# HETEROGENEITY OF ERROR VARIANCES

# IN A

# RANDOMIZED BLOCK DESIGN

By

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## PREFACE

In a randomized block experiment we frequently wish to test the hypothesis that all the treatment means are equal. When we have heterogeneity of error variances, the ratio of the treatment mean square to the error mean square is not distributed as Snedecor's F. An exact method for testing the treatment means equal when we have heterogeneity of error variances has been given by Graybill.

Consider a randomized block experiment with b blocks and $n_1 + n_2$ treatments where the error variance is $\sigma_1^2$ for the first $n_1$ treatments and is $\sigma_2^2$ for the next $n_2$ treatments. The method given by Graybill requires inversion of a matrix of order $n_1 + n_2 - 1$ and is subject to the restriction that $b > n_1 + n_2 - 1$. The method proposed in this paper does not require inversion of a matrix and is subject to the restriction that $b > 2$. In addition, the method proposed in this paper seems to be more powerful than the method proposed by Graybill.

In general, when we have K subsets of treatments such that the first subset has error variance $\sigma_1^2$, the next subset has error variance $\sigma_2^2$, etc., the method proposed in this paper requires inversion of a smaller matrix and is subject to a less stringent restriction than the method proposed by Graybill.

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## INTRODUCTION

Consider a randomized block design with p treatments occurring on each of b blocks. If each of the first $n_1$ treatments have variance $\sigma_1^2$ and each of the next $n_2$ treatments have variance $\sigma_2^2$, etc., and $\sum_{i=1}^{K} n_i = p$, the mathematical model is:

$$Y_{ijk} = \mu + t_{ij} + b_k + e_{ijk} \tag{1.1}$$

$$i = 1, 2, \ldots, K$$

$$j = 1, 2, \ldots, n_i$$

$$k = 1, 2, \ldots, b$$

where the $e_{ijk}$'s are assumed to be normally distributed such that

$$E\, e_{ijk} = 0 \text{ for all } i, j, \text{ and } k,$$

$$E\, e_{ijk}^2 = \sigma_i^2 \text{ for all } j \text{ and } k, \text{ and}$$

$$E\, e_{ijk}\, e_{rmn} = 0 \text{ unless } i = r, j = m, \text{ and } k = n.$$

When $n_i = 1$ for all i the model is $Y_{ik} = \mu + t_i + b_k + e_{ik}$ with the same assumptions as in (1.1). Graybill (2)[1] has discussed the problem of testing $t_1 = t_2 = \ldots t_p$ when $n_i = 1$ for all i. This method involves inversion of a matrix of order p - 1 in the numerical analysis and is valid only if $b > p - 1$. The purpose of this paper is to give a criterion for testing $t_{11} = t_{12} = \ldots = t_{Kn_K}$

[1]Single numbers in parentheses refer to references in bibliography.

for the model in (1.1) where $n_i > 1$ for at least one i. If $n_i > 1$ for at least one i, the restriction $b > p - 1$ can be relaxed somewhat. It is necessary only that $b > K - 1$. The numerical analysis will involve inversion of a matrix of order $K - 1$.

## TEST CRITERION

Consider the $i$ - th subset of observations $Y_{ijk}$ , where $k = 1, 2, \ldots, b$, $j = 1, 2, \ldots, n_i$ . Using these observations, conduct an analysis of variance as below for each subset that $n_i > 1$.

A. O. V. for $i$ - th Subset

| Due to | d. f. | Sum of Squares | |
|---|---|---|---|
| Blocks | $b - 1$ | $n_i \sum_k (y_{i.k} - y_{i..})^2$ | $= A$ |
| Treatments | $n_i - 1$ | $b \sum_j (y_{ij.} - y_{i..})^2$ | $= B$ |
| Error | $(b - 1)(n_i - 1)$ | $\sum_{jk} (Y_{ijk} - y_{i.k} - y_{ij.} + y_{i..})^2$ | $= C$ |

The ratio $\frac{B}{n_i - 1} \Big/ \frac{C}{(b - 1)(n_i - 1)} = F_i$ (where $Y_{ij.}$ indicates summation over k and $y_{ij.}$ indicates the average when summed over k, etc.) is distributed as Snedecor's F with d. f. $(n_i - 1)$ and $(b - 1)(n_i - 1)$ if and only if $t_{i1} = t_{i2} = \ldots = t_{in_i}$. We will have $q - 1$ such analyses, each yielding an F, where $q - 1$ is the number of subsets that $n_i > 1$.

Since $Y_{ijk}$ is a normal variate and since $E(y_{ij.} - E\, y_{ij.}) \cdot (y_{rj.} - E\, y_{rj.}) = 0$ for $r \neq i$, $y_{rj.}$ and $y_{ij.}$ are independent. Therefore $b \sum_j (y_{ij.} - y_{i..})^2$ is independent of $b \sum_j (y_{rj.} - y_{r..})^2$ .

Similarly $\sum_{jk} (y_{ijk} - y_{i.k} - y_{ij.} + y_{i..})^2$ is independent of $\sum_{jk} (y_{rjk} - y_{r.k} - y_{rj.} + y_{r..})^2$. We have, therefore, q - 1 independent ratios, each distributed as Snedecor's F if and only if $t_{i1} = t_{i2} = \ldots = t_{in_i}$ for all i.

If we average our observations within each subset over each block we have:

$$(2.1) \qquad \sum_{j=1}^{n_i} Y_{ijk} / n_i = \mu + \sum_{j=1}^{n_i} t_{ij} / n_i + b_k + \sum_{j=1}^{n_i} e_{ijk} / n_i \; .$$

Denote (2.1) by $B_{ik}$ and let $T_i = \sum_{j=1}^{n_i} t_{ij} / n_i$ and $d_{ik} = \sum_{j=1}^{n_i} e_{ijk} / n_i$.

Then (2.2) $$B_{ik} = \mu + T_i + b_k + d_{ik}$$

From the assumptions in (1.1)

$$E \; d_{ij} = E \sum_{j=1}^{n_i} \frac{e_{ijk}}{n_i} \, ,$$

$$E \; d_{ij} = 0 \, ,$$

$$E \; d_{ij}^2 = \frac{\sigma_i^2}{n_i} \, ,$$

$$E \, (d_{ij} \, d_{is}) = 0 \quad \text{for } j \neq s \; ;$$

$$E \, (d_{ij} \, d_{rj}) = 0 \quad \text{for } i \neq r \; .$$

Thus (2.2) is the model considered by Graybill and we can use Hotelling's $T^2$ to test the hypothesis $H_o : T_1 = T_2 = \ldots = T_K$.

Let $x_{ij} = B_{ij} - B_{Kj}$. Consider $X_j$ a Kx1 column vector with elements $x_{ij}$. $\bar{X} = \sum_j X_j / b$. Then $\frac{(b - K + 1)\, b\, \bar{X}'}{K - 1} \left( \sum_{j=1}^{b} [X_j - \bar{X}] [X_j - \bar{X}]' \right)^{-1} \bar{X}$

which we shall call $F^*$ has Snedecor's F distribution under $H_o$ with $K - 1$ and $b - K + 1$ degrees of freedom (if $b > K - 1$).

Theorem I. $t_{11} = t_{12} = \ldots = t_{1n_1} = t_{21} = \ldots = t_{Kn_K}$ if and only if $T_1 = T_2 = \ldots = T_K$ and $t_{i1} = t_{i2} = \ldots = t_{in_i}$ for all i; $i = 1, 2, \ldots K$.

Proof: 1. If $t_{i1} = t_{i2} = \ldots t_{in_i}$, $T_i = t_{i1} = t_{i2} \ldots = t_{in_i}$. If $T_1 = T_2 = \ldots = T_K$, then $t_{11} = t_{12} = \ldots = t_{1n_1} = t_{21} = \ldots = t_{Kn_K}$. 2. If $t_{11} = t_{12} = \ldots = t_{1n_1} = t_{21} = \ldots = t_{Kn_K}$, then $t_{i1} = t_{i2} = \ldots = t_{in_i}$ for all i and $T_1 = T_2 = \ldots = T_K$.

Theorem II. $B_{mn}$ is independent of $F_i$ for all i.

Proof: Let $(y_{ij.} - y_{i..}) = u_{ij}$ and $(y_{ijk} - y_{i.k} - y_{ij.} + y_{i..}) = v_{ijk}$.

$$\text{Cov } (B_{mn}, u_{ij}) = 0 \text{ for } m \neq i.$$

Let us consider the case when $m = i$.

$$\text{Cov } (B_{in}, u_{ij}) = E\,(e_{i.n})\,(e_{ij.} - e_{i..}),$$

$$= E\ \frac{e_{ijn}^2}{bn_i} - E \sum_j \frac{e_{ijn}^2}{bn_i^2},$$

$$= \frac{\sigma_i^2}{bn_i} - \frac{\sigma_i^2}{bn_i},$$

$$= 0.$$

Also $\quad \text{Cov } (B_{mn}, v_{ijk}) = 0 \text{ for } m \neq i.$

Let us consider the case when $m = i$.

$$\text{Cov}\,(B_{in}\,,\,v_{ijk}) = E\,(e_{i.n})\,(e_{ijk} - e_{ij.} - e_{i.k} + e_{i..})\,,$$

$$= E\,\frac{e_{ijk}^2}{n_i} - E\,\frac{e_{ijk}}{bn_i} - E\,\frac{\sum_j e_{ijk}^2}{n_i^2} + E\,\frac{\sum_j e_{ijk}^2}{bn_i^2}\,,$$

$$= \frac{\sigma_i^2}{n_i} - \frac{\sigma_i^2}{bn_i} - \frac{\sigma_i^2}{n_i} + \frac{\sigma_i^2}{bn_i}\,;$$

$$= 0.$$

Since $B_{mn}$, $u_{ij}$, and $v_{ijk}$ are normal variates and since $\text{Cov}\,(B_{mn}\,,\,u_{ij}) = 0$ and $\text{Cov}\,(B_{mn}\,,\,v_{ijk}) = 0$, $B_{mn}$ is independent of $u_{ij}$ and $v_{ijk}$. Further $B_{mn}$ is independent of any function of $u_{ij}$ and $v_{ijk}$; hence $B_{mn}$ is independent of $F_i$ for all i.

Since F* is a function of $B_{ij}$, F* is independent of each of the (q - 1) F's which we obtained as in the analysis of variance on page 3. We have, therefore, q independent F's, which are simultaneously distributed as Snedecor's F if and only if $H_o$ is true: i.e if and only if $t_{11} = t_{12} = \ldots = t_{1n_1} = t_{21} = \ldots = t_{Kn_K}$. To test $H_o$ requires that we combine q independent tests of significance.

## TESTS OF SIGNIFICANCE

1. Product of Beta Variables (4)

The product of beta variables with parameters $(a_1\ b_1)$ , $(a_2\ b_2)$ . . . . $(a_q\ b_q)$ such that $a_i = (a_{i+1} + b_{i+1})$ is distributed as a beta variable with parameters $(a_q\ ,\ b_1 + \ldots + b_q)$ . Since the transformation $w = m\,F/n \Big/ (1 + mF/n)$ transforms $F\ (m,\ n)$ to a beta variable with parameters $\alpha = m/2$ , $\beta = n/2$ , in some cases we may be able to transform each $F_i$ and $F^*$ to beta variables, form the product, and use Pearson's tables of the incomplete beta function to test $H_o$ .

2. Pearson's $P_\lambda$ Test

If $P_1$ , $P_2$ , . . . , $P_q$ are q independent probabilities then $z_i = -2 \log_e P_i$ is distributed as $\chi^2$ [2]. $P_\lambda$ is therefore distributed as $\chi^2$ [2q].

3. Wilkinson's Methods

Reject $H_o$ if and only if $P_i \leq \alpha$ for r or more of the $P_i$'s where r is a predetermined integer, $1 \leq r \leq q$ , and $\alpha$ is a constant corresponding to the desired confidence level. The q possible choices of r give q different procedures (case 1, case 2, etc.) . Birnbaum (1) indicates that, while there is no single case best for all problems, case 1 seems to be best for this type of problem.

4. Case 1 of Wilkinson's Method

Reject $H_o$ if and only if at least one $F_i > h_i$ where

$P(F_i > h_i \mid H_0) = \alpha$ for all i ; $\alpha$ is predetermined by the desired type I error, i.e. P (I) . P (of rejecting $H_0$ | given $H_0$ is true) equals

$$\sum_{i=1}^{q} P(F_i > h_i) - \sum_{ij} P(F_i > h_i)\, P(F_j > h_j) +$$

$$\sum_{ijk} P(F_i > h_i)\, P(F_j > h_j)\, P(F_k > h_k) - \ldots \pm P(F_1 > h_1)\, P(F_2 > h_2) \cdot$$

$\cdots\ P(F_q > h_q)$ where the second sum is over all combinations of the numbers 1, 2, . . . , q taken two at a time, the third is over combinations of the numbers three at a time, etc. Hence $P(I) = 1 - (1 - \alpha)^q$. For any desired P (I) we can determine $\alpha$.

The power of the test $\beta$ = P (reject $H_0$ | $H_1$) equals

$$1 - \prod_{i=1}^{q} P(F_i < h_i \mid H_1) .$$

5. Comparison of Graybill's Method with the Method Proposed in this Paper.

Let us denote the method proposed in this paper by A and the method proposed by Graybill by B. A comparison of the powers will be made only for case 1 of Wilkinson's methods. For this comparison let us consider the original model (1.1) :

$$Y_{ijk} = \mu + t_{ij} + b_k + e_{ijk}$$

for i = 1, 2, j = 1, 2, . . , $n_i$, $n_1 = n$, $n_2 = 1$, and k = 1, 2, . . . , b.

Method A will be considered first. Using the nb observations $Y_{1jk}$ form the ratio of mean square for treatments to mean square for error. This ratio is distributed as Snedecor's F with n - 1 and (n - 1) (b - 1) d. f. when $t_{11} = t_{12} = \ldots = t_{1n}$.

Consider the means:

$$y_{i.k} = \mu + t_{i.} + b_k + e_{i.k}$$

$$i = 1, 2$$

$$k = 1, 2, \ldots, b .$$

To test the hypothesis $t_{1.} = t_{2.}$ we use the ratio of the mean square for treatments, $\sum_{ik} (y_{i.k} - y_{...})^2$ , to the mean square for error $\sum_{ik} (y_{i.k} - y_{i..} - y_{..k} + y_{...})^2 \Big/ (b - 1)$ . This ratio is distributed as Snedecor's F with 1 and b - 1 d. f. when $t_{1.} = t_{2.}$ .

The power of the test, $\beta_A$ , using method A equals

$$1 - P (F_1 < h_1 \mid H_1) P (F_2 < h_2 \mid H_1) .$$

We can evaluate $\beta_A$ by transforming $F_1$ and $F_2$ to Tang's $E^2$ . Making the transformation $z_i = f_{i1}F_i \Big/ (f_{i2} + f_{i1}F_i)$ , where $f_{i1}$ and $f_{i2}$ are the degrees of freedom for $F_i$ , we have $z_1$ distributed as Tang's $E^2$ with parameters n - 1, (n - 1) (b - 1) , and $\lambda_1$ where $\lambda_1 = b \sum_j (t_{1j} - t_{1.})^2 \Big/ 2\sigma_1^2$ . Also $z_2$ is distributed as Tang's $E^2$ with parameters 1, b - 1, and $\lambda_2$ where $\lambda_2 = \dfrac{bn\ (t_{1.} - t_{21})^2}{2\ (\sigma_1^2 + n\sigma_2^2)}$ .

Hence $P (F_1 < h_1 \mid H_1) = \int_0^{g_1} f(z_1)\, dz_1$ and $P (F_2 < h_2 \mid H_1)$ equals $\int_0^{g_1} f(z_2)\, dz_2$ where $g_1$ and $g_2$ are determined by the transformation

$g_i = f_{i1}h_i \big/ (f_{i2} + f_{i1}h_i)$. Therefore the power of method A equals

$$1 - \int_0^{g_1} f(z_1)\, dz_1 \int_0^{g_1} f(z_2)\, dz_2 \,.$$

For method B Graybill has shown that if we let

$$u_{jk} = Y_{1jk} - Y_{21k}$$

that

$$\bar{\bar{U}}' \left(\sum_k [U_k - \bar{\bar{U}}]\,[U_k - \bar{\bar{U}}]'\right)^{-1} \bar{\bar{U}}\, \frac{(b-n)\,b}{n} = F^* \tag{3.1}$$

where $\bar{\bar{U}} = \sum_k U_k \big/ b$ and $U_k = \begin{bmatrix} u_{k1} \\ u_{k2} \\ u_{k3} \\ \cdot \\ \cdot \\ \cdot \\ u_{kn} \end{bmatrix}$,

is distributed as Snedecor's F with b and b − p d. f. when $t_{11} = t_{12} = \cdot\cdot\cdot\; t_{1n} = t_{21}$.

Therefore $\qquad U_i \sim N\,(\mu^*, A)$

where $\qquad \mu^* = \begin{bmatrix} t_{11} - t_{21} \\ t_{12} - t_{21} \\ \cdot\;\cdot\;\cdot \\ t_{1n} - t_{21} \end{bmatrix}.$

and $\qquad A = (a_{ij})$

where $\qquad a_{ij} = \sigma_1^2 + \sigma_2^2 \quad$ if $i = j$ ,

and $\qquad a_{ij} = \sigma_2^2 \quad$ if $i \neq j$ .

Under $H_1$, $F^*$ is distributed as the non-central F with parameters $n$, $b - n$, and $\lambda_3$ where $\lambda_3 = \dfrac{\mu^{*'} A^{-1} \mu^*}{2}$. To find $\lambda_3$ we must examine the variance-covariance matrix A.

A is a circulant matrix and $A^{-1} = B$ is found to be $(b_{ij})$ where

$$b_{ij} = \frac{\sigma_1^2 + (n - 1)\, \sigma_2^2}{\sigma_1^2\, (\sigma_1^2 + n\sigma_2^2)} \qquad \text{if } i = j\ ,$$

$$b_{ij} = \frac{-\sigma_2^2}{\sigma_1^2\, (\sigma_1^2 + n\sigma_2^2)} \qquad \text{if } i \neq j\ .$$

Then $\lambda_3 = Cb/2$ where C is defined as below.

Writing $t_{21}$ as $t_2$, let

$$C = \sum_{ij} (t_{1i} - t_2)\, b_{ij}\, (t_{1j} - t_2)$$

$$= \sum_{ij} (t_{1j} - t_{1.} - t_2 - t_{1.})\, (t_{1j} - t_{1.} - t_2 - t_{1.})\, b_{ij}$$

$$= \sum_{ij} (t_{1i} - t_{1.})(t_{1j} - t_{1.})\, b_{ij} - 2 \sum_{ij} (t_2 - t_{1.})(t_{1i} - t_{1.}) b_{ij}$$

$$+ (t_2 - t_{1.})^2 \sum_{ij} b_{ij}$$

$$= \sum_{ij} (t_{1i} - t_{1.})\, (t_{1j} - t_{1.})\, b_{ij} + \frac{(t_2 - t_{1.})^2}{\sigma_1^2 + n\sigma_2^2}\ .$$

Let $K = \sum_{ij} (t_{1j} - t_{1.})\, (t_{1j} - t_{1.})\, b_{ij}$ .

$$K = \sum_{i} (t_{1i} - t_{1.})^2\, b_{ii} + \sum_{i} \sum_{j \neq i} (t_{1j} - t_{1.})\, (t_{1j} - t_{1.})\, b_{ij}$$

$$= \sum_i (t_{1i} - t_{1.})^2 \frac{(\sigma_1^2 + [n-1]\sigma_2^2)}{\sigma_1^2 (\sigma_1^2 + n\sigma_2^2)}$$

$$+ \sum_i (t_{1i} - t_{1.}) \sum_{j \neq i} (t_{1j} - t_{1.}) \frac{(-\sigma_2^2)}{\sigma_1^2 (\sigma_1^2 + n\sigma_2^2)}$$

$$= \frac{\sum_i (t_{1i} - t_{1.})^2 (\sigma_1^2 + [n-1]\sigma_2^2) + \sum_i (t_{1i} - t_{1.}) \sigma_2^2}{\sigma_1^2 (\sigma_1^2 + n\sigma_2^2)}$$

$$= \sum_i (t_{1i} - t_{1.})^2 / \sigma_1^2 .$$

Therefore

$$\lambda_3 = \frac{b \sum_i (t_{1i} - t_{1.})^2}{2 \sigma_1^2} + \frac{bn (t_2 - t_{1.})^2}{2(\sigma_1^2 + n\sigma_2^2)} .$$

That is,

$$\lambda_3 = \lambda_1 + \lambda_2 .$$

Power of B $= 1 - \int_0^{g_3} f(z_3)\, dz_3$ where $z_3$ is distributed as Tang's $E^2$ with parameters n, b - n, and $\lambda_3$.

We can now compare the power of method A with the power of method B. In order to use Tang's tables we must compute $\phi_i$

where

$$\phi_i = \sqrt{\frac{2\lambda_i}{f_{i1} + 1}}$$

for $i = 1, 2,$ and $3$.

Since $\lambda_3 = \lambda_1 + \lambda_2$, $\phi_3$ is a function of $\phi_1$ and $\phi_2$. For the special case where n = 3, b = 5, and P(I) = .02, the power of A is compared with the power of B in the table on the following page. The values for the power of B were obtained by double interpolation from Tang's tables.

Comparison of Powers

| $\phi_1$ | $\phi_2$ | A | B |
|---|---|---|---|
| 1 | 1 | 0.118 | 0.053 |
| 1 | 3 | 0.545 | 0.140 |
| 1 | 5 | 0.928 | 0.220 |
| 3 | 1 | 0.823 | 0.113 |
| 3 | 3 | 0.906 | 0.250 |
| 3 | 5 | 0.986 | 0.360 |
| 5 | 1 | 0.999 | 0.360 |
| 5 | 3 | 0.999 | 0.406 |
| 5 | 5 | 1.000 | 0.480 |

These data indicate that, for small samples, method A is more powerful than method B.

## CONCLUSIONS

If an exact method of combining independent tests of significance is used, the method proposed in this paper to test treatment means equal is exact. Its power seems to be better than that of the method proposed by Graybill. It should also be emphasized that the method in this paper requires inversion of a smaller matrix than Graybill's method. In addition, the restriction that b be greater than K is much less stringent than the restriction in Graybill's method that b be greater than p - 1.

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