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# Bear Beta or Speculative Beta?—Reconciling the Evidence on Downside Risk Premium

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## Abstract

This article develops a new approach to explain why risk factors constructed from index option returns are priced in the stock market. We decompose an option-based factor into three main components and identify the one responsible for the beta–return relationship. Applying this method to the bear risk factor proposed by Lu and Murray reveals that the negative correlation between bear betas and stock returns does not reflect systematic risk premia. Instead, it represents an anomaly closely related to the betting-against-beta puzzle. We trace the root of this anomaly to disagreement concerning the aggregate stock market. Our work reconciles the conflicting evidence concerning downside risk by showing that neither *ex-post* nor *ex-ante* downside risk is priced in the cross-section of stocks while making a methodological contribution that facilitates more accurate interpretation of option-based risk factors in future research.

**Keywords:** Downside risk, Disaster risk, Option return, Low-risk anomaly, Mispricing, Disagreement

**JEL Classification:** G11, G12, G13

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## 1. Introduction

The Capital Asset Pricing Model's (CAPM) inability to explain the cross-section of expected stock returns has motivated numerous studies searching for alternative sources of systematic risk. One strand of such research focuses on market betas measured during market downturns. Early studies such as Roy (1952) and Bawa and Lindenberg (1977) have argued that downside betas should play a significant role in determining asset prices. More recently, Ang, Chen, and Xing (2006) show in a model with loss-averse investors that downside betas should be positively correlated with expected stock returns even in the presence of CAPM betas and provide evidence to support this argument. Lettau, Maggiori, and Weber (2014) further show this extends to multiple asset classes. Both studies focus on the contemporaneous relationship between downside betas and asset returns.

The evidence on positive downside risk premium, however, has been called into question by Levi and Welch (2019) and Barahona, Driessen, and Frehen (2021), whose work convincingly demonstrates that downside betas measured in the portfolio formation period fail to predict downside betas in the holding period, and downside betas do not significantly predict stock returns. The lack of persistence in downside beta seems to put the relevance of downside risk in asset management in doubt. In a similar vein, van Oordt and Zhou (2016) and Kapadia *et al.* (2019) go beyond the basic semi-covariance framework to argue that downside risk is not priced with a positive premium.

Motivated by an asset pricing model with time-varying probabilities of negative jumps, Lu and Murray (2019) (hereafter, LM) argue that the standard CAPM needs to be augmented by a factor that captures the risk of deep market declines. Their empirical findings suggest that the exposure to bear market risk is a strong determinant of expected stock returns. Empirically, LM construct an approximate Arrow–Debreu (A–D) security from SPX options to track the risk-neutral probability of the market index falling below a particular threshold on the downside. They show that stocks with high loadings on the A–D security returns tend to earn low expected returns, consistent with a sizable premium being paid to hedge the probability of an upcoming market crash.[1] From the theoretical standpoint, LM's findings seem to point to the importance of *ex-ante* market downside risk, as opposed to the *ex-post*, or realized, downside risk emphasized by Ang, Chen, and Xing (2006), in explaining expected stock returns. Their findings also seem to be free of the two major issues of Ang, Chen, and Xing's (2006) evidence concerning *ex-post* downside risk premium: First, bear betas predict future stock returns; second, bear betas are statistically persistent over time.

The importance of *ex-ante* tail or disaster risk in asset pricing has been well established in theory. Gabaix (2012) and Wachter (2013) argue that time-varying disaster risk provides a unified explanation for several asset pricing puzzles. Before LM's work, another influential study by Kelly and Jiang (2014) utilizes the cross-sectional of stock returns to estimate the shape of the left tail of the stock return distribution. They show that a stock's exposure to this tail risk is negatively correlated with its expected return, pointing to a premium for hedging *ex-ante* tail risk. Taken together, existing evidence appears to strongly support the notion that exposure to *ex-ante* downside risk plays an important role in shaping the cross-section of expected stock returns.

In this work, we provide a careful assessment of the existing evidence. In particular, we use a decomposition method to show that, although the bear risk factor is meant to capture *ex-ante* downside risk, it in fact involves three major sources of systematic risk. The factor can be decomposed into three components,[2] and loadings on different components are associated with drastically different risk exposures and expected returns. Surprisingly, the decomposition reveals that the low average returns of the high bear beta stocks, which are designed to hedge bear market risk, are in fact earned by a group of stocks that carry not only high overall market risk but also countercyclical market risk. These stocks do not

1 In the following, we refer to the time series of the A–D security returns as the bear risk factor and the loading on the factor as bear beta.

2 To perform a complete decomposition of the bear risk factor, we also need a fourth factor, which we call residual component. We show that this component accounts for a small portion of the variations of the bear risk factor and has no pricing power. In addition, its economic meaning is not as clear as the other components. To save space, we do not emphasize it in our exposition.

hedge against downside risk at all, be it *ex-ante* or *ex-post*. If anything, we find that they are particularly exposed to realized downside risk. Therefore, the bear risk premium is not a compensation for systematic risk and should be viewed as an asset pricing anomaly.

A joint analysis of the bear risk factor and the tail risk factor reveals little evidence that exposures to these factors help select stocks that hedge any form of jump or disaster risk. Therefore, we argue that no existing evidence convincingly supports a rational asset pricing theory where investors pay a premium for stocks that hedge some form of systematic risk on the downside or in the left tail.

The anomaly we discover shares the same spirit as Kapadia *et al.* (2019), but the overlap between the two anomalies is not particularly large, which suggests that negative relationships between risk and reward could arise for various reasons. Empirical evidence on negative risk–reward relationships is important to our understanding of the limitations of rational expectations theory. The most widely studied example is perhaps the betting-against-beta (BaB) puzzle popularized by Frazzini and Pedersen (2014). We demonstrate that the component of the bear risk factor that is responsible for its pricing power actually generates a deepened version of the BaB puzzle. Controlling for the BaB factor does not resolve this puzzle, nor do several existing resolutions for the BaB puzzle. Our analysis suggests that the aggregate disagreement channel proposed by Hong and Sraer (2016) provides the best fit for the current anomaly. In particular, our regression models in Section 8.1 imply that if there were no aggregate disagreement in the market, then the anomaly would have changed sign and become consistent with canonical asset pricing theory; hence, the puzzle can be resolved.

To dissect the bear risk factor, our empirical design starts from the fact that option returns are driven by changes both in the underlying asset price and implied volatilities (IVs). Therefore, risk factors constructed from index option returns can be decomposed into innovations driven by these two forces. More specifically, the bear risk factor tracks the probability of the market index falling below a predetermined threshold, and this probability changes for two main reasons.

First, when the index level falls (rises), the likelihood of it falling below the threshold by the option expiration day increases (decreases), assuming that the market index return distribution remains unchanged. In addition, due to the convex relationship between option prices and the underlying price, the impact of a market loss is larger than the impact of a market gain of the same magnitude, leading to an asymmetric exposure of the factor return to the market return. Despite the different empirical implementations, this type of variation is conceptually close to the *ex-post* downside risk studied by Ang, Chen, and Xing (2006).

Second, holding the index level constant, the prospect of entering a bear market could change when the index return distribution changes. Since the distribution of the underlying asset return enters its option prices via IVs, this component can be captured by fixing the index level and only allowing the IVs to drive the bear risk factor. This component reflects changes in *ex-ante* downside risk.[3]

When calculating an approximate A–D security return,[4] we re-calculate the option prices at the end of each return period using the Black and Scholes (1973) formula by letting

3 In Section 1 of the Supplementary Appendix of this paper, we also design bear risk factors that are only exposed to *ex ante* downside risk even without decomposition and arrive at similar conclusions based on those factors.

4 An A–D security is approximated by a bearish spread that involves buying an OTM put and writing a further OTM one.

the index level stay constant and the IVs take their actual values at the end of the period. The A–D security returns based on such fictitious ending option prices constitute the "IV-driven bear risk factor," representing changes in the probability of entering a bear market due to changing index return distribution. The other component is calculated in a similar manner but with the IVs fixed and the index level allowed to change, and it is called the "index-driven bear risk factor."

Although the index-driven factor is highly correlated with the index return itself, they differ in two important aspects. First, the factor is a convex function of the index return, inheriting the convexity of the option pricing function. Second, the factor return, just like any option return, is a scaled version of the index return. Consider an at-the-money (ATM) put option whose delta is fixed at $-0.5$. As its IV increases, the same market return leads to option returns of smaller magnitudes. To separate these two effects, the index-driven component is further decomposed into a "linear" component and a "non-linear" component by separating the first-order term from the higher-order ones in the Taylor series. The linear component models the scaling effect, and the nonlinear one models the convexity.

We show that the linear component of the index-driven bear risk factor is of particular importance. As explained in detail in Section 4.3, CAPM augmented by this linear component amounts to a time-varying market beta model, where stocks with high loadings on the linear component tend to carry high systematic risk when the option-implied market volatility is high. To the extent that IVs tend to increase as the market declines, these stocks can be particularly sensitive to market downturns. Consistent with this intuition, we find that high linear-beta stocks tend to carry high market risk when the VIX index is high, and they have performed particularly poorly during the market downturns in our sample. Importantly, it turns out that these high linear-beta stocks earn low average returns and account for the majority of the bear risk premium, a situation clearly at odds with most asset pricing models populated by rational agents. This finding shows that the bear risk premium is not a compensation for bear market or downside risk; instead, it reveals a striking high-risk-low-return anomaly.

In contrast to the linear factor, loadings on the IV-driven factor and the nonlinear index-driven factor do not significantly predict stock returns, which rules out two potential explanations of the bear risk premium: First, the IV-driven component's lack of pricing power suggests that changes in the probability of a future market downturn that is solely caused by the changes in the index return distribution are not highly priced. In particular, *ex-ante* downside risk exposure is not price. Second, the nonlinear index-driven component's lack of pricing power confirms that the premium is not a compensation for protection against realized downside risk. Regarding the second point, the nonlinear component is highly convex in the underlying index return; therefore, stocks with high and positive loadings on this factor can be expected to have relatively low downside betas. As we show in detail in Section 4.2, the nonlinear component indeed generates such favorable asymmetric market betas, both in the portfolio formation period and, to a lesser degree, in the holding period. However, we find no evidence that this factor is priced in the stock market, which further confirms that *ex-post* downside risk is not priced in the cross-section of stocks.

Our factor decomposition is similar to the one adopted by Israelov and Kelly (2017), who focus on improving forecasts of option prices. It is also closely related to the research of Cremers, Halling, and Weinbaum (2015), who develop tradable option factors for gamma and vega risks. In our decomposition, the components related to these risks are not tradable. We argue that there is a tradeoff in choosing between tradable and nontradable factors. On the one hand, our decomposition method is more flexible and produces factors that are conceptually clean and allow us to reach non-ambiguous conclusions regarding the source of the pricing power. On the other hand, we would not be able to compare the risk premia earned in the equity market and the options market. However, since our focus here is to understand the relationship between factor loadings and expected stock returns, and the evidence clearly shows that the relationship does not reflect a systematic risk premium, it would make little sense to compare the "risk premia" measured from the two markets. Our research points to the importance of correctly understanding a beta–return relationship before questioning the consistency of pricing across different markets. If we cannot be sure of the source of the systematic risk embedded in the factor that causes its pricing power, then comparing a cross-sectional risk premium and its factor counterpart would not be a meaningful exercise.

Although we do not prioritize tradability in our factor design, it is important to point out that the component, linear index-driven factor, that actually accounts for all the pricing power of the bear risk factor is the only tradable component in the decomposition. It amounts to investing in the market index with a time-varying weight that increases with the implied market volatility—the opposite of the volatility-managed portfolio proposed by Moreira and Muir (2017).

More generally, our analysis demonstrates that, although certain option-based factors carry clear economic intuition on their own, using them as asset pricing factors to explain expected stock returns is far from straightforward. Failing to recognize the exact source of risk being priced could lead to erroneous interpretations that represent the polar opposite of the truth. Recently, forceful strides have been made in the asset pricing research on the front of correctly assessing the statistical significance of empirical findings (see Harvey, Liu, Zhu, 2015 and Chordia, Goyal, and Saretto, 2020). Our work serves as a reminder that correctly interpreting a statistically significant result is just as important to asset pricing research.

While our main objective is to provide a better understanding of the evidence on the downside risk premium, we also add new evidence to the growing literature on the low-risk anomalies, where prominent examples include the BaB anomaly and idiosyncratic risk anomaly. Both market beta and idiosyncratic risk are salient features of stocks, and can easily attract investors seeking leverage or lottery-like stocks and cause mispricing. It is interesting to find that a subtle feature such as the loading on the linear component has even stronger pricing power than CAPM beta. It appears that some investors fail to understand the risk dynamics of certain stocks and are surprised by their poor performance during market downturns.

The rest of the article is organized as follows. Section 2 reviews related literature. Section 3 describes data sources and introduces the factor decomposition. Section 4 identifies the component of the bear risk factor that is responsible for its pricing power. Sections 5 and 6 provide detailed analyses of the risk exposures of the portfolios sorted on the bear risk factor and its components. Section 7 analyzes bear beta portfolio's ability to hedge jump risks. Section 8 examines whether existing resolutions of the BaB puzzle help explain the current anomaly. Section 9 provides further discussions and robustness tests. Section 10 concludes.

## 2. Related Literature

Our article contributes to the broad literature on asset pricing factors. In particular, a growing group of studies propose factors constructed from option prices. Ang *et al.* (2006) show that betas with respect to changes in the VIX index negatively predict stock returns. Barras and Malkhozov (2016) provide an interesting comparison between the volatility risk premium measured in the stock market and that in the options market. Barinov (2018) argues that volatility risk exposure can help explain the low returns of lottery-like stocks. Jones (2006) uses the exposure to volatility risk and jump risk to explain the average returns of index options. Chang, Christoffersen, and Jacobs (2013) show that changes in risk-neutral skewness are also priced in the stock market. Cremers, Halling, and Weinbaum (2015) construct a jump risk factor by taking long–short positions in a pair of straddles to offset their vega risks and show that this factor is priced in the stock market. Most closely related to our work is Lu and Murray (2019), who propose a bear risk factor and find that bear betas negatively predict stock returns. Option-based factors are not only important in understanding asset pricing theories, but also in evaluating hedge fund performances, as hedge fund returns tend to be option-like. Glosten and Jagannathan (1994) are among the first to introduce this approach. Fung and Hsieh (2015) suggest using look-back straddle returns as the benchmark to evaluate the performances of trend followers. Agarwal, Arisoy, and Naik (2017) propose a volatility-of-volatility factor and show that the exposure to it is an important determinant of hedge fund returns.

Risk-neutral distributions extracted from option prices are also widely studied outside the framework of asset pricing factors. Examples of methodological contributions include Bakshi and Madan (2000), Bakshi, Kapadia, and Madan (2003), Carr and Wu (2008), Chang *et al.* (2011), and Buss and Vilkov (2012). Risk-neutral moments are often shown to be good predictors of aggregate market returns. Bollerslev, Tauchen, and Zhou (2009) show that the variance risk premium positively predicts market returns. Bollerslev, Todorov, and Xu (2015) and Andersen, Fusari, and Todorov (2015) show that the tail risk premium predicts market returns. Fan, Xiao, and Zhou (2020) show that the skewness and kurtosis risk premia predict market returns. Bakshi, Panayotov, and Skoulakis (2011) show that the forward variances predict market returns and economic activities.

Risk-neutral moments are also strong predictors in the cross-section. Goyal and Saretto (2009) show the difference between the IV and historical volatility of a stock negatively predict straddle returns. An *et al.* (2014) and Cao *et al.* (2021) show predictive power on stock returns and bond returns, respectively.

Our work is also closely related to another strand of research on asset pricing factors that develops variants of CAPM in an attempt to restore the theoretically appealing beta–return relationship. In particular, Roy (1952) and Bawa and Lindenberg (1977) provide an early theoretical framework to emphasize the importance of downside risk. Ang, Chen, and Xing (2006) examine the relationship between downside betas and average stock returns and argue that downside betas are associated with a positive risk premium. Lettau, Maggiori, and Weber (2014) document the downside risk premium in multiple asset classes. However, Levi and Welch (2019) and Barahona, Driessen, and Frehen (2021) show that semicovariance-based downside betas are not persistent; therefore, stocks with low past downside betas do not perform particularly well during subsequent market downturns. This transient nature of downside betas makes it questionable whether the contemporaneous relationship between downside betas and average returns can be interpreted as a risk premium. van Oordt and Zhou (2016) and Kapadia *et al.* (2019) construct alternative downside risk measures and show that the downside risk premium is close to zero and negative, respectively.

Our article develops a decomposition of a risk factor in order to reach a deeper understanding of the empirically observed beta–return relationship. This approach is inspired by previous studies such as that of Campbell and Vuolteenaho (2004), which decomposes the market return into a cash-flow component and a discount-rate component. In options research, Israelov and Kelly (2017) adopt a similar decomposition to our first-tier one to better forecast option price distributions. Schneider (2019) provides a decomposition to better understand what drives the aggregate market returns and finds downside risk to be particularly important.

We also contribute to the large literature on low-risk anomalies. Well-known examples in this category include the idiosyncratic volatility puzzle in Ang *et al.* (2006) and the BaB anomaly in Frazzini and Pedersen (2014). Schneider, Wagner, and Zechner (2020) argue that these anomalies share the same root of coskewness and can be explained by the *ex-ante* skewness factors.

## 3. Data Sources and Factor Construction

### 3.1 Data Sources

The information on SPX options from January 1996 to December 2017 is from OptionMetrics. Because we require 12 months of returns in each regression, the option factor betas are available from December 1996 to December 2017. Stock prices, returns, and trading volumes are from The Center for Research in Security Prices. Information on firm fundamentals is from Compustat. The asset pricing factors in the Fama–French–Carhart model are downloaded from Kenneth French's website. We also obtained several asset pricing factors directly from the authors who proposed the factors. These include the risk-neutral skewness factor in Chang, Christoffersen, and Jacobs (2013) provided by Peter Christofferson, the tail risk factor in Kelly and Jiang (2014) provided by Bryan Kelly, the BaB factor in Frazzini and Pedersen (2014) provided by Andrea Frazzini, the jump risk factor in Cremers, Halling, and Weinbaum (2015) provided by David Weinbaum, the MAX factor in Bali *et al.* (2017) provided by Turan Bali, the safe-minus-risky (SMR) factor in Kapadia *et al.* (2019) provided by Nishad Kapadia, and the *ex-ante* skewness factors in Schneider, Wagner, and Zechner (2020) provided by Paul Schneider. We obtained the dividend yields and credit spreads from Amit Goyal's website, and these data are developed by the authors for Welch and Goyal (2007). We thank these authors for generously sharing their data.

### 3.2 The Bear Risk Factor

We construct the bear risk factor following the method of LM. To approximate an Arrow–Debreu security that captures the probability of the S&P 500 index falling below a predetermined threshold, a bearish spread is constructed at the end of each trading day as follows. It includes long positions in 1-month SPX put options whose strike prices fall between 0.75 and 1.25 standard deviations (SDs) below the fair price of an S&P 500 futures contract expiring on the same day as the options. The fair price of the futures contract is given by $S_t \cdot e^{(r-y)\cdot(T-t)}$, where $S_t$ is the S&P 500 index level, $r$ and $y$ are the annualized interest rate and dividend yield of the index, and $T-t$ is the time to expiration expressed as a fraction of a year. The SD of index returns is calculated using the VIX index as $VIX_t/100 \cdot \sqrt{T-t}$. When multiple available strike prices fall into the range, the options are weighted by their trading volume on the day. In a similar manner, the bearish spread also involves short positions in SPX puts whose strike prices fall between 1.5 and 1.75 SDs below the fair price of the S&P 500 futures. Each bearish spread is held for five trading days, leading to overlapping 5-day returns at the daily frequency.

## 3.3 Factor Decomposition

We first describe how to decompose the return of a single put option and then extend the method to decompose bearish spread returns.

### 3.3.a *Put options*

We begin by decomposing the return of a put option into an index-driven component, $r_{\text{Put,Index}}$, and an IV-driven one, $r_{\text{Put,IV}}$. A 5-day return of the option can be written as:

$$r_{Put} = \frac{P_{t+5}}{P_t} - 1 = \frac{BS(S_{t+5}, K, IV_{t+5}, r_{t+5}, y_{t+5}, T-t-5)}{BS(S_t, K, IV_t, r_t, y_t, T-t)} - 1$$

where $P$ denotes the put option prices, BS$(\cdot)$ refers to the Black–Scholes formula, S denotes the index levels, $K$ is the strike price, IV denotes the Black–Scholes IVs, $r$ denotes the risk-free rates, $y$ denotes the dividend yields, and $T$ represents the expiration day.

The index-driven component is obtained by replacing $IV_{t+5}$ with $IV_t$ in the ending price $P_{t+5}$, that is,

$$r_{\text{Put,Index}} = \frac{\text{BS}(S_{t+5}, K, \mathbf{IV_t}, r_{t+5}, y_{t+5}, T-t-5)}{\text{BS}(S_t, K, IV_t, r_t, y_t, T-t)} - 1.$$

One can certainly set the values of all the other inputs to their starting values and only let the index level drive the return. But our analysis shows that doing so has little impact on the results.

Similarly, the IV-driven component is obtained by keeping the index level constant, that is,

$$r_{\text{Put,IV}} = \frac{\text{BS}(\mathbf{S_t}, K, IV_{t+5}, r_{t+5}, y_{t+5}, T-t-5)}{\text{BS}(S_t, K, IV_t, r_t, y_t, T-t)} - 1$$

Because $r_{\text{Put,Index}}$ and $r_{\text{Put,IV}}$ do not form a complete decomposition of $r_{\text{Put}}$, we denote the residual variation by $r_{\text{Put,Residual}}$, which is equal to $r_{\text{Put}} - r_{\text{Put,Index}} - r_{\text{Put,IV}}$. This component captures the contribution from the interaction between the changes in the index level and IV.

Next, we further decompose $r_{\text{Put,Index}}$ into a linear component, $r_{\text{Put},L}$, and a nonlinear one, $\text{r}_{\text{Put,NL}}$. The linear component is given by $r_{\text{Put},L} = \Delta_t \cdot (S_{t+5} - S_t)/P_t$, where $\Delta_t$ is the put option delta at the end of Day $t$. The nonlinear component is defined as the difference between $r_{\text{Put,Index}}$ and $r_{\text{Put},L}$. In this decomposition, both $r_{\text{Put},L}$ and $r_{\text{Put,NL}}$ are only driven by the contemporaneous index return, and $r_{\text{Put,NL}}$ is responsible for all the convexity.

### 3.3.b *Tradability*

In the four-way decomposition of the tradable factor $r_{\text{Put}}$ into $r_{\text{Put,IV}}$, $r_{\text{Put,L}}$, $r_{\text{Put,NL}}$, and $r_{\text{Put,Residual}}$, the only tradable component is $r_{\text{Put,L}}$, because it is a scaled index return, and the scaling factor is known at time $t$. As we show in Section 4.3, investing in $r_{Put,L}$ is equivalent to scaling up the weight in the market index when the implied market volatility goes up.

### 3.3.c *Bearish spreads*

The above decompositions can be applied to bearish spread returns. Since the price of a bearish spread is the weighted sum of several put option prices, its return from $t$ to $t+5$ can be written as: $r_{\text{Bear}} = \frac{\sum_i w_i P_{i,t+5}}{\sum_i w_i P_{i,t}} - 1$, where w's are the weights based on trading volume. The index-driven component, $r_{\text{Bear,Index}}$, is obtained by replacing each $P_{i,t+5}$ with $\text{BS}(S_{t+5}, K_i, \mathbf{IV_t}, r_{t+5}, y_{t+5}, T-t-5)$. The IV-driven component, $r_{\text{Bear,IV}}$, is obtained by replacing $P_{i,t+5}$ with $\text{BS}(\mathbf{S_t}, K_i, \text{IV}_{t+5}, r_{t+5}, y_{t+5}, T-t-5)$. The residual component $r_{\text{Bear,Residual}} = r_{\text{Bear}} - r_{\text{Bear,IV}} - r_{\text{Bear,Index}}$.

The linear component, $r_{\text{Bear,L}}$, is defined as $r_{\text{Bear,L}} = \frac{\sum_i w_i \Delta_{i,t}(S_{t+5} - S_t)}{\sum_i w_i P_{i,t}}$ and the nonlinear component, $r_{\text{Bear,NL}}$, is the difference between $r_{\text{Bear,Index}}$ and $r_{\text{Bear,L}}$.

### 3.3.d *Summary statistics*

The summary statistics of the option-based factors are reported in Table I, Panel A. The top half of the panel covers the bear risk factor and its components, and the bottom half covers ATM SPX put option returns and their components. Both the bearish spreads and ATM SPX put options have low average returns at −9.13% and −7.21% per five trading days, which reflects investors' willingness to pay a premium to hedge against market downturns or changing investment opportunities. The second row shows that the IV-driven component only accounts for a small portion of the variations in the full bear factor, but its contribution to the average return is important. Rows 3–5 show that the index-driven component derives most of its average return and volatility from its linear component. The sixth row shows that the residual component has the lowest contribution to the variation in the bear risk factor. The next six rows show similar patterns for put returns and their components.

In the regression models that calculate the loadings on these factors, the factors are first divided by their SDs and then multiplied by the SD of the market return in order to be comparable to the market factor itself.

The upper half of Panel B reports the correlation coefficients between the bear factor, its four components, and the market portfolio return, denoted by $r_m$. The correlation between the bear factor return and market return is fairly strong at −0.804, which is mainly accounted for by the index-driven component, especially its linear component. The second column shows that the full bear risk factor is highly correlated with its linear index-driven component, while the correlations with the other two components are much weaker. The lower half of the panel shows the same pattern for put returns and their components.

Together, Panels A and B show that the linear components are the most important source of variations in the bear factor and the put factor, but they do not account for a proportionally large part of the average returns.

Panel C reports the coefficients of regressing each factor return on the market portfolio return and its square. This is done to investigate the convexity of the relationships between the option-based factors and the market factor. In the upper half of the panel, the first column shows that the bear factor is decreasing and convex in the market return, and the next two columns show that these properties are shared by both the index-driven and IV-driven components. The further decomposition of the index-driven component shows that its

**Table I.** Summary statistics

This table reports the summary statistics and other basic properties of the option-based factors studied in the article. The main factors are the bear risk factor, $r_{\text{Bear}}$, and the put factor, $r_{\text{Put}}$. The former consists of returns on bearish spreads formed using SPX put options, and the latter consists of returns on ATM SPX put options. Subscripts IV, Index, L, NL, and Residual indicate the IV-driven, index-driven, linear, nonlinear, and residual components of the full factor, respectively. Panel A reports averages, SDs, and the 25th, 50th, and 75th percentiles of the factor returns. The values are reported in percentage term. Panel B reports the Pearson correlation coefficients between the factors and their correlations with the market returns. Panel C reports the coefficients of regressing the factor returns on the market return and its square. In the parentheses are Newey and West (1987) $t$-statistics with four lags.

Panel A: Option-based risk factors

| Factors | Mean | SD | 25th Pct. | Median | 75th Pct. |
|---|---|---|---|---|---|
| $r_{\text{Bear}}$ | −9.13 | 74.41 | −56.59 | −28.78 | 14.99 |
| $r_{\text{Bear,IV}}$ | −14.66 | 15.89 | −24.02 | −15.33 | −6.02 |
| $r_{\text{Bear,Index}}$ | −4.37 | 72.99 | −49.91 | −22.36 | 20.34 |
| $r_{\text{Bear},L}$ | −4.42 | 59.22 | −44.09 | −8.97 | 30.24 |
| $r_{\text{Bear,NL}}$ | 0.03 | 27.88 | −11.37 | −6.13 | 3.91 |
| $r_{\text{Bear,Residual}}$ | 9.9 | 9.79 | 5.66 | 8.63 | 13.09 |
| $r_{\text{Put}}$ | −7.21 | 58.58 | −47.25 | −21.21 | 18.39 |
| $r_{\text{Put,IV}}$ | −8.5 | 10.68 | −14.32 | −9.1 | −3.58 |
| $r_{\text{Put,Index}}$ | −4.96 | 56.22 | −43.43 | −17.35 | 20.25 |
| $r_{\text{Put},L}$ | −4.36 | 51.2 | −39.21 | −8.17 | 26.35 |
| $r_{\text{Put,NL}}$ | −0.6 | 14.42 | −7.73 | −4.22 | 2.47 |
| $r_{\text{Put,Residual}}$ | 6.26 | 5.48 | 3.85 | 5.31 | 7.8 |

Panel B: Factor return correlations

| | $r_m$ | $r_{\text{Bear}}$ | $r_{\text{Bear,IV}}$ | $r_{\text{Bear,Index}}$ | $r_{\text{Bear},L}$ | $r_{\text{Bear,NL}}$ |
|---|---|---|---|---|---|---|
| $r_{\text{Bear}}$ | −0.804 | | – | – | – | – |
| $r_{\text{Bear,IV}}$ | −0.182 | 0.415 | – | – | – | – |
| $r_{\text{Bear,Index}}$ | −0.819 | 0.983 | 0.274 | – | – | – |
| $r_{\text{Bear},L}$ | −0.888 | 0.910 | 0.181 | 0.932 | – | – |
| $r_{\text{Bear,NL}}$ | −0.244 | 0.625 | 0.334 | 0.624 | 0.297 | – |
| $r_{\text{Bear,Residual}}$ | 0.295 | −0.424 | −0.561 | −0.440 | −0.328 | −0.454 |

| | $r_m$ | $r_{\text{Put}}$ | $r_{\text{Put,IV}}$ | $r_{\text{Put,Index}}$ | $r_{\text{Put,L}}$ | $r_{\text{Put,NL}}$ |
|---|---|---|---|---|---|---|
| $r_{\text{Put}}$ | −0.852 | | – | – | – | – |
| $r_{\text{Put,IV}}$ | −0.219 | 0.427 | – | – | – | – |
| $r_{\text{Put,Index}}$ | −0.861 | 0.991 | 0.322 | – | – | – |
| $r_{\text{Put,L}}$ | −0.889 | 0.952 | 0.226 | 0.968 | – | – |
| $r_{\text{Put,NL}}$ | −0.180 | 0.463 | 0.444 | 0.441 | 0.201 | – |
| $r_{\text{Put,Resicual}}$ | 0.162 | −0.335 | −0.720 | −0.306 | −0.203 | −0.465 |

Panel C: Factor returns regressed on market returns

| | $r_{\text{Bear}}$ | $r_{\text{Bear,IV}}$ | $r_{\text{Bear,Index}}$ | $r_{\text{Bear},L}$ | $r_{\text{Bear,NL}}$ | $r_{\text{Bear,Residual}}$ |
|---|---|---|---|---|---|---|
| Intercept | −0.10 | −0.14 | −0.06 | −0.01 | −0.05 | 0.10 |
| | (−7.81) | (−44.81) | (−4.65) | (−1.45) | (−6.28) | (54.3) |

(continued)

**Table I.** Continued

Panel C: Factor returns regressed on market returns

| | $r_{\text{Bear}}$ | $r_{\text{Bear,IV}}$ | $r_{\text{Bear,Index}}$ | $r_{\text{Bear},L}$ | $r_{\text{Bear,NL}}$ | $r_{\text{Bear,Residual}}$ |
|---|---|---|---|---|---|---|
| $r_m$ | −23.14 | −1.14 | −23.10 | −21.18 | −1.93 | 1.10 |
| | (−30.96) | (−7.81) | (−30.69) | (−34.86) | (−4.29) | (8.71) |
| $r_m^2$ | 85.12 | 5.48 | 85.86 | 3.09 | 82.68 | −6.11 |
| | (3.52) | (2.73) | (3.56) | (0.17) | (6.47) | (−2.81) |
| $R^2$ | 0.682 | 0.036 | 0.708 | 0.788 | 0.296 | 0.097 |
| | $r_{\text{Put}}$ | $r_{\text{Put,IV}}$ | $r_{\text{Put,Index}}$ | $r_{\text{Put},L}$ | $r_{\text{Put,NL}}$ | $r_{\text{Put,Residual}}$ |
| Intercept | −0.07 | −0.09 | −0.05 | −0.01 | −0.03 | 0.06 |
| | (−6.62) | (−37.25) | (−4.73) | (−1.69) | (−7.76) | (50.97) |
| $r_m$ | −19.80 | −0.91 | −19.22 | −18.51 | −0.71 | 0.33 |
| | (−34.61) | (−8.67) | (−34.94) | (−35.79) | (−3.1) | (5.81) |
| $r_m^2$ | 54.19 | 8.01 | 50.46 | 2.59 | 47.87 | −4.28 |
| | (2.8) | (3.42) | (2.73) | (0.16) | (6.61) | (−3.49) |
| $R^2$ | 0.747 | 0.062 | 0.761 | 0.790 | 0.307 | 0.042 |

convexity comes from the nonlinear component. The lower half of the panel shows the same patterns for put returns.

## 4. Tracing the Source of the Bear Factor's Predictive Power

LM show that bear betas negatively predict stock returns and interpret this relationship as evidence supporting the prediction of a theoretical model, where rational and risk-averse investors pay a premium to hedge against bear market risk. Their argument, although very compelling, is not conclusive because the bear risk factor is a convolution of several sources of variations, and the risk premium-based interpretation implies that loadings on some of these variations should be primarily responsible for the beta–return relationship. We examine this issue using the decompositions introduced in Section 3.3.

### 4.1 Index-Driven Component versus IV-Driven Component

We first replicate the main results of LM regarding the bear risk factor and then re-evaluate the results after replacing the bear risk factor with either its IV-driven component or index-driven component. The IV-driven component captures time variations in the risk-neutral distribution of the S&P 500 index returns. Therefore, loadings on this component potentially reflect the stocks' ability to hedge against such variations. If the bear risk premium can be largely attributed to this component, then it is consistent with investors paying a premium to hedge changing investment opportunities. On the other hand, if the index-driven component is priced, then we must consider alternative explanations. Considering that the index-driven component is a convex function of the index return, one possibility is that stocks with high loadings on this component provide a good hedge against realized downside market risk.

To form bear beta portfolios, we sort stocks into deciles based on their bear betas at the end of each month. The bear beta of a stock is calculated by running the following two-factor[5] regression model using daily observations of overlapping 5-day returns from the prior 12 months:

$$r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Bear},i} \cdot r_{\text{Bear}} + e_i \quad (1)$$

where $r_i$ is the stock return, $r_m$ is the market portfolio return and $r_{\text{Bear}}$ is the bear factor return. The bear factor is scaled to have the same SD as the market factor, and the same treatment applies to all the option-based factors throughout the paper. We require at least 180 daily observations in each regression and also apply the Bayes shrinkage method to reduce estimation errors. This part of the analysis is meant to replicate the findings of LM, so we follow their empirical methods closely.

The average monthly returns and Fama–French–Carhart alphas of the value-weighted portfolios are reported in Table II, Panel A. The first two rows are for the 1-month holding period, and the next two are for the 12-month holding period. All the average returns are reported as percentages per month. For the 12-month holding period, we construct twelve sets of decile portfolios at the end of Month $t$ using betas measured in Months $t-12$ to $t-1$, $t-13$ to $t-2$, etc., and then take the average of the twelve portfolio returns in Month $t+1$ for each decile. The resulting portfolio returns are not overlapping. This approach has been adopted by previous studies such as Jegadeesh and Titman (1993).

Column high-minus-low (H−L) reports the returns of the H−L portfolio, and the last column shows their $t$-statistics. The first two rows show that the high bear beta portfolio underperforms the low bear beta portfolio by 0.99% per month, and the difference in alphas is 1.23% per month. The spread in returns diminishes over the 12-month holding period but remains significant.

After confirming LM's main finding, we turn to the first decomposition. The bear factor in Equation (1) is replaced by one of its components at a time,[6] which leads to

$$r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Bear,Index},i} \cdot r_{\text{Bear,Index}} + e_i \quad (2)$$

$$r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Bear,IV},i} \cdot r_{\text{Bear,IV}} + e_i \quad (3)$$

$$r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Bear,Residual},i} \cdot r_{\text{Bear,Residual}} + e_i \quad (4)$$

The results for the index-driven factor are reported in Table II, Panel B and those for the IV-driven factor in Panel C. The index-driven factor delivers nearly identical results as the bear factor itself, while betas with respect to the IV-driven factor do not significantly predict stock returns.[7] In addition, Panel F confirms that the residual component has no pricing power. The lack of abnormal returns associated with the IV-driven component suggests that an Intertemporal Capital Asset Pricing Model-like model is unlikely to be the

5 It is necessary to include the market factor in the regression because the bear risk factor is highly correlated with the market factor and using the bear factor by itself leads to similar results as the CAPM. As we discuss our factor decompositioin, the intuition will become clear as to what the option-based factor brings into the model.

6 We consider models with more than two factors in Section 9.2. Doing so does not change our conclusions.

7 The result for the IV-driven component may seem at odds with existing findings on the volatility risk premium. Part of the difference can be explained by the different empirical settings adopted here. We provide some reconciliation with the existing studies in Section 9.2

**Table II.** Monthly returns of bear beta portfolios

This table reports the average monthly returns ($\overline{R}$) and Fama–French–Carhart four-factor alphas ($\alpha$) of the value-weighted decile portfolios sorted on the loadings on the bear risk factor (Panel A) or on one of its components (Panels B–F). The sample period is from January 1996 to December 2017. All the values are reported as percentages per month. At the end of each month, the factor loadings of a stock are calculated by running two-factor regression models using the prior 12 months' 5-day returns observed at the daily frequency. The two factors in each model are the market factor and one of the option-based factors. The Bayesian shrinkage method is then applied to reduce the errors in the estimated loadings. Stocks are sorted into decile portfolios based on their loadings on the option-based factor, and each portfolio is held for either one or 12 months. Returns of the 12-month holding period do not overlap. At the end of Month $t$, we form twelve sets of decile portfolios using the loadings calculated from $t-11$ to $t$, $t-12$ to $t-1$, $t-13$ to $t-2$, etc., respectively. This leads to twelve returns in Month $t+1$ for each portfolio, which are averaged to produce the Month $t+1$ return for the portfolio. Column "H−L" contains the H−L portfolio returns formed by buying Portfolio 10 and shorting Portfolio 1, and Column "$t$-stat" contains their Newey–West $t$-statistics with one lag.

| Holding period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | H−L | $t$-Stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Bear beta portfolios | | | | | | | | | | | | | |
| 1 month | $\overline{R}$ | 1.27 | 1.10 | 0.84 | 0.77 | 0.82 | 0.69 | 0.68 | 0.60 | 0.63 | 0.28 | −0.99 | −2.31 |
| | $\alpha$ | 0.49 | 0.37 | 0.15 | 0.02 | 0.05 | −0.11 | −0.13 | −0.27 | −0.35 | −0.74 | −1.23 | −3.60 |
| 12 months | $\overline{R}$ | 1.03 | 0.91 | 0.80 | 0.75 | 0.70 | 0.66 | 0.65 | 0.61 | 0.50 | 0.24 | −0.78 | −2.33 |
| | $\alpha$ | 0.24 | 0.20 | 0.14 | 0.10 | 0.06 | 0.00 | −0.05 | −0.13 | −0.29 | −0.60 | −0.84 | −3.27 |
| Panel B: Index-driven bear beta portfolios | | | | | | | | | | | | | |
| 1 month | $\overline{R}$ | 1.11 | 1.10 | 0.83 | 0.82 | 0.73 | 0.69 | 0.69 | 0.74 | 0.68 | 0.17 | −0.95 | −2.14 |
| | $\alpha$ | 0.31 | 0.38 | 0.09 | 0.10 | −0.04 | −0.06 | −0.13 | −0.17 | −0.23 | −0.84 | −1.15 | −3.15 |
| 12 months | $\overline{R}$ | 1.04 | 0.93 | 0.80 | 0.73 | 0.68 | 0.66 | 0.65 | 0.55 | 0.45 | 0.28 | −0.77 | −2.13 |
| | $\alpha$ | 0.26 | 0.22 | 0.13 | 0.10 | 0.03 | 0.00 | −0.03 | −0.20 | −0.34 | −0.58 | −0.84 | −2.99 |
| Panel C: IV-driven bear beta portfolios | | | | | | | | | | | | | |
| 1 month | $\overline{R}$ | 0.86 | 0.84 | 0.76 | 0.81 | 0.73 | 0.67 | 0.65 | 0.94 | 0.89 | 0.70 | −0.16 | −0.55 |
| | $\alpha$ | 0.03 | 0.04 | 0.02 | 0.12 | −0.01 | −0.14 | −0.11 | 0.15 | 0.02 | −0.29 | −0.31 | −1.09 |
| 12 months | $\overline{R}$ | 0.55 | 0.62 | 0.68 | 0.75 | 0.70 | 0.73 | 0.78 | 0.76 | 0.87 | 0.76 | 0.21 | 1.09 |
| | $\alpha$ | −0.29 | −0.11 | −0.01 | 0.07 | 0.05 | 0.05 | 0.11 | 0.02 | 0.09 | −0.10 | 0.19 | 0.98 |
| Panel D: Linear index-driven bear beta portfolios | | | | | | | | | | | | | |
| 1 month | $\overline{R}$ | 1.14 | 0.91 | 0.86 | 0.75 | 0.68 | 0.69 | 0.69 | 0.71 | 0.56 | 0.30 | −0.83 | −1.64 |
| | $\alpha$ | 0.40 | 0.15 | 0.16 | −0.01 | −0.01 | −0.12 | −0.11 | −0.16 | −0.39 | −0.76 | −1.16 | −3.13 |
| 12 months | $\overline{R}$ | 1.09 | 0.91 | 0.80 | 0.72 | 0.72 | 0.57 | 0.61 | 0.59 | 0.65 | 0.43 | −0.66 | −1.63 |
| | $\alpha$ | 0.34 | 0.22 | 0.08 | 0.09 | 0.09 | −0.10 | −0.08 | −0.21 | −0.10 | −0.44 | −0.77 | −2.67 |
| Panel E: Non-linear index-driven bear beta portfolios | | | | | | | | | | | | | |
| 1 month | $\overline{R}$ | 0.92 | 0.68 | 0.71 | 0.98 | 0.94 | 0.60 | 0.81 | 0.68 | 0.81 | 0.87 | −0.05 | −0.14 |
| | $\alpha$ | −0.02 | −0.20 | −0.08 | 0.21 | 0.20 | −0.14 | 0.02 | −0.14 | −0.01 | −0.04 | −0.02 | −0.06 |
| 12 months | $\overline{R}$ | 0.76 | 0.68 | 0.72 | 0.76 | 0.70 | 0.75 | 0.55 | 0.84 | 0.77 | 0.73 | −0.03 | −0.12 |
| | $\alpha$ | −0.04 | −0.08 | 0.00 | 0.06 | 0.03 | 0.05 | −0.21 | 0.08 | −0.03 | −0.11 | −0.07 | −0.30 |

(continued)

**Table II.** Continued

| Holding period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | H–L | *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel F: Residual component bear beta portfolios | | | | | | | | | | | | | |
| 1 month | $\overline{R}$ | 0.72 | 0.89 | 0.82 | 0.69 | 0.66 | 0.80 | 0.79 | 0.79 | 0.89 | 0.76 | 0.04 | 0.15 |
| | $\alpha$ | −0.25 | 0.02 | 0.01 | −0.10 | −0.10 | 0.05 | 0.10 | 0.06 | 0.11 | −0.10 | 0.15 | 0.52 |
| 12 months | $\overline{R}$ | 0.84 | 0.86 | 0.83 | 0.84 | 0.78 | 0.75 | 0.76 | 0.70 | 0.68 | 0.58 | −0.27 | −1.38 |
| | $\alpha$ | −0.07 | 0.03 | 0.07 | 0.12 | 0.08 | 0.05 | 0.05 | −0.02 | −0.11 | −0.32 | −0.24 | −1.25 |

explanation for the bear risk premium. As a result, we focus on understanding why the index-driven component is priced for the rest of the article.

Because the bearish spreads are constructed from put options, we expect their return to be decreasing and convex in the index return. This is verified by the regressions in Table I, Panel C, which show that while both the index- and IV-driven components are decreasing and convex in the market return, the magnitudes of the coefficients are far larger for the index-driven component. The convexity of the index-driven component implies that stocks that are positively correlated with this component likely have lower downside market betas than upside betas, at least during the portfolio formation periods, and the downside risk literature suggests that such stocks could be priced at a premium if investors exhibit loss aversion. Therefore, the bear risk premium could be a form of downside risk premium. To examine this possibility, we use another decomposition to isolate the convexity of the index-driven factor and see if it is truly responsible for the risk premium.

### 4.2 Linear versus Non-linear Component

We further separate the index-driven component of the bear risk factor into a linear component, $r_{\text{Bear},L}$, and a nonlinear one, $r_{\text{Bear,NL}}$, following the procedure described in Section 3.3.c. Because the nonlinear component is dominated by the quadratic term in the underlying index return, it should account for most of the convexity in the index-driven component. Table I, Panel C confirms that $r_{\text{Bear},L}$ is mostly linear in the market return and $r_{\text{Bear,NL}}$ is highly convex. If the convexity of the factor is responsible for the bear risk premium, then we expect the premium to be associated with the loadings on $r_{\text{Bear,NL}}$.

Factor loadings are measured using the following regression models:

$$r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Bear},L,i} \cdot r_{\text{Bear},L} + e_i \tag{5}$$

$$r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Bear,NL},i} \cdot r_{\text{Bear,NL}} + e_i \tag{6}$$

Stocks sorted into deciles using $\beta_{\text{Bear,L,i}}$ and $\beta_{\text{Bear,NL},i}$ following the same procedure as before. Table II, Panels D and E report the average returns and alphas of the portfolios sorted on $\beta_{\text{Bear},L,i}$ and $\beta_{\text{Bear,NL},i}$, respectively.

Panel D shows that the portfolio returns generated by the linear component are similar to those generated by the index-driven component and the original bear risk factor, while Panel E shows that loadings on the nonlinear component do not significantly predict stock returns.

Taken together, Table II clearly shows that the only component that is responsible for the bear risk premium is the linear portion of the index-driven component of the bear risk factor, and the risk premium is unrelated to changes in index return distribution and the convexity of the risk factor.

### 4.3 Interpreting the Linear Component

Now that the linear component is responsible for the bear risk premium, we are faced with two obvious questions: First, what is the economic meaning of the linear component? Second, does this negative beta–return relationship reflect compensation for systematic risk?

To simplify the discussion, we show in Appendix A that all the empirical patterns generated by the bear risk factor and its components can be replicated using ATM SPX put option returns and its components as the factors. The additional complexity of the approximate A–D securities plays no role in explaining stock returns. Therefore, it is sufficient to study ATM put returns as the factor to understand the bear risk premium, and we proceed to explain the intuition of the linear component using single put option returns.

In short, the linear component is a scaled version of the market index return and the scaling factor is an increasing function in the option IV. To see this, consider the linear component of a 5-day put option return: $r_{\text{Put,L}} = \Delta_t \cdot (S_{t+5} - S_t)/P_t$, where $\Delta_t$ is the put option delta on Day $t$, S denotes the index level, and $P_t$ is the put option price on Day $t$. Replacing the option price and delta by their Black–Scholes formulas, we can write the factor return as:

$$
\begin{aligned}
r_{\text{Put,L}} &= \frac{-N(-d_1)(S_{t+5} - S_t)}{N(-d_2)Ke^{(r-y)(T-t)} - N(-d_1)S_t} \\
&= \frac{-N(-d_1)(S_{t+5} - S_t)/S_t}{N(-d_2)\dfrac{K}{S_t}e^{(r-y)(T-t)} - N(-d_1)} \\
&= -\frac{(S_{t+5} - S_t)/S_t}{\dfrac{N(-d_2)}{N(-d_1)}\dfrac{K}{S_t}e^{(r-y)(T-t)} - 1} \\
&= -\frac{1}{\dfrac{N(-d_2)}{N(-d_1)}\dfrac{K}{S_t}e^{(r-y)(T-t)} - 1} \cdot r_{\text{SP}} \\
&:= f(\sigma_t) \cdot r_{\text{SP}}
\end{aligned} \tag{7}
$$

where $r_{\text{SP}}$ is the S&P 500 index return from $t$ to $t+5$, $N(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution, $r$ is the interest rate, $y$ is the dividend yield, $d_1 = \frac{\ln(S_t/K)+(r-y+\sigma_t^2/2)(T-t)}{\sigma_t\sqrt{T-t}}$, $d_2 = d_1 - \sigma_t\sqrt{T-t}$, and $\sigma_t$ is the IV of the option at time $t$.

The above equation shows that the linear component of the put return is a scaled version of the S&P index return. At the daily frequency, the time variations in the scaling factor, $f(\sigma_t)$, is mainly driven by $\sigma_t$ and $K/S_t$. Because we choose ATM puts to construct the factor, $K/S_t$ is typically close to 1. Assuming that $K = S_t$, we have $f(\sigma_t) = \frac{-1}{N(-d_2)/N(-d_1)e^{(r-y)(T-t)}-1}$. To gain insight, we further assume that $r = y$. Then we have $N(-d_2)/N(-d_1) = N(\sigma_t\sqrt{T-t}/2)/N(-\sigma_t\sqrt{T-t}/2)$. Considering that $N(\cdot)$ is a CDF function, we can see that $N(\sigma_t\sqrt{T-t}/2)/N(-\sigma_t\sqrt{T-t}/2)$ increases in $\sigma_t$. Therefore, $f(\sigma_t)$ also increases in $\sigma_t$.

In the data, the interest rate and the dividend yield are not always equal, which complicates the properties of $f(\sigma_t)$. In Appendix B, we show numerically that the function is indeed increasing for the parameter values observed in the data.

Taken together, the two-factor model that actually delivers the bear risk premium is:

$$\begin{aligned} r_i &= a_i + \beta_{m,i} \cdot r_m + \beta_{Put,L,i} \cdot r_{Put,L} + e_i \\ &= a_i + \beta_{m,i} \cdot r_m + \beta_{Put,L,i} \cdot f(\sigma_t) \cdot r_{SP} + e_i, \end{aligned} \tag{8}$$

where $f(\sigma_t)$ is increasing in the IV $\sigma_t$ observed at time $t$ and all the returns are from $t$ to $t+5$.

Because the approximate A–D security is a portfolio of put options, Equation (5) can be transformed in a similar fashion and the scaling factor would be the weighted sum of the scaling factors of the individual options involved.

To summarize, our analysis so far shows that the factor model responsible for the observed bear risk premium is in fact a time-varying beta model, where the variation in beta is positively correlated with the IV of the index option. Stocks with high $\beta_{\text{Bear},L,i}$ (or equivalently, $\beta_{\text{Put},L,i}$) have market risk exposures that increase with market volatility. Such stocks are typically considered risky, and risk-averse investors should arguably demand higher returns on them, which implies that $\beta_{\text{Bear},L,i}$ should "positively" predict stock returns. In a model where investors do not care about the time variations in beta, $\beta_{\text{Bear},L,i}$ should not predict returns. However, Table II, Panel D shows that $\beta_{\text{Bear},L,i}$ negatively predicts returns, inconsistent with both scenarios, suggesting that the beta–return relationship here may not reflect compensation for systematic risk.

Before we can reach a conclusion, the intuition gained from Equation (8) needs to be verified empirically. In addition, the expected return of a portfolio is also affected by its average risk exposure, not just how it varies with market volatility. For this, we provide a comprehensive analysis of the factor loadings of the portfolios during the formation period and holding period in the next two sections.

## 5. Formation-Period Factor Loadings

Formation-period factor loadings are reported in Table III, and they are calculated as follows. First, we calculate the betas for each stock at the end of each month by running regressions using the overlapping 5-day returns over the lagged 12 months, which is the same empirical settings for estimating Equation (1) and its variants. Second, we calculate the value-weighted average betas for each portfolio at the end of each month. Finally, we calculate the time-series average of each beta for each portfolio. The $t$-statistics reported in the last column of Table III are for the time-series averages of the H−L portfolios. We focus on three issues concerning the holding-period betas. First, the overall market risk carried by each portfolio. Second, the asymmetry in upside and downside market betas. Finally, the relationship between the market risk of the portfolios and market volatility.

### 5.1 CAPM beta

Panel A is for the portfolios sorted on LM's original bear betas, and all the portfolios in this panel are sorted on bear betas regardless of which average beta is being calculated. The first two rows report the average betas from the two-factor model in Equation (1). Because the bear betas, $\beta_{\text{Bear}}$, are used to sort the portfolios, they naturally increase from Portfolios 1 to 10. Interestingly though, the market betas, $\beta_m$, also increase in the same order. It is

**Table III.** Bear beta portfolios' factor loadings in formation periods

This table reports the formation-period factor loadings of the portfolios sorted on various betas: bear betas (Panel A), betas on the index-driven component (Panel B), betas on the IV-driven component (Panel C), betas on the linear index-driven component (Panel D), betas on the non-linear component (Panel E), and betas on the residual component (Panel F). At the end of each month, the factor loadings of a stock are calculated by running regression models using the prior 12 months' 5-day returns observed at the daily frequency. Then we calculate the value-weighted cross-sectional average betas for each portfolio. The time-series averages of these monthly portfolio betas are reported in the table. The first two rows in Panel A report betas ($\beta_{Bear}$ and $\beta_m$) from the two-factor model used to calculate bear betas. The third row reports one-factor $\beta_{CAPM}$. The regression setting for $\beta_{CAPM}$ is the same as the two-factor model, except that the bear risk factor is not included. The fourth (fifth) row is for one-factor $\beta_{CAPM}$ calculated within the sub-sample where market excess returns are negative (positive). The last column contains the *t*-statistics for the long–short portfolio betas. They are calculated using the Newey–West method, with eleven lags to account for the fact that the adjacent 12-month windows for beta calculations overlap by 11 months.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | H−L | *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear}$ | −0.59 | −0.33 | −0.19 | −0.08 | 0.01 | 0.11 | 0.21 | 0.33 | 0.49 | 0.84 | 1.43 | 19.07 |
| $\beta_m$ | 0.39 | 0.61 | 0.71 | 0.84 | 0.96 | 1.10 | 1.26 | 1.48 | 1.77 | 2.34 | 1.95 | 16.30 |
| $\beta_{CAPM}$ | 0.97 | 0.92 | 0.89 | 0.91 | 0.94 | 1.00 | 1.07 | 1.16 | 1.30 | 1.54 | 0.57 | 7.58 |
| $\beta_{MarketDown}$ | 1.27 | 1.05 | 0.96 | 0.94 | 0.94 | 0.96 | 1.00 | 1.05 | 1.13 | 1.21 | −0.05 | −0.39 |
| $\beta_{MarketUp}$ | 0.60 | 0.76 | 0.81 | 0.89 | 0.97 | 1.07 | 1.19 | 1.35 | 1.58 | 2.06 | 1.47 | 15.35 |
| Panel B: Index-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear,Index}$ | −0.71 | −0.39 | −0.22 | −0.09 | 0.02 | 0.13 | 0.25 | 0.40 | 0.60 | 1.03 | 1.74 | 15.67 |
| $\beta_m$ | 0.31 | 0.56 | 0.71 | 0.83 | 0.96 | 1.13 | 1.31 | 1.57 | 1.90 | 2.55 | 2.24 | 14.57 |
| $\beta_{CAPM}$ | 0.99 | 0.92 | 0.91 | 0.91 | 0.94 | 1.00 | 1.07 | 1.19 | 1.33 | 1.56 | 0.57 | 6.50 |
| $\beta_{MarketDown}$ | 1.27 | 1.05 | 0.97 | 0.94 | 0.94 | 0.96 | 1.01 | 1.08 | 1.16 | 1.25 | −0.03 | −0.18 |
| $\beta_{MarketUp}$ | 0.61 | 0.75 | 0.82 | 0.89 | 0.97 | 1.07 | 1.20 | 1.38 | 1.62 | 2.12 | 1.52 | 15.49 |
| Panel C: IV-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear,IV}$ | −0.33 | −0.19 | −0.12 | −0.07 | −0.02 | 0.02 | 0.07 | 0.12 | 0.19 | 0.32 | 0.66 | 24.55 |
| $\beta_m$ | 1.09 | 0.99 | 0.94 | 0.93 | 0.93 | 0.95 | 0.98 | 1.05 | 1.14 | 1.33 | 0.23 | 4.43 |
| $\beta_{CAPM}$ | 1.19 | 1.04 | 0.97 | 0.95 | 0.94 | 0.95 | 0.96 | 1.01 | 1.09 | 1.24 | 0.05 | 1.06 |
| $\beta_{MarketDown}$ | 1.36 | 1.14 | 1.02 | 0.98 | 0.94 | 0.94 | 0.94 | 0.98 | 1.02 | 1.15 | −0.21 | −2.68 |
| $\beta_{MarketUp}$ | 1.15 | 1.03 | 0.96 | 0.97 | 0.95 | 0.99 | 1.00 | 1.06 | 1.13 | 1.33 | 0.18 | 2.26 |
| Panel D: Linear index-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear,L}$ | −1.03 | −0.50 | −0.22 | −0.01 | 0.17 | 0.37 | 0.58 | 0.85 | 1.22 | 2.00 | 3.03 | 14.62 |
| $\beta_m$ | −0.12 | 0.40 | 0.70 | 0.94 | 1.18 | 1.46 | 1.76 | 2.14 | 2.69 | 3.71 | 3.83 | 13.97 |
| $\beta_{CAPM}$ | 0.96 | 0.91 | 0.92 | 0.94 | 0.98 | 1.05 | 1.12 | 1.22 | 1.37 | 1.57 | 0.61 | 6.39 |
| $\beta_{MarketDown}$ | 0.88 | 0.89 | 0.91 | 0.94 | 0.99 | 1.08 | 1.16 | 1.29 | 1.45 | 1.71 | 0.83 | 6.46 |
| $\beta_{MarketUp}$ | 0.85 | 0.87 | 0.90 | 0.95 | 1.00 | 1.09 | 1.19 | 1.31 | 1.51 | 1.86 | 1.01 | 7.62 |
| $\beta_{HighVIX}$ | 0.97 | 0.91 | 0.90 | 0.94 | 0.96 | 1.04 | 1.13 | 1.26 | 1.45 | 1.82 | 0.84 | 5.13 |
| $\beta_{LowVIX}$ | 1.08 | 0.97 | 0.96 | 0.95 | 0.98 | 1.05 | 1.11 | 1.19 | 1.29 | 1.34 | 0.26 | 1.79 |

(continued)

**Table III.** Continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | H−L | *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel E: Non-linear index-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{\text{Bear,NL}}$ | −0.42 | −0.25 | −0.17 | −0.11 | −0.06 | −0.01 | 0.04 | 0.10 | 0.18 | 0.34 | 0.76 | 24.13 |
| $\beta_m$ | 1.08 | 1.00 | 0.96 | 0.94 | 0.94 | 0.94 | 0.97 | 1.05 | 1.13 | 1.35 | 0.27 | 3.35 |
| $\beta_{\text{CAPM}}$ | 1.15 | 1.05 | 0.99 | 0.95 | 0.94 | 0.94 | 0.96 | 1.02 | 1.10 | 1.28 | 0.12 | 1.99 |
| $\beta_{\text{MarketDown}}$ | 1.88 | 1.45 | 1.25 | 1.12 | 1.03 | 0.94 | 0.89 | 0.87 | 0.82 | 0.74 | −1.14 | −9.38 |
| $\beta_{\text{MarketUp}}$ | 0.56 | 0.75 | 0.81 | 0.86 | 0.91 | 0.96 | 1.05 | 1.17 | 1.35 | 1.78 | 1.22 | 15.75 |
| Panel F: Residual component bear beta portfolios | | | | | | | | | | | | |
| $\beta_{\text{Bear,Residual}}$ | −0.40 | −0.23 | −0.14 | −0.08 | −0.02 | 0.03 | 0.08 | 0.15 | 0.23 | 0.40 | 0.80 | 23.35 |
| $\beta_m$ | 1.36 | 1.15 | 1.05 | 0.99 | 0.96 | 0.93 | 0.92 | 0.92 | 0.98 | 1.08 | −0.29 | −5.24 |
| $\beta_{\text{CAPM}}$ | 1.25 | 1.09 | 1.02 | 0.97 | 0.95 | 0.94 | 0.94 | 0.96 | 1.04 | 1.19 | −0.06 | −1.22 |
| $\beta_{\text{MarketDown}}$ | 1.16 | 1.03 | 0.97 | 0.95 | 0.94 | 0.95 | 0.97 | 1.01 | 1.14 | 1.37 | 0.21 | 2.67 |
| $\beta_{\text{MarketUp}}$ | 1.35 | 1.14 | 1.06 | 1.02 | 0.98 | 0.96 | 0.96 | 0.95 | 1.01 | 1.14 | −0.21 | −2.78 |

important to keep in mind that the two factors are strongly negatively correlated. Therefore, the rankings of the two betas have opposite implications: the high bear beta of Portfolio 10 implies countercyclicality, while its high market beta implies cyclicality. Because the bear factor is scaled to have the same SD as the market factor, the betas on the two factors can be compared with each other. Column H−L shows that the spread in bear betas (1.43) is lower than the spread in market betas (1.95); hence cyclicality prevails, and the long–short portfolio carries positive market risk. To confirm this net effect, we also report in the third row the one-factor CAPM beta ($\beta_{\text{CAPM}}$), of each portfolio, and they indeed increase from Portfolios 1 to 10. This is interesting because the H−L portfolio is meant to hedge bear market risk, but it ends up carrying positive market risk, making its negative return puzzling.

Comparing the $\beta_{\text{CAPM}}$ in Panels B and C, we can see that the positive market risk of the H−L portfolio sorted on bear betas can be mostly attributed to the index-driven component of the bear risk factor. The third rows of Panels D and E further reveal that the linear index-driven component is the one responsible. In particular, the third row of Panel D shows that stocks with high linear-component beta, $\beta_{\text{Bear},L}$, carry higher market risk than their low-beta counterparts, and the H−L portfolio has a CAPM beta of 0.61. Together, Panels C–F of Tables II and III show that the H−L portfolio with the highest market risk also has the lowest expected return.

The formulation in Section 4.3 further suggests that high linear-component beta stocks should carry even higher market risk when SPX options' IVs are high. To examine this property, we split each 12-month formation period into two equal halves based on the rankings of daily VIX levels, and then calculate one-factor CAPM betas ($\beta_{\text{CAPM}}$) within each half. These split-sample $\beta_{\text{CAPM}}$ are reported in the last two rows of Panel D. From the H−L column, we can see that the long–short portfolio has a market beta of 0.84 during the high-VIX period and only 0.26 during the low-VIX period.

The Panel Ds of Tables II and III show that stocks with high linear-component betas have been exposed to high average market risk and particularly high market risk in volatile markets. Yet, they delivered low expected returns. While this pattern is reminiscent of the BaB puzzle, it also has distinct features. In BaB, high-beta stocks and low-beta ones have similar average raw returns, so the long–short portfolio has a near-zero average return but a negative CAPM alpha. Here, even the average return of the high-risk-minus-low-risk portfolio is negative and its alpha even more so, suggesting a further departure from the rational framework. We provide a more detailed characterization and explanation of the current anomaly in Section 8.

The last three rows in Panel F show that loadings on the residual component do not generate meaningful spreads in market risk exposure.

### 5.2 Upside Beta versus Downside Beta

Turning to the asymmetry in betas, the last two rows of Table III, Panel A report the market risk of the bear beta portfolios measured during up- and down markets, respectively. To calculate the upside (downside) betas, we run the one-factor CAPM using observations where 5-day market excess returns are positive (negative). Comparing the last two rows in the panel, we can see that the high bear beta portfolio (i.e., Portfolio 10's) upside beta (2.06) is higher than its downside beta (1.21), while the low bear beta portfolio exhibits the opposite pattern (0.60 for the upside versus 1.27 for the downside). Column H−L confirms that the long–short portfolio has a much smaller downside beta (−0.05) than upside beta (1.47), which seems to be consistent with the notion that high bear beta stocks are good hedges against downside risk.

The asymmetry is not surprising given that the bear risk factor return is highly convex in the underlying index return as shown in Table I, Panel C. OLS regressions would assign high bear betas to stocks whose returns exhibit the same convexity during the formation period. While the asymmetric betas seem attractive to loss-averse investors, it is unclear if the asymmetry persists into the holding period, considering that much of the stock return convexity can be caused by idiosyncratic returns and is not meant to be persistent. We examine this issue in the next section.

The last two rows of Panel B show that the index-driven component is responsible for most of the asymmetry in upside and downside betas observed in Panel A. The patterns in Panel C are qualitatively similar to the previous two panels, but the magnitudes are much smaller. Panel E further shows that the index-driven component generates the asymmetry mainly via its nonlinear component as the H−L portfolio sorted on this component has an upside beta of 1.22 and downside beta of −1.14 in the formation period. In contrast, Panel D only shows a minor asymmetry generated by the linear component.

Because the asymmetry in market betas is primarily associated with the loadings on the nonlinear index-driven component, and these loadings do not predict stock returns, it is clear that the return predictability is not related to in-sample asymmetry in market betas. In other words, investors do not pay a premium for stocks that exhibit low downside risk in the past.

## 6. Holding-Period Factor Loadings

The formation-period betas of the portfolios reveal two important patterns: First, stocks with high overall market betas and countercyclical betas are the ones earning low expected returns. Second, the asymmetric market betas are delivered by the nonlinear component and do not predict stock returns. Next, we examine whether these patterns hold for the holding-period betas. In addition, we discuss whether the factor loadings in Equations (1)–(6) are persistent.

Table IV reports holding-period betas of the portfolios. Considering that the formation-period betas are calculated using overlapping 5-day returns, we use the same setting to calculate holding-period returns. Specifically, after forming the portfolios at the end of each month, we calculate the value-weighted 5-day returns of each portfolio in the following month. Then we run the same regression models as in Table III to calculate the holding-period betas.[8]

## 6.1 CAPM beta

The first two rows of Table IV, Panel A report betas from the two-factor model in Equation (1). The patterns are qualitatively similar to those in the formation period, though Column H−L shows that the spreads in betas shrink considerably post-formation. The bear beta of the H−L portfolio decreases from 1.43 to 0.18 and its market beta from 1.95 to 0.69. Interestingly, although both betas decrease, their difference remains virtually unchanged from formation periods to holding periods, leading to a very persistent $\beta_{\mathrm{CAPM}}$, which is 0.56 in the holding periods and 0.57 in the formation periods. The same pattern can be observed in Panel D for the linear component. Therefore, even in the holding period, high bear beta and linear-component beta stocks carry high overall market risk.

The last two rows of Panel D report the holding-period $\beta_{\mathrm{CAPM}}$ within high-VIX and low-VIX periods, respectively. We assign each month in the holding period into the high- or low-VIX subsample by ranking its average VIX against the preceding 12 months used for beta calculations. If the average VIX of the holding month ranks above (below) the median of the 13 months, then the holding month is classified as a high-VIX (low-VIX) month. $\beta_{\mathrm{CAPM}}$ are then calculated within each subsample. Betas of the H−L portfolio show that sorting on linear-component betas still leads to a higher spread in $\beta_{\mathrm{CAPM}}$ amid high market volatility, though the difference between high-VIX and low-VIX betas (0.58 versus 0.43) is considerably smaller than the formation-period counterpart.

## 6.2 Upside Beta versus Downside Beta

The last two rows of Panel A show that, in contrast to the formation period, the H−L portfolio's downside market beta (0.60) becomes higher than its upside market beta (0.54) in the holding period. Therefore, the relatively low downside beta of the H−L portfolio sorted on bear betas during the formation period is likely completely driven by idiosyncratic returns. Some stocks' total returns happen to exhibit convexity during the regression window, giving them high betas on the convex option factor. But since such convexity is observed mostly by chance, it disappears in the holding period.

A broad examination of all the portfolios in Tables III and IV confirms a common pattern: the asymmetry in formation-period market betas matches the convexity of the factors themselves very well, but such asymmetry contains little information about the asymmetry during the holding period. In most cases, the difference between upside and downside betas switches sign or practically disappears when moving from the formation to holding period. The only exception occurs in the non-linear index-driven component (Panel E), where portfolio H−L's upside beta remains higher than the downside beta in the holding period, though the magnitude of the difference shrinks considerably.

8 Many previous studies use contemporaneous daily returns together with a number of lags to calculate Dimson (1979, p. 54) betas. In untabulated results, we find that using Dimson betas with four lags leads to similar results as ours.

**Table IV.** Bear beta portfolios' factor loadings in holding periods

This table reports the holding-period factor loadings of the portfolios sorted on various betas: bear betas (Panel A), betas on the index-driven component (Panel B), betas on the IV-driven component (Panel C), betas on the linear index-driven component (Panel D), betas on the non-linear component (Panel E), and betas on the residual component (Panel F). At the end of each month, stocks are sorted into deciles based on their option-factor betas and held for the following month, during which value-weighted, overlapping 5-day portfolio returns are calculated at the daily frequency. The holding period factor loadings are calculated by regressing the portfolio returns on the factor returns using the entire sample period. The first two rows in each panel contain betas from the two-factor model with the market factor and one of the option-based factors. The third row is for one-factor $\beta_{CAPM}$. The fourth (fifth) row is for one-factor $\beta_{CAPM}$ calculated within the sub-sample where market excess returns are negative (positive). The last column reports the *t*-statistics of the factor loadings of the long–short portfolios adjusted for heteroskedasticity.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | H−L | *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear}$ | −0.03 | −0.02 | −0.03 | −0.02 | −0.02 | 0.00 | 0.02 | 0.09 | 0.16 | 0.16 | 0.18 | 3.99 |
| $\beta_m$ | 0.98 | 0.93 | 0.86 | 0.89 | 0.94 | 0.99 | 1.10 | 1.27 | 1.49 | 1.67 | 0.69 | 11.63 |
| $\beta_{CAPM}$ | 1.00 | 0.94 | 0.88 | 0.91 | 0.96 | 1.00 | 1.09 | 1.21 | 1.36 | 1.55 | 0.56 | 15.49 |
| $\beta_{MarketDown}$ | 0.94 | 0.91 | 0.84 | 0.91 | 0.97 | 1.02 | 1.12 | 1.22 | 1.38 | 1.52 | 0.60 | 8.46 |
| $\beta_{MarketUp}$ | 0.98 | 0.98 | 0.91 | 0.92 | 0.98 | 0.97 | 1.07 | 1.23 | 1.39 | 1.54 | 0.54 | 7.10 |
| Panel B: Index-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear,Index}$ | −0.03 | −0.03 | −0.03 | −0.02 | −0.01 | 0.00 | 0.01 | 0.13 | 0.15 | 0.18 | 0.20 | 3.81 |
| $\beta_m$ | 1.02 | 0.93 | 0.89 | 0.90 | 0.94 | 0.99 | 1.08 | 1.35 | 1.49 | 1.69 | 0.67 | 10.20 |
| $\beta_{CAPM}$ | 1.03 | 0.95 | 0.91 | 0.92 | 0.95 | 1.00 | 1.07 | 1.25 | 1.37 | 1.55 | 0.52 | 13.58 |
| $\beta_{MarketDown}$ | 0.98 | 0.87 | 0.92 | 0.91 | 0.95 | 1.02 | 1.10 | 1.28 | 1.37 | 1.52 | 0.57 | 7.61 |
| $\beta_{MarketUp}$ | 1.02 | 0.99 | 0.91 | 0.92 | 0.98 | 0.98 | 1.05 | 1.30 | 1.38 | 1.52 | 0.48 | 5.94 |
| Panel C: IV-Driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear,IV}$ | −0.02 | −0.02 | 0.00 | −0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.03 | 1.58 |
| $\beta_m$ | 1.18 | 1.03 | 0.95 | 0.94 | 0.95 | 0.96 | 0.95 | 1.01 | 1.10 | 1.23 | 0.05 | 1.95 |
| $\beta_{CAPM}$ | 1.19 | 1.04 | 0.95 | 0.94 | 0.95 | 0.96 | 0.95 | 1.01 | 1.10 | 1.23 | 0.05 | 1.79 |
| $\beta_{MarketDown}$ | 1.18 | 1.05 | 0.92 | 0.94 | 0.95 | 0.97 | 0.95 | 0.99 | 1.08 | 1.19 | 0.01 | 0.11 |
| $\beta_{MarketUp}$ | 1.15 | 1.01 | 0.96 | 0.95 | 0.96 | 0.95 | 0.95 | 1.05 | 1.13 | 1.19 | 0.03 | 0.73 |
| Panel D: Linear index-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{Bear,L}$ | −0.04 | −0.01 | −0.04 | 0.02 | −0.01 | 0.02 | 0.03 | 0.13 | 0.20 | 0.32 | 0.35 | 4.71 |
| $\beta_m$ | 1.00 | 0.94 | 0.91 | 0.98 | 0.99 | 1.09 | 1.15 | 1.33 | 1.55 | 1.82 | 0.82 | 8.59 |
| $\beta_{CAPM}$ | 1.03 | 0.94 | 0.94 | 0.97 | 1.00 | 1.07 | 1.13 | 1.22 | 1.38 | 1.55 | 0.52 | 12.29 |
| $\beta_{MarketDown}$ | 0.95 | 0.92 | 0.90 | 0.99 | 0.99 | 1.13 | 1.13 | 1.26 | 1.40 | 1.55 | 0.62 | 7.56 |
| $\beta_{MarketUp}$ | 1.10 | 1.00 | 0.98 | 0.97 | 1.02 | 1.00 | 1.11 | 1.18 | 1.33 | 1.48 | 0.37 | 4.03 |
| $\beta_{HighVIX}$ | 1.00 | 0.94 | 0.94 | 0.99 | 1.00 | 1.07 | 1.17 | 1.23 | 1.40 | 1.58 | 0.58 | 10.25 |
| $\beta_{LowVIX}$ | 1.10 | 0.94 | 0.96 | 0.95 | 0.97 | 1.04 | 1.07 | 1.20 | 1.33 | 1.52 | 0.43 | 7.75 |

(continued)

**Table IV.** Continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | H−L | *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel E: Non-linear index-driven bear beta portfolios | | | | | | | | | | | | |
| $\beta_{\text{Bear,NL}}$ | 0.01 | 0.00 | −0.01 | −0.01 | 0.01 | −0.01 | 0.01 | 0.01 | 0.02 | 0.03 | 0.02 | 0.88 |
| $\beta_m$ | 1.21 | 1.05 | 0.97 | 0.92 | 0.91 | 0.91 | 0.95 | 1.08 | 1.14 | 1.35 | 0.13 | 3.46 |
| $\beta_{\text{CAPM}}$ | 1.21 | 1.05 | 0.98 | 0.92 | 0.91 | 0.92 | 0.95 | 1.08 | 1.14 | 1.34 | 0.12 | 3.16 |
| $\beta_{\text{MarketDown}}$ | 1.21 | 1.04 | 0.99 | 0.91 | 0.88 | 0.92 | 0.94 | 1.11 | 1.13 | 1.30 | 0.08 | 0.89 |
| $\beta_{\text{MarketUp}}$ | 1.15 | 0.99 | 0.93 | 0.90 | 0.93 | 0.90 | 0.99 | 1.11 | 1.20 | 1.41 | 0.25 | 3.50 |
| Panel F: Residual component bear beta portfolios | | | | | | | | | | | | |
| $\beta_{\text{Bear,Residual}}$ | −0.01 | −0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.04 | 0.05 | 2.39 |
| $\beta_m$ | 1.25 | 1.09 | 1.02 | 0.98 | 0.95 | 0.93 | 0.93 | 0.95 | 1.04 | 1.18 | −0.07 | −2.47 |
| $\beta_{\text{CAPM}}$ | 1.24 | 1.08 | 1.02 | 0.98 | 0.95 | 0.93 | 0.93 | 0.95 | 1.04 | 1.19 | −0.06 | −2.09 |
| $\beta_{\text{MarketDown}}$ | 1.21 | 1.05 | 1.01 | 0.98 | 0.98 | 0.90 | 0.92 | 0.94 | 1.07 | 1.18 | −0.04 | −0.69 |
| $\beta_{\text{MarketUp}}$ | 1.19 | 1.11 | 1.07 | 0.98 | 0.92 | 0.95 | 0.95 | 0.96 | 0.99 | 1.16 | −0.03 | −0.61 |

From a practitioner's point of view, this evidence might be encouraging, as it offers some hope in predicting the symmetry in market betas. Nevertheless, the nonlinear component is not priced in stocks, so it confirms that investors do not pay a premium for asymmetric betas.

Focusing on the linear index-driven component in Table IV, Panel D, we can see that the H−L portfolio's downside beta (0.62) exceeds its upside beta (0.37) during the holding period. We argue that this is consistent with the property of the linear component discussed in Section 4.3, where we show that the high linear-component beta stocks carry particularly high market risk when the index option's IV is high. It is well known that the VIX index always spikes up during market crashes, which means that the high linear-component beta stocks' market risk exposure tends to increase during market crashes. In addition, the high market risk should also increase the discount rates of these stocks and further depress their prices. The opposite can be said about the low linear-component beta stocks. This panel confirms the two unattractive aspects of the H−L portfolio sorted on the linear-component betas: It carries high overall market risk and its market risk is even higher during market downturns. The asymmetry in betas is likely masked by in-sample fitting during the formation period.

Examining the first two rows of Panel D in Tables III and IV, we can see that the factor loadings on the two-factor model involving the linear component are statistically persistent, and the *t*-statistic of the linear bear factor bear is 4.71 for the H−L portfolio. Panels C, E, and F in Table IV show that the factor loadings on the IV-driven, nonlinear index-driven, and residual components are insignificant for the H−L portfolio during the holding period. Therefore, out of the four components of the bear risk factor, only loadings on the linear factor are persistent.

Overall, the persistence in factor loadings is relatively weak with the exception of the loadings on the market factor. As pointed out by Barahona, Driessen, and Frehen (2021), rational investors need to know the post-formation betas to make portfolio decisions and price assets accordingly. Therefore, a predictable large spread in port-formation betas is particularly important under a rational framework. On the other hand, models involving agents who are not fully rational could lead to predictions regarding lagged betas as such agents may mistakenly extrapolate past betas into the future.

### 6.3 Is Downside Risk Priced in the Cross-Section of Stocks?

Our research sets out to reconcile the conflict in the evidence concerning the downside risk premium in the stock market and shows that all the evidence in fact points in the same direction: downside risk is not priced in the stock market.

While high loadings on the nonlinear component of the bear risk factor seem to predict lower downside betas than upside betas, these loadings do not predict stock returns. Loadings on the IV-driven component are not persistent and do not predict stocks returns, so it is not the changing forward-looking probability of a market downturn that is driving the bear risk premium.

Finally, the H−L portfolio sorted on the linear-component betas has high average market risk, countercyclical market risk, and asymmetric market risk tilting toward the downside, and yet it has delivered a low average return, which is clearly an asset pricing anomaly. Therefore, the bear risk "premium" does not prove that investors pay a premium to hedge downside or bear market risk.

## 7. Dissecting Jump Risk

Our analysis so far reveals that bear beta portfolios do not provide a good hedge for market downside risk. In this section, we provide further analysis to examine whether such portfolios hedge specific types of market jump risk. Baker *et al.* (2020) use *Wall Street Journal* articles to classify daily jumps in the stock market index since 1900 into categories such as "monetary policy and central banking" and "regulation." Their data allow us to examine whether the bear beta portfolio is particularly good at hedging the risk within certain categories. We focus on two questions. First, does the bear beta portfolio hedge the *ex ante* jump risk within certain categories? If the return of the long–short bear beta portfolio tracks the probability of an upcoming market crash as hypothesized by LM, then we expect the portfolio return to negatively predict a market crash index, which is equal to the market return if a crash occurs and zero otherwise. Second, does the bear beta portfolio hedge the *ex post* jump risk of certain categories? This refers to the contemporaneous correlation between the bear beta portfolio return and the jump index. If the portfolio provides a good hedge, then the correlation should be negative. Besides the bear risk factor, Kelly and Jiang (2014) also propose a measure of the fatness of the left tail and show that the exposure to this factor explains expected stock returns in the cross-section. Their tail risk measure can be viewed as a measure of *ex ante* jump risk as it models the shape of the distribution. Therefore, we examine the hedging performance of the long–short portfolio sorted on their tail risk betas to provide a more comprehensive view on this topic. Kelly and Jiang's measure also allows a longer sample period (1964–2011) for robustness check.

The results are presented in Table V. Panel A explores the hedging of *ex ante* jump risk, where we consider three types of market jumps: Macro, Non-Macro, and Earnings Related. Besides the jump measure of Baker *et al.* (2020), we also consider one based on an index of real economic activities—the Aruoba–Diebold–Scotti Business Conditions Index. To measure jumps, we use unexpected quarterly changes in the index, where an autoregressive model is used to filter out the unexpected changes. This jump index is equal to the

**Table V.** Jump risk exposure of the hedging portfolios

Panel A reports the regression coefficients of predicting future market negative jumps with portfolio returns or the VIX index. "Raw Long-Short Portfolio Return as Regressor" uses the monthly return of a long–short portfolio to predict the downside jumps over the following month. In the case of the ADS index, the predicted horizon is the following 3 months, where the regression is still run at the monthly frequency. The long–short portfolios are sorted on bear betas or the tail risk betas as in Kelly and Jiang (2014). The dependent variables in the first six columns are the monthly accumulations of the negative jumps identified and categorized by Baker *et al.* (2020). ADS index refers to the quarterly innovations in the Aruoba–Diebold–Scotti Business Conditions Index. "Market-Hedged Long-Short Portfolio Return as Regressor" uses the residuals of regressing the long-short portfolio returns on market return as the predictor. "VIX as Regressor" uses the month-end VIX level as the predictor. Panel B reports the coefficients of regressing the long–short portfolio returns on contemporaneous market returns and jump returns. The upper half uses monthly returns and the lower half uses daily returns.

Panel A: Predict jump risk

| Portfolio sorted on | Regressor | Macro | | Non-macro | | Earnings | | ADS index | |
|---|---|---|---|---|---|---|---|---|---|
| | | Coef | *t*-stat | Coef | *t*-stat | Coef | *t*-stat | Coef | *t*-stat |
| | Raw long–short portfolio return as regressor | | | | | | | | |
| Bear beta | Intercept | −0.011 | −4.087 | −0.009 | −3.966 | −0.004 | −3.345 | −0.073 | −1.745 |
| | Portfolio return | 0.067 | 2.534 | 0.056 | 1.775 | 0.020 | 1.117 | 0.599 | 1.325 |
| Tail beta | Intercept | −0.012 | −3.986 | −0.011 | −3.941 | −0.005 | −3.347 | −0.088 | −1.763 |
| | Portfolio return | 0.050 | 0.843 | 0.060 | 1.172 | 0.021 | 0.715 | 0.773 | 1.103 |
| | Market-hedged Long–short portfolio return as regressor | | | | | | | | |
| Bear beta | Intercept | −0.012 | −4.104 | −0.010 | −3.879 | −0.004 | −3.233 | −0.079 | −1.743 |
| | Portfolio return | −0.021 | −0.426 | −0.021 | −0.540 | −0.009 | −0.563 | −0.167 | −0.306 |
| Tail beta | Intercept | −0.012 | −3.927 | −0.011 | −3.899 | −0.005 | −3.315 | −0.089 | −1.752 |
| | Portfolio return | 0.032 | 0.627 | 0.045 | 0.994 | 0.015 | 0.560 | 0.616 | 1.054 |
| IV-driven bear beta | Intercept | −0.012 | −4.095 | −0.010 | −3.872 | −0.004 | −3.246 | −0.079 | −1.741 |
| | Portfolio return | 0.015 | 0.321 | 0.032 | 0.628 | 0.043 | 1.637 | 0.701 | 0.890 |
| | VIX as regressor | | | | | | | | |
| | Intercept | 0.038 | 2.932 | 0.029 | 3.072 | 0.014 | 2.874 | 0.319 | 1.507 |
| | VIX | −0.002 | −3.383 | −0.002 | −3.564 | −0.001 | −3.138 | −0.019 | −1.649 |

Panel B: Hedging contemporaneous jump risk

Monthly Returns

| | | Macro | | Non-macro | | Earnings | |
|---|---|---|---|---|---|---|---|
| Bear beta | Intercept | −0.018 | −3.912 | −0.018 | −3.918 | −0.017 | −4.022 |
| | Jump | −0.192 | −1.667 | −0.177 | −1.320 | −0.310 | −1.230 |
| | Market return | 0.746 | 5.351 | 0.745 | 5.149 | 0.731 | 5.418 |
| Tail beta | Intercept | −0.004 | −1.144 | −0.002 | −0.688 | −0.003 | −0.788 |
| | Jump | −0.088 | −0.690 | 0.007 | 0.050 | −0.029 | −0.101 |
| | Market return | 0.127 | 1.574 | 0.098 | 1.089 | 0.105 | 1.137 |

(continued)

**Table V.** Continued

Panel B: Hedging contemporaneous jump risk

Daily Returns

| | | Macro | | Non-macro | | Earnings | |
|---|---|---|---|---|---|---|---|
| Bear beta | Intercept | −0.001 | −3.617 | −0.001 | −3.708 | −0.001 | −3.330 |
| | Jump | −0.080 | −1.329 | −0.129 | −2.056 | 0.039 | 0.384 |
| | Market return | 0.498 | 14.329 | 0.503 | 14.316 | 0.482 | 14.840 |
| Tail beta | Intercept | 0.000 | −0.593 | 0.000 | −0.066 | 0.000 | −0.446 |
| | Jump | 0.017 | 0.370 | 0.172 | 3.925 | 0.137 | 2.762 |
| | Market return | 0.067 | 2.944 | 0.044 | 2.014 | 0.062 | 2.934 |

change if the change is 1.5 SDs below zero and zero otherwise. Further details about the jumps are provided in Appendix C.

The regression model in Panel A is $\text{Jump}_{t+1} = a + b \cdot \text{Portfolio Return}_t$, where $\text{Jump}_{t+1}$ is the jump index accumulated in Month $t+1$, and $\text{Portfolio Return}_t$ is the long–short portfolio return in Month $t$. In the upper half of the panel, we use the raw portfolio returns as the regressor. In the lower half, the regressor is the market hedged portfolio return, which is the residual of regressing the raw return on the market return. Forecasting 1-month ahead jumps is in line with the choice of time-to-expiration in constructing the bear risk factors. If the portfolio indeed tracks the probability of the jump risk, then we expect coefficient b to be negative. The upper half of the panel shows that for both portfolios and all the jump types, coefficient b is positive. Therefore, the raw returns of these portfolios do not seem to hedge against ex ante jump risk. The lower half shows that coefficient b turns negative for the bear beta portfolio when the portfolio is market-hedged. However, the coefficients are fairly insignificant. For comparison, we show the coefficients of predicting these jumps using end-of-month VIX levels at the bottom of Panel A. As expected, VIX is a good indicator of the probabilities of these jumps. The weak predictive power of the long–short portfolio makes it very unlikely that rational investors would pay such a high premium to hold the long–short portfolio.

It is also puzzling that the long–short portfolio sorted on the loadings on the IV-driven component of the bear risk factor still carries the wrong sign, as shown in the table, because, after all, the IV-driven component is specifically designed to capture ex ante bear market risk. The lack of predictive power from the portfolio associated with this component raises the concern that the weak predictive power from the bear beta portfolio is likely spurious.

Panel B reports the coefficients of the contemporaneous regressions. The upper part is for monthly regressions and the lower part is for daily. The regression model is $\text{Portfolio Return}_t = a + b \cdot \text{Jump}_t + c \cdot \text{Market Return}_t$. Because the Jump term is equal to market return when a jump is identified and zero otherwise, $b + c$ can be interpreted as the loading of the portfolio on the market during a jump. If the portfolio is a good hedge of ex post risk, then we expected $b + c$ to be negative. At the monthly frequency, the negative coefficients on Jump suggest that the bear beta portfolio has lower market exposure during a jump than during a normal period. However, values of $b + c$ remain highly positive, suggesting that this hedging portfolio is still positively exposed to *ex post* jump risk. It is unclear why rational investors would be willing to earn a highly negative expected return to hold such a portfolio. Similar arguments can be made about the tail risk portfolio.

The lower half of the panel runs the regressions using daily returns. Overall, there is still little evidence that these portfolios hedge *ex post* jump risk. Therefore, our analysis in this section confirms the conclusion reached in the previous section: stock portfolios sorted on bear or tail risk exposure do not provide a meaningful hedge for market risk on the downside during the holding period.

## 8. Resolving the Puzzle

The current anomaly is related to the BaB anomaly because both entail a negative relationship between systematic risk and expected returns. Considering that a rich literature has been developed to resolve the BaB puzzle, we next investigate whether the existing solutions can help understand the current anomaly. Overall, we find that the aggregate disagreement channel proposed by Hong and Sraer (2016) is likely the main cause of the current anomaly. In particular, we find that the anomaly would not only disappear but also change sign if there were no disagreement about the aggregate stock market, which suggests that the mispricing caused by disagreement overshadows the positive risk–reward relationship predicted by canonical asset pricing models. We also consider skewness preference as a potential explanation of the anomaly and leave two other considerations—mispricing (Liu, Stambaugh, and Yuan, 2018 and conditional CAPM (Boguth *et al.* (2011))—to the Supplementary Appendix.

In this section, we focus on the H−L portfolios sorted on the loadings with respect to the full bear risk factor, and these loadings are denoted by $\beta_{\text{Bear}}$. Conceptually though, the linear component of the ATM put return is easier to interpret as shown in Equation (8), and most of our arguments speak directly to this component. In Section 7 of the Supplementary Appendix of this article, we confirm that the analyses in the current section indeed lead to qualitatively and quantitatively similar results when applied to portfolios sorted on the linear component of index put returns.

### 8.1 Aggregate Disagreement

Hong and Sraer (2016) argue that stocks with high market betas can be overpriced if there is disagreement about the market return, for such disagreement gives rise to particularly strong disagreement about high-beta stocks, which, coupled with short-sale constraints, could lead to overpricing.

As shown in Sections 5 and 6, stocks with high loadings on the linear component of bear risk factor tend to have high market betas, especially when the market is volatile. If investors are more likely to disagree about future market returns in a volatile market, then stocks with high linear-component betas should be particularly susceptible to disagreement-induced overpricing. If the current anomaly is indeed driven by disagreement, then we expect the return of the long–short portfolio to be particularly low when the aggregate disagreement level is high. In addition, Hong and Sraer (2016) show that the effect of aggregate disagreement on overpricing is particularly strong among speculative stocks, that is, stocks whose systematic risk is high relative to their idiosyncratic risk. We test these two hypothesis in a regression setting.

Following the steps described in Hong and Sraer (2016), we calculate the aggregate disagreement and stock-level speculativeness. The monthly aggregate disagreement measure is the beta-weighted cross-sectional average of the SDs of analyst forecasts regarding long-term earnings growth rates. These SDs are reported in the I/B/E/S unadjusted summary file. Each month, stocks whose $\hat{\beta}/\hat{\sigma^2}$ ratios are above(below) the The New York Stock Exchange (NYSE) median are assigned to the speculative(non-speculative) group, where $\hat{\beta}$ is the estimated market beta and $\hat{\sigma^2}$ is the estimated idiosyncratic variance.

We examine whether the aggregate disagreement predicts the return of the H−L portfolio sorted on bear betas. In addition, we formed the H−L portfolio using speculative stocks and non-speculative stocks separately and compare the predictive power of disagreement for these two types of stocks.

The regression coefficients are reported in Table VI, Panel A, where results for the portfolios containing speculative and non-speculative stocks are in the first and last two columns, respectively. We consider three return horizons: 1, 6, and 12 months. Since the aggregate disagreement is measured at the monthly frequency, predicting 6- and 12-month ahead returns leads to overlapping observations. Lags in the Newey–West $t$-statistics are set to five and eleven to account for the overlap.

The first two columns in Panel A show that, when the portfolio is made of speculative stocks, aggregate disagreement negative predicts future H−L portfolio returns sorted on bear betas, meaning the negative risk–reward relationship is particularly strong following high disagreement. The positive intercepts in these regressions suggest that if there were no disagreement about the market, then the positive risk–reward relationship would have been restored.

The results for non-speculative stocks are clearly weaker than those for speculative ones, consistent with Hong and Sraer (2016)'s prediction. Overall, the evidence in Panel A strongly supports disagreement-induced overpricing being the main explanation of the anomaly associated with bear betas.

A key feature of Hong and Sraer (2016) is the tent-shaped relationship between expected return and market beta. To examine this relationship, we plot the beta–return relationship in Figure 1. The setting here is similar to Figure 7 in Hong and Sraer (2016), except that our portfolios are sorted on bear betas. Both the post-ranking average returns and market betas are measured using 12-month returns. Our figure does not show a pronounced tent shape. However, it is important to consider the range of market betas, as indicated by the horizontal axis. Because we do not sort directly on market betas, we do not get the full range in Hong and Sraer (2016). The lowest post ranking beta in the figure is around 0.8, which roughly corresponds to the location of the top of the tent in Hong and Sraer (2016)'s Figure 7. Therefore, we only get to observe the downward sloping portion of the relationship.

To further add to the evidence regarding disagreement, we consider another aspect of the model in Hong and Sraer (2016). The irrational investors in the model need to agree to disagree with each other and not learn from the asset prices and correct their beliefs. This can be viewed as a form of overconfidence. In other words, investors not only need to start with different opinions, but also need to be overconfident enough to hold on to these opinions. It is reasonable to assume that investors are more likely to be overconfident when the marketwide sentiment is high. Therefore, we argue that the negative relationship between risk and expected return is likely to be strong during high sentiment periods. In Table VI, Panel B, we examine how the marketwide sentiment measure of Baker and Wurgler (2006) predicts bear portfolio returns. Consistent with my argument, we find that the sentiment negatively forecasts the portfolio returns at all three horizons.

**Table VI.** Aggregate disagreement

Panel A reports the coefficients of regressing the returns of the H−L portfolio sorted on bear betas on lagged aggregate disagreement. The portfolio is formed using only speculative stocks in the first two columns and non-speculative stocks in the last two. For the definitions of aggregate disagreement and speculative stocks, we follow Hong and Sraer (2016). Each month, the aggregate disagreement measure is given by the beta-weighted cross-sectional average of the SDs of analyst forecasts regarding long-term earnings growth rates. These SDs of forecasts are obtained from the I/B/E/S unadjusted summary file. Speculative(non-speculative) stocks are those whose $\hat{\beta}/\hat{\sigma^2}$ ratios are above(below) the NYSE median, where $\hat{\beta}$ is the estimated market beta and $\hat{\sigma^2}$ is the estimated idiosyncratic variance. These time-series regressions use observations at the monthly frequency. We consider three holding periods: 1, 6, and 12 months. In other words, we use the aggregate disagreement in Month $t$ to predict portfolio returns in Months $t+1$, $t+1$ to $t+6$, and $t+1$ to $t+12$, respectively. The 6- and 12-month holding periods result in overlapping returns, and the numbers of lags used to calculate their Newey–West $t$-statistics are 5 and 11, respectively. Panel B reports the regression coefficients of predicting the portfolio returns using the market sentiment measure of Baker and Wurgler (2006). Panel C reports the average return of the portfolio within each disagreement–sentiment quadrant.

Panel A: Predict bear beta portfolio return with aggregate disagreement

| Return Horizon | | Speculative Stocks | | Non-speculative Stocks | |
|---|---|---|---|---|---|
| | Regressor | Coef | tstat | Coef | tstat |
| 1 month | Intercept | 0.061 | 1.732 | −0.002 | −0.085 |
| | Aggregate disagreement | −0.012 | −1.700 | −0.001 | −0.238 |
| 6 months | Intercept | 0.311 | 2.474 | 0.102 | 1.247 |
| | Aggregate disagreement | −0.062 | −2.551 | −0.030 | −1.956 |
| 12 months | Intercept | 0.450 | 2.537 | 0.202 | 1.390 |
| | Aggregate disagreement | −0.090 | −2.692 | −0.052 | −1.971 |

Panel B: Predict bear beta portfolio return with sentiment

| | | | | | |
|---|---|---|---|---|---|
| 1 month | Intercept | 0.000 | 0.016 | −0.008 | −2.084 |
| | Sentiment | −0.019 | −1.507 | −0.009 | −1.074 |
| 6 months | Intercept | −0.012 | −1.006 | −0.056 | −3.730 |
| | Sentiment | −0.097 | −2.748 | −0.047 | −2.384 |
| 12 months | Intercept | −0.019 | −0.863 | −0.070 | −1.737 |
| | Sentiment | −0.139 | −3.676 | −0.082 | −2.675 |

Panel C: Bear beta portfolio return within disagreement–sentiment quadrants

| Stock Type | Disagreement/sentiment | 1 month | | 6 months | | 12 months | |
|---|---|---|---|---|---|---|---|
| | | $\overline{R}$ | tstat | $\overline{R}$ | tstat | $\overline{R}$ | tstat |
| Speculative stocks | Low/low | −0.474 | −0.596 | −0.103 | −0.055 | 0.009 | 0.004 |
| | Low/high | 0.466 | 1.087 | −0.101 | −0.078 | −1.043 | −0.495 |
| | High/low | 0.273 | 0.569 | −0.567 | −0.543 | 0.781 | 0.439 |
| | High/high | −2.327 | −1.363 | −13.106 | −3.653 | −21.196 | −4.643 |
| Non-speculative stocks | Low/low | −1.234 | −1.386 | −6.525 | −3.591 | −8.356 | −3.426 |
| | Low/high | −1.299 | −2.367 | −5.898 | −4.028 | −9.333 | −4.124 |
| | High/low | −0.120 | −0.197 | −1.994 | −1.206 | 1.196 | 0.288 |
| | High/high | −1.568 | −1.007 | −13.856 | −4.792 | −22.050 | −4.605 |

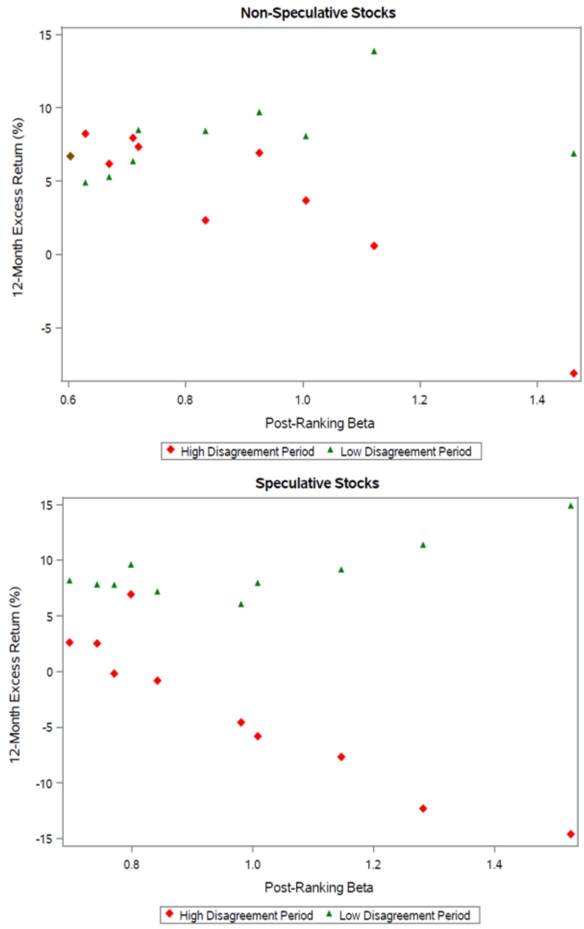


**Figure 1.** Speculative stocks versus non-speculative stocks. This figure plots the relationship between the 12-month returns of the decile portfolios sorted on $\beta_{\text{Put},L}$ and their post-formation market betas. The sample period is sorted into quartiles based on aggregate disagreement and the high-disagreement (low-disagreement) period refers to the top (bottom) quartile. The monthly aggregate disagreement measure is the beta-weighted cross-sectional average of the SDs of analyst forecasts of long-term earnings growth rates reported in I/B/E/S unadjusted summary file. Speculative(non-speculative) stocks' $\hat{\beta}/\hat{\sigma^2}$ ratios are above (below) the NYSE median, where $\hat{\sigma^2}$ is the estimated idiosyncratic variance.

Next, we divide the sample period into four quadrants based on the aggregate disagreement and sentiment to examine the performance of the long–short bear beta portfolio within each quadrant. The average portfolio returns formed on speculative and non-speculative stocks are reported separately. Panel C of the table shows that the lowest return of the portfolio typically follows the months in the high sentiment and high disagreement quadrant, suggesting the interaction between divergence in opinions and overconfidence is an important cause of the anomaly.

## 8.2 Preference for Skewness

In a recent study, Schneider, Wagner, and Zechner (2020) propose a novel explanation for several low-risk anomalies. They show that factors formed on *ex ante* skewness can explain the abnormal returns. We add their factors to the Fama–French–Carhart four-factor model and calculate the alphas of the H−L portfolios formed on bear betas with the 1-month holding period. To distinguish the current anomaly from the BaB anomaly, we also include the BaB factor of Frazzini and Pedersen (2014).

The regression coefficients are reported in Table VII, where all the alphas are measured at the monthly frequency and reported as percentages. The first column only includes the four factors and the BaB factor, and the significant alpha confirms that the current anomaly is not a restatement of the BaB anomaly.

The next three columns introduce the Skew factor (SK), Lower-Skew-Minus-Upper-Skew factor, and Lower-Skew and Upper-Skew factors (LSK and USK), respectively. Column (4) shows that including both LSK and USK factors reduces the magnitude of the

**Table VII.** Controlling for *ex ante* skewness, FMAX, and SMR factors

This table reports the coefficients of regressing the monthly excess returns of the H−L portfolio sorted on bear betas on the Fama–French factors ($r_m$, SMB, and HML), Carhart factor (UMD), BaB factor (BaB), *ex ante* skewness factors (SK, LUSK, USK, and LSK), MAX factor (FMAX), and SMR factor. *t*-statistics adjusted for heteroskedasticity are in the parentheses.

| | [1] | [2] | [3] | [4] | [5] | [6] | [7] |
|---|---|---|---|---|---|---|---|
| $\alpha$ | −1.03 | −1.08 | −1.02 | −0.85 | −0.73 | −0.89 | −0.69 |
| | $(-2.8^{***})$ | $(-2.68^{***})$ | $(-2.5^{**})$ | $(-2.16^{**})$ | $(-2.08^{**})$ | $(-2.28^{**})$ | $(-1.81^{*})$ |
| $r_m$ | 0.41 | 0.29 | 0.18 | 0.14 | 0.10 | 0.31 | 0.00 |
| | $(4.87^{***})$ | $(2.67^{***})$ | (1.54) | (1.37) | (1) | $(3.15^{***})$ | (0.01) |
| SMB | 0.61 | 0.32 | 0.16 | 0.16 | 0.25 | 0.36 | −0.02 |
| | $(5.06^{***})$ | $(2.07^{**})$ | (1.03) | (1.01) | $(1.72^{*})$ | $(2.23^{**})$ | (−0.11) |
| HML | −0.46 | −0.67 | −0.53 | −0.51 | −0.25 | −0.43 | −0.37 |
| | $(-2.77^{***})$ | $(-3.66^{***})$ | $(-2.59^{**})$ | $(-2.48^{**})$ | (−1.65) | $(-2.23^{**})$ | $(-1.95^{*})$ |
| UMD | 0.01 | 0.21 | 0.24 | 0.24 | −0.02 | 0.04 | 0.17 |
| | (0.05) | (1.64) | $(1.89^{*})$ | $(1.85^{*})$ | (−0.2) | (0.36) | (1.4) |
| BaB | −0.32 | −0.12 | −0.10 | −0.09 | −0.02 | −0.27 | 0.04 |
| | $(-2.15^{**})$ | (−0.83) | (−0.65) | (−0.62) | (−0.12) | $(-1.72^{*})$ | (0.21) |
| SK | | −0.14 | – | – | | | – |
| | | $(-2.37^{**})$ | – | – | – | – | – |
| LUSK | – | – | −0.14 | – | – | – | – |
| | – | – | $(-3.04^{***})$ | – | – | – | – |
| USK | – | – | – | 0.09 | – | – | 0.03 |
| | – | – | – | (0.43) | – | – | (0.16) |
| LSK | – | – | – | −0.39 | – | – | −0.24 |
| | – | – | – | $(-2.03^{**})$ | – | – | (−1.16) |
| FMAX | – | – | – | – | 0.55 | – | 0.33 |
| | – | – | – | – | $(3.79^{***})$ | – | $(1.88^{*})$ |
| SMR | – | – | – | – | – | −0.24 | −0.08 |
| | – | – | – | – | – | $(-2.67^{***})$ | (−0.79) |
| Adj. $R^2$ | 0.451 | 0.495 | 0.524 | 0.531 | 0.504 | 0.489 | 0.550 |

$^{*}p < 0.1$,
$^{**}p < 0.05$,
$^{***}p < 0.01$.

alpha of the H−L portfolio by 18 bps per month compared to Column (1), but the alpha remains significant. Therefore, the theory of Schneider, Wagner, and Zechner (2020) provides a partial explanation of the anomaly.

Bali *et al.* (2017) show that a factor based on past maximum stock returns can help resolve the BaB puzzle, and they attribute the BaB anomaly to lottery preference. Column (5) reports the alpha after including the MAX factor. Compared to Column (1), the MAX factor reduces the magnitude of the alpha by 30 bps per month, suggesting that lottery preference plays an important role in the current anomaly.

Kapadia *et al.* (2019) form a portfolio that longs stocks with low systematic risk and shorts stocks with high systematic risk and find that this portfolio has a high expected return. Clearly, their finding shares the same spirit as ours in documenting a negative risk–reward relationship. To examine the relation between the two anomalies, we include their SMR portfolio in the factor model and report the coefficients in Column (6). Compared to Column (1), the magnitude of the alpha drops by about 14 pbs to −0.89%, suggesting some degree of overlap between the two anomalies.

Finally, Column (7) shows that including LSK, USK, MAX, and SMR in the same factor model barely further reduces the alpha compared to the model that only includes FMAX (Column (5)), which suggests that there is also some overlap among the explanations of Schneider, Wagner, and Zechner (2020), Bali *et al.* (2017), and Kapadia *et al.* (2019), and their common component is likely related to skewness or lottery preference.

## 9. Discussion and Robustness Analysis

In this section, we first discuss the implications of the time-varying market beta model implied by Equation (8). Then, we conduct two robustness analyses. In the first robustness analysis, we include all the three components of the bear risk factor in the same factor model to estimate stocks' factor loadings and then use these loadings to sort stocks into portfolios. In the second one, we use Fama and MacBeth (1973) cross-sectional regressions to ensure that our results are robust to the inclusion of various control variables. In the Supplementary Appendix, we also confirm that our conclusions are valid in all the liquidity and size subsamples.

### 9.1 Time-Varying Betas and Conditional CAPM

Equation (8), $r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{Put,L,i} \cdot f(\sigma_t) \cdot r_{SP} + e_i$, implies a form of time-varying betas on stocks. To calculate the betas implied by this equation, we estimate $\beta_{m,i}$ and $\beta_{Put,L,i}$ at the end of Month $t$ for stock i using the lagged 12 months. Then, we evaluate $f(\sigma_t)$ using the marketwide information at the end of month $t$. Finally, we compute the dynamic beta for stock i at the end of Month $t$ as: $\beta_{i,\text{dynamic}} = \hat{\beta_{m,i}} + \hat{\beta_{\text{Put},L,i}} \cdot f(\sigma_t)$. Equipped with these betas, we examine the relationship between betas and expected stock returns, proxied by stock returns in Month $t+1$, using Fama–MacBeth regressions. we also perform the same exercise with simple $\beta_{\text{CAPM}}$ estimated using 12-month rolling windows. This beta is called static beta in the table. Table VIII, Panel A reports the estimated lambdas for the dynamic and static beta. The static beta delivers a negative market risk premium of −14.1 bps per month, while the dynamic beta generates a positive market risk premium of 22.1 bps per month. Although the latter risk premium is still not statistically significant and is only one-third of the empirical market risk premium over the sample (64 bps per month), it does represent an economically meaningful improvement over the traditional method to measure betas. This encouraging finding points to

**Table VIII.** Dynamic beta

Panel A reports the estimated market risk premium from the cross-section of stocks. Dynamic betas are estimated based on the equation: $r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\text{Put},L,i} \cdot f(\sigma_t) \cdot r_{\text{SP}} + e_i$. To calculate the dynamic betas, we estimate $\beta_{m,i}$ and $\beta_{\text{Put},L,i}$ at the end of Month *t* for stock i using the lagged 12 months. Then, we evaluate $f(\sigma_t)$ using the marketwide information at the end of Month *t*. Finally, we compute the dynamic beta for stock i at the end of Month *t* as: $\beta_{i,\text{dynamic}} = \hat{\beta_{m,i}} + \hat{\beta_{\text{Put},L,i}} \cdot f(\sigma_t)$. The static betas are estimated using the same 12-month windows with the standard CAPM model. The risk premium is estimated using Fama–MacBeth regressions of Month $t+1$ stock returns on Month *t* betas. Panel B reports the regression coefficients of three long–short anomaly returns—momentum, profitability, and investment—on the Fama–French three factors, or the three factors augmented by long–short portfolio sorted on the dynamic betas.

Panel A: Estimate market risk premium from the cross-section

| Beta | $\hat{\lambda}$ | tstat |
|---|---|---|
| Dynamic beta | 0.221 | 0.858 |
| Static beta | −0.141 | −0.481 |

Panel B: Explain anomaly returns

| | Momentum | Momentum | Profitability | Profitability | Investment | Investment |
|---|---|---|---|---|---|---|
| $\alpha$ | 1.30 | 1.11 | 0.72 | 0.65 | 0.37 | 0.38 |
| | (2.87) | (2.23) | (3.05) | (2.68) | (1.98) | (2.02) |
| Mkt-Rf | −0.70 | −0.28 | −0.51 | −0.37 | −0.16 | −0.18 |
| | (−3.66) | (−1.79) | (−7.68) | (−4.14) | (−2.62) | (−2.89) |
| SMB | 0.13 | 0.18 | −0.64 | −0.62 | 0.17 | 0.16 |
| | (0.61) | (0.82) | (−7.13) | (−7.32) | (1.82) | (1.75) |
| HML | −0.64 | −0.63 | 0.34 | 0.34 | 0.58 | 0.58 |
| | (−1.91) | (−1.91) | (3.21) | (3.32) | (5.4) | (5.37) |
| H−L | – | −0.32 | – | −0.11 | – | 0.02 |
| | – | (−2.27) | – | (−2.27) | – | (0.38) |
| Adj. $R^2$ | 0.150 | 0.199 | 0.571 | 0.588 | 0.308 | 0.309 |

the potential benefit of modeling time-varying betas as a function of market information that is available at a reasonably high frequency. This method could yield timely measures of systematic risk and improve pricing performance.

In Panel B, we add the long–short portfolio sorted on the dynamic betas to the Fama–French three-factor model and examine its ability to explain the abnormal returns of three prominent anomalies that remain strong in the sample period, namely, momentum, profitability, and investment. The anomaly portfolios are the long–short value-weighted decile portfolios obtained from Ken French's data library. Overall, adding the new factor has little impact on the abnormal returns of the anomalies, suggesting that time-varying betas associated with IV are not the main contribution to these anomalies.

## 9.2 Alternative Factor Models

In calculating the betas with respect to the option-based factors, we follow LM's setting and use two-factor models that include the market factor and one option-based factor at a

**Table IX.** Including the three components of bear risk factor in one model

This table reports the average returns and Fama–French–Carhart alphas of the value-weighted stock portfolios sorted on betas with respect to the three components of bear risk factor returns. Stocks are sorted into deciles using each beta; only the results for the H−L portfolios are reported for brevity. Stock betas are calculated following the regression model: $r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\mathrm{Vol},i} \cdot \text{volatility factor} + \beta_{\mathrm{Bear},L,i} \cdot r_{\mathrm{Bear},L} + \beta_{\mathrm{Bear,NL},i} \cdot r_{\mathrm{Bear,NL}} + e_i$, where $r_i$ is the return of stock i, $r_m$ is the market return, and $r_{\mathrm{Bear},L}$ and $r_{\mathrm{Bear,NL}}$ are the linear and nonlinear index-driven components of bear risk factor. The volatility factor is chosen to be the IV-driven component of the bear risk factor to produce the results in the first two columns and the changes in VIX to produce the last two columns. The regression models are estimated using the prior 12 months of overlapping 5-day returns at the end of each month. Columns titled "H−L" are returns or alphas. Columns titled "*t*-Stat" are Newey–West *t*-statistics with one lag.

| Return Horizon | | IV-driven bear Factor as Vol factor | | Change in VIX as Vol factor | |
|---|---|---|---|---|---|
| | | H−L | *t*-Stat | H−L | *t*-Stat |
| H−L portfolio sorted on $\beta_{\mathrm{Vol}}$ | | | | | |
| 1 month | $\overline{R}$ | −0.28 | −0.99 | −0.58 | −1.29 |
| | $\alpha$ | −0.49 | −1.73 | −0.95 | −2.69 |
| 12 months | $\overline{R}$ | 0.10 | 0.54 | −0.24 | −0.69 |
| | $\alpha$ | 0.03 | 0.14 | −0.42 | −1.80 |
| H−L portfolio sorted on $\beta_{\mathrm{Bear},L}$ | | | | | |
| 1 month | $\overline{R}$ | −0.80 | −1.62 | −1.14 | −2.28 |
| | $\alpha$ | −1.10 | −3.13 | −1.48 | −4.24 |
| 12 months | $\overline{R}$ | −0.55 | −1.37 | −0.65 | −1.63 |
| | $\alpha$ | −0.61 | −2.18 | −0.74 | −2.61 |
| H−L portfolio sorted on $\beta_{\mathrm{Bear,NL}}$ | | | | | |
| 1 month | $\overline{R}$ | 0.07 | 0.20 | 0.02 | 0.06 |
| | $\alpha$ | 0.20 | 0.64 | 0.18 | 0.64 |
| 12 months | $\overline{R}$ | −0.09 | −0.38 | −0.01 | −0.04 |
| | $\alpha$ | 0.02 | 0.08 | 0.11 | 0.55 |

time. A potential concern is whether the results change if we include all the three components of an option-based factor in the same model to form a four-factor model and calculate the factor betas all at once. The regression model can be written as

$$\begin{aligned} r_i = a_i + \beta_{m,i} \cdot r_m + \beta_{\mathrm{Bear,IV},i} \cdot r_{\mathrm{Bear,IV}} \\ + \beta_{\mathrm{Bear},L,i} \cdot r_{\mathrm{Bear},L} + \beta_{\mathrm{Bear,NL},i} \cdot r_{\mathrm{Bear,NL}} + e_i, \end{aligned} \tag{9}$$

where $r_i$ is the return of stock i, $r_m$ is the market return, $r_{\mathrm{Bear},L}$ and $r_{\mathrm{Bear,NL}}$ are the linear and nonlinear components of the bear risk factor, and $r_{\mathrm{Bear,IV}}$ is the IV-driven component.

The resulting portfolio returns and alphas are reported in the first two columns of Table IX. For brevity, we only include the results on the H−L portfolios. Consistent with our analysis using two-factor models, only the portfolio sorted on $r_{\mathrm{Bear},L}$ has significant abnormal returns.

So far in the article, we use the IV-driven option returns as the factor that captures volatility shocks. This is somewhat different from the common practice in previous studies such as Ang *et al.* (2006), where changes in volatilities are treated as a factor. To understand the differences between the two choices, we replace $r_{Bear,IV}$ in Equation (9) by changes in the VIX index and report the results in the last two columns of Table IX. Interestingly, the long–short portfolio sorted on VIX betas has a significant alpha of −0.95% per month for the 1-month holding period, which confirms the existing findings in the volatility risk premium literature. This result also shows that the empirical measures of the volatility risk premium are somewhat sensitive to the choice of the volatility risk factor. Finally, it is worth noting that the alphas of the portfolio sorted on $r_{\text{Bear},L}$ also become more significant in this setting, which means once the volatility risk premium is better accounted for, the current puzzle becomes even deeper.

### 9.3 Cross-sectional Regressions

To control for other well-established stock return predictors, we run cross-sectional regressions of monthly stock returns on their option-factor betas and control variables, implementing the method of Fama and MacBeth (1973). The regression coefficients are reported in Table X. The main predictors include the bear beta and the betas with respect to the three components of the bear risk factor calculated using Equation (9). There are two sets of control variables. Control 1 includes one-factor CAPM beta, market capitalization, book-to-equity ratio, lagged 12-month stock return, idiosyncratic volatility, Amihud illiquidity, annual growth in total assets, operating profitability, downside market beta, VIX beta, and risk-neutral skewness beta. Control 2 includes co-skewness beta, tail risk beta, and jump risk beta. Due to space limit, the table omits the coefficients on the control variables, which can be found in the Supplementary Appendix. Further details of variable construction are also in the Supplementary Appendix. The regressions listed under the same column have the same control variables even though we only label the controls for the first set of regressions. All the independent variables are winsorized at 0.5% and 99.5% levels.

The top section of the table reports the coefficients of predicting 1-month-ahead stock returns. The option-factor betas are available from December 1996 to December 2017, which determines our full sample period used in Models 1, 2, 4, and 5. The control variables in Control 2 are provided by the authors of the original papers and have shorter sample periods than ours. Models 3 and 6 include these controls, and the sample period for these two models is December 1996 to December 2007. Models 1, 2, and 3 show that bear beta is a significant predictor in the full sample with Control 1. Models 4, 5, and 6 show that betas with respect to the linear component are significant in the full sample but not in the shorter sample, while the other two components are always insignificant.

The next three sections of the table report the results of predicting stock returns that are 3, 6, and 12 months ahead. In predicting 3- and 6-month ahead stock returns, both bear betas and betas on the linear component are significant in all the settings. Interestingly, Models 4 and 5 show that the coefficient on the beta with respect to the nonlinear component can be significant when it is jointly used with the other predictors, but only at the 6-month horizon. This probably should only be considered weak evidence for supporting downside risk premium given the lack of robustness. The results on predicting 12-month-ahead returns show that all the option-factor betas become insignificant. Overall, the evidence in this table confirms the findings based on portfolio analysis.

## 10. Conclusion

Implementing a three-way decomposition of index option returns, we show that only betas on one of the three components of the bear risk factor are significantly correlated with expected stock returns. Because the component responsible for the pricing power is a linear

**Table X.** Cross-sectional regressions

This table reports the coefficients of the cross-sectional regressions of monthly stock returns on the option-factor betas and control variables using the method of Fama and MacBeth (1973). The cross-sectional regressions are performed at a monthly frequency. The main predictors include the bear beta and the betas with respect to the three components of the bear risk factor—$\beta_{\text{Bear,IV}}$, $\beta_{\text{Bear},L}$, and $\beta_{\text{Bear,NL}}$. There are two sets of control variables. Control 1 includes one-factor $\beta_{\text{CAPM}}$, market capitalization, book-to-equity ratio, lagged 12-month stock return, idiosyncratic volatility, Amihud illiquidity, corporate investments, operating profitability, downside market beta, VIX beta, risk-neutral skewness beta. Control 2 includes co-skewness beta, tail risk beta, and jump risk beta. All the independent variables are winsorized at 0.5 and 99.5% levels. *t*-statistics based on the Newey–West method with twelve lags are in the parentheses.

| | [1] | [2] | [3] | [4] | [5] | [6] |
|---|---|---|---|---|---|---|
| 1 month | | | | | | |
| $\beta_{\text{Bear}}$ | −0.30 | −0.24 | −0.20 | – | – | – |
| | (−1.65) | (−2.55$^{**}$) | (−1.41) | – | – | – |
| $\beta_{\text{Bear,IV}}$ | – | – | – | 0.22 | 0.25 | 0.45 |
| | – | – | – | (0.76) | (1.3) | (1.54) |
| $\beta_{\text{Bear},L}$ | – | – | – | −0.22 | −0.16 | −0.11 |
| | – | – | – | (−2.19$^{**}$) | (−2.62$^{***}$) | (−1.18) |
| $\beta_{\text{Bear,NL}}$ | – | – | – | 0.07 | −0.10 | −0.13 |
| | – | – | – | (0.28) | (−0.54) | (−0.5) |
| Control 1 | – | X | X | – | X | X |
| Control 2 | – | – | X | – | – | X |
| 3 months | | | | | | |
| $\beta_{\text{Bear}}$ | −0.40 | −0.32 | −0.40 | | | |
| | (−2.45$^{**}$) | (−4.02$^{***}$) | (−3.53$^{***}$) | | | |
| $\beta_{\text{Bear,IV}}$ | – | – | – | 0.19 | 0.23 | 0.07 |
| | – | – | – | (0.7) | (1.42) | (0.37) |
| $\beta_{\text{Bear},L}$ | – | – | – | −0.25 | −0.19 | −0.22 |
| | – | – | – | (−2.85$^{***}$) | (−3.82$^{***}$) | (−3.11$^{***}$) |
| $\beta_{\text{Bear,NL}}$ | – | – | – | −0.10 | −0.21 | −0.19 |
| | – | – | – | (−0.43) | (−1.48) | (−0.89) |
| 6 months | | | | | | |
| $\beta_{\text{Bear}}$ | −0.53 | −0.41 | −0.38 | – | – | – |
| | (−2.85$^{***}$) | (−4.14$^{***}$) | (−2.3$^{**}$) | – | – | – |
| $\beta_{\text{Bear,IV}}$ | – | – | – | 0.05 | 0.04 | 0.01 |
| | – | – | – | (0.18) | (0.26) | (0.06) |
| $\beta_{\text{Bear},L}$ | – | – | – | −0.25 | −0.20 | −0.18 |
| | – | – | – | (−2.33$^{**}$) | (−3.27$^{***}$) | (−2.47$^{**}$) |
| $\beta_{\text{Bear,NL}}$ | – | – | – | −0.45 | −0.53 | −0.33 |
| | – | – | – | (−1.9$^{*}$) | (−3.4$^{***}$) | (−1.19) |
| 12 months | | | | | | |
| $\beta_{\text{Bear}}$ | −0.14 | −0.05 | −0.19 | – | – | – |
| | (−1.16) | (−0.61) | (−1.44) | – | – | – |
| $\beta_{\text{Bear,IV}}$ | – | – | – | −0.06 | 0.01 | −0.12 |
| | – | – | – | (−0.31) | (0.09) | (−0.57) |
| $\beta_{\text{Bear},L}$ | – | – | – | −0.04 | 0.00 | −0.05 |
| | – | – | – | (−0.5) | (0.02) | (−0.74) |
| $\beta_{\text{Bear,NL}}$ | – | – | – | −0.04 | 0.08 | −0.05 |
| | – | – | – | (−0.2) | (0.6) | (−0.22) |

$^{*}p < 0.1$,
$^{**}p < 0.05$,
$^{***}p < 0.01$.

function in the underlying index return conditioning on the lagged IVs, the beta–return relationship cannot be attributed to premia associated with downside risk protection or time-varying investment opportunities. The market risk exposures of the stocks with high loadings on the linear component are not only high on average but also increasing in the index options' IVs. The low expected returns of such stocks, therefore, can be viewed as a variation of the well-known BaB puzzle.

Following Levi and Welch (2019) and Barahona, Driessen, and Frehen's (2021) refusals of *ex post* downside risk being priced in the cross-section of stocks, we overturn Lu and Murray's (2019) interpretation of their findings as support for the pricing of *ex ante* downside risk in the cross-section. In addition, we find no evidence that the bear risk factor or the tail risk factor of Kelly and Jiang (2014) help select stocks with downside or tail risk hedging properties. Therefore, the observed relationship between factor betas and expected returns does not fit rational frameworks where investors pay a premium to hold stocks that hedge against these risks. We claim that the empirical literature has yet to produce evidence to support the pricing of any form of downside risk in the cross-section of stocks.

Consistent with the view of Levi and Welch (2019) and Barahona, Driessen, and Frehen (2021), we find that the asymmetry in upside and downside market betas is very difficult to predict, though we do find moderate predictability of this asymmetry using loadings on the linear and non-linear index-driven components of the bear risk factor or put option returns, which suggests that future empirical studies could potentially develop techniques to better forecast the asymmetry. Such predictability would allow asset managers to more precisely tailor the risk profile of their portfolios.

The abnormal return associated with the linear factor is an interesting asset pricing puzzle in its own right. Although it is conceptually related to the BaB puzzle, controlling for the BaB factor does not affect the abnormal return. After exploring various resolutions, we find that disagreement about aggregate market returns is likely the most important contributor to the anomaly.

Although this article focuses on analyzing the downside and bear risk premia, the research methodology we develop will benefit future research utilizing risk factors constructed from option returns. Option-based factors are not only important for testing asset pricing theories but also for evaluating the performance of financial institutions such as hedge funds. Therefore, a clear understanding of why betas with respect to these factors predict stock returns is important to correctly assess fund performance.

## Supplementary Material

Supplementary data are available at *Review of Finance* online.

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## Authors' Contribution

Tong Wang is the sole author of the paper. He conceived and carried out the empirical analysis and wrote the manuscript.

## Conflict of Interest Statement

None declared.

## References


Agarwal, V., Arisoy, Y. E., and Naik, N. Y. (2017): Volatility of aggregate volatility and hedge fund returns, *Journal of Financial Economics* 125, 491–510.

An, B. J., Ang, A., Bali, T. G., and Cakici, N. (2014): The joint cross section of stocks and options, *The Journal of Finance* 69, 2279–2337.

Andersen, T. G., Fusari, N., and Todorov, V. (2015): The risk premia embedded in index options, *Journal of Financial Economics* 117, 558–584.

Ang, A., Chen, J., and Xing, Y. (2006): Downside risk, *The Review of Financial Studies* 19, 1191–1239.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006): The cross-section of volatility and expected returns, *The Journal of Finance* 61, 259–299.

Baker, M. and Wurgler, J. (2006): Investor sentiment and the cross-section of stock returns, *The Journal of Finance* 61, 1645–1680.

Baker, S. R., Bloom, N., Davis, S. J., Kost, K., Sammon, M., and Viratyosin, T. (07 2020): The unprecedented stock market reaction to COVID-19, *The Review of Asset Pricing Studies* 10, 742–758.

Bakshi, G. and Madan, D. (2000): Spanning and derivative-security valuation, *Journal of Financial Economics* 55, 205–238.

Bakshi, G., Kapadia, N., and Madan, D. (2003): Stock return characteristics, skew laws, and the differential pricing of individual equity options, *Review of Financial Studies* 16, 101–143.

Bakshi, G., Panayotov, G., and Skoulakis, G. (2011): Improving the predictability of real economic activity and asset returns with forward variances inferred from option portfolios, *Journal of Financial Economics* 100, 475–495.

Bali, T. G., Brown, S. J., Murray, S., and Tang, Y. (2017): A lottery-demand-based explanation of the beta anomaly, *Journal of Financial and Quantitative Analysis* 52, 2369–2397.

Barahona, R., Driessen, J., and Frehen, R. (2021): Can unpredictable risk exposure be priced?, *Journal of Financial Economics* 139, 522–544.

Barinov, A. (2018): Stocks with extreme past returns: lotteries or insurance? *Journal of Financial Economics* 129, 458–478.

Barras, L. and Malkhozov, A. (2016): Does variance risk have two prices? Evidence from the equity and option markets, *Journal of Financial Economics* 121, 79–92.

Bawa, V. S. and Lindenberg, E. B. (1977): Capital market equilibrium in a mean-lower partial moment framework, *Journal of Financial Economics* 5, 189–200.

Black, F. and Scholes, M. (1973): The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.

Boguth, O., Carlson, M., Fisher, A., and Simutin, M. (2011): Conditional risk and performance evaluation: volatility timing, overconditioning, and new estimates of momentum alphas, *Journal of Financial Economics* 102, 363–389.

Bollerslev, T., Tauchen, G., and Zhou, H. (2009): Expected stock returns and variance risk premia, *The Review of Financial Studies* 22, 4463–4492.

Bollerslev, T., Todorov, V., and Xu, L. (2015): Tail risk premia and return predictability, *Journal of Financial Economics* 118, 113–134.

Buss, A. and Vilkov, G. (2012): Measuring equity risk with option-implied correlations, *The Review of Financial Studies* 25, 3113–3140.

Campbell, J. Y. and Vuolteenaho, T. (2004): Bad beta, good beta, *American Economic Review* 94, 1249–1275.

Cao, J., Goyal, A., Xiao, X., and Zhan, X. (2021): Implied volatility changes and corporate bond returns, *Management Science* (Forthcoming).

Carr, P. and Wu, L. (2008): Variance risk premiums, *The Review of Financial Studies* 22, 1311–1341.

Chang, B. Y., Christoffersen, P., Jacobs, K., and Vainberg, G. (2011): Option-implied measures of equity risk, *Review of Finance* 16, 385–428.

Chang, B. Y., Christoffersen, P., and Jacobs, K. (2013): Market skewness risk and the cross section of stock returns, *Journal of Financial Economics* 107, 46–68.

Chordia, T., Goyal, A., and Saretto, A. (2020): Anomalies and false rejections, *The Review of Financial Studies* 33, 2134–2179.

Cremers, M., Halling, M., and Weinbaum, D. (2015): Aggregate jump and volatility risk in the cross-section of stock returns, *The Journal of Finance* 70, 577–614.

Dimson, E. (1979): Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 7, 197–226.

Fama, E. F. and MacBeth, J. D. (1973): Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 81, 607–636.

Fan, Z., Xiao, X., and Zhou, H. (2022): Moment risk premia and stock return predictability, *Journal of Financial and Quantitative Analysis* 57, 67–93.

Frazzini, A. and Pedersen, L. H. (2014): Betting against beta, *Journal of Financial Economics* 111, 1–25.

Fung, W. and Hsieh, D. A. (2015): The risk in hedge fund strategies: theory and evidence from trend followers. *The Review of Financial Studies* 14, 313–341.

Gabaix, X. (2012): Variable rare disasters: an exactly solved framework for ten puzzles in macro--finance, *The Quarterly Journal of Economics* 127, 645–700.

Glosten, L. and Jagannathan, R. (1994): A contingent claim approach to performance evaluation, *Journal of Empirical Finance* 1, 133–160.

Goyal, A. and Saretto, A. (2009): Cross-section of option returns and volatility, *Journal of Financial Economics* 94, 310–326.

Harvey, C. R., Liu, Y., Zhu, H. (2015): . . .and the Cross-section of expected returns, *The Review of Financial Studies* 29, 5–68.

Hong, H. and Sraer, D. A. (2016): Speculative betas, *The Journal of Finance* 71, 2095–2144.

Israelov, R. and Kelly, B. T. (2017): Forecasting the distribution of option returns. *SSRN:* https://ssrn.com/abstract=3033242 (accessed 7 September 2017).

Jegadeesh, N. and Titman, S. (1993): Returns to buying winners and selling losers: implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Jones, C. S. (2006): A nonlinear factor analysis of s&p 500 index option returns, *The Journal of Finance* 61, 2325–2363.

Kapadia, N., Ostdiek, B. B., Weston, J. P., and Zekhnini, M. (2019): Getting paid to hedge: why don't investors pay a premium to hedge downturns? *Journal of Financial and Quantitative Analysis* 54, 1157–1192.

Kelly, B. and Jiang, H. (2014): Tail risk and asset prices, *Review of Financial Studies* 27, 2841–2871.

Lettau, M., Maggiori, M., and Weber, M. (2014): Conditional risk premia in currency markets and other asset classes, *Journal of Financial Economics* 114, 197–225.

Levi, Y. and Welch, I. (2019): Symmetric and asymmetric market betas and downside risk, *The Review of Financial Studies* 33, 2772–2795.

Liu, J., Stambaugh, R. F., and Yuan, Y. (2018): Absolving beta of volatility's effects, *Journal of Financial Economics* 128, 1–15.

Lu, Z. and Murray, S. (2019): Bear beta, *Journal of Financial Economics* 131, 736–760.

Moreira, A. and Muir, T. (2017): Volatility-managed portfolios, *The Journal of Finance* 72, 1611–1644.

Newey, W. K. and West, K. D. (1987): A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Roy, A. (1952): Safety first and the holding of assets, *Econometrica* 20, 431–439.

Schneider, P. (2019): An anatomy of the market return, *Journal of Financial Economics* 132, 325–350.

Schneider, P., Wagner, C., and Zechner, J. (2020): Low-risk anomalies? *The Journal of Finance* 75, 2673–2718.

van Oordt, M. R. C. and Zhou, C. (2016): Systematic tail risk, *Journal of Financial and Quantitative Analysis* 51, 685–705.

Wachter, J. A. (2013): Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance* 68, 987–1035.

Welch, I. and Goyal, A. (2007): A comprehensive look at the empirical performance of equity premium prediction, *The Review of Financial Studies* 21, 1455–1508.

# Appendix

## A. Replace the Bear Factor with ATM Put Returns

So far in the article, we have focused on LM's bear risk factor. The fact that it involves long and short positions in multiple options with different strike prices makes interpreting our results less straightforward. Before proceeding with further analyses, we show that using returns on single ATM SPX puts as the factor generates nearly identical results as using the bear risk factor. Therefore, the added complexity of the bearish spreads is not necessary for understanding the bear risk premium. Using simple put returns as the factor not only permits easier interpretations of our findings, but also makes our conclusions generally applicable to any risk factors constructed from option returns.

To compute the put option returns, we select the monthly SPX put contract that expires in the following month and has the strike price closest to the index level. Table A1 reports the returns and betas of the portfolios sorted on betas with respect to the put returns. For brevity, only the results for the H−L portfolios are shown. The settings of the regressions are similar to Models (1)–(5), with the bear factor and its components replaced by the put

**Table A1.** Replace bear risk factor with put returns

The first three panels of this table report the returns and factor loadings of the stock portfolios sorted on the betas with respect to SPX put option returns and their two components. The table only reports the results for the H−L portfolios. Panel A reports the average portfolio returns and Fama–French–Carhart alphas. In the first (middle, last) two columns, stocks are sorted on betas with respect to SPX put returns (the index-driven component of put returns, the IV-driven component of put returns). Panel B reports the formation period factor loadings of the put-beta portfolios. $\beta_{\text{Put}}$ and $\beta_m$ are calculated using the two-factor models. $\beta_{\text{Put}}$ is replaced by $\beta_{\text{Put,Index}}$ and $\beta_{\text{Put,IV}}$ in the middle and last two columns, respectively. $\beta_{\text{CAPM}}$ is the one-factor CAPM beta. $\beta_{\text{MarketDown}}$ and $\beta_{\text{MarketUp}}$ are the $\beta_{\text{CAPM}}$ measured when market excess returns are negative and positive, respectively. Panel C reports the holding period factor loadings. Panel D reports the returns and alphas of the bear beta portfolios formed using double-sorts to control for put betas. Columns titled "*t*-Stat" are Newey–West *t*-statistics with one lag in Panels A and D, and eleven lags in Panel B. The *t*-statistics in Panel C are adjusted for heteroskedasticity.

Panel A: Put-beta portfolio returns

| Holding Period | | $\beta_{\text{Put}}$ | | $\beta_{\text{Put,Index}}$ | | $\beta_{\text{Put,IV}}$ | | $\beta_{\text{Put},L}$ | | $\beta_{\text{Put,NL}}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | H−L | *t*-Stat | H−L | *t*-Stat | H−L | *t*-Stat | H−L | *t*-Stat | H−L | *t*-Stat |
| 1 month | $\overline{R}$ | −1.02 | −2.14 | −1.10 | −2.26 | −0.07 | −0.21 | −1.01 | −1.96 | 0.09 | 0.24 |
| | $\alpha$ | −1.27 | −3.43 | −1.31 | −3.35 | −0.19 | −0.58 | −1.36 | −3.54 | 0.17 | 0.51 |
| 12 months | $\overline{R}$ | −0.82 | −2.08 | −0.81 | −2.05 | 0.27 | 1.18 | −0.78 | −1.89 | 0.02 | 0.09 |
| | $\alpha$ | −0.94 | −3.20 | −0.93 | −3.11 | 0.17 | 0.73 | −0.94 | −3.32 | 0.01 | 0.07 |

Panel B: Formation-period betas

| | $\beta_{\text{Put}}$ | | $\beta_{\text{Put,Index}}$ | | $\beta_{\text{Put,IV}}$ | | $\beta_{\text{Put},L}$ | | $\beta_{\text{Put,NL}}$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| Beta | H−L | *t*-Stat | H−L | *t*-Stat | H−L | *t*-Stat | H−L | *t*-Stat | H−L | *t*-Stat |
| Option beta | 1.87 | 17.71 | 2.13 | 15.08 | 0.69 | 21.19 | 3.18 | 44.42 | 0.69 | 83.77 |

(continued)

**Table A1.** Continued

Panel B: Formation-period betas

| Beta | $\beta_{\text{Put}}$ H–L | $\beta_{\text{Put}}$ *t*-Stat | $\beta_{\text{Put,Index}}$ H–L | $\beta_{\text{Put,Index}}$ *t*-Stat | $\beta_{\text{Put,IV}}$ H–L | $\beta_{\text{Put,IV}}$ *t*-Stat | $\beta_{\text{Put},L}$ H–L | $\beta_{\text{Put},L}$ *t*-Stat | $\beta_{\text{Put,NL}}$ H–L | $\beta_{\text{Put,NL}}$ *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_m$ | 2.55 | 16.26 | 2.80 | 14.90 | 0.37 | 5.58 | 4.04 | 43.31 | 0.21 | 6.45 |
| $\beta_{\text{CAPM}}$ | 0.63 | 7.04 | 0.63 | 6.69 | 0.16 | 2.88 | 0.63 | 18.48 | 0.10 | 3.66 |
| $\beta_{\text{MarketDown}}$ | 0.60 | 3.52 | 0.62 | 3.56 | 0.04 | 0.51 | 0.80 | 16.60 | −0.51 | −12.28 |
| $\beta_{\text{MarketUp}}$ | 1.40 | 10.08 | 1.37 | 9.61 | 0.45 | 4.64 | 0.92 | 18.56 | 0.78 | 20.35 |

Panel C: Holding-period betas

| Beta | $\beta_{\text{Put}}$ H–L | $\beta_{\text{Put}}$ *t*-Stat | $\beta_{\text{Put,Index}}$ H–L | $\beta_{\text{Put,Index}}$ *t*-Stat | $\beta_{\text{Put,IV}}$ H–L | $\beta_{\text{Put,IV}}$ *t*-Stat | $\beta_{\text{Put,L}}$ H–L | $\beta_{\text{Put,L}}$ *t*-Stat | $\beta_{\text{Put,NL}}$ H–L | $\beta_{\text{Put,NL}}$ *t*-Stat |
|---|---|---|---|---|---|---|---|---|---|---|
| Option beta | 0.28 | 4.85 | 0.29 | 4.72 | 0.01 | 0.31 | 0.38 | 4.97 | 0.04 | 1.27 |
| $\beta_m$ | 0.80 | 10.24 | 0.79 | 9.57 | 0.17 | 5.51 | 0.87 | 8.75 | 0.11 | 2.74 |
| $\beta_{\text{CAPM}}$ | 0.56 | 14.05 | 0.54 | 13.14 | 0.16 | 5.57 | 0.54 | 12.69 | 0.10 | 2.65 |
| $\beta_{\text{MarketDown}}$ | 0.59 | 7.52 | 0.62 | 7.58 | 0.09 | 1.47 | 0.64 | 8.06 | 0.06 | 0.65 |
| $\beta_{\text{MarketUp}}$ | 0.50 | 5.93 | 0.45 | 5.15 | 0.17 | 2.76 | 0.36 | 3.91 | 0.24 | 3.54 |

Panel D: Double-sorted bear beta portfolios controlling for put betas

| Holding period | | Sorted on $\beta_{\text{Bear}}$ | | Sorted on $\beta_{\text{Bear,Index}}$ | | Sorted on $\beta_{\text{Bear,IV}}$ | | Sorted on $\beta_{\text{Bear,L}}$ | | Sorted on $\beta_{\text{Bear,NL}}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Controlling for $\beta_{\text{Put}}$ | | Controlling for $\beta_{\text{Put,Index}}$ | | Controlling for $\beta_{\text{Put,IV}}$ | | Controlling for $\beta_{\text{Put,L}}$ | | Controlling for $\beta_{\text{Put,NL}}$ | |
| | | H–L | *t*-Stat | H–L | *t*-Stat | H–L | *t*-Stat | H–L | *t*-Stat | H–L | *t*-Stat |
| 1 month | $\overline{R}$ | −0.16 | −0.59 | −0.06 | −0.20 | −0.01 | −0.02 | 0.10 | 0.38 | −0.17 | −0.64 |
| | $\alpha$ | −0.09 | −0.38 | 0.03 | 0.12 | −0.04 | −0.16 | 0.12 | 0.42 | −0.26 | −0.95 |
| 12 months | $\overline{R}$ | 0.01 | 0.06 | −0.01 | −0.07 | 0.12 | 0.55 | 0.04 | 0.30 | 0.03 | 0.15 |
| | $\alpha$ | 0.14 | 0.81 | 0.12 | 0.65 | 0.19 | 0.95 | 0.12 | 0.83 | 0.01 | 0.07 |

factor and its components. Panel A reports average returns and alphas. Comparing these values with the corresponding values in the last two columns of Table II, we can see that put betas generate similar return spreads as bear betas do, and the index-driven component still accounts for most of the abnormal returns. Panels B and C report the portfolios' risk exposures, and the patterns are very similar to those in the last two columns of Tables III and IV.

To further confirm the equivalence of bear betas and put betas in predicting stock returns, Table A1 Panel D reports the returns of the bear beta portfolios that are double-sorted in order to control for put betas. Specifically, stocks are first sorted into deciles by their put betas, and then further sorted into deciles by bear betas within each put-beta decile. Finally, stocks belonging to the same bear beta decile are collected across the put-beta deciles and aggregated into one portfolio. For the two components of bear betas, we use the corresponding components of put betas as the controls. Panel D shows that controlling for put betas largely subsumes the predictive power of bear betas; therefore, it is sufficient to study put betas in order to understand why bear betas predict stock returns.

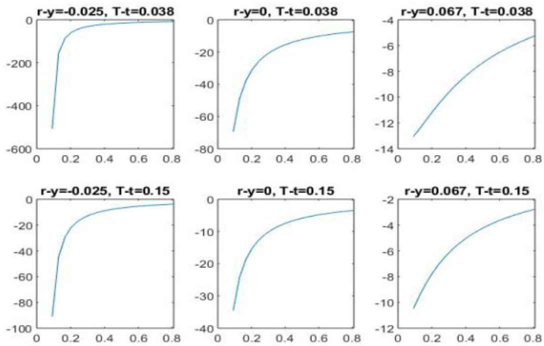
## B. the Linear Component of the Put Option Return

Section 4.3 shows that the linear component of the put return can be written as $r_{\text{Put,L}} = f(\sigma_t) \cdot r_{S\&P500}$, where $f(\sigma_t) = -\frac{1}{\frac{N(-d_2)K}{N(-d_1)S_t}e^{(r-y)(T-t)}-1}$, $d_1 = \frac{ln(S_t/K)+(r-y+\sigma_t^2/2)(T-t)}{\sigma_t\sqrt{T-t}}$, $d_2 = d_1 - \sigma_t\sqrt{T-t}$, and $\sigma_t$ is the IV of the option at time t. Because we choose ATM options, we assume that $S_t = K$ (hence, $ln(S_t/K) = 0$), which allows further simplification of $f(\sigma_t)$:

$$\begin{aligned} f(\sigma_t) &= -\frac{1}{\frac{N(-d_2)}{N(-d_1)}\frac{K}{S_t}e^{(r-y)(T-t)} - 1} \\ &= -\frac{1}{\frac{N(-d_2)}{N(-d_1)}e^{(r-y)(T-t)} - 1} \\ &= -\frac{1}{\frac{N\left(-\frac{(r-y-\sigma_t^2/2)(T-t)}{\sigma_t\sqrt{T-t}}\right)}{N\left(-\frac{(r-y+\sigma_t^2/2)(T-t)}{\sigma_t\sqrt{T-t}}\right)}e^{(r-y)(T-t)} - 1} \\ &= -\frac{1}{\frac{N\left(\sigma_t\sqrt{T-t}/2 - \frac{r-y}{\sigma_t\sqrt{T-t}}\right)}{N\left(-\sigma_t\sqrt{T-t}/2 - \frac{r-y}{\sigma_t\sqrt{T-t}}\right)}e^{(r-y)(T-t)} - 1} \end{aligned}$$

If we can assume that $r = y$, then the expression above can be further simplified to $f(\sigma_t) = -\frac{1}{\frac{N(\sigma_t\sqrt{T-t}/2)}{N(-\sigma_t\sqrt{T-t}/2)}-1}$, which increases in $\sigma_t$ because the ratio between the two CDF functions increases in $\sigma_t$. In our sample period, $r-y$ fluctuates between a minimum of −2.5% and a maximum of 6.7%, and its median is close to zero at 0.25%. Next, we show numerically that the function is increasing for the two extreme values of $r-y$. In addition, we examine the sensitivity of the monotonicity of $f(\sigma)$ to $T-t$. Because we choose monthly option contracts that expire in the following month, their time-to-expiration is always between 2 and 8 weeks. In annual terms, $T-t$ is between 0.038 and 0.15. The following figure plots the function $f(\sigma_t)$ for six situations from the combinations of $r-y \in \{-0.025, 0, 0.067\}$ and $T-t \in \{0.038, 0.15\}$. The range of $\sigma_t$ is chosen to match the range of VIX observed in the sample period—from 9.1% to 80%.

The figure shows that $f(\sigma_t)$ is an increasing function in $\sigma_t$ for all the parameter values, although the shape of the function changes with the values of the parameters.




## C. Categorized Jumps

First, we include all the negative daily jumps categorized as "monetary policy and central banking" or "macroeconomic news & outlook" to form a Macro Jump Index. There are eighty-five such jumps between 1996 and 2017, with an average jump size of −3.6%. For the period of 1964–2011, there are 106 such jumps, with an average size of −3.6%. Second, we group all the other jumps into a Non-Macro Jump Index. There are seventy such jumps between 1996 and 2017, with an average size of −3.7%, and ninety-one jumps between 1964 and 2011, with an average size of −3.9%. Finally, out of all the non-macro jumps, we examine those labeled as "corporate earnings & outlook." There are thirty-four jumps between 1996 and 2017, with an average size of −3.45%, and thirty-nine jumps between 1964 and 2011, with an average size of −3.47%. While there are several other categories within non-macro jumps, the number of observations is fairly small for each category, and we do not consider them separately.

ADS index refers to the quarterly innovations in the Aruoba–Diebold–Scotti Business Conditions Index. The innovations are the residuals of the following AR model: $ADS_{t+90} - ADS_t = a + b \cdot ADS_t + c \cdot ADS_{MA90}$, where $ADS_{MA90}$ is the 90-day moving average of the index. This model accounts for the obvious mean-reverting nature of the index.