COMPOSITE FORECASTING OF THE ANNUAL AVERAGE PRICE OF BEEF CATTLE RECEIVED BY FARMERS IN THE UNITED STATES

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PREFACE

Econometric and Autoregressive Integrated Moving Average forecasts of the annual average price of beef cattle received by farmers in the United States were made over the period from 1966 through 1985. These forecasts from the two models were combined into composite forecasts to improve forecasting accuracy. The period for the analysis of the accuracy of the individual and combined forecasts was 1976 through 1985.

The composite forecasts improved forecasting accuracy in most cases, and the method of combining forecasts provides a useful tool for devloping an accurate set of annual cattle price forecasts.

I am very thankful to my adviser, Dr. John R. Franzmann, for his help, advisement, and encouragement throughout the Doctoral program. I am also thankful for the assistance from my other committee members; with special thanks to Dr. Clem Ward for his guidance during my teaching practicum, Dr. Kim Anderson for his encouragement and motivation, and Dr. Linda Willson for her time spent conducting an independent study with me.

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INTRODUCTION

Problem Statement

During the early 1980's, cattle producers faced conditions of falling prices coupled with difficulty in financing and thus, many producers were unable to survive the price fluctuations associated with the cattle cycle. The U.S.D.A. estimated that between 1984 and 1985, the number of operations with beef cattle dropped by 47,100, and inventories of cattle fell by about 4 million head. One reason for the financial stress within the cattle industry is that many cattle producers make production decisions based on recent price changes, yet production lags create a situation where price may be moving in the opposite direction by the time production has been adapted to the price changes. This is evidenced by the fact that inventories usually continue to rise several years after prices have peaked.

According to Keith (1976, p. 11), a survey of Oklahoma cow-calf operators "seems to portray the cow-calf man as an unrelenting optimist." In a 1974 survey of Oklahoma cow-calf operators, Keith found that 59 percent of the cow-calf operators believed prices would remain stable over the next year, (1974-1975), 24 percent believed prices would recover within the next year, while only 3 percent believed that prices would spiral downward over the next 3 to 5 years. As can be seen in Figure 1, real prices received by beef cattle farmers did not begin to recover until 1978.

Some producers hold slaughter cattle during periods of increasing prices, hoping to receive higher returns, but do not sell until prices already have begun a sharp decline. In doing so, these producers forego marginal returns and increase their marginal costs, selling at a lower price and putting marginally more expensive weight on the cattle. Collectively, these actions of cattle producers result in a glut of cattle for slaughter at heavier slaughter weights after prices have begun to fall.

Resources continue to be applied to a product with diminishing marginal returns. As a result, allocative inefficiency occurs from a lack of understanding of the operative forces during the downswing of the cattle cycle. An improper allocation of resources also occurs during an upswing of prices. After a period of low prices, herd liquidation leads to a low supply of feeder cattle. Therefore, in the early stages of the upswing of the cattle price cycle, as prices are rising, feedlots are operating at levels well below capacity, and only modest returns are received. From the foregoing logic it can be seen that

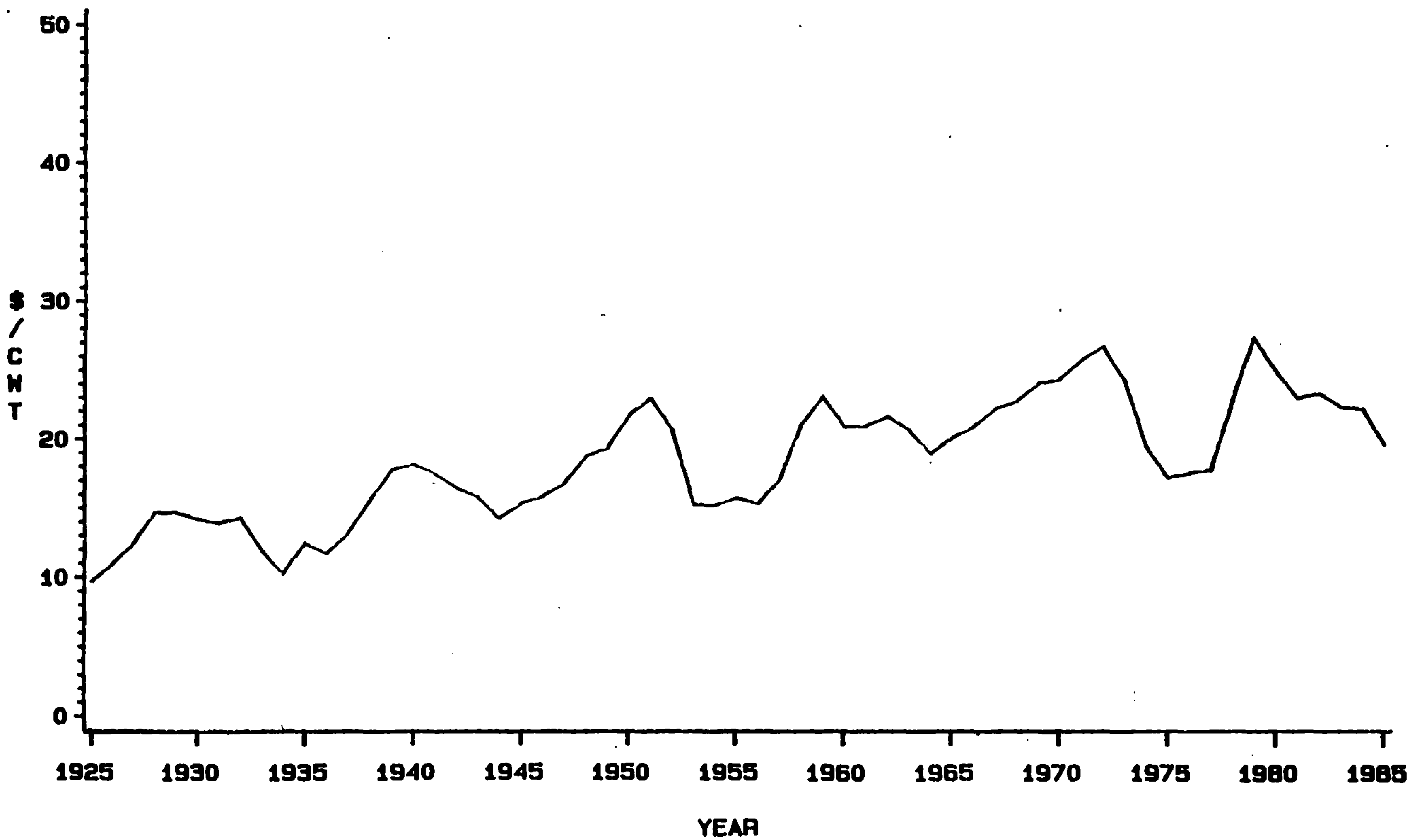
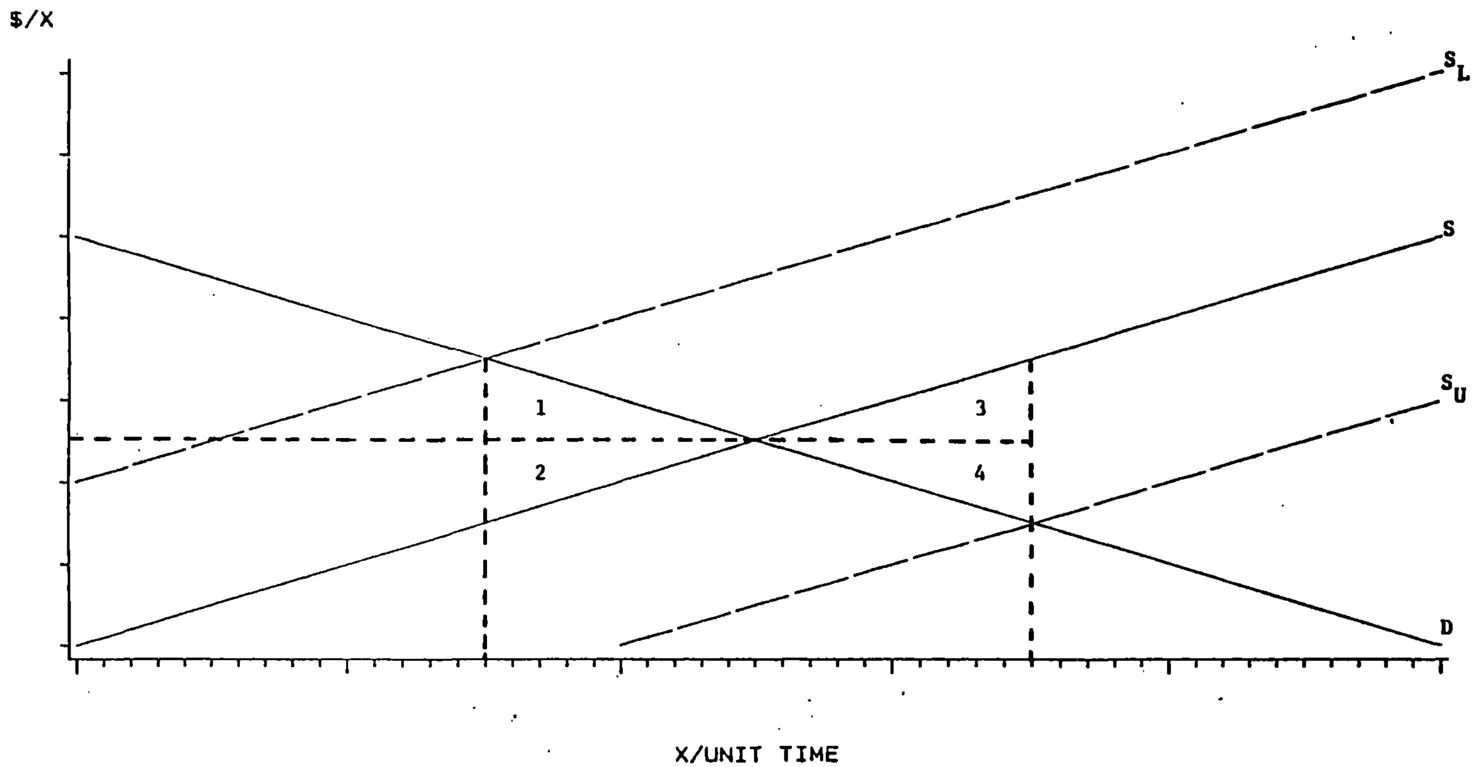


Figure 1. Average Deflated Price of Beef Cattle Received by Farmers in the United States

allocative inefficiencies occur during both the upswing and downswing phases of the cattle cycle. Figure 2 shows hypothetical social costs of the cyclical nature of supply. The curves S_U and S_L represent an outward shift of the supply curve and an inward shift of the supply curve respectively. Areas 1 and 2 represent costs to society when output is low, which could be illustrated by the case when feedlots must operate at below capacity due to a lack of feeders. In this case the supply of feeders is shifted inward and the consumer, the feedlot operator, loses revenues equivalent to area 1, while the producer, for example, the cow-calf operator, loses revenues equivalent to area 2. Areas 3 and 4 represent social costs when supply is increased, area 3 being the loss to consumers of the product and area 4 being the loss to producers of the product. This case is analagous to when operators must begin slaughtering breeding stock and calves due to falling prices. The outward shift in the supply curve would have occurred from some change in supply determinants, such as decreased feed prices.

According to Brandt and Bessler, there are inherent uncertainties within the livestock industry coupled with an inelastic demand curve for farm livestock. Given these conditions they state (1981, p. 3):

> Sensible decision making thus requires knowledge (expectations or beliefs) about the likelihood of many alternative outcomes.



Source: Tweeten, Luther G. Foundations of Farm Policy.

Figure 2. Hypothetical Social Costs of Uncertainty of Supply

Certainly outcomes, such as producer profits, are dependent on how well producer's expectations of product prices coincide with actual prices. The problem is then how to develop accurate expectations of future cattle prices, given their cyclical behavior. One possible means is by developing a set of accurate price forecasts based on past and currently available information.

Behavior of the Cattle Cycle

Unfortunately for forecasting purposes, the cattle price cycle has not shown a fixed periodicity, nor a time symmetry between the upswing phase and the downswing phase. Cycles in numbers have occurred from 1912-1928, 1928-1938, 1938-1949, 1949-1958, 1958-1967, and 1967-1979. Peaks in numbers occurred in 1890, 1904, 1918, 1934, 1945, 1955, 1965, and 1975. In general, peaks in cattle cycle numbers occur approximately 1 to 2 years after prices have peaked. Accumulation phases have varied from 6 to 8 years, and liquidation phases have been even more variable in length, ranging from 3 to 10 years. While liquidation phases within the cycle have tended to become shorter, the liquidation phase of the most recent cycle was two years longer than the previous cycle. Beale, Hasbargen, Ikerd, Murfield, and Petritz (1983, p. 1) noted the variablility in the period of the past 5 cycles, two of which were 9 years long, one which was 10 years in length, an 11 year cycle, and the last cycle, completed in the beginning of 1979, which was 12 years long. As can be seen in Figure 1, peaks in deflated prices received by farmers for beef cattle and troughs in total slaughter took place in 1928, 1940, 1949, 1959, 1971, and 1979, while troughs in deflated prices and peaks in slaughter occurred in 1934, 1944, 1953, 1964, and 1975. Deflated prices have ranged from about four cents per pound from peak to trough in 1940 to 1944, to over ten cents per pound between the price peak and trough in 1975 to 1979. Such variability in cycle length and relative length of phases within each cycle, underscores the difficulty of forecasting cyclical price patterns.

Objectives

With more accurate information concerning price expectations and improved knowledge of where cyclical turning points in prices will occur, producers would be able to make better management decisons regarding the size of their breeding herds including decisions relating to expansion and reduction of herd size. Of particular importance to long-term producer decisions is the ability to correctly identify significant turning points within the cattle price cycle. The objective of this study is to develop a model, or combination of models, capable of accurately forecasting live cattle cash prices throughout the cattle price cycle. Price forecasts from various types of forecasting techniques, and combinations of these techniques will be analyzed in an attempt to find the most accurate set of forecasts of beef cattle prices. Consequently, an additional objective of this study will be to test the relative forecasting accuracy of various types of forecasts and combinations of forecasts, including forecasts from an econometric and an ARIMA model.

Past Studies of the Cattle Cycle

Generally, studies of the cattle cycle have been of three types: qualitative or descriptive studies, econometric studies, and non-structural mechanical studies or those which use only past and present values of the specified variable to formulate forecasts.[1] Qualitative studies of the cattle cycle include Hopkins (1926), Lorie (1947), Burmeister (1949), and DeGraff (1960). While Hopkins and Burmeister hypothesized that the cattle cycle is caused by factors external to the cattle industry, Lorie sought to discount the exogenous theory of causation, emphasizing the importance of the biological process of cattle breeding and raising in determining the length and amplitude of the cattle cycle. Ehrich (1966), as have more recent studies of the cattle cycle, incorporated both the theory of exogenous causation and endogenous causation of the cattle cycle into an econometric study of the cattle cycle. Ehrich assumed supplies were fixed due to the production lag involved in breeding and raising cattle, and therefore hypothesized the price of beef steers to be determined by a demand relation. Non-structural mechanical studies include Franzmann and Walker (1972) who employed harmonic regressions to predict monthly feeder, slaughter, and wholesale beef cattle prices throughout the price cycle, which was assumed to be ten years in length. Cattle cycle studies often employ all three of these types of techniques in their analyses, but the majority of cattle cycle studies since the early 1950's have employed econometrics as the primary means of analysis.[2]

According to Brandt and Bessler (1979, p. 6), econometric models provide a tool for analyzing prices by using information about "relevant supply and demand factors which together determine market price and quantity." Structural models estimate the relationship of present prices to present and past values of exogenous variables. Given that this relationship can be expected to hold in the future, then structural models furnish a means for forecasting future values of the dependent variable. If a complete set of information about these structural relationships is unavailable, then non-structural models exist as an alternative, relating present values of prices to past values of prices. Given the complexity of the cattle industry, the forecaster may not be able to identify all of the relevant structural relationships in order to forecast prices. Yet, structural models may provide a useful set of forecasts. In fact, if both types of models supply information which is independent of the other type of model, then the forecaster can gain information by employing both types of models. Granger and Newbold (1972) proposed that the greatest benefits would arise from combining very different types of techniques, particularly econometric and statistical techniques.

Methodology

One possible means of achieving more accurate price forecasts is the employment of composite forecasting techniques. Composite forecasting techniques use a combination of forecasts developed from alternative forecasting methods. This study will employ a structural model, a time series model or non-structural model, and a composite of these two types of models to make one step ahead annual forecasts of the price of beef cattle received by farmers. The structural model will be of single equation form, relating price to quantities produced and specified demand determinants. The time series model employed will be an autoregressive integrated moving average (ARIMA). The forecasting adequacy of the individual and composite forecasting techniques will be evaluated by several criteria, including the mean squared error and turning point errors produced by each technique.

FOOTNOTES

[1]Bessler and Brandt (1979) consider four classifications of commodity forecasting: structural mechanical, non-structural mechanical, structural non-mechanical, and non-structural non-mechanical. Structural models are those which incorporate supply and demand factors, while non-structural models reflect only past values of the variable to be forecasted. Mechanical models are built, then require no additional human intervention or judgement.

[2]Norblom (1982) gives a cumulative chronology of cattle cycle literature.

CHAPTER TWO

EXPLANATIONS OF THE CATTLE PRICE CYCLE

Definition of the Cattle Cycle

The cattle cycle has several alternative definitions. Various studies define the cattle cycle as the cycle in inventory numbers.[1] Gruber (1965, p. 1) defines the "cattle cycle" as consisting of three separate cycle groups: (1) the cattle inventory cycle, (2) the cattle price and income cycle, and (3) the cattle slaughter and import cycle. Although there are several alternative definitions of the cattle cycle, Breimyer (1962, p. 2) states, "In so far as cyclical trends in inventories, slaughter, and prices are causally linked, it makes no difference by what term the cattle cycle is described." While the primary focus of this study will be to forecast cattle prices and cyclical price behavior, the fact that the three cycles are causally linked necessitates examination of the cyclical behavior of slaughter and inventories in order to formulate a structural model to forecast prices.

Theories of Causation of the Cattle Cycle

Efforts have been made to explain the cattle cycle in causal terms. These efforts have taken two divergent paths, exogenous causality and endogenous causality. The theory of exogenous causality proposes that the value of cattle is affected primarily by influences exogenous to the industry, such as changes in demand, and not changes in cattle numbers. Hopkins felt the irregularity of the length and amplitude in cattle price cycles indicated exogenous causality of the cycle, contrary to the cobweb theory. Hopkins (1926, p. 351) states:

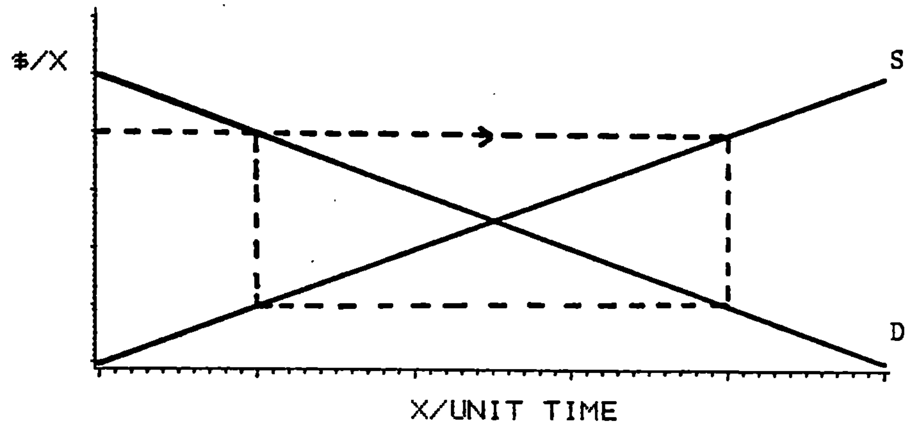
> Granting that adjusting cattle production requires a long period and does not establish the theory that cattle price and production cycles are to be explained by an inherent and self- perpetuating tendency of producers to over and under-produce.
>
> The cattle cycles of the past 60 years are apparently due to forces from outside of the cattle industry, but these forces or conditions which have caused the major crises in the cattle industry do not seem to be related to any regularly recurrent phenomena.

Hopkins lists several exogenous factors which are purported to generate price cycles, including the general level of business activity, wars, expansion of grazing territories, and profitability of alternative enterprises. Similarly, Burmeister (1949, p. 9) sought to explain "the distinctive feature of each cycle in numbers with reference to unusual conditions that have affected the cattle industry at various times..." While Burmeister described the effect of individual events in the economy and development of the industry upon cycles in cattle numbers, he did acknowledge that periodic changes in cattle numbers were partially affected by the biological characteristics of cattle raising. Lorie (1947, p. 50) questions the great importance placed by Hopkins on the effect of business activity upon cattle cycles, noting the marked lack of synchronization between business activity and cattle prices. With regard to Hopkin's linkage of specific economic and physical events to cattle number cycles, Lorie (1947, p. 51) states, "It seems probable that a more consistent and convincing explanation of fairly regular fourteen-to sixteen-year cycles in cattle numbers can be found."

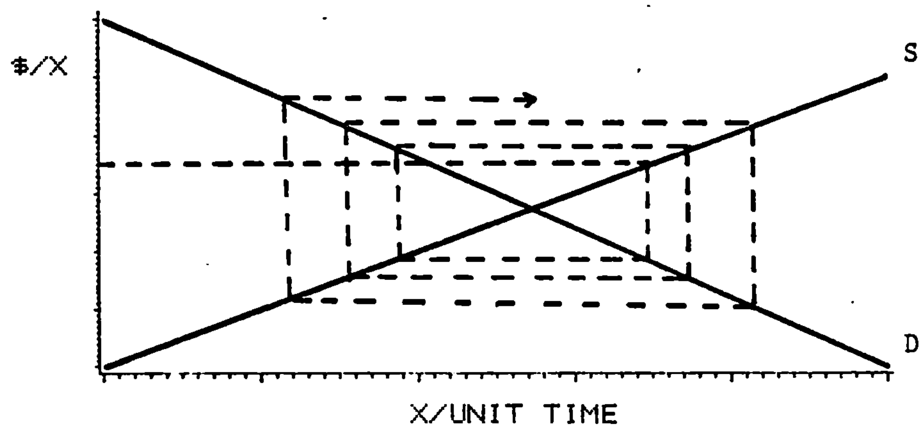
This fair "regularity" in cycles leads to the second theory of causation of the cattle cycle. The endogenous theory of causation of the cattle cycle postulates that the cattle cycle is caused primarily by factors within the cattle industry, and more specifically, by the biological process of raising cattle. DeGraff (1960, p. 42), while acknowledging the influence of factors such as changes in demand or feed supplies upon the initiation of a cycle, proposes that:

> The reason why a cycle follows its standardized pattern is found, not in economics, but in biology. Changes in cattle production, whatever caused their beginning are converted into a cyclical pattern by the natural biology of the cattle species.

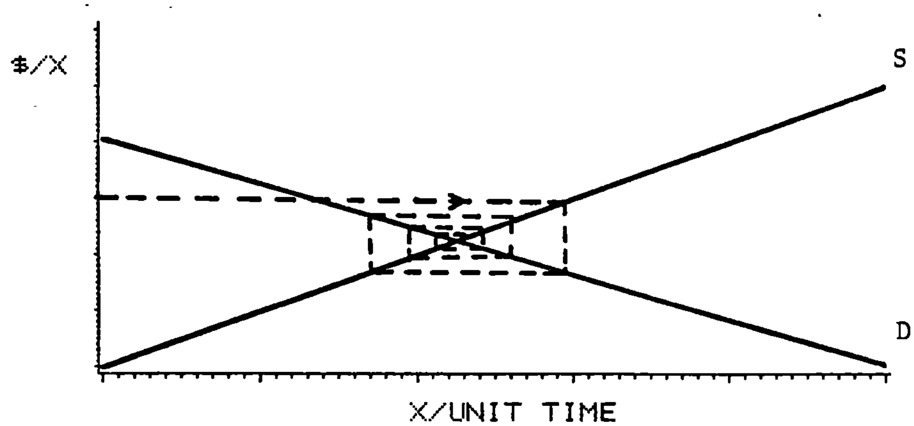
Ezekiel (1938) incorporated the role of prices along with the biological processes causing cycles into the cobweb theory, as may be seen in Figure 3. While producers respond



Case 1



Case 2



Case 3

Figure 3. The Three Cases of the Cobweb Theory

to changes in prices of a commodity, the production of the commodity requires at least one period to be realized once production plans are made. Therefore, responses in quantity to prices are determined by the length of the production process. Case 1 of the cobweb theorem shows the effect of a production lag on prices when supply and demand for the commodity both have the same own price elasticity. In this case there will be no divergence or convergence of prices through time. Case 2 of the cobweb theory shows that when demand is more elastic than supply, prices will tend to converge towards a long-run equilibrium. Consequently, if demand is elastic relative to supply, cyclical behavior will tend to become less evident through time. Yet in Case 3, when demand is inelastic relative to supply, prices will tend to converge away from a long-run equilibrium. Ezekiel noted that the cobweb theorem contained very rigid assumptions, and building hypothetical cycles determined by fixed length of production lags would result in cycles which were much more regular in length than actual cycles. When Ezekiel compared actual deflated prices for cattle with cycles based upon the fixed production periods suggested by the cobweb theorem, the cyclical patterns of the actual data were much more irregular in length and amplitude. Yet, the trend toward a shortening of the cattle cycle over the past 60 years would tend to suggest that in the long-run the cattle price cycle might follow the covergent path of Case 2 of the cobweb theory suggested by Ezekiel.[2]

Later attempts to explain cyclical behavior through the cobweb theorem include the study by Talpaz (1974). Talpaz extended Ezekiel's model to include a demand curve which relates current prices to current market output and a supply curve which relates current output to past prices. Therefore, Talpaz's model extended the static Marshallian supply and demand curves to express the hog cycle as a linear combination of several decomposable hog cycles. According to Talpaz (1974, p. 48):

> This model reflects an integrated multifrequency decision process resulting from the feedback of the production response to the price ratio signal through fixed multiple production lags.

The model incorporated the Cobweb Theorem, the Harmonic Motion, and the Distributed Lags Model.

Lorie (1947) attempted to formulate a theory explaining cyclical fluctuations in cattle which would consider the interrelationships between value, marketings, and number on farms. Lorie also incorporated into the theory the effects of certain exogenous factors upon the model, such as changes in the tastes of consumers or changes in weather. Lorie noted that a change in tastes, specifically an increase in demand, would affect marketing directly, rather than production, but, except for a time lag between the rise in value and the accumulation process, the effect would be the same as with a weather disturbance. According to Lorie (1947, p. 54):

> The subsequent development of the cycle would be the same in the two cases, except the so-called "equilibrium level" itself would be changed if the

> initial disturbance were a change in demand. Consequently, in this latter case the cyclical fluctuations would be around a different equilibrium position.

Lorie states that the varying effects of different exogenous factors result because the response to price changes is less rapid than the response of prices to changes in other factors. Underlying this statement is the concept that although different exogenous factors may have varying stimuli upon the cycle, a given pattern or chain of events will result due to the limitations of the production process. The cycle is therefore affected by both endogenous and exogenous factors.

Phases of the Cattle Cycle

If the exogenous shock is such that it raises the value of beef cattle above equilibrium, cattle slaughter will be reduced, further pushing prices up. This reduction in slaughter occurs for two primary reasons. Cattle feeders, who are buying and selling on a cyclically rising market, are able to outbid packers for veal calves, therefore calf slaughter is reduced. More importantly, beef-cow producers expand herds for breeding purposes in order to increase their potential production of feeder calves in 1 to 3 years. By holding back more cows and heifers, slaughter is reduced even further. Thus during this initial phase, which Beale et al. term the acceleration phase, prices are increasing, slaughter is decreasing, and inventories are rebuilt.[3]

Eventually, following herd build-up, as cows are past breeding, calves reach yearling age, and some firms reach carrying capacity limits, sales are generated from these herds lowering the rate of price increases. Even after prices have peaked, numbers of animals still rise. Beale et al. state that cattle numbers reach their peak about two years after prices peak. Reductions in herds mark the deceleration stage. With falling prices, producers decrease breeding stock by culling cows and placing more heifers on feed. During the deceleration stage feeder supply available for slaughter increases, as well as a greater availability of calves and "nonfed" steers and heifers. The increases in slaughter relative to an inelastic short-run demand drive prices down even further.

Slaughter of breeding stock and, particularly, calves result in smaller potential supplies of slaughter cattle in the future. With a decrease in slaughter, prices begin to rise. Beale et al. refer to this stage in the cattle cycle as the "turnaround stage". They distinguish between it and the "rapid growth" or "acceleration stage" because during the turnaround stage cattle feeders may receive only moderate returns due to high feedlot capacity relative to available feeder supplies.

From the above description of the cattle cycle, it becomes apparent that cattle slaughter plays an important role in determining prices and that in the short-run, prices are based on current quantities. While this is true, producers must base inventory and slaughter decisions largely on past prices or results of past production decisions, such as resulting producer incomes. Although price cycles are partially determined by slaughter cycles, price cycles tend to be more irregular in amplitude and length than slaughter number cycles, indicating the responsiveness of prices to exogenous economic factors

Changes in the Cattle Industry

Several important changes have taken place within the cattle industry since the 1940's which have the potential to affect the regularity of the price cycle. These changes include:

(1) an increase in the efficiency of slaughter and marketing

(2) an increase in cattle feeding

(3) changes in marketing structure, specifically more direct sales to packers

(4) a gradual inward shift in the consumer demand for beef, resulting partially from changes within the poultry industry and the increased demand for poultry.

An increase in the efficiency of slaughter and marketing has resulted in the ability to adapt slaughter and marketing to changes in the prices of cattle more rapidly. The increased ability to adapt slaughter and marketing is a primary cause of the general trend toward shortening of the liquidation phase of the cattle cycle. When prices of cattle begin to fall, and more feeders are available for slaughter, slaughter and marketing facilities are more able to accomodate to the increased numbers. Pyne (1980, p. 16) notes that while the length of the liquidation phase has shortened in general, the accumulation phase has remained fairly stable. The stability of the accumulation phase can be attributed to its higher degree of linkage to the relatively fixed biological process of cattle breeeding and growth.

A trend of rising numbers of cattle being fed relative to nonfed has occurred since the 1950's. The result is a relative increase in fed steer and heifer production. Pyne (1980, p. 28) writes on the effect of this change on the steer slaughter cycle:

> Concurrently, the growth of large-scale feedlots appears to be a major factor affecting steer slaughter...a major part of the inquiry into steer slaughter is that as a result of this structural change it exhibits no noticeable relationship to inventory numbers or to other slaughter rates.

In conjunction with a greater number of fed steers and heifers, the number of calves slaughtered, once an important source of non-fed beef, has shown a decline. This is a natural conclusion since more calves are fed to maturity for slaughter. Also the number of culled dairy cows, another source of non-fed beef, has decreased as dairy cow numbers have trended downward.

The beef marketing structure has changed such that there are more direct sales to packers. Crom et al. state that by 1970, direct sales accounted for 65.3 percent of packer purchases. Yet prior to World War II, most sales were nondirect (through terminal markets and auctions). While there are many beef producers, there are relatively few buyers or packers, thus the buyer has a bargaining advantage. Therefore, demand factors may have become of greater importance in determining the cattle price cycle particularly with respect to the steer cycle, steers being the primary source of beef consumed.

Development of new technologies of production and marketing strategies within the poultry industry have likely had an effect upon the demand for beef. Specifically, innovations, such as those to improve feed conversion rates for broilers, have lowered the costs of poultry production. Additionally, marketing strategies, such as branding of poultry products may have helped to promote competition between poultry products and higher quality cuts of beef. While changes in the poultry industry have likely had an effect upon the beef cattle industry, past studies have found ambiguous results for price flexibilities relating beef prices to quantities of chicken consumed.[4]

FOOTNOTES

[1]These studies include Breimyer (1955), Burmeister (1949), and Pyne (1980).

[2]Ezekiel devised synthetic time series given certain production lags. He examined three cases: the case where supply and demand are of equal elasticity, the case where supply elasticity is greater than demand elasticity, and the case where the elasticity of demand is greater than the elasticity of supply. Notably, when the elasticity of demand is greater than the elasticity of supply then the cycle would undergo convergent fluctuations over time, and when the elasticity of supply is greater than the elasticity of demand then the cycle would become divergent through time.

[3]Beale et al. (1983) break down the traditional accumulation and liquidation phases into the rapid growth, deceleration, and turnaround stages. The rapid growth and turnaround stages are within the traditional inventory accumulation phase, while the deceleration stage includes the end of the accumulation phase and the liquidation phase.

[4]Crom et al. (1973, p. 109) review price flexibilities found in past studies, which showed a negative flexibility between the price of beef and pork quantities, yet showed a positive flexibility between beef prices and chicken quantities.

CHAPTER THREE

COMPOSITE FORECASTING

The Combination of Forecasts

Bates and Granger (1968) suggest that individual forecasting techniques may produce information independent of other individual forecasting techniques. This implies that more information may be gained through a combination of forecasts. Given this implication, Granger and Newbold (1972) proposed that the greatest benefits would arise from combining very different types of forecasting techniques, particularly econometric and statistical techniques. Newbold and Granger (1974) examined forecasts more fully using univariate time series. Their study focused on the combination of such techniques for various reasons. According to Newbold and Granger, these techniques are often quick and inexpensive to operate and often adequate structural data may be unavailable. Additionally, univariate forecasting procedures may be used as a means of comparison for more elaborate techniques.

The methods of combining forecasts may vary according to forecasting needs and the consistency of individual forecasting performance. Three composite methods were selected to generate composite forecasts of cattle prices: 1) a simple average, 2) an adaptive weighting scheme, and 3) an unrestricted linear combination of forecasts. The first two weighting methods assume a linear combination of forecasts with the weights summing to one, as suggested by Bates and Granger (1969).

The general form for composite forecasting with forecasts from K techniques, according to Bates and Granger (1969), is:

$$c_{T+1} = \sum_{i=1}^{K} w_{i,T+1} \, f_{i,T+1} \tag{3.1}$$

where

c_{T+1} = the composite of the K forecasting methods in the T+1 forecast period

$w_{i,T+1}$ = the weight applied to f_i, $\sum_{i=1}^{K} w_{i,T+1} = 1$

$f_{i,T+1}$ = the T+1 forecast from the ith forecasting technique.

This form is evaluated because a linear combination of unbiased forecasts will result in an unbiased composite forecast.[1] Sufficient conditions for a combined forecast bias of zero are that each of the K forecasts has zero mean error and forecast weights that sum to one. The third composite method does not restrict the weights to sum to one. The second two composite techniques, unlike a simple average, use information concerning past forecast error histories to formulate the composite weights. According to Bates and Granger (1969, p. 45):

> Though the combined forecast formed by giving equal weights to each of the individual forecasts is acceptable for illustrative purposes, one would wish to give greater weight to the set of forecasts which seemed to contain lower (mean-square) errors.

A Simple Average of Forecasts

Perhaps the simplest method of composite forecasting is the naive approach of taking an average of forecasts from the individual techniques. An average of forecasts provides a simple and inexpensive means of combining forecasts which perform consistently through time. The formula for a simple average of alternative forecasting techniques is:

$$c_{T+1} = \sum_{i=1}^{K} w_{i,T+1} \; f_{i,T+1} \qquad (3.2)$$

where

$$w_{i,T+1} = 1/K.$$

A simple average will of course give equal weight to each forecast regardless of the accuracy of the forecast. Brandt and Bessler (1981) found that a simple average of econometric and ARIMA models produced lower mean absolute percentage errors, mean forecast errors, and turning point errors than did either of the individual methods when applied to quarterly cattle price data. Harris and Leuthold (1983) additionally applied a simple average composite of econometric and ARIMA models to quarterly farm price of cattle. They noted that the composite produced higher root mean squared errors than the ARIMA alone, but lower than the econometric model. Although this was the case, the ARIMA model alone did not perform well in terms of indicating turning points in the data.

Adaptive Weighting Schemes

Other more sophisticated means of combining forecasts include methods which base forecast weights on the historical forecast's performance. If constant forecasting performance for each technique over time cannot be assumed, the weights may be adapted to account for more recent error histories. Brandt and Bessler (1981) suggest an adaptive weighting method to allow for the inclusion of v error histories in the calcluation of the composite weight:

$$w_{i,T+1} = \frac{\sum_{i=j}^{K} \left(\sum_{t=T-v}^{T} e^2_{j,t} \right)}{(K-1) \sum_{i=1}^{K} \left[\sum_{t=T-v}^{T} e^2_{i,t} \right]} \qquad (3.3)$$

where

$i=1...K$,

and $w_{i,T+1}$ is the weight applied to the ith forecast method in period T+1, $e_{i,t}$ is the error made by forecast method i in period t, K is the number of forecasting methods, v is the number of periods selected to include in calculating the adaptive weights, and T is the total number of periods for which historical errors are available. Therefore, if the number of forecasting techniques combined is two, as will be in the case of this study, then the weight applied on forecasting technique one would be:

$$w_{1,T+1} = \frac{\sum_{t=T-v}^{T} e^2_{2,t}}{\sum_{t=T-v}^{T} e^2_{1,t} + \sum_{t=T-v}^{T} e^2_{2,t}} \qquad (3.4)$$

As in the previous methods of combining forecasts, the weights must sum to one, but this method allows for the selection of the number of error histories to include in calculating the composite weight rather than including the entire forecast error history. The method of including v periods of the error history to calculate the weights on the forecasts does have the disadvantage that the method does not adapt to the possibility that one or more of the techniques may become worse over the v periods. One possible solution is weighting the forecast error histories by an exponential decay to give more recent forecast errors greater importance relative to more distant error histories in determining the weights on the individual forecasts in the composite. Thus, if forecasts from one of the techniques became much worse over the v error histories relative to the other technique, then less importance (a smaller composite weight) would be placed on the forecasts from that technique in calculating the composite forecast. If the correlation between past forecast errors is assumed to be zero, as in the adaptive weighting scheme proposed by

Brandt and Bessler, then the modified adaptive weight would be:

$$w_{i,T+1} = \frac{\sum_{i=j}^{K} \left(\sum_{t=T-v}^{T} a^t e_{j,t}^2 \right)}{(K-1) \sum_{i=1}^{K} \left(\sum_{t=T-v}^{T} a^t e_{i,t}^2 \right)} \tag{3.5}$$

where

$i=0$ to n, $0<n<T$.

The modified adaptive weight is similar to the one proposed by Brandt and Bessler, by allowing the forecaster to consider only v periods, but it also allows more recent forecast errors to be given more importance in determining composite weights than distant ones. Notably, if the smoothing factor, a, is greater than 1, then recent error histories are weighted more heavily than more distant error histories. As a declines toward 1, past errors are given increasingly more importance in determining composite weights, and if a=1, all v error histories are given equal importance. If a is less than 1, distant error histories are given more importance in determining weights than recent error histories. Therefore, from Equation (3.5), it may be seen that if a is greater than 1, and the j...K forecasting techniques begin to perform poorly in more recent forecasts (produce higher sum of squared errors), then more weight will be given to the ith technique.

Unrestricted Linear Combination of Forecasts

Granger and Ramanathan (1984, p. 200) challenged the restriction of convexity of weights, stating "...there is nothing sacred about the weights adding up to unity, although that seems to be the common practice." They evaluate three methods of determining appropriate forecast weights. These methods included an unrestricted linear combination of forecasts without an intercept, a restricted linear combination of forecasts, with the weights summing to one, as suggested by Bates and Granger, and an unrestricted linear combination of forecasts with an intercept.

The first method was an unrestricted combined forecast with no intercept. In other words, the weights were determined by regressing the actual data upon the forecasts from the individual techniques and forcing the intercept to be equal to zero. Unfortunately, in this case the composite forecast errors may not average to zero. To illustrate this problem, if the composite forecast is found by:

$$F\hat{\alpha}, \tag{3.6}$$

F being an nxK matrix of forecasts and α being a Kx1 vector of composite weights, then the composite error will be:

$$\hat{e}_A = x - F\hat{\alpha}. \tag{3.7}$$

The value for $\hat{\alpha}$ must be chosen to minimize:

$$(x - F\hat{\alpha})'(x - F\hat{\alpha}). \tag{3.8}$$

where x is the actual price series to be forecast.

This value will be:

$$\hat{\alpha} = (F'F)^{-1}F'x, \qquad (3.9)$$

and the composite forecasts would be:

$$\hat{x}_A = F\hat{\alpha} = F(F'F)^{-1}F'x, \qquad (3.10)$$

and the sum of squared errors would be:

$$Q_A = \hat{e}_A'\hat{e}_A = (x - \hat{x}_A)'(x - \hat{x}_A)$$
$$= x'x - x'F\hat{\alpha}. \qquad (3.11)$$

Even if each forecast is unbiased, $1'(x - f_j) = 0$, which means that $(1'x)1' = 1'F$, and therefore, $(1'x)1'\hat{\alpha} = 1'F\hat{\alpha}$, in order for the combined forecast to have zero error mean, $1'x$ would have to be zero or $1'\hat{\alpha}$ would have to be equal to one. This is due to the fact that in order to have a zero error mean $1'F\hat{\alpha} = 1'x$, and since $(1'x)1'\hat{\alpha} = 1'F\hat{\alpha}$, then $(1'x)1'\hat{\alpha} = 1'x$.

With the second method tested by Granger and Ramanathan and employed by Rausser and Just, the weights were constrained to sum to unity and the regression was performed without an intercept. If each individual forecast is unbiased, then this weighting scheme will produce an unbiased composite, but if one or more of the forecasts is biased, then the combined forecast may not yield errors which average to zero.[1] Additionally, constraining the weights to sum to one will produce a larger mean-squared error. They represented this case as minimizing $(x - F\hat{\beta})'(x - F\hat{\beta})$ with respect to $\hat{\beta}$ subject to the constraint $1'\hat{\beta} = 1$. The first order condition for minimization:

$$F'x - F'F\hat{\beta} - \lambda 1 = 0, \qquad (3.12)$$

which yields the estimate for β:

$$\hat{\beta} = (F'F)^{-1}F'x - \lambda(F'F)^{-1}$$

$$= \hat{\alpha} - \lambda(F'F)^{-1}1, \qquad (3.13)$$

the value for λ being:

$$\lambda = 1'\hat{\alpha} - 1/1'(F'F)^{-1}1.$$

Since the value for $\hat{e}_B'\hat{e}_B$ is:

$$(x - F\hat{\beta})'(x - F\hat{\beta}),$$

then the sum of squared errors may be rewritten as:

$$Q_B = Q_A + \lambda^2[1'(F'F)^{-1}], \qquad (3.14)$$

by substituting in the value for $\hat{\beta}$ shown in (3.14). Thus, Q_B is greater than Q_A, and the mean-squared error is increased by constraining the composite weights to sum to one.

Granger and Ramanathan also examined the case where there are no restricions on the weights and a constant term is added. The combined forecast is then:

$$\hat{X}_C = \hat{S}_0 1 + F\hat{S} \qquad (3.15)$$

where

$\hat{X}_C$ = a 1xn vector of composite forecasts

$\hat{S}_0 1$ = the constant term when the forecasts are regressed on the data values to be forecast

F = an nxK matrix of the forecast values from the various techniques

$\hat{S}$ = the weights applied to the K forecasts.

The solutions for S and S_0 found by minimizing:

$$(x - \hat{S}_0 1 - F\hat{S})'(x - \hat{S}_0 1 - F\hat{S}), \qquad (3.16)$$

may be given by:

$$\hat{S} = \hat{\alpha} - \hat{S}_0 \ (F'F)^{-1}F'1$$

$$\hat{S}_0 = (1'x - 1'F\ )/n$$

$$= 1'\hat{e}_A \ / \ [n - 1'F(F'F)^{-1}F'1]. \qquad (3.17)$$

where

$\hat{\alpha}$ = a Kx1 vector of weights for f_j's

or

$= (F'F)^{-1}F'1$

x = a 1xn vector of the values to be forecasted.

The combined forecast is $\hat{x}_C = \hat{S}_0 1 + F\hat{S}$, therefore the forecast error is:

$$\hat{e}_C = x - \hat{x}_C = x - \hat{S}_0 1 - F\hat{S}$$

$$= \hat{e}_A - \hat{S}_0[I - F(F'F)^{-1}F']1, \qquad (3.18)$$

and the sum of squared error is:

$$Q_C = Q_A - 2\hat{S}_0 1'[I - F(F'F)^{-1}F']\hat{e}_A$$

$$+ \hat{S}_0{}^2 1'[I - F(F'F)^{-1}F']1. \qquad (3.19)$$

Given that $F'e_A = 0$, then:

$$Q_C = Q_A - (1'\hat{e}_A)^2/ \ [n$$

$$- 1'F(F'F)^{-1}F'1']. \qquad (3.20)$$

Hence, the value for Q_C will be less than the value for Q_A. This method was employed, according to the Granger and Ramanathan, because it produced the lowest mean squared error of the three methods tested and generated an unbiased combined forecast regardless of whether or not the individual forecasts were unbiased. Granger and Ramanthan (1984, p. 201) therefore recommend, "The common practice of obtaining a weighted average of alternative forecasts should be abandoned in favor of an unrestricted linear combination including a constant term."

The weighting schemes evaluated in this study will be the simple average of the two forecasts, the adaptive weighting scheme, and the unrestricted linear combination of forecasts including a constant term. The first weighting scheme will be included primarily because of its simplicity of calculation. The method of adaptive weights will be tested because it permits adaptation of weights on forecasts which do not perform uniformly through time by allowing the weights to place greater importance on more recent error histories. Finally, the unrestricted linear combination of forecasts will be examined due to the fact that this method will produce a minimum variance set of unbiased composite weights.

FOOTNOTES

[1] If each f_i is an unbiased forecast of x, $E[f_i] = E[X]$, $\sum_{i=1}^{K} w_i = 1$, and $w_i > 0$, then:

$$
\begin{aligned}
E[x] &= E[\sum_{i=1}^{K} w_i f_i] \\
&= \sum_{i=1}^{K} w_i \ E[f_i] \\
&= \sum_{i=1}^{K} w_i \ E[x] \\
&= E[x],
\end{aligned}
$$

but if any one of the forecasts, f_i, is not an unbiased forecast of x, for example $E[x] = E[f_K]$, then:

$$
\begin{aligned}
E[x] &= E[\sum_{i=1}^{K} w_i f_i] \\
&= E[\sum_{i=1}^{K-1} w_i f_i + w_K f_K] \\
&= \sum_{i=1}^{K-1} w_i E[f_i] + w_K E[f_K] \\
&= \sum_{i=1}^{K-1} w_i E[x] + w_K (E[x] + (BIAS)) \\
&= \sum_{i=1}^{K-1} w_i E[x] + w_K (BIAS) \\
&= E[x] + w_K (BIAS)
\end{aligned}
$$

CHAPTER FOUR

THE ECONOMETRIC MODEL

Selection of a Model Specification

The cattle cycle has been econometrically modelled in several studies, with varying degrees of complexity. These studies have modelled different aspects of the cattle cycle, encompassing the inventory, slaughter, and price cycles. The primary focus of the econometric model employed in this study will be to forecast the annual average price of beef cattle received by farmers. Due to the fact that only one variable, farm beef cattle price, will be forecasted, the model will be kept relatively simple. Cromarty and Myers (1975) wrote in an analysis of commodity price forecasting models:

> ...emphasis must be placed on understanding the market structure which generated the pricing problem and on specifying a model that will correctly identify the two or three factors influencing the system. Models that serve this purpose are not only easier to understand, but they generally lead to better forecasts and policy prescriptions.

furthermore, they continue:

> Despite strong personal attachments to particular estimating techniques, there is much to be gained from keeping models simple and working with partial systems of equations.

A tradeoff may exist between including all relevant variables and keeping the model simple, but certainly, as Cromarty and Myers indicated, occasions exist when the marginal contributions to forecasting performance gained by applying more complex models may be small or nonexistent.

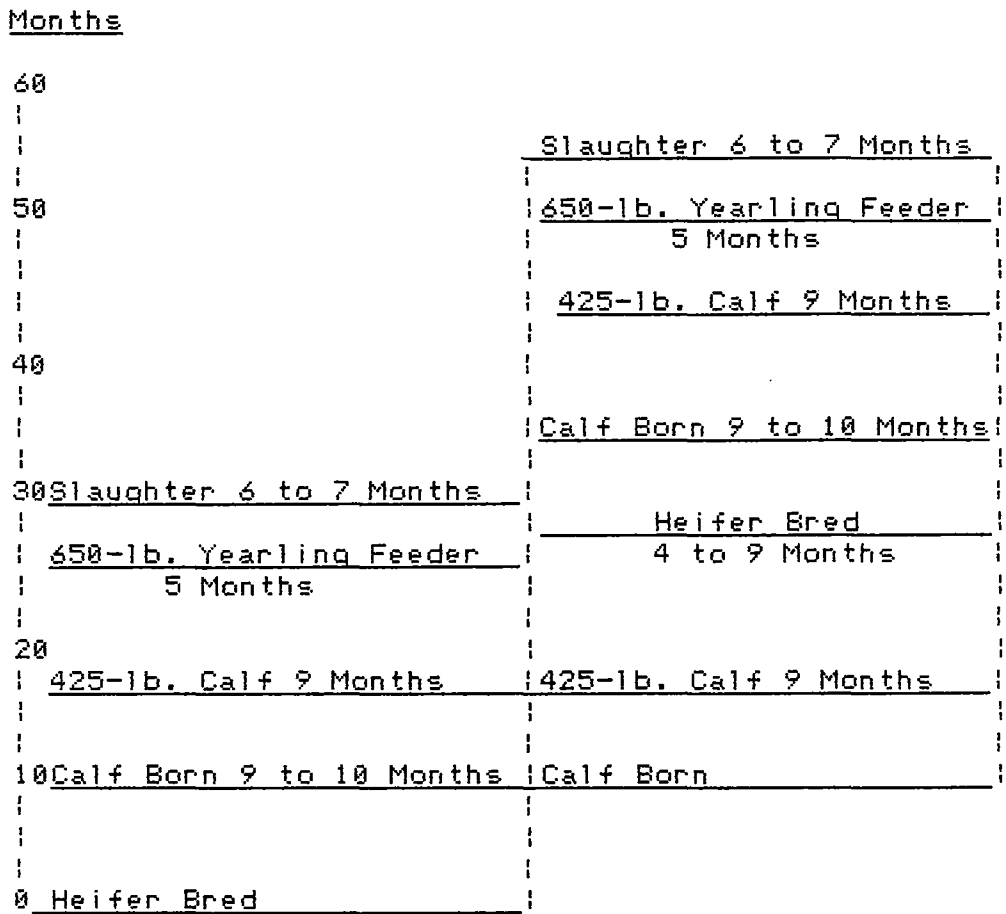
Another consideration in selecting an econometric model, which is perhaps coincidental with determining the desired complexity of the model, is the type of econometric model to be employed. Econometric models of the cattle cycle have been of two basic types: simultaneous systems of equations and recursive systems of equations. Studies which have employed simultaneous systems of equations have generally estimated supply and demand equations for beef, assuming that the demand for beef at the farm level is derived from the retail demand for beef. Recursive models, on the other hand, postulate the supplies of beef as predetermined due to the production lag from conception to retail marketing.

Past Econometric Studies

Wallace and Judge (1959) consider an extensive simultaneous systems of equations to model the beef and pork sectors. The supply and demand at the retail, wholesale, and farm levels were simultaneously determined. Exogenous variables used to determine the supply of beef at the farm level were January 1 inventories of beef cattle and dairy cows, available feedgrains in the previous year, range conditions, and time. Wallace and Judge noted that the sign on the coefficient for range conditions was often contrary to logic perhaps due to the subjective nature of the data used to represent the variable. The demand function at the farm level was a derived demand, and the farm price of beef was postulated to be determined simultaneously with the retail price of beef, farm price of pork, and farm production of beef. Exogenous factors in the demand for beef at the farm level were wage rates of slaughtering facilities to reflect marketing costs, and time as a proxy for technological change.

Similarly, Gruber (1965) employed a simultaneous equations approach to modelling the cattle cycle. Gruber modelled the inventory cycle, the price and income cycle, and the slaughter and import cycle. Current values of cows and heifers over two years kept on farms, calves kept as young heifers, calves and heifers raised, calves available, current slaughter prices and current slaughter were all considered to be simultaneously determined. Gruber did postulate January 1 inventories as predetermined. Unlike Wallace and Judge, who linked the farm and retail levels in estimating supply and demand at the farm level, Gruber linked the slaughter price received by farmers to current cattle slaughter, average liveweight of slaughter, lagged slaughter price, lagged net imports of cattle, hay production, corn price, total disposable personal income, and the supply of other meats. While Gruber tested a dummy variable to shift the intercept in an attempt to account for different behavior near turning points in the cycle, the gains in information from including the dummy variable were negligible. Wallace and Judge assumed farm supplies to be predetermined, employing January 1 inventories as an exogenous variable. Langmeier and Thompson (1967), like Gruber, hypothesized that the weight of fed beef slaughtered, the supply of nonfed beef, imports, per capita demand for beef, fed beef margins, and non-fed beef margins were determined simultaneously. Only the number of fed beef slaughtered was specified in single equation form due to the fact that slaughter numbers were considered to be a function of January 1 inventory, which is, in turn, a function of lagged economic and non-economic variables.

The second type of econometric model which has been employed in previous studies is a recursive system. Recursive systems of equations make the assumption that current prices are determined by current quantities, and current quantities are determined by past prices or production decisions. Current quantities would necessarily be based on past prices due to the fact that a production lag exists. If the time increments examined in an analysis of the cattle cycle, are less than the average period of time it takes from conception of a calf until it reaches the consumer, such as with annual data, then the analysis must include a production lag. This production lag averages 26 to 28 months, as is shown in Figure 4. Since current prices



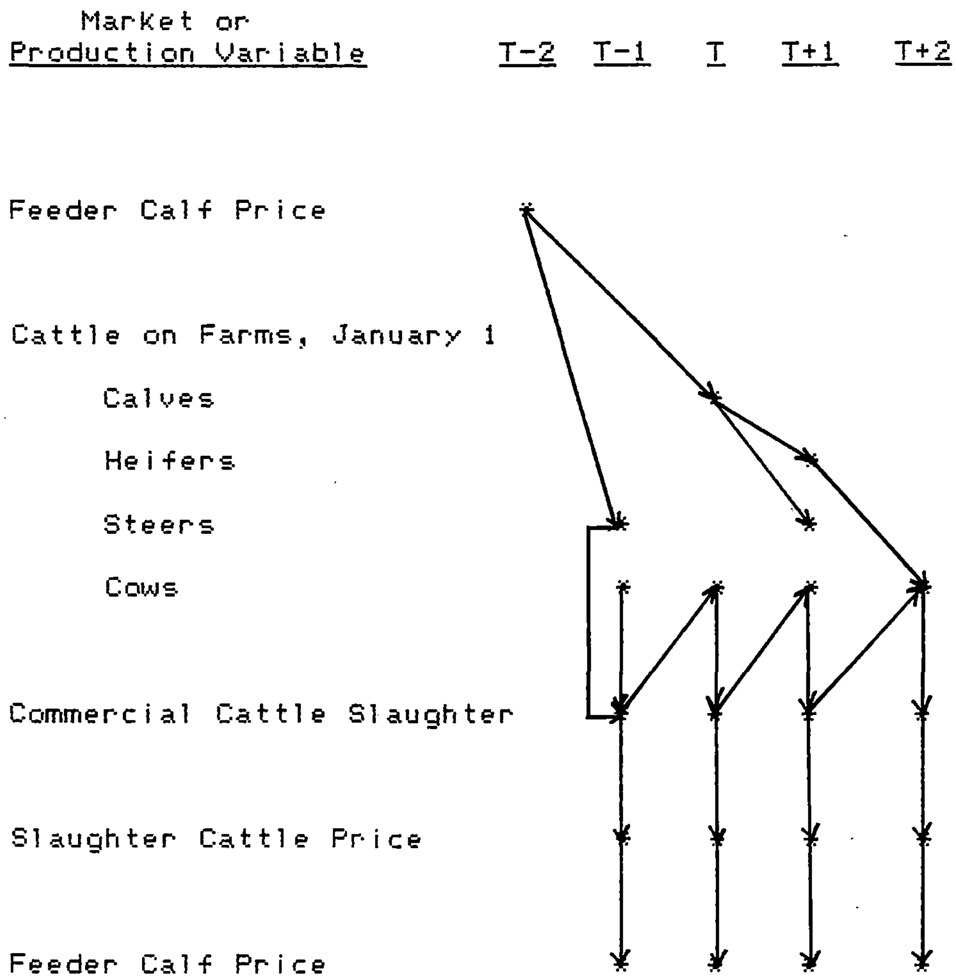
Source: Stillman, Richard P., *A Quarterly Model of the Livestock Industry*. Technical Bulletin, No. 1711, Economic Research Service, U.S.D.A., Washington, D.C., December 1985, p. 2.

Figure 4. Biological Lags in the Beef Production Process

are based on current quantities and not vice-versa, and current quantities are based on past prices or production decisions, a simultaneous system of equations is not needed. Ordinary least squares may be used to estimate the slaughter price equation, the quantity slaughtered equation, and the inventory equations.

Maki (1959) hypothesized a chain of market and production variables, linking feeder calf prices, cattle inventories, cattle slaughter, and slaughter prices, as shown in Figure 5. Notably, slaughter prices are directly caused by commercial cattle slaughter levels within the same year, and slaughter cattle prices affect feeder calf prices within the same year also. The two primary sources of cattle slaughter are steers and cows. A modification to the flow chart might be made for present use. Namely, heifer slaughter has increased through time, so that an arrow might be drawn from inventory of heifers to commercial slaughter within the same time period. Maki hypothesized commercial slaughter to be a function of the change in trend from year to year for the three previous years, of beef cows on hand and of steers on hand January 1. Inventories are in turn affected by feeder calf prices.

Ehrich (1967) also employed a recursive system of equations to model the cattle cycle. In Ehrich's study determinants of prices were specified in demand relations, because quantities supplied were considered to be



Source: Maki, Wilbur R., "Decomposition of the Beef and Pork Cycle," Journal of Farm Economics. 44: 739.

Figure 5. Internal Mechanism of the Beef Cycle

predetermined, as in most recursive models. According to Ehrich (p. 11):

> There are fairly rigid physical limitations on growth which cause a high degree of correspodence between cattle on farms on January 1 and slaughter during the year.

Ehrich does note:

> Quantities supplied may have been influenced by current prices in some instances. For example, low prices may have induced some producers to hold cattle on feed longer than normally, perhaps into a new year, in anticipation of favorable price developments. Such adjustments were probably minimal, and will be ignored in the present study.

Ehrich made the assumption that farm prices are derived from retail prices by a constant marketing margin, therefore farm prices were related to the slaughter of live animals and to variables which were chosen to reflect consumer demand. Specifically, steer prices were evaluated as a function of steer and heifer slaughter, cow slaughter, and demand determinants such as the price of pork and disposable personal income. Unlike Maki's approach, which assumed that (p. 621):

> ...forecasts of livestock prices at the primary market, or a farm level depend on forecasts of the consumer demand for meat and on the price spreads between different marketing levels.

Ehrich did not evaluate marketing margins in his study. While Maki related farm prices to wholesale and retail prices explicitly, Ehrich acknowledged the relationship between retail prices and farm prices by hypothesizing that the factors which affect retail prices would also affect farm prices. Although Ehrich included variables to reflect consumer demand, they were not found to be highly significant in explaining deflated steer prices. In particular the coefficient on per capita disposable income was found to be not statistically significant.

A second set of relationships specified factors believed to influence levels of inventory. January 1 inventories of beef steer, heifers, and calves were hypothesized to be a function of lagged feeder prices and lasts year's January 1 inventory of beef calves. As in Maki's study past inventory relations and past feeder prices were postulated to determine current production. Similarly, Reutlinger (1966) hypothesized steer slaughter in year t to be a function of a beef corn price ratio in year t-1 and January 1 inventories of beef cows and heifers in year t-1.

Several studies have employed recursive models similar to that of Ehrich's study, including Keith (1976), Pyne (1980), and Stillman (1985). All of these models made the commmon assumption that current prices are determined by current quantities, and current quantities are determined by past prices and production decisions.

Keith modified Ehrich's annual model into a quarterly model through the use of dummy variables, but the modelling of slaughter prices upon per capita steer and heifer production, per capita cow production, and per capita disposable income was quite similar to Ehrich's model. Keith found that the coefficient on per capita disposable income to be significant over the 1959 to 1974 period contrary to Ehrich's findings over the 1944 to 1964 period. This could possibly be an indication of the increasing importance of demand factors in determining cattle prices.

Quarterly prices were linked to retail beef prices, fed cattle marketings, and nonfed steer and heifer slaughter by Stillman (1985). Steer and heifer slaughter were viewed as previously determined, while retail prices represented the marginal revenue on the processor's output. Retail prices were hypothesized to be a function of per capita beef, pork, and broiler consumption and per capita disposable income. Fed cattle marketings were considered to be a function of total cattle on feed, while commercial steer and heifer slaughter were considered to be a function of corn price, a distributed lag of steer prices, and steers and heifers greater than 500 pounds.

In summary, studies employing recursive models have contained some common factors. Namely, current prices are hypothesized to be a function of current quantities supplied and demand factors. Current quantities supplied are postulated to be determined by inventories or production decisions, with inventories determined by past prices or producer incomes.

The Hypothesized Model

The model hypothesized in this study will follow the recursive approach suggested by past studies, such as that of Ehrich, that current farm prices are determined by current quantities slaughtered and demand determinants, while current quantities slaughtered are determined by past inventory and production decisions. The hypothesized model is:

$$FBP_t = f(ICA_{t-1}, FCRAT_{t-1}, TDPI_{t-1}, PHF_{t-1}, PCF_{t-1}) \quad (4.1)$$

The variable symbols, variable definitions, and expected signs of the regression coefficients in the postulated model are shown in Table I.

The dependent variable, the annual average price received by farmers for beef cattle in the United States, is an average price for all fifty states. The term "beef cattle" includes steers, bulls, heifers, and cows, and excludes only beef calves. Calves are defined as animals under 500 pounds or under 2 years of age. The term "beef cattle" is also exclusive of dairy animals. The annual average price received by farmers for beef cattle was selected as the dependent variable series for several reasons. An aggregated price series for the United States was employed so an overall measure of the prices received by farmers could be analyzed, and additionally so that aggregate slaughter and inventory data could be employed. A series such as the average price of steers received by farmers in the United States would perhaps have provided a more accurate depiction of the cycle in beef cattle prices, due to the fact that steers are the primary source of fed beef. However, this series was unavailable prior to 1938 and unduly restricted the data for analysis. The behavior

TABLE I

VARIABLES INCLUDED IN THE ECONOMETRIC MODEL

Symbol	Definition	Expected Sign
FBP_t	the annual average price received by farmers for beef cattle in the United States in year t, deflated by the index of prices received by farmers for all farm products, 1967=100, dollars/hundredweight.	
ICA_{t-1}	the inventory of calves on farms in the United States on January 1, lagged by one year, 000 head.	−
$FCRAT_{t-1}$	the ratio of the annual average price paid by farmers for feeder cattle in the United States to the average price of corn received by farmers in the United States, lagged by one year, dollars per hundredweight/dollars per bushel.	+
$TDPI_{t-1}$	total disposable personal income in the United States, lagged by one year, deflated by the index of consumer prices, 1967=100, dollars.	+
PHF_{t-1}	the annual average price received by farmers for hogs in the United States, lagged by one year, deflated by the index of prices received by farmers for all farm products, 1967=100, dollars per pound.	+
PCF_{t-1}	the annual average price received by farmers for chickens in the United States, lagged by one year, deflated by the index of prices received by farmers for all farm products, 1967=100, cents per pound.	+

of the two series, the average price of beef cattle and the average price of steers, coincide closely until the late 1970's, when the average price of steers becomes less cyclical.[1]

Prices of beef cattle are often hypothesized to be negatively correlated with quantities slaughtered (Ehrich, Maki, Pyne, Stillman). This relationship is hypothesized because slaughter cattle are a nonstorable commodity with predetermined production. Thus, given that storage and net imports or exports are negligible, the quantity of slaughter will be a good measure for consumption at the farm level. The one year lag of inventory of calves on January 1 was selected as a measure of quantity slaughtered within this study because it is highly correlated with slaughter within the following year, as may be seen in Figures 6 and 7, and allows a lagged quantity measure to be used within the model.[2] Were a lagged quantity measure not used, a forecast of slaughter in time t would have to be made. The consequences of forecasting an explanatory variable are discussed later within this chapter. It may be noted that the inventory measure is a January 1 figure which has been lagged by one period. The average price of beef cattle is a year-end average. The lag between the inventory measure and the price of beef cattle received by farmers is therefore essentially a two year lag. It may be recalled from Figure 3 that the approximate average production and marketing period for beef cattle is 26 to 28 months. The hypothesized

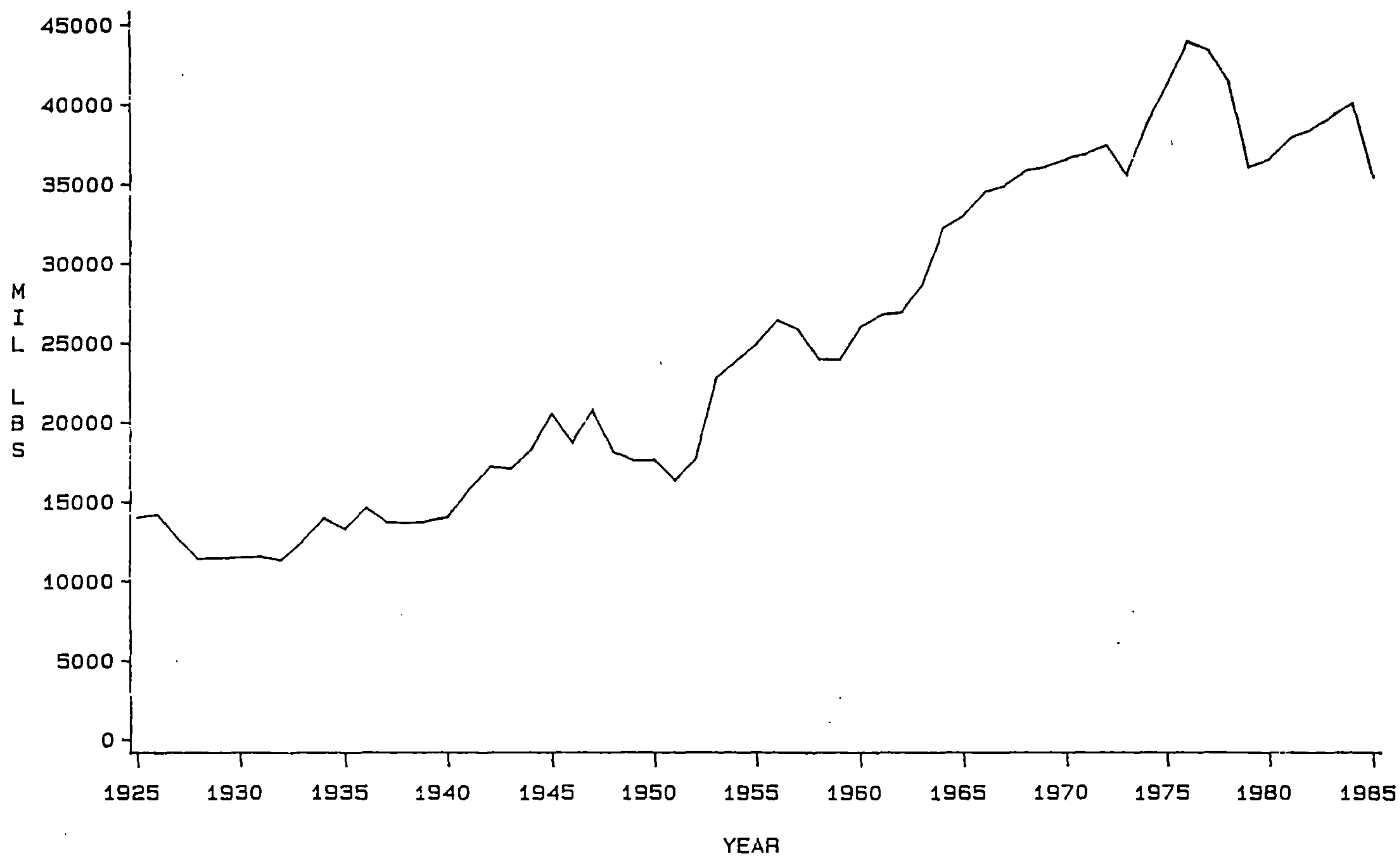


Figure 6. Slaughter of Beef Cattle in the United States

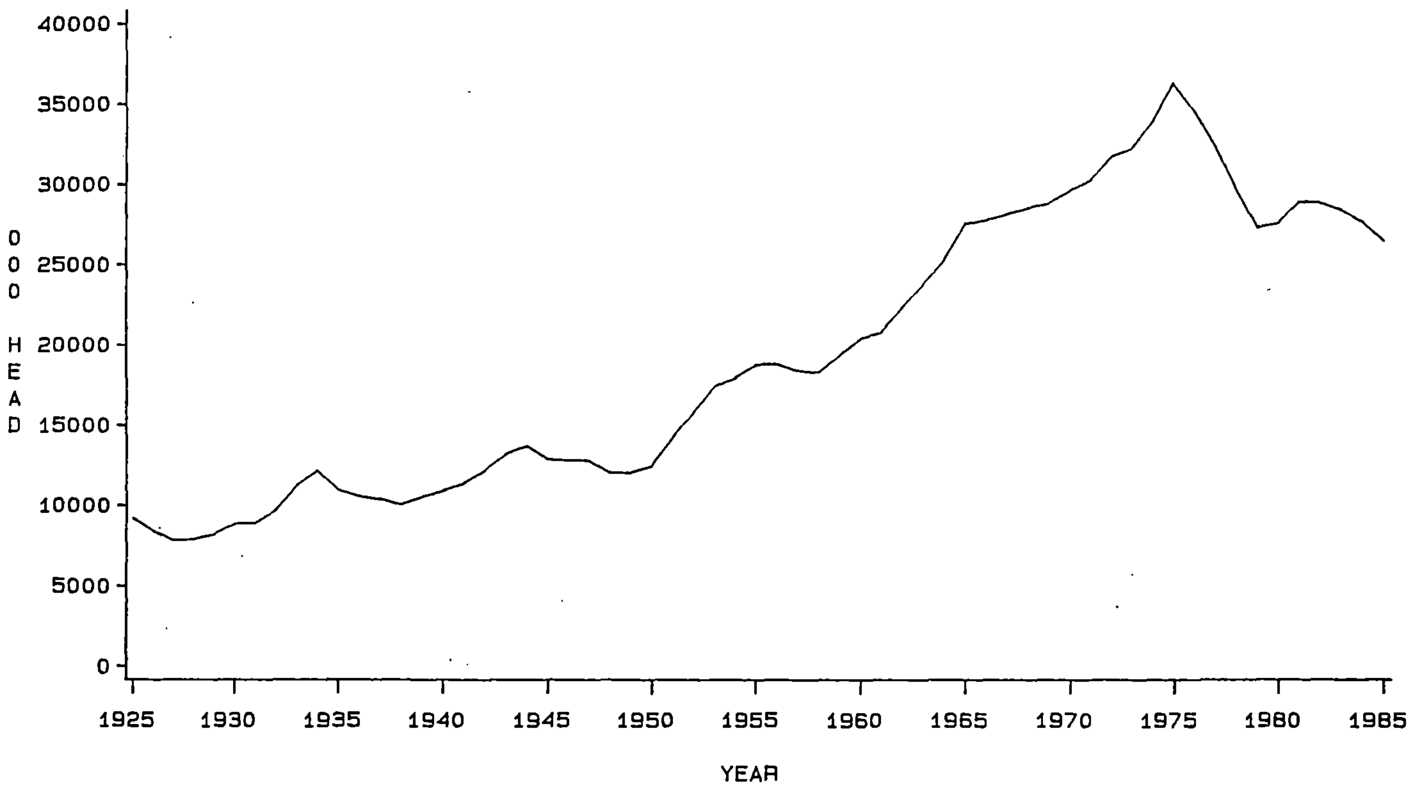


Figure 7. January 1 Inventories of Calves in the United States

sign for the coefficient relating the price received by farmers to the lagged inventory of calves is negative. According to economic theory, given a downwardly sloping demand curve, the hypothesized sign will be negative because the relationship between the price of beef cattle and the lagged inventory of calves is a price-quantity demanded relationship. As the quantity of beef cattle slaughtered increases, the price of beef cattle declines, and as the quantity of beef cattle slaughtered decreases, the price of beef cattle will increase. Since the quantities available for slaughter are relatively fixed by earlier production decisions, primarily be inventories of calves, the price of beef cattle will have a negative relationship to the lagged inventory of calves on farms.

The coefficient on the ratio of the annual average price of feeder cattle paid by farmers in the United States to the average price of corn received by farmers in the United States, or the "feeder cattle ratio" is postulated to have a positive sign in relation to the price of beef cattle received by farmers. The behavior of feeder prices coincides closely with the behavior of beef cattle prices.[3] Low feeder prices are indicative of large supplies of feeders, which can be translated into large potential supplies for slaughter in year t. When the price of feeder cattle is low compared with a relatively high price of corn, more cattle are slaughtered, lowering the current price of beef cattle. Conversely, during periods of low corn prices or increasing feeder prices, more cattle are fed longer. Increasing feeder prices provide a signal to operators of higher future slaughter prices may be possible, so producers may hold onto cattle. Falling corn prices provide less expensive feed prices, so cattle feeders have a greater incentive to feed cattle to heavier weights. As may be seen by comparing Figure 8 and Figure 1, the peaks and troughs in the feeder cattle ratio often precede peaks and troughs in beef cattle prices by 1 to 2 years.

Total disposable personal income in the United States is hypothesized to have a positive relationship to the price of beef received by farmers. If beef is a normal good, as consumer income increases, the demand for beef at the consumer level will increase. Given fixed supplies, the quantity demanded will increase with an increase in demand. As the quantity demanded at the consumer level increases, the quantity demanded at the farm level will also increase. Therefore a rise in consumer incomes will increase the demand at the farm level, and raise the farm price of beef. Consumer income was lagged at the risk of introducing some misspecification into the model. Nevertheless, the cost in terms of forecasting error was assumed to be less than if consumer incomes were forecasted. One reason that the misspecification may not be great is that, given increases in the demand for beef, the only market means by which the price could remain constant, would be if supplies increased by an amount proportional to the increases in demand. Since

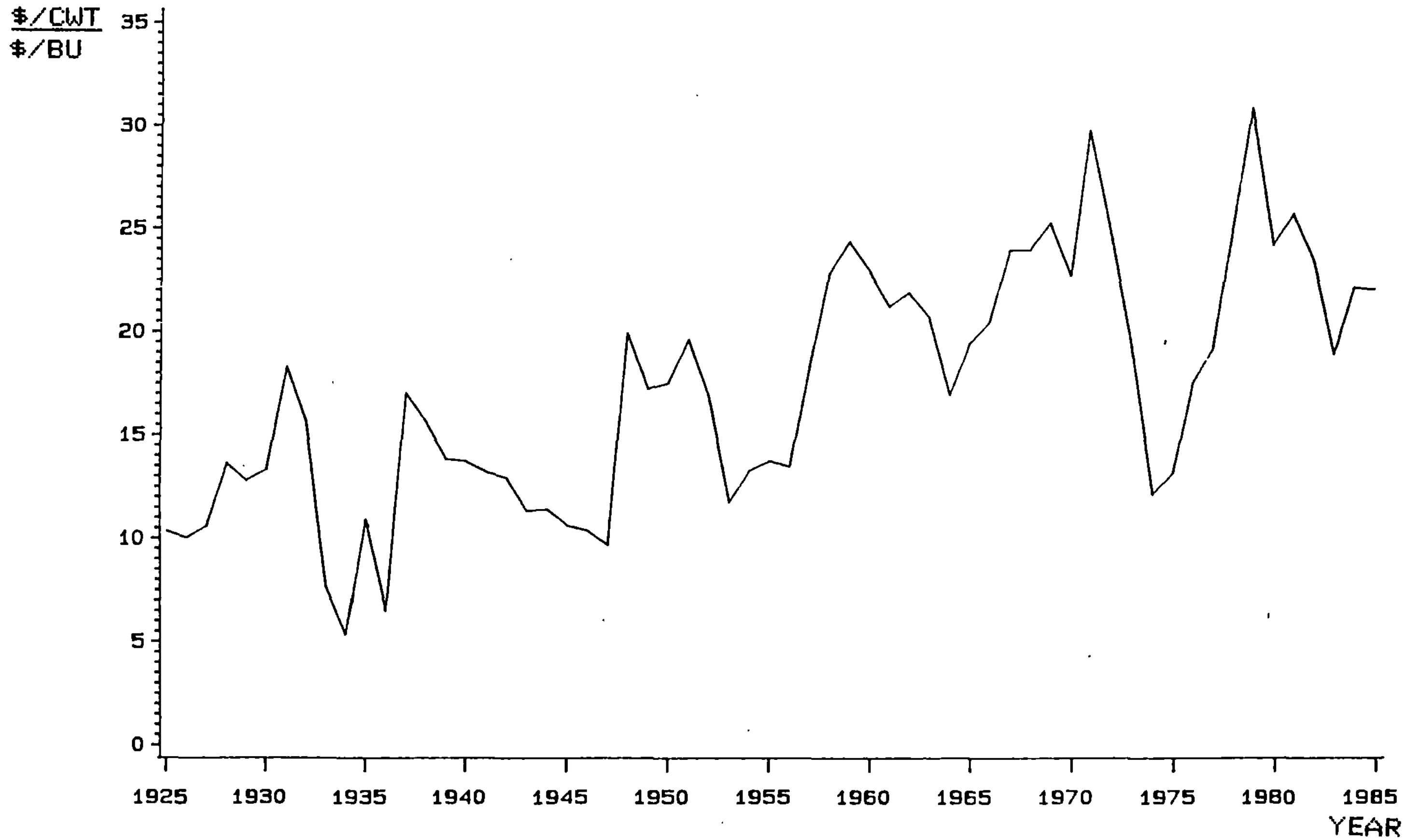


Figure 8. The Feeder to Corn Price Ratio

supplies are relatively fixed during a one year period, the period by which income has been lagged, the effect of increased consumer incomes still will be to increase beef prices in the short-run.

The annual average price received by farmers for hogs in the United States is postulated to have a positive relationship to the price of beef cattle received by farmers. A positive relationship exists because the products, beef and pork, are substitutes. As the price of hogs increases, the quantity demanded of pork falls. If the price of pork increases relative to the price of beef, the demand for beef will increase, thus increasing the price of beef cattle in the short-run. Conversely, as the price of pork falls, the quantity demanded of pork will rise. As the quantity demanded of pork rises, the demand for beef falls, and the price of beef decreases.

Similarly, the coefficient on the price of chicken is hypothesized to have a positive sign because beef and chicken are substitutes. Therefore, as the price of chicken increases, the quantity demanded of chicken will fall, and the demand for beef will be shifted outward, thus raising the price of beef.

The price of hogs and the price of chickens are both farm level prices. While series depicting the retail price of pork and broilers might have provided a more accurate picture of the effects of the changes in prices of demand substitutes upon the price of beef cattle, the farm price series was the only series consistently available throughout the time span of the study.

The prices included in the models, and the income data were all adjusted for the effects of inflation by dividing the farm level prices by the Index of Prices Received by Farmers for All Commodities, while consumer income was divided by the Consumer Price Index.[4] These indexes were used because the price series should be deflated by the series which is calculated at the same market level, unless some measure of marketing margins is included. Although the farm prices, which were adjusted by a farm price index, were included in the calculation of the price index, potentially biasing the estimates downward slightly, the individual prices of each commodity make up a small component of the total index, with the result that the bias can be assumed to be small.[5] A "real" or deflated price of farm beef is forecasted because producers and other forecast users need information concerning "real" expected prices in order to make long term production and planning decisions. A decision based on nominal price forecasts or expectations, which include inflationary trends could lead to "real" losses.

The Classical Linear Regression Model

Given that a single equation model is employed to describe the variation in the price of beef cattle received by farmers then the theoretical model may be written in the generalized form for the $t^{\underline{th}}$ observation:

$$Y_t = \beta_1 + \beta_2 X_{2,t} + \beta_3 X_{3,t} + \beta_i X_{i,t} + \ldots + \beta_K X_{K,t} + \epsilon_t \qquad (4.2)$$

where

Y_t = the observation for the dependent variable at time t.

$X_{i,t}$ = the independent or explanatory variables at time t

ϵ_t = the error term at time t

β_i = the unknown parameters

$t = 1 \ldots T$

$i = 1 \ldots K$

or the model may be represented in matrix notation:

$$Y = X\beta + \epsilon$$

where

Y = a Tx1 vector of dependent variable observations

X = a TxK matrix of independent variable observations

β = a Kx1 column vector of unknown parameters

ϵ = a Tx1 column vector of errors

The assumptions of a multiple linear regression model are that (1) the X's are nonstochastic or fixed in repeated sampling, (2) no exact linear relationship exists between two or more of the exogenous or predetermined variables, (3) the error term has an expected value of zero:

$$E[\epsilon_t] = 0, \quad t = 1...T \tag{4.3}$$

(4) the error term has a constant variance for all observations:

$$E[\epsilon_t \epsilon_s] = \sigma^2, \quad t = s$$
$$t = 1...T$$
$$s = 1...T$$

(5) errors corresponding to different observations are uncorrelated:

$$E[\epsilon_t \epsilon_s] = 0, \quad t \neq s \tag{4.4}$$
$$t = 1...T$$
$$s = 1...T$$

and (6) the error variable is distributed normally, so that:

$$\epsilon \sim N(0, \sigma^2), \tag{4.5}$$

where 0 is the expected value, or mean of the error term, and σ^2 is the variance of the error term which is given by:

$$\sigma^2 = (Y-X\beta)'(Y-X\beta). \tag{4.6}$$

The estimated multiple linear regression model may be expressed as:

$$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_{2,t}X_{2,t} + \hat{\beta}_{3,t}X_{3,t} + ...\hat{\beta}_{k,t}X_{k,t}, \tag{4.7}$$

where:

$\hat{Y}_t$ = the estimate of the $t^{\underline{th}}$ observation of the dependent variable

$X_{i,t}$ = the independent or explanatory variables

$\hat{\beta}_i$ = the estimated parameters that relate the dependent variable to the independent variables

$i = 1 \ldots K$

$t = 1 \ldots T$

The estimated variance is:

$$s^2 = \frac{\hat{\epsilon}'\hat{\epsilon}}{T-K} \tag{4.8}$$

where

$$\hat{\epsilon} = Y - X\hat{\beta}.$$

The model may be estimated by least-squares estimation, so that the vector of estimated parameters, β, are obtained so as to minimize the sum of squared errors, $\hat{\epsilon}'\hat{\epsilon}$. If $\hat{\epsilon}'\hat{\epsilon}$ is:

$$\begin{aligned}(Y - X\hat{\beta})'(Y - X\hat{\beta}) \\ = Y'Y - \hat{\beta}'X'Y - Y'X\hat{\beta} + \hat{\beta}'X'X\hat{\beta} \\ = Y'Y - 2\hat{\beta}'X'Y + \hat{\beta}'X'X\hat{\beta}.\end{aligned} \tag{4.9}$$

The first order conditions for minimization of the sum of squared errors:

$$\frac{\partial\hat{\epsilon}'\hat{\epsilon}}{\partial\hat{\beta}} = -2X'Y + 2X'X\beta = 0, \tag{4.10}$$

may be solved for the least squares estimate of β:

$$\hat{\beta} = (X'X)^{-1}X'Y. \tag{4.11}$$

Using the Gauss-Markov theorem, the estimate β can be shown to be BLUE; a best, linear, unbiased estimator. The estimate $\hat{\beta}$ is an unbiased estimator since:

$$\begin{aligned}E[\hat{\beta}] = E[(X'X)^{-1}X'Y] = E[(X'X)^{-1}X'(X\beta+\epsilon)] \\ = E[(X'X)^{-1}(X'X)\beta + (X'X)^{-1}X'\epsilon]\end{aligned}$$

$$= \beta + E[(X'X)^{-1}X'\epsilon]$$

$$= \beta + (X'X)^{-1}X'E[\epsilon]$$

$$= \beta. \tag{4.12}$$

Therefore, as long as the expected value of the errors is zero, then β will be an unbiased estimator. The estimate β can also be shown to be best in that it has the minimum variance of all unbiased estimators. If:

$$b = (A + C)Y = AY + CY = \beta + CY$$

$$= (A + CY)X\beta + (A + C)\epsilon \tag{4.13}$$

where $A=(X'X)^{-1}X'$, and is fixed in repeated sampling, and C is an arbitrary matrix. Given that b is unbiased:

$$E[b] = (X'X)^{-1}X'X\beta + CX\beta$$

$$= (I + CX)\beta$$

$$= \beta \tag{4.14}$$

If E[b] is equal to β, then CX must equal zero.
Since $AX = (X'X)^{-1}X'X = I$, an identity matrix, then $b - \beta = (A + C)\epsilon$. The variance of b will be:

$$E[(b - \beta)(b - \beta)']$$

$$= E\{[(A + C)\epsilon][(A + C)\epsilon]'\}$$

$$= \sigma^2[(X'X)^{-1}X'X(X'X)^{-1}$$

$$+ CX(X'X)^{-1} + (X'X)^{-1}X'C' + CC'$$

$$= \sigma^2[(X'X)^{-1} + CC']$$

$$= Var(\beta) + \sigma^2 CC' \tag{4.15}$$

If CC' is a positive semidefinite matrix, from Equation (4.15) it can be seen that Var(b)>Var(β), therefore the ordinary least-squares estimator β is the minimum variance estimator of all unbiased estimators. The variance-covariance about the estimate $\hat{\beta}$ will be:

$$\text{Var-Cov } [\hat{\beta}] = E\{(\hat{\beta} - E[\hat{\beta}])(\hat{\beta} - E[\hat{\beta}])'\}$$

$$= E\{(\hat{\beta} - \beta)(\hat{\beta} - \beta)'\}$$

$$= E\{[(X'X)^{-1}X'u][(X'X^{-1}X'u]\}$$

$$= \sigma^2(X'X)^{-1}X'X(X'X)^{-1}$$

$$= \sigma^2(X'X)^{-1}I_T \qquad (4.16)$$

which may be estimated by $s^2(X'X)^{-1}I_t$. Therefore, the standard error about the estimate is:

$$\sigma_\beta = \sigma^2(X'X)^{-1} \qquad (4.17)$$

and can be estimated by:

$$s_{\hat{\beta}} = s^2(X'X)^{-1} \qquad (4.18)$$

The standard errors of the coefficients may then be used to find t values:

$$t = \frac{\hat{\beta}_i - \beta_i}{s_{\hat{\beta}}}, \; i = 1 \ldots k. \qquad (4.19)$$

The t-statistics can be used to test the null hypothesis:

$$H_0\text{: } \beta_j = 0.$$

If the calculated t is greater than the tabulated t with T-k degrees of freedom, at some given probability level, then the null hypothesis must be rejected at that probability level.

The value for R^2 can be calculated by:

$$R^2 = \frac{RSS}{TSS} = 1 - \frac{\Sigma\epsilon^2}{\Sigma(Y_i - Y)^2} \qquad (4.20)$$

The F-statistic may be used to test the joint hypothesis that none of the regression coefficients are significantly different from zero:

$$H_0: \beta_2 = \beta_3 = \ldots = \beta_k = 0.$$

The F-statistic can be calculated as:

$$F = \frac{R^2}{1-R^2} \; \frac{T-k}{k-1} . \tag{4.21}$$

If the multiple linear regression model is employed for the purposes of unconditional forecasting, or forecasting when all of the explanatory variables are known with certainty, then the estimated coefficients from the model in Equation (4.2) are used to generate forecasts of the dependent variable:

$$\begin{aligned} \hat{Y}_{T+1} &= \hat{\beta}_1 + \hat{\beta}_2 X_{2,T+1} + \hat{\beta}_3 X_{3,T+1} \\ &\quad + \ldots + \hat{\beta}_k X_{k,T+1}. \end{aligned} \tag{4.22}$$

or:

$$\hat{Y}_{T+1} = X_{T+1} \hat{\beta}.$$

where X_{T+1} is a 1xk vector of all k independent variables at time T+1.

The forecast error is:

$$\begin{aligned} \hat{e}_{T+1} &= \hat{Y}_{T+1} - Y_{T+1} \\ &= (X_{T+1}\hat{\beta} - \epsilon_{T+1} - X_{T+1}\beta). \\ &= -\epsilon_{T+1} - X_{T+1}\beta + X_{T+1}(X'X)^{-1}X'Y, \end{aligned} \tag{4.23}$$

and since $Y = X\beta + \epsilon$, then:

$$\begin{aligned} e_{T+1} &= -\epsilon_{T+1} - X_{T+1}\beta + X_{T+1}(X'X)^{-1}X'(X\beta + \epsilon) \\ &= -\epsilon_{T+1} - X_{T+1}\beta + X_{T+1}\beta \\ &\quad + X_{T+1}(X'X)^{-1}X'\epsilon \end{aligned}$$

$$= -\epsilon_{T+1} + X_{T+1}(X'X)^{-1}X'\epsilon. \qquad (4.24)$$

It can be noted that the forecast error comes from two sources (Pindyck and Rubinfeld, p. 205): (1) the random nature of the additive error process in a linear regression model that forecasts will deviate from true values even if the model is specified correctly and its parameters are known with certainty and (2) the process of estimating the regression parameters introduces errors because estimated parameter values are random variables which may deviate from the true parameter values. The forecast error is distributed normally because it is a linear function of β, and ϵ_{T+1} which are distributed normally. Furthermore, assuming that the $\hat{\beta}$'s are unbiased estimators of the true population parameters, then the forecast error will have an expected value of zero:

$$E[\hat{e}_{T+1}] = E[(\hat{\beta}-\beta)]X_{T+1} + E[(-\epsilon)] = 0. \qquad (4.25)$$

The variance of the forecast error is:

$$\sigma^2{}_f = E[\hat{e}^2{}_{T+1}]$$
$$E[\epsilon^2{}_{T+1}] - 2X_{T+1}(X'X)^{-1}X'E[\epsilon\epsilon_{T+1}]$$
$$+X_{T+1}(X'X)^{-1}X'E[\epsilon\epsilon']X(X'X)^{-1}X_{T+1}'. \qquad (4.26)$$

If the error terms are assumed to be uncorrelated, and recalling that $E[\epsilon\epsilon'] = \sigma^2 I$, the the variance of the forecast error can be simplified to:

$$\sigma^2{}_f = \sigma^2 + \sigma^2 X_{T+1}(X'X)^{-1}X'IX(X'X)^{-1}X_{T+1}'$$
$$= \sigma^2(1 + X_{T+1}(X'X)^{-1}X_{T+1}'). \qquad (4.27)$$

The 95 percent confidence interval about a one step-ahead forecast therefore is:

$$Y \pm t_{(\alpha=.05/2)}\sqrt{s^2[1 + X_{T+1}(X'X)^{-1}X_{T+1}]}. \qquad (4.28)$$

Problems With Employing Econometric Models for Forecasting Purposes

In employing econometric models to forecast prices a problem arises because current values of quantities and demand factors are used to explain current values of price, while in reality the values of these explanatory variables might not be available at the time the forecast was made.

Two possible solutions to this problem have been presented in past econometric studies of the cattle cycle. Ehrich (1967), Keith (1976), Pyne (1980), and Stillman (1985) attempted to forecast the values of the exogenous variables into the future, and then used these to forecast within the estimated equation explaining price. Another possible solution, suggested by Bessler and Brandt (1981) is to lag the values of the explanatory variables. Both of these proposed solutions are not without problems.

If the values of the explanatory variables are forecasted into the future, quantities for period T+1 are forecasted and these forecasts are employed in a price equation which was estimated over T periods in order to generate price forecasts for period T+1. Since predicted values of the explanatory variables have been used, the conditional forecasts of the dependent variable, Y, will be less reliable than when the explanatory variables are fixed in repeated sampling, and the confidence intervals for the forecasted errors will be increased. This result can be shown with the model:

$$\hat{Y}_{T+1} = \hat{X}_{T+1}\hat{\beta}, \tag{4.29}$$

when:

$$\hat{X}_{T+1} = X_{T+1} + u_{T+1}.$$

The assumptions of this model also include:

$$u_t \sim N(0, \sigma_u^2)$$

$$E[\epsilon_t, u_t] = 0.$$

The forecast error when the values of the X's in period T+1 must be forecasted is:

$$\begin{aligned}\hat{e}_{T+1} &= (X_{T+1} + u_{T+1})\hat{\beta} - \epsilon_{T+1} - (X_{T+1})\beta \\ &= -\epsilon_{T+1} - X_{T+1}\beta + X_{T+1}(X'X)^{-1}X'(X\beta + \epsilon) \\ &\quad + u_{T+1}(X'X)^{-1}X'(X\beta + \epsilon) \\ &= -\epsilon_{T+1} - X_{T+1}\beta + X_{T+1}(X'X)^{-1}X'X\beta \\ &\quad + X_{T+1}(X'X)^{-1}X'\epsilon + u_{T+1}(X'X)^{-1}X'X\beta \\ &\quad + u_{T+1}(X'X)^{-1}X'\epsilon \\ &= -\epsilon_{T+1} + X_{T+1}(X'X)^{-1}X'\epsilon \\ &\quad + u_{T+1}(X'X)^{-1}X'X\beta \\ &\quad + u_{T+1}(X'X)^{-1}X'\epsilon. \end{aligned} \tag{4.30}$$

The expected value of the forecast error will still be equal to zero assuming that the $\hat{\beta}$'s are unbiased estimators of the true parameters and the forecasts of the X's are unbiased:

$$\begin{aligned}E[\hat{e}_{T+1}] &= E[-\epsilon_{T+1}] + X_{T+1}(X'X)^{-1}E[X'\epsilon] \\ &\quad + E[u_{T+1}](X'X)^{-1}X'X\beta \\ &\quad + u_{T+1}(X'X)^{-1}E[X'\epsilon] \\ &= 0. \end{aligned} \tag{4.31}$$

If the values for X must be forecasted then, the variance of the forecast error will be:

$$E[\hat{e}^2_{T+1}] = \sigma^2 + \sigma^2 X_{T+1}(X'X)^{-1}X_{T+1}' + \beta(X'X)^{-1}\beta'\sigma_u^2 + \sigma^2(X'X)^{-1}\sigma^2_u. \quad (4.32)$$

It can be seen that the variance of the forecast error increases by the terms $\beta(X'X)^{-1}\beta'\sigma_u^2$ and $\sigma^2(X'X)^{-1}\sigma^2_u$. The consequences of using predicted values of X's is that, while the parameters may be significant and a good fit may be indicated, the forecasts may not be very accurate.

A second solution to the problem of obtaining values for explanatory variables included in the model is to use lagged values of the explanatory variables. This technique was employed by Bessler and Brandt (1981) to forecast quarterly steer prices. While lagging the exogenous variables provides a solution which is relatively easy to employ, it may result in the misspecification of the model. For shorter time intervals between prices, such as quarterly data in which the value of the exogenous value is close to what it was in the lagged period, the concern over misspecification might be diminished relative to longer time intervals between prices which are less correlated. Returning to the theoretical framework behind recursive models, which states that current prices are based on current quantities , it becomes apparent that an equation which hypothesizes that current prices are determined by last year's quantities may be misspecified. The result of this misspecification will be that forecasts produced by this type of model will generally lag behind major price turns, introducing positive serial correlation of the residuals.

An alternative to the approaches of including forecasted explanatory variables or lagged explanatory variables would be to include variables which predict the explanatory variables directly in the equation of interest, rather than forecasted values of the explanatory variables. For example, many of the recursive models employ slaughter in period t as a explanatory variable for price in time t. Slaughter in time t is then hypothesized to be caused by inventories, which are predetermined, and by lagged prices. The approach described above would directly include inventories and lagged prices rather than slaughter, as was done in this study. The benefits of this approach are twofold: the explanatory variables are fixed in repeated sampling so the confidence intervals on the forecast errors are not widened, and the model does not solely include lagged explanatory variables which in economic theory should not be lagged.

The Estimated Model

The estimated model for 1925 to 1965, using ordinary least squares was:

$$\hat{FBP}_t = \underset{\substack{[5.09957016] \\ (3.404)}}{17.3609792} - \underset{\substack{[.0002388861] \\ (-3.477)}}{.000830617}\ ICA_{t-1}$$

$$+ .00004112942 \, RTDPI_{t-1}$$
$$[.00001159181]$$
$$(3.548)$$

$$+ .2844346 \, FCRAT_{t-1} + .0510765 \, PHF_{t-1}$$
$$[.08290662] \qquad [.12642747]$$
$$(3.431) \qquad (.404)$$

$$- .1658367 \, PCF_{t-1} \qquad (4.33)$$
$$[.08953385]$$
$$(-1.852)$$

$k = 5$ $\qquad$ $R^2 = .7917$

$N = 40$ $\qquad$ $DW = 1.1218$

$SSE = 97.57061$

$MSE = 2.869724$

$DFE = 34$

The R^2 value of .7917 suggests that approximately 79.17 percent of the variation in the real price of beef cattle received by farmers in the United States is explained by the variation in the independent variables over the period of 1925-1965. The calculated F was equal to 21.53, while the tabulated F with 5 and 34 degrees of freedom at the .05 probability level is approximately 2.49, therefore the null hypothesis, H_0: $\beta_2 = \beta_3 = \ldots \beta_K = 0$, must be rejected.

The calculated t values indicated that all of the regression coefficients were siginificant at the 5 percent probability level except for the regression coefficient on the price of hogs received by farmers and the regression coefficient on the farm price of chickens. The tabulated t value with 34 degrees of freedom, at the 5 percent probability level is approximately 2.033. Thus, the regression coefficients for the intercept, the regression coefficients on ICA_{t-1}, $RTDPI_{t-1}$, and $FCRAT_{t-1}$ were all greater than 2.033, and could not be accepted as being equal to zero.

The regression coefficients are interpreted in terms of the change in the dependent variable resulting from a change in the independent variable. For example, the regression coefficient on the one year lag of the January 1 inventories indicates that a one unit change in inventories (000,000 head) resulted in a .83 unit change in the price of beef cattle ($/cwt) in the opposite direction. The coefficient on total disposable personal income shows that a one unit change in total personal income (000 $) resulted in a .04 unit change in the price of beef cattle received by farmers. The regression coefficient on the feeder cattle ratio indicates that a one unit change in the ratio [($/cwt)/($/bu)], resulted in a .28443 unit change in the price of beef cattle. The regression coefficent on the price of hogs received by farmers indicates that a one unit change in the price of hogs will result in a .05107 unit change in the price of beef cattle. Finally, the regression coefficient on the price of chickens indicated that a one unit change in the price of chickens would result in a -.16583 unit change in the price of beef cattle. The signs on all of the estimated regression coefficients were consistent with those hypothesized according to economic theory except for the sign on the coefficient for the price of chickens. This may be due to the variable specification for the price of poultry. In addition, other studies during this time period have also found an ambiguous relationship between beef and poultry.[6]

It may be noted that the calculated Durbin Watson statistic:

$$DW = \frac{\sum_{t=2}^{T} (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2}{\sum_{t=1}^{T} \hat{\epsilon}^2_t} \qquad (4.34)$$

was 1.1218, indicating possible positive serial correlation of the residuals. The null hypothesis is that the no serial correlation is present or:

$$\rho = 0.$$

If the test statistic is less than the tabulated d_l, then the null hypothesis that no first order autocorrelation of the resuduals exists must be rejected. When DW is greater than d_u, the null hypothesis cannot be rejected. If DW lies between d_u and d_l, the results are inconclusive. Due to the fact that the calculated DW = 1.1218, was less than d_l = 1.22, for K = 5 parameters, and N = 40 observations, the null hypothesis that $\rho = 0$ was rejected at the 5 percent level. Thus, the assumption:

$$E[\epsilon_t \epsilon_s] = 0$$

of the classical linear model was violated. Violation of this assumption leads to biased estimates of the standard errors. The estimates of the standard errors will tend to be underestimated, therefore the t-statistics will be overestimated. With overestimates of the calculated t-statistics, the chance of committing a Type I error, or rejecting a true null hypothesis is increased. If the errors contain serial correlation, then the error structure will actually be:

$$\epsilon_t = \rho_1\epsilon_{t-1} + \rho_2\epsilon_{t-2} + \ldots + \rho_p\epsilon_{t-p} + u_t \qquad (4.35)$$

$$= \sum_{p=0}^{\infty} \rho^p u_{t-p}$$

where:

$$E[u_t] = 0$$

$$E[u_t u_s] = \sigma_u^2, \; s = t$$

$$= 0, \; s \neq t$$

$$E[\epsilon_{t-1} u_t] = 0.$$

The variance of the errors may then be expressed as:

$$E[\epsilon_t\epsilon_t] = \sigma^2$$

$$= Var[u_t] + Var[\rho u_{t-1}] + Var[\rho u_{t-2}]$$

$$+ \ldots + Var[u_{t-p}]$$

$$= \sigma^2[1/(1-\rho^2)], \qquad (4.36)$$

and the covariance of the errors is:

$$E[\epsilon_t\epsilon_{t+r}] = \rho^r\sigma_\epsilon^2$$

$$= \frac{\rho^r\sigma^2_u}{1 - \rho^2} \qquad (4.37)$$

Thus, the variance-covariance matrix with first order autocorrelation of the residuals is:

$$E[\epsilon\epsilon'] = \sigma_\epsilon^2 \, \Omega$$

$$= \sigma_\epsilon^2 \begin{bmatrix} 1 & \rho & \rho^2 \dots & \rho^{T-1} \\ \rho & 1 & \rho \dots & \rho^{T-2} \\ & & . & \\ \dots & \dots & \dots & \dots \\ & & . & \\ & & . & \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} \dots & 1 \end{bmatrix} \quad (4.38)$$

The appropriate technique when positive serial correlation is present is generalized least squares, or GLS, because GLS uses information concerning the true error structure to find estimates for β. Namely, if we know what ρ is, then GLS techniques can be used to transform the data so that the variance-covariance matrix of the transformed errors is σ^2 I.

The GLS estimator for $\hat{\beta}$ is:

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y. \quad (4.39)$$

The estimated variance-covariance for $\hat{\beta}_{GLS}$ will be:

$$s_{\hat{\beta}}^2 = s^2(X'\Omega^{-1}X)^{-1} \quad (4.40)$$

where:

$$s^2 = \frac{\hat{\epsilon}\Omega^{-1}\hat{\epsilon}}{T-k}$$

and:

$$\hat{\epsilon} = Y - X\hat{\beta}_{GLS}.$$

The data must be transformed in such a way that the variance of the transformed errors will be σ^2I. If a TxT matrix, H, exists such that:

$$H\Omega^{-1}H = I \tag{4.41}$$

then H can be used to transform the data to produce residuals with the variance $\sigma^2 I$. Equation (4.41) can be rewritten as:

$$\Omega = H^{-1}(H')^{-1} = (H'H)^{-1} \tag{4.42}$$

The matrix, H, in the case of first order autocorrelation of the residuals would be:

$$H = \begin{bmatrix} 1-\rho^2 & 0 & 0 & \ldots & 0 & 0 \\ -\rho & 1 & 0 & \ldots & 0 & 0 \\ 0 & -\rho & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & -\rho & 1 \end{bmatrix}$$

Thus, the model may be transformed as:

$$HY = HX\beta + H\epsilon \tag{4.43}$$

or:

$$W = Q\beta + V$$

where:

$$W = HY$$

$$Q = HX$$

$$V = H\epsilon$$

$$\beta = (Q'Q)^{-1}QW = (X\Omega X)^{-1}X'\Omega^{-1}Y.$$

The error term will meet the assumption of the classical linear model since:

$$E[V'V] = E[V\epsilon\epsilon'V'] = \sigma^2 H\Omega H' = \sigma^2 I, \tag{4.44}$$

so that V is an efficient estimator. Furthermore, if ρ is known with certainty, then $\hat{\beta}_{GLS}$ is an unbiased estimator.

If ρ is not known with certainty, then it must be estimated. The value for ρ may be estimated by applying Ordinary Least Squares to the data and obtaining the values for the estimated residuals. The calculated value for ρ is:

$$\hat{\rho} = \frac{\sum_{t=2}^{T} \hat{\epsilon}_t \hat{\epsilon}_{t-1}}{\sum_{t=2}^{T} \hat{\epsilon}_{t-1}} \qquad (4.45)$$

The calculated value for $\hat{\rho}$ was -.38150032, and is used to generate H. The Y vector and the X matrix can then be transformed by H. The resulting transformed data will be:

$$W_1 = Y_1\sqrt{1-\rho^2}$$
$$W_t = Y_t - \rho Y_{t-1}, \; t = 2 \ldots T$$
$$Q_{1i} = X_{1i}\sqrt{1-\rho^2}, \; i = 1 \ldots k$$
$$Q_{ti} = X_{ti} - \rho X_{t-1,k}, \; t = 2 \ldots T$$
$$i = 1 \ldots k.$$

The estimates resulting from the application of the Prais-Winsten method will be consistent and assymptotically efficient.

The model was re-estimated using the Prais-Winsten procedure to correct for first order autocorrelation of the residuals. The re-estimated model was as follows:

$$\hat{FBP}_t = 19.0240769 \quad - .000897857 \; ICA_{t-1}$$

[5.150097021] [.00028458971]

(3.694) (-3.155)

$$+ .00004682572 \text{ RTDPI}_{t-1}$$
$$[.00001361366]$$
$$(3.440$$

$$+ .1823968 \text{ FCRAT}_{t-1} + .0256857 \text{ PHF}_{t-1}$$
$$[.08268221] \qquad [.12048931]$$
$$(2.206) \qquad (.213)$$

$$- .1764108 \text{ PCF}_{t-1} \qquad (4.46)$$
$$[.09219294]$$
$$(-1.913)$$

R^2 = .88336

K = 6

N = 39

SSE = 77.95274

MSE = 2.362204

DFE = 33

The R^2 value of .8836 suggested that approximately 88.36 percent of the variation in the dependent variable, the farm price of beef cattle, was explained by the variation in the independent variables. The calculated F of 27.55 was greater than the tabled F at a 5 percent significance level, which is 2.50, therefore the null hypothesis, $H_0: \beta_2 = \beta_3 = \ldots = \beta_K = 0$ had to be rejected.

The calculated t values, shown in the parentheses below the regression coefficients in Equation (4.46), were all significantly different from zero at the five percent probability level except for the coefficient on the price of hogs and the coefficient on the price of chickens.

Except for the sign on the coefficient for the price of chickens, the signs on all of the estimated regression coefficients were in agreement with those postulated by economic theory. Similar results were found by Ehrich over the period of 1944-1964, who found a negative relationship between the price of fed cattle and the quantity of steers and heifers slaughtered. Stillman (1985) found that the coefficient on fed cattle marketings and nonfed steer and heifer slaughter, both being measures of available slaughter, showed negative signs in relation to steer prices. Stillman also found that feeder prices in year t-1 showed a positive relationship with steers and heifers greater than 500 pounds in year t, between 1955 and 1981, indicating that higher feed prices lead to a higher number of animals placed in feedlots and fed to maturity rather than being slaughtered at less than 500 pounds. Stillman's findings for the relationship between quantities and feeder prices coincided with Reutlinger's (1966) findings that a beef corn price ratio lagged by one year would have a positive relationship with cattle slaughter. While Ehrich found the sign on the coefficient for personal income to be positive, as suggested by economic theory, the coefficient was not significantly different from zero. Ehrich hypothesized that this effect might be due to increased consumer incomes resulting in greater demand for services at the retail level, so that an increase in consumer incomes would have little effect on the farm level demand for beef. Additionally, Ehrich did not find the regression coefficient on either the price of live pork or the price of broilers to be significantly different from zero over the period of his study.

To ensure that no higher order autocorrelation of the residuals existed, the autocorrelation of the residuals were calculated and plotted. All of the autocorrelations fell within two standard errors, therefore no significant higher order autocorrelation was found at the five percent significance level. Additionally, a plot of the squared residuals versus the dependent variable did not show any identifiable patterns indicating heteroscedasticity of the residual variance.

Having determined the model to be adequate for the purposes of forecasting, the model was used to make a one step ahead forecast of the price of beef cattle. The model, which was estimated over the period of 1925-1965, was updated and re-estimated with each additional year through 1985, and used to make a series of one step ahead forecasts. These one step ahead forecasts will be presented in Chapter Six. The regression coefficients, standard errors of the coefficents, t values for the coefficients, R^2 values, and other relevant regression statistics are presented in Table II. Each of the updated models showed a low Durbin-Watson statistic when estimated with OLS, therefore the estimated models shown in Table II have been corrected for first order autocorrelation of the residuals. For each of these models a check was also made for higher order autocorrelation of the residuals, and the squared residuals were plotted to check for heteroscedasticity, which did not show evidence of either problem within the residuals.

As may be seen in Table II all of the signs on the regression coefficients were in agreement with those hypothesized by economic theory, for each of the updated models from 1966 through 1985, except for the sign on the regression coefficient for the price of chickens. While direct comparisons between each updated model may be made, some trends in the significance of certain esimated parameters may be observed. The coefficients on the price of chickens were not found to be significantly different from zero until the model estimated for 1974. In 1974 and beyond, the absolute value of the coefficient inceased while the standard error of the coefficient remained fairly close to values in the earlier years, thus producing a larger calculated t statistic. The coefficient on the January 1 inventories of calves was significantly different from zero at the 5 percent probability level until 1983, beyond which the sign on the coefficient was still in agreement with economic theory but was insignificant. The regression coefficient tended to diminish in magnitude with each updated model, while the standard errors of the regression coefficient also dropped. The regression coefficients on

TABLE II

TABLE OF REGRESSION COEFFICIENTS AND RELEVANT REGRESSION STATISTICS

Year	β_0, SE_β, t	β_1, SE_β, t	β_2, SE_β, t	β_3, SE_β, t	β_4, SE_β, t	β_5, SE_β, t	R^2	MSE	Est. Autoreg.
1965	19.0240769 [5.15009702] (3.694)	-.000897857 [.0002845897] (-3.155)	.00004682572 [.00001361366] (3.440)	.1823968 [.08268221] (2.206)	.0256857 [.12048931] (.213)	-.1764108 [.09219294] (-1.913)	.8336	2.362204	-.38150032 [.16091194] (-2.370)
1966	17.4198015 [4.55909387] (3.821)	-.00081159 [.0002547681] (-3.185)	.00004456997 [.00001313131] (3.393)	.1884138 [.08122855] (2.320)	.0555996 [.11146660] (.499)	-.1608851 [.08878790] (-1.812)	.8375	2.320725	-.38847746 [.15802878] (-2.458270)
1967	16.9077038 [4.37376529] (3.866)	-.000794347 [.0002497306] (-3.181)	.00004466788 [.00001301492] (3.432)	.1882421 [.08028125] (2.345)	.0623142 [.10919301] (.571)	-.1551349 [.08697372] (-1.784)	.8458	2.266397	-.39208610 [.15549630] (-2.52151)
1968	16.9451172 [4.26396133] (3.974)	-.000795523 [.0002454755] (-3.241)	.00004463975 [.00001282213] (3.481)	.1876995 [.07870455] (2.385)	.0627265 [.10739834] (.584)	-.1556786 [.08522978] (-1.827)	.8551	2.20344	-.39231997 [.15330480] (-2.559085)
1969	16.4997081 [4.16460353] (3.962)	-.000793009 [.00004566586] (-3.270)	.00004566586 [.00001253915] (3.642)	.1886732 [.07803996] (2.418)	.0569811 [.10612025] (.537)	-.1465786 [.08309657] (-1.764)	.8651	2.168651	-.38747327 [.15155629] (-2.556629)
1970	16.3388297 [3.99464257] (4.090)	-.00078975 [.0002385623] (-3.310)	.00004572562 [.00001237153] (3.696)	.189817 [.07673492] (2.474)	.0588867 [.10412086] (.566)	-.1445606 [.08111461] (-1.782)	.8754	2.113161	-.38739994 [.14955384] (-2.590371)
1971	15.8654069 [3.97355581] (3.993)	-.000792709 [.0002379657] (-3.331)	.00004739227 [.00001223765] (3.873)	.184739227 [.07636887] (2.387)	.0585442 [.10440205] (.561)	-.1362591 [.08077283] (-1.687)	.8838	2.122579	-.38214032 [.149797512] (-2.582463)
1972	15.7707554 [3.91541509] (4.028)	-.000794498 [.0002358953] (-3.368)	.0000478877 [.00001205778] (3.972)	.1860112 [.07405972] (2.512)	.0493618 [.09954484] (.496)	-.1327089 [.0791047] (1.678)	.8957	2.071984	-.38601243 [.14585904] (-2.646476)
1973	16.4055976 [3.83719652] (4.275)	-.000798758 [.000234086] (-3.412)	.00004658239 [.00001193023] (3.905)	.1971267 [.07356385] (2.680)	.0409408 [.09929055] (.412)	-.1426465 [.07806163] (-1.827)	.8981	2.075608	-.37668570 [.14467018] (-2.903755)
1974	17.6356311 [4.12690705] (4.273)	-.000640576 [.0002419078] (-2.648)	.00003148037 [.0000117084] (2.689)	.2954503 [.07644670] (3.865)	.0704651 [.11088132] (.635)	-.1984364 [.081555305] (-2.433)	.8709	2.574335	-.31372869 [.14651299] (-2.141303)

TABLE II (Continued)

Year	β_0, SE_β, t	β_1, SE_β, t	β_2, SE_β, t	β_3, SE_β, t	β_4, SE_β, t	β_5, SE_β, t	R^2	MSE	Est. Autoreg.
1975	17.6083191 [4.11059996] (4.284)	-.000647267 [.0002422631] (-2.672)	.00003037887 [.00001160643] (2.617)	.3164716 [.06854902] (4.617)	.0865835 [.10734529] (.807)	-.2050805 [.08108907] (-2.529)	.8699	2.536623	-.32497069 [.14422157] (-2.253274)
1976	17.6094909 [3.70734971] (4.750)	-.000646579 [.0002198974] (-2.940)	.0000303609 [.00001106279] (2.744)	.3159413 [.06770424] (4.666)	.0866637 [.10008807] (.866)	-.2050825 [.07749602] (-2.646)	.8700	2.477496	-.32672785 [.14248197] (-2.293117)
1977	17.9413836 [3.81315438] (4.705)	-.000592846 [.0002221878] (-2.668)	.0000255562 [.00001102399] (2.318)	.3411625 [.06947954] (4.910)	.0970471 [.10435733] (.930)	-.2196775 [.0789417] (-2.783)	.8561	2.680908	-.29561486 [.14240879] (-2.075819)
1978	18.0465619 [3.77014687] (4.787)	-.000617932 [.000203203] (-3.041)	.00002708667 [.00000964025] (2.810)	.335475 [.06734184] (4.981)	.0908700 [.10139251] (.896)	-.2181091 [.07800713] (-2.796)	.8599	2.623219	-.30058079 [.14062370] (-2.137483)
1979	18.0317192 [3.72912428] (4.835)	-.000600496 [.0001723923] (-3.483)	.00002619125 [.0000079066] (3.13)	.3356167 [.06658069] (5.041)	.0915942 [.10019626] (.914)	-.2187010 [.07715669] (-2.835)	.8725	2.56818	-.30150539 [.13907709] (-2.167901)
1980	16.1443673 [3.93822743] (4.099)	-.000360212 [.0001597218] (-2.255)	.00001414944 [.00000714522] (1.980)	.3434914 [.07123008] (4.822)	.1689341 [.10468617] (1.614)	-.2122651 [.08186663] (-2.593)	.8554	2.988104	-.26984190 [.13898331] (-1.941542)
1981	15.7626085 [3.85441422] (4.089)	-.00033156 [.0001496098] (-2.149)	.00001206059 [.00000639215] (1.887)	.3488015 [.06893132] (5.060)	.1851846 [.09995197] (1.853)	-.2116258 [.08193271] (-2.583)	.8579	2.93791	-.28840331 [.13678700] (-2.108412)
1982	15.7619073 [3.84092830] (4.104)	-.000297405 [.0001460035] (-2.037)	.00001062052 [.00000610184] (1.741)	.3494758 [.06865452] (5.090)	.1905490 [.09927690] (1.919)	-.2141734 [.08161920] (-2.624)	.8595	2.913531	-.29048630 [.13532315] (-2.146612)
1983	15.7544244 [3.86616953] (4.075)	-.000247905 [.0001418345] (-1.748)	.00000832791 [.00000586511] (1.420)	.3560368 [.06867957] (5.184)	.1731891 [.09896558] (1.750)	-.2108494 [.08222053] (-2.564)	.8575	2.939008	-.29756783 [.13368483] (-2.225891)
1984	15.7628437 [3.79127585] (4.158)	-.000248733 [.0001320169] (-1.884)	.00000837355 [.00000515631] (1.624)	.3557245 [.06501632] (5.471)	.1729757 [.09705297] (1.782)	-.2108622 [.08141784] (-2.590)	.8592	2.88263	-.29745612 [.13239799] (-2.246681)
1985	15.6196864 [3.79221161] (4.152)	-.000195802 [.00009915024] (-1.975)	.00000562196 [.0000256415] (2.193)	.3696254 [.06111188] (6.048)	.1794511 [.09605801] (1.868)	-.2173178 [.08015139] (-2.711)	.8582	2.851593	-.29462156 [.13126368] (-2.244502)

total disposable personal income were statistically significant from zero at the five percent probability level in each updated model until 1980, and the coefficient was again significant in 1985. The regression coefficient tended to decrease in magnitude through time. The regression coefficients on the feeder cattle ratio were also significantly different from zero in each of the updated models, the calculated t values on this estimated coefficient tended to increase with each updated model. While the standard error of the estimated coefficient remained fairly stable through time, the magnitude of the estimated coefficient increased with the annual updating. The regression coefficients on the price of hogs, while showing the appropriate sign according to economic theory were not significantly different from zero in any of the estimated models. While this is true, the t values did tend to increase with the updating of the model.

The regression R^2 for each of the models fell between .8 and .9; the R^2 with the highest values occurring between 1971 and 1974. The MSE tended to be slightly higher compared with other years between 1980 and 1983. The estimate of ρ used in the transformation of the data to correct for first order autocorrelation of the residuals tended to drop with the updating of the model.

While the relationship between the price of cattle and the various predetermined variables tended to change through time, the changes were very gradual. For example the inventories of calves tended to decrease in importance in explaining the price of beef cattle over the period of study, and the feeder cattle ratio tended to increase in importance in explaining the price of beef cattle. A lengthy time period is covered in the model and the updated model, thus some structrural change might be expected. While this is true, the regression R^2 and the MSE remained fairly stable indicating an adequate fit. The updated models were used therefore to make one step ahead forecasts from 1966-1985.

FOOTNOTES

[1]The estimate of the correlation between the deflated average price of choice steers at Chicago and the deflated price of beef cattle received by farmers over the period of 1925-1948 is .7304 and between the deflated price of choice steers at Omaha and the deflated price of beef cattle received by farmers between 1948 and 1983 is .5067.

[2]The estimate of the correlation between the January 1 inventories of calves and the slaughter of beef cattle over the period of 1925-1985 is .9860.

[3]The estimate of the correlation between average feeder prices and average beef cattle prices over the period of 1925-1985 is .9839.

[4]The estimate of the correlation between the CPI and the FPI over the period of 1925-1985 is .9613.

[5]See Foote (1958, pp. 28,33) and Tomek and Robinson (1981, pp. 321-322) for criteria used in deciding when to deflate and which deflator should be used.

[6]Other studies with ambiguous findings for the sign on the regression coefficient for poultry prices include Ehrich, Brandt and Bessler, and Stillman, which did not find the coefficient relating cattle prices to various measures of poultry prices or quantities to be significant.

CHAPTER FIVE

THE TIME SERIES MODEL

Introduction to Time Series Models

In addition to the econometric model, a time series technique was developed to forecast cattle prices. Time series techniques are useful for the purpose of forecasting when the data have a strong correlation between observations at different time periods. Under such conditions, conclusions about the future behavior of the series may be inferred from past values of the variable. According to Pindyck and Rubinfeld (1981, p. 493), time series techniques differ from merely extrapolating into the future in that they assume that "the series to be forecasted have been generated by a stochastic (or random) process with structures that can be characterized or described." The time series technique used in this study was an autoregressive integrated moving average or ARIMA technique. ARIMA models, as the name implies, can accomodate data containing both autoregressive and moving average processes.

Past studies have used ARIMA models for the purposes of economic forecasting, and in particular, for generating commodity price forecasts. These include Leuthold (1970), Brandt and Bessler (1981), Standaert (1981), Harris and Leuthold (1983), and Granger and Newbold (1984). These studies employed shorter term data, analyzing daily, weekly, monthly, and quarterly data. One reason ARIMA models have not been previously used to analyze cyclical behavior is the lack of annual data covering an adequate time span. Other studies of the cattle cycle, including the work of Franzmann and Walker (1972) and Helmers and Held (1977), have employed techniques such as trend models.

Autoregressive models express the value of the variable at time t, or X_t, as a linear combination of past X values in the form:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \Phi_3 X_{t-3} + \Phi_p X_{t-p} + \epsilon_t + \delta , \qquad (5.1)$$

where the model is an autoregressive model of order p, AR(p), and:

$\Phi_1 \ldots \Phi_p$ are the autoregressive parameters

ϵ_t is a random error process at time t

δ is a constant.

The model may also be written more compactly as:

$$\Phi(B)z_t = \epsilon_t \qquad (5.2)$$

where

$$\Phi(B) = 1 - \sum_{j=1}^{p} \Phi_j B^j$$

and

$$z_t = X_t - \mu .$$

B is a backshift operator so that $B^j X_t = X_{t-j}$ and z_t is the deviations of the data series from its mean.

The random error process, ϵ_t, has the properties:

$$E[\epsilon_t] = 0$$

$$E[\epsilon_t \epsilon_{t-K}] = 0, K \neq 0$$

$$E[\epsilon_t X_{t-i}] = 0, i = 1...p$$

$$Var[\epsilon_t] = (\Phi(B))^2 \, Var \, X_t = \sigma^2_\epsilon.$$

While autoregressive models express the value of the variable at time t as a linear combination of past values of the variable, moving average models express X_t as a linear combination of past errors:

$$X_t = \mu + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \ldots - \theta_q \epsilon_{t-q}. \quad (5.3)$$

where

μ is the true mean of the process

ϵ is the random error process

θ_1 ... θq are the moving average parameters.

The model may also be written as:

$$z_t \; (1 - \theta B) \epsilon_t \quad (5.4)$$

where

$$(\theta B) = \sum_{j=0}^{q} \theta_j B^j, \; \theta_0 = 1.$$

B is a backshift operator, so that $B^j \epsilon_t = \epsilon_{t-j}$

The random disturbances, or each ϵ_t is assumed to be normally distributed with a mean of zero and variance σ^2_ϵ and $E[\epsilon_t \epsilon_{t-K}] = 0$ for $K \neq 0$. Box and Jenkins (1970, p. 46) state:

> ...ϵ_t may be regarded as a series of shocks which drive the system. It consists of a sequence of uncorrelated random variables with mean zero and constant variance,...

ARIMA models include both autoregressive and moving average terms, in which case, the model may be written:

$$X_t = \phi_1 X_{t-1} + \dots + \phi_{t-p} X_{t-p} + \delta - \epsilon_t - \theta_1 \epsilon_{t-1} - \dots - \theta_q \epsilon_{t-q} , \quad (5.5)$$

or:

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) X_t = \delta + (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q)\, \epsilon_t, \quad (5.6)$$

which can be expressed as:

$$\Phi(B) X_t = \delta + (B)\epsilon_t, \quad (5.7)$$

or:

$$\Phi(B) z_t = (B)\epsilon_t \quad (5.8)$$

where the model is an ARMA(p,q). Again, the random errors, ϵ_t are assumed to be independent, normally distributed variables with means of zero and variance σ^2_ϵ.

Properties of Autoregressive and Moving Average Processes

A premise of employing time series models for the purposes of forecasting is that past behavior of the series can be used to infer information about future behavior of the series. In order to simplify the process of modelling, the requirement of stationarity of the series is made. Stationarity implies that a set of fixed coefficients can be used to model the series. The series will then have a constant mean level and Pindyck and Rubinfeld (1981, p. 497) state that "the probability of a given fluctutation in the process from the mean level is assumed to be the same at any given time," so that, "the stochastic properties of the stationary process are assumed to be invariant with respect to time."

Given that an autoregressive process is stationary, the mean will be invariant through time, so:

$$E(X_t) = E(X_{t-1}) = \ldots = \mu. \tag{5.9}$$

The mean can therefore be written as:

$$\mu = \Phi_1\mu + \Phi_2\mu + \ldots + \Phi_p\mu + \delta \tag{5.10}$$

or

$$\mu = \frac{\delta}{1 - \Phi - \Phi_2 - \ldots - \Phi_p}.$$

If the mean is to be invariant through time, then a neccessary condition for stationarity is:

$$\Phi_1 + \Phi_2 + \ldots + \Phi_p < 1. \tag{5.11}$$

Furthermore, to ensure stationarity of an autoregressive process, if X_t is a stationary process, then $\Phi^{-1}(B)$ must converge, where:

$$z_t = \Phi^{-1}(B)\epsilon_t, \tag{5.12}$$

and

$$z_t = X_t - \mu.$$

If the value $\Phi(B) = 1 - \sum_{j=1}^{p}\Phi_j B^j$ is expressed as:

$$(1-G_1B)(1-G_2B)\ldots(1-G_pB), \tag{5.13}$$

then:

$$z_t = \Phi^{-1}(B)\epsilon_t = \sum_{i=1}^{p} \frac{K_i}{(1-G_i B)}\epsilon_t \ . \qquad (5.14)$$

Therefore, if $\Phi^{-1}(B)$ is convergent for $|B| \leq 1$, then $|G_i| < 1$, $i=1...p$, or, the roots of $\Phi(B) = 0$ must lie outside the unit circle.

For a stationary series, an autoregressive process can be expressed as a moving average process:

$$z_t = \Phi^{-1}(B)\epsilon_t = \theta(B)\epsilon_t , \qquad (5.15)$$

The mean of the stationary process, z_t, is therefore:

$$E[z_t] = E[\epsilon_t] = \mu = 0 \ , \qquad (5.16)$$

and the variance of z_t is:

$$Var[z_t] = \sigma_\epsilon^2 \sum_{j=0}^{\infty} \theta_j^2 \qquad (5.17)$$

If the process in Equation (5.15) is to have a finite variance, the weights, θ_j, must decrease, so that the series on the right will converge. If the weights do not decrease as j increases, Equation (5.17) shows that the variance will increase as j increases.

The autocovariance, $COV[z_t, z_{t+\tau}] = E[(Z_t - \mu_z)(Z_{t+K} - \mu_z)]$, of an autoregressive process is:

$$\gamma_0 = \Phi_1\gamma_1 + \ldots + \Phi p\gamma_p + \sigma_\epsilon^2$$

$$\gamma_1 = \Phi_1\gamma_0 + \ldots + \Phi p\gamma_{p-1}$$

$$\vdots \qquad (5.18)$$

$$\gamma_p = \Phi_1\gamma_{p-1} + \ldots + \Phi_p\gamma_0 \ .$$

For lags greater than p:

$$\gamma_k = \phi_1\gamma_{k-1} + \ldots + \phi_p\gamma_{k-p}, \quad k>p. \tag{5.19}$$

Dividing (5.18) through by γ_0, produces the theoretical autocorrelations:

$$\begin{aligned}
\rho_0 &= \phi_1\frac{\gamma_1}{\gamma_0} + \ldots + \phi_p\frac{\gamma_p}{\gamma_0} + \frac{\sigma_\epsilon^2}{\gamma_0} \\
&= 1 = \phi\rho_1 + \ldots + \phi_p\rho_p + \frac{\sigma_\epsilon^2}{\sigma_z^2} \\
\rho_1 &= \frac{\gamma_1}{\gamma_0} = 4_1\frac{\gamma_p}{\gamma_0} + \ldots + \phi_p\frac{\gamma_p}{\gamma_0} \\
&= \phi_1 + \ldots + \phi_p\gamma_{p-1} \\
&\;\;\vdots \\
\rho_p &= \frac{\gamma_p}{\gamma_0} = \phi\frac{\gamma_0}{\gamma_0} + \ldots + \phi_p\frac{\gamma_p}{\gamma_0} \\
&= \phi\gamma_{p-1} + \ldots + \phi_p\gamma_p.
\end{aligned} \tag{5.20}$$

These equations are often referred to as the Yule-Walker equations.

If a moving average process can be inverted to a purely autoregressive process, then:

$$z_t = \theta(B)\epsilon_t \tag{5.21}$$

can be rewritten as:

$$\theta^{-1}(B)z_t = \epsilon_t$$

or:

$$\epsilon_t = z_t + \theta z_{t-1} + \theta^2 z_{t-2} + \ldots$$

It can be noted that if $|\theta|\geq 1$, the weights will diverge in the expansion. Thus, if $\pi_j = -\theta^j$, and

$$z_t = - \theta z_t - \theta^2 z_{t-2} \ldots + \epsilon_t, \tag{5.22}$$

the weights on past values would increase as j increases. The inverted expansion in Equation (5.22) must form a convergent series where $|\theta|<1$ in order to avoid increasing the weights progressively on lags in the further distant past. The invertiblity condition is independent of the stationarity condition, and is applicable to nonstationary linear models.

If z_t is to be invertible, then $\theta^{-1}(B)$ must converge. Expressing $\theta(B)$ as:

$$\theta(B) = \prod_{j=1}^{q}(1-H_jB) \tag{5.23}$$

gives:

$$\theta^{-1}(B) = \sum_{j=1}^{q} \frac{M_j}{(1-H_jB)} \tag{5.24}$$

which will converge if $H_j<1$, $j=1,2,\ldots,q$. The roots of $\theta(B) = 0$ are H_j^{-1}, so that for a moving average process to be invertible, the roots of:

$$\theta(B) = 1 - \theta B - \theta_2 B^2 - \ldots - \theta_q B^q = 0 \tag{5.25}$$

lie outside the unit circle.

For a moving average process, the mean or expected value for z_t will be:

$$E[z_t] = E[\theta(B)\epsilon_t] = \theta(B)E[\epsilon_t] = 0. \tag{5.26}$$

The variance of z_t is:

$$E[z_t z_t] = (\theta(B))^2\sigma_\epsilon^2$$
$$= (1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2)\sigma_\epsilon^2. \tag{5.27}$$

The autocovariance for the series is:

$$\gamma_k = E[z_t z_{t-k}] = \sigma^2 \sum_{i=0}^{q-k} \theta_i \theta_{i+k}$$

$$= (-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2}$$

$$+ \dots + \theta_{q-k}\theta_q)\sigma_\epsilon^2. \qquad (5.28)$$

For lags greater than q, $\gamma_k = 0$. Since $\rho_k = \gamma_k/\gamma_0$, a moving average process will therefore have autocorrelation coefficients of zero for all lags greater than q.

As with autoregressive models, mixed ARMA models have the requirement for stationarity that the roots of $\Phi(B) = 0$ lie outside the unit circle. For invertibility, the roots of $\theta(B) =$ must also lie outside the unit circle.

The mean or expected value for a mixed ARMA process will be:

$$E[z_t] = E[\Phi^{-1}(B)\theta(B)\epsilon_t] = 0. \qquad (5.29)$$

The variance for an ARMA process is:

$$\gamma_0 = \Phi + \dots \Phi_p\gamma_p + \sigma_\epsilon^2 - \theta_1\gamma_{z\epsilon}(-1) - \dots$$

$$- \theta_q\gamma_{z\epsilon}(-q). \qquad (5.30)$$

The autocovariance for a mixed process may be expressed as:

$$\gamma_k = \Phi_1 + \dots + \Phi\gamma_{k-p} + \gamma_{z\epsilon}(k) - \theta\gamma_{z\epsilon}(k-1)$$

$$- \dots - \theta_q\gamma_{z\epsilon}(k-q), \qquad (5.31)$$

where $\gamma_{z\epsilon}(k)$ is the cross covariance function between z and ϵ:

$$\gamma_{z\epsilon}(k) = E[z_{t-k}\epsilon_t]. \qquad (5.32)$$

The value of z_{t-k} will only depend on shocks up to time t-k so:

$$\gamma_{z\epsilon}(k) = 0, \; k>0$$

$$\gamma_{z\epsilon}(k) \neq 0, \; k\leq 0.$$

For lags greater than q+1,

$$\gamma_k = \phi\gamma_{k-1} + \ldots + \phi_p\gamma_{k-p}, \tag{5.33}$$

so:

$$\rho_k = \phi\rho_{k-1} + \ldots + \phi_p\rho_{k-p}, \; k\geq q+1, \tag{5.34}$$

or

$$\phi(B)\rho_k = 0. \tag{5.35}$$

This implies that q autocorrelations will depend on the q moving average parameters and the p autoregressive parameters.

Employing Time Series Models

Time series modelling entails the following four steps:

(1) Identification

(2) Estimation

(3) Diagnostic Checking

(4) Forecasting.

The identification process involves specifying tentative values for the order of the autoregressive process, p, the moving average process, q, and d, the degree of differencing necessary to achieve stationarity of the series. Possible values for p, q, and d may be determined by examining the sample autocorrelation function and the partial autocorrelation function of the time series.

The autocorrelation function provides a measure of the correlation between observations in different time periods. The autocorrelation with lag k is:

$$\rho_k = \frac{E[(z_t - \mu_z)(z_{t+k} - \mu_z)]}{E[z_t - \mu_z)^2]\ E[(z_{t+k} - \mu_z)^2]}$$

$$= \frac{\mathrm{Cov}(z_t, z_{t+k})}{\sigma_{zt}\ \sigma_{zt+k}} = \frac{\gamma_k}{\gamma_0} \qquad (5.36)$$

If the process is stationary and homogeneous, then the variance at time t will be the same as the variance at time t+k, so the autocorrelation may be written:

$$\rho_k = \frac{E[(z_t - \mu_z)(z_{t+k} - \mu_z)]}{\sigma_z^2} = \frac{\gamma_k}{\gamma_0} \qquad (5.37)$$

The sample autocorrelation, the estimate of the theoretical autocorrelation function is:

$$r_k = \hat{\rho}_k = \frac{\sum_{t=1}^{T-k} (z_t - z)(z_{t+k} - z)}{\sum_{t=1}^{T} (z_t - z)^2} \qquad (5.38)$$

According to Box and Jenkins (1970, p. 34), the variance for the estimated autocorrelation coefficient of a stationary Normal process was approximated by Bartlett as:

$$\mathrm{var}[r_k] \approx 1/T \sum_{-\infty}^{+\infty} (\rho_v^2 + \rho_{t+k}\rho_{v-k}$$

$$- 4\rho_k\rho_v\rho_{v-k} + 2\rho_v^2\rho_k^2). \qquad (5.39)$$

The variance for the estimated autocorrelations at lags K greater than q will then be:

$$var[r_K] \approx 1/T(1 + 2\sum_{v=1}^{q} \rho^2_v) \tag{5.40}$$

due to the fact that for processes for which the autocorrelations are zero for v>q, the terms in Equation (5.39), excluding the first term, will disappear. The hypothesis that the true order of a MA process is q can be tested, by whether or not the calculated autocorrelation coeficients for lags K>q are significantly different from zero. Specifically, if any $|r_K|$, K>q, is greater than:

$$1.96[1/(T \ (1 + 2\sum_{i=1}^{q}\rho_v^2))^{.5}] \tag{5.41}$$

then we can be 95 percent confident that the estimated autocorrelation coefficient is not equal to zero.

In order to test the joint hypothesis that all of the autocorrelation coefficients are zero, the Box-Pierce Q statistic may be used, where:

$$Q = T \sum_{k=1}^{K} \rho_k^2 \tag{5.42}$$

The Q statistic is approximately distributed as Chi-squared with K degrees of freedom. Thus, if the calculated Q statistic is greater than the tabulated Chi-squared value with K degrees of freedom, then it can be concluded that these K autocorrelations jointly are significantly different from zero at a selected probability level.

The partial autocorrelation function, as explained by Makridakis and Wheelwright (1978, p. 692):

> is used to identify the extent of the relationship between current values of a variable with earlier values of that same variable (values for various time lags) while holding the effects of all other time lags constant.

Recall that the covariance for k lag displacement may be expressed as:

$$\gamma_k = \Phi_1\gamma_{k-1} + \Phi_2\gamma_{k-2} + \ldots + \Phi_p\gamma_{k-p} \qquad (5.43)$$

for $k>0$,

and dividing through by γ_0 produces:

$$\rho_k = \Phi_1\rho_{k-1} + \Phi_2\rho_{k-2} + \ldots + \Phi\rho_k. \qquad (5.44)$$

If Φ_{kj} is the $j^{\underline{th}}$ coefficient in an autoregressive process of order k, then:

$$\rho_j = \Phi_{k1}\rho_{j-1} + \Phi_{k(k-1)}\rho_{j-k+1} + \Phi_{kk}\rho_{j-k} \qquad (5.45)$$

for $j = 1, 2, \ldots, k$,

which gives the Yule-Walker equations:

$$\begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \Phi_{k1} \\ \Phi_{k2} \\ \cdot \\ \cdot \\ \cdot \\ \Phi_{kk} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdot \\ \cdot \\ \cdot \\ \rho_k \end{bmatrix} \qquad (5.46)$$

which may also be written as:

$$P_k \, \Phi_k = \rho_k.$$

When these equations are solved for $k=1,2,3,\ldots,$ values for Φ_{11}, Φ_{22}, ... Φ_{kk} may be found. In particular the value for Φ_{kk} is:

$$\Phi_{kk} = \frac{\begin{vmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_1 \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_2 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & \rho_k \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{vmatrix}} \qquad (5.47)$$

The quantity Φ_{kk}, a function of the lag k, is called the partial autocorrelation function.

According to Box and Jenkins (1970, p. 65), given an autoregressive process of order p, the variance of the partial autocorrelations of order $k>p$ will be distributed approximately independently with variance:

$$\text{var}[\Phi_{kk}] \approx 1/n. \qquad (5.48)$$

where n is the number of observations. Therefore, if the estimated partial autocorrelation coefficient for lag $k>p$ is greater than:

$$1.96[1/(n)^{.5}], \tag{5.49}$$

then we can be 95 percent confident that the partial autocorrelation coefficient is not equal to zero.

Before tentative values for p and q may be determined, the degree of differencing necessary to obtain stationarity of the series must be identified. A stationary mixed autoregressive moving average process of order (p,0,q) will have an autocorrelation function which satisifies the condition:

$$\Phi(B)\rho_K = 0, \; k>q \tag{5.50}$$

Thus, the autocorrelation function should approach zero as k grows larger. If the autocorrelation function diminishes quickly, and drops off nearly linearly, then the underlying stochastic process should be treated as nonstationary, and differenced by d, the number of differences taken to achieve stationarity of the series. Once d is determined, the resultant series, w_t, which is z_t differenced d times, or $w_t = \Delta^d z_t$, may be used to calculate the sample autocorrelation and partial autocorrelation functions.

As seen in Equation (5.47), the values for Φ_{KK} may be found from the Yule-Walker equations, thus for an autoregressive process of order p, the partial autocorrelation function will be nonzero for $k \leq p$ and zero for $k>p$. Therefore, for an autoregressive process of order p, the partial autocorrelation coefficients will be significant at lags up to p, and decline for lags $k>p$.

Due to the fact that each value contains information from all of the past values of the series, as seen by:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots + \phi_p z_{t-p} + \epsilon_t + \delta, \qquad (5.51)$$

for a purely autoregressive process, the autocorrelation coefficients of the sample autocorrelation function will decline gradually. For example, the autocorrelation function for an AR(1) process has a value of $\rho_0 = 1$, and $k \geq 1$, declines geometrically, as seen by:

$$\gamma_k = \frac{\gamma_k}{\gamma_0} = \phi_1{}^k. \qquad (5.52)$$

The autocorrelation function for a moving average process, unlike that for an autoregressive process, will diminish quickly. Specifically, since:

$$\rho_k = \frac{-\theta_k + \theta_1\theta_{k+1} + \ldots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \ldots + \theta_q^2}, \qquad (5.53)$$

$$k = 1 \ldots q$$

$$= 0, \; k > q$$

Thus, the autocorrelation function for a moving process of order q will drop off sharply at lags $k>q$.

Conversely, the partial autocorrelation function for a moving average process will tend to be dominated by a damped exponential pattern. For example, the partial autocorrelation function for an MA(1) process is:

$$\phi_{kk} = -\theta_1^k(1 - \theta_1^2)/(1 - \theta_1{}^{2(k+1)}). \qquad (5.54)$$

The autocorrelation function for a mixed ARMA model will show a dampened sine wave or exponential decay pattern if $q-p<0$, since the autocorrelation function will be dominated by an autoregressive process. But if $q-p\geq 0$, there will be $q-p+1$ values which do not follow this pattern. The partial autocorrelations of a mixed process will tend to show a damped sine wave or exponential decay due to the fact that for:

$$\epsilon_t = \theta^{-1}(B)\phi(B)z_t, \tag{5.55}$$

θ^{-1} is an infinite series in B. The partial autocorrelation function of a mixed process will then be infinite in extent, and show a damped pattern.

Subsequent to the selection of values for p, q, and d for an ARIMA model:

$$\phi(B)\,\Delta^d z_t = \phi(B)w_t = \theta(B)\epsilon_t \tag{5.56}$$

where:

$$\phi(B) = 1 - \sum_{i=1}^{p}\phi_i B^i$$

$$\theta(B) = 1 - \sum_{i=1}^{q}\theta_i B^i$$

the estimates for the autoregressive and moving average parameters may be obtained. The estimates for the autoregressive and the moving average parameters are chosen so as to minimize the sum of squared errors between w_t and $\hat{w}_t$.

If the error series is written as:

$$\epsilon_t = \theta^{-1}(B)\phi(B)w_t, \tag{5.57}$$

then the sum of squared errors may be expressed as:

$$S(\Phi, \Theta) = \sum_t \epsilon_t^2. \qquad (5.58)$$

The estimates Φ and Θ are chosen to minimize the sum of squared errors in Equation (5.58).

Given that moving average terms are present, then ϵ_t, expressed as a function of parameters, will be nonlinear in parameters, therefore an iterative method of nonlinear estimation must be employed.

If the assumption is made that the ϵ's are independently and normally distributed, then the probability function for them can be approximated by:

$$P(\epsilon_1 \ldots \epsilon_n) \approx \sigma_\epsilon^{-n} \exp(-\sum_{t=1}^{T} \epsilon_t^2 / 2\sigma_\epsilon^2) \qquad (5.59)$$

Thus, the log likelihood function is:

$$\ln L = -T\ln \sigma_\epsilon - (\Sigma\epsilon_t^2)/2\sigma_\epsilon^2. \qquad (5.60)$$

The conditional log likelihood function is given by:

$$L_*(\Phi, \Theta, \sigma_\epsilon) = -T\log\sigma_\epsilon - \frac{S_*(\Phi,\Theta)}{2\sigma_\epsilon^2} \qquad (5.61)$$

where:

T is the number of observations

$\Phi = (\Phi_1, \ldots \Phi_p)$

$\Theta = (\Theta_1, \ldots \Theta_q)$

The log likelihood function is said to be conditional because the sum of squared errors $S_*(\Phi, \Theta)$ is conditional on past unobservable values $w_0, w_{-1}, \ldots w_{-p+1}, \epsilon_0, \epsilon_{-1}, \ldots \epsilon_{-q+1}$. Due to the fact that the least squares estimates depend on past unobservable values of w_t and ϵ_t, values for w_0, w_{-1}, ... must be chosen to initialize the series. The values for w_0, ... w_{-p+1} and ϵ_0, ... ϵ_{-q+1} may be set to their unconditional expected values. If the unconditonal values ϵ_0, ... ϵ_{-q+1} are all 0, and $\delta=0$, the unconditional values for w_0, ... w_{-p+1} will also be 0. An alternative is to determine conditional expected values for w_0, ... w_{-p+1} which are conditional on the estimated values of ϵ_1 ... ϵ_t. The procedure for initializing a conditional least squares estimation is to set w_0, ... w_{-p+1} and ϵ_0, ... ϵ_{-q+1} to 0. The ARIMA model is then estimated to minimize $S(\Phi, \Theta)$ conditional on the 0 values. The estimated model is then used to backcast the values for w_0, ... w_{-p+1}.

Since the differenced series, w_t, is stationary with respect to time, the series may be written:

$$\Phi(F)w_t = \Theta(F)\epsilon_t \tag{5.62}$$

where F is a forward shift operator, so:

$$Fw_t = w_{t+1}.$$

Using this forward shift operator, Equation (5.57) can be rewritten as:

$$w_t = \Phi^{-1}(F)\Theta(F)\epsilon_t. \tag{5.63}$$

This equation can be used to solve for w_0, ... w_{-p+1} from the estimated values of ϵ_1, ... ϵ_T. A set of least squares estimates for Φ and Θ are found by minimizing $S(\Phi, \Theta, \sigma_\epsilon)$ conditional on w_0, ... w_{-p+1}, where:

$$S_*(\Phi, \Theta, \sigma_\epsilon) = \sum_{t=1}^{n} \epsilon_t^2(\Phi\ \Theta|\ w_*, \epsilon_*, w). \tag{5.64}$$

New values for $w_0, w_1, \ldots w_{-p+1}$ may be estimated from Equation (5.63), and the process repeated until the estimates for Φ and Θ converge. Pindyck and Rubinfeld (1981, p. 553) state:

> If the time series is short (relative to p and q), some gain in efficiency would probably result from the use of conditional expected values of $w_0, \ldots, w_{-p+1}$.[1]

Before estimation techniques can be used, initial guesses for the parameters must be made. Recalling from the Yule-Walker equations that:

$$\rho_1 = \phi_1 + \phi_2\rho_2 + \ldots + \phi_p\rho_{p-1}$$

$$\rho_2 = \phi_1\rho_1 + \phi_2 + \ldots + \phi_p\rho_{p-2}$$

$$\vdots$$

$$\rho_p = \phi_1\rho_{p-1} + \phi_2\rho_{p-2} + \ldots + \phi_p$$

By replacing the theoretical autocorrelations with the estimated autocorrelations r_k, initial estimates for Φ can be found. Namely, if:

$$\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \phi_p \end{bmatrix} \quad r = \begin{bmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ r_p \end{bmatrix} \quad R = \begin{bmatrix} 1 & r_1 & r_2 & \cdots & r_{p-1} \\ r_1 & 1 & r_1 & \cdots & r_{p-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \\ r_{p-1} & r_{p-2} & r_{p-3} & \cdots & 1 \end{bmatrix}$$

Φ may be found by:

$$\Phi = R^{-1} r, \tag{5.65}$$

for the estimates.

If the model contains moving average parameters, then the model can be represented as:

$$\theta^{-1}(B)\Phi(B)w_t = \epsilon_t \tag{5.66}$$

which is nonlinear in parameters. The model's parameters must then be estimated by nonlinear estimation techniques.

Equation (5.66) may be linearized with the first two terms in a Taylor series. Values for the errors, which are conditional on w, Φ, and θ, $[\epsilon_t]$, may be expanded with a Taylor series about initial guesses for the parameters (Φ, θ):

$$[\epsilon_t] = [\epsilon_t | w, \beta_0] + \sum_{i=1}^{p+q}(\beta_i - \beta_{i,0}) \left.\frac{\partial[\epsilon_t]}{\partial \beta_i}\right|_{\beta=\beta_0}$$

$$+ .5 \sum_{i=1}^{p+q}(\beta_i - \beta_{i,0})^2 \left.\frac{[\epsilon_t]}{\partial \beta_i^2}\right|_{\beta=\beta_0}$$

$$+ \ldots \tag{5.67}$$

If we let:

$$z_{i,t} = -\left.\frac{\partial \epsilon_t}{\partial \beta}\right|_{\beta=\beta_0}, \tag{5.68}$$

and:

$$[\epsilon_{t,0}] = [\epsilon_t | w, \beta_0], \tag{5.69}$$

then, substituting (5.68) and (5.69) into (5.67):

$$[\epsilon_t] = [\epsilon_{t,0}] - \sum_{i=1}^{p+q}(\beta_i - \beta_{i,0}) z_{i,t} \tag{5.70}$$

approximately.

This may be rewritten as:

$$[\epsilon_{t,0}] + \sum_{i=1}^{p+q}\beta_{i,0} z_{i,t} = \Sigma\beta_i z_{i,t} + [\epsilon_t] \tag{5.71}$$

$[\epsilon_{t,0}]$ representing the error generated by the initial guess for β_0. The values for β_i can be estimated then via ordinary least squares regression, where:

$$Y = Z\beta + [\epsilon] \tag{5.72}$$

where

$$Y = \begin{bmatrix} [\epsilon_{1,0}] + \sum_{i=1}^{p+q} \beta_{i,0} Z_{i,1} \\ \dots\dots\dots\dots\dots\dots \\ [\epsilon_{T,0}] + \sum_{i=1}^{p+q} \beta_{i,0} Z_{i,T} \end{bmatrix}$$

$$Z = \begin{bmatrix} z_{1,1} & z_{2,1} & \cdots & z_{p+q,1} \\ \dots & \dots & \dots & \dots \\ z_{1,T} & z_{2,T} & \cdots & z_{p+q,T} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_{p+q} \end{bmatrix}$$

and $[\epsilon]$ is a Tx1 vector of unobservable error terms.

A new Taylor series of $[\epsilon_t]$ can then be constructed around the estimate of β, in order to generate a further estimate of β. The process is repeated until:

$$\beta_k - \beta_{k-1} \approx 0,$$

where k is the number of iterations necessary for convergence.

Once a model has been estimated, the appropriateness of this model may be determined by the process of diagnostic checking. Diagnostic checking may employ various tools to evaluate the appropriateness of a model, but the residuals are the most commonly used tool to check the appropriateness of an estimated model. If the random error terms, ϵ_t, in the actual process are normally distributed and independent of each other:

$$\epsilon_t \sim N(0, \sigma_\epsilon^2)$$

$$E[\epsilon_t, \epsilon_{t+k}] = 0,$$

and the model which has been estimated is appropriate, then the errors ϵ_t should also have these properties. If the estimated residuals have properties close to the theoretical residuals, the estimated residuals will be nearly uncorrelated with each other. In order to test for the correlation between residuals in different time periods, the sample residual autocorrelation function may be calculated and examined.

The sample autocorrelation function of the residuals should be close to 0 for displacement $k \geq 1$, where:

$$r_k = \frac{\sum_t \hat{\epsilon}_t \hat{\epsilon}_{t-k}}{\sum_t \hat{\epsilon}_t^2}. \tag{5.73}$$

The standard deviation of the sample residual autocorrelation coefficients may be approximated by $1/(t)^{.5}$, so that if the coefficient is greater than two standard deviations, we would be 95 percent confident that the true residual autocorrelation is not zero.

As with the actual data series, the Q statistic may be used to test the joint hypothesis that the residual autocorrelation coefficients are zero:

$$Q = t \sum_{k=1}^{K-q-p} r_k^2 \tag{5.74}$$

Since Q is the sum of K-q-p squared independent variables with normal distributions, means of zero, and variance of 1/t, t = T-d, then Q will be distributed as Chi-squared with K-q-p degrees of freedom. The calculated Q may then be compared with the tabulated value of $x^2(K-q-p)$ to test the hypothesis that the residual autocorrelation coefficients are zero.

If the autocorrelations do not show any significant spikes, and the value of Q is not significant at the prescibed probability level, then the model may be determined to be adequate for the purposes of forecasting.

Given the model:

$$w_t = \Phi_1 w_{t-1} + \ldots + \Phi_p w_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \ldots - \theta_q \epsilon_{t-q} + \delta \tag{5.75}$$

and:

$$z_t = \Sigma^d w_t, \tag{5.76}$$

which may be expressed as $w_t = \theta^{-1} B(1-B)^{-d} \theta(B) \epsilon_t = \psi(B)\epsilon + \delta$, in order to obtain a forecast for w_{T+1}, Equation (5.75) can be modified as:

$$w_{T+1} = \Phi_1 w_T + \ldots + \Phi_p w_{T-p+1} - \theta_1 \epsilon_T$$

$$- \dots - \theta_q \epsilon_{T-q+1} + \delta. \qquad (5.77)$$

which may be rewritten as:

$$w_{T+1} = \delta + \psi_0 \epsilon_{T+1} + \sum_{j=0}^{\infty} \psi_{1+j} \epsilon_{T-j}.^2$$

The forecast $w_T(1)$ is then calculated by taking the conditional expected value of w_{T+1}:

$$\hat{w}_T(1) = E[w_{T+1} | w_T, w_{T-1}, \dots, w_1]$$
$$= \phi_1 w_T + \dots + \phi_p w_{T-p+1} - \theta_1 \epsilon_T$$
$$- \dots \theta_q \epsilon_{T-q} + \delta. \qquad (5.78)$$

or

$$\hat{w}_T(1) = \sum_{j=0}^{\infty} \psi_{1+j}{}^* \epsilon_{T-j}$$

where $\psi_{1+j}{}^*$ is the optimal weight to minimize the mean squared forecast error.

The error for a one period ahead forecast is:

$$e_T(1) = w_{T+1} - w_T$$
$$= \psi_0 \epsilon_{T+1} + \sum_{j=0}^{\infty} (\psi_{1+j} - \psi_{1+j}{}^*) \epsilon_{T-j} \qquad (5.79)$$

For a one period ahead forecast the variance will then be:

$$E[(e_T(1)^2] = (\psi_0{}^2) + \sigma_\beta^2 + \sum_{j=0}^{\infty} (\psi_{1+j} - \psi_{1+j}{}^*) \qquad (5.80)$$

since $E[\epsilon_i \epsilon] = 0$.

Given that the optimal weights are chosen, then their expected values will be equal to the true weights, and the expected values of $\epsilon_{T+1} \dots \epsilon_{T+1}$ are equal to zero. Then the variance for a one period ahead forecast will be:

$$E[e_T(1)^2] = \psi_0{}^2 \sigma_\epsilon{}^2 = \sigma_\epsilon{}^2 \qquad (5.81)$$

due to the fact that $\psi_0 = 1$. The forecast error variance for a one period ahead forecast will therefore be the variance of the error term:

$$\sigma_\epsilon^2 = \frac{S(\phi,\ \theta)}{T-p-q} = \frac{\sum_{t=1}^{T} \hat{\epsilon}_t^{\,2}}{T-p-q} \tag{5.82}$$

According to Box and Jenkins (1970, p. 156), for a desired probability level, α, and each lead time L, the confidence interval about the forecast is:

$$z_T(L) \pm u_{\alpha/2}(1 + \sum_{j=1}^{L-1} \psi^2{}_j)^{.5}\sigma_\epsilon \tag{5.83}$$

where $u_{\alpha/2}$ is the deviation exceeded by $\alpha/2$ of the unit Normal distribution. The confidence interval about a one period ahead forecast will then be:

$$z_T(1) \pm u_{\alpha/2}\ \sigma_\epsilon \tag{5.84}$$

which can be estimated by:

$$z_T(1) \pm u_{\alpha/2}\ S_\epsilon \tag{5.85}$$

Thus, the 95 percent confidence interval about a one period ahead forecast will be:

$$z_T(1) \pm 1.96\ (1)\ S_\epsilon = z_T(1) \pm (1.96)\ S_\epsilon.$$

Identification of a Tentative Model

The data over the period of 1925-1965 was analyzed in order to specify a tentative model. The sample autocorrelation and partial autocorrelation functions were calculated and plotted. The first two autocorrelations exceeded two standard errors of the estimate, with the estimate at lag one equal to .82238 and the estimate at lag two equal to .59495. Also, the autocorrelations tended to exhibit a trend, changing from positive to negative values at lag fourteen, as can be seen in Table III. The values below the estimated autocorrelation coefficients are the standard errors of the coefficients. The calculated Q statistic up to 24 lags was 119.21 which exceeded the tabulated Chi-squared value with K=24 degrees of freedom at the five percent significance level, or 36.42. As a result, the joint null hypothesis that all of the autocorrelations are equal to zero had to be rejected. The partial autocorrelation coefficients showed a strong spike at lag one, with a value of .82238, while none of the other partial autocorrelations showed a strong pattern. As may be seen in Table III, most of the other partial autocorrelations were very close to zero.

Since the autocorrelations showed a trending pattern and the partial autocorrelations exhibited a strong spike at lag one with little distinguishable pattern at higher lags, the data was tentatively identified as either an AR(1) process or as nonstationary. When an autoregressive parameter was estimated at lag one, the estimate was very close to one, with a value of .99652. The residual autocorrelation function still showed two values which were greater than two standard errors at lag one and at lag five. Similarly, the partial autocorrelation showed a strong spike at lag five. Since the autocorrelation at lag one still exceeded two standard errors of the estimate, and the

TABLE III

THE SAMPLE AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS OF THE ORIGINAL SERIES (1925-1965)

Autocorrelations

Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Lag 11	Lag 12
.82238 [.156174]	.59495 [.239543]	.40551 [.273218]	.24719 [.287522]	.13472 [.297522]	.14458 [.29266]	.23488 [.294169]	.33258 [.295897]	.38287 [.30041]	.38591 [.30926]	.34633 [.320613]	.20522 [.331749]

Lag 13	Lag 14	Lag 15	Lag 16	Lag 17	Lag 18	Lag 19	Lag 20	Lag 21	Lag 22	Lag 23	Lag 24
.04993 [.340463]	-.10456 [.343457]	-.18357 [.343634]	-.18800 [.344409]	-.14867 [.346787]	-.09498 [.349264]	-.02359 [.350804]	-.03023 [.351431]	-.09242 [.35147]	-.14931 [.351533]	-.22791 [.352125]	-.31050 [.353666]

Partial Autocorrelations

Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Lag 11	Lag 12
.82238	-.25134	-.00016	-.06767	.01133	.26772	.16691	.08767	-.02942	-.01463	.01759	-.23715

Lag 13	Lag 14	Lag 15	Lag 16	Lag 17	Lag 18	Lag 19	Lag 20	Lag 21	Lag 22	Lag 23	Lag 24
-.02953	-.22268	.04482	.01789	-.07699	-.03335	.01778	-.16015	-.00344	.06171	-.07227	-.05102

S.E. [ϕkk] x $1/\sqrt{n}$ = [.156173] autocorrelations of the original series showed a trend, the data was determined to be nonstationary, and a first difference was required. The sample autocorrelation and partial autocorrelation functions were calculated and plotted. The values for the estimates of the autocorrelations and the partial autocorrelations of the differenced data, along with the standard errors of the estimates are shown in Table IV. A strong spike occurred at lag five in both the autocorrelations and the partial autocorrelations. No other pattern was distinguishable in the partial autocorrelations, but the autocorrelations did seem to show a damped sine wave pattern, with the autocorrelations alternating between positive and negative values with every five or six lags. While there were no further spikes in the autocorrelations which were greater than two standard errors at lags other than lag five, the calculated Q statistic was 78.52 which is greater than the tabulated Chi-squared value of 36.42 for the five percent confidence level with K=24 degrees of freedom. Therefore, the joint hypothesis that all of the autocorrelation coefficients are equal to zero was rejected. This combination of patterns in the autocorrelation and the partial autocorrelations would tend to indicate an autoregressive process with a parameter at lag five, which may be expressed as:

$$(1-B)(X_t - \Phi X_{t-5}) = \delta + \epsilon_t \qquad (5.86)$$

TABLE IV

THE SAMPLE AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS OF THE DIFFERENCED SERIES (1925-1965)

Autocorrelations

Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Lag 11	Lag 12
.30089	-.14487	-.05704	-.18435	-.57118	-.44822	.03914	.15853	.14599	.28499	.39229	.13088
[.158114]	[.171834]	[.17486]	[.175325]	[.180106]	[.220795][	.242478]	[.242636]	[.245212]	[.247375]	[.255451]	[.270092]

Lag 13	Lag 14	Lag 15	Lag 16	Lag 17	Lag 18	Lag 19	Lag 20	Lag 21	Lag 22	Lag 23	Lag 24
-.04800	-.28233	-.27380	-.17198	-.11645	-.06605	.25861	.30478	.03231	.09091	.10268	-.07204
[.271673]	[.271885]	[.279118]	[.2185754][	.28833]	[.289503]	[.28988]	[.295591]	[.303346]	[.303432]	[.304112]	[.304977]

Partial Autocorrelations

Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Lag 11	Lag 12
.30089	-.25884	.08923	-.27402	-.51700	-.32815	-.08362	-.12650	-.09109	-.16096	-.00217	-.01627

Lag 13	Lag 14	Lag 15	Lag 16	Lag 17	Lag 18	Lag 19	Lag 20	Lag 21	Lag 22	Lag 23	Lag 24
.18171	-.27980	-.01510	.04962	.07636	-.11061	.18045	-.11838	-.06459	.17270	-.00913	.08396

S.E. [ϕkk] x $1/\sqrt{n}$ = [.158113]

or

$$(1-B)(1 - \Phi B^5)X_t = \delta + \epsilon_t \qquad (5.87)$$

An autoregressive parameter at lag five within the differenced data could be explained by the cyclical nature of cattle prices. Namely, the period between a peak and trough in the cattle cycle averages between three and ten years, and the period between a trough and peak in the cycle averages about four to six years. If the differenced series are examined in Figure 8, large spikes clearly occur with about five or six years between peak and trough. While the autocorrelations at other lags were not significant, a decaying sine wave pattern was evident. The peaks and troughs in the sine wave pattern of the residual autocorrelations occurred at approximately lags 5, 11, 14 or 15, and 20, which is indicative of the traditional cyclical pattern. Had a longer time span of data been available perhaps these values would have been significantly different from zero.

The Estimated Model

After the model was identified for the period of 1925-1965, it was estimated with the following results:

$$(1-B)(1-.603838B^5)X_t = .420514 + \epsilon_t \qquad (5.88)$$

$\hat{\mu} = .262912$, $t = 1.77$

$\hat{\Phi}_1 = -.603838$, $t = -4.47$

$\hat{\delta} = .420514$

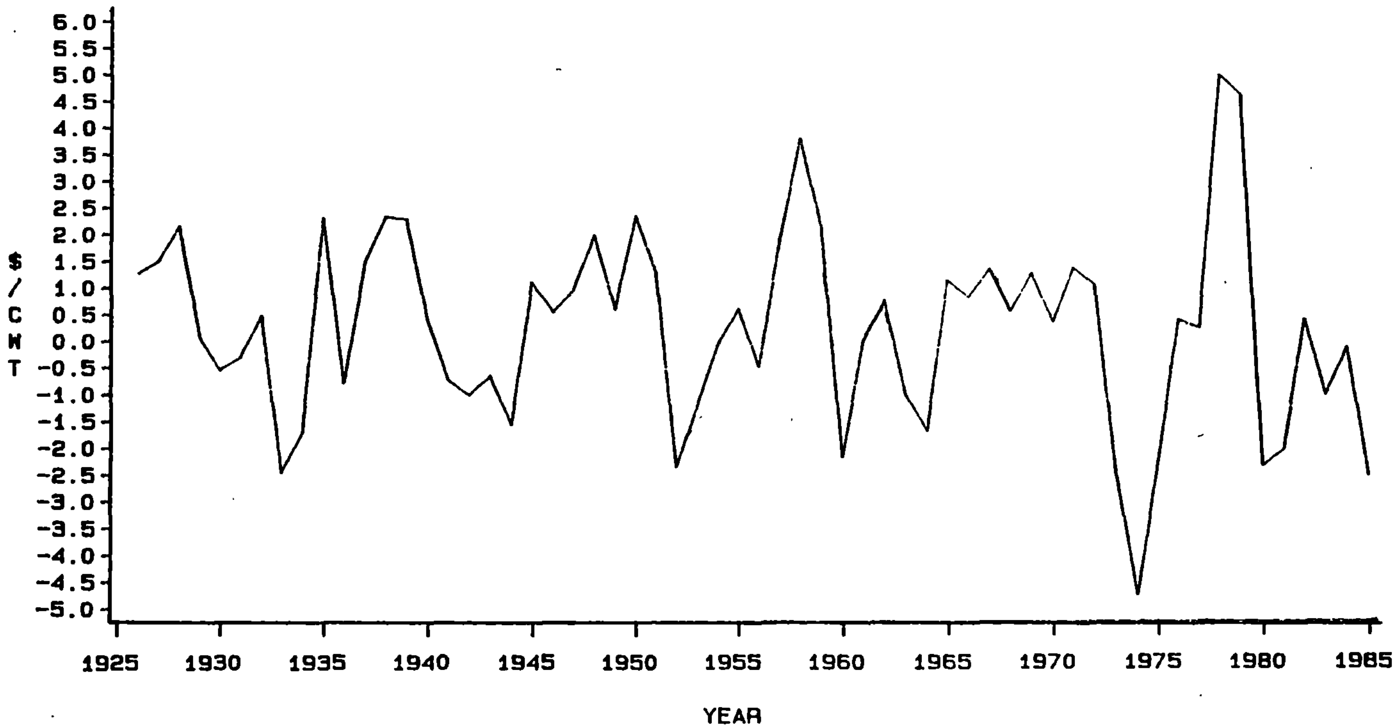


Figure 9. The Differenced Data Series (1925-1985)

The estimate of the variance was 2.085281 and the standard error was 1.44442334.

The autocorrelation function of the residuals was calculated and plotted. All of the residual autocorrelations were less than two standard errors of the estimate and showed no apparent patterns. Furthermore, the calculated Q statistic to 24 lags was 31.52, while the tabulated Chi-squared value for the five percent significance level with K-p-1=22 degrees of freedom is approximately 33.915, therefore the joint null hypothesis that all of the residual autocorrelations are equal to zero could not be rejected. Consequently, the model was determined to be adequate for the purposes of forecasting. The forecasts from the ARIMA model will be presented in Chapter Six.

The ARIMA model, like the econometric model was updated over the period of 1966-1985. The estimates of the autoregressive parameters are presented in Table V along with some relevant measures of fit. The estimated autoregressive parameter at lag five was significantly different from zero at the five percent significance level in each of the updated models. The estimate of the parameter for each of the models fell within a range between .49 and .69. The estimate for the mean of the series fell between .19 and .29. The estimate of the mean was significantly different from zero, from 1967 through 1973, although the calculated t values were not much larger than

TABLE V

THE ESTIMATES OF THE AUTOREGRESSIVE PARAMETERS AND OTHER RELEVANT MEASURES OF FIT

Year	$\hat{\phi}$	$\hat{\mu}$	δ	S^2	S	Q	Tabled $t_{\alpha=.05}$	Tabled χ^2
1965	-.603838 [.135072] (-4.47)	.292192 [.148227] (1.77)	.420514	2.08581	1.44423	31.52	2.025	33.915
1966	-.604781 [.133382] (-4.53)	.268337 [.144334] (1.86)	.430622	2.03581	1.42662	32.37	2.023	
1967	-.599218 [.133087] (-4.50)	.290182 [.142822] (2.03)	.464064	2.0302	1.42485	36.24	2.021	
1968	-.593376 [.1330701][(-4.54)	.281854 .13998] (2.01)	.4491	1.98746	1.40977	37.04	2.019	
1969	-.590075 [.127074] (-4.64)	.278816 [.13692] (2.04)	.443339	1.9411	1.39323	37.56	2.018	
1970	-.5859 [.125414] (-4.67)	.287345 [.134258] (2.14)	.455701	1.90302	1.3795	39.81	2.017	
1971	-.579619 [.125219] (-4.63)	.307738 [.133155] (2.31)	.486109	1.90159	1.37898	37.97	2.016	
1972	-.568195 [.124486] (-4.56)	.32702 [.132363] (2.47)	.512831	1.89646	1.37712	31.71	2.015	
1973	-.573809 [.128265] (-4.47)	.28825 [.134456] (2.14)	.45365	2.01418	1.41922	34.86	2.014	
1974	-.608758 [.139272] (-4.37)	.227385 [.142039] (1.60)	.365809	2.39205	1.54663	25.19	2.013	
1975	.609644 [.141261] (-4.32)	.195447 [.142704] (1.37)	.31458	2.46114	1.5688	24.06	2.012	

TABLE V (Continued)

Year	$\hat{\phi}$	$\hat{\mu}$	δ	S^2	S	Q	Tabled $t_{\alpha=.05}$	Tabled χ^2
1976	-.601893 [.139694] (-4.31)	.206752 [.140959] (1.47)	.331195	2.42738	1.558	25.25	2.011	
1977	-.599055 [.138188] (-4.34)	.213073 [.138562] (1.54)	.340715	2.38394	1.544	26.43	2.010	
1978	-.664257 [.137421] (-4.83)	.252877 [.135017] (1.87)	.420853	2.5109	1.58458	25.52	2.009	
1979	-.694481 [.125119] (-5.55)	.262754 [.130454] (2.01)	.445232	2.47736	1.57396	26.29	2.008	
1980	-.630939 [.129947] (-4.86)	.2616179 [.1418] (1.56)	.352574	2.77169	1.66484	26.30	2.007	
1981	-.634382 [.130656] (-4.86)	.191326 [.140907] (1.36)	.312699	2.80296	1.67421	30.79	2.006	
1982	-.634251 [.129484] (-4.90)	.19397 [.138355] (1.40)	.316996	2.75297	1.65921	31.10	2.005	
1983	-.588801 [.121415] (-4.85)	.213763 [.1409] (1.52)	.339627	2.75338	1.65933	34.78	2.004	
1984	-.543121 [.116454] (-4.66)	.236708 [.144477] (-1.64)	.365269	2.78105	1.66765	37.68	2.003	
1985	-.491694 [.119306] (-4.12)	.187252 [.154165] (1.21)	.279322	3.1948	1.73767	35.23	2.002	

the tabulated values. The years betweem 1967 and 1973 were upswing years of the cattle price cycle. The variance of the error term ranged between 1.89 and 3.01. The variance decreased with the updating of the model between 1965 and 1972, and increased between 1973 and 1985. This perhaps could be explained by the somewhat irregular pattern of the downswing in the last cycle. The calculated Box-Pierce Q statistic to 24 lags was significant between 1967 and 1971, 1973, and 1983 through 1985, but other than an autocorrelation of borderline significance at lag two, none of the other residual autocorrelations were significantly different from zero.

FOOTNOTES

[1]While some gains in efficiency may be expected with small samples, the statistical properties of the estimates are based on assymptotic results and the small sample properties of the estimates are unknown. Ansley and Newbold (1980, p. 164) did investigate the finite sample properties of maximum likelihood, conditional least squares, and exact least squares estimators for autoregressive moving average models. In 1000 replications of autoregressive models run on sample sizes of 50 observations, Ansley and Newbold found that for values of Φ in the range of -.40 to -.75 conditional least squares and maximum likelihood estimators produced the same forecast mean squared error when used to make one step ahead forecasts, outperforming exact least squares.

[2]An (L) period ahead forecast would be expressed as:

$$w_{T+l} = \delta + \psi_0 \epsilon_{T+l} + \psi_1 \epsilon_{T+l-1} + \cdots + \psi_{l-1} \epsilon_{T+1} + \sum_{j=0}^{\infty} \psi_{l+j} \epsilon_{T-j}$$

CHAPTER SIX

FORECASTS: ANALYSIS AND RESULTS

Introduction

The purpose of this chapter is to present the forecasts from the individual models and the various composite forecasting methods. The period of primary interest is 1976-1985 when forecasts were available for the individual models and the composites. Additionally, this chapter will seek to present some measures of relative forecasting accuracy produced by each of these models over the selected time period. Forecasting accuracy is the primary criterion in the selection of a "superior" model or method in a study seeking to improve forecasting performance. The question then arises regarding what are adequate measures of forecasting accuracy. Certainly, the definition of an accurate forecast may depend on the forecast user's needs. Brandon (1983, p.189) wrote:

> The necessity of having an efficient benchmark is required by the nature of measuring forecast accuracy. There are numerous measures of accuracy: mean error, mean absolute deviation, root mean square error, mean absolute percentage error, Theil's measure of inequality, coefficient of variation, and coefficient of determination. Each has specific advantages and disadvantages based on its mathematical properties.

> Accuracy of forecasts is thus, in some sense, a subjective choice. What is most accurate for one purpose need not be for another purpose.

For example, some forecast users may need accurate information concerning turning points in the data, with little interest in the ability of a model to pinpoint a specific level in the data. This may often be the case with users of price forecasts who are primarily interested in changes in market direction. On the other hand, some forecast users may be more interested in the accuracy of the level of the forecast. For example, producers who might wish to base expected returns on a predicted price would be interested in the accuracy of forecasting a given price level. These different information requirements would necessitate the use of different measures of forecasting accuracy. Therefore, a number of price forecasting accuracy measures will be examined. Makridakis and Wheelwright (1978, p. 568) state:

> In spite of the fact that accuracy is given prime importance as a factor in the selection process, little work has been done to develop a framework for measuring and evaluating accuracy issues.

Most forecasting accuracy measures fall under the catergory of descriptive measures, such as those listed by Brandon.

Perhaps the most commonly used measure of forecasting accuracy is mean squared forecast error, which can be expressed as:

$$MSFE = \frac{\sum_{i=1}^{n} (X_i - F_i)^2}{n} \quad (6.1)$$

where

X_i is the actual value

F_i is the forecast for the value of the variable

n is the number of data values or forecast values available

Granger and Ramanathan (1984), Brandon (1983), Newbold and Granger (1972), and Brandt and Bessler (1981) employ mean squared error as a tool for comparison among various forecasts. According to Theil (1971, p.4):

> A linear loss function would be inappropriate since each unit of loss (inaccuracy) would be treated similarly, or stated differently, each unit of marginal loss is assumed to be constant.

The mean squared error does have the properties that it attributes more weight to large errors than small ones, and is symmetrical in that it gives equal weight to over and under forecasts. This descriptive measure would be more applicable to a situation where the costs from an over forecast and an under forecast are about the same. Unfortunately, this may not always be the case when employing price forecasts for the purposes of production planning. For example, if a price forecast is used to calculate an expected return with a given set of expected production costs, the costs resulting from an overly optimistic forecast of prices could be much higher than with an underforecast.

If the costs from consistently over or under forecasting are greater than the opposite case, the forecast user might wish to know whether or not the model consistently over or under forecasts. The mean forecast error, unlike the mean squared error is a linear loss function, and provides a measure of forecast bias. The mean forecast error is:

$$MFE = \frac{\sum_{i=1}^{n}(F_i - X_i)}{n} \tag{6.2}$$

If the mean forecast error is greatly different from zero, forecasting bias may be indicated. For example, when the mean forecast error is positive overforecasting may be indicated. Another method of measuring forecast bias is described by Dhyrmes, et al (1972, p. 313)

> ... we regress actual values on the predicted values and test whether a series and the resulting equations have zero intercepts and slopes not significantly different from one.

Bessler and Brandt employed this technique to test for bias of forecasts. If either the intercept was found to be significantly different from zero, or the slope coefficient was found to be significantly different from one, the forecast were found to be biased on the average over that time period. While Brandt and Bessler found each of the forecasts to be biased, they suggested that a lower mean square error might be a more desirable forecasting goal than an unbiased forecast.

Another alternative to using MSFE as measure of forecasting accuracy is the Theil U coefficient. The advantage of the Theil U coefficient is that it allows for the comparison of formal forecasting methods and naive forecasts. A naive forecast may be defined as a forecast which is based on the assumption that the best forecast of next period's observation is this period's observation. Thus, the forecast for X_{T+1} would be X_T. The Theil U coefficient also has the property, like MSFE, that large errors are given more weight than small errors. The Theil U coefficient uses a measure of relative change rather than an absolute measure which MSFE uses, as can be seen by:

$$U = \sqrt{\frac{\sum_{i=1}^{n-1}(FPE_{i+1} - APE_{i+1})^2/(n-1)}{\sum_{i=1}^{n-1}(APE_{i+1})^2/(n-1)}} \tag{6.3}$$

where

$$FPE_{i+1} = \frac{F_{i+1} - X_i}{X_i}$$

= forecasted relative change

and

$$APE_{i+1} = \frac{X_{i+1} - X_i}{X_i} = \text{actual relative change}$$

The Theil U coefficient can be rewritten:

$$U = \sqrt{\frac{\sum_{i=1}^{n-1}\left(\frac{F_{i+1} - X_i - X_{i+1} + X_i}{X_i}\right)^2 \Big/ (n-1)}{\sum_{i=1}^{n-1}\left(\frac{X_{i+1} - X_i}{X_i}\right)^2 \Big/ (n-1)}} \quad (6.4)$$

and simplified to:

$$U = \sqrt{\frac{\sum_{i=1}^{n-1}\left(\frac{F_{i+1} - X_{i+1}}{X_i}\right)^2 \Big/ (n-1)}{\sum_{i=1}^{n-1}\left(\frac{X_{i+1} - X_i}{X_i}\right)^2 \Big/ (n-1)}} \quad (6.5)$$

The Theil U coefficent may be interpreted as:

when $U=1$, the naive method performs as well as the forecasting method

when $U<1$, the forecasting method performs better than the naive method. The smaller U is, the better the forecasting method is compared with the naive method. If the forecasts are perfect, then the statistic will be zero.

when $U>1$, the naive method outperforms the forecasting method.

The mean absolute percentage error also serves as an alternative measurement of point forecasting accuracy. The mean absolute percentage error is computed by:

$$MAPE = \frac{\sum_{i=1}^{n} |PE_i|}{n} \qquad (6.6)$$

where:

$$PE_i = \frac{X_i - F_i}{X_i} (100)$$

The mean absolute percentage error has the specific advantage that it expresses forecasting accuracy in terms of average percentage error. It does not give greater weight to large errors than smaller errors, but does treat positive and negative errors the same due to the fact that an absolute value is used.

If the forecast user is more concerned with the accuracy of correctly forecasting turning points in the data, the appropriate type of measure is some type of tracking signal. Tracking signals include various indicators such as the number of times a change in price direction is forecasted when no change in price direction occurs or the number of times no change in price direction is forecasted when a change in price direction actually occurs. Most often these types of results are simply presented in tabular form as by Brandt and Bessler (1981, p. 55).

Since annual forecasts of beef cattle prices may be employed for numerous purposes, various measures of forecasting accuracy for each of the forecasts will be presented. These measures include mean squared forecast error (MSFE), mean forecast error (MFE), mean absolute percentage error (MAPE), the Theil U coefficient, and turning point errors.

The Econometric and ARIMA Forecasts (1966-1985)

A series of one step ahead forecasts were generated by updating the econometric and ARIMA models over the period of 1966 through 1985. These one step ahead forecasts, along with the actual values for deflated farm beef cattle prices are presented in Table VI. The errors and squared errors resulting from each of the series of forecasts are also shown in Table VI. Additionally, a series of simple average composite forecasts, their errors and squared errors are presented in Table VI for the purposes of comparison with the econometric and ARIMA forecasts. A simple average composite of the econometric and ARIMA forecasts is compared with the forecasts from the individual techniques over the period of 1966 through 1985 since no forecast error histories are necessary in order to calculate the simple average composite. Table VII contains the mean squared forecast errors, the mean forecast errors, mean absolute percentage errors, and Theil U coefficients for the econometric, ARIMA, and simple average composite forecasts over 1966-1985. Table VIII shows a summary of turning point errors for each of the forecasts.

From Table VII, it can be seen that the ARIMA forecasts produced lower mean squared errors, mean forecast errors,

TABLE VI

THE ECONOMETRIC, ARIMA, AND SIMPLE AVERAGE COMPOSITE FORECASTS, ERRORS, AND SQUARED ERRORS

		Forecasts:			Errors:			Squared Errors:		
Year	Actual	Econometric	ARIMA	Average	Econometric	ARIMA	Average	Econometric	ARIMA	Average
1966	20.9632	19.2780	20.5896	19.9338	-1.6852	-.3736	-1.0294	2.8398	.1395	1.0596
1967	22.3000	21.0775	20.9350	21.0053	-1.2225	-1.3650	-1.2937	1.4945	1.8632	1.6737
1968	22.8293	22.5343	23.3666	22.9505	-1.2950	.5373	.1212	.0870	.2886	.0146
1969	24.0809	22.9946	24.2881	23.6413	-1.0863	.2072	-.4395	1.1800	.0429	.1932
1970	24.4144	23.8265	23.8547	23.8406	-.5879	-.5597	-.5738	.3456	.3132	.3292
1971	25.7750	23.9333	24.4008	24.1671	-1.8417	-1.3742	-1.6080	3.3918	1.8884	2.5855
1972	26.8000	25.6757	25.4863	25.5810	-1.1243	-1.3137	-1.2190	1.2640	1.7258	1.4859
1973	24.2768	25.4316	27.0121	26.2219	1.1548	2.7353	1.9451	1.3335	7.4818	3.7832
1974	19.4992	25.3999	24.0123	24.7061	5.9007	4.5131	5.2069	34.8182	20.3680	27.1118
1975	17.2469	18.9441	19.6620	19.3031	1.6972	2.4151	2.0561	2.8804	5.8327	4.2277
1976	17.6440	18.2957	16.7321	17.5139	.6516	-.9119	-.1301	.4247	.8315	.0169
1977	17.8701	21.5162	17.3582	19.4372	3.6460	-.5119	1.5671	13.2940	.2620	2.4558
1978	22.8235	23.1584	19.7224	21.4404	.3348	-.3101	-1.3581	.1121	9.3092	1.8444
1979	27.3813	27.8670	26.4179	27.1424	.4850	-.9640	-.2395	.2353	.9292	.0573
1980	25.0200	30.9674	29.3913	30.1793	5.9473	4.3713	5.1593	35.3715	19.1082	26.6188
1981	22.9894	25.2849	25.1221	25.2035	2.2954	2.1327	2.2141	5.2693	4.5484	4.9022
1982	23.3911	25.3737	23.1586	24.2661	1.9825	-.2325	.8750	3.9307	.0540	.7657
1983	22.3610	25.2000	20.5664	22.8832	2.8390	-1.7946	.5222	8.0599	3.2205	.2726
1984	22.2179	22.8819	20.0166	21.4493	.6640	-2.2013	-.7687	.4408	4.8457	.5908
1985	19.6853	17.7552	23.8660	20.8106	-1.9301	4.1807	-1.1253	3.7252	17.4782	1.2663

TABLE VII

FORECASTING ACCURACY MEASURES FOR THE ECONOMETRIC, ARIMA, AND SIMPLE AVERAGE COMPOSITE FORECASTS (1966-1985)

Forecast	MSE	MFE	MAPE	THEIL U
Econometric	6.0249	.8913	8.6494	.9975
ARIMA	5.0265	.3219	8.1946	.9263
Average	4.0627	.6066	6.1835	.6545

TABLE VIII

TURNING POINT ERRORS OF THE ECONOMETRIC, ARIMA, AND SIMPLE AVERAGE FORECASTS (1966-1985)

	Changes in Price Direction					
	Forecasted					
Actual	Econometric		ARIMA		Simple Average	
	C	NC	C	NC	C	NC
C	3	2	0	5	0	5
NC	2	11	6	7	4	9

C = Change NC = No Change

mean absolute percentage errors, and Theil U coefficients than did the econometric forecasts. While the ARIMA forecasts produced lower values for these measures of forecasting accuracy, the econometric model performed better in terms of accurately indicating turning points within the data, as seen in Table VIII. Thus, the choice of a "superior" forecast between the econometric and ARIMA forecasts would certainly depend upon forecasting needs. The simple average composite gave a lower forecast mean squared error, mean absolute percentage error, and Theil U coefficient than either of the forecasts from the individual models. As would be expected, the mean forecast error of the simple average composite fell between the mean forecast errors of the econometric and ARIMA forecasts. It may be noted that the simple average composite did not improve the number of turning points which were accurately forecasted by the ARIMA model alone, but did improve the number of no changes which were accurately forecasted.

Beyond 1973, the econometric model performed much better than the ARIMA model or the simple average composite in forecasting turning points correctly. Yet, the econometric model produced larger values for all of the other measures of forecasting accuracy. One reason for these reults may be that the econometric model tended to overforecast after 1973. The econometric model may have proven to be superior over the ARIMA model in indicating turning points because it uses lagged exogenous information to determine prices. For example, the econometric model employed in this study uses lagged January 1 inventories of calves on farms as a predictor of prices, with large inventories serving as an indicator of possible low prices in the future. Given that the relationship between the inventories of calves and beef cattle prices is fairly stable, then information concerning changes in January 1 inventories would give an accurate indication regarding future changes in beef cattle prices.

The ARIMA model assumes that the structure of the prices remains somewhat stable, so that it can be characterized by a set of fixed coefficients, relating prices in different time periods, while the econometric model assumes that the relationship between the dependent variable, price, and the independent variables is stable. Therefore, as long as the relationship between the dependent and the independent variables remained stable through time, a given pattern of prices need not exist for the econometric model to perform well in indicating turning points, as it needs to exist for an ARIMA model to perform well in indicating turning points. Harris and Leuthold (1983, p. 53) found similar results with the ARIMA model which generated a lower root mean square error than did the econometric model, but the ARIMA model did not indicate turning points in quarterly live cattle prices as well as their econometric model. Brandt and Bessler (1981, p. 38,43), on the other hand, found that the ARIMA model which they employed forecasted monthly cattle price levels and turning points better than the econometric model. Both the econometric model and the ARIMA model missed the turning points at 1975-1976 and 1978-1979. It may be noted that the downswing between 1972 and 1975 was shorter than past downswings in the cycle, and the upswing was only four years long between 1975 and 1979.

The Individual and Composite Forecasts (1976-1985)

The econometric and ARIMA models were, as previously stated, updated annually and used to make one step ahead forecasts over the period of 1966 through 1985. In addition several composite forecasts were made which required forecast error histories. These included the unrestricted linear combination of forecasts and the adaptive weighting schemes. Since these composites required past forecast error histories, a second analysis period between 1976 and 1985 was also considered. This time period was chosen because it would allow for several forecast histories (from 1966-1975) to be used in calculating the composites, and would allow ten periods for composite forecasts to be made. As with the econometric and ARIMA forecasts over 1966-1985, the relative forecasting accuaracy of the individual and composite forecasts were evaluated by various forecasting performance measures. The performance measures included mean squared forecast error, mean forecast error, mean absolute percentage error, the Theil U coefficient, and turning point errors. The actual values, the econometric, ARIMA, and composite forecasts are shown in Table IX. As may be seen in Table IX, the econometric model tended to overforecast from 1976 through 1985, while the ARIMA model did not. The simple average of forecasts and the unrestricted linear combination of forecasts helped to offset the overforecasts made by the econometric model, with the unrestricted linear combination of forecasts actually showing a tendecy to overforecast. The composite forecasts generated by the adaptive weighting scheme became much more variable as the number of error histories included in the calculation of the composite weight was decreased and as the value for the smoothing factor, a, became larger.

The simple adaptive weighting scheme composite weights, w_1, on the econometric forecasts are shown in Table X. The weight on the ARIMA forecasts are $1-w_1$. As may be seen in Table X, as the number of error histories, v, was increased, the weights became less variable from one year to the next, particularly with the smaller values of a. The relative stability of the composite weights when a=1, and as v increased towards 10 periods occured because a longer error history is taken into consideration, so that one or two larger values of the squared errors will have less overall effect in determining the composite weight. For example, when only two or three error histories are considered in a composite weight, and a large weight is

TABLE IX

THE ECONOMETRIC, ARIMA, AND COMPOSITE FORECASTS (1976-1985)

Year	Actual	Econometric	ARIMA	Simple Average	Unrestricted Linear Combination
1976	17.6440	18.2957	16.7321	17.5139	16.9271
1977	17.8701	21.5162	17.3582	19.4372	17.5762
1978	22.8235	23.1584	19.7224	21.4404	19.7618
1979	27.3819	27.8670	26.4179	27.1424	25.7425
1980	25.0200	30.9674	29.3913	30.1793	28.9006
1981	22.9894	25.2849	25.1221	25.2033	24.0784
1982	23.3911	25.3737	23.1586	24.2661	22.7004
1983	23.3610	25.2000	20.5664	22.8832	21.0486
1984	22.2179	22.8819	20.0166	21.4493	20.5334
1985	19.6853	17.7552	23.8660	20.8106	21.7573

Adaptive Weighting

a = 1.0

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	17.7787	17.3732	17.4563	17.4634	17.4522	17.4523	17.4425	17.4447	17.4507	17.4293
1977	20.1104	20.1376	19.0833	19.2982	19.3161	19.2859	19.2863	19.2603	19.2661	19.2814
1978	19.8378	20.0223	20.7692	20.9465	21.1177	21.1378	21.1293	21.1310	21.1155	21.1203
1979	27.8497	27.0215	27.0399	27.1319	27.0197	27.0768	27.0821	27.0748	27.0750	27.0681
1980	30.6489	30.9156	30.0768	30.0945	30.1843	30.0537	30.1144	30.1199	30.1117	30.1119
1981	25.1792	25.1807	25.1955	25.1834	25.1841	25.1887	25.1862	25.1905	25.1910	25.1906
1982	24.1848	23.9735	23.9905	24.1612	24.0141	24.0226	24.0774	24.0410	24.0952	24.1013
1983	20.6292	22.1114	22.1753	22.2104	22.5609	22.2815	22.2996	22.4161	22.3672	22.4801
1984	20.8346	20.6312	20.9102	20.9864	21.0054	21.1980	21.0508	21.0611	21.1280	21.1104
1985	18.2648	20.8907	21.4515	21.3168	21.5774	21.5424	21.1757	21.4956	21.4765	21.3519

TABLE IX (Continued)

a = 1.2

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	17.7787	17.3839	17.4519	17.4569	17.4505	17.4506	17.4464	17.4472	17.4490	17.4435
1977	20.1104	20.1370	19.1190	19.2927	19.3052	19.2877	19.2878	19.2770	19.2790	19.2834
1978	19.8378	19.9938	20.5930	20.8535	20.9876	21.0029	21.0023	21.0035	20.9982	20.9999
1979	27.8497	27.0842	27.0961	27.1513	27.0520	27.0892	27.0882	27.0882	27.0882	27.0858
1980	30.6489	30.9099	30.1497	30.1605	30.2119	30.0955	30.1369	30.1369	30.1325	30.1325
1981	25.1792	25.1804	25.1912	25.1838	25.1842	25.1864	25.1875	25.1875	25.1877	25.1876
1982	24.1848	23.9798	23.9935	24.1135	24.0258	24.0303	24.0412	24.0412	24.0642	24.0666
1983	20.6292	22.0305	22.1524	22.1804	22.4244	22.2655	22.3283	22.3283	22.3196	22.3673
1984	20.8346	20.6574	20.8759	20.9626	20.9767	21.1003	21.0298	21.0298	21.0584	21.0606
1985	18.2648	20.7332	21.2167	21.1720	21.4648	21.4416	21.3945	21.3945	21.3864	21.3402

a = 1.4

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	17.7787	17.3942	17.4510	17.4547	17.4506	17.4507	17.4487	17.4490	17.4480	17.4480
1977	20.1104	20.1363	19.1538	19.2971	19.3062	19.2950	19.2951	19.2900	19.2923	19.2923
1978	19.8378	19.9729	20.4589	20.7492	20.8546	20.8662	20.8681	20.8689	20.8676	20.8676
1979	27.8497	27.1382	27.1459	27.1794	27.0921	27.1155	27.1169	27.1146	27.1136	27.1136
1980	30.6489	30.9044	30.2120	30.2187	30.2481	30.1465	30.1701	30.1713	30.1687	30.1687
1981	25.1792	25.1802	25.1885	25.1837	25.1839	25.1850	25.1847	25.1856	25.1856	25.1856
1982	24.1848	23.9857	23.9970	24.0849	24.0295	24.0320	24.0442	24.0385	24.0488	24.0488
1983	20.6292	21.9580	22.1253	22.1485	22.3248	22.2296	22.2349	22.2611	22.2623	22.2816
1984	20.8346	20.6776	20.8520	20.9383	20.9489	21.0299	20.9897	20.9922	21.0050	21.0090
1985	18.2648	20.5935	21.0152	21.0189	21.3201	21.3049	21.1792	21.2713	21.2678	21.2505

a = 1.6

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	17.7787	17.4038	17.4552	17.4550	17.4523	17.4523	17.4513	17.4515	17.4517	17.4511
1977	20.1104	20.1357	19.1876	19.3078	19.3145	19.3070	19.3070	19.3044	19.3047	19.3053
1978	19.8378	19.9569	20.3563	20.6436	20.7263	20.7350	20.7374	20.7379	20.7371	20.7374
1979	27.8497	27.1850	27.1901	27.2106	27.1356	27.1500	27.1506	27.1492	27.1492	27.1488
1980	30.6489	30.8991	30.2656	30.2696	30.2864	30.2007	30.2148	30.2152	30.2136	30.2136
1981	25.1792	25.1801	25.1865	25.1833	25.1834	25.1840	25.1839	25.1843	25.1844	25.1843
1982	24.1848	23.9913	24.0009	24.0673	24.0305	24.0320	24.0384	24.0359	24.0399	24.0402
1983	20.6292	21.8926	22.0952	22.1146	22.2462	22.1864	22.1896	22.2033	22.2060	22.2141
1984	20.8346	20.6937	20.8353	20.9155	20.9234	20.9775	20.9551	20.9565	20.9625	20.9652
1985	18.2648	20.4687	20.8400	20.8687	21.1572	21.1572	21.0737	21.1261	21.1246	21.1246

TABLE IX (Continued)

a = 1.8

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	17.7787	17.4131	17.4587	17.4570	17.4551	17.4551	17.4551	17.4546	17.4547	17.4545
1977	20.1104	20.1351	19.2203	19.3225	19.3275	19.3221	19.3220	19.3207	19.3209	19.3211
1978	19.8378	19.9443	20.2768	20.5439	20.6087	20.6151	20.6172	20.6175	20.6172	20.6173
1979	27.8497	27.2260	27.2294	27.2420	27.1789	27.1878	27.1880	27.1871	27.1871	27.1869
1980	30.6489	30.8940	30.3119	30.3144	30.2385	30.2534	30.2617	30.2618	30.2608	30.2608
1981	25.1792	25.1800	25.1852	25.1829	25.1829	25.1833	25.1833	25.1835	25.1835	25.1835
1982	24.1484	23.9965	24.0048	24.0309	24.0309	24.0318	24.0354	24.0342	24.0360	24.0361
1983	20.6292	21.8337	22.0628	22.1800	22.1800	22.1410	22.1430	22.1506	22.1528	22.1565
1984	20.8346	20.7068	20.8236	20.9011	20.9011	20.9378	20.9249	20.9257	20.9286	20.9301
1985	18.2648	20.3567	20.6863	20.9898	20.9898	20.9835	20.9398	20.9701	20.9699	20.9673

a = 2.0

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	17.7787	17.4218	17.4583	17.4600	17.4586	17.4586	17.4583	17.4583	17.4583	17.4583
1977	20.1104	20.1346	19.2519	19.3397	19.3435	19.3396	19.3396	19.3887	19.3886	19.3389
1978	19.8375	19.9342	20.2144	20.4539	20.5046	20.5094	20.5110	20.5112	20.5112	20.5119
1979	27.8497	27.2623	27.2645	27.2721	27.2198	27.2253	27.2254	27.2248	27.2248	27.2247
1980	30.6489	30.8892	30.3524	30.3537	30.3588	30.3019	30.3068	30.3068	30.3062	30.3062
1981	25.1792	25.1799	25.1841	25.1825	25.1825	25.1827	25.1827	25.1828	25.1828	25.1828
1982	24.1848	24.0015	24.0087	24.0469	24.0314	24.0320	24.0341	24.0335	24.0343	24.0344
1983	20.6292	21.7794	22.0287	22.0431	22.1216	22.0954	22.0967	22.1012	22.1028	22.1045
1984	20.8346	20.7176	20.8154	20.8777	20.8823	20.9075	20.8999	20.9003	20.9018	20.9026
1985	18.2648	20.2555	20.5502	20.5954	20.8273	20.8232	20.7969	20.8154	20.8152	20.8141

TABLE X

THE COMPOSITE WEIGHTS FOR THE ADAPTIVE WEIGHTING SCHEME

a = 1.0

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	.669410	.410030	.463214	.467717	.460540	.460660	.454379	.455796	.459639	.445909
1977	.661926	.668467	.414487	.466589	.470887	.463616	.463710	.457479	.458861	.462532
1978	.019330	.073830	.294416	.346763	.397318	.403261	.400746	.401256	.396672	.398096
1979	.988095	.416549	.429270	.492778	.415321	.454729	.458394	.453337	.453475	.448698
1980	.797940	.967175	.434948	.446175	.503196	.420311	.458803	.462305	.457110	.457223
1981	.350740	.360099	.451032	.376597	.381084	.409442	.393949	.420222	.423239	.421085
1982	.463285	.367924	.375575	.452638	.386220	.390082	.414814	.398381	.422825	.425616
1983	.013565	.333451	.347245	.254803	.430454	.370154	.374066	.399199	.388650	.413024
1984	.285500	.214516	.311886	.338491	.345119	.412316	.360945	.364548	.387916	.381759
1985	.916601	.486886	.395115	.417152	.374510	.380244	.440251	.387897	.391020	.411410

a = 1.2

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	.669410	.416916	.460377	.463069	.459464	.459529	.456886	.457382	.458524	.454988
1977	.661926	.668306	.423487	.465252	.468276	.464046	.464091	461480	.461957	.463022
1978	.019330	.065398	.242352	.319309	.358904	.363424	.363253	.363597	.362047	.362537
1979	.988095	.459867	.468031	.506158	.437643	.463287	.465261	.462585	.462618	.460920
1980	.797940	.963531	.481230	.488088	.520706	.446857	.471235	473116	.470311	.470324
1981	.350740	.358567	.425006	.379436	.381865	.395482	.390202	402091	.403365	.402880
1982	.463285	.370750	.376917	.431116	.391539	.393531	.404800	398464	408849	.409940
1983	.013565	.315989	.342284	.348344	.400986	.366712	.368745	380256	.378377	.388671
1984	.285560	.223656	.299932	.330186	.335096	.378248	.351896	353644	.363609	.364390
1985	.916601	.512665	.433538	.440843	.392931	.396731	.431915	.404442	.405759	.413331

TABLE X (Continued)

a = 1.4

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	.669410	.423446	.459790	.462172	.459564	.459599	.458354	.458553	.458947	.457882
1977	.661926	.668153	.431853	.466326	.468504	.465824	.465845	.464620	.464809	.465167
1978	.019330	.059225	.202756	.288502	.319625	.323046	.323616	.323839	.323279	.323468
1979	.988095	.497081	.502432	.525565	.465274	.481420	.482388	.480819	.480813	.480152
1980	.797940	.960042	.520739	.524972	.543653	.479211	.494146	.494951	.493292	.493273
1981	.350740	.357465	.407888	.378383	.379801	.386803	.385055	.390213	.390716	.390604
1982	.463285	.373413	.378527	.418196	.393191	.394302	.399821	.397263	.401509	.401908
1983	.013565	.300335	.336451	.341445	.379503	.358959	.360099	.365757	.366011	.370185
1984	.285500	.230711	.291590	.321710	.325384	.353668	.339619	.340513	.344971	.346364
1985	.916601	.535527	.466517	.465907	.416619	.419094	.439669	.424608	.425169	.427999

a = 1.6

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	.669410	.429647	.460852	.462379	.460628	.460647	.460005	.460095	.460248	.459878
1977	.661926	.668007	.439973	.468900	.470501	.468689	.468698	.468153	.468153	.468288
1978	.019330	.054511	.172453	.257313	.281743	.284308	.285004	.285145	.284931	.285010
1979	.988095	.529396	.532934	.547088	.495278	.505259	.505687	.504721	.504706	.504429
1980	.797940	.956696	.554724	.557322	.567978	.513592	.522497	.522804	.521789	.521767
1981	.350740	.356636	.396099	.376051	.376929	.380773	.380195	.382497	.382700	.382671
1982	.463285	.375928	.380259	.410247	.393640	.394298	.397195	.396070	.397871	.398022
1983	.013565	.286222	.329946	.334145	.362527	.349635	.350313	.353288	.353850	.355602
1984	.285560	.236321	.285759	.313724	.316492	.335359	.327572	.328048	.330144	.331068
1985	.916601	.555939	.495175	.490482	.443266	.444860	.456929	.448357	.448599	.449679

TABLE X (Continued)

a = 1.8

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	.669410	.435542	.462249	.463633	.462396	.462407	.462051	.462094	.462160	.462015
1977	.661926	.667868	.447838	.472429	.473627	.472341	.472343	.471997	.472037	.472093
1978	.019330	.050794	.148981	.227854	.246990	.248897	.249509	.249597	.249511	.249547
1979	.988095	.557721	.560051	.568726	.525160	.531307	.531471	.530865	.530851	.530728
1980	.797940	.953485	.584157	.585708	.591683	.546996	.552277	.552359	.551733	.551716
1981	.350740	.355988	.387660	.373476	.374046	.376281	.376095	.377180	.377266	.377258
1982	.463285	.378305	.382035	.405313	.393815	.394226	.395840	.395299	.396111	.396172
1983	.013565	.273433	.322945	.326531	.348239	.339832	.340256	.341914	.342387	.343169
1984	.285500	.240889	.281670	.306615	.308713	.321508	.317026	.317289	.318321	.318840
1985	.916601	.574276	.520331	.513693	.470670	.471693	.478847	.473814	.473920	.474343

a = 2.0

Year	v = 1	v = 2	v = 3	v = 4	v = 5	v = 6	v = 7	v = 8	v = 9	v = 10
1976	.669410	.441155	.464479	.465563	.464454	.464660	.464451	.464474	.464504	.464442
1977	.661926	.667735	.455444	.476572	.477481	.476533	.476533	.476331	.476351	.476376
1978	.019330	.047787	.130542	.201283	.216263	.217674	.218155	.218211	.218174	.218191
1979	.988095	.582751	.584257	.589526	.553407	.557222	.557265	.556881	.556871	.556813
1980	.797940	.950402	.609802	.610671	.613881	.577768	.580923	.580916	.580528	.580516
1981	.350740	.355469	.381420	.371040	.371426	.372791	.372738	.373281	.373320	.373317
1982	.463285	.330556	.383810	.402282	.39404[illegible]	.394308	.395255	.394973	.395363	.395390
1983	.013565	.261789	.315601	.318700	.335655	.329994	.330269	.331241	.331581	.331952
1984	.285500	.244680	.278796	.300535	.302136	.310953	.308281	.308431	.308962	.309240
1985	.916601	.590838	.542606	.535203	.497264	.497924	.502234	.499198	.499198	.499417

given to the most recent error history, if that squared error is large is for the forecast from the $j^{\underline{th}}$ technique (the ARIMA forecast) then w_1 will increase dramatically. Also, as a increased above one, the variability of the composite weight also increased, since the weight adapted greater to more recent error histories. It is interesting to note that when a=1 and v increased towards 10, the composite weights, w_1, for the adaptive scheme were not greatly different from the value of .5 for w_1 in a simple average composite. The values for w_1, when a=1 and v=10, for example, ranged from about .38 to .46. This would tend to suggest that the econometric and ARIMA forecasts in terms of resulting squared forecast errors were not greatly different from each other. On the other hand, the composite weights generated by an unrestricted linear combination of forecasts did tend to vary much more than the adaptive weighting schemes using ten error histories even though the smallest number of observations used to calculate the unrestricted linear combination of forecasts was ten observation in 1976. The variability of these composite weights can be seen in Table XI. Yet, in each case the ARIMA model was weighted more heavily than the econometric forecast, with the weight on the econometric forecast actually being negative in 1976 through 1978. By re-examining Table VI, it can be seen that the econometric model over forecasted between 1973 and 1975, with a large over forecast in 1974 at a turning point. If the weights on

TABLE XI

COMPOSITE WEIGHTS FOR THE UNRESTRICTED LINEAR COMBINATION OF FORECASTS (1976-1985)

Year	Intercept	Econometric	ARIMA
1976	2.6752	-.2967	1.1762
1977	3.9176	-.1359	.9553
1978	3.6147	-.0511	.8787
1979	1.2195	.2742	.6390
1980	-.2181	.3577	.6138
1981	5.3375	.0802	.6652
1982	5.6254	.0744	.6557
1983	5.4962	.0943	.6405
1984	5.5204	.1867	.5413
1985	4.1822	.3412	.4221

the econometric and ARIMA forecasts are summed, their sums range between .72 in 1984 and .97 in 1980. The sum of these weights were always less than one, which was found to be the case by Granger and Ramanathan.

The mean squared forecast errors produced by each of the different types of forecasts is shown in Table XII. Each of the composite forecasts resulted in a smaller mean squared forecast error than either the econometric or ARIMA model alone. The forecasts with the lowest mean squared forecast error were those generated by the unrestricted linear combination of forecasts. The simple average composite produced the next lowest mean squared forecast error. Figure 10 shows the mean squared forecast errors from the adaptive weighting schemes tended to produce lower mean squared forecast errors when longer forecast error histories were used to calculate the composite weights. Also, as Figure 10 shows, placing more importance on recent error histories by increasing the value for the smoothing factor, a, did not appear to reduce the resulting mean squared forecast errors.

Table XIII shows that the ARIMA model alone produced the smallest mean forecast error in absolute terms, while the econometric forecasts had the largest mean forecast error. All of the mean forecast errors were positive except for the mean forecast errors produced by the unrestricted linear combination of forecasts. Figure 11 seemed to

TABLE XII

MEAN SQUARED FORECAST ERROR (1976-1985)

Econo-metric	ARIMA	Simple Average	Unrestricted Linear Combination		Adaptive Weighting					
					a =1.0	1.2	1.4	1.6	1.8	2.0
7.0863	6.0587	3.9543	3.8234	v=1	5.8210	5.8210	5.8210	5.8210	5.8210	5.8210
				2	5.7124	5.6783	5.6523	5.6327	5.6180	5.6702
				3	4.1505	4.2368	4.3182	4.3927	4.4605	4.5223
				4	4.1016	4.1574	4.2199	4.2871	4.3559	4.4238
				5	4.2134	4.2464	4.2901	4.3390	4.3926	4.4477
				6	4.0045	4.0787	4.1535	4.2291	4.3043	4.3776
				7	3.9843	4.0675	4.1488	4.2280	4.3047	4.3784
				8	4.0826	4.1188	4.1753	4.2414	4.3115	4.3818
				9	4.0662	4.1104	4.1706	4.3487	4.3098	4.3808
				10	4.0333	4.0955	4.1642	4.2360	4.3087	4.3803

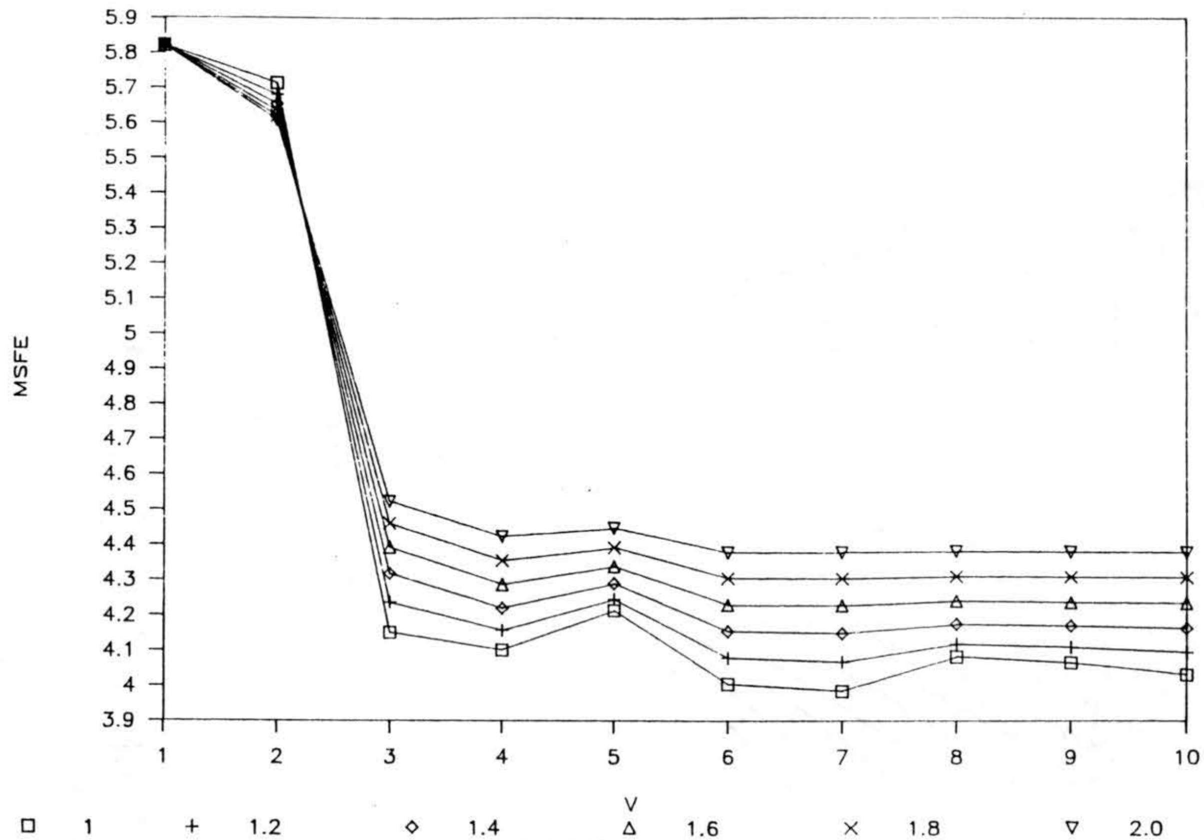


Figure 10. The Mean Squared Forecast Errors From the Adaptive Weighting Scheme

TABLE XIII

MEAN FORECAST ERROR (1976-1985)

Econo-metric	ARIMA	Simple Average	Unrestricted Linear Combination		Adaptive Weighting					
					a =1.0	1.2	1.4	1.6	1.8	2.0
1.6916	.1017	.8966	-.2356	v=1	.3934	.3934	.3934	.3934	.3934	.3934
				2	.6874	.6706	.6557	.6423	.6302	.6192
				3	.6764	.6455	.6215	.6025	.5872	.5744
				4	.7408	.7143	.6889	.6652	.6436	.6244
				5	.8408	.7699	.7375	.7072	.6794	.6546
				6	.7855	.7566	.7271	.6988	.6727	.6491
				7	.7460	.7346	.7147	.6918	.6686	.6467
				8	.7851	.7550	.7258	.6980	.6722	.6489
				9	.7893	.7579	.7275	.6990	.6727	.6492
				10	.7861	.7583	.7282	.6994	.6730	.6493

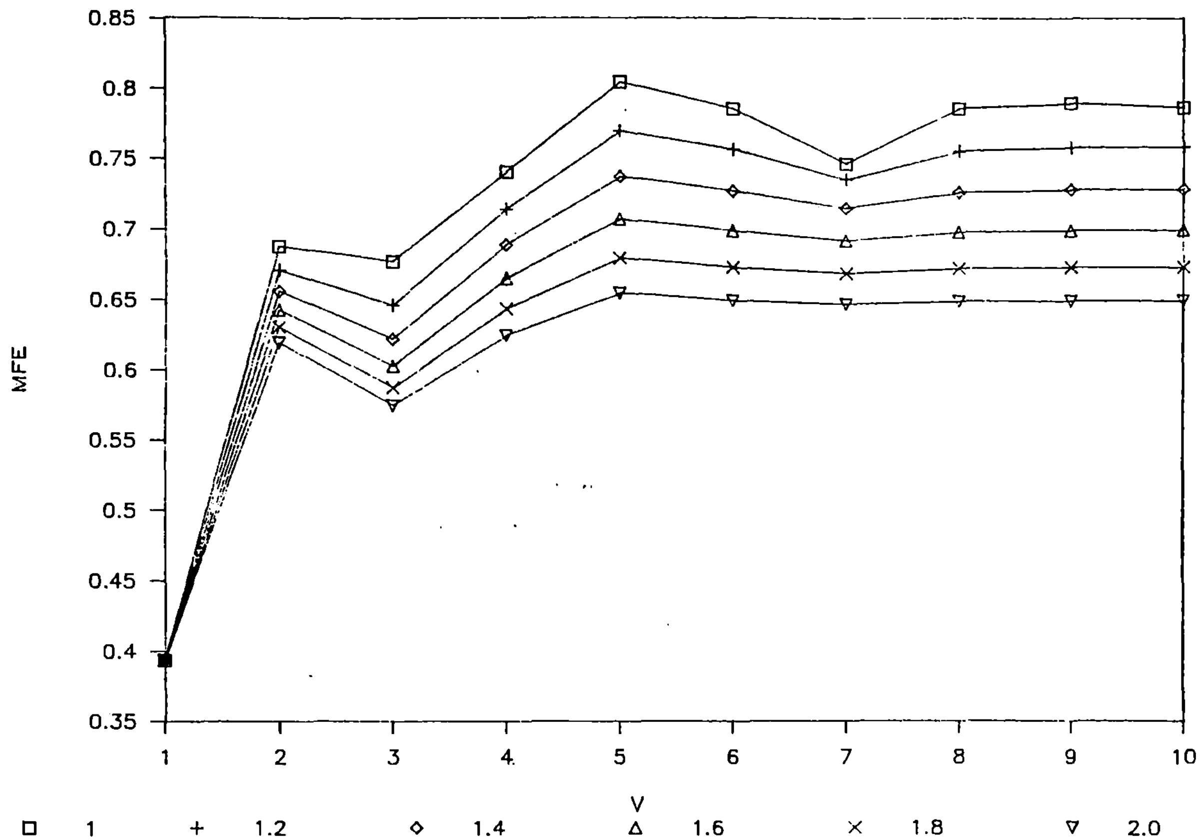


Figure 11. The Mean Forecast Errors From the Adaptive Weighting Scheme

indicate that the mean forecast errors did become slightly smaller as v decreased and as a increased.

The mean absolute percentage errors presented in Table XIV showed result similar to the mean squared forecast errors, with all of the composite forecasts producing smaller mean absolute percentage errors than the econometric or ARIMA forecasts alone. The unrestricted linear combination of forecasts did not produce the lowest mean absolute percentage error as it did the mean squared forecast error. The mean absolute percentage errors, like the mean squared forecast errors, did appear to decrease as v increased and as a decreased towards one. Figure 12 presents the MAPE versus the number of error histories with varying values for a.

Table XV shows that the individual forecasts and the composite forecasts all produced Theil U coefficients less than one, indicating that all were superior to a naive forecast with respect to forecasting relative changes. The Theil U coefficients for each of the composite forecasts were lower than those for the econometric or ARIMA forecasts. The Theil U statistics appeared to decrease slightly as v increased and to decrease as a decreased towards one, as can be seen in Figure 13.

Although the use of composite forecasting appeared to improve the accuracy of point forecasting over the econometric or ARIMA forecasts, composites did not improve upon the capabilities of the forecasts to indicate turning

TABLE XIV

MEAN ABSOLUTE PERCENTAGE ERROR (1976-1985)

Econometric	ARIMA	Simple Average	Unrestricted Linear Combination		Adaptive Weighting a =1.0	1.2	1.4	1.6	1.8	2.0
9.5056	9.1834	6.1835	7.2281	v=1	8.4693	8.4693	8.4693	8.4693	8.4693	8.4693
				2	7.7779	7.7057	7.6416	7.5845	7.5334	7.4873
				3	6.6159	6.6302	6.6364	6.6376	6.6362	6.6337
				4	6.5773	6.5683	6.5659	6.5684	6.5736	6.5800
				5	6.6789	6.6298	6.6166	6.4009	6.5101	6.6546
				6	6.4273	6.5028	6.5560	6.5908	6.6120	6.6242
				7	6.3534	6.4507	6.5222	6.5696	6.5989	6.6160
				8	6.4831	6.5003	6.5520	6.5875	6.6097	6.6226
				9	6.4481	6.4978	6.5479	6.5843	6.6076	6.6213
				10	6.4669	6.4644	6.5309	6.5770	6.6043	6.6198

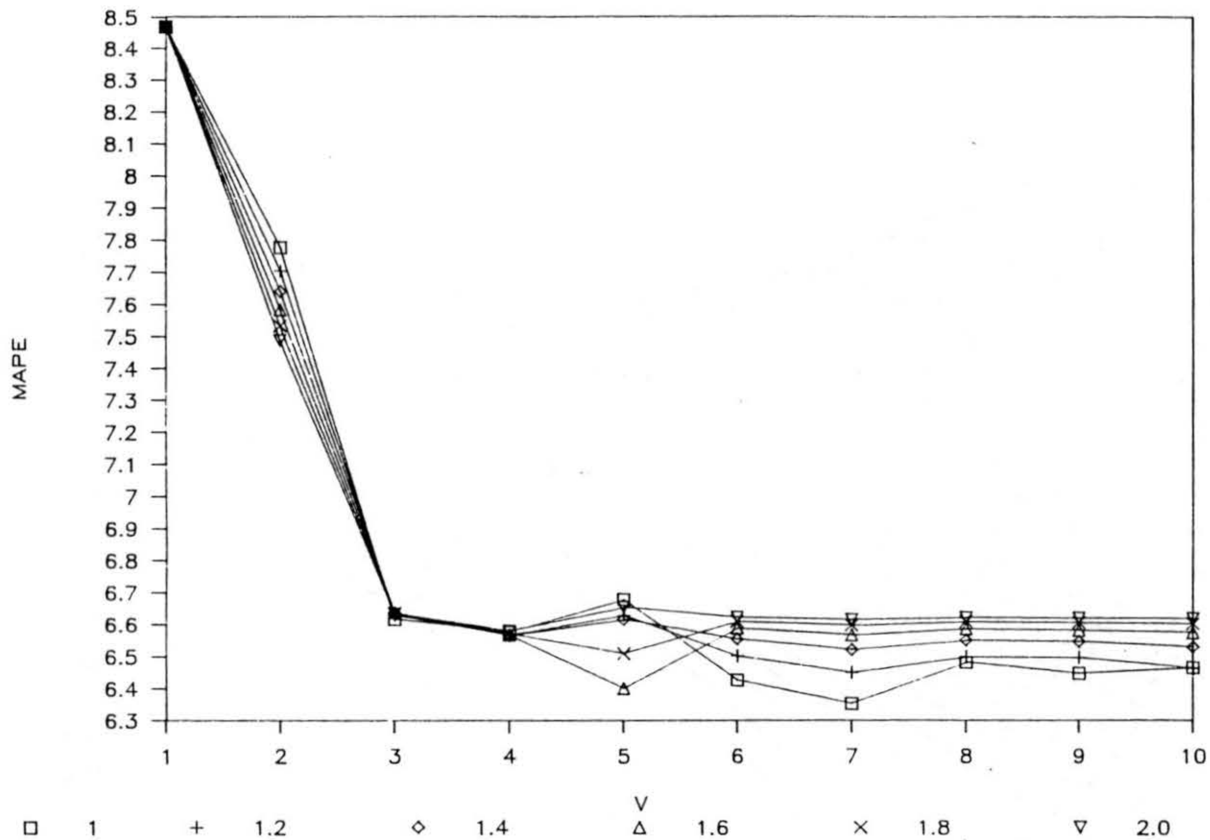


Figure 12. The Mean Absolute Percentage Errors From the Adaptive Weighting Scheme

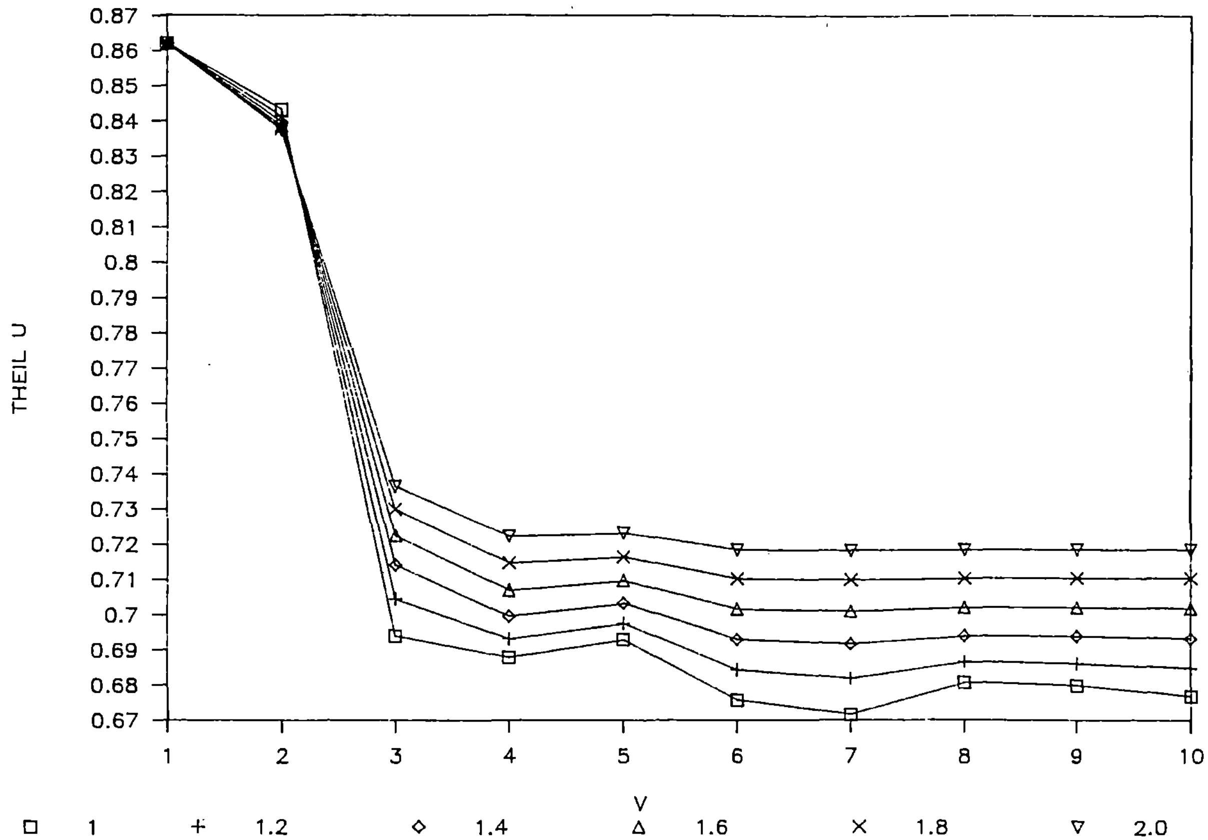


Figure 13. The Theil U Coefficients From the Adaptive Weighting Scheme

TABLE XV

THEIL U COEFFICIENTS (1976-1985)

Econo-metric	ARIMA	Simple Average	Unrestricted Linear Combination	Adaptive Weighting						
					a =1.0	1.2	1.4	1.6	1.8	2.0
.9453	.8983	.6545	.7240	v=1	.8620	.8620	.8620	.8620	.8620	.8620
				2	.8433	.8413	.8398	.8387	.8379	.8374
				3	.6939	.7045	.7141	.7225	.7299	.7364
				4	.6879	.6931	.6996	.7070	.7147	.7223
				5	.6930	.6976	.7034	.7098	.7165	.7233
				6	.6757	.6844	.6930	.7017	.7102	.7184
				7	.6716	.6819	.6917	.7010	.7099	.7182
				8	.6809	.6868	.6942	.7023	.7105	.7185
				9	.6799	.6862	.6939	.7021	.7104	.7185
				10	.6767	.6847	.6932	.7018	.7103	.7184

points. Table XVI shows the turning point errors generated by each of the forecasts, individual and composite. The econometric forecasts were much better at signalling turning points than any of the other forecasts, individual or composite. The econometric model accurately forecasted changes or no changes in the direction of prices 75 percent of the time. Only one change in the direction of price was forecasted when no change occurred, and one actual change in the direction of price was missed, so the econometric model missed one turning point over the 1976-1985 period. The ARIMA model did not accurately forecast any of the changes in price direction accurately, and only forecast three no changes in price direction accurately over 1976-1985. Furthermore, the econometric model accurately forecasted the movement in the direction of prices 77.77 percent of the time over the period of 1966-1985, while the ARIMA model accurately forecasted price direction only 38.88 percent of the time. The composites, in most cases were not any worse than the ARIMA model at forecasting price direction, but did not improve upon the ARIMA model's capabilities to forecast turning points. This may have been because when the ARIMA model missed the turning point, its error was large enough to offset the econometric model's correct forecast of directional change.

TABLE XVI

TURNING POINT ERRORS (1976-1985)

Actual	Econometric		ARIMA		Simple Average		Unrestricted Linear Combination of Forecasts	
	C	NC	C	NC	C	NC	C	NC
C	2	1	0	3	0	3	0	3
NC	1	4	2	3	2	3	2	3

<u>Actual</u>	<u>Adaptive Weighting: a=1.0, a=1.2, a=1.4, a=1.6, a=1.8, a=2.0</u>																			
	v = 1		2		3		4		5		6		7		8		9		10	
	C	NC	C	NC	C	NC	C	NC	C	NC	C	NC	C	NC	C	NC	C	NC	C	NC
C	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3
NC	5	1	4	1	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3

C = Change   NC = No Change

Summary and Conclusions

The results of this study indicated that users of forecasts of annual beef cattle prices who are interested in point accuracy of forecasts would perhaps benefit from using a combination of forecasts. Furthermore, even if a set of forecast error histories are unavailable, the forecast user might be advised to use a simple average of forecasts, particularly when the individual methods employed are highly varied in their assumptions. While a large number of forecast periods were not available for analysis, other studies such as Brandt and Bessler and Granger and Ramanathan have tended to support these findings. The findings in this study do indicate, contrary to Brandt and Bessler's findings, that when forecast error histories are available, the forecaster should make use of the greatest number of forecast error histories available in order to formulate a composite forecast.

While the findings from this study implied that the forecast user who is interested in the point accuracy of forecasts might benefit from using a combination of forecasts, the combination of forecasts did not show any improvement in forecasting turning points over either of the individual forecasts. Had the turning point performance of the two types of models been more similar perhaps the results would have been different. The fact that the econometric model performed much better than the ARIMA model in terms of signalling turning points could be indicative of either a misspecified ARIMA model or that the relationships between the annual average farm price of beef cattle and the independent variables remained fairly stable while the structure of the price series itself was gradually changing. Unfortunately there were no strong patterns in the residual autocorrelations to support either one of these possible answers over the other.

The data for 1985 were used to make forecasts of the 1986 annual average price of beef cattle received by farmers with the econometric model, the ARIMA model, a simple average of the two, and an unrestricted linear combination of the two forecasts. These forecasts are shown in Table XVII. It can be noted that these forecasts all were in the 20 to 21 dollars per hundredweight range, while the average deflated price for 1985 was about 19.68 dollars per hundredweight.

TABLE XVII

FORECASTS FOR THE DEFLATED ANNUAL PRICE OF BEEF CATTLE RECEIVED BY FARMERS IN 1986

Econometric	ARIMA	Simple Average	Unrestricted Linear Combination of Forecasts
19.97	20.96	20.47	19.85

The forecasts in Table XVII can be reinflated with the average value of 330 for the Farm Price Index (1967=100) from January through August of 1986 as estimates of the nominal values. The forecasts in nominal terms are 65.90, 69.16, 67.55, and 65.50 dollars per hundredweight, respectively. The average nominal actual price, reinflated with the estimate of 330 for the index over this period, was 65.53 dollars per hundredweight.

With the financial stress felt currently and in recent years within the beef cattle industry, an accurate expectation of product prices has become of even greater importance in long-term production planning. Given the possibility of lower returns resulting from inaccurate expectations of future prices, more accurate forecasts of prices are needed. Composite forecasting provides one possible means of reducing large forecast errors. This study showed that point forecasting accuracy could be improved with composite forecasting, and that in most cases the composite forecasts did at least as well as the worst of the individual techniques in indicating turning points. Further research in this area might include the formulation of confidence intervals about the individual and composite forecasts. Certainly producers who need to evaluate price risk would find this type of information to be useful in production planning.

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