## PERFORMANCE OF MEMBERS REINFORCED WITH

## SMOOTH WELDED WIRE FABRIC UNDER

STATIC AND REPEATED LOADS

By

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iii

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## TABLE OF CONTENTS

Chapte	r		Page
I.	INTRODUC	TION	. 1
	1. 2.	Statement of the Problem	. 1 . 3
II.	LITERATU	RE REVIEW	. 4
III.	EXPERIME	NTAL PROGRAM	. 33
	1. 2. 3.	Specimens	<ul> <li>33</li> <li>34</li> <li>36</li> </ul>
IV.	EXPERIME	NTAL RESULTS	. 42
	1. 2. 3.	Width of Cracks	. 42 . 61 . 63
۷.	ANALYSIS	AND DISCUSSION OF RESULTS	. 67
	1. 2. 3. 4.	Maximum Width of Cracks.Average Width of Cracks.Crack SpacingsSplice Tests	• 67 • 79 • 86 • 86
VI.	SUMMARY A	AND CONCLUSIONS	. 92
	1. 2. 3.	Summary	• 92 • 93 • 96
A SELE	CTED BIBL	IOGRAPHY	. 98

# LIST OF TABLES

Table		Pa	age
I.	Outline of Specimens	•	35
II.	Age, Strength and Slump of Beams	•	37
III.	Strength Properties of Reinforcements	•	39
IV.	Crack Widths at the Level of Reinforcement at First Load Cycle	•	57
V.	Crack Widths at the Level of Reinforcement at f = 40 ksi for Beams Subjected to Repeated Loads	•	58
VI.	Crack Widths at the Extreme Tensile Fiber at First Load Cycle	•	59
VII.	Crack Widths at the Extreme Tensile Fiber at f = 40 ksi for Beams Subjected to Repeated Loads	• ,	60
VIII.	Number of Cracks and Crack Spacing in Constant Moment Region	•	62
IX.	Splice Tests		64
Χ.	Comparison of Maximum Crack Widths Between Observed Values and Values Predicted by Equations (2.18) and (2.19) at First Load Cycle	•	76
XI.	Comparison of Maximum Observed Widths of Crack and Widths Predicted by Equation (2.26) Under 100 or More Repeated Loads at f = 40 ksi	•	80
XII.	Cumulative Damage in Fatigue of Beams Reinforced With Smooth Welded Wire Fabric	• 、	91

V

## LIST OF FIGURES

Figu	lire	age
1.	Effective Concrete Area	13
2.	Average Stress-Strain Curves for Reinforcement	38
3.	Schematic of Setup	40
4.	Comparison of Maximum Crack Widths of Beams Subjected to Static Loading	44
5.	Comparison of Maximum Crack Widths of Beams Subjected to Repeated Loading	50
6.	Maximum Crack Widths at the Level of Reinforcement Under Static Loads	69
7.	Maximum Crack Widths at the Extreme Tensile Fiber Under Static Loads	70
8.	Increase in Maximum Crack Widths at the Level of Reinforcement Under Repeated Loads	73
9.	Increase in Maximum Crack Widths at the Extreme Tensile Fiber Under Repeated Loads	74
10.	Average Crack Widths at the Level of Reinforcement Under Static Loads	81
11.	Average Crack Widths at the Extreme Tensile Fiber Under Static Loads	82
12.	Increase in Average Crack Widths at the Level of Reinforce- ment Under Repeated Loads	84
13.	Increase in Average Crack Widths at the Extreme Tensile Fiber under Repeated Loads	85
14.	S-N Relationship for $4x12:W40xW20$ with R = 0.25	88
15.	Modified Goodman Diagram for 4x12:W40xW20	89

## LIST OF SYMBOLS

$A = A_e/m$	- Average effective concrete area around a reinforc-
	ing bar, sq. in.
A <sub>c</sub>	- Cross-sectional area of concrete, sq. in.
$A_e = 2b'(h-d)$	- Effective area on concrete in tension, sq. in.
A <sub>l</sub>	- Area of longitudinal wire, sq. in.
A s	- Area of tensile reinforcement, sq. in.
At	- Area of transverse wire, sq. in.
Ъ	- Width of beam at compression side of beam, in.
Ъ'	- Width of beam at centroid of tensile reinforcement,
	in.
<sup>c</sup> 1, <sup>c</sup> 2	- Coefficients dependent on distribution of bond
	stress, bond strength and tensile strength of con-
	crete
С	- Coefficient dependent on surface roughness
C <sub>s</sub>	- Coefficient related to S <sub>t</sub>
Cavg	- Average distance between cracks, in.
cmax	- Maximum distance between two cracks, in.
c <sub>min</sub>	- Minimum distance between two cracks, in.
D	- Nominal diameter of reinforcement, in.
Dl	- Longitudinal wire diameter, in.
Dt	- Transverse wire diameter, in.
đ	- Affective depth of reinforcement, in.

e	- Factor determining the diameter of the concrete area
	affected by the extension of the reinforcement
E s	- Modulus of elasticity of steel, ksi
f'c	- Compressive strength of concrete, ksi
f	- Steel stress at crack section, ksi
fsl	- Steel stress at the limiting stage of crack
	formation, ksi
ft	- Tensile strength of concrete
h	- Overall depth of beam, in.
$h_1 = (1-k)d$	- Distance from the centroid of the reinforcement to
	the neutral surface, in.
$h_2 = h - kd$	- Distance from the extreme tensile face to the
	neutral surface, in.
I <sub>c</sub>	- Moment inertia of gross concrete section
K	- Ratio of total perimeter to total area of steel
$K_1 = 2ru E_s/f_t$	- Constant
к <sub>2</sub>	- Constant = 47,5000 ksi for deformed bars; 29,700
	ksi for smooth bars (recommended by CEB)
k	- Distance from neutral surface to compressive face
	divided by effective depth of beam
m	- Number of tensile reinforcing bars
N	- Number of cracks in the constant moment region
N	- Fatigue life of a specimen tested at the stress level
n	- Ratio of modulus of elasticity of steel to that of
	concrete
<sup>n</sup> i	- Number of cycles at a stress level
r	- Factor reflecting distribution of bond stress

viii

S

Sl

s<sub>t</sub>

t<sub>b</sub>

te

ts

11

w

w<sub>b</sub>

<sup>w</sup>ba

w<sub>b</sub>

w<sub>s</sub>

<sup>w</sup>sa

w s

z

- Stress level
- Spacing of longitudinal wires, in.
- Spacing of transverse wires, in.
- Thickness of concrete bottom cover measured from the extreme tensile fiber to the center of the lowest wire or bar, in.
- The effective thickness of side cover measured from the extreme tensile fiber to the center of the nearest bar or bars, in.
- Thickness of concrete side cover measured from the center of the outer wire or bar, in.

- Bond strength

Average tensile crack widths in the concrete, in.
Maximum measured crack width, at the extreme tensile fiber, in constant moment region, in.
Average crack width at the extreme tensile fiber, in.
Maximum calculated crack width, at the extreme tensile fiber, in constant moment region, in.
Maximum measured surface crack width at the level of reinforcement in constant moment region, in.
Average surface crack width at the level of reinforcement in the constant moment region, in.
Maximum calculated surface crack width at the level of reinforcement in constant moment region, in.
Maximum calculated surface crack width at the level of reinforcement in constant moment region, in.
A quantity limiting distribution of flexural reinforcement

ix.

 $\beta = h_2/h_1$ 

∆w

εs

φ

Σ

face to the neutral surface to the distance from the centroid of the reinforcement to the neutral surface Increase in maximum calculated crack width at the

- Ratio of the distance from the extreme tensile

extreme tensile fiber in constant moment region, in. - Increase in maximum calculated crack width at the

level of reinforcement in constant moment region, in.

- Average steel strain
- Ratio of the assumed effective area to the fully developed area of concrete (see Figure 1)
- $\rho = A_{s}/bd$   $\rho_{e} = A_{s}/A_{e}$   $\rho_{f} = \frac{A_{s}}{b'd/4}$
- Reinforcement ratio

## - Effective reinforcement ratio

- Ratio of the area of steel reinforcement to the area of concrete in tension surrounding the rein-forcement

- Sum of the bar perimeters

**x** .

#### CHAPTER I

#### INTRODUCTION

### 1. Statement of the Problem

The proper design of reinforced concrete members requires adequate strength and serviceability. Strength criteria include shear strength, flexural strength and development of the reinforcement in regions of bar termination, splices, or high moment gradient. Durability and deflection are serviceability criteria. Flexural strength of reinforced concrete members is greatly influenced by the tensile strength characteristic of the reinforcement, while the style of the reinforcement is seldom of significance.

Smooth or deformed welded wire fabric has been commonly used to reinforce slabs for years, and a number of investigators have conducted experiments to determine the crack control, anchorage and splice characteristics of small size welded wire fabric less than 0.5 in. in diameter (1, 2, 3, 4, 5). Recent changes of the ASTM specifications permit smooth welded wire fabric to be larger than 0.5 in. in size.

A number of investigations in recent years have been concerned primarily with the flexural cracking of reinforced concrete members. Cracking in reinforced concrete members is expected under service loads and cannot be economically eliminated unless prestressing is utilized. Various empirical equations have been proposed by investigators to

predict crack widths under static loads (6, 7, 8, 9, 10, 11, 12). Only one equation has been proposed to predict crack width under loads repeated 100 or more times (13).

The splice characteristics of large smooth welded wire fabric under static loads have been studied (14). However, no previous research has been performed on the crack control characteristics of smooth welded wire fabric of sizes larger than 0.5 in. in diameter under static loads, nor has any previous research been performed on the influence of repeated loading on crack widths.

The importance of cracking in reinforced concrete members under static and repeated loads has become increasingly evident during the past 25 years. As designers have gone from allowable stress to ultimate strength design methods, there has been a definite tendency to use materials with higher strengths, which has led to correspondingly higher stresses under service conditions.

Splicing of tensile reinforcement is often required to transfer tensile stress from one wire or bar to another wire or bar. When a splice is accomplished by lapping the reinforcement, the force in the reinforcement must be transferred through the concrete which surrounds the reinforcement in the region of the splice. This force transfer is related to the bond and mechanical anchorage between the reinforcement and the concrete.

Welded wire fabric consists of longitudinal and transverse wires. The spacing of longitudinal and transverse wires which can be adjusted to fit a given situation is not less than 2 in. and seldom greater than 12 in. Intersections are electrically welded together between longitudinal and transverse wires.

Under the action of repeated loading, it is likely that (1) bond between reinforcement and the surrounding concrete will be destroyed; (2) steel stress will increase due to increased internal crackings adjacent to the reinforcement; (3) the crack mechanism in concrete at the level of reinforcement will increase due to tensile stress in the reinforcement; (4) in areas of increasing moment the welded intersection between longitudinal and transverse wires will fail in fatigue; or (5) the longitudinal wires will fail in fatigue. For the first three cases, the crack widths would be expected to be increased; for the last two cases, total collapse of the slab would be possible.

### 2. Object and Scope

The primary object of this study is to determine the crack control properties of large diameter smooth welded wire fabric when used to reinforce concrete beams subjected to static and repeated loads. As a secondary objective, the fatigue characteristics of lap splices of smooth welded wire fabric are also studied. It is intended to compare the results obtained with concrete beams reinforced with high strength smooth welded wire fabric to those obtained with similar beams reinforced with high strength deformed bars.

This study is based upon the results from tests of 19 flexural beams reinforced with different styles of high strength smooth welded wire fabric and high strength deformed bars. Two of the beams were intentionally reinforced with moderately rusted smooth welded wire fabric; a third beam was reinforced with greased smooth welded wire fabric. Normal weight concrete was used for all specimens.

#### CHAPTER II

#### LITERATURE REVIEW

The cracking of concrete has been studied since the beginning of this century. Modern design concepts permit cracking in nonprestressed reinforced concrete structures because it cannot be economically eliminated under service loads. This chapter summarizes the results of research in this area.

Berry and Goldbeck (15) studied the effect of repeated loads on the strength and elastic properties of reinforced concrete beams as early as 1908. In tests that involved as many as 1.75 million load repetitions, they found that neither the static ultimate strength nor the maximum deflection under ultimate load of specimens was influenced. Hairline flexural cracks grew deeper as the number of load repetitions increased, but no cracks extended beyond the neutral axis before one million repetitions. The width of the cracks was not studied. Bond between steel and concrete was not appreciably affected, which was apparent from the difficulty of removing the steel from the breaking beams. The position of the neutral axis was not changed by repeated loads. The most creep of concrete at the level of reinforcement occurred in the first few thousand repeated loads. The creep of concrete on the compression side of the reinforced member was relatively large for the first few thousand repeated loads and increased with stress applied and number of repetitions.

Probst (16) studied the formation of cracks in reinforced concrete beams under repeated loads and the influence of these cracks on load capacity. He found that loads repeated at less than half of the ultimate static load had no influence on the ultimate static capacity of the beams. Well-aged concrete did not have as much permanent deflection as did younger concrete. Because of shrinkage, the width of cracks was larger in aged reinforced concrete beams than in younger beams. He concluded that repeated loadings which produced stresses and strains below the critical level did not affect the ultimate load. Critical stress was considered to be produced at about 60 percent of the ultimate compressive strength. Probst also concluded that there was no permanent set of deflection if the beams were not loaded beyond the elastic range; however, creep was not considered. Crack widths increased as load was applied to the beams and decreased when load was removed.

Both Berry and Probst found that under the usual conditions of service, the number of cracks was increased slightly. Probst also found that the width of cracks was increased considerably by repeated loading. The increases were greatest for the first few repetitions of load, after which both the number and width of cracks changed by only small amounts.

In 1934, Mylrea (3) studied the properties of cold drawn electrically welded wire fabric and its effectiveness in 14 concrete slabs after researchers questioned the use of wire fabric as reinforcement. He considered fabrics of 4x12:W017xW009 and 4x6:W047xW02. Fabric style 4x12:W017xW009 indicates that the longitudinal wires are spaced 4 in, on centers, and transverse wires are spaced 12 in. on centers; the crosssectional areas of the longitudinal and transverse wires are 0.017 sq. in. and 0.009 sq. in., respectively. He found that welded wire fabric can be

expected to develop its full ultimate strength in reinforced concrete slabs. The electrical welding of the wire intersections did not reduce the ultimate strength of the individual wires.

Watstein and Parsons (17) sought to determine the factors governing the width of cracks and their spacing in symmetrically reinforced concrete cylinders subjected to axial tension. They used smooth round bars, different styles of deformed bars, one webbed (Isteg) bar, and threaded bars. Surface characteristics of the reinforcement and the reinforcement ratio were variables in their tests. Each cylinder contained only one reinforcement, which was symmetrically placed. Their tests were performed in an effort to verify the results of previous investigations which had indicated that the spacing and width of cracks were nearly proportional to the ratio of the diameter of the reinforcement to the reinforcement ratio, D/p (18, 19, 20). Available information had indicated that the rate of increase in crack width with increase in steel stress was independent of the compressive strength of the concrete. The first crack developed at a stress approximately equal to the modulus of rupture of the concrete. Deformed bars were more effective in controlling the initial and final widths of cracks than were smooth bars; deformed bars were also superior to smooth bars in controlling the enlargement of the cracks caused by repeated and sustained loads. Based on their experimental results, an equation was proposed by Watstein and Parsons to predict the average width of crack at the surface of an axially reinforced concrete cylinder:

$$w = C_1 \frac{D}{\rho} [f_s \times 1000 - C_2 (\frac{1}{\rho} + n)]$$
(2.1)

where

w = average tensile crack widths in the concrete, in.;  $C_1$  and  $C_2$  = coefficients, dependent on distribution of bond stress, bond strength and tensile strength of concrete; D = nominal diameter of reinforcement, in.;  $\rho = A_s/A_c$ , reinforcement ratio;  $A_s$  = area of tensile reinforcement, sq. in.;  $A_c$  = cross-sectional area of the concrete, sq. in.;  $f_s$  = steel stress at crack section, ksi; n = ratio of modulus of elasticity of steel to that of concrete.

For a given steel stress and a given style of reinforcement, the most prominent factor affecting crack width and spacing was the ratio  $D/\rho$ . The crack width was independent of the compressive strength of the concrete.

Watstein and Seese (21) studied the influence of various kinds of deformations on crack widths in reinforced concrete cylinders subjected to static and repeated tensile loading. They found the greatest crack widths occurred in the cylinders reinforced with smooth bars. They concluded that at a given steel stress, the width of cracks decreased with an increase in bond efficiency, and the width of cracks was proportional to the spacing of the cracks. After 19 applications of 40 ksi stress, the width of cracks increased by 15.3 percent in cylinders reinforced with smooth bars, compared to 7.9 percent in cylinders reinforced with deformed bars.

Wästlund and Jonsson (22) considered the influences of the diameter and surface roughness of reinforcement, the reinforcement ratio, and strength of concrete on the formation of cracks in T-beams. They

studied seven bridges in service and also conducted laboratory tests on T-beams. They found that:

 At a given steel stress the width of cracks increased almost linearly with the bar diameter for beams of different dimensions and amounts of reinforcement.

2. For a given beam cross section, reinforcement diameter and steel stress, the width of cracks decreased slightly when the area of the steel was increased.

3. Given equal variables of diameter, reinforcement ratio, and strength of concrete, wider beams had considerably larger crack widths, while an increase in beam depth had no effect on the width of cracks.

4. Crack width decreased with an increase in the surface roughness of reinforcing bars.

5. Concrete strength did not exert any noticeable influence on the width of cracks.

Although welded wire fabric reinforcement was widely used, little was known about its bond properties until 1952. From pull-out tests Anderson (4) discovered that end slip-resistance of welded wire fabric was proportional to the weld strength of the transverse wires. In his study he used 6xl2:W054xW054, 6x6:W054xW054 with ultimate strength of 85 ksi. His study also included other fabric styles of wire sizes ranging from W021 to W086 with specified minimum ultimate strength of 70 ksi. The specified minimum yield strength was 56 ksi. Anderson concluded that for proper anchorage it would be necessary to select an appropriate ratio of transverse to longitudinal wire size, and to specify minimum weld shear strengths.

Carlton and Senne (5) also studied the bond distribution and proper spacing of longitudinal and transverse wires in welded wire fabric. Reinforcement in two-way slabs consisted of 6x12:W05xW05, 6x12:W05xW04 and 4x8:W05xW05. They concluded that a close spacing of transverse wires helped control the width and spacing of cracks and suggested that this fact could be used to advantage for slabs subjected to severe repeated loading or freezing and thawing.

Clark (7) studied cracking in reinforced concrete flexural slabs and beams. His test variables were the width, depth and span of the slabs, bar size and reinforcement ratio. The members were loaded at quarter points, and the width of cracks was measured in the region of constant moment. He measured the width and spacing of cracks at computed steel stresses ranging from 15 to 45 ksi. The reinforcement was deformed intermediate grade steel bar meeting ASTM A 15-54 T for billet steel bars; the deformations of the reinforcing bars met ASTM A 305-53 T.

Using Equation (2.1), developed by Watstein and Parson, and the results from his own experiment, Clark evaluated the coefficients  $C_1$  and  $C_2$ . He found the following empirical values:

$$C_1 = 2.27 \times 10^{-8} (h-d)/d$$

and

$$C_{2} = 56.6$$

where

h = overall depth of beam, in.;

d = effective depth of reinforcement, in.

With these values and with the modular ratio, n, assumed equal to 8, Clark expressed Equation (2.1) as

$$w_{ba} = 2.27 \times 10^{-8} \left(\frac{h-d}{d}\right) \frac{D}{\rho} [f_s \times 1000 - 56.6 \left(\frac{1}{\rho} + 8\right)]$$
 (2.2)

where

wba = average crack width at the extreme tensile fiber, in.; h = overall depth of beam, in.; d = effective depth of reinforcement, in.; D = nominal diameter of reinforcement, in.; ρ = A<sub>s</sub>/bd, reinforcement ratio; A<sub>s</sub> = area of tensile reinforcement, sq. in.; b = width of beam at compression side of beam, in.;

 $f_{s}$  = steel stress at crack section, ksi.

Clark pointed out that the average width of tensile cracks was proportional to the product of  $D/\rho$ , the ratio of diameter of the bar to the reinforcement ratio, and (h-d)/d, the ratio of thickness of concrete cover to the effective depth of the reinforced member. The width of cracks was found to be proportional to the increase in steel stress beyond the steel stress which existed at the time of initial cracking. The average spacing of cracks reduced rapidly with an increase in steel stress immediately after the initiation of cracking. At a steel stress of approximately 30 ksi, the average crack spacing approached a constant value. Clark also found that the maximum width of cracks averaged 1.64 times the mean width of cracks for steel stresses between 15 and 40 ksi.

Hajnal-Kónyi (23) carried out preliminary experiments on beams reinforced with plain round and twisted deformed (Tentor) bars under repeated loading. The stress varied between 1.26 and 17.91 ksi in the beam with plain bars and between 2.37 and 32.12 ksi in the beam with Tentor bars. The maximum width of crack in the beam reinforced with plain bars increased from 0.002 in. after the first application of load to 0.008 in. after 2,000,000 applications of load, but the maximum width of crack in the beam reinforced with Tentor bars remained constant at 0.004 in.

Shearcroft (23) studied the effects on crack formation produced by additional percentages of small wires or smooth bars accompanying large size bars as reinforcement. His study involved both square twisted bars and plain round bars with small wires and smooth bars used as crack controlling reinforcement. From his results he developed an equation for maximum crack widths for steel stress between 20 and 40 ksi:

$$w_{b}^{*} = \frac{1.18 \text{ f}_{s} \times 10^{-3}}{K}$$
(2.3)

where

w = maximum calculated crack width at the extreme tensile fiber, in constant moment region, in.;

f = steel stress at crack section, ksi;

K = ratio of total perimeter to total area of steel.

Square twisted bars and wires were selected as the crack controlling reinforcement because of the high bond capacity which can be developed.

Abeles (23) investigated a number of problems concerning width of cracks under repeated loading. He concluded that the maximum width of cracks depends mainly on the percentage of reinforcement, the distribution of the steel, and the maximum stress produced by the repeated loading.

In 1958, Chi and Kirstein (24) introduced a concept regarding the formation of cracks in reinforced concrete beams by subjecting them to pure bending and using semi-empirical equations to predict the average minimum spacing and the average width of cracks in the reinforced concrete beams. In their experiment, the variables were beam depth, width, span length, and reinforcement ratio. The reinforcing bars were intermediate grade steel which met ASTM A 15-54 T requirements for billet steel bars and the deformations complied with ASTM A 305-53 T specifications.

From the results of their experiment, Chi and Kirstein derived two semi-empirical equations:

average 
$$c_{\min} = 5\phi D$$
 (2.4)

and

$$w_{sa} = 5\phi D \left[ \frac{f_s \times 1000 - \frac{2500}{\phi D}}{E_s} \right]$$
 (2.5)

where

c = minimum distance between two cracks, in.;

- D = nominal diameter of reinforcement, in.;
- w = average surface crack width at the level of reinforcement in the constant moment region, in.;
  - f = steel stress at crack section, ksi;
- E = modulus of elasticity of steel, ksi.

The value of  $\phi$  and, therefore, the predicted values of spacing and width of cracks varied with the thickness of cover. The average minimum spacing of cracks was proportional to the product of the diameter of reinforcing bar and the parameter  $\phi$ , which was dependent on the arrangement and diameter of reinforcement. The average width of cracks at the steel was the product of average minimum spacing of cracks and a computed steel strain. For computed steel stresses ranging from 15 to 40 ksi,







Assumed Effective Area =  $A_e$ 

0 Fully Developed Area =  $e^2 A_s$ 



the ratio of maximum to average width of crack varied from 1.08 to 2.64 with an average value of 1.63. This ratio was not constant at any particular steel stress. Compressive concrete strength had no effect on crack widths.

High strength steel as concrete reinforcement received some attention by researchers and by the construction industry in the United States in the early 1900's. However, intensive research in this area was not begun in America until the 1940's when the use of high strength steel reinforcing bars was already becoming well accepted in Europe. "High strength steel" refers to steel having a yield strength in excess of 50 ksi. High strength steel normally is produced either by metallurgical means, which consists of relatively high carbon content steel, or by cold working an ordinary grade of steel (27). Although the use of high strength steel will allow higher working stresses, precautions must be taken to avoid excessive crack widths, deflection and fatigue.

Wästlund (25), in reviewing European research on cracking of reinforced concrete, summarized that "The main purpose of a crack formula is to serve as a guide in ensuring that the structure shall be sound with respect to crack formation." He observed most existing crack spacing formulas to be of the following type:

$$c_{avg} = C_1 + C_2 \frac{f_t^A e}{u\Sigma_o}$$
 (2.6a)

where

c<sub>avg</sub> = average distance between cracks, in.; C<sub>1</sub>, C<sub>2</sub> = coefficients, dependent on distribution of bond stress, bond strength and tensile strength of concrete;

f<sub>+</sub> = tensile strength of concrete;

A = effective area on concrete in tension, sq. in.;

u = bond strength;

 $\Sigma_{2}$  = sum of the bar perimeters.

Wastlund also observed the mean crack width, w sa, to be the product of average crack spacing and steel strain at crack section:

$$w_{sa} = c_{avg} \frac{\dot{t}_s}{E_s}$$
(2.6b)

where

w = average side crack width at the level of reinforcement in the constant moment region, in.;

f = steel stress at crack section, ksi;

E<sub>c</sub> = modulus of elasticity of steel, ksi.

In 1945, Jonsson, Ostermann and Wästlund used the following equation to predict maximum crack width under pure bending (23):

$$w_{b}^{*} = CD \left[ \frac{I_{c}f_{s}}{dA_{s}h_{2}E_{s}} \right]^{2/3}$$
(2.7)

where

w = maximum calculated crack width at the extreme tensile fiber in constant moment region, in.;

C = coefficient dependent on surface roughness;

- D = nominal diameter of reinforcement, in.;
- I = moment inertia of gross concrete section, in.<sup>4</sup>;
- $f_{c}$  = steel stress at crack section, ksi;

d = effective depth of reinforcement, in.;

A = area of tensile reinforcement, sq. in.;

h<sub>2</sub> = distance from the extreme tensile face to the neutral surface, in.; E = modulus of elasticity of steel, ksi.

The parameter C was taken equal to 0.23 and 0.16 for plain and deformed (Kam steel 40) bars, respectively. Wästlund (25) reported that Equation (2.7) has been used in Sweden for the design of all highway bridges since 1947. For the dead load, the maximum crack width calculated from this equation must not exceed 0.012 in.; for the sum of the dead load and half the live load, maximum crack width is limited to 0.016 in.

Mathey and Watstein (26) studied the properties of reinforced concrete beams containing high strength deformed steel bars. They determined the effect of higher steel stresses on the resisting moment, width of cracks, strain in the concrete and steel, deflection, load carrying capacity, and the mode of failure of beams. Six types of reinforcement were used with nominal yield strengths ranging from 40 to 100 ksi. The reinforcement ratio was inversely proportional to the yield strength, thus providing equal resistance to yielding under pure bending. The average width of cracks in the region of constant bending moment increased with decreased ratio of reinforcement and increased steel stresses. The resulting average width of cracks compared favorably with Clark's formula, Equation (2.2). The ratio of maximum to average crack width was about 1.6 compared to Clark's ratio of 1.64.

Guralnick (27) investigated flexural strength, shear strength, load deflection, and cracking characteristics of concrete T-beams reinforced with high strength deformed bars of high carbon content. All reinforcement had a nominal yield strength of 80 ksi, and the surface deformations conformed to ASTM A 305 specifications. He found that the width of flexural cracks was proportional to the steel stress, and that the

width of cracks varied with the ratio of surrounding tensile concrete area to total bar perimeter.

From a design standpoint, reinforced concrete members are required to have adequate strength and serviceability. Although durability and deflections are both serviceability criteria which are influenced by cracking, any attempt to limit crack widths in design is usually an attempt to insure necessary durability. Hognestad (28) reasoned that high strength is more advantageous than normal strength steel primarily because the tonnage cost per unit yield strength decreases as yield strength increases. While the unit cost of high strength steel is higher, its use may reduce the cost of a structure by reducing the steel tonnage, placing costs and, possibly, steel congestion. A reduction in the size of major members is often possible, which also reduces dead loads. Of course, the amount of cost reduction depends on the type of structure involved.

The principal objective of Hognestad's study (8) was to explore the assumptions and calculations involved with the European Concrete Committee (CEB) crack equation (29):

$$\sigma_{s}^{*} = \frac{D}{\rho_{e}} \frac{f_{s}}{K_{1}}$$
(2.8)

where

w = maximum calculated surface crack width at the level of reinforcement in constant moment region, in.;

D = nominal diameter of reinforcement, in.;

 $\rho_{p}$  = effective reinforcement ratio;

 $f_s = steel stress in reinforcement at a crack section, ksi;$ 

 $K_1 = \text{constant taken as equal to } 2ru E_c/f_t;$ 

r = factor reflecting distribution of bond stress;

u = bond strength;

E = modulus of elasticity of steel, ksi;

f<sub>+</sub> = tensile strength of concrete.

For  $\rho_{c}$  ranging from 0.02 to 0.20, Equation (2.8) became (30):

$$w_{s}^{*} = [4.5 + \frac{0.40}{\rho_{e}}] D \frac{r_{s}}{K_{2}}$$
 (2.9)

where

 $K_2 = \text{constant taken as 47,500 ksi for deformed bars and 29,700 ksi for smooth bars (recommended by CEB).}$ 

In Hognestad's experiment, 29 of 36 concrete beams were reinforced with modern American deformed bars which met the deformations requirements of ASTM A 305. Bar diameter, beam geometry, and reinforcement cover were the major variables. Bar type, concrete strength and reinforcement ratio were secondary variables.

From the results of his study, Hognestad indicated that crack width was proportional to steel stress for all bar types. Crack width was proportional to bar diameter for plain bar and old-type deformed bars, but less dependent on bar diameter for modern deformed bars. Modern American bars were highly effective in crack control. For beams reinforced with modern American deformed bars, the width of crack was less dependent on the effective reinforcement ratio,  $\rho_e$ , than is indicated by the CEB equation. The width of cracks was independent of concrete strength, width and depth of beam, but strongly influenced by the concrete cover. The best crack control was observed when the reinforcing bars were well distributed over the effective concrete area. Crack width was not directly proportional to the crack spacing. The CEB crack width Equation (2.9) predicted the width of cracks fairly well for beams reinforced with modern American deformed bars, but the CEB crack width Equation (2.8) was not accurate for beams reinforced with modern American deformed bars. The maximum crack widths were 1.36, 1.50, 1.49, and 1.42 times the average crack width at steel stresses of 20, 30, 40, and 50 ksi, respectively.

Gaston and Hognestad (31) studied one full-scale and two model roof girders. By Equation (2.9), the calculated ratio of the crack widths of the models to the full-scale girder was 0.43. The observed ratios were 0.42 for average crack width and 0.30 for maximum crack width. The models' geometric-scale ratio was 0.38. They found that Equation (2.9) accurately predicted the crack width for both models and the full-scale girder.

Kaar and Mattock (9) investigated the cracking behavior of girder and slab specimens having cross sections of the type used in bridge construction under static and repeated loads. All deformed bars used in the study met ASTM A 431-59 T strength requirements and ASTM A 305 deformation requirements. The maximum crack widths increased most rapidly during the first ten thousand applications of load. The width and spacing of cracks at a given steel stress were proportional to the average area of concrete surrounding each reinforcing bar; also, the width of cracks was proportional to the steel stress.

Based on these results, Kaar and Mattock modified Equation (2.9) to predict crack width as a function of average area of concrete surrounding each reinforcing bar and steel stress:

$$w_s^* = 0.067 \sqrt{A} f_s \times 10^{-3}$$
 (2.10)

where

- s = maximum calculated surface crack width at the level of reinforcement in constant moment region, in.;
- A = average effective concrete area around a reinforcing bar, sq. in.;

 $f_s$  = steel stress in reinforcement at a crack section, ksi. This equation overestimated the crack width for large values of A. From their experimental results, Kaar and Mattock derived a similar equation with a different coefficient:

$$w_s^* = 0.115 \sqrt[4]{A} f_s \times 10^{-3}$$
 (2.11)

This equation was effective for predicting maximum surface crack widths at the level of reinforcement for A values ranging from 3.0 to 50.0 sq. in.

The maximum width of crack at the extreme tensile face can be expressed by modification of Equation (2.11) (10):

$$w_b^* = 0.115 \sqrt[4]{A} f_s \beta \times 10^{-3}$$
 (2.12)

where

- w<sup>\*</sup> = maximum calculated crack width, at the extreme tensile fiber, in constant moment region, in.;
- A = average effective concrete area around a reinforcing bar, sq. in.;
- f = steel stress at crack section, ksi;
- $\beta$  = ratio of the distance from the extreme tensile face to the neutral surface to the distance from the centroid of the reinforcement to the neutral surface.

Equation (2.12) was included in a report by Kaar and Hognestad (10) on a study of flexural crack control for the slab flanges of T-beams subjected to negative bending moments. All bars used in the study conformed to ASTM A 305 requirements for deformations. The observed maximum crack widths were in reasonable agreement with the values computed by Equation (2.12) at service load.

Kaar and Hognestad (10) confirmed the results of Kaar and Mattock (9); i.e., the maximum widths of cracks at the level of the centroid of the reinforcing steel and at the extreme tensile face are functions of the average area of concrete surrounding each reinforcing bar and the steel stress. To achieve crack control in design of T-beam flanges, the reinforcing steel should be well distributed and the smallest practical lateral spacing should be used between individual reinforcing bars.

Atlas et al. (1) studied all available test data related to the behavior of one-way concrete floor slabs reinforced with welded wire fabric; this data included the material properties of the reinforcement and the spacing and width of cracks. They also tested one-way slabs reinforced with welded wire fabric which conformed to ASTM A 185 requirements. Fabric styles with longitudinal wires ranging from 0.135 in. to 0.490 in. in diameter were considered.

Atlas et al. found crack spacing and steel stress to be the major factors influencing crack widths. Crack spacings were influenced by the area of concrete surrounding each reinforcing bar, the diameter of the longitudinal wires, the effective reinforcement ratio, and the spacing of transverse wires. The effective reinforcement ratio was considered to be the ratio of the area of tensile reinforcement to the average effective concrete area around a reinforcing bar.

The average spacing of cracks in a slab subjected to static loading was expressed as

$$c_{avg} = C_{s} (3.0 + 0.40 S_{t})$$
 (2.13)

where

c<sub>avg</sub> = average distance between cracks, in.; C<sub>s</sub> = coefficient related to S<sub>t</sub>; S<sub>t</sub> = spacing of transverse wires, in.; and, specifically,

> for  $S_t = 12$  in.,  $C_s = 1 + 0.024$  (D/ $\rho_e - 43$ ); for  $S_t = 6$  in.,  $C_s = 1 + 0.008$  (D/ $\rho_e - 33$ ); for  $S_t = 3$  in.,  $C_s = 1$ ;

where

W

D = nominal diameter of reinforcement, in.;

 $\rho_e$  = effective reinforcement ratio.

Broms (6) observed that crack spacing of concrete flexural members reinforced with one modern deformed bar decreased with increasing stress in the reinforcement until a stress of approximately 30 ksi was reached. The average crack spacing was found to be twice the thickness of concrete side cover, which was approximately 30 percent higher than Broms' analytical crack spacing of one and one-half times the thickness of concrete side cover.

Broms assumed that the elongation of the concrete between two tensile cracks at the level of reinforcement was small and negligible compared to the elongation of the reinforcement. Therefore, the average crack width of concrete members reinforced with one modern deformed bar was considered to be equal to the product of average measured crack spacing and average strain:

$$sa = 2t \varepsilon$$
 (2.14)

where

- w = average surface crack width at the level of reinforcement in the constant moment region, in.;
  - t = thickness of concrete side cover measured from the center of the outer wire or bar, in.;
  - $\varepsilon_{z}$  = average steel strain.

This equation is applicable when the nominal steel stress exceeds 20 ksi and when the thickness of concrete side cover ranges from 1.25 to 3.0 in. This equation is independent of the ratio and size of the reinforcement as well as the tensile and compressive strengths of the concrete.

Broms (6) also studied internal crack formation by injecting resin into tension specimens. He observed that internal cracks did not always penetrate to the surface and such cracks had a maximum width near the surface of the reinforcement.

For concrete members reinforced with two or more modern bars, Broms and Lutz (32) modified Equation (2.14) by replacing the thickness of concrete side cover by  $t_e$ , the effective thickness of the concrete side cover, the distance from the surface to the center of the nearest bar or bars. With this modification, the average width of crack in a concrete beam reinforced with two or more modern bars was expressed as

$$v_{sa} = 2t \varepsilon_{s}$$
(2.15)

where

- w = average surface crack width at the level of reinforcement in the constant moment region, in.;
- t = the effective thickness of side cover measured from the extreme tensile fiber to the center of the nearest bar or bars, in.;

 $\varepsilon_{z}$  = average steel strain.

This equation depends on the effective thickness of the concrete side cover, but is independent of shape and size of the member and of the size and ratio of reinforcement.

From Broms and Lutz's (32) results, the maximum crack width could be calculated approximately as 1.5 to 2.0 times the average crack width, which leads to the following equation of maximum side crack width.

$$w_{s}^{*} = 4t_{e}\varepsilon_{s}$$
(2.16)

where

w = maximum calculated surface crack width at the level of reinforcement in constant moment region, in.;

te = the effective thickness of side cover measured from the
 extreme tensile fiber to the center of the nearest bar
 or bars, in.;

 $\varepsilon_{c}$  = average steel strain.

Therefore, the predicted surface crack width is small if the reinforcement is distributed uniformly with a large number of small diameter bars placed as close to the surface as possible.

Resin was injected into certain specimens in Broms and Lutz's (32) experiment. After the resin had hardened completely, the members were cut apart and the internal cracks were examined. They observed that a large number of small internal cracks developed close to the reinforcement but that these cracks did not extend to the surface.

Abeles (33) reported that both the deflection and width of cracks in prestressed and reinforced concrete members increase with a decrease in the reinforcement ratio and with sustained and repeated loads.

Kaar (34) reported the results from a model study of T-beams with sizes ranging from 1/4 to full scale. Under the same maximum steel stress of 30 ksi, the crack patterns were the same but the number of cracks was greater for the larger members. There were a few exceptionally wide cracks on full-size beams, but not on small-scale models. The crack widths were proportional to the square root of the scale factor. The majority of the crack widths observed in this study validated Equation (2.12). The even distribution of reinforcing steel and the smallest practical spacing between individual reinforcing bars controlled the crack width.

Borges (35) indicated the average crack width was equal to the product of average crack spacing and the difference in strains of steel and concrete. The average crack spacing was a function of the thickness of concrete cover. He derived an equation of the following form which was the basis for the Comité Européen du Béton-Fédération Internationale de la Précontrainte (CEB-FIP) International Recommendations:

$$w_{s}^{*} = \frac{1}{E_{s}} \left[2.5t_{b} + 0.067 \frac{D}{\rho}\right] \left[f_{s} - \frac{0.107}{\rho}\right]$$
 (2.17)

where

ws = maximum calculated surface crack width at the level
of reinforcement in constant moment region, in.;
Es = modulus of elasticity of steel, ksi;
tb = thickness of concrete bottom cover measured from the
extreme tensile fiber to the center of the lowest
wire or bar, in.;

D = nominal diameter of reinforcement, in.;

 $\rho$  = reinforcement ratio;

 $f_s$  = steel stress in reinforcement at a crack section, ksi. To avoid excessive crack widths in concrete members reinforced with small ratios of reinforcement at high steel stress levels, the equation indicates that more bars of smaller diameter are necessary.
Lutz, Sharma and Gergely (36) reported the crack widths in reinforced concrete beams under sustained loading. The major increase in crack width occurred during the first month after load application; thereafter, the rate of increase decreased. The width of cracks increased as a result of (1) the increase in steel stress at the sections between surface cracks caused by increased internal cracking, and (2) the creep of concrete in the compression zone of the beam causing a lowering of the neutral axis of the beam, a decreased internal lever arm, and, therefore, an additional strain of the tension reinforcement.

Gergely and Lutz (12) made a statistical analysis of data obtained from the results of Broms (6), Clark (7), Hognestad (8), Kaar and Mattock (9), Kaar and Hognestad (10), and Rüsch and Rehm (11). Data used in their analysis came from investigations which utilized high strength steels which satisfied the ASTM A 305 standard on bar deformations. They concluded that the steel stress, effective area of concrete, the number of bars, and the side or bottom covers were major variables. From the analysis of all data, they suggested two equations to predict maximum surface crack width at the level of reinforcement and at the extreme tensile fiber, respectively:

$$w_{s}^{*} = 0.076 \frac{\frac{3}{v_{t}} \frac{1}{A}}{1 + \frac{2}{3} \frac{t_{s}}{h_{1}}} f_{s} \times 10^{-3}$$
 (2.18)

and

$${}^{*}_{b} = 0.076 \sqrt[3]{t_{b}A} \beta f_{s} \times 10^{-3}$$
 (2.19)

where

w

s = maximum calculated surface crack width at the level of reinforcement in constant moment region, in.;

- t = thickness of concrete side cover measured from the center of the outer wire or bar, in.;
- A = average effective concrete area around a reinforcing bar, sq. in.;
- h<sub>1</sub> = distance from the centroid of the reinforcement to the neutral surface, in.;
- f = steel stress, calculated by elastic cracked section
   theory, ksi;
- w = maximum calculated crack width, at the extreme tensile fiber, in constant moment region, in.;
- t = thickness of concrete bottom cover measured from the extreme tensile fiber to the center of the lowest wire or bar, in.;
  - $\beta$  = ratio of the distance from the extreme tensile face to the neutral surface to the distance from the centroid of the reinforcement to the neutral surface.

Equation (2.19) has been adopted by the American Concrete Institute for code provisions related to crack control (40).

Nawy (37) had indicated that the control of cracking in concrete structures is as important as the control of deflections because high strength reinforcement permits higher allowable steel stresses and, therefore, larger crack widths. Researchers generally agree that crack width is a function of crack spacing, steel stress, the thickness of concrete cover, and distribution of the reinforcement. In 1968, Nawy used Equations (2.12), (2.18) and the following equation to predict width of cracks in beams.

$$w_b^* = 3.3 t_b \frac{f_s}{E_s} \beta$$
 (2.20)

where

w<sup>°</sup> = maximum calculated crack width, at the extreme tensile fiber, in constant moment region, in.;

- t = thickness of concrete bottom cover measured from the
   extreme tensile fiber to the center of the lowest wire
   or bar, in.;
- f = steel stress at crack section, ksi;
- E\_ = modulus of elasticity of steel, ksi;
  - $\beta$  = ratio of the distance from the extreme tensile face to the neutral surface to the distance from the centroid of the reinforcement to the neutral surface.

Equation (2.20) was recommended by the Cement and Concrete Association. Based on the results from 107 beams, Nawy found that Equation (2.19) is more accurate than Equations (2.12) and (2.20) for predicting width of cracks in beams with high steel stress.

Rahman, Gupta and Gadh (38) report that the crack control characteristics of cold twisted ribbed-Torsteel is superior to plain round steel reinforcement. The width of cracks measured in constant moment regions of beams increased in proportion to the steel stress; the crack spacing tended towards a limiting minimum value. At the same level of steel stress the ratios of maximum to average crack widths were 1.37 and 1.74 for ribbed-Torsteel and plain bar, respectively.

Srinivasa Rao and Subrahmanyam (39) studied concrete one-way slabs reinforced with welded wire fabric. Using statistical methods, an equation was developed to predict the average and maximum crack widths at any level of steel stress:

$$w_{sa} = c_{avg} [\varepsilon_{s} - 0.4 \times 10^{-3}]$$
 (2.21)

where

w<sub>sa</sub> = average surface crack width at the level of reinforcement in the constant moment region, in.; c<sub>avg</sub> = average distance between cracks, in.; ε<sub>a</sub> = average steel strain.

Average crack spacing can be calculated as

$$c_{avg} = \left[0.63 - 0.24 \frac{f_s}{f_{sl}}\right] \left[ \frac{t_b S_l}{10.03 \sqrt[4]{n} \frac{D_t^2}{S_t^2}} \right]$$
(2.22)

where

c<sub>avg</sub> = average distance between cracks, in.; f<sub>s</sub> = steel stress in reinforcement at a cracked section, ksi; f<sub>sl</sub> = steel stress at the limiting stage of crack formation, ksi; t = thickness of concrete bottom cover measured from the

t = thickness of concrete bottom cover measured from the
 extreme tensile fiber to the center of the lowest wire
 to bar, in.;

D<sub>\_</sub> = transverse wire diameter, in.;

S<sub>+</sub> = spacing of transverse wires, in.;

 $D_0 = 1$ ongitudinal wire diameter, in.

From the results of their test data, maximum crack width was given

as

(2.23)

where

s = maximum calculated surface crack width at the level of reinforcement in constant moment region, in.;

w = average surface crack width at the level of reinforcement in the constant moment region, in.

They concluded that the width of crack was a function of the longitudinal and transverse wire diameters and spacings, as well as a function of steel stress.

Lloyd, Rejali and Kesler (2) studied the crack control in one-way slabs reinforced with either deformed wire, deformed bar, or deformed welded wire fabric. Deformed bars satisfied ASTM A 432-65 strength requirements and ASTM A 305-65 deformation requirements. Deformed wires satisfied the ASTM A 496-64 requirements; and the majority of the deformed welded wire fabric exceeded the ASTM A 467-64 requirements. However, some styles of fabric had low weld strengths.

The maximum crack widths measured in the investigation on slabs were in agreement with the results predicted by Equations (2.18) and (2.19). Additional data were taken from Atlas et al. (1) for slabs reinforced with smooth welded wire fabric. Statistical analysis at the 0.95 confidence level indicated that deformed bars, deformed wires, deformed welded wire fabric and smooth welded wire fabric control crack widths equally well in one-way slabs.

ACI 318-71 (40) transformed Equation (2.19) to the following form:

$$z = f_{s} \sqrt[3]{t_{b}A}$$
(2.24)

where

- z = a quantity limiting the distribution of flexural reinforcement;
- f = steel stress in reinforcement at a cracked section, ksi;

The maximum allowable value of z is equal to 175 kips per inch for interior exposure, which corresponds to the crack width limit of 0.016 in. if  $\beta$  is taken equal to 1.20 for beams. For exterior exposure, the

maximum value of z is 145 kips per inch, which corresponds to a crack width of 0.013 in.

ACI Committee 224 (41) reported that the steel stress, the thickness of the concrete cover and area of concrete surrounding each reinforcing bar are important variables for control of cracking.

Maximum permissible crack widths at the tensile face of reinforced concrete structures were recommended by ACI Committee 224 (41) according to given exposure conditions as follows:

Exposure Condition	Maximum Allowable Crack Width
Dry air or protective interior structures	0.016 in.
Humidity, moist air and soil, or exterior structures	0.012 in.
Deicing chemicals	0.007 in.
Seawater and seawater spray and wetting and drying or aggressive agents	0.006 in.
Water retaining or watertight structures	0.004 in.

Equation (2.17), modified by a safety factor of 1.33, was recommended by CEB-FIP for prediction of the crack width for nonrepeated loads (13):

$$w_{s}^{*} = [1.5 t_{b} + 0.16 \frac{D}{\rho_{f}}] [0.069 f_{s} - \frac{0.03}{\rho_{f}}] \times 10^{-3}$$
 (2.25)

where

- w = maximum calculated surface crack width at the level s of reinforcement in constant moment region, in.;

D = nominal diameter of reinforcement, in.;

p = ratio of the area of steel reinforcement to the area
of concrete in tension surrounding the reinforcement;

 $f_s$  = steel stress in reinforcement at a crack section, ksi. In some cases in this study, the width of surface cracks has been observed to increase as much as 100 percent under sustained loading (42). For loads repeated more than 100 times at maximum steel stress, the maximum width of crack at the level of gross reinforcement is given as (13):

$$w_s^* = 0.069 (1.5 t_b + 0.016 \frac{D}{\rho_f}) f_s \times 10^{-3}.$$
 (2.26)

#### CHAPTER III

#### EXPERIMENTAL PROGRAM

#### 1. Specimens

Fourteen beams were tested to study flexural crack control, and five beams were tested to study fatigue characteristics of splices. All beams were 11 ft. long and 8 in. wide. The overall depth of the beams varied from 7 to 16 2/3 in. All flexural crack control specimens has a reinforcement ratio of 0.0075; all splice specimens had a total depth of 12 in. All beams had the same thickness of bottom cover of 2 in., measured from the extreme tensile fiber to the center of lowest wire or bar.

Eight of fourteen flexural crack control specimens were reinforced with either smooth welded wire fabric or deformed bars with the spacing of longitudinal reinforcement at 4-in. centers. The remaining flexural crack control specimens were reinforced with either smooth welded wire fabric having a single longitudinal wire or one deformed bar to represent an 8-in. longitudinal reinforcement spacing. Two of the flexural crack specimens were reinforced with moderately rusted smooth welded wire fabric; another was reinforced with greased smooth welded wire fabric.

Three specimens reinforced with 4x12:W40xW20 smooth welded wire fabric contained a splice at midspan; the sheets of fabric were overlapped 43 in., measured between the outer ends of the longitudinal wires.

The longitudinal wires extended beyond the outermost cross wires 6 in., resulting in an overlap of 31 in. measured between the outermost cross wires. The transverse wires on both sheets of reinforcement were above the longitudinal wires.

Two specimens reinforced with two No. 6 bars contained a 33-in. lap splice of both bars at midspan.

Two of the splice specimens, one with fabric reinforcement and one with deformed bar reinforcement, contained vertical, double leg No. 3 stirrups spaced at 6 in. in the region of the splice.

The lengths of the splices were based on recent test data and code requirements for the development of the yield strength of the reinforcement (14, 40).

To eliminate the possibility of shear failures, vertical, double leg, No. 3 stirrups with nominal yield strength of 40 ksi were placed in the shear spans of all specimens.

Details of the beams are given in Table I.

#### 2. Materials

#### 2.1. Concrete

The concrete used in this study was obtained from a local readymix plant. Type 1 portland cement, river sand from a local source and crushed 1-in. maximum size limestone coarse aggregate were used for all concrete. A nominal 3 ksi mix was used for all beams; however, the small size of the batches made it difficult to obtain consistently uniform concrete. For this reason the plant was requested to supply concrete in a relatively dry condition so that water could be added just

TAB	LΕ	Ι	J.

Beam	Reinfo	rcement Style <sup>a</sup>	Area of Steel, A sq.in.	Reinforce- ment Ratio, p	Width, b in.	Total Depth, h in.	Effective Depth, d in.	Bottom Cover, t <sub>b</sub> in.	Side Cover, t in.		
1		4x12:W40xW20	0.80	0.0075	8	15.33	13.33	2	2		
2	Rusted	4x12:W40xW20	0.80	0.0075	8	15.33	13.33	2	2		
3	Rusted	4 x 6:W40xW20	0.80	0.0075	8	15.33	13.33	2	2		
4		4x12:W30xW20	0.60	0.0075	8	12.00	10.00	2	2		
5		4:No. 5	0.62	0.0075	. 8	12.33	10.33	2	2		
6		4 x 6:W40xW20	0.80	0.0075	8	15.33	13.33	2	2		
7		4:No. 6	0.88	0.0075	8	16.67	14.67	2	2		
8		4x12:W30xW20	0.30	0.0075	8	7.00	5.00	2	4		
9		8:No. 5	0.31	0.0075	8	7.12	5.12	2	4		
10		8x12:W40xW20 <sub>1</sub>	0.40	0.0075	8	8.67	6.67	2	. 4		
11		4x12:W40xW20 <sup>D</sup>	0.40	0.0075	-8	8.67	6.67	- 2	4		
12		8:No.6	0.44	0.0075	8	9.33	7.33	2	4		
13		$4 \ge 6:W40 \ge 0^{D}$	0.40	0.0075	8	8.67	6.67	2	4		
14	Greased	4x12:W30xW20	0,60	0.0075	8	12.00	10.00	2	2		
15		4:No. $6^{c}_{1}$	0.88	0.0110	8	12.00	10.00	2	2		
16		4:No. 6 <sup>c, d</sup>	0.88	0.0110	8	12.00	10.00	2	2		
17		4x12:W40xW20 <sup>d</sup> , <sup>e</sup>	0.80	0.0100	8 .	12.00	10.00	2	2		
Í8		$4x12:W40xW20^{e}$	0.80	0.0100	8	12.00	10.00	2	2		
19		4x12:W40xW20 <sup>e</sup>	0.80	0.0100	8	12.00	10.00	2	2		

OUTLINE OF SPECIMENS

<sup>a</sup>Style designation: 4x12:W40xW20, numbers ahead of colon are longitudinal and transverse wire spacings, and numbers after colon are the longitudinal and transverse wire areas in hundredths of sq. in. 4:No. 5, number before colon is longitudinal bar spacing, and numbers after colon is the bar size.

<sup>b</sup>One longitudinal wire.

<sup>C</sup>Lap splices of 33 in.

<sup>d</sup>With stirrups at the region of splices.

<sup>e</sup>Lap splices of 43 in.

prior to casting to obtain suitable workability. The age, strength and slump of concrete for all beams are given in Table II.

#### 2.2. Reinforcement

Two types of reinforcements were used in this study: smooth welded wire fabric and high strength deformed bars. The fabric styles used in the study were 4x12:W40xW20, 4x6:W40xW20, 8x12:W40xW20 and 4x12: W30xW20. Fabric style 4x12:W40xW20 indicates that the longitudinal wires are spaced 4 in. on centers and that the transverse wires are spaced 12 in. on centers; the cross-sectional areas of the longitudinal wires are 0.40 sq. in., and the transverse wires are 0.20 sq. in.

A summary of the strength properties of the reinforcements is given in Table III, and average stress-strain curves are given in Figure 2. The fabric exceeded the requirements of the ASTM Standard Specifications for Welded Steel Wire Fabric for Concrete Reinforcement, A 185-70, except 4x6:W40xW20 did not meet the weld shear strength requirement of 35 ksi. All fabric styles had a strength of at least 73 ksi at the strain of 0.0035. The deformed bars met the ASTM Standard Specification for Deformed Billet-Steel Bars for Concrete Reinforcement with 60 ksi Minimum Yield Strength, A 615-68.

#### 3. Experimental Procedure

All beams and control cylinders were cast in accordance with pertinent ASTM Specifications. The beams were cast in plywood forms; the inner faces of these forms were covered with formica in order to obtain smooth surfaces. Control cylinders were cast in cardboard or vertical steel molds. The beams and control cylinders were removed from the forms

# TABLE II

Beam	Age of Concrete at Time of Test, Days	Compressive Concrete Strength, ksi	Slump, in.
1	26	4.56	3
2	28	4.60	3
3	27	4.05	5
4	21	3.95	5
5	28	3.59	5
6	21	3.58	5
7	21	4.32	6
8	40	4.30	3
9	36	4.81	1
10	41	4.84	1
11	22	5.73	2
12	28	6.10	2
13	23	5.46	4
14	15	3.66	6
15	25	4.30	2
16	32	4.85	2
17	22	4.51	1
18	28	4.50	1
19	51	7.00 <sup>a</sup>	2

# AGE, STRENGTH AND SLUMP OF BEAMS

a Estimated strength.





24 hours after casting and were cured under wet burlap for six days. Thereafter, the beams and control cylinders were stored in the laboratory under ambient conditions until the time of test.

#### TABLE III

Reinforcement	Yi Stre k	eld ength si	Ultimate Strength,	Weld Shear Strength,
Style	$\varepsilon = 0.0035$	$\varepsilon = 0.005$	ksi	ksi
8x12:W40xW20	78.0	82.0	87.0	44.3
4x12:W40xW20	77.0	82.0	90.0	38.6
4x6:W40xW20	76.0	80.0	90.0	31.6
4x12:W30xW20	73.0	78.0	86.0	45.7
No. 5	62		105.0	
No. 6	62		100.0	

## STRENGTH PROPERTIES OF REINFORCEMENTS

Source: J. P. Lloyd. "Splice Requirements for One Way Slabs Reinforced with Smooth Welded Wire Fabric." Technical Report No. R(S)4, School of Civil Engineering, Oklahoma State University (June, 1971), p. 27.

Beams were tested as shown in Figure 3. The beams were simply supported by steel rollers on a 10-ft. span and loaded by two equal loads 3 ft. from the supports. The loads were applied by a servo controlled hydraulic system and maintained constant while the crack widths were measured. Surface cracks were marked to indicate the crack pattern each time before crack data were recorded.



# Figure 3. Schematic of Setup

Control cylinders for flexural crack control specimens were tested in order to precalculate the loads required to produce steel stresses of 10, 20, 30 and 40 ksi. The width of cracks was measured by an optical comparator with a sensitivity of 0.001 in. During the first cycle of load, measurements were taken at the lateral faces of the beams at the level of the reinforcement and at the extreme tensile fiber. Measurements were taken at loads which produced calculated steel stresses of 20, 30, and 40 ksi. Thereafter, the beams were subjected to sinusoidally repeated loading at a frequency of 120 cycles per minute. The load amplitude was set to produce steel stresses which varied between 10 and 40 ksi. Width of cracks was measured at the end of 2, 10, 100, 10,000, and 100,000 cycles under a static load corresponding to a calculated steel stress of 40 ksi.

The strength of control cylinders for splice specimens was used to calculate the load necessary to produce a steel stress of 60 ksi in the reinforcement. During the first cycle, load was applied in 10 equal increments producing the steel stress of 6 ksi. After the first cycle, the beams were loaded sinusoidally at a frequency of 120 cycles per minute. Initially, the repeated load caused the stress to vary between 6 and 30 ksi. Loading was applied in blocks of 10,000 cycles. After each block of cycles was applied, the load was adjusted so that the maximum steel stress was increased 3 ksi; the minimum stress was maintained at 6 ksi. This procedure was continued until failure occurred.

#### CHAPTER IV

#### EXPERIMENTAL RESULTS

## 1. Width of Cracks

Widths of surface cracks at the level of reinforcement and at the extreme tensile fiber were obtained from direct measurement while the beams were subjected to a static load of the desired magnitude. During the first cycle of loading, the width of cracks was measured at the loads which produced the steel stresses of 20, 30 and 40 ksi. Thereafter, the beams were subjected to repeated load for which the steel stress was varied between 10 and 40 ksi. The widths of cracks were obtained at the static load which produced a steel stress of 40 ksi after 2, 10, 100, 1,000, 10,000 and 100,000 cycles of repeated loads.

Maximum widths of cracks at the level of reinforcement and at the extreme tensile fiber obtained during the first cycle of load are plotted in Figure 4. Figures 4(a) and 4(b) show the widths of cracks of beams reinforced with 4x12:W40xW20 (beam 1), 4x6:W40xW20 (beam 6) and 4:No. 6 (beam 7). These diagrams indicate that 4:No. 6 bar controls the width of cracks better than 4x12:W40xW20 or 4x6:W40xW20; also, 4x6:W40xW20 controls cracking better than 4x12:W40xW20.

Figure 4(c) and 4(d) compare the crack widths of beams reinforced with 4x12:W40xW20 (beam 1), rusted 4x12:W40xW20 (beam 2), rusted 4x6: W40xW20 (beam 3) and 4x6:W40xW20 (beam 6). These figures show the

influence of a moderate amount of rust and the spacing of transverse wires. The amount of rust appears to have no significant influence on width of cracks. However, a smaller spacing of transverse wires appears to provide better crack control.

Figure 4(e) and 4(f) show the widths of cracks of beams reinforced with 4x12:W30xW20 (beam 4), greased 4x12:W30xW20 (beam 14) and 4:No. 5 (beam 5). The wires of beam 14 were greased in order to destroy the bond between the concrete and steel. The width of cracks on the beam reinforced with greased wires is approximately 2.5 times those on the beam reinforced with ungreased wires. At steel stresses of 30 and 40 ksi, the crack widths of beam 5, reinforced with deformed bars, appear to be smaller than that in beam 4, reinforced with smooth welded wire fabric. At the steel stress of 20 ksi, the crack width is insignificantly larger in beam 5.

Figures 4(g) and 4(h) show the crack widths of beams reinforced with 4x12:W30xW20 (beam 8) and 8:No. 5 (beam 9). Beam 8 contained one longitudinal wire to represent 8-in. spacing. The 8:No. 5 bar is shown to provide better crack control than 4x12:W30xW20.

Figures 4(i) and 4(j) show the widths of cracks of beams reinforced with 8x12:W40xW20 (beam 10), 4x12:W40xW20 (beam 11), 4x6:W40xW20 (beam 13) and 8:No. 6 bars (beam 12). Beams 10, 11 and 13 contained one longitudinal wire to represent 8-in. spacing. The 8:No. 6 bar appears to be a better crack controller than smooth welded wire fabrics. Figures 4(i) and 4(j) also indicate that 6-in. spacing of transverse wires controls the widths of cracks better than does 12-in. spacing.

In summary, the results shown in Figure 4 indicate that deformed bars provide slightly better crack control than smooth welded wire



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(b) Maximum Crack Widths at the Extreme Tensile Fiber

Figure 4. Comparison of Maximum Crack Widths of Beams Subjected to Static Loading



(d) Maximum Crack Widths at the Extreme Tensile Fiber

Figure 4. (Continued)





Figure 4. (Continued)

















Figure 4. (Continued)

fabric, and also that 6-in. spacing of transverse wires results in smaller widths of cracks than does 12-in. spacing, but the amount of rust has no influence in crack control. However, greased smooth welded wire fabric results in larger crack widths than ungreased.

Maximum widths of cracks at the level of reinforcement and at the extreme tensile fiber subjected to repeated loading are plotted in Figure 5. Figures 5(a) and 5(b) compare the widths of cracks of beams reinforced with 4x12:W40xW20 (beam 1), 4x6:W40xW20 (beam 6) and 4:No. 6 (beam 7). Figures 5(c) and 5(d) compare the widths of cracks of beams reinforced with 4x12:W40xW20 (beam 1), rusted 4x12:W40xW20 (beam 2), 4x6:W40xW20 (beam 6) and rusted 4x6:W40xW20 (beam 3). Figures 5(e) and 5(f) compare the widths of cracks of beams reinforced with 4x12:W30xW20 (beam 4), greased 4x12:W30xW20 (beam 14) and 4:No. 5 (beam 5). Figures 5(g) and 5(h) show the widths of the cracks of beams reinforced with 4x12:W30xW20 (beam 8) and 8:No. 5 (beam 9). Figures 5(i) and 5(j) show the widths of cracks of beams reinforced with 8x12:W40xW20 (beam 10), 4x12:W40xW20 (beam 11), 4x6:W40xW20 (beam 13) and 8:No. 6 (beam 12). As explained previously, beams 8, 10, 11 and 13 contained only one longitudinal wire to represent 8-in. spacing.

In summary, Figure 5 indicates that the widths of cracks increase with repetition of load, but the spacing of transverse wires and rust have no influence on increased width of crack. Also, the deformed bars provide better crack control than smooth welded wire fabrics.

Summaries of maximum and average surface widths of cracks at the level of reinforcement and at the extreme tensile fiber in the constant moment region are given in Tables IV, V, VI and VII.



Figure 5. Comparison of Maximum Crack Widths for Beams Subjected to Repeated Loading











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The maximum and average surface widths of cracks at the level of reinforcement obtained during the first cycle of load at calculated steel stresses of 20, 30 and 40 ksi are shown in Table IV. As would be expected, both maximum and average widths of cracks increase with increased steel stresses.

In Table V are given the maximum and average surface widths of cracks at the level of reinforcement after various numbers of load cycles; the calculated steel stress was 40 ksi at the time of crack measurement. Both maximum and average widths of cracks increase with the number of load repetitions.

Table VI gives maximum and average widths of cracks at the extreme tensile fiber obtained during the first load cycle at steel stresses of 20, 30 and 40 ksi. Maximum and average widths of cracks increase as steel stresses increase.

In Table VII are given the maximum and average widths of cracks at the extreme tensile fiber after various numbers of load cycles; the calculated steel stress was 40 ksi at the time of crack measurement. Maximum and average widths of cracks increase with the number of load repetitions.

Normally, the crack width at the extreme tensile fiber was greater than the crack width at the level of reinforcement. In some instances this was not the case--often as a result of the branching of a single crack into two cracks between the level of the reinforcement and the extreme tensile surface.

#### TABLE IV

## CRACK WIDTHS AT THE LEVEL OF REINFORCEMENT AT FIRST LOAD CYCLE

	Maximu	m Crack Wid	th, in	Averag	Average Crack Width, in.				
Beam	20 ksi	30 ksi	40 ksi	20 ksi	30 ksi	40 ksi			
1	0.008	0.011	0.014	0.0059	0.0088	0.0113			
2	0.006	0.012	0.015	0.0049	0.0074	0.0104			
3	0.006	0.010	0.016	0.0046	0.0085	0.0123			
4	0.006	0.010	0.014	0.0046	0.0075	0.0098			
5	0.007	0.009	0.012	0.0044	0.0070	0.0098			
6	0.006	0.010	0.012	0.0055	0.0075	0.0096			
7	0.005	0.008	0.011	0.0038	0.0054	0.0072			
8 <sup>a</sup>	0.007	0.010	0.015	0.0050	0.0074	0.0102			
9	0.002	0.006	0.010	0.0020	0.0044	0.0069			
10	0.006	0.015	0.024	0.0038	0.0094	0.0116			
11 .	0.007	0.011	0.013	0.0045	0.0080	0.0102			
12	0.004	0.007	0.010	0.0034	0.0060	0.0076			
13	0.005	0.014	0.015	0.0040	0.0068	0.0097			
14	0.016	0.024	0.035	0.0118	0.0180	0.0210			

<sup>a</sup>Beam accidentally loaded before testing.

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## TABLE V

# CRACK WIDTHS AT THE LEVEL OF REINFORCEMENT AT f = 40 ksiFOR BEAMS SUBJECTED TO REPEATED LOADS<sup>S</sup>

	Maximum Crack Width, in.						Average Crack Width, in.					
Beam	2 cyc.	10 cyc.	100 cyc.	1,000 cyc.	10,000 cyc.	100,000 cyc.	2 cyc.	10 cyc.	100 cyc.	1,000 cyc.	10,000 cyc.	100,000 cyc.
1	0.015	0.017	0.018	0.020	0.021	0.022	0.0123	0.0134	0.0141	0.0145	0.0153	0.0163
2	0.016	0.018	0.018	0.020	0.022	0.025	0.0111	0.0123	0.0126	0.0135	0.0146	0.0166
3	0.017	0.019	0.020	0.021	0.022	0.025	0.0141	0.0157	0.0169	0.0181	0.0185	0.0199
4	0.015	0.016	0.017	0.018	0.020	0.024	0.0116	0.0124	0.0132	0.0136	0.0144	0.0169
5	0.013	0.013	0.015	0.018	0.020	0.025	0.0108	0.0116	0.0123	0.0130	0.0138	0.0145
6	0.015	0.017	0.018	0.020	0.023	0.025	0.0107	0.0118	0.0123	0.0127	0.0138	0.0148
7	0.012	0.013	0.014	0.015	0.017	0.018	0.0080	0.0088	0.0095	0.0103	0.0110	0.0115
8 <sup>a</sup>	0.016	0.016	0.017	0.020	0.021	0.025	0.0111	0.0119	0.0126	0.0141	0.0151	0.0161
9	0.011	0.012	0.013	0.014	0.014	0.020	0.0074	0.0080	0.0088	0.0092	0.0097	0.0116
10	0.025	0.025	0.025	0.026	0.027	0.027	0.0133	0.0137	0.0144	0.0161	0.0168	0.0177
11	0.015	0.016	0.017	0.020	0.020	0.020	0.0115	0.0128	0.0137	0.0147	0.0150	0.0156
12	0.012	0.013	0.014	0.015	0.015	0.017	0.0086	0.0093	0.0099	0.0106	0.0109	0.0111
13	0.016	0.018	0.018	0.019	0.019	0.020	0.0107	0.0117	0.0125	0.0132	0.0136	0.0152
14	0.039	0.041	0.045	0.047	0.048	0.050	0.0224	0.0236	0.0259	0.0271	0.0279	0.0301

<sup>a</sup>Beam accidentally loaded before testing.

				· · ·					
	Maximu	n Crack Wid	th, in	Average Crack Width, in.					
Beam	20 ksi	30 ksi	40 ksi	20 ksi	30 ksi	40 ksi			
1	0.009	0.013	0.016	0.0073	0.0103	0.0130			
2	0.007	0.012	0.016	0.0055	0.0083	0.0108			
3	0.007	0.011	0.018	0.0051	0.0093	0.0137			
4	0,006	0.012	0.016	0.0054	0.0085	0.0120			
5	0.007	0.010	0.012	0.0053	0.0076	0.0102			
6	0.007	0.011	0.015	0.0060	0.0078	0.0109			
7	0.006	0.008	0.010	0.0044	0.0059	0.0067			
8 <sup>a</sup>	0.013	0.014	0.015	0.0094	0.0110	0.0106			
9	0.002	0.007	0.012	0.0013	0.0053	0.0079			
10	0.008	0.015	0.025	0.0030	0.0080	0.0162			
11	0.007	0.012	0.015	0.0050	0.0082	0.0112			
12	0.004	0.009	0.012	0.0035	0.0076	0.0080			
13	0.007	0.010	0.014	0.0047	0.0072	0.0108			
14	0.020	0.029	0.040	0.0140	0.0228	0.0240			

# CRACK WIDTHS AT THE EXTREME TENSILE FIBER AT FIRST LOAD CYCLE

TABLE VI

<sup>a</sup>Beam accidentally loaded before testing.

## TABLE VII

# CRACK WIDTHS AT THE EXTREME TENSILE FIBER AT ${\tt f_S}$ = 40 KSI FOR BEAMS SUBJECTED TO REPEATED LOADS

	Maximum Crack Width, in.							Average Crack Width, in.				
Beam	2 cyc.	10 cyc.	100 cyc.	1,000 cyc.	10,000 cyc.	100,000 cyc.	2 cyc.	10 cyc.	100 cyc.	1,000 cyc.	10,000 cyc.	100,000 cyc.
1	0.017	0.018	0.020	0.022	0.023	0.026	0.0143	0.0150	0.0160	0.0167	0.0173	0.0191
2	0.018	0.020	0.020	0.020	0.023	0.025	0.0117	0.0133	0.0143	0.0142	0.0148	0.0157
3	0.020	0.020	0.022	0.025	0.028	0.030	0.0161	0.0179	0.0188	0.0204	0.0215	0.0247
4	0.018	0.020	0.021	0.022	0.025	0.035	0.0138	0.0154	0.0161	0.0169	0.0186	0.0235
5	0.015	0.017	0.018	0.020	0.021	0.029	0.0116	0.0130	0.0138	0.0152	0.0162	0.0195
6	0.018	0.020	0.021	0.022	0.023	0.027	0.0126	0.0140	0.0148	0.0152	0.0162	0.0180
7	0.011	0.012	0.014	0.016	0.017	0.021	0.0074	0.0082	0.0090	0.0095	0.0102	0.0113
8 <sup>a</sup>	0.016	0.020	0.021	0.022	0.025	0.028	0.0109	0.0119	0.0130	0.0141	0.0157	0.0166
9	0.014	0.016	0.016	0.018	0.018	0.025	0.0090	0.0098	0.0106	0.0112	0.0119	0.0135
10	0.025	0.027	0.030	0.030	0.031	0.033	0.0175	0.0182	0.0202	0.0212	0.0222	0.0235
11	0.016	0.018	0.020	0.021	0.023	0.026	0.0126	0.0146	0.0164	0.0170	0.0180	0.0202
12	0.013	0.013	0.014	0.016	0.017	0.018	0.0090	0.0098	0.0106	0.0111	0.0116	0.0124
13	0.015	0.018	0.019	0.020	0.025	0.030	0.0120	0.0142	0.0157	0.0167	0.0192	0.0230
14	0.045	0.045	0.048	0.050	0.052	0.055	0.0267	0.0293	0.0308	0.0315	0.0328	0.0352

<sup>a</sup>Beam accidentally loaded before testing.

#### 2. Crack Spacing

Crack spacing is the distance between two adjacent flexural cracks in the constant moment region. The average crack spacing is the average of all crack spacings in the constant moment region. Although crack spacings are not considered to be of direct importance, a knowledge of crack spacing permits estimation of the width of cracks in beams. It has been observed that at a given steel stress, the presence of a large number of closely spaced cracks results in a small width of cracks. The style and surface characteristics of reinforcement can influence the spacing and, therefore, the width of cracks. Crack spacing data are presented in Table VIII for calculated steel stresses of 20, 30 and 40 ksi, measured during the first load cycle.

For beams reinforced with smooth welded wire fabric, it was observed that the number and, therefore, the spacing of cracks did not change when the steel stress increased from 30 to 40 ksi. This indicates that at a stress of approximately 30 ksi, the crack spacing and the number of cracks become constant, and at higher stress levels the width of cracks is influenced primarily by steel stress.

At the steel stress of 40 ksi, beams reinforced with deformed bars had a smaller average crack spacing than beams reinforced with smooth welded wire fabric. It was also observed that beams reinforced with smooth welded wire fabric styles having a 6-in. spacing of transverse wires had a smaller average crack spacing than that which occurred in beams reinforced with fabric having a 12-in. spacing of transverse wires.

Only beam 12 had an increase in the number of cracks, with reduced crack spacing, at 100,000 cycles of load. The number and spacing of
TABLE	VIII

NUMBER OF CRACKS AND CRACK SPACING IN CONSTANT MOMENT REGION

		$f_s = 20 \text{ ksi}$				$f_s = 30 \text{ ksi}$			<del>.</del>	$f_s = 40 \text{ ksi}$		
Beam	N <sup>a</sup>	c <sub>min</sub>	c c <sub>max</sub>	d c <sub>aver</sub>	N	c min	c max	c aver	N	b c <sub>min</sub>	c max	d c <sub>aver</sub>
1	15	9.25	13.25	11.23	15	9.25	13.25	11.23	15	9.25	13.25	11.23
2	15	6.50	15.12	11.10	18	6.00	15.00	8.88	18	6.00	15.00	8.88
3	15	7.75	14.00	10.48	. 15	7.75	14.00	10.48	15	7.75	14.00	10.48
4	15	6.00	14.50	9.94	15	6.00	14.50	9.94	15	6.00	14.50	9.94
5	18	6.25	12.75	9.04	18	4.75	10.50	7.35	18	4.75	10.50	7.35
6	12	6.75	14.25	10.17	18	5.00	10.00	7.35	18	5.00	10.00	7.35
7	15	5.75	14.25	9.67	24	2.00	9.50	5.98	26	2.00	9.50	5.46
8 <sup>e</sup>	15	4.00	11.00	7.96	15	4.00	11.00	7.96	17	4.00	10.75	7.18
9	5	13.25	15.75	14.50	21	2.50	9.25	6.30	24	2.50	9.25	6.40
10	11	2.00	13.50	9.69	11	2.00	13.50	9.69	15	2.00	13.50	9.46
11	15	7.50	11.25	9.35	15	7.50	11.25	9.35	15	7.50	11.25	9.35
12	. 11	10.50	16.50	12.72	14	5.50	12.75	9.75	. 22 <sup>f</sup>	4.00	10.75	6.88
13	. 7	8,50	30.50	22.62	18	4.00	12.25	8.17	. 18	4.00	12.25	8.17
14	7	11.75	22.50	15.06	7	11.75	22.50	15.06	10	10.75	13.50	11.95

<sup>a</sup>Number of cracks in the constant moment region.

<sup>b</sup>Minimum distance between two cracks, in.

<sup>C</sup>Maximum distance between two cracks, in.

 $^{\rm d}_{\rm Average}$  distance between cracks, in.

<sup>e</sup>Beam accidentally loaded before testing.

<sup>f</sup>After 100,000 cycles, N = 23;  $c_{min} = 2.25$  in.;  $c_{max} = 10.50$  in.;  $c_{avg} = 6.54$  in.

cracks remain unchanged for each of the other beams under repeated load. Therefore, repeated loading does not influence the number and spacing of cracks.

## 3. Splice Tests

Five beams were tested to study the behavior of concrete beams with lapped splices subjected to repeated loads. Two of these beams were rainforced with two No. 6 bars having a 33-in. lapped splice at midspan which conformed to Class C splice of ACI Code (40). One of these two beams had stirrups in the region of the splice to provide an additional resistance to splitting failure. The other three beams were reinforced with 4x12:W40xW20 smooth welded wire fabric. Each of these three beams contained a 31-in. splice measured between the outermost transverse wires in the splices. The longitudinal wires extended 6 in. beyond the outermost transverse wires, resulting in a total lap length of 43 in. One of these three beams also had stirrups in the region of the splice.

All beams were initially loaded in increments of 6 ksi until a maximum steel stress of 60 ksi was reached. Thereafter, the beams were subjected to sinusoidal repeated loading to cause the stress to vary between 6 and 30 ksi. The load was applied in blocks of 10,000 cycles; each block of cycles had a larger maximum load increased by 3 ksi. This procedure was continued until failure occurred. A summary of splice tests, loading, and mode of failure is given in Table IX.

Beam 15, which was reinforced with No. 6 bars without stirrups, failed by splitting of the concrete in the splice; however, beam 16, with stirrups in the lap region, failed by crushing of the concrete in the

TAB	LE	IΧ

SPLICE TESTS

B(	eam	Lapped Splices, in.	Maximum Designed Loads, kips	Minimum Applied Loads, kips	Maximum Applied Loads, kips	Cycles of Loading	Mode of Failure
						- r	
	15	33	25.45	2.54	12.72	50,000 <sup>e</sup>	
				2.54	13.99	50,000	
			r.	2.54	15.27	50,000 <sup>0</sup>	
				2.54	16.54	10,000	
				2.54	17.81	10,000	
				2.54	19.08	10,000	
				2.54	20.36	10,000	
e al e				2.54	21.63	10,000	
				2.54	22.90	10,000	· ·
~				2.54	24.17	4,658	Splitting
	16	33 <sup>.d</sup>	25.75	2.57	12.87	10,000	
$   _{L^{\infty}(\mathbb{R}^{n})} =    _{L^{\infty}(\mathbb{R}^{n})}$				2.57	14.16	10,000	
				2.57	15.45	10,000	
		• *		2.57	16.73	10,000	
				2.57	18.02	10,000	
				2.57	19.31	10,000	
				2.57	20.60	10,000	
		i'		2.57	21.88	10,000	
				2.57	23.17	10,000	
1 - A				2.57	24.46	10,000	
				2.57	25.75	10,000	
				2.57	27.03	10,000	
		· .		2.57	28.32	10,000	-
• *				2.57	29.61	e	Compression
1.	7	43d	23 47	2 35	11 74	10 000	
	,		23.47	2.35	12 91	10,000	
••				2.35	14 08	10,000	
				2 35	15 26	10,000	
				2.35	16 43	10,000	
- 	· .			2.55	17 60	10,000	
				2.55	18 78	10,000	
				2.55	10.70	10,000	
				2.35	21 12	10,000	
				2.35	22.12	10,000	
				2.55	22.50	1 3/6	Fatione

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Be <b>am</b>	Lapped Splices, in.	Maximum Designed Loads, kips	Minimum Applied Loads, kips	Maximum Applied Loads, kips	Cycles of Loading	Mode of Failure
18	43 <sup>b</sup>	23.34	2.33 2.33 2.33 2.33 2.33 2.33 2.33 2.33	11.67 12.84 14.00 15.17 16.34 17.50 18.67 19.84 21.00 22.17 23.34	10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 2,038	Fatigue
19	43 <sup>b</sup>	24.13	2.41 2.41 2.41 2.41 2.41 2.41 2.41 2.41	12.07 13.27 14.48 15.69 16.89 18.10 19.30 20.51 21.72	10,000 10,000 10,000 10,000 10,000 10,000 10,000 5,000 <sup>f</sup>	Fatigue

TABLE IX (Continued)

<sup>a</sup>Loads which produced a steel stress of 60 ksi.

<sup>b</sup>Without stirrups at the region of splices.

<sup>C</sup>To expedite the experiment, repeated load was limited to 10,000 cycles after the third application on Beam 15.

<sup>d</sup>With stirrups at the region of splices.

<sup>e</sup>Static compression failure while adjusting machine for fatigue test at 29.61 kips.

<sup>f</sup>Estimated.

compression zone. Beams 17, 18 and 19, which were all reinforced with smooth welded wire fabric, failed by fatigue of reinforcement near the welded intersections within the splice region.

## CHAPTER V

### ANALYSIS AND DISCUSSION OF RESULTS

### 1. Maximum Width of Cracks

The maximum width of crack in reinforced concrete members is more important than the average width of crack because of the localized nature of damage caused by exposure to a severe, aggressive environment. Investigators who have attempted to determine which parameters are important in controlling the width of cracks in reinforced concrete members have generally agreed that steel stresses and the number and arrangement of reinforcing bars are the most important variables which influence the maximum width of cracks. In this experiment, the width of cracks in beams reinforced with either smooth welded wire fabric or deformed bars, under the influence of static and repeated loads, was studied.

Beams 2 and 3 contained reinforcement stored for a period of time in a moist chamber to produce a moderately thick layer of rust on the surface. It was expected that the rust would either increase the surface roughness and thereby possibly increase the bond strength or, because of its loose dusty surface, lessen the bond strength. However, the results indicate that the influence of the rust was not significant from either standpoint. Beam 14 contained greased reinforcement in order to prevent the bond between concrete and steel. When this specimen was fabricated, data had already been obtained showing significant increases in crack

width under repeated loading. It was anticipated that this specimen might be useful in determining whether the increases in crack width were related to a breakdown of bond between the concrete and the steel under the action of repeated load. The results from this specimen indicate that although the cracks were initially quite large, their width increased in the same manner as those in the beams with ungreased reinforcement. Therefore, the increasing width of cracks under the action of repeated load does not appear to be strongly related to bond.

It therefore appears that the increase in crack widths under the action of repeated load may be the result of creep in the concrete in the compression zone as well as shrinkage throughout the beam. The shrinkage in the tension zone may intensify the presence of flexural cracks. The pulsating width of the cracks as a result of the repeated load may have facilitated the movement of air within the crack space and, therefore, relatively high rates of moisture loss within the concrete near the cracks.

Figure 6 shows the maximum width of cracks at the level of reinforcement obtained from beams reinforced with smooth welded wire fabric or deformed bars during the first load cycle. The crack width data are plotted against the parameter used by Gergely and Lutz (12) in Equation (2.18). In Figure 7 the maximum width of cracks at the extreme tensile fiber are plotted against a parameter which was used by Gergely and Lutz (12) in Equation (2.19). Because of the large number of variables and limited number of specimens, it is not possible to analyze the influence of each variable with a high level of confidence. For this reason, data are considered in two groups: beams reinforced with smooth welded wire



Figure 6. Maximum Crack Widths at the Level of Reinforcement Under Static Loads

**69** .



Figure 7. Maximum Crack Widths at the Extreme Tensile Fiber Under Static Loads

fabric and beams reinforced with deformed bars. Data from beam 14 with greased reinforcement are plotted but were excluded from the analysis.

In Figures 6 and 7, the maximum width of cracks for beams reinforced with smooth welded wire fabric and for beams reinforced with deformed bars are compared to each other and to the results predicted by Equations (2.18) and (2.19), which were developed by Gergely and Lutz. The lines shown in Figures 6 and 7 for smooth welded wire fabric and deformed bars are based on least square linear regression.

Based on the regression analysis, the maximum widths of cracks at the level of reinforcement and at the extreme tensile fiber for beams reinforced with smooth welded wire fabric subjected to static load can be expressed as

$$w_{s}^{*} = 0.127 \frac{\sqrt[3]{t_{s}}}{1 + \frac{2}{3} \frac{t_{s}}{h_{1}}} f_{s} \times 10^{-3}$$
(5.1)

and

$$w_b^* = 0.084 \quad {}^{3}\sqrt{t_b A} \quad \beta f_s \propto 10^{-3}$$
 (5.2)

where

Similarly, for beams reinforced with deformed bars, these maximum widths of cracks can be expressed as

$$w_{s}^{*} = 0.086 \frac{\sqrt[3]{t_{s}^{A}}}{1 + \frac{2}{3} \frac{t_{s}}{h_{1}}} f_{s} \times 10^{-3}$$
 (5.3)

$$w_b^* = 0.050 \quad {}^3\sqrt{t_b A} \quad \beta f_s \ge 10^{-3}$$
 (5.4)

Statistical analysis at 0.95 confidence level indicates that there are significant differences between the slopes of the regression lines for smooth welded wire fabric and deformed bar in both figures. There is no significant difference between the slopes of the regression lines for deformed bar and Gergely and Lutz's Equation (2.18) in Figure 6. Also, there is no significant difference between the slopes of the regression lines for smooth welded wire fabric and Gergely and Lutz's Equation (2.19) in Figure 7.

The increase in maximum widths of cracks at the level of reinforcement and at the extreme tensile fiber are plotted against cycles of applied load in Figures 8 and 9, respectively. Again, data are divided into two groups: data from beams reinforced with smooth welded wire fabric and data from beams reinforced with deformed bars. The lines which are shown in Figures 8 and 9 for smooth welded wire fabric and deformed bars are based on least square linear regression. Statistical analysis at 0.95 confidence level indicates that there is no significant difference in the slopes of the regression lines.

A casual study of Figure 5 reveals that there is no obvious parameter which plays a dominant role in the rate at which the width of cracks increases with cycles of load. However, in several cases there was a slightly higher rate of increase during the first ten cycles of load than during subsequent cycles. In view of the limited amount of data and the number of variables involved in the research, it seems justified to express the change in crack width as a linear function of the logarithm of the cycles of load.

and



Figure 8. Increase in Maximum Crack Widths at the Level of Reinforcement Under Repeated Loads



Figure 9. Increase in Maximum Crack Widths at the Extreme Tensile Fiber Under Repeated Loads

From the results of this analysis, equations predicting increase in maximum width of cracks at level of reinforcement and at the extreme tensile fiber for beams reinforced with either smooth welded wire fabric or deformed bars subjected to repeated loads can be expressed as

$$\Delta w_{s}^{*} = 0.0017 \log (cycles)$$
 (5.5)

and

$$\Delta w_{\rm b}^{*} = 0.0023 \, \log \, (\rm cycles) \tag{5.6}$$

where

Δw = increase in maximum calculated surface width of crack at level of reinforcement in constant moment region, in.; Δw = increase in maximum calculated width of crack at extreme tensile fiber in constant moment region, in.

For smooth welded wire fabric, the spacing of transverse wires was either 6 or 12 in. As can be seen in Figures 4 and 5, the results indicate that reinforcement with a 6-in. spacing of transverse wires provides better crack control than does reinforcement with a 12-in. spacing under static loads. The spacing of transverse wires does not appear to influence the width of cracks under repeated loads.

In summary, the results from this experiment indicate that deformed bar provides a better crack control than does smooth welded wire fabric under static loads. Repeated loads increased the width of cracks in a similar manner regardless of the style of reinforcement or other crack control parameters.

The observed maximum widths of cracks at the level of reinforcement and at the extreme tensile fiber under static and repeated loads are compared to the values predicted by Equations (2.18) and (2.19). In Table X, the observed maximum widths of cracks at the level of

# TABLE X

	f <sub>s</sub> = 20 ksi						
Beam	Maximum Side Crack Width in.	Maximum Calc. Side Crack Width in.	Maximum Bottom Crack Width in.	Maximum Calc. Bottom Crack Width in.			
1	0.008	0.0042	0.009	0.0058			
2	0.006	0.0042	0.007	0.0058			
3	0.006	0.0042	0.007	0.0058			
4	0.006	0.0041	0,006	0.0062			
5	0.007	0.0041	0.007	0.0062			
6	0.006	0.0042	0.007	0.0059			
7	0.005	0.0043	0.006	0.0057			
8 <sup>c</sup>	0.007	0.0044	0.013	0.0095			
9	0.002	0.0044	0.002	0.0095			
10	0.006	0.0049	0.008	0.0086			
11	0.007	0.0049	0.007	0.0086			
12	0.004	0.0051	0.004	0.0083			
13	0.005	0.0049	0.007	0.0086			
14	0.016	0.0040	0.020	0.0062			

# COMPARISON OF MAXIMUM CRACK WIDTHS BETWEEN OBSERVED VALUES AND VALUES PREDICTED BY EQUATIONS (2.18) AND (2.19) AT FIRST LOAD CYCLE

	f <sub>s</sub> = 30 ksi					
Beam	Maximum Side Crack Width in.	Maximum Calc. Side Crack Width in.	Maximum Bottom Crack Width in.	Maximum Calc. Bottom Crack Width in.		
1	0.011	0.0064	0.013	0.0088		
2	0.012	0.0064	0.012	0.0088		
3	0.010	0.0063	0.011	0.0088		
4	0.010	0.0061	0.012	0.0093		
5	0.009	0.0061	0.010	0.0092		
6	0.010	0.0063	0.011	0.0088		
7	0.008	0.0064	0.008	0.0086		
8 <sup>c</sup>	0.010	0.0066	0.014	0.0142		
9	0.006	0.0067	0.007	0.0142		
10	0.015	0.0074	0.015	0.0129		
11	0.011	0.0074	0.012	0.0129		
12	0,007	0.0077	0.009	0.0125		
13	0.014	0.0074	0.010	0.0129		
14	0.024	0.0061	0.029	0.0093		

	(inded)	
f_=	= 40 ksi	
ximum	Maximum	Maximum
z. Side	Bottom	Calc. Bottom
c Width	Crack Width	Crack Width
in.	in.	in.
0085	0.016	0.0117
0085	0.016	0.0117
	0.019	0 0117

TABLE

Maximum Мах Side Calc Crack Width Crack Beam in. j 1 0.014 0.0 2 0.015 0.0 3 0.016 0.0085 0.018 0.0117 4 0.014 0.0081 0.016 0.0124 0.012 0.0082 0.012 0.0123 5 6 0.012 0.0084 0.015 0.0117 0.0086 0.010 0.0115 7 0.011 8<sup>C</sup> 0.015 0.015 0.0088 0.0190 0.010 0.0089 0.012 0.0190 9 10 0.025 0.0172 0.024 0.0099 0.0099 0.015 0.0172 11 0.013 12 0.010 0.0102 0.012 0.0167 13 0.015 0.0099 0.014 0.0172 14 0.040 0.0124 0.035 0.0081

<sup>a</sup>Calculated from Equation (2.18).

<sup>b</sup>Calculated from Equation (2.19).

<sup>C</sup>Beam accidentally loaded before testing.

reinforcement and at the extreme tensile fiber under static loads also are compared to the values predicted by these equations. In Table XI, the observed maximum widths of cracks at the level of reinforcement are compared to the values predicted by Equation (2.26) for members subjected to 100 or more repeated loads.

## 2. Average Width of Cracks

As was mentioned previously, the maximum crack width is considered more significant than the average crack width or the spacing of cracks. However, the maximum crack width is not an absolute value--at the very least it is strongly related to the length of a specimen available for observation. Provided sufficient data could be obtained, probability would be of great use in predicting a maximum crack width. A study of average crack width and spacing data is useful in helping to substantiate conclusions made in the analysis of maximum crack width data.

Average width of cracks in reinforced concrete members is the sum of all widths of cracks in the constant moment region divided by the number of cracks. The average widths of cracks at the level of reinforcement and at the extreme tensile fiber of beams reinforced with smooth welded wire fabric and deformed bars under static loads are plotted in Figures 10 and 11, respectively. In Figure 10, the average widths of cracks at the level of the reinforcement are plotted against the parameter used by Gergely and Lutz (12) in Equation (2.18) to predict maximum width at the same level. Similarly, in Figure 11 the average widths of cracks at the extreme tensile fiber are plotted against the parameter used by Gergely and Lutz (12) in Equation (2.19) for the prediction of crack width at the extreme tensile fiber. The data are

# TABLE XI

	······································	Ma	Maximum Side Crack Width, in.				
Beam	Max. Calc. Side Crack Width, in.	100 Сус.	1,000 Cyc.	10,000 Cyc.	100,000 Сус.		
1	0.0188	0.018	0.020	0.021	0.022		
2	0.0188	0.018	0.020	0.022	0.025		
3	0.0188	0.020	0.021	0.022	0.025		
4	0.0174	0.017	0.018	0.020	0.024		
5	0.0174	0.015	0.018	0.020	0.025		
6	0.0188	0.018	0.020	0.023	0.025		
7	0.0193	0.014	0.015	0.017	0.018		
8 <sup>a</sup>	0.0174	0.017	0.020	0.021	0.025		
9	0.0174	0.013	0.014	0.014	0.020		
10	0.0188	0.025	0.026	0.027	0.027		
11	0.0188	0.017	0.020	0.020	0.020		
12	0.0193	0.014	0.015	0.015	0.017		
13	0.0188	0.018	0.019	0.019	0.020		
14	0.0174	0.045	0.047	0.048	0.050		

# COMPARISON OF MAXIMUM OBSERVED WIDTHS OF CRACK AND WIDTHS PREDICTED BY EQUATION (2.26) UNDER 100 OR MORE REPEATED LOADS AT f = 40 KSI

<sup>a</sup>Beam accidentally loaded before testing.



Figure 10. Average Crack Widths at the Level of Reinforcement Under Static Loads





considered in two groups: beams reinforced with smooth welded wire fabric and beams reinforced with deformed bars. Least square linear regression lines show that the smooth welded wire fabric is less effective in controlling crack widths than deformed bars. Statistical analysis at 0.95 confidence level indicates that there is significant difference in the slopes of the regression lines in both figures.

The increase in average width of cracks at the level of reinforcement and at the extreme tensile fiber of beams reinforced with smooth welded wire fabric and deformed bars subjected to repeated loads are plotted in Figures 12 and 13, respectively. Both Figures 12 and 13 show the increase in average width of cracks versus cycles of loading. Again, data are divided into two groups; beams reinforced with smooth welded wire fabric and beams reinforced with deformed bars. The increase in average widths of cracks of beams reinforced with smooth welded wire fabric and deformed bars are compared to each other. The lines shown in Figures 12 and 13 are based on least square linear regression. Statistical analysis at 0.95 level of confidence indicates that there is significant difference in the slopes of the regression lines.

The data obtained from beam 14, which contained greased reinforcement, strongly suggests that the bond between clean smooth welded wire fabric and concrete is quite substantial--even if it is not so effective as that provided by deformed bar reinforcement. The average crack width at the level of the reinforcement for beam 14 was approximately 0.005 in. larger than would have been predicted from the elongation of steel and the number of cracks, if the concrete were assumed to be unstrained at the level of the steel. This suggests that shrinkage of the concrete plays a significant role in the width of surface cracks. Also, it may



Figure 12. Increase in Average Crack Widths at the Level of Reinforcement Under Repeated Loads



Figure 13. Increase in Average Crack Widths at the Extreme Tensile Fiber Under Repeated Loads

be that there were internal cracks formed which never reached the surface under static loads, but which closed under the influence of repeated loads.

### 3. Crack Spacings

A knowledge of crack spacing allows an estimation of the width of cracks for reinforced concrete members. From this experiment, it has been found that the average crack spacing of beams reinforced with deformed bars was 6.52 in. At a steel stress of 40 ksi, the average crack spacings were 8.66 and 9.72 in. respectively, for beams reinforced with smooth welded wire fabric with 6-in. and 12-in. spacings of transverse wires. This further substantiates the findings based on crack width data that the types of reinforcement and the spacing of the transverse wires influence crack widths.

## 4. Splice Tests

The two beams reinforced with deformed bars performed satisfactorily under repeated loads. One beam with stirrups in the lap region was failed by a static load producing a steel stress of 69 ksi while the machine was being adjusted for block of load cycles; this failure was a result of crushing of the concrete in the compression zone. The beam without stirrups failed by splitting under repeated loads at a steel stress of 57 ksi. The remaining three beams reinforced with smooth welded wire fabric also performed well under repeated loads. These beams failed by fatigue of the longitudinal wires at or near the welded intersections within the lap region. Two beams failed at a steel stress of 60 ksi; one of these had stirrups in the lap region. The third beam failed at a steel stress of 54 ksi. The absence of splitting failures in the two beams reinforced with smooth welded wire fabric and without stirrups in the region of the splice indicates that the stirrup reinforcement was unnecessary for the conditions used in this study.

An S-N<sub>i</sub> plot represents fatigue life for a specimen at any stress level under a specific ratio of minimum to maximum stresses. S represents the maximum stress level on the specimen and N<sub>i</sub> represents the expected life of the specimen which would take the load. Figure 14 shows the S-N<sub>i</sub> plot for samples of wire removed from 4x12:W40xW20 fabric and tested under repeated loading in air (43).

The phenomenon of cumulative damage under repeated loads is assumed to be related to the net energy absorbed by the specimen. The number of load cycles applied at a given stress level may be expressed as a percentage of the useful life already expended. When the total damage reaches 100 percent, the fatigue specimen should fail.

The Miner rule assumes that each cycle in a constant stress fatigue test uses the same percentage of the fatigue life of a specimen. If different maximum stress levels are applied within one test, failure will occur when (44)

$$\sum_{i=1}^{n} \frac{n_{i}}{N_{i}} = 1$$
 (5.7)

where

n<sub>i</sub> = the number of cycles at a stress level; N<sub>i</sub> = the fatigue life of a specimen tested at the stress level.

Figure 15 shows a modified Goodman Diagram illustrating the fatigue life of a 4x12:W40xW20 fabric from which the Miner rule was applied.



Figure 14. S-N<sub>i</sub> Relationship for 4x12:W40xW20 With R = 0.25





This plot is simplified because of the absence of large quantities of fatigue data for the reinforcement.

The results of cumulative damage in fatigue tests given in Table XII indicate that the sums of the ratios of the number of cycles at stress levels to the fatigue life of a specimen were not equal to unity when fatigue failure occurred. This means that either the Miner rule does not represent the actual behavior of a beam reinforced with smooth welded wire fabric, and gives unconservative results, or that the fatigue characteristics of the reinforcement are modified by the internal conditions in the beam, or that insufficient data existed to properly define the Modified Goodman diagram.

TAB	LE	XII

Beam	Maximum Stress at Inner Layer of Steel, ksi	Applied Cycles, <sup>n</sup> i	Fatigue Life Cycles, <sup>N</sup> i	n <u>i</u> Ni	$\sum \frac{n_i}{N_i}$
17	34 37 40 44 47 51 54 57 61 64 67	10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 1,346	800,000 650,000 500,000 370,000 290,000 220,000 180,000 140,000 100,000 80,000 64,000	0.012 0.015 0.020 0.027 0.034 0.045 0.056 0.071 0.100 0.125 0.021	0.526
18	34 37 40 44 47 51 54 57 61 64 67	10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 10,000 2,038	800,000 650,000 370,000 290,000 220,000 180,000 140,000 100,000 80,000 64,000	0.012 0.015 0.020 0.027 0.034 0.045 0.056 0.071 0.100 0.125 0.032	0.537
19	34 37 40 44 47 51 54 57 61	10,000 10,000 10,000 10,000 10,000 10,000 10,000 5,000	800,000 650,000 370,000 290,000 220,000 180,000 140,000 100,000	0.012 0.015 0.020 0.027 0.034 0.045 0.056 0.071 0.050	0.330

# CUMULATIVE DAMAGE IN FATIGUE OF BEAMS REINFORCED WITH SMOOTH WELDED WIRE FABRIC

<sup>a</sup>Interpolated from Figure 15.

<sup>b</sup>Estimated.

## CHAPTER VI

# SUMMARY AND CONCLUSIONS

### 1. Summary

The width of cracks is subject to relatively large scatter. A thorough understanding of crack control is helpful to design safe and economical structures. The investigation reported herein was conducted to obtain information about crack widths in beams reinforced with smooth welded wire fabric of diameter larger than 0.5 in., with transverse wires spaced at 6 or 12 in. The results are assumed to be applicable to one-way slabs as well.

Nineteen beams reinforced with either smooth welded wire fabric or deformed bars were tested. All reinforcement met or exceeded pertinent ASTM Specifications except one: fabric style 4x6:W40xW20 did not meet the weld shear strength requirement. The beams were 8 in. wide and 11 ft. long, with depths ranging from 7 to 16 2/3 in. All beams were simply supported on a 10-ft. span. Two equal loads were applied 3 ft. from the supports. All beams had a cover thickness of 2 in., measured from the extreme tensile fiber to the center of the longitudinal steel. For crack control specimens, the reinforcement ratio was 0.0075. For splice specimens the overall depth was kept constant at 12.0 in.

Ten beams were reinforced with smooth welded wire fabric and four were reinforced with deformed bars to determine the crack control

characteristics of the reinforcement. Crack widths and crack spacing, both at the level of reinforcement and at the extreme tensile fiber in the constant moment region, were obtained for calculated steel stresses of 20, 30 and 40 ksi for the first cycle of loading. Width of cracks and crack spacing were also obtained after 2, 10, 100, 1,000, 10,000 and 100,000 cycles of sinusoidally repeated loading in which the calculated steel stress varied between 10 and 40 ksi. Maximum widths of cracks at the level of reinforcement and at the extreme tensile fiber for smooth welded wire fabric and deformed bars obtained during the first cycle of loading were compared to the maximum crack widths predicted by Equations (2.18) and (2.19), which were developed by Gergely and Lutz (12). For repeated loading, maximum width of cracks at the level of reinforcement was compared to the maximum crack predicted by Equation (2.26), presented by CEB-FIP (13).

Three beams reinforced with smooth welded wire fabric, and two beams with deformed bars, were tested to determine the characteristics of smooth welded wire fabric with lapped splices under repeated loads. These beams were initially subjected to static loads at a maximum calculated stress of 60 ksi. After the first cycle, loading was repeated sinusoidally. The load amplitude and level were reset after blocks of 10,000 cycles until the beams reached failure.

#### 2. Conclusions

Statistical analysis at 0.95 level of confidence indicates that there is significant difference between the crack widths obtained from beams reinforced with smooth welded wire fabric and beams reinforced with deformed bars. However, there is no significant difference between

the observed width of cracks at the level of reinforcement for beams reinforced with deformed bars and the width predicted by Gergely and Lutz's Equation (2.18). Likewise, there is no significant difference between the observed width of cracks at the extreme tensile fiber of beams reinforced with smooth welded wire fabric and the width predicted by Gergely and Lutz's Equation (2.19). Equation (2.26) was presented by CEB-FIP for predicting maximum width of cracks in members subjected to 100 or more repeated loads. However, the observed values were greater than the predicted values for maximum crack widths at the level of reinforcement for beams with smooth welded wire fabric. Equation (2.26) was fairly accurate when deformed bars were used.

Based on the results obtained from the 14 beams in the crack control specimens it may be concluded that:

1. For similar conditions of diameter, spacing and cover of reinforcement, deformed bars result in smaller, more closely spaced cracks than does smooth welded wire fabric.

2. The width of cracks obtained with smooth welded wire fabric were somewhat smaller in cases where the transverse wires were spaced at 6 in. rather than 12 in.

3. The presence of a moderately heavy layer of rust neither adds to nor detracts from the serviceability of smooth welded wire fabric; however, a coating of grease on the surface of the fabric resulted in extremely large crack widths.

4. The width of flexural cracks increases when a beam is subjected to repeated load.

5. The equation presented by CEB-FIP is satisfactory in predicting the width of cracks for beams reinforced with deformed bars, but it is not satisfactory for smooth welded wire fabric.

6. Designers using large diameter smooth welded wire fabric in flexural applications should anticipate larger crack widths than would occur under similar conditions with deformed bars, deformed wire reinforcement, or smaller, and more common, sizes of smooth welded wire fabric.

7. Data suggest that the following crack width expression can be used by the designer for the maximum crack width at the extreme tensile fiber for large diameter, smooth welded wire fabric under static load conditions:

$$w_b^* = 0.084 \sqrt[3]{t_b A} \beta f_s \times 10^{-3}$$

Similarly, maximum width of crack at the level of reinforcement for large diameter, smooth welded wire fabric under static load can be predicted by

$$w_{s}^{*} = 0.127 \frac{\sqrt[3]{t_{s}^{A}}}{1 + \frac{2}{3} \frac{t_{s}}{h_{1}}} f_{s} \times 10^{-3}$$

8. From this experiment, two equations are derived to predict increase in maximum width of cracks at the extreme tensile fiber and at the level of reinforcement for beams reinforced with either smooth welded wire fabric or deformed bars subjected to repeated loads:

$$\Delta w_b^* = 0.0023 \log (cycles)$$
$$\Delta w_s^* = 0.0017 \log (cycles)$$

Based on the results obtained from the five splice beams in splice tests, it may be concluded that:

1. All beams are 100 percent Code splice because they did not fail at static loads which produce the design steel stress of 60 ksi.

2. Beams with stirrups in the lap region did not fail by splitting.

3. Beams reinforced with smooth welded wire fabric failed by fatigue of the wire rather than by splitting.

4. Miner rule does not represent the actual cumulative damage in fatigue of beams reinforced with smooth welded wire fabric subjected to variable loads, or the fatigue characteristics of the reinforcement are modified by the internal conditions in the beams.

3. Suggestions for Further Work

More research is needed to study width of cracks under repeated loads. The effects of creep, shrinkage and bond on the width of cracks must be included in any equation adopted by ACI Code to predict the width of cracks. During this study many topics were noted which could lead to further investigations. Some suggestions are:

1. Compare the effects of different frequencies of load on the width of cracks.

2. Examine load-deflection behavior under repeated loads.

3. Determine the effects of creep, shrinkage and bond on crack width.

4. Study the effect of variation of beam parameters on crack width.

5. Study the width of cracks with varied spacing and different area of longitudinal wires.

6. Determine the effects of repeated loads on width of cracks in beams reinforced with smaller diameters of smooth and deformed wire fabric.

7. Determine the fatigue characteristics of spliced reinforcement.
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