Developing Content Knowledge of Constant Rate of Change

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Introduction

The goal of this research is to effectively communicate the transition between a well-known tiered model of mental conceptualizations of rate of change previously reported in literature in order to aid students' critical thinking skills and teachers' abilities to support students' learning of this foundational concept (rate of change).

Questions

What conceptions of constant rate of change and slope does a pre-service secondary teacher (Samantha) develop through her participation in a teaching experiment?

What experiences contributed to particular modifications/ reorganizations of Samantha's scheme for constant rate of change and slope?

Methods

Interviews were conducted with a pre-service secondary teacher to assess her rate of change conceptualization when presented problems of varying difficulty.

Results

Teaching Episode 1: *internalized ratio* to *interiorized ratio*

Q: What does it mean to say that Clara rides her bike at a constant speed of 12 miles per hour?

A: Every hour, she goes 12 miles. … You go 12 miles in an hour and then like you're going multiples of 12, I guess, for every hour. So you'd take 12 times one, 12 times two. So it'd be 12 times *x* … where *x* would be your time in hours.

Teaching Episodes 2-5: solidification of *interiorized ratio*

Q: So what if she rides … *d* miles. How long would it take her to ride *d* miles?

A: $d/9^{ths}$ of an hour.

Q: Why? …

A: I want to just say **because they're equivalent fractions**. … I mean, so *d* over nine taking your distance and putting it over nine **that's going to be an equivalent fraction to your fraction of your time**. (*pause*) Because you want them to be the same …

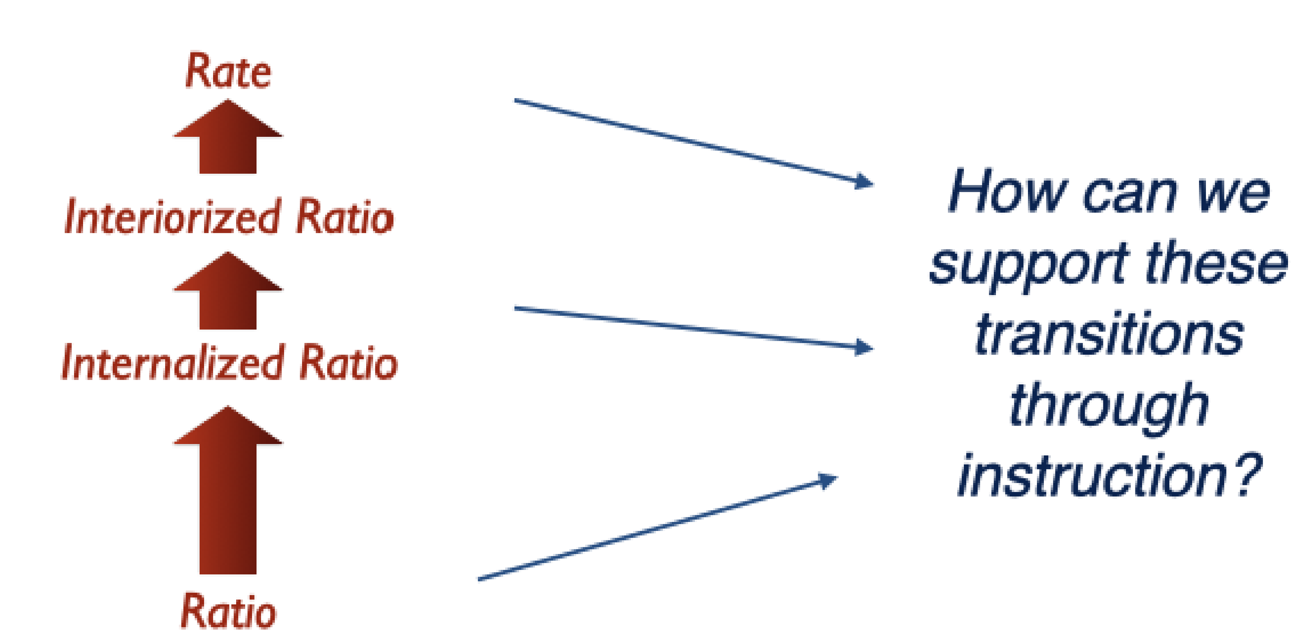
Teaching Episodes 6-8: worked towards constructing conceptualization of *rate*

Q: Can you express your reasoning in the context of "*a* varies at a constant rate of *r* with respect to *b*"?

A: *They would need to understand that it varying at a constant rate means that like* ***any change in a is going to be r times as large as the change in b****. …* ***They have to think the change in a is r times as large as the change in b or change in b is $1/r^{th}$ the change in a****. They have to be able to think about both of those.*

Level
Smooth continuous covariation
Chunky continuous covariation
Coordination of values
Gross coordination of values
Precoordination of values
No coordination

Thompson & Carlson (2017)



Thompson & Thompson (1992)

Conclusions

This illustrates the transition of approaches from a pre-service teacher.This highlights how pedagogical and mathematical content knowledge must be integrated in their preparation. However, it also shows how we must be conscientious of how we present mathematics to students.

Currently, the analysis of these interviews is being used to write an article for submission to a journal. In the future, we will analyze the effects of these "intervention interviews" in the classroom.

Literature cited

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. Compendium for Research in Mathematics Education (pp. 421-456). Reston, VA: National Council of Teachers of Mathematics.

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