AN APPROACH TO OPTIMIZE THE MAINTENANCE PROGRAM FOR A REPAIRABLE SYSTEM

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AN APPROACH TO OPTIMIZE THE MAINTENANCE

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NOMENCLATURE

$A(t)$	Instantaneous Availability
$A_{\tau}(\tau)$	Average uptime availability
$A_{SS}(\infty)$	Steady state availability
A_i	Inherent availability
A_a	Achieved availability
A_o	Operational availability
C&R	Check & Repair maintenance action
C_{AI}	Cost of an air interrupt
C_{CR}	Cost of a Check & Repair maintenance action
C_{ENG}	Cost of an IDG being out-of-service due to an engine change
C_{FA}	Cost of IDG failure while operational
C_{FFR}	Cost of a premature removal, includes labor and flight delay and cancellation costs
C_{INV}	Cost of IDG being out-of-service in inventory
C_{LRR}	Cost of labor to remove and replace an IDG
C_{MEL}	Cost associated with an IDG being inoperative and placarded per the Minimum Equipment List
C_{MR}	Cost of a minimal repair occurring at failure
C_{NFF}	Cost of a No Fault Found (NFF) maintenance action

C_{OVH}	Cost of an IDG overhaul maintenance action
C_{SR}	Cost of an IDG scheduled removal and overhaul
C_{TFS}	Cost of IDG being out-of-service due to shipping time
C_{USOVH}	Cost of an unscheduled (at failure) overhaul
CU	Cost Unit, defined as the cost for a scheduled removal of an IDG
CU/FH	Cost Unit per flight hour
$E[\cdot]$	Expected value of the random variable in brackets
FD	Flight Day, 1 FD = 11.39 flight hours
FH	Flight Hour, one hour of IDG operation
FOM	Force of mortality (hazard rate for parts)
HPP	Homogeneous Poisson process
h(x)	Hazard rate for parts
IDG	Integrated drive generator
MEL	Minimum Equipment List
MTBF	Mean time between failure
MTBUR	Mean time between unscheduled removals
MLE	Maximum likelihood estimate
MMD	Mean maintenance downtime
MOI	Mandatory overhaul interval or T for Type II' Policy
MTBM	Mean time between maintenance
MTTR	Mean time to repair
N(t)	Number of arrivals (failures) on (0,t]

NFF	No Fault Found maintenance action
NHPP	Nonhomogeneous Poisson Process
PDF	Probability density function
PR	Premature Removal, due to either a confirmed or unconfirmed failure
ROCOF	Rate of occurrence of failures (see $\rho(t)$)
$r_X(x)$	Mean residual life of a part
S_q	Start of observation interval for the q-th IDG
t	Total system age, measured in appropriate units, e.g., total operating time, etc., a real variable
T_q	End of observation interval for the q-th IDG
T	Replacement interval for Type II and II' maintenance policies
T*	Optimum replacement interval for Type II and II' maintenance policies
τ_{TSOq}	Flight hours for the q-th IDG since overhaul
τ_{TTq}	Total flight hours of the q-th IDG since original purchase date
X	Time to part failure (random variable)
x	Age of part as measured in appropriate units, e.g., time, etc., a real variable
X_{iq}	Time to the i-th failure for the q-th system
x_{iq}	Time elapsed since the most recent failure, (i-1), for the q-th system
Y_{iq}	Interarrival time between the (i-1)st and i-th failure for the q-th system
β	Shape parameter of the NHPP power-law intensity (peril rate)
$\overline{\beta}$	Unbiased estimate of the shape parameter
$\widetilde{\beta}$	Conditional MLE of β

λ	Scale parameter of the NHPP power-law intensity (peril rate)
$\rho(t)$	Peril rate or the ROCOF of the NHPP
$u(t)$	Intensity function for NHPP, same as peril rate [1]
$v(t)$	ROCOF for a stochastic point process
$\hat{g}$	MLE of dummy parameter g
$\tilde{g}$	Natural estimate of a dummy parameter g
$\chi^2_{df,\gamma}$	Chi-square critical value for df degrees of freedom and γ significance level
C^2_M	Parametric Cramer-Von Mises statistic

CHAPTER 1

INTRODUCTION

1.1 Repairable Systems and Maintenance

A repairable system is one that can be repaired so that it can perform all of its required functions after it has failed. A repair is a corrective maintenance action performed on the system. Maintenance can be classified as either preventive (scheduled) or corrective [1]. Corrective maintenance occurs when the system fails and must be repaired. Preventive maintenance is scheduled and performed with a deteriorating system where the cost of a field failure is much greater than the cost of the scheduled preventive maintenance. Preventive maintenance should have the effect of reducing the system's rate of change of failures. If preventive maintenance is done incorrectly, the rate of change of failures may increase after the maintenance is performed. The cost-to-benefit ratio must be clearly understood before preventive maintenance is undertaken.

In this Thesis, the system under study is an aircraft integrated drive generator (IDG). The IDG provides primary electrical power to the aircraft. Two types of time (or age) based replacement preventive maintenance policies are investigated. The first policy is a Type II; here, the IDG is overhauled when it has reached a prespecified number of operational hours. The second policy is a Type II'; here, the IDG is overhauled at the first failure, provided that it has reached a prespecified number of operational hours. There is a distinct difference between an overhaul and a repair. A repair has no significant effect on the rate of change of failures for the IDG. An overhaul which requires the unit to be completely disassembled, inspected, repaired, and reassembled, has a significant effect on the rate of change of failures for the IDG. In both the Type II and II' maintenance policies, the IDG is repaired, not overhauled, if it fails before the prespecified number of flight hours have been reached. The principal difference between the Type II and Type II' policies is that, the Type II policy requires an operational IDG be removed from service for an overhaul at a prespecified number of flight hours, whereas, the Type II' policy requires the IDG to have failed and reached a prespecified number of flight hours before it is overhauled. This Thesis investigates the dominant IDG failure modes, and using the failure data, determines the optimal replacement intervals for the Type II and II' maintenance policies.

1.2 Problem Statement

This thesis addresses the problem of determining the optimal overhaul interval for the Type II and Type II' maintenance policies and determines the effects of the current maintenance policy on the integrated drive generator. To optimize each maintenance policy, the following issues are investigated:

1. Identification of the dominant failure modes of the IDG.
2. The effect the current maintenance program has on the long-term reliability of the IDG.
3. The optimal mandatory overhaul interval for a Type II' maintenance policy.

4. The optimal overhaul interval for a Type II maintenance policy.

The results will identify and quantify the effects of the current and an optimal IDG maintenance policy.

1.3 Research Methodology

To parameterize the IDG failure data, the nonhomogeneous Poisson process with a power law intensity function is used. The NHPP approach is commonly used when dealing with repairable systems. The failure data are left-truncated to remove the effects of infant mortality after an overhaul. The resulting parameters that quantify the peril rate for the IDG can be used to evaluate the current maintenance program and to optimize the Type II policy. This approach could not be used to optimize the Type II' maintenance policy since the theory for the Type II' policy has not been fully developed in the case of the NHPP with a power law intensity function. For this reason, a Monte Carlo simulation approach is used. The simulation does allow for more information, e.g., state holding times and engine changes, to be used in the development of the maintenance model. This has a direct effect on the operational and maintenance costs that accompany adoption of a new maintenance policy for the actual IDG population.

1.4 Organization of Thesis

Chapter II is a literature review of repairable systems and the applicable preventive maintenance policies. Statistical methods, Monte Carlo simulation, and the definitions of availability are also covered in this chapter.

Chapter III presents the dominant failure modes of the IDG and quantifies the peril rate using the natural and nonhomogeneous Poisson process with power law intensity function estimates.

Chapter IV illustrates the use of a Monte Carlo simulation to determine the optimal overhaul interval for a Type II' maintenance policy.

Chapter V demonstrates an analytical approach to determine the optimal overhaul interval for a Type II maintenance policy.

Chapter VI contains concluding remarks, summary, and areas for further work.

CHAPTER II

BACKGROUND AND LITERATURE REVIEW

2.1 Repairable Systems

2.1.1 Definitions

The following definitions are necessary to understand repairable systems and the difference between a repairable system and a non-repairable part [2].

1. *Part.* An item which is not subject to disassembly and, hence, is discarded the first time it fails.

2. *Socket.* A circuit or equipment position which, at any given time, holds a part of a given type.

3. *System.* A collection of two or more sockets and their associated parts, interconnected to perform one or more functions.

4. *Non-repairable System.* A system that is discarded the first time that it ceases to perform satisfactorily.

5. *Repairable System.* A system which, after failing to perform at least one of its required functions, can be restored to perform all of its required functions by any method, other than replacement of the entire system.

The definition of a repairable system is worded to include the possibility that no parts are replaced after a failure. For example, the system might be repaired by making necessary adjustments. The aircraft IDG is an example of a repairable system. The "parts" that make up the IDG would be, for example, gears, housing, pumps, stator, etc.

2.1.2 Models for Parts

To better understand a repairable system, the terminology and relationships for a part must be discussed [2,3]. Let X be defined as a random variable that is the time to failure for the part. The failure distribution function for X is defined as $F_X(x) \equiv \Pr\{X \leq x\}$ assuming F_X to be absolutely continuous, then the hazard rate or force of mortality (FOM) is defined as

$$h_X(x) \equiv \frac{f_X(x)}{1-F_X(x)} \equiv \text{FOM} \qquad 2.1.2.1$$

where,

$$f_X(x) \equiv \frac{dF_X(x)}{dx} \qquad 2.1.2.2$$

$$1-F_X(x) \equiv R_X(x) \equiv \Pr\{X>x\}. \qquad 2.1.2.3$$

The reliability or survivor function, $R_X(x)$, is given by the relationship

$$R_X(x) = \exp\left\{-\int_0^x h_X(y)dy\right\}. \qquad 2.1.2.4$$

The mean time to failure (MTTF) of a population of parts with failure distribution function F_X and PDF f_X is

$$E[X] = \int_0^\infty x f_X(x)dx \equiv \mu. \qquad 2.1.2.5$$

The mean residual life function of a part, $r_X(x)$ or m.r.l.f., is the expected additional time to failure given survival to x. The mean residual life function [4,5] is given by the relationship

$$r_X(x) \equiv E[X - x | X > x] \equiv \frac{\int_x^{\infty} R_X(y)dy}{R_X(x)} = \frac{\int_x^{\infty}[1 - F_X(y)]dy}{[1 - F_X(x)]}. \qquad 2.1.2.6$$

2.1.3 Models for Repairable Systems

Compared with the large amount of literature on the subject of units, i.e., parts, that fail only once and are discarded (catastrophic failure model), there is a much smaller amount of literature on the reliability of repairable systems [6]. As a result of the reliability of repairable systems being an extension of the reliability of parts, the terminology can be very misleading. Quoting form Ascher and Feingold [2, page 133], "... that the prevalent terminology could scarcely be more misleading if it had been designed to mislead -- specifically, it has engendered such deep-seated misconceptions that it is extraordinarily difficult to supplant it with improved nomenclature." This section attempts to clarify the terminology used in the study of repairable systems.

A renewal process is used to model a system's reliability if it is restored to the level of a brand new system after repair. The phrase *same-as-new* is used to describe the result of a renewal process. Since a repaired system is in the same condition as a new system, a renewal process cannot be used to model a system that is experiencing reliability growth or deterioration.

Systems for which reliability does not change after a repair, the appropriate model is the nonhomogenous Poisson process (NHPP). The term *same-as-old* or *minimal repair* is used to describe a system that is in the same general condition after a repair as before the failure, except that the system can continue to operate after repair.

The mathematics of a stochastic point process is used extensively to describe repairable systems. A point process [7] is intended to model a probabilistic experiment which places points on the time axes. These points (often called arrivals), might be successive failures of a repairable system. Let N(t) be the number of arrivals on (0,t]. For example, N(t)=N(0,t], N(a,b]=N(b)-N(a) and N{a} is the number of arrivals at point "a".

Formally, a *stochastic point process* {N(t)} is a collection of usually interrelated random variables (failure times), each labeled by a point t on the positive line and such that $N(t_2)-N(t_1) = N(t_1, t_2]$. The quantity $N(t_1, t_2]$ has probability one of being a finite nonnegative integer for all $t_2>t_1 \geq 0$. A point process has a finite number of arrivals (failures) on a finite interval with a positive probability. Assume N(0)=0. A point process is said to have no simultaneous arrivals (failures) if, with probability one, each jump of N(t) is of unit magnitude. This excludes the possibility of having more than one failure at any instance in time.

The *failure intensity function* or *intensity function,* denoted by u(t), is defined as

$$u(t) \equiv \lim_{\Delta t \to 0} \frac{\Pr\{N(t+\Delta t) - N(t) \geq 1\}}{\Delta t} . \qquad 2.1.3.1$$

A point process is said to be *orderly* or *regular* if

$$\Pr\{N(t+\Delta t) - N(t) \geq 2\} = o(\Delta t), \qquad 2.1.3.2$$

that is, if independent failures cannot occur simultaneously. The time t, called the *global time*, is the cumulative time since initial startup of the system, not the reset time used in renewal theory after each failure. The quantity $u(t)\Delta t$ is approximately the probability that a failure will occur in the interval $(t, t+\Delta t)$.

For a stochastic point process, the rate of occurrence of failures (ROCOF) is defined as

$$v(t) \equiv \frac{d}{dt} E[N(t)]. \tag{2.1.3.3}$$

For an orderly nonstationary point process, the intensity function u(t) and the ROCOF v(t) are equal, if they exist. The intensity function can be defined for any stochastic point process, not just the NHPP. The intensity functions u(t) and the ROCOF v(t) are the same as the *peril rate* $\rho(t)$ for the NHPP. The terms failure or hazard rate, which are used to describe non-repairable part failures, should not be used to describe the intensity function, ROCOF or the peril rate for repairable systems.

A decreasing peril rate with respect to time implies that the probability of failure during a fixed time interval is decreasing. This is the case for a system experiencing reliability growth. Reliability growth occurs when a system's design is steadily improved, thus making the system more reliable as time progresses. Reliability growth can also occur in operating systems when the maintenance program is sufficiently improved so that the reliability of the system improves. For systems that have an increasing peril rate with respect to time, reliability decreases with time. Here the probability of failure during a fixed time interval increases with time. Reliability deterioration indicates a system experiencing wearout. Deteriorating systems are candidates for preventive maintenance programs.

For a given global time t, the time interval from t to the previous failure is the *backward recurrence time,* B(t), as shown in Figure 1. The backward recurrence time is defined as

$$B(t) \equiv t - X_{N(t)} \qquad 2.1.3.4$$

where $X_{N(t)}$ is the failure time X_i (i=1,2,... final failure) just preceding the global time t. The time interval from time t to the next failure is the *forward recurrence time*, W(t). The forward recurrence time is defined as

$$W(t) \equiv X_{N(t)+1} - t \qquad 2.1.3.5$$

where $X_{N(t)+1}$ is the failure time after X_i. Another quantity of interest is the *instantaneous mean time between failure* (IMTBF). The IMTBF is defined as

$$\mathrm{IMTBF}(t) \equiv \frac{1}{u(t)} = \frac{1}{\rho(t)}. \qquad 2.1.3.6$$

If the system is either deteriorating or experiencing reliability growth, IMTBF is time dependent. For a system described by a NHPP the peril rate is a time dependent quantity, therefore, the IMTBF is also a time dependent quantity.

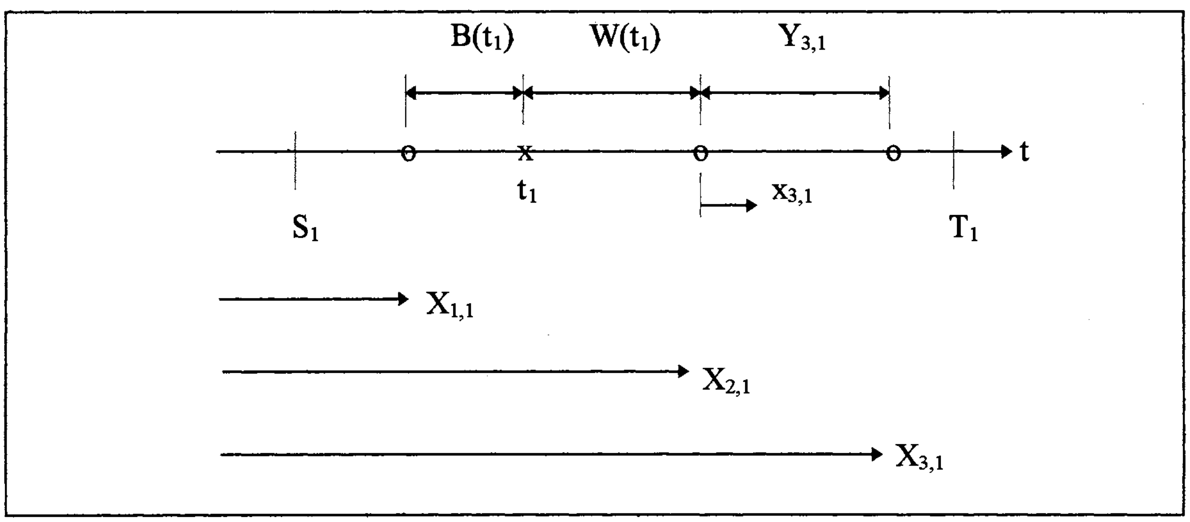

Figure 1. Sample path of a Stochastic Point Process

Figure 1 shows a sample realization for a stochastic point process; X_{iq} measures the total time from zero to the i-th failure for the q-th system and is called an *arrival time*. To clarify the notation a comma is used between the subscripts on the arrival time indices, for example, $X_{1,1}$ has a comma so that it is not confused with X_{11}. The quantity X_{iq} is the same as $X_{i,q}$; however, in this case the comma is not required. X_{iq} is a random variable (RV) contrary to the situation where p parts are tested without replacement; in general there is no upper bound on i. The real variable t measures the total time since the start up of the process. The term global time is used for t. When k copies of a system are under study, t is measured independently for each copy. Y_{iq}, $i=1,2,\ldots N_q$, $q=1,2,\ldots k$, is the interarrival time between the (i-1)th and the i-th failures for the q-th system. The real variable x_{iq} measures the time elapsed since the most recent failure. The term *local* time is used for x_{iq}. The x_{iq}'s are analogous to x for part models but t has no direct analogy in part modeling. The RV, $N_q(t)$, is defined as the maximum value of i for which $X_{iq} \leq t$, i.e., $N_q(t)$ is the number of failures that occur during (0,t]. $\{N_q(t),\ t \geq 0\}$ is the integer valued counting process that includes both the number of failures in (0,t], $N_q(t)$, and the instants X_{1q}, X_{2q}, ..., at which they occur.

The quantity S_1, or in general S_q, is the value of t for which the first (q=1) system's observation period begins. T_1 is the value of t for which the first (q=1) observation interval ends. Usually, a system cannot be monitored for an indefinite period of time. Therefore, the observation period is a finite period of time where the failure times are recorded for the system under study.

2.2 Nonhomogeneous Poisson Process

To develop the NHPP, the homogeneous Poisson process (HPP) must first be explained [2,8]. The HPP can be defined as a nonterminating sequence of independent and identically exponentially distributed Y_{iq}'s. More formally, the counting process $\{N_q(t),\ t \geq 0\}$ is said to be HPP if

(a) $N(0)=0$,

(b) $\{N_q(t),\ t \geq 0\}$ has independent increments, and

(c) the number of events (failures) in any interval of length (t_2-t_1) has a Poisson distribution with mean $[(\rho)(t_2 - t_1)]$. For all $t_2>t_1 \geq 0$ and $j \geq 0$

$$\Pr\{N(t_2) - N(t_1) = j\} = \frac{\exp(-(\rho)(t_2 - t_1))\{(\rho)(t_2 - t_1)\}^j}{j!}. \qquad 2.2.1$$

From condition (c) it follows that $E\{N(t_2-t_1)\}=[(\rho)(t_2 - t_1)]$ where the constant, ρ, is the rate of occurrence of failures (ROCOF). The HPP is characterized by times between failures that are independent and identically distributed with an exponential distribution. Since HPP has stationary, independent increments, then $v(t)= \rho = 1/E[X]$. From this definition, the reliability function is given by

$$R(t_1, t_2) = e^{-(\rho)(t_2 - t_1)}. \qquad 2.2.2$$

The NHPP differs from the HPP only in that the ROCOF varies with time rather than being constant. For the NHPP conditions, (a) and (b) are retained, and condition (c) becomes (c'). The number of failures in any interval (t_1,t_2) has a Poisson distribution with mean

$$\int_{t_1}^{t_2} \rho(t)dt. \qquad 2.2.3$$

For all $t_2 > t_1 \geq 0$ and $j \geq 0$,

$$\Pr\{N(t_2) - N(t_1) = j\} = \frac{\exp\left(-\int_{t_1}^{t_2} \rho(t)dt\right)\left\{\int_{t_1}^{t_2} \rho(t)dt\right\}^j}{j!}. \qquad 2.2.4$$

From condition (c') it follows that

$$E[N(t_2) - N(t_1)] = \int_{t_1}^{t_2} \rho(t)dt. \qquad 2.2.5$$

The reliability function for the NHPP is

$$R(t_1, t_2) = \exp\{-\int_{t_1}^{t_2} \rho(t)dt\}. \qquad 2.2.6$$

The change in statement (c') results in the Y_i's that are neither independent nor identically distributed; neither are they independent samples from any other single distribution. Therefore, any statistical techniques which are based on the assumption that the data are independent and identically distributed (IID) cannot be validly applied to an NHPP. However, due to condition (b), the NHPP retains the independent increments property. If a counting process has independent increments, then the number of failures in an interval is not influenced by the number of failures which occurred in any "strictly earlier" interval, i.e., with no overlap. For the NHPP model of a repairable system, the time variant ROCOF, $\rho(t)$, is referred to as the peril rate rather than the hazard or failure rate for non-repairable parts.

2.2.1 Power Law Process

The term Power Law Process refers to a specific mathematical form of the intensity function for a NHPP. The intensity function u(t), same as $\rho(t)$ in this case, is given by

$$\rho(t) = u(t) = \lambda\beta t^{\beta-1}, \quad t > 0 \qquad 2.2.1.1$$

where $\lambda,\beta > 0$ and t is the age of the system [2]. The parameter β is called the shape parameter, and λ is the scale parameter. The mathematical form for the intensity function (called power law) is the same as the failure rate for a Weibull distribution.

The NHPP with a power law intensity function is commonly referred to as a "Weibull Process". The "Weibull Process" is not the same as a Weibull distribution. This terminology leads to confusion with the Weibull distribution. The Weibull distribution is used for time to first failure of a non-repairable system. The hazard rate is a relative rate of failure for non-repairable systems and the intensity function is an absolute rate of failure for repairable systems.

The intensity function u(t) given in 2.2.1.1 is only one mathematical form of the intensity function; other forms [9,10] are in use. The power law mean value function for the NHPP with intensity 2.2.1.1 is

$$E[N(t)] = \lambda t^{\beta}, \quad t > 0. \qquad 2.2.1.2$$

This is the expected number of failures for a system during its age (0,t). The probability that a failure will occur during the interval (t, t+Δt) is approximately u(t)Δt.

The NHPP reduces to the homogeneous Poisson process when $\beta = 1$. This is the case where the intensity function does not change as the system ages. For $\beta > 1$, the intensity function u(t) is strictly increasing. In this case, the system has a wearout characteristic and the intervals between successive failures X_i and X_{i-1} are stochastically decreasing. When the intervals between successive failures are stochastically decreasing, the expected number of failures in at least one suitably chosen later interval must be strictly greater than the expected number in an initial interval of the same length. For $\beta <$ 1, the intensity function is strictly decreasing. This occurs during the debugging phase of a system's development, or for mature systems this occurs as a result of improved maintenance practices. As a result of the associated increased reliability, the intervals between successive failures X_i - X_{i-1} will stochastically increase.

The NHPP is used to analyze the reliability of a system based on failure data obtained from k copies of the system operating under the same environmental conditions. It is assumed that the failures for each of the k systems is governed by a peril rate given by $\rho(t) = \lambda\beta t^{\beta-1}$. The values of λ and β will be estimated based on the data for the k systems. The system is observed continuously from time S_q to time T_q, (q = 1,2,...,k). Over the interval $[S_q, T_q]$, let N_q be the total number of failures experienced by the q-th system and let X_{iq} be the time at which the i-th failure occurs, (i = 1,2,...,N_q; q = 1,2,...,k).

For the observation interval $[S_q, T_q]$, the times S_q,T_q, , q = 1,2,...,k, may be observed failure times for the q-th system. The time of the last failure determines if the data are time or failure truncated. If the last failure occurs at the end of the observation interval, $X_{N_q,q} = T_q$, then the data on the q-th system are said to be failure truncated. In this case, T_q, is a random variable with N_q fixed. If the last observed failure occurs prior to the end of the observation interval, $X_{N_q,q} < T_q$, the data on the q-th system are said to be time truncated. For time truncated data, N_q is a random variable. To illustrate the notation, in Figure 2 the values for S_q, T_q, N_q, and X_{iq} are shown for three IDGs.

The method of *Maximum Likelihood Estimates* (MLE) [11] is used to determine estimates of λ and β for a homogeneous population of k systems. Let the q-th system be observed from time S_q to T_q, $(q = 1,2,\ldots,k)$. The MLE of λ and β are $\hat{\lambda}$ and $\hat{\beta}$ respectively and are given by

$$\hat{\lambda} = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k} (T_q^{\hat{\beta}} - S_q^{\hat{\beta}})} \qquad 2.2.1.4$$

$$\hat{\beta} = \frac{\sum_{q=1}^{k} N_q}{\hat{\lambda} \sum_{q=1}^{k} (T_q^{\hat{\beta}} \ln T_q - S_q^{\hat{\beta}} \ln S_q) - \sum_{q=1}^{k} \sum_{i=1}^{N_q} \ln X_{iq}}. \qquad 2.2.1.5$$

In equation 2.2.1.5, when $(0 \cdot \ln 0)$ is encountered, it is set equal to zero. Since these equations cannot in general be solved for $\hat{\lambda}$ and $\hat{\beta}$, an iterative approach is used.

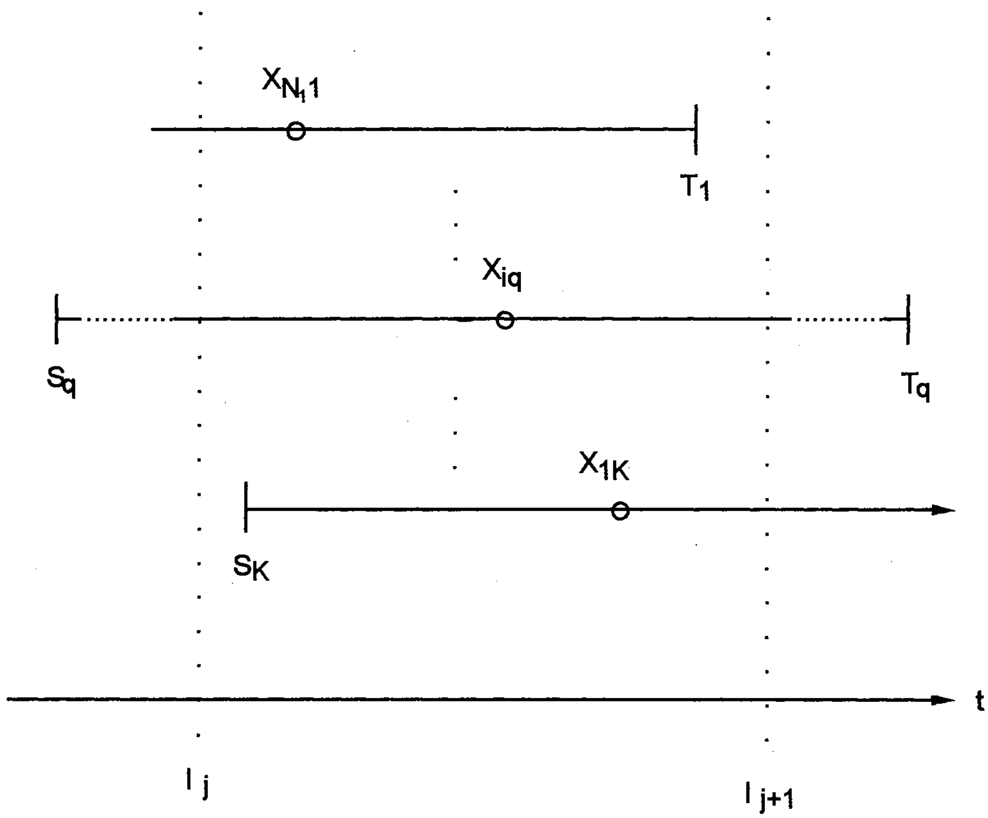

Figure 2. Maintenance Time Line for Three IDGs

Once the estimates of λ and β are obtained, they are used to estimate the peril rate. The peril rate based on MLE is given by

$$\hat{\rho}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}. \qquad 2.2.1.6$$

In addition to the MLE estimate of the peril rate, the nonparametric natural estimate of the peril rate may be calculated [12]. The natural estimate of the peril rate is given by

$$\tilde{\rho}_j(t) = \frac{N_j}{\Delta t_j}, \qquad 2.2.1.7$$

where N_j is the number of failures in each observation cell interval and Δt_j is given by

$$\Delta t_j = \sum_{i \in [I_j, I_{j+1}]} [\min(T_i, I_{j+1}) - \max(I_j, S_i)] . \qquad 2.2.1.8$$

Note that since equation 2.2.1.8 sums over all k systems, therefore, the total number of units operational during the j-th observation cell are taken into account. A typical observation cell is shown in Figure 2. An observation cell (or frequency cell) is the time interval between I_j and I_{j+1}. The number and width of the observation cells is determined by the preference of the investigator. In this study the natural estimate of the peril rate is determined by using five equally spaced observation cells. This choice results in at least five failures per observation cell so that the Chi-Squared goodness-of-fit test can be used for the NHPP estimate.

The terms observation interval and observation cell should not be confused. The observation interval $[S_q, T_q]$ is the length of time over which the failures and subsequent maintenance actions are observed. The observation cell $[I_j, I_{j+1}]$ is a subdivision of the observation interval used for the purpose of failure data analysis.

2.2.2 Comparison of Hazard and Peril Rates

Since the hazard rate for parts and the peril rate for repairable systems are very similar concepts, it is necessary to clearly distinguish between the two [6]. When a part is tested, usually several parts are placed on test at the same time. In this case, it is reasonable to assume that the random lifetimes of the parts on test are independent and identically distributed (IID). The goal of the test is to determine the PDF that best describes the lifetimes. The lifetimes are observed from the shortest to longest. For a simple repairable system, the times to failure and repair form a sequence of numbers from shortest to the longest. However, in the case of a repairable system, the IID assumption is not valid. With a repairable system, reliability growth or deterioration with time causes the intrinsic properties of the system to change. In other words, the time between the third and fourth failures is not independent of the time between the second and third failures. Also, the times between failures are not identically distributed since the system changes in time.

The quantity $\rho(t)dt$ is interpreted as the unconditional probability that a failure, not necessarily the first, occurs in (t,t+dt). This is in contrast to the interpretation of $h_X(x)dx$ which is the conditional probability of first and only failure in (x,x+dx], given survival to x. The survival condition to x is essential since it is meaningless to consider the probability of failure of a part after time x, if it has already failed and been discarded before that time.

2.3 Statistical Tests

2.3.1 Cramer-Von Mises Goodness-of-Fit Test

The Cramer-Von Mises (CVM) test provides a formal basis for evaluating the NHPP power law intensity function model goodness-of-fit. Let the available data for the q-th system over the interval $[0,T_q]$, with successive failure times be $0<X_{1,q}<X_{2,q}<\ldots<X_{N_q,q}\leq T_q$, (q=1,...,k). For the CVM test [11,13] the following steps are undertaken:

Step 1. For failure truncated data, $X_{N_q,q}=T_q$, let $M_q = N_q - 1$. For time truncated data, $X_{N_q,q}\leq T_q$, let $M_q = N_q$. Next calculate M using

$$M = \sum_{q=1}^{k} M_q \,. \qquad 2.3.1.1$$

Step 2. For each system divide each successive failure time by the corresponding end of the observation interval time T_q, $i=1,\ldots,M_q$.

$$Y_{iq} = \frac{X_{iq}}{T_q}, \quad i=1,\ldots,M_q, \ (q=1,\ldots k) \qquad 2.3.1.2$$

Step 3. Calculate the unbiased estimate $\bar{\beta}$ of β from

$$\bar{\beta} = \frac{M-1}{\sum_{q=1}^{k}\sum_{i=1}^{M_q} \ln(\frac{T_q}{X_{iq}})}. \qquad 2.3.1.3$$

Step 4. Treat the M Y_{iq}'s as one ordered group from the smallest to the largest. Call the ordered values $Z_1, Z_2, \ldots, Z_M$, where, $Z_1 < Z_2 < \ldots < Z_M$.

Step 5. Calculate the parametric Cramer-Von Mises statistic [11,13]

$$C_M^2 = \frac{1}{12M} + \sum_{j=1}^{M}\left[Z_j^{\bar{\beta}} - \frac{2j-1}{2M}\right]^2. \qquad 2.3.1.4$$

Step 6. For the desired significance level and the M value, there is a corresponding critical value that appears in reference [11].

Step 7. If the calculated C_M^2 (from step 5) is less than the critical value from step 6 then the hypothesis must be accepted that the failure times for the k systems follow a NHPP with intensity function

$$\rho(t) = u(t) = \lambda\beta t^{\beta-1}. \qquad 2.3.1.5$$

2.3.2 Chi-squared Goodness-of-Fit Test

Unlike the Cramer-Von Mises test, the Chi-squared test does not require that the observation interval start times S_q be 0 for each of the k systems. The Chi-squared test [14,15] uses the fact that the expected number of failures for a system over an interval (a,b) is estimated by

$$\hat{\lambda} b^{\hat{\beta}} - \hat{\lambda} a^{\hat{\beta}}. \qquad 2.3.2.1$$

The steps for the Chi-squared Test are as follows:

Step 1. Use the MLE technique to estimate the values of λ and β. Represent the MLE values of λ and β by $\hat{\lambda}$ and $\hat{\beta}$ respectively.

Step 2. Divide the observation interval into at least three cell intervals. The lengths of the cell intervals do not have to be equal. Let d represent the number of cell intervals.

Step 3. Calculate the expected number of failures in each of the cell intervals, $\theta(j)$. Where $\theta(j)$ is given by

$$\theta(j) = \sum_{i=1}^{k} \left\{ \hat{\lambda}[\min(T_i, I_{j+1})]^{\hat{\beta}} - \hat{\lambda}[\max(I_j, S_i)]^{\hat{\beta}} \right\}. \qquad 2.3.2.2$$

For example, in Figure 2, since S_K is greater than I_j, then the cell interval begins at S_K rather than I_j for this system. If the number of expected failures is not at least five, then the interval cell should be lengthened so that the expected number of failures is at least five. It has been found empirically that the expected number of failures per interval cell must be at least five for the Chi-squared goodness-of-fit test to be valid [16].

Step 4. Determine the number of observed failures N(j) in the cell intervals. Using Figure 2 as an example, there are three failures in this cell interval.

Step 5. Compute the χ^2 statistic [14,15]

$$\chi^2 = \sum_{j=1}^{d} \frac{[N(j) - \hat{\theta}(j)]^2}{\hat{\theta}(j)}. \qquad 2.3.2.3$$

Step 6. Compare the computed value of χ^2 with the tabulated percentiles [14] for a Chi-squared variate, using df = d-2 degrees of freedom. Values of χ^2 from step 5 that are greater than the χ^2 percentiles from Chi-squared tables indicate that the observed data contradicts the power law model.

2.3.3 Test for Trend and Confidence Bounds

The conditional MLE of β, $\tilde{\beta}$, is used to test for trend and to construct conditional confidence bounds on the true value of β [2,17]. To accomplish this, it is observed that

$$U = \frac{2M\beta}{\tilde{\beta}} \qquad 2.3.3.1$$

is distributed as a Chi-squared random variable with 2M degrees of freedom. The conditional MLE of β, $\tilde{\beta}$, is given by

$$\tilde{\beta} = \frac{\sum_{q=1}^{k} M_q}{\sum_{q=1}^{k} \sum_{i=1}^{M_q} \ln\left(\frac{T_q}{X_{iq}}\right)}, \qquad 2.3.3.2$$

where M_q=N_q if the data for the q-th system are time truncated and M_q=N_q-1 if the data are failure truncated. M is defined as

$$M = \sum_{q=1}^{k} M_q. \qquad 2.3.3.3$$

M is the effective total number of failures for the k systems. To test for trend, use the null hypothesis of an HPP or $\beta = 1$. This reduces equation 2.3.3.1 to the test statistic referenced in MIL-HDBK-189 [13]. To reject the null hypothesis of HPP, the value of U must be greater or smaller than the critical values given for Chi-squared distributed data with 2M degrees of freedom at a given significance level. These relationships can also be written [11] in terms of the unbiased estimate of β given by

$$\bar{\beta} = \frac{M-1}{M}\tilde{\beta}. \qquad 2.3.3.4$$

The unbiased estimate of β is used to construct the confidence intervals on the shape parameter β. The exact $(1-\alpha)100$ percent lower and upper confidence bounds on β are

$$\beta_L = \bar{\beta}\frac{\chi^2(\alpha/2, 2M)}{2(M-1)} \qquad 2.3.3.5$$

$$\beta_U = \bar{\beta}\frac{\chi^2(1-\alpha/2, 2M)}{2(M-1)}, \qquad 2.3.3.6$$

where $\chi^2(\gamma, 2M)$ is the γ-th percentile for the Chi-squared distribution with 2M degrees of freedom.

2.4 Example of a Repairable System

An example of a repairable system is presented to illustrate the material previously developed. The example is from L.H. Crow [17]. Suppose there are k=3 systems observed over the interval [0,T]; that is, the data are time truncated with T_q=200, q=1,2,3. The failure data were simulated on a computer with $\lambda = 0.6$ and $\beta = 0.5$. The results are given in Table I.

TABLE I

SIMULATED FAILURE DATA SET

System 1 X_{i1}	System 2 X_{i2}	System 3 X_{i3}
4.3	0.1	8.4
4.4	5.6	32.5
10.2	18.6	44.7
23.5	19.5	48.4
23.8	24.2	50.6
26.4	26.7	73.6
74.0	45.1	98.7
77.1	45.8	112.2
92.1	75.7	129.8
197.2	79.7	136.0
	98.6	195.8
	120.1	
	161.8	
	180.6	
	190.8	

Using the failure data presented in Table I, the MATLAB computer program THPERIL.M (Appendix D.1), and the input data set RMCROWT1.DAT (Appendix D.3), the systems can be analyzed. The numerical values presented in this example are from the output of the program THPERIL.M and agree with the values presented in reference 17. The output from program THPERIL.M is given in Appendix D.2. The λ and β parameters are calculated iteratively using equations 2.2.1.4 and 2.2.1.5. The results are $\hat{\lambda} = 0.4605$ and $\hat{\beta} = 0.6153$. Since beta is less than one, this indicates a system showing reliability growth with time.

The natural estimate of the peril rate is determined by applying equations 2.2.1.7 and 2.2.1.8 to the failure data. To illustrate the calculation of $\tilde{\rho}_j(t)$, the results of program THPERIL.M presented in Appendix D.2 can be used. The program divides the failure data into five equally spaced intervals. In this example the first interval is [0,40). During this interval there are 14 failures with three operational units. Using equation 2.2.1.7 the natural estimate of the peril rate becomes

$$\tilde{\rho}_1(t) = \frac{N_1}{\Delta t_1} = \frac{14}{40+40+40} = 0.1167, \quad \text{for } 0 \le t < 40.$$

The natural and NHPP estimates of the peril rate are given in Figure 3. The IMTBF defined in equation 2.1.3.6 is shown in Figure 4.

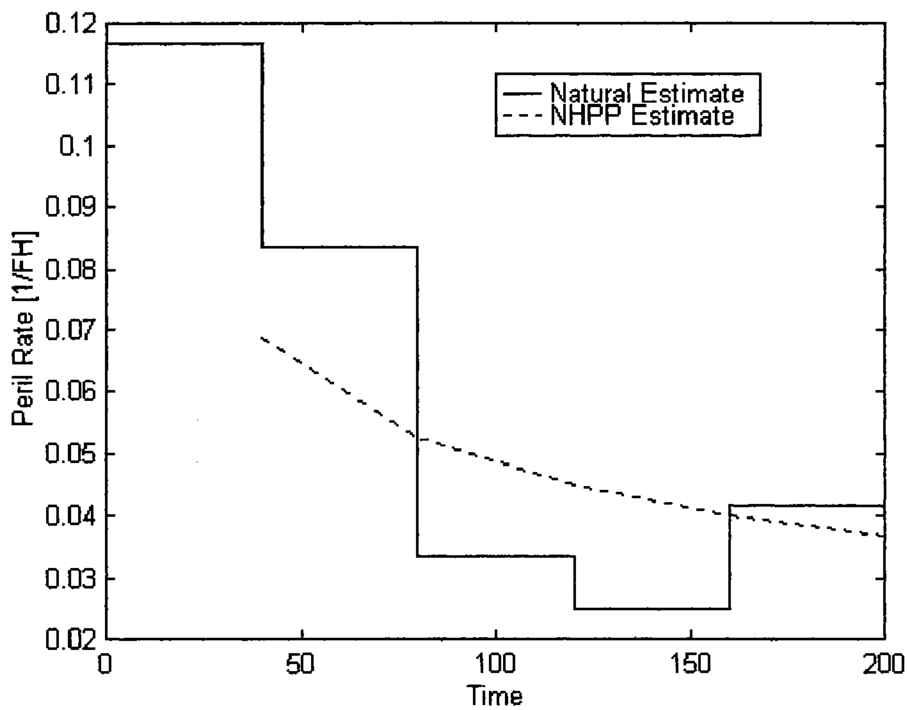

Figure 3. Peril Rate for Example Data Set

To test the hypothesis that the failure data follow a NHPP with a power law intensity function, the Chi-squared and Cramer-Von Mises goodness-of-fit tests are used. First consider the Chi-squared goodness-of-fit test. For example, the expected number of failures during the second cell interval [40,80) is determined by evaluating equation 2.3.2.2 for j=2, this is

$$\theta(2) = [\hat{\lambda}(80)^{\hat{\beta}} - \hat{\lambda}(40)^{\hat{\beta}}] + [\hat{\lambda}(80)^{\hat{\beta}} - \hat{\lambda}(40)^{\hat{\beta}}] + [\hat{\lambda}(80)^{\hat{\beta}} - \hat{\lambda}(40)^{\hat{\beta}}].$$

The expected number of failures during the second interval cell is θ (2)=7.1128. Once the θ (j)'s are calculated the Chi-squared statistic can be calculated using equation 2.3.2.3. This becomes

$$\chi^2 = \frac{(14-13.37)^2}{13.37} + \frac{(10-7.11)^2}{7.11} + \frac{(4-5.8)^2}{5.8} + \frac{(3-5.09)^2}{5.09} + \frac{(5-4.62)^2}{4.62} = 2.65.$$

The calculated value of the Chi-squared statistic must now be compared with the critical values for the Chi-squared Distribution given in reference 14. For three degrees of freedom, (df=5-2=3) at the 0.05 significance level, the critical value of the Chi-squared distribution is 7.81. Since 2.65<7.81, the hypothesis is satisfied that the failure data follows a NHPP with a power law intensity function.

The Cramer-Von Mises goodness-of-fit test can also be used to test the NHPP hypothesis for the failure data points. The procedures to use this test are outlined in section 2.3.1. The first parameter calculated is M, which is 10+15+11=36. since the data are time truncated, M is the number of total failures. Using equation 2.3.1.3, the unbiased estimate of β is $\overline{\beta}$=0.5982. The value of the Cramer-Von Mises statistic is found using equation 2.3.1.4. Evaluating this equation yields C_{36}^2=0.0695. The critical values for C_M^2 are found in Table 2 of reference 17. The critical value at 0.05 level of significance for M=36 is 0.213. Since 0.0695 is less than 0.213, the hypothesis can be accepted that the data are NHPP with a power law intensity at this significance level.

The failure data are now tested for trend. To accomplish this, the statistic U defined by equation 2.3.3.1 is evaluated. The value of the conditional MLE of β , $\tilde{\beta}$, is found by rearranging equation 2.3.3.4. The statistic U is calculated using

$$U = \frac{2M\beta}{\left(\frac{M}{M-1}\right)\bar{\beta}} = \frac{2(M-1)\beta}{\bar{\beta}} = \frac{2(36-1)(0.6153)}{0.5982} = 72.$$

The value of U must now be compared with $\chi^2(\gamma = 0.95, df = 2*36)$. Since many [8,14,16,18] Chi-squared tables do not cover 72 degrees of freedom, an approximation must be used. The equation used in the approximation of the percentiles of the Chi-squared distribution is given by [18]

$$\chi^2 = df\left[1 - \frac{2}{9df} + z\left(\frac{2}{9df}\right)^{1/2}\right]^3 \qquad 2.4.1$$

where z is a standardized normal random variable. For a 0.05 significance level, z=1.645 [18]. Using z=1.645 and df=72, χ^2 using equation 2.4.1 is 92.8. Since U=72 is less than 92.8, it is concluded that the failure data shows trend with a β less than one.

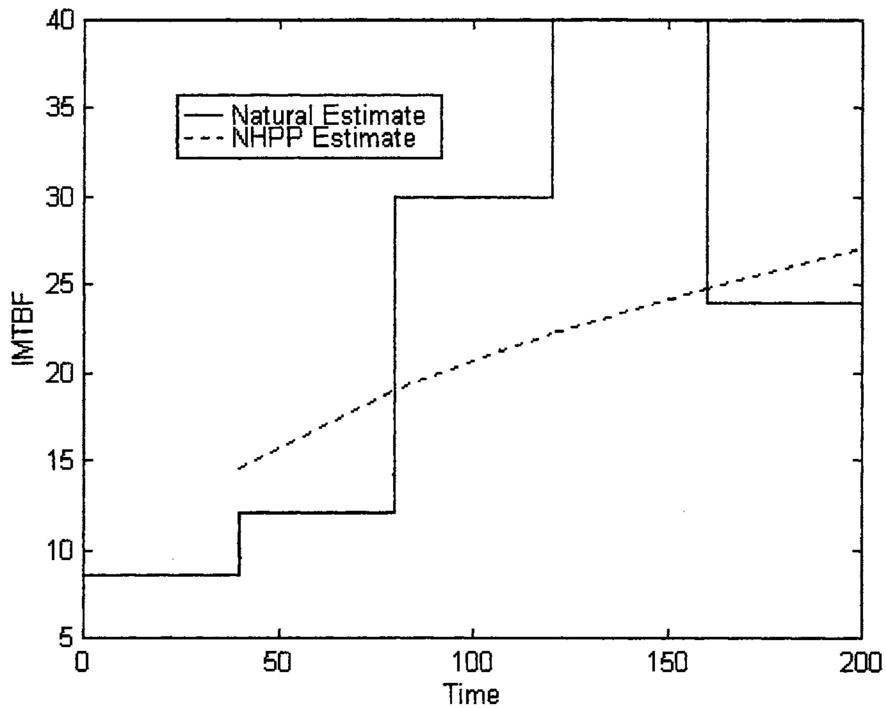

Figure 4. IMTBF for Example Data Set

The lower and upper confidence bounds for β are calculated using equations 2.3.3.5 and 2.3.3.6, respectively. For a significance level of α=0.1 and M=36, the values of χ^2 become $\chi^2(.1/2,2*36) = 53.4$ and $\chi^2(1-.1/2,2*36) = 92.8$. These χ^2 values were calculated using the approximation equation 2.4.1 and z=0.645 for χ^2=92.8 and z=-1.645 for χ^2=53.4. Using these Chi-squared values, $\overline{\beta}$=0.5982 and M=36 in equations 2.3.3.5 and 2.3.3.6, the lower and upper confidence bounds on β are β_L=0.456 and β_U=0.793.

2.5 Type II Maintenance Policy

The Type II maintenance policy was introduced by Barlow and Hunter [19] in 1960. This policy calls for a planned replacement (overhaul) of a system after some prespecified number of system operating hours, regardless of the number of intervening failures. It is assumed that after each failure the system is only minimally repaired, therefore, the ROCOF is unchanged by the repair. An operational system is removed from service to be overhauled when it has reached a prespecified number of hours. Instantaneous repair is also assumed. The Type II policy is the same as a hard-time replacement policy commonly used in the airline industry.

Since scheduled overhauls occur at times T, 2T, 3T,..., the problem reduces to selecting T to minimize the overall maintenance cost function. The long-run expected cost per unit time is given by [20]

$$C(T) = \frac{C_{MR}E[N(T)] + C_{SR}}{T}, \qquad 2.5.1$$

where E[N(T)] is the expected number of minimal repairs over the interval and is given by

$$E[N(T)] = \int_0^T \rho(u)du. \qquad 2.5.2$$

C_{MR} is the cost of a minimum repair performed after a failure. This cost does not include the cost of an overhaul. C_{SR} is the cost of a scheduled overhaul. Combining the power law peril rate given by equation 2.2.1.6 and equations 2.5.1 and 2.5.2, the long-run expected cost per unit time becomes

$$C(T) = \frac{C_{MR}\lambda T^{\beta} + C_{SR}}{T}. \qquad 2.5.3$$

To minimize C(T), set the derivative of equation 2.5.1 with respect to T equal to 0. The resulting equation is

$$\int_0^T (\rho(T) - \rho(u))du = \frac{C_{SR}}{C_{MR}}. \qquad 2.5.4$$

This equation has a unique solution provided the peril rate is strictly increasing to ∞ as $t \to \infty$. When the peril rate is specifically in the form

$$\rho(t) = \lambda\beta t^{\beta-1}, \beta > 1, \qquad 2.5.5$$

the optimal replacement interval T* becomes

$$T^* = \left[\frac{C_{SR}}{\lambda(\beta-1)C_{MR}}\right]^{1/\beta}. \qquad 2.5.6$$

2.6 Type II' Maintenance Policy

The Type II' maintenance policy was first introduced by Makabe and Morimura [21] in 1963. The Type II' policy is : "Perform preventive maintenance at the next failure after T operating hours [22]." Preventive maintenance can consist of system replacement or overhaul. A Type II' maintenance policy is the same as a mandatory soft-time overhaul interval maintenance policy. The soft-time overhaul policy is commonly used in the airline maintenance industry. The original papers by Makabe and Morimura were primarily concerned with the comparison of Type I through V maintenance policies.

In 1977 Muth [4] extended the earlier work of Makabe and Morimura. Muth assumes:

1. A replacement resets the age of the system to 0.
2. A repair does not change the age of the system.
3. When a system fails it is repaired if $t<T$, or replaced if $t>=T$.

In the above, t is the system age, and T is the overhaul interval. The long-run expected cost per unit time is given by

$$C(T) = \frac{C_{USOVH} + C_{MR} H(T)}{T + r(T)}. \qquad 2.6.1$$

Where, C_{USOVH} is the cost of an overhaul at failure given the t>T, C_{MR} is the cost of a minimal repair, and H(T) is defined as

$$H(T) = E\{N_i\} = \int_0^T h(x)dx. \qquad 2.6.2$$

In this equation, h(x) is a nonhomogeneous Poisson process rate function, and N_i is the number of failures that occur in (t_i, t_i+T). The quantity r(T) is the mean residual life function (m.r.l.f.) defined in equation 2.1.2.6.

The shortcoming with Muth's approach is that the m.r.l.f. does not accurately represent the expected time remaining to the next failure when the system has age T. The m.r.l.f. is a concept based on a lifetime distribution function and is used for a parts model to estimate the remaining life of the part once it has reached a specific age. The m.r.l.f. is not defined for a repairable system. Since a repairable system's failure time is more accurately characterized by a NHPP than a distribution function, a Monte Carlo simulation will be needed to determine the IDG's optimum replacement interval, T*, under a Type II' maintenance policy.

2. 7 Monte Carlo Simulation

Monte Carlo simulation [23] has been successfully used in the past to model repairable systems. Kumamoto et al. [24] investigated the use of a state-transition Monte

Carlo method to estimate the unreliability of large repairable systems. Roberts and Mann [25] modeled failure data with Crow's nonhomogeneous Poisson process (NHPP). The expected number of failures predicted for the system by the Crow model was compared with predictions using a Monte Carlo simulation that utilized Weibull parameters for the major components of the system. Calabria et al. [26] used Monte Carlo simulation to assess the performance of point maximum likelihood estimators for parameters, e.g., mean number of failures and failure rate in a nonhomogeneous Poisson process, for in-service failure count data.

2.8 Availability

Reliability is a measure of the probability that the system has operated successfully over the time interval from 0 to t. The definition of reliability does not take into account the maintenance program of the system. Once a system is fully operational and is subject to a maintenance program, the total costs of the system is determined by the operational and maintenance costs. Availability combines a measure of the maintainability and reliability and is widely used to measure the effectiveness of maintained systems.

Availability is defined as the probability that the system is operating successfully at any point in time under stated conditions [3], or it is defined as the ratio of uptime to total time [27]. Based on this broad definition, availability can be more exactly defined by six definitions [27]. The first three depend on the time interval considered. The last three definitions depend on the type of downtime. The six definitions for availability are:

1. *Instantaneous Availability*, A(t), is defined as the probability that the system is operational at any random time t. This is the same as pointwise availability defined in Barlow and Proschan [20].

2. *Average Uptime Availability*, $A_\tau(t)$, is the proportion of time in a specified interval (0, τ) that the system is available for use, and it is given by

$$A_\tau(\tau) = \frac{1}{\tau}\int_0^\tau A(t)dt. \qquad 2.8.1$$

This is the same term as interval availability defined by Barlow and Proschan [20].

3. *Steady State Availability*, $A_{SS}(\infty)$, is defined when the time interval considered is very large and is given by

$$A_{SS}(\infty) = \lim_{\tau \to \infty} A(\tau) \qquad 2.8.2$$

This is the same as the limiting interval availability defined by Barlow and Prochan [20].

4. *Inherent Availability*, A_i, is defined by

$$A_i = \frac{MTBF}{MTBF + MTTR} \qquad 2.8.3$$

where MTBF = mean time between failure and MTTR = mean time to repair. This definition includes only corrective maintenance downtime and excludes ready time, preventive maintenance downtime, logistic (or supply) time, and waiting or administrative downtime.

5. *Achieved Availability*, A_a, includes corrective and preventive maintenance downtime and is a function of the frequency of maintenance and the mean maintenance time. It is defined by

$$A_a = \frac{\text{MTBM}}{\text{MTBM} + \text{MMD}} \qquad 2.8.4$$

where MTBM = mean time between maintenance and MMD = mean maintenance downtime resulting form both corrective and preventive maintenance actions. This definition excludes logistic time and waiting or administrative downtime.

6. *Operational Availability*, A_o, includes ready time, logistic time, and waiting time or administrative time and is expressed by

$$A_o = \frac{\text{MTBM} + \text{ready time}}{(\text{MTBM} + \text{ready time}) + \text{MMD} + \text{delay time}} \qquad 2.8.5$$

where ready time = operational cycle - MTBM - MMD - delay time.

To describe the availability of a system, it is necessary to specify three things: the component(s) failure process; the repair or maintenance process; and system configuration [28] which describes how the components are functionally connected and the rules of operation. The effects of these three items must be investigated before a meaningful availability model is applied.

CHAPTER III

REDUCTION OF FAILURE DATA

This chapter begins by investigating the dominant failure modes of the IDG. An understanding of the dominant failure modes is important to improve the intrinsic reliability [29] of a system. Next, the failure data is characterized using the NHPP and natural estimate methods. Due to the "bathtub" shape of the peril rate, the failure data are left-truncated to remove the infant mortality effects. A quantification of the peril rate is necessary to develop an optimal maintenance program.

3.1 Integrated Drive Generator

The purpose of the integrated drive generator is to provide primary electrical power for the aircraft. The IDG is broken down into two main subsystems (see Figure 5): the constant speed drive (CSD) and the generator.

The CSD is a hydromechanical device whose major components are an axial gear differential, a mechanical governor, a charge pump, a scavenge/inversion pump, two hydraulic pump and motor assemblies, and an electrically actuated input shaft disconnect. The generator converts the 12,000 rpm mechanical output power of the CSD into 3 phase

400 Hz electrical power. The generator is a brushless, three-stage, rotating rectifier type of generator. The rotor assembly contains the exciter rotor, permanent magnet generator rotor, main generator rotor, and rotating rectifier assembly. The stators for the permanent magnet generator, main generator, and exciter are mounted in the IDG housing. The permanent magnet generator provides a signal used for system control, protection, and generator excitation. External to the IDG is the generator control unit (GCU). The GCU provides excitation to the generator and in conjunction with switches on the flight deck panels, provides control, protection, and metering for the generator and load buses.

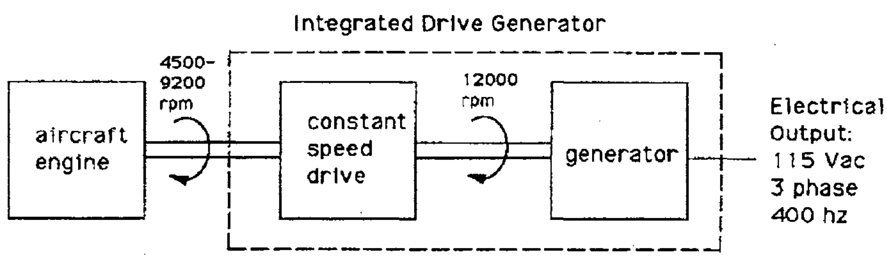

Figure 5. Integrated Drive Generator

3.2 Description of Failure Data

To increase the mean time between failure (MTBF) of a population of systems there are two fundamental approaches. The first is to improve the intrinsic reliability of the individuals in the population; this is accomplished through implementation of product improvement service bulletins and improved repair procedures. The second approach is to modify the scheduled maintenance interval so that for a deteriorating system, the average age of the population decreases, thus increasing the MTBF. An investigation of the dominant fault modes provides information useful in improving the intrinsic reliability of the IDG. The dominant fault modes are identified in this section. This information is useful to focus engineering efforts on the most common or costly problems.

Appendix A contains a list of 84 maintenance events that occurred on 61 separate IDG's [30]. The first column, labeled TRK#, is the unique tracking number of the IDG; this gives each IDG a distinct label. The second column, labeled HOURS SINCE OVH, is the number of flight hours (FH) on each drive since it was overhauled (τ_{TSO_q}). Each IDG in Appendix A has been through at least one overhaul. During an overhaul the IDG is returned to the manufacturer's original fits and tolerances, and the number of hours on the unit is returned to zero. The third column, labeled HOURS SINCE LAST FAILURE, gives the number of flight hours since the unit's last failure. When the HOURS SINCE OVH and HOURS SINCE LAST FAILURE are equal, this represents the first failure of the drive since overhaul. The fourth column, labeled FAILURE MODE, contains the type of failure each drive experienced. A detailed discussion of each failure mode is beyond the scope of this Thesis. The fifth column, labeled TYPE OF MAINT., contains the type of maintenance performed on each drive to return it to service.

The type of maintenance to be performed is largely determined by the mechanic in the repair facility. The least involved type of maintenance is a No Fault Found (NFF). In a NFF, the drive is initially functionally tested, and if all the functional tests are passed, no maintenance is performed, and the drive is returned to inventory for future use. A Check and Repair (C&R) maintenance action occurs when an actual fault is present and can be repaired without a complete overhaul of the IDG. The most extensive form of maintenance is the overhaul (OVH). During an overhaul, a predetermined bill-of-work is performed on the drive to return it to the manufacturer's original fits and tolerances. Unlike the NFF or C&R, the overhaul resets the number of flight hours on the drive to zero ($\tau_{TSO_q} = 0$).

Table II summarizes the data from Appendix A. The most common type of failure and maintenance action is the NFF. The high unconfirmed removal percentage is a result of the troubleshooting time constraints placed on an aircraft mechanic. Since most aircraft maintenance is performed between flights or during overnight maintenance, many components (such as the IDG) are unnecessarily replaced. The most common confirmed failure mode is CARRIER SHAFT ASSY. The most common type of maintenance resulting from this type of failure is an overhaul. Averaging the hours since overhaul in Table II gives 5858 FH. The average FH since overhaul for the CARRIER SHAFT ASSY failure mode is 6680 FH. Since the 6680 FH is significantly higher than the hours since overhaul for the general population, this would indicate that the CARRIER SHAFT ASSY failure mode is associated with a wear out process.

TABLE II

FAILURE MODE ANALYSIS RESULTS

FAILURE MODE	AVG HOURS SINCE OVERHAUL	AVG HOURS SINCE FAILURE	PERCENT	MOST COMMON MAINTENANCE
NFF	5405	3141	29.8	NFF
CARRIER SHAFT ASSY	6680	5633	14.3	OVH
PUMP & MOTOR -ASSY FIXED END	5833	4313	11.9	OVH
STATOR HOUSING ASSY	7008	5589	9.5	C&R
ALL OTHERS	5599	4512	34.5	VARIOUS

3.3 Overview of Field Maintenance Data

Appendix C gives the maintenance history of 41 IDGs. The first column labeled TRAK# is a unique number given to each IDG. The tracking numbers are not sequential so that they remain consistent with reference 30. The next eight columns labeled 1 ST INTVL through 8 TH INTVL are maintenance actions that occurred to each drive. For example, 74-E represents 74 flight hours and an engine change. When an IDG is removed from service because the engine it is mounted to requires maintenance, no IDG maintenance is performed. The "-C" following the number of flight hours is for a C&R maintenance; "-O" is for an overhaul; and "-N" is for a no fault found (NFF) repair. A check and repair (C&R) maintenance action is considered a minimal repair [2]. Each drive was overhauled prior to the 1 ST INTVL flight hours and maintenance action. The column labeled FINAL INTVL is the number of flight hours on a drive from the last maintenance action to the end of the observation interval. These are time truncated data, i.e., the drives had not yet reached the point of an overhaul by the end of the data recording period.

3.4 Reduction of Failure Data

Appendix B gives the maintenance history of 51 IDG's. These are the same IDG's from Appendix A. Only 51 of the 61 entries listed in Appendix A had sufficient maintenance history information to be included in Appendix B. When comparing the times to failure for a specific tracking number between Appendixes A and B, one notices slight differences in the number of flight hours. These differences are a result of two different recording methods for the number of flight hours. The first column in Appendix B, labeled TRK#, is the IDG's unique tracking number. The second column, labeled INITIAL INTVL, is the total number of flight hours the drive has accumulated since the drive was originally purchased. The INITIAL INTVL value is the estimated total number of flight hours from the original purchase date of the drive to the date the data collection effort started. On the average, each drive accumulates 8 FH per day of life. This average includes periods of inactivity for maintenance and inventory. While in service, each drive accumulates 11.39 FH for each calendar day of operation. An R in this or any column in Appendix B indicates that the information is not available.

The third column, labeled 0 TH MAINT, is the first recorded maintenance event at the beginning of the observation interval. The fourth through eleventh columns contain the number of flight hours since the last maintenance event occurred. The letter following the flight hours represents the type of maintenance that occurred. The abbreviations used for each maintenance action are described in section 3.3 of this Thesis.

The last column in Appendix B, labeled FINAL INTVL, is the estimated number of flight hours the IDG experiences since the last maintenance event. This number is determined by taking the difference between the dates the IDG was returned to service and the end of the observation interval multiplied by 11.39 FH/day.

To clarify the information in Appendix B, take the second IDG as an example. On the date the observation period started for drive number 2, it had accumulated approximately 23,904 FH. Since an R appears in the third column, the maintenance event is unknown. After the 0-th maintenance occurred, the drive was placed back in service and accumulated 2943 FH. At the time of this first failure, $\tau_{TT(q=2)} = 23904 + 2943 = 26847$ FH and $\tau_{TSO(q=2)}$ was unknown. The quantity τ_{TT_q} is the total time since the drive was originally placed in service. τ_{TT_q} is not reset to zero by an overhaul. After overhaul $\tau_{TSO(q=2)}$ is set to zero. The drive is returned to service and accumulated an additional 178 FH. At this point the drive is removed from service for an engine change. After the engine's maintenance is completed, the IDG is returned to service with no maintenance being performed on it. Next the IDG operates for 9616 FH, fails, and is overhauled. The drive then accumulates 1830 FH and is removed from the engine due to an apparent failure. The drive is sent to the repair facility where it is tested with no fault found (NFF). Next the drive is returned to service and flies for 262 more hours until the end of the observation interval is reached. At the end of the observation interval, the times on the IDG are: $\tau_{TT(q=2)} = 23904 + 2943 + 178 + 9616 + 1830 + 262 = 38733$ FH and $\tau_{TSO(q=2)} = 1830 + 262 = 2092$ FH. Figure 6 gives a time line for this series of events up to 35,000 FH.

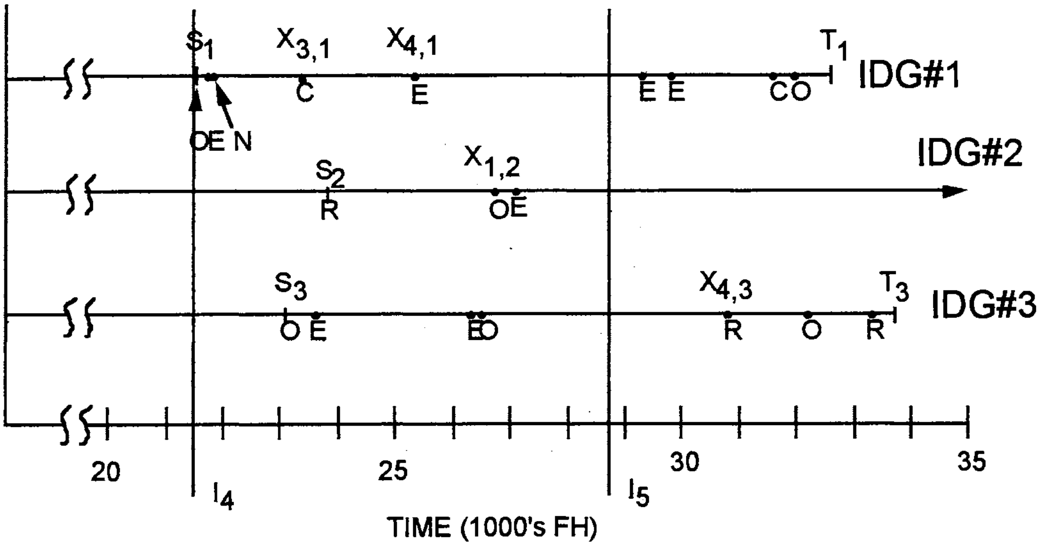

Figure 6. Maintenance Events for the First Three IDG's

3.5 Effect of Overhaul on the IDG

To develop a cost effective maintenance program, it is necessary to know if the system deteriorates or shows reliability growth with time. The time dependence of the peril rate determines if a system is deteriorating or improving with time. If the system's peril rate increases with time, then the system is deteriorating. Reliability growth is indicated by a decreasing peril rate. Once the type of system (deteriorating or improving) is determined, then an optimal maintenance program [2] can be devised to decrease the total maintenance costs and increase the system's availability. In this section, the peril rate of the IDG is investigated. The NHPP with a power law intensity function is used to characterize the failure data. It is assumed that the IDG population is homogeneous, and therefore the failure data can be pooled [31].

Depending on how the failure data are combined, there are two peril rates that are of practical use. The first is the confirmed peril rate, ρ_C. Here the flight hours between removals, shown in Appendix C, for the engine change and the NFF failures are added to the total time to failure. The confirmed peril rate represents only actual or confirmed IDG failures. For example, the IDG with tracking #1 in Appendix C has the following failure data set: $X_{1,1}$=74+138+1444=1656, $X_{2,1}$=1656+2015+3966+712+1697=10046, and $X_{3,1}$=10046+329=10375. The second peril rate takes into account the confirmed failures as well as the NFF failures. This is called the unconfirmed peril rate and is represented by ρ_{UC}. An unconfirmed IDG failure results in a NFF maintenance action. There is a cost associated with a NFF maintenance action. The removal of the IDG for an engine change is not considered in either the confirmed or unconfirmed peril rates. Redoing the previous example for tracking #1, the sequence of failure times become: $X_{1,1}$=74+138=212, $X_{2,1}$=212+1444=1656, $X_{3,1}$=1656+2015+3966+712+1697=10046, and $X_{4,1}$=10046+329=10375.

TABLE III

NHPP PERIL RATES

PERIL RATE	**LEFT-TRUN.**	**BETA**	**LAMBDA**	**CHI-SQ**	**CVM**
CONFIRMED	0	.933	3.78E-4	5.06	.041
UNCONFIRMED	0	.793	1.8E-3	7.04	.035
CONFIRMED	2000	1.3	1.27E-5	7.22	NA
UNCONFIRMED	2000	1.27	2.08E-5	7.75	NA
CONFIRMED	3000	1.74	1.97E-7	4.88	NA
UNCONFIRMED	3000	1.65	5.43E-7	2.89	NA
CONFIRMED	4000	1.80	1.08E-7	1.64	NA
UNCONFIRMED	4000	1.75	2.26E-7	2.39	NA
CONFIRMED	5000	3.06	6.64E-13	1.37	NA
UNCONFIRMED	5000	2.47	2.32E-10	1.19	NA

Table III lists the results of applying equations 2.2.1.4 and 2.2.1.5 to the failure data for confirmed and unconfirmed removals. The second column in Table III, labeled LEFT-TRUN., gives the number of flight hours that the failure data was left-truncated. Left-truncation of the failure data [32] removes the effects of infant mortality. An earlier study by West [30] concluded that the infant mortality effects for an overhauled IDG had the effect of "flattening" the peril rate for the NHPP model. This effect is evident in Table III, the confirmed and unconfirmed peril rates for the case of zero left-truncation results in beta being less than one. A beta value of less than one indicates a system showing reliability growth; this is not the case for the IDG. As the amount of left-truncation increases from 2000 FH to 5000 FH, the beta value increases. In all cases in Table III, the beta value for the unconfirmed peril rate is less than that of the confirmed peril rate. An unconfirmed removal is a result of incorrect field maintenance trouble shooting which is not an age dependent process. This has the effect of reducing the rate of increase of the peril rate, hence beta, for the unconfirmed peril rate.

Figures 7 and 8 illustrate the unconfirmed peril rate with no left-truncation. The unconfirmed peril rate is estimated using the natural and NHPP methods. The NHPP estimate shows the IDG is experiencing reliability growth while the natural estimate shows a "bathtub" [3] shape. The β and λ parameters are estimated using an iterative approach to equations 2.2.1.4 and 2.2.1.5. The program that calculates these parameters is THPERIL.M (Appendix D.1) using the data set P4OVHUC.DAT (Appendix D.6). The tabular output of program THPERIL.M, for Figures 7 and 8, is given in Appendices D.4 and D.5, respectively.

The only difference between Figures 7 and 8 is in the observation cell, $[I_j, I_{j+1}]$, choice. In Figure 7, the data are divided into five equally spaced observation cells over the interval 0 to 15,000 FH. In Figure 8, the data are divided into five equally spaced observation cells over the interval 0 to 13,397 FH. As illustrated in the two figures, the difference in the choice of observation cell widths does make a difference in the natural estimate of the peril rate. The NHPP estimate is not dependent on the observation cell choice and is unchanged between the two figures.

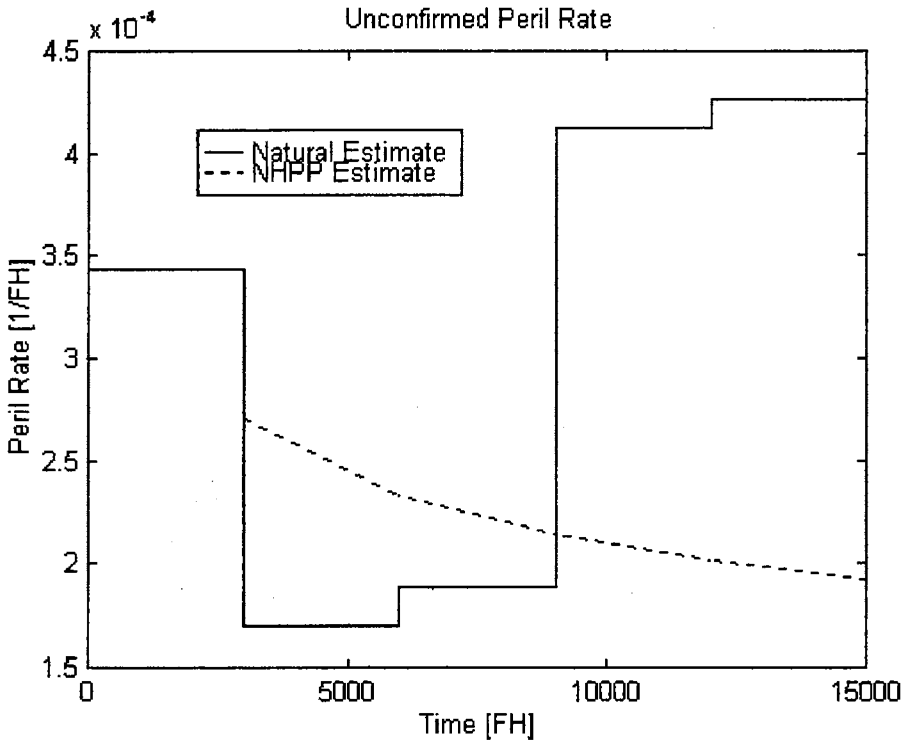

Figure 7. Unconfirmed Peril Rate with no Left-Truncation over the Interval 0 to 15,000 FH

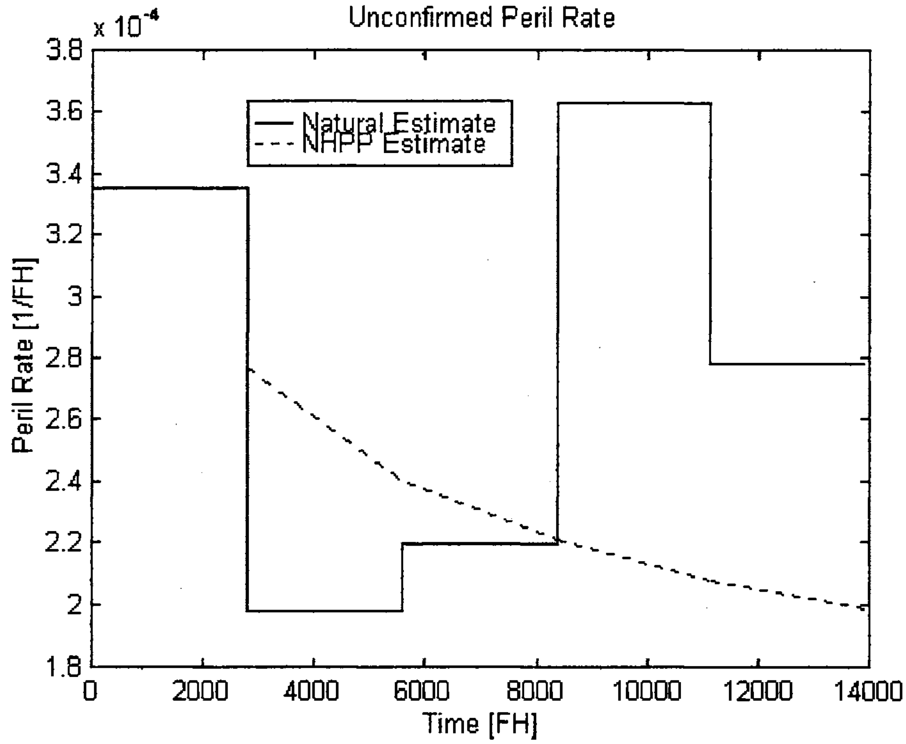

Figure 8. Unconfirmed Peril Rate with no Left-Truncation over the Interval 0 to 13,397 FH

Figures 9 and 10 illustrate the confirmed peril rate for the IDG failure data. The confirmed failure rate is based on actual failures, i.e., C&R or OVH maintenance actions, not unconfirmed failures (NFF). The observation cell width is five equally spaced cells ranging from 0 to 15,000 FH as shown in Figure 9. Figure 10 has five equally spaced cells ranging from 0 to 13,397 FH. Note that both the NHPP and natural estimate of the confirmed peril rates are numerically smaller in all observation cells than the confirmed peril rates shown in Figures 7 and 8. This is to be expected because the unconfirmed peril rate takes into account more maintenance events, i.e., NFF maintenance actions, than the confirmed peril rate.

The computer program used to analyze the failure data is THPERIL.M (Appendix D.1). The data set is P4OVHA.DAT is found in Appendix D.9. The output of program THPERIL.M for the data set P4OVHA.DAT using intervals shown in Figures 9 and 10 is given in Appendices D.7 and D.8, respectively.

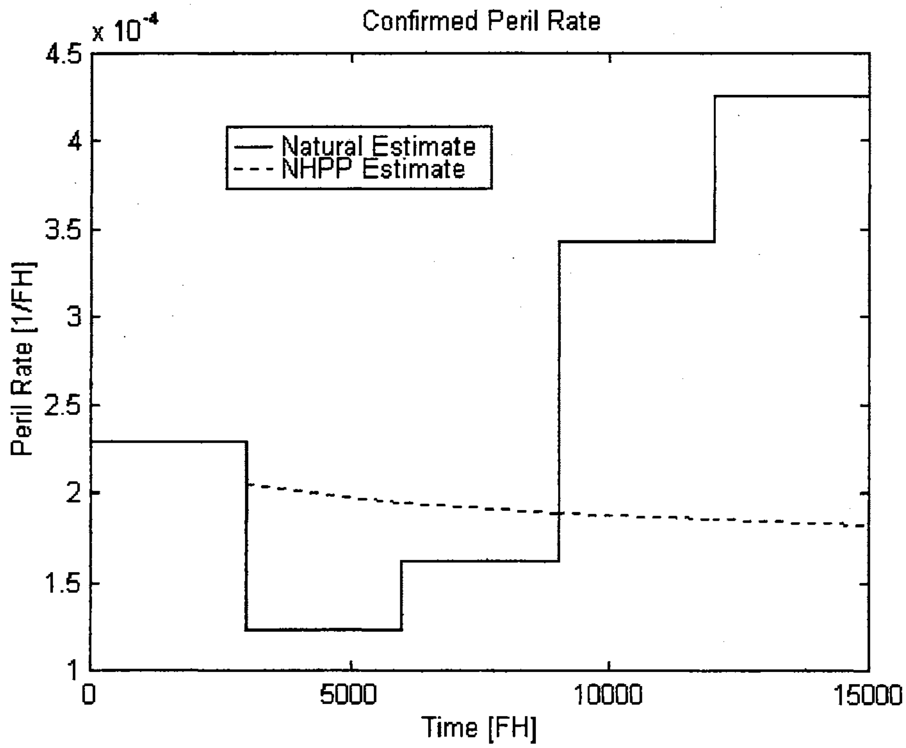

Figure 9. Confirmed Peril Rate with no Left-Truncation over the Interval 0 to 15,000 FH

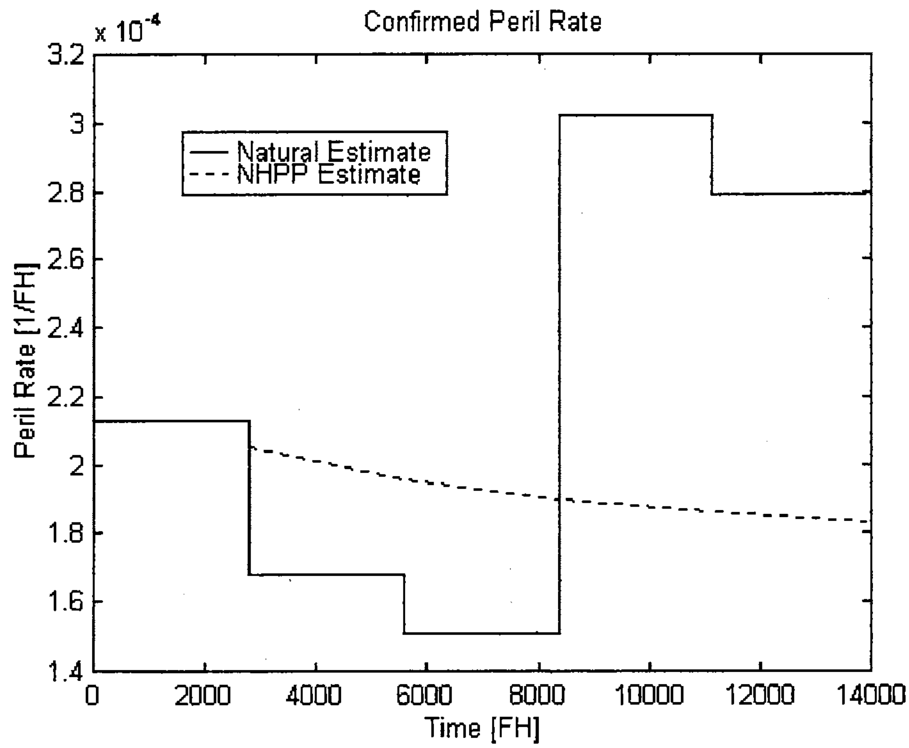

Figure 10. Confirmed Peril Rate with no Left-Truncation over the Interval 0 to 13,397 FH

The NHPP estimate of the peril rate shown in Figures 7 through 10 indicates the IDG is showing reliability growth with the number of flight hours. This contradicts field experience and the natural estimate of the peril rate. To bring the NHPP and natural estimates of the peril rates into agreement, the failure data are left-truncated. Left-truncation involves deleting the failure data that fall below a specified value. Table III gives the parameters values for the confirmed and unconfirmed peril rates. Note that as the amount of left-truncation increases, the β value increases, and the λ value decreases. Left-truncation removes the infant mortality effects from the estimate of the peril rate. Figures 11 and 12 illustrate the effects of left-truncation on the unconfirmed peril rate.

The computer program used to analyze the failure data is THPERIL.M found in Appendix D.1. The 2,000 FH left-truncated data set is P4OVH2KU.DAT listed in Appendix D.11. The output of program THPERIL.M is found in Appendix D.10. For the 5,000 FH left-truncated data, the same program is used. The data set is P4OVH5KU.DAT listed in Appendix D.13. The output of program THPERIL.M is found in Appendix D.12.

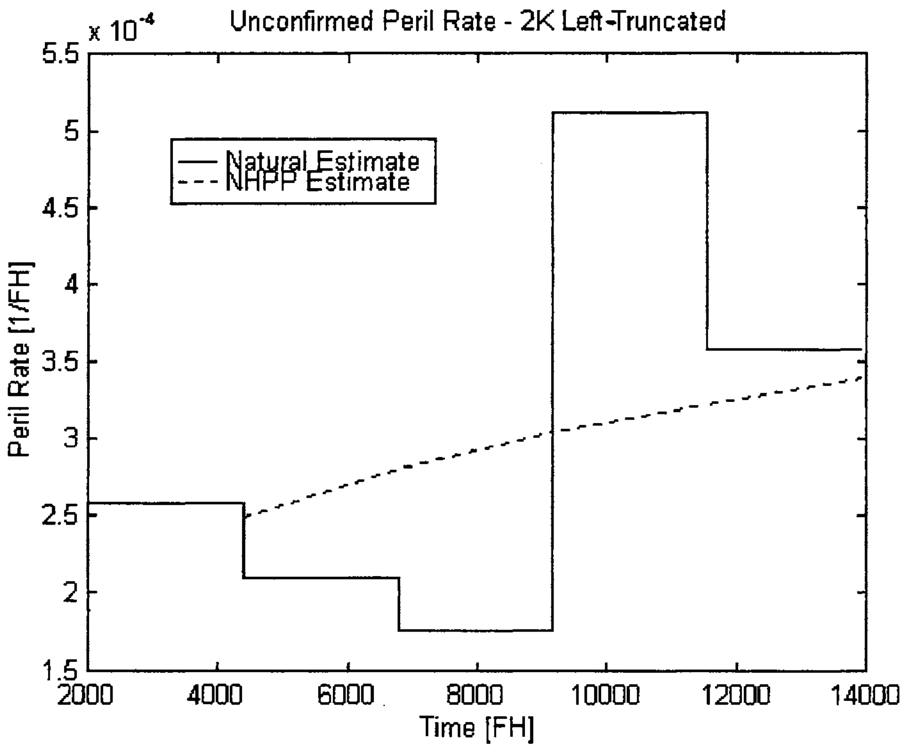

Figure 11. Unconfirmed Peril Rate with 2K FH Left-Truncation over the Interval 0 to 13,397 FH

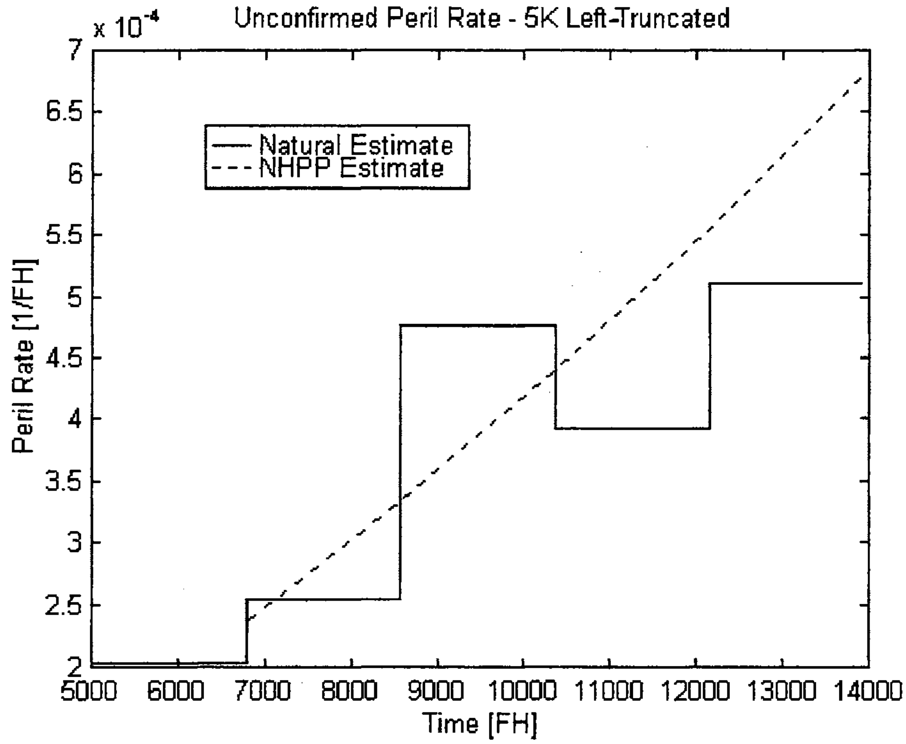

Figure 12. Unconfirmed Peril Rate with 5K Left-Truncation over the Interval 0 to 13,397 FH

3.6 Goodness-of-Fit Tests

When data are fitted to a parameterized model, for example the power law process, it is appropriate to test the compatibility of the model and the data by a statistical goodness-of-fit test. Two statistical tests commonly used are the Cramer-Von Mises (CVM) and the Chi-squared. Crow [17] adapted a parametric Cramer-Von Mises goodness-of-fit test for the multiple system power law process model. This test is appropriate whenever both the observation interval start times for each system are zero and the failure data are complete over the continuous interval $[0,T_q]$ with no gaps in the data. The Chi-squared goodness-of-fit test [14,15,17] is a more general test than the

Cramer-Von Mises test because the Chi-squared test does not require that the observation interval start at zero.

The 5th and 6th columns of Table III give values for the Chi-squared and Cramer-Von Mises statistic respectively. Since the CVM test is only applicable for the case where the starting time is zero, it is only applied to the data sets with zero left-truncation. At 10 percent significance level, M=50, the CVM critical value [17] is 0.173. Since 0.041 and 0.035 are both less than 0.173, then the conclusion is that then NHPP with a power law peril rate provides an adequate model for the failure data. An NA in column six of Table III indicates the CVM test was not applicable for this data set.

The Chi-squared goodness-of-fit test is applied to all ten data sets in Table III. Column five lists the calculated value of the statistic. At 10% significance level with three degrees of freedom, the Chi-squared critical value [14] is 6.25. Not until the left-truncation reaches 3000 FH does the Chi-squared statistic fall consistently below the critical value. This indicates that for the data sets with at least 3000 FH of left-truncation, the NHPP with power law peril rate provides an adequate model for the failure data.

3.7 Effectiveness of the Current Maintenance Program

In this section the peril rate is investigated to determine the effectiveness of the current IDG maintenance program. The current maintenance policy is Type II', i.e., the IDG operates until failure and then the IDG is required to be overhauled if it enters the repair facility with greater than 14,600 FH. The data in Appendix B are reduced in a similar manner as in the previous sections of this chapter. For example, the failure time line for the first IDG is:

Trk#1 S_1=21600, $X_{1,1}$=23256, $X_{2,1}$=31646, $X_{3,1}$=31975, T_1=32635.

In this case, the focus is on the total time of each drive, τ_{TTq}. An overhaul is assumed to not reset the drive flight hours to zero, and the OVH and C&R maintenance actions are considered equivalent. The ENG and NFF maintenance actions are ignored.

The peril rate is estimated using the NHPP MLE technique and the natural estimate methods. The data set P4FAIL.M listed in Appendix D.15 is analyzed using program THPERIL.M. The output of this program is listed in Appendix D.14. The MLE NHPP parameter values are β=1.086 and λ=8.1x10^{-5}. A beta of approximately one indicates a system that is neither improving nor deteriorating with time. Since S_i was not equal to zero in all cases, the CVM test could not be used. The calculated value of the Chi-squared statistic is 3.06; this is well below the 6.25 critical value required for three degrees of freedom at the 10% significance level. This indicates that the power law peril rate provides a good statistical fit for the failure data.

The NHPP and natural estimates of the peril rate for the IDG since original purchase are shown in Figure 13. Definite conclusions about the effectiveness of the current maintenance program cannot be drawn from Figure 13. The natural estimate of the peril rate changes over each interval cell with no trend. The beta value of 1.086 indicates the IDG has nearly a constant NHPP estimate of the peril rate with time.

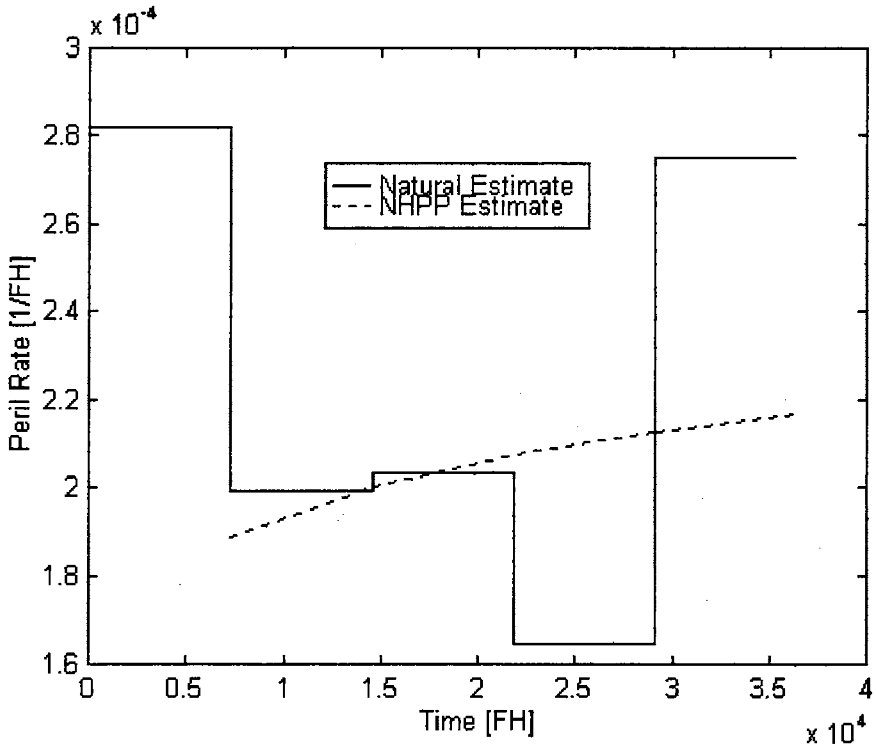

Figure 13. Peril Rate of IDG Since Original Purchase

3.8 Summary

In this chapter it was shown that the IDG failure data peril rate can be parameterized using the NHPP with a power law intensity function. The natural estimate method was used to give a non-parameteric representation of the peril rate. Left-truncation of the failure data is required so that the effects of infant mortality can be eliminated and the NHPP model would accurately represent the IDG as a deteriorating system. The most common failure modes for the IDG are also quantified in this chapter.

CHAPTER IV

TYPE II' MAINTENANCE POLICY

This chapter presents the results of a Monte Carlo simulation that is used to minimize the cost per flight hour for an aircraft integrated drive generator. The computer model of the operational/maintenance cycle requires seven states to adequately describe the system. In the operational state (1), the IDG operates until it is prematurely removed due to a reported failure (state 2); or it is removed from the aircraft for an engine change (state 7). After a reported failure, the IDG enters the repair facility where three possible types of maintenance (states 3-5) can occur. A Type II' maintenance policy requires the IDG to be overhauled once a reported failure has occurred and it has accumulated a prespecified number of flight hours. Then the IDG remains in inventory (state 6) until it is required for service again.

The Monte Carlo simulation models the IDG's under a Type II' maintenance policy. The Type II' policy requires that the IDG be overhauled after a field failure, provided it has accrued a predetermined number of flight hours. One parameter that can be easily changed is the mandatory overhaul interval (MOI). By varying the MOI when an IDG enters the repair facility, the cost per flight hour can be minimized. It is observed that when the optimal MOI is chosen, the mean time between failure and the availability are also maximized.

4.1 Description of Maintenance and Operational Model

Figure 14 is a block diagram maintenance model of the IDG. The IDG is in state one when it is operational on an aircraft. The peril rate (ROCOF) has been parameterized based on the natural estimate method described in Zaino and Berke [12]. As the IDG operates on wing, the peril rate changes depending on the number of flight hours [FH] on the IDG since an overhaul. After each simulated flight day, the state of the IDG is examined to determine if a failure or an engine change has occurred. When a possible failure occurs, the model transitions to the reported failure state (state 2). An engine change (state 7) involves removing the aircraft engine which has an IDG attached to it. During an engine change the IDG receives no maintenance and is returned to service when the engine is placed back on an aircraft.

State two represents the reported failed state. Possible failures that enter state two can either be an unconfirmed or a confirmed failure. An unconfirmed failure occurs when the IDG is removed from operation by a field mechanic as a suspected failure. When the unconfirmed failure enters the repair facility, it is tested, found to be fully functional, and no repair is performed. Confirmed failures require maintenance to be accomplished in the repair facility. When an IDG enters the repair facility for maintenance, there are three possible maintenance activities that can occur: the IDG is tested and no failure is evident (state 5, NFF = No Fault found); a minor repair is accomplished (state 4, C&R = Check and Repair); or a complete overhaul (state 3, OVH) is accomplished. When the failed IDG has significant damage or it has exceeded the current 14,600 FH mandatory overhaul interval, the IDG is overhauled (state 3). If the IDG does not pass the initial visual inspection and functional test, it is then disassembled and repaired. If the damage is minor and does not require complete disassembly of the unit, then a Check and Repair (state 4) is accomplished. A NFF and C&R repair is considered a "minimal" or "bad-as-old" repair [2].

The MOI is referred to in the aviation industry as a "soft-time" overhaul; this implies that the IDG is only overhauled if it has been removed from an aircraft by a reported failure and exceeds the MOI when it enters the repair facility. An operating IDG is not removed from an aircraft when it has exceeded the MOI.

An overhaul resets to zero the number of flight hours on the IDG. Since an overhaul replaces or exchanges a significant number of components in the IDG, it is treated as a "good-as-new" repair [2]. A "good-as-new" repair resets the peril rate to the value before the system goes into initial operation after an overhaul. Since each IDG in this study has been overhauled at least once, "good-as-new" refers to the condition just after an overhaul, not as purchased from the original equipment manufacturer.

After the appropriate maintenance has been performed at the repair facility, the IDG goes into inventory at one of the field stations. The IDG remains idle in inventory until it is needed to replace a failed IDG on an aircraft.

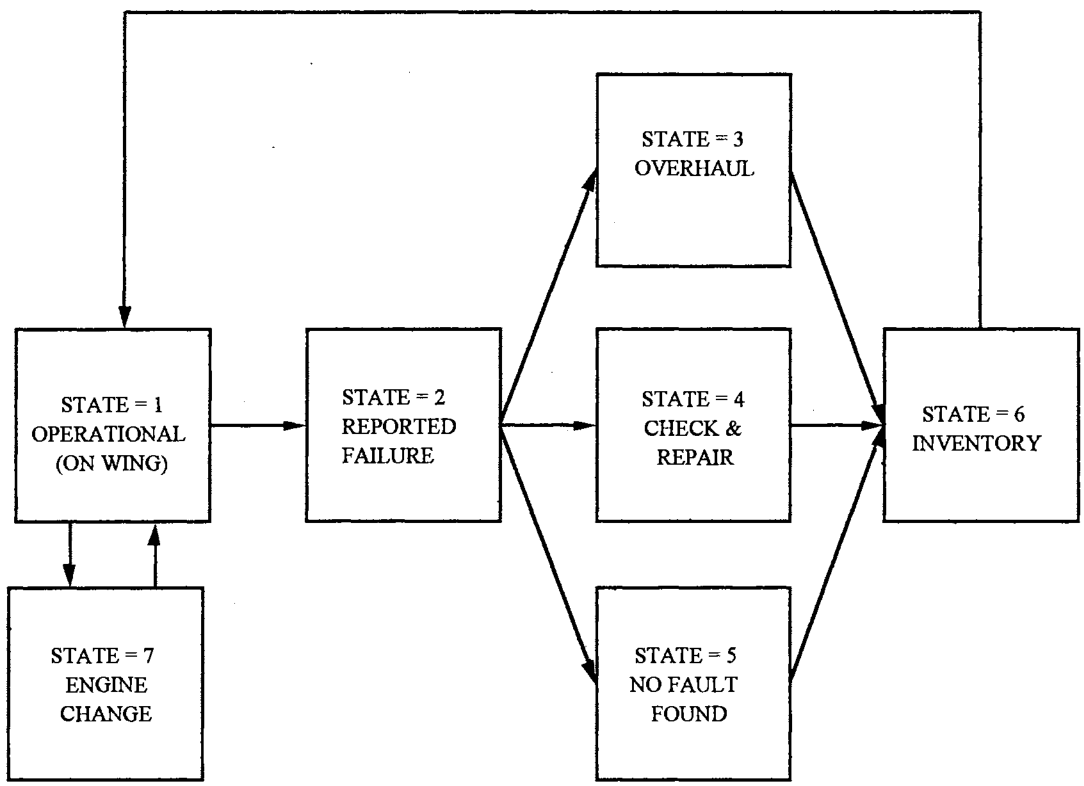

Figure 14. Block Diagram of Operational/Maintenance Model

4.2 State Transition Probabilities and Holding Times

Because the IDG is primarily a mechanical system, it experiences wear which results in an increasing peril rate with time. As a result of the system's properties changing with time, the state transition probabilities are also time dependent. Table IV gives the state-to-state transition probabilities. The probabilities are broken down into 3,000 FH intervals. The following transitions have probability of one regardless of the interval: OVH, C&R and NFF to inventory, inventory to operational, and engine change to operational. The transition probabilities are all derived from the field failure and operational data contained in Appendix C. The actual time the IDG spends in each state is determined by the state holding times.

TABLE IV

STATE-TO-STATE TRANSITION PROBABILITIES

CURRENT STATE	NEXT STATE	FLIGHT INTERVAL	TRANSITION PROBABILITY
1 OPERATIONAL	2 FAILURE	0-3000	.003915
1 OPERATIONAL	2 FAILURE	3001-6000	.002046
1 OPERATIONAL	2 FAILURE	6001-9000	.002148
1 OPERATIONAL	2 FAILURE	9001-12000	.004695
1 OPERATIONAL	2 FAILURE	12001-15000	.004802
1 OPERATIONAL	7 ENG CHANGE	0-3000	.002001
1 OPERATIONAL	7 ENG CHANGE	3001-6000	.001507
1 OPERATIONAL	7 ENG CHANGE	6001-9000	.001228
1 OPERATIONAL	7 ENG CHANGE	9001-12000	.005217
1 OPERATIONAL	7 ENG CHANGE	12001-15000	.0012
2 FAILURE	3 OVH	0-3000	.1111
2 FAILURE	3 OVH	3001-6000	.3684
2 FAILURE	3 OVH	6001-9000	.4286
2 FAILURE	3 OVH	9001-12000	.5
2 FAILURE	3 OVH	12001-15000	1
2 FAILURE	4 C&R	0-3000	.5555
2 FAILURE	4 C&R	3001-6000	.3684
2 FAILURE	4 C&R	6001-9000	.4286
2 FAILURE	4 C&R	9001-12000	.3333
2 FAILURE	4 C&R	12001-15000	0
2 FAILURE	5 NFF	0-3000	.3333
2 FAILURE	5 NFF	3001-6000	.2632
2 FAILURE	5 NFF	6001-9000	.1429
2 FAILURE	5 NFF	9001-12000	.1667
2 FAILURE	5 NFF	12001-15000	0

From the operational state to the failure state, the transition is the result of taking the product of the unconfirmed peril rate and the change in time that was used in the simulation. In the model, the change in time (smallest time increment) is one flight day; as a result of this, the failure probability is the same as the peril rate. The unconfirmed peril rate, or the transition probability from state 1 to sate 2, is not the same as shown in

Figure 7. The values obtained in Table IV are obtained by multiplying the peril rate in Figure 7 by 11.39 FH/FD. This converts the peril rate from the units of failure/FH to failure/FD.

To accurately determine the optimal maintenance program, the state holding times must also be considered. The state holding times are the number of days that the IDG spends in each state, i.e.: operational, reported failure, repair facility (states 3-5), inventory, and an engine change. In most analytical models, the mean time to repair (MTTR) is assumed to be negligible compared to the mean time between failures. In this study, the state holding times were parameterized using a Weibull distribution [33]. The Weibull density function is given by

$$f(t) = \frac{\beta t^{\beta-1}}{\theta^{\beta}} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right] \qquad 4.2.1$$

where β is the shape parameter and θ is the scale parameter. The method of Maximum Likelihood Estimation (MLE) was used to determine the shape and scale parameters for the state holding times. Dodson [33] reports that MLE is a superior parameter estimation technique as compared to probability plotting or hazard plotting. Table V shows the shape parameter, scale parameter, and the calculated statistic for each state's holding time. Field data is used to determine the Weibull parameters. In the model, the simulated state holding times are generated using the Inverse Transform Method [23]. The units for holding times are flight days [FD]. One flight day is equivalent to 11.39 FH.

Note that in Table V the state holding time for the operational state does not have any values. As opposed to the other six states, the holding times for the operational state has not been parameterized with a Weibull distribution. In the operational state, the probability of failure is dependent upon the number of flight hours since overhaul on the IDG. The MATLAB program THSIMEVL.M (Appendix E.6) uses a simulation increment of one flight day and tests for a reported failure. Once the reported failure condition is satisfied, the loop is terminated, and the program goes to the reported failure state (state 2).

The last column in Table V is the calculated statistic. The software provided with reference 33 uses a Hollander-Proschan goodness-of-fit test. Before the Weibull distribution can be used to parameterize the state holding times, the calculated Hollander-Proschan test statistic must fall within the acceptable region of the test. For a significance level of 0.05, the acceptable region is +/- 1.96. Since all the calculated statistics in Table V fall within this range, it can be concluded that the Weibull distribution with the given shape and scale parameters is an adequate model for the state holding times.

TABLE V

COSTS AND HOLDING TIMES FOR EACH STATE

STATE	COST [CU]	HOLDING TIME [FD]	HLD TIME SHAPE PAR.	HLD TIME SCALE PAR.	CALCULATED STATISTIC
1 OPERATIONAL	0	151	-	-	-
2 FAILURE	.615	12.7	.89	12	.549
3 OVERHAUL	.679	7.4	1	7.4	.307
4 CHECK & REPAIR	.456	7.4	1	7.4	.307
5 NO FAULT	.051	7.4	1	7.4	.307
6 INVENTORY	.248	49.6	1.23	53	.028
7 ENGINE CHANGE	.606	126	1.2	134	.399

4.2.1 Costs Associated with Each State

There are many costs associated with the removal of an IDG from an aircraft, e.g., labor, flight delays, flight cancellations, Minimum Equipment List (MEL) items, air interruptions, shipping time and cost, repair facility labor and material, and the time spent out-of-service in inventory. This section looks at the details associated with each of these costs. All costs are per premature removal (PR) and per year.

Cost of a Premature Removal (C_{FFR})

A premature removal is a reported failure that is not a result of a scheduled maintenance program. A premature removal consists of both confirmed and unconfirmed failures. The cost of a premature removal, C_{FFR}, breaks down as follows: The first cost is the labor, C_{LRR}, to remove the failed IDG and replace it with one from inventory. Since it requires four hours for the removal and replacement process, a flight delay or cancellation may occur. The total cost of a premature removal, C_{FFR}, is the sum of the labor cost, possible delay and/or cancellation costs.

Cost of Transit Time (C_{TFS})

After the IDG is removed, it must be shipped to the repair facility. This cost is made up of the cost of shipping and the cost of the IDG being out-of-service. The out-of-service cost is a result of the cost of capital, taxes, and storage for the unit while it is not operational on the aircraft. The total cost of transit from the aircraft to the repair station is the sum of the shipping and out-of-service costs.

Cost of Air-interrupts (C_{AI})

An Air-interrupt (AI) occurs when a critical system fails causing the aircraft to be forced into an unscheduled landing. In the best scenario, the aircraft lands at an unscheduled destination where minimal maintenance is performed, and the flight resumes with minimal delay to the passengers. In the worst case, the aircraft lands at an unscheduled airport, and another aircraft must be ferried in to pick up the passengers so they can resume their flight. This can be more costly if the second aircraft is not available until the next day, and all of the passengers and crew must be placed in a hotel for the night. In any event, the cost of an Air-interrupt is quite high. Since an Air-interrupt does not occur on each flight, the cost of an Air-interrupt, C_{AI}, is found by taking the total yearly Air-interrupt costs associated with the IDG and dividing by the number of yearly premature removals (same as the number of reported failures).

Cost of an MEL item (C_{MEL})

When an IDG fails, it can be immediately replaced, or it can be placarded as inoperative and temporarily placed on the Minimum Equipment List (MEL). The aircraft is allowed to continue revenue flight with one failed IDG. When an IDG is placarded as a result of an MEL item, the aircraft is prevented from International routing and can have a weight restriction. There is a cost associated with the limited routing capabilities of the aircraft and weight restrictions placed on the aircraft. Since an MEL item is not generated at each failure, C_{MEL} is the total MEL yearly cost for the fleet divided by the yearly number of premature removals (reported failures).

Cost of a Repair Facility Visit

A NFF repair (state 5) is the least expensive repair and is represented by C_{NFF}. The cost of a C&R (state 4) is represented by C_{CR}. The cost of the most expensive repair, an overhaul (state 3), is C_{OVH}. In each of these three states, the cost associated with the repair is a combination of labor, material, and the cost of capital for holding the equipment out-of-service.

Cost of Inventory Time (C_{INV})

After the IDG leaves the repair shop, it is held in inventory until it is required to be placed in service. This inventory cost includes the shipping cost to send the IDG back to the aircraft and the cost of out-of-service time ($\tau_{INVENTORY}$) while it waits in inventory.

Cost of an Engine Change (C_{ENG})

Since in the operational state the IDG is always associated with an aircraft engine, the IDG can be taken out-of-service by an engine failure or an engine scheduled maintenance. The costs associated with an engine change (C_{ENG}) are not resulting from maintenance costs; they are the same type of costs as incurred in the inventory state , e.g., taxes, interest on capital, storage, etc., and inventory carrying costs.

Cost of Failure (C_{FA})

When an IDG experiences a premature removal (state 2), either a confirmed or unconfirmed failure, there is a cost (C_{FA}). The cost of a failure is represented by $C_{FA} = C_{FFR} + C_{TFS} + C_{AI} + C_{MEL}$. With each failure, there are the field removal (C_{FFR}) and transit (C_{TFS}) costs. Since an Air-interrupt or MEL item is not associated with each premature removal, this cost is set equal to the average cost per premature removal.

Summary of Cost per State

Because the actual dollar costs associated with each state are proprietary information, the costs have been expressed in terms of a normalized quantity called a Cost Unit [CU]. One Cost Unit is defined as the cost associated with a scheduled removal and overhaul and is given by $C_{SR} = C_{LRR} + C_{TFS} + C_{OHV} + C_{INV} \equiv 1$ CU. A scheduled removal would occur if the maintenance program requires overhaul after a fixed number of flight hours. The costs associated with being in each state are given in Table V.

4.2.2 Illustrative Realization

To better understand how the Monte Carlo simulation works, consider what would be a typical simulated course of events for a single IDG. Assume the maintenance history is: 395-E, 3480-C, 4951-O. The simulation begins by assuming a freshly overhauled IDG is placed on an aircraft. The IDG operates for 395 FH and then is removed from the aircraft because of an engine change. The transition from operational to engine change is determined by the state-to-state transition probabilities given in Table IV. Note that this transition probability is dependent on the total number of flight hours on the IDG. The engine change state holding time (a random variable) is generated using the Weibull parameters given in Table V. Assume that the IDG was held out-of-service for 149 FD because of the engine change. The cost associated with the engine change is a constant 0.606 CU.

After the engine change, the IDG returns to the aircraft where it flies for 3480 FH. The simulation then transitions from operational to the reported failure state (2). Based on the parameters in Table V, the state holding time is 8 FD. The cost associated with the reported failure is a constant 0.615 CU. The transition to the next state (NFF, OVH or C&R) is based on the state-to-state transition probabilities given in Table IV. In this realization of the model, the next state is the Check and Repair state (4).

In the Check and Repair state, the IDG is minimally repaired at a cost of 0.456 CU. The simulated state holding time is 9 FD. After the Check and Repair, the IDG next goes in to the Inventory state (6). The simulated state holding time is 35 FD at a cost of 0.248 CU. After inventory the IDG is returned to the operational state (1).

In the operational state, the IDG flies for 4951 FH and enters the reported failure state (2). This transition is determined as previously described. In state two, the IDG is held 13 FD and 0.615 CU is added to the maintenance cost. The next state is the Overhaul state (3). Here the IDG is completely disassembled, repaired, inspected, reassembled, and tested. The cost of this overhaul is a fixed 0.679 CU. The simulated state holding time is 6 FD. Since the overhaul completely resets the number of operational hours on the IDG to zero, the simulation is terminated at this point. The next IDG is now ready to be simulated.

4.3 Results of Simulation

The current maintenance program for the IDG is to fly the IDG until failure and perform an overhaul at the repair facility when the repair cost exceeds the cost of an overhaul or when greater than 14,600 FH have elapsed since the last overhaul. This type of mandatory overhaul maintenance policy is not the same as the Type I or II maintenance policy discussed in Ascher and Feingold [2]. To optimize the IDG maintenance program, the cost per flight hour must be minimized. The cost per flight hour (CU/FH) is given by

$$CU / FH = \frac{\sum_{q=1}^{k}\sum_{i=1}^{N_s} C_{iq}}{\sum_{q=1}^{k}\sum_{i=1}^{N_s} \tau_{iq}} \qquad 4.3.1$$

where k is the total number of IDGs, N_S is the number of states the IDG passes through before overhaul, C_{iq} is the maintenance cost per state of the q-th IDG in the i-th state, e.g., C_{FA}, C_{OHV}, C_{CR}, C_{NFF}, C_{INV}, and C_{ENG}, and τ_{iq} is the operation time in each state. Note that τ_{iq} will be zero except in the operational state.

To minimize the cost per flight hour, 500 IDGs are simulated at each mandatory overhaul interval. After each 500 IDG simulations, the random number generator seed is changed; this gives 500 different simulations. A total of 1,500 simulations are conducted at each MOI. Figure 15 shows the effect on the cost per flight hour of varying the MOI. The error bars represent +/- one standard deviation for the three 500 IDG simulations. The simulated cost per flight hour (CU/FH) data compares quite well to the field data. The field CU/FH is 5.8% lower than the simulated CU/FH at 14,600 MOI. Since this is the only field CU/FH available, this is the only MOI where the two could be compared. Figure 15 also indicates that for an MOI of 12,000 FH or greater, the CU/FH is flat. This implies that high MOI results in effectively a "fly-until-failure" policy for the IDG. Above 12,000 FH MOI the Type II' policy is ineffective.

The graph in Figure 15 reveals that the cost per flight hour is minimized when the MOI or "soft-time" is set at 6,000 FH. By moving the MOI from 14,600 FH to 6,000 FH, the CU/FH can be reduced by 6.1%. The apparent "oscillations" in the CU/FH curve are due to the random nature of the Monte Carlo simulation. This effect could be minimized by increasing the simulated IDG population. Figure 16 shows the MTBF for the IDG as it depends on the MOI. The MTBF reaches a maximum when the MOI is at 6,000 FH.

There is an 4.6 % increase in the MTBF when the MOI is decreased from 14,600 FH to 6,000 FH.

The MTBF for the field data is 15.9% higher than that of the simulated data. As is the case for the CU/FH, the field and model MTBF number can only be compared at an MOI of 14,600 FH.

The goal of this simulation is to minimize the cost per flight hour of the IDG by varying the MOI rather than to replicate exactly the current field maintenance costs and times. To this end, the simulation is successful. The model is able to predict the relative change in the CU/FH and MTBF as it depends on the MOI. The 6.1% decrease in the CU/FH gives the cost justification necessary to change the maintenance program from an MOI of 14,600 FH to 6,000 FH.

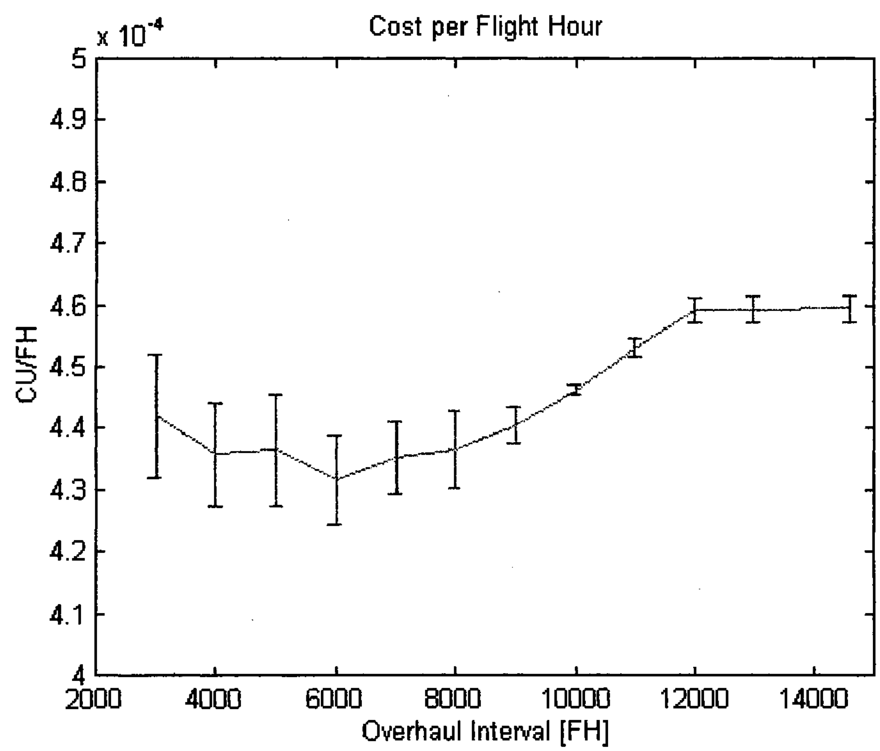

Figure 15. Cost per Flight Hour for Simulated Data

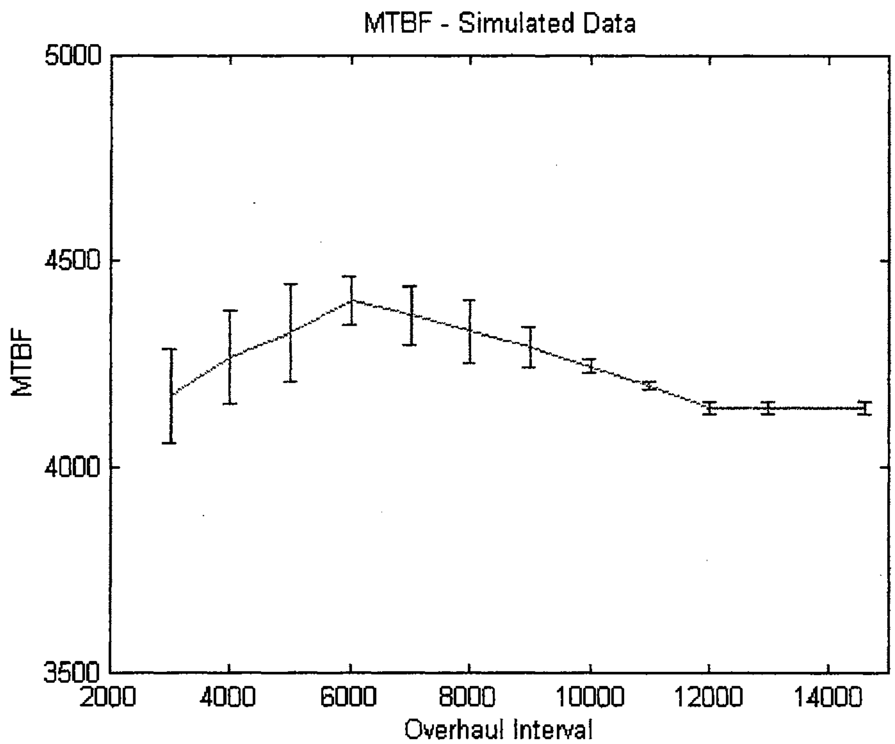

Figure 16. Mean Time Between Failures for Simulated Data

4.4 Availability

In section 2.8 of this Thesis, the availability is defined as: "the probability that the system is operating successfully at any point in time under stated conditions." The definition of availability is dependent upon the system under study. Throughout this Thesis, the word system has meant the IDG. To discuss the availability, the term system must now refer to the aircraft where the IDG is a subsystem. Since an aircraft is not held out-of-service while an IDG is being repaired, it is more appropriate to consider the effects of the IDG maintenance policy on the availability of the aircraft.

From section 2.8, the inherent availability is defined as

$$A_i = \frac{\mathrm{MTBF}}{\mathrm{MTBF} + \mathrm{MTTR}}. \qquad 4.4.1$$

Since the IDG is removed from the aircraft for both confirmed and unconfirmed removals, the MTBF is replaced by the mean time between unscheduled removal (MTBUR). The MTBUR takes into account all reasons for IDG removal except for an engine change. The effects of the engine change are not considered since the engine, not the IDG, is the primary reason for removal. In equation 4.4.1, the MTTR is now the average IDG replacement time rather than the mean time to repair. The average replacement time for the IDG is four hours. The inherent availability now becomes

$$A_i = \frac{\mathrm{MTBUR}}{\mathrm{MTBUR} + 4}. \qquad 4.4.2$$

Figure 17 illustrates the dependence of the availability on the MOI of the Type II' maintenance policy. To generate Figure 17 the MTBUR from Appendix E.9 is used in equation 4.4.2. In Figure 17, the availability peaks when the MOI is 6,000 FH. This result is consistent with the minimization of the CU/FH and maximization of the MTBF at 6,000 FH.

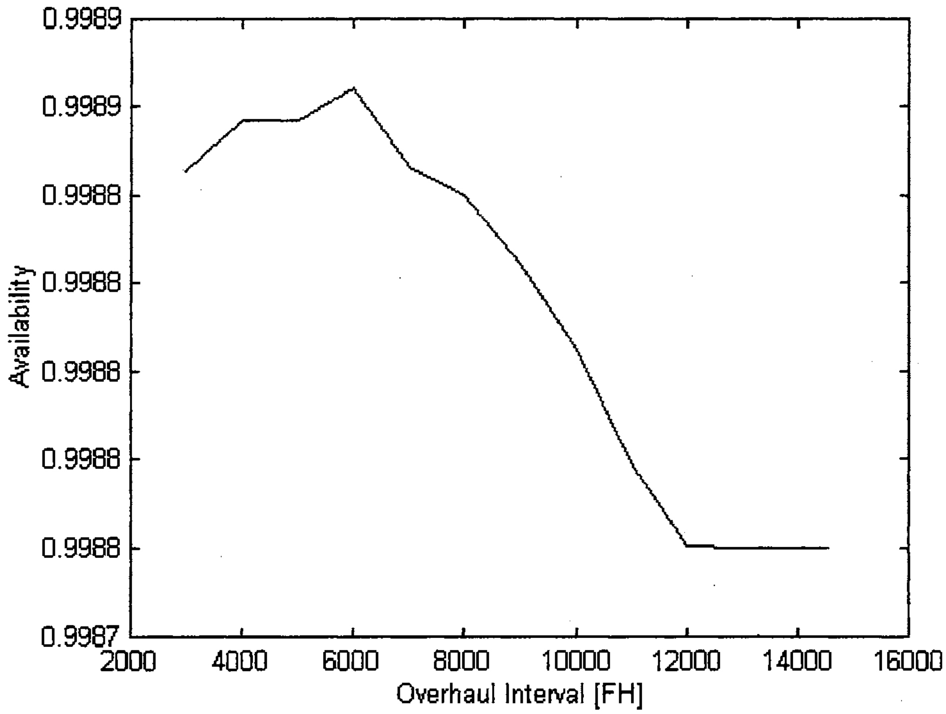

Figure 17. Availability as a Function of the Overhaul Interval

4.5 Simulation Software

All of the software written for the Monte Carlo simulation of the IDG operational and maintenance cycle is written in MATLAB Student Version 4.0 [34]. There are five separate programs that are used in the simulation. The first three programs, THMAINTF.M, THEVALF.M, and THPERIL.M are used to evaluate the field data so that it can be compared with the simulation output. In any simulation, it is necessary where possible [35], to validate the simulation output against known data. The last two programs, THSIMEVL.M and THSIMDAT.M , perform the Monte Carlo simulation and display the results respectively.

4.5.1 Description of Computer Programs

This section describes the input, output, and algorithms in each of the five programs used in the Monte Carlo simulation.

Program THMAINTF.M (Appendix E.2) has as an input data file THMAINTF.DAT (Appendix E.1). This data file has the number of flight hours from each state per IDG for field data. This file is manually produced from the IDG maintenance events data set in Appendix C. The output of program THMAINTF.M is data file THFIELDF.DAT. This data file contains flight hours, maintenance actions and the costs per state. This output file is used as an input for program THEVALF.M.

Program THEVALF.M (Appendix E.3) loads data file THFEILDF.DAT and summarizes the failure data by state and reason for removal. This program calculates the state holding times, CU/FH, MTBUR and MTBF. The output is shown in Appendix E.4. The last step in the program THEVALF.M is to convert the input file to a file format readable by program THPERIL.M. This output file is THPERILF.DAT.

Program THPERIL.M (Appendix D.1) was previously used in Chapter 3 to reduce the failure data to determine the NHPP parameters and the natural estimate of the peril rate. In this application, data file THPERILF.DAT is the input. A tabular listing of the output is shown in Appendix E.5. For the purpose of software validation, it is important to notice that the output in Appendix E.5 is identical to the output in Appendix D.8. The output shown in Appendix D.8 is a result of program THPERIL.M with input file P4OVHA.DAT. Data file P4OVHA.DAT (Appendix D.9) contains failure data for confirmed failures only. The identical outputs in Appendices E.5 and D.8 indicates that programs THMAINTF.M and THEVAL.M are written correctly and do not introduce errors in the failure data.

Program THSIMEVL.M (Appendix E.6) is the Monte Carlo simulation program that models the operational and failure states of the IDG. The inputs to the program are: state-to-state transition probabilities per 3,000 FH interval cell, random number generator seed, cost per state, Weibull parameters used to generate state holding times, MOI, and the number of IDG's in the simulation. A listing of the tabular output of program THSIMEVL.M is shown in Appendix E.7. This particular simulation uses an MOI of 1282 FD which equivalent to 14,600 FH.

Program THSIMDAT.M (Appendix E.8) combines the outputs, i.e., CU/FH, MTBF, and MTBUR, from different runs of program THSIMEVL.M. The tabular output of program THSIMDAT.M is listed in Appendix E.9. The graphical output of program THSIMDAT.M is shown in Figures 15, 16, and 17.

4.6 Summary

This chapter has presented the results of a Monte Carlo simulation of the IDG maintenance and operational cycle. The purpose of the simulation is to minimize the maintenance cost per unit flight hour. The Monte Carlo model consists of seven states. Each state has a maintenance cost, holding time, and a transition probability to the next state. It is shown that the CU/FH minimizes and the MTBUR, MTBF, and availability maximize when the mandatory overhaul interval is set at 6,000 FH.

CHAPTER V

TYPE II MAINTENANCE POLICY

For a repairable system that deteriorates with time, there is an optimal maintenance policy that will minimize the maintenance cost for the system. Before an optimal maintenance policy can be determined, the failure data must be parameterized in some way as to quantify the effects of age on the system. A commonly used [2] approach is to represent the failure data with a nonhomogeneous Poisson process (NHPP) with power law intensity function. This method to represent the failure data can be applied to systems that deteriorate, show reliability growth, and remain constant with age. Since the NHPP is a statistical model for the failure data, goodness-of-fit tests must be used to determine that the model is valid for a given statistical significance level.

By combining an estimate of the parameters used to define the NHPP with maintenance cost data, an optimal maintenance policy can be determined. The maintenance policy under study is a Type II policy [19]. To use this model, the costs of a minimal repair and a scheduled overhaul must be known.

5.1 Mathematical Model for a Type II Maintenance Policy

The Type II maintenance policy was introduced by Barlow and Hunter [19] in 1960. This policy calls for a planned replacement (overhaul) of a system after some prespecified number of system operating hours, regardless of the number of intervening failures. It is assumed that after each failure, the system is only minimally repaired; therefore, the ROCOF is unchanged by the repair. Instantaneous repair is also assumed. The Type II policy is the same as a "hard-time" replacement policy commonly used in the airline industry.

Since an overhaul occurs at times T, 2T, 3T,..., the problem becomes to select T to minimize the overall maintenance cost function. The long-run expected cost per unit time is given by [20]

$$C(T) = \frac{C_{MR}E[N(T)] + C_{SR}}{T}, \qquad 5.1.1$$

where E[N(T)] is the expected number of minimal repairs over the interval, and is given by

$$E[N(T)] = \int_0^T \rho(u)du. \qquad 5.1.2$$

C_{MR} is the cost of a minimum repair performed after a failure. This cost does not include the cost of an overhaul. C_{SR} is the cost of a scheduled overhaul. Combining the power law peril rate given by equations 2.2.1.6, 5.1.1, and 5.1.2, the long-run expected cost per unit time becomes

$$C(T) = \frac{C_{MR}\lambda T^{\beta} + C_{SR}}{T}. \qquad 5.1.3$$

To minimize C(T), set the derivative of equation 5.1.1 is set equal to zero. The resulting equation is

$$\int_0^T (\rho(T) - \rho(u))du = \frac{C_{SR}}{C_{MR}}. \qquad 5.1.4$$

This equation has a unique solution provided the peril rate is strictly increasing to ∞ as $t \to \infty$. When the peril rate is specifically in the form

$$\rho(t) = \lambda\beta t^{\beta-1}, \beta > 1 \qquad 5.1.5$$

then the optimal replacement interval T* becomes

$$T^* = \left[\frac{C_{SR}}{\lambda(\beta-1)C_{MR}}\right]^{1/\beta}. \qquad 5.1.6$$

Since there is a significant cost associated with a NFF maintenance action, the unconfirmed peril, ρ_{UC}, is used rather than the confirmed peril rate in calculation of the optimal replacement interval. The terms overhaul and replacement are considered equivalent in this application of the Type II model.

5.2 Maintenance Costs

Once the parameters have been determined that characterize the failure data, the next step is to determine the costs associated with minimal repair and a scheduled overhaul. During a typical operational and maintenance cycle for an IDG under a Type II maintenance policy, the IDG first goes through a series of operational periods each followed by a minimal repair. After the IDG accumulates a predetermined number of flight hours, it is removed from service and overhauled. The overhauled IDG then goes into inventory until it is placed back in service on an aircraft.

The cost of a scheduled removal and overhaul, C_{SR}, is determined by summing the following individual costs:

1. Cost of labor to remove the IDG and replace it with one from inventory.

2. Shipping and out-of-service costs. The out-of-service cost is a result of the cost of capital, taxes, and storage for the unit while it is not operational.

3. Cost of an overhaul. This includes the cost of labor and materials.

4. Cost of inventory. This includes shipping, storage, and out-of-service time.

Combining these four costs gives the total cost of a scheduled overhaul. Since the actual costs are proprietary information, the cost of a scheduled overhaul is defined as one cost unit, $C_{SR} \equiv 1$ CU.

The computation of the cost of minimal repair, C_{MR}, is more involved than the computation of the cost of a scheduled removal. The cost of minimal repair is broken down as follows:

$$\begin{aligned} C_{MR} &= P(NFF)(C_{LRR} + C_{TFS} + C_{NFF}) \\ &\quad + P(C\&R)(C_{FFR} + C_{TFS} + C_{AI} + C_{MEL} + C_{CR}) + C_{INV}. \end{aligned} \qquad 5.2.1$$

P(NFF) is the probability that the removal will result in a NFF maintenance action. P(C&R) is the probability that the minimal repair will result in a check and repair (C&R) maintenance action. The other costs in equation 5.2.1 have been previously defined. A C&R maintenance action results from a confirmed IDG failure. A typical C&R maintenance action would be the replacement of a leaking output shaft seal. Since this is a confirmed failure, there are additional costs incurred relating to possible aircraft flight delays and cancellations that this failure may have caused. The final term, C_{INV}, is the cost of inventory. The cost of minimal repair written in terms of cost units is C_{MR}=0.969 CU.

5.3 Optimal Replacement Interval

Now that the peril rate has been determined and the maintenance costs are known, the optimal replacement (overhaul) interval for the Type II maintenance policy can be determined. Table VI lists the optimal replacement intervals obtained by substituting the unconfirmed peril rates and cost numbers into equation 5.1.6. Notice that the optimal replacement interval is highly dependent on the amount of left-truncation of the data.

TABLE VI

REPLACEMENT INTERVALS

LEFT-TRUNCATION	**REPLACEMENT INTERVAL**
0	UNDEFINED
2000 FH	13966 FH
3000 FH	8294 FH
4000 FH	7531 FH
5000 FH	6893 FH

A second approach to determine the optimal replacement interval is to determine the long-run expected cost per unit time for the Type II policy (equation 5.1.3). Figure 18 shows the expected range for the cost function as it depends on the replacement interval. The upper curve in Figure 18 is for the 2,000 FH left-truncated data set with both C_{MR} and C_{SR} increased by 10 percent. The lower curve is for the 5,000 FH left-truncated data set with both C_{MR} and C_{SR} reduced by 10 percent. The cost parameters are varied so that when they are combined with the appropriate data set parameters, the result is the worst case for the CU/FH, both high and low. The cost information contained in Figure 18 is valuable to Engineering and Production Management because it allows the impact of the optimal Type II policy to be quantified. Since aircraft maintenance is done at regularly scheduled intervals (e.g., A, B, C checks), this allows the choice of the optimal interval to be adjusted to fit in with existing maintenance intervals.

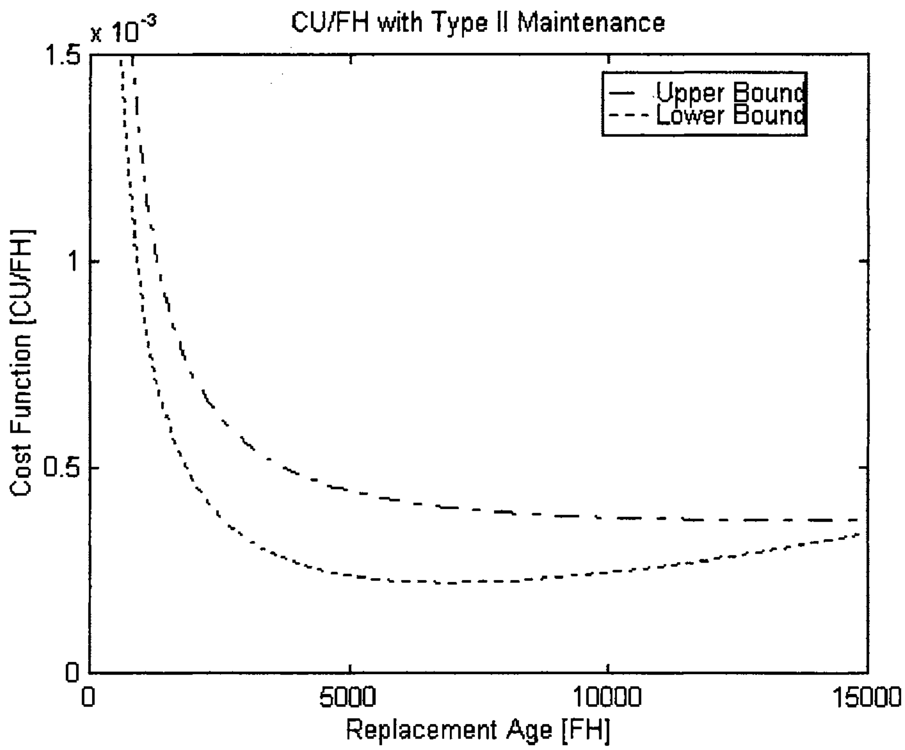

Figure 18. Dependence of the Cost Function on the Replacement Interval

5.4 Summary

To determine the optimal replacement interval for a Type II maintenance policy requires the peril rate and the maintenance costs be known. The peril rate was successfully modeled using the NHPP with power law intensity function. To remove the effects of infant mortality that occur in the IDG after an overhaul, the failure data were left-truncated. Depending on the amount of left-truncation of the failure data, different values were obtained for beta and lambda in the NHPP. Combining the NHPP model parameters with the maintenance costs for a minimal repair and a scheduled removal gives the optimal replacement interval for the Type II maintenance policy. The results of this study indicate that the optimal replacement interval is between 7,000 FH and 9,000 FH. As shown in Figure 18, there is a broad minimum over CU/FH values between 5,000 FH and 10,000 FH. This broad minimum gives management the freedom to choose the replacement interval at the most convenient and cost effective interval.

CHAPTER VI

SUMMARY AND CONCLUDING REMARKS

6.1 Summary

When investigating the failure characteristics of a repairable system, it is necessary to know if the system is deteriorating, remaining constant, or improving with time. This change in the failure characteristics with time is quantified by the peril rate. The peril rate is also called the rate of change of failures (ROCOF). These two terms should not be used interchangeably with the hazard rate. The hazard rate is a term that defines the rate of failure for non-repairable parts. If the peril rate of a system is increasing with time, the system is classified as a deteriorating system. Deteriorating systems are candidates for a preventive maintenance program. The system under study in this Thesis is an aircraft integrated drive generator (IDG). The IDG provides the primary electrical power to an aircraft.

A commonly used method for quantifying the peril rate is the nonhomogeneous Poisson process (NHPP) with a power law intensity function. The NHPP allows a simple mathematical model to be used to quantify the peril rate. The peril rate can also be quantified non-parametrically by using the natural estimate method for the peril rate. The natural estimate takes into account the number of units that fail in a given time interval, given that a certain number of units are available to fail.

Since infant mortality effects are usually present in repairable systems, the failure data may need to be left-truncated to remove these early failures. In the case of the IDG, left-truncation is used to remove the failures that occur in less than 5,000 flight hours. Left-truncation of the failure data gives NHPP parameters that indicate that the IDG is a deteriorating system. This result is consistent with the natural estimate of the peril rate. Once the peril rate is accurately parameterized, then the optimal preventive maintenance program can be developed for the IDG.

An optimal preventive maintenance program minimizes the maintenance cost while maximizing the MTBF for a system. In the airline industry, there are three types of maintenance policies used for components; these are fly-until-failure, Type II and Type II'. The fly-until-failure policy simply requires the unit or system to be replaced at failure. The Type II policy, also called a hard-time replacement policy, requires that the unit be removed from service once it has reached a specified age. At this point, the unit is routed to the repair facility where it is overhauled. An overhaul restores the unit to the fits and tolerances called for in the original equipment manufacturer's maintenance manual. The Type II' policy requires that the unit be overhauled once it has reached a given number of flight hours, and it has already been removed from the aircraft due to a reported failure. The Type II' policy is called a soft time overhaul policy in the airline industry.

To find the optimal Type II policy, an analytical approach is used. The left-truncated NHPP parameters are used with an analytical model for a Type II policy. The optimal removal interval is determined to be in the 7,000 to 9,000 flight hour region. An exact number of flight hours is not given since the optimal interval is dependent upon the values of the NHPP parameters and the costs per maintenance state. Since these quantities are not exact, their variability is taken into account.

The optimal Type II' preventive maintenance policy is found using a Monte Carlo simulation. The Monte Carlo simulation is used since an analytical approach based on the NHPP has not been developed for this model. The operational and maintenance cycle for the IDG is represented by a seven state model. The state transition probabilities and holding times are found using field data. The results of the simulation are that the optimal number of flight hours to overhaul for a failed IDG is 6,000. At the optimal mandatory overhaul interval (MOI), the MTBF and the availability are maximized, and the cost unit per flight hour is minimized.

6.2 Areas for Further Work

The first area of research that requires more work is the development of a Monte Carlo simulation for the Type II maintenance policy. This model would allow the Type II, Type II', and the fly-until-failure policies to be compared. The fly-until-failure policy can be modeled with the Type II' policy simulation by letting the MOI go to a very large number. Using the same modeling technique with the same state transition probabilities and holding times would allow a direct cost unit per flight hour comparison for the three maintenance policies.

A second area that would be interesting and useful to pursue would be to see the evolution in time of the IDG population when the maintenance policy is changed. By having either an analytical or computer model of the IDG population that is time dependent, the number of overhauls, availability, MTBF, and CU/FH could be predicated on a quarterly basis once the maintenance policy is changed. This would give valuable work load and maintenance cost planning information to the responsible management areas. It is vital to know the effects of a change in maintenance policy on future repair facility personnel and material requirements.

A third area of investigation would be to develop a NHPP model that has an intensity function that can model the entire peril rate for a system. This peril rate function would be capable of modeling the infant mortality and wear-out phases of the peril rate. The NHPP with power law intensity function has shown to be insufficient when infant mortality effects are present. Once the parameters for this model have been determined, a preventive maintenance program could be developed based on this formulation of the peril rate. This model of the peril rate could also be used to establish product warrantees.

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APPENDIXES

APPENDIX A

IDG MODES OF FAILURE AND MAINTENANCE

TRK #	HOURS SINCE OVH	HOURS SINCE LAST FAILURE	FAILURE MODE	TYPE OF MAINT.
1	10375	329	CARRIER SHAFT ASSY	OVH
2	12737	12737	CARRIER SHAFT ASSY	OVH
3	5769	5769	PUMP&MOTOR ASSYS: FIXED END	OVH
4	33	33	NFF	NFF
4	4037	4037	PUMP & MOTOR ASSY FIXED END	OVH
4	1306	1273	VARIABLE END	OVH
5	8289	4178	ROTOR BALANCE ASSY	C&R
6	7981	4970	NFF	NFF
6	6903	3892	NFF	NFF
7	12	12	NON-HARDWARE FAILURE	C&R
8	4077	4146	NFF	NFF
9	3633	851	IDG ASSY	C&R
10	3760	3760	CARRIER SHAFT ASSY	C&R
10	3849	89	NFF	NFF
11	3419	2309	CARRIER SHAFT ASSY	OVH
12	1065	1065	NON-HARDWARE FAILURE	C&R
13	9063	1216	MAIN ROTOR	OVH
13	7847	1482	STATOR HOUSING ASSY	C&R
14	3271	3271	PUMP & MOTOR ASSY FIXED END	C&R
15	6360	6360	STATOR HOUSING ASSY	C&R
16	8721	8721	NFF	NFF
17	9897	9892	CARRIER SHAFT ASSY	OVH
18	803	803	IDG ASSY	C&R
18	10946	10946	MAIN ROTOR	OVH
19	9923	6627	CHARGE PUMP	C&R
20	11995	566	NFF	NFF
20	11429	9147	STATOR HOUSING ASSY	C&R
21	9572	9572	CARRIER SHAFT ASSY	C&R
21	9847	275	PUMP & MOTOR ASSY FIXED END	OVH
22	1988	1988	NON-HARDWARE FAILURE MODES	C&R
22	6866	6866	PUMP & MOTOR ASSY FIXED END	OVH
23	8590	8590	CARRIER SHAFT ASSY	OVH
24	9502	9502	NFF	NFF
25	7865	1328	NFF	NFF
26	2680	2680	PUMP & MOTOR ASSY FIXED END	OVH
27	1218	1218	NFF	NFF
27	1223	1223	STATOR HOUSING ASSY	C&R
27	6541	6541	VARIABLE END	OVH
28	10965	5898	NFF	NFF
29	2416	1582	NFF	NFF
29	1070	236	NFF	NFF
30	2011	2011	NFF	NFF
30	6074	6074	VARIABLE END	OVH
31	8680	6661	NON-HARDWARE FAILURE MODES	C&R
32	8333	8333	MAIN ROTOR	C&R
33	4516	4516	NON-HARDWARE FAILURE MODES	C&R
33	6332	1816	NFF	NFF
34	4326	4326	PUMP & MOTOR ASSYS FIXED END	OVH
35	4731	4839	IDG ASSY	C&R

TRK #	HOURS SINCE OVH	HOURS SINCE LAST FAILURE	FAILURE MODE	TYPE OF MAINT.
35	3902	4010	ROTOR BALANCE ASSY	C&R
36	2406	2406	CARRIER SHAFT ASSY	OVH
36	1457	1457	NFF	NFF
36	2303	2301	VARIABLE END	OVH
37	1000	1000	PUMP & MOTOR ASSYS FIXED END	OVH
37	7082	4354	ROTOR BALANCE ASSY	OVH
38	1587	177	CARRIER SHAFT ASSY	OVH
39	13234	11459	NFF	NFF
40	8952	8952	CARRIER SHAFT ASSY	OVH
41	2167	2167	NFF	NFF
42	658	658	NFF	NFF
42	7560	7560	PUMP & MOTOR ASSYS - FIXED END	OVH
43	9446	3819	NFF	NFF
43	12971	7344	PUMP & MOTOR ASSY - FIXED END	OVH
44	6210	6210	VARIABLE END	OVH
45	2503	2503	CARRIER SHAFT ASSY	OVH
46	343	343	ELECTRICAL HARNESS	C&R
47	2650	390	NFF	NFF
47	2260	2260	NON-HARDWARE FAILURE MODE	C&R
48	7099	1149	NFF	NFF
48	5950	5950	NFF	NFF
49	11589	11589	ROTOR BALANCE ASSY	C&R
50	8276	347	ELECTRICAL HARNESS	C&R
50	7929	7929	NON-HARDWARE FAILURE MODES	C&R
51	5470	3425	NFF	NFF
52	8580	7080	STATOR HOUSING ASSY	C&R
53	7838	6636	STATOR HOUSING ASSY	C&R
54	7	7	STATOR HOUSING ASSY	C&R
55	12778	12778	STATOR HOUSING ASSY	OVH
56	2050	2050	NFF	NFF
57	8424	7428	IDG ASSY	OVH
58	2479	2479	ROTOR BALANCE ASSY	C&R
59	5511	5511	PUMP &MOTOR ASSY:CONTROL UNIT	C&R
60	10164	10164	VARIABLE END	OVH
61	6365	6365	CARRIER SHAFT ASSY	OVH

APPENDIX B

IDG MAINTENANCE HISTORY -DATA SET

TRK #	INITIAL INTVL	0 TH MAINT	1 ST INTVL	2 ND INTVL	3 RD INTVL	4 TH INTVL	5 TH INTVL	6 TH INTVL	7 TH INTVL	8 TH INTVL	FINAL INTVL
1	21600	O	74-E	138-N	1444-C	2015-E	3966-E	712-E	1697-C	329-O	660
2	23904	R	2943-O	178-E	9616-O	1830-N					262
3	23088	O	450-E	2842-E	90-O	4417-R	1352-O	1124-R			467
4	22104	R	1087-O	2950-O	33-E	344-R	929-O	3126-E			672
5	R	R	2130-N	1981-O	201-E	64-C	980-E	300-E	2433-C	2415-E	3371
9	R	R	306-R	401-O	2782-N	144-C	7376-O				
12	R	R	5685-O	622-N	833-C	4912-O					R
13	R	R	1482-C	854-E	362-O	595-C	2557-E				1948
14	R	R	962-R	277-O	218-E	1654-E	160-C	7265-O			
15	R	R	317-E	128-E	817-O	323-N	1956-O	1651-E	1168-C		9317
16	19320	R	3310-O	372-E	8349-E	1375-C	2956-E	737-O			
17	18496	R	2082-E	2868-O	5-N	418-E	1937-E	2587-O	1024-E	1189-N	706
18	18712	R	10946-O	803-C	3839-O						729
19	14768	R	3296-C	2534-O	931-N	2238-E	924-C	2319-O			2096
20	21016	0	344-C	9147-C	566-N	922-O					1720
21	17416	R	390-O	126-E	8411-E	645-C	275-O				3793
22	R	R	1494-E	2568-E	2804-O	1988-C	2829-O				R
23	18592	R	213-E	229-C	902-E	1451-E	4296-O				2403
24	R	R	1029-O	309-N	21-N	101-N	17-C	1413-E	6612-E	1498-O	R
26	18200	R	2680-O	1031-E	1493-E						1150
27	14696	O	5897-E	644-O	1218-E	5-C	153-C	1635-O			456
28	14328	R	1128-R	3936-E	3-C	5898-O					5239
29	13176	R	834-O	231-N	5-N	0-N	28-N	2416-E			5137
32	R	R	3609-O	2256-E	3698-N	2379-C	142-E	1825-O			R
33	9144	R	4516-C	1816-E							1048
34	10232	R	3681-O	2808-E	1515-O						581
35	12176	R	108-O	3902-C	829-C						7483
36	11894	R	1838-E	2256-O	1456-N	847-O	2406-O				626
37	10592	R	28-O	223-N	2477-C	4354-O	1000-O	5365-N			6003
40	9184	R	1087-E	1524-O	307-C	3607-E	333-C	2094-O			3075
41	9760	R	3683-E	57-E	345-O	2167-N					2916
42	10208	R	3075-O	7560-O	658-N						3611
43	6720	R	2778-O	3668-C	707-E	2793-N	3525-O				3303
44	7888	R	9590-O	90-N	3849-E	2271-O					2210
46	R	R	4507-E	1839-O	343-C	3322-E					7255
47	6408	R	828-O	1432-C	35-E	85-E	3-E	26-E	241-N		7996
48	5296	R	1868-O	5950-N	1149-N	4490-C					2346
49	6576	R	2643-O	4739-E	4207-C	2212-E					2779
50	7608	R	1012-O	4818-E	2040-E	1071-C	347-C	4663-O			
51	7096	R	929-C	1116-O	759-C	1466-E	1200-N	4374-C	2888-C		1731
52	11136	R	5-C	1120-E	3877-O						1822
53	R	R	794-E	326-O	38-C	44-C	2608-E	4028-C	2505-N		3428
54	4442	R	2095-C	151-O	4808-O	7-C	4662-E				1936
55	7016	R	4137-E	8641-O	1341-N	76-C	2021-E	507-N	288-E		535
56	8200	R	5977-O	2050-N	862-C						3964
57	8464	R	1996-C	6424-O							3258
58	R	R	296-C	119-E	16-E	1067-E	1970-O	2479-C			6321
59	0	R	1962-E	2922-N	407-N	213-C					1857
61	0	R	6354-0								205
62	15504	R	1974-O	1390-C	3173-C	1328-N	1292-E	6055-O			0
63	6584	R	8860-E	2371-O	11121-N	235-E	876-O				0

APPENDIX C

IDG MAINTENANCE EVENTS AFTER OVERHAUL - DATA SET

TRAK #	1 ST INTVL	2 ND INTVL	3 RD INTVL	4 TH INTVL	5 TH INTVL	6 TH INTVL	7 TH INTVL	8 TH INTVL	FINAL INTVL
1	74-E	138-N	1444-C	2015-E	3966-E	712-E	1697-C	329-0	
2	178-E	9616-O							
3	450-E	2842-E	90-O						
3	4417-R	1352-O							
4	2950-O								
4	33-E	344-R	929-O						
5	201-E	64-C	980-E	300-E	2433-C	2415-E			3371
9	2782-N	144-C	7376-O						
12	622-N	833-C	4192-O						
13	595-C	2557-E							1948
14	218-E	1654-E	160-C	7265-O					
15	323-N	1956-O							
15	1651-E	1168-C							9317
16	372-E	8349-E	1375-C	2956-E	737-O				
17	5-N	418-E	1937-E	2587-O					
18	803-C	3839-O							
19	931-N	2238-E	924-C	2319-O					
20	344-C	9147-C	566-N	922-O					
21	126-E	8411-E	645-C	275-O					
22	1988-C	2829-O							
24	309-N	21-N	101-N	17-C	1413-E	6612-E	1498-O		
27	1218-E	5-C	153-C	1635-O					
32	2256-E	3698-N	2379-C	142-E	1825-O				
34	2808-E	1515-O							
35	3902-C	829-C							7483
36	1456-N	847-O							
36	2406-O								
37	223-N	2477-C	4354-O						
37	1000-O								
40	307-C	3607-E	333-C	2094-O					
42	7560-O								
43	3668-C	707-E	2793-N	3525-O					
44	90-N	3849-E	2271-O						
46	343-C	3322-E							7255
47	1432-C	35-E	85-E	3-E	26-E	241-N			7996
48	5950-N	1149-N	4490-C						2346
49	4739-E	4207-C	2212-E						2779

TRAK #	1 ST INTVL	2 ND INTVL	3 RD INTVL	4 TH INTVL	5 TH INTVL	6 TH INTVL	7 TH INTVL	8 TH INTVL	FINAL INTVL
50	4818-E	2040-E	1071-C	347-C	4663-O				
51	759-C	1466-E	1200-N	4374-C	2888-C				1731
53	38-C	44-C	2608-E	4028-C	2505-N				3428
54	4808-O								
54	7-C	4662-E							1936
55	1341-N	76-C	2021-E	507-N	288-E				535
56	2050-N	862-C							3964
58	2479-C								6321
62	1390-C	3173-C	1328-N	1292-E	6055-O				
63	11121-N	235-E	876-O						

APPENDIX D

COMPUTER SOFTWARE AND DATA SETS FOR DATA REDUCTION

APPENDIX D.1

LISTING OF MATLAB PROGRAM THPERIL.M

```
% file: thperil.m, 12/23/95
whitebg
% Parameter estimation of IDG failure data.
% Data reduction per Crow(1990).
%**********************************************
% Variables:
% k = number of systems
% nmax = maximum number of intervals in a data row
% t = stop time for observation of system
% s = start time for observation of system
%**********************************************
clear variables
date  % output date
%*** Next four lines are changed for each new data set **
fid=fopen('rmcrowt1.dat','r');
load rmcrowt1.dat
fprintf(' Data file: rmcrowt1.dat\n')
ydata=rmcrowt1;  % transfer from mcperil to ydata
[k,nmax]=size(ydata);
s=ydata(:,1); s=s'; %start of interval
t=ydata(:,2); t=t'; % end of interval
%**********************************************
% Since the data is loaded in a rectangular array
% there are extra zeros appended to the end of each
% data row, the zeros must be removed
```

```
%***********************************************
x=zeros(k,nmax-2);
for kk=1:k,
   nl=0;  %sets the length of data row to zero (initially)
   for nn=3:nmax,
      if ydata(kk,nn)>0,
         x(kk,nn-2)=ydata(kk,nn);
         nl=nl+1;
      end
  n(kk)=nl;
  end
end
beta=1; lamda=1; %initial guesses
for j=1:20,
   %lamda first
   ttl=sum(n);
   bbl=sum((t.^beta)-(s.^beta));
   lamda=ttl/bbl;
   % calculate beta
   bbb1=lamda*sum(t.^beta.*log(t)-s.^beta.*log(s));
   bbb2=0;
   for kk=1:k,
      for nn=1:max(n),
         if x(kk,nn)>0,
            bbb2=bbb2+log(x(kk,nn));
         end
      end
   end
   beta=ttl/(bbb1-bbb2);
end
fprintf(' \n')
fprintf(' MLE of lamda and beta\n')
   lamda,beta
fprintf(' Minimum Start Time %g\n',min(s))
fprintf(' Maximum Time of last observation %g\n',max(t))
%***********************************************
% estimation of the peril rate
%***********************************************
%tmax=15000; % end of observations
%tmin=0; % start of observations
tmax=max(t);
tmin=min(s);
nintvs=5; % number of intervals
tauintv=(tmax-tmin)/nintvs;
for kk=1:nintvs,
```

```
   nfail(kk)=0; nop(kk)=0;
end
% count the number of units operative in each interval
j=1; % interval counter
for tau=tmin:tauintv:tmax-tauintv,
   tottj(j)=0;   % set time per interval timer to zero
   for kk=1:k,
      if s(kk)<=tau+tauintv & t(kk)>tau,
         nop(j)=nop(j)+1;   % count number operative
         % count op time in each intv
         tottj(j)=tottj(j)+(min(t(kk),tau+tauintv)-max(tau,s(kk)));
      end
   end
   j=j+1; % increment interval counter
end
j=1;
for tau=tmin:tauintv:tmax-tauintv,
   for kk=1:k, % increment unit number
      for nn=1:max(n(kk)), % increment thru failures
         if x(kk,nn)<=tau+tauintv & x(kk,nn)>tau,
            nfail(j)=nfail(j)+1;
         end
      end
   end
   j=j+1;
end
tt=tmin:tauintv:tmax;
clc
fprintf(' Time Intervals\n')
tt
fprintf(' Number that fail in each interval\n')
nfail
fprintf(' Number operational in each interval\n')
nop
fprintf(' Natural estimate of peril rate')
p=nfail./(tottj)
fprintf(' NHPP estimate of peril rate')
clf reset
axis('square')
pp(6)=p(5);  %   transfer peril rate data to new array for plot
pp(1:5)=p(1:5);
[ttstrs,pstrs]=stairs(tt,pp);  %   generates arrays for plots
pnhpp=beta*lamda*ttstrs.^(beta-1);   %NHPP peril rate
plot(ttstrs,pstrs,'k-',ttstrs(2:length(ttstrs)),pnhpp(2:length(ttstrs)),'k:')
xlabel('Time')
```

```
ylabel('Peril Rate [1/FH]')
legend('k-','Natural Estimate','k:','NHPP Estimate')
print
pause
%**********************************************
% Goodness of Fit Test using Chi-Square Distribution
%**********************************************
chi2=0;
for j=1:nintvs,  % nintvs is the same as "d" in Crow (1990)
   theta(j)=0;
   for kk=1:k,
      startintv=tmin+(j-1)*tauintv; %start of interval
      endintv=tmin+j*tauintv; %end of interval defined by j value
      if s(kk)<=endintv & t(kk)>startintv,
         % startintv is the start of observation, s(kk), if it is within
         % the interval defined by j
         if s(kk)>=startintv & s(kk)<=endintv,
            startintv=s(kk);
         end
         % endintv is the end of observation, t(kk), if it is within
         % the interval defined by j
         if t(kk)>=startintv & t(kk)<=endintv,
            endintv=t(kk);
         end
         theta(j)=theta(j)+(lamda*endintv^beta-lamda*startintv^beta);
      end
   end
   chi2=chi2+((nfail(j)-theta(j))^2)/theta(j);
end
clc
fprintf(' Chi-Square Goodness of Fit Parameter\n')
theta,chi2
%**********************************************
% Cramer-von Mises Test
%**********************************************
if sum(s) < .1, % only perform test on data that starts at 0
   % step 1
   m=0;
   for kk=1:k,
      if x(kk,n(kk))==t(kk)   % failure truncated data
         mq=n(kk)-1;
      end
      if x(kk,n(kk))<t(kk)   % time truncated
         mq=n(kk);
      end
```

```
      m=m+mq;
    end
    m
    % step 2
    for kk=1:k,
      for nn=1:nmax-2,
        y(kk,nn)=x(kk,nn)/t(kk);
      end
    end
    % step 3, unbiased estimate of beta
    blnx=0;
    for kk=1:k,
      for nn=1:n(kk),
        blnx=blnx+log(t(kk)/x(kk,nn));
      end
    end
    bbeta=(m-1)/blnx;
    %step 4, first make z array which contains all y's
    knz=0;
    for kk=1:k,
      for nn=1:n(kk),
        knz=knz+1;
        z(knz)=y(kk,nn);
      end
    end
    % now sort z from smallest to largest
    zz=sort(z);
    % step 5, calculate the parametric Cramer-von Mises statistic
    csum=0;
    for j=1:m,
      csum=csum+(zz(j)^bbeta-(2*j-1)/(2*m))^2;
    end
    fprintf(' Cramer von-Mises Test\n')
    c2m=(1/(12*m))+csum
    bbeta
end
%**********************************************
% Calculation of the instantaneous MTBF
%**********************************************
clf
mtbf=pp.^(-1);  %   MTBF is reciprocal of peril rate
[ttstrs,mtbfstrs]=stairs(tt,mtbf);
mtbfnhpp=(beta*lamda*ttstrs.^(beta-1)).^(-1);
plot(ttstrs,mtbfstrs,'k-',ttstrs(2:length(ttstrs)),mtbfnhpp(2:length(mtbfnhpp)),'k:')
xlabel('Time')
```

```
ylabel('IMTBF ')
legend('k-','Natural Estimate','k:','NHPP Estimate')
pause
%print
```

APPENDIX D.2

EXAMPLE OUTPUT FOR PROGRAM THPERIL.M

The following is the output of program THPERIL.M when the input file was RMCROWT1.DAT. The two plots that the program generates are not included.

```
EDU» thperil

ans =

23-Dec-95

 Data file: rmcrowt1.dat

 MLE of lamda and beta

lamda =

    0.4605


beta =

    0.6153

 Minimum Start Time 1e-010
 Maximum Time of last observation 200
 Time Intervals

tt =

    0.0000   40.0000   80.0000  120.0000  160.0000  200.0000

 Number that fail in each interval
```

nfail =

14 10 4 3 5

Number operational in each interval

nop =

3 3 3 3 3

Natural estimate of peril rate

p =

0.1167 0.0833 0.0333 0.0250 0.0417

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

13.3721 7.1128 5.8050 5.0912 4.6187

chi2 =

2.6531

m =

36

Cramer von-Mises Test

c2m =

0.0695

bbeta =

0.5982

APPENDIX D.3

LISTING OF DATA FILE RMCROWT1.DAT

The following is a listing of the input data set named RMCROWT1.DAT. This data originates from Table 1 on page 389 of reference 17.

```
1e-10 200 4.3 4.4 10.2 23.5 23.8 26.4 74 77.1 92.1 197.2 0 0 0 0 0
1e-10 200 .1 5.6 18.6 19.5 24.2 26.7 45.1 45.8 75.7 79.7 98.6 120.1 161.8 180.6 190.8
1e-10 200 8.4 32.5 44.7 48.4 50.6 73.6 98.7 112.2 129.8 136 195.8 0 0 0 0
```

APPENDIX D.4

OUTPUT OF PROGRAM THPERIL.M

WITH INPUT P4OVHUC.DAT

```
EDU» thperil

ans =

6-Feb-96

 Data file: p4ovhuc.dat

 MLE of lamda and beta

lamda =

    0.0018


beta =

    0.7928

 Minimum Start Time 1e-010
 Maximum Time of last observation 13937
 Time Intervals
```

tt =

0 3000 6000 9000 12000 15000

Number that fail in each interval

nfail =

45 19 14 18 4

Number operational in each interval

nop =

45 40 31 22 10

Natural estimate of peril rate
p =

1.0e-003 *

0.3437 0.1796 0.1886 0.4122 0.4215

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

45.2735 26.7538 16.8111 9.2427 1.9188

chi2 =

13.2735

m =

69

Cramer von-Mises Test

c2m =

0.0350

bbeta =

0.5909

APPENDIX D.5

OUTPUT OF PROGRAM THPERIL.M1

WITH INPUT P4OVHUC.DAT

```
EDU» thperil

ans =

6-Feb-96

 Data file: p4ovhuc.dat

 MLE of lamda and beta

lamda =

   0.0018

beta =

   0.7928

 Minimum Start Time 1e-010
 Maximum Time of last observation 13937
 Time Intervals

tt =

  1.0e+004 *

   0.0000    0.2787    0.5575    0.8362    1.1150    1.3937
```

Number that fail in each interval

nfail =

41 20 16 18 5

Number operational in each interval

nop =

45 41 31 23 10

Natural estimate of peril rate
p =

1.0e-003 *

0.3353 0.1975 0.2194 0.3626 0.2779

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

42.8916 25.9978 16.7818 10.6563 3.6724

chi2 =

7.0443

m =

69

Cramer von-Mises Test

c2m =

0.0350

bbeta =

0.5909

APPENDIX D.6

LISTING OF DATA FILE P4OVHUC.DAT

This data set contains all unconfirmed failures with no truncation of the data.

```
1E-10 10375 212 1656 10046 10375 0 0
1E-10 9794 9794 0 0 0 0 0
1E-10 3382 3382 0 0 0 0 0
1E-10 2950 2950 0 0 0 0 0
1E-10 9764 265 3978 0 0 0 0
1E-10 10302 2782 2926 10302 0 0 0
1E-10 6367 622 1455 6367 0 0 0
1E-10 5100 595 0 0 0 0 0
1E-10 9297 2032 9297 0 0 0 0
1E-10 2279 323 2279 0 0 0 0
1E-10 12136 2819 0 0 0 0 0
1E-10 13789 10096 13789 0 0 0 0
1E-10 4947 5 4947 0 0 0 0
1E-10 4642 803 4642 0 0 0 0
1E-10 6412 913 4093 6412 0 0 0
1E-10 10979 344 9491 10057 10979 0 0
1E-10 9457 9182 9457 0 0 0 0
1E-10 4817 1988 4817 0 0 0 0
1E-10 9971 309 330 431 448 9971 0
1E-10 3011 1223 1376 3011 0 0 0
1E-10 10300 5954 8333 10300 0 0 0
1E-10 4323 4323 0 0 0 0 0
1E-10 12214 3902 4731 0 0 0 0
1E-10 2303 1456 2303 0 0 0 0
1E-10 2406 2406 0 0 0 0 0
1E-10 7054 223 2700 7054 0 0 0
1E-10 1000 1000 0 0 0 0 0
1E-10 6341 307 4247 6341 0 0 0
1E-10 7560 7560 0 0 0 0 0
1E-10 10693 3668 7168 10693 0 0 0
1E-10 6210 90 6210 0 0 0 0
1E-10 10920 343 0 0 0 0 0
1E-10 9818 1432 1822 0 0 0 0
1E-10 13935 5950 7099 11589 0 0 0
1E-10 13937 8946 0 0 0 0 0
1E-10 12939 7929 8276 12939 0 0 0
```

1E-10 12418 759 3425 7799 10687 0 0
1E-10 12651 38 82 6718 9223 0 0
1E-10 4808 4808 0 0 0 0 0
1E-10 6605 7 0 0 0 0 0
1E-10 4768 1341 1417 3945 0 0 0
1E-10 6876 2050 2912 0 0 0 0
1E-10 8800 2479 0 0 0 0 0
1E-10 13238 1390 4563 5891 13238 0 0
1E-10 12232 11121 12232 0 0 0 0

APPENDIX D.7

OUTPUT OF PROGRAM THPERIL.M

WITH INPUT P4OVHA.DAT

```
EDU» thperil

ans =

6-Feb-96

 Data file: p4ovha.dat

 MLE of lamda and beta

lamda =

  3.7760e-004


beta =

   0.9335

 Minimum Start Time 1e-010
 Maximum Time of last observation 13937
 Time Intervals

tt =

         0      3000      6000      9000     12000     15000
```

Number that fail in each interval

nfail =

30 14 12 15 4

Number operational in each interval

nop =

45 40 31 22 10

Natural estimate of peril rate
p =

1.0e-003 *

0.2291 0.1323 0.1617 0.3435 0.4215

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

29.0648 21.3588 14.4654 8.3278 1.7833

chi2 =

11.0870

m =

44

Cramer von-Mises Test

c2m =

0.0413

bbeta =

0.6217

APPENDIX D.8

OUTPUT OF PROGRAM THPERIL.M

WITH INPUT P4OVHA.DAT

```
EDU» thperil

ans =

6-Feb-96

 Data file: p4ovha.dat

 MLE of lamda and beta

lamda =

  3.7760e-004


beta =

   0.9335

 Minimum Start Time 1e-010
 Maximum Time of last observation 13937
 Time Intervals

tt =

  1.0e+004 *

   0.0000    0.2787    0.5575    0.8362    1.1150    1.3937

 Number that fail in each interval

nfail =

   26    18    11    15     5
```

Number operational in each interval

nop =

45 41 31 23 10

Natural estimate of peril rate
p =

1.0e-003 *

0.2126 0.1777 0.1508 0.3022 0.2779

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

27.2674 20.5457 14.2865 9.5089 3.3915

chi2 =

5.0643

m =

44

Cramer von-Mises Test

c2m =

0.0413

bbeta =

0.6217

APPENDIX D.9

LISTING OF DATA FILE P4OVHA.DAT

This data set contains confirmed failures only. NFF failures are not included in this data set.

```
1E-10 10375 1656 10046 10375 0 0 0
1E-10 9794 9794 0 0 0 0 0
1E-10 3382 3382 0 0 0 0 0
1E-10 2950 2950 0 0 0 0 0
1E-10 9764 265 3978 0 0 0 0
1E-10 10302 2926 10302 0 0 0 0
1E-10 6367 1455 6367 0 0 0 0
1E-10 5100 595 0 0 0 0 0
1E-10 9297 2032 9297 0 0 0 0
1E-10 2279 2279 0 0 0 0 0
1E-10 12136 2819 0 0 0 0 0
1E-10 13789 10096 13789 0 0 0 0
1E-10 4947 4947 0 0 0 0 0
1E-10 4642 803 4642 0 0 0 0
1E-10 6412 4093 6412 0 0 0 0
1E-10 10979 344 9491 10979 0 0 0
1E-10 9457 9182 9457 0 0 0 0
1E-10 4817 1988 4817 0 0 0 0
1E-10 9971 448 9971 0 0 0 0
1E-10 3011 1223 1376 3011 0 0 0
1E-10 10300 8333 10300 0 0 0 0
1E-10 4323 4323 0 0 0 0 0
1E-10 12214 3902 4731 0 0 0 0
1E-10 2303 2303 0 0 0 0 0
1E-10 2406 2406 0 0 0 0 0
1E-10 7054 2700 7054 0 0 0 0
1E-10 1000 1000 0 0 0 0 0
1E-10 6341 307 4247 6341 0 0 0
1E-10 7560 7560 0 0 0 0 0
1E-10 10693 3668 10693 0 0 0 0
1E-10 6210 6210 0 0 0 0 0
1E-10 10920 343 0 0 0 0 0
1E-10 9818 1432 0 0 0 0 0
1E-10 13935 11589 0 0 0 0 0
1E-10 13937 8946 0 0 0 0 0
```

```
1E-10 12939 7929 8276 12939 0 0 0
1E-10 12418 759 7799 10687 0 0 0
1E-10 12651 38 82 6718 0 0 0
1E-10 4808 4808 0 0 0 0 0
1E-10 6605 7 0 0 0 0 0
1E-10 4768 1417 0 0 0 0 0
1E-10 6876 2912 0 0 0 0 0
1E-10 8800 2479 0 0 0 0 0
1E-10 13238 1390 4563 13238 0 0 0
1E-10 12232 12232 0 0 0 0 0
```

APPENDIX D.10

OUTPUT OF PROGRAM THPERIL.M

WITH INPUT P4OVH2KU.DAT

```
EDU» thperil

ans =

7-Feb-96

 Data file: p4ovh2ku.dat

 MLE of lamda and beta

lamda =

  2.0783e-005


beta =

   1.2679

 Minimum Start Time 2000
 Maximum Time of last observation 13937
 Time Intervals

tt =
```

1.0e+004 *

0.2000 0.4387 0.6775 0.9162 1.1550 1.3937

Number that fail in each interval

nfail =

22 14 9 17 5

Number operational in each interval

nop =

40 33 24 20 10

Natural estimate of peril rate
p =

1.0e-003 *

0.2575 0.2086 0.1767 0.5123 0.3573

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

19.4013 17.7684 14.8716 10.3608 4.6078

chi2 =

7.7534

APPENDIX D.11

LISTING OF DATA FILE P4OVH2KU.DAT

2000 10375 10046 10375 0 0 0 0
2000 9794 9794 0 0 0 0 0
2000 3382 3382 0 0 0 0 0
2000 2950 2950 0 0 0 0 0

2000 9764 3978 0 0 0 0 0
2000 10302 2782 2926 10302 0 0 0
2000 6367 6367 0 0 0 0 0
2000 9297 2032 9297 0 0 0 0
2000 2279 2279 0 0 0 0 0
2000 12136 2819 0 0 0 0 0
2000 13789 10096 13789 0 0 0 0
2000 4947 4947 0 0 0 0 0
2000 4642 4642 0 0 0 0 0
2000 6412 4093 6412 0 0 0 0
2000 10979 9491 10057 10979 0 0 0
2000 9457 9182 9457 0 0 0 0
2000 4817 4817 0 0 0 0 0
2000 9971 9971 0 0 0 0 0
2000 3011 3011 0 0 0 0 0
2000 10300 5954 8333 10300 0 0 0
2000 4323 4323 0 0 0 0 0
2000 12214 3902 4731 0 0 0 0
2000 2303 2303 0 0 0 0 0
2000 2406 2406 0 0 0 0 0
2000 7054 2700 7054 0 0 0 0
2000 6341 4247 6341 0 0 0 0
2000 7560 7560 0 0 0 0 0
2000 10693 3668 7168 10693 0 0 0
2000 6210 6210 0 0 0 0 0
2000 13935 5950 7099 11589 0 0 0
2000 13937 8946 0 0 0 0 0
2000 12939 7929 8276 12939 0 0 0
2000 12418 3425 7799 10687 0 0 0
2000 12651 6718 9223 0 0 0 0
2000 4808 4808 0 0 0 0 0
2000 4768 3945 0 0 0 0 0
2000 6876 2050 2912 0 0 0 0
2000 8800 2479 0 0 0 0 0
2000 13238 4563 5891 13238 0 0 0
2000 12232 11121 12232 0 0 0 0

APPENDIX D.12

OUTPUT OF PROGRAM THPERIL.M

WITH INPUT DATA P4OVH5KU.DAT

```
EDU» thperil

ans =

8-Feb-96

 Data file: p4ovh5ku.dat

 MLE of lamda and beta

lamda =

  2.3200e-010


beta =

   2.4656

 Minimum Start Time 5000
 Maximum Time of last observation 13937
 Time Intervals

tt =

  1.0e+004 *

   0.5000    0.6787    0.8575    1.0362    1.2150    1.3937

 Number that fail in each interval

nfail =

    8    8   13    6    4

 Number operational in each interval

nop =

   23   19   17   11    8

 Natural estimate of peril rate
p =
```

1.0e-003 *

0.2036 0.2546 0.4756 0.3932 0.5036

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

7.5084 8.8808 10.4384 7.5316 4.7866

chi2 =

1.1889

APPENDIX D.13

LISTING OF DATA FILE P4OVH5KU.DAT

```
5000 10375 10046 10375 0 0 0 0
5000 9794 9794 0 0 0 0 0
5000 10302 10302 0 0 0 0 0
5000 6367 6367 0 0 0 0 0
5000 9297 9297 0 0 0 0 0
5000 13789 10096 13789 0 0 0 0
5000 6412 6412 0 0 0 0 0
5000 10979 9491 10057 10979 0 0 0
5000 9457 9182 9457 0 0 0 0
5000 9971 9971 0 0 0 0 0
5000 10300 5954 8333 10300 0 0 0
5000 7054 7054 0 0 0 0 0
5000 6341 6341 0 0 0 0 0
5000 7560 7560 0 0 0 0 0
5000 10693 7168 10693 0 0 0 0
5000 6210 6210 0 0 0 0 0
5000 13935 5950 7099 11589 0 0 0
5000 13937 8946 0 0 0 0 0
5000 12939 7929 8276 12939 0 0 0
5000 12418 7799 10687 0 0 0 0
5000 12651 6718 9223 0 0 0 0
5000 13238 5891 13238 0 0 0 0
5000 12232 11121 12232 0 0 0 0
```

APPENDIX D.14

OUTPUT OF PROGRAM THPERIL.M

WITH DATA FILE P4FAIL.DAT

```
EDU» thperil

ans =

16-Jan-96

 Data file: p4fail.dat

 MLE of lamda and beta

lamda =

  8.1083e-005


beta =

   1.0857

 Minimum Start Time 1e-010
 Maximum Time of last observation 36419
 Time Intervals

tt =

  1.0e+004 *

   0.0000    0.7284    1.4568    2.1851    2.9135    3.6419

 Number that fail in each interval

nfail =

    6   23   34   18   11
```

Number operational in each interval

nop =

9 23 32 21 11

Natural estimate of peril rate
p =

1.0e-003 *

0.2820 0.1990 0.2033 0.1648 0.2751

NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

3.7812 22.6576 34.1162 22.9214 8.5578

chi2 =

3.0612

EDU»

APPENDIX D.15

LISTING OF DATA FILE P4FAIL.DAT

21600 32635 23256 31646 31975 0 0 0
23904 35790 33698 0 0 0 0 0
23088 33830 26470 32239 0 0 0 0
22104 31245 23191 26141 27447 0 0 0
19320 36419 22630 32726 36419 0 0 0
18496 31312 23446 28393 0 0 0 0
18712 35029 29658 30461 34300 0 0 0
14768 29106 18064 20598 24691 27010 0 0
21016 33715 21360 30507 31995 0 0 0
17416 31056 17806 26988 27263 0 0 0
18592 28086 19034 25683 0 0 0 0
18200 24554 20880 0 0 0 0 0

14696 24704 14696 21237 22460 22613 24248 0
14328 29404 18267 24165 0 0 0 0
13176 21827 14010 0 0 0 0 0
9144 16524 13660 0 0 0 0 0
10232 18817 13913 18236 0 0 0 0
12176 24498 12284 16186 17015 0 0 0
11894 21413 16078 18381 20787 0 0 0
10592 30042 10620 13320 17674 18674 0 0
9184 21211 11795 12102 16042 18136 0 0
9760 18928 13845 0 0 0 0 0
10208 25112 13283 20843 0 0 0 0
6720 23494 9498 13166 20191 0 0 0
7888 25898 17478 23688 0 0 0 0
6408 17054 7236 8668 0 0 0 0
5296 21099 7164 18753 0 0 0 0
6576 23156 9219 18165 0 0 0 0
7608 21559 8620 16549 16896 21559 0 0
7096 21559 8025 9141 9900 16940 19828 0
11136 17960 11141 16138 0 0 0 0
4442 18101 6537 6688 11496 11503 0 0
7016 22578 19794 21211 0 0 0 0
8200 21053 14177 17089 0 0 0 0
8464 20142 10460 16884 0 0 0 0
1e-10 7361 5504 0 0 0 0 0
1e-10 6559 6354 0 0 0 0 0
15504 30716 17478 18868 22041 30716 0 0
6584 30047 17815 30047 0 0 0 0

APPENDIX E

COMPUTER SOFTWARE AND DATA SETS FOR MAINTENANCE POLICIES

APPENDIX E.1

LISTING OF DATA FILE THMAINTF.DAT

1	74	7
1	138	5
1	1444	4
1	2015	7
1	3966	7
1	712	7
1	1697	4
1	329	3
2	178	7
2	9616	3
3	450	7
3	2842	7
3	90	3
4	2950	3
5	201	7
5	64	4
5	980	7
5	300	7
5	2433	4
5	2415	7
5	3371	1
6	2782	5
6	144	4
6	7376	3
7	622	5
7	833	4

7	4912	3
8	595	4
8	2557	7
8	1948	1
9	218	7
9	1654	7
9	160	4
9	7265	3
10	323	5
10	1956	3
11	1651	7
11	1168	4
11	9317	1
12	372	7
12	8349	7
12	1375	4
12	2956	7
12	737	3
13	5	5
13	418	7
13	1937	7
13	2587	3
14	803	4
14	3839	3
15	931	5
15	2238	7
15	924	4
15	2319	3
16	344	4
16	9147	4
16	566	5
16	922	3
17	126	7
17	8411	7
17	645	4
17	275	3
18	1988	4
18	2829	3
19	309	5
19	21	5
19	101	5
19	17	4
19	1413	7
19	6612	7
19	1498	3

20	1218	7
20	5	4
20	153	4
20	1635	3
21	2256	7
21	3698	5
21	2379	4
21	142	7
21	1825	3
22	2808	7
22	1515	3
23	3902	4
23	829	4
23	7483	1
24	1456	5
24	847	3
25	2406	3
26	223	5
26	2477	4
26	4354	3
27	1000	3
28	307	4
28	3607	7
28	333	4
28	2094	3
29	7560	3
30	3668	4
30	707	7
30	2793	5
30	3525	3
31	90	5
31	3849	7
31	2271	3
32	343	4
32	3322	7
32	7255	1
33	1432	4
33	35	7
33	85	7
33	3	7
33	26	7
33	241	5
33	7996	1
34	5950	5
34	1149	5

34	4490	4
34	2346	1
35	4739	7
35	4207	4
35	2212	7
35	2779	1
36	4818	7
36	2040	7
36	1071	4
36	347	4
36	4663	3
37	759	4
37	1466	7
37	1200	5
37	4374	4
37	2888	4
37	1731	1
38	38	4
38	44	4
38	2608	7
38	4028	4
38	2505	5
38	3428	1
39	4808	3
40	7	4
40	4662	7
40	1936	1
41	1341	5
41	76	4
41	2021	7
41	507	5
41	288	7
41	535	1
42	2050	5
42	862	4
42	3964	1
43	2479	4
43	6321	1
44	1390	4
44	3173	4
44	1328	5
44	1292	7
44	6055	3
45	11121	5
45	235	7

45 876 3

APPENDIX E.2

LISTING OF PROGRAM THMAINTF.M

```
%thmaintf.m, 2/3/96
% Program converts field IDG failure data to a format
% ready for use by program theval.m
% tsov  Time since overhaul, reset with each overhaul [days]
% tcal  Calendar time, an overhaul does not reset [days]
% all time is in days, 1 day = 11.39 FH
cfa=.6146;    % cost of a field failure in cost units [CU]
covh=.679;  % cost of an IDG overhaul [CU]
ccr=.456;    % cost of a Check and Repair [CU]
cnff=.051;  % cost of a No Fault Found [CU]
cinv=.248;  % cost of time in inventory [CU]
ceng=.6065;  % cost of an engine change [CU]
copr=0;       % cost of operational state
tfail=12.7;  % number of days in failure state
trepair=7.4;  % number of days in repair facility
tinv=49.6;  % number of days in inventory
teng=126;  % number of days in engine change
% input field data file
fid=fopen('thmaintf.dat','r');
load thmaintf.dat
mcmaint=thmaintf
cprint=0;
[nrows ncols]=size(mcmaint)
for k=1:nrows,
  trk=mcmaint(k,1);  % tracking number
  timefh=mcmaint(k,2);  % flight hours in operational state
  state=mcmaint(k,3);  % state unit is in
  if state==1  % operational state ***************
    cprint=cprint+1;
    tcal=timefh/11.39;
    tsov=timefh/11.39;
    mcfield(cprint,1:5)=[trk 1 tsov tcal copr];
  elseif state==3  % overhaul state **************
    % operational state
    cprint=cprint+1;
    tcal=timefh/11.39;  % increment calendar time [days]
```

```
    tsov=timefh/11.39;  % increment tsov [days]
    mcfield(cprint,1:5)=[trk 1 tsov tcal copr];
    % failure state
    cprint=cprint+1;
    tcal=tfail;
    tsov=0;
    mcfield(cprint,1:5)=[trk 2 tsov tcal cfa];
    % overhaul
    tcal=trepair;
    cprint=cprint+1;
    mcfield(cprint,1:5)=[trk 3 0 tcal covh];
    % inventory state
     tcal=tinv;
    cprint=cprint+1;
    mcfield(cprint,1:5)=[trk 6 0 tcal cinv];
  elseif state==4  % C&R state *********************
    % operational state
    cprint=cprint+1;
    tcal=timefh/11.39;  % increment calendar time [days]
    tsov=timefh/11.39;  % increment tsov [days]
    mcfield(cprint,1:5)=[trk 1 tsov tcal copr];
    % failure state
    cprint=cprint+1;
    tcal=tfail;
    tsov=0;
    mcfield(cprint,1:5)=[trk 2 tsov tcal cfa];
    % C&R state
    tcal=trepair;
    cprint=cprint+1;
    mcfield(cprint,1:5)=[trk 4 0 tcal ccr];
    % inventory state
    tcal=tinv;
    cprint=cprint+1;
    mcfield(cprint,1:5)=[trk 6 0 tcal cinv];
  elseif state==5  % NFF state ******************
    % operational state
    cprint=cprint+1;
    tcal=timefh/11.39;  % increment calendar time [days]
    tsov=timefh/11.39;  % increment tsov [days]
    mcfield(cprint,1:5)=[trk 1 tsov tcal copr];
    % failure state
    cprint=cprint+1;
    tcal=tfail;
    tsov=0;
    mcfield(cprint,1:5)=[trk 2 tsov tcal cfa];
```

```
    % NFF state
    tcal=trepair;
    cprint=cprint+1;
    mcfield(cprint,1:5)=[trk 5 0 tcal cnff];
    % inventory state
     tcal=tinv;
    cprint=cprint+1;
    mcfield(cprint,1:5)=[trk 6 0 tcal cinv];
  elseif state==7   % engine change ******************
    % operational state
    cprint=cprint+1;
    tcal=timefh/11.39;  % increment calendar time [days]
    tsov=timefh/11.39;  % increment tsov [days]
    mcfield(cprint,1:5)=[trk 1 tsov tcal copr];
    % engine change state
    cprint=cprint+1;
    tcal=teng;
    tsov=0;
    mcfield(cprint,1:5)=[trk 7 tsov tcal ceng];
  end
end
mcfield
% output data file to disk
fid=fopen('thfieldf.dat','w')
save 'thfieldf.dat' mcfield -ascii
status=fclose(fid);
```

APPENDIX E.3

LISTING OF PROGRAM THEVALF.M

```
% thevalf.m, 2/3/96
% This program takes the data generated by program
% thmaintf.m and calculates statistics
load thfieldf.dat
dat=thfieldf;
[nrows,ncols]=size(dat);  % determine size of data file
% take data from dat and puts in individual arrays
trk=dat(:,1);   % tracking number from column 1
state=dat(:,2);  % state from column 2
dtsov=dat(:,3);  % incremental change in time since ovh - col 3
dtcal=dat(:,4);  % incremental change in calendar time - col 4
```

```
cost=dat(:,5);  % cost of state - column 5
s=date
% Determine the percent of NFF, C&R, and OVH failures
novh=0; nnff=0; nccr=0; neng=0;
for j=1:length(state)
  if state(j)==3  % number of overhauls
    novh=novh+1;
  elseif state(j)==4  % number of C&R
    nccr=nccr+1;
  elseif state(j)==5  % number of NFF
    nnff=nnff+1;
  elseif state(j)==7  % number of eng changes
    neng=neng+1;
  end
end
nfails=novh+nccr+nnff;  % total number of failures
clc
fprintf(' Summary - Types of Repairs\n')
fprintf('\n')
fprintf(' STATE     # EVENTS      PERCENT\n')
fprintf(' -----     --------      -------\n')
fprintf('  OVH         %d           %3.1f\n',novh,(novh/nfails)*100)
fprintf('  C&R         %d           %3.1f\n',nccr,(nccr/nfails)*100)
fprintf('  NFF         %d           %3.1f\n',nnff,(nnff/nfails)*100)
% Determine percent types of removals, ie, eng chg or failure
pengchg=(neng/(neng+nfails))*100;  % percent eng changes
pfails=(nfails/(neng+nfails))*100;  % percent failures
fprintf('\n')
fprintf(' Summary - Types of Removals\n')
fprintf('\n')
fprintf(' REASON FOR\n')
fprintf(' REMOVAL   # EVENTS      PERCENT\n')
fprintf(' -------   --------      -------\n')
fprintf(' ENG CHG     %d           %3.1f\n',neng,pengchg)
fprintf(' FAILURE     %d           %3.1f\n',nfails,pfails)
% Average times in each state
nopr=0; nfail=0; novh=0; nccr=0; nnff=0; ninv=0; neng=0;
for j=1:length(state)
  if state(j)==1  % operation
    topr(nopr+1)=dtcal(j);  % transfers cal time data to array
    nopr=nopr+1;  % increment op state counter
  elseif state(j)==2  % failure state
   tfail(nfail+1)=dtcal(j);  % transfers cal times to array
   nfail=nfail+1;  % counts the # of times in failure state
  elseif state(j)==3  % overhaul
```

```
    tovh(novh+1)=dtcal(j);
    novh=novh+1;
   elseif state(j)==4  % C&R
    tccr(nccr+1)=dtcal(j);
    nccr=nccr+1;
   elseif state(j)==5  % NFF
    tnff(nnff+1)=dtcal(j);
    nnff=nnff+1;
   elseif state(j)==6  % inventory
    tinv(ninv+1)=dtcal(j);
    ninv=ninv+1;  % sums number of visits to inventory
   elseif state(j)==7  % engine change
    teng(neng+1)=dtcal(j);
    neng=neng+1;
   end
end
% calculate and display averages for each state
fprintf('\n')
fprintf('       Holding Times for each State\n')
fprintf('\n')
fprintf(' STATE       AVERAGE[DAYS]  VARIANCE[DAYS]\n')
fprintf('------       -------------  --------------\n')
fprintf(' OPERATION   %4.1f           %8.1f\n',mean(topr),std(topr)^2)
fprintf(' FAILURE     %4.1f           %8.1f\n',mean(tfail),std(tfail)^2)
fprintf(' OVERHAUL    %4.1f           %8.1f\n',mean(tovh),std(tovh)^2)
fprintf(' C&R         %4.1f           %8.1f\n',mean(tccr),std(tccr)^2)
fprintf(' NFF         %4.1f           %8.1f\n',mean(tnff),std(tnff)^2)
fprintf(' INVENTORY   %4.1f           %8.1f\n',mean(tinv),std(tinv)^2)
fprintf(' ENG CHG     %4.1f           %8.1f\n',mean(teng),std(teng)^2)
% Cost per flight hours
fprintf('\n')
fprintf(' Cost per flight hour= %g [CU/FH]\n',sum(cost)/(sum(topr)*11.39))
% Calculation of MTBUR - Mean Time Between Unscheduled Removals
% An unscheduled removal is for failure only
% MTBUR = (total flight hours)/(total failures)
mtbur=(sum(topr)*11.39)/nfail;
fprintf('\n')
fprintf(' MTBUR= %g [FH]\n',mtbur)
% MTBF=(total flight hours)/(total confirmed failures)
mtbf=(sum(topr)*11.39)/(novh+nccr);
fprintf('\n')
fprintf(' MTBF = %g [FH]\n',mtbf)
% Number of units total
fprintf('\n')
fprintf(' Number of units= %g\n',max(trk))
```

```
%************************************************
% This section converts the data into a format so
% it can be read by program mcperilf.m. This program
% calculates the peril rate.
%************************************************
% Calculate the cumulative time since OVH (cumtsov)
cumtsov=zeros(max(trk),6);
for ltrk=1:max(trk)  % loop thru each trk #
  ntrk=0;  % counts numer of times for each trk #
  nfail=0;  % counts number of failures per trk #
  tsf=0;  % time since failure
  for ldat=1:length(state)  % loop thru each row
    if trk(ldat)==ltrk
      ntrk=ntrk+1;
      onetrk(ntrk,1:3)=[ltrk state(ldat) dtsov(ldat)];
    end
  end
  % extract failures from onetrk() and write to cumtsov()
  % array onetrk(,) contains:
  % row1 = tracking #, row2 = state, row3 = operational time
  for lonet=1:ntrk
    % keep adding operational times until C&R or OVH
    tsf=tsf+onetrk(lonet,3);
    if ((onetrk(lonet,2)==3)|(onetrk(lonet,2)==4))
      % stop adding operation time at OVH or C&R
      nfail=nfail+1;
      cumtsov(ltrk,nfail)=tsf;
      tsf=0;  % reset time since failure
    end
  end
  lastint(ltrk)=0;  %  array contains final oper. intvr
  if onetrk(ntrk,2)==1  % final event is operation state
    lastint(ltrk)=tsf*11.39;
  end
  cumtsov(ltrk,1:nfail)=cumsum(cumtsov(ltrk,1:nfail));
end
% Write data to output file for program mcperil.m
fid=fopen('mcperil.dat','w')
cprint=0;
% convert cumtsvo() from flight days to flight hours
cumtsov(:,:)=11.39*cumtsov(:,:);
for ltrk=1:max(trk)
  % count the number of nonzero entries in cumtsov()
  lnonz=0;
  for j=1:6
```

```
    if cumtsov(ltrk,j)>0
      lnonz=lnonz+1;
    end
  end
  % output format depends on # of nonzero numbers in cumtsov()
  if lnonz==1
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-10,cumtsov(ltrk,1)+lastint(ltrk),cumtsov(ltrk,1),0,0,0,0,0];
  elseif lnonz==2
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),0,0,0,0];
  elseif lnonz==3
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),0,0,
0];
  elseif lnonz==4
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),cum
tsov(ltrk,4),0,0];
  elseif lnonz==5
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),cum
tsov(ltrk,4),cumtsov(ltrk,5),0];
  elseif lnonz==6
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),cum
tsov(ltrk,4),cumtsov(ltrk,5),cumtsov(ltrk,6)];
  end
end
% write output file
fprintf('\n')
fprintf(' Output data file (thperilf.dat)\n')
mcperil
fid=fopen('thperilf.dat','w');
save 'thperilf.dat' mcperil -ascii
status=fclose(fid);
```

APPENDIX E.4

OUTPUT OF PROGRAM THEVALF.M

WITH INPUT DATA FILE THFIELDF.DAT

EDU» THEVALF

s =

23-Feb-96

Summary - Types of Repairs

STATE	# EVENTS	PERCENT
OVH	31	31.0
C&R	44	44.0
NFF	25	25.0

Summary - Types of Removals

REASON FOR REMOVAL	# EVENTS	PERCENT
ENG CHG	48	32.4
FAILURE	100	67.6

Holding Times for each State

STATE	AVERAGE[DAYS]	VARIANCE[DAYS]
OPERATION	197.3	41738.3
FAILURE	12.7	0.0
OVERHAUL	7.4	0.0
C&R	7.4	0.0
NFF	7.4	0.0
INVENTORY	49.6	0.0
ENG CHG	126.0	0.0

Cost per flight hour= 0.000433264 [CU/FH]

MTBUR= 3641.2 [FH]

MTBF = 4854.93 [FH]

Number of units= 45

fid =

3

Output data file (thperilf.dat)

APPENDIX E.5

TABULAR OUTPUT OF PROGRAM THPERIL.M

WITH INPUT FILE THPERILF.DAT

EDU» thperil

ans =

23-Feb-96

Data file: thperilf.dat

MLE of lamda and beta

lamda =

3.7760e-004

beta =

0.9335

Minimum Start Time 1e-010
Maximum Time of last observation 13937
Time Intervals

tt =

```
  1.0e+004 *

    0.0000    0.2787    0.5575    0.8362    1.1150    1.3937

 Number that fail in each interval

nfail =

    26    18    11    15     5

 Number operational in each interval

nop =

    45    41    31    23    10

 Natural estimate of peril rate
p =

  1.0e-003 *

    0.2126    0.1777    0.1508    0.3022    0.2779

 NHPP estimate of peril rate Chi-Square Goodness of Fit Parameter

theta =

   27.2674   20.5457   14.2865    9.5089    3.3915


chi2 =

    5.0643


m =

    44

 Cramer von-Mises Test

c2m =

    0.0413
```

```
bbeta =

   0.6217
```

APPENDIX E.6

LISTING OF PROGRAM THSIMEVL.M

```
%thsimevl.m, 2/13/96
% Program to Monte Carlo model
% the operational/maintenance cycle of the IDG
% Definitions of variables
% tsov  Time since overhaul, reset with each overhaul [days]
% tcal  Calendar time, an overhaul does not reset [days]
% all time is in days, 1 day = 11.39 FH
c=fix(clock)  % date and time
%nseed=1;  % random number generator seed
rand('seed',nseed)  % seed number for the random generator
cfa=.6146;   % cost of a field failure in cost units [CU]
covh=.679;   % cost of an IDG overhaul [CU]
ccr=.456;    % cost of a Check and Repair [CU]
cnff=.051;   % cost of a No Fault Found [CU]
cinv=.248;   % cost of time in inventory [CU]
ceng=.6065;  % cost of an engine change [CU]
copr=0;      % cost of operational state
beta2=.89;  % beta of holding time in failure state [days]
theta2=12.01569; % theta of holding time in failure state [days]
beta3=1.001; % beta of holding time in OVH,C&R,NFF states [days]
theta3=7.4167; % theta of holding time in OVH,C&R,NFF states
beta6=1.234; % beta of holding time in inventory state [days]
theta6=53.065; % theta of holding time in inventory state [days]
beta7=1.209;  % beta of holding time in eng change state [days]
theta7=134.4;  % theta of holding time in eng change state [days]
% transition probabilities over the interval 0-3000 FH
pr1fail=.003915;  % failure rate [1/FD]
pr1eng=.002001;   % eng change rate [1/FD]
pr1ovh=.1111;   % probability of an overhaul
pr1crr=.5555;   % probability of a C&R
pr1nff=.3333;   % probability of a NFF
% transition probabilities over the interval 3001-6000 FH
```

```
pr2fail=.002046;
pr2eng=.001507;
pr2ovh=.3684;
pr2crr=.3684;
pr2nff=.2632;
% transistion probabilities over the interval 6001-9000 FH
pr3fail=.002148;
pr3eng=.001228;
pr3ovh=.4286;
pr3crr=.4286;
pr3nff=.1429;
% transition probabilities over the interval 9001-12000 FH
pr4fail=.004695;
pr4eng=.0005217;
pr4ovh=.5;
pr4crr=.3333;
pr4nff=.1667;
% transition probabilities+ over the interval 12001-15000 FH
pr5fail=.004802;
pr5eng=.0012;
pr5ovh=1;
pr5crr=0;
pr5nff=0;
%tsoft=1282;  % soft time overhaul interval [FD]
numidgs=500;  % number of IDGs to simulate
tobslim=1317;  % time [FD] that limits observation interval
cprint=0;
for k=1:numidgs
   tcal=0; tsov=0; state=1; totcal=0;
   while (tsov<=tobslim)  % limit observations to 10,000 FH
     if state==1   % operational **************************
       deltat=1;  %   time interval = 1 FD
       tfail=0;  % time since last fail = operation time
       for tsov=tsov:deltat:1317
         tfail=tfail+deltat;
         if tsov>=0 & tsov<263   % 0 to 3000 FH peril rate
           prfail=pr1fail;
           preng=pr1eng;
         elseif tsov>=263 & tsov<527   % 3001 to 6000 FH
           prfail=pr2fail;
           preng=pr2eng;
         elseif tsov>=527 & tsov<790   % 6001 to 9000 FH
           prfail=pr3fail;
           preng=pr3eng;
         elseif tsov>=790 & tsov<1054   % 9001 to 12000 FH
```

```
      prfail=pr4fail;
      preng=pr4eng;
    elseif tsov>=1054 & tsov<=1317  % 12001 to 15000 FH
      prfail=pr5fail;
      preng=pr5eng;
    end
    tcal=tfail;  % calendar time = time since ovh
    rtest=rand;
    if rtest>=0 & rtest<=prfail
      % failure state
      % write results to output file
        cprint=cprint+1;
      trk(cprint)=k;
      states(cprint)=state;
      dtsov(cprint)=tfail;
      dtcal(cprint)=tcal;
      cost(cprint)=0;
      totcal=tfail+totcal;  % increment total cal time
      state=2;
      break
    elseif rtest>prfail & rtest<=(prfail+preng)
      % write results to output file
        cprint=cprint+1;
      trk(cprint)=k;
      states(cprint)=state;
      dtsov(cprint)=tfail;
      dtcal(cprint)=tcal;
      cost(cprint)=0;
      totcal=tfail+totcal;  % increment total cal time
      state=7;
      break
    else
      state=1;  % remain operational
    end
  end
elseif state==2,  % failure state *******************
  r=round(theta2*((-log(1-rand))^(1/beta2)));
  if r == 0  % check for case when r=0
    r=1;  % minimum time in a state is 1 day
  end
  totcal=totcal+r;
    % write results to output file
    cprint=cprint+1;
  trk(cprint)=k;
  states(cprint)=state;
```

```
dtsov(cprint)=0;
dtcal(cprint)=r;
cost(cprint)=cfa;
  % determine next state
% determine the transistion prob. depending on tsov
if tsov>=0 & tsov<263   % 0 to 3000 FH peril rate
  pr2to3=pr1ovh;
    pr2to4=pr1crr;
elseif tsov>=263 & tsov<527   % 3001 to 6000 FH
  pr2to3=pr2ovh;
  pr2to4=pr2crr;
elseif tsov>=527 & tsov<790   % 6001 to 9000 FH
  pr2to3=pr3ovh;
  pr2to4=pr3crr;
elseif tsov>=790 & tsov<1054   % 9001 to 12000 FH
  pr2to3=pr4ovh;
  pr2to4=pr4crr;
elseif tsov>=1054 & tsov<=1317   % 12001 to 15000 FH
  pr2to3=pr5ovh;
  pr2to4=pr5crr;
end
if tsov>=tsoft   % soft-time overhaul
    state=3;  % OVH state
  else
    r=rand;
  if r<=pr2to3
    state=3;  % OVH state
  elseif (r>pr2to3)&(r<=(pr2to3+pr2to4))
        state=4;  % C&R state
  else
    state=5;  % NFF state
  end
  end
elseif state==3   % overhaul state *****************
  tsov=0;  % reset FH to zero on overhaul
r=round(theta3*((-log(1-rand))^(1/beta3)));
if r == 0  % check for case when r=0
  r=1;  % minimum time in a state is 1 day
end
  totcal=totcal+r;  % increment cal time
% write results to output file
  cprint=cprint+1;
trk(cprint)=k;
states(cprint)=state;
dtsov(cprint)=0;
```

```
dtcal(cprint)=r;
cost(cprint)=covh;
  % since ovh, start with new IDG
  break
elseif state==4   % C&R state *********************
r=round(theta3*((-log(1-rand))^(1/beta3)));
if r == 0   % check for case when r=0
  r=1;   % minimum time in a state is 1 day
end
totcal=totcal+r;   % increment cal time
% write results to output file
  cprint=cprint+1;
trk(cprint)=k;
states(cprint)=state;
dtsov(cprint)=0;
dtcal(cprint)=r;
cost(cprint)=ccr;
  % next state is inventory
  state=6;
elseif state==5   % NFF state *******************
r=round(theta3*((-log(1-rand))^(1/beta3)));
if r == 0   % check for case when r=0
  r=1;   % minimum time in a state is 1 day
end
  totcal=totcal+r;   % increment cal time
% write results to output file
  cprint=cprint+1;
trk(cprint)=k;
states(cprint)=state;
dtsov(cprint)=0;
dtcal(cprint)=r;
cost(cprint)=cnff;
  % next state is inventory
  state=6;
elseif state==6   % inventory state *************
  r=round(theta6*((-log(1-rand))^(1/beta6)));
  if r == 0   % check for case when r=0
    r=1;   % minimum time in a state is 1 day
  end
    totcal=totcal+r;   % increment cal time
  % write results to output file
    cprint=cprint+1;
  trk(cprint)=k;
  states(cprint)=state;
  dtsov(cprint)=0;
```

```
          dtcal(cprint)=r;
          cost(cprint)=cinv;
            % next state is operational
            state=1;
          elseif state==7   % engine change state *******
          r=round(theta7*((-log(1-rand))^(1/beta7)));
          if r == 0   % check for case when r=0
            r=1;  % minimum time in a state is 1 day
          end
            totcal=totcal+r;   % increment cal time
          % write results to output file
            cprint=cprint+1;
          trk(cprint)=k;
          states(cprint)=state;
          dtsov(cprint)=0;
          dtcal(cprint)=r;
          cost(cprint)=ceng;
            % next state is operational
            state=1;
      end
    end
end
nseed
tsoft
tobslim
% ***************************************************
% This part of the program takes the data generated
% and calculates statistics
%****************************************************
% Determine the percent of NFF, C&R, and OVH failures
novh=0; nnff=0; nccr=0; neng=0;
for j=1:length(states)
  if states(j)==3   % number of overhauls
    novh=novh+1;
  elseif states(j)==4   % number of C&R
    nccr=nccr+1;
  elseif states(j)==5   % number of NFF
    nnff=nnff+1;
  elseif states(j)==7   % number of eng changes
    neng=neng+1;
  end
end
nfails=novh+nccr+nnff;   % total number of failures
clc
fprintf(' Summary - Types of Repairs\n')
```

```
fprintf('\n')
fprintf(' STATE     # EVENTS      PERCENT\n')
fprintf(' -----     ---------     -------\n')
fprintf('  OVH        %d          %3.1f\n',novh,(novh/nfails)*100)
fprintf('  C&R        %d          %3.1f\n',nccr,(nccr/nfails)*100)
fprintf('  NFF        %d          %3.1f\n',nnff,(nnff/nfails)*100)
% Determine percent types of removals, ie, eng chg or failure
pengchg=(neng/(neng+nfails))*100;  % percent eng changes
pfails=(nfails/(neng+nfails))*100;  % percent failures
fprintf('\n')
fprintf(' Summary - Types of Removals\n')
fprintf('\n')
fprintf(' REASON FOR\n')
fprintf(' REMOVAL   # EVENTS       PERCENT\n')
fprintf(' -------   ---------      -------\n')
fprintf(' ENG CHG     %d           %3.1f\n',neng,pengchg)
fprintf(' FAILURE     %d           %3.1f\n',nfails,pfails)
% Average times in each state
nopr=0; nfail=0; novh=0; nccr=0; nnff=0; ninv=0; neng=0;
for j=1:length(states)
  if states(j)==1  % operation
     topr(nopr+1)=dtcal(j); % transfers cal time data to array
     nopr=nopr+1;  % increment op state counter
  elseif states(j)==2  % failure state
    tfail(nfail+1)=dtcal(j);  % transfers cal times to array
    nfail=nfail+1;  % counts the number times in failure state
  elseif states(j)==3  % overhaul
    tovh(novh+1)=dtcal(j);
    novh=novh+1;
  elseif states(j)==4  % C&R
    tccr(nccr+1)=dtcal(j);
    nccr=nccr+1;
  elseif states(j)==5  % NFF
    tnff(nnff+1)=dtcal(j);
    nnff=nnff+1;
  elseif states(j)==6  % inventory
    tinv(ninv+1)=dtcal(j);
    ninv=ninv+1;  % sums number of visits to inventory
  elseif states(j)==7  % engine change
    teng(neng+1)=dtcal(j);
    neng=neng+1;
  end
end
% calculate and display averages for each state
fprintf('\n')
```

```
fprintf('       Holding Times for each State\n')
fprintf('\n')
fprintf(' STATE       AVERAGE[DAYS]   VARIANCE[DAYS]\n')
fprintf('------       -------------   --------------\n')
fprintf(' OPERATION    %4.1f           %8.1f\n',mean(topr),std(topr)^2)
fprintf(' FAILURE      %4.1f           %8.1f\n',mean(tfail),std(tfail)^2)
fprintf(' OVERHAUL     %4.1f           %8.1f\n',mean(tovh),std(tovh)^2)
fprintf(' C&R          %4.1f           %8.1f\n',mean(tccr),std(tccr)^2)
fprintf(' NFF          %4.1f           %8.1f\n',mean(tnff),std(tnff)^2)
fprintf(' INVENTORY    %4.1f           %8.1f\n',mean(tinv),std(tinv)^2)
fprintf(' ENG CHG      %4.1f           %8.1f\n',mean(teng),std(teng)^2)
% Cost per flight hours
fprintf('\n')
fprintf(' Cost per flight hour= %g [CU/FH]\n',sum(cost)/(sum(topr)*11.39))
% Calculation of MTBUR - Mean Time Between Unscheduled Removals
% An unscheduled removal is for failure only
% MTBUR = (total flight hours)/(total failures)
mtbur=(sum(topr)*11.39)/nfail;
fprintf('\n')
fprintf(' MTBUR= %g [FH]\n',mtbur)
% MTBF=(total flight hours)/(total confirmed failures)
mtbf=(sum(topr)*11.39)/(novh+nccr);
fprintf('\n')
fprintf(' MTBF = %g [FH]\n',mtbf)
% Number of units total
fprintf('\n')
fprintf(' Number of units= %g\n',max(trk))
%**************************************************
% This section converts the data into a format so
% it can be read by program thperil.m, which
% calculates the peril rate.
%**************************************************
% Calculate the cumulative time since OVH (cumtsov)
cumtsov=zeros(max(trk),6);
for ltrk=1:max(trk)  % loop thru each trk #
  ntrk=0;  % counts numer of times for each trk #
  nfail=0;  % counts number of failures per trk #
  tsf=0;  % time since failure
  for ldat=1:length(states)  % loop thru each row
    if trk(ldat)==ltrk
      ntrk=ntrk+1;
      onetrk(ntrk,1:3)=[ltrk states(ldat) dtsov(ldat)];
    end
  end
  % extract failures from onetrk() and write to cumtsov()
```

```
  % array onetrk(,) contains:
  % row1 = tracking #, row2 = state, row3 = operational time
  for lonet=1:ntrk
    % keep adding operational times until C&R or OVH
    tsf=tsf+onetrk(lonet,3);
    if ((onetrk(lonet,2)==3)|(onetrk(lonet,2)==4))
      % stop adding operation time at OVH or C&R
      nfail=nfail+1;
      cumtsov(ltrk,nfail)=tsf;
      tsf=0;  % reset time since failure
    end
  end
  lastint(ltrk)=0;  % contains final operational interval
  if onetrk(ntrk,2)==1  % final event is operation state
    lastint(ltrk)=tsf*11.39;
  end
  cumtsov(ltrk,1:nfail)=cumsum(cumtsov(ltrk,1:nfail));
end
cprint=0;
% convert cumtsvo() from flight days to flight hours
cumtsov(:,:)=11.39*cumtsov(:,:);
for ltrk=1:max(trk)
  % count the number of nonzero entries in cumtsov()
  lnonz=0;
  for j=1:6
    if cumtsov(ltrk,j)>0
      lnonz=lnonz+1;
    end
  end
  % output format depends on # of nonzero numbers in cumtsov()
  if lnonz==1
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-10,cumtsov(ltrk,1)+lastint(ltrk),cumtsov(ltrk,1),0,0,0,0,0];
  elseif lnonz==2
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),0,0,0,0];
  elseif lnonz==3
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),0,0,
0];
  elseif lnonz==4
    cprint=cprint+1;
```

```
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),cum
tsov(ltrk,4),0,0];
  elseif lnonz==5
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),cum
tsov(ltrk,4),cumtsov(ltrk,5),0];
  elseif lnonz==6
    cprint=cprint+1;
    mcperil(cprint,1:8)=[1e-
10,max(cumtsov(ltrk,:)+lastint(ltrk)),cumtsov(ltrk,1),cumtsov(ltrk,2),cumtsov(ltrk,3),cum
tsov(ltrk,4),cumtsov(ltrk,5),cumtsov(ltrk,6)];
  end
end
% write output file
fid=fopen('thperils.dat','w');
save 'thperils.dat' mcperil -ascii
status=fclose(fid);
```

APPENDIX E.7

OUTPUT DATA FOR SAMPLE RUN

OF PROGRAM THSIMEVL.M

```
EDU» thsimevl

c =

      1996          2          23          5          3          8


nseed =

   1


tsoft =

      1282
```

tobslim =

1317

Summary - Types of Repairs

STATE	# EVENTS	PERCENT
OVH	500	38.4
C&R	505	38.8
NFF	297	22.8

Summary - Types of Removals

REASON FOR REMOVAL	# EVENTS	PERCENT
ENG CHG	557	30.0
FAILURE	1302	70.0

Holding Times for each State

STATE	AVERAGE[DAYS]	VARIANCE[DAYS]
OPERATION	196.9	42167.6
FAILURE	12.6	187.8
OVERHAUL	6.7	44.9
C&R	7.4	46.0
NFF	7.3	39.6
INVENTORY	51.2	1581.1
ENG CHG	120.2	9888.9

Cost per flight hour= 0.000460872 [CU/FH]

MTBUR= 3202.79 [FH]

MTBF = 4149.29 [FH]

Number of units= 500

APPENDIX E.8

LISTING OF PROGRAM THSIMDAT.M

```
% file: thsimdat.m, date: 2/17/96
% This program averages and plots the results of the
% Monte Carlo simulation program 'thsimevl.m'
% Variables:
% stime()= Soft time overhaul interval [FD]
% cufh()= Cost per unit flight hour [CU/FH]
% mtbur()= Mean Time Between Unscheduled Removal [FH]
% mtbf()= Mean Time Between Failure [FH]
whitebg
%**************************************************
% figure - Cost per Unit Flight Hour for Simulated
% Data
%**************************************************
stime(1)=263;  % 3000 FH overhaul interval
cufh(1,1:3)=[.000450501 .000430939 .000444769 ];
mtbur(1,1:3)=[3381 3516 3486];
mtbf(1,1:3)=[4060 4285 4169];
stime(2)=351;
cufh(2,1:3)=[0.000444344 .00042752 .000435504];
mtbur(2,1:3)=[3410 3567 3509];
mtbf(2,1:3)=[4141 4365 4286];
stime(3)=439;
cufh(3,1:3)=[0.000444835 .000426967 .000437373];
mtbur(3,1:3)=[3402 3579 3506];
mtbf(3,1:3)=[4228 4454 4287];
stime(4)=527;
cufh(4,1:3)=[0.000439828 .000427582 .000426977];
mtbur(4,1:3)=[3439 3542 3573];
mtbf(4,1:3)=[4340 4462 4406];
stime(5)=615;
cufh(5,1:3)=[.000441943 .000431967 .000431388];
mtbur(5,1:3)=[3375 3491 3526];
mtbf(5,1:3)=[4286 4400 4415];
stime(6)=702;
cufh(6,1:3)=[0.000443431 .000431623 .000434083];
mtbur(6,1:3)=[3343 3472 3519];
mtbf(6,1:3)=[4241 4382 4357];
stime(7)=790;
cufh(7,1:3)=[0.000443952 .000438328 .000439334];
```

```
mtbur(7,1:3)=[3329 3408 3457];
mtbf(7,1:3)=[4234 4329 4306];
stime(8)=878;
cufh(8,1:3)=[.000446676 .000446613 .000445242];
mtbur(8,1:3)=[3303 3333 3392];
mtbf(8,1:3)=[4221 4252 4250];
stime(9)=966;
cufh(9,1:3)=[.000454153 .000451394 .000453569];
mtbur(9,1:3)=[3245 3275 3298];
mtbf(9,1:3)=[4191 4186 4208];
stime(10)=1054;
cufh(10,1:3)=[.000460755 .000456935 .000460091 ];
mtbur(10,1:3)=[3204 3222 3242];
mtbf(10,1:3)=[4150 4148 4124];
stime(11)=1141;
cufh(11,1:3)=[.000460813 .000456994 .000460156];
mtbur(11,1:3)=[3203 3221 3241];
mtbf(11,1:3)=[4150 4148 4123];
stime(12)=1282;
cufh(12,1:3)=[.000460872 .000457046 .000460223];
mtbur(12,1:3)=[3202 3221 3241];
mtbf(12,1:3)=[4149 4147 4123];
% calculte averages
for kk=1:12
  avgcufh(kk)=mean(cufh(kk,:));
  stdcufh(kk)=std(cufh(kk,:));
  avgmtbur(kk)=mean(mtbur(kk,:));
  stdmtbur(kk)=std(mtbur(kk,:));
  avgmtbf(kk)=mean(mtbf(kk,:));
  stdmtbf(kk)=std(mtbf(kk,:));
end
% plot data
errorbar(stime*11.39,avgcufh,stdcufh)
axis([2000 15000 4e-4 5e-4])
title('Cost per Flight Hour')
xlabel('Overhaul Interval [FH]')
ylabel('CU/FH')
%print
pause
%**********************************************
% figure - Mean Time Between Unscheduled Removal
% for Simulated Data
%**********************************************
errorbar(stime*11.39,avgmtbur,stdmtbur)
axis([2000 15000 2500 4000])
```

```
title('MTBUR - Simulated Data')
xlabel('Overhaul Interval [FH]')
ylabel('MTBUR')
%print
pause
%*********************************************
% figure - Mean Time Between Failures for
% Simulated Data
%*********************************************
errorbar(stime*11.39,avgmtbf,stdmtbf)
axis([2000 15000 3500 5000])
title('MTBF - Simulated Data')
xlabel('Overhaul Interval')
ylabel('MTBF')
%print
pause
clf;
%*********************************************
% write data to screen
%*********************************************
disp('Overhaul          Standard')
disp('Interval          Deviation')
disp('FH     CU/FH      CH/FH')
fprintf('\n')
for kk=1:12
  fprintf('%4.0f   %6.6f    %7.7f\n',stime(kk)*11.39,avgcufh(kk),stdcufh(kk))
end
fprintf('\n')
disp('Overhaul        Standard        Standard')
disp('Interval        Deviation       Deviation')
disp('FH     MTBUR    MTBUR   MTBF     MTBF')
fprintf('\n')
for kk=1:12
  fprintf('%4.0f   %4.0f    %4.0f    %4.0f
%4.0f\n',stime(kk)*11.39,avgmtbur(kk),stdmtbur(kk),mtbf(kk),stdmtbf(kk))
end
%************************************************************
% figure  - Availability
%************************************************************
avail=avgmtbur./(4+avgmtbur);
plot(stime*11.39,avail,'k-')
%axis([2000 15000 .9987 .999])
xlabel('Overhaul Interval [FH]')
ylabel('Availability')
pause
```

```
%print
% write availability results to screen
fprintf('\n')
disp('Overhaul')
disp('Interval')
disp('FH        Availability')
fprintf('\n')
for kk=1:12
  fprintf('%5.0f     %7.7f\n',stime(kk)*11.39,avail(kk))
end
```

APPENDIX E.9

TABULAR OUTPUT OF PROGRAM THSIMDAT.M

EDU» thsimdat

Overhaul Interval FH	CU/FH	Standard Deviation CH/FH
2996	0.000442	0.0000101
3998	0.000436	0.0000084
5000	0.000436	0.0000090
6003	0.000431	0.0000073
7005	0.000435	0.0000059
7996	0.000436	0.0000062
8998	0.000441	0.0000030
10000	0.000446	0.0000008
11003	0.000453	0.0000015
12005	0.000459	0.0000020
12996	0.000459	0.0000020
14602	0.000459	0.0000020

Overhaul Interval FH	MTBUR	Standard Deviation MTBUR	MTBF	Standard Deviation MTBF
2996	3461	71	4060	113
3998	3495	79	4141	114
5000	3496	89	4228	117
6003	3518	70	4340	61
7005	3464	79	4286	71

7996	3445	91	4241	75
8998	3398	65	4234	50
10000	3343	45	4221	17
11003	3273	27	4191	12
12005	3223	19	4150	14
12996	3222	19	4150	15
14602	3221	20	4149	14

Overhaul Interval FH	Availability
2996	0.9988456
3998	0.9988569
5000	0.9988570
6003	0.9988643
7005	0.9988466
7996	0.9988401
8998	0.9988242
10000	0.9988048
11003	0.9987792
12005	0.9987603
12996	0.9987599
14602	0.9987598

APPENDIX F

COMPUTER PROGRAM TO PLOT

THE CU/FH FOR A TYPE II POLICY

```
% file: thhardt.m, 2/25/96
% hard time replacement policy
whitebg
cmr=.969;  % cost of a minimal repair
csr=1;  % cost of a scheduled removal
cmrup=1.0659;  % cost of minimal repair plus 10%
csrup=1.1;  % cost of scheduled removal plus 10%
cmrlow=.8721;  % cost of a minimal repair less 10%
csrlow=.9;  % cost of scheduled removal less 10%
lamda2k=2.08e-5;  % 2k left truncated data
beta2k=1.27;  % 2k left truncated data
lamda5k=2.32e-10;  % 5k left truncated data
beta5k=2.47;  % 5k left truncated data
tau=100:100:15000;
% hard time replacement plot
costht2k=(cmrup*lamda2k*tau.^beta2k+csrup)./tau;
costht5k=(cmrlow*lamda5k*tau.^beta5k+csrlow)./tau;
plot(tau,costht2k,'k-.',tau,costht5k,'k:')
axis([0 15000 0 .0015])
legend('k-.','Upper Bound','k:','Lower Bound')
title('CU/FH with Type II Maintenance')
xlabel('Replacement Age [FH]')
ylabel('Cost Function [CU/FH]')
print
```

2

VITA

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Doctor of Philosophy

Thesis: AN APPROACH TO OPTIMIZE THE MAINTENANCE PROGRAM FOR A REPAIRABLE SYSTEM

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