UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

COMPONENT IMPORTANCE BALANCING OF MULTI-COMMODITY NETWORKS

A THESIS

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE

By

MICK BARLACH CHRISTENSEN
Norman, Oklahoma
2022

COMPONENT IMPORTANCE BALANCING OF MULTI-COMMODITY NETWORKS

A THESIS APPROVED FOR THE
SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

BY THE COMMITTEE CONSISTING OF

Dr. Kash Barker, Chair

Dr. Andrés González

Dr. Rui Zhu

To everyone who lent me their patience, time, and love.

Acknowledgements

First and foremost, I thank Dr. Kash Barker for advising me on this thesis. Not only has he greatly improved the quality of work that would otherwise have been provided in this paper, he is also the main reason for me deciding to do a thesis at all. Had it not been for an email written at the end of my first semester of my accelerated degree, things would likely have looked much different for me. He is passionate about the College and ISE and it shows, and it inspires.

Second, I would like to thank Dr. Andrés González for the great help he provided in the thinking stages of the model development. His great knowledge is an immense gift to the School.

Third, a thank you to Dr. Rui Zhu for agreeing to join my committee, and to Dr. Jonas Johansson for collaborating and sharing the data used in this paper.

Finally, a big thank you to all the friends and educators I have met during my studies at OU and in Oklahoma. The past five years have been a mix of great and troubling times, but the Mick at this end of the experience is much stronger than the one first experiencing the sweltering heat and humidity back in August 2017.

Mick Christensen

Table of Contents

List of Tables

List of Figures

Abstract

Determining which network components to focus on in case of a disruption can be determined by the component with the greatest importance to the network. This study extends a balancing component added to a list of optimization targets in multi-state system to a multi-commodity network. A bi-objective optimization problem is developed in which one objective seeks to minimize the difference in a flow-based component importance measure across one-at-a-time interdiction scenarios by adding capacity to components, while the other minimizes unmet demand to encourage flow through the network. The optimization approach is applied to a multi-commodity Swedish railway network, where a set of link capacity increases per commodity are suggested based on the desire to balance the network.

Chapter 1.0 Introduction and Motivation

Understanding which critical infrastructure systems need more attention and resources is a growing and important research area. Prioritizing these resources can enhance national security and reduce risk in an increasingly interconnected and interdependent system [1, 2]. These critical infrastructures include transportation, energy, water, and communications and provide vital services to a nation. The severity of disruptions to these infrastructures is exemplified by the costs of downtime and recovery. Directly after Hurricane Harvey hit Texas in August 2017, 10% of the entire United States trucking industry was affected, with 25% of the regional trucking industry still affected a month later [3]. Along with this comes issues of decrease in supply and rising costs of transportation, eventually reaching the consumers. Threats to critical infrastructure, including physical and climate threats, are of growing interest for policy makers and researchers alike, with an interest to "safeguard and strengthen the systems" [4]. The resilience of transportation infrastructure is of particular importance to the successes of other infrastructures due to its role in the supply chain and is under increasing stress with rising requirements for maintenance and declining performance [5, 6].

There are several different definitions of resilience in various domains. For a transportation network, one way is as a metric for measuring a system's performance in the event of a disruption, which can be represented graphically as seen in Figure 1 [7]. The figure is split in two after a disruptive event occurs: vulnerability and recoverability. Vulnerability is defined as the inability of the system to sustain its performance immediately after a disruption, while recoverability is defined as the ability of that disrupted system to recover in a timely manner [7-9].

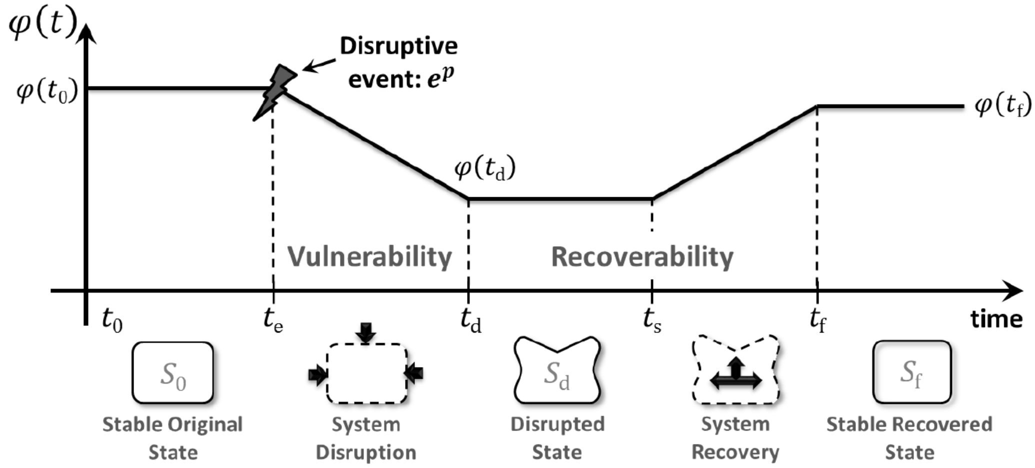

Figure 1. Network performance, $\varphi(t)$, over different network states.

The primary approaches for network vulnerability analysis have been broadly classified as scenario-specific, strategy-specific, simulation, and mathematical modeling methodologies [10]. Interdiction is a common theme for all the methodologies, where a network component is disrupted and its effect on the system is evaluated. Strategy-specific methodologies follow a hypothesized sequence of disruptions under the assumptions of weaker or more at-risk areas [11]. Simulation methodologies and mathematical modeling assessments are useful when little is known about the state of the network or when identifying the scenario with the greatest risk to the network [12, 13]. Scenario-specific methodologies will be utilized in this paper and measures the impact of removing one link at a time.

The performance of a network can be described using importance measures. Graph-theoretic measures, such as centrality-based measures, i.e., closeness, betweenness, straightness, and information centrality, have been developed to classify the performance of a network [14]. However, using flow-based network vulnerability measures is a newer research area that can quantify the impact of disruption to an individual component on the efficiency and connectivity of a network [15, 16]. Hence, flow-based measures may be more useful for decision making purposes, utilizing network efficiency measures such as the N-Q method for capturing the demands, flows, costs, and behavior of networks to assess the importance of network components, or quantifying multi-industry operability after a drop in commodity flow [17, 18].

When evaluating the network using importance measures, certain links may prove to be more important to the performance of the network. Under a disruptive scenario, these links would tend to have a greater impact on the network performance if sufficient redundancy is not available in the network. Therefore, it is a desirable property of a network system to avoid bottlenecks or overly high-performing components in addition to optimizing other stakeholder targets. Prior work has been done in designing systems such that each component has similar importance values throughout the system [19-21]. However, to the author's knowledge, this approach has not been extended to multi-commodity networks utilizing flow-based importance measures.

1.1 Research Focus

Transportation networks contain multiple commodities that should be routed to the customer on time. Since multiple types of commodities need to be routed on each link and throughout the network, certain links may be more important in a network than others. This study considers the links throughout a network and seeks to spread out the importance of links to create a more resilient network.

Through the balancing of network components in a multi-commodity network, the model seeks to provide decision makers with information of which links require more attention in the event of a network disruption. An assumption of this approach is that each link has an equal chance of being disrupted; thus, networks without likely sequences of disruptions, such as in the case of strategy-specific interdiction, can be evaluated. The network vulnerability importance measure utilized in this section was developed in a prior study and involves the unmet demand throughout the network [22].

The remainder of this paper is arranged as follows. Section 2 provides definitions and notation for multi-commodity networks, the network component importance measure, an extension of the balanced system method, an initializing optimization model, and the balanced optimization model. Section 3 introduces a Swedish railway case study and demonstrates the proposed network reduction methodology. Section 4 offers the results of the model and the decision analysis, while Section 5 offers conclusions.

Chapter 2.0 Proposed Methodology

In this section, the methodology used to define multi-commodity network flow, balanced importance measures, multi-objective optimization, the initial optimization model, and the balancing optimization model are described.

2.1 Multi-Commodity Network Flow

This paper extends a model that adapted the traditional minimum-cost multi-commodity network flow problem in which the goal is to minimize the cost of transportation while meeting all demand. For the adapted model, the constraint that all demand must be met was slackened, while the cost minimizing objective was replaced by an unmet demand minimizing objective meant to measure the network's ability to reroute commodities [18, 22].

Let a network be denoted by $G = (N, A)$, where N is a set of n nodes (or vertices) and A is a set of links (or edges). Additionally, let there be $k = 1, \dots, K$ commodities. Then, the capacity of link (i, j) from node i to node j for commodity k is represented by c_{ij}^k. Each commodity has a set of supply nodes in the set S^k represented by s_i^k with supply values of λ_i^k and demand nodes D^k represented by d_j^k with demand values of μ_j^k. It is an assumption of the model that a node can be a supply node for a commodity and a demand node for another, but not both a supply and demand node for the same commodity.

Let the set of interdicted links, L, be a subset of all links, A. Then, if the link $(i, j) = l$ in interdiction scenario $l \in L$, $c_{ij}^k = 0$ and $c_{ji}^k = 0$ for all commodities $k \in K$. That is, no flow of commodities is allowed from node i to node j and vice versa for that interdiction scenario. Then, let α_{ij}^{k} be the percent increase in capacity across link $(i, j) \in A$ used to reroute the flow of commodities that is blocked from the interdiction of link l.

2.2 Importance Measure Balancing

This section describes the balanced optimization component for a multi-state system and extends it to a multi-commodity network.

2.2.1 Multi-State System Balancing

In system design, it is a desirable property to avoid bottlenecks or overly high-performing components in addition to optimizing economic or safety goals. One way to achieve this is to design a system such that each component has similar importance values [19-21]. To achieve this, an importance measure balancing component σ_I can be added to the optimization targets as in Eq. (1):

$$\sigma_I = \sqrt{\overline{I^2} - \bar{I}^2} \tag{1}$$

The balancing component, σ_I, is calculated by the difference in the average squared importance, $\overline{I^2}$, and the square of the average importance, $\bar{I}^2$, of component j for generic importance measure I, I_j. The functions in Eqs. (3)(4) below show the calculation of $\overline{I^2}$ and $\bar{I}$, respectively:

$$\overline{I^2} = \frac{1}{n}\sum_{j=1}^{n} I_j^2 \tag{2}$$

$$\bar{I} = \frac{1}{n}\sum_{j=1}^{n} I_j \tag{3}$$

Then, if $\sigma_I = 0$, the importance of all $j = 1,2, \ldots, J$ system components are equal and the system is perfectly balanced with respect to the importance measure I, which meets the desired quality of the system.

2.2.2 Multi-Commodity Network Balancing

The multi-state system balancing method is extended to multiple commodities in an interdicted network. First, an importance measure I_l^k is defined as the importance of interdicted link $l \in L$ to the network for commodity $k \in K$. The average importance across interdiction scenarios, $\overline{I^k}$, can then be defined by the sum of the importance of interdicted link l, I_l^k, for all interdicted links $l \in L$, divided by the number of interdicted links as in Eq. (4) below:

$$\overline{I^k} = \frac{1}{|L|} \sum_{l \in L} I_l^k \tag{4}$$

The balancing component in this case, then, is calculated by the difference between the importance of interdicted link l, I_l^k, and the average of all interdicted links' importance, summed across all commodities $k \in K$ and interdicted links $l \in L$ as shown in Eq. (5) below:

$$\sum_{k \in K} \sum_{l \in L} \left| \overline{I^k} - I_l^k \right| \tag{5}$$

Thus, the balancing term in Eq. (5) will seek to balance the importance for all interdicted links and commodities.

The importance measure used in this model is the sum of the total unmet demand proportion for all commodities k for all interdiction scenarios l as shown in Eq. (6) below:

$$I_l^k = \sum_{j \in D^k} \frac{\mu_j^k - \left(\sum_{i:(i,j)\in A} x_{ijl}^k - \sum_{i:(i,j)\in A} x_{jil}^k\right)}{\mu_j^k}; \quad \forall k \in K, l \in L \tag{6}$$

The proportion of unmet demand for commodity k in interdiction scenario l, I_l^k, is calculated by the difference in demand of commodity k at node j, μ_j^k, and the flow of commodity k remaining in node j, standardized by μ_j^k. The flow of commodity k remaining in demand node j is calculated as the difference between the flow of commodity k in and out of the node. This quantity is then summed and minimized across all demand nodes $j \in D^k$ for all commodities $k \in K$ and interdiction scenarios $l \in L$.

2.3 Multi-Objective Optimization

As the importance measure used for this study is minimizing unmet demand and subtracting it from the average as shown above, the simplest solution is for no flow to go through the network; hence, the difference in importance, $\overline{I^k} - I_l^k$, will be 0 for all interdicted links $l \in L$ and commodities $k \in K$. To encourage demand through the network, a second objective, minimizing total unmet demand for all interdiction scenarios l and commodities k, defined in Section 2.4, will need to be put in place.

There are multiple ways to deal with multi-objective problems, such as the global criterion approach, goal programming, weighted sum method, or ϵ-constraint method [23, 24]. The global criterion approach is an a-priori method, which searches for a solution as close to a predetermined ideal vector as possible. Goal programming assigns some weight to objectives and applies deviational variables to get close to desired values without strictly enforcing constraints. The weighted sum method assigns a weight to each objective based on its importance relative to the other objectives. However, these weights are usually arbitrary, and a Pareto-frontier is often estimated by varying the weights of the objectives, which is then evaluated by decision-makers. The ϵ-constraint method deals with multi-objective optimization problems by writing all objectives but one as constraints [25, 26] and a variation of it will be utilized for this study.

For a multi-objective optimization problem with objectives $f_1(x), f_2(x), \dots, f_M(x)$, choose an objective to be the main objective, $f_q(x)$. This is usually the objective most important to the stakeholders. This objective is minimized as in Eq. (7) below:

$$minimize: \;\; f_q(x) \tag{7}$$

Then, let the optimization problem be subject to the following constraints in Eq. (8):

$$f_i \leq \epsilon_i; \quad (i = 1,2, q-1, q+1, \dots, M) \tag{8}$$

The ϵ_i in Eq. (8) are limits to the values of the other objectives, i.e., objective f_i cannot be less than a certain threshold ϵ_i. In the case of a bi-objective problem, such as the one utilized in this paper, this would be formulated as in Eqs. (9)(10):

$$minimize: \;\; f_1(x) \tag{9}$$

$$subject\ to\ f_2 \leq \epsilon_2 \tag{10}$$

2.4 Initial Interdiction Optimization Model

This section defines the decision variables, objectives, and constraints of the linear programming model that minimizes unmet demand for each interdiction scenario to gain baseline values.

2.4.1 Decision Variables and Objectives

The optimization model can be formulated as a linear programming model with a set of decision variables x_{ijl}^{k}, the flow of commodity $k \in K$ across link $(i,j) \in A$ in interdiction scenario $l \in L$.

After the variables are defined, the unmet demand minimizing function is added as an objective as shown below in Eq. (11):

$$minimize: \quad \sum_{k \in K} \sum_{j \in D^k} \frac{\mu_j^k - \left(\sum_{i:(i,j)\in A} x_{ijl}^k - \sum_{i:(i,j)\in A} x_{jil}^k\right)}{\mu_j^k}; \quad \forall l \in L \tag{11}$$

The objective is to minimize the sum of total unmet demand percentage for all commodities k across all interdiction scenarios l. This is calculated in the same manner as the importance measure in Eq. (6), and is performed iteratively across all interdiction scenarios $l \in L$. The values of unmet demand at node $j \in D^k$ for all commodities k in interdiction scenario l, seen in Eq. (12) below, are then stored as parameters to be used as minimum values constraints in the next optimization model.

$$\epsilon_l = \sum_{k \in K} \sum_{j \in D^k} \frac{\mu_j^k - \left(\sum_{i:(i,j)\in A} x_{ijl}^k - \sum_{i:(i,j)\in A} x_{jil}^k\right)}{\mu_j^k}; \quad \forall l \in L \tag{12}$$

2.4.2 Model Constraints

The flow of commodity k across link (i,j) for all interdiction scenarios l, x_{ijl}^{k}, is subject to the capacity constraint shown in Eq. (13) below:

$$x_{ijl}^{k} \leq c_{ij}^{k}; \quad \forall (i,j) \in A, k \in K, l \in L \tag{13}$$

The constraint in Eq. says the flow, x_{ijl}^{k}, cannot exceed the capacity at that link, c_{ij}^{k}.

$$-\sum_{j:(i,j)\in A} x_{ijl}^{k} + \sum_{j:(j,i)\in A} x_{jil}^{k} \leq \mu_{i}^{k}; \quad \forall i \in D^{k}, k \in K, l \in L \tag{14}$$

$$\sum_{j:(i,j)\in A} x_{ijl}^{k} - \sum_{j:(j,i)\in A} x_{jil}^{k} \leq \lambda_{i}^{k}; \quad \forall i \in S^{k}, k \in K, l \in L \tag{15}$$

$$\sum_{j:(i,j)\in A} x_{ijl}^{k} - \sum_{j:(j,i)\in A} x_{jil}^{k} = 0; \quad \forall i \in N \backslash \{D^{k}, S^{k}\}, k \in K, l \in L \tag{16}$$

The constraints in Eqs. (14)(15)(16) above balance the flow across the network. Eq. (14) sets the difference in flow in and out of demand node $j \in D^{k}$ for commodity k to no more than the demand of commodity k at node j, μ_{j}^{k}. Eq. (15) sets the difference in flow out and in of supply node $j \in S^{k}$ for commodity k to no more than the supply of commodity k at node j, λ_{j}^{k}. Eq. (16) ensures that the flow into node i equals the flow out of node i for all nodes i in N, excluding all supply and demand nodes for commodity k.

$$x_{ijl}^{k} = 0; \quad (i,j) = l \quad \forall l \in L, k \in K \tag{17}$$

$$x_{jil}^{k} = 0; \quad (i,j) = l \quad \forall l \in L, k \in K \tag{18}$$

The constraints shown above in Eqs. (17)(18) prevent flow across interdicted link (i,j) in interdiction scenario l for all commodities k. This constraint is performed iteratively for each interdicted link $l \in L$.

Finally, all flow variables are nonnegative as shown in the constraint in Eq. (19) below:

$$x_{ijl}^{k} \geq 0; \quad \forall (i,j) \in A, k \in K, l \in L \tag{19}$$

2.5 Balanced Optimization Model

This section defines the decision variables, objectives, and constraints of the importance balancing linear programming model.

2.5.1 Decision Variables and Objectives

The formulation for the flow decision variables, x_{ijl}^{k}, is the same as in section 2.4. However, a second set of variables representing the percentage increase in capacity for link $(i, j) \in A$ for commodity $k \in K$, α_{ij}^{k} is added.

After the variables are defined, the importance measure balancing function is added as an objective as shown below in Eq. (20):

$$minimize: \sum_{k \in K} \sum_{l \in L} \left| \bar{I}^{k} - I_{l}^{k} \right| \qquad (20)$$

Since this problem is to be formulated as a linear problem, however, the importance measure, I_{l}^{k}, and average importance measure across interdiction scenarios, $\bar{I}^{k}$, are replaced by a placeholder variable, U_{l}^{k}, as shown in Eq. (21). Two constraints are then added in Eqs. (22)(23) that constrain the value of U_{l}^{k} to $\left| \bar{I}^{k} - I_{l}^{k} \right|$:

$$minimize: \sum_{k \in K} \sum_{l \in L} U_{l}^{k} \qquad (21)$$

$$I_{l}^{k} - \bar{I}^{k} \leq U_{l}^{k}; \quad \forall l \in L, k \in K \qquad (22)$$

$$-\left(I_{l}^{k} - \bar{I}^{k} \right) \leq U_{l}^{k}; \quad \forall l \in L, k \in K \qquad (23)$$

The two constraints in Eqs. (22)(23), in combination with the objective function which seeks to minimize the differences in Eq. (21), ensure that:

1. When $I_{l}^{k} - \bar{I}^{k} > 0$, U_{l}^{k} is minimized to $I_{l}^{k} - \bar{I}^{k}$.
2. When $I_{l}^{k} - \bar{I}^{k} < 0$, U_{l}^{k} is minimized to $-\left(I_{l}^{k} - \bar{I}^{k} \right)$.

3. When $I_l^k - \overline{I^k} = 0$, U_l^k is minimized to 0.

$$minimize: \sum_{l\in L}\sum_{k\in K}\sum_{j\in D^k}\frac{\mu_j^k - \left(\sum_{i:(i,j)\in A} x_{ijl}^k - \sum_{i:(i,j)\in A} x_{jil}^k\right)}{\mu_j^k} \tag{24}$$

The second objective, minimizing the sum of total unmet demand percentage for all commodities k and all interdiction scenarios l as shown in Eq. (24), is then added as a constraint in the next section as an ϵ constraint.

2.5.2 Model Constraints

The flow of commodity k across link (i,j) for all interdiction scenarios l, x_{ijl}^k, is subject to the capacity constraint shown in Eq. (25) below:

$$x_{ijl}^k \leq \left(1 + \alpha_{ij}^k\right) \times c_{ij}^k; \quad \forall (i,j) \in A, k \in K, l \in L \tag{25}$$

The constraint in Eq. (25) says the flow cannot exceed the sum of the original and added capacity. The added capacity on link (i,j) for commodity k can be limited by financial or physical constraint to the network and is subject to the constraints in Eqs. (26)(27):

$$\alpha_{ij}^k \leq p^k; \quad \forall (i,j) \in A, k \in K \tag{26}$$

$$\sum_{(i,j)\in A} \alpha_{ij}^k \times c_{ij}^k \leq \alpha_{max}^k; \quad \forall\, k \in K \tag{27}$$

The constraint in Eq. (26) ensures the percent added capacity on link (i,j) for commodity k, α_{ij}^k, does not exceed a maximum percentage increase of commodity k, p^k, for all links $(i,j) \in A$ and commodities $k \in K$. The constraint in Eq. (27) says the sum of all added capacity on link (i,j) for commodity k cannot exceed an allotted increase, α_{max}^k, for all commodities $k \in K$. The allotted increase, α_{max}^k, is defined as the total additional capacity available to the network for commodity k.

$$\sum_{k\in K}\sum_{j\in D^k}\frac{\mu_j^k-\left(\sum_{i:(i,j)\in A}x_{ijl}^k-\sum_{i:(i,j)\in A}x_{jil}^k\right)}{\mu_j^k}\leq\epsilon_l;\quad\forall l\in L \tag{28}$$

The flow constraints for this optimization model are the same as those of the previous model in in Eqs. (14)(15)(16). Additionally, the unmet demand values achieved from the previous model, ϵ_l, are added as maximum values for the unmet demand at node for all nodes $j \in D^k$ and commodities k for interdiction scenario l as in Eq. (28) above.

The interdiction constraints and nonnegativity constraints provided in the previous model in Eqs. (17)(18)(19) are all applied to this model, and an additional nonnegativity constraint is added to the percent added capacity variable as in Eq. **(29)** below:

$$\alpha_{ij}^k \geq 0;\quad\forall(i,j)\in A,k\in K \tag{29}$$

Gurobi optimization software has been shown to efficiently find feasible and optimal solutions to linear problems, including multi-commodity network flow problems [27]. The Gurobi Optimizer found a solution using the dual simplex method and the python code for the initial and balancing model can be found in Appendix A and Appendix B, respectively.

Chapter 3.0 Network Application

In this section, the balanced network model is applied to the Swedish railway system. The system transported freight across 1363 stations (nodes) and 1438 bidirectional tracks (links), which was aggregated from public sources in 2012 [28]. The network has previously been studied in the context of vulnerability analysis [22, 29].

3.1 Swedish Railway Network Data

The freight moved in the system is summarized in Table 1 and consists of 20 commodities. As the data were compiled and aggregated to avoid disclosure of sensitive information, only aggregated cargo routes with origin and destination nodes are given, the number of which are summarized in Table 2. To overcome this and estimate link capacities and the supply/demand of nodes, a method developed in a previous study was utilized [22]. Supply and demand values for each commodity were distributed across the nodes utilized for each cargo route, proportional to the number of routes running through that node with that commodity. The commodity specific capacities of each link were estimated based on the freight movement in the network with adjustment factors to produce some slack.

Table 1. Total amount demanded of commodity type, k, and the baseline unmet demand, sorted by kTon.

Commodity	Commodity Group Name	Demand (kTons)	% of Total	Cumulative % of Total	Baseline Unmet Demand
3	Ore	29,427	46.41%	46.41%	0.000%
19	Unidentifiable goods	9,738	15.36%	61.76%	0.000%
1	Agriculture, Forrest, Fishing	8,463	13.35%	75.11%	0.822%
6	Wood, Cork, Pulp, Paper	4,701	7.41%	82.52%	0.190%
10	Fabricated metal products	4,017	6.33%	88.86%	1.209%
7	Petroleum products	1,410	2.22%	91.08%	0.000%
8	Chemicals, rubber, plastics	1,257	1.98%	93.07%	0.000%
16	Equipment for transportation	1,187	1.87%	94.94%	0.000%
14	Return materials and recycling	1,125	1.77%	96.71%	0.000%
12	Transport equipment	894	1.41%	98.12%	0.000%
9	Other non-metallic mineral	449	0.71%	98.83%	0.000%
2	Coal, Crude oil, Natural gas	280	0.44%	99.27%	0.000%
4	Food, Beverage, Tobacco	250	0.39%	99.67%	0.490%
11	Machinery and equipment	95	0.15%	99.82%	0.000%
13	Furniture, Other manufactured	59	0.09%	99.91%	0.804%
18	Loader and grouped goods	39	0.06%	99.97%	0.495%
20	Goods not in group of 1-19	18	0.03%	100.00%	0.297%
5	Textile, leather	1	0.00%	100.00%	1.255%
15	Post and packages	0	0.00%	100.00%	0.000%
17	Moving Goods, vehicles for repair	0	0.00%	100.00%	0.000%
T	Total	63,410	100.00%	100.00%	0.305%

Table 2. Source and sink nodes for each commodity in the aggregated network.

Commodity	Commodity Name	No. of Supply Nodes	No. of Demand Nodes
1	Agriculture, forest, fishing	228	284
2	Coal, crude oil, natural gas	27	19
3	Ore	210	262
4	Food, beverage, tobacco	281	366
5	Textile, leather	240	262
6	Wood, cork, pulp, paper	245	276
7	Petroleum products	198	217
8	Chemicals, rubber, plastics	186	187
9	Other non-metallic mineral	270	258
10	Fabricated metal products	216	193
11	Machinery and equipment	263	251
12	Transport equipment	240	269
13	Furniture, other manufactured	248	239
14	Return materials and recycling	256	380
15	Post and packages	0	0
16	Equipment for transportation	238	260
17	Moving goods, vehicles for repair	0	0
18	Loader and grouped goods	287	241
19	Unidentifiable goods	293	267
20	Goods not in group of 1-19	227	195

The demand for the top three commodities exceeds 75%, where the top commodity, ore, represents around 46% of all demanded goods in kTon. The baseline case, i.e., the undisrupted network, has some unmet demand due to the cargo route aggregation and network estimation, shown in Table 1. Two commodities, 15 and 17, have no demand and no flow through the network. As such, the range of the number of supply and demand nodes for each commodity differs significantly among some of the commodities. This can be due to regionalization, such as in the case of commodity 2, which is centered in the northern region of the network. A visualization of the network is shown in Figure 2 and a comparison of network flow between commodities 1 and 2 are shown in Figure 3 on the next page [22, 28].

Figure 2. Visual representation of the Swedish railway network.

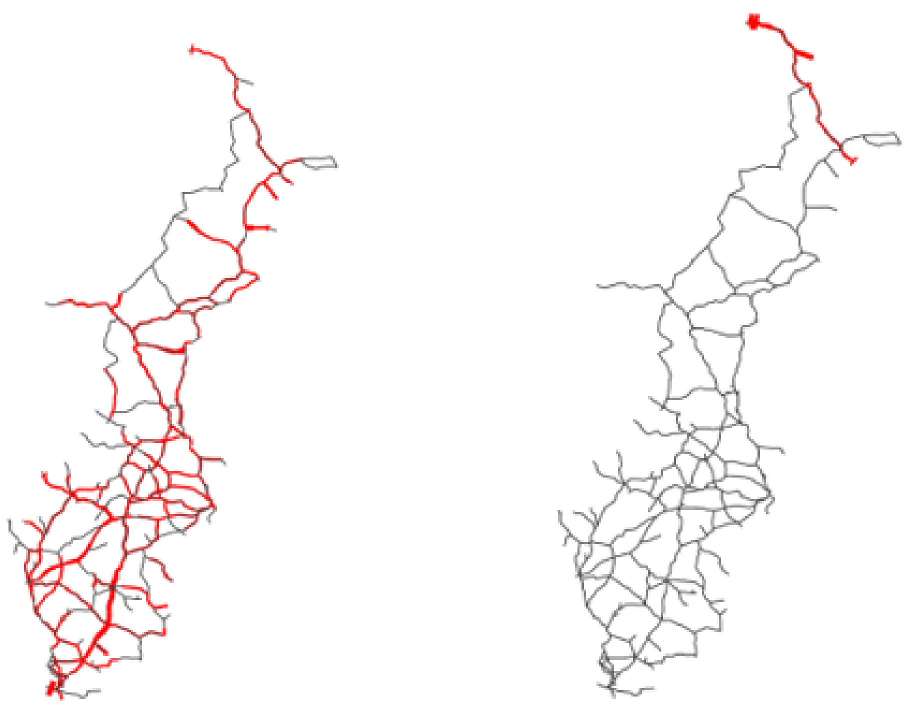

Figure 3. Examples of baseline flow (in red) for Commodity 1: Agriculture, Forest, Fishing (left) and Commodity 2: Coal, Crude Oil, Natural Gas (right) for the Swedish railway network

3.2 Data Selection Methodology

3.2.1 Instrumentation

The data were reduced and output to csv files. The problem was written in Python version 3.7 with the Gurobi Optimizer Python interface. All code was run on an 8-core 2.50 GHz processor with 32 gigabytes of memory.

3.2.2 Data Reduction

The full Swedish railway network dataset is large. If the full network is to be used, with 2878 links, 1439 interdicted links, and 20 commodities, the number of flow variables alone, not considering added capacity variables, will exceed 82,000,000. This is an exceedingly large problem for the instrumentation available. Therefore, the network has been reduced for this paper to allow for faster processing times while still achieving the goals of the model.

First, commodities 5, 15 and 17 are removed from the problem since they only provide 1, 0, and 0 kTon demand in the network, respectively. Second, since the optimization model adds a percentage of the current capacity of a link, any links with a capacity of 0 for all commodities would be unchanged and are, therefore, removed. Third, any of the nodes that are not included in the reduced list of links are removed. This reduces the number of links from 2878 to 2264, nodes from 1363 to 1085, and commodities to 17. When all 2264 bidirectional links are interdicted, the total number of flow variables is reduced to around 43,000,000.

The amount of additional capacity that can be added to the network will depend on the budget and importance the stakeholders put on each commodity. To approximate these numbers, it is assumed each link can only be increased by the average unmet demand percentage per commodity across all interdiction scenarios seen in Table 3 below. Additionally, the total network capacity can only be increased by half of the average unmet demand in kTons per commodity across all interdiction scenarios. The unmet demand for the 100 links with greatest impact on unmet demand after interdiction, summed across all commodities, can be found in Appendix C.

Table 3. Summary of the unmet demand across all interdiction scenarios per commodity.

Commodity	Commodity Group Name	Demand (kTons)	Max. Unmet Demand	Average Unmet Demand
1	Agriculture, Forrest, Fishing	8,463	5.88%	3.56%
2	Coal, Crude oil, Natural gas	280	31.88%	0.33%
3	Ore	29,427	5.82%	0.13%
4	Food, Beverage, Tobacco	250	6.68%	0.65%
5	Textile, leather	1	7.96%	3.52%
6	Wood, Cork, Pulp, Paper	4,701	3.07%	0.13%
7	Petroleum products	1,410	7.09%	0.17%
8	Chemicals, rubber, plastics	1,257	8.29%	0.11%
9	Other non-metallic mineral	449	7.50%	1.31%
10	Fabricated metal products	4,017	2.97%	0.12%
11	Machinery and equipment	95	3.85%	0.20%
12	Transport equipment	894	3.17%	0.87%
13	Furniture, Other manufactured	59	16.90%	3.58%
14	Return materials and recycling	1,125	2.43%	0.07%
16	Equipment for transportation	1,187	4.09%	0.57%
18	Loader and grouped goods	39	2.82%	0.13%
19	Unidentifiable goods	9,738	3.67%	0.12%
20	Goods not in group of 1-19	18	5.88%	3.56%
T	Total	63,410		

Chapter 4.0 Results and Analysis

This section applies the optimization models defined in Section 2 to the Swedish railway network described in Section 3. The first subsection touches on the nature of the network, the next on the product of the balanced optimization model, followed by a comparison of the original network and the balanced network.

4.1 Nature of the Data

Since the capacity constraints were aggregated and estimated by dividing the number of trains on a link by the total number of trains on that link, the capacity values for the different links are very similar [22]. As such, the effect of interdicting a link is very similar for many of the links and may cause a similar effect on the network flow.

4.2 Balanced Model Results

A summary of the values of U, the difference between the average importance of all interdicted links for commodity k and each individual link's importance for commodity k, resulting from the balancing optimization model is seen in Table 4 below. The values of U indicate the ability of the model to minimize the difference in importance across the network while meeting the unmet demand values found in the initial model. The average deviation from the average was below 1% for all commodities, even with some large maximum deviations, such as 32% for Commodity 2 as seen in Table 5 below. In this instance, as seen in Figure 3, Commodity 2 is centralized in the north, resulting in no change when unused links in the south are disrupted, lowering the average despite a large maximum value. The large maximum U values correlate closely to the maximum observed unmet demand before capacity was added as seen it Table 5.

Table 4. Summary of U over 1132 interdicted links for all commodities.

Commodity	Commodity Group Name	Sum of U in %	Average U in %	Min. U in %	Median U in %	Max. U in %	Standard Deviation of U in %
1	Agriculture, Forrest, Fishing	204%	0.18%	0.00%	0.11%	2.25%	0.29%
2	Coal, Crude oil, Natural gas	707%	0.62%	0.02%	0.33%	31.55%	1.90%
3	Ore	240%	0.21%	0.00%	0.13%	5.69%	0.39%
4	Food, Beverage, Tobacco	306%	0.27%	0.00%	0.16%	6.01%	0.59%
6	Wood, Cork, Pulp, Paper	377%	0.33%	0.00%	0.21%	4.43%	0.45%
7	Petroleum products	228%	0.20%	0.00%	0.13%	2.94%	0.30%
8	Chemicals, rubber, plastics	312%	0.28%	0.00%	0.17%	6.91%	0.53%
9	Other non-metallic mineral	213%	0.19%	0.00%	0.11%	8.18%	0.42%
10	Fabricated metal products	182%	0.16%	0.00%	0.09%	6.18%	0.38%
11	Machinery and equipment	218%	0.19%	0.00%	0.12%	2.84%	0.28%
12	Transport equipment	349%	0.31%	0.00%	0.20%	3.65%	0.44%
13	Furniture, Other manufactured	126%	0.11%	0.00%	0.06%	2.29%	0.21%
14	Return materials and recycling	117%	0.10%	0.00%	0.06%	13.36%	0.65%
16	Equipment for transportation	140%	0.12%	0.00%	0.07%	2.36%	0.20%
18	Loader and grouped goods	144%	0.13%	0.00%	0.07%	3.52%	0.27%
19	Unidentifiable goods	234%	0.21%	0.00%	0.13%	2.69%	0.29%
20	Goods not in group of 1-19	218%	0.19%	0.00%	0.12%	3.56%	0.36%

Table 5. Maximum unmet demand vs. maximum and average U for all commodities.

Commodity	**Commodity Group Name**	**Max. Unmet Demand**	**Max. U in %**	**Average U in %**
1	Agriculture, Forrest, Fishing	5.88%	2.25%	0.18%
2	Coal, Crude oil, Natural gas	31.88%	31.55%	0.62%
3	Ore	5.82%	5.69%	0.21%
4	Food, Beverage, Tobacco	6.68%	6.01%	0.27%
6	Wood, Cork, Pulp, Paper	7.96%	4.43%	0.33%
7	Petroleum products	3.07%	2.94%	0.20%
8	Chemicals, rubber, plastics	7.09%	6.91%	0.28%
9	Other non-metallic mineral	8.29%	8.18%	0.19%
10	Fabricated metal products	7.50%	6.18%	0.16%
11	Machinery and equipment	2.97%	2.84%	0.19%
12	Transport equipment	3.85%	3.65%	0.31%
13	Furniture, Other manufactured	3.17%	2.29%	0.11%
14	Return materials and recycling	16.90%	13.36%	0.10%
16	Equipment for transportation	2.43%	2.36%	0.12%
18	Loader and grouped goods	4.09%	3.52%	0.13%
19	Unidentifiable goods	2.82%	2.69%	0.21%
20	Goods not in group of 1-19	3.67%	3.56%	0.19%

The summary of the added commodity per commodity is presented in Table 6 below. In total, 780 capacity increases were implemented across all 17 commodities. The average capacity increase for each commodity was near the maximum allowable percentage capacity increase. All the link-commodity pairs with added capacity can be seen in Appendix D.

Table 6. Summary of the added capacity per commodity in the balanced network.

Commodity	No. of Links with Added Capacity	Average Added Capacity %	Max. Added Capacity %
1	46	3.55%	3.56%
2	2	0.11%	0.11%
3	54	0.13%	0.13%
4	52	0.64%	0.65%
6	51	3.47%	3.52%
7	45	0.13%	0.13%
8	53	0.17%	0.17%
9	47	0.11%	0.11%
10	55	1.05%	1.31%
11	46	0.12%	0.12%
12	46	0.20%	0.20%
13	46	0.78%	0.87%
14	49	3.11%	3.58%
16	48	0.07%	0.08%
18	45	0.57%	0.57%
19	43	0.13%	0.13%
20	52	0.12%	0.12%
Total	780		

The links with increased capacity are shown in Figure 4, **Figure 5**, and **Figure 6** below.

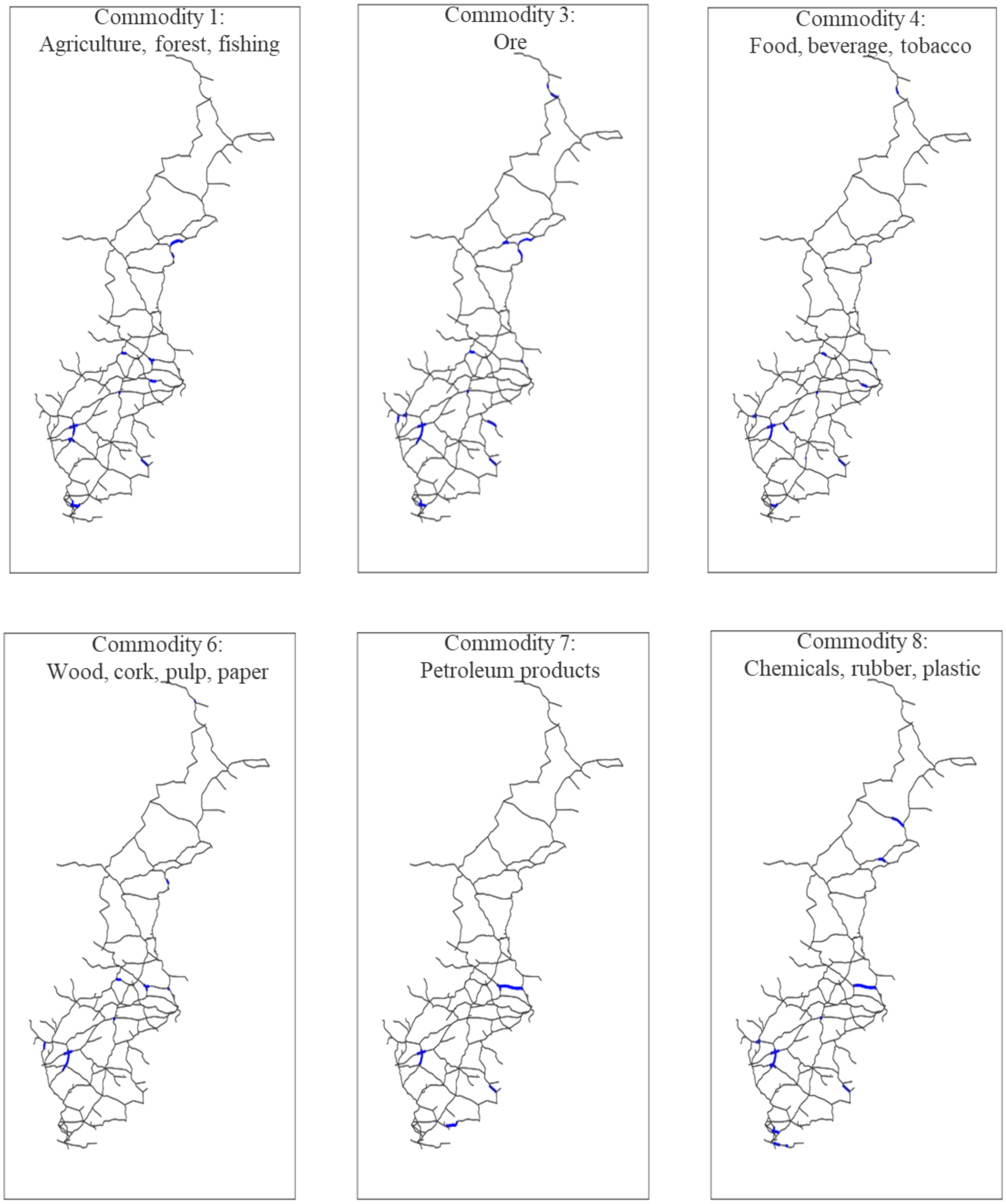

Figure 4. Added capacity to links for commodities 1-8, excluding commodities 2 and 5.

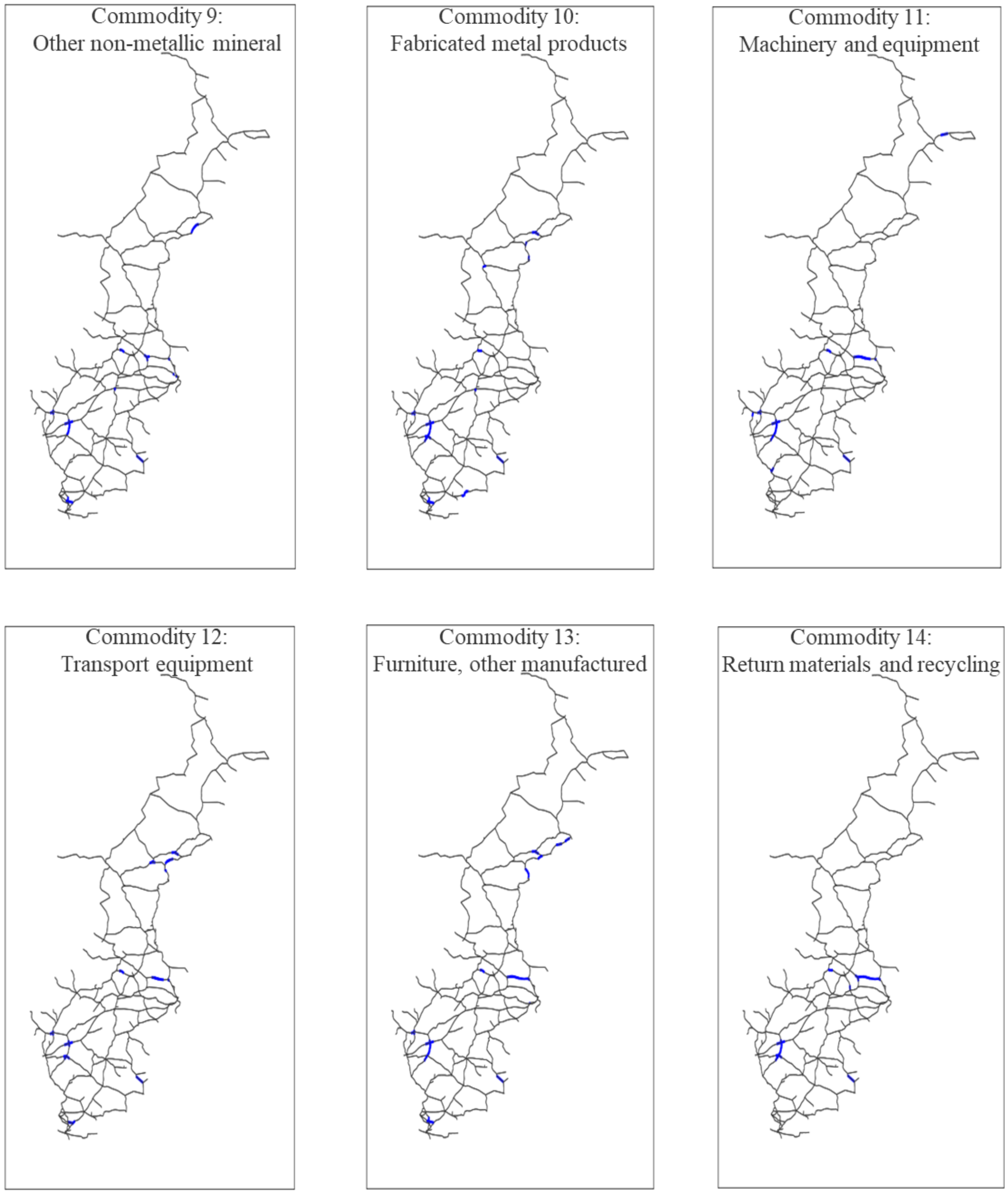

Figure 5. Added capacity to links for commodities 9-14.

Figure 6. Added capacity to links for commodities 16-20, excluding commodity 17.

4.3 Network Balance Comparison

The U values when applying the unmet demand minimizing model to the initial network and the balanced network are shown in Table 7 below. The values are much larger than those of the balanced model; therefore, they have been formatted in decimal form instead of in percentages.

Table 7. Initial U values vs. balanced U values.

Commodity	Commodity Group Name	Initial Average U	Balanced Average U	Initial Standard Deviation of U	Balanced Standard Deviation of U
1	Agriculture, Forrest, Fishing	244.79	246.70	0.392744684	0.395
2	Coal, Crude oil, Natural gas	20.73	20.73	0.063871179	0.064
3	Ore	201.17	201.11	0.428902229	0.429
4	Food, Beverage, Tobacco	385.78	384.85	0.609413633	0.608
6	Wood, Cork, Pulp, Paper	447.55	447.98	0.706132123	0.706
7	Petroleum products	120.93	120.91	0.219821647	0.220
8	Chemicals, rubber, plastics	448.98	448.87	0.945551184	0.946
9	Other non-metallic mineral	223.36	223.30	0.364276773	0.364
10	Fabricated metal products	162.52	161.68	0.247344503	0.247
11	Machinery and equipment	223.69	223.60	0.424359977	0.424
12	Transport equipment	425.53	425.42	0.80458732	0.805
13	Furniture, Other manufactured	147.93	147.85	0.25723058	0.257
14	Return materials and recycling	112.44	111.84	0.267432659	0.268
16	Equipment for transportation	321.57	321.49	0.578579418	0.579
18	Loader and grouped goods	122.90	122.76	0.212311066	0.212
19	Unidentifiable goods	192.62	192.56	0.301925117	0.302
20	Goods not in group of 1-19	212.19	212.16	0.421199248	0.421
T	Total	4014.69	4013.82	1.990	2.692

As seen in Table 7, there is an improvement in the balance of the network after capacity has been added to the network. However, the difference for many of the commodities is negligible, and for those with higher U values, an increase in capacity for links without current capacity may be considered for further improved performance. Nonetheless, a set of links have been identified as candidates for increased capacity under the current budget constraints placed on the network, and the commodities that may require a greater budget have been identified for further consideration.

Chapter 5.0 Conclusion

The work in this thesis aimed to improve network resilience reduce vulnerability by balancing a network using flow-based importance measures. In doing so, the components that would leave the network the most vulnerable if disrupted could be identified and allow stakeholders to focus on the selected components within a limited budget. The concept of a balancing component importance in a multi-state system was extended to a multi-commodity network utilizing flow-based measures, something that has not been done before to the author's knowledge. To do this, an ϵ-constraint model was developed that optimized a bi-objective problem. First, unmet demand across all demand nodes and commodities for all interdiction scenarios was minimized in the network's baseline form. Then, limited additional capacity was added to the network and the previous unmet demand values were applied as minimum values to encourage flow through the network. Finally, the difference in unmet demand across all interdiction scenarios was minimized, leaving the network optimally balanced within the limited budget. The components and their added capacities were reported and acted as recommendations for where to increase the capacity.

As a limitation for this work and an area for future study, the commodities could be ranked by their importance to the network or the industry/economy they are in. Additionally, different costs of adding capacity to the network could be compared to the cost of not meeting demand instead of only considering a maximum allowable capacity increase, making a recommendation even more robust. Finally, link criticality may differ for an n-at-a-time link interdiction analysis strategy compared to the one-at-a-time strategy used in this work, and could be considered for future work [30].

References

[1] S. M. Rinaldi, J. P. Peerenboom, and T. K. Kelly, "Identifying, understanding, and analyzing critical infrastructure interdependencies," *IEEE*, vol. 21, no. 6, pp. 11-25, December 2001 2001, doi: 10.1109/37.969131.

[2] (2019). *A Guide to Critical Infrastructure Security and Resilience*

[3] F. T. Intelligence, "Hurricane Harvey Disrupting Freight Transportation in Gulf Region," ed, 2017.

[4] (2021). *A Proclamation on Critical Infrastructure Security and Resilience Month, 2021*. [Online] Available: https://www.whitehouse.gov/briefing-room/presidential-actions/2021/10/29/a-proclamation-on-critical-infrastructure-security-and-resilience-month-2021/

[5] C. Wan, Z. Yang, D. Zhang, X. Yan, and S. Fan, "Resilience in transportation systems: a systematic review and future directions," *Transport Reviews*, vol. 38, no. 4, pp. 479-498, 2018, doi: https://doi.org/10.1080/01441647.2017.1383532.

[6] (2019). *Freight Facts and Figures*. [Online] Available: https://data.bts.gov/stories/s/Freight-Transportation-System-Condition-Performanc/vvk5-xjjp

[7] D. Henry and J. E. Ramirez-Marquez, "Generic metrics and quantitative approaches for system resilience as a function of time," *Reliability Engineering and System Safety*, vol. 99, pp. 114-122, 2012, doi: doi:10.1016/j.ress.2011.09.002.

[8] H. Baroud, K. Barker, J. E. Ramirez-Marquez, and C. M. Rocco, "Inherent Costs and Interdependent Impacts of Infrastructure Network Resilience," *Risk Analysis*, vol. 35, no. 4, pp. 642-662, 2015, doi: 10.1111/risa.12223.

[9] S. Hosseini, K. Barker, and J. E. Ramirez-Marquez, "A review of definitions and measures of system resilience," *Reliability Engineering and System Safety*, vol. 145, pp. 47-61, 2016, doi: http://dx.doi.org/10.1016/j.ress.2015.08.006.

[10] A. T. Murray, T. C. Matisziw, and T. H. Grubesic, "A methodological overview of network vulnerability analysis," *Growth and Change*, vol. 39, no. 4, pp. 573-592, 2008.

[11] R. Albert, H. Jeong, and A.-L. Barabási, "Error and attack tolerance of complex networks," *nature*, vol. 406, no. 6794, pp. 378-382, 2000.

[12] R. L. Church, M. P. Scaparra, and R. S. Middleton, "Identifying critical infrastructure: the median and covering facility interdiction problems," *Annals of the Association of American Geographers*, vol. 94, no. 3, pp. 491-502, 2004.

[13] T. C. Matisziw, A. T. Murray, and T. H. Grubesic, "Exploring the vulnerability of network infrastructure to disruption," *The Annals of Regional Science*, vol. 43, no. 2, pp. 307-321, 2009.

[14] P. Crucitti, V. Latora, and S. Porta, "Centrality measures in spatial networks of urban streets," *Physical Review E*, vol. 73, p. 036125, 2006, doi: https://doi.org/10.1103/PhysRevE.73.036125.

[15] Z. Xu, J. E. Ramirez-Marquez, Y. Liu, and T. Xiahou, "A new resilience-based component importance measure for multi-state networks," *Reliability Engineering & System Safety*, vol. 193, p. 106591, 2020.

[16] M. Ouyang, L. Hong, Z.-J. Mao, M.-H. Yu, and F. Qi, "A methodological approach to analyze vulnerability of interdependent infrastructures," *Simulation Modelling Practice and Theory*, vol. 17, 5, pp. 817-828, 2009.

[17] A. Nagurney and Q. Qiang, "A network efficiency measure with application to critical infrastructure networks," *Journal of Global Optimization*, vol. 40, no. 1, pp. 261-275, 2008.

[18] M. Darayi, K. Barker, and J. R. Santos, "Component Importance Measures for Multi-Industry Vulnerability of a Freight Transportation Network," *Networks and spatial economics*, vol. 17, no. 4, pp. 1111-1136, 2017.

[19] E. Zio and L. Podofillini, "The use of importance measures for the optimization of multi-state systems," *Eksploatacja i Niezawodnosc–Maintenance and Reliability*, vol. 2, pp. 33-36, 2006.

[20] E. Zio and L. Podofillini, "Importance measures and genetic algorithms for designing a risk-informed optimally balanced system," *Reliability Engineering & System Safety*, vol. 92, no. 10, pp. 1435-1447, 2007, doi: doi:10.1016/j.ress.2006.09.011.

[21] L. Podofillini and E. Zio, "Designing a risk-informed balanced system by genetic algorithms: Comparison of different balancing criteria," *Reliability Engineering & System Safety*, vol. 93, no. 12, pp. 1842-1852, 2008, doi: doi:10.1016/j.ress.2008.03.015.

[22] M. G. Whitman, K. Barker, J. Johansson, and M. Darayi, "Component importance for multi-commodity networks: Application in the Swedish railway," *Computers & Industrial Engineering*, vol. 112, pp. 274-288, 2017, doi: http://dx.doi.org/10.1016/j.cie.2017.08.004.

[23] L. Xiujuan and S. Zhongke, "Overview of multi-objective optimization methods," *Journal of Systems Engineering and Electronics*, vol. 15, no. 2, pp. 142-146, 2004.

[24] G. Chiandussi, M. Codegone, S. Ferrero, and F. E. Varesio, "Comparison of multi-objective optimization methodologies for engineering applications," *Computers & Mathematics with Applications*, vol. 63, no. 5, pp. 912-942, 2012.

[25] G. Mavrotas, "Effective implementation of the ε-constraint method in multi-objective mathematical programming problems," *Applied mathematics and computation*, vol. 213, no. 2, pp. 455-465, 2009.

[26] R. T. Marler and J. S. Arora, "The weighted sum method for multi-objective optimization: new insights," *Structural and multidisciplinary optimization*, vol. 41, no. 6, pp. 853-862, 2010.

[27] Gurobi Optimization Inc., "Gurobi Optimizer Reference Manual," 2022. [Online]. Available: https://www.gurobi.com/documentation/9.5/refman/index.html.

[28] J. Johansson. *SwRailwayData_20141012*.

[29] M. McCarter, K. Barker, J. Johansson, and J. E. Ramirez-Marquez, "A bi-objective formulation for robust defense strategies in multi-commodity networks," *Reliability Engineering and System Safety*, vol. 176, pp. 154-161, August 2018 2018, doi: https://doi.org/10.1016/j.ress.2018.04.011.

[30] D. L. Alderson, G. G. Brown, W. M. Carlyle, and L. A. Cox Jr, "Sometimes there is no" most-vital" arc: Assessing and improving the operational resilience of systems," *Military Operations Research*, vol. 18, no. 1, pp. 21-37, 2013.

Appendix A

Appendix A. Unmet Demand Minimizing Model Code.

```
# Model the data:
    # Set of nodes, links, commodities, interdicted links,
    # supply/demand/transshipment node values, and capacity values.
    # Duplicate the sets as tuplelists/tupledicts for quicksum() function.

import csv
from gurobipy import *
import pandas as pd

def Min_Unmet_Demand(li_in,lj_in, k_in):
    # Create optimization model
    m = Model('bal_net_flow')

    # Decision variables - flow for each commodity
    x = m.addVars(A, name = "x") # flow variables
    mu_var = m.addVars(N, name = "mu_var") # demand variable for easier calculation

    m.update()

    # Constraint 3: Capacity constraint for link (i,j) for commodity k
    m.addConstrs(
        (x[i, j] <= c[i, j][k_in] for i, j in A), "cap")

    m.addConstrs(x[i,j] >= 0 for i,j in A)

    # Constraint 5: Flow balance constraint for link (i,j) for commodity k

    m.addConstrs(
        (quicksum(x[i1, j] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2] for j, i2 in tuple_A.select(j, '*')) <=
        D[k_in] for j in D[k_in].keys())
        )

    m.addConstrs(
        (quicksum(x[i1, j] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2] for j, i2 in tuple_A.select(j, '*')) >=
        0 for j in D[k_in].keys())
        )

    m.addConstrs(
        (quicksum(x[i1, j] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2] for j, i2 in tuple_A.select(j, '*')) ==
        mu_var[j] for j in D[k_in].keys())
        )

    m.addConstrs(
        (-quicksum(x[i1, j] for i1, j in tuple_A.select('*', j)) +
        quicksum(x[j, i2] for j, i2 in tuple_A.select(j, '*')) <=
        S[k_in] for j in S[k_in].keys())
        )

    m.addConstrs(
        (-quicksum(x[i1, j] for i1, j in tuple_A.select('*', j)) +
        quicksum(x[j, i2] for j, i2 in tuple_A.select(j, '*')) >=
        0 for j in S[k_in].keys())
        )

    m.addConstrs(
        (quicksum(x[i1, j] for i1, j in tuple_A.select('*', j)) -
```

```
        quicksum(x[j, i2] for j, i2 in tuple_A.select(j, '*')) ==
        0 for j in T[k_in].keys())
        )

    # Constraints 6 and 7: Interdiction constraints
    m.addConstr(
        (x[li_in, lj_in] == 0), "interdicted_1")

    m.addConstr(
        (x[lj_in, li_in] == 0), "interdicted_2")

    m.update()

    # Set minimized unmet demand objective
    Z = quicksum((D[k_in][j] - mu_var[j])/D[k_in][j] for j in D[k_in].keys())

    m.update()

    # Set objective
    m.setObjective(Z, GRB.MINIMIZE)

    m.update()
    # Solve the model
    m.optimize()

    return m.objVal

# Store unmet demand values
unmet_demand = {}
for k in K:
    for li, lj in L:
        unmet_demand[li, lj, k] = Min_Unmet_Demand(li, lj, k)

```

Appendix B

Appendix B. Balancing Network Model Code.

```
# Model the data:
    # Set of nodes, links, commodities, interdicted links,
    # supply/demand/transshipment node values, and capacity values.
    # Duplicate the sets as tuplelists/tupledicts for quicksum() function.
    # Add maximum added capacity parameters.
    # Import epsilon values/minimum unmet demand values.

import csv
from gurobipy import *
import pandas as pd
import time

def Balance_Network(k_in):

    # Create optimization model
    m = Model('bal_net_flow')

    # Decision variables - flow and unmet demand for each commodity
    x = m.addVars(A, L, name = "x") # flow variables
    alpha = m.addVars(A, name = "alpha") # added capacity variables
    U = m.addVars(L, name = "U") # placeholder variable for objective linearization
    I_bar = m.addVar(vtype = GRB.CONTINUOUS, name = "I_bar") # Average importance measure

```

```
    m.update()

    # Equation 12: Capacity constraint for link (i,j) for commodity k
    m.addConstrs(
        ((1 + alpha[i, j]) * c[i, j][k_in] - x[i, j, li, lj] >= 0 for i, j in A for li, lj in L), "cap")

    # Equation 13: Added capacity constraint
    m.addConstrs(
        (alpha[i, j] <= p[k_in] for i, j in A), "p^k"
        )

    m.addConstr(
        (quicksum(alpha[i, j] * c[i, j][k_in] for i, j in tuple_A.select('*', '*')) <= alpha_max[k_in]), "max_alpha"
        )

    # Equations 15-17: Flow balance constraints for link (i,j) for commodity k
    m.addConstrs(
        (quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) <=
        D[k_in][j] for j in D[k_in].keys() for li, lj in L)
        )
    m.addConstrs(
        (quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) >=
        0 for j in D[k_in].keys() for li, lj in L)
        )

    m.addConstrs(
        (-quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) +
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) <=
        S[k_in][j] for j in S[k_in].keys() for li, lj in L)
        )

    m.addConstrs(
        (-quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) +
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) >=
        0 for j in S[k_in].keys() for li, lj in L)
        )

    m.addConstrs(
        (quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) ==
        0 for j in T[k_in].keys() for li, lj in L)
        )

    # Equations 18-19: Interdiction constraints
    m.addConstrs(
        (x[li, lj, li, lj] == 0 for li, lj in L), "interdicted_1")

    m.addConstrs(
        (x[lj, li, li, lj] == 0 for li, lj in L), "interdicted_2")

    # Epsilon constraint
    m.addConstrs(
        (quicksum((D[k_in][j] - (quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*'))))/D[k_in][j] for j in D[k_in].keys()) <= unmet_demdf[(unmet_demdf['li']==li) & (unmet_demdf['lj']==lj) & (unmet_demdf['k']==k_in)]['unmet_demand'].values[0] for li, lj in L)
        )

    # Constraints for average importance values
```

```
    m.addConstr(
        (I_bar == 1/(len(L)) * quicksum((Dk[k_in] - quicksum(quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) for j in D[k_in].keys())) / Dk[k_in] for li, lj in L))
        )

    # Linearization constraints for objective function
    m.addConstrs(
        (((Dk[k_in] - quicksum(quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) for j in D[k_in].keys())) / Dk[k_in]) - I_bar <= U[li, lj] for li, lj in L)
        )

    m.addConstrs(
        (-(((Dk[k_in] - quicksum(quicksum(x[i1, j, li, lj] for i1, j in tuple_A.select('*', j)) -
        quicksum(x[j, i2, li, lj] for j, i2 in tuple_A.select(j, '*')) for j in D[k_in].keys())) / Dk[k_in]) - I_bar) <= U[li, lj] for li, lj in L)
        )

    m.update()

    # Equation 7: Objective function
    Z = quicksum(U[li, lj] for li, lj in L)

    # Set objective
    m.setObjective(Z, GRB.MINIMIZE)

    m.update()

    # Solve the model
    m.optimize()

    # Write file of added demand values
    name = 'added_demand_eps_com_{}.csv'.format(k_in)
    f = open(name,'w', newline='')
    header = ['i', 'j', 'k', 'added_demand']
    writer = csv.writer(f)
    writer.writerow(header)
    for i, j in A:
        line = [i,j, k_in, alpha[i, j].X]
        writer.writerow(line)
    f.close()

    # Write file of minimized importance values
    name = 'U_eps_com_{}.csv'.format(k_in)
    f = open(name,'w', newline='')
    header = ['li', 'lj', 'k', 'U']
    writer = csv.writer(f)
    writer.writerow(header)
    for li, lj in L:
        line = [li,lj, k_in, U[li, lj].X]
        writer.writerow(line)
    f.close()

    return m.objVal

U_sum = {}
for k in K:
    U_sum[k] = Balance_Network(k)
```

Appendix C

Appendix C. The top 100 interdicted links (i, j) and their respective unmet demand

Node i	Node j	Unmet Demand	Node i	Node j	Unmet Demand
9	8	36.698%	128	82	12.136%
10	9	30.135%	30	19	12.101%
1002	1001	29.458%	32	31	12.072%
1026	1002	25.135%	85	84	11.984%
1001	1000	23.551%	52	51	11.890%
11	10	22.911%	46	45	11.783%
57	56	22.755%	30	17	11.769%
56	55	21.672%	62	58	11.760%
55	54	21.003%	1036	1035	11.757%
3	2	20.273%	65	64	11.649%
33	19	20.202%	58	57	11.376%
16	2	20.064%	45	44	11.141%
4	3	19.965%	48	47	11.098%
5	4	19.940%	87	86	11.054%
6	5	19.917%	700	699	11.051%
54	53	19.592%	1022	1021	11.009%
1084	1025	18.678%	66	65	11.000%
61	36	18.621%	15	14	10.919%
1038	1037	18.445%	888	887	10.914%
12	11	18.442%	460	459	10.909%
13	12	18.432%	88	87	10.889%
34	33	18.201%	461	460	10.751%
38	37	18.088%	89	88	10.719%
14	13	17.943%	31	20	10.612%
704	694	17.790%	705	704	10.573%
1039	1038	17.587%	21	20	10.546%
37	36	17.374%	47	46	10.532%
1026	1025	17.006%	90	89	10.501%
459	414	16.253%	421	420	10.461%
130	129	16.142%	90	64	10.406%
41	40	15.992%	22	21	10.196%
39	38	15.752%	51	50	10.190%
1000	999	15.725%	414	354	10.119%
61	60	15.628%	422	421	10.057%
18	16	14.878%	67	66	10.057%
131	130	14.610%	903	757	9.750%
40	39	14.253%	49	48	9.418%

128	127	14.241%	69	68	9.387%
1037	1036	13.937%	86	85	9.332%
7	6	13.873%	23	22	9.261%
50	49	13.862%	24	23	9.190%
8	7	13.782%	25	24	9.166%
43	42	13.542%	762	761	9.163%
44	43	13.483%	991	990	9.123%
52	31	13.424%	761	760	8.992%
84	83	13.314%	463	462	8.967%
83	82	12.841%	113	112	8.965%
42	41	12.746%	68	67	8.948%
60	53	12.257%	996	995	8.890%
18	17	12.142%	1023	1022	8.881%

Appendix D

Appendix D. Full list of link-commodity pairs with added demand.

Node *i*	Node *j*	Commodity	Added Capacity %	Node *i*	Node *j*	Commodity	Added Capacity %
33	43	1	3.56%	1021	1022	10	1.31%
120	121	1	3.56%	1022	1021	10	0.35%
173	130	1	3.56%	1022	1023	10	1.31%
229	228	1	3.56%	1023	1081	10	1.31%
241	242	1	3.56%	1024	1025	10	1.31%
341	361	1	3.56%	1077	1087	10	1.31%
341	342	1	3.56%	1146	1147	10	1.31%
342	341	1	3.56%	1158	1133	10	1.31%
342	343	1	3.56%	1240	1243	10	1.31%
343	342	1	3.56%	1289	1315	10	1.31%
343	344	1	3.56%	1324	1325	10	1.31%
344	345	1	3.56%	1325	1326	10	1.31%
344	343	1	3.56%	1326	1327	10	1.31%
345	344	1	2.98%	1328	1327	10	1.31%
345	346	1	3.56%	120	121	11	0.12%
346	345	1	3.56%	173	130	11	0.12%
361	360	1	3.56%	223	224	11	0.12%
539	538	1	3.56%	229	228	11	0.12%
563	556	1	3.56%	346	345	11	0.12%
569	570	1	3.56%	456	402	11	0.12%
600	599	1	3.56%	504	510	11	0.12%
634	633	1	3.56%	538	539	11	0.12%

669	825	1	3.56%	555	548	11	0.12%
677	676	1	3.56%	563	556	11	0.12%
686	685	1	3.56%	664	642	11	0.12%
696	742	1	3.56%	667	668	11	0.12%
769	768	1	3.56%	678	679	11	0.12%
789	677	1	3.56%	685	709	11	0.12%
800	809	1	3.56%	691	686	11	0.12%
815	814	1	3.56%	692	691	11	0.12%
912	913	1	3.56%	692	693	11	0.12%
933	932	1	3.56%	693	694	11	0.12%
944	1303	1	3.56%	694	695	11	0.12%
970	1009	1	3.56%	695	696	11	0.12%
1009	970	1	3.56%	789	677	11	0.12%
1020	1019	1	3.56%	800	809	11	0.12%
1021	1020	1	3.56%	815	814	11	0.12%
1022	1021	1	3.56%	873	879	11	0.11%
1023	1006	1	3.56%	884	1015	11	0.12%
1023	1081	1	3.56%	914	1008	11	0.12%
1077	1087	1	3.56%	914	913	11	0.12%
1158	1133	1	3.56%	933	932	11	0.12%
1240	1243	1	3.56%	944	1303	11	0.12%
1243	1240	1	3.56%	949	983	11	0.12%
1258	1289	1	3.56%	1009	970	11	0.12%
1289	1315	1	3.56%	1015	1016	11	0.12%
14	30	2	0.11%	1016	1017	11	0.12%
33	43	3	0.13%	1017	1018	11	0.12%
34	45	3	0.13%	1018	1019	11	0.12%
38	39	3	0.01%	1019	1020	11	0.12%
39	38	3	0.13%	1020	1021	11	0.12%
173	130	3	0.13%	1021	1022	11	0.12%
265	351	3	0.13%	1022	1023	11	0.12%
345	346	3	0.13%	1023	1006	11	0.12%
347	241	3	0.13%	1024	1025	11	0.12%
456	402	3	0.13%	1077	1087	11	0.12%
457	456	3	0.13%	1158	1133	11	0.12%
538	539	3	0.13%	1240	1243	11	0.12%
539	538	3	0.13%	1243	1240	11	0.12%
669	825	3	0.13%	1315	1289	11	0.12%
685	709	3	0.13%	130	173	12	0.20%
686	691	3	0.13%	173	130	12	0.20%

692	691	3	0.13%	229	228	12	0.20%
696	742	3	0.13%	239	171	12	0.20%
815	814	3	0.13%	241	242	12	0.20%
884	1015	3	0.13%	242	243	12	0.20%
912	913	3	0.13%	243	244	12	0.20%
913	914	3	0.13%	265	351	12	0.20%
914	1008	3	0.13%	345	346	12	0.20%
919	912	3	0.13%	347	241	12	0.20%
933	932	3	0.13%	351	265	12	0.20%
944	1303	3	0.13%	538	537	12	0.20%
949	983	3	0.13%	538	539	12	0.20%
970	1009	3	0.13%	569	570	12	0.20%
1009	970	3	0.13%	623	633	12	0.20%
1015	1016	3	0.13%	628	625	12	0.20%
1015	884	3	0.13%	667	668	12	0.20%
1016	1017	3	0.13%	685	709	12	0.20%
1016	1015	3	0.13%	692	693	12	0.20%
1017	1016	3	0.13%	693	694	12	0.20%
1017	1018	3	0.13%	757	758	12	0.20%
1018	1017	3	0.13%	800	809	12	0.20%
1018	1019	3	0.13%	815	814	12	0.20%
1019	1018	3	0.13%	837	838	12	0.20%
1019	1020	3	0.13%	884	1015	12	0.20%
1020	1019	3	0.13%	912	913	12	0.20%
1020	1021	3	0.13%	914	913	12	0.20%
1021	1020	3	0.13%	933	932	12	0.20%
1021	1022	3	0.13%	944	1303	12	0.20%
1022	1021	3	0.13%	949	983	12	0.20%
1022	1023	3	0.13%	970	1009	12	0.20%
1023	1006	3	0.13%	1009	970	12	0.20%
1024	1025	3	0.13%	1019	1018	12	0.20%
1077	1087	3	0.13%	1020	1019	12	0.20%
1146	1147	3	0.13%	1021	1020	12	0.20%
1158	1133	3	0.13%	1022	1021	12	0.20%
1240	1243	3	0.13%	1023	1006	12	0.20%
1241	1198	3	0.13%	1023	1081	12	0.20%
1243	1240	3	0.13%	1077	1087	12	0.20%
1289	1315	3	0.13%	1146	1147	12	0.20%
1315	1289	3	0.13%	1159	1146	12	0.20%
37	38	4	0.17%	1240	1243	12	0.20%
130	173	4	0.65%	1243	1240	12	0.20%

229	228	4	0.65%	1258	1289	12	0.20%
361	360	4	0.65%	1274	1250	12	0.05%
473	476	4	0.65%	1315	1289	12	0.20%
504	509	4	0.65%	229	228	13	0.87%
514	513	4	0.56%	246	245	13	0.87%
538	539	4	0.65%	341	342	13	0.64%
667	668	4	0.65%	342	343	13	0.64%
669	825	4	0.65%	343	344	13	0.64%
677	676	4	0.65%	344	345	13	0.64%
685	709	4	0.65%	345	346	13	0.87%
686	685	4	0.65%	351	265	13	0.87%
757	758	4	0.65%	458	496	13	0.74%
789	677	4	0.65%	538	537	13	0.87%
800	809	4	0.65%	600	599	13	0.87%
815	814	4	0.65%	642	664	13	0.87%
884	1015	4	0.65%	685	709	13	0.87%
914	913	4	0.65%	686	685	13	0.87%
933	932	4	0.65%	686	691	13	0.12%
944	1303	4	0.65%	691	692	13	0.12%
970	1009	4	0.65%	692	693	13	0.12%
1008	914	4	0.65%	693	694	13	0.87%
1009	970	4	0.65%	694	695	13	0.87%
1015	1016	4	0.65%	695	696	13	0.87%
1015	884	4	0.65%	696	742	13	0.87%
1016	1017	4	0.65%	815	814	13	0.87%
1016	1015	4	0.65%	870	875	13	0.09%
1017	1016	4	0.65%	914	913	13	0.87%
1017	1018	4	0.65%	933	932	13	0.87%
1018	1017	4	0.65%	944	1303	13	0.87%
1018	1019	4	0.65%	970	1009	13	0.87%
1019	1018	4	0.65%	1008	914	13	0.87%
1019	1020	4	0.65%	1009	970	13	0.87%
1020	1019	4	0.65%	1015	884	13	0.87%
1020	1021	4	0.65%	1016	1015	13	0.87%
1021	1020	4	0.65%	1017	1016	13	0.87%
1021	1022	4	0.65%	1018	1017	13	0.87%
1022	1021	4	0.65%	1019	1018	13	0.87%
1022	1023	4	0.65%	1020	1019	13	0.87%
1023	1006	4	0.65%	1021	1020	13	0.87%
1023	1081	4	0.65%	1022	1021	13	0.87%
1024	1025	4	0.65%	1023	1006	13	0.87%

1064	1065	4	0.65%	1024	1025	13	0.87%
1077	1087	4	0.65%	1077	1087	13	0.87%
1084	1085	4	0.65%	1158	1133	13	0.87%
1146	1147	4	0.65%	1240	1243	13	0.87%
1158	1133	4	0.65%	1243	1240	13	0.87%
1240	1243	4	0.65%	1289	1315	13	0.87%
1243	1240	4	0.65%	1315	1289	13	0.87%
1258	1289	4	0.65%	1328	1327	13	0.87%
1315	1289	4	0.65%	120	121	14	3.58%
33	43	6	3.02%	229	228	14	3.58%
173	130	6	3.52%	539	538	14	3.58%
229	228	6	3.52%	548	555	14	3.58%
246	223	6	3.52%	563	556	14	3.58%
341	342	6	3.52%	569	570	14	3.58%
342	343	6	2.92%	677	676	14	3.58%
343	344	6	3.52%	685	709	14	3.58%
344	345	6	2.73%	686	691	14	0.48%
345	346	6	2.96%	691	692	14	0.48%
539	538	6	3.52%	692	693	14	0.48%
548	555	6	3.52%	693	694	14	3.58%
556	563	6	3.52%	694	693	14	3.58%
569	570	6	3.52%	694	695	14	0.21%
585	579	6	3.52%	695	696	14	1.96%
600	599	6	3.52%	696	742	14	3.58%
669	825	6	3.52%	722	723	14	0.45%
685	709	6	3.52%	727	728	14	0.75%
696	742	6	3.52%	728	727	14	0.95%
757	758	6	3.52%	757	758	14	3.58%
800	809	6	3.52%	815	814	14	3.58%
815	814	6	3.52%	871	869	14	3.58%
884	1015	6	3.52%	884	1015	14	3.58%
912	913	6	3.52%	944	1303	14	3.58%
913	914	6	3.52%	1009	970	14	3.58%
914	1008	6	3.52%	1015	1016	14	3.58%
919	912	6	3.52%	1015	884	14	3.58%
933	932	6	3.52%	1016	1017	14	3.58%
944	1303	6	3.52%	1016	1015	14	3.58%
1009	970	6	3.52%	1017	1016	14	3.58%
1015	1016	6	3.52%	1017	1018	14	3.58%
1015	884	6	3.52%	1018	1017	14	3.58%
1016	1017	6	3.52%	1018	1019	14	3.58%

1016	1015	6	3.52%	1019	1018	14	3.58%
1017	1016	6	3.52%	1019	1020	14	3.58%
1017	1018	6	3.52%	1020	1019	14	3.58%
1018	1017	6	3.52%	1020	1021	14	3.58%
1018	1019	6	3.52%	1021	1020	14	3.58%
1019	1018	6	3.52%	1021	1022	14	3.58%
1019	1020	6	3.52%	1022	1021	14	3.58%
1020	1019	6	3.52%	1022	1023	14	3.58%
1020	1021	6	3.52%	1023	1081	14	3.58%
1021	1020	6	3.52%	1024	1025	14	3.58%
1021	1022	6	3.52%	1146	1147	14	3.58%
1022	1021	6	3.52%	1158	1133	14	3.58%
1022	1023	6	3.52%	1240	1243	14	3.58%
1023	1006	6	3.52%	1243	1240	14	3.58%
1024	1025	6	3.52%	1250	1249	14	3.58%
1077	1087	6	3.52%	1315	1289	14	3.58%
1158	1133	6	3.52%	14	15	16	0.05%
1243	1240	6	3.52%	229	228	16	0.08%
1315	1289	6	3.52%	341	361	16	0.08%
229	228	7	0.13%	345	344	16	0.04%
685	709	7	0.13%	361	360	16	0.08%
686	691	7	0.13%	669	825	16	0.08%
691	692	7	0.13%	800	809	16	0.08%
692	693	7	0.13%	815	814	16	0.08%
693	694	7	0.13%	884	1015	16	0.08%
694	695	7	0.13%	912	913	16	0.08%
695	696	7	0.13%	913	914	16	0.08%
696	742	7	0.13%	914	1008	16	0.08%
815	814	7	0.13%	914	913	16	0.08%
884	1015	7	0.13%	919	912	16	0.08%
914	913	7	0.13%	933	932	16	0.08%
933	932	7	0.13%	944	1303	16	0.08%
1008	914	7	0.13%	949	983	16	0.08%
1009	970	7	0.13%	970	1009	16	0.08%
1015	1016	7	0.13%	1008	914	16	0.08%
1015	884	7	0.13%	1009	970	16	0.08%
1016	1017	7	0.13%	1015	1016	16	0.08%
1016	1015	7	0.13%	1015	884	16	0.08%
1017	1016	7	0.13%	1016	1017	16	0.08%
1017	1018	7	0.13%	1016	1015	16	0.08%
1018	1017	7	0.13%	1017	1016	16	0.08%

1018	1019	7	0.13%	1017	1018	16	0.08%
1019	1018	7	0.13%	1018	1017	16	0.08%
1019	1020	7	0.13%	1018	1019	16	0.08%
1020	1019	7	0.13%	1019	1018	16	0.08%
1020	1021	7	0.13%	1019	1020	16	0.08%
1021	1020	7	0.13%	1020	1019	16	0.08%
1021	1022	7	0.13%	1020	1021	16	0.08%
1022	1021	7	0.13%	1021	1020	16	0.08%
1022	1023	7	0.13%	1021	1022	16	0.08%
1023	1006	7	0.13%	1022	1021	16	0.08%
1077	1087	7	0.13%	1022	1023	16	0.08%
1158	1133	7	0.13%	1023	1006	16	0.08%
1240	1243	7	0.13%	1023	1081	16	0.08%
1243	1240	7	0.13%	1024	1025	16	0.08%
1315	1289	7	0.13%	1077	1087	16	0.08%
1320	1321	7	0.13%	1146	1147	16	0.08%
1321	1320	7	0.13%	1158	1133	16	0.08%
1322	1319	7	0.13%	1240	1243	16	0.08%
1322	1323	7	0.13%	1243	1240	16	0.08%
1323	1322	7	0.13%	1252	1253	16	0.08%
1323	1324	7	0.13%	1281	1282	16	0.08%
1324	1323	7	0.13%	1282	1289	16	0.08%
1324	1325	7	0.13%	1289	1315	16	0.08%
130	173	8	0.17%	31	32	18	0.57%
176	177	8	0.17%	130	173	18	0.57%
229	228	8	0.17%	229	228	18	0.57%
265	351	8	0.17%	241	347	18	0.57%
346	345	8	0.17%	242	241	18	0.57%
456	402	8	0.17%	345	346	18	0.57%
457	456	8	0.17%	351	265	18	0.57%
569	570	8	0.17%	361	360	18	0.57%
600	599	8	0.17%	456	402	18	0.57%
669	825	8	0.17%	502	503	18	0.57%
685	709	8	0.17%	539	538	18	0.57%
686	691	8	0.17%	669	825	18	0.57%
691	692	8	0.17%	685	709	18	0.57%
692	693	8	0.17%	686	685	18	0.57%
693	694	8	0.17%	696	742	18	0.57%
694	695	8	0.17%	800	809	18	0.57%
695	696	8	0.17%	884	1015	18	0.57%
800	809	8	0.17%	919	912	18	0.57%

815	814	8	0.17%	933	932	18	0.57%
884	1015	8	0.17%	944	1303	18	0.57%
912	913	8	0.17%	949	983	18	0.57%
919	912	8	0.17%	1008	914	18	0.57%
933	932	8	0.17%	1009	970	18	0.57%
944	1303	8	0.17%	1015	1016	18	0.57%
949	983	8	0.17%	1015	884	18	0.57%
970	1009	8	0.17%	1016	1017	18	0.57%
1009	970	8	0.17%	1016	1015	18	0.57%
1015	1016	8	0.17%	1017	1016	18	0.57%
1015	884	8	0.17%	1017	1018	18	0.57%
1016	1017	8	0.17%	1018	1017	18	0.57%
1016	1015	8	0.17%	1018	1019	18	0.57%
1017	1016	8	0.17%	1019	1018	18	0.57%
1017	1018	8	0.17%	1019	1020	18	0.57%
1018	1017	8	0.17%	1020	1019	18	0.57%
1018	1019	8	0.17%	1020	1021	18	0.57%
1019	1018	8	0.17%	1021	1020	18	0.57%
1019	1020	8	0.17%	1021	1022	18	0.57%
1020	1019	8	0.17%	1022	1021	18	0.57%
1020	1021	8	0.17%	1022	1023	18	0.57%
1021	1020	8	0.17%	1023	1006	18	0.57%
1021	1022	8	0.17%	1073	1072	18	0.57%
1022	1021	8	0.17%	1077	1087	18	0.57%
1022	1023	8	0.17%	1158	1133	18	0.57%
1023	1006	8	0.17%	1240	1243	18	0.57%
1023	1081	8	0.17%	1243	1240	18	0.57%
1077	1087	8	0.17%	120	121	19	0.13%
1158	1133	8	0.17%	173	130	19	0.13%
1240	1243	8	0.17%	239	171	19	0.13%
1243	1240	8	0.17%	244	245	19	0.13%
1289	1315	8	0.17%	245	246	19	0.13%
1343	1344	8	0.17%	246	223	19	0.13%
1343	1342	8	0.17%	265	351	19	0.13%
1344	1343	8	0.08%	341	342	19	0.13%
17	16	9	0.11%	342	343	19	0.13%
120	121	9	0.11%	343	344	19	0.13%
130	173	9	0.11%	344	345	19	0.13%
230	231	9	0.09%	345	346	19	0.13%
231	232	9	0.09%	346	345	19	0.13%
232	233	9	0.11%	473	476	19	0.13%

233	234	9	0.11%	538	539	19	0.13%
347	241	9	0.11%	555	548	19	0.13%
538	539	9	0.11%	569	570	19	0.13%
563	556	9	0.11%	618	626	19	0.13%
600	599	9	0.11%	623	633	19	0.13%
664	642	9	0.09%	628	625	19	0.13%
669	825	9	0.11%	669	825	19	0.13%
685	709	9	0.11%	686	685	19	0.13%
696	742	9	0.11%	694	695	19	0.13%
800	809	9	0.11%	696	742	19	0.13%
884	1015	9	0.11%	789	677	19	0.13%
914	913	9	0.11%	800	809	19	0.13%
933	932	9	0.11%	815	814	19	0.13%
944	1303	9	0.11%	912	913	19	0.13%
949	983	9	0.11%	913	914	19	0.13%
970	1009	9	0.11%	933	932	19	0.13%
1008	914	9	0.11%	970	1009	19	0.13%
1009	970	9	0.11%	1009	970	19	0.13%
1015	1016	9	0.11%	1017	1018	19	0.13%
1015	884	9	0.11%	1018	1019	19	0.13%
1016	1017	9	0.11%	1020	1019	19	0.13%
1016	1015	9	0.11%	1023	1006	19	0.13%
1017	1016	9	0.11%	1023	1081	19	0.13%
1017	1018	9	0.11%	1077	1087	19	0.13%
1018	1017	9	0.11%	1123	1122	19	0.03%
1018	1019	9	0.11%	1146	1147	19	0.13%
1019	1018	9	0.11%	1158	1133	19	0.13%
1019	1020	9	0.11%	1243	1240	19	0.13%
1020	1019	9	0.11%	1315	1289	19	0.13%
1020	1021	9	0.11%	229	228	20	0.12%
1021	1020	9	0.11%	230	231	20	0.12%
1021	1022	9	0.11%	231	232	20	0.12%
1022	1021	9	0.11%	232	233	20	0.12%
1022	1023	9	0.11%	233	234	20	0.12%
1023	1006	9	0.11%	234	235	20	0.12%
1024	1025	9	0.11%	265	351	20	0.12%
1077	1087	9	0.11%	346	345	20	0.12%
1158	1133	9	0.11%	456	402	20	0.12%
1240	1243	9	0.11%	457	456	20	0.12%
1243	1240	9	0.11%	538	539	20	0.12%
1289	1315	9	0.11%	540	541	20	0.09%

130	173	10	1.31%	569	570	20	0.12%
229	228	10	1.31%	669	825	20	0.12%
241	347	10	1.31%	685	709	20	0.12%
242	241	10	1.31%	686	685	20	0.12%
243	242	10	1.31%	757	758	20	0.12%
244	243	10	1.31%	769	768	20	0.12%
245	244	10	1.31%	789	677	20	0.12%
276	275	10	1.31%	815	814	20	0.12%
341	361	10	1.31%	884	1015	20	0.12%
342	341	10	0.05%	914	913	20	0.12%
343	342	10	0.05%	933	932	20	0.12%
344	343	10	0.05%	944	1303	20	0.12%
345	344	10	0.05%	1008	914	20	0.12%
346	345	10	0.27%	1009	970	20	0.12%
351	265	10	1.31%	1015	1016	20	0.12%
361	360	10	1.31%	1016	1017	20	0.12%
456	402	10	1.31%	1017	1018	20	0.12%
473	476	10	1.31%	1018	1019	20	0.12%
539	538	10	1.31%	1019	1020	20	0.12%
600	599	10	1.31%	1020	1021	20	0.12%
669	825	10	1.31%	1021	1022	20	0.12%
800	809	10	1.13%	1022	1023	20	0.12%
884	1015	10	1.31%	1023	1006	20	0.12%
914	913	10	1.31%	1024	1025	20	0.12%
944	1303	10	1.31%	1077	1087	20	0.12%
970	1009	10	1.31%	1158	1133	20	0.12%
1008	914	10	1.31%	1194	1193	20	0.12%
1009	970	10	1.31%	1240	1243	20	0.12%
1015	1016	10	1.31%	1243	1240	20	0.12%
1015	884	10	0.35%	1275	944	20	0.12%
1016	1017	10	1.31%	1275	1276	20	0.12%
1016	1015	10	0.35%	1276	1275	20	0.12%
1017	1016	10	0.35%	1276	1277	20	0.12%
1017	1018	10	1.31%	1277	1276	20	0.12%
1018	1017	10	0.35%	1277	1278	20	0.12%
1018	1019	10	1.31%	1315	1289	20	0.12%
1019	1018	10	0.35%	1324	1325	20	0.12%
1019	1020	10	1.31%	1325	1324	20	0.12%
1020	1019	10	0.35%	1325	1326	20	0.12%
1020	1021	10	1.31%	1327	1326	20	0.12%
1021	1020	10	0.35%				