# MONETARY AGGREGATION: A COMPARISON STUDY OF SIMPLE SUM AND DIVISIA MONEY MEASURES

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MONEY MEASURES

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# CHAPTER ONE

# INTRODUCTION

## 1.1 Background

The past two decades generated a great deal of interest in the use of weighted monetary aggregation, the reason for this being that the traditional way of measuring money is considered flawed and thus may have made money no longer to be a viable policy tool. As Kohn notes, "...M1, which used to be considered the most reliable aggregate guide, has become so interest sensitive it can no longer serve as a useful target for policy." (1990, p. 3). Given the fact that the traditional form of a simple summation of monetary assets has produced monetary aggregates that have become increasingly difficult to track for any meaningful monetary policy, central banks have been forced to abandon monetary targeting and instead adopt interest rate targeting. For example, in the United Kingdom (UK), the Thatcher government abandoned the use of monetary targets in 1985, and the Bank of England stopped publishing M1 and M3 altogether in 1989 because it considered the data too distorted by financial innovation (Chrystal and McDonald (1994), p. 80). Currently, Alan Greenspan, the Fed chairman, is more inclined towards 'short term interest rates', rather than, 'money' targeting.

Much of the difficulty encountered in attempts to track monetary aggregates can be explained by the revolutionary changes that have occurred in the financial markets over the past two decades. These changes, mostly in the form of financial innovations and deregulations, have made it especially challenging for the monetary authorities to control the supply of money. For example, prior to 1980s the Fed was able to use M1 and M2 as intermediate targets in conducting its monetary policy. As financial innovations and deregulations took effect it became increasingly difficult for the Fed to target these monetary aggregates and thus was forced to de-emphasize monetary targeting.

The monetary index number theoretical studies initiated by Barnett (1980), are an attempt to salvage 'money' as the main policy targets for guiding monetary policy. The basic motivation for monetary index studies is that the traditional way of measuring money is considered to be severely flawed. Thus, any attempts to employ current measures of money in any monetary study would produce erroneous results. For example, monetary index adherents believe that the perceived instability and breakdown in empirical relationships in the money demand function in the early 1970's was not due to 'money' as such, but by the way money was measured.[1]

Traditional monetary aggregates, referred to as the simple-sum (SS), are obtained by adding dollar-for-dollar quantities of various monetary items. The major implication of the SS aggregation method is that monetary items like demand deposits and time

[1]Goldfeld (1976), in what has been referred to as 'The Case of the Missing Money', first noted the problem when he observed the conventional specification systematically over-predicting actual money balances in the period between 1972-1974. This was followed by a surprising shift in 1981-1982 when the money demand function exhibited extended periods of under-prediction as velocity fell considerably (see Goldfeld and Sichel (1990), p. 300).

deposits are viewed as being perfect substitutes. In contrast, weighted monetary aggregation methods do not view monetary items as perfect substitutes. as each component is assigned a weight according to the degree of 'moneyness'. Lindsey and Spindt (1986, p. 1) explain that monetary indexes are designed to measure aggregate monetary quantities in an environment characterized by a variety of assets, and by allowing for graded differences the indexes can adjust automatically for changes in payment methods or in the menu of financial assets.

Two of the weighted monetary indexes that have received consideration are the monetary quantity index (MQ) inspired by Spindt (1985) and Divisia Quantity index (DI) pioneered by Barnett (1980). Each index uses the equation of exchange as the basic building block, however, MQ takes a narrow view of money as a medium of exchange, while DI *a la* Friedman (1956) takes a broader view of money as providing a wide array of services beyond means of payment.

Lindsey and Spindt (1986, p. 2) indicate that monetary indexes and the traditional aggregates are similar in that the growth rate of each can be thought of as a weighted average of the growth rates of its components. But, for the traditional aggregates these weights are simply quantity shares of components in the total aggregate. The MQ uses as weights the shares of final product transactions financed by each component, while the weights for DI are shares of the total value of monetary services accounted by each component. Since these weights differ, monetary indexes differ from traditional aggregates and from each other.

This paper considers DI and SS in comparing the performance of weighted monetary indexes and the traditional aggregates under various performance criteria. Theoretically, a strong case has been made for the superiority of DI over the SS monetary aggregates. However, no general agreement on the empirical superiority of DI has been reached yet.

## 1.2 Objective of the Research Study

The central focus of this research is to determine empirically whether DI aggregates give more satisfactory answers than the traditional SS aggregates. To achieve this objective, the standard procedures in (money-income models and money demand models) are used to determine the viability for the use of any monetary aggregate as a monetary policy variable. In addition, I have incorporated a rational expectation model to compare the performance of the various monetary aggregates.

By their construct, both DI and SS monetary aggregates are different. And, no doubt Barnett and other monetary indexing adherents have rigorously established the theoretical justification for the use of weighted monetary aggregation. Barnett has argued that: "Except for monetary aggregates, most of the data provided by governmental agencies are constructed in accordance with aggregation number theory" (1982, p. 688). Indeed, economic indices like the CPI and GNP deflators are widely used as aggregate economic measures. And maybe the time has come, as Barnett has advocated, for index number weighted monetary aggregates to be adopted as official measures of monetary stock.

From the review of literature, the theoretical case for the use of weighted monetary aggregates seems to be overwhelming. However, in empirical practice there does not seem to be much improvement over the traditional SS aggregates. Theoretical flaws notwithstanding, the Fed has continued using SS aggregates to measure the official money supply. It maybe that, the body of evidence for use of weighted monetary aggregates is not robust enough to warrant a change in the current money measures. As Goldfeld has pointed out, "On the whole, while promising, the verdict on the DI approach is still out, either as an explanation of instability or for use in policy process" (1989, p. 140). Needless to say, 'paradigm-shifts' always tend to raise more questions than they answer. And, any economic theory or model, until it has withstood the test of time, should always be put under the microscope. Just because DI monetary aggregates perform well in the current environment is not a guarantee they will perform equally as well under a different environment.

Indeed, DI aggregation offers a potential line of research that may have some bearing on the viability of 'money' in monetary policy. It should be noted that, prompted by the financial innovations, the Federal Reserve's shift adjustment of M1-B in 1981 is an example of a strategy towards monetary components weighting according to their degree of 'moneyness'.[2] M1-B, which excludes demand deposits held by foreign commercial banks and institutions, includes interest-earning checkable deposits at all depository institutions - i.e. negotiable order of withdrawal (NOW), automatic transfer

[2]For a detailed discussion of the redefinition of monetary aggregates see Simpson (1980).

from savings (ATS) accounts, and credit union share draft balances - plus demand deposits at thrift institutions (Simpson (1980), pp. 97-98).

Because DI are an alternative to SS aggregates it would be important to compare them to see how any analysis of the effects of monetary policy might be affected by the method of aggregation. Chapter 2 describes the simple sum and indexed methods of aggregation ; Chapter 3 presents literature review on monetary aggregation; Chapter 4 presents a graphical comparison and correlation of SS and DI; Chapter 5 presents various model selection tests in the context of St. Louis reduced-form equation; Chapter 6 presents comparisons of performance of SS and DI in the context money demand model; chapter 7 introduces a model of rational expectations; and finally, chapter 8 presents summary and general conclusions.

# CHAPTER TWO

# MONETARY AGGREGATION

## 2.1 Simple Sum (SS) Aggregation

SS aggregation is derived from the classical's Quantity Theory of Money (QTM) where the main function of money was for transaction purposes. Money was narrowly defined. Currency and demand deposits were considered money and what could not be used directly to facilitate transactions was excluded from the definitions of money. The current broader definition of money are more inclusive.

SS aggregates are obtained by addition of the dollar amounts of each monetary component. Thus:

$$(1) \qquad M=\sum_{i=1}^{n} x_i ;$$

where $x_i$ is the monetary component *i* of subaggregate M1. Under such a structure, equation (1) represents an index of the stock of nominal money where monetary components are dollar-for-dollar perfect substitutes. In the narrowest sense, SS is appropriate because of a fixed exchange rate of units between currency and demand deposits. However, the broader the definition the more inappropriate SS aggregation becomes. Table 1 articulates the various simple sum components of money.

TABLE 1

COMPONENTS OF SS MONETARY ITEMS[3]

1. *Currency*: Coins and notes in circulation.
2. *Demand Deposits*: Non-interest bearing checking accounts at commercial banks, the government, and foreign governments.
3. *Traveler's Checks*: Checks issued by non-banks (such as American Express).
4. *Other Checkable Deposits*: Interest earning checking accounts such as NOW and ATS (automatic transfers from savings).

$$M1 = (1) + (2) + (3) + (4)$$

5. *Overnight Repurchase Agreement* (RP or REPO): Borrowing of a bank from a non-bank customer by selling a security with a promise to repurchase at a fixed price the following day.
6. *Overnight Eurodollars*: Interest paying deposits maturing the following day held in foreign branches of U.S. banks (especially in the Caribbean).
7. *Money market mutual fund* (MMMF) shares: Interest-earning checkable deposits in mutual funds that invest in short-term assets.
8. *Money market deposit accounts* (MMDAs): MMMFs run by banks and insured up to $100,000.
9. *Savings Deposits*: Deposits, at banks and thrift institutions, not transferable by check.
10. *Small time deposits*: Interest earning deposits, less than $100,000, with a specific maturity date.

$$M2 = M1 + (5) \text{ thru } (10)$$

11. *Large-denomination time deposits*: Interest-earning deposits of more than $100,000.
12. *Term repurchase agreements*: REPOs sold by thrift institutions for longer than overnight.

$$M3 = M2 + (11) + (12) + \text{MMMFs held by institutions}$$

13. *Other Eurodollar deposits*: More than overnight Eurodollars.
14. *Savings bonds*: U.S. government bonds, typically sold to small savers.
15; *Banker's acceptances*: Obligations of banks arising mainly from international trade.
16: *Commercial paper*: Short-term liabilities of corporations.
17. *Short-term Treasury securities*: Less than 12 months U.S. Treasury securities.

$$L = M3 + (13) \text{ thru } (17)$$

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[3]Source: See *Federal Reserve Bulletin*, which reports the data and definition in each monthly issue.

For many years only demand deposits at commercial banks were checkable . But today, other financial institutions offer a wide spectrum of checkable deposits without any clear-cut dividing line. Goldfeld and Sichel (1990, p. 315) offer an explanation that some deposit accounts may limit the number of checks written while other accounts permit periodic withdrawals at a charge. And therefore, it is not obvious whether such deposits should be included in the transaction-based definition of money. Moreover, it can also be argued that some components of M2 like MMDAs, MMMFs, and RPs, which are excluded in M1 belong in a transactions measure.

Take for instance RPs, which emerged as a popular device for corporate cash management in the 1970s as a way to convert non-interest earning demand deposits into interest-earning assets. Viewed this way, RPs and demand deposits are essentially perfect substitutes. Indeed, some authors who have argued that redefining narrow money to include RPs could shed some light on the missing money puzzle of the 1970s. For example, Goldfeld and Sichel (1990, p. 315) point out that attempts to consider both instruments as perfect substitutes appeared to have led to dramatic improvements in the forecasting of money demand for 1974-1976. However, Fackler and McMillin (1983, p. 441) note that, a log-level demand function with money defined as M1 plus RPs and M1 plus RPs and MMMFs such as estimated by Porter, Simpson, and Mauskopf (1979), still overpredicts money demand.

Thornton and Yue (1992, p. 36), argue that the theoretical justification of SS began to weaken when it was recognized that demand deposits paid an implicit interest rate, for example through free checking accounts. Now, a whole range of assets which can be used for transactions yield an interest rate and could thus be chosen as a form of store of value as well. For example, interest payments on NOW accounts makes it difficult to distinguish money held for transactions from money held for savings.

In consumer demand theory, SS aggregation is tantamount to treating different monetary components - currency, demand deposits, savings deposits, time deposits, etc. - as perfect substitutes. It is mainly for this reason that SS has been criticized for improper aggregation of assets with differing degrees of liquidity. Only perfect substitutes can be combined as a single commodity. According to Chrystal and McDonald (1994, p. 75), there is an overwhelming body of evidence showing that monetary items are not perfect substitutes and that there is a low degree of substitution between monetary components

## 2.2 The Demand Theory of Monetary Indexation

DI aggregation relies on consumer demand theory and treats money as a commodity held for the flow of utility generating monetary services they provide. The aim of DI aggregation is to construct an index number of monetary services which could capture the transaction services yielded by a range of financial assets: in other words, construct an index of monetary services from a group of monetary assets where the monetary service flow per dollar of the asset held - income effect - can vary from asset to asset. According to Belongia and Chrystal (1991, p. 497), SS index basically suffers from two deficiencies in construction:

(1) Non-weak separability - for example, a function separable from another function - a condition required by aggregation theory.

(2) An equal weighing of non-perfect substitutes.

Barnett (1980) explains the concept of aggregation theory in the following way:

> If the concept of money has to have any meaning then it follows that an aggregate of monetary assets must exist which is treated by the economy *as if* it were a single good, which we thereby call 'money'. Such an aggregate is a function (of its component monetary quantities) which is separable from the economy's structure. *That* concept of money is the subject of aggregation theory and is the concept relevant to policy, since both aggregation theory and policy postulate the appearance of a monetary aggregate as a *meaningful* stably defined variable in the economy's structure. Without the appropriate [weak] separability conditions, any aggregate is inherently arbitrary and spurious and does not define an economic variable. [*italics* appear in the original]. ( p. 13).

Barnett further observes that, when a functional quantity aggregate exists for a consumer, that aggregate itself must possess the known properties of a utility function - homotheticity and weak separability; and, "...when the aggregate quantity index is held constant, 'the utility of money' is necessarily held constant independent of its composition" Barnett (1980, p. 13). According to Thornton and Yue, "...in continuous time DI generates such a monetary aggregate and it is consistent with any unknown utility function implied by their data; in static time DI is in the class of superlative index numbers" (1992, p. 37). They conclude that, "SS index do not possess these desirable properties and thus they have no basis in either consumer demand theory or aggregation theory" (p. 37).

In aggregation theory a quantity index should measure the income effect (welfare or service flow changes) of a relative price change but should be unresponsive to pure substitution effect at constant utility which the index should internalize (Barnett 1980, p. 12). For example, a change in interest rates would cause DI to change only when there is an income effect or when there is a relative price change in utility (monetary service flow). DI completely internalizes substitution effects. In contrast, SS index does not. Changes in interest rates would cause a shift in SS even where there has not been any change in the utility level, hence no change in the monetary service flows.

A. <u>Consumer</u> <u>Demand</u> <u>Theory</u>

Barnett, Fisher, and Serletis (1992, pp. 2093-2103) illustrate the microeconomics derivation of DI monetary aggregation

Consider a consumer's intertemporal utility function

(2) $u = U(c, L, x)$

where $c$ = vector of the services of consumption goods

$L$ = leisure time

$x$ = vector of monetary assets which provide services such as convenience, liquidity, and information.

The optimization problem requires a two-stage procedure. In the first stage the consumer allocates expenditures among broad categories, and then in the second stage, expenditures are allocated within each category (Serletis (1987), p. 171). Thus, in stage one equation (2) is maximized subject to a constraint of

(3) $\mathbf{q}'c + \pi' x + wL = y$

where $y$ = expenditure income

$\mathbf{q}$ = vector of prices of $c$

$\pi$ = vector of monetary asset user costs (or rental prices)

w = shadow price of leisure.

The $i^{th}$ component of $\pi$ is given by,

(4) $\pi_i = p\left[\dfrac{R - r_i}{1+R}\right]$

where $r_i$ = expected nominal yield on the $i^{th}$ asset

$R$ = expected yield on an alternative asset (benchmark asset)

$p$ = the true cost of living index.

Equation (4) measures the opportunity cost- at the margin -of the monetary asset. The two-stage optimization procedure is possible only if the utility function (2) is homothetically weakly separable. Thus, (2) can be written as

(6) $u = U\,[c, L, f(x)]$

where $f$(x) is the monetary services aggregator function (quantity index) which is assumed to satisfy the usual regularity conditions.[4]

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[4]Weak separability requires that $[(\partial U/\partial x_i) / (\partial U/\partial x_j)]/\partial\phi = 0$, for $i \neq j$ and $\phi$ is any component of $\{c, L\}$. This condition implies that under weak separability the marginal rate of substitution between any two monetary assets is independent of the values of $c$ and $L$.

In stage two,

(7) $\quad \text{Max} f(x)$ subject to $\pi' x = m$

where $m$ is the total expenditures on monetary services and $f(x)$ is the monetary services aggregator or what Barnett et al (1992, p. 2095) refers to as an "*economic* (or functional) monetary *index*". Writing the Lagrangian

(8) $\quad L(\lambda, x) = f(x) - \lambda\,(\pi' - m),$

with the first-order condition

(9) $$\frac{\partial f}{\partial x_i} dx_i - \lambda \pi_i dx_i = 0$$

which implies

(10) $$\lambda \pi_i = \frac{\partial f}{\partial x_i}, \qquad i=1,\ldots\ldots\ldots,n.$$

And, a total differential of (7)

(11) $$d f(x) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} dx_i,$$

By substitution,

(12) $$d f(x) = \sum_{i=1}^{n} \lambda \pi_i \; dx_i$$

where $\lambda$= Lagrange multiplier

$\pi_i$ = user cost

$dx_i$ = quantity changes.

Equation (11) states that the growth rate of the aggregate $d f(x)$ is equal to the share-weighted average of the growth rate of the component quantity $\lambda \pi_i$. This result implies that the DI quantity index, $f(x)$, should only respond to income effects (welfare or service flow changes,) - a shift of the budget line - but be unresponsive to pure substitution effect at a constant utility - a movement along a given utility or indifference curve. Thus. a proper aggregation over the money market, should completely internalize the substitution effect and therefore eliminate the destabilizing effects of velocity in the money market. This is one of the main attractions provided by DI.[5]

B. Financial Firm Demand Theory

In the DI formulation financial intermediaries play the key role in the production of aggregated monetary goods (see Hancock (1984, 1985), Barnett et al (1996), and Fixler and Zieschang (1996)). The approach used is to model financial intermediaries as profit maximizing firms in the business of buying and selling financial assets in the neoclassical sense. Financial intermediaries produce liabilities such as demand deposits and time deposits as outputs by employing financial (e.g cash) and non financial factors (e.g labor, capital, and materials) as inputs (see Barnett et al (1996, p. 2-3). Thus, production occurs with both monetary and nonmonetary goods.

Following is a notationally simplified version of Barnett et al (1996, p. 4) model where the firm maximizes a variable profit function at the begining of period $t$

(13) $\pi_t = (1 + r_{t-1})X_{t-1} - X_t + C_{t-1} - C_t$

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[5] Barnett (1980) notes two major observations: an ideal index number represents a theoretically attractive alternative to a fixed weight or SS aggregations, and that consumer theory of utility maximization is consistent with theory that generates the ideal index numbers.

$$+\sum_{i=1}^{I} [D_{i,t} - (1+h_{i,t-1})D_{i,t-1}] - \sum_{j=1}^{J} w_{j,t} z_j - I_t$$

where $\pi_t$ = variable profits in period $t$

$r_{t-1}$ = portfoilio rate of return in time period $t-1$ (unknown in period $t$)

$X_t$ = nominal balances of the asset (loan) portfolio

$D_{i,t}$ = nominal balances (deposits) of the $i^{th}$ produced account type

$C_t$ = nominal (cash) excess reserve balances

$w_j$ = payment to $j^{th}$ input factor

$z_{j,t}$ = quantity of the $j^{th}$ real input (including labor)

$I_t$ = expenditure on investments

Equation (13) indicates that financial intermediaries derive most of their income from loan and investment portfolios. As Barnett et al explains, the first two terms of equation (13) represent the change in variable profit during period $t$, the third and the fourth terms represent the change in the nominal value of excess reserves, the fifth term represents the change in the firm's variable profits from the change in the issuance of produced financial liabilities, the sixth term consists of payments to real inputs, and the last term is the expenditure on investments.

The dynamics in the model is introduced through capital stock $K_t$ which follows the neoclassical growth path

(14) $K_t = I_{t-1} + (1-\delta) K_{t-1}$

where $\delta$ = rate of capital accumulation

$I_{t-1}$ = gross investment at time period $t-1$ which becomes productive in period $t$.

The objective

$$(15) \quad \text{Max } \pi_t \qquad \text{s.t } X_t = \sum_{i=1}^{I} [(1\text{-}rr_{i,t})D_t] - C_t - \sum_{j=1}^{J} w_{j,t}z_{j,t} - I_t$$

where $rr_{i,t}$ = required reserve ratio on $i^{th}$ liability

$\sum_{i=1}^{I} D_t$ = total deposits allocated to required reserves, excess reserves, payments for real inputs, investment in capital, and investment in loans.

By substituting (14) and (15) into (13) to eliminate investment in capital goods and loans, the variable profit function becomes

$$(16) \quad \pi_t = \sum_{i=1}^{I} [(1+r_{t-1})(1\text{-}rr_{i,t-1}) - (1+h_{i,t-1})]D_{i,t-1} + rr_{i,t} D_t$$

$$- r_{t-1} C_{t-1} - (1+r_{t-1}) \sum_{j=1}^{J} w_{j,t-1} z_{j,t-1}$$

$$+ (1\text{-}\delta)(1+r_{t-1})K_{t-1} - (1+r_{t-1})K_{t-1}$$

The financial firm maximizes the expected value of the discounted intertemporal utility of its variable profit stream, subject to its technological constraint. The optimization problem now becomes

$$(17) \quad \text{Max} \quad \mathrm{E_t}\left[\sum_{s=t}^{\infty}(1/1+\mu)^{(s-t)}U(\pi_s)\right]$$

$$\text{s.t.} \quad \Omega(y_{1,s},\ldots, y_{I,s}, C_s, z_{1,s},\ldots,z_{J,s}, K_s) = 0 \quad \forall s \geq t$$

where $\mathrm{E_t}$ = expectation at time $t$

$\mu$ = subjective rate of time preference

U = utility function exhibiting Hyperbolic Absolute Risk Aversion (HARA)[6]

$\pi_s$ = variable profit at time $s$

$\Omega$ = transformation function.

From the convexity requirement, the partial derivatives of the transformation function with respect to inputs and outputs are

$$\partial\Omega/\partial C_s \leq 0,\ \ \partial\Omega/\partial K_s \leq 0,\ \ \partial\Omega/\partial z_{j,s} \leq 0 \qquad \forall j = 1,\ldots,J$$

and

$$\partial\Omega/\partial y_{i,s} \geq 0 \quad \forall_i = 1,\ldots,I.$$

## 2.3 Aggregation of Monetary Goods on the Demand side

If money is to be a viable policy tool, then a properly weighted monetary aggregate is required. Barnett's DI, an asset weighted index, and Spindt's MQ, a velocity weighted index, are two such aggregates which represent theoretical meaningful alternatives to the theoretically flawed SS index (Serletis (1988), p. 352). Each index is calculated by using the Fisher Ideal index (see Barnett (1980), pp. 37-45):

$$(18) \qquad Q^F{}_t = Q^F{}_{t-1} \left[ \frac{\sum_{i=1}^{N} \pi_{it} m_{it} \sum_{i=1}^{N} \pi_{i,t-1} m_{it}}{\sum_{i=1}^{N} \pi_{it} m_{i,t-1} \sum_{i=1}^{N} \pi_{i,t-1} m_{i,t-1}} \right]^{1/2}$$

where, $Q^F$ = Fisher Ideal index

[6]The HARA class functions are represented by

$$U(\pi_t) = \frac{1-\rho}{\rho}\left(\frac{h}{1-\rho}\pi_t + d\right)^{\rho}$$

where $\rho$, $h$, and $d$ are parameters to be estimated (Barnett et al (1996), p. 6).

$m_{it}$ = quantity of the $i^{th}$ asset

$\pi_{it}$ = associated weight which is the user cost for $i^{th}$ asset in DI, and turnover rate in MQ.

Thus, Barnett and Spindt differ in their choice of the $\pi_{it}$. In Barnett's framework, $\pi_{it}$'s are the 'user' cost for $i^{th}$ asset, which is the interest forgone by holding asset $i$ as opposed to an alternative higher-yielding non-monetary asset, for example non-human capital. In contrast to Barnett's approach, Spindt considers the $\pi_{it}$'s as turnover rates of monetary asset $i$ during period $t$, instead of user costs. Spindt employs the equation of exchange to derive MQs:

(19) $$\sum_i^n m_i v_i = \text{PQ},$$

where $v_i$ is the net turnover rate or velocity of the $i^{th}$ asset. It is these $v_i$'s that are used as the weights in (18). According to Serletis (1988, p. 352), the use of turnover rates instead of user costs makes MQ to be inconsistent with the existing aggregation and index number theory. Goldfeld and Sichel (1990, p. 317) have also pointed out some flaws with MQ. They argue that, gross turnover rates are unavailable for some assets like currency and MMMFs and, moreover, even where available, gross turnover rates would reflect a large volume of transactions, for example financial transactions and payments for intermediate goods, not reflected in GNP. Therefore, to move from gross turnover to net turnover would require many assumptions. But the major drawback to the use of MQ in money demand functions is that data on MQ series are only available beginning 1970, and thus cannot be used to solve the 'missing money' puzzle.

Barnett (1980, p. 38-39) has noted that Tornquist (1936) and subsequently Theil (1967), advocated a quantity index number which Barnett refers to as Tornquist-Theil Divisia Index:

$$(20) \quad Q^T{}_t = Q^T{}_t \cdot \prod_{i=1}^{N} \left[ \frac{m_{it}}{m_{i,t-1}} \right]^{\frac{(s_{it}+s_{i,t-1})}{2}}$$

where $Q^T$ = Tornquist-Theil Divisia Index

$$s_{it} = \frac{\pi_{it} m_{it}}{\sum_{k=1}^{N} \pi_{kt} m_{kt}} .$$

In logarithm form (20) becomes

$$(21) \quad \log Q^T{}_t - \log Q^T{}_{t-1} = \sum_{i=1}^{N} s^*_{it}(\log m_{it} - \log m_{i,t-1}),$$

where, $s^*_{it} = \frac{(s_{it} + s_{i,t-1})}{2}$.

DIs are calculated from equation (21). The right-hand side measures the growth rate of the quantity index which equals to the weighted average growth rate of the monetary component as indicated on the left-side of the equation. Also, the weights are the share contributions of each component to the total value of all components. Fisher has indicated that equation (21) "...has the theoretical and statistical backing to measure moneyness" (1992, p. 20). Barnett (1980, p. 39) also note that, the same index number results in (18) and (21). In addition, Barnett (1980, p. 39) points out that, Diewert (1976) has proved that both Fisher Ideal Index in (18) and Tornquist-Theil Divisia index in (20) are Diewert-superlative.[7] Barnett further observes that Tornquist-Theil index is more widely

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[7]Diewert defines an index number to be 'superlative' if it is exact for some aggregator function which can provide a second-order approximation to any linearly homogenous aggregator function. Barnett refers to

used than the Fisher Ideal index, and because of its easier interpretation, Barnett has thus advocated the use of Tornquist-Theil Divisia index to measure the quantity of money at all levels of aggregation higher than M1 (Barnett (1980), p. 39).

### 2.4 Supply Theory of Monetary Indexation

As demonstrated in section 2.2, demand theory can show how consumers and financial intermediaries come about to demand DI money by maximizing their inter-temporal, blockwise, weakly-separable, utility and variable profit functions subject to a budget constraint. Thus, the demand side of DI aggregation presents little if any problem. Such is not the case with the supply side. This is one issue that the DI literature has not been able to address convincingly. It is not yet clear, for example, how the SS reported data are converted to DI data by economic agents as they adapt to their perceived money changes coming from their altered DI situation. Or in other words, what is the supply of DI money mechanism?

Since the central banks to a large extent control the supply of money the role of money creation process is either ignored or de-emphasized on the assumption that the supply of money is essentially exogenously determined by the central banks. Nevertheless, when it comes to monetary aggregation, the supply side of money presents special problems. For example, it is especially difficult to derive a function that explains the motivation or behavior of the monetary authorities.

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such an index number Diewert-superlative. He also considers an index number to be exact if it exactly equals the aggregator function whenever the data is consistent with microeconomics maximizing behavior. Barnett (1980, p. 38) has also noted that Hulten (1973) has proved that in continuos time the Divisia index is always exact for any consistent (blockwise homothetically weakly separable) aggregator function.

A. SS supply function

When deriving the supply function of nominal money, two distinctions are made: outside money and inside money. Outside money is the 'high-powered' money or the 'base', which consists of currency and central bank deposits, while the inside money is the 'low-powered' money which consists of private deposits of other banks and depository institutions in excess of their holdings of outside money assets (see Tobin, 1989, p. 159). From these two distinctions a traditional SS money supply curve is derived

(22) $\quad M = m(i, y, rr)B$

where $m$ = money multiplier a function of ( . )

$i$ = interest rates including bank's excess and borrowed reserves, and market, demand and time deposits interest rates which may or may not be determined exogenously, depending upon the regulatory environment and the operating procedures of the central bank.

$y$ = nominal income

$B$ = monetary base

$rr$ = deposits reserve requirement.

Equation (22) states that the supply of money is determined by the public, financial intermediaries, and the Fed.

B. DI supply function

Barnett (1987) and Hancock (1987) attempts to derive a DI money supply function from a neoclassical model of production by a multiproduct financial intermediary which plays a dual role of demanding and producing monetary goods. As demonstrated in Hancock (1987, pp. 202-205), production occurs with monetary goods $m$ and nonmonetary goods $x$

(23) $m = (m_0,\dots,m_{I-1})$ $\quad\quad m_i > 0$ for $i = 0,\dots,I - 1$.

and

(24) $x = (x_1,\dots,x_J)$ $\quad\quad x_i > 0$ for outputs and $x_i < 0$ for inputs, $i = 1,\dots,J$.

Monetary goods are given to the firm which sets the interest rates. Prices of nonmonetary goods are $p = (p_1,\dots,p_J)$. From a transformation function $T(x,m) = 0$, the variable profit function is

(25) $$\pi(p,m) = \max\left\{\sum_{i=1}^{J} p_i x_i, x, m \in S, p > 0\right\},$$

linearly homogeneous in prices, increasing in output prices and decreasing in input prices.

Hancock proceeds to show that if a money index $M(m_o,\dots,m_{I-1})$ exists, then the tranformation function can be written as

(26) $T(x,M) = 0$, where $M$ is a scalar.[8]

Thus

(27) $M(m) = h(x)$, which implies $M(m)/h(x) = 1$.

The variable profit function becomes

(28) $$\pi(p,m) = \max\left\{\sum_{i=1}^{J} p_i x_i : M(m) = h(x)\right\}$$

$$= e(p, M(m)) = e(p,1)\, M(m) = g(p)\, M(m).$$[9]

---

[8]$T$ is continous from above, where $\partial T/\partial x < 0$ and $\partial T/\partial M < 0$.

[9]By duality theorem, given any technology it is possible to derive the cost function (Varian (1992), p. 81).

where $e(.)$ is a minimum expenditure function subject to maximum level of profits (see Varian (1992), p. 104).

Hancock shows that if money index exists

(29) $g(p) = \pi(p,m)/M(m) = \partial\pi/\partial M$, is a variable profit function dependent only on prices $p$ of nonmonetary goods and not on the quantities of monetary goods $m$.

Equation (29) is only posssible only when a money index exists and thus $g(p)$ summarizes the technology of the firm. Without the existence of a money index, $\pi(p,m)$ cannot be decomposed into a product of two functions representing nonmonetary prices and monetary goods. (Hancock (1987), p. 204) notes two special cases: first, if all monetary goods are inputs then $g$ is the marginal and average user return to money; second, if all money goods are outputs, the cost function is $c = -\pi$. Therefore, the marginal cost of producing monetary goods for a money index $M$

$$\partial c/\partial M > 0, \text{ while } \partial\pi/\partial M < 0.$$

It follows that supplies of outputs and demand for inputs are

(30) $x_i = \partial\pi/\partial p_i \qquad i = I,\ldots,J\text{-}1.$

and for the quantities of goods

(31) $\partial\pi/\partial m = r_i \qquad$ for $i = 0,\ldots,I\text{-}1$

where $r_i$ = user return per dollar held.

## 2.5 <u>Aggregation of Monetary Goods on the Production side</u>

To obtain an exact quantity aggregate which is a measure of the financial firm's produced service flow, Barnett et al (1996, pp. 7-9) follow a similar two-stage procedure as illustrated in section 2.2 above. The first stage determines the existence of an exact aggregate from an admissible group that Barnett refers to as 'blockwise weakly separable'. And, the second stage produces that exact aggregate in the manner consistent with microeconomics theory. As a result, output aggregation is applicable to the construction of a neoclassical money supply function for aggregated money (Barnett (1987), p. 121). Thus given

(32) $\mathbf{y} = (y_{1t},\ldots,y_{It})'$ the financial firm's output - liabilities - vector,

and

(33) $\mathbf{x} = (z_{1,t},\ldots,z_{j,t})'$ the input - nonfinancial factors - vector.

The transformation function can be written as

(34) $\Omega(\mathbf{y},\mathbf{x}) = 0$. Weak separabilty condition implies that (34) can be written as[10]

(35) $\Omega(\mathbf{y},\mathbf{x}) = \mathbf{H}(y_o(\mathbf{y}),\mathbf{x})$

where $y_0(\mathbf{y})$ = exact output (monetary) quantity aggregator function a monotonically increasing and strictly concave function of $\mathbf{y}$.[11]

Shortcomings

As appealing as they may appear, DI are not without shortcomings. One problem is how the user cost is to be calculated. Goldfeld and Sichel argue that: "A first issue concerns the own rate where there are measurement difficulties [from] payment of implicit interest via the provision of services and the existence of explicit service

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[10]Weak separability condition requires that $\partial/\partial x_K \left(\dfrac{\partial\Omega(y,x)/\partial y}{\partial\Omega(y,x)/\partial y_j}\right) = 0 \quad i \neq j$.

[11]Monotonicity requires that $\partial\Omega/\partial y \geq 0$ and $\partial\Omega/\partial x \leq 0$.

charges.[In addition] the lack of data makes it hard to evaluate the seriousness of these difficulties" (1990, p. 317). Also of concern is the use of the benchmark yield of the non-monetary asset. Goldfeld and Sichel (1990, p. 316) note that the current practice is to use a corporate bond rate as the benchmark rate, except when the own rates on some $m_{it}$ are higher than the rate used as the benchmark rate. The implication of this is that the user cost of the nonmonetary asset is zero and the monetary services are therefore regarded as nil. Goldfeld and Sichel point out: "Even in less extreme situations, the evidence suggests that interest rate movements can produce anomalous variations in user costs " (1990, pp. 316-317).

Another problem with the DI as pointed out in Fisher (1989, p. 19) is that DI requires the underlying preference function to be homothetic - i.e. have unitary income-elasticities for the various commodity demands. Also, as pointed out above, the supply side of Divisia Index derivation has not been addressed adequately.

Shortcomings notwithstanding, a theoretic case for use of weighted monetary aggregates seems to be overwhelming. Chrystall and McDonald provide a message to researchers: "All those who do applied research using money should take on board the fact that simple sum-measures are substantially distorted and a better measure is likely to be provided by a monetary services index constructed along something like Divisia lines" ( 1994, pp. 107-108).

## CHAPTER THREE

## LITERATURE CONSIDERATIONS

Attempts to construct monetary aggregates by weighing each component according to its degree of 'moneyness' had been suggested by Gurley, Friedman and Schwartz, Chetty, and Diewert (see Spindt (1985), P. 176). However, much credit goes to William A. Barnett for initiating formal theoretical modeling of monetary aggregation in the context of index number theory (see Barnett, 1978, 1980, 1981, 1982, 1987). Notable contributions on the line of monetary component weighting from include: Barnett and Spindt (1979), Spindt (1985), Barnett, Offenbacher, and Spindt (1981, 1984), Fisher and Serletis (1989), and Barnett, Fisher, and Serletis (1992) from the demand side; Hancock (1985, 1987) Barnett and Hahm (1994) Barnett and Ge Zhou (1994a) Barnett, Kirova, and Pasupathy (1996 part 1) from the production side. Barnett (1980) follows in the tradition of Chetty (1976) who estimated a production function for money services in order to aggregate M1 and small time deposits at banks and thrifts, (see Judd and Scadding, 1982). However, Barnett did not directly estimate a production function for money services, but instead employed microeconomics demand theory to derive DI as in Diewert (1976, 1980). These Divisia indices are then used to generate Divisia monetary aggregate measures.

As for empirical studies, Barnett (1980, p. 12) finds stability of velocity with increased levels of DI aggregation, while that of SS destabilized by aggregation beyond an intermediate level. He further observes that the main problem of the money demand (or velocity) shift was primarily due to the long run substitution effect from increased rates on unregulated monetary assets relative to the own rates on rate regulated monetary assets. Barnett (1980, pp. 39-41), has compared M3 velocities of Fisher Ideal index, Tornquist-Theil Divisia index, and SS index, and finds the velocity of SS index continues to decline secularly from 1972.3, while the velocity of the two indices, considered to be Diewert-superlative, not only to be very close but rising. Barnett feels that since the aggregates included many assets subject to government rate regulations, we should expect substitution (disintermediation) to occur out of the aggregates into such substitutes as money market instruments, for example RP's, treasury bills, commercial papers, and money market funds, during periods of rising interest rates and high inflation. If this is true, then velocity should rise. Therefore, the declining velocity of the SS index seems to be misleading.

Barnett (1980, pp. 41-43) has also compared velocities of the Diewert-superlative index and SS index, with a ten-year government bond rate, and finds the variations in the velocity of the Diewert-superlative index to make more economic sense: the interest elasticity of money demand has the right sign. Thus, Barnett feels that, "...internalizing further money market by aggregating over further money market instruments can be expected to further stabilize the velocity of the superlative index" (1980, p. 14). His reasoning: "The substitution effect (*defined* to hold utility constant) of a change in the relative prices of components *within* an aggregate *cannot* change the value of an economic quantity aggregate (utility level)" (p. 41).

Belongia and Chrystal (1991, p. 502) using index number theory for the UK observe DI M4 to have the most appealing long-run characteristics and to be the aggregate most likely to conform to the traditional homogeneity postulate of monetary theory. Chrystal and McDonald (1994, p. 77) compare the performances of DI and SS, in the context of a St. Louis equation, and confirm the general results of DI studies: DI seems to dominate SS more at broader levels than at lower levels. This implies that the problems inherent in the previous money studies may have been due to bad measurement theory rather than to an instability in the link between the money and the economy. As Chrystal and McDonald suggest: "Rather than a problem associated with the *Lucas Critique*, it could instead be a problem stemming from the *Barnett Critique*" (1994, p. 76).

On the causality question, evidence of DI outperforming SS is not as strong yet. Serletis (1988), used Granger-Sims test to study the relationship between SS, DI, and MQ on money, prices, and income. They show that although aggregation theory and index number theory favor DI over SS and MQ, the Granger-Sims test do not reveal a single uniformly best monetary aggregate. However, Divisia M2 (D2) and Divisia L (DL) are seen to perform better.

# CHAPTER 4

## GRAPHICAL ILLUSTRATIONS OF MONETARY AGGREGATES

### 4.1 Relative Levels

The levels of various monetary aggregates are shown in chart 1 to 4. All the series are normalized to equal 100 in 1960.1. As in many economic time series data, the levels of monetary series show a tendency to grow over time by an increasing amounts and thus are better approximated by a convex function than a straight line. Chart 1, which plots the narrow money aggregates, shows a trend similarity of DI and SS until early 1970s when they began to diverge. Financial innovations of early 1970s, for example the introduction of ATS and NOW accounts, much explains for this divergence.

The divergence between broader monetary aggregates, shown in charts 2 to 4, starts from the initial period of study and widening with time, with the greatest divergence occurring in the early 1970s and 1980s when the financial innovations and derugulations are well in place. As has been well documented, the period encompassing early 1970s and 1980s was characterized by inflationary pressures and high interest rates and thus we should expect to have a greater divergence between broader Divisia and simple sum series during this period. Thus, as indicated in Charts 2 through 4 broader DIs show lower trend than their counterpart simple sums largely due to the smaller weights assigned to broader DIs.

Chart 1. Levels of D1 and M1

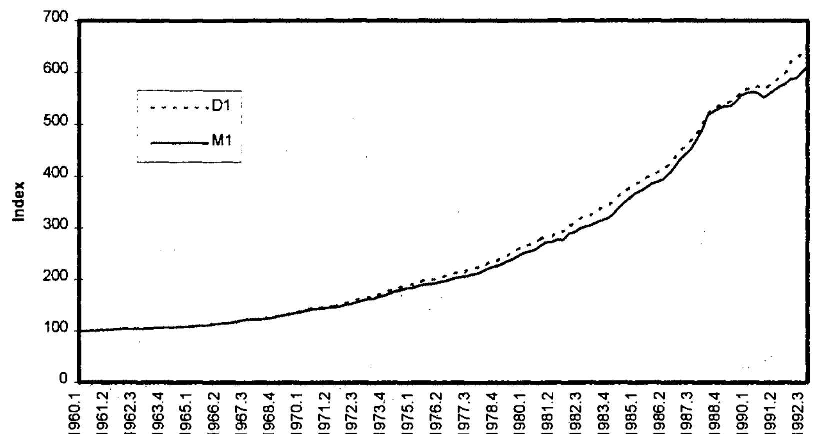
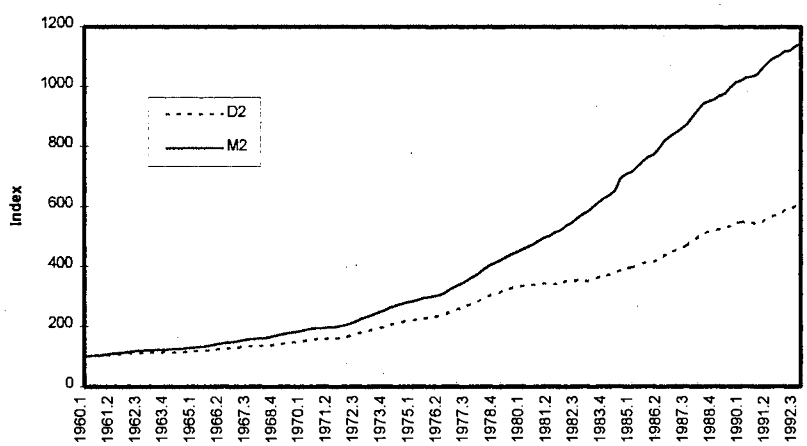
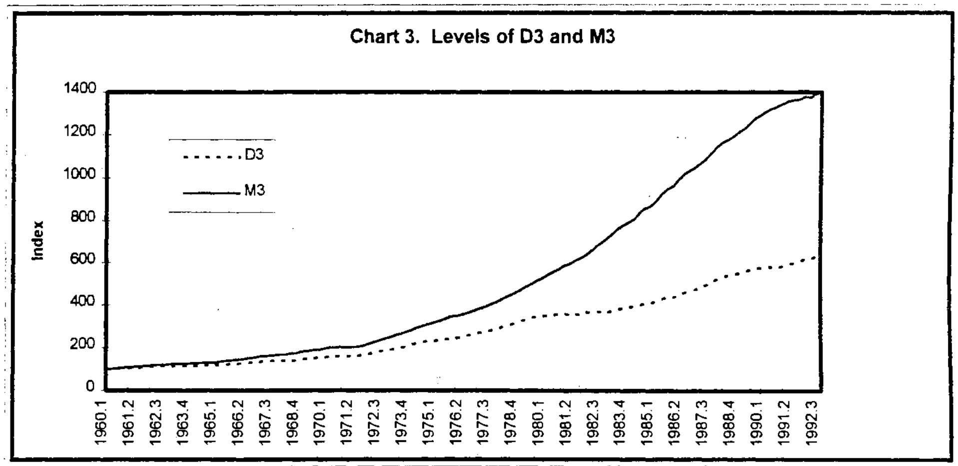
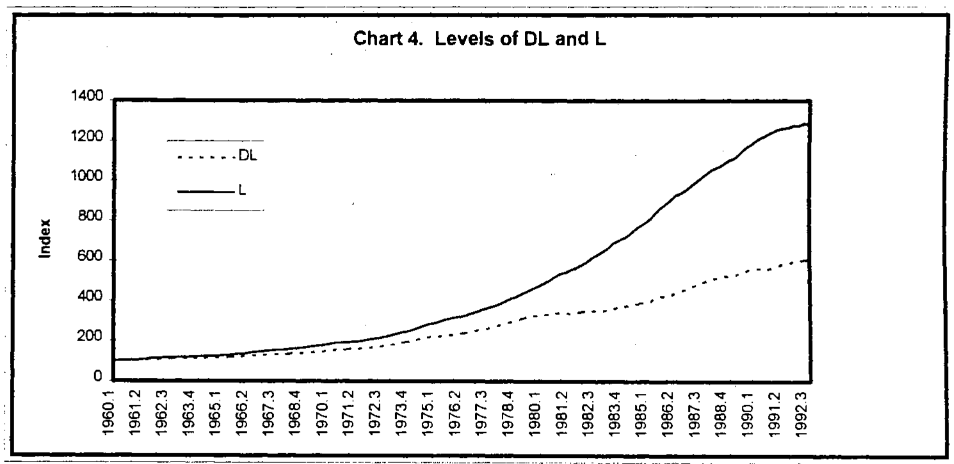



Chart 2. Levels of D2 and M2






Charts 5 and 6 show the levels of broader DI and SS aggregates, respectively. All the levels are essentially the same until early 1970s when they begin to diverge. But, the levels of broader DIs exhibit little differences than their counterpart SS levels. The reason being that DI aggregation gives relatively small weight to less liquid assets that yield higher own rates of return.

**Chart 5. Levels of Broader Divisias**

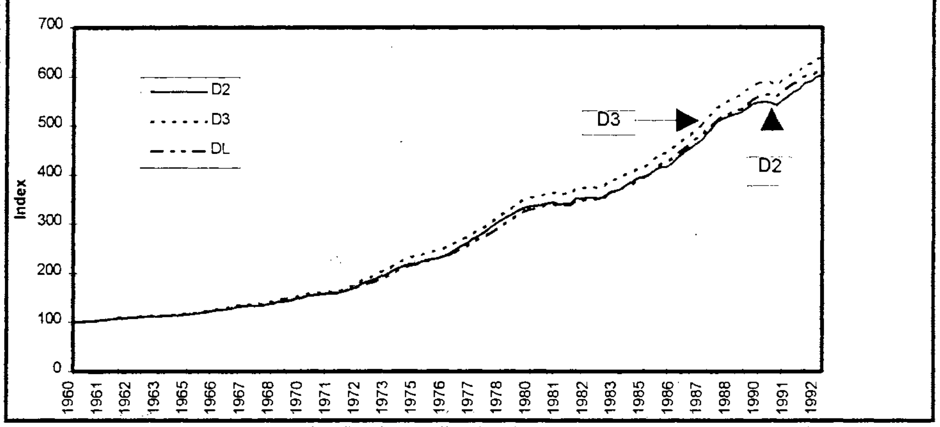
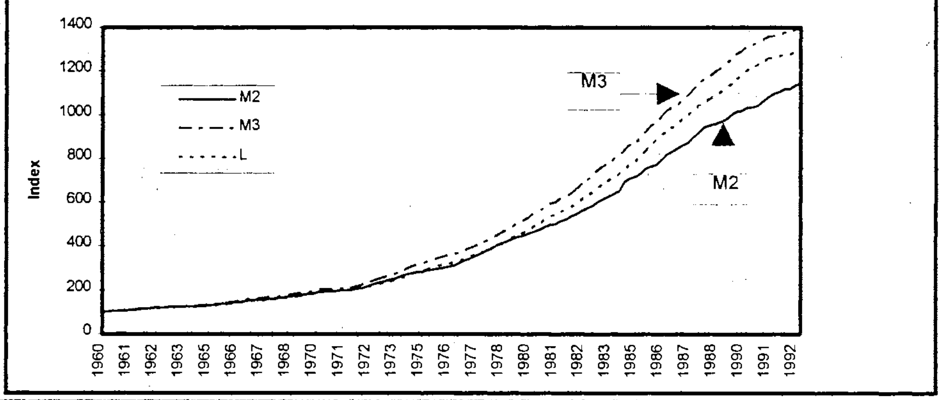



**Chart 6. Levels of Broader SS**



**4.2 <u>Relative</u> <u>Level</u> <u>Growth</u> <u>Rates</u>**

Growth rates of various monetary aggregates are displayed in charts 7 to 10. In contrast to levels of monetary series, percentage growth rates of monetary series display no obvious tendency to rise and fall. As can be observed in Chart 7, both the growth rates of M1 and D1 exhibit a relatively less variation until mid-1980s when the growth rates started to grow more rapidly. Indeed, the growth rates of M1 and D1 were similar until the 1970s, when the growth of ATS and NOW accounts began to accelerate. As noted in Thornton and Yue (1994), the nationwide introduction of NOWs in 1981, tended to increase the growth rate of M1 relative to D1 because the growth rate of NOW accounts gets a smaller weight in the DI measure. Thornton and Yue have also observed that D1 grew more rapidly than M1 due to the rise in the growth rate of currency relative to the growth rate of checkable deposits after the late 1980s.

As for the broader series, there is a much wider divergence between SS and DI series. While the growth rates of broader SS are much higher than those of DI, the growth rates of SS series have shown less variation since 1970. This should not be surprising since interest earning assets are assigned a smaller weight in DI measures. Notice the marked difference between the two series from mid-1970s to mid-1980s. As noted in Lindsey and Spindt (1986), this period was generally characterized by high market rates and an inverted yield curve. Accordingly, the opportunity costs of rapidly growing liquid assets declined as did their weight in the monetary service index. Thornton and Yue also observe that, the much lower growth of the broader DI measures during this period is more consistent with the disinflation of the time than is the growth of SS, whose growth remained fairly rapid. In the subsequent periods, with the decline in market interest rates, in late 1980s, much of the variation in the two series is reduced.

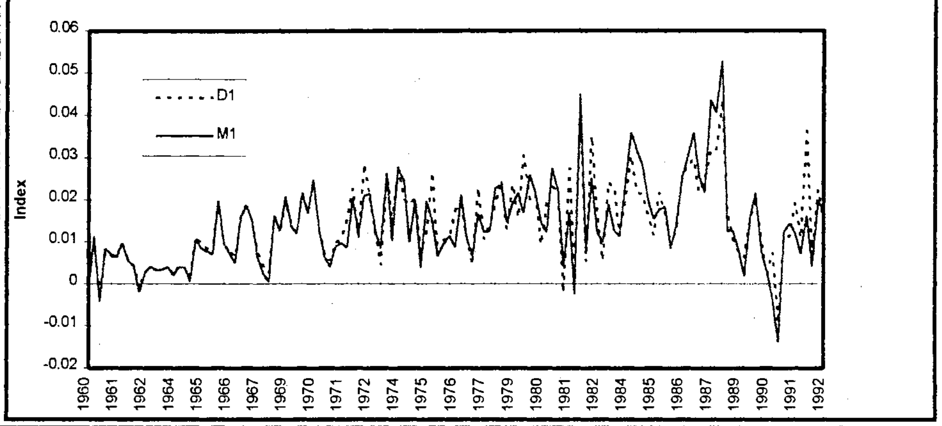



Chart 7. Growth Rates of D1 and M1

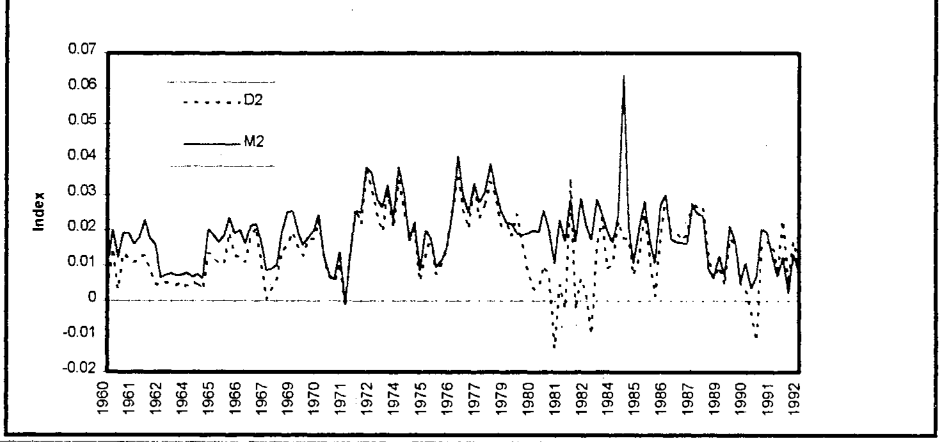
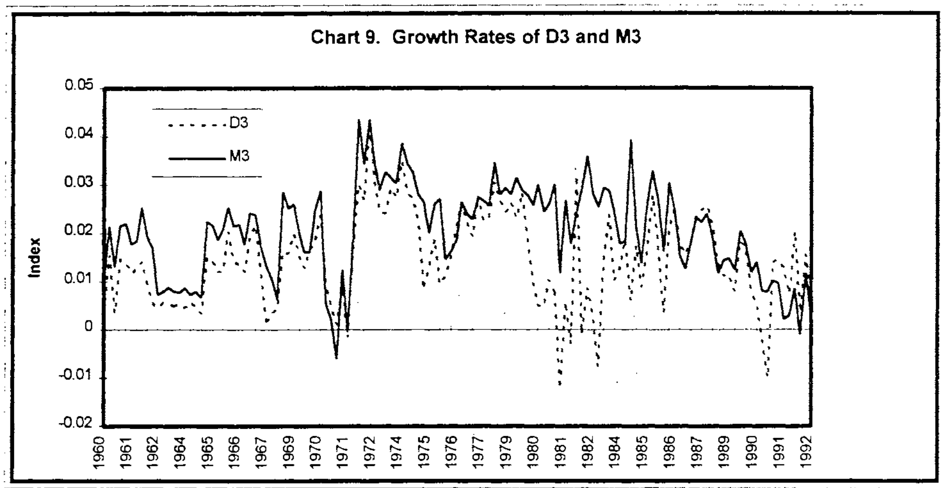
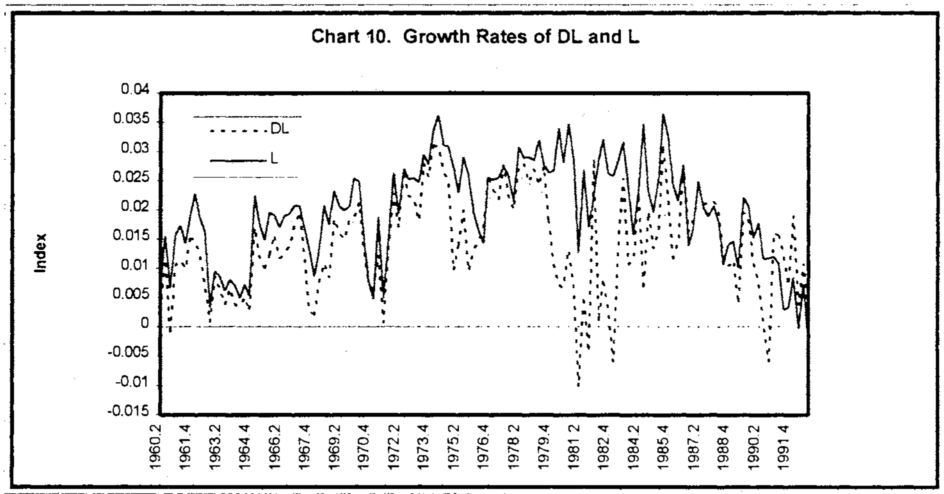



Chart 8. Growth Rates of D2 and M2




Charts 11 and 12, display the growth rates of broader DIs and SS respectively. As noted above, the growth rates of broader SS series show less variation since 1970 than their counterpart DI series. In addition, the growth rates of D3 and DL are similar but differ little from D2.

Chart 11. Growth Rates of Broader Divisias

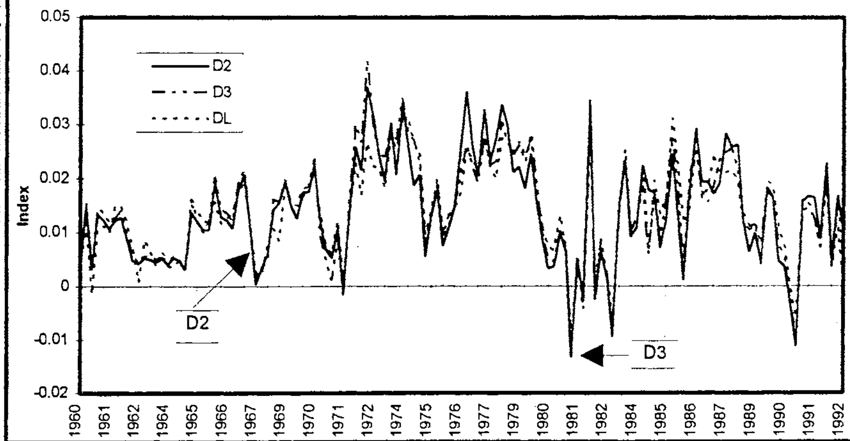
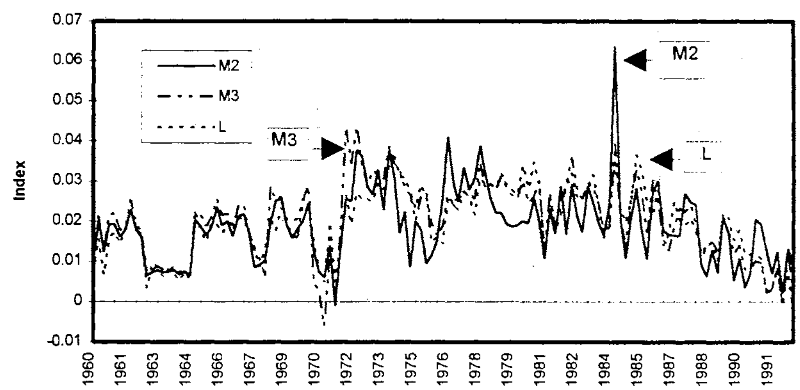



Chart 12. Growth Rates of Broader SS



**4.3 Relative Velocity Levels**

Charts 13 to 16 show the velocity levels of various monetary aggregates. Since velocity of an aggregate is obtained by dividing GNP by the monetary aggregate, the monetary aggregate levels and their velocities should display an identical pattern. Hence, charts 1 to 4, and charts 13 to 16, respectively, are identical.

As pointed out, the differences between SS and DI is much more pronounced in broader than in narrower money series, as indicated in charts 14 to 16. This observation is especially true after 1980 due to financial deregulation which were aimed at making financial institutions more competitive by allowing them to offer competitive market rates for their instruments. The immediate impact of this was to cause a shift of funds from market instruments, such as MMMFs into MMDAs and super NOWs.

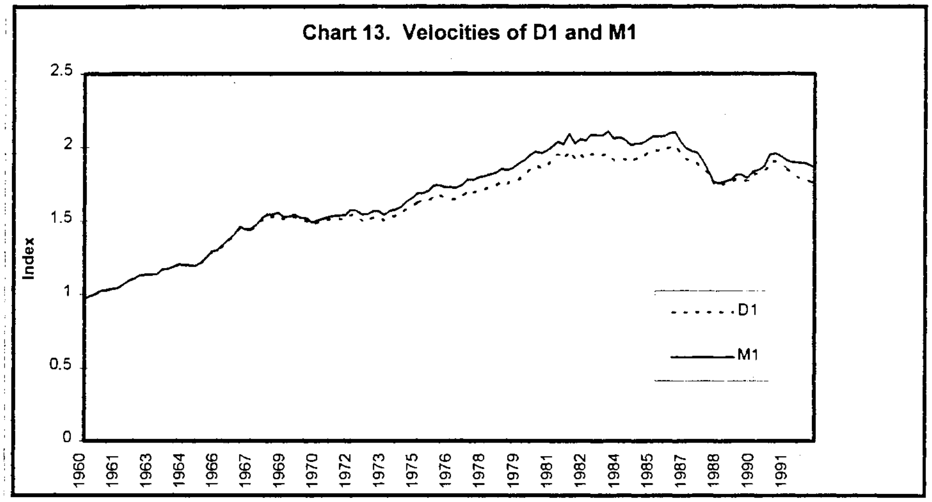


Chart 13. Velocities of D1 and M1
Index
2.5
2
1.5
1
0.5
0
D1
M1
1960
1961
1963
1964
1966
1967
1969
1970
1972
1973
1975
1976
1978
1979
1981
1982
1984
1985
1987
1988
1990
1991

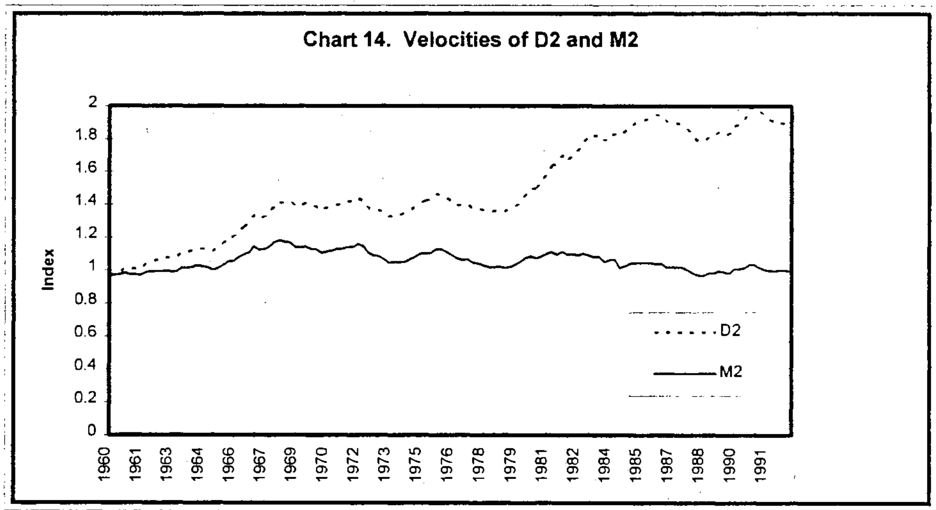
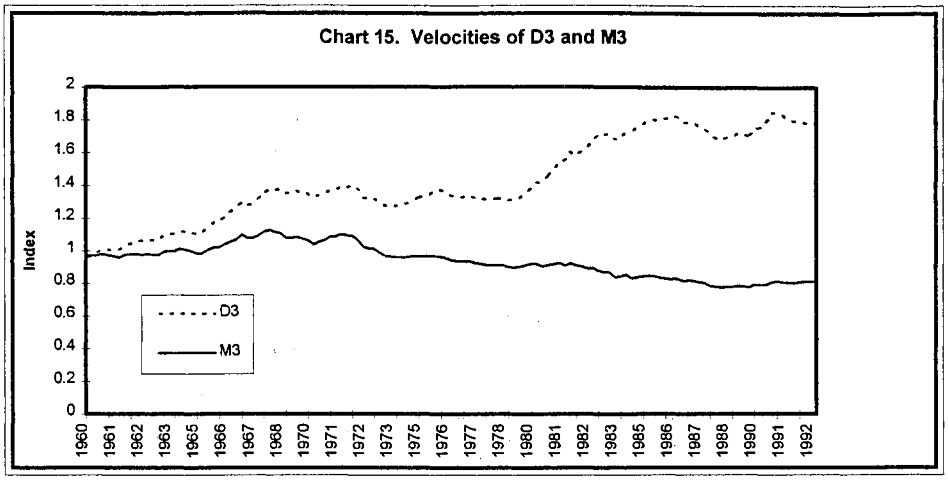
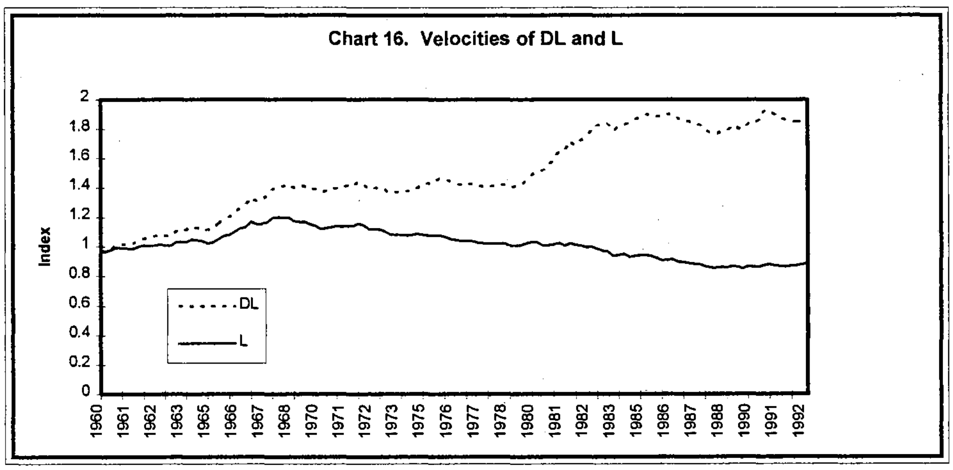


Chart 14. Velocities of D2 and M2
Index
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
D2
M2
1960
1961
1963
1964
1966
1967
1969
1970
1972
1973
1975
1976
1978
1979
1981
1982
1984
1985
1987
1988
1990
1991




Charts 17 and 18 show respective velocities of broader DI and SS series. While the velocity levels of broader DI display a similar and closer upward pattern, the velocity levels of broader SS show more long run stability.

Chart 17. Velocities of Broader Divisias

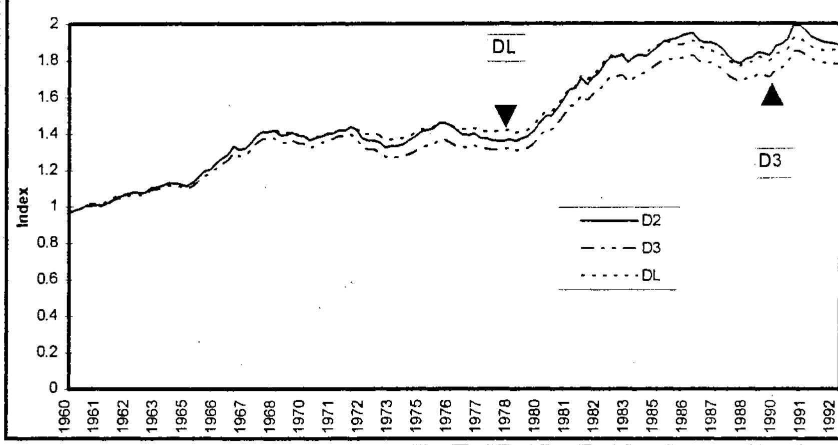
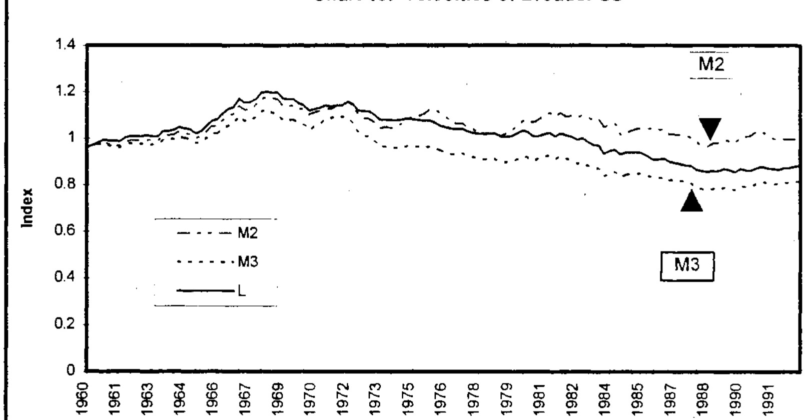



Chart 18. Velocities of Broader SS



## 4.4 Relative Velocity Growth Rates

A comparison of growth rates of velocities of various aggregates is presented in charts 19 to 22. In reiteration to the observation in regard to level growth rates, due to the weighting system in Divisia aggregation, there is a wider disparity between the velocity growth rates of broader than in the narrower money series. Also, for the reasons explained above, this disparity is more pronounced in mid-1970s and mid-1980s, the period characterized by high market yields.

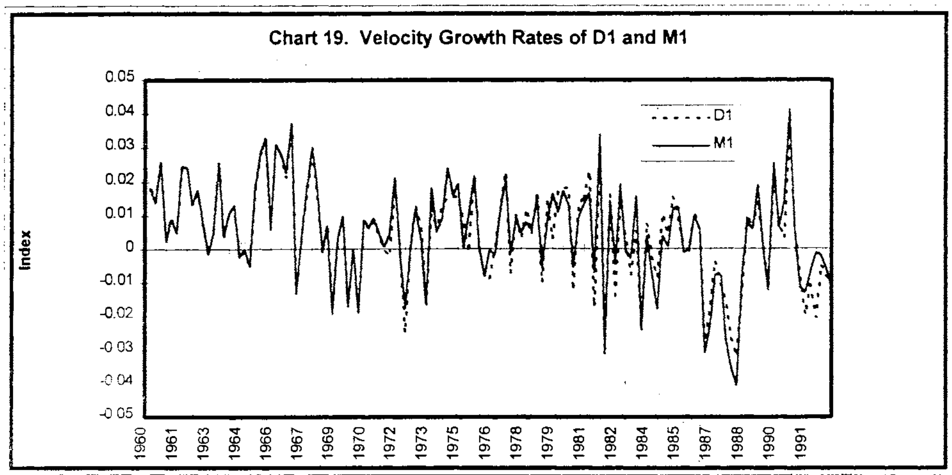


Chart 19. Velocity Growth Rates of D1 and M1
Index
D1
M1
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
1960
1961
1963
1964
1966
1967
1969
1970
1972
1973
1975
1976
1978
1979
1981
1982
1984
1985
1987
1988
1990
1991

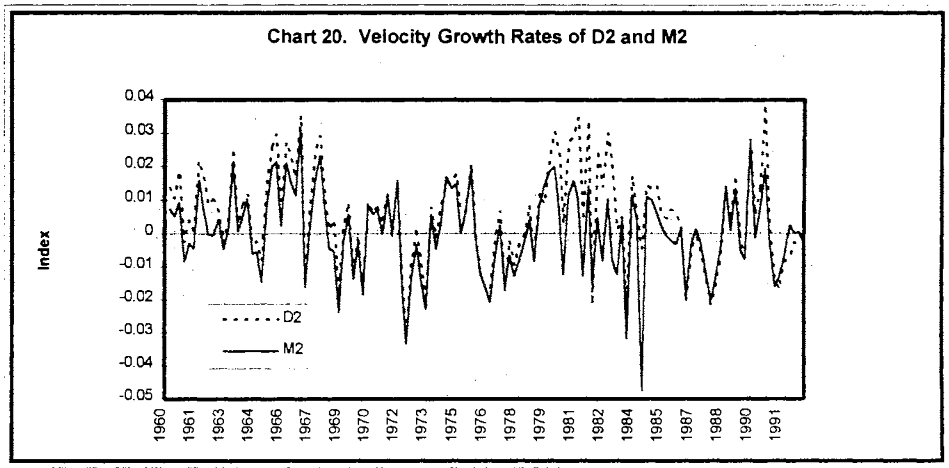


Chart 20. Velocity Growth Rates of D2 and M2
Index
D2
M2
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
1960
1961
1963
1964
1966
1967
1969
1970
1972
1973
1975
1976
1978
1979
1981
1982
1984
1985
1987
1988
1990
1991

Chart 21. Velocity Growth Rates of D3 and M3

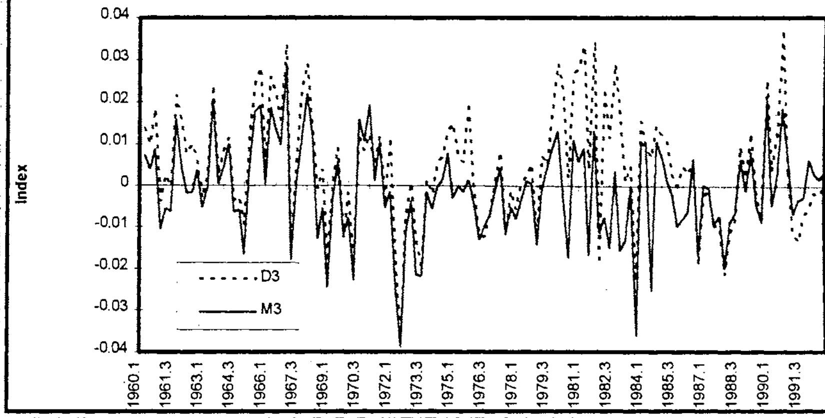
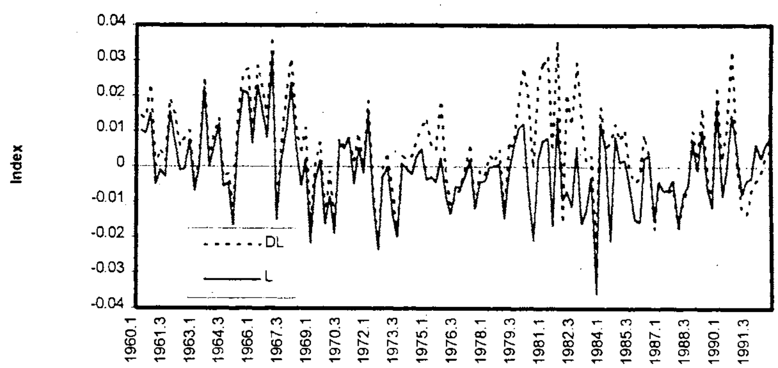
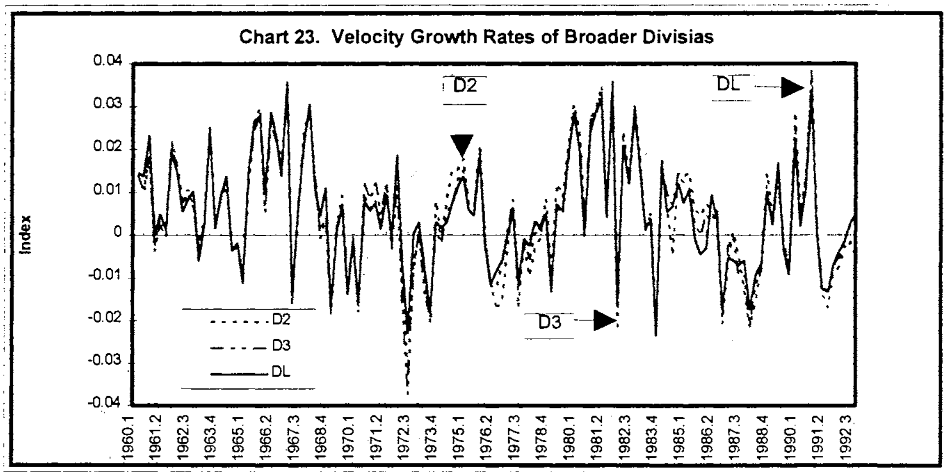
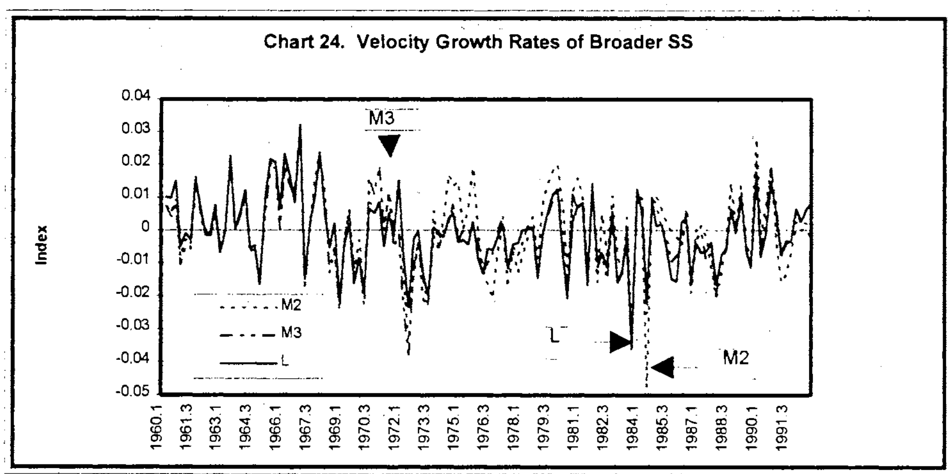



Chart 22. Velocity Growth Rates of DL and L



As indicated in charts 23 and 24 below, growth rates of broader Divisias show a closer pattern than those of the SSs.



## 4.5 Correlation of Divisia and SS Growth Rates

Tables 2, 3 and 4 present correlation coefficients of various monetary aggregate growth rates. As revealed in Table 1 broader Divisias show higher correlation than their counterpart SSs. Divisia aggregation gives smaller weights to less liquid assets that yield high rates of return and thus the weights gets smaller with the level of aggregation. This also means that the levels and hence the growth rates of broader Divisias will differ little. For example, the growth rates of D3 and DL differ little from the growth rate of D2 and thus the correlation between these aggregates is very high. Such a relationship is absent in the case of simple sum measures as seen in Table 3.

Table 4 also confirms what the theory suggests: the differences between DI and SS increases with the level of aggregation. For example, the correlation between the growth rates of D1 and M1 is 0.89 while the correlation between those of DL and L is only 0.61.

TABLE 2

CORRELATION OF QUARTERLY GROWTH RATES

OF DIVISIA MONETARY AGGREGATES

| Aggregates | D1 | D2 | D3 | DL |
|---|---|---|---|---|
| D1 | 1 | | | |
| D2 | 0.6871 | 1 | | |
| D3 | 0.6518 | 0.9588 | 1 | |
| DL | 0.6544 | 0.9153 | 0.9435 | 1 |

TABLE 3

CORRELATION OF QUARTERLY GROWTH RATES

OF SS MONETARY AGGREGATES

| Aggreagtes | M1 | M2 | M3 | L |
|---|---|---|---|---|
| M1 | 1 | | | |
| M2 | 0.5733 | 1 | | |
| M3 | 0.4396 | 0.7993 | 1 | |
| L | 0.4588 | 0.7160 | 0.8887 | 1 |

TABLE 4

CORRELATION OF QUARTERLY GROWTH RATES

OF DIVISIA AND SS MONETARY AGGREGATES

| Aggregates | D1 | D2 | D3 | DL |
|---|---|---|---|---|
| M1 | 0.89 | 0.65 | 0.59 | 0.61 |
| M2 | 0.52 | 0.73 | 0.67 | 0.62 |
| M3 | 0.45 | 0.55 | 0.64 | 0.58 |
| L | 0.46 | 0.46 | 0.53 | 0.61 |

# CHAPTER 5

# MODEL SELECTION TESTS

## 5.1 Intuitions

The task now is to determine whether DI aggregates are empirically more appealing than the SS aggregates. One way to determine this is through the money-income relationship. Essentially, the exercise requires testing hypotheses about the parameters of SS and DI in order to select an appropriate money-income model. Harvey (1990, p. 185) points out that, a typical approach to model selection is by formulating the simplest model possible but with a high predictive power. A high adjusted $R^2$ or equivalently low standard errors is an indication of goodness of fit and thus a temptation to proceed no further. The adjusted $R^2$ assumes that a true model exists and the task is in finding it, taking into account the trade-off between gain in explanatory power and loss in d.f. Harvey, however, warns that, high adjusted $R^2$ can easily be obtained with time series data even though the variances are completely unrelated. Therefore, it is imperative that we should subject our model, simple as it may appear, through a battery of tests before deciding to apply it.

Harvey (1989, pp. 13-14), has presented a list of six criteria for a good model that have been proposed in the econometrics literature:

(a) *Parsimony*: a simple model containing a relatively small number of parameters.

(b) *Data coherence*: a model congruent or consistent with evidence. That is, the model should provide a good fit to the data, and the residuals, as well as being small, should be approximately random.

(c) *Consistency with prior knowledge*: the model should be consistent with any prior knowledge provided by economic theory.

(d) *Data admissibility*: a model should be consistent with theory and should be unable to predict values that violate definitional constraints, for example, negative values.

(e) *Structural Stability*: the parameters of interest should be constant within and out-of-sample. That is, a model should provide a good fit both within and out-of-sample periods.

(f) *Encompassing*: a model is said to encompass all rival models if it can explain the results given by the rival formulations. That is, the rival model contains no information which could be used to improve the preferred model. To be encompassing, a model need not be any more general than its rivals. As Harvey observes: "Indeed the notion of parsimonious encompassing is essential to avoid vacuous formulations." (1990, p. 7).

In the following exercises of choosing between SS and DI, a battery of tests corresponding to the above list of criteria is employed. The procedure is to estimate a model first using SS and then re-estimate the same model using DI to compare the respective performances. In addition to the 'goodness of fit' tests, other model selection tests are: error minimizing tests; non-nested hypotheses tests including Davidson-Mackinnon Cox and J tests; and the Wald and Lagrange Multiplier (LM) tests for the validity of the restrictions imposed on the model.

### 5.2 Error Minimizing Tests

A number of alternative error and information content tests for model selection have been suggested, see for example Judge *et al*. (1985, pp. 242-245), Harvey (1989, pp. 146-189), or Ramanathan (1993, p. 281). The criteria are based on the principle of minimizing prediction error sum of squares (ESS) or minimizing observed likelihood values. As explained in Ramanathan (1993, p. 281), the following tests are based on what he refers to as the mean squared error (ESS/$T$), multiplied by some penalty factor that depends on the model complexity as measured by the number of regression coefficients to be estimated ($k$). These tests are referred to as **Akaike** (1974) **Information Criterion (AIC)**; **Akaike** (1969) **Final Prediction Error (FPE)**; **Hannan and Quinn** (1979) **criterion (HQ)**; **Schwartz** (1978) **Criterion (SC)**; **Shibata** (1981) **criterion (Shibata)**; **Rice** (1984) **criterion (Rice)**; and **Craven-Wahba** (1979) **Generalized Cross Validation(GCV)**. Following is a summary of these criteria statistics:

AIC $\left(\frac{ESS}{T}\right)e^{\frac{2k}{T}}$

Rice $\left(\frac{ESS}{T}\right)\left[\frac{1-2k}{T}\right]^{-1}$

FPE $\left(\frac{ESS}{T}\right)\left[\frac{T+k}{T-k}\right]$

SC $\left(\frac{ESS}{T}\right)T^{\frac{k}{T}}$

GCV $\left(\frac{ESS}{T}\right)\left[\frac{1-k}{T}\right]^{-2}$

Shibata $\left(\frac{ESS}{T}\right)\left[\frac{1+k}{T}\right]$

HQ $\left(\frac{ESS}{T}\right)(\ln T)^{\frac{2k}{T}}$

Charemza and Deadman (1992, pp. 293-295) present the log likelihood statistics referred to as **Akaike** (1973) **Information Criteria (log AIC) and Schwartz** (1978) **criterion (log SC)**:

$$\log \text{AIC} = \frac{[-2\ln(\psi) + 2k]}{T}$$

$$\log \text{SC} = \ln\, \delta^{2} + [k\,.\,\ln(T)]$$

where $\ln(\psi\ )$ is the value of the loglikelihood function of the estimated model and $\delta^{2}$ is an unbiased estimate of the residual variance.

Given competing models, a model with a lower value of criterion statistics is judged to be preferable. An ideal model would be one that obtains the lowest values of all the criteria statistics, however, this may not always happen in practice. In that case, the model that outperforms the other in more of these criteria is preferred (see Ramanathan, 1993, p. 270).

The nominal St. Loius reduced-form equation to be estimated for comparison purposes is:

$$(36) \quad g(Y)_t = \alpha_0 + \sum_{i=1}^{r} \alpha_{1i}\, g(F)_{t-i} + \sum_{i=1}^{s} \alpha_{2i}\, g(M)_{t-i},$$

where Y = nominal GNP

M = SS and DI aggregates

F = government purchases of goods and services, a fiscal variable.

$g$ (.) = annualized growth rate of the argument.

All data are quarterly. Equation (36) is estimated in unrestricted form and in a restricted form. Superneutrality condition is imposed in the restricted case, where the sum of lagged coefficients of money is constrained to be unity. In addition, (36) is estimated with and without lagged values of the dependent variable as regressors. Since most events have effects that persist over time, an appropriate model should include lagged variables. It is in this context that the lagged values of the dependent variable (GNP) are included as a consequence of the theoretical basis of the model.

## Test Results

Tables 5 through 12 present error minimizing criteria results obtained from the estimated regressions. General observation is that the structure of the model obtaining the highest $R^2$ and adjusted $R^2$ also obtains the lowest minimized predicted error sum of squares. For example, in Table 11, the model using M3 has the highest $R^2$ and the lowest error criteria values. On this account, except for M1, SS outperforms DI in all the criteria in Tables 5, 7, 8, 10, and 12.

DI seem to be favored in the unrestricted cases with four and six lags of dependent variable included as regressors. Table 11 shows mixed results. For example, D1 and DL outperform their counterpart M1 and L, while M2 and M3 perform better than D2 and D3. As for the individual models, the model including M3 as money variable seems to do better than the other models in the majority of the cases. Indeed, the best model seem to be one which includes, as regressors, eight lags of the dependent variable and eight lags of M3.

The inclusion of a lagged dependent variable seems to improve on the fit and error test criteria. As for the superneutrality condition, a combination of superneutrality and a lagged dependent variable also significantly improves on the error test criteria. This is especially true with higher lags.

The results obtained in this study seem to contradict those obtained by Chrystal and MacDonald (1994, p. 78). Employing a St. Louis Equation and including a T- bill rate which they had found to be an important variable, find all the broader DI money measures outperforming their SS counterparts under AIC.

To summarize, we can say that, based on the fit and error minimizing test criteria, in the context of reduced-form equation, money indexation offers insubstantial improvements over the SS aggregation. In addition, the imposition of superneutrality condition seems to show some improvements, while the lag structure suggests a lagged dependent variable improves on the fit of the estimated model.

# TABLE 5

$$\text{ERROR CRITERIA: } {}_g(Y)_t = \alpha_0 + \sum_{i=1}^{4} \alpha_{1i\,g}(F)_{t-i} + \sum_{i=1}^{4} \alpha_{2i\,g}(M)_{t-i}$$

(UNRESTRICTED)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.1543 | 0.1630 | 0.1985 | 0.1900 | 0.2114 | 0.2024 | 0.1886 | 0.1802 |
| Adj $R^2$ | 0.0969 | 0.1063 | 0.1441 | 0.1350 | 0.1580 | 0.1483 | 0.1336 | 0.1246 |
| FPE | 0.0132 | 0.0130 | 0.0125 | 0.0126 | 0.0123 | 0.0124 | 0.0126 | 0.0128 |
| LOG AIC | -8.9352 | -8.9456 | -8.9889 | -8.9783 | -9.0052 | -8.9937 | -8.9766 | -8.9663 |
| LOG SC | -8.7336 | -8.7441 | -8.7873 | -8.7768 | -8.8036 | -8.7922 | -8.7750 | -8.7647 |
| GCV | 0.0132 | 0.0131 | 0.0125 | 0.0127 | 0.0123 | 0.0125 | 0.0127 | 0.0128 |
| HQ | 0.0143 | 0.0141 | 0.0135 | 0.0137 | 0.0133 | 0.0135 | 0.0137 | 0.0138 |
| RICE | 0.0133 | 0.0132 | 0.0126 | 0.0127 | 0.0124 | 0.0126 | 0.0128 | 0.0129 |
| SHIBATA | 0.0130 | 0.0129 | 0.0124 | 0.0125 | 0.0122 | 0.0123 | 0.0125 | 0.0126 |
| SC | 0.0162 | 0.0156 | 0.0153 | 0.0154 | 0.0150 | 0.0152 | 0.0155 | 0.0156 |
| AIC | 0.0132 | 0.0130 | 0.0125 | 0.0126 | 0.0123 | 0.0124 | 0.0126 | 0.0127 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.

# TABLE 6

$$\text{ERROR CRITERIA: } {}_g(Y)_t = \beta_0 + \sum_{i=1}^{4} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{4} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{4} \beta_{3i\,g}(M)_{t-i}$$

(UNRESTRICTED)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.2287 | 0.2407 | 0.2427 | 0.2530 | 0.2459 | 0.2605 | 0.2294 | 0.2428 |
| Adj $R^2$ | 0.1475 | 0.1607 | 0.1630 | 0.1744 | 0.1665 | 0.1827 | 0.1483 | 0.1630 |
| FPE | 0.0128 | 0.0126 | 0.0126 | 0.0124 | 0.0125 | 0.0123 | 0.0128 | 0.0126 |
| LOG AIC | -8.9643 | -8.9800 | -8.9827 | -8.9963 | -8.9869 | -9.0065 | -8.9652 | -8.9827 |
| LOG SC | -8.6731 | -8.6800 | -8.6916 | -8.7052 | -8.6957 | -8.7154 | -8.6741 | -8.6916 |
| GCV | 0.0129 | 0.0127 | 0.0127 | 0.0125 | 0.0126 | 0.0124 | 0.0129 | 0.0127 |
| HQ | 0.0144 | 0.0142 | 0.0141 | 0.0139 | 0.0141 | 0.0138 | 0.0144 | 0.0141 |
| RICE | 0.0131 | 0.0129 | 0.0129 | 0..0127 | 0.0128 | 0.0126 | 0.0131 | 0.0129 |
| SHIBATA | 0.0126 | 0.0124 | 0.0123 | 0.0122 | 0.0123 | 0.0120 | 0.0125 | 0.0123 |
| SC | 0.0171 | 0.0168 | 0.0168 | 0.0166 | 0.0167 | 0.0164 | 0.0171 | 0.0168 |
| AIC | 0.0128 | 0.0126 | 0.0126 | 0.0124 | 0.0125 | 0.0123 | 0.0128 | 0.0126 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.

# TABLE 7

$$\text{ERROR CRITERIA: } {}_{g}(Y)_t = \alpha_0 + \sum_{i=1}^{4} \alpha_{1i}\, {}_{g}(F)_{t-i} + \sum_{i=1}^{4} \alpha_{2i}\, {}_{g}(M)_{t-i}$$

(RESTRICTED)**

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.0024 | 0.0466 | 0.1442 | 0.0613 | 0.1342 | 0.0869 | 0.1167 | 0.0889 |
| Adj $R^2$ | -0.0563 | -0.0095 | 0.0938 | 0.0060 | 0.0824 | 0.0323 | 0.0639 | 0.0344 |
| FPE | 0.0153 | 0.0146 | 0.0131 | 0.0144 | 0.0135 | 0.0142 | 0.0137 | 0.0142 |
| LOG AIC | -8.7858 | -8.8311 | -8.9391 | -8.8466 | -8.9136 | -8.8604 | -8.8936 | -8.8626 |
| LOG SC | -8.6067 | -8.6519 | -8.7599 | -8.6674 | -8.7326 | -8.6794 | -8.7126 | -8.6816 |
| GCV | 0.0154 | 0.0147 | 0.0132 | 0.0144 | 0.0135 | 0.0143 | 0.0138 | 0.0142 |
| HQ | 0.0164 | 0.0157 | 0.0141 | 0.0155 | 0.0145 | 0.0153 | 0.0148 | 0.0152 |
| RICE | 0.0154 | 0.0147 | 0.0132 | 0.0145 | 0.0136 | 0.0143 | 0.0139 | 0.0143 |
| SHIBATA | 0.0152 | 0.0145 | 0.0130 | 0.0143 | 0.0134 | 0.0141 | 0.0136 | 0.0141 |
| SC | 0.0183 | 0.0175 | 0.0157 | 0.0172 | 0.0161 | 0.0170 | 0.0165 | 0.0170 |
| AIC | 0.0153 | 0.0146 | 0.0131 | 0.0144 | 0.0135 | 0.0142 | 0.0137 | 0.0142 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.
*Sum of lagged coefficients of money constrained to equal one.

TABLE 8

$$\text{ERROR CRITERIA: } {}_g(Y)_t = \beta_0 + \sum_{i=1}^{4} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{4} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{4} \beta_{3i\,g}(M)_{t-i}$$

(RESTRICTED)*

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.1703 | 0.1927 | 0.1992 | 0.1802 | 0.2266 | 0.2011 | 0.2115 | 0.1933 |
| Adj $R^2$ | 0.0910 | 0.1155 | 0.1226 | 0.1018 | 0.1513 | 0.1233 | 0.1347 | 0.1147 |
| FPE | 0.0136 | 0.0132 | 0.0131 | 0.0134 | 0.0128 | 0.0132 | 0.0131 | 0.0134 |
| LOG AIC | -8.9071 | -8.9345 | -8.9425 | -8.9191 | -8.9625 | -8.9300 | -8.9431 | -8.9202 |
| LOG SC | -8.6384 | -8.6658 | -8.6738 | -8.6503 | -8.6910 | -8.6585 | -8.6715 | -8.6487 |
| GCV | 0.0137 | 0.0133 | 0.0132 | 0.0135 | 0.0129 | 0.0134 | 0.0132 | 0.0135 |
| HQ | 0.0151 | 0.0147 | 0.0146 | 0.0149 | 0.0143 | 0.0148 | 0.0146 | 0.0149 |
| RICE | 0.0138 | 0.0134 | 0.0133 | 0.0137 | 0.0131 | 0.0135 | 0.0133 | 0.0136 |
| SHIBATA | 0.0133 | 0.0130 | 0.0129 | 0.0132 | 0.0126 | 0.0130 | 0.0129 | 0.0131 |
| SC | 0.0177 | 0.0172 | 0.0171 | 0.0175 | 0.0168 | 0.0174 | 0.0171 | 0.0175 |
| AIC | 0.0135 | 0.0132 | 0.0131 | 0.0134 | 0.0128 | 0.0132 | 0.0131 | 0.0134 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.

*Sum of lagged coefficients of money constrained to equal one.

TABLE 9

$$\text{ERROR CRITERIA: } {}_{g}(Y)_t = \beta_0 + \sum_{i=1}^{6} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{6} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{6} \beta_{3i\,g}(M)_{t-i}$$

(UNRESTRICTED)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.2027 | 0.1996 | 0.2080 | 0.2530 | 0.2459 | 0.2605 | 0.2294 | 0.2428 |
| Adj $R^2$ | 0.1173 | 0.1138 | 0.1231 | 0.1744 | 0.1665 | 0.1827 | 0.1483 | 0.1630 |
| FPE | 0.0134 | 0.0135 | 0.0126 | 0.0124 | 0.0125 | 0.0123 | 0.0128 | 0.0126 |
| LOG AIC | -8.9160 | -8.9121 | -8.9827 | -8.9963 | -8.9869 | -9.0065 | -8.9652 | -8.9827 |
| LOG SC | -8.6219 | -8.6180 | -8.6916 | -8.7052 | -8.6957 | -8.7154 | -8.6741 | -8.6916 |
| GCV | 0.0136 | 0.0136 | 0.0127 | 0.0125 | 0.0126 | 0.0124 | 0.0129 | 0.0127 |
| HQ | 0.0151 | 0.0152 | 0.0141 | 0.0139 | 0.0141 | 0.0138 | 0.0144 | 0.0141 |
| RICE | 0.0138 | 0.0138 | 0.0129 | 0.0127 | 0.0128 | 0.0126 | 0.0131 | 0.0129 |
| SHIBATA | 0.0132 | 0.0132 | 0.0123 | 0.0122 | 0.0123 | 0.0120 | 0.0125 | 0.0123 |
| SC | 0.0180 | 0.0181 | 0.0168 | 0.0166 | 0.0167 | 0.0164 | 0.0171 | 0.0168 |
| AIC | 0.0134 | 0.0135 | 0.0126 | 0.0124 | 0.0125 | 0.0123 | 0.0128 | 0.0126 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.

# TABLE 10

$$\text{ERROR CRITERIA: } {}_g(Y)_t = \beta_0 + \sum_{i=1}^{6} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{6} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{6} \beta_{3i\,g}(M)_{t-i}$$

(RESTRICTED)*

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.1703 | 0.1927 | 0.1992 | 0.1802 | 0.2266 | 0.2011 | 0.2115 | 0.1933 |
| Adj $R^2$ | 0.0910 | 0.1155 | 0.1226 | 0.1018 | 0.1513 | 0.1233 | 0.1347 | 0.1147 |
| FPE | 0.0136 | 0.0132 | 0.0131 | 0.0134 | 0.0128 | 0.0132 | 0.0131 | 0.0134 |
| LOG AIC | -8.9071 | -8.9345 | -8.9425 | -8.9191 | -8.9625 | -8.9300 | -8.9431 | -8.9202 |
| LOG SC | -8.6384 | -8.6658 | -8.6738 | -8.6503 | -8.6910 | -8.6585 | -8.6715 | -8.6487 |
| GCV | 0.0137 | 0.0133 | 0.0132 | 0.0135 | 0.0129 | 0.0134 | 0.0132 | 0.0135 |
| HQ | 0.0151 | 0.0147 | 0.0146 | 0.0149 | 0.0143 | 0.0148 | 0.0146 | 0.0149 |
| RICE | 0.0138 | 0.0134 | 0.0133 | 0.0137 | 0.0131 | 0.0135 | 0.0133 | 0.0137 |
| SHIBATA | 0.0133 | 0.0130 | 0.0129 | 0.0132 | 0.0126 | 0.0130 | 0.0129 | 0.0131 |
| SC | 0.0177 | 0.0172 | 0.0171 | 0.0175 | 0.0168 | 0.0174 | 0.0171 | 0.0175 |
| AIC | 0.0135 | 0.0132 | 0.0131 | 0.0134 | 0.0128 | 0.0132 | 0.0131 | 0.0134 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.
*Sum of lagged coefficients of money constrained to equal one.

TABLE 11

$$\text{ERROR CRITERIA: } {}_{g}(Y)_{t} = \beta_{0} + \sum_{i=1}^{8} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{8} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{8} \beta_{3i\,g}(M)_{t-i}$$

(UNRESTRICTED)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.2795 | 0.2843 | 0.2975 | 0.2841 | 0.3090 | 0.2813 | 0.2856 | 0.2906 |
| Adj $R^2$ | 0.2009 | 0.2062 | 0.2209 | 0.2060 | 0.2336 | 0.2029 | 0.2077 | 0.2132 |
| FPE | 0.0121 | 0.0120 | 0.0118 | 0.0120 | 0.0116 | 0.0121 | 0.0120 | 0.0119 |
| LOG AIC | -9.0188 | -9.0255 | -9.0441 | -9.0252 | -9.0606 | -9.0213 | -9.0273 | -9.0343 |
| LOG SC | -8.7215 | -8.7282 | -8.7469 | -8.7280 | -8.7634 | -8.7241 | -8.7301 | -8.7371 |
| GCV | 0.0123 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0122 | 0.0121 |
| HQ | 0.0137 | 0.0136 | 0.0133 | 0.0136 | 0.0131 | 0.0136 | 0.0135 | 0.0135 |
| RICE | 0.0124 | 0.0123 | 0.0121 | 0.0123 | 0.0119 | 0.0124 | 0.0123 | 0.0122 |
| SHIBATA | 0.0119 | 0.0118 | 0.0116 | 0.0118 | 0.0114 | 0.0118 | 0.0118 | 0.0117 |
| SC | 0.0163 | 0.0162 | 0.0159 | 0.0162 | 0.0156 | 0.0163 | 0.0162 | 0.0161 |
| AIC | 0.0121 | 0.0120 | 0.0118 | 0.0120 | 0.0116 | 0.0121 | 0.0120 | 0.0119 |

Note: For convenience all the statistics, except log AIC and log SC. are multiplied by 100.

TABLE 12

$$\text{ERROR CRITERIA: } {}_g(Y)_t = \beta_0 + \sum_{i=1}^{8} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{8} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{8} \beta_{3i\,g}(M)_{t-i}$$

(RESTRICTED)*

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Criteria | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| $R^2$ | 0.2189 | 0.2302 | 0.3302 | 0.2750 | 0.3502 | 0.2880 | 0.2903 | 0.2515 |
| Adj $R^2$ | 0.1094 | 0.1223 | 0.2363 | 0.1734 | 0.2591 | 0.1882 | 0.1908 | 0.1466 |
| FPE | 0.0138 | 0.0136 | 0.0118 | 0.0128 | 0.0115 | 0.0126 | 0.0125 | 0.0132 |
| LOG AIC | -8.8892 | -8.9038 | -9.0430 | -8.9638 | -9.0733 | -8.9819 | -8.9852 | -8.9319 |
| LOG SC | -8.5234 | -8.5380 | -8.6772 | -8.5980 | -8.7075 | -8.6161 | -8.6194 | -8.5661 |
| GCV | 0.0140 | 0.0138 | 0.0120 | 0.0130 | 0.0117 | 0.0128 | 0.0128 | 0.0135 |
| HQ | 0.0160 | 0.0158 | 0.0137 | 0.0148 | 0.0133 | 0.0146 | 0.0145 | 0.0153 |
| RICE | 0.0144 | 0.0142 | 0.0123 | 0.0133 | 0.0120 | 0.0131 | 0.0131 | 0.0138 |
| SHIBATA | 0.0134 | 0.0132 | 0.0115 | 0.0124 | 0.0111 | 0.0122 | 0.0122 | 0.0128 |
| SC | 0.0199 | 0.0196 | 0.0170 | 0.0184 | 0.0165 | 0.0181 | 0.0181 | 0.0190 |
| AIC | 0.0138 | 0.0136 | 0.0118 | 0.0128 | 0.0115 | 0.0126 | 0.0125 | 0.0132 |

Note: For convenience all the statistics, except log AIC and log SC, are multiplied by 100.

*Sum of lagged coefficients of money constrained to equal one.

### 5.2 <u>Forecast Errors Tests</u>

To compare the predictive power of SS money versus DI money in the context of the reduced-form equation, the sample period 1960.2 to 1974.2 is used to forecast the period 1974.3 to 1992.4. The estimated model is one that includes eight lags of dependent and independent variables as regressors since this is the model that emerged as the best model in the error criteria tests.

#### Test Results

A summary of fit and forecast error statistics are presented in tables 13 and 14. The forecast error statistics give an indication of the tracking characteristics of a given monetary variable. The results shown in Tables 13 and 14 are mixed. For example, the findings presented in Table 13, unrestricted estimation procedure, indicate DI aggregates to have better tracking characteristics over their counterpart SSs on the basis of Mean Error criteria, while the reverse is true on the basis of Theil Inequality Coefficient U. On the other hand, Mean Absolute Error and Root Mean Square Error criteria favor neither DI nor SS - the scores are essentially equivalent.

Similar mixed results are also indicated in table 14, the restricted estimation procedure. While SS aggregates show a marked improvement in their tracking ability over the unrestricted estimation procedure, all the aggregates are favored on the basis of Root Mean Square Error criteria, while DIs are favored on the basis of Mean Error criteria. Also, except for M1, SS aggregates show a better performance than DI on the basis of Mean Absolute Error and Theil's criteria.

General conclusions to be observed here is that neither DI nor SS aggregates show superior tracking ability. The results are mixed at best.

TABLE 13

$$\text{FORECAST ERRORS: } {}_g(Y)_t = \beta_0 + \sum_{i=1}^{8} \beta_{1ig}(Y)_{t-i} + \sum_{i=1}^{8} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{8} \beta_{3i\,g}(M)_{t-i}$$

(UNRESTRICTED)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Summary Statistics | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| Mean error | -0.002 | -0.001 | -0.004 | -0.003 | -0.003 | -0.003 | -0.004 | -0.003 |
| Mean absolute error | 0.009 | 0.010 | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 |
| Root mean square error | 0.011 | 0.011 | 0.011 | 0.011 | 0.011 | 0.011 | 0.011 | 0.011 |
| Theil inequality coeff U | 0.752 | 0.764 | 0.745 | 0.779 | 0.754 | 0.778 | 0.766 | 0.798 |
| Fraction of error due to: | | | | | | | | |
| Bias | 0.019 | 0.011 | 0.110 | 0.056 | 0.095 | 0.051 | 0.105 | 0.056 |
| Variance | 0.058 | 0.062 | 0.053 | 0.012 | 0.052 | 0.032 | 0.101 | 0.135 |
| Covariance | 0.923 | 0.928 | 0.836 | 0.932 | 0.854 | 0.917 | 0.794 | 0.809 |

Note: The estimation procedure assumes the estimated coefficients of the regressors are third-degree polynomial-distributed lags with zero restrictions at the end.

TABLE 14

FORECAST ERRORS: $_g(Y)_t = \beta_0 + \sum_{i=1}^{8} \beta_{1i\,g}(Y)_{t-i} + \sum_{i=1}^{8} \beta_{2i\,g}(F)_{t-i} + \sum_{i=1}^{8} \beta_{3i\,g}(M)_{t-i}$

(RESTRICTED)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Summary Statistics | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| Mean error | -0.009 | -0.008 | -0.006 | -0.004 | -0.005 | -0.003 | -0.007 | -0.004 |
| Mean absolute error | 0.013 | 0.012 | 0.010 | 0.012 | 0.010 | 0.011 | 0.011 | 0.012 |
| Root mean square error | 0.016 | 0.015 | 0.013 | 0.015 | 0.013 | 0.014 | 0.013 | 0.014 |
| Theil inequality coeff U | 1.094 | 0.997 | 0.911 | 1.006 | 0.850 | 0.934 | 0.911 | 0.940 |
| Fraction of error due to: | | | | | | | | |
| Bias | 0.317 | 0.307 | 0.179 | 0.063 | 0.156 | 0.035 | 0.265 | 0.076 |
| Variance | 0.002 | 0.055 | 0.006 | 0.019 | 0.009 | 0.000 | 0.017 | 0.018 |
| Covariance | 0.680 | 0.639 | 0.815 | 0.919 | 0.835 | 0.964 | 0.717 | 0.907 |

Note: Note: The estimation procedure assumes the estimated coefficients of the regressors are third-degree polynomial-distributed lags with zero restrictions at the end. In addition, superneutrality is assumed to hold, that is, $\sum_i^8 \beta_{3i} = 1$.

### 5.3 Tests on Restrictions

Steady-state superneutrality implies that the sum of lag coefficients of money are constrained to equal to one, that is $\sum_i^8 \beta_{3i} = 1$. It is then appropriate to test whether the restrictions imposed on the model are valid, or equivalently, whether or not the restrictions contradict the unrestricted model. Since the unrestricted model is, by definition, 'least squares', the imposition of restrictions must lead to some loss of fit. The test then is designed to determine whether the loss of fit is merely due to sampling errors or whether it is so large as to cast doubt on the validity of the restrictions.

To test the validity of restrictions, two test statistics will be used: the Wald - $\chi^2$ and Lagrange Multiplier (LM) asymptotic tests. The Wald - $\chi^2$ statistic is determined in the Shazam program that I have used for this study. A simplified LM statistic is computed as follows (see Harvey (1993), p. 66):

(37) $\mathrm{LM} = T.\, \mathrm{R}^2,$

where $T$ is the number of observations and $\mathrm{R}^2$ is the coefficient of determination. LM statistic is distributed as $\chi^2_{(m)}$ under the null hypothesis, and '$m$' is the number of restrictions. The Wald - $\chi^2$ likelihood statistic is asymptotically distributed as $\chi 2_{(q)}$ under the null hypothesis, where '$q$' is the number of linear hypothesis, in this case $q$=1. Large values of the statistics will imply substantial differences between the most likely values for the estimates suggested by the sample data and the values suggested by the null hypothesis (Ho: $\sum_i^8 \alpha_{3i} = 1$), and we will thus reject the null hypothesis.

<u>Test Results</u>

The computed Wald and LM statistics are presented in Table 24 below. The LM statistics indicate the validity of superneutrality condition for both DI and SS aggregates--all the values are statistically significant at .10 level or better, while the Wald results show only narrow measures M1 and D1 support the neutrality condition. Therefore, we can conclude that, on the basis of Wald and LM statistics, DI aggregates do not explain the superneutrality condition any better than SS aggregates.

TABLE 15

RESTRICTION TESTS: $g(Y)_t = \beta_0 + \sum_{i=1}^{8} \beta_{1i}g(Y)_{t-i} + \sum_{i=1}^{8} \beta_{2i}g(F)_{t-i} + \sum_{i=1}^{8} \beta_{3i}g(M)_{t-i}$

(RESTRICTED)

| Aggregate | Wald- $\chi^2$ | LM |
|---|---|---|
| M1 | 39.071*** | 22.730* |
| M2 | 12.521 | 27.072** |
| M3 | 8.053 | 24.452** |
| L | 20.760 | 25.136** |
| D1 | 32.508*** | 24.403** |
| D2 | 17.406 | 25.752** |
| D3 | 15.895 | 27.044** |
| DL | 17.710 | 21.954* |

Notes: For $m$=14 d.f.:

***$p(\chi 2 > 29.14) = 0.01$

**$p(\chi 2 > 23.68) = 0.05$

*$p(\chi 2 > 21.06) = 0.10$.

$\sum_{i}^{8} \alpha_{3i} = 1.$

### 5.4 Non-Nested Davidson-Mackinnon Cox and J Tests

While, the error minimizing test statistics performed in section 5.2 are appealing in comparing the goodness of fit of competing models, especially in problems where the specifications are based primarily on pragmatic grounds, they do not answer the question as to which of the models is better in a direct comparison with each other, that is, whether one model should be rejected in favor of the other. Such a selection can be made through non-nested hypothesis testing. The test statistics commonly used to test non-nested hypothesis are Davidson-Mckinnon Cox and J tests: see Harvey (1990, pp. 148-149), Charemza and Deadman (1992, pp. 289-292) and Greene (1993, pp. 224-225). The Cox test aims to identify the correct set of regressors, while the J test is a variance encompassing test.

Since, there is no concern with the relative importance of monetary versus fiscal variables, the fiscal and exogenous variables are held constant. Therefore, the two competing linear models to be tested are:

(38) H1: $y_t = \alpha_0 + \alpha_1 m_{DI\,t} + \varepsilon_{at}$, $\qquad \varepsilon_{at} \sim NID(0, \sigma^2)$

(39) H2: $y_t = \beta_0 + \beta_1 m_{SS\,t} + \varepsilon_{bt}$, $\qquad \varepsilon_{bt} \sim NID(0, \sigma^2)$

Where $y_t$ = natural log of GNP

$m_{DIt}$ = natural log of DI

$m_{SS\,t}$ = natural log of SS.

H1 is the null hypothesis that DI are the true regressors in the income model, while H2: is the alternative hypothesis that SS are indeed the true regressors. The roles of the null and the alternative can easily be reversed. Both H1 and H2 are said to be non-nested since neither is a special case of the other or neither model can be obtained from the other.

The Cox procedure for conducting the non-nested hypothesis requires computation of a set of least squares residuals (Greene (1990), pp. 224-225):

(40) $C1 = c12/(vc12)^{1/2}$ and,

(41) $C2 = c21/(vc21)^{1/2}$,

where, $vc12$ = variance in the regression of $y_t$ on $m_{DIt}$,

$vc21$ = variance in the regression of $y_t$ on $m_{SSt}$,

$c12 = n/2 \ln(s2/s21)$ and $c21 = n/2 \ln(s1/s12)$,

where, $s1$ = mean squared residuals in the regression of $y_t$ on $m_{DIt}$,

$s2$ = mean squared residuals in the regression of $y_t$ on $m_{SSt}$,

$s12 = s1 + (1/n)\,\mathbf{b1'X'j2Xb1}$ and $s21 = s2 + (1/n)\,\mathbf{b2'Z'j1Zb2}$,

where, $\mathbf{b1} = \mathbf{(X'X)^{-1}X'y}$ estimated coefficients of DI,

$\mathbf{b2} = \mathbf{(Z'Z)^{-1}Z'y}$ estimated coefficients of SS,

$\mathbf{j2} = \mathbf{I - Z(Z'Z)^{-1}Z'}$ and $\mathbf{j1} = \mathbf{I - X(X'X)^{-1}X'}$

$\mathbf{j2Xb1}$ = residuals in a regression of $y_t$ on DI,

$\mathbf{j1Zb2}$ = residuals in a regression of $y_t$ on SS,

$\mathbf{b1'X'j2Xb1}$ = sum of squared residuals in the regression of $\mathbf{Xb1}$ on SS.

The decision rule is to accept H1: if $|C1| < |C2|$ and conclude that DI are the preferred set of regressors on GNP, otherwise reject H1.[9]

[9]Harvey (1990, p 181) shows that the statistic C1 is asymptotically N(0,1) when H1 is true, and a significant negative value implies a rejection of H1 against H2:. Similarly for H2.

The Davidson-Mckinnon J variance encompassing test consists of first estimating the two competing models, H1and H2, separately to obtain least squares estimates:

$$\bar{y}_1 = \alpha_{t-1}\, m_{DIt} \qquad \bar{y}_2 = \beta_{t-1}\, m_{SSt}.$$

The next step is to run two more OLS with the predicted values $\bar{y}_2$ and $\bar{y}_1$ included in H1 and H2 equations respectively. Thus

(42) HA: $y_t = \alpha_0 + \alpha_1 m_{DIt} + \bar{\alpha}_2\, \bar{y}_{2\,1t} + \varepsilon_{1t}$,

(43) HB: $y_t = \beta_0 + \beta_1 m_{SSt} + \bar{\beta}_2\, \bar{y}_{1\,2t} + \varepsilon_{2t}$.

The test that HA encompasses HB for variance simply consists of testing whether the estimated coefficient of $\bar{y}_2$ is significantly different from zero. Similarly, the test that HB encompasses HA for variance is to test the alternative hypothesis that the estimated coefficient of $\bar{y}_1$=0.

<u>Test Results</u>

Tables 16 through 23 below present results of non-nested hypotheses testing on money-income relationships. Comparisons made on the basis of the Cox test overwhelmingly favors Divisia money over their SS counterparts. As indicated in tables 16 through 19, except for M1, $|C1| < |C2|$, also, the values for C2 show significantly large and negative values which imply a rejection of H2 (Harvey 1990, p. 181).

The results obtained using first differenced natural logs shown in Tables 20 through 23 indicate a strong dominance of DI over SS. Once again, only in the narrow money specification is SS favored over DI. These findings confirm what has readily been observed in DI Studies: DI outperform SS more at broader aggregation levels than at lower levels.

The J-test results are not as clear-cut as the Cox test results. In the natural logs case, the tests show the coefficients of $\bar{y}_2$ not to be significantly different from zero in three situations: when D3 is matched against M2 and M3; and when DL is matched against M3. The coefficient of $\bar{y}_2$ is significantly not different from zero also in three cases: when D1 and D3 is matched against L; and when D2 is matched against M3. On the other hand, the J test favors SS in cases where M1 is matched against DL, that is the coefficient of $\bar{y}_1$ is significantly not different from zero.

The non-nested results obtained above seem to be somewhat in agreement to those obtained in Belongia and Crystal (1991, p. 500). Holding the fiscal variables constant and estimating a St. Louis Equation with contemporaneous plus two and four lags on the regressors, they find a strong dominance of both D1 and DL over the standard aggregates. In addition, their Akaike criterion test results are unambiguously in favor of Divisia money, but, their J-test results in regard to M3 and M3 are inconclusive.

# TABLE 16

## NON-NESTED TESTS: D1 v. SS MONEY (LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D1 | v. M1 | D1 | v. M2 | D1 | v. M3 | D1 | v. L |
| $C_i$ (i=1,2) | -13.780 | 13.755 | 10.887 | -10.975 | 10.245 | -10.342 | 10.918 | -10.944 |
| $\gamma_i$ (i=1,2) | -3.016 | 4.006 | 1.433 | -0.438 | 1.306 | -0.310 | 1.593 | -0.598 |
| | (-9.63) | (12.82) | (26.32) | (-7.99) | (25.15) | (-5.92) | (20.48) | (-7.64) |
| DW | 0.02 | 0.02 | 0.024 | 0.064 | 0.024 | 0.054 | 0.024 | 0.031 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test, and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42 ) and (43).

# TABLE 17

## NON-NESTED TESTS: D2 v. SS MONEY (LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D2 | v. M1 | D2 | v. M2 | D2 | v. M3 | D2 | v. L |
| $C_i$ (i=1,2) | -6.513 | 6.358 | 5.236 | -5.310 | 5.298 | -5.377 | 2.781 | -2.873 |
| $\gamma_i$ (l=1,2) | 0.190 | 0.813 | 1.032 | -0.32 | 1.014 | -1.014 | 0.731 | 0.271 |
| | (3.36) | (14.44) | (13.19) | (-0.41) | (14.16) | (-0.19) | (10.04) | (3.71) |
| DW | 0.044 | 0.020 | 0.044 | 0.060 | 0.044 | 0.054 | 0.044 | 0.032 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test, and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

# TABLE 18

## NON-NESTED TESTS: D3 v. SS MONEY (LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D3 | v. M1 | D3 | v. M2 | D3 | v. M3 | D3 | v. L |
| $C_i$ (i=1,2) | -6.734 | 6.575 | 4.298 | -4.375 | 4.552 | -4.629 | 2.064 | -2.158 |
| $\gamma_i$ (I=1,2) | 0.190 | 0.184 | 0.927 | 0.073 | 0.946 | 0.054 | 0.670 | 0.333 |
| | (3.63) | (15.71) | (12.18) | (0.95) | (13.12) | (0.75) | (9.69) | (4.81) |
| DW | 0.042 | 0.020 | 0.042 | 0.064 | 0.042 | 0.050 | 0.042 | 0.031 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test, and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

# TABLE 19

## NON-NESTED TESTS: DL v. SS MONEY (LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | DL | v. M1 | DL | v. M2 | DL | v. M3 | DL | v. L |
| $C_i$ (i=1,2) | -1.51 | 1.447 | 4.834 | -4.893 | 4.868 | -4.932 | 2.097 | -2.171 |
| $\gamma_i$ (i=1,2) | 0.038 | 0.963 | 1.117 | -0.117 | 1.078 | -0.078 | 0.720 | 0.281 |
| | (0.60) | (15.71) | (11.43) | (-1.20) | (12.35) | (0.89) | (7.73) | (3.01) |
| DW | 0.040 | 0.020 | 0.040 | 0.064 | 0.040 | 0.054 | 0.040 | 0.030 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test, and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

# TABLE 20

## NON-NESTED TESTS: D1 v. SS MONEY (1ST DIFF LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D1 | v. M1 | D1 | v. M2 | D1 | v. M3 | D1 | v. L |
| $C_i$ (i=1,2) | -0.507 | 0.495 | 0.758 | -0.780 | 1.211 | -1.235 | 1.582 | -1.605 |
| $\gamma_i$ (i=1,2) | 0.164 | 0.836 | 0.763 | 0.273 | 0.879 | 0.121 | 1.026 | -0.026 |
| | (0.58) | (2.95) | (5.93) | (1.84) | (1.59) | (1.15) | (9.60) | (-0.24) |
| DW | 1.572 | 1.429 | 1.572 | 1.635 | 1.572 | 1.673 | 1.572 | 1.728 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test, and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

# TABLE 21

## NON-NESTED TESTS: D2 v. SS MONEY (1ST DIFF LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D2 | v. M1 | D2 | v. M2 | D2 | v. M3 | D2 | v. L |
| $C_i$ (i=1,2) | -0.406 | 0.385 | 0.838 | -0.855 | 1.202 | -1.224 | 1.427 | -1.451 |
| $\gamma_i$ (I=1,2) | 0.354 | 0.646 | 0.873 | 0.127 | 0.906 | 0.094 | 0.960 | 0.405 |
| | (2.34) | (4.28) | (5.21) | (0.76) | (7.89) | (0.82) | (9.28) | (0.39) |
| DW | 1.336 | 1.429 | 1.336 | 1.640 | 1.336 | 1.673 | 1.336 | 1.728 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test. and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

TABLE 22

NON-NESTED TESTS: D3 v. SS MONEY (1ST DIFF LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | D3 | v. M1 | D3 | v. M2 | D3 | v. M3 | D3 | v. L |
| $C_i$ (i=1,2) | -0.757 | 0.735 | 0.329 | -0.348 | 0.958 | -0.978 | 1.181 | -1.203 |
| $\gamma_i$ (i=1,2) | 0.241<br>(1.78) | 0.759<br>(5.57) | 0.637<br>(4.23) | 0.363<br>(2.42) | 0.873<br>(5.21) | 0.127<br>(0.76) | 0.915<br>(1.43) | 0.085<br>(0.76) |
| DW | 1.380 | 1.429 | 1.380 | 1.635 | 1.380 | 1.670 | 1.380 | 1.728 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test, and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

TABLE 23

NON-NESTED TESTS: DL v. SS MONEY (1ST DIFF LN)

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | DL | v. M1 | DL | v. M2 | DL | v. M3 | DL | v. L |
| $C_i$ (i=1,2) | -1.175 | 1.154 | -0.094 | 0.075 | 0.536 | -0.557 | 0.949 | -0.968 |
| $\gamma_i$ (i=1,2) | 0.077<br>(0.56) | 0.923<br>(6.76) | 0.467<br>(3.25) | 0.533<br>(3.71) | 0.670<br>(5.69) | 0.301<br>(2.45) | 0.887<br>(6.81) | 0.113<br>(0.87) |
| DW | 1.468 | 1.429 | 1.468 | 1.635 | 1.468 | 1.673 | 1.468 | 1.730 |

Note: t-ratios in parenthesis. $C_i$ is the Cox test. and $\gamma_i$ is the J-test of estimated coefficients of $\bar{y}_2$ and $\bar{y}_1$ in equations (42) and (43).

# CHAPTER 6

# MONEY DEMAND

## 6.1 Money Demand Function

The investigation of causal relationships between economic variables is the bread and butter of econometric analysis. Goldfeld and Sichel (1990) have argued that , "...the demand for money is a critical component in the formulation of monetary policy and a stable demand function for money has long been perceived as a prerequisite for the use of monetary aggregates in the conduct of policy" (p. 300). The tests in the following three sections are designed to compare the performance of Divisia and traditional monetary aggregates in the context of a partial adjustment money demand function. The estimated double-log reduced form equation to be estimated is of the form:

$$\text{(33)} \quad \log (M_t/p^*_t) = \beta_0 + \beta_1 \log (Y_t/p^*_t) + \beta_2 \log (M_{t-1}/p^*_t) + \beta_3 \log\pi_t + \beta_4\, log TB_t + \beta_5 \log RCP_t,$$

where $Y_t$ = nominal GNP

$p^*_t$ = GNP price deflator

$M_t$ = Divisia or SS monetary aggregate

$\pi_t = (P_t/P_{t-1})$ the rate of inflation associated with $p^*_t$

$RCP_t$ = four-to-six months prime commercial paper rate

$TB_t$ = Three-month Treasury bill rate

The inclusion of $\pi_t$ is meant to encompass the real partial adjustment ($\beta_3 = 0$) or the nominal partial adjustment model ($\beta_3 = - \beta_2$) (see Goldfeld and Sichel (1990), p. 302).

## 6.1 Money Demand Parameter Estimates

The results of estimating the standard equation (44), for several sample periods between 1960.1 to 1992.4 are reported in Tables 23 through 26. To correct for first-order serial correlation normally found in the residuals of (44), Maximum Likelihood Estimation (MLE) by Cochrane-Orcutt procedure is used.

For the period before 1974, the estimated money demand model seems to behave rather well for all the aggregates except L and DL. The results are generally sensible with the correct signs on the estimated coefficients and except for the lagged dependent variable, $RCP_t$, and $TB_t$ all the coefficients are significantly greater than zero.

For the period after 1974, the estimated money demand functions seem to deteriorate. For all the aggregates, the coefficient of the lagged dependent variable is large and essentially unity with broader DIs which suggests a mispecified partial adjustment model. Also, the income coefficients appear to be small and in some cases negative while the coefficient of Commercial Paper rate (RCP) shows a wrong sign for all the aggregates. These results may suggest a structural break-down of the money demand function in early 1970s. As for the entire period of estimation 1960-1974, for all the aggregates, only GNP's and inflation's coefficients are significantly different from zero. Nevertheless, the important point to be made here is that none of the DI aggregates shows any improvement on the money demand function after 1974 and estimating the money demand function using DIs appears to substantially reduce the magnitude of the income coefficient.

TABLE 24

MONEY DEMAND ESTIMATES FOR SS M1 AND DIVISIA M1

| Aggregate and Period of Fit | *c* | Lagged Dep var | Real GNP | $\pi_t$ | $RCP_t$ | $TB_t$ | $\rho$ | $R^2$ | SEE |
|---|---|---|---|---|---|---|---|---|---|
| M1: | | | | | | | | | |
| 1960.1-1974.2 | 0.6860 | 0.0012 | 0.8676 | -0.538 | -0.000 | 0.0023 | 0.7574 | 0.83 | 0.005 |
| | (0.179) | (0.005) | (0.012) | (0.194) | (0.012 | (0.012) | (0.086) | | |
| 1960.1-1979.3 | 1.351 | 0.001 | -0.003 | -0.616 | 0.007 | 0.000 | 0.978 | 0.94 | 0.006 |
| | (0.406) | (0.005) | (0.05) | (0.16) | (0.011) | (0.012) | (0.023) | | |
| 1960.1-1992.4 | -0.543 | 0.004 | -0.003 | -0.616 | 0.007 | 0.000 | 0.978 | 0.94 | 0.006 |
| | (0.599) | (0.001) | (0.05) | (0.16) | (0.011) | (0.012) | (0.023) | | |
| 1974.2-1992.4 | -0.222 | 0.946 | 0.038 | -0.271 | 0.052 | -0.523 | 0.225 | 0.99 | 0.009 |
| | (0.169) | (0.023) | (0.024) | (0.305) | (0.020) | (0.02) | (0.113) | | |
| D1: | | | | | | | | | |
| 1960.1-1974.2 | 0.260 | 0.000 | 0.143 | -0.569 | -0.010 | 0.007 | 0.839 | 0.93 | 0.006 |
| | (0.224) | (0.005) | (0.029) | (0.19) | (0.01) | (0.012) | (0.071) | | |
| 1960.1-1979.3 | 1.105 | 0.002 | 0.032 | -0.490 | -0.003 | 0.010 | 0.959 | 0.91 | 0.006 |
| | (0.339) | (0.005) | (0.043) | (0.16) | (0.01) | (0.012) | (0.032) | | |
| 1960.1-1992.4 | -0.652 | 0.002 | 0.264 | -0.495 | -0.002 | 0.948 | 0.993 | 0.99 | 0.009 |
| | (0.557) | (0.007) | (0.068) | (0.20) | (0.01) | (0.013) | (0.010) | | |
| 1974.2-1992.4 | -0.292 | 0.934 | 0.049 | -0.102 | 0.059 | -0.645 | -0.162 | 0.99 | 0.008 |
| | (0.124) | (0.019) | (0.018) | (0.22) | (0.015) | (0.02) | (0.11) | | |

Note: Standard errors are in parentheses.

TABLE 25

MONEY DEMAND ESTIMATES FOR SS M2 AND DIVISIA M2

| Aggregate and Period of Fit | $c$ | Lagged Dep var | Real GNP | $\pi_t$ | $RCP_t$ | $TB_t$ | $\rho$ | $R^2$ | SEE |
|---|---|---|---|---|---|---|---|---|---|
| M2: | | | | | | | | | |
| 1960.1-1974.2 | -3.878 | 0.017 | 0.690 | -0.395 | -0.005 | 0.022 | 0.907 | 0.99 | 0.009 |
| | (0.45) | (0.007) | (0.058) | (0.28) | (0.02) | (0.018) | (0.055) | | |
| 1960.1-1979.3 | -3.873 | 0.018 | 0.691 | -0.376 | -0.013 | 0.024 | 0.898 | 0.99 | 0.008 |
| | (0.32) | (0.007) | (0.042) | (0.22) | (0.02) | (0.016) | (0.049) | | |
| 1960.1-1992.4 | -2.722 | 0.013 | 0.560 | -0.460 | -0.014 | 0.007 | 0.998 | 0.99 | 0.009 |
| | (0.62) | (0.007) | (0.075) | (0.20) | (0.01) | (0.013) | (0.005) | | |
| 1974.2-1992.4 | -0.719 | 0.872 | 0.120 | -0.303 | 0.029 | -0.031 | 0.275 | 0.99 | 0.007 |
| | (0.34) | (0.042) | (0.051) | (0.24) | (0.017) | (0.02) | (0.111) | | |
| D2: | | | | | | | | | |
| 1960.1-1974.2 | -1.033 | 0.007 | 0.314 | -0.453 | -0.007 | 0.013 | 0.926 | 0.98 | 0.007 |
| | (0.41) | (0.006) | (0.053) | (0.23) | (0.02) | (0.015) | (0.049) | | |
| 1960.1-1979.3 | -1.060 | 0.008 | 0.3163 | -0.379 | -0.002 | 0.012 | 0.910 | 0.98 | 0.007 |
| | (0.29) | (0.006) | (0.037) | (0.19) | (0.01) | (0.014) | (0.047) | | |
| 1960.1-1992.4 | -0.733 | 0.008 | 0.273 | -0.417 | -0.002 | 0.004 | 0.977 | 0.97 | 0.009 |
| | (0.46) | (0.008) | (0.057) | (0.21) | (0.01) | (0.014) | (0.019) | | |
| 1974.2-1992.4 | 0.190 | 1.008 | -0.021 | -0.123 | 0.019 | -0.249 | 0.307 | 0.97 | 0.009 |
| | (0.122) | (0.034) | (0.02) | (0.35) | (0.020) | (0.02) | (0.110) | | |

Note: Standard errors are in parentheses.

# TABLE 26

## MONEY DEMAND ESTIMATES OF SS M3 AND DIVISIA M3

| Aggregate and Period of Fit | $c$ | Lagged Dep var | Real GNP | $\pi_t$ | $RCP_t$ | $TB_t$ | $\rho$ | $R^2$ | SEE |
|---|---|---|---|---|---|---|---|---|---|
| M3: | | | | | | | | | |
| 1960.1-1974.2 | -4.838 | 0.020 | 0.816 | -0.281 | -0.004 | 0.025 | 0.903 | 0.99 | 0.011 |
| | (0.55) | (0.009) | (0.072) | (0.35) | (0.02) | (0.022) | (0.056) | | |
| 1960.1-1979.3 | -2.578 | 0.016 | 0.535 | -0.256 | -0.017 | 0.029 | 0.997 | 0.99 | 0.009 |
| | (0.74) | (0.008) | (0.092) | (0.24) | (0.02) | (0.018) | (0.009) | | |
| 1960.1-1992.4 | -2.895 | 0.014 | 0.594 | -0.335 | -0.022 | 0.011 | 0.999 | 0.99 | 0.010 |
| | (0.67) | (0.008) | (0.079) | (0.21) | (0.01) | (0.014) | (0.004) | | |
| 1974.2-1992.4 | 0.966 | 0.977 | -0.005 | -0.099 | 0.024 | -0.017 | 0.263 | 0.99 | 0.005 |
| | (0.243) | (0.025) | (0.04) | (0.19) | (0.012) | (0.01) | (0.111) | | |
| D3: | | | | | | | | | |
| 1960.1-1974.2 | -1.543 | 0.008 | 0.381 | -0.397 | -0.005 | 0.014 | 0.935 | 0.98 | 0.008 |
| | (0.48) | (0.007) | (0.062) | (0.26) | (0.02) | (0.017) | (0.047) | | |
| 1960.1-1979.3 | -1.429 | 0.009 | 0.366 | -0.303 | -0.004 | 0.1560 | 0.925 | 0.99 | 0.007 |
| | (0.32) | (0.006) | (0.041) | (0.19) | (0.01) | (0.014) | (0.043) | | |
| 1960.1-1992.4 | -0.953 | 0.008 | 0.305 | -0.372 | -0.003 | 0.005 | 0.975 | 0.98 | 0.009 |
| | (0.44) | (0.007) | (0.054) | (0.20) | (0.01) | (0.013) | (0.019) | | |
| 1974.2-1992.4 | 0.176 | 1.007 | -0.020 | -0.089 | 0.028 | -0.031 | 0.315 | 0.98 | 0.008 |
| | (0.114) | (0.033) | (0.02) | (0.32) | (0.018) | (0.02) | (0.110) | | |

Note: Standard errors are in parentheses.

TABLE 27

MONEY DEMAND ESTIMATES OF SS L AND DIVISIA L

| Aggregate and Period of Fit | $c$ | Lagged Dep var | Real GNP | $\pi_t$ | $RCP_t$ | $TB_t$ | $\rho$ | $R^2$ | SEE |
|---|---|---|---|---|---|---|---|---|---|
| L: | | | | | | | | | |
| 1960.1-1974.2 | -2.294 | 0.010 | 0.489 | -0.423 | 0.004 | 0.008 | 0.994 | 0.99 | 0.007 |
| | (0.65) | (0.006) | (0.083) | (0.23) | (0.015) | (0.015) | (0.014) | | |
| 1960.1-1979.3 | -1.875 | 0.010 | 0.441 | -0.389 | -0.008 | 0.014 | 0.998 | 0.99 | 0.007 |
| | (0.53) | (0.055) | (0.066) | (0.17) | (0.01) | (0.013) | (0.007) | | |
| 1960.1-1992.4 | -2.479 | 0.009 | 0.538 | -0.446 | -0.011 | 0.000 | 0.999 | 0.99 | 0.008 |
| | (0.58) | (0.006) | (0.067) | (0.18) | (0.01) | (0.012) | (0.003) | | |
| 1974.2-1992.4 | 0.050 | 0.976 | 0.001 | 0.021 | 0.022 | -0.016 | 0.455 | 0.99 | 0.005 |
| | (0.292) | (0.028) | (0.185) | (0.185) | (0.012) | (0.01) | (0.103) | | |
| DL: | | | | | | | | | |
| 1960.1-1974.2 | -8.855 | 0.005 | 0.291 | -0.460 | 0.002 | 0.002 | 0.901 | 0.97 | 0.006 |
| | (0.30) | (0.005) | (0.040) | (0.20) | (0.013) | (0.013) | (0.057) | | |
| 1960.1-1979.3 | -0.794 | 0.007 | 0.282 | -0.370 | 0.004 | 0.005 | 0.893 | 0.98 | 0.005 |
| | (0.21) | (0.005) | (0.027) | (0.15) | (0.011) | (0.011) | (0.050) | | |
| 1960.1-1992.4 | -0.703 | 0.005 | 0.270 | -0.440 | 0.006 | -0.004 | 0.977 | 0.98 | 0.008 |
| | (0.40) | (0.007) | (0.050) | (0.01) | (0.012) | (0.01) | (0.018) | | |
| 1974.2-1992.4 | 0.189 | 1.001 | -0.021 | -0.126 | 0.030 | -0.031 | 0.360 | 0.98 | 0.008 |
| | (0.120) | (0.033) | (0.02) | (0.30) | (0.018) | (0.02) | (0.108) | | |

Note: Standard errors are in parentheses.

### 6.3 Forecast Errors

In comparing the forecasting ability of DI and simple sum monetary aggregates the sample period 1960.1-1974.2 is used to forecast the period 1974.3-1992.4.

A summary of fit and forecast results are presented in table 27. The test results show that, while M3 has the best within-sample fit, the tracking characteristics of Divisia money demand equations, especially broader DIs, are superior than their counterpart SSs as evidenced by smaller forecast errors obtained when the equation is estimated using DIs.

The results, as indicated by Mean Absolute Error and Root Mean Square Errors are in agreement with those obtained in Barnett et al (1984, p. 1064), where they find the values of Root Mean Square Errors and Mean Errors for DIs to be lower than their sum counterparts at all levels of aggregation. Therefore, under forecasting ability tests, broader DI exhibit better tracking ability than their SS counterparts.

TABLE 28

FORECAST ERRORS: MONEY DEMAND EQUATION

| | Monetary Variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Summary Statistics | M1 | D1 | M2 | D2 | M3 | D3 | L | DL |
| Mean Error | -0.0006 | 0.7677 | 0.0815 | -0.0706 | 0.2424 | -0.0710 | 0.2849 | -0.0527 |
| Mean Absolute Error | 0.1096 | 0.0911 | 0.0856 | 0.0778 | 0.2424 | 0.7661 | 0.2849 | 0.0556 |
| Root Mean Square Error | 0.1224 | 0.1048 | 0.1111 | 0.0953 | 0.2690 | 0.0950 | 0.3200 | 0.0776 |
| Theil Inequality coeff U | 9.2960 | 8.7170 | 9.4900 | 7.9100 | 24.787 | 8.6360 | 28.920 | 7.2610 |
| Fraction of Error due to: | | | | | | | | |
| Bias | 0.0000 | 0.0054 | 0.5343 | 0.5496 | 0.8118 | 0.5584 | 0.7925 | 0.4614 |
| Variance | 0.9592 | 0.9391 | 0.4063 | 0.0413 | 0.1766 | 0.0125 | 0.2016 | 0.1115 |
| Covariance | 0.0407 | 0.0555 | 0.0593 | 0.4091 | 0.0116 | 0.4291 | 0.0059 | 0.4271 |

Note: All data are quarterly. The results use parameters estimated for the 1960.1-1974.2 sample period in forecasting from 1974.3-1992.4.

# CHAPTER 7

# RATIONAL EXPECTATIONS: UNANTICIPATED MONEY GROWTH, INCOME, AND PRICE LEVELS

## 7.1 Introduction

This chapter incorporates a linear rational expectations model to look at the empirical relationships between unanticipated money growth, income, interest rates, and prices. The distinction between the possible effects from unanticipated versus anticipated is a topic of much study (see for example, Lucas (1972), Sargent and Wallace (1976), Barro (1977,1978), Mishkin (1982), and Mishkin 1993)). The rational expectations hypothesis assumes that expectations can be modeled as optimal forecasts given all available information. Barro (1977) study the empirical relationship between unanticipated M1 money growth and unemployment in the U.S. from 1941 to 1973. He quantifies his hypothesis by structuring anticipated M1 growth as the amount that could have been predicted based on the historical relation between money growth and a few variables he found to have systematic effects on U.S. money growth - i.e. federal expenditures relative to normal, a lagged unemployment rate, and two lagged values of money growth. Barro's statistical tests confirm the underlying hypothesis that only unanticipated money movements affects the unemployment rate. This observation supports the basic policy premise emphasized in rational expectations hypothesis - the policy ineffectiveness proposition as in Sargent and Wallace (1976), which renders countercyclical stabilization policies irrelevant.

Barro's (1978) study extends his analysis of unanticipated money growth to output and the price level for 1941-1980 period in the U.S. His empirical results lend further support to his earlier findings but also indicate a strong evidence for the homogeneity postulate in his price equation - i.e. a one-to-one contemporaneous link between anticipated money and the price level.

The aim in this section is to construct a simple rational expectations model that compares the performance of DI and SS monetary aggregates in a rational expectations' environment. The process for testing the main hypothesis - that only unanticipated movements in money affect real variables - requires estimating a system of joint equations as outlined in Barro (1977, 1978). The initial equation to be estimated is a money growth equation using the actual money growth (anticipated) as the dependent variable and the relevant regressors that explain the movement of money growth. The key assumption, as perceived in a rational expectations hypothesis, is that the market is using all available information in the formation of expectations about money movements.

The expected values obtained in the money growth equation form the basis of the model. The difference, or residuals, between the anticipated and the expected values of money growth is the unanticipated money growth, the monetary innovations. It is these unanticipated money growth variables that are used as regressors in the subsequent output and price equations in testing the principal hypothesis. Tests of the neutrality condition involve adding current and lagged actual values in the price equation and testing the null hypothesis that their coefficients are equal to zero.

There are several modifications that will be made to Barro's model. First, while Barro (1977, 1978) uses only M1 money growth, this study will use the various SS and DI money growth aggregates to compare their differences in testing the principal hypothesis. Second, while Barro's study covers the period 1940-1980, data availability restricts the study period to 1960-1992. Third, since Barro's study encompasses the war years, some of the variables included in his original study - e.g. military draft - are no longer relevant and therefore are omitted. Therefore, an appropriate output equation similar to Lucas' aggregate supply equation (explained below) and different from Barro (1977, 1978) will be estimated in this model.

A. *Money Growth equation*

The form of the anticipated part of money growth equation as in Barro (1977, p. 104) is

$$(45) \quad GM_t = \alpha_0 + \sum_{i=1}^{2} \alpha_{1i}\, GM_{t-i} + \alpha_2\, FG_t + \alpha_3\, UN_{t-1}$$

where $M_t$ = DI or SS monetary aggregate

$GM_t$ = log ($M_t$) - log ($M_{t-1}$) a measure of average growth rate

$FG_t$ = log ($FG_t$) - [log ($FG$)]$_t^*$ real expenditure of the federal government, where [log ($FG$)]$_t^*$ = β[log ($FG$)]$_t$ - (1- β)[log ($FG$)]$^*_t$ an exponentially declining distributed lag of log $(FG)_{t-1}$.[10]

$UN_t$ = log($U$/1 - $U$) a cyclical variable, where $U$ is the unemployment rate in the total labor force.

Rational expectations implies that anticipation of money growth will be formed optimally using all available information. Thus Equation (45) is used to generate optimal, linear forecasts of anticipated money growth rates $GM^e_t$ which are then used to compute the residuals or unanticipated money growth $GMR_t$[11]

(46) $GMR_t = GM_t - GM^e_t$.

### 7.2 Money Growth Equation Parameter Estimates

The results for estimating (45) using annual observations from 1960 to 1992 are reported in table 29. For the two lagged values of money growth, the estimated results do not show persistent effects of money growth beyond one period. This is indicated by the negative and insignificant coefficient values of ($GM_{t-2}$) for all the aggregates. However, contemporaneous effect seems to be strong especially with broader SS aggregates. The coefficients on lagged unemployment variable ($UN_{t-1}$) while not significantly different from zero, shows the appropriate signs and are comparable to those obtained in Barro (1977, pp. 104-105).

The coefficients of federal variable ($FG_t$) for M1 and D1 show the right signs , however, not as significant as those obtained in Barro (1977, pp. 104-105) which show

[10] Barro (1977, p. 103) uses the value of $\beta = 0.2$.
[11] In rational expectation formulation, $GM^e_t = E(GM_t \mid \phi_{t-1})$, which states that the market incorporates all available information in assessing the probability distribution of all future money growth rates.

the coefficient of the federal variable, estimated only on M1 growth rates, to be 0.082. Surprisingly, the results of the federal variable for all the broader aggregates show negative coefficients.

TABLE 29

PARAMETER ESTIMATES OF ANTICIPATED MONEY GROWTH EQUATION:

$$GM_t = \alpha_0 + \sum_{i=1}^{2} \alpha_{1i}\, GM_{t-i} + \alpha_2\, FG_t + \alpha_3\, UN_{t-1}$$

| Aggregate | Constant | $GM_{t-1}$ | $GM_{t-2}$ | $FG_t$ | $UN_t$ | $R^2$ | $SEE$ |
|---|---|---|---|---|---|---|---|
| M1 | 0.065<br>(0.079) | 0.394<br>(0.180) | -0.008<br>(0.007) | 0.007<br>(0.060) | 0.019<br>(0.079) | 0.31 | 0.028 |
| M2 | 0.171<br>(0.065) | 0.589<br>(0.150) | -0.010<br>(0.006) | -0.012<br>(0.008) | 0.036<br>(0.016) | 0.32 | 0.025 |
| M3 | 0.144<br>(0.079) | 0.712<br>(0.152) | -0.009<br>(0.007) | -0.014<br>(0.018) | 0.027<br>(0.019) | 0.36 | 0.029 |
| L | 0.075<br>(0.065) | 0.873<br>(0.147) | -0.006<br>(0.006) | -0.011<br>(0.008) | 0.011<br>(0.016) | 0.51 | 0.023 |
| D1 | 0.032<br>(0.065) | 0.332<br>(0.181) | -0.007<br>(0.006) | 0.012<br>(0.009) | 0.010<br>(0.016) | 0.38 | 0.022 |
| D2 | 0.125<br>(0.085) | 0.529<br>(0.165) | -0.008<br>(0.007) | -0.008<br>(0.011) | 0.026<br>(0.021) | 0.25 | 0.029 |
| D3 | 0.117<br>(0.087) | 0.535<br>(0.168) | -0.008<br>(0.007) | -0.008<br>(0.012) | 0.022<br>(0.022) | 0.23 | 0.029 |
| DL | 0.095<br>(0.019) | 0.555<br>(0.168) | -0.006<br>(0.006) | -0.006<br>(0.011) | 0.018<br>(0.019) | 0.26 | 0.026 |

Note: Standard errors are in parenthesis.

B. *The Output equation*

The form of the output equation used is

$$(47) \quad \log(y_t) = \beta_0 + \beta_1 \log(y^*_t) + \sum_{i=0}^{3} \beta_{2i}(GM_{t-i} - GM^e_{t-i}) + \varepsilon_t.$$

By substituting (46) into (47)

$$(48) \quad \log(y_t) = \beta_0 + \beta_1 \log(y^*_t) + \sum_{i=0}^{3} \beta_{2i} GMR_{t-i} + \varepsilon_t,$$

where $y_t$ = real GNP in 1987 dollars at time $t$

$y^*_t$ = natural level of real GNP in 1987 dollars at time $t$[12]

$GM_t$ = money growth in time period $t$

$GM^e_t$ = anticipated $GM_t$ conditional on information available in time period $t-1$

$\varepsilon_t$ = a stochastic error term.

The form of equation (48) has been used in rational expectations models (e.g. Sargent and Wallace (1976, p. 170) and Mishkin (1982, p. 23)).

## 7.3 Output Equation Parameter Estimates

The parameter estimates for equation (48) for various monetary aggregates from 1960 to 1992 are presented in table 30. Not surprising, the natural rate output variable

[12]The potential output is determined by using Okun's Law which states that *for every 1% that the actual employment rate exceeds the natural rate, a 2.5% GNP gap occurs* (see McConnell and Brue (1993) p.137). Thus

$$y^*_t = [(u_t - u^*_t)\, 2.5 y_t] + y_t$$

where $u_t$ = rate of unemployment at time period $t$
$u^*_t$ = natural rate of unemployment at time period $t$,
where the estimated values for $u^*_t$ are: 4% for 1960-1969; 5% for 1970-1974; 5.5% for 1975-1979; and 6% for 1980-1992 (see McConnell and Brue (1993, p. 135)).

shows a strong effect on the current period's output. For all the aggregates, the coefficient of $y^*_t$ is close to unit. The results on unanticipated money growth variables (*MGR*) show persistent expansionary effects of unanticipated money growth two periods. Beyond two periods, however, any remnants of expansionary unanticipated money growth has negative though negligible effects on the current output. This may indicate stronger contemporaneous effects of unanticipated money growth on output, as observed in Barro (1978). The point to be noted here is that SS and DI aggregates show significant contemporaneous effects of unanticipated money movement (*GMR*), however, DI show stronger significance for the coefficients of ($GMR_{t-2}$) than their counter part SS.

TABLE 30

PARAMETER ESTIMATES OF OUTPUT EQUATION:

$$\log(y_t) = \beta_0 + \beta_1 \log(y^*_t) + \sum_{i=0}^{3} \beta_{2i}\, GMR_{t-i} + \varepsilon_t,$$

| Aggregate | Constant | $y^*_t$ | $GMR_t$ | $GMR_{t-1}$ | $GMR_{t-2}$ | $GMR_{t-3}$ | $R^2$ | *SEE* |
|---|---|---|---|---|---|---|---|---|
| M1 | 0.220 | 0.971 | 0.158 | 0.195 | 0.183 | -0.006 | 0.99 | 0.021 |
| | (0.600) | (0.044) | (0.143) | (0.157) | (0.149) | (0.005) | | |
| M2 | 0.451 | 0.943 | 0.313 | 0.362 | 0.065 | -0.009 | 0.99 | 0.020 |
| | (0.423) | (0.051) | (0.154) | (0.169) | (0.150) | (0.005) | | |
| M3 | 0.368 | 0.953 | 0.213 | 0.304 | 0.149 | -0.009 | 0.99 | 0.021 |
| | (0.425) | (0.052) | (0.139) | (0.152) | (0.133) | (0.006) | | |
| L | 0.337 | 0.956 | 0.234 | 0.275 | 0.063 | -0.007 | 0.99 | 0.021 |
| | (0.395) | (0.048) | (0.183) | (0.191) | (0.178) | (0.006) | | |
| D1 | 0.210 | 0.972 | 0.109 | 0.176 | 0.261 | -0.006 | 0.99 | 0.021 |
| | (0.390) | (0.047) | (0.177) | (0.200) | (0.189) | (0.005) | | |
| D2 | 0.278 | 0.964 | 0.180 | 0.372 | 0.338 | -0.009 | 0.99 | 0.019 |
| | (0.354) | (0.043) | (0.126) | (0.143) | (0.137) | (0.005) | | |
| D3 | 0.279 | 0.964 | 0.215 | 0.363 | 0.323 | -0.009 | 0.99 | 0.019 |
| | (0.358) | (0.044) | (0.120) | (0.135) | (0.125) | (0.005) | | |
| DL | 0.283 | 0.963 | 0.232 | 0.313 | 0.268 | -0.008 | 0.99 | 0.020 |
| | (0.355) | (0.043) | (0.149) | (0.166) | (0.155) | (0.005) | | |

C. *The Price Equation*

The equation to be estimated similar to Barro's (1978, p. 559)[13]

$$(49) \quad \log (P_t) = \alpha_0 + \alpha_1 \log (M_t) + \sum_{i=0}^{3} \alpha_{2i}\ GMR_{t-i} + \alpha_3 (G/y)_t + \alpha_4\, r_t + \varepsilon_t$$

where $P_t$ = GNP deflator

$G$ = real government purchases of goods and services

$y$ = real GNP in 1987 constant dollars

$r_t$ = Moody's Aaa corporate bond rate

$\varepsilon_t$ = stochastic error term.

## 7.4 Price Equation Parameter Estimates

The estimated coefficients for equation (49) are presented in table 31. Theory suggests that, in the absence of money illusion, the coefficient of log $(M_t)$ should be unity - that is, money movements are reflected fully in the price movements. The results for D1 and M1 are comparable to Barro (1978, pp. 560-562) which support the key hypothesis of a one-to-one contemporaneous link between anticipated money and prices. For the broader measures, DI show estimates for anticipated money that are closer to unity than their counterpart SS. However, a point to be observed is that the empirical results obtained here seem to support the hypothesis of a strong contemporaneous link between money and prices for both DI and SS.

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[13]In the original Barro model, a military variable to account for the effects of military draft was included. However, Barro found the variable to be insignificant in his price equation and thus could be omitted without much loss of fit. Nevertheless, while the military draft variable would may have been important prior to 1970s, it is no longer relevant.

As for the unanticipated money growth variable (*GMR*), several observations are in order: the effect of ($GMR_{t-5}$) on the price movements is inconsequential for all the aggregates; all the other unanticipated money growth variables show the appropriate signs with the magnitude of significance decreasing with increasing lags. This suggests a greater contemporaneous effect of money growth on the price levels. The DI aggregates, however, show estimates that are more significant than those of SS aggregates, suggesting that DI aggregation conform more to the underlying theory than SS aggregation. Barro (1978, p. 564) using M1 as the monetary aggregate, finds all the six of the estimated coefficients of *GMR* variable to be negative and statistically significant.

The estimated coefficients of (*G/y*) variable, which is based on government purchases of goods and services, surprisingly show no effect on the price movements for all the aggregates. This is in contrast to Barro's (1978, p. 566) which shows the estimated coefficient to be positive and significantly different from zero. But, Barro points out that the movement of (*G/y*) was on the downward since 1968. One possible reason to explain the insignificance of the government variable is that *y*, the real GNP has been rising much faster than *G*, which is dominated by military spending, thus the fraction gets ever smaller with time as GNP growth gets larger coupled with defense cut-backs.

The interest rate variable appears to be important in explaining the price level movements. Divisia aggregation theory suggest that interest rates would cause a shift in DI only when there is an income effect. In otherwords, changes in interest rate will change DI only if the change in relative prices results in a change in utility or the flow of services from monetary assets. DI perfectly internalizes pure substitution effect, "...a change in an interest rate will change the aggregate only if it *should* change the aggregate...[hence], the aggregate *will not* change, when it *should* not change." (Barnett and Spindt (1982), p. 7).

Since SS aggregate does not internalize pure substitution effect interest movements would cause SS to shift more than would DI. The results in table 31 seem to support the underlying theory: broader SS aggregates show higher significance of interst rate coefficients than their DI counterparts.

*Test of a unit coefficient on log* ($GM_t$)

The hypothesis of a unit coefficient on log ($M_t$) is essentially a test of homogeneity postulate or the absence of money illusion. As noted above, when the homogeneity postulate is not imposed, the coefficients of DI money seem to conform more to the homogeneity postulate than SS money.

Table 32 show results of re-estimated price equations with the homogeneity postulate imposed by restricting the coefficient of log ($GM_t$) to equal 1. As Barro (1978, pp. 562-563) points out, this restriction amounts to using the negative of the log of real money balances as a dependent variable - that is, log ($P_t$) - log ($GM_t$ ) becomes the effective dependent variable. The results show the first five *GMR* coefficients to be significantly negative while coefficient of $GMR_{t-5}$ is still inconsequential for all the aggregates. But, the point to be made here is that, under the imposed homogeneity condition, all the aggregates show results conforming to the underlying hypothesis.

The $(G/y)$ variable continues to be very insignificant while $r$ becomes less significant for SSs than in the unconstrained state.

TABLE 31

PARAMETER ESTIMATES OF PRICE EQUATION:

$$\log(P_t) = \alpha_0 + \alpha_1 \log(M_t) + \sum_{i=0}^{3} \alpha_{2i}\ GMR_{t-i} + \alpha_3 (G/y)_t + \alpha_4\ r_t + \varepsilon_t$$

| Aggreg. | $\alpha_0$ | $\log M_t$ | $GMR_t$ | $GMR_{t-1}$ | $GMR_{t-2}$ | $GMR_{t-3}$ | $GMR_{t-4}$ | $GMR_{t-5}$ | $(G/y)_t$ | $r_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| M1 | -0.959 | 0.934 | -0.847 | -0.727 | -0.478 | -0.328 | 0.002 | 0 | 0 | -0.071 |
| | (1.173) | (0.252) | (0.309) | (0.301) | (0.294) | (0.312) | (0.007) | (0) | (0) | (0.35) |
| M2 | -0.227 | 0.735 | -0.614 | -0.461 | -0.287 | -0.023 | 0.059 | 0 | 0 | -0.201 |
| | (0.416) | (0.084) | (0.159) | (0.185) | (0.169) | (0.167) | (0.014) | (0) | (0) | (0.215) |
| M3 | -0.325 | 0.582 | -0.384 | -0.253 | -0.040 | 0.794 | 0.059 | 0 | 0 | -0.325 |
| | (0.218) | (0.069) | (0.142) | (0.168) | (0.166) | (0.152) | (0.120) | (0) | (0) | (0.218) |
| L | 0.409 | 0.608 | -0.279 | -0.067 | 0.071 | 0.159 | 0.143 | 0 | 0 | -0.278 |
| | (0.387) | (0.078) | (0.211) | (0.241) | (0.243) | (0.217) | (0.179) | (0) | (0) | (0.233) |
| D1 | -1.488 | 1.044 | -1.063 | -0.761 | -0.424 | -0.389 | -0.272 | 0 | 0 | 0.025 |
| | (1.044) | (0.227) | (0.369) | (0.347) | (0.33) | (0.409) | (0.312) | (0) | (0) | (0.322) |
| D2 | -1.357 | 0.986 | -1.007 | -1.132 | -0.812 | -0.587 | -0.316 | 0 | 0 | -0.018 |
| | (0.534) | (0.111) | (0.161) | (0.180) | (0.170) | (0.174) | (0.135) | (0) | (0) | (0.209) |
| D3 | -1.118 | 0.931 | -0.915 | -1.010 | -0.704 | -0.481 | -0.267 | 0 | 0 | -0.054 |
| | (0.483) | (0.099) | (0.144) | (0.164) | (0.157) | (0.155) | (0.119) | (0) | (0) | (0.199) |
| DL | -1.648 | 1.049 | -1.034 | -1.146 | -0.866 | -0.633 | -0.337 | 0 | 0 | -0.004 |
| | (0.520) | (0.108) | (0.163) | (0.184) | (0.177) | (0.172) | (0.134) | (0) | (0) | (0.206) |

TABLE 32

PRICE EQUATION PARAMETER ESTIMATES:

HOMOGENEITY POSTULATE

| Aggreg | $\alpha_0$ | $GM_t$ | $GMR_t$ | $GMR_{t-1}$ | $GMR_{t-2}$ | $GMR_{t-3}$ | $GMR_{t-4}$ | $GMR_{t-5}$ | $(G/y)_t$ | $r$ |
|---|---|---|---|---|---|---|---|---|---|---|
| M1 | -1.266 | 1.00 | -0.909 | -0.775 | -0.526 | -0.371 | -0.238 | 0 | 0 | -0.050 |
| | (0.123) | (0) | (0.194) | (0.231) | (0.225) | (0.258) | (0.223) | (0) | (0) | (0.335) |
| M2 | -1.528 | 1.00 | -0.872 | -0.681 | -0.543 | -0.198 | -0.048 | 0 | 0 | -0.024 |
| | (0.034) | (0) | (0.177) | (0.215) | (0.190) | (0.198) | (0.168) | (0) | (0) | (0.256) |
| M3 | -1.582 | 1.00 | -0.785 | -0.663 | -0.494 | -0.284 | -0.161 | 0 | 0 | -0.027 |
| | (0.063) | (0) | (0.221) | (0.268) | (0.259) | (0.243) | (0.199) | (0) | (0) | (0.366) |
| L | -1.508 | 1.00 | -0.706 | -0.473 | -0.416 | -0.225 | -0.103 | 0 | 0 | -0.008 |
| | (0.058) | (0) | (0.301) | (0.352) | (0.346) | (0.316) | (0.267) | (0) | (0) | (0.348) |
| D1 | -1.284 | 1.00 | -1.010 | -0.724 | -0.389 | -0.345 | -0.246 | 0 | 0 | 0.006 |
| | (0.101) | (0) | (0.238) | (0.279) | (0.274) | (0.327) | (0.269) | (0) | (0) | (0.296) |
| D2 | -1.426 | 1.00 | -1.020 | -1.146 | -0.824 | -0.598 | -0.323 | 0 | 0 | -0.014 |
| | (0.045) | (0) | (0.112) | (0.137) | (0.132) | (0.142) | (0.120) | (0) | (0) | (0.199) |
| D3 | -1.454 | 1.00 | -0.983 | -1.081 | -0.769 | -0.540 | -0.303 | 0 | 0 | -0.029 |
| | (0.037) | (0) | (0.105) | (0.128) | 0.125) | (0.128) | (0.107) | (0) | (0) | (0.193) |
| DL | -1.415 | 1.00 | -0.987 | -1.098 | -0.822 | -0.593 | -0.313 | 0 | 0 | -0.018 |
| | (0.047) | (0) | (0.117) | (0.142) | (0.140) | (0.139) | (0.117) | (0) | (0) | (0.195) |

# CHAPTER 8

## SUMMARY AND GENERAL CONCLUSIONS

Although aggregation theory strongly favors Divisia quantity index over the simple sum index as a measure of the quantity of an aggregated composite good, the empirical evidence obtained in this study does not show an overwhelming support for the theory. The present paper has systematically compared the empirical performance of SS and DI measures relative to various selection criteria. And, as Barnett et al (1984, p. 1075) found, neither the DI nor the SS uniformly dominate the other relative to all the criteria considered, and no one aggregate was found to be best.

Since by their construct the SS and DI aggregates are different, it makes it necessary that one should compare the empirical performance of both for policy purposes. A strong theoretical case can be made for the use of money indexation as a measure of aggregate money supply. But, empirical evidence obtained in this study does not provide a strong support for the theory. The results in this study are mixed at best. In the context of reduced-form equation, the fit and error minimizing tests show simple-sum measures performing more satisfactorily than their counterpart DI measures. In these tests M3 was perhaps the best aggregate. In the forecast error tests, however, neither SS nor DI show superior tracking ability. And, on the tests on restrictions, on the basis of Wald and LM tests, DI do not explain the superneutrality condition any better than SS aggregates. The non-nested Cox tests strongly suggest broader DI to be more appealing. The J-test, however, do not show as conclusive results as the Cox tests. In some cases, SS seem to dominate DI at both levels of aggregation.

Tests on money demand functions seem to imply a break-down of the model in early 1970s. But, none of the DI seem to make any improvements over the SS aggregates. Broader DI, however, seem to show better forecasting ability than their SS counterparts.

While DI measures are observed to conform more to the homogeneity postulate under the rational expectation hypothesis, DIs do not reveal any superiority over the SSs in the most crucial test - i.e., the effect of unanticipated monetary innovations on output and prices. All the aggregates show strong contemporaneous effects of unanticipated money growth on output and prices.

In sum, empirical evidence obtained in this study are not robust enough to claim a case for choosing either DI or SS over the other. However, as Barnett et al aptly observes; "With so many criteria being considered, the selection of a 'best' aggregate is a hazardous matter" (1984, p. 1076). Indeed, it is.

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