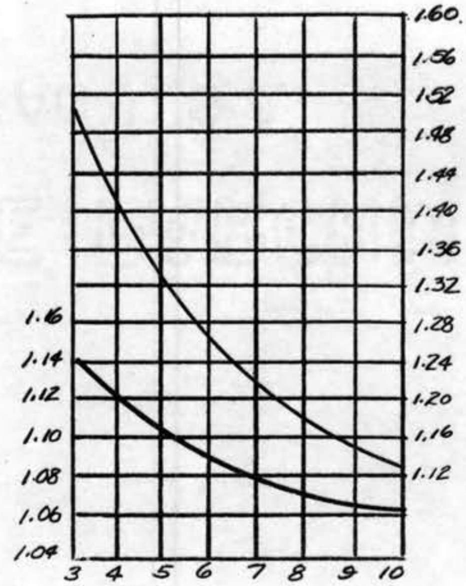


Fig. 40

As an illustration of some of the higher mathematics in the text, the following is an example of term by term integration of a function expanded into a Maclaurin series:

$$\left[\frac{dy}{dr} = \tan\lambda = -\tan\frac{b}{r}\right.$$

The tangent term should now be expanded into a series and integrated term by term.

$$\frac{dy}{dx} = -\frac{b}{r} - \frac{b^3}{3r^2} - \cdots \qquad \text{(f)}$$

$$y = -b \log r + \frac{b^3}{6r^2} + \ldots + C \qquad \text{(g)}$$

When tan λ is less than about 0.4, only the b $\log_e r$ term need be retained. Constant of integration C can be evaluated from the condition that $y = 0$ when $r = r_0$. Then

$$C = b \log_e r_o \qquad \text{(h)}$$

and

$$y = b \log_e \frac{r_o}{r} \qquad \text{(i)}$$

The maximum value of y occurs for $r = r_i$.

$$\text{Hence } y = b \log \frac{r_o}{r_i} - \frac{P}{2 \quad hG} \log \frac{r_o}{r_i} \Big] ^{20}$$

The foregoing, taken verbatim from the text, illustrates quite well the mathematical maturity required by the student in order to read the book. The following, is another verbatim excerpt from the text:

> Summation of the forces in the tangential direction gives
>
> $$T + dT = T + \mu\left[T\, d\theta - \frac{wv^2}{g} d\theta\right]$$
>
> or $$\frac{dT}{T - \frac{wv^2}{g}} = \mu\, d\theta$$
>
> Integration over the active arc from 0 to B gives
>
> $$\log \left[T - \frac{wv^2}{g}\right]_{T_2}^{T_1} = u\theta \Big|_o^B$$
>
> or $$\frac{T_1 - \frac{wv^2}{g}}{T_2 - \frac{wv^2}{g}} = e^{\mu\beta} \qquad (5)$$
>
> where e is 2.718, the base for the system of natural logarithms.] [21]

Again, quoting the text verbatim, the following is given here, since it shows some of the student's mathematical needs.

> Equation (44) can be written
>
> $$A(1 + m_1) = \int_{-h/2}^{h/2} \frac{dv}{1 - \frac{v}{R}_1} \qquad (a)$$

---

20. M. F. Spotts, Design of Machine Elements, Second Edition, (New York, 1953), p. 156.
21. Ibid., p. 190.

When the numerator is divided by the denominator, the following result is obtained.

$$A(1 + m_1) = \int_{-h/2}^{h/2} \left[1 + \frac{v_1}{R} + \frac{v_1^2}{R^2} + \frac{v_1^3}{R^3} + \dots\right] b\,dv$$

This expression should now be integrated term by term and the limits substituted.

$$A(1 + m_1) = b\left[h + \frac{h^3}{12R^2} + \frac{h^5}{80R^4}\right. \qquad \text{(b)}$$

$$\text{or} \qquad m_1 = \left.\frac{h^2}{12R^2} + \frac{h^4}{80R^4}\right] \quad 22$$

The law of cosines occurs in the text, recognizable from the following equation:

$$\overline{A_1 A_2}^2 = r^2 + r_1^2 - 2\, r\, r_1 \cos\beta'.$$

The numerical solution of trangles occurs in several places, as does computation by use of logarithms. The natural base is used frequently, and a table published from which the student may derive values. The student also needs to know the nature of logarithms, and know thoroughly the meaning of base 10, and base e. Trigonometric identities occur in quite a number of places as does radian measure. Also, there are many formulas in which trigonometric functions occur. The sine, cosine, and tangent predominate.

In summary, the text contains very complex mathematics, stressing trigonometry and differential calculus. The following table summarizes the mathematical frequencies more completely:

---

22. Ibid., pp. 398-399.

Table 31. Quantitative Survey of the Mathematics in Design of Machine Elements, Second Edition, by M. F. Spotts.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	202
2. Solution of equations.	13
Trigonometry	
1. Formulas containing trigonometry.	55
2. Use of logarithms.	5
3. Numerical solution of triangles.	
4. Use of radian measure.	10
5. Trigonometric identities.	10
6. Use of vectors.	
7. Inverse functions.	
8. Graphs of trigonometric functions.	4
Differential Calculus	
1. Differentiation of algebraic expressions.	28
2. Differentiation of transcendental expressions.	12
3. Applications of the derivatives.	10
Higher Mathematics	
1. Differential equations, Fourier series, etc.	20

The seventh text in mechanical engineering which is analyzed for mathematical content is Steam Power Stations, Third Edition, by G. A. Gaffert.

This text contains very little mathematics. There are a few algebraic formulas, some numerical computations, and only one place in which calculus occurs. There are a great many graphs of various kinds, however. This is a lengthy book, and in reading it a student would not be handicapped, greatly, if he had no knowledge of mathematics. There are a few places of interest, however, and the following graph is typical.

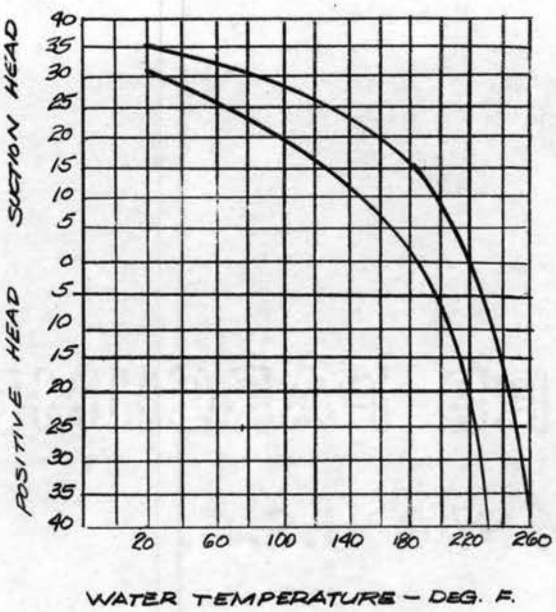


Fig. 41

The following, is the only instance of calculus within the entire text:

$$\int_0^A dA = \frac{C_P\ W}{U}\int_{t_2}^{t_1}\frac{dt_1}{t_s - t_1} \text{ or } A = \frac{C_p W}{U}\log_e\frac{t_s - t_2}{t_s - t_1}\ .$$

If U be considered as independent of temperature difference,

$$UAt = C_p\ W(t_1 - t_2) \text{ or } t_m = \frac{C_p W}{AU}\ (t_1 - t_2).$$

Substituting for A, we have,

$$t = \frac{t_1 - t_2}{\log_e \dfrac{t_s - t_2}{t_s - t_1}} \text{ (logarithmic-mean temperature)}$$

In a few places, the algebraic formulas give rise to some numerical computations. The following are characteristic:

$$h = \frac{(154 + 1.6t)\ V_s^{0.8}}{D}$$

$$= \frac{\left(154 + 1.6\ \dfrac{77 + 147}{2}\right)\ (7.5)^{0.8}}{(1 - 2 \times .065)}$$

$$U = \frac{1}{\left(\frac{1}{2000}\right) + 0.0000915 \times \frac{2 \times 1.00}{1.00 + 0.87}\left(\frac{1}{1718} - \frac{1.00}{0.87}\right)}$$

$$S = \frac{611,000 \times (147 - 77)}{790 \times 27.6} = 1970 \text{ sq. ft.}$$

The following algebraic formulas are taken from the text:

$$\frac{hD}{k} = a\left(\frac{DG}{\mu}\right)^n \left(\frac{CU}{K}\right)^m \left(\frac{D}{N}\right)^e$$

$$\frac{hD}{k} = \phi\left(\frac{D' G_{max}}{\mu}\right)$$

$$h = \frac{1.75 \; (T^{0.3}) \; (G_{max}^{2/3})}{D^{1\,1/3}}$$

In summary, this text contains little mathematics, virtually all of that being in algebraic formulas. The following table summarizes the mathematical frequencies more completely.

Table 32. Quantitative Survey of the Mathematics in Steam Power Stations, Third Edition, by J. A. Gaffert.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	60
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	
2. Use of logarithms.	6
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	
7. Inverse functions.	
8. Graphs of trigonometric functions.	
Integral Calculus	
1. Integration of algebraic expressions.	1
2. Integration of transcendental expressions.	1

Table 32. -- Continued --

Integral Calculus	No. of Occurrences
3. Applications of the definite integral.	
4. Multiple integrals.	

The eighth of the texts in mechanical engineering which is analyzed for mathematical content is Applied Thermodynamics, by V. M. Faires.

This text is replete with mathematics on nearly all levels. As would be expected, however, in a text on this subject, there is no trigonometry. But there is some complex algebra, integral calculus, logarithmic operations, extensive use of graphs, and even some attention given to areas under the curves. Also, the quantity of mathematics is extensive, as there is scarcely a page of the text in which some does not occur.

Consider the following figure, followed by the verbatim statement from the text.

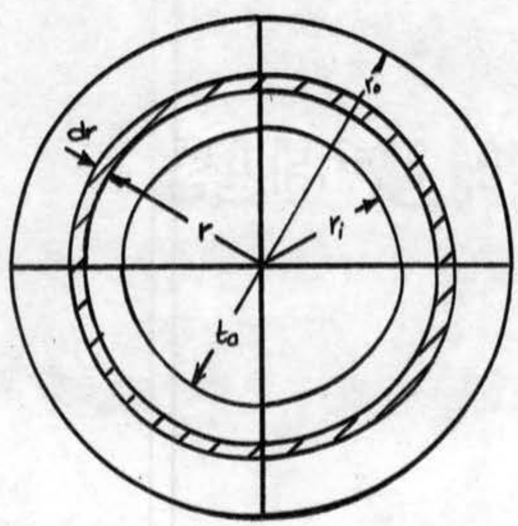


Fig. 42

The area of this thin cylindrical surface is $2\pi rz$. The change in temperature across dr is a differential amount - dt, the negative sign indicating a decreasing temperature with an increasing radius. Thus, Fourier's equation gives

$$Q = k\,2\pi\,rz\left(-\frac{dt}{dr}\right).$$

Separating the variables and integrating, we get

$$Q\int_{r_i}^{r_o}\frac{dr}{r} = 2\pi kz\int_{t_a}^{t_b} - dt,$$

$$Q\log\frac{r_o}{r_i} = 2k\pi z\,(t_a - t_b),$$

or
$$Q = \frac{2\pi kz(t_a - t_b)}{\log\frac{r_o}{r_i}},$$

where r_o is the outside radius of the pipe and r_i is the inside radius.] [23]

The following, is also taken verbatim from the text:

Hence, for the case of the variable specific heats, the isentropic process needs a special handling. In this process, dQ = dU $+ \frac{p\,dV}{J} = 0$, or $dU = -p\frac{dV}{J}$. Since $dU = wc_v$ dT, we have

$$wc_v\,dT = -p\int\frac{dV}{J}. \qquad (f)$$

Substituting $p = \frac{wRT}{V}$ into (f), separating the variables, we find

$$c_v\frac{dT}{T} = -\frac{R}{J}\int\frac{dV}{V} \qquad (g)$$

To integrate the left-hand side of this equation, substitute the proper value of c (see Table V). Using (55a) as an illustration we have $c_v = \alpha V + \beta T + ST^2$. Substituting this value of c_v into (g) and integrating, we get

$$\alpha V\log_e\frac{T_2}{T_1} + \beta(T_2 - T_1) + \frac{S}{2}(T_2^2 + T_1^2)$$
$$= -\frac{R}{J}\log_e\frac{V_2}{V_1} = \frac{R}{J}\log_e\frac{V_1}{V_2}\Big] \qquad (h)$$

[24]

23. V. M. Faires, Applied Thermodynamics (New York, 1949), p. 430.
24. Ibid., p. 214.

From the two foregoing excerpts, it is seen that the author expects some mathematical maturity on the part of the student, as the student is expected to supply all omitted steps.

Most of the other integrations to be performed are, as where these, algebraic, rather than transcendental, although the form $\int \frac{dv}{v} = \log v$ occurs frequently. This is the only transcendental expression to be found. Moreover, the algebraic integrals are principally polynomials. Thus, although integration, and its application, occurs frequently, they are not very complex.

The following are some examples of integral calculus, taken at random from the text:

$$\int_1^2 p\, dV = C\int_{v_1}^{v_2} \frac{dV}{V} = C \log_e \frac{V_2}{V_1} .$$

$$Q = w\int_{T_1}^{T_2} dT ,$$

or

$$Q = cw\int_{T_1}^{T_2} dT = wc(T_2 - T_1) .$$

$$Q = w\int c\, dT = \int_{500}^{2000} (0.2 + 0.00005T)\, dT ,$$

$$= \left[0.2T + \frac{0.00005T^2}{2}\right]_{500}^{2000} = 394 \text{ Btu per lb.} .$$

$$= \int \frac{dQ}{T} = wc\int_{T_1}^{T_2} \frac{dT}{T} = wc \log \frac{T_2}{T_1} .$$

The following figure, and excerpt, are taken verbatim from the text, to show the application of area under a curve.

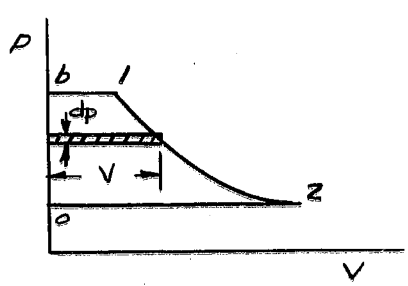


Fig 43

[120. Meaning of $\int V\,dp$. Consider a diagram like that of the figure. The enclosed area is the algebraic sum of the areas under the curves; thus,

$$\text{Area 1-2-a-b} = p_1 V_1 + \int_1^2 p\,dV - p_2 V_2 .$$

Moreover, this area is also equal to $\int_2^1 V\,dp = -\int_1^2 V\,dp$, as seen from the figure. Hence

$$-\int_1^2 V\,dp = p_1 V_1 + \int_1^2 p\,dV - p_2 V_2 . \qquad (r)$$

But from equation (q) we see that the right hand side of (r) is equal to the change of kinetic energy plus the work. Therefore,

$$-\int_1^2 V\,dP = \Delta KE + W \qquad (s)$$

(Reversible process)

If the change of kinetic energy is negligible, as it is in many steadyflow machines, $-\int_1^2 V\,dp = W$, and the area 1-2-a-b represents work.] [25]

The following algebraic formulas are taken at random from the text:

$$\frac{Wv_1^2}{2gJ} + U_1 + \frac{p_1 V_1}{J} + Q = \frac{wv_2^2}{2GJ} + U_2 + \frac{p_2 V_2}{J} + W \quad ,$$

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1-n} \left(\frac{V_1}{V_2}\right)^{n-1},$$

$$T_3 = T_2\left(\frac{V_3}{V_2}\right) = T_1\ r^{k-1} r_c \quad ,$$

$$e = 1 - \frac{1}{r^{k-1}} \quad \frac{r_c^{\,k} - 1}{k(r_c - 1)} \quad .$$

The calculus is more like that occuring in mathematics text books. There are integrations, both algebraic and transcendental. The algebra is rather complex, involving exponents and radicals.

---

25. Ibid., pp. 163-164.

The following is taken verbatim from the text, and is offered here to show the use to which differentials may be put.

[Relation between p, V, and T. During the isentropic process, the pressure, volume, and temperature all vary. The relation between p and V may be determined from the energy relations. Since, by definition, the transferred heat is zero, the simple energy equation is $Q = \Delta U + W = 0$. Applying this equation to a reversible process, we may use $dW = \frac{p\,dV}{J}$ and obtain

$$wc_V\,dT + \frac{pdV}{J} = 0, \text{ or } \frac{-p\,dV}{J} = wc_v\,dT. \qquad (n)$$

Next, differentiate the characteristic equation $pV = wRT$. Since p, V, and T vary, this differentiation gives

$$p\,dV + V\,dp = w\,R\,dT,$$

or

$$dT = \frac{p\,dV + V\,dp}{wR}. \qquad (o)$$

Since we are searching for a relation between p and V, we should eliminate dT from equations (n) and (O). This step gives

$$-\frac{p\,dV}{J} = \frac{c_v\,p\,dV + c_v\,V\,dp}{R}.$$

Multiplying both sides of this equation by R and dividing by pV, we have

$$-\frac{R}{J}\frac{dV}{V} = c_v\frac{dV}{V} + c_v\frac{dp}{p},$$

or

$$-\left(\frac{R}{J} + c_v\right)\frac{dV}{V} = c_v\frac{dp}{p}.$$

Since $\frac{R}{J} + c_v = c_p$, and since $\frac{c_p}{c_v} = k$, we have

$$-k\frac{dV}{V} = \frac{dp}{p}.$$

Inasmuch as k is constant for a perfect gas, we have now arrived at an integrable form. Thus, between any two states 1 and 2,

$$-\int_{V_1}^{V_2} k\frac{dV}{V} = \int_{P_1}^{P_2}\frac{dp}{P},$$

or

$$-k\log_e\frac{V_2}{V_1} = \log_e\left(\frac{V_1}{V_2}\right) = \log_e\frac{p_2}{p_1}.$$

Taking the antilogarithm of the last two expressions, we find

$$\left(\frac{V_1}{V_2}\right)^k = \frac{p_2}{p_1} \qquad (p)$$

or, since states 1 and 2 were chosen at random along any isentropic line,

$$p_1 V_1^k = p_2 V_2^k = p\ V^k = C, \text{ a constant.} \qquad (19)$$

Thus, $p\ V^k = C$ is the equation of an isentropic process for a perfect gas in p-V co-ordinates. The corresponding curve is similar in general appearance to the equilateral hyperbola.][26]

As in the previous excerpts, the authors leave the mathematical details to the student, thus presupposing a certain mathematical maturity.

In summary, this text contains much algebra, differential and integral calculus, much use of logarithms, and some use of graphs. The following table summarizes the mathematical frequencies more completely.

Table 33. Quantitative Survey of the Mathematics in Applied Thermodynamics, by V. M. Faires.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	157
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	2
2. Use of logarithms.	19
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	
7. Inverse functions.	

26. Ibid., pp. 49-51.

Table 32. -- Continued --

Trigonometry	No. of Occurrences
8. Graphs of trigonometric functions.	
Integral Calculus	
1. Integration of algebraic expressions.	35
2. Integration of transcendental expressions.	14
3. Application of the definite integral.	
4. Multiple integrals.	
Higher Mathematics	
1. Differential equations, Fourier series, etc.	1

The ninth text in mechanical engineering which is analyzed for mathematical content is Refrigeration, Second Edition, by J. A. Moyer and R. V. Fittz.

This textbook contains almost no mathematics. There are only a very few places where simple algebra is used, a few formulas occur in which logarithms are employed, but there is no trigonometry or calculus. There are some graphs worth reproducing, such as the following.

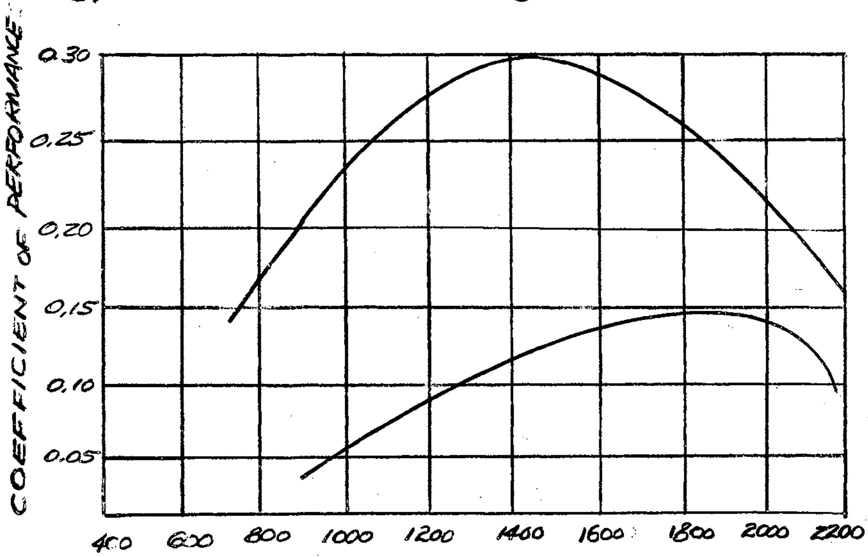


Fig. 44

The following formula is taken from the text:

$$W = P_1 V_1 - P_1 V_1 \log_e \frac{V_1}{V_2} - P_2 V_2 ,$$

which becomes, since $P_1 V_1 = P_2 V_2$,

$$W = - P_1 V_1 \log_e \frac{V_1}{V_2} = P_1 V_1 \log_e \frac{V_2}{V_1} .$$

The following algebraic formulas are taken from the text:

$$E_v = \frac{\left[1 + c\left(1 - \left(\frac{P_3}{P_4}\right)^{1/n}\right)\right] V_s}{V_s}$$

$$= 1 + c\left[1 - \left(\frac{P_3}{P_4}\right)^{1/n}\right] .$$

The following equation is solved for x:

$$79.2x + (40{,}000 - x)\,100 = 40{,}000 \times 90$$
$$79.2x + 4{,}000{,}000 - 100x = 3{,}600{,}000$$
$$20.8x = 400{,}000$$
$$x = 10{,}200 \text{ cu. ft./min.}$$

In summary, there is only a little mathematics in this text, none of it beyond elementary algebra. The following table summarizes the mathematical frequencies more completely:

Table 34. Quantitative Survey of the Mathematics in Refrigeration, Second Edition, by J. H. Moyer and R. V. Fittz.

Algebra	No. of Occurrences
1. Elementary processes and formulas.	47
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	
2. Use of logarithms.	3

The tenth text in mechanical engineering which is analyzed for mathematical content is Steam Turbines, Second Edition, by E. F. Church.

This text, perhaps, contains as great a concentration of mathematics as any of the books reviewed. It is, however, on an elementary level, with only a little calculus. Trigonometry and algebra predominate, with a considerable emphasis on vectors. The following equation is taken from the text, and the maximum obtained by differentiation:

$$\eta = 2P \quad (\cos\alpha - P) \ (1 + \frac{k_b \ \cos\lambda}{\cos\beta}) \ .$$

From this equation may be found the value of P for which the efficiency is a maximun, other conditions remaining constant. Differentiating the expression with respect to P and equating to zero,

$$0 = 2(1 + \frac{k_b \ \cos\lambda}{\cos\beta})(\cos\alpha - 2P) \quad ,$$

from which $P_{max} = \frac{\cos\alpha}{2}$.

For this value of P,

$$\eta_{max} = \frac{\cos^2\alpha}{2} \ (1 + \frac{k_b \ \cos\lambda}{\cos\beta}) \ .$$

The foregoing shows the considerable occurrence of trigonometry, especially the trigonometric functions of sine and cosine. To show the complexity of some of the vector figures, the following is taken from the text:

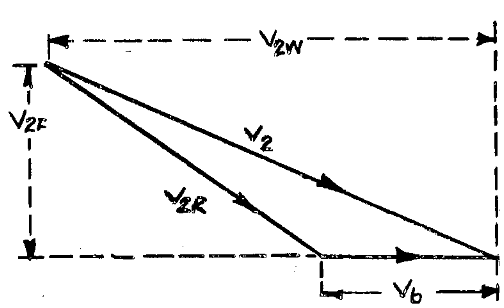


Fig. 45

So much of this text is concerned with trigonometry and vectors, that the following formulas involving trigonometry are taken at random from the text:

$$E_b = \frac{V_b}{g}(V_2 \cos\alpha + V_3 \cos\delta) ,$$
$$V_3 \cos\delta = k_b V_{2r} \cos\lambda - V_b ,$$
$$E_{be} - \frac{V_b}{g}(V_w \cos\alpha + k_b V_{2r} \cos\lambda - V_b) ,$$
$$V_{2r} = \frac{V_2 \cos\alpha - V_b}{\cos\beta} = k_b ,$$
$$\frac{V_b}{g}(V_2 \cos\alpha - V_b)(1 + \frac{k_b \cos\lambda}{\cos\beta}) ,$$
$$\frac{h_2}{h_n} = \frac{V_2 \sin\alpha}{V_{2r} \sin\beta} .$$

Trigonometry is also used in various places in the solution of triangles. For example, consider the following use of the law of sines:

$$\frac{h_3}{h_2} = \frac{V_{2r} \sin\beta}{V_{3r} \sin\lambda} = \frac{6}{5.7} = \frac{861 \sin\beta}{809 \text{ X } 0.783} ,$$

whence, $\sin\beta = 0.775$, and $\beta = 50.8^{\circ}$.

While it is hardly possible to overstress the use of trigonometry in this text, some attention should be given to formulas in which only algebraic expressions occur. The following are taken from the text, and show the use of exponents:

$$v_2 = v_1 (\frac{p_1}{p_2})^{1/R} = v_1 (\frac{1}{r})^{1/R} ,$$
$$r_o = (\frac{2}{k+1}) \cdot \frac{k}{k-1} ,$$
$$pv^{1.135} \; c ,$$
$$V_2 = \sqrt{2g \frac{k}{k-1} p_1 v_1 \left[1 - r_k^{k-1}\right]} .$$

In summary, the text emphasizes algebra, and trigonometry, including vectors. The following table summarizes.

Table 35. Quantitative Survey of the Mathematics in <u>Steam Turbines</u>, <u>Second Edition</u>, by E. F. Church

Algebra	No. of Occurrences
1. Elementary processes and formulas.	99
2. Solution of equations.	
Trigonometry	
1. Formulas containing trigonometry.	39
2. Use of logarithms.	3
3. Numerical solution of triangles.	
4. Use of radian measure.	
5. Trigonometric identities.	
6. Use of vectors.	9
7. Inverse functions.	
8. Graphs of the trigonometric functions.	

The eleventh text in mechanical engineering which is analyzed for mathematical frequency is <u>Heating, Ventilating, and Air-Conditioning Fundamentals</u>, <u>Second Edition</u>, by W. H. Severns and J. R. Fellows.

This text is unusual, in that the mathematics within it, is neither extensive, nor complex. Especially is this true in regard to complexity, as nothing beyond ordinary algebra occurs, not even trigonometry. There are some numerical computations for the student to make, and ratio and proportion is used. The following is taken from the text, and is quite typical:

$$R_a = 240\left(\frac{t_s - t_r}{215 - 70}\right)^{1.3}.$$

where t_s = steam temperature, deg. F. ,

t_w = average temperature of the water, deg. F. ,

t_r = room air temperature, deg. F. .

The text then follows with an application of this formula, and obtains,

$$R_a = 240 \left(\frac{215 - 65}{215 - 70}\right)^{1.3} = 250.7 \text{ Btu.}$$

The student should need to know the use of logarithms, in order to make the above computation, which is derived in the text. The following are formulas taken at random from the text, and are quite typical of those throughout the entire book:

$$E_b = \frac{(h_f + xh_{fg} - h_{f1})\, W_w}{W_f \times F} \times 100 ,$$

$$E = 0.5 \left(\frac{V_d^2 + V_u^2}{V}\right) = 0.5 \left(\frac{30^2 + 24^2}{24^2}\right) = \left(\frac{900 + 576}{576}\right) = 1.3 ,$$

$$U_{cr} = \frac{U_r + U_c}{U_r + \frac{U_c}{r}} ,$$

$$V_e = \sqrt{V^2 - 1.75a} ,$$

$$t_a = \frac{U_1 A_1 t + U_2 A_2 t_o}{U_1 A_1 + U_2 A_2} .$$

There are quite a number of graphs in the text, of which the following is copied, as it typifies them:

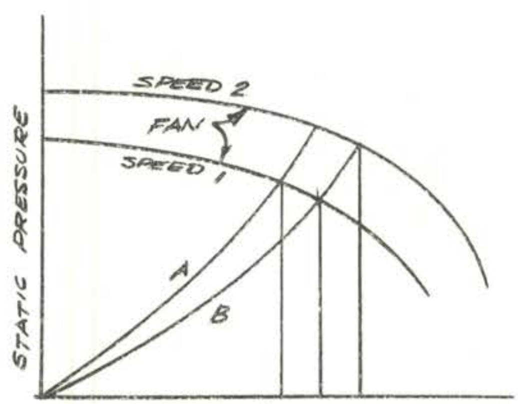


Fig. 46

In summary, the mathematics does not exceed the level of elementary algebra. The following table summarizes the mathematical frequencies more completely:

Table 36. Quantitative Summary of the Mathematics in Heating, Ventilating, and Air-Conditioning Fundamentals, Second Edition, by W. H. Severns and J. R. Fellows.

	No. of Occurrences
Algebra	
1. Elementary processes and formulas.	78
2. Solution of equations.	3
Trigonometry	
1. Formulas containing trigonometry.	
2. Use of logarithms.	5

Since this completes the texts in mechanical engineering, they are now summarized, both quantitatively and qualitatively.

Summary. The occurrence of mathematics within the field of mechanical engineering is not quite so sparse as is that of chemical and petroleum engineering, nor yet so numerous as is that of the fields of electrical and civil engineering. Of the eleven books covered within the chapter, differential calculus occurred only within three of them, and although integral calculus occurred in six of them, it did not occur frequently. Trigonometry occurred with less frequency than in previous chapters. Algebra, of course, was predominant.

The following table gives an overall total, that is, it gives the total number of texts in which the given topic occurred, as well as the total occurrences for all the texts.

Table 37. Quantitative Summary of the Eleven Texts in Mechanical Engineering

	Algebra	No. of Texts Occurring	Total of Occurrences
1.	Elementary Processes and formulas.	11	1071
2.	Solution of Equations.	5	30
	Trigonometry		
1.	Formulas containing trigonometry.	7	142
2.	Use of logarithms.	8	47
3.	Numerical solution of triangles.		
4.	Use of radian measure.	2	69
5.	Trigonometric identities.	2	23
6.	Use of vectors.	3	42
7.	Inverse functions.	1	2
8.	Graphs of trigonometric functions.	2	9
	Differential Calculus		
1.	Differentiation of algebraic expressions.	3	58
2.	Differentiation of transcendental expressions.	2	17
3.	Applications of the derivatives.	2	15
4.	Partial derivatives.		
	Integral Calculus		
1.	Integration of algebraic expressions.	6	60
2.	Integration of transcendental expressions.	4	29
3.	Applications of the definite integral.	1	4
4.	Multiple integrals.		
	Higher Mathematics		
1.	Differential equations, Fourier series, etc.	4	25

CHAPTER III

THE MATHEMATICAL NEEDS OF THE PROFESSIONAL ENGINEER

In keeping with the outline for the plan of this study, a questionnaire was prepared, the results of which are given in this chapter. While the items in the questionnaire are not exhaustive of the topics covered in an undergraduate mathematics curriculum, they nevertheless cover the field rather thoroughly, including spatial geometry, analytic geometry, algebra, trigonometry, and the calculus. The questionnaire was prepared after a careful study of widely used texts in mathematics, and it omits no items consistently stressed.

These questionnaires were sent to two groups. They were sent to members, in the New York City area, of the American Institute of Electrical Engineers, all of whom are professional engineers, although not all, in contrast to the implication of the name, are electrical engineers. Some were also sent to members of the Louisiana Society of Engineers and Surveyors, with as wide a geographical choice as the membership permitted, which of course, included all parts of Louisiana, as well as the surrounding states, and some of the eastern and central states. Each engineer was asked to specify the branch of his employment, that is, whether he was an electrical engineer, a civil engineer, a mechanical engineer, a chemical engineer, or a petroleum engineer.

Since the differences in the responses of these groups were insignificant, the tabulations are not set up by separate groups.

From three hundred twenty-five questionnaires sent, one hundred six were returned. These were divided, as follows: thirty-one electrical engineers, twenty civil engineers, sixteen mechanical engineers, twelve chemical engineers, nine petroleum engineers, and eighteen of doubtful, or unspecified classification.

The following pages contain the letter requesting the cooperation of the engineers, and the questionnaire.

Dear Sir:

I am making a study at Louisiana Polytechnic Institute, Ruston, Louisiana, designed to help co-ordinate the field of mathematics with that of engineering.

As an engineer, in the field, you can aid greatly by filling in the attached questionnaire. I will appreciate your cooperation. If you desire to know the outcome of the project I shall be glad to furnish you any information you desire.

Please be guided by the following instructions: If the topic is used extensively, use number (1) in the parenthesis following; if the topic is used occasionally, use number (2); if never used, use number (3).

I wish to assure you your assistance is very valuable and deeply appreciated.

Yours very truly,

Wallace Herbert,
Associate Professor of
Mathematics

WH:bm

1. Concept of percentage, including per cent of increase and decrease. ( )

2. Use of ratio and proportion. ( )

3. Solution of problems involving physical magnitudes; for example, addition of lengths expressed in feet and inches, calculation of areas, addition or subtraction of angles, and so forth. ( )

4. Scale drawings. ( )

5. Meaning of an approximate number, precision of a measurement, significant digits, and rounding off of numbers. ( )

6. Preparation and interpretation of statistical graphs; for example, bar, circle, and line graphs. ( )

7. Removal of parentheses, brackets, braces, and so forth. ( )

8. Addition, subtraction, multiplication, and division of algebraic fractions. For example, $\frac{x + 2}{3} + \frac{3}{x - 4}$. ( )

9. Addition, subtraction, multiplication, and division of polynomials. For example, $(x^3 + 2x^2 - 3x + 7)(2x - 3)$. ( )

10. Common special products; for example, $a(a + b)$, $(a \pm b)^2$ $(a + b)(a - b)$.

11. Factoring such expressions as $(a^2 + ab)$, $(a^2 \pm 2ab + b^2)$, $(a^2 - b^2)$, and $(ax^2 + bx + c)$. ( )

12. Laws of exponents, including negative and fractional exponents. For example, $x^3 \cdot x^{-2} = x$. ( )

13. Solution of linear equations. For example $2t - 6 = t + 3$. ( )

14. Solution of a pair of linear equations; for example, $(2y - x = 4)$ $(3y - 2x = 6)$ ( )

15. Concept of a function and functional notation. ( )

16. Properties of a linear function; for example, graphical representation, standard form of a linear equation, the slope and y-intercept of a line. ( )

17. Addition, subtraction, multiplication, and division of radicals. For example, $\sqrt{8} + \sqrt{2}$. ( )

18. Addition, subtraction, multiplication, and division of complex numbers. For example, $(8 + 2i)(6 - 3i)$. ( )

19. The standard form ($ax^2 + bx + c = 0$) of a quadratic equation; its graph, the nature of the roots, and expressions for the sum and product of the roots. ( )

20. Solution of a quadratic equation. For example $x^2 - 3x + 2 = 0$. ( )

21. Solution of a system, consisting of a linear and a quadratic equation. For example, $(2x - y = 6)$. $(x^2 + y^2 = 4)$ ( )

22. Solution of pairs of quadratic equations, For example, $(x^2 - y^2 = 6)$ $(x^2 + y^2 = 12)$. ( )

23. Solution of verbal problems by algebraic methods. ( )

24. Solution of equations in which the unknown occurs under a radical sign. For example, $\sqrt{x + 4} = 6$. ( )

25. Use of the binomial theorem. ( )

26. Scientific notation, or standard-form numbers. For example, 2.54×10^3, 1.2×10^{-4}. ( )

27. Computation by means of logarithms. ( )

28. Change of the base of logarithms. ( )

29. Solution of exponential and logarithmic equations. For example, $2^x = 9$. ( )

30. Finding the rational roots of higher degree polynomials. For example, $x^4 + x^3 - x^2 - x = 0$. ( )

31. Rough sketching of the graphs of higher degree polynomials. ( )

32. Approximating the irrational roots of higher degree polynomials. ( )

33. Expansion and use of determinants. ( )

34. Use of arithmetic progressions. For example, $2+4+6+8+ . . .$ ( )

35. Use of geometric progressions, both finite and infinite. For example, $2 + 4 + 8 + 16 + ...$ ( )

36. Concept of inequality, including the symbol, and the properties of inequality. For example $8 > 6$. ( )

37. Use of polygons: triangle, square, parallelogram, trapezoid, hexagon, octagon. ( )

38. Concept of locus, for example, a circle is the locus of all points at a fixed distance from a fixed point. ( )

39. Parallelism and perpendicularity of lines. ( )

40. The Pythagorean theorem. For example, $a^2 + b^2 = c^2$ in a right triangle. ( )

41. Use of polyhedrons: cubes, prisms, and pyramids. ( )

42. Use of cylinders, cones, and spheres. ( )

43. The solution of right triangles, by use of trigonometric functions. ( )

44. Relationships of acute angles of a right triangle: $\sin (90^\circ - A) = \cos A$, and so forth. ( )

45. Solution of verbal problems involving right triangles. ( )

46. Definitions of trigonometric functions of angles greater than 90°. ( )

47. Values of functions of special angles including quadrantal angles, or angles such as 90°, 180°, 270°, and so forth. ( )

48. Fundamental trigonometric identities. For example, $\sin^2 x + \cos^2 x = 1$. ( )

49. Addition identities: $\sin (A + B)$, and so forth. ( )

50. Use of the law of sines: $\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B}$. ( )

51. Use of the law of cosines: $c^2 = a^2 + b^2 - 2ab \cos c$. ( )

52. Use of the law of tangents: $\frac{a - b}{a + b} = \frac{\tan\frac{1}{2}(A - B)}{\tan\frac{1}{2}(A + B)}$ ( )

53. Area formulas: $K = \frac{1}{2} bc \sin A$; $K = x(s - a)(s - b)(s - c)$. ( )

54. Solution of triangles by logarithms. ( )

55. Radian measure of angles. ( )

56. Relation between linear and angular velocity. ( )

57. Graphs of sine and cosine functions. ( )

58. Inverse trigonometric functions. ( )

59. Solution of trigonometric equations. For example, $\sin 2x = 1$. ( )

60. Double-angle identities. For example, $\sin 2A = 2 \sin A \cos A$. ( )

61. Half-angle identities. For example, $\tan \frac{1}{2} x = \frac{1 - \cos x}{\sin x}$ ( )

62. Use of vectors, their components and resultants. ( )

63. Addition and subtraction of vectors. ( )

64. Graphical addition and subtraction of vectors. ( )

65. Use of distance formula in the Cartesian co-ordinate system: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ ( )

66. The polar co-ordinate system. ( )

67. Use of the slope formula in Cartesian co-ordinate system: $m = \frac{y_1 - y_2}{x_1 - x_2}$. ( )

68. Standard forms for equations of straight lines. For example, $y - y_1 = m(x - x_1)$. ( )

69. Standard forms for parabolas. For example, $y^2 = 2px$. ( )

70. Standard forms for ellipses and hyperbolas. For example, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. ( )

71. Use of parameters, parametric equations, families of curves, and so forth. ( )

72. Formulas for dividing a line segment in a given ratio; especially the mid-point formula. ( )

73. Computing areas by the integral calculus. ( )

74. Computing volumes by the integral calculus. ( )

75. First moments, for irregular areas, by integration. ( )

76. Moments of inertia of irregular areas. ( )

77. Arc length for irregular curves, by integration. ( )

78. Maxima and minima. ( )

79. Slope of a curve at any point. ( )

80. The delta or fundamental process for differentiation. ( )

81. Finding work done on a body, with a variable force. ( )

82. Computing surface areas, of surfaces of revolution by integration. ( )

83. The theorem of Pappus. ( )

84. Fluid pressure and force on irregular shaped areas by integration. ( )

85. Centroids of irregular shaped areas. ( )

86. Radius of gyration. ( )

87. Use of three-dimensional Cartesian (rectangular) co-ordinates. ( )

88. Cylindrical co-ordinates. ( )

89. Spherical co-ordinates. ( )

90. Differentiation with respect to time. ( )

91. The laws of falling bodies. ( )

92. Multiple integration. ( )

93. Partial differentiation. ( )

The responses of the engineers to the questionnaire are reported in Table 38. The number of the topic appears at the left, and the number of responses directly beneath the designated choices 1, 2, and 3. The number 1 at the heading of column 1 means that the responders report that the topic is used extensively; the number 2 at the heading of column 2 means that the responders report that the topic is used occasionally; the number 3 at the heading of column 3 means that the responders report that the topic is never used.

Table 38. Frequency of Use of Mathematical Topics by Engineers

Number of Topic	Engineers' Report 1*	2*	3*	Number of Topic	Engineers' Report 1*	2*	3*
1.	90	13	3	47.	56	25	26
2.	90	14	2	48.	57	16	33
3.	88	14	4	49.	30	40	36
4.	84	13	9	50.	46	28	32
5.	89	11	6	51.	26	43	37
6.	70	25	11	52.	16	29	61
7.	40	48	18	53.	11	39	56
8.	33	57	16	54.	4	32	70
9.	18	58	30	55.	63	18	24
10.	30	58	17	56.	37	37	32
11.	24	52	30	57.	58	24	24
12.	49	43	14	58.	26	40	40
13.	38	63	5	59.	28	33	45
14.	35	60	11	60.	12	37	57
15.	60	33	13	61.	15	33	58
16.	68	30	8	62.	76	13	17
17.	30	60	16	63.	72	17	17
18.	27	51	28	64.	60	18	28
19.	13	80	13	65.	15	33	58
20.	26	60	20	66.	31	51	24
21.	19	42	45	67.	35	35	36
22.	20	39	47	68.	16	47	43
23.	38	48	20	69.	30	39	37
24.	28	42	36	70.	18	40	48
25.	40	34	34	71.	39	33	34
26.	60	6	40	72.	14	26	66
27.	30	50	26	73.	28	37	41
28.	11	40	55	74.	19	30	57
29.	35	45	26	75.	26	18	62
30.	6	38	62	76.	19	30	57
31.	14	33	59	77.	3	23	80
32.	4	43	59	78.	30	56	20
33.	28	18	60	79.	29	29	48
34.	18	30	58	80.	18	27	61
35.	21	45	40	81.	11	14	81
36.	45	40	21	82.	14	20	72
37.	25	45	36	83.	13	13	80
38.	22	45	38	84.	8	22	76
39.	33	39	34	85.	13	22	71
40.	70	15	21	86.	17	40	49
41.	21	30	55	87.	35	35	36
42.	20	43	43	88.	17	21	68
43.	64	21	21	89.	20	19	67
44.	49	31	26	90.	30	56	20
45.	43	32	31	91.	19	48	39
46.	56	39	11	92.	15	42	49
				93.	19	36	49

1 Frequently Used. 2 Occasionally Used. 3 Never Used.

These results may be analyzed by obtaining a weighted average for each of the ninety-three topics. On a given topic the number responding as (1) is multiplied by one, the number responding as (2) is multiplied by two, and the number responding as (3), by three. The sum of the three such products is then divided by the total of the responses to the topic. Consider, for example, topic sixteen from Table 38:

$$\begin{array}{ll} 68 \times 1 = 68 & \\ 30 \times 2 = 60 & 68 + 60 + 24 = 152 \\ 8 \times 3 = 24 & 68 + 30 + 8 = 106 \\ \multicolumn{2}{c}{152 \div 106 = 1.43} \end{array}$$

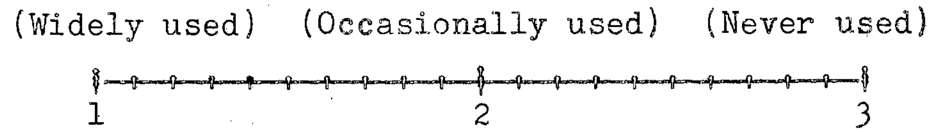
Topic sixteen is thus seen to have a weighted average of 1.43, which will be used as an indication of the importance of the topic involved in the questionnaire.

Table 39 gives such weighted averages for the ninety-three topics.

Table 39. Weighted Averages of the Ninety-three Topics in the Questionnaire

1.	1.18	19.	2.00	37.	2.10	56.	1.95	75.	2.34
2.	1.17	20.	1.94	38.	2.13	57.	1.68	76.	2.36
3.	1.25	21.	2.25	39.	2.01	58.	2.13	77.	2.73
4.	1.29	22.	2.25	40.	1.54	59.	2.16	78.	1.91
5.	1.22	23.	1.83	41.	2.32	60.	2.42	79.	2.18
6.	1.44	24.	2.08	42.	2.22	61.	2.41	80.	2.41
7.	1.79	25.	1.98	43.	1.74	62.	1.44	81.	2.66
8.	1.84	26.	1.81	44.	1.78	63.	1.48	82.	2.55
9.	2.11	27.	1.96	45.	1.89	64.	1.70	83.	2.63
10.	1.86	28.	2.42	46.	1.58	65.	2.41	84.	2.62
11.	2.06	29.	1.92	47.	1.74	66.	1.93	85.	2.54
12.	1.67	30.	2.57	48.	1.77	67.	2.01	86.	2.30
13.	1.69	31.	2.42	49.	2.06	68.	2.25	87.	2.01
14.	1.77	32.	2.52	50.	1.87	69.	2.07	88.	2.48
15.	1.56	33.	2.30	51.	2.10	70.	2.28	89.	2.44
16.	1.43	34.	2.04	52.	2.42	71.	1.95	90.	1.91
17.	1.87	35.	2.18	53.	2.42	72.	2.49	91.	2.19
18.	2.01	36.	1.77	54.	2.62	73.	2.12	92.	2.32
				55.	1.6	74.	2.36	93.	2.25

The figures from Table 39 may be interpreted by means of the following scale:



In weighting the responses, it is clear that a large number indicates a limited use of the topic, whereas a small number indicates extensive use. It is also obvious that the extremes of the average will be 1 (very widely used), and 3 (never used).

If the scale is arbitrarily divided into three segments of equal length, the topics will, of course, fall into three groups. These are designated as A, B, and C. It might be said, roughly then, that in the opinions of those responding to the questionnaire, that the topics in group A contain mathematics very important to the engineer; group B, of moderate importance; and group C, unimportant. The following scale shows the position and numerical range of these three groups.

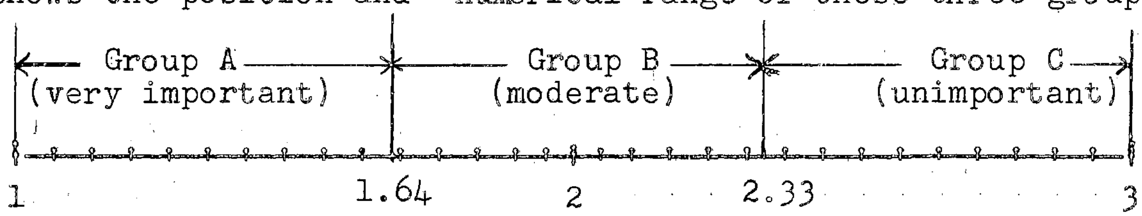


Table 40 shows the topics which the responders to the questionnaire considered very important.

Table 40. Topics Under Group A -- Mathematics Considered by the Responders to the Questionnaire to be Very Important

1. Concept of percentage.
2. Ratio and proportion.
3. Solution of problems involving physical magnitudes.
4. Scale drawings.
5. Meaning of an approximate number.
6. Bar, circle, and line graphs.
15. Functional notation.
16. Properties of a linear function.
40. The Pythagorean Theorem.
46. The use of trigonometric functions of angles greater than ninety degrees.
55. The use of radian measure.
62. The use of vectors.
63. Addition and subtraction of vectors.

Table 41 shows the topics which the responders to the questionnaire considered of moderate importance.

Table 41. Topics Under Group B -- Mathematics Considered by the Responders to the Questionnaire to be of Moderate Importance

7. The use of parentheses, brackets, braces, etc.
8. The use of algebraic fractions.
9. Operations involving polynomials.
10. Common algebraic products.
11. Factoring of standard algebraic expressions.
12. The laws of exponents.
13. The solution of linear equations.
14. The simultaneous solution of equations involving two or more variables.
17. Operations using radicals.
18. The use of complex numbers.
19. The quadratic equation in standard form.
20. The solution of quadratic equations.
21. The solution of a system consisting of a linear and a quadratic equation.
22. The solution of a pair of quadratics.
23. The solution of verbal problems.
24. The solution of algebraic equations involving radicals.
25. The binomial theorem.
26. Scientific notation.
27. Computation by use of logarithms.
29. The solution of exponential and logarithmic equations.
33. The use of determinants.
34. The use of arithmetic progressions.
35. The use of geometric progressions.

Table 41 -- Continued

36. The use of inequalities.
37. The use of polygons.
38. The concept of locus.
39. Parallelism and perpendicularity of lines.
41. The use of polyhedrons.
42. The use of cylinders, cones, and spheres.
43. The solution of right triangles by trigonometry.
44. The relationships of acute angles of a right triangle; e. g., sin (90 - A) = cos A.
45. The solution of verbal problems involving right triangles.
47. The use of quadrantal angles.
48. Fundamental trigonometric identities.
49. Addition identities.
50. The law of sines.
51. The law of cosines.
56. The relation of linear to angular velocity.
57. The application of sine and cosine curves.
58. Inverse trigonometric functions.
59. The solution of trigonometric equations.
64. Graphical use of vectors.
66. The use of polar co-ordinates.
67. The slope formula in Cartesian co-ordinates.
68. Standard forms for equations of straight lines.
69. Standard forms for parabolas.
70. Standard forms for ellipses and hyperbolas.
71. The use of parametric equations.
73. The computation of areas by integration.
78. Maxima and minima.
79. The slope of a curve.
86. Radius of gyration.
87. The use of three-dimensional Cartesian co-ordinates.
90. Differentiation with respect to time.
91. The laws of falling bodies.
92. Multiple integration.
93. Partial differentiation.

Table 42 shows the topics which the responders to the questionnaire considered unimportant.

Table 42. Topics Under Group C -- Mathematics Considered by the Responders to the Questionnaire to be Unimportant

28. Changing of the base of logarithms.
30. Finding the rational roots of higher degree polynomials.
31. Graphing of higher degree polynomials.
32. Finding the irrational roots of higher degree polynomials.
52. The law of tangents.

Table 42 -- Continued

53.	Area formulas; e. g., K = bc sin A.
54.	The solution of triangles by logarithms.
60.	Double-angle identities.
61.	Half-angle identities.
65.	The use of the distance formula in Cartesian co-ordinates.
72.	The formula for dividing a line segment in a given ratio.
74.	Computing volumes by integration.
75.	Computing first moments by integration.
76.	Computing second moments by integration.
77.	Computing arc length by integration.
80.	The delta process of integration.
81.	Finding work done on a body with a variable force.
82.	Computing surface areas by integration.
83.	The theorem of Pappus.
84.	Computing fluid pressure by integration.
85.	Finding centroids by integration.
88.	The use of cylindrical co-ordinates.
89.	The use of spherical co-ordinates.

The opinions of the engineers in the field seem to be fairly consistent with the results obtained from the surveys of the texts. For example, the prevalence of topics in algebra and trigonometry, as contrasted to the dearth of topics in analytic geometry and the calculus, is shown in both the surveys of the texts and in the questionnaire results.

It should be noted that no question found in group A occurs beyond the level of trigonometry; and, of the last twenty-one questions, which are topics of the calculus, none appeared in group A, nine were in group B, and the remaining twelve were in group C. Elementary processes of algebra, the use of radian measure, the application and use of vectors, and the Pythagorean theorem are the topics which most conspicuously corroborate the texts.

CHAPTER IV

SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

Foreword. As a background for the summary and recommendations, it may be well to discuss briefly the role of the engineer in the light of present technology. Engineering is obviously a growing and a rapidly changing field. No engineering school can completely train its students for all the tasks they will be required to undertake; and the engineering curriculum, then, must consist largely of certain broad fields of learning. In his studies in these fields, the student of engineering learns the scope of his problems, that is, the broad principles which he will use as guides, and some, but by no means all, of the specific skills he will need. In no sense is his training like that of, say, a barber, who studies precisely the skills he will practice. Moreover, the engineer does not, as does the physician, practice non-professionally as an intern. His lot is to graduate and to be trained on the job for the skills desired of him.

Thus, although mathematics is an integral part of every major aspect of engineering, one cannot say that the graduates in any given engineering curriculum will all use the same mathematics. In fact, one cannot even maintain that at some time in their professional careers they will all encounter a given topic, say, cylindrical co-ordinates. Indeed, sixty-eight of the one hundred six engineers who filled out the questionnaires testified that they never use such co-ordinates. Perhaps they have even forgotten how to use them. Yet, twenty-one testified that they use these co-ordinates occasionally, and seventeen use them extensively.

Before summarizing the findings of this study, then, it is well to point out that even though the engineering student is guided exclusively, or almost exclusively, by utilitarian objectives in his studies in mathematics, one can not hope to lay down precise, detailed, and unchanging lists of topics in mathematics which all students of engineering should study. On the other hand, without depreciating broad cultural values in mathematics, one should strive to make the mathematics taught to engineering students as practical as possible by continuous study of both engineering textbooks and the reports of engineers in the field.

SUMMARY

Table 43, which follows, combines the findings of Tables 11, 16 , 25, and 37; it presents a summation of the broader facets of the analyses of the textbooks in engineering.

Table 43. Summary of the Number of Engineering Texts Which Present Certain Mathematical Topics and the Total Number of Occurrences of Such Topics in the Texts.

	Algebra	No. of Texts Occurring	Total of Occurrences
1.	Elementary processes and formulas.	33	3306
2.	Solution of equations.	16	185
	Trigonometry		
1.	Formulas containing trigonometry.	26	875
2.	Use of logarithms.	20	209
3.	Numerical solution of triangles.	6	63
4.	Use of radian measure.	11	174
5.	Trigonometric identities.	12	109
6.	Use of vectors.	14	517
7.	Inverse functions.	12	109
8.	Graphs of trigonometric functions.	11	192
	Differential Calculus		
1.	Differentiation of algebraic expressions.	11	135
2.	Differentiation of transcendental expressions.	10	68
3.	Application of the derivatives.	7	52
4.	Partial derivatives.	2	32
	Integral Calculus		
1.	Integration of algebraic expressions.	19	267
2.	Integration of transcendental expressions.	15	157
3.	Application of the definite integral.	5	44
4.	Multiple integrals.	2	11
	Higher Mathematics.		
1.	Differential equations, Fourier series, etc.	16	177

Table 44, which follows, presents a summation of the broader facets of the analysis of the questionnaires. By using the same items for this table as were used for the previous one, it is possible to make a tabular comparison of the textbooks analyzed, with that of the questionnaire. To accomplish this it is only necessary to consolidate some of the questions in the questionnaire. Consider, for example, number one under Algebra: Elementary processes and Formulas; the figures representing this item are arrived at by consolidating questions seven through twelve.

The numbers at the head of the column to the right indicate the same as the responses to the questionnaire; that is, the number (1) indicates that those responding thought the items were of frequent occurrence; the number (2) indicates that they thought the items occurred occasionally; the number (3) indicates that they thought the items did not occur at all.

Table 44. Summary of the Responses to the Questionnaire.

	Algebra	(1) Frequently occurs	(2) Occasionally occurs	(3) Never occurs
1.	Elementary processes and formulas.	32	53	21
2.	Solution of equations.	30	50	26
	Trigonometry			
1.	Formulas containing trigonometry.	56	32	18
2.	Use of logarithms.	20	42	44
3.	Numberical solution of triangles.	37	28	41
4.	Use of radian measure.	50	28	28
5.	Trigonometric identities.	28	32	46
6.	Use of vectors.	69	16	21
7.	Inverse functions.	26	40	40
8.	Graphs of trigonometric functions.	58	24	24
	Differential Calculus			
1 + 2	Differentiation of algebraic and transcendental expressions	24	42	40

Differential Calculus	(1) Frequently occurs	(2) Occasionally occurs	(3) Never occurs
3. Application of the derivative.	30	42	34
4. Partial derivatives.	19	36	49
Integral Calculus			
1. Integration of algebraic expressions.		No data	
2. Integration of transcendental expressions.		No data	
3. Application of the definite integral.	13	22	71
4. Multiple integrals.	15	42	49
Higher Mathematics			
1. Differential equations, Fourier series, etc.		No data	

The following generalized conclusions have been developed:

1. The results of this study indicate that certain areas of mathematics are used almost universally by engineers.

2. This study indicates the tremendous importance of algebraic symbols and language.

3. Trigonometry is quite important to the engineer, but probably less important than algebra.

4. Analytic geometry is important only as a prerequisite to the calculus; it is not a subject that is, of itself, important as far as the engineer is concerned.

5. When the calculus is considered, one seems to have reached the stage at which the specializations inherent in the work of engineers are such that it is difficult, if not impossible, to single out many essential topics.

6. In general the answers to the questionnaires support the engineering textbooks; that is, the subject matter in the text-

books is usually described by the responders to the questionnaires as being extensively used in their work.

RECOMMENDATIONS

In keeping with the purpose of this study and in consideration of the Foreword, and Summary, of this chapter, the following recommendations seem desirable:

1. There is a need for better co-ordination between the departments of mathematics and engineering.
2. In the mathematics curriculum there should be courses designed especially for students other than engineers. Such courses may be college geometry, or history of mathematics. These would ensure a retention of, and emphasis upon, the cultural objectives in the mathematics curriculum.
3. It is recommended that College Algebra be continued in its present form. The inclusion of topics in the algebra, such as permutations and combinations, probability, and the like, while possibly of interest to the student, are strictly not within the engineers' purview. Of much greater value to him would be the mastery of such topics as factoring, addition and multiplication of algebraic fractions, solution of equations, and so forth.
4. The most thorough teaching of the definitions of the trigonometric functions is recommended.
5. It is recommended that very careful consideration be given to the teaching of radian measure. The student will need to know the relation between arc length, radius, and angular velocity; he will need to know these relations, also, when

the time element is involved---i. e. the relation between the radius of a wheel, and linear and angular velocity.

6. It is recommended that the student master the sine and cosine curves. He should be taught, not merely the fundamental, but also the overtones, and be able to compute by inspection the amplitude and period of the curves.

7. Increased attention should be given to the concept and use of vectors. This is, perhaps, the easiest of attainment of any of the recommendations made. In all courses, from the algebra through the calculus, the instructor may merely put greater emphasis upon the scaled drawings. In all such drawings, where magnitude and direction are specified, let the instructor clearly name and label such entities as "vectors."

8. The survey also indicates the importance of the following topics in trigonometry, and it is recommended that they be taught with no less than the usual emphasis: the theorem of Pythagoras; numerical computation of triangles; fundamental identities; trigonometric reductions; formulas for composite angles; the solution of trigonometric equations; inverse functions; and complex numbers.

9. It is recommended that less attention be given to the conic sections, and more time given to the teaching of parametric and transcendental equations, and their curves.

10. The following topics in the calculus were seen to be important, and it is recommended that increased emphasis be given in the teaching of them: rules for differentiation and integration of algebraic and transcendental functions;

applications of first derivatives---i. e. slopes, maxima and minima; differentials and their applications to small errors; the definite integral; the constant of integration; integration by standard devices---i. e. by parts, or trigonometric substitution; and centroids and fluid pressure.

11. It is recommended that students in electrical engineering be given a course within their junior year in which differential equations and Fourier series are offered.

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