CHILDREN'S CONSTRUCTION OF KNOWLEDGE ABOUT FRACTIONS THROUGH WRITING

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CHILDREN'S CONSTRUCTION OF KNOWLEDGE

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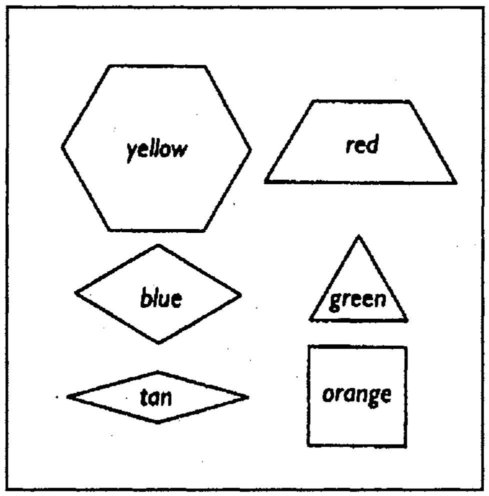


yellow hexagon - a polygon having six sides and six angles.

blue parallelogram - a four-sided plane figure whose opposite sides are parallel and equal.

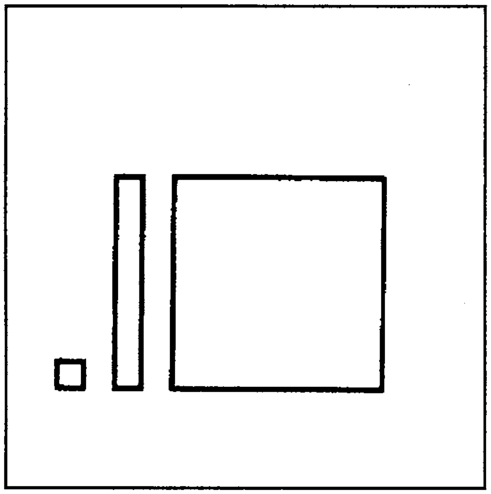
tan parallelogram - a four-sided plane figure whose opposite sides are parallel and equal.

red trapezoid - a four-sided plane figure having only two sides parallel.

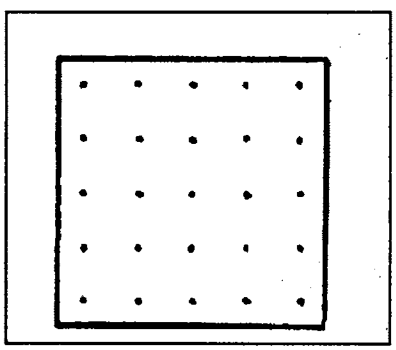
green triangle - a plane figure, bounded by three sides, and having three equilateral angles.

orange square - a parallelogram having four equal sides and four right angles.

Base Ten Blocks - proportional materials used to represent numbers.



Geoboards - a 5-by-5 square array of pegs over which you stretch rubber bands to create polygons. The geoboard provides opportunities to investigate area, perimeter, fractions, geometric properties of shapes, and coordinate graphing.



CHAPTER 1

INTRODUCTION

Vigorous educational reform has been an ongoing focus since the 1980's. On August 26, 1981, the Secretary of Education T. H. Bell created the National Commission on Excellence in Education in order to examine the quality of education in the United Stated and make a report to the nation within 18 months of its first meeting. The report was A Nation At Risk (1983). The study indicated that historically the American people could take pride in the schools and colleges, but education was not continuing to achieve the high expectations in an ever-changing world.

For many years, learning has centered around the "end product", memorizing names of people, places, things, dates formulas, rules, and procedures from the state-adopted textbooks. Testing has been easy. Find out what the children do not know. Schools have produced students who are functional literates, but do not have a clear understanding of what the words mean, or how to solve problems. Change is needed.

Documented testimony of the report, taken from A Nation at Risk, The Full Document (The National Commission On Excellence in Education, 1984, pp. 8-9) included the following:

1. International comparisons of student achievement, completed a decade ago, reveal that on 19 academic tests American students were never first or second and, in comparison with other industrialized nations, were last seven times.
2. Some 23 million American adults are functionally illiterate by the simplest tests of everyday reading, writing, and comprehension.
3. About 13 percent of all 17-year-olds in the United States can be considered functionally illiterate. Functional illiteracy among minority youth may run as high as 40 percent.
4. Average achievement of high school students on most standardized tests is now lower than 26 years ago when Sputnik was launched.
5. Many 17-year-olds do not possess the "higher order" intellectual skills we should expect of them. Nearly 40 percent cannot draw inferences from written material; only one-fifth can write a persuasive essay; and only one-third can solve a mathematics problem requiring several steps.

Even though the average citizen of today is better educated and more knowledgeable than earlier generations, we live in a highly technological society where new commodities are outdated even before they reach the market. Our nation is still not meeting, much less exceeding, the qualifications needed to advance our society and compete adequately with other countries.

The number of Americans who cannot read and write adequately, according to PLUS (Project Literacy US, 1987), is more than twenty-three million; the drop-out rate at some urban high schools is above 50%, contributing to the problem of illiteracy; and one-third of all adult Americans lack the communication skills they needed to function productively (Routman, 1988).

Partly because of A Nation at Risk, the National Council of Teachers of Mathematics decided to write standards for mathematics and recommended major changes for school mathematics in March, 1989. They created a high-quality mathematics education for North American students, K-12, the focus being mathematical power to all students. This includes the ability to explore, reason logically, solve problems, communicate about and through mathematics, and to connect ideas within mathematics and between mathematics and other intellectual activity.

According to the Curriculum and Evaluations Standards for School Mathematics (NCTM, 1989), mathematics teaching for the empowerment of students should make major shifts toward: classrooms as mathematical communities, reasoning, conjecturing, inventing, and connecting mathematics to real world experiences.

As these changes are now being implemented throughout the country, classroom environments are beginning to promote stimulating activities to help students gain mathematical empowerment. Examples of this empowerment include: integrating subject areas to create authentic learning opportunities, more student directed versus teacher directed activities, processes engaging discovery and exploration, and finding connections between real-life experiences and classroom experiences.

The K-4 Standards address the verbs *explore, validate, represent, solve, construct, discuss, use, investigate, describe, develop, and predict*. Activities centered around active processes instead of memorizing and repeating are designed to develop a balanced and enriched curriculum empowering all students mathematically. When children are encouraged to construct their own reasoning about mathematical concepts through dynamic interactions, understanding will occur.

Hands-on activities create concrete opportunities for understanding mathematics concepts. Activities can include playing card games to learn multiplication facts, measuring different objects using string, graphing favorite stories using unifix cubes, predicting number and color using M&M packages, and finding fractional parts with pattern blocks. Students make sense of mathematics when they are actively involved with their peers. As they work through problems and discuss ideas and possibilities, children begin to determine for themselves if something is mathematically correct. As students are engaged in activity, teachers can see what children are doing and how they are interacting with other students. They can ask children questions about how they got an answer. Being a "kid-watcher", a term initiated by Yetta Goodman (1969), can provide the teacher with valuable information about a child's learning. It also lets the teacher know what areas he/she needs to clarify and even change.

In conjunction with such hands-on activities, the emergence of writing is becoming an integral part of the curriculum in mathematics classrooms. Burns (1995) has found two major benefits of using writing.

> It supports students' learning because, in order to get their ideas on paper, children must organize, clarify, and reflect on their thinking. Writing also benefits teachers because students' papers are invaluable assessment resources. Their writing is a window into what they understand, how they approach ideas, what misconceptions they harbor, and how they feel about what they're discovering. (p. 6).

Writing promotes a healthy egocentricity that is natural to active learners. It helps people integrate themselves into their perceptual, intellectual, and imaginative experiences. Language is one way we situate ourselves so that external events have internal meaning. Composing texts is a special way to orient oneself into situations because the texts can be formulated, examined, and revised (Kitagawa, 1993).

> Educational research findings from cognitive psychology and mathematics education indicate that learning occurs as students actively assimilate new information and experiences and construct their own meanings (Case and Bereiter 1984; Cobb and Steffe 1983; Davis 1984;

Hiebert 1986; Lampert 1986; Lesh and Landau 1983; Schoenfeld 1987) (as cited in Professional Standards For Teaching Mathematics, 1991, p.2).

An example of this is children using base ten blocks for dividing. After completing problems together where the groups can be divided equally without regrouping the blocks, a new problem is proposed requiring children to exchange tens to ones before dividing equally. If children have been exposed to these blocks when regrouping was required for addition or subtraction, they can make an easy transition to regrouping with division. As they discuss the problem with their peers, and/or write, they talk about having to exchange in order to continue the process of making equal groups. The children have not memorized a formula for division; they have created a way for the problem to make sense to them. They are becoming autonomous, building a sense of power and ownership over their own learning. Writing facilitates active learning because it helps students develop for themselves the language imperative to their own individual growth. According to Connoly (1989), writing serves to develop:

1. *Abilities* to define, classify, or summarize; to imagine hypotheses and trace inferences; to recognize and evaluate patterns; to establish procedures and analyze problems.

2. *Methods* of close, reactive reading; of recording data; of organizing and structuring; of formulating theories;

and, most important, of recognizing and regulating method itself.

3. *Knowledge* of central concepts in a course; of the broad aims and methods of discipline; of one's own writing, thinking, and learning.

4. *Attitudes* toward learning, knowing, oneself, one's work; toward mistakes and errors; toward the knowledge and opinions of others.

5. *Collaborative learning* by encouraging open exploration within a community of inquiry, rather than isolated competition; promoting connected, not separated, teaching; developing active listening; and locating the motivation for learning neither in the relevance of the subject nor in the performance of the teacher but in the interpersonal dynamic of the learning community itself.

6. *In summary, general capacities for learning* including the ability to question, to wonder, to think for oneself while working with others (pp. 6-7).

Statement of the Problem

According to <u>The Curriculum and Evaluation Standards for School Mathematics</u> (1989, p. 57), the K-4 instruction should help students understand fractions and decimals, explore their relationship, and build initial concepts about order and equivalence. When children possess a sound understanding of fraction and decimal concepts, they can use this knowledge to describe real-world phenomena and apply it to problems involving measurement, probability, and statistics. It is critical in grades K-4 to develop concepts and relationships that will serve as a foundation for more advanced concepts and skills.

The focus of this study is to determine how fourth grade students construct their knowledge of fractions through various types of informal writing. The teacher-researcher planned a five-week unit of instruction using manipulatives and freewriting, metacognitive writing, and reflective writing to learn about fractions and decimals. A final metacognitive assessment was given at the end of the unit to validate student knowledge about fractions and decimals. Students were also asked to write reflectively about their own learning and feelings about the unit of study.

Research Questions

The following questions guide this study:

1. How do students validate their reasoning through writing?
2. How do students view their understanding of fractions through the use of writing?
3. How does the teacher use children's writing to adjust instruction?

Definition of Terms

Freewriting initiates exploration of a question or problem. Examples might include: feelings about their knowledge at the beginning of a concept or predictions about how a concept transfers to another idea. It is ungraded.

Reflective writing concludes a class activity offering students opportunities to question, form opinions, address concerns, and express personal growth.

Metacognitive writing focuses on how one works a problem. Students give sequential steps and their thought processes in solving a problem.

Limitations and Assumptions

It is assumed that the subjects are representative of students in many fourth grade classrooms where the population includes a culturally diverse college community .

This ethnographic study is limited in that all observations and conferences are interpreted from the teacher-researcher's background knowledge and beliefs.

Since students actively participated in self-assessment, the conclusions of the investigation are a result of both student responses and the teacher-researcher.

Summary

The emergence of writing in the mathematics curriculum may create a powerful means of communication. It may be valuable to children's mathematical development of reasoning, problem solving, and confidence building. It may also provide teachers with a more in-depth look at student understanding, development, ability, misconceptions and errors. This study examined one classroom's discovery and investigation of fractions and their process of learning mathematics through informal writing.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

This chapter is devoted to a review of the literature important to this study of writing as a tool for learning mathematics. The review includes three topics: (1) authentic learning (2) authentic learning and writing (3) the relationship of writing to mathematical learning.

Authentic Learning

Authentic learning is an important constructivist concept. Heathcote (1984) describes authenticity as, "the tenuous thread which links all the previous gleanings." Heathcote also believes this can be accomplished through

> the pressure, or the authenticity, of that dramatic moment that creates the new knowledge, that makes new connections, and that suddenly brings connections that have been dormant in my previous knowledge into active use in making sense of new information. Heathcote's study (as cited in Cordeiro, 1992, p. 77).

The emphasis on sharing of authentic skill advocates the presence of self-direction on the part of the learner in true purposeful content. Literacy encourages the learner to become involved in the genuine and serious practice of literate acts and confirm or correct his own work, convictions, or responses (Holdaway, 1979). Students internalize meaning when they are engaged in authentic experiences. For example, instead of introducing multiplication with memorization, real experiences such as grouping beans in cups, connecting unifix cubes in groups, making squares and rectangles on graph paper and discovering factors, lead students to an authentic understanding of multiplication.

Fosnot (1989) defines constructivism in the following four principles:

1. Knowledge consists of past constructions.
2. Constructions come about through assimilation and accommodation.
3. Learning is an organic process of invention, rather than a mechanical process of accumulation.
4. Meaningful learning occurs through reflection and resolution of cognitive conflict and thus serves to negate earlier, incomplete levels of understanding (pp. 19-20).

As children process information within their own framework of development, learning takes place. An emphasis is placed on the student making logical connections of pertinent learning activities instead of just giving verbatim information back to the teacher orally or in written form. Children generally do not learn very well if the teaching is mostly deductive. "Lectures, explanations, definitions, general descriptions, and overviews rarely give people the raw material to construct meaning. We need many examples. This is how we build our understandings of concepts through an inductive thinking process" (Hyde & Hyde, 1991, p. 32).

Fosnot (1989) states, "Learning is an organic process of invention, rather than a mechanical process of accumulation (p. 20). Students involved in stimulating activities assimilate new information and circumstances which empower them with knowledge. Teaching can not be confined into regimented sequential steps. Students need to reconstruct the experiences for themselves, since learning comes from within, not what others give to us.

Integration is an approach to authentic learning and a way of thinking that connects the language processes-reading, writing, speaking, and listening into all areas of the content curriculum. "Integration also means that major concepts and larger understandings are being developed in social contexts and that related activities are in harmony with and important to the major concepts" (Routman, 1991, p. 276).

In regards to authentic math classes, learning occurs naturally in an environment promoting meaningful and relevant activities. The student is motivated intrinsically, is willing to take risks, and to take responsibility for his own learning. NCTM's premise is that **what** a student learns depends to a great degree on **how** he or she has learned it (<u>Curriculum and Evaluation Standards for School Mathematics</u>, p. 5). Activities involving rote memorization or methods encourage students to imitate procedures, and students only appear to achieve concept understanding. However, if ideas are made meaningful and integrated by the child into his or her own existing structure of knowledge, learning occurs.

Evidence shows that the least effective method for mathematics learning is the one that has predominated in most of America's classrooms: lecturing and listening. In such classrooms, learning is primarily watching and imitating, which is a passive activity. Students do not retain information given out in lectures, completed workbook pages, or repetitious homework. Such activities do prepare students for standardized tests because they facilitate lower-order skills. However, they are not successful for higher-order thinking, problem solving, or long-term learning (Everybody Counts, 1989).

"Research in learning shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created" (Everybody Counts, 1989). Mathematics becomes worthwhile to a student when it has been generated through a private commitment that creates new understanding.

"Children learn about math best when the math they meet is embedded in purposeful activities; that is, when the children are immersed in real purposes for using and exploring math" (Baker, Semple, Stead, 1990, pp. 4-5). This is experienced when counting cubes for volume, rolling dice and predicting outcomes, measuring lengths with Cuisenaire rods, and estimating number possibilities with playing cards.

Authentic activities give opportunities for students to become problem solvers, to value mathematics, and to communicate mathematically. Children become mathematically literate and able to work competently through real-world situations. Authentic activities include student exploration of new materials, discussion, questioning, drawing logical conclusions, using models and manipulatives, finding relationships, explaining processes, and analyzing mathematical situations.

Children form beliefs about mathematics and themselves as learners of mathematics. When teachers provide enriching mathematics activities and projects that stimulate children's desire to learn through an integrated curriculum of writing and problem solving, students become mathematically empowered. "Our tactile intelligence, coupled with our visual intelligence, together contribute to the overall learning as our cognition, our mental understanding, grasps and encodes mathematical learning" (Cordeior, 1992, p. 140).

Cooperative learning groups, teacher modeling, child-created problem solving, risk taking, and written and oral sharing need to be a part of the math curriculum as well as the total language program (Routman, 1991). Cooperative groups foster community, an essential element for authenticity. Groups nurture a positive learning environment and encourage communication. Research indicates that working collaboratively with others can increase achievement (Slavin, 1990).

In addition to cooperative learning groups, the role of the teacher is critical in facilitating meaning-making, authentic environments. New goals for students must reflect the importance of mathematical literacy. The K-12 standards articulate five general goals for all students:

(1) that they learn to value mathematics

(2) that they become confident in their ability to do mathematics

(3) that they become mathematical problems solvers

(4) that they learn to communicate mathematically

(5) that they learn to reason mathematically (Curriculum and Evaluation Standards for School Mathematics, p. 5).

The teacher's role is to create an authentic environment in which students feel free to share their ideas and opinions, ask relevant questions, take risks, analyze and hypothesize, and make errors. "Also critical is the inclusion of activities that promote questioning and help students recognize the need to ask questions" (Vacc, 1993).

To accomplish the vision of the NCTM Standards in an authentic classroom, teachers see their role as guiding, helping, and challenging students to develop mathematical power. They change from "tellers" to "questioners and listeners," and their classrooms become intellectual learning communities where collaboration and conjectures are made, arguments are presented, and strategies discussed. Outstanding classrooms are identified by high expectations, challenging work, concentrated endeavors, mutual respect, and opportunities to pursue mathematical literacy for all students. "Mutual respect" means that students with differing abilities are supported, valued, and encouraged (<u>Professional Standards for Teaching Mathematics</u>, 1991).

The National Council of Teachers of English (Tchudi, 1991) supported the 1989 NCTM Report. This report recommended the following similar authentic teaching practices:

1. To empower students:

- as lifelong learners whose command of language is exemplary and who gain pleasure and fulfillment from reading, writing, speaking, and listening.
- as active inquirers, experimenters, and problem solvers who are able to use the arts of language as a means of gaining insight into and reflecting upon their own and others' lives.
- as productive citizens who use language to take charge of their own lives and to communicate effectively with others.
- as theorizers about their own language and learning, able to read, write, and reflect on texts from multiple perspectives.

2. To empower teachers:

- as active learners who serve as coaches, mentors, and collaborative creators of learning experiences rather than as dispensers of information.

- as decision makers in every aspect of schooling.

3. To integrate the arts of reading, **writing**, speaking, and listening throughout the curriculum (pp. 101-102).

Hyde and Hyde (1991) state, "An authentic constructivist mathematical curriculum is filled with realistic situations, phenomena, and interesting, relevant problems. The initial understanding of mathematical concepts students gain through these activities can become the "launching pads" for meaningfully expanding and generalizing these concepts to greater levels of abstraction" (p. 5). Constructivist teachers determine the key concepts of the curriculum at their grade level. They analyze the knowledge, abilities, motivations, interests, and schemata of the children they are teaching. Schema are interpreted as specific knowledge and how it is organized, or the way the information is structured and how it is regularly used. Once analyzed, teachers invent good problem solving activities that bring together the two parts: the domain of mathematics and the world of the students (Hyde, p. 16).

Atmospheres in authentic learning classroom are filled with active process learning by both the teacher and students. They are finding out the why, not just the how. Because of this interaction, students are making connections through communication. NCTM's 1989 <u>New Directions For Elementary School Mathematics</u> says, "a classroom that encourages communicating will be one in which there is a lot of talking, writing, and reading. Students can more readily communicate about something concrete" (p. 8).

Changing the view of assessment is a major aspect of changing the view of mathematics and mathematics learning in authentic classrooms. Evaluation is considered a compelling thrust of curriculum and instruction in this environment. Evaluation must be a partner, not an opponent.

> When assessments of students are used to provide the appropriate curriculum and instruction, then the partnership is sound. Inherent in the previous statement is the need to assess students' learning of the mathematics that is deemed important. That is, if we value reasoning and communicating, then we must include these skills in our evaluation. Thus, there is a need for new approaches and techniques to assessing. We can no longer afford to have mathematics programs that do not enhance the future of all our citizens (New Directions for Elementary School Mathematics, p. 10).

The strength of evaluation lies in the process of becoming--the transformation from what people are to what they come to be. These changes are important not just for students but equally for the professional development of teachers and the powerful nature of the ongoing curriculum (Goodman, 1989). "Authentic assessment tasks highlight the usefulness of mathematical thinking and bridge the gap between school and real mathematics" (Mathematics

Assessment: Myths, Models, Good Questions, and Practical Suggestions, 1991, p. 3).

Piaget introduced the **construction** of autonomy in 1932. Kamii (1985) defines intellectual autonomy as "being governed by oneself and making decisions for oneself" (p. 45). She goes on to state, "The constructivist view is that if children coordinate points of view, or relationships, they will develop their natural intelligence, and this development can tend only toward autonomy" (p. 50). "The socio-affective and intellectual climate of a classroom heavily influences the way children learn or do not learn any academic content" (p. 39). Many teachers of authentic classrooms see autonomy as the goal of education.

Piaget believes that mathematical power leads to acquiring personal "autonomy" (Kamii, 1984). If mathematical power is achieved by helping students develop thought processes that can be used to solve problems and to determine whether solutions are appropriate, evaluation and assessment must be holistic. The focus of learning is the process, not the end-of-unit multiple or true/false test. Meaningful assessment is an accumulation of observation, data collection, conferences, interviews, portfolios, journals, and projects.

"The foremost goal of evaluation is self-evaluation, the analysis of our own attitudes and processes so that we can use the information to promote continued growth and learning" (Routman, 1988, p. 342). Reflection enhances motivation and commitment to personal learning. According to the NCTM Standards, "Students' beliefs exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition" (p. 233). Therefore, authentic classrooms help students become authorities in their own learning processes.

Writing and Authentic Learning

Writing is a natural process that promotes active learning in authentic classrooms. Children develop positive attitudes toward writing when they are immersed in an environment that nurtures expression, voice, ability and individual growth. Children learn language as they interact with others and create for themselves internal meaning.

There are significant benefits for children writing in the elementary school grades. Tompkins (1990) gives six reasons for including writing in the elementary curriculum:

(1) Students need to learn to communicate effectively with others through written language as well as through oral language.

(2) Through writing, students learn and apply language skills.

(3) Students gain valuable knowledge about reading.

(4) Students learn critical thinking skills as they write.

(5) Writing is a valuable tool for learning, and through writing students learn content better.

(6) Writing is fun (pp. 2-3).

Haley-James (1982) stresses that writing is a mode of language that particularly lends itself to the acquisition of new knowledge. Writing requires students to analyze, reflect, and synthesize the material being studied in an authentic, thoughtful, and precise way.

Clay (1975) believes children move through writing stages developmentally, over time. Children who are the same age demonstrate a variety of strengths and needs. Glazer and Brown (1993) list five factors for nurturing this developmental process:

(1) Writing is developmental; progress in writing growth will occur in those areas that are ready to be developed.

(2) Many opportunities for writing must be continuous and ongoing.

(3) Writing growth occurs when teachers act as facilitators and guides, conferencing with students as necessary.

(4) Writing growth occurs when students participate in the assessment process, recognizing their own strengths and needs.

(5) Finally, teachers and children, together, must move back and forth between assessment and instruction in flip-flop activity as the need occurs. (pp. 84-85).

Calkins (1986) has found that guiding students through a writing process is the key to producing not only better writers but better thinkers. Guiding students through the processes of problem solving is essential if they are to become better problem solvers (Ford, 1990). This process helps students focus on the question being asked, look for essential information, and become familiar with the structure of the written problem.

Gere (1985) describes the value of writing as a thinking tool: by forcing a slowdown in thought processes, it frees the brain to play around with ideas and make new discoveries, more fully to integrate new knowledge. When given time to consciously think about their writing, students become more deliberate thinkers and able to assimilate new knowledge.

Gordon and Macinnis (1993) stress three ideas about writing: it clarifies thinking, promotes student learning, and it is a way students reveal their thinking /reasoning to their teachers and demonstrate what they know and do not know. Teachers and students both become a community of learners.

Mumme and Shepard (1990) write that when we ask students to talk or write about their thinking, we are telling them that what they say is important and has merit. When we value their thinking, students gain power and control over their learning. They become empowered.

According to Jensen (1993), four general statements can be made about writing:

(1) writing during the early years is a natural "gateway to literacy";

(2) all children can be writers;

(3) understanding writing and writers means understanding complex and interrelated influences-cognitive, social, cultural, psychological, linguistic, and technological;

(4) we write so that both we and others can know what we think, who we are (p. 290).

Children fully absorbed in practical writing have been described as acting as "true authors," taking full ownership over shape and content of what they write; such writing has been termed the "authentic expression of an individual's own ideas" (Moffett, 1981, p. 89). Authorship leads to meaning. As students establish ownership in their writing, they construct and reconstruct their thought processes, which in turn, develops understanding.

Writing in Mathematics

Mathematics teaching has, until recently, taken children through a textbook chapter by chapter with isolated one-step pencil and paper problems to solve. But with the immersion into relevant and authentic activities, students are making connections between mathematical ideas, skills, and processes in an authentic constructivist environment. Children communicate understanding through their natural language. "Writing is a powerful strategy fostering critical thinking through a language scaffold (Tompkins, 1990). Giving children opportunities to explore and refine their language in a risk-free environment naturally leads to discovery and understanding of their own learning.

A better understanding of students' thinking in mathematics is needed if mathematics education is to be improved. Research has indicated that many traditional tests and activities are not giving a clear enough picture of students' conceptual development in mathematics to assess their progress or misunderstandings. Writing is a simple, inexpensive tool for alternatively evaluating students' progress (Norwood and Carter, 1994).

"Writing experiences are valuable to children's mathematical development and to the building of confidence in doing mathematics" (<u>New Directions for Elementary School Mathematics</u>, p. 17). The more children write, the better they are at refining their ideas and expressing them. "Doing mathematics is much like writing. In each, the final product must express good ideas clearly and correctly, but the ideas must be present before the expression can take form. Good ideas poorly expressed can be revised to improve their form; empty ideas well expressed are not worth revising" (<u>Everybody Counts</u>, p. 44).

Writing provides students a way to extend and strengthen their understanding of concepts. Teachers can also use writing in math as part of the continual assessment process. Journals, portfolios, teacher observation, anecdotal records, and self-evaluation have an important place in math evaluation as well as other language processes. "Having students write about the strategies they used to work through a problem and discuss what they learned makes students aware of their own learning" (Routman 1988, p. 290).

Writing in the mathematics curriculum has at least three significant benefits for elementary students (Hembrow, 1986). It encourages learning of content area information, develops writing fluency and skills, and activates critical thinking skills. Through writing, students are making something that is abstract, concrete. As they write, students explore, arrange, connect, and reflect.

Thompson (1990) had 5th grade students writing in her math class. They wrote for 10 minutes once a week in letter form. The condition was that they must write about math and themselves as mathematicians. Students wrote how they were doing in math, their chronicle of breakthroughs, mastery of new concepts, discoveries of tricks of the math trade, personal satisfactions and accomplishments, and genuine pleasure in mathematics. Thompson wrote brief words of encouragement and made notes to herself about mini-lessons she planned to teach to reinforce concepts. Students took this assignment seriously and wrote as mathematicians, not as just a daily assignment.

Evans (1984) found, after conducting a teacher-researcher project with her fifth-grade math classes, that students who wrote during math learned mathematical concepts better than students who did not write. Her students wrote simulated letters describing procedures for solving math problems, defined and illustrated mathematical terms, and explained errors on homework or quizzes. Interestingly, Evans found that these writing activities were most valuable for her weaker math students.

Kerslake (1986) found that English students of thirteen to fourteen years relied on rote memory of previously learned techniques when working with fractions. She believed the underlying problem is that "with the exception of certain simple examples such as 1/2 and 1/4, fractions do not form a normal part of a child's environment, and the operations on them are abstractly defined" (p. 87). Unfortunately, most approaches do not deal with any type of manipulative material. Students are only interacting with models or pictures in their textbooks. When asked to perform operations with fractions rather than on fundamental concepts as partitioning, order, and equivalence, students are unable to perform the task. Students with answers available can not always judge the reasonableness of their answers and whether their answers make sense. **"Learning to communicate mathematically"** in the NCTM Standards has established a criteria to eliminate this kind of illiteracy. "This is best accomplished in problem situations in which students have an opportunity to read, **write**, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking" (p. 6). When communication is connected with models and manipulatives, mathematical power develops.

Tompkins (1990) believes that "writing-across-the-curriculum activities enhance learning. Students think more deeply about what they are learning and they synthesize and apply this knowledge. Rather than stealing time from instruction, writing across the curriculum makes instruction more meaningful" (p. 363).

Calkins (1986) wants her students to externalize forms of thinking that they will later internalize. Her purpose is not only to prod students to think, but also to teach them "habits of thought". Talking and thinking can also be done through writing, an authentic real-life, activity.

Knowing mathematics is doing mathematics. To learn mathematics, students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, predicting, in short, by being active in the world. Writing is an ideal activity for such processes (Countryman, 1992). With the increased interest in writing to learn mathematics, Rose (1989) states:

> Writing in the mathematics classroom allows students to proceed at their own rate, using their own experiences and language; increases writing fluency; combats passivity; facilitates personal engagement in learning; provides the teacher with a unique diagnostic tool; keeps a record of students' individual travel through their mathematical experiences; and promotes a caring and cooperative atmosphere through writing interaction. Also, as students write expressively to learn mathematics, their writing becomes the transactional record or expression of that process of acquisition.

> When students and teachers become the writers and readers of their own mathematical stories, they regard mathematics and each other in new ways (p. 27).

Conclusions

Teachers who create authentic processes and engage students in active constructivist learning, promote healthy environments conducive to a quality education. By giving children responsibility for their own learning, teachers create opportunities for students to become autonomous. One part of becoming autonomous is developed when children communicate through writing. They gain ownership over what they are learning by constructing and refining their knowledge, and teachers can assess their learning.

In 1989, the NCTM Standards were established to create major changes in our nation's mathematics education. While there had not been extensive research in writing in the primary mathematics curriculum before that time, there is supporting evidence for providing numerous and varied experiences in communication. The use of writing in mathematics K-12 classrooms is an important and valuable component for mathematical empowerment and assessment.

CHAPTER III

METHODOLOGY

This chapter presents an explanation and description of the research study. It includes the methodology implemented, the teacher, subjects, setting, curriculum descriptions, an example of a typical day, data collection procedures, data analyses, and summary.

The focus of this research was to obtain an understanding of fourth grade students' ability to construct their knowledge of fractions through writing; specifically, how students validate their reasoning through writing; how students view their understanding of fractions through the use of writing; and how the teacher uses children's writing to adjust instruction.

This research was a qualitative study that employed an ethnographic approach. Ethnography studies events as they evolve in natural settings and emphasizes context. Qualitative research attempts to be naturalistic and contexts are not constructed or altered. Research takes place in the normal, everyday context of the researched (Kincheloe, 1991). Patterson, Stansell, and Lee (1990) add the following:

> The real quest of a teacher-researcher is a search for those questions and methods that lead to new insights and deeper understanding. The story of such a quest is

> one of re-examination, of re-searching, looking again and again at our present understandings, our data, and the methods by which that data was gathered and reflected upon (p. 8).

Ethnographic research starts with the preference of a topic of interest. In education, this interest may often be how children are learning. "An interest in knowing students: how they learn, how they think about their learning, and ways they make meaning, is one of the main reasons and motives why teachers engage in research" (Fischer, 1996, p. 39). The researcher is interested in understanding and describing a specific identifiable group of people from a personal perspective. "Research that is classroom-based because of its ecological validity makes more significant contributions to our understanding of learners and learning within the context of classrooms than research that is not classroom-based." (Moss, 1990, p. 59). The research is an inquiry of human qualities and abilities in the teacher-researcher's classroom. It enables the teacher-researcher to appraise the significance of the data, reflect on the instruction and activities, enrich his/her own view of educational theories, and as a result, strengthen the quality of the curriculum. Fetterman (1989) states that theories do not need to be elaborate constructs, assumptions or generalizations. They can be personal theories about how a part of the world works. He goes on to say: "Typically, ethnographers do not make a grand theory explicit, because they do not automatically subscribe to one. Usually, ethnographers use theoretical models indirectly linked to grand theories to guide their work" (p. 17). The researcher views the study holistically, recording observations, taking notes, and making connections with people and their environment. He/She is looking for patterns or regularities in an effort to enhance understandings of human thought and action in a social setting. The data is continually being refined as new data is collected. Patterson, Stansell and Lee (1990) state, "Field notes have a dual purpose: to give observation data a tangible form so that it can be analyzed, and to help the researcher recall the entire scene that was observed and not just the specifics that were written down" (p. 12).

As teachers begin the path of research through inquiry, and reflection of their own work, their aims are to:

1) Discuss how reflections on teaching, and the ways teachers think about teaching, constitute a vital and important foundation for doing research.
2) Describe how teachers sort out and identify their research interests, formulate research questions, and develop frameworks that can serve as practical guidelines for their research.
3) Consider how perceptions of the potential role of research are linked to each teacher's hopes for teaching, goals for professional development, and beliefs about how knowledge is constructed (Fischer, 1990, p. 34).

The teacher-researcher tries to include data over a longer period of time. The teacher-researcher is the primary research instrument. Cochran-Smith and Lytle (1993) state, "When teachers themselves conduct research, they make problematic what they think they already know, what they see when they observe their own students as learners, and what they choose to do about the disjunctions that often exist in their classrooms, schools, and communities" (p. 64). This kind of research generates a critical analysis of student learning and teaching through teacher observation. These observations become crucial in the collection and analysis of data. Action research does not require control groups and does not impede the instructional process which is the teacher's fundamental concern (Patterson, 1990).

A teacher-researcher's professional goal is to lead an ongoing inquiry about student learning in his/her classroom. Burnaford (1996) states, "When teachers see themselves as helping to construct knowledge as it is lived daily with children, children may begin to understand that curriculum is developed in and through their own learning as they participate in researching their classroom" (p. 58).

Teacher research involves "a continuing process of self-education," and that the more teachers practice as researchers, "the practice itself is a source of renewal" Schon (1983, p. 299). Schaefer (1967) states that teacher inquiry should be a central focus of the schools. "What could be more engaging than inquiring into the myriad mysteries of the child's world or learning more about ways of fostering the individual student's search for meaning?" (p. 59). Teachers involved in teacher research examine teaching practices and learning opportunities for students.

Other studies that support teacher research include critical pedagogy research [Ellsworth, 1989; Lather, 1991; Lewis, 1989; Miller, 1987]; teaching and learning to teach research [Cochran-Smith, Garfield, & Greenberger, 1992; Dicker, 1990; Kean 1989]; theories of teacher research [Burton, 1991, Hahn, 1991, Queenan, 1988, Schwartz, 1990] (as cited in Cochran-Smith and Lytle, 1993, p.60). Journals pertaining to specific grade levels, content areas, or the teaching profession regarding teacher research are also available. Some include: Cognition and Instruction, Educational Action Research, American Journal of Education, Journal for Research in Mathematics Education, Teaching and Change, Theory into Practice, and Teacher Research. Teacher research is a valuable tool to understand how children construct knowledge and how teachers and students can both reflect on and improve their own learning.

The Teacher

The teacher, also the researcher in this classroom, had been teaching for eighteen years, with the first nine years being a traditional teacher. Her first teaching experience was to fill a maternity leave the last nine weeks of the 1976 school year in a 5th grade class in Olive, Oklahoma. She also finished her Masters of Science Degree at OSU that spring.

Other teaching experiences included: one year substituting in the Midwest City/Del City School System for grades K-8, eight and a half years in the Stillwater Public School System as a 2nd grade teacher, one year as a 4th grade teacher in the Dallas Independent School System, returning back to Stillwater with nine additional years teaching, 2nd grade for six years and three years in 4th grade.

In the summer of 1988, the teacher-researcher also had the opportunity to teach the Primary Math Methods class at Oklahoma State University. She taught this class for five years during the summer. Additionally, she taught the Intermediate Math Methods class the following three summers.

Because of continued educational classes and current professional reading, this teacher developed a constructivist approach to teaching and had incorporated it into her curriculum for the past eight years. An important instructional method for nurturing learning as a constructive process in the classroom is to create a community of scholars, "turning the classroom into a social setting for mutual support of knowledge construction, a setting that could eventually be internalized by the individual students" (Bereiter, 1985, p. 221). The transformation into a constructivist teacher and her quest for a natural research study led the teacher-researcher to research that would be relevant to her students' learning, her teaching, and own personal growth as a professional. "When teachers study and write about their work, they make their own distinctive ways of knowing about teaching and learning more visible to themselves and others" (Cochran-Smith & Lytle, 1993, p. 115).

Subjects

This study involved the teacher-researcher's intact fourth grade classroom. This classroom had 22 students, consisting of twelve males and ten females. Different ethnic backgrounds were represented in this culturally diverse classroom which included: one male Black, two male Hispanic, one female Hispanic, two male Asian, two female Asian, seven male White, and seven female White. All 22 students received parental consent to participate in the study (Appendix A). Data were collected on these students for five weeks during February, March, and April, 1996.

Setting

The setting for this study was one of the six public elementary schools located in a middle-sized Oklahoma community whose population included a major university. The classroom was one of four self-contained fourth grades. Population for this school was approximately 450.

Five five-foot tables replaced individual desks in this classroom. Approximately four to five students occupied the tables, and the mix of students was changed every two weeks. Students were given the opportunity to pick tables according to these stipulations: At least two girls and two boys would occupy each table. Girls and boys would sit diagonally from each other. Each change in seating would require both sexes to sit with someone new even though students would not necessarily move to a new table area.

The reason for using tables instead of individual desks was to emphasize and encourage community. One large container at each table housed the group's individual spirals for literature responses, math, and LAG (language, analogies, geography). These spirals, called response logs, were used to take notes, write personal feelings and opinions, document important information pertinent to each subject, and basically substituted for standard workbooks. Pocket folders for science, math, poetry, social studies, reading, and writing were also placed in the containers. These folders included student produced work, and activity sheets relevant to that specific subject. Two smaller containers at each table were used to hold pencils, crayons, individual chapter books children were currently reading, compasses, rulers, erasers, and glue. Students used materials as they needed them. If one container did not have enough pencils, children freely looked for a pencil in another group's container.

One large couch, two smaller couches, a bench, two over-sized chairs, and many large pillows also occupied the room. Students had the flexibility of working in many areas during the day.

The Curriculum

Authentic experiences are developed according to students' changing needs and interests with the student being the center of the curriculum. Instructional mathematics materials included a variety of hands-on manipulatives and activities which assist students in attaining goals proposed by the NCTM Standards. Students worked with dice, cards, unifix cubes (one inch cubes that can connect to another cube), beans, Cuisenaire rods (colored rods measuring from one to ten centimeters), geoboards, base ten blocks, and patterns blocks to be active in process learning and make mathematics meaningful.

Trade books were used instead of traditional textbooks for reading. These were selected by each student individually according to interest and objectives set by Oklahoma's PASS requirements and the local school system. Students were exposed to all genres and independently read sixteen to forty or more chapter books during the school year.

Hands-on activities in science provided opportunities for discovery, problem solving, inquiry, and the integrated processes of science. Students worked with magnets, batteries, soil, magnifying glasses, and microscopes. They completed many small group activities experimenting, observing, interpreting and testing hypotheses.

Social studies incorporated maps, atlases, encyclopedias, charts, globes, graphs, resource materials and trade books. Students completed research projects and made reports relating to Oklahoma, the regions of the United States, and other continents.

Students also attended music, physical education, computer, guidance, and library class periodically throughout the school year.

A Typical Day

Each morning the students and teacher started their day reading independently for 45 minutes. The students self-selected books according to interest. Book talks and responding to their literature in personal journals followed.

As new ideas and concepts were introduced to students, the teacher modeled reading, writing, working with manipulatives, and her thought processes through oral discussions. Students were encouraged to question, make comments, and work collaboratively, in order to assimilate ideas, create meaning, and be actively engaged in the learning process.

At 10:00 the students went to computer class. Children chose from reading comprehension and vocabulary activities, math, geography, the sciences, social studies, word processing, and keyboarding. The computer choices were manufactured by one computer company and many of the activities were similar to worksheet drills and reinforcement. Students were exposed to a variety of curricular areas, and many selected one of these program according to individual interest during each computer visit.

From 10:30-11:20 time was devoted to writing. Children worked at their own developmental level and choice was still a critical component. During this time students read, wrote, revised, conferenced with their peers and the teacher, and edited with the teacher. Students created poetry, wrote stories for wordless books, patterned after favorite books, researched topics, and in general, developed their writing through a variety of writing genres. Since this curriculum was an integration of different subjects, social studies was incorporated into the writing curriculum from 11:20-12:10. Students were introduced to researching, bibliographies, a thesaurus, and non-fiction literature while studying geography, history, sociology, economics, and global education.

Lunch and recess followed from 12:10-12:50. At 12:50 children stretched out on pillows and listened to different genres of chapter books read aloud by the teacher. During this time there were class discussions about plot, characters, setting, and author's writing style. Predictions were made about the story or characters' actions, children offered advice to characters, and they argued different points of view according to their own beliefs. The teacher read between twenty-five to thirty chapter books during the year.

At approximately 1:40 math began. Hands-on experiences with manipulatives were emphasized. The teacher read and incorporated class discussion with picture books that corresponded with current topics. After reading The Greedy Triangle by Marilyn Burns, students discussed shape, area, mathematical terms, and where these shapes were found in their own world. The teacher-researcher then introduced the students to angles and compasses. Children worked in pairs or table groups and explored math manipulatives, ideas, and solving problems. From the beginning of school students were writing about mathematics in response logs, expressing their ideas, explaining how to solve problems, and reflecting their thoughts.

A twenty minute recess break was at 2:30 and science ended the day. Developing knowledge of and competency in the process of science was created through hands-on activities. Students were given opportunities to observe, classify, communicate, question, infer, make predictions, control variables, formulate hypotheses and experiment in a variety of experiences. When studying plants, students walked around the neighborhood and collected different leaves. They sorted, classified, and identified the leaf varieties. They observed shape, color, texture, and size. They made leaf rubbings, made a class graph of leaf types and explored photosynthesis.

Sometimes there were not natural breaks between core subjects. As much as possible, these subjects were integrated with literature and different kinds of writing and the days were continuations of study, not isolated subjects taught at specific times of the day. When students studied historical fiction the teacher introduced the book Eight Hands Round: A Patchwork Alphabet by Ann Paul. This book explained patchwork quilt making which originated during the first one hundred years after the signing of the Declaration of Independence. The patterns and origins of designs were described in an alphabet book accompanied with illustrations of the quilt pieces. Students extended the reading by creating their own patterns with pattern blocks and then wrote about the different kinds of angles in their creations, which incorporated math and writing. This transferred easily into science when they studied patterns in leaves and plants.

The day ended at 3:40. Children walked or rode bikes home, a few parents waited in cars to pick them up, and a majority of students traveled home on school buses.

Data Collection

The purpose of this study was to use students' writing to obtain an understanding of fourth grade students' ability to construct their knowledge of fractions through writing; specifically, how students validate their reasoning through writing; how students view their understanding of fractions through the use of writing; and how the teacher uses children's writing to adjust instruction.

Approximately forty-five minutes to one hour each day for five weeks were spent teaching a unit on fractions and collecting data from students. All observations and data collected took place during the regular afternoon mathematics time. The last week in February, part of March, and the first week in April, 1996, were used to collect data.

Data collected consisted largely of individual student math response logs, final assessment, and student self-evaluation assessment. Daily audio recordings of oral discussion during lesson activities were also used to incorporate student understanding. The researcher made daily field notes from observations, tape summaries, and reading student response logs. This data was used to help the teacher-researcher initiate oral discussion, interpret student understanding, generate questions, and guide students through the process of thinking, reasoning, and problem solving. Gathering information through a variety of techniques (observation, interview, events, time, and documents) is termed "triangulation" and validates such research (Merriam, 1988).

Federal regulations and Oklahoma State University policy required approval of all research studies that involve human subjects (behind Vita). Permission to conduct qualitative research of fourth grade students learning mathematics through informal writing was approved by the superintendent, principal, and parents of students in the researcher's classroom. Letters were sent to parents of the subjects requesting permission for their children to participate in the study. Parents were familiarized with the intent of the study and were given opportunity to refuse participation or withdraw from the study at any time during the research.

Data Analysis

On a daily basis, the teacher-researcher read through student response logs to find out how children were expressing themselves, how they were thinking and reasoning mathematically, and to decide if they were understanding the concept. When students could adequately demonstrate manipulative use and document valid reasons for solutions, the teacher-researcher continued to the next developmental concept. Each lesson was influenced by student interaction, concept understanding, and ideas or answers appropriately expressed in written form. When the majority of students could successfully manipulate the materials, and validate problem solving reasoning orally and in written expression, the teacher-researcher introduced new concepts or extensions. An example of understanding area using geoboards would be: Students work with geoboards to find an area of six. They would manipulate a rubber band around six nails to create a rectangle. Students could verbalize what they did with the geoboard and a rubber band, and then they would explain in writing something similar to, "I placed a rubber band around six nails to form a rectangle. I know this area is six because each area between nails is considered one, so connecting each of the six nails together would give me an area of six." This would be appropriate validation. Along with discussion and teacher observation, the teacher would know the child was successful.

"During the process of investigation into the child's understanding, the teacher continuously hypothesizes about where the misconceptions may lie and explores the child's reasoning, using various tasks and counter examples. The research problem is 'solved' when the candidate feels confident of a diagnosis" (Fosnot, 1989, p. 22).

Children were evaluated individually depending on their own capabilities. Students who had been exposed to many writing opportunities expressed themselves more fluently than those who had not. Some written validations were short, some were lengthy. Appropriate responses were interpreted individually by what students were capable of doing. "We must tailor mathematics programs to meet the needs of children rather than expecting children to adjust to the demands of a special program" (Dutton, 1991, p. 2).

The teacher-researcher looked for consistencies in whole class writing and in field notes; completing the activity in the same way, such as validating equivalent fractions with the same pattern blocks; documenting in written form the information differently but arriving at the same solution; validating their reasoning with certainty similar to, "I know this answer is correct because..."; using mathematical terms correctly, like hexagon, parallelogram, and decimal; and missing pertinent information for concept understanding such as metacognitive steps to add fractions with unlike denominators. The field notes, which included summaries of response log entries, observations, and lesson tapes, were recorded in a daily journal.

The teacher-researcher evaluated the data each day for the purpose of identifying class consistencies in writing, misconceptions in concept understanding, developmental levels of understanding, and the ways reasoning was being constructed. Data was then sorted based on similar descriptive categories. Categories included: correct terms, manipulative designs in response logs, labeling fractional parts using pattern blocks, explanation of reasoning, sequential organization of their problem solving, attitudes, and concept understanding. These criteria met the NCTM's 1989 "New Goals for Students". Categories were created on a daily basis after the teacher-researcher reviewed daily response logs and listened to audio tapes. The teacher-researcher would determine through these categories if additional teaching for concept understanding was needed, or if the next concept should be introduced. Categories were also reviewed after the study was completed for the purpose of analyzing the data holistically. Cochran-Smith and Lytle (1993) state, "Teacher research is a powerful way for teachers to understand how they and their students construct and reconstruct the curriculum" (p. 51).

The final assessment which included solving problems similar to the daily lesson activities and writing written verifications, along with student self-evaluation sheets, were analyzed separately at the end of the study. They were categorized in the same way daily observations were categorized. Conclusions were drawn from the analysis and comparison of the data. The final assessment was used to determine if accumulated knowledge of the fraction concepts were validated through writing, oral expression, and manipulation of concrete materials. If the majority of students could manipulate materials, be actively involved in oral discussion, and clearly express themselves in written form, it would indicate that students had constructed a knowledge of fractions. The student self-evaluation, which was a critical self-reflection, was used to determine if the majority of students felt they understood the concepts, if the manipulatives use was important for concept understanding, and if the writing was beneficial to their learning. Both the final assessment and self-evaluation helped the teacher-researcher determine student concept understanding, written expression, manipulative use, oral discussion, and how students viewed their own understanding, to successfully construct their knowledge of fractions through writing.

Data taken from the end of the unit assessment had several purposes: to find out if students solved the problems correctly, to find out if students could successfully use the manipulatives that corresponded with each question, and to find out if students could validate their work through writing. Fosnot (1989) states, "Knowledge is constructed in the process of reflection, inquiry, and action, by learners themselves, and thus must be seen as temporary, developmental, and nonobjective" (p. 21).

Students' self-evaluation reflected feelings about their own abilities and understanding. The teacher-researcher used this instrument to explore students' feelings about their own learning, reflections they made about activities or manipulatives, and concerns about concept understanding. Kincheloe (1991) contends that as teachers come to understand how they themselves and their students construct understandings of processes, they can move together toward new frontiers of thinking. Teachers as researchers gain the ability to look at their own practices, question their own theories, and to fully understand their own situations (Carr and Kemmis, 1986). Teacher research allows teachers to establish themselves as communities of researchers devoted to the achievement and enlightenment of their students (Kincheloe, 1989).

Summary

The teacher-researcher employed an ethnographic, classroom study to obtain an understanding of fourth grade students' ability to construct their knowledge of fractions through writing in a natural school setting. She led them through a process of learning about fractions through written validations and explanations, oral discussions, and hands-on experiences with manipulatives.

Data collected addressed student learning, and generated questions for the teacher-researcher in regards to better instructional programs, teacher directed lessons, and her own learning. The teacher-researcher led students through a daily problem solving process on fractions. She initiated discussion, modeled problems, and encouraged group discovery. The teacher-researcher observed peer discussion, manipulation of concrete materials, and validation in response logs. She made field notes of daily audio recordings of oral discussion and math response log entries. This data was used to guide instruction, clear up misconceptions, and help students through the process of thinking, reasoning, and problem solving.

Kincheloe (1991) states, "Qualitative research views experience holistically, as researchers explore all aspects on an experience. The connections which tie experiences together and often provide their significance in human affairs are essential features of holistic qualitative research" (p. 144-145). Cloutheir and Shandola (1993) state that teachers must construct personal knowledge about children's understandings through their own inquiries. As researchers in their own classrooms, teachers are in a wonderful position to determine children's understandings of mathematical concepts. Teachers must access children's prior knowledge in order to link new knowledge to their present conceptual frames.

Kincheloe (1989) expresses these ideas about teacher research:

> The benefits go beyond the effort to escape the blinders of instrumental rationality and to gain insight into the dynamics of their classrooms. When teachers listen to their students and elicit their opinions and perspectives, a variety of benefits are derived. It allows for a healthier, more authentic, teacher-student relationship which inevitably leads to better communication .... The student, and in many cases the teacher, is confirmed, her experiences validated. Teacher-questioning of students induces pupils to organize their thoughts that were previously

unsystematized in order to render them understandable to the teacher (pp. 104-105).

By collecting data from response logs, discussion, audio tapes, and analyzing how children were constructing their knowledge of fractions through writing, the teacher-researcher examined student learning processes.

CHAPTER IV

RESULTS OF THE STUDY

Introduction

The purpose of this study was to obtain an understanding of 4th grade students' ability to construct their knowledge of fractions through writing; specifically, how students validate their reasoning through writing; how students view their understanding of fractions through the use of writing; and how the teacher uses children's writing to adjust instruction.

The Writing Environment

Students in this classroom began writing in all areas of the curriculum at the beginning of the school year. Their exposure to writing in prior years varied. In two of the third grade classrooms at this school, writing centered around teacher directed topics, writing paragraphs, and "easy" report writing. The third teacher had children write self-selected stories with some revising and editing. These students wrote in daily journals, and had some experience writing across the curriculum.

The students in this fourth grade class were encouraged to write freely in their own language, using invented spelling as needed to get the idea down on paper. In regards to writing in the content areas, students wrote to show their understanding of a concept, not to emphasize grammatical construction. This reasoning continued throughout the year. Students' writing abilities were quite varied during the fraction unit. Abilities depended on developmental levels and how much writing exposure students had before they got to fourth grade.

Because of the diversity of students in this elementary school, proficiency levels were quite varied. At the beginning of the year, one third of the students used correct capitalization and punctuation consistently; others did not comprehend these skills at all or used them sporadically. About six students wrote in complete sentences. Most of the students wanted to know "Is this enough?" when asked to support their feelings concerning a particular topic. Generally, students wanted to know what to write about and how long it should be. The teacher-researcher encouraged student expression by asking questions during writing time like: "What do you want to write about?" or "Do you want to talk about how your character feels this way?" When students wrote in their math response log, questions might include: "How would you validate your reasoning?" or "I think you have a good start. Are there other ways you could look at that?"

As a principle guideline for implementing writing to teach mathematics, the researcher established criteria set by the NCTM Standards that emphasized students being mathematically literate, not only during a fraction unit, but in all concept areas in regards to the mathematics curriculum.

Writing in mathematics began the second week of school. On Monday students were given a problem to take home and solve. Problems were first taken from Puzzle of the Week developed by The Whole Math Project in California. The purpose was to develop persistence in problem solving; to develop flexible thinking; to explain thought processes and solution strategies; to write about mathematics. Parents were encouraged to work with the students, ask relevant questions that would help the child find a solution to the problem, and support the child when problems would take several days to complete.

As the year progressed students developed their thought processes and ways to articulate information. This indicated that student writing had improved. They wrote only a few sentences at the beginning of the school year. As they were exposed to many writing activities and knew their ideas and opinions were valued, students wrote longer pieces, documented more information, and gave personal opinions and viewpoints with validated reasoning.

Writing in Mathematics

As children constantly wrote in all areas of the curriculum during the year, it was a natural progression to have students write about how they constructed their knowledge during a fraction unit and how they felt about their own learning.

The intent of the teacher-researcher was to meet national and state guidelines through a variety of resources and materials according to the set timeframe. The philosophy of the teacher-researcher was one in which no set daily concept expectations would be implemented. When children understood the concept and could validate their reasoning through written expression, the teacher-researcher would go on with new ideas or activities. Daily or weekly lessons depended on the students' ability to understand and successfully work the problems and write in their math response logs. The initial goal was to complete the fraction unit in four or five weeks. This study had several interruptions during the unit. The fourth day into the study was designated as Parent-Teacher Conference Day by the local school district, the second full week was Spring Break followed by two days for the teacher-researcher who took Comprehensive Exams. One other day in April was designated as Martin Luther King Day where teachers attended workshops for professional development.

For the purpose of this study, the teacher-researcher implemented the goals set by national and state mandates. NCTM's "Standard 12: Fractions and Decimals" gave several goals for the K-4 mathematics curriculum. It included developing concepts of fractions, mixed numbers, and decimals; developing number sense for fraction and decimals; using models to relate fractions to decimals and to find equivalent fractions; using models to explore operations on fractions and decimals; and applying fractions and decimals to problems situations. Appropriate activities included: paper folding to find fractional parts, using pattern blocks to find fractional relationships, and using base ten blocks to explore decimals.

Oklahoma also developed objectives, P.A.S.S., that were established to ensure a quality curriculum for its K-12 students. The fourth grade criteria for fractions state that the student will:

A. Identify, compare, and order fractional parts and decimal parts.

B. Demonstrate equivalent fractions and mixed numbers.

C. Develop computational skills in adding and subtracting fractions with like denominators and decimals of the same place value.

The researcher's instruction involved students in an active process of learning; developing an understanding of mathematics through exploration, discovery, communication, reasoning, and problem solving, independently and in small groups.

The numbers in the data collected in the math response logs varied each day depending on the number of students absent from school, students pulled out for other special programs, and students choosing not to complete the assignment.

Procedures Used in the Collection of Data

The study was completed by the teacher-researcher in a natural lesson format. Audio tapes were used to record the day's lesson and student responses. Each day usually started with a question for students to solve individually or in small groups. The questions might have been misconceptions or incorrect responses from the previous day's lesson, review or extensions of concepts learned, or explorations of new concepts which were introduced and developed that day. Daily class discussions were interjected into problem solving activities. Discussion also followed activities which allowed students to express how problems were solved in a variety of ways, hear how different students articulate and clarify their reasoning, read solutions from their response logs, make up problems and model them, question, express personal ideas and opinions, and give students time to understand concepts studied.

During lessons, students documented pertinent concept information given by the teacher, solutions and written explanations of problem solving activities individually and as a whole class, feelings, and attitudes about their writing. The teacher-researcher modeled how to work problems with manipulatives and write explanations, led class discussion and questioning, encouraged peer interaction, and observed individual progress. The day's lesson usually ended with problem solving activities using manipulatives that students were to complete on their own or with a partner. Students explained their reasoning, validated by copying designs and labeling, or wrote how they felt about particular concepts or concept understanding in individual response logs.

Since there were 22 students involved in the study, validation included a wide range of responses. The teacher-researcher categorized three areas of development in the results; students who understood the concepts well and fluently wrote explanations, students who had a fairly clear understanding of the concept but needed time to clarify their ideas, and students who had difficulty understanding the concepts or expressing themselves. Additional responses are found in Appendixes C through G.

Each day or evening after school, the teacher-researcher reviewed individual response logs for concept understanding through designs and expression. Misconceptions in designs, labeling, or writing would indicate if additional review was needed. The teacher-researcher would also review audio tapes to make field notes relating to concept understanding, student questions, and lesson format. These notes were used to guide each day's lesson.

Beginning Survey

Students were given a survey at the beginning of the first day to find out what they already knew about fractions (Appendix B). Students were asked to explain equivalent fractions and how a geoboard could be used to find equivalent fractions. Another question asked students to construct different fractional parts using pattern blocks and tell about them. The fourth question asked students to explain how fractions related to decimals and show an example using graph paper. The last question asked students to describe how to add equivalent fractions and demonstrate this process using a manipulative.

Five students told something about fractions on the first question of the survey. Three students referred to half of something; cutting a pie in two pieces, a cup used for measuring one half, and splitting an apple. One student wrote 1/2, 1/4, 1/3, but did not explain how these were to be used or what they meant. One student used the term mixed numbers and said you could add them, but no examples were given. There were no responses which showed how to represent fractions with manipulatives. All the other students said they didn't know what fractions were.

The next three questions were either left blank or had the response "I don't know". Children were told they would be given as much time as needed to finish individually. All finished within ten minutes.

Exploration of Materials

The first day time was also devoted to exploring materials used during the fraction unit. Table groups were given pattern blocks, geoboards, and base ten blocks to explore and become actively involved physically, mentally, and orally. Students made designs, created patterns, imitated creations from their peers, and built sculptures. They talked freely with one another about what they were doing and how they were creating their pieces. The researcher walked around the room, listened to discussion and asked questions. Questions included: "What are you creating?", or "I notice you are making a pattern. How did you decide to put your design together?" The majority of students worked only with the pattern blocks and geoboards.

A few started with the base ten blocks, but within five minutes changed to the pattern blocks and geoboards. All ended the day working with the pattern blocks.

Introductory Group Discussion

The second day began with a group discussion of fractions. Since no one could explain what fractions were in the survey, the researcher gave them a definition that they wrote in their individual math spirals. "Fractions can be used to describe parts of one thing, parts of a group of things, and express same relationships." They were asked where they might find an example of this definition. Responses included cutting a pizza equally, dividing a package of cookies into equal shares, how the day is spent for different curricular activities, and measuring for cooking.

Finding Halves on the Geoboard

The next activity that day was finding halves on the geoboard. Before starting the hands-on activity, the researcher asked, "How will we know when we have created one half?" Through previous experience with this manipulative, students had been exposed to the concept of area, so the question was easily answered. "Since the board is divided into 16 equal squares, you would find one half by having 8 squares on each side". Time was then given for the students to find as many halves as they could on their geoboards and reproduce their findings on dot paper. At the end of the time period, the researcher asked some students to pick one of their interesting or unusual examples. These were copied on an overhead transparency by the researcher. The students were then asked to copy three of the designs and explain if they were or were not halves.

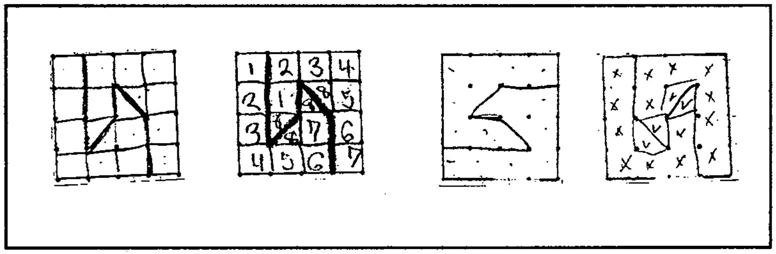


Figure 1. Finding Halves on the Geoboard

Some appropriate responses for finding halves included:

"This is 1/2 because on the right side it has 8 small squares and on the left side it also has 8 small squares."

"This is 1/2 because they have 8 exual squres and the same on both sides only there upside down"

"This is 1/2 because they have 8 squres even thou there different on both sides"

Some responses that need additional information for clarity included:

"It is right because those two triangles make a square if there put together"

"This is an even piece because it also has 8 squares each even though it looks like it doesn't"

Incorrect responses or responses that need time to develop included:

"This is not because there are dots on each side"

"Happy. It has 8"

Using the children's terminology for each individual design, 37 out of 39 designs were correct (See Table I). Two designs were incorrect. One design was divided into four sections and was designated as having 8 sections. The other design divided two pairs of squares in half. The student could not conclude that two squares divided in half would be one, so their answer was incorrect.

TABLE I

FINDING HALVES ON THE GEOBOARD

n=19

Description of 1/2 documentation	Number of Students
Outlined each box in both sections	8
Outlined each box and numbered each box	2
Outlined each box an placed an "x" in each box	1
Outlined each box in only 1 section	4
Outlined only the two sections	3
Could not identify and document 1/2	1

Finding Fourths and Eighths on the Geoboard

The next activity asked students to find examples of fourths and eighths using geoboards similar to finding halves. Only nine people completed this activity (See Figure 2). Appropriate responses for proving fourths included:

"You have 4 parts and in one part it has 4 squares."

"You make sure that there are four sections and that you have four boxs in each section."

"One fourth has four equal squares in each fourth."

"Because in each section of a 4th has 4 squares, and equal to each other."

Responses that need clarity included:

"It is a 4th of a hole."

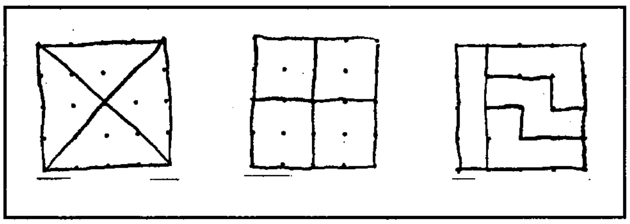


Figure 2. Finding Fourths on the Geoboard

"By showing the other person that there are 4 squares in each part."

There were no responses that showed concept weaknesses. The nine people who responded could verify fluently or that needed a little time refining their answers.

Limited examples were made for eighths (See Figure 3).

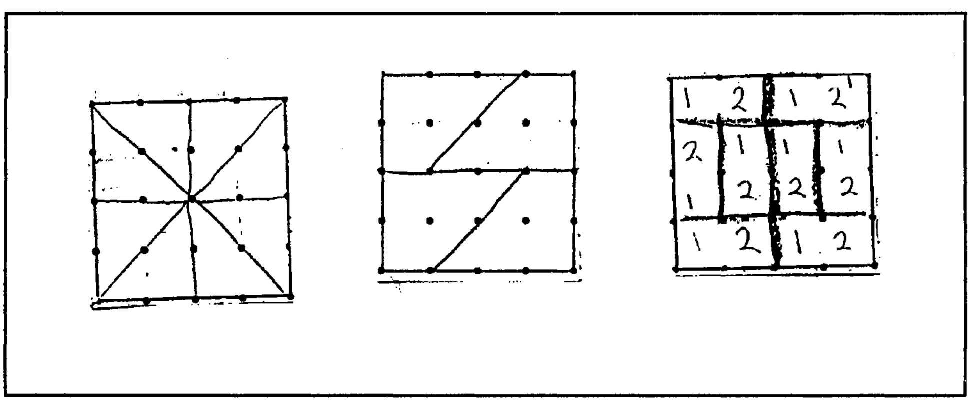


Figure 3. Finding Eighths on the Geoboard

Responses to verify eighths included:

"It has 8 parts and in 1 part it has 2 squares"

"The way you make shure you have one eighths is the same thing as one fourth. It's just that you have eight sections and two in each section."

"One eights has 2 equal squares in each eighth"

"It's an 8th of a hole"

"By showing the other person that there are 2 squares in each part"

"Because in every 1/8 there are 8 sections and in each section there's always 2 squares. Sometimes each 1/8 section are the same shape as the others".

Out of the 18 designs for fourths and eighths and using the student's terminology, all designs were correct (Table II).

Table II. FINDING 1/4 AND 1/8 ON THE GEOBOARD

n=10

Description of 1/4 & 1/8	Number of Students
Outlined and numbered each box in each section . . .	1
Outlined fourths and eighth sections	8
Could not identify and document	1

Paper Folding

The third day students were asked if a fraction could be represented with one sheet of paper. Students answered with ***"one"*** or ***"1/1"***. The researcher asked them to fold their paper into two equal pieces and color one side. When questioned about what fractional part was shaded, the response was 1/2. The teacher-researcher asked if any other fractional part could be represented. The response was ***"1/2 was not shaded"***.

Students continued to fold and orally give the fractional parts; 2/4, 4/8, 8/16, 16/32, 32/64. Children talked about patterns they saw. One example was ***"Both the top and bottom number doubled each time."*** At this time, the terms numerator and denominator were introduced. This was an example of a "teachable moment". Children were ready for new terminology or experiences that related to the current lesson. Other responses included ***"the denominator is twice as big as the numerator", "the paper is getting smaller", "the shaded area is getting smaller"***. When asked about the shaded area compared to the fractional part, the response was ***"as the shaded area gets smaller, the number gets bigger"***. Students decided that they could not fold the paper past 32/64 because of the thickness. When asked if they could tell what fractions would come next in the sequence since they knew the pattern, children responded with ***"64/128 and 128/256"*** respectively. These responses could be done mentally, but students said ***"they needed a calculator to continue the sequence since the numbers were very big"***.

At the end of the activity students were asked to write about the paper folding activity in their math response logs. Several students wanted to complete this as a whole class activity, so the teacher-researcher took their information as a group for documentation. After writing the response on the overhead, all agreed this information was appropriate and correct. The students then copied the information in their spirals. Response was as follows: ***"We folded our paper into 2 equal pieces. We colored 1/2. Fold again = 2/4. Fold again = 4/8 = 8/16 = 16/32 = 32/64."***

The teacher-researcher gave them a problem to solve using mental math, without manipulatives or paper and pencil. "Knowing the pattern in the previous problem, what would the pattern be for continuing "2/3 = ?" Students decided to double the top and bottom number each time; 2/3 = 4/6 = 8/12 = 16/24 = 32/48. Students understood the process and could abstractly apply their knowledge to new situations.

Students were also asked to finish this statement in their math response logs: "I know I have an equivalent fraction when/because..." Some appropriate responses included:

> ***"they are equal parts and they have the same squares shaded in by the amount of the fraction piece".***
>
> ***"thars a shap thats the same size and thar filld in the same aount of spas and thay ar just shapted in drfant fractions"***

Responses that needs more clarification included:

"they are two things but are the same but different

numbers and they are equal parts"

Some responses that need developmental time included:

"I can just see the other parts and divide it into equal parts".

"my parts are equile together".

Some responses referred to a specific part of the paper folding activity. An example of this response included:

"when you use 4/6 and 2/3, 4/6 has 4 squares and if you divide it into 2's to make it equal you shade in two of the three parts you divide it into".

TABLE III. DESCRIBING EQUIVALENT FRACTIONS WITH PAPER FOLDING

n=16	
Fluent written expression for whole concept	3
Needs clarification for whole concept	4
Needs time to develop expression concept	3
Indicated specific idea from concept and expressed fluently	3
Indicated specific idea from concept and needs clarification	3

Six of these students had clarified their thoughts, organized their ideas, and communicated their understanding of equivalent fractions effectively using their own terminology. The only difference is that some students wrote in reference to the whole concept and others gave an example of a specific equivalent. Six students could orally complete the activity with the paper, but not fully articulate their understanding through written expression. Three students needed more time to develop their ideas and write about them because they did not fully clarify their ideas.

Equivalent Fractions

Days 5-7 involved finding equivalent fractions with the same denominator using pattern blocks. Students were asked to find all the equivalent fractions of each individual pattern block using the same colored blocks and record in their spirals using the phrase "I found out..." (Figure 4).

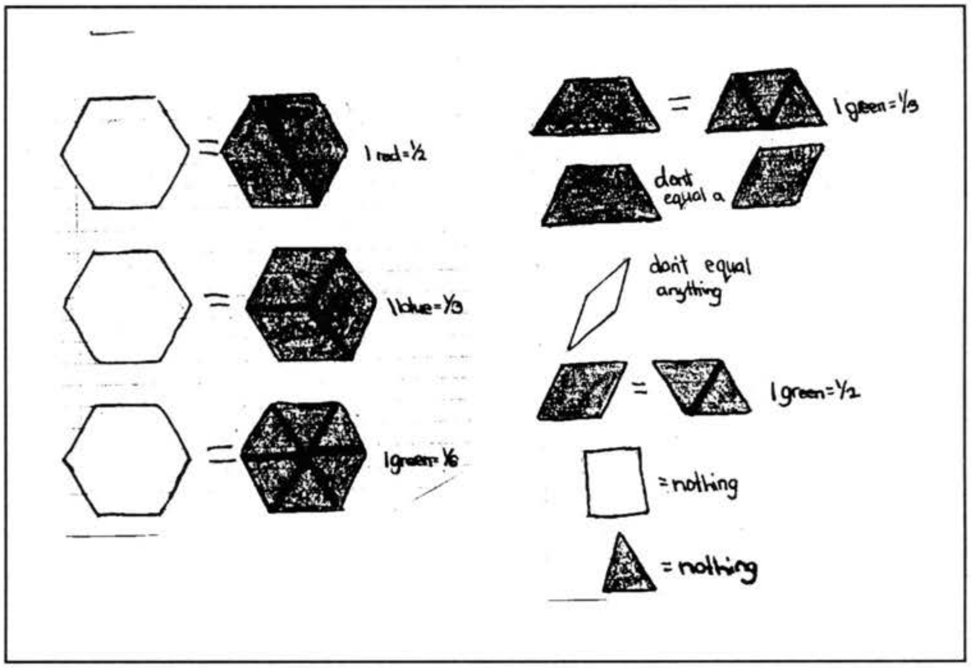


Figure 4. Finding Equivalent Fractions Using Pattern Blocks

Fourteen students found all the equivalent fractions; one hexagon = two red trapezoids, one hexagon = three blue parallelograms, one hexagon = six green triangles; one trapezoid = three green triangles, one blue parallelogram = two green triangles. The same students started with the hexagon and found all the equivalent fractions and then moved in progression by smaller size to the trapezoid and blue parallelogram. Six students worked through the assignment at random and found only partial equivalents.

Ten students also indicated the blocks that did not have equivalent fractions; the orange square, the tan parallelogram, and the green triangle. These findings came from both groups. Some students documented by saying ***"Tan = nothing"*** or ***"1 green = 1 green"***.

Discussion followed this activity. Students explained by telling the colored blocks they used for finding the equivalent fractions. The teacher-researcher introduced correct block names when color was orally presented; hexagon, parallelogram, trapezoid, and triangle. Instead of one yellow = two reds, the students were asked to rephrase with the mathematical terms; one hexagon = two trapezoids.

On Day 8, the teacher-researcher asked students to find equivalent fractions using a combination of blocks. Students reproduced the data in their math response logs. Fourteen students started with the hexagon to find different equivalents, then found equivalents for the trapezoid and blue parallelogram. Three students worked sporadically finding a few equivalents.

Many students got involved in this challenging activity with intensity. Students initially wanted to know how many possibilities there were, but when the teacher-researcher did not give out "the answer", they began to work independently and in pairs. The students who started with the hexagon and found as many ways as they could to

TABLE IV. FINDING EQUIVALENT FRACTIONS WITH PATTERN BLOCKS

n=17

From largest to smallest:	Number of Students
4 combinations for the hexagon	12
3 combinations for the hexagon	2
2 combinations for the trapezoid	11
0 combinations for the trapezoid	3
1 combination for the blue parallelogram	14
Sporadic Combinations:	
3 combinations for the hexagon	1
2 combinations for the hexagon	1
1 combination for the hexagon	1
1 combination for the trapezoid	1
0 combinations for the trapezoid	2
1 combination for the blue parallelogram	3

represent it, found the process of finding the other blocks easier. The other three students who worked randomly through the blocks did not find as many possibilities, were easily frustrated, and quit the task before finding all the solutions. Even though the teacher-researcher went through all the possibilities for each block, these students chose not to write the new information into their response logs. The results are shown in Table IV.

Combining Unlike Denominators to Make a Whole

On Days 9-10, after finding the blocks using the same color combinations, the class discussed the fractional parts of the block combinations and how they would be represented mathematically. Students documented this information together as a group. One example included: ***"Using one yellow hexagon as "1", one equivalent is one blue parallelogram, one green triangle, and one red trapezoid. The parallelogram is equal to 1/3 because it takes 3 parallelograms to equal one hexagon and we have used only one; the triangle is equal to 1/6 because it takes 6 triangles to equal one hexagon and we have used one triangle; the trapezoid is equal to 1/2 because it takes 2 trapezoids to equal one hexagon and we have used only one."*** The final equation for this example was represented as ***"1 = 1/3 + 1/6 + 1/2"***.

Twelve students also documented 1/2 = 3/6 when finding the trapezoid's equivalent using three green triangles, and 1/3 = 2/6 using one blue parallelogram's equivalent, two triangles. This information came from the group who found the equivalents through the patterning.

Comparing Fractions

Comparing fractions was introduced on Days 11-12 . Students were asked to find out which fractional part was more and document with the pattern blocks; 1/2 or 1/3, and 1/6 or 1/3 (See Figure 5).

In the first problem 1/2 or 1/3, fifteen students copied the hexagon and covered the design with one trapezoid to equal 1/2; then students covered one blue parallelogram to represent 1/3.

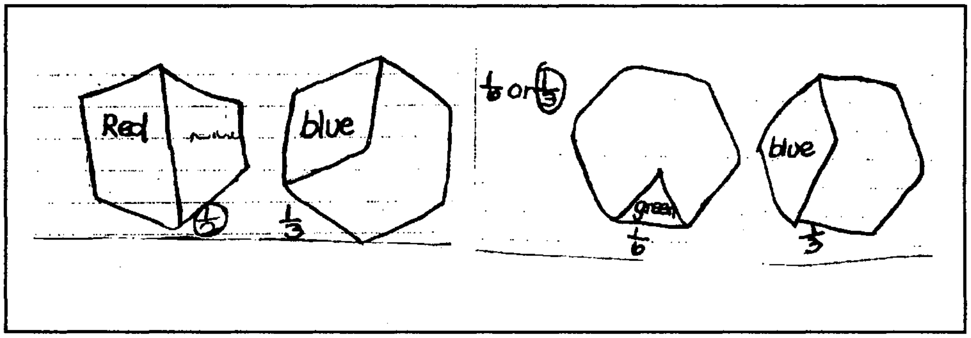


Figure 5. Comparing Fractions Using Pattern Blocks

Correct responses for reasoning were:

"It covers more area."

"Because 1/2 of the yellow is bigger than 1/3 of the yellow."

"The red covers more of the hexagon."

"Because it takes up more room on the hecksagon than the 1/3."

"Because it is half of a yellow and is larger than 1/3 of a whole."

Two responses that needed additional process time were:

"The way I know there right is that when I do that fraction I look at it and make shure I'm right."

"Because it is bigger."

After creating both fractions, they circled the larger fraction. One student had no documentation with the pattern blocks, but responded, ***"1/3 is more than 1/2."*** One student just had the questions written, but did not answer any of the questions. Ten students did not respond in writing.

The process of writing requires gathering, organizing and clarifying thoughts. Drawing and labeling solutions or constructing and labeling solutions, is one way in which students think about problems and solve them. This is one way for students to begin the process of writing and expressing their ideas; it may be a physical representation students need to visualize their solution before they can express it in written form.

When comparing which is larger 1/6 or 1/3, eleven students covered the hexagon with one blue parallelogram and another with one triangle. They circled 1/3 as being a larger fraction. Four students gave written responses. The responses were as follows:

"This is bigger becuase it takes more room on this shape."

"It takes more space because there are less pieces, which makes them big."

"Because it is bigger."

"It covers more area."

Six students did not respond with any documentation to this question.

In summary, we did this question together since only a few students completed the activity and some had difficulty finding and understanding a solution. The researcher modeled the problem on the overhead projector using overhead pattern blocks. She asked students what blocks would be needed to represent sixths. A student said, "***We use 6 triangles to get one hexagon, so we need 1 triangle to equal 1/6***." The teacher-researcher first covered the hexagon with six triangles and then took off five to represent 1/6. Then she asked students how they would represent 1/3. Another student responded, "***You need 3 blues to cover the hexagon, so we should just use one for 1/3***". The researcher followed the child's directions covering the hexagon with three blue parallelograms to represent one and then taking off two of them equaling 1/3. When asked which one was bigger, students indicated and all agreed that 1/3 was larger than 1/6 since it covered more area on the hexagon. Students were encouraged to document this in their individual spirals.

One pattern in the writing process emerged. Students who had difficulty expressing themselves did not write on a consistent basis. Their responses when asked about their validation included, "I forgot", or "I didn't have time." The students who needed the ongoing writing experiences to improve their communication were the ones who found an excuse not to complete the activity. Additional group discussion and modeling may help these children feel confident, individual teacher reinforcement is needed, and parental support to help students complete activities at home would be beneficial.

Fraction Bars

Students were given a duplicated copy of the page "Fraction Bars" from Helping Children Learn Mathematics on Day 13 (Appendix H). Students were asked to formulate ideas, recognize patterns, or generate questions in their spirals individually or collectively. The researcher would then be able to make connections between a new abstract idea that would be addressed and students' existing conceptions about fractions.

Some responses that needed development time included:

"They start big and get smaller."

"The fractions always started with one."

Two examples of a students who clarified their thinking more in-depth included:

"I notice that on the top it has 1,2,3...until that number and under the number it has the same number. I also notice that there are no 7's or 11's."

"On the top, the numbers went from 1-12 and the bodom 2-12. And that it goes from sixths to eighths."

An example of fluent written expression and in-depth observation included:

"I noticed that the bars are the same sizes as each other. What equals each other are 2/4 +2/4 = 4/8, 3/5 + 3/5 = 6/10, 5/5 + 5/5 =10/10. 1/2 is saying

> ***that there is 1 hole but cut into 2 parts. 2/2 is saying that there's 2 parts and there's 2 used. The higher the denominator the less space there is and the lower the denominator the more space there is. It also looks like a peramid from the line in the 1/2 and going down looks like a peramid. There is no straight line down the page. The fractions miss sevenths. You would think that the numbers 2,3,4,5,6,8,9,10,12 are getting larger, but the fractions are getting smaller."***

In summary, only three students wrote single answers for this activity. Many students made comparisons of fractions, found equivalents, described patterns, saw unlike denominators added together to equal a whole, discussed mathematical terminology, made abstract connections, and described fractional parts. When a whole class discussion followed, students were exposed to many facets of this concept that would not ordinarily be introduced in one lesson. Some of the discussion came from knowledge the students already had been introduced to during this unit. Other discussion gave the teacher-researcher information that students had made connections as they constructed their knowledge of fractions and were reasoning about new situations, such as, consecutive smaller fractional parts indicate a larger number that represents them. Students were encouraged to make conjectures, gather evidence, and support their logic through oral and written communication, all of which reflects the new goals documented in the NCTM Standards.

Comparing Fractions with Unlike Denominators

On Day 14, students were asked to compare two fractions and decide which was larger or if they were equal. They were given three problems to solve; 5/6 ? 2/3, 1/2 ? 3/6, and 10 ? 3/4 (See Figures 6,7,8).

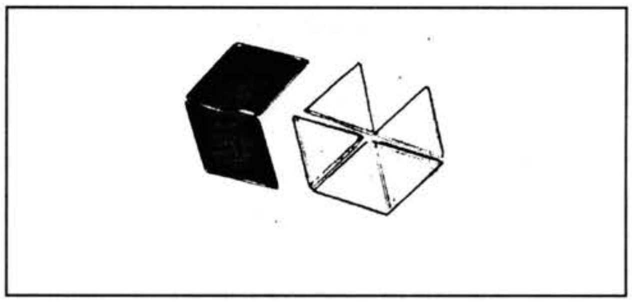


Figure 6. Problem: 5/6 ? 2/3.

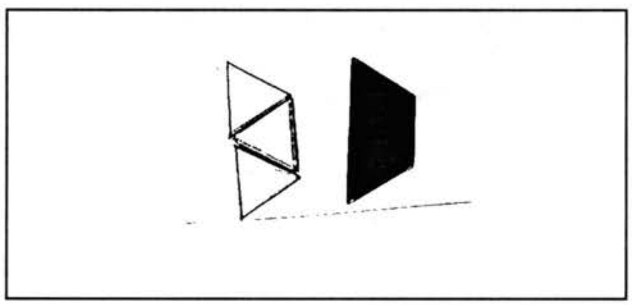


Figure 7. Problem: 1/2 ? 3/6

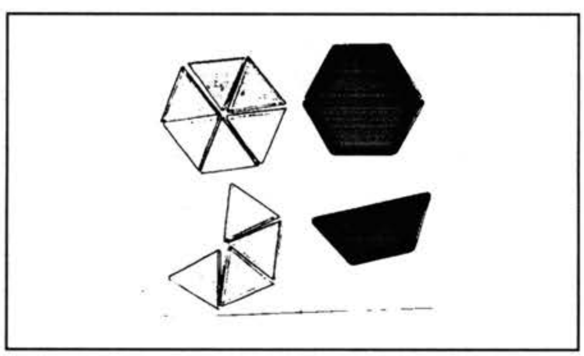


Figure 8. Problem: 10/12 ? 3/4

The teacher-researcher encouraged students to make each fraction pair with the blocks and then solve the problem. All students made the fractions with pattern blocks. Some worked alone while others felt more comfortable working with one other person. In regards to the first problem 5/6 ? 2/3, fourteen students indicated with pattern blocks that 5/6 was larger than 2/3 but did not give a response to verify. Two people said 5/6 was smaller without a written verification. One person documented 5/6 was larger by gluing paper pattern blocks that corresponded with the problem beside his answer. Another person validated 5/6 was larger with the response, ***"5/6 takes more pieces to fill a hexagon so 5/6 is larger"***. One person wrote the problem down but did not answer the question.

Thirteen students answered the second problem, 1/2 ? 3/6, as being equal without written verification or constructing the problem with pattern blocks even though they were used to find an answer. One student indicated 1/2 was greater than 3/6, but did not show why. Another student wrote the problem incorrectly, but the response ***"1/2 is greater than 2/6"*** was correct. Two people showed correct verification with the glued on pattern blocks. Two students left the problem blank.

10/12 ? 3/4 was the last problem to solve. Thirteen students said that 10/12 was the larger fraction and used pattern blocks to prove it, but they had no written verification. One student responded ***"I think 3/4 has more***

area on the hexagon". Three students did not respond. One person said, "***10/12 is less than 3/4***" but did not verify, and one person showed 10/12 greater than 3/4 by the corresponding paper pattern blocks glued to her response log.

In summary, all students were competent making the fractions with pattern blocks. Most students felt confident that their picture examples were sufficient in proving the information without written verification. Students need a variety of ways to prove their reasoning. When diagrams or pictures are accurate and appropriate, children have proven they have a clear understanding of the mathematical idea. Written responses are not always needed.

Smallest to Largest

Students were asked to place three fractions in order from the smallest to the largest on Day 15 (See Figure 9).

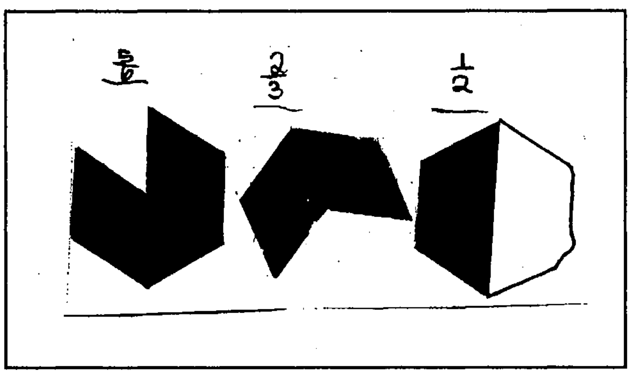


Figure 9. Problem: Smallest to Largest

Given 2/3, 1/2, and 5/6, the teacher-researcher wanted students to verify their reasoning with an explanation. Fluent responses included:

"5/6 is the smallest because it has the least area on the hexagon, 2/3 is the 2nd least fraction because it covers more area than 5/6, and 1/2 is the biggest out of all of them because it covers the most area on the hexagon."

"1/2, 2/3, 5/6 because you need 3/6 to make a whole on 1/2. It takes 2/6 to make 1 on 2/3, and it takes 1/6 to make 1 on 5/6".

Some responses that need a little clarification included:

"5/6 is a bigger number and used up more spaces, 1/2 is a half and 3/6 is a half of the denominator, and 3/4 is a lot smaller than 10/12!"

"1/2, 2/3, 5/6 I know if you put the red over the blue it's needs a triangle. I know that if you put the blue on the green it needs a triangle to. (Examples shown with pattern blocks to verify.)

Some responses that need developmental time included;

"2/3, 1/2, 5/6 Why I am good at this stuff is becuse I listn"

"1/2, 2/3, 5/6 Well I did the objecs and you could see it"

In summary, seven students could give excellent reasons for placing the blocks in order from smallest to largest. Two students did the activity with blocks successfully, but needed more time to express themselves articulately. Two other students were successful working with blocks, but could not write a valid reason for placing the blocks the way they did. The four students who did not respond did not complete the activity that day. Six students were not in school that day.

Mixed Numbers

Students worked with mixed numbers using pattern blocks after comparing fractions on Days 15-16. The class used different numerators and denominators so that students would use a variety of pattern blocks and different fraction possibilities. After exposing the class to many mixed numbers and finding solutions, students were asked to solve five problems on their own or in small groups using the blocks and record in individual spirals.

Fourteen students used trapezoids to find the answer to 5/2 (See Figure 10). They put two pairs of trapezoids together to form two hexagons with one trapezoid left over. Ten of the students wrote "1" under the design when they

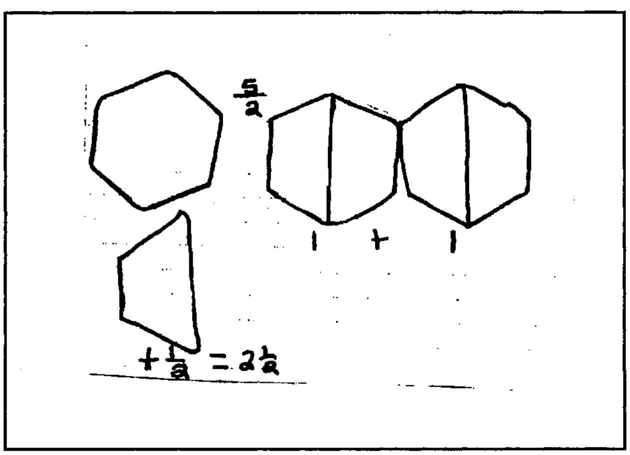


Figure 10. Problem: 5/2 = ?

made one hexagon. Four students only wrote 2-1/2 after placing five trapezoids together. All students found 2-1/2 as their answer. One student did not respond.

Blue parallelograms were used to find 7/3 (See Figure 11). Fourteen students formed two hexagons using three parallelograms each with one left over. Ten students wrote "*1*" under both hexagons. Four students wrote the answer at the end of the blocks. All students found the answer 2 1/3.

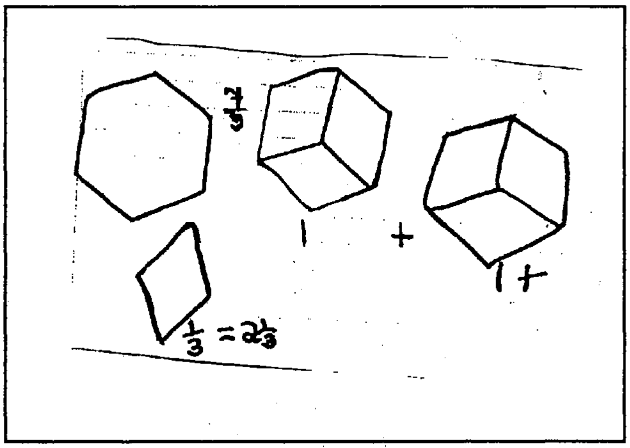


Figure 11. Problem: 7/3 = ?

Students followed the same process for the problem 9/6 (See Figure 12). Eighteen students found the answer 1 1/2

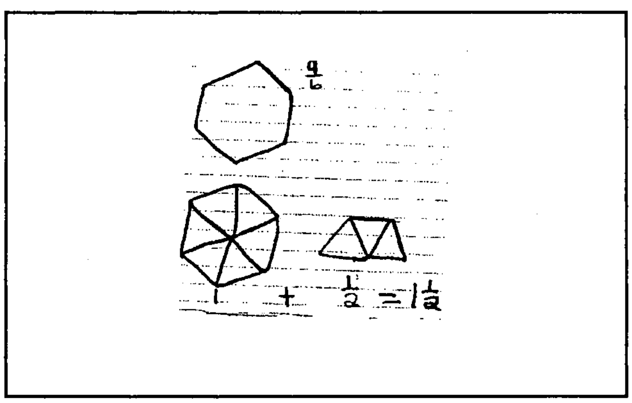


Figure 12. Problem: 9/6 = ?

using the same representation as the previous problems. They formed one hexagon with six triangles and had three triangles remaining.

The teacher-researcher then asked students to compare fractions with the same denominator. Given the problems 5/3, 10/6, and 17/12 (See Figure 13). Twelve students used the pattern blocks and answered the question at the end. Four students continued to write "*1*" under each hexagon when they created it, and two did not respond. All students

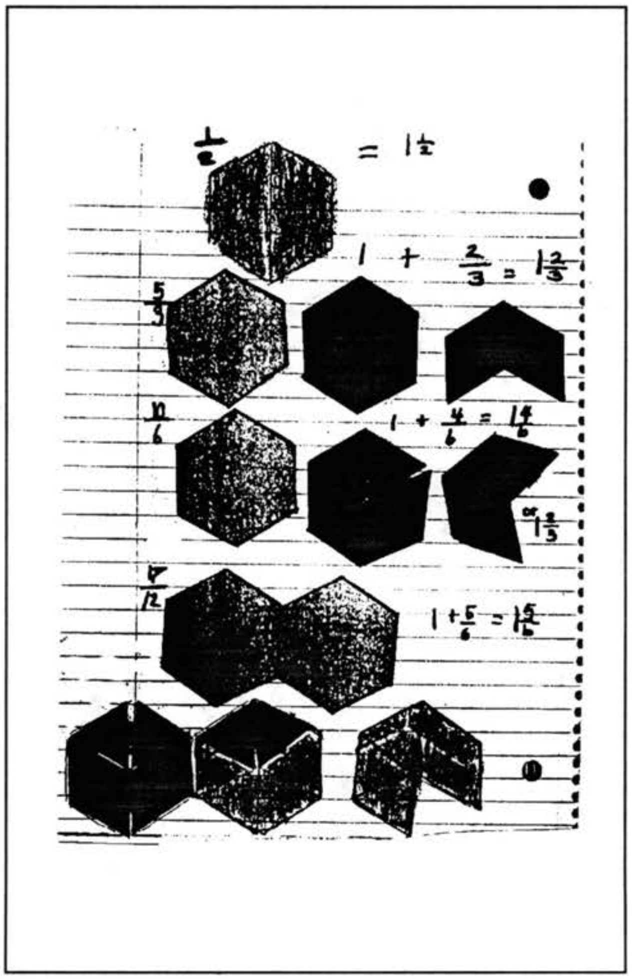


Figure 13. Problem: 5/3; 10/6; and 17/12

except the students who did not respond verified the answers 1 2/3, 1 4/6, and 2 5/12 respectively.

Written verification was added by two people. One person wrote ***"I know the answer because on the top number it shows me how much I need. And on the bottom number it show me how much you need to complete one or two hexagons."*** Another response was, ***"How I know that the bottom number tells me how many it takes to cover the hekagon. The top number tells me how many to take."***

In summary, all students were successful in creating mixed numbers with pattern blocks. Written documentation consisted of placing pattern blocks on the paper and labeling its fractional part and then adding the blocks together to equal the mixed number.

Finding Common Denominators and Adding

On Day 17, six problems of adding fractions without common denominators were given to the students after discussion and exploration of this concept. On the first problem 1/2 + 2/3, students first used one trapezoid to represent 1/2 and two blue parallelograms to represent 2/3 (See Figure 14). Triangles were found to be the common denominator in this problem, so students exchanged the trapezoid with three triangles and the parallelograms with four triangles. The seven triangles equaled 1 1/6. Eleven students exchanged for triangles. Four students kept the original blocks, placed them together to form 5/6 of the hexagon, and then exchanged the parallelogram for two triangles. This created the full hexagon with 1/6, or the triangle remaining.

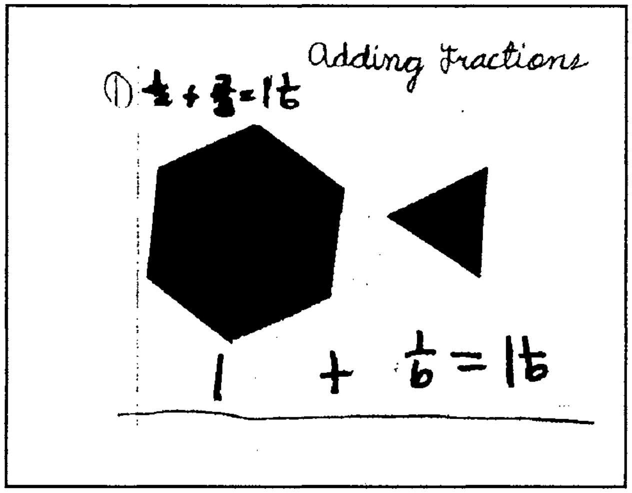


Figure 14. Problem: 1/2 + 2/3

Three students placed two hexagons together and used the original blocks. The blue parallelogram extended into the second hexagon creating 1 1/6.

Three students used the blocks, but only wrote answers beside the problems, so physical verification was not there to determine how the students achieved their answers. They could create concrete evidence, but had not developed the confidence to validate in written form.

The second problem was 2/6 +1/2 (See Figure 15). Twelve students placed two triangles, 2/6, and one trapezoid, 1/2, on top of a hexagon. 1/6 of the hexagon was exposed. Students concluded that 5/6 was the answer. Two of these students also wrote 3/6 on the trapezoid.

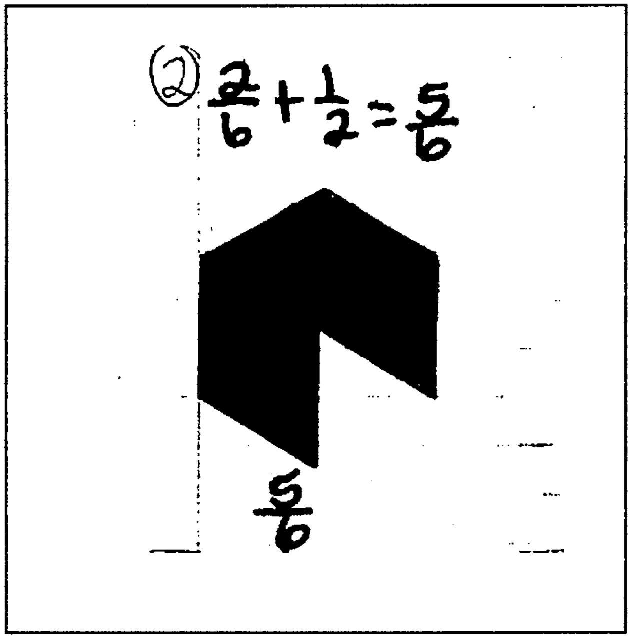


Figure 15. Problem: 2/6 + 1/2

Three students wrote just the problem and answer 5/6. One student wrote the exchanges "***2/6 + 1/2 = 2/6 + 3/6 = 5/6.***"

4/6 +3/2 was the third problem (See Figure 16). Nine students placed two trapezoids together to equal one hexagon. The other half, another trapezoid, created one 1/2 of the second hexagon. Three triangles were placed next to the trapezoid completing the second hexagon. One triangle was left over making 1/6.

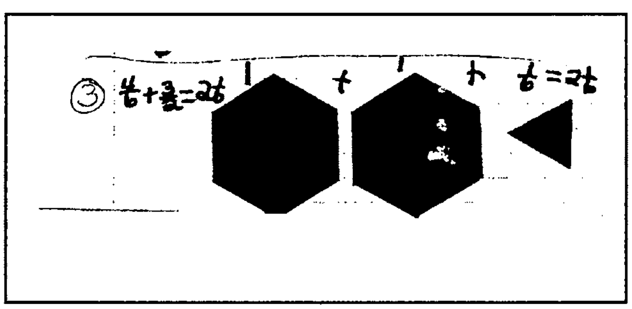


Figure 16. Problem: 4/6 + 3/2

Two students placed triangles and trapezoids on their paper, but a verified answer was not created and a written solution was not found.

One student made three hexagons. One had two triangles covering it, one had two trapezoids covering it, and the other had one trapezoid, two triangles, and one space equal to another triangle was not recognized by the student. The student had to put the pieces together abstractly, but he also got 5/6.

Three students answered 4/6 + 3/2 = 2 1/6. The same student who converted the problem before did it again; 4/6 + 3/2 = 4/6 + 9/6 = 2 1/6.

Problem 4 was 2 1/2 + 1 4/6. Twelve students made this problem using three hexagons, one trapezoid and three triangles to form another hexagon, and one triangle left to make 1/6. All answered 3 1/6.

One student who placed different blocks on the paper without clear verification did the same for this problem.

Four students who only wrote the abstract answer repeated the process and answered 3 1/6.

3 2/3 + 4 1/2 was the fifth problem. Five students used seven hexagons to represent the whole number, a trapezoid to represent the half, and two blue parallelograms to represent 2/3. Their answer was 8 1/6.

Five students used the same process except they exchanged one blue parallelogram to two so they could make eight exact hexagons with one triangle, or 1/6 left over. Their answer was 8 1/6.

Three students did not attempt the problem, while three could abstractly find the correct answer.

The sixth problem was 3 3/6 + 3 1/3. There were eight students who used five hexagons, and one blue parallelogram and three triangles together to form 5/6. Their answer was 5 5/6.

Four students completed the problem abstractly and answered 5 5/6, while three students left the problem blank. One student did not write the problem down, but completed the others correctly.

In summary, most of the students found the correct answer, but there were a variety of solutions. Four students would consistently work the problem abstractly even though they used blocks to originally work the problems. Using blocks for them at this point was not needed. They understood the concept of adding mixed numbers. A few students did not complete the activity and needed additional work time. They had an opportunity to take paper pattern blocks home to finish that evening.

Reflective Writing

Before the teacher-researcher moved into the concept of decimals, she wanted the students to look back on the activities and the writing they did and then discuss the easiest and hardest aspect of fractions up to this point. On the 17th day of the fraction unit some of the responses included:

"I think everything was hard to me because I'm not good at math".

"Adding fractions. I frist i didn't now what to do intill we went through it. The hardest has been equivalent fractions beces I didn't get the which is more part becuse if it wasent for the blcoks I wouldent of got it."

"The easiest thing has been puting numbers in order from smallest to biggest. Because I could use pattern blocks and show how I did the answer. The hardest thing was explaining how I did it because sometimes I always forget how to explain."

"The easiest thing is working with pattern blocks because you can use the pattern blocks to explain are answers instead of writing to explain your answer. Also it helps me with mixed fractions."

In summary, ten students expressed how the manipulatives were needed to get a clear understanding of the mathematical concept. Not any one concept elicited a significant number of responses that were easy or hard. All areas were considered both easy and hard. Twelve out of fifteen students gave reasons for their feelings. Only one student conveyed feelings that he did not understand the concepts.

Decimals

On Day 18, the teacher-researcher introduced the word decimal and asked students where they might see a decimal.

Several students were ready to discuss how it related to money and that using a decimal meant you would be having a part of a dollar or larger bill. One student said decimals were also found on a calculator. The researcher asked if anyone had been to check their temperature with the school nurse. One student said, ***"After she takes your temperature, she writes it on a piece of paper to give to the teacher. If it's too high you go home."***

Words relating to decimal were also introduced. The class discussed a decade being ten years; and a decathlon which is a sport that included ten different events. The teacher-researcher also asked, "If a hexagon is six sides, what do you think a decagon is?" Students quickly answered, ***"a shape with ten sides"***.

When asked how these words would relate to the word decimal, students said it related to ten in some way. The teacher-researcher explained that decimals were numbers that let us show tenths. Base ten blocks were placed on the tables for decimal exploration. Students did not understand how they would be used, but were eager to begin instead of exploring the material again. When asked why, many students said, ***"There wasn't much to do with them"***. So the class moved into an activity.

Instead of working with one cube and working up to a rod as one ten and one flat as ten rods or one hundred, the teacher-researcher started with one flat and explained that it would equal "one". Since it took ten rods to equal one flat, each rod equalled 1/10 of a whole. It took 100 cubes to equal one, so each cube represented 1/100th.

At this time, a look of frustration and lack of understanding was plastered across every child's face in the classroom and ten hands shot up to ask questions. The teacher-researcher asked students to join her in the process of finding how to get from 1/100th to one (1) in the same way they found one to one hundred in the second grade using the base ten blocks. There was a brief discussion about place value; the number of places you would use to the right of the decimal, and why there were no numbers indicated on the left of the decimal. The discussion was brief because without the concrete knowledge of putting the blocks together and understanding this process, the part of writing the numbers as they were created was still too abstract.

The teacher-researcher started with .01 using an overhead projector base ten cube. She asked students to work through the process with her, identifying and writing the decimals as they were created. The teacher-researcher added another cube .01 to the previous cube and asked what they had as a total. Students indicated that they now had .02. The class added cubes until they had to exchange ten cubes for one rod which is one-tenth. The teacher-researcher asked, "Looking at the pattern on your paper, what do you think we should write now?" One child wanted to write .010, but several students said, ***"we would be adding one place value so that couldn't be correct."*** Another child said ***"We have to write it like .10 to keep with the pattern."*** The class continued until they had to exchange again. This time all agreed that .20 was the correct answer since the pattern would stay the same. Instead of going through each added hundredth, the teacher-researcher asked if she could orally give decimals and have students create them and write them. All agreed this would be better and less repetitious. The teacher-researcher gave the decimals .42, .56, .69, 70, .81, and .96 and allowed students time to create, write, and confirm with a partner. During this time, she walked around the room and observed students working the problems. No questions were asked at this time, and all students seemed to be finding the answer easily.

At .96, students were asked once again to add one cube at a time until they reached 100 cubes. This was when students understood the process of adding one block at a time and finding the relationship between the rods and the written decimal form.

Reading and Writing Decimals

On Days 19-20, students blocked off squares 10x10 to form a square using graph paper. As a class students divided the square in ten groups of ten. When asked to shade in 4/10, students filled in four sections and recorded 4/10 and 0.4 to indicate both the decimal and fractional part. Several problems were given. The teacher-researcher continued to observe the students' activity, progress, and question. Individual and class questions included: "What fraction of the square is shaded?" "What part is not shaded?"

Students also shaded parts for hundredths and recorded. The class made a 10x10 square using graph paper. Some students chose to divide their square into 100 squares using a ruler. Others chose to leave the squares unmarked. When asked to show 20/100, all students shaded in 20 squares, but many were unsure of how to write the written form. Some students looked back to their previous recording of hundredths, others questioned each other. As a class, they decided the answer was 20/100 or 0.20. Several problems were completed and all students were successful at the end of the time period.

Students became confused when two possible answers could be used for one problem. Many students could not understand how 40/100 could also be 4/10, and they did not know when they should use one or the other, or if it made any difference. Many problems were completed as a class and the teacher-researcher using the overhead projector base ten blocks. Problems were completed successfully when done together as a class, but many could not do on their own when asked to complete some independently.

In summary, teacher-researcher modeling and class discussion finished the section relating to decimals. The class compared decimals finding which one was greater, and they placed decimals in order from the smallest to the largest. All was completed as a large group activity. Written responses during the decimal part of the study were limited. Many students could not express their ideas fluently. They regressed to single answer reasons, and often did not reveal complete concept understanding.

Assessment and Evaluation

On the 20th day of the study and before the end of the unit assessment and evaluation took place, the teacher-researcher asked students to reflect once more on the writings they did in their math response logs. She asked if they wanted to work through any problems dealing with fractions or decimals.

Students still were not completely sure of changing fractions to decimals using graph paper, so the teacher-researcher did several together as a class. The interest in understanding this concept was not as apparent as when working with concrete materials. The teacher-researcher felt that the students just wanted to move on to another concept even though they did not fully comprehend this part of the study.

On Day 21, the teacher-researcher decided to give the final assessment during the morning so students would be fresh, especially since they would be doing a lot of writing to verify answers. There would also not be any pull-out programs that would conflict with the time allotted for the assessment.

All manipulatives the students had worked with were available at group tables and the teacher-researcher encouraged using the materials to help validate reasoning and working through the problems. No time constraints were implemented during the assessment. The researcher wanted a stress-free environment. She told them, "Do your best and just show me what you know."

The final assessment was divided into five sections (See Appendix I). There were four questions in Section 1. Students were to shade in a fractional part on a 10x10 square, find an equivalent fraction in another 10x10 square, and then validate their reasoning. Section 2 included three parts. Students were asked to make specific fractions with pattern blocks, one to design on their own, label the fractions, and explain how they got their answer. There were four parts to Section 3. Students were asked to make four specific fractions with pattern blocks and one created on their own, label the fractions, place them in order from smallest to largest, and verify their reasoning. Section 4 asked students to make four specific fractions and one of their own choosing using graph paper, name the fractional and decimal part for each problem, and explain their reasoning. The last four questions in Section 5 asked students to label the fractional part of four problems with designated shaded areas on geoboard paper and explain their answers.

Section 1. Showing equivalent fractions with graph paper. Question 1: show 2/4 using graph paper and an equivalent of 2/4 using graph paper. Nineteen students could correctly show 2/4 = 1/2. Examples of written verification showing complete understanding were:

"How I know that this is 2/4 is because I used 4 blocks and colored in 2 blocks. That makes 2/4. How I know that the equivalent fraction of 2/4 is 1/2 is because 1/2 of 4 is 2 and I colored in 2 and that would be 1/2 of 4."

"I got 1/2 because on 2/4, 1/2 of the fourth would look exactly the same as 2/4."

"1/4 is a quarter of the whole and two quarters make a half and two halves make a whole."

One student could show 2/4 = 1/2, but the written responses were only partially correct. Examples were:

"2/4 is like four squares, but only 2."

"I know this is 2/4 because there are 4 squares and 2 colored in. I thought that 1/2 would be 50 because there are 100 squares. Half of 100 is 50 so I thought that would be right."

Two students said 2/4 = 1/2, but did not correctly show an equivalent. One of these students said something about the fraction but failed to discuss why they were equivalent. Her response was, ***"I don't now there were 60/100 left so I just colored in that much."***

One child made no response. This child was in the English As A Second Language Program and was unable to write and clarify his ideas.

Question 2: Show 3/15 and an equivalent fraction of 3/15 using graph paper. Thirteen students verified a written response with complete understanding. Examples:

"You can divide 15 into 3's 5 times. 1/5 is the same as 3/15."

"1/5 is the same as 3/15 because 1/5 out of 15 is 3 so 3/15 is the same as 1/5."

"Because you color in 1 section, 3 boxes out of 5 to get 1/5 = 3/15."

Five students showed an incorrect equivalent, but tried to verify an answer. Examples included:

"I know 3/15 = 12/15 because it covers 3 boxes and there is 15 altogether."

"I know this is 3/15 because you has 15 box at all and in one part you have three box so you called it 3/15. I know 3/5 is the equivalent fraction of 3/15 because you have 3 equal parts and in the three equel part you have 5 in it."

"3/15 = 65/100 Because there is 65 left so I just used that."

Three students did not attempt the problem.

Question 3: Show 6/9 and an equivalent fraction of 6/9 using graph paper. Fourteen could competently validate 6/9 = 2/3. Responses included:

"I know this is 6/9 because there's 9 squares and you color in 6 so it would be 6/9. How I know that the quivalent fraction of 6/9 is 2/3 is because you when you color 6 you split 6 you have 3 boxes and 2 parts of it colored in, so it would be 2/3."

"I got 2/3 because when you fill in 6/9 you can fit 2/3 in it. They cover the exact same amount of space."

"Because you can divide the shape evenly into 3 sections of 3 and color 2 of the sections in you get 2/3."

Three students could not answer the question correctly, but made a response. Their answers included:

"6/9 = 2/28 because it has 9 equal parts and it has 6 box. 2 /28 is equivalent of 6/9 because it has 28 equal parts and in those 28 equal parts it has 2 boxes."

"6/9 = 3/9 I thot if I change it urod it would be the same."

"6/9 6/3 both have three left over in the box."

Three students showed 6/9 twice in each 10x10 square, but did not respond. One student did not attempt the problem.

Question 4. Show 1/4 and show an equivalent fraction of 1/4 using graph paper. Only nine students could correctly validate an equivalent of 1/4. Answers included: 1/4 = 10/40, 1/4 = 3/12, 1/4 = 2/8, 1/4 = 25/100. Examples of responses with complete understanding were:

"1/4 = 2/8. If you looking as the two fractions on graph paper there are the same spaces filled up just twice as big."

"1/4 = 25/100. You can x the numerator and denominator by the same number and get a fraction equivalent to

the one your geting your answer to."

"1/4 = 10/40. I now that this is right because 1/4 you have 4 squares in all and 1 colored in. The equivalent fraction is 10/40 because you 4x10 = 40 so you color in 40 squares. Then 1 x 10 to get 10 of them colored in."

Ten students attempted to find an equivalent. Some of their answers included: 1/4 = 1/4; 1/4 = 95; 1/4 = 4/3; 1/4 = 10; 1/4 = 1/5. Five of these students thought you just had to have one part colored and relate it to any denominator. An example of a response would be: ***"I colored in 1 part of the four so it equals 1 part of the five, so 1/4 = 1/5."***

Two people did not attempt this problem.

Section 2. Creating fractions with pattern blocks. Students were to make 3/3, 1/2, 2/12, 1 2/6, 7/3, one fraction they created on their own, label the fractions, and tell how they got their answer.

Sixteen students were able to make the five fraction problems and identify how they got their answer. Some of their reasoning for 3/3 included:

"This is 3/3 because 3 of them fit in 1 hexagon."

"This is 3/3 because you separate 1 yellow into equal parts and put 3 of the blues blocks on it."

"Because a blue parallelogram is 1/3 of a yellow hexagon. So I fit 3 parallelograms into one yellow hexagon and they fit to make 3/3."

Responses for 1/2 were:

"1 trapezoid is the same as 3/6 and 3/6 is one halfe of a hexagon."

"Half of a hexagon was a red trapazoid so I traced a hexagon over that a trapzaoid and there was my answer."

"How I know this is a 1/2 is because 2 reds equal a hekagon and your only using 1 so it's 1/2 of a hekagon."

Verifications for 2/12 were:

"How I know this is two twleves is that I know that 6 triangles equal 1 hexagon and 6x2 equals twelve."

"This is 2/12 because if you has one hexagon you have to complete one hexegon with 6 green and if you have 2 hexegon you will need 12. If you want to create 2/12 you will need 2 greens."

"Six green triangles fit into one yellow hexagon. I knew 12 triangles would equal two yellow hexagons so I put down two yellow hexagons and 2 green triangles inside one of the yellow hexagons."

Correct written comprehension for 1 2/6 was indicated by these responses:

"This is 1 2/6 because 1 means 1 hole hexagon. 6 greens fit on 1 yellow hexagon so I used 2 greens extra."

"I know this is corect becuse 1 is a hexigon, and 2/6 is 2 more triangles."

"The way I know this is 1 hexagon is a whole and I know that 6 triangles equals 1 hexagon and I'm using 2 triangles together for 1 2/6".

Appropriate written responses for 7/3 included:

"I know this is 7/3 becuse it takes 3 blues to make a hole and I hade to use 7 blues so my ansawr is 7/3."

"First, its a mixed number. What you do is the 7 means you have to use 7 of the same kind and you have to create with those 7 blocks 3 to make a hole like the bottum number says. Then if you add it up you get 7/3."

"The way I know this is 7/3 is that the top number is telling you to use 7 and the bottom number is trying to tell you that you reagarly use 3 to make a whole. I had to think. And I found out that you had to use 7 parallelograms and then put them together."

A response for a fractions they created on their own included:

"1/3. How I know this is 1/3 is because you use 3 to cover 1 hekagon and you only use 1 green so you get 1/3."

"4/6. I know this is corect becuse 1 hexagon uses 6 greens, and 4 of them equals 4/6."

"1 3/3. This is 1 3/3 because 1 means 1 hole and 3 blues fit on 1 yellow hexagon so it also = 2 holes."

Three students did not complete the last two questions of this section, but could correctly validate the first four problems. One student made the correct fractions with pattern blocks and labeled them, but did not write a written validation. One student did not complete this section at all.

Section 3: This part asked students to make 1/2, 2/3, 5/6, 12/12 with pattern blocks and verify the order from smallest to largest in written form.

Eighteen students could identify, correctly place the fractions in order from smallest to largest, and write appropriate reasoning. Examples included:

> ***"1/2 is the smallest because to form a hexagon I need three triangle. 2/3 is the next because to form a hexagon I need two triangles. 5/6 is the next because to form a hexagon I need one triangle. 6/6 makes one full hexagon. And 12/12 is the last because the two hexagons are complete."***
>
> ***"1/2 go's first because it is smaller area than 2/3 and 2/3 is smaller than 5/6 and 5/6 is smaller than 12/12, 13/12 is bigger than all. So 1/2 is the smallest than all."***
>
> ***"I put this in order because every next one covered more space than the other."***

One child wrote, ***"The way I know is that when you look at the shape you can tell if it is the biggest or littlest."*** This student could effectively work through the problems, but had difficulty expressing himself in written form that adequately verified the problem.

Two children who previously did not respond to earlier questions also did not complete this section.

Section 4. Students were asked to make 1/4, 3/10, 4/5, and one fraction they created on graph paper, change them into decimals, and explain their reasoning. Eight students could complete this section with complete understanding. Written validation included:

> ***"1/4 = .25 because it is divided into 4 equal parts so 1/4 of 100 is 25."***
>
> ***"There are 10 tens in a hundred but the fraction only says for 3. So I colored in 3 tens. And I know that it is .3 because if it only has one digit after the decimal it is a ten. Instead of doing .30 I did .3. It's a better way for me."***
>
> ***"50/100 = .50 How I know this is .50 is because your dealing with 100 so you have to have a 2-digit number like 50 and put after the (.) because 50 is not a hole so you put it behind the (.) It is one half."***

Nine students did not understand the concept of changing fractions to decimals, but created reasoning for their answers. Six of these people could identify and validate one or two examples. Some examples of unclear understanding included:

> ***"1/4 = .1 My reasening is that the decimal of 1/4 is .1. Why because I learned to count the squares for the decimal."***

"3/10 = 3/10 because the 3 is on the left side and the 10 is on the right side." (Two people had this reasoning for every problem).

"4/5 = .04 I now this is the right answer because there are 5 squares in all and 4 of them are colored in."

This last student has the fractional part 4/5 correct, but had not realized that the fractional part does not have to be documented as five sections with one part shaded in.

Four students did not attempt this section.

Section 5 asked students to explain four designated fractional parts on geoboard paper. Seventeen students could identify 1/4, 3/8, and 7/16 with correct written validation. The teacher-researcher intended for the last fraction to be considered 1/2 since the paper was divided into two equal sections with eight boxes in each section, one section shaded and the other not shaded. Ten people validated it being 1/2. Six students indicated this would be 8/16 and correctly validated their reasoning.

Responses for the first three questions were very similar. They discussed the number of sections altogether and how many parts were shaded in. Validation for the last answer being 1/2 included:

"There are eight equal squares on each side so it's 1/2."

"Because you need 8 squres in each section to equl a half and there is 8 squers in each section = 1/2."

"It's 1/2 because 8 are shaded and 8 are not shaded."

Reasons for 8/16 were:

"Because there is 16 parts on the geoboard and eight parts are colored in, so you have 8/16."

"Eight squares out of the whole are colored so its 8/16."

"I counted the boxes. I counted just the colerd in ones there is 8 of them. That is how I came up with my ancer 8/16."

"This square is eight sixteenths because eight squares are shaded out of sixteen. It can also be one half because eight squares are left."

The response that showed a little more in-depth thought was this one: ***"1/2, 2/4, 4/8, or 8/16. It is a half because there are eight squares in each."*** This child has internalized this concept, related it to equivalent fractions, and is functioning abstractly. Four students did not complete this section. A summary of the final assessment is shown in Table 5.

Child Self-Evaluation

The last evaluation during this unit of study was completed by twenty students after they had completed the assessment. One student was sick for four weeks off and on during the study and did not feel comfortable taking the assessment until she had more hands-on experience and time to fully understand the concepts.

TABLE 5. FINAL ASSESSMENT SUMMARY

n = 21

Creating and ordering fractions	Students
Correct and valid response	18
Partially correct	0
Tried to explain, but not correct	1
No response	2
Graphing Fractional and Decimal Parts	
Correct and valid response	8
Partially correct	9
Tried to explain, but not correct	0
No response	4
Identifying Fractional Parts on the Geoboard	
Correct and valid response	17
No response	4
Creating Equivalent Fractions Using Pattern Blocks	
Correct and valid response	15
Partially correct	4
Tried to explain, but not correct	2
No response	0
Creating fractions using pattern blocks	
Correct and valid response	16
Partially correct	4
No response	1

There were five questions which asked students to evaluate themselves (See Appendix J). Questions included:

(1) What did you learn about fractions that you didn't already know?

(2) Was there one manipulative that helped you understand fractions the most? Explain.

(3) What tasks were easy, hard, just right, or too difficult? Explain.

(4) Which writing activities helped you the most? the least? Explain.

(5) Does the final assessment accurately show your ability to construct and understand fractions? Why or why not?

To answer question 1, all students listed individual concepts they had studied during the fraction unit even though not every concept was addressed on each child's paper.

Pattern blocks was seen as the manipulative that helped them understand fractions the most with fourteen responses. Three people indicated that all manipulatives were needed, and three people said the geoboards were the most helpful. Reasoning was based on the need ***"to see it to understand it"***.

Easy tasks and the number of responses included: comparing fractions (3), mixed numbers (1), all activities (2), equivalent fractions (3), geoboard activities (2), and all pattern block activities (9). Most of the students' responses exhibited positive feelings of enjoyment when they worked with the pattern blocks.

Just right tasks and the number of responses included: comparing fractions (1), mixed numbers (2), equivalent fractions (2), all the activities (1), and using manipulatives (14). Responses indicated that the manipulative use helped them understand the concept.

Tasks considered too difficult and the number of responses included: mixed numbers (1) and decimals (13). Most of the students said that decimals were hard to learn. Five students felt that it was hard until they learned how, and then it became easier.

Question 4 asked students to discuss the writing activities that helped them the most or least. Responses and the numbers included: taking daily notes (4), explaining how to get an answer (2), make a design and tell about it (1), class discussion and then writing about it (2), all of them (8), no responses (3). There were not any responses for writing that helped students the least. The students who responded felt that writing helped them learn about fractions. The majority of students said that when they had questions, they went back to their previous work and it helped them.

The last question asked students if the final assessment showed their ability to construct and understand fractions. Fourteen students indicated that the test showed their true ability. Several people said they had done their very best, two people said it did even though they did not understand the decimal part, and the others said they had showed the researcher what they could do. Three people answered "***no***" to this question but validated their reasoning by saying they had not done their best. Another "***no***" response indicated that the student had more knowledge but was not asked about it, and two people did not respond to this question.

Summary

Analysis of data indicated that the majority of students successfully made equivalent fractions using graph paper, created fractions using pattern blocks, created and ordered fractions from smallest to largest, designated fractional parts on a geoboard, and wrote written responses that reflected a clear concept understanding of fractions. One section was not as successful. Graphing fractional and decimal parts and verifying answers indicated less than half of the class fully understood this concept. Six students identified and validated one or two examples, and four students made no attempt to work this section.

Through opportunities to observe, model, discuss, manipulate materials, ask questions, reason, solve problems in a variety of ways, students were able to clearly validate their work through informal writing during this fraction unit. Students were allowed time to reflect on their writing, how they solved problems, and then clarify what they knew.

Most students were successful in solving problems when they were able to use concrete materials, but there were still a few students who did not have the conceptual knowledge needed to process information and make connections whether or not they worked with concrete materials. One student clearly needed more time learning the English language since he had difficulty understanding the discussion and assignments.

Writing about their mathematical understanding significantly increased from the beginning survey to the end assessment. Students were able to communicate orally and in written form about their ideas, problem solve, and reason in a variety of activities in conjunction with manipulative use.

The majority of students felt that the assessment showed how they constructed their knowledge of fractions. This unit of study provided a sequence of mathematical concepts developed for appropriate cognitive development. A positive learning environment was created that enabled students to develop their own understanding of fractions through the use of manipulatives, meaningful problem solving, small and large group activities, communication, and writing.

CHAPTER V

SUMMARY AND IMPLICATIONS

The purpose of this study was to obtain an understanding of fourth grade students' ability to construct their knowledge of fractions through writing; specifically, how students validate their reasoning through writing; how students view their understanding of fractions through the use of writing; and how the teacher uses children's writing to adjust instruction. Because this research was designed to generate a broader and deeper understanding of some aspect of people's lives, it required the use of an ethnographic study (LeCompte & Preissle, 1992).

Summary

The teacher-researcher taught a five week fraction unit to one self-contained classroom of 22 students in the Spring of 1996. One-hour time blocks for math were designated for teaching. The unit was interrupted by one local professional day, a parent-teacher conference day, and two days needed by the researcher for doctoral comprehensive exams.

A survey to find out what students already knew about fractions was given on the first day. Only five students made statements about fractions in some way; halves of something, a cup for measuring. A few numerical symbols were used, but not discussed. After exploration of materials, the students worked independently and in small groups through a fraction unit which included: finding halves, fourths, eighths, equivalent fractions, comparing and ordering fractions, mixed numbers, reading and writing decimals, and finding equivalent fractions and decimals.

Students used math response logs to document daily activities, verifications, concepts, processes, reflections, and feelings. At the beginning of the study the pattern of single sentence answers were prominent similar to the example, ***"This is 1/2 because it has equal parts"*** when referring to halves on a geoboard. Some answers did not always fully answer the question, such as, ***"This is not 1/2 because it has 6 on one side and 12 on the other side".*** This student did not explain what "6" meant or what constituted one half. Students had not made any personal connections to the concept, so reasoning at this stage was vague and limited.

Documentation included having students copy the designs of their problem solving on individual response logs, or glue paper pattern blocks of their solutions on response logs. Students labeled them and showed how they related to the problem. An example was: ***"1 hexagon = 2 blue parallelograms and 2 triangles"*** when finding equivalent fractions. The majority of students could easily verify this way of reasoning throughout the study. Because of varying developmental levels, a few children validated with limited written expression, while others continued to clarify their ideas fluently. Sometimes children felt that their labeling was significant verification, especially since a variety of writing was used during the study which asked students to give detailed explanations of their reasoning.

By the time equivalent fractions were discussed, students were starting to make critical observations and verbalize in greater detail. One example was during comparisons of equivalent fractions. One child wrote, ***"Thars a shap thats the same size and thar filld in the same amount of spas and thay ar just shapted in drfant fractions."*** This student discussed the concept of equivalent areas with different denominators even though these mathematical terms had not been introduced yet. It was evident that children with this kind of response were analyzing the problem, organizing information, and developing the background to be able to formulate theories based on accumulated knowledge.

At the beginning of the study, students had difficulty finding equivalent fractional parts using unlike denominators. The teacher-researcher modeled how to do more examples using overhead pattern blocks, and they completed the activity together. Background knowledge was still very narrow. Students needed continual reinforcement and more interaction with the manipulatives.

During freewriting about fraction bars, seventeen students wrote more than single sentence statements. Nine students gave explicit detail as in the following example:

> ***"I notice that on 1/2 it has a two on the bottom, and on 1/3 there is a 3, and it keeps on goin like that. I also saw that on halves and up to twelfths it looks like this 1/3, 2/3, 3/3. Also I notices that on halves and up to twelfths that like you have fourths and the last fraction will be 4/4."***

Comparing fractions with unlike denominators, ordering fractions from smallest to largest, mixed numbers, finding common denominators and adding followed. Students were competent making and verifying fractions with unlike denominators. They transferred the blocks on to paper, labeled them, and through observations, expressed their equivalence. But only seven students could successfully order the blocks and give written validation during the initial activity. The researcher reviewed this concept until students were confident ordering and giving written reasons.

The majority of students were successful making and verifying mixed numbers and finding common denominators and adding them. They created the fractions, traced them on their response logs, and labeled them for verification.

Students were varied in their feelings about each concept studied. Not any one concept elicited a significant number of responses that were easy or hard. Only one person stated that he did not understand the concepts. This student came into fourth grade feeling like a failure.

Positive remarks about his successes were freely given out by the teacher, but his attitude about his learning never changed.

Reading and writing decimals, and changing decimals to fractions were difficult for the majority of students. Not as much time was given to this concept and the activities were completed totally by group activities and discussions. Students could shade in fractional parts of tenths and hundredths on graph paper. They could answer teacher-researcher directed questions when working as a whole, but students floundered when asked to solve problems on their own and explain the reasoning. Writing was mostly symbolic relationships, such as, 4/10 = 40/100, with limited written expression. Many students could create fractional problems on graph paper, but could not transfer this information into a decimal. These students need more hands-on experience, a variety of activities, and more time to make this abstract concept relevant. More time to internalize this concept would give students an opportunity to clarify their thoughts and written expression.

The majority of students successfully completed sections 1, 2, 3, and 5 on the final assessment which reflected a clear concept understanding of fractions. Less than half of the class fully understood graphing fractional and decimal parts and could not give adequate answers for verification. Students need additional time to process this concept and be involved in more concrete activities to develop understanding. The self-evaluation, also verified the frustration students had with this concept. Using their terminology, thirteen students stated that ***"decimals were hard to learn."***

Sixteen to eighteen students in each area of the assessment, except decimals, were able to solve the problems using concrete manipulatives and verify in written form. Mathematical understanding of fraction concepts as shown through metacognitive writing, free writing, and reflective writing, significantly increased for these students during the unit of study.

Students spent from one hour to three hours completing the final assessment and self-evaluation. A small recess during the three hour time span gave students time to refresh their bodies and take a break in the writing process. The teacher-researcher did not believe the assessment and evaluation would take as long as it did for some students. When asked about their feelings, they stated, ***"We don't care how long it takes. We want to do our best".*** Most of the students finished the final assessment in 1 1/2 hours. The self-evaluation took approximately twenty minutes for most students. The students who did not spend as much time writing during the study, also did not spend as much time on the assessment and self-evaluation.

Eighteen students indicated in the self-evaluation a variety of writing activities were beneficial. The majority of students felt that documenting information in their response logs was the most helpful for them to refer back to when they had questions. Other writing included: note taking, explaining how to get an answer, and class discussion before writing. Eight students felt that all the writings were beneficial. This substantiates the reason for writing in a variety of ways and it establishes personal ownership. Composing processes take time to develop. As we communicate with others, reason, and solve problems, connections are made and mathematical writing continues to develop.

Implications

Based on the documentation of activities, observation, response logs, assessment, and self-evaluation, several implications for teaching writing in mathematics emerged.

1. Before students can write about their learning in mathematics, they first have to make connections with materials that are concrete. Exploration time is needed to give students an opportunity to investigate, discover, discuss, create patterns and designs using manipulatives; to become familiar with the materials before instruction begins; to make connections between concrete materials and mathematical concepts. Students have to become personally involved to make it meaningful. Reflective writing about fractions indicated the usefulness of manipulatives when solving problems. By the end of the study the majority of students wrote how manipulatives helped them solve the problems, and show how to get an answer, and without them ***"you can't think"***, as stated by one child.

"Programs that provide limited developmental work, that emphasize symbol manipulation and computational rules, and that rely heavily on paper-and-pencil worksheets do not fit the natural learning patterns of children and do not contribute to important aspects of children's mathematical development" (Curriculum and Evaluation Standards for School Mathematics, p. 16).

2. Since instruction and assessment are closely linked together, authentic assessment includes a variety of documents that assess students' learning. Appropriate developmental instruction provides opportunities to deepen students' understanding of mathematical concepts and make math personally meaningful and relevant. The use of manipulatives and hands-on activities during instruction fosters mathematical power. Appropriate instruction also includes students' attitudes, behaviors, oral and **written** communication. The teacher-researcher felt that students played a major role in their own learning by committing themselves to being actively involved. Twenty students stated in the final assessment that it was an authentic document that revealed active student involvement, students' mathematical knowledge about fractions through writing. These students knew that the final metacognitive assessment was just one of the ways to evaluate their growth and understanding through previous class discussions. The other assessments came from observation, peer interaction, discussion, written responses, and manipulating materials. Collins (1988) states:

> Inquiry teaching forces students to actively engage in articulating theories and principles that are critical to deep understanding of a domain. The knowledge acquired is not simple content, it is content that can be employed in solving problems and making predictions. That is, inquiry teaching engages the student in using knowledge, so that it does not become 'inert' knowledge like much of the wisdom received from books and lectures Collins' study (as cited in Jaworski, 1994, p. 10).

3. Writing in mathematics class, whether it is freewriting, metacognitive, or reflective, is beneficial for some students in their reasoning, problem solving, clarifying one's thinking, and understanding.

Several patterns emerged for some students in their writing, the first being developmental. At the beginning of the study all students documented information with single sentence answers using their own natural language and spelling. Identifying and labeling was successful for most students, and some chose to write their verifications that way consistently. The top level of successful writing included: complete responses, identifying all the important aspects of the problem, using clear and fluent explanations, and often times going beyond the requirements with additional information. Students at the next level could give supporting evidence but not include all of the characteristics mentioned above. Students writing at the lower level may have contained incomplete information or diagrams and unclear explanations.

Many students could correctly use mathematical terms during oral discussion, and many would express them when writing verifications. When first using the pattern blocks, student called the blocks by their colors. On the final assessment, over half of the students verified their reasoning using the terms: hexagon, trapezoid, triangle, and parallelogram. These students were making connections and becoming mathematically literate.

The majority of students were sequential in their writing. When finding all the blocks to equal a hexagon, students logically went through all the possibilities for one block before moving on to another. When discussing their findings, many indicated the pattern they found. Once they knew what one block represented, they could easily trade it for its equivalent. Moving sequentially through concept development enhances a knowledge-based foundation and lays the groundwork for inquiry and additional concept extensions orally and in written expression.

As the unit continued, most students took more time developing their thoughts and expressing themselves logically and reasonably. Answers became longer and validations became more refined. Writing indicated understanding of mathematical ideas through coherent and clear written expression. For some, fluent writing increased. But more significantly, for the majority of students, it enhanced student learning. An example of this was one student's response to placing the blocks in a specific order. She wrote, ***"The order should be 1/2, 2/3, 5/6 because you need 3/6 to make a whole on 1/2. It takes 2/6 to make 1 on 2/3, and it takes 1/6 to make 1 on 5/6".*** This child has realized the need for a common denominator, sixths; she tells what is needed in each block to create a whole; and she has internalized that if you need more space to make a whole you have a smaller area. Verbal discussion later confirmed this statement.

At the beginning of the study, a few students did not always want to take the time to write according to their capabilities. The need to "hurry and get finished" was the goal. As students established ownership of their own learning, writing became a useful tool, making meaning out of concept activities. This is one example of the difference in writing taken from the beginning and the end:

"This is right because they have equal squares" to...

"Because there is 16 parts on the geoboard and eight parts are colored in, so you have 8/16 or 1/2."

"Writing in math class gives children the opportunity to reflect on their work, think about mathematical ideas, and deepen their understanding" (Burns, 1995, p. 179). It also helps the teacher assess student understanding and instructional effectiveness.

Some students could manipulate the materials successfully, verbally express their reasons for validation, but not clarify their thoughts fluently in written form.

These students worked well with a partner, together organizing their ideas and then writing verifications. Class discussion and verification also give students opportunities to hear others articulate their thoughts and see how they begin the process of expressing their ideas and feelings in written form.

A few students still need more time making connections and many opportunities to articulate their ideas. An example of a student needing more time to develop their writing included: ***"Happy. It is 8"*** when verifying equal parts on the geoboard, instead of, ***"This is 1/2 because there are 16 squares on the geoboard and each half will have 8 squares".*** These students' responses were consistently limited. I observed these students closely, watched them work with the materials, listened to their interactions with other children, and asked them relevant questions that kept me current in their personal growth. These students need additional time, review, instructional techniques, manipulatives use, writing opportunities, and peer interaction to understand a concept. The Professional Standards for Teaching Mathematics, 1991 states:

> All students engage in a great deal of invention as they learn mathematics; they impose their own interpretation on what is presented to create a theory that makes sense to them. Students use new information to modify their prior beliefs. As a consequence, each

student's knowledge of mathematics is uniquely personal" (p. 2).

4. When abstract concepts do not have concrete manipulatives to help children develop understanding, personal meaning is not likely to occur. Students have difficulty when working with abstract concepts. Making picture representations of 1/4 on graph paper and converting it to its equivalent decimal is not concrete for students. On the other hand, all of the other concepts that were studied in the unit could be visually represented with manipulatives. Writing and using pattern blocks allowed students to actually create the fractions and make connections with the abstract written form. Geobands were placed and easily moved on the geoboard to concretely make fractional parts.

Writing can not assist in validating processes not internalized. It will only be abstract symbol relationships. Writing is a form of communication; gaining access to student thinking. When students have misconceptions about mathematical concepts, writing will show that students have limited knowledge in that area. Those students who consistently write fluently, revert back to short answer reasons, and narrow explanations until they make mathematical connections. Piaget stated that learning resulted from children's actions related to their external work. Bauersfeld (1985) continues this theory and indicates that learning comes from a variety of internal and external sources. He states,

> Teacher and students act in relation to some matter meant, usually a mathematical structure as embodied or modelled by concrete action with physical means and signs. What he/she tries to teach cannot be mapped, is not just visible, or readable, or otherwise easily decodable. There is access only via the subject's active internal construction mingled with these activities. That is why the production of meaning is intimately and interactively related to the subjective interpretation of both the subject's own action as well as the teacher's and the peers' perceived actions in specific situations Bauerfeld's study (as cited in Jaworski, 1994, p. 27-28).

5. Sharing writing helps students to know and understand the advantages of different methods of solving problems. The whole unit of study encouraged investigating and writing fractions as a class and in small groups. When the teacher-researcher asked the question, "How do you know 5/6 is larger than 2/3", there were opportunities for multiple answers and ideas. This type of open-ended questioning facilitated oral discussion. Many children expressed the same idea, yet stated it differently. Answers included, ***"With 5/6 you have five triangles that cover the area and 2/3 means only four triangles would cover it"***, and ***"It only takes one more triangle to cover the hexagon with 5/6, and it would take two with 2/3"***.

> When students work, discuss, and write together, they have more opportunities to verbalize their thoughts, get reactions from others to their thinking, and hear other points of view. Listening to other students' ideas can help those who don't understand gain new perspectives on the topic. Explaining their reasoning can help students who do understand to cement and even extend their understanding (Burns, 1995, p. 147).

6. Children need to have a voice in their own learning and assessment. When students are actively involved in hands-on activities and making sense of math through writing, it is natural for them to become involved in creating part of the curriculum and evaluating their own learning. Students were given opportunities to make problems for the class to solve and write about during class discussion. They created problems to solve using pattern blocks, geoboards, and base ten blocks, and then modeled how to do them using the overhead manipulatives. Students worked in small groups solving problems, explaining how to work problems, and discussing how to express their ideas in written form. They had an opportunity to create their own examples on the final assessment and to show written validation of their learning. Having an opportunity to write a self-evaluation also included them in having a voice. When students are allowed and encouraged to write about their own learning, it lets them know the teacher values their opinions and feelings. Students gain confidence because they know they are significantly involved

in their own learning and assessment. Klum (1994) reports that students base the way they think about their abilities on their beliefs and attitudes about mathematics. When we help students intently look at their own mathematical thinking processes, we can enhance awareness of their abilities and cultivate positive feelings toward the subject.

Further Activities

Developing an effective, positive learning environment that is sensitive to every child's abilities, learning styles, interests, and demographics is a continuous endeavor for every educator. Keeping current professionally in teacher research, educational reading, professional development, attending conventions, and finding new materials requires teachers to be actively involved on a continuing basis. The following implications for future work are made and based on the documentation in this study and the researcher's journey towards life-long learning.

1. Conduct further research on 5th grade students' writing in reference to understanding decimals. Another year's exposure to this abstract concept would give students more opportunities to reason, problem solve, communicate, and make mathematical connections using a variety of manipulatives and instruction. The 5th grade <u>P.A.S.S.</u> curriculum emphasizes comparing fractions to decimals, decimals to fractions, and ordering decimals and fractions as students develop number sense and theory. It also includes demonstrating the use of common percents which is

beneficial in seeing this fraction/decimal relationship. Involving students in computers and calculators will strengthen computation and problem solving activities.

2. Investigate students in this study again in a later grade and document their developmental progress. As students continually develop and modify conceptions about their learning, data would indicate how developmental levels would change over time. Another perspective would be comparing these same students' writing: those placed in constructivist classrooms that continued to write as part of their mathematics curriculum, and students who were not.

3. Provide yearly staff development training demonstrating appropriate writing activities that correspond with content areas. Teachers need workshops to show them how to implement a variety of writing activities that enhance the child's knowledge and understanding in all areas of the curriculum. Taking into account how children construct their knowledge and that children develop at different rates, examples of grading holistically should be included in the staff development workshops. The uses of metacognitive writing, freewriting, and reflective writing were just several ways children can communicate and express themselves. Other writings could include: poetry, patterning after predictable books, describing what it takes to be a good partner (Burns, 1995), creating puzzle clues and exchanging with another classroom, and writing invitations to parents for a "parent-child math night" at school.

4. Foster a community of math writers in a whole school setting. Children come to kindergarten excited about learning. As children are immersed into literacy acquisition, teachers can become facilitators nurturing a community of learners in the classroom. As a school project, teachers could establish a math portfolio in kindergarten. The writing placed in the portfolio could be contributed by both the teacher and the student. The portfolio then passes through each grade level with continued writings being placed inside. At the end of the 5th grade, the portfolio is bound and given to the child. Not only would it be a wonderful memento for the parent, but an excellent developmental process of the child's mathematical growth over six years.

As teacher-researchers we are continually striving to provide a curriculum and classroom environment that is exciting and enriching to students. Incorporating writing into all areas of the curriculum strengthens the child as a whole. Learning is seen as a continuum, not isolated subjects with designated times for finishing. When we establish that philosophy, we are creating an enjoyable path for life-long learning that children will want to take.

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APPENDIXES

APPENDIX A

PARENTAL CONSENT FORM

I, ________________________ give my permission for my child, ______________________, who is a student in Mrs. Sallee's fourth grade class at Westwood Elementary School, to participate in the research study: A Fourth Grade Classroom's Understanding of Fractions Through Informal Writing, conducted by Adeana Sallee from Oklahoma State University. I understand that information gained for this study will be confidential and the identity of my child will remain anonymous. I understand that I have the right to withdraw my child from the study at any time and that the study will result in no cost to me. I understand that I will receive a copy of this form to keep and that my child will be verbally advised of the study.

I may contact Adeana Sallee, Dr. Gretchen Schwarz, or University Research Services, 305 Whitehurst Hall, Oklahoma State University, Stillwater, Oklahoma, 74078, at any time regarding the study.

____
Parent Signature

____
Date

APPENDIX B

BEGINNING SURVEY

Please tell me what you know about fractions.

1) What are fractions?
2) How could a geoboard be used to find equivalent fractions?
3) Using pattern blocks, construct different fractional parts. Copy them on this paper and tell about them.
4) Explain how fractions relate to decimals.
5) Using the attached graph paper, show an example of how fractions relate to decimals.
6) Describe how to add equivalent fractions. What manipulative would help you demonstrate the concept? Give an example.
7) How would you order fractions from smallest to largest?
8) What else do you know about fractions?

APPENDIX C

Responses for Finding Halves

"This is not 1/2 because on the left side it has 10 and the right has 6 squares"

"This is 1/2 because it has two equal parts"

"This is not because it has 6 on one side and 12 on the other side"

"This is not 1/2 because "

"The first one is wrong because it does not have eight squares"

"This is 1/2 because if you turn one side on the other side there exactle the same"

APPENDIX D

Responses for Paper Folding

"if you have 4/6 and you have another name for it but it must be the same size and it must be the same size that is colored in".

"you have 2/4 because there are four suqers and 2 are collored in".

"because 1/4 equals 1/2" (This person colored 2 out of four boxes in one example to show 1/4 and 2 out of four boxes to show 1/2)

"if you have 20 boxes and you color 5 in you just need to know how many times the number 5 will go into 20".

"you have 2 rows of 15 boxes you can color in the same amount of boxes on each row and have different numbers of what it is like".

"they are equal".

"when I have equll peces of any #".

APPENDIX E

Responses for Fraction Bars

"I notice that on halves they were split into halves. And they did that for every number. Like twelves were split into twleves. I notice that the number 11 isn't betwen tenths and twelves. And the number 7 isn't between sixths, ninths."

" I notice 12/12 equals one hole. I saw that 10/10's equals one hole. (This student continued the pattern with all the fraction bars) The student concluded with "Every two numbers equals one half."

"I noticed that on 1/2 it has a two on the bottom, and on 1/3 on the bottom there is a 3, and it keeps on going like that. I also saw that on halves and up to twelfths it looks like this 1/3, 2/3, 3/3. Also I noticed that on halves and up to twelfths that like you have fourths and the last fraction will be 4/4. Also I noticed that there were not sevenths and elevenths".

"There are many fractions that are the same 2/4 = 1/2, 3/6 = 2/4, 3/6 = 4/8, 4/8 = 5/10 and 5/10 = 6/12. There is a pattern like 1/2 2/4 3/6 4/8 5/10 6/12. If you see the top numbers it is just 1 2 3 4 5 6. If you see the bottom numbers you see that it is always times two 1/2 2/4 3/6 4/8 5/10 6/12".

"6/12 = 1/2, 5/10 = 1/2, 4/8 = 1/2, 3/6 = 1/2, 2/4 =

1/2. On the nineths it dosn't have a half in lease you cut the 5th square in half and the same with fifths and thirds. The denominator is counting up to how ever many squares are in the bar. At the end of the bar it has the same numbers. From havles to eights it would make a peramid if you cut it out."

"I found out the numinator starts with a lower number on the top instead of the higer number exapt when the numbers are equal."

"I notice the bars are splet into groups and they have # in them. I notes it tells the anser."

"The number on the right are the same on the bottom and top. You can make them the same if you take the lines out of the middles".

"I sey no seven and no eleven"

"I see that all the numbers on the end have the same number like 2/2. I see halves, thirds, forthes, fithes, sexths. It goes that way to twelve."

2/12 = 1/6, 2/3 + 1/4 + 1/9 = 1 hole; 2/3 + 1/4 + 1/9 = 1 hole; 1/2 + 1/3 + 1/6 = 1 hole. Thay look like steps bigst to smallst.

APPENDIX F

Responses About Smallest to Largest

"5/6, 2/3, 1/2" was shown with pattern blocks. This person showed the smallest fraction piece left uncovered, so the fraction was backwards.

"2/3, 1/2, 5/6 because if you cut a pie in hafe and you keep one half you don't have as much as you would have if you kept 2/3 or 5/6 of the pie".

" 1/2, 2/3, 5/6 Why?" Three children then proved the problem with paper pattern blocks.

"1/2, 2/3, 5/6 The way I know is that when you do 2/3 and 1/2 you see that 2/3 is bigger".

"1/2 - this takes only half the space of the hexagon, 2/3 - this takes a triangle more than the 1/2, 5/6 - and still, this one takes another triangle more.

"1/2, 2/3, 5/6 How I know that 5/6 was the most was because 5/6 takes up more room on the hexagon then 2/3 or 1/2. 2/3 is more then 1/2 because it takes up more room on the hexagon then 1/2. So that lives 1/2 to have the most amount of room on the hexagon."

"1/2, 2/3, 5/6 I now it because 1 red = 3 green, 2 blue = 4 green, 5 green = 5 green."

APPENDIX G

Responses about Reflective Writing

"The easiet thing is adding."

"I think adding numbers is the easiest".

The est think has been equflant frachins because you do just about the same think on both sides. The harst thing has ben mex up nabers."

"So far the easiet thing is the geoboards because all we did was find all the diffent way to make 1/2, 1/4, 1/6, and 1/8 with rubber bands. The hardest part is comparing fractions because as soon as I found the shape it is hard to explain."

"The easiest thing for me is mixed numbers because I am really starting to understand when you have to split stuff apart and seeing what they equal."

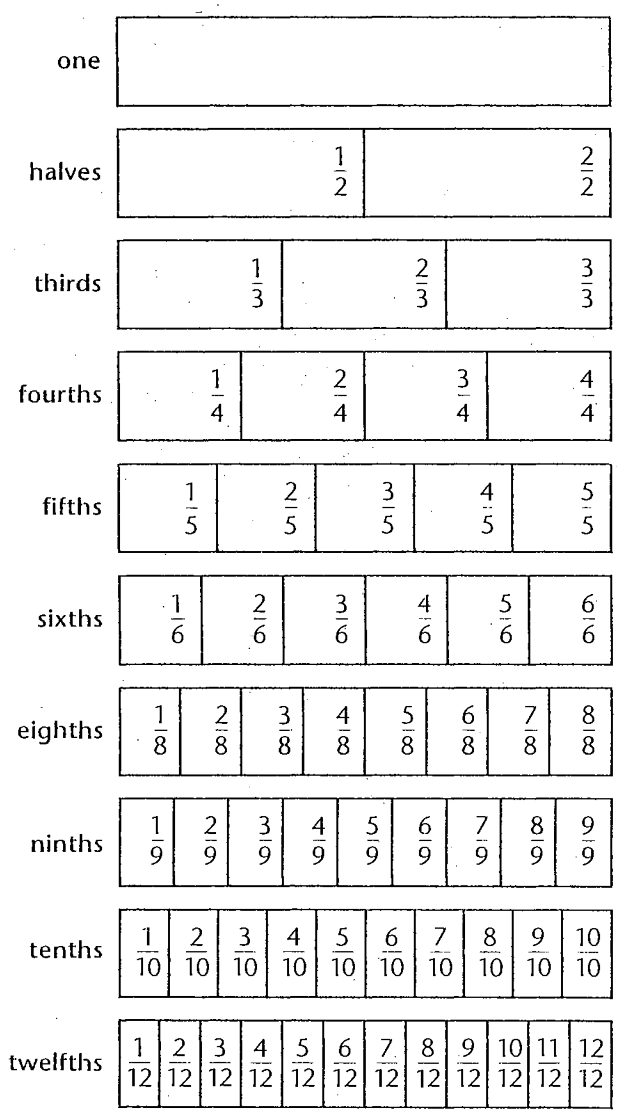
"So far the easiest part is working with pattern blocks to figuer it out. The hardest is two hexigons and mixed blocks"

"The easiest think was understanding which fraction were bigger. I enjoyed the mixed numbers and they were 2nd easiest. They seemed fun to add up and they were. I understood it well"

"The easy fraction is 1/2 because it's more bigger. The hardest is mixed numbers because without patern blocks you can't think"

APPENDIX H

Fraction Bars



APPENDIX I

FINAL ASSESSMENT

Fractions

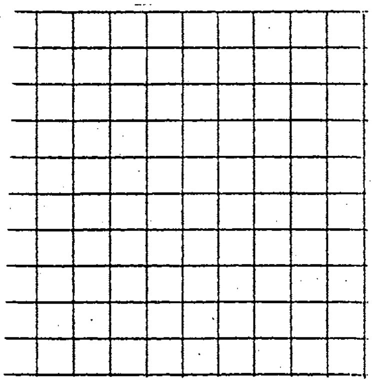
Name________________

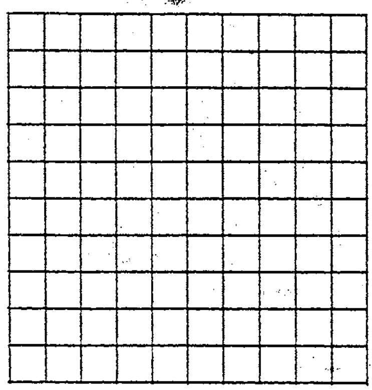
Demonstrate how to find an equivalent fraction using graph paper.

1) Show 2/4.

An equivalent fraction of 2/4 would be ______

Explain how you can verify this.

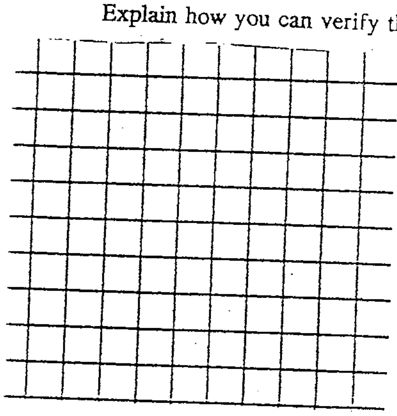


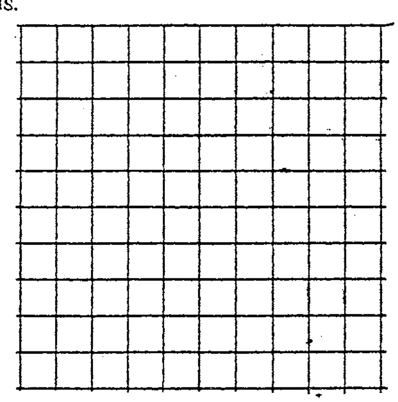


2) Show 3/15.

An equivalent fractions of 3/15 would be ______

Explain how you can verify this.

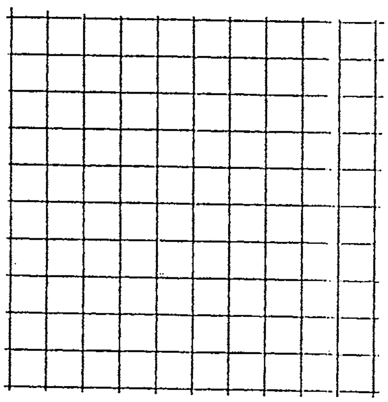


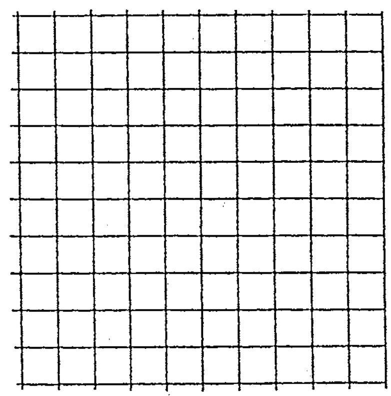


3) Show 6/9.

An equivalent fraction of 6/9 would be ______

Explain how you can verify this.

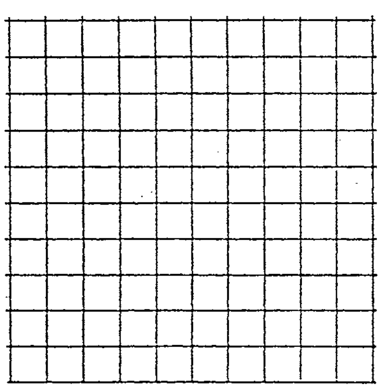


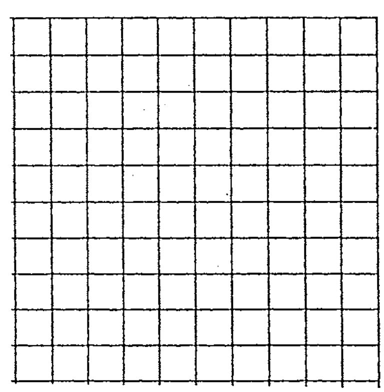


4) Show 1/4.

An equivalent fraction of 1/4 would be______

Explain how you can verify this.





5-10) Create these fractions with your pattern blocks.

3/3, 1/2, 2/12, 1 2/6, 7/3, one you create

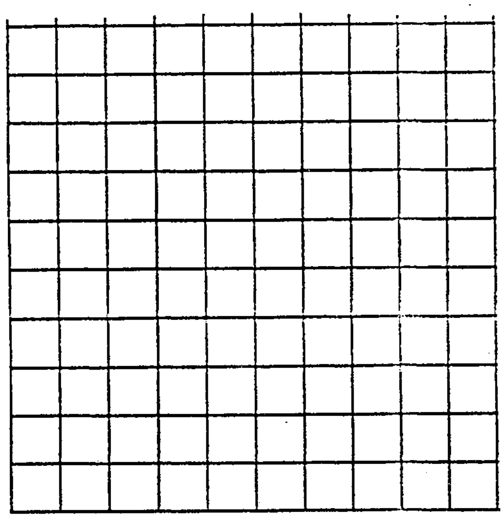
Label your fraction. How did you get your answer?

11-15) Using pattern blocks, create these fractions in order from smallest to largest.

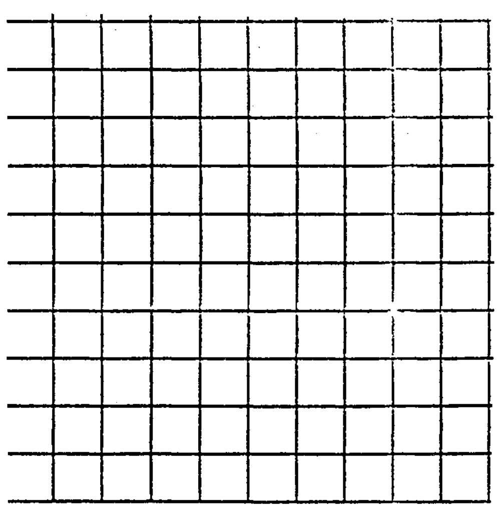
1/2, 2/3, 5/6, 12/12, Verify your reasoning.

16-20) Using graph paper, name the fractional and decimal parts for each problem.

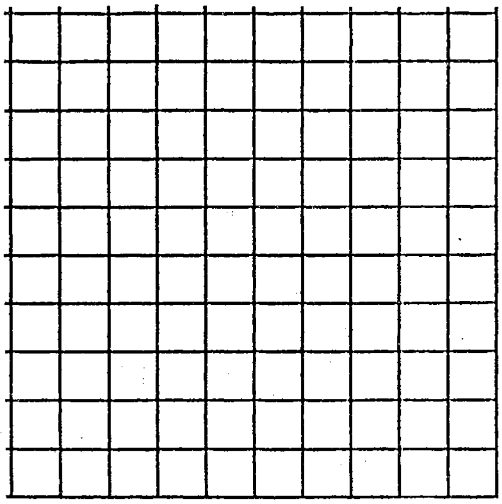
1/4, 3/10, 4/5, one you create Explain your reasoning.



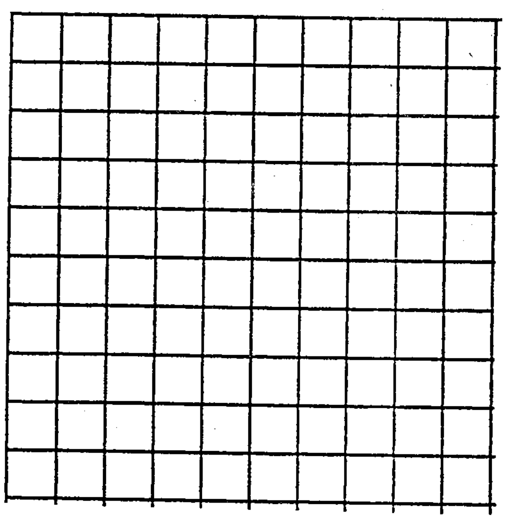
1/4 = ____



3/10 = ____



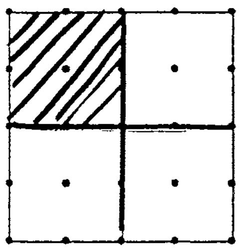
4/5 = ____



____ = ____

21-24) What is the fractional part of these areas on you geoboard?

Explain your answer. (Shaded part)



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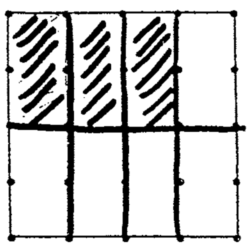

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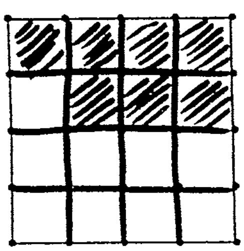

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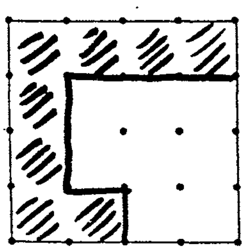

= ___

APPENDIX J

CHILD SELF-EVALUATION

1) What did you learn about fractions that you didn't already know?
2) Was there one manipulative that helped you understand fractions the most? Explain.
3) What tasks were easy, hard, just right, or too difficult? Explain.
4) Which writing activities helped you the most? the least? Explain.
5) Does the final assessment accurately show your ability to construct and understand fractions? Why or why not?

VITA

Adeana Blakley Sallee

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Doctor of Education

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OKLAHOMA STATE UNIVERSITY
INSTITUTIONAL REVIEW BOARD
HUMAN SUBJECTS REVIEW

Date: 02-02-96 **IRB#:** ED-96-065

Proposal Title: A FOURTH GRADE CLASSROOM'S UNDERSTANDING OF FRACTIONS THROUGH INFORMAL WRITING

Principal Investigator(s): Gretchen Schwarz, Adeana Sallee

Reviewed and Processed as: Exempt

Approval Status Recommended by Reviewer(s): Approved

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Provisions received and approved.

Signature: John H. Wyckoff
Chair of Institutional Review Board

Date: February 12, 1996