BAYESIAN ESTIMATION OF A REGIONAL PRODUCTION FUNCTION FOR OKLAHOMA MANUFACTURING

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CHAPTER I

INTRODUCTION TO THE STUDY

Problem Statement

Regional general equilibrium models (CGE) are influential tools in policy analysis. However, some serious questions have been raised about the empirical foundations of these models. The main focus of the critique has to do with the specification of production functions (Rickman and Partridge, 1998). Researchers often specify Constant Elasticity of Substitution (CES) or Cobb-Douglas (CD) production functions in empirical models. The CD production function restricts the elasticities of substitution to be unity, whereas CES permits different, but constant elasticities of substitution across industries. Flexible functional forms (FFF) such as the translog do not impose a priori restrictions on elasticities and are capable of modeling inputs as substitutes and complements. This suggests that they are more suitable for specification of the production sector in a CGE model.

Applied regional general equilibrium models (CGE) also have been criticized for their reliance on calibration. During calibration some parameters such as elasticities are chosen from the literature, others are set arbitrarily and the remainder are set to values that replicate the benchmark data. This may lead to specification problems, especially if the elasticity estimates used have been estimated under a partial equilibrium framework or another framework that is inconsistent with the CGE model under study.

Given the paucity of data at the regional level, it is hardly surprising that there have not been many attempts to estimate regional elasticities. Although a few have employed cross-sectional data to estimate elasticity parameters at the regional level, others have used pooled data to estimate the parameters. Although more precise, there is little reason to believe that these estimates are any less biased than those plucked from other empirical literature.

There has been no attempt to estimate flexible production functions at the state level. One reason could be that the estimated production function might not be well behaved at low levels of aggregation, which in turn could lead to convergence problems for the numerical solutions. One simple soultion would be to estimate a flexible production function at a state level by ensuring that all theoretical properties of the function are imposed so that it is well behaved. Also, if there is information on elasticity parameters from national studies, one could incorporate this as prior information in estimating regional production function. The use of prior information of this type is expected to improve estimator precision; if the prior information is good, only a small amount of bias may be introduced. One advantage of the above approach is that a regional CGE modeler can use this specification for modeling the production sector. This production function would represent the underlying technology for the economy under study. The researcher will not have to resort to ad hoc calibration techniques in the specification of important elasticities.

Several alternative methods for imposing concavity and monotonicity restrictions have been developed in consumer-producer demand models. Conventionally, the way to deal with inequality restrictions is to model the problem at hand as a quadratic program. This can result in estimates which are on the boundary of the constraint. The Bayesian approach provides an alternative to nonlinear programming. The set of inequality restrictions are thought of as priors under this approach, which lead to a truncated posterior. Bayesian point estimates are interior, moving closer and closer to the boundary as data disagrees more and more with the restrictions (Griffiths ,1988). Also, with small sample you get exact finite sample inference. More recent works have emphasized the non-classical approach; e.g. Chalfant and White (1988), Terrell (1996) and Dashti (1996) used Bayesian methods to impose restrictions. The Bayesian approach is well suited to regional analysis, as there is paucity of data at regional level. This can be overcome by specifying informative priors from national data for estimates like elasticities of substitution.

Objectives of the Study

The overall objective of the study is to use Bayesian methodology to estimate a regional translog cost function representing manufacturing in Oklahoma. Specific objectives are:

1. To estimate the translog cost function by imposing monotonicity and concavity restrictions.
2. Based on predictive accuracy and posterior probabilities, choose between the Translog and Cobb-Douglas functional form.

CHAPTER II

PRODUCTION THEORY

Intoduction

The foundation for production theory models is the production function, which summarizes the technological base underlying decisions of the firm. When behavioral assumptions such as cost minimization or profit maximization are added to this, one can construct an explicit optimization problem for the firm using these behavioral assumptions as the goal and the production function as the constraint. Solving the first order conditions of such a model allows us to define a function directly representing the optimal choices of the firm. The resulting cost function is "dual" to the production function in the sense that it is explicitly based on the technological structure underlying the production function relationship, so the technology itself may implicitly be reproduced by a reverse optimization process

For example, cost minimization requires in graphical terms, identifying the scale expansion path as a set of tangencies of isocost curves, defined by the given relative input prices and the isoquants. The shape of the path depends on the shape of the isoquants, which in turn is determined by the form of the production function. Therefore, information on scale expansion paths (SEP) supported by different input price combinations could, in reverse, allow us to trace out the form of the isoquants. Since the scale expansion path provides the basis for the dual cost function, this means the cost function could in principle be used to trace out the shape of the production function. The fundamental idea of duality theory is that technology represented by the production function Y(v, t), where t is a time counter, Y is the output and v is a vector of variable inputs, can be represented equivalently in other forms, most of which also incorporate optimizing behavior. One of these forms is the cost function which reflects the minimum total cost of producing a particular output level, Y, given the production technology and input prices. Since the production function is by definition a constraint on this minimization problem, the cost function not only reflects cost minimization but also completely describes the technology of the firm. In reverse, it implies that in certain circumstances the cost function can be used to reproduce the original production function. Consider the basic cost function, the long run function TC (p, Y, t). Since it is founded on SEP, the optimized (cost minimizing) levels of all inputs are incorporated in the function. Constructing a model based on this function, therefore allows us to explore the dependence of input demand and therefore cost on any exogenous variable included as an argument of the function (Morrison, 1999).

In particular, demand equations for factors of production are derived from this function using Shephard's Lemma (a duality result) to obtain long- run optimal demand for inputs j; ∂ TC (p_j, p, Y, t) / ∂ pj = v_j (p_j; p, Y, t), where p here excludes the price of input j. This procedure graphically implies identifying a point on the TC function, by finding for a particular p_l, say, given p_k, Y, and t, what the associated point on the SEP would be (assuming inputs are K and L). The resulting input demand function includes all arguments other than p_j as shift parameters. More formally the cost function is defined as

$$TC(Y, p, t) = \min_v \{p^T v: Y(v, t) \geq Y^*\} \quad (1)$$

Where $(p_1, p_2 \ldots p_j) = p >> 0_J$ is a vector of J positive input prices, >> implies strictly positive, Y* is a particular output level, no monopsony exists (input prices are exogenous), and the "T" superscript indicates a transposed vector. If a solution to the cost minimization problem (which involves solving first order conditions for all inputs in the v vector as well as satisfying the Y (v, t) ≥ Y* constraint) exists, we can state a number of regularity conditions that the resulting function must satisfy:

1. TC (•) is a nonnegative function (TC (Y, t, p) ≥ 0).
2. TC (•) is positively linear homogeneous in input prices for any fixed output level.
3. TC (•) is increasing in input prices $\partial TC / \partial p_j > 0$.
4. TC (•) is a concave function of p ($\partial^2 TC / \partial p_j^2 \leq 0$).
5. TC (•) continuous in p and continuous from below in Y.
6. TC (•) is nondecreasing in Y for fixed p ($\partial TC / \partial Y \geq 0$).
7. TC (•) meets the symmetry requirement. By the symmetry of the Hessian matrix, it must be that $\partial^2 TC / \partial p_i p_j = \partial^2 TC / \partial^2 p_j p_i$, and similarly for the cross-derivatives with respect to any two arguments of the function.

Most of these conditions are quite intuitive. (1) simply reflects the fact that costs are positive for productive firms, if not, production would be infinite if output price were positive. (2) indicates that units of measurement for cost do not matter; if all prices increase by particular factor, minimum possible total cost of production for a given output level must change by same proportion. This is the homogeneity restriction. (3) implies that if a price increases costs must also increase. If this is not the case costs could not have been minimized in the first place. This is the monotonicity restriction. (4) Implies that as input price rises minimum costs also go up, but this increase will not be proportional because substitution occurs. (5) in effect says that inputs must be divisible. This is required for derivatives to be defined for analysis. (6) ensures that marginal cost as well as total cost is positive, which is necessary for optimization to make sense. (7) requires that substitution matrix is negative semi-definite. It ensures that consistent choices are made. A sufficient condition for negative semi-definite substitution matrix is that eigenvalues must all be less than or equal to zero. If symmetry is falsified, cost is not minimized (Morrison, 1999).

For empirical purposes the choice of the function to use for analysis is important, since, it determines what type of equations represent the production technology and behavior, and what economic performance indicators may be generated directly from these equations.

A production function might initially be thought to be most useful to specify and work with, since, it directly represents the production technology. However, as the basis for representing the behavior and thus responses of the firms, the production function has shortcomings. The main problem with the production function specification involves the endogeniety versus exogeniety of right-hand side variables. Essentially, if the behavioral assumptions are incorporated, the production function framework results in a system of pricing equations for estimation that are implicitly based on the premise that the firm decides what prices to pay for given quantities of inputs.

More specifically, in the production function specification the first-order conditions underlying the system of estimating equations stem from the profit maximizing equalities $p_j = p_Y * MP_j \equiv p_Y * \partial Y (v, t) / \partial v_j$, where $p_Y * MP_j = VMP_j$ is the value of the marginal product. Thus the production function is used to define the marginal products, and the profit maximization assumption is imposed to generate the estimating system.

However, in the above scenario the input price p_j turns up on the left-hand side of the equation for estimation and the marginal product expression appears on the right hand side. This implies that v_j's are exogenous and prices endogenous, whereas one usually would think firms work the other way around - they observe the market price and choose input demand.

It seems more natural to represent firm's behavior in terms of costs. If estimation is based on cost function, the input demand functions can be derived directly from Shephard's Lemma. Input demand is therefore by construction the endogenous variable, which facilitates using these expressions as the basis for a system of estimation equations. Further, if one wishes to represent input demand elasticities, this implies specifying the model in terms of input demands. Use of cost function also avoids the issues of long-run behavior. With constant returns to scale and perfect competition profit maximization in the long run is not defined although cost minimization for a given output level is.

The conventional approach in production economics has been to posit an explicit production function, and then solve the cost-minimizing problem, to derive factor demand functions. This approach has been criticized on the grounds that it imposes quite restrictive assumptions regarding elasticity of substitution parameters, or leads to algebraic expressions for factor demands that involve empirically unmanageable parameters that characterize the underlying production function. This is especially true when one wishes the production function to be flexible in the sense that it provides a second-order differentiable approximation to an arbitrary twice continuously differentiable function.

However, there is an alternative approach to study production relationships invoking the duality theorems. These duality theorems, in addition to their theoretical attractiveness, have resulted in very pragmatic formulation of production functions. Specifically, this approach utilizes Shephard duality and Shephard Lemma. Shepherd duality ensures the existence of a unique production function given the existence of a unique cost function with certain regularity conditions, and the latter require that we then estimate the relevant parameters by simple regression.

CHAPTER III

REVIEW OF LITERATURE AND RELATED RESEARCH

Introduction

There have been few attempts at estimating regional production functions for the United States. One reason for the lack of studies attempting to estimate production functions at the regional level is the lack of regional data. In this section we provide a general overview of the literature dealing with production functions. Next we present a summary of the few attempts at estimating production functions at the regional level

The estimation of production functions has a long history. In the 1920s Charles Cobb and Paul Douglas (1928) used what has become known as the Cobb-Douglas (CD) production function, to test marginal productivity theory. Although the CD function was useful for labor value share application that was of interest to Cobb and Douglas, other economists who were more interested in measuring substitution elasticities among inputs found the CD functional form too restrictive. For the CD case, the elasticity of substitution always equals one. In a very important paper, Kenneth J. Arrow, Hollis B. Chenery, Bagica Minhas, and Robert Solow (1960) proposed an extension of CD function, referred to as Constant Elasticity of Substitution (CES) production formulation. The CES functional form provided an elasticity of substitution which was constant, but was not constrained to unity.

Around this time one sees efforts by Earl Heady and his associates to empirically estimate production functions. Heady et al. (1961) designed crop, seed, and fertilizer experiments for a large number of crops and then used these data and the least squares method to estimate input output relationships with alternative functional forms. Heady et al. wanted to include in their experiments input combinations which resulted in negative marginal products. The CD form only permits positive marginal products. Heady et al. experimented with Taylor's series expansion as polynomial approximations to unknown algebraic forms. They introduced the second degree polynomial in logarithms that added quadratic and cross-terms to CD function. This form was dubbed the translog function a decade later by Christensen et al (1971). They also reported least squares estimates of a square root transformation that included as a special case the generalized linear production function introduced by Diewert (1971).

The efforts by Arrow et al. and Heady et al. all involved attempts to generalize the restrictive form of CD. It is noted that CD and CES functional forms satisfy requirements of global regularity but make strong assumptions about input substitution. Uzawa (1964) showed that the CES form had limited flexibility because it could not realize arbitrary elasticities of substitution. Interest then focused on functional forms which were locally flexible but not globally regular such as the generalized Leontief (GL) and the translog. GL and translog functional forms use duality between production and cost functions and permit arbitrary elasticities of substitution among several inputs.

Price shocks of early 1970s led to a systematic study of relationship between energy and other inputs in the production process. The seminal paper by Berndt and

Wood (1975) use the translog functional form in focusing on cross-substitution possibilities between energy and other inputs to characterize the structure of technology in U.S. manufacturing. They find energy and labor to be substitutes, and energy and capital to be complements. Additional studies conducted by Griffin and Gregory (1976) and Pindyck (1979) find capital and energy to be substitutes. Evidence from engineering studies also tend to report the capital-energy substitutability. This capital-energy controversy has still not been resolved in the production literature. If capital and energy are complements, higher energy prices reduce capital investment and potential GNP. If on the other hand, capital and energy are substitutes, higher energy prices encourage capital investment and increase potential GNP.

The literature at this stage tends to focus on reconciling the above capital and energy controversy. Berndt and Wood (1979) wrote a paper to point out that engineering studies usually include only energy and capital as inputs and focus on movement of production along a capital isoquant. They point out that with only two inputs, capital and energy can not be complements. They suggest that price elasticity be divided into gross and net price elasticity. Using their earlier data, Berndt and Wood (1979) find that net price elasticity dominated gross price elasticity, and thus the finding of energy-capital complementarity. Griffin (1981) refutes the arguments presented in Berndt and Wood (1979) paper and helps to reconcile the capital-energy substitutability finding. He points out that Berndt and Wood (1979) used time series data on U.S. manufacturing that had little energy price variation, while pooled international data provided greater variation. Cross-sectional international data are more likely to provide long-run elasticities, while time series data used by Berndt and Wood (1979) are more likely to provide short-run elasticities.

Berndt and Wood (1981) reply by pointing out that the Griffin paper did not include materials as factor of production. With fewer inputs, the finding of substitution is more likely. The quality of data used was also questioned. The capital price in many countries had to be adjusted for different tax structures and capital stock was measured as value added residual. Norsworthy and Malimquist (1983) use the translog cost function to extend the analysis beyond the first energy price shock. They compare U.S. and Japanese labor productivity, and perform different tests of input aggregation. They point out the capital-energy complementarity in agreement with Berndt and Wood finding.

Improvements to Translog Estimation

Improvements to the translog estimation was suggested by Woodland (1979). He points out that the standard share equation specification neglects the requirement that shares assume a value between zero and one. As an alternative, the Dirichlet distribution that automatically limits share values is suggested. A comparison using data from three empirical studies and two specifications provide evidence for continued use of the normal distribution.

Considine and Mount (1984) use a logit specification that limits possible shares to values between zero and one. The disadvantage of this approach is that symmetry is imposed only on one observational point. They use expanded Berndt and Wood (1975) data for years 1947 through 1981 to estimate translog and logit specifications. The translog suffers from concavity voilations at twenty-one of the thirty-five observations and positive own price elasticities for the years 1958 through 1981. The logit specification has negative own price elasticities for all years but suffers from concavity violation at the mean of the cost share, so concavity is violated at all observations. A dynamic logit model is also specified and no concavity violation is found at the cost share mean and thus concavity is satisfied for all observations. At the mean cost shares, energy and capital are substitutes while energy and labor, and labor and capital are complements in both the short and long run.

Chavas and Segerson (1987) examine the stochastic specification used to estimate share equations and propose an alternative minimum distance estimator. This estimator is invariant to the deleted share equation and is based on a random objective function. They conclude that for the almost ideal demand system and the translog functional form, assuming homoskedasticity can be restrictive and cause biased hypothesis testing.

Considine (1990) uses a iterative nonlinear estimation procedure to impose symmetry for all observations of the logit model. Using the Berndt and Wood data no concavity violations are found. Capital and labor, and energy and labor are substitutes, while energy and capital are complements.

Kim (1992) estimates a translog production function with variable returns to scale. Inverse input demand functions are written using the translog share equations form. These are estimated using the logistic-normal distribution so that shares will be distributed in the unit interval. Using the Berndt and Wood data he finds that constant returns to scale assumption is rejected.

Clark and Youngblood (1992) estimate a translog function as a cointegrated process using Canadian agriculture data. After testing for unit roots they find that factor prices, shares, and output are cointegrated implying that technical change is neutral. A traditional autocorrelation corrected translog with a time proxy for technical change is estimated for comparison. The cointegrated model satisfies the theoretical concavity and monotonicity requirements better than the traditional translog. The cointegrated model finds that all inputs are substitutes, while the translog finds that land and fertilizer are complements.

Development of Alternative Functional Forms

In an attempt to improve the estimation of production functions and criticism of translog functional form has led to the development of alternative functional forms. The different approaches to production or dual cost function estimation attempt to provide good approximation using economic theory, while permitting flexible substitution and complementary relations among inputs. Generally, the more parameters a functional form has the greater its flexibility, the more information it can provide about substitution and complementary relations among the inputs, and the more closely theoretical requirements can be met. Berndt, Morrison, and Watkins (1981) divide the efforts to estimate production functions into three generations.

First Generation Models

The first generation is characterized by single-equation dynamic models with fixed lag structures and limited interaction among the inputs. The CD and CES are examples of this.

Second Generation Models

Second generation models are characterized by functional forms that recognize the inter-relatedness of factor demands such as the translog and generalized Leontief. These forms are termed as flexible functional forms. Flexible functional forms allow a general specification of interactions among arguments of the function such as input substitution. More specifically flexible functional forms allow data to identify patterns among the arguments of the function since by definition they impose no a-priori restrictions on these interactions. Flexible functional forms provide a second order approximation to arbitrary twice continuously differentiable functions.

Further efforts to improve on flexible functional forms has led to the development of semi-nonparametric functional forms. Semi-nonparametric functional forms use additional terms of series expansion to asymptotically eliminate specification error bias. One usually increases the number of expansion terms used as the sample size increases. These functional forms constrain curvature conditions to be satisfied. This forces data to satisfy properties that must hold for our production theory model to hold. Such functional forms include the symmetric Mcfadden, the generalized Box-Cox, the Fourier, and the minflex Laurent. Gallant (1981) uses the Fourier series expansion to create a functional form that will approximate any functional form to a know bound. Barnett, Geweke, and Wolf (1991) have made an important contribution by using Muntz-Szatz series expansion to create their asymptotically ideal model. They find that the generalized Leontief functional form, which is often used as an alternative to translog, is a member of Muntz-Szatz expansion.

Third Generation Models

Third generation models deal with fixity of certain inputs. An important assumption implicit under the formulation of translog and generalized Leontief has been that all inputs adjust instantaneously to their long-run equilibrium values. Specifically, researchers make a distinction between variable and quasi-fixed inputs, where the latter adjust only partially to their full equilibrium level within one period. Besides making a distinction between fixed and variable inputs, this approach analytically derives the optimal transition path from short to long-run. Third generation dynamic models are based on optimizing agents with dynamically changing functions. Input adjustment of quasi-fixed factors have endogenous time varying speeds of adjustment to their equilibrium levels instead of being exogenous and fixed. Short-run demand equations depend on the price of output and variable inputs while holding the quasi-fixed inputs constant. The transition from short to long-run for the variable input includes not only adjustment of quasi-fixed factors, but also incorporates economically optimal variation in their rates of utilization. The dynamic path of adjustment to long-run equilibrium is based on economic optimization at each point in time so the short and long-run are clearly defined. Berndt, Morrison, and Watkins (1981) provide a third generation model using partial adjustment and static expectations. Epstein (1981) develops the theoretical basis for dynamic factor demand models. A practical procedure for generating a large class of dynamic factor demand functional forms is described. Kokkelenberg and Bischoff (1986) discuss what they call fourth generation factor demand models. These models replace the static expectations specification used in earlier generation with stochastic expectations.

To model expectations stochastically requires one to posit an explicit process under which expectations were formulated. Rational expectation paradigm is a popular approach adopted in the literature.

Rational Expectations Paradigm. Rational expectation hypothesis posits that individuals use all information when forming expectations about the future and thus do not make systematic forecasting errors. Rational expectations is commonly used as a "black box" that provides an instantaneous outcome for a complicated unexplained underlying process.

Rational expectations models generally assume a linear time series model of factor prices with a quadratic production technology so a closed-form solution to the dynamic factor demand functions can be found. Attempts to estimate productions with rational expectations can be divided into three approaches. The first approach uses an explicit analytic solution to firm's optimization problem with expectations based on an autoregressive model and is explored by Hansen and Sargent (1980). Epstein and Yatchew (1985) use this approach with a simpler functional form and Berndt and Wood (1975) data extended to 1977. They find data to be inconsistent with the functional form and evidence that higher-order parameter adjustment terms are needed. Kokkelenberg and Bischoff (1986) use the approach with quarterly manufacturing data from 1959 through 1977 and find evidence of energy-capital complementarity. They use ARIMA equations to supply future exogenous variables.

The second approach to estimating production with rational expectations uses parameters from initial planning period Euler equations. Actual observed values are used for expectations and an instrumental variable estimation technique is used. Greater functional form flexibility is possible, but the estimation procedure is not efficient. This approach is explored by Kennan (1979), Hansen (1982) and Hansen and Singleton (1982). Using the technique Pindyck and Rotemburg (1983) found energy and capital to be complements in the short-run with an elasticity close to that found by Berndt and Wood. In the long-run energy and capital are substitutes with elasticities close to that found by Pindyck (1979), and Griffin and Gregory (1976). They warn that with stochastically evolving prices, intermediate-and long-run elasticities must be interpreted with caution.

The third approach suggested by Purcha and Nadiri (1984) uses a numerical algorithm that solves the firm's optimization problem at each iterative step. A certainty equivalence feedback control policy is used instead of stochastic closedloop feedback control policy of first two approaches.

Fourth Generation Models

Fourth generation dynamic production and cost models use rational expectations to model the formulation of expectations by assuming that agents maximize the value of some control function. To do so in the long-run requires knowledge about future prices or resource scarcity.

Autocorrelation Correction

The translog and generalized Leontief specification assume a complete adjustment of factor prices. Time series data used in empirical studies usually violates the assumption of uncorrelated disturbances. Autocorrelation correction can be viewed as a convenient simplification of a dynamic model with a complex lag structure.

In a simultaneous equation system the disturbance term of one equation can be correlated to itself and the disturbance terms of the other equations. A matrix of autoregressive factors can be used in a manner analogous to the single equation to correct the problem. For a single equation the Cochrane-Orcutt or Hildreth-Lu autocorrelation correction procedures are often used. The problem becomes more difficult when the system of simultaneous equations is singular as in the case of the translog share equations.

Berndt and Wood (1975) discuss the autocorrelation correction for a singular system of equations. They prove that the column of the matrix of autoregressive factors must add to an unknown constant or the result will not be invariant to the equation deleted. If a single adjustment factor is used for each equation then the adjustment factors must be identical. When the matrix of autoregressive factors is nondiagonal, additional assumptions must be made to recover the matrix of autoregressive factors for the original share equations.

Anderson and Blundell (1982, 1983) consider the estimation of singular equation system using a general specification that incorporates the static, first differences, partial adjustment, and vector autoregressive models. They observe that when systems of demand equations are estimated, the theoretical requirements of symmetry and homogeneity, are often rejected and they believe that the difficulty exists because the dynamic structure of the model has not been correctly specified. The general specification they suggest can be used with the translog and other functional forms. In empirical tests using the Berndt and Savin (1975) data and Canadian data for nondurable goods they find support for the vector autoregressive model.

Kumbhakar and Seeletis (1990) use a translog specification and the method of Anderson and Blunndell (1982, 1983) to test a sequence of nested dynamic specifications. Using the data of Norsworthy and Malmquis (1983) and computing elasticities at the mean of the data, they found that capital-energy and capital-labor pairs are substitutes, while energy and labor are complements.

Friesen (1992) uses the Anderson and Blundell (1982,1983) autocorrelation approach with a translog error-correction model and Berndt and Wood (1975) data extended through 1981. She tests several dynamic translog models holding various combinations of inputs as quasi-fixed. The quasi-fixed factors enter the translog cost share equation as a quantity instead of a price, and the cost of the factor is not included in the cost of production. The factors not held as quasi-fixed are assumed to fully adjust each period. Symmetry and homotheticity are tested using maximum likelihood values. While her error-correction model dominates other functional forms, it is unable to produce satisfactory long-run elasticity estimates. Positive own-price elasticities are found for many data points. She notes that prices were more stable before 1970s and the additional observations may be the reason for the theoretical violations.

Comparison of Alternative Functional Forms

Several functional forms have been reviewed above. How does one choose among competing functional forms? For example, let us look at three forms: the generalized

Leontief (GL), the translog (TL), and the generalized Cobb-Douglas (GSD). By definition all three have attractive local property and a researcher is not clear as to how to choose among them. A few studies have discussed the issue of choosing among the wealth of different productions that have been proposed. Wales (1977) performed a Monte Carlo study to investigate the ability of GL and TL forms to represent two-commodity homothetic preferences exhibiting constant elasticity of substitution. He finds TL to perform better in some cases while GL performs better in others. Berndt, Darrough, and Diewert (1977) used Canadian expenditure data to estimate three-commodity nonhomothetic GL, TL and GCD forms. On the basis of better fit and conformity with neoclassical restrictions, they concluded that the TL form was the better form for their data set. Tsurumi and Tsurumi (1976) explore Bayesian estimation of the CES production function. A Bayesian highest posterior density interval inference is made to examine the validity of the CD representation. The method is applied to micro and macro data on two Japanese manufacturing plants. Two cases out of five tested in their study reject the CD representation. They recommend to test the validity of CD form in micro and macro studies by testing it against other alternative functional forms. Rossi (1985) presents a Bayesian approach to choosing between two non-nested multivariate regression systems. The Bayesian approach involves the calculation of posterior probabilities of alternative hypothesis and formation of posterior odds ratio. Odds ratio are applied to make a choice between TL and Fourier flexible functional form. They apply this procedure to U.S. manufacturing cost data and the estimated odds ratio favor the Fourier flexible form over the translog form.

The above papers have employed statistical analysis as the basis for choosing among alternative functional forms. Implicit in these statistical investigations is the recognition that the ability of a function to perform well over a range of data points, in addition to the base point, is an important criterion to be used in choosing among flexible forms. Caves and Christensen (1980) address this issue and provide global regularity regions for the GL and TL forms. Recently, Terrell (1996) uses Gibbs sampling to impose the regularity condition on three flexible functional forms: generalized Leontief, symmetric generalized McFadden and translog. Using Berndt and Wood (1977) data he finds that all three functional forms do not meet the regularity conditions. Using an informative prior defined as a prior that incorporates inequality restrictions from theory, he finds that imposing monotonicity and concavity over larger ranges of prices restricts the functional form in a manner similar to global restrictions. He finds that imposing restrictions over smaller range of prices gives one reasonable results without any loss of flexibility.

Hedonic Cost Functions

The translog and generalized Leontief specifications include a single output measure. Spady and Friedlaender (1978) have extended the above specification by integrating the hedonic approach, which emphasizes attributes of outputs, with the flexible functional form literature. They note that in many industries, physical output varies with respect to attributes or qualities. They construct a hedonic measure of output as a function of output characteristics and call it "effective" or "quality- adjusted" output. They argue that failure to take these output characteristics into account can create serious specification errors. They examine regulated trucking in U.S., where ton-miles is the conventional measure of output. They distinguish the ton-mile of short-haul, small-load, less-than-truckload traffic from those of the long haul, large load, truckload traffic, not by treating them as separate outputs, but rather specifying an effective output that depends on generic measure of physical output and on the attributes of this output.

Other Alternatives Formulations

Another alternative is the estimation of frontier production or cost functions that originated with Farrell (1975). When estimating frontier cost or production functions all residuals are typically constrained to be negative. Firms are depicted as approaching but unable to reach a production possibilities frontier because of inefficiencies. This approach has primarily been used in efforts to study the efficiency of different industries. Varian (1990) suggests testing departures from optimizing behavior to see if they are significant in the economic sense instead of statistical sense. He discusses procedures for testing significance in economic sense by using a money metric goodness-of-fit measure and provides an example using aggregate consumer demand. Antle (1983) suggests a flexible moment-based approach to estimating production and cost functions. Affixing an additive or multiplicative error term to a deterministic production function imposes arbitrary restrictions on the moments. McElroy (1987) criticizes studies of production for adding linear error terms after specifying a deterministic model. She proposes general error models. General error models with an additive error is specified and estimated using Berndt and Wood (1975) data. Results are similar to that of Berndt and Wood (1975) study and are found except for a thirty-seven percent higher partial elasticity of substitution between capital and labor. A specification test shows the translog additive general error models is superior to the standard translog specification.

Regional Studies

Studies by Gallaway (1963), Scully (1971), Alperovich (1980) and Hunt and Emerson (1982) are the few examples of regional studies. Gallaway tested and advanced the hypothesis that persistence of a steady 20 percent unadjusted wage differential between northern and southern manufacturing workers could be explained by the differences in production functions. Conclusions were that data could not support the hypothesis. A different conclusion was obtained by Scully (1971), who tested the same hypothesis for the period 1907–1946, and concluded that the two regions operated on different production functions. Alperovich (1980) tests the hypothesis that regional production functions are different due to regional differences in technological agglomeration economies, technical quality of capital etc. Using data from 1950-1969, for nine U.S. regions, they estimate a Constant-Elasticity-Substitution production function. Their conclusion is that elasticities of substitution are different across regions, but with the exception of few industries, these differences are not significant, implying uniformity across regions.

The above studies have been criticized for employing restrictive functional forms for estimating production functions and being limited to the two input case. The translog cost model, a flexible functional form for estimating cost function is estimated by Hunt and Emerson (1982). They estimate a three input cost function for U.S. manufacturing using cross-sectional data by state for 1976. They perform checks for monotonicity and concavity restrictions and find that they are met. They conclude that there is significant North-South regional variation in production functions.

CHAPTER IV

METHODOLOGY

Bayesian Paradigm

In this section we provide a very brief overview of Bayesian modeling. For illustration purposes consider a simple linear regression model:

$$Y = X\beta + e, \qquad e \sim N(0, \sigma^2 I_T), \tag{2}$$

where Y and e are T x 1 vectors, and X is a T x K matrix of rank K containing the fixed regressors, X_1, X_2, ...X_k. Under the Bayesian paradigm, the parameters of the model, $\theta = [\beta\ \sigma^2]$, are treated as random variables having probability distributions. These distributions are used to summarize the status of knowledge about the model parameters. A probability distribution, $g(\theta)$, employed by a researcher to summarize his or her knowledge of θ before observing sample observations on X and Y is called a prior distribution. Different researchers may have different knowledge about the parameters, and the knowledge may be subjective, reflecting prior beliefs. When prior information is not available, a "flat" or diffuse prior is adopted. Once Y is observed, a Bayesian researcher revises the distribution of the parameters by combining the prior distribution with the information contained in the sample, using Bayes' theorem. Denote the distribution of the sample observations Y given the parameters by $f(Y \mid \theta)$, the joint distribution of the data parameters by $h(\theta, Y)$, and the marginal distribution of the data by

$f(Y)$. The posterior distribution of parameters given the data is denoted by $p(\theta \mid Y)$. The joint distribution can also be written as:

$$h(\theta, Y) = f(Y \mid \theta)\, g(\theta) = p(\theta \mid Y)\, f(Y) \tag{3}$$

from which Bayes' theorem is obtained as follows:

$$p(\theta \mid Y) = f(Y \mid \theta)\, g(\theta) \,/\, f(Y) \tag{4}$$

or because $f(Y)$ has no operational significance,

$$p(\theta \mid Y) \propto f(Y \mid \theta)\, g(\theta). \tag{5}$$

It is customary, noting the functional equivalence of $f(Y \mid \theta)$ and the likelihood function $L(\theta \mid Y)$, to express the posterior distribution as

$$p(\theta \mid Y) \propto L(\theta \mid Y)\, g(\theta). \tag{6}$$

This shows that posterior distribution combines the likelihood function with the prior distribution.

Bayesian Approach to Inequality Restrictions

To ensure the regularity of the translog function estimated in this paper, we impose monotonicity and concavity restrictions. These restrictions involve inequality constraints. We follow a Bayesian approach to impose these restrictions. We describe this approach below.

Following the discussion of the above section, the first step of the Bayesian approach is to define a prior density function, over the vector of parameters, β, call it $p(\beta)$. This prior contains all the information available about the parameter before estimation. If there is no prior information available about parameters, a diffuse or non-informative prior is specified. In this case $p(\beta)$ is defined to be proportional to a constant,

$p(\beta) \propto c$. On the other hand, if strong prior knowledge about elasticities and parameters exist, a prior incorporating this information can be specified. A second step in Bayesian analysis is to specify a likelihood function $L(\beta)$. Bayes theorem shows how to combine prior and sample information to obtain the posterior distribution of parameters in a given data set Y. The essential idea is that the posterior assessment is a blend of the new information and the prior assessment. Next, define an indicator function which is defined over the region consistent with inequality restrictions:

$$h(\beta) = \begin{cases} 1 & \text{if } \partial c / \partial p \geq 0,\ \partial^2 c / \partial p \partial p' \text{ is N.S.D.} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

This indicator function is one if monotonicity and concavity are satisfied, otherwise it is assigned a value of zero. Bayes theorem is used to combine the likelihood function, prior information and the above indicator function given in (7) to derive the posterior distribution

$$f(\beta \mid y) = \begin{cases} p(\beta)\, L(\beta \mid y) & \text{if } \partial c / \partial p \geq 0, \partial^2 c / \partial p \partial p' \text{ is N.S.D.} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The above posterior density is only defined over the range where the cost function satisfies monotonicity and concavity. We say that the posterior is truncated as it cannot take values outside the range defined by the indicator function.

Model For Incorporating Informative Priors

From the above discussion we gather that using the Bayesian approach one can impose inequality restrictions using either diffuse or informative priors. In this section, we posit a general model where we can incorporate diffuse and informative priors. We consider the estimation of this model within the context of a set of seemingly unrelated regression (SUR) equations. The general form of the model is given as follows:

$$y_j = Z_j \beta + \varepsilon_j \tag{9}$$

where y is a $T \times 1$ vector, Z_j is a $T \times K_j$ matrix of explanatory variables, β_j is $K_j \times 1$ vector of unknown parameters, ε is a $T \times 1$ vector, and $j = 1, \dots, m$. Define Z as follows :

$$Z' = [Z_1 \dots\dots\dots Z_m].$$

The distributional assumptions for the above model is

$$\varepsilon \mid Z \sim \mathrm{N}(0, H^{-1} \otimes I_T). \tag{10}$$

The $m \times m$ matrix H is the precision matrix of the disturbance vector ε_t.

The prior distribution of ε is given as follows:

$$\beta \sim N(\underline{\beta}, \underline{H}_\beta^{-1}). \tag{11}$$

$$\beta \mid (H, y, Z) \sim N(\overline{\beta}, \overline{H}_\beta^{-1}), \tag{12}$$

where $\overline{H}_\beta = \underline{H}_\beta + Z'(H \otimes I_T)Z$, $\qquad \overline{\beta} = \overline{H}_\beta^{-1}\left[\underline{H}_\beta \beta + Z'(H \otimes I_T)y\right]$

and $\overline{H}_\beta$ and $\overline{\beta}$ represent the posterior precision and mean respectively.

Denote W to represent the Wishart distribution, and assume the prior distribution of precision matrix, H, to be independent of β. Then, the prior distribution of H is given as follows:

$$H \sim \mathrm{W}(\underline{S}^{-1}, \underline{v}), \tag{13}$$

where $\underline{S}$ is the sum-of-squares and cross products for the sample and $\underline{v}$ represents the degrees of freedom. The conditional distribution for H is given as follows:

$$H \mid (\beta, \mathrm{y}, \mathrm{Z}) \sim \mathrm{W}\left[(\underline{\mathrm{S}} + S)^{-1}, \underline{v + T} \right] \tag{14}$$

In the above framework, we can set-up our model to represent complete ignorance (diffuse) of the parameters of the model by specifying precision matrix to be of very small magnitude or we can make use of information available about the parameters from prior studies to set up the precision matrix.

To complete the modeling exercise, we combine the SUR model described above with the Bayesain approach to imposing inequality restrictions, to impose monotonicity and concavity restrictions. To obtain point estimates of parameters of the posterior density under a quadratic loss function, one needs to find the mean of the truncated distribution for the parameter vector. While these calculations are straightforward, the integrals over the posterior density cannot be obtained analytically. Although it is impossible to sample directly from a joint posterior density of an SUR, a Gibbs sampling algorithm can be used to simulate posterior distribution of mean and precision for the model described above. The Gibbs sampling algorithm is discussed below.

Gibbs Sampling

Gibbs sampling is a Markov Chain Monte Carlo simulation tool for approximating joint and marginal distributions by sampling from conditional distributions. Gibbs sampler essentially breaks the "curse of dimensionality" by replacing draws from the high-dimensional joint distribution with draws from low dimensional conditional densities. The Gibbs sampler is made operational with starting values of the model unknowns. Next the sampler updates the components of the model by drawing from the conditional densities. This updating is done repeatedly to obtain a set of updated vectors. This sequence provides a trajectory for a stationary Markov chain whose stationary distribution is precisely the desired joint distribution. In fact, the sequences associated with any particular vector converge in distribution to the marginal distribution of that component (Casella and George, 1992).

Specifically, suppose we are given the joint density of k random variables, $f(z_1, z_2, \ldots z_k)$, and that we are interested in obtaining characteristics of the marginal density

$$f(z_t) = \int \ldots \int f(z_1, z_2, \ldots\ldots, z_k)\, d z_1 \ldots d z_{t-1} d z_{t+1} \ldots\ldots d z_k, \tag{15}$$

such as mean or variance. However, the joint density may not be given, or even if the joint density is given, integration of the above joint density may be difficult to perform. This is usually termed the "curse of dimensionality". The Gibbs sampler as mentioned earlier breaks this curse of dimensionality. If we are given a complete set of conditional densities, $f(z_t \mid z_{j \neq t})$, t = 1, 2,....k, with $z_{j \neq t} = \{z_1, \ldots\ldots z_{t-1}, z_{t+1}, \ldots. z_k\}$, then the Gibbs sampling technique allows us to generate a sample $z_1^j, z_2^j, z_3^j \ldots\ldots\ldots, z_k^j$, from the joint density $f(z_1, \ldots\ldots, z_k)$ without requiring that we know either the joint density or the marginal densities $f(z_t)$, t = 1, 2,.....k. Below we present the basic steps for implementing the Gibbs sampler. Given an arbitrary starting set of values ($z_2^0, \ldots\ldots z_k^0$),

1. Draw z_1^1 from $f(z_1 \mid z_2^0, \ldots\ldots z_k^0)$.
2. Then draw z_2^1 from $f(z_2 \mid z_1^1, z_3^0, \ldots\ldots z_k^0)$.
3. Then draw z_3^1 from $f(z_3 \mid z_1^1, z_2^1, z_4^0 \ldots\ldots z_k^0)$.

.

.

k. Finally draw z_k^1 from $f(z_k \mid z_1^1 \ldots\ldots z_{k-1}^1)$.

Steps 1 through k can be iterated J times to get $(z_1^j, z_2^j, z_3^j \ldots\ldots\ldots, z_k^j)$, j = 1,2,...J. Gelfand and Smith (1990) have shown that the joint and marginal distributions of generated $(z_1^j, z_2^j, z_3^j \ldots\ldots\ldots, z_k^j)$ converge at an exponential rate to the joint and marginal distributions of $z_1, z_2, z_3, \ldots\ldots z_k$, as $J \to \infty$. Thus the joint and marginal distribution of $z_1, z_2, z_3, \ldots\ldots z_k$ can be approximated by the empirical distribution of M simulated values $(z_1^j, z_2^j, z_3^j \ldots\ldots\ldots, z_k^j)$ (j = L + 1, L + M), where L is large enough that the Gibbs sampler has converged. For example, the mean of the marginal distribution of z_i may be approximated by $\frac{\sum_{j=1}^{M} z_i^{L+j}}{M}$ and the marginal distribution may be approximated by the empirical distribution of $(z_1^{L+1}, z_2^{L+2}, \ldots\ldots\ldots, z_i^{L+M})$.

In context of our modeling exercise, we need the conditional densities for the SUR model to make the Gibbs sampler operational. Specifically, we need the conditional densities for the posterior mean and the precision. These are given above in (12) and (14), respectively.

Model Formulation:Translog

Consider the transcendental logarithmic cost function introduced by Christensen-Jorgenson-Lau (1971, 1973). The translog function is viewed as a quadratic logarithmic approximation of an arbitrary function. Letting p_i denote the price of input i, q denote output, the translog function with n inputs is as follows:

$$\ln c(p, y, t) = a_0 + \sum_{i=1}^{n} a_i \ln p_i + a_q \ln q + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln p_i \ln p_j + \sum_{i=1}^{n} a_i \ln p_i \ln q$$

$$+ 1/2\, a_{qq} \ln q \ln q + a_q\, t \ln q + 1/2\, a_{tt}\, t^2, \qquad (16)$$

where

$$a_{ij} = a_{ji} \text{ for all } i, j$$

$$\sum_{i=1}^{n} a_i = 1, \ \sum_{j=1}^{n} a_{ij} = 0 \ (i = 1..., n)$$

$$\sum_{i=1}^{n} a_{iq} = 0, \sum_{i=1}^{n} a_{it} = 0.$$

The above conditions are imposed to ensure that the translog cost function is homogeneous of degree one with respect to factor prices and has a symmetric hessian matrix.

For the purposes of this paper, we assume constant returns to scale and weak separability of materials inputs from the three other inputs namely capital (K), labor (L), and energy (E). The assumption of constant returns to scale is well established in the literature and the latter assumption is necessitated by the lack of suitable data on heterogeneous input materials (Jorgenson, 1974). These above assumptions simplify the modeling process by letting us ignore information on output and price of materials. Our simplified cost function looks as follows:

$$\ln c(p) = a_0 + \sum_{i=1}^{n} a_i \ln p_i + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln p_i \ln p_j, \tag{17}$$

where

$$a_{ij} = a_{ji} \text{ for all } i, j$$

$$\sum_{i=1}^{n} a_i = 1, \ \sum_{j=1}^{n} a_{ij} = 0 \ (i = 1..., n).$$

As before, the above conditions are imposed to ensure that the translog cost function is homogeneous of degree one with respect to factor prices and has a symmetric hessian matrix. Logarithmic differentiation of (17) with respect to input prices and application of Shephard's Lemma give rise to the following share equation (S_i):

$$S_i = \frac{\partial \ln c}{\partial \ln pi} = a_i + \sum_j a_{ij} \ln p_j. \tag{18}$$

The homogeneity and symmetry conditions are easily verifiable. At any given price, monotonicity and concavity conditions can be verified based on the restrictions derived from the translog cost function. Monotonicity in input prices is equivalent to non-negative shares for the translog. This is given by the following inequality:

$$S_i = \frac{\partial \ln c}{\partial \ln pi} = a_i + \sum_j a_{ij} \ln p_j \geq 0 \text{ for all inputs i.} \tag{19}$$

The necessary condition for concavity requires that the matrix of second derivatives with respect to input prices be negative. This condition is equivalent to the restriction that the matrix of substitution elasticities be negative semi-definite (N.S.D.). Negative definiteness implies that alternating minors change signs. For any cost function the matrix of elasticity of substitution is singular, so negative semidefiniteness requires checking the first n-1 principal minors of n dimensional matrix (Caves et al, 1988). These lead naturally to inequality restrictions on elasticities. Restrictions that second-derivative of cost function be negative semidefinite is equivalent to elasticity of substitution matrix being negative semidefinite. The restrictions implied on elasticities are as follows:

$$\sigma_{ii} < 0 \text{ and } \sigma_{ii}\,\sigma_{jj} \geq \sigma_{ij}^{\,2} \tag{20}$$

Allen-Uzawa (σ) elasticities of substitution and own-price (ε) elasticities can be computed at the mean shares(s_i and s_j) to study interrelationships between inputs. These are given as follows:

$$\sigma_{ii} = \frac{s_i^2 - s_i + a_{ii}}{s_i^2} \qquad i = 1, 2, 3, \text{ but } i \neq j \tag{21}$$

$$\sigma_{ij} = \frac{s_i s_j + a_{ij}}{s_i s_j}$$

$$\varepsilon_{ii} = \frac{s_i^2 - s_i + a_{ii}}{s_i} \qquad i = 1, 2, 3, \text{ but } i \neq j \tag{22}$$

$$\varepsilon_{ij} = \frac{s_i s_j + a_{ij}}{s_i}$$

Model Formulation: Cobb-Douglas

In this section we describe the Bayesian model used to estimate the CD cost function for Oklahoma. There are instances when we have prior knowledge about the sign /magnitude of the coefficients of a model. Naturally these will be in the form of inequality restrictions. For the CD cost function we have prior information on labor and capital share given by $0 < \beta_L < 1$ and $0 < \beta_K < 1$. Below we present a model where these constraints will be embodied in a linear regression model. Consider a model posited as follows:

$$Y = X\beta + \varepsilon; \qquad \varepsilon \mid X \sim N(0, h^{-1} I_T) \tag{23}$$

and

$$\beta \sim N(\underline{\beta}, \underline{H}^{-1}), \quad \underline{s}^2 h \sim \chi^2(\underline{v}) \tag{24}$$

The model with linear inequality constraints is (23) and (24) subject to

$$a < D\beta < w$$

where D is k X k nonsingular matrix, and $-\infty \leq a_j < \mathrm{w_j} \leq \infty \, (j = 1,\ldots..k)$.

In this form a model may have from one to k linear combinations of coefficients that are constrained. Next define,

$$\gamma = \mathrm{D}\beta, \quad \mathrm{X}^* = X\,\mathrm{D}^{-1}, \quad \underline{\gamma} = D\underline{\beta}, \quad \underline{\mathrm{H}}_{\gamma} = D'^{-1}\underline{\mathrm{H}}\,\mathrm{D}^{-1}$$

then (23) and (2 4) can be transformed and expressed as

$$Y = X^*\gamma + \varepsilon, \qquad \varepsilon \mid X^* \sim N(0, h^{-1} I_T) \qquad (25)$$

$$\gamma \sim N(\underline{\beta}, \underline{\mathrm{H}}_{\gamma}^{-1}), \quad \underline{s}^2 h \sim \chi^2(\underline{v})$$

$$a < \gamma < w. \qquad (26)$$

For the above model conditional distributions are given as follows:

$$\gamma \mid (h, y, X^*) \sim N(\overline{\gamma}, \overline{\mathrm{H}}_{\gamma}^{-1}), \qquad (27)$$

$$\left[\underline{s}^2 + (y - X^*\gamma)'(y - X^*\gamma)\right] h \mid (\gamma, y, X^*) \sim \chi^2(T + \underline{v}) \qquad (28)$$

where $\overline{H}_{\gamma} = \underline{\mathrm{H}}_{\gamma} + h\,X^{*'}X^*, \mathrm{g} = \left(X^{*'}X^*\right)^{-1} X^{*'}y, \overline{\gamma} = \overline{\mathrm{H}}_{\gamma}^{-1}\left(\underline{\mathrm{H}}_{\gamma}\gamma + h\,X^{*'}X^* g\right).$

From (26) and (27) it is clear that the conditional posterior distribution of a single coefficient γ_j is university normal, subject to inequality restrictions. To describe the university normal distribution, let $\overline{H} = [\overline{h}_{ij}]$. From standard results in multivariate normal distribution theory, it can be shown that the precision of this university normal distribution is $\overline{h}_{ij}$, and mean is $\overline{\gamma}_j + \overline{h}_{jj}^{-1} \sum_{\substack{i=1 \\ i \neq j}}^{k} \overline{h}_{ji} (\gamma_i - \overline{\gamma}_i)$:

$$\gamma_j \big| [\gamma_i (i \neq j), h, y, X] \sim N\left[\overline{\gamma}_j + \overline{h}_{jj}^{-1} \sum_{\substack{i=1 \\ i \neq j}}^{k} \overline{h}_{ji} (\gamma_i - \overline{\gamma}_i), \overline{h}_{jj}^{-1}\right] \tag{29}$$

This setup provides a K+1 – block Gibbs sampler for the posterior distribution.

Again in context of out modeling exercise, we need the conditional densities for constrained OLS regression model to make the Gibbs sampler operational. Specifically, we need the conditional distributions for the posterior mean and precision. These are given as above in (26) and (27), respectively.

Construction of Data Sets

In this section we describe the construction of the three data sets employed in this paper. The three data sets provide information about U.S. manufacturing. Specifically, they provide information on capital, labor and energy. We use these three sets of national data to formulate our national priors. These are labeled as BLS, BW and BWX and are presented in Tables I –III. Next we describe the construction of manufacturing data set for Oklahoma manufacturing. This is presented in Table IV and summary statistics for national and regional data are presented in Tables V-VIII. We also describe the construction of price of capital and capital stock series for Oklahoma. National data sets are described first, and Oklahoma data set description follows.

Description of BLS Data

The BLS has produced data for U.S. manufacturing described in a paper by Gullickson and Harper (1987). The data was produced to provide various multifactor productivity indexes. Annual price and quantity data for output, capital, labor, nonenergy materials, and purchased business services are available from 1949 through 1996 for aggregate manufacturing and twenty two-digit SIC (20-39) manufacturing industries. Below we provide a brief description of how individual data series for capital stock, labor, and energy were constructed.

Capital Stock

The perpetual inventory method is used to compute the quantity of capital. First, real investment data from national income and product accounts is classified by industry and allocated to 47 different kinds of assets. Next, an age/efficiency function for each type of asset is estimated from data on its service life. The age/efficiency functions are used to determine the weight given to each type of asset in past years. A new asset starts with a weight of one that gradually declines with time and eventually reaches zero. For most assets, a concave age/efficiency function is used that features a slow decline during the first years. The stock of each asset for each sector at the end of period is equal to weighted sum of all past investments measured in real dollars. Capital price is constructed from the implicit rental values of services provided by each type of asset in each sector using data on payments to capital. The rental price of an asset is equal to the rate of return on the asset plus the rate of depreciation minus the capital gains.

Labor

Labor input and price data are from Current Employment Statistics survey of establishments, supplemented by the Current Survey of households. Data about the number of jobs, hours worked, and salary are available. Labor quantity is measured as paid hours of all persons engaged in the sector without making a distinction among workers with different skills or wages.

Energy

Energy input quantity and cost are primarily based on information from the Census of Manufactures taken from every five yeas. Quantities are interpolated between census years using Annual Survey of Manufacturing. Prices are constructed by using the survey data used to produce the Producer Price index. Tornqvist aggregation, which uses cost shares instead of BTU weights, is used to combine the different kinds of fuels.

Description of Berndt and Wood (BW) Data

Berndt and Wood (1975) construct data for KLEM for period 1947-1971. Below we present a brief description of how individual series for capital, labor, and energy were constructed.

Capital

Capital input is based on quantities of nonresidential structures and producer's durable equipment. A quantity index is constructed by Divisia aggregation of structures and equipment. The rental price of capital is constructed by following the steps outlined in Christensen-Jorgenson (1969) taking into account variations in effective tax rates and rates of return, depreciation, and capital gains.

Labor

The quantity of labor input is Divisia index of production and nonproduction labor manhours, adjusted for quality changes using the educational attainment index. Measure of the value of the labor services is total compensation to employees in U.S. manufacturing , adjusted for the earnings of proprietors. Price of labor is constructed as adjusted total labor compensation divided by the quantity of labor input.

Energy

Annual quantity indexes of energy are calculated by using interindustry flow tables presented in Faucett (1973). These tables measure flows of goods and services from 25 producing sectors to 10 consuming sectors and five categories of final demand, in both current and constant dollars. Based on these tables, annual quantity indexes of energy as Divisia quantity indexes of coal, crude petroleum, refined petroleum products, natural gas, and electricity purchased by establishment in U.S. manufacturing. Value of energy purchases is computed as sum of current dollar purchases of these five energy types. The price of energy is computed as the value of total energy purchases divided by quantity of energy.

Description of Berndt and Wood Extended (BWX) Data

Most of the studies dealing with estimation of production function at the national level have used the Berndt and Wood (1975) annual manufacturing data for the years 1947 through 1971. Recently, this data set has been extended by Berndt and Wood (1986) to include the years through 1981. We consider this extended data set to be more relevant to be used in the formulation of national prior, as it matches most of time period for Oklahoma data under study: 1970-1989. The construction of individual data series is described in Berndt and Wood (1986a) and here we present a brief discussion of how individual series for capital, labor, and energy were constructed.

Capital

Capital input price and quantity inputs are based primarily on data from BEA, Capital Stock Study. The capital input quantities are based upon equipment and structures investment expenditures and prices from BEA capital stock study. Perpetual inventory method is used to construct the capital stock series. Capital service price is calculated by using the cost of capital approach of Christensen and Jorgenson (1969). Given estimates for capital stock and service price for equipment and structures, an aggregate price and quantity series is constructed by a Tornqvist price index of the equipment and structures components.

Labor

Labor input price and quantity series are based upon estimates of total compensation and manhours worked for production and non-production, respectively. Data for production workers is taken directly from, National Income and Product Accounts (NIPA). Data on compensation of non-production workers is also taken from NIPA sources, with manhours provided by the Bureau of Labor Statistics Monthly Review. Aggregate labor input price and quantity is calculated from a Tornqvist price index of production.

Energy

Energy input prices and quantities include aggregate fuel and electricity used in Heat, Light, and Power (HLP) applications, and fuel (primarily crude oil, gases, and coke) used as feedstock input in the production process. The HLP input price and quantities are based on the Census Bureau's Census of Manufactures (CM) and Annual Surveys (AS), and the Producer Price Indices (PPI) for various fuel types. Further, the feedstock component of the total energy is calculated from data on refinery input of crude oil, natural gas liquids, and coke not reported as HLP. Crude oil input is taken from Energy Information Administration (EIA). Natural gas liquids are taken from American Petroleum Institute (API). Coke input quantities are taken from EIA. Price Data is taken from CM /ASM respectively. Total feedback price and quantity inputs are estimated by a Tornqvist price index of crude oil, natural gas liquids, and coke prices. Finally, total energy input prices and quantities are estimated by a Tornqvist price index of HLP and feedstock prices.

Description of Oklahoma Data

We estimate our model for manufacturing sector of Oklahoma over 1970-1989. Data from 1979-81 is not available and we don't include these three years in our sample. Data for input prices and cost shares of labor, energy and capital are constructed as follows. The price of labor per man-hour is computed as total labor cost (payroll) divided by total man-hours worked. Sources for cost of labor and total man-hours are different issues of the Annual Survey of Manufacturing (ASM) and the Census of Manufacturing (CM). The price of energy is calculated as total cost of fuels and electric energy divided by gross millions of BTUs consumed. Energy consumption data is taken from Energy Information Administration (EIA). EIA collects this data by conducting various surveys and defines manufacturing consumption sector under the industrial sector which also include mining, construction, agriculture, fisheries and forestry. Manufacturing makes up the largest part of this sector. We are compelled to use this data as a proxy for energy consumption in manufacturing sector as ASM stopped reporting estimates for cost of fuel in 1980's. The price of capital is constructed following the approach of Christensen and Jorgenson (1969). We describe below the construction of price and capital stock series.

Price of Capital.

Unlike labor input, for which wage rate data are typically available, the one period user cost of capital is seldom observed. With a few exceptions, a firm usually purchases capital and consumes it entirely by itself. One typically infers indirectly, the user cost of capital that firm implicitly charge themselves to use their own capital inputs. If the secondhand market is assumed to be perfect and firms are indifferent between renting and owning capital, the implicit user cost of capital that firms charge themselves must equal the price that firms could fetch, were they to rent their capital to others. Jorgenson and Hall (1967), pioneering study on effects of taxes on investment, emphasized that the rental price of the capital must incorporate at least four effects. First, there is the opportunity effect of having funds tied up in plant and equipment. Let the asset price of capital be P_t, and the current one-period interest rate yield be r_t. The opportunity cost of capital in this context is given by $r_t * P_t$. Second, assuming capital decays at a constant one-period rate of δ %, the renter would need to compensate the owner for the depreciation, and this will equal $\delta * P_t$. Thirdly, durable goods experience price changes that can result in capital gains or losses to their owners. Following Jorgenson and Hall formulation, the user cost of capital is calculated as the sum of the above three effects, given by,

$$C_t = P_t (r_t + \delta - \Delta P_t / P_t) \tag{30}$$

The above equation should also be adjusted to take into account the effects of various taxes. The implicit assumption being that, firms are unable to shift taxes forward to consumers and their user costs are affected by the taxes. One commonly used formulation used by researchers to incorporate taxes is a slightly revised version of the above equation given by,

$$C_t = TX_t [P_t r_{t-1} + \delta . P_t - \Delta P_t], \tag{31}$$

where TX_t is the effective rate of taxation on capital income given by,

$$TX_t = 1 - \gamma . Z_t / 1 - \gamma, \tag{32}$$

where γ is the effective corporate income tax rate, and Z_t is the present value of depreciation deductions for tax purposes on a dollar's investment over the lifetime of the good.

Capital Stock.

Data for Oklahoma capital stock is constructed using a variant of the perpetual inventory model. Perpetual inventory methodology requires an investment series that dates back to hundred or more years. Regional data sources are scant and usually do not go back that far. Schnorbus and Giese (1989) have developed a methodology suited for regional construction of capital stock. In this paper, we employ their methodology, and present the details below. Capital stock as defined here implies fixed business investment which incorporates expenditures on nonresidential structures (plant) and producers' durable equipment. Further the focus is on construction of net capital stock which takes into account depreciation as opposed to gross capital stock, which does not deduct depreciation.

The perpetual inventory method involves summing of past gross investments less depreciation. The sum is than adjusted by a price deflator, which converts historical dollars to constant dollars. One works with real capital stock as it reflects more accurately the actual change in net capital stock. The net capital stock than involves the cumulative value of the past real gross investments less cumulative depreciation. Mathematically, one can represent the construction of net capital stock (RNK) as follows:

$$RNK_t = \sum_{t=1}^{n} ((HGI - D) \ / \ PGI)_t \tag{33}$$

The above is composed of three parts: firstly, a historical dollar gross investment time series (HGI); secondly, annual depreciation (D); and thirdly, a price deflator which is a ratio of historical to constant dollars (PGI).

The new methodology employs a variant of the perpetual inventory method as it avoids some problems one encounters when building a regional capital stock series. Specifically, the first problem is that perpetual inventory methodology assumes that no capital goods are purchased prior to the first year of investment used. Researchers' mitigate this problem by extending their gross investment series as far back as possible.

The second problem is that the allocation of capital goods to specific manufacturing industries is permanent. No account is taken for transfers of capital goods from one industry to another or for the reclassification of an establishment to a different industry. The capital stock series used in this paper are not disaggregated by industry, and avoids the above problem.

The third problem has to do with the depreciation pattern. The primary source of the problem is that there are a wide variation in service lives and depreciation pattern among types of capital goods and paucity of data to determine these variations. The problem of estimating service lives is mitigated in part by disaggregating the capital stock series into different categories of asset types as BLS and BEA do. Because of lack of regional data, we separate the capital goods into plant and equipment and use average of the BEA service lives.

The fourth problem is the uncertain magnitude of the "values" of the net capital stock series in constant dollars and their intertemporal comparability. Problems arise from two assumptions that are made when historical dollar values are converted into constant dollars. The first assumption called into question is that old and new capital goods are materially the same. Because of the potential differences in old and new capital goods, the accuracy of the use of 'value' as a proxy for quantity is uncertain. The second assumption called into question is that any changes in technology and productive capability is reflected only in changes in real cost. Thus costless improvements in capital goods are excluded. If these improvements are significant, the perpetual inventory method would underestimate the 'value' of capital stock in later years. Thus setting a benchmark against which constant dollar capital stock can be compared is made difficult. These problems, however, can and have been mitigated by adjusting price deflators for quality changes.

Our task of overcoming this problem was simplified as we used BLS historical and constant dollar series which have been adjusted for price differences among asset types and to extent possible, quality changes.

Reverse Perpetual Inventory Methodology: Construction of Capital Stock

The reverse perpetual inventory methodology for construction of capital stock differs from the traditional perpetual inventory methodology. Instead of building capital stock estimates from a gross investment series, Schnorbus and Giese (1989) recommend to begin by picking a base year and then using perpetual inventory methodology in reverse to calculate regional net capital stock. We pick 1986 as the base year and estimate regional net capital stock for this year and then use the perpetual inventory methodology in reverse to calculate 1985 to1970 estimates and forward to calculate 1987 to1989 estimates. The reason for this different approach is that it overcomes some of the estimation problems with the standard perpetual inventory methodology. First, this approach does not require a regional gross investment series that extends back to the early 1900s and thus is not hampered by regional data limitations. Also, any estimation errors in this new method will accumulate over a relatively shorter period of time and will be compounded for earlier years. In contrast, with the perpetual inventory methodology, any estimation errors will be magnified over a relatively long time period and will be compounded for most recent years. The model used to implement the reverse perpetual inventory methodology is presented below.

Construction of Real Net Capital Stock (RNK).

The construction of real net capital stock involves using equation (33) and (34) below to construct net capital stock series. Essentially, one starts off with estimated RNK for 1986 and than subtracts from it real gross investment less cumulative depreciation to get estimates of years 1970 to 1985. For years 1987 and beyond, one adds to the 1986 RNK, gross investment less cumulative depreciation. The depreciation pattern, which accounts for capital consumption, is an essential element in calculating net capital stock estimates. We follow the straight-line depreciation pattern, which implies that physical deterioration of the capital good is linear, that is, equal dollar depreciation across the asset's service life. The depreciation rate used is .033 and .058 for plant and equipment, respectively.

$RNK_{i,\,86\text{-}t}$ = real net capital stock in *i* for *r* for year 1986-t.

For years 1970 to 1985

$$RNK^{r}_{i,86-t} = RNK^{r}_{i,86} - \sum_{t=1}^{16} [RGI^{r}_{i,86-t+1} - (d\,i * RBV^{r}_{i,86-t})] \tag{34}$$

For years 1987 and beyond

$$RNK^{r}_{i,86-t} = RNK^{r}_{i,86} + \sum_{t=-1}^{-3} [RGI^{r}_{i,86-t} - (d\,i * RBV^{r}_{i,86-t-1})] \tag{35}$$

where i = type of capital good (plant or equipment)

r = region (Oklahoma)

$RNK^{r}_{i,86}$ = 1986 estimated net capital stock of *i* in region *r*.

RGI^{r}_{i} = $HGI^{r}_{i,}$ / PGI^{r}_{i} real gross investment of *i* in *r* .

$HGI^r_{i,}$ = historical dollar gross investment of i in r.

PGI^r_i = estimated regional gross investment price deflator.

di = estimated depreciation rate for i.

RBV^r_i = estimated real gross book value of in r.

To make the above model operational, one needs the estimates of Oklahoma's net capital stock (RNK). Unfortunately, no government data is available at the regional level and one solution is to allocate real national net capital stock data across the region. To estimate the regional share of national net capital stock, we use regional and national ASM / CM data on gross book value of depreciable assets. The implicit assumption is that the regional share of national gross book value and net capital stock are commensurate.

Construction of Oklahoma Net Capital Stock

In this section, we present the model used to estimate Oklahoma's RNK for 1986 and the RBV time series which are needed to make equations (33) and (34) operational. The equation for construction of Oklahoma's RNK is given as follows:

$$RNK^r_{i,86} = RNK^{u.s}_{i,86} * (RBV^r_{i,86} / RBV^{u.s.}_{i,86}) \qquad (36)$$

where

$$RNK^{u.s.}_{i,86} = \sum_{j=20}^{39} [w^r_{j,86} * RNK^{u.s}_{ij,86}]$$

$$w^r_{j,86} = (EMP^r_{j,86} \ / \ EMP^r_{86}) / (EMP^{u.s.}_{j,86} / EMP^{u.s.}_{86})$$

j = two-digit standard industrial classification (SIC) code.

$EMP_{i,86}$ = 1986 production worker employment in industry j (in r or in U.S.).

EMP_{86} = 1986 total production worker employment (in r or in U.S.)

$RNK^{u.s.}_{ij,86}$ = 1986 BEA real national net capital stock of i in industry j.

$RBV^{r}_{i,86}$ = 1986 estimated real gross book value of i in region r.

$$RBV^{r}_{i,70+t} = RBV^{r}_{i,70} + \sum_{i=1}^{16}[(NETD_{i,70+t} * RGI^{r}_{i,\ 70+t})] \quad (37)$$

where $RBV^{r}_{i,\ 70} = HBV^{r}_{i,\ 70}\ /\ PBV^{r}_{i,\ 70}$

$$HBV^{r}_{i,\ 70} = ave[\ HGI^{r}_{i}\ /\ HGI^{r}\ ] * HBV^{r}_{70}$$

ave = average ratio between 1971-1973

HGI^{r}_{i} = ASM / CM historical dollar gross investment of i in r.

HGI^{r} = ASM / CM total historical dollar gross investment in r.

HBV^{r}_{70} = 1970 CM total historical dollar book value in r.

$PBV^{r}_{i,70}$ = 1970 estimated regional book value price deflator.

$\%\ NETD\, i = (RGI^{u.s.}_{i} - RDISC^{u.s.}_{i})\, /\, RGI^{u.s}_{i}$ = Average % of real national gross investment in i remaining after discards.

$RGI^{u.s}_{i}$ = BEA real national gross investment in i.

$RDISC^{u.s.}_{i}$ = BEA real value of discards in i.

$RGI^{r}_{i} = HGI^{r}_{i}\ /\ PGI^{r}_{i}$

HGI^{r}_{i} = ASM / CM historical dollar gross investment in i in r.

PGI^{r}_{i} = estimated regional gross investment price deflator for i.

Construction of Real Book Value

$$RBV^{u.s.}_{i,86} = \sum_{j=20}^{39} [\ w^{r}_{j,86} * RBV^{u.s.}_{ij,86}] = \text{1986 estimated real book value of } i \text{ in} \quad (38)$$

the U.S. adjusted for *r's* industry's mix.

where

$$w^{r}_{j,86} = (\ EMP^{r}_{j,86}\ /\ EMP^{r}_{86}\ )/(\ EMP^{u.s.}_{j,86}\ /\ EMP^{u.s.}_{86})$$

$$RBV^{u.s.}_{ij,86} = RBV^{u.s.}_{ij,70} + \sum_{i=1}^{16} [(\%\ NETD_{i,70+t} * RGI_{ij,70+t}\ )] = \text{Estimated real}$$

national gross book value of *i* in *r*.

$$RBV^{u.s.}_{ij,70} = HBV^{u.s.}_{ij,70}\ /\ PBV^{u.s.}_{ij,70} \quad (39)$$

$HBV^{U.S.}_{ij,70}$ = 1970 estimated historical dollar national gross book value for *i* in industry *j*.

$PBV^{U.S.}_{ij,70}$ = 1970 estimated national book value price deflator for *i* in industry *j*.

$RGI^{U.S.}_{ij,70+t}$ = BEA real national gross investment of *i* in *j*.

$HBV^{u.s.}_{ij,70} = ave(\ HGI^{u.s.}_{ij}\ /\ HGI^{u.s.}_{i}\ ) * HBV^{u.s.}_{70}$ = estimated 1970 historical gross book value .

ave = average 1971-73

$HGI^{u.s.}_{ij}$ = ASM / CM historical national dollar gross investment of *i* in *j*.

$HGI^{u.s.}_{i}$ = ASM / CM historical national gross investment of *i* in U.S.

$\%\ NETD_{i}$ = same as before.

Construction of Price Deflators

The selection of deflators can pose some difficulty. As pointed out earlier, the major problem with national deflators is how to account for new assets and significant quality changes. Schnorbus and Giese (1989) recommend using BEA historical and constant dollar series which have already been adjusted for price differences among asset types and to the extent possible, quality changes. Adjustments are made to account for regional differences in industry mix. Below we present the estimation procedure to calculate different price deflators.

Gross Investment for years 1970-1986. Estimated regional gross investment price deflator for i.

$$PGI^{r}_{i,86-t} = \sum_{j=20}^{39} [v^{r}_{j,86-t} * PGI^{u.s.}_{ij,86-t}] = v^{r}_{j,\,86-t} = EMP^{r}_{i,86-t} / EMP^{r}_{86-t} \tag{40}$$

$EMP^{r}_{j,86-t}$ = BLS production worker employment in industry j in r.

EMP^{r}_{86-t} = BLS total production worker in r.

$PGI^{u.s.}_{ij,86-t} = HGI^{u.s.}_{ij,86-t} / RGI^{u.s.}_{ij,86-t}$ = estimated national gross investment price deflator.

$HGI^{u.s.}_{ij,86-t}$ = BEA historical national gross investment of i in industry j.

$RGI^{u.s.}_{ij,86-t}$ = BEA real national gross investment of i in industry j.

Gross Investment For years 1987 and beyond.

$$PGI^{r}_{i,\,86-t} = ave(\ PGI^{r} / PGI^{u.s.}\ )*ave\ (\ PGI^{u.s} / DPGI^{u.s}\ )* DPGI^{u.s}_{i,\,86-t} \tag{41}$$

ave = three year average of 1983- 1985 ratios.

$PGI^{r}_{i,86-t}$ = same as before.

$DPGI^{u.s.}_{i,86-t}$ = national industry implicit price deflator for non-residential fixed investment in i.

Gross Book Value. Estimated regional gross book value price deflator for i.

$$PBV^{r}_{i,70} = \sum_{j=20}^{39} [\, v^{r}_{j,70} * PBV^{u.s}_{ij,70} \,] = v^{r}_{j,70} = \text{same as before.} \tag{42}$$

$PBV^{u.s}_{ij,70} = HGK^{u.s}_{ij,70} / RGK^{u.s}_{ij,70}$ = 1970 national book value price deflator of i in industry j.

$HGK^{u.s.}_{ij,70}$ = 1970 BEA historical dollar national gross capital stock in i in j.

$RGK^{u.s.}_{ij,70}$ = 1970 BEA real national gross capital stock in i in j.

Procedure for Translog Model

Description of the methodology is facilitated by first defining the data generating process. Assuming that input prices can be treated as exogenous, the input share equations are assumed to form a seemingly unrelated regression with normally distributed errors and a contemporaneous covariance matrix. The input equations can be written as a system as follows:

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{bmatrix} \begin{bmatrix} \beta 1 \\ \beta 2 \\ \beta 3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{43}$$

(3T x 1) (3T x 12) (12 x 1) (3T x 1)

The matrix dimensions appear underneath each matrix. X's are a matrix of factor prices, e_i's are the error vectors and S_i are vectors of cost shares. Note that in the above system only n-1 of the share equations are independent since ($\sum_i S_i = 1$). Estimation is carried out by using n-1 share equations. To ensure regularity of the cost function homogeneity and symmetry conditions are imposed on the above system.

Share equations will be estimated as a system. We estimate share equations with homogeneity and symmetry restrictions imposed, at the national level. These estimates give us the prior mean and precision. We set up the model for informative priors by combining these national priors with Oklahoma data. Gibbs sampling is used to generate from conditional densities of mean and precision given in (12) and (14). Only those samples are accepted that meet the monotonicity and concavity restrictions.

Procedure for Cobb Douglas Model

To be consistent we estimate the CD cost function for Oklahoma based on an informative prior. This informative prior again is based on national data. This is the approach that was followed for modeling technology under the translog system. We use the constrained linear regression model for incorporating this prior information. To get the prior mean and precision based on the national data, one can resort to simple ordinary least square estimation of the CD cost function. The mean vector of coefficients and inverse of covariance matrix are the prior mean and precision. The CD cost function for our three inputs where we impose the returns to scale is given as follows:

$$\ln (C / P_e) = \beta_1 + \beta_k \ln (P_k / P_e) + \beta_l \ln (P_l / P_e) \qquad (44)$$

We estimate the above model with ordinary linear regression and get highly implausible estimates for the coefficients and some of the estimates have the wrong sign. Labor and capital inputs are highly correlated leading to a multicollinearity problems, which can be attributed for the nonsensical results. One way to deal with the above problem is to introduce prior information of some sort. We have prior information about coefficients.

We know that $0 < \beta_k < 1$ and $0 < \beta_l < 1$. The Normal distribution is not able to handle these constraints as the range is from $-\infty$ to $+\infty$ We follow a simple procedure outlined in Judge et al (1988). Setting the prior variances of coefficients carefully, one can make the probability of them lying outside the range 0 to 1 very small. The first step is to specify our prior means for the coefficients. Then one sets a confidence interval around these priors. Using the BLS data as an example, our prior mean for coefficients, returns to scale, and a 50% confidence interval around theses priors are as follows:

$$E[\beta_1] = 2, \; E[\beta_k] = .27, \; E[\beta_l] = .69 \text{ and } E[\beta_k + \beta_l] = .96$$

$$\begin{aligned} 5 &< \beta_1 < 10 \\ 0 &< \beta_k < 1 \\ 0 &< \beta_l < 1 \\ 0 &< \beta_{l} + \beta_l < 1 \end{aligned} \tag{45}$$

To calculate the individual elements in the covariance matrix Σ_b one resorts to properties of the normal distribution. For a standard normal variable z we know that

$$P(-.67 < z < .67) = .5$$

We can reformulate it in these terms

$$P\left(-.67 < {(\beta_k - .27)}\Big/{\sqrt{\text{var}(\beta_k)}} < .67\right) = .5 \tag{46}$$

Substituting the values of our prior means in the above one solves for the variance of β_1, β_k, and $\beta_{k} + \beta_l$. To estimate the prior covariance's (cov) we assume that cov (β_1, β_k) = cov $(\beta_1, \beta_l) = 0$. To find covariance between β_k and β_l, we use the following formulation:

$$\text{var}(\beta_k + \beta_l) = \text{var}(\beta_k) + \text{var}(\beta_l) + 2\,\text{cov}(\beta_k, \beta_l) \tag{47}$$

From the above one can solve for cov ($\beta_{k,} \beta_l$). The prior mean and covariance matrix with the individual elements derived above is as follows:

$$E(\beta) = \underline{\beta} = (2.0, .27, .69) \qquad \Sigma_{\beta} = \begin{bmatrix} 19.98 & 0 & 0 \\ 0 & .1624 & .3968 \\ 0 & .3968 & 1.0604 \end{bmatrix} \tag{48}$$

Following similar procedure we get the prior covariance for BW national data. These are given as follow:

$$E(\beta) = \underline{\beta} = (5.0, .16, .70) \qquad \Sigma_{\beta} = \begin{bmatrix} 222.6 & 0 & 0 \\ 0 & .056 & .242 \\ 0 & .242 & 1.12 \end{bmatrix} \tag{49}$$

To obtain point estimates for the CD function, the following steps are followed. A prior mean and precision is posited. Constrained linear regression model setup is combined with Gibbs sampling to impose prior information that takes the form of inequality restrictions

CHAPTER V

FINDINGS

Results and Discussion

A system of share equations based on the translog cost function is estimated for each of the three sets of national manufacturing data. These three data sets are BLS, BW and BWX. As noted earlier, in the above system only n-1 of share equations are independent. We estimate the system of share equations for capital and labor and drop the energy equation. The energy share coefficients are recovered using aggregation conditions. We impose homogeneity and symmetry restrictions on the system. The coefficient estimates from this exercise are presented in the first column of Table IX-XI under "US".

Looking at the national estimates one finds that all the coefficients have correct signs. The capital share equation for all three sets of data has a positive own-price coefficient and the magnitude varies from a low of .04 for BLS data to a high of .09 for BWX data. A positive coefficient implies that as the price of capital rises, the share of capital in total cost also rises. The coefficient on price of labor is negative and the magnitude ranges from -.01 to -.06. The negative coefficient implies that as the price of labor rises the share of capital in the total cost goes down.

Looking at the labor share equation we see a similar pattern. The own-price coefficient is positive and ranges from .01 for BLS and .13 for BWX data. Again the interpretation is that a rise in price of labor will increase the share of labor in the total cost. The coefficient on price of capital is negative and its magnitude ranges from -.01 for BLS to -.06 for BWX data. The negative coefficient implies that as the price of capital rises the share of labor in the total cost will go down.

The estimates from these three sets of national data are used as priors for the mean. Using these alternative priors will give us a way of checking the robustness of priors. For prior information to have any meaningful input in our modeling exercise, it should satisfy the monotonicity and concavity restrictions. We compute the national elasticities of substitution for the three sets of national data and check these elasticities to check their conformity with concavity restriction. We also evaluate predicted/ fitted shares to see if they conform with monotonicity restriction. We find that data agree with these restrictions. The national elasticity of substitution estimates are presented in Table XIII. The estimated elasticities have correct signs and reasonable magnitudes. For all three datasets, capital and labor and labor and energy pair tend to be substitutes. Capital and energy pair tend to be complements for all the data sets.

Next in our modeling exercise we use the Oklahoma manufacturing data and estimate a system of share equations. These estimates are termed the unrestricted estimates. Again capital and labor share equations are estimated and the energy equation is dropped. Homogeneity and symmetry is imposed on the data. The estimates are presented in the second column under "Oklahoma" in Table IX- XI. These are the same for the three tables and are presented for comparison purposes.

The unrestricted estimates based on Oklahoma data have incorrect signs, which makes the interpretation nonsensical. The own-price coefficient in the capital share equation has a negative sign. The interpretation would be that as capital price rises the share of capital in total cost goes down. Similarly, the coefficient on price of labor is positive. We see the same pattern for the labor share equation. Next we will like to test if these estimates meet the theoretical restriction of the production theory: monotonicity and concavity. We find that the Oklahoma model meets the monotonicity restriction but fails to meet the concavity restriction. Given the nonsensical coefficient estimates combined with violations of concavity, it is quite clear that using this data will give us nonsensical elasticity estimates.

We have a very limited information set in terms of a small data set (17 observations) collected at a very disaggregated level. With small samples prior information becomes important. To complete the modeling exercise we combine the model for incorporating an informative prior with the Bayesian approach to inequality restrictions, to impose monotonic and concavity restrictions. The priors means and precisions ($\underline{\beta}$, $\underline{H_{\beta}}$ and $\underline{S}$) for BLS, BW and BWX national manufacturing data which cover the period 1949-1981,1947-1971, and 1947-1981 are estimated using the SUR estimator. The priors for ($\underline{\beta}$, $\underline{H_{\beta}}$ and $\underline{S}$) are the estimated mean, inverse of the estimated covariance matrix of the SUR estimator, and T times the contemporaneous variance-covariance matrix, respectively.

Combining the above priors with Oklahoma data for the 1970-1989 sample period, the posterior distribution of the parameters is simulated using the Gibbs sampler. To simulate $\beta \mid H$ we draw from the conditional posterior mean in equation (12). The monotonic and concavity of the estimated equations are checked using the conditions described in (19) and (20). If satisfied, S is computed and $H \mid \beta$ is drawn from its conditional distribution using equation (14). Note, in the computations the concavity of the cost function was not verified for each combination of shares found in our data. Rather, it was checked at the sample means. A total of 100,000 successful draws are made and the first 20,000 of those are discarded. The sample means of the shares were used for the computations. We label models using prior information from BLS as Model 1, ones based on BW and BWX as Model 2 and Model 3, respectively. The means of posterior for Model 1-3 are presented in the fourth column in Table IX-XI under "Informative Prior". The estimates of elasticity's of substitution and price elasticity's for Model 1-3 are presented in Table XIV and Table XV, respectively.

As expected, we find that the means of the posterior for Model 1-3 have correct signs. These estimates are termed as restricted estimates. These restricted estimates, because they combine the information at the national level with unrestricted estimates, and give more weight to the national estimates by making the precision bigger, give us the correct expected signs. These are based on an informative prior, and are also forced to satisfy the monotonic and concavity restrictions. For example, looking at the capital share equation the own-price coefficient is positive for all the models and ranges from .02 for BWX and .03 for BLS. Similarly, the coefficient on price of labor is negative for all the models. We see that the restricted estimates are much closer to the national estimates as opposed to unrestricted estimates. This is due to the shrinkage that occurs towards the national estimates when the Bayesian approach is used. For Model 2, which is based on BWX data, we find a peculiar result. Recovered energy estimates don't ever meet monotonic restrictions. BWX and Oklahoma data by themselves meet the monotonicity restriction for energy share equation, but when we combine the national and regional data using the Bayesian framework, the energy share equation predicts negative shares. We drop this model and from hereon focus on Models 1 and 3.

Elasticities of substitution and price elasticities have correct signs and reasonable magnitudes. For example, the elasticity of substitution for the K-L pair ranges from .83 for Model 2 to .99 for Model 3. K-L pair are substitutes for all the models, whereas, L-E pair are substitute in Model 1 and Model 3, but are a complement in Model 2. K-E pair are substitutes in Model 2 and Model 3, but are complements in Model 1. In terms of interpretation an elasticity of substitution of .83 for K-L pair implies that if the relative price of capital/labor rises by 1 % the rate of substitution between K and L will be .83 %.

We carry out an another modeling exercise similar to the one discussed above, but where we use an "uninformative" or "diffuse" prior as opposed to an "informative" prior. Essentially, we are positing a scenario where we possess some prior information, but we are not very confident as to the accuracy of this information. To achieve this we set the precision to be very small in our model setup, and hence posterior is determined mostly by the actual sample. The idea is to see if we can get good estimates for mean and elasticities if we just impose monotonicity and concavity restrictions on Oklahoma model. If that is the case, then we can disregard the model based on "informative" prior. The means of the posterior from this exercise are presented in the fifth column in Tables IX-XI under "Diffuse Prior".

The estimates based on the "diffuse" prior have incorrect signs. For example, looking at the share equation for capital we see that the coefficient on own-price is negative and coefficient on price of labor is positive. Elasticity of substitution estimates under the "diffuse" prior have correct signs but unreasonable magnitudes. For example, the estimate for $\sigma_{kk} = -10.07$ for the BWX and $\sigma_{kk} = -10.36$ for the BLS data sets. Similarly, the estimate for $\sigma_{kl} = 2.10$ and $\sigma_{kl} = 12.16$ for the BWX and the BLS data sets, respectively. Based on the fact that under the "diffuse prior" setting, coefficient estimates have incorrect signs and elasticity's have unreasonable magnitudes, we conclude that the model under" informative prior" or the restricted model is far better. Information obtained from the national data is very helpful in getting elasticity estimates that are sensible.

We also estimate an alternative CD functional form. The CD cost function for Oklahoma is estimated initially by just using the regional data. Estimates from this exercise, which are also the predicted shares, are presented in the third column of Table XII under " Model 6". The estimates have correct signs but the magnitudes are nonsensical. The capital share is predicted to be .11, and labor share is at .366. This will translate into an energy share of .52. One way to get better estimates for CD cost function for Oklahoma is to combine the prior information at the national level with the regional data. We also have some prior information about magnitudes of shares. Using the Bayesian methodology described earlier, we estimate two sets of CD cost function. Model 4 is based on BLS prior, and Model 5 is based on BW prior. Looking at these estimates we see that the predicted sharers have not really improved. This could be because we have used a small precision for national data. This embodies our belief that we are not very sure if the regional estimates are very close to national estimates. We use these set of estimates to carry out forecasting experiments which are discussed on the next page.

We are also interested in seeing if the restrictions of monotonicity and concavity are supported by our data. We estimate posterior probabilities that the restrictions hold by computing the ratio of number of restrictions that meet the restrictions over total number of replications. For Model 1, the monotonicity restriction is met for all replications, and for concavity, 82047 replications out of 117953, meet the restriction. This translates into a posterior probability of .69 that the restrictions hold. For Model 3, all replications meet the concavity and monotonicity restrictions. These results suggest that these systems of equation are well behaved with respect to neoclassical restrictions, at least at the sample mean of the data. These results are tabulated in Table XVI.

We are also interested in checking our specification of the functional form for our data. The translog flexible function form collapses to the Cobb-Douglas form for elasticities of substitution equal to unity for all input combinations. To see if our data matches the CD technology, we build a 95% confidence interval for cross elasticities of substitution. If the values of σ_{kl}, σ_{ke}, and $\sigma_{le} = 1$ fall in the confidence interval, we conclude that data supports the CD representation of technology, otherwise, the conclusion is that the translog function is supported by the data. Specifically, we take the 80,000 simulated values of elasticities arranged in a column, and chop off 2.5 % of the top and bottom. We sort this column in an ascending order and this gives us the 95% confidence interval. We pick the first and the last value of the interval to specify the width of the interval.

The results from this exercise are presented in Table XVII. Looking at this table one sees that for σ_{kl} the confidence interval is the range of .87 to 1.10 for Model 1. We see that $\sigma_{kl}=1$ falls in this interval. This is also true for the Model 3. The conclusion is that for capital and labor, the data seems to point out that CD is a better representation. For capital and energy, and labor and energy, we see that for both models the confidence interval is such that the value of one does not fall in it. Also note that for both the models we see that σ_{le} confidence interval is very precise, and ranges from 1.075 to 1.097 and .49 to .62 for Model 1 and Model 3, respectively. This lends support to the translog representation of the technology. Based on above analysis, we conclude that estimated translog specification is supported by our data.

We perform a test of the predictive accuracy between the translog and the CD functional form, as a way of choosing among them. Estimation of the CD form is carried out by using Bayes method where we use diffuse and informative priors. Informative priors are based on BLS and BW national data sets. The models based on informative prior are labeled Model 4-5. Model 4 and 5 are based on BLS and BW respectively, whereas Model 6 is based on regional data. The coefficients of the CD form are estimated by imposing constant returns to scale. We use our posterior estimates from Model 1, 3, 4, 5 and 6 estimated now with 13 observations, to predict K, L and E shares. We calculate root mean square errors (RMSE) for out-of-sample forecast based on 4 observations. Results are tabulated in Table XVIII.

The RMSE for Model 1 and 3 for capital share is .09 and .05, which is considerably smaller than .121 and .232 for Model 5 and 6. Model 4's RMSE for capital share is .06 and it the only predicted share that even comes close to beating RMSE of Model 1 and 3. When it comes to predicting labor and energy share, Model 1 and 3 again beat Model 5-6. The average RMSE for Model 1 and 3 is .072. Compare this to the average of .247, .281 and .438 for Models 4-6 respectively. Based on the predictive accuracy, we again find support for Tran slog functional form specification. Model 1 beats Model 3 in predicting labor share, but Model 3 outperforms Model 1 in predicting capital and energy share.

One of the motivations for this study was to provide the regional CGE modelers with a flexible functional form which could compete with the CD form, which is the default technology posited for modeling the production sector in many CGE models. A natural question that arises is, does it matter if you model your production sector using the CD or the translog technology? To really answer this question one would have to run a CGE model with the two alternative technologies and pick a criteria to compare their performance. One criteria could be the predicted level of impact on cost minimizing demands when input prices are increased separately.

Since we don't have a CGE model at hand we propose a short-cut that gives us at least a perspective as to what to expect if we had one. The short-cut is to perform a simulation in a partial equilibrium framework. The exercise involves tracing the impact of separate increases in input prices on the cost minimizing demands for inputs. This is done for the CD and the translog technology. We expect that we will get different impacts depending on functional form used. If we do, that implies that it matters as to what functional form is posited in the production sector.

For the translog technology, the cost minimizing demand for capital, labor and energy are given as follows:

$$X_I = (C / P_i) * (a_1 + a_{11} \text{ in } P_{ike} + a_{12} \text{ in } P_l + a_{13} \text{ in } P_{er}) \quad (50)$$

$i = K, L, E$ and $C = (a_0 * P_{ike}^{\ a_1} * P_l^{\ a_2} * P_{er}^{\ a_3} * (P_k^{(.5 * a_{11})})^{\ln P_k} * (P_k^{\ a_{12}})^{\ln P_l} * (P_k^{\ a_{13}})^{\ln Pe}$

$(P_l^{(.5 * a_{22})})^{\ln P_l} * (P_l^{\ a_{23}})^{\ln P_e} * (P_e^{(.5 * a_{33})})^{\ln P_e})$

In the above cost function all parameters except for the constant term are available from the estimated share equations. The constant term (a_0) is recovered by using the OLS solution.

For CD specification the cost minimizing demands are given as follows:

$$X_i = (\beta_i * C) / P_i \quad (51)$$

$i = K, L, E$ and $C = \beta_1 * (P_k)^{\beta_2} * (P_l)^{\beta_3} * (P_e)^{\beta_4}$

For example to trace the effect of an increase in input prices (K, L, E) on demand for capital, we separately increase the price of each input and calculate the new demand for capital each time. Carrying out this exercise tells us how demand for capital responds in percentage terms to increases in price of capital, labor and energy. We calculate these direct and cross effects of simulated input increase for all our models with the results tabulated in Table XIX.

As expected, the effect of an increase in an input's own price is negative for all the simulations. We find that capital and labor are substitutes. Labor-energy and capital-energy pair are substitutes for all the models. For Model 1 capital-energy relationship is ambiguous. Further looking at the simulated impacts one sees that depending on the functional form the percentage of the impacts is different. These impacts are generally very small with one or two exceptions. For example comparing Model 1 and Model 4 we see that for energy demand there is big difference in impacts. A 1 % increase in price of capital input reduces energy demand by 32% in Model 1, whereas in Model 4 which is CD form based on BLS informative prior, we see this impact to be an increase of .29%.

From above one draws the conclusion that the choice of functional form does matter in terms of the differences of impact. Our conclusion is based on simple simulations under a partial equilibrium framework. Further work will involve seeing if these results also hold for CGE models.

CHAPTER VI

CONCLUSION

Summary

Regional CGE models have been criticized for employing restrictive production functions. Flexible functional forms such as the translog do not impose a priori restrictions on elasticities and are capable of modeling inputs as substitutes and complements. Further CGE models have also been attacked for using calibration. In calibration, one selects estimates of elasticities from literature at the national level. This may lead to specification problems.

A researcher trying to estimate a regional translog production function at a state level faces two problems. First, there is paucity of data at the regional level. Second, it is possible that production function might not be well behaved at low levels of aggregation, which in turn, may lead to convergence problems for numerical solutions.

This paper proposes a solution to the above problems. We estimate a regional translog cost function for Oklahoma manufacturing by ensuring that all theoretical properties of the function are imposed so it is well behaved. This implies that the production function meets monotonicity and concavity constraints. The latter involve inequality constraints that are difficult to impose. A Bayesian approach is used to impose inequality constraints.

The first step in the Bayesian approach involves positing a prior density function over the parameters of interest. The prior information can be informative e.g. magnitudes of elasticities at the national level or it can be uninformative or diffuse representing no prior information about parameters. The second step is to specify a likelihood function. Using Bayes theorem, one combines the likelihood function and the prior to obtain posterior distribution of the parameters. Following the steps outlined above, we carry out the estimation of translog cost function under three sets of prior information.

We use the Bayesian procedure described above to estimate a system of share equations based on translog cost function for Oklahoma Manufacturing. We impose homogeneity and symmetry on the data. The share equations for labor and capital are estimated and the energy estimates are recovered by using aggregation conditions. For prior information we use three sets of national data. The first data set is compiled by BLS, and second and third data sets are constructed by Berndt and Wood as described earlier. Estimation is carried out by simulating the posterior distribution of the parameters by using Gibbs sampler. To ensure regularity of the cost function, only those draws are accepted that meet the monotonic and concavity constraints. Results from models based on the above informative prior give us estimates with correct signs. The elasticities of substitution and price elasticities have correct signs and reasonable magnitudes.

We also use a "diffuse prior" setup to simulate the posterior distribution of the parameters. Essentially, we put less weight on the national priors and just as before we impose the monotonicity and concavity restrictions. The means of the posterior under this framework have incorrect signs and the elasticity magnitudes are unreasonable. The estimate for σ_{kk} = -10.07 for BWX and σ_{kk} = –10.36 for BLS data. Similarly, the estimate for σ_{kl} = 2.101 and σ_{kl} = 12.16 for BWX and BLS data set.

In judging the adequacy of the above models, we used two types of criteria: internal and comparative. By internal criteria we mean that consistency of our data is checked with the econometric and theoretical assumption of our models. Tests based on comparing different models are referred to as comparative. To see if our data meets the internal criteria, we estimate posterior probabilities that monotonicity and concavity restrictions hold for our sample. We find strong evidence based on posterior probabilities, that these restrictions are supported by our data. Comparative criteria are used to compare our specification of production technology, translog versus the traditional CD form. Posterior probabilities again provide strong support for the translog specification. Again models can also be compared in terms of the forecast performance. A model may do well in meeting theoretical restrictions, but may give significantly poorer forecasts than a simple model may lead us to question the validity of the model accuracy. Predicitive accuracy between the translog and CD functional form is carried out and we find that the translog form outperforms the CD in predicting out-of-sample shares. We also perform a simulation where we compare the impact on cost minimizing demands of separate increases in input prices. We perform this for the translog and the CD technology. The conclusions that we draw from this exercise are that different functional forms give you different levels of impacts. It will be worthwhile to explore this issue in a CGE framework. Based on the above internal and comparative criteria, we conclude that the translog specification is supported by the data.

Given the paucity and disaggregated nature of the data at the state level, it is not surprising that there are hardly any attempts by researchers to estimate state level production functions. We have shown how a well-behaved regional production function can be estimated at a state level. Our primary motivation in estimating the translog cost function for Oklahoma was to provide regional CGE model builders access to FFF functional form with its associated elasticities. Besides the energy input, we find that using two sets of priors seems to give us similar results as far as elasticity of substitution and price elasticities are concerned. This can be taken as a rough proof that our specification of the Bayesian model is robust in terms of the subjective priors.

Our results are also useful for a researcher in another state who might be interested in investigating the input substitution possibilities for his region. If the region has the same manufacturing mix as Oklahoma, he should be able to use our estimates as a proxy for elasticitiy estimates, or priors for elasticities for his state. If the manufacturing mix is very different the researcher can still use our results. In this paper we have essentially developed a new methodology to estimate elasticities at a regional level, where because of the disaggregated nature of the data it is not possible to derive meaningful conclusions about the relationships between the inputs. This new methodology combines the information available on the elasticities at the national level with the regional information using a Bayesian approach. The Bayesian approach essentially shrinks our regional estimates towards the national estimates.

We also compare the national elasticity of substitution estimates with the regional estimates. We find that Oklahoma elasticity estimates for capital and labor are close to their national counterparts. For labor and energy and capital and energy our estimates provide information about input substitution which is quite different from what is available at the national level. At the national level capital and energy are complements for all the models and labor energy are substitutes. At the Oklahoma level we find that capital and energy tend to be substitutes for Model 2 and Model 3, and complement for Model 1. Labor and energy pair tend to be substitutes for Model 1 and Model 3, and tend to be complements for Model 2.

There is some conflicting information about the elasticities from the above models. One way to resolve the conflicting information about substitution relationships between capital and energy and labor and energy is to resort to model choice. We pick Model 1, based on BLS national prior, as our best model. We pick this model as the elasticities from this model closely match the national counterpart. Further prior information is more closely matched for Model 1. The sample for national prior in Model 1 ranges from 1949-1981 which closely matches the Oklahoma sample ranging from 1970-1989. Model 3 on the other hand is composed of national prior that ranges from 1947-1971. Model 2 is based on the national prior ranging from 1947-1981, and it also had problems with meeting monotonicity restriction.

Our results are also useful for policy analysis. Input substitution possibilities have major growth implications. If capital and energy are complements, higher energy prices will dampen demand for both capital and energy. Conversely, if energy and other inputs are substitutes, then rising energy prices will stimulate demand for both capital and labor. Similarly, capital-labor substitutability facilitates a movement towards a more labor intensive, production process in the case of higher capital prices. Easier input substitution possibilities are more adaptable to changes in input mix brought about by resource price fluctuation. On the other hand, if the input substitutability is limited, reduced availability of one input, can slow growth in employment and output

SELECTED BIBLIOGRAPHY

Bairam, E.I. (1994). Homogenous and Nonhomogenous Production Functions: Theory and Applications. Vermont: Ashgate Publishing Company.

Baffes, J. and Vasavada, U. (1989). On the Choice of Functional Forms in Agricultural Production Analysis. Applied Economics, 21, 1053-1061.

Berndt, E.R. (1991). The Practice of Econometrics: Classical and Contemporary. Reading, MA: Addison Wesley Publishing Company.

Chalfant, J.A and Wallace, N.E (1992). Bayesian Analysis and Regularity Conditions on Flexible Functional Forms: Application to the U.S. Motor Carrier Industry. In Griffiths (Ed.) Reading in Econometric Theory and Practice.

Chung, J.W. (1994). Utility and Production Functions: Theory and Applications. Cambridge, MA: Blackwell Publishers 1994.

Chambers, R.G. (1988). Applied Production Analysis: A Dual Approach. Cambridge, U.K: University Press.

Cornes, R. (1992). Duality and Modern Economics. Cambridge University Press.

Dorfman, J.H. (1997). Bayesian Econometrics through Numerical Methods: A Guide to Econometric and Decision–Making with Prior Information. New York: Springer-Verlag.

Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B (1995). Bayesian Data Analysis. NY: Chapman and Hall.

George, E. I. and Blattberg, R.C. " Shrinkage Estimation of Price and Promotional Elasticities: Seemingly Unrelated Equations" Journal of American Statistical Association, 86, 304-313, 1991.

Hey, J.D. (1983). Data in Doubt: An Introduction to Bayesian Statistical Inference for Economists. Oxford: Martin Robertson and Company Ltd.

Heathfield, D.F. and Wike, S. (1987). An Introduction to Cost and Production Functions. Atlantic Highlands, NJ: Humanities Press International.

Intriligator, M.D. (1978) Econometric Models, Techniques, and Applications. Englewood Cliff, NJ: Prentice-Hall.

Kim, J.K. and Nelson, C.R. (1999). State-Space Models with Regime Switching: Classical and Gibbs Sampling Approximations with Applications. Cambridge, MA: MIT Press.

Lee, P.M. (1989). Bayesian Statistics: An Introduction. NY: Oxford University Press.

O' Hagan, A. (1994). Kendall Theory of Statistics. Bayesian Inference, Volume 2B, Cambridge University Press.

Takayama, A. (1985). Mathematical Economics. Cambridge University Press.

Usher, D. ed. (1980) The Measurement of Capital. Chicago: University of Chicago Press.

Varian, H.R. (1984). Microeconomic Analysis. NY: Norton Publishing.

Zellner, A. (1971). An Introduction to Bayesian Inference in Econometrics. NY: John Wiley and Sons, Inc.

Zellner, A and Richard, J. F (1973). Use of Prior Information in the Analysis and Estimation of Cobb-Douglas Production Function Models" International Economic Review, 14, 107-119.

REFERENCES

1. Alperovich, G. "Regional Elasticities of Substitution. " Journal of Regional Science, 20, 503-512, 1980.

2. Anderson, G.J. and Blundell, R.W. " Estimation and Hypothesis Testing in Dynamic Singular Equation Systems." Econometrica, 50, 1559-71, 1982.

3. Antle, J.M. "Testing the Stochastic Structure of Production: A Flexible Moment-based Approach." Journal of Business and Economic Statistics, 1, 192 –201, 1983.

4. Arrow, K.J., Chenery, B.M, Minhas, B. and Solow, R.M. " Capital – Labor Substitution and Economic Efficiency." Review of Economics and Statistics, 43, 225-54,1961.

5. Barnett, W.A., Geweke, J and Wolfe, M. " Seminonparametric Bayesian Estimation of the Asymptotically Ideal Production Model." Journal of Econometrics, 49, 5-50, 1991.

6. Bureau of Economic Analysis (U.S. Department of Commerce). Fixed Reproducible Tangible Wealth in the United States, 1925-1989, Washington, D.C., 1999.

7. Berndt, E. and Wood, D. " Technology, Prices, and the Derived Demand for Energy." Review of Economics and Statistics, 57, 376-384, 1975.

8. Berndt, E.R. and Savin, N.E. " Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances." Econometrica, 43, 937-57, 1975.

9. Berndt, E.R.., Morrison, C.J., Watkins, C.G. "Dynamic Models of Energy Demand: An Assessment and Comparison." In Ernst R. Berndt and Barry C. Fields, eds., Modeling and Measuring Natural Resource Substitution, Cambridge, Mass.: MIT press, 209-229.

10. Berndt, E.R., Darrough, M. N. and Diewert, W.E.. " Flexible Functional Forms and Expenditure: An Application to Consumer Demand Functions." International Economic Review, 18, 651-75,1977.

11. Caves, D.W. and Christensen, L.R. " Global Properties of Flexible Functional Forms." American Economic Review, 70, 422-432, 1980.

12. Casella, G. and George, E. " Explaining the Gibbs Sampler." American Statistician,

46, 167-174, 1992.

13. Chalfant, J. and White, K. " Estimation of Demand Systems with Concavity and Monotonicity Constraints. " Working Paper No. # 454, Department of Agriculture and Resource Economics, University of California, Berkley, 1988.

14. Chavas, J.P and Segerson, K. "Stochastic Specification and Estimation of Share Equation Systems." Journal of Econometrics, 35, 337-58,1987.

15. Christensen, L., Jorgenson, D.W., and Lau, L.J. "Conjugate Duality and the Transcendental Logarithmic Production Function. " Econometrica, 39, 255-256, 1971.

16. Christensen, L., Jorgenson, D.W., and Lau, L.J. "Transcendental Logarithmic Production Frontiers." Review of Economics and Statistics, 55, 28-45, 1973.

17. Clark, J.S. and Youngblood. "Estimating Duality Models with Biased Technical Change: A Time Series Approach." American Journal of Agriculture Economics, 74, 353-60, 1992.

18. Cobb, C. and Douglas, P.H. " A Theory of Production." American Economic Review, 18, 139-65, 1928.

19. Considine, T. "Symmetry Constraints and Variable Returns to Scale in Logit Models." Journal of Business and Economic Statistics, 8, 347-53, 1990.

20. Considine, T. and Mount,T. "The Use of Linear Logit Models for Dynamic Input Demand Systems." Review of Economics and Statistics, 66, 434-43, 1984.

21. Dashti, I.M. "Imposing Monotonicity and Concavity Restrictions into Stochastic Cost Frontiers: A Bayesian Approach." Unpublished dissertation, Kansas State University, 1996.

22. Diewert, W.E. " An Application of the Shepard Duality Theorem: A Generalized Linear Production Function." Journal of Political Economy, 79, 482-507,1971.

23. EIA. "State Energy Data Report 1997: Consumption Estimates." Energy Information Administration, U.S. Department of Energy, Washington, DC, 1999.

24. Epstein, L.G. "Duality Theory and Functional Forms for Dynamic Factor Demands." Review of Economic Studies, 48, 81-95, 1981.

25. Epstein, L.G. and Yatchew, A.J. " The Empirical Determinants of Technology and Expectations: A Simplified Procedure." Journal of Econometrics, 27, 235-58,1985.

26. Farrell, M.J. " The Measurement of Productive Efficiency." Journal of Royal Statistical Society, part 3, 253-90, 1957.

27. Friesen, J. "Testing Dynamic Specification of Factor Demand Equations for U.S. Manufacturing." Review of Economics and Statistics, 74, 240-50, 1992.

28. Jack Faucett Associates. " Data Development for I-O Energy Model: Final Report" Jack Faucett Associates, Inc May, 1973.

29. Gallaway, L. "The North-South Wage Differential. " Review of Economics and Statistics, 41, 264- 272, 1963.

30. Geweke, J. " Exact Inference in Inequality Constrained Normal Linear Regression Model. " Journal of Applied Econometrics, 1, 127-141, 1986.

31. Geweke, J. Applied Econometrics II Lecture Notes. Winter Term, 1998.

32. Gallant, A.P. " On the Bias in Flexible Functional Forms and an Essentially Unbiased Form: The Fourier Flexible Form." Journal of Econometrics, 15, 211-245, 1981.

33. Gelfand, A.E., and Smith, A.F.M. "Sampling Based Approaches to Calculating Marginal Densities." Journal of American Statistical Association, 85, 398-409, 1990.

34. Griffin, J. and Gregory, P.R. "An Intercountry Translog Model of Energy Substitution Responses." American Economic Review, 66, 845-57, 1976.

35. Griffin, J. "Engineering and Econometric Interpretations of Energy-Capital Complementarity: Comment." American Economic Review, 71, 1100-04, 1981.

36. Griffiths, W.E. " Bayesian Econometrics and How to Get Rid of Those Wrong Signs." Review of Marketing and Agricultural Economics, 56, 36-55, 1988.

37. Gullickson, W. and Harper, M.J. "Multifactor Productivity in U.S. Manufacturing, 1947-1983." Monthly Labor Review, October, 1987.

38. Hansen. "Large Sample Properties of Generalized Methods of Moments Estimators." Econometrica, 1029-54, 1982.

39. Hansen, L.P. and Singleton, K. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." Econometrica, 50, 1269-86.

40. Hansen, L.P and Sargent, T.J. " Formulating and Estimating Dynamic Linear Rational Expectations Models." Journal of Economic Dynamics and Control, 2, 7- 46,

1980.

41. Heady, E.O. and Dillon, J.L.. Agricultural Production Functions, Ames, Iowa, Iowa State University Press, 1961.

42. Hunt, T.L. and Emerson, M.J. " Elasticities of Substitution in Regional Production Functions. " Review of Regional Studies, 4, 12-17, 1982.

43. Jorgenson, D.W. " Investment and Production: A Review." in M.D. Intriligator and D.A. Kendrick (eds.). Frontiers of Quantitative Economics, Vol. III, Amsterdam: North-Holland Publishing, 1974.

44. Judge, C.G, Hill, C.R., Griffith, W.E., Lutkepohl, H. and Lee, T.C. (1988). Introduction to the Theory and Practice of Econometrics. John Wiley and Sons, Inc.

45. Kennan, J. "The Estimation of Partial Adjustment Models with Rational Expectations." Econometrica, 47, 1441-55, 1979.

46. Kim, H. Y. "The Translog Production Function and Variable Returns to Scale." Review of Economics and Statistics, 74, 546-52,1992.

47. Kokkelenberg, E.C. and Bischoff, C.W. "Expectations and Factor Demand." Review of Economics and Statistics, 68, 423-31, 1986.

48. Kumbhkar, S.C and Serletis, A. "KLEM Substitutability: A Dynamic Flexible Demand System." Applied Economics, 22, 275-83, 1990.

49. McElroy, M.B. "Additive General Error Models for Production, Cost, and Derived Demand or Share Systems." Journal of Political Economy, 95, 737-57, 1987.

50. Morrison, C.J. (1999). Cost Structure and the Measurement of Economic Performance: Productivity, Utilization, Cost Economies, and Related Performance Indicators. Norwell, MA: Kluwer Academic Publishers.

51. Norsworthy, J.R. and Malmquist, D.H. "Input Measurement and Productivity Growth in Japanese and U.S. Manufacturing: American Economic Review, 73, 947-67, 1983.

52. Pindyck, R and Rotemberg, J.J. "Dynamic Factor Demands, Energy use and the Effects of Energy Price Shocks." American Economic Review, 73, 1983.

53. Pindyck, R. "Interfuel Substitution and the Industrial Demand for Energy: An

International Comparison." Review of Economics and Statistics, 61, 169-79, 1979.

54. Purcha, I.R. and Nadiri, M.I. " A Comparison of Alternative Methods for Estimation of Dynamic Factor Demand Models Under Nonstatic Expectations." Journal of Econometrics, 33, 187-211,1986.

55. Rickman, D.S and Partridge, M.D. "Regional Computable General Equilibrium Modeling: A Survey and Critical Appraisal. " International Regional Science Review, 21, 3-50, 1998.

56. Rossi, P.E. " Comparison of Alternative Functional Forms in Production." Journal of Econometrics, 30, 345-361, 1985.

57. Scully, G.W. " The North-South Manufacturing Wage Differential 1869-1919. " Journal of Regional Science, 11, 235-52, 1971.

58. Spady, R.H. and Friedlaender, A.F. " Hedonic Cost Functions for the Regulated Trucking Industry." Bell Journal of Economics, 9, 145-204,1978.

59. Schnorbus, R.H. and Giese, A.S., "A New Approach to Regional Capital Stock Estimation: Measurement and Performance." Federal Reserve Bank of Chicago Working Paper # WP-1989-6, 1989.

60. Terrell, D. "Incorporating Monotonicity and Concavity Conditions in Flexible Functional Forms." Journal of Applied Econometrics, 11, 179-94, 1996.

61. Tsurumi, H. and Tsurumi, Y. " A Bayesian Estimation of Macro and Micro CES Production Function." Journal of Econometrics, 4, 1-25, 1976.

62. U.S. Department of Commerce, Bureau of the Census. U.S Annual Survey of Manufacturing, 1970 -1989,Washington, D.C.: U.S. Government Printing Office.

63. U.S. Department of Commerce, Bureau of the Census. U.S. Census of Manufactures, 1967, 1972, 1977, 1982, 1987, 1992, Washington, D.C., U.S. Government Printing Office.

64. Uzawa, H. "Duality Principles in the Theory of Cost and Production." International Economic Review, 5, 216-220,1964.

65. Varian, H.R. "Goodness-of-Fit in Optimizing Models." Journal of Econometrics, 46, 125-40, 1990.

66. Wales, T. "On the Flexibility of Flexible Functional Forms: An Empirical Approach" Journal of Econometrics, 5, 183-193,1977.

67. Woodland, A.D. " Stochastic Specification and the Estimation of Share Equations." Journal of Econometrics, 10, 361-383, 1979.

Table I

BLS KLEM Data 1949-1981.

YEAR	Capital Share	Labor Share	Price of Capital	Price of Labor	Price of Energy
1949	0.283	0.683	30.8	8.0	15.1
1950	0.299	0.669	36.8	8.5	15.6
1951	0.298	0.671	39.1	9.2	15.9
1952	0.272	0.696	36.0	9.9	16.2
1953	0.259	0.709	36.0	10.6	17.5
1954	0.259	0.710	34.0	11.0	16.0
1955	0.286	0.683	41.0	11.4	15.6
1956	0.266	0.701	38.4	12.1	16.1
1957	0.263	0.704	37.9	12.8	16.7
1958	0.252	0.714	34.2	13.5	16.3
1959	0.273	0.694	41.9	14.2	16.9
1960	0.257	0.710	38.8	14.7	17.1
1961	0.260	0.706	38.6	15.0	17.1
1962	0.270	0.697	42.5	15.5	17.1
1963	0.280	0.687	45.5	16.1	17.1
1964	0.282	0.686	47.3	16.8	16.5
1965	0.298	0.671	52.1	17.1	16.3
1966	0.290	0.680	52.0	17.9	16.3
1967	0.275	0.694	47.0	19.0	16.3
1968	0.275	0.695	48.5	20.4	16.1
1969	0.255	0.715	45.3	21.9	16.3
1970	0.235	0.733	39.3	23.4	17.4
1971	0.256	0.709	43.9	24.9	18.9
1972	0.262	0.704	48.3	26.2	20.5
1973	0.254	0.712	50.4	28.5	22.7
1974	0.228	0.729	46.2	32.1	29.9
1975	0.261	0.690	52.5	35.1	38.7
1976	0.265	0.683	59.4	38.3	45.3
1977	0.269	0.667	65.9	41.6	53.0
1978	0.263	0.682	69.3	45.2	60.4
1979	0.246	0.696	68.2	49.8	68.3
1980	0.233	0.703	65.3	55.8	82.6
1981	0.246	0.687	73.0	60.9	95.9

Table II

Berndt and Wood KLEM Data 1947-1971.

YEAR	Capital Share	Labor Share	Price of Capital	Price of Labor	Price of Energy
1947	0.150	0.725	1.0000	1.0000	1.0000
1948	0.150	0.717	1.0027	1.1545	1.3025
1949	0.129	0.728	0.7437	1.1558	1.1966
1950	0.145	0.721	0.9249	1.2353	1.2144
1951	0.144	0.728	1.0487	1.3378	1.2517
1952	0.136	0.740	0.9974	1.3794	1.2791
1953	0.132	0.747	1.0065	1.4345	1.2750
1954	0.150	0.723	1.0875	1.4536	1.3035
1955	0.145	0.730	1.1031	1.5112	1.3427
1956	0.128	0.745	0.9960	1.5818	1.3715
1957	0.136	0.734	1.0632	1.6464	1.3801
1958	0.158	0.715	1.1561	1.6738	1.3933
1959	0.163	0.718	1.3075	1.7343	1.3675
1960	0.152	0.728	1.2541	1.7828	1.3802
1961	0.154	0.725	1.2632	1.8197	1.3763
1962	0.145	0.737	1.2652	1.8853	1.3768
1963	0.147	0.735	1.3229	1.9337	1.3473
1964	0.143	0.742	1.3279	2.0099	1.3896
1965	0.145	0.745	1.4065	2.0553	1.3863
1966	0.144	0.750	1.4510	2.1344	1.4010
1967	0.143	0.751	1.3861	2.2061	1.3919
1968	0.149	0.748	1.4990	2.3386	1.4338
1969	0.141	0.756	1.4495	2.4641	1.4648
1970	0.134	0.756	1.3246	2.6053	1.4590
1971	0.123	0.759	1.2017	2.7602	1.6468

Table III

Berndt and Wood KLEM Data 1947-1981.

YEAR	Capital Share	Labor Share	Price of Capital	Price of Labor	Price of Energy
1947	0.150	0.733	0.510	0.324	0.575
1948	0.133	0.738	0.419	0.361	0.743
1949	0.150	0.717	0.427	0.372	0.690
1950	0.196	0.683	0.647	0.390	0.705
1951	0.131	0.745	0.438	0.426	0.733
1952	0.152	0.732	0.544	0.447	0.750
1953	0.159	0.728	0.610	0.471	0.753
1954	0.174	0.708	0.635	0.483	0.770
1955	0.156	0.725	0.592	0.501	0.796
1956	0.163	0.721	0.652	0.528	0.816
1957	0.154	0.724	0.609	0.553	0.827
1958	0.176	0.705	0.660	0.569	0.830
1959	0.185	0.703	0.767	0.592	0.818
1960	0.161	0.727	0.668	0.609	0.819
1961	0.166	0.721	0.690	0.620	0.822
1962	0.158	0.732	0.698	0.635	0.824
1963	0.150	0.740	0.670	0.657	0.816
1964	0.149	0.744	0.690	0.680	0.833
1965	0.160	0.738	0.775	0.697	0.798
1966	0.156	0.746	0.781	0.723	0.800
1967	0.154	0.748	0.745	0.746	0.807
1968	0.168	0.738	0.838	0.799	0.808
1969	0.147	0.758	0.743	0.850	0.831
1970	0.148	0.752	0.731	0.904	0.889
1971	0.156	0.740	0.779	0.952	0.958
1972	0.179	0.721	1.000	1.000	1.000
1973	0.173	0.721	1.073	1.061	1.154
1974	0.164	0.668	1.152	1.167	2.152
1975	0.153	0.656	1.054	1.291	2.558
1976	0.174	0.637	1.377	1.404	2.748
1977	0.159	0.643	1.368	1.525	3.063
1978	0.159	0.651	1.468	1.652	3.282
1979	0.152	0.632	1.552	1.807	4.316
1980	0.165	0.583	1.831	2.005	6.287
1981	0.147	0.586	1.653	2.200	7.749

Table IV

Oklahoma KLEM Data 1970-1989.

YEAR	Capital Share	Labor Share	Price of Capital	Price of Labor	Price of Energy	Estimated Capital Stock
1970	0.166	0.770	11.54	3.47	0.75	942.98
1971	0.171	0.761	12.21	3.38	0.80	968.53
1972	0.162	0.771	12.91	4.01	0.85	997.89
1973	0.154	0.776	13.40	4.31	0.96	1036.11
1974	0.158	0.742	14.24	4.53	1.36	1094.31
1975	0.165	0.727	15.98	4.80	1.70	1131.38
1976	0.168	0.706	17.84	5.37	1.81	1167.93
1977	0.162	0.729	19.11	5.98	2.25	1210.89
1978	0.165	0.712	20.63	6.51	2.49	1376.52
1982	0.164	0.674	30.96	7.21	4.65	1562.52
1983	0.162	0.663	31.12	10.28	5.06	1534.29
1984	0.159	0.664	31.14	10.92	5.07	1538.14
1985	0.154	0.627	31.38	11.82	5.21	1493.00
1986	0.173	0.683	31.75	12.36	4.6	1555.69
1987	0.189	0.680	32.38	12.84	3.78	1626.31
1988	0.246	0.647	33.00	13.16	3.51	2285.31
1989	0.237	0.655	33.80	13.66	3.61	2310.03

TableV

Summary Statistics for BLS KLEM Data (N = 33).

Variable	Mean	S.D.	Minimum	Maximum
S_k	.2707	.0211	.228	.326
S_l	.686	.0218	.63	.733
P_l	36.92	29.45	8	98.4
P_k	52.44	17.11	30.8	94.4
P_e	28.29	20.37	15.1	95.9

Table VI

Summary Statistics for BWX Data: 1947-1981 (N = 35).

Variable	Mean	S.D.	Minimum	Maximum
S_k	.159	.013	.1308	.196
S_l	.707	.046	.583	.758
P_l	.857	.49	.324	2.2
P_k	.853	.364	.419	1.83
P_e	1.54	1.64	.575	7.74

Table VII

Summary Statistics for BW Data: 1947-1971 (N = 25).

Variable	Mean	S.D.	Minimum	Maximum
S_k	.159	.013	.1308	.196
S_l	.707	.046	.583	.758
P_l	.857	.49	.324	2.2
P_k	.853	.364	.419	1.83
P_e	1.54	1.64	.575	7.74

Table VIII

Summary Statistics for Oklahoma KLEM Data (N = 17).

Variable	Mean	S.D.	Minimum	Maximum
S_k	.1738	.0267	.154	.246
S_l	.706	.047	.627	.776
P_l	7.947	3.182	3.47	13.66
P_k	23.14	8.88	11.54	33.8
P_e	2.85	1.67	.75	5.21

Table IX

Model 1. Estimates of the Share Equations for the U.S and Oklahoma using SUR, Bayes Estimates using Gibbs Sampling under Informative and Diffuse priors. Based on BLS Data: 1949-1981.

Coefficient	US	Oklahoma	Informative Prior	Diffuse Prior
Constant	.2322	.2781	.2202	.346
δ_{kk}	.0445	-.1351	.0357	-.1696
δ_{kl}	-.0174	.1754	-.0095	.1421
Constant	.711	.4136	.7266	.426
δ_{kl}	-.0174	.1754	-.0095	.1421
δ_{ll}	.0101	-.0883	.0026	-.038

Table X

Model 2. Estimates of the Share Equations for the U.S and Oklahoma using SUR, Bayes Estimates using Gibbs Sampling under Informative and Diffuse priors. Based on BWX Data: 1947-1981.

Coefficient	US	Oklahoma	Informative Prior	Diffuse Prior
Constant	.1681	.2781	.1794	.3365
δ_{kk}	.0919	-.1351	.0267	-.1608
δ_{kl}	-.060	.1754	-.0208	.1350
Constant	.7402	.4136	.7538	.433
δ_{kl}	-.060	.1754	-.0208	.1350
δ_{ll}	.1302	-.0883	.1075	-.033

Table XI

Model 3. Estimates of the Share Equations for the U.S and Oklahoma using SUR, Bayes Estimates using Gibbs Sampling under Informative and Diffuse priors. Based on BW Data: 1947-1971.

Coefficient	US	Oklahoma	Informative Prior	Diffuse Prior
Constant	.1605	.2781	.1459	.2882
δ_{kk}	.0659	-.1351	.0202	-.0645
δ_{kl}	-.0323	.1754	-.0001	.0781
Constant	.7159	.4136	.7283	.6631
δ_{kl}	-.0323	.1754	-.0001	.0781
δ_{ll}	.0644	-.0883	.0338	-.0644

Table XII

Model 4, 5 and 6: Bayes Estimates of Cobb-Douglas Cost Function for Oklahoma using Gibbs sampling under informative prior, based on BLS and BW data: 1949-1981 and 1947-1971, and Diffuse Prior.

Coefficient	Model 4	Model 5	Model 6
Constant	7.07	6.94	7.05
β_k	.094	.158	.1101
β_l	.363	.351	.366

Table XIII

Estimates of National Elasticities of Substitution for Different Models

Elasticity Estimate	BLS	BWX	BW
σ_{kk}	-2.13	-1.68	-2.724
σ_{ll}	-.45	-.177	-.287
σ_{kl}	.90	.464	.712
σ_{ee}	-5.24	-.87	-2.81
σ_{ke}	-.50	.49	-.577
σ_{le}	.785	.258	.66

Table XIV

Estimates of Oklahoma Elasticities of Substitution for Different Models

Elasticity Estimate	Model 1	Model 2	Model 3
σ_{kk}	-3.571	-3.869	-4.084
σ_{ll}	-.4128	-.2019	-.3401
σ_{kl}	.9224	.8302	.999
σ_{ee}	-5.940	-.9418	-3.589
σ_{ke}	-.2451	.7196	.0448
σ_{le}	1.080	-.0156	.6052

Table XV

Estimates of Price Elasticities for Different Models

Elasticity Estimate	Model 1	Model 2	Model 3
ε_{kk}	-.6204	-.6724	-.7097
ε_{ll}	-.2911	-.1423	-.2398
ε_{lk}	.1603	.1443	.1736
ε_{kl}	.6518	.5854	.7045
ε_{ee}	-.7191	-.1140	-.434
ε_{ek}	-.0425	.1251	.0077
ε_{ke}	-.0296	.0871	.0054
ε_{le}	.1603	-.0018	.0732
ε_{el}	.7616	-.011	.4267

Table XVI

Posterior Probabilities that Concavity and Monotonicity Hold.

Restrictions	Concavity	Monotonicity
Model 1	.69	1
Model 3	1	1

Table XVII

Confidence Interval for Cross Elasticities of Substitution: Choice between CD and Translog.

Elasticities	σ_{kl}	σ_{kl}	σ_{kl}
Model 1	.87 to 1.10	-.28 to .12	1.07 to 1.09
Model 3	.90 to 1.07	-.06 to .62	.49 to .62
CD vs Translog	CD	Translog	Translog

Table XVIII

Out of Sample Forecast Comparison: RMSE.

Cost Share	Model 1	Model 3	Model 4	Model 5	Model 6
K	.090	.05	.06	.14	.232
L	.012	.107	.315	.304	.656
E	.115	.06	.368	.42	.426
Average	.072	.072	.247	.281	.438

Table XIX

Simulation of 1% Increase in Input Prices and Their Impact on Input Demands: Translog and CD specifications.

	Model 1			Model 3			Model 4			Model 5			Model 6		
	P_k	P_l	P_e	P_k	P_l	P_e	P_k	P_l	P_e	P_k	P_l	P_e	P_k	P_l	P_e
X_k	-.40	.39	.61	-.42	.69	.06	-.74	.48	.78	-.69	.76	1.1	-1.14	.34	.68
X_l	.53	-.35	.67	.27	-.38	.19	.40	-.74	.39	.12	-.61	.86	.12	-.64	.88
X_e	-32.3	3.07	-2	.3	.66	-1.2	.29	.44	-.58	.09	.33	-.58	.29	.50	-.50

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VITA

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