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Development of a Topology Optimization Method for the Design of Ground Heat Exchangers

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ABSTRACT

A new method for sizing vertical ground heat exchangers is proposed using topology optimization to reduce the number of boreholes required to fulfill the cooling and heating demand. The ASHRAE sizing equation is adapted to formulate a topology optimization problem to minimize the number of boreholes required in a bore field. The results show that topology optimization can help reduce the number of boreholes required when compared to a sizing performed on a regular grid using conventional sizing methods. These optimized configurations show smaller spacings between the boreholes located on the perimeter and larger spacings between the boreholes located in the center of the bore field.

INTRODUCTION

The design phase is a crucial part of the installation of a ground heat exchanger (GHE). Many aspects must be considered such as the heating and cooling loads of the building, the properties of the ground and the operating conditions. The operating conditions are usually a constraint that is imposed on the temperature of the fluid when entering or leaving the borehole. Multiple boreholes are often required to satisfy all the design parameters. However, boreholes interfere with each other, which may lead to a decrease in the performance of the GHE and an increase in the required borehole length. A higher drilling length in the GHE tends to increase the initial investment cost, which is one of the biggest obstacles for a wider use of GHEs. Working on ways to reduce these expenses by minimizing the total drilling length is a task that could be beneficial for the adoption of the technology on a larger scale.

Existing design methods usually evaluate the minimal required total drilling length given a regular and already planned GHE configuration. An example is ASHRAE's sizing method, as modified by Ahmadfard & Bernier (2018):

$$L_{tot} = \frac{q_a R_{ga,g} + q_m R_{gm,g} + q_h R_{gh,g} + q_h R_{gh,g}}{(T_m - T_g)_{ref}} \tag{1}$$

where L_{tot} is the total drilling length, q_a, q_m and q_h are respectively the mean annual ground load, the mean monthly ground load for the design month, and the hourly peak ground load for the design month, $R_{ga,g}$, $R_{gm,g}$, and $R_{gh,g}$ are the respective thermal resistances for each load which are evaluated using g-functions, T_m is the mean fluid temperature inside the borehole, and T_g is the undisturbed ground temperature. Both T_m and T_g are imposed in that method, thus the notation "$( )_{ref}$" in equation 1.

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Previous research has identified different design strategies to optimize bore field configurations. Cimmino & Bernier (2014) studied the effect of adding or removing boreholes by modifying the spacing between the boreholes in a regular configuration. Guo et al. (2017) studied the impact of the ground temperature variation when displacing boreholes from the center of the bore field to its perimeter. Spitler et al. (2020) analyzed strategies such as irregular spacings and configurations that wrap around the buildings. These previous investigations on the optimization of GHE configurations result in similar conclusions: increasing the spacing with longer boreholes and a density of boreholes that is higher on the perimeter and lower in the center leads to savings in total drilling length and to more effective bore fields. Most of the research made on the subject proposed design strategies that compare advantageously with a base case, usually a regular configuration. The question remains as to how close these strategies are to the optimal layout.

Automated methods to optimize GHEs have been developed over the years. Beck et al. (2013) proposed to optimize both the positioning of the boreholes and their heat extraction rates using linear programming and evolutionary computation. They found that optimizing a combination of both parameters produced minimal differences in the results. Bayer et al. (2014) developed a method to balance the workloads of each borehole in a bore field by removing the least effective ones. More recently, Edigi et al. (2021) proposed a method for evaluating bore field configurations for a fixed number of boreholes that relied on minimizing the sum of squares of the temperature difference in the ground due to the long-term operation of the bore field using the steepest descent method. Even though the objective function of that problem is not a condition that is usually used when designing GHEs, it led to results similar to previous studies. Cook (2021) developed the GHEDT program. The tool searches through pre-defined configurations via various design routines to find a combination of configurations that will optimize the bore field. These methods all converged to configurations where the perimeter is denser in boreholes than the center.

As mentioned by Sigmund & Maute (2013), topology optimization originated for mechanical design applications but has since been used in many other fields of study. The idea behind topology optimization is to find the placement of material that will give the best structural performance. Most topology optimization approaches are density based and are performed on a prescribed domain. This domain is usually divided in elements and a design variable (ρ) that can take either the value of 0 (void) or 1 (solid) is introduced. The optimization process evaluates which elements will contain material to meet the constraints. However, this is a discrete problem, and it is difficult to solve it directly (Sigmund & Maute, 2013). The problem is usually reformulated into a continuous one which allows to use more efficient gradient-based methods. The continuous problem takes the following form:

$$\begin{cases} \text{Minimize:} \quad f_0(\boldsymbol{\rho}) \\ \text{Subject to:} \quad f_i(\boldsymbol{\rho}) \leq 0 \text{ for } i = 1, \ldots, m \\ 0 \leq \rho_j \leq 1 \text{ for } j = 1, \ldots, n \end{cases} \tag{2}$$

where f_0 is the cost-function (i.e. the function to minimize), f_i are the constraints, m is the number of constraints, n is the number of elements in the domain, and $\boldsymbol{\rho}$ is the vector of design variable.

Even though the solving process is more efficient with the continuous formulation, the problem does not always converge to values of 0 or 1, which may create non-feasible solutions. A method to facilitate the convergence to values of 0 or 1 is the Simplified Isotropic Material with Penalization (SIMP) (Bendsoe, 1989), where a penalization term (p) is introduced in the problem formulation. For $p = 1$, the solution is the same as if no penalization was introduced. For $p > 1$, intermediate values of ρ are penalized. As pointed out by Rozvany (2001) and Sigmund & Maute (2013), the optimization problem is solved repeatedly by slowly increasing the values of p which leads to better results.

The placement of the boreholes that compose a GHE has proven to be a parameter that influences the design process because of the thermal interactions between the boreholes. Strategically placing them can lead to significant savings in drilling length, which should usually help reduce the cost of investment of GHEs. This paper proposes a topology optimization method for the design of ground heat exchangers to minimize the number of boreholes in a GHE by strategically placing them inside the available area, adapting the modified ASHRAE sizing method.

METHODOLOGY

Problem Formulation

A problem of the form of equation 2 must be formulated to apply topology optimization for the design of GHEs. The objective of the proposed method is to minimize the number of boreholes of a given length inside a prescribed domain. The problem is constrained by the maximum temperature difference between the undisturbed ground and the fluid circulating in the boreholes, as in the modified ASHRAE sizing equation (equation 1). Contrary to many topology optimization problems, the domain is discretized in points instead of elements as only the coordinates of the boreholes are needed. The optimization determines which combination of boreholes of length L minimizes its number, where L is a fixed parameter. This procedure is explained in details in the next section. With respect to equation 2, the proposed optimization problem can be written as follows:

$$\begin{cases} \text{Minimize: } f_0(\boldsymbol{\rho}) = \sum_{i=1}^{n} \rho_i \\ \text{Subject to: } f_1(\boldsymbol{\rho}) = (T_m - T_g) - (T_m - T_g)_{ref} \leq 0 \\ 0 \leq \rho_i \leq 1 \text{ for } i = 1, \dots, n \end{cases} \tag{3}$$

where ρ_i represents the fraction of a borehole of length L on every point inside the domain (which would ideally only take the value of 0 or 1), and $(T_m - T_g)_{ref}$ the imposed temperature constraint. As previously mentioned, gradient-based optimization methods are efficient, but require a continuous problem formulation, which is why ρ_i is continuous in equation 3.

The modified ASHRAE sizing equation has to be modified to evaluate $(T_m - T_g)$ while taking into account the design variable. The temperature is evaluated as follows:

$$T_m - T_g = \frac{q_a R_{ga,g} + q_m R_{gm,g} + q_h R_{gh,g} + q_h R_b}{\sum_{i=1}^{n} \rho_i \cdot L} \tag{4}$$

with:

$$R_{ga,g} = \frac{g(t_f) - g(t_f - t_1)}{2\pi k_s} \tag{5}$$

$$R_{gm,g} = \frac{g(t_f - t_1) - g(t_f - t_2)}{2\pi k_s} \tag{6}$$

$$R_{gh,g} = \frac{g(t_f - t_2)}{2\pi k_s} \tag{7}$$

where k_s is the ground thermal conductivity, L the individual length for a borehole, and g the g-functions evaluated at timesteps t_f, $t_f - t_1$ and $t_f - t_2$, with $t_1 = 10$ years, $t_2 = t_1 +$ 1 month and $t_f = t_2 + 6$ hours. The g-functions are evaluated by superposition of the finite line source (FLS) solution:

$$g(t) = \frac{\boldsymbol{\rho}^T [b_{ij} \cdot h_{ij}] \boldsymbol{\rho}}{\sum_{i=1}^{n} \rho_i^p} \tag{8}$$

with:

$$h_{ij} = \frac{1}{2L} \int_{1/\sqrt{4\alpha t}}^{\infty} \frac{1}{s^2} \exp\left(-d_{ij}^2 s^2\right) I_{ls}(Ls, Ds) ds \tag{9}$$

$$I_{ls}(Ls, Ds) = 2 \cdot ierf(Ls) + 2 \cdot ierf(Ls + 2Ds) - ierf(2Ls + 2Ds) - ierf(2Ds) \tag{10}$$

where $[b_{ij} \cdot h_{ij}]$ is the array of thermal response factors for a borehole positioned on the j-th node on a borehole positioned on the i-th node, multiplied by a constant. The h_{ij} factors are evaluated using the FLS model as proposed by Claesson & Javed (2011), with d_{ij} representing the radial distance between the i-th and the j-th borehole (with $d_{ii} =$

r_b), D the buried depth of the boreholes, and α the ground thermal diffusivity. The b_{ij} factor is added to ensure a minimal spacing between the boreholes in the solution. It acts as a soft constraint that takes either the value of 5 if $d_{ij} < B_{min}$ or 1 if $d_{ij} \geq B_{min}$, where B_{min} is the imposed minimal spacing. The value of 5 has shown sufficient to fulfill the imposed minimal spacing constraint for the different cases tested. It has however been chosen arbitrarily and is subject to further research. The penalization is added in equation 8. Having the penalization at this specific position was found to be the most efficient way of obtaining discrete values of the design variable.

Optimization Procedure

The available domain is first discretized using the pygmsh 7.1.17 Python module (Schlömer, 2022) with the default Frontal-Delaunay algorithm and is then reloaded with the trimesh 3.11.2 Python module (Dawson-Haggerty et al., 2022) with processing for future manipulations of the grid. The positions are determined with more precision with a finer discretization, at the cost of increasing calculation time. The array of thermal response factors is evaluated for every point in the domain using pygfunction 2.1.0 and the method of similarities (Cimmino, 2018a, 2018b). At this point, it is assumed that there is a borehole on every point of the discretization. The initial value of $\boldsymbol{\rho}$ is generated randomly to avoid the convergence of the solution to a local minimum.

The method of moving asymptotes (MMA) (Svanberg, 1987, 2002) as implemented in the NLopt 2.7.0 Python module (Johnson, 2020) is used to perform the optimization. The MMA requires the evaluation of the derivatives of the cost-function and the constraints functions with respect to the design variable on every point in the domain. The derivatives of the cost function in equation 3 are given by:

$$\frac{\partial(f_0)}{\partial\rho} = [1, \dots, 1]^T \tag{11}$$

and the derivatives of the constraint function in equation 4 are given by:

$$\frac{\partial(f_1)}{\partial\rho} = \frac{\partial(T_m - T_g)}{\partial\rho} = \frac{q_a\frac{\partial R_{ga,g}}{\partial\rho} + q_m\frac{\partial R_{gm,g}}{\partial\rho} + q_h\frac{\partial R_{gh,g}}{\partial\rho}}{L\sum_{i=1}^{n}\rho_i} - \frac{q_a R_{ga,g} + q_m R_{gm,g} + q_h R_{gh,g} + q_h R_b}{L\left(\sum_{i=1}^{n}\rho_i\right)^2} \tag{12}$$

The ground thermal resistance requires the evaluation of the derivatives of the g-function, given by:

$$\frac{\partial g}{\partial\rho} = \frac{1}{\sum_{i=1}^{n}\rho_i^p}\left(-p\boldsymbol{\rho}^{p-1}g + (\mathbf{b}\circ\boldsymbol{h})\boldsymbol{\rho} + \boldsymbol{\rho}^T(\boldsymbol{b}\circ\boldsymbol{h})\right) \tag{13}$$

The process is repeated for values of p ranging from 1 to 3 by increments of 0.05. The limit of 3 was found to provide configurations that contain almost exclusively values of 0 or 1 for the design variable. Gradual refinement of the grid is also proposed to reduce the calculation time. The grid is refined using trimesh around the points where $\rho \geq 0.001$ in the previous solution. The penalization process is repeated starting from $p = 1$ to $p = 3$ using the new refined grid.

CASE STUDY

The medium office from the commercial building library of the U.S. Department of Energy located in the city of International Falls, MN, is chosen for the case study (DOE & PNNL, 2020). The heating and cooling loads are evaluated, and it is assumed that this demand is met by a ground source heat pump system with a COP of 3 in both modes. The three ground loads are then evaluated and multiplied by a factor of 4, requiring a larger bore field. A large imbalance in cooling was found for the ground loads, therefore the sizing is only performed in cooling mode. An additional constraint could be added to equation 3 to also cover the heating mode.

The study is performed on an L-shaped area of 6800 m2. A first sizing is done with the modified ASHRAE sizing method using a regular configuration and serves as the base case (case 1). A target length of around 125 m was aimed and multiple combinations of spacings were tested to find the minimum total drilling length. These combinations were tested manually, which means that it is possible that a better configuration could have been achieved. A spacing of 10 m in the x direction and 5 m in the y direction was finally chosen. Another sizing is performed using the topology optimization method, with boreholes of the same length as the ones evaluated with the first method (case 2). It is then proposed to size the bore field using topology optimization by increasing the individual length of the boreholes (cases 3 and 4). The purpose of these last two cases is to analyze the behavior of the topology optimization method when increasing the individual length, and how it compares to previous studies that analyzed this parameter. A minimal spacing of 2.5 m is imposed for all three sizings done using topology optimization, and the initial grid is refined two times starting from a grid where the longest distance between two consecutive points is 5 m. The parameters used for the sizing are presented in Table 1. The value of $T_{m,ref}$ used in constraint f_1 from equation 3 is evaluated at 37.5°C. This results in a value of 23.5°C for the $\left(T_m - T_g\right)_{ref}$ constraint.

Table 1. Parameters Used in the Simulation

Bore field parameters		
Borehole buried depth (D)	4	m
Borehole radius (r_b)	0.075	m
Borehole thermal resistance (R_b)	0.2	m-K/W
Ground properties		
Thermal diffusivity (α)	1.0×10^{-6}	m^2/s
Thermal conductivity (k_s)	2.0	W/m-K
Undisturbed ground temperature (T_g)	14	°C
Annual ground load (q_a)	108.60	kW
Monthly ground load (q_m)	255.72	kW
Hourly ground load (q_h)	773.36	kW
Fluid properties (propylene-glycol 20% concentration)		
Flow rate ($\dot{V}_f$)	0.05	L/s per kW of peak load
Density (ρ_f)	1008	kg/m^3
Specific heat capacity ($c_{p,f}$)	4014	J/kg-K
Entering fluid temperature ($T_{o,f}$)	40	°C

RESULTS AND DISCUSSION

Figure 1 presents the comparison for sizings done using the methods described in the previous section. These are filtered results, meaning that only the locations where $\rho_i \geq 0.01$ are shown. Figure 2 presents the results for the sizing of case 1. The complete grid after refinement is shown, and the shaded points represent locations where the value of ρ_i is approximately 0. A summary of the four cases is presented in Table 2.

The optimized configurations using topology optimization are in accordance with previous studies and present a combination of the design strategies identified in the literature. The configurations are usually denser on the perimeter and have spacings that increase in the middle of the bore field. The method can achieve significant savings as case 2 presents savings of 9.8% compared to case 1. As previous studies showed, increasing the maximum length of the individual boreholes (cases 3 and 4) usually tends to higher savings in the total drilling length. The perimeter is still filled with boreholes, and the density of boreholes at the center is reduced. The spacing between boreholes is also increased, both on the perimeter and in the center.

The penalization introduced in equation 8 is not sufficient to ensure that $\boldsymbol{\rho}$ converges to values of 0 and 1. The total drilling length in Table 2 for cases 2, 3 and 4 is not equal to the product of the number of boreholes and the individual borehole length. This causes errors in the calculation of the total drilling length since boreholes that are only partially present are accounted for in the cost function which can lead to two phenomena. The first one is when the value of ρ on a given point is significant depending on the case but not equal to 1. For example, case 2 has a point with a value of

$\rho \approx 0.76$. The other case is when multiple values of ρ are small but not equal to 0. These additional lengths are small for a single point in the domain but become significant when summed for the entire domain.

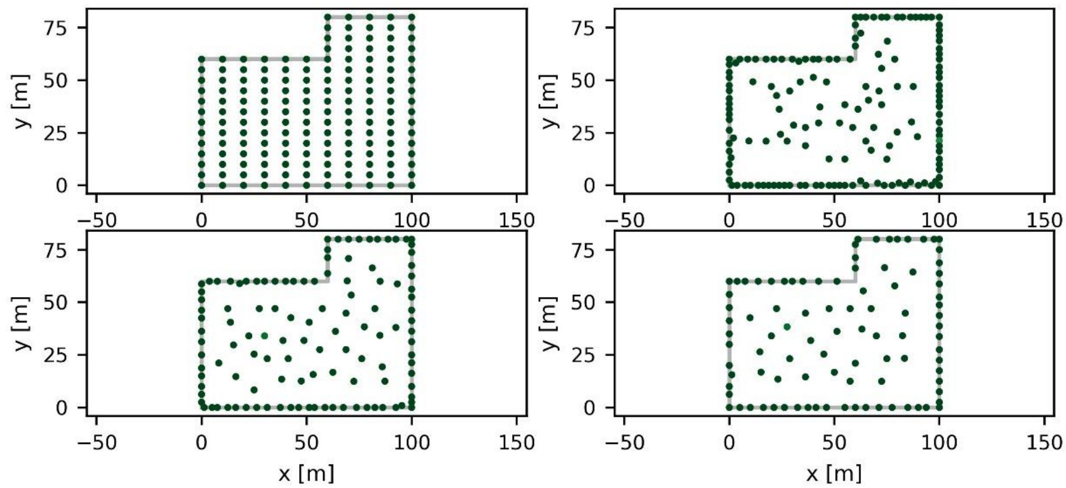

Figure 1 Bore field sizing filtered results case 1 (top left), case 2 (top right), case 3 (bottom left), case 4 (bottom right)

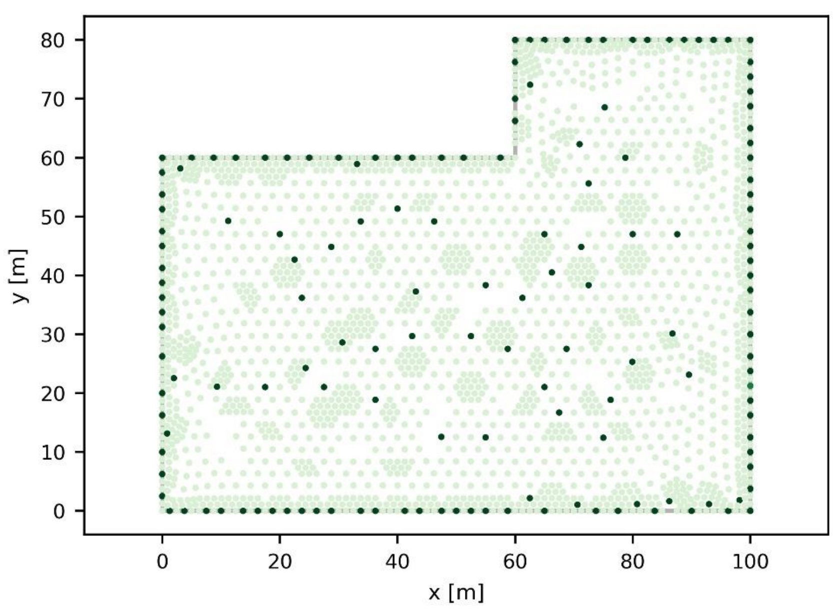

Figure 2 Detailed sizing results with topology optimization for case 2

The optimized configurations are complex and impractical for real applications. The boreholes in the middle are scattered in what seems a random configuration. Since the optimization procedure relies on g-functions, it is the borehole density (e.g. in boreholes per square meter) across the domain that affects the temperature difference and thus affects the presence or absence of boreholes in a given region of the domain. There might be multiple configurations that will satisfy the design criteria with similar total drilling lengths. A "feasibility" constraint could be considered in future work, but this necessitates a quantitative criterion compatible with the problem formulation of equation 2.

Table 2. Sizing Results

Case	Method	Number of boreholes	Individual borehole length (m)	Total drilling length (m)	Savings (%)
1	Modified ASHRAE sizing method	163	127.3	20 750	0.0
2	Topology optimization	150	127.3	18 714	9.8
3	Topology optimization	117	150.0	17 528	15.5
4	Topology optimization	94	175.0	16 426	20.8

The time consumption of the method also represents one of its limits. The method may take a couple of hours for large bore fields that are finely discretized. The optimization method however does not require supervision. Once all the design parameters are entered, the method produces the result automatically. On the other hand, ASHRAE's sizing method takes a couple of seconds, and the same can be said for some of the more recent sizing tools, such as GHEDT.

CONCLUSION

Borehole placement is an important aspect to consider when designing a GHE. Previous research has shown the effect of placement on the total drilling length, and many strategies have been explored to optimize bore field configurations. This paper has proposed a new method to optimize GHEs based on topology optimization.

A case study has been presented, which compared this new optimization method with one that is used frequently when designing bore fields. It has been shown that this method can reduce the total drilling length using efficient and robust optimization algorithms. The configurations obtained are in accordance with the more recent research made on the optimization of bore fields: boreholes should be more densely placed on the perimeter and more coarsely in the center. The current state of the method shows that directly applying topology optimization can lead to savings in total drilling length, but that the resulting configurations are complex. It should however help uncover new design strategies that have not been explored yet. The results can also serve to produce reference lower bounds on the total drilling length to compare other design methods. It is finally worth noting that this paper proposes only one formulation for the design problem. As long as the objective function and the constraints are in respect with the problem formulation in equation 2, different types of optimizations, e.g. techno-economic optimization, could be performed.

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NOMENCLATURE

α	=	Ground thermal diffusivity (m^2/s)
ρ	=	Design variable (-)
ρ_f	=	Fluid density (kg/m^3)
b_{ij}	=	Distance multiplying factor
B_{min}	=	Imposed minimal spacing (m)
$c_{p,f}$	=	Fluid specific heat capacity (J/kg-K)
d_{ij}	=	Radial distance between the i-th and the j-th borehole (m)
D	=	Borehole buried depth (m)
f_0	=	Cost-function
f_1	=	Constraint function

$g(t)$	=	g-function evaluated at time t (-)
$\boldsymbol{h}$	=	Array of ground thermal response factors (-)
$ierf$	=	Error function integral (-)
k_s	=	Ground thermal conductivity (W/m-K)
L_{tot}	=	Total bore field drilling length (m)
L	=	Individual borehole length (m)
p	=	Penalization value (-)
q_i	=	Ground loads (W)
R_b	=	Borehole thermal resistance (m-K/W)
r_b	=	Borehole radius (m)
$R_{gi,g}$	=	Ground thermal resistance evaluated using g-functions (m-K/W)
T_m	=	Mean fluid temperature (°C)
T_g	=	Undisturbed ground temperature (°C)
ΔT	=	Temperature variation constraint (°C)
$T_{o,f}$	=	Entering fluid temperature (°C)
$\dot{V}_f$	=	Fluid flow rate (L/s per kW of peak load)

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