ECONOMIC ANALYSIS OF EXTERNAL

QUALITY LOSSES

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NOMENCLATURE

$\mathbf{A}$	Cost associated to an undesired product characteristic's deviation Δ around the target
$\mathbf{A_L}$	Cost associated to an undesired product characteristic's deviation Δ_L (on the lower side) around the target
ANIB	A type of loss minimization objective: Asymmetric Nominal-is-better
$\mathbf{A_U}$	Cost associated to an undesired product characteristic's deviation Δ_U (on the upper side) around the target
$\mathbf{b_y}$	Bias (relative to its target) of quality characteristic $\mathbf{y}$
$\mathbf{b_y(t)}$	A quality characteristic's mean bias as a function of time
$\mathbf{E[L(y)]}$	Expected loss associated to the quality characteristic $\mathbf{y}$
$\mathbf{E[L(y;t)]}$	Expected loss associated to $\mathbf{y}$, as a function of time
$\mathbf{f_y(y)}$	Probability density function of $\mathbf{y}$
$\mathbf{F_y(y)}$	Cumulative probability function of $\mathbf{y}$
HIBFT	A type of loss minimization objective: Higher-is-better with a finite target
HIBUT	A type of loss minimization objective: Higher-is-better with an undetermined target
$\mathbf{K}$	A positive definite matrix that represents the losses incurred when $\mathbf{Y}$ deviates from τ in the multidimensional TLF
$\mathbf{k}$	Constant in the TLF, equals $\mathbf{A/\Delta^2}$
$\mathbf{k_L}$	Constant in the TLF, equals $\mathbf{A/\Delta_L{}^2}$
$\mathbf{k_U}$	Constant in the TLF, equals $\mathbf{A/\Delta_U{}^2}$
LIB	A type of loss minimization objective: Lower-is-better

L(y) Loss function of a quality characteristic **y**

L(y) Loss function of **y** as a function of time

L(Y) Loss function of a set of quality characteristics **Y**

L[Y(t)] Loss function of **Y** as a function of time

L[Y(X;t)] Loss function of a set of quality characteristics **Y** as a function of a set of process parameters **X** and of time

PW_L Present worth of expected external quality losses

PWL_M Present worth of expected external quality losses due to mean/bias

PWL_V Present worth of expected external quality losses due to variance(s)

r Discount rate

SNIB A type of loss minimization objective: Symmetric Nominal-is-better

TLF Taguchi Loss Function, equivalent to a quadratic loss function

t Time

X Vector representing process parameters

X_M Process parameters that affect only the mean of a product characteristic

X_V Process parameters that affect only the variance of a product characteristic

X_{MV} Process parameter that affect both the mean and the variance of a product characteristic

X_O Process parameters non-relevant for a product's quality

y A generic product characteristic

Y A set of **p** product characteristics

Y(X) A set of **p** product characteristics as a function of a set of process parameters **X**

GREEK SYMBOLS

Δ	Magnitude of a product characteristic's deviation around its target
Δ_L	Magnitude of a product characteristic's deviation around its target's lower side
Δ_U	Magnitude of a product characteristic's deviation around its target's upper side
μ_y	Mean of the product characteristic y
μ_L	Partial mean: conditional mean of y provided that $y < \tau$
μ_U	Partial mean: conditional mean of y provided that $y > \tau$
$\mu_y(t)$	Mean of y as a function of time
μ_Y	Mean vector of Y
$\mu_Y(X)$	Mean vector of Y as a function of X
$\Sigma(t)$	Variance-covariance matrix of Y as a function of time
$\Sigma(X)$	Variance-covariance matrix of Y as a function of X
$\sigma_y{}^2$	Variance of product characteristic y
$\sigma_y{}^2(t)$	Variance of product characteristic y as a function of time
σ_{ij}	Covariance between product characteristics i and j
$\sigma_{ij}(t)$	Covariance between product characteristics i and j as a function of time
τ	A quality characteristic target (desired value) or a vector of targets (multidimensional case)

CHAPTER I

THE RESEARCH PROBLEM

1.1 INTRODUCTION

Industrial competitiveness demands from managers of productive systems an increasing awareness of quality, as seen by the customers. This reality has been steadily gaining recognition, as evidenced by the longest-tenured automotive chief executive in the world (20 years), Eberhard von Kuenheim, chairman of the board of BMW AG. Speaking at Quality Forum VII, von Kuenheim said: "...quality is a multifaceted criterion for judging a product, and consequently, the manufacturer" (Bemowski, 1991). Von Kuenheim's statement pinpoints two fundamental aspects of quality that are relevant to this research: (1) a complex, multidimensional nature, and (2) strategic importance, the latter mainly due to the fact that a product's quality eventually affects its manufacturer.

Since a manufacturer's decisions that affect a product's quality ultimately revert back to them, it is important to know: (1) the impact of the product's quality performance on its users, and (2) the effect of these decisions on the product's cost of manufacturing. In this regard, it is important for the manufacturer to be able to answer questions such as:

- How can a product's quality performance be modeled?
- How does quality performance determine External Quality Losses, EQL (losses incurred by the product's user)?
- What is the effect of the mean and variability of the product's features on EQL?
- How can quality performance degradation be included in the analysis of EQL?
- If there are interdependencies among product characteristics that influence EQL, how can they be handled?
- How can the cost of external quality losses for a product's user be estimated?
- What is the influence on EQL of financial factors, such as discount rate and planning horizon?
- Can the parameters of the production process affecting EQL be identified? Can their effects be estimated?
- For alternative settings of the process parameters, how can the combination that minimizes EQL be determined? How can the combination that minimizes the sum of EQL and costs of manufacturing be determined?
- How robust is this combination of process parameters to changes in financial factors, in product's features, and/or process parameters?

This research develops the concepts and tools to perform the economic analysis of EQL, which allows one to address the above set of questions.

This chapter discusses the conceptual framework to be used in this research (Section 1.2), defines the role of external quality losses within this framework (Section 1.3), describes the linkage between product quality and its production process (Section 1.4), and identifies the objective and subobjectives of this research. Section 1.6 shows the different tasks that constitute this research.

1.2 CONCEPTUAL FRAMEWORK

In this research, the process of manufacturing a product, the production process, is conceived as the transformation of a set of inputs into a set of outputs. The process inputs correspond to a set **X** of different production factors, known as process parameters. The outputs, on the other hand, are related to product characteristics, or the set **Y** of a product's features that determine the product quality performance.

The manufacturer has the ability to affect a product's quality performance by intervening on the production process. In this research, any deliberate intervention on the process is called design. In this context, design is referred to as one of two kinds of activities: (1) design of a new process, and (2) intervention on an existing process. Any design activity affects both the process and the product. However, according to the way design materializes, the intervention might emphasize (Figure 1.1) either: (1) Process Design, or intervention

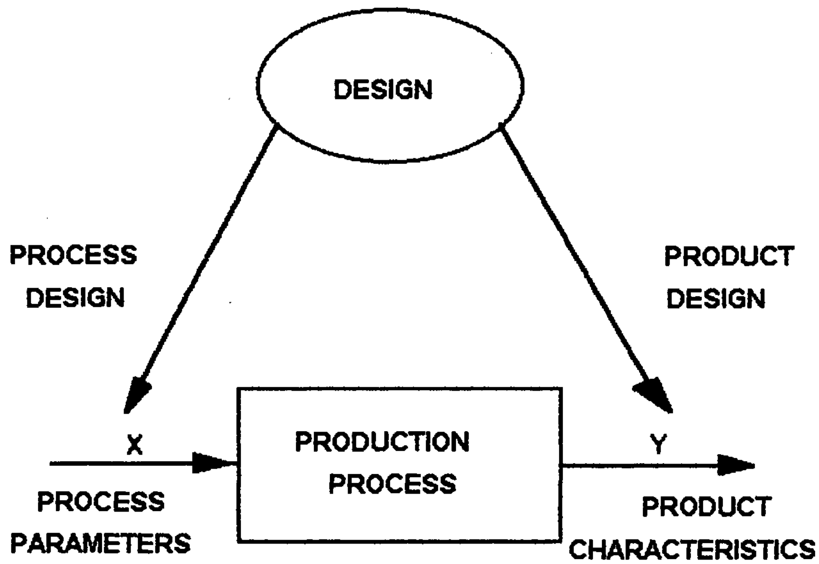

FIG. 1.1 DESIGN FOR QUALITY, PROCESS AND PRODUCT

centered on the process parameters, and (2) Product design, or intervention focused on the product's characteristics.

Recognizing the possible different levels of intervention, Taguchi (1986, 1987) introduces a general methodology for design, which consists of three steps: (1) system design, (2) parameter design, and (3) tolerance design. When Taguchi's methodology is applied for process design, its three steps are outlined as follows:

1. System Design: to determine the basic configuration of the production process (Mayer and Benjamin, 1992). It consists of the selection of the set of process parameters **X** which characterize the process, such as the types of machinery, materials, production methods, and the profile of man-power required to move a workpiece from partial completion to a more advanced stage of completion (Taguchi, et al., 1989).
2. Parameter Design: the operating levels (values) of the process parameters defined in the previous step are selected in order to minimize product variation, while still hitting some desired target.
3. Tolerance Design: to determine the allowance ranges for changes in operating conditions of process parameters. It consists of specifying tolerances, or allowed deviations, on process parameters, in relation to the levels determined during the parameter design step. Tolerance design is not a factor in this research.

As for product design, the three steps can be described as:

1. System Design: denotes the development of a basic product prototype design that performs the desired and required functions of the product. (Taguchi, et al., 1989).

2. Parameter Design: is aimed at determining the optimal levels for the different product characteristics so that quality is optimized. For assembly products, this step also focus on the features of the different product components as to guarantee the optimum performance of the whole product.

3. Tolerance Design: defines allowable ranges for the different product characteristics.

Under either of the two types of design (process design or product design), the relevance of design for quality, as discussed in Section 1.1, reaches beyond the walls of the manufacturing organization.

Juran equates a product's quality to its "fitness for use (Juran and Gryna, 1993)." The level of such a fitness determines the product quality performance, which can be expressed in terms of the product characteristics. However, as a consequence of product use by the user, the levels of the different product characteristics change with time, and so then does the product quality performance. Information on quality performance, as well as on its change over time, represent a crucial input for design activities. Figure 1.2 extends an overall framework that includes all of the concepts discussed so far. It also points out the scope of this research.

Three different types of cash flow implications can be associated with the conceptual framework discussed in Figure 1.2:

- Incremental Manufacturing Cost (IMC), is the set of incremental expenses incurred by the manufacturer due to differences from one design (process and/or product) to another. Some examples include incremental costs due to alternative process equipment and/or alternative product materials.

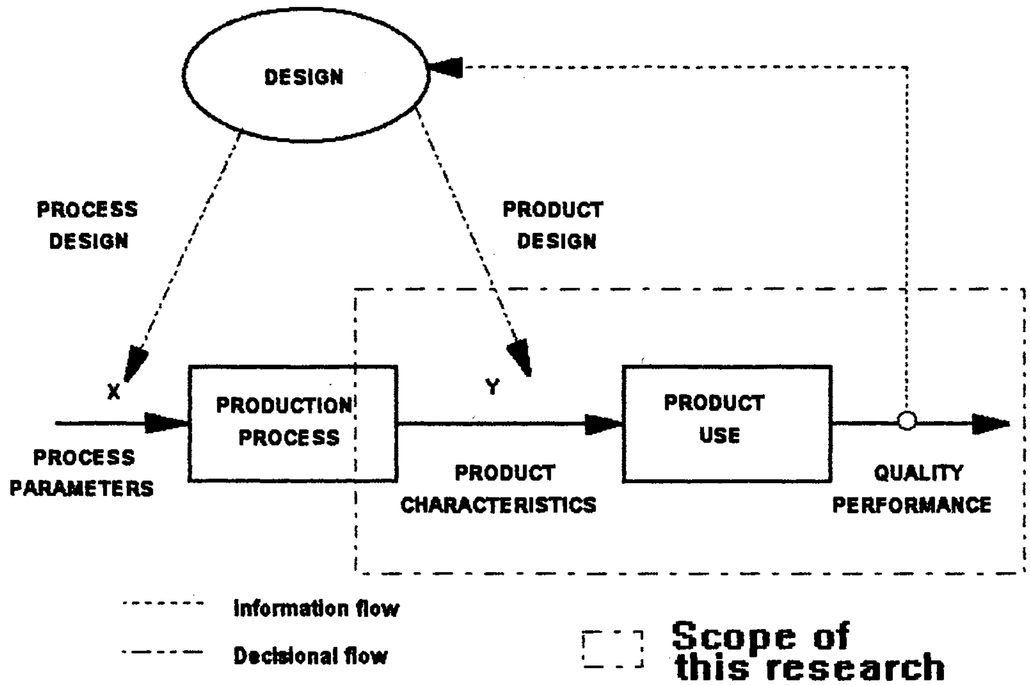

FIG. 1.2 DESIGN AND QUALITY PERFORMANCE

- Internal Quality Losses (IQL) are also losses incurred by the manufacturer, but they directly relate to product characteristics. Some examples are costs derived from scrap, rework, product appraisal, etc.
- External Quality Losses (EQL), are costs (often hidden, but sometimes large) incurred by the user as a consequence of the discrepancy between a product's intended use and its quality performance.

These three concepts can be expressed in monetary terms. This provides the design activities with a clear measure of their outcomes. A monetary type of measure represents a requisite for (1) justification of a design intervention, and (2) an evaluation of its consequences. Furthermore, a monetary measure is helpful in defining the objectives to be achieved from some specific design activity. Typical objectives include minimization of either one of the described costs, or of any combination of them. Some examples include minimization of EQL, minimization of the combination of IMC, IQL, and EQL, etc.

Each of these costs, however, occurs at different points in time. The economic analysis requires incorporating the time value of money, which considers the timing of the different costs (White, et al., 1989; Park and Sharp-Bette, 1990).

This research assumes that the overall objective of a design intervention is the optimization of the present worth of the sum of some combination of IMC, IQL, and/or EQL. The use of the present worth optimization criterion has some advantages, such as: (1) it is superior to other financial measures, such as return on investment, internal rate of return, and payback period (Spitzer, 1993), and (2) it is equivalent to an approach based on the maximization of the utility (Bussey and Eschenbach, 1992).

The major thrust of this research focuses on the lower right part of Figure 1.2, associated with product characteristics, their change over time, and quality performance. That is, modeling external quality losses (EQL).

1.3 EXTERNAL QUALITY LOSSES

The concept of quality loss is central in the quality engineering field. In fact, Taguchi (1986) defines quality as "the loss imparted to society from the time a product is shipped." Taguchi's definition of quality, although controversial, gives rise to the notion of external quality losses, or losses derived from poor quality incurred by someone other than the manufacturer. This idea extends the traditional view, which only considers internal quality losses. These are expenses incurred by the manufacturer due to poor quality, such as rework, scrap, quality inspection, etc. These expenses can be (and usually are) accounted for in the cost of manufacturing.

Unlike the other two types of costs, estimation and modeling of external quality losses has received little attention and is rarely accounted for in determining losses. An exception is provided by Taguchi (1986, 1987), who introduces a Loss Function, $\mathbf{L(y)}$, as a means to express "a monetary evaluation of the quality of products," in terms of a certain (one) product characteristic $\mathbf{y}$. The loss function suggested by Taguchi, which is now also called the Taguchi Loss Function (TLF), has a quadratic form, and is given by:

$$\mathbf{L(y) = k(y-\tau)^2} \qquad (1.1)$$

where $\boldsymbol{\tau}$ is the desired level for $\mathbf{y}$, or its target, and it expresses the product users' expectations on the product. In this expression, $\mathbf{k}$ is a constant that can be estimated by a simple economic argument. The TLF represents a conceptual tool that can be used to quantify costs as a product characteristic varies from its target even when it is within specs (Sullivan, 1987).

Due to the random nature of both $\mathbf{y}$ and $\mathbf{L(y)}$, a measure of quality losses can be associated with the expected value of $\mathbf{L(y)}$. This expected value, as shown in Chapter 2, is a function of the mean and the variance of $\mathbf{y}$. This fact is particularly useful, since quality performance, and therefore external quality losses, can be described in terms of:

- Product characteristic centering: the bias, or difference between the center of $\mathbf{y}$ and its target affects the satisfaction level for the user.
- Product characteristic variability, an intrinsic measure of product consistency (Grego, 1993). The variance is used for this measure.

The characterization of quality performance in terms of a product's mean and variance is consistent with that usually employed in quality engineering. For instance, it has been used for the development of the process capability index $\mathbf{C_{pm}}$, related to the mean squared error of the product characteristic, which Hsiang and Taguchi (1985), and Chan, et al. (1988) independently propose.

Yet, the way the Taguchi Loss Function is formulated in equation (1.1) does not include the effect of product use in quality performance. Product use causes change in the product characteristic **y** (a random variable). This change over time can be modeled by means of a stochastic function **y(t)**. This stochastic function is assumed to be fully described by its first two moments, and therefore, external quality losses will be decomposed into two types of losses (Figure 1.3):

- Quality losses attributable to the variance(s) of the product characteristic(s).
- Quality losses due to mean/bias.

For the case of multiple product characteristics, an additional decomposition can be done for both types of quality losses (those due to variance, as well as those due to bias). Each type of loss further involves aggregate losses:

- Due to different product characteristics (taking each of them alone).
- Resulting from interdependencies among product characteristics.

The present worth of external quality losses is defined as the sum of the present worth of quality losses due to the variance(s) and quality losses due to mean/bias.

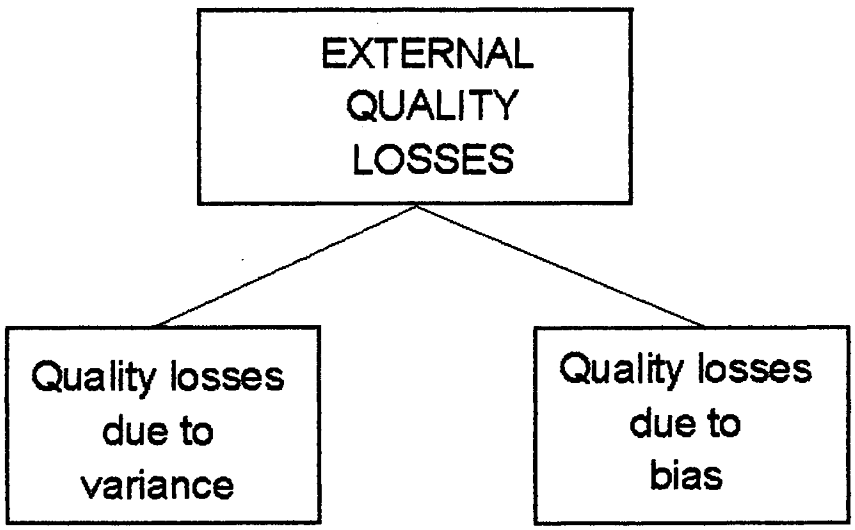

FIG. 1.3 TYPES OF EXTERNAL QUALITY LOSSES

1.4 LINKAGE BETWEEN PRODUCT AND PROCESS

A particular design objective might require intervention on the process. In this regard, a further insight on process parameters is required. External quality losses can be disaggregated into losses associated with the mean, and losses attributable to the variance. Therefore, for a certain product characteristic **y**, the process parameters can be classified as the following four types:

$\mathbf{X_M}$: parameters affecting only the mean/bias of **y**.

$\mathbf{X_V}$: parameters affecting only the variance of **y**.

$\mathbf{X_{MV}}$: parameters affecting both the mean and the variance of **y**.

$\mathbf{X_O}$: parameters that do not affect **y**, and hence the corresponding EQL.

It is important to remark that besides being a quality driver, a certain process parameter can be a manufacturing cost driver as well, i.e., a factor that can cause a change in the cost of a manufacturing activity (Raffish and Turney, 1991; Cooper, 1989). For example, suppose that a certain part can be manufactured from three different types of material (plastic, steel and aluminum) and the part's quality depends on the diameter of a hole that is to be drilled in it. In this example, the type of material has a dual role:

1. As a quality cost driver, it determines

- Mean and variance of the diameter to be drilled, and
- Performance degradation with time of the diameter's mean and variance.

2. As a manufacturing cost driver, it influences

- Technology to utilize, and
- Materials cost.

1.5 PROBLEM STATEMENT

The Taguchi Loss Function model formulated in equation (1.1) overlooks the effect of product use on quality performance. In doing so, it does not reflect a product's quality as perceived by its user. Thus, it fails to meet its intended purpose: to provide "a monetary evaluation of the quality of products" (Taguchi, 1986, 1987). However, the Taguchi Loss Function is potentially useful to model external quality losses, since it: (1) has a simple analytical expression, (2) captures randomness in a simple way (as a function of only the mean and the variance of a product characteristic), and (3) can express quality performance in monetary terms.

This research deals with the extension of the Taguchi Loss Function in order to make it a tool appropriate for manufacturing system design and process parameter design optimization.

1.6 RESEARCH OBJECTIVE AND SUBOBJECTIVES

The objective of this research is to extend the Taguchi Loss Function model so it can effectively provide a monetary evaluation of external quality losses of products. Such an extension includes consideration of: (1) the different types of loss minimization objectives, (2) product use and degradation over time, (3) the time value of money, and (4) simultaneous modeling of multiple product characteristics.

This objective requires accomplishing several subobjectives:

1. Extend the Taguchi Loss Function as applied to different types of loss minimization objectives.

   This subobjective is accomplished when, for each of the loss minimization objectives, expressions of present worth of expected: (1) losses, (2) losses due to variance, and (3) losses due to mean/bias, are obtained. The following tasks are required:

1.1 Examine the different types of loss minimization objectives:

- Lower-is-better.
- Higher-is-better.
- Symmetric nominal-is-better.
- Asymmetric nominal-is-better.

1.2 Determine expressions of expected loss for each of the loss minimization objectives.

1.3 Express the present worth of expected external losses in terms of losses due to variance and losses due to mean/bias of product characteristics (for each of the loss minimization objectives).

2. Introduce different product use models into the extended Taguchi Loss Function model.

This subobjective is accomplished when expression of losses due to variance and losses due to mean/bias are obtained for different types of variance drift and mean drift. This requires, for each of the loss minimization objectives, performing the following tasks:

2.1 Determine present worth of expected losses due to variance, when the variance:

- Shows no drift.
- Has a linear drift over time.

2.2 Determine present worth of expected losses due to mean/bias, when the mean:

- Shows no drift.
- Has a linear drift over time.
- Has a quadratic drift over time.

2.3 Perform sensitivity analysis of losses (due to variance and due to mean/bias) to changes in conditions (variance and mean) at the time the product is shipped to the user.

3. Extend the Taguchi Loss Function model to the multivariate case, when **p** different product characteristics are considered.

This subobjective is accomplished when expressions of present worth of expected: (1) losses, (2) losses due to variance, and (3) losses due to mean/bias, are obtained. The following tasks are required:

3.1 Express the present worth of expected external losses in terms of losses due to variance and losses due to mean/bias of product characteristics for each of the loss minimization objectives.

3.2 Examine the role of interdependencies among product characteristics for external quality losses.

3.3 Illustrate developments using a hypothetical example of a product for which quality is defined by two product characteristics ($\mathbf{p}$=2).

4. Prepare a simple concept illustration to show how the TLF model of external quality losses may be used.

This subobjective is achieved with the identification of extensive future research opportunities. This requires the following tasks:

4.1 Develop hypothetical example with product/process linkage known in advance.

4.2 Determine process settings that optimize "true" expected external quality losses.

4.3 Determine quality performance levels for observations generated by Montecarlo simulation.

4.4 Evaluate expected external quality losses using results of this research.

4.5 Compare and contrast results.

Table 1.1 presents the sections in this document each of the subobjectives is dealt with.

TABLE 1.1 RESEARCH SUBOBJECTIVES AND DOCUMENT SECTIONS

SUBOBJECTIVE	SECTION
1.1	3.2
1.2	3.3
1.3	3.4
2.1	4.2, 4.3, 4.4
2.2	4.2, 4.3, 4.4
2.3	4.2, 4.3, 4.4
3.1	6.2, 6.3
3.2	6.2, 6.3
3.3	6.2, 6.3
4.1	5.2
4.2	5.2
4.3	5.3
4.4	5.2
4.5	5.2

CHAPTER II

LITERATURE REVIEW

2.1 INTRODUCTION

Edwin L. Artzt, chairman of the board and chief executive of Procter and Gamble, states "Consumers still want good performance. But quality now has a stronger value-for-the-money component,...the consistent quality of our brands -and the trust and loyalty this quality creates -is the foundation of our business" (Kelly, 1992). Statements such as Artzt's evidence the wide recognition of the strategic importance of quality. Such a relevance involves the need of sound and effective quality-related decision making.

This chapter reviews literature relevant to quality-related decision making from the conceptual framework adopted in this research. Figure 2.1 points out the scope of this research. Three basic study subjects can be identified within this scope: (1) design for quality, (2) linkage between process and product

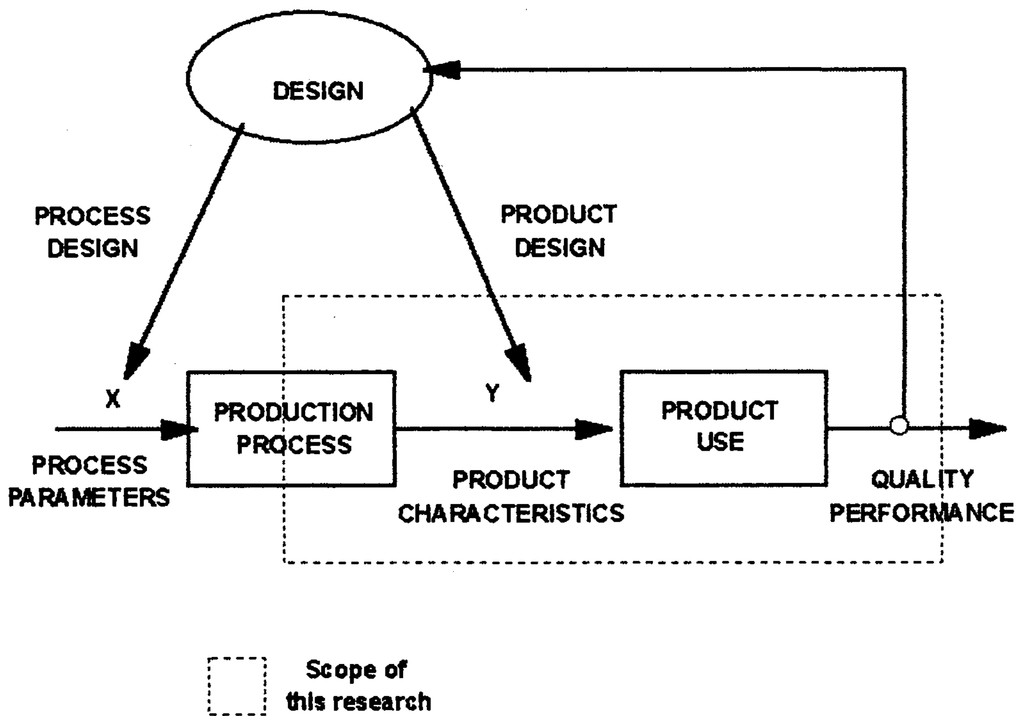

FIG. 2.1 SCOPE OF THIS RESEARCH

characteristics, and (3) quality performance and product use. The present chapter reviews literature on each of these subjects in Subsections 2.2, 2.3, and 2.4, respectively. Section 2.5 presents a review summary.

2.2 DESIGN FOR QUALITY

The quality movement is pioneered by Shewhart (1931), who recognizes an intimate relationship between a product's quality and its value. In fact, Shewhart relates the value of a product to its usefulness, esteem, and exchangeability. Quality is then a product's intrinsic property directly related to its value. This fact has a double sided connotation: (1) quality is a property built into the product (Deming, 1982) (which in many cases might mean that achievement of a higher quality comes with additional cost for the manufacturer), and (2) lowering (improving) product quality performance always implies reducing (enhancing) its value for the user (which definitely has effects on the manufacturer) (Shewhart, 1931; Hill, 1992). The manufacturer of a certain product has the ability to determine its quality by mastering the different production factors from which the product is generated.

Taguchi, et al. (1989) call the overall quality system an integrated set of interacting activities aimed at achieving controlled production of products with superior quality. This system is to act on the different phases of a product's life cycle: (1) product planning,

(2) product design, (3) process design, (4) production, and (5) service after purchase. Taguchi and Wu (1979) refer to activities performed during product and process design as off-line quality control, whereas they call on-line quality control those activities performed during actual production. Using this terminology, this research involves only off-line quality control activities.

Taguchi (1986, 1987) introduces a general methodology for design, which consists of three steps: (1) system design, determines the basic configuration of the product/process, (2) parameter design, determines the operating levels of the process parameters defined in the previous step, and (3) tolerance design, defines allowable ranges for the different product characteristics and/or process parameters.

Increasing competitive pressure is forcing organizations to pursue reducing the elapsed time between a product's initial conception and its availability at the market (Vesey, 1992). Reduction of time-to-market is achieved by increasing the overlaps among the different phases of the product life cycle. This requires, in many instances, simultaneously performing product design and process design, a major tenet of simultaneous, or concurrent engineering (Perry, 1990; Fabrycky and Blanchard, 1991; Kim, 1991; Kim and Ooi, 1991).

A remarkable gap in the quality engineering literature is the lack of an approach that allows simultaneously dealing with product and process design. The conceptual framework used in this research lays the foundation of such an approach. Under this framework, design is defined as any deliberate intervention on the production process. In particular, design for quality has the purpose of building quality into a product, as a means to provide (Case and Bigelow, 1992) superior:

• Value as viewed by the customer and the marketplace.

• Company performance through productivity and effectiveness indicators.

Thus, design for quality aims at increasing the product value, and simultaneously, at improving company performance.

2.3 PRODUCT AND PROCESS LINKAGE

The conceptual framework used in this research identifies the outputs from a production process as the set of product characteristics that determine product quality performance. The inputs to the production process correspond to different process parameters. A causal relationship can be established between product and process.

Quality is characterized in terms of means and variances of product characteristics. This section reviews literature on the linkage between process parameters and mean and variance of one product characteristic. Literature related to multiple product characteristics is practically null.

The linkage between product and process can be represented by a functional relationship. The inherent variation of any production process renders product characteristics with a random nature. Statistics is an appropriate field to deal with the linkage between product and process.

Suppose a product characteristic is to be observed on $\mathbf{n}$ different units. If the $\mathbf{n}$ observable values can be represented by $\mathbf{Y}$, and $\mathbf{X}$ is an $\mathbf{nxp}$ matrix that represents the corresponding settings for the process parameters, then the general linear model is a natural way to represent a relationship between product and process. The general linear model is given by (Graybill, 1976):

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (2.1)$$

where $\boldsymbol{\beta}$ is a $\mathbf{p x 1}$ vector of coefficients expressing the way each of the process parameters influences the product characteristic (for example, if a process parameter has a corresponding coefficient equal to zero, then that parameter does not affect the value of $\mathbf{y}$). In this equation, $\boldsymbol{\varepsilon}$ is an $\mathbf{nxq}$ random vector, which traditionally is assumed to hold the following assumptions: (1) normally distributed, (2) $\mathbf{E}[\boldsymbol{\varepsilon}] = \mathbf{0}$, and (3) $\mathbf{COV}[\boldsymbol{\varepsilon}] = \boldsymbol{\Sigma}$ (homogeneous variance).

An empirical way to improve a product's quality is by experimentation, i.e., by (1) introducing changes in the process parameters, (2) observing the effects on the product, (3) examining the influence of process parameters on quality, and (4) setting process parameters at levels yielding a desired quality. A systematic way to plan and perform experimentation is provided by the area of statistics known as designed experimentation (Cochran and Cox, 1957; Montgomery, 1984; Schmidt and Launsby, 1991).

Designed experimentation for quality purposes has been applied under different approaches: (1) traditional, (2) robust design, and simultaneous modeling of mean and variance. Literature related to each of these approaches is reviewed in the following subsections.

2.3.1 Traditional Approaches

Traditional industrial designed experimentation assumes homogeneity of variance, i.e., that the variance of observed values of product characteristics does not depend on the levels of the process parameters. As a consequence, traditional applications of designed experimentation have been centered on the mean. Thus, experimentation for quality purposes has been mainly aimed at achieving a product characteristic target.

For example, Eibl, et al. (1992) perform sequential experimentation using Response Surface Methodology (Box and Wilson, 1951), for a parameter design problem aimed at achieving a target of 0.8 mm in a painting operation that originally yielded coating thickness varying between 2mm and 2.5 mm. This example illustrates a strategy commonly employed for sequential experimentation, which implements several experiments to successively achieve different goals (Pfeifer, 1989): (1) screening of process parameters, (2) analysis of their interactions, and (3) refined modeling for optimization of product characteristics.

The major deficiency of traditional experimentation is that it requires adoption of classic assumptions of the general linear model, namely (1) homogeneity of variance, (2) normality, and (3) linearity of the model.

To deal with variance heterogeneity, Bartlett (1947) introduces <u>variance stabilizing transformations</u>, or transformations of observed values of product characteristics that yield a transformed variable for which traditional assumptions of the general linear model hold. Box and

Cox (1964) introduce a family of power transformations that include both the log transformation and no transformation as special cases. Box and Cox transformations generate a transformed variable for which classic assumptions of the general linear model hold. Nelder and Lee (1990) state that a single transformation cannot necessarily produce such a type of variable.

2.3.2 Robust Design

Realizing the weakness of the assumption of homogeneous variance when using designed experimentation for quality improvement, Taguchi (1986, 1987) examines product variation, and identifies three different sources for it: (1) the process, (2) components assembly, and (3) environment. As a consequence, Taguchi introduces three different problems into designed experimentation, all of them under the general title of Robust Design (Box, 1988; Montgomery, 1990): (1) Closeness to target, or achieving minimum dispersion for the product characteristic with its location adjusted to the target, (2) Robustness to transmitted variation, or minimizing product sensitivity to variation transmitted from its components, and (3) Robustness to environmental variation, or minimizing product sensitivity to environmental fluctuations.

Taguchi's insights on product variation represent a major engineering contribution, so that he is widely recognized as influential in the increasing awareness of the relevance of designed experimentation to reduce variation (Hunter, 1985; Box, 1988; Dehnad, 1989; Shoemaker, et al., 1991; Nair, 1992).

Literature on robust design largely concentrates on parameter design, and is concerned with development of univariate quality performance measures based on a single statistic that could replace simultaneous analysis of mean and variance (Grego, 1993).

Taguchi states that the goal of parameter design is to choose the settings of the process parameters $\mathbf{X}$ that minimize the expected or average loss caused by the deviations of the output from target. However, when Taguchi applies designed experimentation, he recommends maximizing a different measure, called <u>signal-to-noise ratio</u> (SN ratio).

If $\mathbf{X_M}, \mathbf{X_{MV}}$, and $\mathbf{X_V}$ denote the sets of process parameters affecting mean, mean and variance, and variance, respectively, then to solve the parameter design problem, Taguchi develops a two-step approach: (1) find the settings for $\{\mathbf{X_{MV}}, \mathbf{X_V}\} = \{\mathbf{X^*_{MV}}, \mathbf{X^*_V}\}$, i.e., that maximize the SN ratio, and (2) adjust $\mathbf{X_M}$ to $\mathbf{X^*_M}$, while $\{\mathbf{X_{MV}}, \mathbf{X_V}\}$ is fixed at $\{\mathbf{X^*_{MV}}, \mathbf{X^*_V}\}$, i.e., set the so-called "adjustment parameters" (parameters affecting only the mean) at some convenient level.

This approach, according to Taguchi, applies for any of the cases of loss minimization objectives for the product characteristic (lower-is-better, higher-is-better, or nominal-is-better). The only difference for each case is in the way the signal-to-noise ratio is defined. However, Taguchi does not justify the use of SN ratios, nor provide the connection between a SN ratio and the expected loss. León, et al. (1987) show that only for the nominal-is-better case (bilateral symmetric tolerancing), and under very specific conditions, minimizing the expected loss is equivalent to maximizing the corresponding SN ratio.

Box (1988) criticizes the use of SN ratios, which he considers as a "portmanteau" criteria, and proves them to be (1) an inadequate summary of data, and (2) extremely inefficient measures.

Recognizing the common situation in which the variance is dependent on the mean, León, et al. (1987) introduce the concept of **PerMIA** (Performance Measure Independent of Adjustment), a univariate performance measure independent of the mean over the set of factors affecting the variance ($\mathbf{X_{MV}}$ and $\mathbf{X_V}$). The resulting approach for parameter design consists of two steps similar to those in Taguchi's approach, the only difference being in the maximization of the PerMIA, instead of the SN ratio.

The basic advantage of León's approach over Taguchi's arises from the fact that PerMIAs are performance measures more general than the SN ratios. They can be derived directly from knowledge of the loss function and of the general function for the model describing the performance of the product characteristic. Further generalizations of this approach are given by Nair and Pregibon (1986), and Box (1988).

2.3.3 Simultaneous Models of Mean and Variance

Simultaneous approaches intend to explicitly model both the mean and the variance of a product characteristic. This has been done by either of two types of models: (1) those based on response surface methodology (Vining and Mayers, 1990), and (2) generalized linear models (Nelder and Lee, 1991; Grego, 1993).

In any case, it is assumed that the product characteristic is associated with a model given as:

$$\mathbf{Y} = [X_M, X_{MV}, X_V]\beta + \varepsilon \qquad (2.14)$$

where $\mathbf{Y}$ is the vector of responses, and ε is a vector of random variables with means of zero and

$$\mathrm{VAR}[\varepsilon_i] = \exp\{\gamma'[X_{MV}, X_V]\} \qquad (2.15)$$

A major drawback of these types of approach is that they tend to be deemed as rather complex for industrial experimenters without sound statistical backgrounds.

2.4 QUALITY PERFORMANCE AND PRODUCT USE

For Viswanadham and Narahari (1992), any type of design (deliberate intervention on some product or on its production process) has the goal of improving the process output regarding one or more performance measures. Performance measures, then, are instruments for goal-setting purposes. They represent means to: (1) justify a potential intervention on the process, and (2) evaluate effects of such an intervention.

Section 2.2 emphasizes that design for quality has the overall purpose of building quality into a product. As a consequence, the product value increases and company performance improves. Then, a quality performance measure appropriate for design purposes should reflect (1) the level at which the product meets its user's expectations, and (2) the contribution of quality to overall company performance. Each of these two issues imply a series of difficulties to determine quality performance measures. Difficulties arising from the user's expectations include the following:

- Notions about what determines quality are varied, have a heterogeneous nature, change with time, and differ from place to place (Table 2.1).
- The impact of a certain product goes beyond its user, reaching a wide spectrum of individuals, who can be considered its customers (Juran and Gryna, 1993). Taguchi (1986, 1987) identifies the whole society as the generic customer of any product.
- Understanding and interpreting user needs and expectations, i.e., "listening to the voice of a customer," are not simple tasks, and in most cases require systematic methods to achieve them (Kogure and Akao, 1983; Sullivan, 1986; Gopalakrishnan, et al., 1992; Havener, 1993).
- From the user's perspective, product quality has a multifaceted nature. Quality is perceived as a composite of properties which are often interrelated (Garvin, 1984; Garvin, 1987; Raiman, 1991).

As for the need for a quality performance measure to evaluate the contribution of quality to the company's overall performance, some hurdles include:

- The need to link quality performance with measures used to evaluate company performance, mainly those of a financial nature.

TABLE 2.1 DEFINITION OF QUALITY BY CUSTOMERS OF THREE DIFFERENT COUNTRIES

Responses mentioned by at least 10% of respondents to open-ended questions about what determines quality (in descending order of frequency):

USA	GERMANY	JAPAN
1. Well-known name	1. Price	1. Well-known name
2. Word of mouth	2. Well-known name	2. Performance
3. Past experience	3. Appearance	3. Easy to use
4. Performance	4. Durability	4. Durability
5. Durability	5. Past experience	5. Price
6. Workmanship	6. Quality itself	
7. Price		
8. Manufacturer's reputation		

No. of interviews: USA 1,008
Germany 1,446
Japan 1,000

(Taken from: Ryan, J. "Different Lands, Different Views," Quality Progress, V. 24, N. 11, pp. 25-29).

- Evaluation of impact of interventions on the process requires deep knowledge on the linkage between product and process.
- Agents participating in interventions on the process (upper management, middle management, lower management, and operators) "speak" different languages (Juran, 1989).

A major benefit of good quality performance measures is that they can be helpful to mend the perception commonly found among managers (ASQC/The Gallup Organization, 1989) that quality is an amorphous concept with no obvious, significant cash flow implications (Spitzer, 1993).

Literature on three different types of quality performance measures is reviewed in the following subsections: (1) capability indices, (2) Taguchi Loss Function, and (3) performance degradation models.

2.4.1 Capability Indices

Quality control has provided a set of quality performance measures known as <u>capability indices</u>, which are based on the proportion of nonconforming product a process yields, i.e., product outside specification limits (Kane, 1986; Pearn, et al., 1992; Rodriguez, 1992). The popularity of these indices is attributable to different factors, such as: (1) they are simple to compute, and (2) they are appealing to quality and engineering personnel, since they are based on the traditional engineering concept of specifications limits (Johnson, 1992).

The use of capability indices has been related to quality control purposes: (1) monitor industrial processes, and (2) compare different processes (Kushler and Hurley, 1992). However, misuse and abuse happens (Gunter, 1989) due to ignorance: (1) they require a process under statistical control, (2) they are based on the normal distribution, in a context in which departures from normality are hard to detect. Furthermore, they are unitless measures difficult to comprehend (Franklin and Wasserman, 1992), and there is not a basis to determine an optimum value for them (Taguchi, et al., 1989).

As quality performance measures for quality design, capability indices have two major deficiencies: (1) their significance is not comprehended by management, and in many instances, by production engineers, (2) they fail to reflect the quality level of products shipped to customers (Taguchi, et al., 1989; Taguchi and Clausing, 1990), and (3) they do not consider how product use alters quality performance.

2.4.2 Taguchi Loss Function

Taguchi (1986, 1987) recommends adoption of money-related performance measures for design for quality. For that matter, he suggests using a loss function. The basic underlying idea is that the cost of poor quality, or loss, is minimized at some product characteristic nominal value, or target. There is an ever-increasing loss with any undesired departure from the target. This clearly differs from the traditional method of quantifying quality, which Ross (1989) calls the "goalpost" syndrome, in which if the product is within specifications, everything is fine and no cost is incurred.

The concept of a loss function can be found in many different contexts, particularly in the form of a quadratic loss function. It constitutes the building block for least squares estimation theory, initially developed by Gauss in the early 1800's (Netter, et al., 1989). It is employed in Bayesian Decision Theory and Estimation (Ferguson, 1967; Berger, 1985), in control theory (Åström, 1970), and for the definition of learning rules in neural networks (Widrow, 1962; Zurada, 1992). Taguchi employs a quadratic loss function, since this particular type of function (Taguchi, 1986, 1987; Taguchi, et al., 1989):

1. Derives (as an approximation) from the expansion in a Taylor series of the loss function about the target value, when terms with powers higher than 2 are neglected.
2. Can be easily related to statistical concepts such as squared deviation of a random variable about a certain point.
3. Possesses a relative analytical simplicity.

Literature on quality engineering uses the term Taguchi Loss Function (TLF) as synonymous with a quadratic loss function.

For a product characteristic **y** with bilateral (symmetric) tolerancing, the TLF, **L(y)**, is defined as:

$$\mathbf{L(y) = k(y-\tau)^2} \qquad (2.3)$$

where τ is the target value, and **k** is a constant that can be determined from the following economic argument. If a certain loss A is known to be associated with a deviation Δ from the product characteristic's target. Then:

$$\mathbf{k} = \mathbf{A}/\Delta^2 \qquad (2.4)$$

The expected loss, $\mathbf{E[L(y)]}$ is given by:

$$\mathbf{E[L(y)]} = \mathbf{k}\ \mathbf{E[(y-\tau)^2]} \qquad (2.5)$$

or, equivalently, from the expansion of the expected value of $(\mathbf{y}-\tau)^2$:

$$\begin{aligned}\mathbf{E[L(y)]} &= \mathbf{k\{E[y^2]-2\tau E[y]+\tau^2\}} \\ &= \mathbf{k\{\sigma_y{}^2+(E[y]-\tau)^2\}} \qquad (2.6)\end{aligned}$$

The first term in the above equation is associated with the variance of the product characteristic, whereas the second term is related to the square of the bias or offset of the mean relative to the target. Literature on the TLF concentrates on bilateral symmetric tolerancing.

For the purpose of extending the TLF for simultaneously handling a set of product characteristics (which are represented by $\mathbf{Y}$, a $\mathbf{px1}$ vector), Taguchi, et al. (1989) introduce a multi-dimensional loss function, $\mathbf{L(Y)}$ defined as the summation of the losses associated with each individual product characteristic $\mathbf{y_i}$, i.e.:

$$\mathbf{L(Y)} = \sum_{i=1}^{p} \mathbf{L(y_i)} \qquad (2.7)$$

this formulation assumes a lack of interdependencies among product characteristics.

In a very generic way, Pignatiello (1993) extends the above concepts to include a set of process parameters, denoted by $\mathbf{X}$, a $\mathbf{kx1}$ vector, that influence the vector of product characteristics denoted as $\mathbf{Y}$. A vector of target values is denoted by $\boldsymbol{\tau}$, which includes the target of each of the characteristics. A $\mathbf{pxp}$ positive definite matrix $\mathbf{K}$ represents the losses incurred when $\mathbf{Y}$ deviates from $\boldsymbol{\tau}$. The loss function, as a function of $\mathbf{X}$, is defined as:

$$\mathbf{L[Y(X)] = [Y(X)-\tau]' \; K \; [Y(X)-\tau]} \qquad (2.8)$$

Given the quadratic form for the loss function, its expected value is (Graybill, 1976, p.139):

$$\mathbf{E\{L[Y(X)]\} = trace[K\Sigma(X)]+[\mu(X)-\tau)]'K[\mu(X)-\tau]} \qquad (2.9)$$

where $\boldsymbol{\mu}\mathbf{(X)}$ denotes the means vector of $\mathbf{Y}$ as a function of $\mathbf{X}$, and $\boldsymbol{\Sigma}\mathbf{(X)}$ is the variance-covariance matrix of $\mathbf{Y}$ as a function of $\mathbf{X}$.

Literature on TLF:

- Has been abundant in journals related to quality engineering and management, particularly during the middle and late 1980's and has a distinctive applications-oriented nature (Sullivan, 1984; Pignatiello and Ramberg, 1985; Barker, 1986; Kackar, 1986; Byrne and Taguchi, 1987; Sullivan, 1987; Maghsoodloo, 1990). Journal of Quality Technology has a special issue on Taguchi's methodology (Journal of Quality Technology, 1985). Technometrics (Nair, 1992) publishes discussions from an experts panel. Dehnad (1989) provides a comprehensive compilation of articles on the topic.

- Is permeating titles in different engineering areas, involving quite diversified applications, such as: improvement of the manufacturing process of optical fibers (Kackar and Shoemaker, 1986); determination of tolerance specification of robot kinematic parameters (Liou, et al., 1993), tuning of parameters in the design of mechanical devices (Otto and Antonnson, 1993 (a and b); process improvement in the milling industry (Tuck, et al., 1993); design of propulsion systems (Unal, et al., 1993).

In all cases, however, the literature concentrates on the TLF as a support for robust design, which in fact neglects the TLF and employs signal-to-noise ratios.

Although the TLF is supposedly intended to model external (user's) quality losses, nearly all examples found in the literature deal with internal costs only. When the examples include external costs, they do so referring only to losses incurred by the producer, such as customer returns, warranty claims, and product recalls (Campanella, 1990).

Furthermore, literature on the TLF completely ignores changes of quality performance over time.

2.4.3 Performance Degradation Models

Juran provides a widely accepted definition of quality as "fitness for use" (Juran, 1980; Juran and Gryna, 1993). The fact that such a fitness evolves over time has been largely overlooked. This is tantamount as neglecting the product's performance in the field. A comprehensive definition and modeling of a product's quality has to include the change over time of its quality performance, as a consequence of product use, which instead requires defining the evolution of product characteristics stochastically over time. For this purpose, concepts involved in Performance-Degradation Models, a particular type of model developed for reliability analysis, can be extended to model and understand the behavior of product characteristics over time.

The basic idea of a degradation model is that the variable under analysis, $\mathbf{y}$, exhibits a degradation path over time. The path of $\mathbf{y}$ is a function of (1) fixed effects, which describe population characteristics, and (2) random effects, which describe an individual unit's characteristics. Lu and Meeker (1993) describe a general path model, in which the sample path for the **i**th unit at time $\mathbf{t_j}$ is given by:

$$\mathbf{y_{ij} = \eta_{ij} + \varepsilon_{ij} = \eta(t_j;\phi,\Theta_i) + \varepsilon_{ij}} \quad (2.10)$$

where $\mathbf{t_j}$ = time of the jth measurement; $\mathbf{\varepsilon_{ij}}$ = measurement error with variance $\boldsymbol{\sigma^2_\varepsilon}$; η_{ij} = actual path of the ith unit at time $\mathbf{t_j}$ with unknown parameters; $\boldsymbol{\phi}$ = vector of fixed-effect parameters, common for all units; Θ_i = vector of the ith unit random-effect parameters, representing individual unit characteristics.

Performance-degradation models are intended to estimate measures relevant for systems reliability such as time-to-failure distribution, and first crossing time. Kapur and Lamberson (1977) relate performance degradation (due to stress and strength change) to aging, cyclic damage, and cumulative damage; Nelson (1981) employs these models to determine the distribution of time to failure (dielectric breakdown) of electric insulation devices; Bogdanoff and Kozin (1985) evaluate cumulative damage derived of metal crack growth due to fatigue; Nelson (1990) describes basic ideas on accelerated-test degradation models; Lu and Meeker (1993) examine the time course of drug concentrations in biological systems and develop statistical methods for using degradation measures to estimate a time-to-failure distribution for a broad class of degradation models.

Performance degradation models can be helpful for modeling quality performance and its change over time. However, a major difficulty involved is that the way they have been developed assumes knowledge of the distributional properties of the product characteristic under analysis, as well as on their behavior over time.

2.5 SUMMARY

There is an increasing awareness of the strategic relevance of quality for overall organizational performance. Effective management (planning and control) of design requires quality performance measures that (1) reflect quality from the user's viewpoint, and (2) provide decision makers with a clear picture of the outcomes of design activities.

The literature reviewed:

- Overlooks the effect of product use on quality performance. As a consequence, it fails to effectively consider quality as viewed by the product user.
- Provides quality performance measures with a strong technical connotation. Decision makers usually have difficulties to (1) understand them, and (2) use them as a means to plan and control design for quality.

CHAPTER III

PRESENT WORTH OF QUALITY LOSSES

(ONE PRODUCT CHARACTERISTIC)

3.1 INTRODUCTION

The purpose of this chapter is to introduce, for a single product characteristic, the present worth criterion as a means to provide a monetary evaluation of external quality losses.

Use of the present worth criterion has some advantages: (1) it is superior to other financial measures, such as return on investment, internal rate of return, and payback period (Spitzer, 1993), (2) it is in tune with the shareholders' wealth maximizing objective (Arcelus and Srinivasan, 1993), and (3) it is equivalent to an approach based on the maximization of utility (Bussey and Eschenbach, 1992).

The Taguchi Loss Function (TLF) is used in this research as the building block to model instantaneous external quality losses. The TLF is introduced in Section 3.2 as a tool to (1) model quality performance at an arbitrary time, and (2) classify product characteristics into five different types, according to their quality loss minimization objectives. Section 3.3 incorporates randomness of a product characteristic, so that the expected loss under the different loss minimization objectives can be determined. Section 3.4 extends the TLF by introducing the discounted cash flow viewpoint that allows determining the present worth of expected quality losses. For each of the product characteristic types, two issues are particularly addressed: (1) knowledge requirements on the probability distribution of the product characteristic in order to determine its corresponding present worth of expected quality losses, and (2) feasibility to break down such a present worth into two parts, one related to the product characteristic's variance, and the other part, related to its mean. A chapter summary is presented in Section 3.5.

3.2 EXTERNAL LOSS MINIMIZATION OBJECTIVES

The overall purpose of design for quality involves the optimization of quality losses. A concept fundamental for this research is that of a loss function, which allows one to (1) model external quality losses, and (2) determine external loss minimization objectives to design for quality. This section discusses how a specific type of loss function, known as the Taguchi Loss Function (TLF) can be used in this context.

Subsection 3.2.1 examines the concept of loss function, as a means to provide a monetary evaluation of undesirable quality levels. From the loss function paradigm, the two dominant philosophies of quality are

(1) a traditional philosophy, and (2) the one that supports the TLF. These philosophies are examined in Subsection 3.2.2.

A cautionary remark is that the loss functions to be discussed in this section completely ignore the effect of product use on its quality performance. This issue is dealt with in Section 3.4.

3.2.1 Loss Function

Juran's widely accepted definition of quality as "fitness for use" (Juran, 1993) implicitly involves a product's user who imposes, on a product characteristic, a target, or ideal state. From this standpoint, the quality of a certain product is related to undesirable deviations of a product characteristic $\mathbf{y}$ around its target τ: the smaller an undesirable deviation of $\mathbf{y}$ about τ, the better the quality, and vice versa.

Design for quality (any deliberate intervention on the production process for quality purposes) deals with the risk of having undesirable deviations of a product characteristic $\mathbf{y}$ about its target, τ. Characterization and measurement of such a risk (Fishburn, 1984) calls for a close look at these undesirable deviations around τ. These deviations can be either one of the following two types (Figure 3.1)

1. Unilateral: the set of undesirable deviations is defined only on one side around the target. Two cases are included

- The set of undesirable deviations is defined for the interval of values of $\mathbf{y}$ larger than the target, i.e., for $\mathbf{y} > \tau$ (Figure 3.1 (a)).

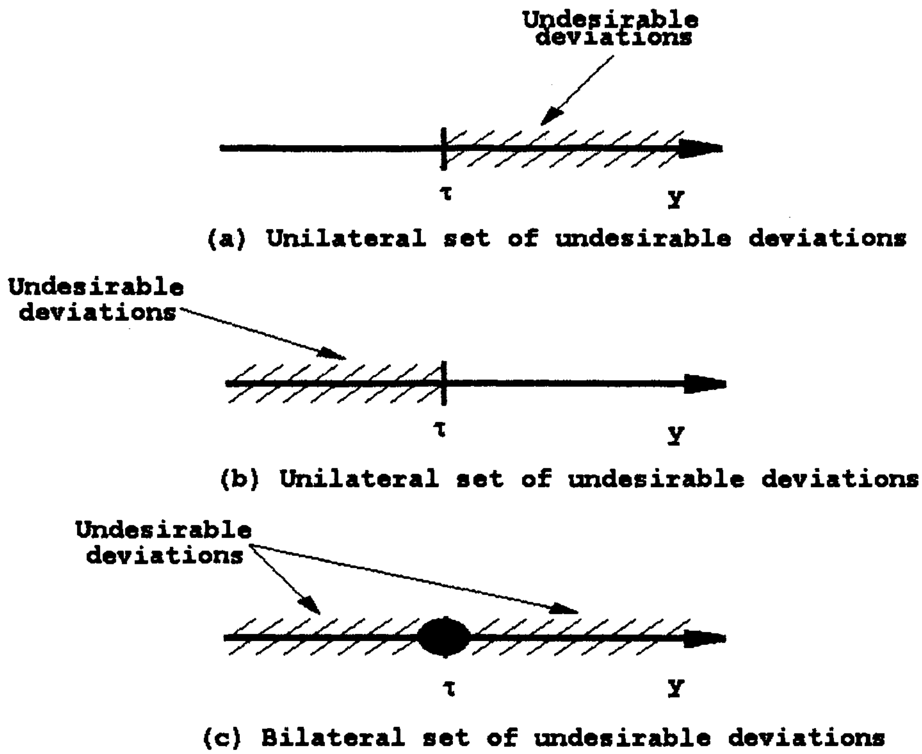

FIG. 3.1 TYPES OF UNDESIRABLE DEVIATIONS AROUND A PRODUCT CHARACTERISTIC'S TARGET

- The set of undesirable deviations is defined for the interval of values of $\mathbf{y}$ on the left side of the target, i.e., for $\mathbf{y} < \tau$ (Figure 3.1(b)).

2. Bilateral: deviations on either side of the target are undesirable, therefore, any $\mathbf{y} \neq \tau$ is undesirable (Figure 3.1(c)).

The overall purpose of design for quality is to optimize quality losses. The concept of loss function allows linking external quality losses and undesired deviations of some product characteristic around its target. Kackar (1985) defines the loss function, $\mathbf{L(y)}$, associated with a certain product characteristic $\mathbf{y}$, as "...the loss in terms of dollars suffered by an arbitrary user of the product at an arbitrary time during the product's life span due to deviation of $\mathbf{y}$ from τ." A loss function then yields a monetary measure of costs associated with undesired quality levels of a product characteristic.

Kackar's definition captures a key aspect of the way loss functions are commonly defined: they are restricted to some certain instant time. By doing so, the effect over time of product use on quality performance is ignored. Such an effect is dealt with in Section 3.4.

3.2.2 Loss Function and Quality Philosophies

A common industrial practice related to design for quality consists of providing specifications on $\mathbf{y}$. A specification is defined by Bemowski (1992) as "a document that states the requirements to which a given product or service must conform." For a given product characteristic, the requirements are usually expressed in terms of its nominal value (assumed equivalent to the target of **y**), and tolerances around it (Kackar, 1985). A specification, then, defines a tolerance interval that relates the target of **y** and one (for the bilateral type of undesirable deviations, two) tolerance limit(s).

For the tolerance philosophy, a product's quality is considered satisfactory if **y** lies within the "acceptance" or tolerance interval; otherwise, its quality is deemed as unsatisfactory. This philosophy implicitly assumes a loss function in which, (1) as long as the product characteristic is within the "acceptance" interval defined by the product tolerance(s), there is a zero loss, and (2) otherwise, there is a loss of **A** monetary units. The tolerance philosophy, also known as the "goalpost" philosophy (Ross, 1979), overlooks the fact that quality is a function of (undesirable) deviations of any size around the target, regardless of what is stated in the specifications for **y**.

In contrast to the above philosophy, Taguchi (1986, 1987) realizes that an appropriate loss function must penalize any (undesirable) deviation of a product characteristic around its target. Then, he introduces a quadratic loss function into the field of quality engineering, which has been also called the Taguchi Loss Function (TLF). The Taguchi Loss Function (TLF) associated with a certain product characteristic **y**, **L(y)**, provides a "monetary evaluation of the quality of products" (Taguchi, et al., 1989) and therefore can be used for modeling external quality losses.

Depending on the way undesired deviations around the target are defined for a certain product characteristic, use of the TLF allows identifying different external loss minimization objectives. Loss minimization objectives commonly found in industry include

1. Lower-is-better (LIB): the set of undesirable deviations is unilateral, and includes deviations larger than the target. The TLF (Figure 3.2 (a)) is defined as

$$\begin{aligned} \mathbf{L(y)} &= \mathbf{k(y-\tau)^2} \quad \text{if } \mathbf{y > \tau} \\ &= \mathbf{0} \qquad\qquad \textbf{otherwise} \end{aligned} \tag{3.1}$$

   where $\mathbf{k}$ is a constant that provides monetary connotation to the TLF. The constant $\mathbf{k}$ can be determined from the following economic argument: if a loss $\mathbf{A}$ (in monetary units) is associated with some undesirable deviation Δ (in the units of $\mathbf{y}$), then $\mathbf{k}$ is given by

$$\mathbf{k = A/\Delta^2} \tag{3.2}$$

   This loss minimization objective is commonly applied to non-negative product characteristics: (1) with a zero target, and (2) which are deemed as "undesirable" for some specific situation, such as friction, impurity, noise level, etc.

2. Higher-is-better with a finite target (HIBFT): the set of undesirable deviations is unilateral, and includes deviations on the left side of the target, which has a finite value. The corresponding TLF (Figure 3.2 (b)) is

$$\begin{aligned} \mathbf{L(y)} &= \mathbf{k(y-\tau)^2} \quad \text{if } \mathbf{y < \tau} \\ &= \mathbf{0} \qquad\qquad \textbf{otherwise} \end{aligned} \tag{3.3}$$

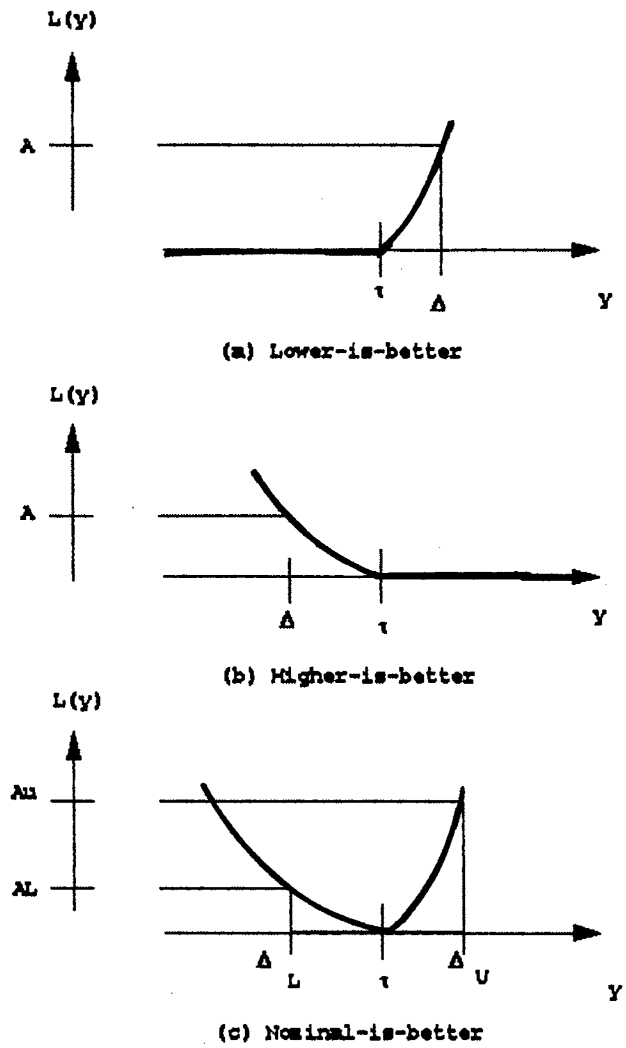

FIG. 3.2 SOME LOSS MINIMIZATION OBJECTIVES UNDER THE TAGUCHI LOSS FUNCTION

The constant **k** can be determined based on an argument similar to the one used in the LIB objective. This objective applies to highly desirable product characteristics with a well-determined, bounded target such as purity, fuel efficiency, etc.

3. Higher-is-better with an undetermined target (HIBUT): product characteristics with this type of objective involve an undetermined target, which approaches ∞. Examples include non-negative product characteristics such as materials strength, adhesive strength, efficiency, etc.

A reciprocal transformation of the product characteristic (given by the variable $\mathbf{z = 1/y}$) allows dealing with the target undetermination, while still having a quadratic loss function applicable. The variable $\mathbf{z}$ is a product characteristic in itself. For any value $\mathbf{c}$ defined on the domain of $\mathbf{y}$, the variable $\mathbf{z}$ has a loss function that satisfies

$$\mathbf{L(z=1/c) = L(y=c)} \qquad (3.4)$$

Furthermore, $\mathbf{z}$ is a product characteristic which: (1) has a zero target, and (2) has a LIB loss minimization objective. Using results for product characteristics with an LIB objective, it can be seen that the TLF of $\mathbf{z}$ is

$$\mathbf{L(z) = kz^2} \qquad (3.5)$$

4. Asymmetric nominal-is-better (ANIB): the set of undesirable deviations is bilateral. Different TLFs apply on each of the sides around the target (Figure 3.2 (c)):

$$\begin{aligned} L(y) &= k_L(y-\tau)^2 \qquad \text{if } y < \tau \\ &= k_U(y-\tau)^2 \qquad \text{if } y > \tau \end{aligned} \qquad (3.6)$$

The constants k_L and k_U can be determined by applying equation (3.2) to deviations on each side around the target and their corresponding losses, i.e., $k_L = A_L/\Delta_L{}^2$, and $k_U = A_U/\Delta_U{}^2$.

Examples of product characteristics with this type of loss minimization include product dimensions such as length of a shift, size of a screw thread, and diameter of a gear. It is assumed that a loss depends on the deviation's: (1) magnitude, and (2) side around the target.

5. Symmetric nominal-is-better (SNIB): the set of undesirable deviations is bilateral. There is a single TLF that applies on both sides around the target:

$$L(y) = k(y-\tau)^2 \qquad (3.7)$$

The constant k can be determined by the same argument that leads to equation (3.2).
Product characteristics with this objective are similar to those described for the ANIB, however, deviations around the target are

penalized depending exclusively on their magnitude, regardless of the side they are on.

An important remark about the constants in equations (3.1) through (3.7), i.e., $\mathbf{k}$, $\mathbf{k_L}$ and $\mathbf{k_U}$ is that, in this research, they are used to represent the instantaneous loss to the product user associated with some undesired deviation ($\mathbf{\Delta}$, $\mathbf{\Delta_L}$, and $\mathbf{\Delta_U}$, respectively). In contrast, all of the literature reviewed (See Subsection 2.4.2) employs these constants to represent the total loss to the manufacturer caused by out-of-specifications conditions (where $\mathbf{\Delta}$, $\mathbf{\Delta_L}$, and $\mathbf{\Delta_U}$, are considered as specification limits). Common manufacturer losses identified from the literature include aggregated costs such as manufacturing, product replacement and repair.

3.3 EXPECTED LOSS AND CLASSIFICATION OF PRODUCT CHARACTERISTICS

As a consequence of the variation inherent in any production process, a product characteristic has a random nature. When the random nature of such a characteristic, $\mathbf{y}$, is considered, it is implicitly assumed that its associated external quality loss is random as well. This loss has an expected value $\mathbf{E[L(y)]}$.

The expected loss, $\mathbf{E[L(y)]}$, as used in this research, is a quality performance measure that expresses a monetary evaluation of the instantaneous risk associated with undesirable deviations around the target.

The purpose of this section is two-fold: (1) to introduce a classification of product characteristics, and (2) to present expressions for the expected loss (derived from using the TLF) for each of the product characteristic classes to be identified.

Depending on the knowledge on the probability distribution of the product characteristic required to determine the expected loss, product characteristics can be grouped into the three classes described below.

Class I: includes product characteristics having LIB, HIBFT, or SNIB loss minimization objectives. To determine $\mathbf{E[L(y)]}$, all the knowledge required on the product characteristic's probability distribution consists of its mean and its variance.

Class II: is associated with product characteristics with the HIBUT loss minimization objective. The expected loss is expressed in terms of the mean and the variance of the transformed product characteristic, i.e., of the reciprocal $\mathbf{z=1/y}$. No further knowledge on the probability distribution of $\mathbf{z}$, other than its variance $\boldsymbol{\sigma_z}^2$, and its mean, $\boldsymbol{\mu_z}$, is required to determine the expected loss. However, determining $\boldsymbol{\sigma_z}^2$ and $\boldsymbol{\mu_z}$ requires knowing the probability distribution of $\mathbf{y}$.

Class III: is related to product characteristics having an ANIB loss minimization objective. To determine the expected loss, full knowledge on the probability distribution of $\mathbf{y}$ is required.

Expressions of the expected loss for the above classes are developed in the following three subsections.

3.3.1 Class I: LIB, HIBFT, and SNIB Objectives

The expected loss for a product characteristic $\mathbf{y}$ in this class is given by

$$\mathbf{E[L(y)] = kE[(y-\tau)^2] = k[\sigma_y{}^2+(\mu_y-\tau)^2]} \qquad (3.8)$$

where μ_y is the mean of $\mathbf{y}$, $\sigma_y{}^2$ denotes its variance, and τ represents its target. This equation can be rewritten in terms of the bias, $\mathbf{b_y}$, or the average deviation of $\mathbf{y}$ around its target, as

$$\mathbf{E[L(y)] = k[\sigma_y{}^2+b_y{}^2]} \qquad (3.9)$$

An examination of equations (3.8) and (3.9) indicates that, for product characteristics within this class:

1. To determine $\mathbf{E[L(y)]}$, it suffices with the product characteristic's variance and mean/bias. No further knowledge of its probability distribution is required.
2. The expected loss is the combination of two additive expected losses: (1) losses due to variance of the product characteristic, and (2) losses due to its mean/bias.

3.3.2 Class II: HIBUT Objective

This class involves product characteristics for which high values are desirable, and the target is assumed to approach ∞. To handle the target undetermination, a variable $\mathbf{z}$ (the reciprocal of $\mathbf{y}$) is defined, such that $\mathbf{z} = \mathbf{1/y}$. The variable $\mathbf{z}$ can itself be considered as a product characteristic with a lower-is-better type of loss minimization objective.

The one-to-one correspondence between $\mathbf{y}$ and $\mathbf{z}$ implies that the loss associated with $\mathbf{y}$ is identical to that of the corresponding $\mathbf{z=1/y}$, i.e.,

$$\mathbf{E[L(y)] = E[L(z=1/y)] = k[\sigma_z^2 + \mu_z^2]} \qquad (3.10)$$

The constant $\mathbf{k}$ can be determined applying equation (3.2) to the variable $\mathbf{z}$, i.e.:

$$\mathbf{k} = \frac{\mathbf{A}}{\Delta^2} \qquad (3.11)$$

where $\mathbf{A}$ is the loss associated with a deviation Δ of $\mathbf{z}$ around its target (zero). As stated earlier, no further knowledge on the probability distribution of $\mathbf{z}$, other than its variance σ_z^2, and its mean, μ_z, is required to determine the expected loss. However, to determine μ_z and σ_z^2, it is required to know the probability density function (pdf) of $\mathbf{y}$, namely $\mathbf{f_Y(y)}$. Being $\mathbf{z}$ is a function of $\mathbf{y}$, its mean and variance are given by (Hines and Montgomery, 1980):

$$\mu_z = \int_Y \frac{1}{y} f_Y(y)\ dy \qquad (3.12)$$

and

$$\sigma_z^2 = \int_Y \frac{1}{y^2} f_Y(y)\ dy - \mu_z^2 \qquad (3.13)$$

Thus, to determine the expected loss of a product characteristic with an HIBUT objective, full knowledge of the probability distribution of $\mathbf{y}$ is required. It will be shown in Subsection 3.4.3, though, that approximations for σ_z^2, μ_z and $E[L(y)]$ can be obtained if all the information available on $\mathbf{y}$ consists of its variance, σ_y^2, and its mean, μ_y.

3.3.3 Class III: ANIB Objective

The TLF for the ANIB loss minimization objective is given in equation (3.6). Its expected value, $E[L(y)]$ is (Kackar, 1985):

$$\begin{aligned} E[L(y)] = \ & k_L E[(y-\tau)^2 \mid y \leq \tau] F_y(\tau) \\ & + k_U E[(y-\tau)^2 \mid y > \tau][1 - F_y(\tau)]\} \end{aligned} \qquad (3.14)$$

or, equivalently

$$E[L(y)] = k_L \int_{-\infty}^{\tau} (y-\tau)^2 f_y(y)\, dy + k_U \int_{\tau}^{\infty} (y-\tau)^2 f_y(y)\, dy \qquad (3.15)$$

Equations (3.14) and (3.15) imply that determining $E[L(y)]$ for a Class III product characteristic requires full knowledge of its probability distribution.

3.4 PRESENT WORTH OF EXPECTED QUALITY LOSSES

This section examines and extends the concept of expected loss, from the standpoint of a discounted cash flow framework. As a result, the present worth of expected quality losses is introduced as a means to perform the translation of quality performance measures into financial measures.

Use of discounted cash flows (which lead to the present worth of quality losses) presents the advantages of being a framework that is common in studies related to long-term financial decisions (Arcelus and Srinivasan, 1993).

Subsection 3.4.1 discusses an extension of the Taguchi Loss Function which allows one to: (1) incorporate the effect of product use on quality performance, and (2) adopt a cash flow viewpoint to determine the present worth of quality losses. Subsections 3.4.2 through 3.4.4 develop expressions of the present worth of expected quality losses for the different classes of product characteristics identified in Section 3.2.

3.4.1 Extending The TLF From A Cash Flow Viewpoint

The loss function **L(y)**, as stated in Section 3.2, relates to the deviation of the product characteristic **y** from its own target at an arbitrary time. However, as a consequence of product use, the product characteristic is a feature that can be changing over time, and so can its loss. Thus, $\mathbf{y}$ is a function of time that can be expressed as $\mathbf{y(t)}$, and the corresponding loss function can be denoted as $\mathbf{L(y;t)}$.

The economic analysis of external quality losses can be done by means of a discounted cash flow viewpoint. Such a viewpoint (White, et al., 1989) considers the time value of money, by assuming that the present value of external quality losses depends on the time the user incurs them.

The loss $\mathbf{L(y;t)}$ can be considered as a continuous cash flow stream (of instantaneous payments) that occurs during a planning horizon $\mathbf{(0,T)}$. For a discount rate $\mathbf{r}$, and assuming continuous compounding (Tanchoco, et al., 1981; Park and Sharp-Bette, 1990), the present worth of quality losses is given by

$$\int_0^T \mathbf{L(y;t)}\ \mathbf{e^{-rt}dt} \qquad (3.16)$$

The loss $\mathbf{L(y;t)}$ is a cash flow stream with an expected value $\mathbf{E[L(y;t)]}$. This is a case in which, according to Young and Contreras (1975), "the expected present worth of a series of payments is equal to the sum of the present worth of the expected payments." Then, the present worth of expected external quality losses, $\mathbf{PW_L}$, for the time interval $\mathbf{(0,T)}$ can be defined as

$$\mathbf{PW_L} = \int_0^T \mathbf{E[L(y;t)]}\ \mathbf{e^{-rt}dt} \qquad (3.17)$$

The following subsections examine $\mathbf{PW_L}$ for product characteristics under each of the different loss minimization objectives.

3.4.2 Present Worth of Quality Losses: Class I Product Characteristics

It is shown in Section 3.3 that the adoption of a Taguchi Loss Function for Class I product characteristics (those having LIB, HIBFT, or SNIB loss minimization objective) makes the expected loss to be the combination of two (additive) expected losses: (1) losses due to variance of the product characteristic, and (2) losses due to its mean/bias. Based on this fact, the expected loss, as a function of time, $\mathbf{E[L(y;t)]}$ can be seen as the combination of two additive cash flow streams. This, in turn implies that the present worth of expected external quality losses can be written as

$$\mathbf{PW_L = PWL_V + PWL_M} \qquad (3.18)$$

where $\mathbf{PWL_V}$ and $\mathbf{PWL_M}$ represent, respectively, the present worth of expected external quality losses due to variance, and the one due to mean/bias.

Equation (3.18) shows that for Class I product characteristics, it is possible to provide a monetary evaluation of the effect of the product characteristic's variance and of the mean on the (total) present worth of external quality losses.

For that matter, let $\boldsymbol{\sigma_y^2(t)}$ and $\boldsymbol{\mu_y(t)}$ be the variance and the mean of $\mathbf{y}$ as functions of time. Likewise, the bias (difference between the mean and the target) is also a function of time, defined as: $\mathbf{b_y(t)} = \boldsymbol{\mu_y(t)} - \tau$. The present worth of expected losses due to the variance of $\mathbf{y}$, $\mathbf{PWL_V}$, is given by

$$\mathbf{PWL_V} = \int_0^T k\ \sigma_Y^2(t)\ e^{-rt}dt \qquad (3.19)$$

and the present worth of expected losses due to mean/bias, $\mathbf{PWL_M}$, is

$$\mathbf{PWL_M} = \int_0^T k\ b_Y^2(t)\ e^{-rt}dt \qquad (3.20)$$

or, equivalently,

$$\mathbf{PWL_M} = \int_0^T k\ (\mu_Y(t)-\tau)^2 e^{-rt}dt \qquad (3.21)$$

3.4.3 Present Worth of Quality Losses: Class II Product Characteristics

Target undetermination represents the central issue for a Class II product characteristic (HIBUT loss minimization objective). Subsection 3.3.2 introduces the reciprocal transformation of the product characteristic as a means to handle this issue while still being able to use a quadratic loss function. The transformed variable $\mathbf{z = 1/y}$ corresponds to a product characteristic with an LIB loss minimization objective.

The expression for the expected loss given in equation (3.10) can be extended to include time. Once this is done, equation (3.17) yields the present worth of expected losses, $\mathbf{PW_L}$, for a product characteristic with an HIBUT objective:

$$PW_L = \int_0^T k[\sigma_z^2(t)+\mu_z^2(t)] \; e^{-rt}dt \qquad (3.22)$$

where $\sigma_z^2(t)$ and $\mu_z^2(t)$ denote the mean and the variance (as a function of time) of the transformed variable z, respectively. Equation (3.22) shows that PW_L consists of two parts, one related to $\sigma_z^2(t)$ and the other, related to $\mu_z^2(t)$. However, the variable z (and therefore $\sigma_z^2(t)$ and $\mu_z^2(t)$) in many cases is meaningless.

The parameters $\sigma_z^2(t)$ and $\mu_z^2(t)$ are defined as

$$\mu_z(t) = \int_{-\infty}^{\infty} \frac{1}{y} f_Y(y;t) \; dy \qquad (3.23)$$

and

$$\sigma_z^2(t)=\int_{-\infty}^{\infty} \frac{1}{y^2} f_Y(y;t)dy-\mu_z(t)^2 \qquad (3.24)$$

These two equations reveal that, in order to determine $\mu_z(t)$, and $\sigma_z^2(t)$, it is required to know the stochastic process underlying y, $f_Y(y;t)$. To overcome such a requirement, the expected loss can be approximated based on expansions of Taylor series for $\mu_z(t)$ and $\sigma_z^2(t)$ (Benjamin and Cornell, 1970). These expansions provide the following approximations:

$$\mu_z(t) \approx \frac{1}{\mu_Y(t)} \qquad (3.25)$$

and

$$\sigma_z^2(t) \approx \frac{4\sigma_y^2(t)}{\mu_y^4(t)} \qquad (3.26)$$

which, substituted into equation (3.22) yield the following approximated value:

$$PW_L \approx k\int_0^T \left[\frac{4\sigma_y^2(t)+\mu_y^2(t)}{\mu_y^4(t)}\right] e^{-rt}dt \qquad (3.27)$$

The way PW_L is defined in this equation precludes separating the interacting effects of $\mu_y(t)$ and $\sigma_y^2(t)$ on PW_L. Then, unlike Class I case, for Class II product characteristics, a meaningful decomposition of PW_L into variance-related and mean-related parts is not possible.

3.4.4 Present Worth of Quality Losses: Class III Product Characteristics

A central issue involving the expected loss of a Class III product characteristic is the loss function asymmetry. This issue implies that two undesired deviations (although having the same magnitude) have different losses if they are on different sides of the target.

Extending the expression of the expected loss (equation (3.15)) to include time, and substituting it into equation (3.17), yields the present worth of expected losses:

$$PW_L = k_L \int_0^T \int_{-\infty}^{\tau} [(y-\tau)^2 f_y(y;t)\,dy]\, e^{-rt} dt$$
$$+ k_U \int_0^T \int_{\tau}^{\infty} [(y-\tau)^2 f_y(y;t)\,dy]\, e^{-rt} dt \qquad (3.28)$$

Equation (3.28) can be expressed in terms of <u>partial moments</u>, or moments defined over a partial domain of **y**, in contrast to the full domain considered for the traditional statistical moments (Winkler, et al., 1972). Two partial domains of **y** arise from equation (3.28): one defined for values of **y** smaller than τ, and the other for values of **y** larger than τ.

<u>Partial means</u> (Buck and Askin, 1986) for the lower side around the target, $\mu_L(t)$, and for the upper side around it $\mu_U(t)$, are respectively defined by

$$\mu_L(t) = \int_{-\infty}^{\tau} y\, f_y(y;t)\, dy \qquad (3.29)$$

and

$$\mu_U(t) = \int_{\tau}^{\infty} y\, f_y(y)\, dy \qquad (3.30)$$

The sum of these partial means is equal to the mean of **y**:

$$\mu_y(t) = \mu_L(t) + \mu_U(t) \qquad (3.31)$$

In a similar fashion, the partial second order moment (with respect to the origin) for the lower side around the target, $SOM_L(t)$, is (Winkler, et al., 1972)

$$SOM_L(t) = \int_{-\infty}^{\tau} y^2 \; f_y(y;t) \; dy \qquad (3.32)$$

whereas the partial second order moment for the interval $y>\tau$ is

$$SOM_U(t) = \int_{\tau}^{\infty} y^2 \; f_y(y;t) \; dy \qquad (3.33)$$

The sum of these partial moments is equal to the second order moment (over the full domain) of **y**:

$$E[y^2;t] = SOM_L(t) + SOM_L(t) \qquad (3.34)$$

Algebraic manipulation of equation (3.28) using partial moments leads to the following expression of PW_L:

$$PW_L = \int_{0}^{T} [A(t)+B(t)+C(t)+D] \; e^{-rt}dt \qquad (3.35)$$

where

$$\begin{aligned} A(t)&=k_L SOM_L(t)+k_U SOM_U(t) \\ B(t)&=-2\tau[k_L\mu_L(t)+k_U\mu_U(t)] \\ C(t)&=\tau^2[(k_L-k_U)F_y(\tau;t)] \\ D &= k_U\tau^2 \end{aligned} \qquad (3.36)$$

It can be seen that

- Part $A(t)$ is a function of only partial second order moments.
- Part $B(t)$ is a function of only partial means.

- Part **C(t)** is a function of the pdf of **y**, and therefore, it depends on both (full domain) mean and variance, and
- Part **D(t)** is independent of distributional parameters.

Then, it can be concluded that for a Class III product characteristic, it is not possible to separate the interacting effects of $\boldsymbol{\mu_y(t)}$ and $\boldsymbol{\sigma_y^2(t)}$ on $\mathbf{PW_L}$ (see equation (3.36)). As a result, it is not possible to break down $\mathbf{PW_L}$ into variance-related and mean-related parts; with the exception being the trivial case of non-drift of both mean and variance of the product characteristic.

3.5 SUMMARY

This chapter introduces the present worth of expected quality losses, $\mathbf{PW_L}$, as a monetary evaluation of external quality losses. The Taguchi Loss Function (TLF) is used to model instantaneous external quality losses.

A classification of product characteristics is made based on the knowledge of their probability distribution required to determine $\mathbf{PW_L}$. Expressions of $\mathbf{PW_L}$ for each class of product characteristics are discussed. It is shown that, for Class I product characteristics, $\mathbf{PW_L}$ can be (1) broken down into two parts: one part is variance-related, and the other is related to the mean/bias of the product characteristic, and (2) determined based only on the product characteristic's variance and mean.

CHAPTER IV

EXTERNAL QUALITY LOSSES UNDER DIFFERENT TYPES OF PRODUCT USE MODELS

4.1 INTRODUCTION

The purpose of this chapter is to examine the effect of product use on the present worth of expected quality losses.

Product use induces degradation on the product's quality performance. As a result, a product characteristic's variance, and/or its mean, change over time. Specific types of variance change over time to be examined include

1. Constant variance. The variance of the product characteristic, $\sigma_y^2(t)$, remains constant, equal to the variance at the time the product exits the production line (assumed to be identical to the time the product is shipped to the user, and defined as $t=0$), i.e.,

$$\sigma_y^2(t)=\sigma_y^2(0) \qquad t\geq 0 \qquad (4.1)$$

2. Linear variance drift. The variance of the product characteristic is assumed to increase at a constant rate $\mathbf{v_1}$ per time unit

$$\sigma_y^2(t)=\sigma_y^2(0)+v_1 t \qquad t\geq 0 \qquad (4.2)$$

The types of change over time to consider for the mean are

1. Constant mean. The mean of the product characteristic, as a function of time $\mu_y(t)$, remains constant, equal to the mean at time $t=0$, i.e.

$$\mu_y(t)=\mu_y(0) \qquad t\geq 0 \qquad (4.3)$$

2. Linear mean drift. The rate of change for the mean is $\mathbf{m_1}$ units per time unit. This yields

$$\mu_y(t)=\mu_y(0)+m_1 t \qquad t\geq 0 \qquad (4.4)$$

3. Quadratic drift. The mean $\mu_y(t)$ includes a quadratic component in time, with an associated constant $\mathbf{m_2}$

$$\mu_y(t)=\mu_y(0)+m_1 t+m_2 t^2 \qquad t\geq 0 \qquad (4.5)$$

Subsections 4.2 through 4.4 investigate the present worth of expected external quality losses under the above described drifts for each of the different classes of product characteristics. A chapter summary is in Subsection 4.5.

4.2 CLASS I LIB, HIBFT, AND SNIB OBJECTIVES

According to Subsection 3.4.2, the present worth of expected quality losses for Class I product characteristics is

$$PW_L = k\int_0^T [\sigma_Y^2(t) + b_Y^2(t)]\ e^{-rt}dt \qquad (4.6)$$

This equation shows that PWL_V represents the sum of two discounted cash flow streams (1) present worth of quality losses due to the variance, PWL_V, given by

$$PWL_V = k\int_0^T \sigma_Y^2(t)\ e^{-rt}dt \qquad (4.7)$$

and (2) present worth of quality losses due to mean/bias, PWL_M

$$PWL_M = k\int_0^T b_Y^2(t)\ e^{-rt}dt \qquad (4.8)$$

Both equations (4.7) and (4.8) involve a discounting scheme in which: (1) the cash flow stream is described by a continuous function, and (2) the discounting is done on a continuous basis. Laplace transforms can be used to handle such a type of discounting schemes (Buck and Hill, 1971 and 1975; Tanchoco, et al., 1981; Park and Sharp-Bette, 1990). Appendix 1 introduces a simplified approach to handle equations (4.7) and (4.8) which takes advantage of the polynomial expressions describing the types of drift considered in this research.

Using this approach, an examination of PWL_V and PWL_M is done in Subsections 4.2.1 and 4.2.2, respectively. The examination includes, for the corresponding present worth: (1) analysis of conditions leading to its minimization, and (2) sensitivity analysis.

4.2.1 Losses Due to Variance

Examination of PWL_V for the different types of variance drift is given below.

Constant Variance

The variance is assumed constant and identical to the variance at the moment the product is shipped to the user, $\sigma_y^2(0)$. Substitution of this constant into equation (4.7) yields

$$PWL_V = k\sigma_y^2(0)\int_0^T e^{-rt}dt \qquad (4.9)$$

using notation and factors defined in Appendix 1, equation (4.9) becomes

$$PWL_V = k\sigma_y^2(0)I(0,r,T) \qquad (4.10)$$

Values of $I(0,r,T)$ for different combinations of r and T are in Table A2.1, Appendix 2.

From equation (4.10), it can be concluded that PWL_V is minimized at $\sigma_y^2(0)=0$.

Two sources of uncertainty can be identified for $\mathbf{PWL_V}$ (1) the financial environment, and (2) the production process. Sensitivity analysis of $\mathbf{PWL_V}$ to changes in $\mathbf{r}$ can be done examining the ratio between $\mathbf{PWL_V}$ (as a function of a change in $\mathbf{r}$) and some base value, $\mathbf{PWL_{V(B)}}$ (White, et al., 1989; Eschenbach and McKeague, 1989; Eschenbach and Gimpel, 1990). A change of $\mathbf{j}$**x100%** in the discount rate $\mathbf{r}$ makes the ratio $\mathbf{PWL_V/PWL_{V(B)}}$ to be

$$\frac{\mathbf{PWL_V}}{\mathbf{PWL_{V(B)}}} = \frac{\mathbf{I(0, (1+j)r, T)}}{\mathbf{I(0, r, T)}} \qquad (4.11)$$

From this ratio, it can be concluded that the effect of a change of $\mathbf{j}$**x100%** in $\mathbf{r}$ makes the present worth to be

$$\mathbf{PWL_V} = \frac{\mathbf{I(0, (1+j)r, T)}}{\mathbf{I(0, r, T)}}\, \mathbf{PWL_{V(B)}} \qquad (4.12)$$

Figures 4.1 and 4.2 display the "spider plots" (Eschenbach and Gimpel, 1990) that show the ratio $\mathbf{PWL_V/PWL_{V(B)}}$ for $\mathbf{T}$=1 and $\mathbf{T}$=5 years, respectively, for different values of $\mathbf{j}$. Joint examination of the spider plots and of equation (4.12) allows one to conclude that $\mathbf{PWL_V}$ (1) decreases with an increment in $\mathbf{r}$, and (2) decreases with $\mathbf{T}$.

As for sensitivity of $\mathbf{PWL_V}$ to changes in $\boldsymbol{\sigma_Y^2(0)}$, the ratio $\mathbf{PWL_V/PWL_{V(B)}}$ as a function of a change of $\mathbf{j}$**x100%** in the variance $\boldsymbol{\sigma_Y^2(0)}$ is

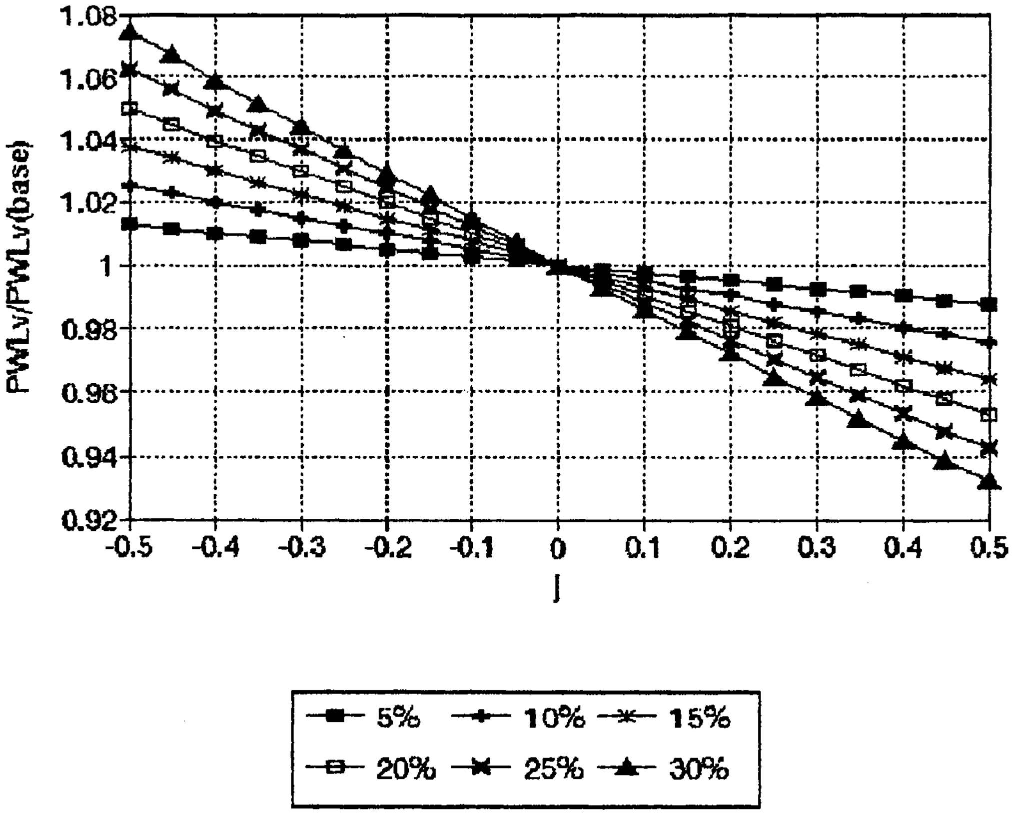

FIG. 4.1 SENSITIVITY OF PWL_V TO CHANGES IN r

(T = 1 year)

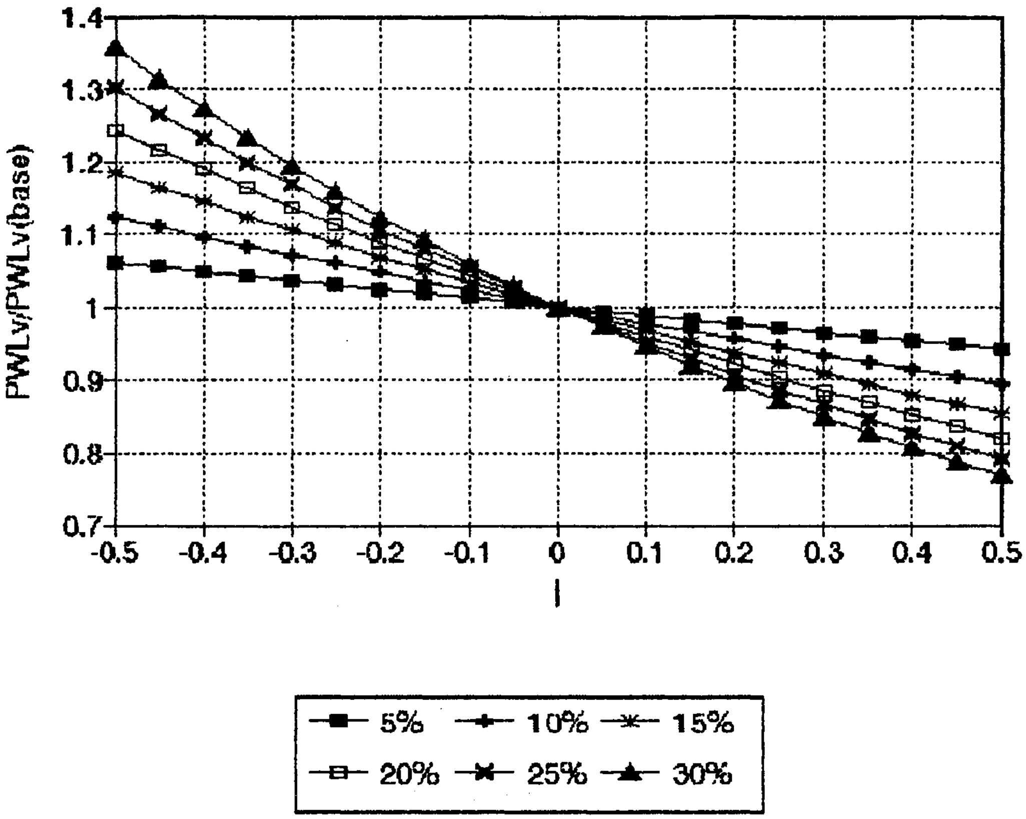

FIG. 4.2 SENSITIVITY OF PWL_V TO CHANGES IN r

(T = 5 years)

$$\frac{PWL_V}{PWL_{V(B)}} = 1+j \qquad j>-1 \qquad (4.13)$$

$$= 0 \qquad \text{elsewhere}$$

Then,

$$PWL_V = (1+j)PWL_{V(B)} \qquad (4.14)$$

This equation describes a straight line which indicates that a reduction (increase) of **jx100%** in $\sigma_y^2(0)$, the variance at $t=0$ induces a reduction (increase) of **jx100%** in PWL_V.

Table 4.1 presents, for a constant variance, a list of possible strategies that allow to reduce PWL_V.

Linear Variance Drift

Under this type of drift, the variance as a function of time is

$$\sigma_y^2(t)=\sigma_y^2(0)+v_1t \qquad t\geq 0, \qquad (4.15)$$

which substituted into equation (4.7) yields the following expression of the present worth of expected quality losses due to the variance

$$PWL_V = k[\sigma_y^2(0)I(0,r,T)+v_1I(1,r,T)] \qquad (4.16)$$

Values of $I(0,r,T)$ and $I(1,r,T)$ for different combinations of r and T are found in Appendix 2, Tables A2.1 and A2.2, respectively.

TABLE 4.1 STRATEGIES TO MINIMIZE PWL_V (CONSTANT VARIANCE DRIFT)

1. Minimize $\sigma^2_Y(0)$, trying to reach

$$\sigma^2_Y(0) = 0$$

2. Increase **r**.

3. Reduce **T**.

According to equation (4.16), PWL_V decreases with a decrement in either $\sigma_y^2(0)$ or in v_1.

Uncertainty of PWL_V, as defined in equation (4.16) derives from (1) the financial environment (reflected through the parameters r and T, (2) the production system (which yields $\sigma_y^2(0)$), and (3) product use (reflected through the parameter v_1).

The partial derivative $\delta PWL_V/\delta r$ can be used to examine the sensitivity of PWL_V to changes in r. Using results from Appendix 1

$$\frac{\delta PWL_V}{\delta r} = -k[\sigma_y^2(0)I(1,r,T)+v_1 I(2,r,T)] \qquad (4.17)$$

As shown in Appendix 1, a change of jx100% in r results in the following (approximated) PWL_V

$$PWL_V \approx jr\frac{\delta PWL_V}{\delta r}+PWL_{V(B)} \qquad (4.18)$$

This equation shows that PWL_V can be reduced by increasing r.

Sensitivity of PWL_V to a change of jx100% in $\sigma_y^2(0)$, depends on the ratio $PWL_V/PWL_{V(B)}$ given by

$$\frac{PWL_V}{PWL_{V(B)}} = j\frac{k\sigma_y^2(0)I(0,r,T)}{PWL_{V(B)}} + 1 \qquad j > -\frac{PWL_{V(B)}}{k\sigma_y^2(0)} \qquad (4.19)$$

$$= 0 \qquad \text{otherwise}$$

Then,

$$PWL_V = jk\sigma_y^2(0)I(0,r,T) + PWL_{V(B)} \qquad (4.20)$$

this equation indicates that reduction of PWL_V can be achieved by reducing $\sigma_y^2(0)$.

Likewise, sensitivity of PWL_V to a change of jx100% in v_1 is related to the ratio

$$\frac{PWL_V}{PWL_{V(B)}} = j\frac{kv_1 I(1, r, T)}{PWL_{V(B)}} + 1 \qquad j > -\frac{PWL_{V(B)}}{kv_1} \tag{4.21}$$

$$= 0 \qquad \text{otherwise}$$

Then, $PWL_{V(B)}$, as a function of j is

$$PWL_V = jkv_1 I(1, r, T) + PWL_{V(B)} \tag{4.22}$$

PWL_V is then directly proportional to j. Thus, PWL_V can be reduced by decreasing v_1.

A list of possible strategies to reduce PWL_V is presented in Table 4.2.

4.2.2 Losses Due to Mean/Bias

Subsection 3.4.2 shows that, for Class I product characteristics, the present worth of expected losses due to mean/bias is given by

$$PWL_M = \int_0^T k\, b_y^2(t)\, e^{-rt} dt \tag{4.23}$$

TABLE 4.2 STRATEGIES TO MINIMIZE PWL_V (LINEAR VARIANCE DRIFT)

1. Minimize $\sigma^2_Y(0)$, trying to reach

$$\sigma^2_Y(0) = 0$$

2. Minimize v_1.
3. Increase r.
4. Reduce T.

or, equivalently in terms of the mean

$$\mathbf{PWL_M} = \int_0^T k\ (\mu_y(t)-\tau)^2\ e^{-rt}dt \qquad (4.24)$$

Analysis of $\mathbf{PWL_M}$ under different types of mean drift is given below.

Constant Mean

Equation (4.3) describes this type of mean drift. It states that the mean, as a function of time, $\mu_y(t)$ remains constant (shows no drift as a consequence of product use), and is equal to the mean at the moment the product is shipped to the user, $\mu_y(0)$. It follows that the bias, $b_y(t)$, remains constant, and identical to $b_y(0)$. Therefore, equation (4.23) becomes

$$\mathbf{PWL_M} = k\ b_y^2(0)I(0,r,T) \qquad ,t\geq 0 \qquad (4.25)$$

Values of $I(0,r,T)$ for different combinations of r and T are in Table A2.1, Appendix 2.

From equation (4.25), it can be concluded that the minimum for $\mathbf{PWL_M}$ occurs at $b_y(0)=0$. This means that, for a Class I product characteristic with a non-drifting mean, the user's quality losses due to the mean are minimized when the production system outputs a product characteristic on target. This result is totally coherent with the existing quality philosophies, which agree on the fact that setting a production process as to yield $b_y(0)=0$ reduces quality losses to the user.

Two sources of uncertainty for PWL_M can be identified from equation (4.20) (1) the financial environment (the discount rate r and the planning horizon T), and (2) the production system (reflected in $b_y(0)$).

A change of $j \times 100\%$ in the discount rate makes the ratio $PWL_M/PWL_{M(B)}$ to be

$$\frac{PWL_M}{PWL_{M(B)}} = \frac{I(0, (1+j)r, T)}{I(0, r, T)} \qquad (4.26)$$

Being this ratio identical to that for a non-drifting variance (equation (4.11)), similar conclusions can be reached. Reduction of PWL_M can be achieved by (1) increasing r, or (2) decreasing T.

Table 4.3 summarizes the possible strategies to reduce PWL_M for the case of a constant mean/bias.

Linear Mean Drift

Under the linear mean drift, the mean as a function of time is

$$\mu_y(t)=\mu_y(0)+m_1 t \quad t\geq 0, \qquad (4.27)$$

and the bias is

$$b_y(t)=b_y(0)+m_1 t \quad t\geq 0, \qquad (4.28)$$

TABLE 4.3 STRATEGIES TO MINIMIZE PWL_M
(CONSTANT MEAN)

1. Set $\mathbf{b_y}(0)$ at

$$b^*_y(0) = 0$$

2. Increase $\mathbf{r}$.

3. Reduce $\mathbf{T}$.

Substituting this value of $b_y(t)$ into equation (4.23) yields the present worth of expected losses due to mean/bias given by

$$PWL_M = k[b_y^2(0)I(0,r,T)+2b_y(0)m_1I(1,r,T) +m_1^2I(2,r,T)] \qquad ,t\geq 0 \qquad (4.29)$$

Values of $I(0,r,T)$, $I(1,r,T)$, and $I(2,r,T)$ for different combinations of r and T can be found in Appendix 2.

From equation (4.29), it can be concluded that, for product characteristics with LIB or HIBFT loss minimization objectives, reduction of PWL_M can be done by reducing $b_y(0)$ and/or reducing m_1.

In the particular case of a product characteristic with an SNIB objective, values of $b_y(0)$ and m_1 that minimize PWL_M can be found from partial derivation of equation (4.24). The optimum for $b_y(0)$ occurs at

$$b_y^*(0) = -m_1\frac{I(1,r,T)}{I(0,r,T)} \qquad (4.30)$$

In contradiction with modern quality philosophies (Wheeler and Chambers, 1992), equation (4.30) shows that (at least for a linear mean drift), setting a production process as to yield $b_y(0)=0$ does not minimizes quality losses to the user.

Likewise, the value of m_1 that minimizes PWL_M is

$$m_1^*=-b_y(0)\frac{I(1,r,T)}{I(2,r,T)} \qquad (4.31)$$

Uncertainty of PWL_M is related to different sources (1) the financial environment (discount rate r and planning horizon T), (2) the production system (reflected in the parameter $b_y(0)$), and (3) product use (as expressed by m_1).

Sensitivity of PWL_M to changes in r is related to the partial derivative $\delta PWL_M/\delta r$. Using results from Appendix 1, it can be shown that

$$\frac{\delta PWL_M}{\delta r} = -k[b_y^2(0)I(1,r,T)+2b_y(0)m_1I(2,r,T) + m_1^2I(3,r,T)] \tag{4.32}$$

which, for a base value $PWL_{M(B)}$, implies that an increase of $j\times100\%$ in r yields the following approximated value of PWL_M

$$PWL_M \approx jr\frac{\delta PWL_M}{\delta r}+PWL_{M(B)} \tag{4.33}$$

Equation (4.33) implies that an increase in r results in a decrement in PWL_M.

As for the sensitivity of PWL_M to changes in $b_y(0)$, the ratio $PWL_M/PWL_{M(B)}$, as a function of a change of $j\times100\%$ in $b_y(0)$ is a parabola given by

$$\frac{PWL_M}{PWL_{M(B)}} = 1+j\ \frac{2k[b_y^2(0)I(0,r,T)+m_1b_y(0)I(1,r,T)]}{PWL_{M(B)}} + j^2\ \frac{kb_y^2(0)I(0,r,T)}{PWL_{M(B)}} \tag{4.34}$$

which implies

$$PWL_M = PWL_{M(B)}+k\{2j[b_y^2(0)I(1,r,T)+b_y(0)m_1I(1,r,T)]$$
$$+j^2b_y^2(0)I(0,r,T)]\} \qquad (4.35)$$

The ratio $PWL_M/PWL_{M(B)}$, as a function of a change of magnitude jx100% in m_1, provides a sensitivity measure for PWL_M to changes in m_1. The ratio is given by

$$\frac{PWL_M}{PWL_{M(B)}} = 1+j\,\frac{2k[b_y(0)m_1I(1,r,T) + m_1^2I(2,r,T)]}{PWL_{M(B)}}$$
$$+j^2\frac{km_1^2I(2,r,T)}{PWL_{M(B)}} \qquad (4.36)$$

Then, PWL_M is

$$PWL_M = PWL_{M(B)}+k\{2j[b_y(0)m_1I(1,r,T)+m_1^2I(2,,r,T)]\} \qquad (4.37)$$

Table 4.4 presents a list of possible strategies to minimize PWL_M under a linear mean drift.

Quadratic Mean Drift

The mean under a quadratic mean drift is given by

$$\mu_y(t)=\mu_y(0)+m_1t+m_2t^2 \qquad t\geq 0, \qquad (4.38)$$

TABLE 4.4 STRATEGIES TO MINIMIZE PWL_M (LINEAR MEAN DRIFT)

PRODUCT CHARACTERISTIC WITH LIB OR HIFT LOSS MINIMIZATION OBJECTIVE

1. Minimize $b_y(0)$.
2. Minimize m_1.
3. Increase r.
4. Reduce T.

PRODUCT CHARACTERISTIC WITH SNIB LOSS MINIMIZATION OBJECTIVE

1. Set $b_y(0)$ at

$$b^*_y(0) = -\frac{[m_1 I(1, r, T) + m_2 I(2, r, T)]}{I(0, r, T)}$$

2. Set m_1 at

$$m_1^* = -\frac{[b_y(0) I(1, r, T) + m_2 I(3, r, T)]}{I(2, r, T)}$$

3. Increase r.
4. Reduce T.

and the bias is

$$b_y(t)=b_y(0)+m_1t+m_2t^2 \qquad t\geq0, \qquad (4.39)$$

which substituted into equation (4.23) yields the present worth of expected losses due to mean/bias

$$PWL_M = k\{b_y^2(0)I(0,r,T)+2b_y(0)m_1I(1,r,T)+[m_1^2+2b_y(0)m_2]I(2,r,T)$$
$$+2m_1m_2I(3,r,T)+m_2^2I(4,r,T)\} \qquad ,t\geq0 \qquad (4.40)$$

For product characteristics with LIB or HIBFT loss minimization objectives, PWL_M can be reduced by decreasing $b_y(0)$, m_1, and/or m_2.

For a product characteristic with an SNIB objective, values of $b_y(0)$, m_1, and m_2 that minimize PWL_M can be found from the corresponding partial derivatives. The optimum for $b_y(0)$ occurs at

$$b^*_y(0)= -\frac{[m_1I(1, r, T) + m_2I(2, r, T)]}{I(0, r, T)} \qquad (4.41)$$

which shows that setting a process on target does not minimizes user's losses.

The optimum for m_1 occurs at

$$m_1^*= -\frac{[b_y(0)I(1, r, T) + m_2I(3, r, T)]}{I(2, r, T)} \qquad (4.42)$$

The optimum for m_2 is at

$$m_2^* = -\frac{[b_y(0)I(2, r, T) + m_1 I(3, r, T)]}{I(4, r, T)} \qquad (4.43)$$

Three different sources account for the uncertainty of PWL_M (1) the financial environment, (2) the production system, and (3) product use. The partial derivative of PWL_M to r can be used to examine the sensitivity of PWL_M to changes in r. Using results from Appendix 1

$$\frac{\delta PWL_M}{\delta r} = -k\{b_y^2(0)I(1,r,T)+2b_y(0)m_1 I(2,r,T)+[m_1^2+2b_y(0)m_2]I(3,r,T)$$
$$+2m_1 m_2 I(4,r,T)+m_2^2 I(5,r,T)\} \qquad (4.44)$$

which allows one to determine the effect of a jx100% change in r, given a base value, as

$$PWL_M = jr\frac{\delta PWL_M}{\delta r}+PWL_{M(B)} \qquad (4.45)$$

This equation indicates that increasing r decreases PWL_M.

To examine the sensitivity of PWL_M to a change of jx100% in $b_y(0)$, the ratio $PWL_M/PWL_{M(B)}$ yields a parabolic function on j

$$\frac{PWL_M}{PWL_{M(B)}} = 1+j\frac{2kb_y(0)[b_y(0)I(0, r, T) + m_1 I(1, r, T) + m_2 I(2, r, T)]}{PWL_{M(B)}}$$
$$+j^2\ \frac{kb_y^2(0)I(0, r, T)}{PWL_{M(B)}} \qquad (4.46)$$

Thus,

$$PWL_M = PWL_{M(B)}+k\{2jb_y(0)[b_y(0)I(0,r,T)+m_1I(1,r,T)+m_2I(2,r,T)]$$
$$+j^2[b_y^2(0)I(0,r,T)\} \qquad (4.47)$$

which indicates that reducing $b_y(0)$ leads to a reduction in PWL_M.

A change of $jx100\%$ in m_1 yields

$$\frac{PWL_M}{PWL_{M(B)}} = 1+j\frac{2k[b_y(0)m_1I(1,r,T)+m_1^2I(2,r,T)+m_1m_2I(3,r,T)]}{PWL_{M(B)}}$$
$$+j^2\ \frac{km_1^2I(2,r,T)}{PWL_{M(B)}} \qquad (4.48)$$

Then,

$$PWL_M = PWL_{M(B)}+k\{2j[b_y(0)m_1I(1,r,T)+m_1^2I(2,r,T)+m_1m_2I(3,r,T)]$$
$$+j^2[m_1^2I(2,r,T)\} \qquad (4.49)$$

Likewise, a change of $jx100\%$ in m_2 yields

$$\frac{PWL_M}{PWL_{M(B)}} = 1+j\frac{2k[b_y(0)m_2I(2,r,T)+m_2^2I(4,r,T)+m_1m_2I(3,r,T)]}{PWL_{M(B)}}$$
$$+j^2\ \frac{km_2^2I(4,r,T)}{PWL_{M(B)}} \qquad (4.50)$$

Then, a change of $jx100\%$ in m_2 yields

$$PWL_M = PWL_{M(B)}+k\{2j[b_y(0)m_2I(2,r,T)+m_2{}^2I(4,r,T)+m_1m_2I(3,r,T)] +j^2m_2{}^2I(4,r,T)\} \quad (4.51)$$

From equations (4.49) and (4.51), it can be concluded that reducing m_1 and/or m_2 leads to a decrement in PWL_M.

Table 4.5 presents a list of possible strategies to minimize PWL_M.

4.3 CLASS II HIBUT OBJECTIVE

As discussed in Subsection 3.4.3, losses associated with a product characteristic **y** having a HIBUT objective can be handled using the transformed variable $z=1/y$. The present worth of expected external quality losses is

$$PW_L = \int_0^T k[\sigma_z{}^2(t)+\mu_z{}^2(t)]\ e^{-rt}dt \qquad (4.52)$$

The parameters $\sigma_z{}^2(t)$ and $\mu_z{}^2(t)$ are defined as

$$\mu_z(t) = \int_{-\infty}^{\tau} \frac{1}{y}\ f_Y(y;t)\ dy \qquad (4.53)$$

and

$$\sigma_z{}^2(t) = \int_{-\infty}^{\tau} \frac{1}{y^2}\ f_Y(y;t)dy-\mu_z(t)^2 \qquad (4.54)$$

TABLE 4.5 STRATEGIES TO MINIMIZE PWL_M (QUADRATIC MEAN DRIFT)

PRODUCT CHARACTERISTIC WITH LIB OF HIFT LOSS MINIMIZATION OBJECTIVE

1. Minimize $b_y(0)$.
2. Minimize m_1.
3. Minimize m_2.
4. Increase r.
5. Reduce T.

PRODUCT CHARACTERISTIC WITH SNIB LOSS MINIMIZATION OBJECTIVE

1. Set $b_y(0)$ at

$$b^*_y(0) = -\frac{[m_1 I(1, r, T) + m_2 I(2, r, T)]}{I(0, r, T)}$$

2. Set m_1 at

$$m_1^* = -\frac{[b_y(0) I(1, r, T) + m_2 I(3, r, T)]}{I(2, r, T)}$$

3. Set m_2 at

$$m_2^* = -\frac{[b_y(0) I(2, r, T) + m_1 I(3, r, T)]}{I(4, r, T)}$$

4. Increase r.
5. Reduce T.

Commonly, equations (4.52), (4.53), and (4.54) require numerical integration. If such is the case, use of the second order moment of $\mathbf{z}$, defined as

$$\mathbf{E[z^2(t)] = \sigma_z{}^2(t) + \mu_z{}^2(t)} \qquad (4.55)$$

$$= \int_{-\infty}^{\infty} \frac{1}{y^2} f_Y(y;t)\,dy$$

to determine $\mathbf{PW_L}$

$$\mathbf{PW_L = k\int_0^T E[z^2(t)]e^{-rt}dt} \qquad (4.56)$$

reduces the number of numerical integrations required to perform.

Usually, the parameters of $\mathbf{z}$ ($\mathbf{\mu_z(t)}$, $\mathbf{\sigma_z{}^2(t)}$, and $\mathbf{E[z^2(t)]}$) do not have a closed form when expressed as functions of the parameters of $\mathbf{y}$. As a result, equations (4.52) and (4.53) do not present an analytically suitable expression. Numerical evaluation of $\mathbf{PW_L}$ under this circumstance requires some sort of discretization of time (Appendix 3 presents a procedure to evaluate the present worth of some cash flow function using time discretization). Appendix 4 presents a **BASIC** program to evaluate $\mathbf{PW_L}$ for a product characteristic which (1) has an HIBUT loss minimization objective, (2) is normally distributed, with mean and variance changing over time according to the drifts considered in this research. The user inputs $\mathbf{r}$, $\mathbf{T}$, $\mathbf{\mu_y(0)}$, $\mathbf{\sigma_y{}^2(0)}$, and the drift parameters for the mean and the variance of the (original) product characteristic $\mathbf{y}$.

An alternative way to evaluate PW_L (particularly useful if $f_Y(y;t)$ is unknown) uses approximations for $\mu_z(t)$, $\sigma_z^2(t)$ and based on Taylor series expansions

$$\mu_z(t) \cong \frac{1}{\mu_Y(t)} \qquad (4.57)$$

and

$$\sigma_z^2(t) \cong \frac{4\sigma_Y^2(t)}{\mu_Y^4(t)} \qquad (4.58)$$

which yields the following approximation for PW_L

$$PW_L \cong k\int_0^T \frac{4\sigma_Y^2(t) + \mu_Y^2(t)}{\mu_Y^4(t)} e^{-rt} dt \qquad (4.59)$$

This approximation does not require knowledge on the stochastic distribution of y.

An examination of PW_L (as given in equations (4.52), (4.56), or (4.59)) allows one to conclude that it is not possible, for a product characteristic with a HIBUT objective, to (1) break down the (total) present worth of expected losses into one part related to the variance of y, and the other part related to its mean, and (2) use formal techniques to perform sensitivity analysis of PW_L to the different uncertainty sources parameters (An exception being cases involving non-drifting mean, when using the approximation for PW_L).

4.4 CLASS III ANIB OBJECTIVE

Subsection 3.4.4 shows that the present worth of expected external quality losses for a product characteristic having an ANIB objective is

$$PW_L = \int_0^T [A(t)+B(t)+C(t)+D]\ e^{-rt}dt \qquad (4.60)$$

where

$$A(t)=k_L SOM_L(t)+k_U SOM_U(t)$$
$$B(t)=-2\tau[k_L\mu_L(t)+k_U\mu_U(t)]$$
$$C(t)=\tau^2[(k_L-k_U)F_y(\tau;t)]$$
$$D = k_U\tau^2 \qquad (4.61)$$

Definitions of partial moments in equation (4.61) are

$$\mu_L(t) = \int_{-\infty}^{\tau} y\ f_y(y;t)\ dy \qquad (4.62)$$

$$\mu_U(t) = \int_{\tau}^{\infty} y\ f_y(y;t)\ dy \qquad (4.63)$$

$$SOM_L(t) = \int_{-\infty}^{\tau} y^2\ f_y(y;t)\ dy \qquad (4.64)$$

and

$$SOM_U(t) = \int_{\tau}^{\infty} y^2\ fy(y;t)\ dy \qquad (4.65)$$

Calculation of partial moments is illustrated in Appendix 5 for the specific case of a normally distributed random variable.

Examination of equation (4.60) allows one to conclude that

1. The definition of partial moments implies knowledge on the stochastic distribution of $\mathbf{y}$, i.e., on $\mathbf{f_y(y;t)}$ as a prerequisite to determine $\mathbf{PW_L}$.
2. The loss asymmetry for a product characteristic with an ANIB objective precludes separating the interacting effects of $\boldsymbol{\mu_y(t)}$ and $\boldsymbol{\sigma_y^2(t)}$ on $\mathbf{PW_L}$. As a result, it is not possible to break down $\mathbf{PW_L}$ into variance-related and mean-related parts.

Then, for a product characteristic having an ANIB objective, it is not possible to (1) break down the total loss into one part related to the variance of $\mathbf{y}$, and the other part related to its mean, and (2) use formal techniques with a general applicability to perform sensitivity analysis of $\mathbf{PW_L}$ to the different uncertainty sources parameters. Sensitivity analysis has to be performed on a case-by-case basis, according to the specific problem situation.

4.5 SUMMARY

This chapter examines, for the different classes of product characteristics, the effect of product use on the present worth of expected external quality losses. Product use is modeled by assuming some specific types of variance and mean drift.

It is shown, for Class I product characteristics, that $\mathbf{PW_L}$ can be broken down into two parts (1) one related to the product characteristic's variance, and (2) other part related to its mean/bias. Analytical expressions to examine sensitivity of $\mathbf{PW_L}$ to changes in different variables are developed. It is found that achieving a product on target (at time $\mathbf{t}$=0) does not minimize the present worth of quality losses (unless there is a constant, non-drifting mean).

CHAPTER V

EXAMPLE AND ANALYSIS OF RESEARCH RESULTS

5.1 INTRODUCTION

This chapter illustrates and examines the use of results from previous chapters to design for quality by means of a hypothetical example.

Section 5.2: (1) presents the hypothetical example (including a model of the product/process linkage), (2) examines the example using both prevailing approaches to design for quality, as well as an approach based on results from this research, and (3) compares approaches. For the same example, Section 5.3 assumes ignorance about the linkage between the product and the production process. This requires statistical fitting of drift models. The section examines the importance of knowledge on product/process linkage. Results derived from the fitted models are compared with those from the "true" conditions (derived in Section 5.2). Section 5.4 presents the chapter summary.

5.2 HYPOTHETICAL EXAMPLE: THE "TRUE" MODEL

Assume a product for which quality is defined in terms of a single product characteristic **y**. The product characteristic has: (1) a symmetric nominal-is-better loss minimization objective, (2) zero target, and (3) mean and variance drifts which are linear, and defined as

$$\mu_y(t) = m_0 + m_1 t$$
$$(5.1)$$
$$\sigma^2_y(t) = v_0 + v_1 t$$

Section 4.2 shows that (since the product characteristic is Class I) no further assumptions on its probability distribution are required to use results from this research.

Furthermore, consider a planning horizon of 5 years, a 10% discount rate, and constant **k** (from the **TLF**) equal to 1.

As discussed in Section 1.4, the linkage between the product characteristic and the process is described by the effect the production parameters have on the product characteristic. For the example, assume the production process can be defined in terms of eight different controllable production parameters, namely X_1, X_2, X_3, X_4, X_5, X_6, X_7, and X_8. However, the drift parameters are functions of only some of the process production parameters, as given by

$$m_0 = -2X_2 + 5X_4 - 3X_7 \qquad (5.2)$$

$$m_1 = X_2-2X_5 \qquad (5.3)$$

$$v_0 = \exp[X_2+X_4] \qquad (5.4)$$

$$v_1 = 4+2X_1+X_2 \qquad (5.5)$$

If the production parameters that determine the product's quality are treated as binary variables that can take the values -1 or 1, then 32 different alternative production settings, or (production alternatives) exist. Table 5.1 shows these alternatives, along with their corresponding drift parameters.

Design for quality relates to the selection of production alternatives that yield a desired product's quality performance. In Subsection 5.2.1, the best production alternatives (for the example) are identified, from the viewpoint of prevailing approaches. The same is done in Subsection 5.2.2, now from the viewpoint of minimization of the present worth of expected external quality losses (i.e., using results from this research). Subsection 5.2.3 compares both types of approaches.

5.2.1 Design For Quality From The Viewpoint Of Prevailing Approaches

As discussed in Section 2.3, designed industrial experimentation is used in industry to analyze the linkage between process and product. Two approaches are common: (1) traditional, and (2) robust design. In a generic way, it can be said that the first type of approaches selects production settings so that the product is on target. The second type considers the product characteristic's variance as well (Hunter, 1985; Nair and Pregibon, 1986, 1987; Box and Bisgaard, 1987; Pignatiello,

TABLE 5.1 PRODUCTION ALTERNATIVES AND THEIR DRIFT PARAMETERS

ALT	x_1	x_2	x_4	x_5	x_7	v_0	v_1	m_0	m_1
1	1	1	1	1	1	7.39	7	0	-1
2	1	1	1	1	-1	7.39	7	6	-1
3	1	1	1	-1	1	7.39	7	0	3
4	1	1	1	-1	-1	7.39	7	6	3
5	1	1	-1	1	1	1	7	-10	-1
6	1	1	-1	1	-1	1	7	-4	-1
7	1	1	-1	-1	1	1	7	-10	3
8	1	1	-1	-1	-1	1	7	-4	3
9	1	-1	1	1	1	1	5	4	-3
10	1	-1	1	1	-1	1	5	10	-3
11	1	-1	1	-1	1	1	5	4	1
12	1	-1	1	-1	-1	1	5	10	1
13	1	-1	-1	1	1	0.14	5	-6	-3
14	1	-1	-1	1	-1	0.14	5	0	-3
15	1	-1	-1	-1	1	0.14	5	-6	1
16	1	-1	-1	-1	-1	0.14	5	0	1
17	-1	1	1	1	1	7.39	3	0	-1
18	-1	1	1	1	-1	7.39	3	6	-1
19	-1	1	1	-1	1	7.39	3	0	3
20	-1	1	1	-1	-1	7.39	3	6	3
21	-1	1	-1	1	1	1	3	-10	-1
22	-1	1	-1	1	-1	1	3	-4	-1
23	-1	1	-1	-1	1	1	3	-10	3
24	-1	1	-1	-1	-1	1	3	-4	3
25	-1	-1	1	1	1	1	1	4	-3
26	-1	-1	1	1	-1	1	1	10	-3
27	-1	-1	1	-1	1	1	1	4	1
28	-1	-1	1	-1	-1	1	1	10	1
29	-1	-1	-1	1	1	0.14	1	-6	-3
30	-1	-1	-1	1	-1	0.14	1	0	-3
31	-1	-1	-1	-1	1	0.14	1	-6	1
32	-1	-1	-1	-1	-1	0.14	1	0	1

1988; Tribus and Szonyi, 1989; Vining and Myers, 1990; Shoemaker, et al., 1991; Box and Jones, 1992). Both types of approaches are limited to the time the product exits the production line (i.e., $\mathbf{t}$=0).

A traditional designed experimentation approach ranks alternatives based only on the value of the mean/bias at time 0. From this viewpoint, for SNIB product characteristics (such as the example's): (1) a production alternative with a zero bias is always preferred over an alternative with a bias different from zero, and (2) two alternatives with equal absolute value of $\mathbf{m_0}$ have the same ranking. Table 5.2 shows the ranking of alternatives these type of approaches would yield. Eight of the alternatives are equally ranked at the top.

Robust design approaches are intended to look at the mean ($\mathbf{m_0}$) and the variance ($\mathbf{v_0}$) simultaneously. From this viewpoint, the example's 32 alternatives can be classified into six groups. These groups are shown in Table 5.3. One can conclude that: (1) group I (alternatives 14, 16, 30, and 32) dominates the rest of the groups (it has both minimum bias, as well as minimum variance), and (2) group III dominates group VI (both have the same variance, the former has lower bias), (3) group IV dominates group V (having both the same bias, the former has lower variance), and (3) there is no clear dominance among groups II, III, and IV (they present different means and variances, and a dominance rule has not been defined).

TABLE 5.2 RANKING OF ALTERNATIVES ACCORDING TO TRADITIONAL DESIGNED EXPERIMENTATION APPROACHES

RANKING	ALTERNATIVES
1	1, 3, 14, 16, 17, 19, 30, 32
2	6, 8, 9, 11, 22, 24, 25, 27
3	2, 4, 13, 15, 18, 20, 29, 31
4	5, 7, 10, 12, 21, 23, 26, 28

TABLE 5.3 RANKING OF ALTERNATIVES ACCORDING TO ROBUST DESIGN EXPERIMENTATION APPROACHES

GROUP	m_0	v_0	ALTERNATIVES
I	0	0.14	14, 16, 30, 32
II	0	7.39	1, 3, 17, 19
III	$\lvert 4 \rvert$	1	6, 8, 9, 11, 22, 24, 25, 27
IV	$\lvert 6 \rvert$	0.14	13, 15, 29, 31
V	$\lvert 6 \rvert$	7.39	2, 4, 18, 20
VI	$\lvert 10 \rvert$	1	5, 7, 10, 12 21, 23, 26, 28

5.2.2 Design For Quality Based On Minimization Of PW_L

Using results from Chapter 4, equivalent present worth of external quality losses due to variance PWL_V, and due to mean PWL_M (as functions of the process parameters) are

$$PWL_V = k\{\exp[X_2+X_4]I(0,10\%,5)+[4+2X_1+X_2]I(1,10\%,5)\} \qquad (5.6)$$
$$= 3.9347\exp[X_2+X_4]+9.0204[4+2X_1+X_2]$$

and

$$PWL_M = k\{(-2X_2+5X_4-3X_7)^2(0,10\%,5)+2(-2X_2+5X_4-3X_7)(X_2-2X_5)I(1,10\%,5)$$
$$+(X_2-2X_5)^2I(2,10\%,5)\} \qquad (5.7)$$
$$= \{3.9347(-2X_2+5X_4-3X_7)^2+2(9.0204)(-2X_2+5X_4-3X_7)(X_2-2X_5)$$
$$+28.7754(X_2-2X_5)^2\}$$

Values of the factors **I(0,10%,5)**, **I(1,10%,5)**, and **I(2,10%,5)** can be found in Appendix 2.

According to equation (4.6) the two above expressions can be combined to yield the equivalent present worth of external quality losses PW_L, as

$$PW_L = PWL_V + PWL_M \qquad (5.8)$$

Table 5.4 shows, for the example's 32 alternatives, the equivalent present worth of losses: (1) due to variance PWL_V, (2) due to mean/bias PWL_M, and (3) total PW_L. It can be seen that PW_L is minimized at

TABLE 5.4 PRESENT WORTH OF LOSSES FOR THE 32 ALTERNATIVES

ALT	PWL_V	PWL_M	PW_L
1	$ 92.22	$ 28.78	$ 120.99
2	92.22	62.18	154.40
3	92.22	258.98	351.19
4	92.22	725.36	817.58
5	67.08	602.65	669.73
6	67.08	163.89	230.97
7	67.08	111.22	178.30
8	67.08	105.44	172.52
9	49.04	105.44	154.48
10	49.04	111.22	160.26
11	49.04	163.89	212.93
12	49.04	602.65	651.69
13	45.63	725.36	771.00
14	45.63	258.98	304.61
15	45.63	62.18	107.81
16	45.63	28.78	74.41
17	56.13	28.78	84.91
18	56.13	62.18	118.31
19	56.13	258.98	315.11
20	56.13	725.36	781.50
21	31.00	602.65	633.65
22	31.00	163.89	194.89
23	31.00	111.22	142.22
24	31.00	105.44	136.44
25	12.96	105.44	118.40
26	12.96	111.22	124.18
27	12.96	163.89	176.85
28	12.96	602.65	615.61
29	9.55	725.36	734.91
30	9.55	258.98	268.53
31	9.55	62.18	71.73
32	9.55	28.78	38.33

alternative 32 (for which $\mathbf{PW_L}$ is \$38.33/product unit). Alternative 32 is then the best production alternative.

Observe that: (1) four alternatives minimize $\mathbf{PWL_V}$ (alternatives 29, 30, 31 and 32), and (2) four alternatives minimize $\mathbf{PWL_M}$ (1, 16, 17, and 32). This shows that minimizing either $\mathbf{PWL_V}$ or $\mathbf{PWL_M}$ does not imply the global minimization of $\mathbf{PW_L}$. The presence of the top ranked alternative (32) in both sets of alternatives is merely circumstantial. This fact should not be considered as a condition for an alternative to be dominant.

To examine the effect of the time value of money, ranking of alternatives under diverse values of the planning horizon $\mathbf{T}$ and the discount rate $\mathbf{r}$ can be considered. Table 5.5 shows the top 10 alternatives that result from varying either $\mathbf{T}$ or $\mathbf{r}$. It is seen that (1) alternative 32 is consistently the dominant alternative, however (2) the ranking of alternatives varies from one set of conditions to another. For example, observe that the runner up alternative for: (1) $\mathbf{r}$=5% is different for $\mathbf{T}$=5 years (alternative 31) than the one for $\mathbf{T}$=3 years (alternative 25), and (2) $\mathbf{T}$=5 years changes for different discount rates (alternative 31 is the runner up for $\mathbf{r}$=5% and $\mathbf{r}$=10%, whereas alternative 16 is the runner up for $\mathbf{r}$=15%). This shows that the time value of money is a factor that cannot be excluded for the economic analysis of external quality losses.

Section 4.2 presents strategies to optimize $\mathbf{PWL_V}$ and $\mathbf{PWL_M}$. To illustrate these strategies using the example, assume it is desired to minimize $\mathbf{PWL_M}$. From equation (4.30) (since $\mathbf{b_y(0)}=\mathbf{m_0}=0$), the present worth of losses due to mean is minimized when the value of $\mathbf{m_0}$ is

TABLE 5.5 EFFECT OF TIME VALUE OF MONEY: ALTERNATIVES RANKING FOR DIFFERENT VALUES OF T AND r

RANK	T=5 yrs, r=5%	T=3 yrs, r=5%	T=5 yrs, r=10%	T=3 yrs, r=10%	T=5 yrs, r=15%	T=3 yrs, r=15%
1	32	32	32	32	32	32
2	31	25	31	25	16	25
3	16	16	16	16	31	16
4	17	24	17	24	17	24
5	15	17	15	17	15	17
6	18	9	18	9	25	9
7	26	8	25	8	1	8
8	1	1	1	1	18	1
9	25	31	26	31	24	31
10	23	30	24	30	26	30

$$b^*_y(0) = m^*_0 = -m_1 I(1,10\%,5)/I(0,10\%,5)$$
$$= -m_1(9.0204)/(3.9346)$$
$$= -2.29m_1$$

Table 5.6 shows, for each of the 32 alternatives: (1) the values of $\mathbf{b_y^*(0)}$ that optimize $\mathbf{PWL_M}$, (2) the original values of $\mathbf{PWL_M}$ and $\mathbf{PW_L}$ (taken from table 5.4), (3) the values of $\mathbf{PWL_M}$ and $\mathbf{PW_L}$ associated with $\mathbf{b_y^*(0)}$, and (4) the difference between $\mathbf{PWL_M}$ and $\mathbf{PWL_M^*}$, i.e., the savings derived of making the production process to yield $\mathbf{b_y^*(0)}$. Possible reductions in $\mathbf{PW_L}$ are significant, in absolute terms (savings for the user of up to $652.50 can be achieved for alternatives 4, 13, 20, and 29) as well as relatively speaking (for example, even for alternative 32, the top ranked alternative, $\mathbf{PW_L}$ can be reduced up to 54%).

To illustrate the way the production process can be set to achieve some desired $\mathbf{PW_L}$, consider the following specific situation: (1) the actual process settings correspond to some production alternative, say alternative 11, and (2) it is desired to minimize $\mathbf{PW_L}$ by changing only the mean at time zero. The optimum value of $\mathbf{m_0}$ for alternative 11 (from Table 5.6, and considering that the target is zero) is $\mathbf{m_0^*=b_y^*(0)}=-2.29$. Equation (5.2) yields

$$m_0^* = -2X_2+5X_4-3X_7 \qquad (5.2)$$
$$-2.29 = -2X_2+5X_4-3X_7$$

This equation, along with equations 5.3 through 5.5 describe the linkage between the process and the product's quality. After examining these equations, one can conclude that (to leave

TABLE 5.6 OPTIMUM VALUES OF PWL_M

ALT	b*	PWL_M	PW_L	PWL_M*	PW_L*	SAVINGS
1	2.29	$28.78	$120.99	$8.10	$100.31	$20.68
2	2.29	62.18	154.40	8.10	100.31	54.08
3	-6.88	258.98	351.19	72.86	165.08	186.12
4	-6.88	725.36	817.58	72.86	165.08	652.50
5	2.29	602.65	669.73	8.10	75.17	594.56
6	2.29	163.89	230.97	8.10	75.17	155.80
7	-6.88	111.22	178.30	72.86	139.94	38.36
8	-6.88	105.44	172.52	72.86	139.94	32.58
9	6.88	105.44	154.48	72.86	121.90	32.58
10	6.88	111.22	160.26	72.86	121.90	38.36
11	-2.29	163.89	212.93	8.10	57.13	155.80
12	-2.29	602.65	651.69	8.10	57.13	594.56
13	6.88	725.36	771.00	72.86	118.50	652.50
14	6.88	258.98	304.61	72.86	118.50	186.12
15	-2.29	62.18	107.81	8.10	53.73	54.08
16	-2.29	28.78	74.41	8.10	53.73	20.68
17	2.29	28.78	84.91	8.10	64.23	20.68
18	2.29	62.18	118.31	8.10	64.23	54.08
19	-6.88	258.98	315.11	72.86	129.00	186.12
20	-6.88	725.36	781.50	72.86	129.00	652.50
21	2.29	602.65	633.65	8.10	39.09	594.56
22	2.29	163.89	194.89	8.10	39.09	155.80
23	-6.88	111.22	142.22	72.86	103.86	38.36
24	-6.88	105.44	136.44	72.86	103.86	32.58
25	6.88	105.44	118.40	72.86	85.82	32.58
26	6.88	111.22	124.18	72.86	85.82	38.36
27	-2.29	163.89	176.85	8.10	21.05	155.80
28	-2.29	602.65	615.61	8.10	21.05	594.56
29	6.88	725.36	734.81	72.86	82.42	652.50
30	6.88	258.98	268.53	72.86	82.42	186.12
31	-2.29	62.18	71.73	8.10	17.65	54.08
32	-2.29	28.78	38.33	8.10	17.65	20.68

unchanged drift parameters other than the mean at time zero), X_2 and X_4 have to be kept at their actual values (-1 and 1, respectively). The only process parameter left to change then is X_7. The value of X_7 that minimizes PW_L (or equivalently, as seen in Table 5.6, that allows achieving PW_L=\$57.13) is

$$\begin{aligned} X_7^* &= (2.29-2X_2+5X_4)/3 \\ &= 3.10 \end{aligned}$$

5.2.3 Comparison Of Approaches

Prevailing approaches used to design for quality (discussed in Subsection 5.2.1) look at the quality of products only at time t=0. They ignore the possibility of having a product characteristic with a mean and/or a variance changing over time. As a result, these types of approaches fail to consider (1) product degradation over time, and (2) the time value of money.

Using concepts from this research, these approaches assume (1) non-drifting mean, (2) non-drifting variance, and (3) r=0. Under these conditions, Table 5.7 presents, for each of the 32 example's alternatives values of present worth of: (1) losses due to variance PWL_V, (2) losses due to mean/bias PWL_M, and (3) total losses, PW_L. Note that alternatives that minimize only PWL_M coincide with the top alternatives identified in Table 5.2 (i.e., those obtained using traditional designed experimentation approaches). Likewise, the four alternatives that minimize PW_L coincide with those identified in Table 5.3 as group I (i.e., the top alternatives from a robust design

TABLE 5.7 PRESENT WORTH OF LOSSES FOR THE 32 ALTERNATIVES

(non-drifting mean and variance, r=0%)

ALT	PWL_V	PWL_M	PW_L
1	36.95	0.00	36.95
2	36.95	180.00	216.95
3	36.95	0.00	36.95
4	36.95	180.00	216.95
5	5.00	500.00	505.00
6	5.00	80.00	85.00
7	5.00	500.00	505.00
8	5.00	80.00	85.00
9	5.00	80.00	85.00
10	5.00	500.00	505.00
11	5.00	80.00	85.00
12	5.00	500.00	505.00
13	0.68	180.00	180.68
14	0.68	0.00	0.68
15	0.68	180.00	180.68
16	0.68	0.00	0.68
17	36.95	0.00	36.95
18	36.95	180.00	216.95
19	36.95	0.00	36.95
20	36.95	180.00	216.95
21	5.00	500.00	505.00
22	5.00	80.00	85.00
23	5.00	500.00	505.00
24	5.00	80.00	85.00
25	5.00	80.00	85.00
26	5.00	500.00	505.00
27	5.00	80.00	85.00
28	5.00	500.00	505.00
29	0.68	180.00	180.68
30	0.68	0.00	0.68
31	0.68	180.00	180.68
32	0.68	0.00	0.68

viewpoint). Traditional designed experimentation approaches yield the same results as the minimization of $\mathbf{PWL_M}$ under a non-drifting mean. Robust design approaches are equivalent to an approach that minimizes $\mathbf{PW_L}$ under non-drifting mean and variance.

In contrast, the proposed approach to design for quality (i.e., based on the minimization of $\mathbf{PW_L}$) considers both product degradation and the time value of money. An additional advantage is that (unlike the prevailing approaches for design for quality) its units can be expressed in monetary terms.

5.3 FITTING OF DRIFT MODELS

This section examines consequences of lack of knowledge on the linkage between the production process and the product's quality. The example from the previous section is used. It is assumed (1) the mean and variance drifts are linear (equation 5.1), and (2) drift parameters are those modeled in equations (5.2) through (5.5). However, these parameters are assumed unknown and are to be estimated. The resulting estimates are used to fit models for the drifts of the mean and the variance.

The section does not intend to examine appropriateness or statistical validity of sampling and estimation procedures used. Its purpose is to provide an insight into effects of the uncertainty regarding the production process/product quality linkage.

It is convenient to remark that practical applications of the results of this research are dependent on estimation of the (variance and mean) drifts models. This fact might be, in some cases, associated with limitations of varied nature, such as cost, time constraints, and technical difficulties.

Subsection 5.3.1 generates a set of data from which drift parameters are estimated. Subsection 5.3.2 uses these parameters to fit drift models and compare them with the "true" models presented in Section 5.2.

5.3.1 Data Generation And Estimation Of Drift Parameters

Knowledge on the linkage between the process and the product can be gained from:

- Designed experimentation, to determine the values of the mean and the variance at time zero, i.e., to estimate the intercept of the corresponding drift model.
- Life testing, to determine models for the mean and the variance drifts. For the example, it allows one to determine values for the slope of both drifts.

Data to estimate the mean and the variance at $\mathbf{t}$=0 were obtained using Monte Carlo simulation. The data were generated: (1) assuming a normally distributed product characteristic with a mean given by equation (5.2) and a variance given by equation (5.4), and (2) using a 2^{8-4} fractional factorial experiment with four replicates at each of the 16 factor combinations (the experiment incorporates three extra variables not affecting any of the drift parameters: $\mathbf{X_3}$, $\mathbf{X_6}$, and $\mathbf{X_8}$).

Each of the factor combinations represents one production alternative. Table 5.8 shows (1) the 16 treatment combinations (equivalent to the alternatives' production settings), and (2) the 64 experimental observations, four for each of the alternatives.

Data to estimate the slopes of mean and variance drifts were obtained from a Monte Carlo simulated life test. The life test was generated: (1) assuming the product characteristic $\mathbf{y}$ is normal, with mean and variance given by equations (5.1) through (5.5), and (2) using the 16 treatment combinations (production alternatives) from the designed experiment. Equally spaced observations were made for each of the 16 treatment combinations (at years 1, 2, and 3), with four replicates per year. For each production alternative, Table 5.9 shows (1) values of production settings ($\mathbf{X_1}$, $\mathbf{X_2}$, ..., $\mathbf{X_8}$), (2) observation time (in years), and (3) its four observations per year.

From the designed experiment, the mean at time zero was estimated using least squares on the experiment observations. The resulting estimator (for a 5% significance level) is

$$\hat{\mathbf{m}}_0 = -1.7596\mathbf{X}_2+4.7053\mathbf{X}_4-2.9868\mathbf{X}_7 \qquad (5.9)$$

This estimator can be compared to the true mean drift intercept, given in equation (5.2) as $\mathbf{m}_0=-2\mathbf{X}_2+5\mathbf{X}_4-3\mathbf{X}_7$.

To estimate the variance at time zero, least squares estimation on the natural log of the variances for each of the 16 treatment combinations was performed. The estimator (for a 5% significance level) is

TABLE 5.8 DATA FROM THE DESIGNED EXPERIMENT

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	y_1	y_2	y_3	y_4
-1	-1	-1	-1	-1	-1	-1	-1	-0.477	0.164	-0.174	0.505
1	-1	-1	-1	1	1	1	-1	-6.274	-6.230	-6.542	-5.483
-1	1	-1	-1	1	-1	1	1	-3.903	-4.255	-2.721	-1.911
1	1	-1	-1	-1	1	1	1	-10.249	-11.015	-8.287	-10.974
-1	-1	1	-1	1	-1	1	1	-5.537	-5.967	-5.733	-5.663
1	-1	1	-1	-1	1	-1	1	-0.475	-0.479	-0.869	-0.386
-1	1	1	-1	-1	1	1	-1	-7.905	-9.990	-10.224	-8.360
1	1	1	-1	1	-1	-1	-1	-3.900	-4.684	-3.446	-3.826
-1	-1	-1	1	-1	1	1	1	2.943	3.250	4.565	3.024
1	-1	-1	1	1	1	-1	1	10.001	10.068	9.145	8.742
-1	1	-1	1	1	-1	1	-1	-3.746	2.892	-0.482	-2.951
1	1	-1	1	-1	1	-1	-1	5.858	3.390	5.488	2.537
-1	-1	1	1	1	1	-1	-1	7.511	9.568	9.266	9.700
1	-1	1	1	-1	-1	1	-1	5.136	2.783	2.157	3.370
-1	1	1	1	-1	-1	-1	1	8.436	5.707	11.678	4.621
1	1	1	1	1	1	1	1	1.434	-2.611	4.237	-1.845

TABLE 5.9 DATA FROM THE LIFE TEST

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	t	y_1	y_2	y_3	y_4
-1	-1	-1	-1	-1	-1	-1	-1	1	4.37	-2.56	0.26	3.50
-1	-1	-1	-1	-1	-1	-1	-1	2	1.85	-0.06	1.44	3.21
-1	-1	-1	-1	-1	-1	-1	-1	3	5.14	-3.21	-2.86	7.20
1	-1	-1	-1	1	1	1	-1	1	-9.62	-10.74	-9.46	-11.56
1	-1	-1	-1	1	1	1	-1	2	-13.06	-14.76	-15.51	-10.40
1	-1	-1	-1	1	1	1	-1	3	-16.41	-11.52	-23.81	-16.99
-1	1	-1	-1	1	1	-1	1	1	-5.01	-5.72	-6.75	-4.40
-1	1	-1	-1	1	1	-1	1	2	-8.56	-7.62	-5.30	-4.18
-1	1	-1	-1	1	1	-1	1	3	-8.29	-4.57	-7.63	-9.12
1	1	-1	-1	-1	-1	1	1	1	-3.48	-6.79	-8.07	-4.42
1	1	-1	-1	-1	-1	1	1	2	-0.86	-1.15	-6.96	0.78
1	1	-1	-1	-1	-1	1	1	3	6.86	-1.42	-10.56	-4.13
-1	-1	1	-1	1	-1	1	1	1	-9.19	-9.65	-8.68	-7.26
-1	-1	1	-1	1	-1	1	1	2	-7.48	-10.13	-13.08	-13.61
-1	-1	1	-1	1	-1	1	1	3	-14.88	-14.72	-17.06	-14.76
1	-1	1	-1	-1	1	-1	1	1	-1.77	-0.27	-1.20	-2.37
1	-1	1	-1	-1	1	-1	1	2	-0.69	-0.04	1.50	2.42
1	-1	1	-1	-1	1	-1	1	3	8.07	5.38	3.82	4.88
-1	1	1	-1	-1	1	1	-1	1	-7.15	-6.13	-6.95	-9.60
-1	1	1	-1	-1	1	1	-1	2	-2.45	-5.91	-2.89	-1.98
-1	1	1	-1	-1	1	1	-1	3	-2.04	0.32	0.38	0.41
1	1	1	-1	1	-1	-1	-1	1	-6.47	-5.17	-8.97	-2.29
1	1	1	-1	1	-1	-1	-1	2	-7.81	-9.79	-3.00	-5.89
1	1	1	-1	1	-1	-1	-1	3	-3.46	-0.09	-9.35	-8.95
-1	-1	-1	1	-1	1	1	1	1	6.89	4.12	3.04	2.03
-1	-1	-1	1	-1	1	1	1	2	1.86	8.34	2.28	8.52
-1	-1	-1	1	-1	1	1	1	3	3.58	7.86	3.99	6.62
1	-1	-1	1	1	-1	-1	1	1	6.80	9.63	8.29	2.67
1	-1	-1	1	1	-1	-1	1	2	3.99	3.55	7.52	2.95
1	-1	-1	1	1	-1	-1	1	3	2.74	3.55	6.33	-2.13

TABLE 5.9 DATA FROM THE LIFE TEST

(Continued)

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	t	y_1	y_2	y_3	y_4
-1	1	-1	1	1	-1	1	-1	1	2.14	5.96	-1.76	2.34
-1	1	-1	1	1	-1	1	-1	2	0.80	1.48	-1.54	-5.09
-1	1	-1	1	1	-1	1	-1	3	-3.52	-1.68	1.17	-4.67
1	1	-1	1	-1	1	-1	-1	1	10.53	10.03	10.96	4.68
1	1	-1	1	-1	1	-1	-1	2	8.19	12.54	10.77	5.32
1	1	-1	1	-1	1	-1	-1	3	17.38	20.13	13.06	8.33
-1	-1	1	1	1	1	-1	-1	1	9.02	7.95	4.45	5.59
-1	-1	1	1	1	1	-1	-1	2	7.87	3.95	0.57	6.25
-1	-1	1	1	1	1	-1	-1	3	-3.99	-5.57	-1.55	2.95
1	-1	1	1	-1	-1	1	-1	1	4.61	5.77	5.94	8.05
1	-1	1	1	-1	-1	1	-1	2	5.19	6.96	6.52	2.11
1	-1	1	1	-1	-1	1	-1	3	11.92	1.51	5.58	16.42
-1	1	1	1	-1	-1	-1	1	1	9.91	8.65	8.84	10.38
-1	1	1	1	-1	-1	-1	1	2	10.03	12.88	13.57	12.23
-1	1	1	1	-1	-1	-1	1	3	17.21	12.23	15.12	12.28
1	1	1	1	1	1	1	1	1	-5.58	4.55	-3.42	-3.71
1	1	1	1	1	1	1	1	1	2.39	3.04	-3.39	-8.53
1	1	1	1	1	1	1	1	3	-9.11	5.99	5.32	-8.30

$$\hat{v}_0 = \exp[1.1270X_2+1.0628X_4] \qquad (5.10)$$

which again, can be compared to the true variance drift intercept, given in equation (5.4) as $v_0=\exp[X_2+X_4]$.

From the life test, estimates for the slopes of the mean and the variance drift were obtained. The slope of the mean drift m_1 was estimated from the difference between observed values of y for two consecutive years. Least squares estimation was performed on these differences to get an estimator of m_1 (on the process parameters, X_1, X_2,..., and X_8). The estimator (for a 5% significance level) is

$$\hat{m}_1 = 0.95740X_2-2.0600X_5 \qquad (5.11)$$

which can be compared to the true mean drift slope, given in equation (5.3) as $m_1=X_2-2X_5$.

A similar procedure was used to estimate the model for the slope of the variance drift, v_1. The estimator obtained from backward elimination (Netter, et al, 1989),for a 5% significance level is

$$\hat{v}_1 = 4.3725+2.7658X_1+0.66925X_2 \qquad (5.12)$$

This estimator can be compared with the true variance drift slope, $v_1=4+2X_1+X_2$ (equation 5.5).

5.3.2 Model Fitting

Equations (5.9) and (5.11) can be combined to obtain the fitted model for the mean drift

$$\mu_y(t) = \hat{m}_0 + \hat{m}_1 \tag{5.13}$$
$$= (-1.7596X_2+4.7053X_4-2.9868X_7)$$
$$+(0.95740X_2-2.0600X_5)t$$

Likewise, from equations (5.10) and (5.12), the fitted model for the variance can be obtained as

$$\sigma^2_y(t) = \hat{v}_0+ \hat{v}_1 \tag{5.14}$$
$$= \exp[1.1270X_2+1.0628X_4]$$
$$+ (4.3725+2.7658X_1+0.6693X_2)$$

These two fitted models can be used to estimate the present worth of expected quality losses using results from Chapter 4. For each of the 32 example's alternatives, Table 5.10 presents (1) drift parameter estimates, obtained from equations (5.9) through (5.12), and (2) the estimated present worth of losses due to the variance $\mathbf{PWL_V}$, due to mean/bias $\mathbf{PWL_M}$, and total $\mathbf{PW_L}$.

Discrepancies among true and estimated values are presented in Table 5.11. This table shows the percent error of estimated present worth of losses ($\mathbf{PW_{Lest}}$) relative to its true value ($\mathbf{PW_{Ltrue}}$). The absolute error was (1) less than 10% for 23 of the 32 production

TABLE 5.10 ESTIMATED PRESENT WORTH OF LOSSES

ALT	v_0	v_1	m_0	m_1	PWL_V	PWL_M	PW_L
1	8.93	7.81	-0.0	-1.1	105.58	35.81	141.38
2	8.93	7.81	5.93	-1.1	105.58	55.45	161.03
3	8.93	7.81	-0.0	3.02	105.58	259.76	365.34
4	8.93	7.81	5.93	3.02	105.58	723.41	828.99
5	1.07	7.81	-9.5	-1.1	74.62	574.50	649.12
6	1.07	7.81	-3.5	-1.1	74.62	151.77	226.39
7	1.07	7.81	-9.5	3.02	74.62	98.98	173.60
8	1.07	7.81	-3.5	3.02	74.62	120.25	194.88
9	0.94	6.47	3.48	-3.0	62.04	120.25	182.30
10	0.94	6.47	9.45	-3.0	62.04	98.98	161.02
11	0.94	6.47	3.48	1.1	62.04	151.77	213.81
12	0.94	6.47	9.45	1.1	62.04	574.50	636.54
13	0.11	6.47	-5.9	-3.0	58.79	723.41	782.21
14	0.11	6.47	0.04	-3.0	58.79	259.76	318.55
15	0.11	6.47	-5.9	1.1	58.79	55.45	114.25
16	0.11	6.47	0.04	1.1	58.79	35.81	94.60
17	8.93	2.28	-0.0	-1.1	55.68	35.81	91.49
18	8.93	2.28	5.93	-1.1	55.68	55.45	111.13
19	8.93	2.28	-0.0	3.02	55.68	259.76	315.44
20	8.93	2.28	5.93	3.02	55.68	723.41	779.09
21	1.07	2.28	-9.5	-1.1	24.73	574.50	599.22
22	1.07	2.28	-3.5	-1.1	24.73	151.77	176.49
23	1.07	2.28	-9.5	3.02	24.73	98.98	123.71
24	1.07	2.28	-3.5	3.02	24.73	120.25	144.98
25	0.94	0.94	3.48	-3.0	12.15	120.25	132.40
26	0.94	0.94	9.45	-3.0	12.15	98.98	111.13
27	0.94	0.94	3.48	1.1	12.15	151.77	163.91
28	0.94	0.94	9.45	1.1	12.15	574.50	586.64
29	0.11	0.94	-5.9	-3.0	8.90	723.41	732.31
30	0.11	0.94	0.04	-3.0	8.90	259.76	268.66
31	0.11	0.94	-5.9	1.1	8.90	55.45	64.35
32	0.11	0.94	0.04	1.1	8.90	35.81	44.70

TABLE 5.11 PERCENT ERROR OF ESTIMATED PRESENT WORTH OF LOSSES

ALT	PW_{Lest}	PW_{Ltrue}	% ERROR
1	141.38	120.99	16.9%
2	161.03	154.40	4.3%
3	365.34	351.19	4.0%
4	828.99	817.58	1.4%
5	649.12	669.73	-3.1%
6	226.39	230.97	-2.0%
7	173.60	178.30	-2.6%
8	194.88	172.52	13.0%
9	182.30	154.48	18.0%
10	161.02	160.26	0.5%
11	213.81	212.93	0.4%
12	636.54	651.69	-2.3%
13	782.21	771.00	1.5%
14	318.55	304.61	4.6%
15	114.25	107.81	6.0%
16	94.60	74.41	27.1%
17	91.49	84.91	7.7%
18	111.13	118.31	-6.1%
19	315.44	315.11	0.1%
20	779.09	781.50	-0.3%
21	599.22	633.65	-5.4%
22	176.49	194.89	-9.4%
23	123.71	142.22	-13.0%
24	144.98	136.44	6.3%
25	132.40	118.40	11.8%
26	111.13	124.18	-10.5%
27	163.91	176.85	-7.3%
28	586.64	615.61	-4.7%
29	732.31	734.91	-0.4%
30	268.66	268.53	0.0%
31	64.35	71.73	-10.3%
32	44.70	38.33	16.6%

alternatives, (2) between 10 and 20% in 8 cases, (3) larger than 20% for 1 alternative.

5.4 SUMMARY

A hypothetical example is used to illustrate and examine basic results from previous chapters. Results from this research are compared with those that would be obtained from common approaches to design for quality. It is shown that the latter overlook (1) product's quality degradation over time, and (2) time value of money. These makes them inappropriate to model external quality losses. These two issues, on the other hand, are incorporated into developments from these research.

Data based on the hypothetical example were generated using Monte Carlo simulation. These data were used to estimate the variance and mean drift models. It is seen that practical applications of the results of this research are dependent on estimation of these drifts models, which can be difficult due to limitations of varied nature, such as cost, time constraints, and technical difficulties.

CHAPTER VI

PRESENT WORTH OF EXTERNAL QUALITY LOSSES

(MULTIPLE PRODUCT CHARACTERISTICS)

6.1 INTRODUCTION

This chapter discusses the economic analysis of external quality losses for a product in which its quality performance is defined in terms of multiple product characteristics. The discussion is limited to consider Class I product characteristics (i.e., those having either one of the following loss minimization objectives: lower-is-better, higher-is-better with a finite target, or symmetric nominal-is-better). The resulting multidimensional TLF has an analytic and exact closed form, which does not require knowledge on the probability distribution of the individual product characteristics. This allows one to: (1) determine the present worth of expected total losses and break it down into losses due to mean/bias and due to variance, and (2) examine the role of interdependencies among product characteristics.

Section 6.2 extends the Taguchi Loss Function (TLF) so it can provide a monetary evaluation of the instantaneous loss of a product with **p** different Class I product characteristics. Section 6.3 examines the equivalent present worth of external quality losses for a product for which quality losses are described by the multidimensional TLF. Concepts and expressions developed on Sections 6.2 and 6.3 are illustrated by means of an example. The example assumes a hypothetical product for which quality is determined by two product characteristics. Section 6.4 presents the chapter's summary.

6.2 MULTIDIMENSIONAL TAGUCHI LOSS FUNCTION

The TLF is a a quadratic loss function **L(y)**, intended to provide a monetary evaluation of a product's quality (Taguchi, 1986; 1987). This function can be extended for a product for which quality is determined by a set **Y** of **p** different product characteristics. The multidimensional loss function, **L(Y)** is defined as the quadratic form expression (Pignatiello, 1993):

$$\mathbf{L(Y)} = [\mathbf{Y}-\boldsymbol{\tau}]'\mathbf{K}[\mathbf{Y}-\boldsymbol{\tau}] \qquad (6.1)$$

where $\boldsymbol{\tau}$ is the **px1** vector of targets of the product characteristics. The matrix **K** is a **pxp** symmetric positive definite matrix representing the losses incurred when **Y** deviates from $\boldsymbol{\tau}$.

An alternative expression of the multidimensional loss function (in terms of the biases of the product characteristics) is

$$\mathbf{L(Y)} = \mathbf{L(B)} = \mathbf{B'K\,B} \qquad (6.2)$$

in which $\mathbf{B}$ denotes the $\mathbf{p x 1}$ vector of biases, i.e., $\mathbf{B} = \mathbf{Y} - \boldsymbol{\tau} = (\mathbf{b_1}, \mathbf{b_2}, .., \mathbf{b_p})'$. This expression is used in this chapter.

Without loss of generality and only for illustrative purposes, assume all the product characteristics have a symmetric nominal-is-better loss minimization objective (the results of the chapter hold for all Class I product characteristics). Let the $\mathbf{p}$ different biases represent the axes of a hyperplane on a $\mathbf{p}$-dimensional space. In such a space, equation (6.2) defines a paraboloid centered at the origin. To illustrate this, Figure 6.1 shows $\mathbf{L(B)}$ for a product for which quality is determined by two product characteristics. Contours of the paraboloid on the bidimensional plane (spanned by $(\mathbf{b_1}, \mathbf{0})$ and $(\mathbf{0}, \mathbf{b_2})$) represent concentric ellipses (Figure 6.2). The ellipse's principal axis is rotated an angle $\boldsymbol{\alpha}$ relative to the axis of abscissas. Appendix 6 analyses ellipses in more detail.

Consider the particular case of $\mathbf{p}=1$ in either equation (6.1) or (6.2). The resulting expression becomes identical to that of a product for which quality is defined in terms of one single product characteristic (presented in Section 3.2): the matrices become scalars such that $\mathbf{b_{11}}$ and $\mathbf{k_{11}}$ are equivalent to the product characteristic's bias ($\mathbf{b_y}$) and the parameter $\mathbf{k}$, respectively.

As with the unidimensional TLF, the expected value of $\mathbf{L(Y)}$ represents a monetary evaluation of the risk, at a certain time instant, associated with undesirable deviations around the target. The expected loss $\mathbf{E[L(Y)]}$, determined from the properties of quadratic forms of random vectors (Graybill, 1976), is

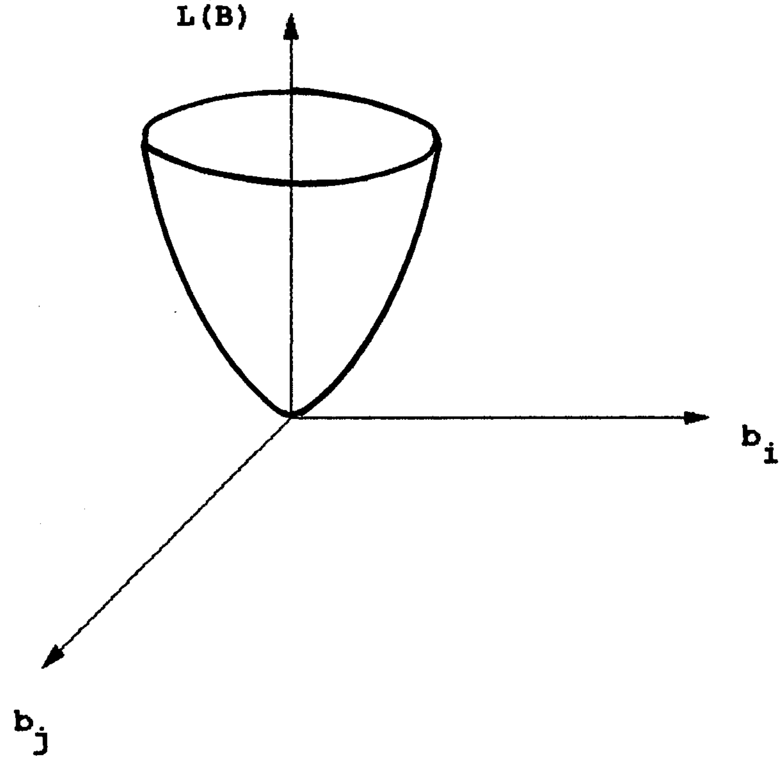

FIG. 6.1 LOSS FUNCTION FOR TWO PRODUCT CHARACTERISTICS

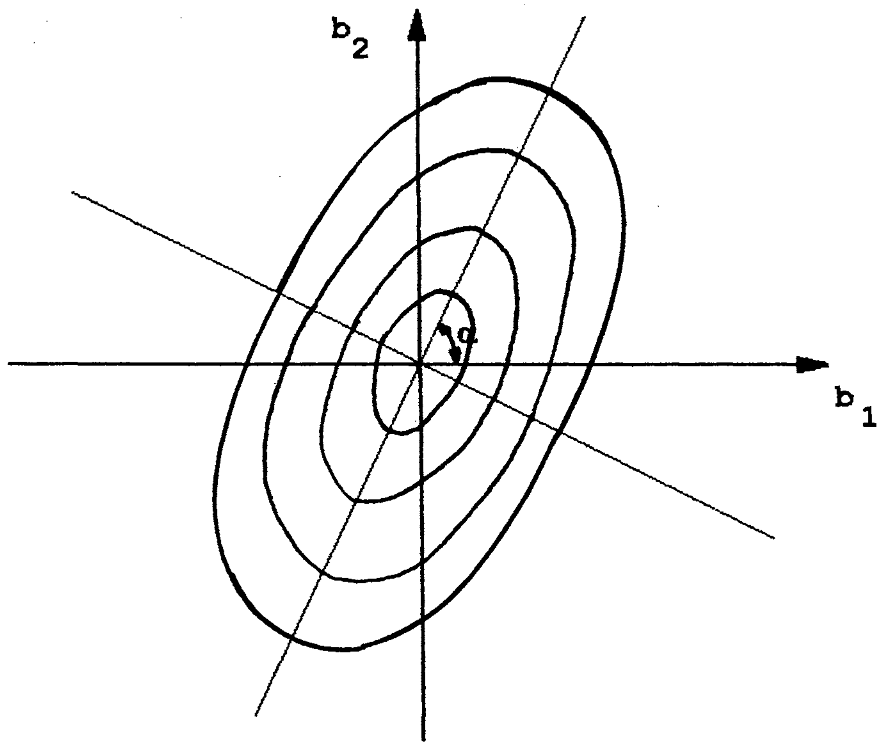

FIG 6.2 CONTOURS OF LOSS FUNCTION FOR TWO PRODUCT CHARACTERISTICS

$$E[L(Y)] = \text{trace}[K\Sigma]+[\mu_Y-\tau]'K[\mu_Y-\tau] \qquad (6.3)$$

$$= E[L(B)] = \text{trace}[K\Sigma]+B'K\,B$$

where μ_Y denotes the means vector of **Y**, and Σ is the variance-covariance matrix of **Y.** The **trace** operator denotes the matrix operator defined as the sum of the diagonal elements of a certain matrix (Graybill, 1983).

In summation notation, equation (6.3) is written as

$$E[L(B)] = \sum_{i=1}^{p} \sum_{j=1}^{p} k_{ij}\sigma_{ij}+\sum_{i=1}^{p} \sum_{j=1}^{p} k_{ij}b_i b_j \qquad (6.4)$$

In this expression, the **ij**-th entry of matrices **K** and Σ are respectively represented by k_{ij} and σ_{ij}. The **i**-th element of the vector **B** is denoted by b_i. An inspection of equation (6.4) allows one to conclude that (as in the case of one product characteristic) the expected loss is the combination of two additive expected losses: (1) aggregated losses associated to the variance-covariance matrix (first term), and (2) aggregated losses associated to the means/biases vector (second term).

Expanding the summations in the above equation, and after algebraic manipulation, the expected loss can be written as

$$E[L(B)] = \sum_{i=1}^{p} k_{ii}\sigma_{ii}+\sum_{i=1}^{p} k_{ii}b_i^2 \qquad (6.5)$$

$$+2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij}\sigma_{ij}+2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij}b_i b_j$$

After regrouping terms, this expression becomes

$$E[L(B)] = \sum_{i=1}^{p} k_{ii}(\sigma_{ii}+b_i^2) + 2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij}(\sigma_{ij}+b_ib_j) \qquad (6.6)$$

This equation shows that, from the viewpoint of interdependencies among product characteristics, the expected loss can be broken down in two parts: (1) aggregated losses from each of the product characteristics taken alone (first term), and (2) losses derived from interdependencies among product characteristics (second term). Taguchi et al. (1989, p.18) state that $E[L(B)]$ is given by the sum of all losses due to individual product characteristics. Such a statement neglects the effect of interdependencies among product characteristics. That is equivalent to overlooking the second term in equation (6.6).

An alternative way of equation (6.6) considers the partial correlation coefficient ρ_{ij} (between product characteristics i and j, where $i \neq j$) as

$$E[L(B)] = \sum_{i=1}^{p} k_{ii}(\sigma_{ii}+b_i^2) + 2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij}[(\rho_{ij}\sigma_{ii}\sigma_{jj})^{1/2}+b_ib_j] \qquad (6.7)$$

Table 6.1 presents the different types of aggregate losses that constitute the expected loss $E[L(y)]$.

Consider the particular case of p=1 in equation (6.7). The expected loss is identical to that of a product for which quality is defined in terms of one single product characteristic (presented in Section 3.3). A comparison between equations (6.7) and (3.9) allows one to conclude that: (1) k_{ii} and σ_{ii} are equivalent to k and σ_y^2 (for a

TABLE 6.1 CLASSIFICATION OF AGGREGATE LOSSES

LOSSES DUE TO:	PRODUCT CHARACTERISTICS TAKEN ALONE	INTERDEPENDENCIES AMONG PRODUCT CHARACTERISTICS
VARIANCES/ COVARIANCES	$\sum_{i=1}^{p} k_{ii}\sigma_{ii}$	$2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij}\sigma_{ij}$
MEANS/BIASES	$\sum_{i=1}^{p} k_{ii}b_i^2$	$2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij}b_ib_j$

single product characteristic) respectively, and (2) $\mathbf{b_i}$ represents the bias of the product characteristic.

As shown in equations (6.5) through (6.7), aggregated losses derived from interdependencies among product characteristics depend on two types of parameters (see equations 6.5 through 6.7): σ_{ij} (or equivalently, ρ_{ij}) and $\mathbf{k}_{ij}$ ($\mathbf{i} \neq \mathbf{j}$). These parameters constitute the elements of the matrices Σ and $\mathbf{K}$, respectively. These matrices are further discussed in Subsections 6.2.1 and 6.2.2. Concepts and use of these matrices are illustrated in Subsection 6.2.3 using as an example a hypothetical product for which quality is determined by two product characteristics.

6.2.1 Interdependencies Among Product Characteristics: Matrix Σ

The matrix Σ is known as the covariance matrix. It is constituted by the generic element σ_{ij}, or covariance between the pair $\mathbf{i}$, $\mathbf{j}$ of product characteristics. The matrix is symmetric (i.e., $\sigma_{ij} = \sigma_{ji}$), idempotent, and positive definite. It has two types of elements:

- Diagonal elements, σ_{ii}, represent the variance of the $\mathbf{i}$-th product characteristic.
- Off-diagonal elements, σ_{ij}, denote the covariance between the $\mathbf{i}$-th and the $\mathbf{j}$-th product characteristic.

The covariance σ_{ij} is a measure of statistical interdependency between two product characteristics. It expresses the degree of "codispersion" (Ditlevsen, 1981) between the $\mathbf{i}$-th and the $\mathbf{j}$-th product characteristic, in such a way that if:

- A deviation of the **i**-th product characteristic above (below) its target is coupled with a deviation of the **j**-th product characteristic above (below) its own target, then σ_{ij} is positive.
- A deviation of the **i**-th product characteristic above (below) its target is coupled with a deviation of the **j**-th product characteristic below (above) its own target, then σ_{ij} is negative.
- The deviations of a pair of product characteristics around their respective targets are uncorrelated, then σ_{ij} is zero.

Both the variance and the covariance are statistical measures of the variability of product characteristics which depend on the linkage between the product and the process.

6.2.2 Interdependencies Among Product Characteristics: Matrix K

According to Pignatiello (1993), the matrix **K** represents the losses incurred when **Y** (a vector) deviates from τ (a vector). The matrix (which generic elements are denoted by k_{ij}) is symmetric (i.e., $k_{ij} = k_{ji}$), idempotent, and positive definite.

The matrix diagonal elements, k_{ii}, are related to one single product characteristic. They are equivalent to the constant **k** used in the unidimensional TLF. Then, according to equation (3.8), if a loss **A** is associated with some undesirable deviation Δ_i of the **i**-th product characteristic around its target, k_{ii} is

$$k_{ii} = A/\Delta_i^2 \qquad (6.7)$$

Consider the bidimensional plane spanned by the vectors $(\mathbf{b_i},\mathbf{0})'$ and $(\mathbf{0},\mathbf{b_j})'$ where $\mathbf{i}\neq\mathbf{j}$. On such a plane, the expression of the loss function (equation 6.2) defines a set of concentric elliptic contours. Each of these contours describe an <u>isopotential</u> or <u>indifference curve</u> (combinations of $\mathbf{b_i}$ and $\mathbf{b_j}$ on the ellipse that provide the user with the same dissatisfaction level, equivalent to $\mathbf{L}$ dollars) (Leftwich, 1985; Nicholson, 1992).

The indifference curve for which $\mathbf{L}=\mathbf{A}$, is described by an ellipse which: (1) passes through the points $(\Delta_i,\mathbf{0})$ and $(\mathbf{0},\Delta_j)$, (2) has a principal axis that passes through the origin and is rotated α radians (relative to the plane's axes) (Figure 6.3):

$$k^2{}_{ii}b^2{}_i+2k_{ij}b_ib_j+k^2{}_{jj}b^2{}_j = A \qquad (6.8)$$

As indicated in Appendix 6, the ellipse's principal axis is rotated through an angle of radian measure α. Let λ_1 and λ_2 be the eigenvectors of the matrix $\mathbf{K}$ such that $\lambda_1\leq\lambda_2$. Then, the direction of the ellipse's principal axis coincides with that of the eigenvector associated with λ_1. The angle α corresponds to the angle between the $\mathbf{b_i}$ axis and the ellipse's principal axis.

6.2.3 Example: Interdependencies Among Product Characteristics

Suppose a product's quality is determined by the product characteristics, $\mathbf{y_1}$ and $\mathbf{y_2}$. Both product characteristics follow a symmetric nominal-is-better loss minimization objective.

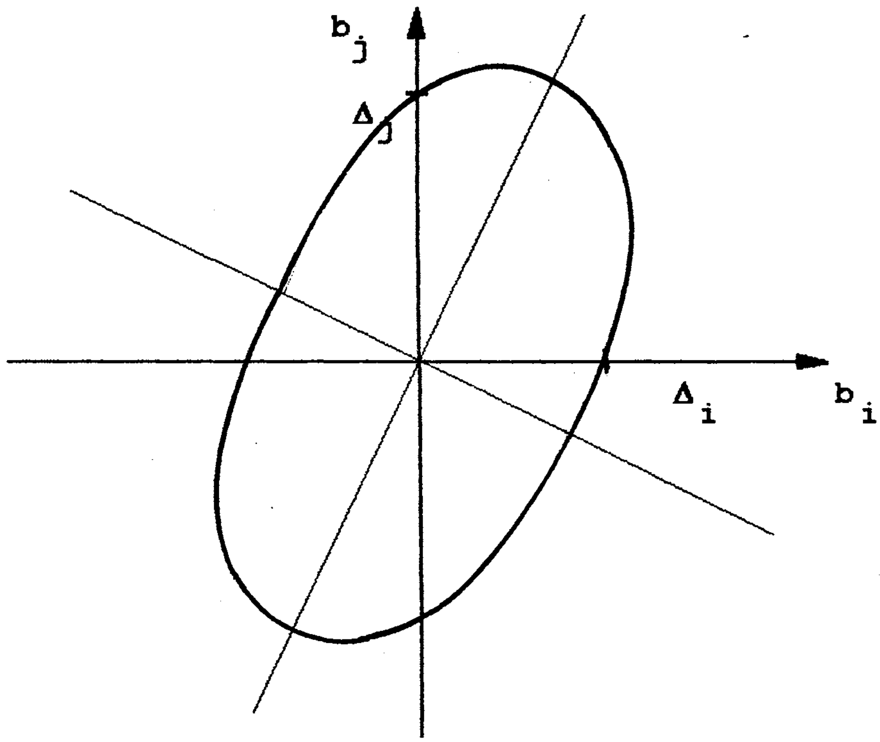

FIG. 6.3 ELLIPTICAL INDIFERENCE CURVE FOR L=A

The elliptical indifference curve for a loss of **L=$80** is

$$17b^2_1-12b_1b_2+8b^2_2=80 \qquad (6.9)$$

The matrix **K** of the quadratic form is then

$$\mathbf{K} = \begin{bmatrix} 17 & -6 \\ -6 & 8 \end{bmatrix} \qquad (6.10)$$

The matrix eigenvalues are $\lambda_1=5$ and $\lambda_2=20$. As discussed in Appendix 6, the ellipse's principal axis has the same direction as the eigenvalue associated with λ_1. Using results from the same appendix, the rotation angle α (angle between the ellipse principal axis and the axis b_1) can be determined to be 1.107 radians. On the new coordinate system (in which the ellipse's principal axis is the axis of the abscissas), the ellipse's equation is

$$b_1'^2+4b_1'b_2'+8b_2'^2=16$$

and its vertices are at **(4,0)** and **(0,2)** (Figure 6.4).

According to equation (6.6) (see Table 6.1), aggregated instantaneous losses due to:

- Variances, and product characteristics taken alone, are

$$k_{11}\sigma_{11}+k_{22}\sigma_{22} = 17\sigma_1^2+8\sigma_2^2$$

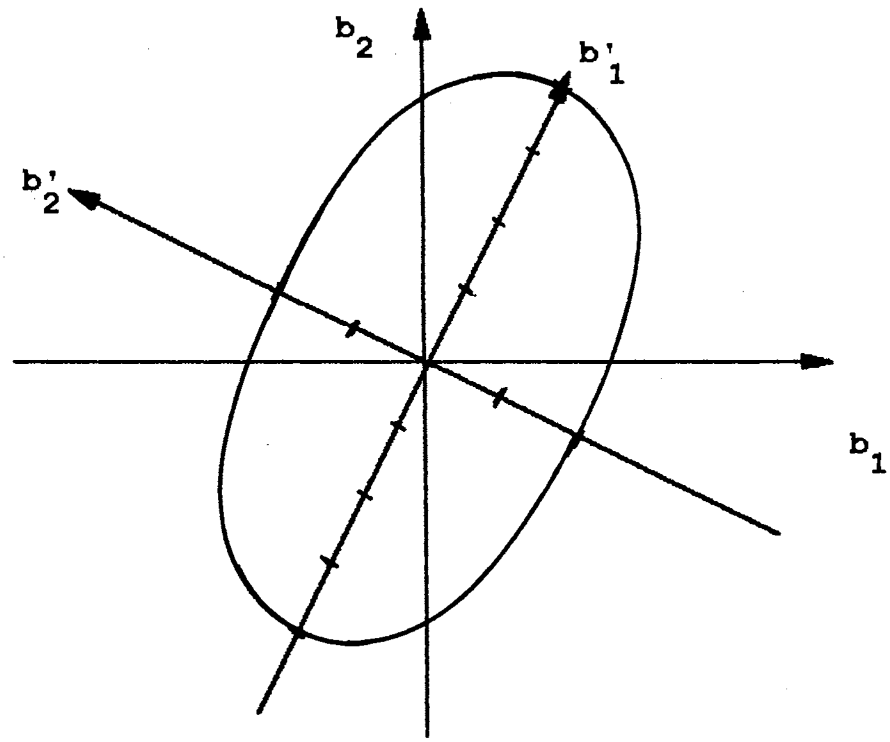

FIG. 6.4 COORDINATE SYSTEMS FOR ELLIPTICAL INDIFERENCE CURVES

where σ_1^2 and σ_2^2 represent the variances of the corresponding product characteristics.

- Means/biases, and product characteristics taken alone, are

$$k_{11}b_1^2+k_{22}b_2^2 = 17b_1^2+8b_2^2$$

- Covariances, and interdependencies among product characteristics are

$$\begin{aligned} 2k_{12}\sigma_{12} &= 2k_{12}\rho_{12}(\sigma_{11}\sigma_{22})^{1/2} \\ &= -24\rho_{12}(\sigma_{11}\sigma_{22})^{1/2} \end{aligned}$$

Consider the effect of the statistical correlation between product characteristics: (1) a negative value of ρ_{12} increases the (total) expected loss, and (2) a positive value of ρ_{12} decreases it.

- Means/biases, and interdependencies among product characteristics are

$$2k_{12}b_1b_2 = -24b_1b_2$$

6.3 PRESENT WORTH OF QUALITY LOSSES

The economic analysis of external quality losses can be done by extending the loss function concept (equation 6.2), to a function that changes over time $\mathbf{L(Y;t)}$. This function represents a cash flow stream, the expected value of which is given by

$$E[L(Y;t)] = \mathrm{trace}[K\Sigma(t)]+[\mu_Y(t)-\tau]'K[\mu_Y(t)-\tau] \qquad (6.12)$$
$$= \mathrm{trace}[K\Sigma(t)]+B'(t)KB(t)$$

This expected cash flow stream can be used to determine the present worth of expected external quality losses, PW_L for the time period $(0,T)$:

$$PW_L = \int_0^T E[L(Y;t)]e^{-rt}dt \qquad (6.13)$$
$$= \int_0^T \{\mathrm{trace}[K\Sigma(t)]+[\mu_Y(t)-\tau]'K[\mu_Y(t)-\tau]\}e^{-rt}dt$$
$$= \int_0^T \{\mathrm{trace}[K\Sigma(t)]+B'(t)\ K\ B(t)\}e^{-rt}dt$$

This equation can be expressed as

$$PW_L = \int_0^T \mathrm{trace}[K\Sigma(t)]e^{-rt}dt$$
$$+ \int_0^T [\mu_Y(t)-\tau]'K[\mu_Y(t)-\tau]e^{-rt}dt \qquad (6.14)$$

In this equation, the first and the second terms are respectively associated with: (1) quality losses due to covariances (and variances), PWL_V, and (2) quality losses due to biases/means, PWL_M. Then, PW_L is the addition of PWL_V and PWL_M , i.e.:

$$PW_L = PWL_V + PWL_M \qquad (6.15)$$

The expected present worth for each of the two types of quality losses is further developed in the Subsections 6.3.1 and 6.3.2. The resulting expressions are illustrated in Subsection 6.3.3, which extends the example presented in Subsection 6.2.3.

6.3.1 Quality Losses Due to Covariances

Expansion of the first term in equation (6.14) yields the following expression for the equivalent present worth of aggregated losses due to covariances:

$$\begin{aligned} \mathbf{PWL_V} &= \sum_{i=1}^{p} \sum_{j=1}^{p} k_{ij} \int_0^T \sigma_{ij}(t)\, e^{-rt}dt \qquad (6.16) \\ &= \sum_{i=1}^{p} k_{ii} \int_0^T \sigma_{ii}(t)\, e^{-rt}dt \\ &\quad + 2 \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij} \int_0^T \sigma_{ij}(t)\, e^{-rt}dt \end{aligned}$$

In terms of the partial correlation coefficients between product characteristics $\mathbf{i}$ and $\mathbf{j}$, $\rho_{ij}(t)$ (where $i \neq j$) $\mathbf{PWL_V}$ can be expressed as

$$\begin{aligned} \mathbf{PWL_V} &= \sum_{i=1}^{p} k_{ii} \int_0^T \sigma_{ii}(t) e^{-rt}dt \qquad (6.17) \\ &\quad + 2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} k_{ij} \int_0^T \rho_{ij}(t)\,[\sigma_{ii}(t)\sigma_{jj}(t)]^{1/2} e^{-rt}dt \end{aligned}$$

In this expression, the first term evaluates the present worth of losses due to variances and from product characteristics taken alone. The second term expresses the present worth of aggregated losses due to covariances from product characteristics.

The effect of the correlation between a pair of product characteristics depends on $\mathbf{k_{ij}}$. For a positive $\mathbf{k_{ij}}$, (1) a positive value of ρ_{ij} increases $\mathbf{PWL_V}$ (relative to the $\mathbf{PWL_V}$ for non correlated product characteristics), and (2) a negative value of ρ_{ij} decreases $\mathbf{PWL_V}$. Likewise, if k_{ij} is negative: (1) a positive value of ρ_{ij} decreases $\mathbf{PWL_V}$ (relative to the $\mathbf{PWL_V}$ for non correlated product characteristics), and (2) a negative value of ρ_{ij} increases $\mathbf{PWL_V}$.

6.3.2 Quality Losses Due To Biases/Means

The equivalent present worth of aggregated quality losses due to means/biases can be obtained expanding the second term in equation (6.14):

$$\begin{aligned} \mathbf{PWL_M} &= \int_0^T \mathbf{B' \, K \, B} \, e^{-rt} dt \qquad (6.18) \\ &= \sum_{i=1}^{p} k_{ii} \int_0^T b_i^2(t) e^{-rt} dt \\ &\quad + 2\sum_{i=1}^{p} \sum_{j=1}^{p} k_{ij} \int_0^T b_i(t) b_j(t) \, e^{-rt} dt \end{aligned}$$

As in the case of losses due to variances, the expected quality losses due to biases/means can be broken down in two parts: one related to each of the quality characteristic taken alone, and the other part associated with the interdependencies between pairs of quality characteristics.

6.3.3 Example: Present Worth Of Expected External Quality Losses

Consider the product in Subsection 6.2.3. Assume a discount rate r=10% and a planning horizon T=5 years. Suppose the product characteristic y_1 has the following mean and variance drifts

- Mean/bias drift:

$$b_1(t) = 1+.5t$$

- Variance drift:

$$\sigma_1(t)^2 = \sigma_{11}(t) = 2+.25t$$

The product characteristic y_2 has the following mean and variance drifts

- Mean/bias drift:

$$b_2(t) = .5+.25t$$

- Variance drift:

$$\sigma_2(t)^2 = \sigma_{22}(t) = 1+.125t$$

To simplify the forthcoming analysis, assume the product characteristics' correlation does not change over time, i.e., the partial correlation $\rho_{12}(t)=\rho$ is constant.

From equation (6.17), the present worth of expected quality losses due to covariances is

$$PWL_V = k_{11} \int_0^T \sigma_{11}(t)\ e^{-rt}dt + k_{22} \int_0^T \sigma_{22}(t)\ e^{-rt}dt$$
$$+2k_{12}\int_0^T \rho\,[\sigma_{ii}(t)\sigma_{jj}(t)]^{1/2}\ e^{-rt}dt$$

$$= 17[2*I(0,10,5)+.25*I(1,10,5)]+8[I(0,10,5)+.125*I(1,10,5)]$$

$$-12\rho\int_0^5 (2+.5t+t^2)^{1/2}e^{-rt}dt$$

$$= 172.04+40.48-42.96\rho = 212.52-42.96\rho$$

From equation (6.18), the present worth of expected quality losses due to means/biases is

$$PWL_M = k_{11}\int_0^T b_1{}^2(t)e^{-rt}dt+k_{22}\int_0^T b_2{}^2(t)e^{-rt}dt$$

$$+2k_{12}\int_0^T b_1(t)b_2(t)\ e^{-rt}dt$$

$$= 17\int_0^T (1+t+.25t^2)e^{-rt}dt+8\int_0^T (.25+.25t+.0625t^2)e^{-rt}dt$$

$$+2(-6)\int_0^T (.5+.5t+.125t^2)\ e^{-rt}dt$$

$$= 342.38+40.28-120.9 = 382.66-120.9 = \$261.76/\text{unit}$$

Table 6.2 breaks down the present worth of expected losses. The total for the first column (product characteristics taken alone) is the value of $\mathbf{PW_L}$ that would be obtained following Taguchi et al. (1989). In general, overlooking the effect of interdependencies among product characteristics leads to inappropriate estimation of $\mathbf{PW_L}$. For the example, Table 6.3 presents, for different values of ρ: (1) the value of $\mathbf{PW_L}$ considering interdependencies among product characteristics, (2) the value of $\mathbf{PW_L}$ as per Taguchi et al., i.e., overlooking interdependencies, and (3) the resulting overestimation percent error of $\mathbf{PW_L}$.

TABLE 6.2 EXAMPLE: PRESENT WORTH OF LOSSES

PW OF LOSSES DUE TO:	PRODUCT CHARACTERISTICS TAKEN ALONE	INTERDEPENDENCIES AMONG PRODUCT CHARACTERISTICS	TOTAL:
VARIANCES/ COVARIANCES	212.52	-42.96ρ	212.52-42.96ρ
MEANS/BIASES	382.66	-120.90	261.76
TOTAL:	595.18	-120.90-42.96ρ	474.28-42.96ρ

TABLE 6.3 CONSEQUENCES OF OVERLOOKING INTERDEPENDENCIES AMONG PRODUCT CHARACTERISTICS

ρ	PW_L CONSIDERING INTER-DEPENDENCIES	PW_L WITHOUT CONSIDERING INTER-DEPENDENCIES	% OVERESTIMATION OF PW_L
-1.0	517.24	595.18	15.1%
-0.5	495.76	595.18	20.1%
0.0	474.28	595.18	25.5%
0.5	452.80	595.18	31.4%
1.0	431.32	595.18	38.0%

6.4 SUMMARY

This chapter provides the basis to perform the economic analysis of quality losses for a product. This is done by extending the Taguchi Loss Function (TLF) to handle different Class I product characteristics simultaneously (i.e., those having either one of the following loss minimization objectives: lower-is-better, higher- is-better with a finite target, or symmetric nominal-is-better). From such an extension, expressions to determine the present worth of expected quality losses are developed.

It is shown that external quality losses can be break down into different types of losses. External quality losses are subject to a double classification. They can be due to the product characteristics': (1) variances/covariances, or their (2) means/biases. Losses can also be derived from: (1) product characteristics taken alone, or (2) interdependencies between pairs of product characteristics.

A hypothetical product with two product characteristics illustrates the results from this chapter.

CHAPTER VII

SUMMARY, CONTRIBUTIONS AND FUTURE RESEARCH

7.1 INTRODUCTION

This research provides an extension to the Taguchi Loss Function model to effectively evaluate external quality losses of products. A summary of the research, as well as its contributions, is presented in Section 7.1. Section 7.2 outlines areas of future research.

7.2 SUMMARY AND CONTRIBUTIONS

Decision making related to industrial product and process design is commonly focused only on the manufacturer. As a consequence, selection criteria for product/process improvement alternatives are usually based only on costs such as incremental manufacturing cost (incremental expenses incurred by the manufacturer due to differences from one design to another) and internal quality losses (losses incurred by the manufacturer directly related to product characteristics). Overlooking External Quality Losses (EQL, or costs often hidden, but sometimes large, incurred by the user as a consequence of the discrepancy between a product's intended use and its quality performance) reverts back ultimately to the manufacturer in a (sometimes drastically) negative way.

Chapter 1 introduces a conceptual framework to design (intervention on a process/product). Unlike traditional frameworks, limited to process and product design, this framework incorporates external quality (after the product is shipped to the user) as well. The suggested framework points out the relevance of product use for quality performance (as a consequence of product use, a product's quality performance changes over time).

The major contribution of this research is the provision of a means to evaluate external quality losses considering product use and degradation over time: the present worth of expected external quality losses, $\mathbf{PW_L}$. For that matter, two concepts are brought together: (1) the present worth criterion, and (2) the Taguchi Loss Function (TLF), a quadratic function intended to quantify quality costs as a product varies from its target. The research extends the TLF model so as to consider: (1) the different product characteristics' type of loss minimization objectives, (2) product use and degradation over time, (3) the time value of money, and (4) simultaneous modeling of multiple product characteristics. Unlike other quality performance measures (reviewed in Chapter 2), $\mathbf{PW_L}$ is a monetary measure that can be combined with other costs relevant for product/process design (such as incremental manufacturing cost, and internal quality losses).

Chapters 3 through 5 analyze $\mathbf{PW_L}$ for a product for which quality is defined in terms of a single product characteristic. Chapter 6 deals with the more general case of multiple product characteristics.

In Chapter 3, the expected value of the TLF is extended from the standpoint of a discounted cash flow framework. It is seen as a continuous cash flow stream (of instantaneous losses) that occurs during a planning horizon $\mathbf{(0,T)}$, under a (continuous compounding) discount rate $\mathbf{r}$. The present worth of such a cash flow stream represents $\mathbf{PW_L}$.

To examine $\mathbf{PW_L}$, different types of loss minimization objectives commonly found in industry are discussed (the types are differentiated according to the way undesired deviations around the product characteristic's target are defined). The types include: lower-is-better (LIB), higher-is-better with a finite target (HIBFT), higher-is-better with an undetermined target (HIBUT), symmetric nominal-is-better (SNIB), and asymmetric nominal-is-better (ANIB). From these types, a classification of product characteristics is introduced. It is based on the minimum knowledge on the probability distribution of the product characteristic required to determine $\mathbf{PW_L}$: (1) for Class I product characteristics (include LIB, HIBFT, and SNIB loss minimization objectives), it suffices with the product characteristic's mean and variance, (2) for Class II product characteristics (HIBUT loss minimization objective), an approximation to $\mathbf{PW_L}$ can be obtained based only on the product characteristic's mean and variance, and (3) for Class III product characteristics (ANIB loss minimization objective), full knowledge on the probability distribution of the product characteristic is required. Expressions to determine $\mathbf{PW_L}$ for all of the different classes are developed.

It is shown that (unlike the other two classes), for Class I product characteristics, it is possible to breakdown $\mathbf{PW_L}$ into two parts: (1) one part related only to the product characteristic's variance, $\mathbf{PWL_V}$, and (2) another part related only to the product characteristic's mean, $\mathbf{PWL_M}$. Such a breakdown of $\mathbf{PW_L}$ is consistent with a common characterization of a product's quality (in terms of a product characteristic's mean and variance). As a result, one can conclude that such a characterization is appropriate for a Class I product characteristic. Then, use of $\mathbf{PW_L}$ (via $\mathbf{PWL_V}$ and $\mathbf{PWL_M}$) allows one to provide a monetary evaluation of the separate effects of its mean and its variance on the overall product's quality.

The effects of product use on $\mathbf{PW_L}$ are examined in Chapter 4. To model product use, different types of change over time are considered for a product characteristic's variance and mean. They include, for the variance: (1) constant (non-drifting) variance, and (2) linear drift. For the mean: (1) constant (non-drifting) mean, (2) linear drift, and (3) quadratic drift.

Under each of these drifts and for Class I product characteristics: (1) expressions to determine $\mathbf{PW_L}$, $\mathbf{PWL_V}$, and $\mathbf{PWL_M}$ are developed, (2) sensitivity analysis of these present worth values is discussed, and (3) strategies to minimize them are presented.

A major contribution to the field of engineering economy is the development of an approach to determine the present worth of a polynomial (continuous) cash flow function. A major advantage of this approach over the usual one, based on Laplace transforms, is its ease of implementation. Tables for different combinations of the discount rate $\mathbf{r}$ and the planning horizon $\mathbf{T}$ are developed, following a format traditional in the field for discrete cash flow schemes. Both the approach and the tables can be used to determine $\mathbf{PW_L}$, $\mathbf{PWL_V}$, and $\mathbf{PWL_M}$ for Class I product characteristics in a very simple way.

From the development of strategies to minimize $\mathbf{PWL_M}$, it is shown that (at least for SNIB product characteristics with a non-constant mean), setting a process at target does not minimize quality losses to the user. This finding is particularly relevant because it contradicts modern quality philosophies, that advocate achieving a product on target (at time $\mathbf{t}$=0).

Expressions to perform sensitivity analysis of $\mathbf{PWL_V}$, and $\mathbf{PWL_M}$ are developed for Class I product characteristics. Exact expressions are presented to analyze their sensitivity to changes in each of the corresponding (variance or mean) drift parameters (one at a time). Approximations are obtained for sensitivity to changes in $\mathbf{r}$.

As for Class II product characteristics, an expression to determine $\mathbf{PW_L}$ is presented. It is applied to develop a **BASIC** program that evaluates $\mathbf{PW_L}$ for a normally distributed product characteristic with variance and mean subject to the types of drift considered in the research. To circumvent the requirement of knowledge on the probability distribution, an approximation to $\mathbf{PW_L}$ is introduced. All it requires are the variance and mean drift models. However, the approximation results in an analytical expression for which sensitivity analysis is rather cumbersome.

For Class III product characteristics, an expression to determine $\mathbf{PW_L}$ is developed. Due to the asymmetry of the loss function around the product characteristic's target, the expression requires the use of partial moments. It implies the need of knowing the probability distribution of the product characteristic. Therefore, determining $\mathbf{PW_L}$ (for a Class III product characteristic) and performing its sensitivity analysis have to be done on a case-by-case basis, according to the specific problem situation.

In Chapter 5, a comparison between: (1) an approach to design for quality based on minimization of $\mathbf{PW_L}$, and (2) prevailing approaches is done. A product in which the product/process linkage is known is postulated. The problem of selecting production alternatives is examined from the viewpoint of: (1) traditional approaches of designed experimentation, (2) robust design, and (3) minimization of $\mathbf{PW_L}$. The latter provides results that are consistent with those of the other approaches in cases of product characteristics with non-drifting variance and mean. In cases in which product degradation over time and the time value of money are important, though, the use of $\mathbf{PW_L}$ is shown to be superior to the other approaches because: (1) it represents an unambiguous criterion to selecting among production alternatives, and (2) it is expressed in monetary terms (this is a relevant feature because, as stated earlier, this allows one to combine EQL with other types of costs when making decisions for product/process improvement). The example illustrates the serious consequences of inadequate statistical estimation of (variance and mean) drift parameters.

Chapter 6 extends the basic results of the previous chapters to the case in which a product's quality is defined in terms of multiple product characteristics, all of them Class I (i.e., having any of the following loss minimization objectives: LIB, HIBFT, or SNIB). Restricting the product characteristics to be Class I results in a multidimensional TLF with an analytic and exact closed form, which does not require knowledge on the probability distribution of the individual product characteristics. This allows one to: (1) determine the present worth of expected total losses and break it down into losses due to means/biases and due to variance, and (2) examine the role of interdependencies among product characteristics.

The multidimensional TLF is presented as a quadratic form expression which is a function of the product characteristics' means/biases. As in the unidimensional case, the expected value of the TLF is seen as a continuous cash flow stream (of instantaneous losses) over a planning horizon $(0,T)$, under a (continuous compounding) discount rate r. The present worth of such a cash flow stream represents PW_L. PW_L is shown to be the aggregation of present worth of losses subject to a double classification: (1) losses due to variances/covariances and those due to means/biases, and (2) losses due to product characteristics taken alone as opposed to losses due to interdependencies among product characteristics. The latter shows the inadequacy of an approach found in the literature which perceives EQL as the sum of losses due to product characteristics, disregarding the possible interdependencies among them.

7.3 FUTURE RESEARCH

Possible future work related to extensions of this research are as follows:

- Examine the problem of selection of production alternatives in which external quality losses are combined with costs incurred by the manufacturer (incremental manufacturing costs, internal quality losses).
- Integrate discrete time losses (such as those derived from product maintenance, repair and/or replacement) on models for $\mathbf{PW_L}$.
- Introduce types of variance and mean drifts other than those considered in this research and examine their effects on $\mathbf{PW_L}$. Particularly, consider models that include the exponential function, since they might be suitable to: (1) model features likely to be appear on variance/mean drifts, such as a saturation level (the variance/mean reaches some threshold after some time), and an S shape for the drifts, and (2) be easily implemented, with the possibility of creating tables of factors to determine the present worth in a simple way.
- Investigate means to determine values of parameters affecting $\mathbf{PW_L}$ that are related to the product's user, such as $\mathbf{r}$, $\mathbf{k}$, $\boldsymbol{\tau}$. Examine the potential use of methods and techniques developed to "listening to the voice of a customer," such as Quality Function Deployment, and customer surveys.
- Develop statistical procedures adequate to estimate drift parameters.
- Develop expressions to determine $\mathbf{PW_L}$ for a product for which quality is a function of multiple product characteristics, which are not necessarily Class I.
- For the multidimensional case, investigate methods to estimate the elements of the matrices $\boldsymbol{\Sigma}$ and $\mathbf{K}$.

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APPENDICES

APPENDIX 1: PRESENT WORTH OF A POLYNOMIAL (CONTINUOUS) CASH FLOW FUNCTION

A1.1 INTRODUCTION

This appendix investigates the equivalent present worth of a cash flow stream described by a polynomial continuous cash flow function, under continuous compounding. Section A1.2 discusses a simplified procedure to determine such a present worth. Section A1.3 introduces a procedure to perform sensitivity analysis of the resulting present worth to changes in the discount rate.

A1.2 PROCEDURE TO DETERMINE THE PRESENT WORTH

Let $\mathbf{F(t)}$ be a polynomial function that describes a continuous cash flow stream over a planning horizon ranging from $\mathbf{t}=0$ through $\mathbf{t=T}$, such that

$$F(t)=c_0+c_1t+c_2t^2+\ldots+c_nt^n \qquad 0\le t\le T \qquad \text{(A1.1)}$$

where $\mathbf{c_0,c_1,\ldots,c_n}$ are constants. The equivalent present worth, $\mathbf{P}$, is (Park and Sharp-Bette, 1990):

$$P=\int_0^T (c_0+c_1t+c_2t^2+\ldots+c_nt^n)e^{-rt}dt \qquad \text{(A1.2)}$$

where r is the (continuous compounding) discount rate. Using the linearity properties of the integral, equation (A1.2) becomes

$$P=c_0\int_0^T e^{-rt}dt+c_1\int_0^T te^{-rt}dt+c_2\int_0^T t^2e^{-rt}dt+\ldots+c_n\int_0^T t^ne^{-rt}dt \qquad \text{(A1.3)}$$

Let $I(m,r,T)$ denote the integral corresponding to the m-th power term in equation (A1.3), i.e.,

$$I(m,r,T)=\int_0^T t^me^{-rt}dt \qquad \text{(A1.4)}$$

It has been suggested the use of Laplace transforms for handling integrals such as $I(m,r,T)$ (Buck and Hill, 1971 and 1975; Park and Sharp-Bette, 1990). A simpler approach takes advantage of the recursive properties of integrals with the form of $I(m,r,T)$ (Abramowitz and Stegun, 1984):

For $m=0$:

$$I(0,r,T)=\frac{1-e^{-rT}}{r} \qquad \text{(A1.5)}$$

and for $m>0$:

$$I(m,r,T)=\frac{1}{r}[-T^me^{-rT}+mI(m-1,r,T)] \qquad \text{(A1.6)}$$

Equation (A1.2) can then be written as

$$P=c_0I(0,r,T)+c_1I(1,r,T)+...+c_nI(n,r,T) \qquad (A1.7)$$

Appendix 2 presents tables of $I(m,r,T)$ for m=0,1,2,3, and 4, and different combinations of r and T. These tables allow a straightforward calculation of P.

Example: Determine the equivalent present worth for $F(t)=3+4t+5t^2$. Assume that r=10% and T=5. Then,

$$P=3I(0,0.1,5)+4I(1,0.1,5)+5I(2,0.1,5)$$

Using tables from Appendix 2:

$$P=3(3.93469)+4(9.020401)+5(28.77536)$$

$$=\$191.76$$

A1.3 SENSITIVITY ANALYSIS

Investigation of the effect on P derived from changes in r can be done using a partial derivative measure sensitivity analysis (Jae, et. al., 1993). Such an approach is based on the total differential of P, dP (Leithold, 1981) , defined as a function of the variables r, T, $c_0, c_1, ..., c_n$:

$$dP = \frac{\delta P}{\delta r}dr+\delta P\frac{\delta P}{\delta T}dT+\frac{\delta P}{\delta c_0}dc_0+... \qquad (A1.8)$$

Particularly relevant to some results from Chapter 3, is the sensitivity analysis of P to changes in r. For that matter, assume that the other variables remain constant. Then, equation (A1.8) becomes

$$dP = \frac{\delta P}{\delta r} dr \qquad (A1.9)$$

The partial derivative $\delta P/\delta r$ is

$$\frac{\delta P}{\delta r} = c_0 \frac{\delta}{\delta r} I(0,r,T) + c_1 \frac{\delta}{\delta r} I(1,r,T) + \ldots + c_n \frac{\delta}{\delta r} I(n,r,T) \qquad (A1.10)$$

It can be shown that, for $m=0,1,\ldots,n$:

$$\frac{\delta}{\delta r} I(m,r,T) = -I(m+1,r,T) \qquad (A1.11)$$

Then, equation (A1.10) can be rewritten as

$$\frac{\delta P}{\delta r} = -[c_0 \frac{\delta}{\delta r} I(1,r,T) + c_1 \frac{\delta}{\delta r} I(2,r,T) + \ldots + c_n \frac{\delta}{\delta r} I(n+1,r,T)] \qquad (A1.12)$$

Given some base value, P_B, a change of $j \times 100\%$ in the discount rate r is associated with a value P_{NEW}. This value can be approximated using "quasilinearization" in the vicinity of P_B (Brown, 1983). This method consists of using increments, instead of differentials in equation (A1.9). Its use is recommended only for small values of j. The approximated value of P_B is then

$$P_{NEW} \approx \frac{\delta P}{\delta r} (r_{NEW} - r) + P_B \qquad (A1.11)$$

$$= \frac{\delta P}{\delta r} jr + P_B$$

where $\delta P/\delta r$ is as defined in equation (A1.10). Equation (A1.11) allows one to conclude that decreasing (increasing) r leads to an increase (decrement) in P_{NEW}.

Example: To illustrate the importance of using the quasilinear approximation discussed above for small values of j only, consider the following example (which will produce a large approximation error). For the previous example, the effect of a change in the discount rate from 10% to 9% (j=10%) on the equivalent present worth can be examined from the partial derivative:

$$\frac{\delta P}{\delta r} = -[3I(1,0.1,5)+4I(2,0.1,5)+5I(3,0.1,5)]$$

$$= -[3(9.020401)+5(28.77536)+5(105.09735)]$$

$$= -696.42$$

Then, from equation (A1.11):

$$P_{NEW} \approx \frac{\delta P}{\delta r} jr+P_B$$

$$= (-696.42)(0.1)(0.1)+191.76$$

$$= \$184.79$$

This result can be compared with the "true" value, which can be determined from equation (A1.7):

$$P_{NEW} = 3I(0,0.09,5)+4I(1,0.09,5)+5I(2,0.09,5)$$

$$= 3(4.0264)+4(9.3135)+5(29.8473)$$

$$= \$198.57$$

The approximation yields in this case a 6.9% error. From this example, it is clear that the approximation method can be used to provide an idea about the direction of the change of $\mathbf{P_{NEW}}$ to changes in $\mathbf{r}$. Its use is not recommended to approximate the value of $\mathbf{P_{NEW}}$, unless the increment $\mathbf{j}$ is very small (and therefore, expected to yield values of $\mathbf{P_{NEW}}$ in the vicinity of $\mathbf{P_B}$.

APPENDIX 2: TABLES OF FACTORS FOR THE PRESENT WORTH OF A POLYNOMIAL CASH FLOW FUNCTION

A2.1 INTRODUCTION

Appendix 1 discussed a procedure to determine the equivalent present worth of a polynomial continuous cash flow function. Such a procedure involves the use of factors defined as

$$\mathbf{I(m,r,T)=\int_0^T t^m e^{-rt} dt} \qquad \text{(A2.1)}$$

This appendix presents tables of **I(m,r,T)** for different combinations of **r** (continuous compounding discount rate) and **T** (planning horizon). Values of **m** included are 0 through 4. The tables presented are developed using the recursive property of equation (A2.1) (see Appendix 1), which can be implemented on a spreadsheet.

TABLE A2.1 VALUES OF I(0,r,T)

$$I(0,r,T) = \int_0^T e^{-rt}dt$$

T	5%	10%	15%	20%	25%	30%
0	0	0	0	0	0	0
0.25	0.2484	0.2469	0.2454	0.2439	0.2423	0.2409
0.5	0.4938	0.4877	0.4817	0.4758	0.4700	0.4643
0.75	0.7361	0.7226	0.7094	0.6965	0.6839	0.6716
1	0.9754	0.9516	0.9286	0.9063	0.8848	0.8639
1.25	1.2117	1.1750	1.1398	1.1060	1.0735	1.0424
1.5	1.4451	1.3929	1.3432	1.2959	1.2508	1.2079
1.75	1.6756	1.6054	1.5392	1.4766	1.4174	1.3615
2	1.9033	1.8127	1.7279	1.6484	1.5739	1.5040
2.25	2.1281	2.0148	1.9097	1.8119	1.7209	1.6361
2.5	2.3501	2.2120	2.0847	1.9673	1.8590	1.7588
2.75	2.5693	2.4043	2.2534	2.1153	1.9887	1.8726
3	2.7858	2.5918	2.4158	2.2559	2.1105	1.9781
3.25	2.9997	2.7747	2.5723	2.3898	2.2250	2.0760
3.5	3.2109	2.9531	2.7230	2.5171	2.3326	2.1669
3.75	3.4194	3.1271	2.8681	2.6382	2.4336	2.2512
4	3.6254	3.2968	3.0079	2.7534	2.5285	2.3294
4.25	3.8288	3.4623	3.1426	2.8629	2.6176	2.4019
4.5	4.0297	3.6237	3.2723	2.9672	2.7014	2.4692
4.75	4.2281	3.7811	3.3972	3.0663	2.7801	2.5316
5	4.4240	3.9347	3.5176	3.1606	2.8540	2.5896

TABLE A2.2 VALUES OF I(1,r,T)

$$I(1,r,T) = \int_0^T te^{-rt}dt$$

T	5%	10%	15%	20%	25%	30%
0	0	0	0	0	0	0
0.25	0.0310	0.0307	0.0305	0.0302	0.0300	0.0297
0.5	0.1229	0.1209	0.1189	0.1170	0.1151	0.1132
0.75	0.2743	0.2676	0.2610	0.2546	0.2484	0.2424
1	0.4836	0.4679	0.4527	0.4381	0.4240	0.4104
1.25	0.7494	0.7191	0.6901	0.6625	0.6361	0.6109
1.5	1.0703	1.0186	0.9697	0.9234	0.8796	0.8382
1.75	1.4448	1.3638	1.2879	1.2168	1.1501	1.0875
2	1.8715	1.7523	1.6416	1.5388	1.4433	1.3545
2.25	2.3492	2.1818	2.0277	1.8860	1.7554	1.6351
2.5	2.8764	2.6499	2.4434	2.2551	2.0832	1.9262
2.75	3.4519	3.1546	2.8860	2.6432	2.4235	2.2247
3	4.0743	3.6936	3.3529	3.0475	2.7737	2.5280
3.25	4.7425	4.2651	3.8417	3.4656	3.1313	2.8338
3.5	5.4552	4.8671	4.3501	3.8951	3.4941	3.1403
3.75	6.2112	5.4977	4.8762	4.3340	3.8602	3.4457
4	7.0092	6.1552	5.4178	4.7802	4.2279	3.7486
4.25	7.8482	6.8378	5.9732	5.2321	4.5955	4.0477
4.5	8.7271	7.5439	6.5406	5.6879	4.9618	4.3421
4.75	9.6445	8.2720	7.1183	6.1464	5.3256	4.6307
5	10.5996	9.0204	7.7048	6.6060	5.6858	4.9131

TABLE A2.3 VALUES OF I(2,r,T)

$$I(2,r,T) = \int_0^T t^2 e^{-rt} dt$$

T	5%	10%	15%	20%	25%	30%
0	0	0	0	0	0	0
0.25	0.0052	0.0051	0.0051	0.0050	0.0050	0.0049
0.5	0.0409	0.0401	0.0394	0.0387	0.0379	0.0372
0.75	0.1367	0.1329	0.1293	0.1257	0.1223	0.1189
1	0.3211	0.3093	0.2980	0.2871	0.2767	0.2666
1.25	0.6213	0.5930	0.5660	0.5404	0.5160	0.4928
1.5	1.0636	1.0057	0.9512	0.8999	0.8515	0.8059
1.75	1.6732	1.5676	1.4691	1.3772	1.2915	1.2114
2	2.4744	2.2970	2.1330	1.9816	1.8416	1.7122
2.25	3.4905	3.2104	2.9542	2.7198	2.5053	2.3089
2.5	4.7436	4.3230	3.9421	3.5969	3.2841	3.0004
2.75	6.2553	5.6484	5.1043	4.6160	4.1777	3.7840
3	8.0458	7.1990	6.4470	5.7788	5.1847	4.6561
3.25	10.1348	8.9856	7.9749	7.0856	6.3023	5.6119
3.5	12.5410	11.0179	9.6915	8.5354	7.5269	6.6463
3.75	15.2823	13.3044	11.5988	10.1264	8.8540	7.7533
4	18.3757	15.8527	13.6979	11.8556	10.2786	8.9269
4.25	21.8374	18.6689	15.9891	13.7197	11.7951	10.1607
4.5	25.6830	21.7587	18.4716	15.7142	13.3977	11.4483
4.75	29.9272	25.1263	21.1437	17.8345	15.0801	12.7834
5	34.5839	28.7754	24.0032	20.0753	16.8361	14.1595

TABLE A2.4 VALUES OF I(3,r,T)

$$I(3,r,T) = \int_0^T t^3 e^{-rt} dt$$

T	5%	10%	15%	20%	25%	30%
0	0	0	0	0	0	0
0.25	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009
0.5	0.0153	0.0150	0.0147	0.0144	0.0141	0.0139
0.75	0.0768	0.0745	0.0723	0.0702	0.0681	0.0661
1	0.2402	0.2308	0.2218	0.2132	0.2049	0.1969
1.25	0.5806	0.5524	0.5256	0.5001	0.4760	0.4530
1.5	1.1920	1.1228	1.0579	0.9968	0.9394	0.8854
1.75	2.1864	2.0392	1.9024	1.7751	1.6566	1.5464
2	3.6930	3.4104	3.1504	2.9109	2.6905	2.4875
2.25	5.8568	5.3554	4.8987	4.4826	4.1033	3.7574
2.5	8.8382	8.0022	7.2484	6.5686	5.9552	5.4016
2.75	12.8118	11.4861	10.3030	9.2467	8.3034	7.4605
3	17.9656	15.9487	14.1671	12.5928	11.2008	9.9694
3.25	24.5002	21.5367	18.9458	16.6794	14.6956	12.9581
3.5	32.6280	28.4010	24.7430	21.5755	18.8307	16.4506
3.75	42.5723	36.6951	31.6610	27.3456	23.6435	20.4650
4	54.5666	46.5751	39.7993	34.0495	29.1658	25.0141
4.25	68.8542	58.1979	49.2545	41.7414	35.4234	30.1048
4.5	85.6868	71.7211	60.1192	50.4702	42.4363	35.7393
4.75	105.3245	87.3022	72.4817	60.2791	50.2193	41.9147
5	128.0349	105.0974	86.4257	71.2056	58.7813	48.6240

TABLE A2.5 VALUES OF I(4,r,T)

$$I(4,r,T) = \int_0^T t^4 e^{-rt} dt$$

T	5%	10%	15%	20%	25%	30%
0	0	0	0	0	0	0
0.25	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
0.5	0.0061	0.0060	0.0059	0.0058	0.0056	0.0055
0.75	0.0460	0.0446	0.0432	0.0419	0.0406	0.0394
1	0.1918	0.1840	0.1765	0.1694	0.1625	0.1559
1.25	0.5794	0.5501	0.5222	0.4959	0.4709	0.4472
1.5	1.4268	1.3406	1.2597	1.1839	1.1127	1.0460
1.75	3.0520	2.8380	2.6395	2.4552	2.2842	2.1254
2	5.8889	5.4197	4.9889	4.5932	4.2299	3.8962
2.25	10.5022	9.5661	8.7156	7.9429	7.2406	6.6022
2.5	17.6019	15.8681	14.3097	12.9087	11.6487	10.5153
2.75	28.0552	25.0322	22.3437	19.9520	17.8236	15.9289
3	42.8990	37.8841	33.4711	29.5865	26.1655	23.1518
3.25	63.3506	55.3718	48.4247	42.3732	37.0996	32.5018
3.5	90.8174	78.5668	68.0126	58.9152	51.0694	44.2996
3.75	126.9056	108.6646	93.1145	79.8508	68.5306	58.8630
4	173.4290	146.9840	124.6771	105.8483	89.9444	76.5016
4.25	232.4156	194.9660	163.7081	137.5985	115.7723	97.5122
4.5	306.1140	254.1714	211.2712	175.8092	146.4700	122.1743
4.75	396.9997	326.2782	268.4798	221.1984	182.4822	150.7471
5	507.7794	413.0775	336.4908	274.4885	224.2383	183.4660

APPENDIX 3: PRESENT WORTH OF A CONTINUOUS CASH FLOW FUNCTION WITH TIME DISCRETIZATION

A3.1 INTRODUCTION

This appendix presents a procedure to determine the equivalent present worth of a continuous cash flow **F(t)** in which time is discretized. Reasons to discretize might include: (1) absence of a closed form expression for **F(t)**, (2) lack of a closed form for the present worth equivalent expression, and (3) need of increased speed of computation, when either **F(t)** or its equivalent present worth have a complex analytical expression.

Section A3.2 introduces the procedure. Section A3.3 presents the **QUICK BASIC** listing of program **DISCRET** that implements the discussed procedure.

A.3.2 DISCRETIZATION OF A CASH FLOW FUNCTION

Suppose it is desired to discretize some continuous cash flow function **F(t)**, defined over a time period **(0,T)** (in years). The resulting discretization will be expressed in terms of monthly intervals. There are **T*12** such monthly intervals.

Month **m** (equivalent to time **t=m/12** years) is assumed to have a constant cash flow, **CF(m)**, equal to the value of the cash flow function at the end of that month. The equivalent present worth, $\mathbf{F(m)}$, at the beginning of month $\mathbf{m}$ (coincident with the end of month $\mathbf{(m-1)}$) is

$$\mathbf{F(m)=I(0,r,1/12)CF(m)} \qquad \text{(A3.1)}$$

The factor $\mathbf{I(0,r,1/12)}$ is defined in Appendix 1.

The equivalent present worth of $\mathbf{F(m)}$, at $\mathbf{t}=0$, is

$$\begin{aligned}\mathbf{P(m)} &= \mathbf{F(m)e^{-r(m-1)/12}} \\ &= \mathbf{I(0,r,1/12)CF(m)e^{-r(m-1)/12}} \qquad \text{(A3.2)}\end{aligned}$$

Thus, the total present worth for the whole period $\mathbf{(0,T)}$ is

$$\mathbf{P=I(0,r,1/12)\sum_{m=1}^{T*12} CF(m)e^{-r(m-1)/12}} \qquad \text{(A3.3)}$$

A3.2 PROGRAM LISTING

Program **DISCRET** performs the above discretization procedure for a second order polynomial cash flow function:

$$\mathbf{CF(t)=c_0+c_1t+c_2t^2} \qquad \text{(A3.4)}$$

The user is asked to input: (1) discount rate, (2) planning horizon (in years), and (3) value of coefficients in equation (A3.4). The program provides the value of **P**. The **QUICKBASIC** program listing is given below.

```
'*************************************************************************
'*                  DISCRETIZATION OF A CASH FLOW FUNCTION                *
'*                          IN MONTHLY PERIODS                             *
'*************************************************************************
COLOR 6, 1: CLS
  COLOR 4, 1: LOCATE 3, 27: PRINT "FINANCIAL PARAMETERS": COLOR 6, 1
  LOCATE 6, 19: INPUT "Planning Horizon (years): ", T
  LOCATE 8, 23: INPUT "Discount Rate (0.--): ", R
  COLOR 4, 1: LOCATE 11, 26: PRINT "CASH FLOW FUNCTION DATA": COLOR 6, 1
  LOCATE 13, 34: INPUT "Intercept: ", C0
  LOCATE 15, 20: INPUT "First Order Coefficient: ", C1
  LOCATE 17, 19: INPUT "Second Order Coefficient: ", C2
DEF FNCF (MONTH, C0, C1, C2) = C0 + C1 * MONTH / 12 + C2 * (MONTH / 12) ^ 2
PC0 = (1 - EXP(-R / 12)) / R
SUMCF = 0
FOR MONTH = 1 TO T * 12
 SUMCF = SUMCF + FNCF(MONTH, C0, C1, C2) * EXP(-R * (MONTH - 1) / 12)
NEXT MONTH
P = PC0 * SUMCF
COLOR 4, 1: LOCATE 21, 23: PRINT "PRESENT WORTH ==> "; P
```

APPENDIX 4: ESTIMATION OF PRESENT WORTH OF LOSSES FOR NORMAL PRODUCT CHARACTERISTIC WITH HIBUT OBJECTIVE

A4.1 INTRODUCTION

This appendix presents a program, **HIBNOR**, that implements results from Section 4.3 for the specific case of a normally distributed product characteristic. The program evaluates the present worth of expected losses for a product characteristic which is: (1) Class II (higher is better loss minimization objective with undetermined target), and (2) normally distributed.

The program is listed in Subsection A4.2, and supports the different types of variance and mean drifts considered in this research. To determine the expected loss at a certain time instant, the program performs numerical integration using the trapezoidal rule. Time is discretized following the procedure discussed in Appendix 3.

The user is required to input: (1) financial parameters (**T** and **r**), (2) the constant **k**, (3) mean drift parameters, and (4) variance drift parameters.

A4.2 PROGRAM LISTING

```
'*******************************************************************
'*               CLASS II PRODUCT CHARACTERISTIC:                  *
'*                      HIBUT OBJECTIVE                            *
'*               ESTIMATING PWL BASED ON Z=1/Y                     *
'*          WHEN Y IS NORMAL, MEAN =Mu(0)+m1t+m2t^2                *
'*                    AND VAR=V(0)+V1*t                            *
'*******************************************************************
COLOR 6, 1: CLS
COLOR 4, 1: LOCATE 3, 27: PRINT "FINANCIAL PARAMETERS": COLOR 6, 1
  LOCATE 6, 16: INPUT "Planning Horizon (years): ", T
  LOCATE 8, 20: INPUT "Discount Rate (0.--): ", R
COLOR 4, 1: LOCATE 12, 27: PRINT "USER'S WANTS": COLOR 6, 1
  LOCATE 14, 30: INPUT "Constant K: ", K
CLS
COLOR 4, 1: LOCATE 4, 33: PRINT "MEAN PARAMETERS": COLOR 6, 1
  LOCATE 6, 34: INPUT "Intercept: ", MU0
  LOCATE 8, 20: INPUT "First Order Coefficient: ", M1
  LOCATE 10, 19: INPUT "Second Order Coefficient: ", M2
COLOR 4, 1: LOCATE 13, 30: PRINT "VARIANCE PARAMETERS": COLOR 6, 1
  LOCATE 15, 34: INPUT "Intercept: ", V0
  LOCATE 17, 20: INPUT "First Order Coefficient: ", V1

DEF FNMUZ (MU, SIGMA, S) = EXP(-.5 * S ^ 2) / (SQR(2 * 3.1416) * (MU + S * SIGMA))
DEF FNSQZ (MU, SIGMA, S) = EXP(-.5 * S ^ 2) / (SQR(2 * 3.1416) * (MU + S * SIGMA) ^ 2)
CLS
PC0 = (1 - EXP(-R / 12)) / R
PW = 0
FOR MONTH = 1 / 12 TO T STEP 1 / 12
  MU = MU0 + M1 * MONTH + M2 * MONTH ^ 2
  VAR = V0 + V1 * MONTH: SIGMA = SQR(VAR)
  GOSUB MSEZ
  LOSSZT = K * MSEZT
  PW = PW + LOSSZT * EXP(-R * (MONTH - 1) / 12)
NEXT MONTH
  PW = PC0 * PW
CLS
COLOR 4, 1: LOCATE 10, 23: PRINT "PRESENT WORTH ===> ", PW
COLOR 6, 1
END

MSEZ:
  A = -4: B = 4: N = 80
  W = (B - A) / N: MSEZT = 0
  FOR I = 1 TO N
    LL = A + (I - 1) * W: UL = A + I * W
    MSEZT = MSEZT + W * (FNSQZ(MU, SIGMA, LL) + FNSQZ(MU, SIGMA, UL)) / 2
  NEXT I
RETURN
```

APPENDIX 5: PARTIAL MOMENTS FOR A NORMALLY DISTRIBUTED RANDOM VARIABLE

A5.1 INTRODUCTION

Evaluation of the present worth of expected losses for a product characteristic with an ANIB loss minimization objective implies use of partial moments around the target. Partial moments are moments defined over a partial domain of a random variable $\mathbf{y}$ (in the context of this research, a product characteristic).

This appendix adapts expressions from Winkler, et. al. (1972) to determine partial moments of a product characteristic normally distributed.

A5.2 PARTIAL MOMENTS

Let $\mathbf{y}$ be a normally distributed product characteristic with mean μ_y and variance σ_y^2. Using notation from Section 3.3, the partial mean for the lower side around the target, μ_L, is

$$\mu_L=-\sigma_y^2 f_y(\tau)+\mu_y F_y(\tau) \qquad \text{(A5.1)}$$

where $f_y(\tau)$ denotes the probability density function of $\mathbf{y}$ evaluated at the target, and $F_y(\tau)$ is the cumulative distribution evaluated at the target.

The partial mean for the upper side around the target, μ_U, can be found by substraction:

$$\mu_U=\mu_y-\mu_L \qquad (A5.2)$$

The partial second order moment for the lower side around the target, SOM_L is

$$SOM_L=-\sigma_y^2(\tau+\mu_y)f_y(\tau)+(\mu_y^2+\sigma_y^2)F_y(\tau) \qquad (A5.3)$$

The partial second order moment for the upper side around the target, SOM_U can be found by substraction:

$$SOM_U=E[y^2]-SOM_L=\sigma_y^2+\mu_y^2-SOM_L \qquad (A5.4)$$

The semivariance, s_L^2 for the lower side around the target is

$$s_L^2=(\tau+\mu_y+2)f_y(\tau)+(2\mu_y^2-2\mu_y+\sigma_y^2)F_y(\tau) \qquad (A5.5)$$

The semivariance, s_U^2 for the upper side around the target can be determined by substraction:

$$s_U^2=\sigma_y^2-s_L^2 \qquad (A5.6)$$

APPENDIX 6: MULTIDIMENSIONAL TLF AND ITS ELLIPTICAL CONTOURS

A6.1 INTRODUCTION

This appendix presents different properties of ellipses useful for the multidimensional TLF discussed in Chapter 6. Subsection A6.2 describes the relationship of ellipses and the multidimensional TLF. Subsection A6.3 analyzes ellipses.

A6.2 MULTIDIMENSIONAL TAGUCHI LOSS FUNCTION

Consider a product for which quality is determined by multiple different product characteristics. Without loss of generality, assume all the product characteristics to be symmetric nominal-is-better. The loss function is Taguchi type (quadratic loss function), $\mathbf{L(Y)}$. From equation (6.1), a multidimensional loss function, $\mathbf{L(Y)}$ is defined as the quadratic form expression:

$$\mathbf{L(Y) = [Y-\tau]' \; K \; [Y-\tau]} \qquad (A6.1)$$

where $\boldsymbol{\tau}$ is the $\mathbf{px1}$ vector of targets of the $\mathbf{p}$ product characteristics. The matrix $\mathbf{K}$ is a $\mathbf{pxp}$ symmetric positive definite matrix representing the losses incurred when $\mathbf{Y}$ deviates from $\boldsymbol{\tau}$. This follows from the fact that $\mathbf{L(B)}>0$ for any vector $\mathbf{(y_1, y_2, \ldots, y_p)' \neq (0, 0, \ldots, 0)'}$.

If $\mathbf{B}$ denotes the $\mathbf{2x1}$ vector of biases, i.e., $\mathbf{B} = \mathbf{Y} - \boldsymbol{\tau}$, then the loss function (in terms of the product characteristics' biases) is:

$$\mathbf{L(B) = L(Y) = B' \, K \, B} \qquad \text{(A6.2)}$$

A6.2 ELLIPTICAL CONTOURS OF THE MULTIDIMENSIONAL TLF

Consider a three dimensional space which includes the plane with axis $(\mathbf{b_i},\mathbf{0})'$ and $(\mathbf{0},\mathbf{b_j})'$. On such a space, $\mathbf{L(B)}$ defines an elliptic paraboloid centered at the origin (Figure 6.1). The contours of the paraboloid are shown in Figure 6.2. They describe concentric ellipses with the principal axis rotated an angle of α radians. Then, a loss of $\mathbf{L(B)=L}$ dollars is associated with the ellipse:

$$k_{11}b^2{}_1+2k_{12}b_1b_2+k_{22}b^2{}_2=L \qquad \text{(A6.3)}$$

This ellipse can be expressed in its <u>canonical form</u>. The canonical form is defined on a new coordinate system $(\mathbf{b'_i},\mathbf{b'_j})'$ such that the cross terms of the ellipse equation vanish. The canonical form of equation (A6.3) is:

$$\lambda_1 b'^2{}_1+\lambda_2 b'^2{}_2=L \qquad \text{(A6.4)}$$

On the new coordinate system, the ellipse: (1) is centered at the origin, (2) has the vertices: $(\sqrt{L / \lambda_1},0)$, and $(0,\sqrt{L / \lambda_2})$, and (3) has its principal axis coincident with the plane's abscissas axis.

The coefficients λ_1 and λ_2 correspond to the eigenvalues or characteristic values of the matrix **K** (Yefimov, 1964; Schmidt and Davis, 1981). The eigenvalues are the roots of the characteristic equation defined as:

$$|\mathbf{K}-\lambda\mathbf{I}|=0 \qquad \text{(A6.5)}$$

Solving for λ in the characteristic equation yields the following two roots:

$$\lambda_1=[k_{11}+k_{22}-\sqrt{(k_{11}-k_{22})^2+4k^2{}_{12}}\,]/2 \qquad \text{(A6.6)}$$

and

$$\lambda_2=[k_{11}+k_{22}+\sqrt{(k_{11}-k_{22})^2+4k^2{}_{12}}\,]/2 \qquad \text{(A6.7)}$$

For ellipses with **K** definite positive, their eigenvalues (Yefimov, 1964): (1) are such that $\lambda_1>0$, and $\lambda_2>0$, and (2) define circular contours of $\mathbf{L(B)}$ if $\lambda_1=\lambda_2$. Note that the way equations (A6.6) and (A6.7) are defined imply that $\lambda_1<\lambda_2$. Then, the principal axis of the ellipse has the same direction as the eigenvector associated with λ_1. Furthermore, if $\mathbf{v}_1(\lambda_1)$ and $\mathbf{v}_2(\lambda_1)$ are the abscissa and ordinate, respectively of such eigenvector on the original coordinate system, the angle of rotation α is given by:

$$\alpha=\tan^{-1}\left(\frac{v_2(\lambda_1)}{v_1(\lambda_1)}\right) \qquad (A6.8)$$

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