NOZZLE DESIGN

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PREFACE

The applications of the nozzle have been widely developed in the steam turbine power plant field. In the continuous-combustion gas turbine air is used as the working substance in the nozzle.

The mechanical construction of the nozzle is very simple. It is merely a circular passage of varying cross-sectional area. But considering the deviation of air from perfect gas behavior and frictional effects, the design of a real nozzle is a difficult problem.

In particular, frictional effects in the nozzle are quite difficult to take into account. The present paper suggests an easier systematic method to design a real nozzle with considerable precision.

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LIST OF ABBREVIATIONS AND SYMBOLS

a	acoustic velocity, ft/sec
A	cross-sectional area of nozzle, sq in.
A_t	cross-sectional area of throat, sq in.
A/A_t	ratio of cross-sectiona area at any point to the cross-sectional area of throat
c_p	specific heat of air at constant pressure=0.24Btu/lb-F
$\tilde{c}_p$	variable specific heat at constant pressure, Btu/lb-F
c_v	specific heat of air at constant volume=0.17Btu/lb-F
$\tilde{c}_v$	variable specific heat at constant volume, Btu/lb-F
D	diameter of cross-sectional area, in.
E_f	energy loss due to friction, Btu/lb
g	acceleration due to gravity=32.2ft/sec^2
h	enthalpy of air=u+pv, Btu/lb
h_o	enthalpy of air at inlet of the nozzle, Btu/lb
$h_{o'}$	enthalpy of air at point of reference=0Btu/lb
Δh	enthalpy change
J	a conversion factor, 778 ft-lb/Btu
k	constant specific-heat ratio, $c_p/c_v = 1.4$
$\tilde{k}$	variable specific-heat ratio, $\tilde{c}_p/\tilde{c}_v$
K.E.	kinetic energy, $V^2/2g$ ft-lb/lb
L	location along the length of nozzle, in. or ft
L_t	location of the throat, % L
n	exponent of polytropic process
p	static pressure, lb/in.^{2}
p_o	static pressure of air at the inlet of nozzle, lb/in.^{2}

p_o'	static pressure at point reference = 1 atm
p_t	critical pressure at the throat, $lb/in.^2$
Q	heat flow, Btu/lb
R	gas constant of air = 53.35 ft-lb/lb R
r_c	critical pressure ratio
s	entropy of air at inlet of nozzle, Btu/lb R
s_o	entropy of air at point of nozzle, Btu/lb R
s_o'	entropy of air at point of reference, Btu/lb R
T	temperature of air, F or R
T_o	temperature of air at inlet of nozzle, F or R
$T_{o'}$	temperature of air at point of reference, F or R
u	internal energy of air, Btu/lb
v	specific volume of air at inlet of nozzle, ft^3/lb
V	velocity of air stream, ft/sec
W	rate of flow of air, lb/sec
W/A	rate of flow of air per sq in. of cross-sectional area, $lb/hr\ in.^2$
η	nozzle efficiency, percent
τ	compressibility, $in.^2/lb$

CHAPTER I

INTRODUCTION

1. CONVERGENT-DIVERGENT NOZZLE

Air is commonly used as the working substance in the nozzle. The nozzle is a device which is designed for the expansion of the working substance with an increase of velocity. Hence, the thermal energy is thereby changed into kinetic energy, or in other words, the exit velocity of the fluid is produced entirely from the transformation of energy.

As shown in Fig. 1, when air enters the nozzle, the pressure decreases from initial pressure p_o (i.e., the stagnation pressure at which the velocity is equal to zero) to same lower value p_t at section t. In this subsonic or subcritical region 0-t, the pressure decreases, the velocity increases, the area decreases, and the specific volume increases. The velocity and the specific volume continue to increase throughout the convergent-divergent nozzle. But in the subsonic region, the velocity increase rate is higher than that of the specific volume.

When the rate of velocity increase is at exactly the same rate as the specific volume increase, this condition is denoted as the critical point. By the continuity equation, the ratio of the velocity of air to its specific volume is identical to the mass rate of air per unit cross-sectional area. Thus for a certain amount of air flow, the area of the nozzle is a minimum at the critical point, and we call this critical point the "throat".

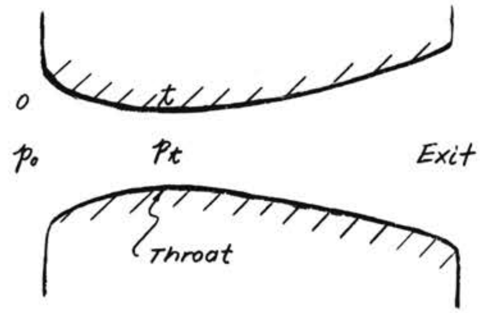

Fig. 1. Convergent-divergent Nozzle.

In the supersonic expansion (supercritical region), the rate of increase of specific volume of air is higher than that of the velocity, and a diverging section is required: (1) to accommodate the flow at the greatly increased specific volume, and (2) to control and guide the expansion for maintaining the velocity higher than the acoustic velocity after the throat.

2. COMPRESSIBLE FLOW

In this treatment, the flow of air will be considered as one-dimensional flow with two pertinent characteristics. One of these two characteristics is its compressibility, and the other is its behavior in accordance with perfect gas law. The compressibility of air τ is defined as the inverse of its bulk modulus.

Hence: τ = 1/Bulk modulus

= Unit strain/Change of unit stress

= $-(dv/v)/vdp$ in.^{2}/lb

The compressibility also varies with the specified process and the temperature concerned. But in the nozzle, air merely undergoes an expansion process, and therefore its compressibility has a positive value. Now the air is treated as a perfect gas and it undergoes an isentropic expansion process in accordance with pv^k=constant. Differentiating the equation, it becomes

$$v^k dp + kv^{k-1} p dv = 0$$

and multiplying by $1/pv^k$,

$$dp/p + k \cdot dv/v = 0$$

Then $$\tau_{ad} = -dv/vdp = +1/kp$$

Therefore the compressibility of air for an isentropic expansion is equal to $-1/kp$.

3. BOUNDARY LAYER

The layer of air between the wall of the nozzle and the location where the main stream velocity is attained is called the boundary layer.

A. The frictional force is mainly due to the viscous effects which are substantially limited to the thin layer close to the system boundaries. The outside of the layer is called the core, and the flow of air is exactly the same as the inviscid flow passing the same boundaries, so that there is no frictional force in the core.

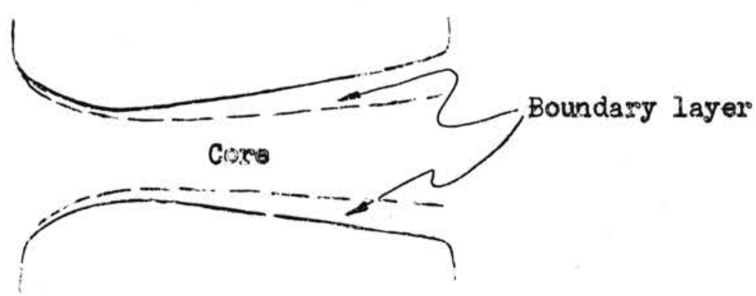

Fig. 2. Boundary Layer in the Nozzle.

B. The flow of air outside the boundary layer is either subsonic or supersonic. The pressure change across the inside viscous layer is quite small, and the velocity distribution equation is changed from an elliptic form to a parabolic form. This means that the laminar flow has been changed into turbulent flow, as shown in Fig. 3.

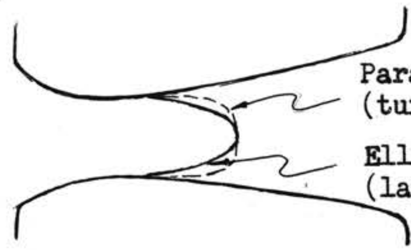

Fig. 3. Laminar Flow and Turbulent Flow.

C. When the effects of the compressibility of air are taken into account and the flow outside the boundary layer is subsonic, an interaction is found. The effect of the laminar flow in the core, will produce some acceleration of the main flow. Any turbulence such as caused by roughness of the wall, will tend to thicken the boundary layer at that point, but it will also produce some further acceleration. This further acceleration will in turn reduce the boundary-layer thickness and therefore reduce the effects of the disturbance. Laminar flow and turbulent flow are both a function of Reynold's number. By experiments, the range of Reynolds number below approximately 2,000 yields laminar flow and above 2,000 yields turbulent flow.

CHAPTER II

IDEAL NOZZLE DESIGN WITH CONSTANT SPECIFIC HEAT

1. REVERSIBLE ADIABATIC NOZZLE

The time required for air to pass through the nozzle is extremely small, since the velocity is so high. The inner wall surface of the nozzle is small as compared to the large quantities of air passing through it, therefore, the expansion of air in a nozzle is essentially adiabatic even though there is some temperature difference between the wall and air.

In the treatment of the ideal nozzle, the flow of air through the nozzle is reversible as well as adiabatic, and thus there is no change of entropy. This nozzle is called a reversible adiabatic nozzle.

Thus if the expansion of the air in a nozzle is shown on an entropy-enthalpy diagram, it will be a vertical line, as shown in Fig. 4.

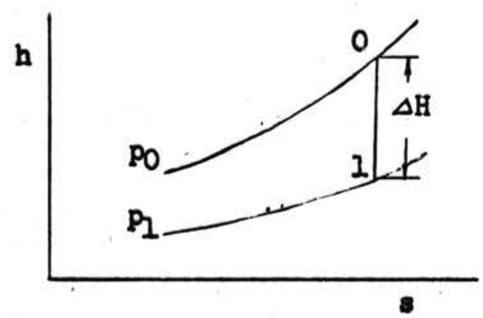

Fig. 4. Isentropic Expansion on h--s Diagram.

2. PROCEDURE

In order to obtain the shape of a nozzle and to analyse the effects due to variable specific heat and friction, one problem will be handled by three different methods as in the procedure of this chapter and in Chapters III, and IV. The design conditions were selected as follows: the initial pressure ahead of the nozzle is 300 lb/in.^{2} abs, the initial temperature 700 F, the final pressure is 1 in. Hg abs, and the flow is to be 1 lb/sec. Assume a constant-entropy expansion and constant pressure drops for equal increments of length. By considering constant specific heat and perfect-gas behavior, an air nozzle will be designed as follows:

A. Entropy. Choose the temperature and the pressure of air at point of reference to be 0 F and 1 atm respectively. Since

$$dQ = dh - vdp$$
$$= c_p dT - pT/J \cdot dp/p$$

the entropy at the point of inlet

$$s_o = \int dQ/T = \int_{T_{o'}}^{T_o} c_p dT/T - \int_{T_{o'}}^{T_o} (R/J)dp/p$$
$$= c_p \ln(T_o/T_{o'}) - (R/J)\ln(p_o/p_{o'})$$
$$= 0.24\ln(700+460)/460 - (53.35/778)\ln(300/14.7)$$
$$= 0.0155 \text{ Btu/lb R}$$

B. Specific Volume. According to the perfect-gas law, the volume of air at inlet,

$$v_o = RT_o/p_o$$
$$= 53.35 \cdot 1160/300 \cdot 144$$
$$= 1.431 \text{ ft}^3/\text{lb}$$

And the volume of air any point,

$$v = (p_o/p)^{1/k} \cdot v_o$$
$$= 300^{1/1.4} \cdot 1.431/p^{0.715}$$
$$= 84.5p^{-0.715}$$

C. Critical Pressure Ratio. For isentropic expansion, the critical pressure ratio can be easily found by

$$r_c = p_t/p_o = (2/k+1)^{k/k-1}$$
$$= (2/1.4+1)^{1.4/1.4-1} = 0.5283$$

D. Critical Point. The critical point is located at the point of critical pressure, and it can be estimated by proportion

$$Lt/0.1L = 141.5/29.95$$

Therefore $$L_t = 0.472\ L$$

E. Temperature. With the same method of (B), the temperature at any point is

$$T = pv/R = 2.7\ pv$$

F. Enthalpy. Assume the enthalpy at 0 F, $h_{o'}$ equals to zero, and specific heat at constant pressure c_p equals to 0.24 Btu/lb. Then the enthalpy of air at any point referred to 0 F,

$$h - h_{o'} = c_p(T - T_o')$$

$$= 0.24\ T(F)$$

G. Enthalpy Change. The enthalpy change is

$$\Delta h = ho - h$$

H. Velocity. The velocity of air is needed to calculate the rate of air flow. Since the inlet area of the nozzle may be considered as infinite, the initial velocity can be assumed to be zero. Thus the velocity at any point can be calculated from the energy equation of steady flow, so that

$$V = \sqrt{2gJ\Delta h}$$

$$= 223.9\sqrt{\Delta h}$$

I. The Rate of Flow per Unit Area. Using the equation of continuity, the rate of flow per unit area can be readily found by

$$W/A = V/v \cdot 3600/144$$

$$= 25\ V/v\ \text{lb/in.}^2\ \text{hr}$$

J. Cross-sectional Area. From the term I, the cross-sectional area at any point can be also calculated, i.e.,

$$A = W/(W/A) = 3600/(W/A)$$

where $W = 3600$ lb/hr.

K. Diameter. The diameter of cross-sectional area at any point can be obtained by the relation

$$D = (4A/\pi)^{1/2}$$
$$= 1.128\, A^{1/2}$$

L. The Ratio of Cross-sectional Area at Any Point to the Throat Area. The ratio of cross-sectional area at any point to the throat area will be

$$A/A_t = A/0.212$$

where A_t equals to 0.212 in.^{2}

3. CALCULATED RESULTS

The results of calculations described above are presented in Table 1. Those results which will indicate the performance of the nozzle, are plotted on the following figures:

(1). The static pressure in the air stream (Fig. 5).

(2). The specific volume of the air (Fig. 6).

(3). The temperature of the air (Fig. 7).

(4). The velocity of the air stream (Fig. 8).

(5). The rate of flow per square inch of cross-sectional area (Fig. 9).

(6). The cross-sectional area of the air stream (Fig. 10).

(7). The diameter of the air stream (Fig. 11).

(8). The ratio of cross-sectional area of the air stream to the throat area (Fig. 12).

All these curves shown in the figures are represented by solid lines.

CHAPTER III

IDEAL NOZZLE DESIGN WITH VARIABLE SPECIFIC HEAT

1. SPECIFIC HEAT OF AIR

The specific heat of air is the amount of heat required to raise the temperature of 1 Fahrenheit degree of 1 lb of air. Not only do perfect gases follow the equation of state, pv=RT, but also the specific heat of the perfect gas is constant. Unfortunately, this is true only at low temperature for real gases.

Since the specific heat of air is not a constant but is influenced by both pressure and temperature, the perfect-gas law does not hold exactly for air. Some important relations between the specific heat and temperature have been found as follows:

Sweigert and Beardsley specific heat equations for air

$$c_p = a - b/T + c/T^2 \qquad (a)$$

or

$$c_p = a - b/T^{1/2} + c/T \qquad (b)$$

where a, b, c are constant.

J. Smallwood specific heat equations for air

$$c_p = 0.342 - 1.25T^{1/2} - 82.4T^{-1} + 31{,}200T^{-2}$$

The specific heat at constant pressure is not truly constant, therefore, the specific heat ratio is also not constant. From the Sweigert-Beardsley equations and the J. Smallwood equation, the specific heat at constant pressure is predominately the function of temperature only. Now the variable specific-heat ratio $\widetilde{k}$ obtained from Table 2 of "Thermodynamic Properties of Air" by Keenan &. Kays is used to estimate the deviation of air from the perfect-gas law, so that an actual contour of the nozzle may be designed.

2. PROCEDURE

In the present design, the problem has the same assumptions as in Chapter II, except using variable specific heat.

A. Entropy. The entropy change of air at any point is measured in the same manner in Chapter II. A.

B. Specific Volume And Temperature of Air. The initial specific volume has the same value calculated in Chapter II. B. At temperature 1,160 R, the variable specific heat ratio k is equal to 1.3704 read from Table 2 of "Thermodynamic Properties of Air". Therefore,

$$\begin{aligned} v_1 &= (p_o/p_1)^{1/k} \cdot v_o \\ &= (300/270.05)^{1/1.3704} \cdot 1.431 \\ &= 1.545 \text{ ft}^3/\text{lb} \end{aligned}$$

And

$$\begin{aligned} T_1 &= p_1 v_1/R \\ &= 270.05 \cdot 144 \cdot 1.545/53.35 \\ &= 1{,}127 \text{ R} \end{aligned}$$

In the same manner, the specific volume and temperature of air at point 2 can be estimated and so on. They are calculated in more detail between the pressure 30.45 psia to 0.491 psia in which the value of $\tilde{k}$ is greater than 1.4.

C. Critical Pressure Ratio. Since the critical pressure ratio was estimated in Chapter II, at which the temperature is about 920 R, the value of $\tilde{k}$ is approximately equal to 1.3820. Therefore the critical pressure ratio

$$\begin{aligned} r_c &= p_t/p_o = (2/1+k)^{k/k-1} \\ &= (2/1.3820+1)^{1.382/1+1.382} \\ &= 0.53 \end{aligned}$$

D. Critical Point. By proportion, the critical point can be found as follow:

$$p_t = 0.53\ p_o = 159 \text{ psia}$$

Therefore $L_t/0.1L = (300-159)/(300-270.05)$

and $L_t = 0.47\ L$

E. Enthalpy. Since the enthalpy of air at point of reference is assumed to be 0, the enthalpy of air at any point can be found by

$$h - h_{o'} = \tilde{c}_p(T - T_{o'})$$

$$h = \tilde{c}_p T(F)$$

where $\tilde{c}_p$ is variable specific heat at constant pressure which can be obtained from Table 2 of "Thermodynamic Properties of Air" at different temperature.

F. Enthalpy Change. The enthalpy change is

$$\Delta h = h_o - h$$

G. Velocity. The velocity of air at any point can be found in the same manner as described in Chapter II-H, i.e.,

$$V = 223.9\sqrt{\Delta h}$$

H. Flow Rate per Unit Area, Cross-sectional Area, Diameter and the Area Ratio. The rate of flow per unit area, the cross-sectional area of the nozzle, the diameter of area, and the ratio of cross-sectional area at any point to the throat area are calculated by the same methods described in Chapter I-I, J, K, & L.

3. CALCULATED RESULTS

The results obtained from the above procedure, are presented in Table 2. These results indicate the deviation of the performance from the perfect-gas nozzle and are plotted on figures as follows:

(1). The static pressure in the air stream (Fig. 5).

(2). The specific volume of the air (Fig. 6).

(3). The temperature of the air (Fig. 7).

(4). The velocity of the air stream (Fig. 8).

(5). The rate of flow per square inch of cross-sectional area (Fig. 9).

(6). The cross-sectional area of the air stream (Fig. 10).

(7). The diameter of the air stream (Fig. 11).

(8). The ratio of cross-sectional area of the air stream to the throat area (Fig. 12).

All these curves shown in the figures are represented by dashed lines.

CHAPTER IV

REAL NOZZLE DESIGN WITH FRICTION

1. ADIABATIC FLOW WITH FRICTION

The stream friction is due to the action of viscosity in the boundary. The turbulence of the air in the laminar boundary layer grows downstream along the nozzle, and consequently, changes the characteristic of laminar flow into a turbulent boundary layer.

Frictional forces will be experienced by the air stream from two different sources; one is caused by the air flowing against the inner walls of the nozzle, and the other one is produced internally by the molecular motion and molecular collisions of the air. As a result the final enthalpy in the flow with friction will be increased over that in the case of the ideal flow. In addition, the entropy of the air increases.

Some useful relations for adiabatic flow with friction will be derived. Suppose

h_o = the enthalpy of unit pound of air at initial state,

h_{10} = the final enthalpy of unit pound of air at state 10, and

h_{10}' = the final enthalpy of unit pound of air at state 10', for actual flow, thus

$(h_o - h_{10})$ = the enthalpy decrease of ideal flow per pound of air, and

$(h_o - h_{10'})$ = the enthalpy decrease of actual flow per pound of air.

It is known that the enthalpy decrease of ideal flow is larger than the enthalpy decrease of actual flow due to frictional effects, i.e.,

$$(h_0 - h_{10}) > (h0 - h_{10'})$$

Since the initial state h_0 remains the same,

$$h_{10}' > h_{10}$$

So it can be imagined that the difference of final enthalpy from the actual flow to the ideal flow will be the energy dissipated in friction.

In order to analyse the frictional effects, the nozzle is divided into a number of infinitely small sections. The relation between the actual enthalpy change and the ideal enthalpy change in each section shown in Fig. 13-b, is numerically

$$E_f = dh_f - dh_i \tag{1}$$

where E_f stands for the loss in enthalpy change due to friction.

For perfect gases, the dynamic equation for flow with friction is

$$vdp + \frac{dV^2}{2g} + 4f(\frac{V^2}{2g})\frac{dx}{D} = 0 \tag{2}$$

where f is the friction coefficient for the nozzle passage, and the term $4f(V^2/2g)(dx/D)$ is really the equvilent of frictional loss represented by E_f.

The energy equation for adiabatic flow is

$$(h_2 - h_1) + \left[(V_2^2/2g)-(V_1^2/2g)\right] = 0$$

or

$$dh_f = -dV^2/2g \tag{3}$$

Substituting Eq. (3) in Eq. (2),

$$E_f = dh_f - vdp \tag{4}$$

Equating Eq. (1) and Eq. (4),

$$dh_f - dh_i = dh_f - vdp \tag{5}$$

Nozzle efficiency is defined as the ratio of actual change of kinetic energy to isentropic change of kinetic energy, and consequently it can be measured by the ratio of actual enthalpy change to the ideal enthalpy change, i.e.,

$$\eta = \Delta h_f/\Delta h_i = dh_f/dh_i \tag{6}$$

Also

$$dh_f = c_p dT \tag{7}$$

Substituting Eq. (6) and (7) into Eq. (5),

$$dh_f(1 - dh_i/dh_f) = dh_f - vdp$$

$$c_p dT(1 - 1/\eta) = c_p dT - vdp$$

$$c_p dT = \eta vdp \tag{8}$$

Since

$$pv = RT$$

and

$$c_p = Rk/k\text{-}1$$

Eq. (7) becomes

$$c_p dT = c_p d(pv/R)$$

$$= (k/k\text{-}1)(pdv+vdp) \tag{9}$$

Equating Eq. (8) and (9),

$$(k/k\text{-}1)(pdv+vdp) = \eta vdp$$

$$(k/k\text{-}1)pdv = (\eta\text{-}k/k\text{-}1)vdp$$

$$(k/k\text{-}1)/(\eta\text{-}k/k\text{-}1) \cdot dv/v = dp/p$$

Integrating this equation,

$$(k/\eta k\text{-}\eta\text{-}k)\ln v + \ln c = \ln p$$

$$(k/k\text{-}\eta k\text{-}\eta)\ln^{-1} + \ln c = \ln p$$

$$\ln p - \ln v^{-k/k-\eta k+\eta} = \ln c$$

$$pv^{k/k-\eta k-\eta} = C \tag{10}$$

which shows the process that a real adiabatic flow undergoes.

The adiabatic flow with friction is an irreversible process, and the equation for the polytropic process $pv^n = C$, will be taken instead of the equation for isentropic process $pv^k = C$. Now, compare the polytropic process $pv^n = C$ with Eq. (10), and it indicates the exponent

$$n = k/k\text{-}\eta k+\eta \tag{11}$$

So the relationship between the exponent of polytropic process n, and the exponent of isentropic process k, can be found, if the nozzle efficiency has been taken into account.

2. PROCEDURE

A. Nozzle Efficiency. Since the frictional force is a function of Mach number and also a function of velocity, and the inlet velocity V_0 can be assumed to be zero, there is no frictional effect at the inlet of the nozzle.

For convenience, the nozzle is divided into ten sections, and each section has a different nozzle efficiency due to varying frictional effects. Since the nozzle efficiency is a function of velocity change, the nozzle efficiency is decreased from subsonic region to supersonic region. By experiments, the nozzle efficiency usually exceeds 85 percent and may easily be as high as 90 percent. At present, the nozzle efficiencies are assumed from 95 percent in the first section to 86 percent in the exit, and are listed in Table III.

B. Specific Volume. Eq. (10) gives

$$p_o/p_1 = (v_1/v_o)^{k/k-\eta k+k}$$

or

$$v_1 = v_o(p_o/p_1)^{1-\eta(\frac{k-1}{k})} \tag{12}$$

Since the pressures at each state are fixed by assumption and v_o is known by calculation, the specific volume v_1 of air at the end of section 0-1, can be found by Eq. 12 with nozzle efficiency η_{0-1} equal to 0.95. Thus from Eq. (12)

$$v_1 = 1.431(300/270.05)^{1-0.95(1.4-1)/1.4}$$

$$= 1.545 \text{ ft}^3/\text{lb}$$

In like manner, when v_1 is obtained, the specific volume of air at state 2 can be also calculated by Eq. (12) with nozzle efficiency

η_{1-2} equal to 94 percent; thus

$$v_2 = 1.545(270.05/240.10)^{1-0.94(0.286)}$$
$$= 1.684 \text{ ft}^3/\text{lb}$$

Likewise the specific volumes of air at state 3, 4, ... 10, are found.

C. Temperature. By the perfect-gas law, the temperature of air at each state with corresponding specific volume found in B, is obtained from

$$T = pv/R$$
$$= 144 \cdot pv/53.35$$
$$= 2.7\ pv$$

D. Critical Pressure. Even with frictional effects, the occurrence of critical pressure may be estimated within the section 4-5. Since the nozzle efficiency is assumed to be 91 percent for the section 4-5, the air will undergo a polytropic process $pv^n = C$, where n equals to 1.351 if $\eta = 0.91$. The properties v_4, T_4, p_4 are calculated, and V_4 is known also. The velocity of approach for this section is finite, therefore, the critical pressure should be found as follow:

Assume any point x is within section 4-5. So

$$(V_x^2/2g - V_4^2/2g) + (h_x - h_4) = 0$$
$$V_x^2/2g = (h_4 - h_x) + V_4^2/2g$$
$$V_x^2 = \sqrt{2gJc_p(T_4 - T_x) - V_4^2}$$
$$= \sqrt{2gJc_p}\sqrt{T_4}\sqrt{(1 - T_x/T_4) - V_4^2/2gJc_pT_4} \qquad (13)$$

If $pv^k = C$

$$T_x/T_4 = (p_x/p_4)^{k-1/k} \qquad (14)$$
$$v_x/v_4 = (p_x/p_4)^{-1/k} = (p_4/p_x)^{1/k} \qquad (15)$$

Let $p_x/p_4 = r$, thus Eq. 15 becomes

$$v_x = v_4 \cdot r^{-1/k}$$
$$= (RT_4/p_4)r^{-1/k} \qquad (16)$$

and Eq. 14 becomes

$$T_x/T_4 = r^{k-1/k} \tag{17}$$

Substituting Eq. 17 into Eq. 13,

$$V_x = \sqrt{2gJc_p \cdot T_4}\sqrt{1 - r^{k-1/k} - V_4^2/2gJc_p \cdot T_4}$$

According to continuity equation

$$W_x/A_x = V_x/v_x$$

and from Eq. 17 and 16,

$$W_x/A_x = \frac{\sqrt{2gJc_p \cdot T_4}\sqrt{1-r^{k-1/k} - V_4^2/2gJc_p \cdot T_4}}{(RT_4/p_4)r^{-1/k}}$$

$$= (\sqrt{2gJc_p} \cdot p_4/R \cdot \sqrt{T_4}) \cdot r^{1/k} \cdot \sqrt{(1-r^{k-1/k}) + V_4^2/2gJc_pT_4}$$

$$= (\sqrt{2gJc_p} \cdot p_4/R\sqrt{p_4v_4/R}) \cdot \sqrt{r^{2/k} - r^{2/k \cdot k-1/k} + (V_4^2/2gJc_pT_4)r^{2/k}}$$

$$W_x/A_x = (2gJc_p/R)^{1/2} \cdot (p_4/v_4)^{1/2} \cdot \left[(1+V_4^2/2gJc_pT_4)r^{2/k} - r^{k+1/k}\right]^{1/2}$$

At critical point, W_x/A_x is a maximum. Since $(2gJc_p/R)^{1/2} \cdot (p_4/v_4)^{1/2}$ is constant, the third term should be maximum.

Then

$$\frac{d}{dr}\left[(1 + V_4^2/2gJc_pT_4)r^{2/k} - r^{k+1/k}\right]$$

$$= (1 + V_4^2/2gJc_pT_4) \cdot \frac{2}{k} \cdot r^{2/k-1} - (k+1/k)r^{\frac{k-1}{k}-1}$$

$$= 0$$

So

$$(1 + V_4^2/2gJc_pT_4) \cdot \frac{2}{k} \cdot r^{2-k/k} = (k+1/k)r^{1/k}$$

$$r^{1-k/k} = (k+1)/2(1+V_4^2/2gJc_pT_4)$$

$$r = \left[\frac{2(1 + V_4^2/2gJc_pT_4)}{k + 1}\right]^{k/k-1}$$

where $k = \tilde{n} = 1.351$, $V_4 = 1323$ ft/sec, and $T_4 = 1014$ R.

Therefore, the critical pressure ratio

$$r_c = \left[\frac{2(1 + 1323^2/2 \cdot 32.2 \cdot 778 \cdot 0.24 \cdot 1014)}{1.351 + 1}\right]^{1.351/1.351-1}$$

$$= 0.9154$$

and the critical pressure

$$p_c = r_c \cdot p_4 = 165 \text{ psia}$$

E. Critical Point. The critical point is found by proportion

$$L_c = 0.1 \cdot L(300-165)/(300-270.05) = 0.451 \text{ L}$$

Hence, the Mach number at the throat for real flow is

$$M_f = 1423/(32.2 \cdot 1.4 \cdot 53.35 \cdot 992)^{1/2} = 0.922$$

while the Mach number at the throat for ideal flow is

$$M_i = 1530/(32.2 \cdot 1.4 \cdot 53.35 \cdot 965)^{1/2} = 1.000$$

F. Enthalpy, Enthalpy Change, Rate of Flow, Cross-sectional Area, and Diameter. Those values are also obtained with the same methods as in Chapter II-F, G, I, J, and K.

G. Area Ratio. The throat area calculated from F, is equal to 0.2248 in.^{2}, and thus the area ratio at any point is

$$A/A_t = A/0.2248$$

H. Entropy. The initial entropy has been calculated in Chapter II-A, and for any polytropic process $pv^n = C$, the entropy change is

$$s_2 - s_1 = (c_p - nc_v/n-1) \cdot \ln(T_1/T_2)$$

Using variable nozzle efficiency, the entropy at state 3 must be calculated after s_2 was obtained, i.e.,

$$s_3 = (c_p - \tilde{n}c_v/\tilde{n}-1) \cdot \ln(T_2/T_3) + s_2$$

In the same manner, the entropy at each state is found.

3. CALCULATED RESULTS

The results by using the above procedure, are listed in Table III. These results will indicate the performance of the real nozzle with frictional effect. They are plotted on the following figures:

(1). The static pressure in the air stream (Fig. 5).

(2). The specific volume of air (Fig. 6).

(3). The temperature of air (Fig. 7).

(4). The velocity of the air stream (Fig. 8).

(5). The rate of flow per square inch of cross-sectional area (Fig. 9).

(6). The cross-sectional area of the stream (Fig. 10).

(7). The diameter of the air stream (Fig. 11).

(8). The ratio of cross-sectional area of the air stream to the throat area (Fig. 12).

All these curves are shown in the figures by heavy lines.

TABLE I

Calculated Results for Ideal Nozzle Design with Constant Specific Heat

L	s	p	v	T	h	Δh	V	W/A	A	D	A/A_t
Position L	Entropy Btu/lbR	Pressure psia	Volume ft^3/lb	Temp R	Enthalpy Btu/lb	Enthalpy change Btu/lb	Velocity ft/sec	Rate of flow lb/hr $in.^2$	Area $in.^2$	Diameter in.	Area ratio -
0.000	0.0155	300.00	1.431	1,160	168.0	0.0	0	0	∞	∞	∞
0.100		270.05	1.537	1,124	159.5	8.5	652	10,610	0.3392	0.656	1.503
0.200		240.10	1.673	1,088	150.0	17.2	928	13,880	0.2592	0.569	1.196
0.300		210.15	1.843	1,045	140.4	27.6	1,176	15,940	0.2258	0.537	1.064
0.400		180.20	2.060	1,001	129.8	38.2	1,385	16,810	0.2140	0.522	1.010
0.472		158.50	2.253	965	121.3	46.7	1,530	16,960	0.2120	0.519	1.000
0.500		150.25	2.345	951	117.8	50.2	1,585	16,900	0.2130	0.520	1.005
0.600		120.30	2.742	891	103.4	64.6	1,800	16,400	0.2194	0.528	1.035
0.700		90.35	3.380	825	87.6	80.4	2,005	14,820	0.2328	0.550	1.145
0.800		60.40	4.510	735	66.0	102.0	2,260	12,530	0.2765	0.600	1.355
0.900		30.45	7.320	602	34.1	133.9	2,590	8,850	0.3945	0.721	1.920
1.000		0.49	140.050	186	-65.8	233.8	3,420	611	5.9000	2.740	27.800

TABLE II

Calculated Results for Ideal Nozzle Design with Variable Specific Heat

L	S	p	v	T	$\widetilde{k}$	$\widetilde{c}_p$	h	Δh	V	W/A	A	D	A/A_t
sition L	Entropy Btu/lbR	Pressure psia	Volume ft^3/lb	Temp R	Specific heat ratio	Specific heat at const p	Enthalpy Btu/lb		Velocity ft/sec	Rate of flow $lb/hr\,in.^2$	Area $in.^2$	Diameter in.	Area ratio –
.0000	0.0155	300.000	1.431	1160	1.3704	0.25346	177.40	0.00	0	0	∞	∞	∞
.1000		270.050	1.545	1127	1.3724	0.25244	168.40	9.00	672	10,870	0.3310	0.650	1.560
.2000		240.100	1.684	1092	1.3746	0.25136	158.80	18.60	964	14,300	0.2600	0.576	1.225
.3000		210.150	1.857	1054	1.3774	0.25028	148.70	28.70	1,200	16,100	0.2240	0.535	1.056
.4000		180.200	2.090	1015	1.3800	0.24905	138.60	38.80	1,390	16,700	0.2160	0.525	1.019
.4705		159.000	2.290	982	1.3820	0.24810	129.30	48.10	1,553	16,950	0.2123	0.520	1.000
.5000		150.250	2.385	967	1.3830	0.24768	125.50	51.90	1,614	16,920	0.2130	0.521	1.005
.6000		120.300	2.800	908	1.3865	0.24602	110.30	67.70	1,830	16,330	0.2200	0.530	1.037
.7000		90.350	3.440	840	1.3900	0.24436	92.70	84.70	2,060	14,970	0.2400	0.553	1.132
.8000		60.400	4.580	748	1.3941	0.24237	69.70	107.70	2,325	12,670	0.2840	0.603	1.340
.9000		30.450	7.500	617	1.3987	0.24050	37.70	139.70	2,645	8,820	0.4080	0.722	1.928
.9250		22.960	9.170	568	1.3996	0.24004	25.90	151.50	2,752	7,500	0.4800	0.782	2.262
.9500		15.570	12.150	508	1.4010	0.23965	11.50	166.90	2,890	5,950	0.6050	0.880	2.852
.9750		7.980	19.500	420	1.4016	0.23934	-9.58	186.98	3,060	3,920	0.9200	1.083	4.330
.9875		4.727	28.400	363	1.4020	0.23930	-23.25	200.65	3,165	2,790	1.2900	1.282	6.090
.0000		0.491	143.500	190	1.4020	0.23920	-64.70	242.10	3,480	605	5.9500	2.755	28.050

TABLE III

Real Nozzle Design with Friction

L	$\tilde{\eta}$	$\tilde{n}$	s	p	v	T	h	Δh	V	W/A	A	$\frac{A_r - A_i}{A_i}$	D	A/A_t
position	Eff.	Exponent	Entropy	Pressure	Volume	Temp.	Enthalpy	Enthalpy change	Velocity	Rate of flow	Area		Diameter	Area ratio
L	percent		Btu/lbR	psia	ft^3/lb	R	Btu/lb	Btu/lb	ft/sec	$lb/hr in.^2$	$in.^2$	%	in.	-
0.000			0.01550	300.00	1.431	1,160	168.0	0.0	0	0	∞	-	∞	∞
	95	1.374												
0.100			0.01645	270.05	1.545	1,127	160.0	8.0	633	10,240	0.3510	3.10	0.6675	1.560
	94	1.369												
0.200			0.01700	240.10	1.684	1,092	151.5	16.5	908	13,500	0.2660	4.00	0.5820	1.185
	93	1.363												
0.300			0.01783	210.15	1.860	1,055	143.0	25.0	1,118	15,040	0.2390	5.50	0.5520	1.063
	92	1.357												
0.400			0.01887	180.20	2.084	1,014	133.0	35.0	1,323	15,880	0.2268	5.98	0.5370	1.008
0.451	91	1.351	0.01976	165.00	2.265	985	126.0	42.0	1,423	16,000	0.2248	—	0.5350	1.000
0.500			0.02020	150.25	2.385	967	121.6	46.4	1,524	15,980	0.2260	6.10	0.5360	1.002
	90	1.346												
0.600			0.02208	120.30	2.810	911	108.3	59.7	1,730	15,400	0.2339	6.60	0.5470	1.039
	89	1.341												
0.700			0.02511	90.35	3.428	837	90.5	77.5	1,968	14,340	0.2510	7.60	0.5655	1.115
	88	1.336												
0.800			0.02897	60.40	4.632	756	71.0	97.0	2,195	11,850	0.3040	10.03	0.6225	1.352
	87	1.331												
0.900			0.03606	30.45	7.750	637	42.5	125.5	2,505	8,070	0.4460	12.99	0.7550	1.982
	86	1.327												
1.000			0.08091	0.49	174.400	231	-55.0	223.0	3,340	478	7.5200	27.45	3.0950	33.400

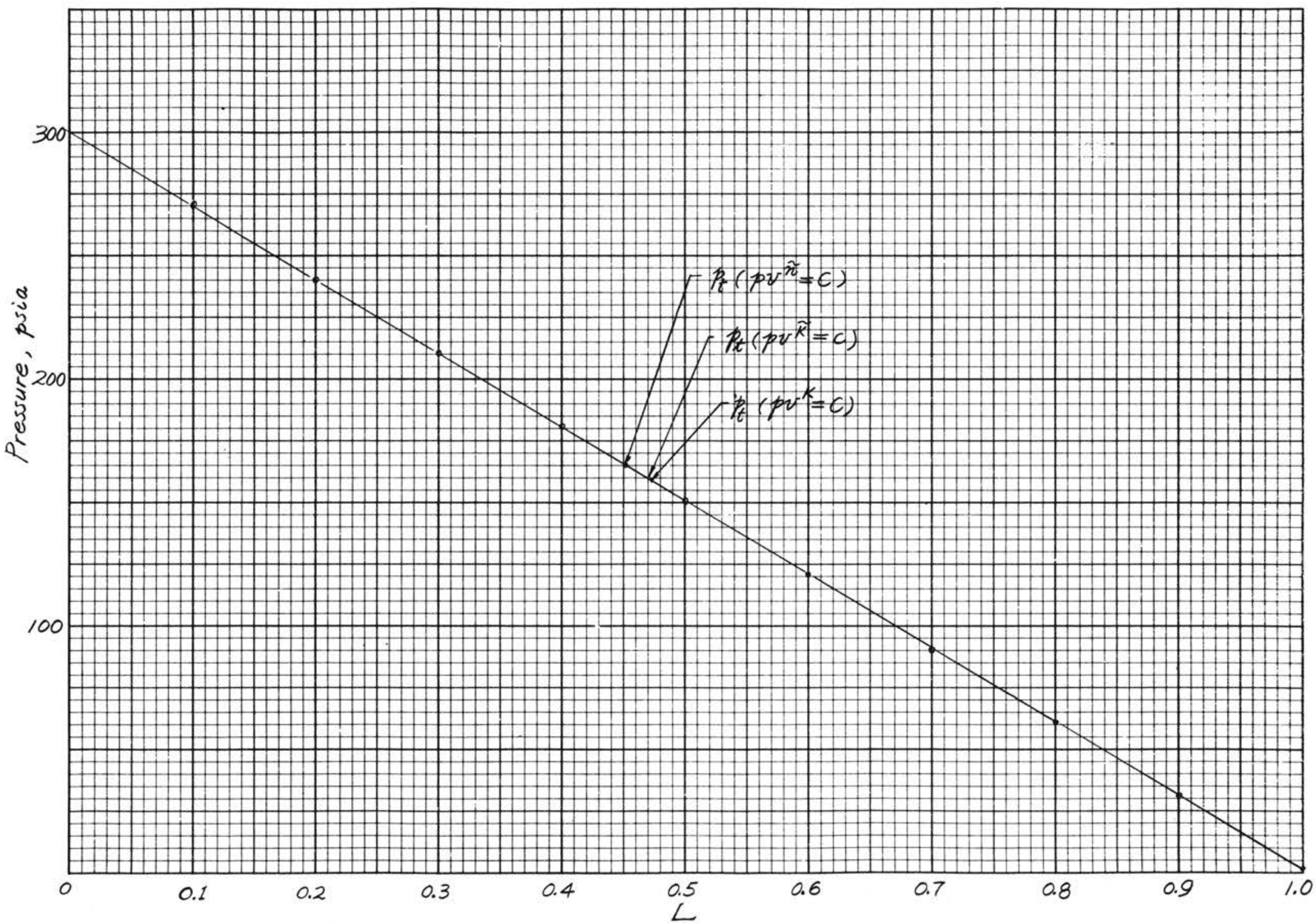

Fig. 5 The Static Pressure in the Air Stream

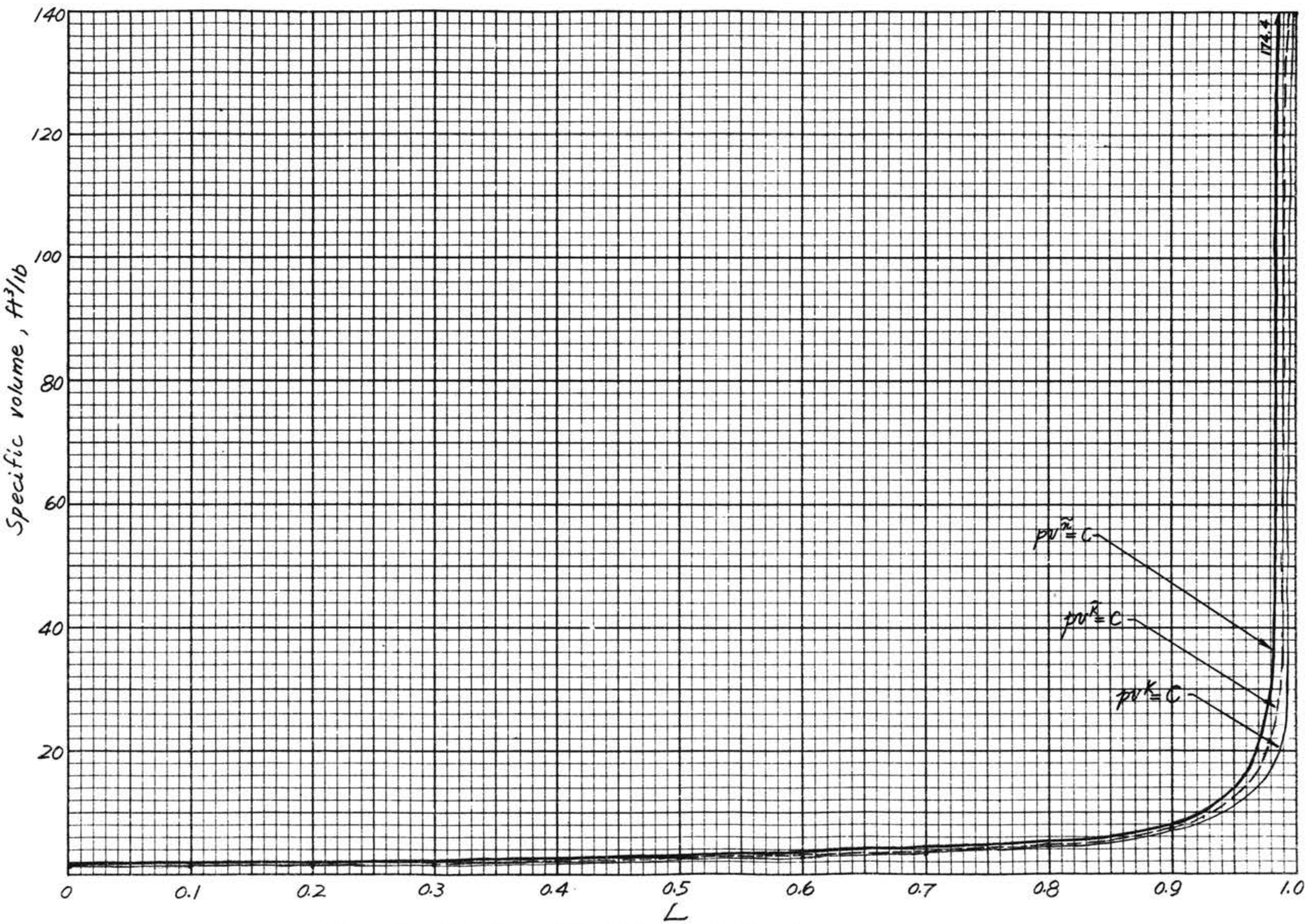

Fig. 6. The Specific Volume of the Air.

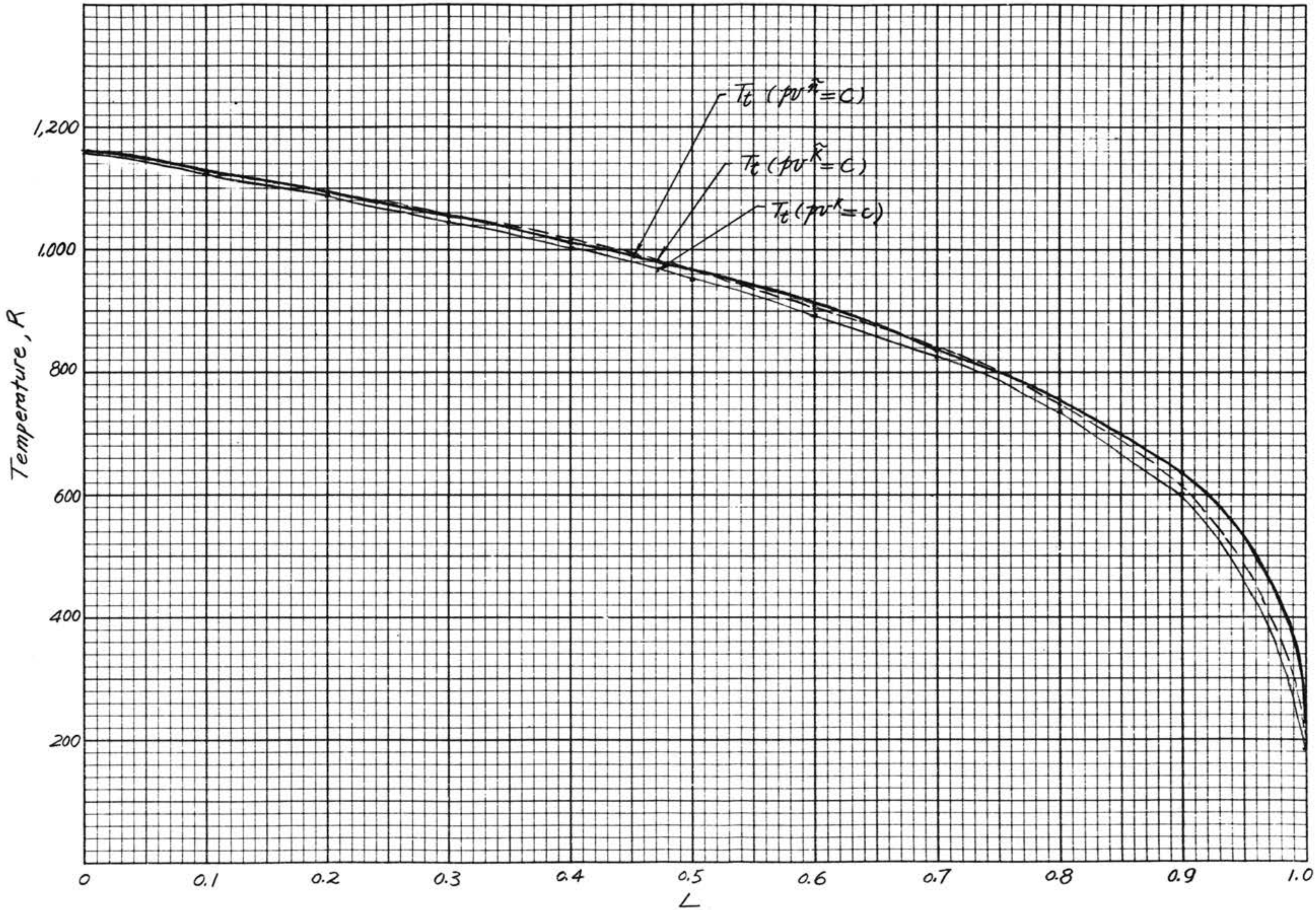

Fig. 7. The Temperature of the Air.

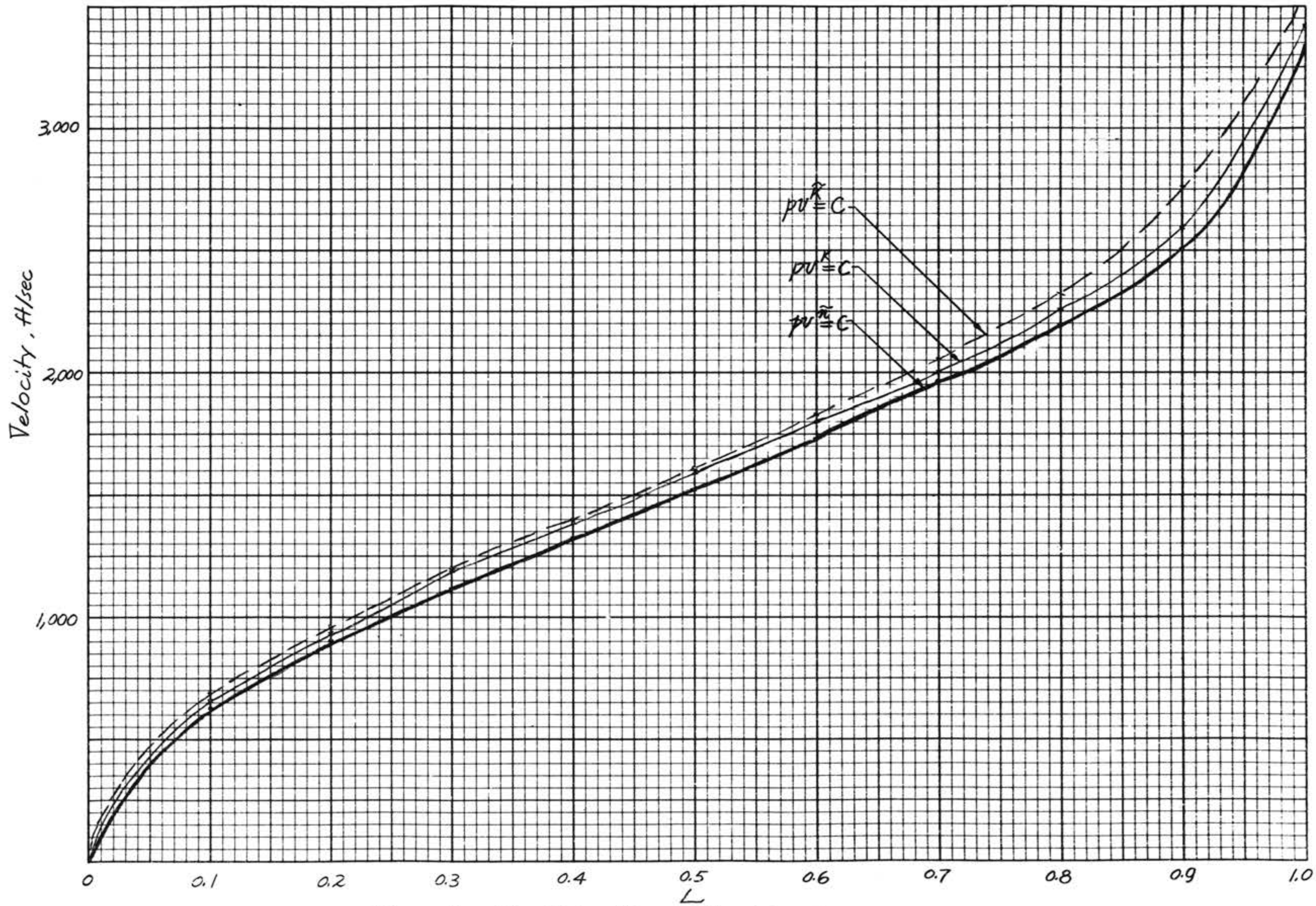

Fig. 8. The Velocity of the Air Stream.

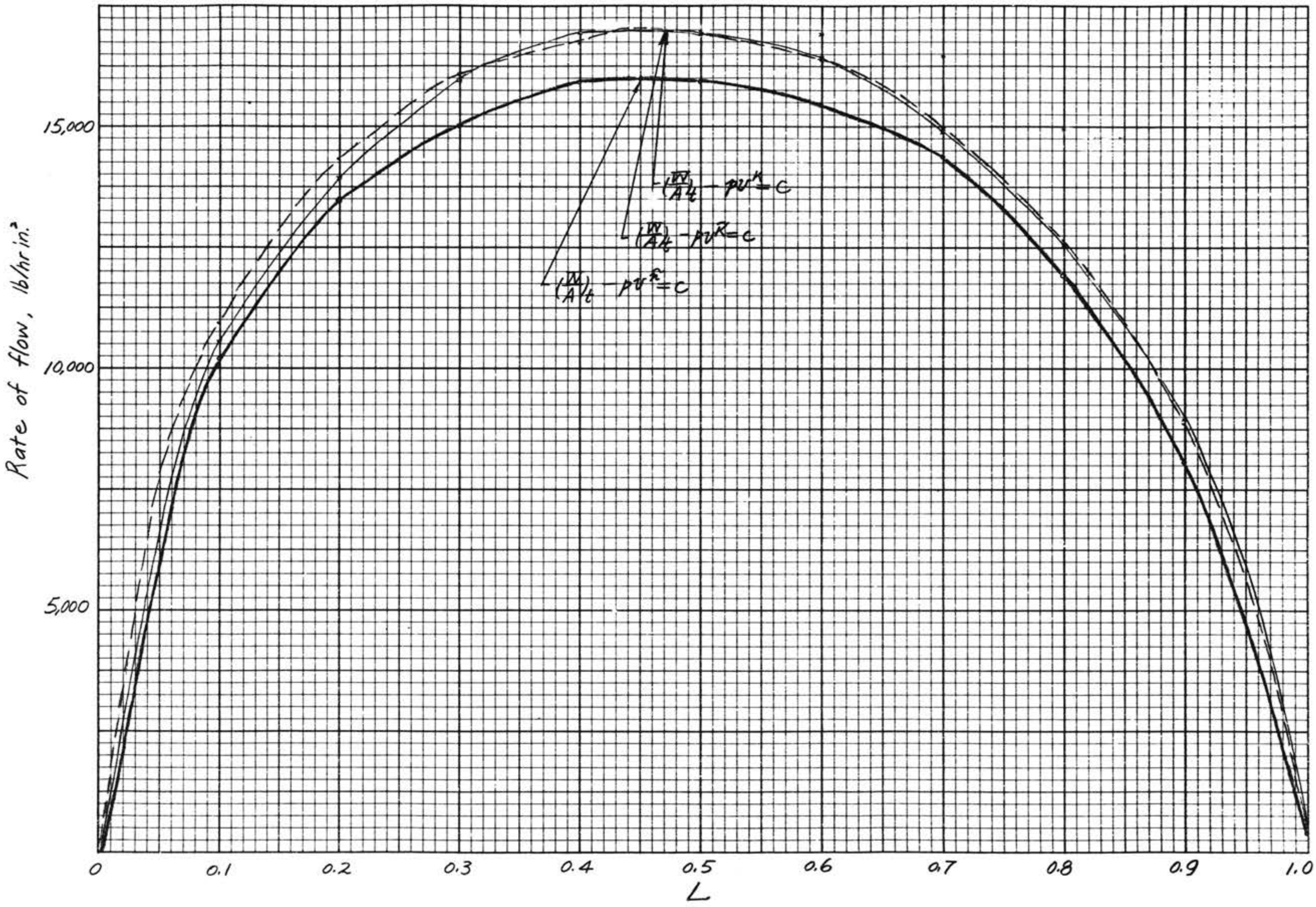

Fig. 9. The Rate of Flow per Sq In. of Cross-sectional Area.

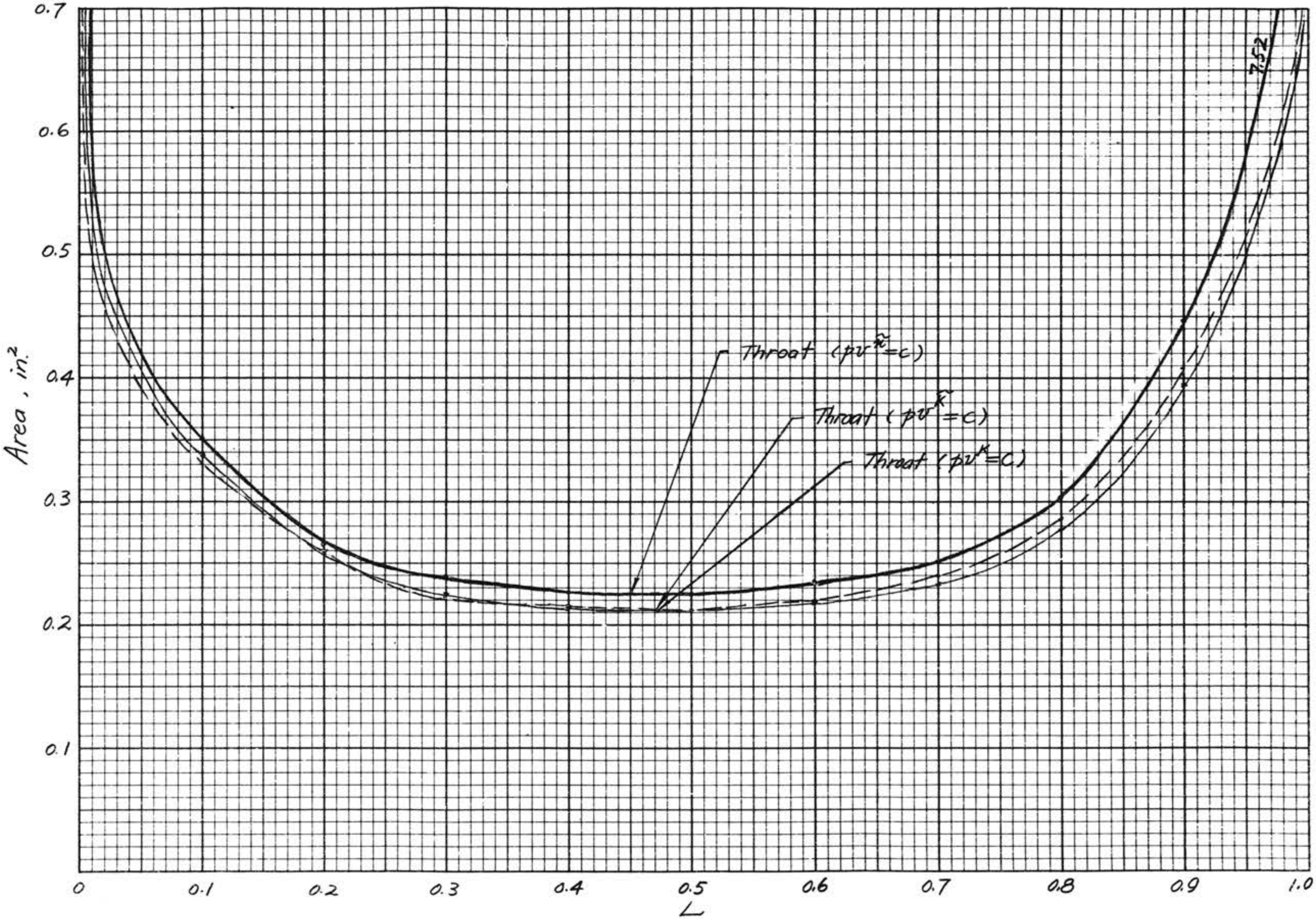

Fig. 10. The Cross-sectional Area of the Air Stream.

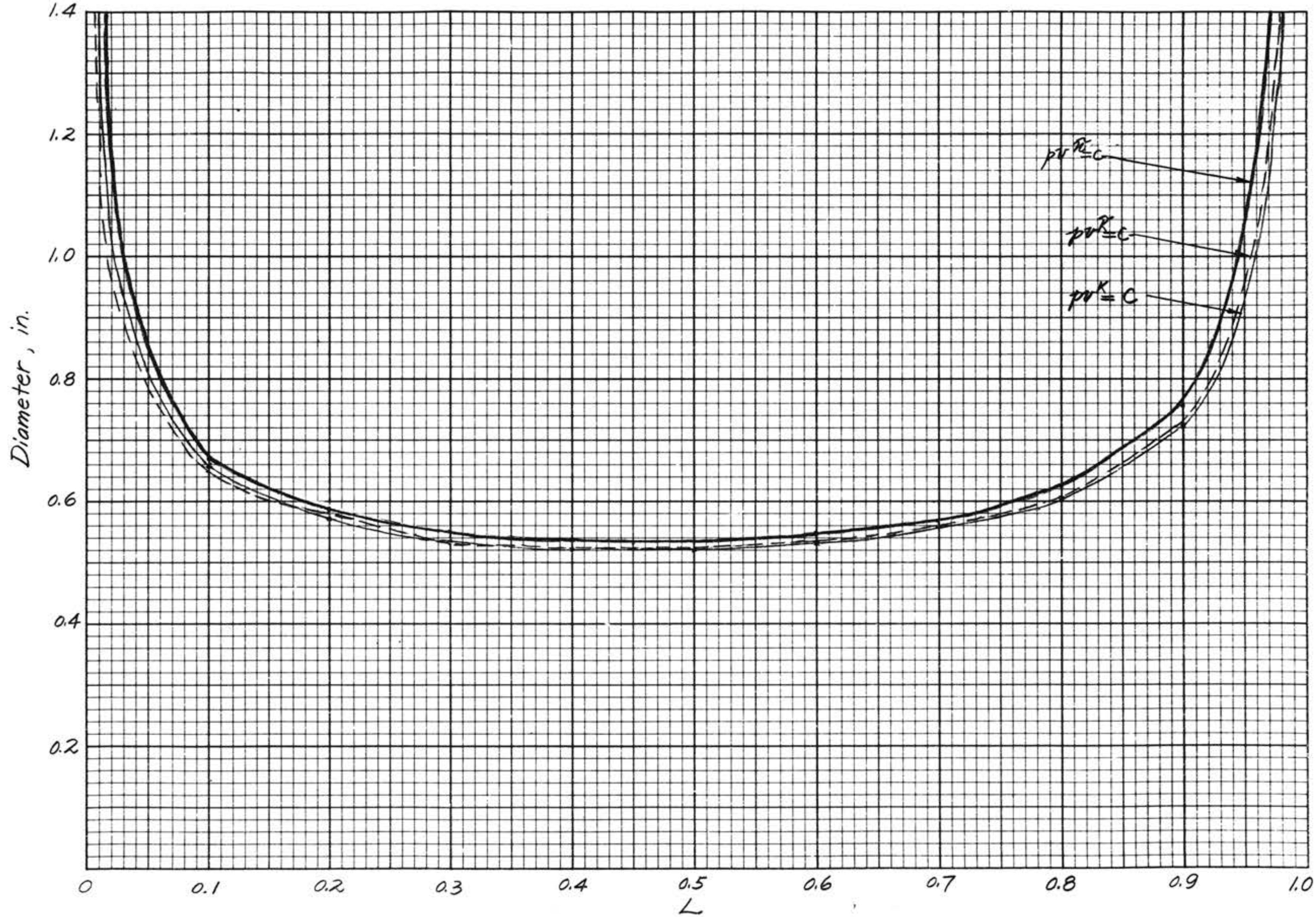

Fig. 11. The Diameter of the Air Stream.

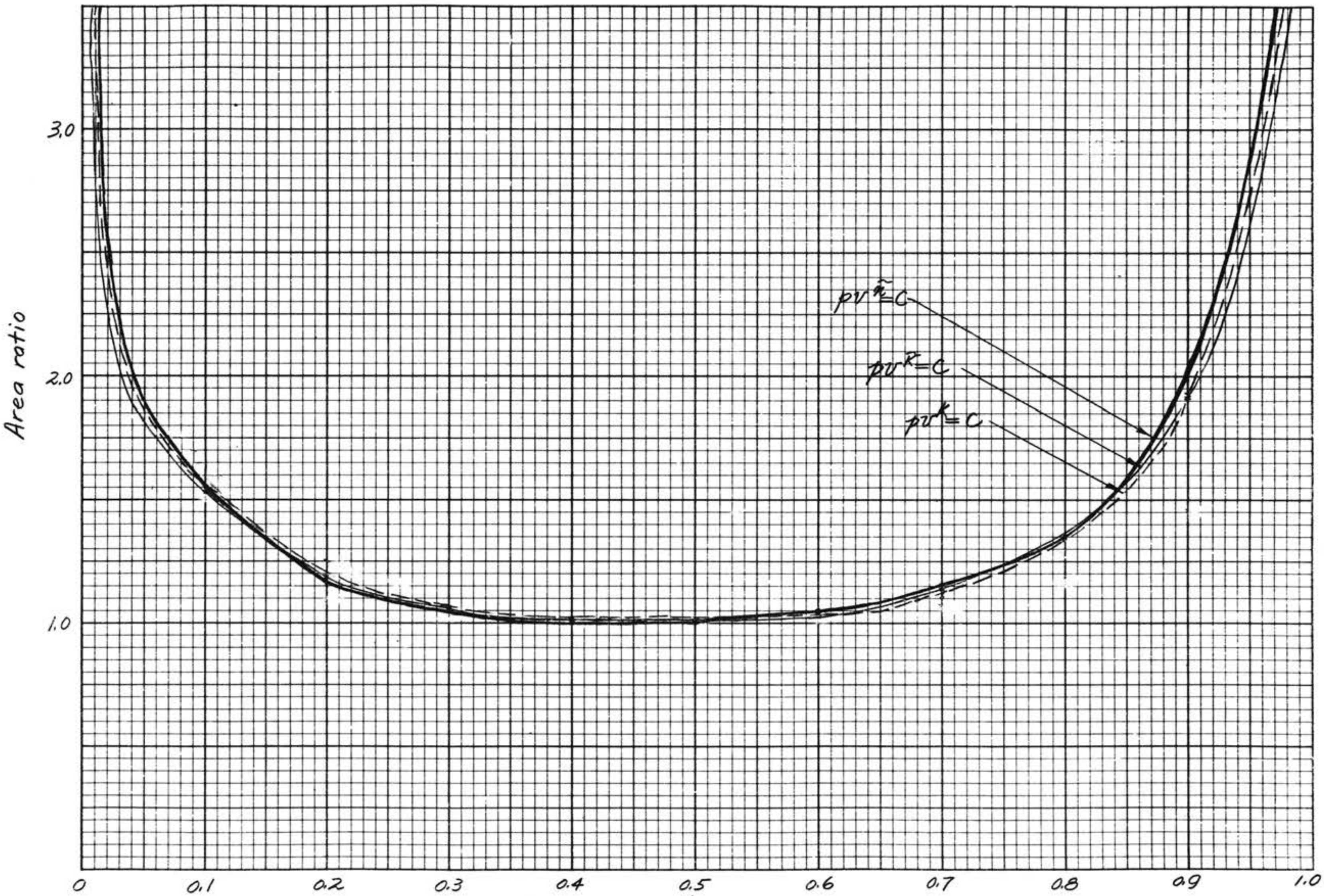

Fig. 12. The Ratio of Cross-sectiona Area of the Air Stream to the Throat Area.

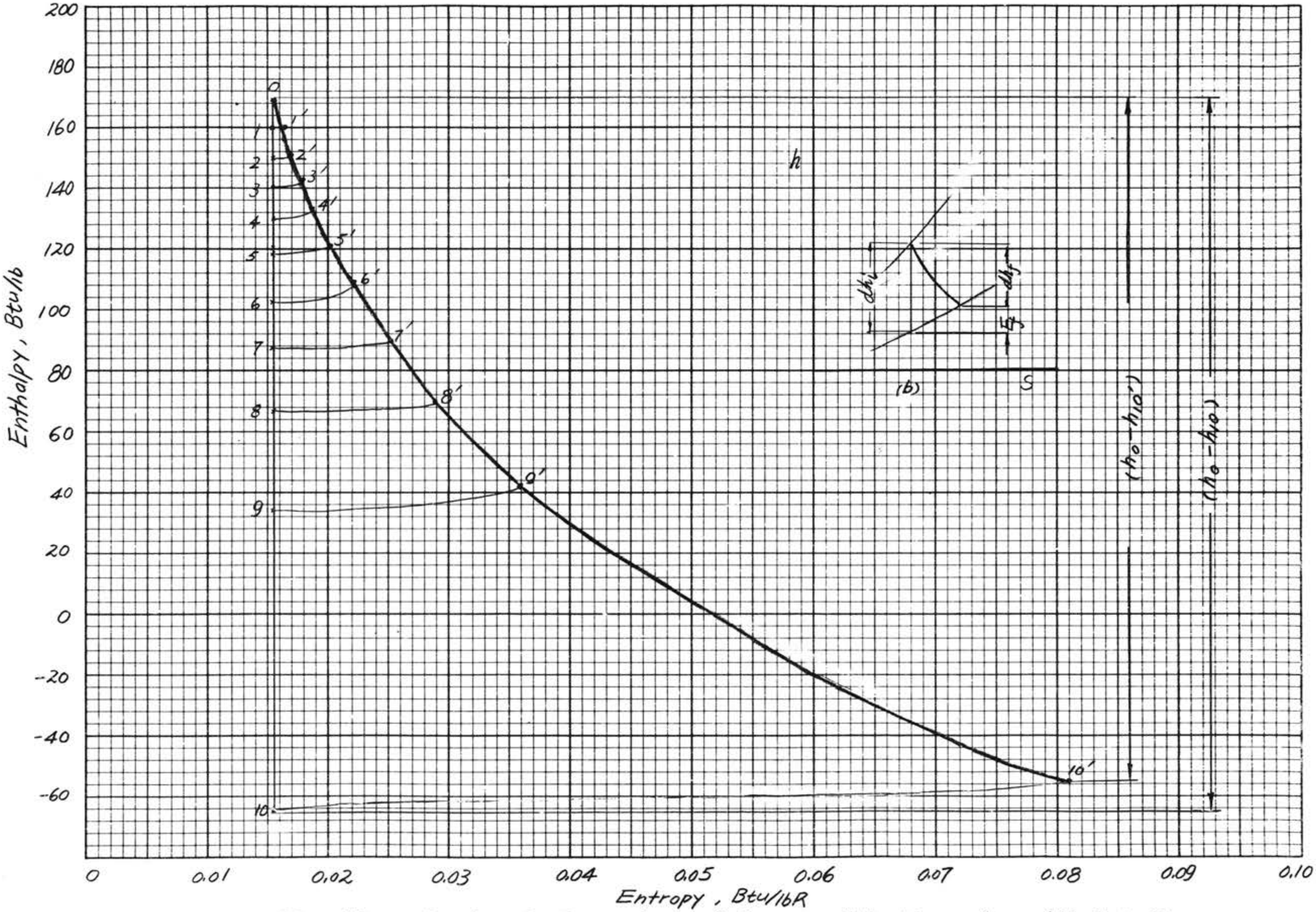

Fig. 13. Isentropic Expansion and Irreversible Expansion with Friction.

CHAPTER V

CONCLUSIONS

1. DEVIATIONS OF A REAL NOZZLE FROM THE IDEAL NOZZLE

A. The departure of the behavior of the air from the perfect-gas law necessites a change of nozzle cross-sectional area. The nozzle designed with variable specific heat, requires larger area in the early sections than that of the nozzle designed with constant specific heat, and both are very similar in the later sections. This is due to the fact that the variable specific heat is much lower than the constant specific heat used in these early sections, but it is very little larger or smaller than the constant specific heat in those later sections. Since the specific heat is proportional to the temperature, the greater the variation of specific heat, the greater is the deviation of specific volume for the same pressure. Therefore, the changes of the nozzle shape are mainly dependent on the variations of specific heat.

B. The design of a real nozzle shows an extraordinary contour compared to the ideal nozzle designed with constant or variable specific heat. In the present design, the frictional effects have been taken into account first by introducing assumed variable nozzle efficiencies. Since the nozzle efficiency decreases from the inlet to the exit, the air will undergo a polytropic process $pv^{\tilde{n}}=C$ with decreasing numerical value of exponent $\tilde{n}$ as tabulated in Table 3. Because these variable exponents are less than constant specific heat ratio k, the specific volume in the polytropic process $pv^{\tilde{n}}=C$ is larger than the specific volume in the isentropic process $pv^k = C$ with the same pressure. It follows that the corresponding temperature in the polytropic process is also greater than the temperature in the isentropic process at the same pressure. The higher temperature also can explained since the actual enthalpy decrease, which is the product of ideal enthalpy decrease multiplied by the nozzle efficiency, is less than the ideal enthalpy decrease: because there has been a smaller conversion of thermal energy to kinetic energy. The actual temperature at a certain pressure is higher than the ideal temperature at that same pressure for same initial temperature.

As the value of exponent n is decreased by the decreasing of nozzle efficiency, the rate of specific-volume change increases from subsonic region to supersonic region accompanying with some increasing order of rate of temperature drop, while the static pressure is equally decreased. Since the velocity increase is a function of square root of temperature drop, the rate of specific volume increase is faster than the rate of velocity increase along the nozzle. From the continuity equation, the cross-sectional area of the nozzle is directly proportional to the ratio v/V. Therefore the increase of the cross-sectional area of actual nozzle is greater than the increase of ideal nozzle cross-sectional area.

In conclusion, the frictional effects result in an increase in the specific volume, temperature and enthalpy of the air throughout the nozzle, and a larger area of nozzle is needed. Since the enthalpy drop is greater in the supersonic region than that of subsonic region, the real nozzle necessitates a larger divergent section than that of the ideal nozzle as shown in Fig. 10. For this reason, the maximum enlargement of nozzle area occurs at the exit, and it is approximately 27.45% greater than the exit area of ideal nozzle calculated in Table 3. The deviation of specific volume, temperature, velocity, and rate of flow per unit area in the real nozzle, are shown in Fig. 6, 7, 8, 9, 10 and 11.

C. Considering frictional effects, the air undergoes a polypic process in which the exponent n is less than the exponent of isentropic process k. Since the critical pressure is greater than that of ideal nozzle, the critical point is further upstream than that of ideal nozzle, and the velocity at this point does not become sonic as denoted in Chapter IV-2.

2. ENTROPY INCREASE DUE TO FRICTION

A. The flow with friction is irreversible, and isentropic flow exists no longer. For equal increments of length the pressure ratio has a minimum value at the exit section. Therefore the temperature drop is a maximum in the exit, and the entropy increases sharply during that time as shown in Fig. 13.

B. The difference of enthalpy between h_{10} and $h_{10'}$ represents the amount by which the final kinetic energy in the actual flow fails to attain that reached in the ideal flow. This amount of enthalpy change is 16.4% of isentropic change in enthalpy for this problem.

3. RECOMMENDATIONS FOR FURTHER INVESTIGATION

With the variable nozzle efficiency considered, the specific volume of air is calculated after the specific volume of air at earlier state was found. Therefore each property of air has been continuously taken considering frictional effects, and consequently this method of design secures a quite precise shape of real nozzle.

For further study, since the nozzle efficiency varies with the pressure ratio, temperature ratio, or specific volume ratio, the equation of energy and momentum will be needed in those many sections to account more correctly for frictional effects. Also the viscosity can not be included in one-dimensional treatment. The advanced concepts of fluid flow and friction at high Mach numbers should be recommended for designing an exact real nozzle.

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