

SEISMIC MULTIPLES ATTENUATION USING  
RADON TRANSFORM AND PRINCIPAL  
COMPONENTS

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Title of Study: SEISMIC MULTIPLES ATTENUATING USING RADON  
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**Abstract:** Seismic data is often contaminated with noise that must be attenuated before making reliable seismic interpretations. Seismic noise can be either random noise, which cannot be correlated between seismograms, or coherent noise, which shows patterns in the seismic gathers. Seismic multiples and ground rolls are good examples of coherent noise. My research focuses on optimizing some existing methods for multiples attenuation. My first project focused on optimizing the Radon Transform method to better attenuate seismic multiples. The Radon Transform method entails applying Normal Move Out (NMO) to the seismic Common Depth Point (CDP) gathers using the velocities of the primary signals to remove travel time delay with increasing offset and flatten seismic events. The NMO corrected CDP gathers are transformed to radon domain (intercept time ( $\tau$ ) – curvature ( $q$ )), where different seismic multiples and primaries are separated based on their curvatures. In the ( $\tau$  –  $q$ ) domain, NMO corrected CDP using primaries velocities depicts near zero curvature for primaries and positive curvatures for multiples. However, using primaries velocities for NMO often results in less distinction between primaries and multiples based on their move out in the ( $\tau$  –  $q$ ) domain. Thus, I used the intermediate velocities between primaries and multiples for the NMO correction of the CDP gathers input to the  $\tau$ - $q$  domain, which resulted in a more significant separation between multiples and primaries and improved multiples removal in the  $\tau$ - $q$  domain. The results showed better multiples removal as compared to using the conventional velocity radon, where CDP gathers are NMO corrected using the primaries velocities. Although applying multiples velocities seem to render more primaries-multiples move out, testing this method on more synthetic and real seismic gathers showed mixing of the primaries and multiples energy at the near offsets in the radon space. Thus, my second project used coherency as the foundation for attenuating multiples. I used the multiples velocities for NMO correction and the singular value decomposition (SVD) for attenuating multiples in the time domain. Using multiples velocity, the NMO flattened coherent multiples attributed to dominant principal components and can be separated from the unflatten primaries. Principal components attributing to multiples are selected and composed back to seismic traces. The selected multiples are then removed from the original data using simple subtraction. Results of multiples attenuation using Radon Transform and principal components methods were compared. The principal components method seemed to be more effective in multiples attenuation. This may be due to the principal component method improves the separation between flattened multiples and primaries and does not require data transformation to another domain, which often produces artifacts. Both methods opt for an unconventional velocity selection approach, resulting in enhanced performance of the parabolic radon and principal components methods for attenuating seismic multiples.

## TABLE OF CONTENTS

Project summary .....	iv
Table of Content.....	v
List of Figures .....	vii
1. Project Description .....	1
1.1 Motivation .....	1
1.2 Objectives.....	3
1.3 Significance of Project.....	4
1.4 References .....	5
2. Paper 1: Elimination of Multiples from Marine Seismic data using the Primary-Multiple Intermediate Velocities in the $\tau$ - $q$ domain .....	7
2.1 Abstract .....	7
2.2 Introduction.....	8
2.3 Parabolic Radon Transform .....	10
2.4 Application of Primary Multiple Intermediate Velocity to Synthetic Data.....	13
2.5 Radon Application to Synthetic Data Using the Intermediate Velocity Approach	14
2.6 Radon Application to Marine Data Using the Intermediate Velocity Approach....	16
2.7 Discussion and Conclusions .....	25
2.8 References.....	26
3. Paper 2: Seismic Multiples Attenuation Using Principal Components.....	30
3.1 Abstract.....	30
3.2 Introduction.....	31
3.3 Theory .....	32
3.3.1 Parabolic Radon.....	32
3.3.2 Singular Value Decomposition & Karhunen Loeve Transform.....	34
3.4 Application and Results.....	36

3.4.1 Application of Parabolic Radon.....	36
3.4.2 Attenuating Multiples Using Principal Components.....	39
3.5 Discussion.....	41
3.6 Conclusions.....	44
3.7 References.....	45

## LIST OF FIGURES

### Paper 1:

Figure 2.1: A synthetic CDP gather before NMO correction (a) after NMO correction using primaries velocity (b) and after NMO correction using multiples velocity (c).....	14
Figure 2.2: A synthetic gather with primary event flattened using primary velocity leaving multiples under-corrected (a), muting of positive q in radon space (b), filtered gather with remnant multiples(c) .....	15
Figure 2.3: A synthetic gather with primary event over-corrected and multiples under-corrected using intermediate velocity (a), muting of positive q in radon space (b), filtered gather (c).....	16
Figure 2.4: Semblance plot (a) and super gathers (b), with primaries velocity picked and NMO applied. Velocity trends of water bottom multiples and inter-bed multiples are indicated by the red arrow in the semblance display.....	17
Figure 2.5: Semblance plot (a) and super gathers (b), with intermediate velocity picked and NMO applied. Velocity trends of water bottom multiples and inter-bed multiples are indicated by the red arrow in the semblance display.....	18
Figure 2.6: Original CDP with NMO correction applied using primaries velocity (a), Radon Transform with-50 to 3000 p-values (b) and inverse radon transform back to CDP(c) after muting multiples.....	19
Figure 2.7: Original CDP with NMO corrected using primary velocity (5a), Radon Transform with p values model associated to multiples (5b) and inverse radon transform back to CDP with multiples only (5c).....	20
Figure 2.8: Original CDP with NMO correction applied using primaries velocity (a), Radon Transform with-3000 to 3000 p-values (b) and inverse radon transform back to CDP(c) .....	21
Figure 2.9: Zoomed Original CDP with NMO corrected using intermediate velocity (8a), Radon panel (8b) and inverse radon transform back to CDP (8c). Notice intermediate velocity picked, results in least q values around 0 in radon space ( $\tau$ -q).....	22
Figure 2.10: Original CDP with NMO corrected using multiples velocity (9a), Radon panel ( $\tau$ -q) with positive q selected to model multiples (9b) and inverse radon transform back to CDP with multiples only (9c).....	23
Figure 2.11: Original Semblance (10a), Semblance created after conventional radon depicting artifacts and remnant multiple energy (red arrows-10b), semblance created after multiples modeled filtering. Multiples are removed (10c) .....	23

Figure 2.12: Original shot gather depicting surface multiples (11a-red arrow), shot gather filtered using conventional radon, depicting remnant multiples (11b-red arrows) and shot gather filtered using multiples modeled radon, with multiples removed (green arrow-11c) .....	24
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**Paper 2:**

Figure 3.1: Semblance velocity plot(left) with primary velocity picked and Normal Move-out (NMO) corrected (flattened) Seismic Gather with primary velocity .....	37
Figure 3.2: Normal Move-out (NMO) flattened Seismic Gather (left), its corresponding Radon panel with mute applied(center), and the filtered Seismic Gather(right).....	38
Figure 3.3: Semblance velocity plot (left) showing artifacts (red arrows) of Radon filtered Seismic Gather (right) using primary velocity,.....	38
Figure 3.4: Semblance velocity with multiples velocity picked and Normal Move-out (NMO) corrected (flattened) Seismic Gather with multiples velocity .....	40
Figure 3.5: Selection of three times gates for applying Eigen Filter .....	40
Figure 3.6: Eigen Filtered Semblance velocity plot with. multiples removed (red arrows). Radon filtered Seismic Gather (right) using multiples velocity, .....	41
Figure 3.7a: Original Shot Gather.....	43
Figure 3.7b: Conventional Radon Gather with remnant Multiples (red arrows), short offset multiples (blue arrows) and artifacts (yellow arrows) .....	43
Figure 3.7c: Eigen Filter Gather, preserving primaries and Removing multiples .....	43



## **CHAPTER I**

### **1. PROJECT DESCRIPTION**

#### **1.1 MOTIVATION**

Multiples are coherent noises that obscure the primary seismic signals and need to be attenuated or completely eliminated during the analysis of seismic data. Multiples, as the name implies, are repeated reflections within the geological layers that arrive at delayed times, obscuring the primary signals. Multiples were recognized in the early days of seismic surveys. Because of nature of wave-propagation, multiples are unavoidable byproduct of seismic survey (Ellsworth, 1948). The different types of multiples including surface-related multiples, peg-leg multiples and interbred multiples may have different ray geometry, but all have a delayed arrival time with respect to their primaries. The second noise associated to seismic acquisition is random noise. Random noise is incoherent noise and depicts random pattern in a recorded seismogram. This type of noise is associated with acquisition environs, equipment etc. The noise is particularly challenging where the recorded signal is weak. One such acquisition is microseismic, where recorded seismic wavefield triggered from fractures/cracks is feeble and is badly obscured by random noise. Many methods were proposed to separate and attenuate multiples. Taner (1980) showed that data transformed in the domain of intercept ( $t$ ) and ray parameter ( $p$ ), can show better periodicity and separation of the multiples. Beside periodicity, multiples have different

move-out times from primaries. In seismic signal processing, move-out, is the delay of the arrival time of an event due to the increase of recording offset. Move-out is velocity dependent and increases with offsets. Methods such as stacking (Mayne,1962), FK Filtering (Ryu,1982) and Parabolic Radon (Hampson,1986) utilize move out as a measurement to separate multiples from primaries. The most sought method in the industry is that proposed by Hampson (1986). Hampson proposed that primary event when NMO corrected (offset delayed time subtracted) using primary velocities, the multiples depict parabolic curvature (Hampson, 1986), which is the move-out property of multiples. Parabolic radon transform, which is arguably, the most widely used technique in the Industry was successful in removing multiples provided the velocities are estimated correctly (Foster and Mosher, 1992; Hampson, 1986; Russell et al., 1990; Sacchi and Porsani, 1999). Input to radon entails a perfect transformation, of CDP gathers from t-x domain into  $\tau$ -p domain. Multiples are muted in the  $\tau$ -p domain and muted gathers are transformed back to the t-x domain to obtain multiples-free CDP gathers.

Although Radon transform is the preferred method to attenuate surface as well as inter-bed multiples, it has its limitations. Often, CDP gathers are NMO corrected using primary velocity. This leaves the multiples in positive curvature in radon space. This practice has potential to have lesser move out or time difference between corrected (flattened) primaries and multiples. Therefore, separation of primaries and multiples is not aptly carried out. My work attempts to enhance primary multiples separation by using intermediate primaries-multiples velocities. It is demonstrated that this enhanced separation can optimize use of radon for efficient multiples removal.

The second method used for multiples removal is application of principal components. Principal component applies linear transformation to a set of observations in data, so that coherent observations are separated from incoherent observations. Therefore, the method provides an efficient way of selecting observations of maximum coherency. Application of normal move out correction using proper velocity functions, flatten events depicting observations of maximum coherency. Application of multiples velocity instead of primaries, results in more separation between primaries and multiples. This leads to better coherency of flatten multiples, and thus the dataset becomes more pertinent to principal component analysis. My research shows that principal component method can perform better than radon, as it requires no transformation of the data in different domains. Since velocity semblances are used for picking velocity, emphasis is drawn on relation of principal component and semblance panels, which represents high correlated data points. Therefore, applying multiples velocity provides mean to target higher eigenvalues, which leads to a better estimate and subsequent removal of multiples.

## **1.2 OBJECTIVES**

1. Improving the performance of the parabolic radon method in separating and eliminating multiples by using Intermediate multiples-primaries velocity for the NMO correction of the seismic gathers input to the parabolic radon method.
2. Principal Component filtering using Karhunen Loeve Transform to eliminate seismic multiples

### **1.3 SIGNIFICANCE OF THE PROJECT**

My work focuses on improving the present method, and introducing a new method for effective attenuation of incoherent seismic noise. The significance of my work is to introduce new idea of using intermediate velocity for optimum application of radon. Based on my experiments, I have used non-traditional method of using multiples velocity and used principal components to remove seismic multiples

The first phase of my work focuses on the application of the radon in.. I have outlined the challenges in using the conventional radon which inhibit optimum utilization of this method. I introduced the idea of using intermediate velocity. This increases primary and multiples separation, which can be exploited in radon. The intermediate velocity is approximately half lower than primaries velocities and half higher than the multiples at a particular time. Intermediate velocities are used to NMO correct CDP gathers, and are transformed into radon space for modeling multiples. The modeled multiples are transformed back in time space domain to be subtracted from the original data. This approach is compared with the conventional radon attenuation practice, where modeled multiples are estimated using primary velocity for radon transform. Our modified radon transform is tested over synthetic gathers and marine seismic gathers acquired at shallow water bottom.

The second project focuses on principal components as an effective tool to attenuate multiples. Emphasis is given on using multiples velocity, which separate primaries from multiples more. Another significant feature of this method is that it does not require transformation. The velocities are selected with the aid of semblance formation in processing. Semblance is correlation, which in essence is covariance matrix containing

eigenvalues divided by the variance. Therefore, selection of velocity using semblance can be effectively translated into principal components analysis. The Principal component approach is compared with conventional Parabolic Radon multiples removal method. The comparison shows that multiples principal component filtering has potential to outperform parabolic radon method, if good estimate of multiples velocity is made.

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## CHAPTER II

### **Paper 1: Elimination of multiples from marine seismic data using the primary-multiple intermediate velocities in the $\tau$ - $q$ domain**

#### **2.1 ABSTRACT**

Removing seismic multiples is an important part in seismic data processing and is often carried out using the Radon transform (*intercept time* ( $\tau$ ) and *curvature* ( $q$ ) domain). In this method, the normal moveout (NMO)-corrected CDP gathers using primary (signal) velocity are transformed into the  $\tau$ - $q$  domain where multiples can be separated from primaries, based on their curvatures, and muted. A drawback of using the primary velocity for NMO correction is that primaries and multiples often exhibit similar curvature in the  $\tau$ - $q$  domain, particularly at near offsets. We propose using velocity function intermediate between primaries and multiples for the NMO correction of the CDP gathers as input to  $\tau$ - $q$  domain to enhance primaries-multiples separation. The primary-multiple intermediate velocity approach is applied to synthetic and real short-streamer marine seismic data. A semblance-weighted radon transform is used to reduce smearing in the radon space. The results showed more primary-multiple separation and better multiple removal.

## 2.2 Introduction

Coherent signals in the seismic data are categorized as desired signal, often referred as primary reflections, and the undesired noise, which masks the primary signals and includes ground roll, mode-converted waves and multiples. Seismic data processing is meant to remove the undesired recorded signals while preserving desired signals. Multiples, as the name implies, are multiple reflections within one or many layers and appear at delayed times and obscure primary signals. Surface related multiples, peg leg multiple, interbred multiples may have different ray geometry, but all have one common characteristic, that they exhibit a delayed time with respect to their primary.

Many methods were proposed to separate and attenuate multiples in seismic data, including stacking (Mayne, 1962), FK Filtering (Ryu, 1982) and Parabolic Radon (Hampson, 1986). Among these methods, parabolic radon is the most widely used. Hampson (1986) pointed out that when a CDP gather is NMO-corrected using the primaries velocities, multiples depict a parabolic curvature, the move-out property of multiples. Input to parabolic radon entails a perfect transformation of CDP gathers from  $t$ - $x$  domain into  $\tau$ - $q$  domain. Multiples are muted in the  $\tau$ - $q$  domain and muted gathers are transformed to time-space domain to obtain multiples-free CDP gathers. Parabolic radon transform is reasonably successful in multiples elimination if correct primaries velocities are used (Foster and Mosher, 1992; Hampson, 1986; Russell et al., 1990; Sacchi and Porsani, 1999). Advancement in Radon algorithms has been made to improve computational performance and efficiency (B. Ursin et al., 2009; Gholami and Sacchi, 2017).



Although the Radon transform is mostly used to attenuate surface as well as inter-bed multiples, it has some limitations. When the CDP gathers are NMO-corrected using the primary velocity, primaries and multiples often demonstrate similar curvatures in the  $\tau$ - $q$  domain, especially at short offset land and marine acquisitions and consequently separation of multiples becomes challenging. In this manuscript, we investigate application of intermediate velocity function between primaries and multiples for NMO correction of data input to  $\tau$ - $q$  domain. The proposed approach has potential to enhance the separation between primaries and multiples.

Radon panel construction is prone to artifacts and smearing. The horizontal smear is due to near-offset energy spread out, and the oblique smear is truncation at far-offset (Z Cao & Bancroft, 2006). The smearing in the Radon domain decreases the ability to separate multiples energy from primaries. In order to reduce the smearing in radon space, semblance-weighted radon was used in this paper. The semblance of weighted Radon Transform concentrates seismic energy along trajectories where seismic events are present and reduces energy concentration, where seismic events do not exist (Z Cao & Bancroft, 2006). The semblance-weighted Gauss-Seidel Radon method was introduced by Bradshaw and Ng (1987) and Ng and Perz (2004). For a particular time, the intermediate velocity function is approximately half lower than the primaries velocity and half higher than the multiples velocity. Unlike, conventional input to radon, which entails primary velocities applied to CDP gathers, application of intermediate velocities results in over-corrected (negative  $q$ ) primaries, and under-corrected (positive  $q$ ) multiples. This enhances the separation between primaries and multiples and improves multiple removals in radon space. We compare our results with the conventional radon attenuation practice, where

multiples are modeled using the primaries velocity. We have tested this approach on synthetic seismic gathers and real marine seismic gathers acquired from relatively shallow water and contains strong water bottom multiples.

### 2.3 Parabolic Radon Transform

Hampson (1986) indicated that when a CDP gather is NMO-corrected using the primaries' velocity, the multiples in the data depict residual parabolic curvature ( $q$ ). These multiples can, therefore be separated from primaries using the Parabolic radon transform of the NMO-corrected CDP gather (Verschuur, 2013). The Parabolic radon (eq. 1) is expressed as (Verschuur, 2013):

$$M(q, \tau) = \int_{-\infty}^{+\infty} d(x, t = \tau + qx^2) dx \quad (1)$$

Where  $q$  is the slope of curvature and  $\tau$  is the two-way intercept time at the apex of the reflections in t-x (Verschuur, 2013). The under corrected multiples with positive curvature are muted in the  $\tau$ - $q$  domain and an inverse radon transform only transfers the remaining primaries into the t-x domain. In the context of radon transform equivalent to move-out or time difference between the flattened and unflatten events.

The parabolic radon transform equation (eq. 1) can be expressed in frequency domain (eq. 2), where the wave field is decomposed into plane waves, after which the result is inverse transformed from frequency to time (Verschuur, 2006):

$$M(q, f) = \int_{-\infty}^{+\infty} d(x, f) e^{(-2\pi f q x^2)} dx \quad , \quad (2)$$

where  $f$  is the frequency,  $q$  is the curvature, and  $x$  represents the offset.

In order to minimize the amplitude smearing on the conventional Radon panels, a least-squares formulation of the Radon methods was proposed by Thorson and Clarerbout (1985) and Hampson (1986). Hampson (1986) used a least-square method in minimizing difference between the original and re-constructed data. This minimization is quantified using the root mean square (RMS) difference of the two data sets. This processes is carried out in the frequency domain to be computationally efficient. The Radon panel consists of all possible curvatures and offsets, computed at each frequency component. The damped least square solution (eq.3) is then calculated (Verschuur, 2013):

$$M = (LL^H + \mu I)^{-1} Ld, \quad (3)$$

where  $M$  is the transformed data and  $d$  is the original data,  $L = e^{-2\pi f q x^2}$ , and  $\mu$  is the damping factor for stable inversion.

In this paper, we use the semblance-weighted radon (Bradshaw & Ng, 1987), which first transforms the data into  $\tau$ - $q$  domain and runs a coherency scan to plot major clusters of energy into a new radon panel. The semblance of Common Depth Point in radon form is calculated in (eq. 4).

$$S(\tau, q) = \frac{\sum_l \sum_h d(x, t = \tau + qx^2, h) dx}{N_h \sum_l \sum_h d^2(x, t = \tau + qx^2, h) dx} \quad (4)$$

Where  $S$  is the semblance in the  $\tau$ - $q$  domain,  $l$  is a window size and is usually a wavelet length;  $N_h$  are trace numbers involved in calculating semblance.

Semblance has an important property that its value do not depend on amplitudes of the input dataset and has 0-to-1 range, that indicates poorest-to-best fit of the proposed trajectory respectively (Bradshaw and Ng, 1987). The weighted Radon transform (eq. 5) for the parabolic trajectory is defined as follows:

$$M(\tau, q) = S(\tau, q) \sum_h d(x, t = \tau + qx^2, h) dx \quad (5)$$

The transform undergoes repeated coherency scan from high-energy clusters to lower energy in  $\tau$ - $q$  domain using Gauss-Seidel sparse matrix, for making the radon panel. The process is iterative until the convergence is achieved. This prevents smearing in radon space.

### **Seismic velocity estimations for multiples and primaries**

As seismic velocities of primaries and multiples are key parameter in designing the Radon transform, we have tested the constant velocity stacks and semblance plots methods for velocity analysis to select optimum velocity functions for data input to the Radon transform. Constant velocity stacks (CVS) is a well-established method for picking velocities, in which different constant velocity functions are applied to seismic events occurring at different times in the same CDP. The correct velocity functions corresponding to seismic event flattens these events. Higher velocities normally correspond to deeper

events. Ideally, primaries with a specific velocity at different times are flattened and multiples are under-corrected. However, using CVS, where a single CDP is subjected to different velocity functions, enables primary events and their respective multiple both flattens. The multiples naturally appear at a delayed time. Both of these events depict coherency, which are shown in semblance plots. Thus, good uniform constant velocity stacks representing the local geology help discern primaries from multiples. Seismic semblance plot is a coherency tool for finding the maximum amplitude event. Semblance is normalized by a cross correlation function and displays the signal strength over the receivers based on targeted lag pattern (Taner & Koehler, 1969). This power is sum of the squared amplitude estimated in time windows and is displayed as velocity semblance (Taner & Koehler, 1969).

#### **2.4 Application of primary-multiple intermediate velocity approach to Synthetic data**

For better primaries and multiples distinction in the  $t-x$  and  $\tau-q$  domain, accurate velocity estimates are important. Using multiples velocities for NMO correction yields greatly over-corrected primaries and a better separation from multiples, even at the near offsets. We have applied NMO corrections to synthetic gathers using primaries and multiples velocities (Fig. 2.1). The NMO-corrected gather using the primaries velocities showed under corrected multiples with a maximum move out of 350 ms (Fig 2.1b). The NMO-corrected gather using the multiples velocity showed overcorrected primaries with a maximum move out of 550 ms (Fig 2.1c). This means a better separation between primaries and multiples is achieved when multiples velocities are used for NMO correction (Fig. 2.1c).

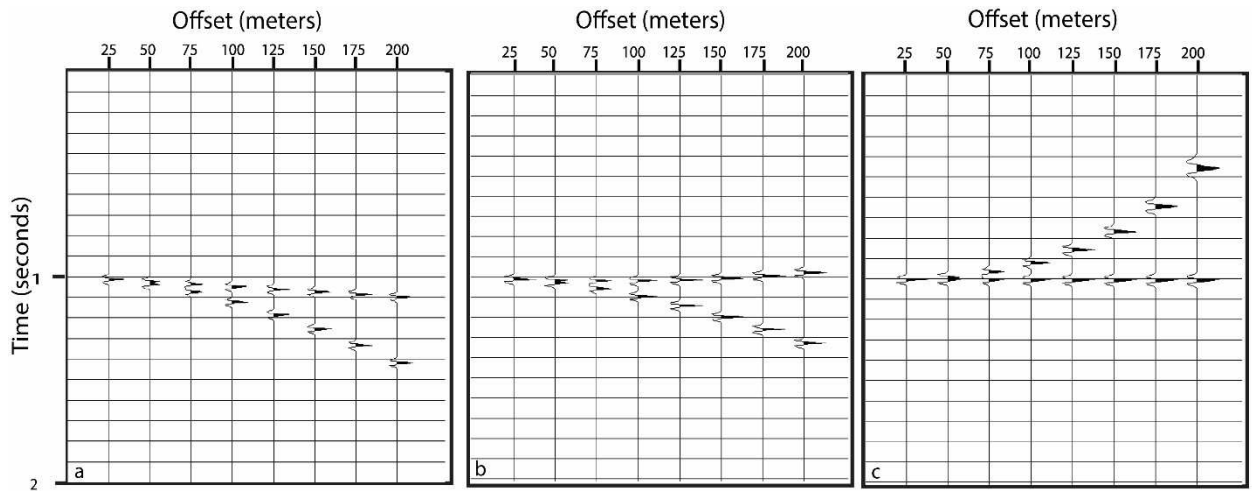


Figure 2.1: A synthetic CDP gather before NMO correction (a) after NMO correction using primaries velocity (b) and after NMO correction using multiples velocity (c).

## 2.5 Radon Application to Synthetic Data using the intermediate velocity approach

The conventional radon filtering uses the primaries velocities for NMO correction and therefore, cannot separate multiples effectively, particularly at near offsets (Fig. 1b). Using multiples velocity will result in better separation, and preserves most of the multiples as flat events, along with the near offset primaries (fig. 1c). In this case, both near offset primaries and multiples are associated with near-zero curvature in radon space. This potentially leads to losing primary events when muting multiples in radon space. Therefore, a better alternative is to use intermediate velocities between primaries and multiples. This practice still ensures better separation between primaries and multiples, and results in fewer primary-multiples event, falling near the smaller  $q$  region in radon space.

A synthetic example of radon application, when primary velocities are used for the NMO correction is shown in figure (2.2). In this case, the primary event is corrected, whereas multiples remain under corrected. The move out difference between the two events at 300

m offset is 720 m. The positive curvatures ( $q$ ) of multiples are muted, and transformed back in T-X domain. Some of the positive  $q$  values are kept intact, as they account for inaccurate RMS velocities that leaves primary events under corrected. Moreover, as the move out between primaries and multiples is less at near offset's, muting smaller  $q$ 's inevitably removes primaries. It is evident, that this practice of preserving primaries results in preserving remnant multiple energy (Fig. 2c).

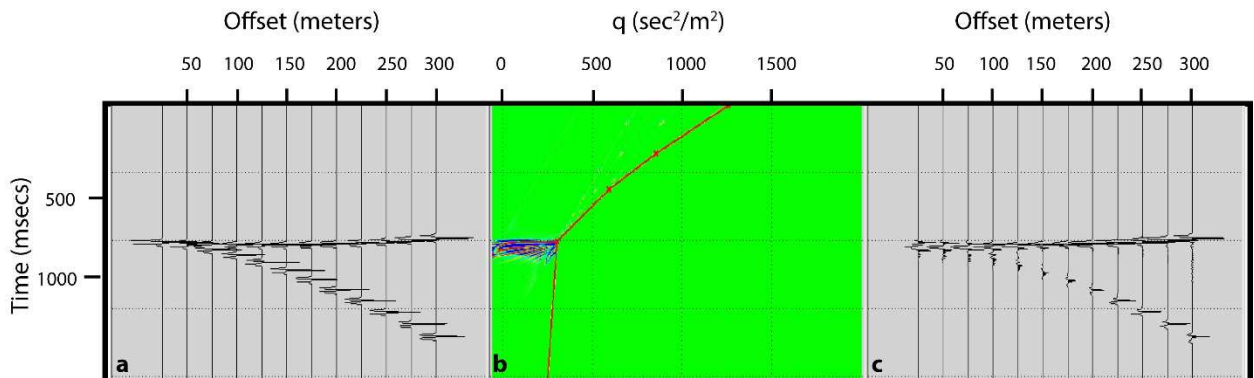


Figure 2.2: A synthetic gather with primary event flattened using primary velocity leaving multiples under-corrected (a), muting of positive  $q$  in radon space (b), filtered gather with remnant multiples(c)

Radon multiples attenuation of the above synthetic gather (Fig.2.2) using intermediate velocities for NMO correction is displayed (Fig. 2.3). In this case, over-corrected primaries and under-corrected multiples are observed. The move out difference between primary and multiple events at 300 m offset is 824 ms, which is 104 ms more than where using primary velocity for NMO correction. Since using intermediate velocities for NMO does not result in flattening the primaries or the multiples, it is expected that least amount of energy falls near 0  $q$ 's in the  $\tau$ - $q$  domain, which allows muting more near zero  $q$ 's. However, in order to preserve the very near offset primaries, 0  $q$  and smaller positive  $q$  values will not be

mutated, which results in a better removal of multiples and preservation of more primaries (Fig. 2.3c).

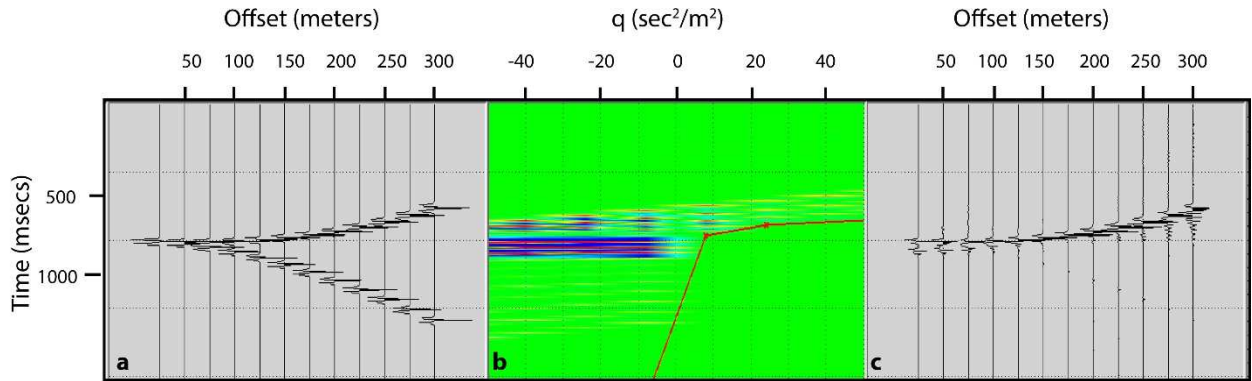


Figure 2.3: A synthetic gather with primary event over-corrected and multiples under-corrected using intermediate velocity (a), muting of positive  $q$  in radon space (b), filtered gather (c)

## 2.6 Radon Application to Marine Data using the intermediate velocity approach

The seismic dataset used to test the Radon transform parameters generated from a short streamer acquisition designed to image only a shallower part of the sub sea. The streamer length is 3200 m with the shot and receiver intervals at 25 m. If reliable velocity functions are used to create semblances, using NMO-corrected super-gathers (multiple CDP's merged together), multiples are differenced from primaries by observing velocity trends in the semblance plots.

The utilized 2D shallow marine seismic data is “Mobil Viking Graben Line 12”, an open source data released for the 1994 SEG workshop, SEG file publication, No 4 (Keys and Foster, 1998).



The process of selecting the appropriate NMO velocities was the same by evaluating both the CVS and semblance. Because, the tested data is relatively shallow, the difference between the primaries and multiples is less pronounced. The dominant multiples energy is at water velocity of 1500 m/s (Fig. 2.4).

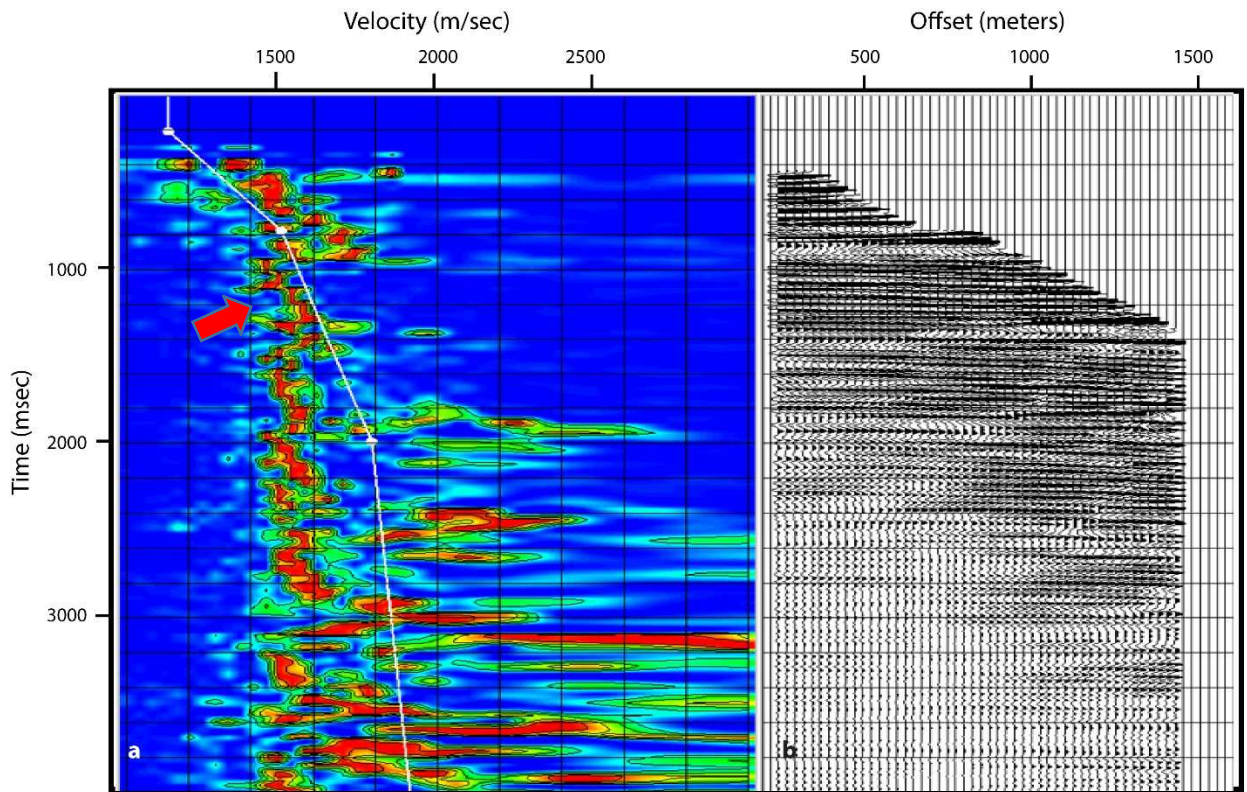


Figure 2.4: Semblance plot (a) and super gathers (b), with primaries velocity picked and NMO applied. Velocity trends of water bottom multiples and inter-bed multiples are indicated by the red arrow in the semblance display.

The strong water bottom reflections are associated with strong multiples, which can be seen appearing below the primaries in the semblance plot. Also interbed multiple energy forming coherent events are also seen (Fig.2.4 and Fig.2.5). NMO correction using the picked velocity functions is applied to the tested CDP gathers that flatten primaries and left multiples under-corrected, depicting positive curvature. The NMO corrected gathers are

modelled inside the  $\tau$ - $q$  domain by predicting curvature range and interval. Accuracy of the  $\tau$ - $q$  model was tested by transforming back modeled gathers from radon space to t-x domain and subtracting from the original gathers. The process is repeated iteratively until the difference between the modeled gathers and original gathers is minimum and the final model gather attained the optimum curvature interval, and range. Fewer  $q$ -values lead to inaccurate transformation of CDP gathers from t-x domain to  $\tau$ - $q$  domain. An optimum  $q$ -values range of -50 to 3000 with an increment of 12  $q$ . Therefore, a total number of 250  $q$  values with a maximum data frequency range of 100 Hz was used to construct a new  $\tau$ - $q$  domain to transform the data and prevent aliasing (Fig.2.6).

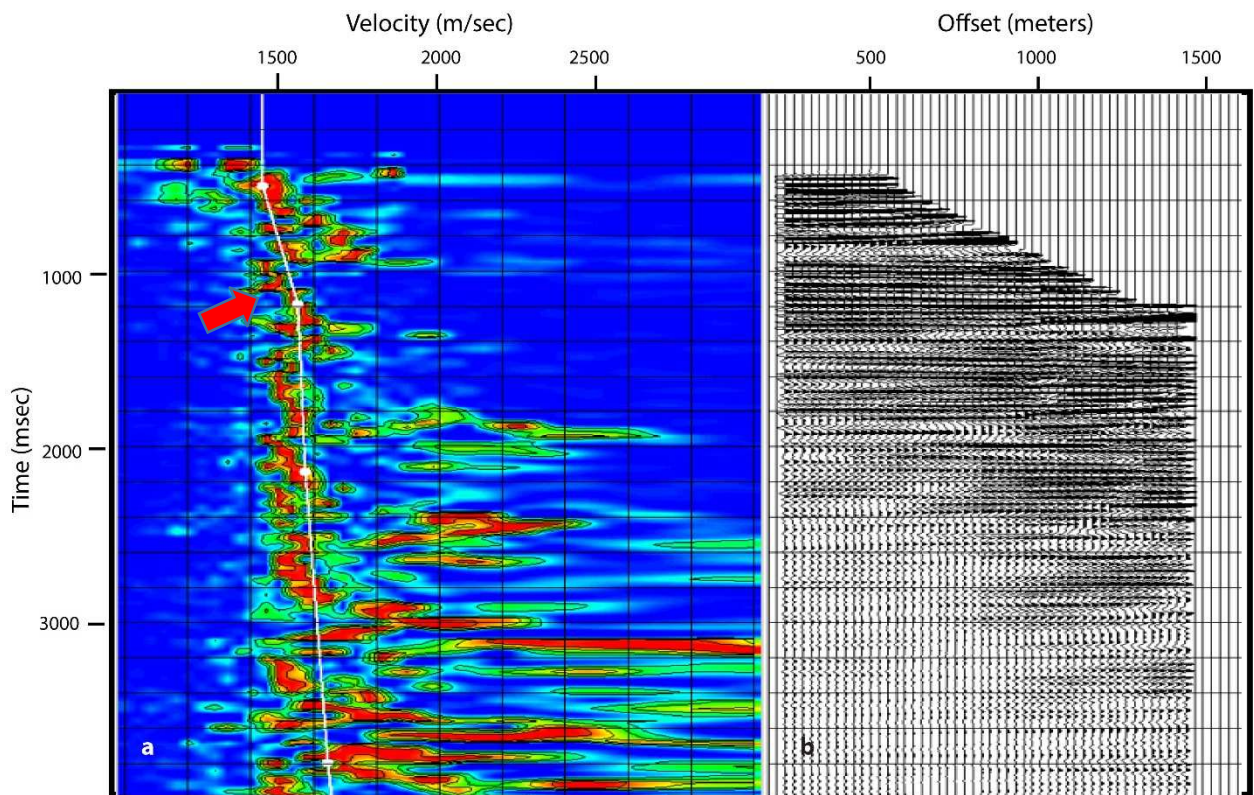


Figure 2.5: Semblance plot (a) and super gathers (b), with intermediate velocity picked and NMO applied. Velocity trends of water bottom multiples and inter-bed multiples are indicated by the red arrow in the semblance display.



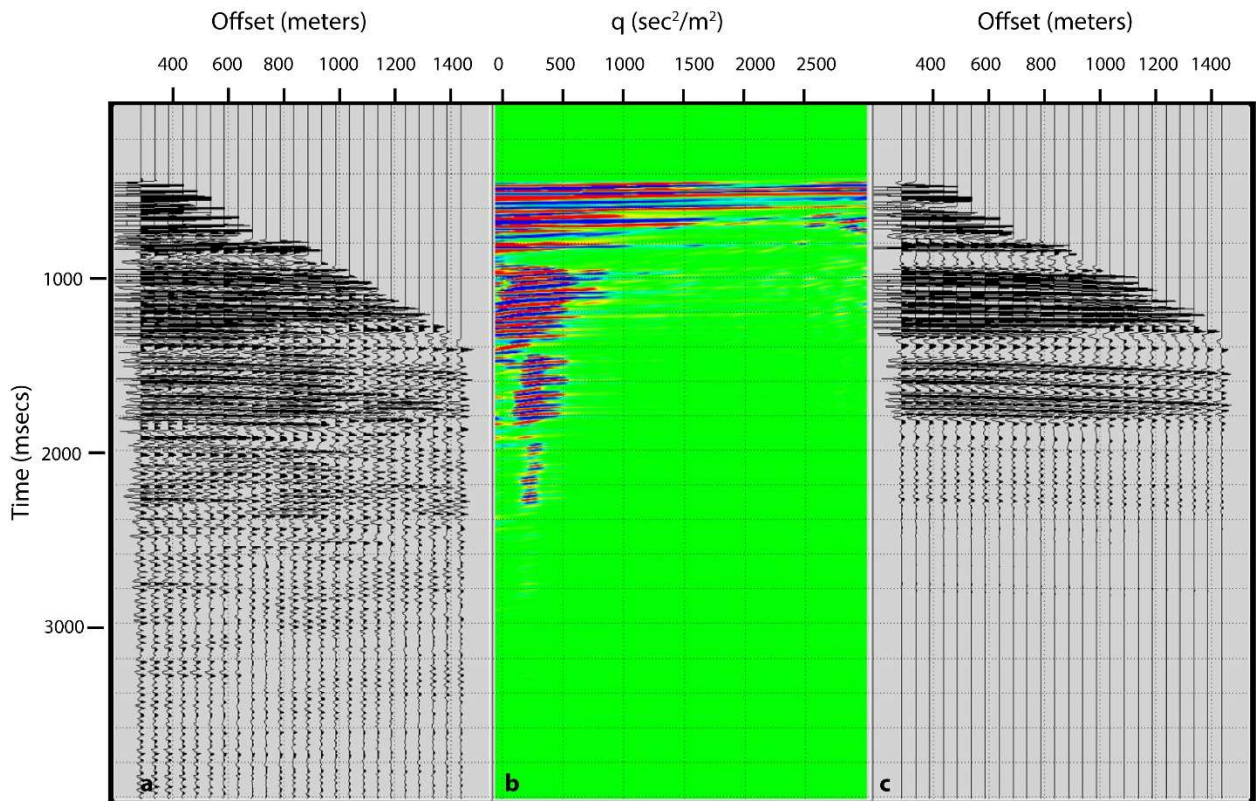


Figure 2.6: Original CDP with NMO correction applied using primaries velocity (a), Radon Transform with 50 to 3000 p-values (b) and inverse radon transform back to CDP(c) after muting multiples.

The final model is used in the  $\tau$ - $q$  domain to select curvature associated with multiples. Multiples can be muted in the radon space. However, it is better to model the multiples in radon space and transform back to t-x domain to be subtracted from the original gathers. This helps in mitigating artifacts created, while muting multiples in  $\tau$ - $q$  domain.

Since primaries velocity is used, multiples depict positive  $q$ -values. The positive  $q$ -values are selected within the  $\tau$ - $q$  domain (Fig. 2.7). Primaries and multiples are hard to differentiate at shallow depths, therefore  $q$ -values associated to shallow events were not removed in order to avoid muting primaries along with multiples. Because it is often difficult to reach velocity functions that completely flatten all the primary events, some

residual move out result in under correction (positive curvature). It is common practice not to mute zero and some positive curvatures also for deeper events, irrespective of move-out difference between primaries and multiples. Therefore, we select maximum  $q$  values at shallower time, and taper it down to small  $q$  values at greater depths (Fig. 2.6). The modeled positive  $q$ -values (Fig. 2.7) are transformed from the  $\tau$ - $q$  domain to the t-x domain to be subtracted from the original gathers. As discussed in our synthetic example, this practice is prone to preserve some multiples energy.

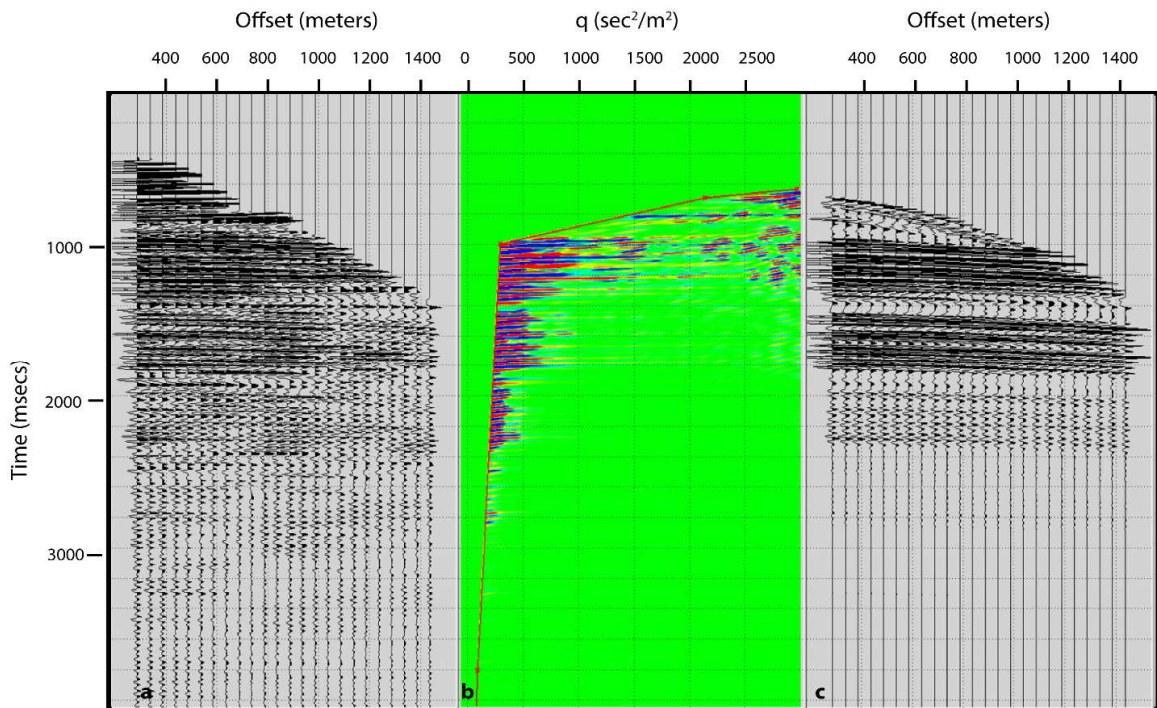


Figure 2.7: Original CDP with NMO corrected using primary velocity (5a), Radon Transform with p values model associated to multiples (5b) and inverse radon transform back to CDP with multiples only (5c)

The intermediate velocity between primaries and multiples is used for enhanced separation between primaries and multiples, in preparing CDP gathers, as input to radon. With the intermediate velocity applied, the primaries are overcorrected depicting negative curvature,

and multiples are under corrected with positive curvature. This new velocity is then used for NMO correction on CDP gathers, and are tested. Testing entailed making best estimate of  $q$  values ranges and interval. Again, the transformation is validated by subtracting the original NMO corrected CDP gather with this new velocity, from the gather obtained after transformed back from the radon space. In our case,  $q$  range of -3000 to 3000 was used with a total  $q$  values of 250, and frequency range of 100 Hz (Fig. 2.8). An important aspect of this approach is that intermediate velocity leads to least amount of  $q$  values fall in the zero curvature region (Fig. 2.9). This prevents modeling near zero curvatures  $q$  as there are primary and multiples event at shorter offsets with almost no curvature difference.

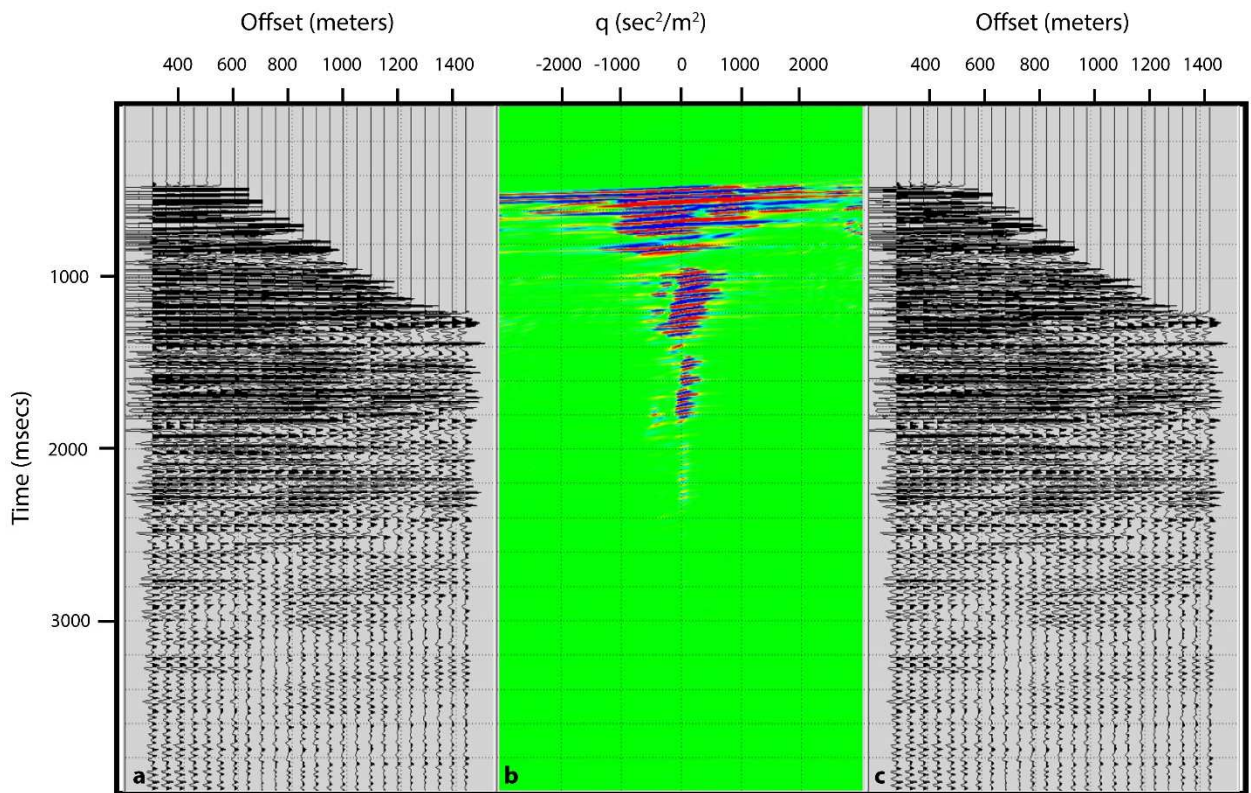


Figure 2.8: Original CDP with NMO correction applied using primaries velocity (a), Radon Transform with -3000 to 3000 p-values (b) and inverse radon transform back to CDP(c)



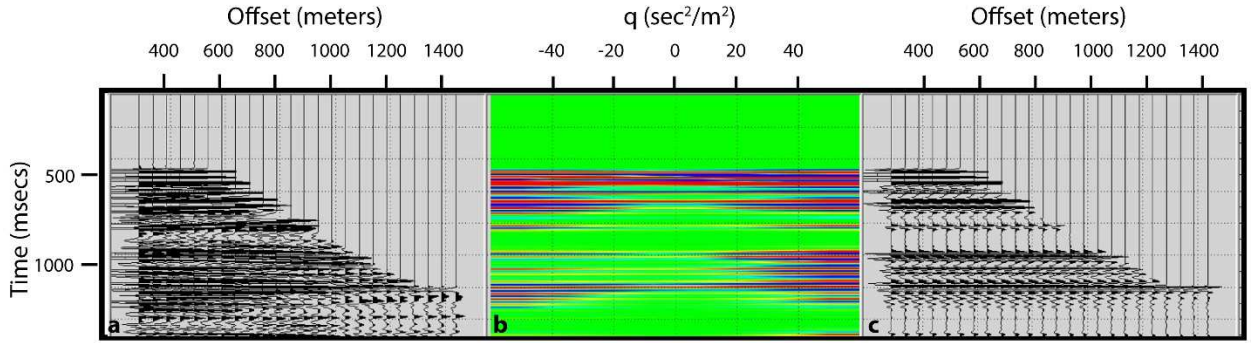


Figure 2.9: Zoomed Original CDP with NMO corrected using intermediate velocity (8a), Radon panel (8b) and inverse radon transform back to CDP (8c). Notice intermediate velocity picked, results in least  $q$  values around 0 in radon space ( $\tau$ - $q$ ).

Once perfect  $q$  range and increments are estimated, we select positive  $q$  values associated to multiples. Since using intermediate velocity assists in better separation, we are able to use more near  $q$  values to be added as multiples. However, for near offset, we keep the practice of not modeling zero  $q$  values, as in conventional radon approach discussed above. The modeled multiples are shown (Fig.2.10). The multiples models in both cases are transformed back in time space domain and are subtracted from the original CDP gathers. The two datasets are then analyzed by inspecting their semblances (Fig. 2.11) and shot gathers (Fig. 2.12).

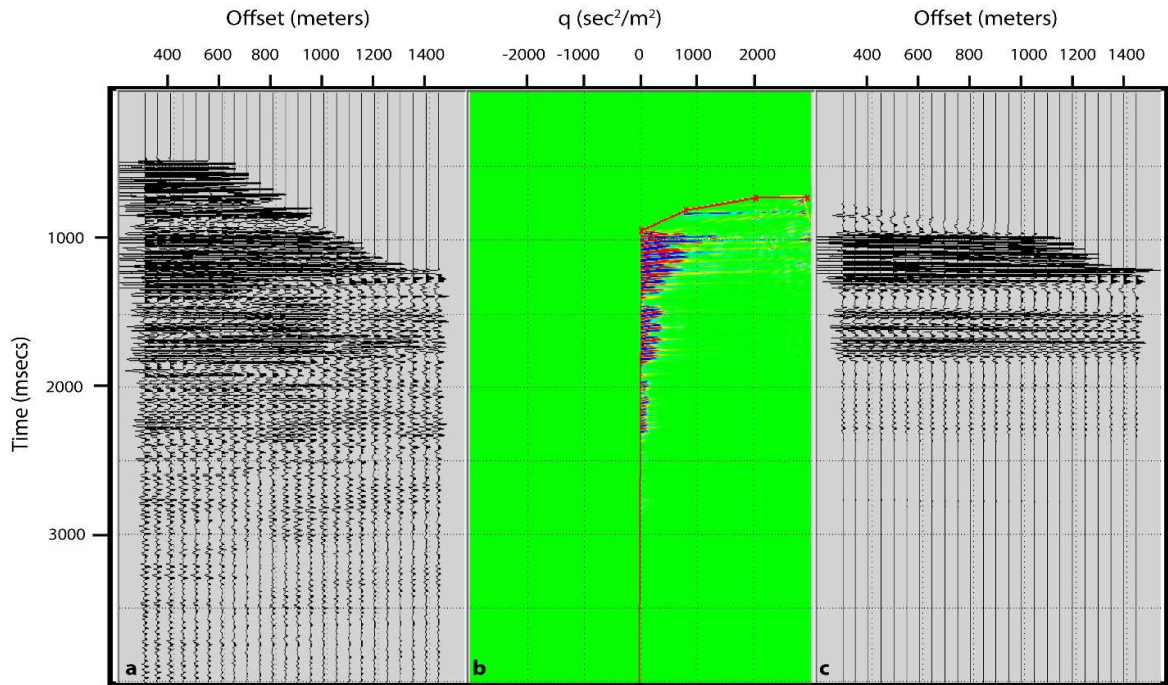


Figure 2.10: Original CDP with NMO corrected using multiples velocity (9a), Radon panel ( $\tau$ - $q$ ) with positive  $q$  selected to model multiples (9b) and inverse radon transform back to CDP with multiples only (9c).

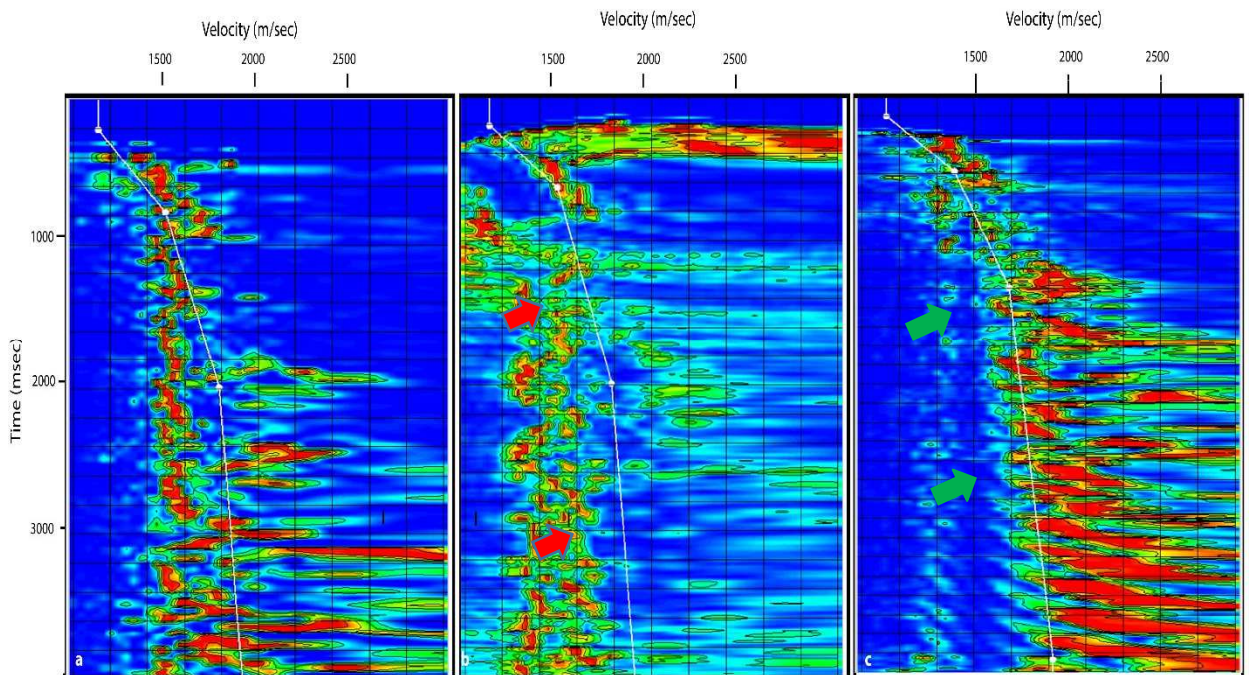


Figure 2.11: Original Semblance (10a), Semblance created after conventional radon depicting artifacts and remnant multiple energy (red arrows-10b), semblance created after multiples modeled filtering. Multiples are removed (10c)

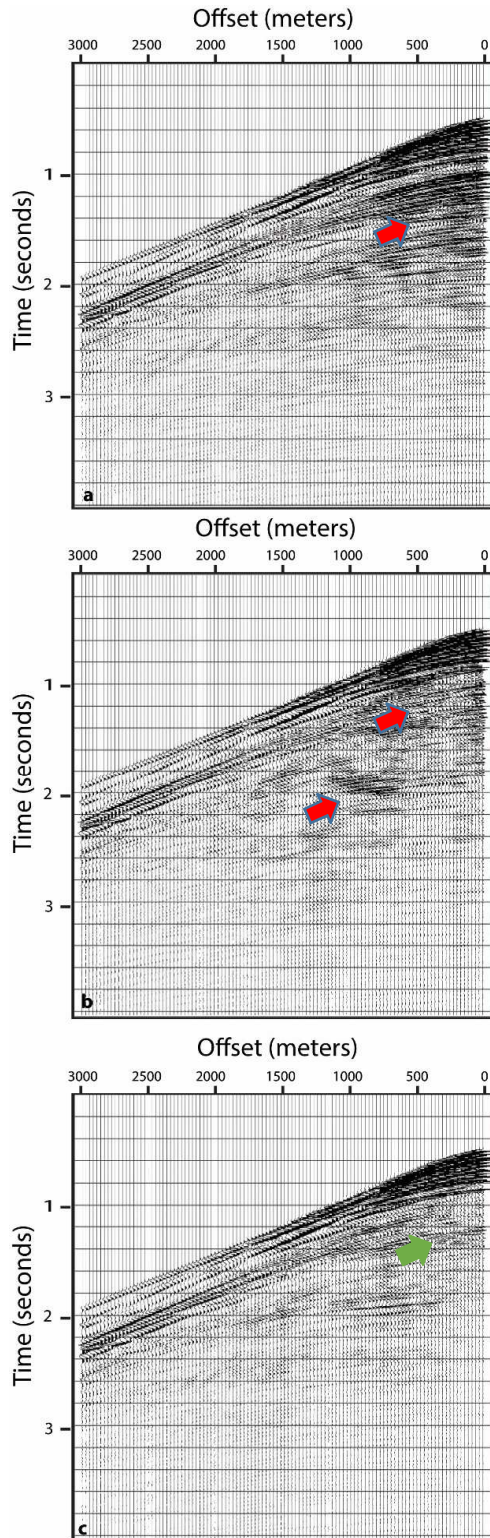


Figure 2.12: Original shot gather depicting surface multiples (11a-red arrow), shot gather filtered using conventional radon, depicting remnant multiples (11b-red arrows) and shot gather filtered using multiples modeled radon, with multiples removed (green arrow-11c)



## 2.7 DISCUSSION & CONCLUSIONS

Parabolic radon is velocity and curvature dependent. The best estimate of velocities and curvature are typically achieved by constructing reliable seismic semblance plots, for identifying primaries and multiples events. Semblance windows should be sufficiently sampled to distinguish noise, in this case multiples from primaries. The weighted semblance radon can provide high resolution and is often preferred. For short-streamer data sets, the standard radon muting practice preserved shallower events with larger positive  $q$  values, and muted deeper events with more move-out or positive  $q$  values. Applying primaries velocities to shallower marine data with a short streamer length, results in smaller move out time between primaries and multiples. Therefore, smaller  $q$  values are not modeled, which results in preserving smaller positive  $q$ -values even at depth. As demonstrated from synthetic data examples, this practice is prone to preserving significant multiple energy. However, the second approach in which CDP gathers are NMO-corrected using intermediate velocity functions between primaries and multiples, showed more distinction between over corrected primaries and under corrected multiples. Subsequently better modeling of multiples is attained with relatively lesser potential to include primaries, in radon space. This approach results in a better attenuation of multiples, especially for short offset acquisitions.

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## CHAPTER III

### **Paper 2: Seismic Multiples Attenuation using Principal Components**

#### **3.1 Abstract**

Seismic datasets are imprinted with multiples energy. Multiples are attenuated using different established methods in the industry. A careful estimate of multiples and their subsequent subtraction is common practice in preserving primaries and aid in the interpretation of seismic data. Most established methods use two characteristics of multiples, delayed arrival time with respect to primaries, and periodicity. An attempt is made to use coherency as the foundation for attenuating multiples. Karhunen Loeve Transform is used on dataset, using multiple velocity. Principal components attributing to multiples are selected and composed back to seismic traces. These multiples are then subtracted from the original gather. The results are compared with conventional radon method. The method offers an advantage over the radon, because no transformation has taken place, which often results in inaccurate curvature values of the seismic gathers in the dataset, and leaves artifacts on real gathers. The method can be applied in windows, with varying principal component filtering criteria, thus providing opportunity to pick the proportion of principal components associate with multiples with varying coherency. This approach can further optimize practice of separating primaries from multiples at different offsets, as varying percentage of principal components for each window can be selected.

Since original multiples have been selected from the dataset without any additional process, the method does not entail designing filters or subsequent adaptive subtraction. The method has successfully attenuated both surface and interbed multiples.

### **3.2 Introduction**

In seismic acquisition, our primary goal is to enhance signal to noise ratio, and make seismic datasets interpretable. During the course of acquisition, seismic coda comprises of signals and noises. The noises can be categorized as coherent and in-coherent noise. Seismic multiples is a coherent noise, which has undergone multiple reflections of seismic waves within layers. Multiples can be further categorized into different types, based on their ray path and period of occurrence. The most common multiples are surface multiple, peg leg multiples and interbed multiples. Separation and ultimately attenuating multiples without compromising signal is the ultimate goal in achieving a reliable dataset. In advanced seismic processing, seismic multiples are used to image sub salt bodies, by using **there** multiple reverberating imprints as a pattern for imaging. Nevertheless, estimating and separation of multiples is still very much sought.

The most prominent multiple attenuation methods attempt to exploit the periodicity and delayed time (move-out) of multiples. The periodicity can be used to estimate lags of multiples which then be used as convolution problem. Deeper depth ocean floor arrests the use of deconvolution approach (Abbasi & Jaiswal, 2013). The moving streamer in recording deep seismic surveys, impedes accurate estimate of lags. The move-out approach is velocity depended. This normally employs radon space transformation, which has

inherent inefficiencies. These include inaccurate parabolic and hyperbolic transformations, which are not consistent with the real gathers attributing to geological inhomogeneity, anisotropy etc. As a result, artifacts are bound to be added to real seismic gathers upon multiple subtraction. Moreover, not all gathers can be tested for accurate transformations. Thus, the same transformation parameters are applied on all gathers in 2D or 3D datasets, leading to inaccuracy. Surface Related Multiple Elimination (SRME) is a data driven surface multiple elimination method. It does not depend upon velocity. However, as the method entails multiple convolution processes to mimic surface multiples reflections, the estimated multiples have to be adaptively subtracted from the real gathers. Adaptive subtraction requires careful estimates of windows and filters. The filters often over estimate primaries and also kill signals. Moreover, SRME is only specific to surface multiples. This work uses the concept of Singular Value Decomposition (SVD) and employs Karhunen Loeve Transform (KLT) to attenuate all types of multiples, which does not depend on periodicity, move out difference and adaptive subtractions. The result is compared with Parabolic Radon.

### **3.3 Theory**

#### **3.3.1 Parabolic Radon**

Parabolic Radon is proposed by Hampson (1986) suggesting that after correcting the event using Normal Move out (NMO) equation (offset delayed time subtracted, and bring all traces at 0 offset), the multiples depict parabolic curvature. This delayed move out or curvature of multiples can be estimated to subtract multiples. Hampson (1986), introduced this transform as an effective tool for multiple attenuation. Below is the equation (eq.1) from (Verschuur, 2013):



$$M(q, \tau) = \int_{-\infty}^{+\infty} d(x, t = \tau + qx^2) dx \quad (1)$$

$q$  is the curvature and  $t$  is the two-way intercept time at the apex of reflections in space-time domain. Curvature is the degree to which a trajectory in space is curved. Positive and Negative curvature corresponds to anticlinal and synclinal shapes, respectively. The anticline is represented by under-corrected multiples, whereas NMO corrected and over-corrected primaries have 0 or negative curvature, respectively. The under corrected multiples with positive curvature, are removed in Radon Space and then reverse transformed into the time domain.

The parabolic radon transform equation (eq. 1) in frequency domain (Verschuur, 2006):

$$M(q, f) = \int_{-\infty}^{+\infty} d(x, f) e^{(-2\pi f q x^2)} dx \quad (2)$$

where  $f$  is the frequency,  $q$  is the curvature, and  $x$  represents the offset.

The conventional Radon panels suffers from smearing amplitudes and therefore a least-squares version of Radon methods was proposed by Thorson and Clarerbout (1985) and Hampson (1986) to reduce effect of smearing. This process is also carried out in frequency domain. The NMO corrected common depth point (CDP) transformed to Radon panel represents the input curvature ranges and offsets for each frequency component. The damped least squares re-representation of Radon (eq.3) is given by (Verschuur, 2006.):

$$M = (LL^H + \mu I)^{-1} Ld, \quad (3)$$

where  $M$  is the transformed data and  $d$  is the original data,  $L = e^{-2\pi f q x^2}$ , and  $\mu$  is the damping factor for stable inversion.

An advanced version of radon the semblance-weighted radon (Bradshaw & Ng, 1987) is used in this paper. It transforms the input data in series of steps into  $\tau$ - $q$  domain based on coherency scan to plot major clusters of energy into a new radon panel. The process is repeated until all data is correctly transformed into  $\tau$ - $q$  domain. The semblance bases radon transform (eq.4) is calculated in (Bradshaw & Ng, 1987):

$$S(\tau, q) = \frac{\sum_l \sum_h d(x, t = \tau + qx^2, h) dx}{N_h \sum_l \sum_h d^2(x, t = \tau + qx^2, h) dx} \quad (4)$$

Where  $S$  is the semblance in the  $\tau$ - $q$  domain,  $l$  is a window size and is usually a wavelet length;  $N_h$  is the number of traces involved in calculating the semblance.

### 3.3.2 Singular Value Decomposition & Karhunen Loeve Transform

Singular Value Decomposition (SVD) is a well-known method that has been used in many branches of science. In linear algebra,  $A_{m \times n}$  matrix can be expressed (eq.5) as (Kirlin, 2001):

$$A = U \Sigma V^H \quad (5)$$

Where  $U_{m \times m} = AA^*$  and  $V_{n \times n} = A^*A$  are unitary matrices and  $\Sigma$  is the rectangular diagonal matrix. This diagonal matrix contains the singular values. In seismic we establish connection between SVD which is the decomposition of rectangular matrix to using KL transform, which is a method of selecting or choosing subset of our desired dataset in terms of principal components. Diagonal matrix  $\Sigma$  only give singular values. Thus, for vector  $A$  if  $r$  singular vectors are used for reconstructing and filtering noises. Then  $A_r$  is expressed (eq.6) as (Kirlin, 2001):

$$A_r = U_r \Sigma V_r^H \quad (6)$$

Where  $U_r$  and  $V_r$  are subset eigenvectors from  $U$  and  $V$  associated with the largest magnitude singular value of  $\Sigma$ . The square of the singular values gives the eigenvalues.

Therefore, singular values corresponding to highest Eigenvalues are used to reconstruct  $A_r$ .

In our application if we choose a window of 10 sample and 5 traces, this is converted into 50 x1 vector. This vector can then be transformed into covariance matrix, to estimate the principal components. The principal components are diagonal elements of covariance matrix (Kirlin, 2001):

$$\text{Cov}(A) = 1/n(AA^H) = U = V \quad (7)$$

Where  $n$  are number of samples and  $A^H$  is the complex conjugate transpose.

Another way of achieving this is using KL transform. It decomposes an image into principal components that are ordered on the basis of spatial correlation (Jiao, Negut, & Link, 1999). Any number of columns  $X_i$  in data matrix  $A$ , can be written as linear combination of singular vectors  $u$  (Kirlin, 2001):

$$X_i = U U^H X_i \quad (8)$$

$U^H$  applies on any  $X$  constitutes KL transform. The vector  $U^H X$  contains the principal components which can be filtered or selected to re construct the signal. The coherent events constitute the first few eigenvalues, in the covariance matrix. Principal component analysis finds a new set of orthogonal axes that have their origin at the data mean and that are rotated so that the data variance is maximized (Guo, Marfurt, & Liu, 2009). In land surveys, clear

reflections are often lacking and masked by ground roll (Alam & Jaiswal, 2015). The Principal component filtering was successfully used in this study for filtering ground roll.

### **3.4 Application & Results**

The seismic dataset is a 2D marine survey. The water bottom can be regarded as shallow, with an approximate depth ranges from 375 meters to 475 meters. The shot and receiver interval are at 25 meters. It appears that reflections are mostly prominent until 2600 msec. Constant velocity stacks (CVS) are used, using velocity range of 1000 to 3000 m/sec. This corresponds to our best estimate of the local geology. 11 CVS are prepared for velocity distribution for a particular CDP. This is aided by making semblance plots. A good separation of multiples and primaries are seen in 200 semblance windows. This means that 200 semblance lateral windows are estimated between the lowest and highest RMS velocity; in the constant velocity stacks.

#### **3.4.1 Application of Parabolic Radon**

The dataset is transformed into Common Depth Point (CDP) domain and NMO corrected. (Hampson, 1986) proposed the multiples depict parabolic curvature after application of NMO on the primaries. The dataset was tested for both hyperbolic and parabolic transforms, but in our case, the parabolic transform appears to me the most accurate transformation. The NMO corrected CDP gathers (Fig.3.1) were tested for accurate transformation, and appropriate  $q$  value range and increment. A  $q$  value range of  $-50$  to  $3000$ , with  $q$  increment of  $40$  was selected. The dataset is muted using conventional muting pattern (Fig 3.2). At shallower times, positive  $q$  values are kept, but the mute gate kept of muting more positive  $q$  values with increasing times. This is to ensure that primaries are

preserved at shallower times, since move-out difference between primaries and multiples are minimal at shallower times. The muted multiples are kept as model, and then subtracted from the original gathers. The filtered gathers have attenuated some multiple energy, but remnant multiples are still present (Fig 3.3). Also, it is evident that some artifact was added in gathers, which are conspicuous on semblance, which is typical in radon.

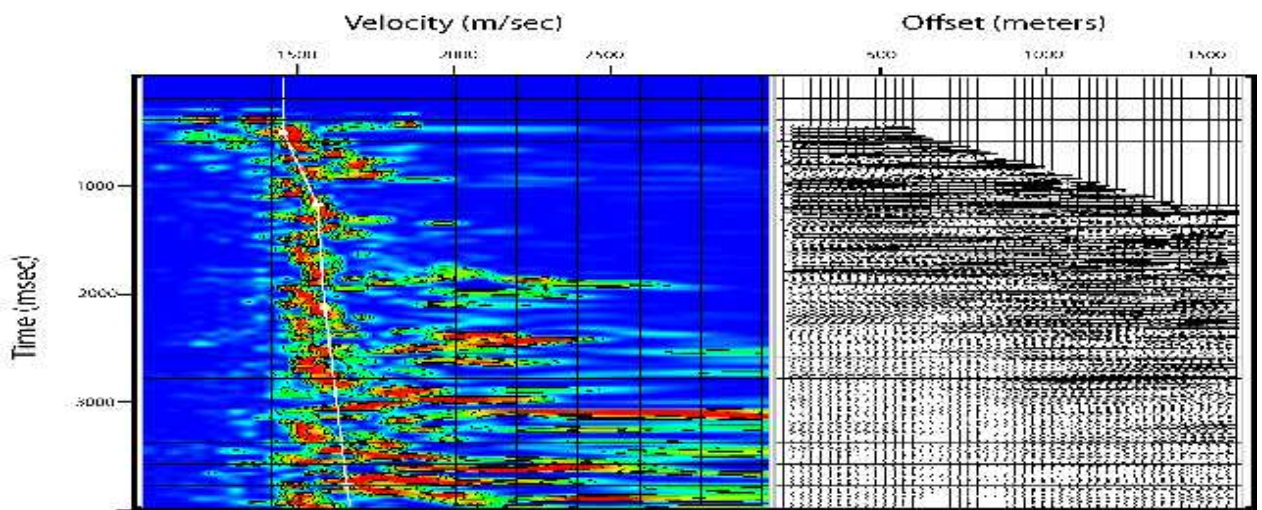


Figure 3.1: Semblance velocity plot(left) with primary velocity picked and Normal Move-out (NMO) corrected (*flattened*) Seismic Gather with primary velocity

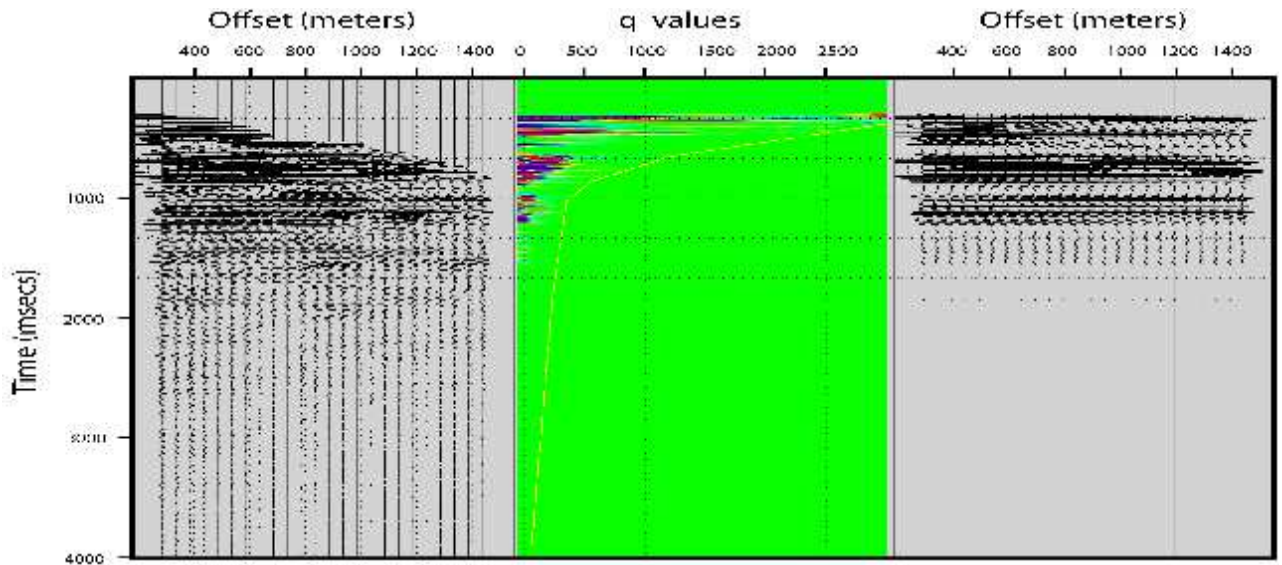


Figure 3.2: Normal Move-out (NMO) flattened Seismic Gather (*left*), its corresponding Radon panel with mute applied(*center*), and the filtered Seismic Gather(*right*)

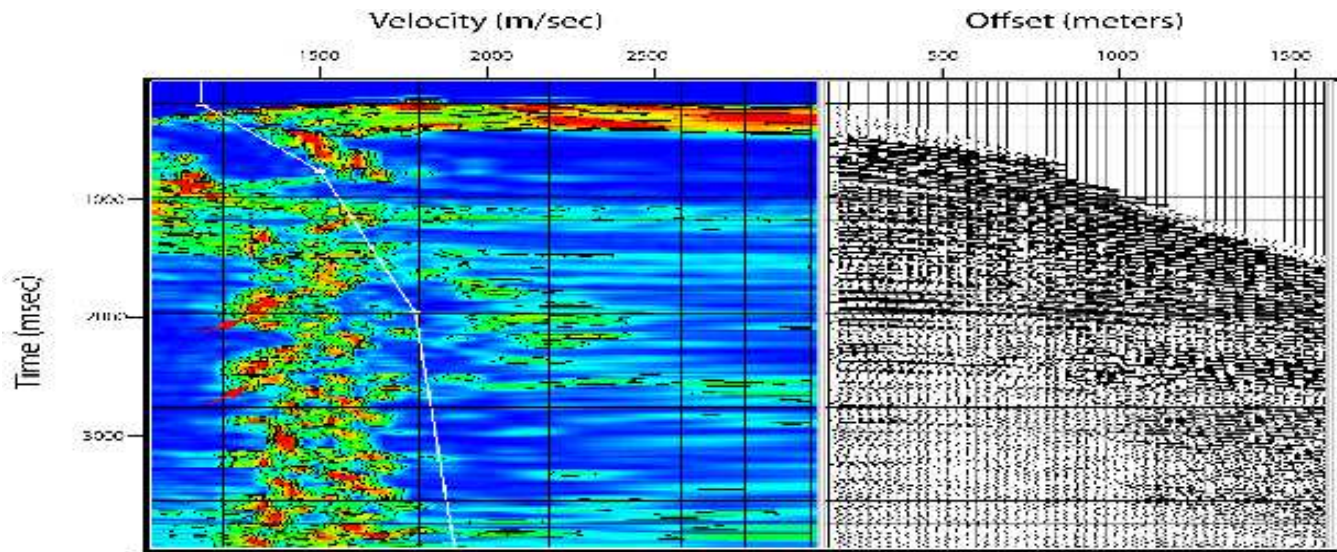


Figure 3.3: Semblance velocity plot (*left*) showing artifacts (red arrows) of Radon filtered Seismic Gather (*right*) using primary velocity,

### **3.4.2 Attenuating Multiples using Principal Components**

The dataset is pre-conditioned to estimate Root Mean Square (RMS) velocity for NMO correction. Super-gathers are formed. The multiples RMS velocities for the datasets are picked. Since strong water bottom reflections also preserve strong water bottom multiples, multiples are conspicuous in semblance plot which appear right at the bottom of the primaries (Fig 3.4). Also, interbed multiple energy forming coherent events are also visible in the semblance. This resulted NMO corrected multiples are flattened and the primaries remained over-corrected. The CDP domain dataset with multiple velocity applied is now used for the eigenvalue decomposition. For the present dataset three gates were selected for estimating the principal components Fig (3.5). An overlap of 50 msec is taken into account for smooth operation of the filter. It is worthwhile to note that for shorter offsets, even smaller gates can be used to give higher degree of variance, and thus becomes more sensitive to smaller change in coherency, which is normal feature, as move-out difference between primaries and multiples is smaller at shallower depths. Each window represents a matrix and is used to estimate eigenvalues or principal components as discussed above. In this work, only flattened (NMO corrected) multiple coda is estimated, therefore a very small percentage of principal components are kept. This selection is iterative. The iteration is based on qualitatively determining, if the selected principal components only constitute multiple energy, with the least amount primary energy leaked into these components. In this dataset only 1% of the principal components are kept in each of the gates. The eigenvalues for the respective eigenvectors are picked and the data is reconstructed. The reconstructed dataset for all the 3 gates represents multiple seismic coda. The estimated multiples are then subtracted from the dataset to obtain the filtered dataset (Fig. 3.6)



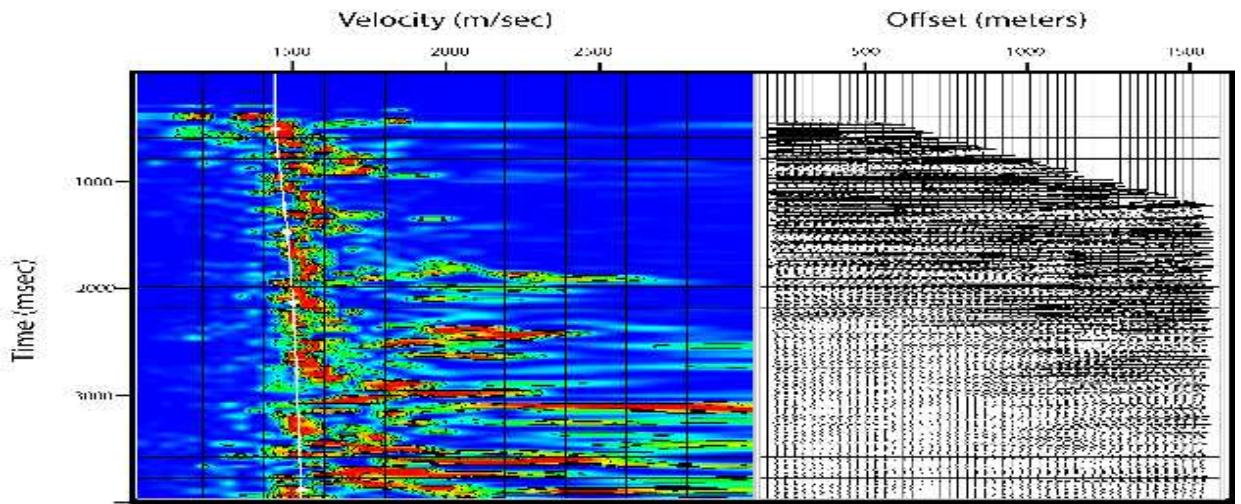


Figure 3.4: Semblance velocity with multiples velocity picked and Normal Move-out (NMO) corrected (*flattened*) Seismic Gather with multiples velocity

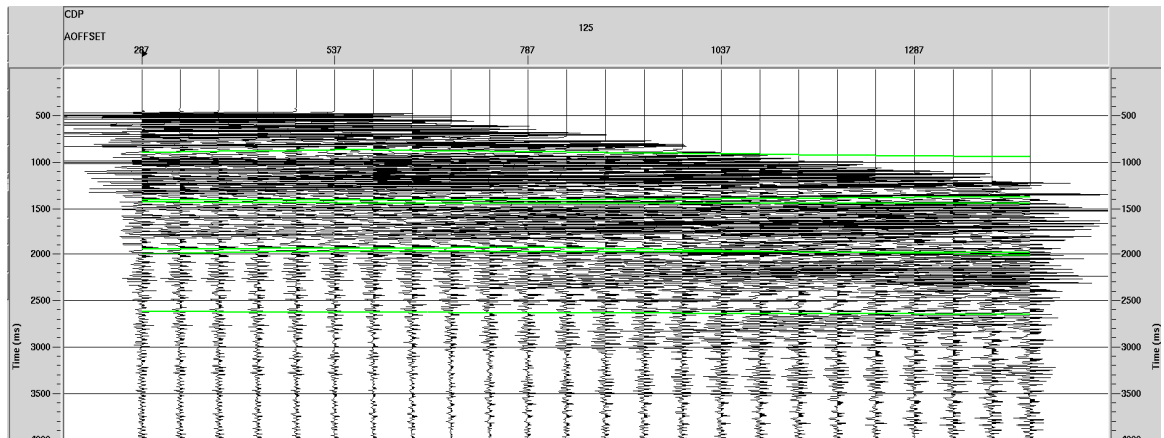


Figure 3.5: Selection of three times gates for applying Eigen Filter



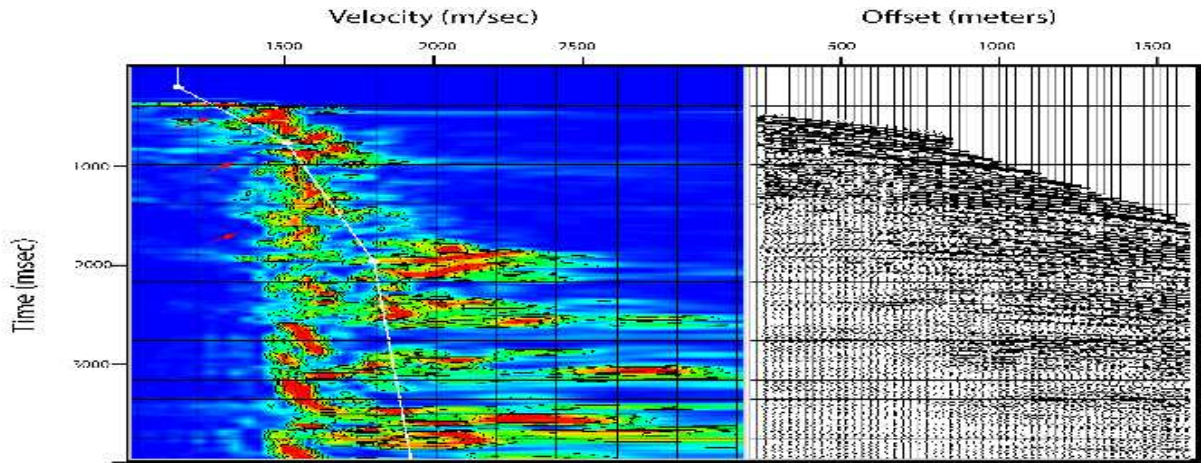


Figure 3.6: Eigen Filtered Semblance velocity plot with multiples removed (red arrows). Radon filtered Seismic Gather (right) using multiples velocity,

### 3.5 Discussion

The result of the parabolic radon multiple attenuation is compared to that of the filtered dataset using principal components. Due to the change in geology, heterogeneity and anisotropy, it is difficult to estimate radon which maps seismic into parabolic curves. Therefore, subtracting the multiple model from real gathers in radon space produces artifacts. Also, it is virtually impossible to test Radon transform quality for every gather, in 2D or 3D datasets. A model radon, which has a specific number of  $q$  values and range, may not be entirely accurate for other gathers. When using these  $q$  values for multiple attenuation, artifacts of transformation are almost inevitable.

It is evident that Principal component filtering has rendered better results. It is common practice to keep positive  $q$  values at shallower depth, which subsequently preserves multiple energy. Also, at deeper depth, some positive  $q$  values are kept, to compensate for the inaccurate primary velocities, which results in preserving remnant multiple energy. By

using multiple velocity and selecting their respective principal components using KL transform, the multiples are well separated from primaries and are subtracted from the original gathers. Also, short period multiples at shallower depths have been effectively attenuated (Fig 3.6). Fig 3.7 displays original shot gather (Fig.3.7a), conventional radon filtered gather (Fig. 3.7b) and the principal component filtered gather Fig. 3.7c. The principal component filtering Fig 7c has shown better results with the least remnant multiple energy, and preserved primaries. Similar to the radon transformation, principal component filtering depends on velocity. It is a normal practice to pick velocity at a much finer interval throughout the survey. This leads to more control over the datasets. Therefore, good control over velocities will help distinguish primaries from multiples. The principal component filtering can be used to render optimum result in multiple attenuation. Moreover, principal component filtering ensures multiples attenuation of surface and inter-bed multiples.

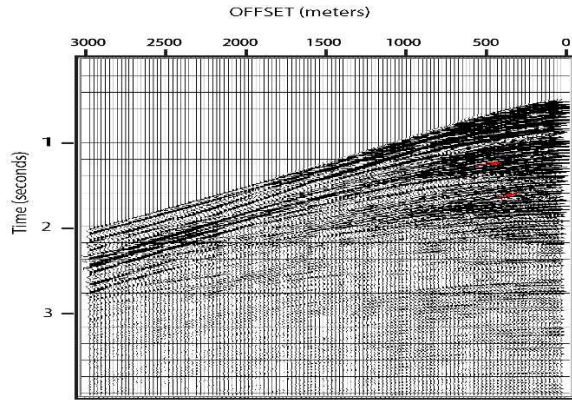


Figure 3.7a: Original Shot Gather

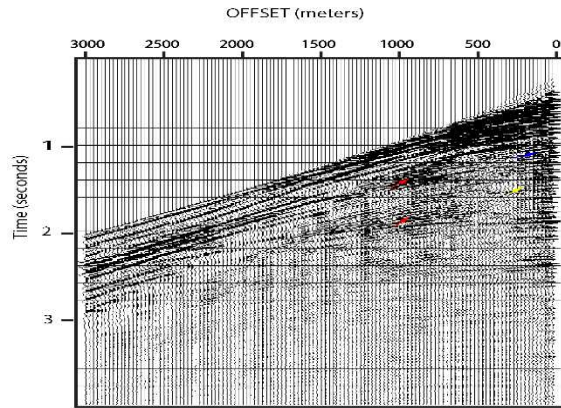


Figure 3.7b: Conventional Radon Gather with remnant Multiples (red arrows), short offset multiples (blue arrows) and artifacts (yellow arrows)

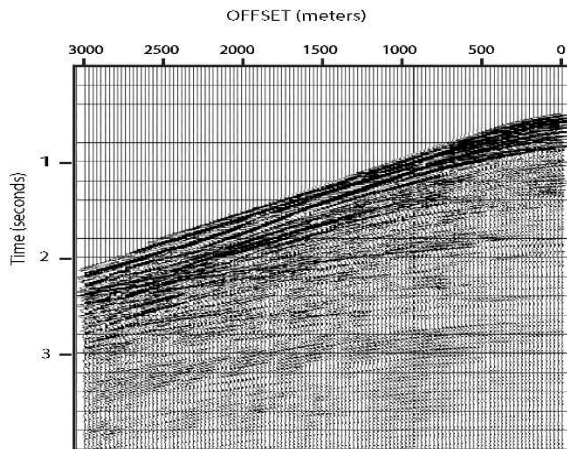


Figure 3.7c: Eigen Filter Gather, preserving primaries and Removing multiples

### **3.6 Conclusion**

In our work we have compared conventional parabolic radon with principal component filtering using Karhunen Loeve Transform for multiple attenuation. Both depends on velocity. However, radon transformation, is prone to inaccuracies due to inaccurate hyperbolic/parabolic transformations, which does not match original data, pertaining to inhomogeneities, anisotropy etc. Also, it is impossible to ensure accurate  $q$  values estimation for all gathers in for the entire 2D or 3D datasets. Principal Component filtering using KL transform is velocity dependent and can be more effective in attenuating multiples, provided if multiples are identified. It is common practice to identify multiples using velocity semblances, and therefore a finer velocity semblance sampling is carried out, to delineate coherent events. A better interpretability of these coherent events, becomes basis for effective use of preserving or filtering seismic coda using principal components. Since principal component filtering attenuated surface and interbed multiples, it appears to be a more effective, less onerous and cost-effective method.

### 3.7 References

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## VITA

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