

Long-term sustainable operation of hybrid geothermal systems through optimal control

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ABSTRACT

Hybrid geothermal systems such as hybrid GEOTABS typically comprise a geothermal heat pump that supplies the main building thermal energy needs, complemented by a fast-reacting supplementary production and/ or emission system for the peak building thermal loads. Optimal predictive controllers such as Model Predictive Control (MPC) are desired for these complex systems due to their optimized and automated energy savings potential (while providing the same or better thermal comfort) thanks to system integration and their anticipative action. However, the predictions of these controllers are typically limited to a few days. Consequently, the controller is unaware whether abusive energy injection/extraction into/ from the soil will deplete the source over the years. This paper investigates in which cases the long-term dynamics of the borefield ought to be included in the MPC formulation. A simulation model of a hybrid GEOTABS system is constructed. Different borefield sizes, ground imbalance loads, and electricity/gas ratios are evaluated. The model control inputs are optimized to minimize the energy use in 5 years through (i) a reference Optimal Control Problem (OCP) for the 5 years, solved in hourly time-steps and (ii) an MPC control with a prediction horizon of 1 week. The obtained results reveal that MPC can be up to 20% far from the true optimal, especially in the cases where the borefield is undersized and there is a large cost gap between the different energy systems.

INTRODUCTION

Optimal control of buildings relies on predicting their behaviour using techniques such as mathematical modelling, data science or a combination of both (Arroyo, (2022)). One common feature of these approaches is that they optimize towards future system behaviour, typically for a few days. However, the time constant of the ground dynamics in ground source heat pump systems is too large to be captured by these prediction windows. Consequently, the controller is unaware of whether (i) is achieving an optimal solution and (ii) will deplete the ground-source in the long term, thereby hampering the sustainable use of the ground.

The borefield long-term dynamics can be well predicted using its characteristic thermal response function or g-function (Eskilson (1987)). Thus, this article follows a physics-based Model Predictive Control (MPC) approach, eliminating the need for large training datasets. MPC uses a mathematical model of the building envelope and the energy system along with forecasted disturbances (weather and occupancy) to predict the building dynamic behavior and the energy system efficiencies over a prediction horizon of typically a few days. An optimization problem finds the optimal control input sequence that minimizes an objective function (energy use, CO2 emissions, operational costs...) while a set of thermal comfort and technical constraints is enforced. Only the first inputs of the sequence are applied as the optimization problem is recursively solved at each control time step.

Few authors have proposed formulations to incorporate the borefield long-term dynamics. Verhelst (2012) and Jorissen (2018) introduced a penalty cost to the dominant ground load in the objective function to reduce the borefield yearly Iago Cupeiro Figueroa (iagocupeiro@protonmail.com) is a former researcher (now building control expert at DeltaQ NV/SA) and Lieve Helsen is professor at the Mechanical Engineering Department in KU Leuven.

imbalances. However, the determination of such a penalty relies on trial-and-error. Antonov (2014) imposed yearly cyclic conditions on the ground temperatures in an optimal control problem (OCP) of one year. Still, the approach can be sub-optimal because (i) a yearly thermal imbalance can be allowed depending on the borefield size and (ii) the freedom of MPC to take decisions is more limited due to the ground temperatures constraints. Cupeiro Figueroa et al. (2020) extended the MPC objective function with a shadow cost that estimates the overall yearly objective. The performance of the approach depends on a set of long-term predictions of the building heating and cooling loads. The contribution of this paper is to take a step back and investigate in which cases the long-term dynamics of the borefield ought to be included in the MPC formulation.

METHODOLOGY

To that end, we construct a model of a building equipped with a hybrid geothermal system using verified high-fidelity models from the Integrated District Energy Assessment Simulations (IDEAS) Modelica library (Jorissen et al. 2018). The model equations are then reformulated into a non-linear optimization problem using Toolchain for Automated Control and Optimization (TACO) (Jorissen et al. 2019). The optimization problem aims at finding the control inputs that minimize a target function while keeping thermal comfort in the building. This optimization problem is solved in hourly time-steps over a 5-year period (i) once covering the whole 5-year period (OCP) and (ii) recursively each hour for the upcoming week (receding horizon, MPC). Two target functions are evaluated: minimization of the energy costs and minimization of the operational costs. In addition, the optimization is solved for different borefield sizes and different levels of borefield imbalance. The results of both approaches are then compared for each specific case.

Case study

The building is modelled as a hybrid GEOTABS (Himpe et al. (2018)) represented in Figure 1, i.e. it comprises a geothermal heat pump (GEO) that supplies low temperature heating and uses passive cooling for high temperature cooling through a thermally activated building structure (TABS). The hybrid character lies in the extension with a fast-reacting auxiliary production and/or emission system for the peak periods. In this case, an additional gas boiler for heat production and an active chiller for cold production are installed. The building can also choose between thermally activating the building structure (TABS) through embedded pipes and air conditioning as the slow and fast-reacting emission systems. Thus, the building case study is hybrid both at the production and emission sides. However, the gas boiler and the chiller can only supply energy to the fast-reacting emission system whereas the TABS are only fed by the ground source heat pump (GSHP) or passive cooling heat exchanger.

The control input variables include the control of the production systems: u_1 for the GSHP, u_2 for the gas boiler, and u_3 for the chiller; the valve openings that control the mass flow rates (and in consequence the heat/cold supply) to the different emission systems u_4 , u_5 , u_6 and u_7 and the 3-way valve that determines the TABS mode u_8 . Since non-linear optimization problems do not accept binary inputs, this 3-way mixing valve needs to be considered as modulating in the simulation study. However, in practice, this valve is expected to be on/off to choose between heating or cooling mode in TABS. All pumps and fans work at a constant pressure head.

Table 1 lists the equations of the components presented in Figure 1. The HVAC equipment is sized to supply the design heat and cold loads, computed according to the standard ISO 12831-1:2017. For specific details on the design nominal values and the used fit coefficients, we refer to the work of Cupeiro Figueroa (2021). The building is modelled as a single space of 1200 m² using the IDEAS library *RectangularZone* component, which assumes a perfectly mixed air volume within a rectangular construction geometry subjected to dynamic heat transfer relations. The outer walls, roof/floor and glazing U-values are 0.21, 0.18 and 1.3 W/(m²K) respectively. Internal walls are also included to account for the internal

thermal mass of the building. The building is assumed to be located in Brussels, Belgium, and as such a typical meteorological year weather file is used. Equation 1 shows the GSHP correlations which assume a linear COP and condenser power, where A_i are the fit coefficients, and $T_{eva,in}$ and $T_{con,in}$ are the inlet working fluid temperatures of the evaporator and condenser and T_{eva,nom} and T_{con,nom} are the nominal temperatures of the evaporator and condenser, respectively. The gas boiler (Equation 2) and the chiller (Equation 3) are assumed to supply/extract heat in linear relation to their modulation signal. While the gas boiler is assumed to have a 100% efficiency, the chiller performance is dependent on the outdoor temperature T_{out} , with B_i being the fit coefficients. Although in this case all pumps are considered to work at a constant pressure head, the pressure drops that some of them need to overcome can vary as a function of the valves opening u4, u5, u6, and u7. For the hydraulic schematic in Figure 1, it is trivial that only the passive cooling pump power $\dot{W}_{pum,pc}$ and the heat pump condenser pump power $\dot{W}_{pum,pc}$ are affected by these valve openings. The 3-way valve opening *u₈* affects the TABS heating and cooling pumps, but their sum remains constant. The heat exchanger is based on the NTU-epsilon method for counter-flow heat exchangers as formulated in Equation 5, where C_{min} and C_{max} are the minimum and maximum heat capacity rates, C_r is the heat capacity ratio, NTU is the number of transfer units, \mathbf{E} is the effectiveness of the heat exchanger and $T_{hex, hot, in}$ and $T_{hex, cold, in}$ are the inlet temperatures in the heat exchanger at the hot and the cold side, respectively. The circulation pump model corresponds to a slightly modified version of the IBPSA library model based on similarity laws (Wetter (2013)). A flow coefficient k correlates the pump pressure head Δp and the corresponding mass flow rate \dot{m} (Equation 6a). The pump electric power \dot{W}_{pum} cubically increases with the mass flow rate, with \dot{m}_{nom} being the nominal mass flow rate of the pump and $\dot{W}_{pum,nom}$ the corresponding nominal power (Equation 6b). The borefield model corresponds to the analytical long-term model used by Cupeiro Figueroa et al. (2020), which is based on a radial resistance-capacitance network to model the borehole short-term heat transfer and the temporal superposition of the g-function and a continuous load-shifting algorithm formulation to model the ground long-term heat transfer.



Figure 1 Schematic presentation of the considered hybrid GEOTABS case, main energy flows, and fluid network pressure drops.

Component	Component Equations					
Zone	IDEAS Zone model. See description by Jorissen et al. (2018)					
	$\dot{Q}_{con} = u_1 \dot{Q}_{con,nom}$	(1a)				
Ground source heat nump	$COP = A_1 + A_2(T_{eva,in} - T_{eva,nom}) + A_3(T_{con,out} - T_{con,nom})$	(1b)				
Ground source near pump	$\dot{W}_{com} = \dot{Q}_{con}/COP$	(1c)				
	$\dot{Q}_{eva} = \dot{Q}_{con} - \dot{W}_{com}$	(1d)				
Cashoilar	$\dot{Q}_{boi} = u_2 \dot{Q}_{boi,nom}$	(2a)				
Gas boller	$\dot{W}_{boi}{=}\dot{Q}_{boi}$	(2b)				
	$\dot{Q}_{chi} = u_3 \dot{Q}_{chi,nom}$	(3a)				
Chiller	$EER = B_1 + B_2 T_{out}$	(3b)				
	$\dot{W}_{chi} = -\dot{Q}_{chi} / EER$	(3c)				
Valvos	$\dot{m}_i = u_i \dot{m}_{nom}$	(4a)				
Valves	$\dot{m}_i = u_i \dot{m}_{nom,1} + (1 - u_i) \dot{m}_{nom,2}$	(4b)				
	$\dot{C}_r = \dot{C}_{min} / \dot{C}_{max}$	(5a)				
	$NTU = UA/\dot{C}_{min}$	(5b)				
Heat exchanger	$1 - e^{-NTU(1-\dot{c}_r)}$	(E -)				
	$\epsilon - \frac{1}{1 - \dot{C}_r e^{-NTU(1 - \dot{C}_r)}}$	(50)				
	$\dot{Q}_{hex} = \varepsilon \dot{C}_{min} (T_{hex, hot, in} - T_{hex, cold, in})$	(5d)				
Bump/fep	$\Delta p = (\dot{m}/k)^2$	(6a)				
r unip/ tan	$\dot{W}_{pum} = (\dot{m}/\dot{m}_{nom})^3 \dot{W}_{pum,nom}$	(6b)				
Borefield	Borefield analytical model. See description by Cupeiro Figueroa et al. (2020)					

Table 1. Model equations of HVAC components presented in Figure 1. Specificdesign details can be found in Cupeiro Figueroa (2021)

Considered cases

The methodology is applied for three different ground loads representing a different level of building and borefield imbalance, manipulated by varying the occupancy internal gains \dot{Q}_{occ} of the building: an extraction-dominated case (ED), an injection-dominated case (ID) and a balanced case (B) are considered. The borefields (with H_b the borehole length) are sized using the modified ASHRAE equation (Ahmadfard and Bernier (2018)) to keep the average fluid temperature between 2 °C and 18 °C using ground loads computed from a previous dynamic simulation with MPC in the building but assuming a perfect source (i.e., infinite energy can be extracted from the field) as the borefield. Size 1 uses the three load pulses, corresponding to an oversized borefield that can cover all ground loads. Size 2 applies the monthly load pulse of the dominant side, corresponding to a slightly undersized borefield. Size 3 only considers the monthly load pulse of the non-dominant side, corresponding to an extremely undersized borefield. For the balanced case, only two sizes are considered since the difference between monthly loads is negligible. The borefield is a 2x2 squared borefield with a relative distance between boreholes of 6 m, an undisturbed ground temperature T_s of 10 °C, a ground thermal conductivity k_s of 2 W/(mK), a ground thermal diffusivity a_s of 9.26E-7 m²/s and a borehole thermal resistance R_b of 0.1 mK/W. The considered cases and the ground loads resulting from the MPC dynamic simulation are summarized in Table 2. For a detailed description of the other borefield parameters (which are kept constant), we refer to Cupeiro Figueroa (2021).

	Extraction-do	minated (ED)	Injection-don	ninated (ID)	Balanced (B)		
${\dot Q}_{ m occ}[{ m kW}]$	0		7.5		3.8		
Size 1 – H _b [m]	122.0		169.8		114.3		
Size $2 - H_b$ [m]	57.2		59.0		37.5		
Size 3 – H _b [m]	12.9		18.0		N/A		
	Extraction	Injection	Extraction	Injection	Extraction	Injection	
q _h [kW]	-13.5	11.9	-13.1	21.2	-13.2	17.5	
q _m [kW]	-6.3	1.5	-2.0	6.5	-4.2	4.1	
$q_y [kW]$	-2.0 0.0		0.0	2.1	0.0	0.0	

Table 2. Summary of the considered borefield cases and used ground loads in theASHRAE equation.

OPTIMIZATION SPECIFICATIONS

A closed-loop non-linear optimization problem is set up using the same building model equations, i.e. there does not exist model mismatch, and state updates are not necessary. In addition, perfect weather predictions are considered, eliminating uncertainties of any kind. The initial conditions set the building states at 20 °C, the borefield states at the undisturbed ground temperature, and the weather conditions at the beginning of the heating season. The optimization problem is subjected to the model equations and the target function can be formulated as:

$$\min_{u_{k}} J = \min_{u_{k}} \sum_{k=1}^{N} \left[c_{el} \dot{W}_{com,k} + c_{g} \dot{W}_{boi,k} + c_{el} \dot{W}_{chi,k} + c_{el} \dot{W}_{pum,pc,k} + c_{el} \dot{W}_{pum,hp,k} + \sum_{j=1}^{n_{s}} \sigma_{j,k} s_{j,k} \right] \Delta t_{k}$$
(7)

where the terms c_{el} and c_g represent the electricity and gas price respectively. For the minimization of the energy use case, these terms are set to 1, whereas for the minimization of the operational costs the considered electricity/gas price ratio is 5:1. Instead of hard constraints, a set of n_s quadratic penalization costs $s_{j,k}$ is introduced to ensure feasibility, and weighted with a scaling factor $\sigma_{j,k}$ to speed up the optimization, including:

- 1. The violation of the lower comfort bound, set at 21 $^{\circ}$ C and with a $-5 ^{\circ}$ C setback during night.
- 2. The violation of the upper comfort bound, set at 25 °C and with a +5 °C setback during night.
- 3. The GSHP lower temperature safety constraint, set at 0 °C.
- 4. The GSHP upper temperature safety constraint, set at 40 °C.
- 5. A dew point temperature protection for TABS imposed on the water temperature to avoid condensation of air humidity on the surfaces.
- 6. A constraint to avoid the use of the passive cooling heat exchanger for heating the building.

The number of steps N depends on whether an OCP or MPC is formulated. The former solves the optimization problem in one shot for a number of steps N=43800 (5 years in hourly steps), whereas the latter recursively solves the optimization problem at each hourly step for a number of steps N=14 using an increasing time-step Δt_k that covers a period of one week.

RESULTS AND DISCUSSION

Target: minimizing energy use

Since we consider the OCP to be the best obtainable result, an MPC efficiency is defined as the ratio between the total energy use of the MPC and the total energy use of the OCP. Table 3 compares these efficiencies for each borefield size and imbalance load using the energy use minimization formulation. As expected, the cases where the borefield is oversized present almost 100% efficiency since there is no risk of depleting the ground source. The small loss of optimality is caused due to the accumulation of minor errors caused by not considering the full horizon. For the extraction-dominated and balanced cases, there is a substantial drop in the MPC efficiency for the undersized borefields. Surprisingly, the efficiency losses in the injection-dominated cases are not as considerable as in the other two cases and especially given the energy use difference between the passive cooling pump and the active chiller. This effect is later explained.



 Table 3. MPC efficiency for each considered case when minimizing the energy use.

All cases where the MPC efficiency is lower than 95% are further analyzed in Table 4. Figure 2 shows the evolution of the building's main energy flows over the 5 years for Size 2 in the extraction-dominated case. Since the borefield is undersized, both optimizations deliberately recirculate heat from the building (from the GSHP and external gains) to

the source-side circuit to keep using the GSHP over the gas boiler. This behavior is optimal since the energy use of the gas boiler is in the order of 5 times higher due to the GSHP COP, thus covering all heating needs by the gas boiler would imply an increase in energy use. The heat re-circulation is achieved using the 3-way mixing valve that regulates the amount of heat/cold that is supplied to the TABS, and which connects the heating and cooling circuits. This behavior is largely magnified in Size 3. However, the OCP limits this behavior and shifts the thermal regeneration of the ground in summer by working at the lower comfort limit as shown in Figure 4, therefore minimizing further the use of the gas boiler.



Figure 4 Temperature evolution of the building space at the end of the cooling season for Case ED – Size 2. Black dashed lines represent the comfort boundaries.

		ED –	Size 2	ED – Size 3		ID – Size 3		B –	Size 2
	[MWh]	ОСР	MPC	ОСР	MPC	ОСР	MPC	ОСР	МРС
Heat pump (w/	Q	107.3	99.7	117.0	117.6	13.2	12.8	52.0	48.9
pump)	Ŵ	22.3	20.8	24.5	23.5	3.55	3.47	11.2	10.6
Gas boiler	Q	11.1	20.7	60.0	78.1	0.01	1.56	1.01	5.37
Gas boller	Ŵ	11.1	20.7	60.0	78.1	0.01	1.56	1.01	5.37
Passive cooling	Ľ	28.2	15.0	82.4	87.0	47.1	51.9	55.8	51.0
pump	Ŵ	1.06	1.00	1.18	1.18	1.97	2.13	1.62	1.55
Activo chillor	Q	0.00	0.00	0.00	0.47	65.4	59.3	1.33	0.89
Active chiller	Ŵ	0.00	0.00	0.00	0.08	10.1	9.52	0.23	0.16
Recirculated heat	Q	3.34	6.53	65.9	74.1	0.98	2.78	2.30	4.31
Total energy use	Ŵ	34.5	42.5	85.7	102.9	15.6	16.7	14.1	17.7

Table 4. Performance results for the cases with MPC efficiency < 95%.</th>

For the injection-dominated case, the OCP energy use of passive cooling is lower than MPC, but due to a higher heat recirculation by the latter one as shown in Figure 3. This heat recirculation eventually decreases the passive cooling capacity for summer, and in turn, the same temperature behaviors as shown in Figure 4 are observed. This also causes a marginal increase in the energy use of the chiller. Nevertheless, the main source of energy use savings comes from eliminating the need for the gas boiler. Size 3 is undersized for both heating and cooling and the OCP gives preference to the heating side due to the larger differences in energy use between the GSHP and the gas boiler compared to passive

cooling and the chiller. In addition, the cost of regenerating the soil to have more cooling capacity for the next season using the GSHP is much higher than the cost of regenerating the soil to have more heating using the passive cooling pump. Due to these energy use differences, the balanced undersized case also shows similar behavior to the extractiondominated cases, giving more priority to use the gas boiler as little as possible.

Target: minimizing operational costs

Table 5 compares the MPC efficiencies for each borefield size and imbalance load using the operational costs minimization formulation. All MPC efficiencies are above 95%. The energy use differences between the GSHP and the gas boiler are offset by the electricity/gas price ratio considered. This highlights that the design of the building energy system must go hand in hand with these ratios. The concept of "undersized" borefield is correlated to these factors.

	COSIS.						
	Extraction-dominated (ED)	Injection-dominated (ID)	Balanced (B)				
Size 1	98.5%	99.2%	98.6%				
Size 2	98.9%	95.9%	98.1%				
Size 3	97.1%	96.6%	N/A				

Table 5. MPC efficiency for each considered case when minimizing the operationalcosts.

CONCLUSION

This paper presents an MPC benchmarking study against its reference OCP in a modular hybrid GEOTABS building case. Results show that the performance of MPC is close to the optimal solution in the cases where the borefield is sized to cover all building's loads. However, for undersized borefields that aim at exploiting the hybrid characteristics of the building, there is a substantial loss of performance by the MPC since it does not account for the long-term effects. When a high electricity/gas price ratio is included in the optimization, the MPC efficiency increases in all cases. This is logical since all borefields were designed based on the ground energy loads and not the cost of energy. Next steps towards a performance analysis for different electrity/gas and COP ratios as well as design criteria are discussed in Cupeiro Figueroa (2021) and left for future work.

In general, due to the low cost of passive cooling, it is beneficial to provide extra cooling in summer to regenerate the ground source and avoid the use of the less efficient gas boiler in winter. On the other hand, providing extra heating through the GSHP does not seem to be a cost-effective way of regenerating the ground in the long-term due to its similar costs compared to the active chiller and the ground losses. A cheap regeneration solution (from the operational perspective) such as solar collectors or sewer water recovery may be of consideration for future study. Long-term MPC formulations can increase the MPC efficiency (Cupeiro Figueroa and Helsen (2022)) and should be further explored when these seasonal energy systems are installed.

NOMENCLATURE

α	=	Thermal diffusivity (m ² /s)	MPC	=	Model Predictive Control
Δ	=	Associated unit increase or difference (-)	OCP	=	Optimal Control Problem
3	=	Effectiveness (-)	р	=	Pressure (Pa)
Ċ	=	Heat capacity flow (W/K)	Ż	=	Heat flow rate (W)

	COP	=	Coefficient of performance (-)	R	=	Thermal resistance $(K/(mW))$
	EER	=	Energy efficiency ratio (-)	Т	=	Temperature (°C or K)
	GSHP	=	Ground source heat pump	TABS	=	Thermally activated building structure
	IDEAS	=	Integrated District Energy Assessment Simulations	TACO	=	Toolchain for Automated Control and Optimization
	k	=	Thermal conductivity (W/(mK)) or flow coefficient (-)	u	=	System input (-)
	'n	=	Mass flow rate (kg/s)	Ŵ	=	Energy use (-)
Subsc	ripts					
	b	=	Borehole	hex	=	Heat exchanger
	bf	=	Borefield	hp	=	Heat pump
	boi	=	(Gas) boiler	max	=	Maximum

DI	_	Doreneid	np	_	ricat pump
boi	=	(Gas) boiler	max	=	Maximum
chi	=	Chiller	min	=	Minimum
con	=	Condenser	nom	=	Nominal
com	=	Compressor	occ	=	Occupancy
eva	=	Evaporator	pc	=	Passive cooling

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