SELECTING t – BEST OF SEVERAL BIRNBAUM – SAUNDERS POPULATIONS BASED ON THE PARAMETERS

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Introduction and Literature Review

Section 1.1 Introduction

People everywhere everyday are faced with making choices or decisions at work and in their daily lives. Ranking and selection procedures can be used to make educated decisions. Ranking and selection procedures are used instead of traditional hypothesis testing on the population parameter of interest because traditional hypothesis testing only detects if there are differences between the populations and does not actually select the best populations as defined by some criterion. Applications of this theory in different disciplines are shown through the following examples :

- The owner of an automotive store is interested in carrying only two or three brands of automotive oil from the different possible brands. He will want to ensure that he selects the two best selling brands of oil.

- A store may also be interested in carrying the two best brands of spark plugs or serpentine belts based on which work the longest or most times until failure.

- A pharmaceutical company is interested in keeping only the three or four best pain relievers that they manufacture. They are interested in comparing the speed and / or length their pain relievers perform.

- A medical researcher may be testing the current treatments for a certain disease to determine the one, two or possibly three best treatments available on the market.

In some of the scenarios, the order of the t-best choices does not matter such as the automotive parts or the pain relievers. In the last scenario, order would in fact be important. You would be most interested in picking the one - best or possibly the two – best treatment(s), if you or someone you knew was in need of the treatment. We can consider the different choices in each of the scenarios as populations; i.e. there are k different populations and we want to select the t-best.

In the last two scenarios, the lifetimes of the pain relievers and the survival times of the patients may follow the probability distribution that was developed in 1969 by Birnbaum and Saunders. The Birnbaum-Saunders distribution has many applications in survival analysis, reliability and life-testing. Therefore, engineering and medical fields are a few places where this distribution is of most interest. Desmond (1986) showed that the Birnbaum-Saunders distribution can be written as a mixture of the Inverse Gaussian distribution and its reciprocal with mixing probability equal to $\frac{1}{2}$. See Chhikara and Folks (1989) for more about the Inverse Gaussian distribution.

There have been many articles published separately on ranking and selection procedures and the Birnbaum-Saunders distribution; but there is currently no literature available on ranking and selection procedures for the Birnbaum-Saunders distribution.

Section 1.2 Ranking and Selection

Bechhofer (1954) developed a procedure for selecting the t-best normal populations out of k independent normal populations with unknown variances. The method that he used is referred to as the indifference zone formulation. This procedure is the one that will be used in this dissertation. Other references on ranking and selection include Gibbons et al. (1977), Gupta and Panchapakesan (1979), and Bechhofer et al. (1995).

Section 1.3 Birnbaum - Saunders Distribution

Birnbaum and Saunders (1969 a, b) introduced a new fatigue life distribution. For complete samples, they derived properties and considered estimation of the parameters. Engelhardt et al. (1981) considered confidence intervals and tests of hypotheses and gave large sample approximations for the distributions of the maximum likelihood estimators. They also mentioned that the scale parameter β, which is the median of the distribution, corresponds to a typical number of cycles until failure occurs. Padgett (1986) considered Bayes estimation on reliability of the Birnbaum-Saunders distribution. Desmond (1986) looked at the relationship between the Inverse-Gaussian and the Birnbaum-Saunders distributions and introduced another derivation of the distribution. Chang and Tang (1993) discussed reliability bounds and critical time for the Birnbaum-Saunders distribution. Chang and Tang (1994 a,b) developed percentile bounds, tolerance limits and discussed a graphical analysis for the Birnbaum-Saunders distribution. Desmond

(1995) also developed shortest prediction intervals for the Birnbaum-Saunders distribution. Dupuis and Mills (1998) looked at the robust estimation for the Birnbaum-Saunders distribution. McCarter (1999) considered estimation and prediction for the Birnbaum-Saunders distribution using Type II-censored samples.

Chapter 2

Ranking and Selection According to the Parameter β

Section 2.1 Birnbaum-Saunders Background

Birnbaum and Saunders (1969) developed a two-parameter fatigue life distribution to model failures due to fatigue-crack growth. This distribution was derived from considerations of the physical behavior of the material that was subjected to a cyclically repeated stress pattern. The resulting distribution models the number of cycles needed to force the length of the fatigue crack to grow past a critical length.

The cumulative distribution function (CDF) of the Birnbaum-Saunders distribution is given by :

$$F\,(\,t;\alpha,\beta\,) = \Phi\left[\left(\frac{1}{\alpha}\right)\xi\left(\frac{t}{\beta}\right)\right] \tag{2.1.1}$$

where $t > 0$, $\beta > 0$, and $\alpha > 0$. Also, $\xi(t) = t^{\frac{1}{2}} - t^{-\frac{1}{2}}$ and $\Phi(z)$ is the standard normal CDF. Figure 2.1.1 shows the cumulative distribution functions for $\beta = 50{,}100{,}200{,}500$ with $\alpha = 1$. As β increases it takes longer for the cumulative distribution function to reach 1. Therefore, as β increases the probability that failure would occur at or before time, t, decreases. The probability density function (pdf) has the form :

$$f(t) = \frac{1}{\alpha\beta}\xi'\left(\frac{t}{\beta}\right)\phi\left[\alpha^{-1}\xi\left(\frac{t}{\beta}\right)\right] \tag{2.1.2}$$

where $t > 0$, $\beta > 0$, $\alpha > 0$, $\xi'(t) = \frac{\partial\xi(t)}{\partial t}$, and ϕ is the pdf of the standard normal distribution. The parameter α is a shape parameter. The scale parameter β corresponds, roughly, to a typical number of cycles to failure. β is the median of the distribution which also implies that β is a location parameter. The expected value and variance of T are given by $E(T) = \beta(1 + \frac{1}{2}\alpha^2)$ and $\mathrm{var}(T) = (\alpha\beta)^2\left(1 + \frac{5}{4}\alpha^2\right)$, respectively.

Figure 2.1.2 shows the probability density functions for $\beta = 50,100,150,200,250$ with $\alpha = 1$. For a fixed value of α, as β increases the distribution function becomes flatter. The peak of the probability density function moves to the right (i.e. a larger value of t). Figure 2.1.2 supports the same conclusion as Figure 2.1.1, as β increases the probability that failure would occur at or before time, t, decreases. Figure 2.1.3 shows the probability density functions for $\alpha = 0.1,0.25,0.50,0.75,1.00,1.25$ with $\beta = 100$. As alpha increases the peak of the probability density function moves closer towards 0. Most of the probability is associated with t values closer and closer to 0 as α increases. Figure 2.1.4 shows the probability density functions where α and β are both changing.

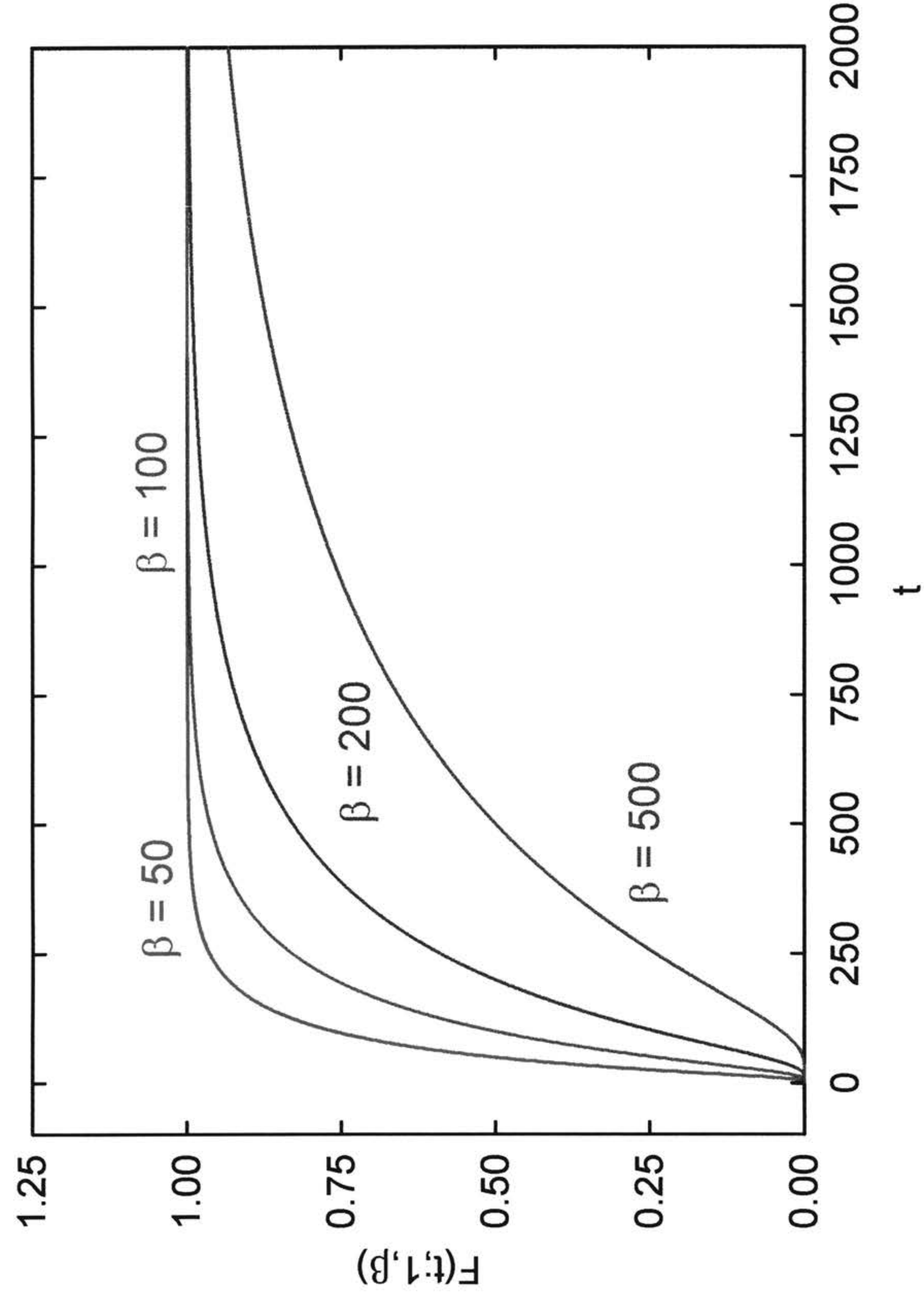

Figure 2.1.1. Birnbaum - Saunders Cumulative Distribution Functions

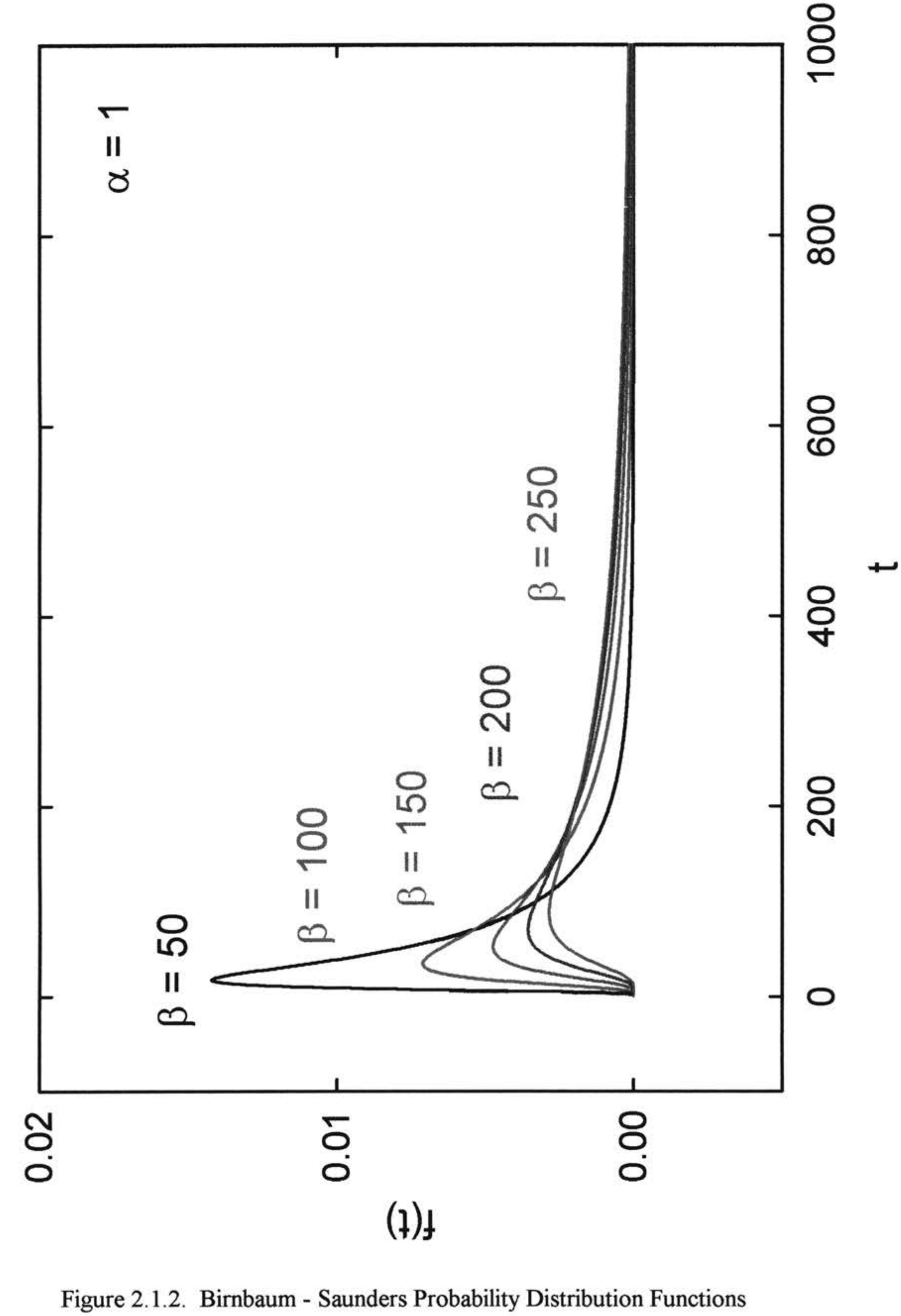

Figure 2.1.2. Birnbaum - Saunders Probability Distribution Functions

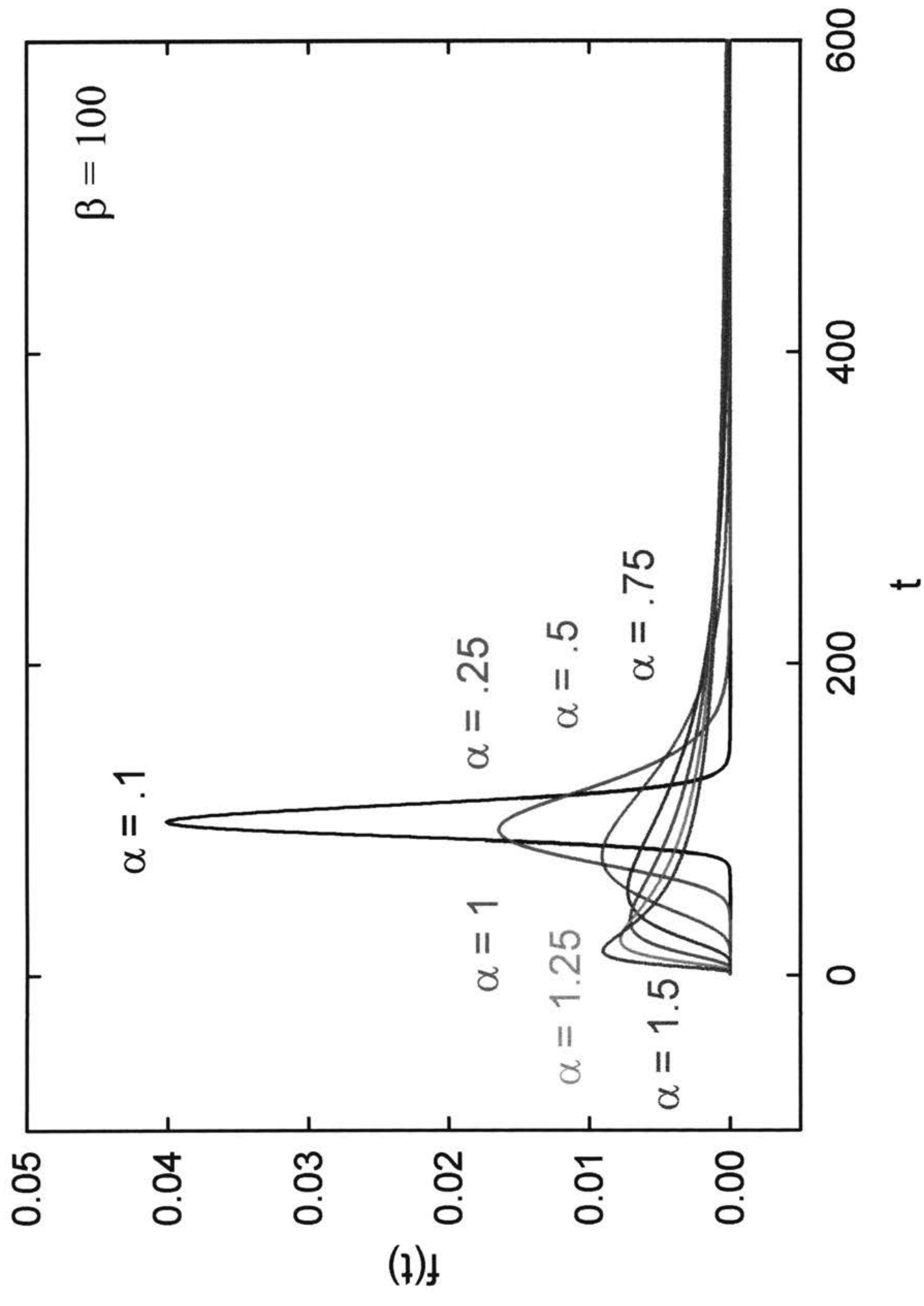

Figure 2.1.3. Birnbaum - Saunders Probability Distribution Function

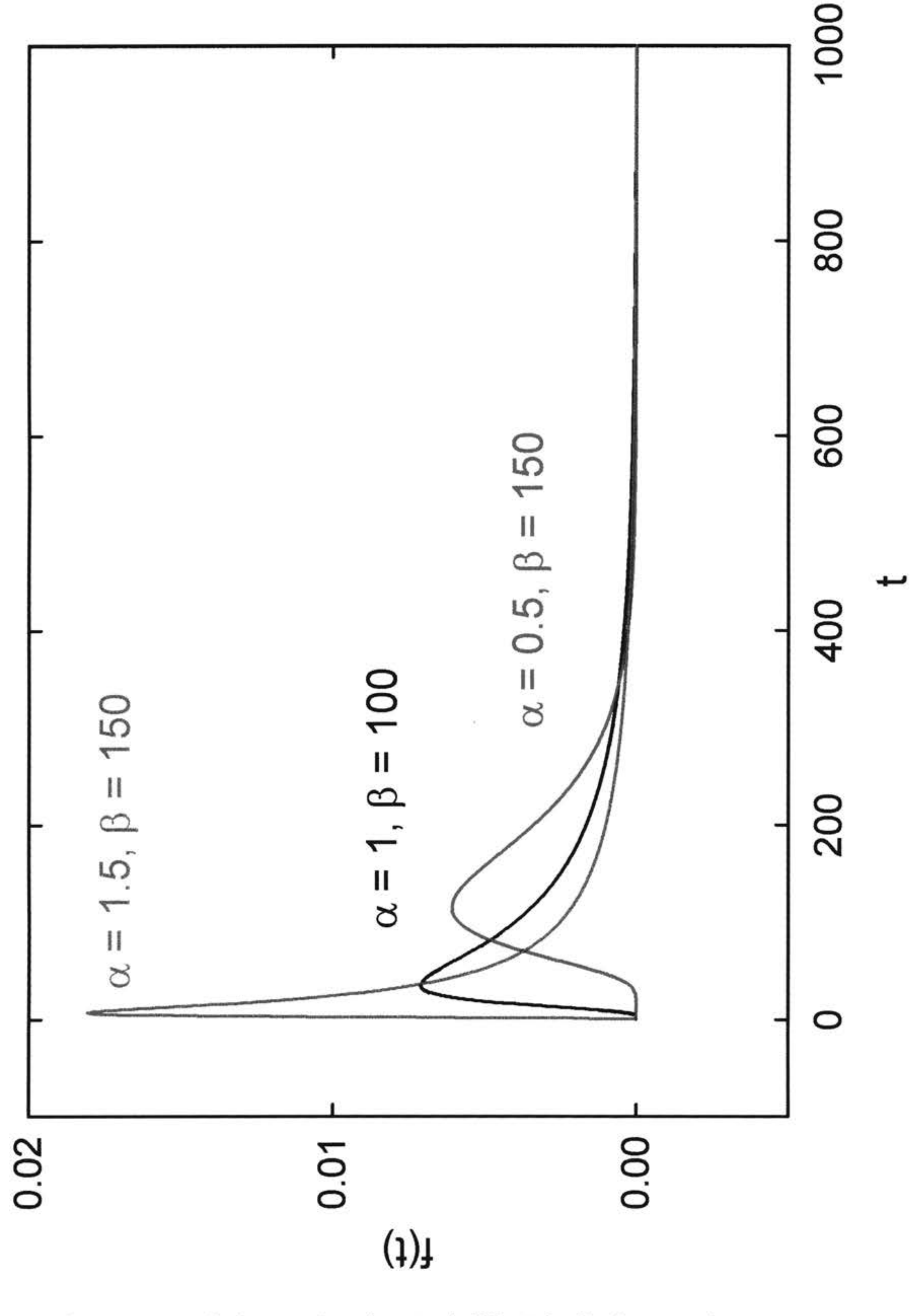

Figure 2.1.4. Birnbaum - Saunders Probability Distribution Functions

Section 2.2 Research Problem

Given k $(k \geq 2)$ independent Birnbaum-Saunders Distributions, (BSD), $\pi_1, \pi_2, \ldots, \pi_k$. Let $\pi_{(i)}$ denote the population having the ith scale parameter $\beta_{[i]}$, where $\beta_{[1]} \leq \beta_{[2]} \leq \cdots \leq \beta_{[k]}$. The population $\pi_{(i)}$ is defined to be better than $\pi_{(j)}$ if $i > j$. The goal is to select the t-best populations with the t largest β parameters, $1 \leq t < k$. Since β is approximately the number of cycles until the fatigue growth crack grows past a critical length then it makes sense to consider the largest β's. In Chapter 4, ranking the shape parameter, α is considered. The goal is to select a group of the t-best ($1 \leq t < k$) populations in an unordered manner when α is assumed known. The choice of any t populations having the t largest parameters is regarded as a correct selection, (CS).

Section 2.3 Basic Results

Before proceeding with the selection procedure, it is useful to note the following results concerning estimators of the parameters for the Birnbaum-Saunders distribution. Let $T_1, T_2, \ldots, T_n$ be a random sample from a Birnbaum-Saunders population.

Theorem 2.3.1 : (Birnbaum and Saunders (1969b)) The maximum likelihood estimator, $\hat{\beta}$, is the unique positive root of

$$x^2 - x\{2H + K(x)\} + H\{\overline{T} + K(x)\} = 0 \text{ where } \overline{T} = n^{-1}\sum_{j=1}^{n} T_j \ ,$$

$$H = n^{-1}\sum_{j=1}^{n} T_j^{-1} \text{ and } K(x) = \left[n^{-1}\sum (x + T_j)^{-1} \right]^{-1} .$$

Theorem 2.3.2 : (Engelhardt et al. (1969)) The distribution of $\frac{\hat{\beta}}{\beta}$ does not depend on β.

Theorem 2.3.3 : (Birnbaum and Saunders (1969b)) The maximum likelihood estimator, $\hat{\alpha}$, is $\hat{\alpha} = \left(\frac{\overline{T}}{\hat{\beta}} + \frac{\hat{\beta}}{H^{-1}} - 2 \right)^{\frac{1}{2}}$ where $\overline{T}, H$, and $\hat{\beta}$ are defined before.

Theorem 2.3.4 : (Engelhardt et al. (1969)) The distribution of $\frac{\hat{\alpha}}{\alpha}$ does not depend on α and β.

There are at least two additional estimators that have been considered by Birnbaum and Saunders (1969) and Desmond (1995) due to the difficulty in computing the MLE's. The two estimators are $\beta' = \frac{\sum T_j^{\frac{1}{2}}}{\sum T_j^{-\frac{1}{2}}}$ and $\widetilde{\beta} = \left(\frac{\sum T_j}{\sum T_j^{-1}} \right)^{\frac{1}{2}}$ and they are both very easy to compute since they are based only on the random sample. $\widetilde{\beta}$ is also known as the "mean mean" estimator.

Theorem 2.3.5 : The distribution of $\frac{\beta'}{\beta}$ does not depend on β.

Proof: $\dfrac{\beta'}{\beta} = \dfrac{\dfrac{\sum (T_j)^{\frac{1}{2}}}{\sum \dfrac{1}{(T_j)^{\frac{1}{2}}}}}{\beta}$

$$= \frac{\sum (T_j)^{\frac{1}{2}}}{\beta^{\frac{1}{2}}\beta^{\frac{1}{2}}\sum \dfrac{1}{(T_j)^{\frac{1}{2}}}} = \frac{\sum \left(\dfrac{T_j}{\beta}\right)^{\frac{1}{2}}}{\sum \left(\dfrac{\beta}{T_j}\right)^{\frac{1}{2}}} = \frac{\sum \left(\dfrac{T_j}{\beta}\right)^{\frac{1}{2}}}{\sum \left[\dfrac{1}{\left(\dfrac{T_j}{\beta}\right)^{\frac{1}{2}}}\right]}$$

Let $U_j = \dfrac{T_j}{\beta}$. The distribution of U_j does not depend on β since

β is a scale parameter. So, the distribution of $\dfrac{\sum (U_j)^{\frac{1}{2}}}{\sum \left[\dfrac{1}{(U_j)^{\frac{1}{2}}}\right]}$ does

not depend on β . Therefore, the distribution of $\dfrac{\beta'}{\beta}$ does not

depend on β as desired.

Theorem 2.3.6 : The distribution of $\dfrac{\widetilde{\beta}}{\beta}$ does not depend on β.

Proof: $\dfrac{\widetilde{\beta}}{\beta} = \dfrac{\left(\dfrac{\sum T_j}{\sum T_j^{-1}}\right)^{\frac{1}{2}}}{\beta} = \dfrac{\left(\sum T_j\right)^{\frac{1}{2}}}{\beta^{\frac{1}{2}}\beta^{\frac{1}{2}}\left(\sum T_j^{-1}\right)^{\frac{1}{2}}} = \dfrac{\left(\dfrac{\sum T_j}{\beta}\right)^{\frac{1}{2}}}{\dfrac{\left(\sum T_j^{-1}\right)^{\frac{1}{2}}}{\beta^{\frac{-1}{2}}}}$

$$= \frac{\left(\frac{\sum T_j}{\beta}\right)^{\frac{1}{2}}}{\left(\frac{\sum T_j^{-1}}{\beta^{-1}}\right)^{\frac{1}{2}}} = \frac{\left(\sum \frac{T_j}{\beta}\right)^{\frac{1}{2}}}{\left(\sum\left(\frac{T_j}{\beta}\right)^{-1}\right)^{\frac{1}{2}}}$$ Let $U_j = \frac{T_j}{\beta}$. The distribution of

U_j does not depend on β since β is a scale parameter. So,

$\frac{\tilde{\beta}}{\beta} = \frac{\left(\sum U_j\right)^{\frac{1}{2}}}{\left(\sum U_j^{-1}\right)^{\frac{1}{2}}} = \left(\sum U_j\right)^{\frac{1}{2}}\left(\sum U_j^{-1}\right)^{-\frac{1}{2}}$ and the distribution of

$\left(\sum U_j\right)^{\frac{1}{2}}\left(\sum U_j^{-1}\right)^{-\frac{1}{2}}$ does not depend on β since the distribution of

U_j does not depend on β. Therefore, the distribution of $\frac{\tilde{\beta}}{\beta}$ does

not depend on β as desired.

Theorem 2.3.7 : (Birnbaum and Saunders (1969b)) $\tilde{\beta}$ is a consistent estimator for β. When $\alpha < \sqrt{2}$, $\tilde{\beta}$ is the same as the MLE, $\hat{\beta}$.

Section 2.4 Probability of Correct Selection and its Minimum

Let $\hat{\beta}_{(i)}$ denote the statistic associated with population $\pi_{(i)}$, $i = 1,\ldots,k$. (From this point forward, this dissertation will use the notation for the MLE, but all results hold for β' and $\tilde{\beta}$.) The ranked $\hat{\beta}$'s are denoted by $\hat{\beta}_{[1]} \le \hat{\beta}_{[2]} \le \ldots \le \hat{\beta}_{[k]}$.

Let $\vec{\beta} = (\beta_{[1]}, \ldots, \beta_{[k]})$ denote a point in the parameter space Ω that is partitioned into a 'preference zone', $\Omega(\delta^*)$, defined by $\Omega(\delta^*) = \left\{ \vec{\beta} : \frac{\beta_{[k-t]}}{\beta_{[k-t+1]}} \leq \delta^*,\ 0 < \delta^* < 1 \right\}$. The complement of $\Omega(\delta^*)$ is called the indifference zone. This dissertation uses the indifference zone approach of Bechhofer (1954). Now consider the following rule, R, for which the probability of correct selection, $P(CS \mid R)$, satisfies $P(CS \mid R) \geq p^*$ for all $\vec{\beta} \in \Omega(\delta^*)$ and fixed α.

Rule R : Select the populations associated with the t largest $\hat{\beta}$ as the t-best populations.

The experimenter specifies in advance the constants δ^* and p^* where $\binom{k}{t}^{-1} < p^* < 1$. If p^* is not assumed greater than $\binom{k}{t}^{-1}$ then the probability of correct selection can be guaranteed by randomly selecting the t best populations.

Now using the results of Section 2.3, the probability of correct selection, $P(CS \mid R)$ is as follows :

$$P(CS \mid R) = P\left[\max_{1 \leq j \leq k-t} \hat{\beta}_{(j)} \leq \min_{k-t+1 \leq i \leq k} \hat{\beta}_{(i)} \right] \tag{2.4.1}$$

$$= P\left[\max_{\substack{1 \leq j \leq k-t \\ j \neq g}} \hat{\beta}_{(j)} \leq \hat{\beta}_{(g)} \leq \min_{k-t+1 \leq i \leq k} \hat{\beta}_{(i)}; g = 1, \ldots, k-t \right] \tag{2.4.2}$$

$$= P\left[\max_{\substack{1 \leq j \leq k-t \\ j \neq g}} \frac{\hat{\beta}_{(j)}}{\beta_{[j]}} \frac{\beta_{[j]}}{\beta_{[g]}} \leq \frac{\hat{\beta}_{(g)}}{\beta_{[g]}} \leq \min_{k-t+1 \leq i \leq k} \frac{\hat{\beta}_{(i)}}{\beta_{[i]}} \frac{\beta_{[i]}}{\beta_{[g]}}; g = 1, \ldots, k-t \right] \tag{2.4.3}$$

Define $V_j = \frac{\hat{\beta}_{(j)}}{\beta_{[j]}}$ and $G_v(v)$ to be the cumulative distribution function of V_j.

$$= P\left[\max_{\substack{1\le j\le k-t \\ j\ne g}} V_j \frac{\beta_{[j]}}{\beta_{[g]}} \le V_g \le \min_{k-t+1\le i\le k} V_i \frac{\beta_{[i]}}{\beta_{[g]}}; g = 1,\ldots,k-t\right]. \quad (2.4.4)$$

Interest is in finding the configuration of the parameters that minimizes $P(CS\,|\,R)$. This configuration of parameters is called the Least Favorable Configuration (LFC). Under the least favorable conditions $\beta_{[1]} = \beta_{[2]} = \cdots = \beta_{[k-t]}$, $\beta_{[k-t+1]} = \cdots = \beta_{[k]}$ and $\beta_{[k-t]} \le \beta_{[k-t+1]}(\delta^*)$ which implies that $\frac{\beta_{[k-t+1]}}{\beta_{[k-t]}} \ge \frac{1}{\delta^*} \ge 1$.

Thus, $$P(CS\,|\,R) \ge P\left[\max_{\substack{1\le j\le k-t \\ j\ne g}} V_j(1) \le V_g \le \min_{k-t+1\le i\le k} V_i\left(\frac{1}{\delta^*}\right); g = 1,\ldots,k-t\right] \quad (2.4.5)$$

$$= \sum_{g=1}^{k-t}\int_0^{\infty}\left[\prod_{\substack{j=1 \\ g\ne j}}^{k-t} G_v(v) \prod_{i=k-t+1}^{k}\left(1 - G_v(v\delta^*)\right)\right] dG_v(v) \quad (2.4.6)$$

$$= \sum_{g=1}^{k-t}\int_0^{\infty} G_v^{(k-t-1)}(v)\left[1 - G_v(v\delta^*)\right]^t dG_v(v) \quad (2.4.7)$$

$$= (k-t)\int_0^{\infty} G_v^{(k-t-1)}(v)\left[1 - G_v(v\delta^*)\right]^t dG_v(v) = P^*(CS\,|\,R), \quad (2.4.8)$$

where $G_v(v)$ is defined to be the cumulative distribution function of the $V_j = \frac{\hat{\beta}_{(j)}}{\beta_{[j]}}$.

So, $$P^*(CS\,|\,R) = P\left[\max_{1\le j\le k-t} V_j \le \frac{1}{\delta^*}\min_{k-t+1\le i\le k} V_i\right]. \quad (2.4.9)$$

Now given k, t, δ^*, and p^* - values, the solution can be obtained by setting $P^*(CS\,|\,R)$ equal to p^* and solving for n. However, this can not be done analytically since the distribution of V_i cannot be obtained in closed form. Therefore, $P^*(CS \mid R)$ has been simulated for various cases in Section 2.6 and large sample approximations will be discussed in Chapter 3.

Section 2.5 Properties of the Probability of Correct Selection

Property 2.5.1: As $\delta^* \to 1$, show that the $P^*(CS \mid R) \to \binom{k}{t}^{-1}$.

Proof: The $P^*(CS \mid R) = (k-t)\int_0^\infty G_v^{(k-t-1)}(v)[1 - G_v(v\delta^*)]^t dG_v(v)$.

And further suppose that $\delta^* \to 1$ then that implies that the

$$P^*(CS \mid R) = (k-t)\int_0^\infty G_v^{(k-t-1)}(v)[1 - G_v(v)]^t dG_v(v).$$

Now let $x = G_v(v)$ then that implies

$$P^*(CS \mid R) = (k-t)\int_0^1 x^{(k-t-1)}[1-x]^t dx.$$

$$= \frac{(k-t)(k-t-1)!\,t!}{k!} = \frac{(k-t)!\,t!}{k!} = \binom{k}{t}^{-1} \text{ as desired.}$$

Property 2.5.2: As $\delta^* \to 0$, show that the $P^*(CS \mid R) \to 1$.

Proof: The $P*(CS \mid R) = (k-t)\int_0^\infty G_v^{(k-t-1)}(v)[1 - G_v(v\delta^*)]^t dG_v(v)$.

And further suppose that $\delta^* \to 0$ then that implies that the

$$P^*(CS \mid R) = (k-t)\int_0^\infty G_v^{(k-t-1)}(v)dG_v(v).$$

Now let $x = G_v(v)$ then that implies $P^*(CS \mid R) = (k-t)\int_0^1 x^{(k-t-1)}(1-x)^0 dx$.

$$= \frac{(k-t)(k-t-1)!}{(k-t)!} = \frac{(k-t)!}{(k-t)!} = 1 \text{ as desired.}$$

Property 2.5.3 : As $n \to \infty$, then $P^*(CS \mid R) \to 1$.

Proof : The proof of this property follows from the normal approximation to $P^*(CS \mid R)$ discussed in Section 2 of Chapter 3.

Property 2.5.3 guarantees that there is a sample size, n, which will guarantee any probability of correct selection.

Section 2.6 Simulations

Fortran programs were written using Monte Carlo methods to simulate probability tables using the estimator, $\hat{\beta}$, for $\alpha = 0.15, 0.25, 0.50, 0.75, 1.0$, $k = 2\ (1)\ 5$, $t = 1\ (1)\ (k-1)$, and $n = 5\ (5)\ 30$. From the literature on the Birnbaum-Saunders distribution reasonable choices for α are less than or equal to 2 and usually only those values less than or equal to $\sqrt{2}$ are used. The tables are located in Appendix A. The tables were constructed by performing 50,000 iterations to calculate the probabilities of correct selection. The

Birnbaum – Saunders populations were generated by an algorithm that was previously used by Desmond (1995). The complete program is located in Appendix H.

To illustrate how these tables are used then consider the following scenario : If a researcher is interested in choosing the 2 "best" populations from 5 populations with $\alpha = 1$ and they further specify that $p^* = 0.90$ and $\delta^* = 0.500$ then according to the table found below from Appendix A they would need to sample 15 from each of the five populations in order to ensure the probability of correct selection to be 0.90, since $k = 5$, $t = 2$, $p^* = 0.90$ and $\delta^* = 0.500$.

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	2	5		.380	.305	.255	.220	.185	.165
		10		.515	.445	.395	.355	.310	.285
		15		.570	***.500***	.450	.415	.370	.345
		20		.610	.550	.500	.460	.420	.390
		25		.640	.580	.540	.500	.470	.435
		30		.660	.610	.570	.530	.490	.460

The simulated probability tables using estimator , β', for $\alpha = 1$, k = 2 (1) 5, t = 1 (1) (k-1), and n = 5 (5) 30 are located in Appendix B.

The simulated probability tables using the estimator , $\widetilde{\beta}$, for $\alpha = 0.15, 0.25, 0.50, 0.75, 1.0, 2.0, 3.0, 4.0,$ and 5.00, k = 2 (1) 5, t = 1 (1) (k-1), and n = 5 (5) 30 are located in Appendix C.

The last two estimators gave very similar values for the probability of correct selection. Therefore, only $\widetilde{\beta}$ has been explored further.

A comparison of the estimators, $\hat{\beta}, \widetilde{\beta}, \beta'$, is given below for the probability of correct selection, $P^*(CS \mid R)$, for the following specificed values : $\alpha = 1$, k=5, t=2, n=30 and $\delta^* = 0.400, 0.500, 0.650, 0.750$.

$P^*(CS \mid R)$

	$\delta^* = 0.400$	$\delta^* = 0.500$	$\delta^* = 0.650$	$\delta^* = 0.750$
$\hat{\beta}$	0.9994	0.9862	0.8296	0.5894
$\widetilde{\beta}$	0.9999	0.9935	0.8703	0.6332
β'	0.9998	0.9889	0.8603	0.6230

Chapter 3

Asymptotic Results of the Estimators for the Parameter β

Section 3.1 Results for the Estimator $\hat{\beta}$

From Chapter 2, Section 4, the probability of correct selection, $P^{*}(CS \mid R)$, was obtained but a closed form of the distribution of V_i does not exist. Therefore, the probability of correct selection, $P^{*}(CS \mid R)$, must be simulated or approximated. Simulations were discussed in Chapter 2 and the tables appear in Appendices A, B, and C. Large sample approximations for the parameter, β, will now be considered.

Before proceeding with the selection procedure, it is useful to note the following asymptotic results concerning the maximum likelihood estimator, $\hat{\beta}$.

Theorem 3.1.1 : (Engelhardt et al., 1981) For n sufficiently large,

$$\hat{\beta} \mathrel{\dot\sim} \mathrm{N}\left(\beta, \beta^2 H^2\left(\alpha^2\right)/n\right) \text{ where } H(u) = \left[\frac{1}{4} + \frac{1}{u} + \mathrm{I}\left(u^{\frac{1}{2}}\right)\right]^{\frac{-1}{2}},$$

$$\mathrm{I}(\alpha) = 2\int_0^{\infty}\left[\left(1+\xi^{-1}(\alpha z)\right)^{-1} - \frac{1}{2}\right]^2 \Phi'(z)dz \text{ and } \xi(t) = t^{\frac{1}{2}} - t^{\frac{-1}{2}}.$$

Theorem 3.1.2 : For n sufficiently large, $\dfrac{\hat{\beta}}{\beta} \mathrel{\dot\sim} \mathrm{N}\left(1, \dfrac{H^2\left(\alpha^2\right)}{n}\right)$ where $H(u)$ is defined as before.

Proof: Suppose $\hat{\beta} \dot{\sim} N(\beta, \beta^2 H^2(\alpha^2)/n)$.

Let $X = \hat{\beta}$ and $Y = \dfrac{\hat{\beta}}{\beta} = \dfrac{X}{\beta}$ then

$$P(Y \le y) = P\left(\frac{X}{\beta} \le y\right) = P(X \le \beta y) \tag{3.1.1}$$

$$= P\left(Z \le \frac{\beta y - \beta}{\sqrt{\beta^2 (H^2(\alpha^2))/n}}\right) \tag{3.1.2}$$

$$= P\left(Z \le \frac{\beta(y-1)}{\beta\sqrt{(H^2(\alpha^2))/n}}\right) \tag{3.1.3}$$

$$= P\left(Z \le \frac{(y-1)}{\sqrt{(H^2(\alpha^2))/n}}\right). \tag{3.1.4}$$

Therefore, $\dfrac{\hat{\beta}}{\beta} \dot{\sim} N\left(1, \dfrac{H^2(\alpha^2)}{n}\right)$ as desired.

Section 3.2 Probability of Correct Selection for Normal Approximation of $\hat{\beta}$

The probability of correct selection must be examined since the distribution of $\dfrac{\hat{\beta}}{\beta}$ is now being approximated by a normal distribution where $\dfrac{\hat{\beta}}{\beta} \dot{\sim} N\left(1, \dfrac{H^2(\alpha^2)}{n}\right)$.

Therefore, from Equation 2.4.9 of Section 2.4 of Chapter 2, the probability of correct selection given the rule R, $P^*(CS \mid R)$ is :

$$P^*(CS \mid R) = P\left[\max_{1\le j\le k-t} V_j \le \frac{1}{\delta^*} \min_{k-t+1\le i\le k} V_i\right] \tag{3.2.1}$$

$$= P\left[\delta^* \max_{1\le j\le k-t} V_j \le \min_{k-t+1\le i\le k} V_i\right] \tag{3.2.2}$$

$$= P\left[\max_{1\le j\le k-t} \delta^* V_j - 1 \le \min_{k-t+1\le i\le k} V_i - 1\right] \tag{3.2.3}$$

$$= P\left[\max_{1\le j\le k-t} \frac{\delta^* V_j - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}} \le \min_{k-t+1\le i\le k} \frac{V_i - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}}\right] \tag{3.2.4}$$

$$= P\left[\max_{1\le j\le k-t} \frac{\delta^*(V_j - 1)}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}} + \frac{\delta^* - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}} \le \min_{k-t+1\le i\le k} \frac{V_i - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}}\right] \tag{3.2.5}$$

$$= P\left[\max_{1\le j\le k-t} \delta^* Z_j + \frac{\delta^* - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}} \le \min_{k-t+1\le i\le k} Z_i\right] \tag{3.2.6}$$

$$\doteq \sum_{g=1}^{k-t} \int_{-\infty}^{\infty} \prod_{\substack{i=1 \\ g\ne j}}^{k-t} \Phi(Z_i) \prod_{i=k-t+1}^{k} \left[1 - \Phi\left(\delta^* Z_j + \frac{\delta^* - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}}\right)\right] d\Phi(z) \tag{3.2.7}$$

$$= (k-t)\int_{-\infty}^{\infty} [\Phi(Z_i)]^{(k-t-1)} \left[1 - \Phi\left(\delta^* Z_j + \frac{\delta^* - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}}\right)\right]^t d\Phi(z) \tag{3.2.8}$$

where $\Phi(z)$ is defined to be the cumulative distribution function of the standard normal distribution. Now given *k, t,* δ^*, and p^* - values, the solution can be obtained by setting $P^*(CS \mid R)$ equal to p^* and solving for *n*.

From Section 2.5 of Chapter 2, Property 2.5.3 states that as $n \to \infty$, then $P^*(CS \mid R) \to 1$. From Equation 3.2.8, as $n \to \infty$, $\dfrac{\delta^* - 1}{\sqrt{\dfrac{H^2(\alpha^2)}{n}}} \to -\infty$, since $\delta^* < 1$.

Therefore, $P^*(CS \mid R) \to (k-t)\int_{-\infty}^{\infty} [\Phi(Z_i)]^{(k-t-1)} (1)^t d\Phi(z) \to 1$ as $n \to \infty$.

Section 3.3 Large Sample Approximations

Fortran programs were written using Monte Carlo methods to calculate probability tables using the estimator, $\hat{\beta}$, for $\alpha = 0.25, 0.50, 0.75, 1.0, 1.25, 1.5$, *k* = 2 (1) 5, *t* = 1 (1) (*k-1*), and $n = 30, 40, 50, 75$. The tables are located in Appendix E.

Section 3.4 Results for the Estimator $\tilde{\beta}$

Before proceeding with the selection procedure, it is useful to note the following asymptotic results concerning the mean - mean estimator, $\tilde{\beta}$.

Theorem 3.4.1 : (Birnbaum and Saunders, 1969 b) For n sufficiently large,

$$\tilde{\beta} \dot{\sim} BS\left(\alpha\theta n^{\frac{-1}{2}}, \beta\right) \text{ where } \theta^2 = \left(1+\frac{3}{4}\alpha^2\right) \Big/ \left(1+\frac{1}{2}\alpha^2\right)^2 .$$ Also,

$$E(\tilde{\beta}) = \beta\left[1+\frac{(\alpha\theta)^2}{2n}\right] \text{ and } Var(\tilde{\beta}) = \frac{(\alpha\theta\beta)^2}{n}\left[1+\frac{5\alpha^2\theta^2}{4n}\right].$$

Theorem 3.4.2 : For n sufficiently large, $\frac{\tilde{\beta}}{\beta} \dot{\sim} BS\left(\alpha\theta n^{\frac{-1}{2}}, 1\right)$ where θ is as above.

Proof : Suppose $\tilde{\beta} \dot{\sim} BS\left(\alpha\theta n^{\frac{-1}{2}}, \beta\right)$. Let $X = \tilde{\beta}$ and $Y = \frac{\tilde{\beta}}{\beta} = \frac{X}{\beta}$

then $$P(Y \le y) = P\left(\frac{X}{\beta} \le y\right) = P(X \le y\beta) \tag{3.4.1}$$

$$= P\left(X \le \frac{n^{\frac{1}{2}}}{\alpha\theta}\xi\left(\frac{y\beta}{\beta}\right)\right) \tag{3.4.2}$$

$$= P\left(X \le \frac{n^{\frac{1}{2}}}{\alpha\theta}\xi\left(\frac{y}{1}\right)\right). \tag{3.4.3}$$

Therefore, $\frac{\tilde{\beta}}{\beta} \dot{\sim} BS\left(\alpha\theta n^{\frac{-1}{2}}, 1\right)$ as desired.

Section 3.5 Probability of Correct Selection for Birnbaum-Saunders Approximation of $\widetilde{\beta}$

The probability of correct selection is examined for the distribution of $\dfrac{\widetilde{\beta}}{\beta}$ which is now being approximated by a Birnbaum-Saunders distribution. Therefore,

$$P^*(CS \mid R) = P\left[\max_{1 \le j \le k-t} Y_j \le \frac{1}{\delta^*} \min_{k-t+1 \le i \le k} Y_i\right] \doteq (k-t)\int_0^\infty G_Y^{(k-t-1)}(y)\left[1 - G_Y(y\delta^*)\right]^t dG_Y(y) \quad (3.5.1)$$

where $G_Y(y)$ is the cumulative distribution function of $Y \dot{\sim} BS\left(\alpha\theta n^{\frac{-1}{2}}, 1\right)$. Now given *k*, *t*, δ^*, and p^* - values, the solution can be obtained by setting $P^*(CS \mid R)$ equal to p^* and solving for *n*.

Section 3.6 Large Sample Approximations

Fortran programs were written using Monte Carlo methods to calculate probability tables using the estimator, $\widetilde{\beta}$, for $\alpha = 0.25, 0.50, 0.75, 1.0, 1.25, 1.5$, $k = 2$ (1) 5, $t = 1$ (1) (*k-1*), and $n = 30, 40, 50, 75$. The tables are located in Appendix F.

Section 3.7 Comparisons of the Simulations and Approximations

A comparison of the estimators, $\hat{\beta}, \tilde{\beta}$, for simulated probabilities and the approximations is given next for the probability of correct selection, $P^{*}(CS \mid R)$, for the following specified values : $\alpha = 1$, k=5, t=2, n=30 and $\delta^{*} = 0.400, 0.500, 0.650, 0.750$.

	$P^{*}(CS \mid R)$			
	$\delta^{*} = 0.400$	$\delta^{*} = 0.500$	$\delta^{*} = 0.650$	$\delta^{*} = 0.750$
simulated $\hat{\beta}$	0.9994	0.9862	0.8296	0.5894
approximated $\hat{\beta}$	0.9988	0.9879	0.8554	0.6155
simulated $\tilde{\beta}$	0.9999	0.9935	0.8703	0.6332
approximated $\tilde{\beta}$	0.9999	0.9939	0.8726	0.6394

Chapter 4

Ranking and Selection According to the Parameter α

Section 4.1 Theory for "Best" Parameter α

When selecting populations according the parameter β, it is most logical to select the t populations with the largest β parameters since β is the median of the distribution. In the case of selection of the t "best" populations with fixed β, according to the parameter α, the choice is much less intuitive. In order to determine whether to choose the t populations with the smallest or largest parameters α, the reliability function, $R(t_0)$, has been investigated. $R(t_0) = P(X > t_0) = 1 - F(t_0; \alpha, \beta)$ where $F(t_0; \alpha, \beta)$ is defined as the cumulative distribution function in Section 2.1 of Chapter 2. If the reliability function were increasing then selecting the t populations with the largest α parameters would be most consistent with what is usually thought of as "best" populations. Conversely, selecting the t smallest α parameters would be considered "best" if the reliability function is decreasing.

It can be shown that the reliability function, $R(t)$, is increasing for $t > \beta$, decreasing for $\beta > t$ and is equal to 0.5 when $t = \beta$ for all α. Furthermore, the mean is an increasing function of α for fixed β, so selecting the populations with the largest parameters α would correspond to selecting the populations with the longest mean time until failure.

Section 4.2 Theory for Selection of α

Given k $(k \geq 2)$ independent Birnbaum-Saunders Distributions, (BSD), $\pi_1, \pi_2, \ldots, \pi_k$. Let $\pi_{(i)}$ denote the population having the *i*th shape parameter $\alpha_{[i]}$, where $\alpha_{[1]} \leq \alpha_{[2]} \leq \ldots \leq \alpha_{[k-t+1]} \leq \alpha_{[k]}$. The population $\pi_{(i)}$ is defined to be better than $\pi_{(j)}$ if $i > j$. Selecting the *t*-best populations with the *t* largest α parameters, $1 \leq t < k$ is considered. The goal is to select a group of the *t*-best ($1 \leq t < k$) populations in an unordered manner. The choice of any *t* populations having the *t* largest parameters is regarded as a correct selection, (CS).

Let $\hat{\alpha}_{(i)}$ denote the maximum likelihood estimator (MLE) of $\alpha_{[i]}$ associated with population $\pi_{(i)}$. The maximum likelihood estimator is computed by the formula,

$\hat{\alpha} = \left(\frac{\overline{T}}{\hat{\beta}} + \frac{\hat{\beta}}{H^{-1}} - 2 \right)^{\frac{1}{2}}$, where $\hat{\beta}$, $\overline{T}$ and H are defined as in Section 2.2 . Define

$W_j = \frac{\hat{\alpha}_{(j)}}{\alpha_{[j]}}$. Due to Engelhardt et al. (1981), the distribution of W_j does not depend on α or β. Let $\vec{\alpha} = (\alpha_{[1]}, \ldots, \alpha_{[k]})$ denote a point in the parameter space Ω that is partitioned into a 'preference zone', $\Omega(\delta^*)$, defined by $\Omega(\delta^*) = \left\{ \vec{\alpha} : \frac{\alpha_{[k-t]}}{\alpha_{[k-t+1]}} \leq \delta^* < 1 \right\}$.

The complement of $\Omega(\delta^*)$ is called the indifference zone. Now consider the following rule, R, for which the probability of correct selection, $P(CS \mid R)$, satisfies $P(CS \mid R) \geq p^*$ for all $\vec{\alpha} \in \Omega(\delta^*)$.

Rule R2 : Select the populations associated with the t largest $\hat{\alpha}$ as the t-best populations.

The experimenter specifies in advance the constants δ^* and p^* where $\binom{k}{t}^{-1} < p^* < 1$.

The probability of correct selection, $P\,(CS \mid R\,2)$ is as follows :

$$P(CS \mid R2) = P\left[\max_{1\le j\le k-t} \hat{\alpha}_{(j)} \le \min_{k-t+1\le i\le k} \hat{\alpha}_{(i)}\right] \tag{4.2.1}$$

$$= P\left[\max_{\substack{1\le j\le k-t \\ j\ne g}} \hat{\alpha}_{(j)} \le \hat{\alpha}_{(g)} \le \min_{k-t+1\le i\le k} \hat{\alpha}_{(i)}; g = 1,\ldots,k-t\right] \tag{4.2.2}$$

$$= P\left[\max_{\substack{1\le j\le k-t \\ j\ne g}} \frac{\hat{\alpha}_{(j)}}{\alpha_{[j]}}\frac{\alpha_{[j]}}{\alpha_{[g]}} \le \frac{\hat{\alpha}_{(g)}}{\alpha_{[g]}} \le \min_{k-t+1\le i\le k} \frac{\hat{\alpha}_{(i)}}{\alpha_{[i]}}\frac{\alpha_{[i]}}{\alpha_{[g]}}; g = 1,\ldots,k-t\right] \tag{4.2.3}$$

$$= P\left[\max_{\substack{1\le j\le k-t \\ j\ne g}} W_j \frac{\alpha_{[j]}}{\alpha_{[g]}} \le W_g \le \min_{k-t+1\le i\le k} W_i \frac{\alpha_{[i]}}{\alpha_{[g]}}; g = 1,\ldots,k-t\right]. \tag{4.2.4}$$

Interest is in finding the configuration of the parameters that minimizes $P(CS \mid R)$. This configuration of parameters is called the Least Favorable Configuration (LFC). Under the least favorable conditions $\alpha_{[1]} = \alpha_{[2]} = \cdots = \alpha_{[k-t]}$, $\alpha_{[k-t+1]} = \cdots = \alpha_{[k]}$ and

$\alpha_{[k-t]} \le \alpha_{[k-t+1]}(\delta^*)$ which implies that $\dfrac{\alpha_{[k-t+1]}}{\alpha_{[k-t]}} \ge \dfrac{1}{\delta^*} \ge 1$.

Thus, $$P(CS \mid R) \ge P\left[\max_{\substack{1\le j\le k-t \\ j\ne g}} W_j(1) \le W_g \le \min_{k-t+1\le i\le k} W_i\left(\frac{1}{\delta^*}\right); g = 1,\ldots,k-t\right] \tag{4.2.5}$$

$$= \sum_{g=1}^{k-t}\int_0^\infty \left[\prod_{\substack{j=1 \\ j\ne g}}^{k-t} G_w(w) \prod_{i=k-t+1}^{k}\left(1 - G_w\left(w\delta^*\right)\right)\right] dG_w(w) \tag{4.2.6}$$

$$= \sum_{g=1}^{k-t} \int_0^{\infty} G_w^{(k-t-1)}(w)\left[1 - G_w(w\delta^*)\right]^{(t)} dG_w(w) \quad (4.2.7)$$

$$= (k-t)\int_0^{\infty} G_w^{(k-t-1)}(w)\left[1 - G_w(w\delta^*)\right]^{(t)} dG_w(w) = P^*(CS \mid R2), \quad (4.2.8)$$

where $G_w(w)$ is defined to be the cumulative distribution function of the $W_j = \dfrac{\hat{\alpha}_{(j)}}{\alpha_{[j]}}$.

So,
$$P^*(CS \mid R2) = P\left[\max_{1 \le j \le k-t} W_j \le \frac{1}{\delta^*} \min_{k-t+1 \le i \le k} W_i\right]. \quad (4.2.9)$$

Now given *k, t,* δ^*, and p^* - values, the solution can be obtained by setting $P^*(CS \mid R2)$ equal to p^* and solving for *n*. Since the distribution of W_i cannot be obtained in closed form, the $P^*(CS \mid R2)$ has been simulated and approximated and large sample approximations will be discussed in Chapter 5.

Section 4.3 Simulations

Fortran programs were written using Monte Carlo methods to simulate probability tables for $\hat{\alpha}$ and for β unknown, *k*=2 (1) 5, *t*=1 (1) (*k-1*), and *n*=5 (5) 30. Furthermore, instead of using $\hat{\beta}$ to estimate $\hat{\alpha}$, the estimator $\tilde{\beta}$ has been used in place of $\hat{\beta}$ to compute the estimates for alpha. For all of the reasons mentioned in Chapter 2, the estimator is a reasonable choice to use and was also used previously by Desmond (1995) in his prediction intervals. The estimator from now on will be referred to as $\tilde{\alpha}$. These tables are located in Appendix D. These probability tables were constructed for selecting *t* populations out of k populations with the largest parameters, α. But, these tables can also be used to select the t populations with the smallest parameters, α, since selecting the t populations with the smallest parameters, α, is equivalent to selecting the k-t populations with the largest parameters α. For example, if the experimenter wants to select the two populations out of five with the smallest parameters, α, then the researcher would use the table in Appendix D for selecting the three populations with the largest parameters α out of five populations.

Chapter 5

Asymptotic Results of the Estimator for the Parameter α

Section 5.1 Results for the Estimator $\hat{\alpha}$

From Chapter 4, Section 2, the probability of correct selection, $P^*(CS \mid R2)$, was obtained but a closed form of the distribution of W_i does not exist. Therefore, the probability of correct selection, $P^*(CS \mid R2)$, must be simulated or approximated. Simulations were discussed in Chapter 4 and the tables appear in Appendix F. Large sample approximations for the parameter, α, will now be considered.

Before proceeding with the selection procedure, it is useful to note the following asymptotic results concerning the maximum likelihood estimator, $\hat{\alpha}$.

Theorem 5.1.1 : (Engelhardt et al., 1981) For n sufficiently large,

$$\hat{\alpha} \mathrel{\dot{\sim}} \mathrm{N}\left(\alpha, 2^{-1}\alpha^2 / n\right).$$

Theorem 5.1.2 : For n sufficiently large, $\dfrac{\hat{\alpha}}{\alpha} \mathrel{\dot{\sim}} \mathrm{N}\left(1, \dfrac{1}{2n}\right)$.

Proof: Suppose $\hat{\alpha} \mathrel{\dot{\sim}} \mathrm{N}\left(\alpha, 2^{-1}\alpha^2 / n\right)$.

Let $X = \hat{\alpha}$ and $Y = \dfrac{\hat{\alpha}}{\alpha} = \dfrac{X}{\alpha}$ then

$$P(Y \le y) = P\left(\frac{X}{\alpha} \le y\right) = P(X \le \alpha y) \tag{5.1.1}$$

$$= P\left(Z \le \frac{\alpha y - \alpha}{\sqrt{2^{-1}\alpha^2 / n}}\right) \tag{5.1.2}$$

$$= P\left(Z \le \frac{\alpha(y-1)}{\alpha\sqrt{1/2n}}\right) \tag{5.1.3}$$

$$= P\left(Z \le \frac{(y-1)}{\sqrt{1/2n}}\right). \tag{5.1.4}$$

Therefore, $\dfrac{\hat{\alpha}}{\alpha} \dot{\sim} \mathrm{N}\left(1, \dfrac{1}{2n}\right)$ as desired.

Section 5.2 Probability of Correct Selection for Normal Approximation of $\hat{\alpha}$

The probability of correct selection must be examined since the distribution of $\dfrac{\hat{\alpha}}{\alpha}$ is now being approximated by a normal distribution where $\dfrac{\hat{\alpha}}{\alpha} \dot{\sim} \mathrm{N}\left(1, \dfrac{1}{2n}\right)$. Therefore, from Equation 4.2.9 of Section 4.2 of Chapter 4, the probability of correct selection given the rule R, $P(CS \mid R2)$, is :

$$P(CS \mid R2) = P\left[\max_{1 \le j \le k-t} W_j \le \frac{1}{\delta^*} \min_{k-t+1 \le i \le k} W_i\right] \tag{5.2.1}$$

$$= P\left[\delta^* \max_{1\le j\le k-t} W_j \le \min_{k-t+1\le i\le k} W_i\right] \quad (5.2.2)$$

$$= P\left[\max_{1\le j\le k-t} \delta^* W_j - 1 \le \min_{k-t+1\le i\le k} W_i - 1\right] \quad (5.2.3)$$

$$= P\left[\max_{1\le j\le k-t} \frac{\delta^* W_j - 1}{\sqrt{\frac{1}{2n}}} \le \min_{k-t+1\le i\le k} \frac{W_i - 1}{\sqrt{\frac{1}{2n}}}\right] \quad (5.2.4)$$

$$= P\left[\max_{1\le j\le k-t} \frac{\delta^* (W_j - 1)}{\sqrt{\frac{1}{2n}}} + \frac{\delta^* - 1}{\sqrt{\frac{1}{2n}}} \le \min_{k-t+1\le i\le k} \frac{W_i - 1}{\sqrt{\frac{1}{2n}}}\right] \quad (5.2.5)$$

$$= P\left[\max_{1\le j\le k-t} \delta^* Z_j + \frac{\delta^* - 1}{\sqrt{\frac{1}{2n}}} \le \min_{k-t+1\le i\le k} Z_i\right] \quad (5.2.6)$$

$$\doteq \sum_{g=1}^{k-t} \int_{-\infty}^{\infty} \prod_{\substack{i=1 \\ g\ne j}}^{k-t} \Phi(Z_i) \prod_{i=k-t+1}^{k} \left[1 - \Phi\left(\delta^* Z_j + \frac{\delta^* - 1}{\sqrt{\frac{1}{2n}}}\right)\right] d\Phi(z) \quad (5.2.7)$$

$$= (k-t)\int_{-\infty}^{\infty} [\Phi(Z_i)]^{(k-t-1)} \left[1 - \Phi\left(\delta^* Z_j + \frac{\delta^* - 1}{\sqrt{\frac{1}{2n}}}\right)\right]^t d\Phi(z) = P^*(CS \mid R2) \quad (5.2.8)$$

where $\Phi(z)$ is defined to be the cumulative distribution function of the standard normal distribution. Now given *k, t,* δ^*, and p^* - values, the solution can be obtained by setting $P^*(CS \mid R2)$ equal to p^* and solving for *n*.

From Section 2.5 of Chapter 2, Property 2.5.3 states that as $n \to \infty$, then

$P^{*}(CS \mid R2) \to 1$. From Equation 3.2.8, as $n \to \infty$, $\dfrac{\delta^{*}-1}{\sqrt{\dfrac{1}{2n}}} \to -\infty$, since $\delta^{*} < 1$.

Therefore, $P^{*}(CS \mid R2) \to (k-t)\int_{-\infty}^{\infty} [\Phi(Z_i)]^{(k-t-1)}(1)^{t}\, d\Phi(z) \to 1$ as $n \to \infty$. Also, Property 2.5.1 and Property 2.5.2 from Section 2.5 of Chapter 2 hold for α as they do for β.

Section 5.3 Large Sample Approximations

Fortran programs were written using Monte Carlo methods to calculate probability tables using the estimator, $\hat{\alpha}$, for k = 2 (1) 5, t = 1 (1) (k-1), and $n = 30,40,50,75$. The tables are located in Appendix G.

Section 5.4 Comparisons of the Simulations and Approximations

A comparison of the estimator, $\hat{\alpha}$, for simulated probabilities and the approximations is given next for the probability of correct selection, $P^{*}(CS \mid R2)$, for the following specified values : k=5, t=2, n=30 and $\delta^{*} = 0.400, 0.500, 0.650, 0.750$.

$P^{*}(CS \mid R2)$

	$\delta^{*} = 0.400$	$\delta^{*} = 0.500$	$\delta^{*} = 0.650$	$\delta^{*} = 0.750$
simulated $\hat{\alpha}$	1.00000	0.99902	0.94616	0.75954
approximated $\hat{\alpha}$	0.99996	0.99870	0.94894	0.77323

Chapter 6

Conclusions and Future Work

Section 6.1 Conclusions

In this dissertation, two main goals were accomplished. The first goal was ranking and selection of Birnbaum-Saunders populations according to the scale parameter, β. The indifference zone approach was used as the selection criteria for the 'best' populations due to Bechhofer (1954). Using this procedure, a probability of correct selection, $P(CS \mid R)$, was obtained and Monte Carlo simulations and large sample approximations tables were computed using Fortran 77 programs. These tables can be used to determine the size that would be needed from each population to sample to ensure a correct selection at a certain probability level.

The second goal was ranking and selection of Birnbaum-Saunders populations according to the shape parameter, α. The indifference zone approach again was used. Also, a statement regarding the probability of correct selection was obtained where Monte Carlo simulations and large sample approximations tables were computed using Fortran 77 programs. Again, tables for the determination of the smallest sample size needed from each of the populations to ensure a correct selection of a certain probability were obtained.

During the course of this dissertation, the author has found that the mean – mean estimator, $\widetilde{\beta}$, is preferred over the maximum likelihood estimator, $\hat{\beta}$, because of the ease in computation. Also, the simulated and approximated probabilities for the estimator are higher than those obtained from the maximum likelihood estimator.

Section 6.2 Future Work

There are many topics that can be explored further. First, with the Birnbaum-Saunders distribution is to consider ranking and selection procedures with censored samples using Type-II censored samples estimators introduced by McCarter (1999). Gupta (1965) considered subset selection procedures that can possibly be applied to develop procedures for the Birnbaum-Saunders distribution and procedures from Tong (1969) on comparisons with a control can also be explored.

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Appendices

Appendix A **Probability Tables for Estimator** $\hat{\beta}$

Appendix B **Probability Tables for Estimator** β'

Appendix C **Probability Tables for Estimator** $\tilde{\beta}$

Appendix D **Probability Tables for Estimator** $\tilde{\alpha}$

Appendix E **Probability Tables for Estimator** $\hat{\beta}$ **Large Samples**

Appendix F **Probability Tables for Estimator** $\tilde{\beta}$ **Large Samples**

Appendix G **Probability Tables for Estimator** $\hat{\alpha}$ **Large Samples**

Appendix H **Fortran Programs for Simulations**

Appendix A Probability Tables for Estimator $\hat{\beta}$

Table 1. A

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 0.15$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
2	1	5		.925	.885	.855	.830	.805	.785
		10		.945	.915	.895	.875	.855	.840
		15		.955	.930	.915	.900	.880	.865
		20		.960	.940	.925	.910	.900	.885
		25		.965	.945	.930	.920	.910	.895
		30		.965	.950	.935	.925	.915	.905

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	1	5		.880	.850	.820	.800	.775	.755
		10		.915	.890	.870	.855	.835	.820
		15		.930	.910	.895	.880	.865	.850
		20		.940	.920	.905	.895	.880	.870
		25		.945	.930	.915	.905	.890	.885
		30		.950	.935	.920	.910	.900	.890

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	1	5		.895	.860	.830	.810	.785	.770
		10		.925	.900	.880	.860	.845	.830
		15		.940	.915	.900	.885	.870	.860
		20		.945	.930	.915	.900	.890	.875
		25		.950	.935	.920	.910	.895	.885
		30		.955	.940	.925	.915	.905	.895

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	2	5		.870	.835	.810	.790	.770	.750
		10		.905	.885	.865	.850	.830	.820
		15		.920	.905	.885	.875	.860	.850
		20		.930	.915	.900	.890	.875	.865
		25		.940	.920	.910	.900	.890	.880
		30		.940	.930	.920	.910	.900	.885

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	2	5		.895	.860	.830	.810	.785	.765
		10		.925	.900	.880	.860	.840	.830
		15		.940	.915	.900	.885	.870	.860
		20		.945	.925	.915	.900	.890	.875
		25		.950	.935	.920	.910	.900	.885
		30		.955	.940	.925	.915	.900	.895

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	3	5		.800	.845	.820	.800	.775	.760
		10		.915	.890	.870	.855	.840	.825
		15		.930	.910	.890	.880	.870	.850
		20		.940	.920	.905	.895	.880	.870
		25		.945	.930	.915	.905	.890	.880
		30		.950	.935	.920	.910	.900	.890

Table 1. A
(continued)

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.15$

p^*

k	t	n	.80	.90	.95	.975	.99	.995
5	1	5	.870	.840	.815	.790	.770	.755
		10	.905	.885	.865	.850	.830	.815
		15	.925	.905	.890	.875	.860	.845
		20	.935	.915	.900	.890	.880	.865
		25	.940	.925	.910	.900	.890	.880
		30	.945	.930	.920	.910	.900	.885

p^*

k	t	n	.80	.90	.95	.975	.99	.995
5	3	5	.855	.825	.800	.785	.760	.745
		10	.895	.875	.855	.840	.825	.810
		15	.915	.895	.880	.870	.850	.845
		20	.925	.910	.895	.885	.870	.860
		25	.930	.915	.905	.895	.885	.875
		30	.935	.925	.910	.900	.895	.885

p^*

k	t	n	.80	.90	.95	.975	.99	.995
5	2	5	.855	.825	.800	.780	.760	.745
		10	.895	.875	.855	.840	.820	.810
		15	.915	.895	.880	.870	.855	.845
		20	.925	.910	.895	.885	.870	.860
		25	.930	.915	.905	.895	.880	.875
		30	.935	.925	.910	.900	.895	.885

p^*

k	t	n	.80	.90	.95	.975	.99	.995
5	4	5	.870	.840	.815	.795	.770	.750
		10	.905	.885	.865	.850	.830	.815
		15	.925	.905	.890	.875	.860	.850
		20	.935	.915	.900	.890	.875	.865
		25	.940	.925	.910	.900	.890	.880
		30	.945	.930	.920	.910	.900	.890

Table 2. A

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ **given values of k, t, δ , p^* and** $\alpha = 0.25$

k	t	n		.80	.90	.95	.975	.99	.995
2	1	5		.875	.820	.770	.735	.690	.665
		10		.910	.865	.830	.805	.770	.745
		15		.920	.890	.860	.835	.805	.790
		20		.935	.905	.880	.855	.830	.815
		25		.940	.910	.885	.865	.845	.825
		30		.940	.915	.890	.870	.850	.835

k	t	n		.80	.90	.95	.975	.99	.995
4	1	5		.810	.760	.725	.690	.650	.630
		10		.860	.825	.790	.765	.740	.720
		15		.885	.855	.830	.805	.785	.765
		20		.900	.870	.850	.830	.810	.790
		25		.905	.880	.860	.845	.820	.810
		30		.910	.885	.865	.845	.830	.815

k	t	n		.80	.90	.95	.975	.99	.995
3	1	5		.830	.780	.740	.705	.670	.645
		10		.880	.840	.805	.785	.760	.735
		15		.900	.865	.840	.815	.790	.775
		20		.910	.880	.855	.840	.815	.800
		25		.920	.890	.870	.850	.825	.815
		30		.920	.895	.875	.855	.840	.820

k	t	n		.80	.90	.95	.975	.99	.995
4	2	5		.790	.745	.710	.680	.640	.620
		10		.850	.810	.785	.765	.735	.715
		15		.875	.845	.820	.800	.780	.760
		20		.890	.860	.840	.820	.800	.785
		25		.900	.875	.855	.835	.820	.800
		30		.900	.880	.860	.840	.820	.810

k	t	n		.80	.90	.95	.975	.99	.995
3	2	5		.830	.780	.740	.700	.670	.640
		10		.880	.840	.805	.780	.750	.730
		15		.900	.865	.840	.815	.795	.775
		20		.910	.880	.860	.840	.815	.800
		25		.920	.890	.870	.850	.830	.815
		30		.920	.895	.875	.855	.835	.820

k	t	n		.80	.90	.95	.975	.99	.995
4	3	5		.810	.760	.720	.685	.650	.630
		10		.860	.825	.790	.770	.745	.725
		15		.885	.855	.830	.810	.785	.765
		20		.900	.870	.850	.825	.805	.790
		25		.905	.880	.860	.840	.820	.810
		30		.910	.885	.865	.850	.830	.815

Table 2. A
(continued)

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ **given values of k, t, δ , p^* and $\alpha = 0.25$**

				p^*					
k	t	n		.80	.90	.95	.975	.99	.995
5	1	5		.790	.745	.710	.680	.650	.625
		10		.850	.810	.785	.765	.730	.715
		15		.875	.845	.820	.800	.775	.760
		20		.890	.865	.840	.825	.800	.785
		25		.905	.880	.860	.840	.825	.805
		30		.905	.880	.860	.840	.825	.805

				p^*					
k	t	n		.80	.90	.95	.975	.99	.995
5	3	5		.770	.725	.690	.665	.630	.610
		10		.830	.800	.770	.750	.725	.705
		15		.860	.830	.810	.790	.770	.750
		20		.875	.850	.830	.815	.795	.775
		25		.885	.865	.845	.830	.810	.795
		30		.890	.870	.850	.835	.815	.805

				p^*					
k	t	n		.80	.90	.95	.975	.99	.995
5	2	5		.770	.725	.690	.665	.610	.610
		10		.830	.800	.770	.750	.725	.705
		15		.860	.830	.810	.790	.770	.750
		20		.875	.850	.830	.815	.795	.780
		25		.885	.865	.845	.830	.810	.795
		30		.890	.870	.850	.835	.820	.800

				p^*					
k	t	n		.80	.90	.95	.975	.99	.995
5	4	5		.795	.750	.710	.685	.650	.625
		10		.850	.815	.785	.765	.730	.710
		15		.875	.845	.820	.800	.775	.760
		20		.890	.865	.840	.825	.800	.785
		25		.900	.875	.855	.835	.815	.805
		30		.905	.880	.860	.845	.825	.815

Table 3. A

For MaximumLlikelihood Estimator Selecting the t-best : Complete Sample Case
Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.50$

k	t	n		.80	.90	.95	.975	.99	.995
2	1	5		.770	.670	.600	.545	.490	.450
		10		.830	.755	.695	.655	.605	.570
		15		.860	.795	.750	.705	.660	.630
		20		.870	.810	.760	.725	.680	.650
		25		.875	.825	.780	.735	.700	.675
		30		.890	.840	.795	.765	.720	.695

k	t	n		.80	.90	.95	.975	.99	.995
4	1	5		.660	.585	.530	.485	.440	.400
		10		.750	.690	.640	.600	.560	.530
		15		.735	.730	.690	.655	.620	.590
		20		.800	.750	.710	.675	.640	.610
		25		.815	.780	.725	.695	.660	.610
		30		.835	.790	.755	.725	.690	.610

k	t	n		.80	.90	.95	.975	.99	.995
3	1	5		.695	.610	.550	.500	.450	.420
		10		.775	.710	.660	.615	.575	.545
		15		.810	.755	.710	.675	.635	.605
		20		.825	.770	.725	.690	.650	.625
		25		.840	.785	.760	.725	.675	.645
		30		.855	.810	.770	.740	.700	.680

k	t	n		.80	.90	.95	.975	.99	.995
4	2	5		.630	.565	.510	.465	.430	.400
		10		.725	.665	.625	.585	.550	.520
		15		.765	.715	.675	.645	.610	.585
		20		.780	.730	.695	.660	.630	.600
		25		.800	.750	.705	.695	.655	.620
		30		.815	.775	.740	.710	.680	.660

k	t	n		.80	.90	.95	.975	.99	.995
3	2	5		.695	.610	.550	.505	.450	.420
		10		.775	.710	.660	.615	.575	.545
		15		.810	.755	.710	.675	.635	.605
		20		.825	.770	.725	.690	.650	.625
		25		.835	.790	.740	.715	.675	.660
		30		.850	.805	.765	.735	.700	.675

k	t	n		.80	.90	.95	.975	.99	.995
4	3	5		.660	.580	.525	.480	.435	.405
		10		.750	.690	.640	.605	.560	.535
		15		.785	.730	.690	.655	.620	.590
		20		.800	.750	.710	.675	.635	.610
		25		.810	.770	.735	.685	.655	.640
		30		.830	.785	.750	.720	.690	.670

Table 3. A
(continued)

For MaximumLlikelihood Estimator Selecting the t-best : Complete Sample Case
Finding the smallest n required for $P(CS \mid R) \geq p^*$ ***given values of k, t,*** δ , p^* ***and*** $\alpha = 0.50$

k	t	n		.80	.90	.95	.975	.99	.995
5	1	5		.635	.565	.510	.470	.425	.400
		10		.730	.670	.620	.590	.550	.520
		15		.770	.720	.680	.645	.605	.580
		20		.785	.73	.695	.665	.625	.600
		25		.800	.765	.720	.680	.640	.625
		30		.820	.780	.745	.715	.680	.660

k	t	n		.80	.90	.95	.975	.99	.995
5	3	5		.600	.535	.490	.450	.410	.390
		10		.700	.645	.605	.570	.540	.510
		15		.745	.695	.660	.630	.595	.570
		20		.760	.715	.680	.650	.615	.590
		25		.785	.740	.725	.670	.640	.625
		30		.800	.760	.725	.700	.670	.645

k	t	n		.80	.90	.95	.975	.99	.995
5	2	5		.600	.530	.485	.450	.410	.390
		10		.700	.645	.605	.570	.535	.510
		15		.745	.695	.660	.630	.595	.570
		20		.760	.715	.680	.645	.610	.590
		25		.780	.745	.700	.660	.635	.605
		30		.800	.760	.725	.700	.670	.645

k	t	n		.80	.90	.95	.975	.99	.995
5	4	5		.640	.570	.515	.470	.430	.400
		10		.730	.670	.625	.590	.545	.520
		15		.770	.720	.680	.645	.610	.585
		20		.785	.740	.700	.665	.630	.605
		25		.795	.755	.720	.690	.645	.620
		30		.815	.775	.740	.710	.680	.660

Table 4. A

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.75$

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
2	1	5	.685	.560	.475	.415	.350	.315
		10	.760	.670	.595	.540	.480	.450
		15	.785	.695	.630	.575	.515	.485
		20	.800	.710	.650	.600	.540	.500
		25	.840	.760	.710	.665	.620	.585
		30	.850	.780	.730	.690	.640	.615

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	1	5	.545	.455	.395	.345	.295	.265
		10	.660	.580	.520	.475	.430	.400
		15	.680	.610	.560	.515	.470	.445
		20	.705	.630	.580	.540	.490	.465
		25	.760	.700	.655	.620	.580	.550
		30	.775	.720	.675	.640	.600	.580

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
3	1	5	.590	.490	.420	.370	.310	.290
		10	.690	.610	.550	.500	.450	.420
		15	.715	.640	.580	.540	.485	.455
		20	.735	.660	.600	.560	.510	.480
		25	.785	.720	.675	.635	.590	.570
		30	.800	.740	.695	.660	.615	.590

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	2	5	.510	.430	.375	.330	.285	.260
		10	.630	.560	.505	.465	.420	.390
		15	.650	.590	.540	.500	.460	.430
		20	.675	.610	.560	.525	.480	.455
		25	.735	.680	.640	.605	.565	.540
		30	.755	.700	.660	.630	.590	.565

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
3	2	5	.590	.490	.420	.370	.315	.285
		10	.690	.610	.545	.500	.450	.415
		15	.710	.635	.580	.535	.480	.455
		20	.735	.660	.600	.560	.510	.480
		25	.790	.725	.670	.635	.590	.560
		30	.800	.740	.695	.660	.615	.590

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	3	5	.545	.455	.390	.345	.295	.270
		10	.660	.585	.525	.480	.435	.400
		15	.680	.610	.555	.515	.470	.440
		20	.700	.630	.580	.540	.490	.465
		25	.760	.700	.650	.615	.570	.545
		30	.775	.720	.675	.640	.605	.580

Table 4. A
(continued)

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 0.75$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	1	5		.515	.430	.375	.330	.290	.260
		10		.630	.560	.505	.465	.420	.390
		15		.660	.595	.540	.505	.460	.430
		20		.680	.615	.565	.525	.480	.455
		25		.740	.685	.640	.605	.560	.535
		30		.755	.700	.660	.630	.590	.565

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	3	5		.470	.400	.350	.310	.270	.250
		10		.595	.530	.480	.445	.400	.375
		15		.625	.565	.520	.485	.445	.420
		20		.645	.585	.540	.505	.465	.440
		25		.710	.660	.620	.590	.550	.530
		30		.730	.680	.645	.615	.575	.550

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	2	5		.470	.400	.350	.310	.270	.250
		10		.595	.530	.480	.445	.400	.375
		15		.625	.565	.520	.485	.440	.420
		20		.645	.590	.540	.505	.465	.440
		25		.710	.660	.620	.590	.550	.525
		30		.730	.680	.645	.615	.575	.550

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	4	5		.520	.440	.380	.335	.290	.260
		10		.630	.560	.505	.465	.420	.390
		15		.660	.590	.540	.505	.460	.435
		20		.680	.615	.565	.525	.480	.450
		25		.740	.680	.640	.605	.565	.540
		30		.760	.710	.665	.630	.590	.570

Table 5. A

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 1$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
2	1	5		.610	.470	.380	.320	.260	.220
		10		.720	.600	.520	.450	.390	.360
		15		.750	.650	.570	.510	.450	.410
		20		.780	.680	.610	.560	.510	.470
		25		.800	.710	.640	.590	.530	.490
		30		.810	.730	.670	.620	.570	.530

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	1	5		.460	.360	.300	.260	.210	.180
		10		.590	.500	.430	.390	.350	.310
		15		.630	.550	.500	.450	.400	.370
		20		.670	.600	.540	.490	.450	.410
		25		.700	.630	.570	.530	.490	.460
		30		.710	.650	.600	.560	.520	.480

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	1	5		.500	.400	.330	.270	.220	.200
		10		.630	.530	.460	.430	.360	.320
		15		.670	.580	.520	.470	.420	.380
		20		.710	.630	.570	.510	.470	.430
		25		.730	.660	.600	.550	.510	.470
		30		.750	.680	.620	.580	.530	.500

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	2	5		.420	.340	.280	.240	.200	.170
		10		.560	.480	.420	.380	.340	.300
		15		.600	.530	.470	.430	.390	.350
		20		.640	.570	.520	.480	.430	.390
		25		.670	.610	.560	.520	.470	.450
		30		.690	.630	.580	.540	.510	.480

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	2	5		.500	.400	.330	.270	.230	.200
		10		.630	.530	.460	.410	.360	.330
		15		.670	.580	.510	.470	.430	.380
		20		.710	.630	.560	.520	.470	.430
		25		.740	.660	.600	.550	.510	.480
		30		.750	.680	.630	.580	.530	.500

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	3	5		.460	.360	.300	.250	.210	.180
		10		.590	.500	.440	.390	.340	.320
		15		.630	.550	.490	.440	.400	.370
		20		.670	.600	.540	.490	.440	.410
		25		.700	.630	.570	.530	.490	.460
		30		.720	.650	.600	.560	.520	.490

Table 5. A
(continued)

For $\hat{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 1$

k	t	n	.80	.90	.95	.975	.99	.995
5	1	5	.420	.340	.280	.240	.200	.180
		10	.560	.480	.420	.375	.330	.300
		15	.610	.530	.475	.430	.390	.360
		20	.650	.580	.530	.485	.430	.410
		25	.675	.610	.560	.520	.480	.450
		30	.695	.630	.585	.545	.510	.480

k	t	n	.80	.90	.95	.975	.99	.995
5	3	5	.380	.310	.260	.220	.190	.165
		10	.515	.445	.390	.355	.320	.280
		15	.560	.500	.450	.410	.370	.345
		20	.610	.550	.500	.460	.420	.390
		25	.640	.580	.535	.500	.470	.435
		30	.665	.610	.570	.530	.490	.455

k	t	n	.80	.90	.95	.975	.99	.995
5	2	5	.380	.305	.255	.220	.185	.165
		10	.515	.445	.395	.355	.310	.285
		15	.570	.500	.450	.415	.370	.345
		20	.610	.550	.500	.460	.420	.390
		25	.640	.580	.540	.500	.470	.435
		30	.660	.610	.570	.530	.490	.460

k	t	n	.80	.90	.95	.975	.99	.995
5	4	5	.430	.350	.290	.245	.205	.180
		10	.560	.480	.420	.375	.340	.305
		15	.600	.530	.470	.435	.390	.355
		20	.650	.580	.520	.480	.435	.400
		25	.680	.610	.560	.520	.480	.455
		30	.700	.640	.590	.550	.510	.475

Appendix B Probability Tables for Estimator β'

Table1.B

For Estimator β' Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1$

K	t	n		p^* .80	.90	.95	.975	.99	.995
2	1	5		.615	.475	.390	.325	.260	.220
		10		.710	.595	.510	.450	.390	.355
		15		.755	.655	.580	.520	.455	.425
		20		.790	.695	.625	.575	.520	.485
		25		.805	.720	.655	.605	.550	.510
		30		.810	.745	.675	.635	.575	.550

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	1	5		.460	.370	.300	.260	.215	.185
		10		.580	.495	.430	.385	.355	.320
		15		.640	.565	.505	.460	.415	.380
		20		.685	.610	.555	.515	.470	.435
		25		.710	.640	.590	.555	.510	.475
		30		.730	.665	.620	.580	.535	.510

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	1	5		.510	.405	.330	.260	.220	.210
		10		.620	.530	.455	.405	.355	.330
		15		.680	.595	.530	.480	.430	.400
		20		.720	.640	.580	.535	.490	.450
		25		.745	.670	.615	.570	.520	.490
		30		.760	.690	.635	.595	.550	.520

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	2	5		.425	.340	.280	.240	.205	.185
		10		.550	.470	.415	.370	.340	.310
		15		.610	.540	.485	.445	.395	.375
		20		.655	.585	.535	.495	.450	.420
		25		.685	.620	.575	.535	.495	.465
		30		.705	.650	.600	.565	.530	.495

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	2	5		.510	.405	.330	.260	.220	.210
		10		.620	.530	.455	.405	.355	.330
		15		.680	.595	.530	.480	.430	.400
		20		.720	.640	.580	.535	.490	.450
		25		.745	.670	.615	.570	.520	.490
		30		.760	.690	.635	.595	.550	.520

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	3	5		.460	.365	.300	.250	.220	.185
		10		.585	.500	.435	.390	.345	.320
		15		.640	.560	.505	.460	.415	.380
		20		.680	.610	.555	.510	.460	.430
		25		.710	.640	.590	.545	.495	.475
		30		.730	.660	.615	.580	.535	.510

Table1.B
(continued)

For Estimator β' Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k,t, δ , p^* and $\alpha = 1$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	1	5		.430	.345	.285	.240	.205	.195
		10		.555	.470	.415	.370	.345	.310
		15		.620	.545	.490	.445	.405	.375
		20		.660	.595	.540	.495	.455	.420
		25		.695	.625	.575	.535	.495	.460
		30		.715	.655	.605	.575	.520	.495

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	3	5		.385	.310	.260	.220	.195	.170
		10		.510	.440	.390	.355	.330	.290
		15		.575	.515	.465	.425	.390	.355
		20		.620	.560	.515	.475	.435	.410
		25		.655	.600	.555	.515	.475	.450
		30		.680	.625	.580	.545	.510	.480

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	2	5		.385	.305	.260	.220	.185	.165
		10		.510	.440	.390	.360	.310	.285
		15		.575	.515	.465	.425	.375	.355
		20		.620	.560	.505	.480	.440	.410
		25		.655	.600	.550	.515	.475	.450
		30		.680	.625	.580	.550	.510	.460

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	4	5		.435	.350	.295	.245	.205	.190
		10		.555	.475	.415	.375	.345	.315
		15		.615	.540	.495	.445	.400	.375
		20		.655	.595	.535	.495	.455	.420
		25		.690	.620	.575	.535	.495	.470
		30		.715	.655	.605	.565	.530	.500

Appendix C Probability Tables for Estimator $\tilde{\beta}$

Table 1.C

For Estimator $\tilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k,t, δ , p^* and $\alpha = 0.15$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
2	1	5		.925	.885	.855	.830	.805	.785
		10		.945	.915	.890	.875	.855	.840
		15		.955	.930	.915	.900	.880	.870
		20		.960	.940	.925	.910	.895	.885
		25		.965	.945	.930	.920	.905	.895
		30		.970	.950	.940	.925	.910	.900

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	1	5		.880	.850	.820	.800	.775	.755
		10		.915	.890	.870	.850	.830	.820
		15		.930	.910	.895	.880	.865	.850
		20		.940	.920	.905	.895	.880	.870
		25		.945	.930	.915	.905	.890	.885
		30		.950	.935	.925	.915	.900	.895

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	1	5		.895	.860	.830	.810	.785	.770
		10		.925	.900	.880	.860	.840	.830
		15		.940	.920	.900	.885	.870	.860
		20		.945	.930	.910	.900	.885	.875
		25		.950	.935	.920	.910	.895	.890
		30		.955	.940	.930	.920	.905	.900

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	2	5		.870	.835	.810	.790	.770	.750
		10		.905	.885	.865	.850	.830	.820
		15		.920	.905	.885	.875	.860	.850
		20		.930	.915	.900	.890	.875	.865
		25		.940	.925	.910	.900	.890	.880
		30		.945	.930	.920	.910	.900	.890

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	2	5		.895	.860	.830	.810	.785	.770
		10		.925	.900	.880	.860	.840	.830
		15		.940	.920	.900	.885	.870	.860
		20		.945	.930	.910	.900	.885	.875
		25		.950	.935	.920	.910	.895	.890
		30		.955	.940	.930	.920	.905	.900

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	3	5		.880	.845	.820	.800	.775	.760
		10		.915	.890	.870	.855	.835	.820
		15		.930	.910	.890	.880	.865	.855
		20		.940	.920	.910	.895	.880	.870
		25		.945	.930	.915	.905	.890	.880
		30		.950	.935	.925	.915	.900	.895

Table 1.C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k,t, δ , p^* and $\alpha = 0.15$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	5		.870	.840	.810	.790	.770	.755
		10		.905	.885	.865	.850	.830	.815
		15		.925	.905	.890	.875	.860	.850
		20		.935	.915	.900	.890	.880	.865
		25		.940	.925	.910	.900	.890	.880
		30		.945	.930	.920	.910	.900	.890

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	5		.855	.825	.800	.780	.760	.740
		10		.895	.875	.855	.840	.820	.810
		15		.915	.895	.880	.870	.850	.845
		20		.925	.910	.895	.885	.870	.860
		25		.930	.920	.905	.895	.885	.875
		30		.945	.925	.915	.905	.895	.885

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	5		.855	.825	.800	.780	.760	.745
		10		.895	.875	.855	.840	.825	.810
		15		.915	.895	.880	.870	.855	.845
		20		.925	.910	.895	.885	.870	.860
		25		.930	.920	.905	.895	.885	.875
		30		.940	.925	.915	.905	.895	.885

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	5		.870	.840	.810	.795	.770	.750
		10		.905	.885	.865	.850	.830	.815
		15		.925	.905	.890	.875	.860	.850
		20		.930	.915	.900	.890	.880	.870
		25		.940	.925	.910	.900	.890	.880
		30		.945	.930	.920	.910	.900	.890

Table 2. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.25$

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
2	1	5	.875	.820	.770	.735	.695	.665
		10	.905	.865	.830	.805	.770	.750
		15	.925	.890	.860	.840	.810	.790
		20	.935	.905	.880	.860	.835	.820
		25	.940	.915	.890	.870	.850	.835
		30	.945	.920	.900	.880	.860	.850

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
3	1	5	.835	.780	.740	.705	.670	.645
		10	.880	.840	.810	.780	.750	.735
		15	.900	.865	.840	.820	.795	.775
		20	.915	.885	.860	.840	.820	.800
		25	.920	.895	.875	.855	.835	.820
		30	.930	.905	.885	.870	.850	.840

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
3	2	5	.830	.780	.740	.705	.670	.645
		10	.880	.840	.810	.780	.750	.730
		15	.900	.865	.840	.820	.790	.775
		20	.915	.885	.860	.840	.820	.800
		25	.920	.895	.875	.855	.835	.820
		30	.925	.905	.880	.870	.850	.835

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	1	5	.810	.760	.720	.690	.655	.630
		10	.860	.825	.790	.770	.740	.725
		15	.885	.855	.830	.810	.785	.770
		20	.900	.875	.850	.830	.810	.795
		25	.910	.885	.865	.850	.830	.815
		30	.915	.895	.875	.860	.840	.830

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	2	5	.790	.745	.710	.680	.645	.620
		10	.850	.815	.785	.760	.735	.720
		15	.875	.845	.820	.800	.780	.760
		20	.890	.865	.840	.825	.800	.790
		25	.900	.880	.860	.840	.820	.810
		30	.910	.885	.870	.855	.835	.825

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	3	5	.810	.760	.720	.690	.650	.630
		10	.865	.825	.795	.770	.740	.725
		15	.885	.855	.830	.810	.785	.765
		20	.900	.870	.850	.830	.810	.795
		25	.910	.885	.865	.850	.825	.815
		30	.915	.895	.875	.860	.840	.830

Table 2. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ , p^{*} and $\alpha = 0.25$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	5		.795	.750	.710	.680	.650	.625
		10		.850	.815	.785	.760	.730	.715
		15		.875	.845	.820	.800	.780	.760
		20		.895	.865	.840	.825	.805	.790
		25		.905	.880	.860	.840	.820	.810
		30		.910	.890	.870	.855	.835	.825

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	5		.770	.725	.695	.665	.635	.615
		10		.830	.800	.770	.750	.720	.705
		15		.860	.835	.810	.790	.770	.750
		20		.880	.855	.830	.815	.795	.780
		25		.890	.870	.850	.835	.815	.800
		30		.900	.880	.860	.850	.830	.815

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	5		.770	.730	.695	.665	.635	.615
		10		.830	.800	.770	.750	.720	.710
		15		.860	.830	.810	.790	.770	.755
		20		.880	.855	.835	.815	.795	.780
		25		.890	.870	.850	.835	.815	.800
		30		.900	.880	.860	.850	.830	.820

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	5		.795	.750	.710	.680	.650	.625
		10		.850	.815	.790	.760	.735	.715
		15		.875	.845	.820	.800	.780	.760
		20		.890	.865	.840	.825	.800	.790
		25		.905	.880	.860	.840	.825	.810
		30		.910	.890	.870	.855	.840	.825

Table 3. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.50$

k	t	n	p^* .80	.90	.95	.975	.99	.995
2	1	5	.770	.670	.605	.550	.490	.460
		10	.830	.755	.700	.650	.600	.570
		15	.860	.795	.745	.705	.660	.630
		20	.880	.820	.780	.745	.705	.680
		25	.890	.835	.795	.765	.725	.700
		30	.900	.850	.810	.780	.750	.725

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	1	5	.665	.590	.530	.490	.435	.410
		10	.750	.685	.635	.600	.560	.530
		15	.790	.735	.690	.660	.620	.600
		20	.815	.770	.730	.700	.665	.640
		25	.835	.790	.755	.730	.695	.670
		30	.845	.805	.770	.745	.715	.695

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	1	5	.700	.615	.555	.505	.455	.425
		10	.780	.710	.660	.620	.570	.545
		15	.815	.755	.710	.675	.635	.610
		20	.835	.785	.745	.715	.675	.650
		25	.850	.805	.770	.740	.705	.685
		30	.865	.820	.785	.760	.725	.705

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	2	5	.630	.565	.510	.470	.425	.400
		10	.725	.670	.620	.585	.550	.520
		15	.770	.720	.680	.650	.615	.590
		20	.795	.750	.715	.685	.650	.630
		25	.815	.775	.740	.715	.685	.665
		30	.830	.790	.760	.740	.710	.690

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	2	5	.700	.615	.555	.505	.455	.425
		10	.775	.710	.660	.620	.570	.545
		15	.815	.755	.710	.675	.635	.610
		20	.835	.785	.745	.710	.670	.650
		25	.850	.805	.770	.740	.705	.685
		30	.865	.820	.785	.760	.725	.705

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	3	5	.660	.585	.530	.485	.435	.410
		10	.750	.690	.640	.600	.560	.535
		15	.790	.735	.690	.660	.625	.600
		20	.815	.765	.730	.700	.660	.640
		25	.830	.790	.750	.725	.690	.670
		30	.845	.805	.770	.745	.720	.695

Table 3. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ , p^{*} and $\alpha = 0.50$

k	t	n		p^{*} .80	.90	.95	.975	.99	.995
5	1	5		.640	.565	.515	.475	.430	.400
		10		.730	.670	.625	.590	.550	.520
		15		.775	.720	.680	.650	.610	.590
		20		.800	.755	.720	.690	.655	.630
		25		.820	.780	.745	.715	.680	.660
		30		.835	.795	.760	.735	.710	.690

k	t	n		p^{*} .80	.90	.95	.975	.99	.995
5	3	5		.600	.540	.490	.455	.410	.380
		10		.700	.645	.600	.570	.530	.505
		15		.745	.700	.660	.635	.600	.580
		20		.775	.735	.700	.670	.640	.620
		25		.800	.760	.730	.700	.670	.650
		30		.815	.780	.750	.725	.695	.675

k	t	n		p^{*} .80	.90	.95	.975	.99	.995
5	2	5		.600	.535	.490	.455	.415	.385
		10		.700	.645	.600	.570	.530	.510
		15		.745	.700	.660	.635	.600	.575
		20		.775	.735	.700	.675	.640	.620
		25		.795	.760	.730	.700	.670	.650
		30		.815	.780	.750	.720	.695	.680

k	t	n		p^{*} .80	.90	.95	.975	.99	.995
5	4	5		.640	.570	.515	.475	.430	.400
		10		.730	.670	.625	.590	.545	.520
		15		.770	.720	.680	.650	.610	.590
		20		.800	.755	.720	.690	.650	.630
		25		.820	.780	.745	.715	.685	.665
		30		.835	.795	.765	.740	.710	.690

Table 4. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t , δ , p^{*} and $\alpha = 0.75$

k	t	n		p^* .80	.90	.95	.975	.99	.995
2	1	5		.690	.565	.480	.420	.355	.320
		10		.770	.670	.595	.540	.475	.445
		15		.805	.720	.655	.610	.550	.515
		20		.830	.755	.700	.650	.600	.570
		25		.845	.770	.720	.680	.630	.600
		30		.855	.790	.740	.700	.655	.630

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	1	5		.550	.435	.380	.335	.295	.285
		10		.660	.580	.520	.475	.430	.400
		15		.710	.640	.590	.550	.500	.475
		20		.745	.680	.635	.595	.550	.525
		25		.770	.710	.670	.635	.585	.560
		30		.785	.730	.690	.655	.620	.590

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	1	5		.595	.495	.425	.375	.320	.295
		10		.695	.610	.550	.500	.445	.420
		15		.745	.670	.615	.570	.520	.490
		20		.770	.705	.655	.615	.570	.540
		25		.795	.730	.685	.650	.600	.580
		30		.810	.750	.710	.675	.630	.605

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	2	5		.515	.440	.380	.335	.295	.285
		10		.630	.560	.505	.465	.420	.390
		15		.680	.620	.570	.535	.490	.470
		20		.720	.660	.620	.580	.540	.510
		25		.745	.690	.650	.615	.580	.550
		30		.765	.715	.675	.645	.605	.585

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	2	5		.590	.500	.425	.375	.320	.305
		10		.695	.610	.550	.495	.445	.415
		15		.740	.670	.615	.565	.520	.500
		20		.770	.705	.655	.615	.565	.540
		25		.795	.730	.685	.645	.600	.580
		30		.810	.750	.710	.675	.630	.600

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	3	5		.550	.460	.395	.350	.300	.290
		10		.660	.580	.525	.480	.435	.405
		15		.710	.640	.590	.550	.505	.475
		20		.745	.680	.630	.595	.550	.520
		25		.765	.710	.665	.630	.590	.560
		30		.785	.730	.690	.655	.620	.595

Table 4. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.75$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	5		.520	.440	.380	.340	.295	.285
		10		.630	.560	.505	.465	.415	.390
		15		.690	.625	.575	.535	.500	.465
		20		.725	.665	.620	.585	.540	.515
		25		.750	.695	.650	.620	.575	.550
		30		.770	.720	.675	.645	.605	.580

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	5		.480	.410	.360	.320	.290	.285
		10		.595	.530	.480	.445	.400	.375
		15		.655	.595	.550	.520	.475	.450
		20		.690	.640	.600	.565	.525	.500
		25		.720	.670	.635	.600	.560	.540
		30		.745	.695	.660	.625	.590	.570

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	5		.475	.405	.355	.315	.290	.285
		10		.595	.530	.480	.445	.400	.375
		15		.655	.600	.550	.520	.475	.450
		20		.695	.640	.600	.565	.525	.500
		25		.720	.670	.630	.600	.560	.540
		30		.740	.695	.660	.625	.590	.570

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	5		.525	.440	.380	.335	.295	.290
		10		.630	.560	.510	.465	.420	.390
		15		.690	.625	.575	.535	.490	.465
		20		.725	.665	.620	.585	.540	.510
		25		.750	.695	.650	.620	.580	.555
		30		.770	.720	.680	.645	.605	.585

Table 5. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, and p^* and $\alpha = 1$

k	t	n	p^* .80	.90	.95	.975	.99	.995
2	1	5	.620	.480	.390	.325	.270	.225
		10	.710	.600	.515	.455	.390	.355
		15	.760	.660	.590	.530	.470	.430
		20	.790	.700	.630	.560	.520	.490
		25	.805	.720	.660	.610	.560	.520
		30	.815	.745	.680	.635	.590	.550

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	1	5	.470	.375	.310	.265	.210	.190
		10	.585	.500	.435	.390	.345	.320
		15	.650	.570	.510	.465	.420	.390
		20	.690	.605	.560	.515	.470	.435
		25	.715	.650	.595	.555	.515	.480
		30	.735	.670	.630	.585	.540	.510

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	1	5	.515	.410	.345	.280	.235	.215
		10	.630	.530	.460	.415	.360	.325
		15	.685	.600	.535	.485	.440	.400
		20	.720	.640	.580	.535	.490	.455
		25	.750	.670	.620	.575	.530	.495
		30	.760	.700	.645	.600	.560	.520

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	2	5	.425	.345	.285	.245	.220	.180
		10	.550	.475	.415	.375	.350	.310
		15	.620	.545	.495	.450	.410	.375
		20	.655	.590	.540	.500	.460	.425
		25	.690	.630	.575	.540	.495	.470
		30	.710	.650	.605	.575	.530	.500

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	2	5	.510	.405	.335	.270	.230	.210
		10	.625	.530	.460	.410	.355	.325
		15	.685	.600	.535	.485	.440	.400
		20	.720	.640	.580	.535	.485	.455
		25	.740	.675	.615	.575	.530	.500
		30	.760	.695	.645	.605	.555	.520

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	3	5	.460	.370	.310	.250	.220	.190
		10	.590	.500	.440	.390	.350	.335
		15	.650	.570	.515	.470	.420	.390
		20	.685	.615	.560	.515	.470	.435
		25	.715	.645	.595	.555	.510	.480
		30	.735	.670	.620	.585	.540	.515

Table 5. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, and p^* and $\alpha = 1$

k	t	n	.80	.90	.95	.975	.99	.995
5	1	5	.435	.345	.275	.240	.205	.190
		10	.560	.475	.415	.375	.340	.315
		15	.620	.550	.495	.455	.400	.375
		20	.665	.595	.545	.505	.460	.430
		25	.695	.630	.575	.540	.495	.470
		30	.715	.655	.605	.575	.530	.500

k	t	n	.80	.90	.95	.975	.99	.995
5	3	5	.385	.310	.260	.225	.195	.170
		10	.515	.440	.395	.355	.330	.290
		15	.580	.520	.470	.435	.390	.355
		20	.630	.570	.520	.485	.440	.415
		25	.660	.600	.560	.525	.480	.455
		30	.685	.630	.585	.555	.515	.485

k	t	n	.80	.90	.95	.975	.99	.995
5	2	5	.385	.300	.255	.220	.195	.175
		10	.515	.440	.395	.355	.325	.290
		15	.580	.520	.475	.435	.390	.360
		20	.630	.570	.520	.485	.440	.410
		25	.660	.600	.560	.525	.480	.455
		30	.685	.630	.585	.550	.520	.490

k	t	n	.80	.90	.95	.975	.99	.995
5	4	5	.435	.350	.295	.245	.205	.200
		10	.555	.480	.415	.375	.355	.325
		15	.625	.550	.495	.455	.410	.375
		20	.665	.600	.545	.505	.455	.425
		25	.695	.630	.580	.540	.495	.475
		30	.715	.660	.615	.575	.535	.500

Table 6. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 2.0$

p^*

k	t	n	.80	.90	.95	.975	.99	.995
2	1	5	.450	.295	.210	.155	.105	.090
		10	.590	.440	.350	.280	.220	.190
		15	.650	.520	.430	.370	.300	.260
		20	.695	.570	.490	.430	.370	.330
		25	.720	.605	.525	.465	.400	.365
		30	.740	.635	.560	.495	.440	.400

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	1	5	.285	.195	.140	.105	.075	.060
		10	.425	.330	.260	.220	.180	.150
		15	.505	.410	.345	.300	.250	.220
		20	.560	.470	.405	.360	.310	.275
		25	.595	.510	.450	.405	.350	.320
		30	.620	.540	.485	.435	.385	.355

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	1	5	.330	.225	.160	.120	.085	.070
		10	.480	.365	.295	.240	.190	.165
		15	.555	.445	.375	.325	.270	.235
		20	.600	.500	.435	.380	.325	.290
		25	.640	.540	.475	.425	.365	.335
		30	.660	.575	.510	.455	.405	.370

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	2	5	.245	.170	.125	.095	.070	.055
		10	.390	.300	.245	.205	.170	.145
		15	.470	.385	.330	.285	.245	.210
		20	.525	.445	.385	.340	.285	.260
		25	.560	.485	.430	.390	.340	.310
		30	.595	.515	.465	.420	.375	.345

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	2	5	.330	.220	.160	.120	.085	.070
		10	.475	.365	.295	.240	.190	.160
		15	.555	.450	.375	.320	.270	.235
		20	.600	.500	.435	.375	.325	.290
		25	.635	.540	.475	.420	.365	.340
		30	.660	.575	.510	.460	.405	.370

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	3	5	.280	.190	.140	.105	.075	.060
		10	.430	.335	.270	.220	.180	.150
		15	.505	.410	.345	.300	.260	.220
		20	.555	.470	.405	.355	.300	.270
		25	.595	.510	.445	.400	.350	.320
		30	.620	.540	.485	.435	.390	.360

Table 6. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 2.0$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	5		.250	.175	.125	.095	.070	.060
		10		.395	.305	.245	.205	.170	.140
		15		.480	.390	.330	.285	.235	.210
		20		.530	.450	.390	.345	.295	.265
		25		.570	.490	.430	.385	.335	.310
		30		.600	.520	.465	.420	.370	.340

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	5		.210	.145	.110	.085	.065	.055
		10		.350	.270	.225	.190	.150	.130
		15		.430	.355	.305	.265	.225	.200
		20		.485	.415	.360	.320	.280	.250
		25		.530	.455	.405	.365	.325	.295
		30		.560	.490	.440	.400	.355	.330

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	5		.205	.145	.110	.085	.065	.055
		10		.350	.270	.225	.190	.150	.130
		15		.430	.355	.305	.265	.225	.200
		20		.485	.415	.360	.320	.280	.250
		25		.530	.455	.405	.365	.325	.295
		30		.560	.490	.440	.400	.355	.325

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	5		.250	.170	.130	.100	.070	.060
		10		.395	.310	.250	.205	.170	.145
		15		.475	.390	.330	.285	.240	.210
		20		.530	.450	.390	.340	.290	.260
		25		.570	.490	.430	.385	.340	.310
		30		.600	.525	.465	.425	.380	.350

Table 7. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ , p^{*} and $\alpha = 3.0$

k	t	n		p^* .80	.90	.95	.975	.99	.995
2	1	5		.375	.220	.140	.095	.060	.045
		10		.530	.380	.280	.215	.160	.130
		15		.605	.465	.375	.305	.240	.200
		20		.655	.525	.435	.370	.310	.270
		25		.680	.560	.475	.410	.350	.310
		30		.710	.590	.510	.445	.380	.345

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	1	5		.205	.125	.085	.060	.035	.030
		10		.365	.265	.200	.160	.120	.110
		15		.450	.350	.285	.240	.190	.160
		20		.510	.415	.350	.300	.250	.215
		25		.555	.460	.395	.345	.295	.265
		30		.580	.495	.430	.380	.330	.295

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	1	5		.250	.155	.100	.070	.045	.035
		10		.415	.300	.230	.175	.130	.105
		15		.500	.390	.315	.260	.205	.175
		20		.550	.450	.375	.315	.270	.235
		25		.595	.495	.425	.365	.310	.280
		30		.620	.525	.460	.405	.350	.310

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	2	5		.170	.105	.070	.050	.035	.025
		10		.325	.240	.190	.145	.110	.090
		15		.410	.325	.265	.225	.185	.150
		20		.425	.390	.330	.280	.230	.200
		25		.510	.430	.375	.330	.280	.255
		30		.545	.470	.410	.365	.315	.285

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	2	5		.250	.155	.100	.070	.040	.030
		10		.415	.300	.230	.175	.130	.105
		15		.500	.390	.315	.260	.205	.175
		20		.550	.450	.375	.320	.270	.230
		25		.595	.490	.420	.365	.310	.280
		30		.620	.530	.460	.405	.350	.310

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	3	5		.200	.120	.080	.055	.035	.025
		10		.365	.270	.205	.160	.120	.100
		15		.450	.350	.285	.240	.195	.160
		20		.505	.410	.345	.295	.245	.215
		25		.550	.460	.390	.345	.300	.260
		30		.580	.490	.430	.380	.330	.300

Table 7. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 3.0$

k	t	n	.80	.90	.95	.975	.99	.995
5	1	5	.175	.110	.075	.050	.035	.025
		10	.330	.240	.185	.145	.110	.090
		15	.420	.330	.270	.225	.180	.155
		20	.480	.390	.330	.285	.240	.210
		25	.525	.440	.375	.325	.280	.250
		30	.555	.470	.410	.365	.320	.280

k	t	n	.80	.90	.95	.975	.99	.995
5	3	5	.140	.090	.060	.040	.030	.020
		10	.285	.210	.160	.130	.100	.080
		15	.370	.295	.245	.205	.165	.140
		20	.430	.360	.305	.260	.220	.190
		25	.480	.400	.350	.305	.265	.240
		30	.510	.440	.385	.345	.300	.270

k	t	n	.80	.90	.95	.975	.99	.995
5	2	5	.140	.085	.060	.040	.030	.020
		10	.285	.210	.160	.130	.100	.080
		15	.370	.295	.245	.205	.165	.140
		20	.430	.360	.305	.260	.220	.190
		25	.480	.400	.350	.305	.265	.240
		30	.510	.440	.385	.345	.295	.270

k	t	n	.80	.90	.95	.975	.99	.995
5	4	5	.180	.110	.075	.050	.035	.025
		10	.330	.240	.185	.145	.110	.090
		15	.420	.330	.270	.225	.180	.150
		20	.480	.390	.330	.280	.230	.200
		25	.520	.435	.375	.325	.280	.255
		30	.555	.470	.415	.365	.320	.290

Table 8. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 4.0$

p^*

k	t	n	.80	.90	.95	.975	.99	.995
2	1	5	.330	.175	.105	.065	.040	.025
		10	.505	.345	.250	.185	.130	.100
		15	.585	.440	.340	.275	.210	.175
		20	.635	.500	.410	.345	.280	.240
		25	.665	.540	.450	.385	.320	.280
		30	.695	.570	.485	.425	.355	.320

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	1	5	.165	.095	.055	.035	.020	.015
		10	.335	.230	.170	.130	.090	.075
		15	.420	.320	.260	.205	.165	.135
		20	.485	.390	.320	.275	.215	.190
		25	.525	.435	.370	.320	.270	.240
		30	.560	.470	.410	.355	.300	.270

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	1	5	.210	.115	.070	.045	.025	.020
		10	.385	.270	.195	.145	.105	.080
		15	.475	.360	.285	.230	.180	.150
		20	.530	.420	.350	.295	.235	.210
		25	.575	.470	.395	.345	.285	.250
		30	.605	.505	.435	.380	.325	.290

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	2	5	.135	.075	.045	.030	.020	.015
		10	.290	.210	.155	.115	.085	.065
		15	.385	.295	.240	.195	.155	.130
		20	.445	.360	.300	.255	.205	.175
		25	.490	.410	.350	.305	.255	.225
		30	.525	.445	.385	.340	.290	.260

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	2	5	.210	.115	.070	.045	.025	.015
		10	.385	.270	.195	.145	.100	.080
		15	.475	.360	.285	.230	.180	.150
		20	.530	.420	.350	.295	.240	.210
		25	.575	.470	.395	.340	.285	.250
		30	.600	.505	.435	.380	.320	.290

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	3	5	.165	.090	.055	.035	.020	.015
		10	.335	.235	.175	.130	.090	.070
		15	.420	.325	.260	.210	.165	.135
		20	.480	.385	.320	.270	.215	.190
		25	.530	.430	.370	.320	.265	.235
		30	.560	.470	.405	.355	.305	.270

Table 8. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 4.0$

k	t	n		.80	.90	.95	.975	.99	.995
5	1	5		.140	.080	.050	.030	.020	.015
		10		.295	.210	.150	.115	.085	.065
		15		.395	.300	.240	.195	.155	.130
		20		.455	.365	.305	.255	.205	.180
		25		.500	.410	.350	.300	.255	.230
		30		.535	.450	.390	.340	.290	.260

k	t	n		.80	.90	.95	.975	.99	.995
5	3	5		.105	.060	.040	.025	.015	.010
		10		.250	.180	.130	.105	.070	.055
		15		.345	.270	.215	.175	.140	.115
		20		.405	.330	.275	.235	.190	.170
		25		.450	.380	.320	.280	.235	.210
		30		.490	.415	.360	.320	.275	.245

k	t	n		.80	.90	.95	.975	.99	.995
5	2	5		.105	.060	.040	.025	.015	.010
		10		.250	.180	.130	.105	.070	.055
		15		.345	.270	.215	.175	.140	.115
		20		.405	.330	.280	.235	.195	.170
		25		.450	.380	.320	.280	.235	.210
		30		.490	.415	.360	.320	.275	.245

k	t	n		.80	.90	.95	.975	.99	.995
5	4	5		.140	.080	.050	.030	.020	.015
		10		.300	.210	.150	.115	.085	.065
		15		.390	.300	.240	.195	.150	.125
		20		.455	.365	.300	.255	.205	.180
		25		.500	.410	.350	.305	.255	.230
		30		.530	.450	.390	.345	.295	.265

Table 9. C

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 5.0$

k	t	n	p^* .80	.90	.95	.975	.99	.995
2	1	5	.305	.150	.080	.050	.025	.015
		10	.490	.330	.230	.170	.110	.090
		15	.570	.425	.330	.265	.195	.160
		20	.630	.490	.395	.330	.265	.230
		25	.655	.530	.440	.370	.305	.270
		30	.685	.560	.475	.410	.340	.305

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	1	5	.140	.075	.040	.025	.015	.010
		10	.315	.215	.150	.115	.075	.060
		15	.405	.310	.240	.195	.145	.120
		20	.470	.375	.310	.260	.205	.180
		25	.520	.420	.360	.305	.255	.225
		30	.545	.460	.395	.345	.285	.255

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	1	5	.180	.090	.055	.030	.020	.010
		10	.365	.250	.175	.130	.090	.065
		15	.460	.345	.270	.215	.160	.130
		20	.520	.410	.335	.275	.225	.190
		25	.560	.460	.385	.330	.270	.240
		30	.595	.495	.420	.365	.305	.275

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	2	5	.115	.060	.035	.020	.010	.005
		10	.275	.190	.135	.100	.070	.050
		15	.370	.280	.220	.180	.135	.115
		20	.435	.350	.285	.240	.190	.160
		25	.475	.395	.335	.290	.240	.210
		30	.515	.430	.370	.325	.275	.250

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	2	5	.180	.090	.050	.030	.020	.010
		10	.365	.250	.175	.130	.085	.065
		15	.460	.350	.270	.215	.160	.130
		20	.520	.410	.335	.280	.225	.190
		25	.560	.460	.385	.325	.270	.240
		30	.590	.495	.420	.370	.305	.275

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	3	5	.140	.070	.040	.025	.015	.010
		10	.315	.220	.160	.115	.080	.060
		15	.405	.310	.245	.195	.150	.120
		20	.470	.370	.305	.255	.200	.175
		25	.520	.420	.355	.305	.250	.220
		30	.550	.455	.390	.345	.290	.260

Table 9. C
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Sample Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 5.0$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	5		.115	.060	.035	.020	.010	.005
		10		.280	.190	.135	.100	.070	.050
		15		.380	.290	.225	.180	.135	.110
		20		.440	.350	.290	.245	.190	.165
		25		.490	.400	.340	.290	.240	.210
		30		.520	.440	.375	.325	.275	.245

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	5		.085	.045	.025	.015	.010	.005
		10		.235	.160	.115	.085	.060	.045
		15		.330	.255	.200	.165	.120	.100
		20		.390	.320	.260	.220	.175	.155
		25		.440	.365	.310	.265	.225	.200
		30		.480	.405	.350	.305	.260	.230

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	5		.085	.045	.025	.015	.010	.005
		10		.235	.160	.120	.085	.060	.045
		15		.330	.250	.200	.165	.120	.100
		20		.390	.320	.265	.220	.175	.150
		25		.440	.365	.310	.265	.225	.200
		30		.480	.405	.350	.305	.255	.230

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	5		.115	.060	.035	.020	.010	.005
		10		.280	.190	.135	.100	.070	.050
		15		.375	.285	.225	.180	.135	.110
		20		.440	.350	.290	.240	.190	.165
		25		.485	.400	.335	.290	.240	.215
		30		.520	.440	.375	.330	.280	.250

Appendix D Probability Tables for Estimator $\tilde{\alpha}$

Table1.D

δ^* values when using the estimator $\widetilde{\alpha}$

p^*

k	t	n	.80	.90	.95	.975	.99	.995
2	1	5	.630	.490	.390	.320	.250	.210
		10	.745	.635	.555	.500	.430	.390
		15	.790	.700	.635	.575	.520	.480
		20	.820	.740	.675	.625	.570	.540
		25	.845	.765	.710	.665	.610	.580
		30	.850	.785	.730	.690	.640	.610

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	1	5	.500	.395	.320	.260	.200	.175
		10	.640	.550	.490	.435	.375	.345
		15	.700	.625	.570	.525	.470	.435
		20	.736	.670	.615	.575	.530	.500
		25	.765	.700	.655	.615	.570	.540
		30	.780	.725	.680	.645	.600	.570

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	1	5	.540	.420	.340	.280	.220	.210
		10	.670	.580	.510	.455	.400	.390
		15	.730	.650	.590	.540	.480	.480
		20	.760	.690	.640	.595	.540	.540
		25	.785	.720	.670	.630	.580	.580
		30	.805	.745	.695	.660	.615	.610

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	2	5	.440	.355	.285	.240	.195	.175
		10	.595	.530	.470	.420	.370	.340
		15	.660	.610	.555	.510	.460	.430
		20	.705	.655	.610	.565	.520	.490
		25	.735	.690	.640	.605	.565	.540
		30	.755	.715	.670	.640	.600	.570

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	2	5	.510	.400	.320	.265	.210	.185
		10	.655	.565	.500	.445	.385	.355
		15	.720	.640	.580	.530	.480	.445
		20	.750	.685	.630	.585	.540	.510
		25	.780	.715	.665	.625	.580	.550
		30	.800	.740	.690	.665	.610	.580

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	3	5	.455	.355	.285	.240	.195	.175
		10	.615	.530	.470	.420	.370	.340
		15	.680	.610	.555	.510	.460	.430
		20	.720	.655	.610	.565	.520	.490
		25	.750	.690	.640	.605	.565	.540
		30	.770	.715	.670	.640	.600	.570

Table1.D
(continued)

δ^* values when using the estimator $\tilde{\alpha}$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	5		.475	.380	.305	.250	.195	.180
		10		.620	.535	.470	.425	.370	.340
		15		.680	.610	.560	.510	.460	.430
		20		.720	.660	.610	.565	.520	.490
		25		.745	.690	.640	.605	.560	.530
		30		.765	.710	.670	.630	.590	.565

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	5		.390	.310	.250	.210	.190	.170
		10		.555	.480	.430	.390	.345	.310
		15		.625	.565	.520	.480	.430	.400
		20		.675	.620	.575	.540	.495	.470
		25		.705	.655	.610	.580	.540	.510
		30		.730	.680	.640	.610	.570	.550

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	5		.405	.320	.260	.220	.190	.170
		10		.565	.490	.440	.395	.345	.320
		15		.635	.570	.520	.485	.440	.410
		20		.680	.625	.580	.545	.500	.470
		25		.710	.660	.615	.580	.540	.510
		30		.730	.685	.645	.615	.575	.550

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	5		.420	.330	.265	.220	.190	.170
		10		.580	.505	.450	.405	.355	.325
		15		.655	.590	.540	.495	.450	.420
		20		.700	.640	.590	.555	.510	.480
		25		.730	.675	.630	.595	.550	.525
		30		.755	.700	.660	.625	.580	.560

Appendix E Probability Tables for Estimator $\hat{\beta}$

Large Samples

Table 1. E

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 0.25$

k	t	n	.80	.90	.95	.975	.99	.995
2	1	30	.945	.920	.900	.880	.860	.845
		40	.955	.930	.910	.895	.880	.865
		50	.960	.940	.920	.905	.890	.880
		75	.965	.950	.935	.925	.910	.900

k	t	n	.80	.90	.95	.975	.99	.995
4	2	30	.910	.885	.870	.855	.835	.825
		40	.920	.900	.885	.870	.855	.845
		50	.930	.910	.895	.885	.870	.860
		75	.940	.925	.915	.905	.890	.885

k	t	n	.80	.90	.95	.975	.99	.995
3	1	30	.930	.905	.885	.865	.850	.835
		40	.940	.915	.900	.885	.865	.855
		50	.945	.925	.910	.895	.880	.870
		75	.955	.940	.925	.915	.900	.890

k	t	n	.80	.90	.95	.975	.99	.995
4	3	30	.915	.895	.875	.860	.840	.825
		40	.930	.905	.890	.875	.860	.850
		50	.935	.915	.900	.890	.875	.865
		75	.945	.930	.920	.910	.900	.890

k	t	n	.80	.90	.95	.975	.99	.995
3	2	30	.925	.900	.880	.865	.845	.835
		40	.935	.915	.900	.885	.865	.855
		50	.945	.920	.910	.895	.880	.870
		75	.950	.935	.925	.915	.900	.890

k	t	n	.80	.90	.95	.975	.99	.995
5	1	30	.910	.890	.870	.855	.835	.825
		40	.925	.900	.885	.875	.855	.845
		50	.930	.910	.900	.885	.870	.860
		75	.945	.930	.915	.905	.890	.885

k	t	n	.80	.90	.95	.975	.99	.995
4	1	30	.920	.895	.875	.860	.840	.830
		40	.930	.910	.890	.875	.860	.855
		50	.935	.920	.900	.890	.875	.865
		75	.950	.930	.920	.910	.895	.885

k	t	n	.80	.90	.95	.975	.99	.995
5	2	30	.900	.880	.860	.845	.830	.820
		40	.910	.895	.880	.865	.850	.840
		50	.920	.905	.890	.880	.865	.855
		75	.935	.920	.910	.900	.890	.880

Table 1. E
(continued)

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.25$

				p^*					
k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.900	.880	.860	.845	.830	.815
		40		.910	.890	.880	.865	.850	.840
		50		.920	.900	.890	.880	.865	.855
		75		.935	.920	.910	.900	.890	.880

				p^*					
K	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.910	.890	.870	.855	.835	.820
		40		.920	.900	.885	.870	.855	.845
		50		.930	.910	.895	.885	.870	.860
		75		.940	.925	.915	.905	.890	.880

Table 2. E

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 0.5$

k	t	n		p^* .80	.90	.95	.975	.99	.995
2	1	30		.900	.850	.810	.780	.740	.720
		40		.910	.870	.835	.805	.775	.750
		50		.920	.880	.850	.825	.795	.775
		75		.935	.900	.880	.855	.830	.815

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	2	30		.830	.790	.760	.730	.700	.680
		40		.850	.815	.790	.765	.735	.715
		50		.865	.835	.810	.785	.760	.745
		75		.890	.860	.840	.820	.800	.785

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	1	30		.865	.820	.785	.755	.720	.700
		40		.880	.840	.810	.785	.755	.735
		50		.895	.860	.830	.805	.780	.760
		75		.910	.880	.860	.840	.815	.800

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	3	30		.840	.800	.770	.740	.710	.685
		40		.860	.825	795.	.770	.740	.720
		50		.875	.840	.815	.790	.765	.750
		75		.900	.870	.850	.830	.805	.790

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	2	30		.860	.815	.780	.755	.720	.700
		40		.880	.840	.810	.785	.750	.735
		50		.890	.855	.830	.805	.780	.760
		75		.910	.880	.860	.840	.810	.800

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	30		.840	.800	.765	.735	.700	.680
		40		.855	.820	.790	.770	.740	.720
		50		.870	.840	.810	.790	.760	.745
		75		.890	.865	.840	.825	.800	.790

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	1	30		.850	.805	.770	.745	.710	.690
		40		.865	.830	.800	.775	.740	.725
		50		.880	.845	.820	.795	.770	.750
		75		.900	.870	.850	.830	.810	.790

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	30		.815	.775	.745	.720	.690	.670
		40		.835	.800	.775	.750	.725	.710
		50		.850	.820	.800	.775	.750	.735
		75		.875	.850	.830	.815	.790	.780

Table 2. E
(continued)

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case
Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 0.5$

				p^*					
k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.810	.770	.740	.720	.690	.670
		40		.835	.800	.775	.750	.725	.705
		50		.850	.820	.795	.775	.750	.735
		75		.875	.850	.830	.815	.790	.780

				p^*					
K	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.830	.790	.755	.730	.700	.680
		40		.850	.815	.785	.760	.730	.715
		50		.865	.830	.810	.785	.760	.740
		75		.890	.860	.840	.820	.800	.785

Table 3. E

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.75$

k	t	n		p^* .80	.90	.95	.975	.99	.995
2	1	30		.860	.790	.740	.695	.645	.615
		40		.875	.815	.770	.730	.690	.660
		50		.890	.835	.790	.755	.715	.690
		75		.910	.860	.830	.795	.760	.740

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	2	30		.760	.710	.675	.635	.590	.560
		40		.790	.745	.705	.675	.635	.610
		50		.810	.770	.735	.705	.670	.650
		75		.845	.805	.780	.750	.725	.705

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	1	30		.810	.750	.705	.665	.620	.590
		40		.835	.780	.740	.705	.665	.640
		50		.850	.800	.765	.735	.695	.670
		75		.875	.835	.800	.775	.745	.725

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	3	30		.775	.720	.680	.640	.595	.570
		40		.805	.755	.715	.685	.645	.620
		50		.825	.780	.740	.715	.680	.655
		75		.855	.815	.785	.760	.730	.710

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	2	30		.805	.745	.700	.660	.615	.585
		40		.830	.775	.735	.700	.660	.630
		50		.845	.800	.760	.730	.690	.670
		75		.870	.830	.800	.775	.740	.720

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	30		.775	.720	.680	.640	.600	.570
		40		.800	.755	.715	.680	.640	.620
		50		.820	.775	.740	.710	.670	.650
		75		.850	.815	.780	.755	.730	.710

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	1	30		.790	.730	.690	.650	.605	.580
		40		.815	.765	.725	.690	.650	.625
		50		.830	.790	.750	.720	.680	.660
		75		.860	.820	.790	.765	.730	.715

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	30		.740	.690	.650	.620	.575	.550
		40		.770	.730	.690	.660	.625	.600
		50		.795	.750	.720	.695	.660	.640
		75		.830	.795	.765	.740	.710	.695

Table 3. E
(continued)

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 0.75$

k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.735	.690	.650	.615	.575	.550
		40		.770	.725	.690	.660	.620	.600
		50		.790	.750	.720	.690	.660	.635
		75		.825	.790	.760	.740	.710	.695

k	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.760	.705	.660	.625	.585	.560
		40		.790	.740	.700	.670	.630	.610
		50		.810	.765	.730	.705	.670	.645
		75		.845	.805	.775	.750	.720	.705

Table 4. E

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.00$

k	t	n		.80	.90	.95	.975	.99	.995
2	1	30		.825	.740	.680	.625	.570	.530
		40		.845	.775	.715	.670	.620	.580
		50		.860	.795	.745	.700	.650	.620
		75		.885	.830	.785	.750	.710	.680

k	t	n		.80	.90	.95	.975	.99	.995
4	2	30		.710	.645	.595	.555	.510	.485
		40		.740	.685	.640	.605	.555	.530
		50		.765	.715	.670	.640	.600	.570
		75		.805	.760	.730	.695	.660	.640

k	t	n		.80	.90	.95	.975	.99	.995
3	1	30		.770	.695	.640	.595	.540	.550
		40		.795	.730	.680	.640	.590	.560
		50		.815	.760	.710	.675	.625	.600
		75		.845	.800	.760	.725	.685	.660

k	t	n		.80	.90	.95	.975	.99	.995
4	3	30		.720	.655	.605	.560	.510	.485
		40		.760	.700	.650	.610	.565	.535
		50		.780	.725	.680	.645	.605	.580
		75		.820	.770	.735	.705	.670	.645

k	t	n		.80	.90	.95	.975	.99	.995
3	2	30		.760	.685	.630	.585	.530	.500
		40		.790	.725	.675	.635	.580	.550
		50		.810	.750	.705	.665	.620	.590
		75		.840	.790	.750	.720	.680	.660

k	t	n		.80	.90	.95	.975	.99	.995
5	1	30		.730	.660	.610	.565	.510	.490
		40		.760	.700	.650	.615	.565	.535
		50		.780	.725	.685	.645	600.	.575
		75		.815	.770	.730	.705	.665	.640

k	t	n		.80	.90	.95	.975	.99	.995
4	1	30		.745	.675	.620	.575	.520	.495
		40		.775	.710	.665	.625	.575	.545
		50		.790	.740	.695	.655	.610	.585
		75		.830	.780	.740	.710	.670	.650

k	t	n		.80	.90	.95	.975	.99	.995
5	2	30		.685	.625	.580	.535	.490	.480
		40		.720	.670	.625	.590	.545	.515
		50		.745	.700	.660	.625	.585	.560
		75		.790	.745	.710	.685	.650	.630

Table 4. E
(continued)

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.00$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	30		.675	.620	.570	.530	.490	.480
		40		.715	.660	.620	.585	.540	.510
		50		.740	.690	.650	.620	.580	.555
		75		.785	.740	.710	.680	.650	.625

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	30		.700	.635	.585	.545	.490	.480
		40		.740	.680	.635	.595	.550	.525
		50		.760	.710	.670	.635	.590	.565
		75		.805	.760	.725	.695	.660	.635

Table 5. E

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample ApproximationCase

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.25$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
2	1	30		.795	.705	.630	.575	.505	.465
		40		.820	.740	.675	.625	.565	.525
		50		.840	.765	.705	.655	.600	.565
		75		.870	.805	.755	.715	.665	.635

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	2	30		.670	.595	.540	.490	.440	.400
		40		.705	.640	.590	.550	.495	.465
		50		.730	.675	.625	.585	.540	.510
		75		.775	.725	.685	.655	.610	.585

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	1	30		.735	.655	.590	.540	.480	.435
		40		.770	.695	.640	.590	.530	.500
		50		.790	.720	.670	.625	.575	.540
		75		.825	.765	.720	.685	.640	.610

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	3	30		.680	.605	.550	.500	.440	.400
		40		.720	.650	.600	.555	.505	.470
		50		.745	.685	.635	.595	.550	.520
		75		.790	.740	.695	.660	.620	.590

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	2	30		.720	.640	.575	.525	.460	.425
		40		.755	.680	.625	.580	.525	.490
		50		.780	.710	.660	.615	.570	.535
		75		.815	.760	.715	.680	.640	.610

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	1	30		.690	.615	.555	.510	.450	.410
		40		.725	.660	.605	.560	.510	.475
		50		.750	.690	.640	.600	.550	.520
		75		.790	.735	.690	.660	.620	.590

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	1	30		.710	.630	.570	.520	.460	.420
		40		.740	.670	.620	.575	.520	.485
		50		.765	.705	.650	.610	.560	.530
		75		.805	.750	.705	.670	.625	.600

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	2	30		.640	.575	.520	.475	.420	.395
		40		.680	.620	.570	.535	.485	.450
		50		.710	.655	.610	.575	.530	.500
		75		.755	.710	.670	.640	.600	.575

Table 5. E
(continued)

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.25$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	30		.630	.565	.515	.465	.415	.380
		40		.675	.615	.570	.525	.480	.445
		50		.705	.650	.605	.565	.525	.495
		75		.750	.705	.665	.635	.600	.570

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	30		.655	.585	.530	.480	.430	.375
		40		.695	.630	.580	.540	.490	.455
		50		.725	.670	.620	.580	.540	.505
		75		.770	.720	.680	.650	.610	.580

Table 6. E

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 1.5$

p^*

k	t	n	.80	.90	.95	.975	.99	.995
2	1	30	.775	.675	.595	.535	.460	.415
		40	.805	.715	.645	.585	.520	.480
		50	.820	.740	.675	.625	.560	.530
		75	.850	.780	.730	.685	.630	.600

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	2	30	.640	.560	.500	.445	.390	.345
		40	.680	.605	.550	.505	.450	.415
		50	.705	.640	.590	.550	.500	.470
		75	.755	.700	.655	.620	.575	.550

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	1	30	.710	.620	.555	.495	.430	.390
		40	.745	.665	.600	.550	.490	.450
		50	.765	.695	.640	.590	.540	.500
		75	.805	.740	.695	.655	.605	.575

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	3	30	.640	.560	.500	.445	.390	.345
		40	.690	.615	.560	.515	.455	.420
		50	.720	.655	.600	.555	.505	.470
		75	.770	.710	.665	.630	.580	.555

p^*

k	t	n	.80	.90	.95	.975	.99	.995
3	2	30	.690	.605	.535	.480	.415	.370
		40	.730	.650	.590	.540	.480	.440
		50	.755	.680	.625	.580	.525	.490
		75	.800	.735	.685	.650	.600	.570

p^*

k	t	n	.80	.90	.95	.975	.99	.995
5	1	30	.660	.580	.520	.465	.410	.360
		40	.700	.625	.570	.520	.465	.430
		50	.725	.660	.605	.565	.510	.480
		75	.770	.710	.665	.630	.580	.555

p^*

k	t	n	.80	.90	.95	.975	.99	.995
4	1	30	.680	.600	.530	.475	.415	.370
		40	.715	.640	.585	.535	.475	.440
		50	.740	.675	.620	.575	.520	.485
		75	.780	.725	.675	.640	.590	.575

p^*

k	t	n	.80	.90	.95	.975	.99	.995
5	2	30	.610	.535	.480	.430	.370	.330
		40	.650	.585	.535	.490	.440	.400
		50	.680	.620	.575	.535	.490	.455
		75	.735	.680	.640	.605	.560	.540

Table 6. E
(continued)

For Estimator $\hat{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.5$

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	3	30		.595	.525	.470	.420	.360	.320
		40		.640	.580	.525	.485	.430	.395
		50		.675	.615	.570	.525	.480	.450
		75		.725	.675	.635	.600	.560	.530

K	t	n		p^* .80	.90	.95	.975	.99	.995
5	4	30		.620	.540	.480	.430	.370	.330
		40		.665	.595	.540	.495	.440	.410
		50		.700	.635	.580	.540	.490	.460
		75		.750	.690	.650	.615	.570	.540

Appendix F Probability Tables for Estimator $\tilde{\beta}$

Large Samples

Table 1. F

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 0.25$

k	t	n		.80	.90	.95	.975	.99	.995
2	1	30		.945	.920	.900	.880	.860	.850
		40		.955	.930	.910	.895	.880	.865
		50		.960	.935	.920	.905	.890	.880
		75		.965	.950	.935	.925	.910	.900

k	t	n		.80	.90	.95	.975	.99	.995
4	2	30		.910	.885	.870	.855	.835	.825
		40		.920	.900	.885	.870	.855	.845
		50		.930	.910	.895	.885	.870	.860
		75		.940	.925	.915	.905	.890	.885

k	t	n		.80	.90	.95	.975	.99	.995
3	1	30		.925	.905	.885	.865	.845	.835
		40		.935	.915	.900	.885	.865	.855
		50		.945	.925	.910	.895	.880	.870
		75		.950	.940	.925	.915	.900	.890

k	t	n		.80	.90	.95	.975	.99	.995
4	3	30		.910	.885	.870	.855	.835	.825
		40		.920	.900	.885	.870	.855	.845
		50		.930	.910	.895	.885	.870	.860
		75		.940	.925	.910	.905	.890	.885

k	t	n		.80	.90	.95	.975	.99	.995
3	2	30		.925	.905	.885	.865	.845	.835
		40		.935	.915	.900	.885	.865	.855
		50		.945	.925	.910	.895	.880	.870
		75		.950	.940	.925	.915	.900	.890

k	t	n		.80	.90	.95	.975	.99	.995
5	1	30		.910	.890	.870	.855	.835	.825
		40		.920	.905	.885	.875	.855	.845
		50		.930	.910	.900	.885	.870	.860
		75		.940	.930	.915	.905	.890	.885

k	t	n		.80	.90	.95	.975	.99	.995
4	1	30		.915	.895	.875	.860	.840	.830
		40		.930	.905	.890	.880	.860	.850
		50		.935	.915	.900	.890	.875	.865
		75		.945	.930	.920	.910	.895	.890

k	t	n		.80	.90	.95	.975	.99	.995
5	2	30		.900	.880	.860	.845	.830	.820
		40		.910	.895	.880	.865	.850	.840
		50		.920	.905	.890	.880	.865	.855
		75		.935	.920	.910	.900	.890	.880

Table 1. F
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \ge p^{*}$ given values of k, t, δ , p^{*} and $\alpha = 0.25$

				p^{*}					
k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.900	.880	.860	.845	.830	.820
		40		.910	.895	.880	.865	.850	.840
		50		.920	.905	.890	.880	.865	.855
		75		.935	.920	.910	.900	.890	.880

				p^{*}					
k	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.900	.880	.860	.845	.830	.820
		40		.910	.895	.880	.865	.850	.840
		50		.920	.905	.890	.880	.865	.855
		75		.935	.920	.910	.900	.890	.880

Table 2. F

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 0.50$

p^*

k	t	n		.80	.90	.95	.975	.99	.995
2	1	30		.900	.850	.810	.785	.750	.720
		40		.910	.870	.835	.810	.775	.755
		50		.920	.880	.850	.825	.790	.780
		75		.935	.900	.880	.855	.830	.815

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	2	30		.830	.790	.760	.735	.705	.690
		40		.850	.815	.790	.765	.740	.720
		50		.865	.835	.810	.790	.760	.750
		75		.890	.865	.840	.825	.805	.790

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	1	30		.865	.820	.785	.760	.720	.705
		40		.880	.840	.810	.785	.755	.740
		50		.890	.860	.830	.810	.780	.760
		75		.910	.880	.860	.840	.815	.800

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	3	30		.830	.790	.760	.735	.705	.690
		40		.850	.815	.790	.765	.740	.720
		50		.865	.835	.810	.790	.760	.750
		75		.890	.865	.840	.825	.805	.790

p^*

k	t	n		.80	.90	.95	.975	.99	.995
3	2	30		.865	.820	.785	.760	.720	.700
		40		.880	.840	.810	.785	.755	.740
		50		.890	.860	.830	.810	.780	.760
		75		.910	.880	.860	.840	.815	.800

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	1	30		.830	.790	.760	.735	.700	.690
		40		.855	.820	.790	.765	.740	.720
		50		.870	.835	.810	.790	.760	.750
		75		.890	.865	.845	.825	.800	.790

p^*

k	t	n		.80	.90	.95	.975	.99	.995
4	1	30		.845	.805	.770	.745	.710	.690
		40		.865	.830	.800	.775	.745	.730
		50		.880	.845	.820	.795	.770	.755
		75		.900	.870	.850	.830	.810	.795

p^*

k	t	n		.80	.90	.95	.975	.99	.995
5	2	30		.810	.775	.750	.725	.690	.675
		40		.835	.805	.780	.755	.730	.710
		50		.855	.825	.800	.780	.760	.740
		75		.880	.855	.830	.815	.800	.780

Table 2. F
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 0.50$

p^{*}

k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.810	.775	.750	.725	.690	.675
		40		.835	.805	.780	.755	.730	.710
		50		.850	.820	.800	.780	.755	.740
		75		.880	.855	.830	.815	.800	.780

p^{*}

k	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.810	.775	.750	.725	.690	.675
		40		.835	.805	.780	.755	.730	.710
		50		.850	.820	.800	.780	.755	.740
		75		.880	.855	.830	.815	.800	.780

Table 3. F

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 0.75$

k	t	n		p^* .80	.90	.95	.975	.99	.995
2	1	30		.860	.790	.750	.705	.660	.630
		40		.875	.805	.765	.740	.695	.660
		50		.890	.835	.795	.760	.720	.700
		75		.910	.860	.830	.800	.770	.745

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	2	30		.765	.715	.675	.645	.610	.580
		40		.785	.745	.710	.670	.645	.630
		50		.810	.770	.735	.710	.680	.660
		75		.840	.810	.780	.755	.730	.710

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	1	30		.810	.750	.710	.670	.630	.605
		40		.830	.775	.730	.700	.685	.640
		50		.850	.800	.765	.735	.700	.680
		75		.875	.835	.800	.775	.750	.725

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	3	30		.765	.715	.675	.645	.610	.580
		40		.805	.765	.720	.700	.675	.620
		50		.830	.780	.750	.720	.690	.660
		75		.860	.820	.790	.735	.720	.665

k	t	n		p^* .80	.90	.95	.975	.99	.995
3	2	30		.810	.750	.710	.670	.630	.605
		40		.830	.770	.740	.715	.680	.650
		50		.850	.800	.765	.735	.700	.680
		75		.875	.835	.800	.775	.750	.730

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	1	30		.770	.720	.680	.645	.610	.585
		40		.800	.745	.690	.675	.645	.630
		50		.815	.770	.740	.710	.680	.660
		75		.845	.810	.780	.755	.730	.710

k	t	n		p^* .80	.90	.95	.975	.99	.995
4	1	30		.785	.730	.690	.655	.620	.590
		40		.795	.745	.720	.680	.645	.615
		50		.830	.780	.750	.720	.690	.665
		75		.860	.820	.790	.765	.735	.720

k	t	n		p^* .80	.90	.95	.975	.99	.995
5	2	30		.745	.700	.660	.630	.595	.570
		40		.770	.745	.715	.680	.645	.620
		50		.795	.755	.720	.700	.670	.650
		75		.825	.795	.770	.745	.720	.700

Table 3. F
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ , p^{*} and $\alpha = 0.75$

k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.745	.700	.660	.630	.595	.570
		40		.765	.715	.685	.660	.635	.620
		50		.795	.755	.720	.695	.670	.650
		75		.825	.795	.770	.745	.720	.700

k	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.745	.700	.660	.630	.595	.570
		40		.775	.720	.690	.655	.620	.600
		50		.815	.770	.740	.710	.680	.660
		75		.845	.810	.780	.755	.730	.710

Table 4. F

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.0$

k	t	n	.80	.90	.95	.975	.99	.995
2	1	30	.825	.750	.685	.640	.590	.560
		40	.850	.775	.720	.680	.630	.600
		50	.860	.795	.750	.710	.660	.635
		75	.885	.830	.790	.755	.715	.690

k	t	n	.80	.90	.95	.975	.99	.995
4	2	30	.710	.655	.610	.575	.530	.505
		40	.745	.690	.650	.615	.580	.550
		50	.770	.720	.680	.650	.610	.590
		75	.810	.765	.730	.705	.670	.650

k	t	n	.80	.90	.95	.975	.99	.995
3	1	30	.765	.700	.645	.605	.560	.530
		40	.795	.730	.685	.645	.600	.575
		50	.815	.760	.710	.675	.635	.610
		75	.845	.800	.760	.725	.690	.670

k	t	n	.80	.90	.95	.975	.99	.995
4	3	30	.735	.675	.625	.585	.540	.520
		40	.770	.710	.665	.630	.590	.560
		50	.790	.740	.695	.660	.620	.600
		75	.825	.780	.740	.710	.680	.660

k	t	n	.80	.90	.95	.975	.99	.995
3	2	30	.765	.700	.645	.605	.560	.530
		40	.795	.730	.685	.645	.600	.575
		50	.815	.760	.710	.675	.635	.610
		75	.840	.795	.760	.725	.690	.670

k	t	n	.80	.90	.95	.975	.99	.995
5	1	30	.720	.660	.610	.575	.530	.510
		40	.750	.690	.650	.620	.580	.555
		50	.775	.720	.680	.650	.610	.590
		75	.810	.770	.730	.705	.670	.650

k	t	n	.80	.90	.95	.975	.99	.995
4	1	30	.735	.675	.625	.585	.540	.515
		40	.770	.710	.665	.630	.590	.560
		50	.790	.740	.695	.660	.620	.600
		75	.825	.780	.740	.715	.680	.660

k	t	n	.80	.90	.95	.975	.99	.995
5	2	30	.685	.630	.590	.555	.520	.490
		40	.720	.670	.630	.605	.560	.540
		50	.745	.700	.660	.635	.600	.580
		75	.790	.750	.715	.690	.655	.635

Table 4. F
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case
Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.0$

k	t	n	p^* .80	.90	.95	.975	.99	.995
5	3	30	.685	.630	.590	.555	.520	.490
		40	.720	.670	.630	.600	.560	.540
		50	.745	.700	.660	.635	.600	.580
		75	.790	.750	.715	.690	.655	.640

K	t	n	p^* .80	.90	.95	.975	.99	.995
5	4	30	.720	.660	.610	.575	.530	.500
		40	.750	.695	.650	.620	.580	.550
		50	.770	.720	.680	.650	.610	.590
		75	.810	.770	.730	.705	.670	.650

Table 5. F

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 1.25$

k	t	n		.80	.90	.95	.975	.99	.995
2	1	30		.800	.710	.645	.595	.530	.505
		40		.820	.745	.680	.635	.585	.550
		50		.840	.765	.710	.665	.615	.590
		75		.870	.805	.760	.720	.675	.650

k	t	n		.80	.90	.95	.975	.99	.995
4	2	30		.670	.610	.560	.520	.475	.450
		40		.710	.650	.605	.570	.525	.500
		50		.735	.680	.640	.605	.560	.540
		75		.780	.730	.690	.660	.625	.600

k	t	n		.80	.90	.95	.975	.99	.995
3	1	30		.730	.655	.600	.555	.505	.475
		40		.760	.695	.640	.600	.550	.525
		50		.785	.720	.670	.635	.590	.560
		75		.820	.765	.720	.690	.650	.625

k	t	n		.80	.90	.95	.975	.99	.995
4	3	30		.700	.630	.580	.535	.490	.460
		40		.735	.670	.620	.585	.535	.510
		50		.760	.700	.650	.615	.575	.550
		75		.800	.745	.705	.675	.635	.610

k	t	n		.80	.90	.95	.975	.99	.995
3	2	30		.730	.655	.600	.555	.505	.475
		40		.760	.695	.640	.600	.550	.525
		50		.785	.720	.670	.635	.590	.560
		75		.820	.765	.720	.690	.645	.620

k	t	n		.80	.90	.95	.975	.99	.995
5	1	30		.680	.610	.560	.525	.475	.450
		40		.715	.655	.605	.570	.525	.500
		50		.740	.685	.640	.605	.565	.540
		75		.780	.730	.695	.665	.625	.600

k	t	n		.80	.90	.95	.975	.99	.995
4	1	30		.700	.630	.575	.535	.490	.460
		40		.735	.670	.620	.585	.535	.510
		50		.760	.700	.650	.615	.575	.550
		75		.800	.745	.705	.675	.630	.610

k	t	n		.80	.90	.95	.975	.99	.995
5	2	30		.640	.585	.540	.505	.460	.440
		40		.680	.630	.585	.550	.510	.490
		50		.710	.660	.620	.585	.550	.525
		75		.755	.710	.675	.645	.610	.590

Table 5. F
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ, p^{*} and $\alpha = 1.25$

k	t	n		p^{*} .80	.90	.95	.975	.99	.995
5	3	30		.640	.585	.540	.505	.460	.440
		40		.680	.630	.630	.550	.510	.460
		50		.710	.660	.650	.585	.550	.525
		75		.755	.710	.675	.645	.610	.590

k	t	n		p^{*} .80	.90	.95	.975	.99	.995
5	4	30		.680	.610	.560	.525	.475	.450
		40		.715	.655	.610	.570	.525	.500
		50		.740	.685	.640	.605	.560	.540
		75		.785	.730	.695	.665	.625	.600

Table 6. F

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ , p^* and $\alpha = 1.5$

k	t	n	p^* .80	.90	.95	.975	.99	.995
2	1	30	.775	.680	.610	.560	.500	.465
		40	.805	.715	.655	.605	.545	.515
		50	.820	.740	.680	.635	.580	.550
		75	.850	.785	.730	.690	.640	.615

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	2	30	.640	.575	.525	.485	.440	.410
		40	.680	.620	.570	.530	.490	.460
		50	.710	.650	.605	.565	.525	.500
		75	.755	.700	.660	.630	.590	.570

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	1	30	.705	.625	.565	.520	.465	.435
		40	.740	.665	.610	.565	.515	.485
		50	.760	.695	.640	.600	.550	.525
		75	.800	.740	.695	.660	.615	.590

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	3	30	.670	.595	.540	.500	.450	.420
		40	.710	.640	.585	.545	.500	.470
		50	.730	.670	.620	.580	.535	.510
		75	.775	.720	.675	.645	.600	.580

k	t	n	p^* .80	.90	.95	.975	.99	.995
3	2	30	.705	.625	.565	.520	.465	.435
		40	.740	.665	.610	.565	.520	.485
		50	.760	.695	.640	.600	.550	.525
		75	.800	.740	.695	.660	.615	.590

k	t	n	p^* .80	.90	.95	.975	.99	.995
5	1	30	.650	.580	.525	.485	.440	.410
		40	.690	.620	.570	.535	.490	.460
		50	.715	.655	.610	.570	.525	.500
		75	.760	.705	.665	.630	.590	.570

k	t	n	p^* .80	.90	.95	.975	.99	.995
4	1	30	.670	.595	.540	.500	.450	.420
		40	.710	.640	.585	.545	.500	.470
		50	.735	.670	.620	.580	.535	.510
		75	.775	.720	.680	.645	.600	.580

k	t	n	p^* .80	.90	.95	.975	.99	.995
5	2	30	.610	.550	.500	.465	.420	.400
		40	.650	.595	.550	.515	.475	.450
		50	.680	.630	.585	.550	.510	.490
		75	.730	.680	.645	.615	.580	.555

Table 6. F
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case
Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^* and $\alpha = 1.5$

k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.610	.550	.500	.465	.420	.400
		40		.650	.595	.550	.515	.470	.450
		50		.680	.630	.585	.550	.510	.490
		75		.730	.680	.645	.615	.575	.555

k	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.650	.580	.525	.485	.440	.410
		40		.690	.620	.570	.535	.490	.460
		50		.715	.650	.610	.570	.525	.500
		75		.760	.710	.665	.630	.590	.570

Appendix G Probability Tables for Estimator $\hat{\alpha}$

Large Samples

Table 1. G

For Estimator $\hat{\alpha}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^*$ given values of k, t, δ, p^*

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
2	1	30	.855	.785	.735	.690	.640	.610
		40	.870	.815	.770	.725	.685	.655
		50	.885	.830	.790	.755	.710	.690
		75	.905	.860	.825	.795	.760	.740

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	2	30	.760	.710	.660	.630	.590	.560
		40	.790	.740	.700	.670	.630	.610
		50	.810	.765	.730	.700	.665	.645
		75	.840	.805	.775	.750	.720	.700

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
3	1	30	.810	.750	.700	.665	.620	.585
		40	.830	.780	.740	.700	.660	.630
		50	.850	.800	.760	.730	.690	.665
		75	.875	.830	.800	.775	.740	.720

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	3	30	.770	.720	.670	.635	.590	.565
		40	.800	.750	.710	.680	.640	.615
		50	.820	.775	.740	.710	.670	.650
		75	.850	.810	.780	.755	.730	.705

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
3	2	30	.800	.740	.695	.655	.610	.580
		40	.825	.770	.730	.695	.660	.630
		50	.845	.795	.760	.725	.690	.660
		75	.870	.830	.800	.770	.740	.720

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
5	1	30	.770	.720	.670	.635	.590	.565
		40	.800	.750	.710	.675	.640	.610
		50	.820	.775	.740	.705	.670	.650
		75	.850	.810	.780	.755	.720	.700

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
4	1	30	.790	.730	.685	.645	.600	.570
		40	.810	.760	.720	.690	.645	.620
		50	.830	.780	.750	.715	.675	.655
		75	.860	.820	.790	.765	.730	.710

			p^*					
k	t	n	.80	.90	.95	.975	.99	.995
5	2	30	.740	.690	.650	.615	.570	.545
		40	.770	.725	.690	.660	.620	.600
		50	.790	.750	.720	.690	.650	.630
		75	.825	.790	.760	.740	.710	.690

Table 1. G
(continued)

For Estimator $\widetilde{\beta}$ Selecting the t-best : Complete Large Sample Approximation Case

Finding the smallest n required for $P(CS \mid R) \geq p^{*}$ given values of k, t, δ , p^{*}

k	t	n		.80	.90	.95	.975	.99	.995
5	3	30		.730	.680	.640	.610	.570	.540
		40		.765	.720	.685	.655	.620	.595
		50		.790	.750	.710	.685	.650	.630
		75		.825	.790	.760	.735	.710	.690

k	t	n		.80	.90	.95	.975	.99	.995
5	4	30		.755	.700	.660	.625	.580	.550
		40		.785	.740	.700	.665	.630	.605
		50		.805	.760	.725	.695	.665	.640
		75		.840	.800	.770	.750	.720	.700

Appendix H Fortran Programs for Simulations

Comment : Small Sample Simulations using the maximum likelihood estimator for the parameter beta

```
      integer  iseed, nout, nz,num,itmax,n,k,ii,iter,tbest
      integer d, count
      double precision z(1000),drnnor,t(1000),tpop(1000,1000)
      double precision tsum(1000),hsum(1000),errel,xguess,x(2)
      double precision tbar(1000),h(1000),fnorm,beta(10,50000)
      double precision mmax(50000), mmin(50000),delta,prob
      double precision zone(1000),alpha
      external drnnor, rnset, umach, wrcrn,zplrc
      external dneqnf,fcn
      parameter(num=2,nn=30,kk=5,mm=50000)

      common h(1000),tbar(1000), tpop(1000,1000),ii,n,alpha,zone
      open(unit=15, file='bsalpha25n30.out')
      call umach(2, nout)
c     write(15,*) 'This is a new program'
      do k=2,kk
      do tbest=1,k-1
      do n=5,nn,5
      write(15,*)'k=',k,' n=',n, 't=',tbest
      nz=k*n
```

```
      iseed=123457
      call rnset(iseed)


      do iter=1,mm
      call drnnor(nz,z)


      do j=1,nz
      alpha=dble(0.25)
      zone(j)=z(j)*alpha
      t(j)= (zone(j)**2+zone(j)*sqrt(4.0+zone(j)**2)+2.00)/dble(2.0)
      enddo


      do j=1,k
      do i=1,n
      tpop(j,i)=t(i+(j-1)*n)
c     write(15,*) 'tpop(',j,',',i,')=', tpop(j,i)
      enddo
      enddo



      do j=1,k
      tsum(j)=0.0
      hsum(j)=0.0
```

```
      enddo

      do i=1,k
      do j=1,n
      tsum(i)=tsum(i)+tpop(i,j)
      hsum(i)=hsum(i)+dble(1.0)/tpop(i,j)
      enddo
      tbar(i)=tsum(i)/dble(n)
      h(i)=(dble(n)/hsum(i))
      enddo
      do i=1,k
c     write(15,991) tbar(i),h(i)
      enddo

      do ii=1,k
      xguess = sqrt(tbar(ii)*h(ii))

      errel=.005
      itmax=2500

      call dneqnf(fcn,errel,num,itmax,xguess,x,fnorm)
      beta(ii,iter)=x(1)
c     write(15,*) x(1), fnorm
```

```
        enddo
c       do i=1,k
c       write(15,*)'beta(',i,',',iter,')=',beta(i,iter),
c       enddo
c       the enddo below is for iteration
        enddo

c       Find the maximum and minimum

        do iter=1,mm
        mmax(iter)=beta(1,iter)
        do j=2,k-tbest
        if (beta(j,iter) .gt. mmax(iter)) then
                mmax(iter)=beta(j,iter)
        endif
        enddo
        mmin(iter)=beta(k-tbest+1,iter)
        do i=k-tbest+1,k
        if (beta(i,iter) .lt. mmin(iter)) then
        mmin(iter)=beta(i,iter)
        endif
        enddo
c       write(15,*) 'max=', mmax(iter),'min=',mmin(iter)
```

```
        enddo



        do d=30,100,1
        count=0
        delta=dble(d)/100.0
        do iter =1,mm
        if(delta*mmax(iter).lt. mmin(iter)) then
        count=count+1
        endif
        enddo
        prob=dble(count)/dble(mm)
        write(15,995)delta,prob
        enddo



c       The following are for the k,t, and n loops
        enddo
        enddo
        enddo
991     format('tbar=', f8.4, ' hbar=',f8.4)
```

```
995    format('delta=', f5.3, ' prob=',f10.8)


       end


       subroutine fcn(x,f,num)
       integer num,j
       double precision bk,tpop(1000,1000)
       double precision x(num),f(num),h(1000),tbar(1000)
       common h(1000),tbar(1000), tpop(1000,1000),ii,n
c      common ii,k,h,tbar,tpop,n
       bk=0.0
       do j=1,n
       bk=bk+dble(1.0)/(x(1)+tpop(ii,j))
       enddo
c      write(15,*) 'ii=',ii,'h(ii)',h(ii), 'tbar(ii)',tbar(ii)
       f(1)=x(1)**2-x(1)*(2*h(ii)+10.00/bk)+h(ii)*(tbar(ii)+10.00/bk)
       return
       end
```

Comment : Small Sample Simulations using the estimator betaprime for the parameter beta

```
      integer  iseed, nout, nz,num,itmax,n,k,ii,iter,tbest
      integer d, count
      double precision z(1000), drnnor,t(1000),tpop(1000,1000)
      double precision tsum(1000),hsum(1000),errel,xguess,x(2)
      double precision btilda(10,50000),bprime(10,50000)
      double precision tbar(1000),h(1000),fnorm,beta(10,50000)
      double precision mmax(50000), mmin(50000),delta,prob
      external drnnor, rnset, umach, wrcrn,zplrc
      external dneqnf,fcn
      parameter(num=2,nn=15,kk=5,mm=10)

      common h(1000),tbar(1000)
      common tpop(1000,1000),ii,n,btilda(10,50000),bprime(10,50000)
      open(unit=15, file='bs2betas.out')
      call umach(2, nout)
c     write(15,*) 'This is a new program'
      do k=2,kk
      do tbest=1,k-1
      do n=5,nn,5
      write(15,*)'k=',k,' n=',n, 't=',tbest
```

```
      nz=k*n
      iseed=123457
      call rnset(iseed)

      do iter=1,mm
      call drnnor(nz,z)

      do j=1,nz
      t(j)= (z(j)**2+z(j)*sqrt(4.0+z(j)**2)+2.00)/dble(2.0)
      enddo

      do j=1,k
      do i=1,n
      tpop(j,i)=t(i+(j-1)*n)
c     write(15,*) 'tpop(',j,',',i,')=', tpop(j,i)
      enddo
      enddo

      do j=1,k
      tsum(j)=0.0
      hsum(j)=0.0
      enddo
```

```
      do i=1,k
      do j=1,n
      tsum(i)=tsum(i)+sqrt(tpop(i,j))
      hsum(i)=hsum(i)+dble(1.0)/sqrt(tpop(i,j))
      enddo
      tbar(i)=tsum(i)/dble(n)
      h(i)=(dble(n)/hsum(i))
      bprime(i,iter)=tbar(i)*h(i)

      enddo
c     do i=1,k
c     write(15,991) tbar(i),h(i)
c     enddo

      do ii=1,k
      xguess = sqrt(tbar(ii)*h(ii))
      btilda(ii,iter)= sqrt(tbar(ii)*h(ii))
      errel=.001
      itmax=2500

      call dneqnf(fcn,errel,num,itmax,xguess,x,fnorm)
      beta(ii,iter)=x(1)
```

```
c       write(15,*) x(1), fnorm
        enddo
        do i=1,k
        write(15,*)'beta(',i,',',iter,')=',beta(i,iter),
        write(15,*)'btilda(',i,',',iter,')=',btilda(i,iter),
        write(15,*)'bprime(',i,',',iter,')=',bprime(i,iter),
        enddo
c       the enddo below is for iteration
        enddo


c       Find the maximum and minimum


        do iter=1,mm
        mmax(iter)=beta(1,iter)
        do j=2,k-tbest
        if (beta(j,iter) .gt. mmax(iter)) then
                mmax(iter)=beta(j,iter)
        endif
        enddo
        mmin(iter)=beta(k-tbest+1,iter)
        do i=k-tbest+1,k
        if (beta(i,iter) .lt. mmin(iter)) then
        mmin(iter)=beta(i,iter)
```

```
      endif
      enddo
c     write(15,*) 'max=', mmax(iter),'min=',mmin(iter)
      enddo


      do d=35,80,2
      count=0
      delta=dble(d)/100.0
      do iter =1,mm
      if(delta*mmax(iter).lt. mmin(iter)) then
      count=count+1
      endif
      enddo
      prob=dble(count)/dble(mm)
      write(15,995)delta,prob
      enddo


c     The following are for the k,t, and n loops
      enddo
```

```
      enddo
      enddo
991   format('tbar=', f8.4, ' hbar=',f8.4)
995   format('delta=', f5.3, ' prob=',f10.8)

      end



      subroutine fcn(x,f,num)
      integer num,j
      double precision bk,tpop(1000,1000)
      double precision x(num),f(num),h(1000),tbar(1000)
      common h(1000),tbar(1000), tpop(1000,1000),ii,n
c     common k,h,tbar,tpop,ii,n
      bk=0.0
      do j=1,n
      bk=bk+dble(1.0)/(x(1)+tpop(ii,j))
      enddo
c     write(15,*) 'ii=',ii,'h(ii)',h(ii), 'tbar(ii)',tbar(ii)
      f(1)=x(1)**2-x(1)*(2*h(ii)+10.00/bk)+h(ii)*(tbar(ii)+10.00/bk)
      return
      end
```

Comment : Small Sample Simulations using the estimator betatilda for the parameter beta

```
      integer  iseed, nout, nz,num,itmax,n,k,ii,iter,tbest
      integer d, count
      double precision z(1000), drnnor,t(1000),tpop(1000,1000)
      double precision tsum(1000),hsum(1000),errel,xguess,x(2)
      double precision tbar(1000),h(1000),fnorm,beta(10,50000)
      double precision mmax(50000), mmin(50000),delta,prob
      double precision zone(1000), alpha
      external drnnor, rnset, umach, wrcrn,zplrc
      external dneqnf,fcn
      parameter(num=2,nn=30,kk=5,mm=50000)

      common h(1000),tbar(1000), tpop(1000,1000),ii,n
      open(unit=15, file='bs50smalltilda.out')
      call umach(2, nout)
c     write(15,*) 'This is a new program'
      do k=2,kk
      do tbest=1,k-1
      do n=5,nn,5
      write(15,*)'k=',k,' n=',n, 't=',tbest
```

```
      nz=k*n
      iseed=123457
      call rnset(iseed)

      do iter=1,mm
      call drnnor(nz,z)

      do j=1,nz
                  alpha=dble(0.5)
      zone(j)=z(j)*alpha

      t(j)= (zone(j)**2+zone(j)*sqrt(4.0+zone(j)**2)+2.00)/dble(2.0)
      enddo

      do j=1,k
      do i=1,n
      tpop(j,i)=t(i+(j-1)*n)
c     write(15,*) 'tpop(',j,',',i,')=', tpop(j,i)
      enddo
      enddo

      do j=1,k
```

```
      tsum(j)=0.0
      hsum(j)=0.0
      enddo

      do i=1,k
      do j=1,n
      tsum(i)=tsum(i)+tpop(i,j)
      hsum(i)=hsum(i)+dble(1.0)/tpop(i,j)
      enddo
      tbar(i)=tsum(i)/dble(n)
      h(i)=(dble(n)/hsum(i))
      beta(i,iter)=sqrt(tbar(i)*h(i))
      enddo
c     do i=1,k
c     write(15,991) tbar(i),h(i)
c     enddo

c     do ii=1,k
c     xguess = sqrt(tbar(ii)*h(ii))

c     errel=.01
c     itmax=2500
```

```
c       call dneqnf(fcn,errel,num,itmax,xguess,x,fnorm)
c       beta(ii,iter)=x(1)
c       write(15,*) x(1), fnorm
c       enddo
c       do i=1,k
c       write(15,*)'beta(',i,',',iter,')=',beta(i,iter),
c       enddo
c       the enddo below is for iteration
        enddo

c       Find the maximum and minimum

        do iter=1,mm
        mmax(iter)=beta(1,iter)
        do j=2,k-tbest
        if (beta(j,iter) .gt. mmax(iter)) then
                mmax(iter)=beta(j,iter)
        endif
        enddo
        mmin(iter)=beta(k-tbest+1,iter)
        do i=k-tbest+1,k
        if (beta(i,iter) .lt. mmin(iter)) then
        mmin(iter)=beta(i,iter)
```

```
      endif
      enddo
c     write(15,*) 'max=', mmax(iter),'min=',mmin(iter)
      enddo

      do d=30,50,1
      count=0
      delta=dble(d)/100.0
      do iter =1,mm
      if(delta*mmax(iter).lt. mmin(iter)) then
      count=count+1
      endif
      enddo
      prob=dble(count)/dble(mm)
      write(15,995)delta,prob
      enddo
c     The following are for the k,t, and n loops
      enddo
      enddo
      enddo
991   format('tbar=', f8.4, ' hbar=',f8.4)
995   format('delta=', f5.3, ' prob=',f10.8)
      end
```

Comment : Small Sample Simulations for the parameter alpha using betatilda in the mle expression

```
      integer  iseed, nout, nz,num,itmax,n,k,ii,iter,tbest
      integer d, count
      double precision z(1000), drnnor,t(1000),tpop(1000,1000)
      double precision tsum(1000),hsum(1000),errel,xguess,x(2)
      double precision tbar(1000),h(1000),fnorm,alp(10,50000)
      double precision beta(10,50000),one(10,50000),two(10,50000)
      double precision mmax(50000), mmin(50000),delta,prob
      external drnnor, rnset, umach, wrcrn,zplrc
      external dneqnf,fcn
      parameter(num=2,nn=30,kk=5,mm=50000)

      common h(1000),tbar(1000), tpop(1000,1000),ii,n
      open(unit=15, file='bsamle.out')
      call umach(2, nout)
c     write(15,*) 'This is a new program'
      do k=2,kk
      do tbest=1,k-1
      do n=5,nn,5
```

```
      write(15,*)'k=',k,' n=',n, 't=',tbest
      nz=k*n
      iseed=123457
      call rnset(iseed)

      do iter=1,mm
      call drnnor(nz,z)

      do j=1,nz
      t(j)= (z(j)**2+z(j)*sqrt(4.0+z(j)**2)+2.00)/dble(2.0)
      enddo

      do j=1,k
      do i=1,n
      tpop(j,i)=t(i+(j-1)*n)
c     write(15,*) 'tpop(',j,',',i,')=', tpop(j,i)
      enddo
      enddo

      do j=1,k
      tsum(j)=0.0
      hsum(j)=0.0
```

```
      enddo

      do i=1,k
      do j=1,n
      tsum(i)=tsum(i)+tpop(i,j)
      hsum(i)=hsum(i)+dble(1.0)/tpop(i,j)
      enddo
      tbar(i)=tsum(i)/dble(n)
      h(i)=(dble(n)/hsum(i))
c     alp(i,iter)=sqrt(dble(2.0)*sqrt(tbar(i)/h(i))-dble(2.0))
      enddo
c     do i=1,k
c     write(15,991) tbar(i),h(i)
c     enddo

      do ii=1,k
      xguess = sqrt(tbar(ii)*h(ii))

      errel=.01
      itmax=2500

      call dneqnf(fcn,errel,num,itmax,xguess,x,fnorm)
      beta(ii,iter)=x(1)
```

```
      one(ii,iter)=tbar(ii)/beta(ii,iter)
      two(ii,iter)=beta(ii,iter)/h(ii)
      alp(ii,iter)=sqrt(one(ii,iter)+two(ii,iter)-dble(2.0))
      write(15,*) x(1), fnorm
      enddo
c     do i=1,k
c     write(15,*)'beta(',i,',',iter,')=',beta(i,iter),
c     enddo
c     the enddo below is for iteration
      enddo

c     Find the maximum and minimum


      do iter=1,mm
      mmax(iter)=alp(1,iter)
      do j=2,k-tbest
      if (alp(j,iter) .gt. mmax(iter)) then
            mmax(iter)=alp(j,iter)
      endif
      enddo
      mmin(iter)=alp(k-tbest+1,iter)
      do i=k-tbest+1,k
```

```
        if (alp(i,iter) .lt. mmin(iter)) then
        mmin(iter)=alp(i,iter)
        endif
        enddo
c       write(15,*) 'max=', mmax(iter),'min=',mmin(iter)
        enddo


        do d=20,100,1
        count=0
        delta=dble(d)/100.0
        do iter =1,mm
        if(delta*mmax(iter).lt. mmin(iter)) then
        count=count+1
        endif
        enddo
        prob=dble(count)/dble(mm)
        write(15,995)delta,prob
        enddo
```

```
c     The following are for the k,t, and n loops
      enddo
      enddo
      enddo
991   format('tbar=', f8.4, ' hbar=',f8.4)
995   format('delta=', f5.3, ' prob=',f10.8)
      end


      subroutine fcn(x,f,num)
      integer num,j
      double precision bk,tpop(1000,1000)
      double precision x(num),f(num),h(1000),tbar(1000)
      common h(1000),tbar(1000), tpop(1000,1000),ii,n
c     common k,h,tbar,tpop
      bk=0.0
      do j=1,n
      bk=bk+dble(1.0)/(x(1)+tpop(ii,j))
      enddo
c     write(15,*) 'ii=',ii,'h(ii)',h(ii), 'tbar(ii)',tbar(ii)
      f(1)=x(1)**2-x(1)*(2*h(ii)+10.00/bk)+h(ii)*(tbar(ii)+10.00/bk)
      return
      end
```

Comment : Large Sample Normal Approximations for the parameter beta

```
integer nout
integer num,k,tbest,n,d,kmt,t

double precision delta,alpha
double precision A,B,ERRABS,ERRREL,RESULT,ERREST
double precision pstar,F,H,P,alp,low,high

parameter(alpha=1.0,low=-5.0,high=5.0)

common k,tbest,n,d,kmt,t,delta,alp

Intrinsic DABS,DEXP,SQRT
External umach,dqdags,F,H,P,dnordf

parameter (num=2,nn=75,kk=5)

call umach (2,nout)
open(unit=15,file='bnoapp.out')
```

```
do k=2,kk

do tbest=1,k-1

do n=75,nn,5

do d=30,90,1


kmt=k-tbest

t=tbest

delta=dble(d)/100.0

alp=alpha



write(15,990) k,t,n,alp



A=low

B=high


ERRABS=0.0

ERRREL=0.001


call dqdags (F,A,B,ERRABS,ERRREL,RESULT,ERREST)


pstar=(k-t)*RESULT
```

```
      write(15,995) delta,pstar,error

      enddo
      enddo
      enddo
      enddo

990   format('k= ',i2,' t= ',i2,' n= ',i3,' alpha= ',f9.3)
995   format(' delta=',f9.3, ' prob=',f20.8, ' error=',f10.8 )

      end

* Find the integral desired

      double precision Function F(x)
      integer k,tbest,kmt,t
      double precision delta
      double precision x,DEXP,dnordf,H,P
      common k,tbest,n,d,kmt,t,delta,alp
      Intrinsic DEXP,DSQRT
      External dnordf,H,P
```

```
xd=dble(x)*delta

F=(H(x-1))**(kmt-1)*(1-H((x-1)*delta+delta-1))**t*P(x)


return

end


double precision Function H(x)

double precision x

double precision dnordf

common k,tbest,n,d,kmt,t,delta,alp

Intrinsic DSQRT

External dnordf

H=dnordf(7.5452*x)

return

end

double precision Function P(x)

double precision DEXP,x,pi

common k,tbest,n,d,kmt,t,delta,alp

Intrinsic DSQRT,DEXP

pi=const("PI")

P=3.010099*DEXP(-28.465*(x-1)**2)

return

end
```

Comment : Large Sample Birnbaum-Saunders Approximations for the parameter beta

```
integer nout,IRULE
integer num,k,tbest,n,d,kmt,t

double precision delta,alpha
double precision A,B,ERRABS,ERRREL,RESULT,ERREST
double precision DABS,DEXP,F,G,P,R
double precision dnordf,alp,low,hi
parameter(alp=0.25,low=0.0000000001,hi=500.00)
common k,tbest,n,d,kmt,t,delta,alpha,const

Intrinsic DABS,DEXP,DSQRT
External umach,dqdag,F,G,P,R,dnordf

parameter(num=2,nn=75,kk=5)

call umach(2,nout)
open (unit=15,file='appb4.out')

do k=2,kk
do tbest=1,k-1
```

```
do n=75,nn,5

do d=40,100,1


kmt=(k-tbest)

t=tbest

delta=dble(d)/100.00

alpha=alp



write(15,990) k,tbest,n


A=low

B=hi

ERRABS=0.0

ERRREL=0.001


IRULE=6

call dqdag(F,A,B,ERRABS,ERRREL,IRULE,RESULT,ERREST)

pstar=kmt*RESULT


write(15,995) delta,pstar,ERREST


enddo
```

```
      enddo
      enddo
      enddo

990   format(' k= ',i2,' t= ',i2,' n= ',i3)
995   format('delta=',f9.3,'prob=',f25.8,'error=',f9.8)

      end

      double precision Function F(x)
      integer k,tbest,kmt,t
      double precision x,delta,xd
      double precision DEXP,dnordf,G,P
      common k,tbest,n,d,kmt,t,delta
      Intrinsic DEXP,DSQRT
      External dnordf,G,P,R

      xd=x*delta

      F=(G(x))**(kmt-1)*(1-G(x*delta))**t*P(x)*R(x)

      return
```

```
end

double precision Function G(x)
double precision x
double precision dnordf
common k,tbest,n,d,kmt,t,delta
external dnordf,DSQRT

G=dnordf(9.819805*(x**(0.5)-x**(-0.5)))
return
end

double precision Function P(x)
double precision DEXP,x,pi
common k,tbest,n,d,kmt,t,delta
intrinsic dexp,sqrt
pi=const("PI")
P=1.958768*(x**(-0.5)+x**(-1.5))
return
end

double precision Function R(x)
double precision DEXP,x,pi
```

```
common k,tbest,n,d,kmt,t,delta

intrinsic dexp,sqrt

pi=const("PI")

R=DEXP(-48.21429*(x+x**(-1.0)-2.0))

return

end
```

Comment : Large Sample Normal Approximations for the parameter alpha

```
integer nout

integer num,k,tbest,n,d,kmt,t


double precision delta

double precision A,B,ERRABS,ERRREL,RESULT,ERREST

double precision DABS,DEXP,F,H,P

double precision error,dnordf,low,hi

parameter(low=-100.00,hi=100.00)

common k,tbest,n,d,kmt,t,delta


Intrinsic DABS,DEXP,SQRT

External umach,dqdags,F,H,P,dnordf


parameter(num=2,nn=75,kk=5)


call umach(2,nout)

open (unit=15,file='alpapprox1.out')


do k=2,kk

do tbest=1,k-1
```

```
do n=30,nn,5
do d=40,100,1

kmt=(k-tbest)
t=tbest
delta=dble(d)/100.00

write(15,990) k,tbest,n

A=low
B=hi
ERRABS=0.0
ERRREL=0.001

call dqdags(F,A,B,ERRABS,ERRREL,RESULT,ERREST)
pstar=kmt*RESULT

write(15,995) delta,pstar,error

enddo
enddo
enddo
enddo
```

```
990   format(' k= ',i2,' t= ',i2,' n= ',i3)

995   format('delta=',f9.3,'prob=',f25.8,'error=',f9.8)

      end

      double precision Function F(x)
      integer k,tbest,kmt,t
      double precision x,delta,xd
      double precision DEXP,dnordf,H,P
      common k,tbest,n,d,kmt,t,delta
      Intrinsic DEXP,DSQRT
      External dnordf,H,P

      xd=x*delta

      F=(H(x-1))**(kmt-1)*(1-H(delta*(x-1)+delta-1))**t*P(x)

      return
      end

      double precision Function H(x)
```

```
double precision x
double precision dnordf
common k,tbest,n,d,kmt,t,delta
external dnordf,SQRT
H=dnordf((x)*SQRT(2.0*n))
return
end



double precision Function P(x)
double precision DEXP,x,pi
common k,tbest,n,d,kmt,t,delta
intrinsic dexp,sqrt
pi=const("PI")

P=(sqrt(n/pi))*DEXP(-1.0*n*(x-1)**2)
return
end
```

γ

VITA

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Doctor of Philosophy

Thesis: SELECTING t – BEST OF SEVERAL BIRNBAUM – SAUNDERS POPULATIONS BASED ON THE PARAMETERS

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