A STUDY OF THE EFFECTS OF REFORM TEACHING METHODOLOGY AND VAN HIELE LEVEL ON CONCEPTUAL LEARNING IN CALCULUS

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CHAPTER I

THE RESEARCH PROBLEM

Introduction

Thirteen years have passed since the first calculus reform meeting at Tulane University when many questions were asked regarding the purpose of requiring calculus as part of the degree plan for a wide diversity of disciplines. Several computer scientists and mathematicians contended that with the rapidly changing technological climate a change in the mathematics curriculum was necessary (Conley, 1997). Their recommendation was a move away from calculus and a move toward finite mathematics (Douglas, 1986). The effectiveness of calculus instruction was also an integral component of the discussions. Several complaints were noted in the methods workshop (Douglas, 1986). Among these complaints were:

- Calculus is used to weed out students from mathematics and the sciences.
- Most calculus courses do not develop conceptual understanding.
- Calculus is very narrow in its scope of thinking and expectations of students.
- Calculus as currently taught is irrelevant for nearly 50% of the students taking it.

The majority of the participants at the Tulane conference agreed that changes in both content and teaching must be made (Douglas, 1986). However, there were those participants who believed that sweeping changes in calculus content and teaching were not necessary. The debate continued the following year when the colloquium *Calculus* *for a New Century: A Pump, Not a Filter* convened in Washington, D.C. One of the primary questions that continues to be addressed is the appropriateness of the methodology employed in teaching college students fundamental calculus concepts. Becker (1990) suggests that research at the college level is needed to determine the impact new teaching methods will have on student learning.

Traditionally, calculus has been a course in which an instructor lectures to a class quite large in number without much interactive questioning or discussion (Solow, 1994). The instructor assigns substantial amounts of homework that is usually not graded, and bases the evaluation of the students on exams alone (Tucker, 1995). Gradually calculators have been incorporated into the course content, and, within the last few years, the use of calculus software has been included in the course outline in many institutions. According to Tucker (1995), the use of graphing calculators and mathematical computing software is now so widespread that these items are no longer considered part of the calculus reform effort.

According to Steen (1987), there are three areas related to calculus teaching where significant changes are taking place. The first area is the training of calculus teaching assistants. The second area is the effect that student writing has on achievement in calculus content. The final concern lies in the implementation of a constructivist approach in the teaching and learning of calculus. The primary focus of this study will be in the third area with secondary research directed at the correlation of geometric reasoning to conceptual reasoning in calculus.

Statement of the Problem

This study will focus on calculus instruction at the college level. Most of the research regarding calculus reform deals with the use of calculators and computer software in the calculus classroom. However, very little research has been done with regard to the impact of teaching strategies and learning styles when applied to the teaching of the calculus. One purpose of the study is to provide information as to whether changes in modes of instruction affect the level of learning in a beginning calculus course. Secondly, the study is to provide information as to whether there is a relationship between conceptual understanding in geometry and conceptual understanding in calculus. Specifically, the questions under investigation are:

1) In what way does the use of a constructivist approach to instruction affect student learning of the geometric concept of derivative in a first semester calculus course?
2) What is the relationship between a student's van Hiele level and that student's geometric conceptual understanding of the derivative in a first semester calculus course?

These questions will be investigated using several different instruments. The first instrument to be employed is a placement exam mandatory for all students beginning the mathematics/science/computer science course sequences. The second instrument is a test based on the van Hiele model of geometric understanding, with particular attention paid to problems involving tangent and secant concepts. The students in two different sections of Calculus I will be presented material relating to the first derivative and its geometric interpretation. Section one will be conducted using traditional instructional techniques, which include primarily lecture and demonstration. Section two will be conducted using a constructivist approach to instruction. The primary vehicles for aiding the students in the construction of their knowledge about the first derivative will be group work, discussion, and writing. Both classes will be administered a post-test after completing the section that discusses derivatives. The post-test will contain problems requiring procedural understanding as well as items which will require conceptual understanding. Thus the study will be quantitative in nature. The population for this will be composed of freshman and sophomore Calculus I students enrolled during the 1998 fall semester at a private mid-western university.

Importance of the Study

The primary goal of calculus reform is to encourage students to pursue fields that employ the use of mathematics. A secondary goal of the reform movement is the improvement of students' conceptual understanding, mathematical reasoning skills, and problem-solving abilities. The third goal of the calculus reform movement is for students to see the beauty, power, and pure enjoyment of learning calculus. According to Tucker (1995) in the first assessment study since implementation of the reform, "a key finding of the assessment study has been that *how* calculus is taught has changed more than *what* is taught" (p. 5). Lucas (1998) observed that following the adoption of a reform calculus text for her AP Calculus class, the methodology employed in the classroom changed drastically. By group investigation and discovery, the students became competent conceptual learners in calculus. As a result of these methodology changes, the reported AP Calculus test scores rose substantially. This study will investigate whether the constructivist approach in instruction does indeed aid the calculus reform movement in attaining these goals for student learning.

Definitions

For the remainder of this study, the following terms will be used. The meanings are discussed below.

Let f be a function and suppose that f is defined at the point c. The derivative of a function f for our study will be defined both geometrically and algebraically. Geometrically, the derivative of a function f at any point x is the slope of the tangent line that touches f at x. The idea is represented in Figure 1.1.

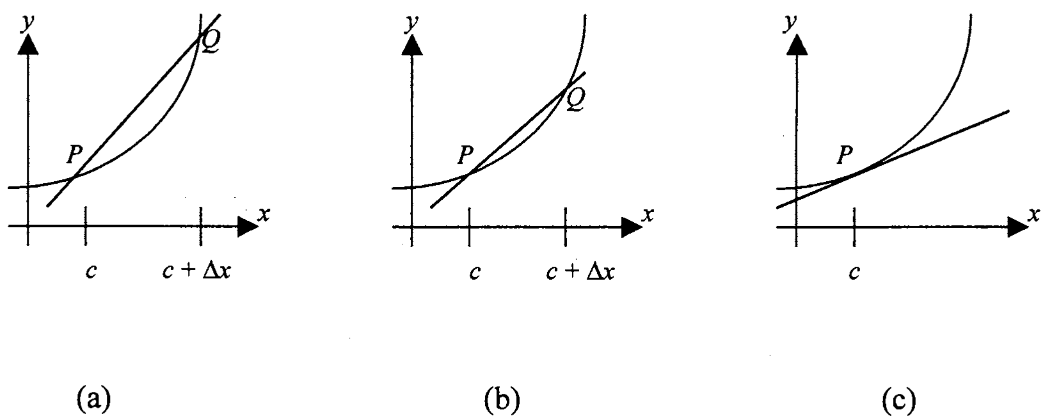

Figure 1.1

In Figure 1.1 (a), the line drawn from point P to point Q forms the secant line PQ. The slope of PQ is found by:

$$m = \frac{f(c + \Delta x) - f(c)}{(c + \Delta x) - c} = \frac{f(c + \Delta x) - f(c)}{\Delta x}.$$

By letting Δx become small, Q moves closer to P as in Figure 1.1 (b), and the secant lines approach the tangent line at $x = c$, as indicated in Figure 1.1 (c). Thus, to find the slope of the tangent line to a curve f at any x in the domain of f, the derivative is defined by

$$\lim \Delta x \to 0 \frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

Constructivism is the educational theory whereby students construct their own knowledge by either a natural ability to think, by learning from the environment, or by a combination of both a natural ability and environmental influences. The result is an autonomous, intellectual student (Kamii, 1985).

A traditional calculus course refers to both the teaching methodology employed in a typical Calculus I course in colleges and universities and the design of the course itself. Traditional methodology usually consists of lectures, demonstrations, homework assignments, and examinations (Tucker, 1995). The design of a traditional calculus class includes lecture sessions aimed at large numbers of students, a textbook with outdated applications and hundreds of rote exercises, and little or no computer/calculator applications (Tucker, 1995).

A reform calculus course refers also to both the teaching methodology employed and the design of the course itself. Though the level or reform varies among colleges and universities involved in the movement, at the very least a reform calculus course uses a text which de-emphasizes integration techniques and encourages graphing calculators. This is coupled with class sizes relatively small in number, usually 25 to 30 students. More radically reformed courses employ computer lab work, cooperative learning, open-ended projects, writing assignments and an increased emphasis on modeling and applications (Steen, 1987).

A van Hiele level is a classification level of geometric thinking established according to the theory of Pierre and Dina van Hiele (Clements, 1992). The van Hiele theory asserts that students progress through sequential, hierarchical levels of thought in the process of learning geometry (Clements, 1992). The van Hiele levels are as follows:

Level 1: Visual. Students identify shapes according to their appearance. They do not attend to the properties nor are they conscious of the properties of the shapes.

Level 2: Descriptive/Analytic. Students recognize and identify shapes according to their properties.

Level 3. Abstract/Relational. Students are able to form abstract definitions and can sometimes provide logical arguments geometrically. They discover properties of classes of geometric objects by informal deduction.

Level 4: Formal Deduction. Students are capable of establishing theorems within an axiomatic system and can construct original proofs. Their reasoning is based on relationships between properties of classes of figures.

Level 5: Rigor/Mathematical. Students can reason formally by manipulating definitions, theorems, and axioms within a mathematical system. Their reasoning is based on relationships between formal constructs.

Procedural knowledge is knowledge that is algorithmic in nature and involves knowledge of rules and symbols (Hiebert & Lefever, 1986). This knowledge includes awareness of surface features, not knowledge of meaning.

Conceptual knowledge is knowledge that involves relationships and assimilation of ideas (Hiebert & Lefever, 1986). This knowledge includes a network or relationships between old and new information.

Assumptions

It will be assumed that the students will respond to the instruments with integrity and that the means of student ability in both sections of Calculus I are relatively the same. This is due primarily to the fact that the students entering the calculus sequence come from comparable mathematical backgrounds and are placed in Calculus I because of their score on the placement exam.

Limitations

This study will be limited in scope since the sample will be drawn from the university's students; thus the sample is not a true random sample. The results of the study will generalize only to colleges or universities that conduct their calculus courses in a similar manner.

Another limitation of the study results from the use of writing as a means of evaluation. Gay's 1990 study acknowledges limitations of test items that include the writing skills of the persons completing the instrument. Since the post-test will include questions which requires a written answer, the lack of ability or experience in providing written answers to mathematics problems may affect the responses to the post-test item.

After the data was collected, the researcher decided that questions 2(b) and 2(c) should be reworded for future reference. Though the results were not significantly affected by omitting the question, it was decided that more mathematical accuracy should be included in the questions. For 2(b) specifically, the researcher suggests that either point *a* or point *b* be fixed so as to not confuse the students when discussing the slope of the secant line. Question 2(c) should be worded so that the slope of the tangent line at a point on the curve is represented as a limit. The question could possibly be restated as follows: Discuss what you are finding if you take the limit of the slope of the secant line as point *b* approaches point *a* on the curve. Question 3 could possibly be even more conceptual in nature if the tangent line is not included in the sketch of the curve. Question 3 would then become one of discussion, as opposed to one of determining a specific value at a point.

Overview

This study will be divided into five chapters. The first chapter will present the statement of the problem, while chapter II will provide a review of the literature that has provided the basis for this study. A brief history of the calculus reform movement and studies conducted within the reform movement will be cited, as well as studies that have investigated students' knowledge of the derivative. Works pertaining to the constructivist theory of learning will also be cited. Studies conducted that compare procedural learning to conceptual learning will be included, and works investigating the van Hiele levels of geometric reasoning will be discussed. Chapter III will discuss the research design to be employed and the processes of collection and analysis of data. The results of the analysis of quantitative data used during the study will be discussed in Chapter IV, and Chapter V will present conclusions and recommendations for future study.

Chapter II

REVIEW OF THE LITERATURE

Introduction

Most of the research regarding calculus reform concerns the use of calculators and computer software in the calculus classroom. In comparison, much less research has been conducted with regard to the impact of teaching strategies and learning styles when applied to the teaching of the calculus. The literature used as a background and basis for this study is categorized as: 1) history of calculus reform, 2) constructivism in mathematics, 3) procedural and conceptual learning in mathematics, and 4) van Hiele theory and learning in geometry. A history of calculus reform is included since the methodologies employed in the study are based on the philosophies of the calculus reform movement. Since constructivism in teaching is at the heart of the reform movement and since two significant components of constructivism in mathematics education are writing and group work, a body of literature on constructivism is included in this study. Constructivism is closely linked to conceptual learning; therefore, a body of literature with a comparison of procedural to conceptual learning is included. Finally, a body of literature that defines and discusses van Hiele levels is included to provide a basis for relating geometric knowledge to geometric knowledge of the derivative.

History of Calculus Reform

The place of calculus in the mathematics curriculum was unquestioned until around 1981, when several computer scientists and mathematicians called for the inclusion of discrete mathematics in the curriculum in place of, or in addition to, calculus. With the ever increasing computational power of the personal computer and scientific calculator, the primary argument was that a mathematics course was needed that dealt with computational theory and issues rather than the continuous cases investigated in calculus (Douglas, 1986). This siege waged upon the study of calculus caused many mathematicians to reflect upon the role of calculus in the college curriculum. After five years of fragmented debate, the first conference on calculus reform convened at Tulane University. Spearheaded by Ronald Douglas, the conference was entitled "*Toward a Lean and Lively Calculus*." Many problems were discussed, such as the high failure rate of students taking calculus, the lack of mathematics majors in the United States, and the lack of personal contact with students taking calculus courses (Douglas, 1986). The discussions held at this conference paved the way for the calculus reform movement.

For nearly two years after that conference there was much activity in the calculus reform arena. The Neal Report to the National Science Board (1986) called for renewed support from the National Science Foundation in the support for undergraduate education. There was also published a mathematics discipline workshop report which called for change in calculus. This report and the call for calculus reform were enthusiastically supported by a small core of mathematicians in all areas of higher education and were also supported by the engineering community.

The calculus reform notion was fueled by a movement in the mathematics community, which was driven by the National Council of Teachers of Mathematics (NCTM). The NCTM was founded in 1920 to assure the place of mathematics in the secondary curriculum, but it was not until the 1980's that it took a leadership role in promotion of mathematics reform (Kilpatrick, 1997). With the publication of Curriculum and Evaluation Standards for School Mathematics, the NCTM sought to provide a vision of the future (Kilpatrick, 1997). The arguments given for reform of school mathematics curriculum, instruction, and assessment rested on the idea that since all "industrialized nations have experienced a shift from an industrial to an information society," (NCTM, pg. 3), the mathematics that students need to know has changed also. A substantial portion of the standards-based reform comes from the position that mathematics has become more computational and less formal. Thus, with the advent of computer software and graphing calculators that can do messy calculations, students can perhaps focus more on the concepts to be learned as opposed to exercise after exercise of meaningless hand calculations. Most observers of mathematics classrooms have been convinced that some type of change was and still is needed (Kilpatrick 1997). This period of reform activity culminated in a formal announcement of the National Science Foundation Calculus program and a colloquium on calculus reform hosted by the National Research Council (Haver, 1998).

A colloquium on calculus reform convened in 1987 in Washington, D.C. and boasted support from 700 participants. This conference, an extension of the Tulane meeting, was entitled "*Calculus for the New Century: A Pump, Not a Filter*." The problems with the current status of the calculus were restated and presented in fuller detail. Issues discussed included the following:

- Since a college education is more commonplace than 40 or 50 years ago, there are many calculus students whose high school mathematics backgrounds are inadequate (Kolata, 1987).
- Different departments want calculus in their degree requirements, thus calculus has been altered to serve these client's disciplines (Steen, 1987).
- Calculus has become a course of mathematical manipulation, with not much emphasis being placed on problem solving and intuition (Steen, 1987).
- During the late sixties and early seventies, there was a change in the complexion of the typical calculus class. More students were embarking upon math-related careers, and there was less money available for instruction. As a result, class sizes increased dramatically, and personal involvement from the instructor was minimized (Steen, 1987).
- Students and faculty alike were generally dissatisfied with the current calculus textbooks, due to their unreadability and lack of word problems (Reed, 1987).

The participants provided many possible solutions to these problems. Some of these solutions included: writing a new breed of calculus textbook; encouraging more personal involvement from the calculus instructor, including collecting and grading homework; integrating graphing calculators and computer software into the calculus course; and introducing new teaching strategies to enhance the learning of calculus.

The Calculus Program had from its beginning the goal of bringing about large-scale improvement in Calculus instruction for all students throughout the nation. The program's primary focus was to raise "students' conceptual understanding, problem-solving skills, analytical and transference skills, while implementing new methods to reduce tedious calculations." (Haver, p. 2). The attack was to be two-directional: high school, college, and university mathematics leaders and professional societies were to be closely involved in all phases of the reform; and calculus reform textbooks and other instructional materials were to be developed, tested, and refined for all students, not just a chosen few. (Haver, 1998).

So the reform movement truly began. Many colleges and universities that believed in the reform movement reduced calculus class sizes to between 20 and 35 students (Solow, 1994). In addition, many universities opted to change Calculus to a four hour per week course, with a day of computer lab work included, thus allowing students the opportunity to use mathematical software to reinforce concepts and numerically perform differentiation and integration. Another technological addition was the requirement of a graphing calculator for use in class with concept investigations and for use on homework (Haver, 1998).

Many calculus instructors began to change their approach to how they presented material. Rather than adhering to a format of standing in front of the room and lecturing to students without any interaction, these instructors began to require that students be involved in and more accountable for their learning. Students were to be required to write about the concepts they were encountering. This was accomplished by writing in journals, homework assignments containing writing sections (Keith, 1994), or by written portions on calculus exams (Lucas, 1998). Students were also required to collaborate with other students to discuss concepts and investigate open-ended problems (Shenk, 1994). Technology was introduced into the calculus courses in the forms of graphing calculators and mathematical computer software (Cannon, 1994). Projects were incorporated into the calculus curriculum at many institutions, with these projects usually consisting of problems that might require two or three weeks to complete (Selden, 1994).

At some institutions, students unaccustomed to the new format were strongly encouraged to meet with the instructor of the reform course prior to enrollment (Simmons, 1994).

One of the most prominent pieces of evidence from the reform movement is a calculus text-book written by Deborah Hughes-Hallett and Andrew Gleason of Harvard (1994). The textbook is often referred to as the Harvard calculus text book. As is the case with many other reform text books, the Harvard text examines the calculus from three perspectives: algebraically, graphically, and numerically (Culotta, 1992). Many of the exercises in the text require numerical or graphical methods to find their solutions and contain more problems that are relevant to modern mathematical trends (Knisley, 1997), thus incorporating technology with mathematical learning (Hughes-Hallett, 1994).

The calculus reform movement also caused the mathematics community as a whole to exchange ideas and to think collectively about the classes that are being offered to students (Ipina, 1994). This type of discourse serves only to strengthen the mathematics curriculum and the bond among mathematics educators.

To say that not all mathematics instructors have been supportive of the reform effort is an understatement. In fact, there has been quite a division among mathematicians as to whether calculus-both content and teaching-should have been changed at all. There are many points of opposition, among them being:

- The reform textbooks do not treat concepts with enough rigor. The reform textbooks do not include enough proof, and the "proofs" that they do contain are not really proofs at all, just heuristic arguments (Wu, 1997).
- Reform textbooks have many exercises that are just too open-ended, and there are not enough good problems contained in them (Wu, 1997).

- Good students are sacrificed in the quest to reach marginal students (Holden, 1998).
- Knowledge of arithmetic and algebra in students starting calculus has fallen, and the reform effort has tried to compensate for this by emphasizing the use of graphing calculators (Askey, 1997).

Wu (1997) is not alone in his criticism of reform textbooks. Weintraub (1997) also bemoans the lack of logical structure in the Harvard text, which is replaced instead with an "incoherent mass of assertions that one has to accept on the authority of the authors (pg. B5). Swann (1997), who believes that texts such as the Harvard text encourage the abandonment of rigor in calculus echoes this sentiment. Askey (1997) believes that there are some mathematics results that are more important than others to know, that mathematical definitions should not be altered in any way without very good reason, and that reform text books do a disservice to students in both these areas. Roberts (1997) warns that though the reform movement calls for a leaner calculus text, leaner does not imply better. Additionally, Balakrishnan (1997) feels that calculus has been tampered with too much already.

Askey (1997) supports Wu's statement that there are not enough good problems in reform texts. He points out that there are many important, rigorous topics that were treated in pre-reform algebra and pre-calculus books that are not discussed even in current calculus texts. Bookman and Friedman (1998) report that many students in reformed calculus courses get very frustrated at not having a way to verify their answers, and many good math students do not get enough practice with symbolic manipulation. The findings from a survey that included student interviews echo this opinion (Holden, 1998). According to this survey, opposition to reform calculus comes primarily from the very top students in the class. These students oppose the notion of being treated as guinea pigs and feel that they are being cheated out of instruction of important mathematical skills.

Wu (1997) joins Askey (1997) in his opposition to constructivist teaching practices because he believes that teachers are not allowed to be leaders and provide insight for students struggling with concepts. Students do not get the appropriate practice with calculations when in the group setting, thus reducing the mathematics abilities of these students. In addition, Wilson (1997) believes that the reliance on computers and calculators has caused students to never learn to perform difficult calculations.

Many calculus instructors that have been using the reform calculus format for some time, however, believe that the change has brought a major improvement with regard to students' conceptual understanding. Maher (1997) suggests that, rather than using time and energy to debate about the reform texts, mathematics educators should instead use that time and energy to research why it is that the reform texts have been so successful. Lucas (1998) reports that with the use of reform materials, her calculus students are better at recognizing the different methods of differentiation and integration, they are more flexible in their thinking with regard to calculus problems, and that overall they are much better problem solvers. Lock (1997) refers to two studies that have compared the calculus skills of traditional calculus students versus reform-calculus students. In both of these studies, the reform-calculus students performed at a higher level than the traditional calculus students did. Garner (1998) provides results in his study that point to reform calculus students having a higher performance on conceptual problems. Russ (1997) suggests that it is not enough that calculus course content is changed; upper level mathematics courses should be altered also to reflect these changes. These differences of perspective will be explored more in the following paragraphs.

Constructivism

In Sinclair's (1987) discussion of constructivism, she states that Piaget (1973) believed students learn first and foremost through actions. Therefore it is behavior that makes some sort of change to a students' knowledge base (Kilpatrick, 1987). And conceptual knowledge is attained only when it has been discussed with and by others (Vergnaud, 1987). Von Glaserfield (1995) assumes that knowledge is not transmitted but instead is constructed. Students have only their personal experiences on which to rely in the constructive process.

Cobb (1991) describes the constructivist perspective of mathematical learning not as a process of internalizing carefully packaged knowledge, but as a matter of reorganizing activity, where that activity can be interpreted broadly to include conceptual activity or thought" (p. 5). Students do not enter a calculus class with blank slates, but rather with some preconception of what mathematics should be. Thus the mathematical images that students construct are more than likely quite different from the ones that instructors are trying to convey to them (Steen, 1987). Therefore, it is imperative that schooling should include some educational component whereby students are involved in the construction of their knowledge (Wheeler, 1987). Barnard (1989) sees the teacher's role as a facilitator and scaffolder by providing the necessary support in the student's construction of knowledge.

Many students see mathematics as little more than a collection of facts, formulas, and algorithms that are to be applied to problems, which are then checked by the teacher or against some answer key (Barton, 1994). Traditional methodology in calculus teaching does not lend itself to the development of intuitive skills, which are imperative in the construction of mathematical knowledge (Stevenson, 1987). The National Research Council (1989) states that very few mathematics teachers have had the experience of constructing their own mathematical knowledge for the subjects they are asked to teach, and inclusion of technology alone will not necessarily ensure that conceptual knowledge is broadened. Porzio (1997) found that the use of a mathematical software package alone did not enhance students' conceptual knowledge. However, with the addition of activities designed for students to explore different representations of the concept of derivative, their conceptual understanding was increased.

Emese (1994) found that most students in his calculus classes (88%) preferred a constructive/discovery style of learning instead of the traditional style of instruction. The results of a study conducted by Frid (1994) demonstrate that students learn calculus concepts by constructing conceptualizations that are "viable models of their experiential world" (p. 93). If students do prefer to construct their knowledge and if an instructor subscribes to a constructivist view of learning, the goal for the instructor becomes finding appropriate activities for students to engage in so the students can construct their own knowledge about the concepts (Cobb, 1991). The two most prominent applications of the constructivist philosophy are cooperative learning and writing.

Cooperative Learning

Much of the construction of knowledge takes place through the interaction among students, with the teacher as a participant in the problem-solving process. By interacting with others, students are able to explain their solutions, to seek clarification, and to resolve conflicts. This leads to opportunities for the students to "reconceptualize a problem and extend their conceptual framework to incorporate the alternative solutions method" (Wood, pg. 245). Stephen Monk, of the University of Washington, has taken this idea of interaction and applied it in the calculus classroom. Monk assigns small learning groups and distributes problems to be solved by the group members. He then navigates the classroom, serving as a resource person rather than the expert. (Cipra, 1987). Monk believes that explaining an idea to another student leads to a deeper understanding of that idea. Dr. Kim Kirpaktrick, of Project NExT, also reinforces group participation in her calculus classes and believes that it enhances student learning and enthusiasm (Shannon, 1998).

There has been limited research done with respect to cooperative learning and calculus, but based on what is available, results indicate that cooperative learning does indeed help students develop conceptual understanding. Oliveros (1997) reported that working in small groups helped students solidify their conceptual understanding of the concept of rate. Schoenfeld (1994 b) encourages student discussion throughout the semester in his problem-solving class, and has found that students internalize their conceptual knowledge when they share their thought processes. Wheatley (1995) designed a four-course program based on collaboration and writing. His findings suggest that, as a result of collaboration with others, students reconceptualized mathematical ideas that were previously misunderstood, their confidence in mathematical knowledge increased, their attitude towards mathematical learning was more positive, and they made connections among algebra, geometry, and calculus. Bonsague (1994) found evidence that his program of intervention and peer tutoring in an academic setting directly affected student performance in calculus, particularly minority students. He believed that the student interaction was a primary reason for the program's success. A study comparing Project CALC students to traditionally taught students found that Project CALC students scored significantly higher on a problem-solving instrument than did the traditionally taught students (Bookman, 1994). Project CALC is a calculus reform project that was developed as a lab-based course strongly emphasizing cooperative learning. Brosnan (1995) reported that the use of cooperative learning groups increased students' conceptual learning in her liberal arts calculus class.

Writing

A major point of emphasis in the National Council of Teachers of Mathematics (NCTM) Standards (1989) is that students learn to communicate their understanding of mathematics in both mathematical and everyday language. The report *Preparing for a New Calculus* (1994) reinforces the idea of incorporating writing in the calculus classroom by stating that rethinking of instructional strategies should involve "increasing expectations for students' abilities to read, write, and speak about mathematics" (p. 60). Writing helps students reflect on their knowledge and thoughts, and causes their implicit knowledge to become explicit. Writing also assists the teacher in assessment of student progress by directing communication from all students in class, by providing information about students' thought processes, beliefs, and errors, and by providing tangible evidence of student's achievements (Masingila, 1996).

Writing allows students to make inferences and relate information to their own environment and experiences, and writing allows students to communicate to the teacher in their own words (Shepard, 1993). In Shepard's (1993) study, it was observed that students using writing assignments spent more time thinking about the problems they were working on than students working on traditional drill and practice problems. Dirkes (1991) studied the use of writing and listing in self-directed problem solving. She found that as college students progressed through the semester, their writing and listing skills improved substantially, and their problem-solving skills increased as well. By requiring students to keep a journal, Wheatley (1995) was able to recognize growth in students' abilities to solve problems.

Keith (1994) uses writing frequently throughout the semester in her calculus class, primarily because

> "frequently we may think we understand something when we only recognize it; we confuse familiarity with understanding. This becomes obvious when we have to explain it in writing (pg. 6)."

Writing in calculus is one of the innovations discussed by Steen in his presentation at the calculus reform colloquium in 1987. David Smith, of Duke University, employs writing assignments with regularity in his calculus classes. His contention is that if students cannot read and interpret the instructions, then they will certainly have difficulty solving the problem. He also maintains that being able to explain the solution process of a problem to another (whether instructor or student) solidifies understanding (Cipra, 1987). Masingila (1996) found that using writing in calculus allows development in mathematical understanding, and through writing (as well as other forms of communication) teachers can assess students' mathematical understanding. Keith (1994) has found that by writing, students are more apt to draw connections, make translations, and summarize what they have learned. Writing assignments also allow Keith to evaluate students' knowledge of a particular math concept when the assignments are used in conjunction with an exam over the same concept. Rose (1994) uses journals in her calculus class, and finds that she is more able to recognize student needs and difficulties, as well as to establish a positive rapport with her students. Stoughton (1994) implements writing in his calculus class by assigning two term papers, one at the beginning of the semester and the other toward the end. The topics are chosen from a list already created by the instructor. Stoughton finds that the students' writing ability and level of mathematical thought is significantly better on the second paper, and most students surveyed feel that writing the papers caused them to ponder more deeply certain mathematical concepts.

Many institutions of higher education now consider writing an important component in the calculus curriculum. According to the report *Assessing Calculus Reform Efforts* by the Mathematical Association of America (1995), 35% of the participants in a survey conducted among universities, 4-year colleges, or community colleges reported substantial use of writing in their calculus classes (Tucker, 1995).

Procedural versus Conceptual Knowledge

One of the most enduring debates in mathematics education is that of conceptual knowledge versus procedural knowledge. Throughout the history of mathematics education, different instructional programs have been implemented which focus on one kind of knowledge as opposed to the other (Hiebert, 1992). Though the arguments for one kind of knowledge or another have been persuasive at times (primarily due to the spokesperson endorsing that particular philosophy), there has not been significant progress in understanding of the issue (Hiebert, 1992). Hiebert (1992) discusses two possible steps to take in understanding the issue of conceptual knowledge versus procedural knowledge.

Instead of trying to decide which type of knowledge is more important, perhaps the first step for the mathematics education community is to understand how conceptual and procedural knowledge are related (Hiebert, 1992). Byrnes and Wasik (1991) present two approaches with regard to how students acquire procedural and conceptual knowledge. The *simultaneous approach* is founded on the idea that conceptual knowledge and procedural knowledge are assimilated at the same time, with conceptual knowledge being both necessary and sufficient for acquisition of procedural knowledge. Therefore, computational mistakes are the result of inadequate conceptual knowledge. The *dynamic approach*, on the other hand, is based on the idea that procedural knowledge and conceptual knowledge act upon and with each other. A well-developed conceptual knowledge base, coupled with general problem-solving skills, help to develop new procedures for solving a specific problem. The learner then discriminates and generalizes to decide in which contexts to apply the procedure. With continued use, the application of the procedure becomes more automatic, and conceptual knowledge is enhanced when one tries to make sense of the results of some procedure. Therefore, conceptual knowledge is necessary but not sufficient in the acquisition of procedural skills. Byrnes' study suggests that children acquire mathematical knowledge via the dynamic approach.

Secondly, Hiebert (1992) suggests that what is needed are more precise definitions of conceptual and procedural knowledge so that specific relationships between them can be established. Several studies offer definitions of both conceptual and procedural knowledge. Skemp (1987) defines conceptual understanding as "knowing what to do and why (p. 153)". Procedural knowledge is based on the usage of procedures without reference to the concept supporting them. Grimison (1992) defines conceptual knowledge as knowledge of process, whereas conceptual knowledge is referred to as knowledge of content.

Before the mathematics reform movement, most mathematics courses consisted of routine exercises, which provided little conceptual understanding (Schoenfeld, 1994a). Many mathematics teachers and students still believe that mathematics teaching and learning consists of memorization of facts and application of facts, and that conceptual knowledge cannot develop until the learner has successfully acquired the individual elements of the concept and the elementary operations and procedures surrounding the concept (Steinbring, 1989). A fifth grade teacher in Putnam's (1992) case study stated that the "hows" of mathematics are what is important, not the concepts. The teacher believed that the conceptual aspect of mathematics is difficult to teach and that students' minds are not mature enough to understand the concept behind the procedure. Schoenfeld (1985) found that most secondary students believed mathematics to be a discipline requiring primarily memorization. These beliefs are also found in a study of fifth graders conducted by Frid and Malone (1994).

The reform movement encouraged mathematics instructors to reevaluate the learning goals for mathematics. Teachers were exhorted to include meaning into the process of teaching from the beginning. To ignore the meaning behind procedures was to deny the learner the opportunity to grow conceptually (Steinbring, 1989). With the inclusion of technology, reformers called for a decrease in the number of procedural problems; they felt that more time should be spent on problems which encourage students to develop strong conceptual understanding (Davis, 1994).

Calculus reformers believe that conceptual understanding is of utmost importance for students. Kaput (1997) writes that teaching methodologies should focus more on conceptual learning so that especially gifted mathematics students will know calculus more deeply, and even those not particularly gifted in mathematics will learn "an even broader and richer mathematics of change and variation (p. 731)." Prados (1997), an engineering professor, believes that in the education of engineers for the 21st century, emphasis must be placed on concepts and not on manipulation of symbols. In a commentary on calculus reform, Ostebee (1997) proposes that more time should be spent in helping students understand the meaning of theorems and deferring their rigorous proofs until later courses in analysis. Ricardo (1997) feels that many of the students that failed his traditionally taught calculus courses did so because of the maze of algebraic manipulations and mounds of trigonometric substitutions, not because of the lack of conceptual understanding of calculus. Pence (1995) found that many first year calculus students do not have strong understanding of algebraic operations, and that this understanding is closely tied to success in calculus.

There are several studies that support the idea of teaching for conceptual understanding. Roddick (1995) compared students from a reform calculus course with students from a traditional calculus background. Her results indicated that reform students were more able to discuss all aspects of a problem, both from a procedural and conceptual standpoint. The reform students were also more confident in how to apply their knowledge to other situations. Tall (1987) found that by using technology and extensive interaction, students were able to develop a better conceptual notion of the geometric interpretation of the derivative. In a study of college mathematics learning conducted by Dubinsky (1994), results indicate that by teaching conceptually, student understanding is increased and problem-solving abilities are acquired. Hagelgans (1997) states that by using a more conceptual approach to teaching calculus, students are more involved in their learning and the attrition rate in her first semester calculus is nearly zero.

Van Hiele Theory of Geometry

The theory of Pierre and Dina van Hiele is founded on the belief that students progress through levels of thought in geometry (van Hiele, 1986). The following characteristics define the van Hiele theory:

- "Learning is a discontinuous process. That is, there are jumps in the learning curve which reveal the presence of discrete, qualitatively different levels of thinking. (Clements, pg. 426)."
- The levels are sequential and follow a hierarchy. Movement from one level to the next is highly dependent upon the instruction that the learner receives; progress from one level to the next is more dependent on instruction than on a learner's age or maturity. (Clements, 1992).
- Implicit conceptual understanding becomes explicit when a student moves up to the next level (Clements, 1992).
- The language used at each level is unique to that level; the development of the structure of the learner's language is a primary factor and crucial to movement through the levels (Clements, 1991).

Several studies indicate that the van Hiele levels are fairly accurate in describing students' geometric conceptual development. Usiskin (1982) found that approximately 75% of secondary students fit the van Hiele model. Burger and Shaughnessy (1986) found that in clinical interviews conducted with students from kindergarten to college, students' geometry behaviors were consistent with the description of the van Hiele levels.

There are no uniform results indicating whether the levels are discrete. One of the main difficulties lies in classifying a student when that student is in transition from one level to another (Fuys, et al, 1988). For many researchers, the fact that these problems exist is evidence enough to question the discreteness of the levels.

Research indicates that students do not reason at the same van Hiele level across different geometry topics. Burger and Shaughnessy (1986) found that students exhibited different characteristics of different levels on varying tasks. Fuys (1988) reported the same results. Students often lapsed to level 1 thinking when encountering a new topic. Fuys also found, however, that students were quickly able to move back to the higher van Hiele level under which they were operating prior to the task.

Most research indicates that there is a hierarchy of levels, progressing from level 1 to level 5 (Clements, 1992). The assignment to a level, however, is not dependent upon age or grade (Burger, 1986). Moreover, progression through the van Hiele levels is expedited through the teaching/learning process (Wirzup, 1976).

There is evidence to support the existence of a level more basic than the van Hiele's level 1 (visual). For example, 34% of the secondary students involved in Usiskin's study (1982) failed to demonstrate cognitive characteristics of even the visual level. Fuys (1988) classifies these students as "weak level 1." Clements (1992) proposes that based on the findings from this and other van Hiele-based research, there exists a level 0, which is also referred to as pre-recognition. At the pre-recognition level, students cannot identify common shapes; learners may identify only a subset of a shape's visual characteristics. For example, they may be able to differentiate between a rectangle and a circle, but not be able to tell the difference between a rectangle and a triangle.

Fuys (1988) and van Hiele (1986) support the reclassification of the original model into a three-level model. The more recent model is characterized as follows:

- Visual (previously Level 1)
- Analytic (previously Level 2)
- Theoretical (previously Level 3-5).

There are some problems with the newer characterization. First, the researchers warn that the 3-level model may not be sufficiently refined to categorize students properly. Secondly, the newer characterization seems to combine some attributes of levels 1 and 2 from the previous model into the visual level of the newer model. Finally, there is a question of whether if, by combining and changing levels, there is indeed a discrete, hierarchical nature to the model. Similarly, there seems to be heavy overlapping between levels with the newer model, thus challenging the notion of hierarchical dependency of the levels (Clements, 1992).

Other research challenges the idea that students operate at only a single van Hiele level. Gutierrez et al (1991) used a vector with four components representing the degree of acquisition for each of the van Hiele levels 1 through 4. Level 5 was not included in the vector because the researchers felt that Level 5 acquisition could not be measured satisfactorily. The study found many students who were simultaneously developing two consecutive levels of reasoning.

There have been many instruments designed in an attempt to assess the van Hiele model of reasoning (Burger and Shaughnessy, 1986; Usiskin, 1982; Fuys, Geddes, Tischler, 1988; De Villiers, 1987). The four primary categories of tests (Gutierrez, 1991) that have been created are:

- Paper and pencil, multiple-choice questions
- Paper and pencil, open-ended questions
- Clinical interviews, open-ended questions
- Learning sequences.

Limited research is available which investigates the relationship between the van Hiele levels in geometry and calculus. Fitzsimmons (1995) paired calculus students according to their van Hiele levels, with some pairs being of the same level and other pairs being separated by one or more levels. The control group consisted of students that were taught traditionally. Results indicated a significant difference in achievement favoring the experimental groups over the control group on the geometric portion of the final calculus examination. Poehl (1997) found that the completion of one nine-week course in AP Calculus increased the level of geometry understanding in students. However, the results of the study also indicated that Usiskin's (1982) van Hiele Geometry test had no construct validity for determining a learner's van Hiele level.

Summary

Historically, the issue of calculus reform has created much controversy in the mathematics community. There seems to be quite a gulf between those instructors and researchers who favor reform and other colleagues whose oppose the idea of changing the way calculus should be taught. Mathematics educators who use the reform techniques believe that the methodologies do indeed make a difference in conceptual learning for students, but little research has been done to support this opinion.

One of the major components of calculus reform is constructivism. This philosophy is based on the idea that students create their knowledge; they are not passive vessels to be filled with ideas. Therefore, it is the responsibility of the instructor to provide opportunities for students to construct this knowledge. Two significant teaching techniques that lend themselves to constructivist ideas are group collaboration and writing.

The underlying motivation for using constructivist methods of teaching is to assist students in developing conceptual knowledge. Though a student can perform well in situations that require only procedural knowledge, that same student may not be equipped with the conceptual knowledge necessary to answer questions which extend basic ideas to a more forma level. Constructivist advocates believe that reform methodologies do enhance a student's conceptual knowledge base.

The van Hiele levels of geometry are measures of conceptual geometry knowledge. Students operate at one of three levels, with level 1 being the most basic level. Level 2 represents an intermediate level of conceptual geometry knowledge, and level 3 is the highest level of conceptual reasoning. Little research has been done that relates van Hiele level to calculus learning.

CHAPTER III

THE RESEARCH DESIGN

Introduction

This study utilized quantitative methods to evaluate the effects that constructivist teaching strategies have on the learning of the concept of derivative in a first semester calculus course. The study also investigated whether there is a relationship between the van Hiele level of a student and that student's conceptual knowledge of the geometric interpretation of the derivative. The study focused on the following research questions:

1) In what way does the use of a constructivist approach to instruction affect student learning of the geometric concept of derivative in a first semester calculus course?
2) What is the relationship between a student's van Hiele level and that student's geometric conceptual understanding of the derivative in a first semester calculus course?

The Sample

The subjects in this study were first semester calculus students at a small private university located in the southwest, with an enrollment of approximately 5,000 students. Enrollment in either section was self-selected by the subjects during a normal enrollment period. The first section, Section 1, met for fifty minutes on Mondays, Tuesdays, Wednesdays, and Fridays at 8:50 a.m. The second section, Section 2, met for fifty minutes on Mondays, Tuesdays, Wednesdays, and Fridays at 4:30 p.m. The set of section one subjects consisted of twenty-two students, all freshmen and sophomores. Of the twenty-two students, 18 were male and 4 were female. The set of twenty-four subjects in section two was similarly composed of freshmen and sophomores. Of the twenty-four students, 17 were male and 7 were female. Both sections did contain additional students, but these students were not included in the study for one of three reasons. (1) Upperclassmen were not included because the researcher limited the sample to freshman/sophomre calculus students at the institution. (2) Some students were not included in the study because they withdrew from the class prior to the administration of either the van Hiele instrument or the post-treatment. (3) Other students, though still enrolled in calculus, were not included because they did not take the van Hiele test. The student population in first-semester calculus consisted of students from one of four areas: pre-medicine, computer science, engineering, or mathematics.

Instruments

Pre-Treatment

Two separate instruments were utilized in the pre-treatment phase of the study. The first instrument that was employed was a placement exam mandatory for all students beginning the mathematics/engineering/science/computer science course sequences. This instrument provided a quantitative measure of the algebraic/mathematical skills of the students in each calculus section, thus revealing whether the two sections were of approximately the same ability level. The test was administered prior to the beginning of the semester to assist students in determining which mathematics course most strongly correlated to their skill level at the time. The exam consisted of thirty-five multiple-choice questions; calculators were not permitted on the placement exam.

The second instrument was a 25-problem, multiple-choice examination designed on the van Hiele model of geometric understanding (Usiskin, 1982). This instrument, employed to determine each student's van Hiele level (Clements, 1992), was administered to each section prior to the beginning of the chapter dealing with derivatives. Based on scores received on each of five sections of the instrument, students were assigned to one of the five van Hiele levels.

Post-Treatment

The post-treatment instrument was prepared by the investigator and contained questions that were either procedural or conceptual in nature. The procedural questions required only calculations or a brief sketch. The conceptual questions required either that students provide a written explanation for their answers, or that they be able to answer questions based on the geometric definition of derivative. A copy of the instrument is included in Appendix C.

Question 1 was conceptual in nature, in that the student was asked to discuss what is meant by the geometric interpretation of the derivative.

Question 2 was divided into three parts. The first part was procedural; the student was asked to provide a sketch of a curve and a secant line to the curve. Parts 2 and 3 were conceptual in nature. Part 2 required an explanation of how the slope of the secant line is changing as points a and b approach each other on a curve. Part 3 asked the student to discuss what is being found when points a and b on the curve coincide.

Question 3 required that the students determine the derivative of a function based on a sketch depicting a function with a line tangent drawn to the function at a given point.

Thus the students were required to apply their conceptual knowledge of what is meant by the derivative of a function at a particular point.

Question 4 was a procedural problem that asked students to only calculate the derivative of a polynomial function at a particular x-value. No explanation or discussion was required, and the students could find the answer easily if they understood how to apply the power rule of differentiation.

The Treatment

Both calculus sections used the same text, Discovering Calculus: A Preliminary Version (Levine, 1994), and the homework assignments from the book were the same for each section. Discussion of the derivative began in chapter 3 of the text, and both sections began chapter 3 during the same week.

The Control Group

In Section 1, the control group, classes were conducted in a traditional manner, with a lecture format being the prevalent teaching methodology employed (Tucker, 1995). On the first day of lessons dealing with the derivative, the instructor in Section 1 began the discussion by stating the formal definition of the derivative. Problems were then assigned which required students to find the derivatives of different functions by using the definition. The second day of instruction involving the derivative began with the instructor presenting a demonstration via the use of a graphing calculator, whose screen was projected onto the front wall of the classroom. Using the graphing calculator, a secant line was drawn to a curve. The instructor demonstrated the changes in the slope of the secant line as the intersection points moved closer and closer together along the curve.

The third day of instruction regarding the derivative, as well as the subsequent class periods, was taught in the traditional lecture format. Differentiation techniques were introduced, and homework was assigned to practice these techniques. Additional homework problems were photocopied from other another text since the instructor felt that there were not enough practice problems in the current text. The graphing calculator was used extensively throughout treatment of the derivative, though primarily in the context of demonstration by the instructor. Homework was not collected; in lieu of homework collection, quizzes were administered by the instructor which lasted approximately 15 to 20 minutes. Problems on these quizzes included homework problems or similar problems. An exam treating differentiation was administered after all sections of chapter 3 had been covered in class.

The Treatment Group

During the coverage of the chapter on derivatives, Section 2 (the treatment group), was taught using constructivist strategies (Steen, 1987). A primary instructional constructivist strategy that was employed was small group collaboration. This teaching strategy, discussed in detail in Chapter II, has been used extensively by Monk (Tucker, 1995). On the first day of treatment of chapter 3, which was an introduction to the derivative, the students were asked to form small groups of two or three. The instructor then drew a sketch of a function on an overhead transparency. Each group was asked to sketch a similar function, then draw a secant line anywhere on the function. The students were then asked to imagine that one point of intersection of the secant line and the curve move along the curve toward the other point of intersection. Each group was then asked to discuss what changes were taking place with regard to the slope of the secant line.

Finally, the students were asked to discuss the situation where the two intersection points actually coincided. After the discussions, the groups were given the assignment of writing a summary of the behavior of the slope of the secant line as one intersection point approached the other point of intersection.

An investigation using the graphing calculator was conducted on day two of the lessons treating the derivative. Students were again asked to form small groups, divided so as to ensure that each group had access to a graphing calculator. The instructor drew a specific function, and then labeled three points a, b, and c on the function, where a corresponded to the smallest x-value, and where c corresponded to the largest x-value. Half of the groups was asked to use points a and b as initial points, and the other half was asked to use points b and c as initial points. Using one of these sets of two points, the students were asked to calculate the slope of the secant line passing through the two points. They were then asked to recalculate the slope of the secant line at certain stages as one point began to move along the curve toward point b. The groups were able to recognize quickly that there existed a limiting value for the slope, regardless of which set of starting points were used, and that the limiting value was the slope of the line tangent to the curve at point b. The instructor sketched a second function, with a jump discontinuity in the graph at $x = 1$. The groups were then asked to discuss whether a tangent line existed for the function as x approached 1 from both the left and right. The class then discussed each group's answers.

On day three of the lessons involving the derivative, the instructor introduced the formal definition of derivative, and asked the class to discuss how the formal definition was related to the investigations that had been conducted in the previous class periods.

After the discussion, the class assignment was to write a paragraph about this relationship.

Students were given homework problems from the text that required finding the derivative by using the formal definition, and subsequent class periods introduced various techniques of differentiation. Quizzes were given that required the students to provide written responses to questions rather than just perform calculations. These writing strategies were based on the work of David Smith of Duke University, who uses writing extensively in his calculus classes with positive results (Cipra, 1987).

Writing assignments, small group collaboration, and class discussions were utilized throughout the treatment of chapter 3. A chapter exam was given after all sections of chapter 3 were discussed.

Collection of the Data

It was not necessary for the researcher to administer the placement examination since the placement test is required for all students preparing to enroll in a math class. However, the results of the placement examination were not immediately available. Permission to use the placement exam scores for all the subjects involved in the study was granted by the placement examination administrator.

The van Hiele geometry test was administered in both calculus sessions by the researcher during a regular class session, within the same week of each other, and prior to the treatment. No calculators were allowed on the exam.

The post-treatment instrument was administered in both calculus sessions by the researcher during a regular class session and within the same week of each other. The post-treatment instrument was given after treatment of chapter 3 in both sections and after the chapter exam was given in both sections. No graphing calculators were allowed on the exam.

Analysis of the Data

The mean of the placement exam scores was calculated for both Section 1 and Section 2. The means were then compared to determine whether the classes were similarly composed.

The van Hiele level instruments were scored by comparison to a multiple choice answer key. The instrument is designed so that the first five questions correspond to van Hiele level 1, questions 6-10 correspond to van Hiele level 2, questions 11-15 correspond to van Hiele level 3, 16-20 correspond to van Hiele level 4, and questions 21-25 corresponding to van Hiele level 5. To determine at which level a particular student was to be assigned, the researcher investigated each block of 5 problems. To be operating at a particular level, the subject must have answered at least four of the five problems in that corresponding block correctly. Otherwise, the subject was assigned to the van Hiele level lower than the level being investigated. For example, if a subject answered 4 out of 5 problems correctly on block 1-5, but answered only 2 problems correctly on block 6-10, then that subject was assigned to level 1. Once two problems in a block were answered incorrectly, the subject was assigned to the lower van Hiele level, and the researcher graded no further on the instrument. Therefore, it is possible that students may have answered 4 or more questions correctly in a block corresponding to a higher van Hiele level. This is a limitation that will be addressed in Chapter V.

After reading the articles by Fuys (1988) and Clements (1992), the recent 3-level van Hiele model that was discussed in Chapter II was utilized in the study. In the 3-level model, levels 1 and 2 remain essentially unchanged, whereas level 3 actually combines all of levels 3,4, and 5. Statistics using both models were calculated and are included in Chapter IV.

The researcher developed the post-treatment instrument; no grading scheme existed prior to the scoring of the instrument. Therefore a rubric was designed to score the post-treatment instrument. In the rubric, procedural problems were graded as either correct or incorrect, and no partial credit was awarded. The instrument contained two problems procedural in nature, and each procedural problem was worth 5 points. All other problems were conceptually oriented problems. Each conceptual problem was worth a total of 10 points, with the number of points being awarded based on the accuracy and completeness of the students' answers.

A measure of reliability for the post-treatment instrument was calculated using the split-half reliability procedure outlined in Gay (1992), and corrected using the Spearman-Brown correction formula. The reliability coefficient that was calculated was not necessarily accurate, primarily because the post-test instrument does not include many problems. The split-half reliability measure tends to provide more accurate measures for longer instruments. However, since the post-treatment was administered only one time, the split-half reliability test was the most valid option.

The grading scheme is presented below in Table I.

TABLE I

RUBRIC FOR SCORING OF POST-TREATMENT INSTRUMENT

Problem Number and Points Possible	Explanation of Solution	Points Awarded
Problem 1; 10 pts.	mention of slope of curve	5
	mention of tangent line to curve	5
	mention of slope of tangent line to curve	8
	complete discussion	10
Problem 2, part (a); 5 pts.	missing parts of graph, besides labels	0
	all parts of graph, only labels omitted	5
Problem 2, part (b); 10 pts.	mention of slope of tangent line to curve	5
	change partially correlates with sketch	5
	mention of approximation of slope of curve	5
	change completely correlates with sketch	10
Problem 2, part (c), 10 pts.	mention of tangent line	3
	mention of derivative of $f(x)$ at $a = b$	6
	mention of slope of curve	6
	mention of slope of tangent line	8
	complete discussion	10
Problem 3; 10 pts.	recognizes points on tangent line	3
	finds equation of tangent line	5
	finds slope, wrong sign	8
	correct slope	10
Problem 4; 5 pts.	incorrect procedure	0
	correct procedure	5

After the post-treatment instruments were scored, totals were calculated for both the procedural and conceptual portions of the test.

Means for section number and van Hiele level were calculated, as well as means for the both the procedural and conceptual portions of the post-treatment instrument. Based on the information gleaned from Stevens (1992), a multivariate analysis of variance (MANOVA) seemed the best statistical approach to take in analyzing the data. Thus, using the SPSS statistical software package, a MANOVA was performed to determine differences on the two dependent variables and the two independent variables. The two independent variables included the placement instrument and the van Hiele instrument, whereas the two dependent variables included the totals of the procedural and conceptual scores of the post-treatment instrument, respectively. The significance of difference was calculated using Roy's Largest Root test.

If there was no interaction between the independent variables, then a one-way analysis of variance (ANOVA) was performed for each independent variable. On the other hand, if an interaction between the independent variables was significant, then two-way ANOVAs were performed using the procedural and conceptual totals in separate analyses. To account for the possibility of multivariate interaction, a discriminant analysis was performed. The eigenvalues were investigated, as were the canonical correlation values.

CHAPTER IV

RESULTS

Introduction

The focus of this study was conceptual knowledge as it pertains to the geometric interpretation of the derivative in a first semester calculus course. Primarily, the researcher sought information as to whether teaching methodology affects conceptual learning of the derivative in calculus. Secondly, the relationship between a student's van Hiele geometry level and the student's ability to reason conceptually was investigated. The results are summarized in two sections: (1) Results of Pre-Treatment Data, and (2) Results of Post-Treatment Data.

Results of Pre-Treatment Data

The placement exam means and standard deviations for each section are summarized in Table II. These statistics were calculated to determine whether the sections were beginning at the same level procedurally. Since the placement exam means of the two sections were not significantly different, no equalization of the sections was necessary.

TABLE II

PLACEMENT EXAMINATION MEANS AND STANDARD DEVIATIONS

	Mean	Standard Deviation	Sample Size
Section 1	19.60	5.23	n = 20
Section 2	19.92	3.60	n = 24

The validity of the exam was not calculated. However, professors in the mathematics education department found that there was a high correlation between the students' scores on their placement examinations and the grades they received in their respective courses. The reliability was evaluated using the split-half reliability procedure. Since there were 35 items on the test and a split-half reliability coefficient was calculated, the coefficient was adjusted used the Spearman-Brown formula (Gay, 1992). The reliability was found to be .85 for the placement examination.

Table III provides a summary for the number of students at each van Hiele level in each section. Note that there were two students who did not take the placement exam, but who did take the van Hiele test. Though the placement exam statistics were affected, the researcher felt it important to include the two van Hiele scores so as to maintain similar sample sizes in both sections.

TABLE III

NUMBER OF STUDENTS AT EACH VAN HIELE LEVEL FOR EACH SECTION AND AVERAGE VAN HIELE LEVEL IN EACH SECTION

	Number at Level 1	Number at Level 2	Number at Level 3	Van Hiele Average	Sample Size
Section 1	5	1	16	2.50	n = 22
Section 2	8	4	12	2.17	n = 24

Results of Post-Treatment Data

As stated in chapter three, the study was designed to investigate the affect of two independent variables (section number and van Hiele level) on two dependent variables (procedural portion of post-treatment test and conceptual portion of post-treatment test). Therefore, the researcher felt that a multivariate analysis of variance (MANOVA) would provide the most information regarding if and how much the independent variables affected the post-treatment instrument, and whether there was an interaction between the two independent variables. Before the MANOVA procedure was applied to the data, the means of the post-treatment instrument were calculated for each section. These statistics aided the researcher in determining whether any relationship appeared to exist between the sections. The means for both the procedural portion and conceptual portion of the exam are summarized in Table IV.

TABLE IV

MEANS AND STANDARD DEVIATIONS ON POST-TREATMENT INSTRUMENT FOR BOTH SECTIONS

	Statistic	PT Procedural	PT Conceptual
Section 1	Mean	9.32	16.14
	Standard Deviation	1.76	9.10
	N	22	22
Section 2	Mean	8.54	16.62
	Standard Deviation	2.32	10.59
	N	24	24

After the means were calculated, the MANOVA procedure was applied to the post-treatment data to determine whether any interaction existed between section number and van Hiele level. The results of the MANOVA procedure are summarized below in Table V.

TABLE V

POST-TREATMENT RESULTS FROM MANOVA

Effect	Multivariate Test	Value	F Value	DF	Prob. >F
Class #	Roys's Largest Root	.067	1.305	2	.283
VH level	Roys's Largest Root	.229	4.570	2	.016*
Class # * VH Level	Roys's Largest Root	.024	.476	2	.625

* indicates significance at $p = .05$

As shown in the table, there was no significant interaction between section number and van Hiele level. Also, there was no significant main effect detected from section number. However, note the F-values for the main effect detected by van Hiele level. For all the multivariate tests, van Hiele level contributed significantly to the scores on the post-treatment instrument. The researcher chose to include all the multivariate tests from the SPSS statistical package since they all produced significance within .05, particularly Roy's Largest Root test. Stevens (1992) noted in his discussion of multivariate tests that Roy's statistic is most powerful when differences among groups are concentrated on the first discriminant function. Therefore, based on the results of the MANOVA, additional tests were found to be necessary. A discriminant analysis was performed after the MANOVA procedure to determine whether the differences in the groups was indeed the concentrated on the first discriminant function, and the results will be summarized later in this section.

Since the main effect of van Hiele level was significant, the researcher felt it was necessary to further investigate this relationship between van Hiele level and the post-treatment instrument. Therefore a one-way analysis of variance (ANOVA) was performed for (1) van Hiele level and the procedural portion of the post-treatment instrument, and (2) van Hiele level and the conceptual portion of the post-treatment instrument. The results are summarized in Table VI.

TABLE VI

ONE WAY ANALYSIS OF VARIANCE

Source	Dependent Variable	DF	F value	Prob. > F
VH Level	PT Procedural	2	1.077	.350
VH Level	PT Conceptual	2	4.538	.017*

* indicates significance at p = .05

As seen in Table VI, there was a significant difference between van Hiele level and the conceptual portion of the post-treatment instrument. An investigation of the correlation between conceptual and procedural performance was conducted and found to be .328, which is considered by Stevens (1992) to be a moderate correlation. Therefore a discriminant analysis was performed to further investigate this correlation.

The first test in the discriminant analysis applied to the data was Box's M test. Using this test, homogeneity of the group covariance matrices was investigated. The F-value was found to be 1.15, with a significance value of .351. Since the F-value was not significant, the assumption of homogeneity of covariance was not violated. Therefore the differences in the group covariance matrices were not significantly different enough to warrant equalization procedures.

The means on the post-treatment instrument for each van Hiele level were compared to determine whether there appeared to be any significant difference between them; they are displayed in Table VII.

TABLE VII

MEANS ON POST-TREATMENT INSTRUMENT FOR EACH VAN HIELE LEVEL

VH Level	Post-Treatment Category	Mean	Standard Deviation
1	PT Procedural	8.08	2.53
	PT Conceptual	12.62	9.12
2	PT Procedural	9.00	2.24
	PT Conceptual	8.40	5.86
3	PT Procedural	9.29	1.78
	PT Conceptual	19.57	9.43

Since it appeared a possible significant difference in the means among the van Hele levels could exist, a test of this difference was performed. The calculation of Wilks' Lambda determines whether a significant difference exists between the means for different groups. Wilks' Lambda was calculated to test for equality among van Hiele level means for conceptual performance and procedural performance, and the results are summarized in Table VIII.

TABLE VIII

TEST OF SIGNIFICANCE AMONG VAN HIELE LEVEL MEANS

	Wilks' Lambda	F-value	Significance
PT Conceptual	.818	4.791	.013*
PT Procedural	.933	1.532	.228

* indicates significance at p = .05

There was a significant difference among the van Hiele levels on the conceptual portion of the post-treatment instrument, but the differences of means was not significant for procedural performance.

The residual test procedure involving Bartlett's Chi-Square tests (Stevens, 1992) was performed to determine the number of significant discriminant functions. Ascertaining the number of discriminant functions assists the researcher in determining which of the dependent variables contributes most significantly to the difference in means between groups. The results for the discriminant functions are summarized in Table IX.

TABLE IX

TEST FOR SIGNIFICANT DISCRIMINANT FUNCTIONS

Test of Function(s)	Wilks' Lambda	Chi-Square	DF	Significance
1 through 2	.785	10.290	4	.036*
2	.961	1.693	1	.193

* indicates significance at p = .05

The maximum number of possible discriminant functions was two, which is the minimum of the number of van Hiele levels and the number of dependent variables. The test of the combination of both discriminant functions (which combined both eigenvalues) yielded a significant χ^2 at the .05 level; thus there was significant overall association. A test of the residual or second discriminant function, with the largest eigenvalue removed, revealed no significant association. Therefore the only significant function is the first function.

The eigenvalues were then investigated to determine which eigenvalue contributed most to the total association. These eigenvalues are representative of the contribution of the dependent variables to the total variance. The eigenvalues are displayed in Table X.

TABLE X

EIGENVALUES

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	.224	84.7	84.7	.428
2	.041	15.3	100.0	.198

Since the eigenvalues additively partition total association and discriminant functions are uncorrelated (Stevens. 1992), the "% of Variance" was calculated by dividing the eigenvalue in question by the sum of both eigenvalues. The first eigenvalue is the primary contributor to the total association (84.7%).

To interpret the first discriminant function, the researcher investigated both the standardized coefficients and the discriminant-variable correlations as suggested by Stevens (1992). The correlations were used to determine the underlying constuct that was represented by the discriminant function. The SPSS procedure empirically clustered the two dependent variables; the next step in the analysis was to determine which dependent variable(s) most significantly defined the first discriminant function. The correlations are summarized in Table XI.

TABLE XI

POOLED WITHIN-GROUPS CORRELATION BETWEEN CANONICAL DISCRIMINANT FUNCTIONS AND DISCRIMINATING VARIABLES

	Function 1	Function 2
PT Conceptual	.996	-.086
PT Procedural	.408	.913

As seen in the table, conceptual performance primarily defined the first discriminant function, with moderate involvement from procedural performance.

Next, the standardized coefficients were examined to determine if there was any redundancy for conceptual performance or procedural performance. In other words, is there any overlap between procedural performance and conceptual performance? The standardized coefficients of the discriminant functions are summarized in Table XII.

TABLE XII

STANDARDIZED CANONICAL DISCRIMINANT FUNCTION COEFFICIENTS

	Function 1	Function 2
PT Conceptual	.966	-.432
PT Procedural	.091	1.055

The analysis revealed that conceptual performance was not redundant because of its coefficient value of .966, whereas procedural performance was redundant (.091).

The last step in the analysis was to investigate the group centroids (means) to determine the relationship between van Hiele level and performance. The results are displayed in Table XIII.

TABLE XIII

UNSTANDARDIZED CANONICAL DISCRIMINANT FUNCTIONS EVALUATED AT GROUP MEANS

VH Level	Function 1	Function 2
1	-.439	-.248
2	-.848	.426
3	.355	.004

The means of the van Hiele levels on the first discriminant function indicated that there was a separation between level 3 and the other two levels.

Since the subject/variable ratio was 23 to 1, there was high reliability in the results from the discriminant analysis (Stevens, 1992). At this point it is important to discuss the validity and reliability of the post-treatment instrument. Though no test of validity was performed on the instrument, content validity was assessed by two mathematics education professors. The reliability of the conceptual portion of the instrument was calculated using the split-half reliability procedure, and was adjusted using the Spearman-Brown correction formula. The reliability was found to be .68. It should be noted, however, that the number of items on the post-treatment instrument was extremely small, thus increasing the possibility of an inaccurate reliability coefficient.

An interpretation of all of the results summarized in this chapter will be presented in Chapter V.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

One of the primary components of calculus reform is a call for a change in teaching methodology. Reform advocates feel that changing the way students are introduced to calculus concepts will enhance their understanding of these concepts. The reform movement also encourages a three-pronged approach to the introduction of a calculus concept: symbolically, numerically, and geometrically. This study was designed primarily to investigate the relationship between teaching modes of instruction and calculus learning. Secondly, the study sought to provide information as to whether there is a relationship between conceptual learning in geometry and conceptual learning in calculus.

The sample for the study included freshman and sophomore students (n = 46) enrolled in beginning calculus in a small, midwestern, private university during the fall semester of 1998. This study used quantitative methods to investigate the following research questions:

1. In what way does the use of a constructivist approach to instruction affect student learning of the geometric concept of derivative in a first semester calculus course?
2. What is the relationship between a student's van Hiele level and that student's geometric conceptual understanding of the derivative in a first

semester calculus course?

The placement examination, which was used as a pre-treatment instrument, was an examination already in place at the university. This instrument, whose main purpose is to help place beginning engineering, mathematics, computer science, or pre-medical students in the appropriate mathematics class, was used in this study to determine whether the composition of the two sections involved in the study was similar. The means for both Section One and Section Two (19.6 and 19.92, respectively) indicated that the student population in each section was comparable. Note that, for the placement examination, section one included four less students in its sample size (n = 20) than in Section two (n = 24). Two of the students who took the van Hiele test and the post-treatment instrument did not take the placement examination. Although this difference in sample size certainly could affect section one's mean on the placement examination, the researcher believed the inclusion of these two students even more important when evaluating the van Hiele instrument and the post-treatment instrument.

The decision to use the van Hiele instrument was based primarily on the desire to determine whether conceptual reasoning ability in geometry translates to conceptual reasoning ability in calculus, and also to determine at what van Hiele levels these calculus students were operating prior to the treatment. The van Hiele instrument consisted of 25 multiple choice questions, with each group of five questions correlating to a higher van Hiele level of learning. Based on the research of Fuys (1988) and van Hiele (1986), students were then classified as belonging to one of three van Hiele categories.

- Level (1) Visual, corresponds to a van Hiele level of 1 on the instrument.
- Level (2) Analytical, corresponds to a van Hiele level of 2 on the instrument.

- Level (3) Theoretical, corresponds to a van Hiele of 3,4, or 5 on the instrument.

An analysis of the data from the van Hiele instrument revealed that there were three more students operating at level one in Section Two than in Section One. There were also three more students operating at level two in Section two than in Section one. However, there were four more students operating at level three in Section One than in Section Two, and this difference accounts primarily for the larger van Hiele level mean in Section One.

After the pre-treatment instruments were given, each section began the chapter that introduced differentiation. Section One (the control group) was taught using traditional teaching methods, which included primarily lecture and demonstration, whereas Section Two (the treatment group) was conducted using reform methodology. Group collaboration, class discussion, and writing were emphasized in the treatment group.

The post-treatment instrument was developed by the researcher. Four of the questions were designed to determine whether students could reason geometrically about the derivative. Two of the questions were strictly procedural in nature, requiring only computational skill and little calculus conceptual knowledge. A rubric was designed to provide a quantitative score for each question on the instrument.

The results of the data analysis provided some insight into the relationship between teaching methodology and conceptual reasoning, as well as the relationship between van Hiele level and conceptual performance.

Conclusions

The following conclusions address the two research questions and provide additional insight into students' conceptual knowledge of the derivative. These conclusions are presented in the context of the limitations of the study.

1. Though Section Two's mean van Hiele level score was lower on the pre-treatment instrument than the mean van Hiele level score for Section One, the post-treatment instrument indicates higher conceptual performance for Section Two. This is of particular interest since there were three students in Section One who were not involved in the study due to either dropping the class or receiving an incomplete. Section Two experienced no attrition, and all students of all van Hiele levels participated in the study.

The differences in the means between the control group and the treatment group might have been significant had these other students been involved in the study.

In the comparison of the procedural performance for the two sections, Section One's scores were significantly higher. This is consistent with previous research cited that compared procedural skills of traditional calculus students to procedural skills of reform calculus students. Since traditionally taught calculus courses tend to emphasize calculational techniques, it is not surprising that the traditionally taught Section One would attain a higher procedural score than that of Section Two, whose basis was more conceptual in nature.

2. According to the results of the MANOVA procedure performed on the data, teaching methodology alone did not significantly affect procedural or conceptual post-treatment performance. Therefore the findings in this study did not support the

reform literature, which purports that reform methodology enhances conceptual ability. As stated previously, however, attrition from Section One for the three-week duration of the study could have greatly affected the outcomes of the MANOVA procedure, as well as the fact that only one chapter was included in the study.

The MANOVA results indicated that regardless of the van Hiele level of a student, teaching methodology combined with van Hiele level did not enhance conceptual performance, nor did it enhance procedural performance. Again, attrition in Section One could have affected these results.

3. The MANOVA revealed that the van Hiele level of a student was significantly related to that student's performance on the post-treatment instrument. After finding this significant difference, two one-way ANOVAs were performed: one between van Hiele level and procedural performance, and the other between van Hiele level and conceptual performance. Results indicated a significant relationship between van Hiele level and conceptual performance on the post-treatment instrument.

An investigation of the means and standard deviations on the post-treatment instrument for each van Hiele level further defined this relationship. Though the procedural means were quite close in value to each other for all three van Hiele levels (8.08, 9.00, and 9.29, respectively), the difference in mean values for conceptual performance was substantial, especially for those students at level 3. The students at van Hiele level 3 scored much higher on the conceptual portion (19.57) of the instrument than those students at either level 1 (12.62) or level 2 (8.40). An item worthy of note is that the students at van Hiele level 1 scored higher on the conceptual portion of the instrument than those students at van Hiele level 2. There were three more level 1 students in the reform section than in the traditionally taught section; and there were three more level 2 students in Section 2 than in Section 1. A possible explanation for this difference in conceptual performance could be that the reform teaching methodology was more beneficial to lower level students, which reinforces results from other research regarding calculus reform.

The calculation of Wilks' Lambda, which provides a statistical comparison of the mean differences for the van Hiele levels on both the procedural and conceptual portions of the post-treatment instrument, revealed a significant difference on the conceptual portion of the instrument. Therefore, it certainly seemed that a student's van Hiele level significantly affected his conceptual performance.

What remained to be investigated was whether there was significant association between conceptual performance and procedural performance. In other words, the researcher sought to determine how much a student's conceptual performance was related to that student's procedural performance.

The discriminant analysis performed on SPSS resulted in two discriminant functions, with only the first function being significant. By examining the correlations between the two resulting discriminant functions and the dependent variables, the correlation between conceptual performance and the first discriminant function was .996, whereas the correlation between procedural performance and the first discriminant function was only .408. This means that conceptual performance primarily defined the first discriminant function, therefore contributing most significantly to the percentage of variance in the first discriminant function.

By inspection of the standardized coefficients for the first discriminant function, the coefficient representing the contribution by conceptual performance was .966, whereas the coefficient representing procedural performance was only .091. These results indicate that given conceptual performance, procedural performance did not contribute substantially in defining the relationship between van Hiele level and performance on the post-treatment instrument. Statistically speaking, procedural performance was a redundant variable.

Combining the information from the coefficient and the discriminant-variable correlations, it can be said that the first discriminant function could be characterized as one that described the relationship between van Hiele level and conceptual performance.

To complete this investigation of the relationship between van Hiele level and conceptual performance, the means of the van Hiele levels on the first discriminant function were calculated using SPSS. The results indicated a separation between those students at level 3 and those students at both levels 1 and 2. Students at van Hiele level 3 tended to score higher on the conceptual portion of the post-treatment instrument.

Based on the results from this study, there is a very significant relationship between van Hiele level and calculus conceptual learning, particularly for those students operating at van Hiele level 3. This important finding suggests a substantial relationship between what students learn in geometry and how well they perform conceptually in calculus. These results provide information that should be utilized by instructors at the high school level, the collegiate level, and by educational researchers.

Recommendations

The following recommendations are offered for mathematics educators who teach calculus and for those interested in further research.

1. The use of reform teaching methodologies should continue to be encouraged, particularly to the benefit of students with more marginal mathematics skills. Ricardo (1997) believed that teaching with a more conceptual basis will benefit those students with weak computational skills, and Roddick (1995) found that students taught using reform methods were more capable of communicating their conceptual knowledge than those students from a traditional calculus background. Lucas (1998) cited the increase in conceptual performance by using reform methodology and Garner (1998) provided results that indicated a significant increase in conceptual understanding by use of reform teaching methods.

2. While implementing reform teaching methods, further research should be conducted that investigates the relationship between reform teaching methods and calculus success, especially as it relates to conceptual knowledge. Haver (1998) indicated the increase in the development of reform materials. This information would be beneficial for all educators involved with calculus, whether in the capacity of teacher or researcher. If instructors and researchers were to be able to review results from research that investigates the contribution these materials are making to the acquisition of calculus conceptual knowledge, teaching methodologies could be enhanced to accommodate this relationship. Though many calculus instructors and researchers feel that reform methods are most beneficial to students (Lucas, 1998; Kilpatrick, 1997), research investigating these methods is still much needed.

3. Further research is recommended with regard to the relationship between van Hiele level and calculus conceptual knowledge. This is particularly important because there is very little research that has been done which investigates this relationship. Fitzsimmons (1995) found that by pairing students according to van Hiele level and encouraging these pairs of students to work together throughout the semester, there was significant improvement in the students' scores on the geometric portion of the calculus final. The scores of these students were significantly higher than the scores of students who were taught using traditional methods. Results cited by Poehl (1995) indicated that students who took a nine-week AP calculus course increased their level of geometric reasoning, but little other research exists which investigates how conceptual understanding in geometry affects conceptual understanding in Calculus.

A more complete understanding of this relationship would be helpful in several ways to instructors of both geometry and calculus. Geometry teachers would be more enlightened as to what areas of geometry should be emphasized so as to enhance students' geometric conceptual development and increase their confidence in their ability to do mathematics. This increase in conceptual ability and confidence could certainly serve to encourage more students to take higher mathematics courses such as calculus.

Calculus instructors could use van Hiele level testing to identify students that need additional assistance in the development of their geometric conceptual knowledge base. Armed with this additional identification of a student's geometri conceptual knowledge, perhaps instructors would be able to help more calculus students experience success. Greater success in calculus could influence attrition rates in that course,causing them to diminish, and encourage more students to choose mathematically related careers.

These are two of the primary goals of the reform movement (Douglas, 1986), and all materials, methods, or devices that will enhance the attainment of these goals should be given serious consideration.

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APPENDICES

APPENDIX A

PLACEMENT EXAMINATION

MATH PLACEMENT TEST

Please read the following instructions carefully:

1. **You may mark on this test. However, only answers on your computer card will be graded.**

2. **USE ONLY A #2 PENCIL.**

3. **Time Limit: 60 minutes.**

4. **Since this is a placement test and not one for which you will receive a grade, blind guessing will reduce the validity and usefulness of the results. Answer only those questions whose answers you know or for which you can make an educated choice.**

1. Which equation illustrates the associative property of addition?

   A. $3 + 5 = 5 + 3$
   B. $2 + (-2) = 0$
   C. $2(3 + 4) = 2(3) + 2(4)$
   D. $(3 +4) + 7 = 3 + (4 + 7)$

2. Solve for x: $3x + 2 = -1$

   A. $\frac{-1}{3}$
   B. $\frac{1}{3}$
   C. -1
   D. 1

3. Solve: $x^2 + 7x = -12$

   A. {3,4}
   B. {-4,-3}
   C. {-4,3}
   D. {-3,4}

4. Solve: $|x - 5| = 2$

   A. {3,7}
   B. {-3,7}
   C. {-7,-3}
   D. (2,5}

5. What is the equation for the line that passes through the points (-1,0) and (0,1)?

   A. $y = -x$
   B. $y = x$
   C. $x + y = 1$
   D. $y = x + 1$

6. What is the equation of the line whose slope is 2 and whose y-intercept is -4?

   A. $y + 2x = -4$
   B. $y = 2x - 4$
   C. $y = -4x + 2$
   D. $y = 12x - 7$

7. If $f(x) = 4 - 3x$, what is $f(1)$?

A. -1
B. 1
C. 7
D. -7

8. Solve this system: $\begin{aligned} x &= y - 2 \\ 2x + 3y &= 0 \end{aligned}$

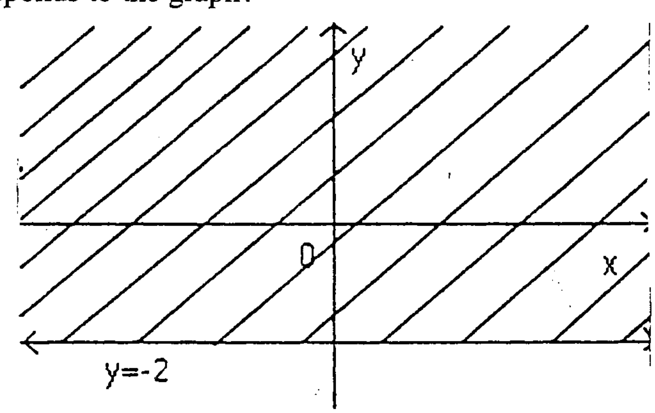
A. $\left\{\frac{6}{5}, \frac{-4}{5}\right\}$
B. $\left\{\frac{14}{5}, \frac{-4}{5}\right\}$
C. $\left\{\frac{-6}{5}, \frac{4}{5}\right\}$
D. $\left\{\frac{-3}{4}, \frac{5}{4}\right\}$

9. $(m + 3n)(2m - n)$

A. $2m^2 + 5mn - 3n^2$
B. $2m^2 - 3n^2 + 6$
C. $2m^2 - 3n^2$
D. $m^2 + 6mn - 3n^2$

10. Factor the following: $a^2 - a - 30$
A. $(a + 5)(a + 6)$
B. $(a - 5)(a - 6)$
C. $(a - 5)(a + 6)$
D. $(a + 5)(a - 6)$

11. Which inequality corresponds to the graph?
A. $x \geq -2$
B. $y \geq -2$
C. $x \leq -2$
D. $y \leq -2$



12. The expression, $3(x+y)+2(x+3y)$, is equivalent to:

A. $5(2x+4y)$
B. $5x+4y$
C. $5x+8y$
D. $5x+9y$

13. The expression, $\frac{8x^3-4x}{4x}$,simplifies to:

A. $2x^2$
B. $8x^3$
C. $2x^2-1$
D. $8x^3-1$

14. Factor the following polynomial expression: $12x^2+60x+75$

A. $3(2x+5)^2$
B. $(6x+25)(2x+3)$
C. $(3x+25)(4x+3)$
D. $(4x+25)(3x+3)$

15. Simplify the following: $\sqrt{48}$

A. $16\sqrt{3}$
B. $4\sqrt{3}$
C. $8\sqrt{6}$
D. $6\sqrt{8}$

16. Simplify $(-4+7i)-(6-2i)$, where $i=\sqrt{-1}$:

A. $-10+5i$
B. $2+5i$
C. $2+9i$
D. $-10+9i$

17. Solve: $3\sqrt{x} = 4$

A. $\pm\frac{16}{9}$
B. $\pm\frac{4}{9}$
C. $\sqrt{\frac{4}{3}}$
D. $\frac{16}{9}$

18. The expression, $\frac{x-3}{5} - \frac{x+2}{10}$, is equivalent to:

A. $\frac{x-4}{10}$
B. $\frac{x-8}{10}$
C. $x - 4$
D. $x - 8$

19. Solve the following for equation for a: $a + \frac{a}{4} = 5$

A. -1
B. 1
C. 4
D. 5

20. Suppose that y varies directly with x and that $y = 50$ when $x = 40$. Find y when $x = 65$.

A. 0.8
B. 32
C. 52
D. 81.25

21. Find the discriminant of $3x^2 + 2x + 2$:

A. $-2 \pm \sqrt{28}$
B. 28
C. $-2 \pm \sqrt{-20}$
D. -20

22. Solve for x in the following equation: $\frac{x}{2}+\frac{x}{5}=1$

A. 10

B. $\frac{10}{7}$

C. $\frac{7}{2}$

D. $\frac{7}{10}$

23. Solve the equation for x: $\frac{1}{x+2}=6$

A. 3

B. $-\frac{11}{6}$

C. 4

D. 7

24. Solve for x in the equation: $\frac{6}{7}x=42$

A. 49

B. 63

C. 56

D. 65

25. The domain of $y=\sin x$ is:

A. $\{-1 \le y \le 1\}$

B. $\{-1 \le x \le 1\}$

C. $\{\text{all real numbers}\}$

D. $\{0 \le y \le 2\pi\}$

26. Find the value of the given trigonometric function for an angle α whose terminal side passes through the point (-3,7).

Cot α =

A. $\frac{3}{7}$

B. $\frac{-7}{3}$

C. $\frac{-\sqrt{58}}{3}$

D. $\frac{-3}{7}$

27. If $\tan A = -\frac{\sqrt{3}}{3}$, and $0^\circ \leq A \leq 180^\circ$, then A =

A. 120°
B. 60°
C. 150°
D. 30°

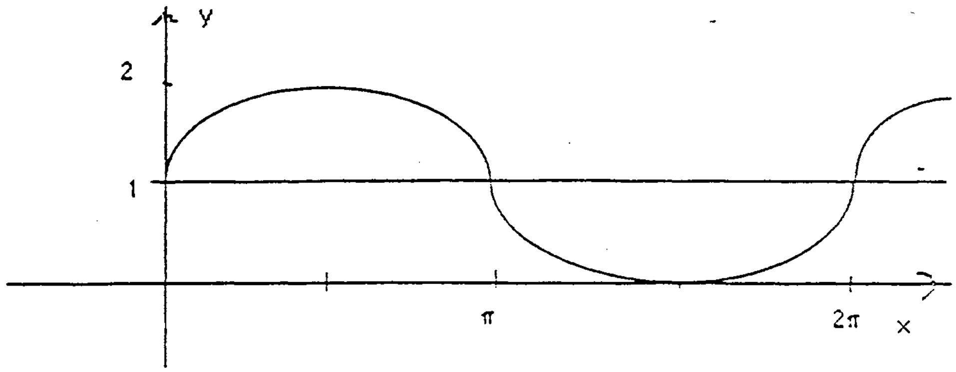
28. The length of the hypotenuse of a 30°-60°-90° triangle is 4 cm. Find the length of the leg opposite the 60° angle.

A. $2\sqrt{3}$ cm
B. 2 cm
C. $2\sqrt{2}$ cm
D. 4 cm

29. Express the following angle measure in degrees: $\frac{7\pi}{10}$ radians

A. 140°
B. 18°
C. 70°
D. 126°

30. Identify the graph.



A. $y = \sin x + 1$
B. $y = \cos x$
C. $y = \sin x$
D. $y = 1 - \cos x$

31. The logarithmic form of $x^2 = 10$ is

A. $\log_x 10 = 2$
B. $\log_{10} x = 2$
C. $\log_2 y = x$
D. $\log_2 x = y$

32. If $2^x = 8$, then $x =$

A. 4
B. $\frac{1}{4}$
C. 1
D. 3

33. $\log_3 5x =$

A. $\log_3 5x + \log_3 x$
B. $\log_x 15$
C. $\log_3 5 - \log_3 x$
D. $\frac{\log_3 5}{\log_3 x}$

34. If $3^x = 7$, then $x =$

A. $\frac{\log_{10} 3}{\log_{10} 7}$

B. $\frac{\log_{10} 7}{\log_{10} 3}$

C. $\log_{10} 3 - \log_{10} 7$

D. $\log_{10} 7 - \log_{10} 3$

35. The exponential form of $\log_4 y = 10$ is

A. $y^{10} = 4$

B. $y^4 = 10$

C. $y = 4^{10}$

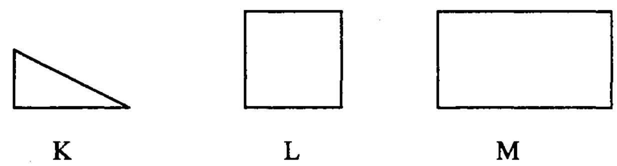
D. $y = 10^4$

APPENDIX B

VAN HIELE GEOMETRY TEST

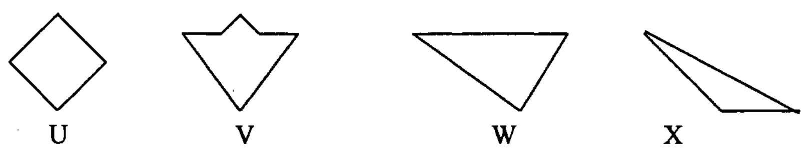
VAN HIELE GEOMETRY TEST

1. Which of these are squares?



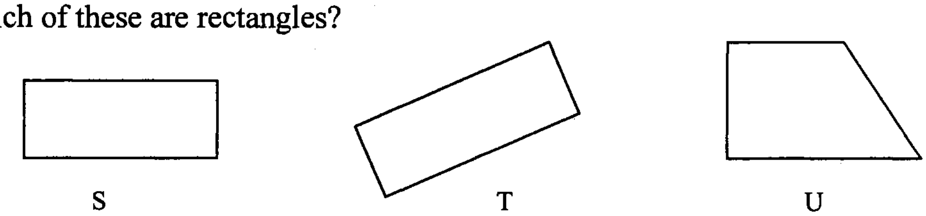
(a) K only
(b) L only
(c) M only
(d) L and M only
(e) All are squares.

2. Which of these are triangles?



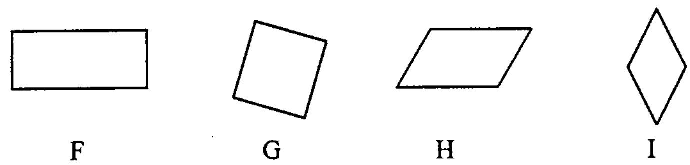
(a) None of these are triangles.
(b) V only
(c) W only
(d) W and X only
(e) V and W only

3. Which of these are rectangles?



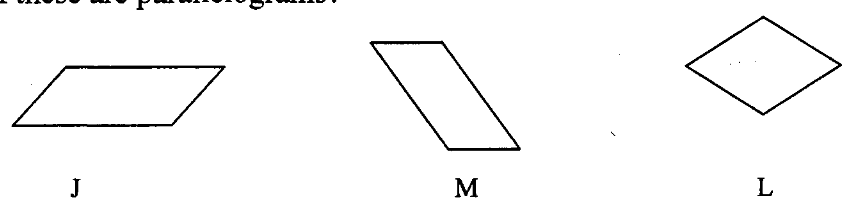
(a) S only
(b) T only
(c) S and T only
(d) S and U only
(e) All are rectangles.

4. Which of these are squares?



(a) None of these are squares.
(b) G only
(c) F and G only
(d) G and I only
(e) All are squares.

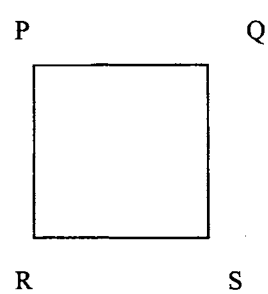
5. Which of these are parallelograms?



(a) J only
(b) L only
(c) J and M only
(d) None of these are parallelograms.
(e) All are parallelograms

6. PQRS is a square.

Which relationship is true in all squares?



(a) $\overline{PR}$ and $\overline{RS}$ have the same lengths.

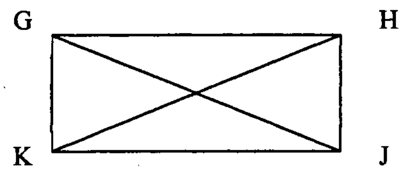
(b) $\overline{QS}$ and $\overline{PR}$ are perpendicular.

(c) $\overline{PS}$ and $\overline{QR}$ are perpendicular.

(d) $\overline{PS}$ and $\overline{QS}$ have the same length

(e) Angle Q is larger than angle R.

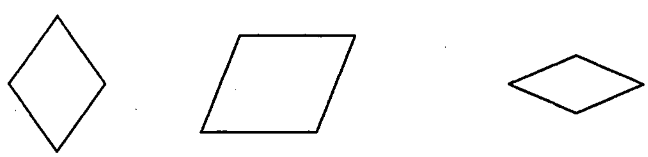
7. In a rectangle, GHJK, $\overline{GJ}$ and $\overline{HK}$ are the <u>diagonals</u>.



Which of (a) - (d) is <u>not</u> true in <u>every</u> rectangle?
- (a) There are four right angles.
- (b) There are four sides.
- (c) The diagonals have the same length.
- (d) The opposite sides have the same length.
- (e) All of (a) - (d) are true in every rectangle.

8. A <u>rhombus</u> is a 4-sided figure with all sides of the same length.

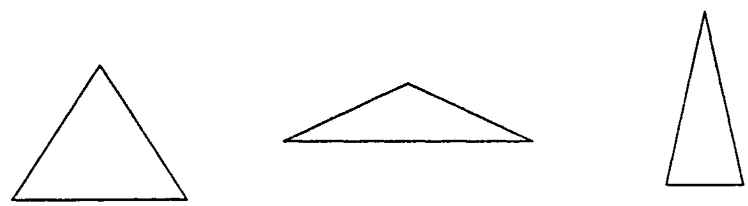
Here are three examples.



Which of (a) - (d) is <u>not</u> true in every rhombus?

- (a) The two diagonals have the same length.
- (b) Each diagonal bisects two angles of the rhombus.
- (c) The two diagonals are perpendicular.
- (d) The opposite angles have the same measure.
- (e) All of (a) - (d) are true in every rhombus.

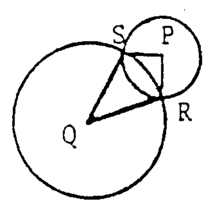
9. An isosceles triangle is a triangle with two sides of equal length.
Here are three examples.

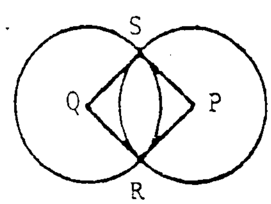


Which of (a) - (d) is true in every isosceles triangle?

(a) The three sides must have the same length.
(b) One side must have twice the length of another side.
(c) There must be at least two angles with the same measure.
(d) The three angles must have the same measure.
(e) None of (a) - (d) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PQRS. Here are two examples.





Which of (a) -(d) is <u>not</u> always true?

(a) PQRS will have two pairs of sides of equal length.
(b) PQRS will have at least two angles of equal measure.
(c) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.
(d) Angles P and Q will have the same measure.
(e) All of (a) - (d) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

(a) If 1 is true, then 2 is true.
(b) If 1 is false, then 2 is true.
(c) 1 and 2 cannot both be true.
(d) 1 and 2 cannot both be false.
(e) None of (a) - (d) is correct.

12. Here are two statements.

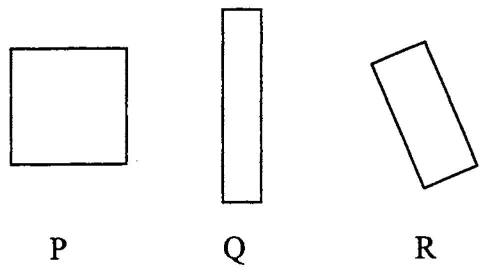
Statement S: ΔABC has three sides of the same length.

Statement T: In ΔABC, $\angle B$ and $\angle C$ have the same measure.

Which is correct?

(a) Statements S and T cannot both be true.
(b) If S is true, then T is true.
(c) If T is true, then S is true.
(d) If S is false, then T is false.
(e) None of (a) - (d) is correct.

13. Which of these can be called rectangles?



(a) All can.
(b) Q only.
(c) R only.
(d) P and Q only.
(e) Q and R only.

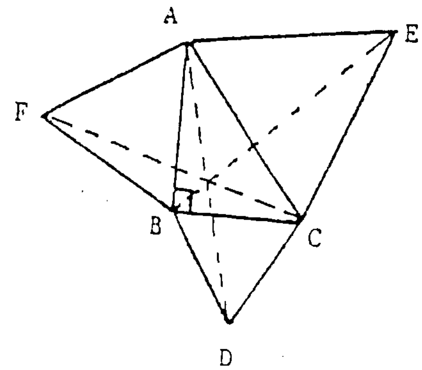
14. Which is true?

(a) All properties of rectangles are properties of all squares.
(b) All properties of squares are properties of all rectangles.
(c) All properties of rectangles are properties of all parallelograms.
(d) All properties of squares are properties of all parallelograms.
(e) None of (a) - (d) are true.

15. What do all rectangles have that some parallelograms do not have?

(a) opposite sides equal
(b) diagonals equal
(c) opposite sides parallel
(d) opposite angles equal
(e) none of (a) - (d)

16. Here is right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(a) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(b) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(c) In any right triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(d) In any triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

(e) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

(a) D implies S which implies R.
(b) D implies R which implies S.
(c) S implies R which implies D.
(d) R implies D which implies S.
(e) R implies S which implies D.

18. Here are two statements.

I. If a figure is a rectangle, then its diagonals bisect each other.

II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(a) To prove I is true, it is enough to prove that II is true.
(b) To prove II is true, it is enough to prove that I is true.
(c) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
(d) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
(e) None of (a) - (d) is correct.

19. In geometry:

(a) Every term can be defined and every true statement can be proved true.

(b) Every term can be defined but it is necessary to assume that certain statements are true.

(c) Some terms must be left undefined but every true statement can be proved true.

(d) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

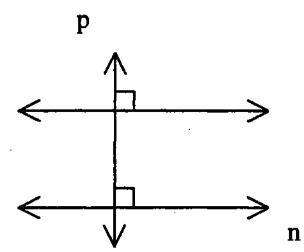
(e) None of (a) - (d) is correct.

20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.
(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.
(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(a) (1) only
(b) (2) only
(c) (3) only
(d) Either (1) or (2)
(e) Either (2) or (3)



21. In F-geometry, one that is different from the one you are use to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are {P,Q}, {P,R}, {P,S}, {Q,S}, And {R,S}.

Q

P

S

R

Here are how the words "intersect" and "parallel" are used in F-geometry.

The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {P,R} are parallel because they have no points in common.

From this information, which is correct?

(a) {P,R} and {Q,S} intersect.
(b) {P,R} and {Q,S} are parallel.
(c) {Q,R}and {R,S} are parallel.
(d) {P,S} and {Q,R} intersect.
(e) None of (a) - (b) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

(a) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
(b) In general, it is impossible to trisect angles using only a compass and a marked ruler.
(c) In general, it is impossible to trisect angles using any drawing instruments.
(d) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
(e) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

(a) J made a mistake in measuring the angles of the triangle.
(b) J made a mistake in logical reasoning.
(c) J has a wrong idea of what is meant by "true."
(d) J started with different assumptions than those in the usual geometry.
(e) None of (a) - (d) is correct.

24. Two geometry books define the word rectangle in different ways.

Which is true?

(a) One of the books has an error.
(b) One of the definitions is wrong. There cannot be two different definitions for rectangle.
(c) The rectangles in one of the books must have different properties from those in the other book.
(d) The rectangles in one of the books must have the same properties as those in the other book.
(e) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I. If p, then q.
II. If s, then not q.

Which statements follow from statements I and II?

(a) If p, then s.
(b) If not p, then not q.
(c) If p or q, then s.
(d) If s, then not p.
(e) If not s, then p.

APPENDIX C

POST-TREATMENT INSTRUMENT

Name____________________
ID Number________________

Derivative Quiz

Instructions: Answer each question as completely as possible.

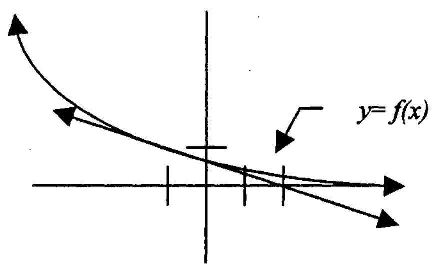
1. We have discussed two different interpretations of the derivative. The first interpretation was the physical interpretation which dealt with the instantaneous rate of change of the dependent variable with respect to the independent variable. The second interpretation was a geometric or graphical perspective. Discuss this geometric interpretation of the derivative.

2. Given a function *f(x)*, suppose you are asked to find the slope of the secant line passing through two distinct points *a* and *b* on *f(x)*.
   (a) Draw a sketch that represents the problem.

   (b) Now suppose in your sketch that *a* and *b* get closer and closer together on the curve. What happens to the slope of the secant line as this happens?

   (c) Discuss what you are actually finding when $a = b$.

3. Given the following sketch, determine the derivative of $f(x)$ at $x = -1$.



4. Find the value of the derivative of $f(x) = 2x^2 - 3x + 1$ at $x = 2$.

APPENDIX D

SCORES ON INSTRUMENTS

SCORES ON INSTRUMENTS
FOR BOTH SECTIONS

SUBJECT	CLASS #	PLACEMENT	VHLEVEL	PTPRO	PTCON
1	1	-	1	10	0
2	1	21	3	10	26
3	1	-	1	5	8
4	1	23	3	10	16
5	1	18	3	10	13
6	1	18	3	5	8
7	1	19	3	10	23
8	1	20	3	10	13
9	1	13	1	10	23
10	1	21	3	10	16
11	1	29	1	10	21
12	1	22	3	10	28
13	1	22	3	10	21
14	1	18	1	10	5
15	1	23	3	10	33
16	1	21	3	5	21
17	1	18	3	10	8
18	1	21	3	10	30
19	1	23	3	10	5
20	1	18	2	10	5
21	1	21	3	10	18
22	1	24	3	10	14
23	2	19	3	5	13
24	2	20	1	10	8
25	2	22	3	10	30
26	2	9	1	5	13
27	2	23	3	10	28
28	2	22	2	10	14
29	2	18	3	10	33
30	2	22	1	10	21
31	2	23	3	10	23
32	2	19	3	10	33
33	2	11	1	5	8
34	2	20	1	5	26
35	2	23	3	10	28
36	2	21	1	10	8
37	2	17	1	10	23
38	2	23	2	10	10
39	2	23	3	10	13
40	2	23	3	10	23
41	2	20	2	10	13

SUBJECT	CLASS #	PLACEMENT	VHLEVEL	PTPRO	PTCON
42	2	22	3	5	0
43	2	19	2	5	0
44	2	19	1	5	0
45	2	22	3	10	26
46	2	18	3	11	5

APPENDIX E

IRB APPROVAL

OKLAHOMA STATE UNIVERSITY
INSTITUTIONAL REVIEW BOARD

DATE: 01-14-99 **IRB #: ED-99-029**

Proposal Title: THE EFFECTS OF CONSTRUCTIVIST TEACHING TECHNIQUES ON THE SPATIALIZATION OF THE DERIVATIVE IN A FIRST SEMESTER CALCULUS COURSE

Principal Investigator(s): Kay Reinke, Mary Lou Miller

Reviewed and Processed as: Expedited

Approval Status Recommended by Reviewer(s): Approved

Signature: Carol Olson

Date: January 14, 1999

Carol Olson, Director of University Research Compliance
cc: Mary Lou Miller

Approvals are valid for one calendar year, after which time a request for continuation must be submitted. Any modification to the research project approved by the IRB must be submitted for approval. Approved projects are subject to monitoring by the IRB. Expedited and exempt projects may be reviewed by the full Institutional Review Board.

VITA

Mary Lou Miller

Candidate for the Degree of

Doctor of Education

Thesis: A STUDY OF THE EFFECTS OF REFORM TEACHING METHODOLOGY AND VAN HIELE LEVEL ON CONCEPTUAL LEARNING IN CALCULUS

Major Field: Curriculum and Instruction

Biographical:

Personal Data: Born in Picher, Oklahoma, November 16, 1959, the daughter of Ralph L. and Virginia L. McKinley.

Education: Graduated from Picher High School, Picher , Oklahoma, in May 1978; Received Bachelor of Science Degree in Education from Oklahoma State University at Stillwater, Oklahoma, in May, 1982; received Master of Science Degree in Mathematics from University of Tulsa at Tulsa, Oklahoma in December 1993, completed requirements for the Doctor of Education Degree from Oklahoma State University at Stillwater, Oklahoma in December 1999.

Professional Experience: High School Mathematics Teacher, Cushing High School, Cushing, Oklahoma, 1982-1986; High School Mathematics Teacher, Catoosa High School, Catoosa, Oklahoma, 1986-1987; Graduate Assistant, Mathematics Department, University of Tulsa, Tulsa, Oklahoma, 1986-1989; Instructor in Mathematics, Missouri Southern State College, Joplin, Missouri, August, 1989 to August, 1993; Graduate Assistant inCurriculum and Instruction, Oklahoma State University, Stillwater, Oklahoma, August, 1993 to August 1996; Instructor of Mathematics, Oral Roberts University, Tulsa, Oklahoma, August 1996 to present.