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QUANTUM FIELD THEORY OF SPINLESS AND SPIN-HALF TACHYONS

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By

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QUANTUM FIELD THEORY OF SPINLESS AND SPIN-HALF

TACHYONS

A dissertation

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CHAPTER 1

INTRODUCTION

For over fifty years after the publication of the special theory of relativity, it was widely assumed and beleived that problems of imaginary energy and causality would rule out the existence of faster than light particles (called tachyons) . Einstein himself made the comment, in his original paper on the special theory that "velocities faster than light have no possibilities of existence". In 1962, Dhar, Bilaniuk, and Sudarshan[1] challenged this assumption. By postulating an imaginary rest mass, they were able to overcome the objections surrounding an imaginary tachyon energy. Such a postulate ruled out accelerating a particle from a speed v less than c, to v greater than c, or vice-versa; but it kept open the possibility of tachyon creation, annihilation or exchange. Since then much has been written on tachyons[1–19], most of the papers dealing with the classical aspects of tachyons. But the classical study of tachyons is very limited and quite unsatisfactory. Feinberg[2] was the first to examine the quantum aspects of tachyons. His paper ruled out the possibility of spinless tachyons being bosons (a marked departure from ordinary particles where spinless mesons are bosons). Sudarshan[3] followed Feinberg, and in his paper reached quite the opposite conclusions, namely that spinless tachyons are bosons. On the assumption that spinless tachyons are participants in radioactive beta decay, Alvager[7] made two attempts to discover tachyons, but without success.

There is a major handicap in developing a tachyon theory. There is no experimental milestone to guide us along. Perhaps that accounts for the widely differing conclusions drawn by different tachyon researchers. In this paper an attempt has been made to develop a scalar tachyon field, that closely resembles the field developed by Sudarshan and Dhar[3]. Later the tachyon spinor field is also developed. We find that in a tachyon spinor field, energy momentum is not an observable. Helicity acquires an added significance, in that it is one of the few physical quantities that are conserved in both tachyon and tardyon spinor fields. We obtain that tachyon exchanges, if they exist, are likely to be long range in nature; and as such are more likely to be found in gravitational and coulomb experiments, than in strong and weak intercation experiments.

CHAPTER 2

CLASSICAL TACHYON THEORY

Einstein's theory of relativity is the only physical theory that appears to place a limit on how fast objects can travel. This apparent restriction centers on two factors. The first has to do with energy expressions becoming imaginary for object velocity $v > c$. The second one involves causality. Let us study both these objections and see how, if at all, we can overcome them.

According to the special theory, the energy of an object of rest mass m, moving with relative velocity v, with respect to an observer is, as measured by the observer, given by

$$E = \frac{mc^2}{\sqrt{1-v^2/c^2}} .$$

If $v > c$, then assuming the rest mass is real, the energy E becomes imaginary. But energy is an observable physical quantity, so it must necessarily be real. Therefore one could argue that faster than light velocities are physically impossible.

But there is a way out, as suggested by Sudarshan[1]. Why not assume that objects moving faster than the speed of light have an imaginary rest mass ? Then the above energy expression becomes real. We must note that the rest mass is not an observable quantity. All physical observers move with $v < c$, and as such can never be in the rest frame of a tachyon. Hence the imaginary rest mass of a tachyon is not a measurable quantity. This approach closely parallels the one used for photons. It would be physically impossible to accelerate a tardyon (an object with $v < c$) to the speed of light, on account of the infinite energy required. Yet photons, moving with the speed of light do exist, although their rest mass is not an observable.

Einsteinian physics splits all particles into three distinct groups, as shown in the diagram on the next page :

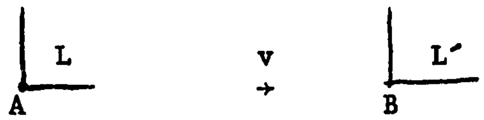
Tardyons , $v < c$;

Photons , $v = c$;

Tachyons , $v > c$.

If a particle belongs to one group, it cannot be accelerated or decelerated into another group. But this does not rule out particles from different groups participating in physical phenomena, as it happens in particle creation, annihilation or exchange.

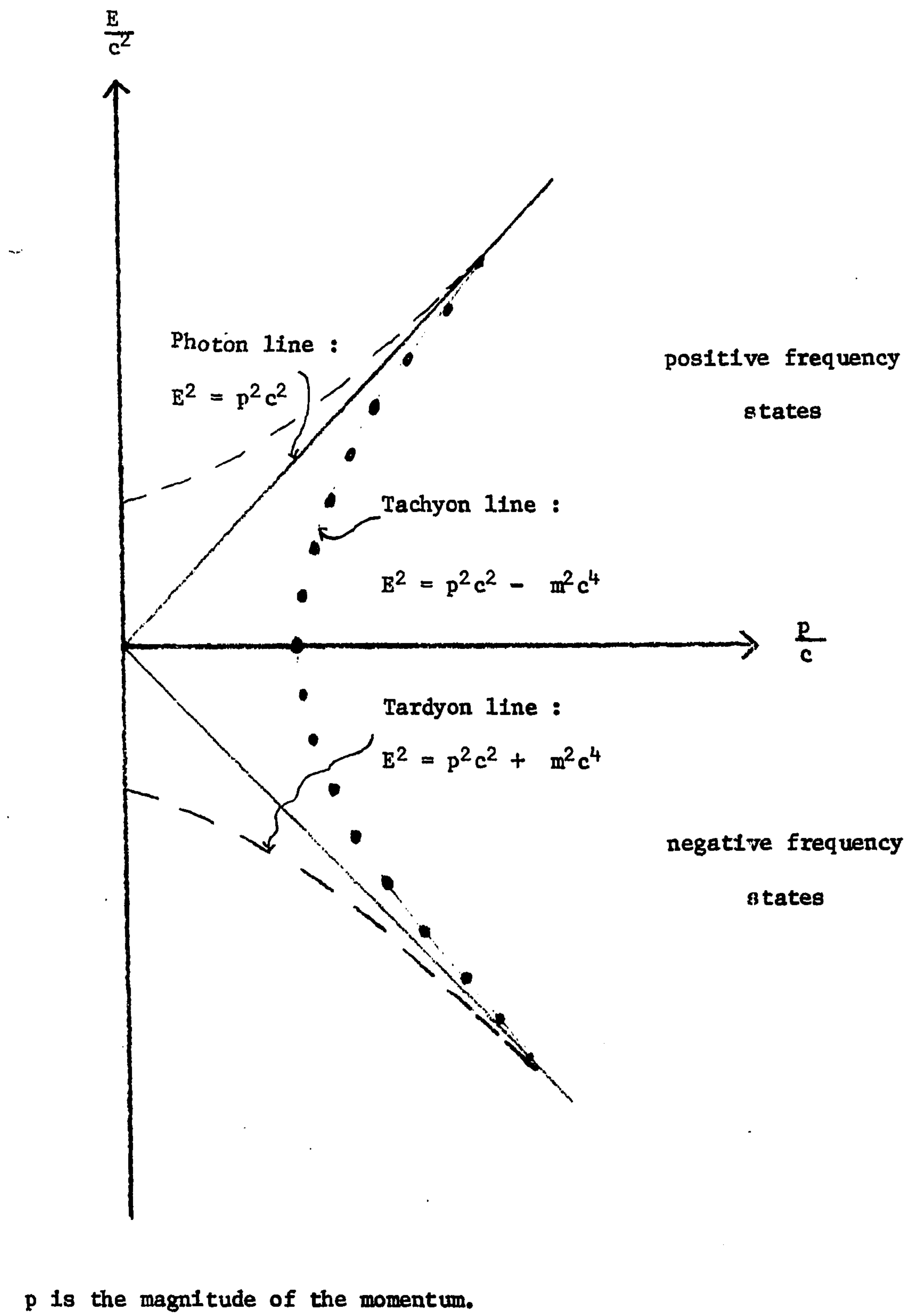
The more serious objection to tachyon existence centers around causality. To appreciate the nature of this difficulty, consider observers L and L′, having a relative speed v between them. Let the origins of L and L′ have emitters (receivers) A and B as shown :



Let us perform a simple experiment involving a tachyon signal sent from A to B. Let B, on receipt of this message, immediately send a tachyon reply back to A. Let us study how L and L′ will observe these message transfers.

1) The first tachyon is emitted at A. To L this event occurs at $x_1 = 0$, $t_1 = t$, and the tachyon has a speed u. As L calculates, L′ ought

Energy-Momentum Diagram



p is the magnitude of the momentum.

to see this event occuring at $x'_1 = -vt\gamma$, $t'_1 = t\gamma$, and the tachyon as having a speed $u' = \dfrac{u-v}{1-uv/c^2}$

2) B absorbs the first tachyon and immediately emits the second tachyon towards A. To L′, the first tachyon has covered a distance $vt\gamma$, with speed u'. As such the time interval between the first tachyon emission and absorbtion is $\Delta t' = \dfrac{vt\gamma}{u'} = \dfrac{vt\gamma(1-uv/c^2)}{(u-v)}$

Therefore $x'_2 = 0$, $t'_2 = t\gamma + \Delta t' = \dfrac{ut}{(u-v)\gamma}$

As L′ calculates, L ought to observe a time interval $\Delta t = \Delta t'(1 + u'v/c^2)\gamma$, as such $x_2 = vt'_2\gamma = \dfrac{uvt}{(u-v)}$, $t_2 = \dfrac{ut}{(u-v)}$

3) A absorbs the second tachyon, which had a speed u'_1 with respect to L′. As L sees it, this tachyon has covered a distance $\dfrac{uvt}{(u-v)}$ with speed $\dfrac{u'_1-v}{(1-u'_1v/c^2)}$

As such the time between the second tachyon emission and absorbtion, as seen by L is $\Delta t_1 = \dfrac{uvt(1-u'_1v/c^2)}{(u-v)(u'_1-v)}$

Therefore to L, the second tachyon has been absorbed at A at the space--time position $x_3 = 0$, $t_3 = \dfrac{ut}{(u-v)} + \dfrac{uvt(1-u'_1v/c^2)}{(u-v)(u'_1-v)}$.

Consider the time difference (t_3-t). Let $u = \dfrac{nc^2}{v}$, $u'_1 = \dfrac{n_1c^2}{v}$,

where n and n_1 are positive numbers.

Then we have $(t_3-t) = \dfrac{\left[(n + n_1)c^2 - nn_1c^2 - v^2\right]}{(nc^2 - v^2)(n_1c^2 - v^2)}v^2t$.

This term becomes negative for $nn_1 > n + n_1 - v^2/c^2$.

Therefore if tachyon speeds u and u'_1 were so chosen as to satisfy the above condition, then an observer could receive a reply to a message, <u>before</u> sending out the message. This is a clear violation of causality.

No one has been able to satisfactorily explain this difficulty. As it stands there is a clear conflict between free tachyon existence and the maintainance of causality. Nevertheless we must note that so far we have studied tachyons, only from a classical viewpoint. All particle physics is basically a statistical quantum mechanical phenomenon. The classical study of tachyons is very limited, and could possibly be very misleading. It is with that thought in mind that we now develop the quantum field theory of tachyons.

CHAPTER 3

REAL SCALAR TACHYONS

The Klein-Gordon equation for a tachyon is $(\Box + m^2)\,\phi(x) = 0$, where m is real, and is the meta mass of a tachyon. Expanding $\phi(x)$ in momentum space, gives *

$$\phi(x) = (2\pi)^{-3/2} \int d^4k \; \phi(k)\; e^{-ikx}\, \delta(k^2 + m^2) \; .$$

Integrating over $|\underline{k}|$, using the Dirac delta property

$$\int f(x)\, \delta\{g(x)\}\, dx = \frac{f(x_o)}{|g'(x_o)|} \; , \text{ where } g(x_o) = 0 \; ,$$

we obtain

$$\phi(x) = \frac{(2\pi)^{-3/2}}{2} \int d\Omega \int_{-\infty}^{\infty} dk_o (k_o^2 + m^2)^{\frac{1}{2}}\, e^{-ikx}\, \phi(k) \; ,$$

where $|\underline{k}| = (k_o^2 + m^2)^{\frac{1}{2}}$, and $\int d\Omega$ is the integral of the solid angle over all directions.

Separating into positive and negative frequency parts, we have

---

* Note : the metric (+ 1, - 1, - 1, - 1) , and natural units in which, $c = \hbar = 1$, are used throughout this paper.

$$\phi(x) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} e^{-ikx} \phi(k)$$

$$+ \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_{-\infty}^0 dk_o (k_o^2 + m^2)^{\frac{1}{2}} e^{-ikx} \phi(k) \quad .$$

In the second integral above, on letting $k \to -k$, we have

$$\phi(x) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} \{ e^{-ikx} \phi(k) + e^{ikx} \phi(-k) \} \quad .$$

We define $\phi^+(x) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} e^{-ikx} \phi(k)$,

and $\phi^-(x) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} e^{ikx} \phi(-k)$, ...2)

so that $\phi(x) = \phi^+(x) + \phi^-(x)$.

Since $\phi(x)$ is a field operator for real tachyons, we obtain

$$\{\phi^+(x)\}^* = \phi^-(x) \; .$$

Commutator Expression

We are interested in the commutator expression $\left[\phi(x),\phi(y)\right]$ for real scalar tachyons. Using eq. 1), we obtain

$$\left[\phi(x),\phi(y)\right] = \frac{(2\pi)}{4}^{-3} \int d\Omega \int d\Omega' \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} \int_0^\infty dk'_o (k'^2_o + m^2)^{\frac{1}{2}}$$

$$e^{-i(kx + k'y)} \left[\phi(k),\phi(k')\right] \quad \text{...3)}$$

where $|\underline{k}| = (k_o^2 + m^2)^{\frac{1}{2}}$, and $|\underline{k}'| = (k_o'^2 + m^2)^{\frac{1}{2}}$.

We do know that

$$\left[\phi(x),\phi(y)\right] = (2\pi)^{-3} \int d^4k\, \varepsilon(k_o)\, e^{-ik(x-y)}\, \delta(k^2 - m^2)$$

for real scalar tardyons,

and

$$\left[\phi_m(x),\phi_n(y)\right] = (2\pi)^{-3}\, g^{mn} \int d^4k\, \varepsilon(k_o)\, e^{-ik(x-y)}\, \delta(k\ )$$

for photons.

Therefore by comparison we assume that for real scalar tachyons,

$$\left[\phi(x),\phi(y)\right] = (2\pi)^{-3} \int d^4k\, \varepsilon(k_o)\, e^{-ik(x-y)}\, \delta(k^2 + m^2) \qquad \ldots 3')$$

where $\varepsilon(k_o) = +1$ for $k_o \geqslant 0$

$= -1$ for $k_o \leqslant 0$

Note that in the limit $m \to 0$, the tachyon commutator expression reduces to the usual photon commutator.

One could well question the Lorentz invariance of eq.3'), on account of the presence of $\varepsilon(k_o)$. To examine this problem in detail, let us refer back to the special theory of relativity, which defines energy-momentum, for a tachyon, by

$$p^\mu = im\frac{dx^\mu}{d\tau},$$

where $d\tau^2 = dt^2 - d\underline{x}^2$ is an invariant.

Tachyons, having space-like momentum, dt is not sign invariant. Therefore, if we want p^{μ} to transform like a four-vector, we will have to take $d\tau$, as $\pm(1-\underline{u}^2)^{\frac{1}{2}}\,dt$, where $\underline{u} \equiv \frac{d\underline{x}}{dt}$, is by definition, the velocity of the particle. Then we have $p^{\mu} = \left(\pm\frac{m}{(u^2-1)^{\frac{1}{2}}}, \pm\frac{m\underline{u}}{(u^2-1)^{\frac{1}{2}}}\right)$.

Consider the case, when the sign in front of the momentum expression, is negative. This implies that an observer may see a tachyon, whose three velocity is in opposite direction, to the direction of the relativistic momentum. One could accept this as another strange characteristic of a tachyon; or one could modify the formalism to remove this particular tachyon behavior. In essence, it is a question of personal philosophy, for at this stage, there is no experimental guidepost to help us out. In this paper, the second approach has been chosen.

Let us consider a one dimensional case of two observers L and L', with relative speed v between them. Let L see a tachyon of energy p_o, momentum p_x, and speed u, as shown,

$$\begin{array}{ccc} L & v & L' \\ * & & * \\ \rightarrow & \rightarrow & \\ p_o, p_x, u. & & \end{array} .$$

If energy momentum were to transform as a four-vector, L would calculate, that L' ought to see the tachyon as having energy p_o', momentum p_x', and speed u', where

$$p_o' = (p_o - vp_x)\gamma,\ p_x' = (p_x - vp_o)\gamma, \text{ and } u' = \frac{u-v}{1-uv}\ ;\ \text{and } \gamma = (1-v^2)^{-\frac{1}{2}}.$$

If $uv > 1$, u and u' will have opposite directions. But for a tachyon $p_x^2 > p_o^2$. Therefore p_x and p_x' are always in the same direction; which implies that p_x' and u' are in opposite directions. To remove this discrepancy, we note that whenever $uv > 1$, we have $p_o < vp_x$;

while whenever $uv < 1$, we have $p_o > vp_x$.

Therefore , if we assume that for $p_o > vp_x$, $p_o' = (p_o - vp_x)\gamma$,

and $p_x' = (p_x - vp_o)\gamma$;

while, for $p_o < vp_x$, $p_o' = -(p_o - vp_x)\gamma$,

and $p_x' = -(p_x - vp_o)\gamma$;

then the relativistic momentum will always have the same direction as that of the three velocity. Also, the energy sign would be the same for all observers.

The assumption made above, implies that energy momentum does not transform as a four vector, but rather as $p^{\mu'} = \pm a^{\mu}{}_{\nu}\, p^{\nu}$; where $a^{\mu}{}_{\nu}$ is the Lorentz matrix. We must bear in mind, that the essential thing in relativity, is that observers be able to communicate with each other. It is not of fundamental importance that energy-momentum be a four vector.

Returning to eq. 3'), on expanding the integral and using the properties of Dirac delta functions, we have

$$\left[\phi(x),\phi(y)\right] = \left(\frac{2\pi}{2}\right)^{-3} \int d\Omega \int_{-\infty}^{\infty} dk_o (k_o^2 + m^2)^{\frac{1}{2}} \varepsilon(k_o)\, e^{-ik_o(t - t')+i(k_o^2+m^2)^{\frac{1}{2}}\hat{\omega}.(\underline{x}-\underline{y})},$$

where $\hat{\omega} = \frac{\underline{k}}{|\underline{k}|}$.

Comparing this last equation with eq.(3), implies

$$\left[\phi(k),\phi(k')\right] = \frac{2\varepsilon(k_o)}{(k_o^2 + m^2)^{\frac{1}{2}}}\,\delta^{(2)}(\hat{\omega} + \hat{\omega}')\,\delta(k_o + k_o') \;. \qquad \ldots 4).$$

On observing the right hand side of 4), we find

$$\left[\phi(k),\phi(k')\right] = -\left[\phi(k'),\phi(k)\right] \;,$$

which implies $\left[\phi(k),\phi(k')\right]$ is a commutator, and <u>not</u> an anti-commutator. The field operators obey Bose-Einstein statistics. Therefore real scalar tachyons behave like real scalar tardyons, in that both are bosons.

In eq. 4), on letting $k' \rightarrow -k'$, the commutator becomes

$$\left[\phi(k),\phi(-k')\right]_- = \frac{2}{(k_o^2 + m^2)^{\frac{1}{2}}}\,\varepsilon(k_o)\,\delta^{(2)}(\hat{\omega} - \hat{\omega}')\,\delta(k_o - k_o') \;. \qquad \ldots 5).$$

If we define $a(\hat{\omega},k_o) \equiv \dfrac{(k_o^2 + m^2)^{\frac{1}{4}}}{\sqrt{2}}\,\phi(k)$, where $|\underline{k}| = (k_o^2 + m^2)^{\frac{1}{2}}$, and $k_o > 0$;

then from eq. 2'), we have

$$a^\dagger(\hat{\omega},k_o) = \frac{(k_o^2 + m^2)^{\frac{1}{4}}}{\sqrt{2}}\,\phi(-k) \;.$$

The commutator expression 5) becomes:

$$\left[a(\hat{\omega},k_o),a^\dagger(\hat{\omega}',k_o')\right]_- = \delta^{(2)}(\hat{\omega} - \hat{\omega}')\,\delta(k_o - k_o') \;.$$

Further eq. 2) reduces to

$$\phi^{+}(x) = \frac{(2\pi)^{-3/2}}{\sqrt{2}} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{1/4} e^{-ikx} a(\hat{\omega},k_o) ,$$

and

$$\phi^{-}(x) = \frac{(2\pi)^{-3/2}}{\sqrt{2}} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{1/4} e^{ikx} a^{\dagger}(\hat{\omega},k_o) . \qquad \ldots 6)$$

Commutator Function

We define the commutator function as $\Delta(x - y) \equiv -i \left[\phi(x),\phi(y)\right]_-$;

Therefore $\Delta(x) = -i(2\pi)^{-3} \int d^4k\, e^{-ikx} \varepsilon(k_o)\, \delta(k^2 + m^2)$.

Integrating first over k_o, and then over the solid angle, we have

$$\Delta(x) = -2\,(2\pi)^{-2} \int_m^\infty d|\underline{k}| \frac{|\underline{k}| \sin(|\underline{k}|x)}{x(\underline{k}^2 - m^2)^{1/2}} \sin\{(\underline{k}^2 - m^2)^{1/2}\, t\} .$$

which means $\Delta(x)\Big|_{t=0} = 0$.

But $\Delta(x)$ being an invariant function, it vanishes for any space-like vector x. This does not imply that tachyons convey messages with $v < c$. For as $\left[\phi(x),\phi(y)\right]_-$ is a c-number, we have

$[\phi(x),\phi(y)]_- = \Phi_o^*[\phi(x),\phi(y)]_-\Phi_o$, where Φ_o is the vacuum state.

Expanding the right hand side of this equation, into positive and negative frequency parts, noting that: $\phi^+(r_1)\,\Phi_o = \Phi_o^*\,\phi^-(r_2) = 0$, we have

$$[\phi(x),\phi(y)]_- = \Phi_o^*\,\phi^+(x)\,\phi^-(y)\,\Phi_o \quad - \quad \Phi_o^*\,\phi^+(y)\,\phi^-(x)\,\Phi_o \,.$$

Therefore $\Delta(x - y)\Big|_{x_o = y_o} = 0$, implies

$$\Phi_o^*\,\phi^+(x)\,\phi^-(y)\,\Phi_o = \Phi_o^*\,\phi^+(y)\,\phi^-(x)\,\Phi_o \,,$$

which means that a transcendental tachyon going from x to y, is equivalent to one going from y to x.

Consider the commutator function

$$\Delta(x) = -i\left(\frac{2\pi}{2}\right)^{-3} \int_{|\underline{k}| > m} \frac{d^3k\, e^{i\underline{k}.\underline{x}}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} \left\{ e^{-i(\underline{k}^2 - m^2)^{\frac{1}{2}}t} - e^{i(\underline{k}^2 - m^2)^{\frac{1}{2}}t} \right\} .$$

Taking the partial differentiation with respect to time, at $t = 0$, of $\Delta(x)$, we obtain

$$\frac{\partial\Delta(x)}{\partial t}\Big|_{t = 0} = -\delta(\underline{x}) + \frac{2(2\pi)^{-2}}{x^3}\{ \sin(mx) - mx\cos(mx) \} .$$

It is worth noting that for real scalar tardyons $\frac{\partial\Delta(x)}{\partial t}\Big|_{t = 0} = -\delta(x)$.

Hence real scalar tachyons cannot be localized in space—a quantum mechanical result in agreement with classical tachyon theory. But because of the factor $1/x^3$, however, $\left.\frac{\partial\Delta(x)}{\partial t}\right|_{t=0} \to 0$, as $x \to \infty$.

Propagator Function

In the quantum field theory of interacting particles, the vacuum expectation value of the chronological product plays a crucial role. It would be appropriate, at this stage to develop the same value for a real scalar tachyon field. By definition, the vacuum expectation value of the chronological product is

$$\Phi_o^* \, T\{\, \phi(x), \phi(y) \,\} \, \Phi_o \equiv \Phi_o^* \, \phi(x)\phi(y) \, \Phi_o \, , \text{ for } x^0 > y^0 \, .$$

$$\equiv \Phi_o^* \, \phi(y)\phi(x) \, \Phi_o \, , \text{ for } x^0 < y^0 \, . \qquad \ldots 7)$$

where $\phi(x)$ and $\phi(y)$ are field operators at x and y respectively . Expanding $\phi(x)$ and $\phi(y)$ into positive and negative frequency parts, and utilising the properties of the vacuum state, we have

$$\Phi_o^* \, T\{\phi(x),\phi(y)\} \, \Phi_o = \Phi_o^* \, \phi^{+}(x) \, \phi^{-}(y) \, \Phi_o \, , \text{ for } x^0 > y^0 \, .$$

$$\Phi_o^* \, \phi^{+}(y) \, \phi^{-}(x) \, \Phi_o \, , \text{ for } x^0 < y^0 \, . \qquad \ldots 8)$$

The commutator function $[\phi^{+}(x),\phi^{-}(y)]_{-}$ being a c-number,

$$[\phi^{+}(x),\phi^{-}(y)]_{-} = \Phi_{o}^{*}\,[\phi^{+}(x),\phi^{-}(y)]_{-}\,\Phi_{o}\,.$$

Utilising the properties of the vacuum state, and the fact that the field operators obey Bose-Einstein statistics, we have

$$[\phi^{+}(x),\phi^{-}(y)]_{-} = \Phi_{o}^{*}\,\phi^{+}(x)\,\phi^{-}(y)\,\Phi_{o}\,.$$

Similarly, it can be shown that

$$[\phi^{-}(x),\phi^{+}(y)]_{-} = -\Phi_{o}^{*}\,\phi^{+}(y)\,\phi^{-}(x)\,\Phi_{o}\,.$$

Feeding these expressions into (8), gives

$$\begin{aligned}\Phi_{o}^{*}\,T\{\phi(x),\phi(y)\}\,\Phi_{o} &= -[\phi^{-}(x),\phi^{+}(y)]_{-}\;, \text{ for } x^{0} < y^{0}\,.\\ &= [\phi^{+}(x),\phi^{-}(y)]_{-}\;, \text{ for } x^{0} > y^{0}\,.\end{aligned}$$

Introducing the step function $\theta(s) = +1$, for $s > 0$.

$= 0$, for $s < 0$.

gives

$$\begin{aligned}\Phi_{o}^{*}\,T\{\phi(x),\phi(y)\}\,\Phi_{o} &= [\phi^{+}(x),\phi^{-}(y)]_{-}\theta(x^{0} - y^{0})\\ &\quad - [\phi^{-}(x),\phi(y)]_{-}\theta(y^{0} - x^{0})\,.\end{aligned}$$

But from the commutator function,we do know that

$$\left[\phi^{-}(x),\phi^{+}(y)\right]_{-} = \frac{(2\pi)^{-3}}{2} \int_{|\underline{k}|\geqslant m} d^3k \, \frac{e^{-ik(x-y)}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} ,$$

and

$$\left[\phi^{+}(x),\phi^{-}(y)\right]_{-} = -\frac{(2\pi)^{-3}}{2} \int_{|\underline{k}|\geqslant m} d^3k \, \frac{e^{ik(x-y)}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} ;$$

where $k_o = (\underline{k}^2 - m^2)^{\frac{1}{2}}$;

which implies

$$\Phi_o^* \, T\{\phi(x),\phi(y)\} \, \Phi_o = -\frac{(2\pi)^{-3}}{2} \int_{|\underline{k}|\geqslant m} d^3k \, \frac{e^{-ik(x-y)}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} \, \theta(y^0 - x^0)$$

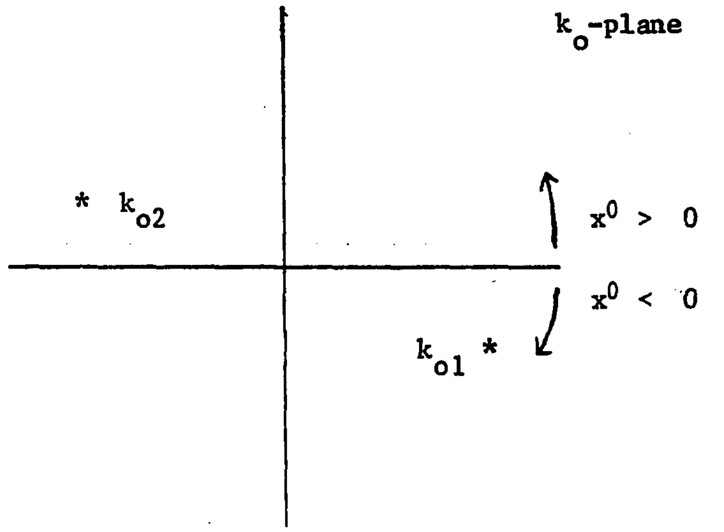
$$-\frac{(2\pi)^{-3}}{2} \int_{|\underline{k}|\geqslant m} d^3k \, \frac{e^{ik(x-y)}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} \, \theta(x^0 - y^0) \quad . \qquad \ldots 9)$$

Let us define a function $\Delta^c(x) \equiv (2\pi)^{-4} \int_{|\underline{k}|>m} \frac{d^4k \, e^{ikx}}{(m^2 + k^2 + i\varepsilon)}$.

$\Delta^c(x)$ has poles in the k_o plane at k_{o1} and at k_{o2} ,where

$$k_{o1} = (\underline{k}^2 - m^2)^{\frac{1}{2}} - \frac{i\varepsilon}{2}(\underline{k}^2 - m^2)^{-\frac{1}{2}}$$

$$k_{o2} = -(\underline{k}^2 - m^2)^{\frac{1}{2}} + \frac{i\varepsilon}{2}(\underline{k}^2 - m^2)^{-\frac{1}{2}}$$



To evaluate $\Delta^c(x)$, we can perform contour integrals in the two distinct cases : a) $x^0 > 0$, b) $x^0 < 0$.

Case a) : $x^0 > 0$, then $\Delta^c(x) = (2\pi)^{-4} \int\limits_{|\underline{k}| \geqslant m} d^3k \int_{-\infty}^{\infty} dk_o \dfrac{e^{-i\underline{k}.\underline{x} + ik_o x^0}}{(k_o - k_{o1})(k_o - k_{o2})}$,

where the contour integration over k_o, involves a pole at k_{o2}. Performing this integration, $\Delta^c(x)$ reduces to

$$\Delta^c(x) = -i\frac{(2\pi)^{-3}}{2} \int\limits_{|\underline{k}| \geqslant m} \frac{d^3k \, e^{-ikx}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} \quad , \text{ where } k_o = (\underline{k}^2 - m^2)^{\frac{1}{2}} \, ;$$

Similarly, for $x^0 < 0$, the pole at k_{o1} is involved, and $\Delta^c(x)$ becomes

$$\Delta^c(x) = -i\frac{(2\pi)^{-3}}{2} \int\limits_{|\underline{k}| \geqslant m} \frac{d^3k \, e^{ikx}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}} \quad , \text{ where } k_o = (\underline{k}^2 - m^2)^{\frac{1}{2}} \quad .$$

Therefore $\Delta^c(x)$ can be written as

$$\Delta^c(x) = -i(2\pi)^{-3} \left\{ \int_{|\underline{k}|\geq m} \frac{d^3k\, e^{ikx}}{2k_o}\, \theta(-x^0) \quad + \quad \int_{|\underline{k}|\geq m} \frac{d^3k\, e^{-ikx}}{2k_o}\, \theta(x^0) \right\}$$

where $k_o = (\underline{k}^2 - m^2)^{\frac{1}{2}}$.

Comparing this expression with 9), implies

$$\Delta^c(x - y) \;=\; +i\, \Phi_o^* \, T\, \{\, \phi(x), \phi(y) \,\}\, \Phi_o \, .$$

Therefore $\Phi_o^* \, T\, \{\, \phi(x), \phi(y) \,\}\, \Phi_o \;=\; -i(2\pi)^{-4} \displaystyle\int_{|\underline{k}|>m} \frac{e^{ikx}\, d^4k}{(m^2 + k^2 + i\varepsilon)}$.

We call $\Delta^c(x - y)$, the propagator function.

Dynamic Variables

Choosing the Lagrangian for the real scalar tachyon as

$$L = \frac{1}{2} \sum_n g^{nn} : \frac{\partial\phi}{\partial x^n} \frac{\partial\phi}{\partial x^n} : + \frac{m^2}{2} : \phi^2 : ,^*$$

* Note : : denotes the normal order product, as in Bogoliubov and Shirkov.[20] The Latin indices have the range 0, 1, 2, 3.

we obtain, on applying the Euler-Lagrange equations, the Klein-Gordon eq. for a tachyon, $(\Box + m^2)\,\phi(x) = 0$. Knowing the Lagrangian, we can find the energy-momentum tensor

$$T^{mn} = g^{mm} g^{nn} : \frac{\partial\phi}{\partial x^m} \frac{\partial\phi}{\partial x^n} : - g^{mn} L .$$

This leads us to the energy-momentum vector $P^n = \int T^{on}\, d\underline{x}$.
Let us consider the energy vector operator $P^o = \int T^{oo}\, d\underline{x}$. Placing in the value for T^{oo} in the expression for P^o, and expanding our field operators into positive and negative frequency parts, P^o takes on the form

$$P^o = \int \frac{1}{2} \{ \sum_n \frac{\partial\phi}{\partial x^n}^+ \frac{\partial\phi}{\partial x^n}^+ - m^2 \phi^+ \phi^+ \}\, d\underline{x}$$

$$+ \int \{ \sum_n \frac{\partial\phi}{\partial x^n}^- \frac{\partial\phi}{\partial x^n}^+ - m^2 \phi^- \phi^+ \}\, d\underline{x}$$

$$+ \int \frac{1}{2} \{ \sum_n \frac{\partial\phi}{\partial x^n}^- \frac{\partial\phi}{\partial x^n}^- - m^2 \phi^- \phi^- \}\, d\underline{x}$$

In the above the normal order product has been removed by making use of the boson property of commutation

Consider the first integral of P^o. On expanding the integral in momentum space, using eq. 6), we find that it contains the factor $-(\sum_n k_n k'_n + m^2)$. Integrating over $\underline{x}$, implies $\underline{k} = -\underline{k}'$, and $k_o = k'_o$, therefore the first integral of P^o vanishes. Similarly, the third integral of P^o also vanishes. Let us consider the second integral of P^o. Expanding in momentum space, this integral takes on the form

$$\left(\frac{2\pi}{2}\right)^{-3} \int d\Omega \int d\Omega' \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{4}} \int_0^\infty dk_o' (k_o'^2 + m^2)^{\frac{1}{4}} \int d\underline{x}\, e^{-i(k - k')x}$$

$$\sum_n (k_n k_n' - m^2)\, a^\dagger(\hat{\omega},k_o)\, a(\hat{\omega}',k_o') \quad .$$

Integrating over $\underline{x}$, using the Dirac delta properties, this integral reduces to

$$\int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} k_o^2\, a^\dagger(\hat{\omega},k_o)\, a(\hat{\omega},k_o) \quad .$$

Therefore the energy operator takes on the form

$$P^o = \int d\Omega \int_0^\infty dk_o k_o^2\, (k_o^2 + m^2)^{\frac{1}{2}}\, a^\dagger(\hat{\omega},k_o)\, a(\hat{\omega},k_o) \quad .$$

Let us define $a_1(\hat{\omega},k_o) \equiv (k_o^2 + m^2)^{\frac{1}{4}}\, k_o^{\frac{1}{2}}\, a(\hat{\omega},k_o)$,

then

$$a_1^\dagger(\hat{\omega},k_o) = (k_o^2 + m^2)^{\frac{1}{4}}\, k_o^{\frac{1}{2}}\, a^\dagger(\hat{\omega},k_o) \ ;$$

and we obtain the final expression for the energy vector operator as

$$P^o = \int d\Omega \int_0^\infty dk_o\, k_o\, a_1^\dagger(\hat{\omega},k_o) a_1(\hat{\omega},k_o) \quad .$$

The product $a_1^\dagger(\hat{\omega},k_o)\ a_1(\hat{\omega},k_o)$ can be interpreted as the average density of particles of energy k_o, space orientation $\hat{\omega}$, and having no charge or spin.

CHAPTER 4

COMPLEX SCALAR TACHYONS

We develop the complex scalar field in analogy to the real scalar field. We will now have two mutually conjugate functions ϕ and ϕ^*, obeying the Klien-Gordon equation. Expanding $\phi(x)$ in momentum space, and then integrating over the radial component $|\underline{k}|$, we have

$$\phi(x) = (2\pi)^{-3/2} \int_{\substack{\text{solid}\\\text{angle}}} d\Omega \int_{-\infty}^{\infty} \frac{dk_0}{2} e^{-ikx} (k_0^2 + m^2)^{1/2} \phi(k)$$

where $|\underline{k}| = (k_0^2 + m^2)^{1/2}$.

Splitting $\phi(x)$ into positive and negative frequency parts, gives

$$\phi(x) = (2\pi)^{-3/2} \int d\Omega \int_0^{\infty} dk_0 e^{-ikx} \frac{(k_0^2 + m^2)^{1/2}}{2} \phi(k)$$

$$+ (2\pi)^{-3/2} \int d\Omega \int_{-\infty}^{0} dk_0 e^{-ikx} \frac{(k_0^2 + m^2)^{1/2}}{2} \phi(k) .$$

In the second integral, on letting $k \to -k$, we obtain

$$\phi(x) = (2\pi)^{-3/2} \int d\Omega \int_0^{\infty} dk_0 \frac{(k_0^2 + m^2)^{1/2}}{2} \{ e^{-ikx} \phi(k) + e^{ikx} \phi(-k) \}$$

Taking the complex conjugate of the last integral, gives us

$$\phi^*(y) = (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_0 \frac{(k_0^2 + m^2)^{1/2}}{2} \{e^{ikx}\phi^*(k) + e^{-ikx}\phi^*(-k)\} .$$

Let us define : $a(\hat{\omega},k_0) \equiv \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} \phi(k)$,

$$b^\dagger(\hat{\omega},k_0) \equiv \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} \phi(-k) ,$$

with $|\underline{k}| = (k_0^2 + m^2)^{1/4}$.

Then we have : $a^\dagger(\hat{\omega},k_0) = \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} \phi^*(k)$,

$$b(\hat{\omega},k_0) = \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} \phi^*(-k) ,$$

and our expressions for $\phi(x)$, $\phi^*(y)$ become

$$\phi(x) = (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_0 \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} \{e^{-ikx} a(\hat{\omega},k_0) + e^{ikx} b^\dagger(\hat{\omega},k_0)\}$$

and

$$\phi^*(y) = (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_0 \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} \{e^{iky} a^\dagger(\hat{\omega},k_0) + e^{-iky} b(\hat{\omega},k_0)\}$$

We can now define $\phi^+(x) \equiv (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_0 \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} e^{-ikx} a(\hat{\omega},k_0)$

$$\phi^-(x) \equiv (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_0 \frac{(k_0^2 + m^2)^{1/4}}{\sqrt{2}} e^{ikx} b^\dagger(\hat{\omega},k_0)$$

Then we have $\phi^{*-}(y) = (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_o \frac{(k_o^2 + m^2)^{1/4}}{\sqrt{2}} e^{iky} a^\dagger(\hat{\omega},k_o)$,

and

$$\phi^{*+}(y) = (2\pi)^{-3/2} \int d\Omega \int_0^\infty dk_o \frac{(k_o^2 + m^2)^{1/4}}{\sqrt{2}} e^{-iky} b(\hat{\omega},k_o),$$

which gives us

$$\phi(x) = \phi^+(x) + \phi^-(x) ,$$

$$\phi^*(y) = \phi^{*+}(y) + \phi^{*-}(y) ,$$

and

$$\{\phi^{\pm}(x)\}^* = \phi^{*\mp}(x).$$

Commutator Expression

To calculate the commutator $[\phi(x),\phi^*(y)]_-$, we utilise the $\phi(x)$ and $\phi^*(y)$ expressions developed above. This gives us

$$\begin{aligned}[\phi(x),\phi^*(y)]_- = \frac{(2\pi)^{-3}}{2} \int d\Omega \int d\Omega' \int_0^\infty dk_o (k_o^2 + m^2)^{1/4} \int_0^\infty dk_o' (k_o'^2 + m^2)^{1/4} \\
\{ e^{-i(kx + k'y)} [a(\hat{\omega},k_o),b(\hat{\omega}',k_o')]_- \\
+ e^{-i(kx-k'y)} [a(\hat{\omega},k_o),a^\dagger(\hat{\omega}',k_o')]_- \quad \ldots (1) \\
+ e^{i(kx - k'y)} [b^\dagger(\hat{\omega},k_o),b(\hat{\omega}',k_o')]_- \\
+ e^{i(kx + k'y)} [b^\dagger(\hat{\omega},k_o),a^\dagger(\hat{\omega}',k_o')]_- \},\end{aligned}$$

But as with the real scalar field, we assume that

$$\left[\phi(x),\phi^*(y)\right]_- = (2\pi)^{-3} \int d^4k \; \varepsilon(k_o) \; e^{-ik(x-y)} \; \delta(k^2+m^2) \; .$$

Expanding this integral into momentum space, and then integrating over the radial component $|\underline{k}|$, we have

$$\left[\phi(x),\phi^*(y)\right]_- = \frac{(2\pi)^{-3}}{2} \int d\Omega \int_0^\infty dk_o (k_o^2+m^2)^{\frac{1}{2}} \; \{ e^{-ik(x-y)} - e^{ik(x-y)} \}$$

Comparing this last equation with eq.(1), gives us

$$\left[a(\hat{\omega},k_o),a^\dagger(\hat{\omega}',k_o')\right]_- = \delta^{(2)}(\hat{\omega}-\hat{\omega}')\;\delta(k_o-k_o')$$

$$\left[b^\dagger(\hat{\omega},k_o),b(\hat{\omega}',k_o')\right]_- = -\delta^{(2)}(\hat{\omega}-\hat{\omega}')\;\delta(k_o-k_o')$$

$$\left[a(\hat{\omega},k_o),b(\hat{\omega}',k_o')\right]_- = 0$$

$$\left[b^\dagger(\hat{\omega},k_o),a^\dagger(\hat{\omega}',k_o')\right]_- = 0 \; .$$

Dynamic Variables

Choosing the Lagrangian for the complex scalar tachyon to be

$$L(x) = \sum_n g^{nn} : \frac{\partial\phi^*}{\partial x^n} \frac{\partial\phi}{\partial x^n} : + m^2 : \phi^*\phi : ,$$

we obtain the energy-momentum tensor

$$T^{mn} = g^{mm} g^{nn} : \frac{\partial\phi^*}{\partial x^m} \frac{\partial\phi}{\partial x^n} + \frac{\partial\phi^*}{\partial x^n} \frac{\partial\phi}{\partial x^m} : - g^{mn} L ,$$

which leads to the energy momentum vector

$$P^n = \int T^{on} d\underline{x} .$$

Consider $T^{oo} = : \frac{\partial\phi^*}{\partial x^o} \frac{\partial\phi}{\partial x^o} + \frac{\partial\phi^*}{\partial x^o} \frac{\partial\phi}{\partial x^o} : - L .$

On placing the value for L, T^{oo} becomes,

$$T^{oo} = \sum_n : \frac{\partial\phi^*}{\partial x^n} \frac{\partial\phi}{\partial x^n} : - m^2 : \phi^*\phi : ,$$

Therefore on expanding into positive and negative frequency parts, the energy operator P^o will take on the form,

$$P^o = \int \{\sum_n \frac{\partial\phi^{*+}}{\partial x^n} \frac{\partial\phi^+}{\partial x^n} - m^2\phi^{*+}\phi^+\} d\underline{x} +$$

$$\int \left\{ \sum_n \frac{\partial\phi^{*-}}{\partial x^n} \frac{\partial\phi^{+}}{\partial x^n} - m^2\phi^{*-} \phi^{+} \right\} d\underline{x}$$

$$+ \quad \int \left\{ \sum_n \frac{\partial\phi^{*-}}{\partial x^n} \frac{\partial\phi^{-}}{\partial x^n} - m^2\phi^{*-} \phi^{-} \right\} d\underline{x}$$

$$+ \quad \int \left\{ \sum \frac{\partial\phi^{-}}{\partial x^n} \frac{\partial\phi^{*+}}{\partial x^n} - m^2\phi^{-} \phi^{*+} \right\} d\underline{x} \ . \qquad ..2)$$

In the above expansion we have utilised the boson property of commutation of the field operators. Let us expand into momentum space the first integral in 2) . Then we have

$$\int d\underline{x} \left\{ \sum_n \frac{\partial\phi^{*+}}{\partial x^n} \frac{\partial\phi^{+}}{\partial x^n} - m^2\phi^{*+}\phi^{+} \right\}$$

$$= \frac{(2\pi)}{2}^{-3} \int d\Omega \int d\Omega' \int_0^\infty dk_0 (k_0^2 + m^2)^{\frac{1}{4}} \int_0^\infty dk_0' (k_0'^2 + m^2)^{\frac{1}{4}} \int d\underline{x}\, e^{-ix(k + k')}$$

$$\left\{ -\sum_n k_n k_n' - m^2 \right\} a(\hat{\omega}, k_0)\, b(\hat{\omega}', k_0')$$

On integrating over $\underline{x}$, we find that the factor $\{ -\sum_n k_n k_n' - m^2 \}$ goes to zero. Therefore the integral in question vanishes. Similarly, we find that the third integral in 2) also vanishes. Let us consider the second integral in 2). Expanding it in momentum space ,we obtain

$$\int d\underline{x} \left\{ \sum_n \frac{\partial\phi^{*-}}{\partial x^n} \frac{\partial\phi^{+}}{\partial x^n} - m^2\phi^{*-}\phi^{+} \right\} =$$

$$\frac{(2\pi)^{-3}}{2} \int d\Omega \int d\Omega' \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{4}} \int_0^\infty dk_o' (k_o'^2 + m^2)^{\frac{1}{4}} \int d\underline{x}\, e^{-i(k - k')x}$$

$$\{ -\sum_n k_n k_n' - m^2 \} a^\dagger(\hat{\omega}', k_o') \, a(\hat{\omega}, k_o).$$

On integrating over $\underline{x}$, the above integral becomes

$$\int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} \; k_o^2 \, a^\dagger(\hat{\omega}, k_o) \, a(\hat{\omega}, k_o) \; .$$

Similarly, the last integral of 2) reduces down to

$$\int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} \; k_o^2 \, b^\dagger(\hat{\omega}, k_o) \, b(\hat{\omega}, k_o) \; .$$

Hence the energy vector operator takes on the form

$$P^o = \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} k_o^2 \{ a^\dagger(\hat{\omega}, k_o) \, a(\hat{\omega}, k_o) + b^\dagger(\hat{\omega}, k_o) \, b(\hat{\omega}, k_o) \}.$$

If we let $a_1(\hat{\omega}, k_o) = (k_o^2 + m^2)^{\frac{1}{4}} k_o^{\frac{1}{2}} \; a(\hat{\omega}, k_o)$

and

$$b_1(\hat{\omega}, k_o) = (k_o^2 + m^2)^{\frac{1}{4}} k_o^{\frac{1}{2}} \; b(\hat{\omega}, k_o) \; , \qquad \ldots 3)$$

then P^o reduces to

$$P^o = \int d\Omega \int_0^\infty dk_o \, k_o \{ a_1^\dagger(\hat{\omega}, k_o) \, a_1(\hat{\omega}, k_o) + b_1^\dagger(\hat{\omega}, k_o) \, b_1(\hat{\omega}, k_o) \} \; .$$

Charge

The current four-vector J^n can be determined from the Lagrangian, and for a complex scalar field, it is

$$J^n = i\, g^{nn} : \{ \phi^* \frac{\partial\phi}{\partial x^n} - \frac{\partial\phi^*}{\partial x^n}\phi \} : .$$

Therefore the charge of the field, which is $Q = \int J^o \, d\underline{x}$, becomes

$$Q = i \int : \{\phi^* \frac{\partial\phi}{\partial x^o} - \frac{\partial\phi^*}{\partial x^o} \phi\} : d\underline{x} .$$

Splitting into positive and negative frequency parts, and removing the normal ordered product, by utilising the boson property of commutation, the charge takes on the form

$$\begin{aligned} Q = {} & i \int \{ \phi^{*+} \frac{\partial\phi^+}{\partial x^o} - \frac{\partial\phi^{*+}}{\partial x^o} \phi^+ \} d\underline{x} \\ & + i \int \{ \frac{\partial\phi^-}{\partial x^o} \phi^{*+} - \phi^- \frac{\partial\phi^{*+}}{\partial x^o} \} d\underline{x} \\ & + i \int \{ \phi^{*-} \frac{\partial\phi^-}{\partial x^o} - \frac{\partial\phi^{*-}}{\partial x^o} \phi^- \} d\underline{x} \\ & + i \int \{ \phi^{*-} \frac{\partial\phi^+}{\partial x^o} - \frac{\partial\phi^{*-}}{\partial x^o} \phi^+ \} d\underline{x} \end{aligned} \qquad \text{...4)}$$

On expanding the first integral of 4) ,into momentum space, we have

$$i \int \{ \phi^{*+} \frac{\partial \phi^{+}}{\partial x^{o}} - \frac{\partial \phi^{*+}}{\partial x^{o}} \phi^{+} \} d\underline{x}$$

$$= \left(\frac{2\pi}{2}\right)^{-3} \int d\Omega \int d\Omega' \int_{o}^{\infty} dk_o (k_o^2 + m^2)^{\frac{1}{4}} \int_{o}^{\infty} dk_o' (k_o'^2 + m^2)^{\frac{1}{4}} (k_o - k_o')$$

$$\int d\underline{x}\, e^{-i(k + k')x}\, b(\hat{\omega}', k_o')\, a(\hat{\omega}, k_o) \, .$$

On integrating over $\underline{x}$, we find that the factor $(k_o - k_o')$ goes to zero . Therfore the entire first integral of 4) vanishes. Similarly, we can show that the third integral of 4) also vanishes. On expanding into momentum space, the second integral of 4), and then integrating over $\underline{x}$, we obtain

$$i \int \{ \frac{\partial \phi^{-}}{\partial x^{o}} \phi^{*+} - \phi^{-} \frac{\partial \phi^{*+}}{\partial x^{o}} \} d\underline{x}$$

$$= - \int d\Omega \int_{o}^{\infty} dk_o (k_o^2 + m^2)^{\frac{1}{2}} k_o\, b^{\dagger}(\hat{\omega}, k_o)\, b(\hat{\omega}, k_o) \, .$$

Similarly, the last integral of 4) reduces to

$$\int d\Omega \int_{o}^{\infty} dk_o (k_o^2 + m^2)^{\frac{1}{2}} k_o\, a^{\dagger}(\hat{\omega}, k_o)\, a(\hat{\omega}, k_o) \, .$$

We thus obtain the charge of the field as proportional to

$$Q = \int d\Omega \int_0^\infty dk_o\, k_o (k_o^2 + m^2)^{\frac{1}{2}} \{a^\dagger(\hat{\omega},k_o)\, a(\hat{\omega},k_o) - b^\dagger(\hat{\omega},k_o)\, b(\hat{\omega},k_o)\}.$$

Introducing $a_1(\hat{\omega},k_o)$ and $b_1(\hat{\omega},k_o)$ as defined in eq. 3), we obtain

$$Q = \int d\Omega \int_0^\infty dk_o \{ a_1^\dagger(\hat{\omega},k_o)\, a_1(\hat{\omega},k_o) - b_1^\dagger(\hat{\omega},k_o)\, b_1(\hat{\omega},k_o) \} .$$

It follows from the structure of P^n and Q, that $a_1^\dagger(\hat{\omega},k_o)$ is the operator for the creation of a particle with energy-momentum k and charge + 1; while $a_1(\hat{\omega},k_o)$ is the annihilation operator for the same. $b_1^\dagger(\hat{\omega},k_o)$ is the creation operator for a particle of energy-momentum k and charge − 1 ; while $b_1(\hat{\omega},k_o)$ is the annihilation operator of the same.

CHAPTER 5

TACHYON SPINOR FIELDS

The tachyon Dirac equation is $\{ \sum_n \gamma^n \frac{\partial}{\partial x^n} + m \} \psi(x) = 0,$

where the γ^n are the Dirac matrices. As $\psi(x)$ also satisfies the Klein--Gordon equation $(\Box + m^2)\, \psi(x) = 0$, we have, on expanding $\psi(x)$ in momentum space,

$$\psi_\alpha(x) = (2\pi)^{-3/2} \int d^4k\, e^{-ikx}\, \phi_\alpha(k)\, \delta(k^2 + m^2) \qquad \ldots 1)$$

where $\phi(k)$ satisfies the Dirac equation in momentum space :

$$(k_n \gamma^n + im)\, \phi(k) = 0 \ .$$

Integrating (1) over the radial component $|\underline{k}|$, using the properties of the Dirac delta functions, we obtain

$$\psi_\alpha(x) = \frac{(2\pi)^{-3/2}}{2} \int d\Omega \int_{-\infty}^{\infty} dk_o (k_o^2 + m^2)^{\frac{1}{2}}\, e^{-ikx} \phi_\alpha(k) \ ,$$

where $|\underline{k}| = (k_o^2 + m^2)^{\frac{1}{2}}$;

Splitting this last integral into positive and negative frequency parts, we have

$$\psi_\alpha(x) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} e^{-ikx} \phi_\alpha(k)$$

$$+ \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_{-\infty}^{0} dk_o (k_o^2 + m^2)^{\frac{1}{2}} e^{-ikx} \phi_\alpha(k) ,$$

In the second integral above, changing $k \rightarrow -k$, gives

$$\psi_\alpha(x) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} \{e^{-ikx} \phi_\alpha(k) + e^{ikx} \phi_\alpha(-k)\}.$$

Introducing the Dirac conjugate $\overline{\psi}(x) \equiv \psi^\dagger(x)\gamma^o$, we obtain

$$\overline{\psi}_\beta(y) = \frac{(2\pi)}{2}^{-3/2} \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)^{\frac{1}{2}} \{e^{iky} \phi_\beta^*(k)\gamma^o + e^{-iky} \phi_\beta^*(-k)\gamma^o\} .$$

Let us define $\sum_{\nu = \pm 1} a_\nu(\hat{\omega},k_o)\, U_\alpha^\nu(\hat{\omega},k_o) = \frac{1}{2} \phi_\alpha(k)$,

and $$\sum_{\mu = \pm 1} b_\mu^\dagger(\hat{\omega},k_o)\, V_\alpha^\mu(\hat{\omega},k_o) = \frac{1}{2} \phi_\alpha(-k) ,$$

where, as usual, $|\underline{k}| = (k_o^2 + m^2)^{\frac{1}{2}}$.

Then we obtain for the conjugate expressions

$$\sum_{\nu = \pm 1} a_\nu^\dagger(\hat{\omega},k_o)\, \overline{U}_\alpha^\nu(\hat{\omega},k_o) = \frac{1}{2} \phi_\alpha^*(k)\gamma^o$$

and
$$\sum_{\mu=\pm 1} b_\mu(\hat{\omega},k_o)\, \overline{V}^\mu_\alpha(\hat{\omega},k_o) = \frac{1}{2}\,\phi^*_\alpha(-k)\gamma^o .$$

Therefore our field operators become

$$\psi_\alpha(x) = (2\pi)^{-3/2}\int d\Omega \int_o^\infty dk_o(k_o^2+m^2)^{\frac{1}{2}}\{e^{-ikx}\sum_\nu a_\nu(\hat{\omega},k_o)\, U^\nu_\alpha(\hat{\omega},k_o)$$
$$+ e^{ikx}\sum_\mu b^\dagger_\mu(\hat{\omega},k_o)\, V^\mu_\alpha(\hat{\omega},k_o)\},$$

and

$$\overline{\psi}_\beta(y) = (2\pi)^{-3/2}\int d\Omega \int_o^\infty dk_o(k_o^2+m^2)^{\frac{1}{2}}\{\, e^{-iky}\sum_\mu b_\mu(\hat{\omega},k_o)\, \overline{V}^\mu_\beta(\hat{\omega},k_o)$$
$$+ e^{iky}\sum_\nu a^\dagger_\nu(\hat{\omega},k_o)\, \overline{U}^\mu_\beta(\hat{\omega},k_o)\}\ldots 2)$$

Let us define

$$\psi_\alpha^+(x) = (2\pi)^{-3/2}\int d\Omega \int_o^\infty dk_o(k_o^2+m^2)^{\frac{1}{2}}\, e^{-ikx}\sum_\nu a_\nu(\hat{\omega},k_o)\, U^\nu_\alpha(\hat{\omega},k_o) ,$$

$$\psi_\alpha^-(x) = (2\pi)^{-3/2}\int d\Omega \int_o^\infty dk_o(k_o^2+m^2)^{\frac{1}{2}}\, e^{ikx}\sum_\mu b^\dagger_\mu(\hat{\omega},k_o)\, V^\mu_\alpha(\hat{\omega},k_o) ,$$

$$\overline{\psi}_\beta^+(y) = (2\pi)^{-3/2}\int d\Omega \int_o^\infty dk_o(k_o^2+m^2)^{\frac{1}{2}}\, e^{-iky}\sum_\mu b_\mu(\hat{\omega},k_o)\, \overline{V}^\mu_\beta(\hat{\omega},k_o) ,$$

and

$$\overline{\psi}_\beta^-(y) = (2\pi)^{-3/2}\int d\Omega \int_o^\infty dk_o(k_o^2+m^2)^{\frac{1}{2}}\, e^{iky}\sum_\nu a^\dagger_\nu(\hat{\omega},k_o)\, \overline{U}^\nu_\beta(\hat{\omega},k_o) ,$$

so that $\psi(x) = \psi^+(x) + \psi^-(x)$, and $\overline{\psi}(y) = \overline{\psi}^+(y) + \overline{\psi}^-(y)$.

Anti-Commutator Expression

Knowing $\psi_\alpha(x)$ and $\overline{\psi}_\beta(y)$, we are now in a position to determine the anti-commutator $\left[\psi_\alpha(x),\overline{\psi}_\beta(y)\right]_+$. From eq. 2), we obtain

$$\left[\psi_\alpha(x),\overline{\psi}_\beta(y)\right]_+ = (2\pi)^{-3} \int d\Omega \int d\Omega' \int_0^\infty dk_o(k_o^2 + m^2)^{\frac{1}{2}} \int_0^\infty dk_o'(k_o'^2 + m^2)^{\frac{1}{2}}$$

$$\{ e^{-i(kx - k'y)} \sum_{\mu,\nu} \left[a_\nu(\hat{\omega},k_o), a_\mu^\dagger(\hat{\omega}',k_o')\right]_+ U_\alpha^\nu(\hat{\omega},k_o)\overline{U}_\beta^\mu(\hat{\omega}',k_o')$$

$$+ e^{-i(kx + k'y)} \sum_{\mu,\nu} \left[a_\nu(\hat{\omega},k_o), b_\mu(\hat{\omega}',k_o')\right]_+ U_\alpha^\nu(\hat{\omega},k_o)\overline{V}_\beta^\mu(\hat{\omega}',k_o')$$

$$+ e^{i(kx + k'y)} \sum_{\mu,\nu} \left[b_\nu^\dagger(\hat{\omega},k_o), a_\mu^\dagger(\hat{\omega}',k_o')\right]_+ V_\alpha^\nu(\hat{\omega},k_o)\overline{U}_\beta^\mu(\hat{\omega}',k_o')$$

$$+ e^{i(kx - k'y)} \sum_{\mu,\nu} \left[b_\nu^\dagger(\hat{\omega},k_o), b_\mu(\hat{\omega}',k_o')\right]_+ V_\alpha^\nu(\hat{\omega},k_o)\overline{V}_\beta^\mu(\hat{\omega}',k_o') \} ;$$

We choose the following anti-commutation rules :

$$\left[a_\nu(\hat{\omega},k_o), a_\mu^\dagger(\hat{\omega}',k_o')\right]_+ = \left[b_\nu^\dagger(\hat{\omega},k_o), b_\mu(\hat{\omega}',k_o')\right]_+ = \delta^2(\hat{\omega} - \hat{\omega}')\, \delta(k_o - k_o')\delta_{\mu\nu},$$

and

$$\left[a_\nu(\hat{\omega},k_o), b_\mu(\hat{\omega}',k_o')\right]_+ = \left[b_\nu^\dagger(\hat{\omega},k_o), a_\mu^\dagger(\hat{\omega}',k_o')\right]_+ = 0 .$$

Note that choosing these rules, implies that the field operators obey Fermi-Dirac statistics.

Then the anti-commutator expression becomes

$$\left[\psi_\alpha(x),\overline{\psi}_\beta(y)\right]_+ = (2\pi)^{-3}\int d\Omega \int_0^\infty dk_o(k_o^2 + m^2)\{ e^{-ik(x-y)}\sum_\nu U_\alpha^\nu(\hat{\omega},k_o)\overline{U}_\beta^\nu(\hat{\omega},k_o)$$

$$+ e^{ik(x-y)}\sum_\mu V_\alpha^\mu(\hat{\omega},k_o)\overline{V}_\beta^\mu(\hat{\omega},k_o)\}; \qquad \ldots 4)$$

To further simplify the anti-commutator expression, we would need to determine the matrices $U^\nu(\hat{\omega},k_o)$ and $V^\mu(\hat{\omega},k_o)$.

Matrix Solutions of the Wave Equations

The matrices $U^\nu(\hat{\omega},k_o)$ and $V^\mu(\hat{\omega},k_o)$ satisfy the following eqs.:

the Dirac equations, $\quad (k_n\gamma^n + im)U^\nu(\hat{\omega},k_o) = 0$,

and

$$(k_n\gamma^n - im)V^\mu(\hat{\omega},k_o) = 0 ;$$

and the helicity eqs., $\quad k_i\sigma^i U^\nu(\hat{\omega},k_o) = \nu k U^\nu(\hat{\omega},k_o)$,

and

$$k_i\sigma^i V^\mu(\hat{\omega},k_o) = \mu k V^\mu(\hat{\omega},k_o) .$$

In solving these equations we will use the following representation for Dirac matrices:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

and $\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$:

If we define $k_1 \equiv (k_o^2 + m^2)^{\frac{1}{2}}\, \hat{\omega}.\hat{i}$,

$k_2 \equiv (k_o^2 + m^2)^{\frac{1}{2}}\, \hat{\omega}.\hat{j}$, and $\overset{1}{U}(\hat{\omega},k_o) = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$,

$k_3 \equiv (k_o^2 + m^2)^{\frac{1}{2}}\, \hat{\omega}.\hat{k}$,

then $(k_n\gamma^n + im)\overset{1}{U}(\hat{\omega},k_o) = 0$, implies

$$\begin{pmatrix} k_o + im & 0 & k_3 & k_1 - ik_2 \\ 0 & k_o + im & k_1 + ik_2 & -k_3 \\ -k_3 & -(k_1 - ik_2) & -(k_o - im) & 0 \\ -(k_1 + ik_2) & k_3 & 0 & -(k_o - im) \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0,$$

while $k_i \sigma^i \overset{1}{U}(\hat{\omega},k_o) = k\overset{1}{U}(\hat{\omega},k_o)$, implies

$$\begin{pmatrix} k_3 & k_1 - ik_2 & 0 & 0 \\ k_1 + ik_2 & -k_3 & 0 & 0 \\ 0 & 0 & k_3 & k_1 - ik_2 \\ 0 & 0 & k_1 + ik_2 & -k_3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = k \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} .$$

From the last two matrix equations, we obtain a general expression for $\overset{1}{U}(\hat{\omega},k_o)$, where

$$\overset{1}{U}(\hat{\omega},k_o) = \left[\frac{k + k_3}{4k}\right]^{\frac{1}{2}} \begin{pmatrix} 1 \\ \dfrac{(k_1 + ik_2)}{(k + k_3)} \\ \dfrac{k_o + im}{k} \\ \dfrac{-(k_o + im)(k_1 + ik_2)}{k(k + k_3)} \end{pmatrix} .$$

Note that $\left[\frac{k + k_3}{4k}\right]^{\frac{1}{2}}$ is the normalization constant.

Similarly for $U^{-1}(\hat{\omega},k_o)$ we get

$$U^{-1}(\hat{\omega},k_o) = \left(\frac{k + k_3}{4k}\right)^{\frac{1}{2}} \begin{pmatrix} -\dfrac{(k_1 - ik_2)}{(k + k_3)} \\ 1 \\ -\dfrac{(k_o + im)(k_1 - ik_2)}{k(k + k_3)} \\ \dfrac{(k_o + im)}{k} \end{pmatrix} .$$

Similarly, on applying the eqs., $(k_n\gamma^n - im)\, V^\mu(\hat{\omega},k_o) = 0$ and $k_i\sigma^i\, V^\mu(\hat{\omega},k_o) = \mu k V^\mu(\hat{\omega},k_o)$, we can show that

$$V^{1}(\hat{\omega},k_o) = \left(\frac{k + k_3}{4k}\right)^{\frac{1}{2}} \begin{pmatrix} -\dfrac{(k_o + im)}{k} \\ -\dfrac{(k_o + im)(k_1 + ik_2)}{k(k + k_3)} \\ 1 \\ \dfrac{(k_1 + ik_2)}{k + k_3} \end{pmatrix} ,$$

and

$$V^{-1}(\hat{\omega},k_o) = \left(\frac{k + k_3}{4k}\right)^{\frac{1}{2}} \begin{pmatrix} -\dfrac{(k_o + im)(k_1 - ik_2)}{k(k + k_3)} \\ \dfrac{(k_o + im)}{k} \\ -\dfrac{(k_1 - ik_2)}{k + k_3} \\ 1 \end{pmatrix} .$$

Note that in all of these above expressions $k = (k_o^2 + m^2)^{\frac{1}{2}}$.

The transcendental frame has an analogous significance for tachyons, as the rest frame has for tardyons. In the transcendental frame, $k_o = 0$, $\hat{\omega}.\hat{k} = 1$, and $\hat{\omega}.\hat{i} = \hat{\omega}.\hat{j} = 0$;
therefore $k_1 = k_2 = 0$, and $k = k_3 = m$.
Then the matrix solutions become:

$$U^1(\hat{\omega},0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -i \\ 0 \end{pmatrix}, \quad U^{-1}(\hat{\omega},0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ i \end{pmatrix}, \quad V^1(\hat{\omega},0) = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 0 \\ 1 \\ 0 \end{pmatrix}, \text{ and } V^{-1}(\hat{\omega},0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i \\ 0 \\ 1 \end{pmatrix}.$$

Knowing the general expressions for the matrices, we can now simplify the anti-commutator expression (4). Using the solutions for $U^\nu(\hat{\omega},k_o)$, we have :

$$\sum_\nu U^\nu(\hat{\omega},k_o)\, \overline{U}^\nu(\hat{\omega},k_o) = \frac{(k_o I - im\gamma^o)\,(k_n\gamma^n + im)}{2(k_o^2 + m^2)} .$$

Similarly, using the solutions for $V^\mu(\hat{\omega},k_o)$, gives us

$$\sum_\mu V^\mu(\hat{\omega},k_o)\, \overline{V}^\mu(\hat{\omega},k_o) = \frac{(k_o I + im\gamma^o)\,(k_n\gamma^n - im)}{2(k_o^2 + m^2)} .$$

Then expression (4) becomes :

$$\left[\psi_\alpha(x),\bar{\psi}_\beta(y)\right]_+ = \frac{(2\pi)^{-3}}{2} \int d\Omega \int_0^\infty dk_o e^{-ik(x-y)} \{(k_o I - im\gamma^o)(k_n\gamma^n + im)\}_{\alpha\beta}$$

$$+\frac{(2\pi)^{-3}}{2} \int d\Omega \int_0^\infty dk_o\ e^{ik(x-y)} \{(k_o I + im\gamma^0)(k_n\gamma^n - im)\}_{\alpha\beta}.$$

In the second integral above, on changing $k \to -k$, we have

$$\left[\psi_\alpha(x),\bar{\psi}_\beta(y)\right]_+ = \frac{(2\pi)^{-3}}{2} \int d\Omega \int_{-\infty}^\infty dk_o e^{-ik(x-y)} \{(k_o I - im\gamma^o)(k_n\gamma^n + im)\}_{\alpha\beta}.$$

Expressing this in four space, noting that by the properties of Dirac matrices :

$$(k_o I - im\gamma^o)(k_n\gamma^n + im) = -k_i\sigma^i\gamma^5(k_n\gamma^n + im),$$

the commutator expression becomes

$$\left[\psi_\alpha(x),\bar{\psi}_\beta(y)\right]_+ = -(2\pi)^{-3} \int d^4k\ e^{-ik(x-y)}\ \delta(k^2 + m^2)\{\hat{\omega}.\vec{\sigma}\gamma^5(k_n\gamma^n + im)\}_{\alpha\beta}\ .$$

Let us define $\Delta'_{\alpha\beta}(x) \equiv -(2\pi)^{-3}\int d^4k\ e^{-ikx}\ \delta(k^2 + m^2)\{\hat{\omega}.\vec{\sigma}\gamma^5(k_n\gamma^n + im)\}_{\alpha\beta}.$

Integrating this expression over k_o, and then taking the value at $t=0$, gives

$$\Delta'_{\alpha\beta}(x)\Big|_{\text{at } t=0.} = -\frac{(2\pi)^{-3}}{2}\int \frac{d^3k\ e^{-i\underline{k}.\underline{x}}}{(\underline{k}^2 - m^2)^{\frac{1}{2}}}\Big[\{\hat{\omega}.\vec{\sigma}\gamma^5\big((\underline{k}^2 - m^2)^{\frac{1}{2}}\gamma^o + k_\alpha\gamma^\alpha + im\big)\}_{\alpha\beta}$$

$$+ \{\hat{\omega}.\vec{\sigma}\gamma^5\big(-(k^2 - m^2)^{\frac{1}{2}}\ \gamma^o + k_\alpha\gamma^\alpha + im\big)\}_{\alpha\beta}\Big].$$

Using matrix calculations, we find that $\Delta'_{\alpha\beta}(x)\Big|_{t=0} \neq 0$, for all α,β.

So the equal time anti-commutator for a tachyon spinor field is not equal to zero. It is also worth noting, that in general, the tachyon spinless commutator $\Delta(x)$, and the corresponding spinor commutator $\Delta'(x)$ are <u>not</u> related by $\Delta'_{\alpha\beta}(x) = \{ \partial_\mu\gamma^\mu - im \}_{\alpha\beta} \, \Delta(x)$. This is in contrast to the relationship that exists between the two commutators in the tardyon case. But in the limit of a massless tachyon, and at time $t = 0$, we do have $\Delta'_{\alpha\beta}(x) = \{ \partial_\mu\gamma^\mu - im \}_{\alpha\beta}\Delta(x)$.

Dynamic Variables

Taking the Lagrangian of a tachyon spinor field to be

$$L = \frac{i}{2} : \sum_n \left\{ \bar{\psi}\gamma^5\gamma^n\frac{\partial\psi}{\partial x^n} - \frac{\partial\bar{\psi}}{\partial x^n}\gamma^5\gamma^n\psi \right\} - im : \bar{\psi}\gamma^5\psi :,$$

we obtain, on applying the Euler-Lagrange equations,

the Dirac equation $$\sum_n \gamma^n\frac{\partial\psi}{\partial x^n} - m\psi = 0 ,$$

and the conjugate eq., $$\sum_n \frac{\partial\bar{\psi}}{\partial x^n}\gamma^n - m\bar{\psi} = 0 .$$

Note that the Lagrangian reduces to zero, for ψ and $\bar{\psi}$ satisfying the field equations. Knowing the Lagrangian, we can obtain the tensor

$$T^{mn} = \frac{ig^{nn}}{2} : \{ \bar{\psi}\gamma^5\gamma^m \frac{\partial\psi}{\partial x^n} - \frac{\partial\bar{\psi}}{\partial x^n}\gamma^5\gamma^m\psi \} :$$

where $\frac{\partial T^{mn}}{\partial x^m} = 0$.

On account of the factor γ^5, T^{mn} transforms under spatial inversion, like an angular momentum, and not like an energy-momentum tensor. As such we cannot identify T^{mn} as the energy-momentum tensor.

Consider the operator $P^o = \int T^{oo}\, d\underline{x}$ .On placing in the value of T^{oo}, this becomes

$$P^o = \frac{i}{2} \int : \bar{\psi}\gamma^5\gamma^o \frac{\partial\psi}{\partial x^o} - \frac{\partial\bar{\psi}}{\partial x^o}\gamma^5\gamma^o\psi : d\underline{x} .$$

Expanding the field operators into momentum space, and then integrating over $\underline{x}$, noting that :

(1) $(2\pi)^{-3} \int d\underline{x}\, e^{i\underline{a}\cdot\underline{x}} = \delta(\,\underline{a}\,)$,

(2) $U^{\mu\dagger}\gamma^5 U^\nu = -\mu\, \delta_{\mu\nu} \frac{k_o}{k}$,

(3) $V^{\mu\dagger}\gamma^5 V^\nu = -\mu\, \delta_{\mu\nu} \frac{k_o}{k}$;

we obtain,

$$P^o = -\int d\Omega \int_0^\infty k_o^2\, (k_o^2 + m^2)^{\frac{1}{2}}\, dk_o : a^\dagger_{-1} a_{-1} - a^\dagger_1 a_1 + b_1 b^\dagger_1 - b_{-1} b^\dagger_{-1} : .$$

Removing the normal ordered product, by utilising the fact that the operators obey Fermi-Dirac statistics, gives

$$P^o = \int d\Omega \int_o^\infty dk_o\, k_o^2\, (k_o^2 + m^2)^{\frac{1}{2}} \{ a_1^\dagger a_1 + b_1^\dagger b_1 - a_{-1}^\dagger a_{-1} - b_{-1}^\dagger b_{-1} \} .$$

Let us define $a_\mu^1(\hat{\omega},k_o) \equiv k_o(k_o^2 + m^2)^{\frac{1}{4}}\, a_\mu(\hat{\omega},k_o)$,

and

$$b_\mu^1(\hat{\omega},k_o) \equiv k_o(k_o^2 + m^2)^{\frac{1}{4}}\, b_\mu(\hat{\omega},k_o) ;$$

then we have

$$P^o = \int d\Omega \int_o^\infty dk_o\, \{a_1^{1\dagger} a_1^1 + b_1^{1\dagger} b_1^1 - a_{-1}^{1\dagger} a_{-1}^1 - b_{-1}^{1\dagger} b_{-1}^1 \} .$$

From the nature of the P^o expression, it is clear that P^o can be identified with the total helicity of the tachyon field. Hence for a tachyon field, total helicity is a conserved quantity.

As discussed earlier in this chapter, the tensor T^{mn} cannot be identified with energy-momentum. This raises an interesting question. The field operators ψ and $\bar{\psi}$ obey the Klein-Gordon equation. The energy-momentum tensor for a complex scalar field is given in Ch. 4, eq.(4). The analogous energy-momentum tensor for a spinor field will be

$$T_1^{mn} = g^{mm} g^{nn} : \frac{\partial\bar{\psi}}{\partial x^m}\frac{\partial\psi}{\partial x^n} + \frac{\partial\psi}{\partial x^n}\frac{\partial\bar{\psi}}{\partial x^m} : - g^{mn} \sum_a g^{aa} : \frac{\partial\bar{\psi}}{\partial x^a}\frac{\partial\psi}{\partial x^a} : - m^2 g^{mn} : \bar{\psi}\psi : .$$

Can the operator $P^o{}_1 = \int T_1{}^{oo} \, d\underline{x}'$, be identified with the energy vector ? On placing the value for $T_1{}^{oo}$ in the expression for $P^o{}_1$, we have, on expanding into positive and negative frequency parts,

$$P^o{}_1 = \int : \{ \sum_n \frac{\partial\bar{\psi}^+}{\partial x^n} \frac{\partial\psi^+}{\partial x^n} - m^2 \bar{\psi}^+ \psi^+ \} : d\underline{x}$$

$$+ \int : \{ \sum_n \frac{\partial\bar{\psi}^-}{\partial x^n} \frac{\partial\psi^-}{\partial x^n} - m^2 \bar{\psi}^- \psi^- \} : d\underline{x}$$

$$+ \int : \{ \sum_n \frac{\partial\bar{\psi}^+}{\partial x^n} \frac{\partial\psi^-}{\partial x^n} - m^2 \bar{\psi}^+ \psi^- \} : d\underline{x}$$

$$+ \int : \{ \sum_n \frac{\partial\bar{\psi}^-}{\partial x^n} \frac{\partial\psi^+}{\partial x^n} - m^2 \bar{\psi}^- \psi^+ \} : d\underline{x} \ .$$

Consider the first integral of $P^o{}_1$. On expanding it into momentum space, we note that this integral contains a factor $(\sum_n k_n k_n' + m^2)$. On integrating over $\underline{x}$, using the Dirac delta properties, this factor goes to zero. Therefore the first integral of $P^o{}_1$ vanishes. In the same way, we find that the second integral of $P^o{}_1$ also gives a zero value.Let us now consider the third integral,

$$\int : \{ \sum_n \frac{\partial\bar{\psi}^+}{\partial x^n} \frac{\partial\psi^-}{\partial x^n} - m^2 \bar{\psi}^+ \psi^- \} : d\underline{x}$$

Expanding in momentum space, and then integrating over $\underline{x}$, this integral becomes

$$2 \int d\Omega \int_0^\infty dk_o (k_o^2 + m^2)\, k_o^2 \,\{ \sum_{\mu,\nu} : b_\mu(\hat{\omega},k_o)\, b_\nu^\dagger(\hat{\omega},k_o) : \overline{V}^\mu(\hat{\omega},k_o)\, V^\nu(\hat{\omega},k_o) \}.$$

But $\overline{V}^\mu(\hat{\omega},k_o)\, V^\nu(\hat{\omega},k_o) = 0$,

therefore the entire integral in question is also zero. Similarly, we get

$$\int :\{ \sum_n \frac{\partial \overline{\psi}^-}{\partial x^n} \frac{\partial \psi^+}{\partial x^n} - m^2\, \overline{\psi}^- \psi^+ \}: d\underline{x} = 0 ,$$

and so $P^o{}_1 = 0$, and the total energy operator of a free tachyon spinor field is just zero.

It should be noted that the Hamiltonian of a Dirac tachyon field is not hermitean. As such the energy of a tachyon is not a real observable for a tachyon spinor field.

CHAPTER 6

TACHYON EXCHANGE SCATTERING

Consider a two particle scattering problem, involving a single meson exchange. From S-matrix theory, the scattering amplitude in the center of mass frame is given by

$$A_1 = \frac{g_1^2}{\mu_1^2 + 2\underline{k}^2(1 - \cos\theta)} ,$$

where g_1^2 is the coupling constant of the field theory,

$\underline{k}$ is the incoming momentum of the particles,

μ_1 is the meson mass,

and θ is the scattering angle.

If the exchange particle were a tachyon, rather than a meson, the scattering amplitude would have been

$$A_2 = \frac{g_2^2}{-\mu_2^2 + 2\underline{k}^2(1 - \cos\theta)} ,$$

where μ_2 is the tachyon meta mass, and $\underline{k}$ is restricted such that $\underline{k}^2 > \mu_2^2$.

We notice that on placing the Yukawa potential $V_1 = \frac{e^{-\mu_1 r}}{r}$,

into the expression for the first Born approximation, reduces the latter to the meson scattering amplitude A_1. Similarly we note that on placing the potential $V_2 = (\cos \mu_2 r)/r$, in the first Born approximation, the latter reduces to the tachyon scattering amplitude A_2. Also, on placing the potential $V_2' = (\sin \mu_2 r)/r$, we obtain the scattering amplitude iA_2.

Just as the Yukawa potential describes meson exchange in strong interactions, we can say that V_2 describes tachyon exchanges. As can be noticed from its form, V_2 goes to zero much slower than the Yukawa potential. Therefore tachyon exchanges are long range in nature. If long range, strong and weak interactions had existed, we would have, most probably, observed them already. As such one can conclude that the place to look for tachyon exchanges would be in either gravitational or coulomb interaction experiments. In this respect it is worth noting that, in the massless tachyon limit, the tachyon potential reduces to the coulomb potential.

CHAPTER 7

CONCLUSION

We have examined the classical difficulties presented by tachyon existence. The problem of imaginary energy has been successfully overcome. But the causality objection still persists. Classically, tachyons raise a perplexing problem. In the transcendental frame, the observer notes that the tachyon has zero energy, a finite non-zero momentum, and that it exists for one particular moment of time. This implies that at one time, the observer finds a tachyon having a finite non-zero momentum; at all other times, he does not observe the tachyon at all. Does this violate the conservation of momentum ?

A quantum field theory of scalar tachyons is developed. The commutator function obtained is found to vanish at all space-like points. This does not imply that tachyons convey messages with a speed less than the speed of light. Rather it means that a tachyon going from the space-time point x to the space-time point y, is equivalent to one going from y to x.

A tachyon spinor theory developed shows some very interesting features. Energy-momentum is not an observable of a tachyon spinor field; as the Hamiltonian of the field is no longer hermitean. Helicity takes on an added significance, as it is the only physical quantity that remains a constant of the motion of the field, for both tachyons and tardyons.

The brief scattering problem discussion implies that tachyon exchanges are long range in nature. The place to look for tachyon exchanges would more likely be in gravitational or coulomb interactions, rather than in strong or weak interactions. This is further reinforced by the fact that for large distances, as well as in the limit of zero tachyon mass interaction, the tachyon potential reduces to the coulomb potential. If tachyon exchanges are found to be present in nature, then it would have vast repercussions in physics. For this would as such return us to the Newtonian concept of action at a distance.

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