COMBINING TWO SAMPLING METHODS, BAYESIAN MULTILEVEL MODELING, AND BUILD-MAINTENANCE SOIL PHOSPHORUS

By

WHOI CHO

Bachelor of Science in Agricultural Economics
Oklahoma State University
Stillwater, OK
2017

Master of Science in Agricultural Economics
Oklahoma State University
Stillwater, OK
2019

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Dissertation Approved:

B. Wade Brorsen, Ph.D.

Dissertation Adviser

Dayton M. Lambert, Ph.D.

John M. Riley, Ph.D.

D. Brian Arnall, Ph.D.

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Name: WHOI CHO

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Title of Study: COMBINING TWO SAMPLING METHODS, BAYESIAN MULTILEVEL MODELING, AND BUILD-MAINTENANCE SOIL PHOSPHORUS

Major Field: AGRICULTURAL ECONOMICS

Abstract: In Chapter I, sometimes two measurements are available to collect soil information: a low-cost noisy measurement and an accurate expensive one. For example, soil testing in a laboratory is expensive and accurate. On-the-go pH meters are available, but they are not as accurate. The question addressed here is what is the best way to combine these measures to guide precision applications? Bayesian Kriging is proposed to estimate the joint spatial distribution considering spatial autocorrelation. This study also obtains the economic optimum ratio of expensive and accurate measurements by maximizing the expected net present value using Bayesian Decision Theory and a grid search procedure.

In Chapter II, absent check plots, near-zero treatment rates, or non-limiting treatment levels, it may be difficult to estimate accurately yield response functions. a Bayesian multilevel modeling approach is proposed to incorporate response parameters from published studies into crop yield response estimation procedures when non-limiting or limiting treatments are omitted in agronomic experiments. A proof-of-concept simulation supplements an empirical application. The simulation investigates the small sample properties of the proposed procedure. The empirical example uses field trial data for a maize planter experiment under different nitrogen (N) fertilizer rates.

In Chapter III, there are two alternative approaches to managing soil phosphorus (P): sufficiency and build-maintenance. Sufficiency seeks to apply the minimum amount of P fertilizer that the crop needs in that year. At higher yield potential or intensive crop rotation, the crops consume more P than applied amounts of P fertilizer with a sufficiency approach. As the soil P level decreases due to higher crop removal, the expected crop yield decreases over time until an equilibrium is reached. The build-maintenance (BM) approach, however, seeks to build and (or) maintain the soil P level for crops so that P is not the limiting nutrient. However, the BM recommendation rate costs more in the short-term because it requires a higher rate than the sufficiency recommendation rate. The producer's long-term returns will differ depending on each approach. This study compares the expected net present values of two alternative recommendation rates with various scenarios.

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CHAPTER I

COMBINING LOW-COST NOISY MEASUREMENTS WITH EXPENSIVE ACCURATE MEASUREMENTS TO GUIDE PRECISION APPLICATIONS

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Abstract

Precision agriculture requires many local measurements. Sometimes two measurements are available: a low-cost noisy measurement and an accurate expensive one. For example, soil testing in a laboratory is expensive and accurate. On-the-go pH meters are available, but they are not as accurate. The question addressed here is what is the best way to combine these measures to guide precision applications? The first step is to estimate the joint spatial distribution of the two measures. The joint distribution is estimated using Bayesian Kriging since it can consider the information when the measures are spatially autocorrelated. The second step is to determine the economic optimum of how many of each measure to use. This study obtained the ratio of expensive and accurate measurements by maximizing the expected net present value using Bayesian Decision Theory and a grid search procedure. To demonstrate the method, a harmonization process that uses no spatial information was compared with Bayesian Kriging using Monte Carlo

data. A wheat production example was used to parameterize the Monte Carlo simulation. Soil pH lab sampling and on-the-go soil pH sensors were simulated as the two different measurements for soil mapping in wheat fields. Bayesian Kriging led to more accurate soil mapping and a higher expected net present value.

Introduction

Precision agriculture depends on measuring attributes at multiple locations in a field. More accurate measurements usually cost more. Low-cost technology examples include on-the-go measurements for soil pH testing (Adamchuk et al. 2004; Schirrmann et al. 2011), quality of sugarcane testing (Nawi et al. 2014), and rising plate meter for estimating forage mass (Cho et al. 2019). These low-cost measurements are less accurate than other expensive technologies such as the laboratory procedures for soil pH (Thomas 1996; Eckert and Sims 2009), sugarcane quality (Purcell et al. 2005), and forage mass (Sollenberger and Cherney 1995). Some producers may utilize both low-cost and expensive measurements. This raises two questions. What methodology of combining information suits this data and what is the optimal allocation of low-cost and expensive measurements?

Previous literature proposed methods to combine spatially heterogeneous data sets from multiple sources. Heuvelink and Bierkens (1992) propose a model averaging method of combining information by taking a weighted average of more accurate information. Various ways of weighting have been developed for efficient averaging of measured information. Ge et al. (2014) utilize the weights derived from the variance-covariance matrix of errors to minimize the variance of the estimation errors. Malone et al. (2014) suggest finding the weights using the covariance of the errors from ordinary-least-squares (OLS) estimation. Caubet et al. (2019) find the weights obtained from OLS estimation can lead to accurate mapping even with a relatively small number of accurate observations.

Harmonization is also a widely used method to combine spatial heterogeneous datasets (Maldaner et al. 2016; Sams et al. 2017; Leroux et al. 2019; Pichon et al. 2019). To harmonize the means, the collected data from multiple sources can be adjusted based on a correction factor, which is the ratio between means of measured values (Maldaner et al. 2016; Sams et al. 2017). Leroux et al. (2019) propose a spatial harmonization method in which different weights are assigned by distance as well as a two-step methodology can also be applied to harmonize heterogeneous data sets considering differences in variance (Leroux et al. 2019). With the two-step method, the noisy data set is scaled with respect to the accurate data set, and then centered to correct the bias (Leroux et al. 2019). Pichon et al. (2019) apply the Leroux et al. (2019)'s two-step harmonization process to two different hand-held sensors.

Past literature including Pichon et al. (2019) did not incorporate cost differences between noisy and accurate measurements. This study goes beyond previous literature by using Bayesian Kriging to estimate the joint distribution of the two measurements. In addition, this study maximizes expected net present value to determine the optimal allocation of information from two different sampling methods. Bayesian Kriging provides spatially smoothed parameter estimates (Park, et al. 2019; Park et al. 2020) and imputes values when observations are unavailable (Cho and Brorsen 2021; Park et al. 2021).

In this paper, the Bayesian Kriging approach is used to estimate a site-specific posterior density when data are obtained from two different sampling methods. This study simulates wheat production and considers two alternative soil pH measurements, which are the laboratory procedures for soil pH (Thomas 1996; Eckert and Sims 2009) and an on-the-go sensor for soil pH (Schirrmann et al. 2011). Based on the estimated soil pH information, aglime is assumed applied to treat low soil pH. Using a Monte Carlo study, this study compares Bayesian Kriging and the harmonization process in Pichon et al. (2019). The goal is to determine the optimal number of accurate measures to maximize expected net present values (NPV) with a 5-year planning horizon. The hypothesis tested is that the Bayesian Kriging method leads to a higher net present value than the method of Pichon et al. The accuracy is measured as the mean absolute errors between true values at each location and the estimates from each method.

Bayesian Kriging

Bayesian Kriging is used to estimate the joint posterior distribution when the values are measured through multiple sources with different accuracy. Two measures are available: expensive but accurate, and low-cost but noisy. In addition, the true measures are assumed to be spatially correlated while measurement errors are not.

Two Different Measurements

The noisy measure is assumed to be biased and have measurement error, which is not spatially correlated. On the other hand, the expensive measure has less measurement error than the noisy measure. The expensive accurate measure and the low-cost noisy measure are represented mathematically as

(1) Expensive measure: $m_s^1 = m_s + \varepsilon_s^1\ ,\ s \in \theta\ ;\ \ \varepsilon_s^1 \sim N(0, \sigma_1^2)$

(2) Low-cost measure: $m_s^2 = \alpha + \beta \cdot m_s + \varepsilon_s^2\ ,\ \ s \notin \theta\ ;\ \ \varepsilon_s^2 \sim N(0, \sigma_2^2)$

where m_s is the true latent value in location ($s = 1, \dots, S$), θ represents the locations where expensive measurements were taken, m_s^1 is the measured value from the expensive measurement that is assumed to be the true value, m_s^2 is the measured value from the low-cost measurements, α is the bias, β is the change in the sensitivity of low-cost measurements, ε_s^1 and ε_s^2 are the random errors that follow $\varepsilon_s^1 \sim N(0, \sigma_1^2)$ and $\varepsilon_s^2 \sim N(0, \sigma_2^2)$, and the variance of the expensive measure's error is smaller than the noisy measure's error variance.

The joint distribution of the two different measurements is

$$(3) \qquad \begin{bmatrix} \boldsymbol{m}^1 \\ \boldsymbol{m}^2 \end{bmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{1} \end{pmatrix} \cdot (\alpha + \beta \cdot \boldsymbol{m}_{\mathrm{T}}) + (1 - \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{1} \end{pmatrix}) \cdot \boldsymbol{m}_{\mathrm{T}} \quad , \quad \begin{pmatrix} \boldsymbol{\sigma}_1^2 \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_2^2 \boldsymbol{I} \end{pmatrix}\right)$$

where $\boldsymbol{m}^1$is a $N_1 \times 1$ vector of measured values from expensive measurements, N_1 is the number of expensive measurements used, $\boldsymbol{m}^2$ is a $N_2 \times 1$ vector of noisy measured values from low-cost measurements, N_2 is the number of low-cost measurements used that excludes the number of expensive measurements used (N_1) from the total number of measurements used (N_{T}); $N_2 = N_{\mathrm{T}} - N_1$, $\boldsymbol{0}$ and $\boldsymbol{1}$ are vectors indicating whether each measurement is biased or not, where a low-cost measurement corresponds to 1, and $\boldsymbol{m}_{\mathrm{T}}$ is a $N_{\mathrm{T}} \times 1$ vector of true values.

The true values $\boldsymbol{m}_{\mathrm{T}}$ are assumed to follow a Gaussian spatial process, so they have a multivariate normal distribution. Spatial dependency is reflected using a simultaneous autoregressive (SAR) precision matrix. The true values $\boldsymbol{m}_{\mathrm{T}}$ are generated from a multivariate Gaussian process ($MVGP$):

(4) $$\boldsymbol{m}_{\mathrm{T}} \sim MVGP(\boldsymbol{m}_a, \boldsymbol{\Sigma}_{\boldsymbol{m}})$$

$$\boldsymbol{\Sigma}_{\boldsymbol{m}} = (\tau(\boldsymbol{I} - \rho\boldsymbol{W}')(\boldsymbol{I} - \rho\boldsymbol{W}))^{-1}$$

where $\boldsymbol{m}_a$ is the vector of whole field average measured value in which each element is the same, $\boldsymbol{\Sigma}_{\boldsymbol{m}}$ is the covariance matrix that uses simultaneous autoregressive form suggested by Poursina and Brorsen (2021), $\boldsymbol{W}$ is the row standardized contiguity matrix between locations, τ is the precision parameter which is same as the inverse variance, and ρ is the spatial correlation parameter, which is between 0 and 1 since positive autocorrelation is a common scenario in soil pH.

Following Bayes Rule, the joint posterior distribution is proportional to the product of the likelihood, process, and prior layers. The likelihood layer specifies the joint distribution of the two different measurements as shown in equation (3). The process layer models the spatial process of the parameters following equation (4). In addition, the prior layer consisted of priors for the parameters of the process layer (whole field average measured value, and Kriging parameters; spatial correlation and precision parameters) as well as the bias and standard deviation of measurement error parameters. The joint posterior distribution of parameters is

(5) $$p(\boldsymbol{m}_{\mathrm{T}}, \boldsymbol{\Theta}|\boldsymbol{M}) \propto p_1(\boldsymbol{M}|\boldsymbol{m}_{\mathrm{T}})p_2(\boldsymbol{m}_{\mathrm{T}}|\boldsymbol{\Theta})p_3(\boldsymbol{\Theta})$$

where p_1, p_2, and p_3 are the densities associated with likelihood, process, and prior layers, respectively, $\boldsymbol{m}_{\mathrm{T}}$ is a vector for Gaussian spatial process of random parameters, $\boldsymbol{\Theta}$ is a vector of hyper parameters, where $\boldsymbol{\Theta} = [m_a, \alpha, \beta, \tau, \rho, \sigma_1, \sigma_2]'$, and $\boldsymbol{M}$ is a matrix of measured values in all locations.

The Hamiltonian Monte Carlo (HMC) algorithm within a Gibbs sampler was used for the Bayesian Kriging estimation. The HMC algorithm is a Markov Chain Monte Carlo (MCMC) method of obtaining the posterior density. MCMC creates the posterior density through numerical integration. The MCMC draws random parameter values from a candidate density and keeps only accepted values. The HMC algorithm is generally faster than Metropolis-Hastings. The rstan package in R (Stan Development Team 2020) performed the Bayesian Kriging estimation.

Harmonization Process

Pichon et al. (2019) used a harmonization process that harmonizes the spatial heterogeneous datasets from two different measurements. The harmonization process assumes the accurate measure (m^1) has a linear relationship with the noisy measure (m^2). The harmonization process requires two steps to transform the noisy measure with respect to the accurate measure. In the first step, a slope parameter was estimated to minimize the difference in variance between accurate and noisy measures. The standard deviation of the noisy data set was divided by accurate data set's standard deviation to calculate a slope parameter. When only one accurate measure is available, this slope parameter would be undefined. Thus, this study substituted one as the slope parameter with only one accurate measure. The noisy measure m^2 was scaled to. The first step was

(6) $$a = \frac{\sigma_{m^2}}{\sigma_{m^1}}$$

where the parameter a is a slope parameter that can be seen as a change in the sensitivity of the measuring system, σ_{m^1} and σ_{m^2} are standard deviations of accurate and noisy data sets, respectively. The intermediate harmonized noisy measure was

(7) $$m^{2\prime} = a \times m^2$$

where $m^{2\prime}$ is the intermediate harmonized noisy measure based on the calculated slope parameter

In the second step, an intercept parameter was estimated to minimize the difference in average values between accurate measure and intermediate harmonized noisy measure ($m^{2\prime}$). The difference between the averages can be represented as

(8) $$b = \overline{m^1} - \overline{m^{2\prime}}$$

while the harmonized noisy measure is

(9) $$m^{2*} = b + m^{2\prime}$$

where $\overline{m^1}$ is the mean of the expensive measures, $\overline{m^{2\prime}}$ is the mean of the intermediate harmonized noisy measures, and the parameter b is the bias correction.

Data and Methods

This study evaluated the performance of the Bayesian Kriging method and the harmonization process, using 30 sets of simulated data. A limited number of replications was used due to the program being computer intensive and that 30 proved adequate to demonstrate the differences. The simulated data sets assume a producer collects soil pH information from fields using two different sampling methods. Each data set simulated a field assuming a 1-hectare square field (100 m ×100 m) with 100 square plots (10 m × 10 m). True soil pH values for each square plot were generated randomly from a multivariate normal distribution given mean soil pH of 5.4. The spatial covariance matrix was the simultaneous autoregressive covariance matrix with spatial correlation parameter, ρ, of 0.8 and precision parameter, τ, of 3. Even though the average soil pH in Oklahoma is 6.4 (Zhang & McCray 2018), this study used a lower soil pH of 5.4 so that applying lime is relevant.

The low-cost measure was assumed to have a bias of 0.52 and a slope of 0.6. Random errors were from a normal distribution with zero mean and a standard deviation of 0.82. These assumptions are the reported on-the-go sensor's mean error, slope, and mean absolute error from Schirrmann et al. (2011). Schirrmann et al. (2011) considered laboratory results as true values that can be compared with the measured values of an on-the-go sensor. The random errors of accurate measurements were drawn from a normal distribution with zero mean and a standard deviation of 0.10. Moreover, this study included sensitivity analysis for the performance of the Bayesian Kriging method and the harmonization process by different error sizes of accurate measurements.

The producer was also assumed to select the center point of the field as much as possible when using expensive measurements. For example, when 1% of expensive measurement was used, the location in row 5 and column 5 point was selected for the expensive measurement. Also, locations in row 3 and column 7 and in row 7 and column 3 points were selected when 2% of expensive measurements were used. This study did not investigate the optimal location for the accurate measurements.

The prior of average soil pH (m_a) was given as $N(5.68, 2)$ based on Schirrmann et al. (2011). For the Kriging parameters, the priors of precision (τ) parameter and spatial correlation (ρ) were $N(3, 2)$ and $U(0, 1)$, respectively. The priors of Kriging parameters used weakly informative priors. The standard deviation of precision parameter prior was given arbitrarily to reduce the probability that the parameter would become negative.[1]

[1] The lower bounds for the spatial correlation, precision, sensitivity change of low-cost measurement, and standard deviation of low-cost measurement error parameters were set to zero, meaning non-negativity

The bias (α) and sensitivity change (β) of low-cost measurement were drawn from $N(0, 1)$ and $N(0.6, 0.2)$, respectively. The prior for the standard deviation of low-cost measurement error (σ_2) parameter was a truncated normal distribution with a mean of 0 and a standard deviation of 1. The standard deviation of expensive measurement error (σ_1) parameter was 0.1. In practice, more accurate priors might be possible, but these are used here to avoid biasing results in favor of Bayesian Kriging.

Convergence of all parameters was checked using trace plots and all posterior densities converged. All 20,000 samples from the posterior distribution were used in calculating the expectation.[2] The joint posterior distribution from Bayesian Kriging was used to determine expected net present value. The optimal percentage of expensive measurements was obtained using a grid search by calculating the net present value for each percentage and selecting the one with the highest net present value.

Expected Net Present Value Maximization

Wheat production was used to parameterize the simulation model. Wheat producers are assumed to use soil sampling and on-the-go soil pH sensors to collect soil pH information on their fields. Soil sampling represents the expensive accurate measurements while values from on-the-go soil pH sensors represent low-cost, noisy measurements. The aglime used to correct low soil pH is often applied every five to six years due to high application costs. Correcting the low soil pH would not generate economic benefits from

constraints. The upper bounds were also set to reduce the program execution time for convergence. This study set the upper bounds to be large enough, with spatial correlation parameter set to 1 and the other three to 10. For the bias parameter, lower and upper bounds set to ±10.

[2] For the Bayesian Kriging estimation, 10,000 iterations each for four MCMC chains were used. The first 5,000 observations were burned in, so 5,000 iterations of each chain, total 20,000 samples were used.

a single year of farming (Cho et al. 2020). Therefore, the expected NPV with a 5-year planning horizon was used. The wheat producer's expected NPV objective function is

(10) $$\max_{\mathrm{A}} E(NPV_{\mathrm{A}})$$

$$s.t.\ \mathrm{A} \in [0,0.1]$$

$$NPV_{\mathrm{A}} = \frac{\sum_{s=1}^{100} \sum_{t=1}^{5} \frac{R_t(M_{s\mathrm{A}})}{(1+d)^t} - r_L \cdot Lime(M_{s\mathrm{A}})}{100} - c_1 \cdot A - c_2$$

$$R_t(M_{s\mathrm{A}}) = p_w \cdot y(pH_{sAt}) - r_N \cdot N$$

where A represents percentage of expensive measurements from 0% to 10%, NPV_{A} is the average of NPV, $M_{s\mathrm{A}}$ is a soil pH of location s, $R_t(M_{s\mathrm{A}})$ is a function of return by the estimated values in year t, d is the discount rate, which was specified to be 3.25%, the interest rate for farm ownership loans reported by the USDA Farm Service Agency (2021), c_1 is the \$10 per sample cost of soil pH lab testing, c_2 is the \$2 per sample cost of on-the-go soil pH sensor, N is the amount of nitrogen fertilizer application that is 140 kilograms per hectare following the fertilizer recommendation corresponding to YG that is 4035 kilograms per hectare (Zhang et al. 2017), and p_w, r_L, and r_N are the price of wheat, lime, and nitrogen fertilizer that are \$0.20 per kilogram, \$48 per metric ton based on 100% effective calcium carbonate equivalent, and \$0.85 per kilogram.

This study used a target pH of 6.5 (Cho et al. 2020). Dynamics of soil pH changes were also considered to calculate the losses from imprecise soil pH information. Wheat grain yields are reduced if the target pH was not reached due to imprecise information. In addition, applying too much aglime could result in unnecessary extra costs. Following Mills et al. (2020), the equations for applying aglime for each location and dynamics of soil pH by year are

(11) $$Lime(M_s) = 24.1584 - 4.0618 \cdot M_s$$

(12) $$pH_{st} = \begin{cases} pH_{s0} - 0.0051 \cdot pH_{s0} \cdot Lime(M_s) + 0.2270 \cdot Lime(M_s) & t = 1 \\ pH_{st-1} + 0.0001 \cdot pH_{st-1} \cdot N - 0.00037 \cdot N & t = 2, 3, \ldots, 5 \end{cases}$$

where $Lime(M_s)$ is the function of the amount of aglime application according to the measured soil pH by each location, pH_{st} is the soil pH in year t, and pH_{s0} is an initial soil pH that is a true value of soil pH from the simulation.

Soil pH was assumed as the only limiting factor for wheat grain yields in order to consider the economic benefit of the accuracy of soil pH mapping. The function of wheat grain yield by soil pH follows the relative yield equation of figure 7-b by Lollato et al. (2019). The equation of wheat grain's yield response to pH is the relative wheat grain yield times yield goal. Following Lollato et al. (2019)'s relative grain yield equation, the wheat yield function by each location's soil pH is

(13) $$y(pH_{st}) = \text{YG} \cdot \min(-61.3 + 62.3 \cdot e^{-1.24pH_{st}}, 1)$$

where $y(pH_{st})$ is a function of wheat grain yield for soil pH in year t, and YG is the yield potential of 4035 kilograms per hectare.

Results

Bayesian Kriging and Harmonization Process

In addition to the true values, the procedure produced three types of measures: the observed value, the Bayesian Kriging measure, and the harmonization measure. The mean absolute errors for each of these three measures are depicted in Figure 1.1. As expected, the error decreased with all measures as more expensive measures were taken. Both Bayesian Kriging and harmonization work well and had less mean absolute errors than observed soil pH. Bayesian Kriging had consistently lower error than harmonization

(Figure 1.1). The paired samples t-test used with the null hypothesis of equal mean absolute errors for Bayesian Kriging and harmonized pH. The null hypothesis was rejected, which the mean of the differences was -0.153 as well as statistically significant (α=0.05, $p < .001$) (Table 1.1).

Bayesian Kriging reduced more variance than harmonization by using spatial information as well as smoothing the values. Moreover, the Bayesian Kriging performance improved as decreasing the error sizes of accurate measurements (Table 1.1). When the error sizes of accurate measurements were 0.05 and 0.01, the means of the difference were -0.152 and -0.150, respectively. The mean differences of mean absolute errors for Bayesian Kriging and harmonized pH were also larger as increasing error size of accurate measurements. When the error size of accurate measurements was 0.50, the mean of the difference was -0.194 as well as rejecting the null hypothesis.

Table 1.2 shows the results of mean absolute errors with a larger variance for the expensive measure and with lower spatial correlation, which are precision parameter of 1 and spatial correlation parameter 0.6, as the sensitivity analysis. Still, Bayesian Kriging had lower error than harmonization, which the mean of the differences was -0.128. However, the overall mean absolute errors of Bayesian Kriging and harmonization were increased because of more noise.

Soil pH maps of estimated soil pH values when 2% expensive measurements were used are shown in Figure 1.2. These are from the first Monte Carlo replication. The observed soil pH and harmonized pH maps are noisy. The overall harmonized pH values were lower than the observed soil pH since the bias was corrected. However, there is no spatial smoothing, so the noisy patterns of the observed pH map still remained on the harmonized pH map. The Bayesian Kriging pH map is smoother than the true soil pH, which is to be expected since an optimal forecast will have smaller variance than the variable being forecasted.

Optimal Choices

The average of expected NPV with a 5-year planning horizon is maximized when 2% of expensive measurements were used (Table 1.3).[3] The expected NPV is $2999 per hectare when the observed soil pH values were used without smoothing or harmonization. The maximum expected NPV is $3236 per hectare with estimated soil pH values by Bayesian Kriging. The estimated soil pH values by Bayesian Kriging also led to $40 per hectare more NPV than soil pH values by harmonization (Table 1.3). Bayesian Kriging always had higher NPV than harmonization, regardless of the percentage of expensive measurements (Figure 1.3).

One caution is that the optimal number of expensive measurements depends heavily on the size of the area. With a larger area there is more benefit to accurate measurements and thus more accurate measurements would be optimal. For example, a sensitivity analysis was conducted assuming a 100-hectare square field with other parameters held constant. In this example, the percentage of expensive measurements for maximum NPV was 10%, the maximum considered.

[3] The percentage of expensive measurements for maximum NPV remained at 2% regardless of error sizes of accurate measurements, although the dollars had changed.

Conclusions

Bayesian Kriging was proposed as a way to combine information from two different measures. Bayesian Kriging smooths parameter estimates across space. Harmonization, an alternative method, can correct bias, but noise remains.

Using wheat fields as an example, this study simulated combining soil pH information from soil lab samples and on-the-go soil pH sensors. Bayesian Kriging had $40 per hectare more NPV using 2% of expensive measurements. The advantage of Bayesian Kriging, in this example, is that there is information in the nearby samples that can be used to reduce error and produce lime applications that avoid yield losses from low pH.

There are many other situations where Bayesian Kriging can be used to combine information from multiple sources. Bayesian Kriging can produce a soil phosphorus map to guide precision phosphorus application. As another example, Bayesian Kriging can generate perennial crop monitoring maps for grape vines and sugarcane quality monitoring maps. It can help predict when the crop will ripen, which can increase profits by determining the appropriate harvest time for growers. However, further research is needed in that the speed of the program is still slow to be applied in practice when fast decision-making may be needed.

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Table 1.1. The Mean Absolute Errors between True Values and Observed, Bayesian Kriging, and Harmonized Soil pH Values by the Percentage of Expensive Measurements by Different Error Sizes of Expensive Measurements

Expensive Measurements (%)	Observed				Bayesian Kriging				Harmonization Process			
Error Size	0.50	0.10	0.05	0.01	0.50	0.10	0.05	0.01	0.50	0.10	0.05	0.01
1	1.609	1.606	1.606	1.606	0.820	0.740	0.733	0.727	1.119	0.999	0.989	0.981
2	1.597	1.591	1.590	1.589	0.667	0.597	0.593	0.592	0.884	0.809	0.807	0.805
3	1.584	1.574	1.573	1.572	0.644	0.594	0.591	0.587	0.848	0.751	0.746	0.744
4	1.575	1.562	1.560	1.559	0.633	0.562	0.555	0.549	0.786	0.690	0.684	0.681
5	1.561	1.546	1.544	1.542	0.618	0.555	0.548	0.544	0.785	0.678	0.671	0.666
6	1.549	1.530	1.527	1.525	0.620	0.547	0.539	0.535	0.795	0.675	0.667	0.662
7	1.537	1.514	1.511	1.509	0.581	0.517	0.511	0.509	0.761	0.647	0.639	0.634
8	1.525	1.499	1.496	1.494	0.565	0.503	0.498	0.496	0.748	0.635	0.627	0.622
9	1.513	1.485	1.481	1.478	0.548	0.493	0.489	0.488	0.728	0.621	0.614	0.609
10	1.503	1.470	1.466	1.463	0.545	0.481	0.476	0.474	0.723	0.612	0.604	0.599

Note. The mean absolute errors from 30 simulation data sets were averaged.

Table 1.2. The Mean Absolute Errors between True Values and Observed, Bayesian Kriging, and Harmonized Soil pH Values by the Percentage of Expensive Measurements by Different Kriging Parameters of Expensive Measurements

Expensive Measurements (%)	Observed		Bayesian Kriging		Harmonization Process	
Plot Size (hectare)[a]	0.01	1	0.01	1	0.01	1
1	1.606	1.617	0.740	1.074	0.999	1.302
2	1.591	1.601	0.597	0.875	0.809	1.087
3	1.574	1.585	0.594	0.854	0.751	1.000
4	1.562	1.571	0.562	0.804	0.690	0.910
5	1.546	1.555	0.555	0.791	0.678	0.884
6	1.529	1.539	0.547	0.765	0.675	0.873
7	1.514	1.524	0.517	0.728	0.647	0.827
8	1.499	1.509	0.503	0.711	0.635	0.810
9	1.485	1.494	0.493	0.692	0.621	0.787
10	1.470	1.480	0.481	0.679	0.612	0.770

Note. The mean absolute errors from 30 simulation data sets were averaged.

[a]A field with 100 plots, each 10 m x 10 m plot, was simulated with spatial correlation parameter of 0.8 and precision parameter of 3. For each hectare plot, a 100-hectare field was simulated with spatial correlation parameter of 0.6 and precision parameter of 1.

Table 1.3: The Average of Expected Net Present Value with a 5-Year Planning Horizon by the Percentage of Expensive Measurements

Expensive Measurements (%)	Observed	Bayesian Kriging	Harmonization Process
0	2999	-	-
	(1429)		
1	2985	3210	3162
	(1650)	(10082)	(18266)
2	2978	3236	3196
	(1684)	(6998)	(8139)
3	2971	3222	3196
	(1669)	(6453)	(7925)
4	2964	3218	3193
	(1733)	(6370)	(7829)
5	2957	3206	3186
	(1718)	(6030)	(7399)
6	2950	3197	3176
	(1756)	(5989)	(7574)
7	2943	3187	3168
	(1791)	(5502)	(6675)
8	2936	3179	3160
	(1797)	(5215)	(6698)
9	2929	3172	3154
	(1794)	(5208)	(6585)
10	2922	3162	3144
	(1803)	(5280)	(6629)

Notes. The unit is dollars per hectare ($/ha). The net present values from 30 simulation data sets were averaged. Numbers in parentheses are the variance of the 30 net present values.

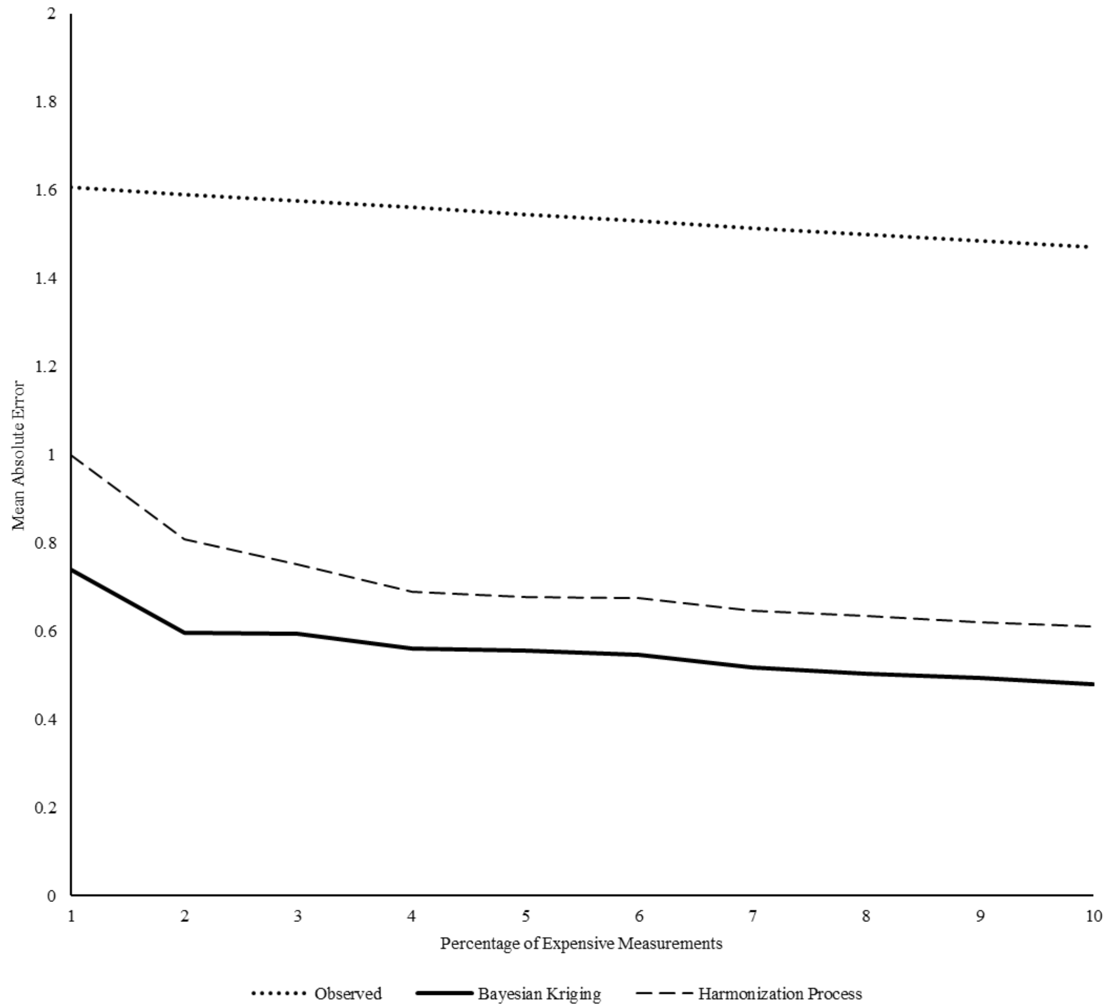

Figure 1.1. The mean absolute errors between true values and observed, Bayesian Kriging, and harmonized soil pH values by the percentage of expensive measurements Note. The mean absolute errors from 30 simulation data sets were averaged.

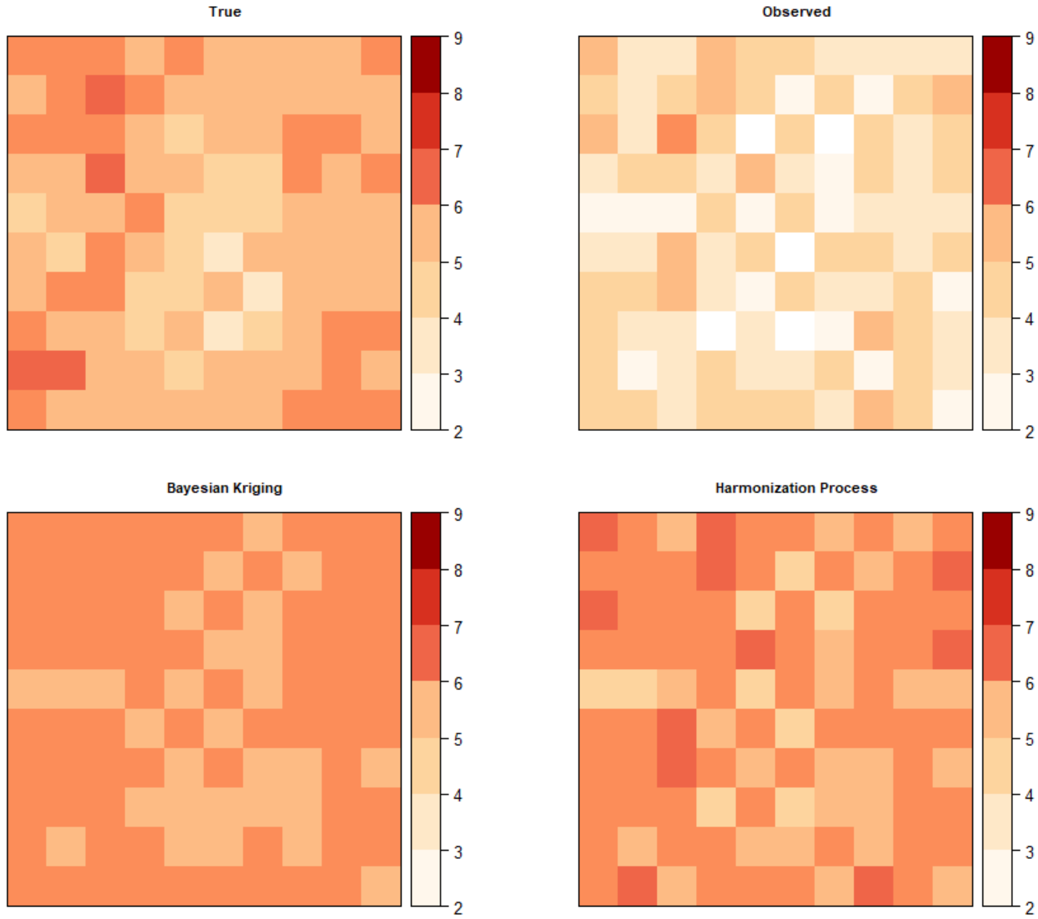

Figure 1.2. The soil pH maps with true, observed, Bayesian Kriging, and harmonization process with a 2% of expensive measurements

CHAPTER II

A BAYESIAN APPROACH FOR SUPPLEMENTING YIELD RESPONSE DATA WITH LIMITED TREATMENT DESIGN

Abstract

Absent check plots, near-zero treatment rates, or non-limiting treatment levels, it may be difficult to estimate accurately yield response functions. This paper proposes a Bayesian multilevel modeling approach to incorporate response parameters from published studies into crop yield response estimation procedures when non-limiting or limiting treatments are omitted in agronomic experiments. A proof-of-concept simulation supplements an empirical application. The simulation investigates the small sample properties of the proposed procedure. The empirical example uses field trial data for a maize planter experiment under different nitrogen (N) fertilizer rates. The planter trial compared mechanical planting methods to methods used in developing countries with limited access to mechanized planter technology. Some experiments had no check plots and all experiments lacked non-limiting fertilizer rates. Linear and quadratic response functions with plateaus are used in the simulation study and empirical application. Monte Carlo results suggest that estimates were closest to true parameter values when priors for optimal N rates from published sources were used. The empirical application found that the Oklahoma State University (OSU) hand planter produced higher yields than mechanical or wooden stick planters did.

Introduction

Researchers sometimes have limited input in the design of yield response experiments. In some cases, agronomic data might lack limiting and non-limiting treatment levels (or 'end-point' treatments), or certain treatments may be omitted from the experimental design. There are several reasons why treatments might be omitted from a field trial. Yield outcomes at 0-fertilizer rates, or at other treatment levels or combinations, may not be central to the primary research question. Researchers may already know the most likely outcomes for check plot yields. Farmers conducting on-farm trials may be reluctant to incur yield losses when no inputs are applied. Cost is another reason why some treatments might be limited. Regardless, if estimating crop yield response to inputs is an important research objective, then under-replication of limiting or non-limiting treatment levels may compromise statistical power and make it difficult to identify important parameters. For example, if non-limiting levels are limited or absent, then finding biologically or economically optimal input levels may be problematic. If empirically determined parameters are required for modeling crop growth, then absence of counterfactual yield outcomes makes yield forecasting difficult and compromise predictive accuracy. Further, under-replication of end-point treatments may produce imprecise parameter estimates and mislead treatment comparisons.

Omission of end-point or other important input levels may also complicate the economic analysis of crop response to inputs. For example, economic analysis of crop yield response to chemical or fertilizer inputs requires knowing the marginal physical product (MPP) of a crop's response to the input. MPP is the physical amount yield changes when an additional unit of input is applied (Heady and Dillon, 1961). The MPP is also the slope of a regression line in statistical models. Estimation of the MPP requires knowing how crops perform at limiting and non-limiting input levels. Researchers typically implement the counterfactual case of limiting inputs on yield by randomly assigning 0-rate replications, or 'check plots', to plots, strips, or blocks. For non-limiting cases, fertilizers or chemicals are applied at rates above an amount believed to maximize yield.

This paper proposes a Bayesian multilevel modeling approach to estimate crop response to inputs using data from agronomic trials with limited or no end-point treatments. Yield response intercepts and optimal N rates for maize from published articles are used to formulate 'hyper-priors' (McElreath, 2020, discussed below) for yield response parameters when an agronomic experiment excludes limiting or non-limiting treatments. The proof-of-concept Monte Carlo simulation examines the small-sample properties of the proposed procedure. An empirical application supplements the Monte Carlo simulation. The empirical example uses data from a N-fertilizer/maize planter experiment. Check plots were unassigned to some treatments. No experiments included non-limiting N rates.

Data

The empirical example uses data from a four-year maize field trial (2013, 2014, 2017 and 2018) conducted by the Plant and Soil Science Department of Oklahoma State University. The objective of the experiment was to compare the effects of N fertilization on maize planted with three methods. The planting methods were an Oklahoma State University (OSU) hand planter (HP), a long wooden stick planter (FP), and a John Deere

2-row MaxEmerge planter (JD). Smallholder farmers in developing countries use FP. HP is a planting method designed to address the FP's shortcomings with respect to N management (Oyebiyi et al., 2019). The JD treatment is the reference technology to which the other planters are compared. Field experiments were conducted at Efaw and Lake Carl Blackwell (LCB) in Stillwater, Oklahoma. The soil classifications of the research plots were an Ashport silty loam (fine silty, mixed, super active, thermic fluventic Haplustolls) at Efaw and a port silt loam (fine silty, mixed, thermic cumulic Haplustolls soil) at LCB.

The experimental design was a randomized complete block with nine treatments, including a 0-N rate check plot for some tillage methods, plus four combinations of seeding and N application methods. The two levels of N fertilizer were 30 and 60 kg N ha^{-1} for all treatments. Treatments were replicated three times for each planter. The highest rate of 60 kg N ha^{-1} is likely non-limiting, as the average optimal N fertilizer rate in Oklahoma is 139 kg N ha^{-1} (Miller et al., 2017). After planting, urea fertilizer (46-0-0) was side-dressed into the soil beside each plant with the HP method. Urea fertilizer was placed on the soil surface on plots planted with the FP method. The JD treatment applied a surface band on top of the ground after planting. Two types of N fertilizer were applied in the JD treatments; urea (46-0-0) and UAN (28-0-0). Check plots were planted with the JD method. The HP and FP treatments did not receive 0-N rates. Additional details of the experiment are summarized in Oyebiyi et al. (2019) and Fornah et al. (2020).

In sum, there were four treatment combinations: 1) HP, 2) FP, 3) JD (the reference treatment), and 4) JD with UAN fertilizer (JDUAN). When, the highest average maize yields were observed at 60 kg ha^{-1} of N under the HP treatment. These yields were

13.29 Mg ha^{-1} for LCB (in 2013) and 8.75 Mg ha^{-1} at Efaw (in 2018). The lowest average maize yields were 2.39 Mg ha^{-1} at LCB (in 2017) under FP and 3.46 Mg ha^{-1} at Efaw (in 2018) with FP when 30 kg ha^{-1} of N was applied (Table 2.1).

Intercepts, optimal N fertilizer rates, and their respective standard errors were collected from 12 studies on maize yield response to N estimated with linear response with plateau (LRP) models (Schmidt et al., 2002; Jaynes, 2011; Shroyer et al., 2011; Gentry et al., 2013; Crozier et al., 2014; Halvorson & Bartolo, 2014; Rajkovich et al., 2015; Kablan et al., 2017; Miller et al., 2017; Alotaibi et al., 2018; Ruark et al., 2018; Cho et al., 2020) (Table 2.2). The average of the maize yield intercepts was 6.12 Mg ha^{-1}, with minimum and maximum values of 2.78 Mg ha^{-1} and 9.33 Mg ha^{-1}. The standard errors for the intercepts reported in these studies ranged between 0.51 Mg ha^{-1} and 4.05 Mg ha^{-1}. The average of the optimal N estimates was 130 kg N ha^{-1}, with a range of 84 kg N ha^{-1} to 195 kg N ha^{-1}. The range of the standard errors for optimal N estimates was 18.24 and 68.64 Mg ha^{-1}. The external data are used to formulate hyper-priors for intercepts, MPP, and optimal N rates. Discussion of the methodology used to incorporate these data into Bayesian estimation procedures follows.

Methods and Procedures

Two response models are considered. The first is a linear response with plateau (LRP). The second is a quadratic response with plateau (QRP). The LRP and QRP models have broad theoretical and practical appeal in the agronomic literature and the economics of crop response to inputs (Lambert & Choi, 2022). The LRP and QRP are based on von Liebig's law of the minimum, which states that plant growth occurs at a constant rate with nutrients contributing to its production in fixed proportions until some factor becomes limiting (Blackman, 1905; Swanson, 1963).

The LRP models yield response to an input as:

(1) $$y = \min(\beta_0 + \beta_1 \cdot X,\ \bar{Y})$$

where y is maize yield, β_0 is an intercept, β_1 is the MPP of plant growth with respect to a 1-unit increase in input X (for example, nitrogen), and $\bar{Y}$ is a yield plateau. The plateau is the highest obtainable yield at the biologically optimal nitrogen rate of $X^* = \frac{\bar{Y}-\beta_0}{\beta_1}$. The parameter X^* is also called a "join-point" because it links the linear response to N to the plateau.

The QRP model is similar to the LRP but it imposes diminishing returns to an input up to the yield plateau. Yield increases at a decreasing rate as more input is applied up to X^*, past which yield plateaus at $\bar{Y}$. The QRP model is:

(2) $$y = \begin{cases} \beta_0 + \beta_1 \cdot X + \beta_2 \cdot X^2 & \text{if } X < X^* \\ \bar{Y} & \text{if } X \geq X^* \end{cases}$$

where X^* is an amount required to achieve the plateau yield. The parameter X^* is also the join-point of the QRP model, and links the slope of the response curve to the plateau, that is, $\bar{Y} = \beta_0 + \beta_1 \cdot X^* + \beta_2 \cdot X^{*2}$.

Both the LRP and QRP require imposing continuity at the join point. This constraint is required to identify the model's parameters. As such, estimation of the LRP and QRP requires using nonlinear least squares (Lambert & Cho, 2022), maximum likelihood (Ouedraogo & Brorsen, 2018; Dhakal et al. 2019), or Bayesian procedures (Moeltner et al., 2021). The approach taken here uses a Bayesian estimation procedure because it is easy to incorporate prior information into the response models.

LRP and QRP Estimation with Priors from External Sources

Bayesian estimation of the LRP and QRP generates posterior distributions of the maize yield parameters including intercepts, MPP, plateaus, and optimal input rates (Moeltner et al., 2021). Point estimates for the response parameters are calculated from the means, medians, or modes of posterior distributions.

Cho et al. (2020) used a Bayesian procedure to estimate an LRP for maize response to nitrogen. That study used priors to delineate lower and upper bounds on response parameters, which imposed theoretical restrictions with respect to parameter signs. For example, MPP and intercept terms are expected to be positive, and the intercept cannot exceed the plateau. The priors used in Cho et al. were from the univariate statistics calculated with the experimental data, from which the minimum, maximum, and average of yields were used as priors to bound parameter signs. Normally, priors should be external to a study and not a function of the data used to estimate parameters. The approach suggested here bypasses this issue by formulating hyper-priors from external response data using a model averaging approach suggested by Gelman et al. (2013) (discussion follows).

First, maize response to an input is assumed to be normally distributed with a mean response of μ_i and a standard deviation of σ. The response model is:

(3) $$y_i|\mu_i, \sigma \sim \mathrm{Normal}(\mu_i, \sigma), \;\; i = 1, \ldots, S$$

where y_i is maize yield from the ith plot and S is the number of observations. The yield plateau ($\bar{Y}$) of equations (1) and (2) are reformulated as a function of an optimal level of input X (for example, applied N fertilizer) and the intercept and MPP response parameters, β_0 and β_1. Formulated this way, the LRP's mean response function is:

$$(4) \qquad \mu_i = \min(\beta_0 + \beta_1 \cdot X_i, \beta_0 + \beta_1 \cdot X^*)$$

where X_i is applied N fertilizer and X^* is the amount of nitrogen required to achieve a biologically optimal yield (the plateau).

The QRP requires a constraint to impose differentiability at the join-point. The constraint results from the first order condition of the QRP, which involves identifying the join-point as $X^* = -\frac{\beta_1}{2 \cdot \beta_2}$. The mean response function for the QRP is:

$$(5) \qquad \mu_i = \begin{cases} \beta_0 + \beta_1 \cdot X_i - \frac{\beta_1}{2X^*} \cdot X_i^2 & \text{if } X_i < X^* \\ \beta_0 + \frac{\beta_1}{2} \cdot X^* & \text{if } X_i \geq X^* \end{cases}.$$

The prior for σ is the half-Cauchy distribution with a lower bound of zero and a standard deviation of 10 for the LRP and QRP models. The half-Cauchy is conducive to modeling residual outliers because of its relatively fat tails. The prior for the MPP is the normal distribution centered on zero and truncated above zero, with a standard deviation of 100. Set this way, the MPP priors are relatively diffuse and weakly informative with no strong assumption about the response parameter's location except that it is positive. A key contribution of the formulation in (4) and (5) is that it is written in terms of the optimal level of nitrogen, which will likely be easier to solicit its distribution from producers than trying to solicit the distribution of other plateau parameters like McFadden et al.'s (2018) specification would require.

Model Averaged Data from Published Sources and Priors

Bayesian estimation procedures provide an efficient method for combining prior information on a parameter's distribution with empirical data (Gelman et al., 2013). The influence of priors on the shape and location of a parameter's posterior distribution is greater when data are sparse, limited in some level of a treatment, or lack sufficient variability. Conversely, when the sample data adequately represents a population's variability, the information contained in a data set "overwhelms" the prior in terms of influence on the posterior distribution.

One approach to incorporate priors from published sources is to use directly the reported averages of point-estimates. An alternative approach is to leverage the variation in parameters reported in published sources to make hyper-priors for the intercepts and join-points. Gelman et al. (2013)'s one-way multilevel normal random effects model (MNRE) procedure, which is used here, incorporates data from external sources into estimation procedures as modeled-averaged hyper-priors. The MNRE model is similar to procedures that treat missing observations as unknown parameters (McElreath, 2020).

Consider, for example, the intercepts (β_{0j}), optimal N-fertilizer rates (X_j^*), and their respective standard errors ($\sigma_{\beta_{0j}}, \sigma_{X_j^*}$) reported in $j = 1$, …, 12 published articles. The averages and standard deviations of point estimates found in these external sources are treated as observed data. These means and variances of these external data are estimated simultaneously with the response function LRP and QRP (Table 2.2). The weighted averages of the external data are subsequently used as hyper-priors for the response parameters' distributions. Figure 2.1 summarizes the combined MNRE and LRP/QRP yield response models, their parameters, hyper-priors, and prior distributions.

The distributions characterizing the modeled averages and the standard deviations of the external response data, β_{0j} (published intercepts) and X_j^* (published optimal N rates), are (respectively):

(6) $$\beta_{0j} | \theta_{\beta_{0j}}, \sigma_{\beta_{0j}} \sim \text{Normal}\left(\theta_{\beta_{0j}}, \sigma_{\beta_{0j}}\right)$$

(7) $$X_j^* | \theta_{X_j^*}, \sigma_{X_j^*} \sim \text{Normal}\left(\theta_{X_j^*}, \sigma_{X_j^*}\right)$$

where $\sigma_{\beta_{0j}}$ and $\sigma_{X_j^*}$ are standard errors from the published data, and the unknown parameters $\theta_{\beta_{0j}}$and $\theta_{X_j^*}$ are linear functions:

(8) $$\theta_{\beta_{0j}} = \bar{\beta}_0 + \sigma_{\bar{\beta}_0} \cdot z_{1j}$$

(9) $$\theta_{X_j^*} = \bar{X}^* + \sigma_{\bar{X}^*} \cdot z_{2j},$$

The $(\bar{\beta}_0, \bar{X}^*)$ parameters are also unknown modeled averages of the published intercept and join-point data, $(\sigma_{\bar{\beta}_0}, \sigma_{\bar{X}^*})$ are modeled standard deviations of these data, and (z_{1j}, z_{2j}) are standardized normal random variables. The priors for the dispersion parameters, $\sigma_{\bar{\beta}_0}$ and $\sigma_{\bar{X}^*}$, are the half-Cauchy distribution, with a lower bound of zero and a standard deviation of 10. The prior for the modeled averages, $\bar{\beta}_0$ and $\bar{X}^*$, are normal, centered on zero with standard deviations of 100. This prior is relatively diffuse, meaning that its location over zero is uncertain.

The parameters $\bar{\beta}_0$, $\bar{X}^*$, $\sigma_{\bar{\beta}_0}$ and $\sigma_{\bar{X}^*}$ enter into the prior distributions of the response parameters (equations 4 and 5) as hyper-priors (Figure 2.1):

(10) $$\beta_0 \sim \text{Normal}\left(\bar{\beta}_0, \sigma_{\bar{\beta}_0}\right)$$

(11) $$X^* \sim \text{Normal}(\bar{X}^*, \sigma_{\bar{X}^*}).$$

The joint-point is calculated as the ratio of the difference between the plateau and intercept divided by the MPP. The normal distribution centered on zero with a standard deviation of 100 is used for the prior of slope parameter.

Estimation

Posterior distributions of the response parameters in equations 4, 5, 8 and 9 are recovered using R-Stan's Hamiltonian Monte Carlo No U-turn Sampler (HMC-NUTS) (Hoffman and Gelman, 2014). The HMC-NUTS procedure exhibits superior convergence properties and typically requires shorter chains to achieve convergence compared to the Gibbs or other standard Metropolis-Hastings samplers (Vehtari et al., 2021). Four chains were run in parallel with a warm-up of 5,000 iterations and an additional 5,000 iterations to generate joint posterior distributions for the MNRE and yield response parameters. Chain convergence was verified using Gelman and Rubin's (1992) diagnostic, $\hat{R}$. $\hat{R}$-statistics approaching one indicate convergence for a given parameter.

Small-Sample Properties of the Proposed Estimator

A Monte Carlo (MC) experiment evaluates the small-sample properties of the proposed procedure. Maize yield response parameters from Boyer et al. (2013) are used to generate yields with a 'true', known model. Boyer et al. (2013)'s intercept, slope, and plateau parameters are 2.58, 0.042, and 9.44 Mg ha^{-1}, respectively. The standard deviation for yield was 2.23 Mg ha^{-1}.

The small-sample properties are investigated at sample sizes $N = 75$ and 750. Three levels of applied nitrogen are 0, 123, and 247 N ha^{-1}. This means, for each treatment and sample size, there are 25 (250) treatment replications for $N = 75$ (750). The proposed MNRE method is compared to a Bayesian procedure that estimates yield response with limited treatment design data (no 0-N rates) but without hyper-priors. Data for the competing, model-averaging approach for hyper-priors, also excludes all 0-N treatments. Thus, N = 50 (500) for the response model with hyper-priors when check plots are deleted from the simulated data.

The MC steps follow. For sample size N = (75, 750) and for MC replication M = 1, 2, ..., 500, the data generating process (d.g.p.) is:

a. Draw random errors for maize yield, $e_i^* \sim \text{Normal}(0,1)$, i = 1,…, N;
b. Simulate N maize yields, $y_i^* = \min(2.58 + 0.042 \cdot X_i, 9.44) + 2.23 \cdot e_i^*$;
c. Estimate $(\hat{\beta}_{0m}, \hat{\beta}_{1m}, \hat{\bar{Y}}_m)$ using the full sample and no hyper-priors;
d. Remove 0-N rate check plots from the simulated data set of size N;
e. Re-estimate $(\tilde{\beta}_{0m}, \tilde{\beta}_{1m}, \tilde{\bar{Y}}_m)$ with $y_{i(-0)}^*, X_{i(-0)}$, using four different hyper-prior scenarios (discussed below), where “-0” indicates removal of 0-N rates from the data;
f. Return to (1) if $m < M$.

Sensitivity of the proposed procedure is evaluated by varying assumptions on the hyper-priors. The four different scenarios include: 1) no hyper-priors (‘reference scenario’), 2) both intercept and optimal N hyper-priors (‘B0 + NSTAR’), 3) intercept hyper-prior only (‘B0’), and 4) optimal N hyper-priors (‘NSTAR’). The reference scenario (the ‘control’) is the case where no hyper-priors are used but 0-N rate treatments are omitted. This case is compared with the other scenarios that used hyper-priors. For the case when no hyper-priors are used, the priors for the intercept and optimal N rate are the normal distribution with a mean of zero and a standard deviation of 100.

Bias and mean squared error (MSE) of the estimators are calculated to compare performance to a ‘reference’ case (that is, no hyper-priors and no 0-N check plots), and under the other assumptions for the hyper-priors. For each response parameter ($\boldsymbol{\theta}$ = intercept, slope, and plateau parameters), average bias is calculated as

(12) $$\boldsymbol{\theta}^{Bias} = \frac{1}{M}\sum_{m=1}^{M}\left(\boldsymbol{\theta}_m - \widehat{\boldsymbol{\theta}}\right)$$

and MSE is:

(13) $$\boldsymbol{\theta}^{MSE} = \frac{1}{M}\sum_{m=1}^{M}\left(\boldsymbol{\theta}_m - \widehat{\boldsymbol{\theta}}\right)^2$$

The bias and MSE are expected to decrease as sample size increases, but there are no expectations on how the different hyper-prior scenarios will affect bias and MSE. It is expected that MSE and bias will be lower for the response models that use hyper-priors even though the priors used have an optimal N level considerably below that of the simulated data.

Empirical Application

The LRP and QRP models developed above are used to compare the effects of N fertilization across three planting methods with under-replication of limiting and non-limiting fertilizer rates. Four different external prior scenarios are considered for each model: 1) no hyper-priors, 2) both intercept and optimal N hyper-priors (‘B0 + NSTAR’), 3) intercept hyper-priors (‘B0’), and 4) optimal N rate hyper-priors (‘NTSAR’). The estimated parameters of the intercept, MPP, and optimal N are compared across treatments.

Under-replicated data also complicates treatment comparisons. Normally, an ANOVA-type analysis would be conducted to statistically compare the intercept, slope, optimal N rates, and plateaus of each treatment. For example, a reference treatment would normally be selected and dummy variables for the other three treatment’s intercepts, slopes, and plateaus would enter the response equations as intercept or slope shifters. Formulated this way, the dummy variables are the average difference from the mean of the reference category. For some treatments, under-replication precludes the identification of intercept and MPP terms, which rules out the use of conventional ANOVA procedures.

Two alternative procedures are used here to address this complication. Both approaches pool the response data to identify treatment effects by including dummy variables. The first procedure compares planting method yields by introducing dummy variables, as follows:

(14) $$y_{ik} = \delta_{0k} \cdot \min(\beta_0 + \beta_1 \cdot X_{ik}, \beta_0 + \beta_1 \cdot X^*) + \varepsilon_{ik}$$

where δ_{0k} is parameter for treatment k. The reference treatment is the JD planter, and its dummy parameter is restricted to be one. Written this way, the intercept, slope, optimal N rate, and plateau are common to all treatments, but the dummy parameters measure the proportional shift in yield caused by a treatment. The null hypothesis is that the product of the treatment dummies with the intercept, MPP, and optimal fertilizer rate estimates are not different from β_0, β_1, or X^*.

The second procedure compares the effect of N treatments on MPP for each planter system. The augmented response model is:

(15) $$y_{ik} = \min(\beta_0 + \beta_1 \cdot X_{ik} + \sum_{k=1}^{3} \delta_{1k} \cdot X_{ik}, \beta_0 + \beta_1 \cdot X^*) + \varepsilon_{ik}$$

where δ_{1k} is dummy parameter for treatment k's MPP. The optimal N rate, intercept, and plateau are common to all treatments, but the MPP is allowed to vary according to the planting method. The JD planter is also the reference technology for this comparison. The priors for the dummy variables are the normal distribution with a mean of zero and a standard deviation of 10.

Results

Small Sample Properties of the Hyper-Prior Estimator

Yield plateau, MPP, and intercept estimates, their bias, and MSE are reported in Table 2.3. Relevant comparisons are the results estimated with hyper-priors and 'no 0-N rates' to the results estimated without hyper-priors and 'no 0-N rates' (cells shaded gray in Table 2.3). Bias and MSE for all parameters decreased as sample size increased, as expected.

Bias and MSE for the intercept terms estimated using only the intercept hyper-prior (B0) were larger than the bias and MSE of the intercept of the reference model (no hyper-priors) for both small (N = 50) and large (N = 500) samples. This result occurs because the intercept term reported in Boyer et al. was lower than the average of the intercept terms from prior studies (Table 2.2) used to formulate a hyper-prior. The same result obtains when the B0 + NSTAR hyper-priors are used. Bias and MSE for the estimator that only used the NSTAR hyper-prior was lowest relative to the reference group and the other hyper-prior scenarios. This finding suggests that, for this specific study and absent 0-N rates, a join-point hyper-prior provides enough information to identify the intercepts and MPP. If a different intercept value that was closer to the average of the published intercepts was used for the MC d.g.p., then the bias and MSE of the B0 and B0 + NSTAR hyper-prior scenarios would likely be lower.

A slightly different pattern was evident for the MPP estimates (Table 2.3). Compared to the reference group's bias and MSE, estimates found using the B0, B0 + NSTAR, and NSTAR hyper-priors were lower. MSE was substantially lower by at least a factor of 92 (= 0.013/0.000141). For the sample size of N = 50, the most precise MPP was estimated with the NSTAR hyper-prior, which had a bias of 0.003 and a MSE of 0.0000847. The result for the MPP was similar at N = 500.

Bias and MSE for the plateaus estimated with the B0 hyper-prior were comparatively larger than the reference group when the sample size was N = 50 (Table 2.3). The B0 + NSTAR and NSATR hyper-prior models performed better than the reference model at the smaller sample size in terms of MSE and bias. Plateau bias and MSE estimated with B0. B0 + NSTAR, and NSTAR were all lower than the reference bias and MSE when N = 500.

The small sample study highlights some important caveats of this procedure. First, success of the procedure, which is measured in terms of bias and MSE, depends on the external data used to develop hyper-priors and how well these priors approximate intercepts, MPP, and optimal N rates recovered from the experimental data. For example, Boyer et al.'s intercept used in the MC d.g.p. was lower than the average of the intercepts retrieved from published studies. Researchers are generally discouraged from using statistics from data as priors. However, if the raw yield average at 0-N is considerably lower than the intercept terms borrowed from external sources, some adjustments are needed. One adjustment would be to eliminate studies that qualitatively differed in some way from the experimental conditions. Second, it is unlikely there is a 'one-size-fits-all' hyper-prior that improves model fit when data are limited treatment design. The luxury an MC study affords in terms of choosing a best-performing estimator based on bias and MSE cannot be extended to empirical analyses. Rather, post-estimation information criterion such as Akaike's Information Criterion, the widely applicable information criterion (WAIC, McElreath, 2020), or some other model performance index would be required to select which hyper-priors contributed to the best-fitting model.

Empirical Application

Estimated yield response curves and the raw data are plotted in Figure 2.2. The 'no hyper-priors' model had steeper MPP estimates than those estimated with hyper-priors. The MPP estimates with the3 B0 hyper-prior also had steeper MPPs. Those two hyper-prior models, which were with no hyper-priors and B0 hyper-prior, over-estimated the yield plateau, as evidenced by their location above the raw data. The NSTAR or B0 + NSTAR hyper-prior models fit better the raw data. The highest level of N fertilizer (60 kg ha^{-1}) used in this experimental data was not enough to reach the join point connecting $N*$ and the plateaus. The $N*$ estimated with the NSTAR and NSTAR + B0 hyper-priors are closer to the average N fertilizer rate for [suppressed for review], which is 139 kg N ha^{-1} (Miller et al., 2017).

Table 2.4 reports the maize yield response parameters of all treatments. The best fitting models were the ones with the smallest WAIC (Watanabe, 2013), which were the QRP model with the B0 + NSTAR hyper priors. The HP intercept estimates is 5.29 Mg ha^{-1}, and the intercept estimates for HP and FP were 5.29 Mg ha^{-1} and 4.55 Mg ha^{-1}, respectively. The JD and JDUAN treatments used the same planter, but their intercept estimates are different, at 5.12 Mg ha^{-1} and 5.23 Mg ha^{-1}, respectively. Estimated optimal

N rates were similar regardless of the treatments, ranging from 120 kg N ha^{-1} to 123 kg N ha^{-1}. For the plateau estimates, HP and JD are 9.14 Mg ha^{-1} and 9.35 Mg ha^{-1}, respectively, which were higher than the two other treatments. The FP and JDUAN plateau estimates are 6.82 Mg ha^{-1} and 6.89 Mg ha^{-1}, respectively.

The B0 + NSTAR hyper priors worked best for each data set so that the B0 + NSTAR hyper-priors were used for pooled-model. The slope and plateau estimates were compared to identify treatment effects on Figure 2.-3 treatment were on Table 2.5. The differences with 90% of confidence intervals by each treatment and JD depicted for each proportional shift and the MPP parameters on Figure 2.3. The HP had significantly positive proportion shift than JD, which would have 17% more maize yields (Table 2.5). The other two treatments, FP (Treatment 2; Broadcast planted with a long wooden stick planter) and JDUAN (Treatment 4; Dribble surface band with UAN using John Deere 2-row MaxEmerge planter), were slightly lower than JD, but they were not statistically lower (Figure 2.3). From the differences of slope dummy parameters, HP had most steeper MPP, which could have more 0.0158 Mg ha^{-1} of maize yield by additional 1 kg of N fertilizer per hectare (Table 2.5). Moreover, the MPP of FP had statistically 0.0142 Mg ha^{-1} of maize yield lower than the MPP of JD. Therefore, this study showed HP have yielded more maize yields than other FP, JD, and JDUAN treatments. That might be because HP could lead efficient N fertilization as preventing loss of N from ammonia volatilization as reducing N fertilizer's exposure to direct heat (Dhillon et al., 2017). At the same time, FP had lowest N use efficiency, which has lower MPP than other treatments.

Conclusions

This paper introduced a Bayesian multilevel modeling approach for estimating yield response to inputs when data was limited in treatment levels. The procedure incorporates data from previous study on maize yield response to inputs. The proof-of-concept exercise used data from a plot trial comparing maize response to N under different planting methods. Two of the treatments planted with a John Deere 2-row MaxEmerge planter received three levels of N, with one of those levels a check plot of 0 applied N. The remaining two treatments with [suppressed for review] hand planter and stick planter did not receive 0-N check plots. The 0-N check plots are important for identifying yield intercepts. These N levels were not enough to identify linear/quadratic response plateau yields. Absence of these treatment levels is it challenging to estimate accurately MPP and yield plateaus.

The problem of limited information was addressed by applying a Bayesian estimation procedure that infilled missing information with hyper-priors based on maize intercept and optimal N rates reported in the literature. An important caveat is the quality, or relevance, of the priors used to condition yield response estimates. For example, if priors taken from experiments located in regions with different weather, soil types, or other factors affecting growing conditions are applied to experiments conducted in a region with substantially different growing conditions, then estimates may lead to inadequate model fits of the data and lead to erroneous treatment comparisons. The onus of judiciously choosing which priors are appropriate for a given experiment lies on the researcher, and the empirical example and the Monte Carlo study both reinforce this conclusion. As demonstrated by the Monte Carlo study, the 'true' parameter values were outside the range of priors retrieved from published articles, and some combinations of priors performed better for small and large samples. The empirical example is also demonstrative of this qualification, showing that while intercept, slope, and plateaus were estimated in all scenarios, the performance of some priors was superior in terms of model fit.

The proposed method also has implications for producers who wish to use their production data to conduct on-farm trials. For example, a producer could use multiple years of their production data to conduct using their typical input application rates and forgo implementing 0-fertilizer check plots. Likewise, a producer could use supplemental data from other farmers or experiments that applied nonlimiting input rates, thereby avoiding the extra costs of applying rates that exceed their usual nutrient management plans. The proposed approach could also be useful in the design of medium- to long-term experiments. Findings from previous trials could be used to fine tune input rates, given target yield goals.

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Table 2.1. Summary of Average Maize Yields by Treatment, Year and Location

			Treatment			
			1	2	3	4
		Nitrogen rate	HP	FP	JD	JDUAN
Year	Place	($kgha^{-1}$)	(N=48)	(N=48)	(N=72)	(N=72)
2013	Efaw	0	-	-	4.40	4.40
		30	7.04	5.89	5.81	5.74
		60	7.75	6.59	6.65	6.16
	LCB	0	-	-	8.73	8.73
		30	9.96	9.40	9.56	9.22
		60	13.29	11.69	11.96	9.24
2014	Efaw	0	-	-	6.15	6.15
		30	7.90	6.11	7.07	7.73
		60	8.75	6.57	8.00	6.66
	LCB	0	-	-	6.91	6.91
		30	7.76	5.33	7.33	8.73
		60	8.74	5.54	8.35	6.16
2017	Efaw	0	-	-	3.35	3.35
		30	4.73	2.88	2.76	2.70
		60	4.63	3.24	3.04	4.07
	LCB	0	-	-	3.90	3.90
		30	5.25	2.39	4.36	4.73
		60	5.23	2.46	4.68	4.66
2018	Efaw	0	-	-	2.95	2.95
		30	5.29	3.46	3.97	4.29
		60	4.95	3.77	5.12	4.58
	LCB	0	-	-	5.84	5.84
		30	7.98	7.08	7.55	6.61
		60	11.17	8.65	8.73	7.12

Notes: The unit is metric tons per hectare ($Mg\ ha^{-1}$). “HP” and “FP” were planted with a hand planter or a long wooden stick, respectively. “JD” and “JDUAN” were planted with a mechanical planter. “JDUAN” applied urea ammonium nitrate fertilizer. “LCB” and “Efaw” are the experimental field.

Table 2.2. Intercepts and Optimal N rates from Previous Studies

		Intercept		Optimal N Fertilizer	
Author	Year	Estimate	Standard Error	Estimate	Standard Error
Schmidt et al.	2002	8.71	2.33	87.43	26.30
Jaynes	2011	5.34	0.56	145.24	18.67
Shroyer et al.	2011	6.74	1.52	88.09	22.66
Gentry et al.	2013	5.33	0.94	192.77	52.50
Croizer et al.	2014	9.33	4.05	126.67	53.93
Halvorson and Bartolo	2014	5.65	0.44	274.00	21.00
Rajkovich et al.	2015	5.73	2.75	135.90	33.01
Kablan et al.	2017	5.37	2.01	195.32	29.89
Miller et al.	2017	6.48	2.90	139.00	68.64
Alotaibi et al.	2018	5.18	1.49	127.08	18.24
Ruark et al.	2018	6.63	3.25	126.50	30.50
Cho et al.	2020	2.78	0.51	83.75	27.66
Average		6.12		130.00	

Notes: Units are metric ton per hectare (Mg ha^{-1}). Units of the optimal N fertilizer rates are in (kg ha^{-1})

Table 2.3. Monte Carlo Simulation Results for Small Samples Properties by External Prior Scenarios

N	Missing 0-N rate?	Priors used?	Intercept Estimate	Intercept Bias	Intercept MSE	Slope Estimate	Slope Bias	Slope MSE	Plateau Estimate	Plateau Bias	Plateau MSE
75	No	No	2.893	0.133	0.184	0.121	0.079	0.014	15.642	5.532	78.853
50[a]	YES	No	3.879	1.119	3.342	0.127	0.085	0.013	16.387	6.277	91.506
	YES	B0	4.916	2.156	9.541	0.119	0.077	0.012	17.250	7.140	117.590
	YES	NSTAR	3.458	0.698	1.545	0.045	0.003	8.47E-05	11.087	0.977	2.731
	YES	BOTH	4.724	1.964	7.919	0.035	-0.007	1.41E-04	10.935	0.825	2.095
750	No	No	2.876	0.116	0.044	0.041	-0.001	3.31E-06	10.111	0.001	0.020
500[a]	YES	No	2.999	0.239	0.152	0.040	-0.002	4.02E-06	10.508	0.398	0.337
	YES	B0	4.508	1.748	6.135	0.028	-0.014	1.97E-04	10.425	0.315	0.226
	YES	NSTAR	2.950	0.190	0.122	0.041	-0.001	3.07E-06	10.444	0.334	0.243
	YES	BOTH	4.416	1.656	5.516	0.029	-0.013	1.76E-04	10.368	0.258	0.159

Notes: Units are metric tons per hectare (Mg ha^{-1}). "MSE" stands for mean squared error. "B0 + NSTAR" means both intercept and optimal N hyper-priors were used. "B0" and "NSTAR" are scenarios that used intercept hyper-priors only and optimal N hyper-priors only, respectively. The base comparisons are missing N rate = YES and prior used = NO (shaded cells). The 0-check plot is omitted (i.e., 25 and 250 observations dropped for N = 75 and 750, respectively). The gray-shaded cells are compared with the other scenarios where hyper-priors were used and 0-N rates omitted.

Table 2.4. Maize Yield Response to Nitrogen with Best Fitting Model and Hyper-Prior Scenarios

Treatment	N	Model	Hyper-priors used?	WAIC		Estimate	Standard Deviation	Effective Sample Size	$\hat{R}$
1 HP	48	QRP	B0 + NSTAR	241.80	Intercept	5.29	1.00	4735	1.00
					Slope	0.06	0.03	4104	1.00
					X*	121.28	17.19	2557	1.00
					Quadratic	-2.74E-04	1.71E-04	1992	1.00
					Plateau	9.10	0.96	6102	1.00
2 FP	48	QRP	B0 + NSTAR	240.90	Intercept	4.55	0.88	4397	1.00
					Slope	0.04	0.02	4261	1.00
					X*	121.81	16.68	3914	1.00
					Quadratic	-1.60E-04	1.72E-04	2154	1.00
					Plateau	6.80	0.83	6519	1.00
3 JD	72	QRP	B0 + NSTAR	343.50	Intercept	5.12	0.45	6901	1.00
					Slope	0.04	0.02	5918	1.00
					X*	122.67	16.44	4920	1.00
					Quadratic	-1.69E-04	7.35E-05	5162	1.00
					Plateau	7.59	0.72	6603	1.00
4 JDUAN	72	QRP	B0 + NSTAR	320.50	Intercept	5.23	0.38	6647	1.00
					Slope	0.03	0.04	533	1.00
					X*	120.34	20.56	1201	1.00
					Quadratic	-1.12E-03	2.42E-02	974	1.00
					Plateau	6.69	0.56	6684	1.00

Notes: Units are metric tons per hectare (Mg ha^{-1}). "MSE" stands for mean squared error. The best fitting hyper-priors were selected having smallest WAIC (Watanabe, 2013). "B0 + NSTAR" means both intercept and optimal N hyper-priors were used. "QRP" stands for the quadratic response plateau functions. "Effective Sample Size" and "$\hat{\boldsymbol{R}}$" is the efficiency and convergence diagnostics for the posterior chains.

Table 2.5. Results of the Maize Yield Response to Nitrogen with Both Intercept and Optimal N Hyper-Priors (N=216)

Procedure		Estimate	Standard Deviation	90% of C. I.	Effective Sample Size	$\hat{R}$
Whole	Intercept	5.14	0.33		4631	1.00
	Slope	0.03	0.01		4399	1.00
	X*	122.11	15.73		5325	1.00
	Plateau	8.80				
Dummy	HP	1.17	0.09	[1.02, 1.32]	6651	1.00
	FP	0.95	0.07	[0.84, 1.06]	6314	1.00
	JD	1.00				
	JDUAN	0.90	0.08	[0.77, 1.03]	7796	1.00
Log Posterior Likelihood		-514.53				
Slope	Intercept	5.14	0.33		4631	1.00
	Slope	0.03	0.01		4399	1.00
	X*	122.11	15.73		5325	1.00
	Plateau	8.80				
Dummy	HP	1.58E-02	9.05E-03	[0.0009, 0.0307]	9433	1.00
	FP	-1.76E-02	9.64E-03	[-0.0335,-0.0017]	10260	1.00
	JD	0.00			7761	1.00
	JDUAN	-1.42E-02	9.99E-03	[-0.0306, 0.0022]	7761	1.00
Log Posterior Likelihood		-512.50				

Notes: The unit is a metric ton per hectare (Mg ha^{-1}).“X*” stands for the optimal N, and the unit of N is a kilograms per hectare (kg ha^{-1}). “LRP” and “QRP” stand for the linear and quadratic response plateau functions, respectively. “Effective Sample Size” and “$\hat{\boldsymbol{R}}$” is the efficiency and convergence diagnostics for the posterior chains.

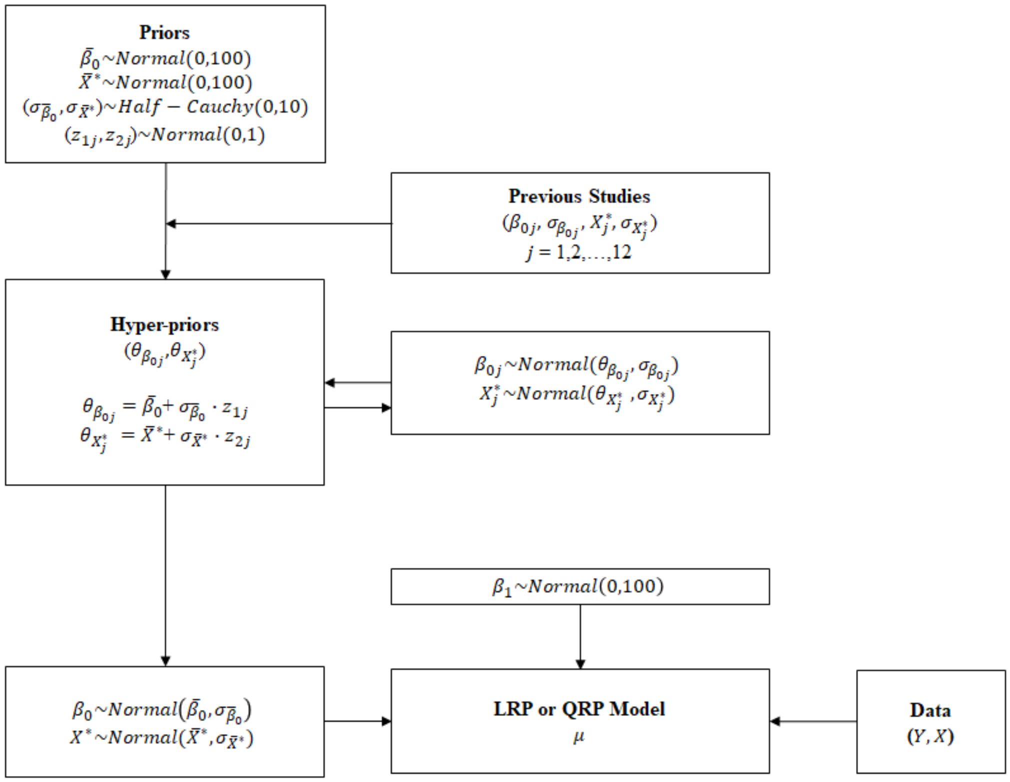

Figure 2.1. Summary of the multilevel normal random effects model (MNRE) and yield response models

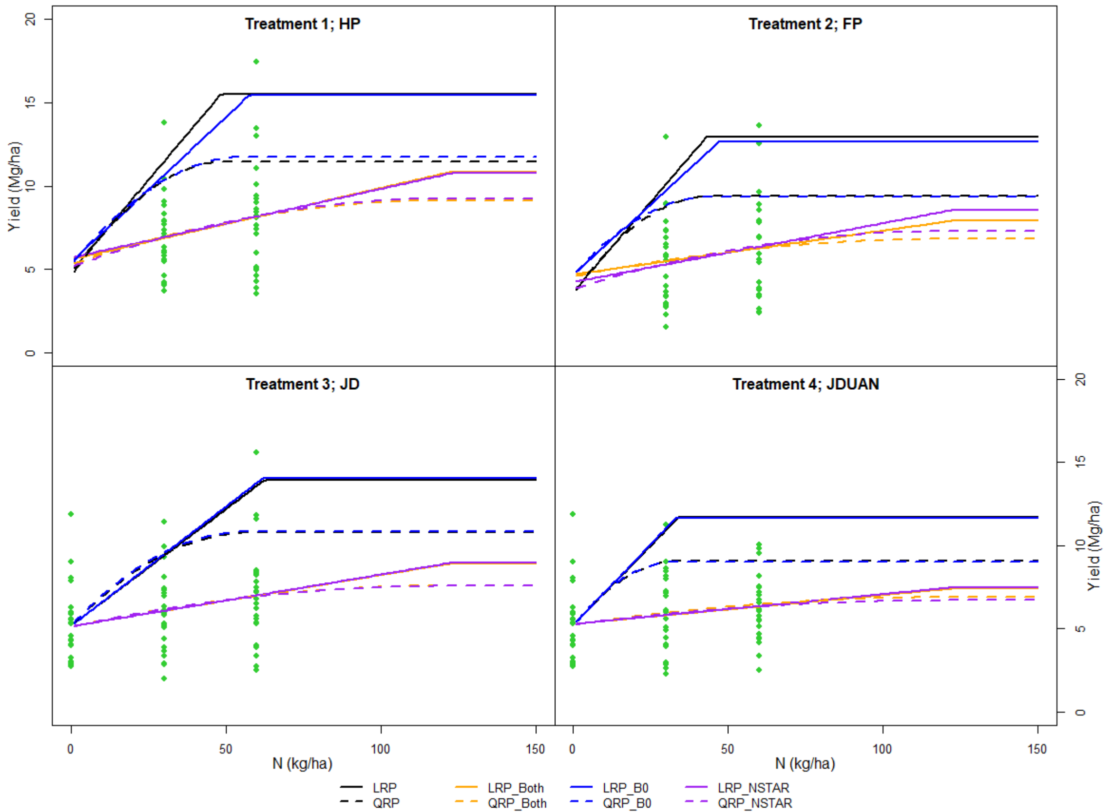

Figure 2.2. Plots of observed maize yields and estimated yield response

Notes. "LRP" and "QRP" stand for linear/quadratic response plateau functions. "Both" means using the both "B0" and "NSTAR" priors that are intercept and optimal nitrogen priors. "HP" and "FP" stand for the treatments that were planted by the hand planter and the long wooden stick, respectively, as well as having no check plots. "JD" and "JDUAN" were planted by a mechanical planter, and "JDUAN" is a treatment that applied urea ammonium nitrate fertilizer.

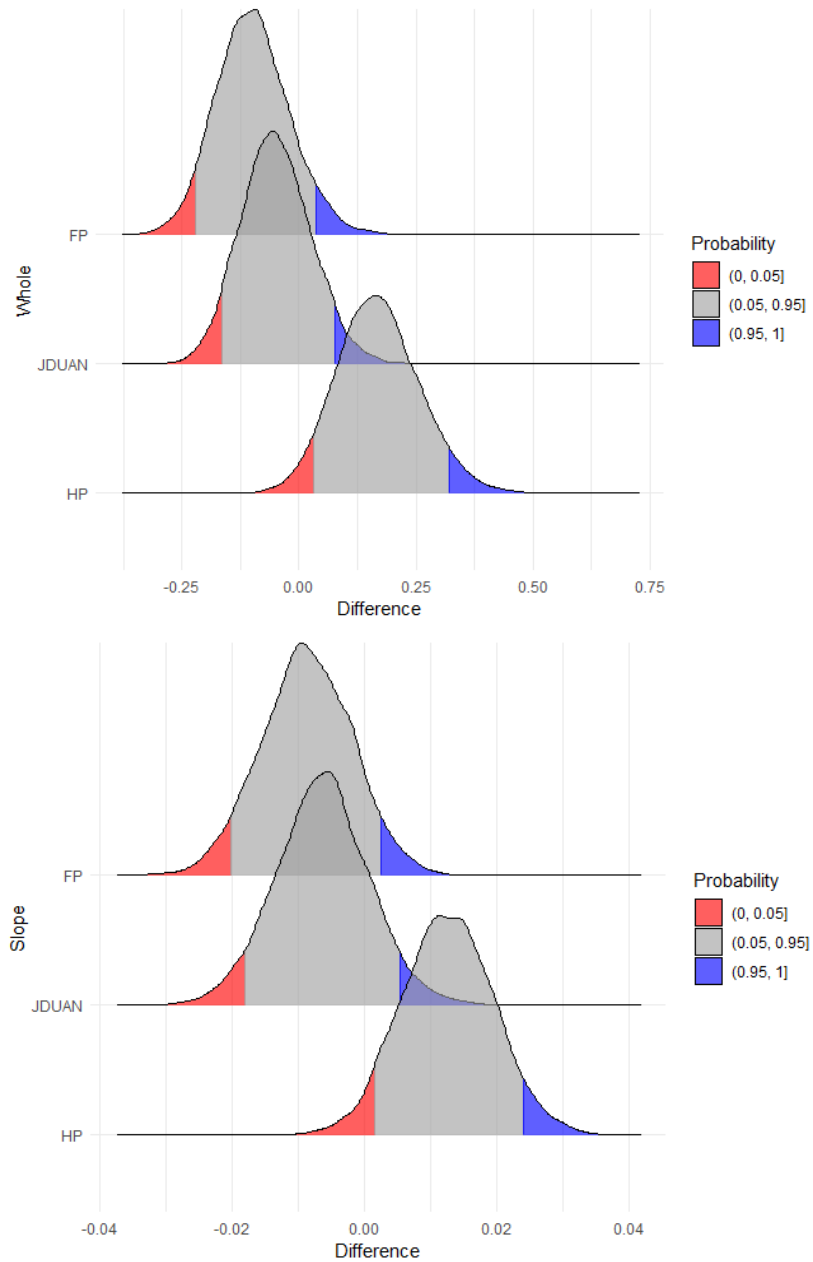

Figure 2.3. Plots of differences between treatments for whole model and slope parameter

Notes. The reference treatment is “JD” planted by a mechanical planter. “HP” and “FP” stand for the treatments that were planted by the hand planter and the long wooden stick, respectively, as well as having no check plots. “JDUAN” was planted by a mechanical planter, but applied urea ammonium nitrate fertilizer unlike “JD”. The 90% of confidence intervals are depicted. The unit is a metric ton per hectare (Mg ha^{-1}) for the proportional shift in yield and the unit of slope is a kilograms per hectare (kg ha^{-1}).

CHAPTER III

SHOULD PHOSPHORUS FERTILIZER RECOMMENDATIONS FOR WHEAT PRODUCTION BE BASED ON SUFFICIENCY OR ON BUILD-MAINTENANCE?

Abstract

There are two alternative approaches to managing soil phosphorus (P): sufficiency and build-maintenance. Sufficiency seeks to apply the minimum amount of P fertilizer that the crop needs in that year. At higher yield potential or intensive crop rotation, the crops consume more P than applied amounts of P fertilizer with a sufficiency approach. As the soil P level decreases due to higher crop removal, the expected crop yield decreases over time until an equilibrium is reached. The build-maintenance (BM) approach, however, seeks to build and (or) maintain the soil P level for crops so that P is not the limiting nutrient. However, the BM recommendation rate costs more in the short-term because it requires a higher rate than the sufficiency recommendation rate. The producer's long-term returns will differ depending on each approach. This study compares the expected net present values (NPV) of two alternative recommendation rates with three different yield potentials, which are 2690, 4035, and 5380 kilograms per hectare, under 4-, 8-, and 20- year planning horizons. With a 20-year planning horizon, BM was always preferred. The sufficiency recommendation rates had higher NPV with the short 4-year planning horizon.

Introduction

Phosphorus (P) is a vital nutrient for crop production and can be a yield limiting factor (Fageria & Baligar, 1999; Hinsinger, 2001; Takahashi & Anwar, 2007). Since P is an immobile nutrient with relatively little leaching by water, P not removed by the harvested crop is carried over to the next year (Sims et al., 2000; Kuo et al., 2005). Many soil testing labs (public university and commercial) offer P fertilizer recommendation rates based on one of the two alternative approaches to managing soil P. One approach is sufficiency that seeks to supply the least amount of P needed to achieve 90 to 95% of maximum yield (Leikam et al., 2003a). Since available P is utilized, a minimal amount of P fertilizer can be applied while satisfying crop uptake. High economic return in a given year can be achieved due to the P fertilizer cost savings (Leikam et al., 2003b). At higher yield potential, the equilibrium soil P level will fall, which requires more P fertilizer and provides reduced crop yields. In the long-term, it is not clear that sufficiency will have the highest economic returns.

The other approach is build-maintenance that seeks to provide abundant nutrients to the soil so that P deficiency does not limit yield (Leikam et al., 1983; Wagar et al., 1986). Build-maintenance recommends building soil P to a target level and maintaining it there. Since it is costly to build up to the target in a single year or two, it is usually recommended to reach the target over four to eight years. Once soil P level reaches the target soil P level, build-maintenance recommends applying P fertilizer at crop removal amounts to maintain the target soil P level. Build-maintenance recommends higher rates than the sufficiency approach until reaching the equilibrium soil P level. Build-maintenance costs more than sufficiency in the short-term. On the other hand, the equilibrium soil P level of sufficiency may have a P deficiency, which would result in differences in crop yields in the long-term.

How best to choose between sufficiency and build maintenance? Two questions are raised for comparison of two approaches. Several studies showed both sufficiency and build-maintenance P recommendation rates have no perceptible crop yield difference in the same year (McCallister et al., 1987; Kaiser et al., 2005; Wortmann et al., 2009). On the other hand, Wagar et al. (1986) showed wheat grain yields increased over 5 years with build-maintenance recommendation rates. Corn yield with build-maintenance recommendation rates also increased 3.3% more than corn yield with sufficiency recommendation rates over 6 years (Wortmann et al., 2018). Hanzra (2021) found build-maintenance recommendation rates built up to sufficient soil P level over 8 years, resulted in 12% more wheat grain yield than yield with the sufficiency recommendation rates.

Many extension papers suggested producers could achieve 100% of yield potential when applying either sufficiency or build-maintenance P recommendation rate (Leikam et al., 2003b; Mallarino et al., 2013; Zhang et al., 2017). In practice, the 100 % of yield potential will not be guaranteed at low soil P level even if P fertilizer is applied (Dodd & Mallarino, 2005; Balemi & Negisho, 2012; Agegnehu et al., 2015). Agegnehu et al. (2015) also showed only 40% of relative wheat grain yield was achieved at low soil P level in the central Ethiopian highlands, despite the application of P fertilizer. There is still considerable uncertainty about crop yield response to P fertilizer under sufficiency and build-maintenance.

Ultimately, it is necessary to know the crop yield differences at the respective equilibrium soil P levels to properly compare the sufficiency and build-maintenance P recommendation rates. This is because both sufficiency and build-maintenance require similar amounts of P fertilizer at the respective equilibrium soil P levels, while their equilibrium levels are different. This study collects meta-data from previous literature to estimate the crop yield differences at the respective equilibrium soil P levels. The percentage of crop yield loss under the sufficiency approach is used for the different yield loss scenarios that can take into account uncertainty.

According to Oklahoma agricultural soil test summary 2014-2017, 54% of sampled fields in Oklahoma were deficient in P and needed to be treated (Zhang & McCray, 2018). Oklahoma producers may want to know which P recommendation approach would be more profitable. The recommended approach could vary depending on their farm plan and field condition such as yield potential and current soil P level. This study aims to guide wheat producers in Oklahoma by comparing the net present value (NPV) of P recommendation with different approaches over varied time-period farm plans. This study considers three different factors of the farm plan: (i) time length of farm plan, (ii) soil P test level in beginning year, and (iii) wheat grain yield potential on the field. Each factor has three different levels; three time-periods, which are considered 4, 8, and 20 years, three soil P test levels in beginning year, which are 5, 15, and 25 Mehlich-3 (M3) ppm, and three wheat grain yield potentials, which are 2690, 4035, and 5380 kilograms per hectare. Ultimately, this study simulates 27 scenarios. The changes of soil P level over time are considered in the simulation. Due to uncertainty about the yield losses under the sufficiency approach, two different yield loss scenarios are considered.

Materials and Methods

Sufficiency Recommendation Rate

In the sufficiency approach, the recommendation rate is the minimum amount of P fertilizer to obtain 90-95% of the wheat yield potential at the soil test level of 15 M3 ppm. Since this approach utilizes the available P in the soil to satisfy the wheat crop uptake, the recommendation rate is determined by the soil P test value only. Warren et al. (2017) assume that the P recommendation rate does not depend on the wheat yield potential unlike mobile nutrients such as nitrogen and sulfur. Oklahoma cooperative extension service by Oklahoma State University recommends different rates by the soil P test value (Macnack et al., 2017; Warren et al., 2017). Following Warren et al. (2017), the Oklahoma state sufficiency recommendation rate (kilograms of P_2O_5 per hectare) is

(1) $$\text{OK State Sufficiecy Rate} = \max(90 - 3.23 \cdot SoilP, 0)$$

where $SoilP$ is the soil P test value in M3 ppm.

Since high yields result in more P being removed with the crop, the equilibrium soil P level would be lowered at higher wheat yield potential. Likewise, the soil P level can keep increasing in low wheat yield potential fields if recommendation rates are only based on the soil P level. Kansas State University agricultural experiment station and Cooperative Extension Service offers the adjusted sufficiency recommendation rate depending on the wheat yield potential as well as soil P level (Leikam et al., 2003a; 2003b). Following Leikam et al. (2003b), the Kansas state sufficiency recommendation rate (kilograms of P_2O_5 per hectare) is

(2) $$\text{K State Sufficiecy Rate} = \max(52 + 0.47 \cdot \text{YG} - 2.20 \cdot SoilP - 0.02 \cdot \text{YG} \cdot SoilP, 0)$$

where YG is the wheat grain yield potential.

Build- Maintenance Recommendation Rate

Build-maintenance seeks to always meet the nutritional needs of wheat by maintaining rich P in the soil. The build-maintenance recommendation rate target to maintain a sufficient soil P level assumed to be 25 M3 ppm (Leikam et al., 2003b; Macnack et al., 2017; Zhang et al., 2017). The recommendation rates depend on the initial soil P, planning years, and wheat yield potential. Initial soil P is used to calculate the total amount of P fertilizer required to reach 25 M3 ppm. The total required amounts of P fertilizer are usually too much for a single year, so that build-maintenance recommendation rates are recommended after dividing total amounts by planning years. In practice, the amounts of P fertilizer for wheat crop should also be applied, additionally. Since different amounts of P will be removed by actual wheat grain yields, the recommendation rates would differ by wheat yield potentials. If reaching 25 M3 ppm or over, the crop removal or less amounts of P fertilizer recommend to maintain the 25 M3 ppm. The build-maintenance (BM) recommendation rate (kilograms of P_2O_5 per hectare) is

$$\text{(3)} \qquad \text{Build} - \text{Maintenance Rate} = \frac{(504 - 18 \cdot SoilP)}{\text{yr}} + \text{P}_2\text{O}_5 \text{ removal}$$

where yr is the build-up planning years that are four and eight years in this study, and wheat grain removes 0.008 kilograms of P_2O_5 per kilogram (Leikam et al., 2003b). In other words, wheat grain takes 22 and 32 kilograms of P_2O_5 when the yield potentials are 2690, and 4035 kilograms per hectare, respectively. Table 3.2 shows the four recommendation rates given the soil P level, which are Oklahoma and Kansas state sufficiency rates, and 4 and 8 years build-maintenance rates.

Soil Phosphorus Level Changes

Generally, the amount of P lost in the runoff is considered inconsequential (Hart et al., 2004; Hussain et al., 2021). Nevertheless, in practice, there are many random effects. For example, sometimes, the soil P level increased even when no P fertilizer was applied on the field (McCallister et al., 1987). Agegnehu et al. (2015) found that a higher proportion of applied P fertilizer was accumulated at higher soil P levels. For simplicity, this study uses a deterministic simulation. This study also assumes that P loss is an immobile nutrients although P can be leached in sandy soils (Wyatt et al., 2019) and can be lost with soil erosion. Therefore, the remaining soil P level in the following year can be estimated based on the initial soil P level in a given year, the amount of fertilizer applied, and the amount removed with the crop. The soil P level in a following year is

(4) $$SoilP_{t+1} = SoilP_t + \frac{R_k - 0.008 \cdot Y_t}{19}$$

where $SoilP_{t+1}$ is the M3 soil P test value following year $t + 1$, and R_k is the recommendation rates by recommendation rate approach k (k = OKSuff, KSuff, BM4, and BM8), and Y_t is the expected wheat grain yield in year t. The 19 kilograms of P_2O_5 per hectare is the amount to change one M3 ppm (Hergert & Shaver, 2009; LaBarge & Lindsey, 2012).

Wheat Grain Yield

This study uses a relative wheat grain yield response function to determine expected wheat grain yield potential. The relative wheat grain yield is expressed as a percentage of how much wheat yield potential can be obtained depending on the soil P test level. The soil 25 M3 ppm is assumed a target level that can get 100% of wheat grain yield potential without P fertilizer application (Leikam et al., 2003b; Macnack et al., 2017).

The crop yields of seven previous studies were collected to compare the crop yield differences at the respective equilibrium soil P levels (Table 3.1). The collected crop yields followed the criteria that the same amounts of P fertilizer were applied at respective equilibrium soil P levels, which are 10-20 ppm for sufficiency and over 25 ppm for build-maintenance. The collected crops varied, which are corn, soybean and wheat, so that the differences in percentage were used. The differences in percentage were tested under the null hypothesis that there is no percentage difference between crop yields at respective equilibrium soil P levels. The meta-analysis showed a significant percentage of crop yield loss under the sufficiency approach, which averaged 10% less at equilibrium soil P level of sufficiency approach than at build-maintenance equilibrium soil P level (Table 3.1).

Macnack et al. (2017) and Zhang et al. (2017) assumed that the expected wheat grain yield would be 100% of yield potential if P fertilizer is applied following the sufficiency recommendation rate. However, as shown in Table 3.1, it is questionable whether 100% of the relative wheat grain yield potential can be achieved when the soil P test level is very low. Vitosh et al. (1995) and Leikam et al. (2003b) recommended applying at least 25 to 50% of recommended rates as a banded P application at low soil P test level. This is because the wheat crop has difficultly to take available nutrients from P deficient soils. In other words, the relative wheat grain yield could not achieve the 100% of yield potential at low soil P level, even if sufficient P fertilizer is applied (Agegnehu et al., 2015).

Therefore, this study assumes the relative wheat grain yields differentiate depending on the soil P test level even if P fertilizer is applied according to the recommendation rate. Our summary of past research showed that equilibrium soil P level for sufficiency achieved 90% of maximum yield. Moreover, following Leikam et al. (2003b)'s statement that "sufficiency recommendation rate can achieve about 90 to 95 percent of maximum yield", two possible percentage of relative wheat grain yield response are assumed as the 90% and 95% at the soil P test level of 15 M3 ppm. The soil P test level of 15 M3 ppm is the equilibrium soil P level. That would be reached and maintained when using the sufficiency approach for a long time in the wheat field with yield potential of 2690 kilograms per hectare. It is also assumed that 100% of relative wheat grain yield response can be achieved at the soil P test level of 25 M3 ppm and the function is linear. Then, at 0 M3 ppm, the 75% and 87.5% of relative wheat grain yield response assume to be yielded even if recommended P rates is applied. The relative wheat grain yield response functions with P fertilizer application are

(5) $$G_t = 0.010 \cdot SoilP_t + 0.750,$$

which is assumed relative wheat grain yield response is 90% at the soil P test level of 15 M3 ppm,

(6) $$G_t = 0.005 \cdot SoilP_t + 0.875,$$

which is assumed relative wheat grain yield response is 95% at the soil P test level of 15 M3 ppm, and the expected wheat grain yield is

(7) $$Y_t = G_t \cdot \mathrm{YG}$$

where G_t is a function of relative wheat grain yield with P fertilizer application, and Y_t is the expected wheat grain yield response function. Figure 3.1 depicts the relative wheat grain yields by soil P test level with and without P fertilizer applied, and the relative wheat grain yield without P fertilizer applied is from Zhang et al. (2017).

Net Present Value

To compare the NPV of P recommendation with different approaches over varied time-period farm plans, the objective function is

(8) $$\max_{k} \ NPV(k, \mathrm{SoilP_0}, \mathrm{YG}) = \sum_{t=0}^{T} \left(\frac{\pi_t(k, \mathrm{SoilP_0}, \mathrm{YG})}{(1+i)^t}\right)$$

$$\mathrm{s.t.}\ k = \{\mathrm{OKSuff, KSuff, BM4, and\ BM8}\}$$

$$\mathrm{SoilP_0} = \{5, 15, 25\}$$

$$\mathrm{YG} = \{2690, 4035, 5380\}$$

$$\pi_t(k, \mathrm{SoilP_0}, \mathrm{YG}) = p \cdot Y_t(k, G_t(\mathrm{SoilP}_t), \mathrm{YG}) - r \cdot R_t(k, \mathrm{SoilP_0}, \mathrm{YG})$$

$$R_t = \begin{cases} R(k, \mathrm{SoilP}_t, \mathrm{YG}) & t = 0, 4, 8, 12, 16 \\ R(k, \mathrm{SoilP}_{t-1}, \mathrm{YG}) & t = \mathrm{others} \end{cases}$$

where NPV is the per hectare net present value of returns (in dollars) by different P recommendation approaches k (k = OKSuff, KSuff, BM4, and BM8), soil P test levels in beginning year $\mathrm{SoilP_0}$, which are 5, 15, and 25 ppm, and wheat grain yield potential, YG, which are 2690, 4035, and 5380 kilograms per hectare, T is the length of the decision maker's planning horizon in years, which are 4-, 8-, and 20-year, π_t is the profit function of wheat production in year t, Y_t and R_t are the functions of wheat yield and recommendation rate in year t, G_t is the function of relative wheat grain yield in year t, p is the price of wheat, \$0.17 per kilogram (USDA, 2021), i is the discount rate, 3% as I nterest rate for farm ownership loan (USDA Farm Service Agency, 2021), and r is the phosphorus fertilizer (P_2O_5) cost, \$0.39 per kilogram (Farmers Coop Association of Snyder, 2021). This study assumed that wheat producer conducts soil test every four years, and that the P recommendation rate changed every four years.

Results

The changes of soil P level are shown in Figure 3.2. BM recommendation rates maintain stable soil P test level over 25 M3 ppm that 100% of relative wheat grain yield response can be achieved. On the other hand, the soil P level converged to an equilibrium level of soil P 8 or more years later, when Oklahoma State or Kansas State sufficiency recommendation rates were applied. The equilibrium levels of soil P with sufficiency approaches were located below 25 M3 ppm and differed by yield goal. Higher yield goal led to convergence to a lower level of soil P.

The NPV by different scenarios is reported in Tables III-3 and III-4. When relative wheat grain yield response was assumed to be 90% at the soil P test level of 15 M3 ppm, Kansas State sufficiency recommendation rates were always the most profitable in the 4-year planning horizon. In the 4-year planning horizon, the build-maintenance recommendation rates with 8-yr building up plan had higher NPV than 4-yr building up plan by saving the costs of P fertilizer. However, the build-maintenance recommendation rates with 4-yr building up plan finishes the four years with a higher soil P than sufficiency and build-maintenance with 8-yr building up plan. In the 4-year planning horizon, the P remaining in the soil is given zero economic value. In 8-year or 20-year planning horizons, build-maintenance recommendation rates had an economic advantage. Build-maintenance recommendation rates could take an advantage of 100% of yield potential. The build-maintenance rates with 4-yr building up plan had an economic advantage over 8-yr building up plan in 8-year or 20-year planning horizons. Since the build-maintenance rates with 4-yr building up plan could build to the soil P test level of 25 M3 ppm in a shorter period than the 8-yr building up plan, it reached 100% of yield potential during more years. Only when the wheat field already had enough soil P and yield potential is 2690 kilograms per hectare, Kansas State sufficiency recommendation rates had higher NPV in the 8-year planning horizon.

On the other hand, the results changed when the relative wheat grain yield response was assumed to be 95% at the soil P test level of 15 M3 ppm (Table 3.4). Kansas State sufficiency recommendation rates had highest NPV in 4-year and 8-year planning horizons. Under this assumption, Kansas State sufficiency recommendation rates were also most profitable in 2690 and 4035 kilograms yield potentials per hectare in 8-year planning horizon. This is because the 5% increment of expected wheat grain yield was not enough to offset the higher P fertilizer costs over the 8 years. The costs of P fertilizers could be a main driving factor for the NPV in the short-term. In the long-term planning horizons, build maintenance recommendation rates are still most profitable due to having higher yields.

When wheat price was assumed to be above average, which is $0.27 per kilogram, sufficiency approach was only most profitable in 4-year planning horizon (Tables 5-6).

Even in 4-year planning horizon, the build-maintenance recommendation rates had NPV approximately the same as the sufficiency recommendation rates when yield potentials are 5380 kilograms per hectare (Table 3.5). When 95% of yield potential was assumed at the soil P test level of 15 M3 ppm, the sufficiency approach still had higher NPV than build-maintenance approach in 4-year planning horizon (Table 3.6). The results consistently show that sufficiency is a short-run strategy that is only optimal due to giving no value to higher levels of soil P at the end of the planning period.

Conclusion and Discussions

This study compared NPV of P recommendation with different approaches over varied time-period farm plans. Two possible relative wheat grain yield responses were assumed to compare due to uncertainty about the respective crop yield response to P fertilizer under sufficiency and build-maintenance. In the short-term, sufficiency approach was more profitable than build-maintenance. Even though sufficiency approach had yield loss from insufficient soil P under most assumptions considered, the cost saving offset the yield losses in a 4-year planning horizon. On the other hand, in a 20-year planning horizon, build-maintenance recommendation rates were always more profitable than the sufficiency recommendation rates. With an 8-year planning horizon, conclusions varied depending on assumptions. Build-maintenance was favored with high crop prices, high yields, assuming only 90% of yield is possible at 15 M3 ppm.

This study had some limitations related to the relative wheat grain yield response function. In practice, the relative wheat grain yield response function after applied P fertilizer could have positive curvature instead of being linear like this study assumed.

This is because the ability of wheat crop to take P nutrients might be better at higher soil P levels (Balemi & Negisho, 2012; Sucunza et al., 2018). Likewise, the effects of applied P fertilizers on wheat grain yield are not as effective as theory, especially at low soil P levels (Agegnehu et al., 2015).   Therefore, the assumption could be too optimistic that P fertilizers could achieve 75% or 87.5% of relative wheat grain yield at the soil P test level of 0 M3 ppm. If the relative wheat grain yield after P applied is lower than this study's assumption, the build-maintenance recommendation rate could be more profitable than sufficiency recommendation rate even in the short-term.

Appendix

Experiments in Oklahoma

Two long-term continuous winter wheat P fertilizer experiments have been conducted in Oklahoma: Magruder plots and experiment 502. The Magruder plots located in Stillwater, OK were initiated in 1892, where had Krikland Silt Loam (fine, mixed, thermic Uderic). Magruder plots have two different P level treatments, which applied 0 kg and 34 kg P_2O_5 per hectare with 0 kg N per hectare for both. Experiment 502 located in Lahoma, OK was established in 1970. Soil at experiment 502 is classified as Grant Silt Loam (fine-silty, mixed, superactive, termic, Udic Arigiustoll). Experiment 502 had 5 different P level treatments, which are 0, 22, 45, 67, and 90 kg P_2O_5 per hectare. All treatments have the same rates of nitrogen and potassium fertilizers, which are 67 kg N per hectare and 67 kg K_2O per hectare.

These real experimental data from two different stations are likely to bring some research questions about the actual wheat grain yield response function corresponding to applying P fertilizer following two different approaches as well as the changes of soil P level. In the Magruder plots, the 34 kg of P_2O_5 led to soil P level being maintained around soil 30 M3 ppm (Figure 3.3) so it is higher than the build-maintenance rate. On the other hand, when no P fertilizer was applied, the soil P level decreased to soil 5 M3 ppm. The wheat grain yield was 11% less with no P applied than when P was applied. Even when nitrogen was constraining yield, P still mattered. It still provides some weak evidence to support this study's assumption that 5 to 10% less wheat grain yield would be achieved at soil 15 M3 ppm than at over soil 25 M3 ppm.

In Experiment 502 plots, even when no P fertilizer was applied and wheat crops were harvested over 40 years, the soil P level maintained or built-up (Figure 3.4). All Experiment 502 plots had over soil 40 M3 ppm, which is higher than the recommended target of 25 M3 ppm. The Experiment 502 plots illustrate the uncertainty surrounding P fertilizer recommendations. Based on current P recommendations, all treatments should have the same yield. Yet, there was a yield boost from the highest levels of P.

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Table 3.1. Crop Yields Differences at Equilibrium Soil P Level of Sufficiency (10-20 ppm) and Build Maintenance (> 25 ppm) From Seven Previous Studies

Author	Year	Region	Duration	Crop	Sufficiency	Build-Maintenance	Difference
McCallister et al.	1987	Nebraska	6-yr	Wheat	2480	3230	23%
Dodd & Mallarino	2005	Iowa	27-yr	Corn	9814	10136	3%
			27-yr	Soybean	2854	2964	4%
Kaiser et al.	2005	Iowa	4-yr	Corn	10880	11930	9%
Slaton et al.	2007	Arkansas	2-yr	Wheat	2421	3161	23%
Wortmann et al.	2009	Nebraska	3-yr	Corn	14080	14120	0%
Wortmann et al.	2018	Nebraska	6-yr	Corn	10290	10450	2%
Singh & Brar	2022	India	10-yr	Corn	4807	5664	15%
			5-yr	Wheat	4264	4724	10%
					Paired t-Test	Mean	10%
						DF	8
						t-value	3.33
						p-value	<.0001

Note. The unit of crop yield is kilograms per hectare (kg ha-1). The Sufficiency and Build-Maintenance applied same amounts of P fertilizer, at respective equilibrium soil P level (10-20 ppm for Sufficiency, and over 25 ppm for Build-Maintenance).

Table 3.2. Phosphorus Fertilizer Rates for Wheat Production Given Soil P Test Levels

Yield Potential	2690 kg ha^{-1}				4035 kg ha^{-1}			
Mehlich-3 ppm	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8
0	90	71	148	85	90	80	160	97
1	85	67	143	83	85	76	155	94
2	81	64	139	81	81	73	150	92
3	76	62	133	78	76	69	144	90
4	72	58	129	75	72	66	140	86
5	67	55	123	73	67	63	134	84
6	63	53	119	71	63	59	130	82
7	59	49	113	68	59	56	124	80
8	56	46	109	65	56	53	120	76
9	52	44	103	63	52	49	114	74
10	48	40	99	60	48	46	110	72
11	45	37	93	58	45	43	104	69
12	41	35	88	55	41	39	100	66
13	38	31	83	53	38	36	94	64
14	35	28	78	50	35	33	90	62
15	33	26	73	48	33	29	84	59
16	29	22	68	45	29	26	80	56
17	27	19	63	43	27	22	74	54
18	24	17	58	40	24	19	69	52
19	21	13	53	38	21	16	64	49
20	19	10	48	35	19	12	59	46
21	17	8	43	32	17	9	54	44
22	15	4	38	30	15	6	49	41
23	13	1	32	28	13	2	44	39
24	11	0	28	25	11	0	39	36
25	10	0	22	22	10	0	34	34
26	8	0	18	20	8	0	29	31
27	7	0	12	18	7	0	24	29
28	6	0	8	15	6	0	19	26
29	4	0	2	12	4	0	13	24
30	3	0	0	10	3	0	9	21
31	2	0	0	8	2	0	3	19
32	1	0	0	4	1	0	0	16
33	0	0	0	2	0	0	0	13
34	0	0	0	0	0	0	0	11
35	0	0	0	0	0	0	0	9
36	0	0	0	0	0	0	0	6
37	0	0	0	0	0	0	0	3
38	0	0	0	0	0	0	0	1
39	0	0	0	0	0	0	0	0

40	0	0	0	0	0	0	0	0

Notes. The unit of P_2O_5 fertilizer rate is kilograms per hectare (kg ha^{-1}). "OKSuff" and "KSuff" mean the sufficiency recommendation rates of Oklahoma State and Kansas State. "BM4" and "BM8" stand for the build-maintenance recommendation rate with the 4-yr and 8-yr building up plans to the target level, respectively.

Table 3.3. Results of Net Present Values by 4-/8-/20-year Planning Horizons When 90% of Relative Wheat Grain Yield Response at the Soil P Test Level of 15 M3 ppm.

			Initial Soil P level											
			5				15				25			
		Years	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8
Yield	2690	5	328	343	288	329	403	416	377	398	478	485	467	467
Potential		10	698	699	714	671	792	794	799	778	886	891	884	884
(kg ha^{-1})		20	1649	1606	1714	1681	1750	1710	1800	1783	1851	1819	1885	1885
	4035	5	531	538	493	524	623	631	596	612	716	721	700	700
		10	1084	1079	1129	1068	1198	1192	1227	1197	1313	1311	1326	1326
		20	2472	2420	2629	2579	2594	2539	2728	2703	2716	2665	2827	2827
	5380	5	730	732	698	721	840	844	816	827	950	954	934	934
		10	1457	1455	1544	1466	1592	1581	1656	1617	1726	1716	1767	1767
		20	3256	3217	3545	3479	3398	3346	3657	3625	3540	3484	3770	3770

Note. The unit of NPV is dollars per hectare ($ ha^{-1}). “OKSuff” and “KSuff” mean the sufficiency recommendation rates of Oklahoma State and Kansas State. “BM4” and “BM8” stand for the build-maintenance recommendation rate with 4-yr and 8-yr building up plans to the target level, respectively.

Table 3.4. Results of Net Present Values by 4-/8-/20-year Planning Horizons When 95% of Relative Wheat Grain Yield Response at the Soil P Test Level of 15 M3 ppm.

			Initial Soil P level											
			5				15				25			
		Years	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8
Yield Potential ($kg\ ha^{-1}$)	2690	5	367	385	317	367	424	439	392	417	480	489	467	467
		10	750	764	742	719	823	836	813	802	895	908	884	884
		20	1722	1713	1742	1727	1800	1793	1813	1806	1878	1874	1885	1885
	4035	5	591	601	536	581	657	668	618	641	723	731	700	700
		10	1171	1178	1169	1137	1255	1261	1248	1231	1338	1346	1326	1326
		20	2611	2593	2670	2646	2700	2681	2749	2737	2790	2773	2827	2827
	5380	5	813	816	755	796	888	895	844	865	963	970	934	934
		10	1585	1588	1598	1555	1679	1680	1682	1661	1773	1775	1767	1767
		20	3477	3463	3599	3565	3578	3557	3684	3668	3678	3656	3770	3770

Note. The unit of NPV is dollars per hectare ($\$\ ha^{-1}$). “OKSuff” and “KSuff” mean the sufficiency recommendation rates of Oklahoma State and Kansas State. “BM4” and “BM8” stand for the build-maintenance recommendation rate with the 4-yr and 8-yr building up plans to the target level, respectively.

Table 3.5. Results of Net Present Values by 4-/8-/20-year Planning Horizons When Above Average Wheat Price Used and 90% of Relative Wheat Grain Yield Response at the Soil P Test Level of 15 M3 ppm.

			Initial Soil P level											
			5				15				25			
		Years	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8
Yield Potential ($kg\ ha^{-1}$)	2690	5	573	584	543	573	669	679	650	665	765	771	757	757
		10	1183	1170	1229	1163	1304	1294	1331	1298	1425	1422	1434	1434
		20	2730	2645	2852	2796	2860	2780	2955	2927	2989	2924	3058	3058
	4035	5	895	898	877	892	1019	1023	1007	1014	1143	1146	1136	1136
		10	1800	1782	1902	1808	1954	1933	2026	1979	2109	2094	2151	2151
		20	4055	3959	4337	4254	4219	4117	4462	4420	4383	4288	4587	4587
	5380	5	1212	1212	1211	1212	1363	1363	1363	1363	1515	1515	1515	1515
		10	2397	2388	2576	2455	2583	2559	2722	2661	2769	2745	2867	2867
		20	5318	5248	5823	5715	5514	5421	5969	5915	5710	5612	6116	6116

Note. The unit of NPV is dollars per hectare ($\$\ ha^{-1}$). Above average wheat price $0.27 per kilogram is used. "OKSuff" and "KSuff" mean the sufficiency recommendation rates of Oklahoma State and Kansas State. "BM4" and "BM8" stand for the build-maintenance recommendation rate with the 4-yr and 8-yr building up plans to the target level, respectively.

Table 3.6. Results of Net Present Values by 4-/8-/20-year Planning Horizons When Above Average Wheat Price Used and 95% of Relative Wheat Grain Yield Response at the Soil P Test Level of 15 M3 ppm.

			Initial Soil P level 5				15				25			
		Years	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8	OKSuff	KSuff	BM4	BM8
Yield Potential ($kg\ ha^{-1}$)	2690	5	634	650	590	635	702	716	674	696	769	777	757	757
		10	1267	1274	1274	1240	1353	1360	1354	1337	1439	1448	1434	1434
		20	2848	2817	2897	2870	2941	2913	2978	2964	3034	3012	3058	3058
	4035	5	990	998	945	983	1072	1082	1041	1059	1154	1161	1136	1136
		10	1940	1940	1968	1918	2044	2043	2059	2034	2148	2149	2151	2151
		20	4278	4239	4404	4361	4390	4347	4495	4474	4501	4462	4587	4587
	5380	5	1344	1346	1301	1331	1440	1445	1408	1423	1536	1541	1515	1515
		10	2602	2602	2663	2597	2723	2717	2765	2732	2844	2838	2867	2867
		20	5674	5644	5910	5853	5803	5762	6013	5984	5932	5888	6116	6116

Note. The unit of NPV is dollars per hectare ($\$\ ha^{-1}$). Above average wheat price $0.27 per kilogram is used. "OKSuff" and "KSuff" mean the sufficiency recommendation rates of Oklahoma State and Kansas State. "BM4" and "BM8" stand for the build-maintenance recommendation rate with the 4-yr and 8-yr building up plans to the target level, respectively.

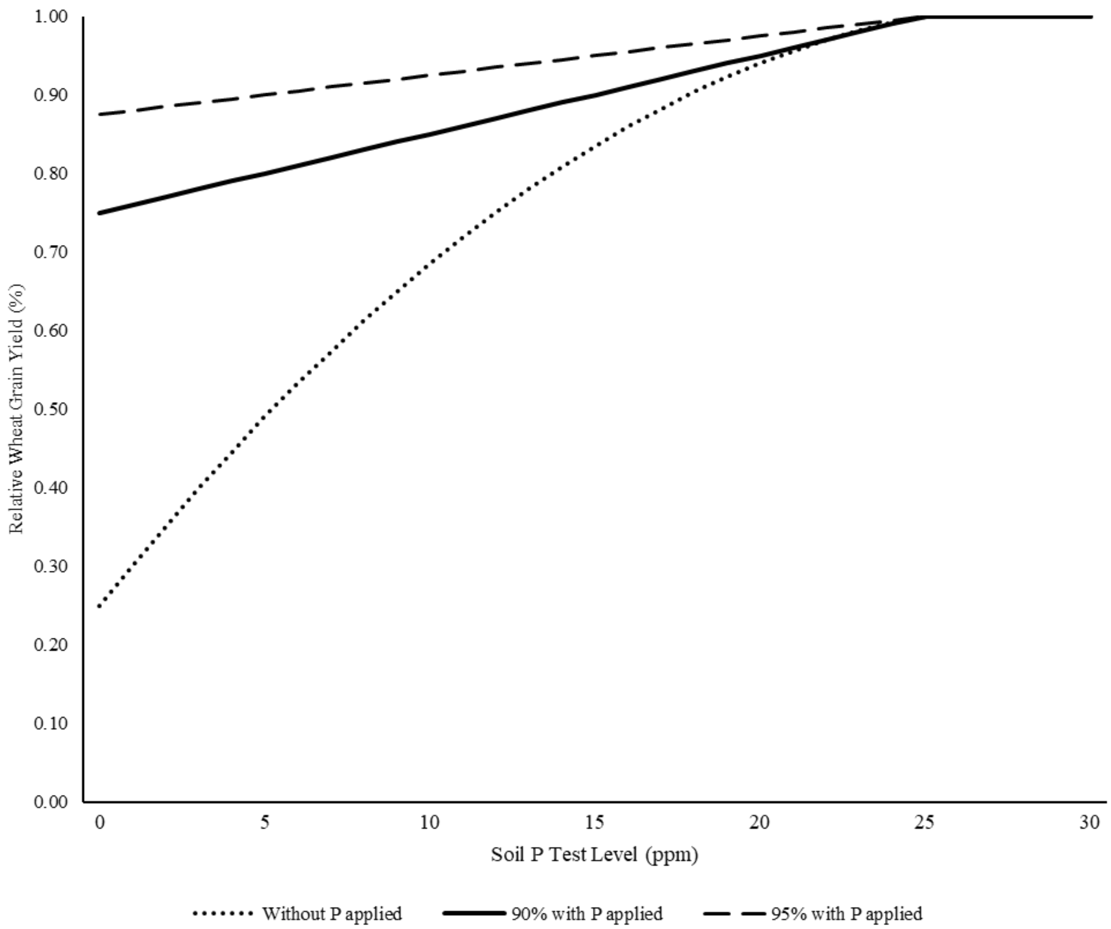


Figure 3.1. The relative wheat grain yield by soil phosphorus test level before/after phosphorus fertilizer applied

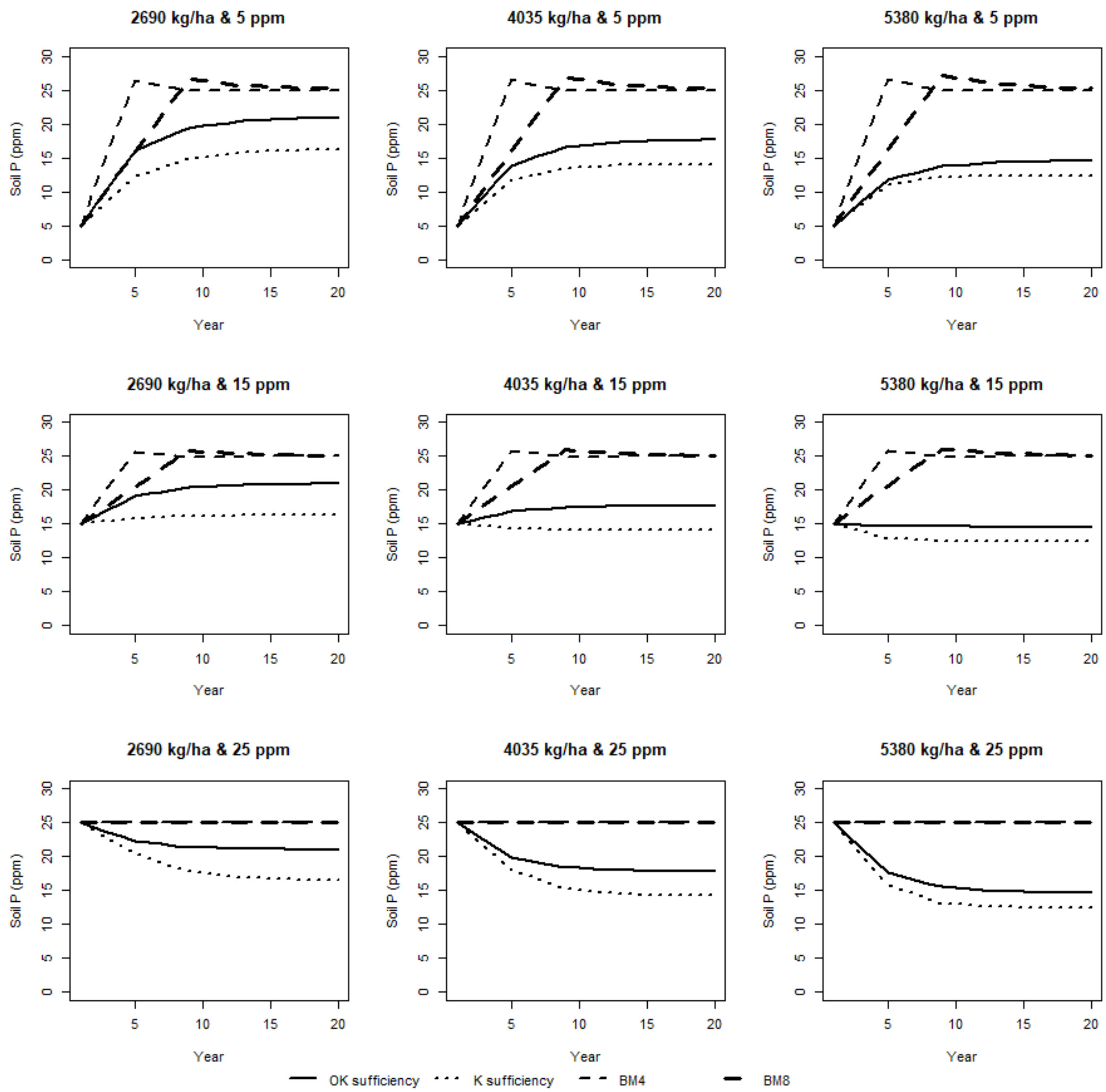


Figure 3.2. Soil P changes by 20-year planning horizon depending on the yield potentials and initial soil P level

Note. "OK sufficiency" and "K sufficiency" mean the sufficiency recommendation rates of Oklahoma State and Kansas State. "BM4" and "BM8" stand for the build-maintenance recommendation rate with the 4-yr and 8-yr plans to build up to the target level, respectively.

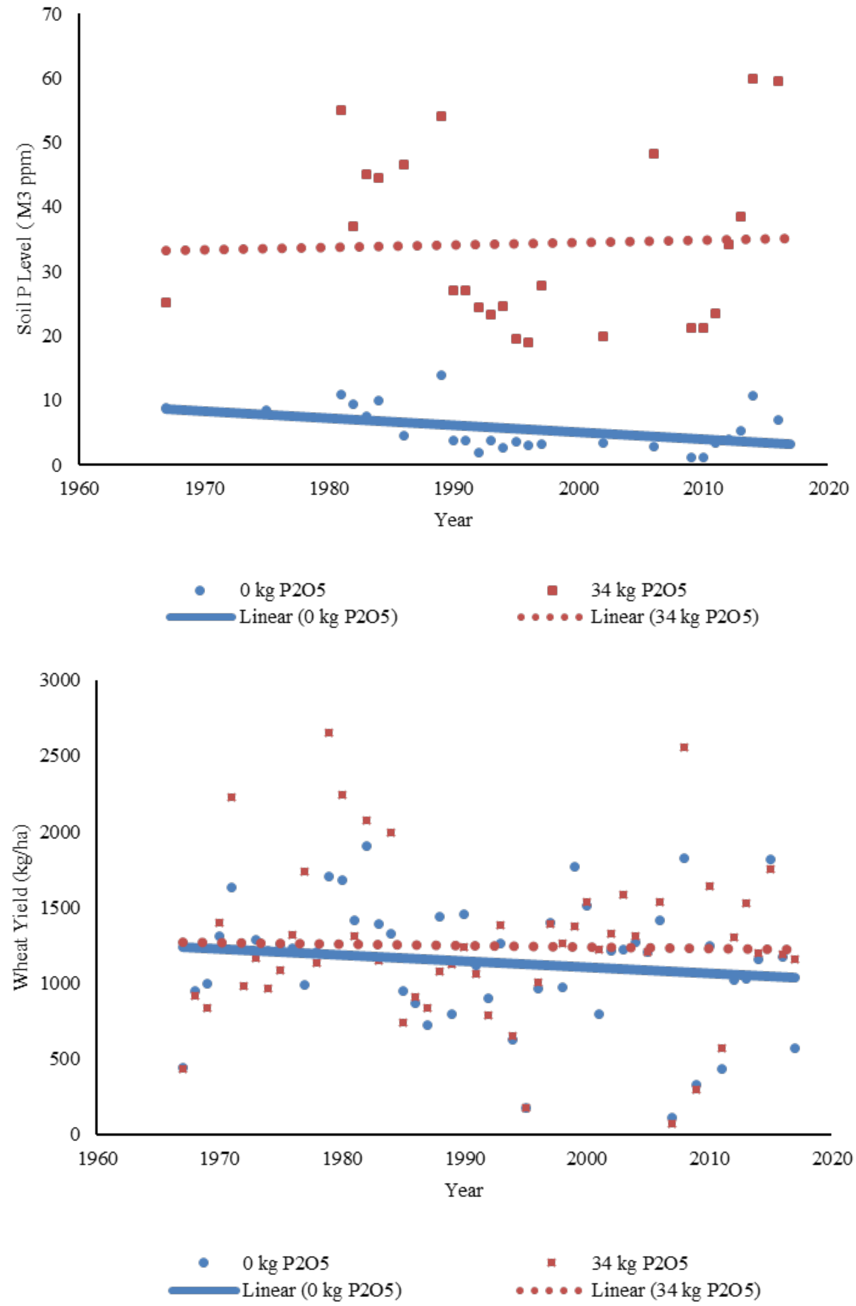
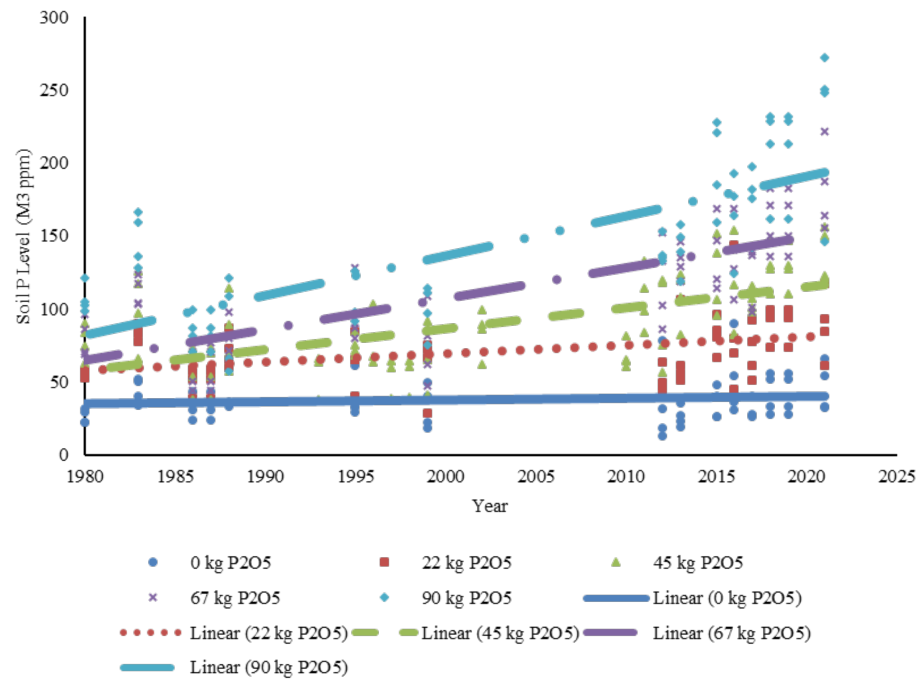


Figure 3.3. Soil P and wheat grain yield changes in Magruder plots in Stillwater, OK Note. Least squares means of soil P level are 5 and 34 ppm for 0 and 34 kg P2O5, respectively. Least squares means of wheat yield are 1143 and 1278 kg ha^{-1}.



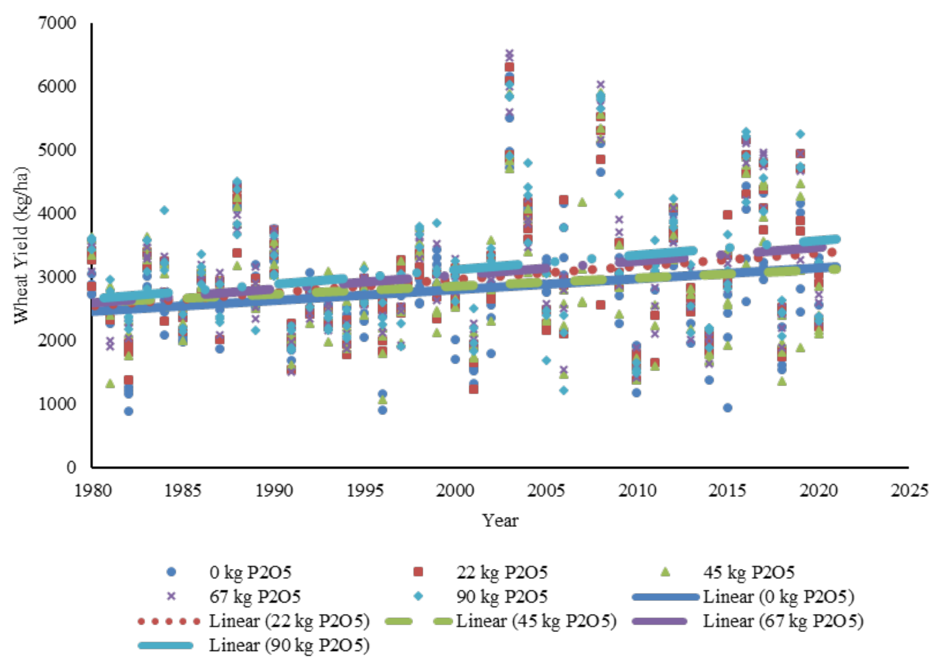


Figure 3.4. Soil P and wheat grain yield changes in Experiment 502 plots in Lahoma, OK Note. Least squares means of soil P level are 39, 98, 140, 172, and 231 ppm for 0, 22, 45, 67, and 90 kg P2O5, respectively. Least squares means of wheat yield are 2757, 2892, 2825, 2959, and 3026 kg ha^{-1}.

VITA

Whoi Cho

Candidate for the Degree of

Doctor of Philosophy

Dissertation: COMBINING TWO SAMPLING METHODS, BAYESIAN MULTILEVEL MODELING, AND BUILD-MAINTENANCE SOIL PHOSPHORUS

Major Field: Agricultural Economics

Biographical:

Education:

Completed the requirements for the Doctor of Philosophy in Agricultural Economics at Oklahoma State University, Stillwater, Oklahoma in July, 2022.

Completed the requirements for the Master of Science in Agricultural Economics at Oklahoma State University, Stillwater, Oklahoma in 2019.

Completed the requirements for the Bachelor of Science in Agricultural Economics at Oklahoma State University, Stillwater, Oklahoma in 2017.

Experience:

Graduate Research Assistant
Department of Agricultural Economics, Oklahoma State University, 2018-2022

Professional Memberships:

Agricultural & Applied Economics Association (AAEA)
Western Agricultural Economics Association (WAEA)
Southern Agricultural Economics Association (SAEA)