SEMI-RIGID CONNECTIONS COMPARED WITH SIMPLY SUPPORTED MEMBERS IN BUILDING FRAMES

By

JOHN KARL PERRY

Bachelor of Science

John Brown University

Siloam Spring, Arkansas

1948

Submitted to the Faculty of the Graduate School of

the Oklahoma Agricultural and Mechanical College

in Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

1952

SEMI-RIGID CONNECTIONS

COMPARED WITH SIMPLY SUPPORTED

MEMBERS IN BUILDING FRAMES

THESIS AND ABSTRACT APPROVED:

J. E. Lothers

Thesis Adviser

Roger L. Flanders

Faculty Representative

D. C. McIntosh

Dean of the Graduate School

PREFACE

The question of semi-rigid connections in building frames as compared with simply supported members first arose in the author's mind while studying semi-rigid connections under Professor J. E. Lothers of the School of Architectural Engineering, Oklahoma A & M College, and again in the study of the design of steel buildings.

The increasing demand for steel in the defense industry causes a shortage of steel for structural purposes. It seems important, therefore, that more accurate design be used in building connections. The present tendency is to assume building connections take no part of the load except that necessary to insure a simply supported member.

A semi-rigid connection is one which allows some rotation of the members, such as beam and column, with respect to each other before taking any of the applied load. This amount of rotation varies with the condition of loading, size and shape of the members and the type of connection used. It is a local weakening between the column and the beam and can be thought of as a concentrated load on the conjugate beam applied at the center of the connection.

Prior to World War I this initial rotation was given very little consideration since there seemed to be a sufficient supply of steel to allow over-design of all members and thus eliminate the effect of local weakening caused by rotation. The amount of steel used during this war caused designers and research men to realize our steel supply was not inexhaustable. These men became interested in the amount of initial rotation as a means of determining the load carrying capacity of semi-rigid connections, and a series of experiments have been conducted in this country and Great Britain[1,2] to determine these properties.

It is rather commonly accepted that a 20% savings of steel can be accomplished by considering the load carried by semi-rigid connections. The author was interested in determining the validity of this statement.

The writter wishes to express his appreciation to Professor Ren G. Saxton, Head of the School of Civil Engineering, Oklahoma A & M College, for his assistance in developing a program of study while at Oklahoma A & M and to Professor J. E. Lothers for his suggestions and criticisms in preparing this paper.

---

[1] Professor Cyril Batho, First, Second, and Final Report, Steel Structures Research Committee, Department of Scientific and Industrial Research. H. M. Stationery Office, London, 1931-1936.

[2] J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 525-596.

TABLE OF CONTENTS

INTRODUCTION

The semi-rigid equations of Maugh[3] and Rathbun[4] are developed in detail and proved identities - Maugh identical to Rathbun.

The chief difference between Maugh and Rathbun is found in the quantities ψ & Z. Rathbun defines Z[5] as "a coefficient of M such that $MZ = \phi =$ angle of rotation of the connection due to moment, M" while Maugh defines ψ[6] as "the slope of the tangent to the curve" or a coefficient of ϕ such that $\phi\psi = M$, moment required to produce an angle rotation ϕ at the connection. From Fig. 1, $\frac{1}{Z}$ equals the slope of the tangent to the curve, therefore, these two quantities are interchangeable.

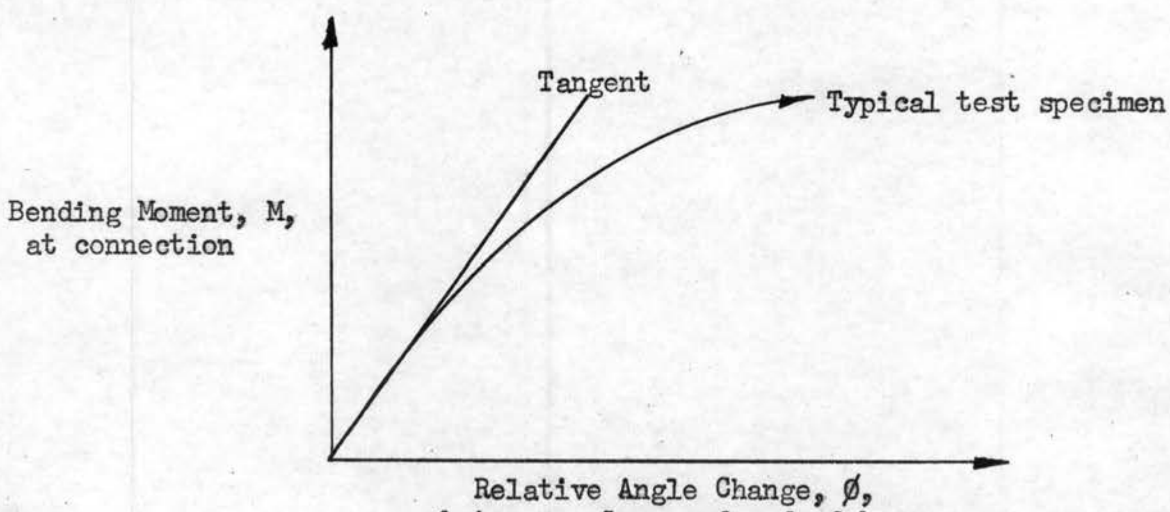


Fig. 1[7]

---

[3] L. C. Maugh, Statically Indeterminate Structures. (New York 1948), pp. 292-296.

[4] J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 549-552.

[5] Ibid., p. 549.

[6] Maugh, Statically Indeterminate Structures, p. 294.

[7] Bruce Johnston and Edward H. Mount, Transactions of the American Society of Civil Engineers, Vol. 107, p. 996.

The maximum moment present at the end of a member is the moment for the completely fixed or rigid condition and the greatest material saving possible is realized by a semi-rigid connection designed for this moment.

In this report a single bay two story bent is analyzed by slope deflection as a rigid frame. The moments from this analysis are taken as the design moments for the semi-rigid connections. Rathbun's equations for semi-rigid connections are used to analyze this frame and these results are checked by Maugh's equations and moment distribution. The moments from this analysis are compared with those of a simply supported member to determine the percentage of steel saved.

THE DERIVATION OF RATHBUN'S EQUATIONS

THE DERIVATION OF RATHBUN'S EQUATIONS

A. Introduction.

Technical literature pertaining to structural design and analysis contains two common difficulties for most readers.

First, there is that of terminology. The reader may be familiar with a term meaning one thing and the author use this same term to designate something different, such as ψ. To some readers this would mean "pounds per square inch", to others it would mean $\frac{\Delta}{L}$ where Δ equals the movement of a member and "L" the length of this same member perpendicular to Δ. Then too, $\frac{\Delta}{L}$ is designated as "R" by still other authors. It can be seen from this that terminology is a major problem. This problem has been greatly reduced by the use of a "glossary" or table of terms preceding technical articles. There still remains a problem, however, if the author takes some generally accepted term with a specific meaning (such as Δ which generally designates deflection) and gives it a new and different meaning.

The second difficulty is that of signs. Some authors take counter-clockwise moment as positive and clockwise as negative. The present tendency is to designate clockwise moment as positive and counter-clockwise as negative. Another sign convention commonly used is the beam convention in which a load that causes a member to bend in such a way as to "hold water" produces positive moment and to "shed water" produces negative moment in the member. These sign conventions can be readily interchanged as follows:

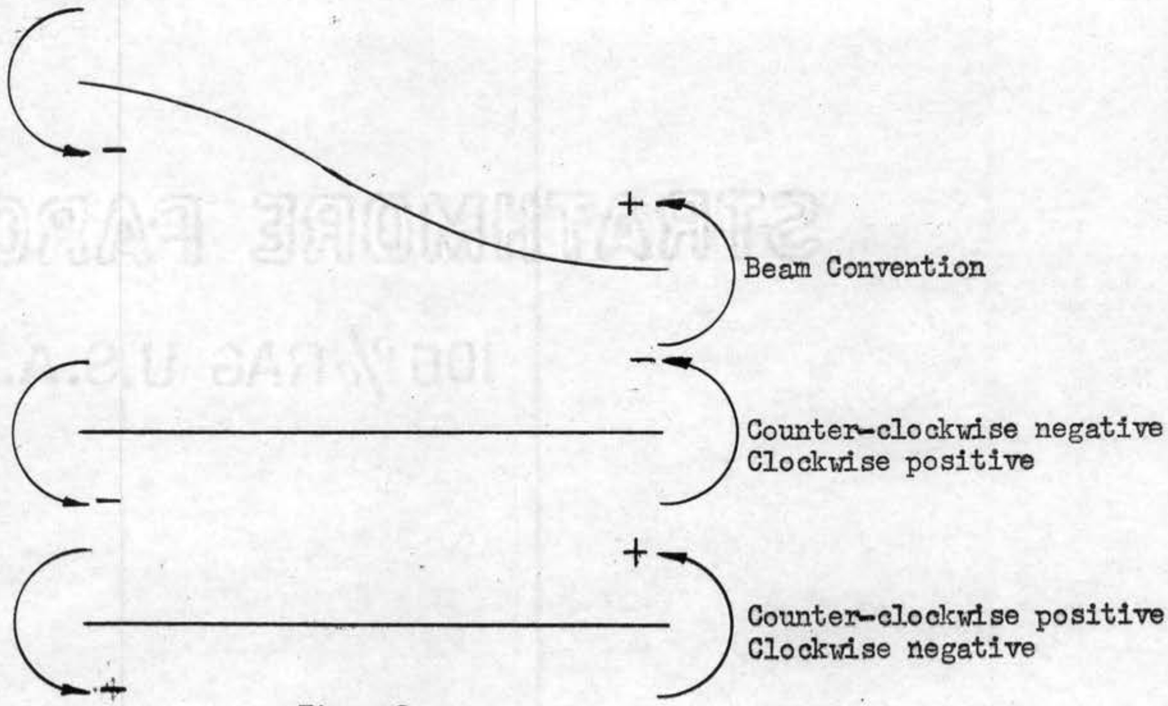


Fig. 2

Difficulty arises, however, if the author changes from one sign convention to another without calling the reader's attention to this change.

"Elastic Properties of Riveted Connections"[8] by J. Charles Rathbun presents this difficulty. Mr. Rathbun changes from one sign convention to another with very little comment, which causes the reader difficulty in deriving his equations. This point was stressed in the discussions of Mr. Rathbun's paper by Mr. Ralph E. Goodwin as follows: "....(Rathbun) flatters the intelligence of his readers when he assumes that the steps in his derivations and his systems of algebraic signs will be self-evident. In problems of this nature the difficulty with algebraic signs becomes almost unsurmountable unless the latter are explicitly defined. The

---

[8] Transactions of the American Society of Civil Engineers, Vol. 101, pp. 525-596.

Any Loading Condition

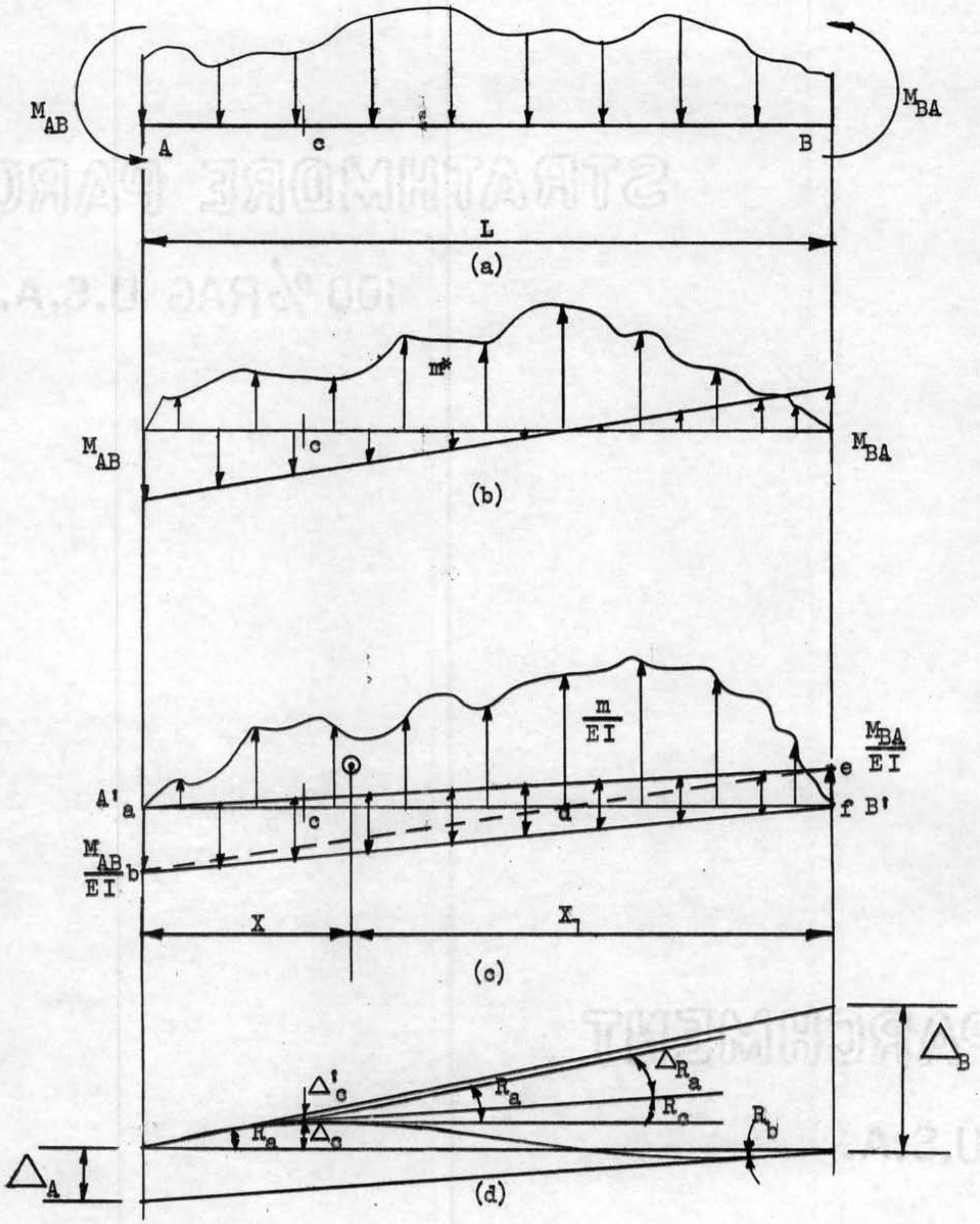


Fig. 3

m - the variable ordinate of the moment diagram due to traverse loads - member is assumed a simple beam.

tendency of experts is to grow so accustomed to their own particular methods and sign conventions that it does not occur to them that these methods and conventions may not be taken for granted by everyone."[9]

Rathbun himself acknowledges this difficulty "the question of signs is one that arises quite often in studies of this nature ...."[10]

Mr. Rathbun uses the beam convention and a moment tending to rotate a member clockwise as negative and counter-clockwise as positive.

B. Elastic Load[11] or Elastic Weights[12] Theory.

A beam "AB" of length "L" loaded with any condition of loading is shown in Fig. 3(a). The moment curve for this beam is shown in Fig. 3(b). Beam A' B' of length "L" loaded with the moment curve of Fig. 3(b), each ordinate of which is divided by EI, is shown in Fig. 3(c). These ordinates now become the loads on beam A' B'. In Fig. 3(c) a straight line is drawn from A' to the maximum ordinate of $\frac{M_{BA}}{EI}$, similiarly a straight line is drawn from B' to $\frac{M_{AB}}{EI}$.

Triangle abde and abfd have a common base and altitude, therefore, are equal in area. Triangle abde minus triangle ade equals triangle abd while triangle abfd minus triangle bdf equals triangle abd. Then triangles ade and bdf are equal in area and abd and edf are increased by these equal quantities to give triangles abfd and adfe. The derived

---

9 Transactions of the American Society of Civil Engineers, Vol. 101, p. 564.

10 Ibid., p. 591.

11 John Benson Wilbur and Charles Head Norris, Elementary Structural Analysis, (New York, 1948), pp. 318-320.

12 Hale Sutherland and Harry Lake Bowman, Structural Theory, (New York, 1944), pp. 182-185.

properties of these triangles give the desired net properties of triangles abd and dfe respectively. This presents a convenient method of using elastic weights without determining points of counter flexure.

The elastic chord or deformed structure of Fig. 3(a) due to the given loads is shown in Fig. 3(d). The following relationships can be derived from Fig. 3(c) and Fig. 3(d).

The second moment-area theorem gives:

$$\Delta_A = \frac{2L}{3}\int_A^B \frac{M_{BA}}{EI}\,dx + \int_A^B \bar{x}\frac{m}{EI}\,dx - \frac{L}{3}\int_A^B \frac{M_{AB}}{EI}\,dx =$$

$$\int_A^B \left[\frac{2L}{3}\frac{M_{BA}}{EI} + \frac{\bar{x}m}{EI} - \frac{L}{3}\frac{M_{AB}}{EI}\right] dx \qquad (1)$$

From the geometry of Fig. 3(d):

$$R_b = \frac{\Delta_A}{L} \qquad (2)$$

Substituting equation 1 in equation 2:

$$R_b = \frac{1}{L}\int_A^B \left[\frac{2L}{3}\frac{M_{BA}}{EI} + \frac{\bar{x}m}{EI} - \frac{L}{3}\frac{M_{AB}}{EI}\right] dx \qquad (3)$$

The reaction of the beam in Fig. 3(c) is:

$$B'_V = \frac{\int_A^B \left[\frac{2L}{3}\frac{M_{BA}}{EI} + \frac{\bar{x}m}{EI} - \frac{L}{3}\frac{M_{AB}}{EI}\right] dx}{L} \qquad (4)$$

Therefore, $R_b = B'_V$, which is the reaction of beam "AB" loaded with $\frac{M}{EI}$ diagram. Similiarly, R_a is equal to the reaction at end A of beam "AB" loaded with the $\frac{M}{EI}$ diagram.

From Fig. 3(c), 3(d) and by first moment-area theorem:

$$R_c = R_a - \Delta_{R_a} \tag{5}$$

$$\Delta_{R_a} = \int_A^c \left[-\frac{M_{AB}}{EI} + \frac{M_{BA}}{EI} + \frac{m}{EI} \right] dx \tag{6}$$

$$R_c = R_a - \int_A^c \left[-\frac{M_{AB}}{EI} + \frac{M_{BA}}{EI} + \frac{m}{EI} \right] dx \tag{7}$$

Therefore, R_c = reaction at end A - area of $\frac{M}{EI}$ diagram between A and c, or R_c = shear at c due to load $\frac{M}{EI}$.

$$\Delta_c = R_a \overline{AC} - \Delta'_c \tag{8}$$

$$\Delta'_c = \int_A^c \left[\frac{M_{AB}}{EI} \text{ x dist to cg} - \frac{M_{BA}}{EI} \text{ x } \frac{\overline{Ac}}{3} - \frac{m}{EI} \text{ x dist to cg} \right] dx \tag{9}$$

$$\Delta_c = R_a \overline{Ac} - \text{moment of area between A and c.} \tag{10}$$

Therefore, Δ_c = the moment of the beam at "c" due to $\frac{M}{EI}$ as the load.

From this it can be seen that the slope and deflection of the elastic curve of a loaded member is equal to the shear and moment respectively of a new member of the same length and with the $\frac{M}{EI}$ diagram of the original as the load on this new member.

Here the beam "AB" has been assumed supported on unyielding supports. The deflection and slope are still equal to the moment and shear of the $\frac{M}{EI}$ loaded member when the supports are yielding if these quantities are measured from the original position of the member.

If the convention used is that upward loads are positive and downward negative, then the positive ordinates of the $\frac{M}{EI}$ loading indicates an upward force and the negative ordinates indicate a negative force. Also, positive bending moment is plotted above the axis and negative below.

From these principles the general slope deflection equations can be developed and in order to keep signs consistent with Rathbun, a moment tending to rotate a member counter-clockwise will be taken as positive and angular rotations will be treated in the same manner.

C. Equations 1 and 2 (The General Slope Deflection Equations).

The general slope deflection equations are developed to present a logical sequence of derivations for Rathbun's equations VI and VII.

Fig. 4(c) gives the following relationships:

(A) $\alpha_a = (\theta_a - R_a)$ (D) $\alpha_b = (\theta_b - R_b)$

(B) $\frac{\Delta_A}{L} = \alpha_b$ (E) $\frac{\Delta_B}{L} = \alpha_a$

(C) $\Delta_A = (\theta_b - R_b)L$ (F) $\Delta_B = (\theta_a - R_a)L$

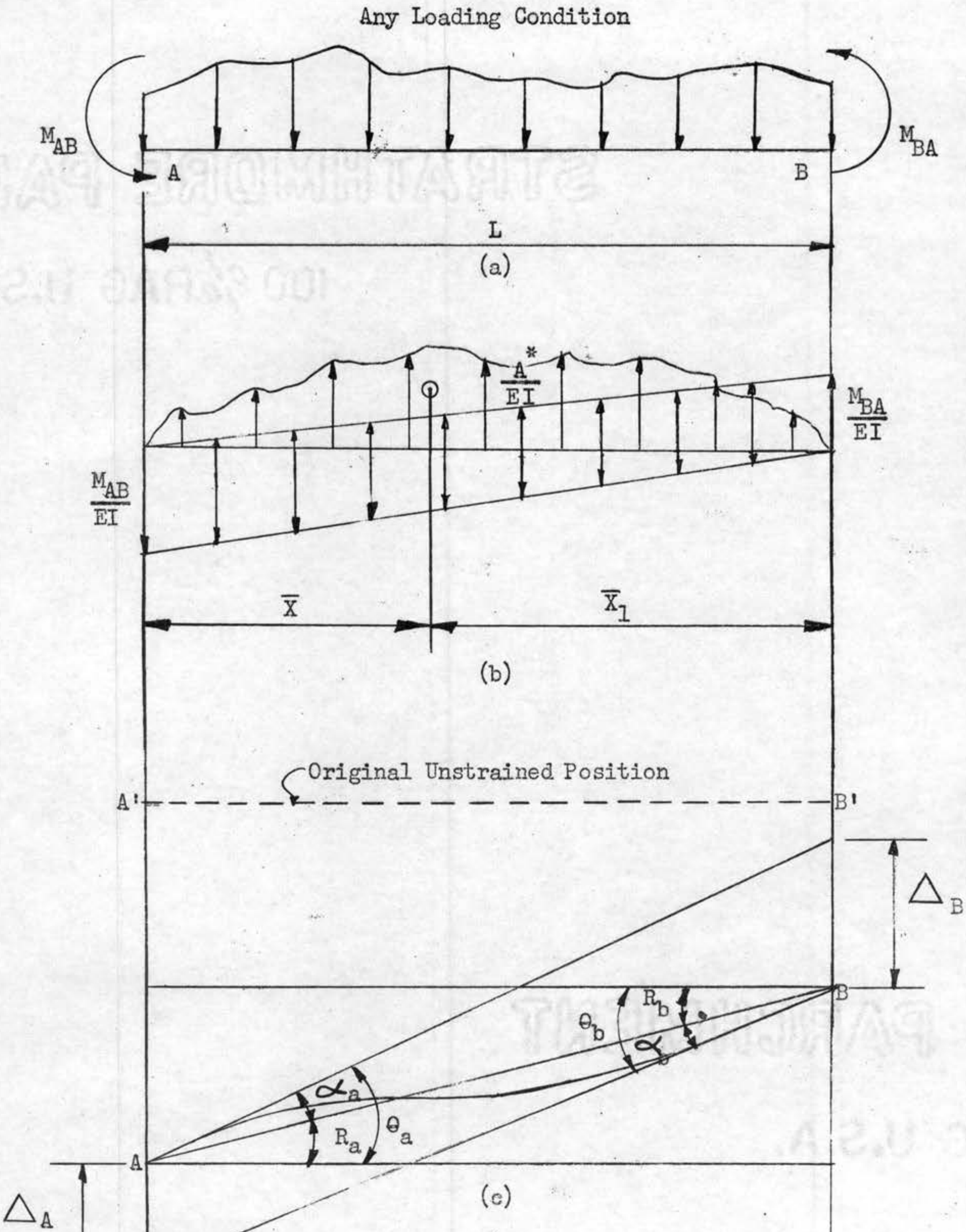


Fig. 4

* A-area of the moment diagram due to traverse loads - member is assumed a simple beam.

The second moment-area theorem gives:

$$\Delta_A = -\frac{M_{AB}}{EI}\frac{L}{2}\frac{L}{3} + \frac{M_{BA}}{EI}\frac{L}{2}\frac{2L}{3} + \frac{A\bar{x}}{EI} = -\frac{M_{AB}L^2}{6EI} + \frac{M_{BA}L^2}{3EI} + \frac{A\bar{x}}{EI} \tag{11}$$

$$\Delta_B = \frac{M_{AB}}{EI}\frac{L}{2}\frac{2L}{3} - \frac{M_{BA}}{EI}\frac{L}{2}\frac{L}{3} - \frac{A\bar{x}_1}{EI} = \frac{M_{AB}L^2}{3EI} - \frac{M_{BA}L^2}{6EI} - \frac{A\bar{x}_1}{EI} \tag{12}$$

Solve (11) and (12) simultaneously for M_{AB}.

Eq. 12 x 2 = Eq. 13

$$2\Delta_B = \frac{2}{3}\frac{M_{AB}L^2}{EI} - \frac{1}{3}\frac{M_{BA}L^2}{EI} - \frac{2A\bar{x}_1}{EI} \tag{13}$$

$$\Delta_A = -\frac{M_{AB}L^2}{6EI} + \frac{L^2}{3}\frac{M_{BA}}{EI} + \frac{A\bar{x}}{EI} \tag{11}$$

Eq. 13 + Eq. 11 = Eq. 14

$$2\Delta_b + \Delta_A = \frac{M_{AB}L^2}{EI}\left[\frac{4}{6} - \frac{1}{6}\right] - \frac{2A\bar{x}_1}{EI} + \frac{A\bar{x}}{EI} \tag{14}$$

Substitute C and F in Eq. 14 and solve for M_{AB}.

$$M_{AB} = \left[2L(\theta_a - R_a) + L(\theta_b - R_b) - \frac{1}{EI}(A\bar{x} - 2A\bar{x}_1)\right]\frac{2EI}{L^2}$$

$$= \frac{4EI}{L}(\theta_a - R_a) + \frac{2EI}{L}(\theta_b - R_b) - \frac{2}{L^2}(A\bar{x} - 2A\bar{x}_1) \tag{15}$$

Let $M_{cA} = \frac{2}{L^2}(A\bar{x} - 2A\bar{x}_1)$, substitute in Eq. 15 and simplify.

$$M_{AB} = \frac{2EI}{L}(2\theta_a + \theta_b - 3R) - M_{cA} \text{ (Rathbun's Eq. 1)} \tag{I*}$$

From Fig. 4(c) $R_a = R_b = R$

Solve Eq. 11 and Eq. 12 simultaneously for M_{BA}.

Eq. 11 x 2 = Eq. 16.

$$2\Delta_A = -\frac{M_{AB}}{EI}\frac{L^2}{3} + \frac{M_{BA}}{EI}\frac{2L^2}{3} + \frac{2A\bar{x}}{EI} \tag{16}$$

$$\Delta_B = \frac{M_{AB}}{EI}\frac{L^2}{3} - \frac{M_{BA}}{EI}\frac{L^2}{6} - \frac{A\bar{x}_1}{EI} \tag{12}$$

Eq. 16 + Eq. 12 = Eq. 17.

$$2\Delta_A + \Delta_B = \frac{M_{BA}L^2}{EI}\left[\frac{4}{6} - \frac{1}{6}\right] + \frac{2A\bar{x}}{EI} - \frac{A\bar{x}_1}{EI} \tag{17}$$

Substitute C and F in Eq. 17 and solve for M_{BA}.

$$M_{BA} = \left[2L(\theta_b - R_b) + L(\theta_a - R_a) + \frac{1}{EI}(A\bar{x}_1 - 2A\bar{x})\right]\frac{2EI}{L^2}$$

$$= \frac{4EI}{L}(\theta_b - R_b) + \frac{2EI}{L}(\theta_a - R_a) + \frac{2}{L^2}(A\bar{x}_1 - 2A\bar{x}) \tag{18}$$

Let $\frac{2}{L^2}(A\bar{x}_1 - 2A\bar{x}) = M_{cB}$, substitute in Eq. 18 and simplify.

$$M_{BA} = \frac{2EI}{L}(2\theta_b + \theta_a - 3R) + M_{cB} \text{ (Rathbun's Eq. 2)} \tag{II}$$

---

* Roman numerals are used to identify Rathbun's equations.

D. Equation 3.

If the right end of a beam "AB" is hinged, then the moment at end "A" (M_A) can be found indirectly by taking the general slope deflection equation for the moment at end "B" (M_B), setting it equal to zero and solving this new equation for θ_b. This value of θ_b can then be substituted into the general slope deflection equation of end "A" and M_A found in terms of θ_a, R and the fixed end moment of the external loads about end "A" that is:

$$M_{BA} = \frac{2EI}{L}(2\theta_b + \theta_a - 3R) + \frac{2}{L^2}(A\bar{x}_1 - 2A\bar{x}) \qquad \text{(II')}$$

Let $M_{BA} = 0$ and substitute in Eq. II'.

$$0 = \frac{2EI}{L}(2\theta_b + \theta_a - 3R) + \frac{2A}{L^2}(\bar{x}_1 - 2\bar{x}) \qquad (19)$$

Solve Eq. 19 for θ_b.

$$\theta_b = -\frac{\frac{A}{LEI}(\bar{x}_1 - 2\bar{x}) - \theta_a + 3R}{2} \qquad (20)$$

$$M_{AB} = \frac{2EI}{L}(2\theta_a + \theta_b - 3R) - \frac{2}{L^2}(A\bar{x} - 2A\bar{x}_1) \qquad \text{(I')}$$

Substitute θ_b of Eq. 20 in Eq. I'.

$$M_{AB} = \frac{2EI}{L}\left[2\theta_a - \frac{\frac{A}{LEI}(\bar{x}_1 - 2\bar{x}) - \theta_a + 3R}{2} - 3R\right] - \frac{2A}{L^2}(\bar{x} - 2\bar{x}_1)$$

$$= \frac{EI}{L}\left[4\theta_a - \frac{A}{LEI}(\bar{x}_1 - 2\bar{x}) - \theta_a + 3R - 6R\right] - \frac{2A}{L^2}(\bar{x} - 2\bar{x}_1) \qquad (21)$$

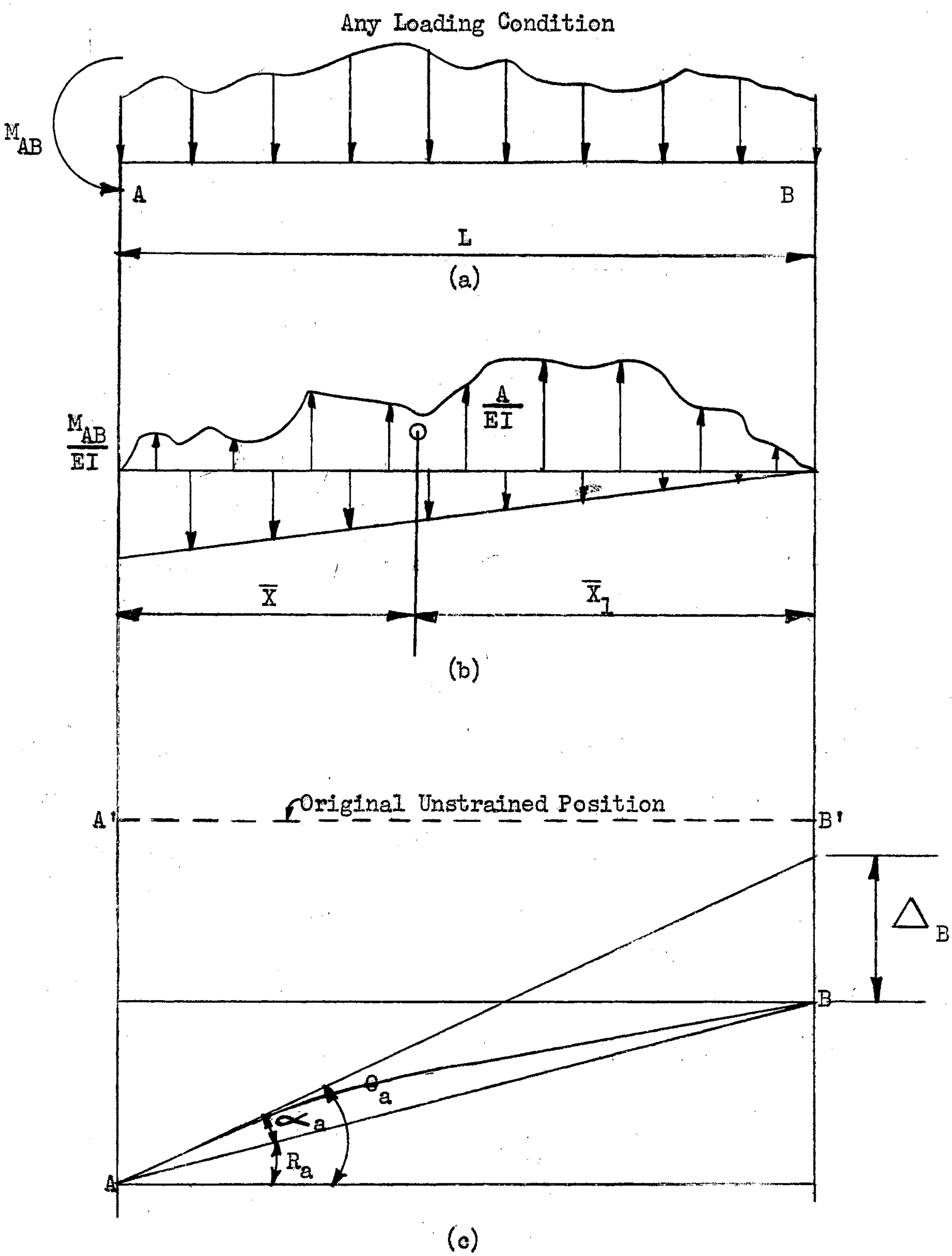

Any Loading Condition
M_{AB}
A
B
L
(a)
$\frac{M_{AB}}{EI}$
$\frac{A}{EI}$
$\bar{X}$
$\bar{X}_1$
(b)
Original Unstrained Position
A'
B'
Δ_B
B
θ_a
α_a
R_a
A
(c)

Fig. 5

$$M_{AB} = \frac{EI}{L}\,(4\theta_a - \theta_a + 3R - 6R) - \frac{A}{L^2}(\bar{x}_1 - 2\bar{x}) - \frac{2A}{L^2}(\bar{x} - 2\bar{x}_1)$$

$$= \frac{EI}{L}(3\theta_a - 3R) + \frac{A}{L^2}(-\bar{x}_1 + 2\bar{x} - 2\bar{x} + 4\bar{x}_1) \qquad (22)$$

$$M_{AB} = \frac{3EI}{L}(\theta_a - R) + \frac{3Ax_1}{L^2} \qquad (III)$$

If the right end of a member is hinged then $M_{BA} = 0$ and from Fig. 5(c).

(G) $\alpha_a = (\theta_a - R_a)$ (J) $\alpha_a = \frac{\Delta_B}{L}$

(H) $\Delta_B = (\theta_a - R_a)L$

$$\Delta_B = \frac{M_{AB}}{EI}\,\frac{L}{2}\,\frac{2L}{3} - \frac{A\bar{x}_1}{EI} \qquad (23)$$

Substitute "H" in Eq. 23.

$$(\theta_a - R_a)L = \frac{M_{AB}L^2}{3EI} - \frac{A\bar{x}_1}{EI} \qquad (24)$$

Solve Eq. 24 for M_{AB}.

$$M_{AB} = \frac{3EI}{L}\,(\theta_a - R_a) + \frac{3A\bar{x}_1}{L^2} \qquad (III)$$

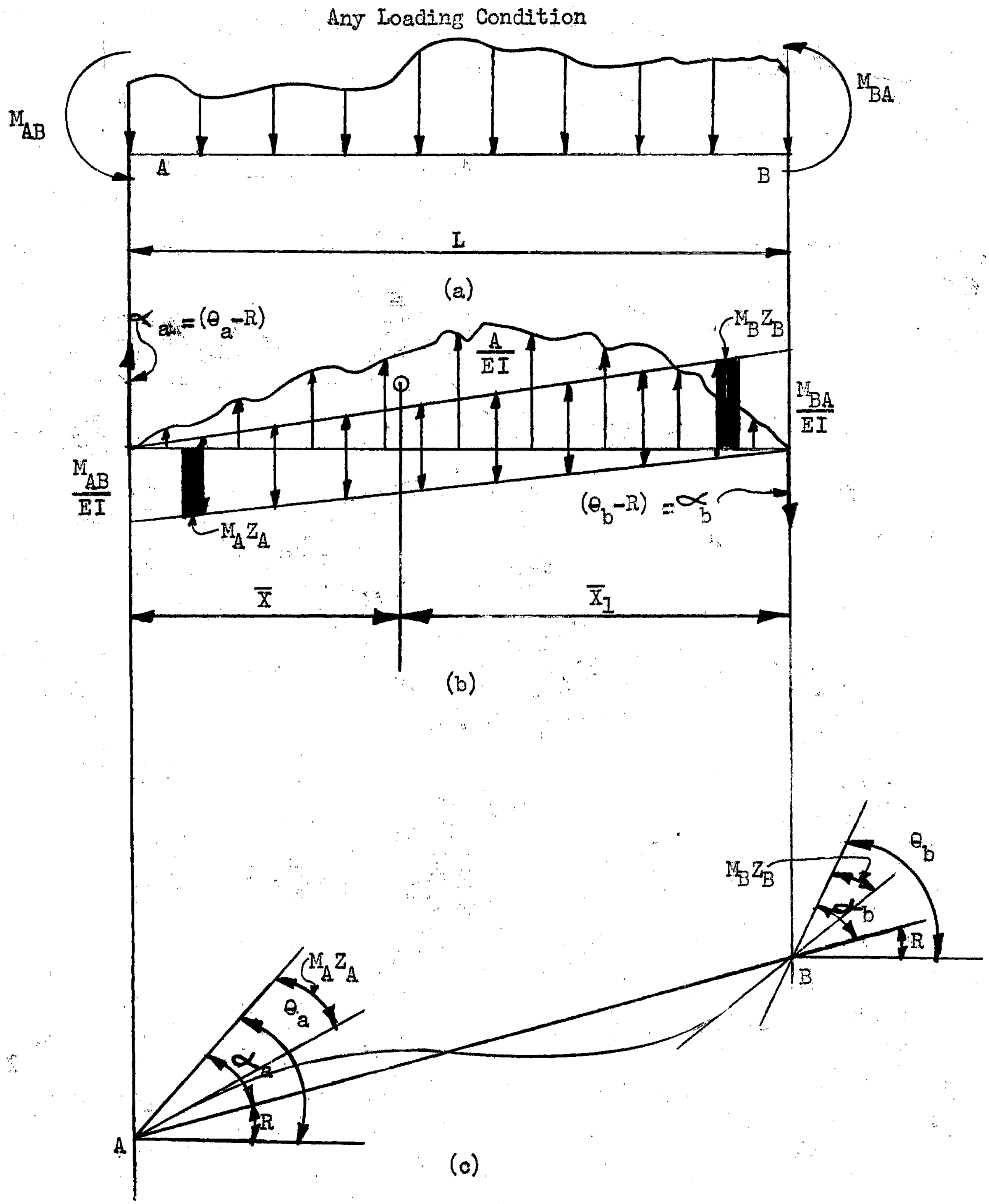

Any Loading Condition
M_{AB}
M_{BA}
A
B
L
(a)
$\alpha_a = (\theta_a - R)$
$\frac{A}{EI}$
$M_B Z_B$
$\frac{M_{BA}}{EI}$
$\frac{M_{AB}}{EI}$
$(\theta_b - R) = \alpha_b$
$M_A Z_A$
$\overline{X}$
$\overline{X}_1$
(b)
θ_b
$M_B Z_B$
α_b
R
B
$M_A Z_A$
θ_a
α_a
R
A
(c)

Fig. 6

E. Equations 10, 11, 12, 13 and 14.

The slope of the elastic chord is the shear in the conjugate beam, therefore, the shear, or reaction, of the conjugate beam is equal to the slope of the elastic chord at the end of the original beam.

$\Sigma M_A = 0$

$$-\frac{M_{AB}}{EI}\frac{L}{2}\frac{L}{3} + \frac{M_{BA}}{EI}\frac{L}{2}\frac{2L}{3} + M_B Z_B L + \frac{A\bar{x}}{EI} - (\theta_b - R)\ L = 0$$

$$-\frac{M_{AB}L^2}{6EI} + \frac{2M_{BA}L^2}{6EI} + M_B Z_B L + \frac{A\bar{x}}{EI} - (\theta_b - R)\ L = 0 \qquad (25)$$

$$-\frac{M_{AB}L^2}{6EI} + \frac{2M_{BA}L^2}{6EI} + \frac{2M_B Z_B L 3EI}{6EI} + \frac{A\bar{x}}{EI} - (\theta_b - R)\ L = 0$$

$$-\frac{M_{AB}L^2}{6EI} + \frac{2M_{BA}L}{6EI}(L + 3EIZ_B) + \frac{A\bar{x}}{EI} - (\theta_b - R)\ L = 0 \qquad (26)$$

Let $L_{2B} = L + 3EIZ_B$

Substitute in Eq. 26.

$$-\frac{M_{AB}L^2}{6EI} + \frac{2M_{BA}LL_{2B}}{6EI} + \frac{A\bar{x}}{EI} - (\theta_b - R)\ L = 0 \qquad (27)$$

$\Sigma M_B = 0$

$$-(\theta_a - R)\ L - \frac{M_{BA}}{EI}\frac{L}{2}\frac{L}{3} + \frac{M_{AB}}{EI}\frac{L}{2}\frac{2L}{3} + M_A Z_A L - \frac{A\bar{x}_1}{EI} = 0$$

$$-(\theta_a - R)\ L - \frac{M_{BA}L^2}{6EI} + \frac{2M_{AB}L^2}{6EI} + M_A Z_A L - \frac{A\bar{x}_1}{EI} = 0 \qquad (28)$$

$$-(\theta_a - R)L - \frac{M_{BA}L^2}{6EI} + \frac{2M_{AB}L^2}{6EI} + \frac{2M_A Z_A L 3EI}{6EI} - \frac{A\overline{x}_1}{EI} = 0$$

$$-(\theta_a - R)\ L - \frac{M_{BA}L^2}{6EI} + \frac{2M_{AB}L}{6EI}\ (L + 3EIZ_A) - \frac{A\overline{x}_1}{EI} = 0 \tag{29}$$

Let $L_{2A} = L + 3EIZ_A$

Substitute in Eq. 29.

$$-(\theta_a - R)\ L - \frac{M_{BA}L^2}{6EI} + \frac{2M_{AB}LL_{2A}}{6EI} - \frac{A\overline{x}_1}{EI} = 0 \tag{30}$$

By changing all signs we get Rathbun's Eq. 10.

$$(\theta_a - R)\ L - \frac{2M_A LL_{2A}}{6EI} + \frac{M_B L^2}{6EI} + \frac{A\overline{x}_1}{EI} = 0 \tag{X}$$

Multiplying Eq. 30 by $\frac{6EI}{L}$ and solving for moment gives Rathbun's Eq. 11.

$$2L_{2A}M_A - LM_B = 6EI\ (\theta_a - R) + \frac{6A\overline{x}_1}{L} = 0 \tag{XI}$$

Multiplying Eq. 27 by $\frac{6EI}{L}$ and solving for moment gives

Rathbun's Eq. 12.

$$2L_{2B}M_B - LM_A = 6EI\ (\theta_b - R) - \frac{6A\overline{x}}{L} = 0 \tag{XII}$$

Eq. 30 x $\frac{2L_{2B}}{L}$ = Eq. 31

$$-(\theta_a - R)\ 2L_{2B} - \frac{2M_{BA}LL_{2B}}{6EI} + \frac{4M_{AB}L_{2A}L_{2B}}{6EI} - \frac{2A\overline{x}_1 L_{2B}}{LEI} = 0 \tag{31}$$

$$-(\theta_b - R)\ L + \frac{2M_B LL_{2B}}{6EI} - \frac{M_{AB}L^2}{6EI} + \frac{A\overline{x}}{EI} \tag{27}$$

Eq. 31 + Eq. 27 = 32.

$$-(\theta_a - R)\,2L_{2B} - (\theta_b - R)\,L + \frac{M_{AB}}{6EI}\,(4L_{2A}L_{2B} - L^2) - \frac{2A\bar{x}_1 L_{2B}}{LEI} + \frac{A\bar{x}}{EI} = 0 \qquad (32)$$

$$M_{AB} = 6EI\,\frac{2L_{2B}(\theta_a - R) + (\theta_b - R)L}{4L_{2A}L_{2B} - L^2} + 6EI\,\frac{2A\bar{x}_1 L_{2B} - A\bar{x}L}{LEI(4L_{2A}L_{2B} - L^2)} \qquad (33)$$

Rathbun's Eq. 13.

$$M_{AB} = 6EI\,\frac{2L_{2B}(\theta_a - R) + (\theta_b - R)L}{4L_{2A}L_{2B} - L^2} + \frac{6A}{L}\,\frac{2\bar{x}_1 L_{2B} - \bar{x}L}{4L_{2A}L_{2B} - L^2} \qquad (XIII)$$

Eq. 27 x $\frac{2L_{2A}}{L}$ = Eq. 34.

$$-\frac{2M_{AB}LL_{2A}}{6EI} + \frac{4M_{BA}L_{2A}L_{2B}}{6EI} - 2(\theta_b - R)L_{2A} + \frac{2A\bar{x}L_{2A}}{LEI} = 0 \qquad (34)$$

$$\frac{2M_{AB}LL_{2A}}{6EI} - \frac{M_{BA}L^2}{6EI} - (\theta_a - R)L - \frac{A\bar{x}_1}{EI} = 0 \qquad (30)$$

Eq. 30 + Eq. 34 = 35.

$$\frac{M_{BA}}{6EI}(4L_{2A}L_{2B} - L^2) - 2(\theta_b - R)L_{2A} - (\theta_a - R)L + \frac{2A\bar{x}L_{2A}}{LEI} - \frac{A\bar{x}_1}{EI} = 0 \qquad (35)$$

$$M_{BA} = 6EI\,\frac{2(\theta_b - R)L_{2A} + (\theta_a - R)L}{4L_{2A}L_{2B} - L^2} - \frac{\frac{6EI}{LEI}\,(2A\bar{x}L_{2A} - A\bar{x}_1 L)}{4L_{2A}L_{2B} - L^2} \qquad (36)$$

Rathbun's Eq. 14.

$$M_{BA} = 6EI \frac{2L_{2A}(\theta_b - R) + (\theta_a - R)L}{4L_{2A}L_{2B} - L^2} - \frac{6A}{L} \frac{2\bar{x}L_{2A} - L\bar{x}_1}{4L_{2A}L_{2B} - L^2} \tag{XIV}$$

F. Equations 4, 5, 6, 7, 8 and 9.

In the slope deflection method, as can be verified by any textbook dealing with the subject, there are three contributing elements that constitute the moment at the end of a member, namely:

1. The slope of tangents to the elastic line at the ends of the member measured from its original position.
2. The rotation of the chord joining the ends of the elastic line.
3. The fixed-end moment from the external loads on the member.

The foregoing equations have been derived by the use of these three elements simultaneously. Each element, however, could have been developed independently and then combined by the laws of superposition to give the same results. Since this is true M_{cA} and M_{cB} can be evaluated from element three.

If in equation I, θ_a, θ_b & $R = 0$, $M_{AB} = -M_{cA}$ or M_{cA} is the "fixed end moment" of the external loads on member AB about end A, since for θ_a, θ_b & R to equal zero the member must be completely fixed against rotation and translation. Similarly, M_{cB} is the "fixed end moment" about end B.

In equation I and II, M_{cA} and M_{cB} are the moments at the support produced by the external loads on member AB, therefore, the resisting or reactionary moments of the beam at A and B respectively, must be $+ M_{cA}$ and $- M_{cB}$.

Letting θ_a, θ_b & $R = 0$ Eq. 15 becomes:

$$M_{cA} = \frac{2A}{L^2}\,(2\bar{x}_1 - \bar{x}) \tag{15'}$$

Let $\bar{x}_1 = L - \bar{x}$ substitute in 15'

$$M_{cA} = \frac{2A}{L^2}\Big[2\,(L - \bar{x}) - \bar{x}\Big] = \frac{2A}{L^2}\,(2L - 3\bar{x}) \tag{37}$$

Using the beam convention

$$M_{cA} = -\frac{2A}{L^2}\,(2L - 3\bar{x}) \text{ (Rathbun's Eq. 6).} \tag{VI}$$

Letting θ_a, θ_b & $R = 0$ Eq. 18 becomes

$$M_{cB} = -\frac{2A}{L^2}\,(2\bar{x} - \bar{x}_1)$$

Let $\bar{x} = L - \bar{x}_1$ and substitute in Eq. 18'.

$$M_{cB} = -\frac{2A}{L^2}\Big[2\,(L - \bar{x}_1) - \bar{x}_1\Big] = -\frac{2A}{L^2}\,(2L - 3\bar{x}_1) \tag{38}$$

The sign of moment on end B of member AB is the same for the beam convention and counter-clockwise positive as can be seen from Fig. 2, therefore,

$$M_{cB} = -\frac{2A}{L^2}\,(2L - 3\bar{x}_1) \text{ (Rathbun's Eq. 7).} \tag{VII}$$

Equations XIII and XIV are the slope deflection equations in terms of Θ_a, Θ_b, R and the resisting moment of a member with riveted, or elastic, connections. By using the beam convention and letting Θ_a, Θ_b & $R = 0$, M_{cA} and M_{cB} can be evaluated.

$$M_{cA} = -\frac{6A}{L} \times \frac{2L_{2B}\bar{x}_1 - L\bar{x}}{4L_{2A}L_{2B} - L^2} \quad \text{(Rathbun's Eq. 8).} \qquad \text{(VIII)}$$

$$M_{cB} = -\frac{6A}{L} \times \frac{2L_{2A}\bar{x} - L\bar{x}_1}{4L_{2B}L_{2A} - L^2} \quad \text{(Rathbun's Eq. 9).} \qquad \text{(IX)}$$

Let $M_{cA} = \frac{6A}{L} \times \frac{2L_{2B}\bar{x}_1 - L\bar{x}}{4L_{2A}L_{2B} - L^2}$

and $M_{cB} = \frac{6A}{L} \times \frac{2L_{2A}\bar{x} - L\bar{x}_1}{4L_{2B}L_{2A} - L^2}$

The moment at the support due to the loads on AB is negative at end A and positive at end B. Substituting into Eq. XIII and XIV, M_{cA} and M_{cB} for their equivalents and following the above sign convention we have:

$$M_{AB} = 6EI\,\frac{2L_{2B}(\Theta_A - R) + L(\Theta_B - R)}{4L_{2A}L_{2B} - L^2} - M_{cA} \quad \text{(Rathbun's Eq. 4)} \qquad \text{(IV)}$$

$$M_{BA} = 6EI\,\frac{2L_{2A}(\Theta_B - R) + L(\Theta_A - R)}{4L_{2A}L_{2B} - L^2} + M_{cB} \quad \text{(Rathbun's Eq. 5)} \qquad \text{(V)}$$

Equations IV and V are identical with equations XIII and XIV when M_{cA} and M_{cB} are substituted from equations VIII and IX. These equations offer a mathematical method of determining the moment reaction of a member when the moment at the support is known.

Equations I, II, IV and V are the obsolescent forms of the slope deflection equations in which the fixed end moment is the moment of the loads about the support. Care must be taken in using these forms that the moment is changed to the reactionary moment. Therefore, the present tendency in "The Transactions of the American Society of Civil Engineers" is to write the slope deflection equations with this reactionary moment thus eliminating the inadvertent use of the moment at the supports.

MAUGH'S EQUATIONS FOR SEMI-RIGID CONNECTIONS

MAUGH'S EQUATIONS FOR SEMI-RIGID CONNECTIONS

Maugh's equations for semi-rigid connections are developed by using the sign convention that moments tending to rotate a member clockwise are positive and angle changes are denoted similarly.

He also uses the more generally accepted method of loading the conjugate beam (i.e. the $\frac{M}{EI}$ diagram acts down on the conjugate beam.)

In Fig. 7(a) and (b) positive unit moments are applied at ends A and B separately. The total effect of these unit moments at A and B are the sum of the effects of the separately applied moments while the effect of any moment at A and B is the effect of the unit moments multiplied by the moment at A and B respectively.

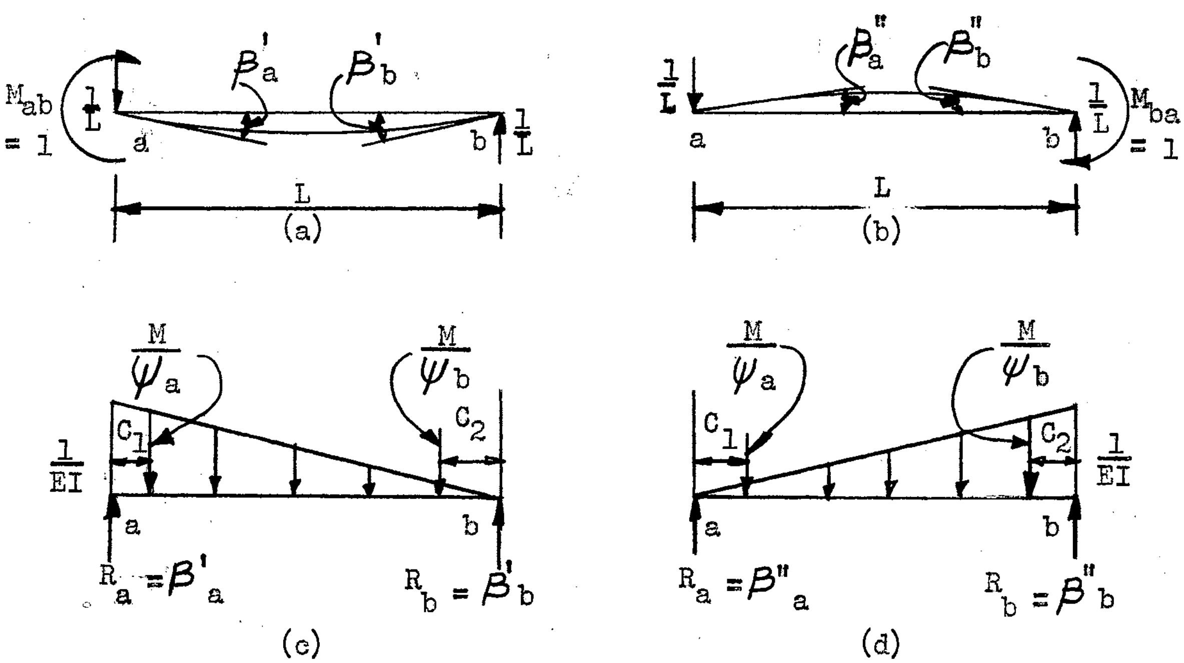


Fig. 7

From Fig. 7(a) and (c)

$\Sigma M_a = 0$

$$-\beta'_b L + \frac{1(0)}{\psi_a} + \frac{(0)L}{\psi_b} + \frac{1}{EI}\frac{L}{2}\frac{L}{3} = 0 \tag{39}$$

$$\beta'_b = -\frac{L}{6EI} \quad \text{(This sign depends on the counter-clockwise angles } \beta'_b\text{.)} \tag{40}$$

$\Sigma M_b = 0$

$$\beta'_a L - \frac{1}{\psi_a}\frac{L}{} - \frac{1}{EI}\frac{L}{2}\frac{2L}{3} = 0 \tag{41}$$

$$\beta'_a = \frac{L}{3EI} + \frac{1}{\psi_a} \tag{42}$$

From Fig. 7(b) and (d)

$\Sigma M_a = 0$

$$-\beta''_b L + \frac{1}{\psi_b}\frac{L}{} + \frac{1}{EI}\frac{L}{2}\frac{2L}{3} = 0 \tag{43}$$

$$\beta''_b = \frac{L}{3EI} + \frac{1}{\psi_b} \tag{44}$$

$\Sigma M_b = 0$

$$\beta''_a L - \frac{1}{EI}\frac{L}{2}\frac{L}{3} = 0 \tag{45}$$

$$\beta''_a = -\frac{L}{6EI} \quad \text{(This sign depends on the counter-clockwise angle } \beta''_a\text{.)} \tag{46}$$

Total angle change = total FEM x angle change due to unit angle change, or:

$$\theta_a = M_{AB}\left[\frac{L}{3EI} + \frac{1}{\psi_a}\right] - \frac{M_{BA}L}{6EI} \tag{47}$$

$$\theta_b = M_{BA}\left[\frac{L}{3EI} + \frac{1}{\psi_b}\right] - \frac{M_{AB}L}{6EI} \tag{48}$$

(K) Let $K = \frac{EI}{L}$

Substitute K in Eq. 47 and Eq. 48.

$$\theta_a = M_{AB}\left[\frac{1}{3K} + \frac{1}{\psi_a}\right] - \frac{M_{BA}}{6K} \tag{49}$$

$$\theta_b = M_{BA}\left[\frac{1}{3K} + \frac{1}{\psi_b}\right] - \frac{M_{AB}}{6K} \tag{50}$$

(L) $$\frac{1}{3K} + \frac{1}{\psi_a} = \frac{1}{3K} + \frac{3K}{\psi_a 3K} = \frac{1}{3K}\left[1 + \frac{3K}{\psi_a}\right]$$

(M) $$\frac{1}{3K} + \frac{1}{\psi_b} = \frac{1}{3K} + \frac{3K}{\psi_b 3K} = \frac{1}{3K}\left[1 + \frac{3K}{\psi_b}\right]$$

Substitute L and M in Eq. 49 and 50.

$$\theta_a = \frac{M_{AB}}{3K}\left[1 + \frac{3K}{\psi_a}\right] - \frac{M_{BA}}{6K} \tag{51}$$

$$\theta_b = \frac{M_{BA}}{3K}\left[1 + \frac{3K}{\psi_b}\right] - \frac{M_{AB}}{6K} \tag{52}$$

$$3K\theta_a = M_{AB}\left[1 + \frac{3K}{\psi_a}\right] - \frac{M_{BA}}{2} \tag{53}$$

$$3K\theta_b = M_{BA}\left[1 + \frac{3K}{\psi_b}\right] - \frac{M_{AB}}{2} \tag{54}$$

(N) Let $1 + \frac{3K}{\psi_a} = C'$

(P) Let $1 + \frac{3K}{\psi_b} = C''$

Substitute N and P in Eq. 53 and Eq. 54.

$$3K\theta_a = M_{AB}C' - \frac{M_{BA}}{2} \tag{55}$$

$$3K\theta_b = M_{BA}C'' - \frac{M_{AB}}{2} \tag{56}$$

Divide Eq. 55 by C'.

$$\frac{3K\theta_a}{C'} = M_{AB} - \frac{M_{BA}}{2C'} \tag{57}$$

Multiply Eq. 56 by 2.

$$6K\theta_b = 2C''M_{BA} - M_{AB} \tag{58}$$

Add Eq. 57 and Eq. 58.

$$\frac{3K\theta_a}{C'} + 6K\theta_b = M_{BA}\left[2C'' - \frac{1}{2C'}\right] = \frac{M_{BA}}{2C'}(4C'C'' - 1) \tag{59}$$

Solve for M_{BA}.

$$M_{BA} = \frac{3K\theta_a}{C'} \frac{2C'}{4C'C'' - 1} + 6K\theta_b \frac{2C'}{4C'C'' - 1} = \frac{6K\theta_a}{4C'C'' - 1} + \frac{12K\theta_b C'}{4C'C'' - 1} \qquad (60)$$

(Q) Let $\frac{6}{4C'C'' - 1} = C_2$

(R) Let $\frac{12C'}{4C'C'' - 1} = C_3$

Substitute Q and R in Eq. 60 and substitute $\frac{EI}{L}$ for K from K

$$M_{BA} = \frac{EI}{L} (C_2\theta_a + C_3\theta_b) \quad \text{(Maugh's Equation.)} \qquad (176b)$$

Multiply Eq. 55 by 2.

$$6K\theta_a = 2M_{AB}C' - M_{BA} \qquad (61)$$

Divide Eq. 56 by C".

$$\frac{3K\theta_b}{C''} = M_{BA} - \frac{M_{AB}}{2C''} \qquad (62)$$

Add Eq. 61 and Eq. 62.

$$6K\theta_a + \frac{3K\theta_b}{C''} = M_{AB}(2C' - \frac{1}{2C''}) = \frac{M_{AB}}{2C''}(4C'C'' - 1) \qquad (63)$$

Solve for M_{AB}.

$$M_{AB} = \frac{6K\theta_a 2C''}{4C'C'' - 1} + \frac{3K\theta_b 2C''}{C''(4C'C'' - 1)} = \frac{12C''K\theta_a}{4C'C'' - 1} + \frac{6K\theta_b}{4C'C'' - 1} \qquad (64)$$

(S) Let $\frac{12C''}{4C'C'' - 1} = C_1$

Substitute S and Q in Eq. 64.

$$M_{AB} = \frac{EI}{L}(C_1\theta_a + C_2\theta_b) \quad \text{(Maugh's Equation.)} \tag{176a}$$

Rathbun lets:

$$L_{2A} = L + 3EIZ_a$$

$$L_{2B} = L + 3EIZ_b$$

And Maugh lets:

$$C' = 1 + \frac{3K}{\psi_a}$$

$$C'' = 1 + \frac{3K}{\psi_b}$$

But:

$$\frac{1}{\psi_a} = Z_a$$

$$\frac{1}{\psi_b} = Z_b$$

By substituting Z_a, Z_b and $\frac{EI}{L}$ in C' and C":

$$C' = 1 + \frac{3EIZ_a}{L}$$

$$C'' \quad 1 + \frac{3EIZ_b}{L}$$

Or:

$$C'L = L + 3EIZ_a = L_{2A}$$

$$C''L = L + 3EIZ_b = L_{2B}$$

Then C' and C" in terms of L_{2A} and L_{2B} becomes:

$$C' = \frac{L_{2A}}{L}$$

$$C'' = \frac{L_{2B}}{L}$$

Solving C_1, C_2 and C_3 in terms of L_{2A} and L_{2B}:

$$C_1 = \frac{\frac{12L_{2B}}{L}}{\frac{4L_{2A}}{L}\frac{L_{2B}}{L} - 1} = \frac{12L_{2B}L}{4L_{2A}L_{2B} - L^2} \tag{65a}$$

$$C_2 = \frac{6}{\frac{4L_{2A}}{L}\frac{L_{2B}}{L} - 1} = \frac{6L^2}{4L_{2A}L_{2B} - L^2} \tag{65b}$$

$$C_3 = \frac{\frac{12L_{2A}}{L}}{\frac{4L_{2A}}{L}\frac{L_{2B}}{L} - 1} = \frac{12L_{2A}L}{4L_{2A}L_{2B} - L^2} \tag{65c}$$

Maugh's equation for the FEM in riveted connections:

$$M'_{Fab} = \frac{1}{6}\left[M_{Fab}\,(2C_1 - C_2) + M_{Fba}\,(2C_2 - C_1)\right] \tag{178a}$$

$$M'_{Fba} = \frac{1}{6}\left[M_{Fab}(2C_2 - C_3) + M_{Fba}(2C_3 - C_2)\right] \tag{178b}$$

Substitute values for C_1, C_2, and C_3 in each of these.

$$M_{AB} = \frac{1}{6}\left[\frac{M_{AB}(2 \times 12L_{2B}L - 6L^2)}{4L_{2A}L_{2B} - L^2} + \frac{M_{BA}(2 \times 6L^2 - 12L_{2B}L)}{4L_{2A}L_{2B} - L^2}\right]$$

$$= L \times \frac{M_{AB}(4L_{2B} - L) + 2M_{BA}(L - L_{2B})}{4L_{2A}L_{2B} - L^2}$$

$$= L \times \frac{2L_{2B}(2M_{AB} - M_{BA}) + L\,(2M_{BA} - M_{AB})}{4L_{2A}L_{2B} - L^2} \tag{66}$$

Since we are dealing with only that portion of the slope deflection equation which pertains to external loads (the FEM of a beam) we can evaluate M_{AB} and M_{BA} by determining the deflection of point A about point B and B about A, setting these deflections equal to zero and solving the resulting equations simultaneously for M_{AB} and M_{BA}.

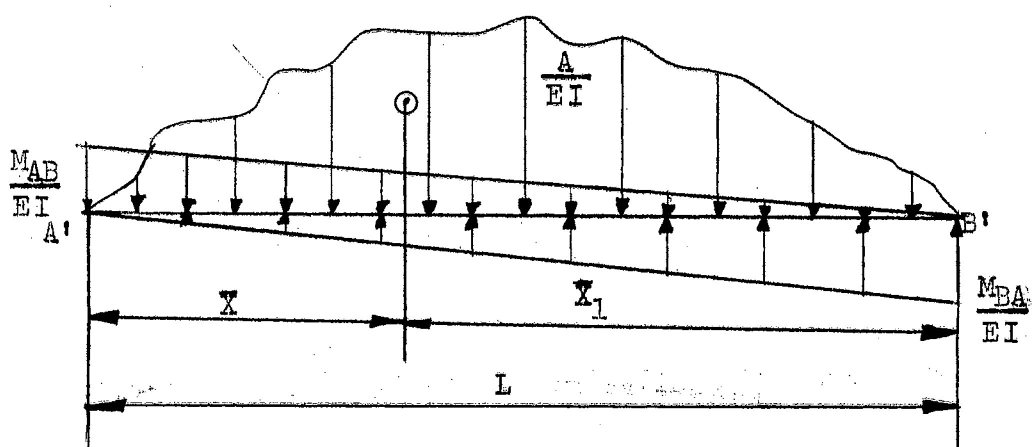


Fig. 8

$$\Sigma M_A = 0$$

$$-\frac{M_{BA}}{EI}\frac{L}{2}\frac{2L}{3} + \frac{M_{AB}}{EI}\frac{L}{2}\frac{L}{3} + \frac{A\bar{x}}{EI} = 0 \qquad (67)$$

$$\Sigma M_B = 0$$

$$-\frac{M_{AB}}{EI}\frac{L}{2}\frac{2L}{3} + \frac{M_{BA}}{EI}\frac{L}{2}\frac{L}{3} - \frac{A\bar{x}_1}{EI} = 0 \qquad (68)$$

Multiply Eq. 68 by 2

$$-\frac{2M_{AB}}{EI}\frac{L}{2}\frac{2L}{3} + \frac{2M_{BA}}{EI}\frac{L}{2}\frac{L}{3} - \frac{2A\bar{x}_1}{EI} = 0 \qquad (69)$$

Add Eq. 69 and Eq. 67.

$$\frac{M_{AB}L^2}{EI}\left[\frac{1}{6} - \frac{4}{6}\right] + \frac{A\bar{x}}{EI} - \frac{2A\bar{x}_1}{EI} = 0 \qquad (70)$$

Solving for M_{AB}.

$$M_{AB} = \frac{2}{L^2}(A\bar{x} - 2A\bar{x}_1) = \frac{2A}{L^2}(\bar{x} - 2\bar{x}_1) \qquad (71)$$

Substitute Eq. 71 in Eq. 67 and solve for M_{BA}.

$$\frac{M_{BA}}{EI}\frac{L^2}{3} = \frac{L^2}{6EI} \times \frac{2A}{L^2}(\bar{x} - 2\bar{x}_1) + \frac{A\bar{x}}{EI} = A\bar{x} - 2\bar{x}_1 A + 3\bar{x}A \qquad (72)$$

$$M_{BA} = \frac{2A}{L^2}(2\bar{x} - \bar{x}_1) \qquad (73)$$

Substitute M_{BA} and M_{AB} in Eq. 66.

$$M_{AB} = \frac{2AL}{L^2} x \frac{2L_{2B}\left[2(\bar{x} - 2\bar{x}_1) - (2\bar{x} - \bar{x}_1)\right] + L\left[2(2\bar{x} - \bar{x}_1) - (\bar{x} - 2\bar{x}_1)\right]}{4L_{2A}L_{2B} - L^2}$$

$$= \frac{2A}{L} x \frac{2L_{2B}(2\bar{x} - 4\bar{x}_1 - 2\bar{x} + \bar{x}_1) + L(4\bar{x} - 2\bar{x}_1 - \bar{x} + 2\bar{x}_1)}{4L_{2A}L_{2B} - L^2}$$

$$= \frac{2A}{L} x \frac{2L_{2B}(-3\bar{x}_1) + L(3\bar{x})}{4L_{2A}L_{2B} - L^2} \qquad (74)$$

$$M_{AB} = -\frac{6A}{L} x \frac{2L_{2B}\bar{x}_1 - L\bar{x}}{4L_{2A}L_{2B} - L^2} \qquad \text{(Rathbun's Eq. 8)} \qquad \text{(VIII)}$$

The moment of the left end of a beam is negative in both the beam and clockwise-positive convention.

$$M_{BA} = \frac{1}{6}\left[\frac{M_{AB}(2 \times 6L^2 - 12L_{2A}L)}{4L_{2A}L_{2B} - L^2} + \frac{M_{BA}(2 \times 12L_{2A}L - 6L^2)}{4L_{2A}L_{2B} - L^2}\right]$$

$$= L x \frac{M_{AB}(2L - 2L_{2A}) + (4L_{2A} - L)M_{BA}}{4L_{2A}L_{2B} - L^2}$$

$$= L x \frac{2L_{2A}(2M_{BA} - M_{AB}) + L(2M_{AB} - M_{BA})}{4L_{2A}L_{2B} - L^2} \qquad (75)$$

Substituting the values for M_{AB} and M_{BA} from Eq. 71 and Eq. 73 in Eq. 75

$$M_{BA} = \frac{L2A}{L^2} x \frac{2L_{2A}\left[2(2\bar{x} - \bar{x}_1) - (\bar{x} - 2\bar{x}_1)\right] + L\left[2(\bar{x} - 2\bar{x}_1) - (2\bar{x} - \bar{x}_1)\right]}{4L_{2A}L_{2B} - L^2}$$

$$= \frac{2A}{L} \times \frac{2L_{2A}(4\bar{x} - 2\bar{x}_1 - \bar{x} + 2\bar{x}_1) + L(2\bar{x} - 4\bar{x}_1 - 2\bar{x} + \bar{x}_1)}{4L_{2A}L_{2B} - L^2}$$

$$= \frac{2A}{L} \times \frac{2L_{2A}3\bar{x} + L(-3\bar{x}_1)}{4L_{2A}L_{2B} - L^2} \tag{76}$$

$$M_{BA} = \frac{6A}{L} \times \frac{2L_{2A}\bar{x} - L\bar{x}_1}{4L_{2A}L_{2B} - L^2} \tag{77}$$

Using the beam convention.

$$M_{BA} = -\frac{6A}{L} \times \frac{2L_{2B}\bar{x} - L\bar{x}_1}{4L_{2A}L_{2B} - L^2} \quad \text{(Rathbun's Eq. 9.)} \tag{IX}$$

Comparing the last terms of Rathbun's equations 13 and 14 with equations VIII and 77 above shows a difference in signs. This is to be expected since Rathbun uses the convention that counter-clockwise movement produces positive moment and Maugh uses the convention that clockwise movement produces positive moment.

The last term of Rathbun's equation 13 can be found by substituting the values of M_{AB} and M_{BA} from equations 15' and 18' in equation 66 as follows:

$$M_{AB} = \frac{L2A}{L^2} \times \frac{2L_{2B}\left[2(2\bar{x}_1 - \bar{x}) - (-)(2\bar{x} - \bar{x}_1)\right] + L\left[2(-)(2\bar{x} - \bar{x}_1) - (2\bar{x}_1 - \bar{x})\right]}{4L_{2A}L_{2B} - L^2}$$

$$= \frac{2A}{L} \times \frac{2L_{2B}(4\bar{x}_1 - 2\bar{x} + 2\bar{x} - \bar{x}_1) + L(-4\bar{x} + 2\bar{x}_1 - 2\bar{x}_1 + \bar{x})}{4L_{2A}L_{2B} - L^2}$$

$$= \frac{6A}{L} \times \frac{2L_{2B}\bar{x}_1 - L\bar{x}}{4L_{2A}L_{2B} - L^2} \tag{66'}$$

The last term of Rathbun's equation 14 is found similarly by substituting M_{AB} and M_{BA} from equations 15' and 18' in equation 75.

APPLICATION

APPLICATION

A. The Rigid Frame.

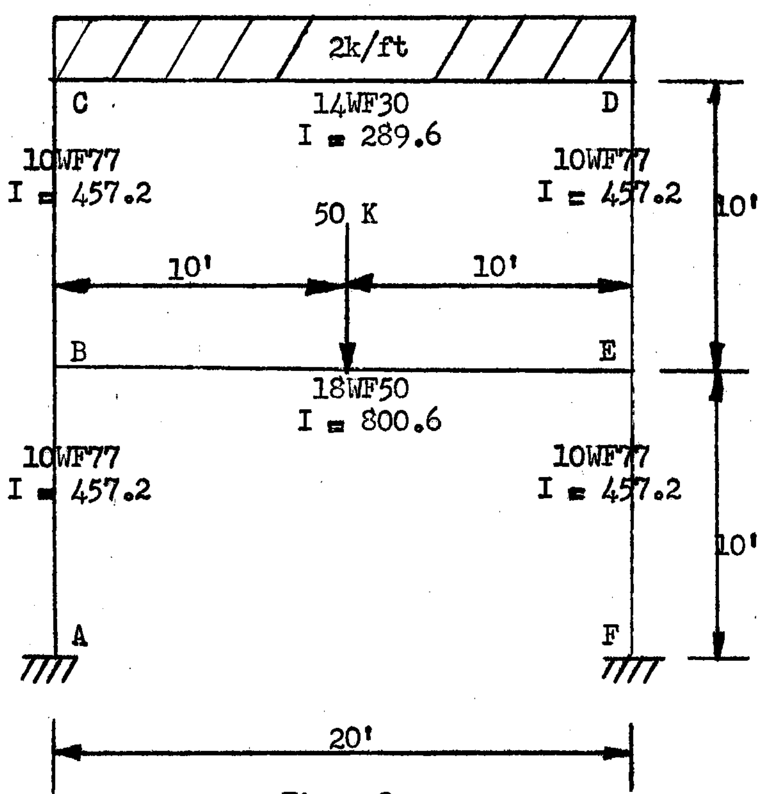


Fig. 9

Joints B, C, D, and E are rigid connections and the bent is rigidly anchored at A and F as shown in details A, B, and C, 7/8" rivets are used throughout.

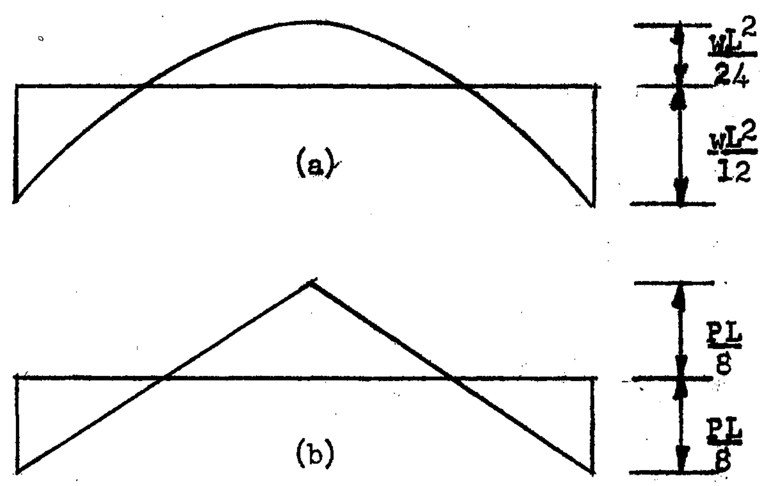


Fig. 10

The moment diagram for complete fixity is shown in Fig. 10(a) and (b). Since joints B, C, D and E are rigid but not fixed they will rotate until part of the fixed end moment of the girder is taken by the columns which reduces the negative moment at the girder ends. Since this is true, members must be designed so as to withstand the maximum moment that will occur. It can be seen that the maximum moment for a uniformly loaded member will be $\frac{wL^2}{12}$ while for a concentrated load some assumption must be made as both positive and negative moments are equal for the fixed end condition. Therefore, assume a total maximum positive moment to be $\frac{PL}{7}$.

The bent is designed by use of these principles as follows:

(CD) $$M_{CD} = \frac{wL^2}{12} = wL^2{}''K$$

$$\frac{I}{c} = \frac{M}{s} = \frac{wL^2}{20} = \frac{2(20)^2}{20} = 40''^3 \qquad \text{Try 14WF30; } \frac{I}{c} = 41.8''^3$$

$$\text{Total } \frac{I}{c} = 40.6''^3 \qquad \text{Then 14WF30 is sufficient}$$

(BE) $$M_{BE} = \frac{PL12''K}{7}$$

$$\frac{I}{c} = \frac{M}{s} = \frac{50(20)12}{(20)7} = 85.8''^3 \qquad \text{Try 18WF50; } \frac{I}{c} = 89.0''^3$$

$$\text{Total } \frac{I}{c} = 86.8''^3 \qquad \text{Then 18WF50 is sufficient}$$

(Columns)

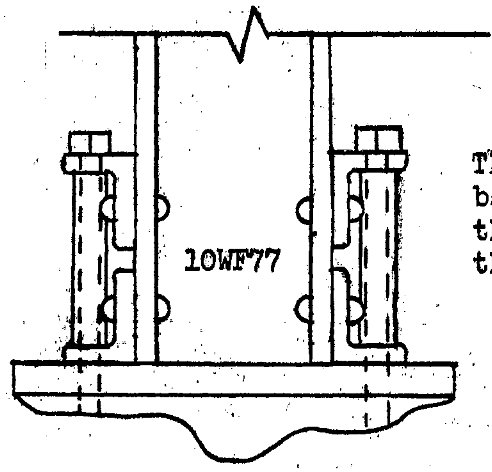
The maximum loaded column is BA and FE.

Try 10WF77

$$1 = \frac{P}{As} + \frac{Mc}{Is} = \frac{45}{22.67(15.96)} + \frac{50(20)1.5}{20(86.1)} = .124 + .87 = .994$$

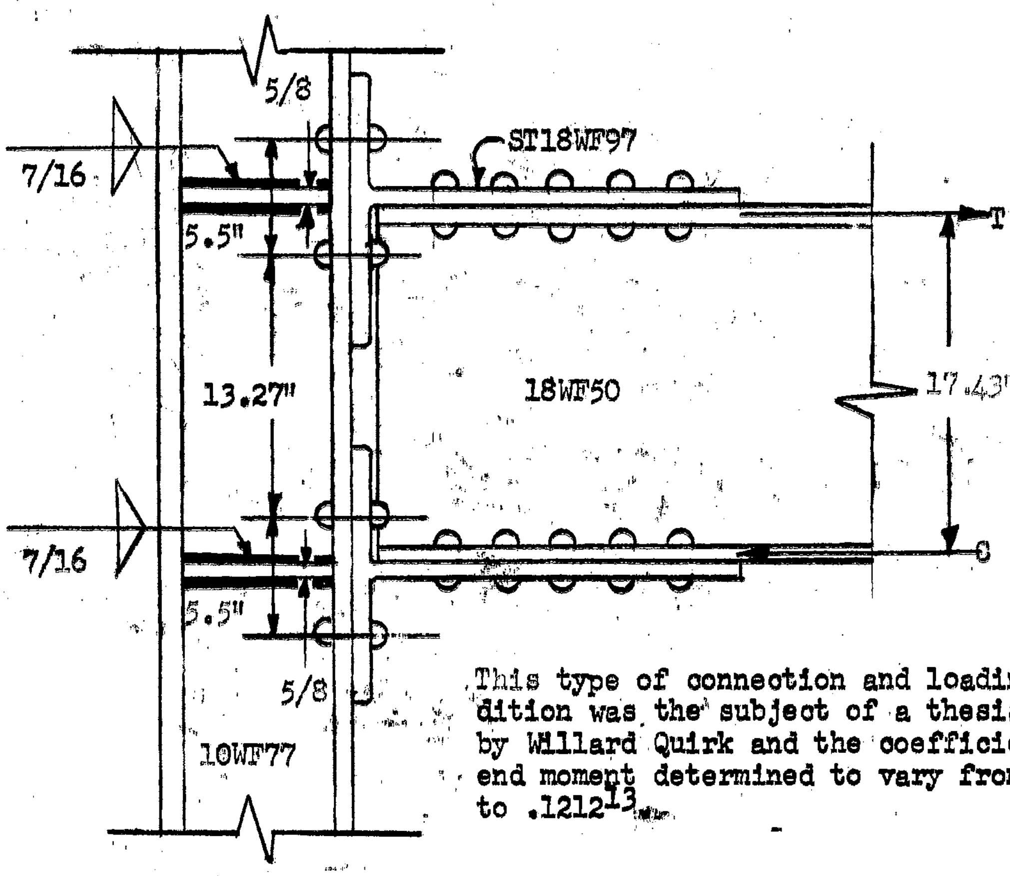
Then 10WF77 is sufficient

Detail "A"



The bent may be rigidly anchored at the base as shown, however, the design of this connection is beyond the scope of this paper and is not included.

Detail "B"



This type of connection and loading condition was the subject of a thesis research by Willard Quirk and the coefficient for end moment determined to vary from .1165 to .1212[13].

---

13 Willard Raymond Quirk, "The Contribution of Column Flanges to the Rotation of Partially Restrained Riveted Connections in Building Frames". Thesis, Stillwater, 1948.

$$V = 25K$$

$$M = 106.11(12) = 1273.3''K$$

$$T = C = \frac{1273.3}{17.43} = 73.0''K$$

Connection to beam

$$\text{No. rivets} = \frac{P}{R} = \frac{73.0}{9.02} = 8.1, \quad \text{use } 10$$

Connection moment

$$M = \frac{T}{2}\,\frac{g}{4} = \frac{Tg}{8}\ ;\ \frac{I}{c} = \frac{M}{s}\ ;\ \frac{I}{c} = \frac{\frac{bh^3}{12}}{c} = \frac{\frac{bt^3}{12}}{\frac{t}{2}} = \frac{bt^2}{6} = \frac{M}{s}$$

$$t = \sqrt{\frac{6M}{sb}} = \sqrt{\frac{6 \times 73 \times 5.5}{8 \times 20 \times 10.62}} = 1.19'' \text{ required, } 1.26'' \text{ furnished.}$$

Connection to Column (1/7 assumption)

$$N^{14} = \frac{1}{ans}\sqrt{\frac{Mans + 2pP^2}{2p}}$$

$$= \frac{1}{.60 \times 2 \times 20}\sqrt{\frac{1273.3 \times .60 \times 2 \times 20 + 2 \times 8.09 \times 25^2}{2 \times 8.09}}$$

$$= 2.09 \text{ required, } 4 \text{ vertical rows furnished.}$$

Detail "C"

Using the proper values the design of this connection is identical with Detail "B", therefore, it is not included.

---

14 J. E. Lothers, Design in Structural Steel.

In the given bent

$$\theta_a = \theta_f = 0$$

$$\theta_b = ? \quad \theta_c = ? \quad \theta_d = ? \quad \theta_e = ?$$

$$\frac{\Delta_{CD}}{L} = \frac{\Delta_{BE}}{L} = 0$$

$$\frac{\Delta_{BC}}{L} = \frac{\Delta_{DE}}{L} = R_2 = ?$$

$$\frac{\Delta_{AB}}{L} = \frac{\Delta_{FE}}{L} = R_1 = ?$$

(A) $\sum M_B = M_{BA} + M_{BE} + M_{BC} = 0$

(B) $\sum M_C = M_{CB} + M_{CD} = 0$

(C) $\sum M_D = M_{DC} + M_{DE} = 0$

(D) $\sum M_E = M_{ED} + M_{EB} + M_{EF} = 0$

(E) $\sum$ M 1st story $= \dfrac{M_{AB} + M_{BA}}{L_{AB}} + \dfrac{M_{FE} + M_{EF}}{L_{EF}} = 0; \quad L_{AB} = L_{EF}$

(F) $\sum$ M 2nd story $= \dfrac{M_{BC} + M_{CB}}{L_{CB}} + \dfrac{M_{ED} + M_{DE}}{L_{DE}} = 0; \quad L_{CB} = L_{DE}$

Writing the slope deflection equations:

$$M_{AB} = \frac{2EI_{AB}}{L}(\theta_b - 3R_1) \quad (78)$$

$$M_{BA} = \frac{2EI_{AB}}{L}(2\theta_b - 3R_1) \quad (79)$$

$$\frac{I_{AB}}{L} = K_2$$

$$M_{BE} = \frac{2EI_{BE}}{L}(2\theta_b + \theta_e) + 125 \quad (80)$$

$$M_{EB} = \frac{2EI_{BE}}{L}(2\theta_e + \theta_b) - 125 \quad (81)$$

$$\frac{I_{BE}}{L} = K_3$$

$$M_{BC} = \frac{2EI_{CB}}{L}(2\theta_b + \theta_c - 3R_2) \quad (82)$$

$$M_{CB} = \frac{2EI_{CB}}{L}(2\theta_c + \theta_b - 3R_2) \quad (83)$$

$$\frac{I_{CB}}{L} = K_2$$

$$M_{CD} = \frac{2EI_{CD}}{L}(2\theta_c + \theta_d) + 66.666 \tag{84}$$

$$M_{DC} = \frac{2EI_{CD}}{L}(2\theta_d + \theta_c) - 66.666 \tag{85}$$

$$\frac{I_{CD}}{L} = K_1$$

$$M_{DE} = \frac{2EI_{DE}}{L}(2\theta_d + \theta_e - 3R_2) \tag{86}$$

$$M_{ED} = \frac{2EI_{DE}}{L}(2\theta_e + \theta_d - 3R_2) \tag{87}$$

$$\frac{I_{DE}}{L} = K_2$$

$$M_{EF} = \frac{2EI_{EF}}{L}(2\theta_e - 3R_1) \tag{88}$$

$$M_{FE} = \frac{2EI_{EF}}{L}(\theta_e - 3R_1) \tag{89}$$

$$\frac{I_{EF}}{L} = K_2$$

Since all members are of the same material, E may be neglected.

Substituting the above values in A through F,

(A') $2K_2(2\theta_b - 3R_1) + 2K_3(2\theta_b + \theta_e) + 125 + 2K_2(2\theta_b + \theta_c - 3R_2) = 0$

(B') $2K_2(2\theta_c + \theta_b - 3R_2) + 2K_1(2\theta_c + \theta_d) + 66.666 = 0$

(C') $2K_1(2\theta_d + \theta_c) - 66.666 + 2K_2(2\theta_d + \theta_e - 3R_2) = 0$

(D') $2K_2(2\theta_e + \theta_d - 3R_2) + 2K_3(2\theta_e + \theta_b) - 125 + 2K_2(2\theta_e - 3R_1) = 0$

(E') $2K_2(\theta_b - 3R_1) + 2K_2(2\theta_b - 3R_1) + 2K_2(\theta_e - 3R_1) + 2K_2(2\theta_e - 3R_1) = 0$

(F') $2K_2(2\theta_b + \theta_c - 3R_2) + 2K_2(2\theta_c + \theta_b - 3R_2) + 2K_2(2\theta_e + \theta_d - 3R_2) + 2K_2(2\theta_d + \theta_e - 3R_2) = 0$

$$K_1 = \frac{I_{CD}}{L_{CD}} = \frac{289.6}{20} = 14.48$$

$$K_2 = \frac{I_{col}}{L_{col}} = \frac{457.2}{10} = 45.72$$

$$K_3 = \frac{I_{BE}}{L_{BE}} = \frac{800.6}{20} = 40.03$$

Substituting the values for K_1, K_2, and K_3 in A' through F' and simplifying

(A") $\theta_b(8K_2 + 4K_3) + \theta_c(2K_2) + \theta_e(2K_3) - 2K_2(3R_1 + 3R_2) + 125 = 0$

$4(91.44 + 40.03)\theta_b + 91.44\theta_c + 80.06\theta_e - 3(91.44)(R_1 + R_2) + 125 = 0$

(B") $\theta_b(2K_2) + \theta_c(4K_1 + 4K_2) + \theta_d(2K_1) - 3R_2(2K_2) + 66.66 = 0$

$91.44\theta_b + 4(60.20)\theta_c + 28.96(\theta_d) - 3(91.44)R_2 + 66.66 = 0$

(C") $\theta_c(2K_1) + \theta_d(4K_1 + 4K_2) + \theta_e(2K_2) - 2K_2 3R_2 - 66.66 = 0$

$28.96\theta_c + 4(60.20)\theta_d + 91.44\theta_e - (3)91.44R_2 - 66.66 = 0$

(D") $\theta_b(2K_3) + \theta_d(2K_2) + \theta_e(8K_2 + 4K_3) - (3R_1 + 3R_2)2K_2 - 125 = 0$

$80.06\theta_b + 91.44\theta_d + 4(91.44 + 40.03)\theta_e - 91.44(3R_1 + 3R_2) - 125 = 0$

(E") $\theta_b(6K_2) + \theta_e(6K_2) - 3R_1(8K_2) = 0$

$3(91.44)\theta_b + 3(91.44)\theta_e - 12(91.44)R_1 = 0$

(F") $\theta_b(6K_2) + \theta_c(6K_2) + \theta_d(6K_2) + \theta_e(6K_2) - 3R_2(8K_2) = 0$

$3(91.44)\theta_b + 3(91.44)\theta_c + 3(91.44)\theta_d + 3(91.44)\theta_e - 12(91.44)R_2 = 0$

This gives six equations with six unknowns. To evaluate these unknowns the Gauss, or "tabulation"[15] method is used for obtaining approximate values. These values are then used in the Iteration[16] method to obtain more accurate values.

---

15 John R. Parcel and George Alfred Maney, <u>Statically Indeterminate Structures</u>. (New York, 1947), pp. 224, 225, 232 and 235.
Hale Sutherland and Harry Lake Bowman, <u>Structural Theory</u>. (New York, 1944), p. 235.

16 <u>Ibid</u>., p. 165 (Parcel & Maney)
<u>Ibid</u>., p. 235 (Sutherland & Bowman)

Table No. 1
Rigid Connections

No.	Oper.	θ_b	θ_c	θ_d	θ_e	R_1	R_2	Const.	Check Term
1	A"	525.88	91.44		80.06	-274.32	-274.32	-125.00	23.74
2	B"	91.44	240.80	28.96			-274.32	- 66.66	20.22
3	C"		28.96	240.80	91.44		-274.32	66.66	153.54
4	D"	80.06		91.44	525.88	-274.32	-274.32	125.00	273.74
5	E"	274.32			274.32	-1097.28			-548.64
6	F"	274.32	274.32	274.32	274.32		-1097.28		0
1'	1÷525.88	1.00	.17		.15	- .52	- .52	- .24	.04
2'	2÷ 91.44	1.00	2.63	.32			-3.00	- .72	.23
3			28.96	240.80	91.44		-274.32	66.66	153.54
4'	4÷ 80.06	1.00		1.14	6.56	-3.42	-3.42	1.56	3.42
5'	5÷274.32	1.00			1.00	-4.00			-2.00
6'	6÷274.32	1.00	1.00	1.00	1.00		-4.00		0
7	2'-1'		2.46	.32	- .15	.52	-2.48	- .48	.19
3			28.96	240.80	91.44		-274.32	66.66	153.54
8	2'-4'		2.63	- .82	- 6.56	3.42	.42	- 2.28	- 3.19
9	2'-5'		2.63	.32	- 1.00	4.00	- 3.00	- .72	2.23
10	2'-6'		1.63	- .68	- 1.00		1.00	- .72	.23
7'	7÷ 2.46		1.00	.13	- .06	.21	- 1.00	- .20	.08
3'	3÷ 28.96		1.00	8.31	3.16		- 9.47	2.30	5.30
8'	8÷ 2.63		1.00	- .31	-2.49	1.30	.16	- .87	- 1.21
9'	9÷ 2.63		1.00	.12	- .38	1.52	- 1.14	- .27	.85
10'	10÷ 1.63		1.00	- .42	- .61		.61	- .44	.14
11	3'-7'			8.18	3.22	- .21	- 8.47	2.50	5.22
12	3'-8'			8.62	5.65	-1.30	- 9.63	3.17	6.51
13	3'-9'			8.19	3.54	-1.52	- 8.33	2.57	4.45
14	3'-10'			8.73	3.77		-10.08	2.74	5.16

11'	11÷ 8.18			1.00	.39	- .02	- 1.03	.30	.64
12'	12÷ 8.62			1.00	.66	- .15	- 1.12	.36	.75
13'	13÷ 8.19			1.00	.43	- .18	- 1.02	.31	.54
14'	14÷ 8.73			1.00	.43		- 1.15	.31	.59
15	12'-11'				.27	- .13	- .09	.06	.12
16	12'-13'				.23	.03	- .10	.05	.21
17	12'-14'				.23	- .15	.03	.05	.16
15'	15÷ .27				1.00	- .48	- .33	.22	.41
16'	16÷ .23				1.00	.13	- .43	.22	.92
17'	17÷ .23				1.00	- .65	.13	.22	.70
18	17'-15'					- .17	.46		.29
19	17'-16'					- .78	.56		- .22
18'	18÷ .17					-1.00	2.70		1.70
19'	19÷ .78					-1.00	.72		- .28
20	18'-19'						1.98		1.74
Gauss Solution		- .22	- .22	.22	.22	0	0		
		- .232	- .217	.219	.234	1st	Approximation		
		- .236	- .214	.218	.235	2nd	"		
		- .236	- .213	.213	.236	3rd	"		
		- .236	- .212	.212	.236	4th	"		
		- .236	- .212	.212	.236	5th	"		

Substituting θ_b, θ_c, θ_d, θ_e, R_1 and R_2 from Table I and the proper K values in equations 78 through 89 gives the end moments for all members of the bent as follows:

$$M_{AB} = 2K_2(\theta_b - 3R_1) = 2 \times 45.72(-.236) = -21.58'K \quad (78')$$

$$M_{BA} = 2K_2(2\theta_b - 3R_1) = 2 \times 45.72(-.236 \times 2) = -43.16'K \quad (79')$$

$$M_{BE} = 2K_3(2\theta_b + \theta_e) + 125 = 2 \times 40.03(-.236 \times 2 + .236) + 125 = 106.11'K \quad (80')$$

$$M_{EB} = 2K_3(2\theta_e + \theta_b) - 125 = 2 \times 40.03(.236 \times 2 - .236) - 125 = -106.11'K \quad (81')$$

$$M_{BC} = 2K_2(2\theta_b + \theta_c - 3R_2) = 2 \times 45.72(-.236 \times 2 - .212) = -62.54'K \quad (82')$$

$$M_{CB} = 2K_2(2\theta_c + \theta_b - 3R_2) = 2 \times 45.72(-.212 \times 2 - .236) = -60.35'K \quad (83')$$

$$M_{CD} = 2K_1(2\theta_c + \theta_d) + 66.66 = 2 \times 14.48(-.212 \times 2 + .212) + 66.66 = 60.52'K \quad (84')$$

$$M_{DC} = 2K_1(2\theta_d + \theta_c) - 66.66 = 2 \times 14.48(.212 \times 2 - .212) - 66.66 = -60.52'K \quad (85')$$

$$M_{DE} = 2K_2(2\theta_d + \theta_e - 3R_2) = 2 \times 45.72(.212 \times 2 + .236) = 60.35'K \quad (86')$$

$$M_{ED} = 2K_2(2\theta_e + \theta_d - 3R_2) = 2 \times 45.72(.236 \times 2 + .212) = 62.54'K \quad (87')$$

$$M_{EF} = 2K_2(2\theta_e - 3R_1) = 2 \times 45.72(.236 \times 2) = 43.16'K \quad (88')$$

$$M_{FE} = 2K_2(\theta_e - 3R_1) = 2 \times 45.72(.236) = 21.58'K \quad (89')$$

Substituting these moments in equations A through F:

(A''') $M_{BA} + M_{BE} + M_{BC} = -43.16 + 106.11 - 62.54 = .41'K \; (4.92''K)$

(B''') $M_{CB} + M_{CD} = -60.35 + 60.52 = .17'K \; (2.04''K)$

(C''') $M_{DC} + M_{DE} = -60.52 + 60.35 = -.17'K \; (2.04''K)$

(D''') $M_{ED} + M_{EB} + M_{EF} = 62.54 - 106.11 + 43.16 = - .41'K$ (4.92"K)

(E''') $M_{AB} + M_{BA} + M_{FE} + M_{EF} = - 21.58 - 43.16 + 21.58 + 43.16 = 0$

(F''') $M_{BC} + M_{CB} + M_{ED} + M_{DE} = - 62.54 - 60.35 + 62.54 + 60.35 = 0$

The errors in A''', B''', C''' and D''' can be reduced by carrying the θ values to four or five decimal places but this is not justified since the I-values of the members are taken from the AISC handbook and are given only to the first decimal place.

It was stated above, since all members were of the same material, E could be neglected. If E were not neglected and all members still of the same material equation B' would be:

(a) $2K_2E(2\theta_c + \theta_b - 3R_2) + 2K_1E(2\theta_c + \theta_d) + 66.66 = 0$

When this equation is substituted into the Gauss solution it takes the form:

(b) $2K_2E(2\theta_c + \theta_b - 3R_2) + 2K_1E(2\theta_c + \theta_d) = - 66.66$

Then the unknown angles and deflections are $E\theta$ and $E\Delta$ = Constant and these constants substituted into the slope deflection equations give the end moments.

But B' substituted in the Gauss solution has the form:

(c) $2K_2(2\theta_c + \theta_b - 3R_2) + 2K_1(2\theta_c + \theta_d) = - 66.66$

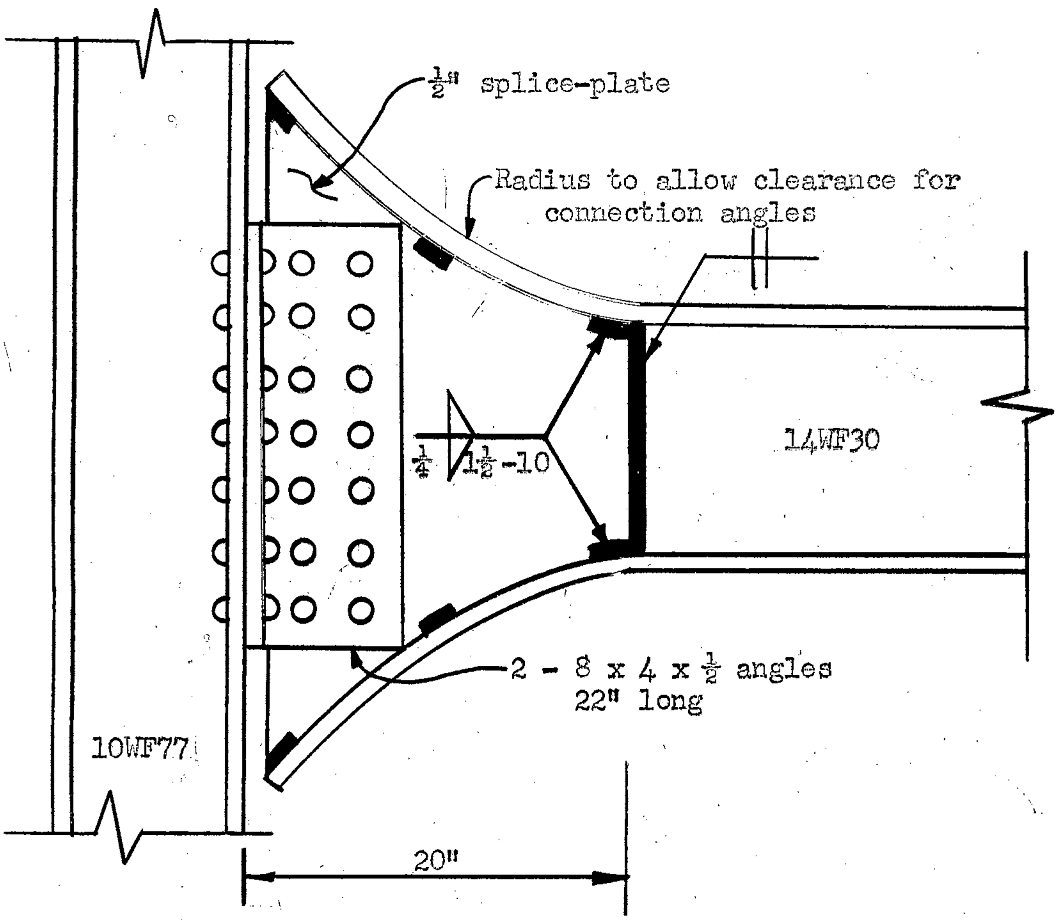
Then for equation c to be valid, θ and Δ must be multiplied by E and although it was neglected in the calculations it must be remembered that the values of the unknowns are understood to be multiplied by E and to obtain their true values must be divided by E.

B. <u>The Semi-Rigid Frame</u> (Rathbun)

Detail "B"

Using the proper values the design of this connection is identical with that at "C" and Z is found to be $\frac{.0089}{E}$.

Detail "C"



Connection to column (1/6 assumption)

$$N^{17} = \frac{1}{ans}\sqrt{\frac{18\ M_{ans} + 5pP^2}{5p}}$$

$$= \frac{1}{.60 \times 2 \times 20}\sqrt{\frac{18 \times 726.2 \times .60 \times 2 \times 20 + 5 \times 3 \times 20^2}{5 \times 3}} = \frac{1}{24}\sqrt{21,360}$$

$= 6.08$ or 7, angle must be 22" long.

[17] J. E. Lothers, <u>Design in Structural Steel</u>

Connection to splice-plate.

$$N^{18} = \frac{1}{2Rpn}\sqrt{2P^2p^2 + 24MRpn}$$

$$= \frac{1}{2 \times 17.5 \times 3 \times 2}\sqrt{2 \times 20^2 \times 3^2 + 24 \times 726.2 \times 17.5 \times 3 \times 2}$$

$$= \frac{1}{210}\sqrt{1,839,200} = 6.45 \text{ or } 7, \text{ angle must be } 22'' \text{ long.}$$

$$n^{19} = \frac{6g(2g + g_1)}{t(4g + g_1)} = \frac{6 \times 2(2 \times 2 + 2.5)}{.5(4 \times 2 + 2.5)} = 14.88''$$

$$y^{19} = \frac{h(nb - \sqrt{nbt})}{nb - t} = \frac{22(14.88 \times 4 - \sqrt{14.88 \times 4 \times .5})}{14.88 \times 4 - .5} = 20.2''$$

$$Z^{19} = \frac{6g^3}{Eht^3y^2} \times \frac{g + g_1}{4g + g_1} = \frac{6 \times 2^3}{E \times 22 \times .5^3 \times 20.2^2} \times \frac{2 + 2.5}{4 \times 2 + 2.5} = \frac{.0183}{E}$$

Splice-plate to beam-web --- a welded connection.

The web thickness **as required** by moment:

$$T^{20} = \frac{1}{2sh^2}\left[\frac{A_wM}{A_f} + \sqrt{\frac{A_wM^2}{A_f} + 4(hV)^2}\right]$$

$$A_f = 2.58 + \frac{.270 \times 13.094}{6} \text{ (1/6 web area resists moment)}$$

$$= 3.169 \text{ sq. in.}$$

$A_w = 3.53$ sq. in.

Use $h = 13''$

$$T = \frac{1}{2 \times 20 \times 13^2}\left[\frac{3.534 \times 672.8}{3.169} + \sqrt{\frac{3.534 \times 672.8}{3.169} + 4(13 \times 16.66)^2}\right]$$

$$= \frac{1}{6760}(1618) = .239'', \ .270'' \text{ furnished.}$$

---

[18] J. E. Lothers, Design in Structural Steel

[19] J. E. Lothers, Transactions of the American Society of Civil Engineers, Vol. 116, p. 485.

[20] J. E. Lothers, Design in Structural Steel.

The web thickness as required by shear:

$$T^{21} = \frac{1}{2Vh^2}\sqrt{\frac{A_w M^2}{A_f} + 4(hV)^2}$$

$$= \frac{1}{2 \times 16.66 \times 13^2}\sqrt{\frac{3.534 \times 672.8^2}{3.169} + 4(13 \times 16.66)^2}$$

$$= \frac{1}{5640}(868) = .154'', \ .270'' \text{ furnished.}$$

Splice-plate to web --- a welded connection.

The web of the 14WF30 beam is cut out 20" from the face of the column and welded to the $\frac{1}{2}$" splice plate, as shown. This constitutes a haunched beam, the analysis of which adds nothing to this problem and is not included; rather the flanges are assumed to extend to the column face and a weld is designed for this condition and applied to the actual case.

$$I = \frac{2bt_f^3}{12} + 2btd^2 + \frac{t_w d^3}{3}$$

$$= \frac{2 \times 6.733 \times .383^3}{12} + 2 \times 6.733 \times .383 \times 6.738^2 + \frac{.5 \times 13.094^3}{3}$$

$$= 607''^4$$

$$P = \frac{19.2 D \ell I}{V \Sigma A\bar{y}} = \frac{19.2 \times .25 \times 1.5 \times 607}{16.66 \times 6.733 \times .383 \times 6.738} = 15.15''$$

Use 3 - $\frac{1}{4}$ x $1\frac{1}{2}$" welds as shown.

---

[21] J. E. Lothers, Design in Structural Steel.

ABC and FED are single members to which the girders CD and BE are connected with web angle connections. Then M_{BE}, M_{EB}, M_{CD}, and M_{DC} are the only equations containing properties of these elastic connections.

$$M_{BE} = 6EI\,\frac{2L_{2E}(\theta_b - R) + L(\theta_e - R)}{4L_{2B}L_{2E} - L^2} + \frac{6A}{L} \times \frac{2L_{2E}\bar{x}_1 - L\bar{x}}{4L_{2B}L_{2E} - L^2} \qquad (90)$$

$$M_{EB} = 6EI\,\frac{2L_{2B}(\theta_e - R) + L(\theta_b - R)}{4L_{2B}L_{2E} - L^2} - \frac{6A}{L} \times \frac{2L_{2B}\bar{x} - L\bar{x}_1}{4L_{2B}L_{2E} - L^2} \qquad (91)$$

$$M_{CD} = 6EI\,\frac{2L_{2D}(\theta_c - R) + L(\theta_d - R)}{4L_{2C}L_{2D} - L^2} + \frac{6A}{L} \times \frac{2L_{2D}\bar{x}_1 - L\bar{x}}{4L_{2C}L_{2D} - L^2} \qquad (92)$$

$$M_{DC} = 6EI\,\frac{2L_{2C}(\theta_d - R) + L(\theta_c - R)}{4L_{2C}L_{2D} - L^2} - \frac{6A}{L} \times \frac{2L_{2C}\bar{x}_1 - L\bar{x}}{4L_{2C}L_{2D} - L^2} \qquad (93)$$

The units of L_{2B}, L_{2C}, L_{2D}, and L_{2E} are length. Therefore, let

$$\frac{I}{4L_{2B}L_{2E} - L^2} = K_4 \text{ and } \frac{I}{4L_{2C}L_{2D} - L^2} = K_5$$

also $4L_{2B}L_{2E} - L^2 = W_1$ and $4L_{2C}L_{2D} - L^2 = W_2$

But $R = 0$

$\bar{x} = \bar{x}_1$ for a uniform load

$\bar{x} = \bar{x}_1$ for a concentrated load at midspan

Substituting these values in equations 90 through 93 and letting $E = 1$:

$$M_{BE} = 6K_4(2L_{2E}\theta_b + L\theta_e) + \frac{6A}{L} \times \frac{(2L_{2E} - L)\bar{x}}{W_1} \qquad (90')$$

$$M_{EB} = 6K_4(2L_{2B}\theta_e + L\theta_b) - \frac{6A}{L} \times \frac{(2L_{2B} - L)\bar{x}}{W_1} \qquad (91')$$

$$M_{CD} = 6K_5(2L_{2D}\theta_c + L\theta_d) + \frac{6A}{L} \times \frac{(2L_{2D} - L)\bar{x}}{W_2} \tag{92'}$$

$$M_{DC} = 6K_5(2L_{2C}\theta_d + L\theta_c) - \frac{6A}{L} \times \frac{(2L_{2C} - L)\bar{x}}{W_2} \tag{93'}$$

Substitute M_{BE}, M_{EB}, M_{CD}, and M_{DC} in A, B, C and D. The slope deflection equations in terms of their elastic properties are:

$$\text{(H)} \quad 2K_2(2\theta_b - 3R_1) + 6K_4(2L_{2E}\theta_b + L\theta_e) + \frac{6A}{L} \times \frac{(2L_{2E} - L)\bar{x}}{W_1} + 2K_2(2\theta_b + \theta_c - 3R_2) = 0$$

$$\text{(J)} \quad 2K_2(2\theta_c + \theta_b - 3R_2) + 6K_5(2L_{2D}\theta_c + L\theta_d) + \frac{6A}{L} \times \frac{(2L_{2D} - L)\bar{x}}{W_2} = 0$$

$$\text{(K)} \quad 6K_5(2L_{2C}\theta_d + L\theta_c) - \frac{6A}{L} \times \frac{(2L_{2C} - L)\bar{x}}{W_2} + 2K_2(2\theta_d + \theta_e - 3R_2) = 0$$

$$\text{(L)} \quad 2K_2(2\theta_e + \theta_d - 3R_2) + 6K_4(2L_{2B}\theta_e + L\theta_b) - \frac{6A}{L} \times \frac{(2L_{2B} - L)\bar{x}}{W_1} + 2K_2(2\theta_e - 3R_1) = 0$$

$$L_{2B} = L_{2E} = L + 3EIZ = 20(12) + 3E800.6\,\frac{(.0089)}{E} = 261.3''$$

$Z_E = Z_B = Z$ – (Z, is in inches, therefore, all dimensions of length are converted to inch units)

$$L_{2C} = L_{2D} = L + 3EIZ = 20(12) + 3E289.6\,\frac{(.0183)}{E} = 255.9''$$

$Z_C = Z_D = Z$

$$K_4 = \frac{800.6}{4(261.3)^2 - \overline{12(20)}^2} = .00372$$

$$K_5 = \frac{289.6}{4(255.9)^2 - \overline{12(20)}^2} = .00142$$

$$W_1 = 4(261.3)^2 - \overline{12(20)}^2 = 215{,}000$$

$$W_2 = 4(255.9)^2 - \overline{12(20)}^2 = 204{,}500$$

$$\bar{x} = \frac{L}{2}$$

Substituting these vales in H, J, K, and L, changing K_1 and K_2 to inch units and simplifying:

$$(H') \quad \theta_b(8K_2 + 6K_4 2L_{2E}) + \theta_c(2K_2) + \theta_e(6LK_4) - 2K_2(3R_1 + 3R_2) + \frac{6A}{L} \times \frac{(2L_{2E} - L)\bar{x}}{W_1} = 0$$

$$4\left[\frac{91.44}{12} + 3(.00372)261.3\right]\theta_b + \frac{91.44}{12}\theta_c + 6(240).00372\theta_e$$

$$- \frac{(91.44)}{12}(3R_1 + 3R_2) + \frac{6(50)20(12)20(12)}{2(4)2} \times \frac{2(261.3) - 240}{(215{,}000)} = 0$$

$$4(10.54)\theta_b + 7.62\theta_c + 6(240).00372\theta_e - (7.62)(3R_1 + 3R_2)$$

$$+ \frac{6(50)20(12)20(12)282.6}{2(4)2 \qquad (215{,}000)} = 0$$

$$(J') \quad \theta_b(2K_2) + \theta_c(4K_2 + 6K_5 2L_{2D}) + \theta_d(6K_5L) - 2K_2(3R_2) + \frac{6A}{L} \times \frac{(2L_{2D} - L)\bar{x}}{W_2} = 0$$

$$\frac{91.44}{12}\theta_b + 2\left[\frac{91.44}{12} + 6(.00142)255.9\right]\theta_c + 6(.00142)240\theta_d$$

$$- \frac{91.44}{12}(3R_2) + \frac{6(2)20(20)12(20)12(2)}{2(8)3} \times \frac{2(255.9) - 240}{(204{,}500)} = 0$$

$$7.62\theta_b + 2(9.80)\theta_c + 6(.00142)240\theta_d - 7.62(3R_2)$$

$$+ \frac{6(2)20(20)12(20)12(2)271.8}{2(8)3 \qquad (204{,}500)} = 0$$

$$(K') \quad \theta_c(6K_5L) + \theta_d(6K_5 2L_{2C} + 4K_2) + \theta_e(2K_2) - 2K_2(3R_2) - \frac{6A}{L} \times \frac{(2L_{2C} - L)\bar{x}}{W_2} = 0$$

$$6(.00142)240\theta_c + 2\left[6(.00142)255.9 + \frac{91.44}{12}\right]\theta_d + \frac{91.44}{12}\theta_e$$

$$- \frac{91.44}{12}(3R_2) - \frac{6(2)20(20)12(20)12(2)}{2(8)3} \times \frac{2(255.9) - 240}{(204,500)} = 0$$

$$6(.00142)240\theta_c + 2(9.8)\theta_d + 7.62\theta_e - 7.62(3R_2)$$

$$- \frac{6(2)20(20)12(20)12(2)271.8}{2(8)3 \qquad (204,500)} = 0$$

$$(L') \quad \theta_b(6K_4L) + \theta_d(2K_2) + \theta_e(8K_2 + 6K_4 2L_{2B}) - 2K_2(3R_1 + 3R_2) - \frac{6A}{L} \times \frac{(2L_{2B} - L)\bar{x}}{W_1}$$

$$6(.00372)240\theta_b + \frac{91.44}{12}\theta_d + 4\left[\frac{91.44}{12} + 3(.00372)261.3\right]\theta_e$$

$$- \frac{91.44}{12}(3R_1 + 3R_2) - \frac{6(50)20(12)20(12)}{2(4)2} \times \frac{2(261.3) - 240}{(215,000)} = 0$$

$$6(.00372)240\theta_b + 7.62\theta_d + 4(10.54)\theta_e - 7.62(3R_1 + 3R_2)$$

$$- \frac{6(50)20(12)20(12)282.6}{2(4)2 \qquad (215,000)} = 0$$

Rewriting E" and F" in inch units

$$(M) \quad 3(7.62)\theta_b + 3(7.62)\theta_e - 7.62(12R_1) = 0$$

$$(N) \quad 3(7.62)\theta_b + 3(7.62)\theta_c + 3(7.62)\theta_d + 3(7.62)\theta_e - 7.62(12R_2) = 0$$

The unknowns are evaluated by the same method as the rigid connections.

Table No. 2
Semi-Rigid Connections

No.	Oper.	θ_b	θ_c	θ_d	θ_e	R_1	R_2	Const.	Check Term
1	H'	42.16	7.62		5.36	- 22.86	- 22.86	-1425.00	-1415.58
2	J'	7.62	19.60	2.04			- 22.86	- 760.00	- 753.60
3	K'		2.04	19.60	7.62		- 22.86	760.00	766.40
4	L'	5.36		7.62	42.16	- 22.86	- 22.86	1425.00	1434.42
5	M	22.86			22.86	- 91.44			- 45.72
6	N	22.86	22.86	22.86	22.86		- 91.44		0
1'	1÷42.16	1.00	.18		.12	- .54	- .54	- 33.80	- 33.58
2'	2÷ 7.62	1.00	2.57	.26			- 3.00	- 99.74	- 98.91
3			2.04	19.60	7.62		- 22.86	760.00	766.40
4'	4÷ 5.36	1.00		1.42	7.86	- 4.26	- 4.26	265.86	267.62
5'	5÷22.86	1.00			1.00	- 4.00			- 2.00
6'	6÷22.86	1.00	1.00	1.00	1.00		- 4.00		0
7	2'-1'		2.39	.26	- .12	.54	- 2.46	- 65.94	- 65.33
3			2.04	19.60	7.62		- 22.86	760.00	766.40
8	2'-4'		2.57	- 1.16	-7.86	4.26	1.26	- 365.60	- 366.53
9	2'-5'		2.57	.26	-1.00	4.00	- 3.00	- 99.74	- 96.91
10	2'-6'		1.57	- .74	-1.00		1.00	- 99.74	- 98.91
7'	7÷ 2.39		1.00	.10	- .05	.22	- 1.02	- 27.58	- 27.33
3'	3÷ 2.04		1.00	9.60	3.74		- 11.20	372.54	375.68
8'	8÷ 2.57		1.00	- .45	-3.06	1.66	.49	- 142.26	- 142.62
9'	9÷ 2.57		1.00	.10	- .38	1.55	- 1.16	- 38.80	- 37.68
10'	10÷ 1.57		1.00	- .47	- .64		.64	- 63.52	- 62.99
11	3'-7'			9.50	3.79	- .22	- 10.18	400.12	403.01
12	3'-8'			10.05	6.80	- 1.66	- 11.69	514.80	518.30
13	3'-9'			9.50	4.12	- 1.56	- 10.04	411.34	413.36
14	3'-10'			10.07	4.38		- 11.84	436.06	438.67

11'	11 ÷ 9.50			1.00	.40	- .02	- 1.07	42.12	42.43
12'	12 ÷ 10.05			1.00	.68	- .16	- 1.16	51.22	51.58
13'	13 ÷ 9.50			1.00	.43	- .16	- 1.06	43.30	43.51
14'	14 ÷ 10.07			1.00	.43		- 1.18	43.30	43.55
15	12'-11'				.28	- .14	- .09	9.10	9.15
16	12'-13'				.25		- .10	7.92	8.07
17	12'-14'				.25	- .16	.02	7.92	8.03
15'	15 ÷ .28				1.00	- .50	- .32	32.50	32.68
16'	16 ÷ .25				1.00		- .40	31.68	32.28
17'	17 ÷ .25				1.00	- .64	.08	31.68	32.12
18	17'-15'					- .14	.40	- .82	- .56
19	17'-16'					- .64	.48		- .16
18'	18 ÷ .14					-1.00	2.86	- 5.86	- 4.00
19'	19 ÷ .64					-1.00	.75		- .25
20	18'-19'						2.11	- 5.86	- 3.75
Gauss Solution		-38.89	-29.55	26.88	30.57	-2.08	-2.78		
		-29.71	-26.78	32.92	34.26	1.14	2.67	1st Approximation	
		-31.24	-26.94	31.37	34.16	.72	1.84	2nd	"
		-31.88	-27.50	30.50	33.72	.46	1.20	3rd	"
		-32.22	-28.02	29.98	33.38	.29	.78	4th	"
		-32.40	-28.40	29.66	33.13	.18	.50	5th	"
		-32.52	-28.64	29.44	32.97	.11	.31	6th	"
		-32.59	-28.81	29.31	32.87	.06	.19	7th	"
		-32.64	-28.92	29.22	32.80	.03	.16	8th	"
		-32.64	-28.94	29.22	32.77	.03	.07	9th	"
		-32.68	-29.03	29.13	32.74	.01	.04	10th	"
		-32.69	-29.05	29.11	32.72	0	.02	11th	"
		-32.70	-29.07	29.10	32.70		0	12th	"
		-32.70	-29.09	29.09	32.70			13th	"
		-32.70	-29.09	29.09	32.70			14th	"

The moment equations for the columns are identical with rigid connections except the angle changes have the values found in Table II. Moment equations of the bent with elastic connections and values of Table II are:

$$K_2 = \frac{45.72}{12} = 3.81 \text{ inch units}$$

$$M_{AB} = 2K_2(\theta_b - 3R_1) = 2 \times 3.81(-32.70) = -249.17''K \qquad (78'')$$

$$M_{BA} = 2K_2(2\theta_b - 3R_1) = 2 \times 3.81\ (-32.70 \times 2) = -498.34''K \qquad (79'')$$

$$M_{BE} = 6K_4(2L_{2E}\theta_b + L\theta_e) + \frac{6A}{L} \times \frac{(2L_{2E} - L)\bar{x}}{W_1} \qquad (90'')$$

$$= 6 \times .00372\ (-32.7 \times 2 \times 261.3 + 240 \times 32.7) + 1425$$

$$= 1425 - 206.26 = 1218.74''K$$

$$M_{EB} = 6K_4(2L_{2B}\theta_e + L\theta_b) - \frac{6A}{L} \times \frac{(2L_{2B} - L)\bar{x}}{W_1} \qquad (91'')$$

$$= 6 \times .00372\ (2 \times 261.3 \times 32.7 - 240 \times 32.7) - 1425$$

$$= -1425 + 206.26 = -1218.74''K$$

$$M_{BC} = 2K_2(2\theta_b + \theta_c - 3R_2) = 2 \times 3.81\ (-32.7 \times 2 - 29.09) \qquad (82'')$$

$$= -719.91''K$$

$$M_{CB} = 2K_2(2\theta_c + \theta_b - 3R_2) = 2 \times 3.81\ (-29.09 \times 2 - 32.7) \qquad (83'')$$

$$= -692.50''K$$

$$M_{CD} = 6K_5(2L_{2D}\theta_c + L\theta_d) + \frac{6A}{L} \times \frac{(2L_{2D} - L)\bar{x}}{W_2} \qquad (92'')$$

$$= 6 \times .00142\ (-29.09 \times 2 \times 255.9 + 240 \times 29.09) + 760$$

$$= 692.65''K$$

$$M_{DC} = 6K_5(2L_{2C}\theta_d + L\theta_c) - \frac{6A}{L} \times \frac{(2L_{2C} - L)\overline{x}}{W_2} \tag{93"}$$

$$= 6 \times .00142\ (29.09 \times 2 \times 255.9 - 29.09 \times 240) - 760$$

$$= -\ 692.65"K$$

$$M_{DE} = 2K_2(2\theta_d + \theta_e - 3R_2) = 2 \times 3.81(2 \times 29.09 + 32.7) \tag{86"}$$

$$= 692.50"K$$

$$M_{ED} = 2K_2(2\theta_e + \theta_d - 3R_2) = 2 \times 3.81\ (2 \times 32.7 + 29.09) \tag{87"}$$

$$= 719.91"K$$

$$M_{EF} = 2K_2(2\theta_e - 3R_1) = 2 \times 3.81\ (2 \times 32.7) \tag{88"}$$

$$= 498.34"K$$

$$M_{FE} = 2K_2(\theta_e - 3R_1) = 2 \times 3.81\ (32.7) \tag{89"}$$

$$= 249.17"K$$

Substituting these moments in equations A through F:

(A_s) $M_{BA} + M_{BE} + M_{BC} = -\ 498.34 + 1218.74 - 719.91 = .49"K$

(B_s) $M_{CB} + M_{CD} = -\ 692.50 + 692.65 = .15"K$

(C_s) $M_{DC} + M_{DE} = -\ 692.65 + 692.50 = -\ .15"K$

(D_s) $M_{ED} + M_{EB} + M_{EF} = 719.91 - 1218.74 + 498.34 = -\ .49"K$

(E_s) $M_{AB} + M_{BA} + M_{FE} + M_{EF} = -\ 249.17 - 498.34 + 249.17 + 498.34 = 0$

(F_s) $M_{BC} + M_{CB} + M_{ED} + M_{DE} = -\ 719.91 - 692.50 + 719.91 + 692.50 = 0$

C. The Semi-Rigid Frame. (Maugh)

As a check and further proof that Rathbun's and Maugh's equations are identical the bent solved by use of Rathbun's equations is solved using Maugh's equations and moment distribution. (Positive moment tends to rotate a member counter-clockwise).

For girder BE:

$$C_1 = \frac{12L_{2E}L}{4L_{2B}L_{2E} - L^2} = \frac{12(261.3)240}{4(261.3)^2 - (240)^2} = \frac{753,000}{215,000} = 3.5 \qquad (65a)$$

$$C_2 = \frac{6L^2}{4L_{2B}L_{2E} - L^2} = \frac{6(240)^2}{4(261.3)^2 - (240)^2} = \frac{345,000}{215,000} = 1.6 \qquad (65b)$$

$$C_3 = C_1 = 3.5 \qquad (65c)$$

$$M'_{BE} = \frac{1}{6}\left[\frac{PL}{8}(2C_1 - C_2) - \frac{PL}{8}(2C_2 - C_1)\right] = \frac{PL}{6(8)}(3C_1 - 3C_2)$$

$$= \frac{PL}{16}(C_1 - C_2) = \frac{PL}{16}(3.5 - 1.6) = \frac{50(20)12}{16}(1.9)$$

$$= 1425''K$$

$$M_{EB} = -1425''K$$

For girder CD:

$$C_1 = \frac{12L_{2D}L}{4L_{2C}L_{2D} - L^2} = \frac{12(255.9)240}{4(255.9)^2 - (240)^2} = \frac{737,000}{204,500} = 3.6 \qquad (65a)$$

$$C_2 = \frac{6L^2}{4L_{2C}L_{2D} - L^2} = \frac{6(240)^2}{4(255.9)^2 - (240)^2} = \frac{345,000}{204,500} = 1.7 \qquad (65b)$$

$$C_3 = C_1 = 3.6 \qquad (65c)$$

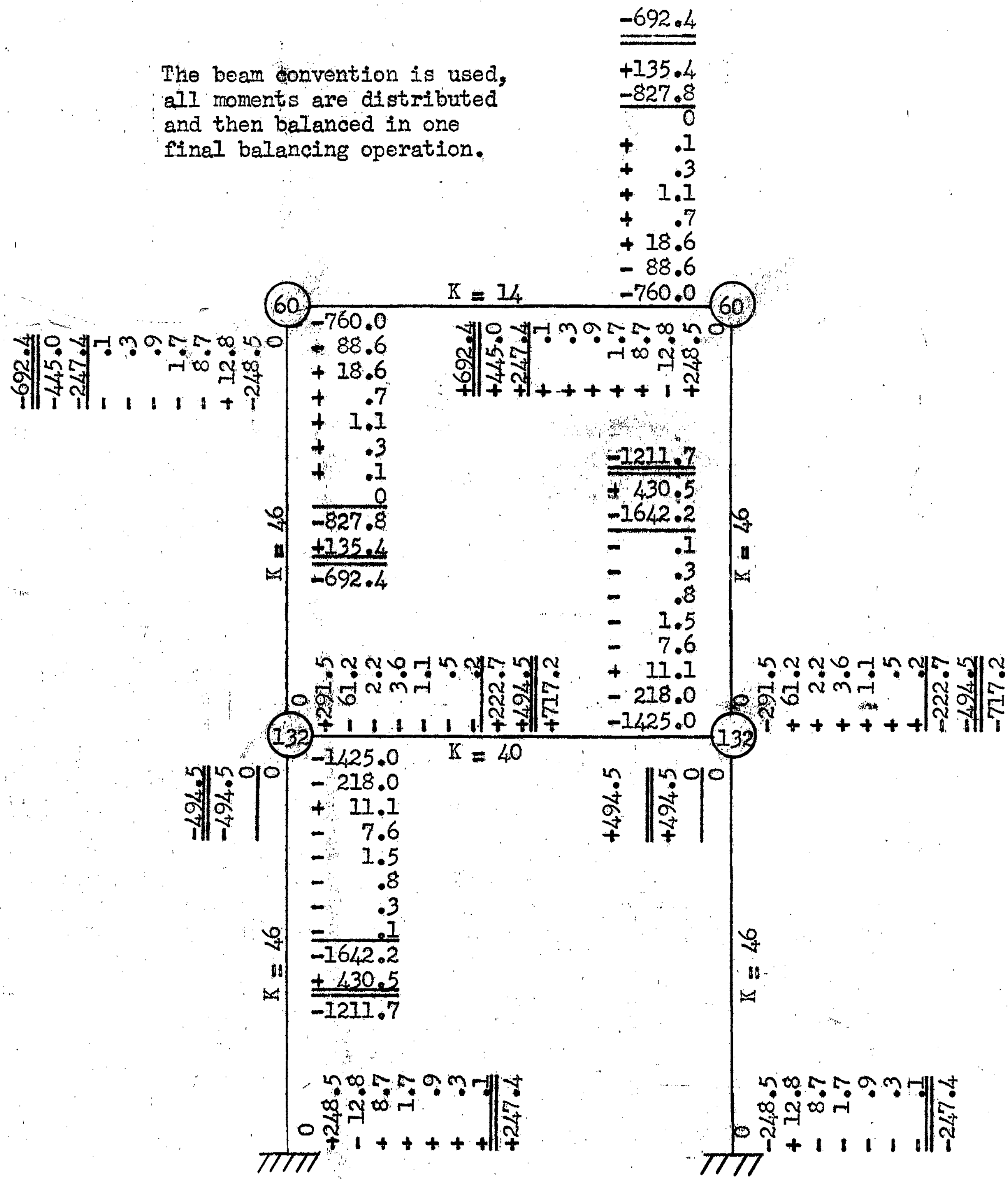
$$M'_{CD} = \frac{1}{6}\left[\frac{wL^2}{12}(2C_1 - C_2) - \frac{wL^2}{12}(2C_2 - C_1)\right]$$

$$= \frac{wL^2}{6(12)}(3C_1 - 3C_2) = \frac{wL^2}{24}(C_1 - C_2) = \frac{wL^2}{24}(3.6 - 1.7)$$

$$= \frac{2(20)20(12)}{24}(1.9) = 760''K$$

$$M'_{DC} = -760''K$$

The beam convention is used, all moments are distributed and then balanced in one final balancing operation.



Comparing these moments with those of slope deflection gives a maximum deviation of less than one per cent.

D. Results.

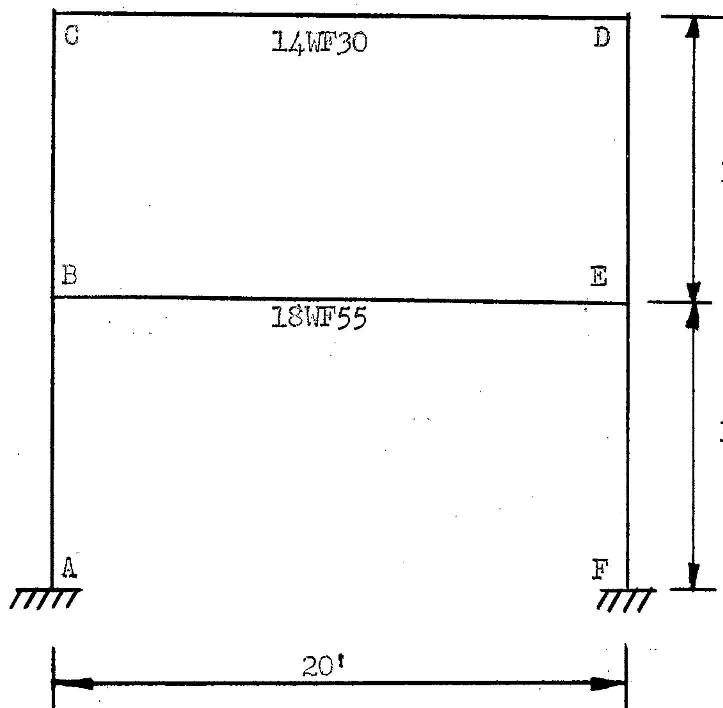


Fig. 11

By present design procedure CD and BE should be $\frac{I}{c} = \frac{1200}{20} = 60''^3$, 16WF40 and $\frac{I}{c} = \frac{3000}{20} = 150''^3$, 21WF73 respectively. Then CD and BE weigh 40(20) = 800# and 73(20) = 1460#. But with semi-rigid connections CD and BE weigh 20(30) = 600# and 55(20) = 1100# respectively. The saving realized is $\frac{200}{800}$ or 25% for CD and $\frac{360}{1460}$ or 24.65% for BE.

(CD) $\frac{wl^2}{8} = \frac{2(20)20(12)}{8} = 1200''K$ (maximum positive moment at mid-span — simply supported member)

Then 692.65 is the design moment.

$$\frac{I}{c} = \frac{M}{s} = \frac{692.65}{20} = 34.632''^3$$

Then 14WF30 is required

(BE) $\frac{PL}{4} = \frac{50(20)12}{4} = 3000''K$ (maximum positive moment at mid-span — simply supported member)

Design moment $= 3000 - 1218.74 = 1781.26''K$

$\frac{I}{c} = \frac{M}{s} = \frac{1781.26}{20} = 89.563''3$

Then 18WF55 is required

Column design

Try 10WF66

$$1 = \frac{P}{A_s} + \frac{Mc}{I_s} = \frac{45}{19.41(15.95)} + \frac{1218.74}{73.7(20)} = .145 + .825 = .97$$

Then 10WF66 is sufficient

The design procedure in building frames, particularly tall buildings, is to make the beam-column connections rigid using welded diaphrams between column flanges and the fire proofing required by building codes. This causes the building to resist wind as a rigid frame. The girders and beams are designed, however, as simply supported members.

The columns required are those necessary to resist the moments and loads from the rigid frame analysis. Their weight $= 20(77) = 1540\#$, while the weight of the column for the semi-rigid frame $= 20(66) = 1320\#$. Each column offers a savings of $\frac{220}{1540}$ or 14.29%.

The total average saving realized from this bent by considering the properties of the semi-rigid connections is 19.55%.

CONCLUSION

It is evident that a material saving can be realized by design if the properties of semi-rigid connections are considered. It appears that twenty per cent is a fairly accurate estimate of this saving as compared with results of the bent analyzed in this report.

The next problem is whether this saving is justified. Disregarding the rapidly diminishing supply of steel, the amount of additional work required to design a semi-rigid frame is very little more than that required by present design practices. The beam-column connections are assumed rigid and this moment determined to design the wind connection. This then gives a design moment for the semi-rigid connection with no additional analysis required.

The only difficulty in using the equations for semi-rigid connections is involved in determining Z. This factor has been developed and determined in terms of the properties of the connection for the web angle connection[22] by Professor J. E. Lothers of the School of Architectural Engineering of the Oklahoma A & M College. Professor Lothers has also computed a table of Z-values[23] for the standard web-angle connections listed in the AISC Handbook. With Z known it becomes a simple matter to use either Rathbun's or Maugh's equations in the analysis of semi-rigid frames.

There remains, however, one important semi-rigid connection for which Z has not been determined and that is the combination of the clip-angle

---

22 J. E. Lothers, Transactions of the American Society of Civil Engineers, Vol. 116, pp. 480-502.

23 Ibid, pp. 488-489.

and web-angle connection. Once this is determined the design of frames with semi-rigid connections involves no more work than for rigid frames.

The properties of semi-rigid connections can be used only when rivet holes are completely filled with properly formed rivets and the joint is tight (i.e. connections are firmly seated against beam and column). This requires rigid inspection and close contact between office and field.

Steel is a natural resource that is rapidly diminishing and, with the increasing demand for military uses, the expense involved in the analysis and control of semi-rigid building frames is more than justified.

BIBLIOGRAPHY

Batho, Cyril. First, Second and Final Report. Steel Structure Research Committee, Department of Scientific and Industrial Research. H. M. Stationery Office, London, 1931-36.

Johnston, Bruce and Mount, Edward H. "Analysis of Building Frames with Semi-rigid Connections." Transactions of the American Society of Civil Engineers. Vol. 107 (1942), 524-530.

Lothers, J. E. Design in Structural Steel. New York: Prentice-Hall, Inc., in process of publication.

Lothers, J. E. "Elastic Restraint Equations for Semi-rigid Connections." Transactions of the American Society of Civil Engineers. Vol. 116 (1951), 480-503.

Maugh, L. C. "Frames with Semi-rigid Connections." Statically Indeterminate Structures. New York: John Wiley & Sons, Inc., 1948, 292-299.

Parcel, John Ira and Maney, George Alfred. Statically Indeterminate Structures. New York: John Wiley & Sons, Inc., 1947.

Quirk, Willard Raymond. "The Contribution of Column Flanges to the Rotation of Partially Restrained Riveted Connections in Building Frames." Thesis. Stillwater: 1948.

Rathbun, J. Charles. "Elastic Properties of Riveted Connections." Transactions of the American Society of Civil Engineers. Vol. 101, 549-552.

Sutherland, Hale and Bowman, Harry Lake. Structural Theory. New York: John Wiley & Sons, Inc., Third edition, 1944.

Wilbur, John Benson and Norris, Charles Head. Elementary Structural Analysis. New York: McGraw-Hill Book Company, Inc., First edition, 1948.

THESIS TITLE: SEMI-RIGID CONNECTIONS COMPARED WITH SIMPLY SUPPORTED MEMBERS IN BUILDING FRAMES

NAME OF AUTHOR: JOHN KARL PERRY

THESIS ADVISER: J. E. LOTHERS

The content and form have been checked and approved by the author and thesis adviser. "Instructions for Typing and Arranging the Thesis" are available in the Graduate School office. Changes or corrections in the thesis are not made by the Graduate School office or by any committee. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

NAME OF TYPIST: Fern B. Hall

* * * * * * * * * * * * * *