VALUING LOAN SALES UNDER VARYING LEVELS OF RECOURSE: A CONTINGENT CLAIMS APPROACH

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INTRODUCTION AND PROBLEM STATEMENT

Financial guarantees or recourse provisions are widespread in financial markets, especially in debt-related contracts. Parent companies routinely guarantee the debt obligations of their subsidiaries. Commercial banks act as guarantors for financial obligations through letters of credit. Insurers guarantee interest and principal on mortgages. The U.S. Government guarantees timely payment of principal and/or interest on certain qualifying mortgages.

In each of these examples, the guarantee gives the lender recourse to someone other than the borrower in case of default. That is, if the borrower defaults, another entity is legally bound to provide payment on the defaulted obligation. As a result, guarantees add value to the particular financial asset because these provisions reduce the holder's risk.

Guarantees are not always explicitly stated, however. Often, it is "understood" or "assumed" that some parent entity will provide a guarantee or recourse even though not obligated to do so. Prime examples of this phenomenon are the quasi-governmental agencies such as the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporate (FHLMC). While the securities issued by these agencies are not actually guaranteed by the "full faith and credit" of the U.S. Government, it is usually assumed that, if they defaulted on their obligations, the Government would provide a guarantee or recourse. The guarantee in this case is not explicitly stated, but implicitly assumed. Nevertheless, such assumed guarantees potentially impact the value of financial obligations. Note that explicit guarantees entail a legally enforceable obligation, while implicit guarantees entail, at best, a moral obligation.

This research explores the provision of both explicit and implicit recourse in a particular type of financial obligation -- commercial loan sales or participations. Such sales frequently contain explicit guarantees. Further, it has been argued (Gorton and Pennacchi 1989, Wall 1991) that in cases where no explicit guarantees are given, implicit ones exist. The loan sales valuation models developed in this research include provisions for both explicit and implicit recourse. It is shown that both contribute to the value of loans sold, but the contribution varies according to the particular conditions of recourse.

The choice of loan sales as a focus of the valuation models is based on the following considerations: (1) that there are regulatory requirements for market value accounting for banks, (2) that recently enacted risk-based capital standards give banks a new incentive for loan sales, and (3) that regulatory treatment of loan sales ignores implicit recourse and the potential risk it introduces into bank balance sheets. Loan sales, then, represent financial instruments which potentially contain implicit recourse and which are important to the asset and liability management of commercial banks. Regulatory considerations of market valuation make a valuation model which accounts properly and fully for risk and recourse necessary and important.

Market Value Accounting

This research is motivated by the recent requirement by regulators of market value accounting by banks (FASB 107). As bank portfolios contain more and more loans purchased from other banks, there will be a growing need for a valuation model applicable to these loans sales.

The requirement of market value accounting has been motivated largely by the problems in the S&L industry. It has been argued (Kane 1985) that a major part of the high cost to taxpayers of the S&L bailout resulted from inefficient imposition by regulators of closure rules. That is, S&L's were often granted "forbearance" and allowed to continue to operate after their net worth had fallen to zero.

A serious flaw in the imposition of closure rules is book value accounting by banks. While mandated by Generally Accepted Accounting Practices (GAAP), book value accounting is often pointed to as problem in need of resolution. GAAP requires that assets and liabilities be recorded at their historical cost. Subsequent decreases in value are addressed through an offsetting depreciation or reserve account, while subsequent increases in value are not realized until disposition of the asset. Using balance sheet data, regulators seek to close insolvent institutions before net worth becomes negative (i.e., when net worth is approximately equal to zero). When an institution is closed, it must be disposed of either by liquidation or, more frequently, by purchase and assumption by some other, more stable institution. If the institution is liquidated, the FDIC pays off the insured depositors and sells off the assets of the closed institution. If, instead, a purchase and assumption is arranged, the purchasing institution assumes all liabilities of the failed institution and purchases selected assets. The purchase of assets is made by sealed bid. Normally, the purchasing institution bids on only the best assets, leaving the "dregs" for the FDIC to liquidate. Both means of closure require the disposition of assets at market value. Thus, insolvency is determined at book value and resolution is accomplished at market value. Because of this, institutions may be found to have market value net-worth less than zero upon liquidation.

Further, GAAP-mandated historical cost accounting may hinder the effectiveness of regulator-required changes in loan loss reserves and in capital accounts. Benston, et.al. (1986) argue that book value accounting confers on bank managers a degree of discretion in the disclosure of information -- either favorable or unfavorable. They point to the 1984 crisis at Continental Illinois as an example of managers not recognizing "bad news" and cite the fact that the bank's unrealized losses had grown to exceed its capital. Market value accounting, on the other hand, would force managers to recognize gains and losses and give a truer picture of the bank's capital position. In this way, the function of capital as a gauge of the need for regulatory discipline would be enhanced. The authors argue that this allows market value accounting to become a form of risk-related premium which substitutes market discipline for regulatory discipline.

Thus, historical cost (book value) accounting generates financial information that does not reflect the current condition of a bank. In addition, with historical cost accounting, managers have considerable discretion in the management of the information reported in the bank's financial statements. The consequence of these conditions is an inefficient system of closure and increased cost to taxpayers. A change from book-value accounting to market value accounting is seen as a way to relieve these problems (see e.g., Benston et.al. 1988, Benston 1990, Mengle 1989, 1990). Altman (1992) argues that market value accounting (MVA) would improve regulatory effectiveness and discipline. MVA, by recognizing the true value of assets and liabilities, could enhance the timeliness of adjustments to loan loss reserves, adjustments to capital accounts, and closure of insolvent institutions.

The potential benefits of MVA have been recognized in a number of regulatory pronouncements. In 1987, the Financial Accounting Standards Board (FASB) proposed requiring companies to report the market values of financial instruments. In 1989, the Shadow Financial Regulatory Committee included in its "Program for Deposit Insurance and Regulatory Reform" the recommendation that reorganization be based on market values as this could be expected to allow regulators to take more informed actions. Also, in 1992, FASB proposed requiring "fair" valuation of many assets, including commercial and industrial loans which are impaired. Most recently, FASB 107 has mandated "fair" valuation of all financial instruments, whether realized or not, in the statement of financial position. This FASB mandate is effective for financial statements for fiscal years ending after December 15, 1992[1]

Resistance to changing to MVA is often based on the assertion that the cost

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[1]The effective date is postponed until December 15, 1995, for entities with less than $150 million in total assets in the current statement of financial position.

will outweigh any benefit. Benston (1990) argues that, for many assets and liabilities, market values are not readily available and cannot be unambiguously determined. Further, "market value" may be construed a number of ways. Market value may be:

1. the value realized if the asset or liability were sold, i.e., exchange value; or
2. the replacement value of the asset or liability; or
3. the present value of the net cash flows expected from the asset or liability, i.e., going concern value.

These suggest that asset values may vary depending upon the circumstances under which they are sold. In that sense, "market value" does not represent an absolute standard and the "correct" market value may change under various circumstances. So, while MVA may offer solutions to some problems, it also creates new problems of its own.

For commercial banks, however, the problems may be less severe than for other non-financial firms. Mengle (1990), Benston (1990), Benston and Kaufman (1988) all argue that the majority of a bank's balance sheet is easily stated at market value and that the proper definition of market value for regulatory purposes is as "going concern" value. In particular, Benston (1990) argues the following:

1. Core deposits may be assumed at or close to market value due to their availability for withdrawal upon demand.
2. Long-term CD's may be valued as the present value of cash flows discounted at the readily-available rate on similar CD's.
3. Traded securities have readily-available market values and non-traded securities may be valued by referring to brokers or by present value calculations using the rate at which similar assets are traded -- that is, using the rate for assets in the same "risk class".
4. Certain standard types of consumer loans (e.g., auto loans, credit card receivables, etc.) may be valued at face value less an allowance for losses based on prior experience.
5. Mortgages may be valued by referring to the yields in the market for

mortgage-backed securities or to the rates on new mortgages with similar durations.

Business or commercial loans, however, present a valuation problem. While a secondary market exists for commercial loans, it is very thinly traded (i.e., transaction volume is low). In such markets, there are potentially large inefficiencies in pricing. Further, commercial loans are often subject to large credit risk specific to the individual borrower. This makes them difficult to value. Mengle (1989, 1990) and Benston (1990) argue that the current practice of maintaining loan loss reserves is very close to a practice of marking loans to market. Loans net of reserves may be at close to market value due to the conscious effort by regulators to ensure that loan loss reserves accurately reflect anticipated losses. However, loans net of reserves are not designed as a means for market value accounting for banks. Further, FASB 107 states that the use of the allowance for loan losses would not provide an acceptable estimate of fair value in most cases because it does not take into account the timing of the expected losses and all the potential losses due to credit risk. However, a market valuation model could well serve as a tool for estimating loan loss reserves, especially if it is accepted that:

$$\text{Face Value} - \text{Market Value} = \text{Loan Loss Reserves}$$

as suggested by Benston (1990).

Also problematic from a valuation standpoint are off-balance sheet activities such as contingent guarantees, commitments, and intangibles. FASB 107 requires "fair" valuation of off-balance sheet activities (OBSAs). But, OBSAs are difficult to value because they have no secondary markets and no specific loan loss requirements to allow estimation of market value. OBSAs totalled $6,347 billion at year end 1990, as compared to $3,389 billion in booked assets for all insured commercial banks (Sinkey 1992, 685). Loan sales (participations) are included in the category of OBSAs as they offer a means of getting an on-balance sheet activity (commercial lending) off the balance sheet. As such, they too are difficult to value, especially considering that they represent sales of commercial loans which are themselves difficult to value.

Risk-Based Capital Standards

Loan sales are not a new phenomenon. It is a long-standing practice for banks to sell all or a portion of the loans they originate. This is particularly true of mortgage loans. In 1970, the Government National Mortgage Association (GNMA) began to market securities backed by pools of home mortgages insured by the Federal Housing Authority (FHA) or guaranteed by the Veteran's Administration (VA). In 1971, the Federal Home Loan Mortgage Corporation (FHLMC) began pooling conventional mortgages (i.e., mortgages without VA or FHA guarantees or insurance) and marketing Participation Certificates (PCs) backed by the pools. In 1981, the Federal National Mortgage Association (FNMA) began pooling FHA and VA mortgages to back the issuance of Mortgage-Backed Securities (MBSs). More recently, Haubrich (1989) and Cantor and Demsetz (1993) have identified an increase during the 1980's in the practice by banks of selling all or a part of the commercial loans they originate.

Banks sell loans in response to a number of motivations including diversification and reduction of funding costs. However, the imposition of risk-based capital standards has added a new impetus for loan sales. The BASLE agreements established a system of calculating bank capital-to-asset ratios which takes into account the risk present in the particular bank's asset portfolio. This system was mandated by U.S. regulators in 1988 to be fully effective by 1992. Under the new rules, each asset category is first multiplied by a "risk factor" and then all are summed to yield a risk-weighted asset total. The capital ratio is then calculated using the risk-weighted asset total and compared to a regulator-determined standard for adequacy. This system is in contrast to the previous one which simply used total assets (unweighted) for capital ratios. There are four general risk classes. They are listed below along with their assigned weights:

Category	Examples of Assets Included	Risk Weight (%)
1	cash and equivalents	0
2	agency and mortgage-backed securities	20
3	single-family mortgages	50
4	commercial and consumer loans	100.

Under the risk-based capital standards, banks may increase their capital ratios by selling loans (risk-weight > 0) for cash (risk-weight = 0) without raising any new capital. Because of this change in capital standards, loan sales of all types may be expected to increase during the 1990's. In other words, the trend in commercial loan sales identified by Haubrich (1989) and Cantor and Demsetz (1993) may be expected to continue. As the loan sales market grows, the need will become more acute for a model which values commercial loan sales.

Regulatory Treatment of Implicit Recourse

The effect of a loan sale is the transfer of the loan's stream of cash flows from the lender/seller bank to the purchaser -- whether a bank, other financial institution, firm, or individual. However, the purchaser is not granted enforceable, legal recourse to the original borrower in case of default. Loan sales contracts often contain recourse provisions between the lender/seller and the purchaser. In the case of sales with full or partial recourse, regulators require the selling bank to include the entire loan amount in its asset totals for capital adequacy measures. Such sales leave all or a part of default risk with the selling bank and must be included in capital ratios in order to accurately account for risk. In the case of no recourse, regulators allow the selling bank to remove the loan entirely from its balance sheet and make no provision for it in capital ratios. Such sales relieve the seller of any default risk. However, it has been argued (Gorton and Pennacchi 1989, Wall 1991) that implicit recourse may exist in sales with no explicit recourse provisions. If this is the case, then lender/seller banks retain a degree of default risk from the loan sale and regulatory standards do not accurately account for this risk.

The above arguments suggest a need for a valuation model for commercial loan sales which takes account of the degree of recourse (either explicit or implicit) present. Chapter II describes analytical tools which are currently in use for valuing loan sales (e.g., intrinsic value, collateral value, expected recovery value, relative value). Each of these is based on subjective judgments and none takes into account recourse. Also described in Chapter II are tools for valuing commercial loans themselves (e.g., matrix pricing, market transaction data). Again, these are not directly applicable to loan sales as they fail to account for recourse.

Other potential sources of valuation tools for loan sales are those in use for mortgage-backed securities issued by GNMA, FHLMC, and FNMA. As described in Chapter II, these mortgage-backed securities have characteristics similar to loan sales. These tools are of two types -- traditional time-value models (e.g., "Average Life", Curly and Guttentag 1974) and contingent claims models. The time-value models, like the current commercial loan and loan sales models, rely on subjective judgments and take no account of recourse. The contingent claims models, however, offer a framework in which recourse may be included and which will allow an analytical rather than "rule of thumb" treatment of valuation. The mortgage-backed security models are not directly applicable to loan sales valuation due to considerations of recourse and guarantee described in Chapter II.

This research develops a valuation model for loan sales based on those developed for GNMA Pass-Through Securities. Specifically, the model is developed using the techniques of contingent claims. The model values commercial loan sales -- in particular, fixed rate, secured, non-amortizing loan sales -- based upon the default and prepayment decisions of the borrower(s) whose loan was sold, subject to the particular recourse provisions of the sale. Sinkey (1992, 559) states that commercial loans are primarily made with fixed-rate, non-amortizing terms. Further, these loans are usually tied to the cash flows from a particular asset or project of the borrower and, therefore, are primarily secured.

The traditional time value approach to valuation anticipates future cash flows and discounts them at some "appropriate" discount rate. Implicit in this process is an assessment of risk and of the degree to which investors are averse to this risk. That is, contracted cash flows are not discounted, but expected cash flows adjusted for the probability of default are discounted. Further, the discount rate is "appropriate" to the extent that it reflects investor attitudes toward risk. The assessments of risk are made at a point in time and applied to the valuation model.

The value of the participations considered in this research, like other fixed-payment securities, is contingent upon interest rates, collateral values, prepayments, and defaults. Time value models take little or no account of these valuation contingencies. By contrast, contingent claims models make no assumptions or assessments regarding individual risk preferences and explicitly allow for the types of contingencies mentioned above. Contingent claims models are built on stochastic diffusion processes. That is, contingent claims models describe the movement of such contingencies as interest rates, collateral value, prepayment, default, etc., through time rather than at a point in time. As with any model, though, a contingent claims model must make certain assumptions. Value is contingent upon an assumed stochastic process for interest rates. Secured participations are also contingent upon collateral value, so a stochastic process is assumed for collateral value. Therefore, the model is contingent upon assumed stochastic processes for short-term interest rates and collateral value. The actual provision of recourse or guarantee is accounted for by the addition of Poisson "jump" processes representing the sudden change in value when actual default or prepayment occurs.

The valuation model is in the form of a partial differential equation whose solution is the value of the particular loan sale. No closed-form solution has been found for the type of differential equation developed, so the technique of finite differences is used to approximate the solution. A set of comparative statics is developed to determine the relative role of each of the model's parameters in the valuation of loan sales. An important empirical question is how well the model describes the prices actually quoted for loan sales. Ideally, the model would be tested by generating prices for loan sales which could be compared to actual prices quoted in the markets. Unfortunately, data necessary for the estimation of the model is proprietary and not readily available to researchers owing to fiduciary responsibilities of the lender to the borrower(s).

The remainder of this research is arranged as follows:

Chapter I	describes the market for loan sales;
Chapter II	reviews the literature concerning the valuation of loan sales and related valuation models;
Chapter III	develops the contingent claims model for valuing secured loan sales under varying levels of recourse;
Chapter IV	develops a testing methodology, estimates the model developed in Chapter III, and performs sensitivity analyses on the results; and
Chapter V	states conclusions and summarizes the research.

CHAPTER I.

THE LOAN SALES MARKET AND MOTIVATIONS FOR LOAN SALES

The Loan Sales Market[2]

The sale of loans by commercial banks is not a new phenomenon. It is a long-standing practice for banks to sell all or a portion of the loans they originate. What makes this a subject of interest is the recent increase in the volume of this activity as documented by Haubrich (1989) and Cantor and Demsetz (1993), the anticipated continued increase due to risk-based capital standards, and regulatory requirement of market value accounting for banks.

The effect of a loan sale is the transfer of the loan's stream of cash flows from the lender/seller bank to the purchasing bank[3]. In a loan sale, the loan contract itself is not transferred. Rather, an additional contract is created between the seller and purchaser. The purchaser does not become a party to the original loan agreement. His or her relationship is solely with the lender/seller bank. Because of this, the loan sale includes no legal claim by the purchaser on the original borrower.

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[2]In order to reduce confusion, the following terminology is used throughout the research:

1. borrower -- This refers to the original borrower in the loan contract. This is the person whose loan is being sold.
2. lender/seller -- These terms are interchangeable and refer to the bank which originated the loan and subsequently sold it.
3. purchaser -- This refers to the bank which purchased the loan contract from the lender/seller bank.

[3]In a sense, a loan sale could be conceived of as a swap of one set of cash flows for another. In the case of an amortizing loan, the sale would be a swap of a single payment (sale price) for a series of payments (borrower scheduled payments). In the case of a single-payment loan, the sale would be a swap of a single payment now (sale price) for a future single payment (borrower scheduled payment).

That is, should the original borrower default, the purchaser of the loan has no legal right to seek restitution from the borrower and the lender/seller must file any and all claims against the borrower. The contract is strictly between the lender/seller bank and the purchaser. Such loan sales are transparent to the borrower as the lender/seller continues to service the loan.

Most loan sales are either loan strips or loan participations. A loan strip is the sale of a short-term portion of a longer-term loan. For instance, a bank might elect to sell one year's cash flows (payments) of a five-year loan. At the expiration of the strip, the selling bank may sell another strip in the same loan, or retain the cash flows from the remaining term for itself. A loan participation, on the other hand, represents the sale of a loan until maturity. In the case of either a strip or a participation, the sale may be in full (100 percent of the cash flows) or in part (< 100 percent of the cash flows). Loan participations and loan strips are in contrast to the securitization of commercial loans. Securitization implies the pooling of a number of loans and the sale of "pass-through" securities backed by the pool. In the same way as a participation, borrower payments are transferred to the purchaser of the "pass-through" securities. But, the risk of a "pass-through" is related to the risk of the pool and any correlation or diversification within the pool. Whereas, the risk of a participation (or strip) is specific to the particular borrower.

The size of the loan sales market is increasing, both in terms of dollar volume (Haubrich 1989, Gorton and Haubrich 1987, Gorton and Pennacchi 1989, Cantor and Demsetz 1993) and in terms of number of banks engaged in loan selling (Pavel and

Phillis 1987, Gorton and Haubrich 1987, Gorton and Pennacchi 1989). Haubrich (1989) reports a 103.4 percent increase in commercial loan sales from December 31, 1985, to June 30, 1988. Cantor and Demsetz (1993) report an 81.5 percent increase from June 30, 1988, to June 30, 1990, and a 16.7 percent increase from June 30, 1990, to June 30, 1992.[4]

Banks of all sizes are eligible to sell commercial loans from their portfolios. However, the market is dominated by large, money center banks. Haubrich (1989) reports that the nine largest banks account for more than half of all commercial loan sales. Cantor and Demsetz (1993) concur noting that the largest 1 percent of all banks account for approximately 90 percent of commercial loan sales. Loan sales by small banks tend to be primarily to upstream correspondents for purposes of covering overlines[5]. Loan sales by large banks tend to be to other U.S. banks and to U.S. branches of foreign banks. In the past, only a small fraction of loan sales have been to finance companies, insurance companies, pension funds, and nonfinancial firms. However, this segment of the purchasing population is increasing.

In the early stages of the development of the loan sales market, the loans sold tended to be loans to investment grade borrowers (BBB or Baa or better with

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[4]Figures reported by Haubrich (1989) were from "the six largest banks in the New York Federal Reserve district, six large banks in the San Francisco district, three in the Minneapolis district and five in each of the other nine districts" (page 40, footnote 2). Cantor and Demsetz quote figures for all banks as reported by the Federal Reserve Board of Governors.

[5]Commercial banks are constrained to lend no more than 15 percent of primary capital to any one borrower and/or his assigns. For small banks this constraint may frequently become binding. As a result, the amount of a loan request over and above the 15 percent limit (i.e., the overline) is sold to another bank, often an upstream correspondent. Overlines are a common motivation for the loan participations discussed in this research.

commercial paper ratings of A1/P1 or A2/P2) and of short maturity (90 days or less, often overnight). In 1986, 70 percent of the loans sold were loans made to investment grade borrowers. This figure fell to 35 percent in 1989, but had risen to 60 percent by 1992 (Cantor and Demsetz 1993, 35). Since 1987, loans sold have been predominately of maturities of one year or more. Today, many are of maturities up to two years, but tend toward shorter maturities.

Motivations for Loan Sales

A number of motivations exist for banks to sell all or a part of the loans they originate. Wall (1991) has identified the following:

1. Diversification -- This motivation is especially important for customer diversification as selling loans allows the bank to avoid credit concentration (i.e., a large number of loans to the same borrower or to borrowers with similar characteristics)
2. Source of funds -- Loan sales may be a source of loanable funds as opposed to reliance only upon deposits. The sale of "booked" loans can be used to provide funds for new loans.
3. Regulatory costs -- Funds from loan sales are non-reservable as opposed to deposits which are subject to fractional reserve requirements. Also, funds from loan sales are not used in the calculation of deposit insurance premiums as are deposits.
4. Risk reduction -- In addition to diversification (#1 above), loan sales decrease interest rate risk and credit risk by adding reviewers (from the loan buyer) to the lending process. This would increase the price received for the loans sold. Benston (1992) argues that, in the case of participations, any gains from this source are offset by moral hazard and adverse selection. That is, the purchaser knows that the seller has an incentive to sell loans of a lower quality and higher risk (especially if the loan is sold without recourse). Further, once the loan is sold, the seller has a reduced incentive to monitor the borrower and pursue any "problems" with the borrower. The purchaser can anticipate this behavior and increase the required return accordingly. This will have the effect of offsetting any gains from additional reviewers. Loan strips, however, are more likely to retain the benefit of risk reduction.

The moral hazard problem is reduced as the sale is only temporary (e.g., a sale of one year's cash flows of a five-year loan).

5. Reduction of funding costs -- In addition to the regulatory costs (#3), the cost of loan sales as a source of funds is related to the level of recourse provided with the loan sale. Loan sales with recourse are a low-risk asset which may be sold to risk averse investors with little discounting to adjust for individual risk tolerance. As the level of recourse is decreased, the discounting by the risk averse purchaser will increase. Further, loan sales with recourse also provide a reduction of the "underinvestment problem" identified by Myers (1977). That is, they reduce the incentive for shareholders to forego positive NPV projects (loans) due to the presence of debt (deposits) in the capital structure. This is discussed more thoroughly in Appendix A.
6. Reduction of capital costs -- Banks may increase their capital ratios by selling undervalued assets (i.e., selling loans), by decreasing their asset base, or by increasing their capital. Loan sales offer a way of decreasing assets (and thereby increasing capital ratios) which is lower in cost than increasing capital. The cost is lower for two reasons:
   A. "Too big to fail" implies 100 percent *de facto* insurance for banks large enough to engage in off-balance sheet activities (Pyle 1985). Selling loans without recourse retains this FDIC subsidy without causing an increase in risk-based capital required. Explicit recourse would cause required capital to increase.
   B. New capital has a negative stock price impact (Myers and Majluf 1984; Polonchek, Slovin, and Sushka 1989; Wall and Peterson 1991).

Of the above motivations, (4) risk reduction, (5) reduction of funding costs, and (6) reduction of capital costs will be affected by the degree of recourse included in the loan sale.

Risk Reduction

The moral hazard and adverse selection problems identified by Benston (1992) are most pronounced in the case of a loan participation without explicit recourse. In such a sale, the selling bank will have realized its required return from the sale and will have "rolled over" the funds into a new loan. The seller's motivation to monitor the borrower for the loan sold will be reduced as it is the purchaser who stands to lose if the borrower defaults. The degree of explicit recourse serves to strengthen the incentive to monitor by the selling bank. The higher the recourse, the higher the potential loss to the selling bank from default and the lower the moral hazard problem. Likewise, the moral hazard problem is less in strips than in participations.

Reduction of Funding Costs

James (1988) showed that loan sales serve to reduce the agency cost of the "underinvestment problem" (Myers 1977). He showed that this is true regardless of the level of recourse offered. Full recourse offers the highest degree of reduction of the agency cost while non-recourse offers the lowest. It can be shown that implicit recourse (Wall 1991) can also serve to reduce agency costs. This property of implicit recourse is demonstrated below.

Capital Costs

Loan participations without (explicit) recourse are treated as transfers for regulatory purposes (Salem 1986, Wall 1991). A participation without recourse is removed from the seller's balance sheet and reduces risk-based assets, thereby increasing regulatory capital ratios. This is not true of loans sold with (full or partial) recourse. While such loans are removed from the seller's balance sheet, regulators require that their full value be included in risk-based asset totals and, therefore, in the bank's capital requirements.

Loan strips, on the other hand, are not treated as transfers. Regulators require a loan to be sold to maturity before it qualifies for removal from the seller's balance sheet. Since buyers of strips are not obligated to continue buying (i.e., "rolling over" the strip), they are not true sales and do not provide the capital cost reduction of participations.

The discussion of this section has shown, then, that the two types of loan sales (i.e., strips and participations) are qualitatively different. Loan strips likely reduce the risk in the lending process more than do participations (motivation #4), but they do not offer a reduction in capital costs (motivation #5) as they are not considered true sales for regulatory purposes. The focus of the remainder of this study will be participations because they are sensitive to the capital-related incentives for loan selling. Henceforth, the terms "participation" and "loan sale" will treated as synonymous and used interchangeably.

Recourse in Loan Sales

As mentioned previously, the loan sales contract includes no recourse between the borrower and the purchaser of the loan. However, recourse is an important consideration between the lender/seller bank and the purchaser of the loan. Most loan sales include no recourse for the purchaser to the lender/seller. That is, in case of borrower default, the purchaser may not pursue the seller for additional repayment. In such cases, accounting and regulatory practices allow the removal of the loan from the originating bank's balance sheet. From a regulatory standpoint, sale of a loan without recourse removes not only the asset from the seller's balance sheet, but removes all residual risk from the sale as well. Some sales, however, contain provisions for recourse -- either full or partial. In these cases, the purchaser may pursue the seller to pay off all or part of the unpaid balance resulting from borrower default. Regulators require that the full value of such loans be included in risk-based asset totals and, therefore, in the bank's capital requirements.

Regulatory practice, then, provides a strong incentive for banks to sell loans without recourse. However, no recourse selling presumes a purchaser willing to accept no recourse. Whether or not a particular loan is marketable without recourse will depend upon the provisions of the loan (e.g., repayment terms, collateral, etc.) as well as the risk aversion of the potential purchaser. Given the incentive toward non-recourse sales, one may wonder why any seller would ever offer recourse. The answer may often be that the particular loan could not be sold without it.

Benveniste and Berger (1987) modeled the characteristics of asset securitization with recourse for commercial banks. They showed that asset securitization with recourse may be the optimal risk-sharing contract under certain levels of risk aversion by the purchaser of the asset-backed security. Further, they argued that asset securitization with recourse may have the effect of improving the quality of the asset portfolios of banks. That is, if the bank's assets may only be securitized with recourse, the bank is given an incentive to only accept low-risk assets due to the default risk retained under recourse. Wall (1991) makes a similar point for participations. Loan sales with recourse increase default risk for the selling bank. But, this is likely offset to a degree as the seller banks have an incentive to originate low-risk loans to sell and to hold in their portfolios. Thus, while loan sales with recourse may result from the characteristics of the particular loan being sold, such sales potentially offer the social benefit of lowering the risk in bank portfolios.

Implicit Recourse

There is some controversy over whether recourse may exist even in those contracts which explicitly offer none. Gorton and Pennacchi (1989) report that their conversations with asset sales bankers at money center banks suggest that, as a matter of course, banks buy back loans. That is, banks provide recourse even when they are not contractually required to do so. Wall (1991) has referred to this practice as "implicit recourse". If such implicit recourse exists and loan sales contain, as a matter of course, implicit guarantees, then the practice of removing loans sold from the balance sheet of the selling bank may be called into question. Such implicit guarantees would create off-balance sheet contingencies for the selling banks and would not remove the residual risk of the loan.

Wall (1991) notes that banks provide the highest recourse possible in order to maximize the price received on a loan sale. He asserts that this may include the granting of implicit recourse. Implicit recourse agreements represent a potentially important facet of the seller's reputation and thus an important determinant of the price received in future loan sales. As such, Wall argues that these agreements are generally honored. Gorton and Pennacchi have conducted studies on the degree to which implicit recourse is involved in loan sales. Their 1989 study suggests that it may be common enough to affect the rate of return loan buyers require. Their 1991 study using data from a specific bank failed to confirm the earlier study which used data averaged from a number of banks.

Related to the question of "implicit recourse" is the question of whether a selling bank would pursue a defaulting borrower for repayment even though the loan itself had been sold off the lender/seller's balance sheet with no recourse. The lack of recourse suggests that the seller has no legal obligation to pursue the borrower. But, Wall's (1991) argument (i.e., the highest recourse possible will be offered) along with the recognition that the seller's reputation may be considered by the purchaser in pricing loans in the future, suggest that the seller may act as "collector" for the buyer. Such a role would preserve the seller's reputation and help ensure future sales.

One could then argue that "no explicit recourse" does not always mean that, in actual practice, none will be offered. That is, a seller will likely stand ready to relieve loss on the part of the buyer either through a repurchase of the loan (implicit recourse) or by acting as a collection agent for the buyer if the reputation cost (i.e., the present value of lost future income from loan sales) exceeds the cost of providing implicit recourse. Thus, when valuing loan sales, those with "no explicit recourse" should be the lowest valued. The values of other types of loan sales will be expected to be higher than the lower bound to the extent the seller's reputation warrants, and as the level of recourse increases.

Regulatory Implications

The preceding discussion suggests that banks have incentives to sell loans without explicit recourse and at the same time to offer implicit recourse. Such recourse may expose banks to a source of off-balance sheet risk not included in regulatory risk measures. In this sense, then, the regulatory practice of taking no account of implicit recourse may be tantamount to ignoring an important source of risk in the banking system. As a result, banks offering implicit recourse benefit from a subsidy from the FDIC since the potential off-balance sheet risk is not "priced" or is incompletely priced. This subsidy gives banks an incentive to provide implicit recourse and represents an agency cost of implicit recourse.

Implicit Recourse and the "Underinvestment Problem"

It is argued, then, that implicit recourse represents an agency cost as it allows loan sellers to exploit a subsidy by the FDIC. It can also be shown, however, that at the same time, implicit recourse offers a reduction in the agency cost resulting from the "underinvestment problem" (Myers 1977). As a result, the two effects are potentially offsetting.

Myers (1977) argued that the presence of debt in a firm's capital structure leads to an incentive for stockholders to forego certain positive NPV projects. This is the "underinvestment problem" and represents an agency cost. Stultz and Johnson (1985) showed that collateralization of debt serves to mitigate at least a part of this agency cost. James (1988) showed that, for banks, loan sales with no recourse and loan sales with explicit recourse (both full and partial) serve the same purpose. The argument put forth by James (1988) is reviewed in detail in Appendix A.

James does not address the issue of implicit recourse (Wall 1991). However, it can be shown, using the framework of James (1988), that funding a loan with the proceeds of a loan sale with implicit recourse also serves to reduce the "underinvestment problem".

Equation 62 (Appendix A) states the James (1988) argument as the difference of three terms:

A. the expected return from the loan funded by the loan sale,
B. the expected payment on new deposits, and
C. the change in value of existing deposits if the new loan is made.

The expression A - B - C, then, is the change in shareholder wealth as a result of the new loan. If (A - B - C) < 0, then a wealth transfer results from the new loan and existing shareholders have an incentive not to make the loan. Note that this incentive exists even if the expected return from the new loan (A) is positive. This is analogous to the "underinvestment problem" of Myers (1977).

James argues that the term (C) is changed by altering the terms of the loan sale, while (A) and (B) are unchanged. The formulation for the wealth transfer under explicit recourse (from Appendix A) is as follows:

$$\int_0^{\bar{s}} \min\left[r_d(1-e)\,,\max\left[\frac{r_d(1-e)+(1-\gamma)\,(r_s)}{L_s}a_1(s)\,,a_1(s)+a_2(s)-r_s\right]\right]f(s)\,ds$$

$$-\int_0^{\bar{s}} \min[r_d(1-e)\,,\ a_1(s)]f(s)\,ds \tag{1}$$

where	s	= state of nature
	$a_1(s)$	= payoff from existing loan in state s
	$a_2(s)$	= payoff from new loan in state s
	A(s)	= total loan payoff in state s
	r_d	= return on existing debt
	r_N^d	= return on new debt
	e	= % of new loan funded from equity
	L_d	= total return on debt
	γ	= the degree of recourse provided.

Equation 1 is James's term (C). Note that $\gamma = 0$ for no recourse, $\gamma = 1$ for full recourse, $0 < \gamma < 1$ for partial recourse. It is argued here that if the loan is funded with a loan sale with only implicit recourse, then the expression becomes:

$$\int_0^{\bar{s}} \min\left[r_d(1-e),\ \max\left[\frac{r_d(1-e)+[1-\gamma(s)](r_s)}{L_s}a_1(s),\ a_1(s)+a_2(s)-r_s\right]\right]f(s)ds$$

$$-\int_0^{\bar{s}} \min[r_d(1-e),\ a_1(s)]f(s)ds\ . \tag{2}$$

Note that the only difference between the implicit and explicit recourse formulations is that for implicit recourse, the recourse term is changed to $\gamma(s)$. This indicates that the degree of recourse is now state-dependent rather than constant over all states (state-independent) as for explicit recourse. If in at least one state $0 < \gamma(s) \leq 1$, then implicit recourse will reduce the underinvestment problem in the same way as explicit recourse. As discussed above, implicit recourse may be viewed as a reputation effect. That is, if a loan seller has provided implicit recourse in the past, it can be expected to provide it again in the future. As a result, the expected implicit recourse may be modeled as:

$$0 < \int_0^{\bar{s}} \gamma(s)ds \leq 1 .$$

The amount of reduction in the "underinvestment problem" agency cost is dependent upon the magnitude of the recourse term -- either γ for explicit recourse or $\gamma(s)$ for implicit recourse. If there exists at least one state of nature for which $\gamma(s) > 0$, then implicit recourse provides more reduction than no recourse. However, unless $\gamma(s) > 0$ for <u>every</u> state of nature, explicit recourse will provide more reduction. This suggests that loan sellers have an incentive to develop a reputation for providing implicit recourse in order to enhance the marketability of the loans they wish to sell. That is, there exists an incentive to provide recourse even if not explicitly required to do so. Provision of implicit recourse potentially reduces an agency cost resulting from the "underinvestment problem" at the same time as it potentially increases an agency cost resulting from regulatory treatment of non-recourse loan sales (i.e., as if implicit recourse did not exist).

CHAPTER II.

EXISTING VALUATION MODELS

Valuation Methods for Loan Sales

While the market for loan sales is currently thinly traded, it is nevertheless an active market. This suggests that methods currently exist for valuation of loan sales. It is well to explore these methods before developing the model of this research. Loan prices are normally quoted on a "percentage of par" basis where "par" is the amount scheduled to be repaid. A buyer, then, is able to use this quoted price to determine if the return it represents is sufficient. The question, of course, is how to arrive at the "percentage of par" which is appropriate. A number of analytical tools (including Intrinsic Value, Collateral Value, Expected Recovery Value, and Relative Value Compared with Alternative Investments) have been suggested for the valuation of loans for sale[6].

1. Intrinsic Value: This tool attempts to determine the value of the loan as a claim against the pool of assets of the borrower and any guarantors. Three steps are required for this process -- (1) determine the value of the assets against which the lender may have claim, (2) determine the position of the lender relative to other claims against the assets, and (3) estimate the costs in both time and money of realizing the lender's claim in case of default. These steps lead to an estimate of the "available asset value". The extent to which this exceeds (or fails to exceed) the par value of the loan in question will determine the

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[6]Ryan, C., "Basic Analytical Tools for Valuing HLT Bank Loans", in J. Carlson and F. Fabozzi, ed.'s, The Trading and Securitization of Senior Bank Loans, (Probus Publishing Company, Chicago, IL), 1992, 137-69. Note that HLT Bank Loans are "Highly Leveraged Transaction" Bank Loans. The term refers to senior bank loans, especially those used as financing for such "highly leveraged" transactions as leveraged buyouts, acquisitions, or recapitalizations. However, the tools are equally applicable to any loans considered for sale.

premium (or discount) necessary in pricing.

2. Collateral Value: This process is similar to that of #1, except the emphasis is on the specific assets pledged to the particular loan of interest. Estimation of the "available collateral value" requires consideration of the type and value of the collateral, the validity of liens and security interests in the collateral, the ability of the lender to foreclose on the collateral, and the importance of the collateral to the ongoing viability of the borrower. The extent to which "available collateral value" covers the amount of the loan (i.e., the extent to which the loan is unsecured) will determine the premium or discount necessary in pricing.
3. Expected Recovery Value: This tool is a risk-adjusted present value calculation of the expected cash flows from the loan.
4. Relative Value Compared with Alternative Investments: This tool is based on a thorough understanding of the loan in question and the comparison with other, more actively traded investments for the purpose of inferring the loan's market value. Choice of the comparable investment(s) is central to the success of this technique. It is suggested that the choice be based both on qualitative measures (e.g., borrower/guarantor characteristics, capital structure of the borrower, collateral, borrowing covenants) and on quantitative measures(e.g., yield-to-maturity, Intrinsic Value, Expected Recovery Value).

Note that Relative Value (tool #4) incorporates much of the analysis suggested by the other three tools. Each of these tools is based largely on "rules of thumb" and subjective judgments. This research develops an objective analytical model appropriate for the valuation of loan sales.

Valuation Methods for Commercial Loans

It has been argued (e.g., Mengle 1989, 1990; Benston 1990; Benston and Kaufman 1988) that most bank assets and liabilities are currently recorded at or near market value -- a conspicuous exception being the loan portfolio. Mengle (1989, 1990) argues that, given certain assumptions about interest rate risk and credit risk, most of the loan portfolio is also already recorded at or near market.

Interest Rate Risk

Fixed rate loans with a maturity of one year or less may be assumed to be at market value. Floating rate loans eligible for repricing within one year may be assumed to be at market value. These assumptions are analogous to that for core deposits as mentioned in the Introduction (Benston 1990). Mengle (1989) shows that this argument would put some two-thirds of bank loan portfolios at market value.

Credit Risk

As Benston (1990) argues (and as stated in the Introduction), certain consumer loans and real estate loans may be readily marked to market value. Mengle (1989) concurs noting that default rates for such loans are well established. According to Berger, et.al., consumer loans are approximately 19 percent of bank loan portfolios and real estate loans are approximately 37 percent of bank loan portfolios.

Even with these assumptions, the category of commercial and industrial loans remains. These loans represent approximately 30 percent of bank loan portfolios and present a potential obstacle to market value accounting for banks. Mengle (1989, 1990) and Benston (1990) argue that the current practice of maintaining loan loss reserves is very close to a practice of marking loans to market. Loans net of reserves may be at close to market value due to the conscious effort by regulators to ensure that loan loss reserves accurately reflect anticipated losses. However, loans net of reserves are not designed as a means for market value accounting by banks. Further, FASB 107 states that the use of the allowance for loan losses would not provide an acceptable estimate of fair value in most cases because it does not take into account the timing of the expected losses and all the potential losses due to credit risk.

Many authors (e.g., Berger et.al. 1991; Altman 1992; Mengle 1989, 1990; Benston 1990) assert that market valuation of commercial and industrial loans must emphasize net present value of expected cash flows. Inherent in such an emphasis, however, are questions of the proper discount rate and the proper calculation of expected future cash flows. Berger et.al. (1991) point out that the two questions should not be intermingled. That is, market valuation should be the net present value of contracted cash flows discounted at a rate which takes account of the particular borrower's credit risk, <u>or</u> it should be the net present value of expected cash flows discounted at the lender's required rate of return. In either case, the problem is not easily resolved. Market-based alternatives exist which will allow a discount rate to be inferred which may be used with contracted cash flows. These alternatives are matrix pricing and market transaction data.

<u>Matrix Pricing</u>

Mengle (1989, 1990) describes matrix pricing as a process of classifying loans by characteristics, relating characteristics to yields, and inferring discount factors for comparable loans. This discount factor is then available for use in net present value calculations. This is a service currently provided by the Loan Pricing Corporation (LPC). LPC has developed a model to relate a rate, expressed as a spread over prime or LIBOR, to such factors as borrower size, risk, location, industry, loan type, purpose, maturity and so forth. Berger et.al. (1991) argue that commercial loans are so idiosyncratic in nature that such efforts are unlikely to capture important public information on the borrower's credit quality. Further, the bank's private information is not available for inclusion in such models.

Market Transaction Data

As mentioned in the Introduction (Benston 1990), assets and liabilities with highly-developed secondary markets are easily marked to market using transaction data. Prime examples would include stocks, Treasury Securities, and home mortgage loans. While there is evidence of a developing and growing secondary market for commercial loans (Haubrich 1989, Gorton and Haubrich 1990), that market is still too small and too thinly traded to offer much help in the area of market valuation.

Altman (1992) has suggested a method for market valuation of loans which he asserts is applicable to loans of all credit quality. His method involves a three-step process:

1. Estimation of default rates and losses associated with known credit standards;
2. Objective measurement of borrowers' credit quality that is consistent with those credit standards; and
3. Modeling the expected cash flows from each loan.

These three steps establish expected cash flows which are then discounted at the lender's cost of capital. This is consistent with the second formulation suggested by Berger et.al. (1991).

Step one (estimation of default and loss rates) is accomplished using mortality rates for publicly traded bonds categorized by bond rating and adjusted for seniority. Step two (measurement of borrower credit quality) is accomplished using the ZETA® Risk Control System (Altman et.al. 1977). This system provides an index which corresponds to an equivalent bond rating. Step three (modeling expected cash flows) is accomplished by combining the information from steps one and two. The contracted cash flows for the loan are decreased by the default rate (step one) appropriate for the borrower (step two) and are then discounted at the lender's cost of capital.

This section has made the argument that most of a bank's portfolio is at or near market value (e.g., Mengle 1989, 1990; Benston 1990; Benston and Kaufman 1988). Altman (1992) has offered a model for those loans which must be marked to market. Since a loan sale is, in essence, the transfer of the cash flows of the loan from seller to buyer, it would seem that these techniques would apply as readily to the loan sale as to the loan itself. However, due to recourse provisions applicable to the sale and <u>not</u> to the original loan, this is not the case. In particular, matrix pricing techniques and Altman's three-step process are based on the credit risk characteristics of the borrower with the understanding that default by the borrower translates directly into loss for the lender. Dependent upon the recourse provision of the particular loan participation, borrower default may mean no loss for the buyer (full recourse) or some loss less than the full participation amount (partial or implicit recourse). With a participation, loss to the buyer is dependent upon the characteristics of the seller, at least in part. A different valuation technique is necessary for valuing the loan participation. This research develops a model which takes account of the recourse provisions and does not rest on estimations of the credit risk of the borrower. Rather, valuation is dependent upon strategic decisions regarding prepayment and default, which may be independent of borrower credit risk.

Valuation Methods for Pass-Through Securities

In 1970, the Government National Mortgage Association (GNMA) began to market securities backed by pools of home mortgages insured by the Federal Housing Authority (FHA) or guaranteed by the Veteran's Administration (VA). These securities are called pass-through's because any and all scheduled payments and unscheduled prepayments are passed directly through to the holders of the securities. GNMA Pass-Through's (hereafter GNMAs) carry an explicit guarantee by the U.S. government. They are backed by the "full faith and credit" of the U.S. Treasury and carry a GNMA guarantee that scheduled payments of principal and interest are "passed through" even if they have not been received from the individual mortgagors. These characteristics may be said to make GNMAs essentially risk-free securities.

In 1971, the Federal Home Loan Mortgage Corporation (FHLMC) began pooling conventional mortgages (i.e., mortgages without VA or FHA guarantees or insurance) and marketing Participation Certificates (PCs) backed by the pools. PCs are pass-through's but FHLMC guarantees only the timely payment of interest and not principal in case of mortgagor delinquency or default. PCs are not risk-free securities and do not carry a U.S. Government guarantee.

In 1981, the Federal National Mortgage Association (FNMA) began pooling FHA and VA mortgages to back the issuance of Mortgage-Backed Securities (MBSs). Like GNMAs, MBSs guarantee timely payment of principal and interest despite borrower delinquency or default. However, unlike GNMAs, they carry no U.S. Government guarantee.

Pass-through's (GNMAs, PCs, MBSs), in effect, represent the sale of a series of cash flows from the original borrower. In this sense, they are identical to loan participations. But they are different in matters of recourse and guarantee. Loan participations carry at best an implicit guarantee of the FDIC (Pyle 1985). That is, while the FDIC does not officially insure off-balance sheet activities, the doctrine of "too big to fail" suggests that even non-liability obligations of an insolvent bank will be guaranteed. This is in contrast to the explicit government guarantees associated with GNMAs and MBSs. Further, purchasers of loan participations must rely on recourse arrangements with the sellers to remedy default by the borrower. This is in contrast to the guarantees of timely principal and interest on GNMAs and MBSs and of timely interest on PCs. So, while pass-through valuation is not identical to participation valuation, it provides an important basis for building a participation valuation model. Figure 1 illustrates the differences between GNMAs and loan sales.

The GNMA valuation literature is the most highly-developed of the three pass-through's (see e.g., Dunn and McConnell 1981a,b; Brennan and Schwartz 1985; Bunce, Macrae, and Szymanoski 1988; Schwartz and Torous 1992; Kish and Greenleaf 1993). Dunn and McConnell (1981a,b) and Kish and Greenleaf (1993) suggest that, in practice, GNMAs are often valued using net present value methods adjusted to account for prepayments by borrowers. They specifically mention the "Average Life" model and the Curley and Guttentag (1974) model (hereafter CG).

"Average Life" Model

This model assumes that scheduled principal and interest are paid on the underlying loan until a period equal to the "average life" of a portfolio of "comparable mortgages" is reached. At that point, the entire principal balance is assumed repaid in full. The average life is usually assumed to be 12 years. The resultant set of cash flows is then used to calculate a yield-to-average-life for a newly-issued security.

Curley and Guttentag (CG) Model

Curley and Guttentag (1974) developed a model to incorporate prepayment probability over the entire life of the loan. A variation of the CG model in widespread use is as follows:

$$M_t = \sum_{j=t+1}^{t+n} \left[(C_j + P_j F_j) e^{y(t-j)}\right] \left[\prod_{k=1}^{j-t} (1 - P_{j-k})\right] \tag{3}$$

where M_t = market value of loan after t months
C_j = scheduled principal and interest payment
F_j = remaining principal
P_{j-k} = conditional probability of prepayment.

The yield-to-maturity on a newly-issued security is found by solving the above for y.

Contingent Claims Models

Each of the models above (i.e., Average Life and CG) is essentially a "rule of thumb" approach to valuation under prepayment risk. Other models have been developed and refined using a contingent claims approach based on option pricing theory and continuous time methods. Such models attempt to capture prepayment trends and the variability of interest rates and often contain explicit specifications to explain prepayment and default.

A contingent claims GNMA model is in the form of a partial differential equation whose solution is the value of the pass-through security. The differential equation is built from the assumed stochastic process for variables upon which value is contingent. This process is described in detail in Chapter III and Appendix B.

Dunn and McConnell (1981b)

This study assumed GNMA pass-through value to be contingent upon the riskless rate of interest (r) and time to maturity (τ). The riskless rate was assumed to follow a mean-reverting stochastic process as follows:

$$dr = k(m - r)dt + \sigma_r\sqrt{r}dz$$

where dr = instantaneous change in r
dt = infinitesimal increment to time
dz = standard Gauss-Wiener process
m = steady state mean of r
k = speed of adjustment factor
σ_r^2 = instantaneous variance of r.

Using this assumed process and the techniques of contingent claims analysis, the authors developed the following valuation model:

$$0 = \tfrac{1}{2}\sigma_r^2 r V_{rr} + [k(m-r)+\lambda r]V_r - rV + \xi(r,t) + V_t + C(t)$$

where V = the value of the Pass-Through
t = time
$C(t)$ = continuous cash flows from the GNMA
λ = the market price of risk
$\xi(\bullet)$ = the jump process for prepayment.

The solution to the equation was approximated using the implicit method of finite differences subject to the following boundary conditions:

$$V(r,t) = 0$$
$$V(,t) = 0$$
$$V(,0) \leq F$$

where F is the remaining principal outstanding.

<u>Brennan and Schwartz (1985)</u>

This study assumed GNMA pass-through valuation to be contingent upon the short-term riskless rate (r) and the consol rate (l). The consol rate was defined as the yield on a security whose maturity is infinite. Both rates were assumed to follow arithmetic Brownian motion, as follows:

$$dr = \beta_r dt + \sigma_r dz_r \qquad dl = \beta_l dt + \sigma_l dz_l$$

where dr = instantaneous change in r
dl = instantaneous change in l
θ_r = trend in r
θ_l = trend in l
σ_r^2 = instantaneous variance of r
σ_l^2 = instantaneous variance of l
dt = infinitesimal increment to time
dz_r = standard Gauss-Wiener process
dz_l = standard Gauss-Wiener process.

The partial differential equation developed as a valuation rule is as follows:

$$\tfrac{1}{2}\sigma_r^2 V_{rr} + \rho_{r,l}\sigma_r\sigma_l V_{rl} + \tfrac{1}{2}\sigma_l^2 V_{ll} + V_r(\theta_r - \lambda_r\sigma_r) + V_l(\theta_l - \lambda_l\sigma_l) + V_t + C - rB = 0$$

where V = value of GNMA pass-through
$\rho_{r,l}$ = correlation coefficient of r with l
λ_l = market price of risk associated with l
λ_r = market price of risk associated with r.

The equation was solved using numerical methods and the following boundary conditions:

$$V(r,l,T) = 0$$
$$V(r,l,r) < \infty .$$

<u>Schwartz and Torous (1992)</u>

This study assumed GNMA value to be contingent upon the riskless rate (r) and the value of the home used as collateral (H). The riskless rate was assumed to follow a mean reverting stochastic process as follows:

$$dr = k(m - r)dt + \sigma_r r dz$$

where dr = instantaneous change in r
dt = infinitesimal increment to time
dz = standard Gauss-Wiener process
m = steady state mean of r
k = speed of adjustment factor
σ_r^2 = instantaneous variance of r.

The mortgaged home (collateral) value was assumed to follow geometric Brownian motion as follows:

$$dH = (\mu - b)Hdt + \sigma_H H dz_H$$

where μ = instantaneous expected housing rate of return
b = housing payout rate
σ_H^2 = instantaneous variance of housing returns.

The partial differential equation resulting from these assumptions was as follows:

$$0 = \tfrac{1}{2}\sigma_r^2 r V_{rr} + \tfrac{1}{2}\sigma_H^2 H^2 V_{HH} + \sigma_H \sigma_r \rho_{Hr} H\sqrt{r} V_{rH} + [k(m-r)+\lambda r]V_r + (r-b)HV_H$$
$$- rV + \xi(r,H,t) + V_t$$

where V = the value of the Pass-Through
t = time
λ = the market price of risk
$\xi(\bullet)$ = the jump processes for prepayment and default.

The solution to the equation was approximated using the implicit method of finite differences subject to the terminal condition: $V(r,H,0) = 0$.

Brennan and Schwartz (1981) showed that prepayment risk and interest rate uncertainty are the major factors influencing pass-through value. Dunn and McConnell (1981a) performed a comparison simulation and sensitivity analysis of the three models above (i.e., Average Life, Curley and Guttentag, and Contingent Claims) and found differences in the values generated by the three to be "significant". While they made no conclusion as to which was "best", their finding along with that of Brennan and Schwartz (1981) suggests that the model which takes the most analytical approach to prepayment and interest rate uncertainty will yield the most reliable result. This suggests that, of the GNMA models cited above (Average Life, CG, contingent claims), the contingent claims approach shows the most promise.

CHAPTER III.

CONTINGENT CLAIMS MODEL FOR VALUATION OF LOAN SALES UNDER VARIOUS LEVELS OF RECOURSE

Models developed here for the valuation of commercial loan participations (fixed-rate, secured, nonamortizing) are based upon those for GNMA Pass-Through Securities. The GNMA models are discussed in Chapter II and in Appendix D. The choice between the two time value models (Average Life and CG) and a contingent claims model may be characterized as a choice between a static approach and a dynamic approach. The time value models approach valuation by anticipating future cash flows and discounting them at some "appropriate" discount rate. Implicit in this process is an assessment of risk and of the degree to which investors are averse to this risk. That is, contracted cash flows are not discounted, but expected cash flows adjusted for the probability of default are discounted. Further, the discount rate is "appropriate" to the extent that it reflects investor attitudes toward risk. The assessments of risk are made at a point in time and applied to the valuation model.

By contrast, contingent claims models make no assumptions or assessments regarding individual risk preferences. They are built on an arbitrage argument whereby a "hedge portfolio" is created which requires no net investment and is formed so that the return on the portfolio is nonstochastic (i.e., without risk). In order to prevent arbitrage profits, the return on the portfolio so formed must be zero. The equilibrium value of the asset is determined by these "no arbitrage" conditions. In contingent claims models, the asset value is viewed as being dependent upon the values of the other assets in the hedge portfolio and "does not depend on investor preferences or knowledge of the expected return on the underlying [asset]" (Merton 1976, 125). Further, contingent claims models are dynamic in that they are built on stochastic diffusion processes. These processes describe the movement of the values of the hedge portfolio components through time rather than at a point in time.

As with any model, though, a contingent claims model must make certain assumptions. The stochastic processes for the assets in the hedge portfolio must be specified. But, more troublesome perhaps, are assumptions of continuous trading and perfect markets. For the purposes of this research, "continuous trading" need not mean that trades actually occur continuously, but rather that there be no impediments to trading. An assumption of continuous trading is necessary as the hedge portfolio weights must be continuously adjusted in order for returns to remain nonstochastic. What Merton (1990) has referred to as the "perfect markets paradigm" of rational behavior and frictionless, competitive, and informationally efficient capital markets is a foundation of all modern finance theory. It is a "necessary evil" whose effect on accuracy varies across applications.

Despite these assumptions, however, contingent claims analysis has proven highly productive in the valuation of a number of assets. It offers more precise theoretical solutions and more refined empirical hypotheses than are possible in static or discrete-time frameworks (Merton 1990, xiv). It has proven quite fruitful in the literature of GNMA valuation and will be utilized for the development of loan sales valuation models.

Development of Valuation Model

The valuation of loan participations is similar to the valuation of GNMAs. Following developments in GNMA valuation (e.g., Dunn and McConnell 1981a,b; Brennan and Schwartz 1985; Schwartz and Torous 1989, 1992) the valuation of loan participations may be approached as an application of contingent claims analysis. The value of the loan participation is contingent upon the value (solvency) of the underlying borrower in the sense that it is the borrower who provides repayment to the purchaser (subject to recourse provisions). In this way, the valuation process is similar to that of risky debt as formulated by Merton (1974). In that paper, Merton showed that the value of risky debt was contingent upon the value of the firm issuing the debt. Further, if the firm were to default, the debtholders would become owners of the firm's assets. In Merton's model, default was equivalent to insolvency.

Valuation of loan participations, on the other hand, is not contingent so much upon overall value of the borrower as upon certain strategic decisions by the borrower -- decisions regarding default and prepayment. Unlike risky debt, default on a loan is not tantamount to insolvency. It may instead represent a strategic decision. Likewise, the borrower may make strategic decisions to prepay loans. It is these decisions (which may be independent of the overall value of the firm) which more directly impact the value of the loan participation. Unlike Merton's (1974) risky debt paradigm, loan default does not give the participation purchaser ownership of all assets of the borrower, but rather ownership of specific assets used as collateral.

Because of these considerations, valuation of loan participations is very much like the valuation of GNMA pass-through securities. Payment on pass-through's is dependent upon an underlying pool of borrowers and their decisions to default or prepay. Default gives lenders ownership of a specific asset (the home used as collateral) rather than all assets of the borrower. Default is not equivalent to insolvency. One important difference between pass-through's and participations is in the area of guarantee. GNMA Pass-Through's (GNMAs) carry an explicit guarantee by the U.S. government. They are backed by the "full faith and credit" of the U.S. Treasury and GNMA guarantees that scheduled payments of principal and interest are "passed through" even if they have not been received from the individual mortgagors. Loan participations carry at best an implicit guarantee of the FDIC (Pyle 1985). That is, while the FDIC does not officially insure off-balance sheet activities, the doctrine of "too big to fail" suggests that even non-liability obligations of an insolvent bank will be guaranteed. The repayment of the loan determines the repayment of the participation, subject to a recourse provision. Figure 1 illustrates the characteristics of GNMA Pass Through Securities and loan participations.

Assumptions

The model development makes the following seven assumptions:

(1) perfect markets;

(2) continuous trading;

(3) the borrower's decision regarding prepayment is based on the "relative loan rate" (i.e., the interest rate on the loan minus the prevailing market rate for similar loans) and on the time to maturity of the bank loan;

(4) the borrower's decision regarding default is based on relative collateral value (i.e., the value of collateral versus the outstanding loan balance);

(5) the instantaneous risk-free rate (r) follows a geometric Wiener process (Dothan 1978)[7], as follows:

$$dr = \sigma_r r dz_r$$

where r = instantaneous risk-free rate
σ_r^2 = instantaneous variance of r
dz_r = standard Gauss-Wiener process;

(6) the value of the underlying collateral (H) follows geometric Brownian motion with drift α and volatility σ, as follows:

$$dH = \alpha H dt + \sigma_H H dz_H$$

where α = instantaneous expected return on collateral
σ_H^2 = instantaneous variance of collateral return
dz_H = standard Gauss-Wiener process;

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[7]Several previous studies (e.g., Dunn and McConnell 1981a,b, Titman and Torous 1989, Schwartz and Torous 1992) have assumed a square-root, mean-reverting process similar to that of Cox, Ingersoll, and Ross (1985):

$$dr = k(m - r)dt + \sigma_r \sqrt{r} dz_r$$

where k = speed of adjustment coefficient
m = long-term mean instantaneous riskless rate
σ_r^2 = instantaneous variance of changes in r
dz_r = standard Gauss-Wiener process.

However, Chan, et.al. (1992) showed that this formulation does not perform well empirically. Their findings suggest that, in modeling interest rates, volatility must be highly dependent upon the level of the interest rate. They suggest the Dothan (1978) model as an alternative as, in their study, it outperformed the Cox, Ingersoll, and Ross (1985) model.

(7) $(dz_r)(dz_H) = \rho_{rH}dt.$

Assumptions (3), (4), and (7) are especially important to the development and will be discussed in detail. Assumption (3) states that the borrower's decision regarding prepayment is based on the "relative loan rate" and on the time to maturity of the bank loan. Relative loan rate (RLR) is defined as the interest rate on the loan minus the prevailing market rate for similar loans. For a given loan rate (r_L) as refinancing rates (r_A) fall, "relative loan rate" increases and vice versa. That is:

$$\begin{aligned} RLR &> 0 \quad \text{when } r_L > r_A \\ RLR &= 0 \quad \text{when } r_L = r_A \\ RLR &< 0 \quad \text{when } r_L < r_A \end{aligned}$$

where RLR = the relative loan rate
r_L = the cost of the existing loan
r_A = the prevailing rate for similar loans.

In particular, relative loan rate (RLR) measures the incentive to prepay, but is subject to the constraints of time to maturity. For loans of very short maturity, the incentive to prepay will be very small even as RLR increases. Likewise, for loans of very long maturity, the incentive to prepay will be great even for very small values of RLR. Between these extremes, the incentive to prepay will be sensitive to changes in RLR. As RLR increases (decreases), the incentive to prepay increases (decreases). For a given RLR, the probability of prepayment will vary between approximately zero and approximately one dependent upon the time to maturity of the loan in question[8]. As a result, the prepayment function may be thought of as being sigmoid in form, similar to the illustration in Figure 2. It is important to note that, for a given r_L, as r_A increases (decreases), RLR decreases (increases), and prepayment decreases (increases). Assumption (4) states that the borrower's decision regarding default is based on relative collateral value (i.e., the value of collateral vs. the outstanding loan balance). In particular, the probability of default is assumed to be non-zero when the value of the collateral is less than the outstanding loan principal. That is,

$$\delta > 0 \text{ when } H(t) < F(t)$$

$$\delta = 0 \text{ otherwise}$$

where δ = probability of default
$H(t)$ = value of collateral at time t
$F(t)$ = outstanding principal at time t.

Assumption (7) states that the correlation between the diffusion processes for the riskless rate and collateral value is $dz_r dz_H = \rho_{rH} dt$. It is important to include this

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[8]This characterization of prepayment is in contrast to the GNMA valuation literature (e.g., Dunn and McConnell 1981a,b; Brennan and Schwartz 1985; Schwartz and Torous 1992) which base prepayment on interest rates and considerations of refinancing long-term mortgages. In particular, Schwartz and Torous (1992) assume:

$$\pi > 0 \text{ when } r_p > r_m$$
$$\pi = 0 \text{ otherwise}$$

where π = probability of prepayment
r_p = rate on the loan
r_m = prevailing market rates.

In the case of long-term mortgages, refinancing to capture lower long-term rates is likely to be the overriding concern in prepayment. For short-term (two years or less) participations, rate-driven refinancing is likely an important a consideration only for those of the longest term. Those of the shortest term (as short as overnight) are not likely to prepay regardless of the difference between current rates and refinancing rates.

correlation as it potentially represents a source of risk for the loan participation. In particular, ρ_{rH} may be interpreted as the correlation between the prepayment decision driven by interest rates and the default decision driven by collateral value.

Define a prepayment function, π, and a default function, δ, as follows:

$$\pi = \pi(r,H,t) \qquad \delta = \delta(r,H,t)$$

where r = instantaneous risk-free rate
H = value of collateral
t = time.

The function $\pi(r,H,t)$ is the rate per unit time at which participations are prepaid, and may be interpreted as the probability of prepayment (conditional on not having previously prepaid). Note that prepayment is taken to mean full prepayment (100 percent of outstanding principal). The function $\delta(r,H,t)$ is the rate per unit time at which participations are defaulted on and may be interpreted as the probability of default (conditional upon not having previously defaulted).

"Basic" Differential Equation

Define the value of the participation as a function of default and prepayment, as follows:

$$P = f(\pi,\delta,t) = g(r,H,t).$$

The participation value is also a function of the contracted cash flow at maturity. This value enters the process through the boundary conditions below. Also, since the model is developed for nonamortizing loans, there are no interim contracted cash flows to be included.

Appendix B demonstrates the development of a "basic" valuation relationship to which recourse provisions may be added. The equation is as follows:

$$\tfrac{1}{2}P_{rr}\sigma_r^2r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2H^2 - P_\tau = 0. \qquad (4)$$

Equation 4 is a second order linear partial differential equation (parabolic) whose solution is P, the value of the participation. Solution of partial differential equations requires initial and boundary conditions. The initial condition prevails when the time variable equals zero. The boundary conditions prevail when the underlying asset value equals zero and when it approaches infinity, respectively. In this case, the conditions are as follows:

1. Lower Boundary Condition

$$P(0,H,\tau) = F$$
$$P(r,0,\tau) = F$$

   Even though $H=0$ suggests $H<F$ and $\delta>0$ (Assumption 4), this does not mean that default is certain ($\delta=1$). The value of the participation is still equal to the contracted outstanding principal.

2. Upper Boundary Condition

$$P(\infty,H,\tau) = 0$$
$$P(r,\infty,\tau) = F$$

   The participation value approaches zero as r gets "very large" due to the present value nature of the valuation process. The participation value approaches F as H gets "very large" because this is the contracted limit regardless of the value of the collateral.[9]

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[9]While these boundary conditions are intuitively reasonable, they are not specifically necessary for the valuation process. Merton (1974, 393) notes that the following regularity conditions are sufficient for the valuation process to proceed:

1. Lower Boundary Condition

$$P(0,H,\tau) \geq 0$$
$$P(r,0,\tau) \geq 0$$

2. Upper Boundary Condition

$$P(\infty,H,\tau) < \infty$$
$$P(r,\infty,\tau) < \infty$$

The boundary conditions cited in this research are consistent with these regularity conditions.

3. Initial Condition

$$P(r,H,0) = F$$

where H is the value of the collateral and F is the principal outstanding. Note that in this formulation, τ = time to maturity. So, when $\tau = 0$ (i.e., at the initial condition), the participation is actually at maturity.

Value With No Explicit Recourse

In the case of no explicit recourse, the value of the participation is contingent only upon repayment by the borrower. If the borrower defaults, the purchaser has recourse neither to the borrower nor to the seller. The "basic" model (equation 4) implicitly takes account of default and prepayment through its functional dependence upon interest rates, r, and collateral value, H. In the event of default, the value of the participation falls to zero due to the lack of recourse. However, in the event of prepayment, the dollars prepaid are passed through directly to the holder of the participation and the value of the participation jumps to its maturity value just as if it were paid as scheduled.

A default function, δ, was defined above. Assume actual default follows a Poisson arrival process. Define u to be the Poisson variable and du to be the instantaneous change in u.

$$du \begin{cases} \xi_u & \text{with probability } \delta dt \text{ of default} \\ 0 & \text{with probability } (1-\delta)dt \text{ of no default} \end{cases}$$

At each instant, t, the variable u is incremented by ξ_u or by zero with probability δ

or (1-δ), respectively. The actual default and resultant change in participation value may be included in the model as a jump process. If default occurs, the participation loses all value (i.e., ξ_u = -P(t)). The expected change in value due to default is

$$\delta dt[\, - P(\tau)\,] + (1-\delta)dt[\, 0\,] = \delta[- P(\tau)]dt.$$

A prepayment function, π, was defined above. Assume prepayment follows a Poisson arrival process. Define q to be the Poisson variable and dq to be the instantaneous change in q.

$$dq \begin{cases} \xi_q \text{ with probability } \pi dt \text{ of prepayment} \\ 0 \text{ with probability } (1-\pi)dt \text{ of no prepayment} \end{cases}$$

At each instant, t, the variable q is incremented by ξ_q or 0 with probability π or (1-π), respectively. The actual prepayment and resultant change in participation value may be included in the model as a jump process. If prepayment occurs, the participation increases in value to the amount prepaid (i.e., ξ_q = F). The expected change in participation value due to prepayment is

$$\pi dt[\, P(\tau) + (F(\tau) - P(\tau)) - P(\tau)\,] + (1-\pi)dt[\, 0\,] = \pi[\, F(\tau) - P(\tau)\,]dt.$$

These processes can be added to equation 65 (Appendix B) as follows:

$$\begin{aligned} dP = [&P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} \\ &+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + \pi(F-P) + \delta(-P)]dt + P_r\sigma_r r dz_r + P_H\sigma_H H dz_H. \end{aligned} \tag{5}$$

As demonstrated in Appendix B, a hedge portfolio may be formed of the loan participation, the underlying collateral, and the risk-free asset. Weights are chosen so that the portfolio return is riskless (nonstochastic) and the net investment required is zero. These weights must be continuously adjusted, hence the assumption of continuous trading (Assumption 2). In order to avoid pure arbitrage, the return on this portfolio must be zero. However, addition of the jump processes which result from prepayment and default and the loan participation's specific recourse provisions make it impossible to form such a hedge portfolio. Therefore, with such jump processes, it is impossible to rely on the "pure arbitrage" valuation relationship.

In the absence of jump processes, a hedge portfolio consisting of the participation and the collateral earns the riskless rate, if continuous adjustment of the hedge portfolio weights is possible. In the presence of jump processes, the continuous adjustment of the weights of the hedge portfolio cannot "neutralize" the impact of discrete jumps. Portfolio proportions are a linear process and the relationship between the participation and the collateral value is a non-linear relationship. Over discrete changes (i.e., Poisson jumps), the return to the hedge portfolio will not be certain, hence, will not equal the riskless rate.

If the risk introduced by the jump processes (i.e., by prepayment, default, and associated recourse) is "nonsystematic", then careful choice of the loan participations included in the hedge portfolio will allow the nonsystematic risk to be diversified. For such a portfolio, then, the return would no longer be zero as in the pure "no arbitrage" case, but rather would be the riskless rate due to the systematic risk (not subject to diversification) in the portfolio.[10] The effects of the addition of the jump

[10]It is important to distinguish between the hedging of risk in the hedge portfolio and the diversification of nonsystematic risk from the jump processes. The hedge portfolio approach relies on equilibrium "no arbitrage" conditions to drive the expected return on the portfolio to zero. The jump process approach relies on the diversification of nonsystematic risk to create a portfolio with

processes on the return on the so-called hedge portfolio are discussed in detail in Appendix B.

The assumption of the nonsystematic or diversifiable nature of the jump processes has often been relied upon in developing contingent claims asset valuation models (e.g., Merton 1976, Dunn and McConnell 1981a, and Schwartz and Torous 1992). Such an assumption is reasonable in this research as well. Recall that Assumption 3 stated that prepayment is a function both of market rates and of time to maturity of the particular loan. That is, changes in market rates relative to the rate on the loan sold may affect the probability of prepayment. Note, however, that changes in market rates will affect different commercial loans differently. The terms of commercial loans are normally negotiated on a loan-by-loan basis. Individual loans will have specific terms not shared by other loans. As a result, a diversified portfolio of loan sales can be constructed by buying loans with differing terms (i.e., both short-term and long-term, tied to projects and collateral in different areas, etc.). In particular, the time to maturity will largely determine the magnitude of changes in prepayment probability resulting from changes in market rates. In this sense, the prepayment risk is idiosyncratic to the particular loan and, therefore, includes a nonsystematic and diversifiable component. Likewise, Assumption 4 stated that default is a function of collateral value relative to outstanding loan principal. It is dependent upon the specific collateral and, therefore, idiosyncratic to the loan and includes a nonsystematic and diversifiable component.

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expected return equal to the riskless rate (dr). Since a hedge portfolio cannot be formed in the presence of jump processes, diversification must be relied on and the portfolio return is nonzero.

Given these arguments for diversifiability of nonsystematic portions of both prepayment risk and default risk, equation 5 becomes:

$$rP = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + \pi(F-P) + \delta(-P) \tag{6}$$

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + \pi(F-P) + \delta(-P) - rP. \tag{7}$$

This is a second order, linear partial differential equation (parabolic) whose solution, P, is the value of the loan participation with no recourse, but taking into account the effect of prepayment.

No closed form solution to this type of equation has been found (Merton 1976, Dunn and McConnell 1981, Schwartz and Torous 1992). However, numerical methods exist which may be used to approximate the solution. These are discussed in Chapter IV.

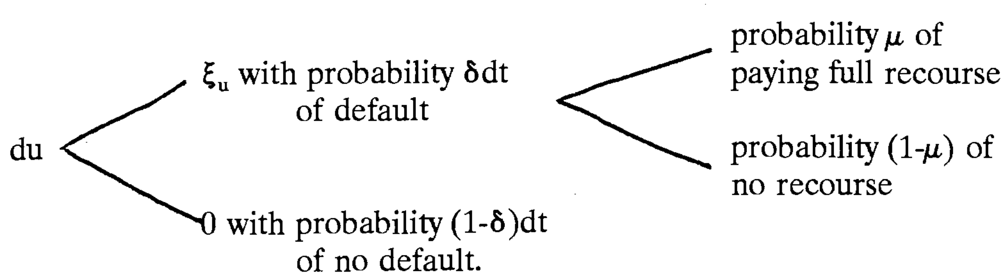
Value with Implicit Recourse (Full or Partial)

Wall (1991) has suggested that recourse may be offered even if there is no explicit recourse provision in the participation agreement. If the purchaser of a participation with no explicit recourse provision has no reasonable expectation that the seller will provide implicit recourse, then the participation should be valued as having no explicit recourse (see above). "No reasonable expectation" may be interpreted as meaning that the seller has not provided implicit recourse in the past. However, if at some point in the past the seller has provided implicit recourse, then it is reasonable to believe that there is some probability that the seller will do so again. Implicit recourse, then, may be viewed as a reputation effect.

Related to the question of "implicit recourse" is the question of whether a selling bank would pursue a defaulting borrower for repayment even though the loan itself had been sold off the balance sheet with no recourse. The lack of recourse suggests that the seller has no legal obligation to pursue the borrower. But, Wall's (1991) argument (i.e., the highest recourse possible will be offered) along with the recognition that the seller's reputation may be considered by the buyer in pricing loans in the future, suggest that the seller may act as "collector" for the buyer. Such a role would preserve the seller's reputation and help ensure future sales.

One could then argue that "no explicit recourse" does not always mean that, in actual practice, none will be offered. That is, a seller will likely stand ready to relieve loss on the part of the buyer either through buying back all or a part of the loan (implicit recourse) or by acting as a collection agent for the buyer if the reputation cost (i.e., the present value of lost future income from loan sales) exceeds the cost of providing implicit recourse. This suggests valuation with "no explicit recourse" as a lower bound on value. Due to implicit recourse of one form or another, values will be expected to be higher than the lower bound to the extent the seller's reputation warrants.

For the purposes of this research, implicit recourse is defined as the seller buying back all or a part of the loan from the purchaser and paying the purchaser all or a part of the contracted maturity amount. That is, implicit recourse is full (or partial) but not certain recourse. If the buyer has a "reasonable expectation" that the seller will provide implicit recourse (i.e., the seller has a reputation of providing recourse when there is no requirement to do so), then the value of the participation is contingent upon decisions of both the borrower and the seller. That is, in this framework, the participation is contingent upon the seller's recourse decision as well as the borrower's default decision.. The implicit recourse decision will only be made in the case of borrower default. The implicit recourse provision may be included in the model by revising the previously defined jump process for borrower default, du, as follows:



This is analogous to the "birth and death" stochastic process discussed in Cox and Ross (1976, 149). It is, however, also possible that the seller will provide implicit recourse for only a portion of the contracted principal. That is, the seller will provide partial implicit recourse. If the degree of implicit recourse is defined as $0 \leq \zeta < 1$, the jump process becomes du: ξ_u with probability δdt of default → probability μ of paying ζ% recourse; probability (1-μ) of no recourse

du: 0 with probability (1-δ)dt of no default.

For implicit recourse, borrower default does not automatically trigger recourse. The actual provision of recourse can be modeled as a dichotomous probability variable based upon the seller's disposition to provide implicit recourse. That is

$0 \leq \mu \leq 1$	=	the probability of providing recourse
$0 \leq (1-\mu) \leq 1$	=	the probability of <u>not</u> providing recourse.

While the seller's reputation is subject to change, it can be expected to change only slowly. As a result, at a point in time or over an infinitesimally small increment of time (dt), it can be thought of as a constant.

If the borrower does not default, then the change in value (du) is zero. If the borrower does default, then the value will change. The amount of the change will depend upon whether or not implicit recourse is actually provided and to what degree. If it is provided, then all or a part of the unpaid balance will be paid. By this definition, then, μ may be viewed as the probability of $0 \leq \zeta < 1$ percent recourse. The expected value of default under implicit recourse is $\delta dt\{ \mu[\zeta F(\tau) - P(\tau)] + (1-\mu)[- P(\tau)] \} = \{\delta\mu[\zeta F(\tau) - P(\tau)] + \delta(1-\mu)[- P(\tau)]\}dt.$ Adding this revision to equation 65 (Appendix B) yields:

$$dP = [P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$
$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + \pi(F-P) + \delta\mu(\zeta F-P) + \delta(1-\mu)(-P)]dt$$
$$+ P_r\sigma_r r dz_r + P_H\sigma_H H dz_H \tag{8}$$

$$dP = [P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$
$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\mu\zeta)(F) + (\pi + \delta)(-P)]dt$$
$$+ P_r\sigma_r r dz_r + P_H\sigma_H H dz_H. \tag{9}$$

Again, assume the risk from the jump processes (i.e., prepayment and default) is diversifiable, then:

$$rP = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$
$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\mu\zeta)(F) + (\pi + \delta)(-P) \tag{10}$$

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$
$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\mu\zeta)(F) + (\pi + \delta)(-P) - rP. \tag{11}$$

This is a second order, linear partial differential equation (parabolic) whose solution, P, is the value of the loan participation with no recourse, but taking into account the effect of prepayment.

No closed form solution to this type of equation has been found (Merton 1976, Dunn and McConnell 1981, Schwartz and Torous 1992). However, numerical methods exist which may be used to approximate a solution. These methods are discussed in Chapter IV.

<u>Value with Explicit Recourse (Full or Partial)</u>

In the case of explicit recourse, the buyer may look to the seller for payment of any unpaid balance resulting from borrower default. This is in contrast to the no-recourse situation discussed above. With full recourse, the seller is responsible for making up the full amount unpaid. With partial recourse, the seller is responsible for making up a portion of the unpaid amount -- dependent upon the particular provisions of the participation agreement. In either case, however, the value of the participation can be expected to change as a result of default and the provision of recourse.

A default function, δ, was defined above. Assume, again, that actual default follows a Poisson arrival process, as follows:

$$du \begin{cases} \xi_u \text{ with probability } \delta dt \text{ of default} \\ 0 \text{ with probability } (1-\delta)dt \text{ of no default} \end{cases}$$

then the actual default and resultant change in participation value may be included in the model as a jump process. The expected change in value due to default is

$$\delta dt[\ P(\tau) + (F(\tau) - P(\tau)) - P(\tau)\ ] + (1-\delta)dt[\ 0\ ] = \delta[\ F(\tau) - P(\tau)]dt.$$

Adding this revision to Equation 65 (Appendix D) yields

$$dP = [P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + \pi(F-P) + \delta(F-P)]dt + P_r\sigma_r r dz_r + P_H\sigma_H H dz_H \quad (12)$$

$$dP = [P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta)(F-P)]dt + P_r\sigma_r r dz_r + P_H\sigma_H H dz_H. \quad (13)$$

Pyle (1985) has suggested that, in the case of insolvency by the selling bank, the FDIC will guarantee even the non-liability obligations of that bank. So, the buyer can expect recourse to be honored either by the seller or by FDIC. For this reason, the risk of the selling bank is not explicitly included in the formulation for recourse.

Unlike the development in Appendix B, a hedge portfolio may not be formed owing to the risk from the jump processes. If it is assumed that both prepayment and default are unique to each borrower (see argument above and Merton 1976, Dunn and McConnell 1981a, Schwartz and Torous 1992), then their risk becomes diversifiable to the buyer of the participation. With this additional assumption, equation 13 becomes:

$$rP = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta)(F-P) \tag{14}$$

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta)(F-P) - rP. \tag{15}$$

This is a second order, linear partial differential equation (parabolic) whose solution, P, is the value of the loan participation with full recourse.

In the case of partial recourse, equation 13 becomes:

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\gamma)(F) + (\pi + \delta)(-P) - rP \tag{16}$$

where γ represents the percent recourse offered ($0 < \gamma < 1$). As equation 16 shows, partial recourse guarantees only a portion of the contracted principal (F). Actual default with partial recourse may mean a loss of value if $(\gamma F-P) < 0$ or an increase in value if $(\gamma F-P) > 0$. By this definition, then, γ is a measure of the percent recourse to be provided with perfect certainty.

No closed form solution to this type of equation has been found (Merton 1976, Dunn and McConnell 1981a, Schwartz and Torous 1992). However, Chapter IV discusses numerical methods which may be used to approximate the solution.

Note that implicit recourse may be taken to mean a $\mu\%$ probability of 100% recourse ($\zeta = 1$) and explicit recourse may be taken to mean a 100% probability of $\gamma\%$ recourse, upon default. If $\mu = \gamma$, then in terms of expectations, these two are equivalent. If, however, implicit recourse is taken to mean a $\mu\%$ probability of $\zeta\%$ recourse, then the two are no longer equivalent. Even if $\zeta = \gamma$, the uncertainty of provision of implicit recourse makes the impact of ζ on value less than that of γ.

Generalization of Models

The development above may be generalized to yield two models: one for the case of no explicit recourse and implicit recourse, and one for explicit recourse (full and partial).

No Explicit Recourse and Implicit Recourse (Full and Partial) Model

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi)(F\text{-}P) + (\delta\mu)(\zeta F\text{-}P) + \delta(1\text{-}\mu)(\text{-}P) - rP \tag{17}$$

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\mu\zeta)(F) + (\pi + \delta)(\text{-}P) - rP \tag{18}$$

Explicit Recourse (Full and Partial) Model

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$

$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\gamma)(F) + (\pi + \delta)(-P) - rP \qquad (19)$$

where
π = the probability of prepayment
δ = the probability of default
γ = the degree of explicit recourse
μ = the probability of providing implicit recourse
ζ = the degree of implicit recourse.

In the case of full explicit recourse, use equation 19 and let $\gamma=1$. In the case of partial explicit recourse, $0<\gamma<1$ equals the degree of explicit recourse offered. Note that γ is constrained to positive values, as a value of zero implies no explicit recourse in which case equation 18 should be used. In the case of no explicit and no implicit recourse, use equation 18 and let $\mu=0$. In the case of no explicit recourse and possible implicit recourse, use equation 18 and let $0<\mu<1$. Note that μ does not equal one as this implies recourse with certainty in which case equation 19 should be used. The degree of implicit recourse equals $0\leq\zeta<1$.

Limitations of Valuation Rules

The valuation rules (equations 18 and 19) developed in this chapter are specific to a certain class of loan sales -- sales of commercial loans which are fixed-rate, secured, and non-amortizing. While this class of loans is the primary one involved in commercial loan sales (Sinkey 1992), valuation of other classes of loan sales will require modification of the equations.

The use of contingent claims analysis in developing the model equations requires assumptions of continuous trading in the loan sales market and of perfect market conditions. Merton (1990) has referred to these assumptions as the "perfect markets paradigm" and has argued that, even though not reflective of actual market conditions, this paradigm has given rise to a highly productive and theoretically rich area of research.

Contingent claims analysis also requires the specification of underlying stochastic processes. In the models developed above, processes were assumed for the instantaneous risk-free rate and for collateral value. For the risk-free rate, a Geometric Wiener Process was assumed. This particular process has been empirically tested by Chan, et.al. (1992). It and several other candidate processes for the risk-free rate were estimated and compared to actual one-month Treasury Bill yields over the horizon 1964 to 1989. The process used in this research was shown to outperform other processes including the Square-Root, Mean Reversion model of Cox, Ingersoll, and Ross (1985).[11] For collateral value, Geometric Brownian Motion with drift α and volatility σ_H was assumed. This process is appropriate as it precludes negative collateral values and has volatility positively related to the level of collateral value, both of which are desirable qualities for a model of collateral values.

The model development above required the formation of "hedge portfolios" which had non-stochastic rates of return. Addition of jump processes for prepayment and default made this impossible. With the inclusion of the jump processes, it was

[11]Models similar to Cox, Ingersoll, and Ross (1985) have frequently been used in the literature on GNMA valuation. See, e.g., Dunn and McConnell (1981a,b), Schwartz and Torous (1992).

assumed that they included nonsystematic risk which was subject to diversification. When prepayment is viewed as a function of time to maturity specific to the particular loan and when default is viewed as a function of the particular collateral of the loan, then it is reasonable to assume that loan participations may be held in well-diversified portfolios. If this is true, then the nonsystematic component of prepayment risk and default risk should not be priced. However, model results are sensitive to the extent to which this diversifiability assumption is true in practice.

Since the valuation model is developed for secured, fixed rate, nonamortizing commercial loan sales, it is necessary to question whether the distribution of such loans is sufficient to allow diversification. The two sources of risk to be diversified are collateral value (related to default) and interest rates and time to maturity (related to prepayment).

Diversification of nonsystematic default risk requires that the loans to be sold be secured by a wide range of collateral. Sinkey (1992) states that the cash flows to repay commercial loans usually are from the specific project(s) financed by the loan. In the same way, the loan is usually secured by assets specific to the project. It may be argued, then, that the wide diversity of companies borrowing from commercial banks and the wide diversity of projects in which these companies are engaged, provide a sufficient distribution for effective diversification.

Diversification of nonsystematic prepayment risk requires that the loans to be sold represent a wide range of maturities. Sinkey (1992) states that commercial loans have a variety of maturities. Short-term commercial loans have maturities in the 30-

to 90-day range, while long-term commercial loans may have maturities as long as 52 to 60 months. This range, and all the potential maturities contained within it, is conducive to diversification. Commercial loans are most frequently fixed rate with the rate pegged to a market-determined yield on other assets (e.g., certificates of deposit) of the same maturity. Assumption 3 states that prepayment is dependent upon the Relative Loan Rate (RLR). Even though commercial loans have a range of interest rates, the RLR of each will indicate that prepayment is potentially more likely as market rates rise and less likely as market rates fall. This is, however, not an obstacle to diversification if it is considered that the effect of market rates on prepayment of commercial loans is idiosyncratic. For loans of very short maturity, the incentive to prepay will be very small even as RLR increases. Likewise, for loans of very long maturity, the incentive to prepay will be great even for very small values of RLR. In this way, the varying propensities to prepay embodied in <u>both</u> the RLR and the time to maturity taken together, suggest a sufficient pool for diversification.

CHAPTER IV.

METHODOLOGY AND TESTING

Numerical Methods

In the case of differential equations for which analytic solutions have not been found, numerical methods are available to approximate the solution process. Methods include finite differences, Monte Carlo simulation, the method of lines, numerical integration, etc. (Ames 1992, Smith 1985, Geske and Shastri 1985, Brennan and Schwartz 1978). Before developing a numerical solution, though, it is necessary to show that analytic solutions to the differential equations developed in Chapter III exist.

Friedman (1964) states the following theorem pertaining to existence and uniqueness of solutions to parabolic differential equations:

> Let (the equation of interest) be a uniformly parabolic equation over its domain $t \in [0,T]$ and assume that the coefficients are sufficiently smooth over that domain that the root condition is satisfied. Then for any sufficiently smooth function f for which
>
> $$\frac{\partial \Xi(x,0)}{\partial t} = 0$$
>
> there exists a unique solution.

This theorem holds for differential equations of order two and higher. The conditions of the theorem are as follows:

1. uniformly parabolic -- Consider a general differential equation

$$Lu = \sum_{i,j=1}^{N} a_{i,j}(x,t)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{N} b_i(x,t)\frac{\partial u}{\partial x_i} + C(x,t)u - \frac{\partial u}{\partial t} = 0$$

   If $a_{i,j}(x,t)$ are continuous functions over the domain and Lu is parabolic in that domain, then this implies that Lu is uniformly parabolic in that domain.

2. root condition -- If the coefficients of Lu as defined in #1 are real numbers, then the root condition is satisfied.

3. sufficiently smooth -- A "sufficiently smooth" coefficient is one which satisfies the root condition in #2. A sufficiently smooth function is one whose coefficients are sufficiently smooth.

4. $\frac{\partial f(x,0)}{\partial t} = 0$ This is satisfied by the initial condition.

The coefficients of interest in Friedman's theorem are those of the cross partial derivative terms. There is only one such term in the equations developed in Chapter III, as follows:

$$P_{rH}\sigma_r\sigma_H r\rho_{rH}$$

and the coefficient portion of this term is

$$\sigma_r\sigma_H r\rho_{rH}.$$

Recall that r and H are variables which follow stochastic diffusion processes. In particular,

$$dr = \sigma_r r dz_r$$

where σ_r^2 = instantaneous variance of changes in r
dz_r = standard Gauss-Wiener process

$$dH = \alpha H dt + \sigma_H H dz_H$$

where α = instantaneous expected return on collateral
σ_H^2 = instantaneous variance of collateral returns
dz_H = standard Gauss-Wiener process.

Merton (1982a) has shown that variables which follow such diffusion processes have continuous sample paths. He also has shown that the standard deviations of such processes (i.e., σ_r, σ_H) are $\Delta(h)$ where h is the increment of time in the model. The asymptotic order symbol $\Delta(h)$ indicates that the standard deviation is bounded as $h \rightarrow 0$. In other words, the standard deviation is proportional to dt and is bounded as dt gets infinitesimally small. For purposes of this model, σ_r, σ_H, and ρ_{rH} are assumed to be constant. This means that the coefficient in question is a product of a continuous variable and constants. This suggests that the coefficient is also continuous and, therefore, satisfies condition #1.

Both r and H are real number variables, so they satisfy condition #2. Therefore, the parabolic differential equations developed in Chapter III satisfy Friedman's theorem. And, given the initial condition of the model, a unique solution to each must exist.

Brennan and Schwartz (1978) demonstrate the use of finite difference methods for pricing contingent claims. The technique has been frequently used in the valuation of GNMA Pass-Through Securities (see e.g., Dunn and McConnell 1981a,b; Brennan and Schwartz 1985; Schwartz and Torous 1989,1992). Geske and Shastri (1985) performed a comparison of a number of approximation techniques for the solution of partial differential equations, in particular, equations for the valuation of put and call options. Their study included the binomial method, explicit finite differences, and implicit finite differences. In terms of convergence of solution and computing time, the explicit finite difference technique was considered "best". As a result, explicit finite difference methods will be used to approximate the solutions of equations 18 and 19.

Finite difference methods approximate solution of a continuous function by dividing its domain into a set of discrete points referred to as a net or mesh. Partial derivatives are then approximated at each point in the net or mesh. The explicit method of Brennan and Schwartz (1978) begins with an initial condition and iteratively calculates the value of the equation at each point in the mesh. In this way, each point is expressed in terms of values at previously-calculated points.

Explicit finite difference methods suggest that

$$P(r,H,t) = W(ih,pq,jn) = W_{i,p,j}$$

where i is the risk-free rate incremented by h, p is the value of the collateral incremented by q, and j is current time incremented by n. The size of the mesh is then determined by h, q, and n.

As before, the development will begin with the basic differential equation without the jump processes. These can then be easily added. Brennan and Schwartz (1978) suggest a log transformation of the stochastic variables r and H. This will yield a differential equation with constant coefficients which will simplify the finite differencing process. In particular:

let $\quad Y = \ln(r) \qquad\qquad$ i.e., $e^Y = r$

$Z = \ln(H)$ i.e., $e^Z = H$

$W(Y,Z,t) = P(r,H,t)$

then it follows that

$$P_r = \frac{\partial W}{\partial Y}\frac{\partial Y}{\partial r} = W_Y \frac{1}{r} = W_Y \frac{1}{e^Y} = W_Y e^{-Y} \tag{20}$$

$$\begin{aligned} P_{rr} &= \frac{d}{dr} P_r = \frac{d(W_Y e^{-Y})}{\partial Y}\frac{\partial Y}{\partial r} \\ &= [W_{YY}e^{-Y} + W_Y(-e^{-Y})]e^{-Y} + (W_{YY} - W_Y)e^{-Y}e^{-Y} \\ &= (W_{YY} - W_Y)e^{-2Y} \end{aligned} \tag{21}$$

$$P_H = \frac{\partial W}{\partial Z}\frac{\partial Z}{\partial H} = W_Z \frac{1}{H} = W_Z e^{-Z} \tag{22}$$

$$\begin{aligned} P_{HH} &= \frac{d}{dH}(W_Z e^{-Z}) = \frac{d(W_Z e^{-Z})}{dZ}\frac{\partial Z}{\partial H} \\ &= [W_{ZZ}e^{-Z} + W_Z(-e^{-Z})]e^{-Z} + (W_{ZZ} - W_Z)e^{-Z}e^{-Z} \\ &= (W_{ZZ} - W_Z)e^{-2Z} \end{aligned} \tag{23}$$

$$\begin{aligned} P_{rH} &= P_{Hr} = \frac{d}{dH}(W_Y e^{-Y}) = \frac{d(W_Y e^{-Y})}{dZ}\frac{\partial Z}{\partial H} \\ &= [(W_Y)(0) + W_{YZ}e^{-Y}]e^{-Z} = W_{YZ}e^{-Y}e^{-Z} \end{aligned} \tag{24}$$

$$\mathbf{P_t = W_t} \tag{25}$$

and, by the definition of τ

$$\mathbf{P_\tau = -W_t}. \tag{26}$$

Equation 65 (Appendix B) becomes, by substitution (before addition of jump processes):

$$P_\tau = P_r\lambda r + P_H\alpha H + \tfrac{1}{2}P_{rr}\sigma_r^2r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2H^2 - rP$$

$$\begin{aligned} -W_t = {} & \lambda rW_Ye^{-Y} + \alpha HW_Ze^{-Z} + \tfrac{1}{2}\sigma_r^2r^2(W_{YY} - W_Y)e^{-2Y} \\ & + \sigma_r\sigma_H\rho_{rH}rHW_{YZ}e^{-Y}e^{-Z} + \tfrac{1}{2}\sigma_H^2H^2(W_{ZZ} - W_Z)e^{-2Z} - rW \end{aligned} \tag{27}$$

$$\begin{aligned} -W_t = {} & \lambda W_Ye^Ye^{-Y} + \alpha W_Ze^Ze^{-Z} + \tfrac{1}{2}\sigma_r^2W_{YY}e^{2Y}e^{-2Y} - \tfrac{1}{2}\sigma_r^2W_Ye^{2Y}e^{-2Y} \\ & + \sigma_r\sigma_H\rho_{rH}W_{YZ}e^Ye^{-Y}e^Ze^{-Z} + \tfrac{1}{2}\sigma_H^2W_{ZZ}e^{2Z}e^{-2Z} - \tfrac{1}{2}\sigma_H^2W_Ze^{2Z}e^{-2Z} - rW \end{aligned}$$

$$\begin{aligned} -W_t = {} & \lambda W_Y + \alpha W_Z + \tfrac{1}{2}\sigma_r^2W_{YY} - \tfrac{1}{2}\sigma_r^2W_Y + \sigma_r\sigma_H\rho_{rH}W_{YZ} + \tfrac{1}{2}\sigma_H^2W_{ZZ} - \\ & \tfrac{1}{2}\sigma_H^2W_Z - rW \end{aligned}$$

$$\begin{aligned} -W_t = {} & (\lambda-\tfrac{1}{2}\sigma_r^2)W_Y + (\alpha-\tfrac{1}{2}\sigma_H^2)W_Z + \tfrac{1}{2}\sigma_r^2W_{YY} + \sigma_r\sigma_H\rho_{rH}W_{YZ} + \tfrac{1}{2}\sigma_H^2W_{ZZ} \\ & - rW. \end{aligned} \tag{28}$$

"Basic" Differential Equation

Appendix C shows the development of the finite difference equation for the "basic" differential equation, as follows:

$$\begin{aligned} W_{i,p,j} = {} & [1/(1 + rn)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ & + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ & + gW_{i-1,p-1,j+1}] \end{aligned} \tag{29}$$

where

$$a = [-(\lambda-½\sigma_r^2)(1/2h) + (½\sigma_r^2)(1/h^2) - (\sigma_r\sigma_H\rho_{rH})(1/hq)]n$$

$$b = [-(\sigma_r^2)(1/h^2) + 2(\sigma_r\sigma_H\rho_{rH})(1/hq) - (\sigma_H^2)(1/q^2) + (1/n)]n$$

$$c = [(\lambda-½\sigma_r^2)(1/2h) + ½(\sigma_r^2)(1/h^2) - (\sigma_r\sigma_H\rho_{rH})(1/hq)]n$$

$$d = [-(\alpha-½\sigma_H^2)(1/2q) - (\sigma_r\sigma_H\rho_{rH})(1/hq) + ½(\sigma_H^2)(1/q^2)]n$$

$$e = [(\alpha-½\sigma_H^2)(1/2q) - (\sigma_r\sigma_H\rho_{rH})(1/hq) + ½(\sigma_H^2)(1/q^2)]n$$

$$f = [(\sigma_r\sigma_H\rho_{rH})(1/hq)]n$$

$$g = [(\sigma_r\sigma_H\rho_{rH})(1/hq)]n.$$

Note that $a + b + c + d + e + f + g = 1$ and, subject to non-negativity, then a through g may be viewed as probabilities and the difference equation may be viewed as a jump process (Cox and Ross 1976).

Value with No Explicit Recourse

In order to account for prepayment by the borrower and the passing of that payment through to the holder of the participation, a jump process was added to the "basic" equation. Adding this jump process to equation 96 (Appendix C) yields:

$$\begin{aligned} W_{i,p,j}(1/n) = {} & aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ & + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ & + gW_{i-1,p-1,j+1} - rW_{i,p,j} + \pi(F-W_{i,p,j}) + \delta(-W_{i,p,j}) \end{aligned} \tag{30}$$

$$\begin{aligned} W_{i,p,j} = {} & [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ & + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ & + gW_{i-1,p-1,j+1} + n\pi F] \end{aligned} \tag{31}$$

where the coefficients a through g are as defined above.

Value with Implicit Recourse (Full or Partial)

With an implicit recourse provision, the jump process must account for the prepayment decision by the borrower and the interrelated default and recourse decisions of the borrower and seller, respectively. This was developed using the "birth and death" stochastic process (Cox and Ross 1976). Addition to equation 96 (Appendix C) yields:

$$\begin{aligned} W_{i,p,j}(1/n) = {} & aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ & + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ & + gW_{i-1,p-1,j+1} - rW_{i,p,j} + (\pi)(F-W_{i,p,j}) \\ & + (\delta\mu)(\zeta F-W_{i,p,j}) + \delta(1-\mu)(-W_{i,p,j}) \end{aligned} \tag{32}$$

$$\begin{aligned} W_{i,p,j} = {} & [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ & + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ & + gW_{i-1,p-1,j+1} + n(\pi+\delta\mu\zeta)F]. \end{aligned} \tag{33}$$

Value with Explicit Recourse (Full or Partial)

With an explicit recourse provision, the jump process added to the finite difference equation must take into account both the possibility of default and of prepayment by the borrower. This addition to equation 96 (Appendix C) yields:

$$W_{i,p,j}(1/n) = aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1}$$

$$+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} + gW_{i-1,p-1,j+1}$$

$$- rW_{i,p,j} + (\pi)(F-W_{i,p,j}) + (\delta)(\gamma F-W_{i,p,j}) \tag{34}$$

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1}$$

$$+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1}$$

$$+ gW_{i-1,p-1,j+1} + n(\pi+\delta\gamma)F] \tag{35}$$

where the coefficients a through g are as defined above. Recall that γ represents the level of explicit recourse. In the case of full recourse, $\gamma = 1$.

Generalization of Models

The finite difference equations above may be generalized to yield two models: one for the case of no explicit recourse and/or implicit recourse (full or partial), and one for explicit recourse (full or partial).

No Explicit Recourse and Implicit Recourse (Full or Partial) Model

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1}$$

$$+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1}$$

$$+ gW_{i-1,p-1,j+1} + n(\pi + \delta\mu\zeta)F] \tag{36}$$

Explicit Recourse (Full or Partial) Model

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1}$$
$$+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1}$$
$$+ gW_{i-1,p-1,j+1} + n(\pi+\delta\gamma)F] \quad (37)$$

where
- π = the probability of prepayment
- δ = the probability of default
- γ = the degree of explicit recourse
- μ = the probability of providing implicit recourse
- ζ = the degree of implicit recourse.

In the case of full explicit recourse, use equation 37 and let $\gamma=1$. In the case of partial explicit recourse, $0<\gamma<1$ equals the degree of explicit recourse offered. Note that γ does not equal zero as this implies no explicit recourse in which case equation 36 should be used. In the case of no explicit recourse and no implicit recourse, use equation 36 and let $\mu=0$. In the case of no explicit recourse and possible implicit recourse, use equation 36 and let $0<\mu<1$. Note that μ does not equal one as this implies recourse with certainty in which case equation 37 should be used. The degree of implicit recourse equals $0\leq\zeta<1$.

Properties of Finite Difference Approximations

As noted by Brennan and Schwartz (1978), finite difference methods may be viewed as restating continuous processes in terms of finite jump processes. This can be seen by examining equations 36 and 37 developed above. Each results in $W_{i,p,j}$ stated as a present value of adjacent points in the mesh plus a constant term including the face value of the participation. The discount rate decreases in size with the addition of more recourse. This would indicate a decrease in value as expected. However, this is subject to an offsetting effect from the face value term which increases with additional recourse.

Ames (1992, 14) has argued that finite difference methods are easily implemented due to high-speed computers, but are "often misapplied and abused" . The problems with application arise from the fact that finite differences are approximations. It is important to ensure that the approximations are reasonable. Two conditions which must be satisfied by a reasonable approximation are convergence and stability.

"Convergence" refers to the convergence of the solution of the approximating difference equations to the solution of the actual differential equation as the size of the net or mesh approaches zero. The Taylor Series development of the approximations to the partial derivatives in the differential equations (Appendix C) involves ignoring terms of a certain order and higher. By ignoring these terms, discretization error is introduced into the iterative explicit process. Discretization error approaches zero as the mesh approaches zero in a convergent finite difference solution.

"Stability" refers to the growth of the discretization error as it is carried forward iteratively through a given mesh size. A stable numerical process should limit the amplification of rounding errors through the iteration process.

Geske and Shastri (1992) explore the convergence and stability considerations of explicit finite difference methods. They suggest that finite difference equations such as those developed in this research are actually approximations for the following equation:

$$\begin{aligned} W_{i,p,j} &= A\,[aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ &\quad + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ &\quad + gW_{i-1,p-1,j+1} + \bigcirc(\Delta r)^2 + \bigcirc(\Delta H)^2 + \bigcirc(\Delta t)] \end{aligned} \tag{38}$$

where A = multiplier dependent upon the recourse provision
$\bigcirc(\cdot)$ = order of error.

This demonstrates that the order of error depends upon $(\Delta r)^2$, $(\Delta H)^2$, and Δt. As $\Delta r \to 0$ and $\Delta H \to 0$ and $\Delta t \to 0$, it follows that $\bigcirc(\Delta r)^2 \to 0$ and $\bigcirc(\Delta H)^2 \to 0$ and $\bigcirc(\Delta t) \to 0$. Therefore, this explicit finite difference method is shown to be convergent.

The authors further show that stability of finite differences is dependent upon the mesh ratio R where

$$R_r = \frac{\Delta t}{(\Delta r)^2} \qquad R_H = \frac{\Delta t}{(\Delta H)^2}\ .$$

In particular, for explicit finite differences to be stable, the following condition must be met:

$$0 < R_r \equiv \frac{\Delta t}{(\Delta r)^2} \le \frac{1}{2}\,\frac{1}{(1-2\theta)}$$

$$0 < R_H \equiv \frac{\Delta t}{(\Delta H)^2} \le \frac{1}{2}\,\frac{1}{(1-2\theta)}$$

where θ is a constant in the interval $0 \leq \theta \leq 1$. In the case of explicit finite differences, $\theta \equiv 0$ and the condition becomes

$$0 < R_r \equiv \frac{\Delta t}{(\Delta r)^2} \leq \frac{1}{2}$$

$$0 < R_H \equiv \frac{\Delta t}{(\Delta H)^2} \leq \frac{1}{2} .$$

The size of the mesh, that is, the sizes of Δh, Δq, and Δn, are chosen as part of the finite differencing application. These so-called step sizes for time and value must be chosen to ensure a stable convergence to a solution. Note that this will also ensure the non-negativity of the coefficients a through g in the finite difference equations above.

Model Estimation

As stated in the Introduction and developed in Chapter III, this research develops a contingent-claims model for the valuation of commercial loan participations (fixed-rate, secured, nonamortizing). As argued in Chapter II, loan participations are similar to GNMA Pass-Through Securities in many ways, but differ specifically in the areas of recourse and guarantee. These differences make it impractical to apply GNMA models directly to the valuation of loan participations.

An important empirical question, then, is how well the models of Chapter III describe the prices actually quoted for loan participations. Ideally, the models would be tested by generating model prices for loan sales which could be compared to actual prices quoted in the markets. Unfortunately, the data necessary for the estimation of the models developed in Chapter III is proprietary and not readily available to researchers owing to fiduciary responsibilities of the lender to the borrower(s). This paucity of data necessitates that assumptions be made about borrower characteristics which calls into question the validity of comparison of model-generated prices with quoted prices. For these reasons, the tests concentrate on the techniques of estimating the models of Chapter IV and on sensitivity analyses of the models' outputs.

Since comparison with market-determined values is not possible, estimation of the models will concentrate on characteristics of the model, comparative statics, and sensitivity analysis. The results will then be used to asses current regulatory policies and make policy statements.

The models are parameterized in the way an institutional user would be required to do. Where available, actual parameter estimates are suggested. Comparative statics are explored. Since no closed-form solution has been found for equations 18 and 19, the finite difference approximation equations 36 and 37 are used for this. Finally, the models are estimated and compared for different levels of recourse. The comparisons concentrate on value under varying recourse levels, and on the sensitivity of value to changes in underlying parameters. The results are reviewed in Chapter IV and assessed for regulatory implications.

Numerical Techniques

Finite difference equations were developed above for valuing loan participations (fixed-rate, secured, nonamortizing) under various levels of recourse. This research uses those equations and numerical techniques to approximate the solutions of the partial differential equations developed in Chapter III.

Finite difference methods approximate solution of a continuous function by dividing its domain into a set of discrete points referred to as a net or mesh. Partial derivatives are then approximated at each point in the net or mesh. The size of the mesh, that is, the sizes of Δh, Δq, and Δn, are chosen as part of the finite differencing application.

Operationally, finite differencing is an iterative process. Each point in the mesh (e.g., $W_{i,j}$ or $W_{i,p,j}$) is the combination of several adjacent points in the mesh (e.g., $W_{i-1,j+1}$, $W_{i,j+1}$, $W_{i+1,j+1}$ or $W_{i-1,p,j+1}$, $W_{i,p,j+1}$, $W_{i+1,p,j+1}$, $W_{i,p-1,j+1}$, $W_{i,p+1,j+1}$, $W_{i+1,p+1,j+1}$, $W_{i-1,p-1,j+1}$), each of which is in turn the result of several adjacent points in the mesh. The solution approximation process begins with the initial conditions and proceeds until the boundary conditions are met. Graphically, this may be represented as shown in Figure 3.

Recall that, in the finite difference approximation, value is represented by $W_{i,p,j}$ where W is the actual participation value and i, p, and j represent increments to the riskless rate, collateral value, and time, respectively. Equations 36 and 37 illustrate how value at a given time increment (j+1) relates to value at a later increment (j).

Assume in Figure 3 that j+1 represents the time increment at maturity (i.e., at j+1, τ=0). In that case, j+1 is the initial condition and W = 1000 for all i,j. In order to value the participation at one time increment back from maturity, use the relationship of equation 36 as follows:

$$W_{i,p,j} = [1/(1+rn+\pi n+\delta n)][a(1000) + b(1000) + c(1000) + d(1000) + e(1000) + f(1000) + g(1000) + n(\pi+\delta\mu\zeta)F].$$

This is the process illustrated in Figure 3. $W_{i,p,j}$ is identified by the dot in the level of the figure marked time increment j. As the equation above shows, that value is determined by the combination of seven values from the previous time increment j+1. The valuation process is repeated for each value in the mesh at time j by shifting the seven values at time j+1. The process is then started over at time j in order to calculate the values at j-1. In this way, a table of values is developed, subject to boundary conditions, for each time increment value. These tables may then be consulted to find the approximate value at a given riskless rate (r), collateral value (H), and time to maturity (τ).

Model Parameters

In addition to setting the mesh size, numerical methods require estimation of the coefficients a - g in the difference equations above. Also, the jump processes for default and prepayment must be estimated. The set of coefficients a - g is dependent upon the characteristics of the underlying stochastic processes for the short-term risk-free rate and for collateral value.

Short-Term Risk-Free Rate

The short-term risk-free rate enters the difference equations through the following three variables:

r	= riskless rate
σ_r^2	= instantaneous variance of changes in r
λ	= market price of interest rate risk.

The riskless rate (r) may be measured as the yield on short-term Treasury Bills. The variance of changes in the riskless rate has been estimated by Chan, et.al. (1992) using the yields on Treasury Bills with one month to maturity for the years 1964 through 1989. For the stochastic process assumed in this research, their estimate of the variance (σ_r^2) was 0.1172.

The market price of interest rate risk (λ) has been estimated in a number of studies. The following estimates were made in studies applicable to this research:

STUDY	λ
Dunn and McConnell (1981a)	0.247
Schwartz and Torous (1992)	0.081
Chan, et.al. (1992)	0.0125.[12]

The estimates vary widely.

[12]These estimates were based on the Cox, Ingersoll, and Ross (1985) formulation of λ, as follows:

$$\lambda = k[\ 1-(m/r_L)\ ] + (\sigma_r^2 r_L)/2km$$

where		
	k	= speed of adjustment coefficient
	m	= long-term mean instantaneous riskless rate
	σ_r^2	= instantaneous variance of changes in r
	r_L	= long-term riskless rate.

The values used by Dunn and McConnell (1981a) were classified as being "similar" to those estimated by Ingersoll (1971). Schwartz and Torous (1992) based their estimates on the analysis of Bunce, MacRae, and Szymanoski (1988).

Collateral Value

Collateral value enters the difference equations through the following:

α = instantaneous expected return on collateral
σ_H^2 = instantaneous variance of collateral returns.

The studies upon which this proposed research is based were of GNMA Pass-Through Securities (GNMAs) each of which is backed by a pool of home mortgage loans. This is not the case for loan participations. Each participation is likely to be backed by a loan secured by unique collateral (or no collateral at all). If, as is likely, participations are held as part of a portfolio of investments, then risk specific to the particular type of collateral may be assumed well-diversified. The expected return on the collateral must be empirically estimated from available market data for the specific collateral type. For purposes of empirical testing, H may be estimated as the appraised value of the collateral backing the loan sold. However, due to the log transformation suggested by Brennan and Schwartz (1978), the actual collateral value (H) does not appear in the valuation equations. Rather, valuation is dependent upon the trend in the collateral value (α) and its variance (σ_H^2). In actual practice, it would be necessary to estimate α and σ_H^2 for the particular type of collateral in question. For purposes of estimating this model, however, it may be assumed that collateral value is increasing with the rate of inflation. With this assumption, α is the rate of inflation and σ_H^2 is the variance of that rate.

It should be pointed out at this point that the difference equations also include a correlation term (ρ_{rH}) between the stochastic processes for the short-term risk-free rate and for collateral value. Schwartz and Torous (1992) assumed that this correlation was zero. If we consider that this is a correlation of unexpected changes in the risk-free rate with unexpected changes in the value of the loan collateral, this assumption seems reasonable. However, the assumption is not necessary for testing the model and is not made in this research. Values between -1 and 1 will be considered.

Prepayment Function

Loan participations are similar to GNMAs in that any prepaid principal is immediately passed on to the holder of the participation. As developed in Chapter III, value is affected by this prepayment risk. A prepayment function, π, was described as the probability per unit time that the borrower would prepay the loan, conditional on not having already prepaid. In particular, the borrower's decision regarding prepayment was assumed to be based on the "relative loan rate" (i.e., the interest rate on the loan minus the prevailing market rate for similar loans) and on the time to maturity of the bank loan. For a given loan rate (r_L), as refinancing rates (r_A) fall, "relative loan rate" increases and vice versa. That is:

$$\text{RLR} > 0 \quad \text{when } r_L > r_A$$

$$\text{RLR} = 0 \quad \text{when } r_L = r_A$$

$$\text{RLR} < 0 \quad \text{when } r_L < r_A$$

where RLR = the relative loan rate
r_L = the cost of the existing loan
r_A = the prevailing rate for similar loans.

In particular, relative loan rate (RLR) measures the incentive to prepay, but is subject to the constraints of time to maturity. For loans of very short maturity, the incentive to prepay will be very small even as RLR increases. Likewise, for loans of very long maturity, the incentive to prepay will be great even for very small values of RLR. Between these extremes, the incentive to prepay will be sensitive to changes in RLR. As RLR increases (decreases), the incentive to prepay increases (decreases). For a given RLR, the probability of prepayment will vary between approximately zero and approximately one dependent upon the time to maturity of the loan in question. As a result, the prepayment function may be thought of as being sigmoid in form, similar to the illustration in Figure 2. This is based on the assumption that the borrower will refinance if a cost savings can be realized by doing so. It is also based on a definition of prepayment as full prepayment. It is important to note that for a given r_L, as r_A increases (decreases), RLR decreases (increases) and prepayment decreases (increases).

A number of studies have addressed estimation of the prepayment function, $\pi(\bullet)$, for GNMAs. Often a "rule of thumb" based on FHA experience is relied upon (Kish and Greenleaf 1993, Dunn and McConnell 1981), or the PSA industry standard prepayment model is used (Kish and Greenleaf 1993, Schwartz and Torous 1992), or the function may be estimated empirically as with maximum likelihood estimation, logit regression, etc. (Schwartz and Torous 1989). Richard and Roll (1989) have tabulated prepayment rates for GNMAs for the years 1979-1988. These are shown in Table I. Note that Table I is segregated by coupon rate and by refinancing rate range. This is consistent with the description of $\pi(\bullet)$ as being partially dependent on the difference between current rate and prevailing market rate. The prepayment rates in this table are for GNMAs which, as has been discussed previously, differ from Participations. In particular, GNMAs are much longer to maturity and the underlying mortgages normally contain prepayment penalties. Participations are backed by loans which may be up to two years to maturity, but tend toward shorter maturities, and which carry no prepayment penalties.

Table II, compiled by the Loan Pricing Corporation shows prepayment rates for a sample of 71 loan syndication transactions with total dollar volume of $36 billion. The loans included in the syndications were originated in 1987 and 1988. The tabulated figures are for repayments and no effort was made to segregate prepayments. Recall that most loans sold tend to be of short maturity (< 2 years) so the high repayment rates likely reflect mostly scheduled repayment rather than prepayment. More specific data regarding commercial loan prepayment is unavailable.

It has been shown (Richard and Roll 1989, Navratil 1985) that, for GNMAs, prepayment rates are their highest in the early months after issue and fall sharply as the underlying loans are more "seasoned". Since participations are so much shorter in maturity than GNMAs, they are expected to "season" much earlier. Even though participations seldom carry prepayment penalties, they may be expected to prepay less readily than GNMAs due to shorter maturities and less time to recoup refinancing costs. These considerations suggest that, since data specific to loan sales is unavailable, GNMA data provides an alternative. GNMA prepayment rates are likely higher than participation prepayment rates, so their use will yield a conservative estimate of loan sale value.

Default Function

Borrower default is passed through in just the same way as borrower prepayment. A default function, δ, was described as the probability per unit time of default, conditional on not having previously defaulted. In particular:

$$\delta > 0 \quad \text{when } H(t) < F(t)$$

$$\delta = 0 \quad \text{otherwise}$$

where δ = probability of default
$H(t)$ = value of collateral at time t
$F(t)$ = outstanding principal at time t.

Because GNMAs are guaranteed by agencies of the Federal Government (GNMA, FHA, VA), they are often valued as default-free securities (Dunn and McConnell 1981, Brennan and Schwartz 1985, Schwartz and Torous 1989). Schwartz and Torous (1992) noted that, even though GNMAs are guaranteed, default decisions by the borrower affect the timing of cash flows. They recognized that this represents risk to GNMA owners and, therefore, impacts value. This is also the case for loan participations which carry at best an implicit FDIC guarantee. Default impacts participation value. As pointed out in Chapter III, in this loan participation framework, default is not necessarily the result of insolvency. But, by the same token, it may be the result of insolvency. An empirical estimate of the default function must take into account both default as a strategic decision and default as an inevitable result of insolvency.

Moody's Bond Rating Service has undertaken analysis of portfolio credit risk for Collateralized Loan Obligations (CLOs). CLOs are equivalent to Collateralized Mortgage Obligations (CMOs) except they are backed by a portfolio of commercial loans. Moody's analysis has included estimates of individual loan default rates for use in estimating portfolio credit risk. These rates are calculated on the basis of bond ratings and are shown in Table III. Note that the Table lists historical rates both with and without "special events". The "special events" are occurrences specific to a particular borrower which precipitated default. In a well-diversified portfolio such "special event" or nonsystematic risk should be absent and the lower default rates used. Use of the higher (*cum* "special event" risk) default figures would yield a more conservative estimate of participation value.

Not all borrowers whose loans are sold have rated debt outstanding. In such cases ratings must be implied. Moses[13] suggests that either a Caa or B3 rating should be inferred if the borrower meets the criteria listed in Table IV.

Recourse Provisions

The impact of the default function and prepayment function is controlled by the recourse provision. Owing to regulatory treatment, it is anticipated that most sales are without explicit recourse. But, it will be important to determine the specific

[13]Moses, L., "Rating Debt Securitized by Bank Loans", in J. Carlson and F. Fabozzi, ed.'s, The Trading and Securitization of Senior Bank Loans, (Probus Publishing Company, Chicago, IL), 1992, 189-227.

recourse provision for each sale in the empirical sample.

In the absence of explicit recourse, implicit recourse may still exist. The relevant variable, μ, will be difficult to estimate. However, a sale which is modeled without implicit recourse will show a value "too low" in comparison to quoted values if implicit recourse actually exists. Values of μ ranging between 0.00 and 1.00 will be considered.

Limitations of Finite Difference Methodology

The numerical methods being used to test the model equations developed in Chapter III are approximations rather than closed-form solutions. Partial derivatives are estimated and evaluated at discrete points rather than continuously over the domain of the valuation functions. As a result, not every possible combination of model parameters is represented. The size of the estimation mesh, and therefore the frequency with which derivatives are estimated, is governed by considerations of convergence and stability. Mesh parameters have been chosen so as to conform to convergence and stability requirements as identified by Geske and Shastri (1992).

Numerical methods approximations require the identification of boundary conditions. In the case of the model equations, the initial condition (i.e., value at maturity when $\tau=0$) was defined to be equal to the principal outstanding. The value of the initial condition may be considered consistent with the argument that the FDIC will guarantee all non-liability obligations of banks even in the case of borrower default at maturity (Pyle 1985). Also, the upper boundary conditions are defined only when r or H become "very large". As such, the boundary is undefined for purposes of approximation and can never be reached. As a result, extreme approximation values (i.e., at the largest values of r and H considered) are unreliable and may have some effect on other adjacent values. For this study, the highest values of r and H reported were sufficiently below the highest inputs to the model to ensure no adverse effects.

Comparative Statics

The models developed in this research are specific to fixed-rate, secured, non-amortizing commercial loan sales. Sinkey (1992) states that these are most often the characteristics of commercial loans originated by commercial banks and thus are the characteristics of the loan sales of this research. Such nonamortizing (so-called "bullet loans") make no principal repayments until maturity. As a result, the loans to be sold must be valued at a discount from par or maturity value. The amount of the discount, then, is dependent upon the various parameters of the model. Since closed-form solutions for the valuation equations have not been found, exact partial derivatives are not available. However, the finite difference approximation equations (equations 36 and 37) are available and allow the approximation of the partial derivatives. These equations are restated below.

No Explicit Recourse and Implicit Recourse (Full or Partial) Model

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} + gW_{i-1,p-1,j+1} + n(\pi + \delta\mu\zeta)F] \tag{36}$$

For ease of exposition, redefine this equation as

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][\bullet].$$

Explicit Recourse (Full or Partial) Model

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} + dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} + gW_{i-1,p-1,j+1} + n(\pi+\delta\gamma)F] \tag{37}$$

For ease of exposition, redefine this equation as

$$W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][\bullet\bullet].$$

The approximate partial derivatives are listed below for both the no recourse/implicit recourse model and the explicit recourse model.

Market Factors

Riskless Rate

The models are developed for fixed-rate, nonamortizing securities. The values of such securities are rate sensitive with an inverse relation to market rates of interest. This demonstrated in the approximate partial derivatives.

No Recourse, Implicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial r} = (1+rn+\pi n+\delta n)^{-1}\left(\frac{\partial \pi}{\partial r}\right)nF - (1+rn+\pi n+\delta n)^{-2}\ n[\bullet] < 0 \qquad (39)$$

Explicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial r} = (1+rn+\pi n+\delta n)^{-1}\left(\frac{\partial \pi}{\partial r}\right)nF - (1+rn+\pi n+\delta n)^{-2}\ n[\bullet\bullet] < 0 \qquad (40)$$

In each case, since by definition all the variables are non-negative and since $\partial\pi/\partial r < 0$, then the derivative is negative.

Variance (Standard Deviation) of the Riskless Rate

The variability of the riskless rate represents a source of risk in the participation. Intuitively, then, as the variance (σ_r^2) or standard deviation (σ_r) increases the value should decrease, and vice versa. However, Cox, Ingersoll, and Ross (1985) argue that bond prices are an increasing, concave function of the variance of the riskless rate. Their reasoning is that increasing variance indicates increasing uncertainty about future real production opportunities, and thus more uncertainty about future consumption. Risk-averse investors would value the contractually guaranteed claim in a bond more highly in such an uncertain world. In the same way, investors in loan participations receive contractually certain cash flows which would be highly valued in a world of variable interest rates.

This derivative is identical for both models.

$$\partial W_{i,p,j}/\partial\sigma_r = (1+rn+\pi n+\delta n)^{-1}\Big[W_{i-1,p,j+1}[(n\sigma_r)(1/2h) + (n\sigma_r)(1/h^2) - (n\sigma_H\rho_{rH})(1/hq)]$$

$$+ W_{i,p,j+1}[-2(n\sigma_r)(1/h^2) + 2(n\sigma_H\rho_{rH})(1/hq)]$$

$$+ W_{i+1,p,j+1}[-2(n\sigma_r)(1/2h) + (n\sigma_r)(1/h^2) - (n\sigma_H\rho_{rH})(1/hq)]$$

$$+ W_{i,p-1,j+1}[-(n\sigma_H\rho_{rH})(1/hq)]$$

$$+ W_{i,p+1,j+1}[-(n\sigma_H\rho_{rH})(1/hq)]$$

$$+ W_{i+1,p+1,j+1}[n\sigma_H\rho_{rH})(1/hq)]$$

$$+ W_{i-1,p-1,j+1}[n\sigma_H\rho_{rH})(1/hq)]\Big] \tag{41}$$

The sign is not readily apparent by inspection.

Market Price of Risk

Since the valuation equations were based in part on the stochastic process governing the riskless rate and since the loan sales being valued are not riskless, a term (λ) was included to account for the pricing by the market of risk. Since λ is explicitly a risk-related term, it is expected that an increase (decrease) in the market price of risk will result in a decrease (increase) in the value of the loan sale.

This derivative is identical for both models.

$$\frac{\partial W_{i,p,j}}{\partial\lambda} = (1+rn+\pi n+\delta n)^{-1}\left(-\frac{n}{2h}W_{i-1,p,j+1} + \frac{n}{2h}W_{i+1,p,j+1}\right) \tag{42}$$

Since all terms are non-negative, the sign is dependent upon the relative sizes of $W_{i-1,p,j+1}$ and $W_{i+1,p,j+1}$. Recall that i was defined as the increment to the riskless rate so that $i-1 < i+1$. An inverse relationship has been established between $W_{i,p,j}$ and the riskless rate. Thus, $W_{i-1,p,j+1} < W_{i,p,j+1}$ and the sign is negative.

Loan-Specific Factors

Collateral Value

The value of the underlying collateral (H) does not appear directly in the finite difference equations. Rather, the parameter α from the stochastic diffusion process governing the movement of collateral value appears. The term α represents the instantaneous trend in the value of the collateral. As stated in the boundary conditions to this model (Chapter III), as $H \rightarrow \infty$, $V \rightarrow F$. In words, as the value of collateral increases, the value of the loan sale increases toward a limit of the contractual principal (F). Since α represents the trend in collateral value, $\alpha > 0$ implies increasing value and $\alpha < 0$ implies decreasing value. This argument suggests that the sign of the derivative is positive. Kau, Keenan, Muller, and Epperson (1992) and Schwartz and Torous (1992) argue that increased collateral value, by decreasing the probability of default, decreases the value of recourse (guarantee, insurance). The negative impact of this decreased recourse value overpowers the positive impact of decreased default probability.

This derivative is identical for both models.

$$\frac{\partial W_{i,p,j}}{\partial \alpha} = (1+rn+\pi n+\delta n)^{-1}\left(-\frac{n}{2q}W_{i,p-1,j+1} + \frac{n}{2q}W_{i,p+1,j+1}\right) \tag{43}$$

The sign is not readily apparent by inspection.

Variance (Standard Deviation) of Collateral Value

As with the riskless rate, variance of collateral value is a source of risk in the valuation process. It is expected that an increase (decrease) in the variance of collateral value (σ_H^2) or standard deviation of collateral (σ_H) will result in a decrease (increase) in the value of the loan sale. However, if the reasoning of Cox, Ingersoll, and Ross (1985) holds here, then the contractual certainty of the payment at maturity would be a source of value to a risk-averse investor in loan participations.

The derivative is identical for both models.

$$\begin{aligned} \partial W_{i,p,j}/\partial\sigma_H = (1+rn+\pi n+\delta n)^{-1} \Big[& W_{i-1,p,j+1}[(-n\sigma_r)(1/hq)] \\ & + W_{i,p,j+1}[2(n\sigma_H\rho_{rH})(1/hq) + 2(n\sigma_H)(1/q^2)] \\ & + W_{i+1,p,j+1}[-(n\sigma_r\rho_{rH})(1/hq)] \\ & + W_{i,p-1,j+1}[(n\sigma_H)(1/2q) - (n\sigma_r\rho_{rH})(1/hq) + n\sigma_H)(1/q^2)] \\ & + W_{i,p+1,j+1}[-(n\sigma_H)(1/2q) - (n\sigma_r\rho_{rH})(1/hq) + n\sigma_H)(1/q^2)] \\ & + W_{i+1,p+1,j+1}[n\sigma_r\rho_{rH})(1/hq)] \\ & + W_{i-1,p-1,j+1}[n\sigma_r\rho_{rH})(1/hq)]\Big] \end{aligned} \tag{44}$$

The sign of the expression is not readily apparent by inspection.

Correlation of the Riskless Rate and Collateral Value

Since both the riskless rate and collateral value are assumed to follow stochastic diffusion processes, it is necessary for valuation to take into account any correlation between these two processes. This correlation is represented by the term ρ_{rH}. Intuitively, correlation between the two processes potentially represents an obstacle to hedging of risk required for the construction of the hedge portfolio which is the basis of the valuation model. In this way, it is expected that an increase (decrease) in ρ_{rH} will result in an decrease (increase) in the value of the loan sale.

The derivative is identical for both models.

$$
\begin{aligned}
\partial W_{i,p,j}/\partial \rho_{rH} = (1+rn+\pi n+\delta n)^{-1} \Big[& W_{i-1,p,j+1}[(-n\sigma_r\sigma_H)(1/hq)] \\
& + W_{i,p,j+1}[2(n\sigma_r\sigma_H\rho_{rH})(1/hq)] \\
& + W_{i+1,p,j+1}[-(n\sigma_r\sigma_H\rho_{rH})(1/hq)] \\
& + W_{i,p-1,j+1}[-(n\sigma_r\sigma_H\rho_{rH})(1/hq)] \\
& + W_{i,p+1,j+1}[-(n\sigma_r\sigma_H)(1/hq)] \\
& + W_{i+1,p+1,j+1}[n\sigma_r\sigma_H\rho_{rH})(1/hq)] \\
& + W_{i-1,p-1,j+1}[n\sigma_r\sigma_H\rho_{rH})(1/hq)]\Big]
\end{aligned}
\tag{45}
$$

The sign is not readily apparent from inspection.

Time to Maturity

In the same way as with the riskless rate, time to maturity has an inverse relation with value. Since the loan sales being valued are nonamortizing, their values are, in a sense, the present values of the maturity payments or "bullet" payments. As a result, the longer the time to maturity, the longer the period over which the present value is taken, and the lower the value. This is reflected in the finite difference equations if it is recognized (Brennan and Schwartz 1978) that each step in the finite difference process is a discounting of the expected value of the asset in question at the previous time step. That is, at each step in the process, the model value ($W_{i,p,j}$) is less than the model value ($W_{i,p,j+1}$). As the process continues backward from the maturity value towards the desired value (i.e., as τ increases), the value will decrease.

So, even though τ does not appear in equations 36 and 37, by construction, its derivative is negative. This is true for both models.

$$\frac{\partial W_{i,p,j}}{\partial \tau} < 0 \tag{46}$$

<u>Prepayment Rate</u>

The valuation models are based in part on the concept of prepayment as a source of risk in loan sales. An increase (decrease) in prepayment (π) results in a decrease (increase) in the value of the loan sale.

No Recourse, Implicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \pi} = (1+rn+\pi n+\delta n)^{-1}\, nF - (1+rn+\pi n+\delta n)^{-2}\, n[\bullet] < 0 \tag{47}$$

By inspection, since $(1+rn+\pi n+\delta n)^{-1} > (1+rn+\pi n+\delta n)^{-2}$ and $nF < [\bullet]$, then the sign of the derivative is ambiguous.

Explicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \pi} = (1+rn+\pi n+\delta n)^{-1}\ nF\ -\ (1+rn+\pi n+\delta n)^{-2}\ n[\bullet\bullet] < 0 \qquad (48)$$

By an argument equivalent to that for the No Recourse/Implicit Recourse Model, the sign of the derivative is ambiguous.

<u>Default Rate</u>

The valuation models of this research are also based in part on the concept of default as a source of risk in loan sales. It is expected that an increase (decrease) in the value of the default term (δ) will result in a decrease (increase) in the value of the loan sale.

No Recourse, Implicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \delta} = (1+rn+\pi n+\delta n)^{-1}\ n\mu\zeta F\ -\ (1+rn+\pi n+\delta n)^{-2}\ n[\bullet] < 0 \qquad (49)$$

By inspection, since $(1+rn+\pi n+\delta n)^{-1} > (1+rn+\pi n+\delta n)^{-2}$ and $n\mu\zeta F < [\bullet]$, then the sign of the derivative is ambiguous.

Explicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \delta} = (1+rn+\pi n+\delta n)^{-1}\ n\gamma F\ -\ (1+rn+\pi n+\delta n)^{-2}\ n[\bullet\bullet] < 0 \qquad (50)$$

By an argument equivalent to that for the No Recourse/Implicit Recourse Model, the sign of the derivative is ambiguous.

Recourse Factors

Probability of Implicit Recourse

This variable is represented by μ and is defined as the probability that the loan will be "bought back" either in full or in part upon borrower default. In that sense, it is the probability of full recourse. Implicit recourse is positively related to value.

No Recourse, Implicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \mu} = (1++rn+\pi n+\delta n)^{-1}\ n\delta\zeta F > 0 \qquad (51)$$

Since all terms in the expression are, by definition, positive, the derivative is positive.

Explicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \mu} = 0 \qquad (52)$$

This derivative is determined by definition of the model.

Degree of Implicit Recourse

This variable is represented by ζ and is defined as the percent of unpaid principal which will be guaranteed by the lender/seller bank upon borrower default and lender/seller decision to provide implicit recourse. The variable ζ is positively related to value.

No Recourse, Implicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \zeta} = (1+rn+\pi n+\delta n)^{-1}\ (n\delta\mu)F \ > 0 \qquad (53)$$

Since all terms in this expression are positive, this derivative is positive.

Explicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \zeta} = 0 \qquad (54)$$

The definition of the model determines this derivative.

Explicit Recourse

This variable is represented by γ and is defined as the percent of unpaid principal which will be guaranteed by the lender/seller bank upon borrower default. Explicit recourse is positively related to value.

No Recourse, Implicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \gamma} = 0 \tag{55}$$

This is true by definition of the model.

Explicit Recourse Model

$$\frac{\partial W_{i,p,j}}{\partial \gamma} = (1+rn+\pi n+\delta n)^{-1}\ (n\delta)F > 0 \tag{56}$$

Since all terms in this expression are positive, this derivative is positive.

It is important to note that, because of the assumptions which form the basis for the valuation relations in this research, it is not necessarily reasonable to isolate the effects of the riskless rate and the prepayment rate. It is assumed that prepayment is determined, in part, by the market rates which are related to the riskless rate. Specifically, Assumption 3 (Chapter III) states that for a given r_L, an increase (decrease) in r_A will result in a decrease (increase) in RLR and a decrease (increase) in prepayment. Because of this assumption, it is expected that, for example, an increase in prepayment will be related to an increase in the riskless rate and a decrease in the participation value.

Tables V and VI show the results of numerical methods approximation of the valuation equations. These results conform to the expected results listed above. The relationships between participation value and the variance of the riskless rate (σ_r^2)

and the variance of collateral value (σ_H^2) are consistent with the argument of Cox, Ingersoll, and Ross (1985). The relationship between collateral value trend (α) and participation value is consistent with the arguments of Kau, Keenan, Muller, and Epperson (1992) and Schwartz and Torous (1992).

Taken in isolation, the effect of increasing prepayment is the opposite of that expected. However, when prepayment and the riskless rate are varied in opposite directions, the expected effect results. This suggests that isolation of prepayment and the riskless rate may be misleading.

By studying the comparative statics of the finite difference equation models, each parameter was evaluated. Table VII contains the partial derivative signs for the market parameters [riskless rate (r), variance of r (σ_r^2), market price of risk (λ)], the loan-specific factors [trend in collateral value (α), variance of collateral value (σ_H^2), correlation of r and H (ρ_{rH}), time to maturity (τ), , default (δ), prepayment (π)], and the recourse factors [probability of implicit recourse (μ), degree of implicit recourse (ζ), degree of explicit recourse (γ)]. As indicated in Table VII, these results are consistent with those reported by Schwartz and Torous (1992), Kau, Keenan, Muller, and Epperson (1992), Titman and Torous (1989), and Dunn and McConnell (1981b). Each of these related studies is reviewed in Appendix D.

Ex Ante, it is expected that both explicit and implicit recourse will impact value. This is borne out by the valuation models. Explicit recourse increases value due to the guarantee it represents. Implicit recourse increases value due to its representation of the reputation of the seller for "buying back" loans. The question becomes one of assessing the relative effects on value of the various recourse provisions. This question is addressed by Figures 4-21. In each of Figures 4-21, all model parameters are as follows, unless noted otherwise:

Market Factors	
risk-free rate	0.05
variance of risk-free rate (σ^2_r)	0.006
market price of risk (λ)	0.247
Loan-Specific Factors	
maturity value	\$1,000.00
collateral value growth rate (α)	0.01
variance of collateral value (σ^2_H)	0.006
correlation of r and H ($\rho_{r,H}$)	0.00
time to maturity	2 years
default rate	0.018
prepayment rate	0.05.

Differences Between 100% and 0% Explicit Recourse Value

Figures 4-12 illustrate the differences in value of two loan sales identical except for the recourse provisions. One has 0% explicit recourse ($\gamma=0$, $\mu=0$) and the other has 100% explicit recourse ($\gamma=1.00$). Each data point shown is the result of:

$$P_{\gamma=1.00} - P_{\gamma=0.00}.$$

Since each point is non-negative, this indicates 100% explicit recourse to be "more valuable" than 0% explicit recourse.

Both models (0% explicit and 100% explicit recourse) exhibited comparative statics with the same signs for each of the model parameters. This being the case, general statements may be made concerning the interpretation of Figures 4-12. If the comparative static relationship was found to be direct (inverse), the sign was positive (negative) and an upward-sloping difference curve suggests that the 100% explicit recourse (0% explicit recourse) model was the more sensitive to the parameter changes. If the comparative static relationship was found to be direct (inverse), the sign was positive (negative) and a downward-sloping difference curve suggests that the 0% explicit recourse (100% explicit recourse) model was the more sensitive to the parameter changes. Table VIII summarizes the interpretation of Figures 4-12.

The 0% explicit recourse value is shown to be more sensitive to changes in the loan-specific terms (α, σ_H^2, τ, δ). The 100% explicit recourse model is shown to be more sensitive to changes in the loan-specific terms (ρ_{rH}, π) and the market factors (r, σ_r^2, λ). Note that each model is sensitive to all the factors to one degree or another. However, the 0% recourse value is more sensitive to those factors which represent default. Recall from Assumptions 4 and 6 (Chapter III) that default (δ) is a function of the stochastic process driving collateral value ($dH = \alpha H dt + \sigma_H H dz_H$). The relative sensitivity is reasonable, then, in the sense that the loan sale which loses all its value upon default (i.e., 0% recourse) should be more sensitive to the factors that drive default than the loan sale which loses only a portion of its value (i.e., positive recourse).

Alternatively, the 100% recourse value is more sensitive to those factors which represent prepayment. Recall from Assumptions 3 and 5 (Chapter III) that prepayment (π) is a function of the relative loan rate (RLR) which derives from the stochastic process driving the riskless rate ($dr = \sigma_r r dz_r$). Further, RLR is driven by the rate on the loan which will include considerations of the market price of risk (λ) and the correlation between rates and collateral (ρ_{rH}). The relativity sensitivity is reasonable, then, since a 100% recourse value need take little account of default. As a result, the greater source of risk is prepayment -- hence the value's greater sensitivity.

Differences Between 50% Explicit Recourse and 50% Implicit Recourse Value

Figures 13-21 illustrate the differences in value of two loan sales identical except for the recourse provisions. One has a 50% probability of a 50% degree of implicit recourse (μ=.50, ζ=.50) and the other has 50% explicit recourse (γ=.50). Each data point shown is the result of:

$$P_{\gamma=.50} - P_{\mu=.50}.$$

Since each point is non-negative, this indicates 50% explicit recourse to be "more valuable" than 50% implicit recourse. Recall that this definition of implicit recourse may be interpreted as a 50% probability that the seller will provide 50% recourse upon borrower default. On the other hand, 50% explicit recourse may be interpreted as a 100% probability that the seller will provide 50% recourse upon borrower default.

Note that both models (explicit and implicit recourse) exhibited comparative statics with the same signs for each of the model parameters. This being the case, general statements may be made concerning the interpretation of Figures 13-21. If the comparative static relationship was found to be direct (inverse), the sign was positive (negative) and an upward-sloping difference curve suggests that the 50% explicit recourse (50% implicit recourse) model was the more sensitive to the parameter changes. If the comparative static relationship was found to be direct (inverse), the sign was positive (negative) and a downward-sloping difference curve suggests that the 50% explicit recourse (50% implicit recourse) model was the more sensitive to the parameter changes. Table IX summarizes the interpretation of Figures 13-21.

The implicit recourse value is shown to be more sensitive to changes in the loan-specific terms (α, σ_H^2, τ, δ). The explicit recourse model is shown to be more sensitive to changes in the loan-specific terms ($\rho_{r,H}$, π) and the market factors (r, σ_r^2, λ). These results are consistent with those reported for the 0% - 100% explicit recourse comparison. In the same way as before, the value with the lesser recourse (i.e., 0% explicit or 50% implicit) is more sensitive to the factors driving default. Likewise, the value with the more recourse (i.e., 100% explicit or 50% explicit) is more sensitive to the factors driving prepayment.

Tables VII, VIII, and IX serve to validate the valuation models and their finite difference approximations. Table VII shows the results of this study to be consistent with those of other similar studies. Tables VIII and IX show the valuation models to be consistent with each other and to exhibit reasonable relationships among their results.

CHAPTER V.

RESULTS AND CONCLUSION

This research has used contingent claims methods to develop models for the valuation of loan participations under differing levels of recourse -- both explicit and implicit. The models were specifically developed for fixed-rate, secured, nonamortizing loan sales. Sinkey (1992) has shown that this description fits the majority of loans being sold as participations. Each model, as shown, is in the form of a differential equation whose solution, P, is the value of the loan sale.

No Explicit Recourse and Implicit (Full or Partial) Recourse Model

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$

$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi)(F-P) + (\delta\mu)(\zeta F-P) + \delta(1-\mu)(-P) - rP \qquad (17)$$

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$

$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\mu\zeta)(F) + (\pi + \delta)(-P) - rP \qquad (18)$$

Explicit Recourse (Full or Partial) Model

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$

$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\gamma)(F) + (\pi + \delta)(-P) - rP \qquad (19)$$

where
- π = the probability of prepayment
- δ = the probability of default
- γ = the degree of explicit recourse
- μ = the probability of providing implicit recourse
- ζ = the degree of implicit recourse.

Solutions to these equations were shown to exist but have not yet been formulated. The method of explicit finite differences was used in Chapter IV to formulate approximations to the valuation rules as follows:

No Explicit Recourse and Implicit (Full or Partial) Recourse Model

$$\begin{aligned} W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][&aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ &+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ &+ gW_{i-1,p-1,j+1} + n(\pi + \delta\mu\zeta)F] \end{aligned} \tag{36}$$

Explicit (Full or Partial) Recourse Model

$$\begin{aligned} W_{i,p,j} = [1/(1 + rn + \pi n + \delta n)][&aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ &+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ &+ gW_{i-1,p-1,j+1} + n(\pi+\delta\gamma)(F)]. \end{aligned} \tag{37}$$

These approximations were then used to value loan sales within ranges of parameter values.

Integral to this research are the various levels of recourse under which loans are sold. A loan sale transfers the loan's cash flows from the lender/seller bank to the purchaser. It does not, however, give the purchaser any recourse to the borrower in the event he or she defaults. Any recourse the purchaser has is to the lender/seller. The particular conditions of this recourse vary from transaction to transaction.

Loans may be sold with an explicit recourse provision. Such explicit recourse may be full or partial. It serves as a guarantee that all or part of the contractual payment to the purchaser will be made by the lender/seller if the original borrower fails to pay. From a regulatory standpoint, such "explicit recourse" sales do not transfer risk from the seller to the purchaser. As a result, the selling bank is required to include the full loan balance in its calculations of risk-based capital ratios. In the model development, explicit recourse was accounted for by the term:

$$\delta(\gamma F - P) \tag{57}$$

where

	$0<\gamma\leq 1$	is the degree of explicit recourse
	δ	is the probability of default
	F	is the contractual payment remaining
	P	is the value of the participation with time to maturity equal to τ.

In this way, explicit recourse may be described as the percentage of recourse which will be provided <u>with certainty</u>.

Often, though, loans are sold with no recourse provision. In such a loan sale, the purchaser has recourse neither to the seller nor the borrower in the case of default. From a regulatory standpoint, such "no recourse" sales completely transfer risk and need not be included in the seller's capital ratio. In the model development, sales without recourse contained the term:

$$\delta(-P). \tag{58}$$

Note that this is equivalent to the explicit recourse formulation with $\gamma = 0$. Again, this may be interpreted as lack of recourse <u>with certainty</u>.

It has, however, been argued (Gorton and Pennacchi 1989, Wall 1991) that loan sellers may provide recourse even when not contractually required to do so.

Wall (1991) has termed this "implicit recourse". From a regulatory standpoint, implicit recourse is not considered. Regulatory measures do not account for the incomplete risk transfer which results from this phenomenon. It is, however, a potential source of default risk to the lender/seller bank. The model development treats implicit recourse as a strategic decision by the seller upon borrower default. This was incorporated into the model by including the terms:

$$\delta\mu(\zeta F - P) + \delta(1 - \mu)(-P) \tag{59}$$

where μ is the probability of implicit recourse and ζ is the degree of implicit recourse. Note that, if $\zeta=1$, this is equivalent to full recourse with probability μ and no recourse with probability $(1-\mu)$. Implicit recourse is modeled, then, as either full or partial recourse under uncertainty.

In Chapter I, the argument was made that loan sales with 0% explicit recourse allow the loan sold to be removed from the seller's balance sheet. In this way, loan sales allow capital ratios to be increased as loans (risk weight > 0) are replaced with cash (risk weight = 0). Risk-weighted capital standards, then, give banks an incentive to sell loans with 0% explicit recourse. It was also argued that banks often buy back loans upon borrower default. That is, banks offer implicit recourse. Regulatory standards take no account of implicit recourse, but it is argued that a reputation for implicit recourse will increase the price for which a bank can sell its loans.

Together, these two arguments suggest that banks have a strong incentive to sell loans with 0% explicit recourse while providing a positive probability of implicit recourse. The valuations resulting from the models of this research were shown to be positively related to both explicit and implicit recourse. The partial derivatives with respect to explicit recourse (γ), the probability of implicit recourse (μ), and the degree of implicit recourse (ζ) were shown in Chapter IV to be positive and Tables V and VI bore this out numerically. However, the current regulatory practice of ignoring the possibility of implicit recourse in sales with no explicit recourse may expose the banking system to a significant source of risk. If banks are routinely providing implicit recourse (as suggested by Wall 1991 and Gorton and Pennacchi 1991), then loan sales with 0% explicit recourse do not transfer all default risk and should not be treated by regulators as if they do. Further evidence of this risk is demonstrated by Figures 4-21. As demonstrated in Chapter IV, the model with 0% explicit recourse and the model with 50% implicit recourse were shown to be more sensitive to default than the 100% or 50% explicit recourse models. However, the models with 100% or 50% explicit recourse were more sensitive to prepayment than the 0% explicit or 50% implicit recourse models. This suggests that loan sales with 0% explicit and 0% implicit recourse are qualitatively different from loan sales with 0% explicit recourse and positive implicit recourse. The regulatory practice of ignoring implicit recourse and treating 0% explicit recourse as a perfect transfer of risk is flawed. Regulatory incentives make implicit recourse sales more likely and the models developed in this research suggest that such sales are more sensitive to default risk than explicit recourse sales.

The BASLE Accord which established risk-based capital standards also established a means for accounting for off-balance sheet risk. Such off-balance sheet items as loan commitments, lines of credit, etc., are multiplied by a conversion factor intended to yield an equivalent on-balance sheet amount. This equivalent amount is then included in the risk-adjusted totals in the same way as other on-balance sheet items. The models of this research suggest that implicit recourse potentially represents a source of off-balance sheet risk. In the same way as other contingent liabilities (commitments, lines of credit, etc.) implicit recourse is not a balance sheet item until exercised. It is the probability of exercise which makes it an on-balance sheet item that gives rise to the need for inclusion in risk-based capital.

Table V and Figures 13-21 suggest that implicit recourse increases the value of a loan participation with no explicit recourse. The reason for this increase in value is the decrease in default risk to the purchaser. It is the seller who takes on this risk. The amount by which implicit recourse increases loan sale value, then, is a function of the risk it represents to the seller.

Referring to equation 18, with no explicit or implicit recourse (i.e., $\mu=0$), the valuation rule is:

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r \rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + \pi F + (\pi + \delta)(-P) - rP. \tag{60}$$

With implicit recourse only, the valuation rule is:

$$0 = P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r \rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 + (\pi + \delta\mu\zeta)(F) + (\pi + \delta)(-P) - rP. \tag{61}$$

If it is assumed for the moment that all terms in the two expressions are equal, then subtracting the two valuation rules yields $\delta\mu\zeta F$. This expression may be viewed as a measure of the expected increase in value of a loan which has been sold as a result of implicit recourse and the decrease in default risk it represents to the purchaser. Recall that loan sales with no explicit recourse allow the entire loan amount to be removed from the seller's balance sheet. Implicit recourse, while treated the same as no explicit recourse for regulatory purposes, increases the value of the loan sale to the purchaser and represents default risk remaining with the seller. The amount $\delta\mu\zeta F$ is potentially a candidate for the on-balance sheet equivalent amount of the off-balance sheet contingent liability represented by implicit recourse. In this way, the term $\delta\mu\zeta F$ is the equivalent on-balance sheet amount required by the BASLE guidelines for inclusion in risk-based totals. Note that $\delta\mu\zeta F$ is a multiple of the outstanding principal, F, and thus functions as an equivalent on-balance sheet amount. The size of the term is dependent upon the probability of default, δ, and the probability of implicit recourse, μ, upon default. It is, then, positively related to the risk inherent in implicit recourse.

At best, $\delta\mu\zeta F$ is an approximation of the difference in value under 0% explicit and implicit recourse and value under positive implicit recourse. The actual difference and, therefore, the actual equivalent on-balance sheet amount, depends upon market conditions. Loan specific factors are not relevant since, for regulatory purposes, the comparison is of the same loan under different recourse provisions.

Figures 22-28 illustrate the value differences between no implicit recourse and positive implicit recourse. That is, each data point represents:

$$P_{\mu>0,\gamma=0} - P_{\mu=0,\gamma=0}.$$

All parameter values are as follows unless noted otherwise:

Market Factors	
risk-free rate	0.05
variance of risk-free rate (σ^2_r)	0.006
market price of risk (λ)	0.247
Loan-Specific Factors	
maturity value	$1,000.00
collateral value growth rate (α)	0.01
variance of collateral value (σ^2_H)	0.006
correlation of r and H ($\rho_{r,H}$)	0.00
time to maturity	2 years
default rate	0.018
prepayment rate	0.05.

Table X summarizes the information contained in Figures 22, 23, and 24. As would be expected from previous results reported in Chapter IV, the loan sale value with the higher recourse is the more sensitive to market factors. Note, however, that in Figures 22, 23, and 24 that the range of differences is very small -- on the order of 15 except for the lowest interest rate shown. The models suggest that, for a given level of implicit recourse, the premium in value over no implicit recourse is relatively constant across a range of market factors (r, σ_r^2, λ).

Regulators are also potentially concerned with the relationship between value with no recourse and value with implicit recourse under extreme market conditions. The results shown in Figures 22, 23, and 24 and Table X assume a very stable interest rate environment ($\sigma_r^2 = 0.006$). In the past decade, however, markets have experienced periods of highly volatile interest rates. Figures 25 and 26 illustrate differences in value for ranges of r and λ given $\sigma_r^2 = 0.500$. Figures 27 and 28 illustrate differences in value for ranges of r and λ given $\sigma_r^2 = 0.900$. These simulate periods of higher volatility in interest rates. Note that, as σ_r^2 increases, the differences also increase slightly over the ranges of λ but decrease slightly over the range of r. The models, then suggest that the difference in value from implicit recourse is stable even in volatile interest rate environments.

Conclusion

The development of the loan sales models in this research was motivated by three factors: (1) regulatory requirements for market value accounting for banks (i.e., FASB 107), (2) recently enacted risk-based capital standards for banks, and (3) regulatory treatment of loan sales without recourse.

FASB 107 potentially will correct a flaw in the system of closure rules for insolvent banks. Currently, closure is determined at book value, while liquidation (or purchase and assumption) is accomplished at market value. This mismatch has led to inefficient closure (i.e., after market value net worth of the bank has become negative), incomplete disclosure of bank condition by managers, and inefficient implementation of regulatory safeguards (e.g., loan loss reserves and capital adequacy). Market value accounting requirements as set forth in FASB 107 will address these problems. The contingent claims valuation models developed in this research provide a means for valuing loan sales using an analytical framework rather than "rule of thumb" models (e.g., Average Life, Curley and Guttentag 1974). The variables in the contingent claims model are readily observable and the underlying stochastic processes have been shown to be reasonable.

Banks sell loans in response to a number of motivations, including diversification, funding costs, regulatory costs, and capital costs. Recently enacted risk-based capital standards have made the capital costs motivation even stronger. The new standards allow banks to sell risky loans (risk weight > 0) for risk-free cash (risk weight = 0). Such a sale decreases the bank's risk adjusted asset total, thereby allowing the selling banks to increase its capital ratios without raising new external capital. This capital effect only results , however, from loan sales with no recourse. For regulatory purposes, no recourse loan sales represent a full transfer of risk from the lender/seller to the purchaser of the loan. As such, the loan is removed from the lender's balance sheet and risk adjusted capital totals. Loan sold with recourse represent an incomplete risk transfer and must still be included in the seller's risk adjusted asset total. Risk-based capital, then, provides a strong incentive for banks to sell loans with no recourse.

It has been argued (Gorton and Pennacchi 1989, Wall 1991) that, in response to borrower default, selling banks often buy back the loans they have sold even when not legally required to do so. That is, they provide implicit recourse when no explicit (contractual) recourse was provided. Regulatory practice currently takes no account of implicit recourse. The existence of implicit recourse suggests that loan sales with no (explicit) recourse do not completely transfer risk and that current capital adequacy rules ignore a potential source of risk on selling banks' balance sheets.

The models developed in this research include provisions for implicit recourse. They treat implicit recourse as a reputation variable for the selling bank. That is, a bank which has provided implicit recourse in the past may be expected by potential purchasers to do so again. The models suggest that this reputation allows sellers to obtain a higher price for loan sales than they would have without the reputation for implicit recourse. This price premium results from the incomplete transfer of risk to the purchaser who is, therefore, willing to pay a higher price to the extent that default risk remains with the lender/seller. Wall (1991) has argued that banks will provide the highest possible recourse in order to maximize price. Capital standards give an incentive for this recourse to be implicit. Tables V, VI, and VII confirm that higher recourse equals higher value, even in the case of implicit recourse.

The regulatory incentive, then, is not only for loan sales without explicit recourse, but also for sales with implicit recourse. This incentive introduces an agency cost as selling banks offer the highest implicit recourse possible. By doing so, they retain risk but this risk is not included in risk based capital. The sellers, then, are enjoying a subsidy from the FDIC while improving their loan sales values.

It was demonstrated (Chapter I) that implicit recourse decreases the agency cost associated with an "underinvestment problem" in banks. To the extent that the increased agency cost from implicit recourse is offset by decreased underinvestment, implicit recourse does not impact bank risk. However, to the extent they are not offsetting, implicit recourse represents a contingent liability for the seller. Risk-based capital standards make provision for such contingent liabilities as letters of credit, loan commitments, etc. The models suggest that such provision should also be made for implicit recourse. It was demonstrated that a given level of implicit recourse adds value to an equivalent no recourse loan sale. The value difference is stable over different levels of interest rate volatility. This suggests that it may be effectively accounted for by regulators.

The models suggest that the value premium resulting from implicit recourse may be illustrated by the term

$$\mu\delta\zeta F$$

where μ = probability of implicit recourse (i.e., reputation)
ζ = degree of implicit recourse ($0 \leq \zeta < 1$)
δ = probability of borrower default
F = outstanding principal.

Like explicit recourse, implicit recourse is only provided if the borrower defaults. Given default (δ), the higher the probability of implicit recourse (μ) and the higher the degree of implicit recourse provided (ζ), the higher the premium in value. That is, as $\mu\delta\zeta F$ increases, the less complete is the transfer of risk from the lender/seller to the purchaser, and the higher the value of the loan sale. The models, then, not only suggest an effect of implicit recourse, but also a way to measure and account for it.

The contribution of this research to the field of Finance is the modeling of implicit recourse and its effects on loan sales value and risk. Other financial assets besides loan sales contain implicit recourse provisions or implicit guarantees, however. This research, then, represents a step toward valuing other assets containing implicit recourse provisions as well.

APPENDIX A

IMPLICIT RECOURSE AND THE "UNDERINVESTMENT PROBLEM"

It has been argued that the presence of debt in a firm's capital structure leads to an incentive for stockholders to forego certain positive NPV projects (Myers 1977, Scott 1977, Stultz and Johnson 1985, James 1990). This so-called "underinvestment problem" is an agency cost associated with debt and applies to both financial and nonfinancial firms. Stultz and Johnson (1985) showed that collateralization of debt serves to mitigate at least a part of this agency cost. James (1988) showed that, under certain circumstances, a bank which funds a new loan entirely with deposits (unsecured debt) will suffer a wealth transfer from current shareholders to depositors. In this case, the bank's shareholders would prefer that the new loan not be made and an underinvestment problem results. In the James (1988) framework, loan sales with recourse serve the same function as collateral in the Stultz and Johnson (1985) argument. That is, they reduce the agency cost.

James (1988) assumed a two-period framework where a bank has one "booked" loan of \$1 and a opportunity to invest in a second loan of \$1. All market participants are assumed to be risk-neutral, markets are assumed to be perfectly competitive, and taxes are ignored. In this framework, James formulated the following expression for the change in value to bank shareholders as a result of funding the new loan with deposits:

$$\int_0^{\bar{s}} a_2(s)f(s)ds \;-\; \int_0^{\bar{s}} \min\left[r_N^d,\ \frac{r_N^d}{L_d}A(s)\right] f(s)ds$$

$$\text{(A)} \qquad\qquad \text{(B)}$$

$$-\left[\int_0^{\bar{s}} \min\left[r_d(1-e),\ \frac{r_d(1-e)}{L_d}A(s)\right] f(s)ds \;-\; \int_0^{\bar{s}} \min[r_d(1-e),\ a_1(s)] f(s)ds\right] \le 0 \qquad (62)$$

$$\text{(C)}$$

where

where	s	= state of nature
	$a_1(s)$	= payoff from existing loan in state s
	$a_2(s)$	= payoff from new loan in state s
	$A(s)$	= total loan payoff in state s
	r_d	= return on existing debt
	r_N^d	= return on new debt
	e	= % of new loan funded from equity
	L_d	= total return on debt.

The above expression states that the change in shareholder value is (A) the expected return from the new loan, minus (B) the expected payment on new deposits, minus (C) the difference between the payoff on existing deposits if the new loan is made (funded by deposits) and the payoff on existing deposits if the new loan is not made. The term labeled (C), then, is the change in value of existing deposits as a result of the new loan. The entire expression (i.e., (A) - (B) - (C)) is the change in shareholder wealth as a result of the new loan. If (A) - (B) - (C) < 0, then the change in value of existing deposits is greater than the benefit to existing shareholders, and the change in shareholder value is negative. Shareholders will then have an incentive not to make the loan even though (A) > 0. Equation 62 demonstrates the "underinvestment problem". James shows that it is (C) which changes as a result of the type of financing used to fund the new loan.

James argues that if a state exists such that:

$$i. \qquad a_1(s) + a_2(s) < r_d^N + (1\text{-}e)r_d$$

$$ii. \quad \int_0^{\bar{s}} \min\left[r_s,\ a_2(s) + \frac{r_s}{L_s}a_1(s)\right] \equiv(s)ds > \int_0^{\bar{s}} \frac{r_d^N}{L_d} A(s) \equiv(s)ds$$

$$\text{where} \quad \begin{array}{lll} r_s & = & \text{return on secured debt} \\ e & = & \text{the percent equity used to fund the loan} \\ A(s) & = & a_1(s) + a_2(s) \\ L_s & = & r_s + r_d(1\text{-}e). \end{array}$$

then the contracted payoff to secured debt (r_s) will be less than the contracted payment on new deposits (r_d^N). In words, these conditions are (i) the actual payoff to the loan portfolio (booked and new) must be less than the promised payoff to depositors, <u>and</u> (ii) the promised payoff to secured debt must be greater than the promised payoff to new depositors if the new loan returns less than the contracted return (i.e., less than r_d^N).

If (A) - (B) -(C) < 0 (i.e., a wealth transfer exists) and conditions i. and ii. are met, then collateralized debt will reduce the wealth transfer. If the loan is funded with collateralized debt rather than with unsecured deposits, (C) becomes:

$$\int_0^{\bar{s}} \min\left[r_d(1-e),\ \max\left[\frac{rd(1-e)}{L_s}a_1(s),\ a_1(s)+a_2(s)-r_s\right]\right] \equiv (s)ds$$

$$- \int_0^{\bar{s}} \min\left[r_d(1-e),\ a_1(s)\right] f(s)ds \tag{63}$$

where r_s = return on existing shares
L_s = total return on shares.

James shows that, if (C) for unsecured debt is compared with (C) for secured debt, the latter is smaller which implies a decrease in the underinvestment problem.

James argues that the payoff characteristics of recourse loan sales are identical to those of collateralized debt. Therefore, if the loan is funded with a recourse loan sale, (C) becomes:

$$\int_0^{\bar{s}} \min\left[r_d(1-e),\ \max\left[\frac{r_d(1-e)+(1-\gamma)(r_s}{L_s}a_1(s),\ a_1(s)+a_2(s)-r_s\right]\right] f(s)ds$$

$$- \int_0^{\bar{s}} \min\left[r_d(1-e),\ a_1(s)\right] f(s)ds \tag{64}$$

where γ is the degree of recourse provided ($\gamma = 0$ for no recourse, $\gamma = 1$ for full recourse, $0 < \gamma < 1$ for partial recourse).[14] In the same way as collateralized debt, all three types of recourse reduce the underinvestment problem.

[14]James used the variable λ to denote the degree of explicit recourse offered. It is changed to γ here for consistency with the terminology of this research.

APPENDIX B

THE "BASIC" DIFFERENTIAL EQUATION

By Itô's Lemma, the value of the participation, P, must follow the process

$$dP = P_r dr + P_H dH + P_t dt + \tfrac{1}{2}[\, P_{rr} dr^2 + 2P_{rH} drdH + P_{HH} dH^2 \,].$$

Assumption 5, Chapter III, defines the stochastic process for the riskless rate (r), as follows:

$$dr = \sigma_r r dz_r$$

$$dr^2 = \sigma_r^2 r^2 dz_r^2$$
$$= \sigma_r^2 r^2 dt.$$

Assumption 6, Chapter III, defines the stochastic process for the collateral value (H), as follows:

$$dH = \alpha H dt + \sigma_H H dz_H$$

$$dH^2 = \alpha^2 H^2 dt^2 + \sigma_H^2 H^2 dz_H^2 + 2\alpha\sigma_H H^2 dt dz_H$$
$$= \quad 0 \quad + \sigma_H^2 H^2 dt \quad + \quad 0.$$
$$= \sigma_H^2 H^2 dt$$

By substitution

$$dP = P_r dr + P_H dH + P_t dt + \tfrac{1}{2}[\, P_{rr} dr^2 + 2P_{rH} drdH + P_{HH} dH^2 \,]$$

$$dP = P_r[\, \lambda r dt + \sigma_r r dz_r \,] + P_H[\, \alpha H dt + \sigma_H H dz_H \,] + P_t dt$$
$$+ \tfrac{1}{2} P_{rr} \sigma_r^2 r^2 dt + P_{rH}[\, \lambda r dt + \sigma_r r dz_r][\alpha H dt + \sigma_H H dz_H]$$
$$+ \tfrac{1}{2} P_{HH} \sigma_H^2 H^2 dt$$

$$dP = P_r[\, \lambda r dt + \sigma_r r dz_r \,] + P_H[\, \alpha H dt + \sigma_H H dz_H \,] + P_t dt$$
$$+ \tfrac{1}{2} P_{rr} \sigma_r^2 r dt + P_{rH}[\, \lambda r \alpha H dt^2 + \lambda r \sigma_H H dt dz_H$$
$$+ \sigma_r r \alpha H dt dz_r + \sigma_r \sigma_H H r dz_r dz_H \,] + \tfrac{1}{2} P_{HH} \sigma_H^2 H^2 dt$$

$$dP = P_r\lambda r dt + P_r\sigma_r r dz_r + P_H\alpha H dt + P_H\sigma_H H dz_H + P_t dt$$
$$+ \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 dt + P_{rH}\sigma_r\sigma_H H r\rho_{rH} dt + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 dt$$
$$dP = [P_r\lambda r + P_H\alpha H + P_t + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH}$$
$$+ \tfrac{1}{2}P_{HH}\sigma_H^2 H^2]dt + P_r\sigma_r r dz_r + P_H\sigma_H H dz_H. \qquad (65)$$

Note that, since the participation is not risk-free, it is necessary to include the term λrdt where λ is the market price of interest rate risk. It is added rather than subtracted because $(\bullet)dt$ is by definition the instantaneous required return. Pricing of risk requires a higher return -- higher by the factor λrdt.

If t = current time

T = maturity

then, the time to maturity

$$\tau = T - t.$$

Given t and T, an increase in t results in an equal decrease in τ. That is,

$$dt = -d\tau$$

so that

$$P_t = -P_\tau$$

and

$$dP = [P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2]dt$$
$$+ P_r\sigma_r r dz_r + P_H\sigma_H H dz_H \qquad (66)$$

which may be redefined as

$$dP = \beta P dt + \gamma P dz_r + \eta P dz_H \qquad (67)$$

where $\beta = \dfrac{P_r\lambda r+P_H\alpha H-P_\tau + \frac{1}{2}P_{rr}\sigma_r^2r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \frac{1}{2}P_{HH}\sigma_H^2H^2}{P}$

$$\gamma = \frac{P_r\sigma_r r}{P} \qquad\qquad \eta = \frac{P_H\sigma_H H}{P}.$$

Following Merton (1973), a hedge portfolio is formed with the following portfolio weights:

W_1 = percentage of collateral

W_2 = percentage of participation

W_3 = percentage of riskless security

where $W_3 = -(W_1 + W_2)$ so that $W_1 + W_2 + W_3 = 0$.

The return on this portfolio may be modeled as

$$dY = W_1(dH/H) + W_2(dP/P) + W_3\,[\lambda rdt + \sigma_r rdz_r]$$

$$dY = W_1\left(\frac{\alpha Hdt+\sigma_H Hdz_H}{H}\right) + W_2\left(\frac{\beta Pdt+\gamma Pdz_r+\eta Pdz_H}{P}\right) + W_3[\lambda rdt + \sigma_r rdz_r]$$

$$dY = W_1\left(\frac{\alpha Hdt+\sigma_H Hdz_H}{H}\right) + W_2\left(\frac{\beta Pdt+\gamma Pdz_r+\eta Pdz_H}{P}\right) - (W_1+W_2)[\lambda rdt+\sigma_r rdz_r]$$

$$dY = \{W_1[\alpha-\lambda r] + W_2[\beta-\lambda r]\}dt + dz_H(W_1\sigma_H+W_2\eta) + dz_r[W_2(\gamma-\sigma_r r) - W_1\sigma_r r].$$

Choose $W_j = W_j^*$, j=1,2,3, such that the coefficients of dz_r and dz_H equal zero. Under these conditions, dY is non-stochastic. Further, since $W_1 + W_2 + W_3 = 0$, no net investment was required. In order to prevent pure arbitrage profits, dY must equal zero.

$$dY = \{W_1[\alpha-\lambda r] + W_2[\beta-\lambda r]\}dt$$
$$+ dz_H(W_1\sigma_H+W_2\eta) + dz_r[W_2(\gamma-\sigma_r r) - W_1\sigma_r r] = 0 \qquad (68)$$

Nontrivial weightings (i.e., $W_1^* \neq 0$, $W_2^* \neq 0$) exist if and only if simultaneously

$$W_1[\alpha - \lambda r] + W_2[\beta - \lambda r] = 0 \tag{69}$$

$$W_1\sigma_H + W_2\eta = 0 \tag{70}$$

$$W_2(\gamma - \sigma_r r) - W_1\sigma_r r = 0. \tag{71}$$

Equations 69, 70, and 71 imply that

$$\frac{\beta - \lambda r}{\alpha - \lambda r} = \frac{\eta}{\sigma_H} = \frac{-(\gamma - \sigma_r r)}{\sigma_r r}.$$

In order for this to hold, the following must hold individually;

$$\text{(A)} \quad \frac{\beta - \lambda r}{\alpha - \lambda r} = \frac{\eta}{\sigma_H} \qquad \text{(B)} \quad \frac{\eta}{\sigma_H} = \frac{-(\gamma - \sigma_r)r}{\sigma_r r}.$$

These are the necessary and sufficient conditions for the portfolio which has no risk and requires no net investment to have a zero expected return as necessary for the absence of arbitrage.

By substitution, (B) equals

$$\frac{\eta}{\sigma_H} = \frac{\gamma}{\sigma_r r} - 1$$

$$\frac{P_H\sigma_H H}{P\sigma_H} = -\frac{P_r\sigma_r r}{P\sigma_r r} + 1$$

$$\frac{P_H H}{P} = \frac{-P_r + P}{P}$$

$$P = P_r + P_H H. \tag{72}$$

This condition shows that the participation value is dependent upon a functional relationship with the short-term riskless rate (r) and the collateral value (H).

By substitution, (A) equals

$$\beta - \lambda r = \frac{\eta[\alpha - \lambda r]}{\sigma_H}$$

$$\beta - \lambda r = \frac{P_H \sigma_H H[\alpha - \lambda r]}{P\sigma_H}$$

$$\frac{P_r\lambda r + P_H\alpha H - P_\tau + \frac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \frac{1}{2}P_{HH}\sigma_H^2 H^2}{P} - \lambda r$$

$$= \frac{P_H H\alpha - P_H H\lambda r}{P}$$

$$P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 - P\lambda r$$

$$= P_H H\alpha - P_H H\lambda r$$

$$P_H H\lambda r + P_r\lambda r - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 - P\lambda r$$

$$= 0. \tag{73}$$

This condition shows the relationship between the participation value and the parameters of the stochastic processes specified for the short-term riskless rate (r) and collateral value (H). Together, these conditions define the valuation relationship upon which this research is based.

Substituting from (B) into (A)

$$P_r\lambda r + P_H\alpha H - P_\tau + \tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 - (P_r + P_H H)\lambda r$$

$$= P_H H\alpha - P_H H\lambda r \tag{74}$$

$$\tfrac{1}{2}P_{rr}\sigma_r^2 r^2 + P_{rH}\sigma_r\sigma_H H r\rho_{rH} + \tfrac{1}{2}P_{HH}\sigma_H^2 H^2 - P_\tau = 0. \tag{75}$$

Equation 75 was stated earlier in Chapter III as equation 4.

Integral to the development above is the assumption that the factors (i.e., r and H) follow continuous stochastic processes with no discrete jumps. This assumption allows the formation of a portfolio which is riskless and requires no net investment, and therefore must have zero return. The riskless portfolio rests on the choice of $W_j = W_j^*$, j = 1,2, such that the coefficients of dz_H and dz_r are always zero, as shown above in equations 71 and 72. If these coefficients are always equal to zero, then the return on the hedge portfolio is nonstochastic.

Following a process analogous to that described above, a hedge portfolio may be formed using two assets (i.e., the participation and the collateral) which earns the riskless rate. the portfolio is not riskless in the way the three-asset portfolio was (i.e., returns are not nonstochastic). Rather, this two-asset hedge portfolio is without nonsystematic risk and earns the riskless rate (r). Likewise, if discrete jumps are introduced into the processes, then the portfolio formed as above is no longer riskless. Consider, the portfolio Y as described above but including a Poisson process which will cause the portfolio's value to jump discretely. The return on this portfolio can be described as follows:

$$dY = \left(W_1[\alpha-\lambda r] + W_2[\beta-\lambda r] + \xi k_Y\right) dt + dz_H \left(W_1\sigma_H + W_2\eta\right)$$

$$dz_r \left(W_2[\gamma-\sigma_r] - W_1\sigma_r r\right) + dq \tag{76}$$

where k_Y = the change in the portfolio value due to the "jump process"
ξ = the probability of the discrete jump occurring
dq = 1 when the jump occurs
0 otherwise.

With this specification, even choosing $W_j = W_j^*$ to make the coefficients of dz_H and

dz_r equal to zero does not leave the portfolio riskless (nor the return nonstochastic) because the stochastic term dq remains.

The return on this portfolio may be described as

$$dY = \left(W_1\ [\alpha - \lambda r]\ +\ W_2\ [\beta - \lambda r]\right)\ dt \tag{77}$$

if the Poisson or jump event does not occur, and as

$$dY = \left(W_1\ [\alpha - \lambda r]\ +\ W_2\ [\beta - \lambda r]\ +\ \xi k_p\right)\ dt \tag{78}$$

if the Poisson or jump event does occur.

The portfolio dynamics described in equation 75 result from the continuous nature of the returns on the assets in the hedge portfolio. The portfolio dynamics of equation 76, on the other hand, are "mixed". That is, equation 76 contains both continuous and jump variables. The continuous variables represent the effect on participation value of marginal changes in the underlying asset values. The jump variables represent the effect on participation value of non-marginal changes in the underlying asset values. If these non-marginal changes are the result of idiosyncratic or asset-specific influences, then the jump variables reflect nonsystematic or diversifiable risk.

For a hedge portfolio containing assets which follow continuous processes (and none which follow jump processes), choosing optimal weights $W_j = W_j^*$, $j=1,2,3$, of the participation, the collateral, and the riskless asset, makes the hedge portfolio return nonstochastic (i.e., the coefficients of dz_H and dz_r are always zero). Choosing optimal weights $W_j = W_j^*$, j=1,2, of the participation and the collateral makes the hedge portfolio return equal to the riskless rate. For a two-asset or three-asset hedge portfolio containing assets which follow continuous processes and assets which follow jump processes, optimal weights do not exist to neutralize risk (i.e., make portfolio returns zero for the three-asset portfolio or the riskless rate for the two-asset portfolio). If, however, the set of assets chosen to make up the hedge portfolio is well-diversified (i.e., the jump processes are contemporaneously independent across assets), then the portfolio return will reflect only systematic risk. If the Capital Asset Pricing Model (CAPM) holds as a description of equilibrium returns, then the beta of such a portfolio is theoretically equal to zero suggesting that its return must equal the riskless rate, r. Thus, if the investor follows the hedge strategy and chooses $W_j = W_j^*$ as above, then he or she will earn a zero return "most of the time" as required for the absence of arbitrage. This is true because "most of the time" the Poisson or jump process does not occur. However, on the "rare" occasions when the Poisson event does occur, the investor will earn a comparatively large gain (or loss depending upon the sign of ξ). The hedge portfolio is no longer riskless and cannot have an expected return equal to zero. If, however, the stochastic term dq represents both systematic and nonsystematic risk, then the return on the portfolio will be equal to the riskless rate to the extent that the nonsystematic risk is diversified.

APPENDIX C

FINITE DIFFERENCE APPROXIMATION

The partial derivatives are approximated by Taylor Series expansion, as follows: Consider the variable $Y = \ln(r)$. The Taylor Series expansions of an increase in Y and a decrease in Y are as follows, respectively

$$W(Y+h,Z,t) = W(Y,Z,t) + hW_Y(Y,Z,t) + \tfrac{1}{2}h^2W_{YY}(Y,Z,t) + (1/6)h^3W_{YYY}(Y,Z,t) + \ldots \tag{79}$$

$$W(Y-h,Z,t) = W(Y,Z,t) - hW_Y(Y,Z,t) + \tfrac{1}{2}h^2W_{YY}(Y,Z,t) - (1/6)h^3W_{YYY}(Y,Z,t) + \ldots \tag{80}$$

Subtraction of equation 80 from equation 79 yields (ignoring terms of second order and higher)

$$W(Y+h,Z,t) - W(Y-h,Z,t) = 2hW_Y(Y,Z,t)$$

$$W_Y(Y,Z,t) = (1/2h)\,[W(Y+h,Z,t) - W(Y-h,Z,t)]$$

$$W_Y = (1/2h)(W_{i+1,p,j+1} - W_{i-1,p,j+1}). \tag{81}$$

Addition of equations 79 and 80 yields (ignoring terms of higher than second order)

$$W(Y+h,Z,t) + W(Y-h,Z,t) = 2W(Y,Z,t) + h^2W_{YY}(Y,Z,t)$$

$$W_{YY}(Y,Z,t) = (1/h^2)\,[W(Y+h,Z,t) - 2W(Y,Z,t) + W(Y-h,Z,t)]$$

$$W_{YY} = (1/h^2)(W_{i+1,p,j+1} - 2W_{i,p,j+1} + W_{i-1,p,j+1}). \tag{82}$$

Consider the variable Z. The Taylor Series expansions of an increase in Z and a decrease in Z are as follows, respectively

$$W(Y,Z+q,t) = W(Y,Z,t) + qW_Z(Y,Z,t) + \tfrac{1}{2}q^2W_{ZZ}(Y,Z,t) + (1/6)q^3W_{ZZZ}(Y,Z,t) + \ldots \tag{83}$$

$$W(Y,Z\text{-}q,t) = W(Y,Z,t) - qW_Z(Y,Z,t) + \tfrac{1}{2}q^2W_{ZZ}(Y,Z,t) - (1/6)q^3W_{ZZZ}(Y,Z,t) + \ldots \tag{84}$$

Subtraction of equation 84 from equation 83 yields (ignoring terms of second order and higher)

$$W(Y,Z+q,t) - W(Y,Z\text{-}q,t) = 2qW_Z(Y,Z,t)$$

$$W_Z(Y,Z,t) = (1/2q)\,[W(Y,Z+q,t) - W(Y,Z\text{-}q,t)]$$

$$W_Z = (1/2q)(W_{i,p+1,j+1} - W_{i,p-1,j+1}). \tag{85}$$

Addition of equation 83 and equation 84 yields (ignoring terms of higher than second order)

$$W(Y,Z+q,t) + W(Y,Z\text{-}q,t) = 2W(Y,Z,t) + q^2W_{ZZ}(Y,Z,t)$$

$$W_{ZZ}(Y,Z,t) = (1/q^2)\,[W(Y,Z+q,t) - 2W(Y,Z,t) + W(Y,Z\text{-}q,t)]$$

$$W_{ZZ} = (1/q^2)(W_{i,p+1,j+1} - 2W_{i,p,j+1} + W_{i,p-1,j+1}). \tag{86}$$

The cross partial derivatives may be found in the same manner. The appropriate Taylor Series expansions are as follows

$$W(Y+h,Z+q,t) = W(Y,Z,t) + hW_Y(Y,Z,t) + qW_Z(Y,Z,t) + \tfrac{1}{2}h^2W_{YY}(Y,Z,t) + \tfrac{1}{2}q^2W_{ZZ}(Y,Z,t) + \tfrac{1}{2}hqW_{YZ}(Y,Z,t) + \ldots \tag{87}$$

$$W(Y\text{-}h,Z\text{-}q,t) = W(Y,Z,t) - hW_Y(Y,Z,t) - qW_Z(Y,Z,t) + \tfrac{1}{2}h^2W_{YY}(Y,Z,t) + \tfrac{1}{2}q^2W_{ZZ}(Y,Z,t) + \tfrac{1}{2}hqW_{YZ}(Y,Z,t). \tag{88}$$

Addition of equation 87 and equation 88 yields (ignoring terms of higher than second order)

$$W(Y+h,Z+q,t) + W(Y-h,Z-q,t) = 2W(Y,Z,t) + h^2W_{YY}(Y,Z,t)$$
$$+ q^2W_{ZZ}(Y,Z,t) + hqW_{YZ}(Y,Z,t)$$
$$W_{YZ}(Y,Z,t) = (1/hq) [W(Y+h,Z+q,t) - 2W(Y,Z,t) - h^2W_{YY}(Y,Z,t)$$
$$- q^2W_{ZZ}(Y,Z,t) + W(Y-h,Z-q,t)]$$

$$W_{YZ} = (1/hq)[W_{i+1,p+1,j+1} - 2W_{i,p,j+1} - (1/h^2)(h^2)(W_{i+1,p,j+1}$$
$$- 2W_{i,p,j+1} + W_{i-1,p,j+1}) - (1/q^2)(q^2)(W_{i,p+1,j+1}$$
$$- 2W_{i,p,j+1} + W_{i,p-1,j+1}) + W_{i-1,p-1,j+1}]$$

$$W_{YZ} = (1/hq)(W_{i+1,p+1,j+1} + 2W_{i,p,j+1} - W_{i+1,p,j+1} - W_{i-1,p,j+1}$$
$$- W_{i,p+1,j+1} - W_{i,p-1,j+1} + W_{i-1,p-1,j+1}). \tag{89}$$

Using the same process for the variable t:

$$W(Y,Z,t+n) = W(Y,Z,t) + nW_t(Y,Z,t) + ½n^2W_{tt}(Y,Z,t) +$$
$$(1/6)n^3W_{ttt}(Y,Z,t) + ... \tag{90}$$

Since the explicit method will be moving forward in time, the partial derivative may be approximated by subtracting W(Y,Z,t) from equation 90 to yield (ignoring terms of second order and higher)

$$W(Y,Z,t+n) - W(Y,Z,t) = nW_t(Y,Z,t)$$
$$W_t(Y,Z,t) = (1/n) [W(Y,Z,t+n) - W(Y,Z,t)]$$
$$W_t = (1/n)(W_{i,p,j+1} - W_{i,p,j}) \tag{91}$$

and by the definition of τ

$$W_\tau = -W_t = -(1/n)(W_{i,p,j+1} - W_{i,p,j}) \quad (92)$$

and $$rP = rW_{i,p,j}. \quad (93)$$

These Taylor Series approximations of the partial derivatives may be substituted into the differential equations developed in Chapter III to yield approximations in terms of finite differences. The development begins with the basic differential equation without the jump processes. These can then be easily added.

"Basic" Differential Equation

Recall the "basic" differential equation from Chapter IV and Appendix B:

$$-P_t = P_r\lambda r + P_H\alpha H + ½P_{rr}\sigma_r^2r^2 + P_{rH}\sigma_r\sigma_H Hr\rho_{rH} + ½P_{HH}\sigma_H^2H^2 - rP. \quad (94)$$

Substituting for the approximations of the partial derivatives:

$$-W_t = W_r\lambda r + W_H\alpha H + ½W_{rr}\sigma_r^2r^2 + W_{rH}\sigma^r\sigma_H Hr\rho_{rH} + ½W_{HH}\sigma_H^2H^2 - rW \quad (95)$$

$$\begin{aligned}
-(1/n)(W_{i,p,j+1} - W_{i,p,j}) = {} & (\lambda - ½\sigma_r^2)(1/2h)(W_{i+1,p,j+1} - W_{i-1,p,j+1}) \\
& + (\alpha - ½\sigma_H^2)(1/2q)(W_{i,p+1,j+1} - W_{i,p-1,j+1}) \\
& + ½\sigma_r^2(1/h^2)(W_{i+1,p,j+1} - 2W_{i,p,j+1} + W_{i-1,p,j+1}) \\
& + \sigma_r\sigma_H\rho_{rH}(1/hq)(W_{i+1,p+1,j+1} + 2W_{i,p,j+1} - W_{i+1,p,j+1} \\
& - W_{i-1,p,j+1} - W_{i,p+1,j+1} - W_{i,p-1,j+1} + W_{i-1,p-1,j+1}) \\
& + ½\sigma_H^2(1/q^2)(W_{i,p+1,j+1} - 2W_{i,p,j+1} + W_{i,p-1,j+1}) - rW_{i,p,j} \quad (96)
\end{aligned}$$

$$\begin{aligned}
(1/n)(W_{i,p,j}) = {} & (\lambda - ½\sigma_r^2)(1/2h)(W_{i+1,p,j+1} - W_{i-1,p,j+1}) \\
& + (\alpha - ½\sigma_H^2)(1/2q)(W_{i,p+1,j+1} - W_{i,p-1,j+1}) \\
& + ½\sigma_r^2(1/h^2)(W_{i+1,p,j+1} - 2W_{i,p,j+1} + W_{i-1,p,j+1}) \\
& + \sigma_r\sigma_H\rho_{rH}(1/hq)(W_{i+1,p+1,j+1} + 2W_{i,p,j+1} - W_{i+1,p,j+1}
\end{aligned}$$

$$
\begin{aligned}
&- W_{i-1,p,j+1} - W_{i,p+1,j+1} - W_{i,p-1,j+1} + W_{i-1,p-1,j+1}) \\
&+ ½\sigma_H^2(1/q^2)(W_{i,p+1,j+1} - 2W_{i,p,j+1} + W_{i,p-1,j+1}) \\
&- rW_{i,p,j} + (1/n)(W_{i,p,j+1})
\end{aligned}
$$

$$
\begin{aligned}
(1 + rn)W_{i,p,j} = {} & W_{i-1,p,j+1}[-(\lambda-½\sigma_r^2)(1/2h) + (½\sigma_r^2 r)(1/h^2) - (\sigma_r\sigma_H\rho_{rH})(1/hq)]n \\
&+ W_{i,p,j+1}[-2(½\sigma_r^2)(1/h^2) + 2(\sigma_r\sigma_H\rho_{rH})(1/hq) - 2(½\sigma_H^2)(1/q^2) + (1/n)]n \\
&+ W_{i+1,p,j+1}[(\lambda-½\sigma_r^2)(1/2h) + ½(\sigma_r^2)(1/h^2) - (\sigma_r\sigma_H\rho_{rH})(1/hq)]n \\
&+ W_{i,p-1,j+1}[-(\alpha-½\sigma_H^2)(1/2q) - (\sigma_r\sigma_H\rho_{rH})(1/hq) + ½(\sigma_H^2)(1/q^2)]n \\
&+ W_{i,p+1,j+1}[(\alpha-½\sigma_H^2)(1/2q) - (\sigma_r\sigma_H\rho_{rH})(1/hq) + ½(\sigma_H^2)(1/q^2)]n \\
&+ W_{i+1,p+1,j+1}[\sigma_r\sigma_H\rho_{rH})(1/hq)]n + W_{i-1,p-1,j+1}[\sigma_r\sigma_H\rho_{rH})(1/hq)]n
\end{aligned}
$$

$$
\begin{aligned}
W_{i,p,j}[1 + rn] = {} & aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\
&+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\
&+ gW_{i-1,p-1,j+1}
\end{aligned}
$$

$$
\begin{aligned}
W_{i,p,j} = {} & [1/(1 + rn)][aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\
&+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\
&+ gW_{i-1,p-1,j+1}]
\end{aligned}
\tag{97}
$$

where

$$a = [-(\lambda-\tfrac{1}{2}\sigma_r^2)(1/2h) + (\tfrac{1}{2}\sigma_r^2)(1/h^2) - (\sigma_r\sigma_H\rho_{rH})(1/hq)]n$$

$$b = [-\sigma_r^2(1/h^2) + 2(\sigma_r\sigma_H\rho_{rH})(1/hq) - \sigma_H^2(1/q^2) + (1/n)]n$$

$$c = [(\lambda-\tfrac{1}{2}\sigma_r^2)(1/2h) + \tfrac{1}{2}(\sigma_r^2)(1/h^2) - (\sigma_r\sigma_H\rho_{rH})(1/hq)]n$$

$$d = [-(\alpha-\tfrac{1}{2}\sigma_H^2)(1/2q) - (\sigma_r\sigma_H\rho_{rH})(1/hq) + \tfrac{1}{2}(\sigma_H^2)(1/q^2)]n$$

$$e = [(\alpha-\tfrac{1}{2}\sigma_H^2)(1/2q) - (\sigma_r\sigma_H\rho_{rH})(1/hq) + \tfrac{1}{2}(\sigma_H^2)(1/q^2)]n$$

$$f = [\sigma_r\sigma_H\rho_{rH})(1/hq)]n$$

$$g = [\sigma_r\sigma_H\rho_{rH})(1/hq)]n.$$

Equation 97 was stated earlier in Chapter IV as equation 29.

APPENDIX D

FINITE DIFFERENCE APPROXIMATION OF PARTIAL DIFFERENTIAL EQUATIONS

In Chapter IV, finite difference approximations were formulated for the valuation models developed in Chapter III. The valuation models were in the form of partial differential equations whose solutions were the loan sale values. No closed form solutions for the equations has been found, hence, the need for approximations. This appendix is included as a brief tutorial over the finite difference approximation method and as a review of studies relevant to this research which have used the method to approximate the solution to partial differential equations.

With the publication and testing of their now-famous option pricing model, Black and Scholes (1972, 1973) not only revolutionized the field of options pricing theory, but also opened a whole new area of study in Finance -- the area of contingent claims analysis. Central to the development of the Black-Scholes Option Pricing Model (BSOPM) is the recognition that the value of a financial option is uniquely determined by the value of the stock underlying that option. That is, the option value is contingent upon the value of the stock and may be modeled as such.

The valuation rule of the BSOPM is in the form of a partial differential equation. That is, the value of the option is modeled as a function of the partial derivatives of that value with respect to the stock value on which it is contingent. A partial differential equation valuation rule is common to all contingent claims analyses. In fact, Merton has identified a Fundamental Partial Differential Equation of Security Pricing, as follows:

$$0 = \tfrac{1}{2}\sigma^2 V^2 F_{VV} + rVF_V - F_\tau - rF \tag{98}$$

where
- F = the value of the security being priced
- V = the value of the underlying asset
- r = the instantaneous riskless rate of interest
- τ = time to maturity
- σ^2 = the variance of r.

and subscripts denote partial derivatives (Merton 1990, 378). This equation is adaptable to various valuation problems and will contain additional terms as the particular security being priced is contingent upon additional underlying assets. However, it is frequently the case that a solution to the partial differential equation has not been found and an approximation must be used. A number of numerical approximation methods are available, including finite differences, Monte Carlo simulation, the method of lines, numerical integration, etc. Brennan and Schwartz (1978) demonstrated that finite difference methods are especially adaptable for pricing contingent claims when the underlying stochastic processes include Poisson or jump variables.

It was recognized by Black and Scholes that their model had applications reaching far beyond option valuation to the general area of corporate liability valuation. A large number of applications have been developed and contingent claims analysis continues to be a rich area of research.[15] Merton (1974) argued that

[15]For an extensive listing of contingent claims studies in the area of corporate liabilities, see Merton (1974). Also, Chan, Karolyi, Longstaff, and Sanders (1992) list a number of studies according to the particular stochastic process used to model the instantaneous riskless rate of interest.

the impact of contingent claims analysis on the field of Finance can be traced to four critical elements:

1. the relatively weak assumptions required for its valid application;
2. the observability or ready estimation of the variables and parameters required as inputs;
3. the computational feasibility of solving the partial differential equations; and
4. the generality of the methodology which permits adaptation to a wide range of finance applications.

The Finite Difference Method

An analytical solution of a partial differential equation is a function of the contingent variables which satisfies the equation at every point in the domains of the contingent variables. Only a limited number of analytical solutions are available, so often solutions are found by an approximation method. A frequently used method, and one well-suited to the types of stochastic processes (i.e., mixed diffusion and jump) of this research is finite differences.

Finite difference approximation is performed in a time-space hyperplane where the space dimension is determined by the number of stochastic variables included in the equation. In the case of equations 18 and 19 , Chapter III, the space dimension is two since two stochastic variables are included -- the instantaneous riskless rate (r) and collateral value (H). The hyperplane is divided into a mesh or lattice. For equations 18 and 19, the mesh is determined by:

$r = i\Delta r = ih$ for the riskless rate
$H = p\Delta H = pq$ for the collateral value
$t = j\Delta t = jn$ for time.

The size of the mesh results from $\Delta r = h$, $\Delta H = q$, and $\Delta t = n$. The actual approximation process, then, relies upon discrete estimates of the continuous partial derivatives at each point in the mesh. The process introduces discretization error at each point in the mesh. The convergence of the process toward a solution and the stability of that solution are integral to the approximation process.

"Convergence" refers to the convergence of the solution of the approximating difference equations to the solution of the actual differential equation as the size of the net or mesh approaches zero. Discretization error approaches zero as the mesh approaches zero in a convergent finite difference solution.

"Stability" refers to the growth of the discretization error as it is carried through the approximation process for a given mesh size. A stable numerical process should limit the amplification of rounding errors through the process.

The size of the mesh, that is, the sizes of Δr, ΔH, and Δt, are chosen as part of the finite differencing application. These so-called step sizes for time and the state space variables must be chosen to ensure a stable convergence to a solution.

The discrete estimates of the partial derivatives result from analyzing the changes in the value of the security being priced for small changes in time or in the stochastic variables in the state space. This analysis may be accomplished in many ways, however, the most common ways are by using forward, central, and backward differences. Using the notation from Appendix C, these differences are illustrated with respect to the rate variable as follows:

forward difference $W_r \approx (1/n)(W_{i+1,p,j} - W_{i,p,j})$

backward difference $W_r \approx (1/n)(W_{i,p,j} - W_{i-1,p,j})$

central difference $W_r \approx (1/2n)(W_{i+1,p,j} - W_{i-1,p,j})$.

These difference equations result from Taylor Series expansions as demonstrated in Appendix C. In the same way, difference equations may be developed for partial derivatives with respect to each of the stochastic state variables and with respect to time.

The solution of the partial differential equation using this method and these difference equations is an iterative process. The time variable is set at an initial value and the difference equations are evaluated according to the initial boundary conditions of the differential equation. Then the time variable is incremented and the difference equations are evaluated from the values at the previous time step. The process is repeated until the desired time increment is reached. In other words, at a given time increment, each difference equation is a function of difference equations at the previous time increment. Equation 97 (Appendix C) illustrates this process and is reproduced below.

$$\begin{aligned} W_{i,p,j} = [1/(1 + rn)][&aW_{i-1,p,j+1} + bW_{i,p,j+1} + cW_{i+1,p,j+1} \\ &+ dW_{i,p-1,j+1} + eW_{i,p+1,j+1} + fW_{i+1,p+1,j+1} \\ &+ gW_{i-1,p-1,j+1}]. \end{aligned} \tag{97}$$

Note that the value at time j is a function of a number of values at time j+1. Likewise, the value at time j-1 is a function of a number of values at time j, and so forth. This process is the essence of finite difference approximation.

It is important to note that the process may proceed either forward or backward in time. That is, time may be incremented from the present forward toward maturity of the security being priced, or it may be incremented from maturity backward toward the present. The forward incrementing process is referred to as "implicit" and the backward incrementing process is referred to as "explicit". Equation 97 illustrates an explicit or backward process. For a simple, one-dimensional state space problem, the two types of processes may be illustrated as in Figure 29. Geske and Shastri (1985) point out that the Binomial Option Pricing Model is actually a special application of an explicit finite differencing scheme.

Relevant Studies

The valuation models of this research are similar to the models developed for the valuation of Government National Mortgage Association (GNMA) Pass-Through Securities. Several GNMA valuation studies are briefly reviewed in Chapter II and repeated below. In addition, several residential mortgage valuation studies are briefly reviewed below. Each of the studies reviewed has followed the contingent claims analysis approach to define partial differential equations as valuation rules.

Schwartz and Torous (1992)

This study developed a valuation rule for GNMA Pass-Through Securities. The authors characterized the value of a Pass-Through Security as being contingent upon the instantaneous riskless rate, the value of the home used as collateral, and the prepayment and default decisions of the borrowers in the underlying mortgages. The partial differential equation developed as a valuation rule is as follows:

$$0 = \tfrac{1}{2}\sigma_r^2 rV_{rr} + \tfrac{1}{2}\sigma_H^2 H^2 V_{HH} + \sigma_H \sigma_r \rho_{Hr} H\sqrt{r}V_{rH} + [k(m-r)+\lambda r]V_r + (r-b)HV_H - rV + \xi(r,H,t) + V_t$$

where

	V	=	the value of the Pass-Through
	r	=	the instantaneous riskless rate
	σ_r^2	=	variance of the riskless rate
	H	=	the value of the collateral
	σ_H^2	=	variance of collateral value
	t	=	time
	b	=	the housing payout rate
	λ	=	the market price of risk
	$\xi(\bullet)$	=	the jump processes for prepayment and default
	k,m	=	coefficients of the Cox, Ingersoll, and Ross (1985) mean-reverting process for r.

The solution to the equation was approximated using the implicit method of finite differences subject to the terminal condition: $V(r,H,0) = 0$.

<u>Kau, Keenan, Muller, and Epperson (1992)</u>

This study developed a valuation rule for fixed-rate residential mortgages. The values of the mortgages were characterized as being contingent upon the instantaneous riskless rate and the value of the home used as collateral. Note that default and prepayment were not specifically addressed here as in Schwartz and Torous (1992). The partial differential equation developed is as follows:

$$0 = \tfrac{1}{2}\sigma_r^2 rV_{rr} + \tfrac{1}{2}\sigma_H^2 H^2 V_{HH} + \sigma_H \sigma_r \rho_{Hr} H\sqrt{r}V_{rH} + k(m-r)V_r + (r-b)HV_H - rV + V_t$$

where

	V	=	the value of the fixed-rate mortgage
	r	=	the instantaneous riskless rate
	σ_r^2	=	variance of the riskless rate
	H	=	the value of the collateral

σ_H^2	=	variance of collateral value
t	=	time
b	=	the housing payout rate (i.e., a "service flow" resulting from use of the house over time)
k,m	=	coefficients of the Cox, Ingersoll, and Ross (1985) mean-reverting process for r.

The solution to the equation was approximated using the explicit method of finite differences subject to the terminal condition:

$$V(H,r,t) = \min(F,H)$$

where F is the outstanding principal at time t.

<u>Titman and Torous (1989)</u>

This study presents a model for pricing non-amortizing commercial mortgages. The authors assume that uncertainty about the mortgage value can be summarized by two state variables -- the instantaneous riskless rate and the value of the mortgaged building. The following partial differential equation was developed as the valuation rule:

$$0 = \tfrac{1}{2}\sigma_r^2 rV_{rr} + \tfrac{1}{2}\sigma_H^2 H^2 V_{HH} + \sigma_H\sigma_r\rho_{Hr}H\sqrt{r}V_{rH} + [k(m-r)+\lambda r]V_r + (r-b)HV_H$$
$$- rV + \xi(r,H,t) + V_t + p$$

where			
	V	=	the value of the Pass-Through
	r	=	the instantaneous riskless rate
	σ_r^2	=	variance of the riskless rate
	H	=	the value of the collateral
	σ_H^2	=	variance of collateral value
	t	=	time
	b	=	the housing payout rate
	λ	=	the market price of risk
	$\xi(\bullet)$	=	the jump processes for prepayment and default
	k,m	=	coefficients of the Cox, Ingersoll, and Ross (1985) mean-reverting process for r

p = the continuous rate of mortgage payment.

The solution to the equation was approximated using the explicit method of finite differences subject to the following boundary conditions:

$$V(H,r,0) = \min(F,H)$$
$$V(,r,t) = H(t)$$
$$V(H,,t) = 0.$$

Brennan and Schwartz (1985)

This study developed a valuation rule for GNMA pass-through securities. The authors assumed value to be contingent upon the short-term riskless rate (r) and the consol rate (l). The consol rate was defined as the yield on a security whose maturity is infinite. The partial differential equation developed as a valuation rule is as follows:

$$\tfrac{1}{2}\sigma_r^2 V_{rr} + \rho_{r,l}\sigma_r\sigma_l V_{rl} + \tfrac{1}{2}\sigma_l^2 V_{ll} + V_r(\beta_r - \lambda_r\sigma_r) + V_l(\beta_l - \lambda_l\sigma_l) + V_t + C - rB = 0$$

where V = value of GNMA pass-through

$\rho_{r,l}$ = correlation coefficient of r with l

β_r = trend in r

β_l = trend in l

σ_r^2 = instantaneous variance of r

σ_l^2 = instantaneous variance of l

λ_l = market price of risk associated with l

λ_r = market price of risk associated with r.

The equation was solved using numerical methods and the following boundary conditions:

$$V(r,l,T) = 0$$
$$V(r,l,r) < \infty.$$

Dunn and McConnell (1981b)

This study developed a valuation rule for GNMA Pass-Through Securities. The authors characterized the value of a Pass-Through Security as being contingent upon the instantaneous riskless rate and prepayments made by the borrowers in the underlying mortgages. The partial differential equation developed as a valuation rule is as follows:

$$0 = \tfrac{1}{2}\sigma_r^2 r V_{rr} + [k(m-r)+\lambda r]V_r - rV + \xi(r,t) + V_t + C(t)$$

where

V	=	the value of the Pass-Through
r	=	the instantaneous riskless rate
σ_r^2	=	variance of the riskless rate
t	=	time
$C(t)$	=	continuous cash flows from the GNMA
λ	=	the market price of risk
$\xi(\bullet)$	=	the jump process for prepayment
k,m	=	coefficients of the mean-reverting process for r.

The solution to the equation was approximated using the implicit method of finite differences subject to the following boundary conditions:

$$V(r,t) = 0$$
$$V(,t) = 0$$
$$V(,0) \leq F$$

where F is the remaining principal outstanding.

TABLE I

PREPAYMENTS FOR GNMAS IN YEARS 1979-1988

		Refinancing Rate Range				
Coupon	Statistic	8.5-9.5%	9.5-10.5%	10.5-11.5%	11.5-12.5%	12.5-13.5%
7.5	Avg. CPR (%)*	6.2	4.9	3.2	4.3	2.8
	Std. Dev (%)	4.5	3.7	2.8	1.6	1.3
8.0	Avg. CPR (%)*	4.7	3.7	2.4	4.4	3.0
	Std. Dev (%)	4.5	3.4	2.5	1.3	1.2
8.5	Avg. CPR (%)*	1.9	2.4	1.6	4.7	3.3
	Std. Dev (%)	3.4	2.4	1.8	2.2	2.0
9.0	Avg. CPR (%)*	2.3	3.2	2.2	3.9	2.6
	Std. Dev (%)	2.9	2.3	1.6	1.2	1.0
9.5	Avg. CPR (%)*	4.5	3.9	2.9	3.6	2.5
	Std. Dev (%)	4.1	3.1	2.0	1.0	0.8
10.0	Avg. CPR (%)*	7.5	4.6	3.6	3.4	2.3
	Std. Dev (%)	4.5	3.5	2.2	2.2	1.5
10.5	Avg. CPR (%)*	18.8	7.1	4.3	2.3	1.6
	Std. Dev (%)	11.1	5.3	3.6	3.6	2.9
11.0	Avg. CPR (%)*	27.2	13.7	7.6	3.5	3.2
	Std. Dev (%)	9.0	5.4	3.6	2.6	1.6
11.5	Avg. CPR (%)*	38.3	25.3	12.5	5.0	2.8
	Std. Dev (%)	8.0	9.6	4.7	2.6	1.6
12.0	Avg. CPR (%)*	41.1	30.3	15.3	4.2	2.4
	Std. Dev (%)	6.4	10.1	5.4	3.1	2.0
12.5	Avg. CPR (%)*	41.8	34.9	18.3	8.2	4.1
	Std. Dev (%)	6.8	11.6	5.6	3.8	3.1
13.0	Avg. CPR (%)*	41.1	38.2	22.5	12.2	4.7
	Std. Dev (%)	7.8	12.7	7.2	5.8	3.7

source: Richard, S., and R. Roll, "Prepayments on Fixed-Rate Mortgage-Backed Securities", Journal of Portfolio Management, Spring 1989, 80.

*CPR = Cumulative average Prepayment Rate for the particular coupon category and for each refinancing rate range

TABLE II

COMMERCIAL LOAN REPAYMENT AND DEFAULT RATES

Broadly Syndicated Highly Leveraged Loans Originated in 1987 and 1988			
Repaid	24.16%	36.00%	62.97%
Outstanding			
Current	68.28%	53.00%	17.82%
Technical Default*	7.56%	7.00%	13.00%
Payment Default	0.00%	4.00%	6.21%

source: Miller, S., "Bank Loans in a Bond Market Context: Nominal Yields and Default Patterns", in J. Carlson and F. Fabozzi, ed.'s, The Trading and Securitization of Senior Bank Loans, (Probus Publishing Company, Chicago, IL), 1992, 226.

*Loans are in violation of covenants but still pay interest to banks
Sample loans: large corporate HLTs of borrowers with sales over $250 million originated in 1987 and 1988.
Sample size: 71 transactions with total dollar volume of $36 billion
Source: Loan Pricing Corporation (Loan Investor Services)
Date: April 1991

TABLE III

INDIVIDUAL LOAN DEFAULT RATES

Rating	Historic 10-Year Default Rates	Historic 10-Year Default Rates Without Special Events
Aaa	1.0%	0.0%
Aa1	1.2	
Aa2	1.4	0.9
Aa3	1.5	(Aa1 to Aa3)
A1	1.7	
A2	1.8	1.1
A3	2.3	(A1 to A3)
Baa1	3.5	
Baa2	4.4	3.4
Baa3	7.5	(Baa1 to Baa3)
Ba1	11.9	
Ba2	16.1	
Ba3	20.6	
B1	25.9	
B2	31.6	
B3	39.6	

source: Moses, L., "Rating Debt Securitized by Bank Loans", in J. Carlson and F. Fabozzi, ed.'s, <u>The Trading and Securitization of Senior Bank Loans</u>, (Probus Publishing Company, Chicago, IL), 1992, 226-7.

TABLE IV

IMPLIED BORROWER BOND RATING

Implied Rating	Necessary Criteria
Caa	Borrower or affiliate is not in reorganization or other insolvency proceedings
	Debt is not in default
Ba	Caa criteria
	Borrower or affiliate has not defaulted on any debt for the past two years
	Borrower has been in business for past 5 years
	Borrower is current on any cumulative preferred dividends
	Fixed-charge ratio* exceeds 1.25 times in each of the past two fiscal years and in the most recent quarter
	Borrower had a net profit before tax in both the past fiscal year and most recent quarter
	Annual financial statements are unqualified and certified by a nationally accredited accounting firm, and quarterly statements are unaudited but signed by a corporate officer

source: Moses, L., "Rating Debt Securitized by Bank Loans", in J. Carlson and F. Fabozzi, ed.'s, The Trading and Securitization of Senior Bank Loans, (Probus Publishing Company, Chicago, IL), 1992, page 198.

*The fixed-charge ratio is defined as earnings before interest, taxes, depreciation and amortization divided by total fixed charges including debt service. It is similar to an interest coverage ratio.

TABLE V

LOAN SALES VALUES, SECURED LOANS: NO EXPLICIT RECOURSE AND IMPLICIT RECOURSE (FULL AND PARTIAL)

default rate	0.018	market price of risk	0.247
prepayment rate	0.05	time to maturity	2 years
risk-free rate	0.05	maturity value	$1,000.00
var of risk-free rate (σ^2_r)	0.006	correl of r,H ($\rho_{r,H}$)	0.00
collat value growth rate (α)	0.01	implicit recourse (μ)	0.50
var of collat value (σ^2_H)	0.006		

These parameter values hold for the table below except as noted.

Market Factors

riskless rate:	0.04	0.05	0.06	0.07	0.08
sale value:	903.0435	886.8183	870.9640	855.4723	840.3222
variance of risk-free rate (σ^2_r):	0.006	0.010	0.050	0.100	0.150
sale value:	886.8183	886.8422	887.0811	887.3798	887.6786
market price of risk (λ):	0	0.247	0.500	0.750	1.00
sale value:	889.7271	886.8183	883.8475	880.9205	878.0021

Loan-Specific Factors

trend in collateral value (α):	-0.50	-0.01	0.00	0.01	0.50
sale value:	907.2837	887.6228	887.2206	886.8183	867.1544
variance of collateral value (σ^2_H):	0.006	0.010	0.050	0.100	0.150
sale value:	886.8183	887.0594	889.4365	892.3223	895.1145
correlation of risk-free rate and collateral value ($\rho_{r,H}$):	-1.00	-0.50	0.00	0.50	1.00
sale value:	886.8892	886.8538	886.8183	886.7828	886.7473
mat'y (yrs):	1/2	1	1 1/2	2	2 1/2
sale value:	970.0188	941.1734	913.4463	886.8183	861.2689

default rate:	0.200	0.150	0.100	0.050	0.018
sale value:	712.4268	753.5232	799.3961	850.7314	886.8183
prepay rate:	0.200	0.150	0.100	0.050	0.018
riskless rate:	0.08	0.07	0.06	0.05	0.04
sale value:	864.0458	870.5116	878.0328	886.8183	882.5014

Recourse Factors

probability of implicit rec:	0.000	0.250	0.500	0.750	0.900
sale value:	879.0117	882.9150	886.8183	890.7216	893.0635
degree of implicit rec:	0.000	0.250	0.500	0.750	0.900
sale value:	879.0117	882.9150	886.8183	890.7216	893.0635

TABLE VI

LOAN SALES VALUES, SECURED LOANS: WITH EXPLICIT RECOURSE (FULL AND PARTIAL)

default rate	0.018	level of recourse (%)	50.0
prepayment rate	0.05	time to maturity	2 years
risk-free rate	0.05	maturity value	$1,000.00
var of risk-free rate (σ^2_r)	0.006	correl of r,H ($\rho_{r,H}$)	0.00
collat value growth rate (α)	0.01	mkt price of risk (λ)	0.247
var of collateral value (σ^2_H)	0.006		

These parameter values hold for the table below except as noted.

Market Factors

riskless rate:	0.04	0.05	0.06	0.07	0.08
sale value:	910.9407	894.6248	878.6819	863.1033	847.8789
variance of risk-free rate (σ^2_r):	0.006	0.010	0.050	0.100	0.150
sale value:	894.6248	894.6488	894.8886	895.1884	894.4883
market price of risk (λ):	0	0.247	0.500	0.750	1.00
sale value:	897.5444	894.6248	891.6431	888.7053	885.7761

Loan-Specific Factors

trend in collateral value (α):	-0.50	-0.01	0.00	0.01	0.50
sale value:	913.6755	895.3738	894.9993	894.6248	876.3204
variance of collateral value (σ^2_H):	0.006	0.010	0.050	0.100	0.150
sale value:	894.6248	894.8493	897.0621	899.7484	902.3481
correlation of risk-free rate and collateral value ($\rho_{r,H}$):	-1.00	-0.50	0.00	0.50	1.00
sale value:	894.6960	894.6604	894.6248	894.5892	894.5536
time to maturity in years:	1/2	1	1 1/2	2	2 1/2
sale value:	972.1435	945.3037	919.4879	894.6248	870.7567

default rate:	0.200	0.150	0.100	0.050	0.018
sale value:	783.7433	809.8304	838.9833	871.6445	894.6248
prepay rate:	0.200	0.150	0.100	0.050	0.018
riskless rate:	0.08	0.07	0.06	0.05	0.04
sale value:	870.4858	877.3541	885.3314	894.6248	907.5095

Recourse Factors

level of recourse(%):	0.10	25	50	75	100
sale value:	879.0430	886.8183	894.6248	902.4314	910.2379

TABLE VII.

SIGNS OF COMPARATIVE STATICS CURRENT STUDY COMPARED TO SIMILAR STUDIES

Parameter	Sign of Partial Derivative C	S	K	T	D
Market Factors					
riskless rate (r)	-	-	-	-	-
variance of r (σ_r^2)	+	n.r.	n.r.	n.r.	+
risk price (λ)	-	n.r.	n.r.	n.r.	-
Loan-Specific Factors					
collat trend (α)	-	+,-*	-	n.r.	n.r.
variance of H (σ_H^2)	+	n.r.	+	+	n.r.
corr of r,H (ρ_{rH})	-	n.r.	-	-	n.r.
maturity (τ)	-	-	-	-	-
default prob (δ)	-	+,-*	n.r.	n.r.	n.r.
prepay prob (π)	-	-	n.r.	n.r.	n.r.
Recourse Factors					
prob of implicit rec (μ)	+	n.a.	n.a.	n.a.	n.a.
degree of implicit rec (ζ)	+	n.a.	n.a.	n.a.	n.a.
explicit rec (γ)	+	n.a.	n.a.	n.a.	n.a.

*For high interest rates and low collateral (mortgaged house) values, the GNMA holder desires default as default triggers payment by the insurer (page 233). Note that in the current study, default does not necessarily trigger full payment by the guarantor, or even payment at all.

n.r. = not reported in the study cited
n.a. = not applicable to the study cited

C = Collier (1994) -- current study
S = Schwartz & Torous (1992) -- GNMA Pass-Through Security valuation
K = Kau, Keenan, Muller, and Epperson (1992) -- fixed-rate residential mortgage valuation
T = Titman and Torous (1989) -- nonamortizing commercial mortgage valuation
D = Dunn and McConnell (1981b) -- GNMA Pass-Through Security valuation

TABLE VIII.

INTERPRETATION OF DIFFERENCE CURVES SHOWN IN FIGURES 4-12

Parameter	Figure	Sign of Comparative Statics	Slope of Curve	Model Showing More Sensitivity
		Market Factors		
r	4	-	-	100% Explicit
σ_r^2	5	+	+	100% Explicit
λ	6	-	-	100% Explicit
		Loan-Specific Factors		
α	7	-	+	0% Explicit
σ_H^2	8	+	-	0% Explicit
ρ_{rH}	9	-	-	100% Explicit
τ	10	-	+	0% Explicit
δ	11	-	+	0% Explicit
π	12	-	-	100% Explicit

TABLE IX.

INTERPRETATION OF DIFFERENCE CURVES SHOWN IN FIGURES 13-21

Parameter	Figure	Sign of Comparative Statics	Slope of Curve	Model Showing More Sensitivity
		Market Factors		
r	13	-	-	50% Explicit
σ_r^2	14	+	+	50% Explicit
λ	15	-	-	50% Explicit
		Loan-Specific Factors		
α	16	-	+	50% Implicit
σ_H^2	17	+	-	50% Implicit
ρ_{rH}	18	-	-	50% Explicit
τ	19	-	+	50% Implicit
δ	20	-	+	50% Implicit
π	21	-	-	50% Explicit

TABLE X.

INTERPRETATION OF DIFFERENCE CURVES SHOWN IN FIGURES 22-24

Parameter	Figure	Sign of Comparative Statics	Slope of Curve	Model Showing More Sensitivity
		Market Factors		
r	22	-	-	50% Implicit
σ_r^2	23	+	+	50% Implicit
λ	24	-	-	50% Implicit

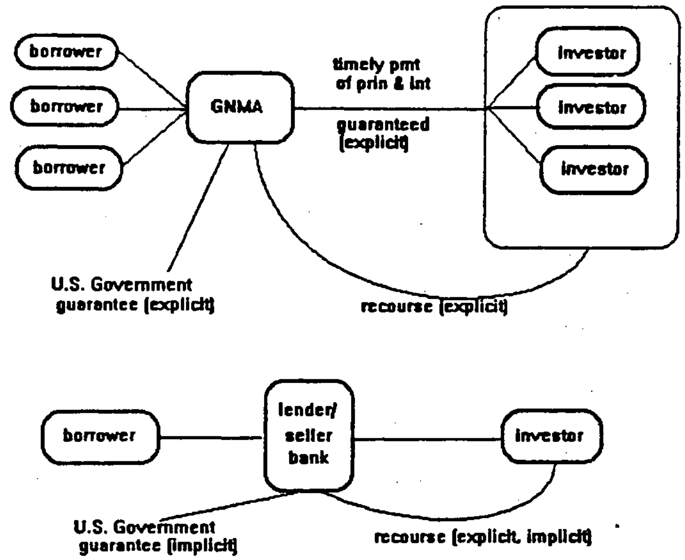


Figure 1. Comparison of GNMA Pass-Through Process with Loan Participation Process

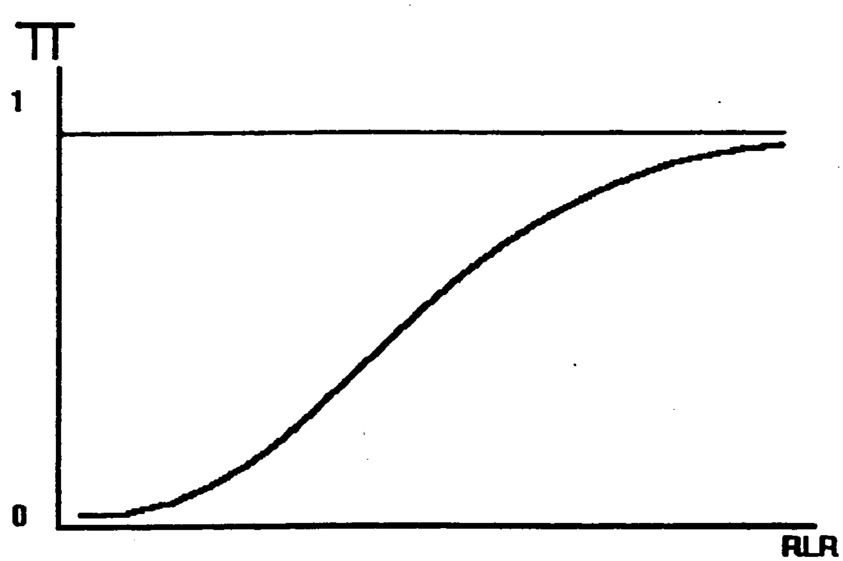


Figure 2. Suggested Sigmoid Form of Prepayment Function

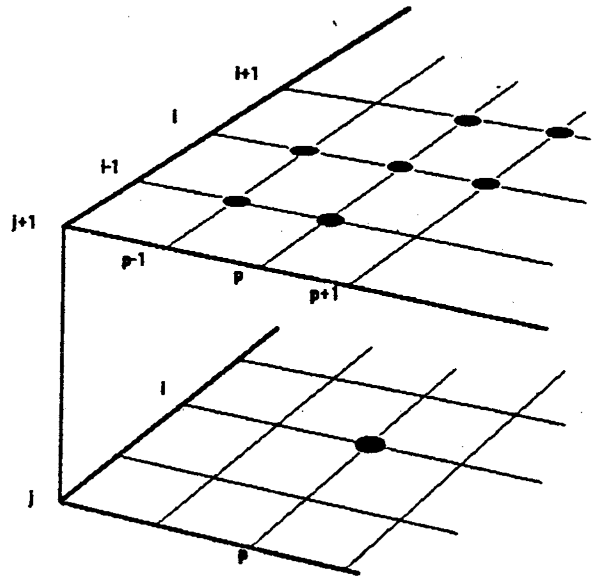


Figure 3. Graphical Depiction of the Process of the Explicit Numerical Method Developed in Chapter IV

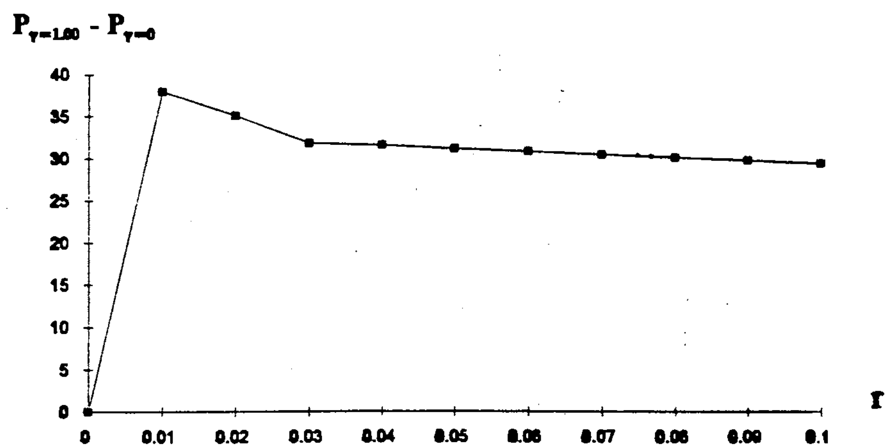


Figure 4. Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Riskless Rate (r)

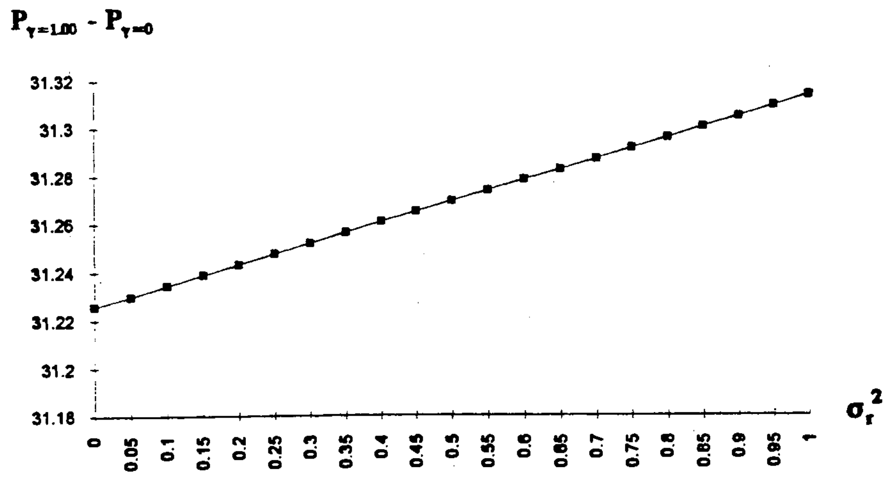


Figure 5. Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Variance of the Riskless Rate (σ_r^2)

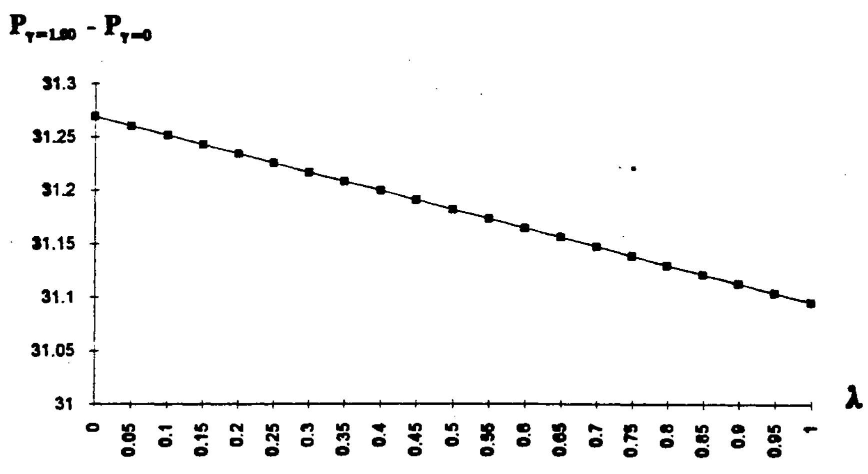


Figure 6. Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Market Price of Risk (λ)

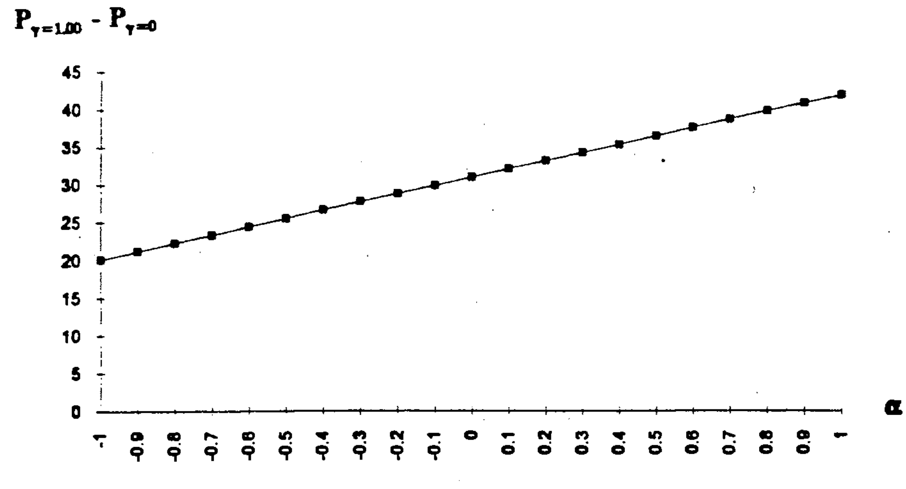


Figure 7. **Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Trend in Collateral Value (α)**

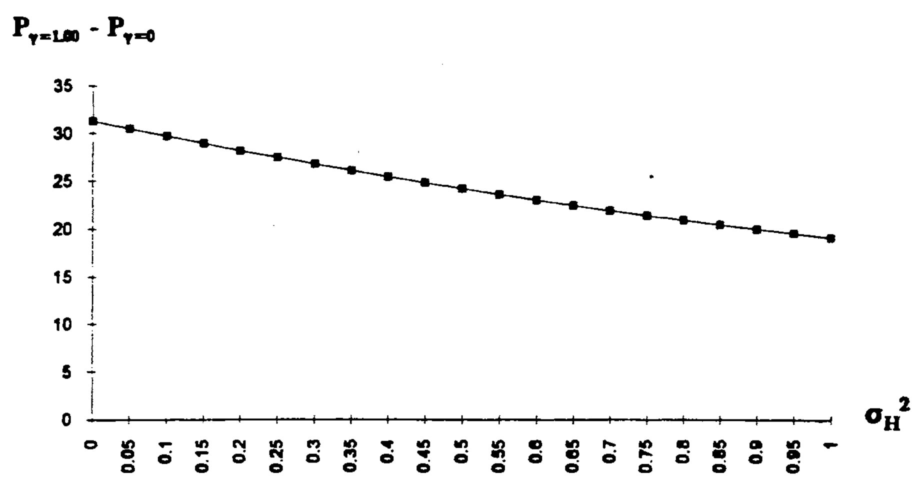


Figure 8. **Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Variance of Collateral Value (σ_H^2)**

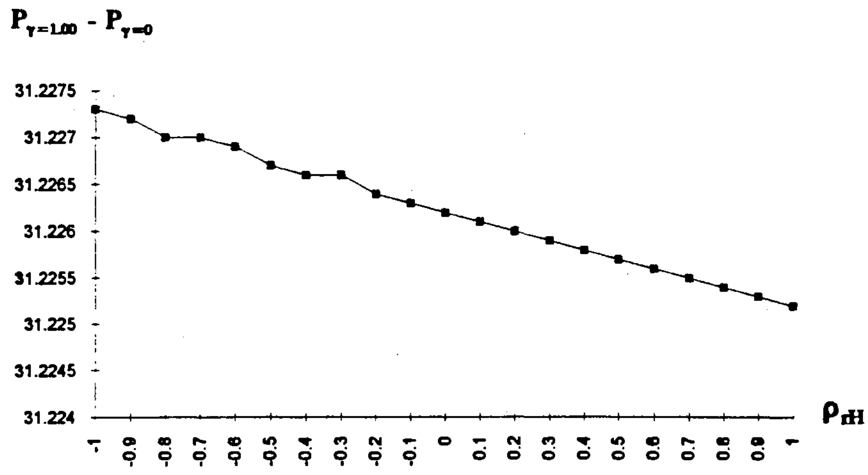


Figure 9. Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Correlation Between the Riskless Rate and Collateral Value (ρ_{rH})

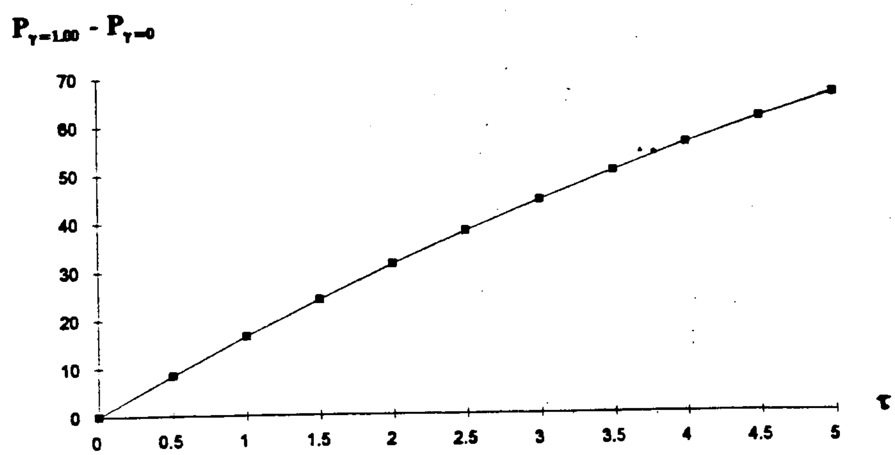


Figure 10. Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Time to Maturity (τ)

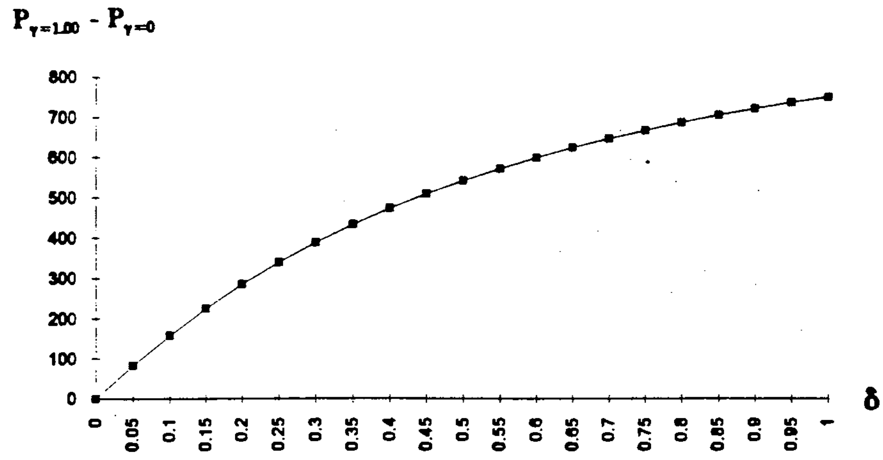


Figure 11. **Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Default Rate (δ)**

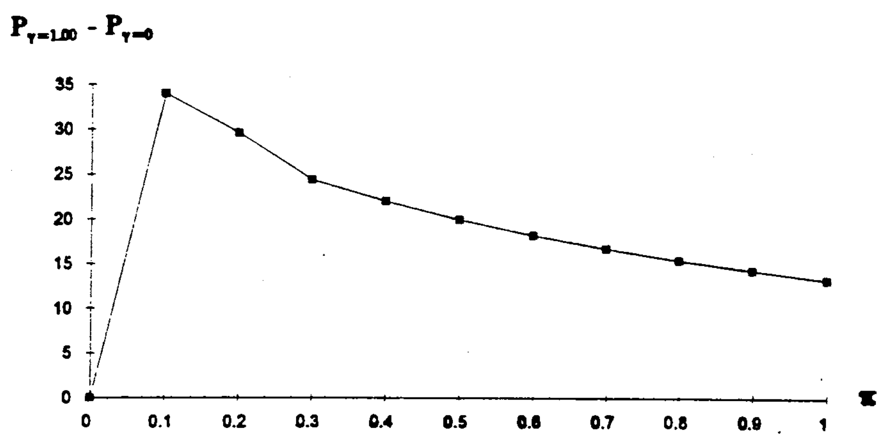


Figure 12. **Valuation Differences Between the 100% Explicit Recourse Model and the 0% Explicit Recourse Model in Response to Changes in the Prepayment Rate (π)**

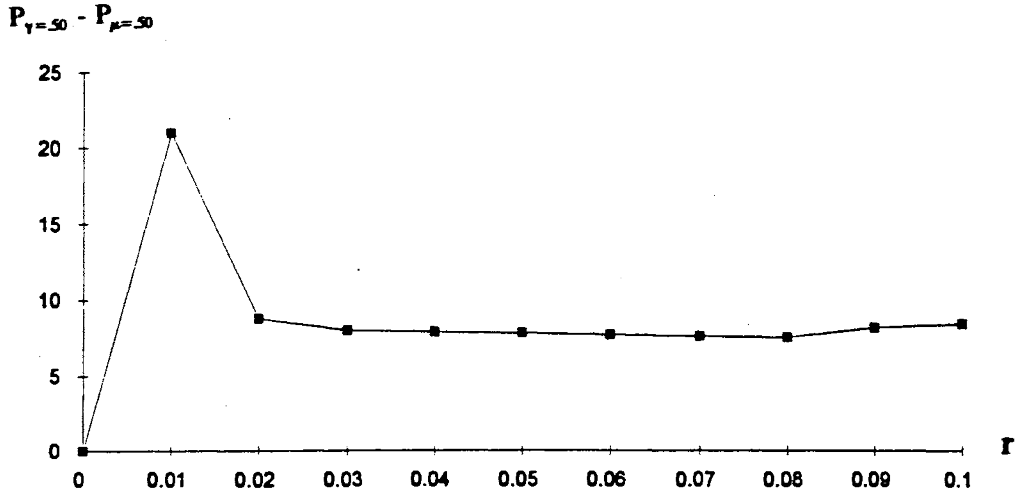


Figure 13. Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Riskless Rate (r)

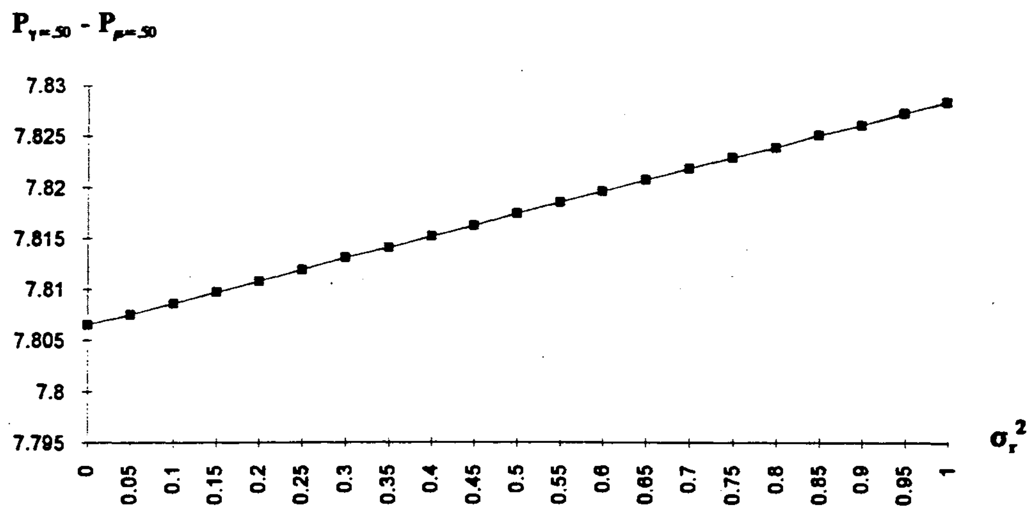


Figure 14. Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Variance of the Riskless Rate (σ_r^2)

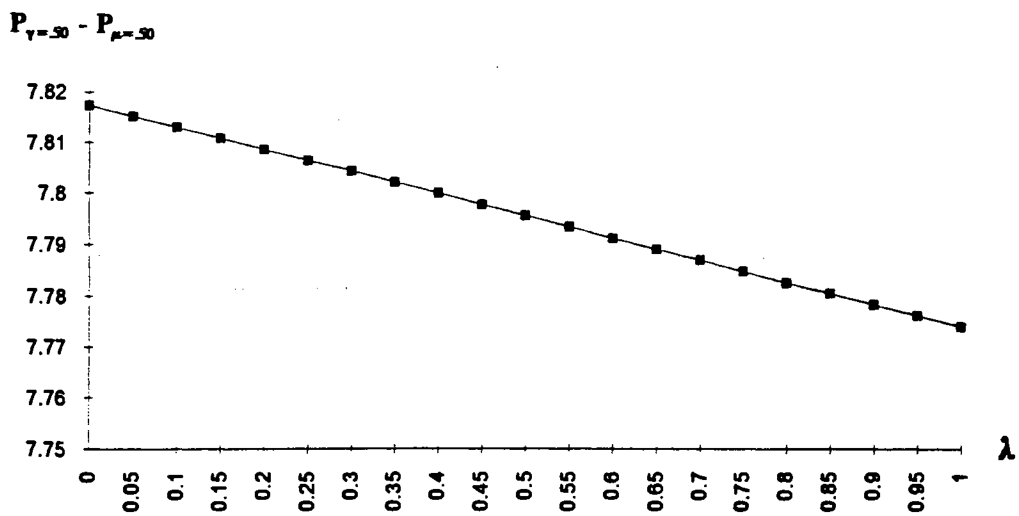


Figure 15. **Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Market Price of Risk (λ)**

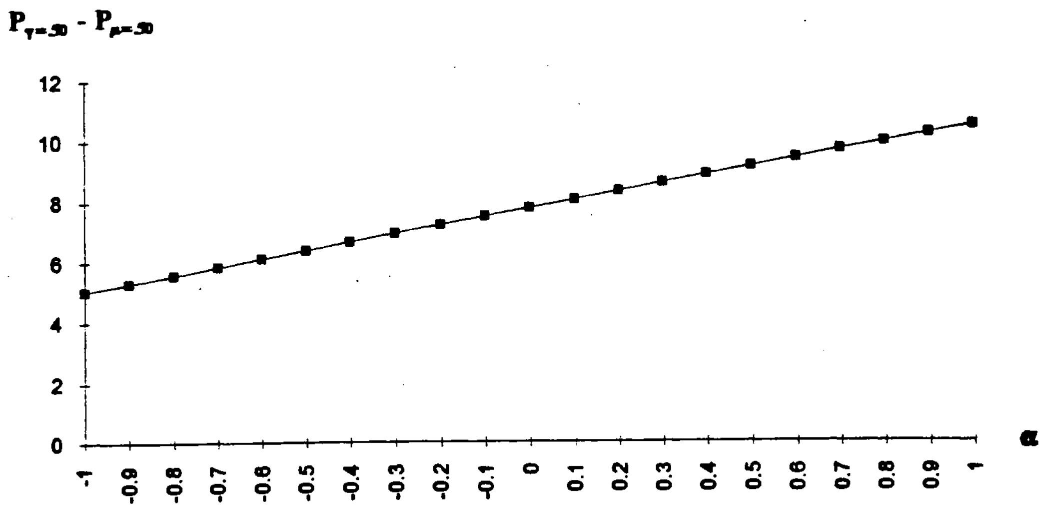


Figure 16. **Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Trend in Collateral Value (α)**

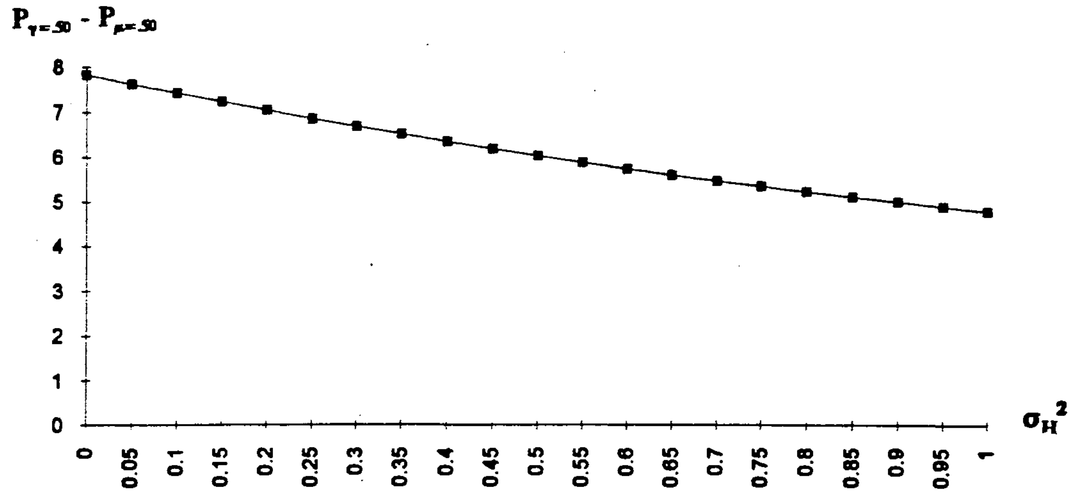


Figure 17. **Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Variance of Collateral Value (σ_H^2)**

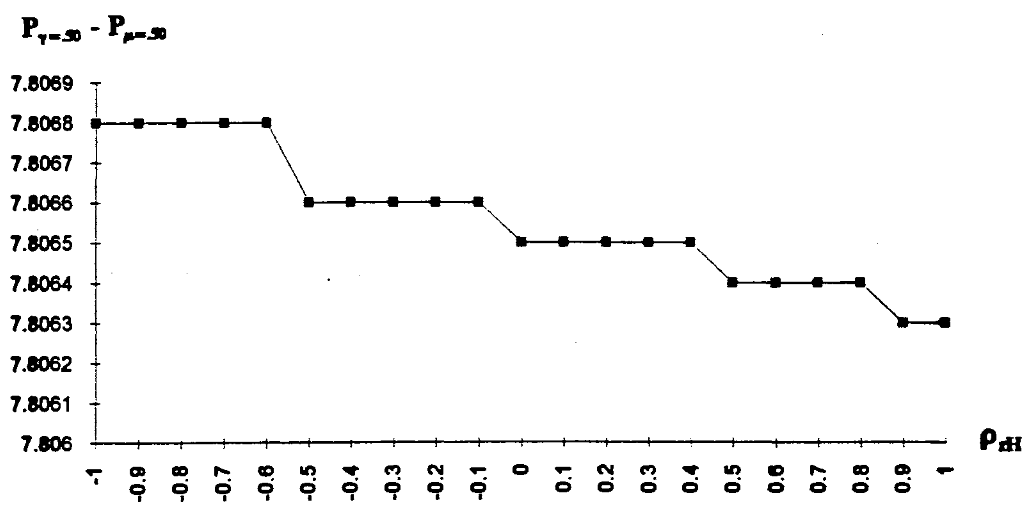


Figure 18. **Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Correlation Between the Riskless Rate and Collateral Value (ρ_{rH})**

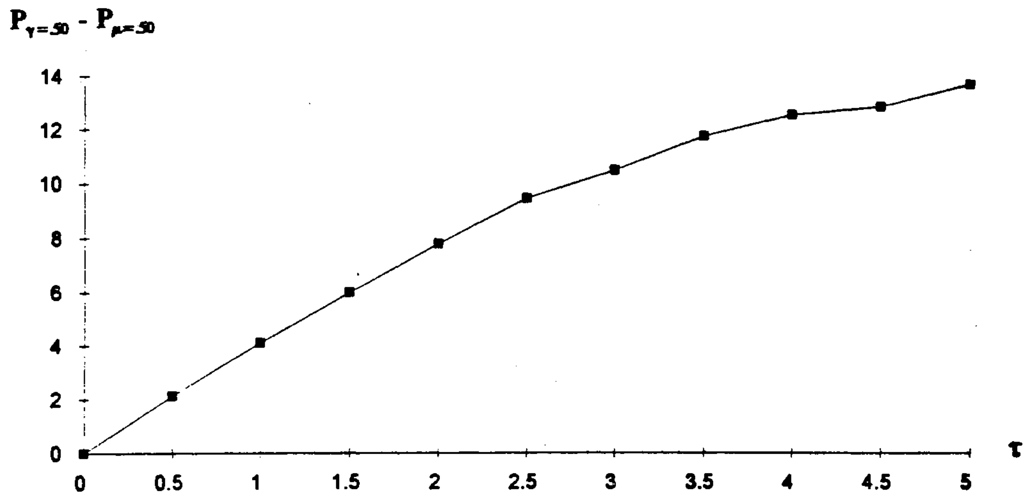


Figure 19. Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Time to Maturity (τ)

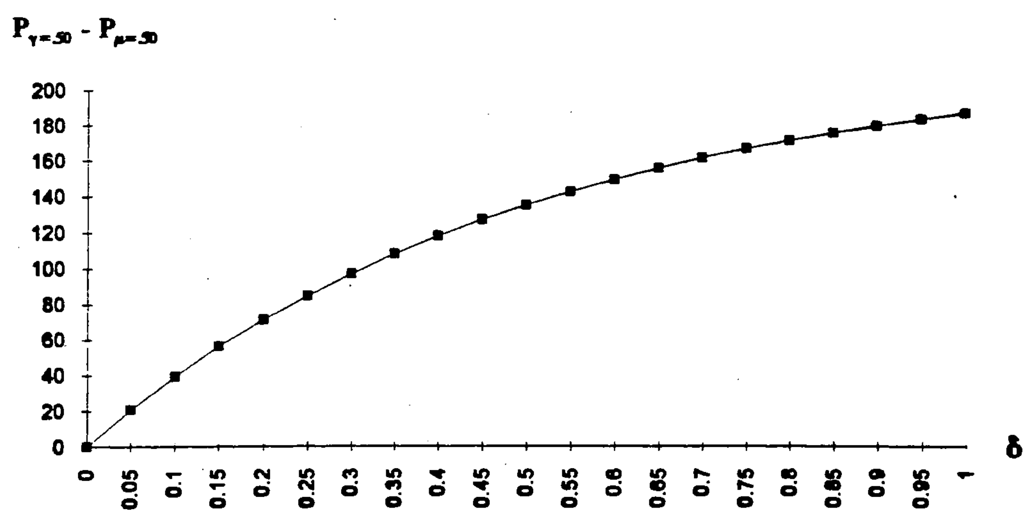


Figure 20. Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Default Rate (δ)

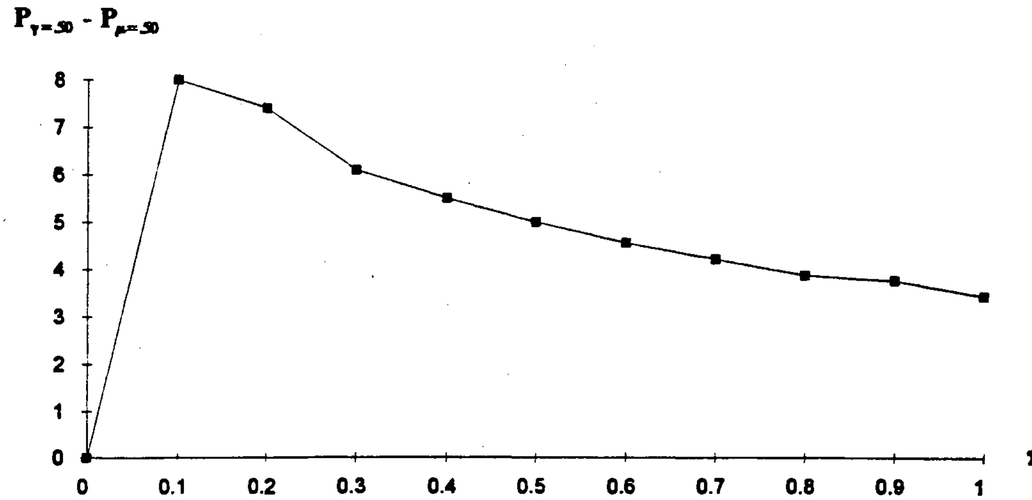


Figure 21. **Valuation Differences Between the 50% Explicit Recourse Model and the 50% Implicit Recourse Model in Response to Changes in the Prepayment Rate (π)**

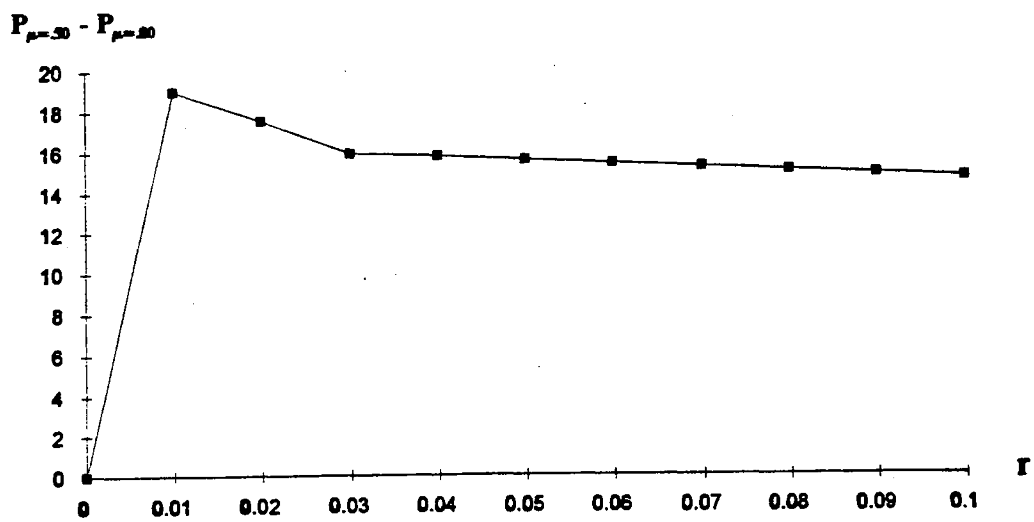


Figure 22. **Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Riskless Rate (r)**

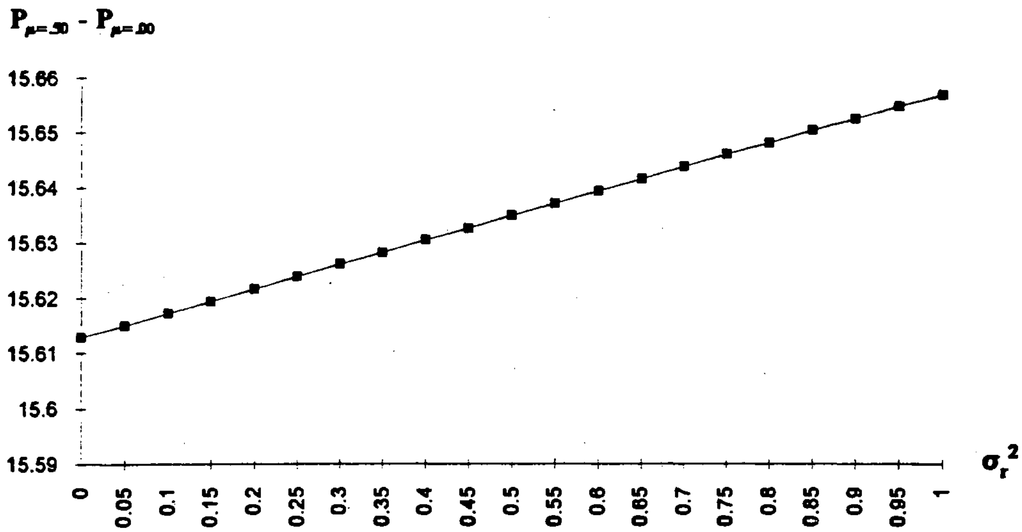


Figure 23. **Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Variance in the Riskless Rate (σ_r^2)**

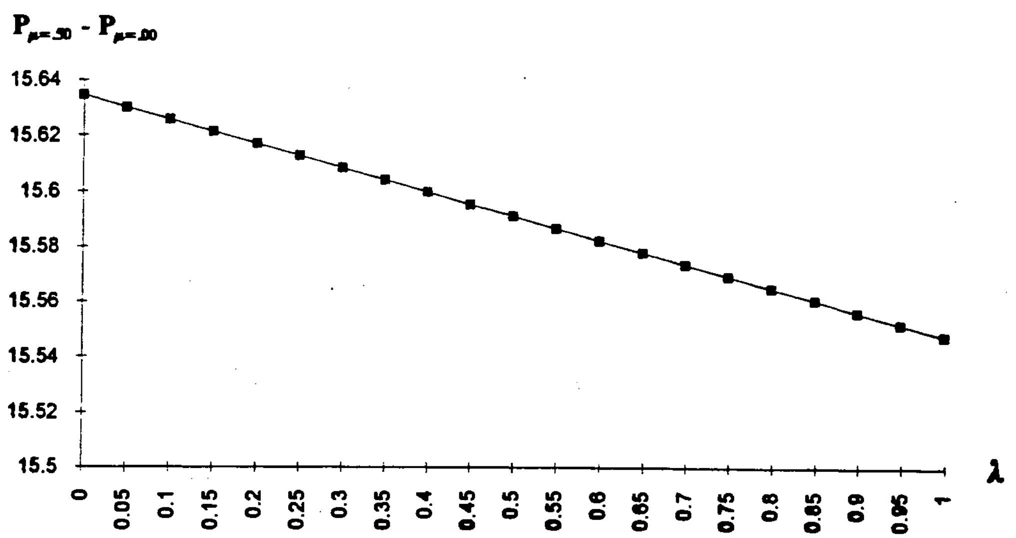


Figure 24. **Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Market Price of Risk (λ)**

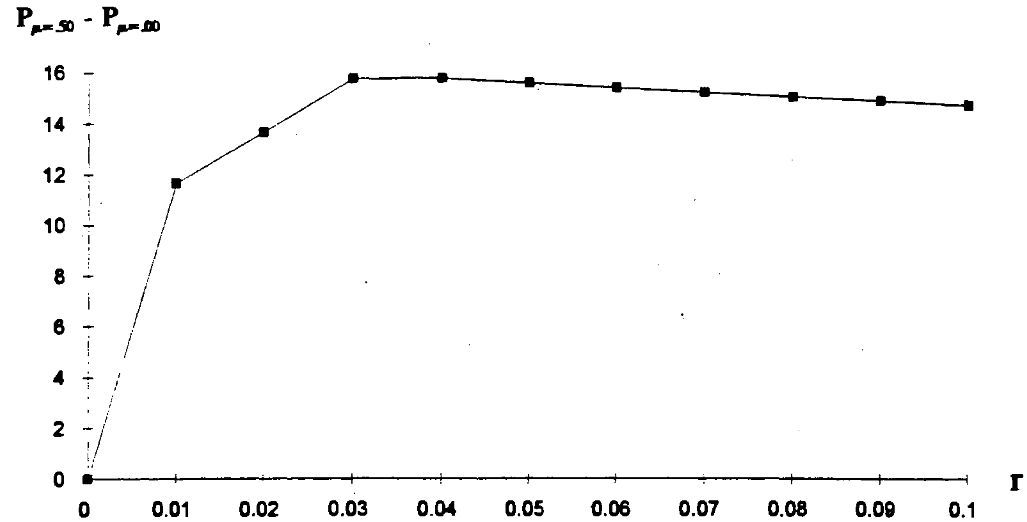


Figure 25. Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Riskless Rate (r) Where $\sigma_r^2 = 0.500$

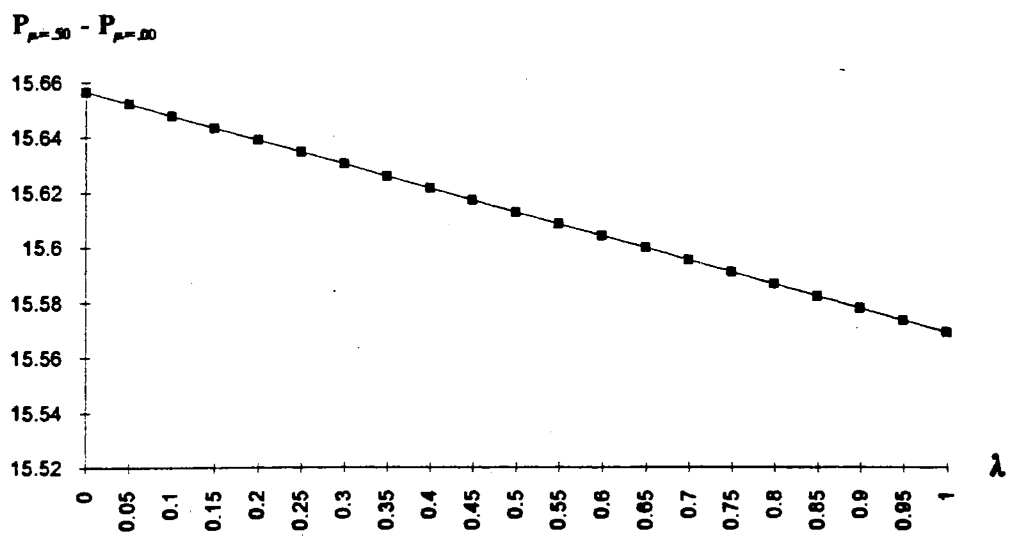


Figure 26. Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Market Price of Risk (λ) Where $\sigma_r^2 = 0.500$

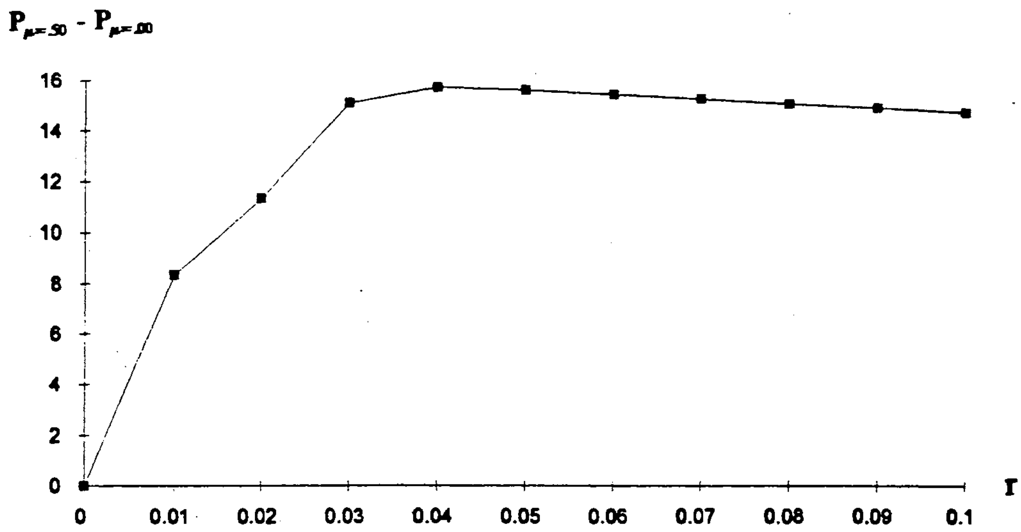


Figure 27. Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Riskless Rate (r) Where $\sigma_r^2 = 0.900$

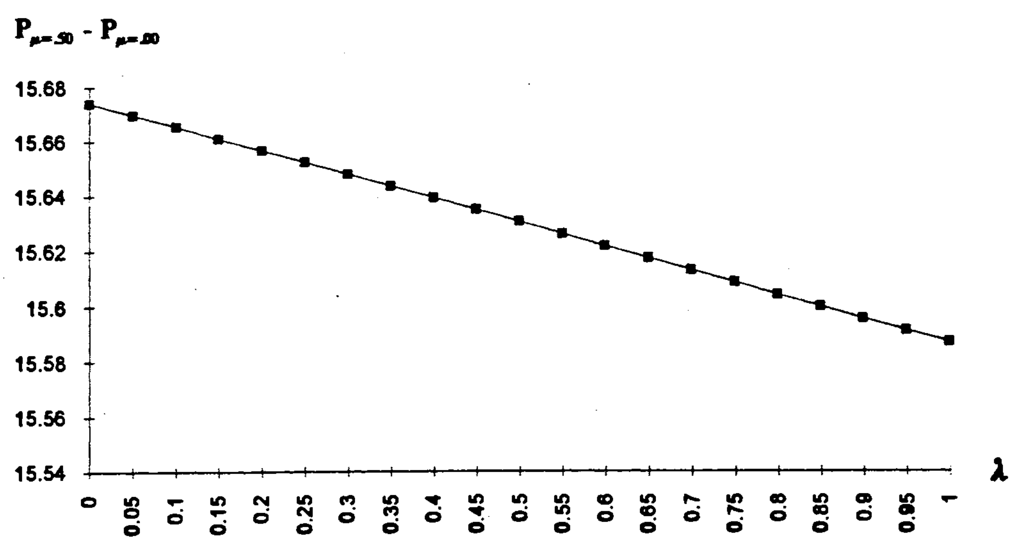


Figure 28. Valuation Differences Between the 50% Implicit Recourse Model and the 0% Implicit Recourse Model in Response to Changes in the Market Price of Risk (λ) Where $\sigma_r^2 = 0.900$

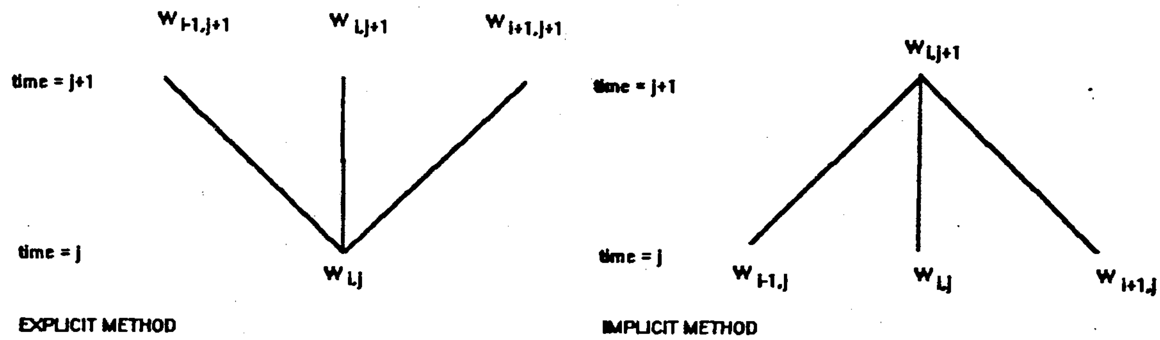


Figure 29. Graphical Depiction of the General Processes of the Explicit and Implicit Numerical Methods Described in Appendix D

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