USING INFORMATIVE BAYESIAN PRIORS AND ON-FARM EXPERIMENTATION TO PREDICT OPTIMAL SITE-SPECIFIC NITROGEN RATES

By

COLE PATTERSON

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Thesis Approved:

Dr. B. Wade Brorsen

Thesis Adviser

Dr. Dayton M. Lambert

Dr. Brian Arnall

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Name: GEORGE PATTERSON

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Abstract: Precise site-specific nitrogen applications will reduce excess nitrogen fertilizer run-off and have corresponding environmental benefits. Adoption of site-specific nitrogen application technology has been slow, and many studies have shown that site-specific nitrogen fertilizer is not yet unambiguously profitable. Most U.S. Corn Belt states now recommend the Maximum Return to Nitrogen (MRTN) method for determining optimal nitrogen rates, which is based on 15 years of on-farm yield response to nitrogen trials. The MRTN method provides a uniform rate recommendation for a region of a state. This study goes beyond the MRTN method by combining the MRTN data, Bayesian methods, and on-farm experimentation to provide site-specific nitrogen recommendations.

On-farm trials are now being used to provide the information necessary for site-specific management. The issue is that recommendations from only a few years of data can be very noisy. Bayesian methods can combine the prior information from MRTN with data from on-farm experiments to provide more accurate site-specific nitrogen rate recommendations. The problem is that the needed models to use as Bayesian priors have not been estimated. This research fills this gap. Utilizing data from the Maximum Return to Nitrogen database, Bayesian estimation is used to estimate production functions that have a time trend to account for increased corn yields over time. The estimated models are then used as an informative prior for yield response estimations using on-farm experimental data. Three years of on-farm experimental data from a single field is used to estimate a spatially varying coefficient model. The model is estimated using two years of data to predict the third year. The predicted spatial variability was small and uncorrelated with spatial variability in the third year. Even though experimentation did not help with variable rate recommendations, it can provide uniform rate recommendations for a specific field.

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CHAPTER I

INTRODUCTION

1.1 Nitrogen Application Methods

Nitrogen fertilizer is necessary to achieve high corn yields (Below, 2008). Historically, a uniform rate of nitrogen was applied across an entire field. The yield goal approach recommended a rate based on expected yields, such as 1.2 pounds of nitrogen per bushel of yield goal (Fernandez et al., 2009). This approach has been largely replaced by the Maximum Return to Nitrogen (MRTN) method that makes a single recommendation for an entire region (Nafziger, 2018). The yield goal approach has become outdated due to modern corn hybrids that greatly improve yields as well as nitrogen use efficiency (Mueller et al., 2019). Soils are not uniform within or across fields and will differ depending on soil type, elevation, erosion, and many other environmental factors. The differences in soil characteristics are one reason why variable rate applications of nitrogen fertilizer might be beneficial for producers.

The technology to apply variable rate nitrogen is included on the newer equipment used by custom applicators. Without an accurate method of estimating optimal nitrogen rates, however, it is questionable that variable rate nitrogen application offers much value to producers. Recent research finds that variable rate nitrogen is not yet unambiguously profitable (Boyer et al., 2011; Stefanini et al., 2015; Larson et al., 2020; Queiroz et al., 2021). The issue is the method for estimating nitrogen rates has not yet been able to catch up to the technology of variable rate applications. This research proposes and tests a method to more accurately estimate variable-rate optimal nitrogen rates.

MRTN recommendations are based on expected profit maximization and estimated corn yield response functions based on data from on-farm research (Nafziger, 2018). Illinois is split into three regions. A different nitrogen rate is recommended for each region. MRTN rates are not site-specific and therefore not beneficial for variable rate applicators (Steinke, 2022). Other studies have used on-farm trials as a way of acquiring information necessary for site-specific management (Bullock et al., 2020). The Data Intensive Farm Management (DIFM) project is a collaboration between researchers and producers across the globe that use on-farm experiments to base farm input management decisions (Bullock et al., 2019). When these on-farm trials are used with only a few years of data, they can produce noisy nitrogen rate recommendations. Combining the extensive MRTN dataset as a prior to be used with experimental on-farm data could reduce the uncertainty and improve the accuracy of optimal nitrogen rate estimates using on-farm experiments.

1.2 Bayesian Methods and Production Functions

Combining MRTN and DIFM data is a Bayesian problem. Bayesian methods can use the data collected on yield response as a prior to estimate more accurate nitrogen rates. Currently, updated stochastic linear plateau model estimates to use as priors do not exist. The current MRTN is in a format that has limited usefulness as a prior. The nitrogen rate calculator only produces optimal nitrogen rates with no measure of uncertainty. MRTN estimations are made using different functional forms for yield response. Typically, a quadratic plateau model is estimated for each site year. Linear, quadratic, and constant functions are sometimes used when they better fit the data than the quadratic plateau. MRTN also does not account for time trends of corn yield response to nitrogen. The MRTN only uses the most recent 15 years of data to estimate regional nitrogen rates. MRTN does not include a time trend when estimating optimal nitrogen rates. However, using the MRTN data to estimate a stochastic linear plateau model with a time trend will provide the priors needed for Bayesian estimation.

The functional form used to estimate yield response functions can lead to very different nitrogen recommendations. Many models are based on von Liebig's Law of the Minimum, which states crop yield is determined by the most limiting essential nutrient. For example, Grimm et al. (1987) found the historical von Liebig model, linear plateau, well represented crop response. Recent research has used stochastic linear plateau functions. Tembo et al. (2008) let the plateau vary stochastically from year to year, while Makowski and Wallach (2002) let all parameters vary. MRTN lets all parameters vary by site year. The more restrictive plateau model is used here as we impose that parameters vary linearly by year. Stochastic linear plateau production functions have been applied to wheat (Brorsen & Richter, 2012), winter rye (Tumusiime et al., 2011), and cotton (Brorsen, 2013). Few applications of stochastic plateau production functions have been used for corn yields (Lambert & Cho, 2022), and most of the research has been outside of major corn production areas (Boyer et al., 2013; Villacis et al., 2020). Estimating stochastic linear plateau models using the MRTN dataset will provide the priors needed for estimating optimal nitrogen rates in the U.S. Corn Belt.

One approach that has been used to estimate spatially varying coefficient production functions is geographically weighted regression (GWR) (Evans et al., 2020; Trevisan et al., 2020; Lambert & Cho, 2022). GWR is often used due to its ability to fit data well. Wheeler and Calder (2007) used a simulation to show that a Bayesian regression model with spatially varying coefficients provided more accurate parameter estimates than that of GWR. Finley (2011) also compared the two approaches and concluded that the Bayesian spatially varying coefficient model has a substantially smaller prediction mean square error. The drawback of using Bayesian methods is that they are computationally heavy and time consuming in comparison to GWR. Using an informative Bayesian prior can improve the accuracy and speed of estimation while also making inference possible that is not possible with GWR. This research goes beyond the previous research on estimating production functions with spatially varying coefficients by using an informative Bayesian prior to estimate a spatially varying plateau model. Estimates are then used to determine optimal nitrogen rates.

Bayesian methods have long been used to estimate yield response functions (Holloway & Paris, 2002; Holloway, 2003; Ouedraogo & Brorsen, 2018; Moeltner et al., 2021). In a simulation study, Lawrence et al. (2015) used Bayesian methods to update the parameters each year, but did not begin with an informative prior. Bullock et al. (2020) stated the greatest value of the on-farm precision experiment came from prior information collected from two previous trials. This information helped producers more accurately estimate a field's optimal uniform application rate. Franz et al. (2020) concluded that among spatial and temporal variables, including soil types, topography, and crop condition, the best predictor of crop yield was historical yield maps. This past research suggests that Bayesian methods are a promising way to reduce the noise in estimates when using only a few years of on-farm experimental data.

1.3 Real-World Application

The methods developed and demonstrated here are intended as a step forward toward designing a system that will be adopted by custom fertilizer applicators, such as Corteva, Bayer Crop Science, Nutrien Ag Solutions, and other custom applicators. More specifically, the MRTN dataset is used to estimate a stochastic linear plateau function that serves as a prior for a spatially varying coefficient model that uses on-farm experimental data to estimate nitrogen rate recommendations.

CHAPTER II

DATA AND METHODS

2.1 Maximum Return to Nitrogen

The Maximum Return to Nitrogen method is a regional approach for estimating corn nitrogen rates for many Midwest states. For example, for Illinois in 2021 the method used 720 corn yield response to nitrogen trials from 15 years, (2006 to 2020), of data. Each year, the oldest year was removed from the dataset as a new year was added to maintain 15 years of data. These trials were conducted with spring application, sidedress application, or split between a preplant application and a sidedress application of nitrogen. No sites were irrigated. In Illinois, MRTN uses mostly QRP models, some quadratic models, a few linear response, and no-response models were estimated for each site year. QRP models are often used because they fit the data well. The MRTN estimations are often kept in place of the actual data. The MRTN approach uses a grid search procedure to determine the optimal level of nitrogen. The data used here includes some older data that are not currently being used by MRTN. Also, the 2010 to 2012 data used in the MRTN were not obtained. The data used here represents only a portion of the MRTN data. Note that MRTN weights each site year equally. Since more data are available from recent years, MRTN is not as slow to adjust to changes over time as it would be if each year had the same number of sites.

The Maximum Return to Nitrogen data used for this research included 2,799 observations of on-farm yield response to nitrogen experiments conducted from 1999 to 2009 and 2013 to 2021 located across the north, central and southern regions of Illinois. The data come from four different projects.

The first project was from 1999 to 2008 at seven different sites. The second project was from 2001 to 2004 and was organized by the Illinois Department of Agriculture. The third project was composed of data from 2006 to 2008, with some additional trials from 2009 and was funded by a fertilizer tonnage fee administered by the Illinois Fertilizer Research and Education Council. The final project was from 2013 to 2021 and was funded by a fertilizer tonnage fee administered by the Illinois Fertilizer Research and Education Council. All projects include corn-soybean rotations. Experiments from 70 Illinois counties out of 102 are in the dataset. Nitrogen rates were not consistent across locations, and six different rates per location were applied. Nitrogen rates ranged from 0 to 382 kilograms per hectare with an average of 147 kilograms per hectare. Typically, 0, 50, 100, 150, 200, and 250 kilograms of nitrogen were applied, and any additional nitrogen applied by the producer was added to these amounts. The yield data are treatment means for each site year, and all yield values were collected with a combine yield monitor, the weigh wagon method, or a small plot combine.

2.2 Production Functions

The data were used to estimate yield response functions. Both stochastic linear response plateau (LRP) and stochastic quadratic plateau (QRP) models were considered. The SAS procedure PROC MCMC was used to estimate the model with Bayesian methods and weakly informative priors. The LRP function assumes that corn yield increases linearly until the plateau is reached. At the plateau, nitrogen no longer affects the yield. The LRP model was then used as an informative prior for estimating a spatially varying coefficient model using on-farm experimental data. The stochastic LRP model with time varying parameters is

$$Y_{itj} = \min[(\beta_0 + \alpha_0 t) + (\beta_1 + \alpha_1 t)N_{itj}, P_0 + \alpha_2 t + u_{it}] + v_t + \gamma_{it} + \varepsilon_{itj} \tag{1}$$

where Y_{itj} is corn yield for the ith location for year t, j treatment, N_{itj} is the nitrogen level, β_0, α_0, β_1, α_1, P_0, and α_2 are parameters to be estimated, and $u_{it} \sim N_i(0, \sigma_u^2), v_t \sim N_i(0, \sigma_v^2), \gamma_{it} \sim N\left(0, \sigma_\gamma^2\right), \varepsilon_{itj} \sim N_i(0, \sigma_\varepsilon^2)$ with all four error terms being independent.

The t is defined as $t = year - 2010$. The estimation procedure used 5,000 observations as burn in, a thinning rate of 20, and 20,000 simulated draws to generate each parameters' posterior distribution.

An adaptive blocked random walk Metropolis algorithm that uses a normal proposal distribution was used. Markov Chain convergence was verified by examining the ODS Graphics feature in the PROC MCMC statement to generate autocorrelation plots for each parameter. With a Markov Chain, each simulated point depends on the previous observation. If autocorrelation is high, then a larger number of samples are needed to explore the parameter space of the posterior distribution.

The priors used with the MRTN data are weakly informative priors:

$$\beta_0 \sim N(147, 10^6),$$

$$\alpha_0 \sim N(0, 10^6),$$

$$\beta_1 \sim N(0.45, 10^6),$$

$$\alpha_1 \sim N(0, 10^6),$$

$$P_0 \sim N(183.6, 10^6),$$

$$\alpha_2 \sim N(0, 10^6).$$

The priors have large variances which means they have little influence on the posterior estimates. An improper inverse gamma distribution is used for variances since α (the first parameter of the distribution) is less than 2.

The stochastic quadratic model with time varying parameters was estimated similarly. Cho et al. (2023) shows how to impose differentiability at the join point by deriving the first order condition of the quadratic response plateau (QRP) model which identifies the join point as $Nstar_{it} = -C_1/(2 \cdot C_2)$. The stochastic QRP model is

(2) $$Y_{itj} = \min\left[\left(C_0 + C_1 \cdot N_{itj} + C_2 \cdot N_{itj}^2\right), \left(C_0 + C_1 \cdot Nstar_{it} + C_2 \cdot {Nstar^2}_{it}\right)\right] + v_t + \gamma_{it} + \varepsilon_{itj}$$

where Y_{it} is corn yield for the ith location for year t for jth treatment, $C_{0t} = \beta_0 + \alpha_0 t$, $C_{1t} = (\beta_1 + \alpha_1 t)$, $C_{2it} = -C_1/(2 * Nstar_{it})$, $Nstar_{it} = Nstar_0 + \alpha_4 t + u_{it}$, N_{itj} is the nitrogen level for the ith location for jth treatment at time t, β_0, α_0, β_1, α_1, $Nstar_0$, and α_2 are parameters to be estimated, $u_{it} \sim N_{ij}(0, \sigma_u^2), v_t \sim N_{ij}(0, \sigma_v^2), \gamma_{it} \sim N_{ij}\left(0, \sigma_\gamma^2\right), \varepsilon_{itj} \sim N_{ij}(0, \sigma_\varepsilon^2)$ with all four error terms being independent, were considered. To impose differentiability in the quadratic plateau model, the left and right derivatives of the function at any point in the domain to be equal. The point at which differentiability needs to be imposed is the join point, namely $Nstar_{it}$.

The Deviance Information Criterion (DIC) is used to determine which model is a better fit for the data. The DIC (Spiegelhalter et al. 2002) is a model assessment tool that is useful in Bayeisan model selection problems where the posterior distributions of the models have been obtained by Markov Chain Monte Carlo (MCMC) simulation. Deviance is defined as:

(3) $$D(\theta) = -2\log\left(p(y|\theta)\right) + C$$

where y are the data, θ are the unknown parameters of the model, and $p(y|\theta)$ is the likelihood function. C is a constant that cancels out in all calculations that compare different models and does not need to be known. Spiegelhalter et al. (2002) gives the deviance information formula as

(4) $$\text{DIC} = \overline{D(\theta)} + p_D = D(\bar{\theta}) + 2p_D$$

where $\overline{D(\theta)}$ is posterior mean of the deviance, $D(\bar{\theta})$ is the deviance evaluated at $\bar{\theta}$, and p_D is the effective number of parameters. While both models being compared have the same number of parameters, they can have a different number of effective parameters. The lower the value, the better the model fits the data.

2.3 Profit Maximization

The optimal level of nitrogen is determined by maximizing expected profit. To perform the optimization, the SAS program PROC NLP is used. Expected profit function is:

(5) $$\max_{N\geq 0} \int E\pi(N|\theta) p(\theta|t)\, d\theta$$

where $E\pi(N|\theta)$ is expected profit given nitrogen and subject to the vectors of relevant parameters of the stochastic plateau, $E\pi = (p \cdot Y_{itj}) - (r \cdot N_{itj})$ where p is the corn price, Y_{itj} is the corn yield from the stochastic plateau model, r is the nitrogen price, and N_{itj} is the amount of nitrogen applied from the stochastic plateau model, $\theta = (B_0, B_1, P_0, \sigma_u^2)$ is the vector of relevant parameters, and $p(\theta|t)$ is the posterior distribution for θ given year t. Brorsen (2013) calculates expected profit as:

(6) $$\max_{N\geq 0} (\frac{1}{n}) \sum_{i=1}^{n} E\pi(N|\tilde{\theta}_i)$$

where $E\pi\left(N|\tilde{\theta}_i\right)$ is the expected profit subject to nitrogen given the estimated parameters. The $\tilde{\theta}_i$ are the MCMC draws from the posterior distribution. For this research, the SAS procedure PROC NLP is used to determine optimal nitrogen. The PROC NLP calculates optimal nitrogen as

(7) $$\max_{N_t\geq 0} E\pi = \frac{1}{n}\sum_{k=1}^{n}(p \cdot ((1-\varphi_k) \cdot (\beta_{0k} + \alpha_{0k}t + (\beta_{1k} + \alpha_{1k}t) \cdot N_t) + \varphi_k \cdot (P_{0k} + \alpha_{2k}t - \sqrt{\sigma_{pk}^2} \cdot \psi_k / \max(\varphi_k, 0.00001)))) - r \cdot N_t$$

where k is a given draw from the posterior, p is corn price, $\varphi_k = \varphi[(\beta_{0k} + \alpha_{0k}t + (\beta_{1k} + \alpha_{1k}t) \cdot N_t) - (P_{0k} + \alpha_{2k}t))/(\sqrt{\sigma_{pk}^2}),0,1]$, φ is the normal cdf of the posterior, $\psi_k = \psi[(\beta_{0k} + \alpha_{0k}t + (\beta_{1k} + \alpha_{1k}t) \cdot N_t) - (P_{0k} + \alpha_{2k}t))/(\sqrt{\sigma_{pk}^2}),0,1]$, ψ is the normal pdf of the posterior, σ_{pk}^2 is the plateau variance, and r is the nitrogen fertilizer price. Other costs are assumed fixed and do not affect optimal nitrogen. For the economic analysis, U.S. prices were used for calculating nitrogen and corn prices. Urea prices were used to determine a nitrogen price of $1.75 /kg by calculating the percent of actual nitrogen is equal to 0.46

multiplied by 2000 pounds of Urea fertilizer is equal to 920 pounds of nitrogen. The cost of urea fertilizer is \$732/ton divided by 920 pounds of nitrogen which provides the cost of nitrogen which \$0.80/lb. or \$1.75/kg. A corn price of \$0.26/kg was used to determine the optimal nitrogen rates. These prices reflect January 2023 market prices for urea fertilizer (Quinn, 2023) in Omaha, Nebraska, and U.S. corn (USAD-NASS, 2023).

2.4 Data Intensive Farm Management

The on-farm experimental data from the DIFM used for the second stage of this research consists of 3,836 observations on a single field in north central Ohio over three years collected with a yield monitor. All collected data were from corn following a soybean rotation. The experimental design was a completely randomized design (CRD) where treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment. The experiment is detailed in Table 1. It is important to note the different planting date of 2019 than the dates of 2017 and 2021. The yield average for 2019 is lower than the averages of 2017 and 2021. The late planting date for 2019 could have contributed to the decrease in yield. Different varieties were used for each year.

Table 1. On-Farm Experimental Data Collected from a Single Ohio Field

	2017	2019	2021
Observations	1,373	1,407	1,056
Planting date	4/22/2017	5/21/2019	4/26/2021
Variety	DKC61-54rib	DKC58-34	p0720q
Yield range (kg /ha)	11,392-17,857	8,238-14,649	8,605-18,187
Yield average (kg /ha)	14,643	11,639	13,416
Nitrogen range (kg /ha)	181-247	166-296	164-329
Nitrogen average (kg /ha)	215	229	248
Seeding range (thousand seed /ha)	74-96	67-100	59-104
Seeding average (thousand seed /ha)	84	86	82

A portion of the field was selected as a test sample for the prediction. Using fewer observations reduces the computational time of the prediction that results in accurate predictions in a fraction of the time compared to using all observations within the field (even with the reduced dataset, the estimation still took over a week for 2021 data). The longitude coordinates of the map were divided by 3, and the latitude coordinates were divided by 4 so that the field was divided into a matrix with 3 columns and 4 rows. One element of the matrix was used for all three years of data. Each cell is approximately 150 sq. meters or 0.03 acres. The width of each cell is roughly 9 meters, but the length of the cells change across the field from 12 to 21 meters in length. The fixed width is due to the width of the grain combine used to collect the data. For each matrix selected there is roughly 2.11 hectares for 2017, 1.52 hectares for 2019, and 1.37 hectares for 2021. Table 2 shows the means from the selected data. The selected data represents the field well, as the means and averages of yield, nitrogen, and seeding rate do not change significantly.

Table 2. On-Farm Experimental Data Selections from Single Ohio Field

	2017	2019	2021
Observations	141	101	91
Yield range (kg. /ha)	11,931-16,646	8,634-12,636	9,639-16,049
Yield average (kg. /ha)	14,567	11,272	13,068
Nitrogen range (kg. /ha)	185-246	176-282	182-320
Nitrogen average (kg. /ha)	218	230	262
Seeding range (thousand seed/ha)	74-95	73-96	59-104
Seeding average (thousand seed /ha)	85	87	81

The area of the field was selected due to having roughly the same number of observations across years. Figure 1 shows the map of the entire field while Figure 2 shows the grids from which a subset of the field was selected. Figures 3-5 show the subset of the field used for each year. Due to missing values and different size grids, the number of observations varied by year. There were 141 observations selected from 1,373 total observations from the 2017 on-farm experimental data. In 2019, 101 observations were selected from 1,407 observations. In 2021, 91 observations were selected from 1,056 observations. The dataset contained missing values which GWR would not be able to estimate without removing the missing values. Missing values were removed from the dataset, but GWR is still impractical to use because the coordinates for each cell changed across years.

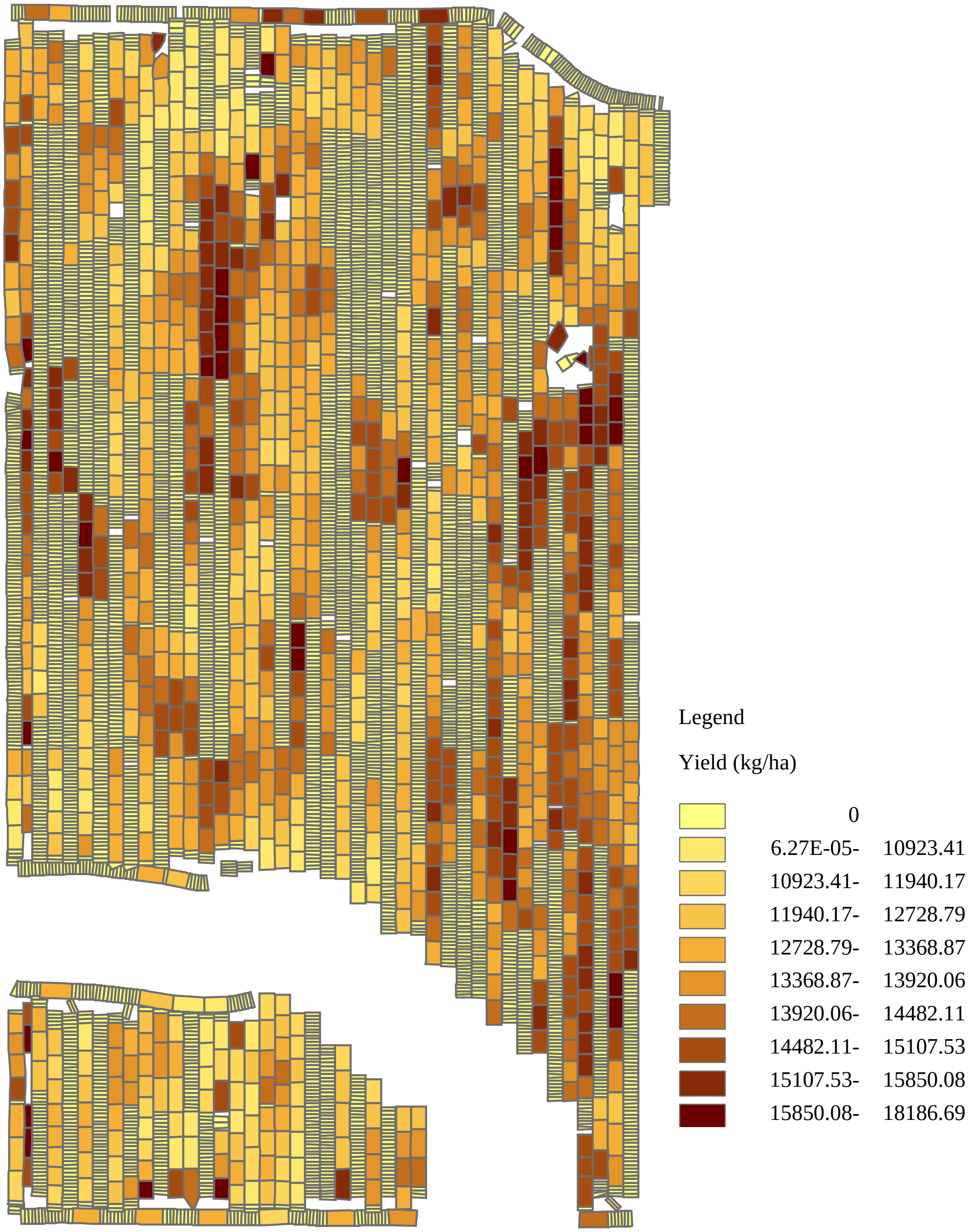


Figure 1. Yield Map of the Entire Field of the 2017 Ohio Experimental Data

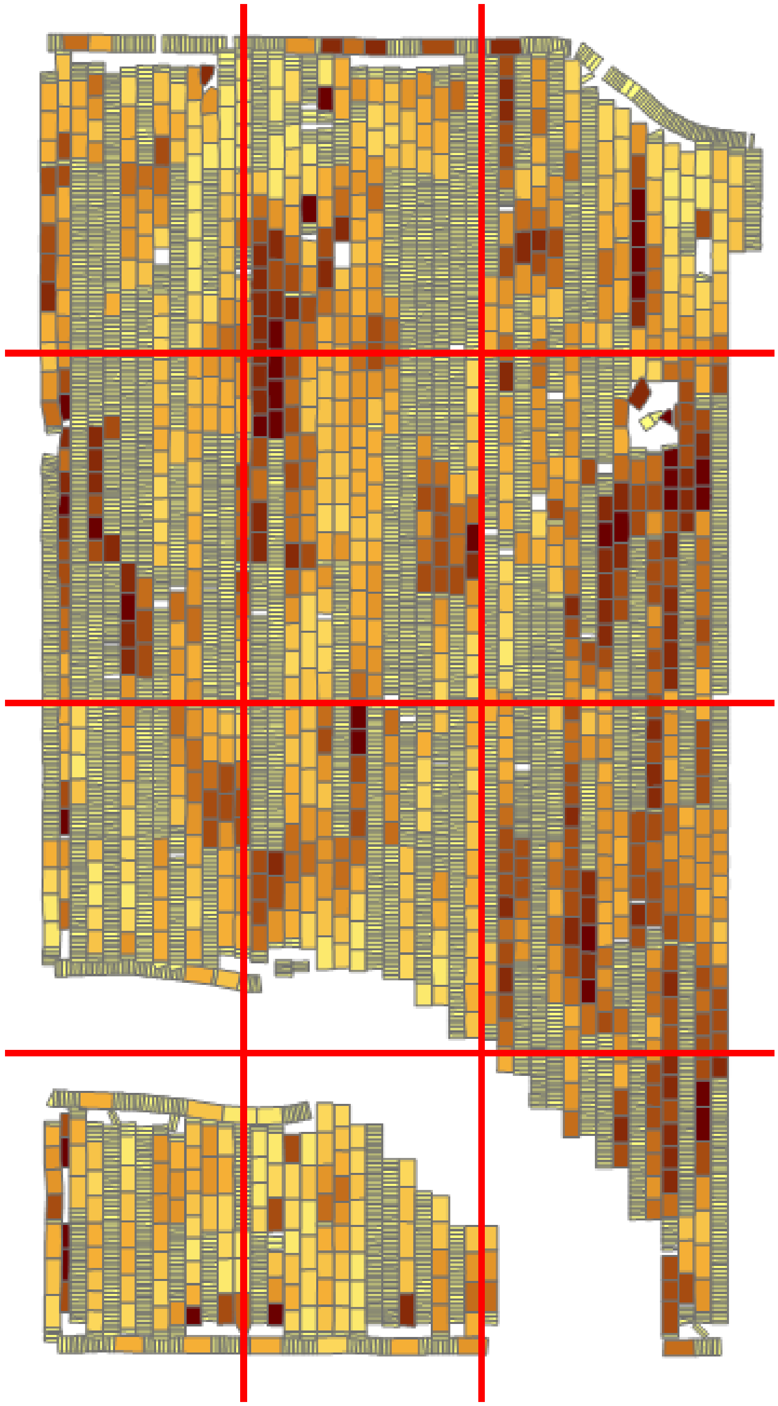


Figure 2. Map of Selection Grid

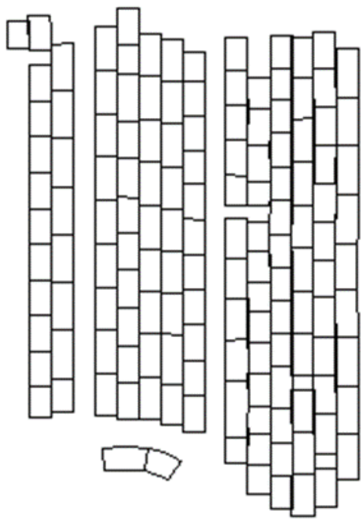


Figure 3. Map of 2017 Selected Data

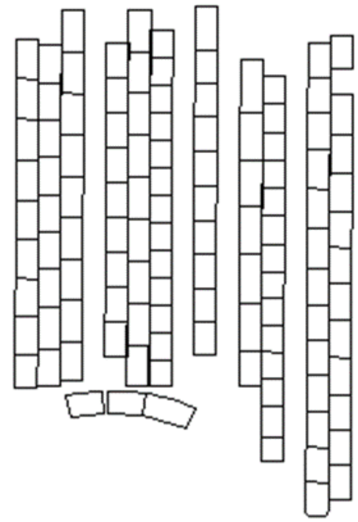


Figure 4. Map of 2019 Selected Data

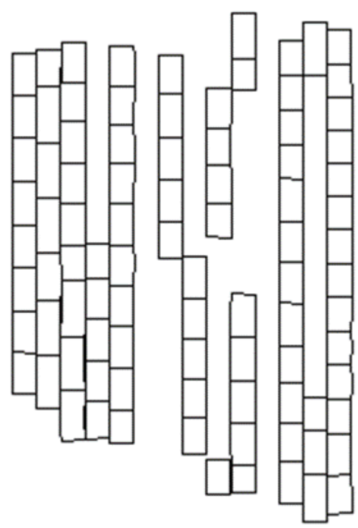


Figure 5. Map of 2021 Selected Data

2.5 Spatially Varying Plateau Model

Using the selected data from the grids shown in figures 3, 4, and 5, a spatially varying plateau model is estimated for each year:

$$(8) \qquad Y_i = \min\left[\beta_0 + \beta_1 N_i + \beta_2 S_i + \beta_3 S_i^2 + \beta_4 S_i N_i, P_i\right] + \varepsilon_i$$

where Y_i is corn yield for the ith location, N_i and S_i are the nitrogen and seeding rate for the ith location respectively. β_0, β_1, β_2, β_3, and β_4 are the corresponding coefficients for intercept, nitrogen, seeding rate, seeding rate squared, and the interaction between nitrogen and seeding rate. The posterior of the stochastic LRP model provided priors for β_0, β_1, β_4, and P_i of the spatially varying coefficients parameters. The informative priors based on the estimated stochastic linear plateau model are:

$$\beta_0 \sim N(6614.5, 281.07),$$

$$\beta_1 \sim N(40.8, 2.51),$$

$$\beta_2 \sim N(0.3, 37.64),$$

$$\beta_3 \sim \mathrm{N}(-0.2, 62.74),$$

$$\beta_4 \sim N(0, 62.74),$$

$$\bar{p} \sim N(11857.5, 284.83)$$

$$\sigma \sim N(0.1, 2)$$

$$sill \sim N(0, 0.21)$$

$$\rho \sim N(0, 0.21)$$

$$nugget \sim N(1,1)$$

Priors are based on the year 2010. All estimated variances were doubled to allow the information from the field to have more impact on the posterior's distribution. $\bar{p}$ is the mean of P_i. We assume that plateau in the field has a spatial behavior, P_i, which can be described through the multivariate normal distribution. Hence,

$$(9) \qquad \boldsymbol{P} \sim MVN(\bar{p}\mathbf{1}, \boldsymbol{\Sigma})$$

where $\boldsymbol{P} = (P_1, P_2, \dots P_n)$ and $\boldsymbol{\Sigma} = cov\big(\, P(\mathrm{s}),\, P(s')\big) = \sigma_p^2 \exp\left(-\frac{d_{s,s'}}{\rho}\right)$

where s and s' are two distinct locations in the field, $P(\mathrm{s})$ is the plateau for location s, σ_p^2 is the variance of the plateau, and $d_{s,s'}$ is the distance between s and s' locations in the field. Bayesian methods are used to fit the model given in equation (7). While the nugget was included in the estimation, it does not appear in the covariance formula. HMC algorithm, which is faster and has a better convergence rate (Carpenter et al., 2017) than Metropolis-Hastings, is used through Stan to obtain Bayesian posterior estimates. Stan uses parallelization which makes it more flexible and faster than using the SAS procedure PROC MCMC. Four chains are simulated with each chain using 2,000 draws as warmup and 5,000 iterations for estimation. The procedure uses parallel computing to reduce the time used to calculate the posterior distribution. The estimation typically takes around 14 hours to complete for one year (for 2021, one chain was slow to converge, and it took over a week). Convergence of the Markov Chain was checked using the Gelman-Rubin statistic which is denoted as:

$$(10) \qquad R = \sqrt{\frac{(d+3)\hat{V}}{(d+1)W}}$$

where $\hat{V}$ is the variance of the Bayesian credible interval, W is the mean empirical variance within each chain, and d is the degrees of freedom estimated by method of moments (Gelman & Rubin, 1992). A Bayesian credible interval is defined as which parameters lie within a given probability range.

Convergence is achieved at a value of R equals 1. The Gelman-Rubin statistic is a ratio which provides a simple summary for any MCMC sampler (Peng, 2022).

The conditional autoregressive (CAR) and simultaneously autoregressive (SAR) models can provide faster computations (Poursina, 2021) than the exponential used here, but CAR and SAR are based on continuity and so are not applicable to data where the locations of the grids change every year.

The posterior from Equation 1 is used as a prior to forecast parameter distribution for 2021 using 2017, 2019, and 2021 data. The posterior predictive distribution is used to forecast parameters for 2021 locations from 2017, 2019, and 2021 data. New coordinates from 2021 are added to forecast the yield value. The posterior predictive distribution is the distribution over new observations given previous observations. The posterior predictive distribution for replications y^{rep} of the original data set y given model parameters θ is defined by:

(11) $$p(y^{rep}|y, x^{rep}) = \int p(y^{rep}|\theta, x^{rep}) \cdot p(\theta|y)\partial\theta$$

where θ is all parameters to be estimated in the model, $p(\theta|y)$ is the posterior distribution, and x^{rep} are nitrogen and seeding rates for the desired year and plot.

2.6 Weighted Average and Mean Squared Error

To obtain forecasts for 2021 given 2017 and 2019 data, the separate estimations for 2017 and 2019 must be combined. Under normality, the Bayesian approach is a weighted average of the two predictions from 2017 and 2019 data where the weights are proportional to the standard deviations of each predicted value. Hence,

(12) $$\mu_i^* = \frac{\left(\frac{\mu_{1i}}{\sigma_{1i}}\right)+\left(\frac{\mu_{2i}}{\sigma_{2i}}\right)}{(1/\sigma_{1i})+(1/\sigma_{2i})}$$

where μ_i^* is the final predicted value of yield or one of the parameters, μ_{1i} is the mean of the predicted values of 2021 yield or one of the parameters based on 2017, μ_{2i} is the mean of the predicted values of

2021 yield or one of the parameters based on 2019, σ_{1i} is the standard deviation from 2017 for 2021 locations, and σ_{2i} is the standard deviation from 2019 for 2021 locations. The weighted average is then compared to actual yield values from 2021.

Mean squared error (MSE) is used to obtain the accuracy of the predicted values for 2017, 2019, 2021, and the weighted average of 2021 predictions from 2017 and 2019 data where MSE is

(13) $$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

n is the number of data points, Y_i is the observed values, and $\hat{Y}_i$ is the predicted values. Furthermore, MSE can be decomposed into bias and variance via following formula:

(14) $$MSE = bias^2 + variance$$

This will allow estimating how much biasedness is in the prediction.

Following the MSE calculations, a linear regression model is used to regress the latent spatial predictions for the 2021 locations from the 2017 and 2019 data against the latent spatial effect in 2021. The estimation will show if the latent spatial process of 2021 is explained by the latent spatial process of 2017 and 2019.

CHAPTER III

RESULTS

3.1 MRTN Data

The annual nitrogen and yield means of the 2,799 observations from the MRTN dataset are shown in Figures 6 and 7. The 2,799 yield response observations from the MRTN dataset are used to estimate the stochastic LRP model that serves as the prior for the spatially varying coefficient model. The average nitrogen application rate, shown in Figure 6, displays an upward trend over time. From 1999 to 2021, the average annual nitrogen rate in the experiments increased by 28 percent. The corn yield in Figure 7 followed the increased nitrogen rate by also increasing over time and increased by roughly 28 percent.

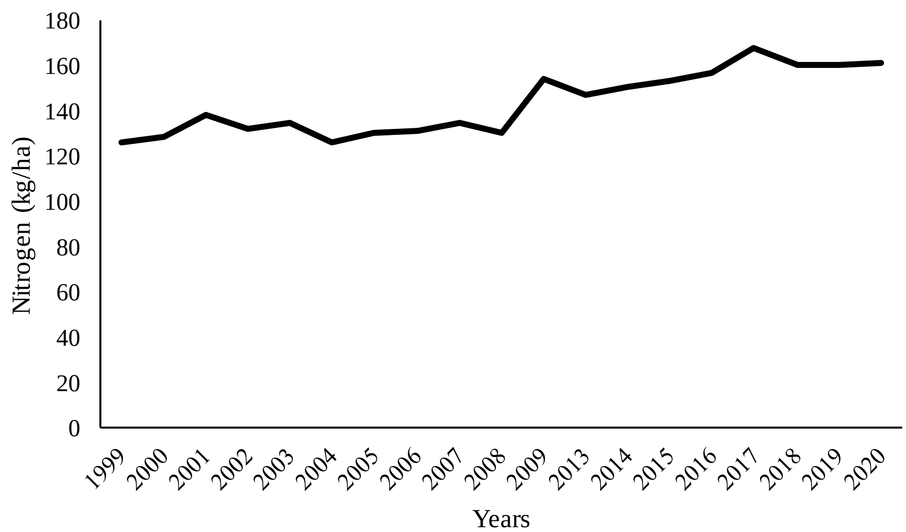


Figure 6. Annual Nitrogen Application Means Across Illinois from MRTN Database

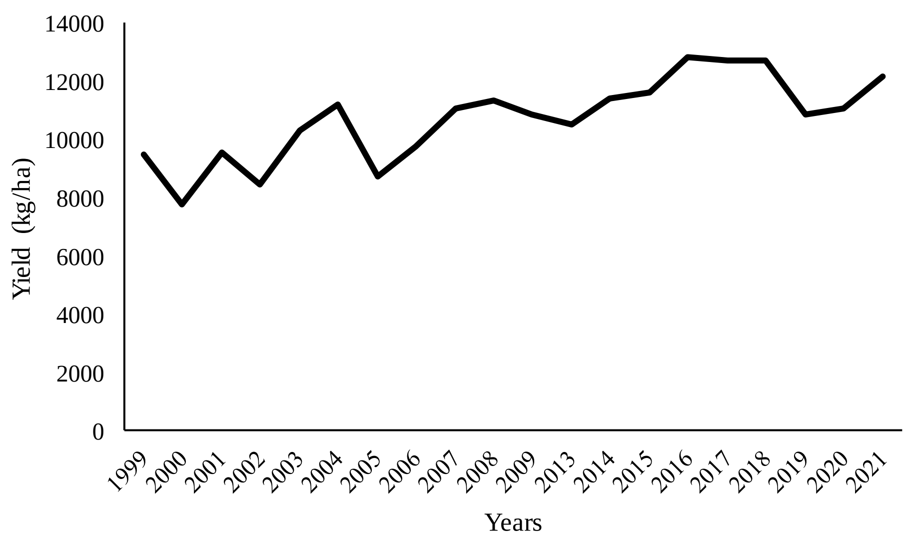


Figure 7. Annual Corn Yield Means Across Illinois from MRTN Database

The mean and standard deviation of the posterior distribution for the parameters of the stochastic LRP model are presented in Table 3 and the estimated stochastic QRP model is in Table 4. Both models show an increase in the optimal nitrogen rate from 2010 to 2021. The stochastic LRP showed an increase of 21% in the optimal nitrogen rate from 2010 to 2021. The stochastic QRP showed an increase of 17% in the optimal nitrogen rate from 2010 to 2021.

3.2 Production Function Estimates

Table 3. Stochastic Linear Plateau Estimates with Time Trends on Illinois Corn Yields (kg /ha), 1999-2021

Parameter	Mean	Standard Deviation
Intercept	6614.5	138.9
Intercept time trend	0.1	17.9
Nitrogen	40.7	1.2
Slope time trend	0.5	0.2
Plateau	11853.4	142.5
Plateau time trend	204.0	20.2
Plateau variance	1728766.0	226626.0
Year random effect	4249889.0	345320.0
Error variance	1393012.0	44255.6
Optimal N 2010 (kg /ha)	160.2	
Optimal N 2021 (kg /ha)	193.3	
Deviance Information Criterion	48083.3	

Note: All values are in kilograms per hectare. The selected price of nitrogen is $1.75/ kg., the price of corn is $0.26/kg. The data are composed of 2799 observations from the Maximum Return to Nitrogen (MRTN) dataset, University of Illinois.

The stochastic LRP model was estimated in terms of the plateau level of yield, P_0. The QRP model was estimated in terms of the plateau level of nitrogen which is denoted in Equation 2 as $Nstar_{it}$. The models can be equivalently estimated with either the plateau yield or the plateau level of nitrogen as the parameter. The parameter, Nstar time trend, directly provides the change in the optimal nitrogen value over time. The estimation shows the plateau nitrogen level increased by 3.4 kg/ha/year.

Table 4. Stochastic Quadratic Plateau Estimates with Time Trends on Illinois Corn Yields (kg. /ha), 1999-2021

Parameter	Mean	Standard Deviation
Intercept	6408.9	214.8
Intercept time trend	4.1	26.5
Nitrogen	45.5	1.0
Slope time trend	1.1	0.1
Nstar	248.9	5.1
Nstar time trend	3.4	0.7
Nstar variance	1912.6	290.5
Year random effect	3655930.0	310663.0
Error variance	1468639.0	45480.1
Optimal N 2010 (kg/ha)	228.5	
Optimal N 2021 (kg/ha)	267.0	
Deviance information criterion	48175.1	

Note: All values are in kilograms per hectare. The selected price of nitrogen is $1.75 / kg, the price of corn is $0.26/kg. The data is composed of 2799 observations from the Maximum Return to Nitrogen (MRTN) dataset, University of Illinois.

Boyer et al. (2013) estimated stochastic LRP and QRP models using corn following soybean yield response to nitrogen that provided similar standard deviations and variances as that in the estimations here. Boyer et al., however, uses corn response data from Tennessee, which could have different yield and nitrogen averages when compared to Illinois corn response data. Boyer et al.'s estimates reveal little difference between the plateau and yield values compared to the research estimates in Table 3. The main difference is Boyer et al. find a lower intercept, which can be explained as Illinois soil typically has a higher level of organic matter that can provide more nitrogen than Tennessee soil.

Utilizing the time trend variable of the stochastic LRP model, the year 2023 optimal nitrogen rate was estimated using January 2023 prices for nitrogen, $1.75/kg, and corn, $0.26/kg (Quinn, 2023; USAD-NASS, 2023). The stochastic LRP model predicted an optimal nitrogen rate of 199 kg of nitrogen /ha. The year 2023 optimal nitrogen rate was estimated using the stochastic QRP which computed an optimal nitrogen rate of 274 kg/ha. Comparatively, the nitrogen rate calculator computed optimal nitrogen rates using the same prices used in our model for the three Illinois regions: North, Central, and South. The calculator estimated an optimal nitrogen rate of

187 kg /ha for the North region, 195 kg /ha for the Central region, and 218 kg /ha for the South region. The stochastic LRP optimal nitrogen rate is far more consistent with the nitrogen rate calculator estimations of optimal nitrogen than the estimation from the stochastic QRP.

The MRTN is based on the quadratic plateau. Why are MRTN optimal nitrogen levels closer to the levels from the linear plateau? One reason is that MRTN does not adjust for a time trend, which makes the MRTN recommendation about 12 % lower. MRTN also includes other functional forms such as quadratic, linear and no response models. Further, the stochastic QRP used here imposes that parameters across space and time are drawn from a normal distribution, which may lead to different estimates.

The deviance information criterion (DIC) values in Tables 3 and 4 are used to determine which model is a better fit for the data. The DIC value for the stochastic LRP model is 48083.3. The DIC value for the stochastic QRP model is 48175.1. The lower value represents a model that is a better fit. The stochastic LRP model had a lower value, which is why it was selected to provide the prior for the spatially varying coefficient model.

Figure 8 plots the stochastic LRP model using posterior means for 2010 and 2021. The figure shows that the intercept, slope, and plateau all increased over time. The intercept increased by 0.02 percent and the slope increased by roughly 13 percent. The plateau increased the most by roughly 28 percent, and thus the optimal nitrogen rate also increased.

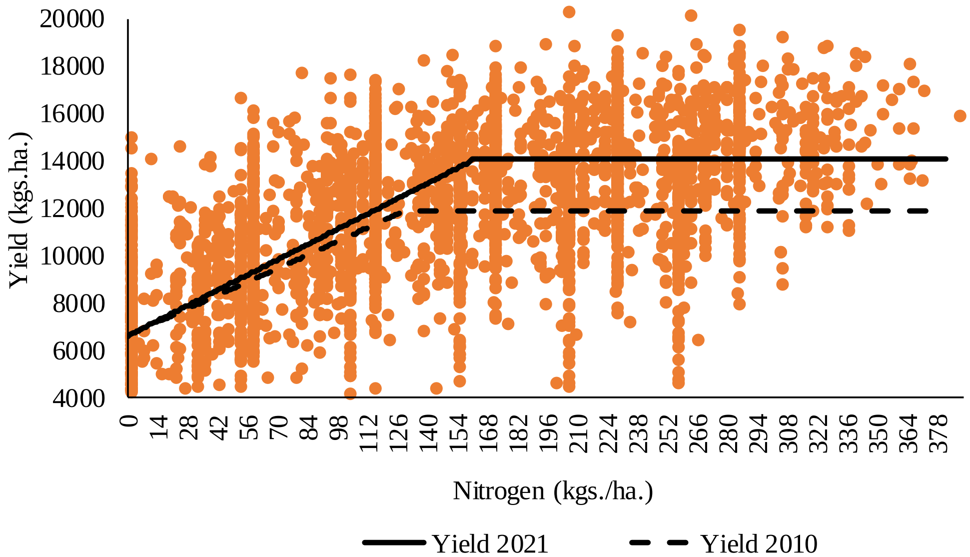
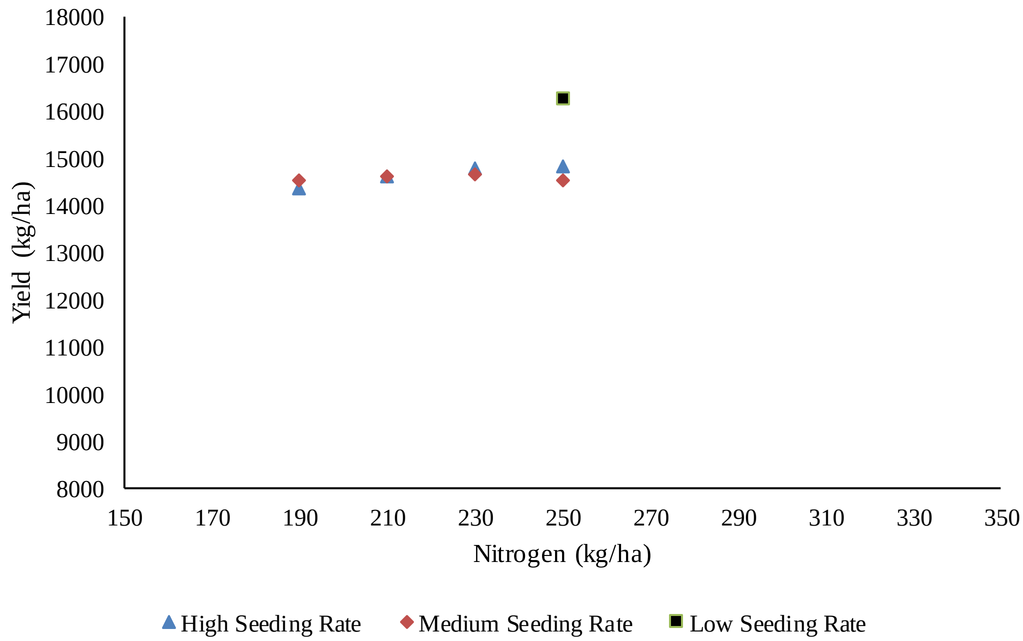


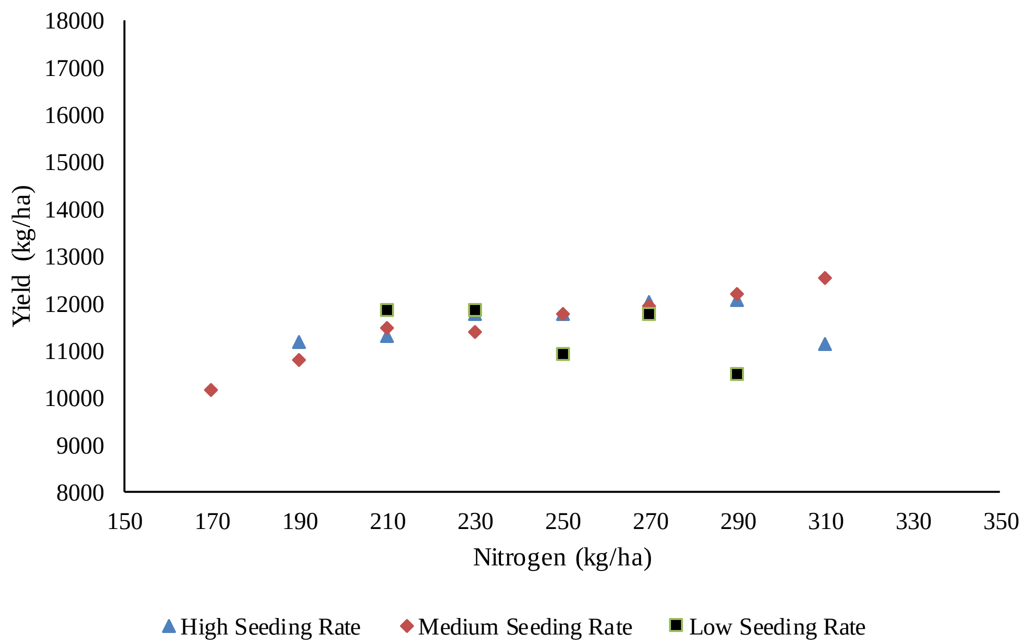
Figure 8. Expected Stochastic Linear Plateau Models for 2010 and 2021

Assefa et al., (2017) studied corn yield data from 1987 to 2015 and concluded that corn yields across the United States increased between 97 and 147 kg/ha/year. There are many reasons as to why corn yields have increased over time. Genetic improvements (Russel, 1991), increased plant densities (Assefa et al., 2018), and earlier planting dates (Tannura et al., 2008) have all been suggested as explanations for increasing corn yields.

3.3 DIFM Data

The mean yield response to nitrogen and seeding rate data for the three years of on-farm experiment from the single north central Ohio field that is used for estimating the spatially varying coefficient model is shown in Figure 9. The mean yield is determined for each seeding rate. The seeding rates are defined as high being between 89,000 and 104,000 seeds/ha, medium being between 74,000 and 88,999 seeds/ha, and low being anything less than or equal to 73,999 seeds/ha. These figures highlight the difficulties in using this data to guide nitrogen and seeding rate recommendations. High seeding rates appear beneficial in 2021 but had no effect in other years. The curves are relatively flat for nitrogen, except for the medium seeding rate in 2019 and the high seeding rate in 2021 where it appears that the plateau was not reached with the highest levels of nitrogen applied.





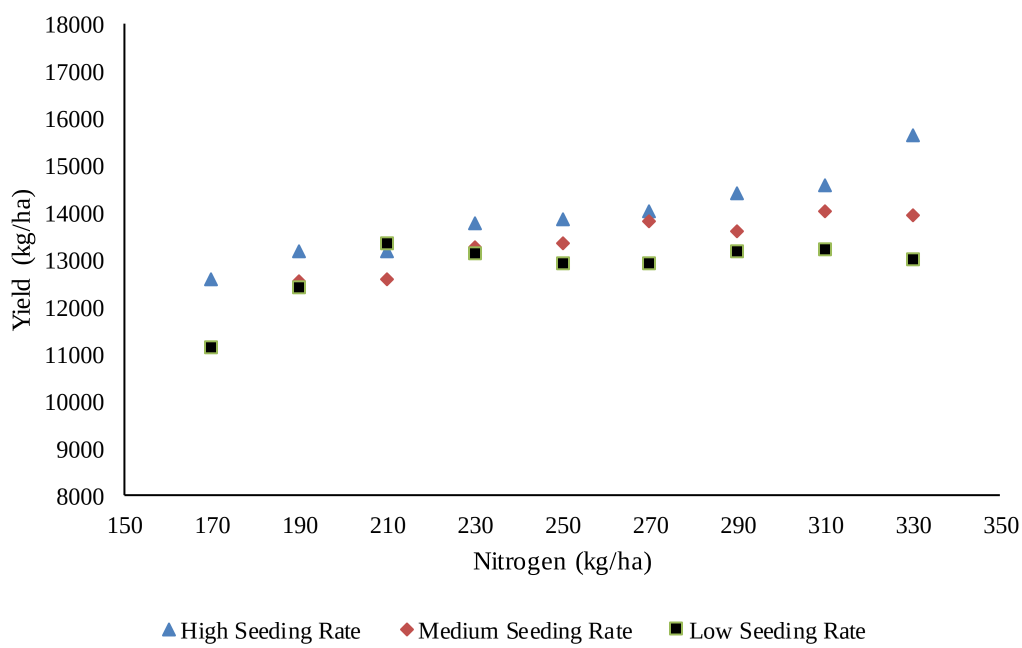


Figure 9. Mean Yield Reponses to Nitrogen and Seeding Rate for 2017 (top), 2019 (middle), and 2021 (bottom)

3.4 Spatially Varying Plateau Estimates

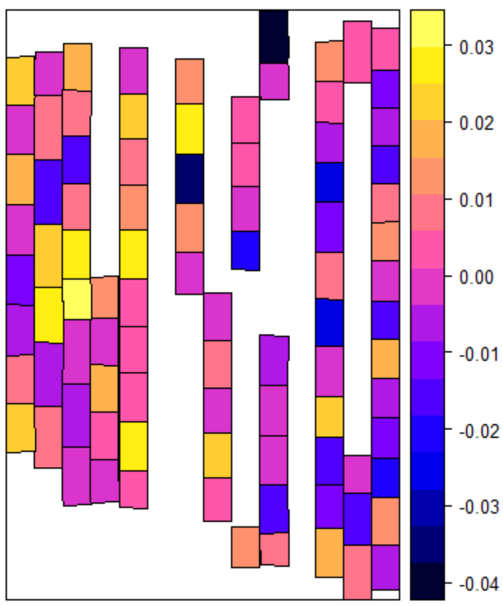
The stochastic LRP model is used as a prior for estimating the spatially varying plateau model. The estimated SVC models are used to predict corn yield response for 2021 selected locations and using response data from 2017, 2019, and 2021. The means and standard deviations are presented in Table 5. The seed and seed squared parameters used weak priors for the seeding rate variable which is why they are not consistent across prediction years. The hyperparameters in the covariance function (nugget, sill and ρ) are almost equal across all three years, which indicates that the spatial correlation remains unchanged overtime. The estimated ρ of 0.2 means that the range extends over 20 percent of the space or 2-3 plots. The estimated sill is small since the data are measured in kg/ha.

Table 5. Means and Standard Deviations of the Estimated Parameters from Parameter Estimates

	2017-2021 Parameter Estimates		2019-2021 Parameter Estimates		2021-2021 Parameter Estimates	
Parameter	Mean	SD	Mean	SD	Mean	SD
Intercept	6625.5	278.4	6477.6	278.2	6610.7	287.3
Slope	40.8	2.5	38.7	2.5	40.8	2.5
Seed	19.0	37.7	8.2	23.3	18.8	37.6
Seed Sq.	-49.7	37.7	-0.5	0.5	-45.6	34.0
Seed and Nitrogen Interaction	57.7	36.5	-0.4	0.1	44.3	38.0
Plateau	14456.5	58.7	11950.3	193.0	12915.6	113.6
sigma	11.0	0.6	12.8	0.8	17.4	1.1
sill	0.2	0.1	0.2	0.1	0.2	0.1
rho	0.2	0.1	0.2	0.1	0.2	0.1
nugget	1.3	0.8	1.3	0.8	1.4	0.8

Note: All values are in kilograms per hectare.

Following the predictions, the output generated latent spatial values. The predicted values of the latent spatial process, however, differ across years (Figures 10-12), although the estimates are quite small. The spatial parameters were used to display the latent spatial process of 2021 locations from 2017, 2019, and 2021 data to create figures 15, 16, and 17. The maps show no distinct pattern across years which means the spatial parameters are negligible and the plateau is not variable across the field, which foretells the finding that precision nitrogen application is of little value in this field.



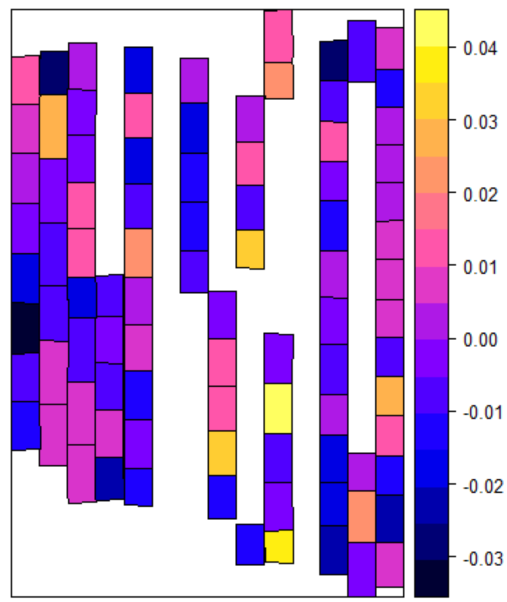


Figure 10. Latent Spatial Process for Predicting 2021 Yield from 2017 (Left) & Figure 11. Latent Spatial Process for Predicting 2021 Yield from 2019 (Right)

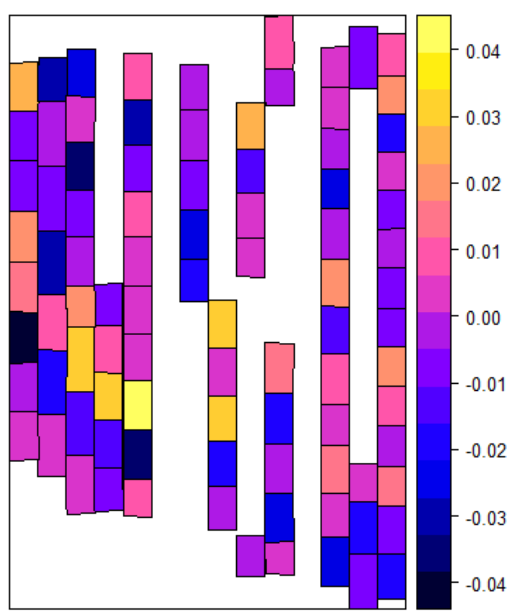


Figure 12. Latent Spatial Process for Predicting 2021 Yield from 2021

MSE values were computed to determine which model was closest to the real experimental data of the prediction year, 2021. The values of the calculation are shown in Table 6. The prediction with the lowest MSE value corresponds to the best fit of the actual data. The weighted average of 2017 and 2019 predictions for 2021 locations produced the lowest MSE value. One can decompose the MSE into bias and variance. Following the MSE calculations, the average yield prediction for all three years was calculated to determine if there is bias in the MSE calculation. We assume the average yield of the field is the true mean for yield variable. The weighted average has the lowest MSE due to having a lower variance as it had slightly more bias than using 2021 to predict 2021.

Table 6. Average Yields and Mean Squared Error Values of Predictions

	2017 to Predict 2021	2019 to Predict 2021	2021 to Predict 2021	Weighted Average of 2017 and 2019 to Predict 2021
Mean Squared Error	904.96	948.75	404.05	360.70
Average Yield of Prediction (kg/ha)	14454.32	11530.38	12895.37	12875.23

Note: Actual yield for 2021 is 13068.00 kg/ha.

The latent spatial predictions for the 2021 locations from the 2017 and 2019 data were regressed against the latent spatial effect in 2021. The regression shows how the latent spatial process of 2021 is explained by the latent spatial process of 2017 and 2019. The R-Squared values in Table 7 show the latent spatial process for 2021 cannot be explained by 2017 and 2019 latent spatial processes.

Table 7. Linear Regression of Predicted Latent Spatial Parameters

Predictions	Parameters	Estimate	SD	t-value
	Intercept	-0.001498	0.001697	-0.883
	Slope	-0.045124	0.118470	-0.381
2021 from 2017 data	R-Squared	0.001627		
	Intercept	-0.001605	0.001714	-0.936
	Slope	0.058617	0.116787	0.502
2021 from 2019 data	R-Squared	0.002822		
	Intercept	-0.001489	0.001704	-0.874
2021 from Weighted	Slope	0.013581	0.169163	0.080
Avg. 2017 and 2019	R-Squared	0.000072		

There is a vast literature on the profitability of variable rate nitrogen applications. Many researchers have shown that applying accurate uniform nitrogen rates for fields with little spatial variability are more profitable than applying variable rate nitrogen applications (Isik & Khanna, 2002; Thrikawala et al., 1999). Variable rate applications have shown to be profitable given sufficient spatial variability within the field (Roberts et al., 2000). The area of the field studied here had little spatial correlation, which reduced the potential benefit of using variable rate nitrogen applications.

The cost of information would be the cost of the entire system required to perform variable rate applications. The cost would include variable rate technology equipment to apply the rates, the technology to collect yield responses, and the analysis needed to estimate the rates.

3.5 Optimal Nitrogen and Seeding Rate

The optimal nitrogen and seeding rates were calculated for each prediction year using the prediction parameter means from the output of the spatially varying coefficient model estimation. A seed corn price of $2.00 per thousand seeds (Lauer & Stanger, 2023), corn price of $0.26 per kilogram (Quinn, 2023), and nitrogen price of $1.75 per kilogram (USAD-NASS, 2023) were used to estimate the profit per hectare. Optimal levels were constrained to be within the range of the data. All optimal levels were corner solutions. Maximum profit for 2017 occurred at 185 kg/ha of nitrogen applied and a seeding rate of 74,000 seed/ha. For 2019, max profit occurred at 282 kg/ha of nitrogen applied and a seeding rate of 73,000 seed/ha. For 2021, max profit occurred at 182 kg/ha of nitrogen applied and a seeding rate of 59,000 seed/ha. Given the varieties used for each year (Table 1), seed corn price was increased to $3.75 per thousand seeds to determine if the optimal nitrogen and or seeding rate changed. No optimal rates changed. What is shown here is that the seeding rates and nitrogen levels used here were so high that almost all the observations were on the plateau. To learn more about optimal levels, lower rates of nitrogen and lower seeding rates would need to be considered.

CHAPTER IV

CONCLUSIONS

The first goal of this research was to develop informative Bayesian priors. The goal was achieved by estimating a stochastic linear plateau model using data from the MRTN database and incorporating a time trend that accounts for increasing corn yields over time. The intercept, slope, and plateau all increased over time. The plateau increased the most by 28 percent, and thus the optimal nitrogen rate also increased. The optimal nitrogen rate from 2010 to 2021 increased by almost 21 percent. The estimated posterior distribution of the stochastic linear plateau is used as a prior for completing the second goal of this research which is to estimate a spatially varying coefficient model to determine accurate site-specific nitrogen rates. After estimating the spatially varying coefficient model, the latent spatial parameters revealed little to no spatial variability across the field which limited the benefit of applying variable rate nitrogen. Poursina (2021) suggests experimenting on only a part of the field, which would increase the importance and dependence upon the priors. The general approach used here might prove useful for determining uniform rate recommendations even if it did not aid in variable rate recommendations. This research is consistent with previous research that found variable rate nitrogen may not be profitable when the only information available is the location of the plot. The value of this research is shown through the high-quality priors created by estimating the stochastic linear plateau model. This research was limited to only using a selected number of observations from a small section of the field due to the computational time limitation.

Another issue is the limited spatial variability within the research field likely contributes to the variable rate nitrogen to not being profitable (Stefanini et al., 2018; Boyer et al., 2012). Another limitation lies within the short ranges of seeding rates and nitrogen rates of the experimental data used to make our predictions. Variable rate nitrogen applications can still be profitable in a field experiment with higher spatial variability. Using a dataset with more observations per location could improve the value and accuracy of the information from the predictions by decreasing the noise associated with few observations. Future research could use a more extensive data set to further improve the predictions.

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APPENDICES

The autocorrelation plots display how convergence was determined for the parameter estimates of the stochastic LRP model (Figures 1A-9A). All plots show a decline to zero as lag is increased. Autocorrelation plots for year random effect and error variance have the lowest autocorrelation.

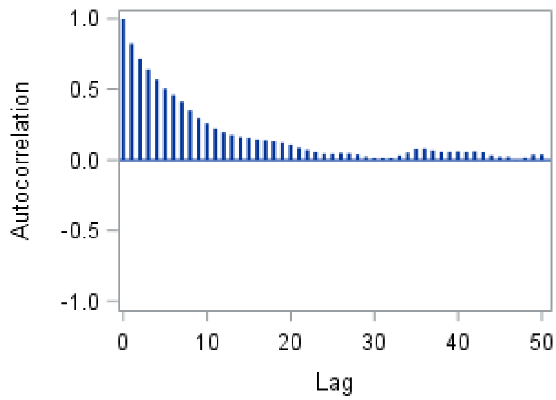

Figure 1A. Intercept Autocorrelation Plot

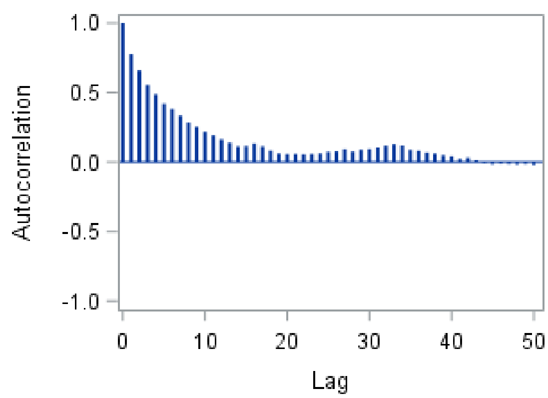

Figure 2A. Intercept Time Trend Autocorrelation Plot

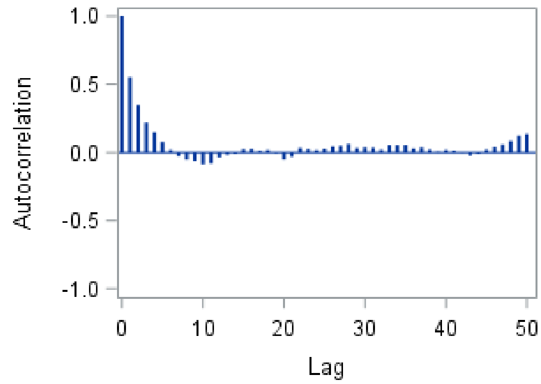

Figure 3A. Nitrogen Autocorrelation Plot

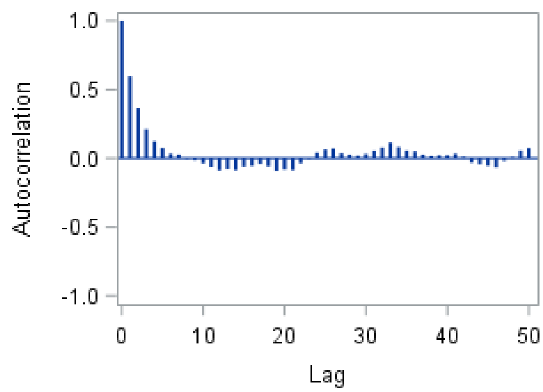

Figure 4A. Slope Time Trend Autocorrelation Plot

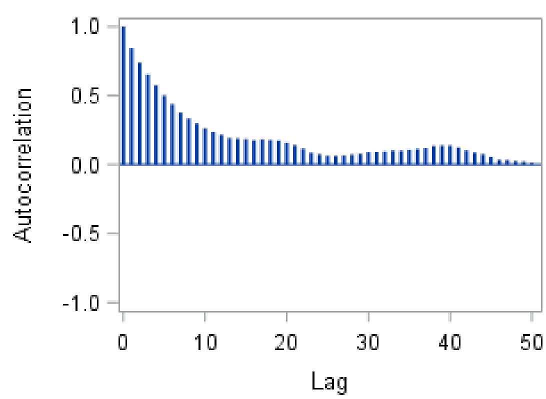

Figure 5A. Plateau Autocorrelation Plot

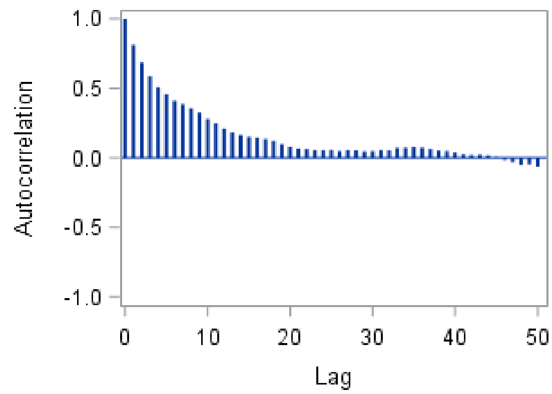

Figure 6A. Plateau Time Trend Autocorrelation Plot

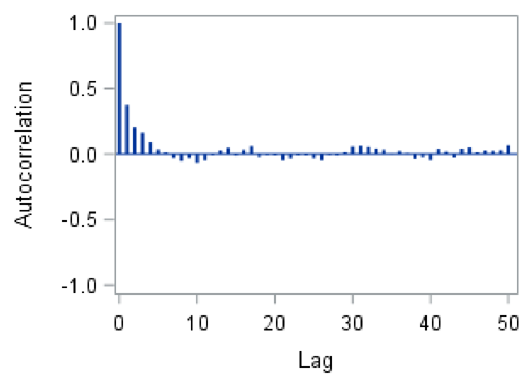

Figure 7A. Plateau Variance Autocorrelation Plot

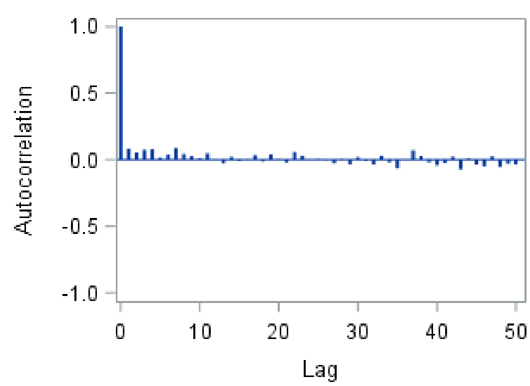

Figure 8A. Year Random Effect Autocorrelation Plot

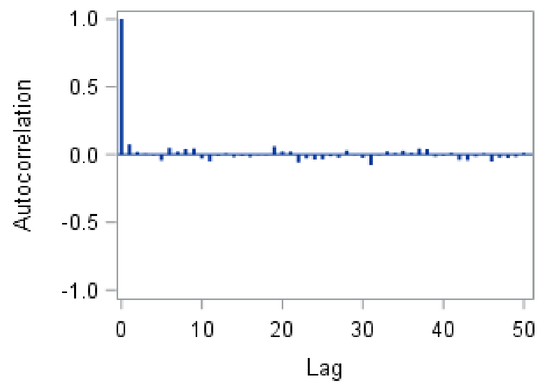

Figure 9A. Error Variance Autocorrelation Plot

VITA

George Coleman Patterson

Candidate for the Degree of

Master of Science

Thesis: USING INFORMATIVE BAYESIAN PRIORS AND ON-FARM EXPERIMENTATION TO PREDICT OPTIMAL SITE-SPECIFIC NITROGEN RATES

Major Field: Agricultural Economics

Biographical:

Education:

Completed the requirements for the Master of Science in Agricultural Economics at Oklahoma State University, Stillwater, Oklahoma in May, 2023.

Completed the requirements for the Bachelor of Science in Agriculture at Abraham Baldwin Agricultural College, Tifton, Georgia in 2021.

Experience:

Graduate Research Assistant, Dept. of Agricultural Economics,
Oklahoma State University
Undergraduate Research Assistant, Dept. of Crop and Soil Sciences,
University of Georgia