BELIEFS: A CASE STUDY ON FORMATION, CHANGE, AND INFLUENCE

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Of all the sections in this dissertation, I believe this section will be the hardest to put into words. I have thought about this page several times throughout this process and have the same lingering questions. How can I truly acknowledge everyone who has helped me through this journey? How do I summarize into a few words on a paper the appreciation I feel towards each and every person who helped me on this journey? Who am I going to forget to acknowledge in this section? So, with these trepidations in mind, I will attempt to place this final piece of the dissertation puzzle.

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Abstract: This study explored the mathematical beliefs of two second-grade teachers. This exploration included 1) the nature of mathematics beliefs and orientation towards teaching and learning held by the teachers, 2) how their beliefs were developed and, when applicable, changed, and 3) how their beliefs influenced their mathematics instruction. It used the work of Green (1971) as its theoretical framework, Ernest (1989) for identification of nature of mathematics beliefs, and Askew et al., (1997) for identification of the orientations towards teaching and learning. This study found that the teachers tended to hold instrumentalist beliefs and transmissive to discovery orientations. It also supported the notion that the teachers' initial beliefs appeared to be developed by their experiences as a K-12 learner and curriculum use and experiences with students tended to be the catalyst for changes in their beliefs. Finally, this study found that the teachers' espoused and enacted beliefs tended to align more than contradict and any contradictions could be because of other factors such as beliefs about their students' abilities or misunderstandings about how to incorporate productive teaching and learning practices (NCTM, 2014). The purpose and results of this study could provide valuable insight on ways to create and implement effective professional development sessions. Additionally, this study suggested that further research should be conducted to determine if the more effective professional development sessions would be sufficient enough to cause changes in in-service teachers or if it needed to be replaced or supplemented with math coaches that work within and with the teachers in the classroom.

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CHAPTER I

INTRODUCTION

As a teacher educator, I am often asked by school districts to provide professional development for their elementary teachers in the area of mathematics. In order to prepare, I start by asking the people charged with setting up the sessions the same question - "What specific elements of elementary mathematics education do the teachers need developed?" They usually give the same response - "whatever you think" or "give us a list and we will pick from it." This interaction often causes me frustration, because I, much like Chen et al. (2014) discovered, believe and have experienced as a receiver and giver of PD that professional development sessions are most effective when they align to the teachers' needs. Nonetheless, I inevitably put together a session that I feel could be generically specific for most of the potential attendees.

Recently, I was asked to provide ongoing PD for a local school. I asked the usual question and got the usual responses. I put together the usual sessions and got the usual results. As I was facilitating our fifth session, I took a step back and began to wonder more deeply about what impact these sessions were having on the teachers. Did they cause them to reflect upon their instruction? Had they caused the teachers to change ineffective teaching practices to research-based approaches that have the potential to enhance student learning and conceptual understanding? Were they aware of their beliefs about mathematics and how mathematics should be taught and learned? These questions then caused me to reflect upon my role in the process. What did I know about these teachers and their beliefs? How would an understanding of their beliefs, how those beliefs formed and changed over the years, and how those beliefs impact their teaching help me develop better and more effective PD sessions? Did I need to be doing something more than just provide ongoing monthly sessions? I wanted to do more with and for these teachers which meant learning more about them. I needed to develop a better understanding of their belief structure and how those organizations impact their mathematical instruction (Cross, 2009).

Statement of the Problem

If my goal was to provide quality professional development that had the potential to develop or enhance effective mathematical instruction in these teachers, I needed to know more about them and, especially, about their beliefs. This understanding was important, because research has supported the notion that instruction is influenced by a teacher's beliefs about what it means to do mathematics and how mathematics should be taught and learned (Beswick, 2005; Cross-Francis, 2015; Lui & Bonner, 2016). However, knowing what their beliefs are was not enough. According to Green (1971), it was also important to explore how these teachers' beliefs were acquired or modified. This was needed because, as I explain in chapter two, beliefs are not independent of each other, but, rather, are parts of a belief system or cluster that have logical or matter of attitude connections. Therefore, any changes to ineffective teaching practices must start with identifying, evaluating and, if needed, modifying belief systems (Crespo, 2003).

Additionally, belief systems can appear to contradict each other (Cross-Francis, 2015; Kennedy, 2005; Speer, 2008). Teachers do not always have a full understanding of their belief structures or know how to follow effective teaching practices. They may have only experienced a traditional style of teaching where the teacher stands at the front of the room and shows the students how to follow procedures and formulas. So, when they are exposed to more non-traditional approaches to teaching, they are uncertain how to put this style of teaching into practice (Crespo & Sinclair, 2008).

For example, it might be suggested in my textbook for me to engage my learners in mathematical discussions with their peers. I believe this would be an effective teaching strategy, so I decided to utilize it in my lesson. If I know what this strategy is or have had adequate training in this practice, I might be able to successfully follow this suggestion. On the other hand, if I have not heard of or experienced this type of teaching, I might believe it is an effective strategy and attempt to use it in my lesson; however, instead of letting the students guide the discussion, ask questions, and provide answers, I take the lead and do all the asking and thinking while still believing I am properly implementing the strategy. I stated that I believed mathematical discussions would be an effective way to teach my students, but my actions, because of my lack of understanding, appeared that I did not follow that belief during my instructional time. This is why it is important for researchers to investigate how these beliefs work together in the classroom; neglecting to follow this step could lead to a biased or misinformed interpretation of the connection between teachers espoused and enacted mathematical beliefs (Copur-Gencturk, 2015).

I must explore teachers' beliefs, how those beliefs were formed and modified, and how they impact their mathematics instruction in order to gain a deeper and more developed understanding of their belief structures and how they are being followed in the classroom. It is only after gaining this understanding will I be able to begin the process of creating a framework for professional development sessions that have the ability to inform and reform educational practices and meet the needs of the teachers in my sessions (Chen et al., 2014; Pajares, 1992).

Purpose of the Study

Research on teachers' mathematical beliefs and their influence on mathematics instruction provides valuable insight (Beswick, 2012; Cross-Francis, 2015; Shirrell et al., 2018); however, I wanted to add to this body of knowledge and gain a better understanding of the influence of beliefs on mathematics instruction for teachers in a rural school in a Midwestern state. These understandings are important because teachers' mathematical beliefs have been shown to influence their instruction which, in turn, impact student learning outcomes (Voss et al., 2013). Thus, the purpose of this study was to explore and describe how mathematical beliefs influence the mathematical instruction of in-service elementary teachers at one rural elementary school. To accomplish this purpose, this study sought to answer the following questions:

1. What were the second-grade elementary teachers' beliefs about the nature of mathematics and their models of teaching mathematics and learning mathematics?
2. How were the belief structures of the second-grade elementary teachers' view about the nature of mathematics and espoused models of teaching mathematics and learning mathematics formed?

3. How did the beliefs about the nature of mathematics and models of teaching and learning mathematics influence the second-grade elementary teachers' mathematical instruction?

Significance of the Study

This study will contribute to the growing body of knowledge about mathematical beliefs and how they influence teacher instruction. The findings of this research could provide valuable insight into how beliefs about teaching and learning mathematics were developed in these teachers, and what, if applicable, were the catalysts for changing any unproductive beliefs into more productive ones. It is hoped that some of these catalysts were the result of PD sessions in which the teachers have engaged; however, if the PD sessions did not bring out positive or long-lasting changes, then these findings will be used to determine what did disturb the teachers' belief structures and how those catalysts could be used to develop more efficient and effective PD for these teachers. Other researchers, mathematics educators, professional development providers, or administrators might be able to find connections between these teachers and their own teachers and, based on the findings of this research, develop ways to modify or create efficient and effective PD sessions as well.

Assumptions, Limitations, and Delimitations

As with any study, it is important to describe the assumptions, limitations, and delimitations that have an impact on the findings. It was assumed that all participants would answer surveys and interview questions in a truthful and candid manner. It was also assumed that participants may alter their mathematical instruction when being observed and that mathematical instruction is not a "single, fixed, objective phenomenon" (Merriam, 1998, p. 202) but, rather, an ever-changing event that can yield different results on different days.

Therefore, I used long-term observations and triangulation to assist me in exploring how beliefs influenced the elementary teachers' mathematics instruction in order to present a holistic view of what was happening in the classroom.

Limitations are areas of which the researcher has no control and, thus, are potential threats to this study (Ellis & Levy, 2009). Potential researcher bias was a limitation of this study. Since I, the researcher, have worked with these teacher participants for the past two years, I might have had biases about the teachers' beliefs, mathematics instruction, and/or classroom management skills which could influence my interpretations and findings. Therefore, I used peer debriefing (having a colleague code the interview transcript, compare his findings to mine, and discuss any discrepancies) and member checking (asking participants to read my analysis of their beliefs and how those beliefs developed and evolved) to build trustworthiness and reduce researcher bias (Merriam, 1998).

Delimitations are factors or variables that are purposefully left out of the study in order to set boundaries and make the study more manageable (Ellis & Levy, 2009). This study was delimited to specific teachers in a second-grade classroom at one rural elementary school in a Midwestern state. This delimitation was chosen a priori in order to answer the identified questions of the study. This study was also delimited to a case study design which entailed a small sample that was purposively chosen. While this design reduced generalizability, it increased my ability to deeply explore the elementary teachers' mathematics instruction and the influences their beliefs had on that instruction.

Organization of the Study

This dissertation study is presented in six chapters. In Chapter I, I provided the introduction to the study, the statement of the problem, the purpose of the study, the research questions addressed in the study, and the assumptions, limitations, and delimitations. Chapter II includes a review of the literature related to the study to provide the reader with the findings of other studies that are related to this study. It is also meant to relate this study to the ongoing dialogue about the topic, fill in the gaps, and extend prior studies (Creswell, 2014, p. 30). In chapter III, the methodology of the study is discussed in detail so that future replications of the study will be possible. This section specifically addresses the design of the research, the participants and setting, procedures used to collect data, instruments used to collect data, data analysis procedures that were implemented, and how trustworthiness, credibility, and ethics were established and maintained. Chapter IV and V contain the results of the two embedded units of analysis used in the case study - Sally (Chapter IV) and Grace (Chapter V). Chapter VI presents the findings of the case study as well as the conclusions, implications, and calls for additional research.

CHAPTER II

LITERATURE REVIEW

The purpose of this chapter is to report the relevant research related to mathematics instruction and beliefs. The research questions guiding this review are:

1. What are elementary teachers' beliefs about the nature of mathematics and their models of teaching mathematics and learning mathematics?
2. How were the belief structures of the second-grade elementary teachers' view about the nature of mathematics and espoused models of teaching mathematics and learning mathematics formed?
3. How do the beliefs about the nature of mathematics and models of teaching and learning mathematics influence the second-grade elementary teachers' mathematics instruction?

Several areas of research related to the current study will be addressed. The sections of this chapter include the research on beliefs, the formation and changes of belief structures, and influences of beliefs. In the beliefs section, I explore the three dimensions of beliefs as outlined by Green (1971), mathematical beliefs identified by Ernest (1989), belief orientations towards teaching and learning (Askew et al., 1997), productive and unproductive beliefs, and constructivists and transmissive beliefs. I also describe what defines high-quality mathematics instruction and how that instruction can be evaluated using Cognitively Guided Instruction and the Mathematics Classroom Observation Protocol for Practices.

Beliefs

Beliefs are "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259), guide teachers' behaviors and thoughts (Capraro, 2005) and affect how and what teachers teach (Haciomeroglu, 2013). They are based more on cognitive factors than attitudes and emotions and, as a result, are harder to change than attitudes or emotions (Philipp, 2007). A belief system is "a metaphor for examining and describing how an individual's beliefs are organized" (Thompson, 1992, p. 130) and can be identified by three dimensions: quasi-logical relation, psychological strength, and cluster location (Green, 1971).

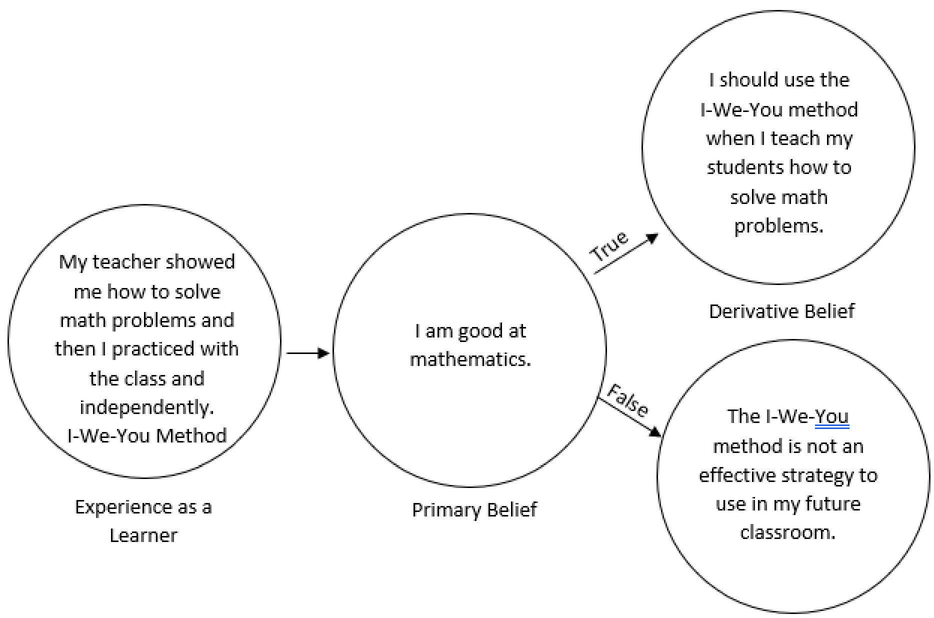
Three Dimensions of Beliefs

In the quasi-logical relation dimension, beliefs are either derivative (derived from another belief) or primary (not derived from other beliefs). This dimension explains what is believed and how it is believed. For example, if my experience as a learner of mathematics was that the teacher directly showed me how to solve problems by modeling the step-by-step procedure to follow, provided time for the class and me to practice using those processes as a group, and then assigned a set of problems for me to work independently (the I-We-You method), and I held a primary belief that I was "good" at mathematics, I might have the derivative belief that this method is an effective instructional strategy to use in my classroom as a teacher (Harbin & Newton, 2013; Maasepp & Bobis, 2015).

If asked to explain why I used the I-We-You strategy in my future classroom, I would follow the cyclical rationalization that my teachers used this method and I am good at mathematics so I should use this strategy as well with my learners. Interestingly, since beliefs are held in true/false dichotomies (Philipp, 2007), the belief that the I-we-you method is effective would only be true (in this example) if the "I am good at mathematics" belief is true; if I did not believe I was good at mathematics, then I would claim that this strategy was not effective (Green, 1971) (see Figure 2.1).

Figure 2.1

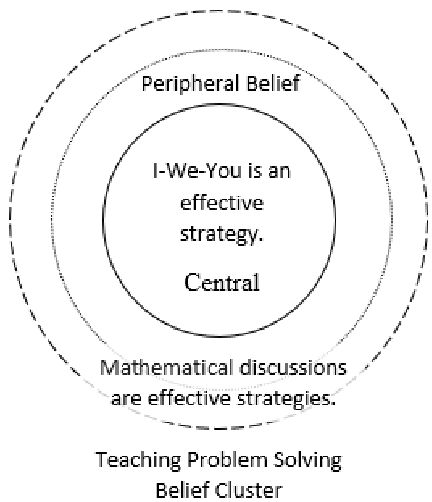
Green's (1971) Quasi-logical Belief Structure



The second dimension, psychological strength, determines a teacher's ability or desire to change beliefs (Green, 1971). In this dimension, beliefs are either central (harder to change) or peripheral (easier to change). Green (1971) uses, what he calls, a metaphor of concentric circles to illustrate this dimension (see Figure 2.2). The central belief sits in the center of the circle; these are the beliefs individuals accept without question, are held most dearly, and are the most difficult to change. Many individuals are not able to nor will they openly debate these beliefs (Green, 1971). As the circles move beyond the center toward the perimeter, the beliefs are held with decreasing strength and become increasingly easier to examine, discuss, and change.

Figure 2.2

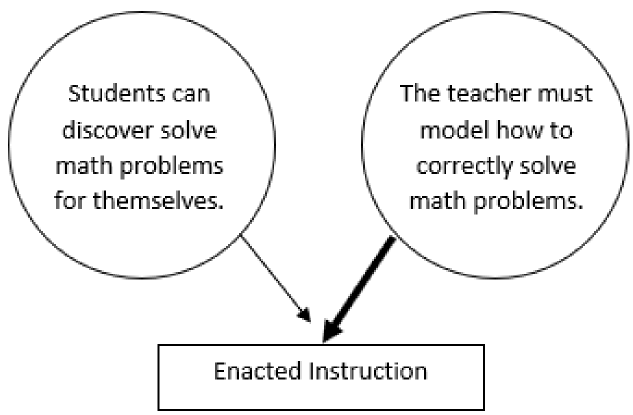
Green's (1971) Psychological Strength



For example, the belief that the I-We-You method of problem solving is an effective instructional strategy could be a central belief held by me as a teacher, especially if it was derived from my early experiences as a learner and my primary belief that I was good at math (Brown, 2005; Harbin & Newton, 2013). If I participate in a professional development session that demonstrates the effectiveness of mathematical discussion in promoting problem solving skills, I might add this view as a peripheral belief to my teaching problem solving belief cluster. If I struggled to use mathematical discussion in my lessons, did not see immediate results, or was shown a different strategy that better aligned to my previous set of beliefs, I might quickly abandon this peripheral belief for an alternative strategy or resort back to how I was taught (Gabriele & Joram, 2007; Linder & Simpson, 2017; Newborn & Cross, 2007). On the other hand, if I encountered the same struggles with the I-We-You method, I could easily rationalize that the students were not ready to learn that concept or were not "paying attention" instead of evaluating the effectiveness of the I-We-You method, because this strategy was centrally believed (Askew et al., 1997). Therefore, the ability to change a belief is not based on "*what* is believed but with *how* it is believed" (Green, 1971, p. 47).

Figure 2.3

Green's (1971) Belief Clusters



Belief clusters are the final dimension of beliefs. Belief clusters sit in isolation from other belief clusters (Green, 1971). This dimension explains how individuals can hold seemingly contradictory beliefs. Teachers with differing belief clusters often are unaware of any possible inconsistencies among them. For example, I might believe students are capable of independently solving mathematical problems. However, I might also believe that I must model mathematical procedures for my students to ensure they know how to properly and quickly solve the problems. These two belief clusters seem to contradict each other: students can independently solve problems but I must specifically show them how. I will tend to use whichever belief cluster is stronger during mathematics lessons. Unless I am given the opportunity to juxtapose the two belief clusters and discover the contradiction, I might never be able to change my beliefs (see Figure 2.3).

Green (1971) also claimed that "beliefs are gathered always as parts of a belief system" (p. 42). This means that beliefs, not to be confused with the belief cluster, do not sit in isolation of each other, but, rather, they exist as sets or groups that relate to one another as the result of logical connections or matters of attitude. One such set or group of beliefs is mathematical beliefs.

Mathematical Beliefs

Mathematical beliefs are held in three distinct belief clusters: the nature of mathematics, teaching mathematics orientations, and learning mathematics orientations (Askew et al., 1997; Ernest, 1989; Lui & Bonner, 2016; Cross, 2009; Philipp, 2007). There are three categories of teachers' beliefs about the nature of mathematics: instrumentalist, Platonist, and problem solving (Ernest, 1989). In the instrumentalist belief cluster, teachers view mathematics as a set of facts, rules, and procedures to be learned and used and different areas of mathematics (number sense, geometry, algebra) are unrelated. Teachers who hold a Platonist mathematical belief view mathematics as an interconnected, unified body of knowledge waiting to be discovered. The final nature of mathematics belief clusters is the problem solving view. In this cluster, teachers believe mathematics is constructed by human endeavors and, therefore, is open to revision and new explorations (Ernest, 1989).

Table 2.1

Summary of Beliefs and Orientations

Beliefs about the Nature of Mathematics (Ernest, 1989)			
	Instrumentalist	Platonist	Problem-Solving
Description	Mathematics as a set of facts, rules, and procedures to be learned and used.	Mathematics as an interconnected, unified body of knowledge waiting to be discovered.	Mathematics is constructed by human endeavors and, therefore, is open to revision and new explorations.
Orientations towards Teaching and Learning Mathematics (Askew et al., 1997)			
Description	Transmission	Discovery	Connectionist
	Emphasis on students' ability to follow routines and procedures rather than on what the students might already know.	Emphasis on student strategies rather than prescribed formulas.	Emphasis on student strategies, but the teacher should help students find the most efficient strategies to use.
	Emphasis on teaching rather than learning.	Emphasis on learning rather than teaching.	Teaching and learning are complimentary.
	Teachers must explain mathematical procedures clearly and logically.	Teachers should incorporate practical experiences, verbal explanations and manipulatives.	Teachers should provide interpersonal activities and have students interact with others.
	Misunderstandings are the result of the students' inability to learn the material, not the teachers' explanation.	Misunderstandings are the result of the student not being ready to learn the material yet.	Misunderstandings should be acknowledged, inspected, and corrected as needed.
	Reinforcement and practice of "correct" methods are needed if students do not understand the learned concepts.	Understanding happens when students work things out for themselves.	Understanding happens when students are challenged and engage in productive struggle.
	Mathematical discussions consist of students answering teacher directed questions, so the teacher can check for understanding.		Mathematical discussions consist of dialogue between the teacher and student and the students with each other.

Teachers can also have different belief clusters or orientations of teaching and learning mathematics (Askew et al., 1997). Askew et al. interviewed and observed 18 teachers and developed a way to characterize teachers' models of sets of beliefs based on their orientations towards teaching and learning mathematics. These orientations included transmission, discovery, and connectionist. Askew et al. (1997) cautioned that these were ideal types and that “no one teacher did, or is ever likely to, fit exactly within the framework of beliefs of any one of the three orientations; many combined several characteristics of two or more orientations” (p. 28). Other research has supported this caution and have found that teachers tend to have and display a combination of beliefs within their belief structures (Aljaberi & Gheith, 2018; Barkatsas & Malone, 2005; Voss et al., 2013).

Teachers with a transmission orientation toward teaching mathematics place emphasis on students’ ability to follow routines and procedures rather than on what the students might already know. Therefore, these teachers rarely use students’ prior knowledge, discoveries, or understandings to build more effective and efficient methods. These teachers also believe it is their responsibility to explain mathematical procedures clearly and logically. If students do not understand these explanations, it is because of the students’ inability to learn the material or their own misunderstandings, not the teachers’ explanations. These students will need more reinforcement and practice of the “correct” method for finding the answers to the mathematical problems given in the day’s lesson (Askew et al., 1997).

Likewise, teachers with a transmission orientation towards learning mathematics place more emphasis on teaching than learning. Interactions between teachers and learners consist of the “I do-We do-You do” process and an exchange of questions and answers so the teacher can check student understanding during the lesson. Since these teachers depend on traditional methods of teaching and learning, mathematical concepts are viewed as discrete packages, similar to Ernest’s (1989) instrumentalist beliefs about the nature of mathematics (Askew et al., 1997).

Teachers with a discovery orientation towards teaching mathematics believe they should incorporate practical experiences, manipulatives, and verbal explanations into the mathematics lessons so students can discover methods and procedures for themselves. However, these are still taught in the traditional discrete packages instead of intertwined with other mathematical concepts and understandings. Teaching and learning are still separate experiences, but learning takes precedence over teaching. Therefore, teachers with a discovery orientation towards learning mathematics believe student strategies are more important than prescribed formulas and understanding happens when students work things out for themselves. If students are not able to learn or discover a mathematical concept, it is because they are not ready to learn the material yet instead of a lack of practice. Teachers with a discovery orientation somewhat mirror Ernest's (1989) Platonist belief about the nature of mathematics (Askew et al., 1997).

The final orientation is connectionist. Teachers with this orientation towards teaching mathematics believe teacher-to-students and student-to-student dialog is essential. These discussions focus around students’ discoveries and understandings of mathematical concepts and procedures. Teachers with this orientation feel it is their responsibility to help students explore their own strategies and determine ways to make them more efficient. They also believe students should be given opportunities to find and interpret the meaning of mathematical problems. Since these teachers view teaching and learning as complementary endeavors, connectionist teachers believe mathematics is learned best when students are given the opportunity to recognize, investigate, and correct their own misunderstandings, interact with others, and challenge themselves to overcome misconceptions and difficulties (Askew et al., 1997). The connectionist orientation aligns closely with Ernest's (1989) problem solving beliefs about the nature of mathematics. These beliefs also have similarities to productive and unproductive beliefs about teaching and learning.

Productive and Unproductive Beliefs

As seen so far, some teachers believe children should be taught following a traditional method. These teachers appear to adhere to the quasi-logical belief that they know how to do mathematics such as adding two-digit numbers with regrouping, because their teachers showed them how to follow the procedure the board; therefore, this style of teaching must be effective (Barkatsas & Malone, 2005; Wilkins, 2008). Unfortunately, however, this model of teaching and learning is unproductive. According to the National Council of Teachers of Mathematics (NCTM, 2014), unproductive beliefs about teaching and learning mathematics include requiring students to memorize and use standard algorithms and exact procedures to solve problems and equation, emphasizing that facts and basic skills must be mastered before students learn how to apply mathematics to real world scenarios or problems, and providing simplistic step by step approaches to problem solving to ensure the mathematics is easy for students and will not cause them to become frustrated or confused. These unproductive beliefs align closely to transmission orientations about teaching and learning.

NCTM (2014) also created a set of productive beliefs about teaching and learning. These include focusing on developing conceptual understanding through problem solving and discussions, utilizing multiple strategies such as general and standard algorithms and processes, providing students with opportunities to explore and make connections to mathematical concepts, and engaging in active dialogue that provides opportunities to represent and justify their understandings as well as listen to and consider the reasonings of their peers. Additionally, teachers with productive beliefs provide students with opportunities to engage in productive struggle and solve meaningful problems. These beliefs focus on engaging students and align to a discovery and connectionist orientation towards teaching and learning.

NCTM (2014) pointed out that their list of productive and unproductive beliefs "should not be viewed as good or bad. Instead, beliefs should be understood as unproductive when they hinder the implementation of effective instruction" (p. 11). This same philosophy holds true for constructivist and transmissive beliefs as well.

Constructivist and Transmissive Beliefs

Another set of beliefs that lead to an understanding of belief structures and their influence on high-quality mathematics instruction is constructivism and transmissive theories about teaching and learning. Voss et al. (2013) claim that there is a close relation "between epistemological beliefs and beliefs about teaching and learning" (p. 253); therefore, constructivist theories about learning co-occur with constructivist beliefs, and transmissive theories co-occur with transmissive beliefs. The constructivist theory states that students approach learning with their own set of prior knowledge and preconceived ideas about mathematics (Voss et al., 2013), and that individuals construct meaning as they participate in life and with the culture in which they live (Merriam, 1998). The constructivist theory has three main types: cognitive, social, and radical; the first two are discussed and used in this study.

Cognitive constructivism is primarily based on the work of Jean Piaget, and social constructivism is primarily based on the work of Lev Vygotsky (McLeod, 2019). In cognitive constructivism, learners are not viewed as empty vessels waiting for the teacher to pour in knowledge but, rather, as creators of their own learning (Fosnot, 1996). Students have cognitive schemas or networks that are formed by making connections between what they already know to what they are currently learning; these connections are important factors in learning and retaining new information (Van de Walle et al., 2018).

Social constructivism is very similar to cognitive as it also assumes that students must engage in meaning-seeking activities to learn; however, it also adds that community and the zone of proximal development (ZPD) are essential to learning as well. Community influences what students think about and how; the ZPD refers to skills that are just beyond the learner's ability to understand. If I am working within my ZPD, a certain activity or concept might be too difficult for me to learn independently; however, my community of peers and more knowledgeable others can scaffold how to understand the problem through the use of manipulatives or by showing me a model or illustration of their thinking to give a platform upon which to construct my own knowledge. As I become more confident in the new knowledge I created, I will no longer need the scaffolds but can share my learning with others who might (McLeod, 2019; Van de Walle et al., 2018).

Constructivism is a theory that informs many teaching strategies that create productive belief structures that lead to effective instruction. These strategies include building new knowledge from prior knowledge, engaging in and learning from productive struggle, and viewing errors as learning opportunities. Constructivism also provides opportunities for students to engage in mathematical discussions and reflective thought. This encourages the use of multiple strategies and honors diversity by showing that all learners are unique and should be valued and included in mathematical discussions (Van de Walle et al, 2018).

The transmissive theory of teaching and learning states that teachers impart their knowledge and understanding to the students who are passive recipients (Voss et al., 2013). In this model, teachers model how to use a procedure or process to solve the given problem, the students independently work sample problems with the teacher monitoring their understanding through questioning, and then students work a problem set independently; in other words, transmissive teaching follows the I-We-You strategy previously discussed. While the teacher does somewhat monitor student understanding through the class instruction, feedback in a transmissive classroom is usually given at the end of the lesson when the teacher assesses the students' independent practice problem (Gravé et al, 2020). The problem with this transmissive style of teaching is it does little to enhance learners' mathematical thinking or understanding (Lee, 2017) and negatively affects number sense and fact recall (Van de Walle et al., 2018).

Several other studies have also been conducted showing the impact of student learning based on whether their teacher followed transmissive or constructivist beliefs about teaching and learning. For example, Meschede et al. (2017) supported their notion that transmissive teaching negatively correlated with professional vision or the teachers' ability to notice or explain events that hinder or foster student learning. Voss et al. (2013) found that teachers who held transmissive beliefs about teaching and learning had lower quality of instruction levels and negatively affected student achievement, whereas teachers with constructivist beliefs had higher quality of instruction levels and positively increased student achievement. Staub and Stern (2002) found that students with teachers who held a constructivist orientation of teaching had larger achievement gains in solving word problems and computational proficiencies when compared to students who had teachers who had a transmissive style of teaching. Finally, results from Blazar's (2015) study indicated that constructivist teaching was positively related to student learning.

Table 2.2

Beliefs and Orientations Connection to Transmissive and Constructivist Theories

Transmissive Theory	Between Theories	Constructivist Theory
Instrumentalist Nature of Mathematics Belief (Ernest, 1989)	Platonist Nature of Mathematics Belief (Ernest, 1989)	Problem Solving Nature of Mathematics Belief (Ernest, 1989)
Transmission Orientation toward Teaching and Learning (Askew et al., 1997)	Discovery Orientation toward Teaching and Learning (Askew et al., 1997)	Connectionist Orientation toward Teaching and Learning (Askew et al., 1997)

Voss et al., (2013) also concluded that beliefs about what it means to do math and the models about teaching and learning mathematics usually align with a constructivist or transmissive belief. In other words, if these orientations were grouped, transmissive beliefs would be categorized with the instrumentalist belief about the nature of mathematics and the transmission orientation of teaching and learning; constructivist beliefs would be associated with the problem solving belief and connectionist orientation. The Platonist belief and discovery orientation would be somewhere between the two (see Table 2.2). It is important to note that Voss et al. (2013), found that transmissive and constructivist beliefs were not "two ends of a one-dimensional continuum and are not mutually exclusive categories but that they are two distinct, negatively correlated dimensions" (p. 257).

Unfortunately, some research has found that some teachers believe they are following constructivism but do not have a full understanding of what it means and fall back into transmissive styles of teaching and learning (Beswick, 2005). Therefore, Voss et al. (2013) concluded that teachers needed to explore their transmissive beliefs and juxtapose those to a constructivist belief system if they were going to make any effective and long-lasting changes. This comparison could lead to discoveries of how their beliefs were formed, if any have changed with experience, and what were the possible catalysts for those changes.

Formation and Change of Beliefs

Green (1971) stated,

> It seems intuitively obvious, furthermore, that the acquisition of beliefs and their modifications is a major concern in the activity of teaching. In attempting to understand that activity, it may be important, therefore, to study how beliefs are acquired or modified. (p. 42)

In other words, in order to truly understand how beliefs influence teachers' mathematics instruction, it is important to explore how those belief structures were formed and what, if applicable, modified any of the teachers' beliefs. This understanding could also be the foundation for developing and implementing effective and focused PD sessions.

Formation of Beliefs

Research appears to support the notion that teachers' experiences as a learner was the most predominant factor that formed their beliefs about the nature of mathematics and their orientations towards teaching and learning mathematics. For example, Barkatsas and Malone (2005) conducted a case study in which they explored Ann's, their participant, espoused and enacted beliefs about mathematics and how mathematics should be taught and learned as well as the factors that influenced or shaped those beliefs. They found that Ann's prior school experiences and her worldviews impacted her beliefs about mathematics, and her prior school experiences and current teaching experiences influenced her beliefs about how mathematics should be taught and learned. Additionally, Harbin and Newton (2013) explored how six teachers' espoused beliefs and views were being enacted in their mathematics instruction and discovered that the predominant influencer of their participants' beliefs and instruction was prior schooling experience. Finally, Maasepp and Bobis (2015) conducted a case study and concluded that their participants', Peter and Lauren, initial beliefs about mathematics and how it should be taught and learned were directly related to their experiences as a learner in the school setting.

These findings suggest that experiences as a learner, especially in primary and secondary educational settings, tend to form mathematical beliefs structures about what it means to do mathematics and how it should be taught and learned. Since these are learned early, they have the potential to be primary beliefs that are centrally believed; this would imply that they would be more difficult to confront, question, or change (Green, 1971). This is why it is important to pay attention to and address how experiences as a learner impact beliefs and, inevitably, teacher practice (Beswick, 2012). If the goal is to change or modify transmissive beliefs into more constructivist ones then knowing what formed teachers' initial beliefs and how those beliefs are held are the first steps needed (Green, 1971). The next step is uncovering effective catalysts for change.

Changing Beliefs

Restructuring a belief system is difficult (Cross, 2009). Teachers often bring deeply ingrained, preconceived ideas of what it means to teach and learn mathematics into their classrooms, which, as previously shown, are usually constructed when they themselves were in school (Beswick, 2012; Cross, 2009; Haciomeroglu, 2013; Harbin & Newton, 2013; Maasepp & Bobis, 2015). Additionally, some teachers have never thought about their mathematical beliefs (what beliefs they hold, why they hold those beliefs, how those beliefs impact their instruction) or were ever given the guidance or opportunity to explore their mathematical beliefs as a pre-service teacher (Beswick, 2012). When teachers become aware of their beliefs, they can start to question those beliefs, make connections to their beliefs and their instructional practice, and, if appropriate, make necessary changes to their belief and practice (Mewborn & Cross, 2007).

These changes do not happen quickly or easily and require time, support, and, often, some type of catalyst (Capraro, 2005). One such catalyst is cognitive dissonance (Hughes et al., 2015; Mewborn & Cross, 2007). Teachers often view early mathematics as simple and believe that it requires little mathematical knowledge to teach it (Chen et al., 2014; Moscardini, 2014). Oftentimes, these teachers feel they do not have strong mathematical abilities, but are able to teach early mathematics because they have memorized and can follow simple formulas, have created and employ mathematical tricks, or have devised or use mnemonic rhymes or chants to remember procedures.

When these teachers are asked to solve a problem without using any formulas, tricks, or procedures, they often face cognitive dissonance (Moscardini, 2014). These teachers know ways to solve the problem but do not understand why these strategies work. They are pressed to think beyond procedures and formulas and must rely on sense-making activities and discussions with their colleagues to “solve” the problem (Moscardini, 2014). This confrontation with their mathematical knowledge, abilities, and/or pedagogical approach causes the teachers to challenge their previously held beliefs of what it means to do, learn, and teach mathematics which can lead to a change in their prior beliefs.

Another type of catalyst that has the potential to change teachers’ mathematical beliefs is an awareness of children’s mathematical thinking (Ambrose, 2004; Lannin & Chval, 2013). Several studies have shown that teachers change their beliefs about teaching and learning mathematics when they are given the opportunity to learn about and observe how children reason mathematically (Ambrose, 2004; Carpenter et al., 1989; Lannin & Chval, 2013; Vacc & Bright, 1999). These experiences cause teachers to question “normally accepted” teaching practices and reflect upon what they believe and “know” about what it means to teach and/or learn mathematics (Philipp, 2007). The teachers must analyze their isolated mathematical beliefs, compare them to other mathematical belief clusters, determine if any inconsistencies between and among them exist, and then, if needed, reconcile the inconsistencies (Green, 1971).

Unfortunately, these beliefs do not change immediately; it sometimes takes two years for teachers to change their beliefs about teaching and learning mathematics, because they must have time to confront their beliefs, change their practice, and witness the effects (increased student achievement and understanding of mathematics) of this change (Fennema et al., 1996; Polly et al., 2014). However, when these changes do occur, teachers tend to keep the new beliefs and practices for several years (Knapp & Peterson, 1995).

The potential for cognitive dissonance and awareness of how children learn mathematics to change transmissive beliefs about teaching and learning mathematics highlights why it is crucial for school districts and administrators to provide formal professional development (PD). PD provides necessary time and guidance for teachers to become aware of, challenge, and revise any mathematical beliefs that could negatively influence mathematical teaching and learning (Voss et al., 2013). School districts and administrators must understand that reforming mathematics requires more than changing teacher practices; it involves changing teacher beliefs about how mathematics should be taught and learned (Shirrell et al., 2018). Therefore, they should provide opportunities and support for teachers to engage in the difficult and time-consuming work of transforming any transmissive mathematical beliefs into constructivist beliefs of teaching and learning mathematics.

Once these beliefs have been evaluated and, if needed, modified, then teachers could start the process of implementing more constructivist and positive beliefs about mathematics and mathematics teaching and learning in their classrooms. Again, this process takes time and requires deliberate reflection and self-appraisal (Barkatsas & Malone, 2005).

Influence of Beliefs on Instruction

In order to start the process of critical reflection and self-appraisal of their instruction, teachers and those positioned to provide guidance and support for the teachers through this process need some way to assess the mathematics instruction and determine if it aligns to a more transmissive or constructivist view of teaching and learning. This can be achieved through the Cognitively Guided Instruction (CGI) Teacher Levels (Fennema et al., 1996) and the Mathematics Classroom Observation Protocol for Practices (MCOP^2) (Zelkowski & Gleason, 2016). However, in order to thoroughly understand how these assessment measures and instruments can be used to identify and modify instructional practice, a definition and description of highly-quality mathematics instruction needs to be given and aligned to the mathematical, productive and unproductive, and constructivist and transmissive beliefs previously outlined.

High-Quality Mathematics Instruction

In 2000, NCTM published a set of *Principles and Standards for School Mathematics.* Their aim was to provide a guide to focus discussions about mathematics in order to improve students' mathematics education. They purposely left curriculum decisions to the local community but provided a set of common language, examples, and recommendations schools and educators could use to build a solid mathematics education program that met the needs of their students. These principles had a great impact on local and national standards and led to many changes in mathematics education (NCTM, 2014).

The principles and standards were updated in 2014 in order to incorporate new research and experiences with effective mathematics programs and address obstacles and unproductive beliefs that the discussions about mathematics education and research uncovered since their original release (NCTM, 2014). These updated guiding principles for school mathematics included areas of teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism. While each of these principles and standards are important, this study will only address teaching and learning and curriculum principles.

Teaching and Learning

Mathematics teaching requires teachers to have a solid understanding of "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395). They also need to know how students learn and progress through mathematical concepts (Daro et al, 2011). The developers of the revised principles and standards were aware of these requirements and aligned their teaching practices to these research findings (NCTM, 2014). These principles include, establishing mathematics goals that focus student learning, implementing tasks that promote reasoning and problem solving, using and connecting mathematical representations, facilitating meaningful mathematical discussions, posing purposeful questions, building procedural fluency from conceptual understanding, supporting productive struggle, and eliciting and using evidence of student thinking (NCTM, 2014). These principles align closely to a constructivist style and a productive belief towards teaching and learning.

Unfortunately, research shows that many mathematics teachers rarely engage their learners, especially low-achieving learners, in high-quality mathematics instruction such as those identified in the principles (Carpenter et al., 2015; Desimone et al., 2005; Polly et al., 2014). These teachers believe these activities are too difficult for their learners and introducing these types of instructional strategies before fact mastery or procedural competency would only cause frustration and increase the likelihood of their learners becoming math anxious (Carpenter et al., 2015; Desimone et al., 2005). Furthermore, some of these teachers may have never experienced this type of teaching and, therefore, are not aware of its ability to engage their learners and develop mathematical understanding (Crespo & Sinclair, 2008). As a result, these teachers tend to follow a more transmissive style of teaching believing they are increasing efficiency and memorization of procedures and facts when they are actually following unproductive beliefs that tend to hinder student learning (Barkatsas & Malone, 2005; Vacc & Bright, 1999; Wilkins, 2008).

Curriculum

Teachers also need to have a firm understanding of how to use curriculum with integrity instead of fidelity, meaning they need to know how to determine if material presented in their textbook is an appropriate task to develop conceptual understanding of the mathematical concept being taught or if the curriculum needs to be modified (NCTM, 2014, 2020). This knowledge of content and curriculum is another essential piece of Ball et al.'s (2008) Domains of Mathematical Knowledge of Teaching. This understanding is essential as not all curriculum is designed from a constructivist framework.

Similar to teacher beliefs, curriculum can either promote transmissive or constructivist views of teaching. Teachers with a more transmissive style of teaching and learning can provide effective mathematics instruction if they follow the lessons and activities outlined in a constructivist curriculum (Hill et al., 2008). These textbooks can show teachers how to effectively engage learners and provide rich mathematical activities. Additionally, constructivist curriculum can explicitly outline the goals and objectives of the lesson and key mathematical ideas for the teachers, which, in return, can have a positive impact on the teacher's quality of mathematics instruction (Charalambous & Hill, 2012; Stein & Kim, 2009). Both the transmissive and constructivist teacher benefit from a constructivist curriculum if the teacher feels comfortable enough to use it (Schoenfeld, 2004). Moreover, teachers who use constructivist curriculum can learn from materials and change unproductive beliefs about teaching and learning as needed (Bay-Williams & Karp, 2010).

On the other hand, curriculum can also be transmissive. Transmissive curriculum can effectively be used by teachers who believe curriculum can and should be supplemented or modified when needed and can identify and properly enrich subpar mathematical lessons, tasks, and problems, as previously mentioned. Unfortunately, transmissive curriculum can prevent students from developing conceptual understanding of mathematics if their teacher or the school system believes curriculum should be followed with fidelity (Charalambous & Hill, 2012; NCTM, 2020; Schoenfeld, 2004). Conversely, if the teacher believes curriculum can and should be modified but does not possess the mathematical knowledge to know how to properly supplement the material or does not know how to identify the insufficient elements of the textbook, mathematics instruction will suffer as well (Hill et al., 2008).

Based on these findings, high-quality mathematics instruction is dependent on a teacher's belief and mathematical knowledge if a transmissive curriculum is used, whereas constructivist-oriented curriculum has the potential to have a positive impact on instruction for teachers with a transmissive or constructivist view toward teaching and learning. These findings show that curriculum "plays a central role in the mathematics education of children and in the mathematics instruction enacted by teachers" (NCTM, 2020, p. 40). Therefore, it is imperative that those involved in the selection and implementation of curriculum need to have a clear understanding of curriculum within and across grade levels, an awareness of the dynamic and ever-changing field of mathematics, and an ability to assess the quality of the materials in question or being used (NCTM, 2014). Additionally, they need to know how to properly use the curriculum in ways that are responsive to the needs of their students and provide time for collaboration and professional learning to learn how to properly use the materials as needed (NCTM, 2020).

Summary

Teachers who display high-quality mathematics instruction know how to engage their learners in mathematical discussion, support and scaffold mathematical dialogue, and ensure mathematical ideas generate from student thought instead of the teacher or textbook. These teachers know how to pose problems for students to solve or provide a mathematical task for students to explore that are problematic, make mathematics the intriguing part of the situation, and provide opportunities to use prior mathematical knowledge to construct new ideas. They provide time and opportunities for classroom discourse and hold students accountable for mathematical understanding and explanation (Munter, 2014).

Teachers who display high-quality mathematics instruction do not follow unproductive beliefs about teaching and learning and curriculum such as 1) focusing on memorization and fact recall, 2) telling students what definitions, procedures, and formulas to use to solve problems, 3) providing step by step directions to make mathematics learning easy and reducing engagement in productive struggle, or 4) following curriculum with fidelity instead of integrity to meet the needs of their students (NCTM, 2014, 2020). These approaches appear effective since it is how mathematics was taught and curriculum was used for many years; however, it does not develop mathematical understanding for learners for several reasons (Pesek & Kirshner, 2002; Schoenfeld, 2004; Van de Walle et al., 2018).

First, it requires students to understand the teacher's explanation instead of the concepts to be learned or understood. Second, it communicates there is only one way to solve or think about a problem, the teacher's way. Third, it places all of the thinking on the teacher and makes the student become a passive learner. Fourth, it decreases the chances that learners will attempt any other problem-solving strategy unless they are explicitly shown how (Van de Walle et al., 2018). Finally, it does not align with the effective instructional strategies identified by NCTM and other researchers (Ball et al., 2008; Daro et al, 2011; Schoenfeld, 2004). Knowing what high-quality mathematics instruction is and is not can provide valuable insight on how to modify unproductive beliefs and determine which beliefs are influencing mathematics instruction.

Assessing High-Quality Mathematics Instruction

Teachers need to engage in self-reflection and analysis in order to identify and modify, if necessary, any unproductive beliefs so they can provide high-quality instruction that has the potential to impact student learning (Blazar, 2015; Meschede et al., 2017; Staub & Stern, 2002; Voss et al., 2013). They can do this by using frameworks and instruments such as the (CGI) Teacher Levels and $MCOP^2$.

Cognitively Guided Instruction (CGI) Teacher Levels

In 1989, Carpenter et al. brought together research about how children learn mathematics and developed a conceptual framework that teachers can use to create an environment where learning math becomes a sense-making activity (Moscardini, 2014). This framework was named Cognitively Guided Instruction (CGI) and was based upon research about children's mathematical thinking which, according to their findings, is initially different from how adults think about mathematics (Carpenter et al., 2015). In 1996, Fennema et al. conducted a three-year longitudinal study of twenty-one teachers who participated in CGI professional development to examine their growth in mathematics instruction. Their findings became a framework for identifying the four levels of a CGI teacher.

Level one (or traditional) teachers follow the I-We-You instructional strategy. The students are taught how to use standard procedures and algorithms to solve problems and are given no or little opportunities to discuss tasks and solutions. These teachers align closely to Ernest's (1989) instrumentalists belief systems and Askew et al.'s (1997) transmission orientations towards teaching and learning mathematics. Level two (or problem poser) teachers provide opportunities for students to solve problems independently; however, these teachers still demonstrate specific methods after the students have had some independent exploration. They may engage students in mathematical discussion, but the discussions are usually centered around the teacher's explanations and/or mathematical procedures. These teachers align to Ernest's (1989) instrumentalist and Platonist belief systems and elements of Askew et al.'s (1997) transmission and discovery orientations towards teaching and learning mathematics. Level one and level two teachers tend to stay at or near the transmissive style of teaching and learning.

Level three (or listener) teachers provide opportunities for students to explore and discuss mathematical problems and tasks. They do not show students specific procedures for problem solving. Additionally, they facilitate mathematical discussions centered around the students' constructed solutions instead of monopolizing the instructional time with their own mathematical understandings. These teachers align to Ernest's (1989) Platonist and problem solving belief systems and elements of Askew et al.'s (1997) discovery and connectionist orientations towards teaching and learning mathematics. Level three teachers tend to stay at or near the constructivist style of teaching and learning.

Finally, level four (knowledge integrator) teachers, similar to level three teachers during mathematics instruction, use what they learned from listening to the students' discussions and mathematical explanations to plan instruction (Carpenter et al., 2015). These teachers have a better understanding of their students' thinking than any other level of teacher and use that understanding to strategically introduce problems or tasks that will provide opportunities for productive struggle and conceptual growth. They "regard their instruction as an opportunity for developing a deeper understanding of children's thinking in general" (Carpenter et al., 2015, p. 205). Level four teachers intently listen to and reflect upon their students' discussions, explanations, and thinking and then use that information to modify and adapt their instruction to meet the needs of their students (Franke & Kazemi, 2001). These teachers align to Ernest's (1989) problem solving belief systems, Askew et al.'s (1997) connectionist orientations towards teaching and learning mathematics, and tend to display the highest levels of the constructivist style of teaching and learning.

Level three and four CGI teachers epitomize high-quality mathematics instruction and meet NCTM (2014) principles and standards by implementing tasks that promote problem solving, facilitate meaningful mathematical discussions, pose purposeful questions, and elicit student thinking. Teachers in this level would tend to score well on Gleason et al.'s (2017) MCOP^2 classroom observation protocol because they know what it means to be a facilitator in the classroom and the importance of student engagement.

Mathematics Classroom Observation Protocol for Practices (MCOP^2)

From 2012 to 2015, the mathematics education faculty from the University of Alabama created a research-based observation protocol for K-16 mathematics teachers known as the Mathematics Classroom Observation Protocol for Practices (MCOP^2) (Zelkowski & Gleason, 2016). This instrument used a holistic approach to assessing the classroom environment and was based on the "Community of Learners" instructional framework by Rogoff et al. (2008) which posits that students and teachers share responsibility for mathematical learning; therefore, it measured two distinct and important factors of mathematical instruction: teacher facilitation and student engagement.

When teachers take on the role as facilitators in the classroom, they provide structure for the lesson and guidance (not solutions) for mathematical problems and tasks.

They create lessons, activities, problems, and discussions within their students' zone of proximal development and know how to scaffold, promote productive struggle, and engage students in high-level thinking problems and activities (Gleason et al., 2017). The student engagement sections of the MCOP^2 evaluated the levels in which students engage in and contribute to the mathematical lessons and instruction; it required students to participate in and persist through mathematical problems, tasks, and discussions that were given and guided by the teacher (Gleason et al., 2017). The two roles, teacher facilitation and student engagement, are dependent on each other (Gleason, Livers, & Zelkowski, 2017). The teacher cannot engage students in high-level thinking activities if students do not participate in the activity. The students cannot contribute to mathematical discourse if the teacher never provides opportunities for the students to discuss the posed problems or mathematical tasks (Zelkowski & Gleason, 2016).

Even though Gleason et al.'s (2017) validation study of their MCOP^2 was listed as one of the top cited articles in the journal *Investigations in Mathematics Learning*, I was not able to find research that supported the claims of the validation study beyond the ones listed in the description of the MCOP^2 (Polly et al., 2020). Despite that fact, a reading through and discussion of the descriptors and performance level listed in the MCOP^2 instrument in conjunction with an analysis of the teachers' self-identified CGI Teacher Level and discussions about productive and unproductive beliefs could prove valuable in future PD sessions as a way to engage the teachers in critical and productive reflection and self-appraisal. These measures could create the cognitive dissonance and awareness of how students learn mathematics that are needed catalysts for change (Ambrose, 2004;

Hughes et al., 2015; Lannin & Chval, 2013; Moscardini, 2015; Mewborn & Cross, 2007).

Alignments and Contradictions

Interestingly, some research has suggested that teachers' espoused and enacted beliefs do not always align with each other meaning a teacher may believe one thing about how a student learns mathematics but then not follow that belief when planning instruction or teaching that student (Cross-Francis, 2015; Kennedy, 2005; Speer, 2008); on the other hand, other research has concluded that teachers' enacted and espoused beliefs were closely aligned (Bray, 2011; Cross, 2009; Wilkins, 2008). As seen throughout this literature review, there are several reasons why either of these phenomena happen.

First, Green (1971) explained that teachers may appear to have a contradiction in their belief structures because certain elements of the structure are stronger than other elements. Second, the teachers' experiences as a learner tended to form primary and central belief structures that are more difficult to analyze and change (Barkatsas & Malone, 2005; Harbin & Newton, 2013; Maasepp & Bobis, 2015). Third, teachers may have constructivist beliefs about teaching but still believe their students are not capable of learning from this style of teaching (Beswick, 2005; Copur-Gencturk, 2015; NCTM, 2014). Next, teachers may not have really reflected upon or evaluated their belief structures and how they influence their instruction (Beswick, 2012). Finally, the pressures of teaching may cause teachers to rely upon unproductive belief structures instead of testing new beliefs they learned either through the curriculum or PD sessions (Harbin & Newton, 2013).

Conclusion

Regardless of the reasons for alignment or contradictions in the teachers' espoused and enacted beliefs, teachers need sustained and continual professional development of their mathematical knowledge for teaching and how students learn (NCTM, 2020). They need time to be involved in activities that disturb their thinking and challenge their beliefs about what it means to do mathematics and how it should be taught and learned (Brown, 2005; Cross, 2009) and to test what they are learning in the classroom to see how it impacts their learners (Mewborn & Cross, 2007). These activities can provide the opportunities for teachers to reflect upon what they believe, how it is believed, and the influence of their beliefs on their mathematics instruction.

In Chapter III, the Methodology utilized to explore these research questions will be discussed. The methodology section will provide a description of the setting and how the participants were chosen. It will also describe how surveys, observations, and interviews were conducted and analyzed. Finally, Chapter III will discuss the philosophical foundation and theoretical framework that guided this study.

CHAPTER III

METHODOLOGY

The overarching purpose of this study was three-fold. First, it sought to describe the mathematical beliefs of second grade elementary teachers who work in an elementary school in a Midwestern state. These beliefs included the nature of mathematics (what it means to do mathematics), model for teaching mathematics (how mathematics should be taught), and model for learning mathematics (how mathematics is learned) (Askew et al., 1997; Ernest, 1989). Second, it explored how these teachers' beliefs originated and evolved over time by developing an understanding of the teachers' quasi-logical belief structure, psychological belief structure, and belief clusters (Green, 1971). This involved discussions and analysis of several variables such as but not limited to curriculum, experiences as a student and with students, colleagues, and professional development sessions. Finally, it examined if and how these mathematical beliefs influenced the teachers' mathematical instruction by using observations and interviews to construct an instructional practice flowchart (Ernest, 1989).

The questions of the study were:

- What were the second-grade elementary teachers' beliefs about the nature of mathematics and their models of teaching mathematics and learning mathematics?

- How were the belief structures of the second-grade elementary teachers' view about the nature of mathematics and espoused models of teaching mathematics and learning mathematics formed?
- How did the beliefs about the nature of mathematics and models of teaching and learning mathematics influence the second-grade elementary teachers' mathematical instruction?

Research Design

This study used a case study design which has unique characteristics. First, the unit of analysis rather than the focus of the study itself defines case study design. This unit of analysis is bounded to a finite scope of persons, groups, events, or time boundaries (Yin, 2018) and is one of the most significant and distinguishable characteristics of the case study design (Merriam & Tisdell, 2016). Second, case study can include both qualitative and quantitative data, because its emphasis is on understanding the phenomenon rather than a particular process (Merriam & Tisdell, 2016; Yin, 2018). Finally, some case studies include and utilize propositions to guide data collection and analysis (Yin, 2018). These propositions are derived from empirical research such as literature, experiences, and/or theories (Baxter & Jack, 2008). The propositions for this study are in Table 3.1.

More specifically, this research was an embedded single-case heuristic case study. A case study "is an in-depth description and analysis of a bounded system" (Merriam & Tisdell, 2016, p. 37) that seeks to answer why and how phenomena occur in an uncontrolled environment (Yin, 2018). Case studies can be further defined by special features and foci such as particularistic (focusing on a particular phenomenon), descriptive (describing a particular event or situation), or heuristic (deepening the understanding of a phenomenon) (Merriam, 1998). This study utilized the heuristic characteristics of the case study design, because it sought to describe, explore, and examine teachers' view about the nature of mathematics, their espoused and enacted models of teaching and learning mathematics, and the formation and influence of those belief structures (Merriam, 1998). In addition, this study used a single-case design (two second grade elementary teachers) with two embedded units of analysis (Sally and Grace, pseudonyms).

Table 3.1

Propositions

Propositions	**Research supporting proposition**
In-service teachers' mathematical views about the nature of mathematics and orientations toward teaching and learning mathematics are independent beliefs and can, therefore, seemingly contradict each other.	(Ernest, 1989; Green, 1971; Voss et al., 2013)
In-service teachers' experiences as a student have a stronger impact on how their beliefs structures are formed than experiences in professional development sessions.	(Beswick, 2012; Cross, 2009; Harbin & Newton, 2013; Maasepp & Bobis, 2015)
In-service teachers' mathematical views about the nature of mathematics and orientations toward teaching and learning mathematics influence their enacted model of teaching and learning mathematics.	(Beswick, 2012; Cross-Francis, 2015; Shirrell et al., 2018)

Theoretical Framework

In order to answer the questions for this study, I needed to understand what teachers believed, how they believed it, and the ways they believed it. Green's (1971) theory on beliefs provided this structure and was the theoretical framework used to guide this study.

There are three dimensions to a belief system: quasi-logical relation (primary or derivative), psychological strength (central or peripheral), and cluster location (Green, 1971). In the quasi-logical relation dimension, beliefs are either derivative (derived from another belief) or primary (not derived from other beliefs). This dimension explains how some beliefs were developed or derived because of their relation to other beliefs. For example, if I believed I learned how to solve mathematical problems because my former teachers modeled the procedures for me, I must believe two things: I can solve mathematical problems and the teacher modeling the procedure taught me how to solve them. Based on this belief, I might derive the best way to teach problem solving is by modeling the procedure and would, therefore, model problem solving procedures to my students. If research suggests modeling procedures do not increase student's conceptual understanding and ability to solve mathematical problems as effectively as providing opportunities for students to discuss invented and/or alternative problem-solving strategies, I may have to re-evaluate all the beliefs I held about problem solving, modeling, and alternative problem-solving strategies.

My ability and willingness to change my beliefs about how to teach problem solving would be determined by the second dimension: psychological strength (Green, 1971). The quasi-logical relation deals with what is believed; the psychological strength addresses how it is believed. In this dimension, beliefs are either central or peripheral. Green (1971) uses the imagery of concentric circles to illustrate this concept (see Figure 3.1). The core circle or central belief sits in the center; these are the beliefs individuals are prone to accept without question and are the most difficult to change. As the circles move beyond the center toward the perimeter, the beliefs are held with decreasing strength and become increasingly easier to discuss and change.

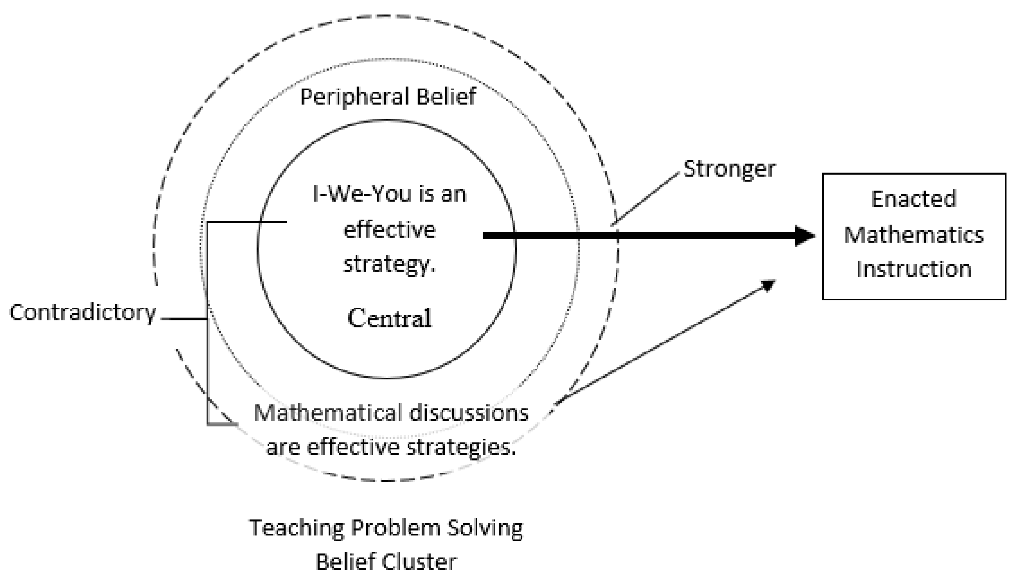
As with the previous example, the belief that I learned how to solve problems because my former teachers modeled the procedures for me could be a core belief I hold. Research on the effectiveness of mathematical discussion in promoting problem solving skills could be added as a peripheral circle to my belief on how problem solving should be taught. If someone questioned my use of modeling procedures, I might not evaluate or change this central belief; however, if someone questioned my use of mathematical discussion to increase problem solving skills, I might abandon this peripheral belief for an alternative strategy. Therefore, the ability to change a belief is not based on "*what* is believed but with *how* it is believed" (Green, 1971, p. 47).

Finally, beliefs are held in isolated, protected clusters away from other belief clusters (Green, 1971). This dimension explains how individuals can hold seemingly contradictory beliefs. Unless forced to do so, these belief clusters do not cross or confront each other. For example, I might have a belief cluster about the effectiveness of modeling mathematical procedures and show students how to solve a problem; however, I may have another belief cluster that students need to participate in inquiry learning and give them a question to explore. These two belief clusters seem to contradict each other: students must be shown how to solve a problem and must be given the opportunity to explore solutions. However, I may hold my beliefs about teaching mathematics through modeling higher than my belief about inquiry learning and would continue to model how to procedurally solve problems. If I do not juxtapose the two clusters, then I will never notice a contradiction between the two.

Green's (1971) theoretical framework (see Figure 3.1) provided the structure I needed to describe what the teachers believed, why they formed those beliefs, and how those beliefs impacted their mathematical instruction. This framework guided my propositions and was the lens I use to collect and analyze data.

Figure 3.1

Green's (1971) Second and Third Dimensions of Belief Systems



Note. Thicker arrow denotes stronger belief.

Setting

The setting for this study was a rural K-8 elementary school in a Midwestern state in the United States of America. Established and built in the 1930s, the small school sits on a county road among modest homes. The original part of the building houses the administration offices (originally the cafeteria and a classroom), a set of bathrooms, narrow hallways, and some classrooms; these classrooms were once eight rooms, but walls were later removed to create four learning spaces. In the 1980s, the school received funding to add more classrooms, a library, a band room, and a gymnasium to the original building. Oddly placed stairs, ramps, and hallway transitions mark the connections of the original building to the 1980 additions. Incidentally, the band room is now a multipurpose resource room and the gymnasium is the school's cafeteria.

The school was able to use other grants and funding to add other buildings to the property for a gymnasium, band room, several portable buildings (currently not being used) and a separate middle school facility. Just recently, the school was able to add an early childhood learning center which houses a preschool (which is not officially part of the school district because of how it is funded) and the kindergarten classes. This building has their own cafeteria and play area. This collection of the 1930s original building with 1980 updates, several portable buildings and separate facilities, and the new early childhood center is how the school stands today and shows the heritage and growth this school has made in the almost 100 years it has been in existence.

The teachers and staff at the school appear to have a very laid-back demeanor about them. During my many visits, I noticed that many of them wear blue jeans and t-shirts (usually with the school logo or something about teaching and learning). There are usually three teachers for each of the preschool to second grade self-contained classrooms (total of twelve during the time of this study). The third and fourth classes are departmentalized into language arts (two teachers for each grade), mathematics (two teachers for each grade), and specials which include science, PE, SIPPS (specialized reading program) and keyboarding. Each of these specials has one teacher for each distinct subject. The middle school is also departmentalized and a variety of teachers who either teach a designated grade and subject (such as 5th grade mathematics), a combination of a subject and/or grade level and an elective such as yearbook, newspaper, science, humanities, social studies, or a specific elective such as band or athletics.

Additionally, the school has a strong focus on reading skills and has several tutors, directors, and reading specialists devoted to enhancing their students' performance in that area. It appeared that little attention had been given to developing mathematical skills beyond the daily allotted class time and providing once a week computer time to practice math skills through online games and lessons such as IXL, Study Island, or SplashLearn. Pre-K to second grade teachers devoted three hours of uninterrupted time every morning to reading and writing lessons and activities; they allot the remaining four hours of the day to lunch, recess, mathematics, science, social studies, PE, library, computer days, and assemblies. However, if a student needs reading remediation, the specialists and directors pull the students in the afternoon outside of the required classroom reading instruction time. This often makes it difficult for me to observe mathematics teaching, because the teachers find it difficult to find a constant time to teach the subject.

The school has a strong Cherokee language focus as many of the teachers and students are members of this tribe. According to the Office of Educational Quality and Accountability (OEQA), of the 605 students enrolled in the site school for the 2018/19 academic school year, 58% of them identified as Native American, 19% as Caucasian, and 10% as Hispanic. For this reason, the teachers and staff decorate the halls and their classrooms with the Cherokee syllabary and posters in the Cherokee language. Additionally, all of the students enrolled at this school qualify for free or reduced lunch, more than one-third (39%) of the students were English Language Learners (ELL), and 18% of the students were in special education classes or pull-out sessions (Office of Educational Quality and Accountability, n.d.).

One of the requirements for teacher educators in my state is to spend ten hours in the field with teachers and students. I chose the school that was a few miles from my home and work to meet this requirement. I wanted to work with this school for a couple of reasons. First, I knew the principal, and we had previously discussed their need for mathematics PD at their school; they had a strong reading program but wanted to look at ways to boost their mathematics scores on the state test. Second, I grew up attending urban and rural schools and always found great joy in the rural setting. I taught at a small school and wanted to continue my educational journey at that type of school. Finally, I was intrigued by this school's Cherokee heritage and wanted to learn how that was incorporated through the building and in the classrooms. I felt this was a great way to add to my understanding about diversity and culture in the school setting. Since I had already built relationships and was the regular mathematics PD provider at this school, I felt this was a good fit to conduct this research; they claimed they were excited about the prospect as well.

Sampling

This study used purposive convenience sampling. The purpose of this study was to gain a deeper understanding of the teachers' beliefs in order to develop ways to help them reflect upon and juxtapose their beliefs and practices against transmissive and constructivist styles of teaching and learning (Voss et al., 2013). I had worked with these teachers in the past, so trust was already built between these teachers and me, especially the second-grade teachers. Since I had established relationships, I felt I could now observe these teachers and ask them questions and they would be less likely to put up too many safe-guards or precautions; hopefully I would get to observe authentic teaching and get true responses to the interview questions. This meant I could dig deeper into their belief systems and gain a richer understanding of their beliefs and how they influence their mathematics instruction compared to me just meeting and observing these teachers for the first time (Merriam & Tisdell, 2016). This understanding could then provide a stronger and more reliable framework upon which I could build effective professional development sessions and guidance for these teachers. The established criteria for the sample in this study included:

- teachers who taught mathematics in the same grade at a K-8 elementary school
- teachers who used the same curriculum and did not supplement or omit most of that curriculum
- teachers with whom I had established relationships and trust

The second-grade teachers at the selected K-8 elementary school site met all of these requirements. Therefore, these teachers became the case for this study, and the individual second-grade teachers were the embedded units of analysis. I will discuss how each teacher met the requirements for the case later in this chapter.

Participants

The participants for this case study were three second-grade teachers who met the criteria for the study, Sally, Grace, and Jenny (pseudonyms). These teachers divided up lesson planning and training by learning about and planning for a particular subject for their grade level; Sally was the mathematics representative and planner for the lessons for the second graders. One of her colleagues and the other participant, Grace, was the representative and planner for science and social studies. Jenny was the second-grade representative and planner for reading was the third participant at the start of the study; however, due to COVID and other complications, she was removed from the study. I have worked with these teachers in the past, all the teachers used the same curriculum and expressed that they trusted me and knew I was there to "help and not judge them"; therefore, they met the criteria for the embedded subunits for the study.

Sally

Sally is an eleven-year veteran teacher at the rural K-8 elementary school. During her teaching career, she has taught students in kindergarten for two years, first grade for one year, and second grade (her current teaching grade) for eight years all at the same school. Sally is very comfortable in a small rural school setting; as a student, she attended a rural elementary school just a few miles away from the school at which she currently teaches. Sally did not venture too far from the area for her upper education either. The high school and university she attended are in the same geographic area and, even though they are considered urban districts, are on the smaller side compared to other high schools and universities in the state.

As a student, Sally had a neutral feeling towards mathematics. She did not feel "really strong in math, necessarily" nor did she feel like she really struggled with math either. Growing up and while taking courses to prepare her to become a teacher, Sally never thought "Yes, I can be a math teacher," but she felt like she was good at second grade math. As a teacher, she did not feel like she had a lot of anxiety dealing with the math she taught. She confessed that when she was exposed to higher levels of math in professional development sessions, however, her math anxiety did increase; she had "no idea where to even start with that type of math." She was comfortable with her math ability to teach students in kindergarten through second grade mathematics but not beyond those grades.

Experience with Sally. I have had the opportunity to work with Sally three years prior to conducting this research. I was asked by the school district to provide some professional development and guidance in the area of mathematics. At the time, the teachers at the school were using a variety of curriculums and/or lessons they found on the internet within and across different grade levels. The school wanted a more unified approach to ensure students were getting a consistent mathematics education from kindergarten through eighth grade. To help with this effort, the school assigned a teacher from each grade to be the mathematics representative. Even though the PD sessions were open to all, the representatives were required to attend the sessions with me and then take the information back to their colleagues in their respective grades. There were three second grade teachers; Sally was their chosen mathematics representative.

In the beginning, we were tasked with finding a curriculum that would meet the needs of each student and teacher in every grade. Sally explained at one of the meetings that during her then five years as a second-grade teacher at the school, they had used Saxon math for one year, GoMath for two years, and a compilation of mixed curriculum the last two years. We explored and evaluated many different curriculums and options. The faculty ultimately decided to adopt Eureka mathematics as their mathematics curriculum for two reasons: 1) it received high rankings, and 2) since everything was available online, the district did not have to buy new textbooks or teacher manuals.

Once the curriculum was selected, I spent the next two years and PD sessions showing teachers how to use research-based strategies to implement the curriculum effectively and use best practices to teach their students mathematics. Our sessions included topics such as: developing a pacing calendar with Eureka, how to use Sprints (Eureka speed drills) successfully, how to convert Application Problems (which second-grade turned into Application Journals) into mathematical discussions, and how to integrate children's literature into the mathematics curriculum. I also attempted to challenge the teachers' beliefs about mathematics by providing activities that would cause cognitive dissonance and require them to reflect upon their teaching practices and mathematical understanding. Beyond my PD sessions, Sally attended other mathematics related professional development workshops conducted by Eureka representatives, Kim Sutton, and other various sessions during her time as a second-grade teacher.

Sally's Classroom. I have had the opportunity to observe Sally many times before and during this research. During each of these observations, she has always appeared to have a calm and confident approach to teaching. She knew her objectives and how she wanted to approach the lesson; her students knew what she expected of them and followed her established classroom norms and protocols. She talked with a moderate tone and reassured her students when they seemed confused or anxious. I have never heard her raise her voice, even when she was upset with their behavior. She listened to her students when they had questions or concerns, and her students, for the most part, listened to her when she gave directions and instructions. It appeared that her students respected her, and she respected them.

Additionally, Sally's classroom was set up for maximum interaction between the students and her and the students with their peers. She put the students' desks in a U-shape around the room which gave her the ability to monitor her students easily and walk around the classroom freely. She had and used a document camera on her desk connected to a SmartBoard at the front of the room to model how to use math manipulatives and solve problems. During most of her lessons, Sally provided opportunities for her students to use the document camera to show or explain their thinking to the rest of the class. She also used her dry erase board to model mathematical procedures and point out common student errors. While most of her classroom walls were covered with reading resources like a word wall, punctuation poster, and reading quotes, she did have one wall devoted to mathematics. It contained a number line, calendar, math vocabulary, 100s chart, and place value information. During the observations, however, I never witnessed her using items from the mathematics wall.

Grace

Grace was a seven-year veteran teacher. She graduated from college with an elementary education degree and quickly added the early childhood certification after her first year of teaching. This was her second year as a second-grade teacher at the rural school in which she currently taught. Prior to that, Grace taught three years at a suburban preschool and two years in a suburban kindergarten classroom. Grace indicated her preference for rural schools over the suburban schools because of her experiences in small schools as a child. She also enjoyed the collaboration she has had with the other teachers at the rural school; she felt more connected to the students and her colleagues at this school than she did her first five years of teaching.

The Eureka mathematics curriculum was introduced to the research site at the same time in which Grace began her teaching career there. At first, Grace struggled with the new teaching position and mathematics curriculum. She admitted that mathematics was never her strong point, and she often struggled with the subject as a student in both P-12 and college classes. Additionally, Grace identified as a good reader who was very artistic and visual. She further added that her reading ability helped her understand and solve mathematical word problems, but her artistic and visual needs were often ignored in her elementary and-high school mathematics classes.

For these reasons, Grace felt she could really relate to her students that were struggling; she knew how they felt and wanted to find ways to help. So, she would often seek the advice of other teachers who used Eureka mathematics and watched several videos on how to teach this new curriculum. She had found one particular YouTuber who showed how to solve all the Eureka homework problems; he was able to give her several different inputs and strategies for teaching and learning mathematics. These resources gave her more confidence to teach students in second grade or below mathematics in ways that they would be able to "understand and master."

Experience with Grace. Professional development sessions with me was another resource Grace used to develop some of her mathematics teaching and understanding.

Grace did not attend the school's mathematical professional development sessions for most of the first year the school started using Eureka mathematics. There were three second grade teachers and each of them took a different subject area on which to focus. Sally was the mathematics representative; the other teacher was the reading representative, and Grace was the social studies and science representative for the second grade. During Grace's first year of teaching at this school, the respective teacher only attended PD sessions that related to either general teaching or their specific subject areas. Therefore, at first Grace only attended PD sessions with me if we discussed general teaching strategies such as using children's literature to teach subject content. Any information I provided was passed on from Sally to the other two teachers. However, toward the end of her first year of teaching and after a few observations and discussions with me, Grace felt more comfortable with the curriculum and me and attended about four mathematics PD sessions beyond the two school required ones.

Grace's Classroom. The desks in Grace's classroom were arranged in a U-shape with a floor mat placed in the gap the desks made. Grace had the students sit on the "carpet" area when she taught her lesson and at their desk when they did their independent work. She felt that the students could see the whiteboard and listen to the lesson better when they were on the floor. When they did the lesson from their desks, the students would complain they could not see or hear, especially since she had a quiet voice, and Grace believed they got too distracted and were too far away. According to Grace, the students had to sit on the floor when she taught so she could maintain control and keep them engaged.

Grace also felt her students' maturity level this year was another reason why her students had to receive their lesson from the mat. Grace believed her students were capable of doing the work required of them, but they lacked drive or motivation. She stated that many had never been pushed at home, came from homes where the grandparents or aunts/uncles were raising them, and/or had a lot of other emotional problems because of their home life. According to Grace, this group of students was more concerned about surviving and what they would go home to than they were about reading or mathematics; they were "in constant fight or flight mode."

For these reasons, she felt they were very needy, did not know how to try, and really needed her to grab their attention and show them what to do. "They needed a lot more attention and reminders" than her group last year; however, when Grace compared last year's and this year's students' scores on the reading and math assessments, she felt they were performing about the same or not very far apart from each other which was not something she expected. Regardless of that finding, Grace still felt her students needed to work on the mat in order to pay attention and understand what to do and how to do it no matter what subject or lesson she was teaching them.

The Curriculum: Eureka Math

According to the publishers of Eureka, Great Minds Inc., Eureka mathematics is a Common Core State Standard (CCSS) aligned curriculum that was written by more than 200 PK-12 teachers and experts and was designed to develop conceptual understanding by using stories and visuals to connect to mathematical concepts. The authors of the curriculum were intentional about having students revisit and apply previously learned concepts as they practiced and focused on the process of problem solving rather than just finding answers (Williams, 2015). They accomplish this task by dividing the lessons up into four different sections: application problems, problem sets, sprints and fluency, and exit tickets. Samples of each type of activity can be found on the Engage^{ny} website www.engageny.org/resource/grade-2-mathematics-module-4/file/116971 starting at page 16.

The application problems (which the second-grade teachers turned into application journals) are word problems given at the beginning or right after the fluency or sprint exercises. These problems require students to apply previously learned concepts or explore new ideas that will be introduced in the lesson. The intention of the application problem is to give students opportunities to practice or discover mathematical concepts or strategies. Additionally, the application problems are also a time for students to interact with their peers and discuss the strategies they used or how they approached the problem. To facilitate this and show teachers how to conduct these math talks, the writers of Eureka included scripted dialogue the teachers could follow to engage their students in the discussion. They also added possible illustrations the students might have used to explain their thinking. However, the publishers of Eureka math advise that their curriculum is not intended to be followed as a script, but rather as a guide to support teachers (Great Minds, Inc., n.d.).

Eureka mathematics also utilizes sprints and fluency, problem sets, and exit tickets to facilitate student learning. The sprints and fluency are designed to build students' fact mastery and number sense abilities. Sprints are timed drills that progress from simple to more complex problems such as addition and subtraction facts, skip counting, and ordering numbers. Fluency activities include such things as choral counting, using manipulatives, or finding patterns. The fluency or sprint activities may or may not connect to the day's problem set or application problem (Great Minds, Inc., n.d.).

According to the publishers, the problem sets provide independent practice for the students and are designed to have multiple entry points to provide opportunities for differentiation and modifications if needed. The curriculum provides vignettes and other resources such as pictures, reusable models, or data sets the teachers can use to conduct the mathematics lessons that connect to the problem sets. The teachers may choose to have students complete the problem sets independently and then share their results with a partner or in a group or complete the work and then go over the solutions as a class. Either way, Eureka also provides exit tickets that are given to students at the end of the lesson so teachers can have a quick and immediate way to check student understanding (Great Minds, Inc., n.d.).

It was very difficult to find research to support the use or the impact of Eureka mathematics on student learning that was independent of the company's findings. The research I found that included Eureka mathematics only looked at how teachers used or viewed the curriculum such as did they follow the curriculum with fidelity or supplement with other materials (Blazar et al., 2020), how it was implemented in the classroom (Walker, 2019), or what were the teachers' perceptions of the curriculum (Guyton, 2021). This is interesting because the curriculum was published in 2013, but there has been little research done on its effectiveness.

Data Collection

This study used both quantitative and qualitative data to describe the teachers' beliefs, examine their belief structures, and explore factors that impacted their enacted models of teaching and learning mathematics to answer the research questions. These data collection techniques aligned well with the theoretical orientation, problem and purpose, and selected sample (Merriam & Tisdell, 2016).

Quantitative Data Collection

The quantitative instruments included the Revised Mathematics Beliefs Scale (RMBS), Cognitively Guided Instruction Teacher levels, and Mathematics Classroom Observation Protocol for Practices ($MCOP^2$). I used these quantitative cross-sectional instruments so I could analyze the numeric description of the teachers' beliefs and opinions (Creswell, 2014). This data was used as a catalyst to start our interviews and later for evidence of the teachers' beliefs and orientations.

Revised Mathematical Beliefs Scale (RMBS.)

In 1989, Peterson et al. developed and used a 48-item questionnaire aimed at determining teachers' pedagogical content knowledge and beliefs about how children learn mathematics, the relationship between skills and problem solving, sequencing of mathematical instruction, and how addition and subtraction should be taught. Participants were asked to use a 5-point Likert-type scale (Strongly agree = 5 to Strongly disagree = 1) to indicate their level of agreement with statements about their mathematical beliefs. Later, Fennema et al. (1990) modified the assessment instrument by removing items specific to addition and subtraction in order to make it usable across varying mathematical topics. The new questionnaire was called the Mathematical Beliefs Scale (MBS) and measured the role of the learner, the relationship between skills and understanding, sequencing of topics, and the role of the teacher (Philipps, 2007).

Researchers began to question if the MBS really measured the four aspects the authors claimed it was measuring, because the test was long and many of the items were repetitive. With this in mind, Capraro (2005) set out to determine if the instrument was reliable and if any of the items could be removed while still obtaining the same information. Capraro administered the Fennema et al. (1990) version of the questionnaire to 123 in-service teachers and 54 preservice teachers. Her findings showed that the questionnaire had a reliability of .68 for the in-service teachers and .86 for the pre-service teachers and a combined reliability of .78 (Capraro, 2005). These reliability scores would be considered fair because they fall within the .67-.80 range (Fisher, 2007).

Next, Capraro utilized factor analysis to determine which items could be removed and which should remain. This process showed that 18 variables explained 46.23% of the variance and were found in only three of the factors: the role of the learner, the relationship between skills and understanding, and the role of the teacher. Capraro renamed these factors to Student Learning, Stages of Learning, and Teacher Practices, respectively. She then developed the Revised Mathematical Beliefs Scale (RMBS) that reduced the 48 items to the 18 variables that were previously identified as having the highest variance. These 18 items divided nicely into the three factors (six items in each factor). The RMBS was administered to a group of pre-service teachers and obtained a reliability of .86 – the same as the original version (Capraro, 2005). The RMBS appeared to be able to accurately and consistently measure a teacher's mathematical beliefs. This questionnaire provided an entry point for the teachers and me to start the process of discussing their mathematical beliefs.

The final version of the RMBS was used in this study. This instrument used the same 5-point Likert-type scale as the original survey and asked the teachers to rate their agreement to statements about their mathematical beliefs that relate to Student Learning, Stages of Learning, and Teacher Practices such as: *children learn math best by attending to the teacher's explanations, mathematics should be presented to children in such a way that they can discover relationships for themselves,* and *children should understand computational procedures before they master them*. Half of the questions are negatively or positively worded. This survey also included some demographic information and provided space for participants to clarify any of their responses if desired.

Cognitively Guided Instruction (CGI) Teaching Levels.

Another quantitative data collection I used was the CGI Teaching Levels. I used the findings and identified CGI Teaching Levels from Fennema et al.'s (1996) research to create a short survey the teachers could use to self-identify their beliefs about teaching and learning mathematics. I divided the summary of the CGI Teacher Levels into two sections (see Figure 3.2) to determine if the teachers strongly or somewhat identified with a particular level. The first statement in each section aligns to a level one CGI teacher; the second and third statements align to a level two and level three/four CGI teacher, respectively. The first section asks the teachers how they feel they teach children mathematics; the second section omits the "children" aspects and asks how the teachers teach mathematics. The teachers were asked to read the statements and circle the statement in each section that best described their belief. Their responses were then used to open the semi-structured interviews. These data were triangulated with other findings such as their responses to the RMBS and interview questions and used to start the process of understanding and analyzing their mathematical belief structures.

Figure 3.2

CGI Teacher Level Identification Chart

Instructions: Circle the statement in each section that best describes their belief about teaching and learning mathematics.

Section 1

a. I explicitly teach children how to do math.
b. I provide opportunities for children to solve problems using their own strategies as well as show the children specific methods to solve problems.
c. I do not show children how to solve problems; they can solve problems without me providing or teaching strategies to them.

Section 2

a. When I teach math, I clearly explain how to solve math problems and then provide opportunities for my students to practice the steps I taught.
b. When I teach math, I first provide opportunities for my students to solve problems using their own strategy. Then, after discussion, I clearly explain how to solve the problem using specific methods.
c. When I teach math, I ensure my students spend most of their time solving and reporting their solutions to a variety of problems. We then compare and contrast the different strategies in class. I rarely model how to solve problems.

Mathematics Classroom Observation Protocol for Practices (MCOP^2).

The final quantitative data collection instrument I used was The Mathematics Classroom Observation Protocol for Practices (MCOP^2). The MCOP^2 was developed by the University of Alabama's mathematics education faculty. This instrument was used in this study for several reasons. First, the developers used Rogoff et al.'s (2008) "Community of Learners" framework which claims that students and teachers share responsibility for mathematical learning to create an instrument that measures two distinct yet dependent factors of teacher facilitation and student engagement (Zelkowski & Gleason, 2016). This approach is what sets the MCOP^2 apart from other observational protocols such as the Reformed Teaching Observation Protocol (RTOP) developed by Sawada et al. (2010) or the Mathematical Quality of Instruction (MQI) developed by the Learning Mathematics for Teaching (LMT) institute and team (2010) (Gleason et al., 2017).

Second, the $MCOP^2$ does not require extensive training or professional development. Instead, the developers of $MCOP^2$ created a descriptors manual that defines operationalized terms and provides deeper descriptions of the 16 indicators. Observers and/or evaluators read the manual in its entirety before observations and refer to the manual when scoring. During the validation study, the $MCOP^2$ had a fair interrater reliability (IRR=0.669 for student engagement, IRR=0.616 for teacher facilitation) even though the users of the instrument had no training (just read the manual), practice, or discussion about the instrument or how they rated the observations (Gleason et al. 2017). Finally, the $MCOP^2$ had a good internal consistency for student engagement and teacher facilitation (α=0.897, α=0.850 respectively) (Zelkowski & Gleason, 2016).

The lack of need for training makes the $MCOP^2$ more manageable and easier to administer when compared to other observation instruments. It can be used for individual, peer, or program evaluations by preservice teachers, in-service teachers, administrators, university faculty, and/or researchers (Gleason et al., 2017) and as a formative assessment for a single observation or a summative assessment for three to six observations (Gleason et al., 2015). The authors of the $MCOP^2$ grant permission for anyone to use the instrument for research purposes, with proper citations, but ask evaluators to request consent from the authors if being used for evaluative purposes (Gleason et al., 2015).

Qualitative Data Collection

Qualitative data were also collected to answer the research questions in this study. According to Patton (2015), qualitative data is an effort to understand the opinions and experiences of the participants. For this study, the effort was focused on gaining a deeper understanding of the teachers' beliefs, how those beliefs were formed, and, if applicable, how and what has caused those beliefs to change. These data collection methods included observations and semi-structured interviews.

The observations in this research were used to determine how the teachers' espoused beliefs impacted their enacted mathematics instruction. In this study, three observations were conducted for each of the teachers. The observations were scheduled two weeks apart from each other so I could have time to transcribe and analyze the observations and interviews between each session. This ensured I was able to focus on factors that appeared to support or contradict the teachers' espoused beliefs. The observations were recorded and transcribed. Later the recording and transcriptions were used to find evidence to support claims I made about the teachers' espoused and enacted belief. Additionally, the $MCOP^2$ recommends using three to six observations in order to provide a summative evaluation of the classroom instruction; by recording the observations, I was able to view all three teaching sessions together and give a more accurate assessment of the teachers' mathematics instruction (Gleason et al., 2015). I did use field notes during the observations so I could ask clarifying questions during the semi-structured interviews that followed that day's lesson (see Table 3.2)

The four semi-structured interviews consisted of questions that were derived from the propositions (Appendix D); however, additional questions were added to each session based on the observations or responses given by the teacher such as why the teacher used a particular phrase or followed a certain procedure for conducting the lesson. The first, second, and third interviews took place two hours after the observations; the fourth interview was a follow-up session given two months after the last observation; it was used to clarify statements or add to my understanding of the teachers' beliefs. For example, after I had completed coding all three interviews and observations, I realized I did not have enough information to ascertain what Sally's and Grace's nature of mathematics beliefs were. So, I utilized the follow-up interview to gain a better understanding of this belief system for both teachers.

Procedures

The data were collected in two phases. In the first phase, before conducting any observations or interviews, participants were asked to complete the RMBS. Once the teachers completed and submitted the RMBS, they were asked to select time slots for each observation and interview. These data were analyzed and calculated so I could gain a preliminary understanding of what the teachers believed. I used that understanding and their responses to develop questions to determine why the teachers made the selections they did and how that might influence their beliefs or the formations of their beliefs.

Phase two of the study included observations and semi-structured interviews with teachers. During the observations, I used field notes to track what was happening and to note potential questions I might ask during the interviews. This process ensured I had an opportunity to clarify what the teachers were thinking and another avenue for us to explore their espoused and enacted beliefs. These field notes consisted of three columns (see Table 3.2).

Table 3.2

Sample Field Notes

Observation	Questions	Notes
Drew out the number of crackers in each of the four boxes.	Why did you model first? Is the normal procedure for problem solving activities? Why did you use that illustration?	Suggested example given by the curriculum to show solutions students might give. The textbook suggested students give their own examples first (did not completely follow curriculum in this example)

The first column was used to record observations such as the activities, student/teacher interactions, and conversations of the participants; the second was used to note potential questions to ask during the interview in order to clarify actions and potentially gather insight as to the purpose or meaning behind the teachers' enacted models of teaching and learning. In the third column, I jotted down notes such as subtle factors (interruptions, nonverbal communications, symbolic meanings of words), my own behaviors, keywords, connections to the propositions, or possible explanations of the observed phenomenon (Merriam & Tisdell, 2016). Additionally, I assumed an "observer as participant" role during the observations, meaning the participants were aware of my presence and purpose in the classroom, but I did not participate in the mathematics lessons or activities (Merriam & Tisdell, 2016). I did occasionally walk about the classroom to observe student work during independent practice, but I did not interact with the students.

Two hours after each observation, I conducted an hour-long semi-structured interview. This interview included responses to the RMBS, questions noted during the observation, and tentative questions that were created based on propositions (Appendix D, E, and F). During the first interview, I asked the teachers to determine with which Cognitively Guided Instruction Teacher level they most identified (see Figure 3.2). After they picked their level, I asked the questions planned for the first semi-structured interview and why they responded the way they did to the questions on the RMBS. Next, I asked some of the questions I noted in the field notes such as why they used a particular word or phrase or why they conducted the lesson the way they did.

After each data collection session, I summarized the observation and transcribed the interview. The summary included adding timestamps in the field notes for important events, noting responses and possible reasonings behind observed behaviors, and adding information about the observation such as curriculum suggestions and clarifications. I used the summaries and transcriptions to create a memo for each teacher and data collection cycle. This memo included notable quotes, possible codes (first cycle only), codes used (second and third cycle, later added to first cycle), summary of their story, and other notes (Appendix G). I used this memo, the respective semi-structured interview questions, and observations to complete the second and third interviews. After I summarized all three observations, coded all three interviews, and outlined the findings, I conducted a follow up interview with each participant to clarify their statements, conduct member checks, and ask additional questions as needed such as a specific definition of what it means to do mathematics. I used member checks throughout the process to ensure I was truly capturing the teachers' words and intentions.

Additionally, after completing the third observation, I reviewed all three video recordings to score each teacher with the $MCOP^2$. The $MCOP^2$ required reviewers to use three or more observations to score participants properly and fairly (Appendix C). This instrument used two subscales of nine items each to measure two distinct factors: teacher facilitation and student engagement (Gleason et al., 2017). Teacher facilitation examined the amount of scaffolding, productive struggle, peer-to-peer discourse, and questioning that occurred during the mathematical instruction and included items such as *the lesson promoted modeling with mathematics* and *the lesson promoted precision of mathematical language*.

Student engagement evaluated whether or not students were given the opportunity to "engage in and contribute to the learning experience" (Gleason et al., 2017, p. 114) and included items such as s*tudents were engaged in mathematical activities* and *students use a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts*. I used all three observations to score each participant (from zero to three) based on the performance descriptors for each item and found the total score for teacher facilitation, student engagement, and the overall performance. The results from the MCOP^2 were used to identify and summarize the teachers' mathematical instruction and determine connections to their espoused and enacted models of teaching and learning mathematics.

Data Analysis

I followed Merriam and Tisdale's (2016) process to analyze interview and observation data. I began by reading the first interview transcript and field notes for each participant. I jotted down notes, comments, and observations that struck me as interesting, relevant, or important to the study, questions, and propositions (Saldaña, 2016). This method, known as open coding by Merriam and Tisdale (2016) and initial coding by Saldaña (2016), provided a starting point for me to start the process of analyzing the data and reflect upon the participants' utterances. Since I wanted to honor the teachers' voice, I also incorporated In Vivo coding into this process by noting words or phrases the participants used in both the interview and mathematical instruction (Saldaña, 2016). The In Vivo coding yielded valuable data such as how many times Sally used the phrase "I never thought about that" or Grace stated that she "followed the book."

Next, I transferred the notes, comments, quotes, and phrases into the memo template (Appendix G). As I organized the data in the memo, I looked for patterns and categories. I used the purpose of the study, research design, and theoretical framework as guides in the data grouping process and constantly checked my own biases by using peer debriefing and member checks to ensure I was not projecting my own beliefs or life experiences onto the data, especially since this was a limitation of the study (Merriam & Tisdale, 2016). I repeated this axial coding process several times with the initial interviews and observations of both participants. I also used this process to determine if I was missing any important information that needed to be asked during the follow-up interview.

Once I compiled a list of categories and themes, I created a tentative coding sheet. I then attempted to code the second and third interview and observation. I revised as needed until I came up with a coding system that could answer the research questions, encompassed all relative data, was sensitive to relative data, and could place relative data into mutually exclusive categories (Merriam & Tisdale, 2016). Next, I had a peer use the code sheet to code the first and third interviews of each participant as they were the ones that yielded the most or summarizing data points. We discussed and modified any codes that seemed ambiguous or confusing to him or that contradicted my coding. We also discussed if the categories made sense in connection to the data and if there were any potential biases on my part (Merriam & Tisdale, 2016). Finally, I gave the participants a copy of the coded interviews and asked them to comment on my interpretation of their responses. I used positive wording and descriptions to ensure the participants did not feel judged or attacked.

After I coded all of the interview data, I read through the participants' responses to the RMBS and interview questions and looked for patterns and connections between the two. I used that analysis and Green's theoretical framework to create a visual representation of the teachers' espoused belief structures. I used Ernest's (1989) framework to define the teachers' nature of mathematical beliefs and Askew et al.'s (1997) models of teaching and learning mathematics to define how the teachers believed mathematics should be taught and how it is learned by students. I used the collected data to support those definitions. I then used the field notes, curriculum, $MCOP^2$ results and Ernest's (1989) flowchart to create a visual representation of the influences and results of the teachers' enacted models of teaching and learning mathematics. I recorded these findings in Chapter IV for Sally and Chapter V for Grace.

I followed this same process for the case analysis; however, instead of looking at Sally's and Grace's responses and instructional practices separately, I unified the findings in order to provide a general explanation of the identified themes and beliefs for the case study itself (Merriam & Tisdale, 2016). This triangulation of the data and pattern matching to the philosophical foundation of constructivism and Green's (1971) theoretical framework provided a means for me to explore, examine, analyze, and discuss the second-grade teachers' beliefs and how they influence their instructional practices (Yin, 2018).

Trustworthiness and Credibility

In order to demonstrate the trustworthiness of the research (or why this study and its findings were worthy of attention) and its credibility (or how the teachers' views and my representation of those views connect), I used triangulation, peer debriefing, and member checking (Erlandson et al., 1993; Lincoln & Guba, 1985). For the data triangulation, I combined multiple data sources in order to enhance validity and develop a thorough understanding of the research questions; I also used this process to test for consistency and/or highlight any possible inconsistencies that needed further investigation (Patton, 2015). I used peer debriefing when I asked another peer who was not associated with the study but had a general understanding of the topic to examine the data and evaluate my conclusions (Onwuegbuzie & Johnson, 2006). I used member checking by asking the participants of the study to determine if the coding accurately depicted their beliefs, explanations, and definitions (Creswell, 2014). This process included providing opportunities for the participants to fact check or challenge the coding and conducting a follow-up interview to verify statements (Erlandson et al., 1993).

Ethics

I have an obligation to protect my research participants and their profession. This develops trust between the participants and me, safeguards against information that might be misleading, and ensures the integrity and rigor of the study (Ary, et al., 2010). I anticipated, addressed, and followed certain ethical standards in the proposal, before starting the research, throughout the data collection, and when reporting and storing the data.

I began by filing an application with the institutional review board (IRB) that outlined the procedures I used and described how I recruited and disclosed information to my participants. My informed consent form identified who I was, my sponsoring institution, the purpose of the study, the benefits for participating in the study, and the level and type of participation expected. I notified the participants of any known risks, guaranteed confidentiality, assured them that they could withdraw from the study at any time without consequences, and provided contact information for questions or concerns (Creswell, 2014).

Beyond the IRB, I also reviewed and adhered to the code of ethics established by professional associations such as the American Psychological Association (APA) or American Educational Research Association (AERA). Furthermore, I attempted to select a site that did not interfere with my objectivity or inhibit full expression of participants in the study. Once the site was selected, I obtained permission from the site superintendent by writing a letter that addressed the time it would take to finish the study and the potential impact and outcomes of the study (Creswell, 2014).

Once the proposal was approved and before data collection commenced, I informed the participants of their rights. They were given the informed consent form that was approved by the IRB and made aware of the purpose of the study and their right to refuse to participate. I did not pressure the participants and made every effort to give the impression that I was not pressuring the participants to sign the consent forms (Creswell, 2014).

During the data collection process, I was aware of the influence on the school I was using to conduct my study and attempted to minimize disruption to this site. For example, I conducted research at times that were convenient for the participants. Additionally, I attempted to avoid deceiving and exploiting my participants and was constantly aware of potential power imbalances and collecting of harmful information about the participants. These issues were addressed by avoiding leading questions, sharing personal opinions, or disclosing information about the participants or others (Creswell, 2014). Lastly, I attempted to respect and take measures to ensure I was protecting the privacy and identity of participants by de-identifying responses made by using pseudonyms and/or coding throughout the study.

Finally, ethical standards were considered when reporting and storing data. I avoided falsifying information by honestly reporting findings and avoided plagiarism by properly citing others' research or seeking permission to reprint or adapt other researchers' work, if applicable. I attempted to use appropriate language and abstained from using terms that were biased or insensitive such as "female mathematician" instead of "mathematician" or "subject" instead of "participant". Raw data were kept for a "reasonable period of time" and then destroyed (Creswell, 2014, p. 100). If requested, I will provide evidence of compliance with ethical standards and practices by filing statements that show no or any potential for conflict of interest (Creswell, 2014).

Summary

A summary of the research questions being studied and the related research instruments and data analysis is provided below:

1. What were the second-grade elementary teachers' beliefs about the nature of mathematics and their models of teaching mathematics and learning mathematics? This was measured with the Revised Mathematical Beliefs Survey and a semi-structured interview. It was analyzed using descriptive statistics and open, In Vivo, and axial coding.
2. How were the belief structures of the second-grade elementary teachers' view about the nature of mathematics and espoused models of teaching mathematics and learning mathematics formed? The data were collected through a semi-structured interview and analyzed using open, In Vivo, and axial coding.
3. How did the beliefs about the nature of mathematics and models of teaching and learning mathematics influence the second-grade elementary teachers' mathematical instruction? This was measured through the Mathematics Classroom Observation Protocol for Practices, field notes, and semi-structured interviews. The curriculum was used to verify lesson objectives and structure. The data was analyzed through descriptive statistics and open, In Vivo, and axial coding.

The embedded units of analysis sections of Sally and Grace are presented in Chapters IV and V, respectively. The findings of the case study are presented in Chapter VI.

CHAPTER IV

SALLY: “IT NEVER OCCURRED TO ME”

Since I had worked with Sally for two years prior to this study, I believed I had a good idea of what her mathematical beliefs were. I had previously observed lessons where she had students use their shoe to measure objects in the classroom and then graph the different answers they found; she taught students how to regroup when adding three digit numbers by modeling the procedure on the board; she called out single digit addition and subtraction facts and had her students write the answers on the dry erase board, show her what they had, then give them a new problem. I assumed I would be able to pinpoint her beliefs and identify how those beliefs were formed. However, as we progressed through the study and truly explored her beliefs, we both discovered some new revelations and insights about her mathematical beliefs and how they were truly impacting her instructional decisions.

Sally’s Mathematical Beliefs

Beliefs guide teachers’ behaviors and impact how and what they teach (Capraro, 2005; Haciomeroglu, 2013). If commonly grouped, unproductive and transmissive beliefs would be grouped with instrumentalist and some Platonist and transmission and some discovery orientations. Productive and constructivist beliefs would be grouped with problem solving and some Platonist views and connectionist and some discovery orientations (Askew et al., 1997; Ernest, 1989). These beliefs combine together to make distinct belief clusters that define and explain Sally's nature of mathematics beliefs and her orientations towards teaching and learning mathematics.

Sally's Nature of Mathematics Beliefs

Sally held an instrumentalist belief with occasional hints of a Platonist belief about the nature of mathematics (Ernest, 1989). Sally's Platonist beliefs could be seen in some of her responses to the Revised Mathematical Belief Scale (RMBS). She agreed that children should find their own solutions, discover their own relationships, and engage in productive struggle and that *time should be spent solving problems before children spend much time practicing computational procedures*; however, she also agreed that *time should be spent practicing computational procedures before children are expected to understand the procedures* (see Table 4.1).

In other words, problem solving was an important and necessary piece of doing math and was more important than practicing computational procedures (Platonist), but practicing procedures or math strategies and learning the mathematical rules needed to happen before students would be able to understand what they were doing or why (instrumentalist). Additionally, Sally believed doing mathematics meant "working problems using math strategies to come to the answer" instead of solving problems in order to uncover mathematical knowledge and develop understanding.

Sally's Model of Teaching Mathematics Orientation

Sally fluctuated between a transmission and discovery orientation of how mathematics should be taught (Askew et al., 1997). She felt she had to give her students verbal explanations of the strategies and procedures (transmission) and by providing students with practical activities designed to help students discover their own methods and procedures (discovery); she also believed these discoveries were best applied by using manipulatives to teach students discrete lessons (discovery) (Askew et al., 1997).

Sally was aware of her oscillation between these two beliefs about teaching mathematics when she self-identified as a CGI Level Two teacher (Appendix A). Level two CGI teachers, or problem posers, provide opportunities for their students to solve problems using their own strategies; however, afterwards they explain or show their students how to solve the problem using specific methods and procedures. She also scored a 4.2 on Factor 3 of the RMBS (see Table 4.1) which meant she believed teachers should facilitate student knowledge but might have to teach procedures and direct student learning along the way, respectively (Capraro, 2005).

Sally explained that when she taught her students mathematics, she often started by giving her students application problems to solve (transmission). They were to solve the word problems on their own at first, and she would not give any guidance or instruction even if she noticed they were making a mistake (discovery). This step was important to Sally, because she wanted to give her students the freedom to work it out on their own using whatever strategy worked best for them in order to see how they would solve the problem (discovery). This teaching practice also gave her students time to "explore their own thinking" (discovery). According to Sally, this exploration involved practicing with manipulatives or trying different strategies to solve the problems in order to see if the strategy worked or not and determine if they needed to try something else (discovery). These strategies were usually taught to the students in previous lessons (transmission).

Once students had the opportunity to explore their thinking, practice using different strategies, and share their method for solving the problem with their classmates (either by verbally explaining how they solved the problem or using the document camera to show their work), Sally would model for the students how to solve the problem using a strategy they had already learned or were going to learn in the day's lesson (transmission) even though she disagreed that *children learn math best by attending to the teacher's explanations* (see Table 4.1). Again, Sally acknowledged her fluctuation between transmission and discovery beliefs about how to teach students mathematics, claiming that she knew students need to explore mathematical concepts on their own, and she was "trying to do that more," but she still had an urge to show them what to do. Sally wanted to follow her discovery beliefs, but her transmission beliefs were very strong.

Sally's Model of Learning Mathematics Orientation

Even though Sally displayed more instrumentalist/transmission orientations in her beliefs about what it means to do mathematics and how mathematics should be taught, Sally did not hold these orientations in her belief about how mathematics is learned. Instead she wavered between discovery and connectionist beliefs. The data revealed that she felt students needed to learn mathematics by using manipulatives and working things out on their own when they were ready to learn the concept (instrumentalist); however, the data also supported the notion that Sally felt children learn mathematics when they interacted with their peers and are challenged to over difficulties (connectionist) (Askew et al., 1997).

Table 4.1

Sally's Response to the Revised Mathematical Belief Scale (RMBS)

Question	Response (1 – 4 pts.)
Factor 1: Student Learning	2.8
- 10. Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, division).	2
- 13. Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.	2
- 16. Time should be spent practicing computational procedures before children are expected to understand the procedures.	2
- 9. Children should not solve simple word problems until they have mastered some number facts.	3
+ 11. Time should be spent solving problems before children spend much time practicing computational procedures.	4
+ 3. Children should be expected to understand how computational procedures work before they master those computational procedures.	4
Factor 2: Stages of Learning	3.5
- 15. Most young children have to be shown how to solve simple word problems.	4
+ 5. Children should understand computational procedures before they master them.	4
- 1. Children learn math best by attending to the teacher's explanations.	4
+ 8. Most young children can figure out a way to solve many mathematical problems without any adult help.	4
- 18. To be successful in mathematics, a child must be a good listener.	2
- 7. Children need explicit instructions on how to solve word problems.	3
Factor 3: Teacher Practices	4.2
+ 6. Teachers should encourage children to find their own solutions to math problems even if they are inefficient.	4
- 14. Teachers should teach exact procedures for solving word problems.	4
+ 2. Mathematics should be presented to children in such a way that they can discover relationships for themselves.	5
+ 4. The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.	4
+ 12. Teachers should allow children who are having difficulty solving a word problem to continue to find a solution.	4
+ 17. Teachers should allow children to figure out their own ways to solve simple word problems.	4

Note. Statements preceded by a "-" were reverse coded.

Most of Sally's discovery beliefs were seen in her responses to the RMBS. On this instrument, Sally strongly agreed *mathematics should be presented to children in such a way that they can discover relationships for themselves* and agreed that *most young children can figure out a way to solve many mathematical problems without any adult help* (see Table 4.1). When her students learned this way, Sally felt they were more confident and demonstrated stronger number sense abilities. Her score of a 2.8 on Factor 1 (Student Learning) and 3.5 on Factor 2 (Stages of Learning) of the RMBS indicated she believed students learn best when they construct their own knowledge and solve real world problems on their own.

Sally's discovery orientations were also seen in many of her responses to the interviews. Beyond finding their own solutions, Sally felt students learn mathematics when they are able to see it through hands-on activities that included using manipulatives (Askew et al., 1997). She felt that when students used place value disks or base ten blocks, they were able to manipulate the objects and see how they break apart or bundle to make the ten or the one. Additionally, when students used ten-frames or illustrated their thinking using the ten-frame format, they were able to see how many more were needed to complete the ten with just a glance. According to Sally, this process is an important part of learning mathematics, because "for some reason it makes sense for them to be able to see and make that connection… it just helps them learn it."

Finally, Sally felt students best learn mathematics when they interact with others (connectionist). Sally always included time for her students to share their work and thinking with their classmates in her lessons, especially when doing application problems from the Application Journal. This sharing was either done by having students show their work on the document camera or work with a partner or share with a partner how they solved the problem. Sally felt that these opportunities were important to student learning because they required students to fully explain how or why they approached the problem the way they did which often seemed to make sense to the other students. The students were able to learn from each other instead of relying on her for all the answers.

Summary of Sally's Beliefs

Like many other teachers, Sally's mathematical beliefs alternated between two different orientations for each belief category (Askew et al., 1997). She demonstrated instrumentalists beliefs about the nature of mathematics and discovery orientations about how mathematics should be taught. She showed discovery and connectionist orientations about the nature of learning mathematics but did not have a strong orientation for either. The RMBS, CGI Teacher level questions, and interviews gave Sally an opportunity to think about her beliefs. These explorations about her beliefs led to questions about how those beliefs were formed, how or if they have changed, and what, if applicable, influenced those changes.

Table 4.2

Summary of Sally's Beliefs

Belief	Transmission / Instrumentalist	Discovery/ Platonist	Connectionist/ Problem Solving
Nature of Mathematics	X	x	
Teaching Mathematics Orientation	X	X	
Learning Mathematics Orientation		X	x

Note. Lower case = hint of the stated belief

Sally's Mathematical Belief Journey

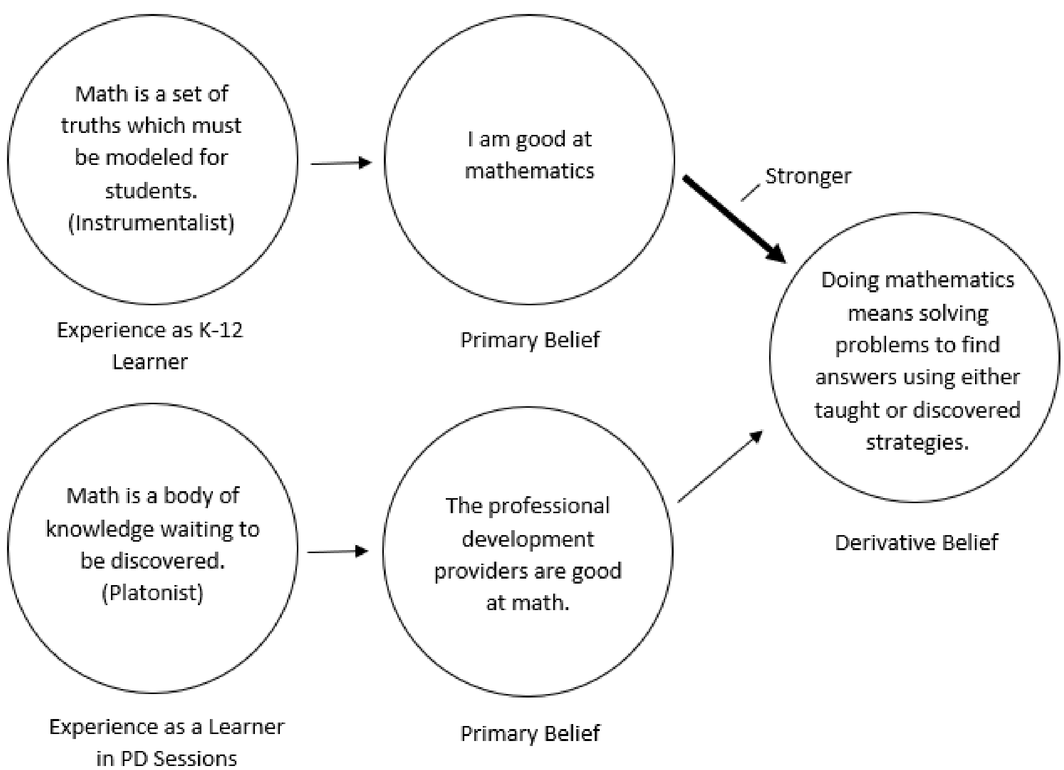
When asked, Sally explained that she felt mathematical beliefs could be changed, because hers had changed "especially within the last two years." She credited PD sessions that made her think about her beliefs and using the Eureka curriculum for those changes. However, were those the only things? Sally added that she often feared and even sometimes caught herself "reverting back" to her old way of teaching. If she felt her beliefs were changing and was excited about those changes, then, as Sally stated, "why am I not doing this as much as I know I should?" These questions were explored in Sally's mathematical belief journey.

Sally's Nature of Mathematics Journey

Interview data showed that Sally's nature of mathematics beliefs were mostly derived from her experiences as a learner in her K-12 and college classes and through various professional development sessions such as Eureka curriculum training, Mathematics with Kim Sutton sessions, and school specific mathematics instruction meetings with me. In her K-12 class experiences, Sally explained that the teacher taught and she learned it; there was not a lot of opportunity for student discovery in those classes. Sally identified this traditional method of teaching mathematics as "workbook, paper, pencil; get on it" (instrumentalist). Sally did not remember a lot of mathematics exploration in her teacher education preparation courses either; instead the instructor passed out the work or manipulatives and then modeled the skill in front of her (instrumentalist). Sally did not learn that students needed opportunities to discover math for themselves until after she became a teacher and attended professional development sessions (Platonist). Sally's Platonists beliefs began here. However, even though she had learned about the importance and effectiveness of providing opportunities for students to explore the math on their own, her experiences as a K-12 or college student were foundational in forming her beliefs about what it meant to do math and those instrumentalist beliefs were stronger and, as Sally explained, was "really where I feel like I was geared to go."

Figure 4.1

Sally's Quasi-logical Belief Structure: Nature of Mathematics



Note. The thickness of the arrow denotes the strength of the influence; thicker arrow means stronger influence.

One reason Sally's instrumentalist beliefs were strong was because of Sally's primary belief that she was "good" at mathematics and felt like she did not have a lot of anxiety dealing with mathematics. Since beliefs are held in true/false dichotomies

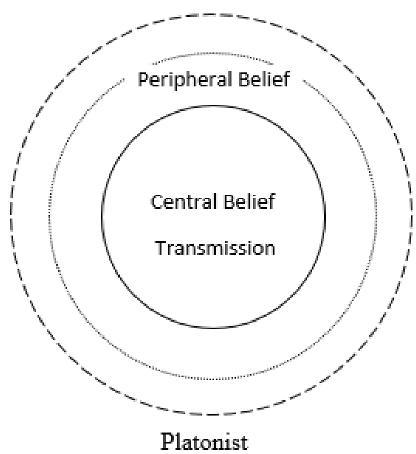
(Philipp, 2007), Sally's belief that she was good at math was a truth to her which, in turn, caused her instrumentalist beliefs to become true as well (Green, 1971). She was good at math because of the way her teachers or instructors taught her. On the other hand, she developed Platonist beliefs because she also believed that the professional development providers, such as myself, were also good at math, so what they were saying must be true as well - students need to discover mathematics for themselves. These two beliefs collided into a derived belief that doing mathematics meant "working problems using strategies to come to answer." However, the instrumentalist beliefs were rooted in her own personal experiences which, again, gave them more strength (see Figure 4.1).

Another reason Sally's instrumentalist beliefs were stronger was because her instrumentalist beliefs were central while her Platonist beliefs were peripheral (see Figure 4.2). Central beliefs are the beliefs individuals accept without question, are held most dearly, and are the most difficult to change; peripheral beliefs are held with decreasing strength and become increasingly easier to examine, discuss, and change (Green, 1971). Prior to attending professional development sessions or participating in this study, Sally had never thought about what it meant to do mathematics; no one had ever "just really made [her] think about that kind of stuff." She accepted her instrumentalist beliefs without question for eight years. These instrumentalist beliefs and her experiences as a learner told her she had to show students how to solve the math problems her "way or the way the book wanted [her] to show them." It had never "really occurred" to her before that there might be another way.

When she learned about this other way - children can discover mathematics for themselves - her thinking was perturbed. She reflected upon this new concept and questioned its validity. Sally felt the claim had some plausibility, especially because it came from an authority (professional development providers), so she placed this Platonist view as a peripheral belief. It was not fully accepted and could be easily abandoned if Sally could not reconcile how to implement the new idea or decided it was not really an effective practice. This explained why Sally would often revert back to her instrumentalists beliefs; those beliefs were foundational and central. They were built on experience and effectiveness; after all, she was good at second grade mathematics. It would take more evidence, a better understanding, and a deeper examination for Sally to be able to trust this new Platonist belief over her central instrumentalist beliefs (Green, 1971).

Figure 4.2

Sally's Psychological Belief Structure: Nature of Mathematics

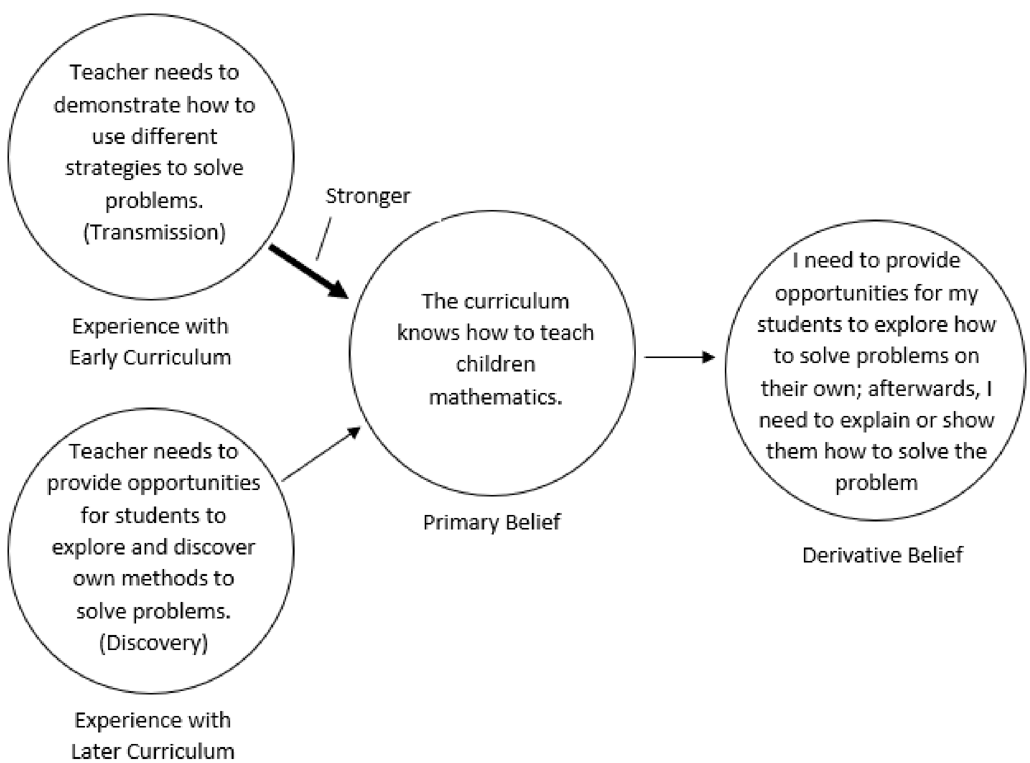


Sally's Model of Teaching Mathematics Journey

A majority of Sally's teaching mathematics orientation stemmed from her experience as an in-service teacher with curriculum. At the beginning of her teaching career, Sally "just followed the curriculum," because the textbook knew how to teach mathematics and had authority over her. The developers of this early curriculum conveyed and solidified some of Sally's transmissive beliefs that teaching children mathematics meant explaining the mathematical concepts to the students, practicing the skill, and then moving on to the next skill. Sally did not remember the textbook providing a lot of opportunities for exploration. As she progressed in her teaching career, Sally used other curriculums that showed some different ways to teach the same mathematical skills; however, the teacher was still expected to model how to solve the problems for the students.

Figure 4.3

Sally's Quasi-logical Belief Structure: Teaching Mathematics Orientation



Note. The thickness of the arrow denotes the strength of the influence; thicker arrow means stronger influence.

When she used these different curriculums, she would sometimes pull in outside resources that she believed might help her students understand the math concept better or build a connection, but she never let her students problem solve on their own without her guidance. Since Sally had a primary belief that curriculum had authority and an inherent knowledge of how mathematics should be taught, she never questioned whether or not the way the curriculum or, consequently, she was teaching children mathematics was effective or not.

Later, Sally's school adopted Eureka Math. While the developers of this curriculum still displayed some embedded transmissive beliefs about teaching mathematics, it reinforced what Sally had learned about from professional development sessions: children can discover how to solve problems without a lot of teacher guidance (Walker, 2019). This new curriculum provided Sally a platform on which to give students "the opportunity to explore their thinking" - the Application Journals. Instead of showing students how to solve the daily word problems, the curriculum suggested that Sally provide opportunities for students to first solve the problems independently and then discuss strategies the students either discovered or used (see Figure 4.3).

Sally believed this new curriculum changed her belief about what it meant to teach mathematics from providing "constant guidance" to "relinquishing control" and giving it to the students. She expressed during the interviews that she first believed she had to be the only one showing students how to solve problems; however, this new curriculum was showing her that math could be taught by giving students "examples of problems to try on their own just to see how they try to work it out." She felt this approach could give her a better “perspective of the students,” because it showed how the students approached problems and which strategy they preferred or knew how to use.

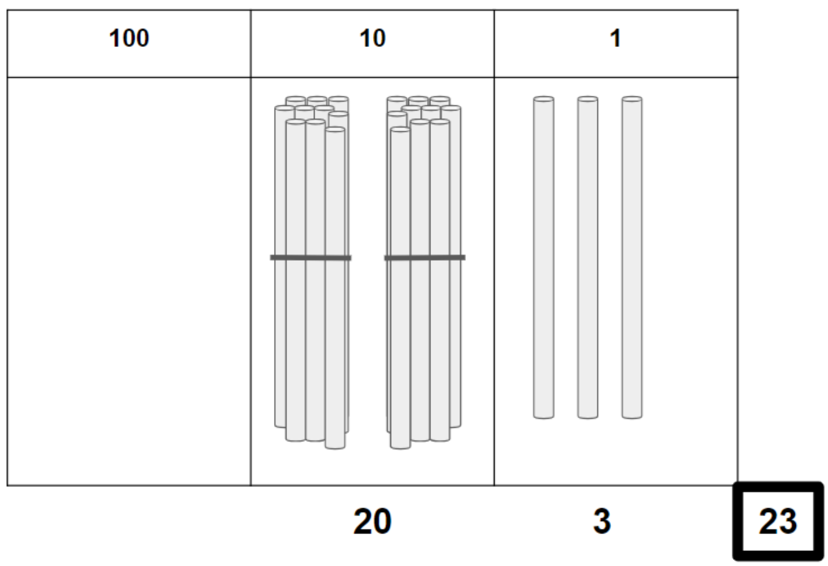
Even though Sally was trying to provide more opportunities for her students to explore mathematics, she still tended to “just fall back into old habits.” This was because her new discovery beliefs were not fully developed and were still peripheral in their psychological strength which meant they were easily challenged and somewhat weak (Green, 1971). The data revealed two reasons for this: 1) lack of confidence and 2) lack of understanding.

When Sally first started using Eureka Math, she did not feel confident with it. She felt more comfortable with the earlier math curriculums she used, because they recommended the teacher teach the way she was taught. She often felt she did not know why Eureka Math was teaching mathematical skills and concepts a certain way. For example, why did they show so many different ways to regroup (e.g. arrow way, place value disks, 100s chart, bundling sticks as seen in Figure 4.4) instead of just the traditional algorithm, and why were students supposed to explore these ideas instead of just learning it and doing it? It did not make sense to her. According to Sally, she agreed with the parents and “hated” this new curriculum as well. Therefore, she placed the discovery teaching style introduced to her in Eureka Math in the outermost peripheral ring of how to teach children mathematics.

After teaching the curriculum for one year and having a new set of students who were exposed to this discovery belief of teaching mathematics found in Eureka Math last year, Sally began to see how the concepts worked together and why giving students the opportunity to independently solve problems without her was important; it started to make “a lot more sense.” There were still certain things that were “cumbersome” or confusing to her, but for the most part she was gaining confidence in her ability to teach, as Sally explained, like the book showed or wanted her to teach. This pushed her discovery belief of how mathematics should be taught a bit closer but not into the central circle; it was still peripheral. Her central psychological strength was held tightly by her foundational and more confident transmission belief.

Figure 4.4

Example of Bundling Sticks

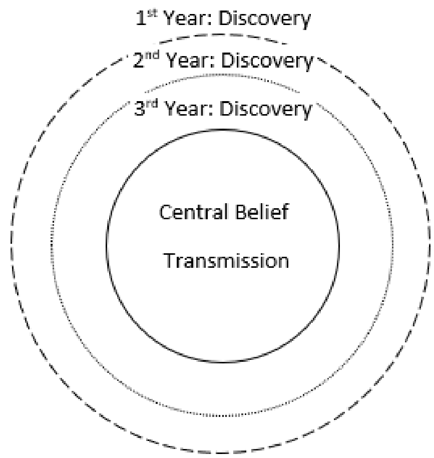


Another reason Sally was not able to change her teaching mathematics orientation from transmission to discovery was because of her lack of understanding of what it truly meant to explore. Sally defined exploration as giving students “the chance to *practice* with manipulatives or *try* different strategies to solve the problem” [emphasis added]. When pressed further, she requested that she not be asked “what else it could mean, because [she could not] think of anything else.” That is what it meant. In Sally’s definition and understanding of exploration, students were not given the opportunity to use the manipulatives to discover their own solutions but, rather, were shown how to use the manipulatives to solve given problems - the same way she learned to use the manipulatives in her teacher preparation courses. Since the curriculum never suggested just giving the student the manipulatives and seeing what they did with them, she never tried it; "it never even occurred to [her] to just give it to them" until she was questioned about it. According to Sally, she could not even imagine what her students would do with this type of instruction. Afterwards, she wanted to know how to give her students more freedom with manipulatives without her showing them what to do or how to do it. She wanted to have a discovery belief about teaching mathematics, but did not understand how to make that happen, especially with manipulatives.

Additionally, Sally used the terms "try," "use," or "teach" several times when referring to "different strategies." In her definition, exploration of strategies meant using different strategies they were shown, not necessarily discovering new strategies for themselves. Sally justified this stance by explaining that if she was not the one showing the students the different strategies, then they were not going to know what to do. "They would go to third grade and bomb that test or something." She wanted the students to discover their own strategies but doing so might mean ultimate failure. This fear kept her from fully letting go and letting her students truly explore and discover their own methods or procedures even though she agreed that *teachers should encourage children to find their own solutions to math problems even if they are inefficient* (see Table 4.3). Unless Sally gained more confidence and a better understanding of the discovery approach to teaching mathematics, this belief would remain centrally transmissive.

Figure 4.5

Sally's Psychological Belief Structure: Teaching Mathematics Orientation

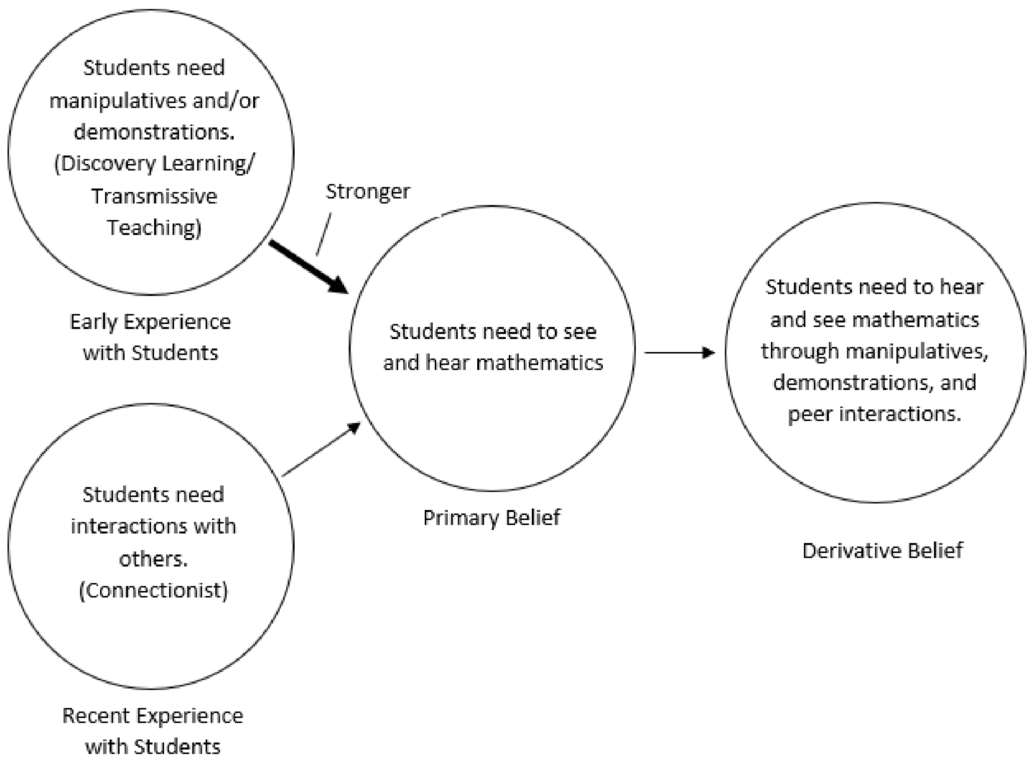


Sally's Model of Learning Mathematics Journey

Sally's final mathematical orientation, learning mathematics, was primarily formed and changed because of her experiences with her students. The data from the interviews supported the notion that Sally has always had a primary belief that students need to see and hear mathematics. She felt it was easier for some of her students to grasp a concept if they could "just see it." For example, when learning to regroup during addition, Sally and her students used straws to show why they had to regroup and move ten ones to the tens column when they added 37 + 48. The seven straws in the ones column combined with the eight straws in the ones column which left the students with fifteen straws in the ones column. Students needed to "bundle" ten of those straws and move them to the tens column. Sally explained that when they did this activity, her students were able to see why they had to regroup when they used the standard algorithm. Even if it felt like a "hassle to pull out all of the manipulatives," her students needed to "put their hands on it and see it." This belief that students needed to see the mathematics through the use of manipulatives provided evidence that she was developing a discovery belief; however, her transmissive teaching beliefs would often combine with this discovery belief about learning mathematics, and Sally would feel that, "by nature," she needed to just show them how use the manipulatives or learn a new strategy.

Figure 4.6

Sally's Quasi-logical Belief Structure: Learning Mathematics Orientation



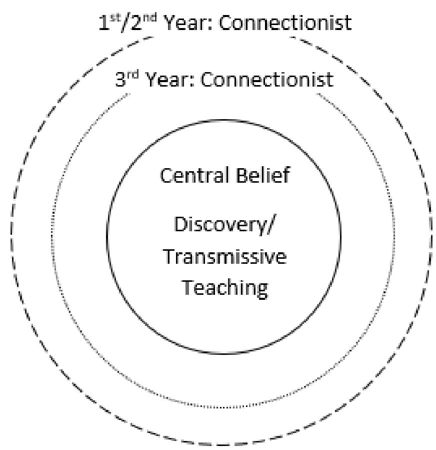
Note. The thickness of the arrow denotes the strength of the influence; thicker arrow means stronger influence.

Later, Sally's central belief that she needed to be the one who showed and explained how to use manipulatives and strategies was challenged by professional development and curriculum. These two "authorities" were now telling that her students could also be the ones showing and explaining how to use the manipulatives or strategies (see Figure 4.6). When Sally first learned about this connectionist idea and throughout the study, she was hesitant to provide these opportunities for "fear of it getting out of control." She reasoned that if she gave her students the chance to talk, they would "get off task or something." Eventually, she realized students should be given the chance to discuss with one another. This connectionist idea was introduced by the curriculum and our professional development sessions which caused it to sit as a peripheral belief on the very outermost circle; however, once she started trying it and observed how this practice increased her students' number sense, this concept began heading towards a more centralized belief and was something she now felt she needed to "let [her students] do more of."

Sally verified this change in her belief and explained that during one of their Application Journal discussions, a "couple of students here or there tried [a strategy] or showed [her] a way to solve a problem she had never thought of." When she gave those students the opportunity to show their classmates how they either created their own strategy or used a learned strategy in a different manner than she had taught them, she was "surprised" that they were "fully able to explain how or why they did what they did" and their explanation and strategy made sense to the other students. This caused her to take a step back. She reflected upon what was happening and thought, "Wow. This is probably what I should be doing." Sally began making a concerted effort to identify students who used a different strategy than the other students and have them show everyone what they did. Additionally, Sally believed that when she "remembered" to provide these opportunities for her students, she felt it "took a burden" off of her and, more importantly, gave her students more confidence. As their confidence increased and they were better able to come up with and explain their strategies, Sally began to have more confidence as well as felt "like maybe [she] did do alright."

Figure 4.7

Sally's Psychological Belief Structure: Learning Mathematics Orientation



Sally's Mathematical Beliefs Influence on Her Instruction

Now that I had a better understanding of Sally's beliefs and how those beliefs were formed, I needed to examine if Sally's espoused beliefs aligned to or contradicted her enacted beliefs. This is important because research has shown that teachers' beliefs influence their instruction, and Sally did show some transmissive views about what it means to do mathematics and how it was taught (Beswick, 2012; Cross-Francis, 2015; Lui & Bonner, 2016). These findings could show me ways to help Sally move across the continuum to a more constructivist style of teaching and learning.

Sally's Nature of Mathematics Influence

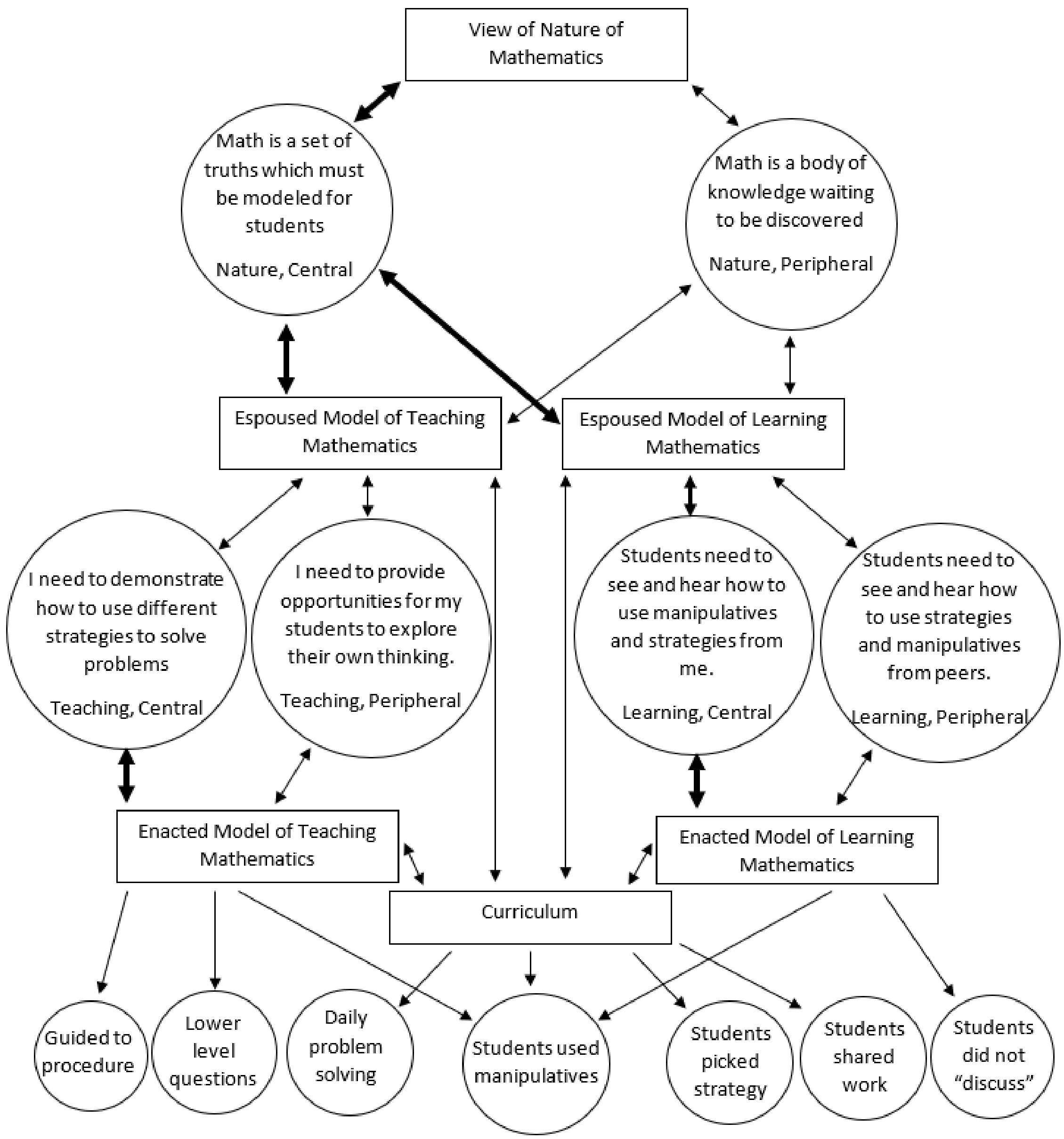
Sally had an espoused instrumentalist/Platonist belief that mathematics was a set of rules and truths that could be discovered by her students but also needed to be explained by her. During the interviews, Sally explained that doing math meant working problems to find answers. Sally often began her lessons by having students complete the daily Application Journal which was an embedded problem-solving exercise found in the Eureka curriculum (sample information is given in Chapter II of this document). However, on the second observation, Sally began by starting with the lesson instead of the Application Journal; she knew the lesson might be lengthy, and they were already running out of time as they still needed to complete a social studies test and the math lesson in less than two hours.

Sally began the math lesson by having her students open their workbooks while she displayed the first problem from the problem set on the document camera which was projected onto the interactive whiteboard. The objective for the day's lesson was to use place value disks to solve two-digit addition problems. Instead of giving students the opportunity to try to solve the problems on their own with the place value disks (the curriculum required manipulative for the current lesson), Sally demonstrated how to use the disks to solve the two-digit addition problems. The students then were asked to use the disks to solve the problems independently.

While the students were working independently, some students were not able to use the disks or solve the problem, so Sally stopped the class and began re-explaining the procedure for using the disks. During the explanation she stated, "Here, I want you to see it my way." Sally used this opportunity and the manipulatives as another procedure for her students to learn instead of as a tool for exploration or perseverance. Later, during the interview, Sally reflected upon the incident and her comment. She questioned her own motives and wondered why she would say something like that. She wanted to let them explore their own strategies and thinking and try to solve problems without her constant control or guidance, but her learned definition that doing math meant showing students how to solve problems had a strong influence on her instruction.

Figure 4.8

Sally's Instructional Practice Flowchart



Note. The thickness of the arrow denotes the strength of the influence; thicker arrow means stronger influence.

This example was one reason why Sally scored a one (on a three-point scale) for *students persevered in problem solving* on the Mathematics Classroom Observation Protocol for Practices (MCOP^2) instrument (see Table 4.3). When Sally's students were confronted with an obstacle in the lesson, less than half of her students persevered in trying to find an entry point, using a different strategy, or continuing to work on the problem (Gleason et al., 2015). This could be because, as her score of one on *the lesson provided opportunities to examine mathematical structures* suggested, Sally tended to give her students some time to examine notations, patterns, generalizations, or conjectures, but she provided too much scaffolding for them to grasp the full meaning of each of these concepts. Sally wanted her students to discover the mathematics for themselves and persevere in their attempts at problem solving, but she often guided them to the procedure or answer herself which mirrored how she learned to do mathematics. Sally's view of the nature of mathematics influenced every other belief and had a big impact on her mathematics instruction (see Figure 4.8).

Sally's Model of Teaching Mathematics Influence

The strong instrumentalist beliefs of Sally permeated much of her model of teaching mathematics orientation which, in turn, influenced how she taught mathematics. This was evidenced in Sally's first and third lesson observations. Sally began these lessons by having students complete the daily application journal problem. According to Eureka, the curriculum used at Sally's school and in Sally's classroom, these application journal problems provide opportunities for students to apply problem solving skills and understandings by either extending concepts learned in previous lessons or introducing new concepts which would be explored in that day's lesson. Sally opened the lesson by reading the problem to the students, discussing the procedure for solving the problem, and then telling the students to solve the problem however they wished. On the first observation, Sally read the problem and asked:

> How many fewer rocks did she collect on Monday than on Tuesday? Remember when we're reading a word problem, we look for those clues and that "how many fewer" is a clue to tell us what we need to do. What do you think we do if we see how many fewer? Are we going to add or subtract?

When the students replied, “subtract”, Sally confirmed their response and told them to solve the problem how they wished. She added they could use their place value disk or 100s chart - resources and strategies they had previously learned to use. One student asked if they could solve the problem vertically. Sally responded, “No, if you were to do it vertically, it would require us to regroup, and we have not learned that yet.”

The third observation had a similar conversation between Sally and her students. Sally read the word problem and then asked:

> Do you think we are going to add or subtract when they give us the total or the whole and then one part? Remember, we talked about this. When they give us the whole and one of the parts, are we adding or subtracting to find the other part?

When the students responded with add, she frowned until they exclaimed “subtract”. She confirmed subtraction was the correct operation and then told them to subtract however they wanted. As she walked around the room and monitored student progress, she would stop and make different announcements such as “the last couple of lessons we were adding. In this one we are subtracting. Make sure you are subtracting” and “if you are using the place value chips, remember when you subtract you make the first number out of the place value chips, and then the second number is what you are crossing out. Remember?" When she noticed several students were still struggling to solve the problem, Sally stated, "Basically everyone is stuck right now. Alright, let's do this together. I am going to show you both vertically and with the place value chips because the lesson is going to ask you to do both" - a teaching habit that often develops persistence issues in students.

Sally had an espoused belief that teaching mathematics meant that she should first provide opportunities for her students to explore how to solve problems by either discovering or using math strategies. According to Sally, this gave the students a chance to examine their thinking and gave her the ability to see what her students could come up with. Then, since some problems might be too difficult or her students might not know what to do without her guidance, Sally would need to show them how to solve the problem her way or the way the book wanted them to solve it.

However, as evidenced in the previous vignette, classroom observations, and her MCOP^2 score, Sally's enacted model of teaching mathematics was opposite; she began by guiding students to the entry point of the problem instead of encouraging them to find their own solution path. For example, at the start of the first and third lesson, Sally explicitly asked the students which operation they should use. To get them to that operation, she used lower order teacher talk meaning that her questions or statements were "knowledge based" and focused "on recall of facts" (Gleason et al., 2015, p. 14) such as asking her students to look at keywords to determine how to solve the problem - a strategy that is not highly effective according to Carpenter et al. (2015). Even though the curriculum provided rich tasks that could have promoted exploration and independent problem solving, Sally reduced her students' potential to discover their own pathways or examine their own thinking by asking questions that encouraged recall of facts, procedures, or tricks (score of one on question nine and eleven on the MCOP^2, respectively).

The observations and MCOP^2 also revealed that Sally often neglected to provide time for students to critically assess mathematical strategies (score of zero on question four) or use students' questions or comments to enhance conceptual mathematical understanding (score of one on question sixteen). On the first observation, Sally told the students they could solve the problem anyway they wanted; however, when a student asked to solve it vertically, she replied they would not know how to regroup. Instead of using that student's question to assess what that student already knew about regrouping, Sally missed this opportunity and shut down the exploration and guided the student back to learned strategies.

She could have encouraged the student to attempt the vertical way. If the student was successful, she could have used that opportunity to ask the student to show the vertical method to the class and then have the students critically assess how that strategy was similar or different to using the place value disks or 100s chart. If the student attempted the vertical method, did not know how to regroup, and got an incorrect answer, as Sally suspected, then Sally could have had the student verify the answer with the place value disks or 100s chart. This exploration, again, could have been shared with the rest of the class, and Sally could have engaged her students in an open discussion which could have led to a better conceptual understanding and connection of the manipulatives and the traditional algorithm. It would have become the students' discovery instead of a taught strategy in a later lesson.

These vignettes and other observations of Sally's mathematics instruction resulted in her receiving an overall score of 1.3 (on a zero to three-point scale) on the Teacher Facilitation subscale of the $MCOP^2$. This subscale score measured Sally's teacher role as "one who provides structure for the lesson and guides the problem-solving process and classroom discourse" (Gleason et al., 2015, p. 3). For the first part of the score, providing structure for the lesson, Sally scored a two. Her lessons did promote conceptual understanding, precision of mathematical language, modeling mathematics, and opportunities to examine mathematical structures (score of two on indicators six, ten, seven, and eight, respectively). However, with the exception of precision of mathematical language, each of those lesson planning components were built-in to the Application Journal section of the curriculum.

When it came to guiding the process and classroom discourse, Sally received a much lower score of 0.75 out of three points. She rarely or occasionally provided opportunities for students to critically assess mathematical strategies or find multiple paths or solutions; her teacher talk consisted of low-level questioning which did not encourage student thinking, and she rarely used students' questions or comments to enhance conceptual understanding (score of zero on indicator four, score of one on indicators nine, eleven, and sixteen, respectively). When she did have classroom discussions, only a few students (a mixture of raised hands on two observations and random calling of students on the other observation) shared their thinking or solution (score of one on question three).

Sally was able to adequately prepare effective mathematics lessons, with the influence of the curriculum, but struggled with facilitating the problem-solving process and student discourse. She believed she was providing opportunities for her students to explore the mathematical concepts and discover the mathematics being taught, but the observations and $MCOP^2$ score revealed that she tended to follow the curriculum even if students were ready to jump ahead, and she guided most of the thinking and exploration. Her experience as a traditional mathematics learner, her central beliefs that mathematics is a set of truths that must be modeled, and her lack of understanding of what it means to explore conflicted with her espoused transmission/discovery belief and revealed a more deeply embedded transmission view of teaching mathematics.

Sally's Model of Learning Mathematics Influence

Sally believed that students need to see and hear mathematics in order to learn it. This meant students needed to see her work problems on the board and show them how to use the manipulatives while she explained what to do. Professional development sessions from me and the curriculum showed her that these demonstrations, explanations, and discussions could also come from classmates. Because of this later change in Sally's belief on how students learn mathematics, Sally did occasionally facilitate student learning by asking students questions to guide them through the lesson and develop their understanding or provide opportunities for students to discuss and show how they solved a particular problem with the rest of the class. Sally's exploration into this type of teaching and learning could be seen in an example from the first observation.

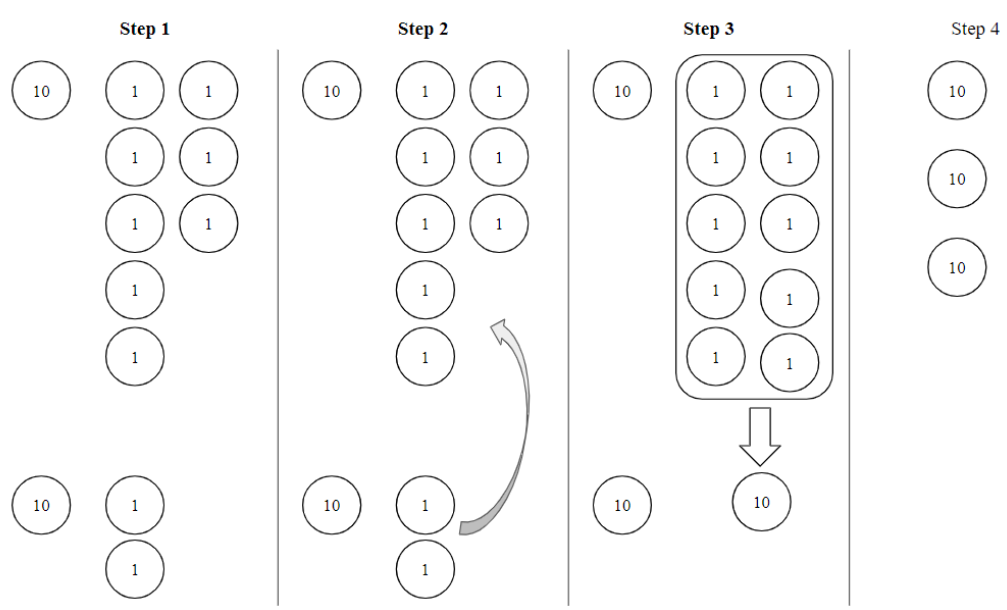
Sally asked the students to answer the following question: what is 13 tens and 2 ones? One student yelled out, "132." Sally asked her how she arrived at her answer, and the student responded she “looked at them and then figured out that it is the same thing.” Wanting more from her students, Sally asked if there was “any other way to figure that out?” Another student raised his hands and explained “there is a thirteen, and then there is a two in the ones.” Sally asked him what he meant, and he was unable to answer. So she asked him “how much is 13 tens?” The first student replied “130, because you count the tens thirteen times.” Sally asked the second student if he agreed; he did and then he stated, “so then just add the two for 132.” Sally did not just give the answer, she asked the students questions to guide them to the answer. When the student did not know, she posed questions that helped him see how to solve it. She knew he knew it; he just needed a little more time and help to get to the solution.

Sally also provided opportunities for her students to show their classmates how they solved a particular problem, especially if it was something that she did not see anyone else doing; she would ask one or two students to go to the board or document camera to show and explain what they did to solve the problem and explain their thinking. For example, during the second observation, many students used their 100s chart to solve the practice problems even though the students had just recently learned how to use the place value disks. Sally noticed one student, Keith (pseudonym), used place value disks. She explained to the class she was excited to see so many students using and understanding how to use the 100s chart, but she wanted to choose someone who did not use the 100s chart to come to the camera and explain how they used the place value chips, so she asked Keith to show and explain his strategy. Sally gave him a problem, “18 add 12,” asked Keith to set it up with the place value disk, and told the rest of the class to give a thumbs up if Keith did indeed model eighteen and twelve.

Keith placed a 10s disk at the top and eight 1s disk beside the 10s disk (see Figure 4.9). When he placed the eight 1s disks under the camera, he followed the 10-frame format by placing five disks in a column and three more disks beside the five (step 1). He then left a small space below the depiction of "18" and placed another 10s disk and two 1s disks to represent "12". After the class gave Keith a thumbs up, Sally instructed him to go ahead and solve the problem. Without speaking, Keith took the two 1s disks at the bottom and placed them in the empty spaces at the top to complete the 10-frame format (step 2). He then pushed all ten 1s disks to the side and replaced them with a 10s disk (step 3). Since Keith did not speak while solving the problem, Sally asked him what he did. He explained, "that made a ten so I replaced them; now I have three 10s. Three 10s is thirty." Sally asked the class if they agreed with what Keith did; they gave him another thumbs up. Sally gave Keith the opportunity to show his work but was unsure how to get him to fully explain what he did.

Figure 4.9

Keith's Demonstration of Adding 18+12



Even though the observations revealed that Sally did provide opportunities for students to share their solution to the application journal problems or practice problems with the entire class, she never had students work with or discuss their solutions with a partner or a small group. When asked why, Sally explained that she would have students work with a partner if the curriculum suggested it; however, if the curriculum did not specifically say to have students work with or discuss strategies used with a partner, she did not provide this opportunity because she did not even think about it. This explanation and the previous classroom observation resulted in Sally receiving a score of one on the MCOP^2 indicator fifteen: *students were involved in the communication of their ideas to others* (see Table 4.3). When students shared their ideas, the classroom discourse was primarily teacher directed and only lasted about five minutes.

The observations and MCOP^2 scores also revealed that Sally did provide times for students to discuss their mathematical thinking and solutions with their classmates, but only when the curriculum called for it and/or only during whole group discussion time. When students did share their solutions, they showed and explained what they did and did not critically assess the mathematical strategy used (score of zero on question four). Additionally, the observations revealed that only one or two students shared their strategies which meant less than half of the students were active in making, exploring, or responding to conjectures made; the rest of the class only engaged with thumbs up or down agreements instead of open discussion (score of one on questions twelve and thirteen). However, most of the *students were engaged in mathematical activities* and Sally's students did use a variety of means to represent concepts (score of two on questions three and two, respectively). These observations resulted in Sally scoring a 1.1

(on a three-point scale) for the Student Engagement subscale of the MCOP^2; this score measured the role of the students in the classroom and their engagement with the learning process.

Sally really believed she should provide opportunities for her students to discuss and explore mathematical concepts together. This connectionist belief was developing as a result of the curriculum and professional development sessions with me. She attempted to tap into this belief by creating a climate of respect for classmates' discoveries or explanations and using appropriate wait-time, but she still struggled with effectively providing opportunities for her students to communicate their ideas to each other. She did display her discovery beliefs by engaging students in problem solving activities, providing opportunities for students to talk in relation to mathematics, and using a variety of means to represent concepts, but these were dictated and expected by the curriculum. In the end, the observations and MCOP^2 revealed Sally believed students could learn from their classmates, but her nature of mathematics instrumentalist beliefs and how she learned mathematics was still very strong which resulted in her ineffectively or occasionally following her connectionist's view of how students learn mathematics.

Additional Findings

Throughout the research, I was always aware of the authority I had in Sally's classroom during her observations and throughout our interview sessions. Sally acknowledged that she taught differently when I was in the room. She was aware she would ask the student more questions such as "how did you get that answer" instead of "just accepting [their answer] and moving on" when I observed her. She also stated that she felt she resorted back to explaining the math to her students more when I was not there compared to when I was. She hoped she did not "do that all the time, like I [hope] I'm not reverting back to just 'Alright, this is how we do it.'" However, she was aware that was a tendency for her.

Table 4.3

Sally's Summative Score for MCOP^2 Based on Three Observations

Factor			Indicator	Score
S		1.	Students engaged in exploration/investigation/problem solving.	1
S		2.	Students used a variety of means (models, diagrams, graphs, concrete materials, manipulatives, etc.) to represent concepts.	2
S		3.	Students were engaged in mathematical activities.	2
S	T	4.	Students critically assessed mathematical strategies.	0
S		5.	Students persevered in problem solving.	1
	T	6.	The lesson involved fundamental concepts of the subject to promote relational/ conceptual understanding.	2
	T	7.	The lesson promoted modeling with mathematics.	2
	T	8.	The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	2
	T	9.	The lesson included tasks that have multiple paths to a solution or multiple solutions.	1
	T	10.	The lesson promoted precision of mathematical language.	2
	T	11.	The teacher's talk encouraged student thinking.	1
S		12.	There were a high proportion of students talking related to mathematics.	1
S	T	13.	There was a climate of respect for what others had to say.	1
S		14.	In general, the teacher provided wait-time.	1
S		15.	Students were involved in the communication of their ideas to others (peer-to-peer).	1
	T	16.	The teacher uses student questions/comments to enhance conceptual mathematical understanding.	1
S		S	Student Engagement Total	1.1
	T	T	Teacher Facilitator Total	1.3

According to Sally, changes in curriculum, PD sessions, and reflections had given her some insight into her belief about who holds the authority in her classroom. She "knew" the authority (the one who has the power to make decisions about what and how to teach) should be her, but she really felt "like we teach to whatever curriculum we are given." She justified this claim by stating:

> We have been flat out told, not in math, but other subjects, "Don't make it your own; follow the script. It's intended to be taught a certain way." So I guess because of that, I don't feel like necessarily I am the authority in the classroom. I am given a script and have to follow it… And, there is a lot of micro-managing us and making sure every second is spent, you know. They're always like it's instructional time, time on task; everything is boom-boom-boom.

Beyond feeling like administration expects her to use the curriculum as intended, Sally admitted that she had always just followed the curriculum throughout her many years of teaching. Through the PD sessions, however, she felt this had been changing for her. In her Eureka PD sessions, she discovered that even Eureka representatives felt it was not possible to "fit everything in." During our sessions, she learned ways to evaluate the curriculum and decide what to include, what to modify, and what to omit. She confessed that leaving out some of the problems was still difficult for her, but she was trying to overcome that thought process. She explained she used to really look at the problem set before the start of each lesson and teach the lesson and show the examples "exactly how they had it." But now, after two years of teaching Eureka, she does not follow the curriculum as much. She still reviews the teacher's guide "as a reference to make sure - 'Oh, this is how they're wanting' or 'What they're wanting them to understand by the end of it?'" but it does not have as much authority as it used to.

Reflection

During the last interview, I asked Sally if there was anything she would like to add to our conversation. She took a moment to think and then responded,

> I want to watch you teach a lesson just to see someone else. There are so many things I never thought about, or as you pointed out, occurred to me. I probably just need to watch you and them. Apparently, my kiddos are doing things I didn't even realize they are doing. And this is why I wanted to do this, so I could think about all this and see what I need to do or remember what to do. I think that will really help me.

Sally realized she was at the brink of moving down the continuum from a transmissive teacher to constructivist educator. The next step was to determine the best way to help her make that transition.

In the next chapter we will explore the embedded unit of Grace followed by the case study in Chapter VI.

CHAPTER V

GRACE: BY THE BOOK

Grace was excited for this opportunity to learn about her mathematical beliefs and how they impacted her teaching. Compared to her first year of teaching second grade, Grace felt she had learned more about her students and was getting more comfortable with the curriculum. She believed those two growth areas and this research could help her become a better teacher by motivating her to reflect upon her beliefs and teaching style and their impact on her students' learning. According to Grace, this process could transform her teaching and her understanding of mathematics, her students, and herself.

Grace's Mathematical Beliefs

Mathematical beliefs are held in three distinct belief clusters: the nature of mathematics, teaching mathematics orientations, and learning mathematics orientations (Askew et al., 1997; Ernest, 1989; Lui & Bonner, 2016; Cross, 2009; Philipp, 2007). In the nature of mathematics categories, teachers can view mathematics as a set of facts and rules that need to be conveyed to students (instrumentalist), as an interconnected, unified body of knowledge waiting to be discovered (Platonist), or human constructed endeavors that are open to revision and new explorations (problem solving) (Ernest, 1989). In the orientations of how mathematics should be taught and learned categories, teachers can believe that teaching are separate experiences and teaching takes precedence over learning (transmission) or learning takes precedence over teaching (discovery), or teachers can view teaching and learning as complementary endeavors (Askew et al., 1997).

Grace's Nature of Mathematics Beliefs

Grace's responses to the Revised Mathematical Belief Scale (RMBS) and interview questions revealed that she held an instrumentalist belief about the nature of mathematics, seeing it as an "accumulation of facts, rules, and skills...that needed to be conveyed to students" (Ernest, 1989, p. 7). On the RMBS, Grace strongly agreed that *children should not solve simple word problems until they have mastered some number facts, time should be spent practicing computational procedures before children are expected to understand the procedures, and most young children have to be shown how to solve simple word problems* (see Table 5.1). Additionally, Grace scored a 2.7 on a five-point scale on Factor 2 of the RMBS which suggested that Grace felt it was necessary for her students to know computational skills before they would be able "to solve even simple word problems" (Capraro, 2005, p. 86). When asked to explain her strong agreement with these statements, Grace explained that her students often struggled and became flustered when it came to solving word problems. If she knew they had mastered their math facts, then she could show them how to solve the problems without needing to wait for them to remember what 5+3 was.

Grace "defined" mathematics as "connecting numbers together to answer questions and solve problems, especially word problems." Again, Grace reiterated that her students struggled when it came to solving those word problems, so she had to give a lot of guidance. "I let them work on their own, but I have to read it to them and sort of give them the steps. They get frustrated easily with it if I don't do that." When asked why, she just reexplained that her students really needed her help to do the math and solve the problems - a task that was accomplished much easier when they knew their math facts.

Grace's Model of Teaching Mathematics Orientation

Grace held a transmission orientation towards teaching mathematics with an occasional hint of a discovery orientation. This meant she believed she should give verbal instruction so her students could understand her, the curriculum, and the mathematical concepts; she should engage her students in question and answer exchanges to check for student understanding; she should use word problems to help her students practice facts and skills and occasionally discover new strategies on their own (hint of discovery orientation, but only for high math achievers); and she should give students many different options or strategies to solve problems, so they could use the one that worked best for them even though it might not be the most appropriate or efficient method (Askew et al., 1997). Grace summarized this orientation when she explained how mathematics should be taught.

> Teach them. Let them problem solve, and then give them the explanation. Explain it your way, but let them let them learn to apply things on their own terms. But then guide them and give them the way the book shows. Give them many different options on how to solve a problem like Eureka Math does. They can show several different ways on how to solve one problem, but that's really good for some kids because they can pick which works.

Grace added that “teach them” meant she needed to activate their prior knowledge which included a discussion on what the problem was asking and the best way to solve it. She would let students suggest different strategies and then led them to or showed them the strategy that was given in the textbook. She did give students the opportunity to try a different strategy if they wanted or come up with their own, because, as seen in her disagreement with the RMBS statement that *children learn math best by attending to the teacher's explanations,* Grace also believed that students sometimes needed to figure out the problem on their own (discovery orientation), especially her “really, really high math achievers.”

Grace felt that there were times where she would explain the strategy to her students until she was “blue in the face,” but some of her students were not going to understand it; they needed to apply it themselves or take it apart and look at it from a different perspective. Sometimes these high achievers were then able to explain their strategy to the rest of the class and another student would have a “light bulb moment.” Grace added that “some kids for sure look to the teacher’s explanations, but other kids might look to another student’s explanation for answers or their own.” Grace felt that this freedom to follow the teacher’s, a classmate’s, or their own strategy gave her students independence and confidence which was something they were lacking - especially her students this school year.

Grace’s self-identification as a Level Two CGI teacher, problem poser, supported her interview statements that students could try to discover their own strategies when trying to solve the application journal word problems, but they still would often require specific instruction from the teacher in order to be successful. She expounded on this concept by explaining that she wanted to see what strategies her students used or came up with first before they tried to solve the word problem independently. This is why she began the lesson by reading the problem to them then asking specific questions on how to solve it. This question and answer session provided opportunities for her students to discover new strategies or activate their prior knowledge about previously learned strategies and then decide which strategies to use based on information they gleaned from either their classmates or her (the curriculum). Grace found it interesting that her students often used the strategy that was given by the curriculum through her or her students' explanations instead of using their own processes.

Her students' tendency to follow the curriculum's suggested strategy was one reason why Grace agreed with the RMBS statement that *teachers should teach exact procedures for solving word problem*, and scored a 3.3/5.0 on Factor 3: Teacher Practices (see Table 5.1). Grace justified her agreement with the statement claiming that her students often felt more confident in their ability to solve problems when they followed the curriculum's strategies which is why she taught exact procedures. Her score of 3.3 supported her belief that her students needed to take control of their learning and apply that learning (discussion of and choice in strategies to use), but she needed to use the curriculum to provide structure and organization in order to direct her students' learning and feelings of success (Capraro, 2005). Grace felt students could learn independent of her, but, as is the case with most teachers who hold a transmission orientation, she tended to place more emphasis on her need to teach instead of her students' ability to learn (Askew et al., 1997).

Table 5.1

Grace's Responses to the Revised Mathematical Belief Scale (RMBS)

Question	Response (1 – 4 pts.)
Factor 1: Student Learning	2.0
- 10. Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, division).	2
- 13. Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.	2
- 16. Time should be spent practicing computational procedures before children are expected to understand the procedures.	1
- 9. Children should not solve simple word problems until they have mastered some number facts.	1
+ 11. Time should be spent solving problems before children spend much time practicing computational procedures.	4
+ 3. Children should be expected to understand how computational procedures work before they master those computational procedures.	2
Factor 2: Stages of Learning	2.7
- 15. Most young children have to be shown how to solve simple word problems.	1
+ 5. Children should understand computational procedures before they master them.	4
- 1. Children learn math best by attending to the teacher's explanations.	4
+ 8. Most young children can figure out a way to solve many mathematical problems without any adult help.	2
- 18. To be successful in mathematics, a child must be a good listener.	4
- 7. Children need explicit instructions on how to solve word problems.	1
Factor 3: Teacher Practices	3.3
+ 6. Teachers should encourage children to find their own solutions to math problems even if they are inefficient.	3
- 14. Teachers should teach exact procedures for solving word problems.	2
+ 2. Mathematics should be presented to children in such a way that they can discover relationships for themselves.	4
+ 4. The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.	4
+ 12. Teachers should allow children who are having difficulty solving a word problem to continue to find a solution.	4
+ 17. Teachers should allow children to figure out their own ways to solve simple word problems.	3

Note. Statements preceded by a "-" were reverse coded.

Grace’s Model of Learning Mathematics Orientation

Grace’s responses to the RMBS statements and interview questions also supported a transmission orientation (students learn by following and remembering strategies, routines, procedures and processes; if students cannot do this, they need more practice) in her belief on how students learn mathematics. This finding was supported by her low score (2.0/5.0) on the RMBS Factor 1: Student Learning. This factor is used to measure how a teacher believes students learn. A high score would indicate that the teacher believes students can construct their own mathematical knowledge; a low score would indicate that the teacher believes students must receive much of their mathematical understanding from their teacher (Capraro, 2005). Grace’s low score would suggest that she believes students learn mathematics from her and the curriculum’s explanations. Additionally, Grace either agreed or strongly agreed to each of the statements that suggested students must either know or practice facts and procedures until they have mastered (or remembered) them. During the interviews, Grace reported that she often had to reiterate procedures and strategies for her students because “they forget things;” she expected them to remember, but they usually did not.

Grace also believed that students learn differently; some were visual and some were auditory. Grace felt that her visual students needed to see the things she wrote on the board as well as see the mathematics through the use of manipulatives. For example, when her students were learning about adding with regrouping, they were able to bundle tens and see the actual explanation of what it meant to regroup. Grace explained that if her students forgot how to regroup, she simply reminded them of the bundling activity, and they could then remember what to do. Their learning appeared to be “based on actions on objects" - a discovery orientation (Askew et al., 1997); however, Grace's clarification of this process showed a belief that this was not because of a discovery orientation, but, rather, because of her belief that her visual students needed to see the concept in order to follow and remember the instructions rather than discover for themselves how and why regrouping occurs.

Additionally, Grace disagreed with the RMBS statement that *to be successful in mathematics, a child must be a good listener* (see Table 5.1). She felt that many of her students were visual learners and, therefore, did not need to be a good listener; they could see the explanation and did not have to completely focus on what Grace was saying. Grace added that she had some students who seem like they never listened to the instruction she gave. They tended to stare at the wall and never seemed to be able to follow along or restate what she just taught. However, when she gave them the problems to work on, they were able to solve it without any help. When asked to explain this phenomenon, Grace was not quite sure. She reckoned they either were listening and just did not appear to be, listened to enough of the lesson to get an idea of what to do, were visual learners who did not need the verbal instruction, or were just good problem solvers. According to Grace, the last group of students, good problem solvers, tended to be her high achievers who did not require much guidance from her in most subjects.

Grace's belief that students learn differently was also seen in her appreciation that the Eureka curriculum taught and encouraged multiple strategies for solving problems and gave her students the ability to "pick which [strategy] works" best for them. She added that this approach was "a pain to teach, but it did make [the students] stronger in the long run." When asked to explain this statement, Grace stated that the curriculum spent a lot of time showing students different ways to solve two-digit addition and subtraction problems with regrouping. Even though this process felt redundant and time consuming, Grace knew that her students needed to be *shown* the different strategies just in case one of the methods they learned was confusing for some of her students (*emphasis added*). Sometimes she would ask one of her students to explain to their classmates how they solved a particular problem if she saw that they were using a previously learned strategy just in case one of their classmates might need to hear a different process in order to solve the problem. According to Grace, the ability to learn about and know how to use *multiple* strategies was essential for her students to learn mathematics.

Summary of Grace's Beliefs

Grace's mathematical beliefs and orientations tended to remain in the traditional or transmissive orientation of teaching. She held an instrumentalist view of mathematics believing that doing mathematics meant accumulating facts, strategies, and processes to solve problems. She also had a transmission-oriented approach to teaching and learning mathematics. She felt it was her responsibility to show students how to solve problems according "to the book" (a statement she used ten times throughout the interviews). This meant she had to teach them many different visual and verbal strategies so they could understand the processes she and the book needed them to learn. If she could not accomplish this task, her high achieving math students might be able to explain their strategy to the rest of the class and, hopefully, a few of them would learn it that way. When her students could remember and follow the taught processes, she knew they had learned the mathematical concepts she was trying to teach.

Table 5.2

Summary of Grace's Beliefs

Belief	Transmission / Instrumentalist	Discovery/ Platonist	Connectionist/ Problem Solving
Nature of Teaching Mathematics	X		
Teaching Mathematics Orientation	X	x	
Learning Mathematics Orientation	X		

Note. Lower case = hint of the stated belief

Grace's Mathematical Belief Journey

When asked, Grace explained that many of her beliefs have changed along her journey from student to teacher. She stated that her beliefs have "definitely evolved" and that her experience teaching her students this year and last has given her a new "outlook on how to solve problems." However, what were the root causes of those changes? What primary and derivative beliefs did she hold and how did she believe them? These questions guided Grace's mathematical belief journey.

Grace's Nature of Mathematics Journey

The interview data supported the notion that Grace's belief that math was a set of facts that needed to be practiced stemmed from her experiences with the Eureka math curriculum and as a student. Grace never felt she was strong in mathematics. As a result, she did not always feel confident that she would be able to teach some mathematical concepts. Grace explained that she needed "some sort of curriculum or something to follow" especially with concepts that were "more difficult to teach" such as regrouping and problem-solving techniques. This belief coupled with her primary belief that the curriculum is the authority in her classroom caused her to follow the curriculum over other potential authorities.

For example, the Eureka curriculum incorporated a Sprint (timed fact mastery drill or worksheet) into every lesson. Even though Grace had attended two professional development sessions that discussed the potential for these timed tests to cause math anxiety in her students and was shown ways to modify and effectively use the Sprints, she stated that she still followed the curriculum's guidance and timed her students on their completion of the set of math facts given to them. Grace justified this practice stating that the curriculum was just trying "to get them faster" so they could solve the problems quicker and easier. The result of these beliefs (Grace is not strong in math; the curriculum knows how to teach math; the curriculum is the authority) led to Grace's derivative instrumentalist belief that mathematics is the accumulation and mastery of facts and skills (see Figure 5.1). They were based on cognitive factors such as Grace's reasoning that she must follow the curriculum since it knew how to teach math and was the authority (Danili & Reid, 2006; Philipp, 2007).

This derivative belief also became or held a central psychological strength (see Figure 5.2); it was a strong belief that Grace really did not want to or know how to explore (Green, 1979). Any attempt to dig deeper into this belief resulted in the same responses and logical reasoning. For example, when Grace was asked if the book, the students, or she was the final authority about what it meant to do math, the following conversation occurred:

> Grace: As long as my students get the right answers, I'm fine with it. But I still want them to know how to solve the problem according to the book. Yeah, I let

> them solve but then go back and teach it by the book and give them the *proper* [emphasis added] processes.
>
> Interviewer: So would you say the book decides what it means to do math in your classroom?
>
> Grace: Well, I like Eureka math because it shows multiple strategies to solve one problem, so [my students] can choose which way they're more comfortable with. And it doesn't have to follow the book exactly. I really am not too picky about how they solve it - which strategy they use. But they usually don't stick to the way they started to solve the problem; they usually go with the lesson in the book. I show them the lesson, and the students usually go with that way.

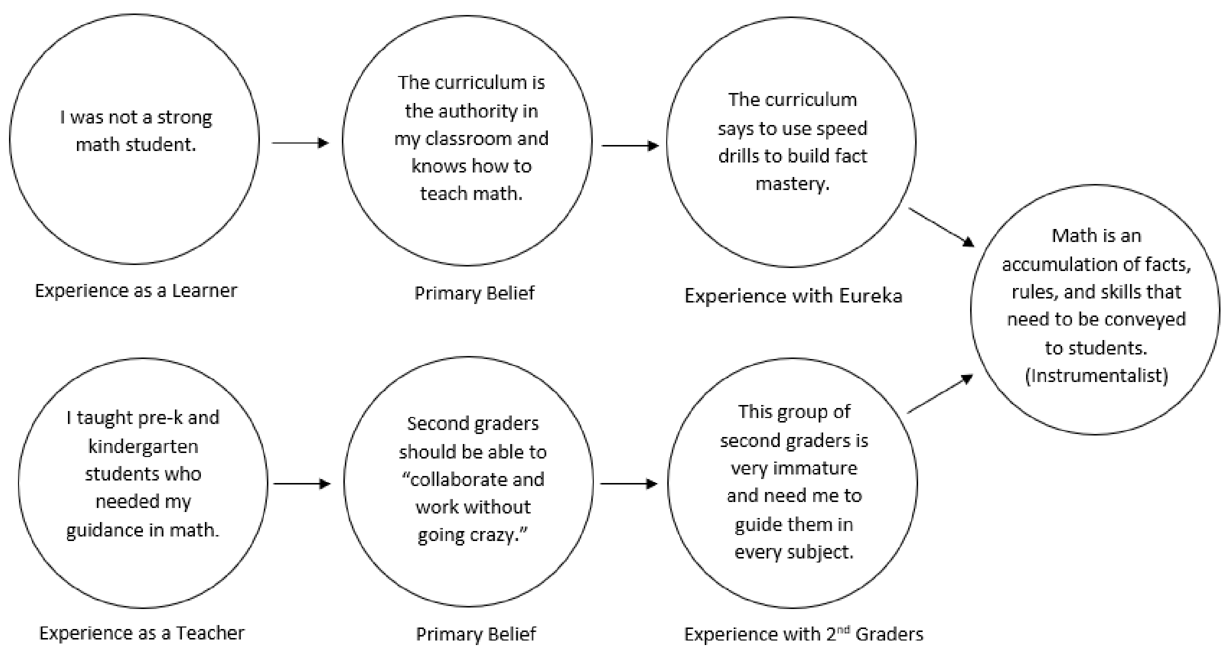
Grace was not able to determine who or what defined what it meant to do math.

The second part of Grace's instrumentalist belief, math needed to be conveyed to students, was formed through her experience with her students. Grace had taught pre-k and kindergarten students five years prior to her two years in the second grade. Grace felt they did not do a lot of complex math in those grades; "it was more counting," and her students were good at that. Grace felt comfortable with this age group, because she knew how they thought, and she could easily guide them and help them learn how to do mathematics.

Grace also held a primary belief that as students got older they did not need "so much individual attention." She thought seconders would be more independent and be able to "collaborate and work without going crazy" compared to her pre-k and kindergarten students; however, this was not her experience. Grace believed her students this year were "very immature" and needed "a lot of guidance with everything." She explained that if she let her students work independently, they would get loud and would be “completely not doing what [was] intended even though [she had] explained it in detail.” When she did give them independent work, some of her students would have “meltdowns,” cry, and get very frustrated, because they would not either know what to do or how to do it. This led Grace to believe she had to show the students how to do math, because they were not mature enough to learn, discover, or explore it without her guidance (see Figure 5.1).

Figure 5.1

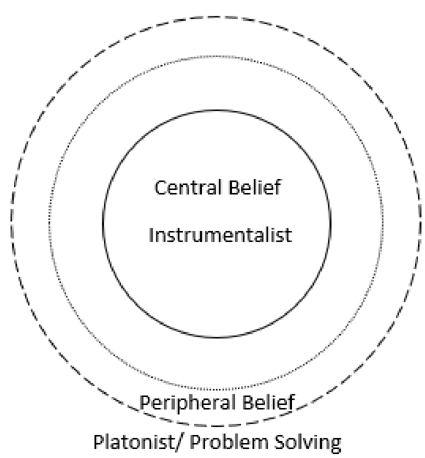
Grace’s Quasi-logical Belief Structure: Nature of Mathematics



Additionally, the interview data supported that conclusion that Grace did not hold any other beliefs about the nature of mathematics. She struggled to define what it meant to do math and never expressed that mathematics was open to discovery or revision (Ernest, 1989). She hinted at the idea that students might be able to explore some mathematical concepts or strategies on their own, but this was usually reserved for her high math achievers. She just knew that her second-graders had "so many fundamental skills that they need to learn" and it was "hard to *drill* that into all 21 of them" [emphasis added]. She needed the curriculum to help her teach those fundamentals skills, and the Sprints to drill those facts. The PD sessions that addressed the ineffectiveness of this skill and drill view of mathematics appeared to have little to no impact on Grace's instrumentalist view which supported the notion that any Platonist or problem solving views about the nature of mathematics held little if any peripheral strength (see Figure 5.2).

Figure 5.2

Grace's Psychological Belief Structure: Nature of Mathematics



Grace's Model of Teaching Mathematics Journey

Grace felt she looked to the curriculum and colleagues to help her know how to teach mathematics to her students and not necessarily any other experiences or resources such as professional development sessions. During the interviews, Grace admitted that at first, she strongly disliked Eureka math. It was "difficult to teach and explain math in a different way," and she had to "really step out of the box" in order to understand it. For example, the curriculum asked her to show her students different ways to solve math problems and even encouraged open exploration and discussion. She had never learned these different strategies nor had she ever experienced exploration and discussion as a student. However, she held a primary belief that the curriculum was the expert and knew how to teach mathematics, and she knew she needed a "guideline" to help her with pacing and teaching certain mathematical concepts. So, as evidenced with her self-identification as a CGI Level Two Problem Poser, she gave the students a problem, discussed different strategies they could use, let them try to solve it, and then showed them the strategy the book wanted them to use.

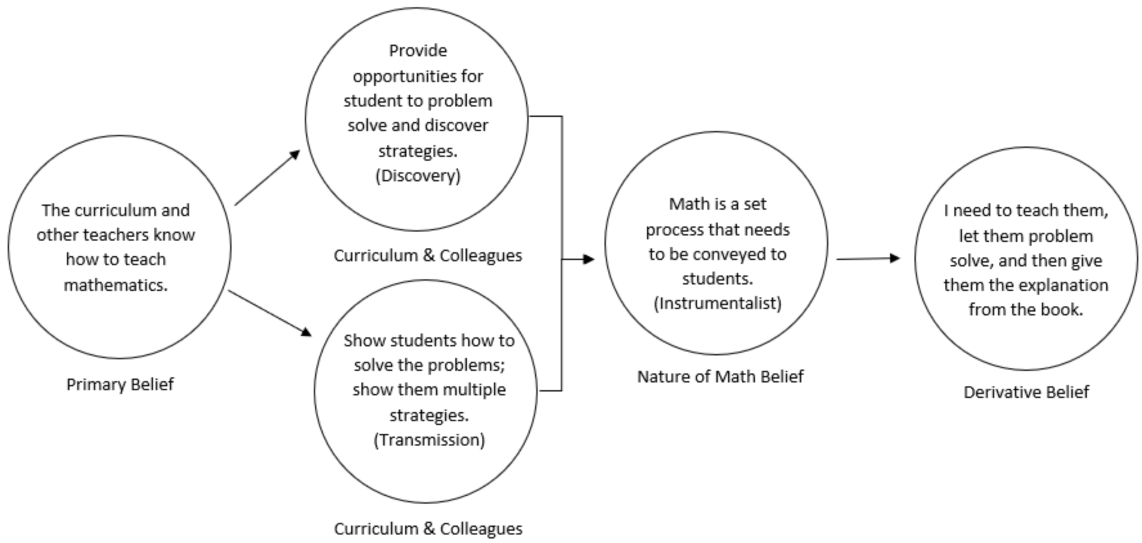
It was during these discussion sessions (her hint of discovery orientation) that some of her students would occasionally show her and the class a different way to solve the problem - a strategy that was not something she remembered teaching them nor did she believe that had learned last year. This would often surprise her and, according to Grace, only happened from her "really high, high math achievers." Grace added that she believed her experience with the Eureka curriculum has helped her become a better mathematics teacher; it gave her the structure and guidance she needed to ensure she was properly teaching her students.

The interviews also revealed that Grace felt her colleagues had a strong influence on her teaching mathematics orientation. Grace explained that she was "trying to learn from other teachers how to improve and how to teach." She would often talk with the other second-grade teachers and ask them questions about teaching and the Eureka math curriculum. In fact, this was a very common practice for the teachers at this school, especially the second-grade teachers. These teachers met once a week to plan and discuss the next week's lesson plans; Sally was responsible for the math, the other teacher took care of the reading, and Grace organized the social studies and science lessons. Grace sought the advice of her colleagues on how to handle a student or how to teach a particular math lesson. When she was not able to have these discussions or if she felt she needed more guidance, Grace sought out other teachers for direction such as one particular teacher who posted all of his Eureka math lessons online. Grace liked his approach because he taught it "the way Eureka showed."

Interestingly, even though the curriculum and her colleagues suggested that Grace give her students the opportunity to explore and discover mathematical strategies for themselves (discovery orientation), Grace still struggled with this. She believed her high achieving students could do this occasionally, but this was more the exception than the rule. She provided the freedom for her students to use whatever strategies they wanted or needed to solve the word problems in the Application Journal (her attempt at the discovery orientation), but always followed that with her own need to show the students how to solve the problem "according to the book." As seen in Figure 5.3, her orientation of mathematics teaching was being filtered through her nature of mathematics belief which resulted in her derived belief that she needed to "teach them" (instrumentalist), "let them problem solve" (discovery), and "then give them the explanation from the book" (transmission). The curriculum and colleagues were guiding her towards a potential discovery belief, but most of that orientation was being filtered out by her nature of math belief.

Figure 5.3

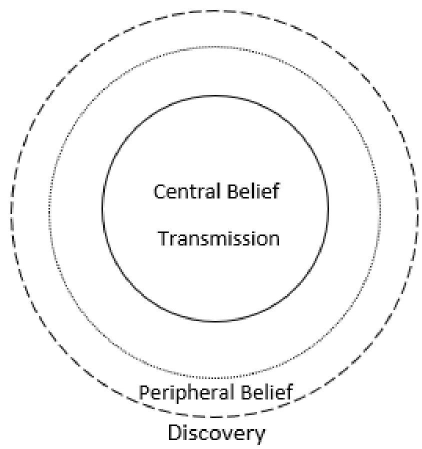
Grace's Quasi-logical Belief Structure: Teaching Mathematics Orientation



This hesitancy towards a discovery orientation could be explained through her psychological belief structure. Grace's transmission orientation towards teaching mathematics held a strong central location while the discovery orientation remained on the outer peripheral ring (see Figure 5.4). Grace believed that her students needed to "sometimes kind of figure it out on their own." If they were not given this opportunity, they tended to become dependent on the teacher. This peripheral belief had a very weak outer peripheral strength that could be easily followed or ignored based on the lesson, student, or any other justifiable rationale (Green, 1971). On the other hand, Grace's belief that she had to teach her students "some sort of process they can use" to solve mathematical problems had a very strong central location kept there by her strong instrumentalist nature of mathematics belief. Since central beliefs are accepted without question and difficult to explore and change, Grace could not see that her desire to create independence in her students through a discovery approach to teaching mathematics was being overpowered by her instrumentalist nature of mathematics belief and her transmissive nature of teaching mathematics orientation. She just knew that she needed to teach them how to "apply things on their own terms, but then guide them and give them the way the book shows."

Figure 5.4

Grace's Psychological Belief Structure: Teaching Mathematics Orientation



Grace's Model of Learning Mathematics Journey

Grace's transmission orientation towards learning mathematics stemmed from her experiences as and with students and with the Eureka math curriculum. Based on the interviews, Grace believed that students, and she, needed a process in order to learn mathematics. If students did not understand the process being taught, they would not be able to successfully learn the mathematical concepts being taught (transmission orientation). This was a derivative belief that came from her experience as a student as evidenced by her claim that learning a process was what helped her in her math classes. If Grace's teachers showed her a process that she understood, she was better able to learn and remember the mathematical concept that was being taught. Unfortunately, however, her teachers usually only demonstrated one strategy or process - if Grace did not understand the taught strategy she did not learn the lesson.

This experience caused Grace to believe her students needed to learn multiple strategies in order to learn mathematics. She understood,

> What it's like to not have an idea at all what the teacher is talking about. And they give you one, just one strategy to solve it. You're just like I can't do that way. So I think from being a struggling math student I understand how my students feel sometimes if *I* [emphasis added] can only show them one thing.

All of her students learned differently and needed different strategies. Unfortunately, however, Grace had never learned different strategies and, therefore, struggled with showing her students different ways to solve problems.

This is why Grace felt she had to follow the textbook. The textbook knew the best and a variety of ways to teach the students how to do the math problems. If the method they learned today did not make sense to the student, then they could use a strategy they learned yesterday or maybe pick up the concept from the process that will be taught tomorrow. This was what was happening to Grace as a teacher and learner of these new strategies. The curriculum was showing her new ways to find answers to math problems. She could now “use the strategies in [her] head more so than [she] used to” and was feeling more confident in her ability to learn and teach mathematics. Her own realization of the power of learning multiple strategies led her to conclude that her students should learn multiple ways to solve problems and then “choose which way they’re more comfortable with.” Since Grace learned these processes through the guidance of the curriculum and not through a discovery approach, her students would need to be taught and shown new strategies through her guidance.

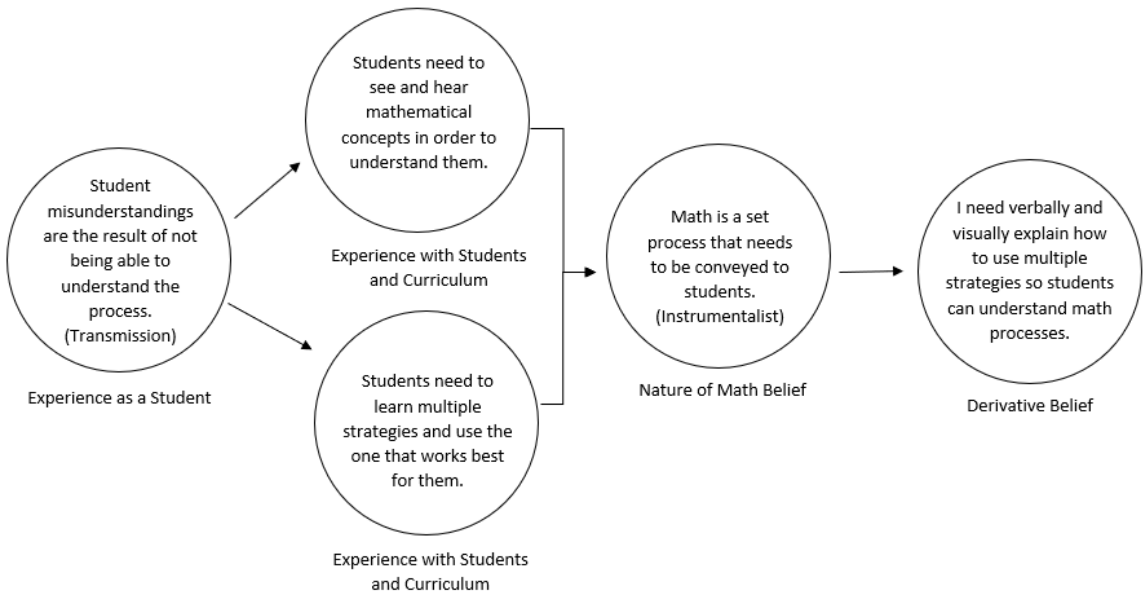
Grace also believed that she struggled with understanding the mathematical process being taught because she was a visual learner but most of her teachers taught the math lesson verbally. This left Grace feeling frustrated and discouraged during math class, because she could not always understand her teachers' verbal explanations; she "couldn't see it." She could empathize with her student who struggled to understand her verbal explanations because she knew what it was "like to not understand what you're hearing at all." Grace expressed that she "would have gone a lot farther in math if I would have had some sort of visual instead of just the number and the problem in front of me. It seemed very abstract."

This was another reason why she began appreciating the Eureka math curriculum; it showed her and the students' visual ways to understand the mathematical concepts. For example, when her students were learning how to add two-digit numbers with regrouping, they were shown how to use place value disks to represent the tens and ones. When the students had ten ones, they could actually see how the ones regrouped into a ten and were then moved to the tens column. According to Grace, they were able to "visualize and count what they were adding" and physically move the numbers around. Grace's belief that many of her second-grade students were visual learners led her to believe that they also needed to "see the math through things like the place value disks. Those things really help most students." This was a strong illustration of the true/false dichotomy of beliefs; according to Grace, it is true that most second-graders are visual so it is true that they must be able to see the mathematics (Phillip, 2007). Grace added

Eureka's inclusion of the visuals has even helped her visualize the problems "a bit more and solve them with more automaticity" which added to the truth that students need to see the math as well.

Figure 5.5

Grace's Quasi-logical Belief Structure: Learning Mathematics Orientation



Grace concluded this discussion claiming that the way students learn math really depended on the student. "I think some students learn more visually from the things I'm writing on the board, and I think other students learn more from hearing it. They sometimes need both." This statement supported the conclusion that, like her teaching mathematics orientation, Grace's transmissive learning mathematics orientation was being filtered through her instrumentalist nature of math belief and was supported by her primary belief that the curriculum was the authority. The students needed to learn mathematics through the multiple visual and verbal processes, and she had to be the one who conveyed and showed her students these processes. She had to guide her students just like the curriculum guided her (see Figure 5.5).

Grace's experiences as a student, with her students, and with the curriculum and her nature of math beliefs had pushed her derivative belief that she needed to verbally and visually explain how to use multiple strategies so her students could understand math processes into a central psychological strength position (Green, 1971). When asked to summarize how she developed this orientation, Grace explained that it evolved from her experiences with the students and curriculum. According to Grace, she has learned more about how her students learn and how to better accommodate and meet their needs. She was not able to elaborate on how she acquired this information but did credit the curriculum with changing her "outlook on how to solve problems and showing me how to use the strategies in my head more so than I used to." She added,

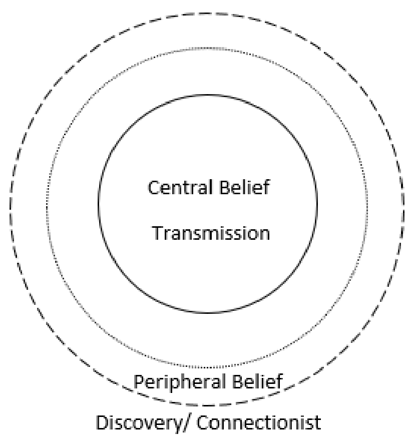
> But now, since I've learned the curriculum, it's like the things that I learned as a child, they aren't important. They aren't even there anymore. I learned this new way so that's how I think now. Now I think like this instead of the way I learned.

The curriculum taught her a new way to approach problems and gave her a new understanding of mathematics which, in turn, has made her "think I am a better teacher now." She could now show her students how to use multiple visual and verbal processes that they could use to learn and understand the mathematical concepts she was teaching.

Interestingly, even though the curriculum did embed a discovery and open communication approach in the Application Problem sections of the lessons, Grace did not always make those connections. For example, during the first observation, Grace began the lesson with the Application Problem. The curriculum provided a script that asked the teacher to read the problem to the students and then have the students discuss potential ways they could illustrate how to solve the problem with their partners. Instead of following this suggested process, Grace showed her students the illustration the textbook included as a possible response the students might give on how they solved the problem. When asked why she showed the students the book's example instead of following the book's suggested instruction, Grace explained she had read the script, but during the lesson, she "jumped to the explanation for some reason, but I don't know why."

Figure 5.6

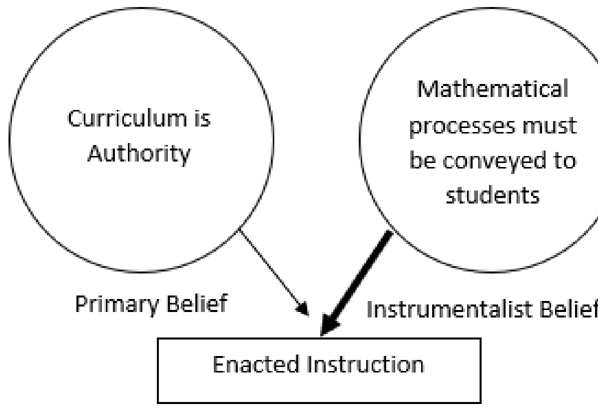
Grace's Psychological Belief Structure: Learning Mathematics Orientation



Grace's inability to explain why this occurred was caused by her psychological belief structure and her belief clusters (Green, 1971). Her transmission orientation was central while any discovery beliefs were peripheral (see Figure 5.6). This meant her transmission orientation was stronger and harder to change than the discovery orientation. Even though she read that she should give her students the opportunity to discuss how they would solve the problem before showing them what to do, the stress and pressure from teaching and being observed appeared to cause her to abandon any discovery suggestions and stick to her transmissive orientation.

Figure 5.7

Grace's Belief Clusters: Primary Belief about Curriculum and Instrumentalist Belief



Note. The thickness of the arrow denotes the strength of the influence; thicker arrow means stronger influence.

Additionally, Grace held an instrumentalist belief that processes needed to be conveyed to students and a primary belief that the curriculum was the authority and should be followed. In this situation, these two belief clusters were now sitting in opposition to each other. According to Green (1971), when this happens, the person will often follow the stronger of the two. As evidenced throughout the discussions about her beliefs and orientations, Grace had a very strong nature of mathematics instrumentalist belief structure. Grace's acknowledgement of reading the curriculum's suggestion and then ignoring it in the lesson suggested that her instrumentalist belief was stronger than her curriculum belief; therefore, she followed this belief cluster and "jumped to the explanation" without even thinking about it (see Figure 5.7). Grace would need to juxtapose these belief clusters in order to understand why she did this and how to reconcile the differences.

Grace's Mathematical Beliefs Influence on Her Instruction

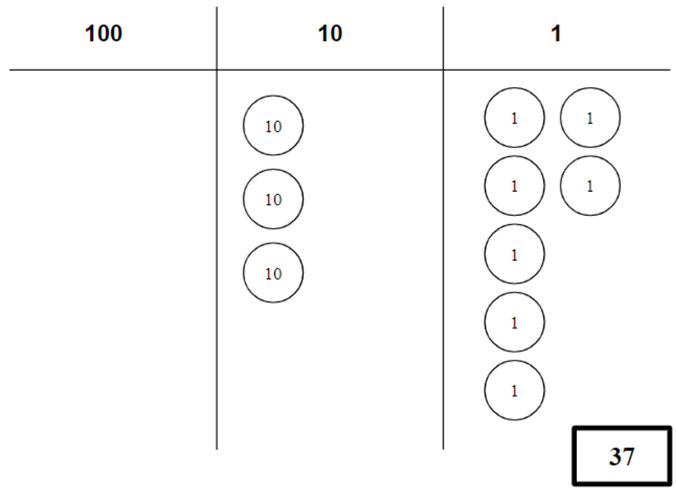
Since mathematics instruction is influenced by teachers' beliefs about what it means to do mathematics and orientations towards the teaching and learning of mathematics, I now want to explore how these beliefs work together or in opposition to each other to influence Grace's mathematics instruction (Copur-Gencturk, 2015).

Grace's Nature of Mathematics Influence

The lesson observations indicated Grace tended to lean on her espoused instrumentalist belief that mathematics is a collection of facts that needs to be conveyed to students. She taught her students how to follow rules and procedures to complete the day's lesson. For example, in the first observation, Grace introduced the math lesson by reminding students how to use place value disks - a strategy that would be needed to complete the problem set correctly. She drew the place value chart and some place value disks in the correct columns on the board (see Figure 5.8). She asked Sam to identify the value of the disks she drew. After Sam correctly identified the amount, Grace proceeded to show how to find the amount, instead of asking Sam how he arrived at his answer, and explained to her students that this was how they would need "to solve today's lesson - just count the disks like I did." Similar scenarios occurred during Grace's second and third observations as well. These repeated instructional methods earned her a score of two out of three points on the Mathematics Classroom Observation Protocol for Practices ($MCOP^2$) item *the lesson promoted modeling with mathematics*, because, according to the performance descriptor, her modeling was turned into a procedure on how to solve the rest of the worksheet (see Table 5.3).

Figure 5.8

Grace's Illustration of Place Value Chart and Place Value Disks



The MCOP^2 also revealed some other influences of Grace's instrumentalist nature of mathematics belief on her instruction. She scored a zero out of three points on *students critically assessed mathematical strategies*, because she was the only one discussing the strategy in relation to the specific problem being solved; she scored a one on *the teacher uses student questions/comments to enhance conceptual mathematical understanding*, because she focused on the procedural rather than conceptual knowledge when her students asked or answered questions. For example, during the second observation, students were asked to find the total number of crackers if there were thirty crackers in four boxes. After giving the students a few minutes to solve the problem, Grace asked Logan for the final amount. He responded with an answer of 120, and Grace asked him, "Why do you think that?" After his response of "they are all 30 so I put them together," Grace explained, "so he did 30, 30, 30, 30 and then added them for 120." Grace showed and focused on the process Logan used to arrive at his answer instead of probing and asking questions to evaluate and build Logan's and his classmates' conceptual understanding of the problem. Throughout each of the observations, Grace's instrumentalist nature of mathematics belief influenced her to be the only one assessing, connecting, or modeling the mathematical concepts.

Grace's Model of Teaching Mathematics Influence

Grace's espoused and enacted model of teaching mathematics also aligned closely with each other; in each of the observations, she taught them the lesson, let them problem solve, and then gave them the solution from the book. Grace rationalized this belief by explaining that her students and she felt more confident when she followed the curriculum's strategies and suggestions for solving the word problems and problem sets. As a result, Grace had the textbook on a chair in the front of the class and often read from it during all three observations, and she used the phrase or phrases similar to "the book wants us to" seven times during her instruction. During the second observation (the question about the number of crackers), Grace explained how Logan solved the problem and added, "let's see what the book says. It tells us to count by tens, so let's do that." The class and Grace choral counted by tens to 120; Grace confirmed that Logan got the same answer as the book and, therefore, was correct. Grace espoused that she believed she should give them the book's explanation, and that was what she did.

Table 5.3

Grace's Summative Score for MCOP² *Based on Three Observations*

Factor			Indicator	Score
S		1.	Students engaged in exploration/investigation/problem solving.	1
S		2.	Students used a variety of means (models, diagrams, graphs, concrete materials, manipulatives, etc.) to represent concepts.	1
S		3.	Students were engaged in mathematical activities.	1
S	T	4.	Students critically assessed mathematical strategies.	0
S		5.	Students persevered in problem solving.	1
	T	6.	The lesson involved fundamental concepts of the subject to promote relational/ conceptual understanding.	
	T	7.	The lesson promoted modeling with mathematics.	2
	T	8.	The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	1
	T	9.	The lesson included tasks that have multiple paths to a solution or multiple solutions.	2
	T	10.	The lesson promoted precision of mathematical language.	2
	T	11.	The teacher's talk encouraged student thinking.	1
S		12.	There were a high proportion of students talking related to mathematics.	1
S	T	13.	There was a climate of respect for what others had to say.	1
S		14.	In general, the teacher provided wait-time.	1
S		15.	Students were involved in the communication of their ideas to others (peer-to-peer).	1
	T	16.	The teacher uses student questions/comments to enhance conceptual mathematical understanding.	1
S		S	Student Engagement Total	0.9
	T	T	Teacher Facilitator Total	1.1

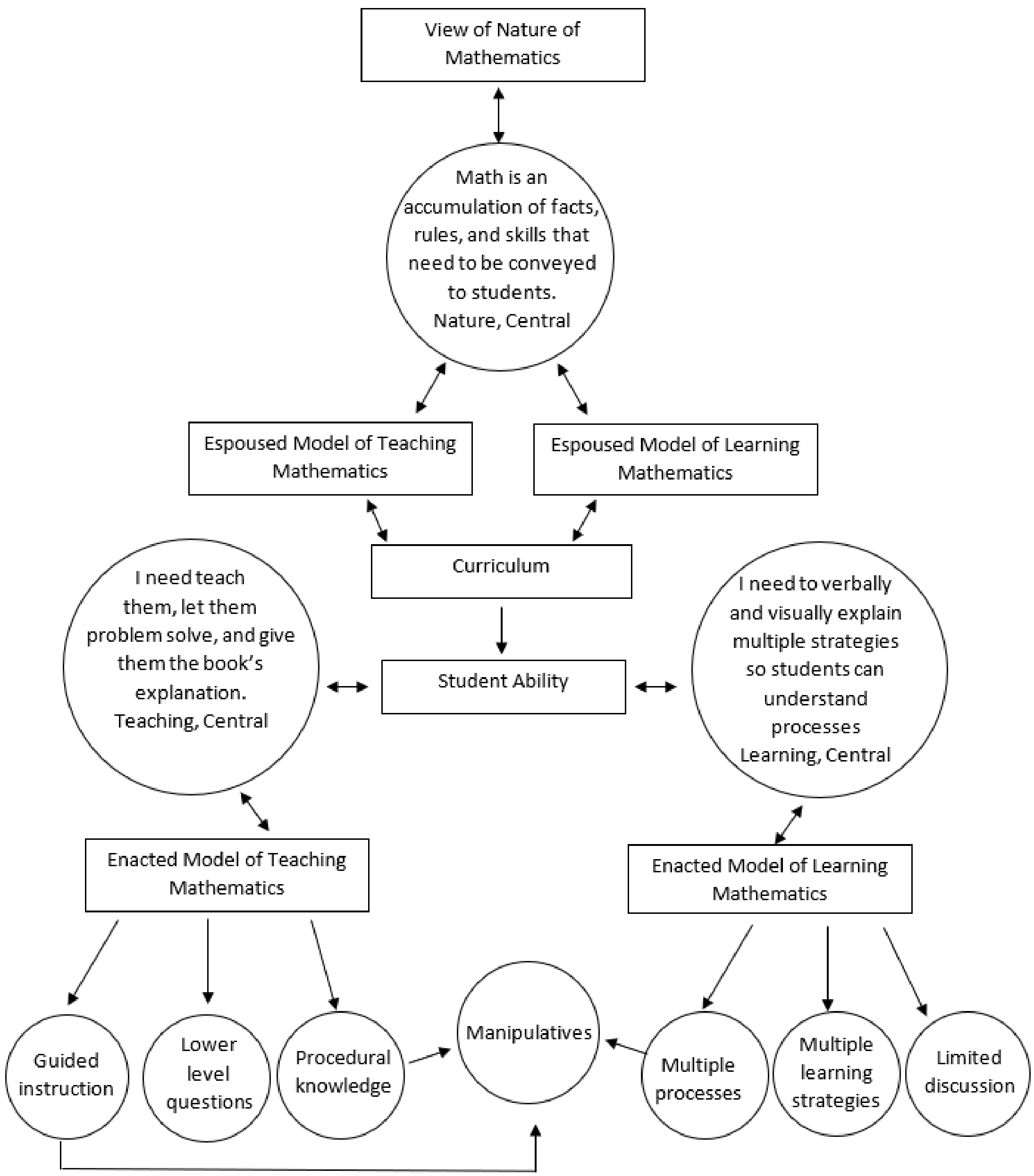
Additionally, Grace credited the curriculum with showing her and her students "several different ways on how to solve one problem." Because of this influence, Grace was often heard telling her students to use any strategy they wanted or felt comfortable with to solve the problems. This is also why she scored a two (out of three points) on the MCOP² indicator: *the lesson included tasks that have multiple paths to a solution or multiple solutions*. If Grace had to create a lesson that was not from the Eureka curriculum, such as the pattern lesson in observation three (discussed in Grace's Model of Learning Mathematics section), she still tried to encourage students to find multiple paths or solutions.

Despite her dependence on the curriculum, Grace occasionally would modify the instruction if another method was more convenient or understandable for her. For example, on the second observation, the curriculum suggested that she model and students use place value disks to practice adding two-digit numbers with regrouping. Grace read this suggestion to her students but then explained that they would "just draw the disks instead of using the chips because it [was] faster." Regardless of this modification, the curriculum still appeared to have a strong influence on her enacted model of teaching mathematics (see Figure 5.9).

Grace's beliefs about her students also had a strong influence on her enacted model of teaching mathematics. Grace claimed that she felt her students needed a lot of guidance. They struggled with understanding how to solve word problems, "even [her] higher-level student," so she had to give them the steps. This was why she felt she had to show her students mathematical generalizations instead of giving them the "opportunity to discover these generalizations themselves" and tended to ask "lower order" questions which resulted in her scoring a one on $MCOP^2$ indicators eight and eleven (Gleason et al., 2015). For example, during the third observation, Grace asked her students what made the pattern she drew on the board a pattern. She asked several students the same question, drawing names from the speaking cup, until one of them gave her the response she was looking for: "it repeats itself." Occasionally, she would ask questions such as "why do you think that?" or "would you like to add anything to your response?" However, the students usually responded with a short answer such as "because that is what it showed" or "just that I added them because that's what I was supposed to do," and Grace accepted these answers without trying to get the student to dig deeper.

Figure 5.9

Grace's Instructional Practice Flowchart



The observations and $MCOP^2$ rubric revealed that Grace's espoused belief that she should teach her students the lesson, let them independently solve problems to apply learned strategies, and then show her students the processes presented in the book aligned to her enacted model of teaching mathematics. According to Grace, this interplay among her teaching, her students learning, and the book guiding the lessons has really helped her become more confident and comfortable with "teaching the material"; she did not feel her beliefs were changing, but she was "learning new techniques and things from teaching the Eureka math."

Grace's Model of Learning Mathematics Influence

The RMBS and interview questions revealed that Grace's espoused model of learning math was students learn math by remembering and using processes she taught. In order for her students to actually learn and remember those processes, she had to explain mathematical procedures both verbally and visually and, according to the book, through a variety of strategies so students could pick whichever strategy worked best for them to solve the problems. In general, Grace's espoused belief aligned closely with her enacted model. She used a lot of illustrations and manipulatives to show students how to solve the Application Journal problem or the day's problem set; verbally, she explained how to use the manipulatives or follow the procedure to solve the problems; she showed students multiple strategies and expressed to students to use whatever process they wanted or remembered when working word problems or problem sets. However, despite the latter of these strategies, Grace still only scored a one out of three points on the $MCOP^2$ indicator two: *students used a variety of means to represent concepts*.

According to the MCOP^2 rubric, in order for a teacher to score a two or higher on indicator two, students must generate or manipulate two or more representations. While Grace gave students the option to use whatever strategy they wanted to solve the problem, she began by showing students how to solve the application problem or the problem set in each of the observations based on the suggestions given in the textbook. For example, in her first lesson, Grace read the problem to the students, drew the illustration from the book that modeled one way to solve the problem, and then told students, "based on these drawings, how are you going to figure out how many crackers are in the four boxes? I want you to figure it out on your own and then we will talk about it." Students copied Grace's illustration and then independently solved the problem. Since the students used the drawing she provided and did not try to find or use another model, Grace's MCOP^2 score for that indicator was a one. They did use the representation, but it was only one way to represent the concept (Gleason et al., 2015).

The observations also supported Grace's score of one on the MCOP^2 indicators twelve, thirteen, and fifteen: *there were a high proportion of students talking related to mathematics, there was a climate of respect for what others had to say*, and *students were involved in the communication of their ideas to others* (respectively). Grace did provide time for students to "talk about" how they solved the problem; however, she only called on one student, Logan, to give a possible solution to the application problem. In observation two, Grace did have students turn and talk to their partner twice how they solved a particular problem she wrote on the board, but she only had one pair come to the board and show their solution. Both of those observations showed that Grace, even though she previously espoused that her students "might look to another student's explanation for answers," tended to have less than half of her students talking about mathematics (indicator twelve), only a few students share their solution while the rest were listening (indicator thirteen), and less than five minutes of the lesson was given to peer-to-peer communication (indicator fifteen) (Gleason et al., 2015).

Interestingly, the third observation about patterns was different. On this particular day, Grace only had ten of her twenty-one students present; the rest were absent because of the town's homecoming parade. So, Grace had every student come to the board and show a pattern. When asked about this incident during the interviews, Grace responded that she usually did not have many students share their solutions because "we [would] end up on a tangent on something completely different, and they [would] want to tell stories." She added, "this group is so hard. If I let them work with each other and collaborate, sometimes it works great and other times it's just total chaos." During the interviews, Grace explained that she was having her students delve "into more of them teaching each other and becoming more comfortable and confident with it," but, according to the observations, this only happened when she had a small group of students. According to Grace, this collaborative learning was something she felt they could do more of later in the year and was working towards, but "right now it's a struggle."

Additional Findings

By the time this research started, Grace felt at ease with me being in her classroom and observing. She felt my presence in her classroom did not change or influence her teaching; she taught the same way no matter who was observing her. Since Grace did not feel she taught differently when I was observing, she never really viewed me as an authority - someone telling her how to run her classroom. Actually, Grace never identified an authority; when asked if she, the curriculum, the administration, colleagues, or I was an authority, she redirected the question commenting that her students can use whatever strategy they want to solve a problem even if it did not follow what the book said. However, the interview data suggested that Grace viewed the curriculum and her colleagues as potentially holding the authority in her classroom.

Throughout the interviews, Grace commented that she showed students how to work the math problems "according to the book" ten times. She even stated that she thought she needed some sort of curriculum or something to follow so she did not have to "fly by the seat of her pants." She has had to make up her own curriculum before during her first few years of teaching and believed that having a curriculum was helpful. When the curriculum lacked or was not clear, Grace turned to her second-grade colleagues for guidance. According to Grace, they were a great resource when she did not understand what the lesson was wanting or how to make her students understand or listen. This guidance and a set curriculum were some of the reasons why she liked teaching at this school - they both showed her "what to teach and how to teach it" and have helped her become a better teacher.

Reflection

At the end of the second interview, I asked Grace how she felt about this process. She responded,

> I like to learn. I try to learn from other teachers how to improve and how to teach. I want to do my best. There is always continual learning as a teacher. You never stop. You can never stop learning. One of the kids asked me if I was an expert

> teacher. I said, no. I can never be an expert teacher because you never stop learning. So, I feel like the more I grow the more I will keep going and going. Learning new techniques and learning things from other teachers. And learning things from teaching the Eureka math. I want to read [your study] and see what I need to learn. And I would like to watch you so I can have different input and strategies. I think I could do more questioning if I could see it.

In chapter VI, the case study analysis and discussion will be presented. Additionally, the implications of the findings and future research will be shared.

CHAPTER VI

CASE STUDY ANALYSIS AND DISCUSSION

The purpose of this study was to explore and describe how mathematics beliefs influence the instruction of these two elementary teachers. An understanding of these belief structures, how they formed and evolved, and how they directly or indirectly impacted the teachers' enacted beliefs about how math should be taught and learned could be used to develop PD sessions that have the potential to change any of these teachers' moderately effective transmissive beliefs and enhance any of their highly effective constructivist beliefs (Askew et al., 1997). To accomplish this purpose, I utilized a heuristic single-case study (second-grade elementary teachers at a particular school) design with two embedded units of analysis (Sally and Grace) to answer the following research questions:

1. What were the second-grade elementary teachers' beliefs about the nature of mathematics and their models of teaching mathematics and learning mathematics?
2. How were the belief structures of the second-grade elementary teachers' views about the nature of mathematics and espoused models of teaching mathematics and learning mathematics formed?

3. How did the beliefs about the nature of mathematics and models of teaching and learning mathematics influence the second-grade elementary teachers' mathematical instruction?

I used both quantitative and qualitative data to answer these questions. The quantitative instruments included the Revised Mathematics Beliefs Scale (RMBS), Cognitively Guided Instruction Teacher levels, and Mathematics Classroom Observation Protocol for Practices ($MCOP^2$). I used this survey, identification statements, and observation instrument to build construct validity by using multiple sources of evidence to reduce subjectivity and support claims I made about the participants' responses to the interviews. I also used qualitative data in the form of semi-structured interviews and observations to capture the teachers' views and understandings. The findings were used to gain a deeper understanding of the teachers' beliefs, the formation of those beliefs, and how those beliefs impacted the teachers' instruction in mathematics.

These Second-grade Teachers' Mathematical Beliefs

The first step in understanding how the teachers' mathematical beliefs influenced their instruction was to explore what these teachers believed about what it meant to do mathematics and how mathematics should be taught and learned. I used Ernest's (1989) conception of the nature of mathematics definitions to determine if the teachers held an instrumentalist, Platonist, or problem-solving view about the nature of mathematics. I used Askew et al.'s (1997) explanation of the orientations or "ideal types" of the teachers' beliefs about how to best teach and learn mathematics to determine if the teachers held a transmission, discovery, or connectionist orientation (p. 28). The instrumentalist belief and transmission orientation were very similar to traditional meanings, teachings, and learning styles, whereas the problem solving and connectionist belief and orientation were more constructivist in nature; the Platonist and discovery belief and orientation were between the traditional and constructivist models.

Nature of Mathematics

The data from the interviews, RMBS, and CGI levels revealed that both Sally and Grace held instrumentalist beliefs about the nature of mathematics meaning they viewed mathematics an accumulation of facts and rules that needed to be conveyed to students (Ernest, 1989). They both agreed with the RMBS statement that students should spend time practicing computational procedures before they are expected to understand how and why those procedures work and that children should be able to recall facts before they understand operations. Additionally, during the interviews, both teachers explained that doing mathematics meant working problems to find answers. These data also showed that Sally had occasional hints of a Platonist belief about the nature of mathematics meaning she occasionally viewed mathematics as a body of knowledge waiting to be discovered.

These findings align with other research about the nature of mathematics beliefs held by in-service teachers. Siswono et al., (2019) conducted a study of 80 primary in-service teachers with a minimum of five years teaching experience and found that these teachers tended to hold an instrumentalist view about the nature of mathematics over a Platonist or problem-solving belief. Other research also concluded that in-service teachers tended to hold traditional beliefs about what it means to do mathematics (Barkatsas & Malone, 2005; Lui & Bonner, 2016; Nisbet & Warren, 2000; Safrudiannur & Rott, 2021). Furthermore, Aljaberi and Gheith (2018) found that 65.7% of the 101 in-service teachers in their study held an in-between traditional and constructive belief about the nature of mathematics just like Sally. This is alarming because these instrumentalist beliefs about the nature of mathematics align closely to The National Council of Teachers of Mathematics (NCTM, 2014) set of unproductive beliefs such as a belief that learning should focus on practicing procedures and basic skills before applying mathematics. Teacher educators, professional development providers, and mathematics coaches need to be aware of the findings from this and other studies about teachers' beliefs toward the nature of mathematics if we want to promote any type of change in beliefs and how those beliefs impact teaching and learning (Cross, 2009)

Orientations Towards Teaching and Learning Mathematics

The data also revealed that Sally and Grace held a transmission and discovery orientation towards teaching mathematics, Sally held a discovery with a hint of connectionist orientation towards learning mathematics, and Grace held a transmission orientation about how students should learn mathematics (see Table 6.1). Again, apart from Grace's orientation about mathematical learning, this in-between and mixture of beliefs aligned with other research about teaching and learning mathematics (Lui & Bonner, 2016; Siswono et al., 2019; Voss et al., 2013). However, Sally's and Grace's teaching orientations and Grace's learning orientation did not align with other research. For example, Aljaberi and Gheith's (2018) discovered that a majority or 77% and 77.4% of their in-service teachers held an in-between constructively inclined orientation towards how mathematics should be taught and how mathematics should be learned, respectively; Scrinzi's (2011) reported that thirteen of the twenty-nine participants agreed with a constructivist belief in regards to mathematics instruction in her study.

Sally and Grace appeared to want to have a strong constructivist orientation towards teaching and learning mathematics. For example, Sally used the phrase "explore their own thinking" four times in the first interview and agreed that children should find their own solutions, discover their own relationships, and engage in productive struggle; Grace talked about giving her students opportunities to practice, think, or figure things out on their own or on their own terms six times in the first and last interview and felt that her students needed to take control of their own learning. Both teachers agreed with the RMBS statements that *the goals of instruction in mathematics are best achieved when students find their own methods for solving problems* and *teachers should allow children who are having difficulty solving a word problem to continue to find a solution*.

However, similar to what Beswick (2005) discovered in her participants, Sally and Grace appeared to have insufficient understandings of constructivism, discovery or connectionist orientations. For Sally, "exploring" meant students were given the opportunity to practice with manipulatives or try strategies *after* they were taught how to use the manipulatives or shown how to use the strategies. The same was true for Grace; "figuring things out on their own" meant using a previously learned strategy when solving the application problems. This insufficient understanding was why both teachers agreed with the CGI Level Two (problem poser) level. This meant they believed they should give students the opportunity to solve problems using their own strategies, but they also needed to go back and explain the procedures or show the students how to solve the problem or, as Grace stated, "Teach them, let them problem solve, then give them the explanation." Sally and Grace both believed they should be facilitators of student learning (score of 4.2 and 3.3 on RMBS, respectively).

They wanted their students to explore and figure out strategies and problems, but they filtered their orientations through their instrumentalist nature of mathematics belief (Cross, 2009) even though they found merit in the idea of the constructivist approach to teaching (Nisbet & Warren, 2000). This finding supported this study's first proposition that the second-grade in-service teachers' nature of mathematics beliefs and how mathematics should be taught and learned orientations were distinct beliefs and could, therefore, seemingly contradict each other (Ernest, 1989; Green, 1971; Voss et al., 2013)

Sally was aware of this conflict and wanted "to know how to give [her students] more freedom" and not revert back to "alright, this is how you do it." However, even though Grace wanted her students to gain independence and see how they solved problems without her, she held onto her unproductive belief that her role was to tell and show students how to solve the problems (NCTM, 2014); she *had* to "guide them and give them the way the book shows" in order for them to be successful and know what to do.

Incidentally, Xie and Cai (2021) supported the notion that teachers with six to ten years of experience in the classroom indicated a strong desire to adopt constructivist views of teaching and learning mathematics, whereas teachers with one to five years of experience did not appear to have as strong of a desire. Sally had been teaching students in the second-grade for eight years and was quite comfortable with the students, content, and curriculum; she may have had instrumentalist and Platonists beliefs about what it meant to do mathematics, but her orientations towards teaching and learning mathematics were not as traditional (Barkatsas & Malone, 2005). Grace, on the other hand, had only been teaching students in the second-grade for two years and, according to her, was just starting to gain a better understanding of how second-grade “minds work," and how to “accommodate students who were struggling.” She also admitted that she was getting more comfortable and familiar with the curriculum and felt that she was just starting to teach more as she believed this year. Perhaps Grace was displaying what Xie and Cai’s (2021) found in their study that new teachers might not be ready take on new beliefs when they are still trying to figure out the content, students, and curriculum of a new grade; it would be interesting to explore any changes Grace may have made with her espoused and enacted beliefs as she gains more experience and exposure to this grade level in future research (NCTM, 2014).

Moreover, Sally and Grace both disagreed with the RMBS statement that *children learn math best by attending to the teacher’s explanations* claiming that students should explain their solutions and strategies to their classmates so other students could see and hear different ways to solve problems. According to Grace, “some kids for sure look to the teacher’s explanations, but other kids might look to another student’s explanation for answers;” however, Grace concluded that it was only her “really, really high math achievers" that could find and explain different strategies to their classmates. Grace’s belief was similar to Torrf’s (2005) finding that teachers believed cognitively demanding tasks, such as explaining different strategies to classmates, should only be given to high-achieving students, and Safrudiannur and Rott’s (2020) finding that teachers tend to have an instrumentalist view of mathematics in their low achieving classes compared to their classes with high achieving students. Even though Sally did not claim that only her high-ability students could explore or explain different strategies to solve problems, she did explain that she was often surprised when one of her students showed her or the class a solution that neither she nor the curriculum had shown before displaying an underestimation of her students' abilities just like Grace.

Finally, both teachers believed students learn mathematics best when they can see the mathematics. For Sally, this meant seeing how other students solved problems (connectionist) and using manipulatives (discovery). She explained that these two strategies were important because "for some reason it makes sense for them to be able to see and make that connection… it just helps them learn it." For Grace, seeing the mathematics meant showing them the steps on the board and using manipulatives (transmission). Grace reckoned that students were either auditory or visual learners, so she needed to utilize both to ensure her students could learn and understand the mathematics lesson she was teaching. Grace explained that if her students forgot something she said, she would remind them of the manipulatives they used during the lesson. She added that "a lot of second-graders are very visual, and they need to see it like the bundles and place value disks. Those things really help most students remember."

Other research echoed Sally's and Grace's beliefs that use of manipulatives should be used in the elementary classroom (Uribe-Flórez & Wilkin, 2017) but for different reasons. Sally and Grace believed the manipulatives provided a way for students to see the mathematics; Swan and Marshall (2010) found that 188 of the 249 in-service teachers in their study had this same belief stating that manipulatives assisted their students with concrete visualizations. Sally believed the manipulatives provided hands-on learning; the same belief was held by 54% of Swan and Marshall's participants. Grace believed the manipulatives were beneficial to her visual learners but not necessarily for her auditory learners; Golafshani (2013) found that some of the teachers in her study viewed the use of manipulatives as an option for some students but not necessarily for all students. Sally believed students became numerate by working and exploring with manipulatives (discovery); Grace believed students became numerate by seeing and learning how to use manipulatives to develop mathematical procedures (transmission) (Askew et al., 1997). Both teachers found benefits to use manipulatives; however, like other teachers, their biggest concern for using or not using manipulatives equated to the time it took to use them or their ability to manage or know how to use them (Golafshani, 2013; Swan & Marshall, 2010; Uribe-Flórez & Wilkin, 2017).

Summary of Beliefs

The heuristic design of this case study provided an opportunity for Sally, Grace, and me to explore the mathematical beliefs and orientations they had about what it meant to do mathematics and how mathematics should be taught and learned. Sally and Grace both held an instrumentalist belief about the nature of mathematics, and Sally had a hint of Platonist belief in this area. Sally and Grace both held a mixed transmission and discovery orientation towards how mathematics should be taught with Sally's discovery orientation being more pronounced than Grace's. The area of major difference was in the teachers' learning mathematics orientation. Grace remained in the transmission stage in this area, whereas Sally displayed discovery with hints of problem solving in her orientation on how students should learn mathematics (see Table 6.1).

Sally and Grace also displayed some productive and unproductive beliefs about teaching and learning mathematics. First, their transmission orientation toward teaching mathematics promoted an unproductive belief that the teacher's role is to tell students how to solve the problem and provide step-by-step instructions to ensure the mathematics is not too difficult (NCTM, 2014). Grace's transmission orientation toward learning mathematics promoted the unproductive belief that the students' role is to memorize information and solve problems; Sally was starting to adopt more productive beliefs that students can learn mathematics through exploration and by being actively involved in the mathematics through interactions with her and their peers (NCTM, 2014). Both teachers held the productive belief that students need to have a range of strategies and approaches when trying to solve problems; however, those were the result of their experiences with their students and the curriculum - which is discussed in the next section.

Table 6.1

Summary of Sally's and Grace's Beliefs

Belief	Transmission / Instrumentalist	Discovery/ Platonist	Connectionist/ Problem Solving
Nature of Mathematics	B	s	
Teaching Mathematics Orientation	B	b	
Learning Mathematics Orientation	G	S	s

Note. B=both Sally and Grace; S=Sally; G=Grace; Lowercase letters indicate a hint of the belief or orientation.

During one of the interviews, Grace expressed that she was glad she was getting the opportunity to participate in this study. She added, "I feel like I'm learning more about myself and about my teaching through it. I really feel like it sums up my philosophy for math." Sally iterated these sentiments and added that it was good that I was "forcing" her to think about her beliefs; she felt she could not always explain her beliefs, but she did know she had "made some changes" in her belief systems because she as "having to think about them" through this study and through our PD sessions. Exploring these changes and formations of Sally's and Grace's belief system was the next thing I needed to explore to gain a better understanding of the teachers' mathematical beliefs and orientation in order to develop effective PD sessions to meet their needs.

These Second-grade Teachers' Mathematical Belief Journey

The data from the interviews revealed that Sally and Grace developed and changed their beliefs based on their experiences as a learner and as a teacher. Those experiences as a teacher broke down into two other factors: experience with their students and their colleagues. These beliefs were either primary (not based on other beliefs) or derivative (formed by other beliefs or experiences) and central (foundational and hard to change) or peripheral (developed and easier to change or remove).

Experiences as a Learner

A major developer of Sally's and Grace's mathematical beliefs was their experiences as a learner, more importantly their experiences as a K-12 learner (Barkatsas & Malone, 2005; Harbin & Newton, 2013; Maasepp & Bobis, 2015). For Sally, these experiences formed her central nature of mathematics instrumentalist belief that math is a set of truths which need to be modeled for students. She stated,

> I think honestly, I was taught explicitly. The teacher taught and that's how I learned it… It was always just workbook, paper, pencil, get on it. So, that is really where I feel like I'm geared to go… That's, I guess, where my mind goes to most of the time.

The way her teachers taught her combined with her primary belief that she was "good at second-grade math" to form a central derivative belief that doing mathematics meant solving problems to find answers.

For Grace, these K-12 experiences developed her central transmission orientation towards learning mathematics that students' misunderstandings were the result of not being able to understand processes. She explained, "I give them the processes to use to solve problems. I think that's what helped me so much was a process." She added,

> I feel like it helps them to have a process. It helps them to create number sentences that go with the word problems which is really difficult for them. If they don't want to use those processes later on, that's okay. I just want the ones that struggle to use the process I showed them right now, because they can't just work on their own independently yet with those types of problems... So, I let them problem solve but then go back and teach it by the book and give them the proper processes.

This belief aligned well with her instrumentalist nature of mathematics belief that mathematics is a set of processes that needs to be properly conveyed to students. These beliefs, along with her beliefs that were developed by her experiences with her students and curriculum (discussed later), led to the transmissive derivative belief that she needed to verbally and visually show students how to use multiple strategies so they can understand the processes of mathematics.

Professional development was another experience the teachers had as learners of mathematics. Grace felt the PD sessions had "some degree, but not really" a lot of influence on the formation or changes to her mathematical beliefs or orientations;

however, Sally attributed changes in her mathematical beliefs to this experience as a learner. When asked if she felt her beliefs have changed over the years, she responded they had. She explained the PD sessions were causing her to think about what she believed and why which, in effect, was moving her to a more Platonist (math is a body of knowledge for students to explore) view about the nature of mathematics.

This finding that variability exists between the teachers (no effect on Grace but spurred changes in Sally) and their experiences in PD aligned to other research. For example, Harbin and Newton (2013) found that PD sessions only appeared to influence the beliefs of one out of the six in-service teachers in their study. Spillane et al.'s (2018) research supported the notion that hours of professional development was not a significant predictor of the formation or potential changes to teachers' belief structures which was seen in Grace. However, Sally had been attending the mathematics session longer than Grace, and PD sessions do not have an immediate impact on teachers' belief (Polly & Hannafin, 2011). Perhaps the longer exposure and continuous reflections about her beliefs promoted initial or stronger changes in Sally when compared to Grace (Cross, 2009). This would be one area to consider when developing PD sessions for these teachers in the future.

The research proposition that *second-grade in-service teachers' experiences as a student have a stronger impact on how their beliefs structures are formed than experiences in professional development sessions* was supported by these findings (Beswick, 2012; Cross, 2009; Harbin & Newton, 2013; Maasepp & Bobis, 2015). As stated, Grace felt the PD sessions had little impact on the formation and changes in her mathematical beliefs, but her experience as a K-12 learner highly influenced her orientation towards how children should learn mathematics. She knew what she needed and tried to give that to her students. For Sally, her nature of mathematics beliefs were formed by her experiences as a K-12 learner. Her new experiences as a learner in PD sessions did have some impact on her nature of mathematics beliefs (instrumentalist to Platonist), but that impact only gave trace evidence of a change in beliefs. Her instrumentalist beliefs were still very strong and very central. Her Platonist beliefs were moving towards a central belief, but that process was slow. She admittedly often abandoned that constructivist belief about what it meant to do math as evidenced by her proclamation that she knew her students could discover how to solve the problems without her, but tended to fall "back into old habits" and "show them how to do it" even though "I know I shouldn't."

Experiences as a Teacher

Another factor that influenced how the teachers' mathematical belief structures were formed and changed was their experiences as teachers which included experiences with their students, colleagues, and the curriculum. An interesting finding in this study was that Sally appeared to align to Barkatsas and Malone's (2005) case study of Ann where Grace only somewhat aligned. For example, Barkatsas and Malone found that Ann's prior school experiences were the main influence on her nature of mathematics beliefs, and her experiences as a teacher were the main influences on her orientation towards teaching and learning mathematics. These same findings were true for Sally. However, Grace's nature of mathematics beliefs and orientation towards how mathematics should be learned were influenced by her experiences as a teacher with a small contribution of her belief that she was not a strong math student (see Figure 5.1);

Grace's orientation toward how mathematics should be learned was influenced by her experiences as a student along with her experiences as a teacher.

Experiences with Students.

One area where Sally and Grace aligned with each other and with research about the formation or development of their beliefs was in their experience with students. For Sally, interactions with her students early in her career showed her that her students needed manipulatives (a discovery orientation) in order to learn how to do mathematics; this belief became stronger throughout the years especially since she had learned new ways to use the manipulatives.

More recently, Sally noticed that when she "gave" her students opportunities to demonstrate their thinking to their classmates, she was often surprised by their ability to 1) discover strategies that neither she nor the curriculum taught them and 2) effectively explain how they solved the problems or discovered the strategy. On top of that, the other students were learning from these explanations and interactions with their classmates (connectionist orientation). These two beliefs filtered through her earlier belief that students need to see and hear mathematics which led to the derivative belief that students learn mathematics when they hear and see mathematics through manipulatives and peer interactions (see Figure 4.5). Unfortunately, the connectionist orientation was still in the peripheral psychological belief structure and was easily abandoned. Sally was aware of this and, when asked how second-graders learned math, stated the students needed "the chance to discuss with one another which" was something she felt she needed to do more of.

Like Sally, Grace also felt her students needed to see and hear the mathematics and valued the use of manipulatives in her classroom. During the first interview, Grace explained that her students “enjoyed” using the place value disks and making the straw bundles when they learned about regrouping and trading for two-digit subtraction. She stated, “they love that because, I think, they are visual and they can really see it. They can see the regrouping for subtraction.” Grace also recognized that when she was a learner in an elementary classroom, she needed to “see the mathematics;” that would have helped her “tremendously” because she was “very visual.” During another interview she added, “I think I would have gone a lot farther in math if I would have had some sort of visual instead of just the number and the problem in front of me.” For these reasons, Grace made sure she told (for her auditory learners) and showed (for her visual learners) how to do and follow the mathematical processes.

However, unlike Sally, Grace's experiences with her students kept her from providing opportunities for her students to communicate and interact with each other (connectionist). Grace had spent her first five years in an early childhood classroom. Her experience with this level of students developed a primary belief that younger children need help and guidance, and older students were more independent. However, according to Grace, the group of second-graders this year were very immature and needed a lot of direction and instruction; she needed “them right in front of” her. Additionally, when she did provide opportunities for her students to share their thinking, it was rarely productive. She explained,

> This year they will have the right answer on the paper, but then when they go up [to the document camera] to do it, they get nervous or they don't explain it very

> easily. Then the rest of the class is lost. We're working on it, though. That's something that is a goal of mine is to get them more involved in explaining things.

On the other hand, she did occasionally have success when her students interacted with their peers, but this was more the exception than the rule. She stated,

> I like that approach. I think that it is great. I think that sometimes, from another student, they can get something that they may not get from me. And sometimes they listen to their friends better than they listen to their teacher. I feel like it is a really useful tool to have, because I have some in here who are such high learners, really, really high math achievers that they can explain something to somebody that might not understand and then that student has a light bulb moment. Any light bulb moment I can get, I will take.

Grace believed her students could interact with each other only if the high achievers were talking because they were the most capable and mature (Torff, 2005).

Cross (2009) also found that experiences with students have the potential to form and change teachers' orientations towards teaching and learning mathematics. Two of the five participants (Ms. Jones and Mr. Simpson) in Cross's study rationalized their beliefs by providing examples of past experiences with their students. However, unlike Sally and Grace, Ms. Jones and Mr. Simpson both had strong constructivist views about teaching and learning mathematics and would often engage their students in discussions and interactions with each other. Additionally, Ms. Jones had been teaching for 30 years, and Mr. Simpson had been teaching for 18 years. Perhaps, as Sally and Grace gain more experience with their second-graders and provide more opportunities for them to interact with each other (something they both claimed they wanted in their classroom), they, like

Ms. Jones and Mr. Simpson, will develop a deeper understanding of how to engage students and do it more often. During the PD sessions, I could possibly increase and vary the ways I show them to engage their learners in mathematical discussion by scaffolding mathematical dialogue more often (Munter, 2014). I could also add more opportunities for the teachers to explore their own students' mathematical reasoning so they could gain a deeper awareness of how their students approach mathematics (Ambrose, 2004; Lannin & Chval, 2013). These two additions could create more, as Sally exclaimed, "Wow, I should be doing it this way" moments in their classrooms and cause changes to their orientations towards teaching and learning mathematics (Philipp, 2007).

Experiences with Curriculum.

Curriculum was another experience in teaching that developed and influenced Sally's and Grace's orientations toward teaching and learning mathematics. As a beginning teacher, Sally looked to the curriculum to show her how to teach her students. She had a primary belief that the curriculum knew how to teach mathematics, so she "just followed" it. Additionally, the curriculum they used during her first six years teaching children in the second-grade aligned to how she was taught as a student, traditionally, so she never questioned; "It had never occurred" to her that "there might be a different way to teach." This transmission orientation towards how mathematics should be taught became stronger as she gained more experience as a teacher and continued to use what appeared to be a transmissive curriculum. These experiences caused Sally to develop a transmission orientation toward teaching mathematics that had a central psychological belief position which made it more difficult to change.

Later, her school adopted the Eureka mathematics curriculum. This new curriculum appeared to be more constructivist in design compared to the previous curriculum Sally's school used and tended to focus on learning and using various strategies instead of just finding answers to problem sets, especially the application problems (Walker, 2019). This new curriculum paired nicely with the lessons Sally was learning in PD sessions, and her views about how mathematics should be taught and learned started changing. According to Sally, she was learning how to relinquish control and give her students a voice in the classroom. This discovery orientation towards teaching mathematics was slowly making its way from a far outer peripheral psychological circle to a more centralized position; however, her strong instrumentalist nature of mathematics was still keeping the discovery orientation in the peripheral circles. It will be interesting to see if more exposure to this curriculum and how it impacts her students' learning will eventually move her discovery orientation into the central circle.

Grace also held a primary belief that the curriculum knew how to teach mathematics. Grace's previous school did not have any school prescribed curriculum and held the unproductive belief that teachers could and should develop their own mathematical lessons that aligned closely with the standards (NCTM, 2020). Grace did not like this approach as she felt she needed some type of "guideline" when planning her lessons. However, at first Grace did not like the new Eureka curriculum the school was using, claiming that it was "difficult to teach and explain math in a different way" from how she had been taught. As she became more familiar with the material, she started to understand and appreciate some of the elements built into Eureka such as the use of manipulatives, showing students how to "see" the mathematics they were learning, and giving them multiple strategies and processes to “find the answers”; according to Grace, even she was learning new ways to approach mathematics and was becoming a stronger teacher and learner of mathematics because of this new curriculum.

Unfortunately, like Sally, Grace filtered much of the curriculum’s instruction through her instrumentalist nature of mathematics views. So instead of fully adopting the curriculum’s more constructivist approach to teaching and learning mathematics, Grace read the word problems to her students “and sort of [gave] them the steps” to solve them, otherwise her students would “get frustrated easily” and ask for her help. Next, she let her students independently work the problem to see what types of strategies they preferred and then explained “how to solve it according to the book.” Even though Grace held a strong primary belief that the curriculum was the authority in her classroom and knew how to teach mathematics better than her, she reverted to her transmissive style of teaching and learning because that belief structure was stronger and more centrally located. I would need to provide opportunities for Grace to juxtapose these two opposing beliefs and cause her to have some cognitive dissonance about what it means to do mathematics in order to potentially move her transmissive teaching and learning approach to a more constructivist one (Moscardini, 2014).

The findings from this study supported much of the research about the use of curriculum in the class and how it can influence teachers’ orientations towards teaching and learning mathematics. Sally and Grace admitted to using and following their curriculum most of the time and that they would often show students how to solve the problems “according to the book” (Copur-Gencturk, 2015). Sally admitted that it was the curriculum that encouraged and led her to provide opportunities for her students to discuss the strategies they used with each other which was moving her to a more constructivist approach to teaching (Schoenfeld, 2004). Grace explained that at first, she did not like the new curriculum and struggled following it (Spillane et al., 2018); however, as she gained more experience and a better understanding of how to implement the lessons, she began to see its merit and followed it with near fidelity; her decisions to modify certain lesson instructions will be discussed in the next section. Both of these teachers eventually viewed the curriculum as a valuable guide in their classroom which in turn was causing their beliefs to be challenged and changed (Bay-Williams & Karp, 2010).

I need to continue to highlight the transmissive and constructivist views of teaching in the curriculum so the teachers can learn to evaluate better what they are using and be able to justify why they should or should not follow the textbook's instructions (Cross, 2009; NCTM, 2014). This could lead them to follow the curriculum with less fidelity and more integrity of implementation or the ability to remain "true to the core ideas of the curriculum while being responsive to local conditions and context" (NCTM, 2020, p. 38). This could also have the potential to cause the teachers to modify their belief that the curriculum holds all the authority in their classroom and, instead, transfer that power to themselves and the needs of their students to ensure they are providing effective mathematics instruction (Griffith et al., 2013; NCTM, 2020).

Summary of Beliefs Journey

As seen in Table 6.2, Sally's and Grace's experiences as a K-12 learner developed many of their transmission and instrumentalist views about what it means to do mathematics and how it should be taught and learned. This was not a surprise as many researchers have also found this connection to teachers' early experiences as learners and their current beliefs (Barkatsas & Malone, 2005; Cross, 2009; Maasepp & Bobis, 2015; Raymond, 1997). Sally was the only one who appeared to have any changes in her beliefs because of her experiences as a learner in PD sessions which contradicts Polly et al. 's (2014) finding which will be discussed later. This could be because Sally had more years of experience and attended more PD sessions when compared to Grace, and research has shown that changes in beliefs, especially through PD sessions, take time (Fennema et al., 1996; Polly et al., 2014).

Many of the changes in the teachers' beliefs occurred because of experiences with their students and the curriculum. Again, these findings were not a surprise. Experience with students and gaining a deeper understanding of how they think and reason mathematically was found to be a catalyst for potential change in teachers (Ambrose, 2004; Lannin & Chval, 2013); however, Sally and Grace are going to have to confront unproductive and fixed mindsets towards the abilities of their student and realize that all students can engage in exploration of mathematical concepts and can effectively share those understandings with their peers if they are going to continue to develop their constructivist styles of teaching and learning and more productive beliefs towards teaching and learning (NCTM, 2014, 2020).

Changes in Sally's and Grace's beliefs because of their experiences with the curriculum also has the ability to continue to move these teachers towards a constructivist style of teaching. Sally and Grace both self-identified as a level two CGI teacher; according to Hill et al. (2008), curriculum with a more constructivist orientation has the potential to promote productive teaching and learning practices and beliefs. However, in order to do this, these teachers are going to have to be able to become more comfortable with the curriculum (Schoenfeld, 2004) and learn how to identify the constructivist elements of Eureka math so they can know what to use and what to modify or disregard - a major step toward using the curriculum with integrity rather than fidelity (NCTM, 2014, 2020).

Table 6.2

Summary of Sally's and Grace's Beliefs Journey

Belief	Teacher	Transmission / Instrumentalist	Discovery/ Platonist	Connectionist/ Problem Solving
Nature of Mathematics	Sally	Experience as Learner (K-12)	Experience as Learner (PD)	
	Grace	Experience as Teacher Experience as Learner		
Teaching Mathematics Orientation	Sally	Experience with Curriculum (before Eureka)	Experience with Curriculum (Eureka)	
	Grace	Experience with Curriculum (before Eureka)	Experience with Curriculum (Eureka)	
Learning Mathematics Orientation	Sally	Experience with Students (early career)	Experience with Students (early career)	Experience with Students (later career)
	Grace	Experience as Learner (K-12)	Experience as Teacher (curriculum & students)	

These Second-grade Teachers' Mathematical Beliefs Influence on Their Instruction?

Some research suggested that teachers' espoused and enacted beliefs do not always align with each other meaning a teacher may believe one thing about how a student learns mathematics but then not follow that belief when planning instruction or teaching that student (Cross-Francis, 2015; Kennedy, 2005; Speer, 2008); on the other hand, other research has concluded that teachers' enacted and espoused beliefs were closely aligned (Bray, 2011; Cross, 2009; Wilkins, 2008). This study supported both of these findings. In general, Sally and Grace appeared to follow their espoused beliefs during their mathematics instruction. When there seemed to be a contradiction between the two, the data supported the notion that it was because the pressures of teaching caused them to "revert" back to their experiences as a learner (Harbin & Newton, 2013). Other times it appeared to be because they filtered their actions through their strong central instrumentalist belief about the nature of mathematics (Beswick, 2012) or their beliefs about their students' abilities (Copur-Gencturk, 2015).

Alignment Between Espoused and Enacted Beliefs

Sally and Grace held the instrumentalist belief that mathematics is a set of truths which *need* to be modeled for children (emphasis added) (Ernest, 1989). This nature of mathematics belief was seen throughout each of Sally's and Grace's teaching lessons. For example, on the second observation, Grace read the problem to the students, showed them how to draw the crackers, and, after asking a student to give a solution to the problem, explained how the called-on student solved the problem instead of letting the student explain his thinking. During one of the lessons, Sally showed her students how to use the place value disks instead of giving them the opportunity to explore, adding that she wanted the student to see the problem her way. According to Sally, that was how she was taught, that was how she learned, and that was where she felt she was "geared to go" (Beswick, 2012; Cross, 2009; Haciomeroglu, 2013; Harbin & Newton, 2013; Maasepp & Bobis, 2015).

Moreover, both teachers self-identified as a Level Two CGI teacher, and both justified that identification because they knew they felt a strong need to show the students how to solve or do the problems. According to the teachers, neglecting to show the students what to do would mean they, Sally and Grace, would have failed their students; those students would get confused and frustrated and not know what to do (Carpenter et al., 2015; Desimone et al., 2005; NCTM, 2014); this could cause the students to go into third grade not knowing anything, and they would bomb the state test next year, a common fear among teachers even in low-stakes testing grades (Abrams, et al., 2003; Gonzalez et al., 2017; Thibodeaux, 2014). The teachers may have felt they were promoting positive and effective learning experiences for their students (Vacc & Bright, 1999); however, they were really utilizing unproductive beliefs that may have had little impact on their students' mathematical thinking or understanding (Lee, 2017; NCTM, 2014).

On the other hand, the teachers did have an espoused and enacted productive teaching and learning belief that students need to see and hear multiple strategies to solve problems; they also both believed that manipulatives and illustrations were a great way to achieve this task (NCTM, 2014). For Sally, this appeared in both her orientation towards teaching mathematics and her orientation towards learning mathematics; for Grace, this was only seen in her orientation towards learning mathematics. During the observations, I witnessed Sally and Grace showing the students how to use place value disks and model their thinking through illustrations. I saw students pull out 100s charts and solve two-digit addition problems with them, use pattern blocks to make different patterns during Grace's third observation, and model their problem-solving strategy on the document camera or Smartboard. Both teachers talked about using bundling sticks and how effective it was for their students to see why you regroup or trade when adding and subtracting multi-digit numbers.

These teachers believed their students were gaining a deeper understanding of what it meant to add and subtract and make patterns because they, both the teachers and the students, were seeing and hearing and using manipulatives to make those connections. This was a catalyst for them to go from how they were taught to a better way to teach even if it meant they were still dependent on their instrumentalist belief that they needed to show the students how to use the different strategies (Lui & Bonner, 2016). According to Grace, "this is how I think now," and these new beliefs were providing new ways for the teachers to implement more productive beliefs in their teaching (NCTM, 2014), especially since they were starting to see how beneficial it was for their students (Beswick, 2012; Gabriele & Joram, 2007).

Contradiction Between Espoused and Enacted Beliefs

Even though Sally and Grace had areas where it appeared that their espoused and enacted beliefs aligned, they also had some areas where the two seemed to be misaligned or contradict each other. For Sally, this was apparent in her Platonist nature of mathematics belief and her orientation towards teaching mathematics. According to

Sally, she wanted her students to explore and discover mathematics for themselves. When her students did this, they were able to examine their thinking, and she was able to see possible flaws in her students' thinking; however, this was not how Sally conducted her lessons. She started each application journal lesson by asking the students how they were going to solve the problem and kept giving them clues until they arrived at the correct operation or process; she showed students how to use the manipulatives and had never even considered they could figure out how to use the manipulatives without her guidance. Finally, she stopped a student from utilizing the vertical method for subtracting, because they had not learned how to use that algorithm yet. She wanted to let her students explore, but her lessons did not support this belief.

For Grace, this contradiction appeared in her belief that she needed to teach her students according to the book. During the lesson about how many crackers were in the box, the curriculum told her to ask students to discuss strategies they would use with a partner then share their solutions with the class. The textbook gave dialogue Grace could follow to facilitate this mathematical discussion and provided possible illustrations students could use. Grace appeared to ignore the suggestions given by the book and used the illustration as a way to show the students how to solve the problem the way the "book wants us to" and was seemingly unaware that that was not the instructions given in the curriculum.

While it appeared that Sally and Grace were going against their espoused beliefs, they were really just following other and stronger beliefs (Copur-Gencturk, 2015; Green, 1971; Linder & Simpson, 2017). According to Green (1971), teachers can hold many different belief clusters. When these belief clusters are being accessed to make instructional decisions, the stronger of the two ultimately takes precedence. Oftentimes, the teachers are not even aware of the conflict. For Sally, her instrumentalist belief was stronger than her newly learned beliefs that students should explore and discover mathematics for themselves. Her transmissive beliefs were rooted in her experiences as a learner, which were shown to be very strong in teachers (Cross, 2009; Maasepp & Bobis, 2015); her Platonist belief and discovery orientation stemmed from her experiences as a learner in PD sessions and through the curriculum and, therefore, were not as strong. Grace's instrumentalist belief, which also developed because of her experiences as a learner, was stronger than her curriculum belief which caused her to follow the book if it aligned to her transmissive style of teaching and learning or modify the lesson until it did (Brown, 2005). Sally still believed her students should discover mathematics for themselves; Grace still believed she needed to follow the book. However, those beliefs were easily abandoned if they kept the teachers from showing their students how to find the answers.

Sally and Grace also believed that their students needed to learn mathematics by seeing and hearing how their peers solved problems; however, during the observations, I rarely witnessed students explaining their processes to their classmates. Sally did select one or two students to show their solutions to the application journal in two of the observations, but no one shared their thinking for the "actual" lesson or when working sample problems before doing the problem set. Grace only had her students demonstrate their thinking twice during all three lesson observations. The first time, two students came to the board to show how they solved the problem but could not verbalize what they did which, according to Grace, caused her to get frustrated with the lesson; the second time, when most of the class was gone for homecoming, all of Grace's student who were present that day shared a pattern on the Smartboard. During the interviews, when we discussed the students sharing their strategies, both teachers acknowledged they knew it was something that was beneficial for the students and something they needed "to do more of," but it was also something that was easily forgotten or sometimes problematic.

A possible cause for this misalignment between wanting their students to interact with their peers and not giving the students opportunities to engage in mathematical discussion could be because of Sally's and Grace's beliefs about their students' abilities (Beswick, 2005). Grace had stated three times in the interviews that her students, especially her students this semester, were very immature. Sally explained that she was often surprised when her students would demonstrate a different way to solve a problem than what she or the book showed them. Both teachers believed their students needed to see and hear how to use strategies and manipulatives from peers, but that belief was not stronger than their belief that their students were immature and probably incapable of accurately explaining their thinking unless they were the "really, really high achievers." I would need to provide opportunities for these teachers to juxtapose these two opposing views about their students in order for them to strengthen the more productive belief that their students can and should be actively involved in justifying their solutions and "considering the reasoning of others" (NCTM, 2014, p. 11).

Summary of Influence of Espoused Beliefs on Instruction

The findings from Sally's and Grace's espoused and enacted beliefs support the first and third proposition of this study. First, the teachers' nature of mathematics belief and orientations towards teaching and learning mathematics did indeed influence their instruction (Beswick, 2012; Cross, 2015; Shirrell et al., 2018). They both believed math was an accumulation of facts that needed to be conveyed to students, and that students needed to see and hear mathematics through their teaching and the use of manipulatives. So they told them (auditory) and demonstrated (visual) how to solve the problems and use the manipulatives to see the solutions. They both believed they were Level Two CGI teachers, so they made sure they let their students "explore" or use "whatever strategy they" wanted to solve the application journal problem initially, but then went back and "showed them how to do it according to the book."

Second, when their espoused and enacted beliefs appeared to be in opposition to each other, it was usually because the teachers were following a stronger belief structure rather than contradicting the weaker belief. This finding supported the first proposition of this study that the *in-service teachers' mathematical views about the nature of mathematics and orientations toward teaching and learning mathematics are independent beliefs and can, therefore, seemingly contradict each other* (Ernest, 1989; Green, 1971; Voss et al., 2013). Furthermore, it appeared that Sally and Grace did not have a full understanding of how to provide opportunities for their students to interact with each other or realize they were filtering their instruction through unproductive beliefs about their students which could be another possible explanation for the perceived contradiction between what they believed and how they taught (Beswick, 2005). Their interview discussions revealed that they truly believed they were being more constructivist in their teaching and learning practices, but their mathematics instruction supported the notion that they were actually following their stronger transmissive beliefs (Cross-Francis, 2015; Kennedy, 2005; Speer, 2008).

This is why it is important for teachers and researchers to consider how all the beliefs are interacting with each other before concluding that teachers' beliefs are not actually influencing their mathematics instruction (Copur-Gencturk, 2015). The teachers and I must be aware of and look at the teachers' entire instructional practice flowchart in order to determine how the belief clusters are interacting with each other before we can make plans to provide effective and purposeful PD sessions that can meet their individual and group needs (Chen, 2014; Cross, 2009).

Implications and Future Research

The purpose of this research was to explore the influence of elementary teachers' beliefs on their mathematics instruction. Since, as a teacher educator, I am often asked to conduct PD sessions for elementary teachers who teach children mathematics, I wanted to use the information I learned from this experience to improve those interactions. I had recently felt my sessions with the teachers at the location of this research were not really meeting the needs of the teachers. I knew that if I wanted to exact change at this school and with these educators, I needed a deeper understanding of their beliefs and how they played out in the classroom.

To accomplish this task, I needed to first discover what beliefs these teachers had about mathematics and how it should be taught and learned. Next, I needed to know how they held these beliefs and which appeared to be stronger belief structures. Once I understood what their beliefs were, I needed to know how those beliefs were formed and how, if applicable, they had changed over time. Even though this study was more qualitative than quantitative and, therefore, is not generalizable, the findings supported much of the research about the role of beliefs on mathematics instruction and can be used by me and other teacher educators or professional development providers to develop experiences that have the potential to change unproductive beliefs teachers may have into productive beliefs that are better aligned to a more productive style of teaching and learning.

The findings from this study has supported claims by other researchers that changing beliefs is a difficult process that requires time and sustained support (Brown, 2005; Cross, 2009; Newborn & Cross, 2007; NCTM, 2014; Polly & Hannafin, 2011). This is why it is important for schools to recognize that a once-a-year PD session on elementary mathematics is not going to produce effective and lasting results for their elementary teachers. If these schools, administrators, and teachers are serious about creating "change so that each and every child can have access to learning environments that are designed as mathematically powerful spaces," then they are going to have to invest more time, energy, and resources into purposeful and meaningful PD sessions (NCTM, 2020, p. 1).

When teachers attend these sessions, they need the opportunity, just like their students, to explore their thinking. This exploration should, as Sally stated, make the teachers "really think about" their beliefs and mathematics instruction and start them on a path of critical reflection and self-appraisal that have the potential to bring about positive changes (Barkatsas & Malone, 2005; Beswick, 2012). To accomplish this, the PD providers need to expose teachers to new ideas in order to cause cognitive dissonance about what it means to do, teach, or learn mathematics (Mewborn & Cross, 2007; Moscardini, 2014). Once the teachers have explored and wrestled with the differences between what they were learning and what they have experienced, they can then start implementing these new ideas in their classrooms in order to gain firsthand experience on how the revisions in their beliefs impact their students which, as shown in this study and other research, has the potential to cause meaningful change (Ambrose, 2004; Carpenter et al., 1989; Lannin & Chval, 2013; Philipp, 2007). These examinations of beliefs, exposures to new ideas, and explorations of new practices in the classroom cannot be accomplished with a one-and-done PD session; they require a cyclical and continuous engagement in these activities for positive changes to occur in the teachers (Brown, 2005; Cross, 2009).

Unfortunately, even with the best intentions, the PD sessions can still fall short of meeting the needs of the individual teachers (Shirrell et al., 2017). For example, Sally had spent several PD sessions with me in the past, but she still had specific needs I had not addressed such as feeling pressure about state testing and how that kept her from fully engaging her students in exploration. Additionally, even if she did attempt to provide opportunities for her students to discover mathematics for themselves, she had an underdeveloped understanding of what it meant for students to explore their thinking. Even though we had talked about it - that is where she learned the concept - she did not know how to accomplish it. This became apparent during the study when she explained the reason she showed her students how to use the manipulatives or used the phrase “let me show you my way” during the actual lesson was because “it had never occurred to her” that the students could explore with the manipulatives or that she was using those types of phrases. She was learning new ideas in the PD sessions, but not how to implement those ideas in the classroom.

The same thing happened for Grace. Even though she had not attended as many PD sessions as Sally, she did attend the mandated school sessions and a few of the after-school sessions we had scheduled throughout the year both before and during the time of the study. Therefore, she was aware of and exposed to the new ideas about teaching and learning mathematics such as giving students time to discuss their mathematical thinking and strategies with each other. She was trying to do this, as evidenced by her having the two students show how they solved their application journal problem on the board or having her students share patterns during the third observation. However, her classroom management skills were lacking because of her inexperience in a second-grade classroom and her preconceived notions about what a second-grader should be able to do. This led Grace not knowing how to assist her students when they tried to explain their thinking on the board or knowing how to ask questions that could lead to mathematical discussion rather than answers. Sally and Grace had different needs which were not being met in the one-size-fits-all PD sessions.

This is why some researchers have determined that instructional coaches might be a better alternative or needed addition to PD sessions (Polly & Hannafin, 2011; Walker, 2019). NCTM (2014) claimed that experienced and inexperienced teachers would greatly benefit from mathematics instructional coaches stating that they could provide the specialized and classroom attention needed to cause effective and lasting changes. These coaches could not only help teachers confront unproductive beliefs but also show them how to implement productive beliefs into their classrooms with their own students. The coaches could observe the teachers teaching and point out misconceptions or ask why they interacted with their students or the curriculum in a particular way (Spillane et al., 2018). They could also model effective mathematics instruction so the teachers could see how their students reasoned mathematically or that all their students were capable of exploring the mathematics - not just the “really, really high achievers.” Both Sally and Grace were aware of the potential benefit of this coaching approach and stated they wanted to see me teach a lesson in their classroom so they could have “different input and strategies,” notice how their students were “doing things” they “didn’t even realize” they were doing, and could “think about all this and see” what they need to be doing to help their students learn the mathematics they were teaching.

This inclusion of mathematics coaches in the school setting is an area of research that could yield interesting information about best ways to help teachers change unproductive and transmissive beliefs about teaching and learning mathematics. This could include exploring effective ways for the teachers to collaborate with the coaches. It could also involve seeing how the lessons I learned through this experience can provide a framework for coaches and me to use to prepare teachers for the growing pains of evaluating and, if needed, developing or changing the strength of productive belief structures. Another area for future research could be exploring the coaches’ beliefs and how they impact instruction they give their teachers. However, for this research to take place, several schools would need to allocate resources for hiring these coaches, time for teachers and coaches to collaborate together, and provide opportunities for the coaches and teachers to engage in effective and meaningful PD sessions (NCTM, 2014). If these changes were to take place and the many areas of research were able to be conducted, the schools, teachers, coaches, and I could potentially be a powerful catalyst for change in the elementary mathematics classroom (NCTM, 2020).

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APPENDICES

APPENDIX A

Letter of Consent

College of Education, Health, and Aviation

CONSENT FORM

The Influence of Beliefs and Contextual Factors on Mathematical Instruction

Background Information

You are invited to be in a research study exploring how mathematical beliefs held by in-service elementary teachers in a local elementary school and contextual factors influence mathematical instruction. You were selected as a possible participant because you teach 2nd grade mathematics at your elementary school. We ask that you read this form and ask any questions you may have before agreeing to be in the study. Your participation is entirely voluntary.

This study is being conducted by Ms. Tonya Garrett, College of Education, Health, and Aviation at Oklahoma State University, under the direction of Dr. Adrienne Sanogo, College of Education, Health, and Aviation at Oklahoma State University.

Procedures

If you agree to be in this study, we would ask you to do the following things:

1. Complete and sign this consent form.
2. If you give your consent to participate, you will be asked to send an "Awareness of Video Recording" letter home with every student in your classroom and have the letters signed by your students' legal guardian and returned to you.
3. After you have collected all the "Awareness of Video Recording" letters and submitted them to the researcher, will be given and asked to complete the Revised Mathematical Belief Scale (RMBS).
4. Once you have submitted your RMBS, you will be asked to schedule three times in which to be observed by the researcher. These observations should take place during your mathematical instruction time and will be videoed. I will use the Mathematics Classroom Observation Protocol for Practices (MCOP^2) to score your mathematical instruction.
5. Next, you will be asked to schedule three interview times. The interviews will be about an hour in length.
6. Throughout the process, you will be asked to read through the researchers notes and verify if the information is accurate.

Participation in the study involves the following time commitment:

1. Mathematical Beliefs Survey: approximately 20 minutes
2. Three observations: length of mathematics lesson times 3
3. Three interviews: one hour each for a total of 3 hours
4. Member checking: one to two hours

Risks and Benefits of being in the Study

There are no known risks associated with this project, which are greater than those ordinarily encountered in daily life.

The benefits which may reasonably be expected to result from this study include a deeper understanding of your mathematical beliefs for teaching and how those beliefs and contextual factors influence your mathematical instruction. We cannot guarantee or promise that you will receive any benefits from this study. More broadly, this study may help the researchers learn more about mathematical beliefs held by in-service elementary teachers in your area. This information can assist researchers in designing and implementing specific and targeted professional development models in order to increase mathematical instruction which could ultimately improve student learning.

Compensation

You will receive no payment for participating in this study.

Confidentiality

The information that you give in the study will be handled confidentially. You will create a pseudonym and this pseudonym will be used throughout the study. The list connecting your name to this code and your pseudonym will be kept in a locked file. When the study is completed and the data have been analyzed, this list will be destroyed.

Voluntary Nature of the Study

Your participation in this research is voluntary. There is no penalty for refusal to participate, and you are free to withdraw your consent and participation in this project at any time. You can skip any questions that make you uncomfortable and remove yourself from the study at any time.

Contacts and Questions

The Institutional Review Board (IRB) for the protection of human research participants at Oklahoma State University has reviewed and approved this study. If you have questions about the research study itself, please contact the Principal Investigator at 918-444-3719, tonya.garrett@okstate.edu. If you have questions about your rights as a research volunteer or would simply like to speak with someone other than the research team about concerns regarding this study, please contact the IRB at (405) 744-3377 or irb@okstate.edu. All reports or correspondence will be kept confidential.

You will be given a copy of this information to keep for your records.

Statement of Consent

I have read the above information. I have had the opportunity to ask questions and have my questions answered. I consent to participate in the study.

Signature:____ Date: _________

Signature of Investigator:____________________________________ Date: _________

APPENDIX B

Revised Mathematical Beliefs Scale

Please read and indicate your level of agreement with each of the following statements. Circle the number that corresponds to your level of agreement. 1= Strongly Disagree, 2=Disagree, 3=Neither Agree or Disagree, 4=Agree, 5= Strongly Agree. In all statements, interpret "student" as a generic term referencing an "average" student at the grade level you teach.	Strongly Disagree 1	Disagree 2	Neither Agree or Disagree 3	Agree 4	Strongly Agree 5
1. Children learn math best by attending to the teacher's explanations.	1	2	3	4	5
2. Mathematics should be presented to children in such a way that they can discover relationships for themselves.	1	2	3	4	5
3. Children should be expected to understand how computational procedures work before they master those computational procedures.	1	2	3	4	5
4. The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.	1	2	3	4	5
5. Children should understand computational procedures before they master them.	1	2	3	4	5
6. Teachers should encourage children to find their own solutions to math problems even if they are inefficient.	1	2	3	4	5
7. Children need explicit instructions on how to solve word problems.	1	2	3	4	5
8. Most young children can figure out a way to solve many mathematical problems without any adult help.	1	2	3	4	5
9. Children should not solve simple word problems until they have mastered some number facts.	1	2	3	4	5
10. Recall of number facts should precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).	1	2	3	4	5

Please read and indicate your level of agreement with each of the following statements. Circle the number that corresponds to your level of agreement. 1= Strongly Disagree, 2=Disagree, 3=Agree, 4= Strongly Agree. In all statements, interpret "student" as a generic term referencing an "average" student at the grade level you teach.	Strongly Disagree 1	Disagree 2	Neither Agree or Disagree 3	Agree 4	Strongly Agree 5
11. Time should be spent solving problems before children spend much time practicing computational procedures.	1	2	3	4	5
12. Teachers should allow children who are having difficulty solving a word problem to continue to find a solution.	1	2	3	4	5
13. Children will not understand an operation (addition, subtraction, multiplication, or division) until they have mastered some of the relevant number facts.	1	2	3	4	5
14. Teachers should teach exact procedures for solving word problems.	1	2	3	4	5
15. Most young children have to be shown how to solve simple word problems.	1	2	3	4	5
16. Time should be spent practicing computational procedures before children are expected to understand the procedures.	1	2	3	4	5
17. Teachers should allow children to figure out their own ways to solve simple word problems.	1	2	3	4	5
18. To be successful in mathematics, a child must be a good listener.	1	2	3	4	5

Use the space below to clarify or elaborate on any of your responses to the statements.

Demographics.

1. What is highest degree you have earned?
   - Bachelors
   - Masters
   - Doctorate

2. List your major for each degree earned.

3. List your current certifications.

4. How many years taught (including the current year)?
   - 1 – 3
   - 4 – 6
   - 7 – 9
   - 10 – 12
   - 13 – 15
   - 16 – 18
   - 19 – 20
   - 21+

5. How many years taught in your current grade (including this year)?
   - 1 – 3
   - 4 – 6
   - 7 – 9
   - 10 – 12
   - 13 – 15
   - 16 – 18
   - 19 – 20
   - 21+

6. In what grade(s) do you currently teach? Check all that apply.
   - □ Kindergarten
   - □ First grade
   - □ Second grade
   - □ Third grade
   - □ Fourth grade
   - □ Fifth grade
   - □ Sixth grade
   - □ Seventh grade
   - □ Eighth grade

APPENDIX C

Mathematics Classroom Observation Protocol for Practices ($MCOP^2$)

1) Students engaged in exploration/investigation/problem solving.

SE	TF	Description	Comments
3	3	Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.	
2	2	Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.	
1	1	Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving while other students watched but did not actively participate.	
0	0	Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.	

2) Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.

SE	TF	Description	Comments
3	3	The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.	
2	2	The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were not explicitly discussed by the teacher or students.	
1	1	The students manipulated or generated one representation of a concept.	
0	0	There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.	

3) Students were engaged in mathematical activities.

SE	TF	Description	Comments
3	3	Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)	
2	2	Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.	
1	1	Most of the students spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of students' mathematical engagement.	
0	0	Most of the students are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.	

4) Students critically assessed mathematical strategies.

SE	TF	Description	Comments
3	3	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.	
2	2	At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.	
1	1	An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one student.	
0	0	Students did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.	

5) Students persevered in problem solving.

SE	TF	Description	Comments
3	3	Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.	
2	2	Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.	
1	1	Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a road block to score above a 0.	
0	0	Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem solving situation no students persevered. That is to say, all students either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy they stopped working.	

6) The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.

SE	TF	Description	Comments
3	3	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.	
2	2	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.	
1	1	The lesson mentions some fundamental concepts of mathematics, but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.	
0	0	The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the "why" behind the procedures.	

7) The lesson promoted modeling with mathematics.

SE	TF	Description	Comments
3	3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).	
2	2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure.); <u>or</u> modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.	
1	1	The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.	
0	0	The lesson does not include any modeling with mathematics.	

8) The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)

SE	TF	Description	Comments
3	3	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.	
2	2	Students are given some time to examine mathematical structure, but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.	
1	1	Students are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.	
0	0	Students are given no opportunities to explore or understand the mathematical structure of a situation.	

9) The lesson included tasks that have multiple paths to a solution or multiple solutions.

SE	TF	Description	Comments
3	3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.	
2	2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; or more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.	
1	1	Multiple solutions and/or multiple paths minimally occur, and are not explicitly encouraged; or a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.	
0	0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.	

10) The lesson promoted precision of mathematical language.

SE	TF	Description	Comments
3	3	The teacher "attends to precision" in regards to communication during the lesson. The students also "attend to precision" in communication, or the teacher guides students to modify or adapt nonprecise communication to improve precision.	
2	2	The teachers "attends to precision" in all communication during the lesson, but the students are not always required to also do so.	
1	1	The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.	
0	0	The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.	

11) The teacher's talk encouraged student thinking.

SE	TF	Description	Comments
3	3	The teacher's talk focused on high levels of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. *Analysis:* examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. Synthesis: requires original, creative thinking. Evaluation: makes a judgment of good or bad, right or wrong, according to the standards he/she values.	
2	2	The teacher's talk focused on mid-levels of mathematical thinking. Interpretation: discovers relationships among facts, generalizations, definitions, values and skills. Application: requires identification and selection and use of appropriate generalizations and skills	
1	1	Teacher talk consists of "lower order" knowledge based questions and responses focusing on recall of facts. Memory: recalls or memorizes information. Translation: changes information into a different symbolic form or situation.	
0	0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.	

12) There were a high proportion of students talking related to mathematics.

SE	TF	Description	Comments
3	3	More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.	
2	2	More than half, but less than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.	
1	1	Less than half of the students were talking related to the mathematics of the lesson.	
0	0	No students talked related to the mathematics of the lesson.	

13) There was a climate of respect for what others had to say.

SE	TF	Description	Comments
3	3	Many students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.	
2	2	The environment is such that some students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.	
1	1	Only a few share as called on by the teacher. The climate supports those who understand or who behave appropriately. Or Some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication.	
0	0	No students shared ideas.	

14) In general, the teacher provided wait-time.

SE	TF	Description	Comments
3	3	The teacher frequently provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
2	2	The teacher sometimes provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
1	1	The teacher rarely provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
0	0	The teacher never provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	

15) Students were involved in the communication of their ideas to others (peer-to-peer).

SE	TF	Description	Comments
3	3	Considerable time (more than half) was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.	
2	2	Some class time (less than half, but more than just a few minutes) was devoted to peer to peer (pairs, groups, whole class) conversations related to the mathematics.	
1	1	The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes.	
0	0	No peer to peer (pairs, groups, whole class) conversations occurred during the lesson.	

16) The teacher uses student questions/comments to enhance conceptual mathematical understanding.

SE	TF	Description	Comments
3	3	The teacher frequently uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.	
2	2	The teacher sometimes uses student questions/ comments to enhance conceptual understanding.	
1	1	The teacher rarely uses student questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task verses conceptual knowledge of the content.	
0	0	The teacher never uses student questions/ comments to enhance conceptual mathematical understanding.	

Additional Notes: Preservice or Inservice. Live or Video. #Students, Grade Level, topic/subject, date, other demographics, school, etc.

Was an indicator marked lower based on teaching practices or student engagement on the MCOP2 that were due to inequity? If yes, which points of the rubric?

APPENDIX D

Semi-Structured Interview #1: Beliefs and How They Originated

Participants will be given a copy of the following statements and asked to circle the statement in each section that best describes their belief about teaching and learning mathematics.

Section 1

a. I explicitly teach children how to do math.
b. I provide opportunities for children to solve problems using their own strategies as well as show the children specific methods to solve problems.
c. I do not show children how to solve problems; they can solve problems without me providing or teaching strategies to them.

Section 2

a. When I teach math, I clearly explain how to solve math problems and then provide opportunities for my students to practice the steps I taught.
b. When I teach math, I first provide opportunities for my students to solve problems using their own strategy. Then, after discussion, I clearly explain how to solve the problem using specific methods.
c. When I teach math, I ensure my students spend most of their time solving and reporting their solutions to a variety of problems We then compare and contrast the different strategies in class. I rarely model how to solve problems.

1. Please explain your choices for each of the two sections.
2. Explain your choice on statement [#] from the Revised Mathematical Beliefs Survey.
3. Describe your beliefs about how mathematics should be taught in a second-grade classroom.
4. How do you feel these beliefs originated?
5. Describe your beliefs about how second graders learn mathematics.
6. How do you feel these beliefs originated?
7. Do you feel your beliefs about how mathematics should be taught could be changed? Why or why not? What could bring about those changes?
8. Do you feel your beliefs about how mathematics is learned could be changed? Why or why not? What could bring about those changes?

APPENDIX E

Semi-Structured Interview #2: Mathematical Instruction and Beliefs

Date:

Lesson:

1. What was (were) the objective(s) of the lesson in the first observation? Second?

2. Do you feel you met the objectives for each lesson? Why or why not?

3. Explain your thinking behind the [*topic of lesson in a particular* video clip] lesson. What influenced your decision to conduct the lesson in this manner?

4. If applicable, describe how your mathematical beliefs helped you teach this lesson.

5. If applicable, describe how your mathematical beliefs may have hindered your teaching of this lesson.

6. How has your belief about teaching mathematics affected your teaching?

7. How has your belief about learning mathematics affected your teaching?

8. Is there anything else you would like me to know about these lessons?

APPENDIX F

Semi-Structured Interview #3: Contextual Factors Influence on Instruction

Date:

Lesson:

1. What was (were) the objective(s) of the lesson?

2. Do you feel you met the lesson objectives? Why or why not?

3. Explain your thinking behind the [topic of lesson in a particular video clip] lesson. What influenced your decision to conduct the lesson in this manner?

4. What role does classroom management decisions play before or during your mathematical instruction?

5. How much time do you allot for math? How does this time allotment impact the decisions you make about your mathematical instruction?

How closely do you follow the lessons outlined in your curriculum? What are the strengths and weaknesses of your curriculum?

APPENDIX G

Memo Template

Memo for Date: Participant and Interview #

Today I Coded:

Name Interview #: Link to Script and Codes

Quotes:

Codes Used:

What's their story (beliefs)?

Nature of Math - espoused

-

Model of Teaching - espoused

-

Model of Learning - espoused

-

Nature of Math - formed/changed

-

Model of Teaching - formed/changed

-

Model of Learning - formed/changed

-

Nature of Math - enacted

-

Model of Teaching - enacted

-

Model of Learning - enacted

-

Other Notes:

VITA

Tonya Garrett

Candidate for the Degree of

Doctor of Philosophy

Dissertation: BELIEFS: A CASE STUDY ON FORMATION, CHANGE, AND INFLUENCE

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Completed the requirements for the Doctor of Philosophy in Education at Oklahoma State University, Stillwater, Oklahoma in May, 2022.

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Completed the requirements for the Bachelor of Science in Elementary Education at Northeastern State University, Tahlequah, Oklahoma in 1999.

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2021- Present	Assistant Professor, Elementary Education Program, Northeastern State University
2017-2021	Director of Assessment, College of Education, Northeastern State University
2016-2017	Instructor, Teacher Education, Northeastern State University

Professional Memberships:
Member, National Council of Teachers of Mathematics
Member, Research Council on Mathematics Learning
Member, School Science and Mathematics Association