THE PRICING OF CURRENCY OPTIONS WITH STOCHASTIC VOLATILITIES

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Submitted to the Faculty of the
Graduate College of the
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in partial fulfillment of
the requirements for
the Degree of
DOCTOR OF PHILOSOPHY
May, 1997

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Thesis Approved:

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PREFACE

This study examines the performance of a stochastic variance option pricing model applied to the valuation of British Pound options traded at Philadelphia Stock Exchange. Option values from Heston's stochastic volatility model and values from the modified Black-Scholes model are compared to option premiums. Pricing biases are then compared for these models. Pricing biases related to option strike prices, time to maturity, volatility, and interest rate differentials are considered. Premium for volatility risk is measured.

I sincerely thank my doctoral committee--Drs. Timothy Krehbiel (chair), John Polonchek, Janice Jadlow, and Wade Brorsen--for guidance and support in the completion of this research. I also thank the Department of Finance for supporting me throughout this research project.

ACKNOWLEDGMENTS

I would like to express my gratitude and appreciation to the people without whose help the preparation of this manuscript would have been difficult if not impossible. Special thanks go to Dr. Timothy Krehbiel, my Ph.D. thesis advisor, whose intelligent supervision, constructive guidance, inspiration and friendship have been invaluable. I would like to thank Dr. John Polonchek, Dr. Janice Jadlow, and Dr. Wade Brorsen for serving on my thesis supervisory committee. Their many constructive comments and suggestions are greatly appreciated.

I wish to express my special appreciation to my wife, Zakia Raana, for her strong encouragement at times of difficulty, love and understanding throughout this whole process.

I would also like to take a moment to express my sincere thanks and appreciation to Dr. Mohammad Saeed, my late eldest brother, and my parents for encouraging me to pursue education to the utmost of my ability and for their support, both emotional and financial, in the pursuit thereof.

Finally, I would like to thank the Department of Finance for supporting me throughout my Ph.D. program.

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CHAPTER 1

INTRODUCTION

Problem Identification

Options on foreign currencies trade actively on the organized exchanges in the U.S., Europe, and Canada. The Philadelphia Stock Exchange, the world's largest currency option market, began trading standardized currency options in 1982, and currently offers standardized options on eight major currencies and two cross exchange rates. Customized currency options on 110 currency pairs were added to the exchange in November 1994. The exchange traded about thirteen million and ten million standardized currency option contracts, respectively, in 1993 and 1994, with option trading volume as high as four billion dollars per day in the underlying value. Monthly trading volume and open interest for standardized options averaged 255 thousand and 263 thousand contracts, respectively, from November 1996 to February 1997.

In addition to exchange-traded options, large money-center banks write currency options directly to their corporate customers, and currency option features frequently appear in foreign currency bonds where the bond holder can choose the currency in which coupons or principal or both are paid. The asymmetric payoff function of currency options affords attractive hedging opportunities for foreign currency risks that are not provided by the symmetric payoff function of currency futures and forward contracts. For these reasons the effective modeling of the valuation of currency options has import beyond the sphere of enlarged traded contracts.

Currency options have been valued using a modified Black-Scholes model where the spot exchange rate replaces the stock price in the standard Black-Scholes model and the foreign interest rate enters as an additional variable. Holding a foreign currency is analogous to giving up return equal to the foreign risk-free rate and, therefore, a foreign currency option is analogous to an equity option on a stock paying a continuous dividend (Garman and Kohlhagen, 1983; Grabbe, 1983; Biger and Hull, 1983).

Recent attempts to use the modified Black-Scholes model in pricing currency options indicate that the model exhibits systematic pricing biases (Bodurtha and Courtadon, 1987a,b; Shastri and Tandon, 1986a; Tucker, 1985; Melino and Turnbull, 1987). In particular, Bodurtha and Courtadon (1987 a,b) have reported that their models tend to overvalue in the money and at the money options, and undervalue out of the money options. The average ratio of the absolute error to the option premium was about 13 percent for both the puts and calls, with about 29 percent underpricing of short-run out-of-the-money calls. The authors linked these biases to the empirical observation that a jump diffusion process is more appropriate to describe exchange rate movements than is a pure diffusion process of the Black-Scholes model. This view was partially supported by Shastri and Wethyavivorn (1987). Ball and Torous (1985) reported, however, no operationally significant price differences for stock options priced via the Black-Scholes model and Merton's (1976) jump-diffusion model.

Ogden and Tucker (1988), examining the relative valuation of American currency spot options, contended that the pricing inconsistencies in their models stem from the unrealistic assumption of constant interest rates. Allowing for stochastic interest rates in a currency option model, Hilliard et al. (1991) showed that the stochastic interest rate model has greater pricing accuracy than the constant interest rate alternative. However, the stochastic interest rate model still systematically underpriced the currency options.

Because the observed biases in the predicted prices of currency and stock options have little changed by using models that incorporated stochastic interest rates and jumps processes, many investigators have argued that the constant variance assumption of the Black-Scholes model--the only unobservable variable in the model--is the leading candidate for the mispricing of currency and stock options (Johnson and Shanno, 1987; Chesney and Scott, 1989; Hull and White, 1987; Wiggins, 1987; Melino and Turnbull, 1990). Along these lines, Melino and Turnbull (1987) and Scott (1987) found evidence that the volatility parameter of the exchange rate movements is highly unstable over time. Boothe and Glassman (1987) reported kurtosis far in excess of the normal distribution in the distributions of daily and weekly exchange rate changes, while McFarland et al. (1982) noted that finite but heteroscedastic variances can generate the fat tail distributions that seem to characterize exchange rate changes.

Hull and White (1987) pointed out that Merton's mixed jump-diffusion process for stock prices is a special type of stochastic volatility problem. Furthermore, Merton (1976) suggested that variable interest rates and discontinuous stock-price sample paths lead to instantaneous implied volatilities that change over time. Changes in interest rate differentials between U.S. and foreign countries cause significant variations in exchange rates (Edwards, 1982; Frankel, 1979). Although Merton's hypothesis has not been empirically examined, it suggests that a stochastic variance currency option model has the potential of capturing the combined impact of stochastic interest rates and exchange rate jumps on the currency option prices.

The differential valuation equation that the option must satisfy under the stochastic volatility option model is well known from the work of Cox, Ingersoll, and Ross (1985a) and Garman (1976). When volatility is random, a riskless hedge cannot generally be formed from only one option and the stock because the stochastic differential for the option contains two sources of risk, namely, the volatility risk and a risk premium on the underlying asset (Wiggins, 1987). A riskless hedge can be formed from the stock and two options, but this hedge does not yield a unique pricing function because volatility itself, or a known function of volatility, is not traded (Scott, 1987). Hence, arbitrage alone cannot lead to a unique option pricing function in a random variance option model.

Equilibrium asset pricing models are required to derive a unique option pricing function. The resulting valuation equation explicitly involves the risk of the hedge portfolio (the price of volatility risk) and the correlation between the stock price and its volatility (Hull and White, 1987; Johnson and Shanno, 1987; Wiggins, 1987; Scott, 1987). The correlation term reflects the skewness of the distribution of the stock prices.

The extant stochastic variance option pricing models differ in their treatment of the risk of the hedge portfolio and the specification of correlation between the stock price and its volatility. In the models of Hull and White (1987), Scott (1987), and Wiggins (1987), the risk of the hedge portfolio is removed from the valuation equation, but for different reasons. Hull and White (1987) assumed that volatility is uncorrelated with aggregate consumption, implying that the volatility has zero systematic risk. Scott (1987) assumed that volatility risk can be diversified away. Wiggins (1987) allowed a zero price for the volatility risk by assuming that the correlation of the market return with the volatility of the stock price is equal to the correlation of the market return with the stock price times the correlation of the stock price with its volatility. The volatility risk was nonzero in Chesney and Scott (1989) and in Stein and Stein (1991). The correlation of the stock price with its volatility was permitted to be nonzero in Wiggins (1987), while it was zero in Hull and White (1987), Stein and Stein (1991), Chesney and Scott (1989), Ball and Roma (1994), and Scott (1987).

The assumptions of a zero correlation and zero price for volatility risk simplify the option valuation with stochastic variance. Risk neutrality prevails when volatility risk is unpriced, thereby allowing the option price to be the present value of the expected terminal option value discounted at the risk-free rate. The calculation of the expected terminal option price involves a conditional distribution function of the terminal stock price which, assuming zero correlation, depends on, inter alia, the distribution of the average stochastic variance (Hull and White, 1987; Scott, 1987; Stein and Stein, 1991). The form of the conditional distribution function under a nonzero correlation remains to be discovered. Hull and White (1987) and Scott (1987) were unable to obtain an analytic form for the distribution of average variance, thereby expressing the option price as the expected Black-Scholes price integrated over the distribution of mean variance. Using Fourier inversion methods, Stein and Stein (1991) did derive an analytic form of the distribution of average variance which they used to derive closed-form and approximate solutions for option prices.

Although the foregoing assumptions of zero correlation and zero price of volatility risk are useful in simplifying the option valuation, they have little empirical support. Merville and Pieptea (1989) found that changes in volatility are strongly correlated across stocks and a marketwide volatility effect existed. Christie (1982) reported that equity volatilities tend to move together. Lamoureux and Lastrapes (1993) provided evidence that the volatility risk is priced and that the market premium for variance risk is time varying. Assuming different values for the price of volatility risk, Melino and Turnbull (1990) found the price of volatility risk to have a significant impact on currency option premiums.

Empirical evidence suggests a nonzero correlation of the stock price (or the exchange rate) to its volatility. Beckers (1983) , Geske (1977), French, Schwert, and Stambaugh (1987), and Christie (1982) support the notion of nonzero, negative relationship between a stock price and its volatility. Similarly, Melino and Turnbull (1990) and Tucker (1985) found the correlation between currency return variance and exchange rate level to be negative. Heston (1993) has noted that the correlation between the spot price and its volatility is necessary to generate skewness in the empirical distribution of spot prices. Skewness affects the pricing of out-of-the-money options relative to in-the-money options. With a zero correlation, stochastic volatility only affects the pricing of near-the-money options relative to far-from-the-money options.

An additional drawback of the existing option pricing models is that they require extensive use of numerical techniques to solve for option prices. While Stein and Stein (1991) and Ball and Roma (1994) provided exact solutions for option prices, their solutions hold only if the correlation of spot price to its volatility is zero. Using Fourier inversion methods, Heston (1993) has provided for the first time a closed-form solution for valuing stock options under stochastic volatility without imposing the restrictions of zero correlation and zero price of volatility risk. This study will use Heston's model in pricing foreign currency options.

Objectives of the Study

This study examines the performance of a stochastic variance option pricing model applied to the valuation of British Pound options traded at Philadelphia Stock Exchange. Option values from Heston's stochastic volatility model and values from the modified Black-Scholes model are compared to option premiums. Pricing biases are then compared for these models. Pricing biases related to option strike prices, time to maturity, volatility, and interest rate differentials are considered.

One approach to implement Heston's model is to estimate the price of volatility risk. A significant contribution of this study is the test of difference from zero of the price of volatility risk.

Using the estimated price of volatility risk, option values are evaluated using Heston's model. The modified Black-Scholes model is also used to produce model values. Pricing biases relative to option premiums are evaluated for both models.

CHAPTER 2

FOREIGN CURRENCY OPTION PRICING MODELS: A REVIEW

It will be assumed throughout this review that trading takes place in continuous time, contracts are default-free, and markets are frictionless. Interest rate parity condition will be assumed for some results.

Notation, Assumptions, and Relations

The following notation is used:

$S(t)$ = the spot price of the deliverable currency (dollars per unit of foreign currency),

$F(t,T)$ = the forward exchange rate at time t for settlement at time T,

$B(t,T)$ = the domestic currency (dollar) price of a bond at time t with maturity T and unit face value,

$B^*(t,T)$ = the foreign currency price of a bond with maturity T and unit face value,

$C(S,T)$ = the price of a European currency call option (dollars per foreign currency unit),

T = the maturity date of the contract,

τ = T-t= the time till expiration of the option

r = the instantaneous domestic (riskless) interest rate,

r^* = the instantaneous foreign (riskless) interest rate,

$N(.)$ = cumulative normal distribution function,

X = the exercise price of the option,

α_c = the expected rate of return on the currency call option,

σ_c = the standard deviation of the call option's rate of return.

The following assumptions and relations are employed:

(1) $F(t,T) = S(t)\, B^*(t,T)/B(t,T)$; (interest rate parity)

(2) $dS/S = \mu dt + \sigma dZ$; (spot exchange rate process)

(3) $dB/B = \mu_1 dt + \sigma_1(t,T)\, dZ_1$; (domestic bond process)

(4) $dB^*/B^* = \mu_2\, dt + \sigma_2(t,T)\, dZ_2$; (foreign bond process)

(5) $dr = \alpha(r,t)\, dt + \sigma_r\, dZ_r$; (domestic interest rate process)

(6) $dr^* = \alpha^*(r^*,t)dt + \sigma_{r*} dZ_{r*}$; (foreign interest rate process)

Currency Option Models with Constant Interest Rates and Constant Volatilities

The standard Black-Scholes formula has the form:

$$C = S\, N(d1) - \exp(-r\,\tau)\, X\, N(d2) \qquad (7)$$

$$d1=\{[\ln(S/X)+(r+(\sigma^2/2))\tau]/\sigma\sqrt{\tau}\}$$

$$d2= d1- \sigma\sqrt{\tau}$$

where S is the stock price and σ is the instantaneous standard deviation of the stock return. This formula does not apply well to foreign currency options, since they involve both domestic and foreign interest rates in ways differing from the assumptions of the Black-Scholes model. Biger and Hull (1987) and Garman and Kohlhagen (1987) have established the isomorphic relationship between a currency option and an option on a stock paying a continuous dividend yield. An investor who wants to hold a foreign currency would always prefer to hold a short-term risk-free foreign bond instead of holding the foreign currency in a non-interest-bearing account. Holding a foreign currency is ,therefore, analogous to giving up return equal to the foreign risk-free rate, and, consequently, the valuation of a currency option is analogous to the valuation of an option on a stock paying a constant dividend rate. The foreign interest rate of the currency option simply replaces the constant dividend yield of the stock option. Merton (1973) developed the valuation formula for the option on a stock having a dividend yield δ:

$$C = \exp(-\delta\,\tau)S\,N(d1) - \exp(-r\,\tau)X\,N(d2) \qquad (8)$$

$$d1=\{[\ln(S/X)+(r-\delta+(\sigma^2/2))\tau]/\sigma\sqrt{\tau}\}$$

$$d2= d1- \sigma\sqrt{\tau}$$

where S is the stock price and σ is the instantaneous standard deviation of the stock return. Equation (8) also provides the valuation formula for a European call option written on a foreign currency when S is the spot exchange rate and $\delta=r^*$. These modifications yield

$$C = \exp(-r^*\tau)S\,N(d1) - \exp(-r\,\tau)X\,N(d2) \qquad (9)$$

$$d1=\{[\ln(S/X)+(r-r^*+(\sigma^2/2))\tau]/\sigma\sqrt{\tau}\}$$

$$d2= d1- \sigma\sqrt{\tau}$$

The hedge ratio for equation (9) is the same as for the standard Black-Scholes model.

Garman and Kohlhagen (1987) and Grabbe (1983) have derived the differential valuation equation which, given the boundary condition, yields equation (9). The risk-adjusted expected returns of securities must be identical in an arbitrage free economy. The application of this condition to the ownership of foreign currency gives

$$[(\mu+r^*-r)/\sigma] =\lambda \qquad (10)$$

where λ represents the excess return per unit of risk and is not dependent on the security considered. The expected return from holding the foreign currency is the drift of the exchange rate, μ, plus the foreign riskless interest rate, r^*, earned from holding the currency in the form of an asset. The no-arbitrage condition implies that the European call option yields

$$[(\alpha_c - r)/\sigma_c]=\lambda \tag{11}$$

Equating (10) and (11), we have

$$(\mu+r^*-r)/\sigma =(\alpha_c - r)/\sigma_c \tag{12}$$

Applying Ito's lemma to C(S,T) and using (2), we have

$$dC =[0.5\sigma^2 S^2 C_{ss} +\mu SC_s -C_\tau]dt+ C_s\sigma SdZ \tag{13}$$

where the subscripts on C denotes partial derivatives. Equation (13) can be written as

$$dC= \alpha_c dt + \sigma_c dz \tag{14}$$

where α_c represents the elements enclosed within the bracket and σ_c equals $C_s\sigma S$. Substituting the values of α_c and σ_c from (14) into (12) yields the fundamental valuation equation for the currency option:

$$(\sigma^2/2) S^2 C_{ss} -rC +(r-r^*)S C_s=C_T \tag{15}$$

where r^* may be considered as the "dividend rate" of the foreign currency. The only difference between equation (15) and the fundamental valuation equation for the Black-Scholes model is the foreign risk-free rate term, r^*. The solution to (15), with the boundary condition C(S,0)=max[0, S-X], yields the currency option valuation formula (9).

Grabbe has provided an alternative way of obtaining equation (9) directly from the Black-Scholes formula (7). The value of a risk-free, foreign currency unit bond under the assumption of constant interest rate is $B^*(t,T)= \exp(-r^*\tau)$. Since the foreign interest rate is the only additional variable in the currency option case compared to the no-dividend stock option case, a new variable $G=SB^*$ may be defined for the currency option situation. Now the currency option model has as many variables as in the stock option model. Replacing S everywhere in the Black-Scholes formula (7) with G results in the currency option formula (9).

The currency option formula can also be expressed in terms of the forward exchange rate. The spot exchange rate can be written using the interest rate parity relationship (1) as

$$S = F(t,T)\, B(t,T)/B^*(t,T) = F \exp(r^*-r)\tau \tag{16}$$

Substituting this value of S into (9) gives

$$C = \exp(-r\,\tau)F\, N(d1^*) - \exp(-r\,\tau)\, X\, N(d2^*) \tag{17}$$

where $d1^*=\{[\ln(F/X)+(\sigma^2/2)\tau]/\sigma\sqrt{\tau}\}$

$$d2^*= d1^* - \sigma\sqrt{\tau}$$

The currency option value depends only upon F and r in equation (17); it does not involve independently S and r^*. The forward rate F reflects both S and r^* in equation (17).

A third currency option formula, besides formulas (9) and (17), can be derived by assuming that F rather than S follows a Geometric Brownian Motion. Following Biger and Hull (1987), the riskless hedge is formed by combining a long position in forward contracts with a short position in call options. The currency option formula under these modifications is

$$C = \exp(-r\,\tau)F\,N(d1^*) - \exp(-r\,\tau)X\,N(d2^*) \quad (18)$$

where $d1^*=\{[\ln(F/X)+(\sigma_F^2/2)\tau]/\sigma_F\sqrt{\tau}\}$

$$d2^*= d1^* - \sigma_F\sqrt{\tau}$$

Equation (18) is identical to equation (17) with σ being replaced by σ_F. Since the foreign interest rate is not assumed to be constant here, equation (18) reflects a situation where the domestic interest rate is constant while the foreign interest rate is variable.

Currency Option Models with Stochastic Interest Rates and Constant Volatilities

Grabbe (1983) and Hilliard et al. (1991) have developed currency option formulas under the assumption of stochastic interest rates. Following Grabbe, let $G=SB^*$ and write the diffusion process for G as

$$dG/G = d(SB^*)/SB^* =(\mu +\mu_2+ \rho_{sB*}\sigma\sigma_2)dt+\sigma dZ +\sigma_2 dZ_2 \quad (19)$$

or $dG/G=\mu_G(t,T)dt +\sigma_G(t,T)dW$ (20)

where ρ_{sB*} is the correlation coefficient between dZ and dZ_2. The currency call option can now be expressed as C(G,B,t). One can form a zero-wealth portfolio V composed of G, B, and C as follows:

$$V= C + bB + eG \quad (21)$$

where $b= -\partial C/\partial B$ and $e= -\partial C/\partial G$

Since call option prices are homogenous in G and B, as in Merton (1973), equation (21) assures that V=0. Applying Ito's lemma to (21) and recognizing that dV is zero yields the valuation equation under the stochastic interest rate:

$(\frac{1}{2}(\phi))-C_\tau=0$ (22)

where $\phi=(C_{GG}G\sigma_G^2+2C_{GB}GB\,\rho_{sB}\sigma_G\sigma_1+C_{BB}B^2\sigma_B^2)$

Invoking the risk-neutrality principal of Cox and Ross (1976) and using the boundary condition $C(S,X,0)=\max(0,S_T-X)$, we have

$$C = G\,N(d3)-XB(t,T)N(d4) \quad (23)$$

or $$C = SB^*(t,T)\,N(d3) - XB(t,T)\,N(d4) \quad (24)$$

where $d3=(\ln(SB^*/XB)+(\sigma^2/2)T)/\sigma\sqrt{\tau}$

$d4=d3-\sigma\sqrt{\tau}$

$\sigma^2=\int(1/\tau)[\sigma^2_G+\sigma_1^2+2\rho_{GB}\sigma_G\sigma_1]du$

Using the interest rate parity relationship (1), equation (23) can be expressed in terms of the forward exchange rate as

$$C = B(t,T)\,[F(t,T)\,N(d3^*) - XN(d4^*)] \quad (25)$$

where $d3^*=[\ln(F/X)+(\sigma^2/2)\tau]/\sigma\sqrt{\tau}$

$d4^*=d3^*-\sigma\sqrt{\tau}$

$\sigma^2=\int 1/\tau(\sigma_F^2)du$

and σ_F^2 is the instantaneous variance of dF/F. The risk adjusted process for dF/F, denoted by an asterisk (*), can be written as

$$dF^*=\sigma dZ+\sigma_1 dZ_1-\sigma_2 dZ_2 \quad (26)$$

Hilliard et al. (1991) have derived the functional forms of the bond price, B(t,T), and integrated variance, σ^2, in equation (24) for the mean-reverting Ornstein-Uhlenbeck (OU) process for r and r^*. The OU processes, respectively, for r and r^* are

$$dr=\alpha(\theta-r)dt+\sigma_r\,dZ_r \quad (27)$$

$$dr^*=\alpha^*(\theta^*-r^*)dt+\sigma_{r*}\,dZ_{r*} \tag{28}$$

where α is a speed-of-adjustment coefficient, θ is the long-run average of r, and σ_r is the constant standard deviation. The parameters denoted by an asterisk apply to the foreign interest rate process. A formal solution of the bond price is

$$B(t,T) = E[\exp(-\int r(s)\,ds)] \tag{29}$$

Vasicek (1977) has provided an analytic solution for the bond price under the assumption of a mean reverting OU process of r. His solution is

$$B(t,T) = A\exp[-rD]=\exp[\ln A-rD] \tag{30}$$

where $D=[1-\exp(-\alpha\tau)]/\alpha$

$$A=\exp[k(D-\tau)-(\sigma_r(D/2))^2/\alpha]$$

$$k=\theta+(\sigma_r\lambda/\alpha)-(\sigma_r/\alpha)^2/2$$

$$\lambda=(\mu_1-r)/\sigma_1$$

where τ=T-t and λ is interpreted as the market price of risk. The solution for $B^*(t,T)$ is the same as for B(t,T) with non-starred parameters in (30) being replaced by their starred counterparts from (28), and μ_1 and σ_1 being replaced by μ_2 and σ_2, respectively. Cox, Ingersoll, and Ross (1985b) have provided a bond pricing formula similar to (30) for the mean-reverting square-root process for r.

Hilliard et al.(1991) show that, using the approximation D=-τ, the variance can be approximated as

$$\sigma^2 = (1/T)[\sigma^2\tau+(\tau^3/3)(\sigma^2_r+\sigma_{r*}^2-2\sigma_{rr})+\tau^2(\sigma_{sr}-\sigma_{sr*})] \tag{31}$$

The closed-form solution of the European call option on a foreign currency in equation (25) is complete once the solutions of B(t,T) and σ^2, respectively, from (30) and (31) are employed.

Fernandez-Nava (1994) has employed the equivalent martingale measure approach of Heath, Jarrow, and Morton (1992) to value spot, forward, and futures options on foreign exchange. A closed-form solution for European option was derived under the assumptions of constant volatilities, binomial poisson risk, and stochastic term structures of interest rates.

Currency Option Models with Stochastic Volatilities and Constant Interest Rates

Advances in the pricing of options with stochastic volatilities are represented by the work of Hull and White (1987), Wiggins (1987), Scott (1987), Chesney and Scott (1989), Stein and Stein (1991), Heston (1993), and Ball and Roma (1994). With the exception of Chesney and Scott (1989), and Heston (1993), all the investigators have examined the valuation of stock options. The currency option pricing models, however, can be easily obtained from the stock option models with few minor modifications to incorporate the foreign interest rates.

Consider a stock call option C with a price that depends on the stock price, S, and its instantaneous variance, $V=\sigma^2$. Hull and White (1987) employed the following processes for S and V:

$$dS=\mu S\,dt+\sigma S\,dZ \tag{32}$$

$$dV=\kappa V\,dt+\xi V dW \tag{33}$$

The Wiener processes dZ and dW have correlation ρ. The call option will be a function of three variables: $C=C(S,V,t)$. Scott (1987), Heston (1993), and others have used a mean-reverting Ornstein-Uhlenbeck process for V instead of the Geometric Brownian Motion in (33).

The stochastic differential equation for the option, based on the Ito's lemma on C(S,V,t), is

$$dC= [C_S\mu S + C_\sigma\kappa V + C_t +0.5\, C_{SS}\sigma^2 S^2 +C_{S\sigma}\rho\xi\sigma SV +0.5C_{\sigma\sigma}\xi^2 V^2]dt$$

$$+C_S\sigma SdZ+ C_\sigma\xi VdW \qquad (34)$$

When volatility is random, a riskless hedge cannot be formed using one option and the stock. The problem arises because equation (34) contains two sources of uncertainty, dZ and dW. Scott has shown that the riskless hedge can be formed with two call options and the stock. This riskless hedge, however, does not lead to a unique option pricing function because the call option value depends upon the price of another option. The latter price cannot be determined since volatility is not a traded asset. Johnson and Shanno (1987) suggest an alternative strategy to eliminate the risk from equation (34). They assumed the existence of a volatility-based asset, and proposed to form a hedge consisting of one share of stock long, $(\partial C/\partial S)^{-1}$ options short, and m shares of the volatility-based asset long. If there is indeed a volatility-based traded asset, all of the intertemporal general equilibrium considerations related to the volatility risk would be impounded in the price of the volatility-based asset, and the options could be priced using a hedge that contains this asset. The problem is that there is no such volatility-based asset. Hence, arbitrage alone is not sufficient to eliminate preferences from the valuation equation.

Equilibrium asset pricing models, based on investors' tolerance for bearing risk, are required to derive a unique option pricing function. Garman (1976) and Cox, Ingersoll, and Ross (1985a) have provided identical general forms of the partial differential equation (PDE) for the stochastic variance problem. Garman shows that a call option, $C(\theta_i,t)$, must satisfy the differential equation:

$$C_t+0.5\Sigma\rho_{ij}\sigma_i\sigma_j C_{ij}-rC = \Sigma\theta_i C_I[-\mu_i+\Phi_i] \quad (35)$$

where θ_i represents the state variables, μ_i is the instantaneous mean for θ_i, and Φ_i is the market risk premium associated with variable θ_i. If variable θ_i is traded, it satisfies the (N+1)-factor CAPM, and its right-hand side element in (35), $\theta_i C_i[-\mu_i+\Phi_i]$, is equal to $-r\theta_i C_i$. The PDE in (35) for the stock option C(S,V,t), which involves two state variables, S and V, of which S is traded, can be written as

$$C_t+(½)[\sigma^2 S^2 C_{SS}+2\rho\sigma^3\xi\, SC_{SV}+\xi^2 V^2 C_{VV}]-rC=-rSC_S -[\kappa -\Phi_V]\sigma^2 C_V \quad (36)$$

where Φ_V is the risk premium for the volatility, and ρ is the correlation between the stock price and its volatility.

Chesney and Scott (1989) have shown that the PDE for the currency option is the same as in (35) with rSC_S being replaced with $(r-r^*)SC_S$ under the currency option to account for the foreign interest rate r^*.

The derivation of an analytic solution to (36) is complicated by the presence of ρ and Φ_V terms. Investigators have used many different sets of assumptions to remove these terms from the PDE. Hull and White (1987) assumed that the volatility is uncorrelated with aggregate consumption, implying that volatility risk is unpriced. Scott assumed that volatility risk can be diversified away. Wiggins assumed that the correlation of the market return with the volatility of the stock price is equal to correlation of the market return with the stock price times the correlation of the stock price with its volatility. This assumption sets a zero price for the risk of a hedge portfolio. Stein and Stein (1991), Heston (1993), and Chesney and Scott (1989) allowed the volatility risk to be nonzero. Wiggins (1987) and Heston (1993) are the only studies in which the correlation between the stock price and its volatility is permitted to be nonzero.

The PDE in (36) is free of risk preferences when Φ_V is zero, and the option value may be derived by using the risk-neutral valuation of Cox and Ross and employing the expected value pricing method of Smith (1976). If Φ_V is nonzero, the Fourier inversion technique of Stein and Stein (1991) and Heston (1993) can be used to estimate the option value. Both approaches require the identification of the conditional distribution function of the terminal stock price or the characteristic function associated with the conditional distribution. This distribution function is unknown for the case of nonzero ρ even when Φ_V is zero.

Assuming that Φ_V and ρ are both zero, Scott (1987) and Hull and White (1987) express the option price as the present value of the expected terminal price of C discounted at the risk-free interest rate. That is, the option price is

$$C(S_t,V_t,t) = \exp(-r(T-t))\int C(S_T, V_T,T)p(S_T|S_t,V_t)dS_T \tag{37}$$

where p(.) is the conditional distribution of S_T given the security price and variance at time t. This distribution depends on the processes driving S and V. Let the mean variance, M, be defined as

$$M = (1/\tau)\int V(t)dt \tag{38}$$

The conditional distribution of S_T may be written as

$$p(.)= \int g(S_T|M)h(M|V_t)dV \tag{39}$$

where g(.) and h(.) are the conditional density functions. Substituting (39) into (37) yields

$$C(S_t,V_t,t)=\int[\exp(-r(\tau)\int C(S_T)g(S_T|M)dS_T]h(M|V_t)dM \quad (40)$$

The term enclosed in brackets is the Black-Scholes price for a call option on a stock with a mean variance M, which will be denoted as C(M) herein. The stock option price can be rewritten as

$$C(S_t,V_t)=\int C(M)\ h(M|V_t)dM \quad (41)$$

$$C(M) = S_t\,N(d5)-\exp(-r(\tau))X\,N(d6) \quad (42)$$

where $d5=[\ln(S_t/X)+(r+(M/2))(\tau)]/\sqrt{M(\tau)}$

$d6= d5-\sqrt{M(\tau)}$

In equation (41), the option price is the expected Black-Scholes price where the expectation is taken over the distribution of the mean variance.

Chesney and Scott (1989) have shown that equation (41) applies as well to the currency option when C(M) is the price of a currency call option from the modified Black-Scholes model of equation (9). Denoting the modified C(M) as C´(M), equation (42) can be written for the currency call option as

$$C'(M)=\exp(-r^*(\tau))S_t\,N(d7)-\exp(-r(\tau))XN(d8) \quad (43)$$

where $d7=[\ln(S_t/X)+(r-r^*+M/2)(\tau)]/\sqrt{M(\tau)}$

$d8=d7-\sqrt{M(\tau)}$

An exact solution for C(.) in (41) is available if the analytic form of the variance distribution h(.) is known. Hull and White (1987) and Scott (1987) were unable to obtain the analytic form of h(.), and thus resorted to Monte Carlo simulation to estimate option prices. Alternatively, Hull and White (1987) have developed a Taylor series approximation of (41) about the point where the volatility is non-stochastic and for the case of a zero mean-reversion in the volatility process. The series solution is

$$C(S,V)= \quad C(E(M))+0.5\, C_{MM}\, var(M) +(1/6)C_{MMM}\, skew(M)+... \qquad (44)$$

where the subscripts on C denote the partial derivatives of C(E(M)) with respect to the mean variance M, E(M) represents the mean, var(M) and skew(M) denote, respectively, the second and third moments of M, and the t subscript has been dropped in (44) to simplify the notation. Hull and White (1987)provide explicit expressions for E(M), var(M), and skew(M).

Using Fourier inversion methods, Stein and Stein (1991) managed to derive an analytic form of the distribution of mean variance h(.) in (41) which they used to obtain an exact solution for C(S,V). Although their closed-form solution is quite cumbersome, it is not computationally demanding.

Ball and Roma (1994) uncover that the Hull and White (1987) approach is closely related to the Stein and Stein approach through the moment generating function (MGF) of the average variance. Hull and White (1987) used the MGF to find the moments of the variance distribution, while Stein and Stein employed the MGF to find the density function of the average variance. Ball and Roma demonstrate that the problem of finding the MGF of the average variance is analogous to the problem of finding the price of a unit value, risk-free bond. The latter problem was solved earlier in equations (29) and (30).

To illustrate the approach of Ball and Roma (1994), the MGF for the mean variance M in (38) can be expressed as

$$H(\delta)= E(\exp(-\delta M))= E[\exp((-\delta/\tau)\int V(t)dt)] \quad (45)$$

or

$$H(\delta)= E[\exp(- \int V^*(t)\, dt)] \quad (46)$$

where $V^*=(\delta/\tau)V$, E is the expectation operator, and V(t) represents the diffusion process of the variance. Following equation (29), the bond price can be expressed as

$$B(r,t,T) = E\,[\exp(-\int r(s)\, ds)] \quad (47)$$

where r(.) represents the diffusion process of the spot interest rate. Since equation (47) is identical in form to equation (46), the bond pricing formula may be viewed as the MGF of the positive diffusions of r(.). Vasicek (1977) and Cox, Ingersoll, and Ross (1985b) have provided analytical solutions to (47), respectively, for the mean-reverting OU and mean-reverting square-root processes of r(.). These solutions apply as well to $H(\delta)$ when the variance follows a mean-reverting OU process or a mean-reverting square-root process.

The foregoing option pricing solutions of Hull and White (1987) and Stein and Stein (1991) break down when the correlation of the stock price to its volatility is nonzero. Heston (1993) has shown that a non-zero value of this correlation is necessary to generate the skewness that characterizes the empirical distribution of exchange rates. His stochastic-variance option pricing model permits a non-zero correlation and non-zero price of volatility risk. Heston's model is presented in the next chapter.

CHAPTER 3

CURRENCY OPTION MODEL AND HYPOTHESIS TESTING

The present study uses Heston's (1993) model to price currency options.

Heston's Stochastic Volatility Model for Options on Stocks and Foreign Currencies

When volatility is stochastic, volatility itself becomes a state variable and a volatility process is specified to capture the stochastic properties of volatility over time. Following Heston, assume that the stock price and volatility of the stock price obey the stochastic processes:

$$dS = \mu S dt + \sqrt{V} S\, dZ_1 \tag{48}$$

$$d\sqrt{V} = -\beta\sqrt{V}\, dt + \delta dZ_2 \tag{49}$$

where S is the stock price, $\sqrt{V}$ is the volatility (standard deviation) of the stock price, μ, β, and δ are fixed constants, and dZ_1 and dZ_2 are Wiener processes. The process dZ_1 has a correlation ρ with process dZ_2. Equation (48) proposes a conditional lognormal diffusion process for the stock price dynamics subject to a stochastic volatility that follows an arithmetic Ornstein-Uhlenbeck process. Stein and Stein (1991), Wiggins (1987), Scott (1987), and Merville and Pieptea (1989) have proposed volatility processes similar to (49). Applying Ito's lemma to (49) shows that the variance V follows the process:

$$dV = [\delta^2 - 2\beta V]dt + 2\delta\sqrt{V}\, dZ_2 \tag{50}$$

Equation (50) can be expressed as the familiar square-root process:

$$dV = \kappa[\theta - V]dt + \sigma\sqrt{V}\, dZ_2 \tag{51}$$

where $\kappa=2\beta$, $\theta=\delta^2/2\beta$, and $\sigma=2\delta$. The process (51) accounts for mean reversion, is always positive for plausible parameter values, and is mathematically tractable, as shown in Cox, Ingersoll, and Ross (1985b). The mean reversion feature of the volatility process has received strong empirical support in equity and currency markets. Merville and Pieptea (1989) report that implied volatilities for the Standard and Poor's 500 stock index and for twenty-five individual stocks from different industries follow a mean-reverting diffusion with noise process. Stein (1989) shows that implied volatility for the Standard and Poor's 100 index is strongly mean reverting. Melino and Turnbull (1990) demonstrate that volatility of the Canada-U.S. currency exchange rate has a strong tendency to revert quickly to the mean level of volatility.

Assume that the domestic interest rate r is constant, so that the current price of a unit discount bond with maturity at time $t+\tau$ is

$$P(t, t+\tau) = \exp(-r\tau) \tag{52}$$

The call option C under processes (48) and (51) will be a function of three variables: $C=C(S,V,t)$. Following Garman (1976) and Cox, Ingersoll, and Ross (1985b), the option must satisfy a partial differential equation (PDE) that includes a market risk premium for volatility since volatility is not a traded asset. The fundamental PDE is:

$$0.5(VS^2C_{SS}) + \rho\sigma\, VSC_{SV} + 0.5(\sigma^2VC_{VV}) + rSC_S$$
$$+\{\kappa[\theta-V]-\lambda(S,V,t)\}C_v - rC + C_t = 0 \tag{53}$$

where the subscripts on C denote the partial derivatives and $\lambda(.)$ is the price of volatility risk. The general equilibrium model of Cox, Ingersoll, and Ross (1985a) shows that, using the Breeden's (1979) consumption process where consumption growth has constant correlation with the spot-asset return, the risk premium $\lambda(.)$ is proportional to V; that is, $\lambda(.)=\lambda V$. In theory the term $\lambda(.)$ can be estimated from one volatility-dependent asset and then used to price other volatility-dependent assets. For instance, $\lambda(.)$ may be measured by using a hedge portfolio that included only at-the-money option and the underlying asset of the option, and then used to price in-the-money and out-of-the-money options on the same asset using a stochastic volatility option pricing model. The estimated premium for volatility risk, however, will differ across markets, so that the premium for volatility risk of the U.S. dollar to British pound exchange rate would differ from that of the U.S. dollar to Canadian dollar exchange rate unless the two exchange rates exhibit exactly the same volatility structure.

A European call option with exercise price K and time to maturity T satisfies equation (53) subject to the boundary conditions:

$$C(S,V,T) = Max(0,S-K) \tag{54a}$$

$$C(0,V,t) = 0 \tag{54b}$$

$$C_S(\infty,V,t) = 1 \tag{54c}$$

$$rSC_S(S,0,t) + \kappa\theta - C_V(S,0,t) - rC(S,0,t) + C_t(S,0,t) = 0 \tag{54d}$$

$$C(S,\infty,t) = S \tag{54e}$$

Conditions (54a) to (54c) are the familiar restrictions on option prices derived using the Black-Scholes model. The stochastic volatility assumption adds restrictions (54d) and (54e) to the model. Proposition (54a) states that at maturity the option will be exercised if it has a positive intrinsic value (S-K>0); otherwise it will be discarded unused. Restriction (54b) states that an option is worthless if the expected stock price is zero at all times including the option's maturity date. Condition (54c) implies that the option's price is directly proportional to the stock price when the stock price is extremely large relative to the exercise price. Restriction (54d) is obtained from equation (53) by setting V equal to zero. Relation (54e) states that the option price is equal to the stock price when the latter is extremely volatile. The solution to PDE in (53), subject to conditions (54a)-(54e), is guessed to be in the form of Black-Scholes formula:

$$C(S,V,t) = SP_1 - K\exp(-r\tau)P_2 \tag{55}$$

where P_1 and P_2 are unknown probabilities analogous to the cumulative normal probabilities under the Black-Scholes model. The first term, SP_1, in equation (55) is the present value of the spot asset upon terminal exercise, and the second term, $K\exp(.)P_2$, is the present value of the exercise price. These terms should satisfy the PDE in (53). It is well known from Cox and Ross (1976) and Ingersoll (1987, p.350) that the cumulative probability functions satisfy the forward Kolomogorov (or Fokker-Planck) equation and the backward Kolomogorov equation. Substituting solution (55) into the PDE in (53) and letting $x=\ln(S)$, Heston (1993) shows that P_1 and P_2 must satisfy the PDEs (i.e. forward Kolomogorov equations):

$$0.5V(\partial^2P_j/\partial x^2)+\rho\sigma V(\partial^2P_j/\partial x\partial V)+0.5\sigma^2V(\partial^2P_j/\partial V^2)$$

$$+(r+\mu_jV)(\partial P_j/\partial x)+(a_j-b_jV)(\partial P_j/\partial V)+\partial P_j/\partial t=0 \tag{56}$$

for probabilities p_1 and p_2 ($j=1,2$),

where $\mu_1=1/2$, $\mu_2=-½$, $a=\kappa\theta$, $b_1=\kappa+\lambda-\rho\sigma$, $b_2=\kappa+\lambda$.

The risk-adjusted equivalents of processes (48) and (51) that lead to PDEs in equation (56) are:

$$dx = (r + \mu_j V)dt + \sqrt{V}\, dZ1 \tag{57}$$

$$dV = (a_j - b_j V)dt + \sigma\sqrt{V}\, dZ2 \tag{58}$$

Given that the option price satisfies the terminal conditions in equation (54a)-(54e), the PDEs in (56) are subject to the terminal condition:

$$P_j(x,V,T;\ln(K)) = 1_{\{x \geq \ln(K)\}} \tag{59}$$

The P_j in (59), interpreted as the risk-adjusted or risk-neutralized probability, is the conditional probability that the option expires in the money:

$$P_j(x,V,T;\ln(K)) = P_r\,[x(T) \geq \ln(K) | x(t)=x,\ V(t)=V] \tag{60}$$

where P_r denotes the conditional probability. A closed-form solution to the probabilities is immediately unavailable. However, their characteristic functions satisfy the same PDEs in (56) and are available in a closed form, as shown in Heston (1993). A characteristic function uniquely determines its probability distribution function. The relationship between P_j and its characteristic functions, f_j, is:

$$f_j = \int \exp(ix\phi)dP_j = \int \exp(ix\phi)g_j dx \tag{61}$$

$$g_j = (2\pi)^{-1}\int \exp(-ix\phi)f_j\, d\phi = (1/\pi)\ \mathrm{Re} \int \exp(-ix\phi)f_j d\phi \tag{62}$$

where $I=\sqrt{-1}$, $dP_j = g_j\, dx$, g_j is the probability density function, and Re is the real part of f_j. The f_j may be expressed in real and imaginary parts. Equations (61) and (62) are generally referred to as the Fourier transform formula and the Fourier inversion formula, respectively.

The exact solution for the characteristic function is

$$f_j(x,V,t;\phi) = \exp\{C(T-t;\phi) + D(T-t;\phi)V + i\phi x\} \tag{63}$$

where

$C(\tau;\phi)=r\phi i\tau+(a/\sigma^2)\{(b_j-\rho\sigma\phi i+d)\tau-2\ln[(1-ge^{d\tau})/(1-g)]\}$

$D(\tau;\phi)=((b_j-\rho\sigma\phi i+d)/\sigma^2)[(1-e^{d\tau})/(1-ge^{d\tau})]$

$g=[b_j-\rho\sigma\phi i+d]/[b_j-\rho\sigma\phi i-d]$

$d=[(\rho\sigma\phi i-b_j)^2-\sigma^2(2\mu_j\phi i-\phi^2)]^{1/2}$

$\tau=T-t$

The cumulative probabilities P_j can be obtained using the inversion formula (62):

$$P_j(x,V,T;\ln(K))=(1/2)+(1/\pi)\int \mathrm{Re}[\{e^{-i\phi\ln(K)} f_j(x,V,T;\phi)\}/i\phi]d\phi \quad (64)$$

where Re denotes the real part and the integral extends from zero to infinity. These probabilities are substituted in equation (55) to solve for the European call option price C(.). The integral in equation (64) can be evaluated using numerical approximations for integrating functions with infinite limits of integration.

Equations (55) and (64) can be employed with minor modifications to estimate the prices of European foreign currency options. Following Grabbe (1983), one can define new variables $G=SB^*$, $x^*=\ln(SB^*)$, and $B^*=e^{-r^*(T-t)}$ for the currency option problem, where r^* is the foreign risk-free interest rate, B^* is the price of foreign unit value bond, and S is now the spot exchange rate. Substituting x^* for x and G for S in equations (64) and (55) yield the modified equation (55) as the pricing formula for a European option on foreign currency.

Implementing the Model for Currency Options

The term $f_j(.)$ in equation (64) is a complex function since it contains complex variables. Further, the denominator of the integrand in equation (64) contains an imaginary variable $i\phi$ which includes the imaginary number I. The term $e^{-i\phi \ln(K)}$ in (64) can be evaluated in sine and cosine functions since $e^{i\phi x}=\cos(\phi x)+i\sin(\phi x)$ and $e^{-i\phi x} = \cos(\phi x)-i\sin(\phi x)$. The probabilities in equation (64) are evaluated using only the real part of the integrand in brackets, Re[.]. This real part was obtained using the symbolic algebra routines in the MAPLE software. The real part, presented in appendix, is very cumbersome and lengthy, but it consists of only sine, cosine, signum, sine function of complex and real variables (csgn), and inverse tangent (arctan) functions. The MAPLE, however, proved quite ineffective and time intensive in solving for the numerical integration of the symbolic solution of equation (64) for given values of variables and parameters. Consequently, a SAS IML program was written for the Trapezoidal Rule of numerical integration. In the IML program, the step size (base) of the trapezoids was 0.1 and the convergence criterion for the integration was 0.0001. The IML program provides the numerical solution of equation (64) in a fraction of a second. A comparison of the IML program with the Simpson's integration rule in MAPLE indicated that, for a range of 1 to 10 for the integration variable ϕ on ten selected British pound currency options, the estimated probabilities, P_j, from the IML program matched with those from the MAPLE Simpson's rule up to three decimal points. However, the numerical integration procedures in MAPLE failed to evaluate the infinite integral of equation (64).

The estimation of probabilities in equation (64) requires the spot exchange rate, option's exercise price and expiration date, domestic and foreign interest rates, volatility estimates, price of volatility risk, and parameter estimates of the spot exchange rate and volatility processes (48) and (49).

Data

The study used daily closing prices from January 1993 to October 1995 for European currency call options written on the British pound. The dollar/British pound exchange rates were selected on the basis of the empirical observation that, among the British pound, Canadian dollar, Japanese yen, Swiss franc, French franc, the British pound appears to have the largest variation in periodic exchange rate variances during 1972 to 1992 period (Madura, pp. 271-74). This observation suggests that the potential impacts of stochastic volatility on option prices are more likely to show up in options written on the British pound than in other options. In addition, Bodurtha and Courtadon (1987b) have shown that the Black-Scholes model performs much worse in pricing high interest-rate currency options, such as options on the British pound, than in pricing low interest-rate currency options. Shastri and Tandon (1986b) have reported similar results for high interest-rate currency options. Since the average British interest rates were higher than the average U.S. interest rates during the sample period, any potential benefits of stochastic volatility option pricing model over the Black-Scholes model in pricing currency options are more likely to be realized in British pound options than in low interest-rate currency options.

The pricing of options on the British pound requires simultaneous data on the spot dollar/British pound exchange rate, on the variables underlying the British pound option contracts, and on the British and U.S. interest rates for default-free claims matching the maturities of the options contracts. The simultaneous option and currency prices were obtained from the transaction surveillance report of the Philadelphia Stock Exchange. The report lists for each option trade the date and time of the trade; maturity, exercise price, and option price; number of contracts traded; and the prevailing bid and ask quotes of exchange rates at the time of the trade.

The European British pound options were identified in the transaction report with symbols CBZ, CBY, CBP, CBX, EPZ, EPU, EPO, EPX, and YPX. Only symbols CBY, CBP, and EPO were applicable before May 1995; other symbols were introduced since to accommodate new strike price intervals. All call options with these symbols were extracted from the transaction data for the period January 1993 to October 1995, providing a total of 879 observations. The sample appears small because, compared to American options, European options are typically a small percentage of the total currency options traded. The sample was further reduced since only the European call options, rather than both put and call options, were retained in the sample.

The resulting call options data included both the month-end options and the mid-month options. The month-end options expire on the last Friday of the expiration month while the mid-month options expire on the third Wednesday of the expiration month. The expiration months are March, June, September, December, plus two near-term months for mid-month options, and only the nearest three months for month-end options. The strike price intervals for British pound options were one cent for the three nearest option expiration months, two cents for the six, nine, and twelve month option expiration dates, and four cents for options expiring in twelve months. Strike price intervals of 2.5 cents also existed before May 1995. One option contract extends the right to purchase 31,250

British pounds. The minimum premium change was 0.01 cent per pound or $3.125 per option contract, and the option premiums were quoted in cents per pound.

To exclude uninformative options records form the sample, the call options which fell in the following categories were discarded:

i) Options with time to expiration of less than 5 calendar days.

ii) Options violating the European boundary conditions, i.e. $c < Se^{-r^*\tau} - Ke^{-r^*\tau}$.

iii) Options with a premia of less than or equal to 0.02 cents.

Criterion (i) is justified because the implied volatility of options with small time to maturity behaves erratically. Criterion (iii) eliminates options which are very thinly traded and are not representative of the market. These criteria removed twelve options from the sample, leaving 867 call options for the analysis.

The yield on the U.S. Treasury bills with maturity closest to the option maturity date was used for the domestic interest rates. The yield was based on the average of the bid and asked discounts on the Treasury bills. The T-bill data were obtained from the Federal Reserve Bank of New York.

Similar data on yields of British Treasury bills were not available. The interest rate parity (IRP) relationship was used to estimate the implied British risk free rate. The interest rate parilty relationship can be written:

$$[(F-S)/S] = [(1+r)/(1+r^*)]-1 \quad (65)$$

where F is the forward (futures) exchange rate of the U.S. dollar to British pound, S is the spot exchange rate, r is the U.S. T-bill rate, and r^* is the British riskfree rate. At any given time t the futures exchange rate that had a maturity date most closely matched to that of the concurrent T-bill was used in the interest rate parity relationship. The futures exchange rate data were supplemented by the forward exchange rates in cases where the futures rates with maturities matching with those of the T-bills were unavailable. The futures exchange rates were obtained from the Futures Industry Institute and the forward exchange rates were hand collected from the Wall Street Journal.

Given the observed values of futures (forward) exchange rate, spot exchange rate, and U.S. T-bill rate in equation (65), the corresponding implied British riskfree rates were estimated. The IRP relationship in equation (65) is typically applied to the forward rates. Since Cornell and Reinganum (1981) and others have shown that the difference between the futures price of a commodity and the corresponding forward price is very small, it is implicitly assumed in equation (65) that the IRP relationship holds also with respect to the futures price.

Estimating the Volatility

The estimation of option prices from the modified Black-Scholes model and Heston's model, which were required to examine the second objective of the study, depends upon, inter alia, the estimates of volatility of exchange rates. Given the current level of volatility, Heston's volatility process in equation (49) employs the mean reverting parameter of the process to pull the current volatility towards its long-run value.

Volatility of asset returns is commonly estimated either by the standard deviation of historical returns or by the implied standard deviation (ISD) of returns. Both measures were employed in this study. Empirical evidence appears to favor an implied volatility estimate over a historical estimate as a predictor of ex-post volatility (Chirac and Manaster, 1978; Merville and Pieptea, 1989). However, Canina and Figlewski (1993) showed that, using S&P 100 index options data, the historical volatility was a better predictor of subsequent realized volatility than was implied volatility. They argued that, for pricing options, the standard deviation of past prices over the average time to maturity of sample options is a better proxy for historical volatility.

Following Canina and Figlewski (1993), the historical volatility for day t was estimated as the annualized sample standard deviation of the natural logarithm of daily closing exchange rate changes, $\Delta \ln S_t$, for the past seventy-five days; the 75 days were the average time to expiration of sample currency options.

The ISD was estimated for both the modified Black-Scholes model and the stochastic volatility model. Although the true volatility is not option dependent, the ISD estimates are option dependent. The regression-based iteration approach of Whaley (1982) was employed to smooth out option-specific errors. The approach uses only the options which share a common maturity, and finds the volatility value that minimizes the sum of squared errors between the model prices and actual prices. The iteration process for finding the implied volatility is based on the regression model:

$$c_j - c_j(\sigma_0) + \sigma_0 (\partial c_j / \partial \sigma_0 |\sigma_0) = \sigma(\partial c_j / \partial \sigma_0 |\sigma_0) + e_j \quad (66)$$

where c_j are the actual option prices, $c_j(\sigma_0)$ are the predicted option prices using σ_0 as an estimate of volatility, and $(\partial c_j / \partial \sigma_0 |\sigma_0)$ is the option vega based on σ_0. The regression (66) yields an estimate of σ, which is then used in another iteration of the regression until the implied volatility estimate changes less than a stipulated amount in absolute value.

Most studies have computed the ISD at the same instant at which the option is priced. Such a contemporaneous estimation of volatility and valuation of options yields a selection bias which simultaneously identifies bid prices as undervalued options and ask prices as overvalued options, as pointed out by Phillips and Smith (1980). Following Whaley (1982), the ISD was estimated at time t-1 to circumvent this problem. Such an estimate is theoretically more plausible than contemporaneous ISD estimate because it does not use information that might be unavailable to option traders at the time of the option transactions.

For the stochastic volatility model, the above procedure requires parameters of the volatility process, volatility risk premium, and option's vega in order to estimate the ISD. Nandi (1996) has derived the stock option's vega for the Heston model:

$$\partial C/\partial \sigma = (S/\pi)P_1 - (K\, e^{-r\tau}/\pi)P_2 \qquad (67)$$

where P_1 and P_2 are probabilities of the Heston model in equation (64), and all other variables are defined as before under the Heston model. To adjust the above vega for the currency option case, the variable S was replaced with $Se^{-*r\tau}$ in the first term and in P_1 and P_2 of equation (67). Given parameters of the volatility process and volatility risk premium, the ISD for the stochastic volatility model was estimated using the foregoing Whaley (1982) approach. Whaley's approach, however, failed to provide fast convergence for a subset of options because the vega-based increments in c_j (.) generated σ's that bypassed the convergence criterion; the approach was also quite sensitive to the initial value of σ. For such situations, the ISD was estimated by subjecting σ to equal increments until the sum of squares between the actual and predicted values was minimized.

Estimating the Price of Volatility Risk

A study of the first objective of this research requires estimates of the price of volatility risk. This price is also an input in the Heston's option pricing model.

When volatility is random and a marketwide volatility effect exists, a hedge portfolio formed from one option and the underlying stock would not be riskless and would not yield a return equal to the riskless interest rate. Heston (1993) has argued that the excess return on the hedge portfolio, the difference between the hedge portfolio's return and the riskless interest rate, is an appropriate measure of the premium for volatility risk. Wiggins (1987) stresses that the volatility risk premium depends upon the beta of the hedge portfolio and thus it can take both positive and non-positive values.

The present study measured the premium for volatility risk as the excess actual return on the hedge portfolio. The price of volatility risk must be independent of the particular asset. The risk premium, therefore, can be measured using an asset that leads to simpler formula for the hedge portfolio. Along these lines, Heston (1993) shows that the Black-Scholes model generates option prices identical to his stochastic volatility model for at-the-money options, and that all option models with the same volatility are equivalent for at-the-money options. The stochastic volatility and the correlation between the asset's return and its volatility affect only the pricing of in-the-money, out-of-the-money, and near-the-money options. Consequently, the excess return on the hedge portfolio formed from the spot asset and an at-the-money option, using the option delta of the modified Black-Scholes model as the hedge ratio, was used as the premium for volatility risk. The value of the hedge portfolio at time t, Q_t, is

$$Q_t = H_t S_t - C_t \tag{68}$$

where S_t is the spot exchange rate, C_t is the price of the at-the-money currency option, and H is the hedge ratio defined as

$$H_t = \partial C/\partial S = e^{-r^*\tau} N(d_1) \tag{69}$$

where $d_1 = \{\ln(S/K) + (r - r^* + (V/2))\tau\}/\sqrt{V}\,\tau$

V= implied volatility

$N(d_1)$ = the cumulative normal distribution function

The variables K, r, r^*, and τ are defined as before. Since the hedge ratio in (69) varies with S, T, and V, it was readjusted daily to maintain the hedge position of the portfolio. Specifically, the value of the hedge portfolio at time t+1 was estimated as

$$Q_{t+1} = H_t S_{t+1} - C_{t+1} \tag{70}$$

The return on the hedge portfolio, R, was then estimated:

$$R_{t+1} = (Q_{t+1} - Q_t)/Q_t \tag{70a}$$

The premium for volatility risk, λ, was estimated:

$$\lambda_{t+1} = \text{Annual } R_{t+1} - r_{t+1} \tag{71}$$

where r is the annual t-bill rate at time t+1. The yield on the the three-month U.S. Treasury bills was used for the t-bill rate. The yield was based on the average of the bid and asked discounts on the Treasury bills.

Estimating the Parameters of the Volatility Process

The estimation of probabilities for the Heston model requires, inter alia, the estimates of parameters κ,θ, and σ of the volatility process, and of the correlation coefficient, ρ, between the volatility and exchange rate processes. One estimation approach is to use the maximum likelihood estimator on the unconditional distribution of security returns which is a function of κ, θ, and σ. Scott (1987) has argued that this unconditional, joint distribution is quite difficult to derive when security returns are autocorrelated as is the case in Heston's model. Further, Bates (1995) has pointed out that the maximum likelihood approach is computationally intensive and often impossible. Alternatively, Nandi (1996) has devised a non-linear least squares procedure to simultaneously estimate the implied volatility and parameters of the volatility process. His procedure is at best cumbersome and computationally intensive; the procedure took 3.5 days to estimate parameters on a non-standard SPARC workstation. Also, Nandi's procedure cannot identify the parameter of the volatility risk premium.

Two other alternatives are to estimate the parameters either from the method of moments, or from the time series of implied volatilities (Scott, 1987; Chesney and Scott, 1989; Melino and Turnbull, 1991; Knoch, 1992). The present study initially employed the method of moments using the approaches of Scott (1987) and Chesney and Scott (1989). However, this methodology yielded negative variances of the parameters and thus was not used in this study. Kearns (1992), Bates (1995), and Wiggins (1987) have pointed out that the parameter estimates based on the method of moments are quite sensitive to the choice of moments. Also, Bates (1995) has argued that the Chesney and Scott approach yields considerable imprecision in parameter estimates.

The present study estimates the volatility parameters using the seemingly unrelated regression (SUR) method on the time series of implied volatilities and exchange rates. The stochastic processes of spot exchange rates (S) and volatility of exchange rates (V) were specified, respectively, in equations (48) and (49):

$$dS=\mu Sdt + \sqrt{V}S\, dZ_1 \qquad (48)$$

$$d\sqrt{V} = -\beta\sqrt{V}\, dt + \delta dZ_2 \qquad (49)$$

Assuming Δt is 1 (one day), a discrete time version of processes (48) and (49) can be written:

$$\Delta\log(s_t) = \mu + \sqrt{V}_t u_t; \qquad u_t \sim N(0,1) \qquad (48a)$$

$$\sqrt{V}_t = \alpha\sqrt{V}_{t-1} + \phi e_t; \qquad e_t \sim N(0,\phi^2) \qquad (49a)$$

where $\alpha=(1/1+\beta)$, ϕ= standard deviation of $e_t=\delta(1/1+\beta)$, and $\Delta S/S=\Delta\log(S)$. The implied volatilities of the modified Black-Scholes model, which were revised daily, were used for V. Equations (48a) and (49a) can be rewritten:

$$\Delta\log(s_t)/\sqrt{V}_t = \mu/\sqrt{V}_t + u_t; \; u_t \sim N(0,1) \qquad (48b)$$

$$\sqrt{V}_t = \alpha\sqrt{V}_{t-1} + \phi e_t; \qquad e_t \sim N(0,\phi^2) \qquad (49b)$$

The equations were estimated for January 1993 to October 1995 as a SUR system using the SAS software, without allowing for intercepts in the equations. The parameter ρ is the correlation between u_t and e_t; SAS supplied its estimate along with the SUR results. Also, SAS supplied the variance of e_t which was used for ϕ^2. Given α and ϕ, the estimates for β and δ were recovered:

$\beta = (1/\alpha)-1;\ \delta = \qquad \phi(1+\beta)$

Given β and δ, the estimates of original parameters were recovered:

$\kappa=2\beta$, $\theta = \delta^2/2\beta$, and $\sigma=2\delta$

The variances of κ, θ, and σ were derived from the variances of β and δ using the large-sample approximation procedure of Kmenta (1971, P. 444):

if $A= f(B_1, B_2,\ldots,B_k)$, then

$\mathrm{Var}(A)= \Sigma[\partial f/\partial B_k]^2\ \mathrm{Var}(B_k) + 2\Sigma[\partial f/\partial B_j][\partial f/\partial B_k]\mathrm{Cov}(B_j,B_k)$

where Var(.) denotes the variance and Cov (.) represents the covariance. The procedure indicated that $\mathrm{var}(\beta)=(-1/\alpha^2)^2*\mathrm{var}(\alpha)$; $\mathrm{var}(\kappa)=4*\mathrm{var}(\beta)$; and $\mathrm{var}(\sigma)=4*\mathrm{var}(\delta)$. Similarly, other variances were derived.

Methods for Testing the Hypothesis of the Study

Hypothesis I. The premium for the volatility risk is non-zero and is directly proportional to the level of volatility.

A t-statistic was used to test if the mean of the volatility-risk premium is statistically different from zero. The relationship between volatility and volatility-risk premium was investigated using the time-series regression:

$$\lambda_t = b_0 + b_1 V_t + e_t \qquad (72)$$

where λ_t is the estimated premium for volatility risk at time t, V_t is the corresponding estimate of the volatility, and e_t is the error term. The premium λ will be directly proportional to the stochastic volatility if $b_0=0$ and $b_1=1$. This hypothesis was tested using the Wald statistic which has an asymptotic $\chi^2(2)$ distribution.

Hypothesis II: The stochastic volatility model corrects the observed biases in pricing foreign currency options.

The pricing accuracy was initially evaluated by comparing the mean squared error, mean relative error, and mean absolute error of the Heston's model with those of the modified Black-Scholes model. Following Melino and Turnbull (1990), Hilliard et al. (1991), and Chesney and Scott (1989), the pricing biases of the stochastic variance and modified Black-Scholes models were investigated using the cross-sectional regressions:

$$C = \alpha_0 + \alpha_1 C^* + e_1 \tag{73}$$

$$C - C^* = \gamma_0 + \gamma_1 (S-K)/K + \gamma_2 T + \gamma_3 V + \gamma_4 (r - r^*) + e_2 \tag{74}$$

where C denotes the observed option price, C^* the model-predicted price, K the exercise price, S the spot exchange rate, T the maturity of the option, $r(r^*)$ the U.S. (U.K.) interest rate, V the estimated volatility, and e_1 and e_2 are the disturbance terms. In equation (73), the model price provides an unbiased estimate of the actual option price if $\alpha_0=0$ and $\alpha_1=1$. This hypothesis was tested using the Wald statistic. The regression (74) is designed to examine whether the pricing errors exhibit the strike price bias, time-to-maturity bias, volatility bias, and interest rate differential bias. The regression is structured to identify if the prediction errors are systematically related to the fundamental inputs of the currency option pricing model. A positive value of γ_1 indicates that, for call options, the model underprices (overprices) an option that is in (out of) the money. A positive value of γ_2 suggests the maturity bias. A positive value of γ_3 points to the existence of the volatility bias. If γ_4 is positive, the degree of mispricing the options is an increasing function of the interest rate differential. If $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$, then the pricing errors are independent of the exercise price, time to maturity, volatility, and interest rate differential. The Wald statistic to test the joint hypothesis of a zero value for all the parameters in regression (74) is asymptotically distributed as $\chi^2(5)$.

Equation (73) was estimated using the ordinary least squares (OLS) method, while equation (74) was estimated using the Generalized Method of Moments in SHAZAM with correction for heteroscedasticity. Because option observations are clustered in time and across cross sections, the residuals of equation (74) may be heteroscedastic as pointed out by Hilliard, et al. (1991) and Scott (1987). Most previous studies have estimated equation (74) using OLS method (Melino and Turnbull, 1990; Nandi, 1996; Chesney and Scott, 1989; Hilliard, et al., 1991; Shastri and Tandon, 1986a; Whaley, 1982). Such an approach makes no distinction between the time series and cross-sectional properties of the sample and treats the sample as a single collection of $N \times T$ observations, where N represents the number of cross sections and T denotes the number of observations over time. The approach seems to stem from the notion that, on a given day, each traded option is distinct in nature and that the same exact option is not traded the next day. For instance, a given option may have the same strike price over time but its time to maturity changes over time. Consequently, the choice of what constitutes a cross section is unclear. Assuming that the cross section is defined, the typical options data give an unbalanced pooled time-series cross-section data structure since some cross sections (options) are traded more frequently than others. Nandi (1996) and Whaley (1982) have handled such a data structure by running separate OLS regressions for each of the several cross sections defined.

The present study makes an attempt to capture the cross-sectional time-series properties of the sample currency options in the estimation procedure. An index variable, which served as a cross-sectional identifier, was created using the option's time to maturity and option's moneyness, the difference between the strike price and spot price. A scale variable distinguished the moneyness classes by assigning a value of 5 to in-the-money options ($S/K \geq 1.02$), 6 to at- or near-the-money options ($0.98 < S/K < 1.02$), and 7 to out-of-the-money options ($S/K \leq 0.98$). A second scale variable differentiated the time-to-maturity classes by assigning a value of 1 to options with maturity less than 30 days, 2 to options with maturity between 30 and 90 days, 3 to options with maturity between 90 and 180 days, and 4 to options with maturity more than 180 days. The two scale variables were then multiplied to create a cross-sectional index variable, providing twelve possible cross sections for the sample data. The creation of moneyness and time-to-maturity scale variables stemmed from the extant empirical evidence that the pricing errors of options strongly depend on the time to maturity and moneyness of options.

The number of observations under each of the twelve cross sections varied in the sample data, yielding an unbalanced pooled cross-section time-series structure. Given such a data structure, equation (74) was estimated using the Error Components Model and Dummy Variable Model routines in LIMDEP software. The models assume that each cross section has its own, distinct intercept but all cross sections share the same slope coefficients. The Error Components Model assumes that the intercepts are drawn from a random sample, whereas the Dummy Variable Model assumes that the intercepts are fixed parameters. Both procedures were used here since the choice between fixed and random effects was unclear a priori. The estimation results of equation (74) using the Generalized Method of Moments, Error Components Model, and Dummy Variable Model are presented in Chapter 4.

CHAPTER 4

EMPIRICAL RESULTS OF CURRENCY OPTION PRICING MODELS

Descriptive statistics of sample currency options are presented in Tables 1 and 2. The mean spot exchange rate differed from the strike exchange rate by only $0.03, suggesting that, on average, the options were trading near the money. The option's time to expiration averaged seventy-five days, and the average implied volatility was 6.3 percent lower than the average historical volatility. In Table 2, changes in daily exchange rates display a small negative mean and negative skewness, but consistently display positive excess kurtosis. Both the Kolmogrov statistics and Jarque-Bera test indicated that the distributions of exchange rates and changes in exchange rates are not normal. The skewness and excess kurtosis features of exchange rate distributions are expected to be captured by the correlation coefficient and volatility of volatility parameters of the stochastic volatility model.

Pricing Performance of the Modified Black-Scholes Model

A pricing error is the difference between actual option price and predicted option price of the model. The relative mean error, mean absolute error, and root mean square error (RMSE) of the Black-Scholes model for the aggregate sample and for the moneyness subsamples are shown in Table 3. The relative mean error is equal to the average of the difference of the actual option price and the model option price divided by the actual option price. The mean absolute error is the average of the absolute values of

Table 1. Descriptive Summary Statistics of the Sample Data on British Pound Currency Options, 1993-95

Variable	Mean	Standard Deviation	Minimum	Maximum
Call price (cents/unit)	1.786	2.13	0.030	14.35
Spot exchange rate ($/pound)	1.54	0.054	1.417	1.650
Exercise exchange rate ($/pound)	1.57	0.074	1.475	2.10
Years to expiration	0.205	0.171	0.016	0.991
Annualized U.S. riskless rate (%)	4.07	1.126	2.08	7.15
Annualized British riskless rate (%)	4.35	1.159	1.714	8.46
Historical volatility (75 days moving average)	0.095	0.031	0.053	0.166
Implied volatility	0.089	0.025	0.010	0.261

Note: The sample has 867 observations.

Table 2. Descriptive Statistics of the Distribution of U.S. Dollar-British Pound Exchange Rate, 1993-95

Statistic	S[a]	Log(S)	Δ (S)	ΔLog(S)
Mean	1.54	0.43	-0.0001	-0.0002
Skewness	0.77	0.60	-0.37	-0.28
Excess kurtosis	1.86	1.34	3.12	2.55
Kolmogorov statistics[b] (W-statistics)	0.94*	0.95*	0.96*	0.96*
Jarque-Bera test[c]	196.1*	85.24*	345.7*	229.19*

[a] The letter S denotes the spot exchange rate.
[b] H_0: the data set is a sample from a normal distribution. An asterisk indicates the rejection of the null hypothesis at the 1 percent significance level.
[c]. The critical chi-square value is 9.21. The null hypothesis is that the data set is a sample from a normal distribution. The test statistic is: $LM= N[(g^2/6) +(k^2/24)]$, where LM denotes the Lagrange Multiplier, and g and k are coefficients of skewness and kurtosis, respectively.

the these relative pricing errors. Thus, the relative mean error and the mean absolute error are both measures of relative pricing error with respect to the actual option price; the former measure works with actual, raw pricing errors while the latter measure works with absolute pricing errors. Following Bodurtha and Courtandon (1987a,b), a call option was considered to be in the money if the ratio of the spot price to exercise price was greater than 1.02; an option was out of the money when the spot price to exercise price ratio was less than 0.98; and, finally, an option was near or at the money when the spot price to exercise price ratio was between 0.98 and 1.02. Small changes in the range of these price ratios did not substantially alter the results.

Positive pricing errors under the relative mean error measure imply option underpricing by the Black-Scholes model, and negative errors imply overpricing. Table 3 shows that, using historical volatility, the Black-Scholes model overprices call options irrespective of their moneyness class, with an aggregate overpricing of nineteen percent. The overpricing of near- or at-the-money options was more severe than the overpricing of in-the-money options, a result consistent with those of Bodurtha and Courtadon (1987a,b). The relative mean error, however, may cancel out large positive errors with large negative errors, yielding a small average error. The mean absolute errors, which circumvented this deficiency, indicate that, on average, the Black-Scholes model resulted in an aggregate mispricing of forty-nine percent. Further, the mean absolute errors were a decreasing function of how deep the option is in the money, a result also reported by Bodurtha and Courtadon (1987a,b). The RMSE value was the highest for in-the-money options, and lowest for out-of-the-money options.

Table 3. Pricing Errors as Differences between Actual Prices and Predicted Prices of the Modified Black-Scholes Model Using Historical and Implied Volatilities , by Moneyness Classes, 1993-95

Statistic	All Options	In-the-Money	Near-the-Money & At-the-Money	Out-of-the-Money
		Historical Volatility		
Relative Mean Error (%)	-19.7	-16.4	-34.0	-1.0
Mean Absolute Error (%)	49.0	21.0	50.0	0.55
RMSE (cents)	0.61	1.05	0.61	0.43
		Implied Volatility		
Relative Mean Error (%)	-4.0	-15.0	-12.0	9.0
Mean Absolute Error (%)	19.0	17.0	20.0	18.0
RMSE (cents)	0.36	0.96	0.21	0.21
N	867	88	443	336

Notes: In-the Money: (Spot/Exercise) $\geq$ 1.02
Near- or At-the-Money: 0.98 < Spot/Exercise <1.02
Out-of-the-Money: Spot/Exercise $\leq$ 0.98
Pricing Error: Actual Price- Model Predicted Price
Relative Mean Error $= (1/N)\ \Sigma\ [(\text{Actual-Predicted})/\text{Actual}]$
Absolute Mean Error $= (1/N)\ \Sigma[(|\text{Actual -Predicted}|)/\text{Actual}]$
RMSE $= [(1/N)\Sigma(\text{Actual-Predicted})^2]^{0.5}$

Using implied volatilities, the aggregate relative mean error dropped to only negative four percent, but out-of-the-money options were now underpriced; the latter result corroborates those of Bodurtha and Courtadon (1987b). The use of implied volatilities lowered the mean absolute error to 19 percent, lowering RMSE to 0.36 cents compared to 0.61 cents under historical volatility. Bodurtha and Courtadon (1987b) have reported mean absolute errors of similar magnitude for high interest-rate currency options which included the options on British pound and Canadian dollars. Overall, the implied volatilities yielded substantially less mispricing, albeit still large errors, than did the historical volatilities. Bodurtha and Courtadon (1987a,b) and Chesney and Scott (1989) have reported similar findings.

Table 4 presents the regression-based tests of whether the Black-Scholes model price is an unbiased estimate of the actual price. Overall, the Black-Scholes model explained seventy-five percent of the actual prices under historical volatility measure, and eighty-eight percent under implied volatility measure. The use of implied volatilities markedly improved the explanatory power of the Black-Scholes model under all moneyness classes. The Black-Scholes model displayed the best performance for near- or at-the-money options with implied volatilities where it explained ninety-five percent of actual prices. Both Heston (1993) and Nandi (1996) have reported that the Black-Scholes model performs quite well in explaining near-the-money options. The slope coefficients of the regressions were close to one, except the slope coefficient for out-of-the-money options under historical volatility. However, the intercepts were statistically different from zero in four of the eight cases. The Wald test statistics shows that, with the

Table 4. Regression Tests for the Relation between Actual Call Prices and Predicted Call Prices from the Modified Black-Scholes Model, by Moneyness Classes, 1993-95

	All Options	In-the-Mney	Near-the-Money & At-the-Money	Out-of-the-Money
		Historical Volatility		
Intercept (α_0)	0.01 (5.03)	0.08 (0.28)	0.07 (1.62)	0.29 (10.48)
Slope (α_1)	0.94 (101.2)	0.96 (23.4)	0.86 (41.5)	0.59 (20.22)
R^2	0.75	0.86	0.79	0.55
Wald-Test (H_0: $\alpha_0 = 0$, $\alpha_1 = 1$)	64.6*	2.15	45.76*	183.1*
		Implied Volatility		
Intercept (α_0)	0.02 (1.68)	0.09 (0.34)	-0.10 (-0.64)	0.10 (5.87)
Slope (α_1)	0.87 (72.29)	0.90 (26.02)	0.91 (130.13)	0.90 (44.36)
R^2	0.88	0.88	0.95	0.82
Wald-Test (H_0: $\alpha_0 = 0$, $\alpha_1 = 1$)	27.35*	2.06	40.23*	34.54*

Notes: An asterisk on a value indicates the rejection of the null hypothesis at the 5% level. The t-values are given in the parenthesis. The above results were based on the regression model:

$$C = \alpha_0 + \alpha_1 C^* + e$$

C= actual call price

C*= predicted call price

exception of in-the-money options, the null hypothesis that the model prices are unbiased estimate of actual prices can be overwhelmingly rejected.

Table 5 presents regression tests for four pricing biases that have been documented in the literature on stock and currency options: the strike-price or moneyness bias, time-to-maturity or maturity bias, volatility bias, and interest rate differential bias. The aggregate results indicate that all of the four biases exist under the historical volatility measure, while three of the four biases, the exception being the maturity bias, exist also under the implied volatility measure. The positive coefficient of time-to-maturity variable under the historical volatility measure indicates that the difference between actual price and model price increases with the time to maturity of the option. Hence, the historical volatility biases the model toward undervaluing long maturity options and overvaluing short maturity options. The regression R^2 indicates that, for the aggregate sample, the four pricing biases can explain forty-three percent of the option mispricings under the historical volatility measure, and only four percent under the implied volatility measure. Consequently, the pricing biases were relatively weaker with the implied volatilities than with the historical volatilities. These findings based on historical volatility corroborate those of Whaley (1982) for stock options, Nandi (1996) for equity index options, Shastri and Tandon (1986a) for currency futures options, and Melino and Turnbull (1990) for currency options.

Since the foregoing studies have relied heavily upon implied volatilities, the results based on implied volatilities should provide a better comparison. However, a complete comparison of present results with those of previous studies is not possible

Table 5. Regression Tests of the Biases of the Modified Black-Scholes Model in Pricing British Pound Currency Options, 1993-95

	All Options	In-the-Money	Near-the-Money & At-the-Money	Out-of-the-Money
		Historical Volatility		
Intercept	0.92	1.07	0.96	0.97
	(16.8)	(2.30)	(15.02)	(20.168)
S-K/K	-2.69	-3.81	-3.62	-3.34
	(-6.43)	(-0.73)	(-2.02)	(-8.29)
T	0.77	0.97	1.34	0.18
	(7.37)	(1.58)	(9.41)	(2.11)
r-r*	-0.04	-0.42	-0.031	-0.002
	(-1.68)	(-2.27)	(-0.88)	(-0.10)
V	-12.95	-13.05	-15.28	-11.57
	(-23.82)	(-3.29)	(-23.34)	(-24.8)
R^2	0.43	0.17	0.58	0.67
Wald-Test	702.44*	20.62*	733.35*	699.1*
(H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)				
		Implied Volatilities		
Intercept	0.09	0.04	0.09	0.23
	(1.50)	(0.08)	(2.78)	(4.83)
S-K/K	-1.26	-4.47	-2.59	-0.21
	(-3.70)	(-0.91)	(-2.80)	(-0.66)
T	0.06	0.25	0.10	0.05
	(0.72)	(0.43)	(1.26)	(1.20)
r-r*	-0.04	-0.23	-0.01	-0.05
	(-1.74)	(-1.17)	(-1.02)	(-2.63)
V	-1.58	-0.38	-2.22	-1.80
	(-2.88)	(-0.08)	(-4.74)	(-3.56)
R^2	0.04	0.03	0.06	0.07
Wald-Test	31.06*	3.48	58.43*	40.27*
(H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)				

Notes:The above results were derived using the regression model:
$C\text{-}C^* = \alpha_0 + \alpha_1\ S\text{-}K/K + \alpha_2\ T + \alpha_3\ r\text{-}r^* + \alpha_4\ V + e$, where C is the actual call price and C* is the model predicted call price.

because no previous study has considered all of the four biases simultaneously. Among the previous investigations, the studies of Chesney and Scott (1989), Melino and Turnbull (1990), and Nandi (1996) are comparable, in overall focus and spirit, to the present study. Chesney and Scott (1989), using currency options, found only the strike-price bias for the Black-Scholes model. Nandi (1996) tested for strike-price and maturity biases and confirmed their existence in stock index options. Melino and Turnbull (1990), examining currency options, reported the existence of strike-price, maturity, and interest rate biases. In contrast, the present study uncovered the existence of volatility bias, strike-price bias, and interest-rate differential bias in currency options. Further, the regression R^2 indicates these biases explained only four percent of the option mispricings, compared to thirty-one percent for similar regressions in Melino and Turnbull (1990), suggesting that these biases were relatively weak in this study.

A Wald test was used to examine the hypothesis that the model is correctly specified. The model was presumed to be correctly specified when all the pricing biases were jointly equal to zero. Table 5 shows that for seven of the eight cases, the exception being in-the-money options with implied volatility, the Wald test overwhelmingly rejected the null hypothesis that the model is correctly specified.

Tests of pricing biases using the dummy variable model and the error components model are presented in Table 6. The results are virtually identical to those discussed above which were generated using the generalized method of moments. The LR-test and F-test in Table 6 evaluates the existence of fixed effects in intercepts of regression equations, while the LM-test examines the presence of random effects in intercepts of

Table 6. Tests of the Pricing Biases of the Modified Black-Scholes Model Using Dummy Variable and Error Components Regression Models, 1993-95

	BS with Historical V		BS with Implied V	
	Dummy Variable Model	Error Cmponents Model	Dummy Variable Model	Error Components Model
S-K/K	-2.75 (-6.5)	-2.75 (-6.5)	-1.29 (-3.9)	-1.29 (-3.8)
T	0.75 (7.2)	0.75 (0.1)	0.05 (0.62)	0.05 (0.63)
r-r*	-0.04 (-1.6)	-0.04 (-1.6)	-0.03 (-1.7)	-0.3 (-1.7)
V	-13.0 (-23.9)	-13.0 (-23.8)	-1.57 (-2.8)	-1.57 (-2.8)
R^2	0.43	0.41	0.038	0.032
LR-Test (χ^2-Test) (H_o: no fixed effects)	5.27 (0.50)	---	5.27 (0.50)	---
F-Test (H_o: no fixed effects)	0.87 (0.51)	---	0.87 (0.51)	---
LM-Test (χ^2-Test) (H_o: no random effects)	---	0.09 (0.75)	---	0.07 (0.78)

regression equations. These three tests suggested that, taking all the cross sections as a group, cross-sectional fixed effects or random effects do not exist.

Pricing Performance of Stochastic Volatility Model

Volatility Risk Premium

Table 7 presents tests of the significance of volatility risk premium. The volatility risk premium, estimated as the difference between the return on a hedge portfolio and the risk-free rate, averaged -5.2 percent. Both a t-test and a non-normal t-test suggested that this estimated volatility risk premium was statistically different from zero at the 1-percent significance level. Melino and Turnbull (1990) and Whaley (1982) have also suggested a negative premium for volatility risk using alternative approaches to measure the risk premium.

Two alternative approaches to estimating the volatility risk premium are either to infer the volatility risk premium from observed option prices, which is analogous to extracting implied volatility from option prices, or to estimate the beta of the hedge portfolio. Using the first approach, Melino and Turnbull (1990) uncovered a negative premium for volatility risk from observed prices of currency options on Canadian dollars. Whaley (1982), using the beta approach, reported a zero beta value for a stock-option hedge portfolio, implying a zero premium for volatility risk. He also found, however, a negative beta for a hedge portfolio when a CRSP equal-weighted market index was used to measure market returns. Using a variant of the beta approach on volatilities of stock returns, Merville and Pieptea (1989) found that the volatilities are correlated across stocks

Table 7. Tests for the Non-Zero Mean of Volatility Risk Premium, 1993-95

	Mean	S.D.	N	t-test[a] (normal)	Modified t-test[b] (non-normal)
Volatility Risk Premium (λ)	-0.052	0.51	867	-3.01*	-3.02*

[a] $t = \sqrt{n}\, x/s$, where x is the mean value, s is the standard deviation (S.D.) of x, and n is the total number of observations. An asterisk on the t-value implies that the null hypothesis of zero mean was rejected at the 1% significance level.
[b] $t_{John} = n[x + (\mu/6s^2 n) + \mu x^2 + /3\, s^4]/s$, where μ is the third central moment. The modified t-value, adapted from Whaley (1982), is the t_{John} value.

and with a market volatility, suggesting a non-zero price of volatility risk. Similarly, Lamareux and Lastrapes (1993) have argued that the volatility risk is priced. However, neither Merville and Pieptea (1989) nor Lamareux and Lastrapes (1993) estimated the size or sign of volatility risk premium.

A direct impact of negative volatility risk premium on option prices can be illustrated using the risk-neutralized or risk-adjusted volatility process of Heston (1993):

$$dV = \kappa^*[\theta^* - V]dt + \sigma\sqrt{V}\,dZ_2 \tag{75}$$

where $\kappa^* = \kappa + \lambda$, $\theta^* = \kappa\theta/(\kappa+\lambda)$, λ is the volatility risk premium, κ is the mean reversion parameter of the true volatility process (51) from Chapter 3, θ is the long-run mean variance of the true volatility process, V is the current variance, and κ^* and θ^* are the risk-adjusted counterparts of κ and θ, respectively. The mean reversion parameter κ^* controls the speed at which the variance drifts toward the long-run mean variance θ^*, and it also determines the relative weight of the current variance and long-run variance on option prices. A negative λ decreases the value of κ^* which in turn increases the value of the average variance θ^*, thereby making the risk-adjusted variance θ^* to be higher than the average variance θ of the true volatility process. An increase in average variance θ^* increases the prices of currency options in Heston's model. This inflationary impact of the negative volatility risk premium on option prices may have contributed toward the overpricing of British pound options by Heston's model, as shown later in Table 11.

An investigation of the possible sources of negative market risk premium for volatility risk requires an explicit description of a representative investor's preferences, endowments, and technology. Assuming logarithmic utility function, Wiggins (1987) has derived equilibrium expressions for volatility risk premium, λ:

$$\lambda = \sigma_M [\rho_{MS}\rho_{s\sigma} - \rho_{M\sigma}]/(1 - (\rho_{s\sigma})^2)^{0.5} \quad (76)$$

where σ_M is the volatility of the market return, ρ_{MS} is the correlation between the asset return and market return, $\rho_{s\sigma}$ is the correlation between the asset return and volatility of asset return, and $\rho_{M\sigma}$ is the correlation between the market return and volatility of asset return. Wiggins demonstrates that the sign of λ depends on the beta coefficient of the hedge portfolio. Thus, a negative volatility risk premium translates into a negative beta for the currency option hedge portfolio, suggesting that the returns on the currency hedge portfolio moves in opposite direction to returns on the market portfolio. A negative beta asset is conceptually like a fire insurance policy as pointed out by Brigham and Gapenski (1996). Although the currency option hedge portfolio is presumed to be riskless in each time period, its payoff actually comes only when the market returns are quite unattractive.

Given that σ_M is positive and $\rho_{s\sigma}$ is -0.14 in this study (i.e. ρ value in Table 10), the sign of λ depends on the sign of $[\rho_{MS}\rho_{s\sigma} - \rho_{M\sigma}]$. Thus, a negative λ will result under three situations: (I) $\rho_{MS}>0$ and $\rho_{M\sigma}>0$; (ii) $\rho_{MS}>0$, $\rho_{M\sigma}<0$, and $\rho_{MS}\rho_{s\sigma} > \rho_{M\sigma}$; and (iii) $\rho_{MS} <0$, $\rho_{M\sigma} >0$, and `$\rho_{MS}\rho_{s\sigma} < \rho_{M\sigma}$. The above equilibrium expression for λ applies as well to non-logarithmic utility functions if the market return accounts for all the contribution of stock's volatility to the market volatility movements.

The finding that the volatility risk premium is negative contradicts fundamentally the assumption of zero price of volatility risk in models of Hull and White (1987), Johnson and Shanno (1987), and others, and the resulting derivations of option pricing formulas in those models.

A regression analysis in Table 8 indicated that the volatility risk premium is positively related to the level of volatility (variance) under both measures of volatility. The Wald statistic, however, rejected the null hypothesis of a direct, proportional relationship between risk premium and the level of volatility. Finally, examination of the intercept and its statistical significance suggest that the volatility risk premium may have more complex, non-linear relationship with volatility than is investigated here.

Sensitivity of Option Prices to Changes in Volatility Risk Premium

Table 9 shows the sensitivity analysis of predicted call option prices from Heston's model to changes in the value of volatility risk premium under varying degrees of moneyness of options. Simulated option prices are for an actual British pound option, which was traded on January 6, 1993 at the Philadelphia Stock Exchange, with an exercise exchange rate of \$1.575, spot exchange rate of \$1.549, and time to expiration of 0.1475 year. Values of default parameters used in the simulations are from Table 11 and are shown in "Notes" to Table 9. These values are also used in subsequent sections for estimating option prices from Heston's model. Given these parameter values, the time to expiration of the option, and the spot exchange rate, option prices are estimated from Heston's model by varying the exercise exchange rate from \$1.375 to \$1.875 in increments of \$0.10 and the volatility risk premium from -9.2% to 8.8% in increments of 2%. The option is near the money at exercise exchange rate of \$1.575, in the money at strike prices of \$1.475 and \$1.375, and out of the money at exercise prices of \$1.675, \$1.775 and \$1.875. The range of exercise exchange rates corresponds approximately to

Table 8. Regression Tests for the Relation between Volatility Risk Premium and Level of Volatility, 1993-95

	Regression Analysis $\lambda_t = b_0 + b_1 V_t + e_t$	
	Implied Volatility[a]	Historical Volatility
Intercept (b_0)	-11.65 (-3.62)	-11.07 (-3.22)
Slope (b_1)	0.043 (2.36)	0.067 (1.96)
R^2	0.10	0.09
Wald-Test (H_0: $b_0 = 0, b_1 = 1$)[b]	5226.4*	1747.6*

a. The implied volatility estimates were used for the volatility variable (V_t).
b. The critical χ^2 value is 9.21. An asterisk indicates the rejection of null hypothesis.

Table 9. Sensitivity of Predicted Option Prices from Heston's Model to Changes in the Value of Volatility Risk Premium

Volatility Risk Premium (λ)	Strike Exchange Rate ($/pound)					
	1.375	1.475	1.575	1.675	1.775	1.875
-0.092	17.923	8.755	2.842	0.798	0.622	0.091
-0.072	17.923	8.754	2.839	0.798	0.622	0.088
-0.052	17.922	8.753	2.837	0.796	0.621	0.081
-0.032	17.922	8.747	2.833	0.795	0.621	0.076
-0.012	17.921	8.744	2.830	0.793	0.620	0.071
0.008	17.921	8.743	2.828	0.792	0.619	0.069
0.028	17.920	8.740	2.825	0.791	0.618	0.065
0.048	17.919	8.739	2.822	0.789	0.618	0.061
0.068	17.918	8.736	2.819	0.788	0.618	0.058
0.088	17.918	8.734	2.817	0.787	0.618	0.055

Notes: The above call option prices are in cents per pound and are based on these parameter values in Heston's model: spot exchange rate ($/pound)=1.549; implied standard deviation=0.11818; time to expiration of option=0.1475 year; U.S. risk free interest rate=2.97%; implied British risk free interest rate=3.10%; mean reversion of volatility (V) process (κ) =0.072; daily long-run mean volatility (θ)=0.006; correlation of Δ S and ΔV (ρ)= -0.142; and volatility of volatility of V process (σ)=0.044. Paramater estimates of volatility process are from Table 11.

the range that actually existed in the sample currency option data of this study.

Simulated option prices in Table 9, expressed in cents per pound, suggest that an increase in volatility risk premium consistently decreases option prices irrespective of the moneyness of options. This result is consistent with the expected impact of volatility risk premium on option prices from Heston's model as discussed in the previous section. However, changes in volatility risk premium have little impact on simulated option prices under varying degrees of moneyness of options. When the spot exchange rate is held at $1.549, an increase in volatility risk premium by 12%, from -5.2% to 6.8%, lowers option price by 0.018 cents for near-the-money option with an exercise exchange rate of $1.575. This price drop translates to $5.62 for one British pound call option contract that extends the right to purchase 31,250 British pounds. Option price decreases only by 0.004 cents from the same 12% increase in volatility risk premium when the strike prices are either $1.375 or $1.775; the price drop for a British pound call contract is $1.25.

Option prices dropped by 0.017 cents and 0.008 cents, respectively, at exercise exchange rates of $1.475 and $1.675 when volatility risk premium increased by 12% from -5.2% to 6.8%. The largest decrease in option price occurred for far out-of-the-money option with an exercise price of $1.875. In this case a 12% increase in volatility risk premium decreases the option price by 0.028 cents, from 0.081 cents to 0.058 cents, or $8.75 for a British pound call option contract. This price drop amounts to 34.5% from the original price of 0.081 cents, or 2.88% for a 1% increase in the volatility risk premium.

Overall, the largest deflationary price impact of increases in volatility risk premium, both in magnitude and in percentage terms, is for far out-of-the-money option. Price for near-the-money option dropped only 0.63%, or by 0.018 cents, when volatility risk premium increased by 12% from -5.2% to 6.8%. Since currency options in the sample data of this study were trading, on average, near the money as shown previously in Table 1, modest changes in volatility risk premium are likely to have little effect on option prices estimated using Heston's model in this study.

Parameter Estimates

Parameter estimates of the volatility and exchange rate processes are reported in Tables 10 and 11. These parameter estimates were derived by first estimating the discrete time volatility and exchange rate processes as a system of seemingly unrelated regressions (SUR), as shown in Table 10. Then, SUR estimates and variances were used to recover estimates and variances of original parameters of exchange rate and volatility processes, shown in Table 11, which appear in Heston's option pricing model.

The model as a whole, shown in Table 10, appears to be well specified judging from the system R^2 and t-ratios for the estimated parameters. All parameter estimates in Table 11 are different from zero at the 1-percent significance level. The mean reversion estimate, κ, suggests that shocks to volatility in the British pound/U.S. dollar exchange rates are short-lived, and there is a tendency for volatility to revert quickly to the mean level of volatility. The half-life of a shock to volatility, $\ln(2)/(\kappa+\lambda)$, is approximately nine days. Melino and Turnbull (1990) have reported the half-life of a shock to Canada-U.S. currency exchange rate as one week, whereas Chesney and Scott (1989) have estimated

Table 10. Seemingly Unrelated Regression Estimates of Exchange Rate and Volatility Processes

Dependent Variables	Independent Variables: $1/\sqrt{V_t}$	Independent Variables: $\sqrt{V_{t-1}}$	System R^2
$\Delta\log(S_t)/\sqrt{V_t}$	0.0022 (5.48)	--	0.89
$\sqrt{V_t}$	--	0.965 (84.34)	0.89

Notes: The t-statistics are given in parenthesis. The above results were based on the model:

(1) $\Delta\log(S_t)/\sqrt{V_t} = \mu\, 1/\sqrt{V_t} + u_t$, $u_t \sim N(0, 1)$

(2) $\sqrt{V_t} = \gamma\, \sqrt{V_{t-1}} + \phi\, e_t$, $e_t \sim N(0, \phi^2)$

where S denotes the British pond to dollar spot exchange rate, V_t is the implied volatility (variance), and log represents the natural logarithm.

Table 11. Parameter Estimates of the Stochastic Volatility Model, 1993-95[a]

Parameter	Estimates	Standard Error
Drift term of the $\sqrt{V}$ process (β)	0.036	0.0119
Mean reversion of the V process (κ)	0.072	0.023
Volatility of volatility for the $\sqrt{V}$ process (δ)	0.022	0.0002
Volatility of volatility of the V process (σ)	0.044	0.0005
Long-run mean volatility, daily (θ)	0.006	0.0003
Long-run mean volatility, annual	0.114	0.057
Correlation of ΔS and ΔV (ρ)	-0.142	0.015[b]
Mean of volatility risk premium (λ), annual	-0.052	-3.01[c]

[a]These parameter estimates and their standard errors, except ρ and λ, were recovered from SUR-based parameter estimates in Table 9.
[b.]The value is the probability significance level, p-value, at which the null hypothesis of zero correlation was rejected.
[C.]This is the calculated t-value for the null hypothesis of zero mean of the volatility risk premium.

the mean half-life of a shock to U.S. dollar/Swiss franc exchange rate as 32.6 days.

The correlation between the innovations to volatility and the level of exchange rates is negative (-0.14), suggesting that depreciation of the British pound tends to precede periods of high volatility. Melino and Turnbull (1990) and Tucker (1985) have also reported negative correlation estimates. The negative, significant correlation is strongly at variance with the assumption of zero correlation in models of Stein and Stein (1991), Hull and White (1987), and others. As discussed by Heston (1993), the correlation captures the skewness of exchange rate distributions, and a negative correlation creates a fat left tail and a thin right tail. Further, a negative correlation decreases the prices of out-of-the-money options relative to in-the-money options.

Pricing Errors of The Stochastic Volatility Model

The relative mean error, mean absolute error, and RMSE of the stochastic volatility model and Black-Scholes model are presented in Table 12. Both models yield virtually identical values for the three measures of pricing errors for near- or at-the-money options irrespective of the measure of volatility. This result agrees with the observation of Heston (1993) that, for at-the-money options, the Black-Scholes formula produces option prices identical to his stochastic volatility model.

The Black-Scholes model outperformed the stochastic volatility model in the aggregate sample and for in- and out-of-the-money options when historical volatilities were used: for the stochastic volatility model, the aggregate RMSE was 11 percent higher and mean absolute error was 8 percent larger. This increased mispricing translates to $22

Table 12. Pricing Errors as Differences between Actual Prices and Predicted Prices of the Stochastic Volatility Model Using Historical and Implied Volatility, by Moneyness Classes, 1993-95

	All Options	In-the-Money	Near- the-Money & At-the-Money	Out-of-the Money
		Historical Volatility		
Relative Mean Error (%)	-23.0	-23.0	-33.0	-4.0
(Black-Scholes Error)	(-19.0)	(-16.0)	(-34.0)	(-1.0)
Mean Absolute Error (%)	53.0	31.0	50.0	58.0
(Black-Scholes Error)	(49.0)	(21.0)	(50.0)	(55.0)
RMSE (cents)	0.68	1.21	0.60	0.62
(Black-Scholes RMSE)	(0.61)	(1.05)	(0.61)	(0.43)
		Implied Volatility		
Relative Mean Error (%)	-7.0	-10.0	-13.0	-4.0
(Black-Scholes Error)	(-4.0)	(-15.0)	(-12.0)	(9.0)
Mean Absolute Error (%)	21.0	20.0	19.0	24.0
(Black-Scholes Error)	(19.0)	(17.0)	(20.0)	(18.0)
RMSE (cents)	0.37	1.03	0.30	0.24
(Black-Scholes RMSE)	(0.36)	(0.96)	(0.31)	(0.21)
N	867	88	443	336

Notes:

In-the Money:	(Spot/Exercise) $\geq$ 1.02
Near- or At-the-Money:	0.98 < Spot/Exercise <1.02
Out-of-the-Money:	Spot/Exercise $\leq$ 0.98
Pricing Error:	Actual Price- Model Predicted Price

per British pound option contract with an average contract value of $558. The use of historical volatility, the 75-days moving average standard deviation, is providing an unfair edge to the Black-Scholes model over the stochastic volatility model. Because the moving average variance generates a gradual, smooth change in volatility over time, it is inconsistent with the mean reverting volatility process of the stochastic volatility model.

Using implied volatilities that were revised daily, the Black-Scholes model performed only marginally better than the stochastic volatility model: the aggregate RMSE and mean absolute errors were 0.36 cents and 0.19 percent, respectively, for the Black-Scholes model, compared to their respective values of 0.37 cents and 0.21 percent for the stochastic volatility model. This increased mispricing for the stochastic volatility model averaged about $11 for one British pound option contract, or about 2 percent of the average premium of one option contract. As expected from the negative correlation between the innovations to volatility and the level of exchange rates, the stochastic volatility model yielded relatively less overpricing for out-of-the-money options than for in-the-money options under both measures of volatility.

The regression tests in Tables 13 and 14 show that the Black-Scholes model is somewhat better than the stochastic volatility model in predicting the actual option prices. The Wald test, however, rejected the null hypothesis that model prices are unbiased estimate of actual prices for both models. Chesney and Scott (1989) have also reported that the Black-Scholes model with daily-revised implied volatilities outperformed their version of the stochastic volatility model for currency options on Swiss franc.

The use of daily-revised implied volatilities, however, is not internally consistent

Table 13. Regression Tests for the Relation Between Actual Call Prices and Predicted Call Prices from the Stochatic Volatility Model, by Moneyness Classes, 1993-95

	All Options	In-the-Money	Near-the-Money & At-the-Money	Out-of-the-Money
		Historical Volatility		
Intercept (α_0)	0.20	2.72	0.12	0.53
	(4.87)	(6.75)	(2.56)	(13.81)
Slope (α_1)	0.76	0.53	0.82	0.46
	(58.73)	(11.0)	(39.4)	(5.91)
R^2	0.72	0.70	0.78	0.49
Wald-Test (H_0: $\alpha_0=0$, $\alpha_1=1$)	404.3*	107*	127.3*	271.4*
		Implied Volatility		
Intercept (α_0)	0.28	3.53	0.09	0.52
	(6.38)	(8.01)	(3.92)	(13.3)
Slope (α_1)	0.77	0.74	0.89	0.79
	(53.60)	(8.17)	(80.15)	(16.35)
R^2	0.86	0.84	0.94	0.81
Wald-Test (H_0: $\alpha_0=0$, $\alpha_1=1$)	79.0*	103.0*	54.04*	81.92*

Notes: An asterisk on a value indicates the rejection of the null hypothesis at the 5% level. The t-values are given in the parenthesis. The above results were based on the regression model:

$$C = \alpha_0 + \alpha_1 C^* + e$$

C= actual call price

C*= predicted call price

Table 14. Comparison of Tests for the Systematic Relation between Actual Prices and Predicted Prices: Black-Scholes Model Versus Stochastic Volatility Model

	BS with Historical V	BS with Implied V	SV with Historical V	SV with Implied V
Intercept (α_0)	0.01 (5.03)	0.02 (1.68)	0.20 (4.87)	0.28 (6.38)
Slope (α_1)	0.94 (101.2)	0.87 (172.23)	0.76 (58.73)	0.77 (53.6)
R^2	0.75	0.88	0.72	0.86
F-Test (H_0: $\alpha_0 = 0$, $\alpha_1 = 1$)	64.6*	27.33*	404.3*	79.0*
N	867	867	867	867

Notes: An asterisk on a value indicates the rejection of the null hypothesis at the 5% level. The t-values are given in the parenthesis. BS denotes the Black-Scholes model while SV represent the stochastic volatility model. The above results stemmed from the regression model:

$$C = \alpha_0 + \alpha_1 C^* + e$$

C= actual call price

C*= predicted call price

with the underlying assumptions of the Black-Scholes model. The Black-Scholes model assumes that the variance rate is constant or, at most, a deterministic function of time. Hence, the Black-Scholes model with a constant variance, such as the average of the implied volatilities for the sample period, should be used in pricing currency options in order to be consistent with the volatility assumption of the Black-Scholes model. But there is little evidence that the industry actually uses such a procedure. The Black-Scholes model with revised estimates of volatility, using either historical volatility or implied volatility, is considered here as a less expensive, simple approximation to the true stochastic volatility option pricing model.

The superior performance of the Black-Scholes model with daily-revised implied volatilities may be attributed to three factors. First, traders and market-makers are generally believed to be using some variants of the Black-Scholes model with implied volatilities that are revised daily, as pointed out by Nandi (1996) and Chesney and Scott (1989). Such variants of the Black-Scholes model may be better approximations to the true underlying currency option pricing model than is Heston's stochastic volatility model. Second, the true underlying volatility process may be approximated better by the structure of daily-revised volatilities implied by the Black-Scholes model than by the mean-reverting volatility process of the Heston's model. Finally, the market may be simply mispricing currency options. The mispricing issue has been the subject of previous market efficiency studies and was clearly beyond the scope of the present study.

Turning to tests of pricing biases, the prices generated from the stochastic volatility model were subject to fewer biases than those generated from the Black-Scholes model, as shown in Tables 15 and 16. All four biases were present in the Black-Scholes model in the aggregate sample under the historical volatility measure, whereas only the maturity and volatility biases existed in the stochastic volatility model. The results were even more favorable to the stochastic volatility model when implied volatilities were used: the stochastic volatility model exhibited only the moneyness bias in the aggregate sample while the Black-Scholes model displayed the moneyness bias, volatility bias, and interest-rate differential bias.

Both models suffered from the same biases for near- or at-the-money options irrespective of the measure of volatility. Further, both models were free of the four pricing biases for in-the-money options when implied volatilities were used. Hence, the major difference between the pricing biases of the two models under the implied volatility measure came from out-of-the-money options. The stochastic volatility model displayed the moneyness, maturity, and volatility biases for these options, whereas the Black-Scholes model exhibited the volatility and interest rate differential biases. Although the volatility bias was common to both models, its direction differed between the two models. The stochastic volatility model tends to underprice out-of-the-money options when volatility is high and overprice options when volatility is low. The results were the exact opposite for the Black-Scholes model. A similar conflict for the moneyness bias existed between the two models for out-of-the-money options when historical volatility was used. These opposing pricing biases for out-of-the-money options in turn affected the aggregate pricing biases between the two models. For instance, the stochastic volatility model tends to underprice in-the-money calls and overprice out-of-

Table 15. Regression Tests of the Biases of the Stochastic Volatility Model in Pricing British Pound Currency Options, 1993-95

	All Options	In-the-Money	Near-the-Money & At-the-Money	Out-of-the-Money
		Historical Volatility		
Intercept	0.94	0.87	1.04	8.57
	(7.23)	(0.70)	(14.68)	(7.33)
S-K/K	0.43	-34.17	-4.34	8.57
	(0.45)	(-2.60)	(-2.18)	(7.33)
T	0.64	2.71	1.19	0.65
	(2.62)	(1.79)	(7.74)	(2.54)
r-r*	0.08	-0.17	-0.01	0.10
	(1.30)	(-0.34)	(-0.36)	(1.47)
V	-13.78	-6.42	-16.07	-8.15
	(-11.06)	(-0.60)	(-22.40)	(-6.04)
R^2	0.16	0.11	0.56	0.41
Wald-Test	233.02*	18.27*	699.29*	283.6*
(H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)				
		Implied Volatilities		
Intercept	0.15	1.01	0.17	-0.05
	(1.04)	(0.66)	(3.00)	(-0.33)
S-K/K	4.18	-6.17	-5.74	13.40
	(3.98)	(-0.37)	(-3.62)	(11.71)
T	0.01	0.65	0.005	0.82
	(0.06)	(0.34)	(0.04)	(3.25)
r-r*	0.08	0.09	-0.02	0.02
	(1.18)	(0.15)	(-0.63)	(0.33)
V	-2.48	-13.50	-3.54	3.39
	(-1.41)	(-0.86)	(-4.42)	(1.86)
R^2	0.04	0.03	0.07	0.17
Wald-Test	5.02*	1.84	60.69*	249.04*
(H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)				

Notes:The above results were derived using the regression model:
$C\text{-}C^* = \alpha_0 + \alpha_1$ S-K/K + α_2 T + α_3 r-r* + α_4 V + e, where C is the actual call price and C* is the model predicted call price.

Table 16. Comparison of Tests of the Pricing Biases in Currency Options: Black-Scholes Model Versus Stochastic Volatility Model

	BS with Historical V	BS with Implied V	SV with Historical V	SV with Implied V
Intercept	0.92	0.09	0.94	0.15
	(16.8)	(1.50)	(7.23)	(2.04)
S-K/K	-2.69	-1.26	0.43	4.18
	(-6.43)	(-3.70)	(0.45)	(3.98)
T	0.77	0.06	0.64	0.01
	(7.37)	(0.72)	(2.62)	(0.06)
r-r*	-0.04	-0.04	0.08	0.08
	(-1.68)	(-1.74)	(1.30)	(1.18)
V	-12.95	-1.58	-13.78	-2.48
	(-23.82)	(-2.88)	(-11.06)	(-1.41)
R^2	0.43	0.04	0.16	0.04
Wald-Test (H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)	702.4*	31.1*	233.0*	55.0*
N	867	867	867	867

Notes: The above results stemmed from the regression model:
$C\text{-}C^* = \alpha_0 + \alpha_1$ S-K/K $+ \alpha_2$ T $+ \alpha_3$ r-r* $+ \alpha_4$ V + e, where C is the actual call price and C* is the model predicted call price.

the-money calls when implied volatilities were used; the exact opposite pricing pattern held for the Black-Scholes model.

In sum, the stochastic volatility model exhibited, compared to the Black-Scholes model, fewer pricing biases as well as different impacts of these biases. Thus, the stochastic volatility model provides a significant improvement over the Black-Scholes model in eliminating some of the well-known systematic pricing biases of the Black-Scholes model.

The pricing biases of the stochastic volatility model of this study differed considerably from those of other stochastic volatility models of previous studies all of which have used implied volatilities. As shown in Table 17, Chesney and Scott (1989) uncovered the moneyness, maturity, and volatility biases in currency option prices generated from their stochastic volatility model. On the other hand, Melino and Turnbull (1990), also examining currency options, found only the interest rate bias in their version of the stochastic volatility model. Scott (1987) reported the moneyness bias for stock option prices generated from his stochastic volatility model. Examining equity index options, Nandi (1996) reported the moneyness bias and maturity bias in prices derived from the Heston's model. He also reported two additional biases, namely, the trading volume bias and the bid-ask spread bias. In contrast, the present study uncovered only the moneyness bias in the aggregate sample, no bias for in-the-money options, moneyness and volatility biases for at-the-money options, and moneyness, volatility, and maturity biases for out-of-the-money options. Such differences in reported pricing biases may be attributed to the fact that these studies either used different option commodities, or used

Table 17. Comparison of Option Pricing Biases from Alternative Stochastic Volatility Option Pricing Models

Biases/Commodity	Melino and Turnbull (1990)	Chesney and Scott (1989)	Nandi (1996)	Scott (1987)	Present Study
Moneyness Bias	No	Yes	Yes	Yes	Yes
Maturity Bias	No	Yes	Yes	NT	No
Volatility Bias	NT	Yes	NT	NT	No
Interest Rate Bias	Yes	NT	NT	NT	No
Option Commodity	C$ Option	SF Option	S&P 500 Index Option	Stock Option	BP Option

Notes: A "No" indicates the bias did not exist whereas "Yes" indicates the presence of the bias. NT means the bias was not tested in the study. BP denotes British pound, C$ denotes Canadian dollar, and SF represents Swiss franc. Results of the present study are for the aggregate sample under the implied volatility measure.

different versions of the stochastic volatility model.

One noticeable finding of the present study is the absence of interest-rate differential bias in prices derived from the stochastic volatility model, which suggests that the payoffs from relaxing the constant interest rate assumption in Heston's model are likely to be minimal. Merton (1976) has suggested that stochastic volatilities result in part from variable interest rates. The non-existence of interest-rate differential bias here supports Merton's conjecture, suggesting that the stochastic volatility process of the model may have captured the impact of variable interest rates on currency option prices.

Results of pricing biases using the dummy variable model and error components model are presented in Table 18. The dummy variable model captures the influence of fixed effects in intercepts of regression equations, while the error components model reflects the influence of random effects on intercepts of regression equations. Both models attempt to capture the separate impacts of time series and cross-sectional properties of the sample data in estimates of regression parameters. The generalized method of moments used earlier makes no distinction between the time series effects and cross-sectional effects. Results from the dummy variable model and error component model are identical to those reported in Table 14 which were estimated using the generalized method of moments. The LR-test, F-test, and LM-test all indicated that cross-sectional random or fixed effects in intercepts do not exit. These findings fail to undermine the normal practice in previous studies of estimating the pricing biases using the ordinary least squares method. This conclusion is, of course, limited to results from the present sample on currency options. Other options data may lead to different findings.

Table 18. Tests of the Pricing Biases of the Stochastic Volatility Model Using Dummy Variable and Error Components Regression Models, 1993-95

	SV with Historical V		SV with Implied V	
	Dummy Variable Model	Error Cmponents Model	Dummy Variable Model	Error Components Model
S-K/K	0.42 (0.44)	0.42 (0.45)	4.08 (3.84)	4.11 (3.88)
T	0.64 (2.74)	0.64 (2.73)	0.004 (0.018)	0.004 (0.017)
r-r*	0.08 (1.31)	0.08 (1.31)	0.08 (1.22)	0.08 (1.20)
V	-13.8 (-11.2)	-13.7 (-11.2)	-2.49 (-1.4)	-2.49 (-1.4)
R^2	0.16	0.16	0.047	0.044
LR-Test (χ^2-Test) (Ho: no fixed effects)	1.49 (0.95)	---	2.45 (0.87)	---
F-Test (Ho: no fixed effects)	0.24 (0.95)	---	0.40 (0.87)	---
LM-Test (χ^2-Test) (Ho: no random effects)	---	0.75 (0.38)	---	0.48 (0.48)

Notes: The t-values of the coefficients and probability values, p-values, of the three tests are given in parenthesis. The LR denotes the Likelihood Ratio test and LM represents the Lagrange Multiplier test. The letter SV denotes the stochastic volatility model.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The modified Black-Scholes model for currency options is known to produce large forecast errors and exhibit systematic biases in pricing options on foreign currencies. The Black-Scholes model assumes that, inter alia, the variance rate of currency exchange rate is constant over time. The central issue in this study was to investigate whether the consideration of stochastic variance in currency exchange rates can improve the pricing of currency options and correct the pricing biases observed for the modified Black-Scholes model. Option values from Heston's stochastic volatility option pricing model and values from the modified Black-Scholes model are compared to option premiums. Pricing biases are then compared for these models. Pricing biases related to option strike prices, time to maturity, volatility, and interest rate differentials are considered.

Heston's model requires estimates of the correlation between innovations to volatility and level of exchange rates, the volatility risk premium, and the parameters of the volatility process, in addition to standard inputs of the Black-Scholes model. The correlation and parameters were estimated using the time series of implied volatilities, and the volatility risk premium was estimated as the difference between the return on a hedge portfolio and the riskfree return. Empirical analysis was conducted using the daily data on European call options written on British pound from the Philadelphia Stock Exchange for the period January 1993 to November 1995.

Results indicate that the volatility risk premium, with an average value of -5.2 percent during the sample period, was statistically different from zero. Further, the volatility risk premium was positively related to the level of volatility. However, the volatility risk premium did not bear direct, proportional relationship with the level of volatility. The finding that the volatility risk premium is negative contradicts fundamentally the assumption of zero price of volatility risk in existing stochastic volatility models and the resulting option pricing formulas in those models.

A negative, significant correlation existed between innovations to volatility and the level of exchange rates. The negative correlation, which captures the skewness of exchange rate distributions, decreases the prices of out-of-the-money options relative to in-the-money options.

Turning to the pricing performance of the model, the Black-Scholes model performed marginally better than the stochastic volatility model when the implied variance was revised daily in the Black-Scholes model. The Black-Scholes model performed significantly better than the stochastic volatility model in pricing currency options when historical volatilities were used, although both models exhibited much larger pricing errors with historical volatilities than with implied volatilities. However, prices generated from both models provided close and similar correspondence to actual prices in the sample for options trading near- or at-the-money when daily-revised implied volatilities were used. Consequently, the main difference between the forecast errors of the two models lies in their relative accuracy in pricing in-the-money and out-of-the-money options. The greater pricing accuracy of the Black-Scholes model with daily-revised implied volatilities may be related to the preponderance of traders who use variants of the Black-Scholes model to price currency options with daily revisions of implied volatilities, and/or the potentially weak correspondence between the true underlying volatility process and the mean reverting volatility process of the Heston's model, or the possible mispricing of British pound options by the market.

Although the Black-Scholes model displayed marginally better pricing performance compared to the stochastic volatility model, its prices were subject to more and stronger systematic pricing biases than were the prices of the stochastic volatility model. The only bias that existed for the stochastic volatility model in the aggregate sample was the moneyness bias when implied volatilities were used. In contrast, the prices from the Black-Scholes model with daily-revised implied volatilities exhibited the moneyness bias, volatility bias, and interest-rate differential bias. Although the moneyness bias was common to both models, its direction differed between the two models. The stochastic volatility model tends to underprice in-the-money calls and overprice out-of-the-money calls. The exact opposite pricing pattern existed for the Black-Scholes model. This conflict remained even when the historical volatilities were used.

Both models exhibited, however, similar pricing biases for near- or at-the-money options irrespective of the measure of volatility. The use of historical volatilities failed to reduce significantly the differences in pricing biases between the two models that were observed using implied volatilities. Consequently, the stochastic volatility model provides a major improvement over the Black-Scholes model in eliminating some of the well-known pricing biases of the Black-Scholes model.

The stochastic volatility model did not exhibit the interest rate differential bias when either the implied volatilities or the historical volatilities were used. The absence of interest rate bias suggests that the payoffs from relaxing the constant interest rate assumption in Heston's model are expected to be small.

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APPENDIX

MAPLE SOLUTION FOR THE REAL PART OF PROBABILITIES

```
>
> assume(g,complex);
> assume(d,complex);
> d:=sqrt((r2*s*phi*I-b)^2-s^2*(2*u*phi*I-phi^2));
```

$$d := \sqrt{(I\,r2\,s\,\phi - b)^2 - s^2\,(2\,I\,u\,\phi - \phi^2)}$$

```
>
> g:=(b-r2*s*phi*I+d)/(b-r2*s*phi*I-d);
```

$$g := \frac{b - I\,r2\,s\,\phi + \sqrt{(I\,r2\,s\,\phi - b)^2 - s^2\,(2\,I\,u\,\phi - \phi^2)}}{b - I\,r2\,s\,\phi - \sqrt{(I\,r2\,s\,\phi - b)^2 - s^2\,(2\,I\,u\,\phi - \phi^2)}}$$

```
>
>
> ep:=(b-r2*s*phi*I+d)*t2-2*ln((1-g*(exp(d*t2)))/(1-g));
```

$$ep := (b - I\,r2\,s\,\phi + \sqrt{\%1}\,)\,t2 - 2\ln\left(\frac{1 - \dfrac{(b - I\,r2\,s\,\phi + \sqrt{\%1}\,)\,e^{(\sqrt{\%1}\;t2)}}{b - I\,r2\,s\,\phi - \sqrt{\%1}}}{1 - \dfrac{b - I\,r2\,s\,\phi + \sqrt{\%1}}{b - I\,r2\,s\,\phi - \sqrt{\%1}}}\right)$$

$$\%1 := (I\,r2\,s\,\phi - b)^2 - s^2\,(2\,I\,u\,\phi - \phi^2)$$

```
> C:= (t2,phi) -> r*phi*I*t2+(a/s^2)*ep;
>
```

$$C := (t2, \phi) \rightarrow I\,r\,\phi\,t2 + \frac{a\,ep}{s^2}$$

```
> D2:=(t2,phi)->(b-r2*s*phi*I+d)*(1-exp(d*t2))/(1-g*exp(d*t2))/s^2;
```

$$D2 := (t2, \phi) \rightarrow \frac{(b - I\,r2\,s\,\phi + d)\,(1 - e^{(d\,t2)})}{(1 - g\,e^{(d\,t2)})\,s^2}$$

```
> f:=(x,v,t2,phi) ->exp(C(t2,phi)+D2(t2,phi)*v+I*phi*x);
```

$$f := (x, v, t2, \phi) \rightarrow e^{(C(t2,\phi) + D2(t2,\phi)\,v + I\,\phi\,x)}$$

```
> exp3:=(exp(-I*phi*ln(K))*f(x,v,t2,phi))/(I*phi);
```

$$exp3 := -\frac{I\,e^{(-I\,\phi\ln(K))}\,e^{\left(I\,r\,\phi\,t2 + \dfrac{a\left(\%2\;t2 - 2\ln\left(\dfrac{1 - \dfrac{\%2\,e^{(\sqrt{\%1}\;t2)}}{b - I\,r2\,s\,\phi - \sqrt{\%1}}}{1 - \dfrac{\%2}{b - I\,r2\,s\,\phi - \sqrt{\%1}}}\right)\right)}{s^2} + \dfrac{\%2\,(1 - e^{(\sqrt{\%1}\;t2)})\,v}{\left(1 - \dfrac{\%2\,e^{(\sqrt{\%1}\;t2)}}{b - I\,r2\,s\,\phi - \sqrt{\%1}}\right)s^2} + I\,\phi\,x\right)}}{\phi}$$

$$\%1 := (I\,r2\,s\,\phi - b)^2 - s^2\,(2\,I\,u\,\phi - \phi^2)$$

$$\%2 := b - I\,r2\,s\,\phi + \sqrt{\%1}$$

```
> result1:=Re(exp3);
```

$$result1 := \Im\left(\frac{e^{(-I\phi\ln(K))}\, e^{\left(I\, r\, \phi\, t2 + \frac{a\left(\%2\, t2 - 2\ln\left(\frac{1-\frac{\%2\, e^{(\sqrt{\%1}\, t2)}}{b - I\, r2\, s\, \phi - \sqrt{\%1}}}{1-\frac{\%2}{b - I\, r2\, s\, \phi - \sqrt{\%1}}}\right)\right)}{s^2} + \frac{\%2\,(1-e^{(\sqrt{\%1}\, t2)})\, v}{\left(1-\frac{\%2\, e^{(\sqrt{\%1}\, t2)}}{b - I\, r2\, s\, \phi - \sqrt{\%1}}\right) s^2} + I\,\phi\, x\right)}}{\phi}\right)$$

$$\%1 := (I\, r2\, s\, \phi - b)^2 - s^2\,(2\, I\, u\, \phi - \phi^2)$$

$$\%2 := b - I\, r2\, s\, \phi + \sqrt{\%1}$$

```
>
>
> result3:=evalc(result1);
```

$$result3 := \Bigg(-e^{(\phi\,(1/2 - 1/2\,\mathrm{signum}(K))\,\pi)}\,\sin(\phi\ln(|K|))\, e^{\Big(}$$

$$\frac{a\left((b + 1/2\sqrt{\%5})\, t2 - \ln\left(\left(\frac{\%16\left(1 + \%10 + \frac{\%9\,\%7}{\%8}\right)}{\%24} + \frac{\%14\,\%13}{\%24}\right)^2 + \left(\frac{\%14\left(1 + \%10 + \frac{\%9\,\%7}{\%8}\right)}{\%24} - \frac{\%16\,\%13}{\%24}\right)^2\right)\right)}{s^2}$$

$$+ \frac{\left(\frac{\%18\,\%16}{\%16^2 + \%14^2} + \frac{\%19\,\%14}{\%16^2 + \%14^2}\right) v}{s^2}\Big)\ \cos\Bigg(r\,\phi\, t2 + a\Bigg(\%17\, t2 - 2\arctan\Bigg($$

$$\frac{\%23\left(1 + \%10 + \frac{\%9\,\%7}{\%8}\right)}{\%21} + \frac{(-1 - \%15 - \%22)\,(\%12 + \%20)}{\%21},$$

$$\frac{(1 + \%15 + \%22)\left(1 + \%10 + \frac{\%9\,\%7}{\%8}\right)}{\%21} + \frac{\%23\,(\%12 + \%20)}{\%21}\Bigg)\Bigg)\Bigg/ s^2$$

$$+ \frac{\left(\frac{\%19\,\%16}{\%16^2 + \%14^2} - \frac{\%18\,\%14}{\%16^2 + \%14^2}\right) v}{s^2} + \phi\, x\Bigg) + e^{(\phi\,(1/2 - 1/2\,\mathrm{signum}(K))\,\pi)}\cos(\phi\ln(|K|))\, e^{\Big(}$$

$$\frac{a\left((b + 1/2\sqrt{\%5})\, t2 - \ln\left(\left(\frac{\%16\left(1 + \%10 + \frac{\%9\,\%7}{\%8}\right)}{\%24} + \frac{\%14\,\%13}{\%24}\right)^2 + \left(\frac{\%14\left(1 + \%10 + \frac{\%9\,\%7}{\%8}\right)}{\%24} - \frac{\%16\,\%13}{\%24}\right)^2\right)\right)}{s^2}$$

$$+\frac{\left(\frac{\%18\,\%16}{\%16^2+\%14^2}+\frac{\%19\,\%14}{\%16^2+\%14^2}\right)v}{s^2}\Bigg)\sin\Bigg(r\,\phi\,t2+a\Bigg(\%17\,t2-2\arctan\Bigg($$

$$\frac{\%23\left(1+\%10+\frac{\%9\,\%7}{\%8}\right)}{\%21}+\frac{(-1-\%15-\%22)\,(\%12+\%20)}{\%21},$$

$$\frac{(1+\%15+\%22)\left(1+\%10+\frac{\%9\,\%7}{\%8}\right)}{\%21}+\frac{\%23\,(\%12+\%20)}{\%21}\Bigg)\Bigg)\Big/s^2$$

$$+\frac{\left(\frac{\%19\,\%16}{\%16^2+\%14^2}-\frac{\%18\,\%14}{\%16^2+\%14^2}\right)v}{s^2}+\phi\,x\Bigg)\Bigg)/\phi$$

$$\%1 := \sqrt{r2^4\,s^4\,\phi^4+2\,r2^2\,s^2\,\phi^2\,b^2-2\,r2^2\,s^4\,\phi^4+b^4+2\,b^2\,s^2\,\phi^2+s^4\,\phi^4+8\,r2\,s^3\,\phi^2\,b\,u+4\,s^4\,u^2\,\phi^2}$$

$$\%2 := 2\,\%1+2\,r2^2\,s^2\,\phi^2-2\,b^2-2\,s^2\,\phi^2$$

$$\%3 := \mathrm{csgn}(-2\,r2\,s\,\phi\,b-2\,s^2\,u\,\phi+I\,r2^2\,s^2\,\phi^2-I\,b^2-I\,s^2\,\phi^2)$$

$$\%4 := \frac{1}{2}\%3\sqrt{\%2}\;t2$$

$$\%5 := 2\,\%1-2\,r2^2\,s^2\,\phi^2+2\,b^2+2\,s^2\,\phi^2$$

$$\%6 := e^{(1/2\sqrt{\%5}\;t2)}$$

$$\%7 := -r2\,s\,\phi-\frac{1}{2}\%3\sqrt{\%2}$$

$$\%8 := \left(b-\frac{1}{2}\sqrt{\%5}\right)^2+\%7^2$$

$$\%9 := r2\,s\,\phi-\frac{1}{2}\%3\sqrt{\%2}$$

$$\%10 := \frac{\left(-b-\frac{1}{2}\sqrt{\%5}\right)\left(b-\frac{1}{2}\sqrt{\%5}\right)}{\%8}$$

$$\%11 := \left(\%10+\frac{\%9\,\%7}{\%8}\right)\%6\,\sin(\%4)$$

$$\%12 := \frac{\%9\left(b-\frac{1}{2}\sqrt{\%5}\right)}{\%8}$$

$$\%13 := \%12 - \frac{\left(-b - \frac{1}{2}\sqrt{\%5}\right)\%7}{\%8}$$

$$\%14 := \%13\ \%6\cos(\%4) + \%11$$

$$\%15 := \left(\%10 + \frac{\%9\ \%7}{\%8}\right)\%6\cos(\%4)$$

$$\%16 := 1 + \%15 - \%13\ \%6\sin(\%4)$$

$$\%17 := -r2\ s\ \phi + \frac{1}{2}\%3\sqrt{\%2}$$

$$\%18 := \left(b + \frac{1}{2}\sqrt{\%5}\right)(1 - \%6\cos(\%4)) + \%17\ \%6\sin(\%4)$$

$$\%19 := \%17\,(1 - \%6\cos(\%4)) - \left(b + \frac{1}{2}\sqrt{\%5}\right)\%6\sin(\%4)$$

$$\%20 := \frac{\left(b + \frac{1}{2}\sqrt{\%5}\right)\%7}{\%8}$$

$$\%21 := \left(1 + \%10 + \frac{\%9\ \%7}{\%8}\right)^2 + (\%12 + \%20)^2$$

$$\%22 := (-\%12 - \%20)\ \%6\sin(\%4)$$

$$\%23 := (\%12 + \%20)\ \%6\cos(\%4) + \%11$$

$$\%24 := \left(1 + \%10 + \frac{\%9\ \%7}{\%8}\right)^2 + \%13^2$$

Notes: In the above derivation, result3 is the real part of the integrand of equation (64), Re[.], in chapter 3. Since some of the symbols used in equation (64) are reserved characters in MAPLE, such symbols were denoted differently in result3, above. The correspondence between the symbols in result3 and those in equation (64) is as follows: r2=ρ; s=σ; phi=ϕ; I=i; b=b_j ; u=u_j ; t2=τ; and D2=D.

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