KINEMATICS OF A VARIABLE NOZZLE USED AS AN EXHAUST VALVE FOR RECOVERY OF POWER FROM INTERNAL COMBUSTION ENGINES

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PREFACE

The constant effort to improve the efficiency of internal combustion engines has led to the investigation of recovery of power from the exhaust gases of reciprocating engines.

The first practical application of the exhaust gases to power recovery was the turbo-supercharger. Although the turbo-supercharger does not contribute power directly to the propeller shaft, it relieves the engine of the burden of mechanical supercharging, thus increasing the net power output. However, its usefulness is limited to high-altitude operation, since the operation of a supercharger at low altitudes usually stresses the engine past the safety requirement.

The next step in power recovery was to gear the exhaust turbine to the propeller shaft; thus, the power recovered by the exhaust turbine could be used at low altitudes without overstressing the cylinders.

In present engines the exhaust gases are expanded through poppet valves and conducted to turbines where the kinetic energy of the gases is converted to mechanical energy.

The purpose of this report is to suggest a method for designing a valve mechanism of the nozzle type to produce an improvement in the expansion process.

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LIST OF SYMBOLS

Upper Case

A = Cross-sectional Area - ft^2

C = Constant

D = Piston Displacement - ft^3

K = Constant

M = Mass - lbs

R = Gas Constant for air - 53.3 ft.lbs/^{o}R

T = Temperature - ^{o}R or ^{o}F

V = Volume - ft^3

W = Work - ft.lbs

Z = The Ratio of Crank Radius Over Connecting Rod Length

Lower Case

a = Sonic Velocity - ft/sec.

g = Acceleration of Gravity 32.2 ft/sec^2

p = pressure - lbs/ft^2

r = ratio

r_c = compression ratio

t = time - seconds

u = Velocity - ft/sec

w = Angular Velocity - r.p.m. or Rads/sec

Greek

α = Angular Constant

γ = Ratio of Specific Heats

θ = Crank Angle - either radians or degrees

μ = Mach Number

ρ = Density lbs/ft^3

Subscripts and Superscripts

0 = Reservoir Condition

1 - 2 - 3 = To distinguish Successive Constants

* = Throat Condition

e = Exit Condition

i = Initial Condition (at the instant flow begins)

Abbreviations

ft = feet

Rad = Radian

Sec = Seconds

lbs = Pounds

rpm = Revolutions per Minute

CHAPTER I

INTRODUCTION AND STATEMENT OF PROBLEM

The reciprocating internal combustion engine, which will be called the engine for simplicity, fails to make full use of the energy in the products of combustion, which for the purpose of this report will be called the gas. At the time the gas is released from the cylinder, its temperature and pressure conditions are above those of the atmosphere. Thus the gas is still capable of expansion which, if controlled, could deliver additional work to the drive shaft. The reciprocating engine cannot develop this work economically because of the large volume required to expand the gas to atmospheric pressure. The mechanism required for the additional expansion would add excessive weight and bulk to the engine, consequently, in the usual engine, some of the energy is lost.

The advent of the gas turbine affords a means for developing useful work from large volumes of gas. The application of the turbine to the task of controlling the expansion of exhaust gases introduces a host of new problems. Some of these problems arise from the fact that the turbine is essentially a constant-speed prime mover and as such requires constant-flow conditions; the engine, however, delivers a pulsating flow of exhaust gas, and these pulsations limit the efficiency of the turbine. Therefore, one of the problems in recovering energy from the exhaust is to minimize the adverse effect of the pulsating flow. Compromises in the design of the turbine wheel can reduce some of the adverse effects of this varying flow. Compromises in the engine design might achieve similar results, however, this report is only concerned with the turbine nozzle design. The engine design and the turbine-wheel design are excluded from consideration.

Since the De Laval nozzle is a constant-flow device it must be designed for a given flow and a given pressure drop, in order to achieve maximum efficiency. Since the output of an impulse turbine depends on the kinetic energy developed in the nozzle, a nozzle subject to variable flow must be made to change its shape and size according to the flow and pressure variations.

In particular this report develops an ideal nozzle which is made to vary in shape and cross-sectional area (called a variable nozzle) in such a manner that the maximum kinetic energy shall be developed in the exhaust gas. The ideal nozzle is then modified because of practical limitations. Lastly the kinematics of a mechanism for the practical variable nozzle are presented. The kinematics are intended to show only that a variable nozzle is mechanically possible.

CHAPTER II

DISCUSSION AND ANALYSIS OF THE PROBLEM

Some of the conditions to be met in this problem are unusual in that the engineer seldom needs to analyze them. This chapter presents a non-technical discussion in preparation for the development to follow.

The first problem which may trouble the reader is the behavior of the gases within the cylinder during the exhaust process. This problem may be clarified by concentrating attention on some fraction of the gas within the cylinder. It may be assumed that this fraction contains only those particles which remain in the cylinder during any one instant of time. If all particles of gas other than those in the assumed fraction escape from the cylinder, the assumed fraction will expand to fill the space previously occupied by the escaped particles. Now the original shape of the assumed fraction is not specified; however, the shape has no effect on the process since the forces transmitted across its boundaries are the same as the forces experienced by every other particle within the cylinder during the process. Thus, though the actual shape is arbitrary as shown in Fig. (II-1)(a), the whole effect would be unchanged if the shape were assumed to be cylindrical as shown in Fig. (II-1)(b) with the boundary remaining a flat circular surface throughout the process. As far as the assumed fraction of the gas is concerned the flat boundary might just as well be replaced by another piston. This concept of a fictitious piston makes it easier to see that the gas within the cylinder follows the same pressure, temperature, and density relations as a process within a closed cylinder. These relations can be found in most text books on thermodynamics.

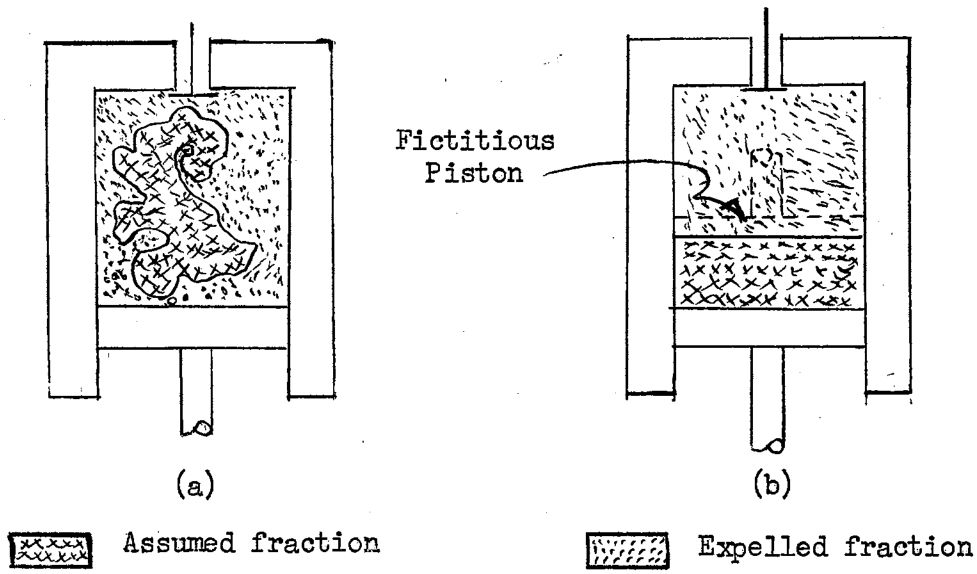

Fig. (II-1) Assumed Division of the Gas.

The next problem is the determination of the relationship between the gas conditions within the cylinder and the flow produced. It has been established that the temperature, pressure, and density within the cylinder are related by known functions; if we can relate any one of these to the flow the immediate problem will be solved. The mass within the cylinder can be determined by the product of density and volume. If this product is differentiated with respect to time, the mass flow (change of mass in the cylinder) is established. Thus the density relates the conditions in the cylinder to the flow produced.

The channel area required for this flow will be determined by DeLaval nozzle theory. For super-critical flow the channel must have a throat followed by a divergent section; for sub-critical flow the channel must converge to the exit section. The flow conditions of the exhaust process change from super-critical at the beginning of the process to sub-critical during the last portion of the process. Therefore, the nozzle must change its shape from convergent-divergent, to convergent in order to expand the flow properly.

The poppet valve which is used in most engines cannot satisfy these requirements for expansion because the divergence cannot be controlled independently from the throat area. This valve has other undesirable features such as sudden changes in the direction of flow, and discontinuities in the channel area.

In order to satisfy the nozzle design a mechanism must be provided which will furnish: (1) independent control of both the throat and exit area, and (2) a smooth channel with minimum changes in the direction of flow. Also since the mechanism is to be used as a valve it must provide a satisfactory seal when closed.

The use of steady-flow relations on the unsteady flow of the exhaust process is justified because the acceleration forces caused by the unsteady motion are small compared to the other forces acting on the flow. This fact has been verified by experiment.[1]

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[1] Tsung-chi Tsu, "Theory of the Inlet and Exhaust Processes of Internal Combustion Engines," NACA-TN No. 1446, p. 40.

CHAPTER III

APPLICATION OF THERMODYNAMIC EQUATIONS TO FLOW FROM A CYLINDER THROUGH A VALVE

One of the objects of this investigation is to incorporate in the design of the exhaust valve the function of a nozzle in such a way that the maximum kinetic energy in the exhaust gas is developed throughout the exhaust process without affecting the normal pressure time relationship within the cylinder.

The solution is based on certain assumptions which simplify the procedure. These assumptions are stated below.

1. Air is the medium; it replaces the burned air-fuel mixture. Although the exhaust contains other gases, air is the principal component and the behavior of the gases is nearly the same as air under the same conditions.
2. Air is a perfect gas; this is generally accepted as approximately true.
3. Isentropic processes; the effects of friction and heat transfer on the exhaust flow and expansion within the cylinder are neglected for the sake of simplicity.
4. Steady flow conditions exist in the nozzle. Actually the flow is unsteady but this assumption introduces negligible errors.[1]

As a result of these assumptions the relationship between pressure,

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[1] Tsun-chi Tsu, "Theory of the Inlet and Exhaust Processes of Internal Combustion Engines," NACA-TN No. 1446, p. 40.

density, and temperature within the cylinder can be expressed as[2]

$$p_o = \rho_o^{\gamma} \text{ x constant} = T^{\frac{\gamma}{\gamma-1}} \text{ x constant} \qquad \text{(III-1)}$$

where p = pressure

ρ = density

T = absolute temperature

$\gamma = \frac{C_p}{C_v} = 1.4$ = ratio of specific heats for air.

The subscript 0 specifies conditions within the cylinder at any given instant of time.

Since the expansion across the nozzle has been assumed isentropic these conditions may be expressed as[3]

$$\left(\frac{p_e}{p_o}\right) = \left(\frac{\rho_e}{\rho_o}\right)^{\gamma} = \left(\frac{T_e}{T_o}\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{(III-2)}$$

where p_e, ρ_e, and T_e are the properties of the gas when expanded isentropically to the external pressure, these shall be called the exit conditions.

The mass rate of flow, the exit cross-sectional area of the channel and the conditions in the cylinder are related by the following function,[4]

$$-\frac{dM}{dt} = A_e \sqrt{\frac{2\gamma}{\gamma-1} p_o \rho_o \left(\frac{p_e}{p_o}\right)^{2/\gamma} \left[1 - \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad \text{(III-3)}$$

where M = mass within the cylinder

t = time

p = pressure

A = exit cross-sectional area of the channel.

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[2] H. W. Liepmann and A. E. Puckett, Introduction to Aerodynamics of a Compressible Flow, p. 12.

[3] Ibid, p. 18.

[4] Ibid, p. 32.

The negative sign before dM/dt indicates that the mass passing through the channel is leaving the cylinder. As the cylinder conditions are related by (III-2) and density is the condition most directly related to mass, it is convenient to rewrite (III-3) thus:

$$-\frac{dM}{dt} = A_e \sqrt{\frac{2\gamma}{\gamma-1} p_e \rho_e} \sqrt{\left(\frac{\rho_o}{\rho_e}\right)^{\gamma-1} - 1} \qquad \text{(III-3)}'$$

Complete expansion at the exit section, when the flow is supercritical, requires a throat in the channel. The conditions in the throat can be determined by the following relations[5]

$$p^* = p_o \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad \rho^* = \rho_o \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \quad T^* = T_o \left(\frac{2}{\gamma+1}\right) \qquad \text{(III-4)}$$

where the asterisk identifies the states in the throat. The size of the throat area may be determined by dividing the mass rate of flow by the product of the throat velocity and density. Thus:

$$A^* = \frac{-dM/dt}{a^* \rho^*} \qquad \text{(III-5)}$$

where a^* is the throat sonic velocity which is identical to the value of the throat velocity.

Since sonic velocity is proportional to the square root of temperature and density is related to temperature by (III-2) sonic velocity in the throat may be expressed as a function of the density thus,

$$a^* = a_e \left(\frac{T^*}{T_e}\right)^{\frac{1}{2}} = a_e \left(\frac{\rho^*}{\rho_e}\right)^{\frac{\gamma-1}{2}} = a_e \rho_e^{\frac{1-\gamma}{2}} \rho^{*\frac{\gamma-1}{2}}$$

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[5] Ibid, p. 31.

By combining this relation with (III-4) and (III-5) yields,

$$A^* = \frac{-dM/dt}{a_e \rho_e^{\frac{1-\gamma}{2}} \rho_*^{\frac{\gamma+1}{2}}} = \frac{-dM/dt}{a_e \rho_e^{\frac{1-\gamma}{2}} \rho_o^{\frac{\gamma+1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma-1}{2(\gamma-1)}}} \qquad \text{(III-6)}$$

Since the exit conditions a_e and ρ_e are constants for any given solution (III-6) may be expressed thus

$$A^* = -K \rho_o^{-\frac{\gamma+1}{2}} \frac{dM}{dt} \qquad \text{(III-6)'}$$

$$\text{where } K = \frac{1}{a_e \rho_e^{\frac{1-\gamma}{2}} \left(\frac{2}{\gamma-1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

The ratio of throat to exit area is obtained by substituting (III-3)' in (III-6) and dividing by A_e. Performing this operation and simplifying yields,

$$\frac{A^*}{A_e} = \sqrt{\frac{\left(\frac{\gamma-1}{2}\right)}{\left(\frac{2}{-1}\right)^{\frac{\gamma+1}{\gamma-1}}} \left(\frac{\rho_e}{\rho_o}\right)^2 \left[1 - \left(\frac{\rho_e}{\rho_o}\right)^{\gamma-1}\right]} \qquad \text{(III-7)}$$

Equation (III-7) determines the relative sizes of the throat and exit area, this term will be called the divergence function.

The mass within the cylinder can be determined by the product of the density and the volume within the cylinder. The volume-time relationship is fixed by the crank linkage; thus, if a density-time relationship is established, the mass rate of flow may be determined by differentiating the product of the volume and density functions.

Thus,

$$\frac{dM}{dt} = \frac{d(\rho_o V)}{d\theta}\frac{d\theta}{dt} = \left[\rho_o \frac{dV}{d\theta} + V\frac{d\rho_o}{d\theta}\right]\frac{d\theta}{dt} \tag{III-8}$$

where $V =$ volume within the cylinder

$\theta =$ crank angle.

This solution depends on the selection of an appropriate density-time relationship. Since the density within the cylinder is related to the pressure by (III-2), the density-time relationship may be derived from the desired indicator card for the engine. This method of solution produces the necessary area-time relationship for a given pressure-time relation.

A solution is also possible by assuming an area-time relationship and deriving the pressure-time relation. If the solution is carried out in this way, it would be convenient to change the form of (III-6)'. This method requires an integration process in which it becomes expedient to separate the variables. As density is determined by the quotient of mass and volume, (III-6)' may be rewritten in the form,

$$A^* = -K\left(\frac{M}{V}\right)^{-\frac{1+\gamma}{2}}\frac{dM}{dt}$$

Since V and A^* are related to time, the variables may be separated thus:

$$\frac{A^*}{V^{\frac{1+\gamma}{2}}}\,dt = -KM^{-\frac{1+\gamma}{2}}\,dM$$

The integration of this function yields

$$\int \frac{A^*}{V^{\frac{1+\gamma}{2}}}\, dt = \frac{2K}{1+\gamma} M^{\frac{1-\gamma}{2}} + C \qquad \text{(III-9)}$$

where C is the constant of integration.

Both of these solutions are simplifications of the problem and may produce some undesirable or impractical values of exit area. In both solutions the flow conditions are determined before the divergence function is applied. Consequently where the effect of this function is large, the exit area-time curve may have undesirable characteristics. This effect will be illustrated in the solution of a typical case.

CHAPTER IV

SOLUTION OF A TYPICAL CASE

A. Statement of Problem.

In order to clarify the procedures established in Chapter III we shall attempt a solution for an arbitrary set of specifications.

A variable nozzle is to be developed which is to act as a valve for the following hypothetical engine.[1]

1. A four stroke, otto-cycle, aircraft engine.
2. Bore 6-inches, stroke 6-inches, connecting rod 12-inches long.
3. Compression ration $r_c = 6.5$.
4. Engine speed $w = 2400$ rpm.
5. Assumed indicator card of Fig. (IV-1).

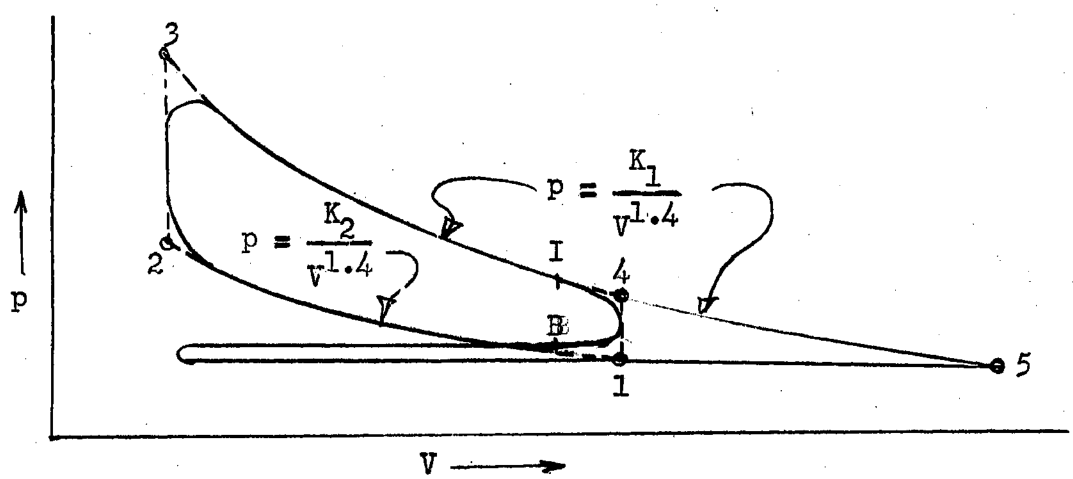

Fig. (IV-1) Assumed Indicator Card.

$p_1 = p_5 = 15$ psia, $T_1 = 60^{\circ}F$, $p_4 = 80$ psia. Exhaust to begin at point I = 135° of crank travel.

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1 These values based on information taken from K. D. Wood, Technical Aerodynamics, Table C.

The dashed lines in Fig. (IV-1) outline the familiar Otto Air Cycle. The area 1451 represents the work which could be recovered by complete expansion of the gas.

B. Investigate the Recoverable Power.

Before attempting the solution it is well to examine the source of the power to be recovered and estimate the amount of energy which could be recovered.

Otto Cycle. The ideal work produced by the Otto Cycle referred to the conditions at point 4 of Fig.(IV-1) may be computed by the following relation.

$$W = p_4 V_4 \frac{\left(1 - \frac{p_1}{p_4}\right)}{(1.4-1)} \left[\left(\frac{V_4}{V_3}\right)^{1.4-1} - 1 \right]$$

Substituting for the values of $\frac{p_1}{p_4}$, $\frac{V_4}{V_3}$ yields

$$W = p_4 V_4 \frac{(1 - \frac{15}{80})}{0.4} \left[6.5^{.4} - 1 \right]$$

$$W = 2.19 \; p_4 V_4$$

The Recovery Process. The area 1451 represents the work which could be developed by complete expansion of the exhaust gas and, therefore, is the equivalent of the maximum kinetic energy which the exhaust gases could attain. This work may be computed by

$$W' = \int_4^5 pdV - p_1 \ (V_5 - V_4)$$

$$= p_4 V_4 \left\{ \frac{1}{1.4-1} \left[\left(\frac{V_5}{V_4} \right)^{(1.4-1)} - 1 \right] - \left(\frac{p_1}{p_4} \right) \left[\left(\frac{V_5}{V_4} \right) - 1 \right] \right\}$$

From assumption No. 5

$$p_4 V_4^{1.4} = p_5 V_5^{1.4}$$

or

$$\frac{V_5}{V_4} = \left(\frac{p_4}{p_5} \right)^{\frac{1}{1.4}} = \left(\frac{80}{15} \right)^{\frac{1}{1.4}} = 3.305$$

$$\left(\frac{V_5}{V_4} \right)^{1.4-1} = (3.305)^{.4} = 1.612$$

thus the recoverable work becomes

$$W' = p_4 V_4 \left\{ \frac{1}{0.4} (1.612-1) - \frac{15}{80} (3.305-1) \right\}$$

$$W' = 1.097 \ p_4 V_4$$

$$\frac{W'}{W} = \frac{1.097}{2.19} \frac{p_4 V_4}{p_4 V_4} = .501$$

This shows that the power can be increased by 50.1 per cent if the entire available energy is recovered.

Now let us consider what happens when the gas is exhausted through a poppet valve. The effect of the sudden changes in area, and direction of flow, make the effect of this valve akin to exhausting through an orifice.

Let us assume the turbulence, induced by the sudden change in section, and the shock losses limit the recoverable kinetic energy to that attained at the critical section. In other words the flow is expanded in an isentropic process to the critical pressure, at which time it crosses the minimum section where the remaining pressure must change to the exhaust pressure.

Figure (IV-2) represents an enlargement of the exhaust process of Fig. (IV-1), where the line $4'$-$5'$ represents the critical pressure or $0.528\ p_{45}$. Thus those particles expelled from the cylinder are able to recover the kinetic energy equivalent of the cross-hatched area $455'4'4$. The lost availability may be represented by area $4'5'14'$. Thus under this assumption the minimum energy lost is

$$W_L = \int_{4'}^{5'} p\,dV - p_1 (V_{5'} - V_{4'})$$

where $p_{4'5'} = 0.528\ p_{45} = 0.528 \dfrac{K_1}{V^{1.4}}$

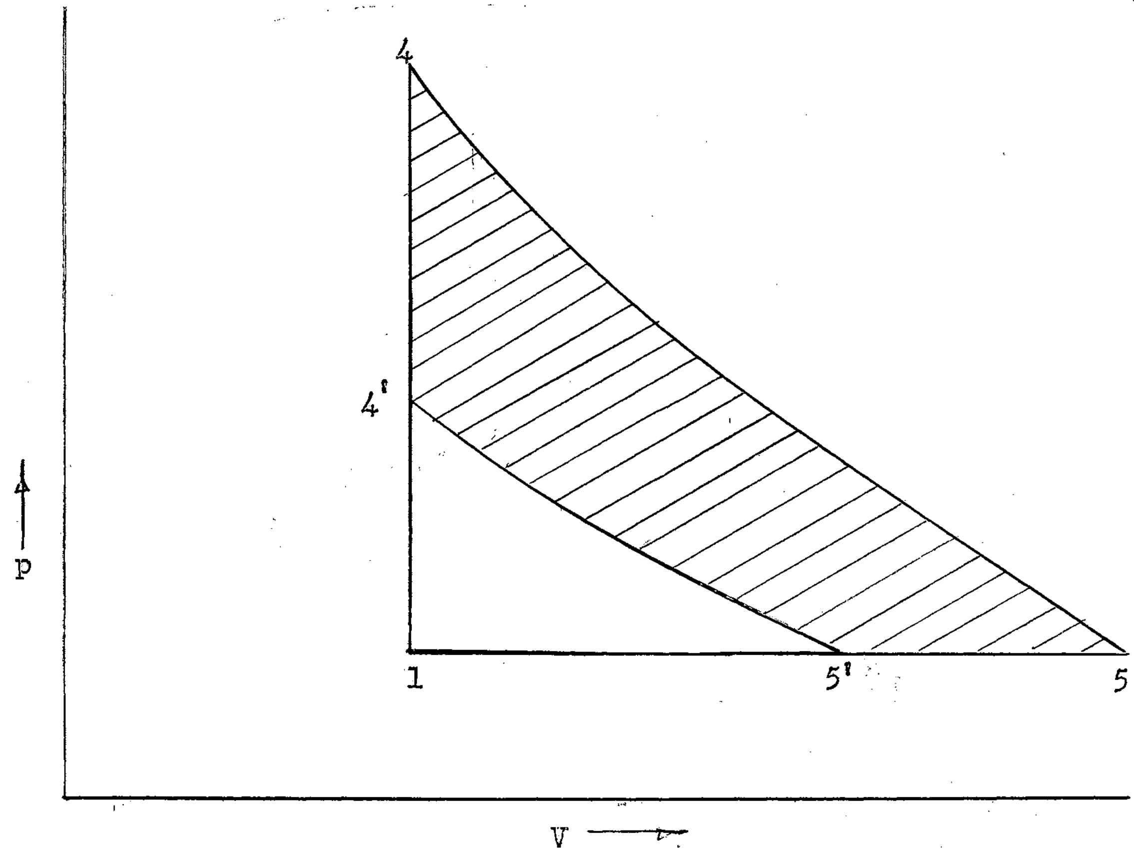

Fig. (IV-2). The Exhaust Process.

The integral becomes

$$\int_{4'}^{5'} p\,dV = .528\ K_1 \int_{4'}^{5'} \frac{dV}{V_{4'5'}^{\ 1.4}} = 0.528\ \frac{p_4V_4}{0.4}\left[\left(\frac{V_{5'}}{V_{4'}}\right)^{0.4} - 1\right]$$

Evaluating the ratio $\frac{V_{5'}}{V_{4'}}$ yields

$$\frac{V_{5'}}{V_{4'}} = 2.092$$

Substituting this value in W yields

$$W_L = .528\ \frac{p_4V_4}{.4} \times (2.092-1) - p_4V_4\ \frac{15}{80} \qquad 2.092-1$$

$$W_L = p_4V_4\ (0.249)$$

The fraction lost becomes

$$\frac{W_L}{W} = \frac{.249}{2.19} = .1137$$

Thus the per cent recovery in this case would be

$$\frac{W'}{W} - \frac{W_L}{W} \times 100 = 38.73 \text{ per cent.}$$

This loss represents 11.37 per cent of the idealized power represented in the original Otto Cycle. This indicates that the power which could be recovered by exhausting through a poppet valve is of the order of 40 per cent of the engine's power, where complete expansion could produce an increase of the order of 50 per cent.

The manufacturers of compound engines using poppet valves, claim recovery factors of the order of 20 per cent. This fact indicates a considerable loss of recoverable power which may be explained as follows. This discrepancy is caused by the effect of wire-drawing, the use of an imperfect gas, and the fact that the turbine can not efficiently recover all the energy of a variable flow; also, it is probable that the poppet valve with its ducting does not deliver the full kinetic energy that is developed at the throat. Even a poorly-designed nozzle could reduce this loss considerably, for all that is required is to have a fairly-continuous channel section.

C. <u>Assume Density Function</u>.

This case will be solved by assuming a characteristic density-time curve and determining the required area-time curve for the flow.

Let us now proceed to establish a density function to fit the general requirements.

Referring to Fig. (IV-3), the portion of the curve from I to B could be approximated by the function

$$\frac{\rho_o}{\rho_e} = \left[K_1 - K_2 \cos\left(\frac{5}{3}\theta - \alpha\right)\right] \qquad \text{(IV-1)}$$

where K_1, K_2, and α are constants to be determined by the conditions to be imposed on the curve. These conditions are the ordinate and slope of the curve at point I, and the ordinate at point B.

It is now necessary to determine these conditions for a density function. Conditions at point I shall be called initial conditions and will be designated by the subscript i, as in Fig. (IV-1).

The initial mass within the cylinder will be assumed the same as that at the beginning of the compression stroke. Thus by the perfect gas law:

$$M_i = M_1 = \frac{p_1 V_1}{RT_1}$$

By assumption No. 5

$$p_1 = 15 \times 144 = 2160 \text{ p.s.f.}$$

$$T_1 = 460 - 60 = 520^\circ R$$

$$V_1 = D\left(\frac{r_c}{r_c - 1}\right) = \frac{6}{12} \times \left(\frac{6}{12}\right)^2 \frac{\pi}{4} \left(\frac{6.5}{6.5-1}\right) = .0982 \times \frac{6.5}{5.5} = 116 \text{ ft}^3$$

where D = piston displacement = .0982 ft^3.

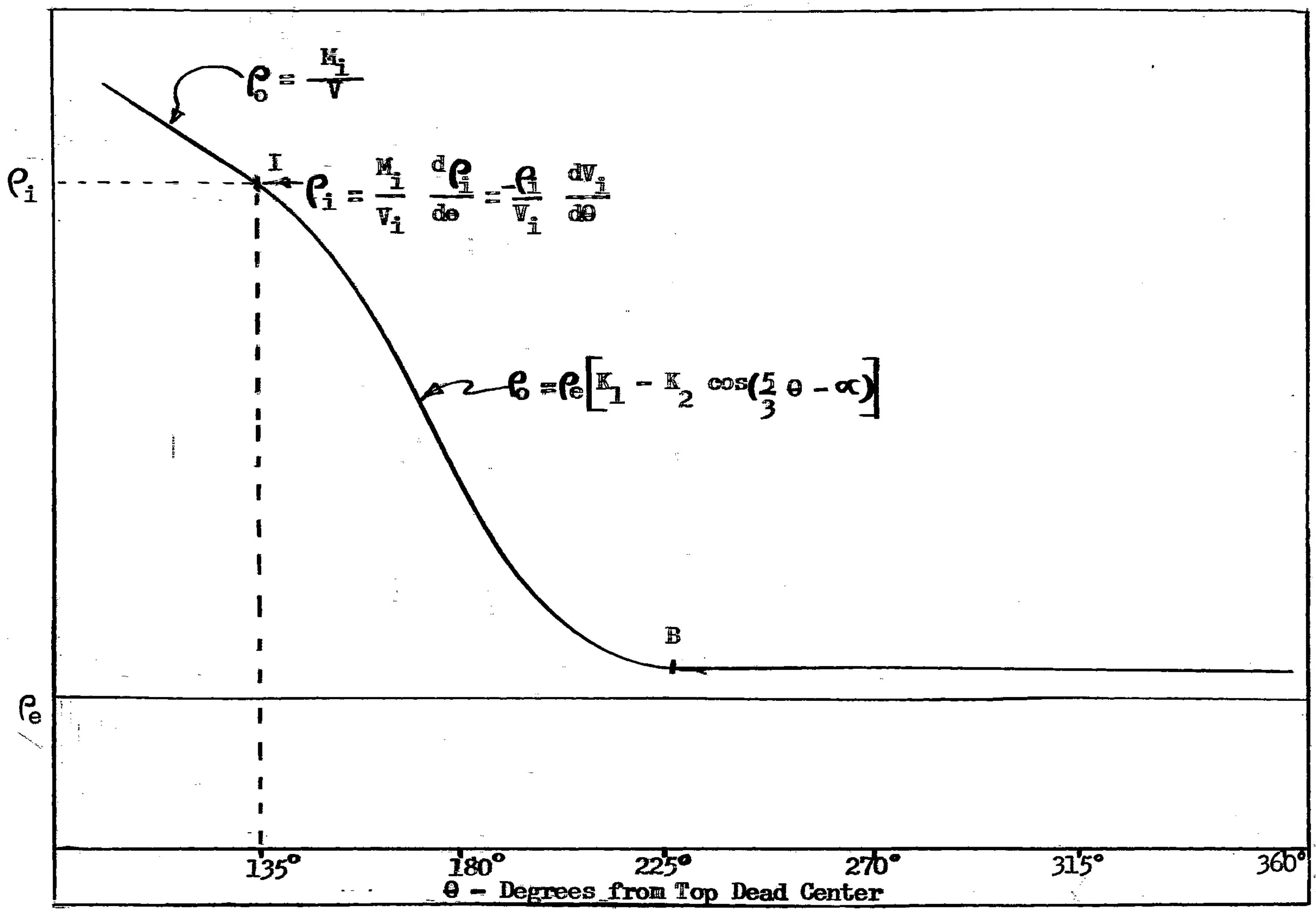

Fig. (IV-3). Assumed Density Relation

$$M_i = \frac{2160 \times .116}{53.3 \times 520} = .00894 \text{ lbs.}$$

The volume relation to crank angle may be expressed thus[2]

$$V = D\left[\frac{1}{r_c - 1} - \frac{1}{2}\left(1 - \cos\theta - \frac{1}{2} Z \sin^2\theta\right)\right] \quad \text{(IV-2)}$$

$$= .0982\left[.182 - \frac{1}{2}\left(1 - \cos\theta - \frac{1}{8}\sin^2\theta\right)\right]$$

where Z is the ratio of the crank radius to the connecting rod length (in this problem $Z = 3''/12'' = \frac{1}{4}$)

Since the initial conditions are at 135° the initial volume is

$$V_i = 0.0982\left[\frac{1}{6.5-1} - \frac{1}{2}(1.770)\right] = 0.0982 \times (.182 - .885)$$

$$V_i = 0.0982 \times 1.067 = 0.1048 \text{ ft}^3$$

Thus the initial density is

$$\rho_i = \frac{M_i}{V_i} = \frac{.00894}{.1048} = 0.0852 \text{ lbs/ft}^3$$

At the initial point the slope of the density curve must be

$$\frac{d\rho_i}{d\theta} = \frac{d\left(\frac{M_i}{V_i}\right)}{d\theta} = -\left(\frac{M_i}{V_i^2}\right)\frac{dV_i}{d\theta} = -\frac{\rho_i}{V_i}\,\frac{dV_i}{d\theta}$$

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[2] The values of this function can be found tabulated for various values of θ and Z in a text by, J. Liston; <u>Aircraft Engine Design</u>, Table 4-3, p. 41.

The derivative of the volume function is[3]

$$\frac{dV}{d\theta} = D\frac{1}{2}\left(\sin\theta - \frac{1}{2} Z \sin 2\theta\right) = \frac{.0982}{2}\left(\sin\theta - \frac{1}{8}\sin 2\theta\right) \quad \text{(IV-3)}$$

Substitution for D, Z, and θ for initial conditions yields

$$\frac{dV_i}{d\theta} = \frac{.0982}{2}(0.582) = .0286 \text{ ft}^3/\text{Rad.}$$

Thus

$$\frac{d\rho_i}{d\theta} = -\frac{0.0852}{0.1048} \times 0.0286 = -\ .0233 \text{ lbs/ft}^3\text{Rad.}$$

The density at B is determined by the pressure desired at B. This pressure will be assumed as 2 psig or $p_b = 17$ psia. As the initial pressure can be determined by

$$p_i = p_1\left(\frac{p_i}{p_1}\right)^{1.4} = p_1\left(\frac{V_1}{V_i}\right)^{1.4} = 80\left(\frac{0.116}{0.1048}\right)^{1.4} = 92 \text{ psia}$$

The density at B is determined by

$$\rho_b = \rho_i\left(\frac{p_e}{p_i}\right)^{\frac{1}{1.4}} = .0852\left(\frac{17}{92}\right)^{\frac{1}{1.4}} = .02556 \text{ lbs/ft}^3$$

where the subscript b denotes a condition at point B.

Now the density of the gas at the exit becomes

$$\rho_e = \rho_i\left(\frac{p_e}{p_i}\right)^{\frac{1}{1.4}} = .0852\left(\frac{15}{92}\right)^{\frac{1}{1.4}} = .0234 \text{ lbs/ft}^3$$

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3 *Ibid*, Table 4-1, p. 39.

Referring all conditions to the exit density we obtain

$$r_i = \frac{\rho_i}{\rho_e} = \frac{.0852}{.0234} = 3.64 \quad r_b = \frac{\rho_b}{\rho_e} = \frac{.02556}{.0234} = 1.092$$

$$\frac{dr_i}{d\theta} = \frac{1}{\rho_e} \frac{d\rho_i}{d\theta} = -\frac{.0233}{.0234} = -.995$$

Now we obtain the conditions to be imposed on (IV-1). Applying these conditions we obtain at $\theta = 135^o$

$$3.64 = \left[K_1 - K_2 \cos\left(\frac{5}{3} 135^o - \alpha\right)\right] = \left[K_1 - K_2 \cos(225^o - \alpha)\right]$$

$$-.995 = -\frac{5}{3} K_2 \sin(225^o - \alpha)$$

The minimum value of this relation is at point B and is to be

$$\frac{\rho_o}{\rho_e} = r_b = 1.092 \text{ where } \cos\left(\frac{5}{3}\theta_b - \alpha\right) = -1$$

$$r_b = 1.092 = K_1 - K_2$$

Solving these equations simultaneously yields

$\alpha = -198.3^o$ $\qquad\qquad$ $\theta_b = 227^o$

$K_2 = 1.345$

$K_1 = 2.437$

Thus from $\theta = 135^o$ to $\theta = 227^o$, (IV-1) becomes

$$\frac{\rho_o}{\rho_e} = 2.437 - 1.345 \cos\left(\frac{5}{3}\theta - 1983\right) \qquad \text{(IV-4)}$$

Differentiating (IV-4) with respect to θ yields

$$\frac{1}{\rho_e}\frac{d\rho_o}{d\theta} = -2.241 \sin\left(\frac{5}{3}\theta - 198.3\right) \qquad \text{(IV-5)}$$

The mass rate of flow may be determined by substituting (IV-3), (IV-4), and (IV-5) in (III-8).

Thus

$$\frac{dM}{dt} = \left[V\frac{dr}{\rho_e d\theta} + \frac{\rho}{\rho_e}\frac{dV}{d\theta}\right]\rho_e w$$

$$= \left[(\text{IV-2})(\text{IV-5}) + (\text{IV-3})(\text{IV-4})\right]\rho_e w \qquad \text{(IV-6)}$$

where the substitutions are indicated by the equation reference number. Following this same procedure the throat area becomes

$$A^* = wK\rho_e^{-1.2}\left(\frac{\rho_o}{\rho_e}\right)^{-1.2}\frac{dM}{dt}$$

$$= wK\rho_e^{-1.2}\,(\text{IV-4})^{-1.2}\,(\text{IV-6}) \qquad \text{(IV-7)}$$

The calculation of the divergence function (III-7) can be simplified by using "Computation Curves for Compressible Fluid Problems"[4]. Once the value of (III-7) is determined for a given point the exit area may be computed by the quotient of (IV-7) and (III-7),

$$A_e = \frac{A^*}{A^*/A_e} = \frac{(\text{IV-7})}{(\text{III-7})} \qquad \text{(IV-8)}$$

---

[4] C. L. Dailey and F. C. Wood, Computation Curves for Compressible Fluid Problems.

Table (IV-1) presents the computations indicated above. Figures (IV-4), (IV-5), (IV-6), and (IV-7) show the values of ρ_o/ρ_e, A^*, A_e, and dM/dΘ as computed in Table (IV-1); these are plotted as a function of the crank angle. The area curves were modified in order to provide more continuous curves. These modifications, and their effect on the density curve, are described by the dashed lines in each Figure.

The density curve represents the original assumption. The area curves constitute the essential information for the valve action required of the nozzle. Though this report is not primarily concerned with the mass rate of flow, the flow has an important bearing on the solution; therefore, this curve is also included.

Table (IV-1)
Calculation of Volume, and Density Curves

Column No.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪
Refer To		(IV-2)	(IV-3)	(IV-2)	(IV-4)	(IV-4)	(IV-4)	(IV-5)	(IV-4)	(IV-4)	(IV-7)
Subject	θ	fθ	dfθ	V	5/3 θ	⑤-1983	cos⑥	sin⑥	1.345⑦	ρ_o/ρ_e	$⑩^{-1.2}$
Dimensions	degrees			$ft^3 x10^{-3}$							
	135	1.067	0.291	1.047	225.0	26.7	.894	.499	1.203	3.640	.212
	140	1.091	0.260	1.070	233.3	35.0	.819	.574	1.102	3.539	.220
	150	1.131	0.196	1.110	250.0	51.7	.620	.660	0.835	3.272	.242
	160	1.159	0.131	1.136	266.7	68.4	.369	.894	0.496	2.933	.275
	170	1.176	0.065	1.152	283.3	95.0	-.087	.995	0.117	2.544	.318
	180	1.182	0.000	1.160	300.0	101.7	-.202	.980	-0.272	2.165	.396
	190	1.176	-0.065	1.152	316.6	118.3	-.473	.880	-0.637	1.800	.994
	200	1.159	-0.131	1.136	333.3	135.0	-.707	.707	-0.953	1.484	.623
	210	1.131	-0.196	1.110	350.0	151.7	-.881	.474	-1.185	1.252	.763
	220	1.091	-0.260	1.070	366.7	168.7	-.979	.202	-1.318	1.119	.874
	225	1.068	-0.291	1.047	375.0	176.7	-.998	.055	-1.341	1.096	.896
	230	1.040	-0.321	1.020	383.3	184.0	1.000	.000	-1.345	1.092	.900
	250	0.988	-0.429	0.979	"	"	1.000	.000	-1.345	1.092	.900
	270	0.744	-0.500	0.729	"	"	1.000	.000	-1.345	1.092	.900
	290	0.566	-0.510	0.555	"	"	1.000	.000	-1.345	1.092	.900
	310	0.474	-0.445	0.436	"	"	1.000	.000	-1.345	1.092	.900
	330	0.264	-0.304	0.259	"	"	1.000	.000	-1.345	1.092	.900
	350	0.191	-0.108	0.183	"	"	1.000	.000	-1.345	1.092	.900
	360	0.182	-0.000	0.177	"	"	1.000	.000	-1.345	1.092	.900

Column 2 — fθ = (0.182 + ½ the value of the piston travel factor as read from Table 4-3, Aircraft Engine Design by J. Liston.

Column 3 — dfθ = ½ the value of the piston velocity factor as read from Table 4-1, Aircraft Engine Design by J. Liston.

Table (IV-1) extended
Calculation of Area Curves

Column No.	12	13	14	15	16	17	18	19
Refer To		(IV-6)	(IV-6)	(V-6)	(IV-7)	Mach Nos.	(III-7)	(IV-8)
Subject	θ	$-V\frac{d\rho}{d\theta}$	$-\rho\frac{dV}{d\theta}$	$\frac{dM}{d\theta}$	A^*	μ	A^*/A_e	A_e
Dimensions	degrees	#/Rad $\times 10^{-3}$	#/Rad $\times 10^{-3}$	#/Rad $\times 10^{-3}$	ft^2 $\times 10^{-2}$			ft^2 $\times 10^{-2}$
	135	2.460	-2.460	0.000	0.0010	1.900	.642	0.0000
	140	3.225	-2.200	1.025	0.2070	1.810	.690	0.2960
	150	3.846	-1.477	2.369	0.5310	1.745	.723	0.7340
	160	5.350	-0.885	4.465	1.1250	1.640	.779	1.4450
	170	6.030	-0.107	5.893	1.1730	1.500	.850	2.0350
	180	6.070	-0.000	6.070	2.2100	1.355	.924	2.3100
	190	5.330	+0.259	5.589	2.5350	1.160	.980	2.5900
	200	4.230	+0.402	4.832	2.7600	0.925	.995	2.7700
	210	2.760	+0.564	3.324	2.5300	0.685	.905	2.8000
	220	1.135	+0.668	1.803	1.6050	0.475	.720	2.2300
	225	0.095	+0.745	0.840	0.7000	0.430	.667	1.0500
	230	0.000	0.794	0.794	0.6550	0.425	.655	1.0000
	250	0.000	1.075	1.075	0.8880	0.425	.655	1.3550
	270	0.000	1.254	1.254	1.0340	0.425	.655	1.5800
	290	0.000	1.278	1.278	1.0030	0.425	.655	1.6090
	310	0.000	1.115	1.115	0.9210	0.425	.655	1.4080
	330	0.000	0.763	0.763	0.6320	0.425	.655	0.9650
	350	0.000	0.274	0.274	0.2260	0.425	.655	0.3450
	360	0.000	0.000	0.000	0.0000	0.425	.655	0.0000

Evaluation of Constants

$w = 2400$ rpm $= 251$ Rads/sec

$$wK\rho_e^{-1.2} = \frac{w\ \rho_e^{-1.2}}{a_e\ \rho_e^{-.2}\left(\frac{.2}{2.4}\right)^3} = \frac{w}{a_e\rho_e\ \left(\frac{1}{1.2}\right)^3}$$

$$a_e = 49.1\sqrt{T_e} = 49.1\sqrt{\frac{p_e}{R\ _e}} =$$

$$a_e = 49.1\ \sqrt{\frac{2160}{53.3 \times .0234}} = 2{,}045 \text{ fps}$$

$$wK\rho_e^{-1.2} = \frac{251}{2045 \times .0234}\ (1.2)^3 = 9.09$$

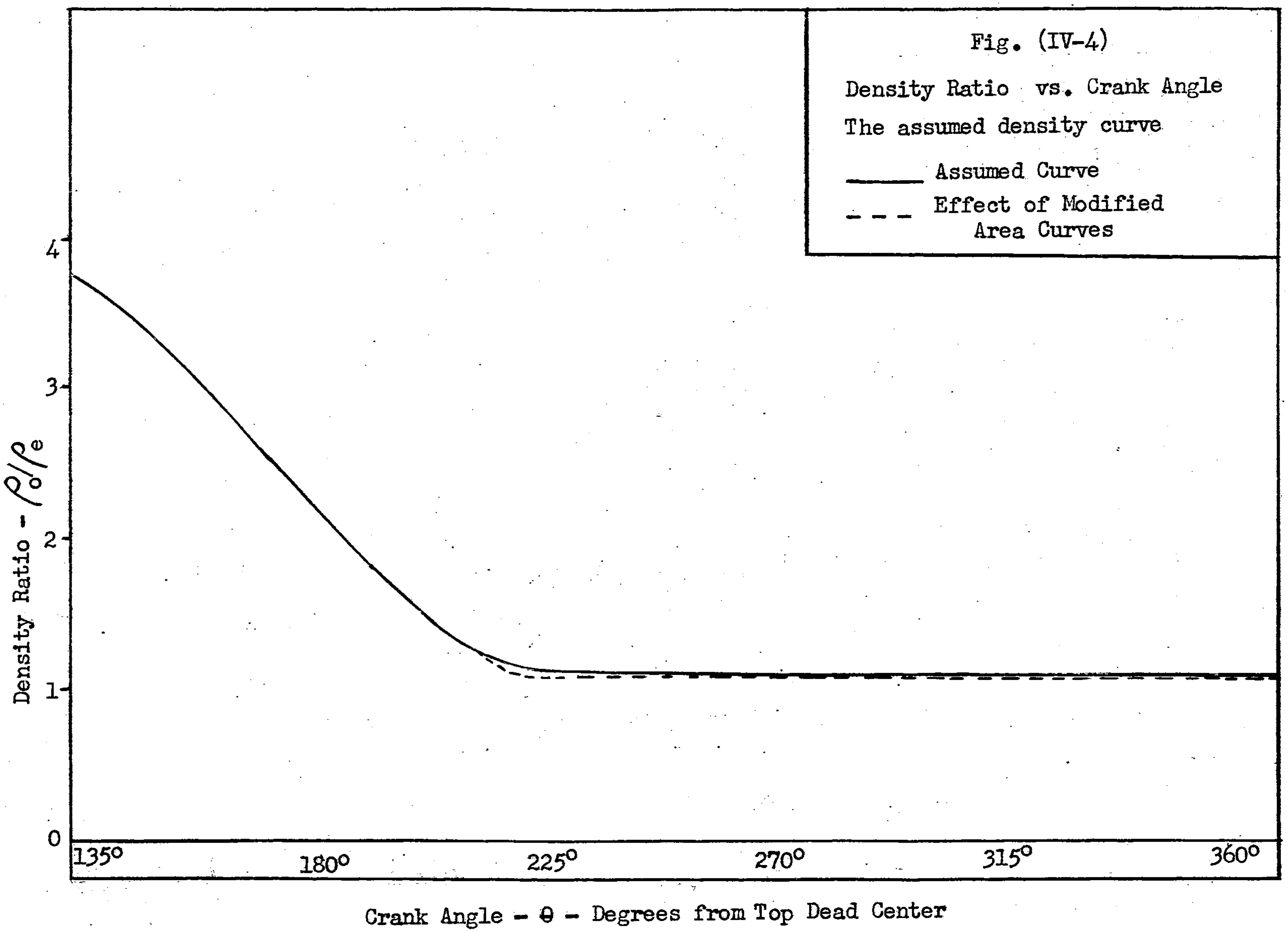

Fig. (IV-4)
Density Ratio vs. Crank Angle
The assumed density curve
Assumed Curve
Effect of Modified Area Curves
Density Ratio - ρ_o/ρ_e
4
3
2
1
0
135°
180°
225°
270°
315°
360°
Crank Angle - θ - Degrees from Top Dead Center

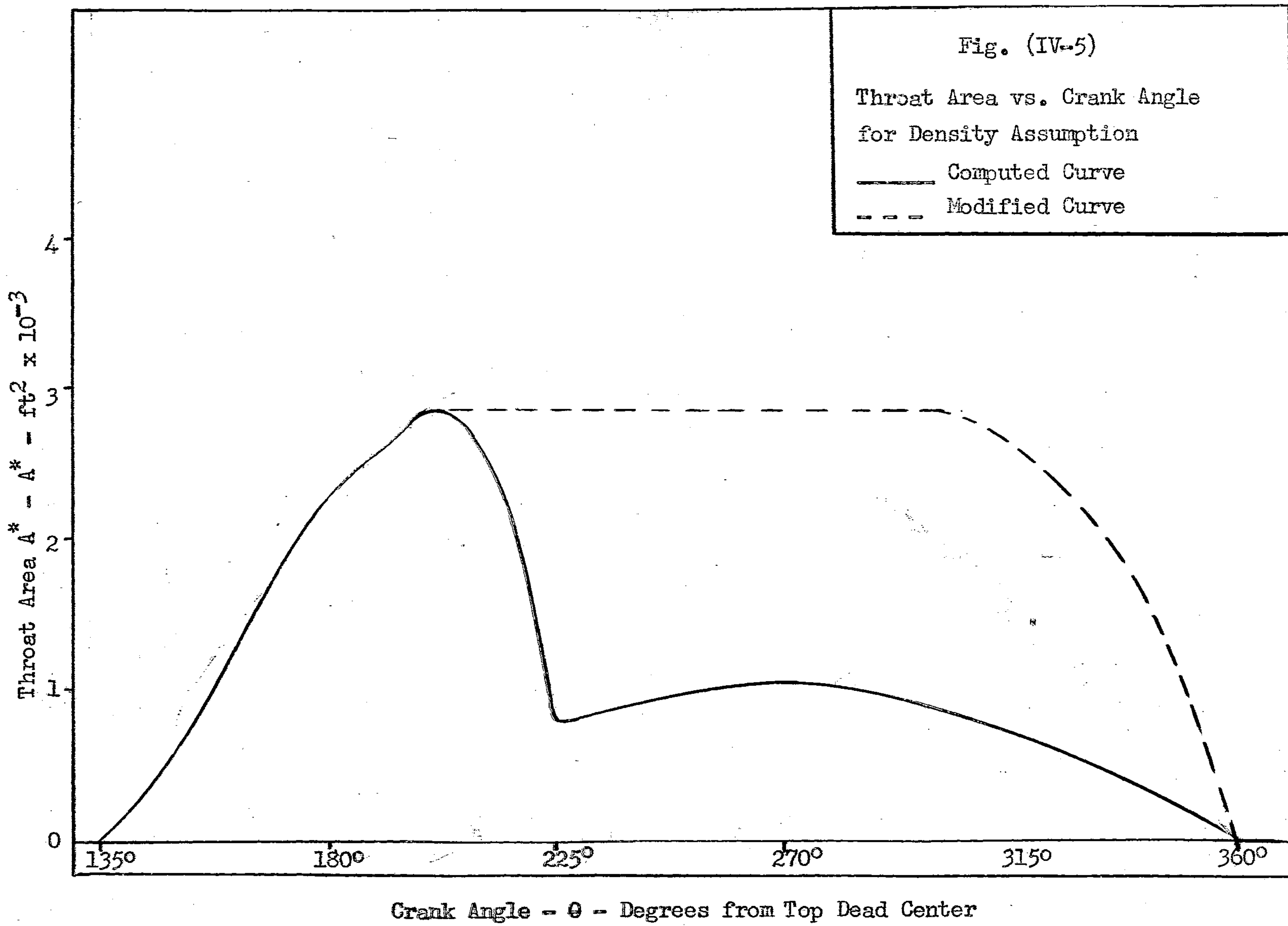
Fig. (IV-5)
Throat Area vs. Crank Angle
for Density Assumption
Computed Curve
Modified Curve
4
3
2
1
0
Throat Area A* - A* - ft² x 10⁻³
135°
180°
225°
270°
315°
360°
Crank Angle - θ - Degrees from Top Dead Center

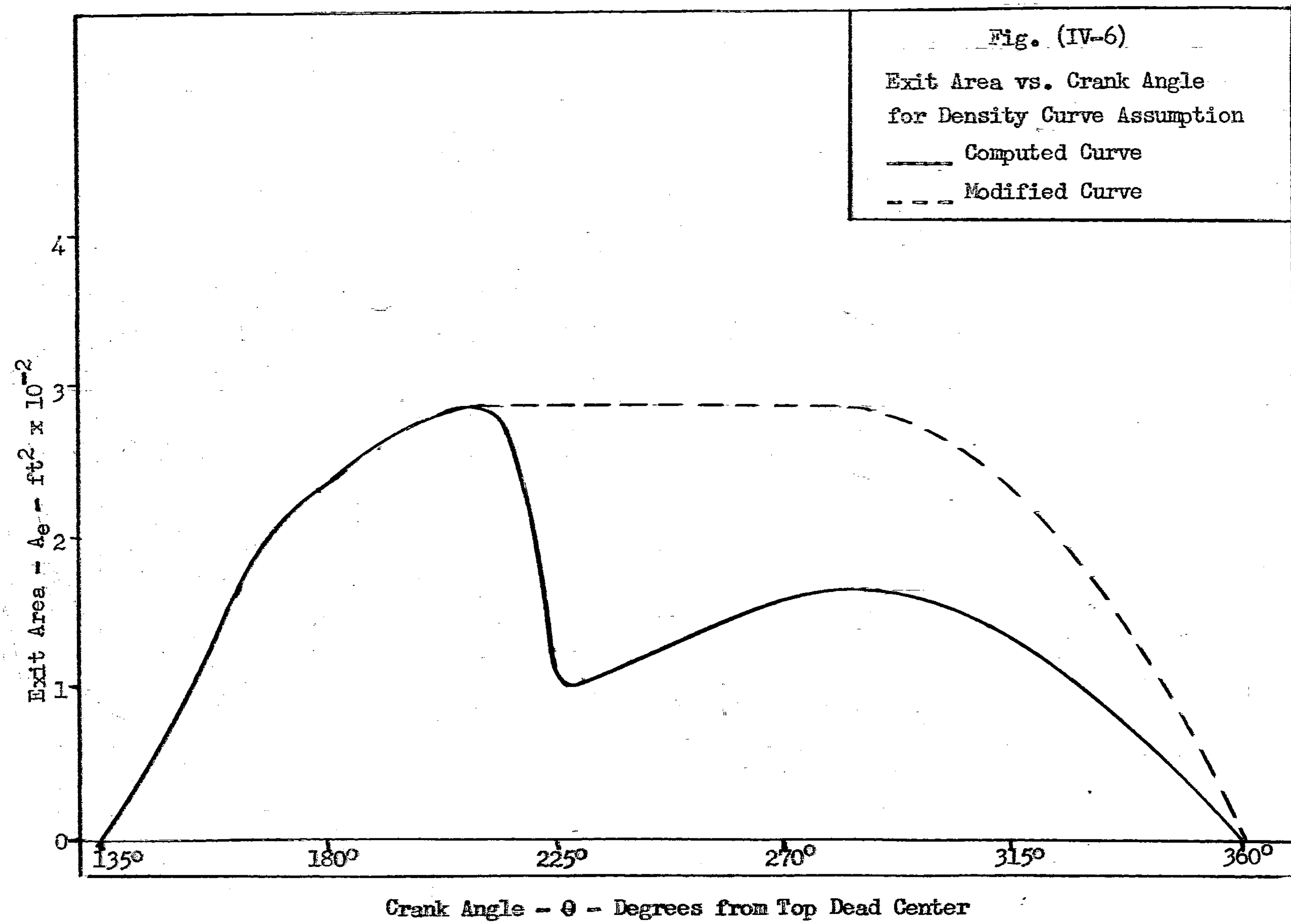

Fig. (IV-6)
Exit Area vs. Crank Angle for Density Curve Assumption

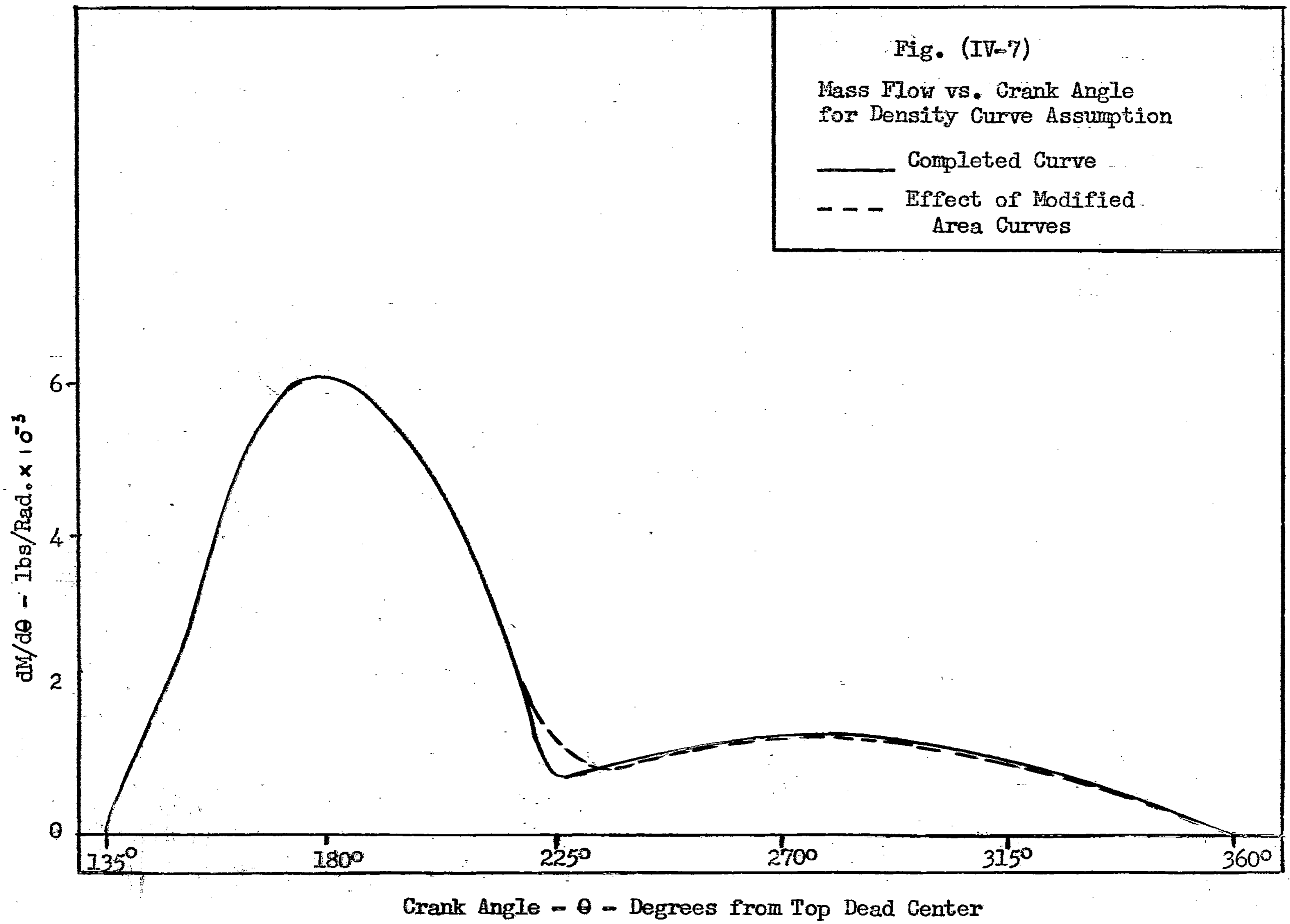
Fig. (IV-7)
Mass Flow vs. Crank Angle
for Density Curve Assumption
Completed Curve
Effect of Modified Area Curves
dM/dθ - lbs/Rad. x 10⁻³
6
4
2
0
135°
180°
225°
270°
315°
360°
Crank Angle - θ - Degrees from Top Dead Center

CHAPTER V

KINEMATIC PRESENTATION

The preceding chapter developed the necessary area-time relations for the nozzle. The remaining task is to demonstrate that a reasonable and practical mechanism can control the throat and exit areas according to the desired values.

This chapter describes only the essential parts of the mechanism. It is only intended to show that such a device is possible. The drawings are schematic and the linkages are simplified for the sake of clarity.

Figure (V-1) is a diagrammatic representation of a mechanism which can perform the required motions. The device also satisfies the requirements of a smooth continuous channel, and a reasonable seal when closed.

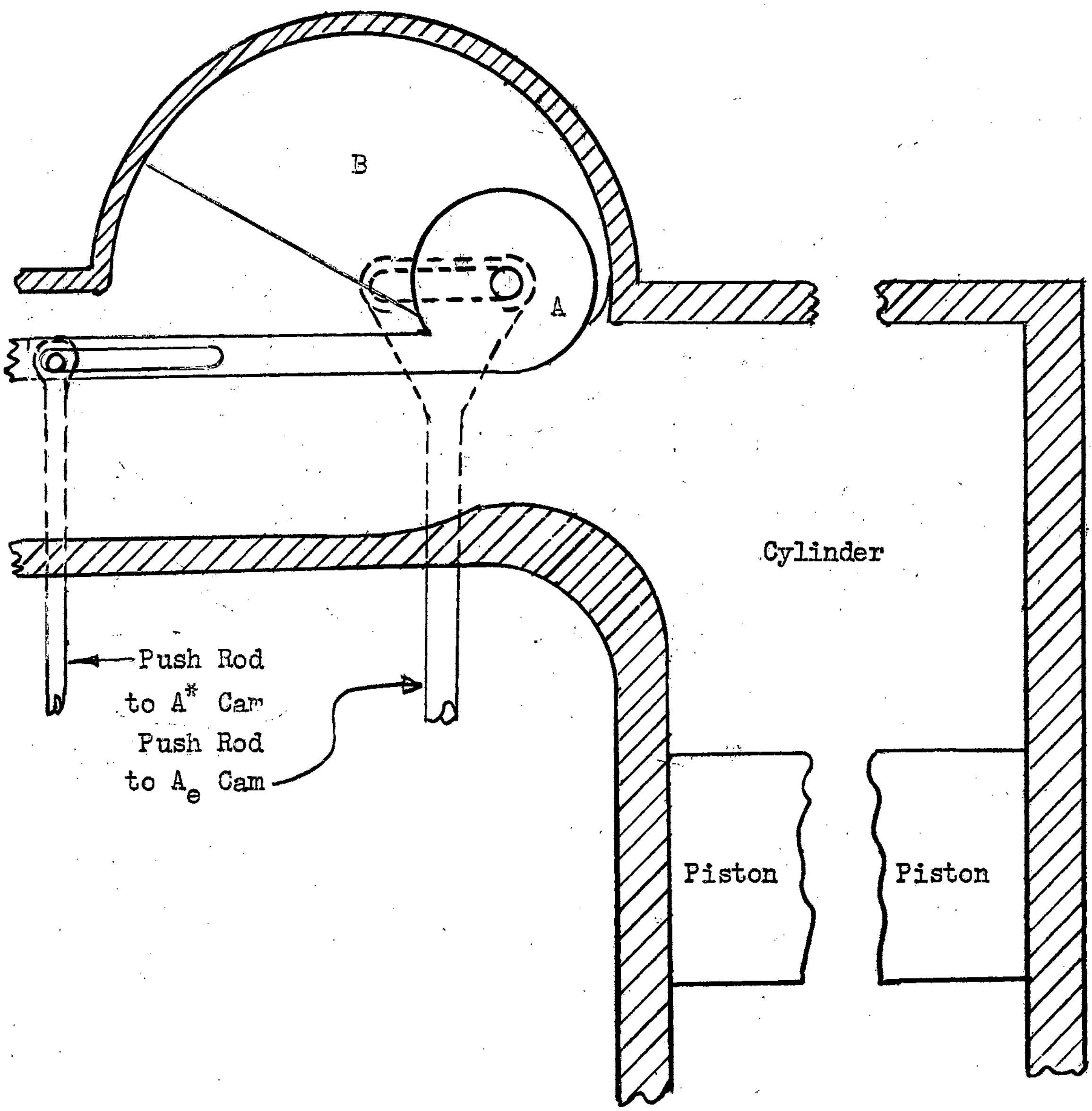

Fig. (V-1). Schematic Diagram of the Valve Mechanism.

The essential parts of this valve are the movable boundary (A) supported by the sliding block (B) which also provides the necessary seal when the valve is closed. The push rod connections must allow for the lateral motion of the movable boundary.

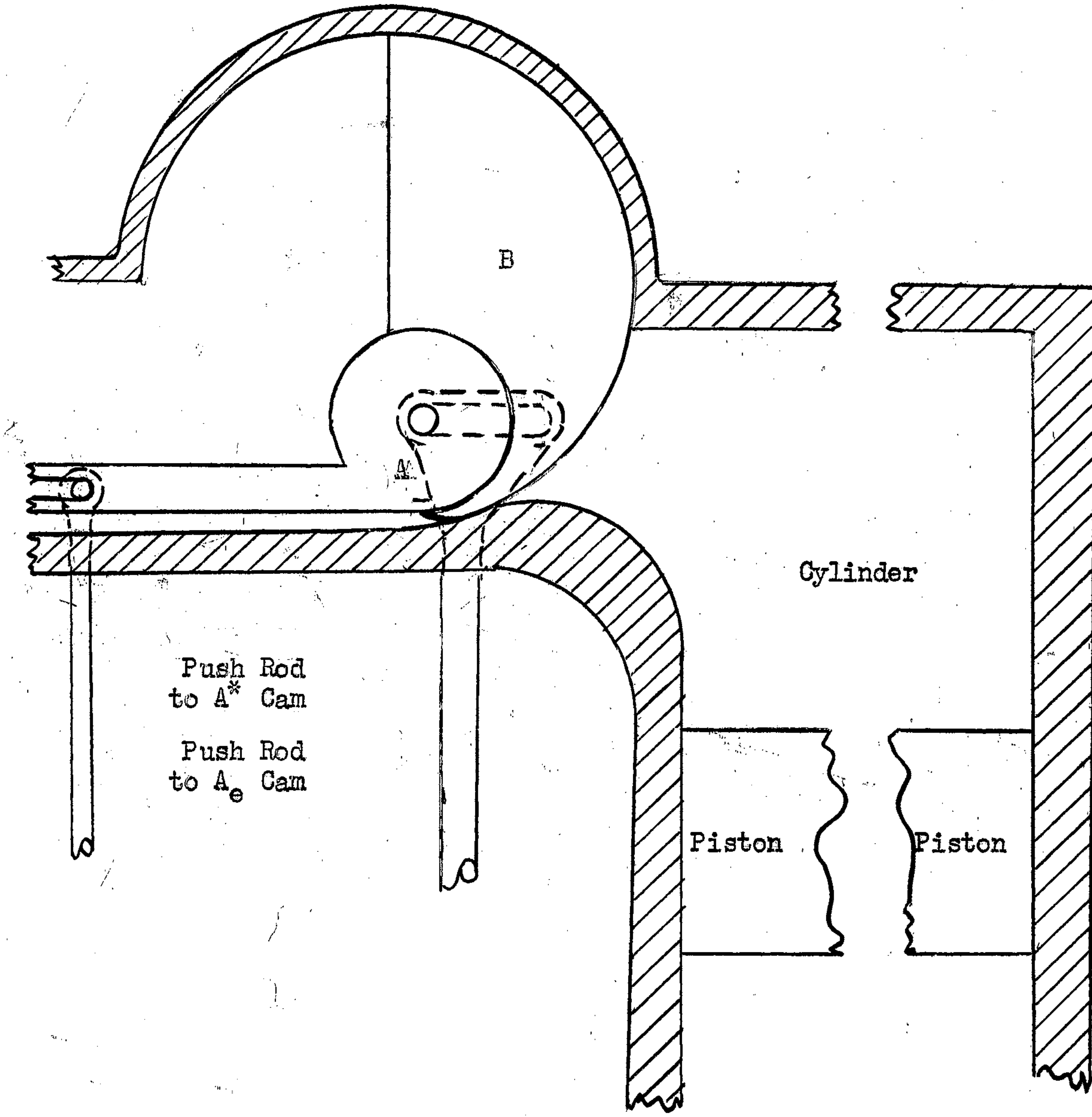

Fig. (V-2). Schematic Diagram of the Valve (closed position).

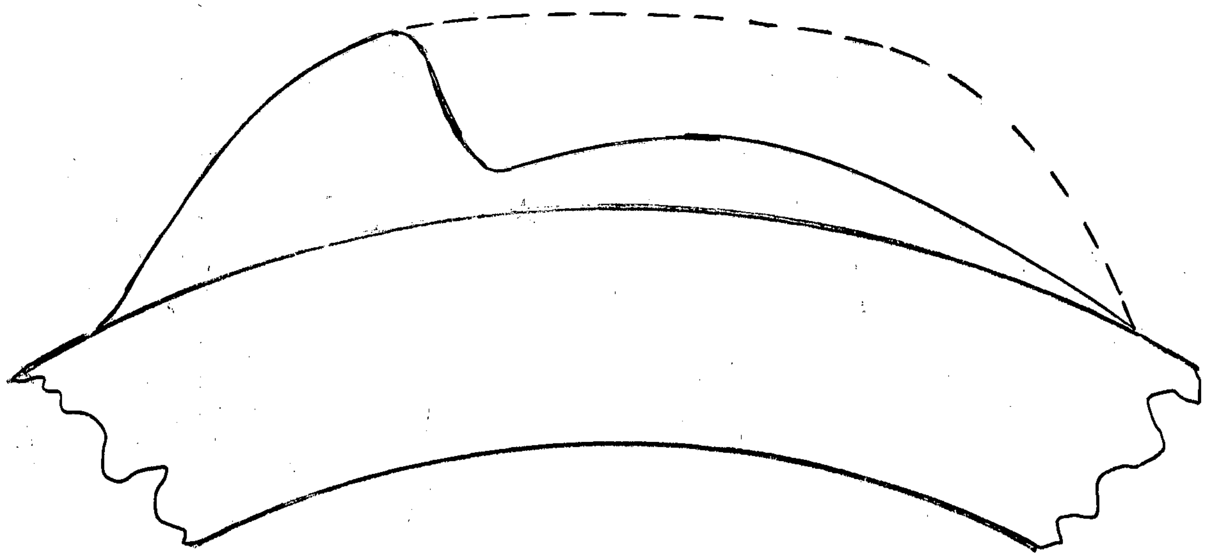

Fig. (V-3). Profile of the Throat Area Cam.

This Figure shows the shape of the throat area cam. The drawing is a segment of a ring type cam. This cam with a knife edge follower controls the throat area A* of the nozzle. The solid line represents the contour needed to satisfy the throat area conditions as computed from the density curve assumption; the dashed outline represents the modified throat area curve.

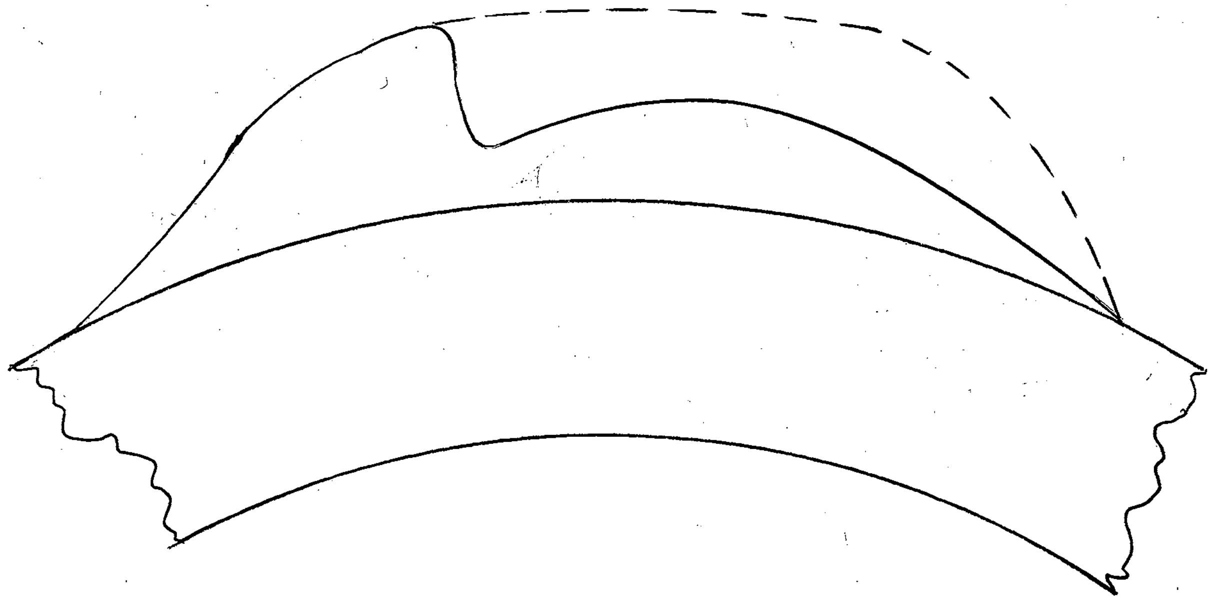

Fig. (V-4). Profile of the Exit Area Cam.

This Figure shows the shape of the exit area cam. The drawing is a segment of a ring type cam. This cam with a knife edge follower controls the exit area A_e of the nozzle. The solid line represents the contour needed to satisfy the exit area conditions as computed from the density curve assumption; the dashed outline represents the modified exit area curve.

CHAPTER VI

ANALYSIS AND CONCLUSIONS

A. Analysis of Area Curves.

Both of the developed area curves show an undesirable discontinuity in the region of 210°. Since the flow is sub-critical in this region there is no need for a throat, the nozzle will expand the gas to the exit conditions without the expanding channel which is required for the super-critical flow. Thus the exact shape of the throat area curve (in the sub-critical region) has little bearing on the objective of this report; but the kinematics of the valve require that the mechanism (which provided the throat for the super-critical region) have a continuous motion. Therefore, an arbitrary curve was selected to suit this requirement.

The exit area curve must also be modified to suit the kinematic requirement of continuity. However, the exit area controls the sub-critical flow; therefore, any changes in this curve will affect the assumed density vs. time relationship. An arbitrary exit area curve for this region was selected to fit the kinematic requirement. A point by point computation was carried out to determine the effect of the new curve on the assumed density-time relationship. It is evident from Fig. (VI-4) that the effect is small. The reason for this is found in the fact that, when the value of the density ratio approaches unity, the density ratio is insensitive to changes in the divergence function.

Although the altered exit area curve produces changes in the shape of the assumed density curve, the total developed kinetic energy remains unchanged.

B. Discussion of Attempted Solutions. This section presents the experience encountered in the several attempts toward a solution. The solutions are classified according to initial assumptions.

1. Gas state assumptions. This type of solution depends on the selection of an appropriate gas condition versus time curve from which the required area curves are computed. Solutions were attempted for each of the following gas properties: (1) pressure, (2) temperature, (3) density. The density solution is the one which is presented in detail in this report. Since density is closely related to mass there are fewer operations required in the solution. This simplifies the calculations. The results obtained with the pressure and temperature assumptions were essentially the same as those obtained from the density assumption.

Several different types of mathematical curves were tried in an attempt to improve the sub-critical solution. It was found that this solution could be improved by increasing the complexity of the curve, but these improvements required a disproportionate amount of labor. The additional labor is not justified in view of the fact that the area curves may be altered in the manner described in the analysis of the area curves.

2. Throat area assumption. This type of solution depends on the appropriate selection of a throat area versus time curve and calculating the required gas properties and exit area.

The approach to this solution is indicated by the development which precedes and includes equation (III-9). This solution produces smoother area curves in the super-critical region, but tends to produce impractical and imaginary values of the exit area in the sub-critical region. In order to avoid this difficulty great care must be exercised to insure that the density ratio is greater than unity at all times.

This solution involves an integration process and, therefore, is more tedious than the solutions based on the gas state assumption. The integration indicated in the left hand term of equation (III-9) can be simplified by assuming a throat area vs. time function of the following form:

$$A^* = V^{\frac{\gamma+1}{2}} (K_1 - K_2 \cos wt - K_3 \cos 2\, wt \, \ldots\ldots$$

$$- K_5 \sin wt - K_6 \sin 2\, wt \, \ldots\ldots)$$

where $K_{1,2,3\ldots}$ = Fourier Coefficients

Thus the fraction $A^*/V^{\frac{\gamma+1}{2}}$ can be integrated directly.

If the throat area curve is carefully selected, this solution can yield better results than the gas state solutions, but is inherently more tedious.

3. Exit area assumption. This solution depends on the appropriate selection of an exit area versus time curve and the calculation of the required gas conditions and throat area.

This solution gives the best results in both the super-critical and sub-critical regions, but requires the integration of equation (III-3). This integration cannot be performed directly and must be solved by means of the calculus of finite differences. The improvement in the results is hardly worth the effort.

This method is suggested if a differential analyser is available for the integration of equation (III-3).

C. Conclusions.

The results of this report indicate that a considerable improvement is possible in the performance of compound engines. Part of this improvement can be effected by developing the maximum kinetic energy in the exhaust gases with an exhaust valve designed like a DeLaval nozzle. The thermodynamic requirements of the nozzle can be solved with a reasonable amount of effort. Finally it is possible to design a mechanism to act as a variable nozzle exhaust valve.

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THESIS TITLE: KINEMATICS OF A VARIABLE NOZZLE USED AS AN EXHAUST VALVE FOR RECOVERY OF POWER FROM INTERNAL COMBUSTION ENGINES

NAME OF AUTHOR: CHARLES H. HUBER

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