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AN EXPERIMENTAL EXAMINATION OF A RELATIONSHIP BETWEEN VERBAL CHAINING AND MATHEMATICAL CONCEPT ACQUISITION IN UNDERGRADUATE COLLEGE STUDENTS: AN ATTEMPT AT A PARTIAL VERIFICATION OF ONE STEP IN GAGNE'S HIERARCHY OF LEARNING TYPES.

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Graduate College

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A dissertation

submitted to the graduate faculty

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Doctor of Philosophy

BY

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Norman, Oklahoma

1972

AN EXPERIMENTAL EXAMINATION OF A RELATIONSHIP
BETWEEN VERBAL CHAINING AND MATHEMATICAL CONCEPT
ACQUISITION IN UNDERGRADUATE COLLEGE STUDENTS:
An attempt at A Partial Verification of One
Step in Gagné's Hierarchy of Learning Types

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CHAPTER 1

INTRODUCTION

Stimulus-response theory

The name "Stimulus-Response Theory" can be used to refer to a class of theories which attempt to explain the behavior of biological organisms. In this study, differences among the theories in the class will not be considered. Furthermore, S-R theory will be considered only as it applies to human subjects. Although this application has always been the most important, a large percentage of research on the theory has been done using animal subjects. The practicability of using animal subjects is high compared to that of using human subjects, and not a small number of investigators have felt that a partial generalization to human behavior was justified. Such generalization becomes very suspect, however, when one considers the objections (by competent statisticians) to generalizing even from one human population to a similar one.

Through the centuries of man's attempts to analyze himself, there has seldom been any doubt that he functions as a perceptive instrument. He has accepted himself as a biological organism with the capability of making decisive responses to both external and internal stimuli. This acceptance evolved into a science approximately a century ago with the aid of such men as Thorndike, Pavlov, and Watson. Since its first formulation, the S-R theory springing from the work of these men has taken a variety of forms and enjoyed a varying degree of popularity. After a very optimistic beginning the theory lost much of its appeal during the period from 1920 to 1940. The reason for this loss of popularity was the appearance of Gestalt Theory, which seemed to many investigators to be more descriptive of human neurological processes. However, Gestalt Theory, which has evolved into present day field theories, suffered from a lack of rigor in basic definition. This seriously limited growth through scientific research methods, and allowed S-R theory to gradually gain back much of its prestige.

Today most learning theories can be classified in two general categories: Cognitive theories, and S-R theories. Both categories assume man to be a perceptive organism functioning in response to stimuli. The basic differences may be stated as follows. In cognitive theories the neurological activity is strongly *goal* centered, whereas in S-R theory the neurological processes may be quite peripheral in relation to a response goal. In a cognitive process, the neural activity sequence need not be directly related to the past experience of the organism, but can proceed according to innate powers. In S-R theory the process is a product of habit -- of previously learned activity which the organism automatically performs in response to given stimuli. In particular, the behavior of a human being, according to S-R theory, is the result of a chain of S-R occurrences beginning with some initial stimulus and containing those responses which characterize the behavior. A given organism will respond to a wide variety (a large set) of stimuli. That set of stimuli overlaps the set of responses of which the organism is capable. Thus each response in a simple S-R occurrence is a potential stimulus, and if we assume habitual responses to given stimuli then the existence of habitual chains follows. That is, there may be established a chain of S-R occurrences which will reoccur in its entirety upon the occurrence of the initial stimulus.

In general, an S-R theory requires the following assumptions:

1. Man is a perceptive organism, capable of making decisive responses to a wide varity of external and internal stimuli.
2. At any one time there are only a finite number of stimuli perceivable to one subject (organism).
3. For any one subject, there is a functional relationship between his universal set of stimuli and his set of responses, and this functional relationship is determined by the subject through a process of extrapolation. An ordered pair (S,R) in this functional relationship has been called an "S-R connection."

As the subject comes into contact with familiar stimuli, he responds according to his established functional relationship. As he comes in contact with a new stimulus, he perceives, relates the perception to his past experience, assigns meaning or worth to the perception -- that is, he assigns a functional value to the stimulus, and thereby determines an S-R connection. This process occurs quite probably on several levels of awareness, from an automatic reaction to a deliberate completely conscious decision.

It is not always easy or even possible to isolate or identify a simple stimulus or a simple response. In fact, one S-R theory assumes that a random sample of all stimuli perceivable by the subject is actually being perceived by the subject at any given time. An alternate assumption is that from the subject's total perception, a dominant element, which may or may not be identifiable, is the stimulus to which the subject will respond. Other stimuli that are present are taken into account as *conditions* under which the stimulus is occurring. The latter assumption is more compatible with the use of S-R theory in this study.

There are also several positions that may be taken with regard to the conditions under which a functional value (response) will actually be assigned (become habitual) to a stimulus. Today most theorists feel that a response must be accompanied by, or followed immediately by, reinforcement in order for it to become the functional value of the stimulus.

Important to this study is the fact that man is apparently immensely superior to animals in his ability to use representative sounds -- spoken words -- and their symbols -- written words. He is also able to experience these words without any sign of overt behavior -- no sound, no written symbol, and no muscular movement. This gives him a large set of internal stimuli, other than kinesthetic, to which S-R theory applies. Through the use of words and symbols, man is able to experience vicariously the important technological advances down through history. In fact, it is an assumption of this study, that most of what man learns (in present day civilized society) is through words.

> Organized, meaningful verbal information is essential for virtually all kinds of learning except the simplest varieties (4:153).*
>
> The vast majority of signs used in ordinary communication are what we may term *assigns* -- their meanings are literally "assigned" to them via association with other signs rather than via direct association with the objects signified. The word *zebra* is understood by most six-year-olds, yet few of them have encountered ZEBRA objects themselves. They have seen pictures of them, been told that they have stripes, run like horses, and are usually found wild (17:8).

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*The notation (n:m) is used hereafter to mean page m of bibliographic reference n.

> . . . , the meanings of assigns develop out of the *context* of primary signs with which they occur. As the child who has learned to read with some facility moves through a story, the matrix of familiar signs limits the possible meanings which the new and unfamiliar words can have. And since the adult story writers are reasonably consistent in the signs they put together (PRIESTS are *kind* and *calm*, LIBERTY is *good* and *free*, VICIOUS is something characteristic of *wild animals*, *bad men*, and *so on*), a reasonably stable assign-meaning develops. Certainly, the vast majority of lexical items employed and understood by adult humans are assigns in this sense (17:287).

In particular, most mathematics is learned through the use of words. Most words in the language of mathematics are assigns. That is, their meaning is given through the use of other words. Thus, in applying S-R theory to the learning of mathematical concepts, it is reasonable to assume that words form both stimulus and response in most S-R occurrences. For example, given the word TWO and the word DOUBLE, either one as a stimulus in the presence of the other will automatically evoke the response "four" in a fifth grade student. It is possible that he learned this response using apples or oranges. In fact it is likely that some students actually *handle* apple and orange objects in the initial stages of learning the above response. And a far greater number are presented with pictures of apples and oranges. But it is a certainty that <u>all</u> students encounter the <u>words</u> along with any objects or pictures. Furthermore, even for students that actually are presented with objects or pictures in the first encounter, the role of the object or picture is taken over thereafter by words. The point is that once a person learns the concept of *two-ness* and the concept of *orange*, he can experience the concept of *two oranges* merely by perceiving the words TWO ORANGES. Thus in most learning that takes place in school, words are more likely to be used than tangible objects.

Gagné's hierarchy of learning types

S-R theory enters into this study as it is embedded in the hierarchy of learning types suggested by Robert M. Gagné. The following categorization of types of learning is based on S-R theory and was presented by Gagné in The Conditions of Learning (4:36).

1. Signal learning: natural automatic responses to certain stimuli, such as an eye blink to a puff of air, or the retraction of a hand from contact with a hot stove.
2. Stimulus-response: conditioned nonautomatic responses to certain stimuli, such as turning on a lamp to obtain more light with which to read, or turning the steering wheel of an automobile to negotiate a curve.
3. S-R chaining: a chain or sequence of S-R occurrences such as the movements involved in starting the engine of a car, or the sequence of thoughts in the mind of a person trying to remember something forgotten.
4. Verbal association: a subcategory of S-R chaining -- a sequence of words or phrases, each of which acts as a stimulus for its successor and response to its predecessor. Such words or phrases might occur in connection with the acquisition of abstract concepts where chains of muscular actions would be insufficient.
5. Discrimination: acquiring the ability to make finer discriminations in responses as, for example, in color identification or musical pitch recognition.
6. Concept learning: recognition or identification of categories such as the category of *evenness* as applied to integers. Gagné writes,

"*The effect of concept learning is to free the individual from control by specific stimuli.*" (4:182)

> A concept is a stored set of connections in which a verbal signal controls the arousal of habits appropriate to new situations, so that the learning of new specific habits is unnecessary (5:43).

7. Rule learning: recognition or identification of relationships between concepts such as the relationship between radius and area of a circular disc.

8. Problem solving: the combining of rules to form new rules, such as using the rules of equations to get the solution to a particular equation, or finding a new rule of equations through the logical combination of known rules.

Gagné has the following to say concerning conditions under which concept learning takes place.

> The specific stimulus objects, to which chains that include a common final link have been previously learned, are presented simultaneously, or in close time succession. Instructions are used to stimulate the learner to recall and reinstate these chains (4:181).

Further conditions include evaluation and reinforcement. As for the internal conditions:

> Prerequisites to the learning of concepts are capabilities that have previously been established by multiple discrimination. A set of verbal (or other) chains must have previously been acquired to *representative* stimulus situations that exhibit the characteristics of the class that describes the concept, and that distinguish these stimuli from others not included in the class (4:180).

The conditions under which S-R chaining takes place are as follows:

Internal Conditions:

> Of utmost importance to the acquiring of chains is the requirement that each individual stimulus-response connection be *previously learned* (4:128).

> If one could make sure that each individual link were fully learned, the additional external cues required could be reduced to an absolute minimum (4:129).

External Conditions:

> Assuming that the links are known, the main condition for the establishment of a chain is getting the learner to *reinstate them* one after the other *in the proper order* (4:129).

The other conditions include contiguity of individual links, repetition, reinforcement, and low interference. It is suggested that the beneficial effects of repetition are from the probability that conditions for learning are not all optimum during any single S-R occurrence, and from the weakening of interference by repetition. "Interference" can be defined as the weakening of one S-R connection by the occurrence of another.

The problem

The purpose of the study was to investigate the applicability of Gagné's hierarchy of learning types to the learning of mathematical concepts. Specifically, the attempt was to detect the presence and effect of S-R chaining, in the form of verbal association, on the learning of algebraic and geometric concepts -- to verify (or refute) that the acquisition of these concepts, by college students, involves the formation of verbal chains, and to present valid statistical evidence having implications regarding the effect of these verbal chains (if they exist) on the student's performance in concept acquisition.

The problem can be put in the form of two questions:

1. Does verbal chaining occur in the process of learning geometric and algebraic concepts?

2. Is the performance of S, in the acquisition of algebraic and geometric concepts, affected by the occurrence of verbal chaining?

Detecting the occurrence of a verbal chain in the neurological processes of S, as S studied a concept, was of key importance in the study. The decision as to whether such a chain occurred was made on the basis of S's self-evaluation of his *thoughts* immediately after he had studied the concept. At that time he was asked to choose the one of the following three statements which was most nearly true.

As I studied the concept,

a. there were definitely no words in my thoughts.

b. there may have been certain words in my thoughts.

c. there were definitely certain words in my thoughts.

A measure of the adequacy of verbal chains was then made by asking S to write a list of words that may have been (or definitely were) "in his thoughts" -- words that helped him to learn the concept. The number of suitable words in this list was taken to be the value of variable V for the subject. It was decided that a word was "suitable" if it could be seen, by E, to be suitable as a link in a verbal chain for the concept. A more complete discussion of the measurement of V may be found in Appendix A.

Two other independent variables in the study are concept type (T), and concept complexity (C). Tasks were chosen (appendix B) so that T and C each had two fixed values: Algebraic or geometric for T, and simple or complex for C.

The dependent variable, performance (P), was defined as the score on a test given to S to determine how well he had learned the concept. The scoring scheme and reliability are given in appendix C.

Hypotheses

S-R theory and Gagné's hierarchy lead to the following hypotheses:

1. Verbal chaining *does* occur in the process of learning geometric and algebraic concepts.
2. An increase in the amount of verbal chaining will increase performance in the acquisition of algebraic and geometric concepts.
3. More verbal chaining will occur in learning complex concepts than in learning simple concepts.

In terms of the variables of the study the hypotheses may be stated as follows:

1. S will choose statement b or c. (page 9)
2. V will be positively correlated with P.
3. V will be positively correlated with C.

Hypothesis 1 follows from the theory since the theory requires S-R chaining in the learning of concepts. The links of the verbal chains appropriate for the concepts of this study are assumed to have been learned by all subjects. It is the *use* of these known links to construct chains in learning the concepts that will be of central interest. In the event that S does not construct verbal chains the interpretation will be that S-R chaining is not a prerequisite of concept learning. A possible exception might occur if all the links in the S-R chains were nonverbal -- e.g., in learning a simple geometric concept the S-R links might all be visual images (4:137). Thus, hypothesis 1 might be expected to be false for simple geometric concepts.

The theory predicts hypothesis 2 since, in this study, the failure on the part of S to use (while studying) appropriate words to describe the concept will be interpreted to mean that there are missing links in his S-R chains for the concept. That is, few appropriate words used by S will indicate poor internal conditions for learning, whereas, if S uses a greater number of appropriate words, the conditions for learning will be considered to be better. Again an exception might be made for simple geometric concepts where visual links are taking the place of verbal links.

The theory would predict, in general, that a complex concept would require more chains and longer chains than a simple concept. This accounts for hypothesis 3.

Related literature

Most of the literature related to Gagné's suggested hierarchy concerns itself with concept acquisition, rule learning, and problem solving, and deals for the most part with *practical* aspects of these subjects rather than *theoretical* aspects. Verbalization enters into some research, but from the aspect of the relative effectiveness of verbal presentation vs. some other type of presentation. The question of *how* (or *whether*) words enter into the neurological activity of S as he learns is not often treated.

Joseph Manning Lane, Jr. (1970) found that supplying verbal mediators or requiring S to find his own verbal mediators produced significantly better performance in paired-associate learning than if S was not given any mediational treatment (14).*

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*The notation (n) is used to refer to bibliographic reference n.

Clark Lewis Wanbold (1970) concluded from a study with educable mentally retarded children that instructing youngsters to formulate verbal mediators or supplying them with verbal mediators facilitated their learning on a paired-associate learning task and also on a subsequent, similar paired-associate task (22).

Murray S. Bartky (1971) found that more solutions were achieved (solving problems of conservation and transitivity) by S's reporting the use of verbal mediation, but results only indicated a slight trend (1).

John Poston Houston, (1965) working with a sample of female undergraduates at the University of Michigan, (in paired-associate learning) concluded strong support for the hypothesis that ease of verbal S-R learning is a direct function of the number of mediating chains existing between the stimulus and the response (10).

In 1962, Gagné and E. C. Smith, Jr. did a research study on the effects of verbalization on problem solving in which S's were requird in one treatment to state verbally a reason for each step in the solution of a problem. The study, done with 9th and 10th grade male subjects, suggested that verbalization during problem solving does increase performance (8). A similar result was obtained by Marks in 1951, when he obtained a correlation of .83 between performance and vocalization by S's (15). However, many studies have been done in which verbalization by S's was not found to be helpful. Katona (1940) found that a method which involved teaching verbal principles in solving matchstick problems was less effective than a method of teaching by example (8). Haslerud and Meyers (1958) found an experimental treatment in which verbally stated principles of solution of cryptograms were given to S's to be less effective for solution of new cryptograms than was a treatment in which S's were required to discover solutions for themselves (8).

Gertrude Hendrix (1947) studied generalization and spoke of "intermediate flashes of unverbalized awareness." (9:200) Her study supported the hypothesis "For generation of transfer power, the unverbalized awareness method of learning a generalization is better than a method in which an authoritative statement of the generalization comes first." (9:198) The study also indicated that verbalization of a generalization immediately after discovery may actually decrease transfer power (9). A neutral result was obtained by Bernard R. Corman, (1957) who wrote "success in verbalizing the rule was uncorrelated with success in solving the problems." (2)

Leonard Theodore Nelson, Jr. (1968) concluded high verbal ability alone did not give significantly better performance in learning the concept of function. Nor did a verbal approach to teaching achieve better results. Approaches considered were verbal, visual-spatial, and numerical. The same categories were considered in ability (16).

A study was done by Raymond W. Kulhavy in 1971 involving language mediation and paired-associate learning in college students. The study suggested that written word mediators for paired-associates (embedding in a sentence, or connecting with a verb) were of no help to S's. Kulhavy conjectured that the result was due to the advanced ability of the college student to provide his own mediational devices (18). (Some earlier studies had indicated that younger children were helped by being given word mediators.)

A study by Lee and Gagné in 1969, involving college students, indicated that a learned concept is more likely to be integrated with others when it is properly labeled in terms of the formal similarity of the two units to be integrated (6). This bears directly on the present study, since the hypothesized verbal chains used during concept acquisition are assumed to involve the integration of known concepts.

Another study by Lee and Gagné in 1970, entitled Effects of Degree of Component Learning on the Acquisition of a Complex Conceptual Rule, gave evidence supporting a mediational account of the process of conceptual rule learning (7).

A study in 1967, of the Effects of Verbalization and Pretraining on Concept Attainment by Children in Two Mediational Categories, supported neither the presence nor absence of verbal mediational processes (21).

B. F. Skinner writes

> The data which give rise to the notion of covert speech can be dealt with as such with the degree of rigor prevailing elsewhere in a science of verbal behavior at the present time (20:435).
>
> The range of verbal behavior is roughly suggested in descending order of energy, by shouting, loud talking, quiet talking, whispering, muttering "under one's breath," sub-audible speech with detectable muscular action, sub-audible speech of unclear dimensions, and perhaps even the "unconscious thinking" sometimes inferred in instances of problem solving. There is no point at which it is profitable to draw a line distinguishing thinking from acting on this continuum. So far as we know, the events at the covert end have no special properties, observe no special laws, and can be credited with no special achievements (20:438).

It is with the existence and effect of the covert end of this continuum that the present study concerns itself, the assumption being that it embodies the form of S-R chaining appropriate to the learning of concepts in Gagné's suggested hierarchy.

CHAPTER II

DESIGN

Experimental design

One hundred and sixty students were taken from seventeen freshmen mathematics classes -- ninety from nine sections of Intermediate Algebra and seventy from seven sections of College Algebra. The students were obtained by asking them to volunteer during class time. Coincidentally, there were an equal number of males and females in the sample.

The subjects came one at a time from their classroom to another room in the same building to participate in the experiment. They were assigned randomly to one of the four tasks (appendix B) by using a table of random numbers. The experimental procedure was as follows.

The subject (S) was ushered into the room and asked to sit down at an empty desk. The room was well insulated from distracting noise, on the ninth floor of the Physical Sciences building on The University of Oklahoma campus. S was then given instructions and information concerning a sheet of paper which was about to be given to him. If S was to be given an algebraic task from the random ordering, he was told that on the sheet were written four sequences, and that he was to study the four sequences silently until he knew the common pattern and could complete such a sequence if given the first number. Care was taken to be sure S knew what was expected of him. He was told to take as much time as he needed and to let the experimenter know when he was through (for timing purposes); then he was given the study sheet (appendix D).

If S was to be given a geometric task from the random ordering, he was told that the sheet about to be given to him contained four examples of a geometric figure -- a geometric figure had been drawn on the sheet four times -- and that he was to study the examples of the figure silently until he felt that he could draw it from memory. Other instructions were the same as for the algebraic tasks.

When S indicated that he was through studying, the time was recorded and he was immediately given sheet 2 which read as follows:

Circle the letter of the statement which is most nearly true.

As I studied the examples, I

a. definitely had no words in my thoughts.

b. may have had certain words in my thoughts.

c. definitely had certain words in my thoughts.

When S completed sheet 2, if he had chosen "b" or "c," he was given sheet 3 upon which he was asked to write a list of words that were (or may have been) in his thoughts while he was studying -- words that helped him "learn the concept." After he completed this list (or immediately after completing sheet 2 if he chose "a" on that sheet) he was given sheet 4 which requested the number of years of mathematics he had taken in school since the eighth grade, his average grade in mathematics and his age.

After completing sheet 4, S was given the test sheet and timed. The tests for each of the four concepts are explained in appendix C.

Sheet 3, on which S listed words (that were in his thoughts as he studied the concept), was expected to act as a deterrent to interference. The repetition of the verbal chain(s) here could have acted against the effects of interference generated on sheet 4. If S did not form verbal chains, or formed inadequate verbal chains, interference was expected to lower his performance. It should be noted that this repetition on sheet 3 lacked some of the beneficial effect of *practice* in that S was repeating what he *believed* (*at that time*) to be the chain(s). It is possible that some interference had already altered his original verbal chain(s) by the time he began the repetition. However, it seems reasonable to assume that S experienced *greater* interference on sheet 4 where he concentrated his attention on things entirely different from the learned verbal chain(s). And it is the effect of this latter interference that the practice on sheet 3 should have helped to combat. Thus a device was built into the experimental design which was expected to magnify any positive correlation that truly exists between V and P.

After all S's had been processed, word counts (the number of suitable words listed by S on page 3) were made independently by three judges. Each subject's word score was then taken to be the average of the three scores given him by the judges. The same procedure was used in grading the geometric tests, the graders being careful not to refer in any way to S's word score while grading his test. The grading scheme on the algebraic tests was considered by the author to be objective enough that this type of average technique was not needed.

It was believed at first that the difficulties to be encountered in obtaining accuracy in the necessary measurements would introduce sufficient inaccuracy that several covariates might be needed to make it statistically possible to display the hypothesized relationships. In particular, *study time* and *test time* were expected to be needed.

Obtaining *age*, *years of mathematics experience*, and *average grade* also served to improve experimental design, as noted above.

A list of variables which were available for statistical processing is given below.

Performance (grader 1)

Performance (grader 2)

Performance (grader 3)

P (average of the three graders)

Word count (judge 1)

Word count (judge 2)

Word count (judge 3)

V (average of the three judges)

Sex

Concept type

Concept complexity

Study time

Test time

Years of school mathematics since the eighth grade

Average grade in mathematics

S's self evaluation of whether he had "words in his thoughts"

Age

Statistical design

A preliminary correlation matrix was computed using all variables. From the results of this matrix, hypotheses 2 and 3 were tested, and reliability coefficients were obtained on the word count and performance. A multiple linear stepwise regression was then run, using performance as the dependent variable. This yielded a correlation coefficient for performance and word count after the variance due to concept complexity had been accounted for. This was used as a further test of hypothesis 2.

As a final statistical procedure related to hypothesis 2, t-tests were made on the differences of performance means for high and low V in each of the four tasks.

Hypothesis 1 was tested using a chi square goodness of fit test on the frequencies of "a," "b," and "c" chosen by S on sheet 2. The theory would predict that if the concept were learned, "b" or "c" would be chosen. Hence, if "a" was chosen, then the concept had not been learned. Thus, there should be no more choices of "a" than there are S's who failed to learn the concept. Since the study time was terminated only after S believed he had learned the concept, a reasonable theoretical model would be zero choices of "a" and one hundred and sixty choices of "b" or "c." However, since validity of the chi square test requires a theoretical frequency of five or more in each cell, the model was altered accordingly -- five choices of "a" and one hundred and fifty five choices of "b" or "c."

CHAPTER III

RESULTS

The attitude of the subjects was excellent, and although frequently little irregularities occurred in the experimental procedure, on the whole the processing of subjects went smoothly with few unexpected disrupting experiences. On three occasions a subject was unable to learn the concept and "gave up" after ten minutes of study. In each of these cases the task was given to the next subject. The only other major irregularity (which occurred twice) came from the misunderstanding of instructions on the part of the subject. Since evaluation of the tests of these two S's would not have been meaningful, at the end of the experiment after all other subjects had been processed, these two tasks were reassigned to two additional subjects.

Distributions of the variables in the study are given in Figures 1 through 7, and a preliminary correlation matrix is given in Table 1.

The correlation of V with C was found to be 0.466 which is significant at the .0001 level. Thus hypothesis 3 is strongly supported.

The correlation of V with P was found to be 0.093 which is not significant even at the .1 level. However, when the effect of complexity is removed, the correlation is much improved. The correlation coefficients and their significance levels are displayed in Table 2 which shows that hypothesis 2 is well supported for simple concepts and well supported for complex concepts.

Less impressive results were obtained by using t-tests on the differences of mean performances between high and lôw V in each of the four tasks. Table 3 displays these results. It is the author's opinion that a reasonable increase in sample size would yield significant differences for these mean performances except for simple geometric tasks where visual links might prevent significance even though the sample size were very large.

In the stepwise regression, after complexity was partialed out, the partial correlation of V with P was found to be 0.37, which is significant at $p < .0001$. Furthermore, V was the second variable to enter the predicting equation -- again adding support to hypothesis 2.

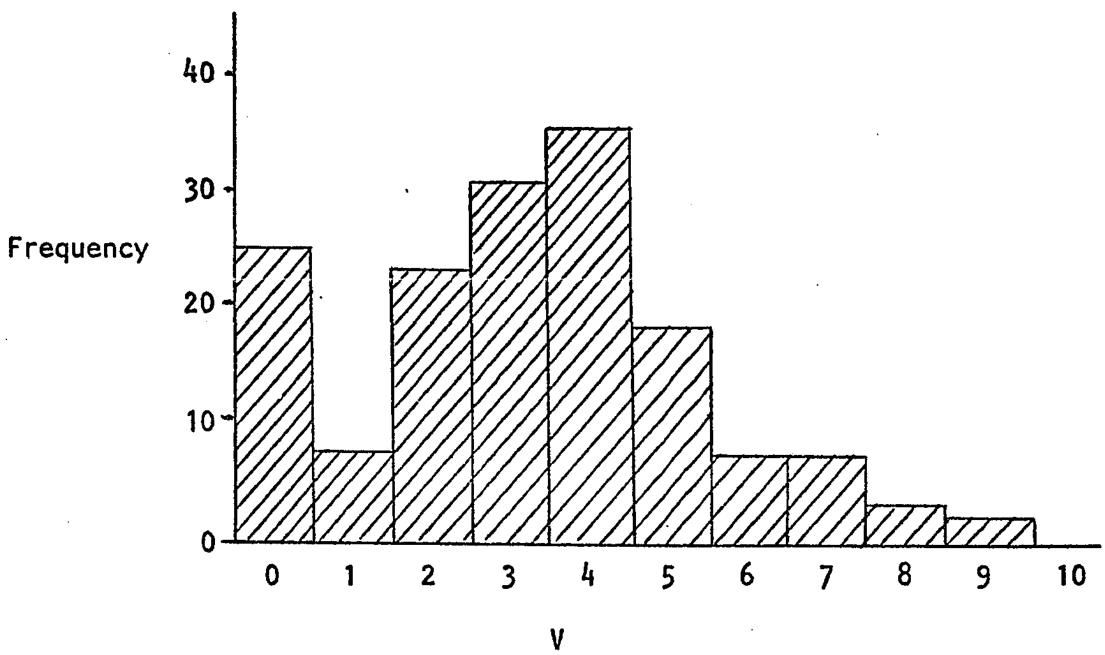
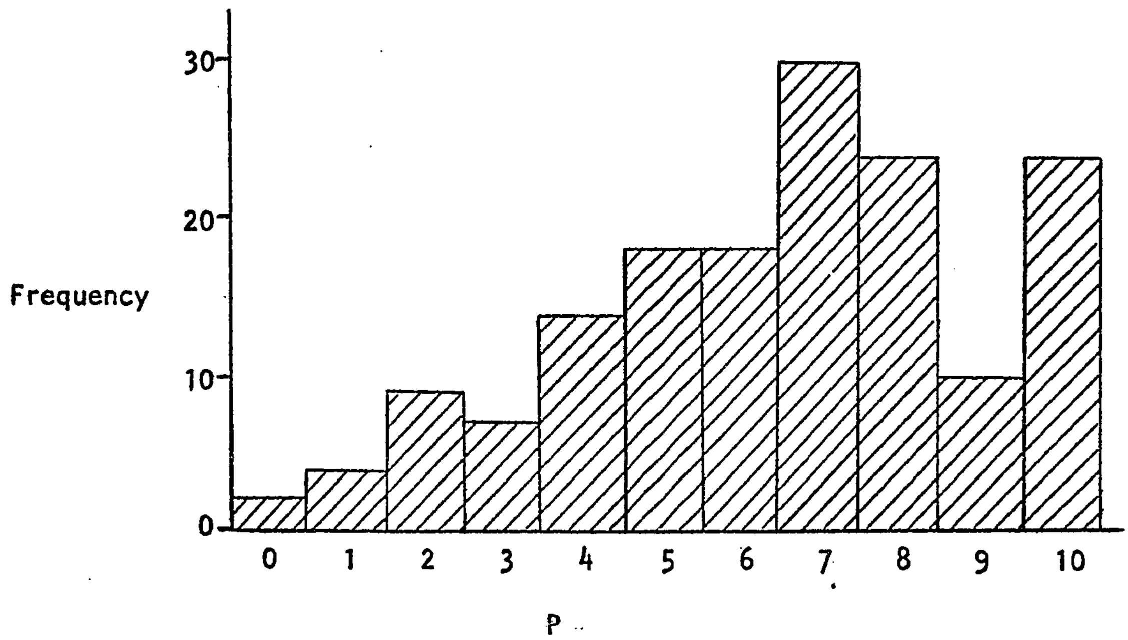


Fig. 1


30
20
Frequency
10
0
0
1
2
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4
5
6
7
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P

Fig. 2


60
40
Frequency
20
0
17
18
19
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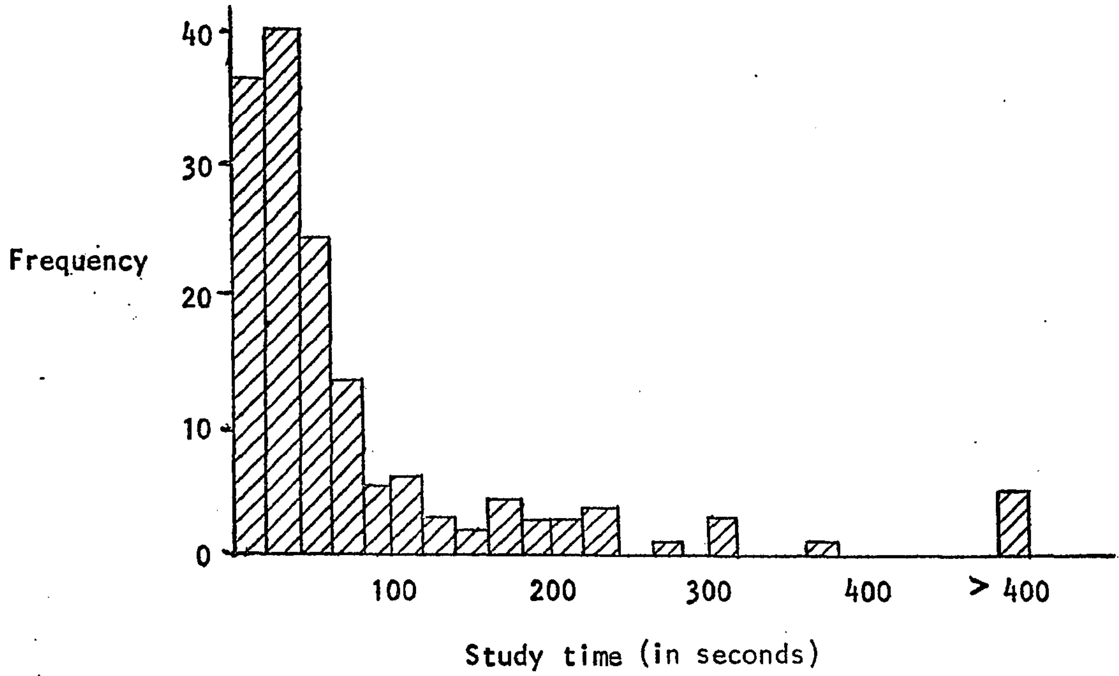
21
22
23
24
>24
Age

Fig. 3


Frequency
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30
20
10
0
100
200
300
400
> 400
Study time (in seconds)

Fig. 4

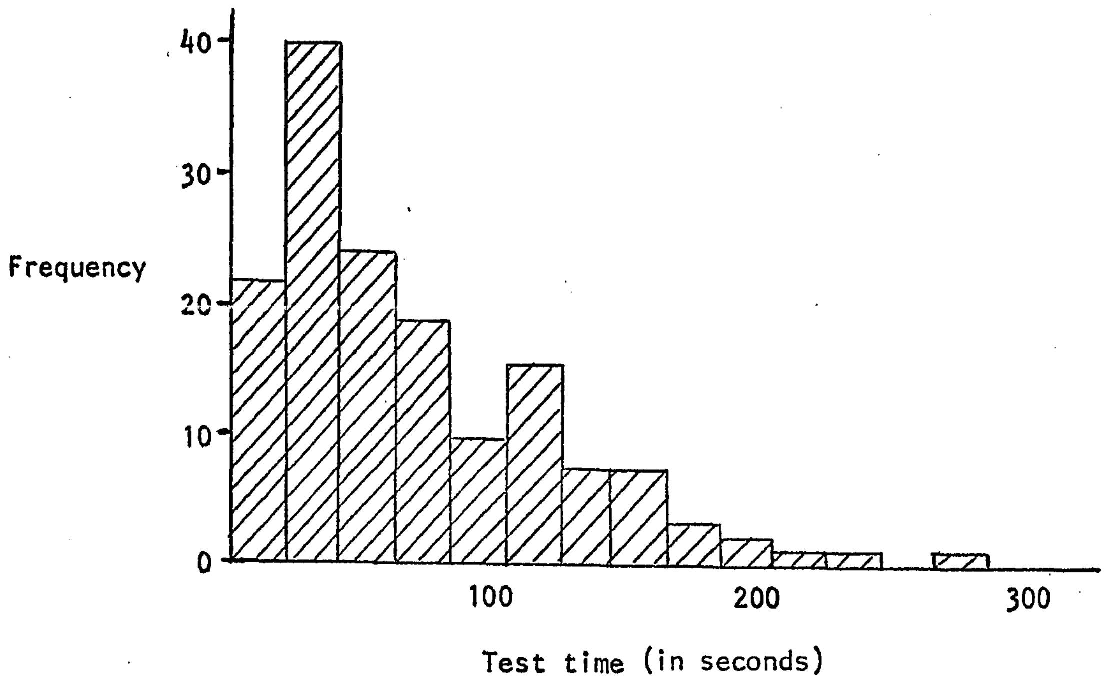

Frequency
40
30
20
10
0
100
200
300
Test time (in seconds)

Fig. 5

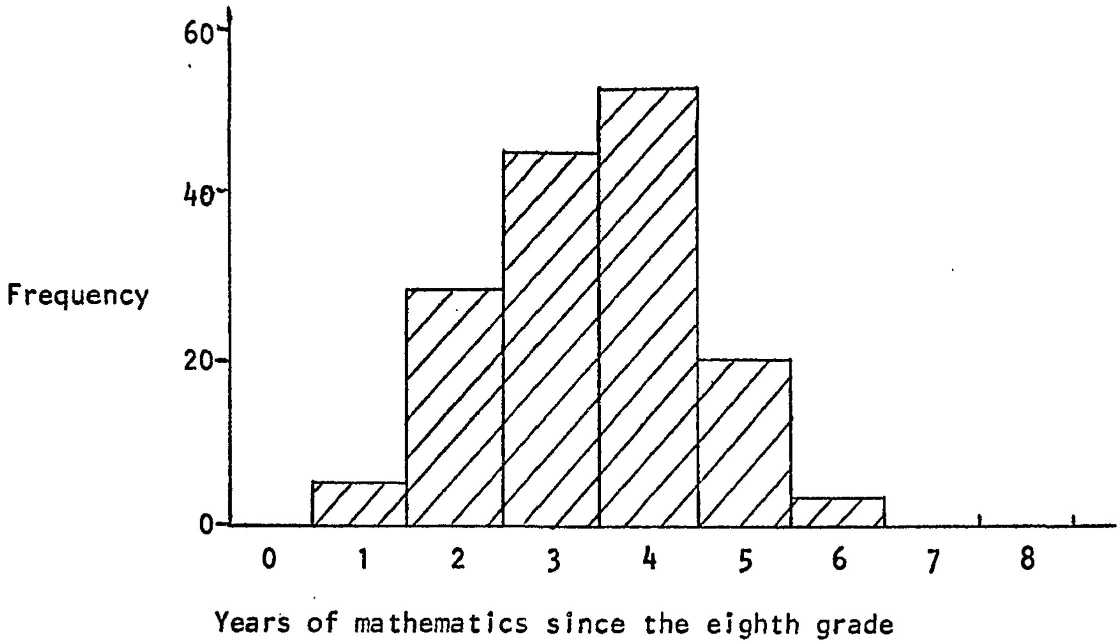

60
40
Frequency
20
0
0
1
2
3
4
5
6
7
8
Years of mathematics since the eighth grade

Fig. 6

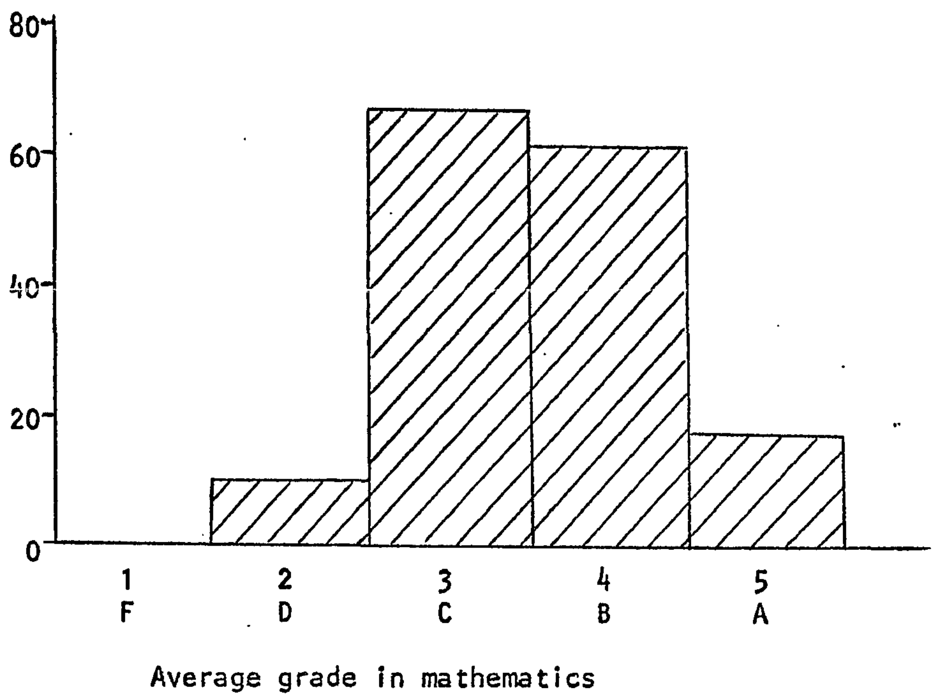

80
60
40
20
0
1
F
2
D
3
C
4
B
5
A
Average grade in mathematics

Fig. 7

TABLE I

CORRELATION MATRIX -- ALL VARIABLES

	P	C	T	study time	test time	yrs. math	ave. math grade	age	sex
V	.093	.466	-.012	.072	.236	.036	.032	-.092	.090
P		-.431	-.067	-.266	-.287	.184	.139	-.089	.127
C			.000	.346	.526	-.114	.040	-.094	.100
T				-.473	-.126	.103	-.071	-.047	-.100
study time					.341	-.024	-.039	.014	-.109
test time						-.012	.050	-.100	.007
years of math							.221	-.221	-.103
ave. math grade								-.262	.357
age									-.165

TABLE 2

CORRELATION OF V WITH P

task	correlation	significance
simple	.35	$p < .001$
complex	.38	$p < .001$

TABLE 3

DIFFERENCES BETWEEN P MEANS FOR HIGH AND LOW V

	P means and sample size					
task	low V	n	high V	n	difference	significance
simple algebraic	8.01	19	8.55	21	0.54	$p < .3$
complex algebraic	4.38	19	6.91	21	2.53	$p < .01$
simple geometric	7.13	20	7.90	20	0.77	$p < .3$
complex geometric	5.17	21	6.25	19	1.08	$p < .2$

Although it was believed initially that study time and test time would have to be used as covariates to get significant correlation of V with P, the effect of these variables was reduced sufficiently when complexity entered the equation that V accounted for more of the remaining variance of P than they did.

Hypothesis 1 fails to hold for the simple geometric task since there were seventeen choices of "a" (by S on sheet 2), fourteen of which were for the simple geometric task. However, if the simple geometric task is omitted from the study, the theoretical model would be five choices of "a" and one hundred and fifteen choices of "b" or "c." Since the experimental frequencies were three and one hundred and seventeen, the fit is excellent. The value of chi square is .83 which is significant at $p > .6$, but the fit is even better than this indicates since the data is actually a better approximation to the true theoretical hypothesis than the model.

CHAPTER IV

CONCLUSIONS AND DISCUSSION

If the grading scheme used in the study is assumed to be a valid measure of concept acquisition, then it is reasonable to conclude strong support for all three hypotheses with the exception that hypothesis 1 fails for simple geometric concepts. This partial failure of hypothesis 1 was not unexpected since this would be a natural place for "visual" chains to occur. It was noted by the author during the experiment that many of the subjects with the simple geometric task would indicate completion of study in less than ten seconds, and most of these S's also indicated that they had no words in their thoughts as they studied. In fact, thirteen of the seventeen such indications on the geometric task terminated their study time in eleven seconds or less, the average study time for those S's indicating no words being nine seconds. The average study time for the other S's on the simple geometric task was twenty seconds. This lends credence to the theoretical explanation of visual chains -- a *photographic* type of acquisition. Thus the experimental results do not give reason to doubt the S-R theory.

It is the conclusion of the author that the study gives strong support to S-R theory of human learning as embodied in the hierarchy of learning types suggested by Gagné. Although wide generalization of the results is not recommended, it is felt that the results could be applicable to college freshmen in the United States. The courses from which the subjects were taken are service courses containing students reasonably representative of the university population. Furthermore, of the seventeen classes used, approximately fifty percent of each participated in the study, and the participating students were not given any preferential treatment in class.

No *causal* relationship can be inferred from the study results. However, the following applications to the classroom situation (involving the learning of algebraic or geometric concepts) are believed to be appropriate.

1. In the teaching of algebraic and geometric concepts, a teacher should take care to help the student build and use an adequate vocabulary, since in the study it was the *use* of *known* verbal chains that correlated positively with performance.
2. The common phenomenon of a student "knowing the concept" but not being able to explain it in words (usually accompanied by a desire on the student's part to discontinue study of the concept) could be taken as a sign that the student needs more study of the concept if he is to use it in subsequent work.
3. Students who actually acquire an adequate vocabulary will be able to progress at a faster rate than those who do not, since a student who acquires an adequate vocabulary will be able to learn concepts of a greater complexity, giving him a more rapid rate of vertical transfer.

The above suggested applications comprise a meager contribution to the solutions of problems which exist -- problems with which educators and teachers are surrounded daily, and for which, far too often, a "common sense" solution is used without question. The dearth of our knowledge of the learning phenomenon in the human organism should warn us that *common sense* might have little or no value. Much more research is needed.

The results of this study include implicitly information about generalization, an area studied by Hendrix (9), whose conclusions do not agree with what one might expect if the present work is accepted. This doesn't constitute a contradiction -- it reflects widely separated approaches, different experimental conditions, and different samples of the population. Her concern (not unlike many other researchers) was with overt verbal behavior rather than the verbal content of S's neural processes during study. The experimental procedure used by Hendrix was not *designed* to discover covert verbal activity. The central consideration was teaching method.

Lack of subjects prevented the author from including in this study a variable *supply* of verbal mediators. Kulhavy's research (19) indicated that supplying verbal mediators did not help college students in paired-associate learning. Whereas, Lane (14) found differently for first graders, as did Wanbold (22), for educable mentally retarded children. Wanbold concluded both increased transfer and retention were obtained by supplying verbal mediation. Again, as with most studies, paired-associate learning was being considered. Early's study (3) on solving verbal arithmetic problems treated sixth grade subjects and compared performance of the same subject on pairs of problems -- one problem in each pair having a "word clue" and the other having no clue. Better performance was noted on problems having a word clue -- especially for "low performers."

Nelson's results (16) seem to be at variance with the present study. (Nelson's dissertation indicated that verbal ability alone did not significantly increase performance.) He worked with subjects of a similar age group on the learning of a mathematical concept. The *apparent* contradiction comes from the assumption that "verbal ability" would be positively correlated with V (the number of appropriate words S reported to be "in his thoughts"). This assumption may, of course, not be realistic, but even if it were, positive correlation of both verbal ability and concept acquisition with V does not imply positive correlation of verbal ability with concept acquisition.

The work done by Bartky (1) differed from the present study in that it treated seven- to eleven-year-old subjects and was concerned with three different approaches to teaching problem solving. Of central interest was the relation between mediational instruction and age. Houston (10) *taught* mediators for paired-associate learning to find the relation of verbal chain size to performance. His study was on the same theoretical level of the present one and indicated compatible results.

Another area not included in the scope of the work at hand is retention. As previously mentioned, Wanbold's results (22) indicated that supplying verbal mediators to educable mentally retarded children facilitated retention of paired-associate learning tasks. Kulhavy (13) found no increase in retention upon supplying mediators to high school juniors and seniors. Kiess (12) required S to report verbally any "natural language mediation" he experienced -- the report being made during exposure to a stimulus item. Exposure time was a variable (2, 3, or 4 seconds). Short term memory correlated positively with the number of mediators. Paivio (18:357) cites research on "imaginal and verbal mediators" in paired-associate learning. He indicates definite support for increased retention from mediation -- especially imaginal mediation. The effect of possible covert verbal activity upon retention of learned mathematical concepts in college students is open to research.

APPENDIX A

The verbal score (V) is an attempt to quantify the degree to which verbal activity is present in S's neural processes during the time S is studying the concept. It was defined in this study as the number of appropriate words S listed as having been "in his thoughts" as he studied. Only words that could reasonably be considered to be related to the concept were counted. Because of the subjective nature of the decision about the "appropriateness" of words, the word list given by each S was examined by three judges. Each judge decided independently on a "word count," and the three counts were averaged for each S. The scores varied from 0.00 to 10.0 with a distribution shown in Figure 1 (page 21).

During the pilot studies, the following guidlines were established, by the author, for accepting or rejecting words from S's list:

1. A word is counted if it is descriptive of some part of the particular concept.
2. A word is rejected if it is descriptive only in a general way. The words *relation*, *sequence*, *pattern*, and *geometric* would fall into this classification.
3. A word is counted only once even though it is used several times by S.
4. A phrase which acts as a unit of speech may be counted as a word -- e.g., *the next number* would be counted as one word whereas the word *number* appearing by itself would be rejected by guideline 2.

5. For geometric tasks, letters or numbers are to be accepted if some part of the figure resembles them.
6. For an algebraic task, numbers which are descriptive of the sequence pattern are to be accepted, but the numbers in one of the example sequences are not (e.g., in the simple algebraic task, the pattern is x, 2x, 2x+1, 2(2x+1), 2(2x+1)+2. The numbers "1" and "2" would be counted, but the numbers in a listing "2, 4, 5, 10, 12" of the first example would not be counted).

Given below are two representative examples from each of the four tasks.

Subjects number 4 and 16 were assigned the simple algebraic task. Their word lists appeared as follows:

S number 4

half, the, number
twice,
one number ahead.

S number 16

multiply
addition
numbers of two
order

All judges gave three points to S number 4. S number 16 was given two points by one judge and three points by the other two judges.

Algebraic complex tasks were assigned to subjects 2 and 14. Their word lists read as follows:

S number 2

twice, twice-plus-one
three-times,

S number 14

double
add 3
double and add 1
subtract 10
triple
add 1

S number 2 was given four points by one judge and five points by the other two. All three judges agreed on seven points for S number 14.

Subjects number 1 and 31 were assigned the geometric simple task. Following is a reproduction of their lists:

S number 1

square
center
½ Length

S number 31

zee
triangle
vee
backward "L"

All judges gave four points to S number 1. S number 31 received four points from two of the judges and five points from the remaining judge.

Geometric complex tasks were assigned to subjects 5 and 8. Their word lists appeared as follows:

S number 5

down
across
box
triangle
dot

S number 8

right angle
dot
straight line

All judges agreed on five points for S number 5. One judge gave S number 8 five points and the other two judges gave three points.

All but ten of the scores less than one came from the simple geometric concept where it is felt that visual mediation may have been occurring. If the simple geometric scores are not considered, the distribution is approximately normal in appearance, skewed slightly to the right.

The reliability for the word count is given in Table 4. The table contains simple correlation coefficients for each of the three judges with the average.

TABLE 4

RELIABILITY OF WORD COUNT

	Judge 2	Judge 3	Average
Judge 1	.962	.946	.985
Judge 2		.957	.988
Judge 3			.982

The author acted as Judge 2. Judge 1 was the author's wife, a pharmacy student at The University of Oklahoma who helped in the processing of pilot study data and the developement of the experimental design. Judge 3 was Mona Sue Corbitt, a medical student at The University of Oklahoma Medical Center. These judges were also the graders of the geometric tasks.

APPENDIX B

The requirements for a concept to be used in the learning tasks were the following:

1. The concept must be unknown to S.
2. It must be mathematical.
3. It must be presentable nonverbally.
4. The difficulty level must be so chosen that variability in performance will be sufficient to allow statistical processing.
5. It must be sufficiently easy that S will believe he has attained the concept after less than ten minutes study.
6. It must be such that S's attainment is measurable.

To satisfy condition one it was necessary to choose either known concepts beyond S's past experience, or nonsense concepts -- concepts consisting of simple combinations of concepts well known to S -- combinations that S would not have encountered before. Having no way to obtain a sample of S's to satisfy adequate similarity of past experience, it was decided that nonsense combinations of concepts should be constructed.

Condition two was satisfied by choosing concepts that consisted of number sequences (algebraic) and geometric figures (geometric).

Condition three was met by presenting S with nonverbal examples, and requiring silence while S was studying.

Conditions four and five proved to be the most difficult to satisfy. Along with the problem of satisfying these conditions came the necessity of properly separating the difficulty of "simple" concepts and "complex" concepts. Satisfying these conditions was accomplished during several pilot projects.

The results of satisfying condition six are shown in Appendix C with the tasks used in the study.

APPENDIX C

Task 1: simple algebraic

The subject was given a study sheet which was blank except for the examples shown in Figure 8.

(2 , 4 , 5 , 10 , 12)

(5 , 10 , 11 , 22 , 24)

(9 , 18 , 19 , 38 , 40)

(12 , 24 , 25 , 50 , 52)

Fig. 8

He was asked to study the examples (silently) until he knew the common "pattern." The test given a few minutes later consisted of asking S to complete the following two sequences in the same pattern as the one in the examples he studied:

(3 , , , ,)

(4 , , , ,)

He was given all the time he needed, both for study and for the test.

Pilot studies indicated that simple right-or-wrong scoring techniques would not give reasonable measures of concept acquisition. Therefore, a scheme involving "approximate answers" was developed. The grading scheme is given in Table 5. The first column lists the four relations in the pattern as they occur from left to right. The second and third columns give the score for correct and approximate answers respectively. The fourth column defines "approximate." S also earned 1.25 points if he identified an exact relation but it was one position out of order -- e.g., in the response "(2, 4, 5, 7, 14)" the relations are, from left to right, *double*, *plus one*, *plus two*, and *double*. The subject would be given 2.5 points for each of the first two relations and 1.25 for each of the last two relations, the last two each being exact relations displaced one position.

Each of the two test sequences was graded by the above scheme and the scores were then averaged to get S's performance score (P).

TABLE 5

GRADING SCHEME FOR SIMPLE ALGEBRAIC TASK

relation	score for correct ans.	score for approx. ans.	variation allowed for approx. ans.
double	2.5	1.25	plus or minus one
plus 1	2.5	1.25	plus one
double	2.5	1.25	plus or minus two
plus 2	2.5	1.25	plus or minus one

<u>Task 2: complex algebraic</u>

The subject was given a study sheet on which were written only the examples in Figure 9. He was asked to study the sheet silently until he knew the common pattern.

(2 , 4 , 7 , 15 , 5 , 15 , 16)
(5 , 10 , 13 , 27 , 17 , 51 , 52)
(9 , 18 , 21 , 43 , 33 , 99 , 100)
(12 , 24 , 27 , 55 , 45 , 135 , 136)

Fig. 9

The test given a few minutes later consisted of asking S to complete the following two sequences in the same pattern as the one in the examples he studied:

(3 , , , , , ,)

(4 , , , , , ,)

S was given all the time he needed, both for study and for the test.

The grading scheme is similar to that used for the simple algebraic task. The point values are given in Table 6.

TABLE 6

GRADING SCHEME FOR COMPLEX ALGEBRAIC TASK

relation	score for correct ans.	score for approx. ans.	variation allowed for approx. ans.
double	1.67	.83	plus or minus one
plus 3	1.67	.83	plus or minus one
double plus 1	1.67	.83	plus or minus two
minus 10	1.67	.83	plus or minus two
triple	1.67	.83	plus or minus five
plus 1	1.67	.83	plus one

Task 3: simple geometric

The subject was given a study sheet on which the examples in Figure 10 were displayed. He was asked to study the examples (silently) until he felt that he could draw the figure "from memory." The test given a few minutes later consisted of asking S to draw the figure twice. He was given all the time he needed, both for study and for the test.

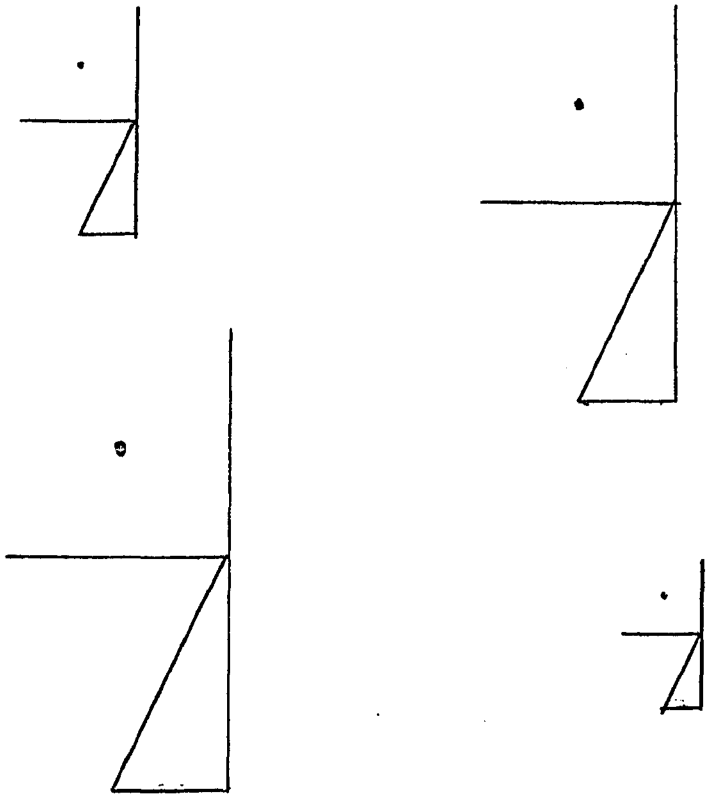


Fig. 10

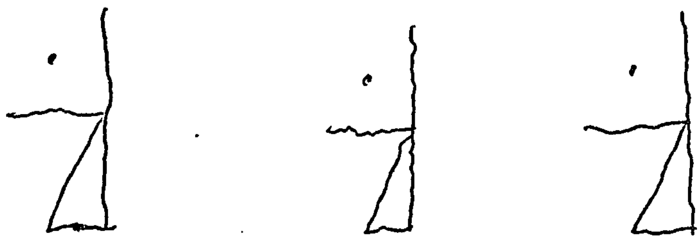
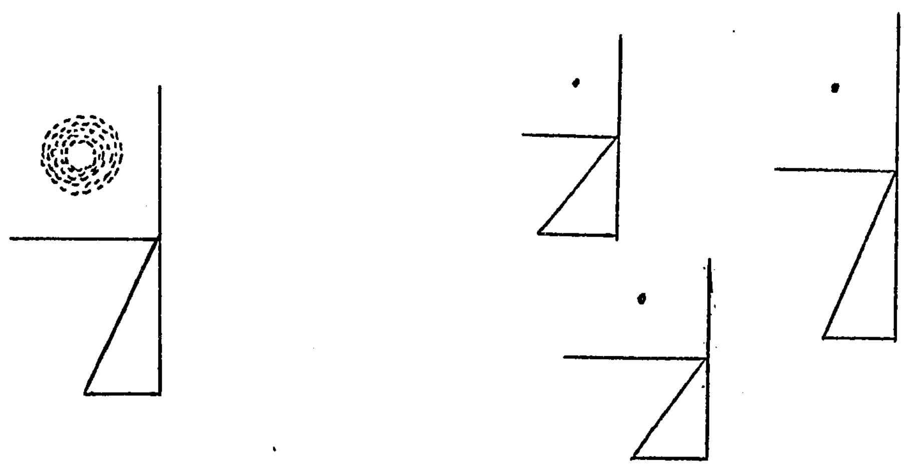


Fig. 11

The following grading scheme was used on each of the two figures drawn, and the grades were averaged.

A score of ten points was given if only "slight" irregularities were present in the drawing. Figure 11 illustrates the author's interpretation of "slight" irregularities. It was attempted here to distinguish between "lack of artistic ability" and "lack of knowledge of the concept."

A score of nine points was given if the dot was placed within the shaded region of Figure 12 or if the proportion was distorted no more than is illustrated in Figure 12.



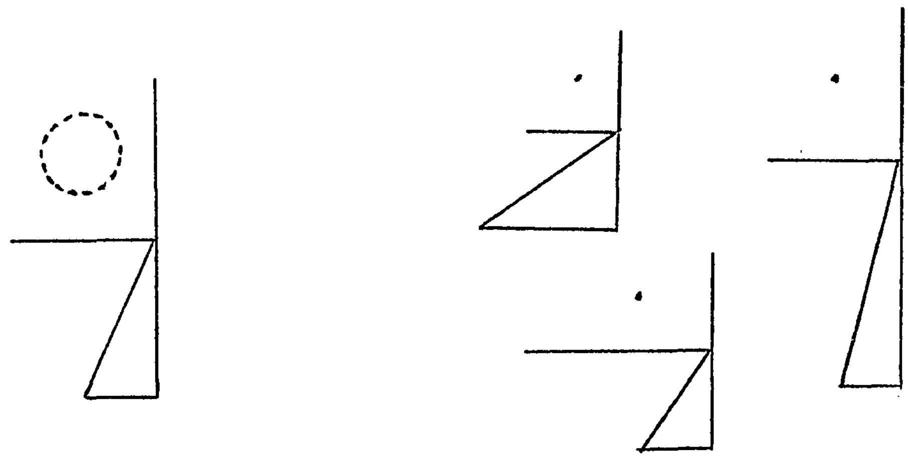
dot placed within the shaded area

proportion

Fig. 12

A score of eight was given if *both* dot placement and proportion were in error within the degree illustrated in Figure 12 or if *either* differed from the true figure in the degree illustrated in Figure 13.

A score of seven was given if either the dot or proportion were in error in the degree illustrated in Figure 13 and the other was in error no more than is illustrated in Figure 12. A score of six was given if both dot and proportion differed from the true figure in the degree illustrated in Figure 13.

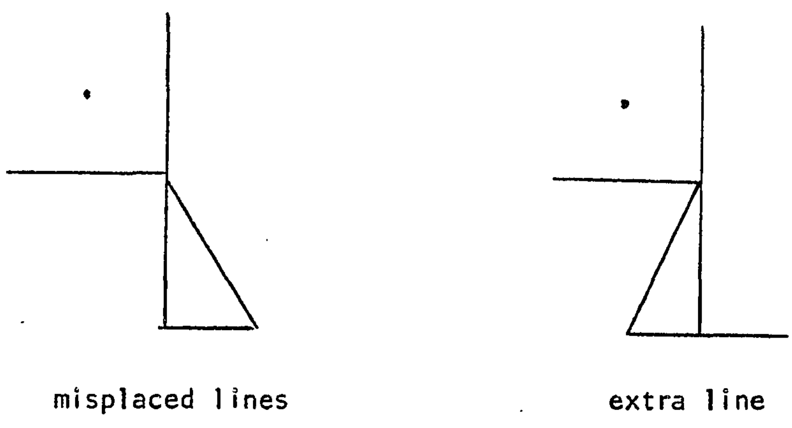


dot placed outside of the circle

proportion

Fig. 13

The above score was reduced by two points for a missing dot and for each missing line. Finally, the score was reduced by one point for each "misplaced" line and each "extra" line. Figure 14 illustrates the interpretation of "misplaced" and "extra" lines.



misplaced lines

extra line

Fig. 14

Although the above grading scheme could be applied to almost all test results in the study, there were a few cases in which the obtained grades seemed undesirable. If the process resulted in a negative score, the subject was given a score of zero (later changed to 0.1 to accommodate a computer program). The scheme also occasionally yielded a score which seemed low (by an intuitive comparison with other test results). In such cases the graders were instructed to make corrections subjectively.

Task 4: complex geometric

The subject was given a study sheet which was blank except for the four examples displayed in Figure 15.

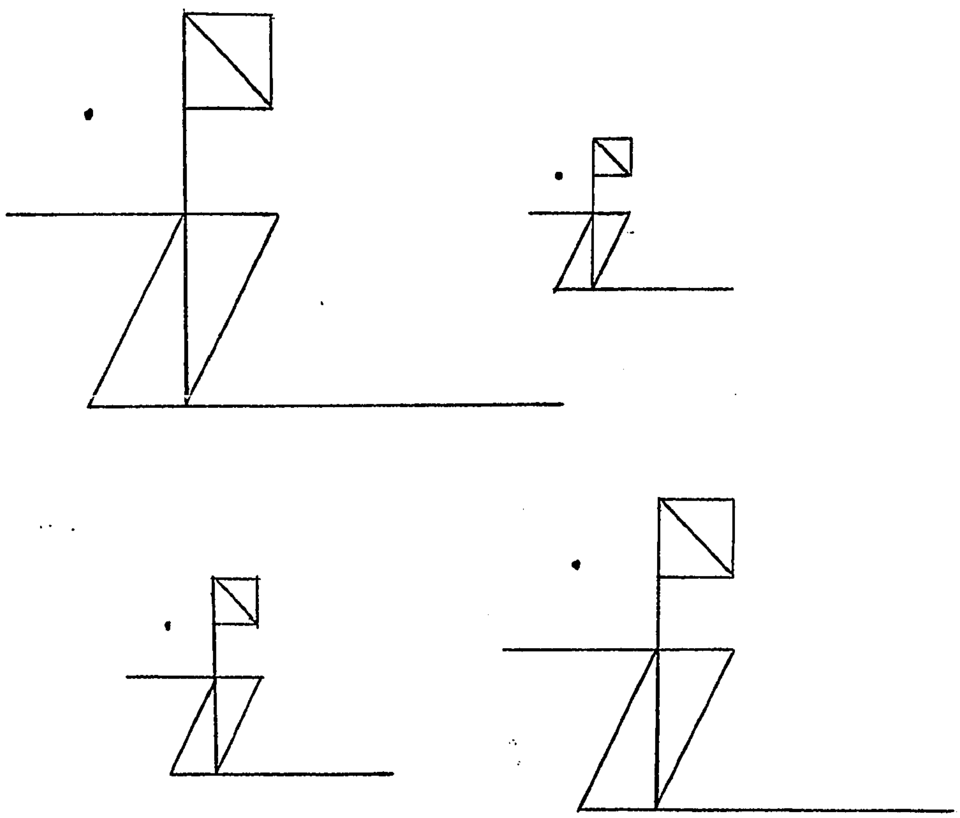


Fig. 15

He was given all the time he needed, both for study and for the test which consisted of asking S to draw the figure twice. The grading scheme was identical to that for task 3. Because of the subjective aspects of this scheme three graders were used. The graders acted independently and the score on each test was taken as the average of the grader's scores on that test. Reliability is shown in Table 7 where correlation coefficients appear. Word count judges 1, 2, and 3 also acted as graders 1, 2, and 3 respectively.

TABLE 7

RELIABILITY OF GEOMETRIC TASK GRADING

	Grader 2	Grader 3	Average
Grader 1	.939	.918	.975
Grader 2		.930	.977
Grader 3			.975

APPENDIX D

(2 , 4 , 5 , 10 , 12)

(5 , 10 , 11 , 22 , 24)

(9 , 18 , 19 , 38 , 40)

(12 , 24 , 25 , 50 , 52)

Algebraic Simple

Sheet 1

(2 , 4 , 7 , 15 , 5 , 15 , 16)

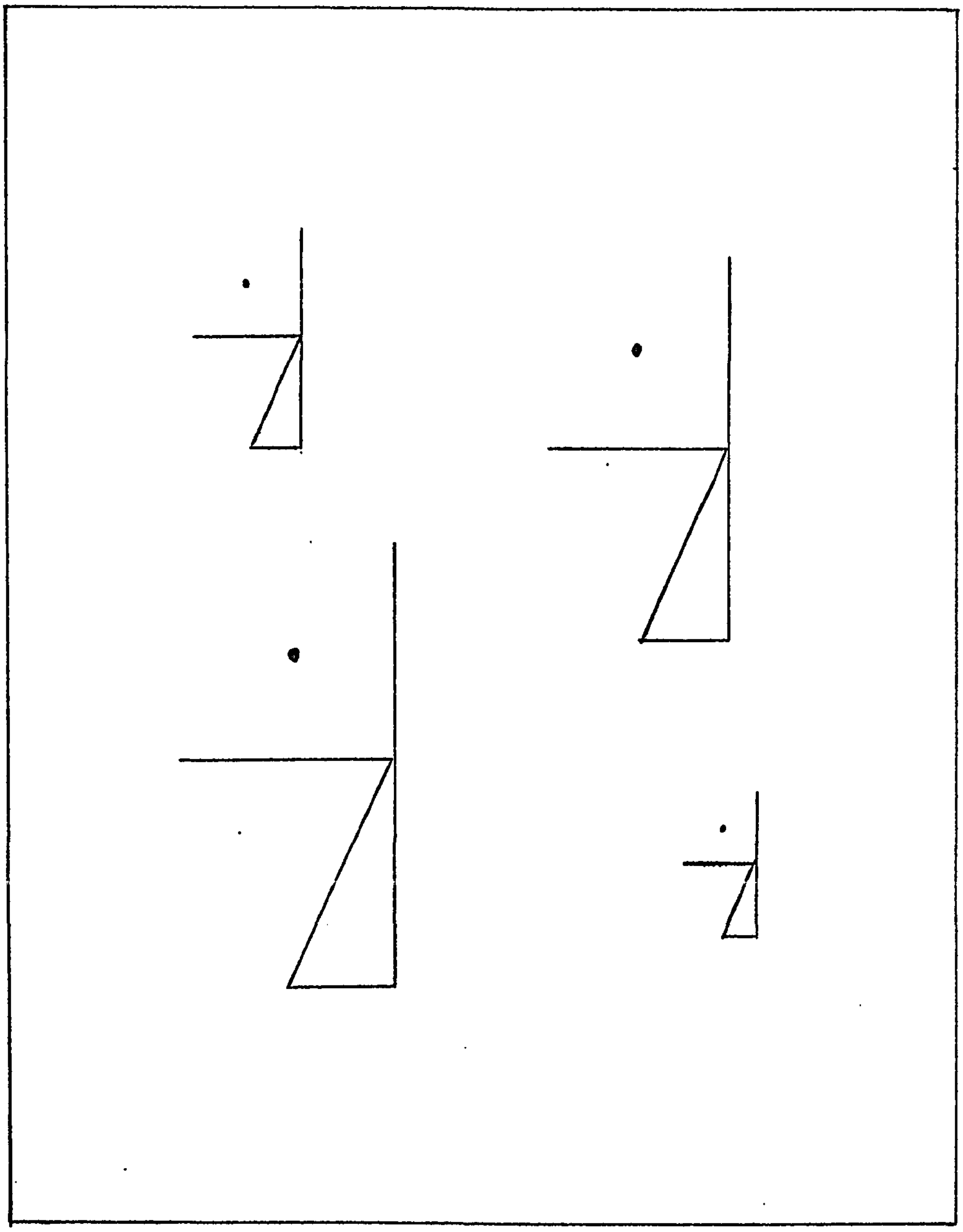
(5 , 10 , 13 , 27 , 17 , 51 , 52)

(9 , 18 , 21 , 43 , 33 , 99 , 100)

(12 , 24 , 27 , 55 , 45 , 135 , 136)

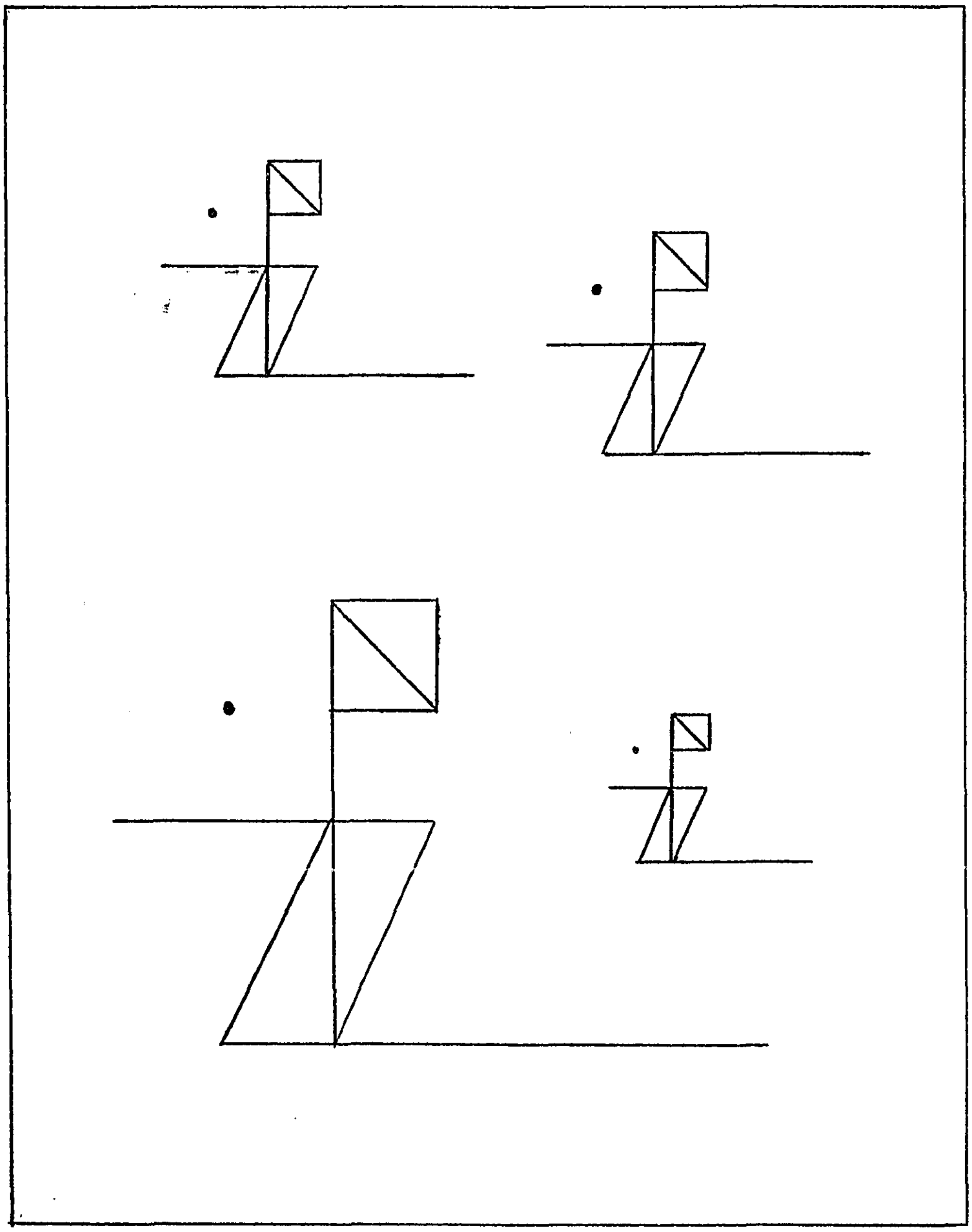
Algebraic Complex

Sheet 1



Geometric Simple

Sheet 1



Geometric Complex

Sheet 1

Circle the letter of the statement which is most nearly true.

As I studied the examples, I

a. definitely had no words in my thoughts.

b. may have had certain words in my thoughts.

c. definitely had certain words in my thoughts.

2

All Tasks

Sheet 2

If you think there were words in your thoughts which helped you learn the concept, please list them here.

3

All Tasks

Sheet 3

Circle the number or letter which best answers the question.

		circle one
1.	How many years of math have you had since the eighth grade?	0 1 2 3 4 5 6 7 8 9
2.	What was your average grade? (approximate)	A B C D F
3.	What is your age?	under 17 17 18 19 20 21 22 23 24 over 24

4

All Tasks

Sheet 4

(3 , , , ,)

(4 , , , ,)

Algebraic Simple

Test Sheet

(3 , , , , , ,)

(4 , , , , , ,)

Algebraic Complex

Test Sheet

Geometric

Test Sheet

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