CONTROLLER DESIGN FOR NONLINEAR SYSTEMS BASED ON SIMULTANEOUS STABILIZATION THEORY AND DESCRIBING FUNCTION MODELS

By

Amir Nassirharand

Bachelor of Science
Oklahoma State University
Stillwater, Oklahoma
1980

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1981

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the Degree of
DOCTOR OF PHILOSOPHY
July, 1986

CONTROLLER DESIGN FOR NONLINEAR SYSTEMS BASED

ON SIMULTANEOUS STABILIZATION THEORY AND

DESCRIBING FUNCTION MODELS

Thesis Approved:

Karl N. Reid
Thesis Adviser

James H. Taylor

James R. Rowland

Atmaram H. Soni

Gary E. Young

Norman N. Durham
Dean of Graduate College

ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to my major advisor Dr Karl N. Reid for his support, guidance and technical critique throughout my graduate program and in preparing the final manuscript of this thesis. His invaluable guidance in teaching me to communicate both orally and in writing is greatly appreciated.

I also wish to thank Dr. James H Taylor, General Electric Corporate Research and Development, for his invaluable comments, suggestions, and support through out this research project. I thank him for allocating his time in discussing technical aspects of the project over the telephone. This was valuable in realizing some of the practical applications of this work.

I thank Dr James R. Rowland, Professor and Chairman, Department of Electrical and Computer Engineering, University of Kansas, for his encouragement, enthusiasm, and technical support.

I thank Dr. Karl Reid, Dr James Taylor, Dr James Rowland, Dr Atmaram Soni, and Dr Gary Young for serving as members of my advisory committee. Dr. Chandler, Department of Computer and Information Sciences, Oklahoma State University, and my colleague, Mr Sankaran Mohan, MAE computer-lab systems manager, School of Mechanical and Aerospace Engineering, were quite helpful with their advice on some of the computer aspects of this project.

I would also like to thank my former supervisor, Mr. Thomas A. Cook, CEAT CAD lab manager and lecturer, School of Mechanical and

Aerospace Engineering, whose understanding made my Ph.D. program much easier. I thank my colleague, Mr. Ronald D. Delahoussaye, CEAT CAD lab assistant manager, for teaching me the fundamental concepts of computer graphics. His efforts in building and advancing the graphics capabilities of the CAD lab was of particular value in speeding the analysis of final results of this thesis work and advancing the quality of the software which I developed

I wish to thank my daughter, Nicole, for her patience which was a definite force towards the completion of this research study. I dedicate this thesis to her. I also wish to thank my parents, Mr. and Mrs Hassan Nassirharand, for their support, encouragement, sincere concern, and patience.

This research project was in part funded by the School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, Oklahoma, 74078

TABLE OF CONTENTS

Chapter		Page
I.	INTRODUCTION	1
	Objectives	1
	Outcome	3
	Contributions	3
	Controller Synthesis Procedure	4
	Software	5
	Demonstration Example Problem	7
II.	LITERATURE SURVEY	9
	Controller Design	9
	Describing Functions	11
	System Identification Theory	15
	Simultaneous Stabilization	16
III.	CONTROLLER SYNTHESIS PROCEDURE	18
	Step 1 Performance Specifications	20
	Step 2 Input/Output Characterization	22
	Step 3 System Identification	24
	Step 4 Controller Parameterization	26
	Simultaneous Stabilization	27
	Definition	29
	Formulation	32
	Step 5 Controller Synthesis	33
	Step 6 Design Validation	36
IV.	DEMONSTRATION EXAMPLE PROBLEM	38
	Problem Statement	41
	Step 1 Performance Specifications	41
	Step 2 Input/Output Characterization	45
	Step 3 System Identification	47
	Analysis of System Identification Results	49
	Step 4 Controller Parameterization	55
	Step 5 Controller Synthesis	56
	Step 6 Design Validation	57
	Dual-Range Linear Controller Design Vs Single-Range Linear Controller Design	60
	Dual-Range Linear Controller Design Vs. Multi-Range Nonlinear Controller Design	63

Chapter Page

V. SUMMARY AND RECOMMENDATIONS.. 65

Summary 65
Recommendations........ 67

A SELECTED BIBLIOGRAPHY 70

APPENDIXES...... 75

APPENDIX A IDENTIFICATION OF SINGLE-INPUT SINGLE-OUTPUT LINEAR SYSTEMS FROM FREQUENCY RESPONSE DATA .. 76

APPENDIX B A SYSTEMATIC PROCEDURE FOR PERFORMING COPRIME FACTORIZATION OVER s. 83

APPENDIX C APPLICATIONS OF A FACTORIZATION APPROACH TO CONTROLLER DESIGN FOR CLOSED-LOOP SYSTEM STABILITY 89

APPENDIX D A TUTORIAL ON DFGEN SOFTWARE UTILITY 100

APPENDIX E A TUTORIAL ON SYSID SOFTWARE UTILITY. 107

APPENDIX F A TUTORIAL ON DRLCD SOFTWARE UTILITY. 152

LIST OF TABLES

Table		Page
I	Frequency Response Data for the SIDF Model Number 4 . .	48
II.	Frequency Response Data for the SIDF Model Number 1. . .	48
III.	Frequency Response Comparison for the SIDF Model Number 4 and the Approximating Open-Loop System, $G_0(s)$	53
IV.	Frequency Response Comparison of the SIDF Model Number 1 and the Approximating Open-Loop System, $G_1(s)$	54
V	Frequency Response Data of Transfer Function $W_1(s)$..	109
VI	SYSID Command Definition	112

LIST OF FIGURES

Figure		Page
1.	A Typical Control System	2
2.	The Systematic Controller Synthesis Procedure	6
3.	User Interface With Software Utilities	8
4	Schematic Model of a Saturation Element	12
5.	Describing Function Model of a Saturation Element.	14
6.	Representation of An Open-Loop System Operating At a Nominal Condition..	28
7.	Open-Loop Pole Location of $T'_0(s)$	30
8.	Representation of an Open-Loop System Operating Away from the Nominal Condition	31
9.	Open-Loop Pole Locations of $T'_1(s)$	31
10.	Position Servo Open-Loop Model Schematic	39
11.	Model of the Servomotor Saturation Effects	40
12.	Structure of the Closed-Loop Feedback System	42
13	Structure of the Nonlinear Controller and the Nonlinear Closed-Loop Feedback System	43
14	Normalized Step Responses of the Nonlinear PID and Plant	44
15.	SIDF Models of the Nonlinear Plant	46
16.	Approximation of the Frequency Response of the Quasilinear Model Number 4 with a Linear Model of an Open-Loop System	50
17.	Approximation of the Frequency Response of the Quasilinear Model Number 1 with a Linear Model of an Open-Loop System..	51
18	Subroutine MOTor-LoaD Describing the Dynamic Behavior of the Nonlinear Plant	58

Figure Page

19. Normalized Step Response Plots of the Synthesized Linear Controller and the Nonlinear Plant 59

20. Linear PID Control of the Nonlinear Plant 61

21 Normalized Step Response Plots of the System with Linear PID Control of the Nonlinear Plant 62

22 Stabilization of A Single Variate Servomechanism.... 91

23. The User-Input Data File 0001DATA*DFGEN IN 104

24 Magnitude Plots of SIDF Models.... 105

25. Phase Plots of SIDF Models... 106

26. SYSID Main Command List 111

27 Example 1 CRT Screen After the Execution of the D Command. 117

28. Example 1 Examination of the Quality of Fit by Viewing the Nyquist Plot 118

29. Example 1 Examination of the Quality of Fit by Viewing the Magnitude Plot. 119

30. Example 1 Examination of the Quality of Fit by Viewing the Phase Plot 120

31 Example 2/Task 4 - CRT Screen After the Execution of the First D Command 123

32. Example 2/Task 4 - CRT Screen After the Execution of the C Subcommand 123

33 Example 2/Task 5 - Examination of the Quality of Fit by Viewing the Nyquist Plot.. 124

34. Example 2/Task 7 - Investigation of the Gain in the Quality of Fit by Increasing Transfer Function Order From Two to Three 125

35. Example 2/Task 8 - Investigation of the Loss in the Quality of Fit by Decreasing the Number of Transfer Function Zeros Via the Visual Examination of the Nyquist Plot. 127

36. Example 2/Task 8 - Investigation of the Loss in the Quality of Fit by Decreasing the Number of Transfer Function Zeros Via the Visual Examination of the Magnitude Plot 128

Figure		Page

37. Example 2/Task 8 - Investigation of the Loss in the Quality of Fit by Decreasing the Number of Transfer Function Zeros Via the Visual Examination of the Phase Plot.. 129

38. Example 2/Task 9 - CRT Screen After the Execution of the F Command...... 131

39. Example 2/Task 9 - CRT Screen After the Execution of the D Command. 132

40. Example 2/Task 9 - CRT Screen After the Execution of the C Subcommand.. 132

41. Example 2/Task 10 - CRT Screen After the Execution of the O Command 133

42. Example 2/Task 10 - CRT Screen After the Execution of the D Command 134

43. Example 2/Task 10 - CRT Screen After the Execution of the C Subcommand. 134

44. Example 2/Task 11 - Visual Examination of the Difference Between the Nyquist Plot of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model.... 136

45. Example 2/Task 11 - Visual Examination of the Difference Between the Magnitude Plot of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model.... 137

46 Example 2/Task 11 - Visual Examination of the Difference Between the Phase Plot of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model. . . 138

47 Example 2/Task 11 - Visual Inspection of the Per Cent Error Between the Magnitude of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model 139

48 Example 2/Task 11 - Visual Inspection of the Per Cent Error Between the Phase of the Approximating Linear Systems, $G_0(s)$, with that of the Quasilinear Model. 140

49. Example 2/Task 11 - Visual Inspection of the Relative Error Between the Magnitude of the Approximating Linear Systems, $G_0(s)$, with that of the Quasilinear Model 141

50 Example 2/Task 11 - Visual Inspection of the Relative Error Between the Phase of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model 142

51. Example 3/Task 5 - CRT Screen After the Execution of the D Command.. 144

Figure | Page

52. Example3/Task 6 CRT Screen After the Execution of the C Subcommand 144

53. Example 3/Task 7 - Visual Examination of the Difference Between the Nyquist Plot of the Approximating System, $G_1(s)$, with that of the Quasilinear Model. 145

54. Example 3/Task 7 - Visual Examination of the Difference Between the Magnitude Plot of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model 146

55. Example 3/Task 7 - Visual Examination of the Difference Between the Phase Plot of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model. 147

56 Example 3/Task 11 - Visual Inspection of the Per Cent Error Between the Magnitude of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model 148

57. Example 3/Task 7 - Visual Inspection of the Per Cent Error Between the Phase of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model 149

58. Example 3/Task 7 - Visual Inspection of the Relative Error Between the Magnitude of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model. 150

59 Example 3/Task 7 - Visual Inspection of the Relative Error Between the Magnitude of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model 151

60. DRLCD Main Command List. 156

61. CRT Screen After the Execution of the H Command 156

62. CRT Screen After the Execution of the H R Command. 157

63 CRT Screen After the Execution of the H.S Command. . . 157

64. CRT Screen After the Execution of the H I Command. 158

65. CRT Screen After the Execution of the H C Command 158

66. CRT Screen After the Execution of the H.D Command. 159

67 CRT Screen After the Execution of the H.T Command. . . . 159

68 CRT Screen After the Execution of the H O Command 160

69. CRT Screen After the Execution of the H V Command . . 160

70. CRT Screen After the Execution of the H Q Command 161

Figure | Page

71. CRT Screen After the Execution of the T.C GO Command........... 163

72. CRT Screen After the Execution of the T.C G1 Command........... 163

73. CRT Screen After the Execution of the C.I Command.............. 165

74. The CRT Screen After the Execution of the T.D CS Command...... 167

75. The CRT Screen After the Execution of the T.D RS Command....... 168

76. Normalized Step Response of the Starting Linear Controller and the Nonlinear Plant....................................... 169

77. Frequency Response Plots and Unit Step Response Plots of GCLD, $\Sigma(C_s, G_0)$, and $\Sigma(C_s, G_1)$.............................. 171

78. The CRT Screen After the Execution of the T.D C Command (Status of the Synthesized Controller)................ 172

79. The CRT Screen After the Execution of the T.D R Command (Status of the Synthesized Parameter r'(s)).................. 173

80. Frequency Response Plots and Unit Step Response Plots of GCLD, $\Sigma(C, G_0)$, and $\Sigma(C, G_1)$................................. 174

81. Normalized Step Response Plots of the Linear Controller and the Nonlinear Plant....................................... 175

NOMENCLATURE

a'	The amplitude of the Excitation Signal
$a_0, a_1, \ldots, a_n$	The coefficients of the Denominator Polynomial of Transfer Function $g(j\omega)$ (See Equations (a.1) and (a.17))
$a_1(s)$	A Transfer Function Defined by Equation (3.12)
A	A Constant System Matrix (See Equations (d.4) and (d.28))
A_0	A Constant Matrix Defined by Equation (d.10) (See Equation (b.30))
A'_0	A Constant Matrix Defined by Equation (b.11) (See Equation (b.31))
$b_0, b_1, \ldots, b_m$	The coefficients of the Numerator Polynomial of Transfer function $g(j\omega)$ (See Equations (a.1) and (a.17))
$b_1(s)$	A Transfer Function Defined by Equation (3.13)
B	A Constant Input Matrix (See Equations (b.4) and (b.28))
$c(s)$	A Stabilizing Controller (See Equation (3.15))
C	A Constant Matrix (See Equation (b.5) and (b 28))
$C_s(s)$	A Transfer Function (The Starting Controller Solution Serving the Controller Synthesis Algorithm)
$d(s)$	A Transfer Function ($d(s)$ and $n(s)$ are Coprime Factors of $G(s)$) (See Equation (3.9))
$d_0(s)$	A Transfer Function ($d_0(s)$ and $n_0(s)$ are Coprime Factors of $G(s)$) (See Equation (3.15))
$d_1(s)$	A Transfer Function ($d_1(s)$ and $n_1(s)$ are Coprime Factors of $G_1(s)$)

$D(s)$	A Transfer Function ($D(s)$ and $N(s)$ are Coprime Factor of $T(s)$)
DFGEN	Describing Function GENerator (A Software Utility)
DRLCD	Dual-Range Linear Controller Design, Also a Software Utility
e	Error signal
$e(\omega)$	An Error Function Defined by Equation (3.8)
E	A Matrix Representing the Least-Squares Error (See Equations (c.9) and (c.13)), Also a Constant Matrix (See Equation (b.5))
E_i	A complex Number which Represents the error in approximating the Frequency Response Datum ($R_i + j\omega_i$) at Frequency ω_i (See Equation (a.4))
E'	A Matrix Representing the Frequency Response Error (See Equations (a.24))
E''	A Matrix Representing the Frequency Response Error (See Equation (a.28))
f	A Dummy Rational Function
f_1	A Dummy Rational Function
f_2	A Dummy Rational Function
f_3	A Dummy Rational Function
f_4	A Dummy Rational Function
f_c	A Constant (f_c = 10 Mewton-Meters)
$f_d(t_i)$	The Digitized and Normalized User-Defined Step Response (See Equation (3 2))

$f_D(e)$	Derivative Gain (See Figure 13)
$f_I(e)$	Integral Gain (See Figure 13)
$f_P(e)$	Proportional Gain (See Figure 13)
$f_r(t_i)$	The Digitized and Normilized Step Response of the Reference Linear Model (See Equation (3 2))
f_v	A Constant (f_v = 0.1 Newton-Meters)
F	An Objective Function Whose Optimized Value Corresponds to the Near-Optimum Linear Controller (See Equation (3.22))
$\bar{F}$	An Auxiliary Constant Matrix Given by (b.23)
F'	A Constant Matrix (See Equation (b.27) and (b.29))
F_0	An Auxiliary Function Defined by Equation (3.23)
F_1	An Auxiliary Function Defined by Equation (3.24)
F_{id}	An Objective Function Defined by Equation (3.2)
g	A Dummy Rational Function
$g(j\omega)$	A Transfer Function (See Equation (a 1) and a 17))
$G(j\omega)$	The Fitted Frequency Response
$G_0(s)$	A Transfer Function, $G_0(s)$ Represents the Dynamic Behavior of the Nonlinear Plant With the Most Gain Amplification Effect
$G_1(s)$	A Transfer Function, $G_1(s)$ Represents the Dynamic Behavior of the Nonlinear Plant With the Least Gain Amplification Effect
$G_a(s)$	An Auxiliary Transfer Function $G_a(s) = \frac{b_1(s)}{a_1(s)}$
$G_d(j\omega)$	The Desired Frequency Response
$G_r(s)$	The User-Defined Reference Linear Model
GCLD	The User-Defined Reference Linear Model (Same as $G_r(s)$)

h	A Dummy Rational Function
H	An Algebraic Ring Consisting of the Transfer Function of Bounded-Input Bounded-Output Stable Systems
i	A Dummy Integer Index
I	The Identity Matrix
Imarg	The Imaginary Part of arg
I_i	The Imaginary Part of the User-Supplied Frequency Response Datum at Desecrate Frequency ω_i (See Equation (a.4))
j	$j = \sqrt{-1}$
J	Load Moment of Inertia
k'	The Total Number of User-Supplied Frequency Response Data
K	A Constant Matrix (See Equations (b.19) and (b.29))
$\bar{K}$	An Auxiliary Constant Matrix (See Equation (b.15))
l	The Total Number of Observed Frequencies
L	Iteration Number (A Subscript)
m	A Dummy Integer Variable (The Number of Zeros of A Transfer Function)
m'	A Dummy Integer Variable
m_1	A Constant (m_1 = 5 Newton-Meters/Volt)(See Figure 11)
m_2	A Constant (m_2 = 1 Newton-Meter/Volt)(See Figure 11)
$M'(j\omega)$	Defined by Equation (a.3)
$M''(j\omega)$	Defined by Equation (a.19)
MIMO	Multiple-Input Multiple Output
MSE	Mean-Square Error
$M_{k'}$	Magnitude at Period k' (See Equation (d.1))
n	An Dummy Integer Variable (The Order of A Transfer Function)

$n(s)$	A Transfer Function ($n(s)$ and $d(s)$ are Coprime Factors of $G(s)$) (See Equation (3.9))
$n_0(s)$	A Transfer Function ($n_0(s)$ and $d_0(s)$ are Coprime Factors of $G_0(s)$)
$n_1(s)$	A Transfer Function ($n_1(s)$ and $d_1(s)$ are Coprime Factors of $G_1(s)$)
$N'(j\omega)$	Defined by Equation (a.3)
$N''(j\omega)$	Defined by Equation (a.19)
$N(s)$	A Transfer Function ($N(s)$ and $D(s)$ are Coprime Factor of $T(s)$)
$p(s)$	A Transfer Function ($p(s)$ and $q(s)$ are Coprime, and They, Together with $n(s)$ and $d(s)$, Satisfy the Bezout Identity) (See Equation (3.10))
$p_0(s)$	A Transfer Function ($p_0(s)$ and $q_0(s)$ are Coprime, and They, Together with $n_0(s)$ and $d_0(s)$, Satisfy the Bezout Identity)
$p_1(s)$	A Transfer Function ($p_1(s)$ and $q_1(s)$ are Coprime, and They, Together with $n_1(s)$ and $d_1(s)$, Satisfy the Bezout Identity)
$P(s)$	A Transfer Function ($P(s)$ and $Q(s)$ are Coprime, and They, Together with $N(s)$ and $D(s)$, Satisfy the Bezout Identity)
P'	A Constant Matrix (See Equation (b.26))
P	A Auxiliary Constant Matrix (See Equation (b.18))
$P_1, \ldots, P_n$	Coefficients of the Characteristic Polynomial of System Matrix A (See Equation (b.13))
$\bar{P}_1, \ldots, \bar{P}_n$	Coefficient of the Polynomial Given by Equation (b.14)
$P'_1, \ldots, P'_2$	Desired Pole Location of(A+BK) (See Equation (b.12))

q^i	A Column Vector Defind by Equation (b.16) (i=1,...,n-1, and n is System Order)
$q_0(s)$	A Transfer Function ($q_0(s)$ and $p_0(s)$ are Coprime, and They, Together with $n_0(s)$ and $d_0(s)$, Satisfy the Bezout Identity)
$q_1(s)$	A Transfer Function ($q_1(s)$ and $p_1(s)$ are Coprime, and They, Together with $n_1(s)$ and $d_1(s)$, Satisfy the Bezout Identity)
Q	A Constant Matrix (See Equation (b.17))
Q(s)	A Transfer Function (Q(s) and P(s) are Coprime, and They, Together with N(s) and D(s), Satisfy the Bezout Identity) (See Equation (b.3))
Q'	A Constant Matrix (See Equation (b.25))
r(s)	A Transfer Function (The Parameter in Parameterizing the Class of All Controllers that Stabilize One Unstable Plant Whose Dynamic Behavior is Adequately by A Linear Model)
r'(s)	A Transfer Function (The Parameter in Parameterizing the Class of All Controllers that Stabilize Two Systems Whose Dynamic Behavior is Adequately Represented by Two Linear Models)
R(s)	The Set of All Rational Function (For $s = j\omega$) with Real Coefficients
Rearg	The Real Part of arg
R_i	The Real Part of the User-Supplied Frequency Datum at Discrete Frequency ω_i (See Equation (a.4)
RIDF	Random Input Describing Function
s	The Complex Frequency $j\omega$

SIDF	Sinusoidal-Input Describing Function
SISO	Single-Input Single-Output
SRLCD	Single-Range Linear Controller Design
SSL	Small-Signal Linearization
SYSID	SYstem IDentification (A Software Utility)
t	Time
T	Period (See Equation (3.4))
T(s)	A Transfer Function Representing the Dynamic Behavior of an Open-Loop System (See Equation (c 1))
$T_0(s)$	A Transfer Function Representing the Specific Dynamic Behavior of an Open-Loop System (See Equation (c.15))
$T_{0d}(s)$	A Transfer Function ($T_{0d}(s)$ and $T_{0n}(s)$ are Coprime Factors of $T_0(s)$) (See Equation (c.18))
$T_{0n}(s)$	A Transfer Function ($T_{0n}(s)$ and $T_{0d}(s)$ are Coprime Factors of $T_0(s)$ (See Equation (c.17))
$T_{0p}(s)$	A Transfer Function ($T_{0p}(s)$ and $T_{0q}(s)$ are Coprime, and They, Together with $T_{0n}(s)$ and $T_{0d}(s)$, Satisfy the Bezout Identity) (See Equation (c.20))
$T_{0q}(s)$	A Transfer Function ($T_{0q}(s)$ and $T_{0p}(s)$ are Coprime, and They, Together with T_{0n}(s0 and $T_{0d}(s)$, Satisfy the Bezout Identity) (See Equation (c.21))
$T_1(s)$	A Transfer Function Representing the Dynamic Behavior of an Open-Loop System (See Equation (c 16))
$T_{1d}(s)$	A Transfer Function ($T_{1d}(s)$ and $T_{1n}(s)$ are Coprime Factors of $T_1(s)$)
$T_{1n}(s)$	A Transfer Function ($T_{1n}(s)$ and $T_{1d}(s)$ are Coprime Factors of T(s))

$T_{1p}(s)$	A Transfer Function ($T_{1p}(s)$ and $T_{1q}(s)$ are Coprime, and They, Together with $T_{1d}(s)$ and $T_{1n}(s)$, Satisfy the Bezout Identity) (See Equation (c.22))
$T_{1q}(s)$	A Transfer Function ($T_{1q}(s)$ and $T_{1p}(s)$ are Coprime, and They, Together with $T_{1n}(s)$ and $T_{1d}(s)$, Satisfy the Bezout Identity) (See Equation (c.22))
$T_a(s)$	A Transfer Function Representing the Specific Dynamic Behavior of the Open-Loop System Defined by Equation (c.23))
$T_{ac}(s)$	A Transfer Function (Representation of A Controller Capable of Stabilizing $T_a(s)$) (See Equation c 21))
$T_{ad}(s)$	A Transfer Function ($T_{ad}(s)$ and $T_{an}(s)$ are Coprime Factors of $T_a(s)$) (See Equation (c 27))
$T_{an}(s)$	A Transfer Function ($T_{an}(s)$ and $T_{ad}(s)$ are Coprime Factors of $T_a(s)$) (See Equation c.26))
$T_{ap}(s)$	A Transfer Function ($T_{ap}(s)$ and $T_{aq}(s)$ are Coprime, and They, Together with $T_{ad}(s)$ and $T_{an}(s)$, Satisfy the Bezout Identity) (See Equation (c 29))
$T_{aq}(s)$	A Transfer Function ($T_{ad}(s)$ and $T_{ap}(s)$ are Coprime, and They, Together with $T_{ad}(s)$ and $T_{an}(s)$, Satisfy the Bezout Identity) (See Equation (c.30))
$T_{ar}(s)$	A Transfer Function (The Parameter in Parameterizing the Class of All Controllers that Stabilize $T_a(s)$)
$T_c(s)$	A Transfer Function of the Controller that stabilizes T(s) (See Equations (c 8) and (c 13))
$T_d(s)$	A Transfer Function ($T_d(s)$ and $T_n(s)$ are Coprime Factors of T(s)) (See Equation (c 4))

$T_g(s)$	The Transfer Function of the Unity Closed-Loop Feedback System Whose Forward Path Transfer Function is $T_c(s)$ $T(s)$ (See Equation (c.9))
T_m	The Applied Torque to the Load (See Equation (4 4) and Figure (10))
$T_n(s)$	A Transfer Function ($T_n(s)$ and $T_d(s)$ are Coprime Factors of $T(s)$) (See Equation (c.3))
T_e	Torque (See Equation (4.3) and Figure (9))
$T_p(s)$	A Transfer Function ($T_p(s)$ and $T_q(s)$ are Coprime, and They, Together with $T_n(s)$ and $T_d(s)$, Satisfy the Bezout Identity) (See Equation (c.6))
$T_q(s)$	A Transfer Function ($T_q(s)$ and $T_p(s)$ are Coprime, and They, Together with $T_n(s)$ and $T_d(s)$, Satisfy the Bezout Identity) (See Equation (c.7))
$T_r(s)$	A Transfer Function The Parameter in Parameterizing the Class of All Controllers that Stabilize Plant $T(s)$ (This Parameterization is given by Equation (c.8))
$T_{sc}(s)$	A Transfer Function (Represents A Controller that Simultaneously Stabilizes the Open-Loop Linear Systems Whose Dynamic Behavior is Adequately Represented by $T_0(s)$ and $T_1(s)$)
$T'_0(s)$	A Dummy Linear Model (See Figure 6)
$T'_1(s)$	A Dummy Linear Model (See Figure 8)
$U_1, \ldots, U_n$	Coefficients of the Characteristic Polynomial of System Matrix A (See Equation (b.21))
$\bar{U}_1, \ldots, \bar{U}_n$	Coefficient of the Polynomial Given by Equation (b.22)
$U'_1, \ldots, U'_n$	Desired Pole Locations of (A + F'C) (See Equation (b.20))

V_{in}	The Input to the Nonlinear Plant (See Figure (10))
V^i	A Row Vector Defined by Equation (b 24) ($i = 1, . . , n - 1$, and n is System Order)
$W(\omega)$	A User Defined Weighing Function (See Equation (3.7))
$W_1(s)$	A Transfer Function Defined by Equation (e 1)
W_0'	The Identity Matrix
W'	A Diagonal Weighting Matrix (See Equation (a 15))
$\underline{x}(t)$	A Vector Representing System States (See Equation (1 1-a))
$\dot{\underline{x}}(t)$	A Vector Representing the Time Rate of Change of Vector $\underline{x}(t)$ (See Equation (1 1-a))
$\dot{x}_1$	Position (See Equation (4 1) and Figure (10))
x_1	Velocity (See Equation (4.1) and Figure (10))
x_2	$x_2 = \dot{x}_1$
$\dot{x}_2$	Acceleration (See Equation (4.2))
$X(t)$	A Vector of Dummy States (See Equation (d 4))
X'	A Matrix Defined by Equation (a.10)
X''	A Matrix Defined by Equation (a.25)
X	A Vector Representing the Time Rate of Change of Vector $X(t)$
$y(t)$	System Output (See Equation (1 1-b))
$Y(t)$	A Dummy Output (See Equation (d.5))

Greek Letters

α	A Weighing Coefficient (See Equation (3.22))
β	A Vector (See Equation (a.11))
β'	A Constant Vector (See Equation (a.26))
δ	A Constant (0.5 Volts), Also See Figure 13
$\delta(t)$	The Unit-Pulse Function
ω	Frequency
ω_n	Natural Frequency
π	A Constant (3.1415927)
$\phi_{k'}$	Phase at Period k' (See Equation (d.2))
$\Sigma(C(s),G(s))$	Representation of a Closed-Loop Unity Feedback System Whose Forward-Path Transfer Function is C(s) G(s). C(s) and G(s) are Dummy Transfer Functions in this Context.
θ	A Vector (See Equation (a.12))
θ_L	A Solution Vector (See Equation (a.14))
θ'	A Vector (See Equation (a.27))
ζ	Damping Ratio

CHAPTER I

INTRODUCTION

A typical closed-loop feedback control system is shown in Figure 1. The plant or process is the physical dynamic system to be controlled The analytical design of a feedback control system usually entails modeling the plant, and designing a controller that causes the system to behave in a desired way. In some cases, the plant may be described adequately by a linear model. In other cases, a nonlinear model may be required Methods are well established for the design of linear controllers for linear plants. A conventional linear controller (e g , a PID controller) may not produce adequate performance over the expected range of operating conditions, especially if the plant is highly nonlinear (1,2) In this case, a nonlinear controller, or a "multi-range" controller, may be preferred. Previously used methods for designing nonlinear controllers are heuristic, in that they rely heavily on the experience and judgment of the designer The need exists for a design method which is systematic and not heuristic, but which yields controller designs which are better than those produced from conventional linear system methods.

Objectives

The primary objective of the work presented herein was to develop a new systematic and algebraic linear controller design procedure for use

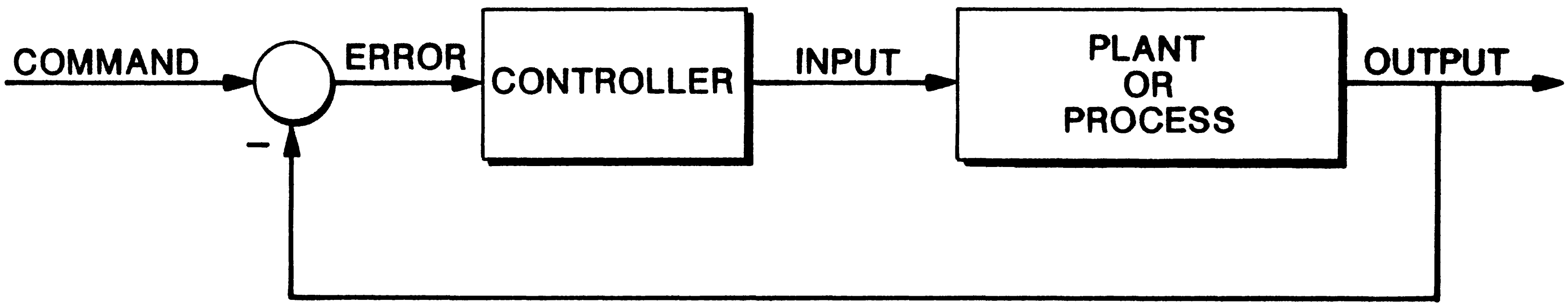


Figure 1   A Typical Control System

with feedback control systems which have highly nonlinear plants. An algebraic procedure minimizes the degree of subjective judgment that has to be employed by the designer to arrive at a practical design, such a procedure can easily be automated on a digital computer.

The secondary objectives were (1) development and selection of a set of appropriate numerical algorithms which would be used to develop the primary software utilities which would be required to implement the controller design procedure and (2) demonstration of the design procedure with a typical problem.

Outcome

The procedure identifies a near-optimum linear controller through the use of the simultaneous stabilization theory (3) and based on sinusoidal-input describing function models of the plant. A near-optimum linear controller is defined herein as a controller which achieves a closed-loop system (i) whose dynamic behavior is relatively insensitive to the amplitude level of the command signal and (ii) whose dynamic behavior satisfies a set of user-defined performance measures in a near-optimum fashion. A robust design is achieved since item (i) above is one of the most important criteria in robustness (1) If the procedure is unable to identify the linear controller which achieves (i) and (ii) above, then an "n-range" ($n > 2$) linear or a nonlinear (1) controller design procedure should be used.

Contributions

Controller design based on the simultaneous stabilization theory, and on two quasilinear models of a nonlinear plant was originally proposed by Taylor (2). The primary contributions of the work presented herein are

(i) The systematic method which utilizes the near-optimum linear controller that forces the dynamic behavior of the closed-loop nonlinear feedback system to behave similar to the dynamic behavior of a user-defined open-loop linear reference model. Technical details of the systematic method is given in section III 5, and technical details of the procedure which may be used to identify the linear reference model is given in section III 1

(ii) The development and linkage of several algorithms and user-friendly software to implement the overall design procedure.

Controller Synthesis Procedure

Execution of the systematic controller synthesis procedure requires the following a priori information.

1) The mathematical model of the nonlinear plant in the state-variable differential equation format. That is,

$$\underline{x}(t) = \underline{f}(\underline{x}(t),u(t)), \text{ and} \tag{1.1-a}$$

$$y(t) = g(\underline{x}(t),u(t)). \tag{1 1-b}$$

2) The operating conditions of the nonlinear plant The operating conditions are defined by (1) the range of the expected level of the excitation command and (2) the range of the excitation frequency of interest

This information is used to obtain the sinusoidal-input describing function models of the plant at the operating conditions of interest If the mathematical model of Equation (1 1) does not exist, it is assumed that the sinusoidal-input describing function models are experimentally obtainable. This may be accomplished by exciting the system with a sinusoid, and measuring the output response with a frequency analyzer.

The user must go through six primary steps to obtain the near-optimum linear controller for a given nonlinear plant A "step" is defined herein as a functional unit composed of a well-defined procedure which is driven by a well-defined input to produce a well-defined output, this output in turn may be the input to the next or other steps

The controller synthesis procedure is composed of the following six primary steps.

1. Define the desired closed-loop system performance specifications, and identify the reference linear model whose static and dynamic behavior matches the desired closed-loop system performance specifications.
2. Characterize the I/O behavior of the nonlinear plant
3. Identify a finite set of linear systems.
4. Classify all stabilizing linear controllers.
5. Search for the near-optimum linear controller
6. Validate design via a digital simulation.

In some cases, the user may have to execute step 1 after input/output characterization task of step 2

The six steps and their interconnections are defined in Figure 2

Software

Three command-driven, user-friendly, and portable software utilities were developed to implement the controller design procedure.

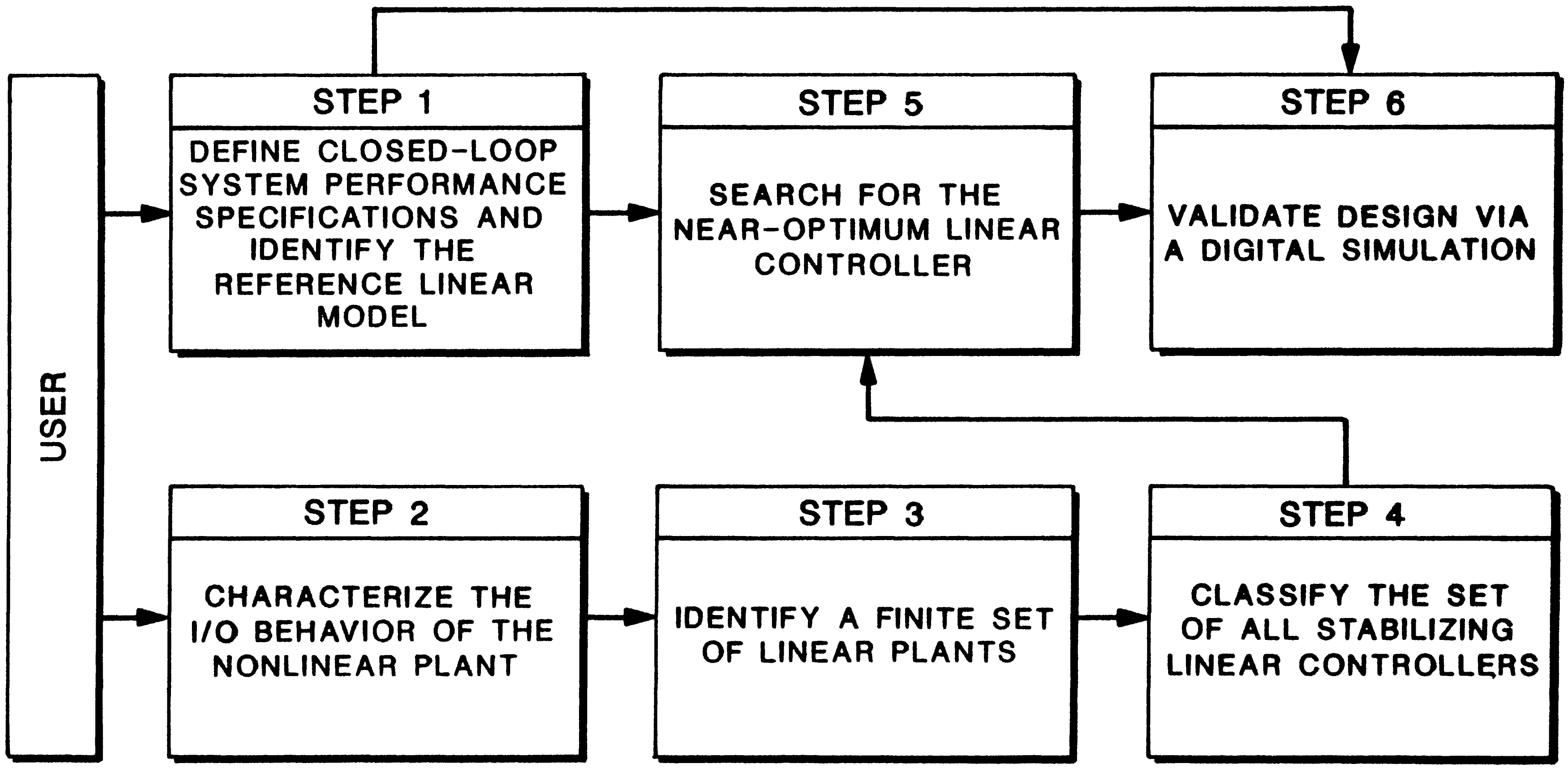


Figure 2 The Systematic Controller Synthesis Procedure

Those are (i) the computer-aided Describing Function model GENerator (DFGEN) software utility, (ii) the computer-aided SYStem IDentification (SYSID) software utility, and (iii) the Dual-Range Linear Controller Design (DRLCD) software utility The algorithms which were used to implement each of these software utilities are discussed in Chapter III. Figure 3 dictates the flow of steps, which were defined in Figure 2, that a user has to undertake and interface with the software utilities.

The three software utilities were written in standard FORmula TRANslation (FORTRAN) 77 The software was designed and developed on a Harris-800 minicomputer and a Tektronix 4115B high resolution raster color graphics terminal

Demonstration Example Problem

A practical position control system, previously used as a demonstration example problem by Taylor (1), is used here also to illustrate the design approach and the typical results that may be achieved (see Chapter IV). It is shown that the performance of a dual-range linear controller is superior to the performance of a single-range linear controller. In this example, the performance of the feedback control system with a dual-range linear controller, is compared to the performance with a multi-range nonlinear controller (1)

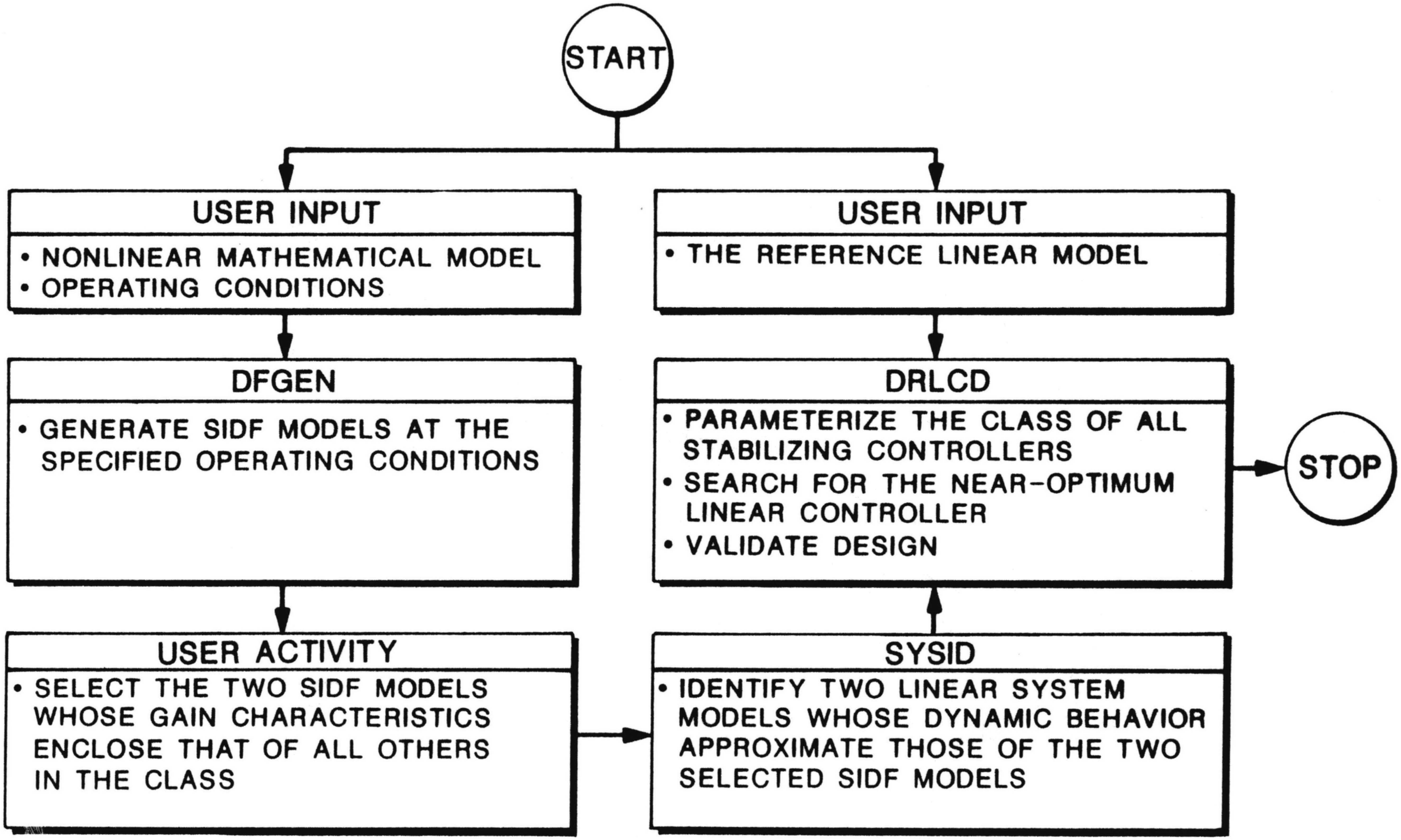


Figure 3. User Interface With Software Utilities

CHAPTER II

LITERATURE SURVEY

The control systems engineer may utilize linear control theory (classical or modern), optimal control theory, or adaptive control theory to obtain a controller that will cause a plant to have satisfactory response. Systems with linear plants have received considerable attention (4-12). Systematic controller design techniques for use with nonlinear plants are at their early stages of development (for example, (1, 2, 13-18)). The optimal (19) or adaptive (20) control laws are difficult or impossible to design for nonlinear plants, and when they can be obtained, usually require a dedicated digital computer for implementation, and all of the states must be available for feedback to the controller. The use of controller design based on either an optimal control theory or an adaptive control theory is justified if the classical control theory is not applicable (19,20)

Controller Design

It is desirable to interface the controller design of nonlinear systems with the rich theory of linear systems The method of characterizing the input/output behavior of the nonlinear plant in the frequency domain is one interfacing tool. A global characterization of nonlinear plants is generally difficult, more importantly, it is impractical.

There are two common approaches for characterizing the input/output behavior of a nonlinear plant The first approach is to linearize the equations of motion around an operating point of interest This characterization technique is referred to as the small-signal linearization (SSL) technique The input/output relation is obtained by replacing each nonlinearity term with a linear term whose gain is the slope of the nonlinearity at the operating point. The SSL method has the following disadvantages (2).

i) The method is not applicable to nonlinear plants which have discontinuous or multivalued nonlinearities (e.g saturation, hysteresis, and backlash)

ii) The input/output behavior of a nonlinear plant is dependent on the amplitude of the input signal Input/output characterization via the SSL technique deletes this essential characteristic of the nonlinear plant.

iii) Controller design techniques for use in nonlinear closed-loop feedback systems, which are based on a small-signal model of the nonlinear open-loop system, are highly sensitive to the operating point of interest. Perturbation of the nonlinear system from its operating point of interest may result in unsatisfactory system behavior

The second approach for characterizing the input/output behavior of a nonlinear plant does not have the above disadvantages In this latter approach, the quasilinear (describing function) model of the nonlinear plant is obtained (21,22) This technique is defined and discussed in the next subsection The development of systematic controller design techniques based on quasilinear models of nonlinear plants has received considerable attention (1,2,14-16,23). The linear controller design technique based on one quasilinear model of the plant has been developed (2), this is referred to as the single-range linear controller design (SRLCD) technique. In fact, this is the "classical industry approach". If the dynamic behavior of the nonlinear plant changes "appreciably" as the amplitude of the command signal changes, the SRLCD technique may fail. An alternative, which is less likely to fail, is to base the controller design on several quasilinear models of the nonlinear plant. Linear as well as nonlinear controller design based on several quasilinear models of the plant was originally suggested by Taylor (2) The Dual-Range Linear Controller Design (DRLCD) technique, which is the controller design technique based on two quasilinear models of the nonlinear plant is developed herein, if satisfactory performance is not obtained, then a linear or nonlinear (1) controller design technique based on more than two quasilinear models of the nonlinear plant may be required.

Describing Functions

In characterizing the input/output behavior of a nonlinear plant, one may replace each nonlinearity with a quasilinear gain which is dependent on the amplitude of the excitation signal. The function corresponding to the quasilinear gain is referred to as the describing function of that nonlinearity, it is based on the form of the excitation signal, which is assumed in advance. For example, consider the nonlinear operation shown in Figure 4 with a sinusoidal input For small input amplitudes there is no saturation, and therefore the gain of the nonlinear operation equals its slope. However, for large input

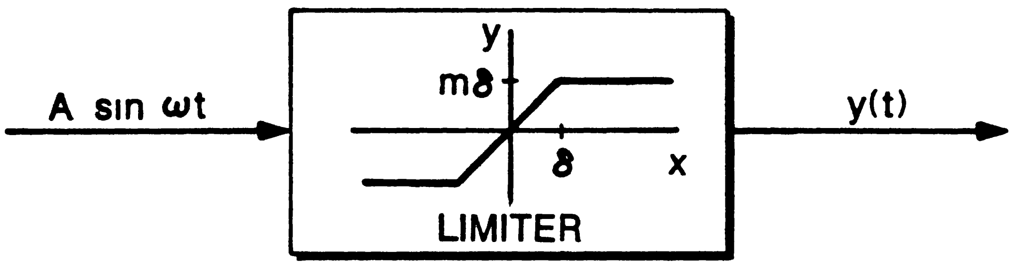


Figure 4 Schematic Model of a Saturation Element

amplitudes there is saturation, and this effect is characterized by a reduced gain as shown in Figure 5 The sinusoidal and random excitation signals are most commonly used, the corresponding describing functions are referred to as (1) sinusoidal-input describing functions (SIDF) and (2) random-input describing functions (RIDF). Although it has been shown that the form of the assumed excitation signal is not of a major concern (2), it has been demonstrated that sinusoidal-input describing function models are more meaningful for controller design applications (14).

Sinusoidal-input describing function (SIDF) models of nonlinear plants do not have the disadvantages of the small-signal models, and they have the following features (2).

i) SIDF models may be used to interface with tools of linear control system analysis and design.

ii) SIDF models approach the SSL models (if they exist) when the amplitude of the excitation signal is small For this reason, the control engineer does not expect to obtain strange results

iii) SIDF models of any plant that can be represented in standard state variable form are obtainable.

iv) SIDF models provide an excellent basis for a robust control system design because they retain amplitude sensitivity characteristics of the nonlinear plant.

The SIDF models of the nonlinear plant at the specific operating conditions of interest are obtained by approximating the gain and phase of the nonlinear plant at discrete frequencies. There are two approaches for obtaining SIDF models The first approach is similar to that used in limit cycle analysis (21,22,24-27), and it involves the

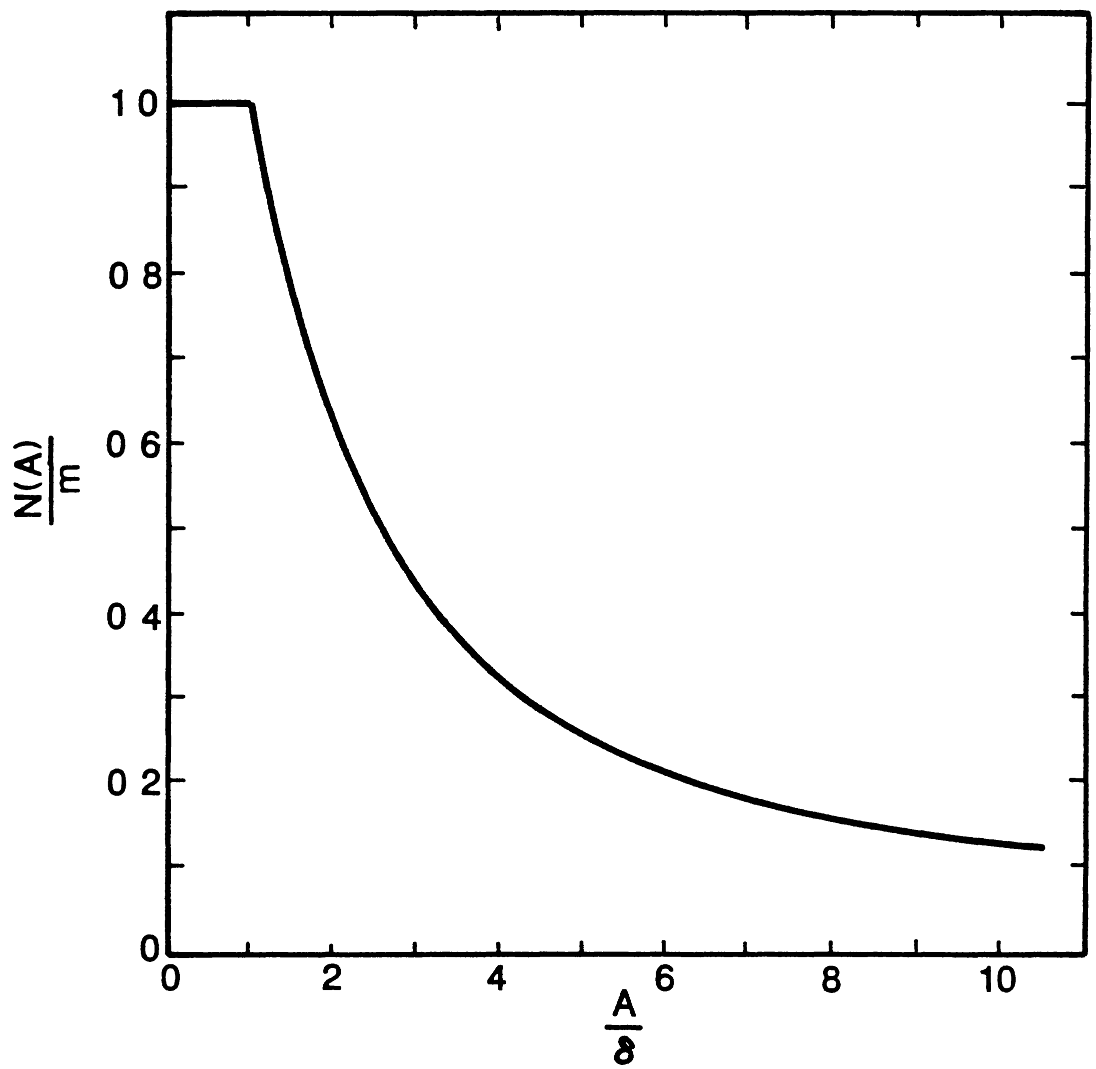


Figure 5   Describing Function Model of a Saturation Element

solution to a set of algebraic nonlinear equations The second approach utilizes direct simulation and evaluation of Fourier integrals. The first method assumes the input to each nonlinearity is nearly sinusoidal. It also assumes that nonlinearities do not produce output signals which are rich in higher harmonics. Such assumptions do not exist in the second approach, the only assumption is that a describing function model is a good representation of the input/output behavior of the nonlinear plant. An algorithm for the second approach has recently been developed (23). Using either approach, an input/output characterization of the following form is obtained.

$$u = a' \cos(\omega t) \tag{2 1}$$

$$y \approx a' \, \mathrm{Re}[G(j\omega, a') \quad e^{j\omega t}] \tag{2.2}$$

System Identification Theory

Early system identification techniques (28,29) may be applied successfully only with low-order systems, and they are difficult to automate on a digital computer. The first class of techniques, graphical techniques (28-29), is restricted to minimum-phase systems, and the accuracy of the results often are questionable (30). The second class of techniques (31-34) is algorithmic and does not require the subjective judgment of the user However, these latter techniques may not be applied if the system poles are close together, and the numerical algorithm is difficult to implement on a digital computer. The third class, fitting techniques (35-40), has received considerable attention. An early contribution (35) was based on the minimization of the squares of the error between the experimental frequency response data and the actual transfer function This method does not give accurate results if the frequency response data is extended into several decades. This deficiency was removed by Sanathanan and Koerner (36), but the solution of a set of linear simultaneous equations for transfer function identification was still required Such a method will fail if the system of simultaneous equations are "ill conditioned". Lawrence and Rogers (39) developed a method based on the work done by Levy (35). This latter method used the sequential fitting of data, and it does not require matrix inversion.

All the system identification methods mentioned so far assume the orders of numerator and denominator polynomials are determined by a priori information The system identification technique developed by Lin and Wu (30), which is based on the work reported in (35) and (37), is applicable to multi-input multi-output linear systems This technique identifies the order of transfer function from a generalized-least squares algorithm (40), and identifies a minimal system via a decomposition theorem, the effect of inexact system parameters is reduced This algorithm does not include user-defined weights, therefore, the method does not allow the user to obtain a different quality of fit in different frequency ranges. Reference (30) is the basis of the system identification technique developed in Chapter III

Simultaneous Stabilization

Simultaneous stabilization theory (3) is based on a factorization approach (5,6,41-44) The approach is to represent (factor) a system transfer function as a ratio of stable transfer functions Factorization theory is primarily used to algebraically constrain the closed-loop poles of a linear system to lie in a specified region of the s-plane Factorization theory is a powerful tool in the sense that it deals with continuous/discrete-time, lumped/distributed, time-invariant/time-varying, and single-input single-output/multi-input multi-output linear systems (5) In the work presented herein, factorization theory is applied to single-input single-output, continuous-time, and linear time-invariant, and deterministic systems. The interested reader may refer to (5,6) for a complete treatment of linear control system design based on a factorization approach

Simultaneous stabilization theory also allows one to parameterize the class of all controllers that stabilize a family of linear plants. This is the primary motivation for the use of this theory herein The result of parameterization is a mathematical relationship which expresses the class of all stabilizing controllers in terms of the coprime factors of the linear plants to be stabilized and another unknown nth-order transfer function r'(s) This unknown transfer function is defined to be the parameter, and the term parameterization arises from the fact that the class of all stabilizing controllers are expressed in terms of the parameter r'(s), which merely has to be a stable rational function. The theory of the simultaneous stabilization, which is presented in (3), is used in this thesis effort to obtain the class of all controllers that stabilize two linear systems.

The theory makes extensive use of abstract algebra (45,46) The notation and terminology used in control system design literature which discusses factorization theory is summarized by Blomberg and Ylinen (47)

CHAPTER III

CONTROLLER SYNTHESIS PROCEDURE

The controller synthesis procedure is composed of the following six primary steps

1 Define the desired closed-loop system performance specifications, and identify the reference linear model whose static and dynamic behavior matches the desired closed-loop system performance specifications.

2 Characterize the I/O behavior of the nonlinear plant.

3. Identify a finite set of linear systems

4 Classify all stabilizing linear controllers

5. Search for the near-optimum linear controller

6. Validate design via a digital simulation

In some cases, the user may have to execute step 1 after input/output characterization task of step 2

Execution of the procedure requires the following a priori information

1) The mathematical model of the nonlinear plant in the state-variable differential equation format (See Equation 1 1)

2) The operating conditions of the nonlinear plant The operating conditions are defined by (1) the range of the expected level of the excitation command and (2) the range of the excitation frequency of interest.

The procedure is applicable to highly nonlinear, single-input single-output, time-invariant, deterministic, and continuous time systems which are representable in the state-variable form.

The first step involves establishment of a set of performance specifications. The linear model of an open-loop system whose static and dynamic performance corresponds to these specifications is to be identified. This linear model serves as a reference model, and it is used for the controller synthesis (step 5) and design validation (step 6)

The second step is to characterize the input-output behavior of the nonlinear plant for various input levels or amplitudes This is accomplished by obtaining the quasilinear models of the nonlinear plant around several pre-defined operating conditions of interest. The two quasilinear models whose gain characteristics enclose those of all others in the class are selected.

The third step is to identify two linear models whose dynamic behavior is similar to the dynamic behavior of the two selected quasilinear models This task provides the basis for applying the simultaneous stabilization theory In step 4, the simultaneous stabilization theory is applied to parameterize the class of all controllers that stabilize the identified linear plants In step 5, the class of all linear controllers are searched for the near-optimum linear controller Finally, in step 6, the controller synthesis is validated via a digital simulation.

If the digital simulation results of step 6 reveal that the dynamic behavior of the nonlinear closed-loop feedback system is appreciably different from the dynamic behavior of the identified linear reference model of step 1, the designer may be justified to design a controller based on more than two quasilinear models of the nonlinear plant.

The synthesized linear controller is required to form a closed-loop system whose dynamic behavior (i) is relatively insensitive to the amplitude level of the command signal and (ii) satisfies a set of user-defined performance specifications in a near-optimum fashion. The controller design procedure does not allow a quantitative measure of system response sensitivity to the level of the step command The sensitivity issue is relative, and the procedure inherently identifies linear controllers that form closed-loop nonlinear feedback systems that are as insensitive to the level of the input command as possible. This inherent feature results whenever the controller synthesis procedure is based on several quasilinear models of a nonlinear plant (1).

Step 1 Performance Specifications

The first step of the design procedure is to specify a set of performance specifications for the closed-loop system either in the time domain or in the frequency domain In either domain, the user is required to identify an nth-order linear model of a system whose dynamic behavior matches the specified performance specifications for the closed-loop system This linear model is designated by the symbol $G_r(s)$ The existence of such a linear model assures that the performance specifications are compatible. For example, it is not realistic to have a second-order linear model of a system whose damping ratio is 0 70, natural frequency 10 0 radians/seconds, and rise-time is 0 01 seconds The reference linear model, $G_r(s)$, is used to synthesize the linear controller for the closed-loop nonlinear feedback system (Step

5), and design validation (Step 6).

Three possible strategies for identification of the reference linear model are given below. Any one of these strategies may be used to identify the reference linear model, $G_r(s)$. The form of $G_r(s)$ is

$$\frac{\sum_{i=0}^{m} a_i s^i}{\sum_{k=0}^{n} b_k s^k}, \quad m \leq n, \tag{3 1}$$

where each of the coefficients a_i and b_k are real numbers.

The first strategy is to construct the desired closed-loop step response This construction depends on the user's definition of a "satisfactory" step response. For example, in a position control system design problem, the user may specify the desired position as a function of time. An nth-order linear model, $G_r(s)$, whose step-response closely approximates the desired step-response of the closed-loop system may be identified. Here, the goal is to appropriately select m, n, a_i, and b_k (see Equation (3.1)). To achieve this goal, one may minimize the following objective function via a gradient search technique

$$F_{id} = \sum_i |f_d(t_i) - f_r(t_i)|^2 \tag{3 2}$$

where,

F_{id} is the objective function to be minimized,

$f_d(t_i)$ is the user-defined discretized time response,

$f_r(t_i)$ is the discretized time response of the as yet unknown reference linear model $G_r(s)$, and

t_i is the user-defined and one-dimensional time array

This strategy was not implemented on a digital computer in this thesis effort.

The second strategy is to construct the desired closed-loop system frequency response. The construction of the desired closed-loop system frequency response depends on the user's definition of a satisfactory frequency response For example, in a position control system design problem, the user may specify the desired closed-loop system gain and phase as a function of frequency to enforce certain cut-off frequency, phase margin, and gain margin Then, the computer-aided system identification software utility, SYSID, may be used to identify the linear system $G_r(s)$ The method of system identification, which is used by SYSID, is given in System Identification section of this chapter Appendix E is designed to tutor the user to use SYSID to identify a linear transfer function from its frequency response information

The third strategy is to define a set of performance specifications either in the time domain or in the frequency domain. Identify a linear model of the system whose dynamic behavior corresponds to the specified performance specifications for the closed-loop system, e.g , a linear second-order system model which has the desired natural frequency and damping ratio This strategy is used in this thesis. The user is not limited in identifying linear second-order models. Meyfarth (48) has presented step, impulse, and frequency response characteristics of linear third-order models in the form of dimensionless response plots for a number of different combinations of system parameters. These response plots may be examined visually to select the reference linear model

Step 2 Input/Output Characterization

This step involves characterization of the input/output behavior of the nonlinear plant around several predefined operating conditions of interest. This characterization is accomplished by obtaining the sinusoidal-input describing function (SIDF) models of the nonlinear plant for several input amplitudes The rationale for this approach of input/output characterization was given in Chapter II.

The SIDF models are obtained by approximating the gain and phase of the nonlinear plant at a number of user-defined input amplitude levels over a range of user-defined frequencies. The two quasilinear models whose gain characteristics enclose that of all others in the class are selected, the controller synthesis is based on these two quasilinear models of the nonlinear plant.

If the mathematical model of the nonlinear plant is available (see Equation (1) in Chapter I), the following approach may be used to compute the SIDF models, otherwise, it is assumed that the SIDF models are experimentally obtainable. Taylor (23) has adapted a Fourier analysis approach to compute the SIDF models. This approach is adapted herein, also, and it is given below.

The plant is excited by a known input of the following general form.

$$u(t) = u_0 + a' \cos(\omega t) \tag{3.3}$$

Where,

u_0 is the DC value of the input signal

and

a' is the amplitude level of the excitation signal

Then, the dynamic equations of motion, which are given by Equation (1.1), are numerically integrated to obtain the output as a function of time, $y(t)$. Then, Fourier integrals for period k' are calculated when $y(t)$ is at steady-state These integrals are given by the following

$$I_{m',k'} = \int_{(k'-1)T}^{k'T} y(t)\, e^{-jm'\omega t} dt \tag{3.4}$$

where

$k' = 1, 2, \ldots,$

$m' = 0, 1, 2, \ldots,$ and

$T = 2\pi/\omega.$

The constant or "DC" component of the response is given by $I_{0,k'}$, and the pseudo-transfer function at discrete frequencies, which is represented by the complex number $G_{1,k'}(j\omega, u_0, a')$, is given by the following relation

$$G_{1,k'}(j\omega,u_0,a') = \omega I_{1,k'}/a'\pi. \tag{3.5}$$

In order to analyze the importance of higher harmanic effects, one may evaluate

$$G_{m',k'}(jm'\omega,u_0,a') = \omega I_{m',k'}/a'\pi, m' = 2,3, . \tag{3.6}$$

For a given excitation amplitude, a', Equation (3.5) at discrete frequencies, over the range of user-defined frequency range of interest is evaluated to obtain one quasilinear model of the nonlinear plant This procedure for various user-defined excitation amplitudes is repeated to obtain a number of quasilinear models of the nonlinear plant The two quasilinear models whose gain characteristics enclose that of all others in the class are selected, and they are set aside for system identification purposes of the next step.

Step 3 System Identification

This step involves the identification of the two linear system models whose dynamic behavior approximate the dynamic behavior of the two selected quasilinear (SIDF) models from the previous step The user may desire to obtain a different quality of fit in different frequency ranges, e.g. the user may wish to sacrifice the quality of fit at high frequencies to gain on the quality of fit at the cross-over. For this reason, a new method for identification of single-input single-output linear, time-invariant, and deterministic systems from frequency response data is developed. This method is described below.

Any linear system model may be represented via a transfer function of the form given by Equation (3.1). The objective is to identify m, n, a_i, and b_k in such a manner that the frequency response of the identified transfer function G(s) approximates the desired frequency response data in a near-optimum fashion. The optimal transfer function to be identified is the one that produces a minimum mean-square error. The mean-square error (MSE) is defined by the following equation.

$$MSE = \int_{\omega} W(\omega)\ e(\omega)\ d\omega \qquad (3.7)$$

where,

$$e(\omega) = |G_d(j\omega) - G(j\omega)|^2 , \qquad (3.8)$$

$G_d(j\omega)$ is the desired frequency response,

$G(j\omega)$ is the fitted frequency response,

$W(\omega)$ is the user-defined weighing function,

and

l is the total number of observed discrete frequencies

For a given m and n, an optimization technique may be used to vary a_i and b_k to minimize MSE The PATRN optimization routine (49), which uses the Hooke and Jeeves (50) algorithm, is used to minimize MSE. The user may desire to adapt other well-known optimization routines to minimize the MSE (51-54).

Most optimization techniques require a starting solution to use as a reference to minimize a given objective function. The starting solution is obtained from the application of the system identification technique of Lin (30). Lin's algorithm uses a generalized weighted least-squares technique to identify linear systems of the general form given by Equation (3 1) with $a_0 = 1$ In this work, the algorithm of Lin was reformulated to allow identification of linear systems of the general form given by Equation (3 1) with $b_0 = 1$ (See Appendix A). This reformulation was necessary to allow identification of type one systems. A computer-aided engineering environment, based on the above system identification techniques, was created via the command-driven software utility SYSID (See Appendix E).

The primary advantages of the above system identification technique with respect to the system identification technique of Lin are (1) the user is able to obtain a better quality of fit in specific frequency ranges and (2) the user is able to perform constraint minimization, for example, it may be desired to constrain the optimization routine to solutions which have no right-half s-plane poles

Step 4 Controller Parameterization

This step involves parameterization of the class of all controllers that simultaneously stabilize the closed-loop systems comprising the two linear system models of the nonlinear plant from the previous step. This parameterization involves developing a mathematical relationship which expresses the class of all stabilizing controllers in terms of the linear system models from the previous step and another as yet unknown stable nth-order transfer function $r'(s)$ This transfer function is defined to be the parameter, and the term parameterization arises from the fact that the class of all stabilizing controllers are expressed in terms of the parameter r'(s). Such a parameterization of the class of all stabilizing controllers provides logical means for obtaining the near-optimum linear controller The simultaneous stabilization theory of Vidyasagar and Viswanadham (3) is adapted to fulfill the purpose of this step of the design procedure. This theory is discussed below.

Simultaneous Stabilization

Consider an example problem involving a one degree-of-freedom electrohydraulic position control system The system is required to have acceptable dynamic performance when operated in the vicinity of two different operating points. A controller may be designed so that the feedback system meets certain user-defined performance measures when operated at a given nominal condition. If the closed-loop system operates with this same controller at a different condition (e.g different temperature and pressure), the dynamic behavior may be sluggish, too lightly damped, or even unstable. A compromise or alternative approach to the design of the controller is required One approach is to design a controller which produces adequate dynamic performance for both expected operating conditions Simultaneous stabilization theory provides a means for identification of the class of all stabilizing controllers (if such a class exists), whether the system is operating at the nominal operating condition, or at other defined conditions.

Assume that the open-loop system operating at a nominal condition may be represented by the stable linear model $T'_0(s)$ shown in Figure 6, the corresponding open-loop pole locations of $T'_0(s)$ shown in Figure

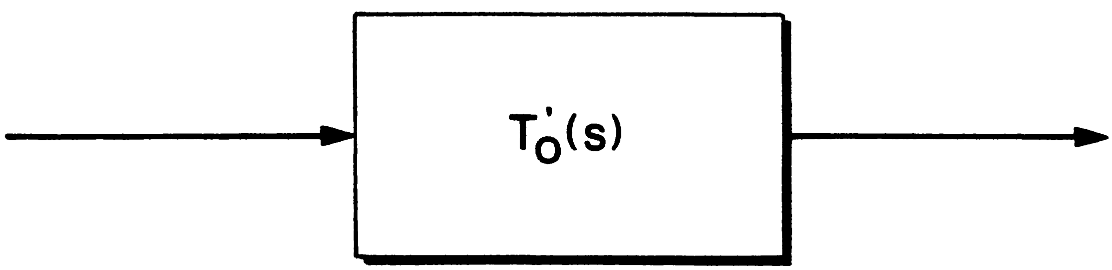


Figure 6 Representation of An Open-Loop System Operating At a Nominal Condition

7. Also assume that the open-loop system operating at another condition may be represented by the unstable linear model $T'_1(s)$ are shown in Figure 8, the corresponding open-loop pole locations of $T'_1(s)$ are shown in Figure 9. The theory of simultaneous stabilization may be used to parameterize the class of all controllers that stabilize both systems. An example problem, which involves design of a controller which would simultaneously stabilize two first-order plants, is given in Appendix C. In this thesis work, the theory is adapted to identify the class of all controllers that stabilize the two identified linear plants of the previous step.

Definition

The notation used by Vidyasagar and Viswanadham (3) is used herein. The set of all rational functions (for $s=j\omega$) with real coefficients is represented by R(s). A rational function in s is defined as the ratio of two polynomials in s. The subset of R(s) consisting of stable rational functions is denoted by the commutative algebraic ring H. An algebraic ring is a non empty set with two binary operations addition and multiplication. If f_1 and f_2 belong to H, it is concluded that $f_3 = f_1 - f_2$ and $f_4 = f_1.f_2 = f_2.f_1$ also belong to H. Therefore, any rational function h in R(s) may be represented as the ratio of two functions ($h = \frac{f}{g}$) whose greatest common divisor is one. Rational functions f and g are defined to be coprime factors of h. In the work presented herein, coprime factorization is performed in the algebraic ring H, and not in the ring of polynomials. Refer to Appendix B for illustration of a systematic technique which would allow coprime factorization of a given rational function in s, the systematic

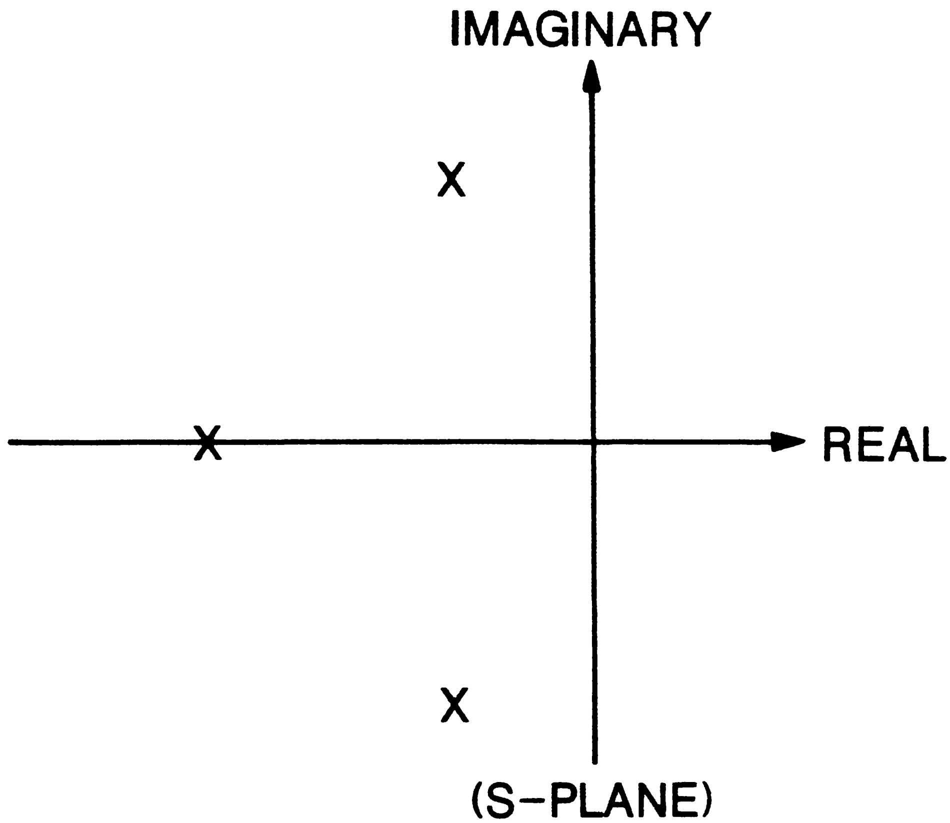


Figure 7 Open-Loop Pole Location of $T'_0(s)$

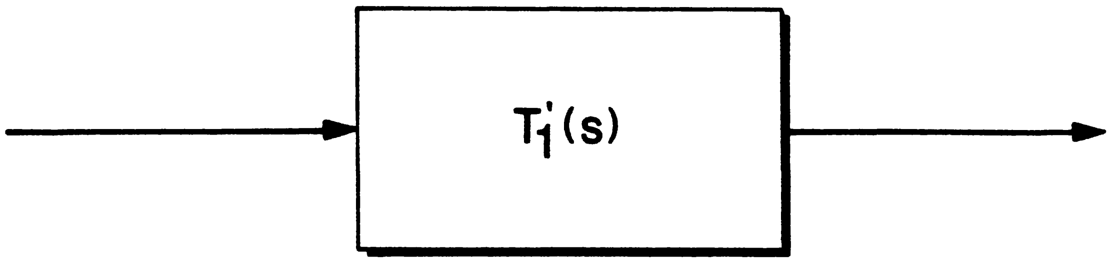


Figure 8 Representation of an Open-Loop System Operating Away from the Nominal Condition

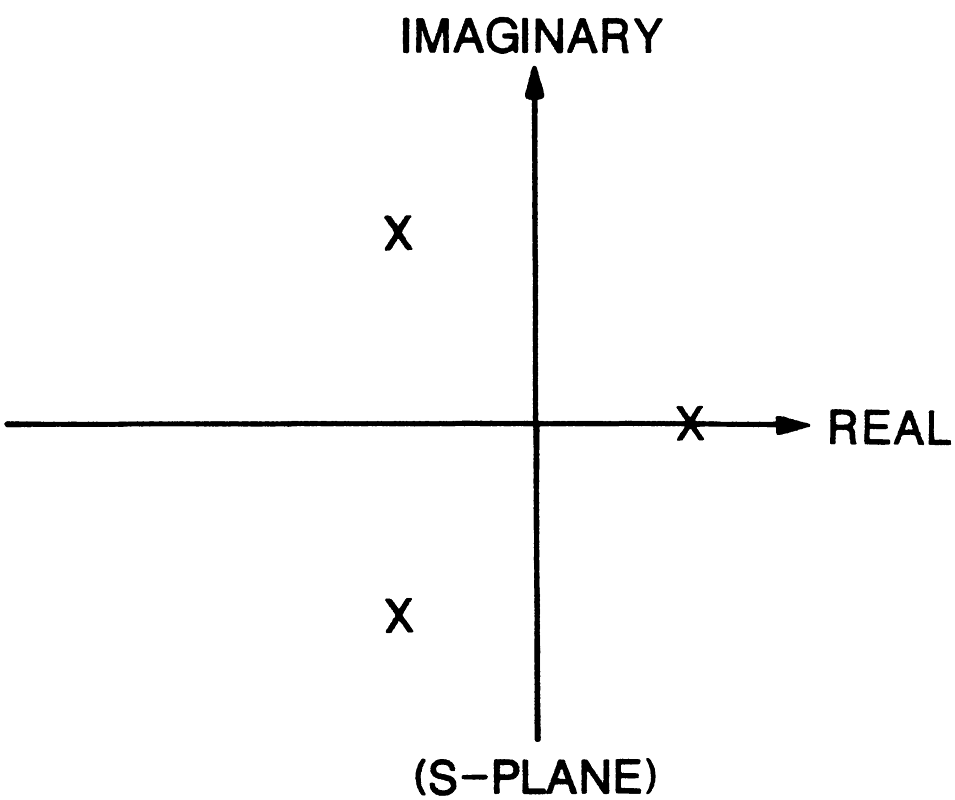


Figure 9 Open-Loop Pole Locations of $T'_1(s)$

technique is algebraic, and it places the poles of the coprime factors in a user-defined region of the s-plane.

Formulation

The class of all controllers that stabilize one plant may be parameterized in terms of a linear stable transfer function, r(s) That is,

$$c(s) = \frac{p(s) + r(s)\, d(s)}{q(s) - r(s)\, n(s)} \tag{3.9}$$

where,

c(s) is the stabilizing controller,

r(s) belongs to the algebraic ring H, and it is defined to be the unknown parameter This unknown parameter may be selected to achieve certain performance criteria (See Example 1 of Appendix C)

$G(s) = \frac{n(s)}{d(s)}$,

n(s) and d(s) are coprime factors of G(s),

the pair n(s) and d(s) belong to H,

the pair p(s),q(s) are coprime, and

the pair p(s),q(s) satisfy the Bezout identity

$$p(s)\, n(s) + q(s)\, d(s) = 1 \tag{3.10}$$

Example 1 of Appendix C illustrates the application of Equation (3.9).

The simultaneous stabilization of L linear plants is given by Vidyasagar and Viswanadham (3) Parameterization of the class of all controllers that stabilize two stable linear system models is given next

Let the pair $(n_i(s),d_i(s))$ correspond to the coprime factors of $G_i(s)$ $(i=0,1)$, and designate the corresponding factors that satisfy the Bezout identity by the coprime pair $(p_i(s),q_i(s))$ Let the algebraic ring M consist of the set of all controllers that strongly stabilize

$$G_a(s) = \frac{b_1(s)}{a_1(s)} \tag{3.11}$$

where

$G_a(s)$ is an auxiliary transfer function.

$$a_1(s) = q_0(s)\, d_1(s) + p_0(s)\, n_1(s), \text{ and} \tag{3.12}$$

$$b_1(s) = -n_0(s)\, d_1(s) + d_0(s)\, n_1(s). \tag{3.13}$$

The set of all controllers that simultaneously stabilize $G_0(s)$ and $G_1(s)$ is given by the following equations.

$$c(s) = \frac{p_0(s) + r'(s)\, d_0(s)}{q_0(s) - r'(s)\, n_0(s)}, \tag{3.14}$$

where $r'(s)$ belongs to the set M. Example 2 of Appendix C is designed to familiarize the reader with the application of the simultaneous stabilization theory

Step 5 Controller Synthesis

This step involves searching the class of all stabilizing controllers, which were parameterized in the previous step, for the near-optimum linear controller, i e., the controller that produces a closed-loop system whose dynamic behavior approximates the dynamic behavior of the identified linear system of step 1

One approach to controller synthesis is to parameterize the class of all controllers that place the closed-loop poles of the linear plants in a specified region of the s-plane. This requires one to obtain the coprime factors of the linear plants whose poles lie in the specified region of the s-plane Then, this class of stabilizing controllers is to be searched for the near-optimum linear controller This is the recommended approach in the open literature (44)

The above approach has the following drawback. Controllers with high order dynamic terms may be designed The lower the order of the coprime factors, the lower is the order of all stabilizing controllers There does not exist an algorithm which would obtain the coprime factors of the lowest possible order In general, the order of a coprime factor is equal to the system order. However, all factors may not have to be of the same order as that of the system. See Appendix B for justification of above.

At this stage of the controller design procedure, the user has obtained two stable linear models whose gain characteristics enclose the gain characteristics of the nonlinear plant at the operating conditions of interest. Since the linear plants are stable, the parameterization of the previous step may further be simplified In this case, transfer functions $n_i(s)$, $d_i(s)$, $p_i(s)$, and $q_i(s)$ may be defined by the following

$$n_i(s)=G_i(s),\ d_i(s)=1,\ p_i(s)=0,\ \text{and}\ q_i(s)=1 \qquad (3\ 15)$$

Where,

$n_i(s)$ and $d_i(s)$ are the coprime factors of $G_i(s)$, $i=0,1$, and

$q_i(s)$ and $p_i(s)$ satisfy the Bezout identity

From the application of the simultaneous stabilization theory, the class of all stabilizing controllers is given by

$$c(s) = \frac{r(s)}{1 - r(s)\,G_0(s)} \qquad (3\ 16)$$

where,

r(s) belongs to the algebraic ring M which consists of the set of all controllers that stabilize $G_a(s) = \frac{b_1(s)}{a_1(s)}$

From Equation (3 11), (3.12), and(3 13),

$$G_a(s) = \frac{b_1(s)}{a_1(s)} = G_1(s) - G_0(s). \qquad (3\ 17)$$

Since both $G_0(s)$ and $G_1(s)$ are strictly proper and stable, their difference is also strictly proper and stable. In this case, $n_a(s)$, $d_a(s)$, $P_a(s)$, and $q_a(s)$ may be defined by the following.

$$n_a(s) = G_a(s),\ d_a(s) = 1,\ P_a(s) = 0,\ \text{and}\ q_a(s) = 1.$$

Where,

$n_a(s)$ and $d_a(s)$ are the coprime factors of $G_a(s)$, and

$q_a(s)$ and $P_a(s)$ satisfy the Bezout identity.

The set of all r(s), which corresponds to the class of all controllers that stabilize $G_a(s)$, is given by Equation (3.9).

That is,

$$r(s) = \frac{r'(s)}{1 - r'(s)\ G_a(s)} \qquad (3\ 18)$$

Substitute (3.18) into (3.16) to obtain the class of all controllers that stabilize both G_0 and G_1.

$$c(s) = \frac{r'(s)}{1 - r'(s)\ G_1(s)} \qquad (3.19)$$

where,

r'(s) belongs to the algebraic ring H (3 20)

and

$G_a(s)$ must be strongly stabilizable, i.e., $G_a(s)$ must be stabilized via a stable r(s). (3.21)

Therefore, the class of all controllers that stabilize $G_1(s)$ subject to the constraints given by Statements (3.20) and (3.21), stabilize both $G_0(s)$ and $G_1(s)$. This set may be searched for the near-optimum linear controller. The search algorithm, Algorithm 1, is given below.

Algorithm 1

1) Assume a stable transfer function c(s).

2) Compute r'(s) using Equation (3.19).

3) If Constraints (3.20) and (3.21) are violated provide a barrier for the optimization routine, and go to step 6, otherwise go to step 4.

4) Compute objective function F

$$F = \alpha F_0 + (1-\alpha)F_1 \qquad (3.22)$$

where

$$F_0 = |G_r(s) - \Sigma(C(s), G_0(s))|^2, \qquad (3.23)$$

$$F_1 = |G_r(s) - \Sigma(C(s), G_1(s))|^2, \text{ and} \qquad (3.24)$$

$$\alpha = 0.5$$

5) If F is optimized then stop here, otherwise go to step 6

6) Vary coefficients of c(s), then go to step 2.

The first item of algorithm 1 requires subjective judgement of the user. In order to remove this undesirable feature of the algorithm, the supplemental algorithm 1.1 was developed. This algorithm is given below.

Algorithm 1.1

a) Let $G_r(s) = \frac{c(s)\ G_0(s)}{1 + c(s)\ G_0(s)}$. (3.25)

b) From (a) above, solve for c(s) as a function of known quantities $G_r(s)$ and $G_0(s)$. That is,

$$c(s) = \frac{G_r(s)}{G_0(s)(1 - G_r(s))} \ . \qquad (3.26)$$

Step 6 Design Validation

Verify controller design via direct simulation of the closed-loop system. A fourth-order Runge-Kutta routine is adapted to accomplish the simulation of the closed-loop feedback system. The user may obtain the normalized step responses of the closed-loop system for a range of step magnitudes. If the normalized step responses are appreciably different from the unit-step response of the reference model $G_r(s)$, then the user is justified to design a linear or a nonlinear (1) controller based on n (n > 2) quasilinear models of the nonlinear plant

CHAPTER IV

DEMONSTRATION EXAMPLE PROBLEM

The primary objectives of this chapter are to (i) apply the controller synthesis approach to a real problem with a highly nonlinear plant, and (ii) compare the performance of the system with the synthesized linear controller to the performance of the system with a linear and a nonlinear controller which has been reported by Taylor and Strobel (1). The example problem under study is of the type encountered in position control. The block diagram of the open-loop system is shown in Figure 10. The mathematical model of the open-loop system (nonlinear plant), is given by Equations (4.1-4.4) given below.

$$\dot{x}_1 = x_2 \tag{4.1}$$

$$\dot{x}_2 = \frac{T_m}{J} , \tag{4.2}$$

where $J = 0.01$ kg-m^2.

The servomotor saturation effects (Figure 11) are modeled by the following relationships (1)

$$\begin{aligned} T_e &= V_{in}\, m_1 && \text{if} \quad |V_{in}| \leq \delta, \\ T_e &= \mathrm{Sign}(V_{in})\ (\delta\, m_1 + m_2\ (|V_{in}| - \delta)) && \text{if} \ |V_{in}| > \delta. \end{aligned} \tag{4.3}$$

where,

$\delta = 0.5$ Volts, $m_1 = 5$ Newton-Meters/Volt, and $m_2 = 1.0$ Newton-Meters/Volt. The servomotor friction characteristics include coulomb

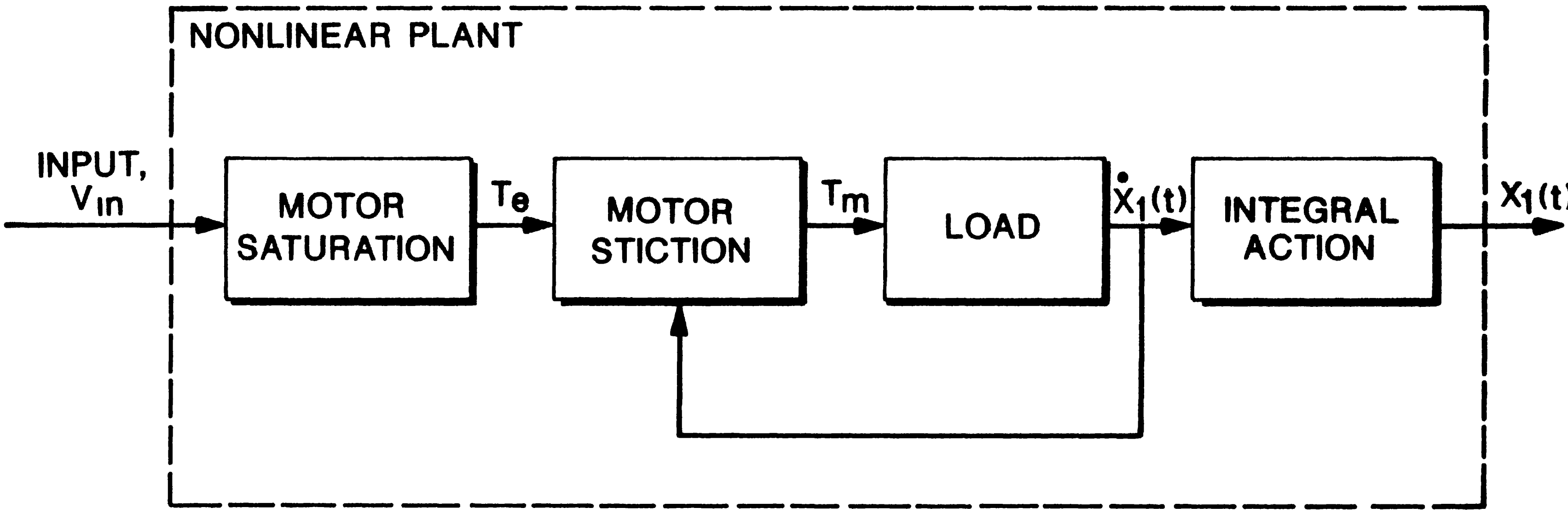


Figure 10 Position Servo Open-Loop Model Schematic

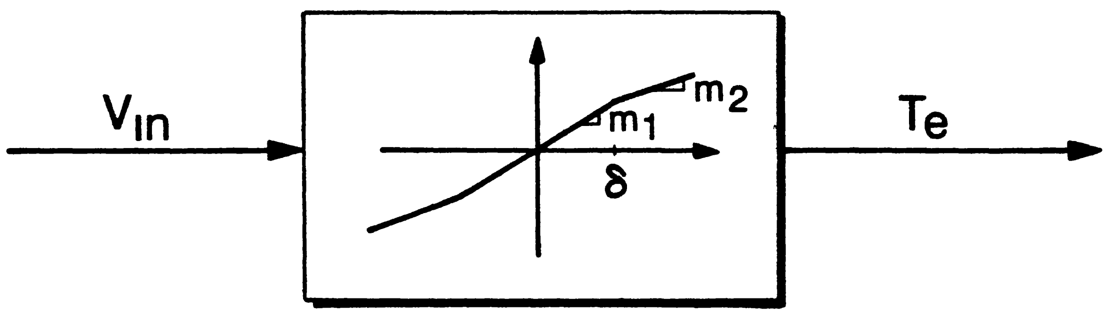


Figure 11 Model of the Servomotor Saturation Effects

friction and viscous friction, these characteristics are defined by the following relationships (1).

$$T_m = \begin{cases} T_e - f_v\dot{x}_1 - f_c \operatorname{sign}(x_1) \text{ if } |T_e| > f_c \\ T_e - f_v\dot{x}_1 - f_c \operatorname{sign}(\dot{x}_1) \text{ if } x_1 \neq 0, \text{ and} \\ 0.0 \text{ if } |T_e| < f_c \text{ and } \dot{x}_1 = 0 \end{cases} \quad (4.4)$$

where,

$$f_v = 0.1 \frac{\text{Newton-meters-seconds}}{\text{radians}} \text{ and } f_c = 1.0 \text{ Newton-meters}$$

Problem Statement

The objective is to synthesize a linear controller for the nonlinear plant of Figure 10. A closed-loop feedback system of the form shown in Figure 12 is to be formed. The resulting feedback system is to be as insensitive to the level of the input command as possible, and it is to satisfy a set of user-defined performance specifications in a near-optimum fashion. The performance measures are specified in Step 1

Step 1 Performance Specifications

The first step is to define a set of performance specifications Taylor and Strobel (1) have synthesized a nonlinear controller for the nonlinear plant of Figure 10, the corresponding nonlinear feedback system is shown in Figure 13. The corresponding normalized step-responses of the closed-loop nonlinear feedback system for a range of step amplitudes is shown in Figure 14. The maximum percent overshoot is about 37, and the two percent settling time is about 0.30 seconds. For comparison purposes, these two time-domain performance specifications are utilized herein, also. A linear n-th order linear model (reference model) of system whose dynamic behavior satisfies these performance

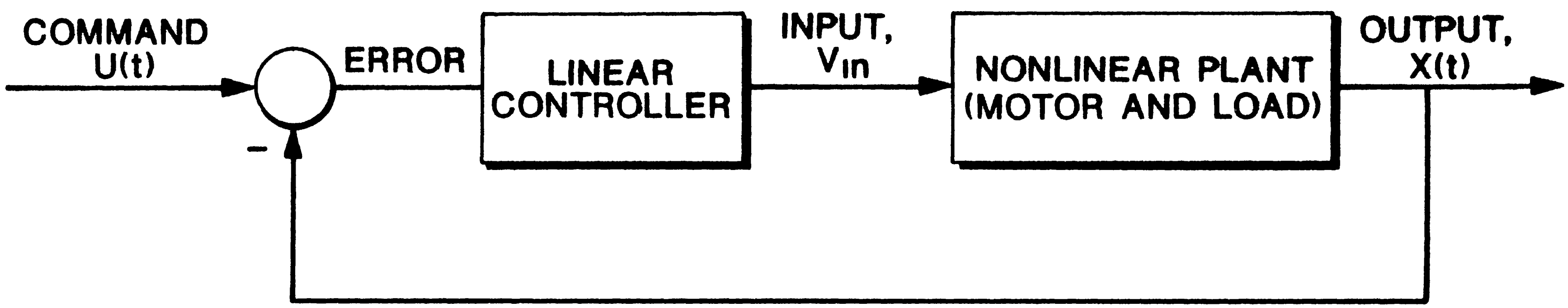
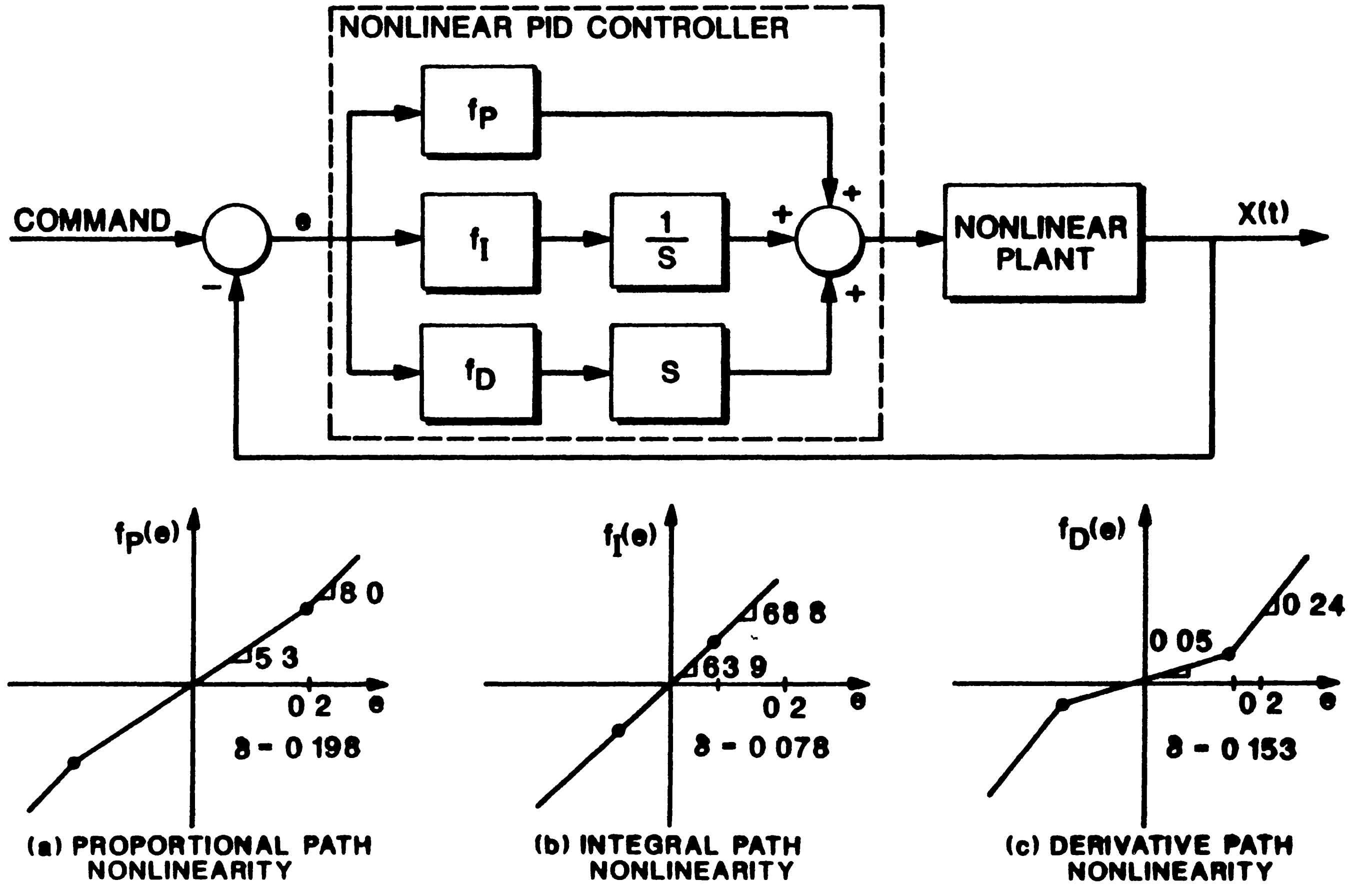
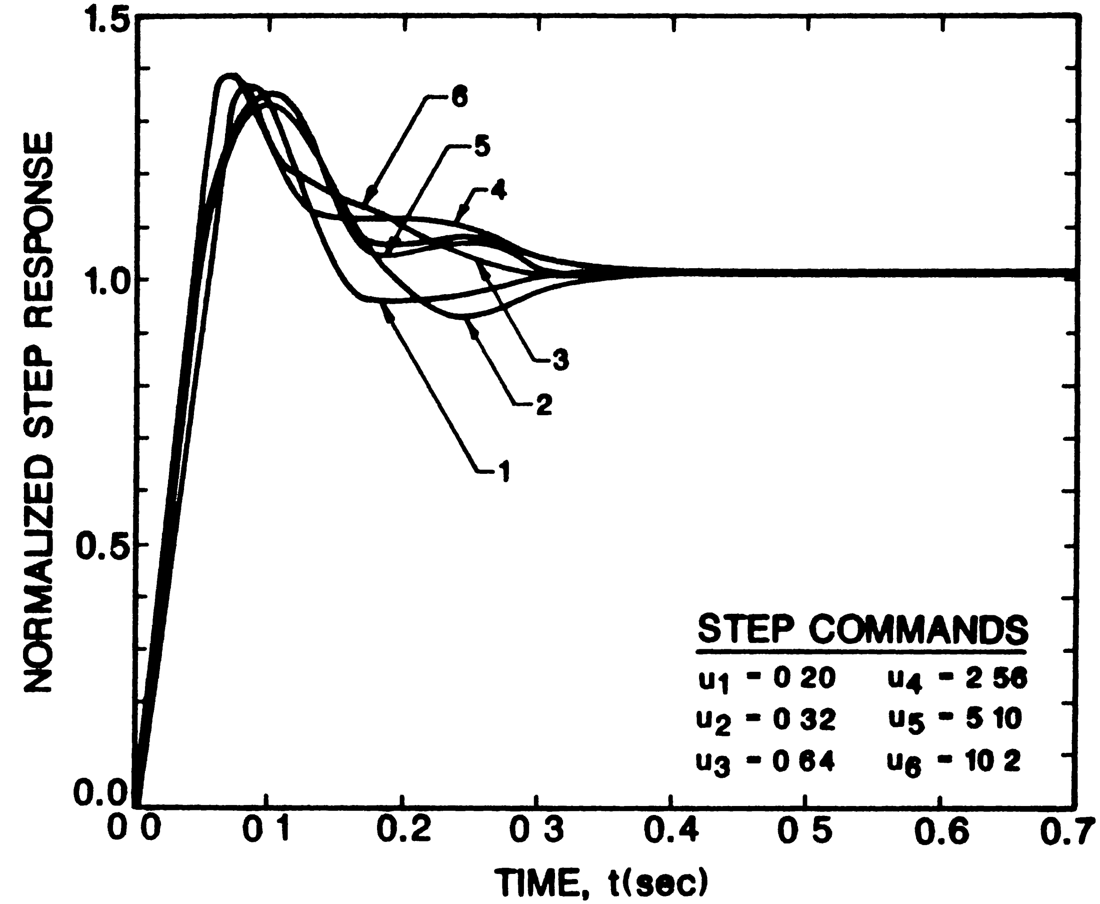


Figure 12 Structure of the Closed-Loop Feedback System



Source Taylor and Strobel (1)

Figure 13 Structure of the Nonlinear Controller and the Nonlinear Closed-Loop Feedback System



Source Taylor and Strobel (1)

Figure 14 Normalized Step Responses of the Nonlinear PID and Plant specifications is to be identified. Three strategies for identification of such a linear model was given in Chapter 3. The third strategy is applicable in this case. A linear second-order model whose damping ratio and natural frequency are 0.375 and 37.0 radians/seconds, would exhibit approximately a 37% overshoot and a 2% settling time of 0.3 seconds when excited with a step input. The reference model is given by

$$G_r(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4.5}$$

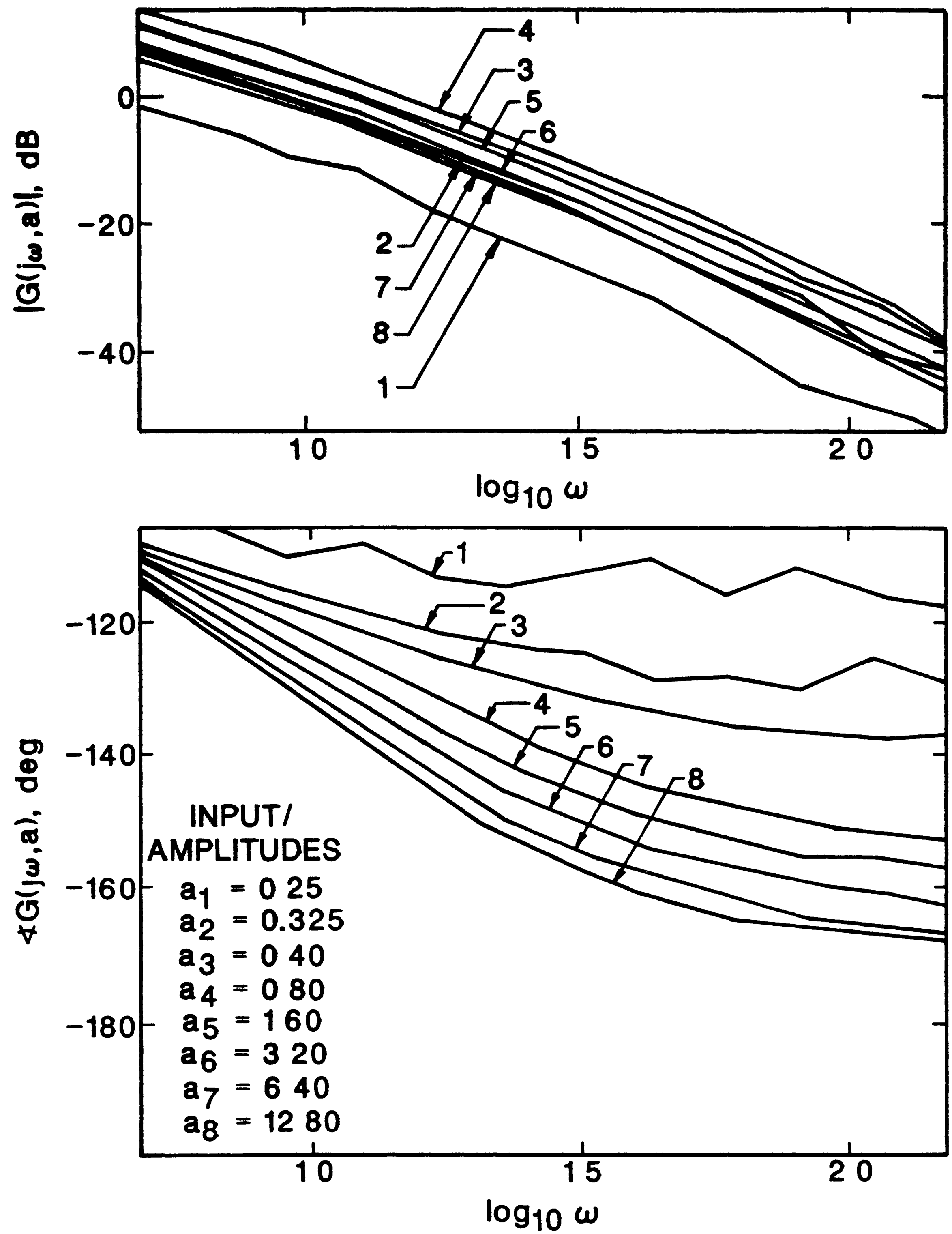
where,

$$\omega_n = 37.0 \frac{\text{radians}}{\text{seconds}} \text{ and } \zeta = 0\ 375.$$

Step 2 through 5 will yield a linear controller which would force the dynamic behavior of the closed-loop nonlinear system to closely approximate the dynamic behavior of the reference model $G_r(s)$

Step 2 Input/Output Characterization

The input/output behavior of the nonlinear plant at several predefined operating conditions of interest is to be characterized. Sinusoidal-input describing function models of the nonlinear plant are obtained using a frequency-domain technique. The nonlinear plant is excited with a sinusoid of an assumed amplitude for a range of user-defined frequencies. The frequency response information is then obtained as outlined in Chapter III. Taylor and Strobel (1) have previously derived these models for eight prespecified operating conditions, the resulting frequency response plots are given in Figure 15, and each curve corresponds to a particular operating condition. The operating conditions are specified in terms of the range of frequency of interest and the range of the level of the excitation signal. In this



Source Taylor and Strobel (1)

Figure 15 SIDF Models of the Nonlinear Plant example, the upper and lower bounds on the frequencies of interest are 5.0 and 150.0 radians/seconds, the input amplitude levels considered herein are 0.25, 0 325, 0.4, 0 8, 1.6, 3.2, 6.4, and 12 8 Volts. The two quasilinear models whose gain variation enclose those of the others are selected for system identification purposes, that is, the curves which are labeled by arabic numerals 1 and 4.

In this thesis effort, the software tool, DFGEN, was developed to automate the generation of the sinusoidal-input describing function models. Appendix D is a tutorial for the use of the software, and the SIDF models shown in Figure 15 are computed therein, also The frequency response data corresponding to SIDF models number 1 and 4 are tabulated in Tables I and II

Step 3 System Identification

Linear systems whose dynamic behavior approximate the dynamic behavior of the open-loop system (nonlinear plant) around the operating conditions of interest are to be identified The dynamic behavior of the nonlinear plant was characterized via describing function models in the previous step The linear models of the system whose dynamic behavior approximate the dynamic behavior of the two selected quasilinear models of the nonlinear plant are designated by $G_0(s)$ and $G_1(s)$, respectively These models were identified via the computer-aided system identification software utility SYSID, technical details of this software utility is based on the algorithm developed in Chapter III The inputs to SYSID were the real and imaginary parts of the frequency response information for the quasilinear models These data are tabulated in Tables I and II The resulting linear approximations are of

TABLE I

FREQUENCY RESPONSE DATA OF THE SIDF MODEL NUMBER 4

Frequency (RAD /SEC.)	Real Part	Imaginary Part
0.5000000E+01	-0.1517412E+01	-0.4025078E+01
0.6811673E+01	-0.1270355E+01	-0.2682782E+01
0 9279777E+01	-0.1011572E+01	-0.1718434E+01
0.1264216E+02	-0.7630895E+00	-0 1038714E+01
0.1722285E+02	-0.5332355E+00	-0 5911674E+00
0.2346329E+02	-0 3481629E+00	-0.3207299E+00
0.3196484E+02	-0 2158670E+00	-0.1693839E+00
0.4354681E+02	-0 1282173E+00	-0 8635313E-01
0.5932532E+02	-0.7160715E-01	-0 4423007E-01
0.8082092E+02	-0.4050847E-01	-0 2152321E-01
0 1101051E+03	-0.2272230E-01	-0 1113880E-01
0 1500000E+03	-0 1141469E-01	-0.5040533E-02

TABLE II

FREOUENCY RESPONSE DATA OF THE SIDF MODEL NUMBER 1

Frequency (Rad /Sec)	Real Part	Imaginary Part
0 5000000E+01	-0.2196103E+00	-0 8066907E+00
0.6811673E+01	-0 1653188E+00	-0 5295195E+00
0 9279777E+01	-0 1183520E+00	-0 3393041E+00
0 1264216E+02	-0.8062143E-01	-0 2129374E+00
0 1722285E+02	-0 5169478E-01	-0 1314640E+00
0 2346329E+02	-0.3190666E-01	-0 7176102E-01
0 3196484E+02	-0.1819518E-01	-0.4177350E-01
0.4354681E+02	-0 1115129E-01	-0.2442076E-01
0 5932532E+02	-0 5783351E-02	-0 1817636E-01
0 8082092E+02	-0 3188869E-02	-0 8117869E-02
0 1101051E+03	-0.2108138E-02	-0 4857120E-02
0.1500000E+03	-0 5426540E-02	-0 2284526E-02

the following form.

$$G_0(s) = \frac{(1.231 + 0.003989\ s)}{s(0.05475 + 0.003761\ s)} \qquad (4\ 6)$$

and

$$G_1(s) = \frac{(1\ 0000 + 0.040314\ s)}{s(0.22521 + 0\ 23641\ s)} \qquad (4.7)$$

Refer to Appendix E for details of using the SYSID software utility in identifying $G_0(s)$ and $G_1(s)$ above. Comparisons of the two quasilinear models (from Step 2) with the two linear models are shown in Figures 16 and 17.

Analysis of System Identification Results

To put the analysis of the system identification results in perspective, consider fitting of an nth-order polynomial with real coefficients to a set of data points whose domain and range consist of real numbers. As the order of the fitted polynomial is increased, the magnitude of the error at discrete data points is expected to decrease This decrease in the magnitude of the error does not necessarily imply that a smooth fit is obtained, and most likely the fitted polynomial will fluctuate around the discrete data points This is undesirable, and there exists a tradeoff between the smoothness (quality) of the fit and the error at discrete data points.

A similar trade-off must also be enforced when identifying a transfer function from a set of frequency response information. In this case, the data is composed of complex numbers, and a two-dimensional fit must be obtained. The user must take the following three key issues in mind while he is identifying a transfer function whose frequency response information approximates that of a quasilinear model These items are

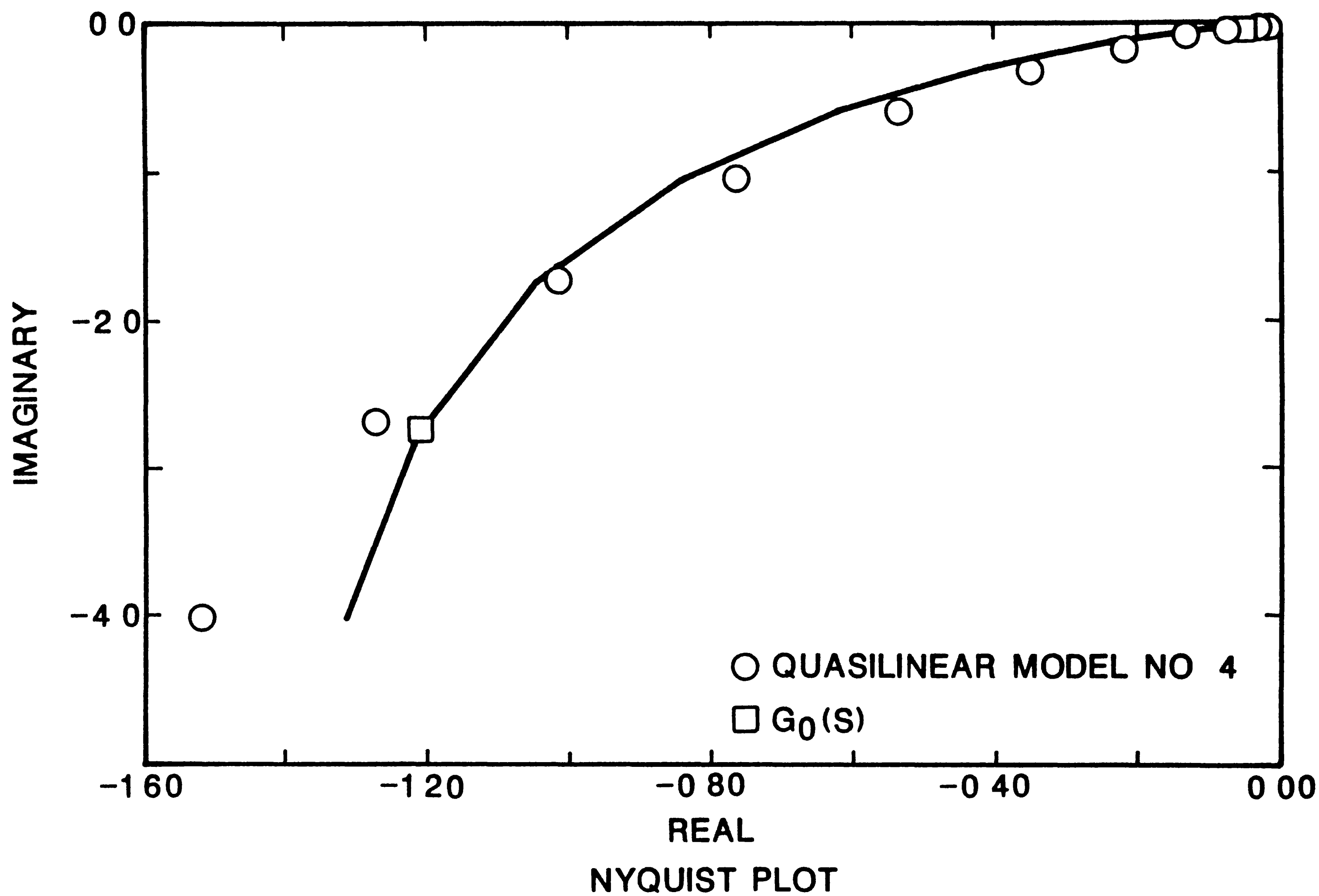


Figure 16 Approximation of the Frequency Response of the Quasilinear Model Number 4 with a Linear Model of an Open-Loop System

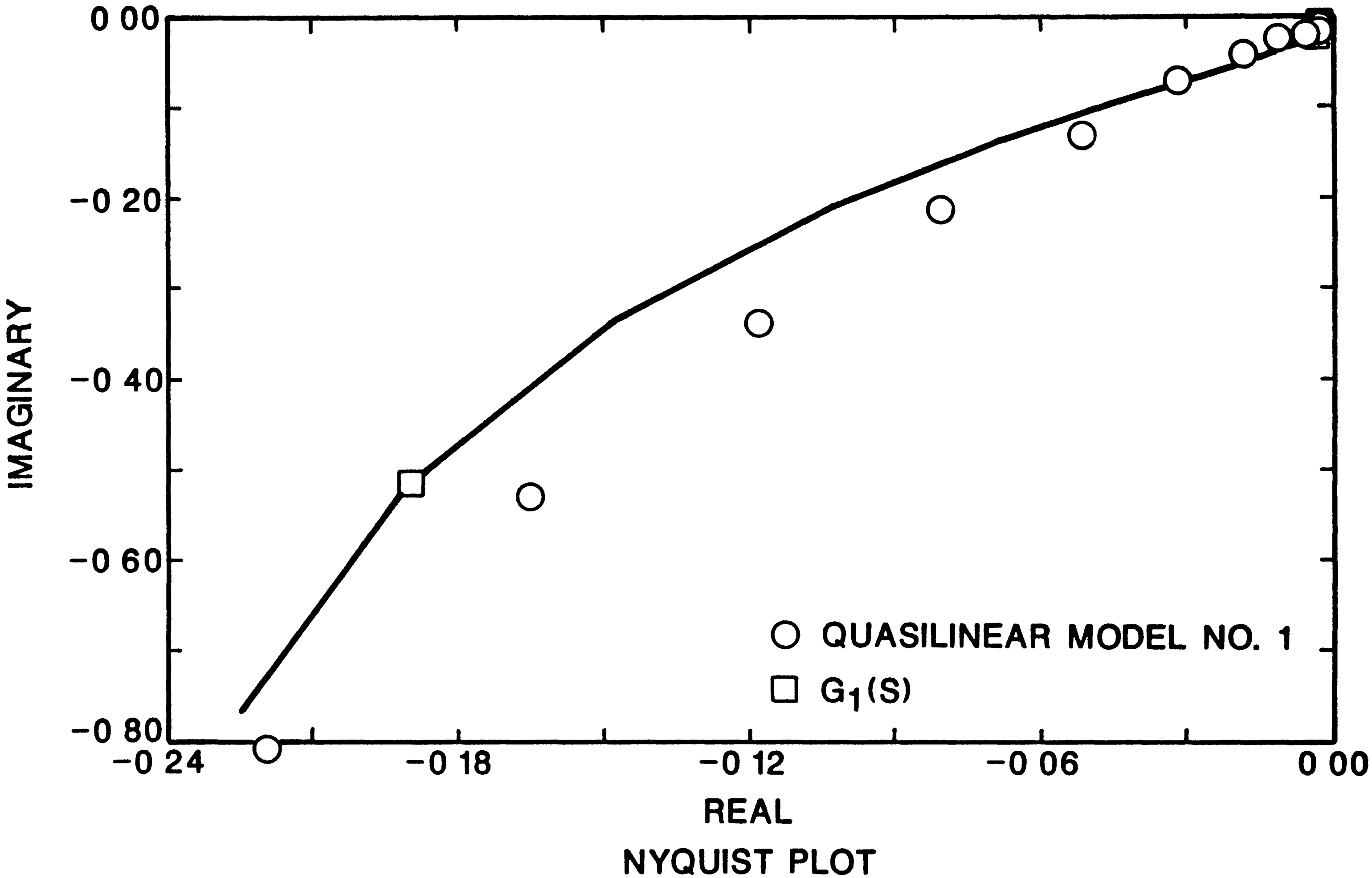


Figure 17 Approximation of the Frequency Response of the Quasilinear Model Number 1 with a Linear Model of an Open-Loop System

(i) The quasilinear models are not linear models, and unlike linear system models (transfer functions) the Hilbert relation between the real and imaginary parts of the frequency response does not hold. Therefore, a "perfect" fit is not possible, and the quality of fit is a function of the dominance of nonlinear effects. For example, the motor stiction effects at small signals are more dominant than those at large signals Therefore, it is expected to obtain a better quality of fit to the frequency response information of the quasilinear models at higher amplitude excitation signals than those at lower amplitude excitation signals. This is clearly apparent from examination of the "%ERROR" columns of Tables III and IV.

(ii) The user may be able to identify transfer functions with high-order dynamic terms that fit the data "perfectly" With reference to the polynomial fitting discussion at the beginning of this subsection, the identification of high-order transfer functions will most likely be undesirable That is, such transfer functions will most likely be unstable, while the actual quasilinear models are representation of a stable nonlinear system when operating at different operating conditions

(iii) If the real or imaginary part of the frequency response data is comprised of numbers with low orders of magnitude (for example, (0 01, 0.0j)), then relatively large percent errors may be expected since percent error calculation would involve division by small numbers. It should be kept in mind

TABLE III

FREQUENCY RESPONSE COMPARISON FOR THE SIDF MODEL NUMBER 4 AND THE APPROXIMATING OPEN-LOOP SYSTEM, $G_0(s)$

REAL PART			IMAGINART PART		
QUASILINEAR MODEL	$G_0(s)$	%ERROR	QUASILINEAR MODEL	$G_0(s)$	%ERROR
-1.51700	-1 31700	13.23	-4 02500	-4.0450	0.501
-1.27000	-1 20800	4 95	-2.68300	-2.7360	1 993
-1 01200	-1.04700	3 46	-1 71800	-1 7560	2 190
-0 76310	-0 83910	9 96	-1 03900	-1.0500	1 091
-0.53320	-0 61330	15 02	-0 59120	-0 5800	1 891
-0 34820	-0 40910	17.50	-0.32070	-0 2990	6 772
-0.21590	-0 25280	17.12	-0 16940	-0.1483	12 430
-0 12820	-0.14790	15.39	-0 08635	-0.0738	14.520
-0 07161	-0 08359	16.73	-0.04423	-0 0384	13.210
-0.04051	-0.04625	14.17	-0 02152	-0.0215	0 324
-0 02272	-0 02529	11.28	-0 01114	-0 0130	6 490
-0 01141	-0.01373	20.31	-0 00504	-0 0084	66 720

TABLE IV

FREQUENCY RESPONSE COMARISON OF THE SIDF MODEL NUMBER 1 AND THE APRROXIMATING OPEN-LOOP SYSTEM, $G_1(s)$

REAL PART			IMAGINART PART		
QUASILINEAR MODEL	$G_1(s)$	%ERROR	QUASILINEAR MODEL	$G_1(s)$	%ERROR
-0 21960	-0.19920	9 30	-0 80670	-0.79310	1 69
-0 16530	-0.18070	9.28	-0 52950	-0 52910	0 87
-0.11840	-0 14470	22 23	-0 33930	-0 34130	0.59
-0 08062	-0.10348	28 30	-0 21290	-0.21590	1 40
-0.05169	-0.06721	30.01	-0 13150	-0 13720	4 400
-0 03191	-0 04065	27.39	-0.07176	-0 89550	24 79
-0.01820	-0.02343	28 78	-0.04177	-0 06047	44 75
-0.01115	-0.01312	17.62	-0.02442	-0.04208	72 31
-0 00578	-0 00722	24 81	-0 01818	-0.02992	64 62
-0.00319	-0.00394	23.39	-0.00812	-0.02157	165.70
-0 00211	-0 00213	1.197	-0.00486	-0.01567	222 70
-0.00543	-0.00115	78 75	-0.00229	-0.01144	400 90

that the quasilinear models are only approximation to a nonlinear model which itself is an approximation for mathematical description of a physical process Therefore, it suffices to identify transfer functions whose frequency response information approximate the frequency response information of the quasilinear models in a near-optimum fashion (See Equation (3.8)). The user should accept the best fit, and continue with the controller synthesis procedure. The results of Step 6 (Design Validation), will assure the user if the final synthesized controller is acceptable or not. If the results are not acceptable, then this method has failed, and an n-range linear or nonlinear (1) controller may be synthesized

Step 4 Controller Parameterization

The class of all controllers that stabilize the identified linear systems of the previous step is to be parameterized This parameterization was developed in Chapter Three, and it is given by Equation (3 14) subject to constraints (4 9) and (4 10), that is

$$c(\, r'(s)\,) = \frac{r'(s)}{I - r'(s)\, G_1(s)} \qquad (4\ 8)$$

where,

r'(s) belongs to the algebraic ring H, (4 9)

and

$G_a(s)$ must be strongly stabilizable, i.e , $G_a(s)$ must be stabilized via a stable r(s) (4 10)

Where,

$G_a(s) = G_1(s) - G_0(s)$, and $G_0(s)$ and $G_1(s)$ are given by Equation (4.6) and (4.7), respectively.

Any r'(s) that satisfies constraints (4.9) and (4.10) may be substituted into Equation (4.8) to obtain a specific common controller which would assure closed-loop system stability whether the plant is represented by $G_0(s)$ or $G_1(s)$. However, any r'(s), that assures closed-loop system stability, may not force the static and dynamic behavior of the closed-loop system to be close to those of the reference linear model which was identified in Step 1. In Step 5, the class of all stabilizing controllers is searched for the controller which could not only achieve simultaneous closed-loop system stability, but it would also achieve the required closed-loop state and dynamic performance (as defined in Step 1) in a near-optimum fashion.

Step 5 Controller Synthesis

The identified class of all stabilizing controllers is to be searched for the near-optimum controller - the controller that forces the dynamic behavior of $\Sigma(c,G_0)$ and $\Sigma(c,G_1)$ to be as close to $G_r(s)$ as possible. With reference to Equation (3.19) and constraints (3 20) and (3.21), the DRLCD, which is the computer-aided Dual Range Linear Controller Design software utility, is used to identify that r'(s) which would define the near-optimum linear controller DRLCD uses Algorithm 1, which was presented in Chapter III, to synthesize the near-optimum linear controller. The primary inputs to DRLCD are linear models of the open-loop systems $G_0(s)$, $G_1(s)$, and $G_r(s)$. These linear models are given by Equations (4.6), (4.7), and (4.5), respectively. The user must also supply a subroutine whose input is the plant input, u(t), and its output is the plant output, y(t). Such a subroutine for this example problem is shown in Figure 18 DRLCD uses Algorithm 1.1, which was presented in Chapter III, to obtain a reliable starting solution for the optimization routine

Technical details of the controller synthesis was presented in Chapter III, and DRLCD software utility was designed and developed to create an environment which man and machine would work together to arrive at a satisfactory solution. Due to the complexity of the algorithms and interfacing of various steps (specially Steps 4, 5, and 6), a detailed methodical approach, which would attempt to illustrate the application of the controller synthesis procedure, may not be effective. Alternatively, Appendix F is designed to illustrate how one may interface with the DRLCD software utility to apply the controller synthesis procedure to arrive at a practical controller For this example problem, the near-optimum linear controller is of the following form (See Appendix F for details)

$$c(s) = \frac{59\ 968 + 4\ 119\ s}{27.750 + 1.000\ s} \qquad (4\ 11)$$

Step 6 Design Validation

The design is to be validated by direct simulation of the synthesized linear controller and the actual nonlinear process (see Figure 10). The normalized step responses for a range of step magnitudes, which correspond to the operating conditions, of interest are obtained, the results are shown in Figure 19 The maximum per cent overshoot is 20 and its two percent settling time is 0 3 seconds, and system response is relatively insensitive to the level of the step input, and its dy-

```
        SUBROUTINE MOTLD(UPR,X,XDOT,IC2)
      DIMENSION X(1),XDOT(1)
      REAL JØ
      DATA JØ,B,FØ,DEL,AK1,AK2,VELL
          /Ø Ø1,Ø 1,1 Ø,Ø 5,5 Ø,1 Ø,Ø Ø/
      DATA ONE/1 Ø/
C--
C----  IMPLEMENT THE INNER LOOP, UP IS THE INPUT TO THE
C----  NONLINEAR PROCESS
C--
      UP=UPR
      VEL=X(IC2+2)
C--
C----  NONLINEAR SATURATION
C--
      AU=ABS(UP)
      SU=SIGN(ONE,UP)
      IF(AU LE DEL)THEN
          TA=UP*AK1
      ELSE
          TA=SU*(AK1*DEL+AK2*(AU-DEL))
      ENDIF
C--
C----  TA IS THE OUTPUT OF THE SATURATION ELEMENT, AND
C----    IT IS ALSO THE INPUT TO THE STICTION ELEMENT
C--
C--
C----  IMPLEMENTATION OF THE COULOMB FRICTION LOGIC
C--
      IF(ABS(TA).GE.FØ)THEN
          TE=TA-SIGN(FØ,VEL)
      ELSE IF ((ABS(VELL) GT Ø Ø) AND (VELL*VEL GT Ø Ø))THEN
          TE=TA-SIGN(FØ,VEL)
      ELSE
          TE=Ø Ø
          X(IC2+2)=Ø Ø
          VEL=Ø Ø
      ENDIF
      VELL=VEL
C--
C---- COMPUTE DERIVATIVES
C--
      XDOT(IC2+1)=X(IC2+2)
      XDOT(IC2+2)=TE/JØ-(B/JØ)*X(IC2+2)
      RETURN
      END
```

Figure 18 Subroutine MOTor-LoaD Describing the Dynamic Behavior of the Nonlinear Plant

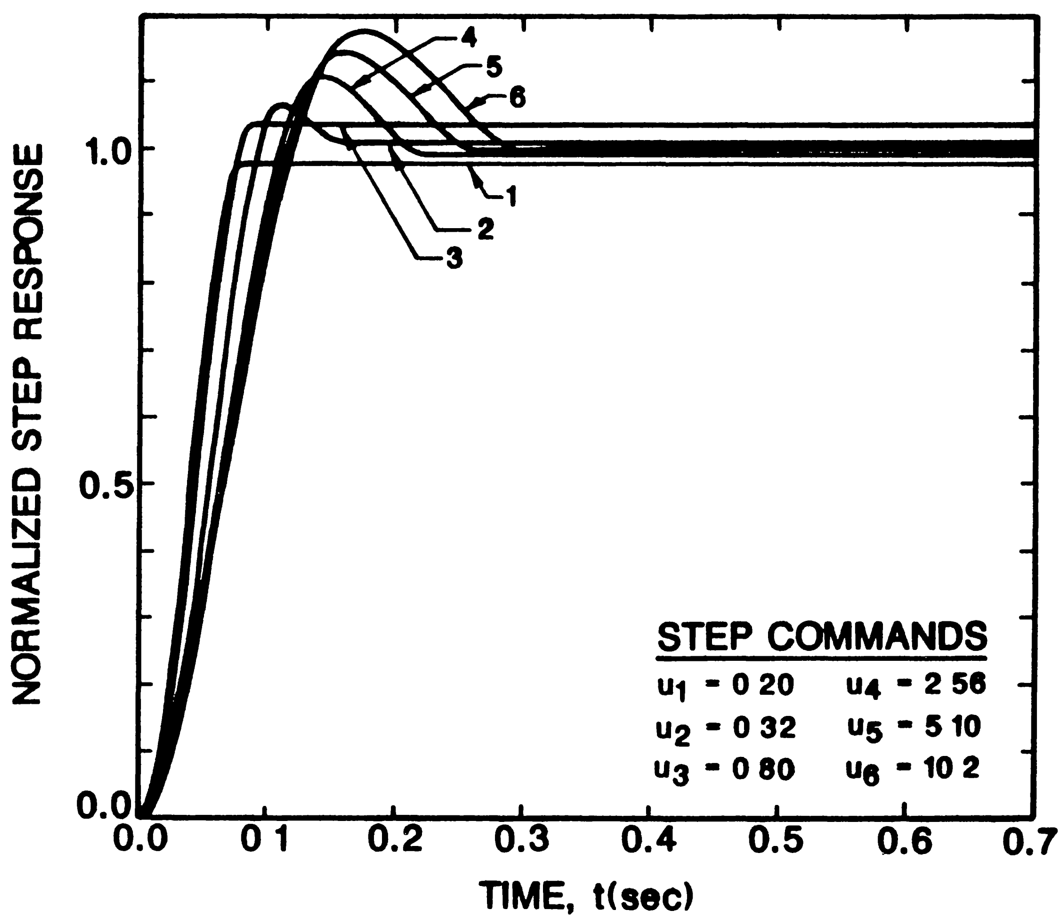


Figure 19 Normalized Step Response Plots of the Synthesized Linear Controller and the Nonlinear Plant

namic behavior is satisfactory, i.e., the maximum percent overshoot is less than 37 percent and the two percent settling time is less than 0.3 seconds for all interested input amplitudes.

Controller Comparison

One of the primary objectives of this chapter was to compare the performance of the system with a linear and a nonlinear controller which has been reported by Taylor (1). This objective is fulfilled in this section.

Dual-Range Linear Controller Design Vs. Single-Range Linear Controller Design

A linear PID controller may be synthesized based on only one sinusoidal-input describing function model of the nonlinear plant of Figure 10 (SRLCD). Quasilinear model number 4 (See Figure 15), which has the highest gain amplification, was selected by Taylor and Strobel (1) to design a linear PID for the example problem The controller configuration and the normalized step responses for a range of input amplitudes are shown in Figures 20 and 21 From the comparisons of Figures 19 and 21 the following conclusions may be drawn

(i) The single-range linear PID controller has formed a closed-loop system which is appreciably more sensitive to the amplitude-level of the command signal Typically, single-range linear controllers designed for use with highly nonlinear plants whose dynamic behavior is appreciably different around the operating conditions of interest, if interest do form closed-loop feedback systems which are

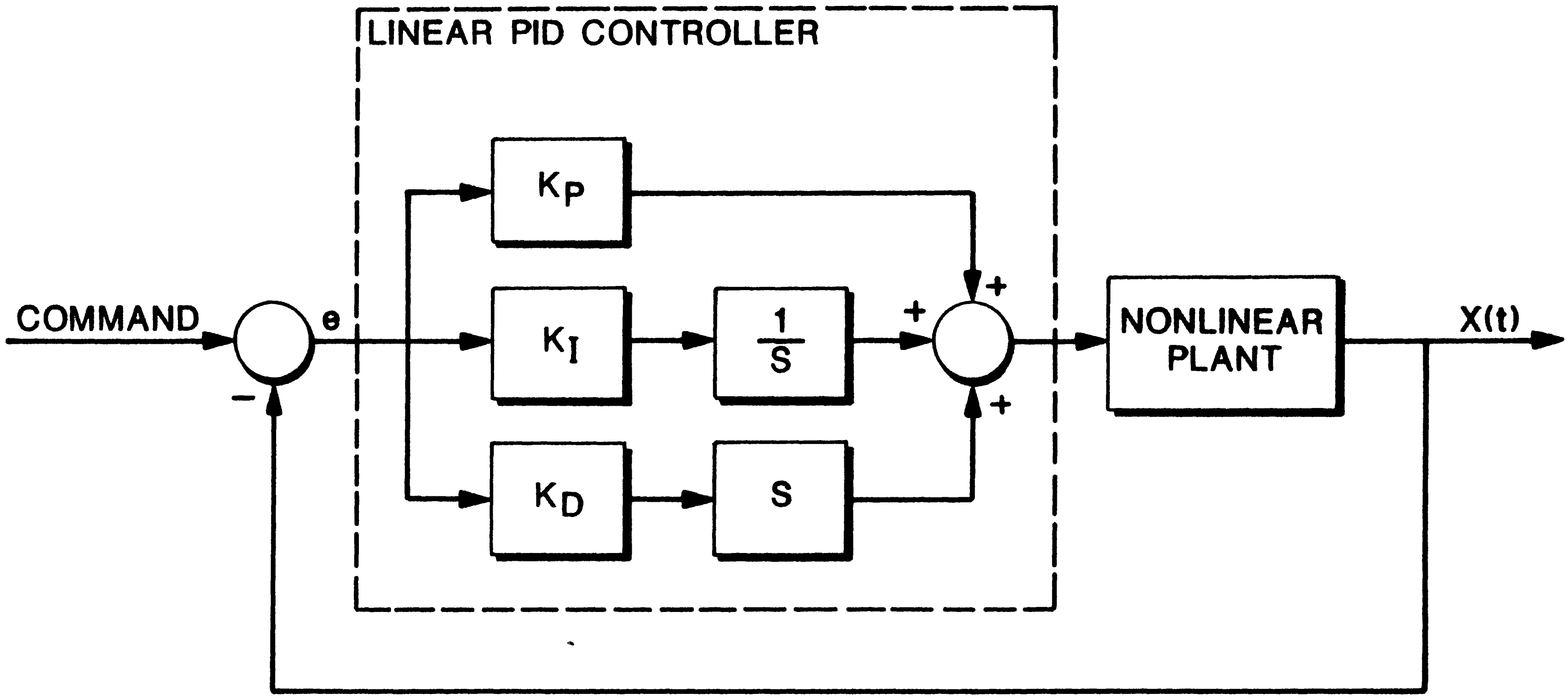
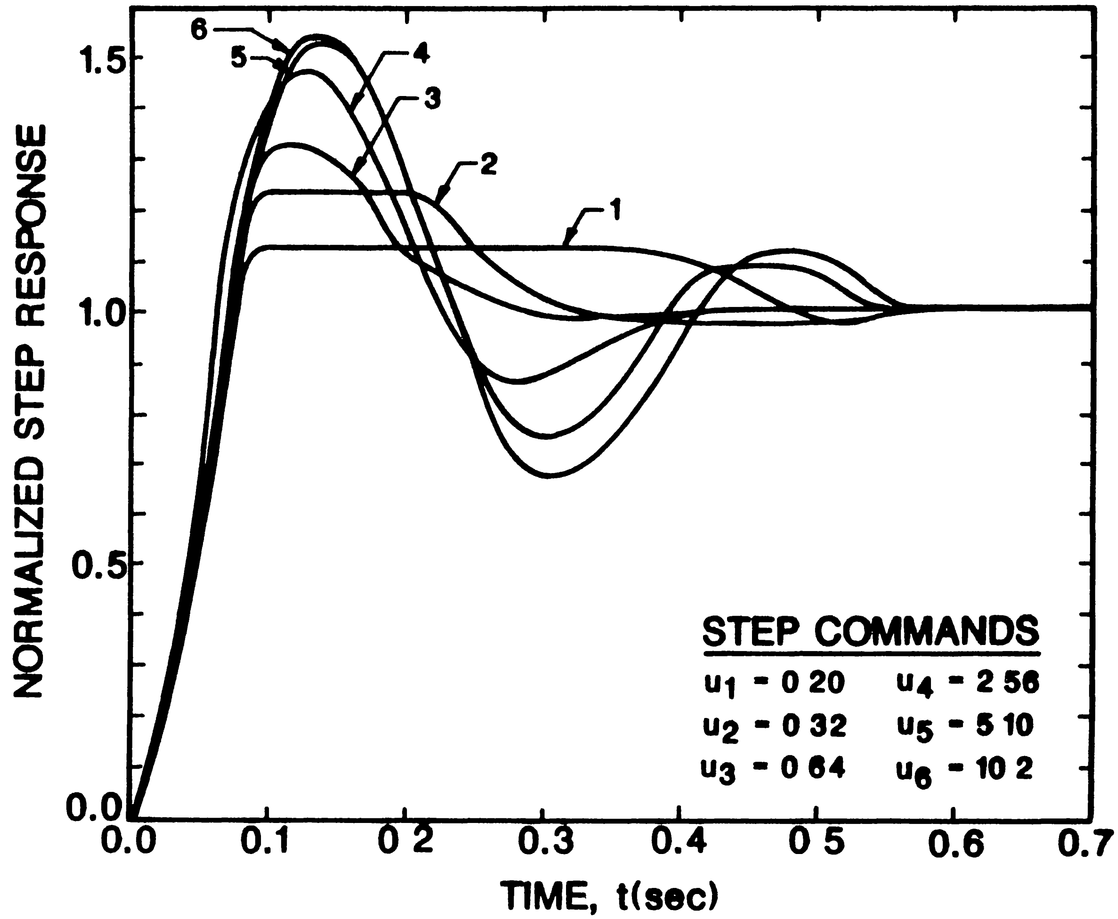


Figure 20   Linear PID Control of the Nonlinear Plant



Source Taylor and Strobel (1)

Figure 21 Normalized Step Response Plots of the System with Linear PID Control of the Nonlinear Plant sensitive to the amplitude-level of the command signal. Since sensitivity issue is an important factor for a robust design, it may be concluded that in dealing with highly nonlinear plants, where robustness is an important performance criterion, a dual-range linear controller design technique is prefered. In any case, the SRLCD technique should proceed the DRLCD technique so the user has confirmed the failure of the SRLCD technique Only in that case, the user can justify the application of the DRLCD technique

(ii) The closed-loop system with the single-range linear PID controller is more sluggish than the closed-loop system with the dual-range linear controller The degree of the closed-loop system stability with the single-range linear PID controller decreases as the amplitude-level of the command signal increases. This is not the case with the closed-loop system with the dual-range linear controller.

(iii) The 2% settling time of the closed-loop system with the single-range linear PID ranges from 0.3-0.6 seconds over the range of amplitude-level of the excitation signal The 2% settling time of the closed-loop system with the dual-range linear controller ranges from 0 08-0.3 seconds over the same range of amplitude-level of the excitation signal.

Dual-Range Linear Controller Design Vs Multi-Range Nonlinear Controller Design

A nonlinear PID controller was designed by Taylor and Strobel (1) based on several quasilinear models of the nonlinear plant The controller configuration and the normalized step responses for a range of input amplitudes are shown in Figures 13 and 14. From the comparisons of Figures 14 and 19 the following conclusions may be drawn

(i) The closed-loop system with the nonlinear controller and the closed-loop system with the synthesized dual-range linear controller are both "equally" insensitive to the amplitude-level of the command signal.

(ii) The closed-loop system with the nonlinear controller has a maximum percent overshort of 37% while the closed-loop system with the dual-range linear controller has a maximum percent overshoot of 20

(iii) The closed-loop system with the nonlinear controller as well as the closed-loop system with the synthesized dual-range linear controller both have a maximum 2% settling time of 0.3 seconds.

CHAPTER V

SUMMARY AND RECOMMENDATIONS

Summary

The goal of this study was to develop a systematic and algebraic controller synthesis procedure for the design of feedback control systems with highly nonlinear, single-input single-output, time invariant, and continuous time plants That goal has been met.

The controller synthesis procedure is based on simultaneous stabilization theory and sinusoidal-input describing function models of the nonlinear plant. The use of SIDF models results in a robust feedback control system which is the least conservative (1) The user provides two types of information (1) a set of performance measures in either the time domain or in the frequency domain, and (2) the mathematical model of the nonlinear plant in standard state-variable differential equation form. The mathematical model of the nonlinear plant is used to generate the SIDF models of the plant at the specified operating conditions of interest. If such models do not exist, the user must obtain the SIDF models via laboratory experiments, exciting the plant with sinusoidal inputs.

Then, upon the execution of the control synthesis procedure given in Chapter III, the near-optimum linear controller is identified. The procedure does not idenfity the optimum controller in the global sense, hence, the term near-optimum controller. The near-optimum linear controller is the controller that forms (i) a closed-loop system whose dynamic behavior is as insensitive to the amplitude level of the input signal as possible, and (ii) a closed-loop system whose dynamic behavior satisfies a set of predefined performance measures in a near-optimum fashion.

The controller synthesis procedure is composed of six primary steps (i) specification of a set of performance measures for the closed-loop system either in the time domain or frequency domain, and identification of the corresponding linear reference model of an open-loop system, (ii) characterization of the input/output behavior of the nonlinear plant by obtaining the quasilinear (describing function) models of the nonlinear plant for several predefined operating ranges of interest, (iii) identification of two linear systems whose input/output behavior approximate the input/output behavior of the two quasilinear plants whose gain characteristics bound those of all others in the class, (iv) parameterization of the class of all controllers that simultaneously stabilize the two identified linear plants, (v) search of the class of all stabilizing controllers for the near-optimum linear controller, and (vi) validation of the design via a digital simulation of the nonlinear feedback system. A computer-aided design (CAD) environment was designed and developed by the author on a Harris-800 minicomputer and a Tektronix 4115B high resolution color graphics terminal to implement steps (ii) - (vi) above.

Since the controller synthesis procedure is both systematic and algebraic, the procedure may be employed without a high level of subjective judgment. The only area calling for user judgment is step (iii), where the user may have to iterate to obtain a suitable fit. The method and the associated software was applied to a practical position control problem of the sort encountered in robotic control. A linear PID controller, based upon a nominal SIDF model of the nonlinear plant, and a nonlinear PID controller, based upon several SIDF models of the nonlinear plant, were synthesized previously for the example nonlinear plant under study (1). The performance of the example system with a linear controller synthesized using the procedure presented herein, was found to be superior to the performance of the system with a linear PID controller, and comparable to the performance of the system with a nonlinear PID controller

Recommendations

There are two primary issues that concern this research project. The first issue is whether a controller design procedure based on two linear models, which are approximations to two SIDF models of a nonlinear plant, in conjuction with the simultaneous stabilization theory is applicable to systems which have highly nonlinear plants Based on the work done in this thesis effort, one may conclude that such a controller design procedure may be applied to form closed-loop systems that satisfy a set of user-defined performance measures in a near-optimum fashion (see Figure 81). The second issue is how should the set of all controllers that stabilize the two linear models be searched for the near-optimum controller The near-optimum controller, in mathematical terms, is defined as the controller that minimizes objective function (3.22) subject to constraints (3 20) and (3.21) This type of constrained minimization has the following difficulties

(1) Due to the presence of the constraints, unconstrained optimization algorithms that minimize a given function either based upon function evaluations (Pattern search (49,50,55)) and/or based on evaluation of the gradients of the function are not effective. Once the constraints are violated, the objective function is set to a large value, and the optimization algorithm stops after a few iterations

(2) The constraints are implicit in the sense that there does not exist a function that would describe the constraints The constraints are tests, once they are failed the objective function takes on a large value. Therefore, many available constraint optimization algorithms can not be applied.

In this thesis effort, the simplex search method (55) was employed to overcome these problems, as recommended by Wright (56). The method forms a regular simplex in the space of the independent variables Then, the objective function at each vertex is evaluated. The vertex with the highest functional value is located and reflected through the centroid to complete a new simplex As long as the performance index decreases smoothly, the iterations move along crabwise This feature of the simplex algorithm will avoid the forbidden zones of independent variables. However, this algorithm is highly sensitive to the starting solution, and satisfactory results may not easily be obtained. For this reason, the optimization which would minimize the objective function given by (3 22) subject to constraints (3 20) and (3.21) require further investigation.

There are other tasks that are recommended for future work Those are given below.

1) Extension of the controller system design procedure from a two model approach to an n-model approach, i.e , base the controller synthesis on n (n>2) SIDF models of the nonlinear plant. There may be cases which gain variations of the SIDF models of the nonlinear plant may require inclusion of more than two SIDF models.
2) Extension of the method and modification of the developed software utilities for use with multiple-input multiple-output nonlinear time-invariant continuous time systems.
3) Extension of the method to permit the solution of the disturbance rejection problem
4) Extension of step 4 of the design procedure to permit coprime factorization in a user-defined region of the s-plane

A SELECTED BIBLIOGRAPHY

(1) Taylor, J. H , and Strobel, K. L., "Nonlinear Control System Design Based on Quasilinear System Models," American Control Conference, 1985.

(2) Taylor, J. H., "A Systematic Nonlinear Controller Design Approach Based on Quasilinear Models," American Control Conference, 1983, WA7-11 45, pp. 141-145.

(3) Vidyasagar, M., and Viswandham, N , "Algebraic Design Techniques for Reliable Stabilization," IEEE Transactions on Automatic Control, Vol AC-27, October 1982, pp. 1085-1095

(4) Chen, C. T., Introduction to Linear System Theory, Holt, Reinehart, and Winstop Inc., New York, 1970.

(5) Vidyasagar, M., Control System Synthesis A Factorization Approach, MIT Press, 1985.

(6) Callier, F M , and Desoer, C. A , Multivariable Feedback Systems, Springer-Verlag, 1982.

(7) Rowland, J. R., Linear Control Systems, John Wiley and Sons, New York, 1986

(8) Rosenbrock, Computer-Aided Control System Design, Academic Press, London, 1974

(9) MacFarlane, A. G. J , Complex Variable Methods for Linear Multivariable Feedback Systems, Taylor and Francisco LTD, 1980

(10) D'Azzo, J. J., and Houpis, C. H , Linear Control System Analysis & Design, Conventional and Modern, McGraw-Hill New York, 1981

(11) Dorf, R. C., Modern Control Systems, Addison-Wesley Publishing Company, Third Edition, Reading Massachusetts, 1980

(12) Kuo, B. C., Automatic Control Systems, Prentice-Hall Inc , Fourth Edition, Englewood Cliffs, N. J , 1982.

(13) Taylor, J. H., "Robust Computer-Aided Design of Multivariable Nonlinear Plants," Application of Multivariable Systems Theory, Manadon, Plymouth, UK, October 1982.

(14) Taylor, J. H., and Strobel, K L , "Applications of a Nonlinear Controller Design Approach Based on Quasilinear System Models," Proc. American Control Conference, San Diego, CA, June 1984.

(15) Suzuki, A., and Hedrick, J Karl, "Nonlinear Controller Design by an Inverse Random Input Describing Function Method," American Control Conference, 1985, FA2-10 15, pp. 1236-1241.

(16) Beaman, Joseph J., "Application of NQG Control with Uncertain and Incomplete Measurements," American Control Conference, 1984, TA10-8 30, pp. 798-802.

(17) Freund, E , "Fast Nonlinear Control with Arbitrary Pole-Placement for Industrial Robots and Manipulators," The International Journal of Robotics Research, Vol 1, No. 1, Spring 1982, pp. 65-78.

(18) Young, G. E., and Rao, S L , "Robust Control of a Nonlinear Process with System Uncertainty and Delay Using Variable Structure," American Control Conference, 1986

(19) Kirk, Donald E , Optimal Control Theory, Prentice-Hall Inc , Englewood Cliffs, N J., 1970

(20) Landau, Yoan D., Adaptive Control, Marcel Dekker, Inc., 1979

(21) Gelb, A , and Vander Velde, W. E , Multiple-Input Describing Functions and Nonlinear System Design, McGraw- Hill, New York, 1968

(22) Atherton, D P., Nonlinear Control Engineering, van Nostrand Reinhold Co , London, 1975

(23) Taylor, J H., "Computer-Aided Control Engineering Environment for Nonlinear Systems," Third IFAC Symposium CAD in Control and Engineering Systems, Lyngby, Denmark, August 1985

(24) Taylor, J. H., "General Describing Function Method for Systems With Many Nonlinearities, With Applications to Aircraft Performance," Joint Automatic Control Conference," San Francisco, 1980

(25) Taylor, J H , "Describing Function Method for Limit Cycle Analysis of Highly Nonlinear Systems," IX Int. Conf. Nonlinear Oscillations, Keiv, USSR, 1981

(26) Taylor, J. H., "Applications of a General Limit Cycle Analysis Method for Multivariable Systems," Chapter 9 of Nonlinear System Analysis and Synthesis Vol. 2- Techniques and Applications, Edited by Ramnath, R. V., Hedrick, J K , and Paynter, H M , The American Society of Mechanical Engineers, 1980

(27) Taylor, J. H., "An Algorithmic State-Space/Describing Function Technique for Limit Cycle Analysis," TIM-612-1, The Analytic Sciences Corp (TASC), Reading, MA, 1975.

(28) Chen, C. F., and Philip, B. L., "Graphical Determination of Transfer Function Coefficient of a System From its Frequency Response," IEEE Transactions on Applied Industry, Vol 83, March 1963, pp. 42-45.

(29) Ausman, J. S., "Amplitude Frequency Response Analysis and Synthesis of Unfactored Transfer Functions," ASME Journal of Basic Engineering, Vol 86, March 1964, pp 32-36

(30) Lin, P. L., and Wu, Y C., "Identification of Multi-input Multi-output Linear Systems From Frequency Response Data," Transactions of the ASME Journal of Dynamic Systems, Measurements, and Control, Vol. 104, March 1982, pp. 58-64.

(31) Dudnikov, E. E., "Determination of Transfer Function Coefficients of a Linear System From the Initial Portion of an Experimentally Obtained Amplitude-Phase Characteristic," Automatic Remote Control, Vol 20, 1959, pp. 552-558

(32) Golant, A. I., and Dudnikov, E. E , "Method of Determining Linear Phase Characteristic," Automatic Remote Control, Vol 25, 1964, pp. 1243-1248.

(33) Chen, C. F., and Philip, B. L , "Accurate Determination of Complex Root Transfer Function From Frequency Response Data," IEEE Transactions on Automatic Control, Vol AC-10, July 1965, pp 356-358

(34) Shieh, L. S., and Navarro, J. M , "Frequency-Variation Method for System Identification," IEEE Transactions on Circuits and Systems, Vol. CAS-21, November 1974, pp. 754-763.

(35) Levy, E. C., "Complex Curve Fitting," IRE Transcations on Automatic Control, Vol AC-4, 1959, pp. 37-43

(36) Sanathanan, C. K., and Koerner, J , "Transfer Function Synthesis as a Ratio of Two Complex Polynomials," IEEE Transactions on Automatic Control, Vol AC-8, January 1963, pp 56-58.

(37) Payne, P A , "An Improved Technique for Transfer Function Synthesis From Frequency Response Data," IEEE Transactions on Automatic Control, Vol AC-15, Aug. 1978, pp 480-483.

(38) Brittingham, J. N., Miller, E. K , and Williows, J L , "Pole Extraction From Real-Frequency Information,", Proceeding of IEEE, Vol 68, February 1980, pp 263-273

(39) Lawrence, P J , and Rogers, G J., "Sequential Transfer Function Synthesis From Measured Data," Proc. IEEE Transactions on Automatic Control, Vol. 126, Jan 1979, pp 104-106.

(40) Hsia, T C , System Identification, Lexinton Books, Lexington, Massachusetts, 1977.

(41) Vidyasagar, M., Schneidir, H., and Francis, B. A , "Algebraic and Topological Aspects of Feedback Stabilization," IEEE Transactions on Automatic Control, Vol. AC-27, No 4, August 1982, pp 880-894.

(42) Nett, C. N , Jacoson, C. A., and Balas, M. J., "A Connection Between State-Space and Doubly Coprime Fractional Representations," submitted for publication in IEEE Transactions on Automatic Control.

(43) Nett, C. N , Jacobson, C. A., and Balas, M. J , "Fractional Representation Theory Robustness With Applications to Finite Dimensional Control of A Class of Linear Distibuted Systems," Submitted for publication in IEEE Transactions on Automatic Control

(44) Desoer, C. A , Liu, R-W, Murray, J , and Sakes, R., "Feedback Systems Design The Fractional Representation Approach to Analysis and Synthesis," IEEE Transactions on Automatic Control, Vol. AC-25, No 3, June 1980, pp 399-412

(45) Jacobson, N., Lectures in Abstract Algebra, Vol. I. Berlin, Germany Springer-Verlag, 1964.

(46) Zariski, O., and Samuel P., Commutative Algebra, Vol I Berlin, Germany Springer-Verlag, 1958.

(47) Blomberg, H., and Ylinen, R , Algebraic Theory for Multivariable Linear Systems, Academic Press, New York, 1983.

(48) Meyfarth, P. F., Dynamic Response Plots and Design Charts for Third-Order Linear Systems, M I T Dynamic Analysis and Control Laboratory, Research Memorandom 7401-3, September, 1958

(49) Chandler, J P., PATRN Optimization Computer Program, UCC WATFIV Library, Oklahoma State University, Stillwater, OKlahoma

(50) Hooke Robert, and Jeeves T A., "Direct Search SOlution of Numerical and Statistical Problems," ACM Transactions on Mathematical Software, August 1961, pp 212-229

(51) More, J. J , B , S Garbow, and K. E Hillstropm, User Guide to MINPACK-1, Argone National Laboratory, Report No ANL 80-74

(52) Kuester, J. L , and Mize, J. H., Optimization Techniques with Fortran, McGraw-Hill, 1973

(53) Hasdorff, L , Gradient Optimization and Nonlinear Control, John Weily & Sons, New York, 1976.

(54) Gerald, F C., Applied Numerical Analysis, Adison-Wesley Publishing Company, Inc., Reading Massachusetts, 1980

(55) Reklaitis, G.V., Ravindran, A, and Ragsdell, K.M., Engineering Optimization, John Wiely and Sons, New York, 1983

(56) Wright, Margaret, Practical Optimization, Academic Press, 1982.

(57) Kummar, S., "Computer-Aided Design of Multivariable Systems," M.S thesis, State University of New York, Stony Brook, 1969.

(58) Chandler, J. P , SMPLX Optimization Computer Program, UCC WATFIV Library, Oklahoma State University, Stillwater, Oklahoma

APPENDIXES

APPENDIX A

IDENTIFICATION OF SINGLE-INPUT SINGLE-OUTPUT LINEAR SYSTEMS FROM FREQUENCY RESPONSE DATA

The primary objectives of this Appendix are to illustrate (i) the system identification of single-input single-output, time invariant, linear, and deterministic systems from frequency response data (30) and (ii) the necessary reformulation of item (i) above which would allow the identification of type one systems as well.

Identification of Type Zero Systems

In this section, the system identification technique of Lin and Wu (30) for finding the transfer function from frequency response data is presented.

Formulation

The formulation of the system identification technique which is given below, is directly taken from the work done by Lin and Wu (30), and no contribution is claimed. A transfer function may be represented by the following equation

$$g(j\omega) = \frac{b_0 + b_1 + \ldots + b_m s^m}{1 + a_1 s + \ldots + a_n s^n},\quad m \leq n. \tag{a.1}$$

To obtain the sinusoidal steady-state response, substitute $s=j\omega$ into Equation (a.1). The following equation is obtained.

$$g(j\omega) = \frac{(b_0 - b_2\omega^2 + b_4\omega^4 - \ldots) + j(b_1\omega - b_3\omega^3 + b_5\omega^5 - \ldots)}{(1 - a_2\omega^2 + a_2\omega^2 - \ldots) + j(a_1\omega - a_3\omega^3 + a^5\omega^5 - \ldots)} \tag{a.2}$$

$$= \frac{N'(j\omega)}{M'(j\omega)}. \tag{a.3}$$

Let

$$g(j\omega_i) = R_i + jI_i + E_i, \tag{a.4}$$

where

R_i is the real part of the user-supplied frequency response datum at discrete frequency ω_i,

I_i is the imaginary part of the user-supplied frequency response datum at discrete frequency ω_i, and

E_i is a complex number which represent the error in approximating the frequency response datum $(R_i + jI_i)$ at frequency ω_i.

Substitute Equation (a.4) into Equation (a.3) to obtain the following equations.

$$b_0 - b_2\omega_i^2 + b_4\omega_i^4 - \ldots = R_i(1 - a_2\omega_i^2 + a_4\omega_i^4 - \ldots) - I_i(a_1\omega_i - a_3\omega_i^3 + a_5\omega_i^5 - \ldots) + E_{R_i}, \text{ and} \tag{a.5}$$

$$b_1\omega_0 - b_3\omega_i^3 + b_5\omega_i^5 - \ldots = I_i(1 - a_2\omega_i^2 + a_4\omega_i^4 - \ldots) + R_i(a_1\omega_i - a_3\omega_i^3 + a_5\omega_i^5 - \ldots) + E_{I_i}. \tag{a.6}$$

Where,

$$E_{R_i} = (\mathrm{Re}E_i)(1 - a_2\omega_i^2 + a_4\omega_i^4 - \ldots) - (\mathrm{Im}E_i)(a_1\omega_i - a_3\omega_i^3 + a_5\omega_i^5 - \ldots) \tag{a.7}$$

$$E_{I_i} = (\mathrm{Im}E_i)(1 - a_2\omega_i^2 + a_4\omega_i^4 - \ldots) + (\mathrm{Re}E_i)(a_1\omega_i - a_3\omega_i^3 + a_5\omega_i^5 - \ldots), \tag{a.8}$$

$\mathrm{Re}E_i$ is the real part of the complex number E_i, and

$\mathrm{Im}E_i$ is the imaginary part of the complex number E_i.

Designate the number of user-supplied frequency response data by the symbol k'. Write Equations (a.5) and (a.6) k' times for $\omega_1, \omega_2, \ldots, \omega_k$, and concatenate the $2k'$ linear equations. The following linear equation would be obtained.

$$X'\theta = \beta + E. \tag{a.9}$$

Where

$$X = \begin{bmatrix} 1 & 0 & -\omega_1^2 & 0 & \omega_1^4 & \cdots & I_1\omega_1 & R_1\omega_1^2 & -I_1\omega_1^3 & \cdots \\ 0 & \omega_1 & 0 & -\omega_1^3 & 0 & \cdots & -R_1\omega_1 & I_1\omega_1^2 & R_1\omega_1^3 & \cdots \\ 1 & 0 & -\omega_2^2 & 0 & \omega_2^4 & \cdots & I_2\omega_2 & R_2\omega_2^2 & -I_2\omega_2^3 & \cdots \\ 0 & \omega_2 & 0 & -\omega_2^3 & 0 & \cdots & -R_2\omega_2 & I_2\omega_2^2 & R_2\omega_2^3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \\ 1 & 0 & -\omega_k^2 & 0 & \omega_k^4 & \cdots & I_k\omega_k & R_k\omega_k^2 & -I_k\omega_k^3 & \cdots \\ 0 & \omega_k & 0 & -\omega_k^3 & 0 & \cdots & -R_k\omega_k & I_k\omega_k^2 & R_k\omega_k^3 & \cdots \end{bmatrix} \quad (a.10)$$

$$\beta = [R_1\ I_1\ R_2\ I_2 \ldots R_{k'}\ I_{k'}]^T \quad (a.11)$$

$$\Theta = [b_0\ b_1 \ldots b_m\ a_1\ a_2 \ldots a_n]^T \quad (a.12)$$

and

$$E = [E_{R_1}\ E_{I_1}\ E_{R_2}\ E_{I_2} \ldots E_{R_k}\ E_{I_{k'}}]^T. \quad (a.13)$$

From the application of the generalized least-squares algorithm (GLSA) (36), the following recursive equation provides an estimate to the unknown vector Θ, and it minimizes the error vector E in a least-square sense.

$$\Theta_L = (X'^T W'_{L-1} X')^{-1} X'^T W'_{L-1} \beta. \quad (a.14)$$

where,

the subscript L corresponds to the iteration number,

W'_0 is the identity matrix, and

$$W'_{L-1} = \mathrm{Diag}[\,|M'_{L-1}(j\omega_1)|^{-2}\,|M'_{L-1}(j\omega_1)|^{-2}\,|M'_{L-1}(j\omega_2)|^{-2}\,|M'_{L-1}(j\omega_2)|^{-2} \ldots |M'_{L-1}(j\omega_{k'})|^{-2}\,|M'_{L-1}(j\omega_{k'})|^{-2}],\ L=1,2,\ldots \quad (a.15)$$

The iterative process is continued until the solution converges.

The following procedure is used to solve for the proper m an n (See Equation (a.1)) if they are not known a priori.

Let

$$e^{m,n} = \sum_{i=1}^{k'} \left| g^{m,n}(j\omega_i) - (R_i + jI_i) \right|^2, \text{ and} \tag{a.16}$$

apply the following algorithm.

Algorithm A.1

(a) set $n' = 1$,

(b) Compute $e^{n',n'}$ and $e^{n'+1,n'+1}$ from Equation (a.16).

(c) If $e^{n',n'} \cong e^{n'+1,n'+1}$, then

set system order $n=n'$, and

let $m'=1$ and go to (d),

otherwise,

increase n' by 1 and go to (b).

(d) Compute $e^{n',n'-m'}$

(e) If the increase from $e^{n',n'-m'+1}$ to $e^{n,n'-m'}$ is large, then

$m = n' - m' + 1$, and stop here.

(f) Increase m' by 1.

(g) If $m'-n'<0$, then set $m=0$ and stop here.

(i) Go to (d)

Identification of Type One Systems

In this section, the system identification technique of Lin and Wu (30) is reformulated to allow identification of type one systems as well. Such a reformulation is necessary, since the constant term of the denominator polynomial of Equation (a.1) is set to 1, therefore, the system identification method, of the previous section is not capable of identifying type 1 systems. Alternatively, a transfer function may be represented by the following equation

$$g(j\omega) = \frac{1 + b_1 s + ... + b_m s^{m'}}{a_0 + a_1 s + ... + a_n s^{n'}}, \quad m' \leq n'. \qquad \text{(a 17)}$$

To obtain the sinusoidal steady-state response, substitute $s = j\omega$ into (a.17). The following equation is obtained.

$$g(j\omega) = \frac{(1 - b_2\omega^2 + b_4\omega^2 - ...) + j(b_1\omega - b_3\omega^3 + b_5\omega^5 - ...)}{(a_0 - a_2\omega^2 + a_4\omega^2 - ...) + j(a_1\omega - a_3\omega^3 + a_5\omega^5 - ..)} \qquad \text{(a.18)}$$

$$= \frac{N''(j\omega)}{M''(j\omega)} \qquad \text{(a.19)}$$

Substitute (a.4) into Equation (a.18) to obtain the following equations.

$$1 - b_2\omega_1^2 + b_4\omega^4 - ... = R_1(a_0 - a_2\omega_1^2 + a_4\omega_1^4 - ...) - I_1(a_1\omega_1 - a_3\omega_1^3 + a_5\omega_1^5 - ...) + E'_{R_1}, \text{ and} \qquad \text{(a.20)}$$

$$b_1\omega_1 - b_3\omega_1^3 + b_5\omega_1^5 - ... = I_1(a_0 - a_2\omega_1^2 + a_4\omega_1^4 - ...) + R_1(a_1\omega_1 - a_3\omega_1^3 + a_5\omega_1^5 - ...) + E'_{I_1} \qquad \text{(a.21)}$$

Where,

$$E'_{R_1} = (ReE_1)(a_0 - a_2\omega_1^2 + a_4\omega_1^4 - ...) - (ImE_1)(a_1\omega_1 - a_3\omega_1^3 + a_5\omega_1^5 - ...), \qquad \text{(a 22)}$$

$$E'_{I_1} = (ImE_1)(a_0 - a_2\omega_1^2 + a_4\omega_1^4 - ...) + (ReE_1)(a_1\omega_1 - a_3\omega_1^3 + a_5\omega_1^5 - ...), \qquad \text{(a.23)}$$

ReE'_1 is the real part of complex number E_1, and

$Im\,E'_1$ is the imaginary part of complex number E_1. Designate the number of user-supplied frequency response data by symbol k'. Write Equations (a.20) and (a.21) k' times for $\omega_1, \omega_2, ..., \omega_k$, and concatenate the 2k' linear equations. The following linear equation would be obtained.

$$X'' \theta' = \beta' + E'. \tag{a.24}$$

$$x = \begin{bmatrix} R_1 & -I_1\omega_1 & -R_1\omega_1^2 & I_1\omega_1^3 & R_1\omega_1^4 & \cdots & 0 & \omega_1^2 & 0 & -\omega_2^4 & \cdots \\ I_1 & R_1\omega_1 & -I_1\omega_1^2 & -R_1\omega_1^3 & I_1\omega^4 & \cdots & \omega_1 & 0 & -\omega_1^3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \\ R_{k'} & -I_{k'}\omega_{k'} & -R_{k'}\omega_{k'}^2 & I_{k'}\omega_{k'}^3 & R_{k'}\omega_{k'}^4 & \cdot\cdot & 0 & \omega_{k'}^2 & 0 & -\omega_{k'}^4 & \\ I_{k'} & R_{k'}\omega_{k'} & -I_{k'}\omega_{k'}^2 & -R_{k'}\omega_{k'}^3 & I_{k'}\omega_{k'}^4 & \cdots & \omega_{k'} & 0 & -\omega_{k'}^3 & 0 & . \end{bmatrix} \tag{a.25}$$

$$\beta' = [1\ 0\ 1\ 0\ .\ \ ]^T, \tag{a.26}$$

$$\Theta = [a_0\ a_1\ a_2\ \ldots\ b_1\ b_2\ ..\ ]^T, \text{ and} \tag{a.27}$$

$$E'' = [E'_{R_1}\ E'_{I_1}\ E'_{R_2}\ E'_{I_2}\ \ldots\ E'_{R_{k'}}\ E'_{I_{k'}}]^T. \tag{a.28}$$

The generalized least-squares algorithm (GLSA) (40) is applied to obtain a vecursive equation similar to the obtained in the prior section (see Equation (a.14) and (a.15)). If integer numbers m' and n' (see Equation (a.17)) are not known in a priori, algorithm B.1, which was presented in the previous section, may be adapted to solve for the proper m' and n'

APPENDIX B

A SYSTEMATIC PROCEDURE FOR PREFORMING COPRIME FACTORIZATION OVER s

The objectives of this Appendix are to illustrate (i) a systematic procedure for performing coprime factorization over s, and (ii) the need for development of an algorithm which would attempt to compute coprime factors of lowest possible order. Objective (i) above is fulfilled via an example problem, and objective (ii) is fulfilled in "Discussion" section of this appendix.

Problem Statement

Consider transfer function T(s) given by Equation (b.1).

$$T(s) = \frac{s + 1}{s^2 - 4} \tag{b.1}$$

It is required to obtain the coprime pairs (N(s), D(s)) and (P(s), Q(s)) having poles in the region $Re \leq -1$, such that

$$T(s) = \frac{N(s)}{D(s)} \quad \text{and} \tag{b.2}$$

$$P(s)\, N(s) + Q(s)\, D(s) = 1 \tag{b.3}$$

Method of Solution

The coprime factorization method of Nett (38) together with the pole-placement algorithm of Kumar (57) are adapted to fulfill the purpose of this example problem.

Coprime Factorization Over s

Consider a system described by the equation

$$X(t) = AX(t) + BU(t) \tag{b.4}$$

$$Y(t) = CX(t) + EU(t) \qquad (b\ 5)$$

where A, B, C, and E are constant matrices of compatible dimensions, and the pairs (A,B) and (A,C) are stabilizable and detectable, respectively Then,

$$N(s) = (C - EK)(sI - A_0)^{-1}B + E, \qquad (b\ 6)$$

$$P(s) = K(sI - A_0^{'})^{-1}F', \qquad (b.7)$$

$$D(S) = 1 - K(sI - A_o)^{-1}B, \text{ and} \qquad (b\ 8)$$

$$Q(s) = 1 + K(sI - A_0^{'})^{-1}(B - F^{'}E) \qquad (b\ 9)$$

Where,

$$A_0 = A - BK, \qquad (b.10)$$

$$A_0^{'} = A - F^{'}C, \text{ and} \qquad (b\ 11)$$

Constant matrices K and F' are selected such that A_0 and $A_0^{'}$ are both Hurwitz A matrix is Hurwitz if all its eigenvalues have negative real parts.

Selection of constant matrix K can be viewed as a problem of arbitrarily assignment of poles of the single-variable dynamical equation (b 4) and (b.5) via state feedback (4) Kumar (57) has developed an algorithm for this purpose This algorithm is given below

Algorithm 2 (Kumar)

(a) Define the desired poles of (A + BK) That is,

$$P_1^{'}, P_2^{'}, \ldots , P_n^{'} \qquad (b\ 12)$$

(b) Find the characteristic polynomial of A That is

$$s^n + P_1 s + \qquad + P_n \qquad (b.13)$$

(c) From (a) above, compute the desired characteristic polynomial That is,

$$s^n + \bar{P}_1 s - \; . \; + \bar{P}_n \qquad (b.14)$$

(d) Compute $\bar{K} = [P_n - \bar{P}_n \;\; P_{n-1} - \bar{P}_{n-1} \; . \; . \; P_1 - \bar{P}_1]$. (b.15)

(e) Compute $q^{n-i} = Aq^{n-i+1} + P_i q^n$. for $i=1,2,. \; ., (n-1)$,

with $q^n = B$ (b.16)

(f) Form $Q = [q^1 \; q^2 \;\; . \;\; q^n]$. (b.17)

(g) Find, $P = Q^{-1}$. (b.18)

(h) $K = \bar{K} \; P$ (b.19)

The following algorithm, which is in parallel with Algorithm 2 above, is developed to utilize the constant matrix F'.

<u>Algorithm 3</u>

(a) Define the desired poles of (A + F'C) That is

$$U_1', \; U_2', \quad , \; U_n' \qquad (b\ 20)$$

(b) Find the characteristic polynomial of A That is,

$$s^n + U_1 s + \ldots + U_n \qquad (b.21)$$

(c) From (a) above, compute the following polynomial

$$s^n + \bar{U}_1 s^{n-1} + \quad + \bar{U}_n \qquad (b\ 22)$$

(d) Compute $\bar{F} = [U_n - \bar{U}_n \; U_{n-1} - \bar{U}_{n-1} \quad U_1 - \bar{U}_1]$ (b 23)

(e) Compute $V^{n-i} = V^{n-i+1} A + V^n U_i$, for $i = 1,2,..$,

$(n-1)$, with $V^n = C$ (b.24)

(f) Form $Q' = [V^1 \; V^2 \; .. \;\; V^n]$. (b.25)

(g) Find $P' = Q'^{-1}$ (b 26)

(h) $F' = \bar{F} \; P'$ (b 27)

For the open-loop system, whose transfer function is given by (d.1), constant matrices A, B, C, and E may be defined as follow.

$$A = \begin{bmatrix} 0.0 & 1\ 0 \\ 4\ 0 & 0\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 \\ 1\ 0 \end{bmatrix},$$

and

$$C = [10 \quad 10],\ E = 0\ 0 \tag{b 28}$$

From the application of Algorithm 2 and Algorithm 3 given above, the constant vectors K and F' are selected to place the poles of A - BK and A - F'C at (-2.0, 0 0j) That is,

$$K = [8\ 0 \quad 4\ 0] \text{ and } F' = [4/3 \quad 8/3]^T \tag{b 29}$$

Substitute (b 28) and (b.29) into (b.10) and (b 11).

$$A_o = \begin{bmatrix} 0\ 0 & 1.0 \\ -4.0 & -4.0 \end{bmatrix}, \text{ and} \tag{b 30}$$

$$A_o' = \begin{bmatrix} -4.3 & -1/3 \\ 4/3 & 8/3 \end{bmatrix} \tag{b 31}$$

To obtain the coprime pairs (N(s), D(s)) and (P(s) and Q(s)), substitute (b.28) - (b 31) into (b 6) - (b 9).

$$N(s) = \frac{s + 1}{(s + 2)^2}, \tag{b 32}$$

$$P(s) = \frac{64}{3(s + 2)}, \tag{b 33}$$

$$D(s) = \frac{s - 2}{(s + 2)}, \text{ and} \tag{b 34}$$

$$Q(s) = \frac{s^2 + 8s + 20/3}{(s + 2)^2} \tag{b 35}$$

The coprime pairs (N(s), D(s)) and (P(s), Q(S)) have poles in the region $R(s) \leq -1$, such that (b 2) and (b 3) are satisfied, and the problem is solved.

The coprime factorization over s, which is given by Equations (b.6)-(b.9) does not attempt to provide the user with the coprime paris (N(s), D(s)) and (P(s), Q(s)) with the lowest possible order For example, P(s) and Q(s) given by Equations (b.34) and (b.35), which was obtained using the proposed method of Nett (42), could have been chosen as the following.

$$P(s) = \frac{16}{3}\text{, and} \tag{b 36}$$

$$Q(s) = \frac{s + 2/3}{s + 2} \tag{b 37}$$

The coprime pairs (N(s), D(s)), given by Equation (b 32) and (b 33), and (P(s), Q(s)), given by either Equation set (b 34) and (b.35) or (b 47) and (b.48), satisfy the Bezout identity (Equation (b 3)) Therefore, it may be concluded that a method is needed which would attempt to identify coprime factors of the least possible order

APPENDIX C

APPLICATIONS OF A FACTORIZATION APPROACH TO CONTROLLER DESIGN FOR CLOSED-LOOP SYSTEM STABILITY

The objectives of this Appendix are the following (i) to illustrate the application of a facterization approach for stabilization of linear, single-input single-output, time-invariant deterministic, and continuous time systems and (ii) to illustrate the application of the simultaneous stabilization theory for linear systems with integrity - linear systems which may become unstable due to structural changes. The objectives are fulfilled by solving two example problems Refer to "Step 4 Controller Parameterization" for definition of notation used herein

Example 1 A Single Variate Servomechanism Problem

Problem Statement

Consider a one degree-of-freedom servomechanism whose dynamic behavior is modeled by the following transfer function (44).

$$T(s) = \frac{s + 1}{s^2 - 4} \qquad (c.1)$$

It is required to design a controller for this unstable plant which will place the poles of the feedback system shown in Figure 22 in the region, Re(s) < -1, and force the system to track a step input.

Method of Solution

The following systematic and algebraic solution method, which is based on a factorization approach, is adapted

a) Identify the desired algebraic ring H In this example problem, the algebraic ring H is defined as the set of all rational functions whose poles lie in the region Re(s) < -1

b) Perform coprime factorization in the algebraic ring H. That

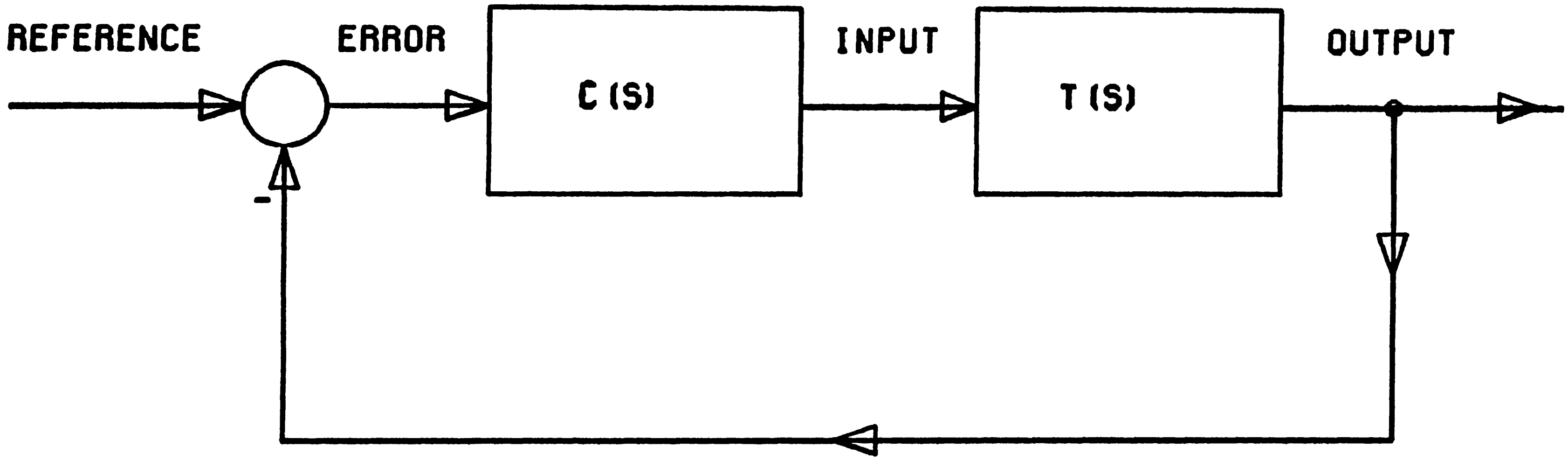


Figure 22 Stabilization of A Single Variate Servomechanism

is, find coprime factors $T_n(s)$ and $T_d(s)$ such that

$$T(s) = \frac{T_n(s)}{T_d(s)} \quad \text{and} \tag{c.2}$$

$T_n(s)$ and $T_d(s)$ belong to H

Those are given below (44)

$$T_n(s) = \frac{s+1}{(s+2)^2} \quad \text{and} \tag{c 3}$$

$$T_d(s) = \frac{s-2}{s+2} \ . \tag{c.4}$$

c) Find $T_p(s)$ and $T_q(s)$, in algebraic ring H, to satisfy the Bezout identity.

$$T_p(s)\ T_n(s) + T_q(s)\ T_d(s) = 1 \tag{c.5}$$

For this example problem, $T_p(s)$ and $T_q(s)$ are of the following form (44)

$$T_p(s) = \frac{16}{3} \quad \text{and} \tag{c 6}$$

$$T_q(s) = \frac{s + 2/3}{s+2} \tag{c.7}$$

d) Parameterize the class of all controllers that stabilize plant T(s). That is given by Equation (3 9). By adapting the notation used in this example problem to this Equation, the following mathematical relationship parameterizes the class of all stabilizing controllers in terms of parameter $T_r(s)$.

$$T_c(s) = \frac{T_p(s) + T_r(s)\ T_d(s)}{T_q(s) - T_r(s)\ T_n(s)} \ . \tag{c 8}$$

The transfer characteristics of the resulting closed-loop system becomes

$$T_g(s) = \frac{T_c(s)\ T(s)}{I + T_c(s)\ T(s)} \ . \tag{c 9}$$

Substitute (c 2) and (c 8) into (c.9) and simplify to obtain the transfer characteristics of the closed-loop system in terms of coprime factors of T(s) and the as yet unknown parameter $T_r(s)$.

$$T_g(s) = T_n(s)\, T_d(s)\, T_r(s) + T_p(s)\, T_n(s) \qquad \text{(c.10)}$$

e) Any transfer function, $T_r(s)$, which belongs to the algebraic ring H, may be substituted in Equation (c.8) to obtain the stabilizing controller By substituting $T_r(s)$ into Equation (c.10), the corresponding closed-loop transfer function would be obtained. Since it is required to design a controller which would not only stabilize the unstable plant but also force the closed-loop system to track a step input, any $T_r(s)$ in algebraic ring H would not be valid In otherwords, those $T_r(s)$, belonging to the algebraic ring H, which force the closed-loop system to track a step command are acceptable. That is,

$$T_g(s = 0) = 1. \qquad \text{(c.11)}$$

Substitute Equations (c.3), (c.4), and (c 6) into (c.10), and evaluate $T_g(s=0)$ The following is obtained

$$T_g(0) = (-\tfrac{1}{4})T_r(0) + \tfrac{4}{3} \qquad \text{(c 12)}$$

From Equations (c.12) and (c 11), it is concluded that all $T_r(s)$, belonging to the algebraic ring H, with $T_r(0) = -\frac{4}{3}$ will stabilize the unstable plant T(s) and force the feedback system to track a step input Therefore, the simplest $T_r(s) = -\frac{4}{3}$

f) Substitute Equations (c.3), (c 4), (c.6), (c 7), and $T_r(s) = -\frac{4}{3}$ into Equation (c.8), to obtain the desired controller

$$T_c(s) = \frac{(20s + 24)\ (s + 2)}{s(3s + 4)} \qquad \text{(c 13)}$$

The transfer function of the resulting closed-loop feedback system is given by the following equation.

$$T_g(s) = \frac{(s + 1)\ (20s + 24)}{3(s + 2)^3} \qquad (c.14)$$

The controller design problem is solved.

Example 2 Simultaneous Stabilization of A Single Variate Linear System With Integrity

Problem Statement

Assume a dynamic open-loop system operating at a nominal condition may be represented by the stable linear model $T_0(s)$ given below

$$T_0(s) = \frac{1}{4(s + 2)} \qquad (c.15)$$

Also assume that the open-loop system operating at another condition may be represented by the unstable linear model $T_1(s)$ given below

$$T_1(s) = \frac{3}{4(s - 2)} \qquad (c\ 16)$$

It is required to design a controller so that the closed-loop system operating with either plant $T_0(s)$ or plant $T_1(s)$ is stable - it is required to design a controller which will simultaneously stabilize $T_0(s)$ and $T_1(s)$.

Method of Solution

The following systematic and algebraic approach is taken to design the controller.

a) Identify the desired algebraic ring H. In this example problem, the algebraic ring H is defined as the set of all rational functions whose poles lie in the region Re(s) < 0

b) Obtain the coprime factors of $T_0(s)$ in the algebraic ring H Those are given below.

$$T_{0n}(s) = \frac{1}{4(s + 2)} \text{ and} \tag{c 17}$$

$$T_{0d}(s) = 1. \tag{c.18}$$

c) Find the coprime pair $T_{0p}(s)$ and $T_{0q}(s)$, in algebraic ring H, to satisfy the Bezout identity

$$T_{0p}(s)\ T_{0n}(s) + T_{0q}(s)\ T_{0d}(s) = 1. \tag{c 19}$$

For this example problem, $T_{0p}(s)$ and $T_{0q}(s)$ may be set to zero and one, respectively. That is,

$$T_{0p}(s) = 0 \text{ and,} \tag{c.20}$$

$$T_{0q}(s) = 1. \tag{c.21}$$

d) Let $T_{1n}(s)$ and $T_{1d}(s)$, belonging to the algebraic ring H, represent the coprime factors of the rational function $T_1(s)$, and let the coprime pair, $T_{1q}(s)$ and $T_{1p}(s)$, belonging to the algebraic ring H, satisfy the Bezout identity

$$T_{1p}(s)\ T_{1n}(s) + T_{1q}(s)\ T_{1d}(s) = 1 \tag{c 22}$$

e) Adapt the notation used in this Appendix, and identify the auxiliary transfer function given by Equation (3.11). That is,

$$T_a(s) = \frac{-T_{0n}(s)\ T_{1d}(s) + T_{0d}(s)\ T_{1n}(s)}{T_{0q}(s)\ T_{1d}(s) + T_{0p}(s)\ T_{1n}(s)} \tag{c.23}$$

Substitute (c 18), (c 20), and (c 21) into (c 23) and simplify, Equation (c 23) becomes

$$T_a(s) = T_1(s) - T_0(s). \tag{c 24}$$

Substitute (c.15) and (c 16) into (c.24).

$$T_a(s) = \frac{s + 4}{2(s^2 - 4)} \qquad (c.25)$$

f) Identify the algebraic ring M consisting of the class of all controllers that strongly stabilize $T_a(s)$. This can be viewed as a problem of reliable stabilization of one unstable plant (See Example 1 of this Appendix) $T_a(s)$ is strongly stabilizable if it can be stabilized by a stable controller

f-a) Identify the desired algebraic ring H. This was accomplished at (a) above.

f-b) Obtain the coprime factors of $T_a(s)$ in the algebraic ring H. Those are given below

$$T_{an}(s) = \frac{s + 4}{2(s + 2)^2} \quad \text{and} \qquad (c\ 26)$$

$$T_{ad}(s) = \frac{s - 2}{s + 2} \qquad (c.27)$$

f-c) Find $T_{ap}(s)$ and $T_{aq}(s)$, in algebraic ring H, to satisfy the Bezout identity.

$$T_{ap}(s)\ T_{an}(s) + T_{aq}(s)\ T_{ad}(s) = 1. \qquad (c.28)$$

For this example problem, $T_{ap}(s)$ and $T_{aq}(s)$ are of the following form.

$$T_{ap}(s) = \frac{16}{3} \quad \text{and} \qquad (c.29)$$

$$T_{aq}(s) = \frac{s + 10/3}{s + 2}\ . \qquad (c.30)$$

f-d) Parameterize the class of all controllers that strongly stabilize plant $T_a(s)$, this identifies the algebraic ring

M, and it accomplishes the purpose of (f) above. By adapting the notation used in this example problem to Equation (3.14), the following mathematical relationship parameterizes the class of all controllers that stabilize $T_{ac}(s)$ in terms of parameter $T_{ar}(s)$.

$$T_{ac}(s) = \frac{T_{ap}(s) + T_{ar}(s)\, T_{ad}(s)}{T_{aq}(s) - T_{ar}(s)\, T_{an}(s)} \; . \qquad (c.31)$$

Substitute Equations (c.26), (c.27), (c.29), and (c.30) in Equation (c.31)

$$T_{ac}(s) = \frac{32(s + 2)^2 + 6\, T_{ar}(s)(s - 2)(s + 2)}{(6s + 20)(s + 2) - 3\, T_{ar}(s)(s + 4)} \; . \qquad (c.32)$$

where $T_{ac}(s)$ strongly stabilizes $T_a(s)$. (c.33)

Equation (c.32) and Constraint (c.33) accomplish the objective of (f) above.

For illustration purposes, let

$$T_{ar} = 2 \; . \qquad (c.34)$$

Substitute Equation (c.34) into Equation (c.32).

$$T_{ac}(s) = \frac{22(s^2 + 64s + 40)}{3s^2 + 13s + 8} \; . \qquad (c.35)$$

To verify that the above $T_{ac}(s)$ strongly stabilizes $T_a(s)$, form $\Sigma(T_a(s), T_{ac}(s))$. That is,

$$\Sigma(T_a(s), T_{ac}(s)) = \frac{T_{ac}(s)\, T_a(s)}{1 + T_{ac}(s)\, T_a(s)} \qquad (c.36)$$

$$= \frac{(s + 4)(11s + 10)}{3(s + 2)^3} \; . \qquad (c.37)$$

Hence, $T_{ac}(s)$ strongly stabilizes $T_a(s)$.

g) Adapt the notation used in this example problem to Equation (3.14), and obtain the class of all controllers that simultaneously stabilize $T_0(s)$ and $T_1(s)$. That is,

$$T_{sc}(s) = \frac{T_{op}(s) + T_{r'}(s)\, T_{od}(s)}{T_{oq}(s) - T_{r'}(s)\, T_{on}(s)} \qquad \text{(c.38)}$$

where $T_{r'}(s)$ belongs to the class M defined in (f) above. Substitute Equation (c.17), (c.18), (c.20), and (c.21) into Equation (c.34).

$$T_{sc}(s) = \frac{4(s + 2)T_{r'}(s)}{4(s + 2) - T_{r'}(s)} \qquad \text{(c.39)}$$

h) Any transfer function, $T_{r'}(s)$, which belongs to the algebraic ring M, may be substituted in Equation (c.39) to obtain the controller that simultaneously stabilizes $T_0(s)$ and $T_1(s)$. The set of all valid $T_{r'}(s)$ is given by (f) above (see Equation (c.32) and Constraint (c.33)), $T_{r'}(s)$ for a specific valid $T_{ar}(s)$ is equal to $T_{ac}(s)$. Such a $T_{ac}(s)$ for $T_{ar}(s) = 2$ is given by (c.35) Therefore,

$$T_{r'}(s) = \frac{22s^2 + 64s + 40}{3s^2 + 13s + 8} \qquad \text{(c 40)}$$

Substitute (c.40) into (c.39) and simplify. The following equation is obtained

$$T_{sc}(s) = \frac{40 + 44s}{3 + 6s} \qquad \text{(c 41)}$$

$T_{sc}(s)$ given by Equation (c 41) is the controller that simultaneously stabilizes both $T_0(s)$ and $T_1(s)$, and the problem is solved.

$$\Sigma(T_0(s), T_{sc}(s)) = \frac{T_0(s)\, T_{sc}(s)}{1 + T_0(s)\, T_{sc}(s)} \qquad \text{(c.42)}$$

$$= \frac{5.5(s + 0.909)}{3(s + 0.743)(s + 3.591)} \qquad (c.43)$$

and

$$\Sigma(T_1(s),\ T_{sc}(s)) = \frac{T_1(s)\ T_{sc}(s)}{1 + T_1(s)\ T_{sc}(s)} \qquad (c.45)$$

$$= \frac{16.5(s + 0.909)}{3(s + 2)^2} \qquad (c.46)$$

APPENDIX D

A TUTORIAL ON DFGEN SOFTWARE UTILITY

The objective of this appendix is to tutor the reader in using the Describing Function model GENerator (DFGEN) software utility, which was developed in this thesis effort, to obtain the sinusoidal-input describing function models of a nonlinear system. The objective is fulfilled by solving one example problem. Refer to "Step 2 Input/output Characterization" section of Chapter III of this thesis for technical details of the DFGEN software utility. It is assumed that the user has access to the Oklahoma State University's College of Engineering, Architecture, and Technology (CEAT) Computer-Aided Design (CAD) and Graphics Research Facility The first-time user of the CEAT CAD lab must consult with the site manager to gain access to the Harris-800 minicomputer. Select one of the available Tektronix 4115B color graphics work stations, and sign-on to the Harris-800 minicomputer

Example

Problem Statement

Consider the nonlinear plant depicted in Figure 10. The corresponding dynamic equations of motion is given by Equations (4 1)-(4 4). Obtain the SIDF models of the nonlinear plant around the following operating conditions (1) excitation amplitudes of 0 25, 0.325, 0.40, 0.80, 1 60, 3.20, 6 40, and 12 8 volts and (2) frequency range of 5 to 150 radians/seconds

Method of Solution

The DFGEN software utility is used to obtain the SIDF models. The user must provide the following information before the execution of the DFGEN software utility

(1) Subroutine MOTLD whose input is the plant input, u(t), and its output is the plant output y(t). Such a subroutine for this example problem is shown in Figure 18. With reference to Figure 18, the variable IC2 is of no significance in this case. The subroutine name must specifically be MOTLD, but the text file name is arbitrary. In this example, the text file name is 0001DRLC*MOTLD SF, and it is shown in Figure 18 Consult with the site manager, and obtain the necessary information which is required to compile the FORTRAN text file 0001DRLC*MOTLD SF, to add the resulting object module to the 0001INST*INSTAL library, and finally to link the 0001QLDF*DFGEN 0 object module to the 0001INST*INSTAL and 0000*LIBRARY libraries to create the executable module 0001QLDF*DFGEN.

(2) Total number of SIDF models to be generated, NDEF (NDEF$\leq$8)

(3) Amplitude of the excitation sinusoids, AMPL(I), I = 1,NDEF

(4) Total number of discrete frequencies, NFREQ

(5) Integration step size, DTPL

(6) Final simulation time, TFINAL

(7) Phase error bound, PHIEPSILON and magnitude error bound, MEPSILON The Fourier integrals are said to have converged when the following conditions are satisfied

$$\frac{|M_{k'} - M_{k'-1}|}{|M_{k'}|} < \text{MEPSILON, and}$$

$$|\phi_{k'} - \phi_{k'-1}| < \text{PHIEPSILON.}$$

(8) An array of discrete frequencies in radians/seconds

For this example problem NDEF=8, AMPL(1)-AMPL(8)=0 25, 0 325, 0 40, 0.80, 1.60, 3 20, 6 40, 12 80, NFREQ=12, NXPL=2, DTPL=0.001,TFINAL=0.8, PHIEPSILON=2 0, and MEPSILON=0.05. The elements of the frequency array are given in Table I The user is required to create the data file 001DATA*DFGEN IN whose first line comprises the value of NDEF and AMPL(1)-AMPL(NDEF), the second line comprises the values of NFREQ, NXPL, DTPL, TFINAL, PHIEPSILON, and MEPSILON, respectively. The third and the following lines of the data file comprise the discrete values of frequency. Figure 23 presents the user-input data file 0001DATA*DFGEN IN for this example problem At this stage, the user may execute the Describing Function model GENerator software utility via the 0001QLDF*DFGEN job control command The software utility will output the numerical data corresponding to the generated describing function models into file 0001DATA*DFGEN OT The first column of this data file is frequency, the second column is the real part of the frequency response, and the third column is the imaginary part of the frequency response. The software utility will also output pseudo Bode plots, these are shown in Figures 24 and 25

8 0.25 0 325 0.40 0.80 1.60 3.20 6.40 12.8

12 1 0.001 0.80 5 0.05

5. 6 811673 9.279777 12.64216 17.22285 23 46329

31.96484 43.64681 59.32532 80.82092 110.1051 150.

Figure 23 The User-Input Data File 0001DATA*DFGEN IN

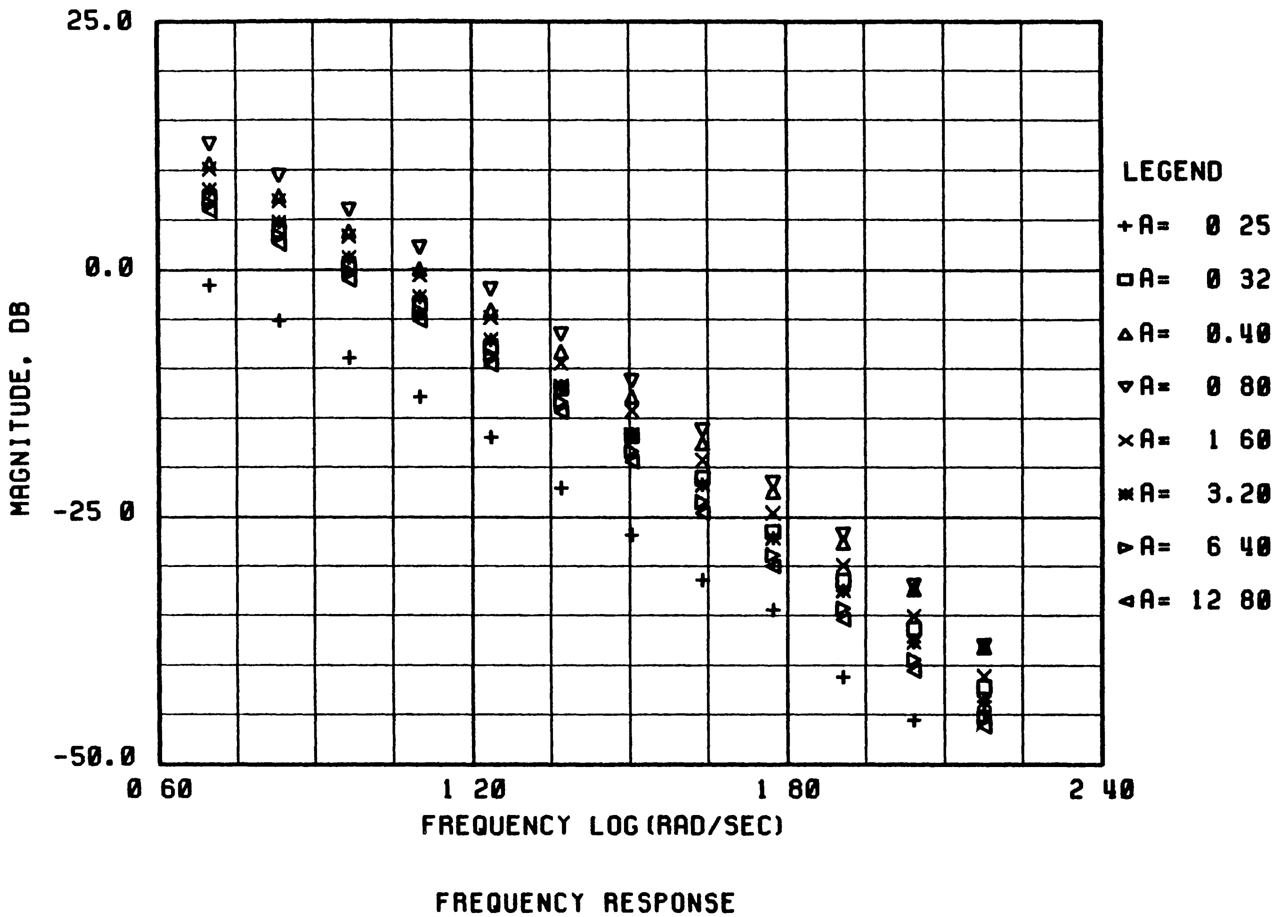


Figure 24 Magnitude Plots of SIDF Models

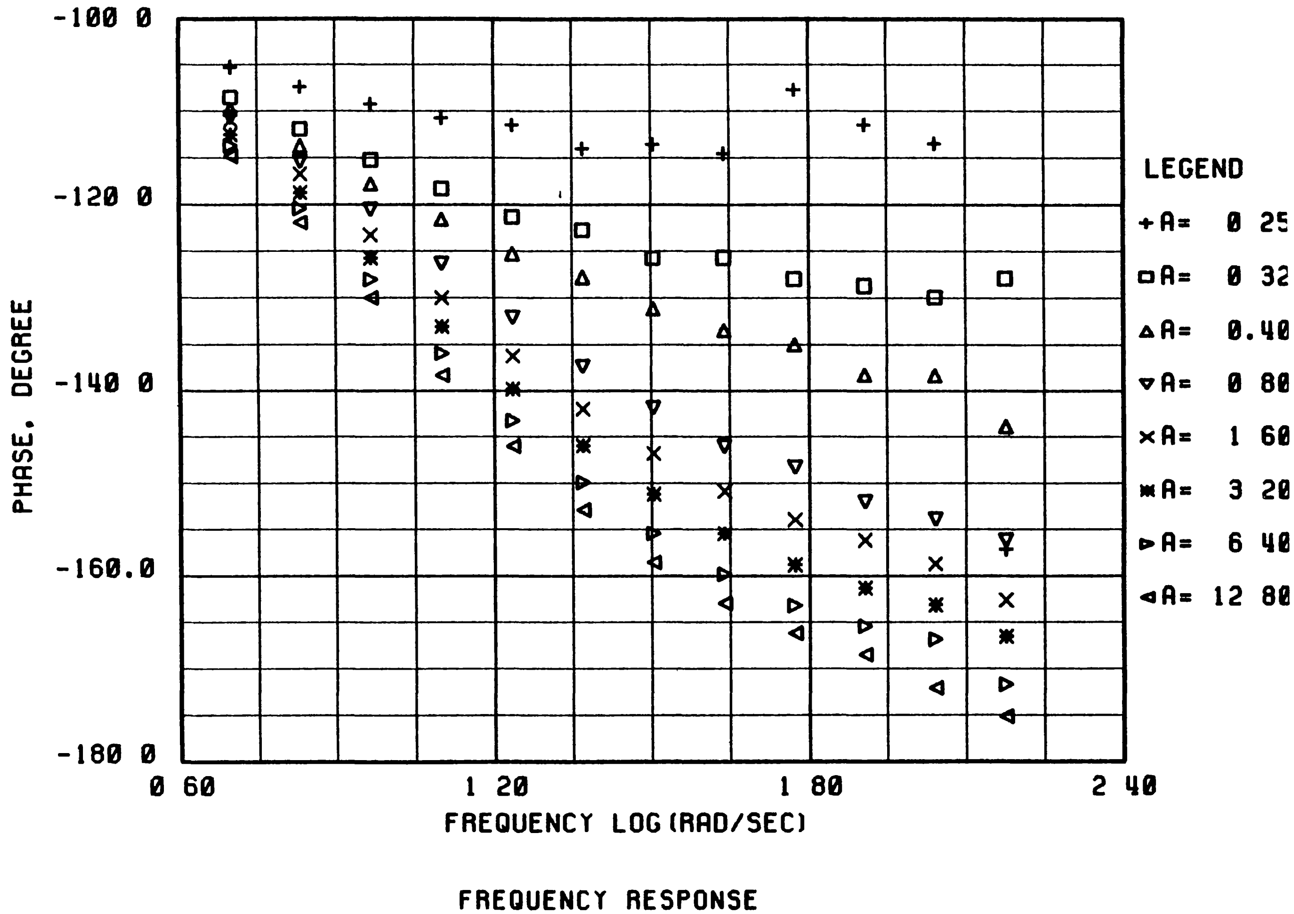


Figure 25 Phase Plots of SIDF Models

APPENDIX E

A TUTORIAL ON SYSID SOFTWARE UTILITY

The objective of this appendix is to tutor the reader in using the SYStem IDentification (SYSID) software utility which was developed in this thesis effort. The objective is fulfilled by solving two example problems. Refer to Appendix B and Chapter III of this thesis for technical details of the identification technique of SYSID.

It is assumed that the user has access to the Oklahoma State University's College of Engineering, Architecture, and Technology (CEAT) Computer-Aided Design (CAD) and Graphics Research Facility. The first-time user of the CEAT CAD lab must consult with the site manager to gain access to the Harris-800 minicomputer. Select one of the available Tektronix 4115B color graphics work stations, and sign-on to the Harris-800 minicomputer.

Example 1

The first-time SYSID user should build his or her confidence by solving a known example problem. Consider an open-loop system whose dynamic behavior is adequately represented by the following transfer function.

$$W_1(s) = \frac{(s + 10.0)(s + 0.5)}{s(s + 1.0)} \qquad \text{(e.1)}$$

To obtain the sinusoidal steady-state response, substitute $s=j\omega$ into Equation (e.1), and evaluate the real and imaginary parts of $W_1(j\omega)$ over a specific range of frequencies. For this example problem, the sinusoidal steady-state response over $0.01 < \omega < 150.0$ radians/seconds is tabulated in Table V.

TABLE V

FREQUENCY RESPONSE DATA OF TRANSFER FUNCTION $W_1(S)$

Frequency (Rad./Sec.)	Real Part	Imaginary Part
1.00000E-02	5.4996	-500 04
1 27961E-02	5.4993	-390 80
1.63741E-02	5.4988	-305.43
2.09526E-02	5 4980	-238.73
2 68113E-02	5.4968	-186 61
3 43081E-02	5 4947	-145 89
4.39011E-02	5 4913	-114.09
5 61765E-02	5 4858	-89 257
7.18843E-02	5 4769	-69.878
9.19843E-02	5 4622	-54 768
0.11770	5.4385	-43 002
0 15062	5 4002	-33 860
0.19273	5.3388	-26 779
0.24662	5.2420	-21 320
0.31558	5 0924	-17 135
0 40382	4 8691	-13 944
0.51674	4.5517	-11.511
0.66122	4.1311	-9 6321
0.84611	3.6225	-8 1283
1 0827	3.0716	-6 8610
1 3854	2 5414	-5 7445
1 7728	2.0862	-4.7460
2 2685	1.7322	-3 8650
2 9028	1.4774	-3.1082
3 7145	1.3041	-2 4757
4 7532	1.1907	-1 9585
6 0822	1.1184	-1.5425
7 7829	1 0731	-1 2112
9.9591	1 0449	-0 94939
12 744	1.0275	-0 74330
16 307	1.0169	-0 58153
20 867	1 0103	-0.45477
26 702	1.0063	-0.35555
34 168	1 0039	-0 27793
43.722	1.0024	-0 21723
55.947	1 0014	-0 16978
71.590	1.0009	-0.13269
91 608	1.0005	-0 10370
117.22	1.0003	-8.10393E-02
150.00	1 0002	-6.33318E-02

Problem Statement and the Method of Solution

Identify a transfer function whose frequency response is as close to the frequency response data given in Table V as possible. The following is a step-by-step guide which would yield identification of such a transfer function. All input/output data files reside in 0001DATA Harris-800 qualifier.

1. Create a data file whose first column is the frequency in radians/seconds, second column is the real part of the frequency response, and the third column is the imaginary part of the frequency response In this example, the data file is named 0001DATA*TEST1

2 Enter the following command from Harris-800 Job Control

1SYID*SYSID

This command executes the computer-aided SYStem IDentification (SYSID) software utility A list of valid commands along with a short description of their functions is displayed (See Figure (26)). The user will exercise all commands by the end of this tutorial. Refer to Table VI for a complete list of commands and their associated functions and syntaxes.

3. Use the **G** command to read in the frequency response data from the hard disc In this case, enter the following command.

G/TEST1/

4 In this example problem, the system type, the number of transfer function poles, and the number of transfer function zeros are known (see Equation (e 1)). Enter the following command

K2,2

```
G ==>  G/XXXXXXXX/ READ DATA FILE XXXXXXXX
R ==>  SET THE CONSTANT TERM OF THE DENOMINATOR POLYNOMIAL TO 1
K ==>  SET THE CONSTANT TERM OF THE NUMERATOR POLYNOMIAL TO 1
D ==>  DISPLAY THE STATUS OF THE IDENTIFIED TRANSFER FUNCTION
F ==>  FIX (FREEZE) TRANSFER FUNCTION COEFFICIENT(S)
P ==>  VIEW BODE PLOTS (MAGNITUDE AND PHASE)
N ==>  VIEW NYQUIST PLOT
E ==>  VIEW ERROR PLOTS
I ==>  TOGGLE PLOT OVERLAY OPTION
S ==>  S.BNE/XXXXXX/ SAVE PLOT DATA FILES
W ==>  SET WEIGHING COEFFICIENTS
O ==>  ACTIVATE THE PATERN OPTIMIZATION ROUTINE
Q==>   QUIT SESSION
```

Figure 26   SYSID Main Command List

TABLE VI

SYSID COMMAND DEFINITION

Command	Subcommand	Function	Syntax/Remarks	Command Example
G	None	To read a data file from disc	G/xxxxxxxx/	G/TEST1/
R	None	To Identify a Type zero system	Rz,p z is the number of zeros p is the number of poles	R1,2
K	None	To identify a type one system	Kz,p	K1,2
D	None	To display the status of the identified transfer function, i.e, transfer function coefficients, zero location(s), and pole location(s)	D	D
F		To fix (Freeze) transfer function coefficient	F	F
	NUM	To fix numerator polynomial coefficients	NUM, b_0, b_1, , b_m, b_0, b_1, ..., b_m are numerator polynomial coefficients in ascending powers of s	NUM,1 0,2.0,
			NUM, D, b_1, . , b_m, To use the present (default) value of a particular coefficient enter a D in the proper location	NUM,D,1.0,2.0,

TABLE VI (Continued)

Command	Subsommand	Function	Syntax/Remarks	Command Example
	DEN	To fix denominator polynomial coefficients	DEN, a_0, a_1, . . ., a_n a_0, a_1, . ., a_n are denominator polynomial coefficients in ascending powers of s	DEN,5.0,6.0,
			DEN, a_0, D, ..., D To use the present (default) value of a paricular coefficient enter a D in the proper location	DEN, 5 5, D, D,
P	None	To view Bode plots	P	P
N	None	To view Nyquist plots	N	N
I	None	To toggle plot overlay option from ON mode to OFF mode and vice versa. Command I only affects the P and the N commands	I	I
E	None	To view error plots. Four sets of plots are displayed (1) The percent error in magnitude (2) The percent error in phase (3) The relative error in magnitude (4) The relative error in phase	E	E

TABLE VI (Continued)

Command	Subsommand	Function	Syntax/Remarks	Command Example
S	None	To save Bode, Nyquist, and error data plot files for future hard-copy reproduction.	S B/xxxxxx/ Data files xxxxx MP and xxxxx PP, which correspond to magnitude and phase plot data files, are created.	S B/BODE/
			S N/xxxxx/ Data File xxxxx NP, which corresponds to Nyquist plot data file, is created.	S.N/NYQUI/
			S E/xxxxx/ The following data files are created (1) xxxxx1OD (2) xxxxx2OD (3)-(6) xxxxx3OP-xxxxx6OP These are plot data files corresponding to the percent error in magnitude, percent error in phase, relative error in magnitude, and relative error in phase, respectively	S.E/ERROR/
			Options of the S command (N,B and E) may be combined in any order	S.EB/OUTPT/ S BNE/OUTPT/

TABLE VI (Continued)

Command	Subsommand	Function	Syntax/Remarks	Command Example
W	None	To enter the weighing coefficients	$W(i,x_i,y_i),W(j,x_j,y_j)$, i and j refer to the ith and jth frequency reponse datum x_i (y_i) refer to the weighting coefficient of the real (imaginary) part of the frequency response datum	W(1,2.0,3.0), W(25,1.5,4.0)
O	None	To Activate the PATRN optimization routine	O	O
Q	None	To Quit Session		

5. To view the status of the identified transfer function, enter the **D** command. Figure (27) is displayed on the screen, and $W_1(s)$ is identified. Hit the enter key to return to the main command list.
6. Enter the **N** command to examine the quality of fit by viewing the Nyquist plot. Figure (28) is displayed. The user is instructed to hit the return key to return to the main command list.
7. Enter the **P** command to view the Bode plots. Figures (29) and (30) are displayed. The user is instructed to hit the return key to return to the main command list.
8. The system identification problem is solved. Enter the **Q** command to quit.

Example 2

In Chapter IV, transfer function $G_0(s)$ was identified to be a linear approximation to the dynamic behavior of the open-loop system (nonlinear plant shown in Figure (10)) when it was excited by a sinusoidal input signal with an amplitude of 0.8 volts. Therein, it was stated that $G_0(s)$ was identified via the SYSID software utility, and it was given by Equation (4.6). The details of identifying $G_0(s)$ via the SYSID software utility is given below.

Problem Statement and the Method of Solution

Identify a transfer function, $G_0(s)$, whose frequency response is as close to the frequency response data given in Table II as possible. The following is a step-by-step guide which would yield identification of such a $G_0(s)$.

FITTED TRANSFER FUNCTION

$$G = \frac{1.000 + 2.100\ S + 0.200\ S\ \hat{}\ 2}{0.1482E-09 + 0.200\ S + 0.200\ S\ \hat{}\ 2}$$

THE ZEROS ARE

Z1 (-0.500, 0.000J) Z2 (-10.000, 0.000J)

THE POLES ARE

P1 (-0.741E-08, 0.000J) P2 (-1.000, 0.000J)

Figure 27 Example 1 CRT Screen After the Execution of the **D** command

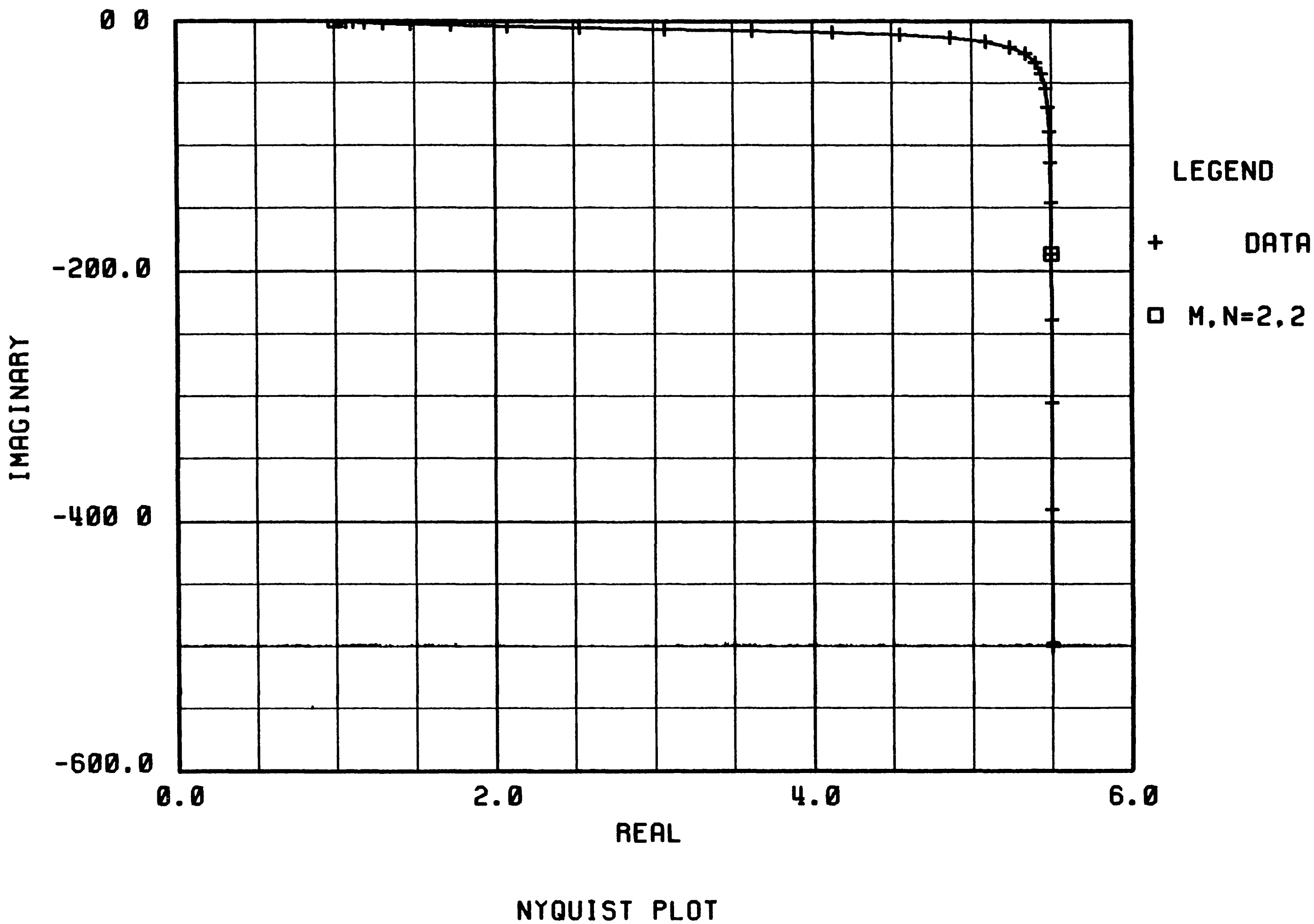
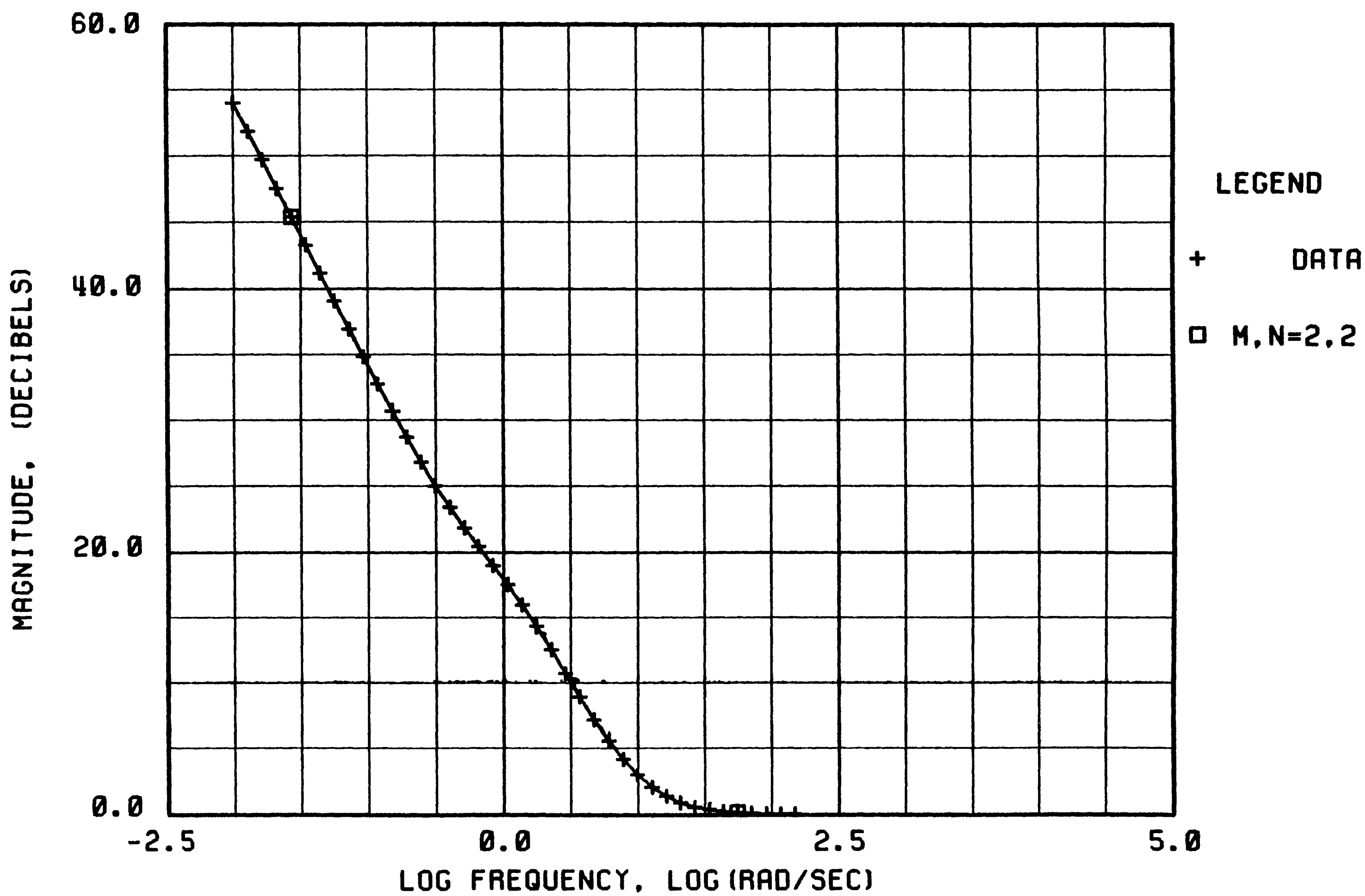
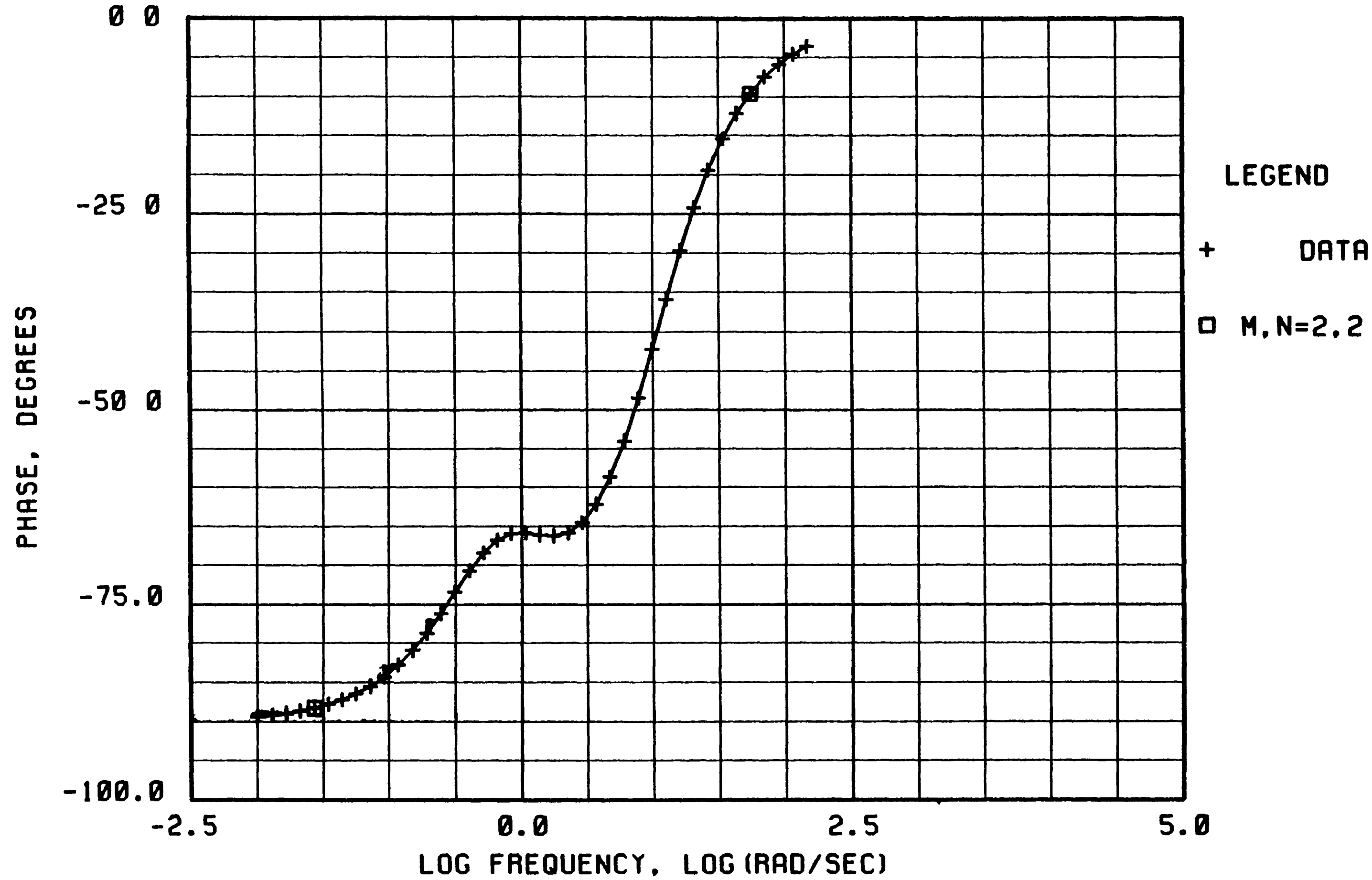


Figure 28 Example 1 Examination of the Quality of Fit by Viewing the Nyquist Plot



SYSTEM IDENTIFICATION FROM FREQUENCY RESPONSE

Figure 29 Example 1 Examination of the Quality of Fit by Viewing the Magnitude Plot



SYSTEM IDENTIFICATION FROM FREQUENCY RESPONSE

Figure 30 Example 1 Examination of the Quality of Fit by Viewing the Phase Plot

1 Create a data file, with the same format as that created in Example 1 above, whose contents are the frequency response information of Table I. In this example, the data file is named TEST2.

2 Enter the following command from Harris-800 Job Control.

1SYID*SYSID

A list of valid SYSID commands will be displayed (See Figure (26)).

3. Enter the following command to read in data file TEST2 from the hard disc

G/TEST2/

4. At this time, the user may start the system identification task. In general, the user may not know the system type, the number of system poles, and/or the number of system zeros In that case, Algorithm a.1, which is given in Appendix A, may be applied in conjunction with both **R** and **K** commands to securely identify transfer function $G_0(s)$. However, for this example problem, Figure (10) may be examined to make the following observations.

   (i) It is apparent that the open-loop nonlinear system is a rate-type system. Therefore, the linear approximation to this nonlinear plant will most likely be a type-one transfer function

   (ii) There exists two integral actions Therefore, the approximating transfer function will at least (or most likely) be a second-order one

Hence, it is safe to start the system identification task by identifying a type-one transfer function with two poles and two zeros whose frequency response approximates the frequency response data given in Table I in a near-optimum fashion. Enter the following command

K2,2

To view the status of the identified transfer function, enter the **D** command. Figure (31) is displayed. Enter the subcommand **C** to display function coefficient with free format. Figure (32) is displayed Hit the return key twice to return to the main command list.

5. Examine the quality of fit by viewing the Nyquist plot Enter the **N** command. Figure (33) is displayed. The user is instructed to hit the return key to continue

6 The user must be certain that there does not exist a third-order transfer function whose frequency response is a "better" approximation to the frequency response data of Table II than that of the identified second-order transfer function. To investigate this matter, enter the following command

K3,3

7 Examine the gain in the quality of fit by viewing the Nyquist plot Enter the **N** command Figure (34) is displayed. The user will be instructed to hit the return key to continue

Examination of Figure (34) suggests that the gain in the quality of fit by increasing the order of the transfer function from two to three is quite small The user may

FITTED TRANSFER FUNCTION

$$G = \frac{1.000 + 0.011\ S + 0.000\ S\ ^\wedge\ 2}{-0.010 + 0.043\ S + 0.003\ S\ ^\wedge\ 2}$$

THE ZEROS ARE

Z1 (-163.451, 0.000j)   Z2 (-216.073, 0.000j)

THE POLES ARE

P1 (0.221, 0.000j)   P2 (-12.995, 0.000j)

Figure 31. Example 2/Task 4 - CRT Screen After the Execution of the First D Command (See item 4 of Example 2)

THE NUMERATOR COEFFICIENTS IN ASCENDING POWERS OF S

1.0000	1.07461E-02	2.83147E-05

THE DENOMINATOR COEFFICIENTS IN ASCENDING POWERS OF S

-9.66395E-02	4.29180E-02	3.35998E-03

Figure 32. Example 2/Task 4 - CRT Screen After the Execution of the C Subcommand

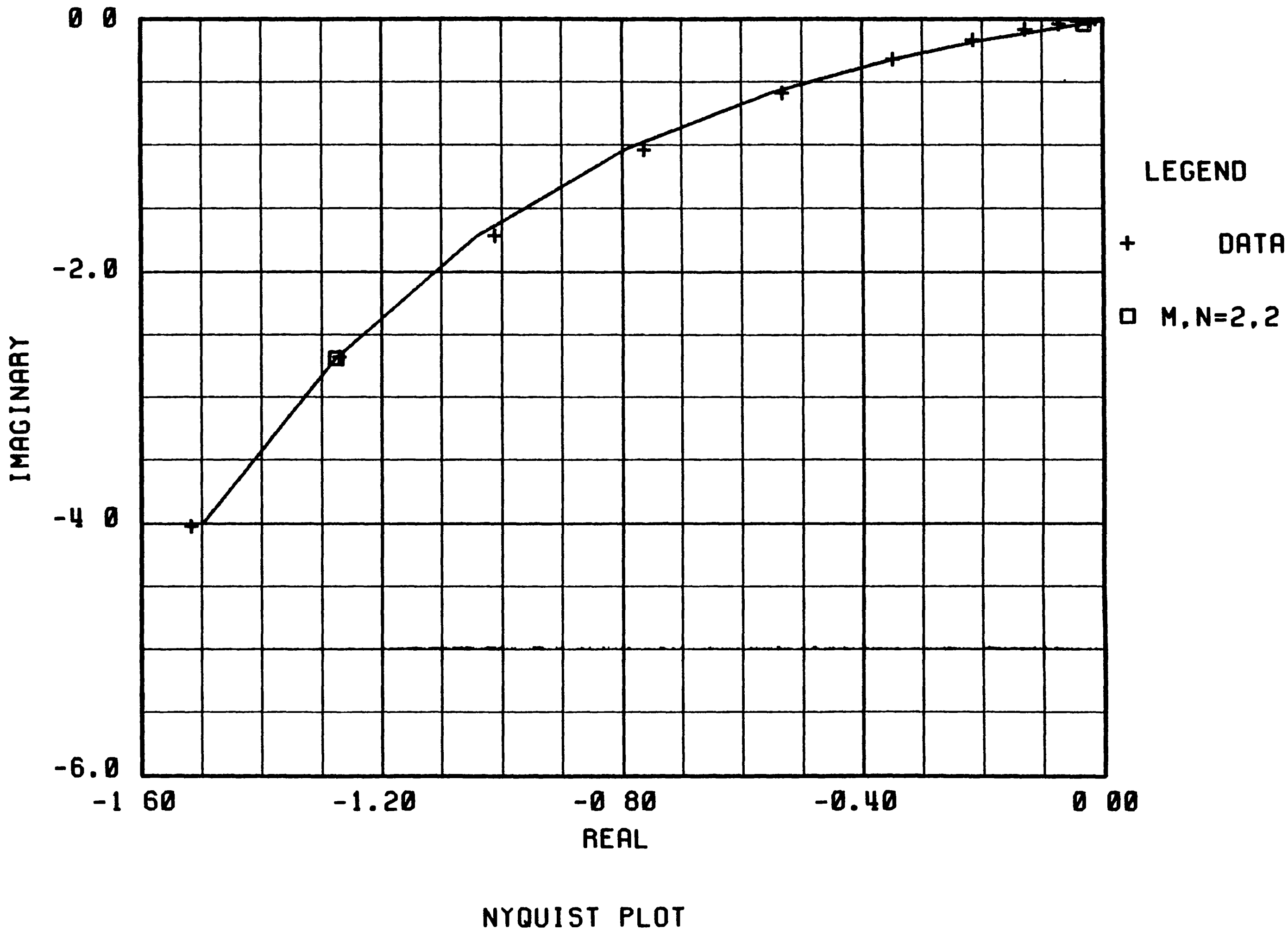


Figure 33 Example 2/Task 5 - Examination of the Quality of Fit by Viewing the Nyquist Plot

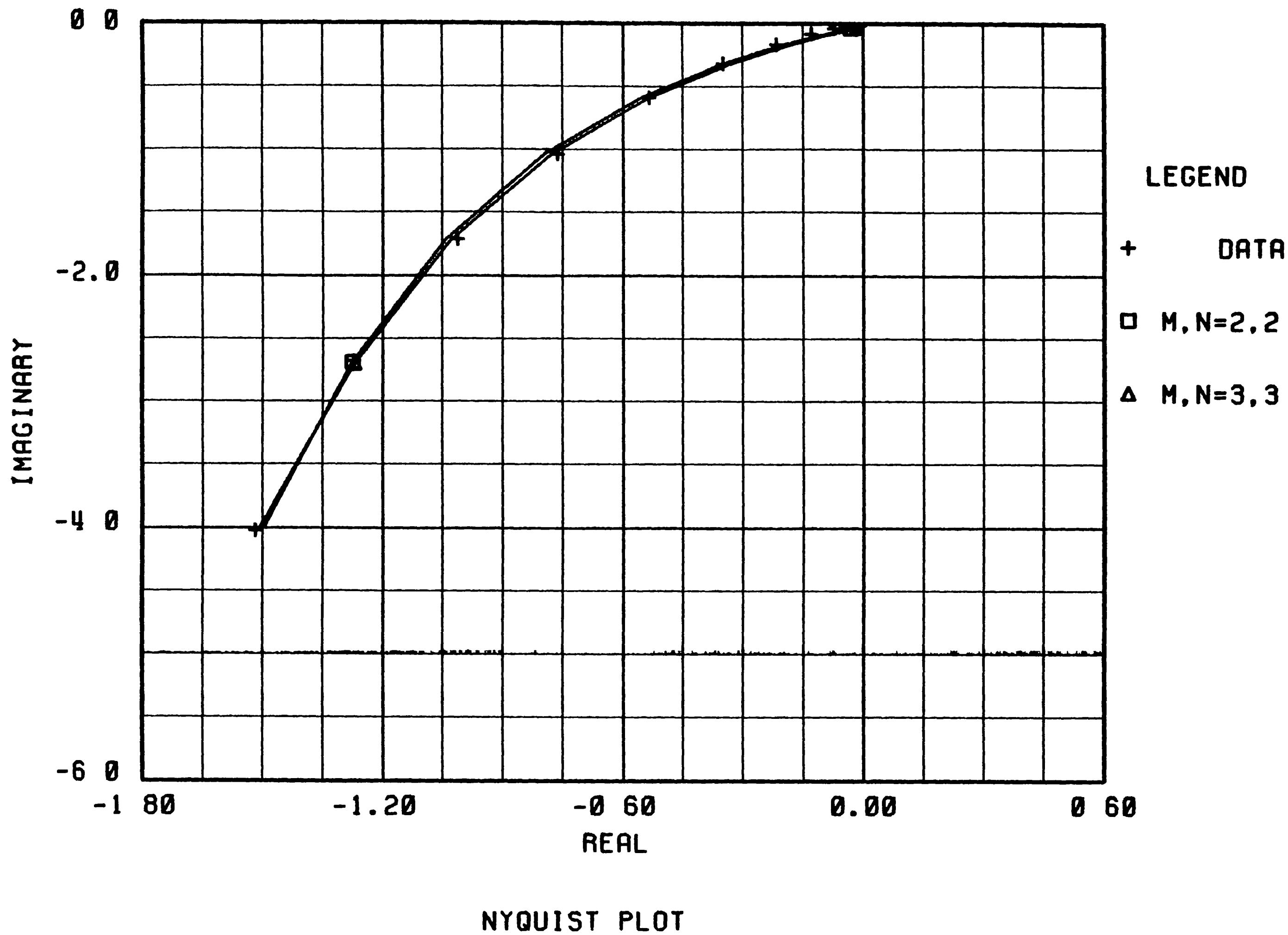


Figure 34 Example 2/Task 7 - Investigation of the Gain in the Quality of Fit by Increasing Transfer Function Order from Two to Three

conclude that, the approximating transfer function is a second-order transfer function

8. Enter the **I** command to toggle the plot overlay option (plot overlay option is off). At this time, the user must determine if he or she can identify a second-order transfer function with only one zero or even with no zero whose frequency response would adequately represent the input frequency response data. Enter the following command

K2,2

Enter the **N** command to view the Nyquist plot, enter the **P** command to view the Bode plots. Identify a second-order transfer function with only one zero by entering the following command

K1,2

Enter the **I** command to toggle the plot overlay option (plot overlay option is on) Enter the **N** command to view the Nyquist plot, enter the **P** command to view the Bode plots Identify a second-order transfer function with no zeros by entering the following command

K0,2

Enter the **N** command to view the Nyquist plot, Figure (35) is displayed Enter the **P** command to view the Bode plots. Figures (36) and (37) are displayed From the examination of these figures, it is apparent that unlike the identified second-order transfer function with no zeros, the loss in the quality of fit by identifying a second-order transfer function with one zero in comparison to the transfer function with two zeros is

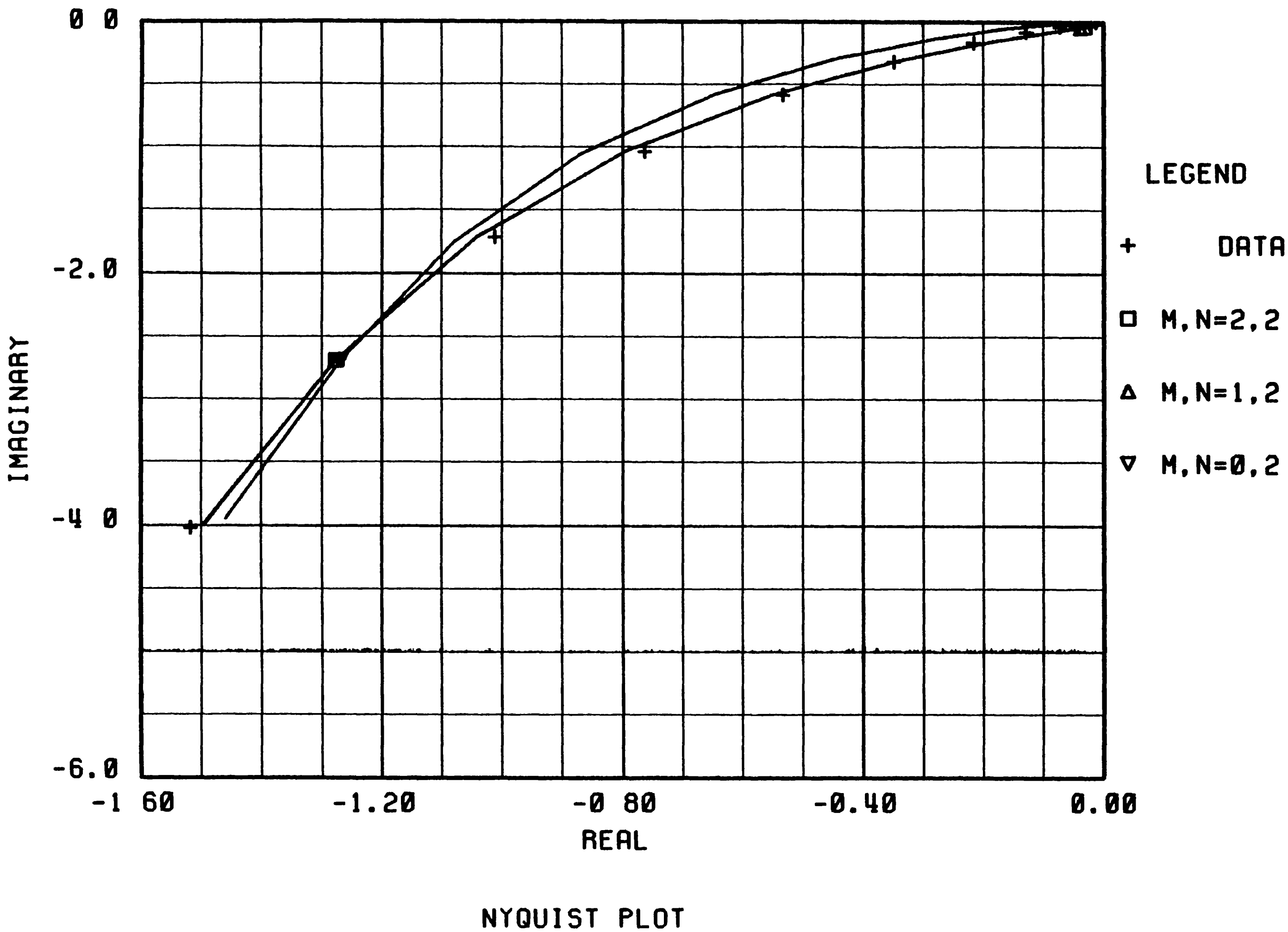
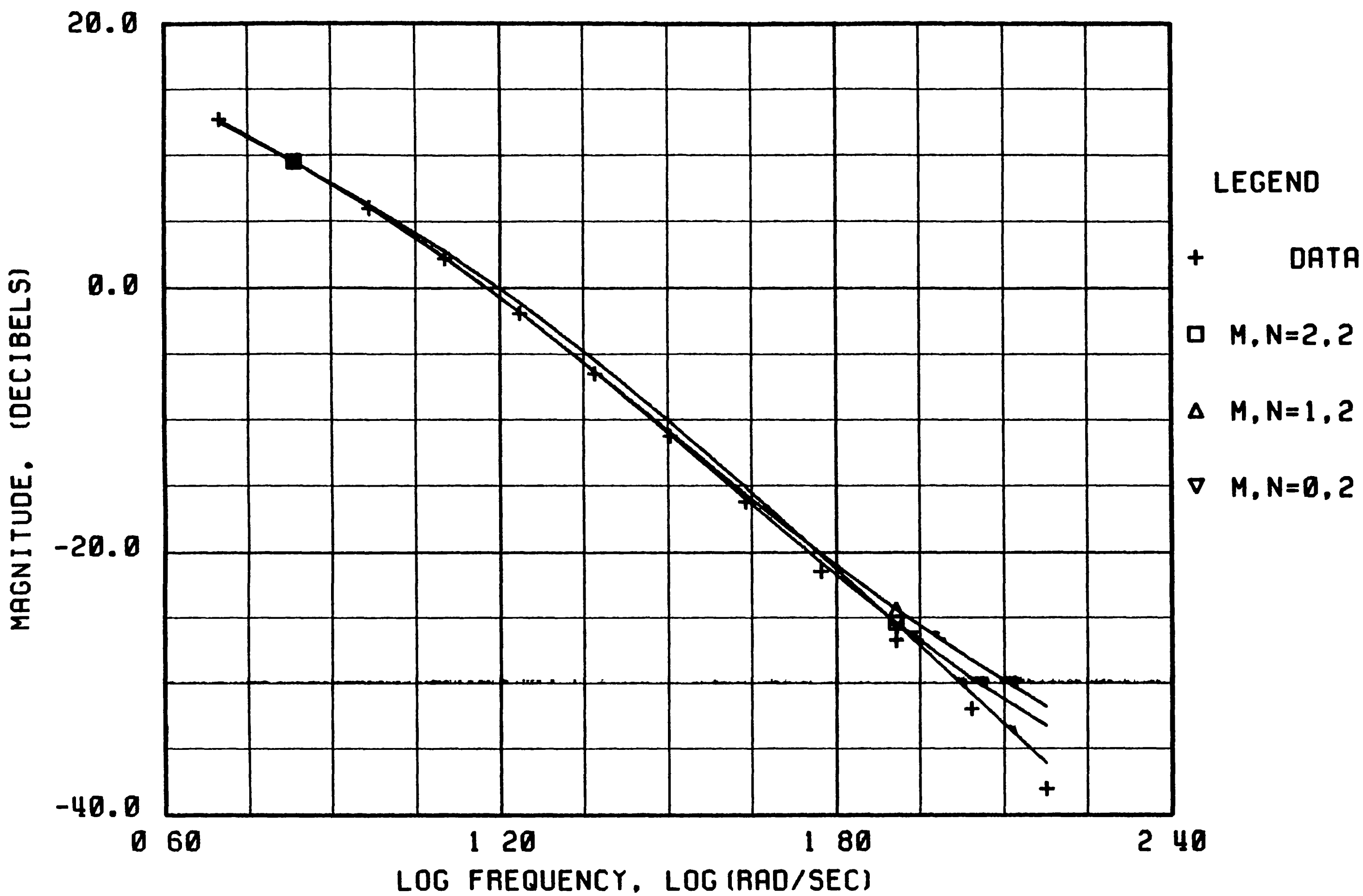


Figure 35 Example 2/Task 8 - Investigation of the Loss in the Quality of Fit by Decreasing the Number of Transfer Function Zeros Via the Visual Examination of the Nyquist Plot



SYSTEM IDENTIFICATION FROM FREQUENCY RESPONSE

Figure 36 Example 2/Task 8 - Investigation of the Loss in the Quality of Fit by Decreasing the Number of Transfer Function Zeros Via the Visual Examination of the Magnitude Plot

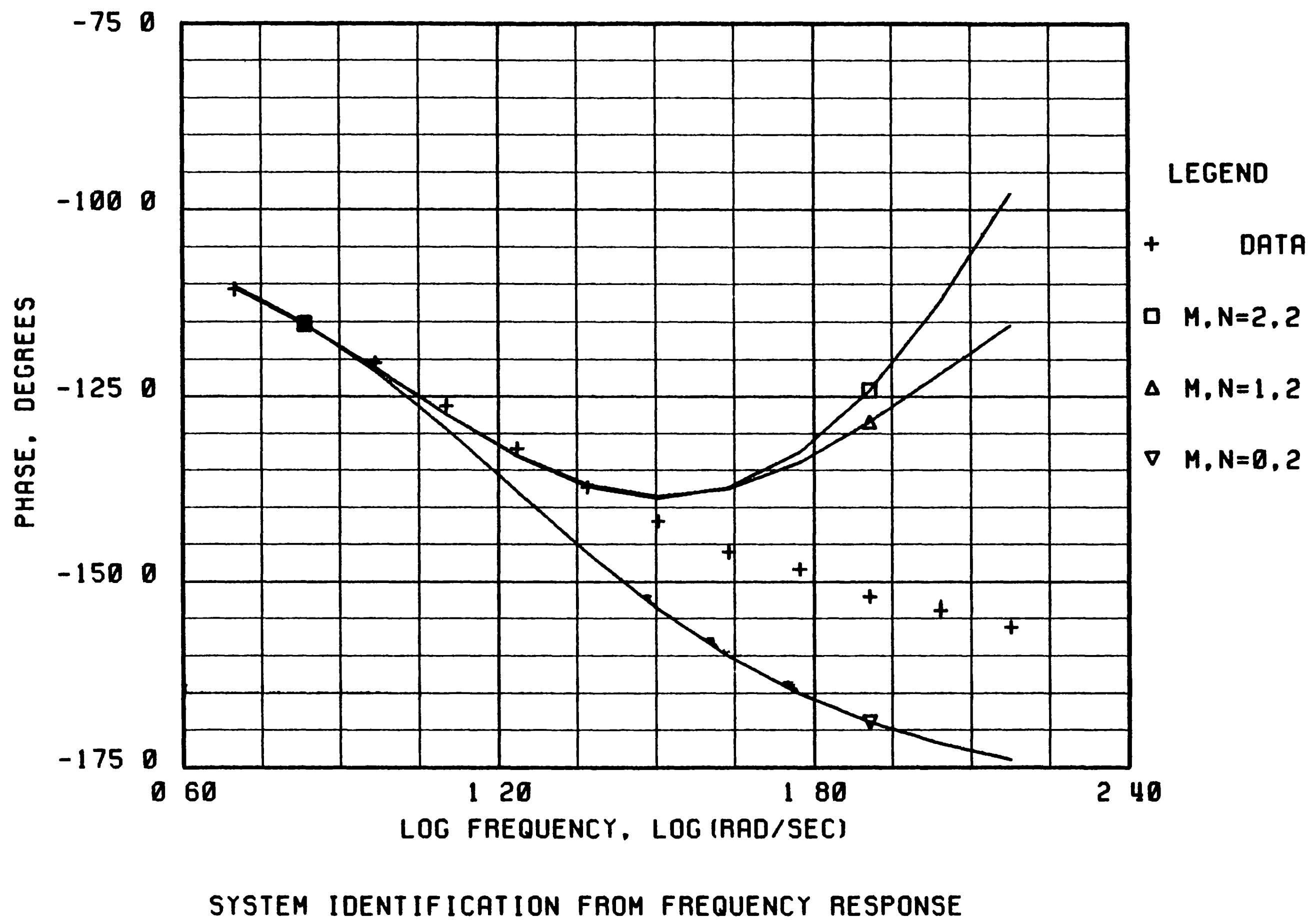


Figure 37 Example 2/Task 8 - Investigation of the Loss in the Quality of Fit by Decreasing the Number of Transfer Function Zeros Via the Visual Examination of the Phase Plot

negligible. Therefore, the identified second-order transfer function with one zero is selected Enter the following command.

K1,2

9. Enter the **F** subcommand to fix the constant term of the denominator to a zero to secure a type-one transfer function. Figure (38) is displayed Enter the following subcommand.

DEN,0.,D,D,

Enter the **RE** subcommand to return to the main command list Enter the **D** command to display the status of the identified transfer function. Figure (39) is displayed Enter the **C** subcommand to display transfer function coefficients with free format Figure (40) is displayed.

10 From examination of the phase plot shown in Figure (37), the user may desire to sacrifice the quality of fit at lower frequencies to improve on the quality of fit at higher frequencies. For example, enter the following "Weight" commands (refer to Table VI for definition of the command syntax).

W(10,20.,20.),W(11,20.,20.),W(12,20.,20.)

Then, enter the **O** command Figure (41) is displayed Enter the **R** subcommand to return to the main command list Enter the **D** command, and then enter the **C** subcommand, Figures (42) and (43) are displayed. Hit the return key twice to return to the SYSID main command list.

11 Examine the quality of fit by the viewing the Nyquist plot, Bode plots, and error plots by entering the **N**, **P**, and the **E**

```
FITTED TRANSFER FUNCTION

                          1.000 + 0.011 S
              G = ---------------------------------
                   -0.009 + 0.043 S + 0.003 S ^ 2

NUM == > MODIFIES THE NUMERATOR COEFFICIENTS,
         NUM, 0., D, 1., WILL SET THE CONSTANT TERM TO 0.0,
                         WILL NOT MODIFY THE S1 COEFFICIENT,
                         WILL SET THE S2 COEFFICIENT TO 1.0,

DEN ==> MODIFIES THE DENOMINATOR COEFFICIENTS
        DEN, 0., D, 1., WILL MODIFY DENOMINATOR COEFFICIENTS LIKE ABOVE

RET ==> RETURN TO THE CALLING ROUTINE

ENTER SUBCOMMAND
```

Figure 38. Example 2/Task 9 - CRT Screen After the Execution of the **F** command

FITTED TRANSFER FUNCTION

$$G = \frac{1.000 + 0.011\ S}{0.000 + 0.043\ S + 0.003\ S \wedge 2}$$

THE ZEROS ARE

Z1 (-87.435, 0.000j)

THE POLES ARE

P1 (0.000, 0.000j) P2 (-12.544, 0.000j)

Figure 39. Example 2/Task 9 - CRT Screen After the Execution of the D Command

THE NUMERATOR COEFFICIENTS IN ASCENDING POWERS OF S ARE

1.0000	1.14371E-02

THE DENOMINATOR COEFFICIENTS IN ASCENDING POWERS OF S ARE

0.0000	4.28745E-02	3.41800E-03

Figure 40. Example 2/Task9 - CRT Screen After the Execution of the C Subcommand

TOTAL NUMBER OF FUNCTION EVALUATIONS 35

$$GS = \frac{1.000 + 0.011 S}{0.000 + 0.043 S + 0.003 S ^ 2}$$

$$G = \frac{1.231 + 0.004 S}{0.004 - 0.055 S + 0.004 S ^ 2}$$

THE VALUE OF THE OBJECTIVE FUNCTION 0.5351302E+00

HIT THE RETURN KEY TO CONTINUE WITH THE OPTIMIZATION ROUTINE

ENTER R TO RETURN TO THE CALLING ROUTINE

Figure 41. Example 2/Task 10 - CRT Screen After the Execution of the O Command

FITTED TRANSFER FUNCTION

$$G = \frac{1.231 + 0.004\ S}{0.000 + 0.055\ S + 0.003\ S\ \hat{}\ 2}$$

THE ZEROS ARE

Z1 (-308.629, 0.000j)

THE POLES ARE

P1 (0.000, 0.000j) P2 (-14.557, 0.000j)

Figure 42. Example 2/Task 10 - CRT Screen After the Execution of the D Command

THE NUMERATOR COEFFICIENTS IN ASCENDING POWERS OF S ARE

1.2312 3.98925E-03

THE DENOMINATOR COEFFICIENTS IN ASCENDING POWERS OF S ARE

0.00000 5.47508E-02 3.76117E-03

Figure 43. Example 2/Task 10 - CRT Screen After The Execution of the C Subcommand

commands, respectively. Figures (44)-(50) are displayed

12 The transfer function whose frequency response approximates the frequency response data given in Table II is identified (See Figures (42) and (43)) This transfer function is designated by the symbol $G_0(s)$, and the system identification of this example problem is solved. Enter the **Q** command to quit.

Example 3

In Chapter IV, transfer function $G_1(s)$ was identified to be a linear approximation to the dynamic behavior of the open-loop system (nonlinear plant shown in Figure (10)) when it was excited by a sinusoidal input with an amplitude of 0.2 mili-volts. Therein, it was stated that $G_1(s)$ was identified via the SYSID software utility, and it was given by Equation (4 7). The details of identifying $G_1(s)$ via the SYSID software utility is given below

Problem Statement and the Method of Solution

Identify a transfer function, $G_1(s)$, whose frequency response is as close to the frequency response data given in Table II as possible The following is a step-by-step guide which would yield identification of such a $G_1(s)$

1. Create a data file, with the same format as that created in step 1 of Example 2 above, whose contents are the frequency response information of Table II In this example, the data file is named 0001DATA*TEST3.
2. Enter the following command from Harris-800 Job Control.

1SYID*SYSID

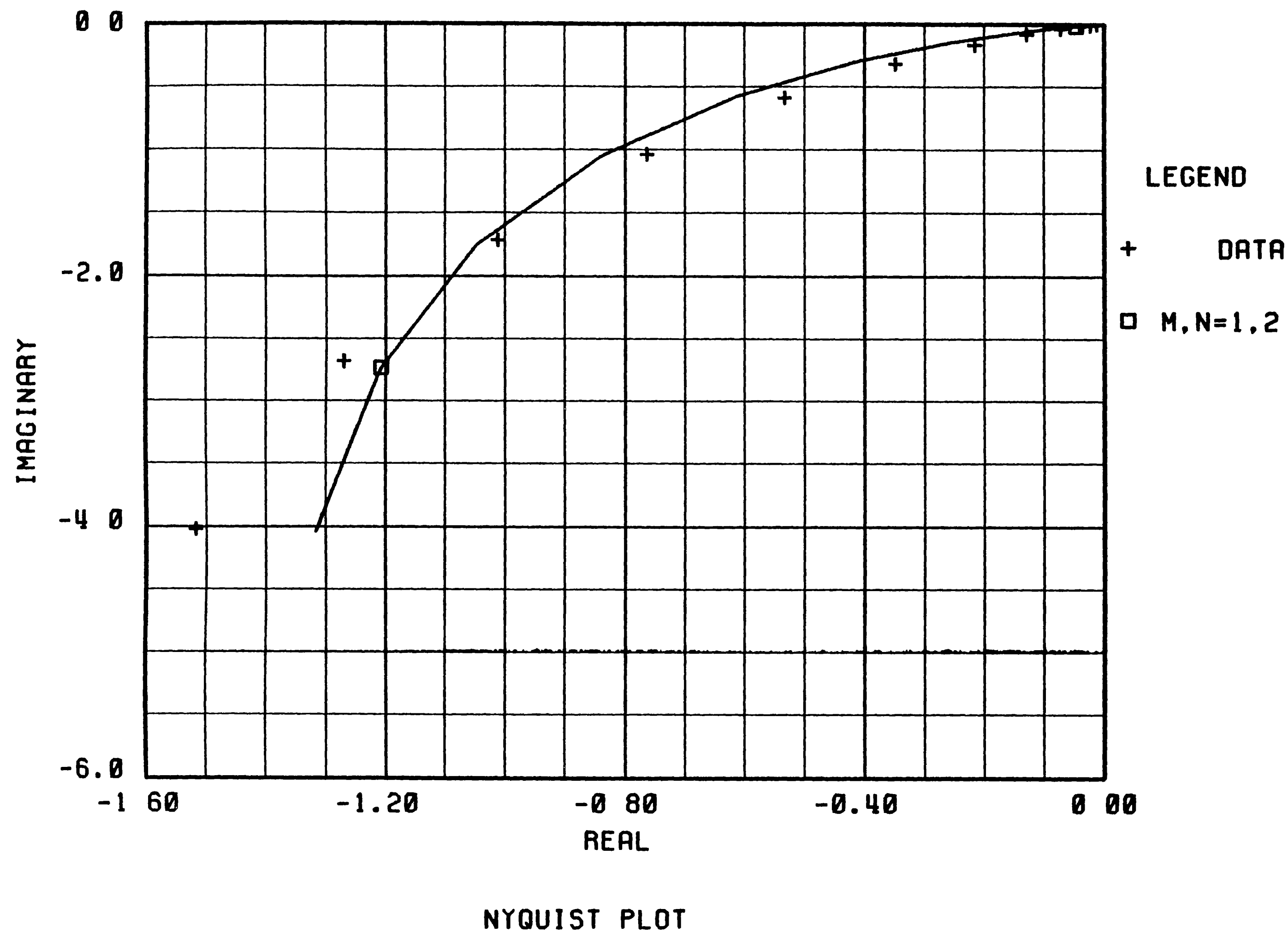


Figure 44 Example 2/Task 11 - Visual Examination of the Difference Between the Nyquist Plot of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model

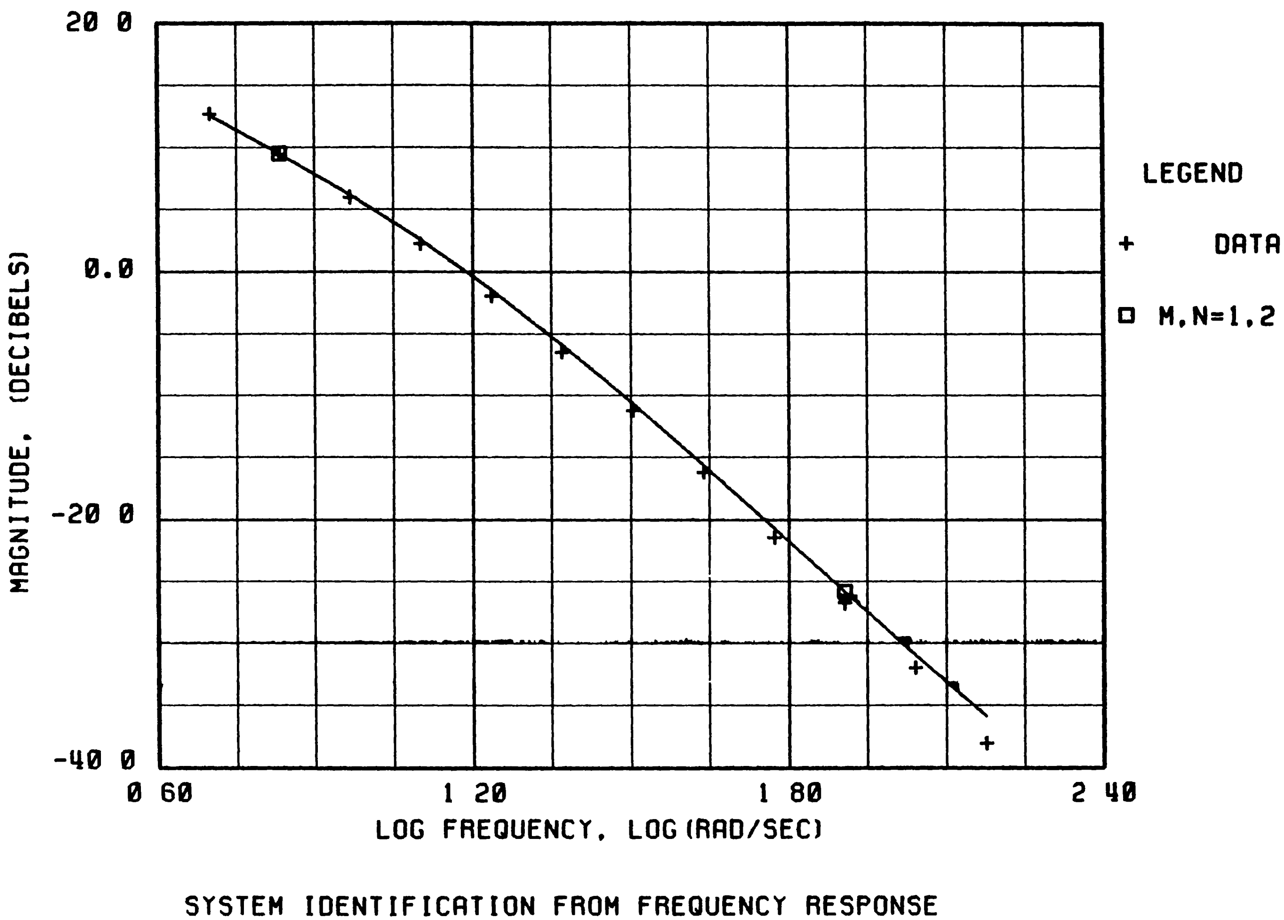
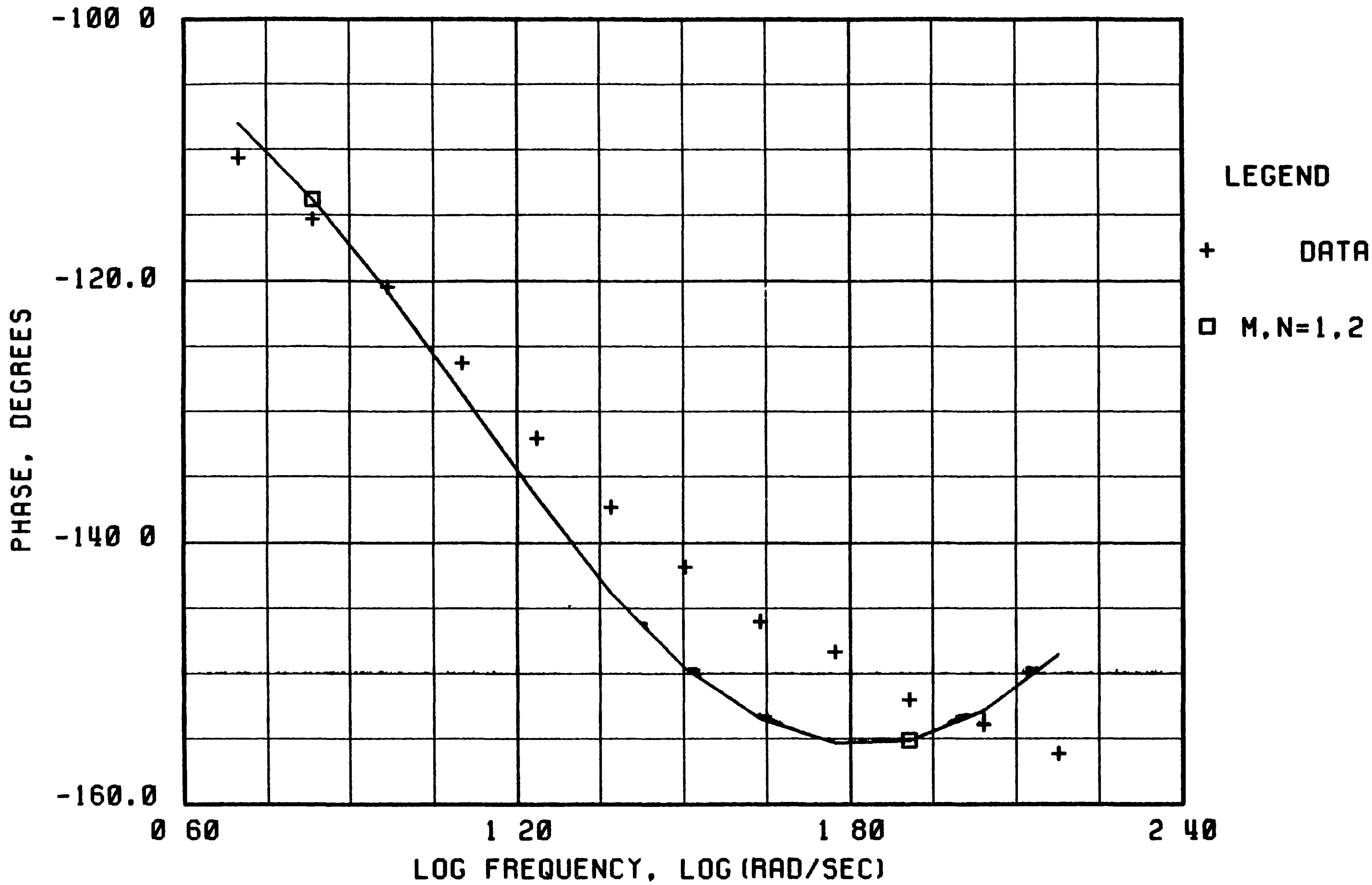


Figure 45 Example 2/Task 11 - Visual Examination of the Difference Between the Magnitude Plot of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model



SYSTEM IDENTIFICATION FROM FREQUENCY RESPONSE

Figure 46 Example 2/Task 11 - Visual Examination of the Difference Between the Phase Plot of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model

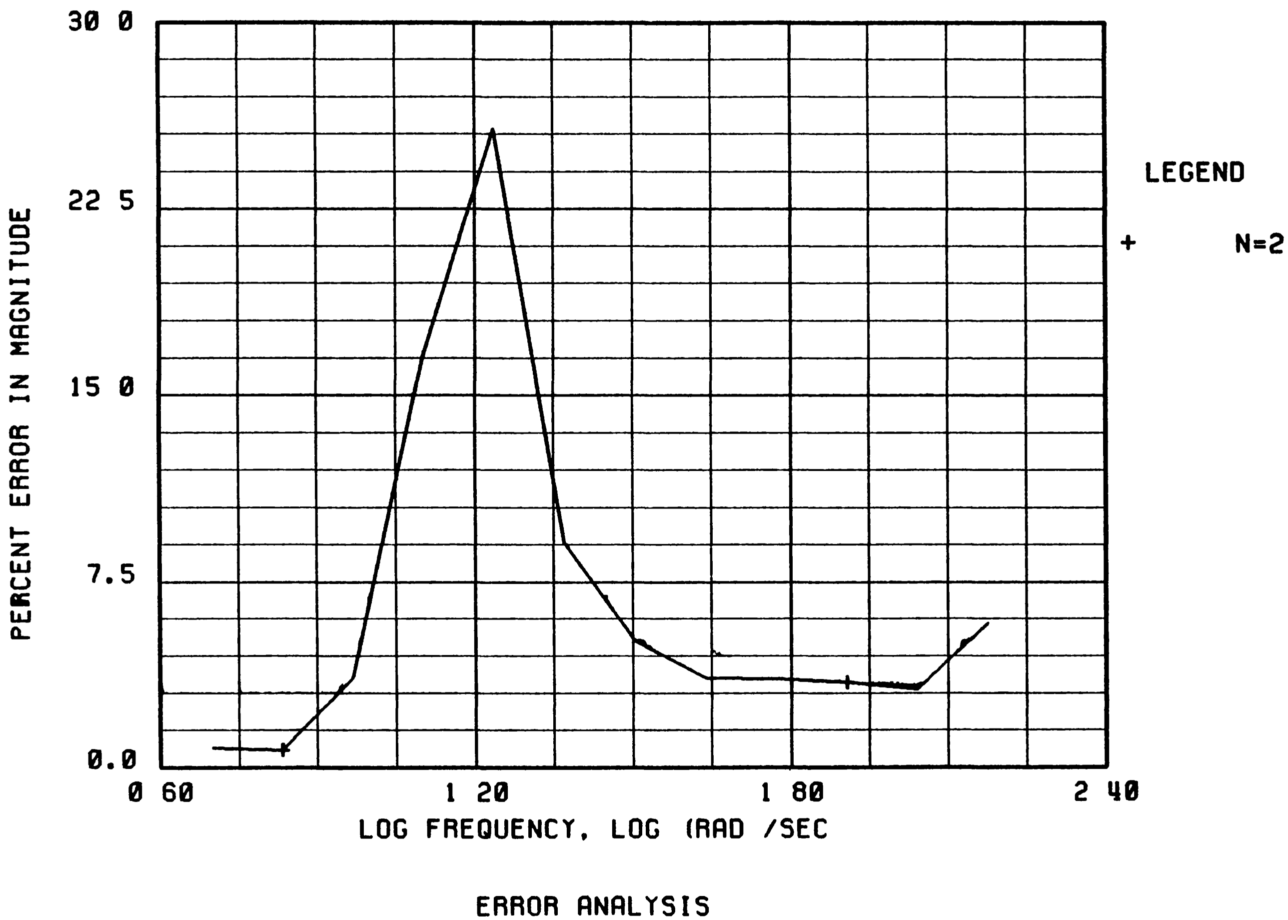


Figure 47 Example 2/Task 11 - Visual Inspection of the Per Cent Error Between the Magnitude of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model

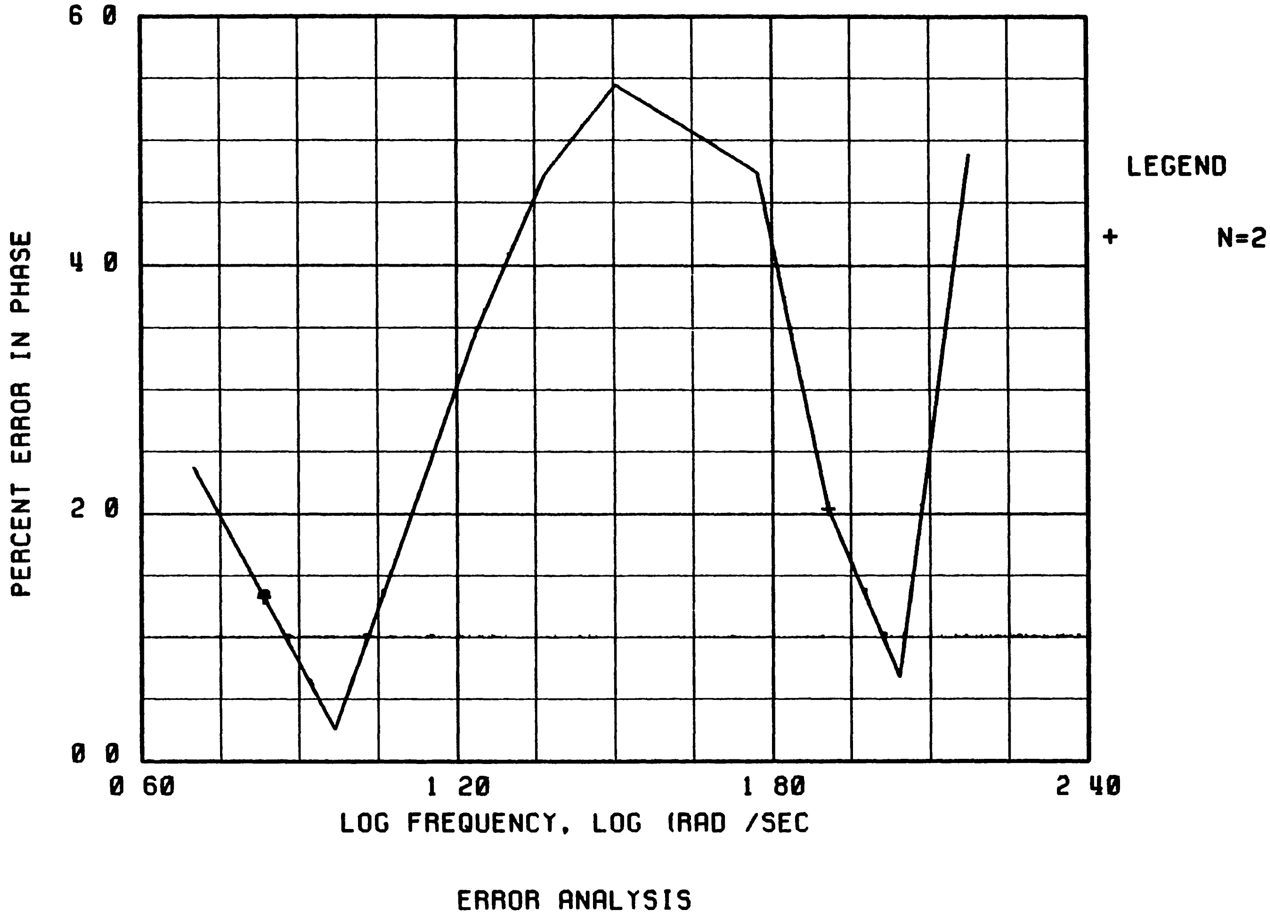


Figure 48 Example 2/Task 11 - Visual Inspection of the Per Cent Error Between the Phase of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model

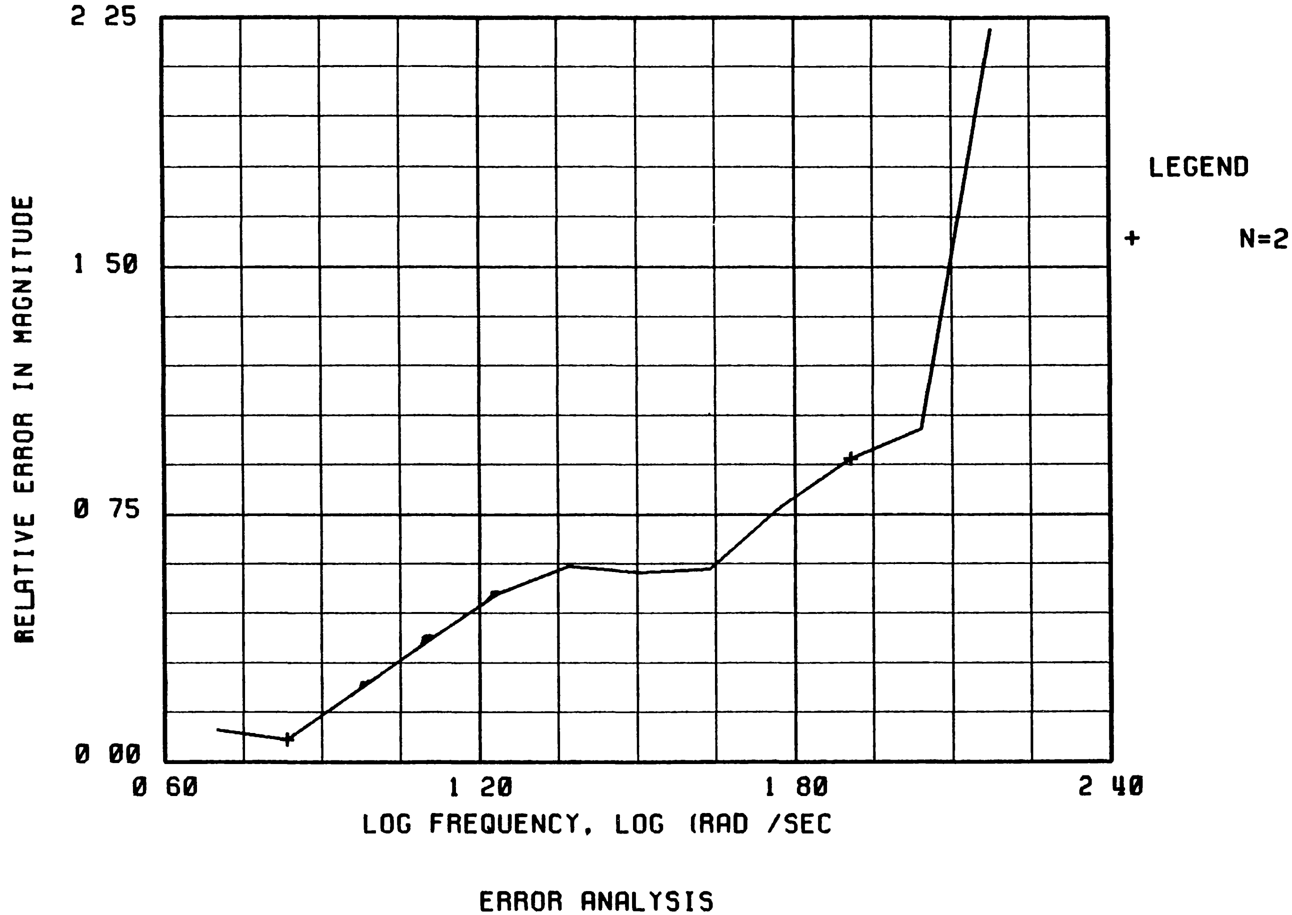


Figure 49 Example 2/Task 11 - Visual Inspection of the Relative Error Between the Magnitude of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model

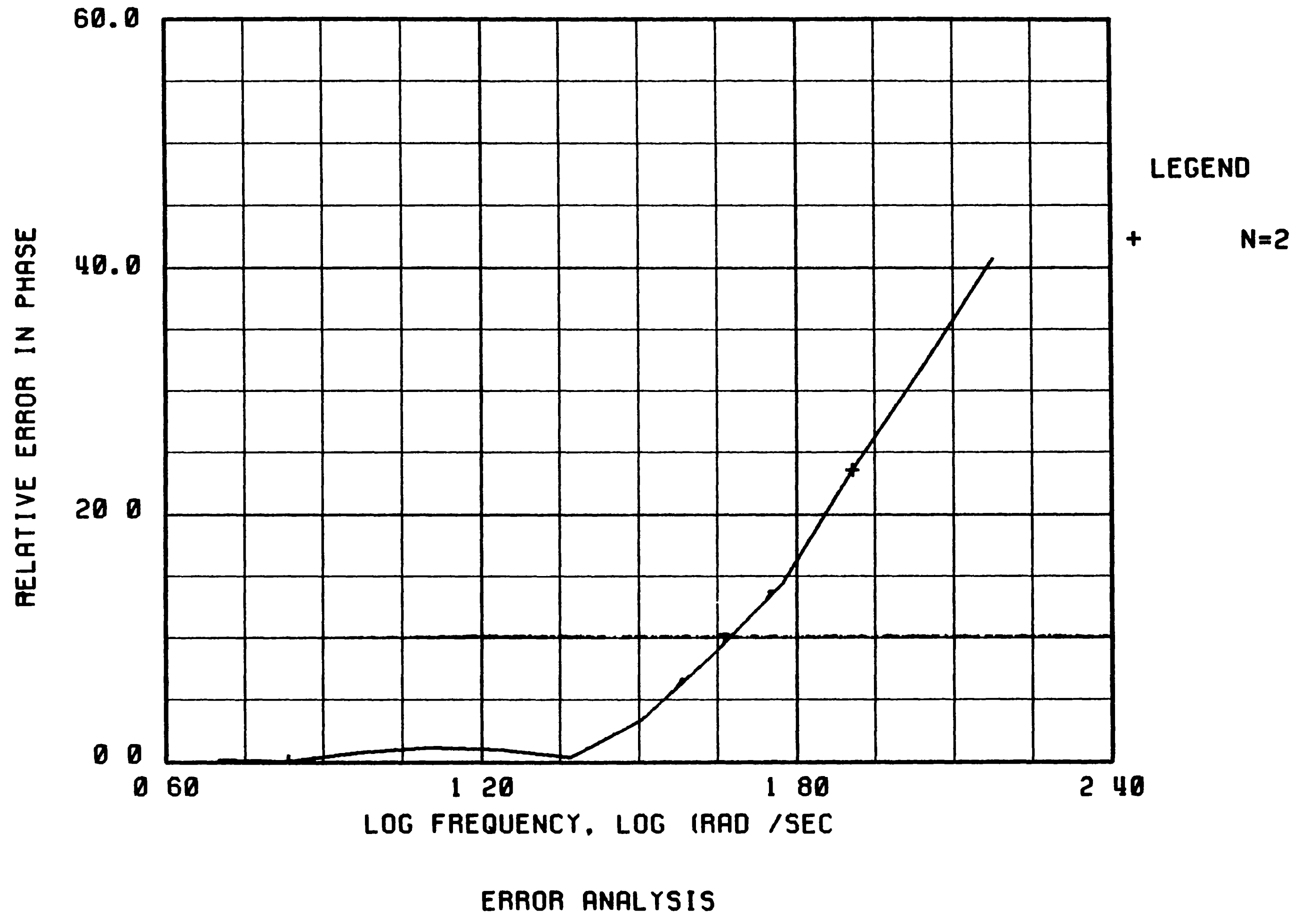


Figure 50 Example 2/Task 11 - Visual Inspection of the Relative Error Between the Phase of the Approximating Linear System, $G_0(s)$, with that of the Quasilinear Model

3. Enter the following command to read the data file TEST3 from the hard disc.

G/TEST3/

4. The user must go through a similar exercise as that given in tasks 4-8 of Example 2 above to verify that (i) the gain in the quality of fit by increasing the transfer function order from two to three is negligible, and (ii) a second-order transfer function with one zero is to be identified. Enter the following command.

K1,2

5. With reference to the task done in step 9 of Example 2 above, use the **F** command to fix the constant term of the transfer function denominator to a zero to secure a type-one system Enter the D command, and then enter the C subcommand, Figures (51) and (52) are displayed. Hit the return key twice to return to SYSID main command list

6 Examine the quality of fit by viewing the Nyquist plot, Bode plots, and error plots by entering the **N**, **B**, and the E commands, respectively Figure (53)-(59) are displayed

7 The transfer function whose frequency response approximates the frequency response data given in Table II is identified (see Figure (51) and (52)) This transfer function is designated by the symbol $G_1(s)$, and the system identification task of this example problem is solved. Enter the **Q** command to quit.

FITTED TRANSFER FUNCTION

$$G = \frac{1.000 + 0.040\ S}{0.000 + 0.225\ S + 0.024\ S\ \hat{}\ 2}$$

THE ZEROS ARE

Z1 (-24.806, 0.000j)

THE POLES ARE

P1 (0.000, 0.000j) P2 (-9.526, 0.000j)

Figure 51. Example 3/Task 5 - CRT Screen After the Execution of the **D** Command

THE NUMERATOR COEFFICIENTS IN ASCENDING POWERS OF S ARE

1.0000 4.03136E-02

THE DENOMINATOR COEFFICIENTS IN ASCENDING POWERS OF S ARE

0.0000 0.22521 2.36410E-02

Figure 52 Example 3/Task 5 - CRT Screen After the Execution of the **C** Subcommand

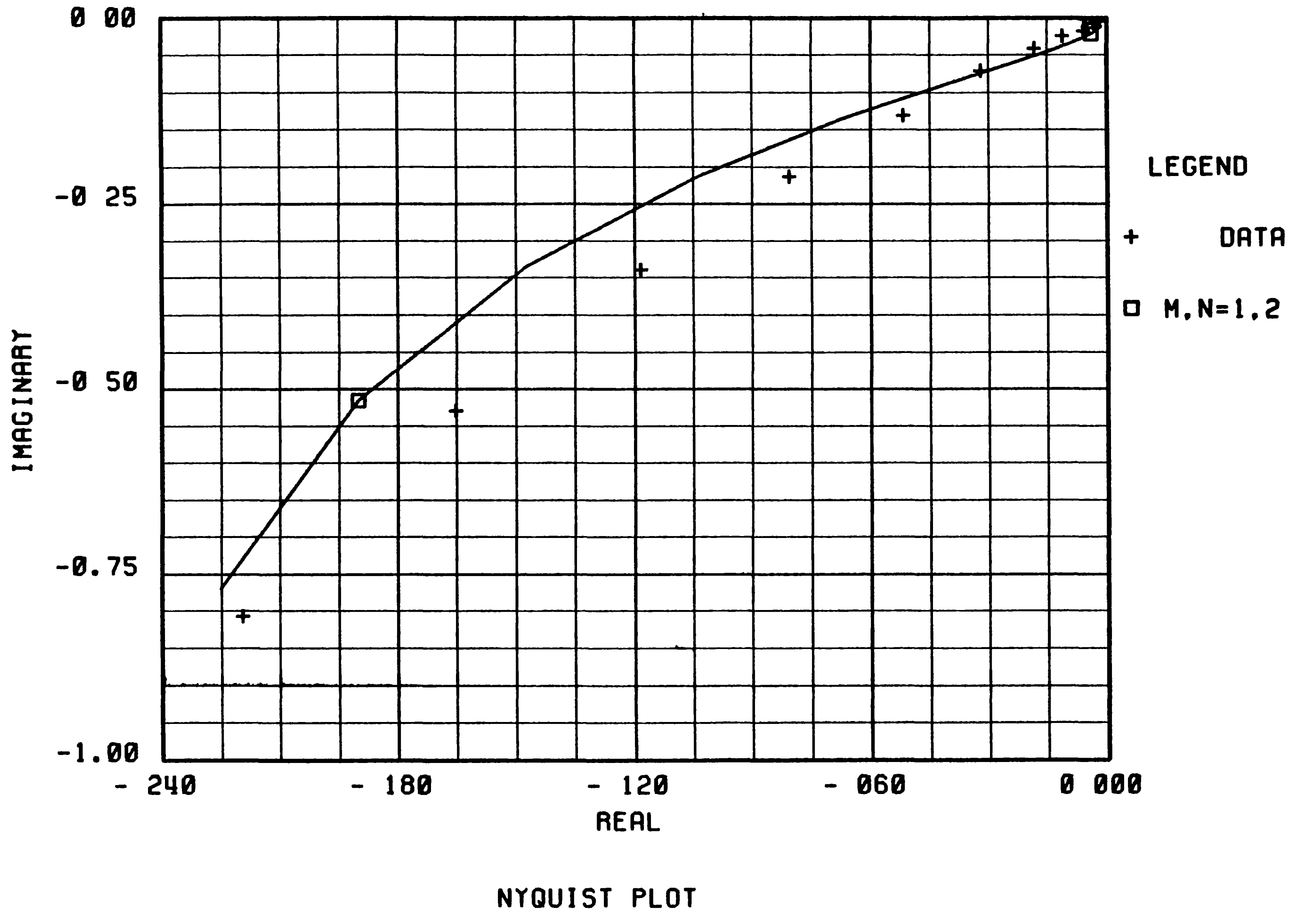
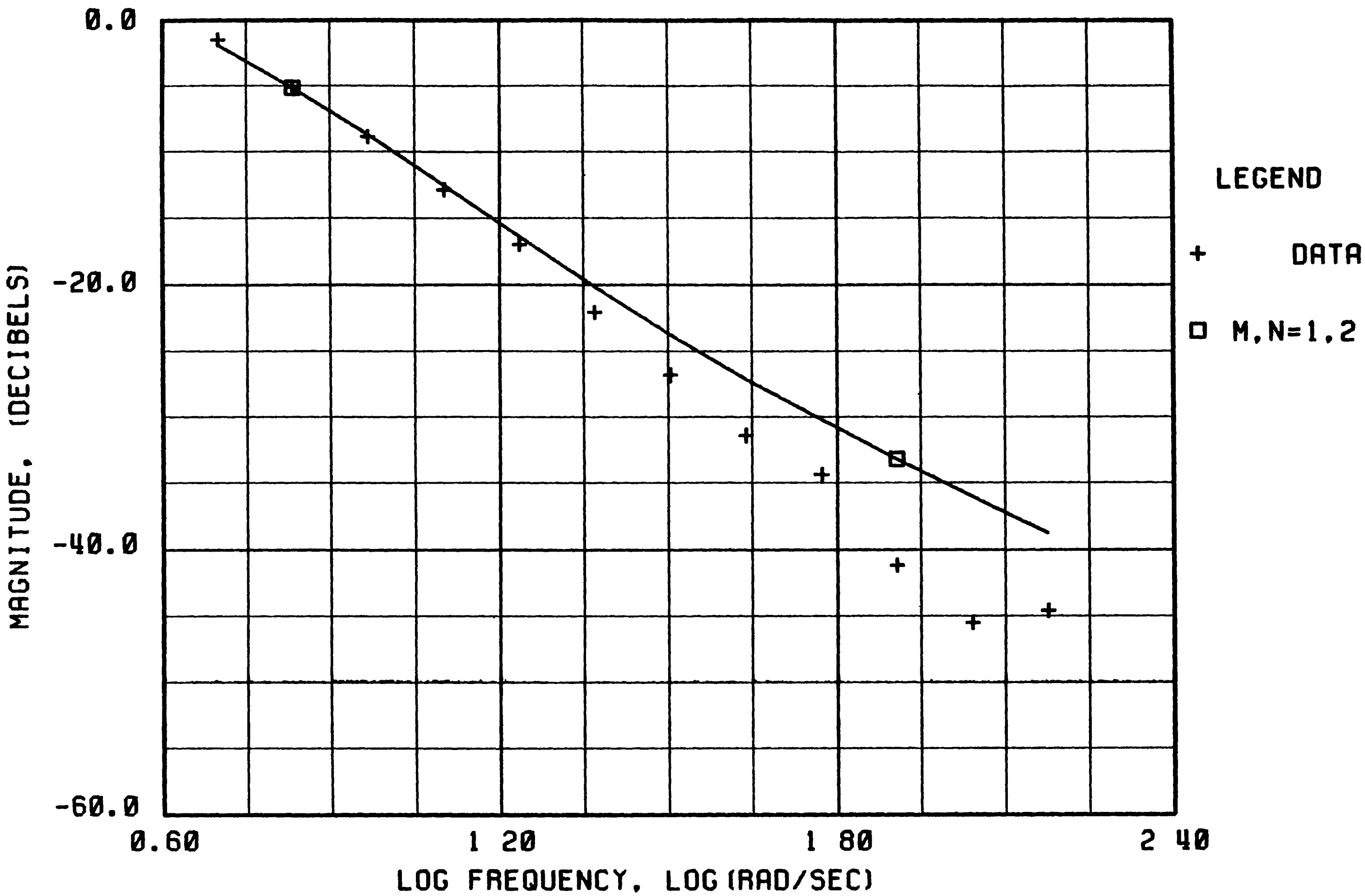


Figure 53 Example 3/Task 7 - Visual Examination of the Difference Between the Nyquist Plot of the Approximating System, $G_1(s)$, with that of the Quasilinear Model



SYSTEM IDENTIFICATION FROM FREQUENCY RESPONSE

Figure 54 Example 3/Task 7 - Visual Examination of the Difference Between the Magnitude Plot of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model

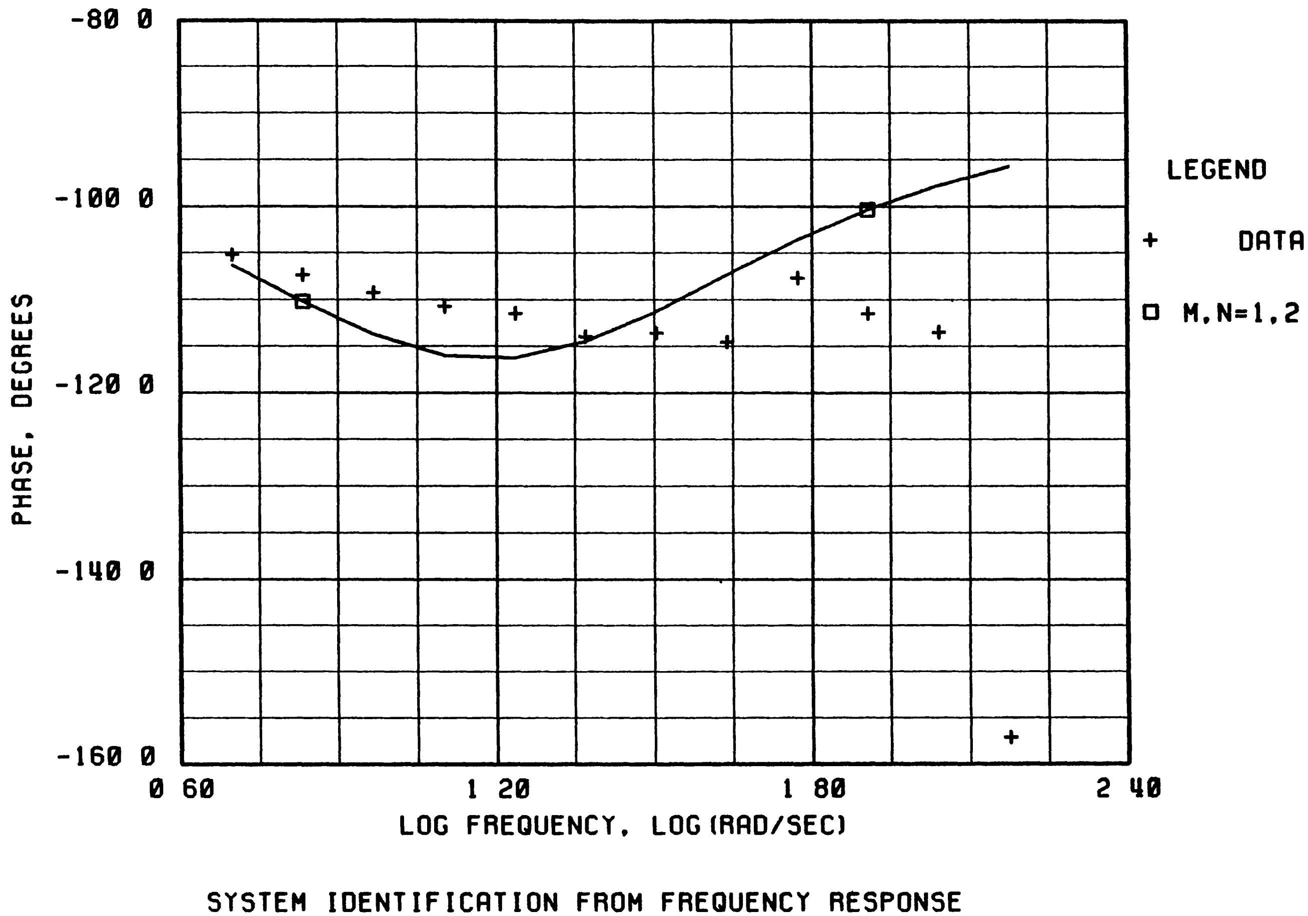
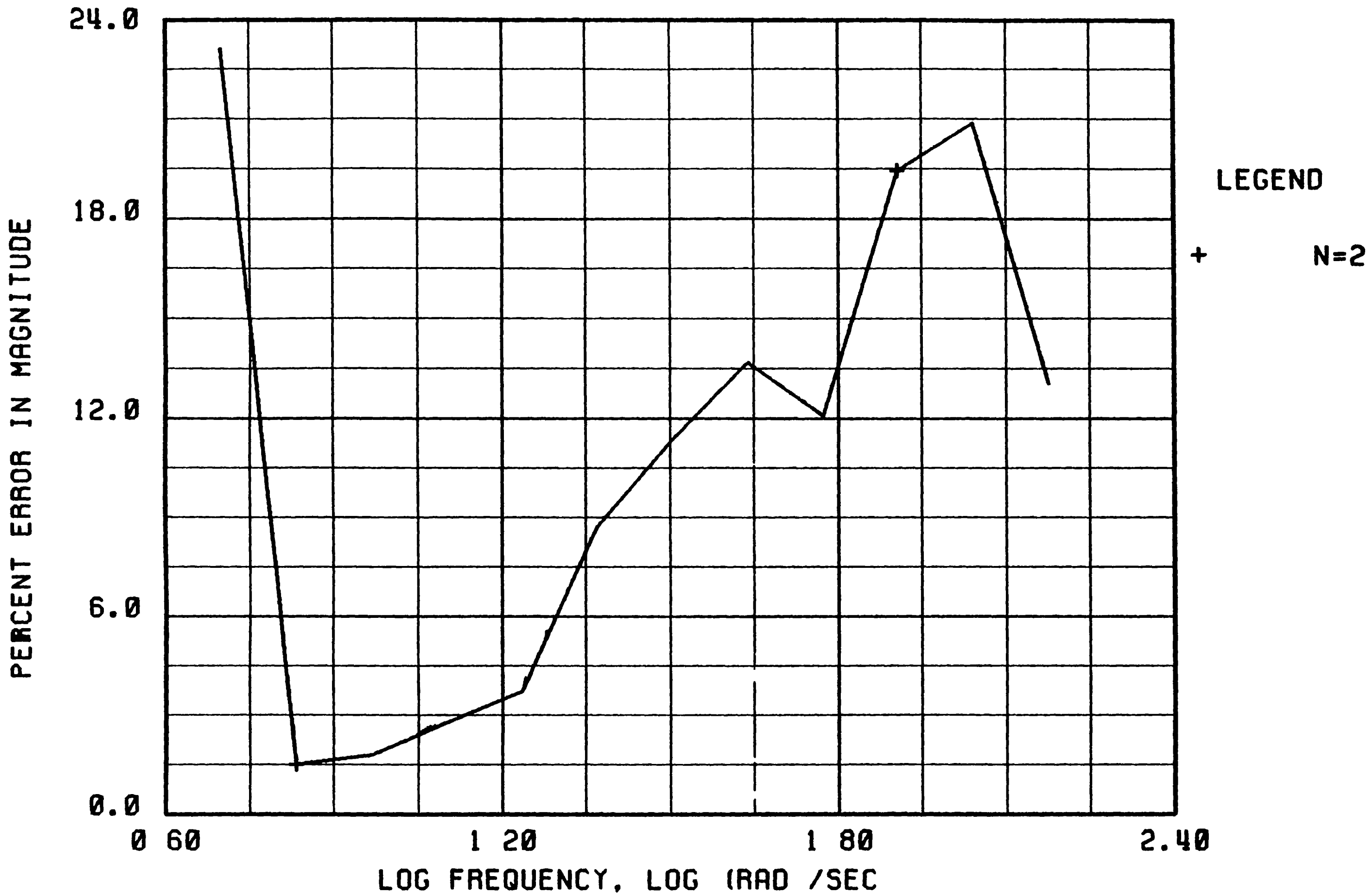
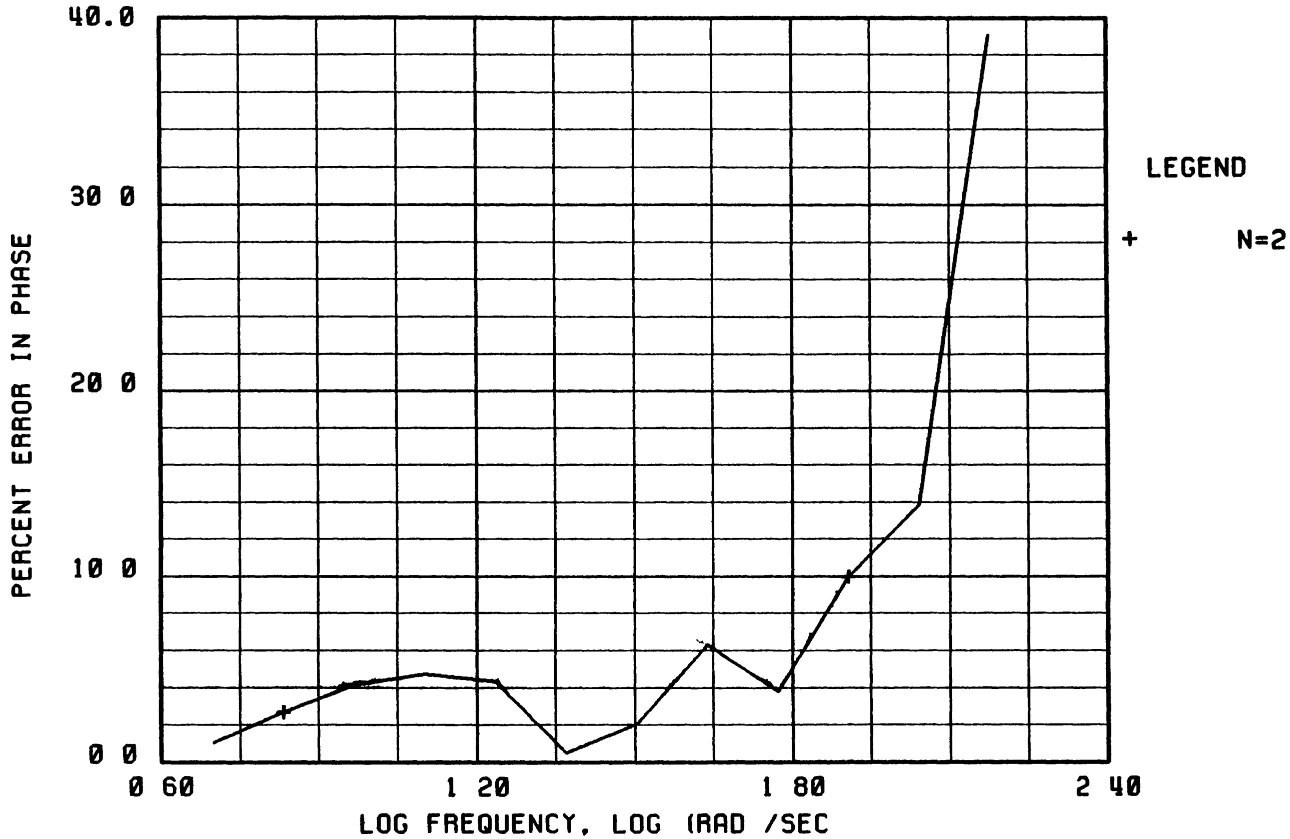


Figure 55 Example 3/Task 7 - Visual Examination of the Difference Between the Phase Plot of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model



ERROR ANALYSIS

Figure 56 Example 3/Task 11 - Visual Inspection of the Per Cent Error Between the Magnitude of the Approximating Linear System, $G_1(s)$, with that of Quasilinear Model



ERROR ANALYSIS

Figure 57 Example 3/Task 7 - Visual Inspection of the Per Cent Error Between the Phase of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model

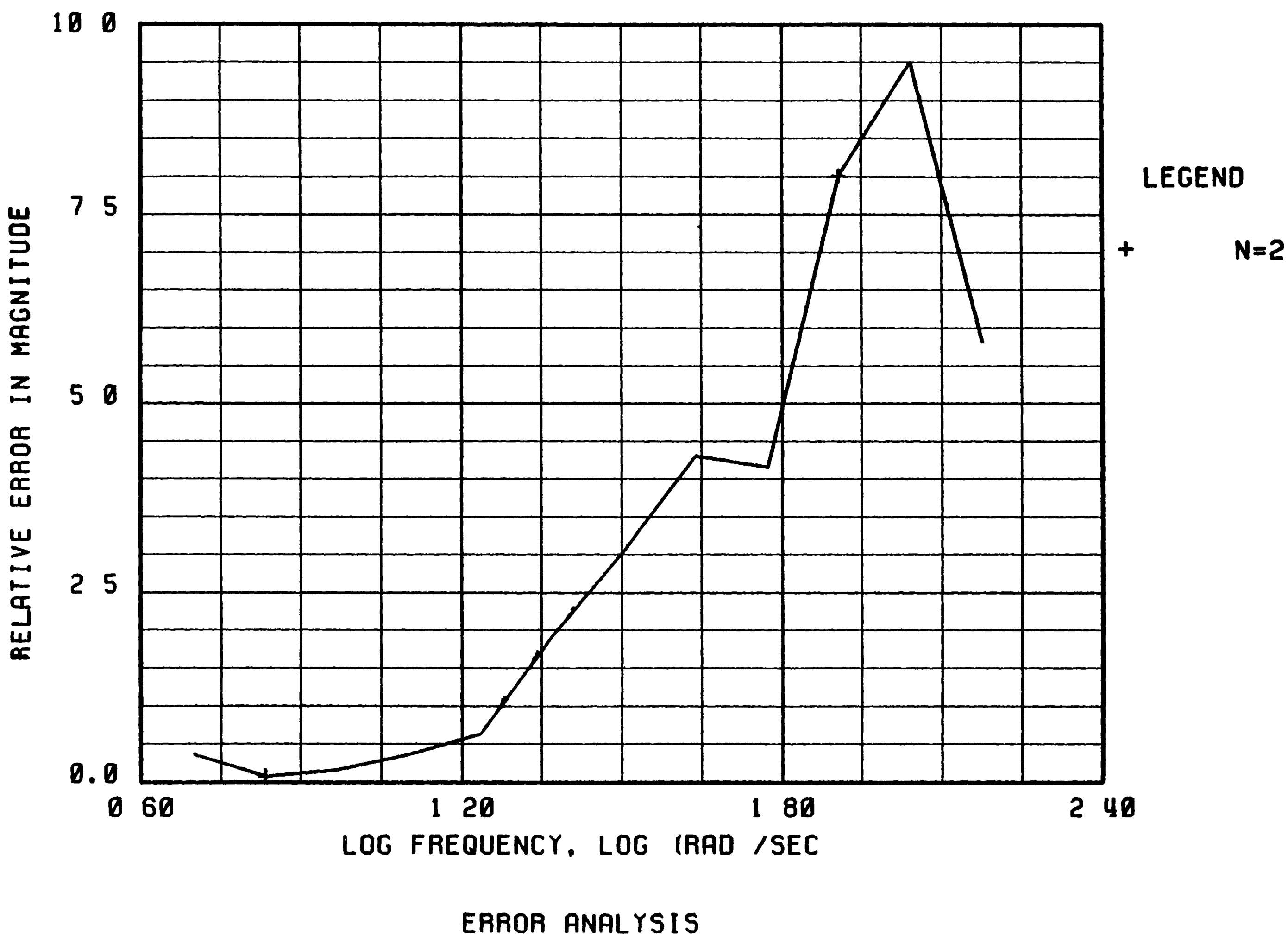


Figure 58 Example 3/Task 7 - Visual Inspection of the Relative Error Between the Magnitude of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model

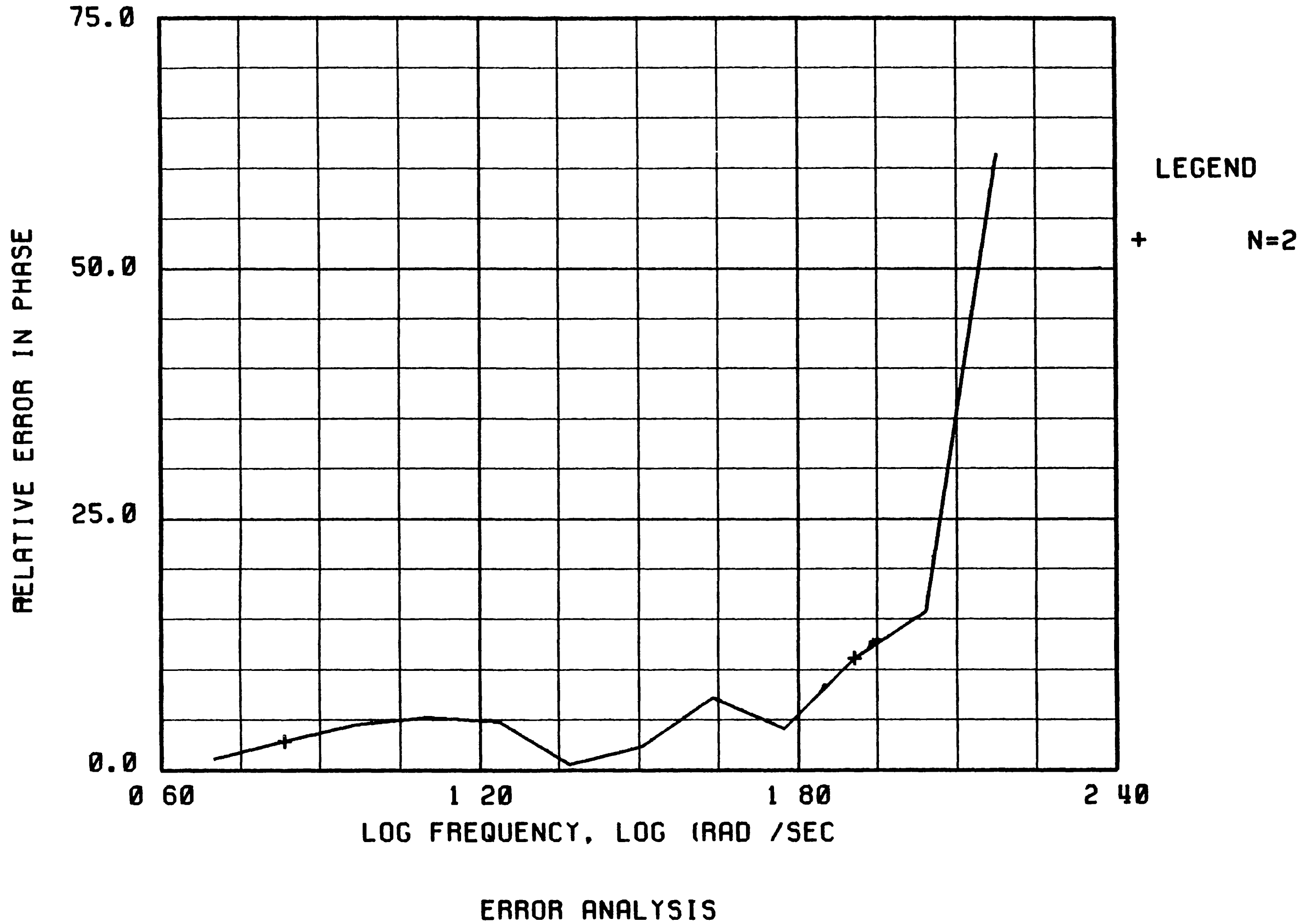


Figure 59 Example 3/Task 7 - Visual Inspection of the Relative Error Between the Magnitude of the Approximating Linear System, $G_1(s)$, with that of the Quasilinear Model

APPENDIX F

A TUTORIAL ON DRLCD SOFTWARE UTILITY

The objective of this appendix is to tutor the reader in using the Dual-Range Linear Controller Design (DRLCD) software utility which was developed in this thesis effort. The objective is fulfilled by solving one example problem It is assumed that the user has access to the Oklahoma State University's College of Engineering, Architecture, and Technology (CEAT) Computer-Aided Design (CAD) and Graphics Research Facility. The first-time user of the CEAT CAD lab must consult with the site manager to gain access to the Harris-800 minicomputer. Select one of the available Tektronix 4115B color graphics work stations, and sign-on to the Harris-800 minicomputer.

In Step 5 (See Figure 2), the search for the near-optimum linear controller must never find itself outside of the user-defined algebraic ring H, therefore, the set of all stabilizing controllers belonging to the algebraic ring H must be defined and available to the search routine. Once the search is completed, the user must validate design via a digital simulation (Step 6). Hence, the command driven DRLCD software utility was designed to automate Steps 4-6 of the developed systematic controller synthesis procedure.

Example

Problem Statement

Design a controller for the nonlinear plant shown in Figure 8 to form a closed-loop feedback system whose structure is defined by Figure 10, the closed-loop feedback system must (i) be as insensitive to the level of the command signal and (ii) behave similarly to the reference linear model, $G_r(s)$, given by Equation (4 5).

Method of Solution

The requirements (i) and (ii) above are both outcomes of the controller synthesis procedure whose technical details are given in Chapter III. This procedure identifies a near-optimum linear controller through the use of the simultaneous stabilization theory and based on two sinusoidal-input describing function models of the plant It is assumed that the user has successfully accomplished the tasks defined in Steps 1-3 or the controller synthesis procedure, and the following primary information is available.

(i) Subroutine MOTLD given by Figure 18

(ii) Two linear approximating models of the nonlinear plant given by Equations (4 6) and (4 7).

(ii) The user-defined reference linear model $G_r(s)$ given by Equation (4.5).

The following is a step-by-step guide in using the DRLCD software utility to design the near-optimum linear controller

1 Create a text file, and type the subroutine that describes the nonlinear plant (motor and load) The subroutine name must specifically be **MOTLD**, but the text file name is arbitrary. In this example, the text file name is **MOTLD·SF**, and it is shown in Figure 18 Consult with the site manager, and obtain the necessary information which is required to compile the FORTRAN text file **MOTLD:SF**, to add the resulting object module to the **0001INST*INSTAL** library, and finally link the **0001DRLC*DRLCD:0** object module to the **001INST*INSTAL**, **0000*RGL**, and **0000*LIBRARY** libraries to create the executable module **DRLCD**

2. Enter the following command from the Harris-800 Job Control

DRLCD

This command executes the command-driven computer-aided Dual-Range Linear Controller Design software utility A list of valid commands along with a short description of their functions is displayed (see Figure (60)). The user will exercise the primary commands by the end of this tutorial. The commands and their proper syntaxes and functions are given below. The syntax of all commands is of the following generic form.

command option <required parameter> <optional parameter>

where,

command is a one letter ascii symbol whose function is to execute a generic task,

option is a one or two letter ascii symbol which executes a specific task,

required parameter is the required information before the task defined by the command can be executed, and

optional parameter specifies a task to be completed in a user-defined (non standard) format

Due to the "complexity" of the command syntaxes, a help command is provided to remind the user of the proper syntax of each command. Enter the **H** command, Figure (61) is displayed At the prompt, hit the return key to return to the main command list Enter the commands **H.R, H.S, H.I, H.C, H.D, H.T, H.O, H.V,** and **H.Q** to become familiar with the syntax of each command (see Figures (62)-(70))

The user must provide the DRLCD input data using the **R** command if there exists a data file with the required DRLCD format. If such a data

```
R ==> TO READ DATA FROM DISC
S ==> TO SAVE DATA ON DISC
I ==> TO GET STARTING SOLUTION
C ==> TO CHANGE/MODIFY DATA
D ==> TO DISPLAY STATUS
T ==> TO MANIPULATE TRANSFER FUNCTION
O ==> TO SYNTHESIZE
V ==> TO VALIDATE DESIGN
Q ==> TO QUIT SESSION
H ==> TO GET HELP

ENTER COMMAND
```

Figure 60 **DRLCD** Main Command List

```
                             H

PURPOSED   TO HELP THE USER WITH DRLCD COMMANDS

SYNTAX   H<.RIDPOS>

REQUIRED PARAMETERS   NONE

OPTIONAL PARAMETERS   NONE

OPTIONS         R DISPLAY R HELP PANNEL
                S DISPLAY S HELP PANNEL
               .I DISPLAY I HELP PANNEL
                C DISPLAY C HELP PANNEL
                D DISPLAY D HELP PANNEL
                T DISPLAY T HELP PANNEL
                O DISPLAY O HELP PANNEL
                V DISPLAY V HELP PANNEL
                Q DISPLAY Q HELP PANNEL

COMMAND H ACCEPTS ONE OPTION, ONLY
```

Figure 61 CRT Screen After the Execution of the **H** Command

R

PURPOSE TO READ DATA FROM DISC

SYNTAX R<.IS> NAME

REQUIRED PARAMETERS NAME

OPTIONAL PARAMETERS NONE

OPTIONS I TO READ DRLCD INPUT DATA
S TO READ STARTING SOLUTION

Figure 62 CRT Screen After the Execution of the **H.R** Command

S

PURPOSE TO SAVE DRLCD INPUT DATA ON DISC

SYNTAX S< DS> NAME

REQUIRED PARAMETERS NAME

OPTIONAL PARAMETERS NONE

OPTIONS D TO SAVE DRLCD INPUT DATA
.S TO SAVE STARTING SOLUTION

Figure 63. CRT Screen After the Execution of the **H.S** Command

I

PURPOSE TO OBTAIN STARTING SOLUTION

SYNTAX I

REQUIRED PARAMETERS NONE

OPTIONAL PARAMETERS NONE

OPTIONS NONE

COMMAND I USES ALGORITHM 1 1 TO GET STARTING

Figure 64 CRT Screen After the Execution of the **H.I** Command

C

PURPOSE TO CHANGE/MODIFY INPUT DATA

SYNTAX C< I> NAME

REQUIRED PARAMETERS NONE

OPTIONAL PARAMETERS NONE

OPTIONS I TO CHANGE DRLCD INPUT DATA

Figure 65. CRT Screen After the Execution of the **H.C** Command

D

PURPOSE TO DISPLAY/VIEW DRLCD STATUS

SYNTAX D<.DS>

REQUIRED PARAMETERS NONE

OPTIONAL PARAMETERS NONE

OPTIONS .D TO DRLCD INPUT DATA
.S TO DISPLAY DRLCD STARTING SOLUTION

Figure 66. CRT Screen After the Execution of the **H.D** Command

T

PURPOSE TO MANIPULATE A SPECIFIC TRANSFER FUNCTION

SYNTAX T< BCRDSF> TRANSFER-FUNCTION-NAME

REQUIRED PARAMETERS TRANSFER-FUNCTION-NAME
VALID REQUIRED PARAMETERS ARE
GO,G1,GCLD,CS,RS,C,AND R

OPTIONAL PARAMETERS OV (OVERLAYS PLOTS)

OPTIONS .B TO BUILD A TRANSFER FUNCTION
.C TO CHANGE/MODIFY TRANSFER FUNCTION COEFF
R TO REMOVE POLES AND ZEROS
.D TO DISPLAY TRANSFER FUNCTION STATUS
.S TO VIEW STEP RESPONSE PLOT
F TO VIEW FREQUENCY RESPONSE PLOTS

Figure 67. CRT Screen After the Execution of the **H.T** Command

O

PURPOSE   TO SYNTHESIZE THE NEAR-OPTIMUM CONTROLLER

SYNTAX   O<.S>

REQUIRED PARAMETERS   NONE

OPTIONAL PARAMETERS   NONE

OPTIONS   S

Figure 68   CRT Screen After the Execution of the **H.O** Command

V

PURPOSE   TO VALIDATE DESIGN

SYNTAX   V

REQUIRED PARAMETERS   ONE OF THE FOLLOWING
(C,G0) (CS,G0) (C,G1) (CS,G1)
(C,NL) (CS,NL)

OPTIONAL PARAMETERS   QU
DIVIDES THE GRAPHICS SCREEN INTO
FOUR QUADRANTS
OV OVERLAYS PLOTS

OPTIONS   NONE

Figure 69. CRT Screen After the Execution of the **H.V** Command

Q

PURPOSE TO QUIT THE DRLCE SESSION

SYNTAX Q

REQUIRED PARAMETERS NONE

OPTIONAL PARAMETERS NONE

OPTIONS NONE

Figure 70 CRT Screen After the Execution of the **H.Q** Command file does not exist, the following procedure should be used to interactiviely create the DRLCD input data file The **T** command may be used to input the two linear approximating models of the nonlinear plant which are designated by symbols $G_0(s)$ and $G_1(s)$ as well as the user-defined reference linear model $G_r(s)$. With reference to Figure 67, enter the following command to input the first approximating linear model.

T.C GO

Figure 71 is displayed Enter the following subcommands (see Equation 4 6)

NUM,1.2312,0.00398925

DEN,0.00547508,0.00376117

RET

The DRLCD main command list should be displayed Similarly, enter the following command to input the second approximating linear model

T.C G1

Figure 72 is displayed Enter the following subcommands (see Equation 4.7)

NUM,1.00,0.0403136

DEN,0.0,0.22521,0.0236410

RET

Similarly, input the reference linear model, which is given by Equation (4 5), using the **T.C GCLD** command

There are other flags and parameters that have to DRLCD With reference to Figure 65, enter the following command.

C.I

```
THE NOMINAL TRANSFER FUNCTION

               GO = 0.000
                    -----
                    0.000

NUM ==> MODIFIES THE NUMERATOR COEFFICIENTS
          NUM, 0.,D,1.,  WILL SET THE CONSTANT TERM TO 0 0,
                WILL NOT MODIFY THE S1 COEFFICIENT,
                WILL SET THE S2 COEFFICIENT TO 1.0,

DEN ==> MODIFIES THE DENOMINATOR COEFFICIENTS
        DEN,0.,D,1., WILL MODIFY DENOMINATOR COEFFICIENTS LIKE ABOVE

RET ==> RETURN TO THE CALLING ROUTINE

ENTER SUBCOMMAND
```

Figure 71. CRT Screen After the Execution of the **T.C G0** Command

```
THE NOMINAL TRANSFER FUNCTION

        G1 = 0.000
             -----
             0.000

NUM ==> MODIFIES THE NUMERATOR COEFFICIENTS
        NUM, 0 ,D,1 , WILL SET THE CONSTANT TERM TO 0.0,
               WILL NOT MODIFY THE S1 COEFFICIENT,
               WILL SET THE S2 COEFFICIENT TO 1 0,

DEN ==> MODIFIES THE DENOMINATOR COEFFICIENTS
        DEN,0.,D,1., WILL MODIFY DENOMINATOR COEFFICIENTS LIKE ABOVE

RET ==> RETURN TO THE CALLING ROUTINE

ENTER SUBCOMMAND
```

Figure 72. CRT Screen After the Execution of the **T.C G1** Command

Figure 73 is displayed. To change the integration step size from zero seconds to 0.05 seonds, enter the following subcommand.

TSTEP(0.05)

Similarly, final simulation time, magnitude of test step inputs, or other variables shown in Figure 73 may be changed accordingly. For example, the following command sets the final simulation time to 8.0 seconds.

TFINAL(8.0)

Several commands may be combined. For example, the following command may be used to modify objective function weighting coefficient, minimum frequency of interest, maximum frequency of interest, number of frequency response data points to be generated for optimization purposes, and maximum number of function evaluations to be made by the optimization routine.

ALPHA(0.5),WMIN(0.05),WMAX(600),NOMEG(120),MAXEV(300)

The value of **NOMEG** should not exceed 120. Enter the following command to set the magnitude of the test step inputs.

USTP(0.25,0.32,0.65,0.80,5.10,10.20)

No more than six values for USTP is allowed. Enter the **R** subcommand to return to the DRLCD main command list.

Controller Synthesis

At this stage, it is assumed that all DRLCD input information is furnished. The user may start the task of controller synthesis. The controller synthesis procedure is to search the class of all controllers that stabilize linear plants $G_0(s)$ and $G_1(s)$ for the near-optimum linear controller - the controller that forces the dynamic behavior of $\Sigma(C,G_0)$

```
INTEGRATION STEP SIZE,          TSTEP = 0.000
INITIAL SIMULATION TIME,        TZERO = 0.000
FINAL SIMULATION TIME,         TFINAL = 0.000
MAGNITUDE OF THE TEST STEP INPUTS,  USTP =

MINIMUM FREQUENCY OF INTEREST     WMIN = 0 000
MAXIMUM FREQUENCY OF INTEREST     WMAX = 0.000
NUMBER OF FREQUENCY RESPONSE DATA POINTS
  TO BE GENERATED FOR OPTIMIZATION  NOMEG = 0
OBJECTIVE FUNCTION WEIGHING COEFF. #1          ALPHA = 0 00
MAXIMUM NUMBER OF FUNCTION EVALUATIONS         MAXEV = 300
```

Figure 73. CRT Screen After the Execution of the **C.I** Command

and $\Sigma(C,G_1)$ to be as close to $G_r(s)$ as possible. The class of all stabilizing controllers is gievn by Equation (4.8) subject to constraints (4.9) and (4.10). With reference to Equation (4.8) and constraints (4.9) and (4.10), the objective is to select that r'(s) from the algebraic ring H which would minimize the objective function given by Equation (3.22). The search for the optimum r'(s) is done via a simplex algorithm (58). Enter the **I** command to obtain the starting solution. Command I uses Algorithm 1.1, which was given in Chapter III, to compute the starting solution. Enter the **T.D CS** command to view the status of the starting controller. Figure 74 is displayed. The starting controller has a high-order dynamic term which may be removed. The starting controller would be given by the following equation.

$$C_s(s) = \frac{59.970 + 4.119\ s}{27.750 + 1.000\ s} \qquad \text{(f.1)}$$

Use the **T.C CS** command to modify the starting solution controller as above. Enter the **T.D RS** command to view the status of the starting solution for parameter r'(s), Figure 75 is displayed. Enter the **V (CS,NL)** command to view the normalized step responses of the closed-loop system (starting controller and the nonlinear plant) Figure 76 is displayed. From the examination of this figure it is concluded that the closed-loop system satisfies the specified performance measures, i.e., percent overshoot is less than 37 and the 2% settling time is 0.3 seconds. In this case, no optimization is required. In order to exercise the optimization algorithm, a different starting solution is selected. Let the corresponding starting solution for $C_s(s)$ be of the following form

THE STRATING CONTROLLER

$$CS = \frac{0.000 + 74.954\ s + 5.149\ s\ ^\wedge 2}{0.000 + 34.166\ s + 1.342\ s\ ^\wedge 2 + 0.004\ s\ ^\wedge 3}$$

THE ZEROS ARE

Z1 (0.000, 0.000J) Z2 (-14.557, 0.000J)

THE POLES ARE

P1 (0.000, 0.000J) P2 (-27.750, 0.000J)

P3 (-308.629, 0.000J)

Figure 74. The CRT Screen After the Execution of the **T.D CS** Command

STARTING SOLUTION

$$RS = \frac{0.000 + 13.506\ s + 2.345\ s\ \hat{}\ 2 + 0\ 97\ s\ \hat{}\ 3}{59.970 + 12.786\ s + 1.047\ s\ \hat{}\ 2 + 0.024\ s\ \hat{}\ 3}$$

THE ZEROS ARE

Z1 (0.000, 0.000j) Z2 (-9.526, 0.000j)

Z3 (-14.559, 0.000j)

THE POLES ARE

P1 (-7.949, 5.112j) P2 (-7.949, -5.112j)

P3 (-28.402, -0.299E-12j)

Figure 75 The CRT Screen After the Execution of the **T.D RS** Command

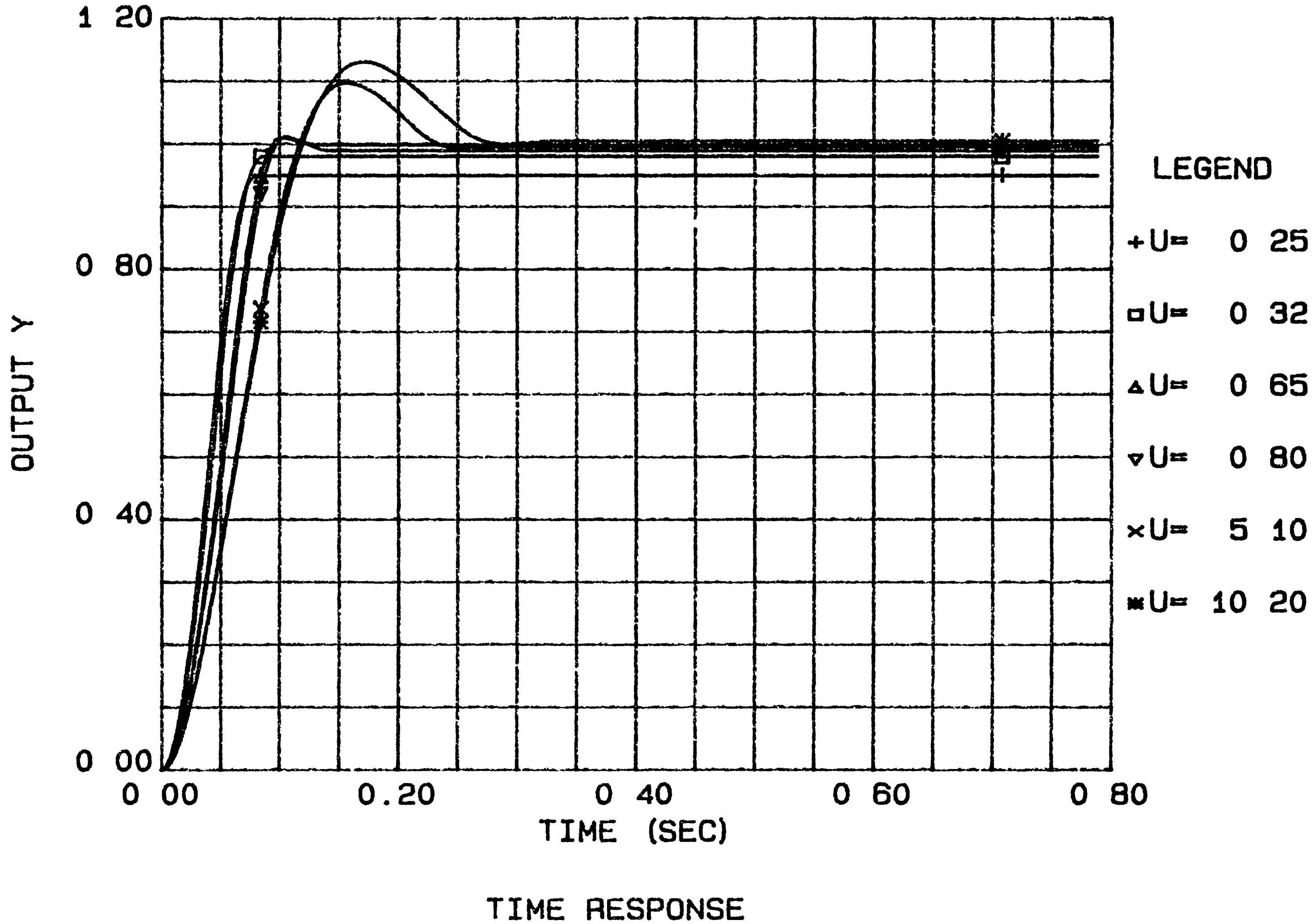


Figure 76 Normalized Step Response of the Starting Linear Controller and the Nonlinear Plant

$$C_s(s) = \frac{4\ 119s + 59.970}{s + 1.0} \quad \text{(f 2)}$$

Examine the relative closeness of $\Sigma(C_s,G_0)$ and $\Sigma(C_s,G_1)$ with respect to the reference linear model by entering the following commands.

T.FS GCLD QU

V.FS (CS,GO) QU OV

V.FS (CS,G1) QU OV

Figure 77 is displayed. Enter the **O.S** command to start the optimization algorithm. Once the main DRLCD command list is displayed, the task of controller synthesis is completed Enter the **T.D C** command to view the status of the synthesized controller, Figure 78 is displayed Enter the **T.D R** command to view the status of the corresponding r'(s), Figure 79 is displayed. Enter the following commands

T.FS GCLD QU

V.FS (C,GO) QU OV

V.FS (C,G1) QU OV

Figure 80 is displayed. From the comparison of Figures 77 and 80, it may be concluded that the controller synthesis algorithm had identified a controller that forces $\Sigma(C,G_0)$ and $\Sigma(C,G_1)$ to be as close to GCLD ($G_r(s)$) as possible. In order to compare the performance of the starting controller in controlling the nonlinear plant output with that of the optimized controller enter the following command

V (CS,NL) (C,NL) QU

Figure 81 will be displayed The plot on the left-hand side of the figure is the simulation of the starting controller with that of the nonlinear plant, and the plot on the right-hand side of the figure is the simulation of the synthesized controller with that of the nonlinear plant. From the examination of this figure, it may be concluded that

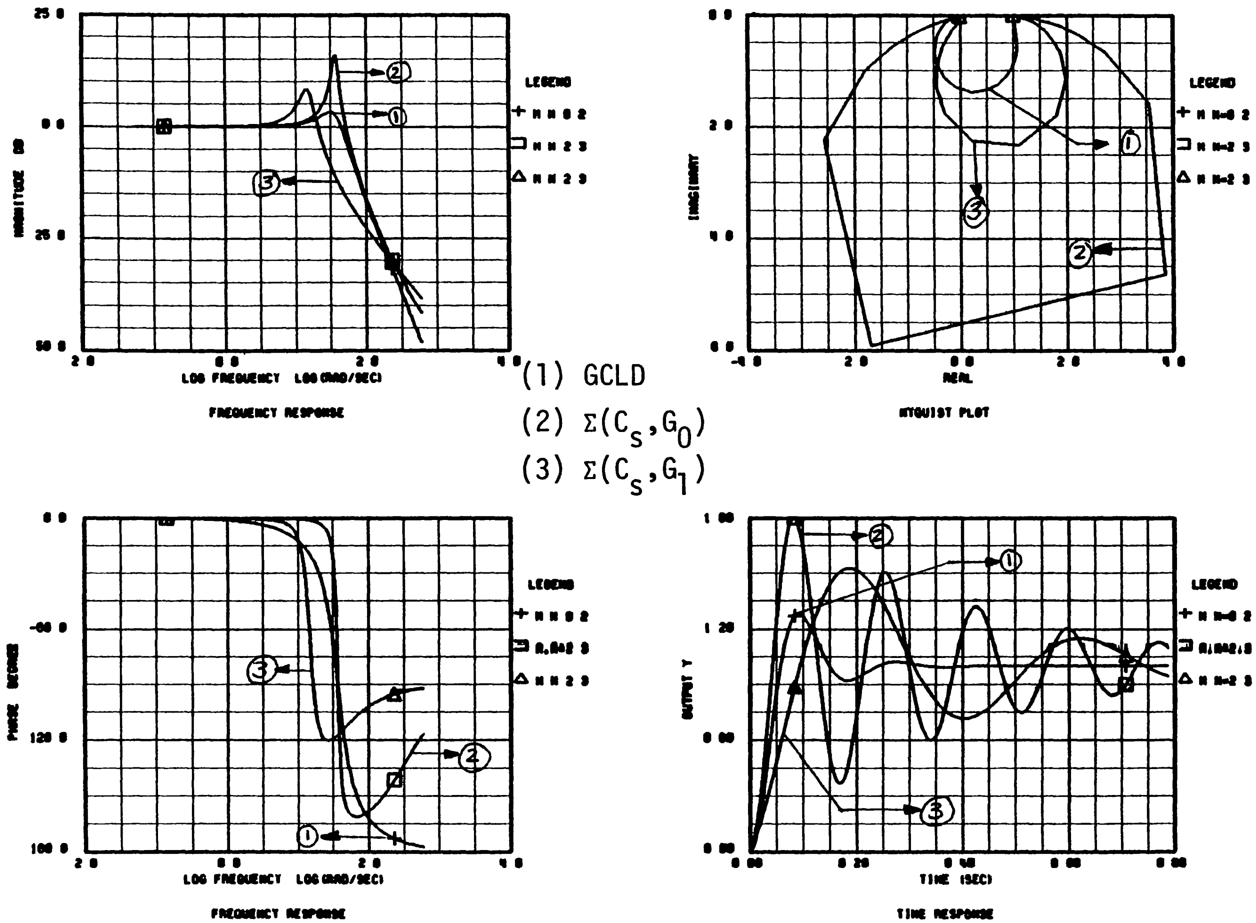


Figure 77 Frequency Response Plots and Unit Response Plots of GCLD, $\Sigma(C_s,G_0)$, and $\Sigma(C_s,G_1)$

THE CONTROLLER

$$C = \frac{0.789 + 17.626\ S}{8.495 + 4.133\ S}$$

THE ZEROS ARE

Z1 (-0.045, 0.000J)

THE POLES ARE

P1 (-2.056, 0.000J)

Figure 78. The CRT Screen After the Execution of the **T.D C** Command (Status of the Synthesised Controller)

PARAMETER R(S)

$$R = \frac{0.000 + 0.178\ S + 3.988\ S\ \hat{}\ 2 + 0.417\ S\ \hat{}\ 3}{0.789 + 19.571\ S + 1.842\ S\ \hat{}\ 2 + 0.098\ S\ \hat{}\ 3}$$

THE ZEROS ARE

Z1 (0.000, 0.000j) Z2 (-0.045, 0.000j)

Z3 (-9.526, 0.000j)

THE POLES ARE

P1 (-0.040, 0.000j) P2 (-9.407, -10.539j)

P3 (-9.407, -10.539j)

Figure 79. The CRT Screen After the Execution of the **T.D R** Command (Status of the Synthesised Parameter r'(s))

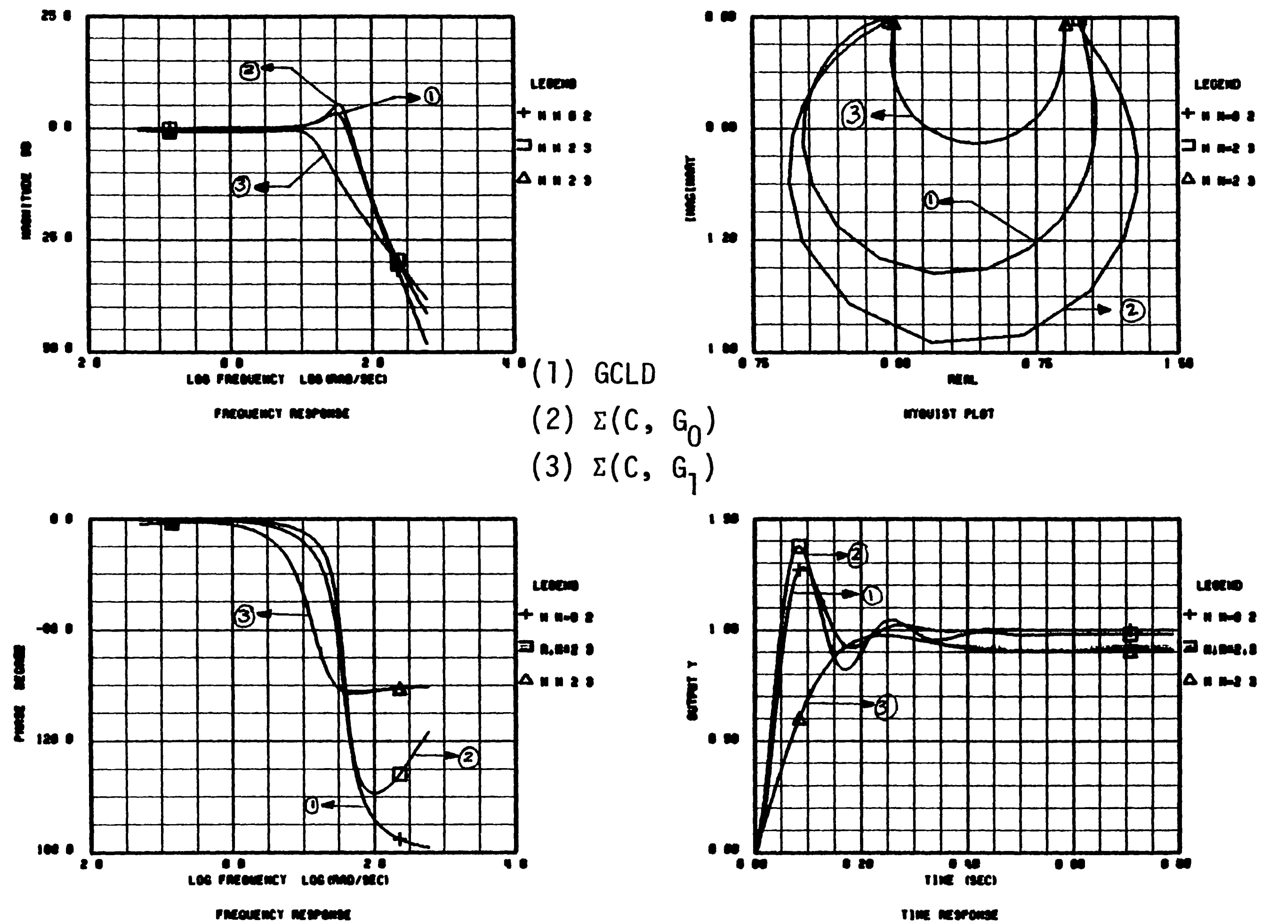


Figure 80 Frequency Response Plots and Unit Step Response Plots of GCLD, $\Sigma(C, G_0)$, and $\Sigma(C, G_1)$

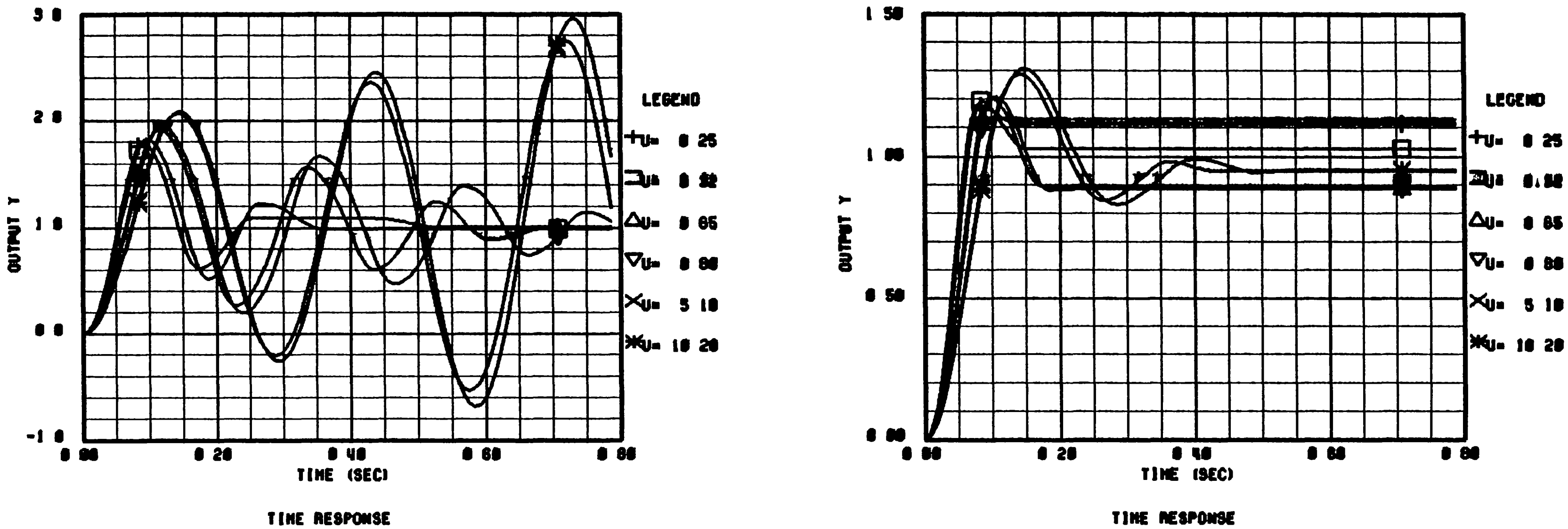


Figure 81 Normalized Step Response Plots of the Linear Controller and the Nonlinear Plant

the controller synthesis algorithm has identified a controller which controls the nonlinear plant output better than that of the starting controller.