# THE EFFECT ON THE UNDERLYING STOCK 

 FROM THE INTRODUCTION OF ANOPTIONS MARKET: A UTILITY
PREFERENCE APPROACH

By
LARRY C. HOLLAND
Bachelor of Science, Chemical Engineering
University of Missouri-Rolla
Rolla, Missouri
1968
Master of Business Administration
University of Tulsa
Tulsa, Oklahoma
1975

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## Larry C. Holland

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Thesis Approved:


## PREFACE

This study was conducted to determine the effect of an options market on the price of the underlying asset. A theoretical model is first developed to show that the introduction of individual options circumvents short sale restrictions, which results in negative abnormal returns. Then, empirical tests of the model show that the data are consistent with the model. In addition, a specific objective of this research is to refine an event study methodology for measuring long term abnormal returns. This methodology adjusts for differences in size (measured by market capitalization) and uses holding period returns for determining multiperiod abnormal returns.

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## I. INTRODUCTION

Exchange traded options were first introduced in 1973. The options market expanded rapidly in the next four years, reaching an option open interest ${ }^{1}$ of four million contracts (see Figure 1). This rapid expansion, however, brought concern to the SEC about potential negative effects on the underlying stocks. As a move of caution, the SEC instituted a moratorium on the introduction of new options from July 15, 1977 to March 26, 1980 in order to provide sufficient time to research the potential effects of options. Research at that time (and subsequently) indicates positive effects from the introduction of options, including an increase in the price of the underlying stock on the introduction date and a subsequent reduction in total variance. The research in this paper is unique in developing and testing a model that posits a long term negative effect on the price of the underlying stock following the introduction of options. This result occurs as options make the market more efficient by circumventing a market inefficiency -- restrictions on selling short individual stocks.

Two main features of the theoretical model in this dissertation are (1) a dispersion of beliefs about the future stock price and (2) a restriction on short sales for a subset of investors while less binding restrictions are imposed on the specialists and market makers supplying options. The first feature means that some investors

[^0]FIGURE 1
Option Open Interest for
All Exchanges -- 1975-1994
(Million Contracts)
Source: Barron's

may desire to hold long positions in the stock while others may desire to hold short positions. The second feature does not allow a subset of investors who desire a short position to sell the stock short. Restrictions on short sales produce an incomplete market when investors with a negative assessment about a stock cannot create an investment position to take advantage of their beliefs. Thus, the stock price with short sale restrictions is higher than the stock price in an unrestricted equilibrium. In the case of short sale restrictions, the availability and use of options eliminates this upward stock price bias and causes a negative price effect along the following line of reasoning (this process is discussed in greater detail later in this dissertation). From a utility preference approach, investors who have short sale restrictions are unable to reach their maximum expected utility if they desire to sell a stock short. When options are available, however, they can reach their maximum level of expected utility by writing call options or purchasing put options. When the market maker (who takes an opposite position) constructs a hedge portfolio to reduce the risk of his position, he sells the underlying stock short (the Federal Reserve regulation on short sales exempts market makers from ordinary restrictions on short sales). An increase in short interest effectively increases the supply of stock available, which places a downward pressure on the stock price. Therefore, the introduction of options causes a negative price effect on the underlying stock by circumventing short sale restrictions.

Because the downward price pressure depends upon the net increase in short interest, the magnitude of the negative price pressure would directly correlate with the
level of option open interest. Thus, the negative price effect on the underlying stock would occur over the development period of the market in the respective option and its measurement should be over a significant period of time (e.g., a one year period following introduction).

To set the stage for development and testing of the model later in this dissertation, the next sections present the implications of this research, a review of short sale restrictions, and a discussion of the linkage between option introduction and the stock price.

## Implications of this Research

The research in this dissertation deals with the question of whether the introduction of an options market has a negative effect on the price of the underlying stock. Although this question is very straightforward, there are several important implications if such a negative effect occurs. First of all, such an effect would be a concern to regulatory agencies. As mentioned earlier, the SEC imposed a moratorium on new option introductions from 1977 to 1980 to specifically determine if options have a negative effect on the underlying stocks. New introductions of options continue today (e.g. the introduction in 1989 of LEAPS, or Long term Equity Anticipation Securities, which are long term options), and regulatory agencies remain concerned about potential negative impacts on existing equityholders. Just as important is a better understanding of the mechanism that results in negative effects.

In addition, this research also provides a theoretical model which can be applied to explore other issues, including the option expiration effect and impacts from the 1987 stock market decline.

A second important implication pertains to an investment strategy for investors. If there is a negative price impact following the introduction of options, then investors should divest of stocks when an option introduction is announced in order to avoid a continuing loss as the options market develops. Of course, if all investors chose to do this, the price of the underlying stock would fall immediately after the announcement in anticipation of future negative price pressures. Some options that are introduced, however, do not develop a significant level of trading even one year after introduction. Thus, the price effect is not totally predictable. Therefore, a continuing negative impact over a five year period is not necessarily inconsistent with an efficient market hypothesis.

A third important implication is a model which demonstrates a link between option introduction and the stock price. Such a linkage is normally assumed away (e.g., the Black-Scholes and binomial option pricing models). Extensions of the research in this dissertation include an adjustment to option pricing during periods of change in option open interest.

## Short Sale Restrictions

There are several restrictions to short sales of a stock that are a key feature of the model developed later. This section discusses the mechanics of short selling and identifies various regulatory restrictions and exemptions.

The short sale of a stock occurs when an investor instructs a broker to sell shares of stock that he does not own. The investor would desire to do this if he believes that the stock price is going to decline in the future. Since the investor does not own the stock, his broker must borrow the stock from some other investor (usually from a margined account that holds a long position). A profit can be made if the price of the stock declines. At that point, the investor could purchase the stock at a lower price and replace the borrowed stock. The net from initially selling at a higher price minus the lower subsequent purchase price creates a profit for the investor.

Often the assumption is made that the full proceeds from the initial short sale are available to the investor prior to closing the short position (by purchasing the stock to replace the borrowed stock). However, the actual process does not allow such access. Regulation T by the Federal Reserve not only restricts access to the proceeds of a short sale but also requires an additional $50 \%$ margin to be deposited with the broker. ${ }^{2}$ Dunkin (1991) further points out that larger investors with more

[^1]than $\$ 250,000$ invested often negotiate to receive interest on the $150 \%$ margin requirement (initial sale plus $50 \%$ margin). Smaller investors normally do not receive interest on their margin accounts. Thus, when interest rates are higher, the disincentive against short sales for smaller investors becomes greater, while there is little change for the larger investors (who are able to receive interest on their margin requirement).

The Federal Reserve, however, specifically exempts registered specialists and market makers from the $150 \%$ margin requirement for short sales in their Regulation T. ${ }^{3}$ This exemption is the basis for the case with no short sale restrictions as part of the development of the theoretical model.

The Securities and Exchange Commission (SEC) also places restrictions on the short sale of stock. Its regulations require the price of the stock to increase before the stock can be sold short. ${ }^{4}$ This is often referred to as the "uptick rule." This particular regulation is designed to prevent the short sale of a stock when the stock price is already declining so that the decline will not be exacerbated. It should be noted, however, that at the present time the uptick rule does not apply to NASDAQ stocks. Again registered specialists and market makers are exempted from the

[^2]restriction (the uptick rule). ${ }^{5}$

In spite of restrictions on short sales for investors who are not market makers, the trend of aggregate short interest over the last fifteen years shows a sharp increase, as illustrated by the short interest on the New York Stock Exchange (NYSE) in Figure 2. Makin (1993) reports that the total outstanding short interest on all exchanges reached a high of 1.4 billion shares in November, 1992. Dunkin (1991) indicated that the short interest had reached a dollar value of about $\$ 30$ billion compared to about $\$ 3$ trillion in "long" stock positions. In a recent Wall Street Journal article, Naik (1993) reports that short interest on the NYSE increased to a record 1.24 billion shares as of mid December, 1993, while the American Stock Exchange short interest increased to 102 million shares. Getler (1993) reports that the short interest on the NASDAQ Stock Market also set a record of 672 million shares in December, 1993. A summation of these short interest statistics indicates that total reported U.S. short interest reached a new record in December, 1993, of 2.01 billion shares.

An increase in the short interest of a stock has the effect of increasing the number of shares available to investors beyond the total outstanding shares shown on the records of the issuing corporation. John Conyers, Chairman of the House Committee on Government Operations, submitted a committee report on short selling that reviewed 695 companies which had relatively high short interest from 1986 to

[^3]FIGURE 2
NYSE Short Interest -- 1975-1994
(Million Shares)
Source: NYSE Fact Books


1990 (see Conyers, 1991). A review of the list for examples of relative short interest (short interest divided by total shares outstanding on record) above $20 \%$ illustrates how extreme the short interest can expand the number of shares available. For example, the short interest for GCA Corporation in April, 1987, was $98 \%$ of shares outstanding, nearly doubling the record number of shares for that company. Hitech Engineering had short interest of $41 \%$ of shares outstanding in September, 1987, and Blockbuster Entertainment had short interest of $24 \%$ of shares outstanding in December, 1990. In a celebrated abuse of short selling by a market maker, Leon Greenblatt of the securities firm, Scattered Corporation, alone sold short over $100 \%$ of the shares of record for LTV Corporation from May through June, 1993 (see Peers and Taylor, 1993).

It should be noted that there are several reasons for the increase in short interest over the past 20 years. Makin (1993) pointed out that much of the growth in short interest occurs with hedging from market neutral managers. Price (1989) also pointed out that short positions have a place in broad portfolio strategies because of their negative correlation with long positions.

In summary, ordinary investors have restrictions regarding the short sale of stock because of the regulatory requirement of a $150 \%$ margin. In contrast, specialists and market makers are exempt from this requirement. This restriction for ordinary investors (but not for market makers) is an important feature of the
mechanism in the theoretical model which results in a negative price effect on the stock underlying an option.

## Option Introduction and Stock Price Link

Another important aspect of this research is the link between the introduction of an option and the underlying stock price. The original development of the Black and Scholes (1973) option pricing model is based upon a portfolio of stocks and riskfree bonds that replicates the payoff structure of an option. The implicit assumption is that an option is a redundant asset and that the creation of an option has no effect on the underlying stock. More specifically, the change in the stock price is assumed to follow the equation:

$$
\begin{equation*}
d S=\alpha S d t+\sigma S d z \tag{1}
\end{equation*}
$$

where $\mathrm{dS}=$ the change in the stock price at any instant,
$S \quad=$ the instantaneous value of the stock price,
$\mathrm{dt}=$ the smallest increment of time,
$\alpha=$ the drift per unit of time, which can be a function of $S$ and $t$,
$\sigma=$ the volatility of the stock price, which can be a function of $S$ and $t$, and
$\mathrm{dz}=\mathrm{a}$ random draw from a normal distribution with a mean zero and a variance of dt .

Note that the change in the stock price is not influenced by the presence or absence of options. The binomial option pricing methodology (see Cox and Rubenstein (1979)) also does not permit options to affect the stock price. In this case, a lattice is first developed showing assumed possible stock prices over time. Next, given the strike price, a lattice of possible call prices is developed recursively using the data in the stock price lattice to create replicating portfolios. However, if the creation of options has an effect on the underlying stock price, the stock price lattice must be adjusted for changes in the stock price when options are introduced before the option price can be determined. Thus, implicitly the Black-Scholes and binomial option pricing models assume that the creation of an options market has no effect on the underlying stock price.

The research in this dissertation develops a model which demonstrates the negative effect that the introduction of options can have on the price of the underlying security when options are used to circumvent restrictions on short sales. This feature is not included in other models and empirical tests in the literature.

In order to provide a background of the existing research, Chapter II reviews the theoretical and empirical literature on the effect of option introduction and short sale constraints. Chapter III then develops a theoretical linkage between the introduction of options and the underlying stock price when there are constraints on short sales. In this chapter, a one-period model which maximizes expected utility with
heterogeneous expectations of the future stock price is developed and includes scenarios that restrict short sales and that introduce options. A numerical example at the end of the chapter illustrates the model results. Chapter IV discusses the hypotheses, data, and methodology used to empirically test the model. Then Chapter V presents the results of the empirical tests. Finally, Chapter VI provides the conclusions derived from this dissertation.

This chapter first reviews the theoretical models in the literature. Then, existing empirical studies are presented. In the third section, the existing literature is compared to the research in this dissertation. Finally, the research studies on downward sloping demand curves for equities is presented.

## Theoretical Models in the Literature

It is surprising that there are very few theoretical models to explain how the introduction of an options market can affect the underlying stock, especially given the abundance of empirical studies reviewed in the next section. In the options literature, the Detemple and Selden (1991) model is the only model that specifically details the effect on the underlying stock from the introduction of options. There are also several models in the short sales literature which, although they do not specifically address an effect from options, can be extended to include such an effect.

Detemple and Selden (1991) develop a theoretical model that predicts a positive effect on the underlying stock when options are introduced. Their model assumes an incomplete market and two investors who have the same belief about the mean of the stock price distribution but have heterogeneous beliefs about the downward potential of the underlying stock price (the upside potential is assumed to be the same). The mean and covariance of the stock distribution are assumed to be identical between the two investors, and only the perceived value of the variance is
allowed to be different. The authors note that the highly specific nature of this heterogeneity is "quite limited" but necessary to arrive at a clear cut result about the effect on the underlying stock. Short sales are specifically not allowed. Using the Capital Asset Pricing Model (CAPM) and a quadratic utility function, they show an inverse relationship between an option strike price and the price of the underlying stock. From this relationship, they graphically illustrate that as the exercise price rises, there is less and less incentive for trading in options. At the limit, when the exercise price converges on the maximum possible stock price, there will be no trading in options. Since this condition is equivalent to a market without options, they conclude that the introduction of options (i.e., an exercise price lower than the maximum stock price) will have a positive effect on the price of the underlying stock. This conclusion in their model is valid as long as the exercise price is above the area of disagreement between the two investors.

There are also a few models in the short sale literature that relate to the model in this dissertation. Miller (1977) develops a model showing that restrictions in short sales can bias the stock price upwards if there are heterogeneous expectations about the future stock price. He hypothesizes that there are more investors than shares of stock, and that the most optimistic investors will purchase one share of the stock. The price of the stock will be above the average assessment of the investors if short sales are restricted (i.e., negative demand eliminated). Also, the greater the dispersion of beliefs about the future stock price, the greater would be the upward price bias. If
short sales are possible, however, the upwards bias in the security price would be mitigated by pessimistic investors selling the stock short. The additional supply of the stock would create a lower stock price closer to the average value expected by all investors.

Jarrow (1980) shows that with more than one asset, if investors also disagree about the covariance matrix of the next period asset prices, the price bias from short sale constraints could be positive, negative, or neutral. He demonstrates this by extending the Lintner (1969) single period, mean-variance model of capital market equilibrium. However, if the price of each risky asset is determined independently or if there is no disagreement about the covariance matrix, short sale restrictions would always cause an upward bias in the stock price.

Diamond and Verrechia (1987) model the price effect from short selling restrictions using a rational expectations approach. They assume investors and market makers are risk neutral and show that there is no upward bias in the stock price from short sale constraints. However, their focus is on differences in the speed of adjustment to new information and whether informed or uninformed traders are more affected rather than the effect of a market restriction on holdings of a stock.

Using a one period model, Figlewski (1981) maximizes expected utility at the end of the period, defined on both mean and variance (stock returns are assumed to be
multivariate normally distributed). He then develops an unrestricted demand curve for stock which is below a demand curve when there are short sale constraints. Given a fixed supply, his model shows a higher stock price when there are short sale constraints and a lower stock price when there are no constraints.

In summary, the theoretical model developed by Detemple and Selden (1991) predicts that the introduction of options will have a positive price effect on the underlying stock. This is opposite to the effect that the model in this dissertation predicts. The models of Miller (1977) and Jarrow (1980) predict a positive bias in the stock price from the constraint on short sales while Diamond and Verrechia (1987) show no such positive bias in the stock price from a constraint in short sales using their rational expectations approach (focusing mainly on an information effect and speed of adjustment issues relating only to short sales). The model developed by Figlewski (1981) also features an upward bias in stock price as a result of short sale restrictions, using an objective of maximizing expected utility.

## Empirical Tests in the Options Literature

Many empirical studies show that the introduction of options affects the price of the underlying stock as well as some of the risk characteristics. For example, Conrad (1989), Detemple and Jorion (1990), Kim and Young (1991), and Haddad and Voorheis (1991) find evidence of an increase in the underlying stock price on the day an option is first introduced, and for a short time surrounding the introduction day
(upto 30 days). Table 1 shows a summary of their results, which generally indicate a maximum positive excess return of about $3 \%$.

In addition to the price effect, there is also empirical evidence of an introduction effect on the risk characteristics of the underlying stock. Whiteside, Dukes, and Dunne (1983), Conrad (1989), Bansal, Pruitt, and Wei (1989), Skinner (1989), Detemple and Jorion (1990), and Haddad and Voorheis (1991) find no change in systematic risk following the introduction of options. However, the mean beta calculated from daily data is slightly lower, but the difference is not statistically different from zero. Although systematic risk (beta) does not decline in a statistical sense, several studies find that the total variance has a statistically significant decline. These studies include Conrad (1989), Bansal Pruitt, and Wei (1989), Skinner (1989), Detemple and Jorion (1990), and Haddad and Voorheis (1991).

Damodaran and Subrahmanyam (1992) provide a survey of the literature on the effects of derivative securities on the markets for the underlying assets. In addition to the price and volatility effects discussed in the previous paragraphs, they cite studies which show a decrease in bid-ask spreads following the introduction of options and review related effects in the futures market.

A related area of the empirical literature focuses on short sales of stock.
Brent, Morse, and Stice (1990) use a regression analysis in an attempt to identify

TABLE 1
Empirical Studies on the Price Effect
of Short Term Options

|  | CAAR $_{0}$ | CAAR $_{-5,+5}$ CAAR $_{-10,+10}$ | Data <br> Period | Number of <br> Options |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Conrad (1989) | 0.31 | 2.75 | 3.14 | $1974-80$ | 96 |
| Detemple and Jorion (1990)* | 0.62 | 2.8 | 3.2 | $1973-86$ | 300 |
| Kim and Young (1991) | 0.37 | 1.39 | 0.58 | $1973-87$ | 249 |
| Haddad and Voorheis (1991) | 0.37 | 0.70 | 0.26 | $1973-86$ | 327 |

*Detemple and Jorion (1990) results are mean adjusted CAARs; CAAR ${ }_{-5,+5}$ and $\mathrm{CAAR}_{-10,+10}$ are estimated from their Figure 2 graph of cumulative mean-adjusted returns (Page 791 in their article). The other three studies use the market model to calculate CAARs.
factors which explain higher recent levels of short interest in stocks. Prominent in their study is a strong positive correlation between option open interest and short interest in the underlying stock. They speculate that arbitrage and hedging activities in the options market relate to a higher short interest level. They also show a month-to-month time series analysis in which fluctuations in option open interest due to periodic option expirations are significantly correlated with changes in short interest in the underlying stock.

More recently, Figlewski and Webb (1993) test the line of reasoning of the Figlewski (1981) model. They focus on a negative information effect from high levels of short sales and the subsequent effect on the returns of the stock. In a regression equation, they find that one of the factors that explains subsequent negative excess returns is a high level of short interest in the previous year. They also include a dummy variable term in their regression to determine any effect from the existence of an options market in the underlying stock. The coefficient on this dummy variable, however, is not significant in explaining subsequent negative excess returns.

In a study which is the direct impetus for this study, Swidler (1988) finds that options mitigate the effect of short interest restrictions from an empirical test of the CAPM model, taking into account the dispersion of analysts forecasts. He indicates that the net effect of a divergence of opinion is ambiguous. Specifically, actual returns are lower than expected (i.e., higher stock price) when there are short sale
restrictions, and this tends to mask a higher required return (i.e., lower stock price) when there is a higher dispersion of analyst forecasts. Swidler then empirically demonstrates that stocks which have options unambiguously show a higher return given a wider dispersion of analysts' forecasts. In contrast, stocks which do not have options do not show a higher return. Swidler concludes that options reduce the effect of short sale restrictions on security returns. In the conclusion of his article, Swidler states (1988, page 33), "the potential impact of option markets on security prices should be the focus of future research." The research in this dissertation is aimed at defining such an impact on the underlying security from the introduction of options.

## Comparison with this Research

A comparison of the theoretical model in this dissertation to the theoretical models in the literature identifies some differences and similarities.

The Detemple and Selden (1991) model indicates a positive effect on the price of the underlying stock when options are introduced, which is opposite to the result of the model in this dissertation. This opposite result occurs because of differences in assumptions and approach. Detemple and Selden assume a highly restrictive form of heterogeneous beliefs in which only the downward potential of the stock is allowed to be different (the upside potential is assumed to be the same). The mean and covariance of the stock distribution are assumed to be identical, and only the perceived value of variance is allowed to be different. With this restrictive definition
of heterogeneous beliefs, comparative statics in their model unambiguously show a positive price effect when options are introduced. If a less restrictive form of heterogeneous beliefs is allowed, the results from this model are not clear cut. In comparison, the model developed in this dissertation has a more general form of heterogeneous beliefs. In this case, each investor can have a different belief about the mean of the future stock price.

A second and more contrasting difference in the two models relates to the basic approach. In the Detemple and Selden (1991) model, the stock and options are considered as separate securities with independent supply and demand. Options are also considered an inside asset, with zero aggregate supply. This means that there must be a bullish call option purchaser for every bearish call option seller. In contrast, the model in this dissertation allows a connection between options and the underlying stock in which they are not independent assets. In this case, options can be created from a position in the underlying stock (i.e., short sales). This means that a balance between bullish and bearish investors is not required. As shown later, bearish investors who decide to sell call options are allowed to do so by market makers who purchase the call options and take a hedged position in the underlying stock (i.e., bullish investors are not required to purchase the written call options). This focus on a connection between options and the underlying stock provides a more direct analysis of the effect options have on the underlying stock. The source of this effect is a restriction on short sales of the stock, which causes the stock price to be
upwardly biased (note that the Detemple and Selden model specifically does not allow short positions in the stock). Options in this case circumvent the short sale restrictions, remove the upward bias, and thus result in a negative effect on the price of the underlying stock. Therefore, differences in assumption and approach account for the differences in the two models.

The models of a stock price bias in the short sale literature generally show consistent results compared to the model developed in this dissertation. The model of Miller (1977), further extended by Jarrow (1980), indicates an upward bias in the stock price when there are short sale restrictions. Jarrow's extension of the model to include more than one stock indicates an ambiguous bias when the stock prices are not independent or when there is disagreement about the covariance matrix. To avoid this ambiguity, the model in this dissertation assumes no disagreement about the covariance matrix. Thus, differences in belief about future stock prices also must be reflected in perfect substitutes.

The Diamond and Verrechia (1987) model shows no upward bias in the stock price. However, they assume that investors and market makers are risk-neutral. Their focus is on differences in speed of adjustment and whether informed or uninformed traders are more affected. In contrast, the model in this dissertation assumes risk-averse utility preferences and the objective of maximizing expected utility. Since the assumptions, approach, and focus of these two models are quite
different, it is not too surprising that the results concerning the stock price bias are also different.

The Figlewski (1981) model is similar to the model developed in this dissertation. Both follow the same general logic at first, but the model in this dissertation adds the feature of an options market. Figlewski points out the need for additional research to take into account the trading of options. In the concluding comments to his article, he states, "One aspect of the problem which we have not considered in this paper is the possibility that pessimistic investors will attempt to circumvent the constraints on short selling in the stock market by trading in options. By buying puts or writing calls an investor can take an options position that is equivalent to a short sale of the stock. As investors do this, the price of the underlying stock will be influenced and their unfavorable information will be incorporated by this indirect means. This mechanism will be examined in future research." ${ }^{6}$ The objective of the research in this dissertation is to model and test the effect on the underlying stock when options are introduced (and restrictions on short sales are circumvented).

Finally, a comparison of the results of existing empirical studies to those in this dissertation contrasts effects measured over different periods of time. The existing empirical studies of the effect on the underlying stock show a maximum

[^4]positive abnormal return of $3 \%$ within 30 days of the introduction of an option on that stock. In contrast to these 30-day results, the research in this dissertation covers a period of one to five years following the introduction of options and finds a significant negative price effect over this longer time frame. This additional period of time is required in order for the options market to become large enough to have a measurable impact on the stock price.

The empirical work of Swidler (1988) finds evidence that short sale restrictions bias the stock price upwards. The model developed in this dissertation demonstrates that such an upward bias is removed as options are used to circumvent short sale restrictions. Brent, Morse, and Stice (1990) also provide corroborative evidence consistent with the model in this dissertation. They show a significant correlation between option open interest and short interest in the underlying stock. The model in this dissertation predicts such a correlation. Last of all, the empirical study of Figlewski and Webb (1993) explain subsequent negative excess returns in a stock in a regression equation with high levels of short interest in the previous year and a dummy variable for the existence of options (a lagged relationship). In contrast, the model and empirical tests in this dissertation focus on the introduction of options and the concurrent effect on short interest and negative excess returns. This coterminous focus on the introduction of options permits a more dynamic and direct analysis of the effect on the underlying stock price.

## Research on Downward Sloping Demand Curve

An important issue affecting the empirical testing of the model is whether the demand curve is downward sloping in both the short and long term. It is often assumed that the demand curve is perfectly elastic (horizontal). Such an assumption is viable if there is a ready availability of perfect substitutes. With such a condition, any change in the supply of an individual stock would not affect the price of that stock as investors shift among perfect substitutes to compensate for the changes in the supply of an individual stock. The model in this dissertation avoids this issue by simplifying to a single stock, which creates a downward sloping demand curve by construction. However, empirical tests of the model occur in a market of multiple stocks, including substitutes (although perhaps not perfect substitutes). These tests implicitly assume a downward sloping demand curve over the long term (one to five years following option introduction) and that changes in supply and demand have an impact on the price of the stock. The price of an individual stock will thus be affected as supply increases from additional short interest (when options circumvent short sale restrictions) and as institutional demand for the stock increases. If in fact the demand curve is horizontal, these changes in supply and demand will not affect the stock price. The following paragraphs address the current research on the slope of the demand curve in the short and long term.

Previous research on whether the demand curve is downward sloping focuses mainly on the effect of block sales of stock and on the impact of listing on the S\&P

500 Index. Scholes (1972) and Mikkelson and Partch (1985) show that there are significant negative abnormal returns on the day of issue of secondary distributions of stock (large block sales). Scholes further finds these abnormal returns are persistent 18 months after the month of sale. Kraus and Stoll (1972) find similar negative abnormal returns on block trades of stock on a down tick (assumed to be seller initiated). Both Mikkelson and Partch (1985) and Kraus and Stoll (1972) also demonstrate a relationship between abnormal returns and the size of the offering, which is consistent with a downward sloping demand curve. However, the same results could also occur if there is a transfer of information with these block sales.

Another empirical approach in which there is no information transfer pertains to the addition of stocks to the S\&P 500 Index. Periodically, some securities are dropped from the S\&P 500 Index because of mergers, acquisitions, and bankruptcies. When new securities are added to the index, investment companies that manage S\&P 500 index funds must purchase these securities to maintain a portfolio that mimics the index. This increase in demand is a shift of the demand curve to the right, which in turn results in a higher stock price if the demand curve is downward sloping (there would be no change in price if the demand curve is horizontal). Shleifer (1986) and Woolridge and Ghosh (1986) find positive abnormal returns on the first trading day after addition to the S\&P 500 Index. In addition, they find that these positive abnormal returns are persistent 21 to 30 days thereafter, especially in more recent subsamples (1980-1983). In contrast, Harris and Gurel (1986) and Lamoreux and

Wansley (1987) find positive abnormal returns on the first trading day after addition, but which gradually disappear after about three to four weeks. Pruitt and Wei (1989) extend the Harris and Gurel (1986) study by showing a significant relationship between positive abnormal returns and a positive change in institutional holdings. This provides evidence of an increase in demand from institutional investors rather than just reshuffling the holdings among index fund and non-index fund institutions.

In summary, the current research in the literature provides support for a downward sloping demand curve in the short term, but ambiguous results concerning a downward sloping demand curve for the longer term (over one month). Empirical results consistent with the model in this dissertation would provide evidence for a downward sloping demand curve over a longer term period, with persistent abnormal returns one to five years after the introduction of options.

## Event Study Methodologies

This section reviews various approaches for calculating abnormal returns using event study methodologies. Recent issues and the current state-of-the-art in event study methodologies are reviewed in Peterson (1989) and Strong (1992). Brown and Warner $(1980,1985)$ classify event study methodologies into three categories: (1) Market model, (2) Comparison period mean adjusted model, and (3) Market adjusted model. A fourth category is a size decile adjusted model, which is a variation of the third category using the return from a portfolio of comparble size securities instead of
a market index as the expected return. The first two methods require an estimation period (generally before the event) to calculate estimated parameters for the model. Then abnormal returns are calculated for a test period, usually at the event date. The following paragraphs review the specific characteristics of these four categories of event study methodologies.

Market Model. Fama, Fisher, Jensen, and Roll (1969) use the ordinary least squares market model to calculate abnormal returns around stock splits as follows:

$$
\begin{gather*}
R_{j t}=\alpha_{j}+\beta_{j} R_{m}+e_{j t}  \tag{2}\\
A R_{j t}=R_{j t}-\alpha_{j}-\beta_{j} R_{m t}  \tag{3}\\
A A R_{t}=\frac{1}{n} \sum A R_{j t} \tag{4}
\end{gather*}
$$

where $\mathrm{AAR}_{\mathrm{t}}=$ average abnormal return at time t ,
$\mathrm{AR}_{\mathrm{jt}}=$ abnormal return for security j at time t ,
$\mathrm{R}_{\mathrm{jt}} \quad=$ return of security j at time t ,
$\alpha_{\mathrm{j}} \quad=$ market model intercept,
$\beta_{\mathrm{j}} \quad=$ market model slope coefficient,
$R_{m t}=$ return of the market at time $t$,
$\mathrm{n} \quad=$ number of securities in the sample, and
$\mathrm{e}_{\mathrm{jt}} \quad=$ the error term of the market model regression.
To use this appproach, the parameters $\alpha_{\mathrm{j}}$ and $\beta_{\mathrm{j}}$ are estimated from the market model
(Equation 2) using data over the estimation period. Then these estimated parameters are used in Equation 3 to calculate abnormal returns in the testing period. However, if abnormal returns exhibit either heteroscedasticity or cross-sectional dependence, the calculation of an average abnormal return is an inefficient estimate. Patell (1976) is generally credited with the standardized market model approach, which standardizes the abnormal returns with the standard error of the estimate from the market model regression for each security. Examples of using the standardized market model approach for measuring abnormal returns with an event study methodology are Masulis (1980) and Dann and Mikkelson (1984).

Mean Adjusted Model. This approach assumes that the expected returns of security j remain constant from the estimation period through the test period. Abnormal returns are calculated using the following equation:

$$
\begin{gather*}
M R_{j t}=\sum_{E P} R_{j t}  \tag{5}\\
A R_{j t}=R_{j t}-M R_{j t}  \tag{6}\\
A A R_{t}=\frac{1}{m} \sum_{T P} A R_{j t} \tag{7}
\end{gather*}
$$

where $\mathrm{MR}_{\mathrm{jt}}=$ mean return for security j over the estimation period,
$\mathrm{R}_{\mathrm{jt}} \quad=$ return of security j at time t ,
$\mathrm{AR}_{\mathrm{jt}}=$ abnormal return for security j at time t ,
$\mathrm{AAR}_{\mathrm{t}}=$ average abnormal return at time t,
$\mathrm{n} \quad=$ number of securities in the sample
EP $=$ the estimation period, and
$\mathrm{TP}=$ the test period.
A mean return is first calculated for each security using data from the estimation period, as in Equation 5. Then abnormal returns are calculated by subtracting the estimation period mean return from the testing period returns, as in Equation 6. An example of the use of the comparison period mean adjusted methodology is Kalay and Lowenstein (1985).

Market Adjusted Model. This approach assumes that the expected return for each security is the return on a market index (such as the CRSP equally weighted index). Abnormal returns are calculated as follows:

$$
\begin{align*}
A R_{j t} & =R_{j t}-R_{m t}  \tag{8}\\
A A R_{t} & =\frac{1}{n} \sum A R_{j t} \tag{9}
\end{align*}
$$

where $\mathrm{AR}_{\mathrm{jt}}=$ abnormal return of security j at time t ,
$\mathrm{R}_{\mathrm{jt}} \quad=$ return of security j at time t,
$\mathrm{R}_{\mathrm{mt}}=$ return of the market at time t,
$\mathrm{AAR}_{\mathrm{t}}=$ average abnormal return at time t , and
$\mathrm{n} \quad=$ number of securites in the sample.
No estimation period is required in this approach since the current market return is used as the expected return. An example of an event study using the market adjusted
approach is Dennis and McConnell (1986).

Size Decile Adjusted Model. The firm size effect indicates that larger firms tend to have lower returns compared to the market and smaller firms tend to have higher returns. This effect has been widely documented (see Banz (1981), Reinganum (1981), and Keim (1983)). The size decile adjusted model first sorts all the securities in the market into deciles based on market capitalization. Then this approach assumes that the expected return for each security is the return of the size decile portfolio which contains that security. Abnormal returns are calculated as follows:

$$
\begin{align*}
A R_{j t} & =R_{j t}-R_{i(j) t}  \tag{10}\\
R_{i(0) t} & =\frac{1}{m} \sum_{k=1}^{m} R_{k t}  \tag{11}\\
A A R_{t} & =\frac{1}{n} \sum_{j=1}^{n} A R_{j t} \tag{12}
\end{align*}
$$

where $\mathrm{AR}_{\mathrm{jt}}=$ abnormal return of security j at time t ,
$\mathrm{R}_{\mathrm{jt}} \quad=$ return of security j at time t ,
$\mathrm{R}_{\mathrm{i}(\mathrm{j}) \mathrm{t}} \quad=$ return of the size decile that contains security j at time t ,
$\mathrm{R}_{\mathrm{kt}} \quad=$ return of security k in the size decile that contains security j at time t ,
$\mathrm{AAR}_{\mathrm{t}}=$ average abnormal return at time t,
$\mathrm{m} \quad=$ number of k securities in the size decile that contains security j , and
$\mathrm{n} \quad=$ number of securities in the sample.

No estimation period is required in this approach since the current return of the appropriate size decile is used as the expected return. Examples of event studies using a size decile adjusted approach are Foster, Olsen, and Shevlin (1984), Dimson and Marsh (1986), and Lakonishok and Vermaelen (1990). Of these three studies, Dimson and Marsh (1986) also incorporate holding period returns into the approach (a buy and hold strategy) and thus provide the basis for calculating abnormal returns in this dissertation. The use of holding period returns is recommended by Conrad and Kaul (1993). They show that measurement errors in observed prices create a cumulative positive bias when returns are added over multiple time periods. This bias is minimized when one holding period is used for the test period under consideration.

Kothari and Wasley (1989) use Monte Carlo simulation to show the level of Type I errors and the statistical power of various event study methodologies, including the market model, market adjusted, and size decile adjusted approaches. They conclude that if the sample is composed of only large (small) firms, the market model and market adjusted approaches have very large Type I errors (2 to 7 times the nominal level of significance) and are misspecified. In constrast, the size decile adjusted approach has acceptable Type I error rates with equal or greater power than alternative testing procedures. For this reason, this methodology is the chosen methodology in this dissertation to determine abnormal returns.

## Summary

The research in this dissertation develops and tests a model that demonstrates the effect of the introduction of options on the underlying security. A positive price effect has been found empirically in previous studies to occur within the first 30 days of the introduction of options. And Detemple and Selden (1991) have developed a model which predicts such a positive price effect. In contrast, this paper develops a model which predicts a negative effect on the price of the underlying stock after the introduction of options. This negative effect occurs because of a direct conversion to short sales from options by means of a hedge portfolio (such a mechanism is not included in the Detemple and Selden (1991) model). The empirical tests in this dissertation show a negative price effect over a one to five year time period following the introduction of options, which is consistent with the theoretical model. Models in the short sale literature generally verify an upward bias in the stock price as a result of constraints in short sales. Figlewski (1981) develops a model which shows a higher stock price when short sales are restricted. However, the theoretical model in this dissertation extends this approach to include the impact of the introduction of options. As part of the development of the model, current research also indicates support for a downward sloping demand curve (at least in the short term), which is a necessary condition for the restriction in short sales to affect the stock price. Finally, the current state-of-the-art in the application of event study methodologies is reviewed.

From this background and review of the literature, this research develops a
theoretical model to demonstrate the linkage between the underlying stock price and the introduction of options when there are constraints on short sales.

The theoretical model includes two points in time. At date 1 , the investors decide on the proportion of wealth to be invested in each investment choice and reach equilibrium by maximizing their expected date 2 utility. After date 1 equilibrium is reached, all choices are locked in until date 2 occurs. At that point in time, the possible state of nature that occurs becomes known and all payoffs are made in cash. All investors have an identical power utility function of the form $U=-W^{-1}$, which exhibits risk-averse behavior ( $\mathrm{U}^{\prime}>0$ and $\mathrm{U}^{\prime \prime}<0$ ) along with decreasing absolute risk aversion and constant relative risk aversion equal to two. ${ }^{7}$

Each investor has an initial endowment, $W_{I}$, at date 1 such that the endowment would be worth $\mathrm{W}_{\mathrm{E}}$ at date 2 if it were invested in a risk-free asset at the risk-free rate of interest, R (defined as 1 plus the interest rate). This endowment can be invested in any combination of a risk-free asset and a risky asset. The risky asset is a stock which at date 2 will either be worth $S_{u}$ if the stock price goes up, or $S_{d}$ if the stock price goes down. Furthermore, these two states of nature are the only two possibilities. State $S_{u}$ is expected to occur with probability $q_{i}$, while state $S_{d}$ is expected to occur with probability $\left(1-q_{i}\right)$. Heterogeneous beliefs for the investors are made possible in that each investor can have a different assessment, $q_{i}$, of the probability of an increase in the stock price, and thus $q_{i}$ is distributed throughout the

[^5]investor population. The level of $q_{i}$ for each investor is also assumed to be independent of the date 1 stock price, $S_{1}$. An appropriate stock price is offered at date 1 such that the investors agree to purchase all the available shares at an equilibrium stock price of $S_{1}$ per share. Their decisions are based on maximizing date 2 expected utility.

The thrust of the theoretical model is to separate the effects of short sale restrictions and the availability of options into three scenarios. Common to all three scenarios is the aggregate demand curve, which is the sum of the positive optimal number of shares demanded by all investors at various date 1 stock prices. The primary differences in the three scenarios relate to the position of the supply curve. All three scenarios include the record number of shares outstanding as a minimum quantity of shares supplied (in the short run, this is represented by a vertical line at the record number of shares outstanding). The three scenarios differ on the number of additional shares supplied from short sales at various date 1 stock prices. Scenario A is an unrestricted equilibrium case. The aggregate supply consists of a fixed number of record shares outstanding plus any additional shares from short selling of the stock. Scenario B imposes short sale restrictions, which shift the supply curve back to a vertical line at the record number of shares outstanding. Scenario C introduces options when short sale restrictions are imposed on all except market makers. This scenario has the effect of shifting the supply curve in Scenario B to the right, as in Scenario A (see Figure 3). As a result, the supply and demand curves in

FIGURE 3
Aggregate Supply and Demand
Curves for Scenarios A, B, and C


Scenario C and Scenario A look similar since the availability of options circumvents the restrictions on short sales. A comparison of Scenario B to Scenario C then shows the effect of the introduction of options on the price of the underlying stock when there are short sale restrictions. The next sections provide a more thorough development for each of the three scenarios and a comparison to show the effect of the introduction of options.

## Scenario A

In the first scenario, there are no short sale restrictions or options. The initial date 1 endowment could be invested in a risk-free asset, resulting in a sure amount of $\mathrm{W}_{\mathrm{E}}$ at date 2. Each investor could also purchase the offered number of shares of stock at a price of $S_{1}$ that yields an expected utility equivalent to $W_{E}$. However, there is a higher expected utility possible. The optimum number of shares can be determined by finding the maximum, date 2 , expected utility for the investors. Appendix A derives the optimal number of shares, $\mathrm{n}_{\mathrm{i}}{ }^{*}$, demanded by an individual investor to be described by the following equation:

$$
\begin{equation*}
n_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{A B}{C}}}{A} \tag{13}
\end{equation*}
$$

where $\quad n_{i}^{*}=$ the optimal number of shares of stock for investor $i$,

$$
\mathrm{W}_{\mathrm{E}} \quad=\quad \text { value of the initial endowment at date } 2,
$$

$A=-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}$,
$B \quad=\quad-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}$,
$\mathrm{C}=\left(-\mathrm{RS}_{1}+\mathrm{S}_{\mathrm{u}}\right)\left(-\mathrm{RS}_{1}+\mathrm{S}_{\mathrm{d}}\right)$,
$\mathrm{q}_{\mathrm{i}} \quad=\quad$ probability for investor i that the stock price will increase at date 2 ,
$\mathrm{R}=\quad$ one plus the interest rate,
$S_{u} \quad=\quad$ stock price if the price increases at date 2 ,
$\mathrm{S}_{\mathrm{d}} \quad=\quad$ stock price if the price decreases at date 2, and
$S_{1} \quad=\quad$ stock price at date 1.
If $n_{i} *$ is greater than zero, this optimum represents the number of shares demanded by investor i at a given stock price, $\mathrm{S}_{1}$. However, if $\mathrm{n}_{\mathrm{i}}{ }^{*}$ is less than zero, this optimum represents the number of shares that investor i desires to sell short. Selling short increases the number of shares supplied above the record number of shares outstanding.

Using Equation 13, an aggregate downward sloping demand curve can be determined by summing all $n_{i} *>0$ for each investor at various levels of $S_{1}$. Note that each investor has his own belief as to the value of $q_{i}$ and, therefore, has a unique $n_{i}{ }^{*}$. If the positive optimal number of shares for each investor is summed over all investors for any choice of $S_{1}$, an aggregate quantity of shares of stock demanded, $N_{D}$, can be calculated by the following equation:

$$
\begin{equation*}
N_{D}=\sum_{n_{i}^{*}>0} n_{i}^{*} \tag{14}
\end{equation*}
$$

A schedule of various levels of $N_{D}$ for corresponding choices of $S_{1}$ determines a normal downward sloping aggregate demand curve for the stock at date 1 .

The supply curve in Scenario A consists of the fixed record number of shares outstanding plus any additional shares sold short. The quantity of shares desired to be sold short is determined by the sum of all negative $n_{i}^{*}$. Thus, the aggregate quantity supplied at a given level of $S_{1}$ is

$$
\begin{equation*}
N_{A}=N_{R}+\sum_{n_{i}^{*}<0}-n_{i}^{*} \tag{15}
\end{equation*}
$$

where $\quad \mathrm{N}_{\mathrm{A}}=$ aggregate quantity of shares supplied in Scenario A, and
$\mathrm{N}_{\mathrm{R}}=\quad$ record number of shares outstanding.

The aggregate supply and demand curves for Scenario A are illustrated in Figure 4.

## Scenario B

In Scenario B, investors are restricted from selling short the underlying stock.
Such a restriction does not allow some investors to reach their maximum expected utility at date 2 . This occurs for any investor that has a value of $q_{i}$ such that $n_{i}{ }^{*}<0$. The particular values of $q_{i}$ that cause $n_{i} *$ to be negative are determined from Equation 2 above. Appendix A shows that the necessary and sufficient condition under which $\mathrm{n}_{\mathrm{i}}^{*}<0$ is shown in the following equation:

FIGURE 4
Aggregate Supply and Demand


$$
\begin{equation*}
q_{i}<\frac{R S_{1}-S_{d}}{S_{u}-S_{d}} \tag{16}
\end{equation*}
$$

where $\quad q_{i}=$ probability for investor $i$ that the stock price will increase at date 2 , $\mathrm{R}=$ one plus the interest rate,
$\mathrm{S}_{\mathrm{u}} \quad=\quad$ stock price if the price increases at date 2,
$\mathrm{S}_{\mathrm{d}} \quad=\quad$ stock price if the price decreases at date 2 , and
$\mathrm{S}_{1} \quad=\quad$ stock price at date 1.
The intuition behind Equation 16 is that an investor would desire to sell the stock short if $\mathrm{RS}_{1}$ is greater than the investor's expected value of the date 2 stock price.

This intuition is more apparent if we rearrange Equation 16 as follows:

$$
\begin{equation*}
q_{i}\left(S_{u}\right)+\left(1-q_{i}\right) S_{d}<R S_{1} \tag{17}
\end{equation*}
$$

Therefore, for every case where $q_{i}$ satisfies the condition in Equation 16, the investor would prefer to sell the stock short. However, since all investors in Scenario B are restricted from short sales, no additional shares will be supplied from short sales, and the supply curve remains a vertical line at the record number of shares outstanding. ${ }^{8}$ This vertical line is to the left of the supply curve in Scenario A.

The aggregate quantity demanded under Scenario B is calculated as before by

[^6]summing only the positive values of $\mathrm{n}_{\mathrm{i}}{ }^{*}$ as follows:
\[

$$
\begin{equation*}
N_{D}=\sum_{n_{i}^{*}>0} n_{i}^{*} \tag{18}
\end{equation*}
$$

\]

This is the same quantity demanded as in Scenario A for a given $S_{1}$. As in Scenario A, a schedule of various levels of $N_{D}$ for corresponding choices of $S_{1}$ determines a normal downward sloping aggregate demand curve for the stock at date 1. Thus, the demand curve in Scenario B is the same as the demand curve in Scenario A.

Note that the market in Scenario B does not fully reflect all the negative information about the stock (as compared to the market in Scenario A) because some investors cannot reach their maximum possible expected utility at date 2 (i.e., some investors are restricted from short sales). As shown later, this causes the date 1 stock price in Scenario B to be higher than the stock price in Scenario A as a result of restrictions on short sales.

## Scenario C: Options Available

This scenario restricts all short sales for ordinary investors as in Scenario B but also makes available the trading of options. However, the market makers who create the options are not subject to the same restriction in short sales (see Chapter I for a description of short sale restrictions for ordinary investors and exemptions for market makers).

Those investors who have a $q_{i}$ which yields a positive $n_{i}^{*}$ will continue to maximize their date 2 expected utility by purchasing the optimal quantities of stock. As in Scenarios A and B, the sum of the positive values of $n_{i}^{*}$ yields the aggregate quantity demanded for each level of $S_{1}$ under Scenario C.

Those investors who have a $q_{i}$ which yields a negative $n_{i}$ * will again be restricted from selling stock short, but they now have another alternative as a result of the availability of options. To see this, it is necessary to develop an arbitrage strategy similar to the binomial option pricing model developed by Cox, Ross, and Rubenstein (1979). They created a portfolio that exactly replicates the payoff of a call option as

$$
\begin{equation*}
C=\Delta S-B \tag{19}
\end{equation*}
$$

where $\mathrm{C}=$ call option price,
$\mathrm{S}=$ stock price,
$B=$ investment in risk-free bonds, and
$\Delta=$ number of shares.

For conceptual simplicity, this formula has been re-stated such that $C, \Delta, S$, and $B$ are all positive and the explicit sign indicates a long or short position. Multiplying all terms by minus one yields

$$
\begin{equation*}
-\Delta S+B=-C \tag{20}
\end{equation*}
$$

Thus, an investor who writes a call option can exactly replicate a short position in $\Delta$
shares of stock and a quantity of B invested in the risk-free asset. In this way, the investor can maximize his date 2 expected utility in spite of the restriction in short sales for ordinary investors. However, the market makers creating the options market must take an opposite position of purchasing a call option. To hedge this position, the market makers must create an arbitrage portfolio, which by definition requires zero investment. Rearranging Equation 20 shows such an arbitrage portfolio, as follows:

$$
\begin{equation*}
C-\Delta S+B=0 \tag{21}
\end{equation*}
$$

Note that the market makers in the market perform a financial intermediation role by selling short $\Delta$ shares of stock in place of the ordinary investor, who is restricted from doing so.

In addition to writing a call option, an investor could purchase a put option, which produces a similar result of circumventing short sale restrictions. A portfolio that replicates the payoff of a put option can be represented as

$$
\begin{equation*}
P=-\Delta_{p} S+B_{p} \tag{22}
\end{equation*}
$$

where $\quad \mathbf{P}=$ put price,

$$
S=\text { stock price, }
$$

$$
\mathrm{B}_{\mathrm{p}}=\text { investment in risk-free bonds, and }
$$

$$
\Delta_{\mathrm{p}}=\text { number of shares of stock. }
$$

Thus, an investor who buys a put option can replicate a short position in $\Delta_{p}$ shares of stock and a quantity $\mathrm{B}_{\mathrm{p}}$ invested in the risk-free asset. As with writing call options, an investor can maximize his date 2 expected utility in spite of a restriction in short
sales for ordinary investors. Again, the market makers creating the options market must take an opposite position of writing a put option. To hedge this position, the market makers must create an arbitrage portfolio. Rearranging Equation 22 shows such an arbitrage portfolio, as follows:

$$
\begin{equation*}
-P-\Delta_{p} S+B_{p}=0 \tag{23}
\end{equation*}
$$

In this case, the market makers write a put, sell short $\Delta_{p}$ shares of stock, and purchase quantity $B_{p}$ of the risk-free asset. Thus, writing a call or purchasing a put both result in the short sale of stock by market makers.

At this point, it should be noted that there are other market processes for supplying options which also result in financial intermediation of short selling. Suppose, for example, that there are both call and put options available. To circumvent short sale restrictions, investors would desire to write call options or purchase put options. Suppose also that the market makers choose to maintain only a limited short position in the underlying stock. As the option market trading expands, the demand for selling calls and buying puts will then decrease the relative price of calls and increase the relative price of puts. If the spread between the relative put price and call price exceeds that defined in the put call parity theorem, an arbitrage portfolio can be formed as follows:

$$
\begin{equation*}
C-P-S+e^{-r t} K=0 \tag{24}
\end{equation*}
$$

where $\mathrm{C}=$ call price, $\mathrm{P}=$ put price, $\mathrm{S}=$ stock price, $\mathrm{r}=$ interest rate, $\mathrm{t}=$ time remaining to maturity, and $\mathrm{K}=$ strike price of the options.

Thus, a third party could buy a call, write a put, sell short stock, and invest in a riskfree asset to create a hedge portfolio. Such a process of supplying options is known as "reverse conversion arbitrage" or simply a "reversal" (see Fullman, p. 187 or Baird, p. 87). Webb (1988) further provides evidence that such a process likely occurs. For example, on the New York Stock Exchange (NYSE) the largest growth in short sales from 1973 to 1985 was in the sector identified as "other NYSE members" (other sectors include "specialists" and "non-members"). The "other NYSE members" sector includes brokerage firms who have seats on both the equity and options exchanges, which allows these firms to have minimum transaction fees for the options and the stock. At the same time, many firms in this sector will have access to margined accounts from which they can borrow shares of stock to be sold short. Thus, the process of reverse conversion arbitrage can be established very profitably among these market participants and creates another market process which circumvents short sale restrictions.

Given a market process for supplying options, Appendix B derives the optimal number of call options, $\mathrm{o}_{\mathrm{i}}{ }^{*}$, written by the investor as

$$
\begin{equation*}
O_{i}^{*}=\frac{-W_{E}+W_{E}^{\prime} \sqrt{1-\frac{E F}{G}}}{F} \tag{25}
\end{equation*}
$$

where $\mathrm{E}=\mathrm{RC}_{1}+\mathrm{q}_{\mathrm{i}}\left(\mathrm{K}-\mathrm{S}_{\mathrm{u}}\right)$,
$\mathrm{F}=\mathrm{RC}_{1}+\left(1-\mathrm{q}_{\mathrm{i}}\right)\left(\mathrm{K}-\mathrm{S}_{\mathrm{u}}\right)$,
$\mathrm{G}=\mathrm{RC}_{1}\left(\mathrm{RC}_{1}-\mathrm{S}_{\mathrm{u}}+\mathrm{K}\right)$,
$C_{1}=$ price of a call option in period 1,
$\mathrm{K}=$ strike price of the call option in period 1 ,
$S_{u}=$ stock price if price increases at date 2,
$\mathrm{R}=$ one plus the interest rate, and
$q_{i}=$ probability for investor $i$ that the stock price will increase at date 2.

Alternatively, Appendix C derives the optimal number of put options, $\mathrm{p}_{\mathrm{i}}$, purchased by the investor as shown in the following equation:

$$
\begin{equation*}
p_{i}=\frac{W_{E}+\sqrt{1-\frac{X Y}{Z}}}{Y} \tag{26}
\end{equation*}
$$

$$
\text { where } \quad \begin{aligned}
\mathrm{X} & =\mathrm{RP} P_{1}+\left(1-\mathrm{q}_{\mathrm{i}}\right)\left(\mathrm{S}_{\mathrm{d}}-\mathrm{K}_{\mathrm{p}}\right) \\
\mathrm{Y} & =\mathrm{RP} P_{1}-\mathrm{q}_{\mathrm{i}}\left(\mathrm{~S}_{\mathrm{d}}-\mathrm{K}_{\mathrm{p}}\right) \\
\mathrm{Z} & =\mathrm{RP} P_{1}\left(\mathrm{RP}_{1}-\mathrm{K}_{\mathrm{p}}+\mathrm{S}_{\mathrm{d}}\right),
\end{aligned}
$$

$P_{1}=$ price of a put option at date 1,
$\mathrm{K}_{\mathrm{p}}=$ strike price of the put option at date 1 ,
$S_{d}=$ stock price if price decreases at date 2,
$\mathrm{R}=$ one plus the interest rate, and
$\mathrm{q}_{\mathrm{i}}=$ probability for investor i that the stock price will increase at date 2 .

Thus, an investor who desires to sell shares of stock short because of a less optimistic view of $q_{i}$ can do so by writing call options or purchasing put options. At the optimal point, an investor can in fact reach the same level of date 2 expected utility as would be possible if there were no restrictions on short sales. This statement must be true since the date 2 payout from the short sale of stock and a positive quantity of the risk-free asset can be exactly replicated by writing a call, as shown in Equation 9, or by buying a put, as shown in Equation 11. If the payout can be exactly replicated for all states of nature, then the investor can reach the same level of date 2 expected utility achievable through short sales by a strategy of writing calls or buying puts.

Since investors can circumvent restrictions on short sales by trading options, the quantity of shares supplied consists of the record number of shares outstanding plus any short sales from market makers. The supply curve for Scenario C is then equivalent to the supply curve in Scenario A. The only difference is that in

Scenario A investors are allowed to sell stock short whereas in Scenario C market makers, in place of the individual investors, intermediate by selling short the same quantity of the stock in place of the individual investors. This intermediation follows directly from the market process of supplying options.

## Comparison of Scenarios B and C

As mentioned previously, a comparison of Scenario B to Scenario C shows the effect of the introduction of options on the price of the underlying stock when there are short sale restrictions. The aggregate demand curve is the same in both scenarios, but the aggregate supply curve is different. In Scenario B, the aggregate supply curve is a vertical line at the record number of shares outstanding. For all values of $S_{1}$, the quantity supplied can be represented by

$$
\begin{equation*}
N_{B}=N_{R} \tag{27}
\end{equation*}
$$

where $\quad N_{B}=\quad$ quantity of shares supplied in Scenario B, and

$$
\mathrm{N}_{\mathrm{R}}=\text { record number of shares outstanding. }
$$

In Scenario $C$, the quantity supplied for each value of $S_{1}$ equals the record number of shares outstanding plus the quantity of shares sold short, as follows:

$$
\begin{equation*}
N_{C}=N_{R}+\sum_{n_{i}^{*}<0}-n_{i}^{*} \tag{28}
\end{equation*}
$$

where $\quad N_{C}=\quad$ quantity of shares supplied in Scenario C,
$N_{R}=$ record number of shares outstanding, and
$\mathrm{n}_{\mathrm{i}}^{*}=\quad$ optimal number of shares for investor i .
Thus, for every case in which there is at least one investor that has a value of $q_{i}$ such that $\mathrm{n}_{\mathrm{i}}{ }^{*}<0$, the quantity supplied in Scenario C will be larger than the quantity supplied in Scenario B (the record number of shares outstanding). A schedule of various levels of $N_{C}$ for corresponding choices of $S_{1}$ then determines a normal upward sloping aggegate supply curve for Scenario C. Thus, the aggregate supply curve for Scenario C will be to the right of the aggregate supply curve for Scenario B.

The intersection of the supply and demand curves determine the equilibrium stock price for Scenarios B and C. These are shown in Figure 5. Note that the equilibrium stock price in Scenario C (when options are available) is less than the equilibrium stock price in Scenario B. Thus, the theoretical model predicts that the introduction of options would result in downward pressure on the stock price when there are restrictions on short sales.

## A Numerical Example

A numerical example will demonstrate some of the dynamics of the model in this paper. Suppose a certain stock will have either a value of 30 or 40 at date 2 . Further assume that there are three investors who believe that the probability of 40 at

FIGURE 5
Aggregate Supply and Demand
Stock Curves for Scenarios B and C

Price

date 2 is $.6, .5$, and .4 respectively. Each investor has an initial wealth such that he would have $\$ 25,000$ at date 2 if he chooses to invest in the risk-free asset at an assumed interest rate of $10 \%$. The optimal number of shares to maximize date 2 expected utility can be calculated for each investor using Equation 13, as follows:

Investor X $\quad(q=.6)$

$$
\begin{equation*}
n_{x}^{*}=\frac{-25000+25000 \sqrt{1-\frac{A B}{C}}}{A} \tag{29}
\end{equation*}
$$

where

$$
\begin{aligned}
& A=-1.1 S_{1}+.6(30-40)=34-1.1 S_{1} \\
& B=-1.1 S_{1}+.6(40-30)=36-1.1 \mathrm{~S}_{1}, \text { and } \\
& C=\left(-1.1 \mathrm{~S}_{1}+40\right)\left(-1.1 \mathrm{~S}_{1}+30\right)=1.21 \mathrm{~S}^{2}-77 \mathrm{~S}_{1}+1200
\end{aligned}
$$

Substituting and simplifying yields the following result:

$$
\begin{equation*}
n_{x}^{*}=\frac{-25000+25000 \sqrt{1-\frac{1.21 S_{1}^{2}-77 S_{1}+1224}{1.21 S_{1}^{2}-77 S_{1}+1200}}}{34-1.1 S_{1}} \tag{30}
\end{equation*}
$$

Investor $\mathrm{Y} \quad(\mathrm{q}=.5)$

$$
\begin{equation*}
n_{y}^{*}=\frac{-25000+25000 \sqrt{1-\frac{A B}{C}}}{A} \tag{31}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{A}=-1.1 \mathrm{~S}_{1}+.5(30-40)=35-1.1 \mathrm{~S}_{1}, \\
& \mathrm{~B}=-1.1 \mathrm{~S}_{1}+.5(40-30)=35-1.1 \mathrm{~S}_{1}, \text { and } \\
& \mathrm{C}=\left(-1.1 \mathrm{~S}_{1}+40\right)\left(-1.1 \mathrm{~S}_{1}+30\right)=1.21 \mathrm{~S}^{2}-77 \mathrm{~S}_{1}+1200
\end{aligned}
$$

Substituting and simplifying yields the following result:

$$
\begin{equation*}
n_{y}^{*}=\frac{-25000+25000 \sqrt{1-\frac{1.21 S_{1}^{2}-77 S_{1}+1225}{1.21 S_{1}^{2}-77 S_{1}+1200}}}{35-1.1 S_{1}} \tag{32}
\end{equation*}
$$

Investor $Z \quad(q=.4)$

$$
\begin{equation*}
n_{z}^{*}=\frac{-25000+25000 \sqrt{1-\frac{A B}{C}}}{A} \tag{33}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{A}=-1.1 \mathrm{~S}_{1}+.4(30-40)=36-1.1 \mathrm{~S}_{1} \\
& \mathrm{~B}=-1.1 \mathrm{~S}_{1}+.4(40-30)=34-1.1 \mathrm{~S}_{1}, \text { and } \\
& \mathrm{C}=\left(-1.1 \mathrm{~S}_{1}+40\right)\left(-1.1 \mathrm{~S}_{1}+30\right)=1.21 \mathrm{~S}^{2}-77 \mathrm{~S}_{1}+1200
\end{aligned}
$$

Substituting and simplifying yields the following result:

$$
\begin{equation*}
n_{z}^{*}=\frac{-25000+25000 \sqrt{1-\frac{1.21 S_{1}^{2}-77 S_{1}+1224}{1.21 S_{1}^{2}-77 S_{1}+1200}}}{36-1.1 S_{1}} \tag{34}
\end{equation*}
$$

For various assumed values of $S_{1}$, the aggregate number of shares demanded is the sum of the positive $\left(n_{i}^{*}>0\right)$ optimal number of shares for all three investors, which is reported in Table 2. Figure 6 shows the aggregate demand curve for Scenarios A, B, and C. The supply curves are determined by summing the record number of shares outstanding and the number of any shares sold short $\left(\mathrm{n}_{\mathrm{i}}^{*}<0\right)$. In this example, an assumption is made that there are 350 record outstanding shares.

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TABLE 2
EQUILIBRIUM DEMAND FOR STOCK
(Numerical Example)

Individual Optimal Shares

| $\mathrm{S}_{1}$ | Investor $X$ $(q=.6)$ |  | Investor Z $(\mathrm{q}=.4)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 32.70 | 16 | -499 | -1024 |
| 32.60 | 72 | -440 | -961 |
| 32.50 | 128 | -381 | -899 |
| 32.40 | 184 | -324 | -839 |
| 32.30 | 239 | -267 | -779 |
| 32.20 | 295 | -211 | -721 |
| 32.10 | 350 | -155 | -664 |
| 32.00 | 405 | -100 | -607 |
| 31.90 | 460 | -45 | -551 |
| 31.80 | 515 | 10 | -495 |
| 31.70 | 571 | 65 | -440 |
| 31.60 | 627 | 120 | -385 |
| 31.50 | 684 | 176 | -330 |
| 31.40 | 742 | 231 | -275 |
| 31.30 | 801 | 288 | -219 |
| 31.20 | 861 | 345 | -164 |
| 31.10 | 921 | 403 | -108 |
| 31.00 | 984 | 461 | -62 |
| 30.90 | 1048 | 521 | 5 |
| 30.80 | 1113 | 582 | 63 |
| 30.70 | 1180 | 644 | 122 |
| 30.60 | 1250 | 708 | 182 |
| 30.50 | 1322 | 774 | 243 |
| 30.40 | 1396 | 842 | 305 |
| 30.30 | 1474 | 912 | 370 |
| 30.20 | 1554 | 985 | 436 |
| 30.10 | 1638 | 1060 | 504 |
| 30.00 | 1726 | 1139 | 575 |


| Aggregate Demand (shares) | Aggregate Supply (shares) |  |  |
| :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Scenario } \\ \text { A } \end{gathered}$ | $\begin{gathered} \text { Scenario } \\ \text { B } \end{gathered}$ | Scenario $\mathrm{C}$ |
| 16 | 1873 | 350 | 1873 |
| 72 | 1751 | 350 | 1751 |
| 128 | 1630 | 350 | 1630 |
| 184 | 1513 | 350 | 1513 |
| 239 | 1396 | 350 | 1396 |
| 295 | 1282 | 350 | 1282 |
| 350 | 1161 | 350 | 1161 |
| 405 | 1057 | 350 | 1057 |
| 460 | 946 | 350 | 946 |
| 525 | 845 | 350 | 845 |
| 636 | 790 | 350 | 790 |
| 748 | 735 | 350 | 735 |
| 860 | 680 | 350 | 680 |
| 974 | 625 | 350 | 625 |
| 1089 | 569 | 350 | 569 |
| 1205 | 514 | 350 | 514 |
| 1324 | 458 | 350 | 458 |
| 1445 | 412 | 350 | 412 |
| 1574 | 350 | 350 | 350 |
| 1758 | 350 | 350 | 350 |
| 1947 | 350 | 350 | 350 |
| 2140 | 350 | 350 | 350 |
| 2339 | 350 | 350 | 350 |
| 2544 | 350 | 350 | 350 |
| 2755 | 350 | 350 | 350 |
| 2975 | 350 | 350 | 350 |
| 3203 | 350 | 350 | 350 |
| 3440 | 350 | 350 | 350 |

FIGURE 6
Aggregate Supply and Demand
Curves for Scenarios A,B, and C
(Numerical Example)


Note that with short sale restrictions as in Scenario B, the aggregate supply curve for the stock is a vertical line (at 350 shares) which is to the left of the aggregate supply curve for Scenario A. This occurs because Investor Y would like to sell short shares of stock when date 1 stock prices are above $\$ 31.80$ and Investor Z would like to sell short shares of stock when date 1 stock prices are above $\$ 30.90$, but both are restricted from doing so in Scenario B.

Under Scenario A, interpolating the data in Table 2 shows that an equilibrim price of $\$ 31.61$ is required for the quantity demanded to equal the quantity supplied at 739 shares. With the short sale restrictions on the investors in Scenario B, an equilibrium price of $\$ 32.10$ is required for the quantity demanded to equal thequantity supplied at 350 shares. Thus, the model predicts that the stock price will have an upward bias if there are short sale restrictions, which is consistent with Miller (1977) and Figlewski (1981).

Finally, with options available, as in Scenario C, along with short sale restrictions investors Y and Z can write call options to achieve maximum expected utility at date 2 , which drives the date 1 stock price down from $\$ 32.10$ to $\$ 31.61$ (as in Scenario A with no restrictions). Table 3 shows the maximum expected utility for each investor in the three Scenarios.

TABLE 3
Equilibrium Number of Shares of Stock or Options and Expected Utility


## Comparative Statics

This section discusses the economic behaviors implied by the comparative static derivatives of the model. Appendix D shows the calculation of the comparative static derivatives, which are presented below. The sign of the following derivatives are unambiguous:

$$
\left.\begin{array}{ll}
\frac{\partial n_{i}^{*}}{\partial W_{E}} & <0 \text { when } n_{i}^{*}<0
\end{array} \right\rvert\, \frac{\partial o_{i}^{*}}{\partial W_{E}}<0 \text { when } o_{i}^{*}<0
$$

The signs of the following derivatives were determined by numerical methods, and apply to the ranges of data within the numerical example:

$$
\begin{array}{ll}
\frac{\partial n_{i}^{*}}{\partial S_{1}}<0 & \frac{\partial O_{i}{ }^{*}}{\partial C_{1}}<0 \\
\frac{\partial n_{i}^{*}}{\partial R}<0 & \frac{\partial O_{i}{ }^{*}}{\partial R}<0 \\
\frac{\partial O_{i}^{*}}{\partial K}<0 & \frac{\partial O_{i}{ }^{*}}{\partial S_{1}}<0
\end{array}
$$

First, the partial derivative of $n_{i}^{*}$ (optimal number of shares) with respect to $S_{1}$ (stock price at date1) is negative. This is logical since the original wealth is invested in an optimal combination of stock and the risk-free asset. Given the same date 2
payoffs, as $S_{1}$ increases, the return on the stock becomes less attractive compared to the return of the risk-free asset. Thus, an investor would desire less stock and more risk-free asset, consistent with the sign of the comparative static derivative.

Likewise, the partial derivative of $n_{i}{ }^{*}$ with respect to $R$ (the return of the risk-free asset) is negative. In this case, as R increases, the risk-free asset becomes more attractive compared to the stock. Thus, an investor would again desire less stock and more risk-free asset, consistent with the sign of the comparative static derivative. This line of reasoning also applies to the partial derivative of $o_{i}^{*}$ (the optimal number of call options) with respect to R .

The partial derivative of $n_{i}{ }^{*}$ with respect to $q_{i}$ (the probability that the stock price will increase at date 2 ) is positive. As an investor increases $q_{i}$, his belief that the stock price will increase at date 2 , the expected return increases. Thus, an investor would desire more stock as he becomes more optimistic of the date 2 stock price, which is consistent with the sign of the comparative static derivative. For the same reason, an investor would also desire more call options as $q_{i}$ increases.

The partial derivative of $n_{i}{ }^{*}$ with respect to $W_{E}$ ( $R$ times the initial endowed wealth) is positive when $n_{i}{ }^{*}$ is positive and negative when $n_{i}{ }^{*}$ is negative (again, the partial derivative of $\mathrm{o}_{\mathrm{i}}{ }^{*}$ with respect to $\mathrm{W}_{\mathrm{E}}$ parallels this discussion). In this case, the wealth level acts as a scaling factor. For example, an increase in wealth would
encourage an investor with a long position in stock to increase his holdings and would encourage an investor with a short position to sell more stock short. However, the level at which an investor chooses zero stock is independent of the wealth level (see Equation 16).

The partial derivative of $\mathrm{o}_{\mathrm{i}}{ }^{*}$ with respect to $\mathrm{C}_{1}$ (the price of a call option at date 1 ) is negative. This means that an investor would desire to write more calls as $\mathrm{C}_{1}$, increases. Conversely, an investor would desire to buy more calls $\mathrm{C}_{1}$ decreases, again consistent with the sign of the comparative static derivative.

Likewise, the partial derivative of $\mathrm{o}_{\mathrm{i}}{ }^{*}$ with respect to K (the strike price) is negative. In this case, as the strike price increases, buying call options becomes less attractive because the date 2 payoff decreases. This is consistent with the sign of the comparative static derivative.

Finally, the partial derivative of $\mathrm{o}_{\mathbf{i}}{ }^{*}$ with respect to $\mathrm{S}_{1}$ is negative. As the stock price increases at date 1 , the price of the call option at date 1 also increases. Thus, an investor would desire a smaller quantity of call options, which is consistent with the sign of the comparative static derivative.

In summary, the intuition for the sign of each of the comparative static derivatives is consistent with the derived equations in Appendix D.

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## Summary

This chapter presents a theoretical model that predicts an upward bias in the stock price when there are restrictions on short sales. When options are available, this bias is removed and there is negative pressure on the price of the underlying stock. This is shown by a comparison of two scenarios -- one with restrictions on short sales and no options market, and one with both restrictions on short sales and with the availability of options. A numerical example then illustrates some of the dynamics of the model. Finally, comparative static derivatives derived in Appendix $D$ are shown to be intuitive.

This chapter contains the design of empirical tests to determine if the theoretical model developed in Chapter III is consistent with empirical data. First, testable hypotheses are identified. Then a methodology to test each of these hypotheses is developed. Finally, specific characteristics of the data to be used in the tests are presented.

## Hypotheses

There are several implications of the model that lead to testable hypotheses.
First, the model predicts that the price of the underlying security will decrease following the introduction of an option on that security. This occurs when investors with assessments of a lower future stock price are restricted from selling the stock short; thus, the stock price prior to an options market is artificially high. When options are introduced, this inefficiency is removed, and the stock price decreases to the equilibrium price for a fully efficient market. The decrease in price predicted by the model leads to the following null and alternative hypotheses:
$\mathrm{H}_{1, \mathrm{O}}$ : There is no effect from the introduction of options.
$\mathrm{H}_{1, \mathrm{~A}}$ : There are negative abnormal returns on the underlying security following the introduction of options.

If abnormal returns are negative and significantly different from zero at the .05 level, the null hypothesis is rejected.

Two additional tests can provide further evidence consistent with the model. The first is a cross-sectional test based on a correlation between the negative abnormal returns and a change in option open interest. The model predicts that a downward influence on the stock price (negative abnormal returns) is a result of the availability of options. When call options are written (or put options purchased), negative abnormal returns should occur, and these two variables should be correlated. This leads to the second hypothesis:
$\mathrm{H}_{2, \mathrm{O}}$ : There is no relationship between abnormal returns and a change in option open interest.
$\mathrm{H}_{2, \mathrm{~A}}$ : There is a negative relationship between abnormal returns and a change in option open interest.

The null hypothesis will be rejected if the regression coefficient is significantly different from zero at the .05 level.

A final test is to verify that the change in option open interest is accompanied by a change in short interest, as predicted by this study's theoretical model. Brent,

Morse, and Stice (1990) have already demonstrated that, in general, such a relationship exists. The test in this dissertation is designed to show that a similar relationship holds for the particular data in this study's sample. Thus, the third hypothesis is:
$\mathrm{H}_{3,0}$ : There is no relationship between a change in short interest and a change in option open interest.
$\mathrm{H}_{3, \mathrm{~A}}$ : There is a positive relationship between a change in short interest and a change in option open interest.

Again, the null hypothesis will be rejected if the correlation is significantly different from zero at the .05 level.

## Methodology

In this section, a specific methodology is outlined to test each of the three hypotheses. An event study methodology will be designed to test the first hypothesis. The second will use a cross-sectional regression of the abnormal returns found in the event study to show a direct relationship to the use of options. Finally, the last hypothesis will use a regression equation to demonstrate a relationship between options and short sales.

Hypothesis 1. The first hypothesis of negative abnormal returns following
option introduction can be tested using an event study methodology with the first date of introduction of options on a particular security as the event date. Event study methodologies compare the actual return for a particular security to an estimate of what the return would have been using a particular model as a standard. Frequently, abnormal returns are calculated by comparing against the market model. The market model parameters are usually estimated over a time period before the event of interest occurs, using the following equation:

$$
\begin{equation*}
R_{j t}=\alpha_{j}+\beta_{j} R_{m t}+e_{j t} \tag{35}
\end{equation*}
$$

where $\quad R_{j t}=$ actual return for security $j$ at time $t$,
$R_{m t}=$ actual return for the market at time $t$,
$\alpha_{j}=\quad$ estimate of market model intercept,
$\beta_{j} \quad=\quad$ estimate of market model slope coefficient, and
$e_{j t} \quad=\quad$ error term of the regression.

$$
\begin{equation*}
A R_{j t}=R_{j t}-\left(\alpha_{j}+\beta_{j} R_{m t}\right) \tag{36}
\end{equation*}
$$

Abnormal returns are then calculated using the following equation: where $A R_{j t}$ represents the abnormal returns for security $j$ at time $t$ and all other terms are as defined in Equation 35.

The model predicts that a negative price effect occurs as options are traded and
short sale restrictions are circumvented. Since an options market for a particular security typically continues to grow after the first introduction, a monthly event study over a one to five year period following introduction will be used to measure the negative price effect on the underlying security. However, there are measurement problems in the testing for abnormal returns over such a long period of time.

First, the estimated market model parameters ( $\alpha_{j}$ and $\beta_{j}$ ) may not remain stationary over a five year period. For example, a downward bias on calculated abnormal returns could occur if the systematic risk as measured by beta is lower after the introduction of options (recall the empirical evidence in Chapter II which indicates a decrease in beta, but not statistically significant). If the estimate for beta is higher than the actual beta, then Equation 36 shows that the calculated abnormal return would be smaller than what actually occurs. Thus, to the extent that beta does in fact decrease, abnormal returns would be biased downward.

A second possible measurement problem is the failure to account for firm size as a proxy for risk. Fama and French (1992) use monthly data from 1963 to 1990 to show that large companies tend to have lower monthly returns while smaller firms tend to have higher returns. They find this to be true even within portfolios constructed from securities with common betas (using beta deciles). The fact that larger firms are more likely to have options introduced than small firms and that larger firms tend to have lower returns even in the same beta deciles could again lead to a
downward bias in the measurement of abnormal returns.

To avoid these test measurement biases, a test procedure used by Dimson and Marsh (1986) is adopted for the methodology to test the first hypothesis. In their study, they conclude that long-term performance measures using the techniques described in Brown and Warner (1980) should be avoided. Instead, they recommend that abnormal returns should be estimated using a methodology which explicitly controls for size.

The specific test used in this dissertation determines long-term abnormal returns by comparing each security's actual return against the return in a size-based control portfolio ${ }^{8}$ as shown in the following equation:

$$
\begin{equation*}
A R_{j s t}=R_{j s t}-R_{i()) s t} \tag{37}
\end{equation*}
$$

$$
\text { where } \begin{aligned}
& A R_{j s t}=\quad \text { abnormal return for security } j \text { from time } s \text { to time } t, \\
& R_{j s t}=\quad \text { return for security } j \text { from time } s \text { to time } t \text {, and } \\
& R_{i(j) j t}=\quad \text { return for size decile i that includes security j from time } \\
& \\
& \\
& s \text { to time } t .
\end{aligned}
$$

Dimson and Marsh (1986) also define their method of determining returns over

[^7]multiple periods. Specifically, they calculate average holding period abnormal returns (AHPARs) rather than cumulative average abnormal returns (CAARs). Most studies compute multiple period CAARs according to the following equations:
\[

$$
\begin{align*}
& C A A R_{s t}=\sum_{\tau=s}^{t} A A R_{\tau}  \tag{38}\\
& A A R_{\tau}=\frac{1}{n} \sum_{j=1}^{n} A R_{j \tau} \tag{39}
\end{align*}
$$
\]

where $\mathrm{CAAR}_{\mathrm{st}}=$ cumulative average abnormal return from time s to time t , $\mathrm{AAR}_{\tau}=$ average abnormal return at time $\tau$,
$\mathrm{AR}_{\mathrm{j} \mathfrak{r}} \quad=\quad$ abnormal return for security j at time $\tau$, and $\mathrm{n} \quad=\quad$ number of securities in the sample.

Dimson and Marsh (1986) point out that such a methodology will introduce a bias if there are price measurement errors from intraspread price fluctuations, price rounding, untimely quotations (including nonsynchronous trading) and source document errors. This bias occurs partly because of the implicit rebalancing within the cumulative approach. Rebalancing involves reducing the holdings in stocks which have apparently appreciated (from a spurious increase) such that a correction the following period is on a smaller holding of the stock. Another cause for the bias is the nonsymmetrical relationship of percentage increases and decreases (e.g., a $100 \%$ increase is reversed by a $50 \%$ decrease in the next period). More recently, Conrad and Kaul (1993) show that low priced stocks in a sample can particularly introduce significant bias (they also use holding period returns to minimize this effect). Therefore, to
compensate for this bias from cumulating abnormal returns, the following equations identify a methodology to calculate average holding period abnormal returns:

$$
\begin{align*}
A H P A R_{s t} & =\frac{1}{n} \sum_{j=1}^{n} H P A R_{j s t}  \tag{40}\\
H P A R_{j s t} & =H P R_{j s t}-A H P R_{i(1) s t}  \tag{41}\\
H P R_{j s t} & =\prod_{\tau=s}^{t}\left(1+R_{j \tau}\right)  \tag{42}\\
A H P R_{i(\eta) s t} & =\frac{1}{m} \sum_{k=1}^{m} H P R_{k s t}  \tag{43}\\
H P R_{k s t} & =\prod_{\tau=s}^{t}\left(1+R_{k \tau}\right) \tag{44}
\end{align*}
$$

where $\mathrm{AHPAR}_{\mathrm{st}}=$ average holding period abnormal return from time s to time t ,
$\mathrm{HPAR}_{\mathrm{jst}}=$ holding period abnormal return for security j from time s to time t ,
$\mathrm{HPR}_{\mathrm{jst}}=$ holding period return for security j from time s to time t , $\mathrm{AHPR}_{\mathrm{ij}) \mathrm{st}}=$ average holding period return for the size decile that contains security j from time $s$ to time $t$,
$H P R_{\text {kst }}=$ holding period return for security k in the size decile that contains the sample security j for time $\tau$,
$\mathrm{R}_{\mathrm{j} \tau} \quad=\quad$ return for security j for time $\tau$,
$\mathrm{R}_{\mathrm{k} \tau} \quad=$ return for security k in the size decile that contains the sample security j for time $\tau$,
$\mathrm{n} \quad=$ number of securities in the sample, and
$\mathrm{m} \quad=\quad$ number of securities in the size decile that contains security j . The methodology used in this dissertation follows the Dimson and Marsh (1986) and Conrad and Kaul (1993) approach of using holding period returns as outlined in Equations 29 through 33.

A finding that $A H P A R_{s t}$ is negative and significantly different from zero provides evidence in favor of the alternate hypothesis, which is consistent with the theoretical model developed in Chapter III.

Hypothesis 2. The second hypothesis is that there is a negative relationship between abnormal returns and a change in option open interest. This hypothesis can be tested with a cross-sectional regression of the abnormal returns from the event study against a change in option open interest. Two other independent variables are added to the regression to account for changes in institutional holdings of the underlying security and the availability of index options.

An increase in institutional holdings would result in a positive effect on the abnormal returns because of higher demand for the underlying stock. Such an increase in demand would occur if institutional investors prefer securities which have options available. Damodaran and Lim (1991) show that institutional holdings increase an average of nearly $15 \%$ of shares outstanding from three quarters before option introduction to three quarters after introduction. Their sample included 200 stocks with options introduced from 1973 to 1983 on the CBOE and AMEX exchanges. An earlier study by Pruitt and Wei (1989) also provides positive evidence that a change in institutional holdings can significantly affect abnormal returns (see Chapter II, page 28 of this dissertation). Thus the effect of any change in institutional holdings must be controlled in the cross-sectional regression. A positive coefficient for the change in institutional holdings would be expected in the crosssectional regression.

The availability of index options provides a mechanism for using options on a portfolio of stocks instead of options on individual stocks. Index options first became available in 1983. The CBOE first introduced the popular S\&P 100 Index Options on March 13, 1983, followed by the S\&P 500 Index Options on July 3, 1983. AMEX also introduced the MMI Index Options on April 19, 1983, and the NYSE introduced the NYSE Composite Index Options on September 23, 1983. Index option open interest quickly expanded to over one million contracts in 1984. Since index options offer an alternate to equity options, the impact of the introduction of equity options
would be less after 1983 as the index option open interest increases. Thus, the coefficient for the change in option open interest in the regression would be expected to be negative (the same sign as expected for individual equity open interest).

The following equation shows the cross-sectional regression equation:

$$
\begin{equation*}
A R_{j s t}=\beta_{1}+\beta_{2}\left(O P E N_{j s t}\right)+\beta_{3}\left(I N D E X_{s t}\right)+\beta_{4}\left(I N S T_{j s t}\right)+e_{j s t} \tag{45}
\end{equation*}
$$

where $\mathrm{AR}_{\mathrm{jst}}=$ abnormal returns for security j from time s to time t , OPEN $_{\mathrm{jst}}=$ change in relative option open interest from time s to t , $\mathrm{INDEX}_{\mathrm{st}}=$ change in index option open interest from time s to t , $\mathrm{INST}_{\mathrm{jst}}=$ change in institutional holdings as a percent of shares outstanding from time $s$ to time $t$, $\beta_{1}=$ regression intercept,
$\beta_{2}=$ regression coefficient for the option open interest term,
$\beta_{3} \quad=\quad$ regression coefficient for the index options term,
$\beta_{4}=$ regression coefficient for institutional holdings term, and $\mathrm{e}_{\mathrm{jst}}=\quad$ regression error term.

Note that a change in relative option open interest is used as an explanatory variable. Relative option open interest equals the open interest divided by shares outstanding. This ratio creates a common basis for this variable across different
securities (correcting for the relative number of shares outstanding). If the regression slope coefficient, $\beta_{2}$, is negative and significantly different from zero, the null hypothesis would be rejected. This would provide evidence consistent with options being used to circumvent short sales and causing negative abnormal returns on the underlying stock. This regression is applied to various holding periods from one to five years following the introduction of options to demonstrate the link between options and the negative abnormal returns.

Hypothesis 3. The third hypothesis is that there is a positive relationship between a change in short interest and a change in option open interest. This hypothesis can be tested with a regression of a change in short interest against an explanatory variable of a change in option open interest. In this case, the regression also needs to take into account changes in the availability of index options and changes in interest rates. The availability of index options provides an alternative to the use of individual equity options in the context of a portfolio of stocks. As in equity options, a positive relationship between short interest and index option open interest is expected. A change in interest rates also needs to be accounted for in the regression equation because this affects the disincentives against short sales for investors regarding the $150 \%$ margin requirement (see Chapter I, pages 6 and 7). Since a higher interest rate increases the disincentive for short sales, a negative relationship is expected between short interest and interest rates. The following equation shows the regression test equation:

$$
\begin{equation*}
\text { SHORT }_{j s t}=\alpha_{1}+\alpha_{2}\left(O P E N_{j s t}\right)+\alpha_{3}\left(I N D E X_{s t}\right)+\alpha_{4} R_{s t}+v_{j s t} \tag{46}
\end{equation*}
$$

where | SHORT $_{\mathrm{jst}}$ | $=$ change in relative short interest from time s to time t, |
| ---: | :--- |
| OPEN $_{\mathrm{jst}}$ | $=$ change in relative option open interest from time s to t, |
| INDEX $_{\mathrm{st}}$ | $=$ change in index option open interest from time s to t, |
| $\mathrm{R}_{\mathrm{st}}$ | $=$ change in interest rates from time s to time t, |
| $\alpha_{1}$ | $=$ regression intercept, |
| $\alpha_{2}$ | $=$ regression coefficient for the option open interest term, |
| $\alpha_{3}$ | $=$ regression coefficient for the index option term, |
| $\alpha_{4}$ | $=$ regression coefficient for the interest rate term, and |
| $\mathrm{v}_{\mathrm{jst}}$ | $=$ regression error term over time s to time t. |

Note that a relative change in option open interest and short interest again is used to provide a common basis across securities with a different number of shares outstanding. In each case, the relative change is determined by dividing by the number of shares outstanding. If the slope coefficient, $\alpha_{2}$, is positive and significantly different from zero, the null hypothesis is rejected. This would provide direct evidence that options are being used to circumvent short sale restrictions. This regression is applied to various holding periods from one to five years following the introduction of options to demonstrate the connection between options and short sales over a long period of time.

## Data

This section provides the source and characteristics of the empirical data used in the tests identified in the methodology section.

Introduction Dates. The dates that individual options were first made available for trading (defined as the introduction dates) were obtained for options on five exchanges -- the Chicago Board Options Exchange (CBOE) ${ }^{9}$, the American Stock Exchange (AMEX), the Philadelphia Exchange (PHLX), the Pacific Stock Exchange (PSE), and the New York Stock Exchange (NYSE). In each case, the introduction date is prior to 1988 to provide five years of trading data through 1992, which allows enough time for the options market in each individual option to develop. Also, the underlying common stock for each individual option must be listed on the CRSP NYSE/AMEX tapes with at least five years of historical data prior to option introduction to allow for parameter estimation in calculating abnormal returns. After eliminating duplicate introduction dates (on more than one exchange), 358 unique introduction dates are obtained. Figure 7 shows the distribution of these introduction dates over the 1973 to 1987 period.

[^8]FIGURE 7
Distribution of 358 Option
Introduction Dates


Short Interest. Short interest data represents the total number of shares sold short at a point in time. Mid-month short interest data for common stock on the New York Stock Exchange (NYSE) are published near the end of each month in Barron's. This data also are reported for each individual stock in the NYSE Daily Stock Price Record, which is the source of short interest data for this study. The short interest for each stock is divided by the total outstanding shares of that stock (contained on the CRSP data tapes) to develop the relative short interest. The relative short interest is determined for the month of introduction and for one, two, three, four, and five year intervals following option introduction for each individual stock. Brent, Morse, and Stice (1990) point out that short interest data are developed from a survey and represent data for the 7th or 8th of each month, although normally reported for the 15th of each month. Therefore, the short interest data are shifted back one month (i.e., the short interest reported in January is more reflective of the December end of month value). The fact that the short interest data represent points one week into the following month is not critical in this study since the data are collected at annual intervals following the introduction of options.

Option Open Interest. The option open interest represents the number of contracts outstanding to purchase or sell 100 share lots of the underlying stock. The data for option open interest include put and call contracts, different strike prices, and different expiration dates. The theoretical model in Chapter III predicts that $\Delta$ shares of stock will be sold short for each option contract, where $\Delta$ is the slope of the option
pricing curve. Figure 8 illustrates the concept of an option delta. Since the majority of these varied contracts are near the money (i.e., the strike price is near the current stock price), they all have a similar effect in terms of resulting short interest.

Therefore, a single number for the option open interest on the options of a particular stock at any point in time can be determined by adding all the open interest for all the varied contracts. Such a single number will be biased to the extent that the distribution of the option contracts changes over time. However, the general trend of the total option open interest will correctly indicate the growth in the use of options for a particular underlying stock.

Also, it should be noted that the total option open interest drops somewhat each time an expiration date for some of the contracts occurs, followed by a recovery in total option open interest as the open interest of the new contracts develops. This process of expiration and subsequent growth in the open interest for new contracts should also be reflected directly in the short interest data and, therefore, should not overly distort the relationship between options and short interest. ${ }^{10}$

Option open interest data are listed in Barron's, which on Monday of each week publishes the open interest for option contracts on various exchanges for the previous Thursday. These data are collected for intervals of one, two, three, four,

[^9]FIGURE 8
The Concept of Option Delta for a Call Option

and five years following the date of option introduction for each individual stock. So that the option open interest data correspond as closely as possible to an end of month value, the particular issue of Barron's chosen are those between the 1st and 7th of the month. Such an issue will list open interest data for the close of the previous Thursday, which corresponds to the last two days of the previous month through the 3rd of the month of the issue date of Barron's. This time period is within three days of the end of the month value for option open interest. Using open interest data within three days of the end of the month maintains the option open interest data approximately seven days before the monthly short interest data.

The relative option open interest is determined by dividing option open interest by the number of shares outstanding. The number of shares outstanding is contained on the CRSP data tapes.

Index Options. Index options represent option contracts on an index rather than an individual stock. Index options data are collected from Barron's using the 8th through 14th weekly issue for each month's data. These data include the month of introduction and one, two, three, four, and five year intervals following the introduction of each individual option.

Institutional Holdings. Institutional holdings data represent the number of shares held by mutual funds, pensions, and other investment companies. These data
are listed in the Standard and Poor's Monthly Stock Guide. These data are collected for the month of the introduction date and for intervals of one, two, three, four, and five years following the introduction of options for each individual stock.

Interest Rates. Data for monthly interest rates are represented by the asking discount rate for 90 -day U.S. Treasury bills. These data are collected for the month of introduction and for intervals of one, two, three, four, and five years following the introduction of options for each individual stock. The data for 90 -day Treasury bills are collected from the Survey of Current Business.

## Summary

The three hypotheses identified in the first section of this chapter are tested using the methodologies and empirical data outlined in the second and third sections. The results of these tests are provided in the next chapter.

This chapter reviews the empirical results of the tests designed in the previous chapter. First, the chosen method of calculating abnormal returns is outlined, and the results are compared to abnormal returns calculated using various other event study methodologies. Next, the empirical results are presented for the tests of the three hypotheses. The conclusions from these empirical tests then are summarized.

## Abnormal Returns

An event study as outlined in Chapter IV is used to determine if there are statistically significant negative abnormal returns following the introduction of options, which would reject Hypothesis 1. This event study methodology subtracts a comparable size decile holding period return from the holding period return for each security in the sample to determine a holding period abnormal return, as shown in the following equation:

$$
\begin{equation*}
H P A R_{j s t}=H P R_{j s t}-A H P R_{i(j) s t} \tag{47}
\end{equation*}
$$

where HPAR $_{\text {jst }}=$ holding period abnormal return for security j from time s to time t ,
$\mathrm{HPR}_{\mathrm{jst}} \quad=$ holding period return for security j from time s to time t , and
$\mathrm{AHPR}_{\mathrm{i}() \text { st }}=$ holding period return for size decile i that includes security j from time $s$ to time $t$.

Size-based deciles are formed at the beginning of each calendar year based on the market capitalization for each security at the end of the previous year. Each security is assigned a portfolio, with portfolio 10 containing the largest firms. Securities maintain the same portfolio assignment for one year; then the process is repeated at the beginning of the next calendar year. Holding period returns are then calculated for each security in a size decile and averaged to determine the holding period return for that decile. A holding period abnormal return (HPAR) is calculated by subtracting the appropriate decile average holding period return (AHPR) from the holding period return (HPR) of each security j . The average holding period abnormal return (AHPAR) is then determined by calculating the mean of the holding period abnormal returns across all n securities in the sample, as shown in the following equation:

$$
\begin{equation*}
A H P A R_{s t}=\frac{1}{n} \sum_{j=1}^{n} H P A R_{j s t} \tag{48}
\end{equation*}
$$

where $A H P A R_{s t}=$ average abnormal return for the sample from time $s$ to time $t$,
$\mathrm{HPAR}_{\mathrm{jst}}=$ holding period abnormal return for security j from time s to time t , and
$\mathrm{n} \quad=$ number of securities in the sample.

The results from this methodology show a $-9.74 \%$ abnormal return over the five years following option introduction, which is significant at the .05 level using a one-tailed cross-sectional t-test. However, there are many other event study
methodologies that could be used to calculate abnormal returns. The following section compares the abnormal returns using 15 different methodologies. The advantages of using the methodology just described and the disadvantages of using other methodologies are presented. The methodology adopted in this dissertation, which is an approach that uses holding period returns and size based decile returns, is identified as method 1.

## Comparison with other Methodologies

Table 4 and Figure 9 show the abnormal returns following the introduction of options using various event study methodologies compared to the preferred methodology used in method 1. Since the test period spans up to five years after introduction, these methodologies are segregated by those which use holding period returns and those which use arithmetic sums to calculate abnormal returns. The following sections compare the results from these methodologies.

Comparison Period Mean Adjusted. This method (identified as method 15) subtracts the average return for a security during the five years before introduction from the actual return each month following the introduction. The sum of the abnormal returns for five years following introduction shows a $-45.08 \%$ price effect. This method of calculating abnormal returns is dependent on the estimation period being "normal." A negative bias can occur if the average size of the sample securities is smaller during the estimation period than during the testing period. In this case, the

TABLE 4
Average Abnormal Returns

## Using Various Event Study Methodologies

|  | $\begin{aligned} & \text { Year } 1 \\ & (1,12) \end{aligned}$ | $\begin{aligned} & \text { Year } 2 \\ & (13,24) \end{aligned}$ | $\begin{aligned} & \text { Year } 3 \\ & (25,36) \end{aligned}$ | $\begin{aligned} & \text { Year } 4 \\ & (37,48) \end{aligned}$ | $\begin{aligned} & \text { Year } 5 \\ & (49,60) \end{aligned}$ | $\begin{gathered} \text { 2-Year } \\ (1,24) \end{gathered}$ | $\begin{gathered} \text { 3-Year } \\ (1,36) \end{gathered}$ | $\begin{aligned} & \text { 4-Year } \\ & (1,48) \end{aligned}$ | $\begin{aligned} & 5 \text {-Year } \\ & (1,60) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HPR of Sample Less |  |  |  |  |  |  |  |  |  |
| 1. HPR of size decile | $\begin{gathered} -1.59 \\ (-1.11) \end{gathered}$ | $\begin{gathered} -2.05 \\ (-1.70) \end{gathered}$ | $\begin{aligned} & -1.22 \\ & (-.84) \end{aligned}$ | $\begin{gathered} -.90 \\ (-.60) \end{gathered}$ | $\begin{aligned} & -1.13 \\ & (-.71) \end{aligned}$ | $\begin{gathered} -3.88 \\ -(1.87) \end{gathered}$ | $\begin{gathered} -4.41 \\ (-1.50) \end{gathered}$ | $\begin{gathered} -7.00 \\ (-1.68) \end{gathered}$ | $\begin{gathered} -9.74 \\ (-1.72) \end{gathered}$ |
| 2. HPR of market (EW) | $\begin{gathered} -6.87 \\ (-4.77) \end{gathered}$ | $\begin{gathered} -6.70 \\ (-5.39) \end{gathered}$ | $\begin{gathered} -4.48 \\ (-3.01) \end{gathered}$ | $\begin{gathered} -1.87 \\ (-1.20) \end{gathered}$ | $\begin{gathered} -1.97 \\ (-1.20) \end{gathered}$ | $\begin{aligned} & -15.00 \\ & (-6.94) \end{aligned}$ | $\begin{aligned} & -25.14 \\ & (-7.93) \end{aligned}$ | $\begin{aligned} & -32.34 \\ & (-7.34) \end{aligned}$ | $\begin{aligned} & -42.60 \\ & (-7.09) \end{aligned}$ |
| 3. Compound size decile | $\begin{gathered} -2.40 \\ (-1.65) \end{gathered}$ | $\begin{gathered} -2.42 \\ (-2.01) \end{gathered}$ | $\begin{gathered} .38 \\ (.26) \end{gathered}$ | $\begin{gathered} -2.33 \\ (-1.53) \end{gathered}$ | $\begin{gathered} -1.69 \\ (-1.04) \end{gathered}$ | $\begin{gathered} -5.56 \\ (-2.66) \end{gathered}$ | $\begin{gathered} -7.25 \\ (-2.41) \end{gathered}$ | $\begin{aligned} & -10.65 \\ & (-2.55) \end{aligned}$ | $\begin{aligned} & -14.24 \\ & (-2.53) \end{aligned}$ |
| 4. Compound market | $\begin{gathered} -8.31 \\ (-5.74) \end{gathered}$ | $\begin{gathered} -8.03 \\ (-6.50) \end{gathered}$ | $\begin{gathered} -6.19 \\ (-4.11) \end{gathered}$ | $\begin{gathered} -3.71 \\ (-2.36) \end{gathered}$ | $\begin{gathered} -3.51 \\ (-2.14) \end{gathered}$ | $\begin{aligned} & -18.50 \\ & (-8.50) \end{aligned}$ | $\begin{aligned} & -30.83 \\ & (-9.60) \end{aligned}$ | $\begin{aligned} & -39.79 \\ & (-8.81) \end{aligned}$ | $\begin{aligned} & -52.82 \\ & (-8.66) \end{aligned}$ |
| 5. Compound EW CRSP | $\begin{gathered} -8.18 \\ (-5.64) \end{gathered}$ | $\begin{gathered} -7.85 \\ (-6.33) \end{gathered}$ | $\begin{gathered} -6.10 \\ (-4.05) \end{gathered}$ | $\begin{gathered} -3.70 \\ (-2.35) \end{gathered}$ | $\begin{gathered} -3.56 \\ (-2.17) \end{gathered}$ | $\begin{aligned} & -18.22 \\ & (-8.33) \end{aligned}$ | $\begin{aligned} & -30.50 \\ & (-9.44) \end{aligned}$ | $\begin{aligned} & -39.64 \\ & (-8.71) \end{aligned}$ | $\begin{aligned} & -52.85 \\ & (-8.58) \end{aligned}$ |
| 6. Compound VW CRSP | $\begin{gathered} -.77 \\ (-.54) \end{gathered}$ | $\begin{gathered} -2.18 \\ (-1.78) \end{gathered}$ | $\begin{gathered} .13 \\ (.08) \end{gathered}$ | $\begin{gathered} -2.79 \\ (-1.82) \end{gathered}$ | $\begin{gathered} -.50 \\ (-.31) \end{gathered}$ | $\begin{gathered} -3.50 \\ (-1.68) \end{gathered}$ | $\begin{gathered} -4.71 \\ (-1.57) \end{gathered}$ | $\begin{gathered} -8.65 \\ (-2.01) \end{gathered}$ | $\begin{aligned} & -10.08 \\ & (-1.74) \end{aligned}$ |

Sum of Sample Returns less Sum of

| 7. Size decile returns | $\begin{aligned} & -1.27 \\ & (-.99) \end{aligned}$ | $\begin{gathered} -1.46 \\ (-1.27) \end{gathered}$ | $\begin{gathered} -.70 \\ (-.56) \end{gathered}$ | $\begin{gathered} -1.40 \\ (-1.10) \end{gathered}$ | $\begin{aligned} & -1.29 \\ & (-.86) \end{aligned}$ | $\begin{gathered} -2.80 \\ (-1.63) \end{gathered}$ | $\begin{gathered} -2.22 \\ (-1.05) \end{gathered}$ | $\begin{gathered} -3.52 \\ (-1.43) \end{gathered}$ | $\begin{gathered} -4.74 \\ (-1.59) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8. Market returns (EW) | $\begin{gathered} -6.32 \\ (-4.90) \end{gathered}$ | $\begin{gathered} -6.84 \\ (-5.80) \end{gathered}$ | $\begin{gathered} -4.74 \\ (-3.69) \end{gathered}$ | $\begin{gathered} -3.12 \\ (-2.34) \end{gathered}$ | $\begin{gathered} -3.04 \\ (-2.00) \end{gathered}$ | $\begin{aligned} & -13.16 \\ & (-7.44) \end{aligned}$ | $\begin{aligned} & -17.90 \\ & (-7.99) \end{aligned}$ | $\begin{aligned} & -21.02 \\ & (-7.86) \end{aligned}$ | $\begin{aligned} & -24.06 \\ & (-7.52) \end{aligned}$ |
| 9. EW CRSP returns | $\begin{gathered} -6.23 \\ (-4.83) \end{gathered}$ | $\begin{gathered} -6.73 \\ (-5.68) \end{gathered}$ | $\begin{gathered} -4.65 \\ (-3.61) \end{gathered}$ | $\begin{gathered} -3.10 \\ (-2.32) \end{gathered}$ | $\begin{gathered} -3.07 \\ (-2.01) \end{gathered}$ | $\begin{aligned} & -12.96 \\ & (-7.29) \end{aligned}$ | $\begin{aligned} & -17.61 \\ & (-7.81) \end{aligned}$ | $\begin{aligned} & -20.72 \\ & (-7.68) \end{aligned}$ | $\begin{aligned} & -23.78 \\ & (-7.37) \end{aligned}$ |
| 10. VW CRSP returns | $\begin{gathered} .32 \\ (.25) \end{gathered}$ | $\begin{aligned} & -1.14 \\ & (-.97) \end{aligned}$ | $\begin{gathered} .36 \\ (.28) \end{gathered}$ | $\begin{gathered} -1.82 \\ (-1.42) \end{gathered}$ | $\begin{aligned} & -.30 \\ & (-.20) \end{aligned}$ | $\begin{gathered} -.82 \\ (-.48) \end{gathered}$ | $\begin{gathered} -.46 \\ (-.21) \end{gathered}$ | $\begin{aligned} & -2.28 \\ & (-.90) \end{aligned}$ | $\begin{aligned} & -2.58 \\ & (-.84) \end{aligned}$ |
| 11. Market model (EW) | $\begin{aligned} & -12.26 \\ & (-6.76) \end{aligned}$ | $\begin{aligned} & -13.82 \\ & (-7.62) \end{aligned}$ | $\begin{aligned} & -11.60 \\ & (-6.40) \end{aligned}$ | $\begin{aligned} & -10.08 \\ & (-5.56) \end{aligned}$ | $\begin{gathered} -9.59 \\ (-5.00) \end{gathered}$ | $\begin{gathered} -26.08 \\ (-10.17) \end{gathered}$ | $\begin{gathered} -37.69 \\ (-11.99) \end{gathered}$ | $\begin{gathered} -47.78 \\ (-13.17) \end{gathered}$ | $\begin{gathered} -57.37 \\ (-14.14) \end{gathered}$ |
| 12. Market model (VW) | $\begin{aligned} & -10.22 \\ & (-6.63) \end{aligned}$ | $\begin{aligned} & -11.35 \\ & (-7.36) \end{aligned}$ | $\begin{aligned} & -12.91 \\ & (-8.37) \end{aligned}$ | $\begin{aligned} & -13.56 \\ & (-8.80) \end{aligned}$ | $\begin{aligned} & -12.16 \\ & (-8.62) \end{aligned}$ | $\begin{aligned} & -21.57 \\ & (-9.89) \end{aligned}$ | $\begin{gathered} -34.49 \\ (-12.91) \end{gathered}$ | $\begin{gathered} -48.05 \\ (-15.58) \end{gathered}$ | $\begin{gathered} -60.22 \\ (-17.46) \end{gathered}$ |
| 13. Standardized (EW) | $\begin{aligned} & -10.66 \\ & (-7.18) \end{aligned}$ | $\begin{aligned} & -12.23 \\ & (-8.93) \end{aligned}$ | $\begin{aligned} & -10.65 \\ & (-7.79) \end{aligned}$ | $\begin{aligned} & -10.30 \\ & (-7.52) \end{aligned}$ | $\begin{gathered} -9.09 \\ (-7.35) \end{gathered}$ | $\begin{gathered} -22.89 \\ (-11.89) \end{gathered}$ | $\begin{gathered} -33.54 \\ (-14.15) \end{gathered}$ | $\begin{gathered} -43.85 \\ (-16.01) \end{gathered}$ | $\begin{gathered} -52.94 \\ (-17.29) \end{gathered}$ |
| 14. Standardized (VW) | $\begin{gathered} -8.77 \\ (-6.51) \end{gathered}$ | $\begin{aligned} & -10.15 \\ & (-7.54) \end{aligned}$ | $\begin{aligned} & -11.73 \\ & (-8.73) \end{aligned}$ | $\begin{aligned} & -13.19 \\ & (-9.81) \end{aligned}$ | $\begin{aligned} & -11.14 \\ & (-9.14) \end{aligned}$ | $\begin{aligned} & -18.93 \\ & (-9.93) \end{aligned}$ | $\begin{gathered} -30.67 \\ (-13.15) \end{gathered}$ | $\begin{gathered} -43.86 \\ (-16.29) \end{gathered}$ | $\begin{gathered} -55.01 \\ (-18.27) \end{gathered}$ |
| 15. Comparison Period | $\begin{gathered} -9.04 \\ (-2.78) \end{gathered}$ | $\begin{aligned} & -15.48 \\ & (-4.75) \end{aligned}$ | $\begin{gathered} -4.74 \\ (-1.46) \end{gathered}$ | $\begin{gathered} -9.35 \\ (-2.87) \end{gathered}$ | $\begin{gathered} -6.45 \\ (-2.06) \end{gathered}$ | $\begin{aligned} & -24.53 \\ & (-5.32) \end{aligned}$ | $\begin{aligned} & -29.27 \\ & (-5.19) \end{aligned}$ | $\begin{aligned} & -38.63 \\ & (-5.93) \end{aligned}$ | $\begin{aligned} & -45.08 \\ & (-6.19) \end{aligned}$ |

## FIGURE 9 <br> Comparison of Abnormal Returns from <br> Various Event Study Methodologies



Years after Introduction
expected returns would be higher during the estimation period (smaller firms) than during the test period (larger firms). Subtracting the larger estimation period returns from the smaller test period returns thus introduces a negative bias. Figure 10 shows that the average portfolio assignment for the sample securities increased from 8.1 five years before introduction to 8.8 during the month of option introduction. This change in average size for the sample causes the abnormal returns calculated using this methodology to be inaccurate.

Market Model. The validity of using the market model to adjust sample security returns depends on the stationarity of beta for each security. Table 5 shows that beta calculated from monthly data for the sample securities increases after the introduction of options. Using data over the five year test period following option introduction, the average beta is .98 (median of .97). In contrast, the average beta using data over the five years before introduction is .93 (median of .92 ). A simple t-test shows that the difference of the mean betas in the five years before and after the introduction date is significantly different from zero at the .05 level (p-value of .014 ). The fact that beta increases for the average sample securities should introduce a positive bias in the calculation of abnormal returns. Recall that Chapter IV discussed an anticipated decline in beta which would have introduced a negative bias (see page 60 of this dissertation for a discussion of the change in beta as measured in short term studies). The result that the market model methodologies actually calculate abnormal returns which are significantly more negative than method 1 likely indicates that a

FIGURE 10
The Change in Average Portfolio Assignment 5 Years Before to 5 Years After Introduction


TABLE 5
Change in Beta and Alpha
After Option Introduction

## Average Beta 5 Years after Introduction . 9762

Average Beta 5 Years before Introduction . 9294

Change in Beta . 0468
T-statistic* (2.464)
p-value* (.0142)

Average Alpha 5 Years after Introduction -. 00338
Average Alpha 5 Years before Introduction . 00700

Change in Alpha -. 01038
T-statistic*
(-13.539)
p-value*
(.0001)

* The t-statistics and p-values test whether the change is significantly different from zero.
failure to adjust for differences in size outweighs any change in beta for long term event studies. Thus, the results from methods 11 through 14 in Table 4, which use the market model, are unreliable.

Market Adjusted Approach. Event study methodologies 8, 9, and 10 in Table 4 use a market adjusted approach. In each case, the abnormal returns are calculated as follows:

$$
\begin{align*}
& C A A R_{s t}=\sum_{\tau=s}^{t} A A R_{\tau}  \tag{49}\\
& A A R_{\tau}=\frac{1}{n} \sum_{j=1}^{n} A R_{j \tau}  \tag{50}\\
& A R_{j \tau}=R_{j \tau}-R_{m \tau} \tag{51}
\end{align*}
$$

where $\mathrm{CAAR}_{\mathrm{st}}=$ cumulative average abnormal return from time s to time t ,
$\mathrm{AAR}_{\tau}=$ average abnormal return at time $\tau$,
$\mathrm{AR}_{\mathrm{j} \tau}=$ abnormal return of security j at time $\tau$,
$\mathbf{R}_{\mathrm{j} \tau} \quad=$ return of security j at time $\tau$, and
$R_{m \tau}=$ return of the market at time $\tau$.
Using the equally weighted (EW) CRSP index return as the market (method 9), the cumulative average abnormal return in the five years following the introduction of options is $-23.78 \%$. Event study method 8 uses an equally weighted market index return which is calculated using the same data screening procedure as used for the
sample (i.e., a requirement of 121 months of non-missing returns and market capitalization data for size decile ranking). The results for method 8 are essentially the same as the results for method 9 (EW CRSP), which indicates that the data screening procedure for the sample does not introduce a selection bias.

Method 10 is a market adjusted approach using the value weighted (VW) CRSP index returns. The value weighted index places a higher weight on larger capitalization securities. To the extent that larger capitalization securities have lower returns, the negative abnormal returns calculated using a value weighted market index will be larger than the abnormal returns calculated using an equally weighted index. Method 10 (VW CRSP) shows a five year abnormal return of $-2.58 \%$, which is larger than the abnormal returns from method 9 (EW CRSP).

To determine whether it is more appropriate to use an equally weighted or value weighted market index, it is necessary to determine the average firm size of the sample. If the average firm size of the sample is larger (smaller) than the average market size, a negative (positive) bias in abnormal returns would be introduced. Table 6 indicates that the average market capitalization of the sample is much larger than the median market capitalization of all securities in the market. This measure of size is determined by assigning all securities a size decile portfolio (portfolio 10 being those securities with the largest capitalization) at the beginning of each calendar year. As shown in Table 6, the sample average portfolio assignment is 8.78 on the

TABLE 6 Change in Average Portfolio Assignment for Sample of 358 Securities
AveragePortfolioAssignment
Year -5 ..... 8.12
Year -4 ..... 8.26
Year -3 ..... 8.42
Year - 2 ..... 8.52
Year -1 ..... 8.67
Month 0 ..... 8.78
Year 1 ..... 8.84
Year 2 ..... 8.84
Year 3 ..... 8.79
Year 4 ..... 8.76
Year 5 ..... 8.71
5 Years After ..... 8.79
5 Years Before ..... 8.40
Change Before and After ..... 39
T-statistic* ..... (7.794)
p-value* ..... (.0001)

* The t -statistic and p -value test whether the change is significantly different from zero.
introduction month, which is much larger than the median security (which has an average portfolio assignment of 5.5). Thus, abnormal returns calculated using an equally weighted market index would have a negative bias. Using a value weighted index would be more appropriate when the average portfolio size of the sample is large. The accuracy of the abnormal returns depends heavily on exactly matching the average capitalization of the sample with the average capitalization size of the weighted market index. Any bias resulting from differences in capitalization size are then magnified when calculating cumulative abnormal returns since a consistent bias would be additive over multiple periods. Therefore it is justified to use a more accurate correction for size differences.

Size Decile Adjusted Approach. Event study method 7 in Table 4 explicitly corrects for differences in market capitalization using the following equations:

$$
\begin{align*}
& C A A R_{s t}=\sum_{\tau=s}^{t} A A R_{\tau}  \tag{52}\\
& A A R_{\tau}=\frac{1}{n} \sum_{j=1}^{n} A R_{j \tau}  \tag{53}\\
& A R_{j \tau}=R_{j \tau}-R_{i\langle j) \tau}  \tag{54}\\
& R_{i(j) \tau}=\frac{1}{m} \sum_{k=1}^{m} R_{k \tau} \tag{55}
\end{align*}
$$

where $\mathrm{CAAR}_{\mathrm{st}}=$ cumulative average abnormal return from time s to time t,

| $\mathrm{AAR}_{\text {г }}$ | $=$ average abnormal return for time $\tau$, |
| :---: | :---: |
| $\mathrm{AR}_{\mathrm{jr}}$ | $=$ abnormal return for security j at time $\tau$, |
| $\mathrm{R}_{\mathrm{j} \mathrm{\tau}}$ | $=$ return for security j at time $\tau$, |
| $\mathrm{R}_{\mathrm{i}(\mathrm{j})}$ | $=$ return for the size decile that contains security j at time $\tau$, |
| $\mathrm{R}_{\mathrm{kr}}$ | $=$ return for security k in the size decile that contains security j |
|  | at time $\tau$, |
| $n$ | $=$ number of securities in the sample, and |
| m | $=$ number of k securities in the size decile that contains security |

In this methodology, the average return for securities in each size decile is first determined for each calendar month. Then the abnormal return is determined by subtracting the return of the appropriate size decile from the return of a sample security for each month of the test period. The cumulative abnormal returns are averaged over the sample and then cumulated over the desired test period. Using method 7 , the average cumulative abnormal return for five years following option introduction is $-4.74 \%$. This method provides a higher abnormal return estimate than the preferred method 1 , and could occur from a positive bias introduced by cumulating monthly abnormal returns rather than determining holding period abnormal returns (see Chapter IV, pages 59 to 62 for a more thorough discussion of this potential bias).

HPR and Compounded Index Approach. Methods 3, 4, 5, and 6 in Table 4
subtract a compounded index from the sample security holding period return to obtain abnormal returns, as in the following equations:

$$
\begin{gather*}
A H P A R=\frac{1}{n} \sum_{j=1}^{n} H P A R_{j s t}  \tag{56}\\
H P A R_{j s t}=H P R_{j s t}-C I_{s t}  \tag{57}\\
H P R_{j s t}=\prod_{\tau=s}^{t}\left(1+R_{j \tau}\right)  \tag{58}\\
C I_{s t}=\prod_{\tau=s}^{t}\left(1+I_{\tau}\right)  \tag{59}\\
I_{\tau}=\frac{1}{m} \sum_{k=1}^{m}\left(1+R_{k \tau}\right) \tag{60}
\end{gather*}
$$

where AHPAR $_{\mathrm{st}}=$ average holding period abnormal return from time s to time t ,
$\mathrm{HPAR}_{\mathrm{jst}} \quad=$ holding period abnormal return for security j form time s to time t ,
$\mathrm{HPR}_{\mathrm{jst}}=$ holding period return for security j from time s to time t ,
$\mathrm{CI}_{\mathrm{st}}=$ compounded index return from time s to time t ,
$R_{j \tau} \quad=\quad$ return of security $j$ for time $\tau$,
$\mathrm{R}_{\mathrm{kr}} \quad=$ return for security k in the index for time $\tau$,
$I_{\tau} \quad=\quad$ return of the index for time $\tau$,
$\mathrm{n} \quad=$ number of securities in the sample, and
$\mathrm{m} \quad=$ number of securities used in the calculation of the index.

Methods 4,5 , and 6 also fail to accurately correct for a difference in firm size. Method 3 uses specific size deciles. All four compound the individual months to obtain a multiple period index. One of the difficulties in calculating abnormal returns with the preferred method 1 is that a holding period return index must be calculated for every specific holding period desired in the research test. As a simplification, one may be inclined to substitute a compounded single period index (as in Equation 48) for the true holding period index. This method of calculating an index incorporates a positive bias in the index by first averaging the returns (which may contain spurious variation) before compounding. Each single period index would then have a potential positive bias along the line of reasoning by Dimson and Marsh (1986) and Conrad and Kaul (1993). Compounding after averaging then tends to exaggerate the single period bias; the longer the time period the greater the bias.

In contrast, the true holding period return index is calculated according to the following equations:

$$
\begin{align*}
H P R I_{s t} & =\frac{1}{m} \sum_{k=1}^{m} H P R_{k s t}  \tag{61}\\
H P R_{k s t} & =\prod_{\tau=s}^{t}\left(1+R_{k \tau}\right) \tag{62}
\end{align*}
$$

where $H P R I_{s t}=$ holding period return index for time $s$ to time $t$,
$H \mathrm{HR}_{\text {kst }}=$ holding period return for security k in the index from time s to time t ,
$\mathrm{R}_{\mathrm{k} \tau}=$ return for security k for time $\tau$, and
$\mathrm{m} \quad=$ number of securities contained in the index.

Using this method for calculating an index incorporates only the bias in a single holding period.

The abnormal returns calculated using methods $3,4,5$, and 6 shown in Table 4, therefore, potentially contain a negative bias because of compounding a single period index. In calculating abnormal returns, since the market index return is subtracted from the sample return, a positive bias in the market index translates into a negative bias in the abnormal returns.

Five year abnormal returns are $-52.82 \%,-52.85 \%$, and $-10.08 \%$ respectively for the compounded equally weighted market index, equally weighted CRSP index, and value weighted CRSP index. The same bias from the failure to accurately account for size differences in the market adjusted approaches (methods 8, 9, and 10) is again reflected in the compounded approaches (methods 4,5 , and 6 ). In the compounded index approaches, the bias appears to be exacerbated, more than doubling the negative abnormal returns. This is also true when size is explicitly taken into account, as in method 3, which uses a compounded size decile index. Again note that a positive bias in the compounded size decile index introduces a negative bias in five year abnormal returns from $-4.74 \%$ to $-14.24 \%$. Therefore, approaches for
calculating abnormal returns which subtract a compounded index from the sample holding period returns should be avoided.

HPR Less EW HPR Index . Method 2 in Table 4 calculates abnormal returns by subtracting an equally weighted holding period return index from each of the sample security holding period returns, as in the following equations:

$$
\begin{align*}
A H P A R_{s t} & =\frac{1}{n} \sum_{j=1}^{n} H P A R_{j s t}  \tag{63}\\
H P A R_{j s t} & =H P R_{j s t}-A H P R_{M s t}  \tag{64}\\
H P R_{j s t} & =\prod_{\tau=s}^{t}\left(1+R_{j \tau}\right)  \tag{65}\\
A H P R_{M s t} & =\frac{1}{m} \sum_{k=1}^{m} H P R_{k s t}  \tag{66}\\
H P R_{k s t} & =\prod_{\tau=s}^{t}\left(1+R_{k \tau}\right) \tag{67}
\end{align*}
$$

where $\mathrm{AHPAR}_{\mathrm{st}}=$ average holding period abnormal return from time s to time t ,
$\mathrm{HPAR}_{\mathrm{jst}}=$ holding period abnormal return for security j from time s to time t ,
$\mathrm{HPR}_{\mathrm{jst}}=$ holding period return for security j from time s to time t ,
$\mathrm{AHPR}_{\text {Mst }}=$ average holding period return for the market index from time s
to time t ,
$\mathrm{HPR}_{\mathrm{kst}}=$ holding period return for security k contained in the market index,
n $\quad=$ number of securities in the sample, and
$\mathrm{m} \quad=$ number of securities contained in the market index.

This method of calculating abnormal returns minimizes the bias from single period variations in return. However, the method does not account for the difference in size between the sample and the index. Method 2 indicates a five year abnormal return of $-42.60 \%$, which is substantially more negative than the $-9.74 \%$ abnormal return from the preferred approach using holding period size decile returns, as in method 1. This substantial difference in abnormal returns highlights the importance of adjusting for a difference in size between the sample and the index when using the market adjusted event study methodology.

HPR Less HPR Size Index. Method 1 in Table 4 is the preferred event study methodology, which is used in this dissertation to test Hypothesis 1. The following equations summarize this methodology:

$$
\begin{gather*}
A H P A R_{s t}=\frac{1}{n} \sum H P A R_{j s t}  \tag{68}\\
H P A R_{j s t}=H P R_{j s t}-A H P R_{i(j) s t} \tag{69}
\end{gather*}
$$

$$
\begin{align*}
H P R_{j s t} & =\prod_{\tau=s}^{t}\left(1+R_{j \tau}\right)  \tag{70}\\
A H P R_{i(j) s t} & =\frac{1}{m} \sum_{k=1}^{m} H P R_{k s t}  \tag{71}\\
H P R_{k s t} & =\prod_{\tau=s}^{t}\left(1+R_{k \tau}\right) \tag{72}
\end{align*}
$$

where $\mathrm{AHPAR}_{\text {st }}=$ average holding period abnormal return from time $s$ to time $t$,


This method is similar to a market adjusted approach, such as method 2, except that size based decile returns are used (instead of a market return) to explicitly adjust for
differences in size. The methodology also minimizes the potential multiple period bias by using holding period returns for both the sample returns and the size based decile returns. This method indicates a $-9.74 \%$ abnormal return in the five year period following the introduction of options.

Conclusions. This section compares the results for calculating abnormal returns using various event study methodologies. These results lead to various conclusions.

First, consistent with Dimson and Marsh (1986), this review shows the importance of an explicit correction for differences in size. The methods which do not correct for size differences (including the market model, comparison period mean adjusted, and equally weighted market adjusted approaches) show a significant negative bias compared to those approaches which adjust for size differences. There are two main reasons for this bias.

The first reason for a negative bias is that the average sample firm size is significantly larger than the average firm in the market, and larger firms tend to have lower returns. For example, average holding period return (AHPAR) methods 1 and 2 and cumulative average abnormal return (CAAR) methods 7 and 8 use an identical methodology except that methods 1 and 7 correct for size differences. For the AHPAR methodology, the correction for size increases the average abnormal returns
by $32.86 \%$ ( $-9.74 \%$ less $-42.60 \%$ ). For the CAAR methodology, the correction for size increases the average abnormal return by $19.32 \%$ ( $-4.74 \%$ less $-24.06 \%$ ). Thus, the correction for size differences in this study likely removes a substantial negative bias in calculated abnormal returns.

Using a value weighted index to adjust for size in the market adjusted event study methodology provides a better estimate of abnormal returns than using an equally weighted approach when the sample firm size is substantially larger than the average firm in the market (as in this study). However, the accuracy of using a value weighted market adjusted approach is heavily dependent on a size match between the sample and the value weighted market average, which is heavily weighted towards larger firms. In this particular study, by coincidence the average weighted size for the value weighted (VW) CRSP index is closer to the average sample size than the equally weighted (EW) CRSP index. For a direct comparison, methods 7 and 10 use an identical CAAR methodology except method 7 adjusts for specific size decile returns while method 10 adjusts for the fixed VW CRSP market index returns. Specific correction for size lowers the abnormal returns by $2.16 \%$ ( $-4.74 \%$ less $-2.58 \%$ ). An approach which uses size decile returns is more appropriate as a general event study methodology because it more accurately accounts for differences in size.

The second reason for a negative bias for differences in size is that the sample itself changes over time. Methodologies which assume parameters remain constant
from the estimation period (before the event) to the testing period (after the event) can introduce a significant bias when the average sample size changes before and after the event. In this particular study, the average size of the sample increased significantly. This causes the introduction of a negative bias (larger firms tend to have lower returns unrelated to any abnormal returns). For example, the four methodologies that use a market model approach (methods 11, 12, 13, and 14) show the lowest abnormal returns of all 15 methods. The comparison period mean adjusted approach is only slightly larger. These "estimation period" methods show abnormal returns which have a significant bias compared to the size decile adjusted HPR approach of method 1 (a bias of $-35 \%$ to $-50 \%$ ).

A second conclusion in comparing methodologies is that methodologies which minimize the multiple period bias (resulting from spurious return variations between periods) show lower abnormal returns than methodologies that do not minimize this bias. As in Conrad and Kaul (1993), this study provides evidence that a positive bias from this effect is being removed from the abnormal returns when a holding period approach is utilized. This is reflected by comparing methods 1 and 7 for size adjusted methodologies and methods 2 and 8 for market adjusted methodologies. With a size adjusted methodology, the holding period return (HPR) approach decreases abnormal returns by $5.00 \%$ ( $-9.74 \%$ less $-4.74 \%$ ) and the market adjusted methodology approach decreases abnormal returns by $18.54 \%$ ( $-42.60 \%$ less $-24.06 \%$ ). Thus, the use of a HPR approach likely removes a positive bias from measurement errors over
multiple periods, which has a significant effect on the calculation of abnormal returns. The approach of using holding period returns therefore is appropriate.

A third conclusion is that the practice of using a compounded index in an HPR approach introduces a significant negative bias and should be avoided. This bias is reflected in a comparison of methods 1 and 3 in the size adjusted methodologies and methods 2 and 4 in the market adjusted methodologies. A bias of $-4.50 \%(-14.24 \%$ less $-9.74 \%$ ) is introduced in the size adjusted methods, and a bias of $\mathbf{- 1 0 . 2 2 \%}$ $(-52.82 \%$ less $-42.60 \%)$ is introduced in the market adjusted methods. Thus, it is inappropriate to use a compounded single period index in the HPR approach to calculate abnormal returns. Instead, a holding period index must be calculated for each holding period used in the test. This adds a level of complexity to the holding period size decile method, but it must be accomplished to maintain the accuracy of the methodology.

Summary. This comparison of 15 event study methodologies shows the effect of various approaches to calculating abnormal returns and leads to conclusions about appropriate adjustments. First of all, when the average sample size is significantly different from the average size in the market, an adjustment for size differences is appropriate. Also, when the average sample size changes before and after the event, an adjustment for size differences is necessary for "estimation period" methods (such as market model approaches and the comparison period mean adjusted approach).

Next, when there are multiple periods in the test period, a holding period return (HPR) approach is justified to minimize measurement error bias. Finally, the use of a compounded single period index should be avoided in the HPR approaches.

As a result of this comparison of event study methodologies applied to the data of this research, abnormal returns in this dissertation will be calculated using a size adjusted holding period return approach. These average holding period abnormal returns (AHPAR) will be used to test Hypothesis 1 and for cross-sectional regressions in testing Hypothesis 2.

## Test of Hypothesis 1

The alternate hypothesis in the first test is that there are negative abnormal returns following the introduction of options. Table 7 shows the abnormal returns for various holding periods after option introduction using the size based decile event study methodology described above. Note that the two year, four year, and five year holding periods have abnormal returns which are negative and significantly different from zero at the .05 level. Among the individual years, only year 2 is negative and significantly different from zero at the .05 level. However, the t-test used to determine statistical significance assumes that abnormal returns are normally distributed. Using the Kolmogorov D test, the hypothesis that abnormal returns are normally distributed is rejected at the .07 level for 1 -year abnormal returns and at less than the .01 level for all other holding periods. Therefore, a nonparametric test is

TABLE 7
Average Holding Period Abnormal Returns (AHPAR) Following the Introduction of Options

| Holding |  | Median | T :Mean $>=0$ | D: Normal |  | Signed Rank S:Mean $>=0$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Periods | AHPAR | HPAR | Prob $<\mathrm{T}^{1}$ | Prob $<\mathrm{D}^{2}$ | Pos:Neg ${ }^{3}$ | Prob $<S^{4}$ |
| 1 Year | -1.59\% | -2.87\% | . 133 | . 0740 | 154:204 | . 0298 |
| 2 Year | -3.88\% | -7.14\% | . 031 | . 0001 | 146:212 | . 0011 |
| 3 Year | -4.41\% | -7.81\% | . 068 | . 0001 | 159:199 | . 0034 |
| 4 Year | -7.00\% | -17.88\% | . 047 | . 0001 | 139:219 | . 0002 |
| 5 Year | -9.74\% | -23.88 \% | . 043 | . 0001 | 133:225 | . 0001 |

Median T:Mean $>=0 \mathrm{D}$ : Normal

| Year 1 | $-1.59 \%$ | $-2.87 \%$ | .133 | .0740 | $154: 204$ | .0298 |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- |
| Year 2 | $-2.05 \%$ | $-3.25 \%$ | .045 | .0033 | $156: 202$ | .0076 |
| Year 3 | $1.22 \%$ | $-1.16 \%$ | .201 | .0040 | $173: 185$ | .4144 |
| Year 4 | $-.90 \%$ | $-5.54 \%$ | .275 | .0001 | $147: 211$ | .0072 |
| Year 5 | $-1.13 \%$ | $-2.91 \%$ | .240 | .0001 | $162: 196$ | .0393 |

1. T represents a t-statistic that tests the hypothesis that the mean of the data is non-negative. Prob $<\mathrm{T}$ indicates the probability that a negative mean could occur by chance.

2 D represents the Kolmogorov D statistic that tests the hypothesis that the HPARs are normally distributed. Prob<D indicates the probability that the sample districtution differs from a normal distribution by chance.

3 Pos:Neg indicates the number of positive and negative HPARs in the sample of 358 securities.
4 The Signed Rank test is a nonparametric procedure which is equivalent to a t-test after a signed rank tranformation of the data (see Conover (1980), pages 335-336). A signed rank tranformation means the data are first sorted by absolute value with the smallest assigned a rank of 1 and the largest assigned a rank of 358 . Then the ranks are assigned the same sign as the original data. S represents a t -statistic that tests the hypothesis that the mean of the signed ranks is non-negative. Prob $<S$ indicates the probability that a negative mean could occur by chance.
appropriate for determining if the abnormal returns are statistically significant. The signed rank test indicates that the number of ranked negative abnormal returns is signicantly different from chance at less than the .05 level for all holding periods except for Year 3 following option introduction. Thus, Hypothesis 1 is rejected based on nonparametric tests, which is consistent with the theoretical model.

Table 8 provides additional insight by showing the growth in relative option open interest for various periods following the introduction of options. On average, the option open interest increases to on $3.11 \%$ of shares outstanding having contracts by the end of the first year after introduction. Surprisingly, the average option open interest does not increase or decrease significantly in any of the next four years. Since the focus of the model is on negative abnormal returns that result from the development of an options market, it is clear that the test for abnormal returns should focus primarily on the first year following introduction. The abnormal return for this one year holding period is $-1.59 \%$, and is significantly different from zero at the .05 level using the signed rank nonparametric test.

A closer look at the first year following introduction reveals that a number of securities have very little option open interest one year after introduction. Figure 11 illustrates the relative option open interest sorted by deciles and shows that approximately half of the sample has less than $1 \%$ of the shares outstanding under an option contract after one year. Table 9 indicates that arbitrarily splitting the sample

TABLE 8
Change in Relative Option Open Interest by Year After Introduction (Sample Size of 358 Securities)

Average<br>Relative Option<br>Open Interest<br>(\% of shares out.)

Change in Average
Relative Option
Open Interest $\quad \mathrm{T}:$ Mean $=0$
(\% of shares out.) $\quad$ Prob $>|T|$

| Year 1 | $3.11 \%$ | $3.11 \%$ | .0001 |
| :--- | :--- | :--- | :--- |
| Year 2 | $3.12 \%$ | $.01 \%$ | .9606 |
| Year 3 | $2.97 \%$ | $-.15 \%$ | .4434 |
| Year 4 | $2.79 \%$ | $-.18 \%$ | .2914 |
| Year 5 | $2.96 \%$ | $.17 \%$ | .5769 |

1 T represents a $t$-statistic that tests the hypothesis that the mean of the data is equal to zero. Prob $>|\mathrm{T}|$ indicates the probability that the actual change in relative option open interest could occur by chance.

FIGURE 11
Relative Option Open Interest
Sorted by Deciles


TABLE 9<br>Statistics from Splitting the Sample Above and Below 1\% Relative Option Open Interest

|  | $<1 \%$ | $>1 \%$ | Total |  |
| :--- | :---: | :---: | :---: | :---: |
|  | - |  |  |  |
| Number | 176 |  | 182 | 358 |
| Average Option Open Interest (1-Year) | $.38 \%$ | $5.75 \%$ | $3.11 \%$ |  |
|  |  |  |  |  |
| AHPAR (1-Year) | $.02 \%$ | $-3.14 \%$ | $-1.59 \%$ |  |
| T: Mean $>=0$, Prob $<\mathrm{T}^{1}$ | .496 | .075 | .133 |  |
| D:Normal, Prob $<\mathrm{D}^{2}$ | .001 | .918 | .074 |  |
| Pos:Neg ${ }^{3}$ | $74: 102$ | $80: 102$ | $154: 204$ |  |
| S: Mean $>=0$, Prob $<\mathrm{S}^{4}$ | .150 | .048 | .030 |  |

$1 \quad \mathrm{~T}$ represents a t-statistic that tests the hypothesis that the mean of the data is non-negative. Prob $<\mathrm{T}$ indicates the probability that a negative mean could occur by chance.

Pos:Neg indicates the number of positive and negative HPARs in the sample of
Pos:Neg indicat
358 securities.
D represents the Kolmogorov D statistic that tests the hypothesis that the HPARs are normally distributed. Prob $<$ D indicates the probability that the sample districtution differs from a normal distribution by chance.

The Signed Rank test is a nonparametric procedure which is equivalent to a $t$ - test after a signed rank tranformation of the data (see Conover (1980), pages 335-336). A signed rank tranformation means the data are first sorted by absolute value with the smallest assigned a rank of 1 and the largest assigned a rank of 358 . Then the ranks are assigned the same sign as the original data. $S$ represents a $t$-statistic that tests the hypothesis that the mean of the signed ranks is non-negative. Prob<S indicates the probability that a negative mean could occur by chance.
above and below a relative option open interest of $1 \%$ of shares outstanding shows the lower (below $1 \%$ ) portion of the sample has an average of $.38 \%$ while the upper portion averages $5.75 \%$ of shares outstanding. The corresponding average abnormal returns are respectively $.02 \%$ and $-3.14 \%$. Again with this subsample, Hypothesis 1 can be statistically rejected at the .05 level using the signed rank nonparametric test, which is consistent with the theoretical model. It should be recognized, however, that changes in institutional holdings and index options could potentially confound this particular test. Thus, the test of Hypothesis 2 is a more definitive test.

## Test of Hypothesis 2

The alternate hypothesis for the second test is that there is a negative relationship between abnormal returns and a change in option open interest. This hypothesis is tested by a cross-sectional regression of one-year abnormal returns against relative option open interest, the change in institutional holdings and the change in index options, as follows:

$$
\begin{equation*}
A R_{j s t}=\beta_{1}+\beta_{2}\left(\text { OPEN }_{j s t}\right)+\beta_{3}\left(I N D E X_{s t}\right)+\beta_{4}\left(I N S T_{j s t}\right)+e_{j s t} \tag{73}
\end{equation*}
$$

where $\quad \mathrm{AR}_{\mathrm{jst}}=$ abnormal returns for security j from time s to time t , $\mathrm{OPEN}_{\mathrm{jst}}=$ change in relative option open interest from time $s$ to $t$, $\mathrm{INDEX}_{\mathrm{st}}=$ change in index option open interest from time s to t,

| $\mathrm{INST}_{\mathrm{jst}}$ | $=$ change in institutional holdings as a percent of shares |
| :--- | :--- |
|  | outstanding from time s to time t, |
| $\beta_{1}=$ | regression intercept, |
| $\beta_{2}=$ | regression coefficient for the option open interest term, |
| $\beta_{3}=$ | regression coefficient for the institutional holdings term, |
| $\beta_{4}=$ | regression coefficient for the index options term, and |
| $\mathrm{e}_{\mathrm{jst}}=$ | regression error term. |

Table 10 shows the results of this regression (for completeness, Appendix E shows the results from cross-sectional regressions using abnormal returns for time periods other than one year). In all four regressions in Table 10, the coefficient on the change in relative option open interest (OPEN) and institutional holdings (INST) are significantly different from zero. Furthermore, the coefficient on the relative option open interest term is negative as predicted by the theoretical model. Likewise, the coefficient on the relative institutional holdings term is positive and significant, as anticipated. The coefficient on the change in index options, however, is not significant. The overall Fstatistic, which tests the hypothesis that the coefficients on all independent variables are jointly different from zero, is 6.844 for regression 3 , which is significant at the .0012 level. Thus, Hypothesis 2 is rejected, which is consistent with the theoretical model developed in this dissertation.

TABLE 10
1 Year Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 1-year time period following option introduction (month 1 to month 12) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST | F-Statistic ${ }^{\text {b }}$ |
| (1) | $\begin{gathered} -.004210 \\ (.7797) \end{gathered}$ | $\begin{gathered} -.375482 \\ (.0199) \end{gathered}$ |  |  | $\begin{gathered} 5.471 \\ (.0199) \end{gathered}$ |
| (2) | $\begin{gathered} -.004100 \\ (.7894) \end{gathered}$ | $\begin{gathered} -.376071 \\ (.0204) \end{gathered}$ | $\begin{aligned} & .001912 \\ & (.9696) \end{aligned}$ |  | $\begin{gathered} 2.729 \\ (.0667) \end{gathered}$ |
| (3) | $\begin{gathered} -.016943 \\ (.2767) \end{gathered}$ | $\begin{gathered} -.379829 \\ (.0174) \end{gathered}$ |  | $\begin{aligned} & .438422 \\ & (.0047) \end{aligned}$ | $\begin{gathered} 6.844 \\ (.0012) \end{gathered}$ |
| (4) | $\begin{gathered} -.016848 \\ (.2880) \end{gathered}$ | $\begin{gathered} -.380338 \\ (.0179) \end{gathered}$ | $\begin{aligned} & .001652 \\ & (.9734) \end{aligned}$ | $\begin{aligned} & .438413 \\ & (.0047) \end{aligned}$ | $\begin{gathered} 4.550 \\ (.0038) \end{gathered}$ |
| (5) | $\begin{gathered} -.016321 \\ (.2611) \end{gathered}$ |  | $\begin{gathered} -.009206 \\ (-.8544) \end{gathered}$ |  | $\begin{gathered} .034 \\ (.8544) \end{gathered}$ |
| (6) | $\begin{gathered} -.028644 \\ (.0547) \end{gathered}$ |  |  | $\begin{aligned} & .434888 \\ & (.0053) \end{aligned}$ | $\begin{gathered} 7.874 \\ (.0053) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

## Test of Hypothesis 3

The alternate hypothesis for the third test is that there is a positive relationship between a change in short interest and a change in option open interest. This hypothesis is tested by a regression of the change in relative short interest against the change in relative option open interest, index option open interest, and interest rates, as follows:

$$
\begin{equation*}
\operatorname{SHORT}_{j s t}=\alpha_{1}+\alpha_{2}\left(\text { OPEN }_{j s t}\right)+\alpha_{3}\left(I N D E X_{s t}\right)+\alpha_{4} R_{s t}+v_{j s t} \tag{74}
\end{equation*}
$$

where SHORT $_{\text {jst }}=$ change in relative short interest from time $s$ to time $t$,
$\mathrm{OPEN}_{\mathrm{jst}}=$ change in relative option open interest from time s to t ,
INDEX $_{\text {st }}=$ change in index option open interest from time s to t ,
$\mathrm{R}_{\mathrm{st}}=\quad=$ change in interest rates from time s to time t ,
$\alpha_{1} \quad=\quad$ regression intercept,
$\alpha_{2} \quad=$ regression coefficient for the option open interest term,
$\alpha_{3}=\quad$ regression coefficient for the index option term,
$\alpha_{4} \quad=\quad$ regression coefficient for the interest rate term, and
$\mathrm{v}_{\mathrm{jst}} \quad=\quad$ regression error term over time s to time t .

Table 11 shows the results from these regressions (again for completeness, Appendix E shows the results from regressions for time periods other than one year). In all four regressions, only the coefficient on the change in relative option open interest is significantly different from zero, and this coefficient is strongly significant

TABLE 11
1 Year Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL | F-Statistic ${ }^{\text {b }}$ |
| (1) | $\begin{gathered} -.001450 \\ (.0348) \end{gathered}$ | $\begin{aligned} & .074425 \\ & (.0001) \end{aligned}$ |  |  | $\begin{aligned} & 103.890 \\ & (.0001) \end{aligned}$ |
| (2) | $\begin{gathered} -.001505 \\ (.0316) \end{gathered}$ | $\begin{aligned} & .074723 \\ & (.0001) \end{aligned}$ | $\begin{gathered} -.000970 \\ (.6702) \end{gathered}$ |  | $\begin{aligned} & 51.916 \\ & (.0001) \end{aligned}$ |
| (3) | $\begin{gathered} -.001441 \\ (.0373) \end{gathered}$ | $\begin{aligned} & .074484 \\ & (.0001) \end{aligned}$ |  | $\begin{aligned} & .0028256 \\ & (.9091) \end{aligned}$ | $\begin{aligned} & 51.807 \\ & (.0001) \end{aligned}$ |
| (4) | $\begin{gathered} -.001496 \\ (.0337) \end{gathered}$ | $\begin{aligned} & .074812 \\ & (.0001) \end{aligned}$ | $\begin{gathered} -.001000 \\ (.6623) \end{gathered}$ | $\begin{gathered} -.0037612 \\ (.8797) \end{gathered}$ | $\begin{aligned} & 34.523 \\ & (.0001) \end{aligned}$ |
| (5) | $\begin{aligned} & .000923 \\ & (.2155) \end{aligned}$ |  | $\begin{aligned} & .001239 \\ & (.6303) \end{aligned}$ |  | $\begin{gathered} .232 \\ (.6303) \end{gathered}$ |
| (6) | $\begin{aligned} & .000808 \\ & (.2763) \end{aligned}$ |  |  | $\begin{aligned} & .014962 \\ & (.5931) \end{aligned}$ | $\begin{gathered} .286 \\ (.5931) \end{gathered}$ |

${ }^{2}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.
at less than the .0001 level. Thus, Hypothesis 3 is rejected, which is consistent with the theoretical model.

## Empirical Test of Subsample

As corroborating evidence, the same two tests on Hypotheses 2 and 3 are run on subsamples of the data. Subsamples include abnormal returns after eliminating the bottom $10 \%$ of the observations which have close to zero option open interest after one year (a sample size of 322) and abnormal returns from observations with relative option open interest above $1 \%$ (a sample size of 182 ). The results are quite similar to the previous tests on the full sample, and are shown in Tables $30,31,32$ and 33 in Appendix F. As a point of comparison, Tables 34 and 35 show these same two tests for observations with relative option open interest below $1 \%$ (a sample size of 176). None of the significant results from previous tests appear in this subsample. Again this points to the importance of option open interest in explaining the negative abnormal returns from the event study.

## Summary

In summary, results of the three empirical tests show significant evidence in support of the theoretical model developed in this dissertation. The first test finds negative abnormal returns that are significantly different from zero using a nonparametric test. The focus is on abnormal returns over a one year period following option introduction because average relative option open interest increases
only in the first year. The calculation of abnormal returns uses a size adjusted holding period return approach. This event study methodology is shown to be more appropriate than various other event study methodologies that do not adjust for size differences or use holding period returns.

The second empirical test uses a cross-sectional regression to show that the abnormal returns are significantly related to changes in relative option open interest and institutional holdings. The coefficient on the change in relative option open interest variable is negative and significantly different from zero, which is consistent with the theoretical model. This provides support for the mechanism in the model that creates an increase in the supply of stock when options are used to circumvent short sale restrictions. The coefficient on the change in relative institutional holdings is positive and significantly different from zero, as anticipated. The fact that these two effects exist also provides evidence consistent with a long term downward sloping demand curve.

The last empirical test also provides evidence linking the change in option open interest with the change in short interest. In this test, a regression of the change in relative option open interest is shown to be positive and significantly different from zero, which again is consistent with the theoretical model. This result provides support for the linkage in the model that an increase in option open interest for a security directly increases the short interest.

In summary, the empirical data are shown to be consistent with the theoretical model developed in this dissertation. These results thus lead to the conclusion that options are used to circumvent short sale restrictions.

The research in this dissertation leads to several conclusions regarding the effect on the price of the underlying stock from the introduction of an options market. The major contributions of this research are summarized in the following paragraphs along with policy implications and recommendations for future research.

First of all, this dissertation develops a theoretical model which demonstrates a linkage between the market for options and the price of the underlying stock. The model shows that an increase in the market for options on a particular stock has a negative effect on the price of that stock. This occurs because short sale restrictions can be circumvented through the use of options. From a utility preference approach, investors who are restricted from selling stock short can reach their optimal level of utility by writing call options or purchasing put options. The market participants, who take an opposite position in these options, hedge their positions by a combination of buying bonds and selling stock short (market makers and specialists have fewer restrictions on short sales). Additional short sales, in effect, shift the supply curve to the right, which places downward pressure on the price of the underlying stock. Thus, according to the model, an increase in an options market for a stock would have a negative effect on the price of the underlying stock.

Empirical tests of the theoretical model in this dissertation provide evidence consistent with the model. First of all, average abnormal returns of $-1.59 \%$ are
measured over the one year period following option introduction. These abnormal returns are measured using a size decile adjusted event study methodology, and are statistically significant at the .05 level using a nonparametric test. A more definitive test is a regression of abnormal returns which includes explanatory variables external to the model. Cross sectional regressions show that relative option open interest and the change in relative institutional holdings are significant variables in explaining abnormal returns. The relative option open interest is negatively related, while the change in institutional holdings is positively related. Furthermore, a change in short interest is shown to be significantly related to a change in option open interest as predicted by the model. These empirical tests provide evidence consistent with the theoretical model developed in this dissertation. Thus, the model developed in this dissertation provides a new approach for explaining the relationship between short interest, option open interest, and the price of the underlying stock.

A second contribution from the research in this dissertation is to provide further support for the Dimson and Marsh event study methodology, which adjusts for differences in size. This methodology is compared and contrasted to other event study methodologies to illustrate the impact of using a technique which adjusts for size differences and which uses holding period returns. The evidence from the research in this dissertation shows that an adjustment for differences in size has a significant impact on the measurement of abnormal returns when the average capitalization is significantly different from the average market size and when the relative
capitalization of the sample changes over time. The evidence from this research also provides support for the contention of Conrad and Kaul (1993) that cumulating single period abnormal returns introduces a positive bias. Consistent with Conrad and Kaul (1993), in this dissertation the use of holding period returns shows the abnormal returns are lower compared to the cumulative approach.

A final contribution of the research in this dissertation is evidence that the long term demand curve for an individual stock is downward sloping. The significance of explanatory variables in the cross sectional regression of abnormal returns would not occur if the demand curve were horizontal. Therefore, the results from this research imply that the dynamics of supply and demand are useful in explaining changes in returns, at least over a one year time period.

Policy implications from the research in this dissertation relate to the market inefficiencies caused by restrictions on short sales. The theoretical model shows that the negative abnormal returns from the introduction of options are associated with short sale restrictions. Regulators now have available additional insight on improving the efficiency of the market. The market can be made more efficient by decreasing the level of restrictions on short sales or by the availability of options to circumvent these restrictions. At the same time, regulators should be aware that removing the market inefficiencies has a negative effect on the price of the underlying stock.

Finally, the research in this dissertation points to the need for additional research in several areas. The model developed in this dissertation may be useful in explaining other market dynamics involving options, such as the role of options in the 1987 stock market decline. This model can also be extended to improve on option pricing models, specifically the binomial option pricing model. Inclusion of a feedback feature that allows the level of option open interest to impact the stock price (which in turn affects the option price), as developed in this dissertation, would refine the binomial option pricing model.

In summary, the research in this dissertation has developed a theoretical model that indicates a linkage between the introduction of options and the price of the underlying stock. Empirical tests show there is a negative effect on the underlying stock following the introduction of individual options, which is directly related to the level of option open interest. Thus, these empirical tests are consistent with the theoretical model.

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Determining the Scenario A Optimal Number of Shares and Critical Value of $q_{i}$

The optimal number of shares for an investor in Scenario A can be determined by finding the maximum date 2 expected utility. Date 2 expected utility can be calculated by

$$
\begin{equation*}
E_{i}\left(U_{2}\right)=q_{i} U_{u}+\left(1-q_{i}\right) U_{d} \tag{75}
\end{equation*}
$$

At date 1 , the investor buys $n_{i}$ shares of stock at stock price $S_{1}$. Cash left at date 1 after this transaction is

$$
\begin{equation*}
\operatorname{Cash}_{1}=\frac{W_{E}}{R}-n_{i} S_{1} \tag{76}
\end{equation*}
$$

Invested at the risk-free rate of interest, this cash becomes at date 2

$$
\begin{equation*}
\text { Cash }_{2}=W_{E}-n_{i} R S_{1} \tag{77}
\end{equation*}
$$

If the stock price is $S_{d}$ at date 2 the wealth level is $W_{d}$, and if the stock price becomes $S_{u}$ the wealth level is $W_{u}$, as follows:

$$
\begin{align*}
& W_{u}=W_{E}-n_{i} R S_{1}+n_{i} S_{u} \\
& W_{d}=W_{E}-n_{i} R S_{1}+n_{i} S_{d} \tag{78}
\end{align*}
$$

From these two wealth levels and a given power function utility preference $\left(\mathrm{U}=-\mathrm{W}^{-1}\right), \mathrm{U}_{\mathrm{u}}$ and $\mathrm{U}_{\mathrm{d}}$ can be calculated as

$$
\begin{align*}
& U_{u}=-\left(W_{E}-n_{i} R S_{1}+n_{i} S_{u}\right)^{-1} \\
& U_{d}=-\left(W_{E}-n_{i} R S_{1}+n_{i} S_{d}\right)^{-1} \tag{79}
\end{align*}
$$

The date 2 expected utility is

$$
\begin{equation*}
E_{i}\left(U_{2}\right)=-q_{i}\left(W_{E}-n_{i} R S_{1}+n_{i} S_{u}\right)^{-1}-\left(1-q_{i}\right)\left(W_{E}-n_{i} R S_{1}+n_{i} S_{d}\right)^{-1} \tag{80}
\end{equation*}
$$

To find the number of shares that yields the maximum expected utility, we take the first derivative with respect to the number of shares, as follows:

$$
\begin{equation*}
\frac{\partial E_{f}\left(U_{2}\right)}{\partial n_{i}}=G_{i}\left[W_{E}+n_{i}\left(-R S_{1}+S_{u}\right)\right]^{-2}\left(-R S_{1}+S_{u}\right)+\left(1-q_{i}\right)\left[W_{R}+n_{i}\left(-R S_{1}+S_{d}\right)\right]^{-2}\left(-R S_{1}+S_{d}\right) \tag{81}
\end{equation*}
$$

Setting this derivative equal to zero yields

$$
\begin{equation*}
\frac{q_{i}\left(-R S_{1}+S_{u}\right)}{\left[W_{E}+n_{i}^{*}\left(-R S_{1}+S_{u}\right)\right]^{2}}+\frac{\left(1-q_{i}\right)\left(-R S_{1}+S_{d}\right)}{\left[W_{E}+n_{i}\left(-R S_{1}+S_{d}\right)\right]^{2}}=0 \tag{82}
\end{equation*}
$$

Multiplying by the product of the denominators yields

$$
\begin{aligned}
q_{i}\left(-R S_{1}+S_{u}\right) & {\left[W_{E}^{2}+2 W_{E} n_{i}^{*}\left(-R S_{1}+S_{d}\right)+\left(n_{i}^{*}\right)^{2}\left(-R S_{1}+S_{d}\right)^{2}\right]+(83) } \\
\left(1-q_{i}\right)\left(-R S_{1}+S_{d}\right) & {\left[W_{E}^{2}+2 W_{E} n_{i}^{*}\left(-R S_{1}+S_{u}\right)+\left(n_{i}^{*}\right)^{2}\left(-R S_{1}+S_{u}\right)^{2}\right]=0 }
\end{aligned}
$$

Collecting terms,

$$
\begin{align*}
& \left(n_{i}^{*}\right)^{2}\left[q_{i}\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)^{2}+\left(1-q_{i}\right)\left(-R S_{1}+S_{d}\right)\left(-R S_{1}+S_{u}\right)^{2}\right]+ \\
& n_{i}^{*} \quad\left[q_{i} 2 W_{E}\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)+\left(1-q_{i}\right) 2 W_{E}\left(-R S_{1}+S_{d}\right)\left(-R S_{1}+S_{u}\right)\right]+ \\
& \quad\left[q_{i} W_{E}^{2}\left(-R S_{1}+S_{u}\right)+\left(1-q_{i}\right) W_{E}^{2}\left(-R S_{1}+S_{d}\right)\right]=0 \tag{84}
\end{align*}
$$

Simplifying,

$$
\begin{gather*}
\left(n_{i}^{*}\right)^{2}\left[\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)\left[-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]+\right. \\
n_{i}^{*}\left[2 W_{E}\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)\right]+  \tag{85}\\
{\left[W_{E}^{2}\left(-2 R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}\right]=0\right.}
\end{gather*}
$$

Solving this quadratic equation for the roots of $\mathrm{n}_{\mathrm{i}}$, with only the positive root meaningful,

$$
\begin{equation*}
n_{i}^{*}=\frac{-2 W_{E}\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)+\sqrt{Z}}{2\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)\left[-R S_{1}+G_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]} \tag{86}
\end{equation*}
$$

where

$$
\begin{gathered}
Z=\left[-2 W_{E}\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)\right]^{2}- \\
4 W_{E}^{2}\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)\left[-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]\left[-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}\right]
\end{gathered}
$$

Factoring out $\left(-\mathrm{RS}_{1}+\mathrm{S}_{\mathrm{u}}\right)\left(-\mathrm{RS}_{1}+\mathrm{S}_{\mathrm{d}}\right)$,

$$
n_{i}^{*}=\frac{-2 W_{E}+\sqrt{4 W_{E}^{2}-4 W_{E}^{2} \frac{\left[-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]\left[-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}\right]}{\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)}}}{2\left[-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]} \text { (88) }
$$

Factoring $4 W_{E}{ }^{2}$ out of the square root radical and dividing all terms by 2 yields

$$
\begin{equation*}
n_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{\left[-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]\left[-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}\right]}{\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)}}}{\left[-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}\right]} \tag{89}
\end{equation*}
$$

Substituting for some of the expressions in the square root radical yields

$$
\begin{equation*}
n_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{A B}{C}}}{A} \tag{90}
\end{equation*}
$$

where

$$
\begin{gather*}
A=-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u} \\
B=-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}  \tag{91}\\
C=\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)
\end{gather*}
$$

To determine which values of $\mathrm{q}_{\mathrm{i}}$ cause $\mathrm{n}_{\mathrm{i}}{ }^{*}$ to be negative, the signs of $\mathrm{A}, \mathrm{B}$, and C can be analyzed. Note first that C must always be negative. This is true because the relationship $S_{d}<\mathrm{RS}_{1}<\mathrm{S}_{\mathbf{u}}$ must be true for all rational investors. Thus ( $-\mathrm{RS} \mathrm{I}_{1}+\mathrm{S}_{\mathrm{u}}$ ) must be positive, and ( $-\mathrm{RS}_{1}+\mathrm{S}_{\mathrm{d}}$ ) must be negative, requiring C to always be negative. Next, observe that if $A>0$, then $B$ must be negative for $n_{i}{ }^{*}<0$. This combination causes the square root term to be less than 1 and the numerator negative while the denominator is positive. Now observe that if $A<0$, then $B$ must again be negative for $\mathrm{n}_{\mathrm{i}}{ }^{*}<0$. This combination causes the square root term to be greater than 1 , and the numerator positive while the denominator is negative. In either case, $\mathrm{n}_{\mathrm{i}}{ }^{*}$ is negative only when $B<0$, or

$$
\begin{equation*}
-R S_{1}+g_{i}\left(S_{u}-S_{d}\right)+S_{d}<0 \tag{92}
\end{equation*}
$$

Solving for $q_{i}$ yields

$$
\begin{equation*}
q_{i}<\frac{R S_{1}-S_{d}}{S_{u}-S_{d}} \tag{93}
\end{equation*}
$$

This is a necessary and sufficient condition for $\mathrm{n}_{\mathrm{i}}{ }^{*}<0$.

The value of $q_{i}$ for which an investor is indifferent between a long and short
position in the stock can be defined as the critical value of $q_{i}$, or $q_{i}$ (crit), as follows:

$$
\begin{equation*}
q_{i(c r i t)}=\frac{R S_{1}-S_{d}}{S_{u}-S_{d}} \tag{94}
\end{equation*}
$$

Comparative static derivatives can then be determined for this critical value of $q_{i}$.

## Partial Derivative with respect to $S_{1}$

Taking the first partial derivative of $q_{i(c r i t)}$ with respect to $S_{1}$ yields

$$
\begin{equation*}
\frac{\partial q_{i(c r i t)}}{\partial S_{1}}=R \tag{95}
\end{equation*}
$$

Since $R$ is always positive, this comparative static derivative is also positive. As $\mathbf{S}_{1}$ increases, the threshold level of $q_{i}$ for which $n_{i}^{*}$ is negative also increases, which provides a driving force for additional short sales. In this way, at the critical value of $q_{i}$, an increase in $S_{1}$ decreases the optimal number of shares, $n_{i}{ }^{*}$. Thus, at the critical value of $q_{i}$, the partial derivative of $n_{i}^{*}$ with respect to $S_{1}$ is negative. This is illustrated graphically in Figure 12.

## Partial Derivative with Respect to $\underline{R}$

Taking the first partial derivative of $q_{i(c r i t)}$ with respect to $R$ yields

$$
\begin{equation*}
\frac{\partial q_{i(c r i t)}}{\partial R}=S_{1} \tag{96}
\end{equation*}
$$

Since $S_{1}$ is always positive, this comparative static derivative is always

FIGURE 12
The Effect of Increasing $S_{1}$ on the Optimal Number of Shares, $n_{i}{ }^{*}$, at $q_{i(\text { crit })}$

positive. As in the previous partial derivative, an increase in $R$ increase $q_{i(c r i t)}$, which in turn decreases $n_{i}{ }^{*}$. Thus, at the critical value of $q_{i}$, the partial derivative of $n_{i}{ }^{*}$ with respect to $R$ is negative.

The optimal number of call options for an investor in Scenario C can be determined by finding the maximum date 2 expected utility. Date 2 expected utility can be calculated by

$$
\begin{equation*}
E_{i}\left(U_{2}\right)=q_{i} U_{u}+\left(1-q_{i}\right) U_{d} \tag{97}
\end{equation*}
$$

Let $o_{i}$ equal the number of call options to be purchased by an investor (if $o_{i}<0$, the investor writes options). At date 1 , the investor writes options on $\mathbf{o}_{\mathbf{i}}$ shares of stock with a call price of $\mathbf{C}_{1}$. Cash left at date 1 after this transaction is

$$
\begin{equation*}
\operatorname{Cash}_{1}=\frac{W_{E}}{R}-o_{i} C_{1} \tag{98}
\end{equation*}
$$

Invested at the risk-free rate of interest, at date 2 this cash becomes

$$
\begin{equation*}
\operatorname{Cash}_{2}=W_{E}-o_{i} R C_{1} \tag{99}
\end{equation*}
$$

If the stock price is $S_{d}$ at date 2 the wealth level is $W_{d}$, and if the stock price becomes $S_{u}$ the wealth level is $W_{u}$, as follows:

$$
\begin{align*}
W_{u} & =W_{E}-o_{i} R C_{1}+o_{i}\left(S_{u}-K\right)  \tag{100}\\
W_{N} & =W_{n}-0 . R C_{1}
\end{align*}
$$

From these two wealth levels and a given power function utility preference $\left(\mathrm{U}=-\mathrm{W}^{-1}\right), \mathrm{U}_{\mathrm{u}}$ and $\mathrm{U}_{\mathrm{d}}$ can be calculated as

$$
\begin{gather*}
U_{u}=-\left[W_{E}-O_{i}\left(R C_{1}-S_{u}+K\right)\right]^{-1}  \tag{101}\\
U_{d}=-\left[W_{B}-O_{i} R C_{1}\right]^{-1}
\end{gather*}
$$

The date 2 expected utility is

$$
\begin{equation*}
E_{i}\left(U_{2}\right)=-G_{i}\left[W_{E}-O_{i}\left(R C_{1}-S_{u}+K\right)\right]^{-1}-\left(1-G_{i}\right)\left[W_{E}-O_{i} R C_{1}\right]^{-1} \tag{102}
\end{equation*}
$$

To find the number of options that yields the maximum expected utility, we take the first derivative with respect to the number of options, as follows:

$$
\frac{\partial E_{1}\left(U_{2}\right)}{\partial o_{1}}=q_{i}\left[W_{B}-O_{1}\left(R C_{1}-S_{\mathrm{u}}+K\right)\right]^{-2}\left(-R C_{1}+S_{\mathrm{u}}-K\right)+\left(1-q_{i}\right)\left[W_{B}-o_{1} R C_{1}\right]^{-2}\left(-R C_{1}(103)\right.
$$

Setting this derivative equal to zero yields

$$
\begin{equation*}
\frac{q_{i}\left(-R C_{1}+S_{u}-K\right)}{\left[W_{E}-O_{i}^{*}\left(R C_{1}-S_{u}+K\right)\right]^{2}}+\frac{\left(1-q_{i}\right)\left(-R C_{1}\right)}{\left[W_{E}-O_{i}{ }^{*} R C_{1}\right]^{2}}=0 \tag{104}
\end{equation*}
$$

Multiplying by the product of the denominators yields

$$
\begin{gathered}
\left.q_{i}\left(-R C_{1}+S_{u}-K\right) \quad\left[W_{E}^{2}-2 W_{E} O_{i}^{*} R C_{1}+\left(O_{i}^{*}\right)^{2}\left(R C_{1}\right)^{2}\right]+\stackrel{+}{(10 \xi}\right) \\
\left(1-q_{i}\right)\left(-R C_{1}\right)\left[W_{E}^{2}-2 W_{E} O_{i}^{*}\left(R C_{1}-S_{u}+K\right)+\left(o_{i}^{*}\right)^{2}\left(R C_{1}-S_{u}+K\right)^{2}\right]=0
\end{gathered}
$$

Collecting terms,

$$
\begin{gathered}
\left(O_{i}^{*}\right)^{2}\left[q_{i}\left(-R C_{1}+S_{u}-K\right)\left(R C_{1}\right)^{2}+\left(1-q_{i}\right)\left(R C_{1}-S_{u}+K\right)^{2}\left(-R C_{1}\right)\right]+ \\
O_{i}^{*}\left[q_{i} 2 W_{E} R C_{1}\left(R C_{1}-S_{u}+K\right)+\left(1-q_{i}\right) 2 W_{E}\left(R C_{1}\right)\left(R C_{1}-S_{u}+K\right)\right] \\
{\left[q_{i} W_{B}^{2}\left(-R C_{1}+S_{u}-K\right)+\left(1-q_{i}\right) W_{E}^{2}\left(-R C_{1}\right)\right]=0}
\end{gathered}
$$

## Simplifying,

$$
\begin{gather*}
\left(O_{i}^{*}\right)^{2} R C_{1}\left(-R C_{1}+S_{u}-K\right)\left[R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)\right]+ \\
O_{i}^{*}\left[2 W_{E}\left(R C_{1}\right)\left(R C_{1}-S_{u}+K\right)\right]+  \tag{107}\\
W_{E}^{2}\left[\left(-R C_{1}\right)+q_{i}\left(S_{u}-K\right)\right]=0
\end{gather*}
$$

Solving this quadratic equation for the roots of $\mathrm{o}_{\mathrm{i}}{ }^{*}$, with only the positive root meaningful,

$$
\begin{equation*}
o_{i}^{*}=\frac{-2 W_{E}\left(R C_{1}\right)\left(R C_{1}-S_{u}+K\right)+\sqrt{Z}}{2\left(R C_{1}\right)\left(-R C_{1}+S_{u}-K\right)\left[R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)\right]} \tag{108}
\end{equation*}
$$

where

$$
\begin{gather*}
Z=\left[2 W_{E}\left(R C_{1}\right)\left(R C_{1}-S_{u}+K\right)\right]^{2}-  \tag{109}\\
4 W_{E}^{2}\left[-R C_{1}+q_{i}\left(S_{u}-K\right)\right]\left[R C_{1}\left(-R C_{1}+S_{u}-K\right)\right]\left[R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)\right]
\end{gather*}
$$

Factoring out $2 \mathrm{RC}_{1}\left(\mathrm{RC}_{1}-\mathrm{S}_{\mathbf{u}}+\mathrm{K}\right)$,

$$
\begin{equation*}
o_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{\left[-R C_{1}+q_{i}\left(S_{u}-K\right)\right]\left[R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)\right]}{\left(R C_{1}\right)\left(-R C_{1}+S_{u}-K\right)}}}{\left[-R C_{1}+\left(1-q_{i}\right)\left(S_{u}-K\right)\right]} \tag{110}
\end{equation*}
$$

Substituting for some of the expressions in the square root radical yields

$$
\begin{equation*}
o_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{E F}{G}}}{-F} \tag{111}
\end{equation*}
$$

where

$$
\begin{align*}
E & =-R C_{1}+q_{i}\left(S_{u}-K\right) \\
F & =R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)  \tag{112}\\
G & =R C_{1}\left(-R C_{1}+S_{u}-K\right)
\end{align*}
$$

To determine the conditions which cause $\mathrm{o}_{\mathrm{i}}{ }^{*}$ to be negative (writing calls), the signs of E, F, and G can be analyzed. Note first that G must always be positive. This is true because the maximum value a call can have is $\left(S_{u}-K\right)$ if the stock price goes up
at date 2. If it is certain that the price will increase, the maximum an investor would pay for a call would be

$$
\begin{equation*}
C_{1}=\frac{S_{u}-K}{R} \tag{113}
\end{equation*}
$$

or, multiplying by $\mathbf{R}$,

$$
\begin{equation*}
R C_{1}=S_{u}-K \tag{114}
\end{equation*}
$$

If there is a finite chance that the price could go down to $S_{d}$ at date 2, a rational investor would value a call option for less than Equation 113, as follows:

$$
\begin{equation*}
C_{1}<\frac{S_{u}-K}{R} \tag{115}
\end{equation*}
$$

or, multiplying by R ,

$$
\begin{equation*}
R C_{1}<S_{u}-K \tag{116}
\end{equation*}
$$

Subtracting $\mathrm{RC}_{1}$ from both sides yields

$$
\begin{equation*}
-R C_{1}+S_{u}-K>0 \tag{117}
\end{equation*}
$$

Since $R C_{1}$ is always positive and $\left(-R C_{1}+S_{u}-K\right)$ is always positive, $G$ must always be positive. Next, observe that if $\mathrm{F}>0$, then E must be negative for $\mathrm{o}_{\mathrm{i}}{ }^{*}$ to be negative. This combination causes the square root term to be greater than 1 and the numerator to be positive while the denominator is negative. Now observe that if $\mathrm{F}<0$, then E must again be negative for $\mathrm{o}_{\mathrm{i}}{ }^{*}$ to be negative. This combination causes the square root term to be less than 1 , and the numerator is negative while the denominator is positive.

In either case, $\mathrm{o}_{\mathrm{i}}{ }^{*}$ is negative only when $\mathrm{E}<0$, or

$$
\begin{equation*}
-R C_{1}+q_{i}\left(S_{u}-K\right)<0 \tag{118}
\end{equation*}
$$

Cox and Rubenstein (1979) also show a hedge portfolio in the one period binomial option pricing model to be

$$
\begin{equation*}
C_{1}=\Delta S_{1}+B \tag{119}
\end{equation*}
$$

where

$$
\begin{align*}
\Delta & =\frac{S_{u}-C_{d}}{(u-d) S_{1}}  \tag{120}\\
B & =\frac{u C_{d}-d C_{u}}{(u-d) R} \tag{121}
\end{align*}
$$

and
$\Delta=$ number of shares of stock in a hedge portfolio,
$B=$ amount of borrowing of the risk free asset in a hedge portfolio,
$C_{u}=$ value of a call if $S_{u}$ occurs, or $\left(S_{u}-K\right)$,
$C_{d}=$ value of a call if $S_{d}$ occurs (zero),
$\mathbf{u}=\mathbf{S}_{\mathbf{u}} / \mathrm{S}_{1}$,
$\mathrm{d}=\mathrm{S}_{\mathrm{d}} / \mathrm{S}_{1}$, and
$\mathbf{R}=1$ plus the interest rate.

Substituting the values of $u, d, C_{u}$, and $C_{d}$ into Equation 119 yields

$$
\begin{equation*}
C_{1}=\frac{\left(S_{u}-K-0\right)}{\left(\frac{S_{u}}{S_{1}}-\frac{S_{d}}{S_{1}}\right) S_{1}} S_{1}+\frac{\frac{S_{u}}{S_{1}}(0)-\frac{S_{d}}{S_{1}}\left(S_{u}-K\right)}{\left(\frac{S_{u}}{S_{1}}-\frac{S_{d}}{S_{1}}\right)(R)} \tag{122}
\end{equation*}
$$

Simplifying yields

$$
\begin{equation*}
C_{1}=\frac{\left(S_{u}-K\right)}{\left(S_{u}-S_{d}\right)} S_{1}-\frac{S_{d}\left(S_{u}-K\right)}{R\left(S_{u}-S_{d}\right)} \tag{123}
\end{equation*}
$$

Factoring out $\left(S_{1}-S_{d} / R\right)$ yields

$$
\begin{equation*}
C_{1}=\left[S_{1}-\frac{S_{d}}{R}\right] \frac{\left(S_{u}-K\right)}{\left(S_{u}-S_{d}\right)} \tag{194}
\end{equation*}
$$

Substituting this value into Equation 118 yields

$$
\begin{equation*}
\left[-R S_{1}+S_{d}\right] \frac{\left(S_{u}-K\right)}{\left(S_{u}-S_{d}\right)}+q\left(S_{u}-K\right)<0 \tag{125}
\end{equation*}
$$

Dividing all terms by $\left(\mathrm{S}_{\mathrm{u}}-\mathrm{K}\right)$ yields

$$
\begin{equation*}
\frac{-R S_{1}+S_{d}}{S_{u}-S_{d}}+q<0 \tag{126}
\end{equation*}
$$

Adding the first term to both sides yields

$$
\begin{equation*}
q_{i}<\frac{R S_{1}-S_{d}}{S_{u}-S_{d}} \tag{127}
\end{equation*}
$$

This is the same necessary and sufficient condition for $\mathrm{n}_{\mathrm{i}}{ }^{*}<0$. Thus, an investor would only want to write call options under the very same conditions he would desire to sell the stock short. This is consistent with the proposition in this dissertation that
options are used to circumvent short sale restrictions.

As in Appendix A, the value of $q_{i}$ for which an investor is indifferent between a long and short position in a call option can be defined as the critical value of $q_{i}$, or $q_{i(c r i t)}$. Setting Equation 118 equal to zero and solving for $q_{i}$ at the critical value yields

$$
\begin{equation*}
q_{i(c r i t)}=\frac{R C_{1}}{S_{u}-K} \tag{128}
\end{equation*}
$$

Comparative static derivatives can then be determined for this critical value of $q_{i}$.

## $\underline{\text { Partial }}$ Derivative with respect to $C_{1}$

Taking the first partial derivative of $q_{i(c r i t)}$ with respect to $C_{1}$ yields

$$
\begin{equation*}
\frac{\partial q_{i(c r i t)}}{\partial C_{1}}=\frac{R}{S_{u}-K} \tag{129}
\end{equation*}
$$

Since $\mathbf{R}$ is always positive, this comparative static derivative is also positive. As $\mathbf{C}_{1}$ increases, the threshold level of $q_{i}$ for which $o_{i}{ }^{*}$ is negative also increases, which provides incentive for writing additional call options. In this way, at the critical value of $q_{i}$, an increase in $C_{1}$ decreases the optimal number of call options, $o_{i}{ }^{*}$. Thus, at the critical value of $q_{i}$, the partial derivative of $o_{i}{ }^{*}$ with respect to $C_{1}$ is negative, as follows:

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial C_{1}}<0 \tag{130}
\end{equation*}
$$

This analysis parallels the derivation of comparative static derivatives in Appendix A.

## Partial Derivative with Respect to $K$

Taking the first partial derivative of $q_{i(c r i t)}$ with respect to $K$ yields

$$
\begin{equation*}
\frac{\partial q_{i(c r i t)}}{\partial K}=\frac{-R C_{1}}{S_{u}-K} \tag{131}
\end{equation*}
$$

Since $C_{1}$ and $R$ are always positive, this comparative static derivative is also negative. Thus, an increase in $K$ decreases $q_{i(c r i t)}$, which in turn increases $o_{i}{ }^{*}$. Therefore, at the critical value of $q_{i}$, the partial derivative of $o_{i}{ }^{*}$ with respect to $K$ is positive, as follows:

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial K}>0 \tag{132}
\end{equation*}
$$

## Relationship to $\mathbf{S}_{1}$

The comparative static derivative in Equation 130 can also be related to $S_{1}$. Equation 119 shows that $C_{1}$ and $S_{1}$ have a positive linear relationship (i.e., $C_{1}$ increases in a linear fashion as $S_{1}$ increases). Using this equation, the following partial derivative quantifies this relationship:

The relationship in Equation 133 can be used to relate the comparative static

$$
\begin{equation*}
\frac{\partial C_{1}}{\partial S_{1}}=1 \tag{133}
\end{equation*}
$$

derivative in Equation 130 to $S_{1}$. Multiplying the partial derivative of $o_{i}{ }^{*}$ with respect to $\mathrm{C}_{1}$ times Equation 133 yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial C_{1}} \frac{\partial C_{1}}{\partial S_{1}}=\frac{\partial o_{i}^{*}}{\partial S_{1}} \tag{134}
\end{equation*}
$$

Since Equation 130 shows that the partial derivative of $o_{i}^{*}$ with respect to $C_{1}$ is negative and Equation 133 shows the partial derivative of $C_{1}$ with respect to $S_{1}$ equals one, the following must also be true at the critical value of $q_{i}$ :

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial S_{1}}<0 \tag{135}
\end{equation*}
$$

## APPENDIX C <br> Determining the Scenario C Optimal Number of Put Options

The optimal number of put options for an investor in Scenario C can be determined by finding the maximum date 2 expected utility. Date 2 expected utility can be calculated by

$$
\begin{equation*}
E_{i}\left(U_{2}\right)=q_{i} U_{u}+\left(1-q_{i}\right) U_{d} \tag{136}
\end{equation*}
$$

Let $p_{i}$ equal the number of put options to be purchased by an investor (if $p_{i}<0$, the investor writes put options). At date 1 , the investor purchases put options on $\mathrm{p}_{\mathrm{i}}$ shares of stock with a put price of $\mathbf{P}_{1}$. Cash left at date 1 after this transaction is

$$
\begin{equation*}
\operatorname{Cash}_{1}=\frac{W_{E}}{R}-p_{i} P_{1} \tag{137}
\end{equation*}
$$

Invested at the risk-free rate of interest, at date 2 this cash becomes

$$
\begin{equation*}
\text { Cash }_{2}=W_{E}-p_{i} R P_{1} \tag{138}
\end{equation*}
$$

If the stock price is $S_{d}$ at date 2 the wealth level is $W_{d}$, and if the stock price becomes $S_{u}$ the wealth level is $W_{u}$, as follows:

$$
\begin{equation*}
W_{d}=W_{E}-p_{i} R P_{1}+p_{i}\left(K_{p}-S_{d}\right) \tag{139}
\end{equation*}
$$

From these two wealth levels and a given power function utility preference $\left(\mathrm{U}=-\mathrm{W}^{-1}\right), \mathrm{U}_{\mathrm{u}}$ and $\mathrm{U}_{\mathrm{d}}$ can be calculated as

$$
\begin{gather*}
U_{d}=-\left[W_{E}-p_{i}\left(R P_{1}-K_{p}+S_{d}\right)\right]^{-1}  \tag{140}\\
U_{u}=-\left[W_{E}-p_{i} R P_{1}\right]^{-1}
\end{gather*}
$$

The date 2 expected utility is

$$
\begin{equation*}
E_{i}\left(U_{2}\right)=-q_{i}\left[W_{E}-p_{i} R P_{1}\right]^{-1}-\left(1-q_{i}\right)\left[W_{E}-p_{i}\left(R P_{1}-K_{p}+S_{d}\right]\right]^{-1} \tag{141}
\end{equation*}
$$

To find the number of options that yields the maximum expected utility, we take the first derivative with respect to the number of options, as follows:

$$
\begin{equation*}
\left.\frac{\partial E_{i}\left(U_{2}\right)}{\partial p_{i}}=q_{i}\left[W_{E}-p_{i} R P_{1}\right]^{-2}\left(R P_{1}\right)+\left(1-q_{i}\right)\left[W_{E}-p_{i}\left[R P_{1}-K_{p}+S_{d}\right]\right]^{-2}\left(R P_{1}+K_{p}-S_{d}\right)\right] \tag{142}
\end{equation*}
$$

Setting this derivative equal to zero yields

$$
\begin{equation*}
\frac{q_{i}\left(R P_{1}\right)}{\left[W_{E}-p_{i}^{*} R P_{1}\right]^{2}}+\frac{\left(1-q_{i}\right)\left(R P_{1}-K_{p}+S_{d}\right)}{\left[W_{E}-p_{i}^{*}\left(R P_{1}-K_{p}+S_{d}\right)\right]^{2}}=0 \tag{143}
\end{equation*}
$$

Multiplying by the product of the denominators yields

$$
\begin{aligned}
& q_{i}\left(R P_{1}\right)\left[W_{E}^{2}-2 W_{E} p_{i}^{*}\left(R P_{1}-K_{p}+S_{d}\right)+\left(p_{i}^{*}\right)^{2}\left(R P_{1}-K_{p}+S_{d}\right)^{2}\right]+(144) \\
& \left(1-q_{i}\right)\left(R P_{1}-K_{p}+S_{d}\right)\left[W_{E}^{2}+2 W_{E} p_{i}^{*}\left(R P_{1}\right)+\left(p_{i}^{*}\right)^{2}\left(R P_{1}\right)^{2}\right]=0
\end{aligned}
$$

Collecting terms,

$$
\begin{align*}
& \left(p_{i}^{*}\right)^{2}\left[q_{i}\left(R P_{1}-K_{p}+S_{u}\right)^{2}\left(R P_{1}\right)+\left(1-q_{i}\right)\left(R P_{1}-K_{p}+S_{d}\right)\left(R P_{1}\right)^{2}\right]+ \\
& p_{i}^{*}\left[q_{i}\left(-2 W_{E}\right)\left(R P_{1}\right)\left(R P_{1}-K_{p}+S_{d}\right)+\left(1-q_{i}\right)\left(-2 W_{E}\right) R P_{1}\left(R P_{1}-K_{p}+S_{d}\right)\right]+ \\
& {\left[q_{i} W_{E}^{2}\left(R P_{1}\right)+\left(1-q_{i}\right)\left(W_{E}\right)^{2}\left(R P_{1}-K_{p}+S_{d}\right)\right]=0} \tag{145}
\end{align*}
$$

Simplifying,

$$
\begin{gather*}
\left(p_{i}^{*}\right)^{2}\left[R P_{1}\left(R P_{1}-K_{p}+S_{d}\right)\left[R P_{1}-q_{i}\left(S_{d}-K_{p}\right)\right]+\right. \\
p_{i}^{*}\left[-2 W_{E}\left(R P_{1}\right)\left(R P_{1}-K_{p}+S_{d}\right)\right]+  \tag{146}\\
{\left[W_{E}^{2}\left(R P_{1}+\left(1-q_{i}\right)\left(S_{d}-K_{p}\right)\right]=0\right.}
\end{gather*}
$$

Solving this quadratic equation for the roots of $p_{i}$, with only the positive root meaningful,

$$
\begin{equation*}
p_{i}^{*}=\frac{2 W_{E}\left(R P_{1}\right)\left(R P_{1}-K_{p}+S_{d}\right)+\sqrt{W}}{2\left(R P_{1}\right)\left(R P_{1}-K_{p}+S_{d}\right)\left[R P_{1}-q_{i}\left(S_{d}-K_{p}\right)\right]} \tag{147}
\end{equation*}
$$

where

$$
\begin{aligned}
& W=\left[2 W_{E}\left(R P_{1}\right)\left(R P_{1}-K_{p}+S_{d}\right)\right]^{2}- \\
& 4 W_{E}^{2}\left[R P_{1}+\left(1-q_{i}\right)\left(S_{d}-K_{p}\right)\right]\left[R P_{1}\left(R P_{1}-K_{p}+S_{d}\right)\right]\left[R P_{1}-q_{i}\left(S_{d}-K_{p}\right)\right]
\end{aligned}
$$

(148)

Factoring out $2 \mathrm{RP}_{1}\left(\mathbf{R P}_{1}-\mathrm{K}_{\mathrm{p}}+\mathrm{S}_{\mathrm{d}}\right)$,

$$
\begin{equation*}
p_{i}^{*}=\frac{W_{E}+W_{E} \sqrt{1-\frac{\left[R C_{1}+q_{i}\left(K-S_{u}\right)\right]\left[R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)\right]}{\left(R C_{1}\right)\left(R C_{1}-S_{u}+K\right)}}}{\left[R C_{1}+\left(1-q_{i}\right)\left(K-S_{u}\right)\right]} \tag{149}
\end{equation*}
$$

Substituting for some of the expressions in the square root radical yields

$$
\begin{equation*}
p_{i}^{*}=\frac{W_{E}+W_{E} \sqrt{1-\frac{X Y}{Z}}}{Y} \tag{150}
\end{equation*}
$$

where

$$
\begin{align*}
X & =R P_{1}+\left(1-q_{i}\right)\left(S_{d}-K_{p}\right) \\
Y & =R P_{1}-q_{i}\left(S_{d}-K_{p}\right)  \tag{151}\\
Z & =R P_{1}\left(R P_{1}-K_{p}+S_{d}\right)
\end{align*}
$$

## Comparative Static Derivatives

## Comparative Static Derivatives for $\underline{n}_{i}{ }^{*}$

Appendix A derives the optimal number of shares of the underlying stock, $\mathrm{n}_{\mathrm{i}}{ }^{*}$, to be

$$
\begin{equation*}
n_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{A B}{C}}}{A} \tag{152}
\end{equation*}
$$

$$
\text { where } \begin{aligned}
A & =-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u} \\
B & =-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d} \\
C & =\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)<0
\end{aligned}
$$

Comparative static derivatives on this optimum will provide additional insight on the number of shares desired to be sold short (i.e., when $n_{i}^{*}<0$ ). To simplify the various partial derivatives, Equation 152 can first be re-arranged. Multiplying both sides by A , adding $\mathbf{W}_{\mathbf{E}}$, and dividing by $\mathbf{W}_{\mathbf{E}}$ yields

$$
\begin{equation*}
\frac{A n_{i}^{*}}{W_{E}}+1=\sqrt{1-\frac{A B}{C}} \tag{153}
\end{equation*}
$$

Squaring both sides of Equation 153 yields

$$
\begin{equation*}
\left(\frac{A n_{i}^{*}}{W_{E}}+1\right)^{2}=1-\frac{A B}{C} \tag{154}
\end{equation*}
$$

Simplifying yields

$$
\begin{equation*}
\frac{A^{2} n_{i}^{* 2}}{W_{E}^{2}}+2 \frac{A n_{i}^{*}}{W_{E}}+1=1-\frac{A B}{C} \tag{155}
\end{equation*}
$$

Subtracting 1 from both sides, adding $\mathrm{AB} / \mathrm{C}$, and dividing by A yields

$$
\begin{equation*}
\frac{A n_{i}^{* 2}}{W_{E}^{2}}+2 \frac{n_{i}^{*}}{W_{E}}+\frac{B}{C}=0 \tag{156}
\end{equation*}
$$

Factoring out the constants yields

$$
\begin{equation*}
\frac{1}{W_{E}^{2}}\left(A n_{i}^{* 2}\right)+\frac{2}{W_{E}}\left(n_{i}^{*}\right)+\frac{B}{C}=0 \tag{157}
\end{equation*}
$$

where $A=-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u}$
$B=-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}$
$C=\left(-\mathrm{RS}_{1}+\mathrm{S}_{\mathbf{u}}\right)\left(-\mathrm{RS}_{1}+\mathrm{S}_{\mathrm{d}}\right)$
Taking a partial derivative with respect to any variable, x , yields

$$
\begin{equation*}
\frac{1}{W_{E}^{2}}\left[A 2 n_{i}^{*} \frac{\partial n_{i}^{*}}{\partial x}+n_{i}^{* 2} \frac{\partial A}{\partial x}\right]+\frac{2}{W_{E}} \frac{\partial n_{i}^{*}}{\partial x}+\frac{C \frac{\partial B}{\partial x}-B \frac{\partial C}{\partial x}}{C^{2}}=0 \tag{158}
\end{equation*}
$$

Simplifying yields

$$
\begin{equation*}
\left[\frac{2 A n_{i}^{*}}{W_{E}^{2}}+\frac{2}{W_{E}}\right] \frac{\partial n_{i}^{*}}{\partial x}+\left[\frac{n_{i}^{* 2}}{W_{E}^{2}}\right] \frac{\partial A}{\partial x}+\frac{1}{C} \frac{\partial B}{\partial x}-\frac{B}{C^{2}} \frac{\partial C}{\partial x}=0 \tag{159}
\end{equation*}
$$

Isolating the coefficient to the first term and substituting for $n_{i}{ }^{*}$ yields

$$
\begin{equation*}
\frac{2}{W_{E}}\left[\frac{A}{W_{E}}+\frac{-W_{E}+W_{E} \sqrt{1-\frac{A B}{C}}}{A}+1\right]=\frac{2}{W_{E}} \sqrt{1-\frac{A B}{C}}>0 \tag{160}
\end{equation*}
$$

Substituting this result into Equation 159 yields

$$
\begin{equation*}
\left[\frac{2}{W_{E}} \sqrt{1-\frac{A B}{C}}\right] \frac{\partial n_{i}^{*}}{\partial x}+\left[\frac{n_{i}^{* 2}}{W_{E}^{2}}\right] \frac{\partial A}{\partial x}+\frac{1}{C} \frac{\partial B}{\partial x}-\frac{B}{C^{2}} \frac{\partial C}{\partial x}=0 \tag{161}
\end{equation*}
$$

Solving for (òn ${ }_{\mathbf{i}}{ }^{*}$ /òx) yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial x}=\frac{W_{E}}{2 \sqrt{1-\frac{A B}{C}}}\left[-\frac{n_{i}^{* 2}}{W_{E}^{2}} \frac{\partial A}{\partial x}-\frac{1}{C} \frac{\partial B}{\partial x}+\frac{B}{C^{2}} \frac{\partial C}{\partial x}\right] \tag{162}
\end{equation*}
$$

This general equation can be used to solve for a particular comparative static derivative by substituting the partial derivatives with respect to a particular variable (in place of $\mathbf{x}$ ). Note that the term outside the brackets is positive, so the sign of the comparative static derivative depends upon the sign of the terms within the brackets.

## 

Given the definitions of $A, B$, and $C$, the following are easily determined:

$$
\begin{gather*}
\frac{\partial A}{\partial q_{i}}=S_{d}-S_{u}  \tag{163}\\
\frac{\partial B}{\partial q_{i}}=S_{u}-S_{d}  \tag{164}\\
\frac{\partial C}{\partial q_{i}}=0 \tag{165}
\end{gather*}
$$

Substituting Equations 163, 164, and 165 into Equation 162 yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial q_{i}}=\frac{W_{E}}{2 \sqrt{1-\frac{A B}{C}}}\left[\frac{n_{i}^{* 2}}{W_{E}^{2}}\left(S_{u}-S_{d}\right)-\frac{1}{C}\left(S_{u}-S_{d}\right)\right] \tag{166}
\end{equation*}
$$

Looking inside the brackets, $\left(S_{\mathbf{u}}-S_{d}\right)$ is positive, the squared term is positive, and the last term in the brackets is positive (because C is always negative). Thus, the term in
the brackets is positive and the term preceding the brackets is positive. Therefore, this partial derivative is always positive, as follows:

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial q_{i}}>0 \tag{167}
\end{equation*}
$$

## Partial Derivative of $\underline{n}_{i}{ }^{*}$ with Respect to $\underline{S}_{1}$

Given the definitions of $\mathbf{A}, \mathrm{B}$, and $\mathbf{C}$, the following are easily determined:

$$
\begin{gather*}
\frac{\partial A}{\partial S_{1}}=-R  \tag{168}\\
\frac{\partial B}{\partial S_{1}}=-R  \tag{169}\\
\frac{\partial C}{\partial S_{1}}=R\left(2 R S_{1}-S_{u}-S_{d}\right) \tag{170}
\end{gather*}
$$

Substituting into Equation 162 yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial S_{1}}=\frac{W_{E}}{2 \sqrt{1-\frac{A B}{C}}}\left[\frac{n_{i}^{* 2}}{W_{E}^{2}} R+\frac{R}{C}+\frac{R B}{C^{2}}\left(2 R S_{1}-S_{u}-S_{d}\right)\right] \tag{171}
\end{equation*}
$$

Factoring R from the last three terms yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial S_{1}}=\frac{R W_{E}}{2 \sqrt{1-\frac{A B}{C}}}\left[\frac{n_{i}^{* 2}}{W_{E}^{2}}+\frac{1}{C}+\frac{B}{C^{2}}\left(2 R S_{1}-S_{u}-S_{d}\right)\right] \tag{172}
\end{equation*}
$$

Looking at the three terms in the brackets, the first term is always positive, the second
is always negative, and the third can be either negative or positive. Thus, it is not clear whether this partial derivative is always positive or negative.

Since this comparative static derivative is intractable, numerical methods are used to provide insight. Figure 13 shows a surface plot of $\mathrm{n}_{\mathrm{i}}{ }^{*}$ for various values of $S_{1}$ and $q_{i}$ from the numerical example. Note that the slope of this surface plot appears to be negative at all points, which leads to the conclusion that the comparative static derivative for $S_{1}$ is negative. Figure 14 shows a surface plot of Equation 172, which indicates that the partial derivative of $n_{i}{ }^{*}$ with respect to $S_{1}$ is negative at least for the parameters chosen in the numerical example, as follows:

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial S_{1}}<0 \tag{173}
\end{equation*}
$$

## $\underline{\text { Partial }}$ Derivative of $n_{i}{ }^{*}$ with Respect to $\underline{R}$

Given the definitions of A, B, and C, the following are easily determined:

$$
\begin{gather*}
\frac{\partial A}{\partial R}=-S_{1}  \tag{174}\\
\frac{\partial B}{\partial R}=-S_{1}  \tag{175}\\
\frac{\partial C}{\partial R}=S_{1}\left(2 R S_{1}-S_{u}-S_{d}\right) \tag{176}
\end{gather*}
$$

FIGURE 13
The Optimal Number of Shares, $\mathrm{n}_{\mathrm{i}}{ }^{*}$, for Various Values of $S_{1}$ and $q_{i}$


FIGURE 14
A Surface Plot of the Partial Derivative of $n_{i}^{*}$ with Respect to $S_{1}$


Substituting into Equation 162 yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial R}=\frac{W_{E}}{2 \sqrt{1-\frac{A B}{C}}}\left[\frac{n_{i}^{* 2}}{W_{E}^{2}} S_{1}+\frac{S_{1}}{C}+\frac{S_{1} B}{C^{2}}\left(2 R S_{1}-S_{u}-S_{d}\right)\right] \tag{177}
\end{equation*}
$$

Factoring out $S_{1}$ yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial R}=\frac{S_{1} W_{E}}{2 \sqrt{1-\frac{A B}{C}}}\left[\frac{n_{i}^{* 2}}{W_{E}^{2}}+\frac{1}{C}+\frac{B}{C^{2}}\left(2 R S-S_{u}-S_{d}\right)\right] \tag{178}
\end{equation*}
$$

Note that the term preceding the terms in the brackets is always positive. Also, the terms in the brackets are identical to the terms in the brackets in Equation 172. Since the sum of the terms in the brackets were shown to be negative in the previous section, the partial derivative of $n_{i}{ }^{*}$ with respect to $R$ must be negative. To verify this conclusion, Figure 15 shows a surface plot of Equation 178, which indicates that the partial derivative of $n_{i}{ }^{*}$ with respect to $R$ is negative at least for the parameters chosen in the numerical example, as follows:

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial R}<0 \tag{179}
\end{equation*}
$$

## Partial Derivative of $n_{i}{ }^{*}$ with Respect to $W_{E}$

Appendix A derives the optimal number of shares of the underlying stock, $n_{i}^{*}$, to be

$$
\begin{equation*}
n_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{A B}{C}}}{A} \tag{180}
\end{equation*}
$$

FIGURE 15
A Surface Plot of the Partial Derivative of $n_{i}^{*}$ with Respect to $R$


$$
\text { where } \begin{aligned}
A & =-R S_{1}+q_{i}\left(S_{d}-S_{u}\right)+S_{u} \\
B & =-R S_{1}+q_{i}\left(S_{u}-S_{d}\right)+S_{d}, \text { and } \\
C & =\left(-R S_{1}+S_{u}\right)\left(-R S_{1}+S_{d}\right)
\end{aligned}
$$

Taking the first partial derivative of Equation 180 with respect to $\mathrm{W}_{\mathrm{E}}$ yields

$$
\begin{equation*}
\frac{\partial n_{i}^{*}}{\partial W_{E}}=\frac{-1+1 \sqrt{1-\frac{A B}{C}}}{A} \tag{181}
\end{equation*}
$$

The sign of this comparative static derivative depends on whether $n_{i}{ }^{*}$ is positive or negative. First consider $n_{i}{ }^{*}<0$, which implies $B<0$. This means that $A B / C$ has the same sign as $A$. If $A<0$, then $A B / C$ is negative and $(1-A B / C)$ is greater than 1 . Therefore, the square root is greater than 1 , the numerator is positive, and the denominator is negative. This determines that the partial derivative is negative. If $A>0$, then $A B / C$ is positive and $(1-A B / C)$ is less than 1 . Therefore the square root is less than 1 , the numerator is negative, and the denominator is positive. Thus, if $n_{i}{ }^{*}<0$, the partial derivative is always negative.

Next consider $\mathrm{n}_{\mathrm{i}}{ }^{*}>0$, which implies $\mathrm{B}>0$. This means $\mathrm{AB} / \mathrm{C}$ is the opposite sign of $A$. If $A<0$, then $A B / C$ is positive and $(1-A B / C)$ is less than 1 . Therefore the square root is less than 1 , the numerator is negative, and the denominator is negative. This determines that the partial derivative is positive. If $A>0$, then $A B / C$ is negative and ( $1-\mathrm{AB} / \mathrm{C}$ ) is greater than 1 . Therefore the square root is greater than 1 , the numerator is positive, and the denominator is positive. Once again, the partial derivative is always positive.

The above results mean that an increase in wealth would encourage an investor who desires a short position to want to sell short more shares, and an investor who holds shares to buy more shares. However, the point at which an investor desires zero shares of stock (the critical value of $q_{i}$ ) is independent of the wealth level. The wealth level thus acts like a scaling factor. The comparative static derivatives for $W_{E}$ are summarized as follows:

$$
\begin{align*}
& \frac{\partial n_{i}^{*}}{\partial W_{E}}>0 \quad \text { when } n_{i}^{*}>0  \tag{182}\\
& \frac{\partial n_{i}^{*}}{\partial W_{E}}<0 \quad \text { when } n_{i}^{*}<0 \tag{183}
\end{align*}
$$

## Comparative Static Derivatives for $\mathbf{o}_{i}{ }^{*}$

Appendix B derives the optimal number of options of the underlying stock, $\mathrm{o}_{\mathrm{i}}{ }^{*}$, to be

$$
\begin{equation*}
o_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{E F}{G}}}{-F} \tag{184}
\end{equation*}
$$

$$
\text { where } \begin{aligned}
\mathrm{E} & =-R C_{1}+\mathrm{q}_{\mathrm{i}}\left(\mathrm{~S}_{\mathrm{u}}-K\right) \\
\mathrm{F} & =R C_{1}+\left(1-\mathrm{q}_{\mathrm{i}}\right)\left(\mathrm{K}-\mathrm{S}_{\mathrm{u}}\right) \\
\mathrm{G} & =R C_{1}\left(-R C_{1}+\mathrm{S}_{\mathrm{u}}-K\right)
\end{aligned}
$$

To simplify the derivation of the various partial derivatives, Equation 184 can first be rearranged. Dividing by $\mathrm{W}_{\mathrm{E}}$, multiplying both sides by -F , and adding 1 to both sides of Equation 184 yields

$$
\begin{equation*}
1-\frac{F o_{i}^{*}}{W_{E}}=\sqrt{1-\frac{E F}{G}} \tag{185}
\end{equation*}
$$

Squaring both sides of Equation 185 yields

$$
\begin{equation*}
\left(1-\frac{F o_{i}^{*}}{W_{E}}\right)^{2}=1-\frac{E F}{G} \tag{186}
\end{equation*}
$$

Simplifying yields

$$
\begin{equation*}
1-\frac{F^{2} O_{i}^{* 2}}{W_{E}^{2}}+2 \frac{F O_{i}^{*}}{W_{E}}=1-\frac{E F}{G} \tag{187}
\end{equation*}
$$

Subtracting 1 from both sides, adding EF/G, and dividing by F yields

$$
\begin{equation*}
\frac{F o_{i}^{*} 2}{W_{E}^{2}}-2 \frac{o_{i}^{*}}{W_{E}}+\frac{E}{G}=0 \tag{188}
\end{equation*}
$$

Taking a partial derivative with respect to any variable, $x$, yields

$$
\begin{equation*}
\frac{1}{W_{E}^{2}}\left[F 2 o_{i}^{*} \frac{\partial o_{i}^{*}}{\partial x}+o_{i}^{* 2} \frac{\partial F}{\partial x}\right]+\frac{2}{W_{E}} \frac{\partial o_{i}^{*}}{\partial x}+\frac{G \frac{\partial E}{\partial x}+E \frac{\partial G}{\partial x}}{G^{2}}=0 \tag{189}
\end{equation*}
$$

Collecting terms yields

$$
\begin{equation*}
\left[\frac{2 F o_{i}^{*}}{W_{E}^{2}}-\frac{2}{W_{E}}\right] \frac{\partial o_{i}^{*}}{\partial x}+\left[\frac{o_{i}^{* 2}}{W_{E}^{2}}\right] \frac{\partial F}{\partial x}+\frac{1}{G} \frac{\partial E}{\partial x}-\frac{E}{G^{2}} \frac{\partial G}{\partial x}=0 \tag{190}
\end{equation*}
$$

Isolating the coefficient to the first term and substituting for $o_{i}{ }^{*}$ yields

$$
\begin{equation*}
\frac{2 F}{W_{E}^{2}}\left[\frac{-W_{E}+W_{B} \sqrt{1-\frac{E F}{G}}}{-F}\right]-\frac{2}{W_{E}}=-\frac{2}{W_{E}} \sqrt{1-\frac{E F}{G}} \tag{191}
\end{equation*}
$$

Substituting this result into Equation 190 yields

$$
\begin{equation*}
\left[-\frac{2}{W_{E}} \sqrt{1-\frac{E F}{G}}\right] \frac{\partial o_{i}^{*}}{\partial x}+\left[\frac{o_{i}^{* 2}}{W_{E}^{2}}\right] \frac{\partial F}{\partial x}+\frac{1}{G} \frac{\partial E}{\partial x}-\frac{E}{G^{2}} \frac{\partial G}{\partial x}=0 \tag{192}
\end{equation*}
$$

Solving for $\left(8 o_{i}{ }^{*} / 8 \mathrm{x}\right)$ yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial x}=\frac{W_{E}}{2 \sqrt{1-\frac{E F}{G}}}\left[\frac{o_{i}^{* 2}}{W_{E}^{2}} \frac{\partial F}{\partial x}+\frac{1}{G} \frac{\partial E}{\partial x}+\frac{E}{F G^{2}} \frac{\partial G}{\partial x}\right] \tag{193}
\end{equation*}
$$

This general equation can be used to solve for a particular comparative static derivative by substituting the partial derivatives with respect to a particular variable (in place of $\mathbf{x}$ ). Note that the term outside the brackets is positive, so the sign of the comparative static derivative depends upon the sign of the terms within the brackets.

## Partial Derivative of $\mathrm{o}_{\mathrm{i}}{ }^{*}$ with Respect to $\mathrm{g}_{\mathrm{i}}$

Given the definitions of $E, F$, and $G$, the following are easily determined:

$$
\begin{align*}
& \frac{\partial E}{\partial q_{i}}=S_{u}-K  \tag{194}\\
& \frac{\partial F}{\partial q_{i}}=S_{u}-K \tag{195}
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial G}{\partial q_{i}}=0 \tag{196}
\end{equation*}
$$

Substituting Equations 194, 195, and 196 into Equation 193 yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial q_{i}}=\frac{W_{E}}{2 \sqrt{1-\frac{E F}{G}}}\left[\frac{o_{i}^{* 2}}{W_{E}^{2}}\left(S_{u}-K\right)+\frac{1}{G}\left(S_{u}-K\right)\right] \tag{197}
\end{equation*}
$$

Looking inside the brackets, $\left(S_{u}-K\right)$ is positive, the squared term is positive, and the last term in the brackets is positive (because G is always positive). Thus, the term in the brackets is positive and the term preceding the brackets is positive. Therefore, this partial derivative is always positive, as follows:

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial q_{i}}>0 \tag{198}
\end{equation*}
$$

## Partial Derivative of $o_{i}^{*}$ with Respect to $C_{1}$

Given the definitions of $\mathrm{E}, \mathrm{F}$, and G , the following are easily determined:

$$
\begin{gather*}
\frac{\partial E}{\partial C_{1}}=-R  \tag{199}\\
\frac{\partial F}{\partial C_{1}}=R \\
\frac{\partial G}{\partial C_{1}}=R\left(-2 R C_{1}+S_{u}-K\right) \tag{201}
\end{gather*}
$$

(200)

Substituting into Equation 193 yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial C_{1}}=\frac{W_{E}}{2 \sqrt{1-\frac{E F}{G}}}\left[\frac{O_{i}^{* 2}}{W_{E}^{2}} R+\frac{R}{G}+\frac{R E}{G^{2}}\left(2 R C_{1}-S_{u}+K\right)\right] \tag{202}
\end{equation*}
$$

Factoring R from the last three terms yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial C_{1}}=\frac{R W_{E}}{2 \sqrt{1-\frac{E F}{G}}}\left[\frac{o_{i}^{* 2}}{W_{E}^{2}}+\frac{1}{G}+\frac{E}{G^{2}}\left(2 R C_{1}-S_{u}+K\right)\right] \tag{203}
\end{equation*}
$$

Looking at the three terms in the brackets, the first term is always positive, the second is always negative, and the third can be either negative or positive. Thus, it is not clear whether this partial derivative is always positive or negative.

Since this comparative static derivative is intractable, numerical methods are used to provide insight. Figure 16 shows a surface plot of $o_{i}^{*}$ for various values of $C_{1}$ and $q_{i}$ from the numerical example. Note that the slope of this surface plot appears to be negative at all points, which leads to the conclusion that the comparative static for $C_{1}$ is negative. Figure 17 shows a surface plot of Equation 203, which indicates that the partial derivative of $o_{i}^{*}$ with respect to $C_{1}$ is negative at least for the parameters chosen in the numerical example, as follows:

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial C_{1}}<0 \tag{204}
\end{equation*}
$$

FIGURE 16
The Optimal Number of Options, $\mathrm{o}_{\mathrm{i}}{ }^{*}$, for Various Values of $\mathrm{C}_{1}$ and $\mathrm{q}_{\mathrm{i}}$


1
qi

FIGURE 17
A Surface Plot of the Partial Derivative
of $\mathrm{o}_{\mathrm{i}}{ }^{*}$ with Respect to $\mathrm{C}_{1}$


1
qi

## Partial Derivative of $0_{i} *$ with Respect to $R$

Given the definitions of $E, F$, and $G$, the following are easily determined:

$$
\begin{gather*}
\frac{\partial E}{\partial R}=-C_{1}  \tag{205}\\
\frac{\partial F}{\partial R}=C_{1}  \tag{206}\\
\frac{\partial G}{\partial R}=C_{1}\left(-2 R C_{1}+S_{u}-K\right) \tag{207}
\end{gather*}
$$

Substituting into Equation 193 yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial R}=\frac{W_{E}}{2 \sqrt{1-\frac{E F}{G}}}\left[\frac{o_{i}^{* 2}}{W_{E}^{2}} C_{1}-\frac{C_{1}}{G}+\frac{C_{1} E}{G^{2}}\left(2 R C_{1}-S_{u}+K\right)\right] \tag{208}
\end{equation*}
$$

Factoring out $\mathrm{C}_{1}$ yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial R}=\frac{C_{1} W_{E}}{2 \sqrt{1-\frac{E F}{G}}}\left[\frac{o_{i}^{* 2}}{W_{E}^{2}}+\frac{1}{G}+\frac{E}{G^{2}}\left(2 R C_{1}-S_{u}+K\right)\right] \tag{209}
\end{equation*}
$$

Note that the term preceding the terms in the brackets is always positive. Also, the terms in the brackets are identical to the terms in the brackets in Equation 203. Since the sum of the terms in the brackets were shown to be negative in the previous section, the partial derivative of $o_{i}^{*}$ with respect to $\mathbf{R}$ must be negative. To verify this conclusion, Figure 18 shows a surface plot of Equation 209, which indicates that the partial derivative of $o_{i}{ }^{*}$ with respect to $R$ is negative at least for the parameters chosen in the numerical example, as follows:

FIGURE 18
A Surface Plot of the Partial Derivative of $o_{i}$ * with Respect to $R$


$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial R}<0 \tag{210}
\end{equation*}
$$

Partial Derivative of $\mathrm{o}_{\mathrm{i}}{ }^{*}$ with Respect to $\mathrm{W}_{\mathrm{E}}$
Appendix B derives the optimal number of options on the underlying stock,
$o_{i}{ }^{*}$, to be

$$
\begin{equation*}
O_{i}^{*}=\frac{-W_{E}+W_{E} \sqrt{1-\frac{E F}{G}}}{-F} \tag{211}
\end{equation*}
$$

$$
\text { where } \begin{aligned}
\mathrm{E} & =-\mathrm{RC} C_{1}+\mathrm{q}_{\mathrm{i}}\left(\mathrm{~S}_{\mathrm{u}}-\mathrm{K}\right) \\
\mathrm{F} & =-R C_{1}+\left(1-\mathrm{q}_{\mathrm{i}}\right)\left(\mathrm{S}_{\mathrm{u}}-\mathrm{K}\right), \text { and } \\
\mathrm{G} & =R C_{1}\left(-R C_{1}+S_{u}-K\right)
\end{aligned}
$$

Taking the first partial derivative of Equation 211 with respect to $\mathrm{W}_{\mathrm{E}}$ yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial W_{E}}=\frac{-1+1 \sqrt{1-\frac{E F}{G}}}{-F} \tag{212}
\end{equation*}
$$

The sign of this comparative static derivative depends on whether $o_{i}{ }^{*}$ is positive or negative. First consider $\mathrm{o}_{\mathrm{i}}{ }^{*}<0$, which implies $\mathrm{E}<0$. This means that $\mathrm{EF} / \mathrm{G}$ has the opposite sign as $F$. If $\mathrm{F}<0$, then $\mathrm{EF} / \mathrm{G}$ is positive and ( $1-\mathrm{EF} / \mathrm{G}$ ) is less than 1 .

Therefore, the square root is less than 1 , the numerator is negative, and the denominator is positive. This determines that the partial derivative is negative. If $\mathrm{E}>0$, then $\mathrm{EF} / \mathrm{G}$ is negative and ( $1-\mathrm{EF} / \mathrm{G}$ ) is greater than 1 . Therefore the square root
is greater than 1 , the numerator is positive, and the denominator is negative. Thus, if $o_{i}^{*}<0$, the partial derivative is always negative.

Next consider $\mathrm{o}_{\mathrm{i}}{ }^{*}>0$, which implies $\mathrm{E}>0$. This means $\mathrm{EF} / \mathrm{G}$ is the same sign as F. If $\mathrm{F}<0$, then $\mathrm{EF} / \mathrm{G}$ is negative and $(1-\mathrm{EF} / \mathrm{G})$ is greater than 1. Therefore the square root is greater than 1 , the numerator is positive, and the denominator is positive. This determines that the partial derivative is positive. If $F>0$, then $E F / G$ is negative and ( $1-\mathrm{EF} / \mathrm{G}$ ) is less than 1 . Therefore the square root is less than 1 , the numerator is negative, and the denominator is negative. Once again, the partial derivative is always positive.

The above results mean that an increase in wealth would encourage an investor who desires to write call options to want to write more calls, and an investor who buys call options to buy more calls. The wealth level thus acts like a scaling factor.

The comparative static derivatives for $\mathrm{W}_{\mathrm{E}}$ are summarized as follows:

$$
\begin{align*}
& \frac{\partial o_{i}^{*}}{\partial W_{E}}>0 \quad \text { when } o_{i}^{*}>0  \tag{213}\\
& \frac{\partial o_{i}^{*}}{\partial W_{E}}<0 \quad \text { when } o_{i}^{*}<0 \tag{214}
\end{align*}
$$

## $\underline{\text { Partial }} \underline{\text { Derivative of } o_{i}}{ }^{*}$ with Respect to $\underline{S}_{1}$

The comparative static derivative in Equation 204 can also be related to $\mathbf{S}_{\mathbf{1}}$. Equation 119 shows that $C_{1}$ and $S_{1}$ have a positive linear relationship (i.e., $C_{1}$ increases in a linear fashion as $S_{1}$ increases). Using this equation, the following partial derivative quantifies this relationship:

$$
\begin{equation*}
\frac{\partial C_{1}}{\partial S_{1}}=1 \tag{215}
\end{equation*}
$$

The relationship in Equation 215 can be used to relate the comparative static derivative in Equation 204 to $\mathbf{S}_{1}$. Multiplying Equation 204 times Equation 215 yields

$$
\begin{equation*}
\frac{\partial o_{i}^{*}}{\partial C_{1}} \frac{\partial C_{1}}{\partial S_{1}}=\frac{\partial o_{i}^{*}}{\partial S_{1}} \tag{216}
\end{equation*}
$$

Since Equation 204 shows that the partial derivative of $o_{i}{ }^{*}$ with respect to $C_{1}$ is negative (at least for the parameters of the numerical example) and Equation 215 shows the partial derivative of $C_{1}$ with respect to $S_{1}$ equals 1 , the partial derivative of $o_{i}^{*}$ with respect to $S_{1}$ must be negative (again, for the parameters of the numerical example), as follows:

$$
\frac{\partial o_{i}^{*}}{\partial S_{1}}<0
$$

## APPENDIX E

## Hypotheses 2 and 3 Regressions

for Various Holding Periods

The tables on the following pages show the results of the same regressions used to test Hypotheses 2 and 3 applied to various holding periods up to five years after the introduction of options. The same tests are also applied to the individual years up to year 5 after option introduction.

TABLE 12
1-Year Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.004210 \\ (.7797) \end{gathered}$ | $\begin{gathered} -.375482 \\ (.0199) \end{gathered}$ |  |  | $\begin{gathered} 5.471 \\ (.0199) \end{gathered}$ |
| (2) | $\begin{gathered} -.004100 \\ (.7894) \end{gathered}$ | $\begin{gathered} -.376071 \\ (.0204) \end{gathered}$ | $\begin{aligned} & .001912 \\ & (.9696) \end{aligned}$ |  | $\begin{gathered} 2.729 \\ (.0667) \end{gathered}$ |
| (3) | $\begin{gathered} -.021709 \\ (.1533) \end{gathered}$ | $\begin{gathered} -.391930 \\ (.0127) \end{gathered}$ |  | $\begin{aligned} & .511479 \\ & (.0001) \end{aligned}$ | $\begin{aligned} & 12.861 \\ & (.0001) \end{aligned}$ |
| (4) | $\begin{gathered} -.021798 \\ (.1595) \end{gathered}$ | $\begin{gathered} -.391465 \\ (.0133) \end{gathered}$ | $\begin{gathered} -.001518 \\ (.9752) \end{gathered}$ | $\begin{aligned} & .511536 \\ & (.0001) \end{aligned}$ | $\begin{gathered} 8.550 \\ (.0001) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 13
2-Year Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 2-year time period following option introduction (month 1 to month 24) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. $P$-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.032008 \\ (.1726) \end{gathered}$ | $\begin{gathered} -.217446 \\ (.5342) \end{gathered}$ |  |  | $\begin{gathered} .387 \\ (. .5342) \end{gathered}$ |
| (2) | $\begin{gathered} -.033952 \\ (.1490) \end{gathered}$ | $\begin{gathered} -.146251 \\ (.6806) \end{gathered}$ | $\begin{gathered} -.076008 \\ (.2609) \end{gathered}$ |  | $\begin{gathered} .828 \\ (.4379) \end{gathered}$ |
| (3) | $\begin{gathered} -.035724 \\ (.1687) \end{gathered}$ | $\begin{gathered} -.223673 \\ (.5237) \end{gathered}$ |  | $\begin{aligned} & .062603 \\ & (.7358) \end{aligned}$ | $\begin{gathered} .250 \\ (.7789) \end{gathered}$ |
| (4) | $\begin{gathered} -.038137 \\ (.1430) \end{gathered}$ | $\begin{gathered} -.152372 \\ (.6687) \end{gathered}$ | $\begin{gathered} -.076918 \\ (.2561) \end{gathered}$ | $\begin{aligned} & .070107 \\ & (.7056) \end{aligned}$ | $\begin{gathered} .598 \\ (.6166) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 14
3-Year Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 3-year time period following option introduction (month 1 to month 36) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. $P$-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.025508 \\ (.4580) \end{gathered}$ | $\begin{gathered} -.627069 \\ (.2924) \end{gathered}$ |  |  | $\begin{gathered} 1.112 \\ (.2924) \end{gathered}$ |
| (2) | $\begin{gathered} -.011646 \\ (.7461) \end{gathered}$ | $\begin{gathered} -.636763 \\ (.2846) \end{gathered}$ | $\begin{gathered} -.087651 \\ (.1971) \end{gathered}$ |  | $\begin{gathered} 1.392 \\ (.2499) \end{gathered}$ |
| (3) | $\begin{gathered} -.053604 \\ (.1718) \end{gathered}$ | $\begin{aligned} & -.647338 \\ & (.2764) \end{aligned}$ |  | $\begin{aligned} & .344895 \\ & (.1384) \end{aligned}$ | $\begin{gathered} 1.661 \\ (.1915) \end{gathered}$ |
| (4) | $\begin{gathered} -.040059 \\ (.3218) \end{gathered}$ | $\begin{gathered} -.658081 \\ (.2680) \end{gathered}$ | $\begin{gathered} -.091293 \\ (.1786) \end{gathered}$ | $\begin{aligned} & .355868 \\ & (.1261) \end{aligned}$ | $\begin{gathered} 1.715 \\ (.1635) \end{gathered}$ |

${ }^{\mathrm{a}}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
$\mathrm{b}_{\text {The }} \mathrm{F}$-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 15
4-Year Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 4-year time period following option introduction (month 1 to month 48) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.053644 \\ (.2608) \end{gathered}$ | $\begin{gathered} -.588052 \\ (.4769) \end{gathered}$ |  |  | $\begin{gathered} .507 \\ (.4769) \end{gathered}$ |
| (2) | $\begin{gathered} -.012248 \\ (.8108) \end{gathered}$ | $\begin{gathered} -.517398 \\ (.5297) \end{gathered}$ | $\begin{gathered} -.171 \\ (.0316) \end{gathered}$ |  | $\begin{gathered} 2.585 \\ (.0768) \end{gathered}$ |
| (3) | $\begin{gathered} -.148609 \\ (.0113) \end{gathered}$ | $\begin{gathered} -.656200 \\ (.4234) \end{gathered}$ |  | $\begin{aligned} & .805186 \\ & (.0060) \end{aligned}$ | $\begin{gathered} 4.078 \\ (.0177) \end{gathered}$ |
| (4) | $\begin{gathered} -.106420 \\ (.0849) \end{gathered}$ | $\begin{gathered} -.587036 \\ (.4722) \end{gathered}$ | $\begin{gathered} -.162 \\ (.0395) \end{gathered}$ | $\begin{aligned} & .780800 \\ & (.0075) \end{aligned}$ | $\begin{gathered} 4.167 \\ (.0064) \end{gathered}$ |

${ }^{\mathrm{a}}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
$\mathbf{b}_{\text {The }}$ F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F -statistics.

TABLE 16
5-Year Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 5 -year time period following option introduction (month 1 to month 60) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P -values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | -. 14177 | 1.503 |  |  | 4.323 |
|  | (.0191) | (.0383) |  |  | (.0383) |
| (2) | -. 02880 | 1.333 | -. 274 |  | 5.981 |
|  | (.6912) | (.0646) | (.0063) |  | (.0028) |
| (3) | -. 25766 | 1.381 |  | . 7789 | 5.669 |
|  | (.0006) | (.0554) |  | (.0088) | (.0038) |
| (4) | -. 14361 | 1.234 | -. 252 | . 71298 | 5.996 |
|  | (.0965) | (.0853) | (.0114) | (.0160) | (.0005) |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

## TABLE 17

Year 2 Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the second year following option introduction (month 13 to month 24) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) |  | . 250055 |  |  | 1.086 |
|  | $(.0898)$ | (.2981) |  |  | (.2981) |
| (2) | -. 019444 | . 230163 | -. 021819 |  | . 663 |
|  | (.1155) | (.3454) | (.6226) |  | (.5159) |
| (3) | -. 020235 | . 251079 |  | -. 01437 | . 548 |
|  | (.1048) | (.2971) |  | (.9054) | (.5783) |
| (4) | -. 019121 | . 231191 | -. 021771 | -. 011944 | . 445 |
|  | (.1318) | (.3442) | (.6239) | (.9092) | (.7209) |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
$\mathrm{b}_{\text {The }}$ F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 18
Year 3 Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the third year following option introduction (month 25 to month 36) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.012915 \\ (.3766) \end{gathered}$ | $\begin{aligned} & .439094 \\ & (.2521) \end{aligned}$ |  |  | $\begin{aligned} & 1.3166 \\ & (.2521) \end{aligned}$ |
| (2) | $\begin{gathered} -.020434 \\ (.2005) \end{gathered}$ | $\begin{aligned} & .430017 \\ & (.2619) \end{aligned}$ | $\begin{gathered} -.049817 \\ (.2430) \end{gathered}$ |  | $\begin{gathered} 1.342 \\ (.2626) \end{gathered}$ |
| (3) | $\begin{gathered} -.005778 \\ (.6990) \end{gathered}$ | $\begin{aligned} & .519727 \\ & (.1757) \end{aligned}$ |  | $\begin{aligned} & .350146 \\ & (.0400) \end{aligned}$ | $\begin{gathered} 2.789 \\ (.0628) \end{gathered}$ |
| (4) | $\begin{gathered} -.014206 \\ (.3783) \end{gathered}$ | $\begin{aligned} & .514315 \\ & (.1797) \end{aligned}$ | $\begin{gathered} -.058963 \\ (.1670) \end{gathered}$ | $\begin{aligned} & .373297 \\ & (.0292) \end{aligned}$ | $\begin{gathered} 2.503 \\ (.0591) \end{gathered}$ |

${ }^{\mathrm{a}}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
$\mathrm{b}_{\text {The }} \mathrm{F}$-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 19
Year 4 Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the fourth year following option introduction (month 37 to month 48) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.006779 \\ (.6516) \end{gathered}$ | $\begin{gathered} 1.230905 \\ (.0070) \end{gathered}$ |  |  | $\begin{gathered} 7.369 \\ (.0070) \end{gathered}$ |
| (2) | $\begin{gathered} -.008220 \\ (.6162) \end{gathered}$ | $\begin{gathered} 1.225831 \\ (.0073) \end{gathered}$ | $\begin{aligned} & .014400 \\ & (.8261) \end{aligned}$ |  | $\begin{gathered} 3.699 \\ (.0257) \end{gathered}$ |
| (3) | $\begin{gathered} -.012212 \\ (.4462) \end{gathered}$ | $\begin{gathered} 1.226208 \\ (.0072) \end{gathered}$ |  | $\begin{aligned} & .14623 \\ & (.3334) \end{aligned}$ | $\begin{gathered} 4.153 \\ (.0165) \end{gathered}$ |
| (4) | $\begin{aligned} & -.013726 \\ & (.4292) \end{aligned}$ | $\begin{gathered} 1.220907 \\ (.0076) \end{gathered}$ | $\begin{aligned} & .015015 \\ & (.8188) \end{aligned}$ | $\begin{gathered} .146567 \\ (.3330) \end{gathered}$ | $\begin{gathered} 2.779 \\ (.0411) \end{gathered}$ |

${ }^{\mathrm{a}}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
${ }^{\mathbf{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 20
Year 5 Cross-Sectional Regressions
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the fifth year following option introduction (month 49 to month 60) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.012877 \\ (.4143) \end{gathered}$ | $\begin{aligned} & .924443 \\ & (.0008) \end{aligned}$ |  |  | $\begin{aligned} & 11.500 \\ & (.0008) \end{aligned}$ |
| (2) | $\begin{gathered} -.008953 \\ (.5917) \end{gathered}$ | $\begin{aligned} & .929315 \\ & (.0007) \end{aligned}$ | $\begin{gathered} -.028008 \\ (.4713) \end{gathered}$ |  | $\begin{gathered} 6.002 \\ (.0027) \end{gathered}$ |
| (3) | $\begin{gathered} -.020679 \\ (.2042) \end{gathered}$ | $\begin{aligned} & .995265 \\ & (.0003) \end{aligned}$ |  | $\begin{aligned} & .231930 \\ & (.0649) \end{aligned}$ | $\begin{gathered} 7.503 \\ (.0006) \end{gathered}$ |
| (4) | $\begin{gathered} -.017018 \\ (.3232) \end{gathered}$ | $\begin{aligned} & .998762 \\ & (.0003) \end{aligned}$ | $\begin{gathered} -.025413 \\ (.5122) \end{gathered}$ | $\begin{aligned} & .228908 \\ & (.0689) \end{aligned}$ | $\begin{gathered} 5.137 \\ (.0017) \end{gathered}$ |

${ }^{\mathrm{a}}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
$\mathrm{b}_{\text {The }}$ F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 21
1-Year Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1-year time period following option introduction (month 1 to month 12) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | -. 001450 | . 074425 |  |  | 103.890 |
|  | (.0348) | (.0001) |  |  | (.0001) |
| (2) | -. 001505 | . 074723 | -. 000970 |  | 51.916 |
|  | (.0316) | (.0001) | (.6702) |  | (.0001) |
| (3) | -. 001441 | . 074484 |  | . 0028256 | 51.807 |
|  | (.0373) | (.0001) |  | (.9091) | (.0001) |
| (4) | -. 001496 | . 074812 | -. 001000 | -. 0037612 | 34.523 |
|  | (.0337) | (.0001) | (.6623) | (.8797) | (.0001) |

${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 22
2-Year Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 2-year time period following option introduction (month 1 to month 24) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | $\begin{gathered} -.000251 \\ (.7083) \end{gathered}$ | $\begin{aligned} & .040227 \\ & (.0001) \end{aligned}$ |  |  | $\begin{aligned} & 16.160 \\ & (.0001) \end{aligned}$ |
| (2) | $\begin{gathered} -.000181 \\ (.7871) \end{gathered}$ | $\begin{gathered} .037679 \\ (.0002) \end{gathered}$ | $\begin{aligned} & .002720 \\ & (.1597) \end{aligned}$ |  | $\begin{gathered} 9.095 \\ (. .0001) \end{gathered}$ |
| (3) | $\begin{gathered} -.000311 \\ (.6449) \end{gathered}$ | $\begin{aligned} & .040493 \\ & (.0001) \end{aligned}$ |  | $\begin{gathered} .0249 \\ (.3128) \end{gathered}$ | $\begin{gathered} 8.591 \\ (.0002) \end{gathered}$ |
| (4) | $\begin{gathered} -.000250 \\ (.7098) \end{gathered}$ | $\begin{gathered} .037279 \\ (.0003) \end{gathered}$ | $\begin{aligned} & .003582 \\ & (.0760) \end{aligned}$ | $\begin{gathered} .0380 \\ (.1387) \end{gathered}$ | $\begin{gathered} 6.818 \\ (.0002) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
${ }^{\mathbf{b}}$ The F -statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

## TABLE 23

3-Year Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 3 -year time period following option introduction (month 1 to month 36) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
|  |  |  |  |  |  |
| (1) | . 001236 | . 002548 |  |  | . 030 |
|  | (.1443) | (.8619) |  |  | (.8619) |
| (2) | . 001044 | . 002682 | . 001214 |  | . 279 |
|  | (.2393) | (.8548) | (.4683) |  | (.7570) |
| (3) | . 001099 | . 002072 |  | . 0197 | . 345 |
|  | (.2033) | (.8877) |  | (.4172) | (.7084) |
| (4) | . 000453 | . 001800 | . 002986 | . 0446 | . 944 |
|  | (.6403) | (.9022) | (.1445) | (.1326) | (.4196) |

${ }^{\mathrm{a}}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 24
4-Year Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 4-year time period following option introduction (month 1 to month 48) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | $\begin{aligned} & .001118 \\ & (.1768) \end{aligned}$ | $\begin{aligned} & .035130 \\ & (.0147) \end{aligned}$ |  |  | $\begin{gathered} 6.012 \\ (.0147) \end{gathered}$ |
| (2) | $\begin{aligned} & .001280 \\ & (.1524) \end{aligned}$ | $\begin{aligned} & .035406 \\ & (.0141) \end{aligned}$ | $\begin{gathered} -.000666 \\ (.6296) \end{gathered}$ |  | $\begin{gathered} 3.116 \\ (.0455) \end{gathered}$ |
| (3) | $\begin{aligned} & .001186 \\ & (.1697) \end{aligned}$ | $\begin{aligned} & .035377 \\ & (.0143) \end{aligned}$ |  | $\begin{gathered} -.0050686 \\ (.7797) \end{gathered}$ | $\begin{gathered} 3.038 \\ (.0492) \end{gathered}$ |
| (4) | $\begin{aligned} & .001728 \\ & (.1071) \end{aligned}$ | $\begin{aligned} & .036635 \\ & (.0117) \end{aligned}$ | $\begin{gathered} -.001531 \\ (.3925) \end{gathered}$ | $\begin{aligned} & -.0179 \\ & (.4474) \end{aligned}$ | $\begin{gathered} 2.268 \\ (.0804) \end{gathered}$ |

${ }^{\mathrm{a}}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
$b_{\text {The }}$ F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 25
5-Year Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 5 -year time period following option introduction (month 1 to month 60) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBLLL |  |
| (1) | $\begin{aligned} & .001789 \\ & (.0110) \end{aligned}$ | $\begin{aligned} & .000556 \\ & (.9473) \end{aligned}$ |  |  | $\begin{gathered} .004 \\ (.9473) \end{gathered}$ |
| (2) | $\begin{aligned} & .002001 \\ & (.0192) \end{aligned}$ | $\begin{aligned} & .000237 \\ & (.9777) \end{aligned}$ | $\begin{gathered} -.000513 \\ (.6607) \end{gathered}$ |  | $\begin{gathered} .099 \\ (.9060) \end{gathered}$ |
| (3) | $\begin{aligned} & .001818 \\ & (.0107) \end{aligned}$ | $\begin{aligned} & .000983 \\ & (.9085) \end{aligned}$ |  | $\begin{gathered} -.0040008 \\ (.7794) \end{gathered}$ | $\begin{gathered} .041 \\ (.9594) \end{gathered}$ |
| (4) | $\begin{aligned} & .002457 \\ & (.0178) \end{aligned}$ | $\begin{aligned} & .001332 \\ & (.8765) \end{aligned}$ | $\begin{gathered} -.001351 \\ (.3946) \end{gathered}$ | $\begin{gathered} -.0152 \\ (.4344) \end{gathered}$ | $\begin{gathered} .270 \\ (.8471) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBLLL is the change in interest rates for 90 -day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 26
Year 2 Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period in the second year following option introduction (month 13 to month 24) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | . 000127 | . 107540 |  |  | 180.965 |
|  | (.7531) | (.0001) |  |  | (.0001) |
| (2) | . 000152 | . 107104 | -. 000478 |  | 91.307 |
|  | (.7122) | (.0001) | (.7461) |  | (.0001) |
| (3) | . 000109 | . 107659 |  | . 0106 | 90.476 |
|  | (.7876) | (.0001) |  | (.5671) | (.0001) |
| (4) | . 000133 | . 107240 | -. 000458 | . 0105 | 60.196 |
|  | (.7472) | (.0001) | (.7567) | (.5728) | (.0001) |

${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 27
Year 3 Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the l-year time period in the third year following option introduction (month 25 to month 36) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | . 000382 | . 048304 |  |  | 10.858 |
|  | (.4941) | (.0011) |  |  | (.0011) |
| (2) | . 000102 | . 048643 | . 001859 |  | 6.083 |
|  | (.8678) | (.0010) | (.2552) |  | (.0025) |
| (3) | . 000184 | . 047054 |  | . 0348 | 6.217 |
|  | (.7519) | (.0015) |  | (.2127) | (.0022) |
| (4) | -. 000471 | . 046768 | . 003384 | . 0600 | 5.346 |
|  | (.4863) | (.0015) | (.0616) | (.0528) | (.0013) |

${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90-day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 28
Year 4 Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period in the fourth year following option introduction (month 37 to month 48) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | . 000860 | . 041391 |  |  | 6.055 |
|  | (.1232) | (.0143) |  |  | (.0143) |
| (2) | . 000936 | . 041658 | -. 000758 |  | 3.069 |
|  | (.1246) | (.0140) | (.7550) |  | (.0477) |
| (3) | . 000835 | . 041579 |  | . 0036465 | 3.029 |
|  | (.1552) | (.0143) |  | (.8897) | (.0496) |
| (4) | . 000920 | . 041729 | -. 000719 | . 0016368 | 2.041 |
|  | (.1634) | (.0142) | (.7753) | (.9521) | (.1078) |

[^10]TABLE 29
Year 5 Regressions
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period in the fifth year following option introduction (month 49 to month 60) against various independent variables. The sample consists of option introduction dates for 358 securities over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | $\begin{gathered} -.000269 \\ (.5924) \end{gathered}$ |  |  |  | $\begin{gathered} 2.037 \\ (.1543) \end{gathered}$ |
| (2) | $\begin{gathered} -.000439 \\ (.4093) \end{gathered}$ | $\begin{gathered} -.012608 \\ (.1476) \end{gathered}$ | $\begin{aligned} & .001212 \\ & (.3278) \end{aligned}$ |  | $\begin{gathered} 1.499 \\ (.2248) \end{gathered}$ |
| (3) | $\begin{gathered} -.000250 \\ (.6252) \end{gathered}$ | $\begin{gathered} -.012494 \\ (.1523) \end{gathered}$ |  | $\begin{aligned} & .0043932 \\ & (.8421) \end{aligned}$ | $\begin{gathered} 1.036 \\ (.3560) \end{gathered}$ |
| (4) | $\begin{aligned} & -.000420 \\ & (.4311) \end{aligned}$ | $\begin{gathered} -.012988 \\ (.1372) \end{gathered}$ | $\begin{aligned} & .001528 \\ & (.2546) \end{aligned}$ | $\begin{gathered} .0148 \\ (.5361) \end{gathered}$ | $\begin{gathered} 1.125 \\ (.3388) \end{gathered}$ |

${ }^{{ }^{\mathrm{a}} \text { Variable definitions are as follows: SHORT is the change in the relative short }}$ interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90-day Treasury bills.
${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

# Hypotheses 2 and 3 Regressions for Various Sample <br> Partitions of Relative Option Open Interest 

Tables 30 and 31 show the tests for Hypotheses 2 and 3 repeated for a subset of the sample, eliminating the bottom $10 \%$ with close to zero relative option open interest. Tables 32 and 33 show these tests using only those observations which have relative option open interest above $1 \%$ one year after the introduction of options. The results from both these regressions are very similar to the regressions for the full sample. As a point of comparison, Tables 34 and 35 show the two tests for those observations with relative option open interest below 1\%. These regressions do not show the same significant results as in the full sample or in subsamples with high option open interest. This points to the importance of option open interest in explaining abnormal returns from the event study.

TABLE 30
1-Year Cross-Sectional Regressions for Subsample of 322
(Top $90 \%$ Relative Option Open Interest)
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The subsample consists of option introduction dates for 322 securities over the 1973-1987 time period after eliminating the bottom $10 \%$ with relative option open interest near zero. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.000737 \\ (.9645) \end{gathered}$ | $\begin{gathered} -.387755 \\ (.0213) \end{gathered}$ |  |  | $\begin{gathered} 5.352 \\ (.0213) \end{gathered}$ |
| (2) | $\begin{gathered} -.000935 \\ (.9552) \end{gathered}$ | $\begin{gathered} -.386085 \\ (.0224) \end{gathered}$ | $\begin{gathered} -.009304 \\ (.8820) \end{gathered}$ |  | $\begin{gathered} 2.679 \\ (.0702) \end{gathered}$ |
| (3) | $\begin{gathered} -.014697 \\ (.3936) \end{gathered}$ | $\begin{gathered} -.387447 \\ (.0202) \end{gathered}$ |  | $\begin{aligned} & .435636 \\ & (.0075) \end{aligned}$ | $\begin{gathered} 6.348 \\ (.0020) \end{gathered}$ |
| (4) | $\begin{gathered} -.014760 \\ (.3933) \end{gathered}$ | $\begin{gathered} -.386836 \\ (.0209) \end{gathered}$ | $\begin{gathered} -.003404 \\ (.9563) \end{gathered}$ | $\begin{aligned} & .435321 \\ & (.0077) \end{aligned}$ | $\begin{aligned} & 4.220 \\ & (.0060) \end{aligned}$ |
| (5) | $\begin{aligned} & -.014418 \\ & (.3583) \end{aligned}$ |  | $\begin{gathered} -.018909 \\ (.7639) \end{gathered}$ |  | $\begin{gathered} .090 \\ (.7639) \end{gathered}$ |
| (6) | $\begin{gathered} -.028091 \\ (.0864) \end{gathered}$ |  |  | $\begin{aligned} & .435897 \\ & (.0079) \end{aligned}$ | $\begin{gathered} 7.150 \\ (.0079) \end{gathered}$ |

[^11]TABLE 31
1-Year Regressions for Subsample of 322
(Top $90 \%$ Relative Option Open Interest)
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The subsample consists of option introduction dates for 322 securities over the 1973-1987 time period after eliminating the bottom $10 \%$ with relative option open interest near zero. P-values are in parentheses.


[^12]TABLE 32
1-Year Cross-Sectional Regressions for Subsample of 182
(> 1\% Relative Option Open Interest)
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The subsample consists of option introduction dates for 182 securities which have relative option open interest above $1 \%$ of shares outstanding over the 1973-1987 time period. P -values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.010721 \\ (.6562) \end{gathered}$ | $\begin{gathered} -.360121 \\ (.0507) \end{gathered}$ |  |  | $\begin{gathered} 3.869 \\ (.0507) \end{gathered}$ |
| (2) | $\begin{gathered} -.009760 \\ (.6886) \end{gathered}$ | $\begin{gathered} -.359689 \\ (.0516) \end{gathered}$ | $\begin{gathered} -.032738 \\ (.7637) \end{gathered}$ |  | $\begin{gathered} 1.970 \\ (.1424) \end{gathered}$ |
| (3) | $\begin{gathered} -.034114 \\ (.1786) \end{gathered}$ | $\begin{gathered} -.340576 \\ (.0605) \end{gathered}$ |  | $\begin{aligned} & .545929 \\ & (.0090) \end{aligned}$ | $\begin{gathered} 5.482 \\ (.0049) \end{gathered}$ |
| (4) | $\begin{gathered} -.033719 \\ (.1898) \end{gathered}$ | $\begin{gathered} -.340489 \\ (.0613) \end{gathered}$ | $\begin{gathered} -.011049 \\ (.9181) \end{gathered}$ | $\begin{aligned} & .544275 \\ & (.0097) \end{aligned}$ | $\begin{gathered} 3.638 \\ (.0140) \end{gathered}$ |
| (5) | $\begin{gathered} -.030388 \\ (.1707) \end{gathered}$ |  | $\begin{gathered} -.034404 \\ (.7539) \end{gathered}$ |  | $\begin{gathered} .099 \\ (.7539) \end{gathered}$ |
| (6) | $\begin{gathered} -.054349 \\ (.0194) \end{gathered}$ |  |  | $\begin{aligned} & .561983 \\ & (.0076) \end{aligned}$ | $\begin{gathered} 7.290 \\ (.0076) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
${ }^{\mathbf{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 33

> 1-Year Regressions for Subsample of 182
> ( $>1 \%$ Relative Option Open Interest)
> Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The subsample consists of option introduction dates for 182 securities which have relative option open interest above $1 \%$ of shares outstanding over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | $\begin{gathered} -.003317 \\ (.0083) \end{gathered}$ | $\begin{aligned} & .080193 \\ & (.0001) \end{aligned}$ |  |  | $\begin{aligned} & 71.754 \\ & (.0001) \end{aligned}$ |
| (2) | $\begin{gathered} -.003369 \\ (.0081) \end{gathered}$ | $\begin{aligned} & .080169 \\ & (.0001) \end{aligned}$ | $\begin{aligned} & .001767 \\ & (.7536) \end{aligned}$ |  | $\begin{aligned} & 35.747 \\ & (.0001) \end{aligned}$ |
| (3) | $\begin{gathered} -.003600 \\ (.0051) \end{gathered}$ | $\begin{aligned} & .079822 \\ & (.0001) \end{aligned}$ |  | $\begin{aligned} & .000406 \\ & (.2824) \end{aligned}$ | $\begin{aligned} & 36.491 \\ & (.0001) \end{aligned}$ |
| (4) | $\begin{gathered} -.003686 \\ (.0047) \end{gathered}$ | $\begin{aligned} & .079772 \\ & (.0001) \end{aligned}$ | $\begin{aligned} & .002490 \\ & (.6602) \end{aligned}$ | $\begin{aligned} & .000425 \\ & (.2646) \end{aligned}$ | $\begin{aligned} & 24.282 \\ & (.0001) \end{aligned}$ |
| (5) | $\begin{aligned} & .001229 \\ & (.3592) \end{aligned}$ |  | $\begin{aligned} & .002138 \\ & (.7474) \end{aligned}$ |  | $\begin{gathered} .104 \\ (.7474) \end{gathered}$ |
| (6) | $\begin{aligned} & .000903 \\ & (.5069) \end{aligned}$ |  |  | $\begin{aligned} & .052074 \\ & (.2415) \end{aligned}$ | $\begin{gathered} 1.381 \\ (.2415) \end{gathered}$ |

[^13]TABLE 34
1-Year Cross-Sectional Regressions for Subsample of 176
( $<1 \%$ Relative Option Open Interest)
Dependent Variable: AHPAR

Below are estimated coefficients for a cross-sectional regression of average holding period abnormal returns (AHPAR) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The subsample consists of option introduction dates for 176 securities which have relative option open interest below $1 \%$ of shares outstanding over the 1973-1987 time period. P -values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | INST |  |
| (1) | $\begin{gathered} -.034406 \\ (.2333) \end{gathered}$ | $\begin{gathered} 9.189081 \\ (.1217) \end{gathered}$ |  |  | $\begin{gathered} 2.419 \\ (.1217) \end{gathered}$ |
| (2) | $\begin{gathered} -.036837 \\ (.2649) \end{gathered}$ | $\begin{gathered} 9.528440 \\ (.1338) \end{gathered}$ | $\begin{gathered} -.008951 \\ (.8785) \end{gathered}$ |  | $\begin{gathered} 1.215 \\ (.2994) \end{gathered}$ |
| (3) | $\begin{gathered} -.034936 \\ (.2266) \end{gathered}$ | $\begin{gathered} 8.296487 \\ (.1680) \end{gathered}$ |  | $\begin{aligned} & .222056 \\ & (.3655) \end{aligned}$ | $\begin{gathered} 1.620 \\ (.2009) \end{gathered}$ |
| (4) | $\begin{gathered} -.036419 \\ (.2706) \end{gathered}$ | $\begin{gathered} 8.510138 \\ (.1875) \end{gathered}$ | $\begin{gathered} -.005474 \\ (.9257) \end{gathered}$ | $\begin{aligned} & .220537 \\ & (.3712) \end{aligned}$ | $\begin{gathered} 1.077 \\ (.3605) \end{gathered}$ |
| (5) | $\begin{aligned} & .003019 \\ & (.8782) \end{aligned}$ |  | $\begin{aligned} & .021920 \\ & (.6906) \end{aligned}$ |  | $\begin{gathered} .159 \\ (.6906) \end{gathered}$ |
| (6) | $\begin{gathered} -.004669 \\ (.8040) \end{gathered}$ |  |  | $\begin{aligned} & .277692 \\ & (.2529) \end{aligned}$ | $\begin{gathered} 1.316 \\ (.2529) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
$\mathrm{b}_{\text {The }} \mathrm{F}$-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

TABLE 35
1-Year Regressions for Subsample of 176
( $<1 \%$ Relative Option Open Interest)
Dependent Variable: SHORT

Below are estimated coefficients for a regression of the change in relative short interest (SHORT) for the 1 -year time period following option introduction (month 1 to month 12) against various independent variables. The subsample consists of option introduction dates for 176 securities which have relative option open interest below $1 \%$ of shares outstanding over the 1973-1987 time period. P-values are in parentheses.

|  | Independent Variables ${ }^{\text {a }}$ |  |  |  | F-Statistic ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Intercept | OPEN | INDEX | TBILL |  |
| (1) | $\begin{array}{r} -.00713 \\ (.4543) \end{array}$ | $\begin{aligned} & .300523 \\ & (.1254) \end{aligned}$ |  |  | $\begin{gathered} 2.371 \\ (.1254) \end{gathered}$ |
| (2) | $\begin{gathered} -.000885 \\ (.4168) \end{gathered}$ | $\begin{aligned} & .324602 \\ & (.1221) \end{aligned}$ | $\begin{gathered} -.000635 \\ (.7427) \end{gathered}$ |  | $\begin{gathered} 1.234 \\ (.2938) \end{gathered}$ |
| (3) | $\begin{gathered} -.000550 \\ (.5604) \end{gathered}$ | $\begin{aligned} & .254504 \\ & (.1916) \end{aligned}$ |  | $\begin{gathered} -.062144 \\ (.0284) \end{gathered}$ | $\begin{gathered} 3.656 \\ (.0279) \end{gathered}$ |
| (4) | $\begin{gathered} -.000869 \\ (.4200) \end{gathered}$ | $\begin{aligned} & .298170 \\ & (.1511) \end{aligned}$ | $\begin{gathered} -.001196 \\ (.5351) \end{gathered}$ | $\begin{gathered} -.064412 \\ (.0246) \end{gathered}$ | $\begin{gathered} 2.557 \\ (.0569) \end{gathered}$ |
| (5) | $\begin{aligned} & .000472 \\ & (.4683) \end{aligned}$ |  | $\begin{aligned} & .000417 \\ & (.8189) \end{aligned}$ |  | $\begin{gathered} .053 \\ (.8189) \end{gathered}$ |
| (6) | $\begin{aligned} & .000408 \\ & (.4957) \end{aligned}$ |  |  | $\begin{aligned} & -.066095 \\ & (.0194) \end{aligned}$ | $\begin{gathered} 5.570 \\ (.0194) \end{gathered}$ |

${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90-day Treasury bills.
${ }^{\mathrm{b}}$ The F -statistic tests the hypothesis that all coefficients are different from zero. The p -values are shown in parentheses under the F -statistics.

## $v$ <br> VITA

Larry C. Holland

Candidate for the Degree of
Doctor of Philosophy

# Thesis: THE EFFECT ON THE UNDERLYING STOCK FROM THE INTRODUCTION OF AN OPTIONS MARKET: A UTILITY PREFERENCE APPROACH 

Major Field: Business Administration
Biographical:
Personal Data: Born in St. Louis, Missouri, on February 9, 1947, the son of James W. Holland and Evelyn T. Holland. Married to Sandra M. Holland and the father of Chelsea C. Holland and Curtis W. Holland.

Education: Graduated from Farmington Senior High School, Farmington, Missouri in May, 1964; received a Bachelor of Science degree in Chemical Engineering from the University of Missouri at Rolla in May, 1968. Received a Masters degree in Business Administration from the University of Tulsa in December, 1975. Completed the requirements for a Ph.D. degree in Business Administration from Oklahoma State University in December, 1994.

Experience: Employed by Oklahoma State University as a lecturer from Fall, 1994 to present and a graduate teaching assistant from Spring, 1992, to Spring, 1994; twenty years of previous experience with various positions in the natural resources industry, including the areas of management information systems, budgeting, corporate planning, productivity improvement, environmental control, and engineering.

Professional Memberships: Financial Management Association, Southwest Financial Management Association, and Southern Financial Association.


[^0]:    ${ }^{1}$ Option open interest is the number of outstanding contracts to buy or sell 100 share lots of the underlying stock.

[^1]:    ${ }^{2}$ See Coyne (1991) or the Code of Federal Regulations 12 CFR 220.5 (b)(1) and 12 CFR 220.18.

[^2]:    ${ }^{3}$ See Federal Reserve Regulation 12 CFR 220.12 (b)(3)(i).
    ${ }^{4}$ See SEC Regulation 17 CFR 240.10a-1 and 240.10a-2.

[^3]:    ${ }^{5}$ See SEC Regulation 17 CFR 240.10a-1(3)(e)(5).

[^4]:    ${ }^{6}$ See Figlewski (1981), page 475.

[^5]:    ${ }^{7}$ Copeland and Weston (1988) note that this power function exhibits intuitively plausible properties consistent with empirical results of Friend and Blume (1975), who use IRS data to estimate that absolute risk aversion decreases with wealth while relative risk aversion is constant at a level of 2.

[^6]:    ${ }^{8}$ If some stockholders choose not to offer for sale the shares they hold when the stock price is too low, it is possible for the supply curve to be kinked at low stock prices (a reservation price effect). As long as this supply curve is not negatively sloped, the effect on the underlying stock price remains directionally the same.

[^7]:    ${ }^{8}$ Dimson and Marsh (1986) and Lakonishok and Vermaelen (1990) use the same test procedure for calculating long-term abnormal returns. These studies show nearly identical results for a size-based control approach (as in Equation 26) and a more involved approach correcting for both differences in size and changes in beta. The methodology used in this dissertation is based on the simpler approach of correcting only for size.

[^8]:    ${ }^{9}$ The Chicago Board Options Exchange is acknowledged for their assistance in supplying introduction dates.

[^9]:    ${ }^{10}$ Brent, Morse, and Stice (1990) demonstrate a relationship between option expiration and a decline in short interest such as described here.

[^10]:    ${ }^{\mathrm{a}}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
    $\mathbf{b}_{\text {The }}$ F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F -statistics.

[^11]:    ${ }^{\mathrm{a}}$ Variable definitions are as follows: AHPAR is the average holding period abnormal return, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and INST is the change in relative institutional holdings.
    ${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

[^12]:    ${ }^{\mathrm{a}}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
    ${ }^{\mathrm{b}}$ The F-statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

[^13]:    ${ }^{\text {a }}$ Variable definitions are as follows: SHORT is the change in the relative short interest, OPEN is the change in relative option open interest, INDEX is the change in index option open interest, and TBILL is the change in interest rates for 90 -day Treasury bills.
    ${ }^{6}$ The F -statistic tests the hypothesis that all coefficients are different from zero. The p-values are shown in parentheses under the F-statistics.

