STRUCTURAL BREAKS AND PURCHASING POWER PARITY FOR THE COUNTRIES IN THE GROUP OF SEVEN

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CHAPTER I.

INTRODUCTION

Many balance of payments and exchange rate determination models are built on the assumption that purchasing power parity (PPP) holds. Therefore, purchasing power parity is one of the most important theories in international finance. New innovations in time series analysis have renewed interest in testing the theory of purchasing power parity. However, much of the evidence using these new innovations, including unit root tests and cointegration techniques, is inconclusive. This study will use the bilateral real exchange rate to test purchasing power parity among the Group of Seven (G-7) countries. The G-7 consists of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States.

The real exchange rate is considered stationary if it returns to its long-run equilibrium value after a shock or short-term deviation. If the real exchange rate fails to return to its long-run value, then a change in the real exchange rate will be permanent. Therefore, for PPP to hold, an autoregression of the real exchange rate must provide evidence that the autoregressive "root" or coefficient is significantly less than one. If there is no significant evidence to reject a unit root, the real exchange rate is nonstationary and the theory of purchasing power parity does not hold.

While long run, low frequency (annual) tests have often found evidence of PPP,[1] shorter time period, high frequency (monthly or quarterly) tests have not[2].

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[1] For example Abuaf and Jorion (1990), Crownover (1994), Diebold, Husted and Rush (1991), Enders (1989), Glen (1992), Johnson (1990), Kim (1990), Perron and Vogelsang (1994).

This study focuses on the relatively short time period of high frequency (monthly data) from 1957 to 1993, to test purchasing power parity. This study will also test for the presence of heteroskedasticity in a procedure which endogenously looks for a structural break while testing the real exchange rate for evidence of a unit root. For comparison purposes, this study includes tests of PPP on low frequency (annual) data for the period 1900-1994.

Objectives of the Study

The objectives of our study are threefold:

1. To determine whether the theory of purchasing power parity holds when structural breaks in the mean and trend are considered for the countries of the Group of Seven.
2. To determinate if purchasing power parity holds with grouped heteroskedasticity and structural breaks in the mean.
3. To determine if the Group of Seven countries' real exchange rates conform to the the theory of purchasing power parity with structural breaks and contemporaneous correlation.

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[2] See for example Abuaf and Jorion (1990), Ardeni and Lubian (1991), Cheung and Lai (1993), Corbae and Ouliaris (1988), Enders (1988), Fisher and Park (1991), Frankel and Rose (1995), Kim and Enders (1991), MacDonald (1993), Manzer (1990), Mark (1990), Phylaktis and Kassimates (1994), Pippinger (1992), and Whitt (1992)..

Plan of the Study

The purpose of this study is to report further evidence on the theory of purchasing power parity. Chapter 2 provides a review of the literature concerning purchasing power parity and unit root tests using the real exchange rate and structural breaks. Chapter 3 explains the methodology and includes the graphs of the real exchange rates for the G-7. Single equation models are included in this Chapter to examine the current methods and endogenously search for structural breaks while testing for a unit root. Further tests are developed for the evidence of a unit root and structural breaks when the Estimated Generalized Least Squares approach is used. Due to the lack of power in single equation unit root tests,[3] multivariate techniques are used to increase the power of these tests. The restricted Seemingly Unrelated Regression estimator is explained as well as the tests for unit roots and contemporaneous correlation. A discussion of the Monte Carlo techniques used to generate the probability distributions for the test statistics concludes Chapter 3. The results and test statistics for the monthly data are presented and discussed in Chapter 4. Chapter 5 examines the annual data and test statistics and contrasts them with the monthly results. Chapter 6 completes the dissertation by summarizing the findings and making some conclusions.

[3] This is discussed in Dickey and Fuller (1979) and later in Chapter 3 of this study.

CHAPTER 2.

PURCHASING POWER PARITY

Introduction

Purchasing power parity (PPP) is the hypothesis that the domestic price level should equal the exchange-rate adjusted foreign price level. Therefore, according to PPP the bilateral exchange rate will adjust to differences in the two countries' price levels. This is often written:

$$(1) \qquad E_t = \frac{P_t^*}{P_t}$$

where E_t is the nominal exchange rate in foreign currency per dollar (domestic currency), P_t is the domestic price level and P_t^* is the foreign price level in period t.

The theory of purchasing power parity is derived from the law of one price. However, the law of one price concerns a single homogeneous good's exchange rate adjusted price in two countries. Apart from transportation costs and tariffs the price of the homogeneous good should be equal in the two countries. Thus, theoretically an identical basket of goods would behave like identical price levels. In reality most countries use very different baskets of goods in constructing price indices. As explained in Froot and Rogoff (1994, p. 5), identical baskets of goods are rarely compared but instead the countries' Consumer Price Indices and Wholesale Price Indices are used. Therefore deviations from PPP are to be expected because of the proxies used to empirically test the theory.

There are two versions of purchasing power parity, absolute and relative PPP. Absolute PPP requires that the price indices in the countries be equal when adjusted for the exchange rate, at each point in time. Relative PPP only requires that changes in the price indices be equal when adjusted by changes in the exchange rate.

There is no reason to expect that absolute PPP should hold. Short run deviations from PPP may be caused by any of a number of factors, for example: i) the use of nontradable goods or goods with high transactions costs included in the price indices, ii) the different weights used in the basket of goods in each country, as well as the consumption patterns for tradable goods, iii) macroeconomic conditions, and iv) variations in product qualities.

For relative PPP, it is only necessary that in the long run, the exchange rate appreciate when the foreign country's price level increases (relative to the domestic price level) and depreciates when the domestic price level increases (relative to the foreign price level).[4] This is consistent with the "slow adjustment" monetary (asset) models of Dornbusch (1976), and Frankel (1979).

Froot and Rogoff (1994, p. 5) state that

> much of [an] economist's faith in PPP derives from a belief that over most of the past century, price level movements have been dominated by monetary factors. If price index movements are dominated by monetary shocks, and if money is neutral in the long run, then it won't matter if the two baskets being compared are not the same; relative PPP should still hold (approximately).

[4] Froot and Rogoff (1994, p.5) state that "the bulk of the empirical literature focuses on testing *relative consumption-based PPP* ... which requires that *changes* in relative price levels be offset by changes in the exchange rate."

In our study, the theory of PPP will be tested with bilateral real exchange rates. We will use unit root tests to determine if the real exchange rate is stationary. If we fail to reject the existence of a unit root for the real exchange rate, long-run mean reversion of the real exchange rate will not hold and PPP will fail.

PPP is an old concept going back to Cassell in the 1920s and has been tested extensively over the last 70 years. Much of the recent literature has focused on the existence of a unit root for the real exchange rate. As shown in Table 1, a number of studies focus on one or more of the econometric or conceptual problems associated with tests of PPP[5]. Others attempt to refine the methods used[6], and still others compare time periods[7] to find evidence in favor of or against the theory of PPP.

We begin by expressing (1) in logarithmic form:

$$(2) \qquad e_t = p_t^* - p_t$$

where lower case letters represent the natural logarithms of the variables in (1). Equation (2) can be used to test absolute PPP. Following Froot and Rogoff (1994, p. 5) an alternative version is relative PPP or:

$$(3) \qquad \Delta e_t = \Delta p_t^* - \Delta p_t$$

where Δ is the first difference operator. For absolute PPP to hold, p_t, and p_t^* must contain an identical bundle of goods and have the same corresponding weights (i.e. consumption patterns and common base periods). The Consumer Price Index, the Producer (or

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[5] For example see Abuaf and Jorion (1990), Crownover (1994) and Diebold, Husted and Rush (1991).
[6] Cheung and Lai (1993) for example look at tradable versus nontradable goods. Fisher and Park (1991) look at testing the null hypothesis of cointegration versus a null hypothesis of no cointegration.
[7] Ardeni and Lubian (1991), Diebold, Husted and Rush (1991), Glen (1992), and Johnson (1990) exemplify this comparison.

Wholesale) Price Index and the Gross Domestic Product deflator are most frequently used to proxy the price level in each country.

Pippenger (1993) explicitly tests absolute and relative PPP for Belgium, Canada, Denmark, France, Germany, Japan, the Netherlands, Norway, Switzerland, the United Kingdom, and the United States. By estimating the bilateral exchange rate and the price level, he finds that absolute PPP does not hold but there is evidence that relative PPP holds. The distinction between which form is tested is important because, Pippenger (1993, p. 48) states, if the error term in (2) is white noise then the real exchange rate follows a random walk. Therefore, any deviation in relative PPP [equation (3)] will not be eliminated over time and PPP would not hold as a long-run equilibrium in its "weak form." He refers to Frenkel (1981) as an example of confusion "regarding the behavior of the real exchange rate." He further states, that "... this seemingly trivial distinction leads to a substantial difference in the implication each equation has regarding the behavior of the real exchange rate" (Pippenger, 1993, p. 48). He argues that estimating (3) is not a valid test of PPP because it assumes a priori that the real exchange rate is nonstationary. However, if a unit root is found then there are serious estimation problems with (3) because the equation is overdifferenced (Pippenger, 1993, pp. 58-59). Thus to consider the distribution of the residuals and the performance of any shocks to the model, we must use the absolute PPP form, equation (2).

If deviations from PPP are due mostly to changes in the relative prices of tradable goods, then the nominal exchange rate should adjust to these changes in prices. The real exchange rates should not have any persistent deviations but tend to return to an equilibrium level. If however the deviations in PPP are due mostly to changes in the relative prices of nontradables or between nontradables and tradables, the real exchange rate could have a permanent change in its equilibrium level. The expectation of next period's real exchange rate is this period's real exchange rate. That is, next period's shock would be just as likely to reinforce as to deviate from the last shock, which is the definition of a martingale. (Davutyan and Pipinger, 1985, p. 1152)

Three Methods of Estimation

There are three ways of testing purchasing power parity in use currently. The first regresses the nominal exchange rate on the domestic and foreign price levels and tests whether the coefficients on the foreign and domestic price levels are significantly different from one. The second method regresses the nominal exchange rate on a constant and the relative price level and tests whether the coefficient on the relative price ratio is significantly different from one. The third method combines the nominal exchange rate, the domestic price level and the foreign price level into a real exchange rate and tests if the residuals are mean reverting.

The first method, estimates the model:

(4) $$e_t = \alpha + \beta p_t^* + \gamma p_t + \varepsilon_t$$

and tests whether $\beta = 1$ and $\gamma = -1$. If the domestic price level increases by more than the foreign price level then the foreign currency should appreciate relative to the domestic currency or the domestic price of foreign exchange increases. Authors using this method include Ardeni and Lubian (1991), Cheung and Lai (1993), Fisher and Park (1991), Kim (1990), MacDonald (1993), and Pippenger (1993).

The second method in effect constrains the coefficients on the price levels to be equal (and opposite in sign), and estimates:

(5) $$e_t = \delta + \theta p_t^{'} + \varepsilon_t$$

where $p_t^{'} = p_t - p_t^{*}$. The hypothesis tested is $\theta = 1$. Authors using this method include Crownover (1994), Davutyan and Pipinger (1985), Frenkel (1981), Johnson (1990), and McNown and Wallace (1989).

The third method of looking at PPP uses the real exchange rate, r_t, defined as:

(6) $$r_t = e_t + p_t - p_t^{*}.$$

The real exchange rate is subjected to unit root and cointegration tests. This is the method used most frequently in the literature probably due to the ease of use in dealing with a single variable. Abuaf and Jorion (1990), Corbae and Ouliaris (1988), Diebold, Husted and Rush (1991), Enders (1988, 1989), Glen (1992), Kim (1990), Kim and Enders (1991), Phylaktis and Kassimatis (1994), Perron and Vogelsang (1992), and Whitt (1992a) are some of the authors using the real exchange rate.

There are several tests for unit roots provided in Engle and Granger (1987). The most widely used are the Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) tests. The ADF test includes "error-correction" or lagged dependent variable terms in the Dickey-Fuller regression. Following Engle and Granger (1987), the ADF regression equation for the real exchange rate is:

(7) $$\Delta r_t = -\rho r_{t-1} + \beta_1 \Delta r_{t-1} + \ldots + \beta_p \Delta r_{t-p} + \varepsilon_t.$$

where the change in the real exchange rate is regressed on its past values and p lags of the first difference of the real exchange rate. The error term ε_t is assumed to be normally and identically distributed. The lag length p is chosen so that the residual is white noise (Granger, 1986, p.218).

The Augmented Dickey-Fuller test (often designated by τ) is used to test for unit roots in the real exchange rate.[8] The distribution of the Augmented Dickey-Fuller τ-test is not the standard Student's t-test, asymptotically or in small samples. Engle and Yoo (1987) calculate the critical values for the test using 10,000 replications. However, according to MacKinnon (1991, p. 271) "Unfortunately, these critical values are based on only 10,000 replications, so that they suffer from considerable experimental error." His critical values are calculated using 25,000 replications.

Researchers studying PPP often fail to reject a unit root using the ADF or DF tests. This was true of authors using any of the three methods above. Failure to reject a unit root means that the real exchange rate is non-stationary and the Gauss-Markov theorem does not hold as its mean is nonstationary. However, as Engle and Granger, (1987, p. 253) explained it was possible that two nonstationary variables may be cointegrated or move together in such a manner that the equilibrium error was stationary. Many of the researchers have used this method known as the Engle/Granger two-step method. The first step is to test for a unit root as detailed above. The second step is to test the residuals from a regression [as for example, in (7)] for a unit root. Rejection of a unit root in the residuals constitutes cointegration in the Engle/Granger method and

[8] Calculations of the ADF statistics is explained in Chapter 3.

therefore a long-run equilibrium (PPP) holds. Other econometric methods have been employed including but not limited to the Phillips and Perron (1988) nonparametric test which uses the same critical values as the ADF test; the Trace and Eigenvalue tests of Johansen (1988), and Johansen and Juselius (1990); and the Bayesian Sims test by Whitt (1992). Table 1 shows the dates, price indices, countries, exchange rate method used, basic tests and conclusion of many of the authors of recent articles using the new time series innovations.

The Real Exchange Rate

Abuaf and Jorion (1990) and Phylaktis and Kassimatis (1994) discuss the behavior of the real exchange rate in the long run and deviations from the long-run real exchange rate. This is the approach we follow here. The real exchange rate r_t in (5) follows a first-order autoregressive process [AR(1)] as follows:

(8) $r_t = c_0 + c_1 r_{t-1} + u_t$

where c_0 and c_1 are constants, and u_t is normally, identically and independently distributed.

Taking the unconditional expectation of (8) and assuming that $|c_1| < 1$ shows that the long-term equilibrium real exchange rate is,

$r = \dfrac{c_0}{(1-c_1)}$. Long-run PPP is violated if either $|c_1| \geq 1$ or c_0 and c_1 are not time invariant constants. If long-run PPP holds then short-run PPP is violated whenever r_t is not equal to its long run value. Finally, they point out that "... if $c_1 < 1$, shocks to the system are corrected at the rate $(1 - c_1)$ per period" (Abuaf and Jorion, 1990, p. 159; Phylaktis and Kassimatis, 1994, p. 479).

Abuaf and Jorion (1990) confirm that the real exchange rate follows a first-order autoregressive process and that the "root" is significantly different from one. The real question, they state, "is whether these deviations [from PPP] tend toward zero when the economic forces such as commodity arbitrage or capital movements are allowed to take full effect" (Abuaf and Jorion, 1990, p. 158). They study the real exchange rate between the U.S. dollar and currencies of ten countries.[9] The WPI is used to construct annual real exchange rates for 1900 - 1972, and the CPI for monthly data between January 1973 through December 1987.

Abuaf and Jorion (1990) argue that the low power of previous unit-root tests may result in failing to reject the random walk model (and thus reject PPP). They point out that Dickey and Fuller (1979, 1981) found that their tests were more powerful using regressions in the levels than using regressions in the first-differences. Abuaf and Jorion (1990) use levels (as does our work) and construct a multivariate model from which they derive Augmented Dickey Fuller tests. They point out that Kendall (1973) found that the usual OLS tests of the autoregressive coefficient are consistent but biased downward of the order $-\frac{(1+3c_1)}{T}$ where T is the sample size.

Abuaf and Jorion (1990) and Phylaktis and Kassimatis (1994) estimate equation (8) and construct two statistics to test for a unit root. The first (ρ_μ) is only valid when there are no lags of the dependent variable present:

(9) $$\rho_\mu = T(\hat{c}_1 - 1)$$

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[9] The ten countries include Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Norway, Switzerland and Britain.

where T is the sample size and $\hat{c}_1$ is the OLS estimate of c_1 in (8)[10]. The second test is the Student's t-test for a unit root:

$$(10) \qquad \tau_{\mu} = (\hat{c}_1 - 1)/[\sigma(\hat{c}_1)]$$

where $\sigma(\hat{c}_1)$ is the OLS standard error of $\hat{c}_1$. The null hypothesis is $c_1 = 1$ in equation (8). However since c_1 is downward biased in finite samples, the (negative) critical values of τ_μ are lower than that of the Student's t distribution.

As was noted previously, these tests may be more powerful when calculated for the variables in level rather than the first-difference form. The power of these tests, however, is still low. Calculations by Dickey and Fuller (1981) found that the probability of rejecting the null at the 5% level is only 19% for the first test and only 12% for the "pseudo t-test" statistic from a sample size of 100 when the true value of the autoregression coefficient is 0.95 (Abuaf and Jorion, 1990, p. 160). Edison, et. al. (1994, p. 17) also cite the low power of the tests as the reason for the lack of evidence for PPP in the post-Bretton Woods era.

To increase the power, Abuaf and Jorion (1990) extend the univariate autoregressions using Seemingly Unrelated Regression (SUR). This allows them to restrict the autoregressive coefficients to be equal across equations and then test that the restricted coefficient is equal to one. As the distribution of these test statistics is unknown they use Monte Carlo simulations to estimate the critical values. They find that the OLS tests are not powerful enough. "...for values of [c_1] such as 0.975, the probability of

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[10] The ρ_μ and τ_μ tests are sensitive to any added independent variables such as a constant or time trend.

rejecting the null is only 16.5% and 10.5%, respectively for ρ_μ and τ_μ" (Abuaf and Jorion, 1990, p. 161). Furthermore, if drift is allowed for (i.e. the intercept is not zero) the power of the tests will be even lower (Abuaf and Jorion, 1990, p. 165). They found also that heteroskedasticity and differing correlation matrices had little effect on the increase in power derived from using the multivariate SUR approach (Abuaf and Jorion, 1990, pp. 167-8).

Next Abuaf and Jorion (1990, p. 169) include lagged values of the difference in the log of the real exchange rates. These are included because of the criticism that the AR(1) specification restricts the dynamics of real exchange rates to only three possibilities: an explosive process, a random walk, or a monotonic adjustment to a constant value. This specification leads to their rejection of the random walk hypothesis at the 5% level. However, this is not a test of the random walk hypothesis since a random walk does not include lagged first-differences of the dependent variable. Furthermore, they use the same Monte Carlo critical values to test the unit root hypothesis in this new model.[11]

Finally, they look at annual data and clearly reject the random walk hypothesis using single-equation models. This refutes Adler and Lehmann (1983) who used OLS on the same data set. When Abuaf and Jorion (1990) use the SUR restricted approach a stronger rejection results. The stronger rejection means that the evidence for long-run PPP is stronger. The calculation of the half-life or speed of adjustment back to PPP after a shock[12] only includes the data subset up through 1972. They find a half-life of 3.3 years

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[11] See Engle and Granger (1987) for examples of the differences in critical values of the Dickey-Fuller and the Augmented Dickey-Fuller tests.

[12] Some authors call these innovations, e.g. Kim and Enders (1991).

for this annual data subset, which is consistent with their monthly results. For monthly data $\hat{c}_1 = .98$ and therefore 34 months are needed for the real exchange rate to move halfway to its long-run value (returning to long-run PPP) following a shock. The annual data coefficient is .8 and therefore there is a half-life of 3.1 years.

Real Exchange Rate Literature

Corbae and Ouliaris (1988) compare the Augmented Dickey Fuller (ADF) test with the Phillips-Perron (1988) test with monthly data from July 1973 to September 1986 for real exchange rates for the G-7 calculated using the CPI. The real exchange rates are indistinguishable from the random walk. They find no tendency for deviations from Purchasing Power Parity to converge.

Diebold, Husted and Rush (1991) use over a hundred years of data from the nineteenth century, (approximately from 1800 to 1913)[13]. They generated 16 real annual rates using various combinations of the CPI and WPI for Belgium, France, Germany, Sweden, United Kingdom, and the US. They challenge the findings by a number of authors[14] that the real exchange rate is approximated by a martingale. If deviations from PPP are highly persistent or even a martingale, then shocks permanently affect the real exchange rate, furthermore these shocks are unpredictable. The authors used the standard unit root tests based on ARIMA models and found little evidence of PPP. Therefore, they posit a more general model where the order of differencing (unit root) need not be one but may be less

[13] The time spans vary from 74 years to 123 years.

[14] Roll (1979), Adler and Lehmann (1983), Darby (1983), Mussa (1986), Diebold (1988), Meese and Rogoff (1988) and Baillie and McMahon (1989) are cited in their work.

than one. This method is called an ARFIMA (autoregressive fractionally integrated moving average) representation. Maximum likelihood estimation of the ARFIMA model is used to challenge the view that a random walk (martingale) is the correct representation for the theory of Purchasing Power Parity. Diebold et. al. find in all 16 cases that PPP holds in the long run for their annual data set with tests based on an ARFIMA representation.

Enders (1988) uses the WPI to compute German, Canadian and Japanese real exchange rates vis-a-vis the U.S. dollar. PPP does not hold for any of the autoregressive models using ADF tests. Using the standard deviation as a measure of variability he finds that the real exchange rate is more volatile in the flexible period and less predictable than in the 1960's fixed rate period.

Enders (1989) found evidence for Purchasing Power Parity in a study of the real exchange rate for the U.S. and Britain between 1862-1914. The data are semi-annual real exchange rates calculated from two different sources by the author. Enders looks at two exchange rate regimes, the flexible period from 1862-1878, and the fixed regime that was reestablished and existed until the beginning of World War I (1879-1914). The paper uses ARIMA, Box-Jenkins and Engle and Granger (1987) two-step methods to test for PPP.

Enders (1989, p. 59) states that even though there are differing views about the reasons for deviations from PPP, there is wide support for the 'stylized facts' of Mussa (1979). "PPP performs best over long periods of time in which there is high inflation, high rates of money growth, and few supply shocks." Enders (1989) considers a time period that did not conform with these "stylized facts" because prices were decreasing, and there were positive supply shocks. The stated goal of the U.S. and U.K. governments was to reestablish the fixed exchange rate that was implemented through a monetary contraction.[15] His results are generally supportive of PPP and show similar performance across regimes. The error correction models were almost identical across the two periods. A one unit deviation from PPP resulted in a 22% adjustment in prices in Britain during the greenback period and a 24.7% correction one period later during the gold standard period. The U. S. prices measured in terms of gold corrected by 33% in the greenback period and 36% in the gold standard period.

Enders believes that the similarity in the two regimes can be explained by agents expectations because the monetary moves were all well known and anticipated. Therefore, he suggests further study with respect to unanticipated versus anticipated real and monetary disturbances from PPP.

Enders and Hurn (1995) study optimum currency areas through common trends in real exchange rates. Their thesis is that real output levels are nonstationary and thus forcing variables which cause real exchange rates to be nonstationary. They develop a theory they call generalized purchasing power parity (G-PPP) to explain the stylized facts of real exchange rates. Using cointegration tests they find that linear combinations exist (G-PPP holds) individually for each of the pacific rim countries and four developed nations. However, as a group G-PPP does not hold between these four nations, Germany, Japan, the United Kingdom, and the United States. They conclude that the real income processes of the larger countries strongly influence each of the pacific rim nations. This

[15] The Resumption Act of January, 1873.

influence is found to be greater than the relationship amongst the pacific rim nations themselves.

Frankel and Rose (1995) use a panel of 150 countries and 45 years of annual data to test for mean-reversion of the real exchange rate. They restrict their study to the post-Bretton Woods era of flexible exchange rates. Their rationale is that real exchange rates act differently depending upon the exchange rate regime. Therefore the speed of adjustment towards PPP may also vary.[16] They use the difference of the real exchange rate (natural log) as a dependent variable. This has lower power when used in a univariate framework, so they use cross section data to increase the power of their tests. Therefore they can focus on short time series, (1973 - 1992) with no change in exchange regimes (or structural shifts) and use tests that are of sufficient power.

They use a White/Huber estimator in all calculations because of the possible presence of heteroskedasticity. They find that the real exchange rate has a mean reversion with a half life of about four years. Additional tests with country specific dummy variables, time specific dummy variables as well as pure time series and cross-section tests, show their results are robust and "quite insensitive" to these changes (p. 14).

Glen (1992) also finds that previous tests of the random walk hypothesis ($c_1 = 1$) to be unconvincing because of their low power. He uses variance ratio tests and autocorrelation tests with long time horizons. While finding little evidence for mean

[16] Mussa (1986) showed that real exchange rates may behave differently depending upon the exchange regime.

reversion with monthly data for a group of countries including the G-7, he does find evidence for Purchasing Power Parity with annual data.

Kim and Enders (1991) use monthly data and the wholesale price index to compute real exchange rates for a group of Pacific Rim countries (Korea, Japan, Thailand and India) vis-a-vis the dollar during 1973-1987. They use a variety of tests including the Dickey-Fuller, Augmented Dickey-Fuller, and Philips-Perron tests on residuals (i.e. the Engle/Granger two-step method). They also use a Vector Autoregression (VAR) approach and compute impulse responses to shocks (real and nominal) to trace the possible sources of these shocks. They find no evidence for Purchasing Power Parity using the two-step cointegration test of Engle-Granger (1987) on the residuals from the unit root equation.

Kim (1990, p. 493) used annual data (1900-1987 for the WPI and 1914-1987 for the CPI) for the real exchange rates of the Group of 5 (G-5). He uses the Phillips and Perron (1988) and Perron (1988) procedures to test for a unit root in the real exchange rate because "They are robust to a wide variety of serial correlation and time-dependent heteroskedasticity, which seem very likely in our data set given the length of time-series, the smoothing inherent in the averaging process, and the regime changes that occurred during the time period." (Kim, 1990, p. 493) However, the Phillips-Perron (1988) test is a nonparametric test which has lower power than the ADF parametric test. Kim (1990) follows the unit root tests with the Engle-Granger (1987) two-step residual based cointegration test. He is able to reject PPP using the real exchange rate for France and Italy but the evidence for Canada, Japan and the U.K. either fails to reject the random walk model or the evidence is mixed. He states (Kim, 1990, p. 497) that this "indicate[s] that neither Alder and Lehman (1983) nor Frankel (1986) can be generalized" (p. 497). While his results are more consistent with Frankel (1986) he recognizes that the choice of price indices matters.[17] Kim (1990) also states that PPP is less likely to hold when productivity growth and demand patterns differ significantly and when the price index weighs nontraded goods significantly.[18] Finally, he criticizes Alder and Lehman (1983) because one of their restrictions (deletions) in their empirical equation "refutes the existence of [the] error correction mechanism and cointegration." Using an error correction model, he concludes that their results are due to a misspecified model with too many lags. Furthermore, his model, in which "the error correction term is invariably significant" suggests that about 30 - 50% of the deviations from PPP are corrected by exchange rate movements within one year (Kim, 1990, pp. 498-9).

Lothian and Taylor (1992) use 200 years of data to examine the real exchange rates for the countries of the United States, the United Kingdom and France. They find that there is significant mean reversion of the real exchange rate in each case. Investigating out-of-sample forecasts for the recent flexible exchange rate period, they provide strong evidence that this period is consistent with the prior century and a half of data. PPP continues to hold as a basic economic long-run equilibrium condition. Even the

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[17] Also shown in McKown and Wallace, (1989).

[18] As does the CPI, a fact also noted by Fisher and Park (1991).

extreme economic and political diversity cannot suppress that the exchange rate gradually reverts towards equilibrium, even during the recent float.

Mark (1990) uses the Engle/Granger two-step with the ADF test to study a group of eight countries during the recent flexible period. He studies three sets of bilateral real exchange rates with the U.S., U.K. and Germany alternating as the domestic country. Canada, Belgium, France Germany Japan and Italy are included in the study. He does not find evidence that PPP holds as a long run concept but notes (Mark, 1990, p. 123) that the monthly data available to him (16 years) and the lack of power as a rationale for the inconsistency of his results.

Perron and Vogelsang (1994) study over 100 years of annual real exchange rates to test for PPP while at the same time searching for a possible structural break in the data. The equation they use for testing PPP is the Augmented Dickey Fuller equation. Successive break points are chosen and a dummy variable is added at each time period to test for a shift in the mean. The maximum of all of the successive tests of this break is then compared to the critical value. They reject the random walk (unit root) hypothesis (find support for PPP) for the U.S./U.K. real exchange rate and find a break in the data in 1943. For the U.S./Finland real exchange rate they find evidence for PPP (reject the unit root hypothesis) and find a structural break in 1937.

Phylaktis and Kassimatis (1994) study the black market real exchange rates of eight Pacific Rim countries (Taiwan, Japan, Korea, Malaysia, Philippines, Singapore, Thailand and Indonesia) relative to the U. S. dollar. They use monthly data from January 1974 through March 1987 and alternatively the CPI and the WPI in calculating the real exchange rate. Like Abuaf and Jorion (1990) they use the Dickey-Fuller test in a system of univariate autoregressions, estimated jointly by GLS. First they test that SUR is more efficient than single-equation estimation and find this to be the case. They restrict all autoregressive coefficients to be equal and test the joint restriction that all eight real exchange rates have a unit root. They find that the Breusch and Pagan (1980) Lagrange Multiplier test for contemporaneous correlation is highly significant, supporting their use of SUR. The restricted coefficient is significantly different from one and they conclude that the random walk model is not appropriate. Furthermore, they find an upward trend in the real exchange rate of 6 countries, (Taiwan, and Japan are excluded). Their empirical analysis shows a real exchange rate depreciation over time in six of the eight countries using the CPI and four the eight with the WPI. Balassa (1964) reasoned an appreciation would occur in the traded goods sector if one country grew faster than the other (a productivity bias, for instance). Phylaktis and Kassimatis find no evidence for this appreciation in the real exchange rate. They note however, that other factors, such as the real interest rate differential [as in Meese and Rogoff (1988)] could have exerted a dominant influence. They also note that the black market rates show a large turbulence during the period (except for the Philippines). As with Abuaf and Jorion (1990), Phylaktis and Kassimatis use the adjustment of Kendall (1973) to take into account small sample bias when estimating the speed of adjustment. The order of bias is $-\frac{(1+3c_1)}{T}$ where c_1 is the coefficient hypothesized to be one and T is the number of observations in the sample.

They estimate the half life of a deviation from equilibrium to be 10 to 13 months. They note that this is much faster than that found in Abuaf and Jorion.

Whitt (1992a) uses Sims (1988) test based upon a Bayesian methods to "discriminate between a unit root and a large but stationary autocorrelation coefficient." (Whitt, 1992a, p. 73) Using five countries and monthly data from June 1973 through December 1989 he finds evidence for a large autocorrelation coefficient. However, the coefficient is significantly different from one, providing evidence for PPP. Furthermore, he finds that with CPI based real exchange rates a trend must often be included. This finding is consistent with the CPI placing greater weight on nontraded goods. Whitt (1992a) also tests long-run annual data since World War II (with relatively few observations) so as to compare his results [using the Sims (1988) test] with those of other authors (e.g. Abuaf and Jorion, 1990). He includes a time trend as in Frankel (1985) to adjust for structural shifts such as a productivity bias (the Balassa effect). Again he finds evidence for a large autocorrelation coefficient that is significantly different from one. Table 1 summarizes this literature on Purchasing Power Parity.

Structural Break Literature

Papers have recently been published using Perron (1993),[19] Zivot and Andrews (1992), which endogeneously test for structural breaks, as well as a paper using the Banerjee, Lumsdaine and Stock (1992) asymptotic distribution theory of testing for an endogenous structural break. Zelhorst and De Haan (1995) apply the Perron (1993)

[19] Zelhorst and Haan (1995) develop the endogenous procedure on their own but it uses Perron (1989) and is essentially the same.

procedure to annual real output data from 1870-1989. They find structural breaks varying from 1913 in Finland to 1946 in Germany. There conclusion is in contrast to Christiano (1992). They find that, for the majority of their 12 industrial countries, the one-time structural break model represents the data better than a unit root model. The unit root null hypothesis is rejected in favor of the one-time structural break.

Culver and Papell (1995) is of interest to this study because they investigate real exchange rates using the Perron (1993) procedure. Sixteen real bilateral exchange rates are included in the study covering the gold standard era of the 18th century.[20] Their investigation focuses on whether the unit-root null hypothesis can be rejected for real exchange rates. Therefore, Culver and Papell (1995) use Augmented Dickey-Fuller tests to eliminate ten cases. They find that of these ten cases a unit root null hypothesis can be rejected at the 1% significance level for eight of them and at the 5% significance level for two more. In the other six cases, they conclude that four of the real exchange rates can be modeled as stationary around a break in the trend.

Alba and Papell (1995) use the structural break tests of Zivot and Andrews (1992) on Gross Domestic Product (GDP) data beginning in the 1950's. Alba and Papell (1992, 265) cite Campbell and Perron's (1991) comment as a rationale for including the structural break:

> Campbell and Perron concede, however, that a longer span of data on real GDP is more likely to be influenced by a major structural shift, which if not accounted for, results in misspecification and biases the test towards the nonrejection of the unit-root hypothesis.

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[20] The first year of the sample varies depending upon the country. The UK and Germany begin in 1792, the USA in 1793, France in 1806, Belgium in 1832 and others still later.

Alba and Perron use Zivot and Andrews (1992) sequential Dickey-Fuller test on Summers and Heston's (1991) data. The data is for per capita and aggregate real Gross Domestic Product for nine developing countries in east and southeast Asia. They find significant structural breaks in most countries are due to changes in government policy. They reject the unit-root hypothesis, finding evidence for trend stationarity with structural shifts in 14 of the 18 cases at the 5% significance level.

Li (1995) uses the Banerjee, Lumsadine and Stock (1992) tests for unit roots in the presence of a structural break. Using the Nelson-Plosser data set Li uses the rolling, recursive and sequential tests to determine if each of the time series is nonstationary (has a unit root) or is stationary around a shifting trend. In some cases, Li corroborates the results of previous studies (unemployment rates for example, are found to be stationary). In one case Li found that Perron (1989) was correct in "picking" the known break date (stock prices). Li did find a break date for other time series, but different dates than Perron (1989) "picked" (nominal GNP and industrial production). Finally, in many cases Li (1995, 513) was unable to reject the unit root hypothesis or found inconclusive results. These results are consistent with Nelson and Plosser (1982), he states. Li concludes that the sensitivity study shows that the test results may be sensitive to the selection of the number of lagged first differences included in the test.

Table 1 A Summary of the literature on PPP

Authors	Years	Type	Price Index	Base Country	Exchange Rate	Tests	Basic Conclusion
Abuaf and Jorion (1990)	1900-1972 1973-1987	Annual Monthly	CPI	G-10	Real	ADF. SURE	PPP holds
Ardeni and Lubian (1991)	1/1957-12/1985	Monthly and Annually	CPI and WPI	G-7, **US** and C, F, I, UK, **US**	Nominal	E/G 2-Step and Variance Ratio	PPP holds
Cheung and Lai (1993)	1/1974-12/1989	Monthly	WPI and CPI	**US**, UK, France, Germany, Swit, Canada	Nominal	E/G 2 Step, Johansen ADF, Z drift	PPP holds
Corbae and Ouliaris (1988)	1/73-9/86	Monthly	CPI	G-7: Japan, **US**, Germany, Italy Japan, Canada, UK	Real	ADF t-test, Phillips/Perron Z-test, Engle/ Granger 2 Step	PPP Fails Unit root in real
Crownover (1994)	1927-1972	Annual	CPI	G-6, **US**, (ex. Japan)	e on p: (p^*/p)	E/G 2 Step Serial Correlation. and Joint Endogeneity.	PPP holds (14/15)
Culver and Papell (1995)	1792-1913	Annual	WPI & CPI	16 Countries	Real	ADF & Trend-Break	PPP Holds
Davutyan/ Pipinger (1985)	1920's and 1970s	Monthly	WPI	G-6, **US** (ex. Italy)	e on p: (p^*/p)	GLS with AR (1)	PPP did not fail "Shocks"
Diebold, Husted and Rush '91	Aprx. 1800-1913	Annual	WPI and CPI	Bel, France, Germany, **US**, Sweden, UK	Real	ARFIMA	PPP holds
Enders (1988)	1/60-4/71 1/73-11/86	Monthly	WPI	Germany, **US** Japan, Canada	Real	ADF, ARIMA E/G 2 Step	PPP Mixed
Enders (1989)	1862-1914	Annual		US, Britain	Real	ADF, E/G 2 Step	PPP holds
Enders and Hurn (1995)	1973-1989	Monthly	WPI	Germany, US UK and Pac Rim	Real	Johansen	GPPP holds
Fisher and Park, (1991)	3/1973-5/1988	Monthly	WPI and CPI	G-10 + 1	Nominal	J_1 test and J_2 test	PPP holds
Frankel and Rose, ('95)	1973-1992	Annual	CPI	150 Countries, **US**	e on p and Real	OLS, Hetero Panel data	PPP holds

Authors	Years	Type	Price Index	Basc Country	Exchange Rate	Tests	Basic Conclusion
Glen (1992)	6/73-12/88 1900-1987	Monthly Annual	CPI	G-7 plus	Real	Variance Ratio autocorrelations	PPP holds
Johnson (1990)	1947-1986 1870-1986	Quarterly Annual	GNP*d*	**US** and Canada	e on *p* : (p*/p)	ADF E/G 2 Step	PPP holds
Kim and Enders(1991)	1973-87	Monthly	WPI	Pacific Rim	Real	E/G 2 Step and VAR	PPP Fails
Kim (1990)	1900-87 1914-87	Annual	WPI CPI	G-5 **US** base	Real and Nominal	PP, E/G 2 Johansen	PPP holds
MacDonald (1993)	1/1974- 6/1990	Monthly	WPI CPI	**US**, C, F, G, J, UK	Nominal	Johansen. Trace and Eigenvalue	PPP holds
Manzur (1990)	3/1973- 4/1986	Quarterly	CPI[21]	G-7	Nominal	Divisia moment	PPP holds
Mark (1990)	6/1973- 2/1988	Monthly	CPI	**US, G, UK,** I, Bel, C, F, J	e on *p*: Real	ADF and E/G 2-Step	PPP does not hold
McKown/ Wallace (1989)	For 3 aprx 76-86 (4 Chile)	Monthly	WPI and CPI	Argentina. Brazil, Israel Chile	e on *p*:	ADF and E/G 2-Step	PPP holds
Perron/ Vogelsang (1992)	1892-1988 1869-1987	Annual	CPI GDPdf	US/UK US/Finland	Real	E/G 2-Step with a Structural Break	PPP holds
Phylaktis/ Kassimatis (1994)	1/1974- 3/1987	Monthly	WPI and CPI	8 countries of the Pacific Rim[22]	Real	SURE	PPP holds
Pippenger (1993)	1/73-6/88	Monthly	WPI	12 countries	Nominal	ADF and E/G 2 Step	PPP holds
Whitt (1992)	6/73-12/89	Monthly	WPI CPI	**US**, UK, F, G Swit, J	Real	Sims test	PPP holds

Note: ADF is the Augmented Dickey-Fuller test for a unit root. E/G 2 Step is the Engle-Granger two step process to test for cointegration. Johansen refers to the maximum likelihood procedure as delineated in Johansen (1988). ARIMA means Autoregressive Integrated Moving Average representation using Box-Jenkins methodology. ARFIMA refers to the ARIMA process that may not have a unit root but a fractional root. J_1 and J_2 are tests for a null hypothesis of cointegration. The Z-test refers to the ρ_μ test of Dickey and Fuller (1979). SURE means a Seemingly Unrelated Regression Estimation process was used.

[21] GDP shares are used for weights vis-a-vis the other currencies.

[22] Taiwan, Japan, Korea, Malaysia, Philippines, Singapore, Thailand and Indonesia.

CHAPTER III

METHODOLOGY, ROLLING, RECURSIVE AND SEQUENTIAL MODELS INVESTIGATING THE UNIT ROOT WITH A TIME TREND

Method

This study will use the models of Sims, Stock and Watson (1990) and Banerjee, Lumsdaine and Stock (1992) to test the hypothesis that the real exchange rate has a unit root. The Sims, Stock and Watson (1990) procedure maintains that the coefficients of stationary variables from an OLS regression converge at a different rate from nonstationary variables[23]. They use a transformation of the regressors to maintain the statistical property of consistency and asymptotic normality. This is necessary when drawing inferences on constants or the nonstationary variables, namely those variables with a unit root and/or a time trend coefficient. Banerjee, Lumsdaine and Stock (1992) use the Sims, Stock and Watson (1990) framework to test for endogenous structural breaks in the data. This study follows their procedure and uses Wald and Quandt Likelihood Ratio tests to identify endogenous structural breaks.

Our study examines the possible presence of heteroskedasticity especially in light of the increase in volatility in real and nominal exchange rates following the collapse of the Bretton Woods agreement. It tests formally for heteroskedasticity and uses the Estimated Generalized Least Squares (EGLS) method to correct for it when present. Monte Carlo simulations are used to find the critical values for the Dickey-Fuller τ-test for a unit root in the presence of heteroskedasticity. Monte Carlo simulations are also used to find the

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[23] For discussion of convergence see Judge et. al. (1985, p. 426).

critical values for the Wald test for a structural break and the Quandt Likelihood Ratio test when a time trend is not included.

Due to the low power of the ADF τ-test in univariate regressions we use multivariate techniques that combine six real exchange rates between the US dollar and the other G-7. We impose the restriction that the estimate of the first-order autoregressive coefficient is the same across countries and test for the validity of the restriction with a Wald test. The restriction increases the power of the Augmented Dickey Fuller unit root test. Once more, Monte Carlo simulations are necessary to obtain critical values of the tests.

Figures 1 through 6 show the natural logarithm of the real exchange rate of the six bilateral exchange rates between the U.S. dollar and the other G-7 currencies. Figures 7 - 11 show the natural logarithm of the real exchange rate of the five bilateral exchange rates between the U.K. and the other G-7 currencies (excluding Germany due to the lack of available data). Two series are shown for each bilateral exchange rate. One employs the Wholesale Price Index (WPI) and the other the Consumer Price Index (CPI) in the construction of the real exchange rates. All subsequent tests of PPP will employ both series of the real exchange rate.

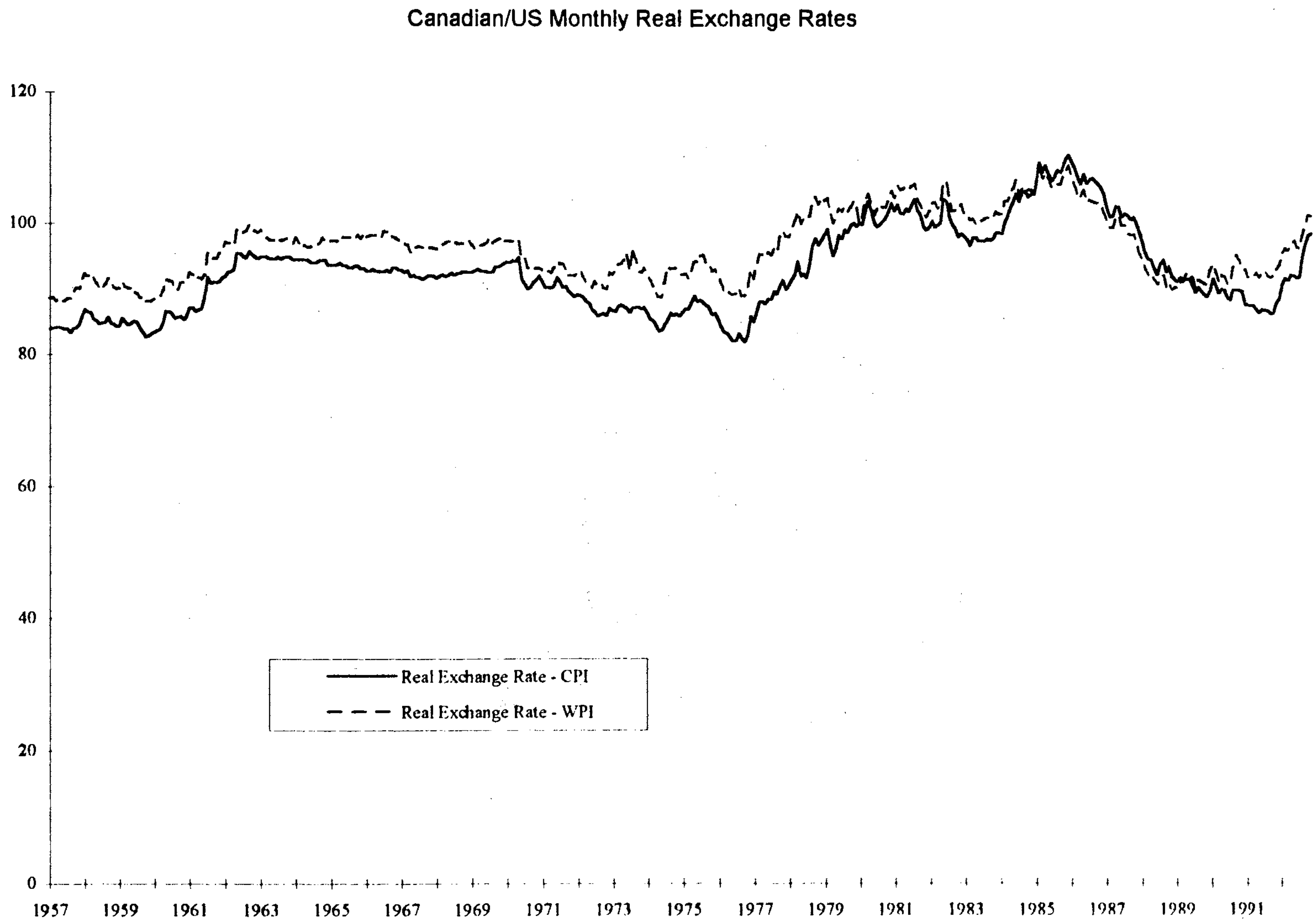

Figure 1: Monthly rates from January 1957 through December 1992.
The real exchange rate is indexed with January 1980 as the base year.

French/US Monthly Real Exchange Rates

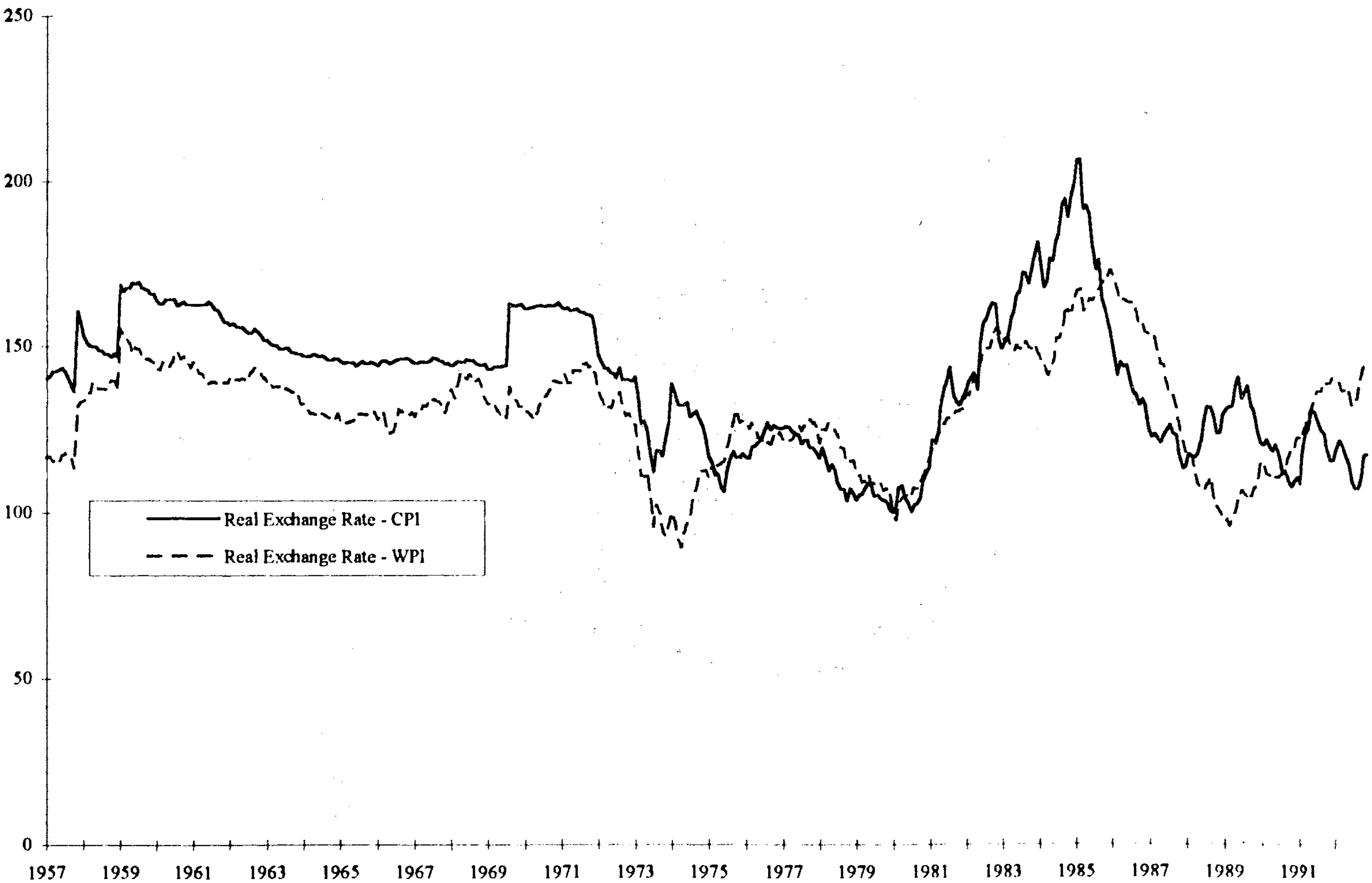

Figure 2: Monthly rates from January 1957 through December 1992.
The real exchange rate is indexed with January 1980 as the base year.

German/US Monthly Real Exchange Rates

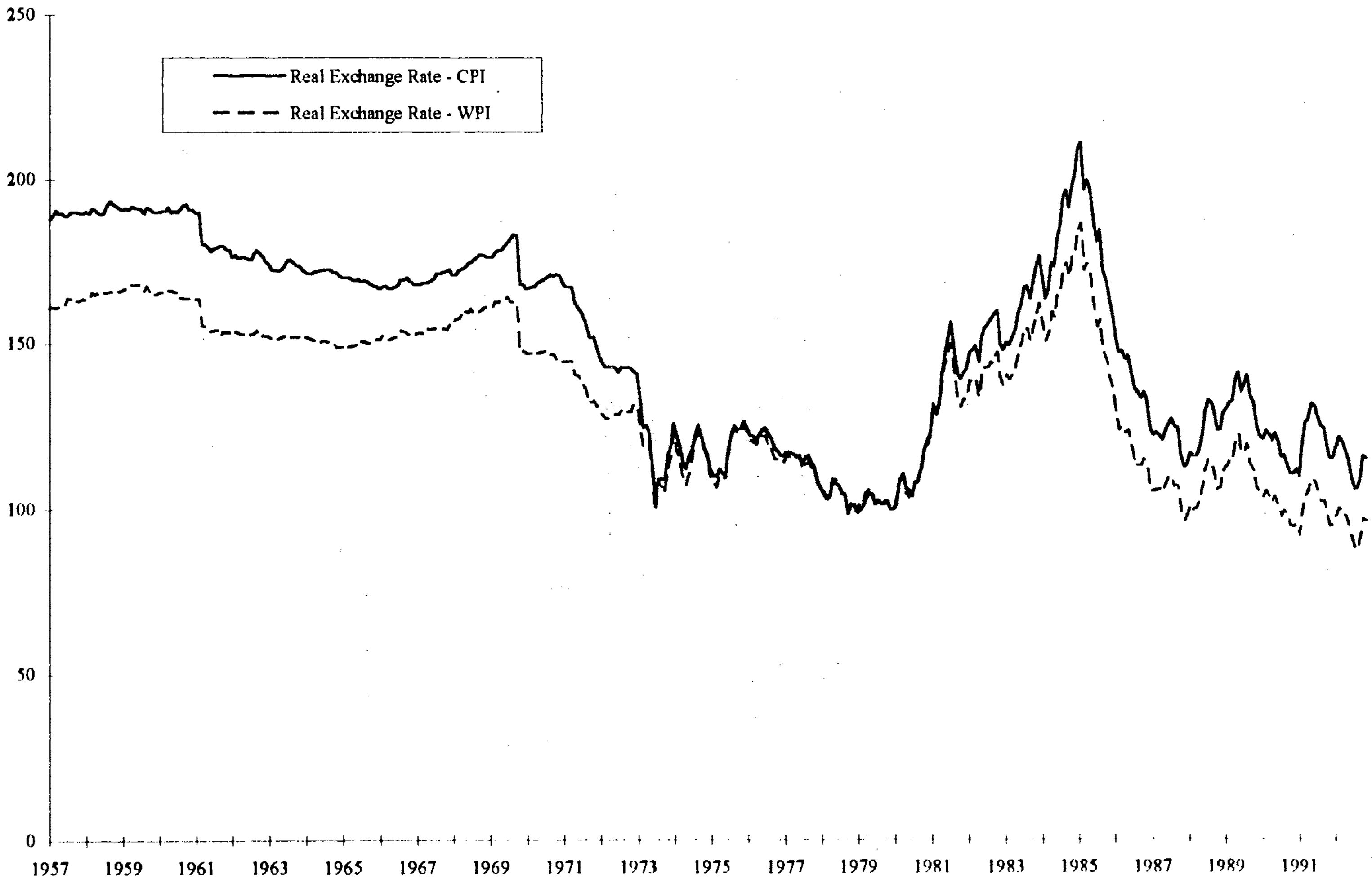

Figure 3: Monthly rates from January 1957 through December 1992.
The real exchange rate is indexed with January 1980 as the base year.

Italy/US Monthly Real Exchange Rates

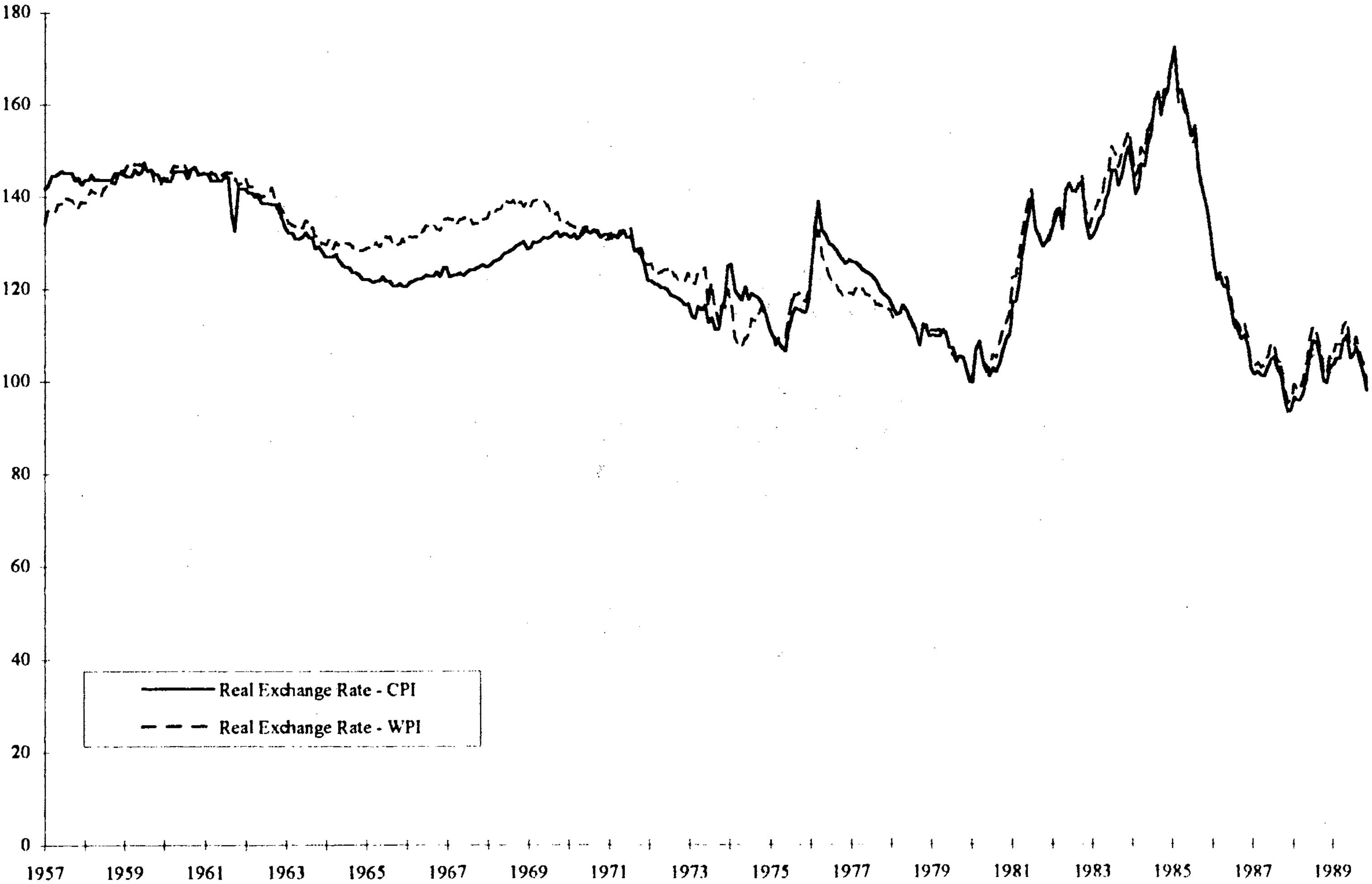

Figure 4: Monthly rates from January 1957 through December 1989.
The real exchange rate is indexed with January 1980 as the base year.

Japanese/US Monthly Real Exchange Rates

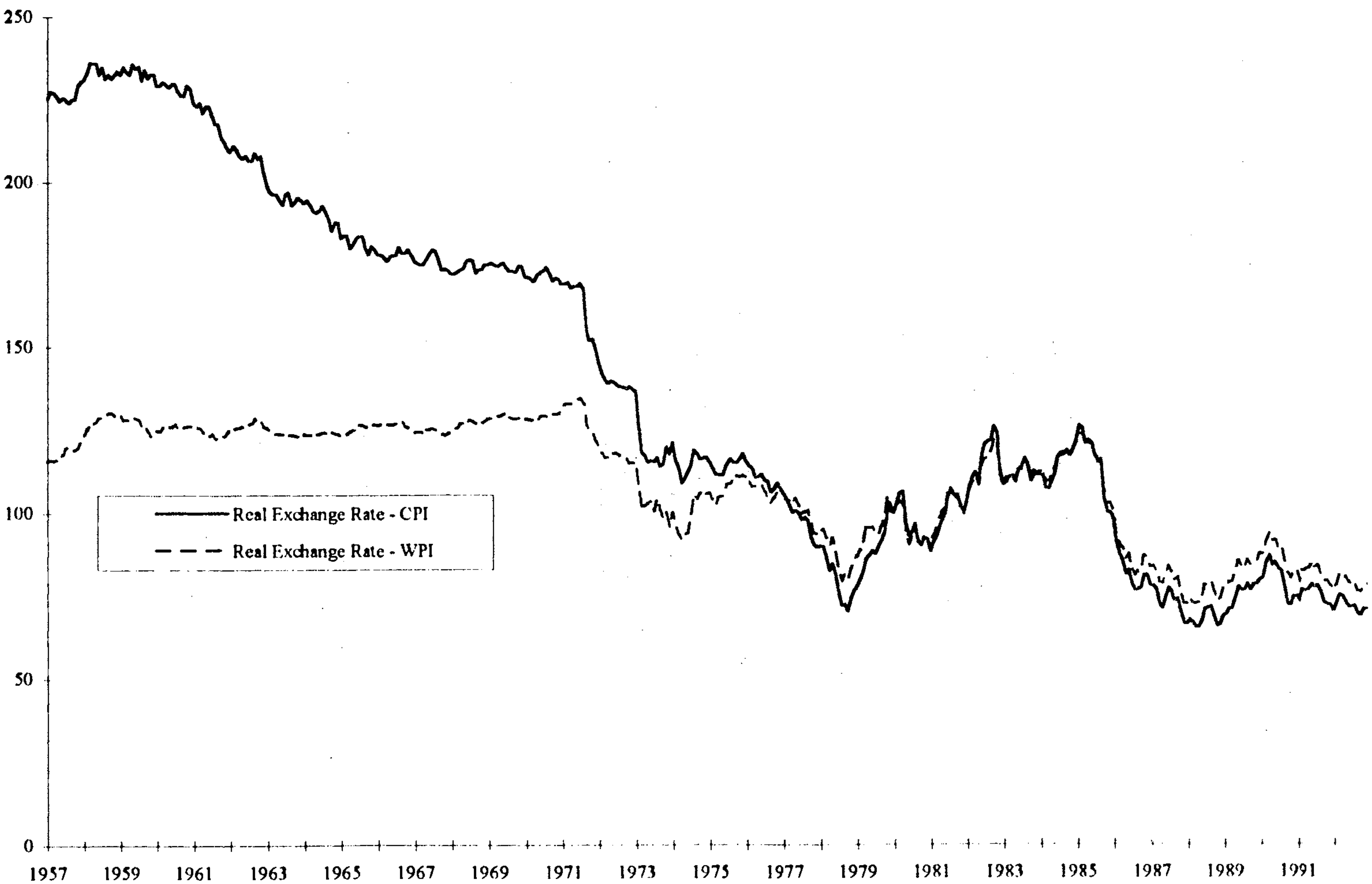

Figure 5: Monthly rates from January 1957 through December 1992.
The real exchange rate is indexed with January 1980 as the base year.

UK/US Monthly Real Exchange Rates

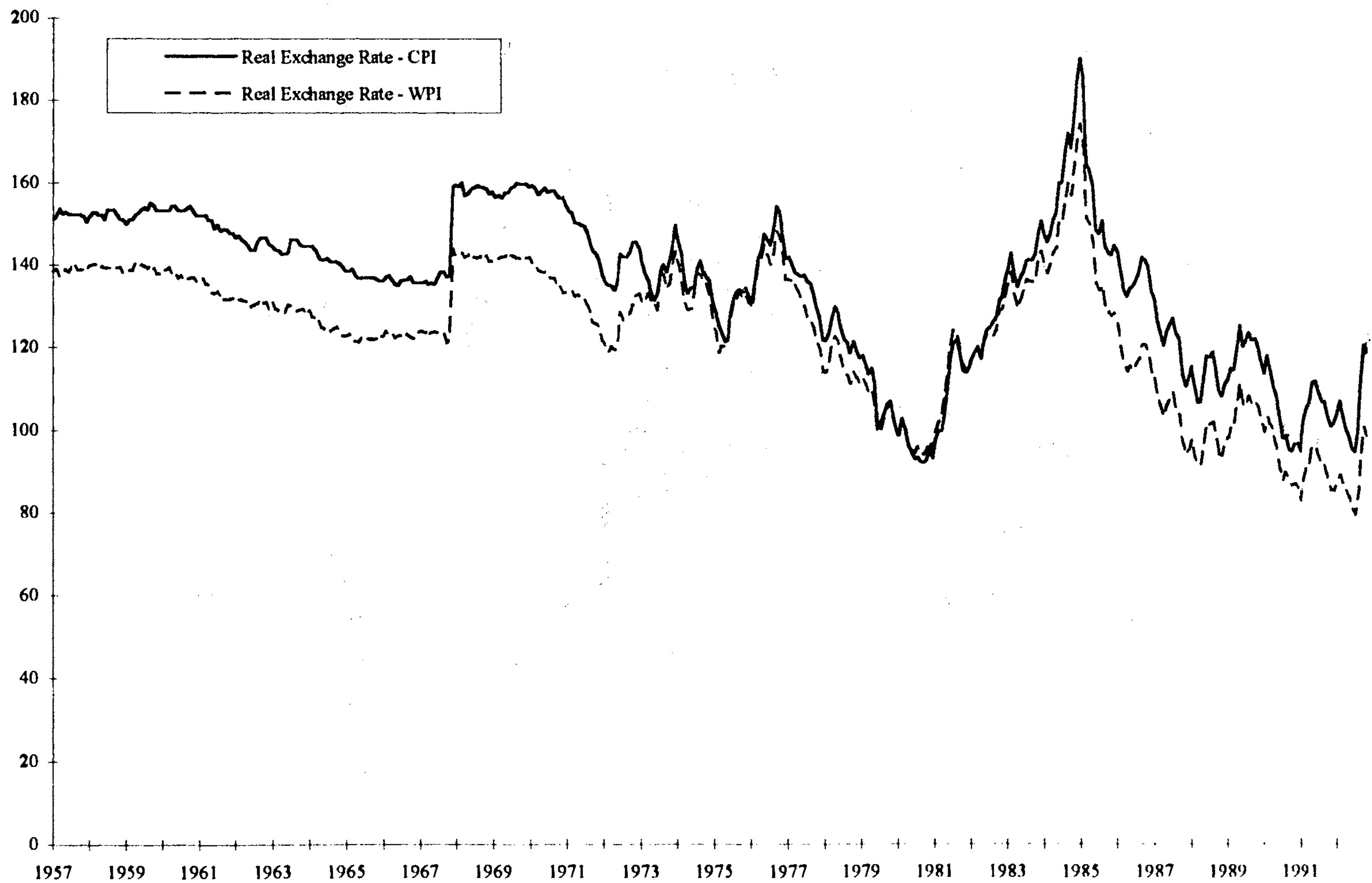

Figure 6: Monthly rates from January 1957 through December 1992.
The real exchange rate is indexed with January 1980 as the base year.

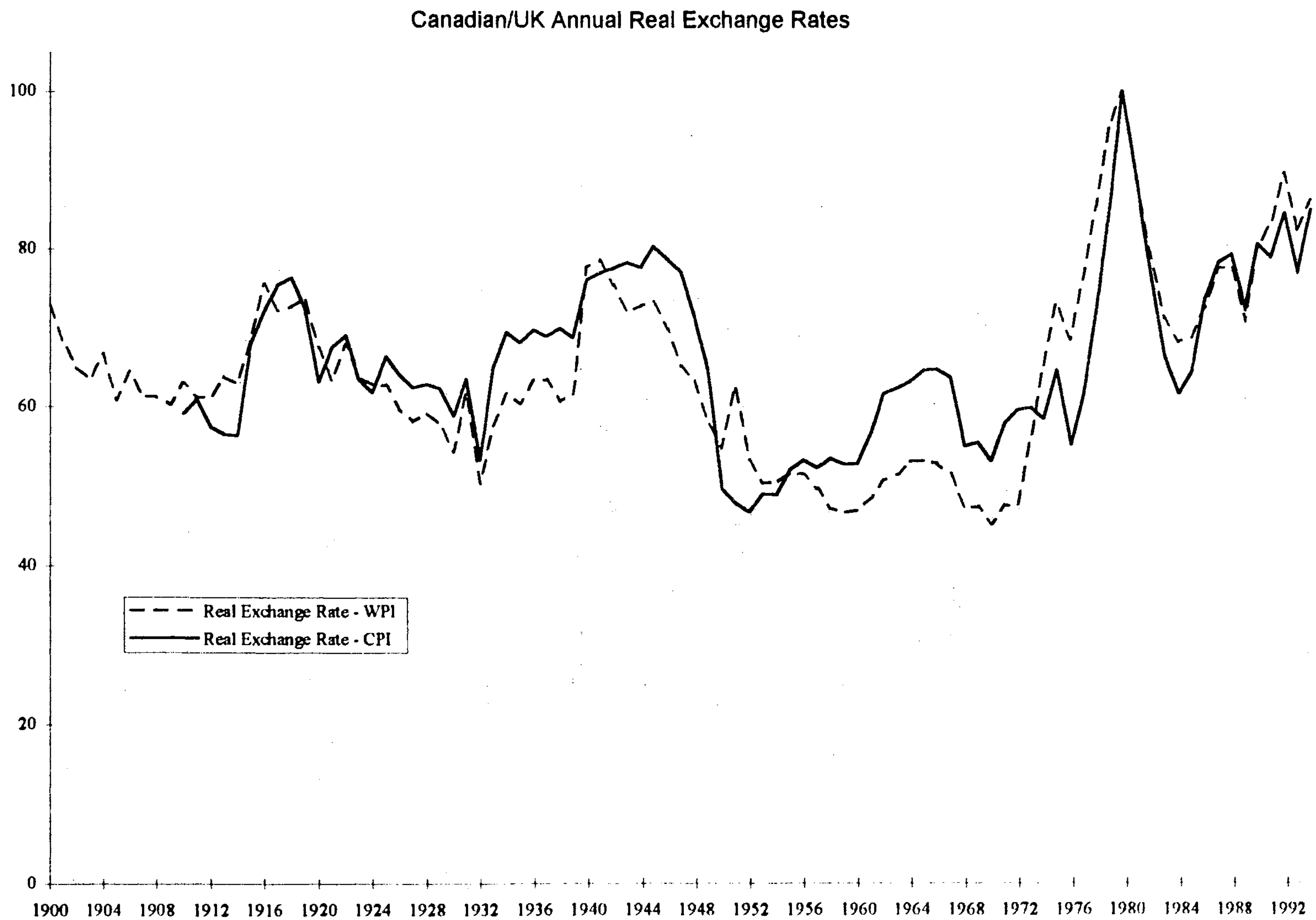

Figure 7: Annual rates from 1900-1994 (WPI), 1910-1994 (CPI).
The real exchange rate is indexed with 1980 as the base year.

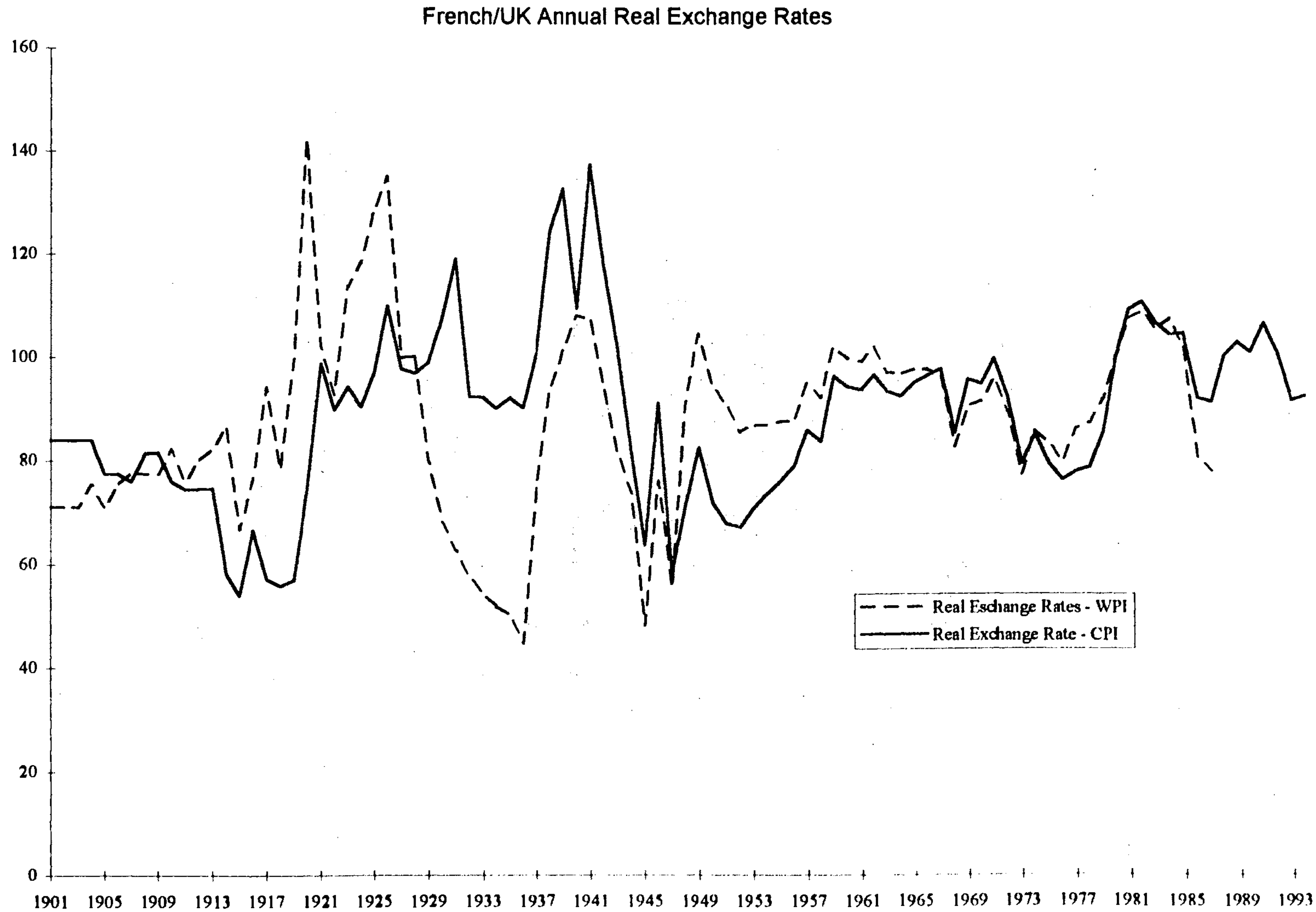

Figure 8: Annual rates from 1900-1994 (CPI), 1900-1987 (WPI).
The real exchange rate is indexed with 1980 as the base year.

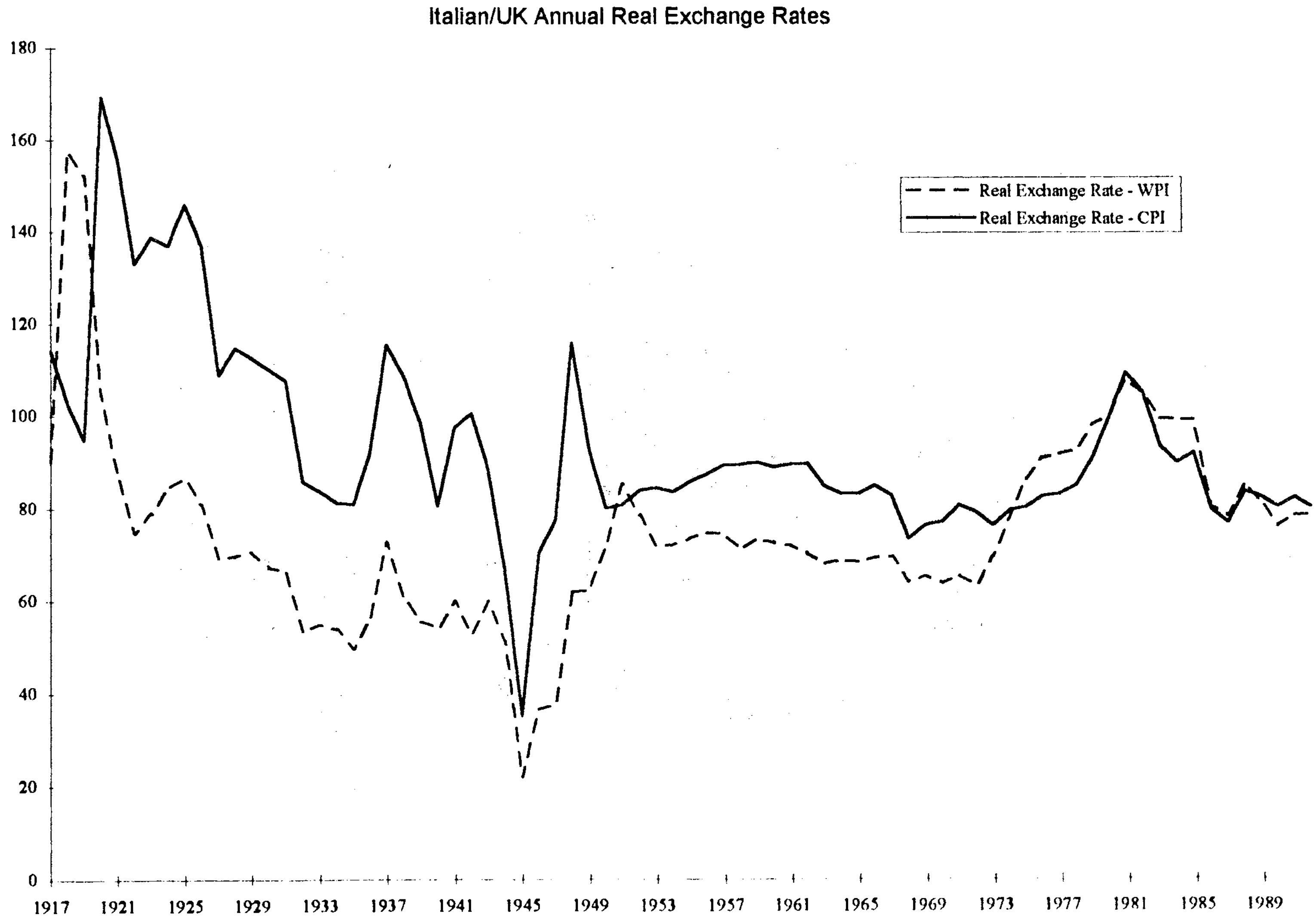

Figure 9: Annual rates from 1917-1992.
The real exchange rate is indexed with 1980 as the base year.

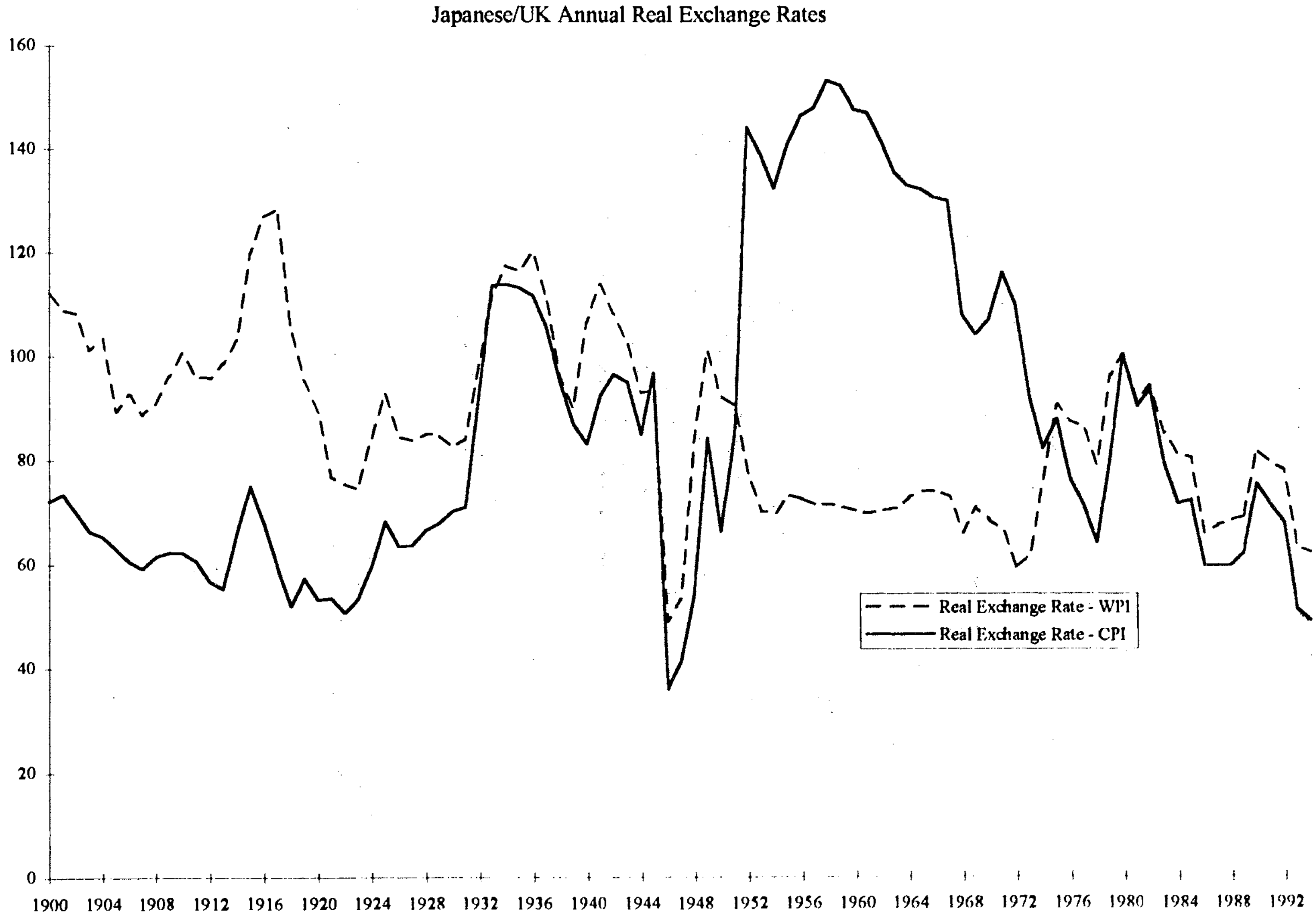

Figure 10: Annual rates from 1900-1994. The real exchange rate is indexed with 1980 as the base year.

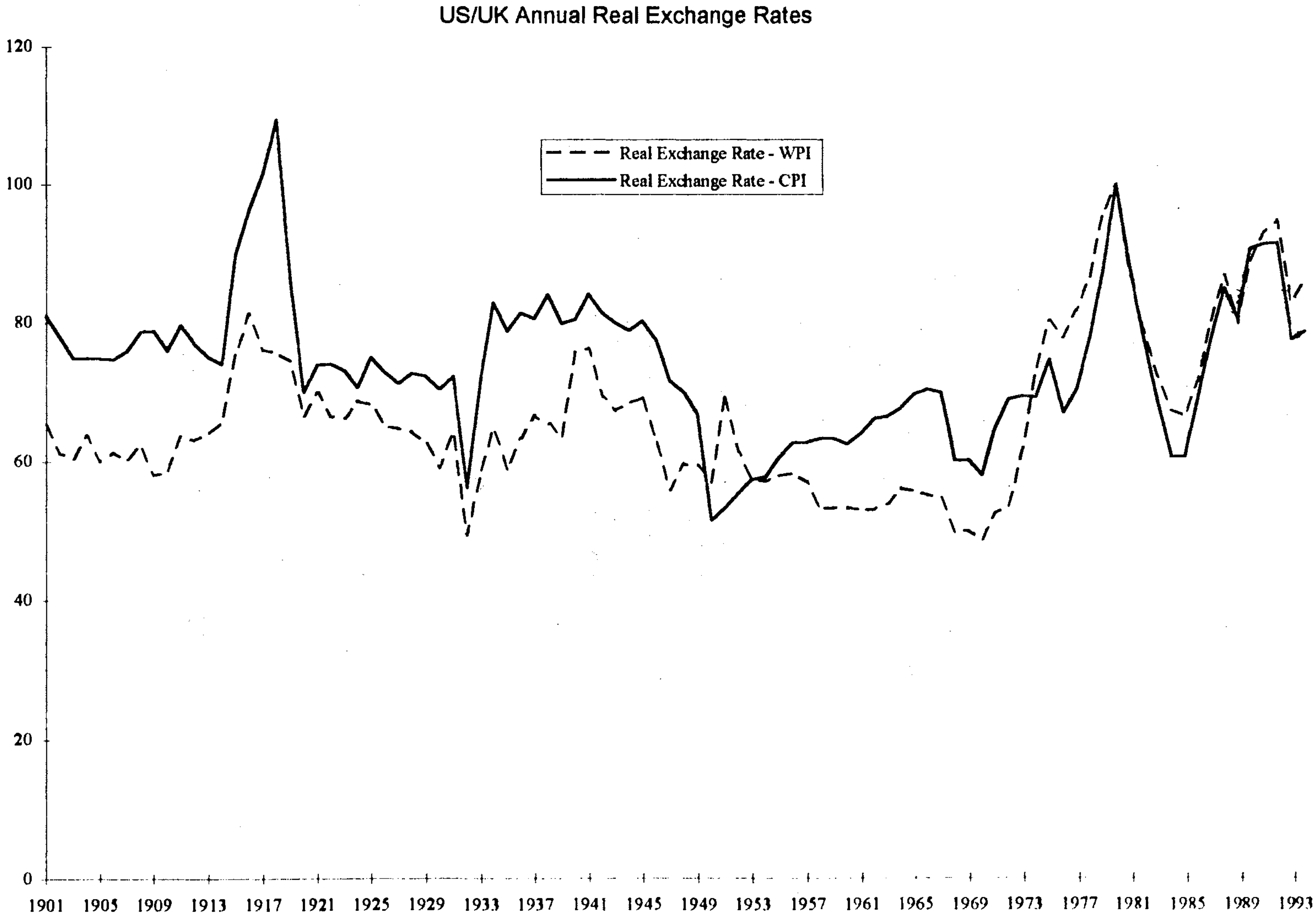

Figure 11: Annual rates from 1900-1994. The real exchange rate is indexed with 1980 as the base year.

Rolling, Recursive, And Sequential Time Trend Models

The following section reviews the unit root tests used by many of the authors discussed in Chapter 2. The next section details the model of Sims, Stock and Watson (1990) applied to the rolling and recursive regressions by Banerjee, Lumsdaine and Stock (1992). The following section delineates the model of sequential testing for a structural break developed by Banerjee, Lumsdaine and Stock (1992).[24] These test procedures are applied to exchange rate tests of Purchasing Power Parity. The next section shows the EGLS estimator and tests of heteroskedasticity. The next two sections discuss multivariate unit root tests and the joint restrictions imposed on the autoregressive coefficients. The final section explains the Monte Carlo simulation procedures.

Unit Roots and the Error Correction Model

The first test we consider is the Augmented Dickey-Fuller (ADF) test, a procedure used by many authors. The ADF test is based on the error-correction model that Engle and Granger (1987) use to test for a unit root. The model includes as many lagged dependent variables (Δr_{t-p}) as necessary to render the residuals white noise. The model is:

$$r_t = \mu_0 + \alpha r_{t-1} + \beta_1 \Delta r_{t-1} + \ldots + \beta_p \Delta r_{t-p} + \varepsilon_t \qquad t = 1, \ldots, T \tag{11}$$

where r_t is the natural log of the real exchange rate, Δr_{t-i} are first differences of the natural log of the real exchange rate ($r_{t-i} - r_{t-i-1}$), μ_0 is the intercept, α is the coefficient tested to be equal to one, the β_i are the coefficients on the first difference terms, T is the number of

[24]The procedure is also explained in Hamilton (1994) Chapters 16 and 17.

observations, and ε_t is the error term which is assumed to be a white noise[25]. Engle and Granger (1987, p. 254) explain that the lagged first difference (or error correction) terms represent the proportion of the disequilibrium in one period that is corrected in the next period. The original version of the Dickey-Fuller (DF) test does not include any autoregressive lags:

$$r_t = \mu_0 + \alpha r_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T \tag{11`}$$

We will use both of these forms in our tests below. Furthermore, the ADF and DF unit root tests in the real exchange rate literature sometimes include a time trend to test the Balassa (1964) hypothesis that there is a productivity bias in favor of tradable goods[26]:

$$r_t = \mu_0 + \alpha r_{t-1} + \beta_1 \Delta r_{t-1} + \ldots + \beta_p \Delta r_{t-p} + \mu_1 t + \varepsilon_t \qquad t = 1, \ldots, T \tag{12}$$

$$r_t = \mu_0 + \alpha r_{t-1} + \mu_1 t + \varepsilon_t \qquad t = 1, \ldots, T \tag{12`}$$

corresponding to the ADF and DF models of (11) and (11`) and t is the time trend.

The Dickey-Fuller or Augmented Dickey-Fuller test statistic is $(\hat{\alpha} - 1) / S_\alpha$ where $\hat{\alpha}$ is the estimate of α, S_α is the standard error of the estimator. We reject the null hypothesis that the test statistic ($\hat{\alpha}$) is equal to one at critical values determined by MacKinnon (1991).

The Rolling and Recursive Unit Root Procedure

Rolling and recursive regressions use the transformation of Sims, Stock and Watson (1990). The Sims, Stock and Watson (1990) method is used in connection with

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[25] White noise means that the error is independently normally distributed with a mean of zero and constant variance.

[26] See for example Phylaktis and Kassimatis (1994).

time series models because the elements of the coefficient matrix do not converge at the same rates. Therefore, when there exists one or more unit roots in the vector autoregression (VAR), the coefficients on any intercepts or time trend and their associated t statistics will typically have nonstandard limiting distributions (p. 114). "The transformation is necessary only when one considers joint hypotheses of parameters converging at different rates." (Li, 1995, p. 503)

The rolling and recursive regressions search for a break in the data by searching for a unit root in subsets of the data set. If the regressions have a unit root in some subsets but then "break" this form and have a stationary root in other subsets then a break has occurred.[27] The rolling regressions procedure tests for a unit root in a fixed subset (δT) of observations ($\delta<1$) and T is the number of observations. The sample is trimmed by $\delta_0 T$ ($\delta_0 = 0.15$) observations at each end. The first subset begins with the first observation of the data set and ends with the fixed number of observations (δT). Each successive subset begins one time period later and performs the unit root test with the fixed number of observations. The recursive regressions procedure tests for a unit root with a subset of the data set that progressively gets larger as the number of observations is increased by one for each test. The first observation in the data set is always in the subset of the data being tested.

[27] Banerjee, Lumsdaine and Stock (1992, p. 272) introduce this test and distribution theory to "recommend" their method to DeLong and Summers (1988) because the subsample dates of DeLong and Summers are data dependent and determined from historical evidence.

The Rolling Regressions Model

The rolling regressions procedure begins at observation $\delta_0 T$ where $\delta_0 = 0.15$ and uses the next 180 observations to test for a unit root. The first observation ($\delta_0 T$) is then dropped, the next observation is added (181st + $\delta_0 T$) and the test is repeated. In our data set with 36 years of monthly data, this provides approximately 120 tests of the null hypothesis of a unit root. Each test uses 15 years of monthly data. The estimated coefficient α [equations 11, 11`, 12, and 12`] is tested for a unit root using the Sims, Stock and Watson (1990) transformation. We follow the notation in Banerjee, Lumsdaine and Stock (1992) to transform equation (12) to produce:

(13) $$r_t = \theta' \mathbf{Z}_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T$$

where

(14) $$\mathbf{Z}_{t-1} = \left[\mathbf{Z}^{1'}_{t-1} Z^2_{t-1} Z^3_{t-1} Z^4_{t-1}\right]'$$

$$\mathbf{Z}^1_{t-1} = \left(\Delta r_{t-1} - \overline{\mu_0}, \ldots, \Delta r_{t-p} - \overline{\mu_0}\right)'$$

$$Z^2_{t-1} = 1$$

$$Z^3_{t-1} = \left(r_{t-1} - \overline{\mu_0}(t-1)\right),$$

$$Z^4_{t-1} = t,$$

$$\overline{\mu_0} = E\Delta r_t = \mu_0\left(1 - \beta(1)\right) \text{ and } \beta(1) = \sum_{i=1}^{p} \beta_i$$

$$\beta = (\beta_1, \ldots \beta_p)$$

The terms in Z_{t-1} include the p lagged first differences of the real exchange rate $Z^1_{t-1}{}'$, a constant Z^2_{t-1}, the centered real exchange rate Z^3_{t-1}, and the time trend Z^4_{t-1}. In addition $\theta = (\theta_1'\theta_2\theta_3\theta_4)'$ and $\theta_1 = \beta', \theta_2 = \mu_0 + (\beta(1) - \alpha)\overline{\mu_0}$, $\theta_3 = \alpha$, $\theta_4 = \mu_1 + \alpha\overline{\mu_0}$.[28] Finally, the residual ε_t is assumed to be a martingale difference sequence.[29]

The rolling OLS estimator of the coefficient vector is:

$$\hat{\theta}(\delta) = (\hat{\theta}_1'\hat{\theta}_2\hat{\theta}_3\hat{\theta}_4)' = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'\mathbf{r} \tag{15}$$

where $\mathbf{r} = (r_1, \ldots, r_{T\delta})'$ and $\mathbf{L} = [\mathbf{Z}_1, \ldots, \mathbf{Z}_{T\delta}]'$ is a (T x K) matrix, $T\delta$ is the number of observations, K is the number of regressors (p +3), and $\mathbf{Z}, \ldots, \mathbf{Z}_{T\delta}$ are matrices defined in (14). The model is estimated after being trimmed by a fraction δ_0 at either end, where $0 \le \delta_0 \le \delta \le 1$ and δ_0 is chosen to allow a tradeoff between finding breaks in the early and later part of the data set and using enough observations in the regressions to support Gaussian approximation (Banerjee, Lumsdaine and Stock, 1992, p. 277)[30]. Therefore, the first

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[28] It should be recalled that α, μ_0 and μ_1 were as defined in (11), (11'), (12) and (12'), and $\beta = \beta_1, \ldots \beta_p$.

[29] A martingale difference sequence is a sequence of scalars that have a mean of zero over all of the variable's values and an expectation of zero given all of the past and present values of the variable. Thus, $\{Y_t\}_{t=1}^{\infty}$ is a martingale difference sequence if $E(Y_t) = 0$ and $E(Y_t \mid Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0$. This condition is stronger than assuming that Y_t is serially uncorrelated. A serially uncorrelated sequence cannot be forecast on the basis of a linear function of its past values. This does not rule out the possibility that higher moments such as $E(Y_t^2 | Y_{t-1}, Y_{t-2}, \ldots, Y_1)$ might depend upon past values. Thus $\{Y_t\}_{t=1}^{\infty}$ may not be independent so Banerjee, Lumsdaine and Stock (1992) add Assumption A which specifies that higher moments must be finite. According to Assumption A: ε_t is a martingale difference sequence and satisfies $E(\varepsilon_t^2 | \varepsilon_{t-1}, \ldots) = \sigma^2$, $E(|\varepsilon_t|^i | \varepsilon_{t-1}, \ldots) = \kappa_i$ (i = 3, 4), and $\sup_t E(|\varepsilon_t|^{4+\gamma} | \varepsilon_{t-1}, \ldots) = \kappa < \infty$ for some $\gamma > 0$.

[30] Remember that δ is the subset of the data over which the regression is run.

regression starts with observation δ_0T and uses δT observations, the next regression begins with [(δ_0+1)T] and includes the next [δT] observations and so forth until the last regression which includes [δT] observations up to observation [(1-δ_0)T]. Results from the usual distribution theory do not apply when the break point is treated as unknown.[31] Furthermore, the Student t-test converges at the rate $T^{1/2}$, but the Sims, Stock and Watson (1990) transformation "pseudo" Student t-tests converge at the rate of $T^{3/2}$ for the time trend (μ_1) and T for the Dickey-Fuller tau test for a unit root (α). The Dickey-Fuller tau test (τ_{DF}) is calculated as

(16) $$\hat{\tau}_{DF}(\delta) = T\left(\hat{\theta}_3(\delta) - 1\right) / \left[\hat{V}_T(\delta)_{33}\ s^2(\delta)\right]^{\frac{1}{2}} \qquad \delta_0 \leq \delta \leq 1 - \delta_0,$$

where $\hat{\theta}_3(\delta)$ is the estimated coefficient of the "centered" first-lag of the dependent variable (Z^3_{t-1}), δ is the fraction of the data after trimming, δ_0 is the amount of trimming required, p is the number of lagged error-correction terms, $\hat{V}_T(\delta)_{33}$ denotes the estimate of the diagonal element of the $V_T(\delta) = (Y_T)^{-1} L' L (Y_T)^{-1}$ matrix corresponding to $\hat{\theta}_3$, and Y_T is a diagonal scaling matrix or $Y_T = \mathrm{diag}(T^{1/2}I_p, T^{1/2}, T, T^{3/2})$, I_p is a p-dimensional identity matrix and $s^2(\delta) = \sum_{t=1}^{[T\delta]} \left[r_t - \hat{\theta}(\delta)' Z_{t-1}\right]^2 / \left([T\delta] - p - 3\right)$. The diagonal scaling matrix is adopted because the elements of θ converge at different rates.

[31] Banerjee, Lumsdaine and Stock (1992, p. 272) state that the usual distribution theory--which is conditional on a nonradom known break point--does not apply. They develop a distribution theory for a series of statistics evaluated over a range of possible break dates. "This permits analyzing the distribution of continuous functionals of these statistics--for example, the maximum of the sequence of unit-root test statistics, one for each possible break date. Christiano (1988) and Evans (1989) recognized the nonstandard nature of these distributions and used numerical simulations to examine extrema of sequences of test statistics."

The ρ_μ test for a unit root is only used in the DF model (i.e. when no autoregressive lags are included in the model) as follows:

(17) $$\rho_\mu = T(\hat{\alpha} - 1) \text{ or } \rho_\mu = T(\hat{\theta}_3(\delta) - 1).$$

This statistic converges in probability to σ^2 [for discussion see Dickey and Fuller (1979)].

Recursive Regressions

Recursive tests use progressively larger samples. The first sample uses the initial [Tδ] observations. In the empirical section $\delta_0 = 0.25$ so 108 observations are used, $\delta = \delta_0$. The sample, δ, is then expanded by one observation to 109 and the presence of a unit root is again tested. The process is repeated until all observations have been included. Therefore, rather than a fixed number of observations (as in the rolling regression procedure) the recursive regressions increase constantly the number of observations. The model is the same as those in (11) and (12) and the transformation is identical to (13) and (14). The Dickey-Fuller tau test is that of equation (16).

The Sequential Regression Model

The sequential regression model also uses an algorithm to search for a potential break endogenously rather than choosing a break point a priori. The model here includes a dummy variable or time trend depending on whether a shift in the mean or trend value is being tested:

(18) $$r_t = \mu_0 + \mu_1\tau_{1t}(k) + \mu_2 t + \alpha r_{t-1} + \beta(L)\Delta r_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T$$

where $\tau_{1t}(k)$ is a dummy variable defined below, t is a variable for time, and $\beta(L)$ is a lag polynomial of known order p with the roots of $1 - \beta(L)L$ outside the unit circle. As in Perron (1989, 1990) and Banerjee, Lumsdaine, and Stock (1992) we consider two cases:

(19) Shift in trend: $\tau_{1t}(k) = (t\text{-}k)\ [1(t > k)]$

(20) Shift in mean: $\tau_{1t}(k) = 1(t > k)$

where $1(\cdot)$ is the indicator function and k is the time period chosen for the break. Testing the hypothesis that $\mu_1 = 0$ with (19) is equivalent to a test for a change in the slope of the trend whereas using (20) tests whether there is a jump or break in the mean of the time series.

Following Banerjee, Lumsdaine and Stock (1992), the model is transformed as in the rolling regressions case. Therefore it is rewritten as:

(21) $$r_t = \theta' \mathbf{Z}_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T$$

where $$\boldsymbol{Z}_{t-1} = \left[\mathbf{Z}_{t-1}^{1'} Z_{t-1}^{2} Z_{t-1}^{3} Z_{t-1}^{4} Z_{t-1}^{5}\right]'$$

$$\boldsymbol{Z}_{t-1}^{1} = \left(\Delta r_{t-1} - \overline{\mu_0}, \ldots, \Delta r_{t-p} - \overline{\mu_0}\right)'$$

$$Z_{t-1}^{2} = 1$$

$$Z_{t-1}^{3} = \left(r_{t-1} - \overline{\mu_0}(t-1)\right),$$

$$Z_{t-1}^{4} = \tau_{1t}(k),$$

$$Z_{t-1}^{5} = t, \text{ and}$$

$$\overline{\mu_0} = E\Delta r_t = \mu_0\left(1 - \beta(1)\right).$$

There are p lagged first differences of the real exchange rate $\mathbf{Z}_{t-1}^{1}{}'$, a constant Z_{t-1}^{2}, the centered real exchange rate Z_{t-1}^{3}, the structural break component Z_{t-1}^{4}, and the time trend Z_{t-1}^{5}. The sequential estimator of the coefficient vector, for $\delta_0 \leq \delta \leq 1 - \delta_0$,[32] is:

[32] Thus δ is a subset of the data set between δ_0 and $1 - \delta_0$.

$$(22) \qquad \hat{\theta}(\delta) = \left(\hat{\theta}_1' \hat{\theta}_2 \hat{\theta}_3 \hat{\theta}_4 \hat{\theta}_5\right)' = (L'L)^{-1}L'\mathbf{r}$$

with $\mathbf{r} = (r_1, \ldots, r_T)'$, $\theta_1 = \boldsymbol{\beta}', \theta_2 = \mu_0 + (\beta(1) - \alpha)\overline{\mu_0}$, $\theta_3 = \alpha, \theta_4 = \mu_1, \theta_5 = \mu_2 + \alpha\,\overline{\mu_0}$ and $\mathbf{L} = [\mathbf{Z}_1, \ldots, \mathbf{Z}_T]'$. The residual, ε_t, is assumed to be a martingale difference sequence. The model in (21) is estimated over the entire sample of T observations. The break point, k, is chosen successively for $k = k_0, k_0+1, \ldots, T - k_0$ where $k_0 = [T\delta_0]$ and δ_0 is chosen so as to balance the requirements detecting a trend sufficiently early and sufficient observations to allow appropriate estimation of the coefficients. In the empirical section δ_0 is set at 0.15. The lag length p is chosen so that the residual is white noise. (Granger, 1986, p. 218) The coefficient of interest for the unit root test (and PPP) is α. The model is estimated with and without a time trend to investigate Balassa's (1964) hypothesis that a traded goods productivity bias can exist (especially if countries' economic growth rates differ) giving rise to a trend in the real exchange rate.

The time period at which the structural break occurs will be chosen on the basis of the supremum of various test statistics. The test statistics computed are a Dickey-Fuller tau test, a Wald test and a Quandt Likelihood ratio test. They are described below and the results of these tests are reported in Chapter 4 for the monthly data set and Chapter 5 for the annual data set..

The Dickey-Fuller tau test for a unit root ($\theta_3 = 0$) is calculated as follows:

$$(23) \qquad \tau_{DF}^{SEQ}(\hat{\theta}) = T\left(\hat{\theta}_3(\delta) - 1\right) / \left[\hat{V}_T(\delta)_{33} s^2(\delta)\right]^{\frac{1}{2}} \qquad \delta_0 \le \delta \le 1 - \delta_0,$$

where $\hat{\theta}_3(\delta)$ is the coefficient of the first-lag of the dependent variable, and the shift parameter occurs at observation [Tδ], p is the number of error-correction terms, $\hat{V}_T(\delta)_{33}$ denotes the diagonal element of the $V_T(\delta)$ matrix corresponding to the unit root coefficient: $V_T(\delta) = (Y_T)^{-1} L'L(Y_T)^{-1}$, $Y_T = \text{diag}(T^{1/2}I_p, T^{1/2}, T, T^{1/2}, T^{3/2})$ in the case of (19) or $Y_T = \text{diag}(T^{1/2}I_p, T^{1/2}, T, T^{3/2}, T^{3/2})$ for (20), I_p is an identity matrix of dimension p and $s^2(\delta) = \sum_{t=1}^{T}\left[r_t - \hat{\theta}(\delta)' Z_{t-1}\right]^2 / (T-p-4)$.

A Wald test-statistic for testing the significance of the structural break ($\theta_4 = 0$) is computed as follows:

$$F_T(\delta) = \left[W\hat{\theta}(\delta) - w\right]' \left[W(L'L)^{-1} W'\right]^{-1} \left[W\hat{\theta}(\delta) - w\right] / s^2(\delta) \tag{24}$$

where W is a (1 x K) matrix of restrictions, K is the number of regressors and w is a (1 x 1) matrix to test the null hypothesis that $\hat{\theta}_4 = 0$.

A Quandt likelihood ratio test is calculated for each successive break as follows:

$$Q_{LR}(\delta) = \left[-2\ln\lambda(k)\right], \text{ where } \lambda(k) = \frac{(\sigma_{1,k})^k (\sigma_{k+1,T})^{T-k}}{(\sigma_{1,T})^T} \tag{25}$$

where $(\sigma_{T_1,T_2})^2$ is the Gaussian maximum likelihood estimator of the regression error variance calculated over observations T_1, ..., T_2. This test is used to test for a break in any or all of the coefficients. "The LR statistic is calculated for each possible break point, and the Quandt LR statistic Q_{LR} is the maximum of these." (Banerjee et. al., 1992, p. 276) The critical values are calculated by Monte Carlo simulation and reported in Banerjee, Lumsdaine and Stock (1992).

The requirement that enough observations be included to support the Gaussian approximation including early and late observations to capture possible breaks early and late in the sample led Banerjee et. al. to "trim" the Q_{LR} statistic by twenty five percent at each end and the $F_T(\delta)$ and τ_{DF}^{SEQ} by 15% at each end. We follow their suggestion.

Heteroskedasticity

As many commentators have pointed out, the possibility of heteroskedasticity may arise when including data from the period prior to and following the breakdown of the Bretton Woods era.[33] In particular the possibility of increased volatility of real and nominal exchange rates after the Bretton Woods period necessitates testing whether the error variance is significantly higher in the post Bretton Woods period. The Breusch Pagan test is used and where a significant difference in error variance is detected the Estimated Generalized Least Squares (EGLS) method is used to correct the estimates. Judge et. al. (1985, p. 429) state that EGLS is almost as efficient as GLS when there are two subperiods. Furthermore, "EGLS dominates LS [Least Squares] over a large range of the parameter space including rather small samples with a moderate degree of heteroskedasticity." The critical values and probability levels for the structural break and unit-root tests of the EGLS estimators are not available and will be obtained via Monte Carlo simulations.

The Likelihood Ratio Tests for Heteroskedasticity

We will use two likelihood ratio tests for heteroskedasticity. The first is a general F-test and the second is a χ^2 test for the specific form of heteroskedasticity we employ.

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[33] For example, see Enders (1988), Frankel and Rose (1995), and Kim (1990).

A general F-test is used to determine if the variances of two subperiods are equal. The null hypothesis is: H_0: $\sigma_1^2 = \sigma_2^2$; the alternative is H_1: $\sigma_1^2 < \sigma_2^2$ where σ_1^2 is the Bretton Woods era variance. The test statistic is:

$$F_{(T_1-1,T_2-1)} = \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2}. \tag{26}$$

This statistic follows an F distribution with T_1-1, T_2-1 degrees of freedom. (Judge et. al. p. 363)

The Breusch-Pagan likelihood ratio test is a regression of the least squares residuals on various explanatory variables determined by the form of heteroskedasticity hypothesized. In our case we will test that the error variance during the Bretton Woods era is different from the flexible exchange rate era. Thus the two explanatory variables are a constant and a dummy variable which is zero prior to and including December 1972 and one thereafter. The Breusch-Pagan statistic is calculated such that:

$$\frac{1}{2}\frac{RSS}{\tilde{\sigma}^4} \sim \chi^2_{s-1} \tag{27}$$

where RSS is the Regression Sum of Squares from the regression of the OLS residuals on a constant and the shift variable, $\tilde{\sigma}^2$ is the maximum likelihood estimator of the error variance or $\tilde{\sigma}^2 = \frac{\sum_{t=1}^{T} \hat{e}_t^2}{T}$ and $\hat{e}_t^2$ are the squared OLS residuals, i.e. $\hat{e}_t = y_t - \mathbf{x}_t' \mathbf{b}$. The test in (27) is distributed as a Chi-square with (s-1) degrees of freedom, where s is the number of regressors in the test (two here).

Estimated Generalized Least Squares

If the variances for the two subperiods (groups) are significantly different, the EGLS estimator can be used to correct for heteroskedasticity. The EGLS estimator of the sequential model in (21) is

$$(28) \qquad \hat{\hat{\boldsymbol{\theta}}}(\delta) = \left[\hat{\hat{\theta}}_1' \hat{\hat{\theta}}_2 \hat{\hat{\theta}}_3 \hat{\hat{\theta}}_4 \hat{\hat{\theta}}_5\right]' = (\mathbf{L}'\hat{\boldsymbol{\Psi}}^{-1}\mathbf{L})^{-1}\mathbf{L}'\hat{\boldsymbol{\Psi}}^{-1}\mathbf{r}$$

where, $\mathbf{r} = (r_1, \ldots, r_T)'$, $\mathbf{L} = [\mathbf{Z}_1, \ldots \mathbf{Z}_T]'$ and $\hat{\boldsymbol{\Psi}}^{-1}$ is a diagonal [T x T] matrix of the variances in two subperiods. The estimated covariance matrix for $\hat{\hat{\boldsymbol{\theta}}}(\delta)$ is

$$(29) \quad \hat{V}_T(\delta) = (L'L)^{-1}L'\hat{\Psi}L(L'L)^{-1} = \text{diag}\,(\hat{\sigma}_1^2 Y_T'(L'\hat{\Psi}^{-1}L)^{-1}Y_T, \hat{\sigma}_2^2 Y_T'(L'\hat{\Psi}^{-1}L)^{-1}Y_T)$$

where $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are the mean squared errors (estimated variances) of σ_1^2 and σ_2^2 respectively and Y_T is the scaling matrix defined in connection with (23) above. The Dickey-Fuller tau test for a unit root is calculated as in (23) except that the estimates of θ and standard errors are those from (28) and (29) respectively:

$$(30) \qquad \tau_{DF}^{EGLS}(\delta) = T\left[\hat{\hat{\theta}}_3(\delta) - 1\right] / \left[\hat{V}_T(\delta)_{33}\hat{\sigma}^2(\delta)\right]^{\frac{1}{2}}$$

where $\hat{\hat{\theta}}_3(\delta)$ is the coefficient of the first lag of the dependent variable, δ is the fraction of the data after trimming, $\hat{V}_T(\delta)_{33}$ denotes the appropriate element of the $\hat{V}_T(\delta)$ matrix in (29) and $\hat{\sigma}^2(\delta) = \sum_{t=1}^{T}\left[r_t - \hat{\theta}(\delta)' Z_{t-1}\right]^2 / (T-K)$, where K is the number of regressors (p -4, p is the number of lagged first difference terms).

The Wald test for a structural break in the EGLS case is

(31) $$F_T(\delta) = \left[W\hat{\hat{\theta}}(\delta) - w\right]' \left[W\left(L'\hat{\Psi}^{-1}L\right)^{-1}W'\right]^{-1} \left[W\hat{\hat{\theta}}(\delta) - w\right] / \hat{\hat{\sigma}}^2(\delta)$$

where $\hat{\hat{\sigma}}^2(\delta) = \{\left[\mathbf{r} - \mathbf{L}\hat{\hat{\boldsymbol{\theta}}}(\delta)\right]' \hat{\Psi}^{-1} \left[\mathbf{r} - \mathbf{L}\hat{\hat{\boldsymbol{\theta}}}(\delta)\right]\} / (T - p - 4)$ and the other variables were defined previously.

Seemingly Unrelated Regression (SUR)

Bilateral exchange rates are rarely determined solely by economic conditions in those two countries but are often affected by the economic conditions in other countries. Therefore we have reason to believe that real exchange rates are contemporaneously correlated. For example, a deviation in the Japanese/US real exchange rate may affect the German/US real exchange rate and the French/US real exchange rate. Our study investigates this contemporaneous correlation with real exchange rate data for the G-7 because of their self-professed goal of exchange rate coordination. A Seemingly Unrelated Regression (SUR) model is used to capture the contemporaneous interrelationship of the G-7's bilateral real exchange rates.

The assumptions in the SUR model, as stated in Judge (1988, p. 445-6), are that: all disturbances have a zero mean, each equation's disturbance variance is constant over time but can vary between equations, the cross equation (contemporaneous) correlation exists, and that the error terms are not autocorrelated or serially correlated. There are two cases in which there is nothing to gain by using SUR: If all contemporaneous correlations are zero then no new information can be gained from using SUR over Generalized Least Squares estimation. Second, there is nothing to gain if the explanatory variables in each equation are numerically identical (Judge et. al., 1988, p. 448)[34]. Least squares applied to each bilateral real exchange rate [35] will give us the best linear unbiased estimator of that real exchange rate. However, if information from other bilateral real exchange rates will help explain the original real exchange rate, then we should incorporate that information. Our model includes six bilateral real exchange rates and thus six model equations.

The test for contemporaneous correlation used for this study is from Breusch and Pagan (1980). The statistic is the sum of the squared correlations across equations from the OLS estimation. For the M = 6 equation case the statistic is:

(37) $$\lambda = T\sum_{i=2}^{M}\sum_{j}^{i-1} r_{ij}^2 = T\sum_{i=2}^{6}\sum_{j=1}^{i-1} r_{ij}^2 ,$$

where the squared correlation is the square of the covariance divided by the two product of the two variances:

(38) $$r_{ij}^2 = \frac{\hat{\sigma}_{ij}^2}{\hat{\sigma}_{ii}\hat{\sigma}_{jj}} .$$

Under the null hypothesis that all of the contemporaneous covariances are zero, λ has an asymptotic χ^2-distribution with M(M-1)/2 or 15 degrees of freedom. The alternative hypothesis is that at least one of the covariances is zero.

Following Judge et. al. 1988, p. 452) we apply SUR to the M equations (11`) to derive the estimator:

(32) $$\hat{\hat{\beta}}_U = [X'(\hat{\Sigma}^{-1} \otimes I)X)^{-1}]X'(\hat{\Sigma}^{-1} \otimes I)r$$

[34] See also Fomby, Hill and Johnson (1994) page 159.

[35] Least squares applied to the whole system is identical to least squares applied to each equation separately. (Judge et. al., 1988, p. 447)

where $\hat{\hat{\beta}}_U$ is the unrestricted SUR coefficient matrix of dimension (MK x 1), X is an (MK x T) matrix of M real exchange rate equations, X also includes an (M x T) vector of dummy variables to indicate the presence of structural breaks, K= K_1, ... , K_m regressors (specific to each equation), and T observations. $(\hat{\Sigma}^{-1} \otimes I)$ is the estimated covariance matrix, where the ijth element σ_{ij} is the estimated covariance between the ith and jth equations given by:

$$\hat{\hat{\sigma}}_{ij} = T^{-1}(r_i - X_i\hat{\hat{\beta}}_i)'(r_i - X_i\hat{\hat{\beta}}_i) \tag{33}$$

where $\hat{\hat{\beta}}' = (\hat{\hat{\beta}}'_1, \hat{\hat{\beta}}'_2, \ldots, \hat{\hat{\beta}}'_M)$.

Next, our monthly model corrects for grouped heteroskedasticity.[36] The volatility during the Bretton Woods era is significantly lower than in the flexible exchange rate period. Therefore, the estimates of $\hat{\hat{\sigma}}_{ij}$ differ for each country (equation) before and after 1973. This means that instead of 6 homoskedastic variances in $(\hat{\Sigma}^{-1} \otimes I)$ there will be 12. $(\hat{\Sigma}^{-1} \otimes I)$ contains 30 covariance terms representing the contemporaneous correlations between each country in the differing periods.

Wald Tests of the Cross-Equation Restriction

We test the cross equation restrictions that the coefficients on the unit root regressor are equal with a Wald F test. The restricted Seemingly Unrelated Regression estimator, $\hat{\hat{\beta}}_R$, is:

[36] Note that we do not apply this heteroskedasticity step to the annual data in Chapter 5.

(34) $$\hat{\hat{\beta}}_R = \hat{\hat{\beta}}_U + \hat{C}W'(W\hat{C}W')^{-1}(w - W\hat{\hat{\beta}}_U)$$

where $\hat{C} = [X'(\hat{\Sigma}^{-1} \otimes I)X)^{-1}]$

and $\hat{\hat{\beta}}_U = [X'(\hat{\Sigma}^{-1} \otimes I)X)^{-1}]X'(\hat{\Sigma}^{-1} \otimes I)r$.

and the variables are as explained above. The restrictions are tested by the Wald F-test:

(35) $$F_{SUR} = \left[W\hat{\theta} - w\right]'\left[W\hat{C}^{-1}W'\right]^{-1}\left[W\hat{\hat{\theta}} - w\right] / J$$

where J (= 5 in our case) is the number of restrictions. This test follows an approximate $F_{(J,MT-K)}$ distribution where K is the average number of regressors in each equation (Judge et. al., 1988, pp. 451-459).

The Student's t-test is:

(36) $$t_{SUR} = \left[W\hat{\hat{\theta}}_i - w\right]'\left[W\hat{C}^{-1}W'\right]^{-1/2}$$

where $\hat{\hat{\theta}}_i$ is the i[th] coefficient that is tested and the other terms are defined above. There is only one restriction for the Student's t-test. Testing that the restricted unit root regressor's coefficient is equal to one corresponds to the Dickey-Fuller unit root tau test discussed above.

Monte Carlo Simulation Procedures

Due to the lack of small sample properties for the distributions of many of the tests in this paper we used Monte Carlo simulations to empirically derive the probability distributions. There are two sets of Monte Carlo simulations in this paper. The first is the univariate procedure for the sequential test Estimated Generalized Least Squares (EGLS) results. The empirical distributions for the second procedure are for the multivariate Seemingly Unrelated Regression results.

For the univariate procedure, the data is generated such that $\mu_0 = 0$, $\mu_1 = 0$, $\mu_2 = 0$, $\alpha = 1$ and $\beta(L) = 0$, in (18) which is repeated below for convenience:

$$(18) \quad r_t = \mu_0 + \mu_1\tau_{1t}(k) + \mu_2 t + \alpha r_{t-1} + \beta(L)\Delta r_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T.$$

There are different assumptions for the annual data and monthly data Monte Carlo simulations. The monthly data simulation was generated from a random walk model with a heteroskedastic variance. The simulated observations are from a heteroskedastic random walk model: $r_t = r_{t-1} + \varepsilon_t$ where ε_t is iid $N(0, \sigma_i^2)$. A $N(0, \sigma_i^2)$ distribution is used to create the heteroskedasticity that is assumed to exist, where i=1,2 corresponds to the homoskedastic variance before January 1973 and the differing homoskedastic variance after January 1973. The first simulated observation is generated with the January 1957 real exchange rate as the lagged regressor, r_{t-1}.[37] The first 68 simulated observations simulated are not used because of possible start-up bias. The data set is formed from the next 432 simulated observations. This data set is then used to simulate the probability distribution for the structural break F-test statistic, the Dickey-Fuller *tau* test statistics and the Quandt Likelihood Ratio test statistic. These probability distributions consist of 500 simulations for each of the four test statistics. The critical values from each probability distribution are then compared with the calculated test statistic results in Chapter 4.

The annual regression Monte Carlo simulations are generated from a random walk model with homoskedastic variance. The simulated observations are from a

[37] However, we could have used any value for the first observation.

homoskedastic random walk model: $r_t = r_{t-1} + \varepsilon_t$ where ε_t is iid $N(0, \sigma^2)$. As in the monthly univariate simulations, the first 50 observations simulated are not used because of possible start-up bias. The data set is formed from the next 95 simulated observations. As before, this data set is then used to simulate the probability distribution for the structural break F-test statistic, the Dickey-Fuller *tau* test statistics and the Quandt Likelihood Ratio test statistic. These probability distributions consist of 2000 simulations for each of the four test statistics. The critical values from each probability distribution are then compared with the calculated test statistic results in Chapter 5.

The annual data and monthly data multivariate simulations are carried out exactly as above. However, in the multivariate case, the six real exchange rate simulated data sets are combined into a single large data set. This data set is then used to simulate the Lagrange multiplier test statistic for contemporaneous correlation, the Dickey-Fuller *tau* test statistics and the structural break and time trend t-test statistics. A probability distribution is generated for each statistic to determine the critical values for the SUR test results in Chapters 4 and 5. Some models include a test for the restriction that the unit root is the same across the equations (countries). A probability distribution and critical values are also simulated for this test statistic.

In the multivariate model the covariance matrix from the data set is used to simulate the data. In this paper we found that using a diagonal covariance matrix or a full covariance matrix did not affect the critical values. This is consistent with Abuaf and Jorion (1990 p. 165) where they found that the structure of the correlations has little effect on the tests.

CHAPTER IV

RESULTS OF THE TESTS OF PURCHASING POWER PARITY USING THE MONTHLY REAL EXCHANGE RATE

Introduction

This chapter presents empirical results from monthly time series that cover a relatively short time span (1957-1993). The next chapter will cover the annual data set that spans a relatively long number of years (1900-1994). Therefore we have results from two data sets: short-term high frequency time series and long-term low frequency data.

The first three sections in this chapter test Purchasing Power Parity with the natural logarithm of six bilateral real exchange rates. There are twelve time series data sets because each bilateral exchange rate is computed with either the CPI or the WPI. The first section will use the Dickey-Fuller (DF), and Augmented Dickey-Fuller (ADF) tests which are the standard tests of a unit root (or random walk) time series. The second section reports the rolling regressions and third the recursive regressions results.

The next sections, the sequential regressions, look at structural breaks in each of the 12 time series. Ordinary least squares is used in the first set of tables. A test for heteroskedasticity at January 1973 is performed to test whether the more flexible exchange rate period following 1973 results in greater exchange rate volatility than the Bretton Woods Era (1957-1973). Estimated Generalized Least Squares is used to correct for the presence of heteroskedasticity. Because standard tables do not cover the nonstationary unit-root case, Monte Carlo simulations are performed to determine the critical values for the heteroskedastic estimators.

Finally the 6 bilateral real exchange rates are stacked into a multi-equation model to test Purchasing Power Parity over the entire Group of Seven countries with SUR. This will increase the power of the unit root test over the univariate case. Again the standard critical values do not apply and Monte Carlo simulations are performed for the probability values.

The Standard Test for a Unit Root

First we test the unit root hypothesis with the Augmented Dickey-Fuller test equation (7). We calculate the values for the natural logarithm of the bilateral real exchange rates for the Group of Seven countries. The United States is the base country for the monthly data, which includes the period from 1957 through 1992 except for Italy which only covers up through 1989.. These results are displayed in the first two columns of Table 2. The first column uses the Consumer Price Index when computing the bilateral real exchange rates whereas the second column uses the Wholesale Price Index. The number of lags is calculated using the highest significant lag order from either the autocorrelation function or the partial autocorrelation function of the first differenced series (up to a maximum lag order of $\sqrt{N}$)[38]. The number of lags differs for each variable and each price index. The Dickey-Fuller tau test for a unit root (τ_{DF}) has critical values from a nonstandard Student's t-distribution. MacKinnon (1991) calculated the critical values using 25,000 replications (and thus are a little more accurate that previous authors[39] that use only 10,000 replications). We reject the null if the tau statistic is less

[38] See page 160 of Shazam User's Reference Manual Version 7.0 (1993).

[39] Many authors use the previous simulations of Engle and Yoo, 1987.

than the critical values of -2.5671, -2.8621, and -3.4335 for the 10%, 5% and 1% significance levels (when a time trend is not included). Failure to reject the null hypothesis (of a unit root) implies that the variable is not stationary and regressions using the variable in this form are "spurious." In that case, purchasing power parity does not hold and the real exchange rate follows a random walk.

The third and fourth columns of Table 2 report the results of unit root tests on the first differences of the natural logarithms of the real exchange rate for the CPI and WPI, respectively.

The results of Table 2 reveal that when using the standard tests the real exchange rate is not stationary in the level form and we cannot reject the null hypothesis of a unit root, (with the exception of the WPI for France). The real exchange rates are stationary in their first differences so the real exchange rate is integrated of order one.

In Table 3 we show the results of the Augmented Dickey-Fuller test where a trend is included as in equation (12). When a time trend is added the critical values for the Dickey-Fuller τ-test are -3.1279, -3.4126 and -3.9638 (for the 10%, 5% and 1% significance levels respectively). The results are identical to those of Table 2: the real exchange rate is stationary when differenced but nonstationary in the level form.

As a further check in testing for a unit root we drop the lagged first difference regressors from the model. This is the original Dickey-Fuller test, equation (7`). In this case the null hypothesis cannot be rejected in all cases for the log of the real exchange rate the variables. The variables are stationary in their first differences as shown in Table 4.

Table 2. Augmented Dickey Fuller tests on the real exchange rates for the G-7, without a time trend

Country	r_t (CPI)	r_t (WPI)	$r_t - r_{t-1}$ (CPI)	$r_t - r_{t-1}$ (WPI)
Canada	-2.03	-2.08	-3.13**	-3.92***
France	-2.19	-3.68***	-4.54***	-3.97***
Germany	-1.70	-1.49	-3.71***	-5.56***
Italy	-2.17	-2.15	-4.47***	-4.08***
Japan	-0.84	-1.01	-4.89***	-5.14***
United Kingdom	-2.24	-1.62	-3.71***	-4.65***

Note: Test statistics less that the critical values of -2.5671, -2.8621, and -3.4335 are rejected for the 10% (*), 5% (**), and 1% (***) significance levels respectively.

Table 3. Augmented Dickey Fuller tests on the real exchange rates for the G-7, with a time trend

Country	r_t (CPI)	r_t (WPI)	$r_t - r_{t-1}$ (CPI)	$r_t - r_{t-1}$ (WPI)
Canada	-2.35	-2.24	-3.06	-3.88**
France	-2.54	-3.62**	-4.54***	-3.97***
Germany	-2.16	-2.39	-3.70**	-5.56***
Italy	-2.67	-2.78	-4.47***	-4.10**
Japan	-2.39	-2.69	-4.89***	-5.15***
United Kingdom	-2.94	-2.41	-3.68**	-4.65***

Note: Test statistics less that the critical values with a trend are -3.1279, -3.4126 and -3.9638 for the 10% (*), 5% (**), and 1% (***) significance levels respectively.

Table 5 shows the results of the Dickey-Fuller test with a trend or equation (8`). When a time trend is added, the null hypothesis cannot be rejected for the log levels of the real exchange rates using either price index.

In conclusion, using standard DF and ADF tests, there is no evidence that

Table 4. Dickey-Fuller test statistics on the real exchange rate without a trend (no lags)

Country	r_t (CPI)	r_t (WPI)	$r_t - r_{t-1}$ (CPI)	$r_t - r_{t-1}$ (WPI)
Canada	-1.62	-2.07	-17.78***	-18.70***
France	-1.52	-1.69	-17.26***	-17.29***
Germany	-1.17	-0.95	-15.40***	-16.13***
Italy	-1.05	-0.85	-15.12***	-15.70***
Japan	-0.50	-0.52	-15.64***	-15.91***
United Kingdom	-1.67	-1.40	-14.71***	-14.92***

Note: Test statistics less that the critical values of -2.5671, -2.8621, and -3.4335 are rejected for the 10% (*), 5% (**), and 1% (***) significance levels respectively.

Table 5. Dickey-Fuller test statistics on the real exchange rate with a trend (no lags)

Country	r_t (CPI)	r_t (WPI)	$r_t - r_{t-1}$ (CPI)	$r_t - r_{t-1}$ (WPI)
Canada	-1.65	-2.10	-17.77***	-18.68***
France	-1.93	-1.69	-17.25***	-17.29***
Germany	-1.51	-1.79	-15.38***	-16.11***
Italy	-1.49	-1.49	-15.11***	-15.70**
Japan	-1.89	-2.33	-15.63***	-15.92***
United Kingdom	-2.15	-2.08	-14.69***	-14.91***

Note: Test statistics less that the critical values with a trend are -3.1279, -3.4126 and -3.9638 for the 10% (*), 5% (**), and 1% (***) significance levels respectively.

Purchasing Power Parity holds for our data with or without a time trend. We fail to reject the hypothesis of a unit root for the level of the real exchange rate. Therefore, we find results similar to other authors using the DF and ADF tests[40], namely that the level of the real exchange rate cannot be distinguished from a random walk.

Rolling and Recursive Regressions

In the previous section, we rejected the hypothesis that the real exchange rate is integrated of order two (a unit root exists for first differences) but failed to reject the single unit root hypothesis, i.e. real exchange rates are integrated of the first order. Therefore, following Banerjee et al. (1992) we model each of the twelve bilateral real exchange rates as containing a single unit-root and apply the rolling and recursive methodology. If we reject the null hypothesis of a unit root that can be seen as evidence in favor of purchasing power parity.

Tables 6 - 9 show the maximum and minimum Dickey-Fuller and Augmented Dickey-Fuller tau tests for rolling and recursive tests of the unit root hypothesis. We report two values, the maximum and minimum tau test statistics, which are the highest and lowest values of this test. They are calculated using equation (16) over the appropriate subset of the sample. Because the critical values reported by MacKinnon (1991) for the Dickey-Fuller τ-test are inappropriate in the case of rolling and recursive regression. Banerjee, et. al. report critical values from Monte Carlo simulations. The critical values depend on the sample size, however not the number of lags. Our sample is 432 observations thus the critical values are those from Monte Carlo simulations reported by Banerjee et. al. (1992) for 500 observations.

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[40] For example see Corbae and Ouliaris (1988), Enders (1988),

Table 6: Rolling regressions *tau* tests for a unit root (four lags)

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-0.25	-2.06	0.20	-2.55
France	-0.34	-2.62	-0.30	-2.14
Germany	0.48	-1.59	0.93	-2.17
Italy	0.25	-1.51	-0.59	-1.61
Japan	0.88	-1.83	0.10	-2.16
United Kingdom	-0.36	-1.64	-0.05	-1.76

Note: The τ^{min} critical values for 500 observations are 4.55, -4.79 and -5.00 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 500 observations are -1.25, -1.47 and -1.62 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

Table 7: Rolling regressions *tau* tests for a unit root (no lags)

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	0.53	-2.53	0.52	-2.54
France	0.32	-1.75	0.32	-1.75
Germany	3.39	-2.04	3.39	-2.04
Italy	0.04	-1.25	0.04	-1.28
Japan	0.77	-1.66	0.77	-1.66
United Kingdom	0.97	-1.40	0.96	-1.40

Note: The τ^{min} critical values for 500 observations are 4.55, -4.79 and -5.00 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 500 observations are -1.25, -1.47 and -1.62 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

Table 6 shows the ADF tests and Table 7 the DF tests (without lags of the dependent variable) for the rolling regression methodology. None of the test statistics are

Table 8: Recursive regressions *tau* tests for a unit root (four lags)

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-0.80	-2.55	-0.07	-2.09
France	-1.90*	-4.96***	-0.39	-4.04*
Germany	0.42	-4.06*	2.06	-2.71
Italy	0.37	-2.90	-0.56	-2.96
Japan	1.07	-4.44***	0.20	-3.50
United Kingdom	0.27	-3.17	-0.08	-3.08

Note: The τ^{min} critical values for 500 observations are -3.88, -4.18 and -4.42 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 500 observations are -1.66, -1.92 and -2.17 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

Table 9: Recursive regressions *tau* tests for a unit root (no lags)

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-0.17	-1.80	-0.33	-1.80
France	0.34	-4.62***	0.34	-4.62***
Germany	5.82	-2.77	5.82	-2.58
Italy	0.36	-2.56	0.36	-2.56
Japan	1.16	-3.42	0.95	-3.40
United Kingdom	0.76	-2.69	0.76	-2.69

Note: The τ^{min} critical values for 500 observations are -3.88, -4.18 and -4.42 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 500 observations are -1.66, -1.92 and -2.17 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

significant at the 10% level. The subsets of the data in the rolling regressions case only span fifteen years which is not considered a long-run by most of the recent authors.[41]

Next we use the recursive method to search the data set for a unit root. Each successive regression increases by one observation. Table 8 shows the ADF tests, Table 9 the DF tests. For France we reject a unit root at the 1% significance level in three cases and 10% in the fourth. In three of the cases the date of the minimum Dickey-Fuller tau test is July 1969. The fourth, when we use the Wholesale Price Index with four lags, has a minimum significant unit root test on November of 1967.[42] A quick glance on page 31 Figure 2, shows a strong spike in July of 1969. Thus the French/U.S. real exchange rate is stationary up to July 1969 but not after.

Japan, for the Wholesale Price Index, is significant at the 1% level on September 1968. This is a puzzling case with no apparent explanation when looking at the data. Furthermore, when the Dickey-Fuller test (without lags) is used, Japan is not significant leading one to believe that perhaps the inclusion of the lags is affecting the significance of the test. The coefficient is 0.9012 and the test statistics are consistently below -4.00 in value until January of 1971.

This reveals some evidence in the form of a stationary time series, stationary up to January 1971. However, the results on the whole do not support PPP. The tests fail to reject the random walk representation of the twelve bilateral real exchange rates.

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[41] See for example Frankel and Rose (1995) and Hakkio and Rush (1991).

[42] The coefficients are 0.8853 and 0.8825 for the WPI (4 lags & no lags), and for the CPI, 0.8926 and 0.8825 (4 lags and no lags) respectively.

Therefore we proceed to test for a structural break that may be present in our data series using the sequential model.

Sequential Regressions

Following Banerjee, Lumsdaine and Stock (1992), there are four statistics of interest to our analysis: the maximum value for the F_T statistic in (24) that tests the hypothesis that $\mu_1 = 0$, F^{max}; the Dickey-Fuller tau statistic in (23) evaluated at period k that maximizes the F_T statistic, τ^{max}; the minimal Dickey-Fuller tau statistic over all k, τ^{min}; and the maximum Quandt Likelihood Ratio statistic, Q_{LR} in (24).

Tables 10 - 13 report these four statistics for the Wholesale Price Indices (WPI) and Consumer Price Indices (CPI) for two examples of a break: a shift in the mean as well as a shift in the trend, shown in equations (19) and (20) respectively. Each model is estimated with four lags of the first differenced autoregressive terms. These tables also show the date (k) at which the maximum F_T statistic (F^{max}) is observed.

The Quandt likelihood ratio statistic is significant for all countries and all real exchange rates (with the possible exception of France for the Wholesale Price Index for the mean shift; it is significant at the 10% level). Banerjee, Lumsdaine and Stock (1992 p. 279) claim that the Quandt likelihood ratio test statistic is a powerful and reliable diagnostic tool for detecting a break over all of the coefficients. However, there are no significant F^{max} statistics at the 5% significance level for the mean or trend shift. This indicates that no shifts (in mean or trend) in the real exchange rate are significantly large enough to be recognized by our test. The exception at the 10% level is Germany when calculated with the CPI. A mean shift in August 1980 is indicated. None of the Dickey-

Table 10. Sequential test statistics using the WPI, for a mean shift, four lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (4)
Canada	10/76	10.23	-3.40	-3.43	73.75***
France	3/80	11.31	-3.77	-3.77	33.36*
Germany	7/80	10.71	-3.78	-3.79	329.02***
Italy	4/85	13.57	-2.24	-3.30	282.82***
Japan	10/85	8.46	-4.11	-4.11	369.41***
United Kingdom	3/85	13.81	-3.42	-3.50	285.09***

Note: The critical values are -4.49, -4.77 and -5.05 for the τ^{max} mean shift statistics and are -4.51, -4.78 and -5.05 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.78, 18.99 and 21.26 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (4) they are 30.72, 34.00 and 37.00. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432, except for Italy which has 396).

Table 11. Sequential test statistics using the CPI for a mean shift, four lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (4)
Canada	11/76	13.51	-3.14	-3.14	68.06***
France	8/80	15.33	-4.25	-4.27	51.11***
Germany	8/80	18.70*	-4.50*	-4.50*	304.06***
Italy	4/85	13.13	-3.19	-3.63	192.77***
Japan	11/78	14.31	-3.49	-4.01	222.29***
United Kingdom	2/81	7.38	-3.16	-3.21	269.85***

Note: The critical values are -4.49, -4.77 and -5.05 for the τ^{max} mean shift statistics and are -4.51, -4.78 and -5.05 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.78, 18.99 and 21.26 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (4) they are 30.72, 34.00 and 37.00. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432).

Table 12. Sequential test statistics using the WPI, for a trend shift, four lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (4)
Canada	3/89	3.18	-0.76	-1.95	74.39***
France	11/88	3.65	-2.50	-2.64	42.31***
Germany	11/83	1.53	-1.86	-2.33	329.20***
Italy	6/83	1.56	-2.88	-2.90	370.02***
Japan	1/84	3.28	-2.70	-3.01	283.99***
United Kingdom	4/84	4.17	-2.79	-2.79	284.14***

Note: The critical values are -4.12, -4.39 and -4.68 for the τ^{max} mean shift statistics and are -4.13, -4.39 and -4.69 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 13.20, 16.04 and 18.58 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (4) they are 30.72, 34.00 and 37.00. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432, Italy has only 396).

Table 13. Test statistics for using the CPI, for a trend shift, four lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (4)
Canada	3/89	3.67	-1.39	-2.33	77.95***
France	5/73	0.83	-3.04	-3.04	51.57***
Germany	4/73	1.52	-2.26	-2.43	304.52***
Italy	11/83	2.70	-2.32	-2.44	193.76***
Japan	9/76	2.01	-3.09	-3.33	223.71***
United Kingdom	7/84	0.74	-3.20	-3.20	269.60***

Note: The critical values are -4.12, -4.39 and -4.68 for the τ^{max} mean shift statistics and are -4.13, -4.39 and -4.69 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 13.20, 16.04 and 18.58 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (4) they are 30.72, 34.00 and 37.00. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432).

Fuller tau statistics evaluated at the break (τ^{max}) are significant at the 5% level, thus we are unable to reject the hypothesis that α in (18) is different from one. The same holds for the minimum *tau* test. Again, at the 10% significance level, the exception is the German case. Therefore, these results suggest that purchasing power parity does not hold in the presence of a structural break.

Tables 14 - 17 report test results with no autoregressive lag terms are included in the model. In the case of the mean break, there are five other cases in addition to the German case discussed above. The maximum F_T statistic (F^{max}) shows a significant break for Italy (WPI and CPI), Japan (CPI), Germany (CPI) and the United Kingdom. Therefore, six of the twelve bilateral real exchange rates exhibit mean shift structural breaks. However, there is no evidence to refute the unit root hypothesis in the mean break model with no lagged first difference terms but a time trend included.

None of the trend-shift statistics are significant. Thus, in all four cases neither the τ^{max} nor the τ^{min} statistics are significant. Therefore we fail to reject the hypothesis that the real exchange rates follow a random walk and find no evidence for PPP.

Table 14. Sequential test statistics using the WPI, mean shift, no lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	11/76	11.08	-3.41	-3.41	74.36***
France	3/80	14.67	-3.19	-3.20	25.77
Germany	4/85	12.79	-1.89	-3.43	335.56***
Italy	4/85	19.34**	-2.06	-2.99	286.20***
Japan	7/85	10.55	-3.50	-3.54	354.27***
United Kingdom	3/85	17.20*	-3.31	-3.39	298.44***

Note: The critical values are -4.49, -4.77 and -5.05 for the τ^{max} mean shift statistics and are -4.51, -4.78 and -5.05 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.78, 18.99 and 21.26 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (0) they are 25.19, 27.80 and 30.42. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432, except for Italy which has only 396).

Table 15. Sequential test statistics using the CPI for a mean shift, no lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	11/76	15.72	-3.12	-3.12	66.12***
France	8/80	17.09*	-3.91	-3.98	42.03***
Germany	8/80	20.26**	-4.29	-4.29	314.02***
Italy	4/85	18.69*	-3.00	-3.40	201.52***
Japan	11/78	21.82***	-3.07	-3.17	235.33***
United Kingdom	2/81	9.25	-2.80	-2.83	278.54***

Note: The critical values are -4.49, -4.77 and -5.05 for the τ^{max} mean shift statistics and are -4.51, -4.78 and -5.05 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.78, 18.99 and 21.26 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (0) they are 25.19, 27.80 and 30.42. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432).

Table 16. Sequential test statistics using the WPI, for a trend shift, no lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	3/89	3.91	-1.33	-2.19	75.17***
France	12/60	6.70	-1.74	-2.26	21.38
Germany	8/83	1.92	-1.97	-2.02	335.81***
Italy	9/83	4.27	-1.92	-2.04	289.69***
Japan	7/82	1.00	-2.52	-2.53	355.56***
United Kingdom	9/83	2.67	-2.62	-2.62	300.63***

Note: The critical values are -4.12, -4.39 and -4.68 for the τ^{max} mean shift statistics and are -4.13, -4.39 and -4.69 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 13.20, 16.04 and 18.58 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (0) they are 25.19, 27.80 and 30.42. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432; 396 for Italy).

Table 17. Sequential test statistics using the CPI, for a trend shift, no lags

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	3/89	5.63	-0.48	-1.72	78.04***
France	11/61	1.10	-1.90	-2.04	42.24***
Germany	3/73	1.23	-1.83	-1.83	314.88***
Italy	9/83	5.00	-2.06	-2.21	176.89***
Japan	4/73	2.15	-2.17	-2.21	236.37***
United Kingdom	3/89	0.65	-1.79	-2.21	278.45***

Note: The critical values are -4.12, -4.39 and -4.68 for the τ^{max} mean shift statistics and are -4.13, -4.39 and -4.69 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 13.20, 16.04 and 18.58 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (0) they are 25.19, 27.80 and 30.42. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). These values are calculated with a sample size of 500 (our sample size is 432).

Estimated Generalized Least Squares

In order to investigate the unit root hypothesis further, we test for the presence of heteroskedasticity. Two tests are used: the likelihood ratio F-test (F_{het}) and the Breusch-Pagan (B.P.) test. The results for the WPI are in Table 18 and for the CPI in Table 19.

Table 18. Heteroskedasticity tests using the WPI

	No Lags, No Trend		No Lags	
Country	F_{het}	B.P.	F_{het}	B.P.
Canada	2.85***	40.61***	2.87***	45.15***
France	1.33**	2.44	1.26**	2.21
Germany	8.76***	104.27***	8.74***	104.16***
Italy	14.09***	144.05***	14.14***	144.18***
Japan	7.64***	108.92***	7.60***	108.66***
United Kingdom	4.27***	73.48***	4.27***	73.46***

Note: The critical F(200,200) values for the F_{het} statistics are 1.26 and 1.39 for the 5% and 1% significance levels respectively. For the Breusch-Pagan Chi-square test with one degree of freedom, they are 3.84 and 6.64 respectively (Johnston, 1984, p.549-553).

With the possible exception of the French (WPI-B.P.) case all of the tests reject the hypothesis of constant error variance at the 1% significance level. The Breusch-Pagan test for the type of heteroskedasticity we have chosen is highly significant. Therefore we use the Estimated Generalized Least Squares method to correct for the presence of this type of grouped heteroskedasticity.

Table 19. Heteroskedasticity tests using the CPI

Country	No Lags, No Trend		No Lags	
	F_{het}	B.P.	F_{het}	B.P.
Canada	2.66***	40.57***	2.65***	40.54***
France	2.04***	20.98***	2.05***	21.35***
Germany	7.18***	90.48***	7.22***	90.59***
Italy	7.03***	108.61***	7.04***	108.77***
Japan	5.28***	87.24***	5.28***	87.30***
United Kingdom	4.98***	83.14***	4.98***	83.16***

Note: The critical F(200,200) values for the F_{het} statistics are 1.26 and 1.39 for the 5% (**) and 1% (***) significance levels respectively. For the Breusch-Pagan Chi-square test with one degree of freedom, they are 3.84 and 6.64 respectively (Johnston, 1984, p.549-553).

Given the presence of heteroskedasticity, the four test statistics that we calculated previously will be utilized in the EGLS method. We examine the case of a break in the mean only. We begin with the case of no autoregressive lags and a time trend, following with the case of no time trend or autoregressive lags. In the case of no lags and no time trend, the only change to equations (18) and (20) is that the four lags of the first difference of the autoregressive lags are restricted to be equal to zero:

(18`) $r_t = \mu_0 + \mu_1 \tau_{1t}(k) + \mu_2 t + \alpha r_{t-1} + \varepsilon_t$ t = 1, ... , T, where we have

(20) Shift in mean: $\tau_{1t}(k)$= 1(t >k).

The transformation is analogous producing the model:

(21`) $r_t = \theta' \mathbf{Z}_{t-1} + \varepsilon_t$ t = 1, ... , T

where $\mathbf{Z}_{t-1} = \left[Z_{t-1}^2 Z_{t-1}^3 Z_{t-1}^4 Z_{t-1}^5\right]'$

$$Z_{t-1}^2 = 1$$

$$Z_{t-1}^3 = \left(r_{t-1} - \overline{\mu_0}(t-1)\right),$$

$$Z_{t-1}^4 = \tau_{1t}(k),$$

$Z_{t-1}^5 = t$, and

$\overline{\mu_0} = E\Delta r_t = \mu_0$.[43]

Thus the time trend, no autoregressive lag model is very similar to those shown previously. When the time trend is restricted to be zero the Dickey-Fuller model changes and we have:

(18``) $r_t = \mu_0 + \mu_1 \tau_{1t}(k) + \alpha r_{t-1} + \varepsilon_t$ t = 1, ... , T, where the

(20) shift in mean: $\tau_{1t}(k)$= 1(t >k).

The transformed model is:

(21``) $r_t = \theta' \mathbf{Z}_{t-1} + \varepsilon_t$ t = 1, ... , T

where $\mathbf{Z}_{t-1} = \left[Z_{t-1}^2 Z_{t-1}^3 Z_{t-1}^4\right]'$

$$Z_{t-1}^2 = 1$$

$$Z_{t-1}^3 = \left(r_{t-1} - \overline{\mu_0}\right),$$

$Z_{t-1}^4 = \tau_{1t}(k)$, and

$\overline{\mu_0} = E\Delta r_t = \mu_0$.

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[43] Note that we kept the same superscripts as before so that the missing regressors are

$Z_{t-1}^1 = \left(\Delta r_{t-1} - \overline{\mu_0}, \ldots, \Delta r_{t-p} - \overline{\mu_0}\right)'$.

In each case the tests for the unit root and the structural break do not change but are repeated here for convenience. The probability distributions and critical values for the test statistics are not available and were simulated. The Monte Carlo simulations are data dependent and thus differ between countries because of the heteroskedasticity structure imposed on the simulated data.

(23) $$\tau_{DF}^{SEQ}(\hat{\theta}) = T\left(\hat{\theta}_3(\delta) - 1\right) / \left[V_T(\delta)_{33} s^2(\delta)\right]^{\frac{1}{2}} \qquad \delta_0 \le \delta \le 1 - \delta_0,$$

(24) $$F_T(\delta) = \left[W\hat{\theta}(\delta) - w\right]' \left[W(L'L)^{-1} W'\right]^{-1} \left[W\hat{\theta}(\delta) - w\right] / s^2(\delta).$$

The results of these tests are shown in Tables 20 - 23. The Quandt Likelihood ratio test Q_{LR} (0) is highly significant in all cases. Banerjee et. al. (1992, p. 278) state that the Q_{LR} typically has the best power against changing autoregressive coefficients (θ_3). Therefore, this test provides significant evidence that there is a shift in the model, a structural change.

The Monte Carlo simulations for Tables 20-23 are calculated such that the data set that is used to compute the simulations is our original data set. In Table 20 four of the six structural breaks are significant, 3 at the 1% and 1 at the 5 % significance level. All of the unit root hypothesis are rejected at the 10% significance level except for the United Kingdom. Six of the twelve are significant at the 5% level or higher.

These statistics are very different from the corresponding homoskedastic model, Table 14. The structural break test statistics are significantly different in most cases. Canada remains insignificant but France (March 1980), Germany (October 1969) and Japan (August 1971) became highly significant from insignificant. Furthermore, after

Table 20. Sequential test statistics using the WPI, for a mean shift, no lags, time trend

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	12/72	7.32 (0.24)	-2.28** (0.03)	-2.98*** (0.01)	74.36*** (0.00)
France	3/80	14.78*** (0.01)	-3.18** (0.04)	-3.22* (0.09)	25.77*** (0.00)
Germany	10/69	18.96*** (0.01)	-1.89* (0.07)	-2.02* (0.08)	335.56*** (0.00)
Italy	3/85	6.01 (0.39)	-1.88* (0.07)	-2.43** (0.04)	289.44*** (0.00)
Japan	8/71	31.78*** (0.00)	-4.09*** (0.00)	-4.24*** (0.00)	354.27*** (0.00)
United Kingdom	3/85	12.03** (0.02)	-3.16 (0.14)	-3.54 (0.19)	114.47*** (0.00)

Note: The critical values for the statistics are dependent upon the country. Probability values are quoted in parentheses below each statistic. The statistics are evaluated at the 10% (*), 5% (**) and 1% (***)significance levels. All probability values were simulated using $r_t = r_{t-1} + \varepsilon_t$ where ε_t is iid N(0,1) and are based on 500 Monte Carlo replications.

Table 21. Sequential test statistics using the CPI for a mean shift, no lags, time trend

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	11/76	12.32*** (0.01)	-2.53 (0.13)	-2.53 (0.13)	66.33*** (0.00)
France	8/80	13.96*** (0.01)	-3.51** (0.03)	-3.56** (0.03)	42.03*** (0.01)
Germany	11/69	21.45*** (0.00)	-0.67 (0.48)	-1.76 (0.38)	314.02*** (0.00)
Italy	4/85	8.51 (0.16)	-1.52 (0.37)	-2.36 (0.15)	175.46*** (0.00)
Japan	9/71	25.82*** (0.00)	-3.83** (0.02)	-3.83** (0.02)	235.34*** (0.00)
United Kingdom	1/71	5.42 (0.36)	-2.23 (0.16)	-2.56 (0.13)	278.54*** (0.00)

Note: The critical values for the statistics are dependent upon the country. Probability values are quoted in parentheses below each statistic. The statistics are evaluated at the 10% (*), 5% (**) and 1% (***)significance levels. All probability values were simulated using $r_t = r_{t+1} + \varepsilon_t$ where ε_t is iid N(0,1) and are based on 500 Monte Carlo replications.

correcting for heteroskedasticity, Italy's structural break test statistic became insignificant. The break for the United Kingdom was unaffected by the correction for heteroskedasticity. The break data for the U.K. is March 1985 in both instances.

The results are similar in the case of CPI-based real exchange rates (Table 21). Except for Italy and the UK there are significant breaks at the 1% level. The Japanese and German break dates coincide with final years of the Bretton Woods era. As in Table (20) the German break data results when the currency was revalued (1969) however not when the Bretton Woods period officially ended. The significance of the break point for the U.K. and Canada depends upon the price index used. France, Germany and Japan have consistently significant break points regardless of the price index used. It is interesting to note that the unit root hypothesis is rejected (at the 5% significance level) in the only two of the cases, France and Japan for the CPI.

The WPI time series is a better indicator of traded goods. This may be reflected in the larger number of significant unit root tests. PPP holds for 5 of the 6 real exchange rate series when the WPI is used. The evidence is weaker for the CPI, as only two of the six cases are significant.

The results when the time trend is excluded are shown in Tables 22 and 23. In Table 22, the results significantly change for France and the U.K. when the time trend is excluded from the model for the WPI. All of the tests become insignificant for France when the time trend is not included. For the U.K. the opposite happens as the unit root is rejected and further evidence is provided for a structural break in March of 1985. Japan is unaffected by the time trend change. The break in August 1971 and rejection of the unit

Table 22. Sequential test statistics using the WPI, for a mean shift, no lags, no trend

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	11/76	3.07 (0.63)	-2.81 (0.11)	-2.81 (0.11)	72.51*** (0.00)
France	3/89	5.17 (0.37)	-1.20 (0.42)	-2.03 (0.23)	19.23* (0.06)
Germany	11/69	23.03*** (0.00)	-2.59 (0.12)	-2.59 (0.12)	345.05*** (0.00)
Italy	4/85	9.08* (0.09)	-1.59 (0.34)	-2.04 (0.20)	284.43*** (0.00)
Japan	8/71	44.72*** (0.00)	-4.60*** (0.00)	-4.68*** (0.00)	354.43*** (0.00)
United Kingdom	3/85	14.26** (0.02)	-3.54** (0.03)	-3.55** (0.03)	299.62*** (0.00)

Note: The critical values for the statistics are dependent upon the country. Probability values are quoted in parentheses below each statistic. The statistics are evaluated at the 10% (*), 5% (**) and 1% (***)significance levels. All probability values were simulated using $r_t = r_{t+1} + \varepsilon_t$ where ε_t is iid N(0,1) and are based on 500 Monte Carlo replications.

root hypothesis are highly significant in all cases. Germany continues to have a break in November 1969 but we fail to reject that the real exchange rate follows a unit root autoregressive representation. Thus there are four significant breaks but only two of the six real exchange rates find evidence for PPP.

In Table 23 all of the unit root hypotheses are rejected at the 10% significance level. PPP holds in all six of the real exchange rates when calculated with the CPI but no time trend is included. Three of the countries have significant structural breaks: Germany in November 1969, Italy in April of 1985 and Japan of 1971. Again when the time trend is dropped from the model the results differ significantly. Canada, France and Italy have significant changes in the structural break test statistic.

Table 23. Sequential test statistics using the CPI for a mean shift, no lags, no trend

Country	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	11/76	5.08 (0.30)	-2.64*** (0.01)	-2.64*** (0.01)	63.87*** (0.00)
France	8/80	6.72 (0.14)	-2.14** (0.03)	-2.59** (0.02)	40.06*** (0.00)
Germany	11/69	17.18*** (0.00)	-2.00* (0.08)	-2.44** (0.03)	310.64*** (0.00)
Italy	4/85	9.69* (0.06)	-2.05* (0.07)	-2.13* (0.09)	174.16*** (0.00)
Japan	9/71	23.11*** (0.00)	-3.71*** (0.00)	-3.74*** (0.00)	232.96*** (0.00)
United Kingdom	1/71	7.59 (0.11)	-2.74*** (0.01)	-2.78** (0.02)	276.25*** (0.00)

Note: The critical values for the statistics are dependent upon the country. Probability values are quoted in parentheses below each statistic. The statistics are evaluated at the 10% (*), 5% (**) and 1% (***)significance levels. All probability values were simulated using $r_t = r_{t-1} + \varepsilon_t$ where ε_t is iid N(0,1) and are based on 500 Monte Carlo replications.

The break date for Japan and Germany are consistent with models using the either price index and a time trend is included or left out. The evidence for a break for these two countries is clear as is the evidence in favor of PPP for Japan. Overall, in 15 of the 24 cases we can reject the unit-root hypothesis at the 10% level. This provides significant evidence in favor of the long-run theory of PPP when using 36 years of monthly data.

Furthermore, in 15 of the 24 cases there is a significant break. However, concluding that the break should be during the collapse of Bretton Woods would be a mistake. The incorporation of heteroskedasticity is clearly an important

The Method of Seemingly Unrelated Regression

The method of seemingly unrelated regression (SUR) is more efficient that OLS equation by equation when it takes advantage of the information from the cross-equation (contemporaneous) correlations. Furthermore, by testing for and imposing the restriction that all of the values on the first-order autoregressive coefficient, α, are equal we will also see a gain in power.

Tables 24-27 report tests of purchasing power parity for the equation by equation OLS cases. The regressors are an intercept, the lagged dependent variable and a time trend. Tables 24 and 26 are estimated without a time trend, whereas Tables 25 and 27 show the results with a time trend. The calculation of the real exchange rate for Tables 24 and 25 uses the Wholesale Price Index, and Tables 26 and 27 uses the Consumer Price Index. For Tables 24 -27 we use the Dickey-Fuller critical values as determined in Monte Carlo simulations by MacKinnon (1991)[44]. The critical values for the rho test (ρ_μ) are reported from Fuller (1976).

The τ_μ test fails to reject the unit root hypothesis at the 10% critical values. Even though the ρ_μ test has greater power, this test also fails to reject the unit root null hypothesis at the 10% level in all cases. These results are consistent with those presented previously. We fail to reject the random walk hypothesis for the time period including the combined Bretton Woods and more recent flexible exchange rate regime.

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[44] See pages 53-54 above for an explanation of these statistics.

Table 24. OLS Estimates without a time trend (WPI)

Country	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0040 (0.0018)	0.9824 (0.0085)	-2.07	-7.60
France	0.0266 (0.0155)	0.9866 (0.0079)	-1.69	-5.79
Germany	0.0038 (0.0054)	0.9942 (0.0061)	-0.95	-2.51
Italy	-0.0038 (0.0556)	0.9934 (0.0078)	-0.85	-2.61
Japan	0.0148 (0.0300)	0.9971 (0.0056)	-0.52	-1.25
United Kingdom	-0.0053 (0.0034)	0.9893 (0.0076)	-1.41	-4.62

Note: The test statistics are taken from MacKinnon (1991). The Dickey-Fuller tau test statistic that is less than the critical values of -2.5671, -2.8621, and -3.4335 are rejected for the 10%, 5% and 1% significance levels respectively (*, **, and ***). For the Rho test the critical values are -14.0, -16.8, and -20.5 at the 10%, 5% and 1% significance levels for a sample size of 500 (Table 8.5.1 in Fuller, 1976).

It is important to note, as was explained earlier, the data sets in the SUR section for some countries do not correspond to those reported earlier. this will produce somewhat different estimates and test statistics.

Table 25. OLS Estimates with a time trend (WPI)

Country	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0040 (0.0018)	0.9808 (0.0091)	1.8 E-6 (3.8 E-6)	-2.10	-8.29	0.47
France	0.0272 (0.0160)	0.9865 (0.0080)	-1.3 E-6 (8.3 E-6)	-1.69	-5.83	-0.16
Germany	0.0164 (0.0096)	0.9847 (0.0086)	-2.0 E-5 (1.2 E-5)	-1.78	-6.61	-1.67
Italy	0.0815 (0.0723)	0.9869 (0.0088)	-9.8 E-5 (5.3 E-5)	-1.49	-5.19	-1.85
Japan	0.1248 (0.0532)	0.9780 (0.0094)	-3.3 E-5 (1.3 E-5)	-2.34	-9.50	-2.54
United Kingdom	-0.0054 (0.0034)	0.9798 (0.0097)	-1.8 E-5 (1.1 E-5)	-2.08	-8.73	-1.64

Note: The test statistics are taken from MacKinnon (1991). The Dickey-Fuller tau test statistic that is less than the critical values of -3.1279, -3.4126 and -3.9638 are rejected for the 10%, 5% and 1% significance levels respectively (*, **, and ***). For the Rho test the critical values are -14.0, -16.8, and -20.5 at the 10%, 5% and 1% significance levels for a sample size of 500 (Table 8.5.1 in Fuller, 1976).

When a time trend is included, there is no significant difference in the results. The real exchange rate calculated with the Wholesale Price Index, Tables 24 and 25 and the Consumer Price Index, Tables 26 and 27, consistently fail to reject the unit root hypothesis. No evidence of purchasing power parity is found in any of the ordinary least squares regressions.

The time trend is significant for Japan for the WPI with a time trend but not for any of the other countries.

Table 26. OLS Estimates without a time trend (CPI)

Country	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0021 (0.0011)	0.9896 (0.0065)	-1.61	-4.49
France	0.0226 (0.0152)	0.9880 (0.0079)	-1.52	-5.18
Germany	0.0041 (0.0046)	0.9936 (0.0054)	-1.18	-2.76
Italy	0.0670 (0.0497)	0.9907 (0.0068)	-1.37	-4.02
Japan	0.0051 (0.0157)	0.9986 (0.0028)	-0.50	-0.60
United Kingdom	-0.0061 (0.0035)	0.9871 (0.0077)	-1.67	-5.57

Note: See the note in Table 24.

Table 27. OLS Estimates with a time trend (CPI)

Country	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0019 (0.0012)	0.9880 (0.0073)	1.9 E-6 (4.0 E-6)	-1.65	-5.18	0.48
France	0.0346 (0.0178)	0.9833 (0.0087)	-1.4 E-6 (1.0 E-5)	-1.92	-7.21	-0.14
Germany	0.0100 (0.0077)	0.9894 (0.0070)	-1.1 E-5 (1.2 E-5)	-1.51	-4.58	-0.95
Italy	0.1170 (0.0639)	0.9843 (0.0086)	-1.3 E-5 (1.0 E-5)	-1.83	-6.78	-1.27
Japan	0.0982 (0.0534)	0.9838 (0.0086)	-5.0 E-5 (2.7 E-5)	-1.89	-7.00	-1.84
United Kingdom	-0.0063 (0.0035)	0.9787 (0.0099)	-1.6 E-5 (1.2 E-5)	-2.15	-9.20	-1.38

Note: See the note in Table 25.

Tables 28-31 report tests of purchasing power parity for the EGLS regressions for the grouped heteroskedasticity cases. The regressors are an intercept, the lagged dependent variable and (in Tables 29 and 31) a time trend. The calculation of the real exchange rate for Tables 28 and 29 use the Wholesale Price Index, and Tables 30 and 31 use the Consumer Price Index.

The critical values for Tables 28-31 are derived from Monte Carlo simulations. The data for each country were separated into two groups, January 1957 through December 1972 and January 1973 through December 1992. The standard deviation from each country was used to simulate the observations for the Monte Carlo procedure. This method creates heteroskedastic observations consistent with the results previously reported and discussed in Enders (1988). Recall that he reported that using the standard deviation as a measure of variability, the real exchange rate was more volatile in the floating period than in the fixed exchange rate period.

Thus the simulations were generated from a random walk model with a heteroskedastic variance. The simulated observations form a heteroskedastic random walk model: $r_t = r_{t-1} + \varepsilon_t$ where ε_t is iid $N(0, \sigma_i^2)$. The $N(0, \sigma_i^2)$ distribution is used to correct for heteroskedasticity, where i=1,2 corresponding to the homoskedastic variance before 1973 and the differing homoskedastic variance after 1973. The first observation is generated with the January 1957 real exchange rate as the lagged value.[45] The first 68 observations simulated are not used because of possible start-up bias. The data set is formed from the next 432 simulated observations.

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[45] However, we could have used any value for the first observation.

Table 28. EGLS unit root tests and estimates, no trend (WPI)

Country	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0039 (0.0017)	0.9817 (0.0083)	-2.20	-7.91
France	0.0866 (0.0252)	0.9566 (0.0127)	-3.42	-18.75**
Germany	0.0002 (0.0054)	0.9987 (0.0055)	-0.24	-0.56
Italy	-0.0163 (0.0540)	1.0020 (0.0073)	0.27	0.86
Japan	0.0103 (0.0297)	0.9981 (0.0054)	-0.35	-0.82
United Kingdom	-0.0049 (0.0032)	0.9882 (0.0080)	-1.48	-5.10

Note: The critical were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0,1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a heteroskedastic $N(0, \sigma_i^2)$ random error. The critical values for ρ_μ are -12.83, -14.73 and -19.19 for the 10%, 5% and 1% significance levels respectively.

The next step is to simulate the EGLS model and generate the τ- and F-tests. Each simulation was replicated 2000 times to generate the sample distribution for our test statistics.

For Table 28, the EGLS model with no time trend using the WPI, the τ-test critical values are -5.71, -6.13 and -7.02 for the 10%, 5% and 1% significance levels, respectively. For the rho test the critical values are -12.83, -14.73 and -19.19. Only the French real exchange rate can reject a unit root and only with the ρ_μ test.

When a time trend is included into the model, Table 29, the τ-test Wholesale Price Index critical values are -4.02, -6.85 and -8.21 for the 10%, 5% and 1% significance

Table 29. EGLS unit root tests and estimates, with a trend (WPI)

Country	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0040 (0.0017)	0.9805 (0.0092)	1.2 E-6 (3.7 E-6)	-2.13	-8.42	0.32
France	0.0965 (0.0257)	0.9528 (0.0128)	-2.1 E-5 (1.1 E-5)	-3.68	-20.39***	-1.94
Germany	0.0177 (0.0098)	0.9839 (0.0088)	-2.4 E-5 (1.2 E-5)	-1.83	-6.96	-2.00
Italy	0.0481 (0.0649)	0.9941 (0.0085)	-5.5 E-5 (3.1 E-5)	-0.69	-2.55	-1.77
Japan	0.1011 (0.0382)	0.9823 (0.0068)	-2.8 E-5 (7.5 E-5)	-2.60	-7.65	-3.73
United Kingdom	-0.0063 (0.0033)	0.9785 (0.0102)	-1.6 E-5 (1.0 E-5)	-2.11	-9.29	-1.60

Note: The critical were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0,1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a heteroskedastic $N(0, \sigma_i^2)$ random error. The critical values for ρ_μ are -8.39, -17.03 and -24.40 for the 10%, 5% and 1% significance levels respectively.

levels. For the rho test they are -8.39, -17.03 and -24.40 at the 10%, 5% and 1% significance levels, respectively. Again, the only significant statistic is the ρ_μ value for France. With the exception of the rho test for France, all of the hypothesis tests fail to reject a unit root for the real exchange rate.

The results are similar for the Consumer Price Index in Tables 30 and 31. As previously mentioned, Dickey and Fuller (1979, 1981) determined that the rho test is more powerful than the τ-test. So while the statistic for France sticks out amongst the other countries, it is not surprising that the rho statistic is significant while the Dickey-Fuller τ-test is not.

Table 30. EGLS Unit root tests and estimates, no trend (CPI)

Country	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0020	0.9887	-1.72	-4.88
	(0.0011)	(0.0066)		
France	0.0656	0.9671	-2.62	-14.21**
	(0.0251)	(0.0126)		
Germany	-0.0007	0.9993	-0.14	-0.30
	(0.0047)	(0.0049)		
Italy	0.0461	0.9936	-1.01	-2.76
	(0.0469)	(0.0064)		
Japan	-0.0078	1.0009	0.37	-0.39
	(0.0144)	(0.0025)		
United Kingdom	-0.0049	0.9875	-1.60	-5.40
	(0.0030)	(0.0078)		

Note: The critical were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a heteroskedastic $N(0, \sigma_i^2)$ random error. The critical values for ρ_μ are -11..93, -14.04 and -18.42 for the 10%, 5% and 1% significance levels respectively.

For Table 30, the Consumer Price Index simulated Dickey-Fuller τ-test critical values are -5.79, -6.35 and -7.31, for the 10%, 5% and 1% significance levels, respectively. None of these statistics is significant at the 10% significance level. For the rho test the critical values are -11.93, -14.04 and -18.42 respectively.

For Table 31, when a time trend is included the τ-test critical values for the Consumer Price Index are -4.23, -7.22 and -8.85 for the 10%, 5% and 1% significance levels, respectively. For the rho test they are -13.46, -17.76 and -25.61 at the 10%, 5% and 1% significance levels respectively. The only significant statistic is the ρ_μ value for France.

Table 31. EGLS unit root tests and estimates, with a trend (CPI)

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0020 (0.0011)	0.9878 (0.0076)	8.8 E-7 (3.9 E-6)	-1.61	-5.27	0.23
France	0.0942 (0.0276)	0.9546 (0.0135)	-3.3 E-5 (1.4 E-5)	-3.36	-19.61**	-2.36
Germany	0.0115 (0.0088)	0.9890 (0.0079)	-1.9 E-5 (1.1 E-5)	-1.39	-4.75	-1.73
Italy	0.1194 (0.0672)	0.9839 (0.0090)	-1.5 E-5 (9.6 E-6)	-1.79	-6.96	-1.56
Japan	0.1296 (0.0555)	0.9790 (0.0089)	-7.0 E-6 (2.7 E-6)	-2.36	-9.07	-2.59
United Kingdom	-0.0061 (0.0031)	0.9789 (0.0100)	-1.4 E-5 (1.0 E-5)	-2.11	-9.12	-1.40

Note: The critical were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0,1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a heteroskedastic N(0, σ_i^2) random error. The critical values for ρ_μ are -13.46, -17.76 and -25.61 for the 10%, 5% and 1% significance levels respectively.

We fail to find any consistent evidence to support PPP except in the case of France.

Restricted Seemingly Unrelated Regressions

Tables 32-35 examine the 1957-1993 monthly data however, following Abuaf and Jorion (1990) these tables assume homoskedastic variances. As discussed in Abuaf and Jorion (1990, p. 165), this study found little difference when the structure of the correlations was changed. The effect on the test using a "full historical matrix" showed little significant difference from a "diagonal matrix" in which the correlations were set to zero.

Tables 32 and 33 replicate the restricted SUR approach of Abuaf and Jorion (1990) however with our longer data set. Their restricted SUR approach assumes a homoskedastic variance for each equation (country) and no trend is included. Therefore, the unit root test will be conducted on the restricted coefficient, β_1. Abuaf and Jorion (1990) find that they can reject the unit root test at the 10% significance level for the restricted SUR model. While this is true for the rho test, they fail to reject the Dickey-Fuller *tau*-test at the 10% significance level. Our results concur as we reject the rho test at the 10% significance level, as in Table 33.

The contemporaneous correlation Breusch and Pagan (1980) Lagrange multiplier test results reject the null hypothesis that all of the across-equation covariances are zero. This supports the decision that Seemingly Unrelated Regression (SUR) estimation should be used. The Breusch-Pagan contemporaneous correlation statistics are 1.42 without a time rend and 0.62 with a time trend for the CPI. Similarly for the WPI the values are 1.07 without and 1.61 with a time trend. The Chi-squared critical values for 5 degrees of freedom is 9.24 at the 10% significance level. Therefore we fail to reject the restrictions in all four cases.

Table 32. Restricted EGLS unit root tests and estimates, no trend (WPI)

	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0014 (0.0006)	0.9935 (0.0021)	-3.06	-2.80
France	0.0118 (0.0043)			
Germany	0.0041 (0.0021)			
Italy	0.0457 (0.0156)			
Japan	0.0327 (0.0120)			
United Kingdom	-0.0033 (0.0014)			

Note: The critical values were computed using $r_t = a + r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a homoskedastic N(0, σ^2) random error.

In Table 32, the Dickey-Fuller τ-test statistic is -3.06 which compares to critical values of -4.30, -4.60 and -5.23 for the 10%, 5% and 1% significance levels, respectively. The rho test statistic is -2.80 which we fail to reject at the 10% significance level. The critical values for the rho test are -5.80, -6.65 and -8.41 for our data set with 432 observations.

Table 33. Restricted EGLS Unit root tests and estimates (CPI)

	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0032 (0.0009)	0.9855 (0.0035)	-4.14	-6.25*
France	0.0281 (0.0069)			
Germany	0.0120 (0.0033)			
Italy	0.1063 (0.0255)			
Japan	0.0780 (0.0188)			
United Kingdom	-0.0064 (0.0017)			

Note: The critical were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a homoskedastic N(0, σ^2) random error.

In Table 33, the τ statistic for the CPI case is -4.14 which is insignificant at the 10% critical value of -4.30 (the 5% and 1% levels are -4.60 and -5.23. The rho test statistic is -6.25 which is significant at the 10% level of -5.80 but not at the 5% or 1% levels of -6.65 and -8.41. Abuaf and Jorion (1990) found a rho test statistic of -4.55 which they reject at the 5% critical value of -4.49 with 180 observations. Therefore, our test statistics for the longer data set are comparable to their results. The Dickey-Fuller tau test was insignificant in both cases. Their restricted "GLS" coefficient was 0.9747, which compares with our coefficient of 0.9855 for the longer combined period from 1957 through 1993. Thus, the half-life for our sample is 48 months which is longer than Abuaf and Jorion (1990) but consistent with other authors such as Frankel and Rose (1995).

Table 34. Restricted EGLS Unit root tests and estimates, with a trend (WPI)

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0044 (0.0011)	0.9800 (0.0042)	-8.5 E-7 (4.1 E-6)	-4.78	-8.64	-0.21
France	0.0416 (0.0085)		-1.4 E-5 (9.5 E-6)			-1.47
Germany	0.0207 (0.0050)		-1.9 E-5 (1.1 E-5)			-1.80
Italy	0.1497 (0.0312)		-1.6 E-5 (8.3 E-6)			-1.93
Japan	0.1135 (0.0236)		-2.9 E-5 (9.8 E-6)			-3.01
United Kingdom	-0.0060 (0.0025)		-1.3 E-5 (9.8 E-6)			-1.28

Note: The critical were computed using $r_t = r_{t-1} + e_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo dynamic simulations..

Tables 34 and 35 show the affect of adding a time trend to Tables 32 and 33. Abuaf and Jorion (1990) do not report tables similar to Tables 32, 33 and 35 but they are added here for completeness. The Dickey-Fuller tau test statistic is -4.78 which compares with the critical values of -5.19, -5.49 and -6.04 for the 10%, 5% and 1% significance levels, respectively. The rho test statistic is -8.64 which we fail to reject at the 10% significance level. The critical values for the rho test for the WPI are -10.36, -11.58 and -13.84. None of the time trends are significant at the 10% level.

These results are consistent with those found in Table 32. The restriction boosts the power of the test but our longer data set includes heteroskedasticity for which we must correct.

Table 35. Restricted EGLS Unit root tests and estimates, with a trend (CPI)

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0024 (0.0010)	0.9828 (0.0036)	3.2 E-6 (3.7 E-6)	-4.77	-7.43	0.86
France	0.0355 (0.0077)		-1.4 E-5 (9.8 E-6)			-1.42
Germany	0.0169 (0.0044)		-1.8 E-5 (9.7 E-6)			-1.84
Italy	0.1275 (0.0269)		-1.4 E-5 (8.5 E-6)			-1.61
Japan	0.1044 (0.0225)		-5.3 E-5 (1.4 E-5)			-3.76*
United Kingdom	-0.0051 (0.0025)		-1.3 E-5 (9.5 E-6)			-1.34

Note: The critical were computed using $r_t = r_{t-1} + e_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo dynamic simulations..

Table 35 shows very similar results to Table 34, the tau statistic for the CPI case is -4.77 which compares to the critical values of -5.23, -5.48 and -6.08 for the 10%, 5% and 1% significance levels, respectively. The rho test statistic is -7.43 which compares with critical values of -9.57, -10.65 and -12.49 for the 10%, 5% and 1% significance levels, respectively. The time trend for Japan is significant at the 5% level, however the other countries are not significant at the 10% significance level.

There is no evidence to reject the unit root hypothesis for the extended cases similar to the Abuaf and Jorion (1990) results using our expanded data set. Only one of the four of the rho test statistics is significant at the 10% level and none of the Dickey-Fuller tau test statistics are significant at the 10% level. Therefore, we impose heteroskedasticity and the structural breaks in the next section.

Table 36. Restricted EGLS Unit root tests and estimates, with structural breaks, no trend (WPI)

	β_0 (SE)	β_1 (SE)	β_2 (SE)	τ - test	Rho Test	t - test (break)
Canada	0.0045 (0.0009)	0.9785 (0.0035)		-6.14***	-9.29***	
France	0.0423 (0.0070)					
Germany	0.0217 (0.0036)		-0.0054 (0.0011)			-4.86***
Italy	0.1591 (0.0259)					
Japan	0.1201 (0.0194)		-0.0071 (0.0010)			-7.13***
United Kingdom	-0.0080 (0.0015)		-0.0056 (0.0031)			-1.81

Note: The critical were computed using $r_t = r_{t-1} + e_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo dynamic simulations. The significance levels are indicated by *, ** and *** for the 10%, 5% and 1% significance levels respectively.

As was the case in Tables 32-35, the unit root coefficients in Tables 34-37 are restricted to be equal, however a heteroskedastic variance is assumed for each equation such that the variance is different before 1973 and after. The structural mean break coefficients, β_2, found in Tables 20-23 are inserted into the SUR model also.

Table 36 shows the restricted EGLS estimates where the lagged autoregressive (unit root) coefficient has been restricted to be equal across the six equations (countries). In addition, structural breaks have been inserted at the appropriate dates where sequential test estimates showed these to be significant.

The F-test for the restriction that all of the coefficients are equal is not rejected. The critical value is 15.14 at the 10% significance level and the F-statistic is 4.34. The Dickey-Fuller τ-test statistic is -6.14. The critical values are -3.94, -4.29 and -4.92 for the

10%, 5% and 1% significance levels, respectively. Thus reject the unit root hypothesis at the 1% significance level. The rho test statistic is -9.29 and the critical values are -5.82, -6.80 and -8.83 for the 10%, 5% and 1% significance levels, respectively.

The two-tailed critical values for Germany's structural break are -2.43, -2.89 and -3.69 for the 10%, 5% and 1% significance levels, respectively. For Japan, the critical values are -2.33, -2.74 and -3.68, respectively. Finally, the critical values for the U.K. are -2.01, -2.42 and -3.23, respectively. Therefore, we are able to conclude that the structural break values for Germany (-4.86) and Japan (-7.13)are highly significant at the 1% level, whereas the break in the mean is not significantly different from zero for the United Kingdom (-1.81).

When a time trend is included for the WPI, the results in Table 37 are tabulated. We fail to reject the restriction that the coefficients are equal across countries. The critical value for the restriction is 17.93 at the 10% significance level. The test statistic is 2.46. The Dickey-Fuller τ-test critical values are -5.77, -6.05, and -6.70 for the 10%, 5% and 1% significance levels respectively. The tau test statistic is -7.31 and thus we reject the unit root hypothesis for the WPI structural break model with a trend at the 1% significance level. The rho test statistic of -12.31 is significant at the 10% level of -12.30 but not at the 5% significance level of -13.57.

Table 37. Restricted EGLS Unit root tests and estimates, with structural breaks, with a trend (WPI)

	β_0 (SE)	β_1 (SE)	β_2 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (break)
Canada	0.0058 (0.0009)	0.9715 (0.0039)		4.7 E-7 (3.9 E-6)	-7.31***	-12.31*	
France	0.0624 (0.0083)		0.0125 (0.0038)	-5.4 E-5 (1.5 E-5)			3.30**
Germany	0.0295 (0.0044)		-0.0086 (0.0016)	1.0 E-7 (1.0 E-5)			-5.35***
Italy	0.2142 (0.0293)			-2.9 E-5 (6.3 E-6)			
Japan	0.1594 (0.0219)		-0.0094 (0.0013)	-2.2 E-6 (7.4 E-6)			-7.06***
United Kingdom	-0.0097 (0.0020)		-0.0065 (0.0038)	-6.4 E-6 (1.1 E-5)			-1.71

Note: The critical were computed using $r_t = r_{t-1} + e_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo dynamic simulations. The significance levels are indicated by *, ** and *** for the 10%, 5% and 1% significance levels respectively.

The structural break Student's t-test two-tailed (positive) critical values are 2.32, 2.84 and 3.72 for the structural breaks for France. Therefore, the break statistic of 3.30 is significant at the 5% level. For Germany the statistic is -5.35 which is significant at the 1% level with negative two-tailed critical values of -2.59, -3.07 and -4.10. The Japanese break test statistic of -7.06 is significant at the 1% level for values of -2.41, -2.88 and -3.72. The United Kingdom however, is insignificant at the 10% level with critical values of -2.29, -2.69 and -3.71 for the 10%, 5% and 1% levels, respectively.

Table 38. Restricted EGLS Unit root tests and estimates, with structural breaks, no trend (CPI)

	β_0 (SE)	β_1 (SE)	β_2 (SE)	τ - test	Rho Test	t - test (break)
Canada	0.0026 (0.0006)	0.9847 (0.0027)		-5.66***	-6.61**	
France	0.0292 (0.0056)					
Germany	0.0145 (0.0029)		-0.0053 (0.0011)			-4.81***
Italy	0.1112 (0.0204)					
Japan	0.0893 (0.0166)		-0.0115 (0.0020)			-5.68***
United Kingdom	-0.0062 (0.0013)					

Note: The Monte Carlo simulations are generated similarly to the previous Tables 36 and 37.

Table 39. Restricted EGLS Unit root tests and estimates, with structural breaks, with a trend (CPI)

	β_0 (SE)	β_1 (SE)	β_2 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (break)
Canada	0.0058 (0.0010)	0.9698 (0.0041)	0.0069 (0.0017)	1.7 E-5 (6.4 E-6)	-7.37***	-13.05**	4.04***
France	0.0647 (0.0088)		0.0076 (0.0036)	-4.7 E-5 (1.4 E-5)			2.12
Germany	0.0311 (0.0046)		-0.0091 (0.0016)	-7.9 E-6 (1.0 E-5)			-5.62***
Italy	0.2246 (0.0308)			-2.7 E-5 (7.4 E-6)			
Japan	0.1850 (0.0257)		-0.0099 (0.0023)	-6.6 E-5 (1.5 E-5)			-4.25***
United Kingdom	-0.0084 (0.0018)			-2.3 E-5 (8.5 E-6)			

Note: See the note in Table 37.

For Table 38, without a time trend, the across equation restriction is not rejected at the 10% significance level critical value of 12.57. The F-test statistic is 4.32. The Dickey-Fuller τ-test statistic is -5.66 and significant at the 1% level when compared with the critical values of -3.65, -4.05 and -4.85 for the 10%, 5% and 1% significance levels, respectively. The rho test statistic is -6.61 which is significant at the 5% level. The critical values are -4.81, -5.66 and -7.92 for the 10%, 5% and 1% significance levels, respectively. Therefore, we reject the unit root hypothesis at the 1% significance level. The structural break critical values for Germany are -2.61, -3.19 and -4.02 for the 10%, 5% and 1% significance levels, respectively. Japan's structural break critical values are -2.37, -2.85 and -3.76.

For Table 39, the restriction is not rejected at the 10% significance level critical value of 13.56. The F-test statistic is 1.70. Therefore, as before the restriction is imposed. The Dickey-Fuller τ test critical values are -5.78, -6.13 and -6.70 for the unit root test at the 10%, 5% and 1% significance levels, respectively. The rho test statistic of -13.05 is significant at the 5% significance level of -12.37 but not at the 1% level of -14.99. There are three significant structural break test statistics for the two-tailed t-tests. Only the critical values for the positive or negative side, depending on the t-test statistic, are reported here. France is not significant at the 10% significance level of 2.12 compared with the critical values of 2.45, 2.80 and 3.52 for the 10%, 5% and 1% significance levels, respectively. However, the Canadian, German, and the Japanese structural breaks are significant at the 1% level (4.04), 1% level (-5.62) and 1% level (-4.25), respectively. The critical values are 2.39, 2.86 and 3.84 for the Canadian structural break. For Germany the critical values are -3.09, -3.74, and -5.22, while for Japan the values are -2.67, -3.12, and -4.12, respectively.

Evidence is provided for the structural break critical values and purchasing power parity, the unit root hypothesis is rejected at high significance levels.

Some evidence of a productivity bias exists for a few countries (e.g. Italy for the WPI). In the significant cases, the negative coefficient indicates that the real exchange rate appreciates which is consistent with the Balassa (1964) reasoning.[46]

The half-life of the real exchange rate is the time it takes for the real exchange rate to return half of the distance back to equilibrium. Taking into account the bias mentioned in Chapter 2,[47] the coefficients vary from 0.994 to 0.9788. The WPI half-life is 57.8 months however when a time trend is included the half-life is only 35.6 months. These are consistent with the periods other authors have found. The CPI without a time trend is 112.7 months which is greater than most authors have found. However, when a time trend is included the half-life is only 32.8 months which is at the low end of most authors estimations.

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[46] See Chapter 2 page 22 for discussion.

[47] See pages 12 and 22.

CHAPTER V

RESULTS OF THE TESTS OF PURCHASING POWER PARITY USING ANNUAL REAL EXCHANGE RATES

Introduction

In this chapter we analyze annual data and contrast it with the previous section of monthly data. The annual data include Canada, France, Italy, Japan, the United Kingdom

Table 40. Annual ADF *tau* Test Statistics, No Trend

Country	r_t (CPI)	p	r_t (WPI)	p	$r_t - r_{t-1}$ (CPI)	p	$r_t - r_{t-1}$ (WPI)	p
Canada	-2.71*	3	-1.62	8	-4.08***	5	-3.33**	5
France	-2.50	6	-3.40**	5	-3.98***	7	-4.62***	2
Italy	-2.60	5	-2.79*	0	-4.32***	4	-6.85***	1
Japan	-2.18	0	-3.03**	2	-4.04***	6	-3.69***	7
United States	-2.98**	0	-2.08	0	-5.89***	2	-5.04***	2

Note: p is the number of significant lags in the Augmented Dickey-Fuller Equation. The test statistics are taken from MacKinnon (1991). The Dickey-Fuller tau test statistic that is less than the critical values of -2.5671, -2.8621, and -3.4335 are rejected for the 10%, 5% and 1% significance levels respectively (*, **, and ***).

Table 41. Annual ADF *tau* Test Statistics, Trend

Country	r_t (CPI)	p	r_t (WPI)	p	$r_t - r_{t-1}$ (CPI)	p	$r_t - r_{t-1}$ (WPI)	p
Canada	-2.72	3	-1.64	8	-4.10***	5	-3.36*	5
France	-2.70	6	-3.48**	5	-3.99***	7	-4.59***	2
Italy	-2.72	5	-2.89	0	-4.34***	4	-6.98***	1
Japan	-2.07	0	-3.93**	2	-4.17***	6	-3.67***	7
United States	-2.96	0	-2.32	0	-5.84***	2	-5.01***	2

Note: p is the number of significant lags in the Augmented Dickey-Fuller Equation. The test statistics are taken from MacKinnon (1991). The Dickey-Fuller tau test statistic that is less than the critical values of -3.1279, -3.4126 and -3.9638 for the 10%, 5% and 1% significance levels respectively.

and the United States from 1900 through 1994 with a few exceptions. The German real exchange rate is missing because the data from an extended period during the war years was not available. Italy does not have data prior to 1917 therefore the observations used in the Seemingly Unrelated Regression model begins in 1917. For the CPI the observations end in 1994 for a total of 78 in the data set. The WPI data set ends with 1988, yielding 72 observations because France significantly changed the goods and services measured by this price index. The annual real exchange rate is calculated with the pound sterling as the base currency instead of the U.S. dollar as in the monthly real exchange rate calculations. Therefore, all of the results except for those detailed above for France (WPI), Italy and SUR use an annual data set of 95 observations.

Tables 40 through 44 exhibit the Augmented Dickey Fuller equation results from equations (11) - without a time trend and (12) - with a time trend. Tables 40 and 41 allow the number of lags to be set according to the highest significant lag order from the autocorrelation function or the partial autocorrelation function (as explained in the monthly results in Chapter IV on page 60). Tables 42 and 43 report the tau-test statistics results for the Dickey-Fuller or "no-lag" cases. Table 44 reports the ρ_{μ} test.

Tables 40 and 41 are consistent for France and Japan: When the lagged first differences are added to make the error term white noise, the CPI based real exchange rates are indistinguishable from a unit root. On the other hand, the WPI based real exchange rates are significant at the 5% level, that is PPP holds for this case.

For the other cases, Canada and the U.S. reject the unit root for the CPI and Italy for the WPI based real exchange rates, respectively. Therefore, for five of the ten cases, PPP holds.

When the lagged first differences are dropped from the model, four more cases are examined: the random walk model or the autoregressive of order one model [AR(1)], and the AR(1) with a time trend model, for the CPI and the WPI. The results from these models are shown in Tables 42 - 45. Table 44 reports the ρ_μ test, which is a more powerful test as established by Abuaf and Jorion (1990, p. 163). The ρ_μ test is only valid when no lags are included in the model. Therefore, Table 44 corresponds to the Dickey-Fuller *tau* test statistics in Tables 42 and 43, respectively.

For Table 42, the French, Italian and U.S. real exchange rates calculated with the CPI without a trend are significant at the 5% level. For the WPI, the French and Japanese real exchange rates reject a unit root at the 5% level and Italy at the 10% level. In Table 43, France and Italy for the CPI based, and France and Japan for the WPI based real exchange rates reject the unit root hypothesis at the 5% significance level. The addition of the time trend causes the breakdown in the PPP hypothesis for the U.S.-CPI based real exchange rate and the Italian-WPI based real exchange rate.

For the rho-test statistic, using annual data, we find that fourteen of the twenty cases are significant at the 10% level and twelve at the 5% significance level. The first two columns are testing PPP against the random walk hypothesis. As seven of the ten cases reject the random walk model, we find good evidence that PPP holds. This is consistent with many of the authors as discussed in Chapter 2.

Table 42. Annual ADF *tau* Test Statistics, No Trend, No Lags

Country	r_t (CPI)	r_t (WPI)	$r_t - r_{t-1}$ (CPI)	$r_t - r_{t-1}$ (WPI)
Canada	-2.37	-1.83	-7.80***	-9.28***
France	-3.25**	-3.56***	-11.60***	-10.80***
Italy	-3.53***	-2.79*	-9.59***	-10.80***
Japan	-2.18	-3.02**	-9.35***	-8.05***
United States	-2.98**	-2.08	-8.26**	-9.54***

Note: The test statistics are taken from MacKinnon (1991). The Dickey-Fuller tau test statistic that is less than the critical values of -2.5671, -2.8621, and -3.4335 are rejected for the 10%, 5% and 1% significance levels respectively (*, **, and ***).

Table 43. Annual ADF *tau* Test Statistics, Trend, No Lags

Country	r_t (CPI)	r_t (WPI)	$r_t - r_{t-1}$ (CPI)	$r_t - r_{t-1}$ (WPI)
Canada	-2.41	-1.94	-7.75***	-9.29***
France	-3.43**	-3.63**	-11.53***	-10.74***
Italy	-3.98***	-2.89	-9.53***	-10.09***
Japan	-2.07	-3.69**	-9.34***	-8.00***
United States	-2.96	-2.32	-8.22***	-9.51***

Note: The test statistics are taken from MacKinnon (1991). The Dickey-Fuller tau test statistic that is less than the critical values of -3.1279, -3.4126 and -3.9638 for the 10%, 5% and 1% significance levels respectively. p is the number of significant lags in the Augmented Dickey-Fuller Equation.

The evidence is far from conclusive, however we find much more support for PPP using long-run, low frequency data. The problem with the power of these tests has been discussed previously so we will just note again here that for this reason, further investigation is warranted.

Table 44. Annual ρ_μ Test Statistics

Country	ρ_μ (CPI)	ρ_μ (WPI)	ρ_μ (CPI), trend	ρ_μ (WPI), trend
Canada	-11.20*	-7.35	-11.56*	-7.81
France	-19.17***	-22.44***	-21.53***	-23.92***
Italy	-21.40***	-14.21**	-27.21**	-14.96**
Japan	-10.05	-17.15**	-10.35	-24.52***
United States	-16.42**	-9.43	-16.74**	-10.82

Note: For ρ_μ the critical values are -11.0, -13.7, and -19.8 at the 10%, 5% and 1% significance levels for a sample size of 100 (Table 8.5.1 in Fuller, 1976).

This led us to investigate the rolling and recursive regression procedures as with the monthly data.

Rolling and Recursive Regressions

The next six tables show the results of the rolling and recursive regressions, respectively, for the cases with four lags and a time trend (Tables 45 and 46), no lags and a time trend (Tables 47 and 48) and no lags and no time trend (Tables 49 and 50).

Only one of the twenty tau-test statistics can the unit root hypothesis be rejected when four lags are included in a rolling regression model (Table 45). In the recursive case (Table 46), we find seven of ten tests that are significant at the 10% level for the CPI but only two of the ten cases for the WPI.

Tables 47 and 48 drop the lags from the Augmented Dickey-Fuller equation (remember this is called the Dickey-Fuller equation) two more cases become significant for the rolling regression model. That is, we can reject the unit root hypothesis for Italy in

Table 45: Rolling Regressions *tau* Tests for a Unit Root, Four Lags

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	0.04	-1.74	-0.07	-1.43
France	-0.18	-1.28	-0.42	-2.19
Italy	0.04	-2.55	-0.01	-6.81***
Japan	-0.32	-1.85	-0.07	-2.01
United States	-0.10	-1.70	-0.20	-2.49

Note: The τ^{min} critical values for 100 observations are -4.71, -5.01 and -5.29 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 100 observations are -1.31, -1.49 and -1.66 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

Table 46: Recursive Regressions *tau* Tests for a Unit Root, Four Lags

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-0.86	-3.06	-0.63	-2.37
France	-1.85	-3.61	-3.42***	-5.84***
Italy	-1.52	-4.09*	-2.21***	-4.22*
Japan	-1.46	-4.55**	-0.70	-4.37*
United States	-0.71	-3.93	-2.27***	-4.85***

Note: The τ^{min} critical values for 100 observations are -4.00, -4.33 and -4.62 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 100 observations are -1.73, -1.99 and -2.21 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

both cases and in Japan for the CPI based real exchange rate. There is still not much support for PPP using the rolling regressions model.

The trend critical values for Tables 45 - 48 are from Banerjee, et. al. (1992) whereas the no-trend critical values for Tables 49 and 50 come from Monte Carlo simulations. These simulations are created exactly as the sequential univariate simulations

Table 47: Rolling Regressions *tau* Tests for a Unit Root, No Lags

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-0.19	-1.59	0.07	-2.40
France	-0.56	-3.84	-0.54	-2.57
Italy	-0.09	-6.51***	0.77	-8.38***
Japan	0.18	-3.43	0.05	-13.74***
United States	-0.09	-2.10	-0.09	-3.14

Note: The τ^{min} critical values for 100 observations are -4.71, -5.01 and -5.29 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 100 observations are -1.31, -1.49 and -1.66 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

Table 48: Recursive Regressions *tau* Tests for a Unit Root, No Lags

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-0.70	-3.32	-1.19	-3.69
France	-0.88	-4.54**	-1.09	-3.61
Italy	-2.67**	-4.48**	-2.08**	-5.27***
Japan	1.14	-3.95	-0.32	-3.82
United States	-1.23	-3.93	-1.85*	-3.51

Note: The τ^{min} critical values for 100 observations are -4.00, -4.33 and -4.62 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 100 observations are -1.73, -1.99 and -2.21 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

a specific break is not inserted into the model or simulation procedure.

The recursive regression model in Table 48, shows an increase in the number of cases for the WPI but a decrease in the CPI when the lagged first differences are dropped. We still do not find consistent significant evidence against the unit root hypothesis with these models. Therefore, we drop the time trend from these models and exhibit the results

Table 49: Rolling Regressions *tau* Tests for a Unit Root, No Lags, No Trend

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	0.55	-2.96	0.37	-1.93
France	-0.32	-2.39	-0.52	-2.32
Italy	1.41	-9.13***	0.32	-8.30***
Japan	1.82	-6.81***	0.37	-2.68
United States	1.87	-2.05	0.48	-2.51

Note: The critical values for the t^{max} statistic are -1.51, -1.92 and -2.57 for the 10%, 5% and 1% significance levels respectively. The critical values for the t^{min} statistic are -4.43, -4.32 and -5.26 for the 10%, 5% and 1% significance levels respectively. The critical values were computed using data generated by $r_t = r_{t-1} + \varepsilon_t$, where ε_t is identically and independently distributed N(0, 1) from 5000 Monte Carlo simulations for a sample size of 95.

Table 50: Recursive Regressions *tau* Tests for a Unit Root, No Lags, No Trend

Country	τ^{max} WPI	τ^{min} WPI	τ^{max} CPI	τ^{min} CPI
Canada	-1.81	-4.51**	-0.99	-6.23***
France	-0.81	-3.63	-0.51	-3.03
Italy	-0.77	-3.85	0.40	-3.31
Japan	-1.19	-4.70**	-0.20	-5.54***
United States	-1.20	-7.12***	-1.46	-3.34

Note: The critical values for the t^{max} statistic are -1.54, -1.90 and -2.55 for the 10%, 5% and 1% significance levels respectively. The critical values for the t^{min} statistic are -3.91, -4.32 and -5.26 for the 10%, 5% and 1% significance levels respectively. The critical values were computed using data generated by $r_t = r_{t-1} + \varepsilon_t$, where ε_t is identically and independently distributed N(0,1) from 6000 Monte Carlo simulations for a sample size of 95.

from the reestimation of these models in Tables 49 and 50.

For Tables 49 and 50, Canada, Japan and the United States in the recursive case and Italy and Japan in the rolling regressions case have significant minimum *tau* statistics. Thus, though there are differences in these six models, we find little conclusive support for PPP. This is not a significant improvement over the basic Augmented Dickey-Fuller and

Table 51. Sequential Test statistics, WPI, for a Mean Shift, Four lags

Country	k	F_T	τ^{max}	τ^{min}	Q_{LR} (4)
Canada	1971	17.72*	-4.06	-4.08	51.88***
France	1927	7.41	-4.75*	-4.78*	73.24***
Italy	1946	9.51	-1.76	-3.14	56.11***
Japan	1946	9.76	-4.64*	-4.64*	45.08***
United States	1973	16.92*	-4.45	-4.45	45.31***

Note: The critical values are -4.52, -4.80 and -5.07 for the τ^{max} mean shift statistics and are -4.54, -4.80 and -5.07 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.20, 18.62 and 20.83 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (4) they are 31.78, 34.56 and 37.25. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992 p. 278). These values are calculated with a sample size of 100 (our sample size is 96).

Dickey-Fuller tau and rho tests in Tables 40 through 44, therefore we continue to investigate for evidence of a structural break within the unit root model using the sequential regressions models as in Chapter 4 using the monthly data.

Sequential Regressions

The sequential regressions are performed as discussed in Chapter 3. The data are trimmed by the same percentages for each test but the data set is smaller at 95 observations and so we use the Banerjee, et. al (1992) critical values for a sample size of 100. There are two countries (Canada and the U.S.) that have significant structural breaks for the WPI at the 10% level. All of the Quandt Likelihood Ratio test statistics are significant at the 1% level. France and Japan can reject the unit root hypothesis.

Table 52 . Sequential Test statistics, CPI, for a Mean Shift, Four lags

Country	k	F_T	τ^{max}	τ^{min}	Q_{LR} (4)
Canada	1949	15.84	-4.51*	-4.51*	39.25***
France	1943	8.76	-4.11	-4.24	59.97***
Italy	1946	7.01	-0.83	-3.46	53.96***
Japan	1947	13.87	-2.66	-2.78	69.29***
United States	1949	16.66*	-5.03**	-5.03**	74.89***

Note: Same as in Table 51.

Table 53. Sequential Test statistics, WPI, for a Mean Shift, No lags

Country	k	F_T	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	1971	20.74**	-4.18	-4.18	63.47***
France	1927	7.76	-4.31	-4.40	36.35***
Italy	1946	7.49	-2.57	-3.51	41.83***
Japan	1946	5.20	-4.41	-4.41	36.92***
United States	1973	18.17*	-4.77*	-4.77*	72.54***

Note: The critical values are -4.52, -4.80 and -5.07 for the τ^{max} mean shift statistics and are -4.54, -4.80 and -5.07 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.20, 18.62 and 20.83 for the 10%, 5% and 1% significance levels respectively. For Q_{LR} (0) they are 23.86, 26.45 and 28.96. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992 p. 278). These values are calculated with a sample size of 100 (our sample size is 96).

For the CPI only one case (the U.S.), is there a significant structural break however, the tau-test statistic also rejects the unit root hypothesis. Canada has a large but barely insignificant structural break and the unit root hypothesis is rejected. While none of the other cases signify a structural break the Quandt test again shows high significance in all cases.

Table 54 . Sequential Test statistics, CPI, for a Mean Shift, No lags

Country	k	F_T	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	1948	14.25	-4.36	-4.36	43.29***
France	1942	9.38	-4.42	-4.55	20.79
Italy	1932	8.96	-5.17***	-5.17***	29.90***
Japan	1947	13.45	-3.28	-3.53	39.51***
United States	1946	6.11	-3.87	-3.87	36.70***

Note: Same as in Table 53.

Table 55. Sequential Test statistics, WPI, for a Mean Shift, No lags, No trend

Country	k	F_T	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	1971	13.10	-3.20	-3.24	32.56***
France	1927	2.25	-3.66	-3.66	37.47***
Italy	1946	4.77	-2.99	-3.27	41.64***
Japan	1944	9.41	-4.36**	-4.38**	24.97*
United States	1973	13.87	-4.05	-4.05	25.45*

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0,1) and are based on 10,000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error. The critical values for F_T are 14.90, 17.32 and 22.09 for the 10%, 5% and 1% significant levels respectively. The τ_{max} and τ_{μ} critical values are not statistically different for a sample size of 96. They are -4.07, -4.35 and -4.93 for the 10%, 5% and 1% significance levels respectively. The critical values for Q_{LR} are 14.18, 16.17 and 21.25 for the 10%, 5% and 1% significance levels respectively.

Table 53 confirms the results in Table 51 that Canada and the U.S. have a significant structural break in 1971 and 1973, respectively. However, Tables 52 and 54 show the U.S. case becomes an insignificant structural break whereas Canada and Japan maintain larger test statistics, however they are still insignificant. In Table 55, when the time trend is dropped from the equation, the Canadian and U.S. structural breaks become

Table 56 . Sequential Test statistics, CPI, for a Mean Shift, No lags, No trend

Country	k	F_T	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	1977	6.37	-3.23	-3.23	13.73*
France	1920	7.75	-4.29*	-4.29*	28.87***
Italy	1932	12.39	-5.18***	-5.18***	51.03***
Japan	1947	3.34	-2.77	-2.77	56.12***
United States	1919	3.84	-3.47	-3.47	37.80***

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 10,000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error. The critical values for F_T are 14.67, 17.13 and 22.68 for the 10%, 5% and 1% significant levels respectively. The τ_{max} and τ_{min} critical values are not statistically different for a sample size of 96 (except at the 10% level). They are (-4.06) -4.05, -4.32 and -4.96 for the 10%, 5% and 1% significance levels respectively. The critical values for Q_{LR} are 12.91, 14.76 and 18.95 for the 10%, 5% and 1% significance levels respectively.

insignificant. For Japan we can reject the unit root hypothesis but none of the other cases are significantly. The same is true in Table 56 where only Italy has a large but insignificant test statistic. The unit root null hypothesis can be rejected for France at the 10% significance level and Italy at the 1% significance level.

The sequential regressions provide addition information in the form of structural breaks but do not provide consistent rejection of the unit root null hypothesis. Therefore, as with the monthly data set, we proceed to use multivariate analysis to increase the power of the tests and find further evidence for purchasing power parity.

Seemingly Unrelated Regression

Tables 57 - 60 show the ordinary least squares results for the annual data. Tables 53 and 54 examine the WPI without and with a time trend, respectively. Tables 55 and 56 show the results of the CPI based real exchange rate, without and with a time trend,

Table 57. OLS Estimates using the WPI without a time trend

	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0438 (0.0274)	0.9144 (0.0505)	-1.70.	-6.16
France	1.5690 (0.5202)	0.7652 (0.0777)	-3.20**	-16.91**
Italy	1.3813 (0.5118)	0.8096 (0.0704)	-2.70*	-13.71**
Japan	1.4719 (0.4559)	0.7565 (0.0749)	-3.25**	-17.53**
United States	0.0496 (0.0256)	0.8777 (0.0597)	-2.05	-8.81

Note: Same as in Table 40 and Table 44.

Table 58. OLS Estimates using the WPI with a time trend

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0170 (0.0395)	0.9158 (0.0506)	0.0005 (0.0005)	-1.66	-6.06	0.87
France	1.5726 (0.5299)	0.7650 (0.0784)	-4.3 E-5 (-0.00093)	-3.00	-16.92**	-0.03
Italy	1.3969 (0.5137)	0.8015 (0.0714)	8.0 E-4 (0.0010)	-2.78	-14.29**	0.80
Japan	1.7024 (0.5115)	0.7253 (0.0812)	-7.7 E-4 (0.00077)	-3.38*	-19.78**	-0.73
United States	0.0220 (0.0345)	0.8733 (0.0597)	5.5 E-4 (0.00046)	-2.12	-9.12	1.27

Note: Same as in Table 41 and Table 44.

respectively. As mentioned previously, the data set for the multivariate regressions include only from 1917 through 1988 for the WPI and 1917 through 1994 for the CPI.

Therefore, the OLS estimates and statistics reported in Tables 40 - 44 will have differing values from those in Tables 57 - 61.

The Dickey-Fuller tau test statistics and rho test statistics are significant for France, Italy and Japan. Therefore, we can reject the unit root null hypothesis for these three countries when a time trend does not exist. Table 58 confirms this result with the ρ_μ test, though some of the less powerful tau-test statistics are insignificant in this case. Furthermore, the time trend is not significant in any of the countries for the Wholesale Price Index in Table 58.

Tables 59 and 60 show that seven of the ten cases have highly significant test statistics for the annual data. These test statistics support the theory of purchasing power parity and reject the unit root hypothesis. As in the WPI case in Table 58, the time trends are not significant in the CPI case either (Table 60).

Overall we find the evidence in Tables 57 through Tables 60 to provided basically the same results as in Tables 40 - 44. It is interesting that the shorter time span in these latter tables provided greater evidence for PPP than in the longer time series represented in the earlier tables. This observation shows that care must be taken to choose the size of the data set when annual frequencies are used.

Table 59. OLS Estimates using the CPI without a time trend

	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0792 (0.0353)	0.8624 (0.0592)	-2.32	-10.73
France	1.8429 (0.4897)	0.7293 (0.0721)	-3.75***	-21.11***
Italy	2.1120 (0.5994)	0.7181 (0.0798)	-3.53***	-21.99***
Japan	0.7911 (0.3626)	0.7911 (0.0588)	-3.55***	-16.29**
United States	0.1052 (0.0349)	0.7903 (0.0647)	-3.24**	-16.36**

Note: Same as in Table 40 and Table 44.

Table 60. OLS Estimates using the CPI with a time trend

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0584 (0.0411)	0.8562 (0.0595)	0.0004 (0.0004)	-2.42	-11.22	0.99
France	1.8362 (0.4955)	0.7310 (0.0737)	-8.2 E-5 (0.0006)	-3.65**	-20.98***	-0.13
Italy	2.7733 (0.7019)	0.6418 (0.0900)	0.0016 (0.0089)	-3.98***	-27.94***	-1.75
Japan	0.7820 (0.3642)	0.8787 (0.0660)	-6.6 E-4 (0.00096)	-1.84	-9.46	-0.68
United States	0.0851 (0.0448)	0.7941 (0.0651)	53.2 E-4 (0.00044)	-3.16*	-16.06**	0.72

Note: Same as in Table 41 and Table 44.

Restricted Seemingly Unrelated Regressions

Table 61. Restricted SUR Estimates using the WPI without a time trend

	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.0972 (0.0191)	0.8077 (0.0320)	-6.01***	-14.62***
France	1.2843 (0.02152)			
Italy	1.3948 (0.2336)			
Japan	1.1605 (0.1953)			
United States	0.0775 (0.0158)			

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

Tables 61 - 68 restrict the coefficients on the autoregressive coefficients to be equal. The method of EGLS is used to estimate the coefficients and test statistics in these tables. The second four tables include the structural breaks found in Tables 53-56.

For the Wholesale Price Index without a time trend (Table 61), the F-test that the autoregressive coefficients are equal has a value of 1.07 and the critical values determined by Monte Carlo simulation are 3.25, 3.90 and 5.50 for the 10%, 5% and 1% significance levels, respectively. Thus, we fail to reject the restriction that the coefficients are equal across the countries at the 10% significance level. The value of the Dickey-Fuller tau test for a unit root is -6.01 when no time trend is included. The critical values for this pseudo t-test are -4.18, -4.49 and -5.09 for the 10%, 5% and 1% significant levels respectively. Therefore, we can reject the unit root hypothesis at the 1% significance level. The rho

Table 62. Restricted SUR Estimates using the WPI with a time trend

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0788 (0.0342)	0.7961 (0.0329)	0.0005 (0.0329)	-6.20**	-15.50***	0.02
France	1.3630 (0.2279)		-2.3 E-5 (-0.00093)			-0.02
Italy	1.4356 (0.2426)		8.1 E-4 (0.0010)			0.81
Japan	1.2581 (0.2108)		-5.1 E-4 (0.00072)			-0.71
United States	0.0508 (0.0291)		5.9 E-4 (0.00046)			1.28

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

Chi-square test confirms this result with the test statistic equal to -14.62 and highly significant when compared with critical values of -5.64, -6.44 and -8.17 at the 10%, 5% and 1% significance levels.

In Table 62, using the WPI and a time trend the F-test for the restriction has a value of 1.40 and the critical values determined by Monte Carlo simulation are 3.39, 4.21 and 6.12 at the 10%, 5% and 1% significance levels, respectively. Thus we fail to reject the restriction that the coefficients are equal across the countries at the 10% significance level. The value of the Dickey-Fuller unit root tau-test is -6.20 when a time trend is included. The critical values for this "pseudo" t-test are -5.66, -5.89 and -6.50 for the 10%, 5% and 1% significant levels, respectively. Therefore, we can reject the unit root hypothesis at the 5% significance level. The rho Chi square test statistic is -15.50 and

Table 63. SUR Estimates using the CPI without a time trend

	β_0 (SE)	β_1 (SE)	τ - test	Rho Test
Canada	0.1078 (0.0182)	0.8126 (0.0270)	-6.94***	-13.49***
France	1.2775 (0.0184)			
Italy	1.4025 (0.2035)			
Japan	1.1515 (0.1675)			
United States	0.0936 (0.0170)			

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

therefore significant at the 1% critical level when compared with the -11.39, -12.46 and -14.85 values for 10%, 5% and 1%, respectively.

None of the time trends are significant. Monte Carlo simulations find that the 10% significance levels are usually much greater than 3.00 or less than -3.00 for a two-tailed test.

Table 63 estimates the model using the Consumer Price Index, without a time trend. This is the case studied by Abuaf and Jorion (1990, p. 170). Their data is from 1901 - 1972 and includes three other countries but is consistent with our results. The F-test that the autoregressive coefficients are equal has a value of 2.68 and the critical values determined by Monte Carlo simulation are 3.35, 3.96 and 5.57 for the 10%, 5% and 1% significance levels. Thus, we fail to reject the restriction that the coefficients are equal across the countries at the 10% significance level. The value of the Dickey-Fuller tau test

Table 64. Restricted SUR Estimates using the CPI with a time trend

	β_0 (SE)	β_1 (SE)	β_3 (SE)	τ - test	Rho Test	t - test (trend)
Canada	0.0848 (0.0303)	0.8065 (0.0285)	0.0005 (0.0004)	-6.79***	-13.93**	1.08
France	1.3298 (0.1949)		-1.9 E-4 (-0.00061)			-0.31
Italy	1.4919 (0.2270)		-7.7 E-4 (0.00079)			-0.97
Japan	1.2142 (0.1803)		-4.4 E-4 (0.00095)			-0.47
United States	0.0782 (0.0312)		3.3 E-4 (0.00044)			0.74

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

for a unit root is -6.94 when no time trend is included. The critical values for this pseudo t-test are -4.10, -4.45 and -5.07 for the 10%, 5% and 1% significant levels, respectively. Therefore, we can reject the unit root hypothesis at the 1% significance level. The rho Chi-square test statistic is -13.49 and highly significant, when compared with critical values of -5.74, -6.52 and -7.95 at the 10%, 5% and 1% significance levels, respectively.

In Table 64, when a time trend is included the F-test for the restriction has a value of 3.74 and the critical values determined by Monte Carlo simulation are 3.82, 4.53 and 6.21 at the 10%, 5% and 1% significance levels, respectively. Thus we fail to reject the restriction that the coefficients are equal across the countries at the 10% significance level. The value of the Dickey-Fuller unit root tau-test is -6.79 when a time trend is included. The critical values for the pseudo tau-test are -5.60, -5.92 and -6.43 for the 10%, 5% and

Table 65. Restricted SUR Estimates using the WPI no time trend and structural breaks

	β_0 (SE)	β_1 (SE)	β_2 (SE)	τ - test	Rho Test	t - test (break)
Canada	0.0891 (0.0185)	0.7847 (0.0317)	0.0806 (0.0198)	-6.79***	-16.79***	4.08***
France	1.4382 (0.2130)					
Italy	1.5618 (0.2312)					
Japan	1.3003 (0.1933)					
United States	0.0726 (0.0152)		0.0656 (0.0178)			3.68***

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

1% significant levels, respectively. Therefore, we can reject the unit root hypothesis at the 1% significance level. The rho Chi square test statistic is -13.93 and therefore significant at the 5% critical level when compared with the -11.42, -12.52 and -15.10 values for 10%, 5% and 1%, respectively.

Table 65 includes the structural breaks identified in Table 53, using the Wholesale Price Index SUR model when no time trend is included (Table 61). The F-test for the existence of a restriction is not rejected at the 10% significance level. The critical values of 3.48, 4.19 and 5.54 compare with the computed F-statistic of 1.94. The Dickey-Fuller tau test critical values are -4.40, -4.69 and -5.19 for the 10%, 5% and 1% significance levels, respectively. The rho test critical values are -6.52, -7.38 and -8.87, respectively. Therefore, we reject the unit root hypothesis at the 1% significance level for these two test statistics of -6.79 and -16.79, respectively. Furthermore, the two structural breaks are

Table 66. Restricted SUR Estimates using the WPI with time trend and structural breaks

	β_0 (SE)	β_1 (SE)	β_2 (SE)	τ - test	Rho Test	t - test (break)	t - test (trend)
Canada	0.1784 (0.0372)	0.7627 (0.0324)	0.1372 (0.0273)	-7.32***	-18.50***	5.02***	-2.72
France	1.5876 (0.2245)						-0.05
Italy	1.6744 (0.2390)						0.87
Japan	1.4677 (0.2076)						-0.87
United States	0.1119 (0.0306)		0.0873 (0.0235)			3.71**	-1.25

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

highly significant, Canada at the 1% level and the U.S. at the 1% level. The t-test statistics are 4.08 and 3.68, respectively. The critical values for the positive side of the two-tailed test are 2.24, 2.68 and 3.63 for Canada, 2.20, 2.62 and 3.42 for the U.S. for the 10%, 5%, and 1% significance levels, respectively.

Table 66 includes the structural breaks identified in Table 54, the Wholesale Price Index SUR model when a time trend is included (Table 62). The restriction is not rejected at the 10% significance level. The critical values of 3.39, 4.14 and 6.02 compare with the computed F-statistic of 1.51. The Dickey-Fuller tau test critical values are -5.81, -6.19 and -6.77 for the 10%, 5% and 1% significance levels, respectively. The rho test critical values are -12.26, -13.70 and -16.52, respectively. Therefore, we reject the unit root hypothesis at the 1% significance level for these two tests. Furthermore, the two structural breaks are highly significant, Canada at the 1% level and the U.S. at the 5%

Table 67. Restricted SUR Estimates using the CPI no time trend and structural breaks

	β_0 (SE)	β_1 (SE)	β_2 (SE)	τ - test	Rho Test	t - test (break)
Canada	0.1125 (0.0188)	0.8045 (0.0283)		-6.91***	-15.24***	
France	1.3329 (0.1924)					
Italy	1.4891 (0.2247)		-0.0315 (0.0349)			-0.90
Japan	1.2018 (0.1754)					
United States	0.0978 (0.0176)					

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

level. The t-test statistics are 5.02 and 3.71 respectively. The critical values for the positive side of the two-tailed test are 2.45, 2.87 and 3.73 for Canada, 2.81, 3.18 and 3.85 for the U.S. for the 10%, 5%, and 1% significance levels, respectively. None of the time trend test statistics are significant for the Wholesale Price Index data.

Table 67 includes the structural breaks identified in Table 55, the Consumer Price Index SUR model when no time trend is included (Table 63). The restriction is rejected at the 1% significance level. The critical values of 3.33, 4.13 and 5.62 compare with the computed F-statistic of 5.83 and therefore the restriction should not be imposed. The case for Italy's structural break nullifies the ability to impose the restriction. The Dickey-Fuller tau test critical values are -4.25, -4.55 and -5.15 for the 10%, 5% and 1% significance levels, respectively. The rho test critical values are -5.92, -6.88 and -8.53, respectively. Therefore, we reject the unit root hypothesis at the 1% significance level for these two

Table 68. Restricted SUR Estimates using the CPI, a time trend and structural breaks

	β_0 (SE)	β_1 (SE)	β_2 (SE)	τ - test	Rho Test	t - test (break)	t - test (trend)
Canada	0.0821 (0.0286)	0.7469 (0.0290)	-0.0748 (0.0197)	-8.73***	-18.22***	-3.79***	-3.52***
France	1.7297 (0.1980)						-0.17
Italy	1.9559 (0.2306)						-1.33
Japan	1.7522 (0.1916)		0.4161 (0.0685)			5.99***	-5.24***
United States	0.1110 (0.0313)						0.66

Note: The critical values were computed using $r_t = r_{t-1} + \varepsilon_t$ where e_t is iid N(0, 1) and are based on 2000 Monte Carlo simulations. The observations were simulated such that each succeeding observation is generated from the prior period plus a N(0, 1) random error.

tests. The Italian structural break is not significant at the 10% level. The two-tailed t statistic is -0.90 and the critical values are -2.18, -2.57 and -3.60 for the U.S. for the 10%, 5%, and 1% significance levels, respectively.

Table 68 includes the structural breaks identified in Table 56, the Consumer Price Index SUR model when a time trend is included (Table 64). The restriction is not rejected at the 10% significance level. The critical values of 3.50, 4.19 and 5.84 compare with the computed F-statistic of 1.20. The Dickey-Fuller tau test critical values are -5.78, -6.08 and -6.63 for the 10%, 5% and 1% significance levels, respectively. The rho test critical values are -11.95, -13.00 and -15.18, respectively. Therefore, we reject the unit root hypothesis at the 1% significance level for these two tests. There are two structural breaks that are significant, Canada at the 1% level and the Japan at the 1% level. The t-test statistics are -3.79 and 5.99 respectively. The critical values for the negative side of the two-tailed test are -2.40, -2.88 and -3.54 for Canada. The critical values for the positive side are 2.23, 2.72 and 3.46 for the Japanese break for the 10%, 5%, and 1% significance levels, respectively. The time trends are insignificant for France, Italy and the United States. The t-test statistics for the time trends for Canada, 3.52, and Japan, -5.24 are significant when compared with their respective critical values of 2.86, 3.42 and 4.39 and -3.06, -3.64 and -4.40 for the 10%, 5% and 1% significance levels.

These last four cases, Tables 65 - 69, provide further evidence in favor of purchasing power parity and against the unit root hypothesis. The significance of the unit root tests increases with the addition of the structural breaks except in the case of the CPI based real exchange rate without a time trend included. The Italian mean shift adds nothing to the model.

For the WPI based real exchange rate, a shift in the mean occurred for Canada and the U.S. whether or not a time trend exists. For the CPI, when a time trend is included, Canada and Japan have significant mean shifts. Interestingly, the time trends for these two countries are significant also.

Taking into account the bias, as mentioned in Chapters 2 and 4, the coefficients vary from 0.788 to 0.848. The half-life of the CPI based real exchange rate is 3.3 to 4.6 years. For the WPI it is 3.6 to 4.1 years, when the structural break models are considered.

Some evidence exists in the annual data for a productivity bias in the CPI based real exchange rate. Canada and Japan have negative coefficients indicating an appreciation in the real exchange rate vis-a-vis the United Kingdom.

CHAPTER VI.

SUMMARY AND CONCLUSIONS

This paper examines real exchange rate data for the Group of Seven countries for evidence of purchasing power parity (PPP). The Group of Seven countries includes Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. Monthly data spanning 1957-1993 and annual data covering 1900-1994 are employed to find evidence for PPP.

Support for purchasing power parity in real exchange rate data is found by testing the hypothesis that the real exchange rate is indistinguishable from the prior period. Testing the null hypothesis that the autoregressive coefficient equals one is called a unit root test. Rejection of the unit root hypothesis constitutes evidence for PPP. Therefore, in each of the models examined below we searched for evidence of PPP by testing for a unit root.

Four types of models are used in our analysis: rolling regression models, recursive regression models, sequential regression models and Seemingly Unrelated Regression (SUR) models. The first three are models that endogenously search for a structural break in the bilateral exchange rates of the Group of Seven countries while simultaneously testing the unit root hypothesis. These first three are univariate equation models whereas the final (SUR) models use all of the countries in a multivariate framework. The SUR models test the unit root hypothesis using Estimated Generalized Least Squares.

The Monthly Results

The three following algorithms of Banerjee, Lumsdaine and Stock (1992) are used to search for structural breaks in the real exchange rates of the Group of Seven.

The rolling regressions models test for a unit root in a fixed subset of the data thus searching for a structural break. We fail to reject the unit root hypothesis for all of the countries using the monthly data set. Therefore, no structural breaks exist. This model adds no new information to the standard Augmented Dickey-Fuller and Dickey-Fuller unit-root tests. We did not find any consistent and clear evidence for PPP using the rolling regressions models.

The recursive regressions models also test a subset of the data for a unit root and thus a break in the data. Significant evidence is available to reject six of the twenty-four hypothesis tests of a unit root using the monthly data set. Therefore, we find some limited support for PPP using the recursive regressions models.

The sequential regressions models use the entire data set for each unit root test and includes a structural break variable sequentially each period to determine endogenously (rather than apriori) whether a break in the data set exists. We are unable to reject the null hypothesis of a unit root in almost all of the cases for the monthly data set. Some evidence of structural breaks is present for the mean or dummy variable type of shift in the intercept. Only two of the twelve bilateral real exchange rates have structural breaks when four lags are included in the equation. However, when no lags are used, six of the twelve bilateral real exchange rates have significant structural (mean) breaks.

Two tests for grouped heteroskedasticity find significant evidence that the variance before 1973 differs from that after 1973. This new specification of the model significantly changes the evidence of structural breaks and unit roots. We find with this new specification that 15 of 24 unit root hypothesis tests are rejected. In addition, there is evidence of structural breaks in 14 of the 24 cases. Therefore, the sequential regressions models with grouped heteroskedasticity find clear evidence for structural breaks and evidence for purchasing power parity in over half of the cases studied when monthly data set is examined.

We agree that the endogenous test for a sequential break introduced by Banerjee, et. al. (1992) is a powerful tool for data analysis. Authors often make a priori decisions about a structural break or choose their data set based upon apriori information. This study concludes that those apriori decisions are often not correct. While Germany and Japan have significant breaks close to the collapse of Bretton Woods, many countries do not have a significant break or at least do not have a break around the end of the Bretton Woods era. Furthermore, correcting for heteroskedasticity is important in real exchange rate time series. The importance of this result is that cointegration and unit root tests require long-run data sets. If a researcher is to properly model the long-run data then structural breaks and changing variances must be included for a properly specified model.

The final Seemingly Unrelated Regression model is employed to take advantage of the contemporaneous correlation that exists between the Group of Seven countries' real exchange rates. We find that the bilateral real exchange rates follow the same pattern. The coefficients on the lagged dependent variable are insignificantly different. An advantage to using this restriction is that we increase the power of our unit root tests. Our monthly data set is approximately double the size of the one employed in a previous study by Abuaf and Jorion (1990). We present an updated version of their model and we include three more cases. We improve their results by adding heteroskedasticity and structural breaks to the model. When the structural breaks found in the sequential regressions model are introduced, the restricted unit root hypotheses are rejected consistently at the 1% significance level. Furthermore, these results are robust to whether or not a time trend is included and to whether the proxy is the Consumer Price Index or the Wholesale Price Index.[48] We find support using a monthly data set for the theory of purchasing power parity and its use as a base for the models of exchange rate determination.

The Annual Results

The annual data set of 95 years is a long-run, low frequency data set. The evidence for PPP is stronger than in the monthly data set using the standard Augmented Dickey-Fuller and Dickey-Fuller equations. Fourteen of twenty tests of the ρ_μ unit root hypothesis are rejected at the 10% significance level.

The rolling and recursive regressions models do not provide much further evidence for structural breaks or PPP. The annual data set for these models fails to reject the unit root hypothesis with a few exceptions.

Using the sequential regressions models, the annual data reject the unit root null hypothesis in only 9 of the 30 cases examined. However, we find that there are 11 of 30

[48] Abuaf and Jorion (1990) only use the CPI in their analysis.

real exchange rates time series that have structural breaks. We find that contemporaneous correlation exists and the five bilateral exchange rates should be considered in an SUR model. As with the monthly data set, due to the low power of the tests we test and find that the restriction that the group of real exchange rates move together is not rejected.

Using the restricted Seemingly Unrelated Regression model with the annual data set we find results similar to those of the monthly data set. The unit root test is rejected at the 1% significance level for all of the cases. Furthermore, inclusion of the structural breaks identified in the sequential regressions model further increases the significance of these tests.

Therefore, by increasing the power of the test through using multivariate analysis we have found consistent and clear evidence that purchasing power parity holds. This evidence is robust with regard to proxy (the WPI or the CPI) and frequency of the data set (monthly or annual). However, it is also clear that choice of the size of the data set is crucial. Including too large of a data set without including a structural break may lead to incorrect conclusions. Similarly, arbitrarily choosing too small of a data set decreases the information to the researcher and may also lead to incorrect conclusions.

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