A CAUSAL MODEL OF TEACHER INFLUENCE ON MATHEMATICAL SELF-CONCEPT AMONG COLLEGE FRESHMEN

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CHAPTER 1
INTRODUCTION

"Tell me and I'll forget. Show me and I may not remember. Involve me, and I'll understand."

—Native American saying.

Anyone who has ever taught, participated in, or even observed a math class has seen this student: flushed, impatient, and discouraged daily by the tortuous rigors of a mathematics class. For forty to forty-five minutes each day, this student feels he or she can do nothing right. Even simple calculations are incorrect; everything becomes a jumble; the mind goes blank, and this student is convinced that there is absolutely no way he or she can pass this mathematics test. This student and countless others may do very well in other classes. However, when it comes to math class, confidence in their ability, also known as math self-concept, seems to lag. To make matters worse, the idea of "not being good at math" has become readily acceptable, even in adults, whereas such distinctions are not made in other fields such as reading, English, or social studies (Skiba, 1990). However, because math is based not just on memorizing, but on understanding and reasoning (more than in classes like history, where memorizing facts and dates is often adequate), people tend to think that math is logical to a few, but incomprehensible to most (Greenwood & Anderson, 1983). This seems to not only justify, but also perpetuate the idea of declaring oneself "not good at math," and results in the continual decline of students' math-self concept throughout their education.
Instructors are central to students' educational experiences. In fact, teachers have been shown to have more influence on student achievement than methods of teaching or curriculum (Austin, Bitner, & Wadlington, 1992). As almost any instructor will verify, a student interested in a subject is more likely to be motivated to spend time working at improving skills and knowledge in that subject, and to enjoy that subject more. This raises the question of what "magical" characteristics or techniques can be implemented by math instructors to raise the level of the students' interest and, more importantly, to foster a positive math self-concept among students, and hopefully encourage students to continue taking mathematics courses?

Important instructional techniques have been outlined in the mathematics reforms set forth by the National Council of Teachers of Mathematics (NCTM). They cover five basic areas: (1) the type of tasks that teachers should pose, (2) the type of discourse teachers should foster in their classroom, (3) the students' role in classroom discourse, (4) promoting mathematical disposition, and (5) the learning environment. Each of these five areas affects what a student learns, how they learn, and most importantly, how they view what they've learned. Therefore, the effects of the instructor, along with the effects of utilizing NCTM teaching techniques on math self-concept will be of primary interest in this study.

While the perceptions a student has of his or her math instructor and instructional techniques are critical, other factors have been significant in examining math self-concept in past research. These factors include gender, interest in math, perceived usefulness of math, number of math classes taken, math ability, and parental
influences (specifically influences of the father).

**Purpose of the Study**

Past studies have looked at innumerable different potential factors which possibly affect math self-concept. However, much of the research has produced results that are inconclusive. In fact, research shows that factors which would seem obvious, such as mathematical aptitude and scores on standardized tests are not always strong predictors of math self-concept (Sax, 1992). Therefore, the purpose of this study was to test a causal model derived from the literature which examines the causal relationship different factors have with students' math self-concept. The variables of interest included in the theorized causal model of math self-concept are gender, fathers' math ability, fathers' education level, number of math classes taken in high school, math achievement, interest in math, perceived usefulness of math, perceptions of math teacher, and utilization of NCTM standards. Specifically, variables presumed to have a direct effect on math self-concept are (1) students' interest in math, (2) gender, (3) math achievement, and (4) perceptions of their most memorable math teacher. Variables believed to exert indirect effects on math self-concept are (1) gender, (2) fathers' math ability, (3) fathers' education level, (4) number of high school math classes taken, (5) degree to which math teachers implemented NCTM standards, (6) students' perceptions of their most memorable math teacher, (7) perceived usefulness of math, and (8) interest in math. Several of the variables have both direct and indirect effects. While previous studies have looked at relationships of various factors with
math self-concept, no studies have examined the causal ordering of the variables. This study investigates the fit of data gathered on college freshmen to the theorized causal model predicting math self-concept.

Figure 1 presents the theorized causal model. Straight, single-headed arrows represent a unidirectional causal path in a path diagram. The arrow originates at the variable exerting the causal influence, and the arrow points toward the variable being affected. Math self-concept is the dependent variable of interest for this study. Interest in math, math achievement and perceived usefulness of math are considered intervening or mediating variables, and gender, fathers' math ability, fathers' education level, number of math classes taken, utilization of NCTM standards, and perceptions of math teacher are considered independent variables.

From the background research conducted, two general research questions were formulated:

**Research Question #1:** Do the data collected on college freshmen fit the theorized causal model?

**Research Question #2:** Does the students' perception of their math instructor and type of instructional methods used to teach mathematics have a significant effect in determining a students' math self-concept, or is math self-concept shaped more by some or all of the following factors: gender, math achievement (measured by the ACT math test score), pre-college enrollment patterns in math classes, fathers' educational level, students' ranking of their fathers' mathematical ability, student interest in math, and perceived future usefulness of mathematics?
Figure 1. Theoretical Structural Model

- Gender
- Fathers' Math Ability
- Fathers' Ed Level
- Number of Math Classes
- NCTM Standards
- Perceptions of Math Teacher

Interest in Math

Math Achievement

Math Self-Concept

Perceived Usefulness of Math
With the exception of fathers' education level, and ranking of fathers' mathematical ability, the relationship between each factor and students' math self-concept should be positive. Past research on female students shows the parental factors (specifically the fathers' education level and math ability) have a negative relationship to students' math self-concept. This is explained by women feeling a pressure to succeed, accompanied by feelings of not living up to their parents' (specifically the fathers') expectations or achievements (Sax, 1992). No studies were located that investigated the effect of parental factors on males students, or that investigated the effects of the mothers' education level or math ability on male or female students.

Each of the factors were selected based on rationale from previous research. While factors such as scores on standardized tests (for the purposes of the this study, the ACT mathematics test) have not been found to be the strongest predictors of math self-concept (Sax, 1992), they each still contribute in past prediction models. The ACT mathematics test scores will serve as the measure of high school mathematics achievement. Pre-college enrollment patterns in mathematics classes has also been shown to affect a student's perception of their mathematical ability. Taking more college-preparatory mathematics classes tends to raise students' math self-concept (Eccles, Meece, & Wigfield, 1990).

Central to this study are the characteristics of teachers. The instructor has been cited as the most pivotal influence on students' interest and motivation in a subject (Froebe, 1996). Therefore, the characteristics of past mathematics instructors would certainly seem to have a significant impact on math self-concept. Teacher
characteristics include the interaction between the student and teacher, the way an instructor managed the class, problem-solving and communication techniques, and encouragement and positive feedback the instructor provided. In this area, probably more than any other, there is research to show that males and females learn and respond differently to different types of instruction and classroom discourse (Hanson, 1992). Research has also shown that there is a difference in how males and females assess their mathematical ability. Females tend to assess their ability lower than males, even when the females have higher grades in mathematics courses.

**Definition of Terms**

*Standardized Math Tests:* Tests such as the ACT, SAT, or state achievement tests, in which the testing conditions and the scoring of the tests are consistent among examinees. These tests have been shown to have very high reliability and moderate validity for predicting freshmen performance in mathematics classes.

*Mathematical Achievement:* A construct measured, for this study, in terms of scores on standardized mathematics tests.

*Mathematics Self-Concept:* Individuals' self-assessment of their mathematical ability as measured by related items concerning confidence in mathematics, enjoyment of math, and past experiences in mathematics.

*Perceived Usefulness of Mathematics:* Individuals' opinion of how important mathematics is to their future, whether the future is school, occupation, or simply life skills.
**Spatial Visualization**: Individuals' ability to picture spatial relationships within and between objects.

**Classroom Discourse**: Considered central to how students learn, discourse is the way ideas are exchanged within a classroom.

**Significance of the Study**

The primary focus of this study examined causal relationships in which students' math self-concept was directly influenced by past instructors. While other research has looked at effects of individual variables, this study examined the causal relationships. This is important because causal modeling moves beyond typical regression analysis and allows the researcher to investigate not only the direct unmediated causal effects of each variable, but also their indirect effects through intervening variables (Braxton, Duster, & Pascarella, 1988).

The effects and implications associated with math self-concept are also of great importance. It is no secret that mathematics achievement scores of students in the United States are on the decline. At all levels, students in the United States seem to be falling behind their counterparts from other countries at all levels (Swetman, 1994). A discovery in the early seventies that women are more likely than men to avoid courses in mathematics made gender differences in mathematics a source of concern and extensive research. However, it is now generally recognized that many highly motivated, bright people, including both men and women, avoid courses and activities that involve quantitative analysis. The immediate result of this trend is low
achievement test scores in the area of mathematics. The long term--and more devastating--result is that these people severely limit their career options and put themselves at a potentially costly disadvantage in their daily lives (Diers & Shodahl, 1984).

Reasons for the decline in mathematics scores have been investigated and reported. Among these reasons are gender, innate mathematical ability, parental expectations, negative attitudes toward mathematics, low number of math courses taken, and increase in mathematics anxiety (Swetman, 1994). While gender, innate mathematical ability, and even parental expectations are characteristics developed before a child enters a formal educational setting, attitude toward mathematics, number of courses taken, and mathematics anxiety are more connected to events in their educational experiences. It is these experiences which will be the focus in this study.

Due to the distress over the national decline in math achievement, and interest in math-related careers, any research which may provide evidence to explain this phenomena would be considered significant. While some factors are beyond control, others are areas that can be examined and possibly improved on. For example, if evidence related to characteristics of instructors and/or type of instruction can be shown to causally affect the way students learn and view mathematics, then specific actions can be taken to optimize instruction for both males and females.
Assumptions

First, since the data involved information from pre-college educational experiences, it was assumed that first-semester freshmen were able to objectively and accurately recall details from their previous mathematics classes. Second, though the majority of freshmen enroll in an orientation class, there was a small percentage of freshmen who did not. Since the sample was drawn exclusively from freshmen orientation classes, it was assumed that the students enrolled in these classes were representative of all freshmen at this university. From comparisons made between the sample and population concerning ACT math test scores, and enrollment percentages by college and gender, there was no evidence that the sample was not representative.

The last assumption concerns the method used for obtaining the sample. Cluster sampling--sampling in which groups, not individuals, are randomly selected--was used to select the sample for this study. While there are numerous advantages to this type of sampling, a drawback is that the chances were greater of selecting a sample that is not representative in some way of the population. However, the assumption was made that a sufficiently large sample size would compensate for this problem.

Limitations

A limitation to be considered was that students participated on a voluntary basis. If the students willing to participate had characteristics important to this particular study which differ from the students who did not wish to participate, this would bias the results. Also, while subjects were asked to answer questions pertaining
to math instructors and math classes based on their most memorable instructor (high school or before), because students surveyed had been attending college classes for several weeks, it is possible that college math experiences could have contaminated results. One other limitation in regard to the survey was the subjects' rating of their fathers' math ability. This rating was obviously a subjective one, and could have been influenced by the students' perception of their own math ability.

Another critical limitation involves the scheduling of classes to survey. While the classes should ideally be chosen randomly, several constraints made this impossible. First, freshmen orientation classes meet only for nine weeks, so the time span in which one could utilize these classes for collecting data was somewhat limited. Second, logistical constraints and permissible times at which the instructors would allow classes to be surveyed had to be taken into account. However, the sample drawn did include representatives from every college on campus. Also, comparisons were made between the participants and the entire freshmen class, which showed the sample to be representative of the target population, based on the characteristics which were compared.

A final limitation is the lack of control over student attrition. While a dropout rate, specifically for the 1997 fall semester, was not available, the average dropout rate for freshmen at Oklahoma State University for the previous ten years is 23.9 percent. If students dropped out during the course of this study this could affect its internal validity.
Organization of Dissertation

Chapter two discusses the pertinent literature and previous research related to math self-concept, gender differences in mathematics, and the important role the instructor plays in a mathematics class.

Chapter three presents the methodology used in the study: sampling techniques, an explanation of each instrument used, a discussion of how the study was implemented, and the type of statistical procedures used. First, an explanation of how the sample of subjects were selected, along with a brief description of the characteristics of the sample, including how large a sample was chosen. Second, this chapter addresses and includes an explanation of each instrument chosen to be used in the study. Those instruments include the ACT mathematics test to assess students' mathematical achievement, the Mathematics Attitude Inventory (MAI) to assess student attitudes concerning numerous different areas, a scale to assess the utilization of NCTM standards, and items to obtain demographic information on subjects. Finally, chapter three includes a discussion of how the study was conducted, including the type of statistical procedures used and how the statistical results were analyzed.

Chapter four will present the results of the analyses; chapter five the summary, conclusions, recommendations.
CHAPTER 2

REVIEW OF THE LITERATURE

A strong background in mathematics is critical for many career and job opportunities in today's increasingly technological society. In fact, there is some indication that high school mathematics coursework is related to confidence and attitudes towards computers since math has been found to be a necessary prerequisite for computer performance (Dambrot & Lindbeck, 1986). However, many academically capable students prematurely restrict their educational and career options by discontinuing their mathematical training early in high school. Only 50% of all high school graduates enroll in mathematics courses beyond the 10th grade (Eccles, Meece, & Wigfield, 1990). Of the 50% who do enroll in advanced math classes, the number of males is significantly higher than the number of females. By the time they reach college, most young women have opted out of mathematics-related programs altogether, a process that begins to be most apparent after high school geometry (Hanson, 1992). Lack of enrollment in mathematics courses obviously leads to lower mathematical ability, but also indicates a negative attitude in general regarding mathematics, and one's own mathematical ability.

This raises several distinct areas in which past studies should be examined: (1) math self-concept in general, (2) factors which impact math self-concept, and (3) the effects instructors have on students' math self-concept. The literature review will conclude with a detailed description of mathematics reforms set forth by the National
Council of Teachers of Mathematics (NCTM). The five basic areas addressed by these reforms are believed to influence students' math self-concept, and therefore are a critical part of this research study.

Math Self-Concept

Self-concept is broadly defined as a person's self-perceptions, formed through experience with and interpretations of one's environment. It is especially influenced by evaluations by significant others, reinforcements, and attributions for one's own behavior and accomplishment (Marsh, 1993). Math self-concept would then be a person's self-perceptions of their mathematical ability, formed by past experiences, and evaluations. These experiences and evaluations involve situations that one would typically associate with formal education and, therefore, it is a fair assumption that for most students, math self-concept is determined throughout their school years. It has been found that low math self-concept has its roots in the teachers and the teaching of mathematics (Williams, 1988). The question of when this occurs and why should be posed. In the National Assessment of Educational Programs (NAEP), 9-year-olds ranked math as their best-liked subject, 13-year-olds ranked it second best, but 17-year-olds placed it as the least-liked subject (Stodolsky, 1985). Somewhere between the ages of 13 and 17, there is a drastic change in the way students feel about mathematics. These are typically the years when students would be taking Algebra I, Algebra II, Geometry, and advanced math classes. It is also in these classes that students would be forced to move from simple concrete ideas, to more abstract notions and thinking.
Other studies confirm this decline in positive attitudes towards math in high school years (Brush, 1981; Crosswhite, 1972). However, even though positive attitudes may decline, there is no decrease across the school years in students' perceptions that mathematics is important (Stodolsky, 1985). So even though students may feel that mathematics are important to their future, they dislike math so intensely that they appear willing to jeopardize their future education and career options by not enrolling in math courses past the mandatory requirements.

Literature has shown that teachers are a major source of influence on student achievement and student attitudes (Flannery, 1993; Froebe, 1996). In fact, many attitudes about math (especially negative ones) can be traced to a particular teacher or class (Martinez, 1987). Students who have a low math self-concept often remember feelings of tension and inadequacy when an instructor told them to raise their hand when they knew the right answer, or even more specific instances when they were told to "stay at the chalkboard until you find your mistake." As students became older, asking a teacher for help on a problem, only to have the teacher work it for them without explanation was a common recollection. The literature (Greenwood, 1984; Lazarus, 1974; Peterson & Fennema, 1985; Williams, 1988) overwhelming implies that math learning--more than most subjects--is largely a function of math teaching. A logical corollary to that statement is that math self-concept may also be a function of the math teacher. Therefore, what goes on in the classroom, in terms of both student-teacher interaction and methods of teaching is of prime importance.

A student's math self-concept is, among other factors, determined in part by
past successes and/or failures. While students who experience success do not always have a positive math self-concept because they attribute that success to other factors, students who experience consistent failure in a mathematics class do indeed appear to understandably have a negative self-concept of their math ability (Bandalos, Thorndike-Christ, & Yates, 1995). Students with a low math self-concept often claim that they feel like they have no control or understanding over what they are doing. In fact, one of the major sources of math anxiety and low math self-concept lies in the impersonal, nongrowth, nonrational methodologies that are characterized by the "explain-practice-memorize" paradigm. This typical math teaching paradigm is based on memorization, not on understanding and reasoning. It promotes and perpetuates the very common perception of mathematics as a subject that appears easy and logical to a few "brains" and incomprehensible to most common folk (Greenwood & Anderson, 1983). Like music, art, and some other fields, math is accepted as a domain in which ability plays a major role and is commonly perceived in a rather dichotomous way: either one has or does not have the ability to do math (Stodolsky, 1985). While this way of thinking is not acceptable in classes such as history or English, it has become culturally acceptable to declare oneself simply "not good at math."

**Gender Differences**

While the downward spiral of students' attitudes towards mathematics involves both males and females, research consistently shows that females exhibit higher levels of math anxiety than men, and that math avoidance is more prevalent among women
(Meece & Wigfield, 1988; Llabre & Suarez, 1985; Tobias, 1993). Also, gender differences in mathematical achievement tend to favor boys. This difference in mathematical achievement between males and females includes grades received in mathematics classes, as well as scores on standardized achievement tests (Callahan & Clements, 1984; Chambers, 1988; Dossey, Lindquist, & Mulis, 1988; Hanna, Kundiger, & Larouche, 1990; Rickman, 1989; Sax, 1992). As previously noted, males take significantly more advanced math courses than females. By the time they reach college most young women have not taken a math class in several years, and have decided against any math-related major. Whereas boys seem to accept mathematics as a means to college and their future goals, girls often see no need for mathematics (Skiba, 1990). Greater expectations typically exist for boys to use mathematics for their later careers, and therefore they tend to take more mathematics courses when they have the choice (Macoby & Jacklin, 1974). The results are quite clear that women seem to be undereducated in mathematics and underrepresented in professions related to math (Greenwood, 1984).

A potential reason may begin in early childhood. Earlier research on gender differences in the field of perception suggested that, as children, males have inherently greater interest in objects and visual patterns, whereas females show more interest in people and facial features (Garai & Scheinfeld, 1968). The toys given to boy and girl babies also tend to differ. Boys are given toys that encourage small motor skills and spatial visualization, skills believed to be necessary for later math success. Girls' toys, on the other hand, often encourage relational or traditionally nurturing activities
More currently there has been research showing that boys are encouraged much more to become adept at using computers. For example, when the parents of a boy and girl buy a new computer, one study found, it usually goes in the boy's bedroom. In education settings, teachers often allow boys to dominate computer classes, while the less-assertive girls, left by the wayside, often don't increase their skills much. This is similar to problems which have historically existed regarding the teaching of math and science to girls. Just how important are computer skills? Even playing computer games helps develop an array of learning skills, such as focusing, concentration and problem-solving. Most importantly, perhaps, it helps children to acquire a familiarity and ease with technology, which is of critical importance in the future job market (Brzowsky, 1998).

There has been a long-standing idea that females excel in areas that rely on verbal skills, while males excel in areas that emphasize mathematical skills. Although there have been studies supporting this theory, there have also been studies that have failed to support this notion. In fact, Marsh (1993) not only found no support for these gender-specific patterns of innate skills, but further found only a small difference in math self-concept for males and females. However, this remains a very commonly accepted opinion. So common in fact, that it has seemed to become a self-fulfilling prophecy (Boekaerts & Seegers, 1996). Differences in how males and females are treated begins at a very young age. Males are encouraged to participate in sports much more than females, and thereby develop more of a sense of competitiveness than females. In many cases where mathematics classrooms foster competitive attitudes,
males would seem to have a definite advantage (Rickman, 1989).

Another important area of research has centered around the difference in the way males and females view their mathematical ability. The research fairly consistently found that men were more likely to rate their mathematical ability higher than females, despite the fact that females sometimes had higher grades (Barnes, Marsh, & Smith, 1985; Boekaerts, & Seegers, 1996; Brahier, 1995; Sherman, 1983). Similarly, in a study which compared college students' math self-concept estimates with their actual SAT math scores, Drew (1992) found women more likely than men to underestimate their math ability. It thus appears that math self-concept may be a function of factors other than actual math ability (Csikszentmihalyi & Schiefele, 1995; Sax, 1992).

Research on mathematics achievement of girls has also surfaced in which several important points indicate a strong pattern of socialization to mathematics success or failure. Throughout their learning girls are encouraged to be passive, caring, to take no risks, and to defer to male voices in the public discussion. They are also given the message that math is for males. Such an orientation obviously has an impact on how they learn and behave in school. Even the discourse mode, and the dynamics of the classroom are oppositional to the way females are socialized to interact and communicate. Males tend to focus on the importance of debates about ideas, while females place importance on mutual support and the building of collaborative knowledge (Hanson, 1992).

For those women who attempt to enter into the discourse as equals by adopting
a male discourse mode, the response is no better. Women are often penalized for attempting to participate in the male domain. Often the perception of behavior is confused with actual behavior, based on sex-role stereotypes. While a male might be called ambitious, assertive, and independent, a woman displaying the same behaviors is often labeled aggressive, pushy, and argumentative. Studies have shown that when women and men exhibit the same behavior, that behavior is devalued for women (Pearson, 1987).

Research has also been conducted which shows that males receive the most positive attention within a class; that males are also pushed to think, to expand ideas, and to defend their positions more than females (Irvine, 1985). A critical study has shown that teachers give the most attention to students whom they perceived as above average, and that it is these students who end up performing better on tests (Leder, 1987). Therefore, if a teacher innately believes that males are better than females at mathematics, the males will be given the most attention, and perform better on tests, whether males are inherently better at mathematics than females or not.

**Parental Influences**

Closely tied with students' math self-concept are the stereotypical beliefs concerning mathematics which are shared among the majority of students, and even society in general, regardless of gender. Research in this area is significantly related to parental and societal factors. An important characteristic affecting the development of women's math self-concept is the level of the father's education and math ability. In a
past study (Sax, 1992), which examined the influence fathers' education and math ability had on the daughters' mathematics self-rating, the resulting regression coefficient was negative. This suggests that women with more highly educated fathers are more likely to have lower mathematics self-ratings regardless of these women's initially higher math abilities. This is an intriguing finding, and suggests that perhaps women with more educated fathers feel a greater pressure to succeed, and might not feel that they are living up to their father's expectations or achievements. Interestingly, this was the only research which looked at the relationship between math self-concept and either parent. No research was found which discussed the mothers' potential influence on a child's math self-concept, or discussed the influence fathers had on sons.

**Number of Math Classes Taken in High School**

Enrollment patterns in math classes is also very important. It is not surprising that one study (Witherspoon, 1993) showed that students who are in a non-college preparatory track (NCP) and took fewer math classes demonstrated lower math achievement than students in a college preparatory track who took more math classes, and that NCP students appeared to depend on a school context for deciding whether situations were mathematical. This indicates that the number of math classes taken not only influences math achievement, but also the ability to determine specific situations where math was useful, which relates to the perceived usefulness of math. Another study (Eccles, Meece, & Wigfield, 1990) which supports these findings also found that students' ratings of the importance of math was directly related to their math course
enrollment. A final study (Csikszentmihalyi & Schiefele, 1995) stated that the number of math classes taken significantly predicted interest in math as well. These studies show evidence of the importance course enrollment in mathematics classes plays in (1) math achievement, (2) perceived usefulness of math, and (3) interest in math.

Interest In Mathematics

Research by Csikszentmihalyi and Schiefele (1995) revealed numerous findings concerning interest in mathematics. Results indicated that interest in mathematics was the strongest predictor of quality of experience in a math class. Specifically, interest showed significant relations to potency, intrinsic motivation, self esteem, and perceptions of skill. Interest also proved to be a moderate and significant predictor of grades. For the purposes of this study, these findings indicate that causal relationships should exist between interest in math and math self-concept, as well as between interest in math and math achievement.

Perceived Usefulness of Math

While it has been stated that students' rating of the importance of math is related to their course enrollment, it has also been shown that teachers can help enhance students' valuing of math. This can be done in several ways, including relating the value of math to students' everyday lives, making math personally meaningful, and counseling students about the importance of math for various careers. In turn, mathematical tasks which are seen as valued and applicable to real world
situations were also found to be positively related to the students' interest in math
(Eccles, Meece, & Wigfield, 1990). Therefore it appears that students' perceived
usefulness of math is related not only to the number of math classes taken, but also to
the instructor, and that the more useful mathematical tasks are, the more interesting
they become to students.

Math Achievement

As previously stated, math achievement has not always been found to be the
strongest predictor of math self-concept (Sax, 1992). However, that is not to say that
math achievement has no impact on math self-concept. For academically-oriented
students (those who attend college) it is highly possible that their perception of their
ability in a subject is closely tied to achievement in that particular subject. Therefore
math achievement is hypothesize to have a significant effect on math self-concept.
Also, according to the American College Testing Program, math achievement (as
measured by the ACT math test) is positively related to the number of math classes
taken. ACT math test scores for students who have completed three years of math are
at least three points higher than students who have not completed three years of math

Instructor Effects

The last area of crucial research deals with the mathematics instructors, and the
effects they can have on students. Not only is the teacher, by far the most significant
influence on students' interest in a subject (Froebe, 1996), but of students who had been discouraged in mathematics, approximately 70% report being discouraged by a teacher (Flannery, 1993). Considering this, the importance of what actually happens in classrooms, particularly in terms of teacher-student interactions and both teacher and student expectations, should not be underestimated. It is within the classroom that sex role expectations and socialization converge to influence both the curriculum and the real experiences of the students (Hanson, 1992). According to Fennema and Peterson (1986) the challenge to educators is complex: encourage girls and women to participate in mathematics, and change the paradigm of discourse that prevents their participation. They further suggest that teachers need to (1) directly encourage autonomous learning behaviors in girls, (2) engage girls in high-level discourse interactions, (3) provide praise and positive feedback for effort and for appropriate strategies, (4) develop strategies for encouraging divergent thinking, and (5) encourage independence. These things are important because past studies have shown a difference in learning styles, achieving styles, and learning strategies for males and females (Brahier, 1995; Martin, 1993; Moore, 1994).

Learning strategies deal with how these assignments are approached. This may include verbal-logical versus visual-spatial strategies (e.g. writing details out or simply picturing objects or equations in one's mind). Learning styles, on the other hand, involve sensory learning modalities such as auditory, visual, kinesthetic, and tactile. Auditory learners are those students who learn best by verbal instruction, tend to talk while they write, and typically find games and pictures distracting. Visual learners
need to see words or pictures, have vivid imaginations, and tend to think in images. Kinesthetic and tactile learners are very similar in that they learn by doing. These students are very "hands-on," and need manipulatives because they tend to remember what was done, not things that were seen or talked about. Learning styles can also involve analytical learners versus global learners. Analytical learners (left brained or convergent thinkers) are organized, and attempt to think through problems in a linear, sequential order. Global learners (right brained or divergent thinkers) are spontaneous, and attempt to think through problems from a more creative, intuitive perspective (Wilson, 1998).

Elliott (1983) compared divergent and convergent thinkers in relation to math anxiety and ability. Mathematics, as it is most typically taught, capitalizes on and rewards convergent, logical, stepwise thinking, while almost totally ignoring the creative, freeflowing, imaginative, divergent thinking. Not surprisingly, learners inclined toward divergent thought grow resentful or develop anxieties about having to obtain a definite answer by following a formal sequence of steps. However, convergent thinkers also develop anxieties and resentments because their lockstep methods (which they grew to expect always to work) begin to fail them when confronted with creative problem-solving tasks. This occurs because few instructors realize that mathematics requires a coupling of both divergent and convergent thinking. Working story problems or application problems are prime examples of the need to combine divergent and convergent thinking. The actual calculations typically use logical steps, thereby invoking convergent thinking. However, setting up the problem
so that calculations can be made involves more creativity and imagination—characteristics of divergent thinking. Both types of thinking are therefore essential to completing the problem.

Failure to teach students how to use both types of thought processes is one instruction-related factor which results in increased math anxiety, and in turn affects students' math self-concept, but it is definitely not the only critical factor. Success in mathematics is related to individual learning styles, and more specifically to the coupling of learning styles with methods of presenting the material. This combination is even more critical than the subject matter itself (Dunn, 1986; Hodges, 1983; Williams, 1988). However, evidence suggests that instruction in math classes tends to be much more homogeneous than instruction in most other subjects. Put simply, there seems to be less variation in how mathematics are taught than in other subjects. This is true starting in the elementary setting all the way through high school. In elementary school, the same teacher teaches multiple subjects, but is still more restrictive in how math is taught as compared to other subjects taught by the same instructor. In the high school settings instructors specialize in one subject. However, math instructors are still more rigid in methods of instruction, and classroom routines that instructors of other subjects. The instructional formats (e.g. seatwork and recitation), the behavior expected from students, and materials used all show greater similarity from day to day within a given math class and across classes of different teachers in different schools and districts than did the same variables in other subjects. Stodolsky (1985) described the typical math instruction as containing six main components: (1) reliance on a
recitation and seatwork pattern of instruction, (2) reliance on teacher presentation of new concepts, (3) textbook centered instruction, (4) textbooks that lack developmental or instructional material for concept development, (5) lack of manipulatives, and (6) lack of social support or small-group work.

Settings similar to the one described can reach only a small percentage of students with diverse learning styles, but even more distressing is that this type of instruction does not in any way encourage students to become independent thinkers. Students often find textbooks basically unintelligible and of little help, and typically assume that the teacher is their only learning resource. This dependence created between the math teacher and math learner over many years is a root problem in attempting to raise students' math self concept, as well as math achievement, because if they don't understand the instruction provided by the teacher, students feel powerless to do anything to help themselves (Stodolsky, 1985).

NCTM Reforms

The effect of the mathematics instructor is not be restricted simply to the interaction they have with students, but also the methods of instruction they employ. In past years, the National Council of Teachers of Mathematics (NCTM) has paved the way for all national standards curriculum reform efforts (Center for Science and Technology, 1994). The NCTM publishes curriculum standards, as well as professional standards for teaching mathematics. These standards represent NCTM's vision of what students should learn in mathematics classrooms. Congruent with aims
and rhetoric of the current reform movement in mathematics education (e.g., National Research Council, 1989, 1990), the Standards is threaded with a commitment to developing the mathematical literacy and power of all students. The vision embodied in the Curriculum and Evaluation Standards is that mathematical reasoning, problem solving, communication, and connections must be central. Computational algorithms, the manipulation of expressions, and paper and pencil drills must no longer dominate school mathematics. Along with all this, teachers must foster in students the disposition to use and engage in mathematics, an appreciation of its beauty and utility, and a tolerance for getting stuck or sidetracked. Teachers must help students realize that mathematical thinking involves dead ends and detours, encourage them to persevere when confronted with a puzzling problem, and help them develop the self-confidence and interest to do so (National Council of Teachers of Mathematics [NCTM], 1991).

The NCTM sets forth guidelines in numerous areas, including (1) the type of tasks that teachers should pose, (2) the type of discourse a teacher should foster in their classroom, (3) the students' role in classroom discourse, (4) promoting mathematical disposition, and (5) the learning environment. These are all areas which are either directly or indirectly related to the instructor and the students' impression of that instructor. Therefore, in examining the effect an instructor has on a students' self-perceived ability in mathematics, it is necessary to explore the extent to which the instructor attempts to implement these reform guidelines, and the students' perceptions of these areas as well.
The first of these areas deals with the type of tasks a teacher should pose. The mathematics tasks in which students engage—projects, problems, constructions, applications, exercises, and so on—and the materials with which they work, frame and focus students' opportunities for learning mathematics in school. Tasks provide the stimulus for students to think about particular concepts and procedures, their connection with other mathematical ideas, and their applications to real-world contexts. Good tasks can help students to develop skills in the context of their usefulness. Students not only learn mathematical techniques, but understand how these techniques are applied to practical situations. Tasks also convey messages about what mathematics is and what doing mathematics entails. Tasks that require students to reason and to communicate mathematically are more likely to promote their ability to solve problems and to make connections. Such tasks can illuminate mathematics as an intriguing and worthwhile domain of inquiry. A central responsibility of teachers is to select and develop worthwhile tasks and materials that create opportunities for students to develop these kinds of mathematical understandings, competence, interests, and dispositions (NCTM, 1991). The teacher of mathematics should pose tasks that are based not only on the knowledge of students' understandings, interests, and experiences; but also on the knowledge of the range of ways that diverse students learn mathematics. These tasks should also develop students' mathematical understanding and skills, stimulate students to make connections and develop a coherent framework for mathematical ideas, call for problem formation, problem solving, and mathematical reasoning, promote communication about mathematics, and represent mathematics as an ongoing

The next two areas of NCTM reforms deal with discourse in the classroom--both that of the teacher and the students. The discourse of a classroom--the ways of representing, thinking, talking, agreeing and disagreeing--is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing. Discourse is both the way ideas are exchanged and what the ideas entail: Who talks? About what? In what ways? What do people write, what do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are valued? Who determines the end of a discussion? The discourse is shaped by the tasks in which students engage and the nature of the learning environment; it also influences them.

Discourse entails fundamental issues about knowledge: What makes something true or reasonable in mathematics? How can we figure out whether or not something makes sense? That something is true because the teacher or the book says so is the basis for much traditional classroom discourse. Another view, the one put forth by NCTM, centers on mathematical reasoning and evidence as the basis for the discourse. In order for students to develop the ability to formulate problems, to explore, conjecture, and reason logically, to evaluate whether something makes sense, classroom discourse must be founded on mathematical evidence.

Students must talk with one another as well as in response to the teacher. When the teacher talks most, the flow of ideas and knowledge is primarily from teacher to student, which leads to students feeling dependent solely on the teacher to understand
mathematics. However, when students make public conjectures and reason with others about mathematics, ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings within an intellectual community. The teacher's role is to initiate and orchestrate this kind of discourse and to use it skillfully to foster student learning. In order to facilitate learning by all students, teachers must also be perceptive and skillful in analyzing the culture of the classroom, looking out for patterns of inequality, dominance, and low expectations that are primary causes of nonparticipation by many students (NCTM, 1991).

While including every student in the discourse of the class is not an easy task, some guidelines set forth by the NCTM include (1) posing questions and tasks that elicit, engage, and challenge each student's thinking; (2) listening carefully to students' ideas; (3) asking students to clarify and justify their ideas orally and in writing; (4) deciding when and how to attach mathematical notation and language to students' ideas; (5) deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty; and (6) monitoring students' participation in discussions and deciding when and how to encourage each student to participate (NCTM, 1991). On the other side of the coin, the students should learn to (1) listen to, respond to, and question the teacher and one another; (2) use a variety of tools to reason, make connection, solve problems, and communicate; (3) initiate problems and questions; (4) make conjectures and present solutions; (5) explore examples and counterexamples to investigate a conjecture; and (6) try to convince themselves and one another of the validity of particular representations,
solutions, conjectures, and answers (NCTM, 1991).

The mathematics teacher is also responsible for creating an intellectual environment in which serious engagement in mathematical thinking is the norm, for the environment of the classroom is foundational to what students learn. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment forms a hidden curriculum with messages about what counts in learning and doing mathematics. These include neatness, speed, accuracy, listening well, being able to justify a solution, and being able to work independently or in a group setting. If students are to learn to make conjectures, experiment with alternative approaches to solving problems, and construct and respond to others' mathematical arguments, then creating an environment that fosters these kinds of activities is essential (NCTM, 1991).

Specifically, the teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems; using the physical space and materials in ways that facilitate students' learning of mathematics; and respecting and valuing students' ideas, ways thinking, and mathematical dispositions. Mathematics teachers should also expect and encourage students to work independently or collaboratively to make sense of mathematics, and to take intellectual risks by raising questions and formulating conjectures (NCTM, 1991).

Finally, if students are to develop a disposition to do mathematics, it is essential
that the teacher communicate a love of mathematics and a spirit of doing mathematics that captures the notion that mathematics is an invention of the human mind.

Techniques used by a mathematics instructor to foster students' mathematical dispositions include the instructor modeling a disposition to do mathematics; demonstrating the value of mathematics as a way of thinking and its application in other disciplines and in society; and promoting students' confidence, flexibility, perseverance, curiosity, and inventiveness in doing mathematics through the use of appropriate tasks and by engaging students in mathematical discourse (NCTM, 1991). These standards for teaching techniques are an essential step to help students regain the interest in math they exhibit at younger ages, and in doing so have a positive impact on math achievement.

Summary

There are numerous factors which historically have been shown to affect students' math self-concept. These factors included gender, fathers' education level and math ability, math achievement, number of math classes taken in high school, perceived usefulness of math, and students' interest in math. While various studies have shown the importance of each of these variables, and their direct and/or indirect influence on math self-concept, a constant throughout the literature involved the importance of the role the math instructor plays, and the ways in which mathematics are taught. The National Council of Teachers of Mathematics has set forth standards which detail the types of reforms necessary to not only teach mathematics, but to
empower students to learn mathematics on their own.

This new set of mathematical standards strives to emphasize problem solving, mathematical thinking, open-ended questions, and discussions about mathematical ideas. Most importantly, these standards promise to reward thinking and originality in place of mindless memorization. The ultimate goal is for students to achieve mastery, and above all, autonomy in doing math, meaning that a student increasingly assumes control over his or her learning. The basic premise of course, being that one can only learn when one feels in control. While the focus of this study is centered around math self-concept, the impact of instructors and instructional techniques (as set forth in the NCTM standards) on math self-concept are of primary interest.
This chapter will begin with an explanation of the characteristics of the population and how the sample was drawn from that population. This will be followed by a discussion of the instruments used, including why they were selected, and evidence of reliability and validity for each. This chapter will conclude with an overview of the research design and procedure selected for this study, and once again, why this particular design is appropriate.

**Subjects**

The population for this study consisted of freshmen students who were attending Oklahoma State University in the first nine weeks of the fall, 1997 semester. Freshmen students were chosen as the target population specifically because this study relies on information based on students' high school math experiences, and therefore it was believed that students who had been out of high school for the most minimal amount of time would have the best recollection of earlier math instructors. The total number of freshmen enrolled at Oklahoma State University for the 1997 fall semester was 3380. Freshmen are defined as students who have 29 or fewer credit hours. This figure included first and second semester freshmen, as well as students transferring from other institutions.

The sampling frame consisted of freshmen who were enrolled in an orientation
course within their respective college at Oklahoma State University or in a course entitled "World of Work." Freshmen orientation classes are designed to acquaint incoming students with various facets of the university (e.g. the library and computer labs), and help them become assimilated into college life. Orientation classes, while not required, are recommended for all incoming freshmen, and therefore are taken by the majority of freshmen. Seventy-three percent (2478 out of 3380) of all freshmen were enrolled in an orientation class. For this study the rationale for sampling from orientation classes rather than from academic courses was as follows. There were numerous classes which historically contain high percentages of freshmen (English Comp I, College Algebra, political science, psychology). However, these classes presented two problems. First, they were not restricted to freshmen only, and therefore sampling from these classes could result in large fluctuations of the actual amount of useable data per class. Second, sampling from these classes would not guarantee a sample representative of all the various colleges at Oklahoma State University. Orientation classes contain only freshmen (so all surveys would represent useable data), and because each college offers their own orientation classes, also provides representativeness from each of the different colleges at Oklahoma State University. The sample also included subjects who were enrolled in a course entitled "World of Work" rather than an orientation class. "World of Work" is a class taken by some freshmen instead of a freshmen orientation class, and accounted for an additional 104 freshmen. Including these classes in the sample served to include part of the overall freshmen population not enrolled in an orientation class.
Cluster sampling was used to obtain the actual sample. Cluster sampling is sampling in which groups, not individuals, are randomly selected. Therefore, as opposed to selecting individual freshmen students, freshmen orientation classes in every college were selected to make up the sample. Logistically, cluster sampling was considered the most convenient method of sampling in terms of being able to explain the study to subjects, administer the survey, and immediately collect completed surveys for groups of freshmen, as opposed to individual freshmen. To select the specific orientation classes to be sampled, each college was contacted to determine who the instructors of orientation classes were. Most colleges only had one or two instructors for orientation classes (the exceptions being the College of Art and Sciences and the College of Engineering). These instructors were contacted to set up dates and times when their orientation classes could be surveyed.

This study used factor analysis which is considered a large-sample procedure, so it was important that the minimum number of subjects in the sample be five times the number of variables being analyzed. It is also recommended that the sample size exceed the five-to-one ratio to allow for surveys which had unanswered items and could not be used for the factor analysis (Hatcher, 1994). There were forty-three items used in the factor analyses, which therefore indicated that a minimum sample size of 215 was required. A sample size of 300 to 400 was decided on in order to meet the above criteria and ensure that orientation classes from each college were represented. Table 1 gives the number of orientation classes offered in each college, and the number surveyed.
Table 1

Total Number of Orientation Classes per College, and Number of Classes Surveyed Per College.

<table>
<thead>
<tr>
<th>College</th>
<th>Number of Orientation Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample</td>
</tr>
<tr>
<td>Agricultural Sciences and Natural Resources</td>
<td>1</td>
</tr>
<tr>
<td>Arts and Sciences</td>
<td>4</td>
</tr>
<tr>
<td>Business</td>
<td>1</td>
</tr>
<tr>
<td>Education</td>
<td>2</td>
</tr>
<tr>
<td>Engineering</td>
<td>2</td>
</tr>
<tr>
<td>Human Environmental Sciences</td>
<td>1</td>
</tr>
<tr>
<td>University Academic Services</td>
<td>2</td>
</tr>
<tr>
<td>World of Work Classes</td>
<td>2</td>
</tr>
</tbody>
</table>

Due to the schedules already set up for the freshmen orientation classes, the classes surveyed were not selected completely at random. However, there is no logical basis for assuming the nonrandom nature of the sampling introduced systematic bias. Characteristics of this sample were compared to the overall population of college freshmen at Oklahoma State University to evaluate the representativeness of the sample. Characteristics which were used to compare the sample to the overall population are found in Table 2 and Table 3. Table 2 includes the enrollment by college and gender for the population, and the numbers of students sampled by college and gender. Table 3 includes the mean ACT math test scores for the overall population.
Table 2

Enrollment by College and Gender of Population (OSU Freshmen), and the Number of Students Sampled by College and Gender.

<table>
<thead>
<tr>
<th>College</th>
<th>AS &amp; NR</th>
<th>A&amp;S</th>
<th>BUS</th>
<th>ED</th>
<th>ENG</th>
<th>HES</th>
<th>UAS</th>
<th>World of Work</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>127</td>
<td>593</td>
<td>265</td>
<td>122</td>
<td>135</td>
<td>154</td>
<td>320</td>
<td>NA</td>
<td>1716</td>
</tr>
<tr>
<td></td>
<td>46%</td>
<td>59%</td>
<td>48%</td>
<td>65%</td>
<td>24%</td>
<td>88%</td>
<td>52%</td>
<td></td>
<td>51%</td>
</tr>
<tr>
<td>M</td>
<td>148</td>
<td>420</td>
<td>285</td>
<td>432</td>
<td>21</td>
<td>295</td>
<td>NA</td>
<td></td>
<td>1664</td>
</tr>
<tr>
<td></td>
<td>54%</td>
<td>41%</td>
<td>52%</td>
<td>76%</td>
<td>12%</td>
<td>48%</td>
<td></td>
<td></td>
<td>49%</td>
</tr>
<tr>
<td>Total</td>
<td>275</td>
<td>1013</td>
<td>547</td>
<td>188</td>
<td>175</td>
<td>615</td>
<td>NA</td>
<td></td>
<td>3380</td>
</tr>
</tbody>
</table>

| Sample |
| F | 9 | 49 | 24 | 33 | 9 | 11 | 23 | 191 |
| | 45% | 62% | 48% | 67% | 27% | 87% | 50% | 57% | 58% |
| M | 11 | 30 | 26 | 16 | 24 | 5 | 11 | 17 | 140 |
| | 55% | 38% | 52% | 73% | 13% | 50% | 43% | 42% |
| Total | 20 | 79 | 50 | 49 | 33 | 38 | 40 | 40 | 331 |

Source for Population Information: OSU Office of Institutional Research

Note: AS & NR = Agricultural Sciences and Natural Resources
A & S = Arts and Sciences
BUS = Business
ED = Education
ENG = Engineering
HES = Human Environmental Sciences
UAS = University Academic Services
Table 3

Mean ACT Math Test Scores for Population and Sample of OSU Freshmen (Standard Deviations in parentheses).

<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th>Males</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22.04</td>
<td>23.56</td>
<td>22.78</td>
</tr>
<tr>
<td>Population</td>
<td>(4.54)</td>
<td>(4.81)</td>
<td>(4.74)</td>
</tr>
<tr>
<td></td>
<td>n = 1509</td>
<td>n = 1430</td>
<td>n = 2939</td>
</tr>
<tr>
<td></td>
<td>22.17</td>
<td>23.99</td>
<td>22.92</td>
</tr>
<tr>
<td>Sample</td>
<td>(5.31)</td>
<td>(4.90)</td>
<td>(5.22)</td>
</tr>
<tr>
<td></td>
<td>n = 182</td>
<td>n = 127</td>
<td>n = 309</td>
</tr>
</tbody>
</table>

Source for Population Information: OSU Office of Institutional Research

While the total percentages of males and females is slightly different for the sample than for the population (58% females and 42% males for the sample versus 51% females and 49% males for the population), the percentages of males and females surveyed within each college are extremely close to that of the overall population. Also, while participation in this study by students in freshmen orientation classes was not mandatory, there were only three students in the 14 classes surveyed who chose not to participate.

In comparing the sample ACT math test scores to the population ACT math test scores, one-sample Z-tests (Bartz, 1988) were used to answer the following questions:

1. Is the mean ACT math test score for the sample significantly different
from the freshmen population mean ACT math test score?

2. Is the mean ACT math test score for the female sample significantly different from the freshmen female population mean ACT math test score?

3. Is the mean ACT math test score for the male sample significantly different from the freshmen male population mean ACT math test score?

In each of the three comparisons, no significant differences ($\alpha = .05$) were found between the population ACT math scores, and the sample ACT math scores. Based on the comparisons made between the sample and the population (involving percentages of males and females in each college, as well as ACT math test scores comparisons), there was no evidence to indicate that the sample was not representative of the population.

**Instruments**

This study required the use of two existing instruments: Mathematics Attitude Inventory (Welch, 1972), and the American College Testing Mathematics Usage Test (American College Testing Program). A scale to measure the extent to which instructors utilized the NCTM standards was developed for this study (NCTM standards utilization scale). Items to obtain demographic information about the subjects were included.

**Mathematics Attitude Inventory.** The first instrument, the Mathematics Attitude Inventory (MAI), measures student perception of mathematics teachers,
anxiety toward mathematics, value of mathematics in society, self-concept in mathematics, enjoyment or interest in mathematics, and motivation in mathematics (Sandman, 1979). Four of the scales from the Mathematics Attitude Inventory were used for this study. These scales included:

1) **Perception of the Mathematics Teacher**: This scale reflects a student's view regarding the teaching characteristics of his or her mathematics teacher.

2) **Value of Mathematics in Society**: This scale reflects a student's view regarding the usefulness of mathematical knowledge.

3) **Self-Concept in Mathematics**: This scale reflects a student's perception of his or her own competence in mathematics.

4) **Enjoyment or Interest in Mathematics**: This scale reflects the pleasure a student derives from engaging in mathematical activities.

Each of these scales have favorable evidence concerning reliability and validity.

Reliability coefficients for each scale, established using Cronbach's Alpha Coefficient on data from 5,034 students (Sandman, 1979), are presented in Table 4.

Table 4

<table>
<thead>
<tr>
<th>Scale</th>
<th>Reliability Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perception of Math Teacher</td>
<td>.83</td>
</tr>
<tr>
<td>Value of Math in Society</td>
<td>.78</td>
</tr>
<tr>
<td>Self-Concept in Math</td>
<td>.84</td>
</tr>
<tr>
<td>Enjoyment/Interest in Math</td>
<td>.89</td>
</tr>
</tbody>
</table>
According to the author (Sandman, 1979), the following steps were taken to build in validity for the original mathematics attitude inventory in assessing attitudes regarding mathematics. For each construct an item pool was developed. Items took the form of statements with which students could either strongly agree, agree, disagree or strongly disagree. Special care was taken to make the items accurately represent the constructs, and to use vocabulary suitable for all secondary levels. The items for each scale were reviewed by several persons, some skilled in test construction and others involved in mathematics education. Comments and criticism received led to deletions and additions of items for each scale. Through this procedure, evidence for content validity was obtained.

An eleven-item scale was developed for this study to measure the extent to which math instructors utilized the standards set forth by the NCTM. These items were written directly from the NCTM's "Standards for Teaching Mathematics." Other math instructors and high school students reviewed these items to make sure they reflected the proposed standards, and that they were understandable (see appendix A for survey instrument). Other items were included to obtain information about gender, number of math classes taken in high school, college major, and various parental factors (i.e. education level of parents and rating of parents math ability). For any item on the survey that dealt with math instructors or math classes, subjects were asked to answer based on the math instructor or math class most memorable to them. Also, items which were negatively worded were reverse coded for analyses.
ACT Math Test. The second instrument, the American College Testing Mathematics Usage Test (ACT math test) scores of the students was used to measure of mathematical achievement. The American College Testing (ACT) Assessment testing program has long been recognized as a reliable and valid method of measuring future success at the post-secondary education level (Wallace, 1972; American College Testing Services, 1997). The ACT consists of four tests of educational development and scholastic ability. The test of interest to this study is the mathematics usage test (ACT math test). This test consists of 60 items reflecting the following categories: 1) pre-algebra (23%), 2) elementary algebra (17%), 3) intermediate algebra (15%), 4) coordinate geometry (15%), 5) plane geometry (23%), and 6) trigonometry (7%) (American College Testing Services, 1997). There has existed some debate as to whether the ACT is considered an ability test or an achievement test. While ability tests measure traits attributable primarily to hereditary factors, achievement tests measure skills developed as a result of environmental stimulation, and are tied to specific educational curricula and courses of academic study. Although there is definitely some overlap in what the ACT Assessment measures (ability versus achievement), because each ACT test is developed to meet careful content specifications developed by subject matter specialists, the ACT Assessment emerges primarily from an educational and curriculum theory perspective (Geisinger, 1984). This indicates that the ACT Assessment can be viewed as an achievement test, and is therefore considered a valid instrument for measuring achievement in the various areas which the tests cover. For the purposes of this study, the area of interest is the
Mathematics Usage test as a measure of achievement in math.

Obtaining evidence for reliability and validity of the ACT has been very extensive with consistently good results. For the Mathematics Usage test, the median score reliability is .89 (American College Testing Services, 1997).

The three types of validity information of consequence for these academic-oriented tests: content validity, predictive validity, and construct validity. However, because this study is not concerned with how math achievement relates to future performance, it is felt that predictive validity is not as important, and therefore the scope of this discussion will be limited to content and construct validity.

The guiding principle underlying the development of the ACT tests is that the best way to predict success in college is to measure as directly as possible the degree to which each student has developed the academic skills and knowledge that are important for success in college. Tasks presented in the ACT tests must therefore be representative of scholastic tasks. In this context, content-related validity is particularly significant (Geisinger, 1984, American College Testing Services, 1997).

The ACT Assessment tests contain a proportionately large number of complex problem-solving exercises and few measures of narrow skills. The tests are oriented toward major areas of college and high school instructional programs. Thus, ACT test scores are directly related to student educational progress (American College Testing Services, 1997).

The test development procedures include an extensive review process with each item being critically examined at least twenty times. Detailed test specifications have
been developed to ensure that the test content is representative of current high school and university curricula. All test forms are reviewed by experts in test construction and subject matter to ensure that they match these specifications. Hence, there is an ongoing assessment of the content validity of the tests during the development process (American College Testing Services, 1997).

The other important type of evidence for validity is construct validity. This is shown in several ways. For example, ACT tests correlate with high school grades, as well as with each other (both with correlation coefficients in the .60 to .70 range). ACT tests also correlate with other educational tests, such as the Scholastic Aptitude Test (SAT), as highly as one would reasonably expect. Research has also shown the ACT to be equally valid for both males and females (Geisinger, 1984; American College Testing Services, 1997).

**Research Design and Procedure**

Classes used for the sample were approved through the various instructors. The actual data collection took approximately one month, in order to survey each class at a time convenient for the instructors. For every class surveyed, a specific script (found in appendix B) was used to introduce the study and request students' assistance. Surveys were then immediately handed out to the entire class and students were allowed time to fill out the surveys until everyone had finished. Students were asked to supply their student identification number for purposes of obtaining students' ACT math test score from official university records. OSU academic advisors suggested that
students' ability to recall this score would not be completely reliable (Leigh Goodson, former admissions counselor for College of Education at OSU, personal communication, September, 1997). ACT math test scores were supplied by the Office of Institutional Research after all of the surveys had been administered.

**Data Analyses**

The goal of this research was to examine the appropriateness of a theoretical causal model of math self-concept. Structural equation modeling with a combination of latent and observed variables (also called manifest variables) was used to analyze the data. Structural equation modeling (most frequently referred to as "LISREL-type" models) is a comprehensive statistical approach to testing hypotheses about relations among observed and latent variables. Structural equation modeling begins with the specification of a model to be estimated. At the most basic level, a model is a statistical statement about the relations among variables.

The SAS procedure CALIS was used to analyze the structural relations among the variables in this study (Hatcher, 1994). It uses a two-step approach to test latent-variable models. With this approach, the first step involves using confirmatory factor analysis to develop an acceptable measurement model. When testing the measurement model, one looks for evidence that indicator variables are reliable and valid measures of the underlying constructs of interest, and that the measurement model demonstrates an acceptable fit to the data. The second step is to use LISREL-type analyses to predict specific causal relationships among latent and/or manifest variables. This is called the
structural model, and the testing of these models allows hypotheses that certain latent variables and/or manifest variables have causal effects on other latent variables and/or manifest variables to be tested.

It is important at this point to define some specific terms. A manifest variable is one that is directly measured or observed in the course of an investigation (e.g. items on a survey), while a latent variable is a hypothetical construct that is not directly measured or observed (e.g. factors formed by items on the survey), and is presumed to be in a causal relationship with one or more manifest variables. Typically (though not always) latent variables are conceptualized as the cause of observed variables which are viewed as indicators. In structural equation modeling a distinction is also made between endogenous variables and exogenous variables. An endogenous variable is one whose variability is predicted to be causally affected by other variables in the model. In the path diagram, any variable that has a straight, single-headed arrow pointing at it is an endogenous variable. Exogenous variables, on the other hand, are constructs that are influenced only by variables that lie outside the causal model. Exogenous variables do not have any straight, single-headed arrows pointing to them (except error terms). This means that the theorist makes no predictions about what influences exogenous variables. In most models, exogenous variables will affect other variables, but by definition, exogenous variables are never affected by other variables in the model (Hatcher, 1994).
Exploratory Factor Analysis. The first type of statistical analysis performed was an exploratory factor analysis. Exploratory factor analysis is typically used when one has obtained measures on a number of variables, and wants to identify the number and nature of the underlying factors that are responsible for covariation in the data. In other words, exploratory factor analysis is appropriate when one wants to identify the factor structure underlying a set of data (Hatcher, 1994). Because confirmatory factor analysis is less appropriate for variable reduction, exploratory factor analysis was utilized simply as a method of decreasing the number of variables to be used in the structural model. The goal was to "pick" the best items to retain, so that the structural model to be used would not be too cumbersome.

Factor analysis is considered a large-sample procedure, so it is important that the minimal number of subjects in the sample be larger than 100, or five times the number of variables being analyzed (Hatcher, 1994). There were 43 items being analyzed, and a sample size of 331 (316 after surveys with missing answers were omitted). This more than meets the 5:1 ratio of sample size to items being analyzed. In considering how many meaningful factors would be retained, there are several criteria that can be used. One of the most common is the Kaiser criterion, also called eigenvalue-one criterion. In using this criterion, the researcher retains any factor with an eigenvalue greater than 1.00. This criterion is very useful in principal components analysis (another variable reduction technique) because each variable contributes one unit of variance to the analysis, and therefore ensures that factors would not be retained that accounted for less variance than had been contributed by one variable. However,
the eigenvalue-one criterion is less appropriate in factor analysis, because each variable does not necessarily contribute one unit of variance to the analysis (Hatcher, 1994).

The second criterion is a scree test. With the scree test, the eigenvalues associated with each factor are plotted and the researcher looks for a break between the factors with relatively large eigenvalues and those with smaller eigenvalues. The factors that appear before the break are assumed to be meaningful and are retained for rotation; those appearing after the break are assumed to be unimportant and are not retained (Hatcher, 1994).

The third criterion is the proportion of variance accounted for. This may involve retaining any single factor which accounts for a certain percentage of the variance (e.g. retaining any factor that accounts for at least 5% of the variance), or retaining factors that cumulatively retain a specific proportion of the variance (e.g. retaining enough factors so that at least 90% of the variance is accounted for) (Hatcher, 1994).

Interpretability is the last criterion, and perhaps the most important to use when solving the number of factors problem. This criterion is defined as interpreting the substantive meaning of the retained factors and verifying that this interpretation makes sense in terms of what is known about the constructs under investigation. There are four rules to follow in doing this:

1. Are there at least three variables (items) with significant loadings on each retained factor?
2. Do the variables that load on a given factor share some conceptual meaning?

3. Do the variables that load on different factors seem to be measuring different constructs?

4. Does the rotated factor pattern demonstrate "simple structure?" Simple structure means that the pattern possesses two characteristics: (a) most of the variables have relatively high factor loadings on only one factor, and near-zero loadings for the other factors, and (b) most factors have relatively high factor loadings for some variables, and near-zero loadings for the remaining variables (Hatcher, 1994).

A combination of these criterion were used to determine the number of factors to be retained.

While evidence of the reliability and construct validity of several scales on the instrument had been previously shown, there were also new items added. While these new items are based on a strong theory, they had yet to be verified through any type of analysis. Therefore, exploratory factor analysis was used to group similar items into a smaller number of factors. The four scales used from the MAI should be definite factors since they have been previously shown to have validity as measures for the constructs of math self-concept, enjoyment/interest in mathematics, value of mathematics in society, and perceptions of math teacher (Sandman, 1983). The items added that address the NCTM standards for teaching mathematics should group into another factor (NCTM standards utilization scale). An exploratory factor analysis was
Figure 2. Theorized Factor Loadings for Exploratory Factor Analysis

- NCTM Standards
- Math Self-Concept
- Perceived Usefulness of Math
- Interest in Math
- Perception of Math Teacher
considered an appropriate initial tool to examine the theoretical five-factor structure of the survey instrument (Figure 2).

An exploratory factor analysis was run to look at how items grouped together, and verify that the hypothesized constructs were actually being measured. In the initial exploratory factor analysis no stipulations were made as to the number of factors to be retained. However, two important specifications were made. First, the analysis was run using the principal factors method for extracting the factors. This extraction method specifies that the number of factors extracted will be equal to the number of variables being analyzed. The first factor can be expected to account for a fairly large amount of the common variance. Each succeeding factor will account for progressively smaller amounts of variance. The second specification was to use a Harris-Kaiser orthoblique rotation (Gorsuch, 1983) versus an orthogonal rotation. Orthogonal rotations can only be used when the factors are assumed to be uncorrelated. In the case of this study, since all of the factors deal with math attitudes, the assumption of uncorrelated factors was not realistic, and therefore called for an oblique rotation. Based on findings in the first exploratory factor analysis, another was run with the specification of 5 factors.

**Model Specification and Identification.** Model specification and identification are critical first steps in structural equation modeling. The type of structural model used for this research study is a nonstandard recursive model. Nonstandard indicates that both latent variables and manifest variables will make up the structural model. A
recursive path model is one in which causation flows in only one direction. In a recursive model, a consequent variable never exerts causal influence on an antecedent variable that first exerts causal influence on it. In other words, recursive models are unidirectional.

Model identification must be done prior to any type of analyses of the model and is quite complex. However, the identification problem is one of the most important concepts. In general, identification refers to having enough information for model estimation, so that the parameters of the model are uniquely determined (Long, 1983). There are three types of possible model identification. A model can be underidentified, just-identified, or overidentified. It should be noted that model identification is a different issue from model estimation. Even with access to the entire population, parameters can not be obtained for underidentified models.

It is highly desirable that a model not be underidentified prior to estimation. Technically, a system is underidentified when the number of independent elements of the variance/covariance matrix is less than the number of model parameters to be estimated. To calculate the number of elements in the variance/covariance matrix the formula \( p(p + 1)/2 \) is used where \( p \) is the number of manifest variables in the model. When a model is underidentified, an infinite number of solutions can be generated to solve for its parameters. For example if PROC CALIS were used to estimate an underidentified model, performing the analysis with one set of starting values might generate one set of parameters, while running the analysis a second time with a different set of starting values might generate a radically different set of parameter
estimates (different values for the same path coefficients, etc.) Results obtained from the analysis of an underidentified model are completely meaningless (Hatcher, 1994).

Parameter estimates such as path coefficients are meaningful only if they are obtained from the estimation of an identified model. A model may be identified either by being just-identified or overidentified. A just-identified model is one in which there are exactly as many elements in the variance/covariance matrix as model parameters to be estimated. Although a just-identified model has the advantage of allowing the estimation of just one unique set of parameters (for a given sample), it has the disadvantage of not allowing any tests for goodness of fit, because just-identified models always provide a perfect fit to the data. It is for this reason that researchers typically need to be sure that their models are overidentified (Hatcher, 1994).

A model is said to be overidentified when the number of elements in the variance/covariance matrix exceeds the number of model parameters to be estimated. As with a just-identified model, the estimation of an overidentified model will result in only one set of parameter estimates for a given sample of data. Overidentified models, however, have an additional property: they can be tested for overall goodness of fit (Hatcher, 1994).

While standard, recursive models are always just-identified or overidentified, nonstandard recursive models are not. These models can be underidentified, and any researcher who estimates an underidentified model may unknowingly obtain meaningless parameter estimates. Fortunately, the CALIS procedure itself can often detect an underidentified model, and alert the researcher to this. Also, one criterion
for overidentification is that the number of independent elements of the variance/covariance matrix should exceed the number of parameters to be estimated. This information can also be obtained in the CALIS output (Hatcher, 1994).

A final test for identification is to repeat the analyses several times, using very different starting values for parameter estimates each time. If PROC CALIS arrives at the same final parameter estimates each time, it is likely (but not certain) that the model is identified.

These last two procedures (comparing sample variances/covariances and model parameters, and conducting analyses with differing starting values) are necessary but not sufficient conditions for demonstrating identification. This means that, if the model fails to pass these tests, it is clearly underidentified, but if it does pass these tests it does not conclusively prove that it is identified. The only way to be sure is to use one of the more time-consuming approaches which involves detailed and rigorous algebraic manipulations of the model's covariance equations to show that each of the parameters can be solved in terms of the population's variances and covariances of the observed variables (Hatcher, 1994).

For this study the conditions of comparing sample variances/covariances and model parameters, as well as conducting analyses with different starting points were used. For the theorized structural model, there were 528 elements of the variance/covariance matrix, and 91 parameters to be estimated. Since the number of elements of the variance/covariance matrix exceeds the number of parameters to be estimated (528 > 91), this meets one of the necessary conditions for overidentification.
Also, 3 additional repetitions of the analysis were run using various starting points for parameter estimates. In each case, the final estimates generated by CALIS were the same. Because both of these necessary conditions were met, and the fact that the CALIS procedure itself did not show any indication of an underidentified model, the model was assumed to be overidentified.

Subsequent analysis was conducted in two parts: a confirmatory factor analyses, and the structural causal model.

**Confirmatory Factor Analysis.** Once the number of observed variables was reduced to a manageable level, the next step was to use the SAS procedure CALIS. While CALIS is used for both confirmatory factor analysis, and structural equation modeling, this is done in two separate steps. The first involves using confirmatory factor analysis, to look at what is called the "measurement model" and assess if the model is acceptable. Testing the measurement model entails looking for evidence that indicator variables or manifest variables (variables that are directly observable—in this case, survey items) really are measuring the underlying constructs of interest, and that the measurement model demonstrates an acceptable fit to the data. These constructs are often called latent variables. The measurement model does not specify any causal relationships among latent variables (factors) of interest; at this stage of the analysis, each latent variable is allowed to correlate freely with every other latent variable. However, because latent variables are hypothetical constructs, they have no established scale or unit of measurement. This is known as a scale indeterminacy problem. Left
unaddressed, this problem makes it impossible to distinguish between situations in which the factor has a large variance and the factor loadings are small, and situations in which the factor variance is small and the loadings are large (Hatcher, 1994). To solve this problem, the variances of each latent variable were fixed at one. This established a scale for the latent variables, and helped ensure that the model was identified.

If the measurement model is not acceptable, then CALIS supplies suggestions (modification indices) as to how the model might be improved. These modifications are then examined along with past research and theory to determine what, if any, improvements to the model should be made. An important note regarding model modifications is that the SAS generated suggestions serve only to fit the model to the available data. One of the goals of factor analysis is to use indicator variables to explain a construct or "unobservable trait." A researcher has to be able to determine if potential modifications are consistent with the theoretical framework provided by past studies in explaining constructs or if they only serve to make interpretation of constructs more difficult.

The basic confirmatory factor analysis model used (Figure 3) consisted of five latent variables or factors (represented with ovals): math self-concept, perception of math teacher, perceived usefulness of math, utilization of NCTM standards, and interest in math. Manifest indicator variables for each latent variable are represented by rectangles. These indicator variables are responses to survey items. math self-concept has 4 manifest variables, perception of math teacher teacher, and perceived usefulness of math have 5 manifest variables a piece, utilization of NCTM standards
Figure 3. Theoretical Measurement Model

- NCTM Standards
- Math Self-Concept
- Perceived Usefulness of Math
- Interest in Math
- Perceptions of Math Teacher
has 7 manifest variables, and interest in math has 6 manifest variables. Every factor is connected to every other factor by a curved, two-headed arrow, meaning that every factor is allowed to covary with every other factor. The "E" terms below each manifest variable are the residual terms associated with each manifest variable.

While the SAS procedure CALIS produces a substantial amount of output, there are some very particular areas to look at. Specifically, a measurement model provides an ideal fit to the data when it displays the following characteristics:

1) The p-value for the model chi-square test should be nonsignificant (should be greater than .05); the closer to 1.00, the better.
2) The chi-square/df ratio should be less than 2.
3) The comparative fit index (CFI) and the non-normed fit index (NNFI) should both exceed .9; the closer to 1.00, the better.
4) The absolute value of the t statistic for each factor loading should exceed 1.96, and the standardized factor loadings should be nontrivial in size.
5) The distribution of normalized residuals should be symmetrical and centered on zero, and relatively few (or no) normalized residuals should exceed 2.0 in absolute value.
6) Composite reliabilities for the latent factors should exceed .70 (.60 at the very least).
7) Variance extracted estimates for the latent factors should exceed .50.
8) Discriminant validity for questionable pairs of factors should be demonstrated through the chi-square difference test, the confidence interval test, or the variance extracted test.

One point to keep in mind however, is that these characteristics represent an ideal model, which will not always be attained even when the measurement model is quite good. A model's fit need not meet all of these criteria in order to be deemed
acceptable. In particular, requiring that the model chi-square be nonsignificant is an excessively strict requirement in most applied situations (Hatcher, 1994).

When implementing the modification indices, the researcher runs a risk of creating a model that will not generalize to other samples or to the population. Two recommendations for modifying models are: (1) to make as few modifications as possible, and (2) make only changes that can be meaningfully interpreted. Also, it has been argued that adding new paths may be somewhat more likely to capitalize on chance characteristics of the sample data and lead to an inaccurate model (Hatcher, 1994). With this in mind, it is typically thought to be safer to drop existing parameters than it is to add new ones. For this reason, the Wald test should be consulted first.

The Wald test estimates the change in the model chi-square that would result from fixing a given parameter at zero. In other words, this test estimates the change in chi-square that would result from eliminating a specific path or covariance from the model. The first parameter listed is the one that would result in the least change in chi-square if deleted, the second parameter would result in the second-least change, and so forth.

In one sense, dropping one of the parameters listed in the Wald test does not really improve the fit of the model (dropping any parameter will almost always increase chi-square, at least to a small degree). However, fixing one of these parameters at zero does free up one degree of freedom for the analysis. Freeing this degree of freedom decreases the ratio of chi-square to df, and this index is one of the informal measures of model fit. In addition, if the ratio can be decreased enough, it is even possible that a
significant chi-square test will become nonsignificant (another indication of model fit) (Hatcher, 1994).

What was essentially being looked for in the Wald test was some indication that a factor loading could be dropped from the model without hurting the model chi-square. Such a finding would possibly mean that an indicator variable was not doing a good job of measuring the factor to which it was assigned. Any suggestions to eliminate a covariance between latent variables is considered inappropriate since all factors are normally allowed to covary during a confirmatory factor analysis.

Next suggestions from the Lagrange multipliers should be examined. The Lagrange multiplier test estimates the reduction in model chi-square that would result from freeing a fixed parameter and allowing it to be estimated. In other words, for a confirmatory factor analysis, the Lagrange multiplier estimates the degree to which chi-square would improve if a new factor loading or covariance were added to the model. The part of the Lagrange multiplier test that is of interest is the rank order of ten largest Lagrange multipliers. Again any modifications must be based on theory, not just suggestions generated by CALIS.

**Structural Analysis.** The second step of the final analysis deals with the "structural model." The structural model is assessed using the SAS procedure CALIS much the same way the measurement model was assessed, with certain modifications to the program. In the measurement model, no causal relationships between latent variables were specified. Instead, each latent construct was allowed to covary
(correlate) with every other latent construct. For the structural model, the measurement model is modified so that it now specifies causal relationships between some of the latent variables. In the case of this particular research study, the model will include causal relationships between a combination of latent variables and manifest variables.

Also, when performing confirmatory factor analysis, it was noted that the variances of the latent exogenous variables were fixed at one in order to solve the problem of scale indeterminacy. However, when performing a LISREL-type analysis, the variances of these latent variables should be free parameters to be estimated. For the structural model, the scale indeterminancy problem is solved another way. By fixing at a value of 1.00 the path from a latent variable to one of its manifest indicators, the unit of measurement for the latent variable becomes equal to the unit of measurement for that indicator variable (minus its error term). For this reason the factor loading of the indicator variable which best represents a latent construct should now be fixed at one. The path to the manifest indicator with the largest factor loading is typically therefore the path chosen to be fixed at one (Hatcher, 1994).

In the analysis of causal models, a distinction is made between direct and indirect effects manifest and latent variables in the structural model have on the endogenous variables in the model. The direct effect, is a directional relation between two variables. Within a model, each direct effect characterizes the relation between what could be considered an independent variable and a dependent variable, although the dependent variable in one direct effect can be the independent variable in another.
Moreover, a dependent variable can be related to multiple independent variables, and an independent variable can be related to multiple dependent variables. The capacity to treat a single variable as both a dependent and an independent variable lies at the heart of the indirect effect. The indirect effect is the effect of an independent variable on a dependent variable through one or more intervening, or mediating, variables. The sum of direct and indirect effects of an independent variable on the dependent variable is the total effect of that independent variable (Hoyle, 1995).

For instance, the study's theoretical causal model in Figure 4 shows math achievement is hypothesized to have only a direct effect on math self-concept. Other variables (such as number of math classes taken in high school) are hypothesized to have only indirect effects on math self-concept. Gender, however, is hypothesized to have a direct effect on math self-concept, as well as an indirect effect on math self-concept (with math achievement serving as the mediating variable). When variables have a direct and indirect effect, the sum of these are considered the total effect. Note that a variable may have multiple indirect effects as does the number of math classes taken.

The theoretical structural model to be tested predicts the following: 1) math self-concept is causally determined by direct paths from interest in math, math achievement, gender and perceptions of high school math teacher, 2) math achievement is directly causally determined by interest in math, number of math classes taken in high school, perception of math teacher, utilization of NCTM standards, and gender, 3) interest in math is directly causally determined by number of math
Figure 4. Theoretical Structural Model

Gender

Fathers' Math Ability

Fathers' Ed Level

Number of Math Classes

Interest in Math

Math Achievement

Math Self-Concept

NCTM Standards

Perceived Usefulness of Math

Perceptions of Math Teacher
classes taken in high school, utilization of NCTM standards, perceived usefulness of math, and the fathers' math ability, and 4) perceived usefulness of math is directly causally determined by teacher characteristics, number of math classes taken in high school, fathers' education level, and the fathers' math ability.

The theoretical model to be tested in this phase of the study is reproduced in Figure 4. Notice that the latent variables are no longer all connected by curved, double-headed arrows. Instead, straight, single-headed arrows are now used to indicate direct causal effects from one variable to another. A distinction should be made about the type of model being used for this study. In causal analysis, one can use either a standard model or a nonstandard model. With a standard model, all constructs that constitute the structural portion of the model (the "structural variables") are represented by latent variable (factor) with multiple manifest indicators. With a nonstandard model, on the other hand, at least one of the constructs that constitute the structural portion of the model is represented as a single manifest variable.

The theorized structural model for this study is a nonstandard model because it has a mixture of latent variables (represented by ovals) and manifest variables (represented by rectangles). Math self-concept, math achievement, perceived usefulness of math, and interest in math are treated as endogenous variables, while gender, number of math classes taken in high school, fathers' education level, fathers' math ability, NCTM standards, and perceptions of math teacher are specified as exogenous variables. An important note involves the choice to make the number of math classes taken in high school an exogenous variable. There has been research
involving high school students which indicates that factors such as gender and teacher characteristics do affect the number of math classes taken in high school. However, since the sample for this study involved college students, it was assumed that their decision to attend college would dictate the number of math classes taken more than any factors within the scope of the actual study. This was reinforced by looking at a frequency distribution of the responses for that particular item. With the choices being zero through five, 99% of the subjects had taken three or more math classes, with 54% taking four math classes. This seems to indicate that the number of math classes are affected by factors outside those included in this study.

While there are additional items to examine when assessing the fit of the structural model to the data, several steps are identical to the process used in assessing the measurement model. These include the following:

1) The p-value for the model chi-square test should be nonsignificant (should be greater than .05); the closer to 1.00, the better.

2) The chi-square/df ratio should be less than 2.

3) The comparative fit index (CFI) and the non-normed fit index (NNFI) should both exceed .9; the closer to 1.00, the better.

4) The absolute value of the t statistic for each factor loading should exceed 1.96, and the standardized factor loadings should be nontrivial in size.

5) The distribution of normalized residuals should be symmetrical and centered on zero, and relatively few (or no) normalized residuals should exceed 2.0 in absolute value.

In addition to the above-mentioned information, another criterion is how parsimonious the theoretical model is. In the broadest sense, the parsimony of a model
refers to its simplicity. The principal of parsimony states that, when several theoretical explanations are equally satisfactory in accounting for some phenomenon, the preferred explanation is the one that is least complicated; the one that makes the fewest assumptions. To evaluate this, we look again at the Covariance Structure Analysis at the parsimonious fit index (sometimes called the parsimonious normed-fit index or PNFI). While there is no firm criterion as to how large the PNFI must be for the model to be acceptable, .60 has been suggested (Hatcher, 1994).

Variables to be examined in the structural model include manifest variables of gender, math achievement, number of high school math classes taken, and various parental factors (including rating of parental mathematics skills and parental level of education). Latent variables (or constructs identified by the confirmatory factor analysis) include math self-concept, enjoyment/interest in mathematics, value of mathematics in society, perceptions of high school mathematics instructors, and the extent to which a math instructor implements NCTM standards.
CHAPTER 4

RESULTS

This chapter will present the results of the statistical analyses used for this study. Specifically the research questions set forth in chapter one will be discussed and statistical evidence provided in answer to these questions. This research study was conducted to examine the effects of the math instructor and type of instructional methods used to teach mathematics on students' math self-concept, relative to other factors of gender, math achievement, number of math classes taken in high school, fathers' education level, students' ranking of their fathers' math ability, interest in math, and perceived usefulness of math in the future.

Sample Adequacy

Before the research questions are discussed however, an issue dealing with the sample should be addressed. Previously, it was stated that the sample was not strictly randomly chosen. Because the sample obtained was not random, one-sample Z-tests were used to answer the following three questions in order to show evidence of representativeness of the sample with the population:

1. Is the mean ACT math test score for the sample significantly different from the freshmen population mean ACT math test score?
2. Is the mean ACT math test score for the female sample significantly different from the freshmen female population mean ACT math test score?

3. Is the mean ACT math test score for the male sample significantly different from the freshmen male population mean ACT math test score?

The resulting z-scores and their corresponding p-values are in Table 5.

Table 5

<table>
<thead>
<tr>
<th>Variable</th>
<th>Z-Score</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT Math (total)</td>
<td>.51</td>
<td>.31</td>
</tr>
<tr>
<td>ACT Math (females)</td>
<td>.38</td>
<td>.35</td>
</tr>
<tr>
<td>ACT Math (males)</td>
<td>1.00</td>
<td>.16</td>
</tr>
</tbody>
</table>

To be considered significantly different, the p-values would have to be less than .05 (the predetermined significance level for this study). As shown in the table, none of the p-values are less than .05. This indicates that the differences in ACT math test scores (overall, for females, and for males), for the sample and the population are not statistically significant. While this does not prove that the sample is representative of the population, it does show evidence of representativeness.
Exploratory Factor Analysis

In analyzing the data, the initial procedure involved conducting an exploratory factor analysis of items 9 through 51 on the survey instrument. These items measured students' beliefs about mathematics and characteristics of math instruction. This served only as a preliminary analysis, and mainly as a variable reduction technique. Criteria used for determining the number of factors to retain included: (1) Kaiser criterion (eigenvalue greater than one), (2) scree test, (3) proportion of variance accounted for, and (4) interpretability. Among these criteria, interpretability was considered the most important.

A principal axis factor analysis with an oblique Harris-Kaiser orthoblique rotation was conducted on items nine through fifty-one from the survey instrument. Models with four, five, and six factors were evaluated against the above criteria. Table 6 shows the eigenvalues along with the proportion of variance explained by each eigenvalue, and the cumulative proportion for the five-factor model.

Table 6

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
<td>12.23</td>
<td>4.14</td>
<td>1.96</td>
<td>.89</td>
<td>.86</td>
</tr>
<tr>
<td>Proportion</td>
<td>.555</td>
<td>.188</td>
<td>.089</td>
<td>.041</td>
<td>.039</td>
</tr>
<tr>
<td>Cumulative</td>
<td>.555</td>
<td>.743</td>
<td>.831</td>
<td>.872</td>
<td>.911</td>
</tr>
</tbody>
</table>
As stated previously, the most important criterion is interpretability. First, there were numerous variables that loaded on more than one factor. When these variables were eliminated, numerous factors had fewer than three significant loadings, where a significant loading was considered .4 or above. Also, when reliabilities were checked for these factors, several had reliabilities under .70 (which is considered the minimal amount needed to show reliability). These findings showed support for a 5 factor solution. Specifying a 5 factor model was not only theoretically sound, but also provided enough factors to account for more than 90% of the cumulative variance, and resulted in a rotated factor pattern where factor loadings made construct (factor) identification clearly evident. Table 7 gives a summary of which items loaded on each factor, and the underlying constructs measured by the factors.

Table 7

Constructs Measured by Each Factor, and Items Loading on Those Factors.

<table>
<thead>
<tr>
<th>Construct</th>
<th>Math Self-Concept</th>
<th>Perception of Math Teacher</th>
<th>Perceived Usefulness of Math</th>
<th>NCTM Standards</th>
<th>Enjoyment/Interest in Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>12</td>
<td>19</td>
<td>10</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>From</td>
<td>22</td>
<td>21</td>
<td>15</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>Survey</td>
<td>26</td>
<td>23</td>
<td>25</td>
<td>37</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>27</td>
<td>35</td>
<td>42</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>39</td>
<td>43</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>48</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>51</td>
</tr>
<tr>
<td>Reliability</td>
<td>.74</td>
<td>.83</td>
<td>.79</td>
<td>.85</td>
<td>.87</td>
</tr>
<tr>
<td>Factor</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
As can also be seen, the reliabilities were calculated for each of the five scales using Cronbach's Coefficient Alpha, and each scale's reliability was over .70.

Factor one was labeled "Math Self-Concept." This factor reflects a student's perception of his or her own competence in mathematics. Any items that were reverse coded are marked with an "r." Items which loaded on this factor, along with the item number and corresponding factor loadings were:

1. I don't do very well in mathematics. (#12 [r], .71)
2. No matter how hard I try, I cannot understand mathematics. (#22 [r], .79)
3. I often think, "I can't do it," when a mathematics problem seems hard. (#26 [r], .70)
4. If I don't see how to work a mathematical problem right away, I never get it. (#44 [r], .57)

Factor two was labeled "Perceptions of Math Teacher." This factor reflects a student's view regarding the teacher characteristics of his or her math teacher. Any items that were reverse coded are marked with an "r." Items which loaded on this factor along with the item number and corresponding factor loadings were:

1. My math teacher knew when we were having trouble with our work. (#19 [r], .45)
2. My math teacher seemed to enjoy teaching mathematics. (#21, .71)
3. My math teacher was willing to give us individual help. (#23, .59)
4. My math teacher knew a lot about mathematics. (#27, .69)
5. My math teacher didn't mind students asking questions. (#29, .49)

Factor three was labeled "Perceived Usefulness of Math" This factor reflects a student's view regarding the usefulness of mathematical knowledge. Any items that were reverse coded are marked with an "r." Items which loaded on this factor along with the item numbers and corresponding factor loadings were:

1. Mathematics is useful for the problems of everyday life. (#10, .63)
2. There is little need for mathematics in most jobs. (#15 [r], .50)
3. Mathematics is helpful in understanding today's world. (#25, .75)
4. It is important to know mathematics in order to get a good job. (#35, .65)
5. You can get along perfectly well in everyday life without mathematics. (#39 [r], .59)

Factor four was labeled "NCTM Standards." This factor reflects the extent to which a student's mathematics instructor implements the standards set forth by the NCTM. Any items that were reverse coded are marked with an "r." Items which loaded on this factor along with the item number and corresponding factor loadings were:

1. My math teacher showed interest in the students. (#11, .43)
2. My math teacher presented material in a clear way. (#13, .52)
3. My math teacher used various methods of instruction based on the range of ways that diverse students learn mathematics. (#37, .74)
4. My math teacher demonstrated practical applications for mathematics.
5. My math teacher gave us time to explore ideas and problems. (#43, .73)
6. My math teacher encouraged us to work in groups. (#48, .63)
7. My math teacher respected students' ideas and answers even if they were incorrect. (#51, .65)

Factor five was labeled "Enjoyment/Interest in Math". This factor reflects the pleasure a student derives from engaging in mathematical activities. Any items that were reverse coded are marked with an "r." Items which loaded on this factor along with the item numbers and corresponding factor loadings were:

1. I like the easy math problems the best. (#9 [r], .40)
2. I would like a job which doesn't use any mathematics. (#14 [r], .54)
3. Mathematics is more of a game than it is hard work. (#18, .68)
4. Mathematics is something I enjoy very much. (#24, .79)
5. I enjoy talking to other people about mathematics. (#34, .86)
6. The only reason I'm taking mathematics is because I have to. (#38 [r], .67)

Confirmatory Factor Analysis (Measurement Model)

The basic confirmatory factor analysis model (Figure 5) used consisted of five latent variables or factors (represented with ovals): math self-concept, perceptions of math teacher, perceived usefulness of math, utilization of NCTM standards, and interest in math. Manifest indicator variables for each latent variable are represented
Figure 5. Theoretical Measurement Model

- NCTM Standards
- Math Self-Concept
- Perceived Usefulness of Math
- Interest in Math
- Perceptions of Math Teacher

Questions and variables:
- q11, q13, q37, q42, q43, q48, q51, q10, q15, q25, q35, q39, q9, q14, q18, q24, q24, q38, q19, q21, q23, q27, q29
by rectangles. These indicator variables are responses to survey items. Math self-concept has four manifest variables, perceptions of math teacher and perceived usefulness of math have five manifest variables a piece, utilization of NCTM standards has seven manifest variables, and interest in math has six manifest variables. Every factor is connected to every other factor by a curved, two-headed arrow, meaning that every factor is allowed to covary with every other factor. The "E" terms below each manifest variable are the residual terms associated with each manifest variable.

Parameter estimates were made using maximum likelihood estimation.

Criteria for assessing model fit includes the following: (1) nonsignificant chi-square, (2) chi-square/df ratio less than two, (3) CFI and NNFI greater than .90, (4) significant t-statistic for each factor loading, (5) symmetrical distribution of normalized residuals, centered at zero, (6) compositie reliabilities for latent factors greater than .70, (7) variance extracted estimates greater than .50, and (7) discriminant validity for questionable pairs of factors. It is also important to remember that a model's fit need not meet all of these criteria in order to be deemed acceptable. The Covariance Structure Analysis contains information about chi-square, the comparative fit index, and the non-normed index. Table 8 displays the information used for making decisions about the model.

As stated previously, it is desirable to have a non-significant chi-square, but this is difficult to attain (Hatcher, 1994). However, even though the chi-square was statistically significant, the chi-square/df ratio was less than two. Also, the CFI and
Table 8

Information about Chi-Square and Fit Indices for Measurement Model.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-square</td>
<td>628.7281</td>
<td>df = 402</td>
</tr>
<tr>
<td>Prob &gt; chi-square</td>
<td>.0001</td>
<td></td>
</tr>
<tr>
<td>chi-square/df ratio</td>
<td>628.7281/402</td>
<td>1.56</td>
</tr>
<tr>
<td>Comparative Fit Index</td>
<td>.9395</td>
<td></td>
</tr>
<tr>
<td>Non-normed Fit Index</td>
<td>.9301</td>
<td></td>
</tr>
</tbody>
</table>

NNFI were both greater than .90. Since these two criteria were met the next piece of information evaluated was the t-statistics for each factor loading.

- T-values for each manifest variable ranged from 7.6 to 18.3. All were well above the required 1.96 needed to meet the .05 significance level. This provided evidence concerning the validity of the manifest variables as indicators of the latent variables.

By subtracting each element of the predicted model matrix from the corresponding element of the original covariance matrix, a residual matrix is obtained. If the theoretical model successfully accounts for the actual relationships between the variables each element of this residual matrix should be zero or near zero (Hatcher, 1994; Bollen, 1989). To simplify interpretation, CALIS prints the normalized residual matrix. The normalized residual matrix consists of values obtained by dividing the residual by the square root of its estimated asymptotic variance (Bollen, 1989). The distribution of normalized residuals is shown in Figure 6. Upon examining the distribution, one can see that it is centered on zero, and while it is not perfectly
symmetric, it does show strong evidence of symmetry. However, there are numerous normalized residuals which exceed 2.0 in absolute value (approximately one standard deviation from the mean). This suggests that modifications to the measurement model may be necessary, and will be addressed.

Figure 6

Distribution of Asymptotically Standardized Residuals for Measurement Model.

<table>
<thead>
<tr>
<th>Standardized Residual Values</th>
<th>Number of Residuals</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.75 to -2.50</td>
<td>4</td>
<td>0.81</td>
</tr>
<tr>
<td>-2.50 to -2.25</td>
<td>9</td>
<td>1.81</td>
</tr>
<tr>
<td>-2.25 to -2.00</td>
<td>11</td>
<td>2.22</td>
</tr>
<tr>
<td>-2.00 to -1.75</td>
<td>18</td>
<td>3.63</td>
</tr>
<tr>
<td>-1.75 to -1.50</td>
<td>19</td>
<td>3.83</td>
</tr>
<tr>
<td>-1.50 to -1.25</td>
<td>20</td>
<td>4.03</td>
</tr>
<tr>
<td>-1.25 to -1.00</td>
<td>31</td>
<td>6.25</td>
</tr>
<tr>
<td>-1.00 to -0.75</td>
<td>21</td>
<td>4.23</td>
</tr>
<tr>
<td>-0.75 to -0.50</td>
<td>31</td>
<td>6.25</td>
</tr>
<tr>
<td>-0.50 to -0.25</td>
<td>32</td>
<td>6.45</td>
</tr>
<tr>
<td>-0.25 to 0.00</td>
<td>37</td>
<td>7.46</td>
</tr>
<tr>
<td>0.00 to 0.25</td>
<td>80</td>
<td>16.13</td>
</tr>
<tr>
<td>0.25 to 0.50</td>
<td>36</td>
<td>7.26</td>
</tr>
<tr>
<td>0.50 to 0.75</td>
<td>25</td>
<td>5.04</td>
</tr>
<tr>
<td>0.75 to 1.00</td>
<td>24</td>
<td>4.84</td>
</tr>
<tr>
<td>1.00 to 1.25</td>
<td>24</td>
<td>4.84</td>
</tr>
<tr>
<td>1.25 to 1.50</td>
<td>23</td>
<td>4.64</td>
</tr>
<tr>
<td>1.50 to 1.75</td>
<td>13</td>
<td>2.62</td>
</tr>
<tr>
<td>1.75 to 2.00</td>
<td>8</td>
<td>1.61</td>
</tr>
<tr>
<td>2.00 to 2.25</td>
<td>8</td>
<td>1.61</td>
</tr>
<tr>
<td>2.25 to 2.50</td>
<td>7</td>
<td>1.41</td>
</tr>
<tr>
<td>2.50 to 2.75</td>
<td>6</td>
<td>1.21</td>
</tr>
<tr>
<td>2.75 to 3.00</td>
<td>7</td>
<td>1.41</td>
</tr>
</tbody>
</table>
The last two parts which were be addressed were the composite reliability for the latent factors, and the variance extracted estimates for the latent factors. Composite reliability and variance extracted are calculated using the following equations:

\[
\text{Composite reliability} = \frac{(\sum L_i)^2}{(\sum L_i)^2 + \sum \text{Var}(E_i)}
\]

where \( L_i \) = the standardized factor loadings for that factor

and \( \text{Var}(E_i) \) = the error variance associated with the individual indicator variables.

\[
\text{Variance extracted} = \frac{\sum L_i^2}{\sum L_i^2 + \sum \text{Var}(E_i)}
\]

Note: The formula for variance extracted differs from composite reliability in that the \( \sum L_i \) term is no longer with parentheses. This means that each factor loading is squared first, and then these squared factor loadings are summed. Because a squared factor loading for an indicator is equivalent to that indicator's reliability, this is equivalent to simply summing the reliabilities for a given factor's indicators (Hatcher, 1994).

Table 9 on the following page presents the standardized factor loadings for each item. From the factor loadings, the indicator reliability is obtained by squaring the factor loadings. The indicator reliability is the percent of variation in the indicator that is explained by the factor that it is supposed to measure, and the error variance is
Table 9

Composite Reliabilities and Variance Extracted Estimates for Measurement Model.

<table>
<thead>
<tr>
<th>Construct and Indicators</th>
<th>Standardized Loadings</th>
<th>Indicator Reliability</th>
<th>Error Variance</th>
<th>Composite Reliability</th>
<th>Variance Extracted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Self-Concept (F1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q12</td>
<td>0.8226</td>
<td>0.677</td>
<td>0.323</td>
<td>0.761</td>
<td>0.642</td>
</tr>
<tr>
<td>q22</td>
<td>0.8533</td>
<td>0.728</td>
<td>0.272</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q26</td>
<td>0.4497</td>
<td>0.202</td>
<td>0.798</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q44</td>
<td>0.4926</td>
<td>0.243</td>
<td>0.757</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher Perceptions (F2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.835</td>
</tr>
<tr>
<td>q19</td>
<td>0.6336</td>
<td>0.401</td>
<td>0.599</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q21</td>
<td>0.7403</td>
<td>0.548</td>
<td>0.452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q23</td>
<td>0.8214</td>
<td>0.675</td>
<td>0.325</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q27</td>
<td>0.5903</td>
<td>0.348</td>
<td>0.652</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q29</td>
<td>0.7484</td>
<td>0.560</td>
<td>0.440</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perceived Usefulness (F3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.790</td>
</tr>
<tr>
<td>q10</td>
<td>0.7110</td>
<td>0.506</td>
<td>0.494</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q15</td>
<td>-0.6236</td>
<td>0.389</td>
<td>0.611</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q25</td>
<td>0.7036</td>
<td>0.495</td>
<td>0.505</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q35</td>
<td>0.6293</td>
<td>0.396</td>
<td>0.604</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q39</td>
<td>0.6080</td>
<td>0.370</td>
<td>0.630</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NCTM Standards (F4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.859</td>
</tr>
<tr>
<td>q11</td>
<td>0.7562</td>
<td>0.572</td>
<td>0.428</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q13</td>
<td>0.7323</td>
<td>0.536</td>
<td>0.464</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q37</td>
<td>0.7140</td>
<td>0.510</td>
<td>0.490</td>
<td></td>
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</tr>
<tr>
<td>q42</td>
<td>0.6806</td>
<td>0.463</td>
<td>0.537</td>
<td></td>
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</tr>
<tr>
<td>q43</td>
<td>0.7223</td>
<td>0.522</td>
<td>0.478</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q48</td>
<td>0.5341</td>
<td>0.285</td>
<td>0.715</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q51</td>
<td>0.6191</td>
<td>0.383</td>
<td>0.617</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest in Math (F5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.878</td>
</tr>
<tr>
<td>q9</td>
<td>0.5174</td>
<td>0.268</td>
<td>0.732</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q14</td>
<td>0.7495</td>
<td>0.562</td>
<td>0.438</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q18</td>
<td>0.6846</td>
<td>0.469</td>
<td>0.531</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q24</td>
<td>0.8688</td>
<td>0.755</td>
<td>0.245</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q34</td>
<td>0.7519</td>
<td>0.565</td>
<td>0.435</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q38</td>
<td>0.8331</td>
<td>0.694</td>
<td>0.306</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
computed by subtracting the indicator reliability from one. The composite reliability is calculated using the previously stated formula. The index is analogous to coefficient alpha, and reflects the internal consistency of the indicators measuring a given factor. Generally .60 or .70 is the minimally acceptable level of reliability for instruments used in research (Hatcher, 1994). Examination of the table shows that composite reliabilities all exceed .70. Finally, the variance extracted estimates were calculated using the previously stated formula. This figure is the amount of variance captured by a given construct. It is desirable that constructs exhibit estimates of .50 or larger, because estimates less than .50 indicate that variance due to measurement error is larger than variance captured by the factor (Hatcher, 1994). For the theorized measurement model, three of the five variance extracted estimates exceed .50.

In looking at the characteristics of an "ideal fit" for the measurement model, the theorized model failed to meet some of these, but met the majority of the criteria including:

1) chi-square/df ratio < 2,
2) CFI and NNFI > .9,
3) t-statistics > 1.96,
4) symmetric residuals centered on zero,
5) composite reliabilities > .70, and
6) most variance extracted estimates > .50.

This suggests that an adequate measurement model has been used, except for having
normalized residuals greater than two, and two variance estimates that are less than 0.50. Therefore, information for model modification from CALIS, which contains two parts—the Wald test (which suggests paths to be dropped) and the Lagrange multiplier (which suggests paths to be added)—was examined.

The Wald test suggested that eleven different parameters could be dropped without causing a significant increase in chi-square. However, each of these paths were identified as covariance between factors. In confirmatory factor analysis, all factors are normally allowed to covary during the analysis, and therefore it would not be appropriate to drop these factor covariances simply because they were nonsignificant.

What was being looked for in the Wald test was some indication that a factor loading could be dropped from the model and do the least amount of change to the model chi-square. Such a finding would possibly show that an indicator variable was not doing a good job of measuring the factor to which it was assigned. However, it comes as no surprise that no factor loadings were reported in the Wald test, because all of the factor loadings were significant when they were reviewed earlier.

Next suggestions from the Lagrange multipliers are examined. The Lagrange multiplier test estimates the reduction in model chi-square that would result from freeing a fixed parameter and allowing it to be estimated. In other words, for a confirmatory factor analysis, the Lagrange multiplier estimates the degree to which chi-square would improve if a new factor loading or covariance were added to the model. The model modifications of additional paths suggested in this section were deemed to be
theoretically unsound. Because the original measurement model supports past research, and was deemed to be adequate based on the majority of criteria for ideal fit, it was therefore concluded that the original measurement model would be used in assessing the next step, which is the structural model.

**LISREL Analysis (Structural Model)**

The structural model (Figure 7) was assessed using the SAS procedure CALIS much the same way the measurement model was assessed. Table 10 contains the information needed for making decisions about the model.

| Information about Chi-Square and Fit Indices for Theoretical Structural Model. |
|---|---|---|---|
| Chi-square = 690.3 | df = 438 | Prob > chi-square = .0001 |
| chi-square/df ratio = 690./438 = 1.56 |
| Comparative Fit Index = .9361 |
| Non-normed Fit Index = .9276 |

As stated previously one hopes for a non-significant chi-square, but does not always attain this. However, even though the chi-square was statistically significant, the chi-square/df ratio was less than two. Also, the CFI and NNFI were both greater than .90.
Figure 7. Theoretical Structural Model

- Gender
- Fathers’ Math Ability
- Fathers’ Ed Level
- Number of Math Classes
- NCTM Standards
- Perceptions of Math Teacher

Factors influencing Math Ability:
- Interest in Math
- Math Achievement
- Math Self-Concept
- Perceived Usefulness of Math
Since these two criteria were met the next pieces of information evaluated were the t-statistics for each path coefficient.

T-values for each manifest variable (and multiple t-values for endogenous variables, depending on how many paths were going to that variable) presented some problems, in that several were nonsignificant (less than 1.96). The nonsignificant t-values included the following path coefficients:

- gender → math self-concept
- gender → math achievement
- fathers' education level → perceived usefulness of math
- perceptions of math teacher → math self-concept

At this point the decision was made to consider modifying the structural model by deleting paths, producing a modified model that would need to be tested with data collected in the future. For possible modifications, the Wald test (which suggests paths to be dropped) was the first to be examined. Not surprisingly, the Wald test suggestions included dropping each of the nonsignificant paths. Subsequent models were tested. In each attempt one additional path was dropped, until the modified structural model contained path coefficients which were all statistically significant. In the modified structural model, a total of four paths were dropped. These modifications were not only the suggestions generated by SAS, but also theoretically sound. The modified structural model was then compared to the original theoretical model using the chi-square difference test to ensure that the modified model was indeed significantly better than the original.
**Modified Structural Model**

The modified structural model eliminated each of the nonsignificant paths discussed previously. Table 11 displays the resulting information used for making decisions about the model.

Table 11

**Information about Chi-Square and Fit Indices for Modified Structural Model.**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-square</td>
<td>646.9</td>
</tr>
<tr>
<td>df</td>
<td>416</td>
</tr>
<tr>
<td>Prob &gt; chi-square</td>
<td>.0001</td>
</tr>
<tr>
<td>chi-square/df ratio</td>
<td>646.9/416</td>
</tr>
<tr>
<td></td>
<td>1.56</td>
</tr>
<tr>
<td>Comparative Fit Index</td>
<td>.9384</td>
</tr>
<tr>
<td>Non-normed Fit Index</td>
<td>.9312</td>
</tr>
</tbody>
</table>

While the chi-square was still significant, the chi-square/df ratio was less than two. Also, the CFI and NNFI were both greater than .90, and actually a little larger than the original structural model. Since these two criteria were met the next piece of information to look at was again the t-statistics for each path coefficient.

T-values for each manifest variables (and multiple t-values for endogenous variables, depending on how many paths there are going to that variable) ranged from 2.4 to 15.2. All were above the required 1.96 needed to meet the .05 significance level. This served to reinforce the acceptability of the modified structural model for the sample data.

The "Distribution of Asymptotically Standardized Residuals" (Figure 8)
is found below. Upon examining the distribution, one can see that it is centered on zero, and while it is not perfectly symmetric, it does show strong evidence of symmetry. However, again there are numerous normalized residuals which exceed 2.0 in absolute value. This is somewhat of a problem, and will addressed.

Figure 8

Distribution Of Asymptotically Standardized Residuals for Modified Structural Model.

<table>
<thead>
<tr>
<th>Standardized Residual Values</th>
<th>Number of Residuals</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.75 to -2.50</td>
<td>3</td>
<td>0.60</td>
</tr>
<tr>
<td>-2.50 to -2.25</td>
<td>7</td>
<td>1.41</td>
</tr>
<tr>
<td>-2.25 to -2.00</td>
<td>11</td>
<td>2.22</td>
</tr>
<tr>
<td>-2.00 to -1.75</td>
<td>14</td>
<td>2.82</td>
</tr>
<tr>
<td>-1.75 to -1.50</td>
<td>17</td>
<td>3.43</td>
</tr>
<tr>
<td>-1.50 to -1.25</td>
<td>16</td>
<td>3.23</td>
</tr>
<tr>
<td>-1.25 to -1.00</td>
<td>31</td>
<td>6.25</td>
</tr>
<tr>
<td>-1.00 to -0.75</td>
<td>18</td>
<td>3.63</td>
</tr>
<tr>
<td>-0.75 to -0.50</td>
<td>38</td>
<td>7.66</td>
</tr>
<tr>
<td>-0.50 to -0.25</td>
<td>28</td>
<td>5.65</td>
</tr>
<tr>
<td>-0.25 to 0.00</td>
<td>30</td>
<td>6.05</td>
</tr>
<tr>
<td>0.00 to 0.25</td>
<td>63</td>
<td>12.70</td>
</tr>
<tr>
<td>0.25 to 0.50</td>
<td>34</td>
<td>6.85</td>
</tr>
<tr>
<td>0.50 to 0.75</td>
<td>35</td>
<td>7.06</td>
</tr>
<tr>
<td>0.75 to 1.00</td>
<td>31</td>
<td>6.25</td>
</tr>
<tr>
<td>1.00 to 1.25</td>
<td>41</td>
<td>8.27</td>
</tr>
<tr>
<td>1.25 to 1.50</td>
<td>20</td>
<td>4.03</td>
</tr>
<tr>
<td>1.50 to 1.75</td>
<td>17</td>
<td>3.43</td>
</tr>
<tr>
<td>1.75 to 2.00</td>
<td>12</td>
<td>2.42</td>
</tr>
<tr>
<td>2.00 to 2.25</td>
<td>6</td>
<td>1.21</td>
</tr>
<tr>
<td>2.25 to 2.50</td>
<td>6</td>
<td>1.21</td>
</tr>
<tr>
<td>2.50 to 2.75</td>
<td>7</td>
<td>1.41</td>
</tr>
<tr>
<td>2.75 to 3.00</td>
<td>6</td>
<td>1.21</td>
</tr>
</tbody>
</table>
Another criterion for evaluating structural models deals with how parsimonious the modified structural model is. This information is found with the comparative fit index and the non-normed fit index, and is called the parsimonious fit index (sometimes called the parsimonious normed-fit index or PNFI). While there is no firm criterion as to how large the PNFI must be for the model to be acceptable, .60 has been suggested (Hatcher, 1994). The PNFI attained was .7573, and therefore indicated an acceptable level of parsimony. This was also an improvement over the PNFI of the original structural model, which was .7429.

However, since there were still some criteria for ideal fit which were not met, looking at the Wald test and the Lagrange multiplier test for further modifications to be made was deemed appropriate. In looking at the Wald test, all of the modifications suggested now only involved covariances between exogenous variables. Covariances are generally estimated for all possible pairs of exogenous F variables in an analysis of this sort, so Wald test results that include these were disregarded.

In looking at the Lagrange multiplier test rank order of ten largest Lagrange multipliers, several options were to add paths between indicator variables that already loaded on the same factor. Only a few options did not involve pairs of indicator variables loading on a common factor, and these options were not consistent with the theoretical basis of the background literature.

Because paths had been deleted to obtain the modified structural model, it was considered a nested model. In general, any model which requires that some function of its free parameters equals another free parameter or equals a constant is nested in the
identical model that has no such restriction. A nested model can be compared to the original structural model to determine significant improvement in model fit by using the chi-square difference test. The chi-square difference test calculates the difference between the two resulting chi-square values (from the original structural model and the modified structural model), and compares it to critical chi-square value obtained from any chi-square table, using the difference in the degrees of freedom associated with the two models as the new degrees of freedom. When the chi-square difference test was performed, a value of 43.4 with 22 degrees of freedom was obtained. This was compared to a critical chi-square value of 33.9 (Bartz, 1988). Since 43.4 is greater than 33.9, this indicates that the modified structural model demonstrated a significant improvement in model fit.

Because of this, and because the majority of the criteria for an adequate model were met, it was decided that the modifications already made were sufficient, and the revised structural model was adequate. The original theorized structural model is presented in Figure 9 (with paths to be deleted represented by dotted lines), while the modified structural model which was used for interpretation is presented in Figure 10. The modified structural model includes path coefficients (all of which were statistically significant) and statistically significant covariances among exogenous variables.

One of the benefits of structural equation modeling is the concept of variables being able to have direct and/or indirect effects on other variables. For this study, the direct effects latent or manifest variables have on math self-concept were examined,
Figure 9. Theoretical Structural Model

Note: Nonsignificant (deleted) paths are indicated by dashed lines; significant paths by solid lines.
Figure 10. Modified Structural Model.
and in addition, the indirect effect that a variable has on math self-concept through a mediating variable was also critical. The direct effect is the path coefficient between the two variables. For instance in Figure 10 the path coefficient from math achievement to math self-concept was .21. This is the direct effect of math achievement on math self-concept. The indirect effects are the product of the path coefficients from one specific variable to the variable of interest. For example in Figure 10 the utilization of NCTM standards has indirect effects on math self-concept through math achievement (as well as through other variables). Therefore the indirect effect of utilization of NCTM standards on math self-concept through math achievement would be the product of the two path coefficients ([.41][.21] = .09) The sum of any direct and indirect effects of a variable on math self-concept makes up the total effects. The direct, indirect, and total effects are summarized in Table 12.

Variables are listed in rank order of the total effects (i.e. interest in math had the largest total effect, perceived usefulness of math had the next largest total effect, etc.). Path coefficients multiplied together to calculate indirect effects, along with the specifics paths of the indirect effects are also included in the table. For instance, interest in math had a direct effect on math self-concept, as well as an indirect effect on math self-concept through math achievement. The sum of the direct and indirect effects resulted in the total effects. According to Asher (1976) and Pedhazur (1982) total effects are the correlation between two variables, and can be decomposed into a sum of direct and indirect effects. Because total effects are correlations, they can be evaluated for statistical significance by comparing them to significant value tables for
Table 12

**Direct, Indirect, and Total Effects on Math Self-Concept**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Direct Effects</th>
<th>Indirect Effects</th>
<th>Paths Effects</th>
<th>Total Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest in Math</td>
<td>0.63</td>
<td>(.51)(.21) = .11</td>
<td>Math Ach.</td>
<td>0.74*</td>
</tr>
<tr>
<td>Perceived</td>
<td>0</td>
<td>(.46)(.63) = .29</td>
<td>Interest</td>
<td></td>
</tr>
<tr>
<td>Usefulness of Math</td>
<td></td>
<td>(.46)(.51)(.21) = .05</td>
<td>Interest, Math Ach.</td>
<td>0.34*</td>
</tr>
<tr>
<td>NCTM Utilization</td>
<td>0</td>
<td>(.28)(.63) = .18</td>
<td>Interest</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.28)(.51)(.21) = .03</td>
<td>Interest, Math Ach.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.41)(.21) = .09</td>
<td>Math Ach.</td>
<td>0.30*</td>
</tr>
<tr>
<td>Number of Math Classes Taken in High School</td>
<td>0</td>
<td>(.19)(.51)(.21) = .02</td>
<td>Interest, Math Ach.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.19)(.63) = .12</td>
<td>Interest</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.18)(.21) = .01</td>
<td>Math Ach.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.29)(.46)(.63) = .08</td>
<td>Usefulness, Interest</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.29)(.46)(.51)(.21) = .01</td>
<td>Usefulness, Interest, Math Ach.</td>
<td>0.24*</td>
</tr>
<tr>
<td>Math Achievement</td>
<td>0.21</td>
<td>0</td>
<td></td>
<td>0.21*</td>
</tr>
<tr>
<td>Perceptions of Math Teacher</td>
<td>0</td>
<td>(.39)(.21) = .08</td>
<td>Math Ach.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.27)(.46)(.63) = .08</td>
<td>Usefulness, Interest</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.27)(.46)(.51)(.21) = .01</td>
<td>Usefulness, Interest, Math Ach.</td>
<td>0.17*</td>
</tr>
<tr>
<td>Fathers' Math Ability</td>
<td>0</td>
<td>(.14)(.63) = .09</td>
<td>Interest</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.14)(.51)(.21) = .01</td>
<td>Interest, Math Ach.</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: * indicates statistical significance at $\alpha = .05$
correlations. Degrees of freedom for this procedure was sample size minus two (Bartz, 1988). Sample size used for the measurement model and structural model (after incomplete surveys were disregarded) was 294, yielding a degrees of freedom of 292. The critical value for statistical significance of correlations using 200 degrees of freedom, and a .05 significance level was .138 (Bartz, 1988). Therefore, all of the total effects shown in Table 12 are statistically significant except for the total effect of fathers' math ability.

While interest in math and math achievement are the only two variables that had a direct effect on math self-concept, there were other variables that had meaningful indirect effects on math self-concept. Despite not having any direct effects on math self-concept, perceived usefulness of math, students' perceptions of the math teacher, utilization of NCTM standards and number of math classes taken in high school all had statistically significant total effects on math self-concept, and in fact, three of the four had larger total effects on math self-concept than math achievement.

**Summary**

The results of the statistical analyses can be summarized in three parts. First, exploratory factor analysis was used to verify what items loaded to each factor. A five-factor model was concluded, with the factors including: (1) math self-concept, (2) perceptions of high school math teacher, (3) perceived usefulness of math, (4) utilization of NCTM standards, and (5) interest/enjoyment in math.

Next, a confirmatory factor analysis was run using proc CALIS to assess the
measurement model. The theorized measurement model (using the five factors described above) met the majority of the criteria needed to indicate an adequate model. These factors (also called latent variables) were then used, along with manifest variables of math achievement, gender, fathers' math ability, fathers' education level, and number of math classes taken in high school to predict math self-concept in a causal structural model. While the majority of fit indices were acceptable, results showed that there were several nonsignificant path coefficients. Subsequent models were tested, dropping one additional path each time, until all path coefficients were statistically significant. This modified structural model was then compared to the theorized structural model using the chi-square difference test to ensure that the new model was indeed significantly better than the original structural model. The modified structural model did meet the majority of fit indices which characterize ideal fit, and therefore was determined to be an adequate model for purposes of interpretation. From the modified structural model, the direct, indirect, and total effects of each factor on math self-concept were then calculated and interpreted.
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This final chapter will serve three important purposes. First, the study and results will be summarized. Second, conclusions based on results from chapter four will be discussed, and third, recommendations for future research will be made.

Summary

This research study centered around the issue of math self-concept. Specifically, what factors causally determine math self-concept (factors of interest included gender, perceptions of math teacher, utilization of NCTM standards, interest/enjoyment in math, perceived usefulness of math, math achievement, and fathers' math ability and education level). While studies have been conducted that have included math self-concept in looking at other variables, no studies have focused on factors that causally determine math self-concept. Also, evidence showing that the utilization of NCTM standards appears to have a significant impact on numerous factors associated with math self-concept helps to show the potential benefits for math instructors to implement these standards into their teaching methods.

The importance of specifically studying causal relationships of math self-concept can be identified in two areas which are of significant concern. First is the fact that students in the United States are scoring much lower on mathematics achievement tests than their counterparts from other countries. The cause of this problem seems to be
that many otherwise highly motivated and highly bright people, including both men and women, seem to avoid taking any mathematics courses past the minimum requirements. As stated previously, approximately half of all high school students graduate without taking a math class past the 10th grade. The second problem is more long term, and it involves these people severely limiting their educational and career options due to the fact that they do not possess the math background vital in so many occupational areas today.

The decline in enrollment in mathematics courses obviously leads to lower mathematical ability, but also indicates a negative attitude in general regarding mathematics. The negative attitude typically translates into math anxiety and/or a low self-concept of one's math ability. For the most part, studies which have included math self-concept have an invariable common thread, and that is the role the math instructor plays. It has been emphasized that teachers are shown to be the prime source of influence on student achievement and student attitudes. In fact, many attitudes about math (especially negative ones) can be traced to a particular teacher or class. The literature overwhelmingly implies that math learning--more than most subjects--is largely a function of math teaching.

This would seem to be true because unlike many classes, math is not something that can simply be memorized. It is essential to have an understanding of the underlying principles, and how they are applied. The problem lies in the typical math teaching paradigm which is based on memorization, not on understanding and reasoning. In past years, the National Council of Teachers of Mathematics has set
forth standards of what (and how) students should learn in a math class. These standards include a commitment to developing mathematical literacy and power to all students, and emphasizing mathematical reasoning, problem solving, communication, and connections, rather than concentrating on paper and pencil drills.

Therefore, the purpose of this study was to examine a theoretically derived causal model of the relationships among identified factors and students' math self-concept. Instructor influences (students' perceptions of the teacher and utilization of the NCTM standards) were of particular interest in their impact on students' math self-concept. Other variables (math achievement, math interest, perceived usefulness of math, gender, number of math classes taken in high school, and fathers' education level and math ability) were included in a theoretically derived order in the causal model.

To obtain information about these factors, a survey instrument was developed. Scales from the Math Attitudes Inventory were used to measure math self-concept, perceptions of a math teacher, perceived usefulness of math, and interest in math. A scale was added to examine to what extent past math teachers had utilized the standards set forth by the NCTM, as well as items to determine gender, number of math classes taken in high school (and which ones), and the mothers' and fathers' educational level and math ability. Also, students were asked to supply their student identification number. Student identification numbers were submitted to the Office of Institutional Research in order to obtain each student's ACT math test score.

While the target population was all freshmen enrolled at Oklahoma State University in the fall of 1997, the sample consisted of students enrolled in freshmen
orientation classes in each of the various colleges at Oklahoma State University or a "World of Work" class for that semester. The final sample size was 331, which is approximately 10% of the overall population.

Once data had been collected, the results of the surveys, along with ACT math test scores were entered into a computer using Excel, and then imported into SAS for the statistical analyses. The results were obtained using the SAS procedure CALIS. CALIS enables the researcher to accomplish two things. First, the researcher can utilize confirmatory factor analysis to check the theorized "measurement model," and second, the researcher can utilize a Lisrel causal analysis to check the theorized "structural model." While the measurement model is essential, the most critical part of this research study is encompassed in the structural model.

The confirmatory factor analysis provided evidence that the indicator variables (items from the survey) were really measuring the underlying constructs of interest, and that the theorized measurement model demonstrated an acceptable fit to the data. Several characteristics displayed when a measurement model provides ideal fit to the data were listed, and it was noted that these characteristics, while ideal, will not always be attained, even when the measurement model is quite good. A decision about the theorized measurement model was made based on two things. First, the majority of the criteria for a good fit were met. Second, the suggestions for modifications were either theoretical unsound or inappropriate changes for a confirmatory factor analysis (i.e. dropping factor covariances because they were nonsignificant). Therefore, it was decided that the measurement model was adequate, and would be used in subsequent
analyses. This measurement model established five latent variables (factors) being measured by the manifest or indicator variables (items on the survey). These latent variables included math self-concept (factor 1), perception of a math teacher (factor 2), perceived usefulness of math (factor 3), utilization of the NCTM standards (factor 4), and interest/enjoyment in math (factor 5).

The next step was to use these latent variables, along with numerous manifest variables, including math achievement (measured by the ACT math test), gender, number of math classes taken in high school, and the fathers' education level and math ability, to examine causal relationships with math self-concept. While items on the survey did inquire about the mothers' education level and math ability, there was no reference in any background research or past literature to indicate that these variables had been shown to be of significance. Therefore, they were not included in the theoretical structural model.

The same steps taken to check the measurement model were used to check the structural model, along with checking the added area of parsimony. The output gave evidence of several nonsignificant paths, and therefore the structural model was modified to eliminate nonsignificant paths (one at a time) until the modified model was shown to be significantly improved over the theoretical model using the chi-square difference test. The paths dropped included:

Gender → Math Self Concept
Gender → Math Achievement
Fathers' Education Level → Perceived Usefulness of Math
Perception of Math Teacher → Math Self Concept

These modifications eliminated gender and fathers’ educational level from the structural model completely, and are therefore not shown in the modified structural model. The elimination of gender from the structural model is contradictory to past research findings, and will be discussed. Besides being shown to be significantly improved over the theoretical model, the decision to accept the modified structural model was made based on two things. First, the majority of the criteria for a good fit was met. Second, any further suggestions for modifications were either theoretical unsound or inappropriate. Therefore, it was decided that the modified structural model was adequate (refer to Figure 10, pg. 92).

Conclusions

Several interesting findings were obtained. While numerous factors were included in the study, the factors that involved the perceptions of the math teacher, and the utilization of NCTM standards and how these influenced math self-concept were of primary interest. The perception of the math teacher did not directly affect math self-concept as originally hypothesized. This seems contradictory to the previous research, which consistently emphasized the influential role of the math instructor on math self-concept. However, both the perceptions of the math teacher, and the utilization of NCTM standards had direct effects on math achievement, as well as perceptions of the math teacher having a direct effect on perceived usefulness of math, and utilization of NCTM standards have a direct effect on interest in math. Therefore teacher variables
do affect math self-concept. The direct effects of these two variables also indicate several things. First, the utilization of NCTM standards directly affecting interest in math as well as math achievement, is a positive sign. As discussed in chapter two, the NCTM standards main goals are to shift from math classrooms dominated by lectures, and paper and pencil drills, to classrooms where students are given tasks with real-world applications which develop mathematical understanding and skills, stimulate students to make connections, promote communication about mathematics, and represent mathematics as an ongoing human activity. The fact that the level of attainment of these goals does have a direct effect on interest in math and math achievement serves to verify the importance which the NCTM has placed on these standards, and furthermore serves to encourage math instructors to put forth the time and effort of implementing these standards in their own classrooms. While the utilization of the NCTM standards does not directly affect math self-concept, it does account for several indirect effects, and has a total effect of .30. This total effect is statistically significant, and actually higher than the total effect of math achievement which has a direct effect on math self-concept.

Second, it is also a positive sign (though not surprising) to see that perceptions of the math teacher have direct effects on math achievement and perceived usefulness of math. The "perceptions of the math teacher" construct included numerous items which focused on student-teacher relations and interaction. This indicates the significance of the role a math instructor plays in communicating the relevance and usefulness of math to their students, and signifies the importance of that student-teacher
relationship on math achievement. Again, while the "perceptions of the math teacher" construct did not have the hypothesized direct effect on math self-concept, it should be noted that it does have a statistically significant total effect on math self-concept.

As far as practical significance, the most important direct effects that the instructor-related factors show are those on math achievement, interest in math, and perceived usefulness of math. It is critical that math instructors realize the impact they can have on students in these areas. From the model, it is obvious that if interest in math, perceived usefulness of math, and math achievement can be improved by an instructor, then they can in turn improve students' math self-concept.

A somewhat surprising finding was, according to the model, that gender did not form a significant causal relationship with math self-concept or math achievement. While this conclusion is a direct contradiction of most findings in previous research, it shows supports for Marsh's study (1993), which found no differences in math achievement for males and females, and only a small difference in math self-concept. A possible explanation for this deals with the sample used. Previous research which was discussed in the review of the literature typically involved students who were in high school, middle school (or junior high) or even elementary school. The sample used in this study involved only students who chose to attend college after high school. Obviously, not all high school students chose to attend college. It is a reasonable assumption that students who do would be those who are, at least to some extent, self-confident in their academic ability. Therefore in this research study dealing with academic achievement and self-concept, a sample of college students would likely differ
from a sample of high school students. Also, of those who do attend college, a number of those go to smaller colleges for several years, therefore making students who start their freshman year at a major 4-year university an even more homogeneous group, and even less similar to a general high school population. This means that it is highly possible that a notable difference exists between students who do go on to college and students who do not. In this case, that difference includes the fact that gender does not play a significant role in determining math self-concept.

An interesting finding centered around the fathers' educational level and math ability. While the fathers' education level did not significantly influence the model, the fathers' math ability did have significant direct effects on interest in math, (which in turn affected math-self concept). This is consistent with findings in the literature that fathers do influence the math-self concept of their children.

The last finding of interest in the structural model involves the number of math classes taken in high school. While it is not surprising that this had a direct effect on math achievement (and therefore an indirect effect on math self-concept), it does lend credibility to the ongoing efforts to raise the number of math classes required for high school graduation, especially since the number of math classes taken in high school also had direct effects on interest in math and perceived usefulness of math. In fact, number of math classes taken in high school contributes its strongest effect on perceived usefulness of math. As with the "perceptions of the math teacher" and "utilization of NCTM standards" constructs, the number of math classes taken in high school, while not directly influencing math self-concept, did have a statistically
significant total effect on math self-concept.

The goal of this study was to determine if the math instructor and type of instructional techniques used had a significant effect in determining students' math-concept, or if math self-concept was influenced more by other factors. This study shows that the math instructor and type of instructional techniques used do have indirect effects on math self-concept, and statistically significant total effects. However, the only factors which directly influenced math self-concept were found to be math achievement and interest in math, with interest in math being the most significant determinant. It should be emphasized again though that the factors which focused on instruction (NCTM standards and perception of math teacher) both directly or indirectly influence interest in math, as well as math achievement. This underscores the importance of the role math instructors play in the classroom.

Recommendations

This section includes six recommendations. Five concerning future research studies, and another concerning the results of this particular study.

Results of this study show evidence of differences in what affects math self-concept among college students, and what previous research has shown to affect math self-concept among students who are high school age or younger. Differences may even exist between students who attend large universities versus students who attend smaller universities (although that is purely conjecture). However, a suggestion for future research would be to compare these groups. If possible, a study starting with
students in high school and following up on what each student did after high school (e.g. attending a major university, attending a junior college or smaller university, attending vo-tech, going directly to work), and compare factors which causally determine math self-concept for each group. Also, because the number of math classes taken in high school had its strongest effect on perceived usefulness of math, it would be interesting for future research to investigate a reciprocal relationship between these two variables. This would help to determine if in fact, the number of math classes taken in high school influences perceived usefulness of math, or if perceived usefulness of math influences the number of math classes taken in high school.

Another recommendation for future research centers around comparing males and females. For this study, sample size prohibited legitimate interpretation of separate models for males and females. For future research it would be beneficial to obtain a large enough sample so separate models for males and females could be interpreted. Closely tied to comparing males and females, is the issue of parental influence. The influence of fathers' math ability and education level has been examined in relation only to daughters. Future research should examine the impact of both the fathers' and mothers' math ability and education level on daughters and sons.

One last recommendation for future research would be to examine each of the NCTM standards separately. For this study, the influence of the NCTM standards as a whole was examined. However, it is entirely possible that specific standards do not exert the same influences over the same variables. Therefore, looking at each standard individually could result in different findings.
While speculations can always be made as to future research studies, the final recommendation deals directly with the results of this study. There were several exogenous variables which would be of interest to most math teachers. Among those are the perceptions of the math teacher, and the utilization of NCTM standards. While it may be somewhat surprising that neither of these had a direct effect on math self-concept, both of them had significant indirect and total effects. It is the opinion of the researcher who happens to also be a math instructor that the effects of the instructor and the type of instruction used cannot be overstated. As shown in the structural model, these things affect interest in math, achievement in math, and perceived usefulness of math (all of which subsequently affect math self-concept).

In regard specifically to the utilization of the NCTM standards, it is shown that this directly affects interest in math. It is essential that math instructors become familiar with these standards which cover the following five areas: (1) the types of tasks that teachers should pose, (2) the type of discourse a teacher should foster in their classroom, (3) the students' role in classroom discourse, (4) promoting mathematical disposition, and (5) the learning environment. Each of these five areas affect what a student learns, how they learn, and most importantly, how they view what they’ve learned. This type of teaching makes the subject matter not only meaningful, but also relevant. It is an essential step to developing students who can compete with their counterparts from other countries by raising scores on standardized math tests, and more importantly, to developing students who can compete in careers by having the mathematical background critical to many occupations.
REFERENCES


I appreciate your taking time out of your schedule to assist in a research project for my doctoral dissertation. Your input will help me determine factors which predict mathematics self-concept. The survey should take about 5 to 10 minutes.

Thank you for your time!

STUDENT I.D. #: __________-__________-__________

Please place a ✓ in the box or space that indicates your answer.

1. Classification?  □ Freshman  □ Sophomore  □ Junior  □ Senior  □ Other
2. Gender?  □ Female  □ Male
3. Number of Math Classes taken in high school (9th-12th grade)?
   □ 0  □ 1  □ 2  □ 3  □ 4  □ 5
   Which ones:  □ General Math  □ Pre-Algebra  □ Algebra I  □ Algebra II
               □ Geometry  □ Trigonometry  □ Calculus  □ Other Advanced Math
4. Father’s education level?
   □ Some High School  □ High School Graduate  □ Trade or Technical School
   □ Some College  □ College Graduate  □ Some Post Graduate  □ Post Graduate Degree
5. Mother’s education level?
   □ Some High School  □ High School Graduate  □ Trade or Technical School
   □ Some College  □ College Graduate  □ Some Post Graduate  □ Post Graduate Degree
6. I would rank my father’s mathematical ability as:
   □ low  □ slightly below average  □ average  □ slightly above average  □ high
7. I would rank my mother’s mathematical ability as:
   □ low  □ slightly below average  □ average  □ slightly above average  □ high
8. What is your major? __________________________

The following statements are about the study of mathematics. Please read each statement carefully and circle the answer which best describes the way you feel about mathematics. For this research project, please answer statements in regard to mathematics teachers or mathematics classes based on the math teacher or math class most memorable to you.

1. STRONGLY DISAGREE
2. DISAGREE
3. AGREE
4. STRONGLY AGREE

9. I like the easy math problems the best.  1  2  3  4
10. Mathematics is useful for the problems of everyday life.  1  2  3  4
11. My math teacher showed interest in the students.  1  2  3  4
12. I don’t do very well in mathematics.  1  2  3  4
13. My math teacher presented material in a clear way.  1  2  3  4
14. I would like a job which doesn’t use any mathematics.  1  2  3  4
15. There is little need for mathematics in most jobs.  1  2  3  4
16. Mathematics is easy for me.  1  2  3  4
17. My math teacher made math interesting.  1  2  3  4
18. Mathematics is more of a game than it is hard work.  1  2  3  4

PLEASE CONTINUE QUESTIONS ON THE OTHER SIDE
19. My math teacher knew when we were having trouble with our work.
20. Most people should study some mathematics.
21. My math teacher seemed to enjoy teaching mathematics.
22. No matter how hard I try, I cannot understand mathematics.
23. My math teacher was willing to give us individual help.
24. Mathematics is something I enjoy very much.
25. Mathematics is helpful in understanding today's world.
26. I often think, "I can't do it," when a mathematics problem seems hard.
27. My math teacher knew a lot about mathematics.
28. I usually understand what we are talking about in mathematics class.
29. My math teacher didn't mind students asking questions.
30. Mathematics is of great importance to a country's development.
31. My math teacher used problem solving activities in the classroom.
32. It is important to me to understand the work I do in mathematics.
33. My math teacher expected us to be able to communicate mathematically.
34. I enjoy talking to other people about mathematics.
35. It is important to know mathematics in order to get a good job.
36. I am good at working mathematics problems.
37. My math teacher used various methods of instruction based on the range of ways that diverse students learn mathematics.
38. The only reason I'm taking mathematics is because I have to.
39. You can get along perfectly well in everyday life without mathematics.
40. My math teacher did most of the talking.
41. I would rather be given the right answer to a math problem than to work it out myself.
42. My math teacher demonstrated practical applications for mathematics.
43. My math teacher gave us time to explore ideas and problems.
44. If I don't see how to work a mathematical problem right away, I never get it.
45. Most of the ideas in mathematics aren't very useful.
46. My math class was one where students felt comfortable raising questions without fear of being embarrassed.
47. My math teacher encouraged us to work independently.
48. My math teacher encouraged us to work in groups.
49. My math teacher encouraged class participation from every student.
50. I felt dependent on my teacher's explanation of a concept in order to understand well enough to complete an assignment.
51. My math teacher respected students' ideas and answers even if they were incorrect.

THANK YOU AGAIN FOR YOUR ASSISTANCE!
APPENDIX B

SCRIPT FOR INTRODUCING SURVEY TO SUBJECTS
My name is Christie Hawkins, and I am a graduate student working on my PhD in Research and Evaluation here at O.S.U. For my dissertation, I am conducting a research study to determine the effect specific factors have on mathematics self-concept. The majority of the factors deal with characteristics of your most memorable math teacher or math class, but other factors include gender, parental background, and past achievement in mathematics. To measure these factors I am asking each of you to fill out a short survey. Your participation in this study is completely voluntary, and you will in no way be penalized for not participating.

When I hand out the survey, you may notice that your student ID number is requested. This information is essential only so that an office on campus can give me math ACT scores. Student ID numbers will never be reported in any way in the research, nor will they be used for anything other than your math ACT score. Also, when the study is completed, all surveys will be destroyed.

While participation is strictly voluntary, it is extremely important in research that a sample be representative of the entire population. The College of __________ makes up a substantial percentage of the study body at O.S.U., and therefore needs to be represented proportionally in this study. I would greatly appreciate your assistance, and would like to thank you for your time. When you have completed your survey please turn them in on your way out.
APPENDIX C

INSTITUTIONAL REVIEW BOARD APPROVAL FORM
Date: 09-19-97

Proposal Title: PREDICTING MATH SELF-CONCEPT IN COLLEGE FRESHMAN (DOCTORAL DISSERTATION)

Principal Investigator(s): Laura Barnes, Christie Hawkins

Reviewed and Processed as: Expedited

Approval Status Recommended by Reviewer(s): Approved

ALL APPROVALS MAY BE SUBJECT TO REVIEW BY FULL INSTITUTIONAL REVIEW BOARD AT NEXT MEETING, AS WELL AS ARE SUBJECT TO MONITORING AT ANY TIME DURING THE APPROVAL PERIOD.

APPROVAL STATUS PERIOD VALID FOR DATA COLLECTION FOR A ONE CALENDAR YEAR PERIOD AFTER WHICH A CONTINUATION OR RENEWAL REQUEST IS REQUIRED TO BE SUBMITTED FOR BOARD APPROVAL.

ANY MODIFICATIONS TO APPROVED PROJECT MUST ALSO BE SUBMITTED FOR APPROVAL.

Comments, Modifications/Conditions for Approval or Disapproval are as follows:

Approval is granted if the principal investigators agree to procedures listed in #2 in memo dated 9/15/97:

Principal Investigators will give list of student #’s to personnel in the Office of Institutional Research to retrieve ACT scores.

Signature: Date: September 19, 1997
Chair of Institutional Review Board
cc: Christie Hawkins
APPENDIX D

VITA
VITA

Mary Christie Hawkins

Candidate for the Degree of

Doctor of Philosophy

Dissertation: A CAUSAL MODEL OF TEACHER INFLUENCE ON MATHEMATICS SELF-CONCEPT AMONG COLLEGE FRESHMEN

Major Field: Applied Behavioral Studies

Biographical:

Personal Data: Born in Woodward, Oklahoma, August 26, 1965, the daughter of John and Mary Ann Hawkins.

Education: Graduated from Woodward High School, Woodward, Oklahoma, in May, 1983; received Bachelor of Science degree in Secondary Education/Math from Oklahoma State University in 1989; received Master of Science degree in Curriculum and Instruction from Oklahoma State University in 1993; complete requirements for the Doctor of Philosophy degree at Oklahoma State University in December, 1998.

Professional Experience: High School Math Instructor, Perkins, Oklahoma, August, 1989 to present; member Phi Kappa Phi Honor Society, August, 1993.